OPEN BOOK DECOMPOSITIONS AND CONTACT SURGERIES

AÇIK KİTAP AYRIŞIMLARI VE KONTAKT AMELİYATLAR

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ABSTRACT

OPEN BOOK DECOMPOSITIONS AND CONTACT SURGERIES

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By Alexander it was shown that an open book decomposition can be found for each closed, oriented 3-manifold. In this thesis, the importance of open book decompositions in terms of contact 3-manifolds will be studied by using surgery techniques. A closed curve in a 3-manifold that does not intersect itself is called a knot. Especially, knots which can be drawn on page of open book decompositions and boundary of open book decompositions will be studied in terms of topology and contact topology. The invariants of contact structures from open book decompositions will be studied.

Keywords: open book, contact structure, Legendrian knot, transversal knot, contact surgery

ÖZET

AÇIK KİTAP AYRIŞIMLARI VE KONTAKT AMELİYATLAR

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Yüksek Lisans, Matematik Bölümü

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Alexander tarafından her kapalı, yönlü 3-manifold için bir açık kitap ayrışımı bulunabileceği gösterilmiştir. Bu tezde açık kitap ayrışımlarının kontakt 3-manifoldlar açısından önemi ameliyat tekniklerini kullanarak çalışılacaktır. 3-manifoldlar içerisinde kendi kendisini kesmeyen kapalı eğrilere düğüm denir. Özel olarak, açık kitap ayrışımlarının sayfası üzerine çizilebilen düğümler ve açık kitap ayrışımlarının sınırı olan düğümler topolojik açıdan ve kontakt topolojik açıdan çalışılacaktır. Kontakt yapıların açık kitap ayrışımlarından elde edilen değişmezleri çalışılacaktır.

Anahtar Kelimeler: açık kitap, kontakt yapı, Legendre düğümü, transvers düğüm, kontakt ameliyat

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1 INTRODUCTION

Contact topology is concerned with geometric structures on smooth manifolds. These natural geometric structures on smooth manifolds are called contact structures. Contact structures on smooth 3-manifolds are 2-plane fields which are maximally non-integrable. Marinet proved that there exists a contact structure on each 3-manifold, [1]. If a 3-manifold includes a contact structure on it, then it is named as a contact 3-manifold. To understand contact manifolds better, their subspaces can be studied. Specifically, knots in contact 3-manifolds can be studied. In contact 3-manifolds, there are two types of knots: Legendrian knots and transverse knots.

A knot which takes place in contact 3-manifolds and is always tangent to the contact planes is named as a Legendrian knot. Legendrian knots have three classical invariants: the knot type, the Thurston-Bennequin invariant and the rotation number.

There are several ways to construct 3-manifolds. Dehn surgery is a way which constructs 3-manifolds. This technique is named after Max Dehn, [2]. A Dehn surgery on knots or links in 3-manifolds consists of removing a tubular neighborhood of a knot or link and gluing it back via a diffeomorphism of solid torus, a new 3-manifold is constructed. Each 3-manifold that are closed and oriented can be achieved by Dehn surgery with the framing (± 1) applied to a link of unknots in S^3. In 1960 Wallace and in 1962 Lickorish proved the theorem above independently, [3], [4].

The surgery which can be done on Legendrian knots is called contact surgery. A contact surgery which is applied to Legendrian knots with respect to contact framing is done by executing Dehn surgery applied to Legendrian knots and extending the former contact structure to the yielding 3-manifold. Like in the case of 3-manifolds, all closed and oriented contact 3-manifolds can be obtained by contact surgery with the framing (± 1) applied to a Legendrian knot in (S^3, ξ_{std}). This theorem was proved by Ding and Geiges, [5].

While executing surgery on 3-manifolds can result in a different 3-manifold, Kirby moves give the same 3-manifold. These moves are named after Robion Kirby, [6]. There exist two types of moves. One move is taking a connected sum with S^3 or cancelling it. The other move is sliding one component of the framed link over one another. These two moves do not change the existing framed link.

Open book decompositions are decompositions of closed and oriented 3-manifolds. These decompositions are a combination of solid tori and surfaces with boundary. Open book decompositions are the main focus of this thesis and will defined precisely.

Existence of open book decompositions of 3-manifolds has been known since 1920 thanks to Alexander, [7]. Another central result of open book decompositions is given by Giroux. Giroux has shown that there is a relevance between contact structures and open book decompositions. A one-to-one correspondence between oriented contact structures on a closed, oriented 3-manifold up to isotopy and open book decompositions of this 3-manifold up to positive stabilizations was shown by Giroux, [8]. This theorem allows us to study contact structures from a completely topological viewpoint.

In this thesis, the relation of open book decompositions and contact structures is analyzed. The proof of Alexander's theorem is examined by surgery techniques and planar open book decompositions. Also, in this thesis invariants of contact structures from open book decompositions are studied. The support genus and the binding number of a contact structure are the invariants. The support genus is the minimal genus for a page of an open book decomposition which is supporting the contact structure. The binding number is the minimum number of boundary components of a page for an open book decomposition which supports the contact structure with genus is equivalent to support genus. These invariants are introduced by Etnyre and Ozbagci, [9].

In 2008, Etnyre and Ozbagci listed all contact 3-manifolds having a supporting open book decomposition with support genus 0 and binding number 2. In 2007, Arikan listed all contact 3-manifolds having a supporting open book decomposition with support genus 0, binding number 3 by using first Chern classes and three dimensional invariants, [10]. In 2007, Baldwin listed all contact 3-manifolds having support genus 1, binding number 1 by using Heegard Floer homology theory, [11]. In 2010, Lekili listed all contact 3-manifolds having support genus 0, binding number 4 by using integral surgery on pure braid closure with 3 component, [12]. In this thesis, contact 3-manifolds which have support genus 0 and binding number equal to 1, and contact 3-manifolds which have support genus 0 and binding number equal to 2 are studied in detail.

In Section 2 Background section, fundamental definitions and examples are given. We give the definitions and examples of topological and differentiable manifolds. Knots which are 1-manifolds are introduced. Also, surfaces that are connected 2-manifolds are introduced. Seifert surfaces for knots and links are constructed. Open book decompositions and abstract open book decompositions are introduced. Additionally, positive stabilization of abstract open books is given. Moreover, surgery techniques and Kirby moves are given with the most essential examples.

In Section 3 Existence of Open Book Decompositions section, two proofs of Alexander's theorem are given. One of the proofs of Alexander's existence theorem is studied in detail. In this proof, a planar open book decomposition is constructed with an explicit monodromy. This technique is introduced in Onaran's paper, [13].

In Section 4 Contact 3-Manifolds section, the definitions of contact 3-manifolds, Legendrian knots and their classical invariants are given. Also, in this section classical invariants of various Legendrian knots are computed. Contact surgery along Legendrian knots are defined and surgery techniques are analyzed.

In Section 5 Open Book Decompositions and Contact 3-Manifolds section, open book decompositions are linked with contact structures. Compatible contact structures are given with examples. Fundamental theorems on open book decompositions and contact structures are given. Additionally, the proof of Cancellation Lemma of Ding and Geiges [14] is given with regard to open book decompositions. In the same section, the support genus invariant and the binding number invariant are introduced. The support genus 0 and the binding number equal to 1 and 2, respectively are studied.

Finally in Section 6 Conclusion section, contact surgery techniques with regard to open book decompositions are analyzed. Some open problems about invariants of contact structures from open book decompositions are listed.

2 BACKGROUND

In this section the basics of knot theory and geometric topology are reviewed. Firstly, manifolds are introduced. After knots and links in 3-manifolds are presented, some properties of them are given. The focus of this thesis, open book decompositions are defined with examples. Also, in this section Dehn surgery technique and Kirby moves are analyzed. For more information see [15], [16] and [17].

2.1 Manifolds

Manifolds are separated into many categories. In this thesis, differentiable manifolds will be the focus. Before introducing to the differentiable manifolds, the definition of topological manifolds will be given.

2.1.1 Topological Manifolds

Definition 2.1.1. [18] Take a topological space M. If M fulfills the conditions, that are mentioned below, then M is called a *topological n-manifold*.

1) The topological space M is a Hausdorff space: That is, M fulfills the Hausdorff separation axiom. Take any two distinct points r, $s \in M$ and their open neighborhoods R and S, respectively. If the intersection of these two neighborhoods $R \cap S = \emptyset$ is empty, then M is named as a *Hausdorff space*.

2) The topological space M is second countable: That is, M has a basis which is countable. There are countably many open subsets $R_1, R_2, ...$ of M such that $r \in R_i \subset R$ for an open set R and a point $r \in R$.

3) The topological space M locally looks like $\mathbb{R}^n$: Take any point $r \in M$ and its open neighborhood R. This open set R is homeomorphic to an open set $S \subset \mathbb{R}^n$. Let $\psi : R \longrightarrow S$ be such a homeomorphism. The image of every point in $\mathbb{R}^n$ is shown below:

$$\psi(p) = (s_1(p), ..., s_n(p)).$$

These ordered n-points are called *the local coordinates* of p, and the neighborhood R of the point r is called a *coordinate neighborhood*. Additionally, $s_1, ..., s_n$ are called *coordinate functions* on R. The pair (R, ψ) is called a *local chart*.

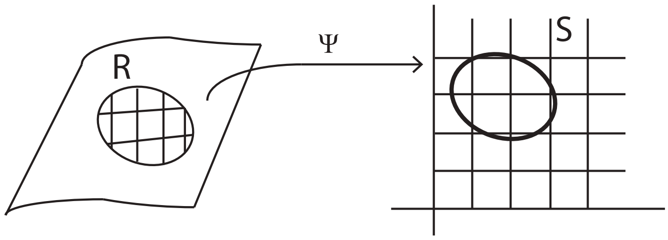

Figure 1: Local chart.

Definition 2.1.2. [18] Take a topological manifold X and open sets R_α for some $\alpha \in I$. If $\bigcup \{R_\alpha\}_{\alpha\in I} = X$, then a family of local coordinate systems $\mathscr{A} = \{(R_\alpha, \psi_\alpha)\}_{\alpha\in I}$ is called an *atlas* of X.

Let S_α presents the image of the homeomorphism ψ_α from R_α into $\mathbb{R}^n$. Coordinate neigborhoods R_α in the atlas $\mathscr{A}$ cover the topological manifold X and on the other side an open set S_α of $\mathbb{R}^n$ identifies every R_α via the homeomorphism ψ_α. So, it can be said that X can be obtained of open sets S_α of $\mathbb{R}^n$ by gluing these parts one by one, [18].

Consider two coordinate neighborhoods R_α, R_β in the atlas $\mathscr{A}$ intersect. Then the coinciding two open sets S_α, S_β in $\mathbb{R}^n$ are to overlap to obtain X.

$$h_{\beta\alpha} = \psi_\beta \circ {\psi_\alpha}^{-1} : \psi_\alpha(R_\alpha \cap R_\beta) \longrightarrow \psi_\beta(R_\alpha \cap R_\beta), \text{ [18]}.$$

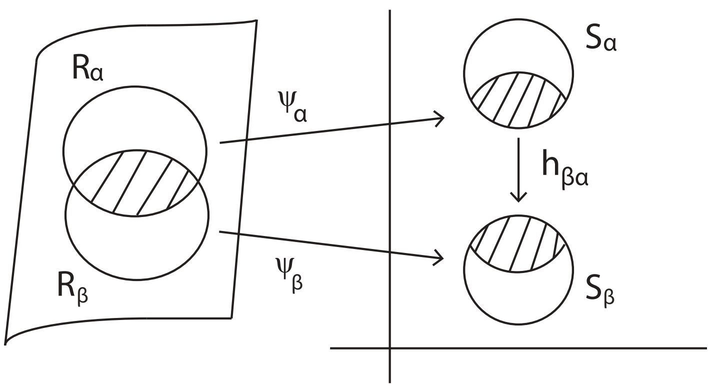

Figure 2: Coordinate change.

This $h_{\beta\alpha}$ is a homeomorphism from an open set of $\mathbb{R}^n$ to $\mathbb{R}^n$ and it is defined as $h_{\beta\alpha} = (h^1_{\beta\alpha}, ..., h^n_{\beta\alpha})$ with n continuous functions $h^i_{\beta\alpha}$. The local coordinates of any point r on $R_\alpha \cap R_\beta$, $(x_1(r), ..., x_n(r))$ with respect to (R_α, ψ_α) and $(y_1(r), ..., y_n(r))$ with respect to (R_β, ψ_β), then there exists a relation $y_i(r) = h^i_{\beta\alpha}(x_1(r), ..., x_n(r))$ between them. Hence, the homeomorphism $h_{\beta\alpha}$ defines the relation between two local coordinates, and it is called the *coordinate change*, [18].

Example 2.1.1. [19] The n-dimensional real space $\mathbb{R}^n$ is the most obvious example for n-manifolds. Also, its open subsets are the examples of n-manifolds.

Example 2.1.2. [19] Another example for n-manifolds is the n-sphere $S^n = \{p \in \mathbb{R}^{n+1} : ||p|| = 1\}$. Stereographic projection of S^n gives a homeomorphism $f_- : S^n \setminus \{(0, ..., 0, 1)\} \longrightarrow \mathbb{R}^n$. There is a neighborhood of p that is homeomorphic to $\mathbb{R}^n$ where $p \in S^n$ and $p \neq (0, ..., 0, 1)$. To show a neighborhood of $(0, ..., 0, 1)$ which is homeomorphic to $\mathbb{R}^n$, compound the reflection in $\mathbb{R}^n \times \{0\}$ with the homeomorphism f_- to achieve $f_+ : S^n \setminus \{(0, ..., 0, -1)\} \longrightarrow \mathbb{R}^n$. Hence, $\mathbb{R}^n$ is homeomorphic $S^n \setminus \{(0, ..., 0, -1)\}$ that is a neighborhood of $(0, ..., 0, 1)$.

Example 2.1.3. [19] The real projective space $\mathbb{R}P^n$ is obtained by identifying antipodal points on S^n. Take a point $[p] \in \mathbb{R}P^n$. Since the n-sphere S^n is an n-manifold, p has a neighborhood V and a homeomorphism φ from V to $\mathbb{R}^n$. Fix $-V = f(V)$, for $f : S^n \longrightarrow S^n$ the antipodal map. So, $-V$ is a neighborhood of $-p$ and $-\varphi = \varphi \circ f$ is a homeomorphism between $-V$ and $\mathbb{R}^n$. After making V smaller, if necessary, the intersection of $f(V)$ and V is empty. Hence, a homeomorphism $[\varphi] : [V] \longrightarrow \mathbb{R}^n$ is obtained. Each of all this kind of homeomorphisms is an element of an atlas for $\mathbb{R}P^n$.

Example 2.1.4. [19] The space $T^n = S^1 \times ... \times S^1$, n-torus, is a quotient space. It is achieved down below:

Let H be a group in $\mathbb{R}^n$. Its generators are translations of distance by 1 through the coordinate axes. Two points $p, r \in \mathbb{R}^n$ are identified if and only if there exists a $h \in H$ so that $h(p) = r$. Represent this quotient map by $g : \mathbb{R}^n \longrightarrow T^n$. Take $[p] \in T^n$ and the sphere V which has radius $\frac{1}{4}$ centered at $p \in \mathbb{R}^n$. For all $h \in H$ the intersection of $h(V)$ and V is empty. From there a homeomorphism $g^{-1}|_{h(V)} : h(V) \longrightarrow V$ is obtained. Each of all this kind of homeomorphisms is an element of atlas for T^n. Hence, T^n is an n-manifold.

2.1.2 Differentiable Manifolds

Definition 2.1.3. Take a topological manifold X. An atlas $\mathscr{A} = \{(R_\alpha, \psi_\alpha)\}$ of X is called a C^∞ *atlas*, if all its coordinate changes $h_{\beta\alpha} = \psi_\beta \circ \psi_\alpha^{-1}$ are C^∞ maps. This means that the atlas $\mathscr{A}$ decides a C^∞ structure on X. A C^∞ *differentiable manifold* is a manifold which has a C^∞ structure on it. It is also called a *smooth manifold*.

Example 2.1.5. The n-dimensional topological manifold $\mathbb{R}^n$ is also an n-dimensional differentiable manifold with the identity map from its open subset to itself.

Example 2.1.6. [19] The n-sphere S^n is an n-dimensional differentiable manifold .

2.2 Knots and Links in 3-Manifolds

Definition 2.2.1. [15] If a subset K of M^3 is homeomorphic to S^1, then K is called a *knot*.

Example 2.2.1. The obvious example for a knot is the unknot. It is sometimes called the *trivial knot* and is illustrated in Figure 3.

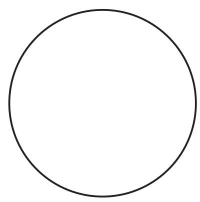

Figure 3: The unknot.

Example 2.2.2. Left-handed trefoil is a non-trivial knot. Also, figure eight knot is a non-trivial knot. These are illustrated in Figure 4.

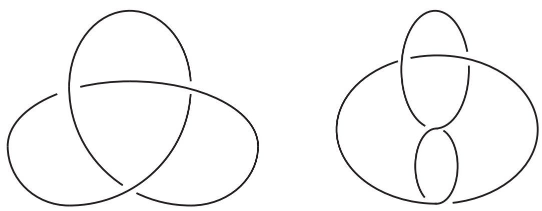

Figure 4: Left-handed trefoil and figure eight knot.

Definition 2.2.2. [15] A *link* is a subset of a 3-manifold which is disjoint union of knots.

Example 2.2.3. Hopf link and King Solomon's link are the examples of links in M^3, and respectively given in Figure 5.

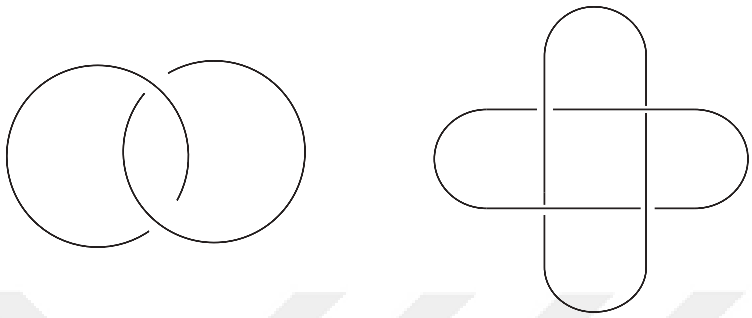

Figure 5: Hopf Link and King Solomon's Link.

Example 2.2.4. In Figure 6, two Hopf links are given. The link in Figure 6(a) is called *positive Hopf link* and the link in Figure 6(b) is called *negative Hopf link*.

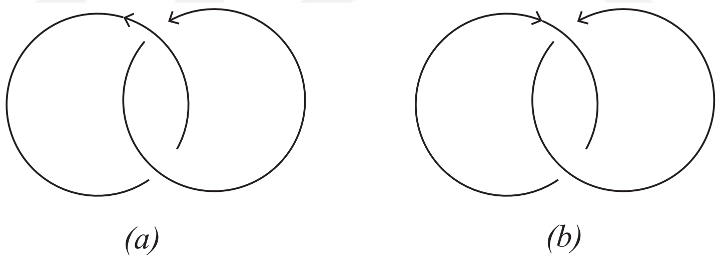

Figure 6: (a) Positive Hopf link, (b) Negative Hopf link.

In the first example of the following subsection, the reason why they are called positive and negative is explained.

2.3 Linking Number

Definition 2.3.1. [15] *Writhe number* is the total number of crossings which are assigned as positive or negative in an oriented knot diagram.

In Figure 7, negative and positive crossings are illustrated.

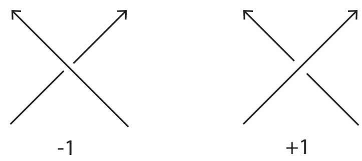

Figure 7: Negative and positive crossings.

Definition 2.3.2. [15] [16] Take two oriented knots K_1 and K_2 in M^3. The *linking number* is described as the half of the total number of signed crossings between K_1 and K_2. It is represented as $lk(K_1, K_2)$.

$$lk(K_1, K_2) = \frac{1}{2}\sum \#\text{Signs}.$$

Example 2.3.1. In Figure 8, two Hopf links are given. In Figure 8(a) there are two crossings in the link diagram which are both positive crossings. So, the linking number is equal to $\frac{1}{2}(+1+1) = +1$, the link is called *positive Hopf link.* There are two crossings in the link diagram which are both negative crossings that can be seen in Figure 8(b). The linking number is equal to $\frac{1}{2}(-1-1) = -1$, the link is called *negative Hopf link.*

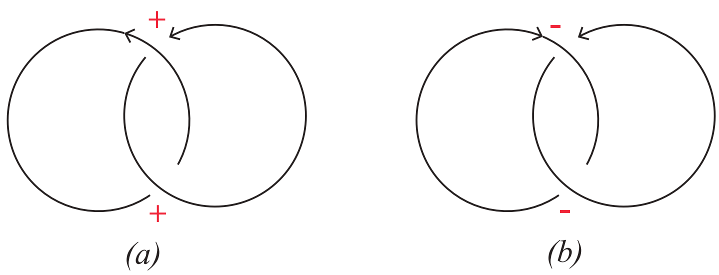

Figure 8: (a) Positive Hopf link, (b) Negative Hopf link.

2.4 Seifert Surfaces for Knots

Connected two manifolds are called *surfaces.* Closed surfaces are surfaces that are compact with no boundary components, [15].

Example 2.4.1. In Figure 9, the 2-sphere S^2, torus T^2 and genus 2-surface are given as the examples of closed surfaces.

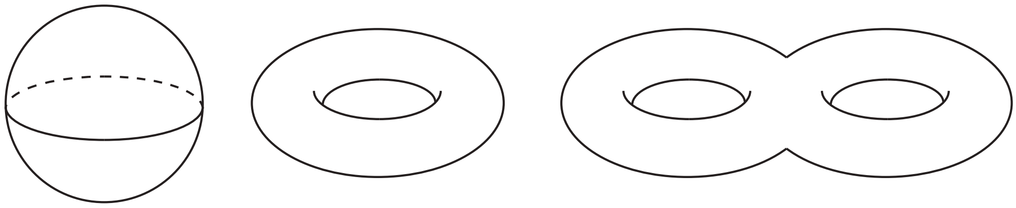

Figure 9: The 2-Sphere, torus and genus-2 surface.

On the other side, there are surfaces which have boundary. They are obtained by taking any surface without boundary and removing the interiors of disks from it. The boundaries of the disks left behind on the surface. So, these circles are now boundaries of the surface. These circles are called *boundary components*, [16]. The number of the removed interiors of the disks is equal to the number of boundary components.

Example 2.4.2. Cylinder and Möbius band are examples of surfaces with boundaries and given in Figure 10. Cylinder is a surface with boundary where its boundary is an unlink with 2 disjoint pieces. Möbius band is surface with boundary where its boundary is the unknot.

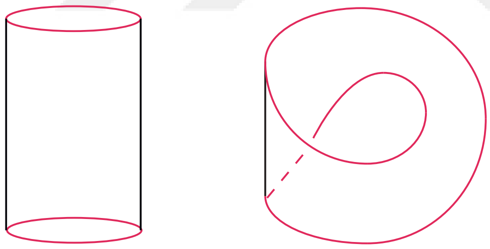

Figure 10: Surfaces with boundary.

Disk and annulus are another examples of surfaces with boundary components. These are obtained by removing point or points from closed surfaces.

Example 2.4.3. Let k be a point on S^2. The 2-sphere which has one boundary component, $S^2 - \{k\}$, is homeomorphic to a disk D^2. It is given in Figure 11.

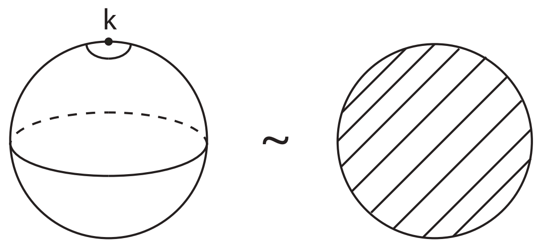

Figure 11: Surface with 1-boundary component.

Example 2.4.4. Let k_1 and k_2 be two points on S^2. The 2-sphere with two boundary components, $S^2 - \{k_1, k_2\}$, is homeomorphic to an annulus.

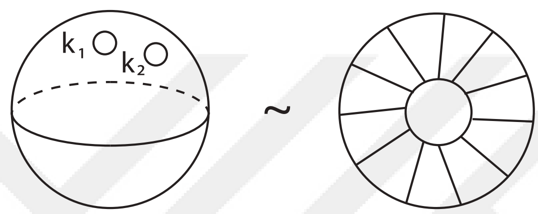

Figure 12: Surface with 2-boundary components.

Pick a normal vector on a given surface. This normal vector's direction is either the inside of the page or outside of the page. If the normal vector is always inside or outside of the page at every point on the surface, then the surface is given an orientation.

Definition 2.4.1. [16] If one can choose a consistent normal vector at each point on a surface, then the surface is called *orientable*. Otherwise, it is called *non-orientable*.

Example 2.4.5. The cylinder is an orientable surface, since one can pick a consistent normal vector at every point on the surface.

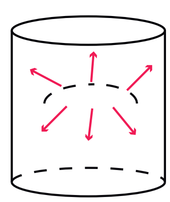

Figure 13: Orientable surface.

Also a 2-sphere, a torus, a disk and a torus with one boundary component are the examples of orientable surfaces.

Example 2.4.6. The Möbius band is a non-orientable surface. Pick a normal vector that pointing outside, as one moves along the surface this vector will eventually turns inside. Since the normal vector is not consistent, the Möbius band is non-orientable.

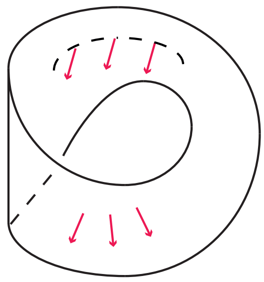

Figure 14: Non-orientable surface.

Definition 2.4.2. [16] A surface is called a *Seifert surface* for a given knot or link if surface's boundary is the given knot or link.

Theorem 2.4.1. [20] *" Every oriented knot or link is boundary of an orientable surface."*

This theorem was proved by Seifert in 1934. This proof is named as *Seifert algorithm*. In his algorithm, Seifert surface can be found of an oriented knot or link easily. Seifert algorithm is given step by step down below.

1) First step of the algorithm is that a given oriented knot or link has to be resolved from all crossings depending on its orientation.

2) From the first step there are set of circles which are called *Seifert circles*. These circles are boundary of disks in $\mathbb{R}^3$.

3) Assume that these disks are separated from each other and at different heights as a third step.

4) In fourth step, these disks will be connected by twisted bands from the crossing points by taking into account of the crossings. Finally, the result is an oriented surface with boundary. The boundary of the constructed surface is the given knot or link, [16], [21].

Example 2.4.7. In Figure 15, Seifert surface for a given figure eight knot is obtained by applying the algorithm to the knot.

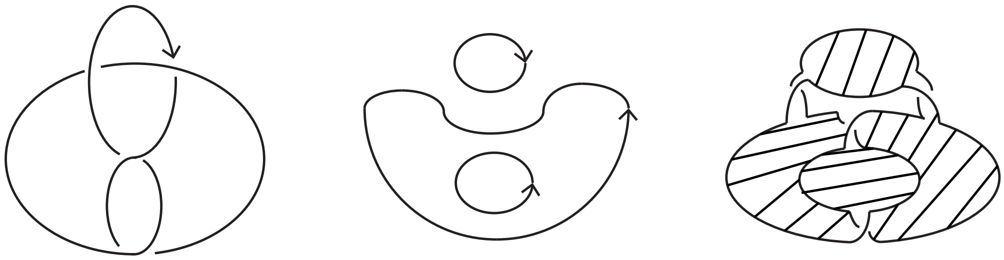

Figure 15: Seifert surface for a figure eight knot.

Example 2.4.8. In Figure 16, Seifert surface for a positive Hopf link is obtained by applying the algorithm to the link.

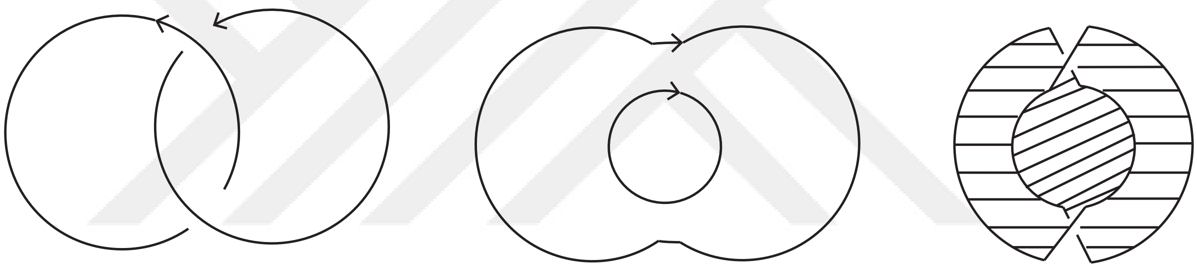

Figure 16: Seifert surface for a positive Hopf link.

2.4.1 Seifert Framing

Let M^3 be an oriented 3-manifold. Take a knot $K \subset M^3$ and a Seifert surface S for the knot.

Definition 2.4.3. [15] *The Seifert* or *surface framing* of the knot K is the trivialization of the normal bundle NK matching to the parallel curve obtained by pushing the knot K along the surface S. By way of explanation, K' is a parallel push off the knot K with $lk(K, K') = 0$ which stays in the surface S. It is denoted by λ_s.

Example 2.4.9. Let K be the unknot. So, the Seifert surface for K is a disk. Let us look at the Seifert framing of K in Figure 17.

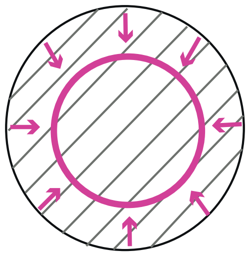

Figure 17: Seifert framing for the unknot along a disk.

2.5 Dehn Twist

Definition 2.5.1. Consider a simple closed curve b on an orientable, closed surface S and the neighborhood $N(b) \cong S^1 \times [0, 1]$ of b. Cut S through the neighborhood of b, twist one of the ends of the neighborhood of b through 2π to the right or to the left and glue back to the surface S, [4]. Right twist is called a *right-handed Dehn twist* and it is represented as t_b. Left twist is called a *left-handed Dehn twist* and it is represented as t_b^{-1}. In Figure 18 and in Figure 19, these operations are shown respectively.

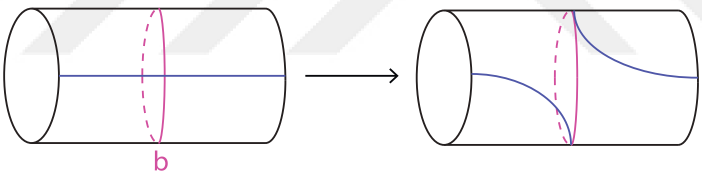

Figure 18: Right-handed Dehn twist.

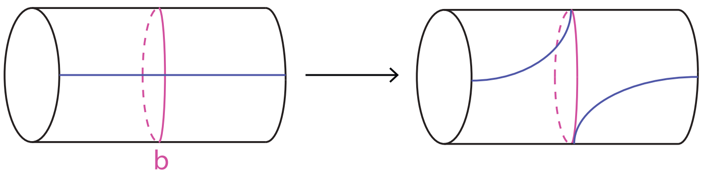

Figure 19: Left-handed Dehn twist.

2.6 Open Book Decompositions

Definition 2.6.1. [15][22] An open book decomposition of M^3 which is a closed, oriented 3-manifold is a triple (S, K, p). In here, K is an oriented knot or link in M^3. It is named as the *binding* of (S, K, p). The map $p : M^3 \backslash K \to S^1$ is a fibration of the $M^3\backslash K$ so that for each $\alpha \in S^1$, $p^{-1}(\alpha)$ is the interior of compact $S \subset M^3$ whose boundary is K. This compact surface is named as the *page* of (S, K, p).

$$\begin{array}{cc} S \hookrightarrow & M^3\backslash K \\ & \downarrow p \\ & S^1 \end{array}$$

The map p is named as *fibration map* and $M^3\backslash K$ fibers over S^1 with fiber S.

Definition 2.6.2. The *genus* of the open book decomposition is the genus of the page. Specifically, open book decompositions with genus zero are called *planar open book decompositions.*

Example 2.6.1. Consider S^3 and remove the unknot from it.

Think S^3 as a union of two solid tori, after all S^3 is a boundary of 4-ball D^4.

Figure 20: An unknot in the 3-Sphere.

Remove tubular neighborhood of U from S^3. Since the tubular neighborhood of U is a solid torus in S^3, remove this solid torus from 3-sphere and left with a solid torus. Pages are disks, binding is the unknot and (D^2, U, p) is an open book decomposition for S^3. The map p, the fibration map, is given down below:

$$p(x, y) = \frac{1}{|x|}(x).$$

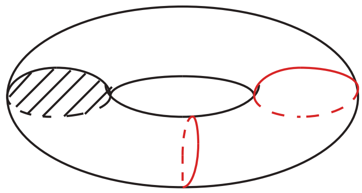

Figure 21: Solid torus.

$$
\begin{array}{ccl}
D^2 \hookrightarrow & S^3 \backslash U \simeq D^2 \times S^1 & \\
& \downarrow p & . \\
& S^1 &
\end{array}
$$

Example 2.6.2. Consider S^3 and remove positive Hopf link from it.

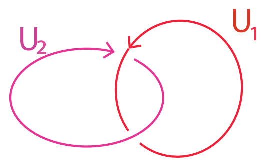

Figure 22: Positive Hopf link in the 3-Sphere.

Think S^3 as a union of two solid torus and think positive Hopf link in S^3. Remove the neighborhood of U_1 from S^3, then left with a solid torus whose inside contains the unknot.

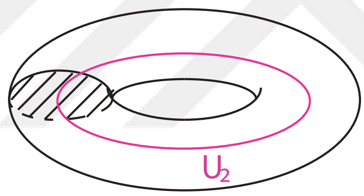

Figure 23: The unknot in solid torus.

Remove tubular neighborhood of the unknot from solid torus. Pages are annulus, binding is the positive Hopf link and $(S, U_1 \cup U_2, p)$ is an open book decomposition for S^3. The map p, the fibration map, is given down below:

$$p(z_1, z_2) = \frac{1}{|z_1 z_2|}(z_1 z_2).$$

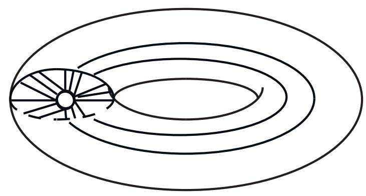

Figure 24: Open book decomposition of S^3.

$$\begin{array}{ccc} S \hookrightarrow & S^3 \backslash (U_1 \cup U_2) & \\ & \downarrow p & . \\ & S^1 & \end{array}$$

Example 2.6.3. Consider S^3 and remove negative Hopf link from it.

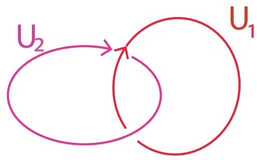

Figure 25: Negative Hopf link in the 3-Sphere.

Think S^3 as a union of two solid torus and think negative Hopf link in S^3. Remove the neighborhood of U_1 from S^3, then left with a solid torus whose inside contains the unknot. Remove tubular neighborhood of the unknot from solid torus. Pages are annulus, binding is the negative Hopf link and $(S, U_1 \cup U_2, p)$ is an open book decomposition for S^3. The map p, the fibration map, is given down below:

$$p(z_1, z_2) = \frac{1}{|z_1 \overline{z_2}|}(z_1 \overline{z_2}).$$

$$\begin{array}{ccc} S \hookrightarrow & S^3 \backslash (U_1 \cup U_2) & \\ & \downarrow p & . \\ & S^1 & \end{array}$$

2.6.1 Abstract Open Book Decompositions

Definition 2.6.3. Take a surface S which is oriented and has a boundary knot or link K. Take a self-diffeomorphism ψ of S such that this map is identity on ∂S. An abstract open book decomposition can be defined as below;

$$M^3 \backslash K = S \times [0,1]/(\psi(x), 0) \sim (x, 1), [22].$$

The pair (S, ψ) is named as an *abstract open book*. The map $\psi : S \longrightarrow S$ is named as the *monodromy* of (S, ψ). The surface is named as the *page* of (S, ψ). The boundary of the surface is named as the *binding* of (S, ψ).

Example 2.6.4. Take the page S as a disk D^2 with boundary the unknot U and the monodromy ψ as the identity map. In this case, (D^2, id) gives an abstract open book decomposition of S^3.

$$M^3 \backslash U = D^2 \times [0,1]/(x,0) \sim (x,1).$$

Example 2.6.5. Take the page S as an annulus which is a Seifert surface for positive Hopf link L_+ and monodromy ψ as a right-handed Dehn twist. In this case, (S, ψ) yields an abstract open book of S^3. It has positive Hopf link as the binding.

$$M^3 \backslash L_+ = S \times [0,1]/(\psi(x),0) \sim (x,1).$$

Example 2.6.6. Take the page S as an annulus which is a Seifert surface for negative Hopf link L_- and monodromy ψ as a left-handed Dehn twist. In this case, (S, ψ) yields an abstract open book of S^3. It has negative Hopf link as the binding.

$$M^3 \backslash L_- = S \times [0,1]/(\psi(x),0) \sim (x,1).$$

Definition 2.6.4. One can define *positive stabilization* of (S, ψ) as follows:

1) Take union of the surface S of the abstract open book and a 1-handle; $S' = S \cup$ 1-handle,

2) By taking a right-handed Dehn twist on a curve b in S' which intersects the co-core of the 1-handle only one time, one can get a new monodromy $\psi \circ t_b$, where t_b refers to a right-handed Dehn twist, [22].

Positive stabilization is shown as $(S', \psi \circ t_b)$. An abstract open book (S, ψ) and its positive stabilization $(S', \psi \circ t_b)$ give diffeomorphic manifolds, [22].

2.7 Dehn Surgery Along Knots

Dehn surgery is a way to build a new topological 3-manifold from a closed orientable 3-manifold and it is named after Max Dehn. For more information see [2], [15], [17] and [23].

To perform Dehn surgery, determine a knot K in the given M^3, take a tubular neighborhood $nbhd(K) \cong S^1 \times D^2$ of the knot and remove it from the 3-manifold M^3.

After this, by gluing the removed neighborhood with a diffeomorphism $\varphi : \partial(S^1 \times D^2) \longrightarrow \partial(M^3 \setminus nbhd(K))$, a new 3-manifold is constructed. It is shown as $M' = (M^3 \setminus nbhd(K)) \cup_\varphi (S^1 \times D^2)$ and M' is achieved by Dehn surgery along K. This new manifold changes depending on diffeomorphism φ.

Let $M^3 = S^3$. A curve on $\partial(S^3 \setminus nbhd(K))$ is determined by integers (p, q) which are relatively prime. The union of a 3-dimensional 2-handle and a 3-handle can be thought as a solid torus. Since, a 3-handle can be glued only one way, it is up to 2-handle to determine the surgery by gluing curve $(\{*\} \times \partial D^2)$ and $\varphi(\{*\} \times \partial D^2) = \alpha \in \partial(S^3 \setminus nbhd(K))$, [17].

Homology class of the curve α; $[\alpha] \in H_1(\partial(S^3 \setminus nbhd(K)); \mathbb{Z}) \cong H_1(T^2; \mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z}$. A curve on $\partial(S^3 \setminus nbhd(K))$ is called a *meridian* on $nbhd(K)$, μ, that performs a generator of $H_1(\partial(S^3 \setminus nbhd(K)))$. Another curve on $\partial(S^3 \setminus nbhd(K))$ is called a *longitude*, λ_s, that corresponds to the zero framing as Seifert framing. These two curves are a basis for $H_1(\partial(S^3 \setminus nbhd(K)))$. So, for coprime p and q,

$$\varphi(\{*\} \times \partial D^2) = \alpha = q\mu + p\lambda_s, \text{ [23]}.$$

This coprime integers (p, q) might be thought as a reduced fraction $\frac{p}{q}$. The surgery coefficient is defined by this rational number $\frac{p}{q} \in \mathbb{Q} \cup \{\infty\}$. This coefficient contains all the information about the surgery. These type of surgeries are called *rational surgery*. An *integral surgery* is a surgery with $q = \pm 1$, [23].

Example 2.7.1. [23] $\infty = \frac{1}{0}$ Dehn surgery along a knot $K \subset S^3$ gives S^3.

2.7.1 Lens Spaces

Definition 2.7.1. [24] The *lens space* $L(p, q)$ can be formed by a Dehn surgery applied to the unknot with the surgery coefficient $-\frac{p}{q}$.

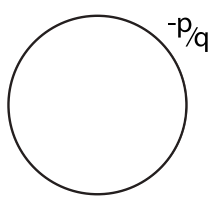

Figure 26: $L(p, q)$.

Example 2.7.2. [24] $L(1,0) = S^3$ and $L(0,1) = S^1 \times S^2$.

Definition 2.7.2. [25] Another definition of the 3-dimensional lens spaces can be given by quotients of the 3-sphere S^3 by $\mathbb{Z}_p$ actions. Here, think the 3-sphere S^3 in $\mathbb{C}^2$. The 3-sphere S^3 is the set of $\{(k_1, k_2) \in \mathbb{C}^2 : |k_1|^2 + |k_2|^2 = 1\}$ and take the cyclic group $\mathbb{Z}_p = \{0, 1, ..., p-1\}$ with p does not have to be prime number. So, $\mathbb{Z}_p$ acts on S^3 freely and it is given down below:

$$s \cdot (k_1, k_2) = (e^{\frac{2\pi i s}{p}} \cdot k_1, e^{\frac{2\pi i s q}{p}} \cdot k_2) = L(p,q) \text{ for some } s \in \mathbb{Z}_p.$$

Example 2.7.3. $L(2,1) = \mathbb{R}P^3$: By using the second definition of the lens space, $L(2,1) = S^3/\mathbb{Z}_2$. This means that S^3, where (k_1, k_2) is identified with $(k_1^{'}, k_2^{'})$.

$$(k_1^{'}, k_2^{'}) = (e^{\frac{2\pi i s}{2}} \cdot k_1, e^{\frac{2\pi i s}{2}} \cdot k_2)\ .$$

Since $s \in \mathbb{Z}_2$, s is either 0 or 1. If $s = 0$, then $(k_1^{'}, k_2^{'}) = (k_1, k_2)$ or if $s = 1$, then $(k_1^{'}, k_2^{'}) = (-k_1, -k_2) = -(k_1, k_2)$. Thus, $L(2,1)$ is the 3-sphere with antipodal points identified. From Example 2.1.3 this is the definition of real projective space $\mathbb{R}P^3$.

The classification of lens spaces was shown by Broody in 1960 up to homeomorphism.

Theorem 2.7.1. [26] *“ Lens spaces $L(p,q)$ and $L(p,q')$ are homeomorphic if and only if $q \equiv \pm q' \mod p$.”*

In 1960 Wallace proved Theorem 2.7.2 and after that in 1962 Lickorish proved the same theorem in a different way.

Theorem 2.7.2. [4] [3] *“ Each closed, oriented M^3 can be achieved by Dehn surgery with the framing (± 1) along a link of unknots in S^3.”*

2.8 Kirby Moves

By using Dehn surgery along knots, a given 3-manifold can be changed into a different 3-manifold, but with Kirby moves situation will be different. By applying Kirby moves on a framed link $\mathcal{K}$ one can get the same 3-manifold again. For further explanation see [27], [24] and [23].

2.8.1 First Kirby Move-K1

First Kirby move is related with taking a connected sum with S^3 or cancelling it from the framed link $\mathcal{K}$. Here, the unknot with framing ± 1 represents S^3, [23].

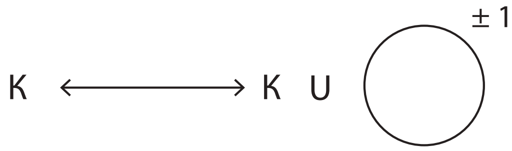

Figure 27: Kirby move-K1.

2.8.2 Second Kirby Move-K2

Second Kirby move is related with component sliding of the framed link $\mathcal{K}$. K_1 and K_2 are two components of the link $\mathcal{K}$. K_1 has n_1 framing and K_2 has n_2 framing which is defined by a longitude K_2'. Define $K_\#$ as a union of K_1 and K_2. $K_\#$ is obtained by connecting K_1 and K_2' via a band connection k. It is shown as $K_\# = K_1 \#_k K_2'$ in Figure 28. The other parts of the link will be the same. This operation is explained as K_1 was slide over K_2. The framing of the component $K_\#$ can be computed according to the formula, $n_1 + n_2 + 2lk(K_1, K_2)$, [23].

By the way, framing of the K_1 does not change while K_2 changes, [23].

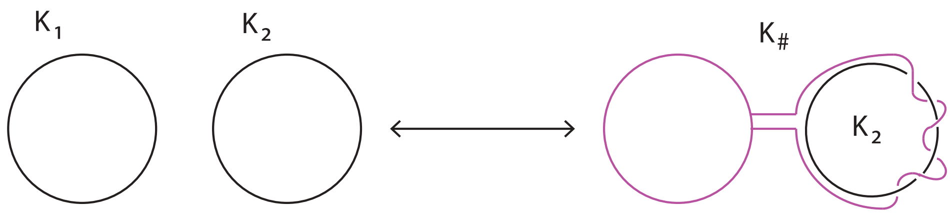

Figure 28: Kirby move-K2.

The *blow-down* operation is defined as putting aside an unknot with framing (± 1). The *blow-up* operation is the inverse of blow-down operation, that is, it is done by adding an unknot with framing (± 1). General situation for these operations are illustrated in Figure 29, [23].

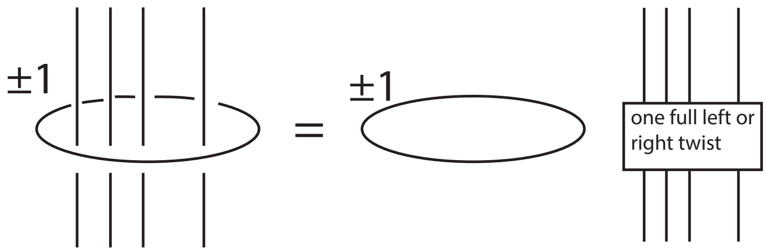

Figure 29: General situation for (± 1) blow-down/up.

Example 2.8.1. In Figure 30 and Figure 31 are illustrated blow-down operation whose final states are S^3.

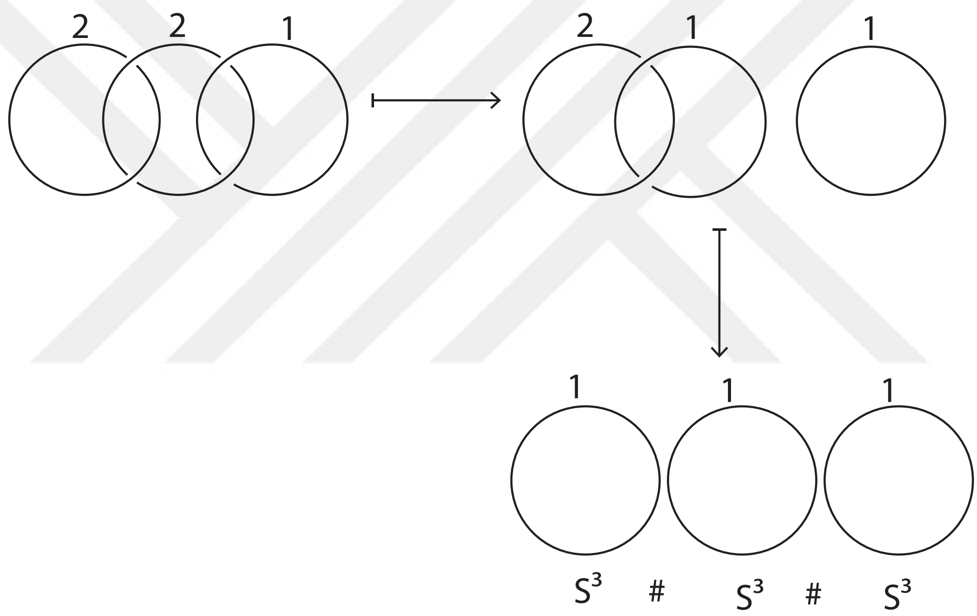

Figure 30: The framed link represents S^3.

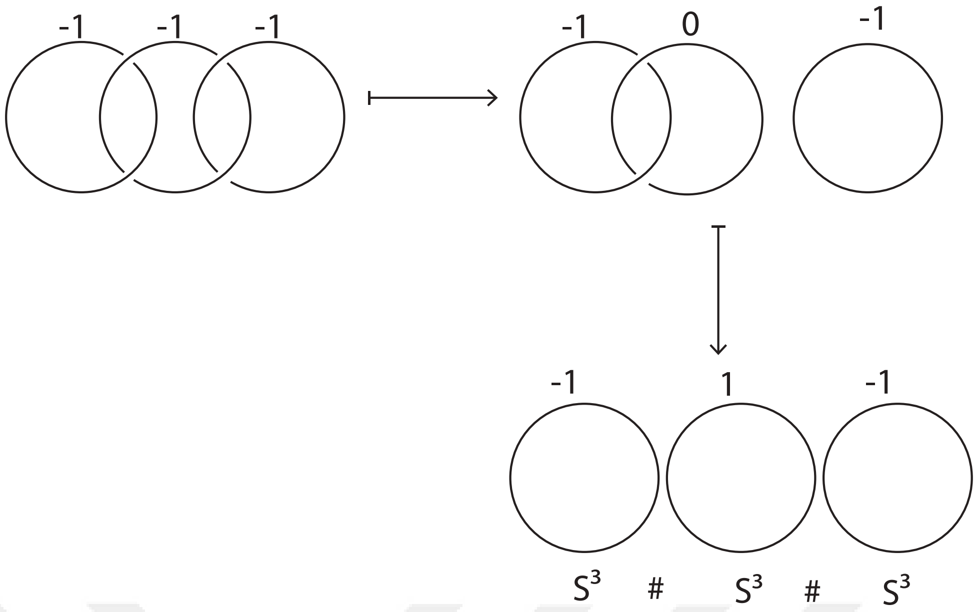

Figure 31: The framed link represents S^3.

3 EXISTENCE OF OPEN BOOK DECOMPOSITIONS

Existence of open book decompositions of 3-manifolds has been known since 1920 thanks to Alexander.

Theorem 3.0.1. [7] *“ Take a* 3*-manifold which is closed and orientable. Then M^3 has an open book decomposition.”*

Proof. In [3] and [4], Wallace and Lickorish showed that all closed, oriented M^3 is achieved by Dehn surgery with the framing (± 1) along a link $L_{M^3} \subset S^3$ of unknots. Additionally, there exists an unknot $U \subset S^3$ so that the link L_{M^3} is braided about the unknot. Also, every component L_k of the link L_{M^3} might be considered to link the unknot absolutely once. This situation can be seen in Figure 32.

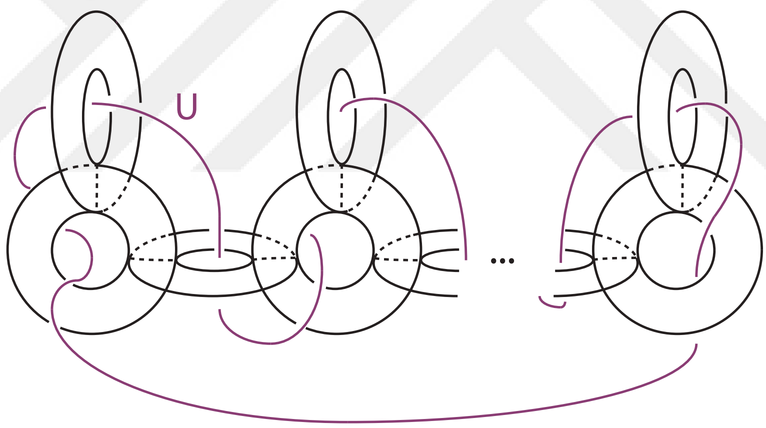

Figure 32: L_{M^3} is braided about U.

Think the trivial abstract open book (D^2, id) of S^3: the pages are disks, the binding is the unknot U and the monodromy is the identity map. Since the unknot U links every component L_k of the link L_{M^3} absolutely one time, every component L_k of the link L_{M^3} punctures each disk transversely one time. After executing (± 1)-surgery along the link L_{M^3}, boundary of every puncture, that is a meridional curve to every component L_k, becomes longitudinal. So, these new punctured disk pages give the page of the open book decomposition of M^3. Therefore, the pages are planar. Moreover, its binding is formed by U and L_{M^3}. $\square$

In the given proof of Alexander's theorem, the monodromy map of the appearing open book decomposition is unknown. To shed light on this monodromy issue, an algorithm for finding the monodromy map of open book decompositions in Theorem 3.0.1 will be studied in detail and this algorithm is in Onaran's paper [13].

3.1 The Algorithm for Planar Open Book Decompositions

Take an oriented and closed M^3 and an abstract open book (S, ψ) of this 3-manifold.

- Consider a knot K in M^3 sitting on the page of (S, ψ). By performing (± 1)-surgery along the knot K, a new 3-manifold is obtained with $(S, \psi \circ t_K^{\mp 1})$. Here, t_K^{-1} refers to left-handed Dehn twist and t_K^{+1} refers to right-handed Dehn twist about K.

As in Figure 33, the link L_{M^3} that is mentioned in the proof of Alexander's theorem can be presented as the closure of an n-braid. In Lemma 3.3 of Onaran's paper [28] it is shown that every link has a pure braid presentation. This means that one can assume the n-braid which is illustrated in Figure 33 is indeed a pure n-braid. Therefore, the n-braid forming the link L_{M^3} can be decomposed with regard to standard generators of the pure n-braid which are given in Figure 34.

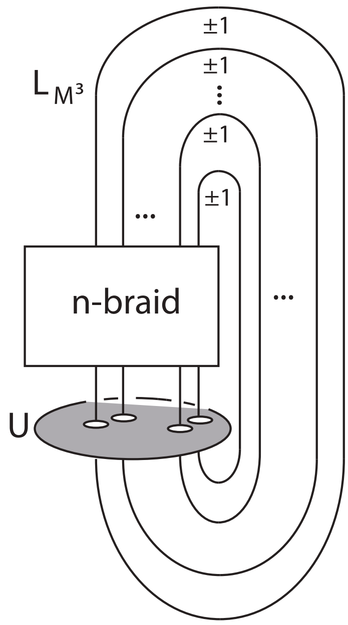

Figure 33: Surgery link.

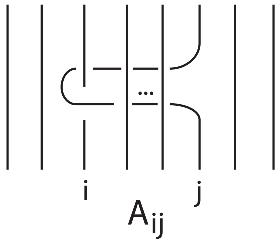

Figure 34: A_{ij},for $1 \leq i \leq j \leq n$.

Theorem 3.1.1. [13] *" Take an oriented, closed 3-manifold M^3 that is achieved by Dehn surgery with the framing (± 1) along a link L_{M^3} of n-unknots in S^3. Then, a planar open book decomposition of M^3 is obtained with its page which is a disk with n-punctures. Also, its monodromy can be achieved from a pure braid presentation of the link L_{M^3}."*

Proof. Begin with a pure braid presentation of the link L_{M^3} as giving in standard generators. From the proof of Alexander's theorem, remember that the unknot U is the boundary of a disk in S^3 and every component of the link L_{M^3} punctures that disk exactly once. By using the algorithm for open book decompositions and open book (D^2, id) of S^3, a planar open book for M^3 will be built.

Eliminate every linking among the components of L_{M^3} via blowing up which is mentioned in Subsection 2.8. Therefore, the yielding unknots with framings (± 1) can be isotoped to place on the punctured disk whose boundary is the unknot U. Specifically, adding a (± 1)-framed unknot to a diagram will add (± 1) to the framing of every corresponding component of the link L_{M^3}. The blowing up operation eliminates full twists. So, the pure braid presentation of the L_{M^3} with regard to standard generators enables us to eliminate all linking. After that, keep on blowing up until all framing coefficients of every component of L_{M^3} is zero. This zero framed L_{M^3} transversely punctures every disk page one time. So, the yielding (± 1) link of unknots can be isotoped onto one of the punctured disk pages. A planar open book decomposition of M^3 is obtained, after executing surgeries by the algorithm 3.1. The pages of this open book are disk which has n-punctures. Its monodromy is a product of left/right-handed Dehn twists about the surgery curves with the framing (± 1) which is on the punctured disk whose boundary is the unknot U. □

The following two examples illustrate the proof of Theorem 3.1.1. In each example the resulting 3-manifolds after surgeries, their open book decompositions and their monodromies are explicitly given.

Example 3.1.1. The $L(2,1)$ can be given by a surgery on the Hopf link.

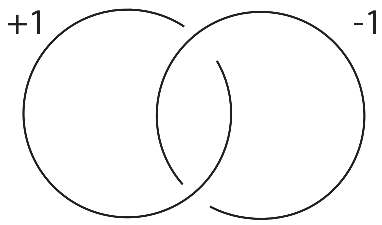

Figure 35: Surgery on the Hopf link.

Let us first see the 3-manifold is $L(2,1)$.

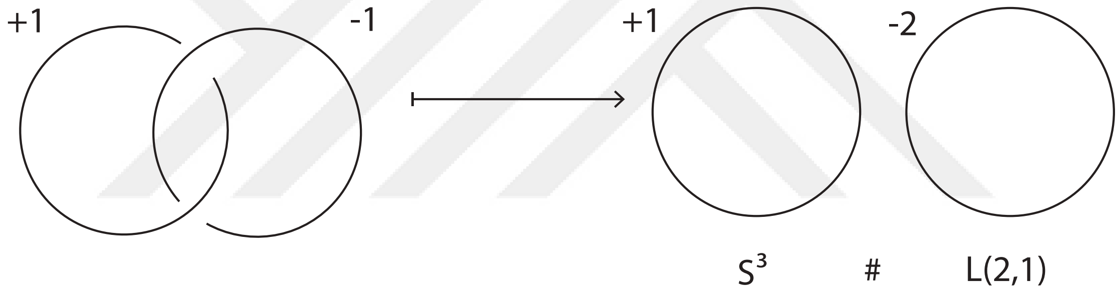

Figure 36: The resulting manifold is $L(2,1)$.

Firstly, demonstrate the Hopf link as a pure 2-braid closure. One can decompose this pure 2-braid with regard to standard generators of the pure braid group on 2-strands. After that, eliminate every linking among the components of the Hopf link via blowing up. Keep on this operation until the framing of every component of the Hopf link is zero. These operations are shown in Figure 37.

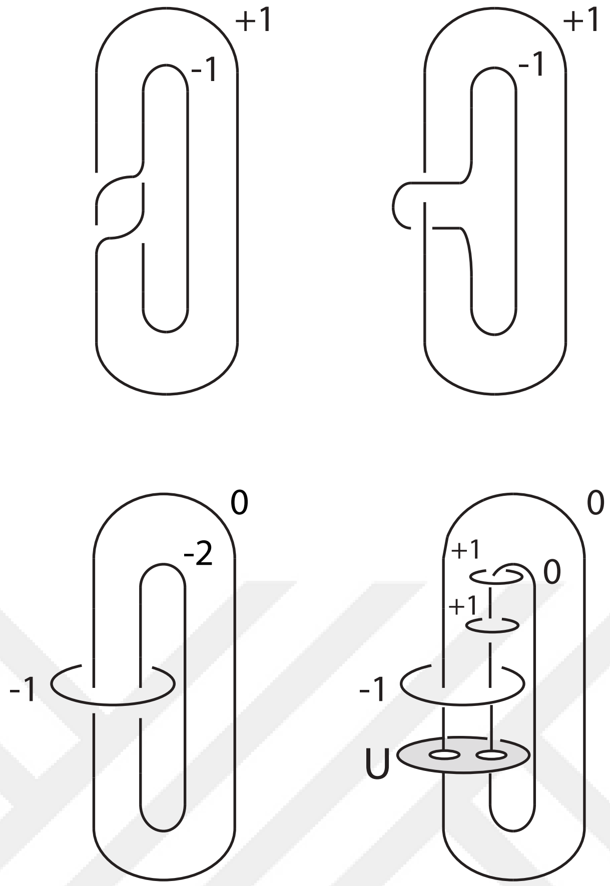

Figure 37: Pure braid presentation of the Hopf link and a way of eliminating linkings.

As can be seen in Figure 37, there is the unknot U which bounds a disk in S^3. That disk is punctured by every 0-framed unknot. Isotope (± 1)-framed unknots onto the punctured disk whose boundary is the unknot U. By the algorithm for open book decompositions, after executing surgeries, a planar open book decomposition (S, ψ) of $L(2,1)$ is obtained. Its page is a disk with two punctures. Every 0-framed unknot is a binding component. Let us call these binding components α_1 and α_2 from outer one to inner one. The monodromy of the open book is $\psi = t_\beta t_{\alpha_2}^{-2}$ where β is the simple closed curve surrounding the holes α_1 and α_2.

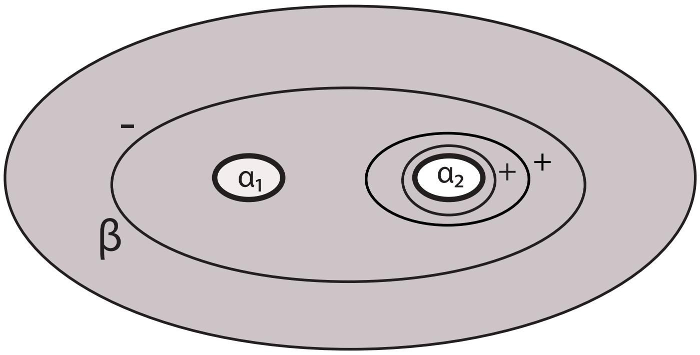

Figure 38: A planar open book for $L(2,1)$.

Example 3.1.2. The $L(3,1)$ can be obtained by a surgery on the link which is illustrated in Figure 39.

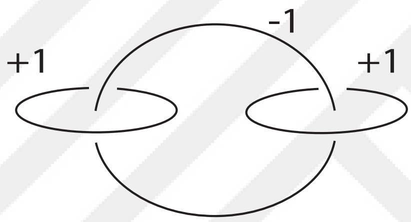

Figure 39: Link with unknots.

Let us first see the 3-manifold is $L(3,1)$.

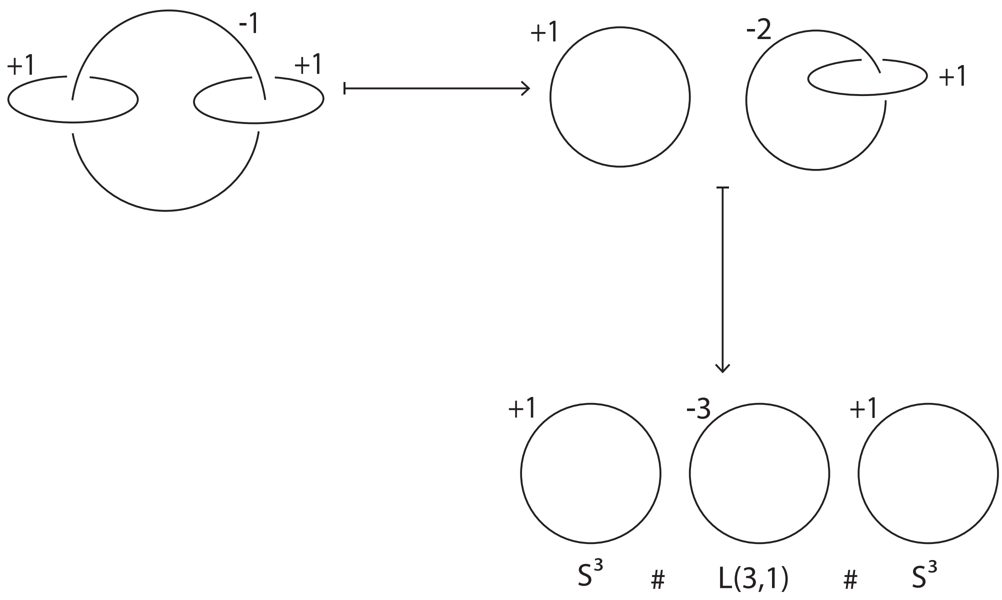

Figure 40: The resulting manifold is $L(3,1)$.

Firstly, demonstrate the given link as a 3-braid presentation. After that, eliminate every linking among the components of the given link via blowing up. Keep on this operation until the framing of every component of the given link is zero. These operations are shown in Figure 41.

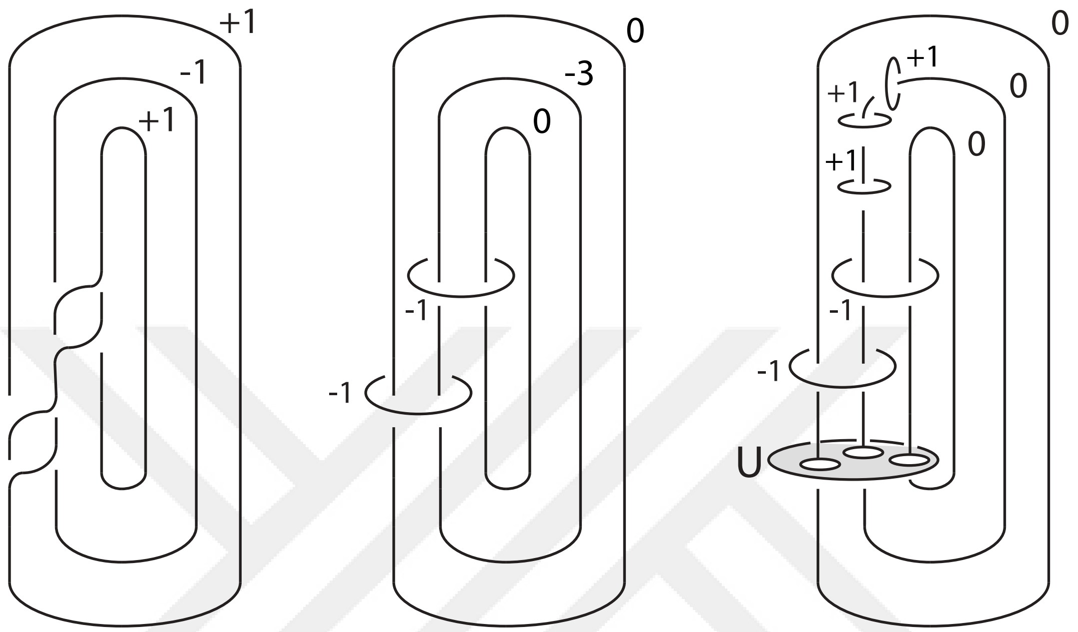

Figure 41: Braid presentation of the given link and a way of eliminating linkings.

As can be seen in Figure 41, there is the unknot U which bounds a disk in S^3. That disk is punctured by every 0-framed unknot. Isotope (± 1)-framed unknots onto the punctured disk whose boundary is the unknot U. By the algorithm for open book decompositions, after executing surgeries, a planar open book decomposition (S, ψ) of $L(3,1)$ is obtained. Its page is a disk with three punctures. Every 0-framed unknot is a binding component. Let us call these binding components α_1, α_2 and α_3 from outer one to inner one. The monodromy of the open book is $\psi = t_\alpha t_\beta t_{\alpha_2}^{-3}$ where α and β are simple closed curves. The curve α surrounds the holes α_1 and α_2. The curve β surrounds the holes α_2 and α_3 .

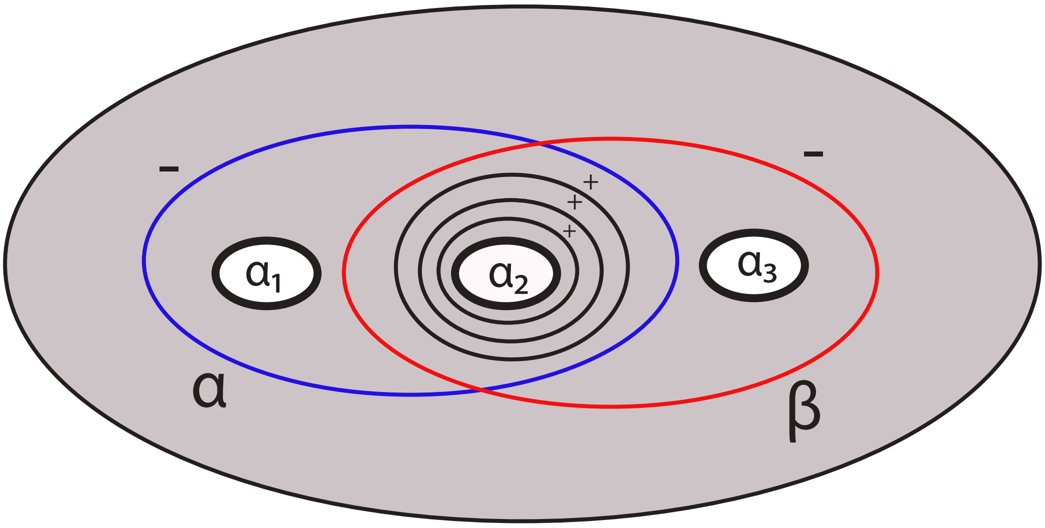

Figure 42: A planar open book for $L(3,1)$.

4 CONTACT 3-MANIFOLDS

4.1 Contact 3-Manifolds

Definition 4.1.1. [29] [30] Take an oriented 3-manifold M^3. Let $\alpha : TM^3 \to \mathbb{R}$ be a 1-form on M^3 and take the set of $\ker\alpha = \{u \in TM^3 | \alpha(u) = 0\}$. If a 1-form α fulfills the conditions $\alpha \wedge (d\alpha) \neq 0$ and locally $\xi = \ker\alpha$, then α is named as a *contact 1-form* on the 3-manifold and ξ is named as a *contact structure* on M^3.

Definition 4.1.2. [29] [30] A *contact* 3*-manifold* M^3 is a manifold that has a contact structure on it. It is denoted by (M^3, ξ).

Example 4.1.1. [29] [30] Let $\alpha_{std} = dz - ydx$ be a 1-form on $\mathbb{R}^3$ with Cartesian coordinates. The 1-form α_{std} is contact, since

$$\begin{aligned} \alpha_{std} \wedge d\alpha_{std} &= (dz - ydx) \wedge d(dz - ydx) \\ &= dx \wedge dy \wedge dz \neq 0. \end{aligned}$$

The contact planes are $\xi_{std} = \ker\alpha_{std} = \left\langle \frac{\partial}{\partial y}, \frac{\partial}{\partial x} + y\frac{\partial}{\partial z} \right\rangle$. In that case, α_{std} gives a contact structure on $\mathbb{R}^3$. This contact structure is called *standard contact structure* on $\mathbb{R}^3$.

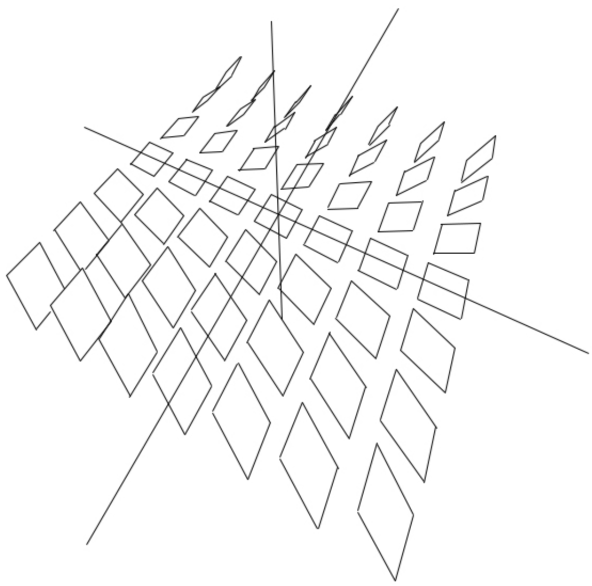

Figure 43: Standard contact structure on $\mathbb{R}^3$.

Example 4.1.2. [30] [17] Take the unit 3-sphere $S^3 \subset \mathbb{R}^4$. Let $\alpha_1 = (x_1dy_1 - y_1dx_1 + x_2dy_2 - y_2dx_2)|_{S^3}$ be a 1-form on S^3 with Cartesian coordinates.

$$\begin{aligned}
\alpha_1 \wedge d\alpha_1 &= (x_1dy_1 - y_1dx_1 + x_2dy_2 - y_2dx_2) \wedge d(x_1dy_1 - y_1dx_1 + x_2dy_2 - y_2dx_2) \\
&= (x_1dy_1 - y_1dx_1 + x_2dy_2 - y_2dx_2) \wedge (2dx_1 \wedge dy_1 + 2dx_2 \wedge dy_2) \\
&= \begin{array}{l} 2x_2dx_1 \wedge dy_1 \wedge dy_2 - 2y_2dx_1 \wedge dy_1 \wedge dx_2 + \\ 2x_1dy_1 \wedge dx_2 \wedge dy_2 - 2y_1dx_1 \wedge dx_2 \wedge dy_2. \end{array}
\end{aligned}$$

A basis for the tangent space of S^3 can be given by $\left\langle \frac{\partial}{\partial x_1} - \frac{x_1}{y_1}\frac{\partial}{\partial y_1}, \frac{\partial}{\partial x_2} - \frac{x_2}{y_2}\frac{\partial}{\partial y_2}, \frac{\partial}{\partial x_1} - \frac{x_1}{y_2}\frac{\partial}{\partial y_2} \right\rangle$. Then, $\alpha_1 \wedge d\alpha_1 \neq 0$ on this basis. In that case, α_1 gives a contact structure on S^3. This contact structure is called the *standard contact structure* on the 3-sphere. This contact 3-manifold is denoted by (S^3, ξ_{std}).

Example 4.1.3. [29] [30] Let $\alpha_2 = dz + xdy$ be a 1-form on $\mathbb{R}^3$ with Cartesian coordinates. The 1-form α_2 is contact, since

$$\begin{aligned}
\alpha_2 \wedge d\alpha_2 &= (dz + xdy) \wedge d(dz + xdy) \\
&= dx \wedge dy \wedge dz \neq 0.
\end{aligned}$$

The contact planes are $\xi_2 = \ker \alpha_2 = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} - x\frac{\partial}{\partial z} \right\rangle$. In that case, α_2 gives a contact structure on $\mathbb{R}^3$.

Example 4.1.4. Let $\alpha_3 = dz + r^2d\theta$ be a 1-form on $\mathbb{R}^3$ with (r, θ, z)-cylindrical coordinates. The 1-form α_3 is contact, since

$$\begin{aligned}
\alpha_3 \wedge d\alpha_3 &= (dz + r^2d\theta) \wedge d(dz + r^2d\theta) \\
&= (dz + r^2d\theta) \wedge (2rdr \wedge d\theta) \\
&= 2rdr \wedge d\theta \wedge dz \neq 0.
\end{aligned}$$

The contact planes are $\xi_3 = \ker \alpha_3 = \left\langle \frac{\partial}{\partial r}, \frac{\partial}{\partial \theta} - r^2\frac{\partial}{\partial z} \right\rangle$. In that case, α_3 gives a contact structure on $\mathbb{R}^3$.

Example 4.1.5. [30] Let $\alpha_{sym} = dz - ydx + xdy$ be a 1-form on $\mathbb{R}^3$ with Cartesian coordinates. The 1-form α_{sym} is contact, since

$$\begin{aligned}
\alpha_{sym} \wedge d\alpha_{sym} &= (dz - ydx + xdy) \wedge d(dz - ydx + xdy) \\
&= 2dx \wedge dy \wedge dz \neq 0.
\end{aligned}$$

The contact planes $\xi_{sym} = \ker \alpha_{sym}$ is spanned by $\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right\}$ when $x = y = 0$, spanned by $\left\{x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}, \frac{\partial}{\partial x} + y\frac{\partial}{\partial z}\right\}$ when $y \neq 0$ and spanned by $\left\{x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}, \frac{\partial}{\partial y} - x\frac{\partial}{\partial z}\right\}$ when $x \neq 0$. In that case, α_{sym} gives a contact structure on $\mathbb{R}^3$. In cylindrical coordinates α_{sym} is equal to $\alpha_{sym} = dz + r^2 d\theta = \alpha_3$.

Example 4.1.6. Let $\alpha_{ot} = \cos r dz + r \sin r d\theta$ be a 1-form on $\mathbb{R}^3$ with (r, θ, z)-cylindrical coordinates. The 1-form α_{ot} is contact, since

$$\begin{aligned}\alpha_{ot} \wedge d\alpha_{ot} &= (\cos r dz + r \sin r d\theta) \wedge d(\cos r dz + r \sin r d\theta) \\ &= (\cos r dz + r \sin r d\theta) \wedge (-\sin r dr \wedge dz + (\sin r + r\cos r) dr \wedge d\theta) \\ &= (1 + \frac{\sin r \cos r}{r}) r dr \wedge d\theta \wedge dz \neq 0\end{aligned}$$

for $r > 0$.

The contact planes are $\xi_{ot} = \ker \alpha_{ot} = \left\langle \frac{\partial}{\partial r}, \frac{\partial}{\partial \theta} - r \tan r \frac{\partial}{\partial z} \right\rangle$. In that case, α_{ot} gives a contact structure on $\mathbb{R}^3$.

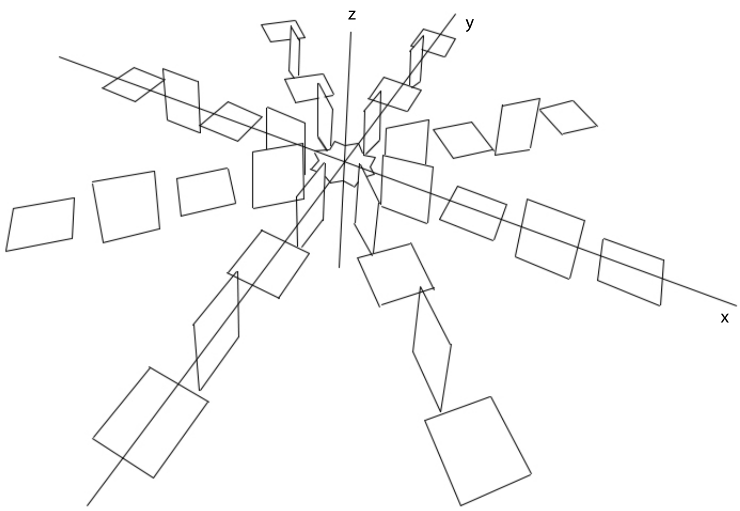

Figure 44: Overtwisted contact structure on $\mathbb{R}^3$.

As a contact 3-manifold, there are two types: tight and overtwisted.

Definition 4.1.3. [29] [30] Take a contact 3-manifold (M^3, ξ) and a disk $D \subset M^3$. The disk D is called *overtwisted disk* if the condition $T_p D = \xi_p$ is fulfilled by $\forall p \in \partial D$.

Definition 4.1.4. [29] [30] [17] If (M^3, ξ) contains an overtwisted disk, then (M^3, ξ) is called *overtwisted*. If not it is called a *tight contact 3-manifold*.

Example 4.1.7. [31] Contact 3-manifolds $(\mathbb{R}^3, \xi_{std})$ and $(\mathbb{R}^3, \xi_{sym})$ are tight contact 3-manifolds, contact 3-manifold $(\mathbb{R}^3, \xi_{ot})$ is overtwisted. The disk $D = \{(r, \theta, z) | z = 0, r \leq \pi)\}$ in $(\mathbb{R}^3, \xi_{ot})$ is an overtwisted disk.

Definition 4.1.5. [29] Take two contact 3-manifolds (M_1, ξ_1) and (M_2, ξ_2). Supposing that there is a diffeomorphism ω from M_1 to M_2 so that $\forall p \in M_1$ $d\omega_p(\xi_p) = \xi'_{\omega(p)}$, then diffeomorphism ω is called a *contactomorphism*. In that case, these two contact manifolds are contactomorphic manifolds.

Example 4.1.8. Contact 3-manifolds $(\mathbb{R}^3, \xi_{std})$ and $(\mathbb{R}^3, \xi_2)$ are contactomorphic manifolds. Let diffeomorphism ω defined as;

$$\begin{aligned} \omega : (\mathbb{R}^3, \xi_2) &\longrightarrow (\mathbb{R}^3, \xi_{std}) \\ \omega(x, y, z) &\longmapsto (y, x, -z) = (x', y', z'). \end{aligned}$$

The diffeomorphism ω is a contactomorphism which equalize the condition $d\omega(\xi_2) = \xi_{std}$.

$$\begin{aligned} d\omega : T\mathbb{R}^3 &\longrightarrow T\mathbb{R}^3 \\ \xi_{2p} &\longmapsto d\omega_p(\xi_2) = \xi_{std\Phi(p)} \end{aligned}$$

$$\begin{aligned} d\omega_{(x,y,z)}\left(\tfrac{\partial}{\partial x}\right) &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ &= \tfrac{\partial}{\partial y'} \\ d\omega_{(x,y,z)}\left(-x\tfrac{\partial}{\partial z} + \tfrac{\partial}{\partial y}\right) &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ x \end{bmatrix} \\ &= \tfrac{\partial}{\partial x'} + x\tfrac{\partial}{\partial z'} = \tfrac{\partial}{\partial x'} + y'\tfrac{\partial}{\partial z'} \end{aligned}$$

The vectors of $\left\{\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y} - x\dfrac{\partial}{\partial z}\right\}$ which produces contact structure ξ_2 is carried to the vectors of $\left\{\dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial x} + y\dfrac{\partial}{\partial z}\right\}$ which produces ξ_{std} by diffeomorphism ω.

Theorem 4.1.1. (Darboux's Theorem, [32]) *" Take a contact structure on M^3. For an arbitrary point $z \in M^3$ there is a neighborhood V of z so that this neighborhood is contactomorphic to a neighborhood Y which is the neighborhood of the point $(0,0,0)$ in $(\mathbb{R}^3, \xi_{std})$."*

As stated in Darboux's theorem locally all contact 3-manifolds look like the same.

4.2 Legendrian Knots

Definition 4.2.1. [29] [33] A given knot L in (M^3, ξ) is named as a *Legendrian knot* if the knot L is everywhere tangent to contact planes. In other words, for an arbitrary point $z \in L$, if the condition $T_z L \subset \xi_z$ is satisfied then L is a Legendrian knot.

Let

$$\begin{aligned} \psi : S^1 &\longrightarrow \mathbb{R}^3 \\ \eta &\longmapsto (x(\eta), y(\eta), z(\eta)) \end{aligned}$$

be a parameterization of L and also ψ be the class of C^1 and an immersion. Furthermore, L is tangent to ξ can be symbolized by $\psi'(\eta) \in \xi_{\psi(\eta)}$ or after all $\xi = \ker(dz - ydx)$ one can get that

$$z'(\eta) - y(\eta)x'(\eta) = 0. \tag{1}$$

Legendrian knots can be illustrated in two ways in $(\mathbb{R}^3, \xi_{std})$ but only one will be used which is front projection, [33].

Definition 4.2.2. [29] [33] Let L be a Legendrian knot in $(\mathbb{R}^3, \xi_{std})$ and

$$\begin{aligned} p : \mathbb{R}^3 &\longrightarrow \mathbb{R}^2 \\ p(x,y,z) &= (x,z) \end{aligned}$$

be a projection map. The image $p(L)$ is named as *the front projection* of L.

Considering ψ parameterizes L then

$$\begin{aligned} \psi_p : S^1 &\longrightarrow \mathbb{R}^2 \\ \eta &\longmapsto (x(\eta), z(\eta)) \end{aligned}$$

parameterizes $p(L)$. Although ψ was an immersion(embedding), ψ_p is not. From the equation 1; $z'(\eta) = y(\eta)x'(\eta)$. This equation implies that when $x'(\eta) = 0$ then $z'(\eta) = 0$. So if ψ_p was an immersion, $x'(\eta)$ must never disappear. In other words, $p(L)$ does not have vertical tangencies, [33].

For an usual C^1 Legendrian embedding in $\mathbb{R}^3$, $x'(\eta)$ can solely disappear at isolated points. Also, there exists a well-defined tangent line as horizontally in the front projection at isolated points . These kind of points are called *generalized cusps*, [33].

These two facts that are mentioned above are characterization of front projections. Especially, any map

$$g : S^1 \longrightarrow xz-plane$$
$$\eta \longmapsto (x_g(\eta), z_g(\eta))$$

that fulfills these facts appear as a Legendrian knot, after all under these facts $y(\eta)$ can be defined always by

$$y(\eta) = \frac{z_g'(\eta)}{x_g'(\eta)}.$$

Then $\psi(\eta) = (x_g'(\eta), y(\eta), z_g'(\eta))$ will be a Legendrian knot, [33].

For a given knot diagram that satisfies the below conditions, presents image of the front projection of a Legendrian knot.

- Vertical tangencies do not exist,
- Generalized cusps are the only singular points,
- At each crossing the slope of the undercrossing is bigger than the overcrossing.

Example 4.2.1. Front diagrams for Legendrian knots are given in Figure 45.

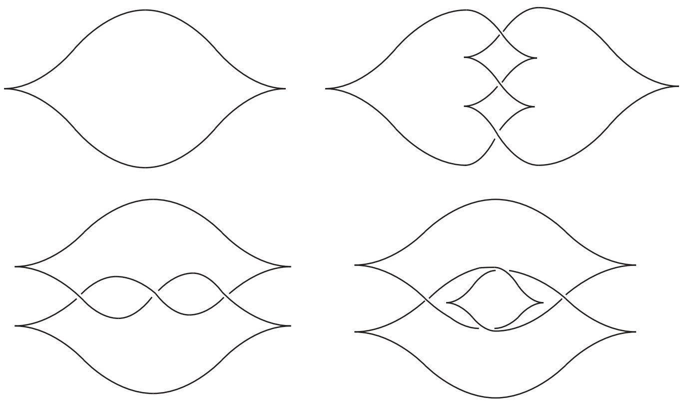

Figure 45: Legendrian unknot, Legendrian left-handed trefoil, Legendrian right-handed trefoil and Legendrian figure eight knot.

Theorem 4.2.1. [33] *" All topological knot type is expressed with a Legendrian knot."*

Proof. Any topological knot type is expressed with a Legendrian knot in $(\mathbb{R}^3, \xi_{std})$ by applying the changes that have been shown in Figure 46 to the topological knot. $\square$

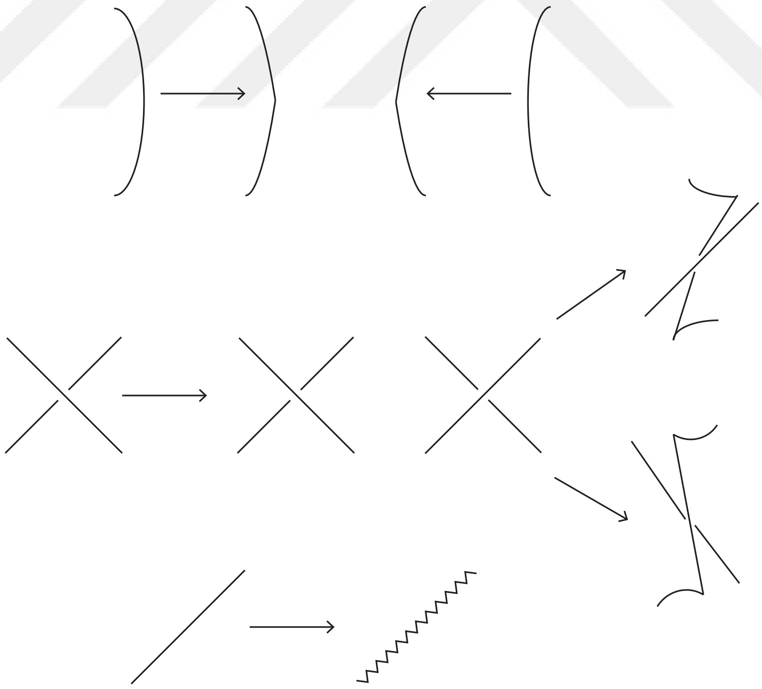

Figure 46: Modifications for a topological knot to be a Legendrian knot.

Example 4.2.2. A trefoil knot in $\mathbb{R}^3$ in Figure 47 is represented as a Legendrian knot in $(\mathbb{R}^3, \xi_{std})$ in the same figure.

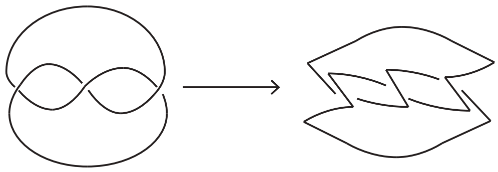

Figure 47: Converting left-handed trefoil into Legendrian left-handed trefoil.

Definition 4.2.3. [33] An isotopy of two Legendrian knots L_1 and L_2 in (M^3, ξ) can be defined by family of diffeomorphisms ψ_k, $k \in [0,1]$, on M^3 such that $\psi_0(L_1) = L_1$ and $\psi_1(L_1) = L_2$.

Theorem 4.2.2. [34] *" The Legendrian isotopic Legendrian knots can be expressed by two front diagrams if and only if these diagrams are associated with regular homotopy and a series of moves shown in* Figure 48*. These moves are called Legendrian Reidemeister moves."*

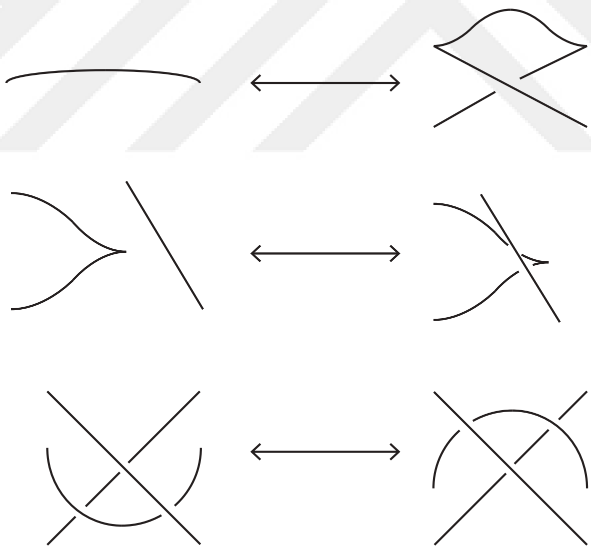

Figure 48: Legendrian Reidemeister moves.

Example 4.2.3. Different front projections of the Legendrian unknot that are shown in Figure 49 are Legendrian isotopic.

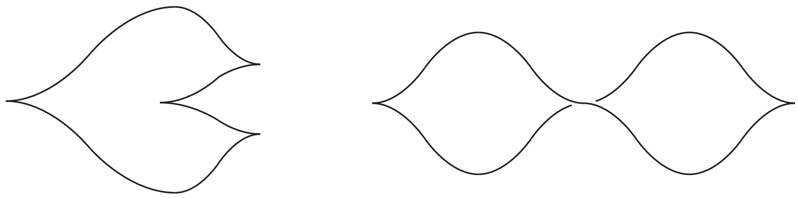

Figure 49: Different front projections of the Legendrian unknot.

Let us show these two projections are Legendrian isotopic. It is illustrated in Figure 50 by using Legendrian Reidemeister moves.

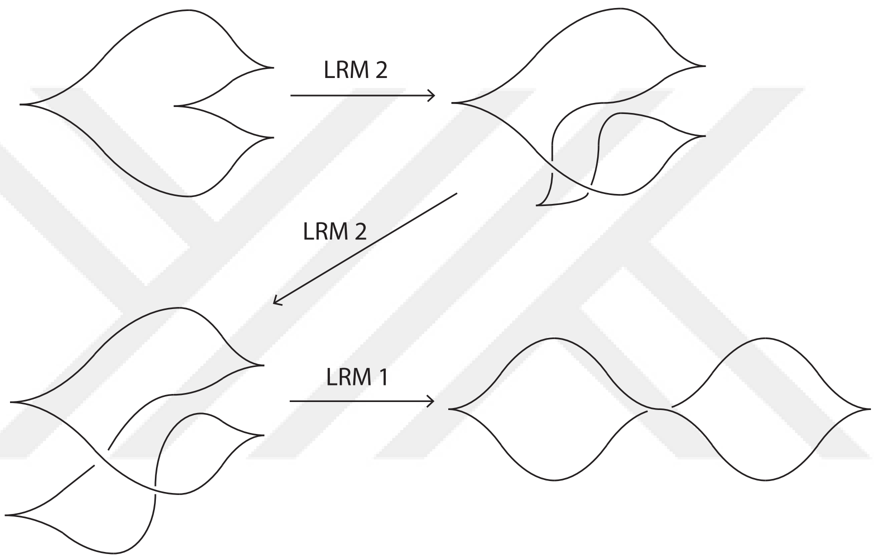

Figure 50: Legendrian isotopic front projections.

4.2.1 Contact Framing

Take a Legendrian knot L in $(\mathbb{R}^3, \xi_{std})$.

Definition 4.2.4. [29] The *contact framing* is the one matching to the parallel curve that is achieved by pushing L in the direction of $\frac{\partial}{\partial z}$ which is transverse to ξ_{std}. It is denoted by λ_l.

Example 4.2.4. Take Legendrian unknot in Figure 51 and vector field $v = \dfrac{\partial}{\partial z}$ which is transverse to ξ_{std}.

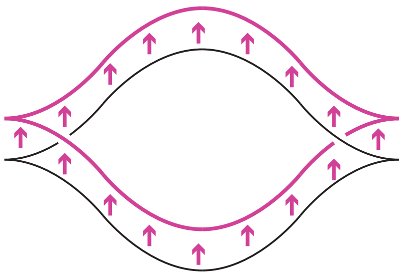

Figure 51: Contact framing for Legendrian unknot in the direction z-axis.

4.2.2 Classical Invariants of Legendrian Knots

4.2.2.1 Knot Type

The topological knot type for Legendrian knots is the most explicit invariant, [33].

Example 4.2.5. Legendrian knots that are given in Figure 52 and Figure 53 are different from each other since their knot types are different.

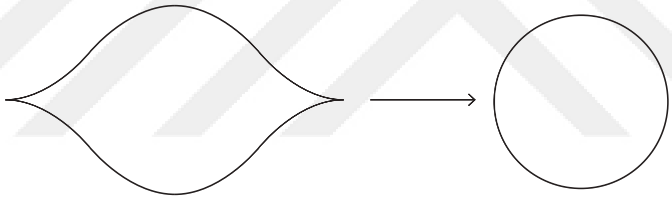

Figure 52: Knot type is the unknot.

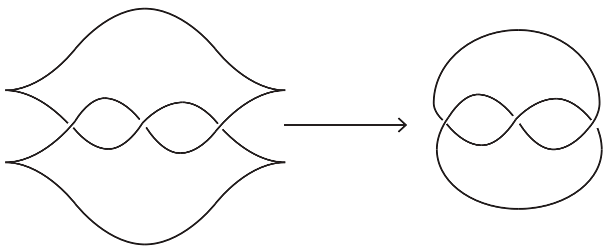

Figure 53: Knot type is the left-handed trefoil.

4.2.2.2 Thurston-Bennequin Invariant

Assume that Legendrian knot L is in $(\mathbb{R}^3, \xi_{std})$ and it is null-homologous. The twisting of contact framing about the Seifert framing of L is named as the *Thurston-Bennequin invariant* of L, represented as $tb(L)$. By way of explanation, choose a vector field which is transverse to ξ_{std} along L. Determine a parallel push off L' of L along this vector field. Then, $tb(L)$ equals to the linking number between L and L'. In this case, vector field can be chosen as $v = \frac{\partial}{\partial z}$. Take parallel push off L in the direction of v, [29], [33]. Since $tb(L) = lk(L, L')$, from the definition of linking number $tb(L) = \frac{1}{2}\sum \#\text{Signs}$. Let λ_s denote Seifert framing and λ_l denote the contact framing of L. Then, $\lambda_s = tb(L) + \lambda_l$.

At each positive (negative) crossing in the front projection of L, two positive (negative) crossings of L and L' will occur. At each positive (negative) cusp of L, a left crossing of L and L' will occur. So,

$$\begin{aligned} tb(L) &= \frac{1}{2}\sum \#\text{Signs} \\ &= \frac{1}{2}(2\text{writhe}(p(L)) - \text{total cusps in } p(L)) \\ &= \text{writhe}(p(L)) - \frac{1}{2}(\text{total cusps in } p(L)), \text{ [33]}. \end{aligned}$$

Example 4.2.6. The front projection of a Legendrian unknot L and its parallel push off L' in the direction of $\frac{\partial}{\partial z}$ are given in Figure 54. The linking number between them is $lk(L, L') = -1$, that is $tb(L) = -1$. On the other hand, since the writhe number of L is zero and there are two cusps on the front projection of L, from the formula

$$tb(L) = \text{writhe}(p(L)) - \frac{1}{2}(\text{total cusps in } p(L)) = 0 - \frac{1}{2} \cdot 2 = -1.$$

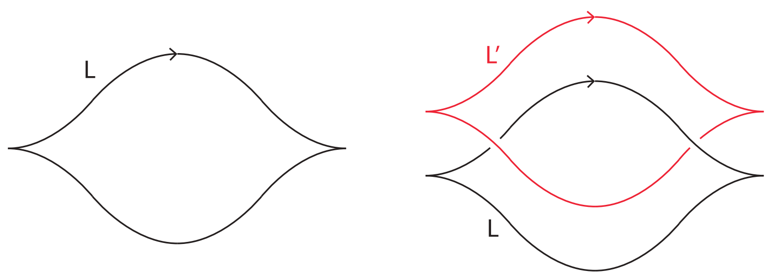

Figure 54: Legendrian unknot and its parallel push off.

By reversing the orientation, the writhe number does not change. When the given orientation is reversed, all of the orientations will be reversed on the strings. So, signs of the crossings will be the same. Thus, the Thurston-Bennequin invariant does not vary by the given orientation.

4.2.2.3 Rotation Number

Assume that Legendrian knot L is in $(\mathbb{R}^3, \xi_{std})$ and it is null-homologous. The *rotation number* of L, represented as $r(L)$, is the winding number about the vector field that produces a trivialization of ξ_{std}. This vector field can be $v = \frac{\partial}{\partial y}$ and take a nonzero vector field u which is tangent to L. Thus, to find the rotation number one should count how many times u passes v. By way of explanation, calculating a signed sum of how many times u and v pointing in the same direction. Its sign is described as $(+1)$ when u passes v counterclockwise. Its sign is described as (-1) when u passes v clockwise. The vector field u will be pointing in the direction of $\pm v = \pm\frac{\partial}{\partial y}$ at the cusps. At down cusps the intersection will be positive and at up cusps the intersection will be negative. Furthermore, considering the rotation number counts the number of times u intersects both v and $-v$, so multiply by $\frac{1}{2}$ to get $r(L)$, [33]. Then in the front projection,

$$r(L) = \frac{1}{2}(D - U), \text{ [33]}.$$

Let D denote the total down cusps. Let U denote the total up cusps.

Example 4.2.7. The Legendrian unknot, that is given in Figure 55, has one up cusp and three down cusps due to the orientation. So, the rotation number is equal to $rot(L) = \dfrac{1}{2}(3 - 1) = 1$.

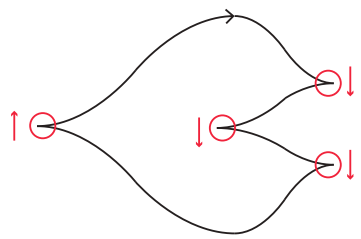

Figure 55: Up cusp and down cusps of Legendrian unknot.

By reversing the orientation which is given in Figure 55, then the rotation number will change sign. In other words, the up cusp will be down cusp and the down cusps will be up cusps since the orientation is reversed. So, the rotation number of the given Legendrian unknot is -1. Thus, the rotation number is orientation dependent.

Example 4.2.8. Legendrian right-handed trefoil, Legendrian left-handed trefoil and Legendrian figure eight knot are given in Figure 56, Figure 57 and Figure 58, respectively. Let us calculate their classical invariants.

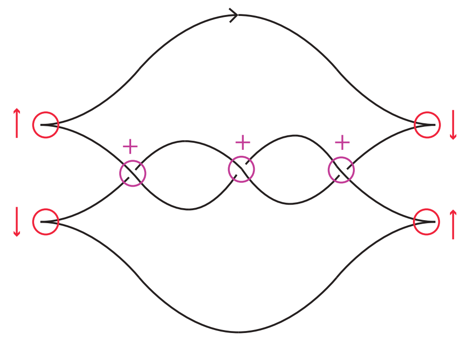

Figure 56: Crossings and cusps on Legendrian right-handed trefoil.

The writhe number of a Legendrian right-handed trefoil is 3 in Figure 56. There are four cusps. Two of them are up cusps and the other two are down cusps.

$$\begin{aligned} tb(L) &= \text{writhe}(p(L)) - \frac{1}{2}(\text{total cusps in } p(L)) \\ &= 3 - \frac{1}{2} \cdot 4 = 1 \quad \text{and} \\ rot(L) &= \frac{1}{2}(D - U) = \frac{1}{2}(2 - 2) = 0. \end{aligned}$$

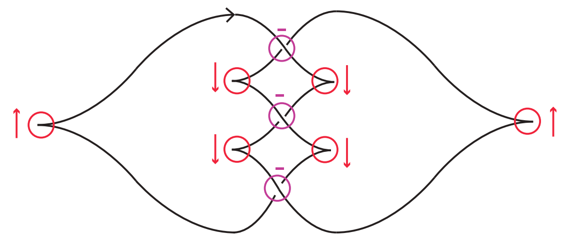

Figure 57: Crossings and cusps on Legendrian left-handed trefoil.

The writhe number of a Legendrian left-handed trefoil is -3 in Figure 57. There are six cusps. Two of them are up cusps and the other four are down cusps.

$$\begin{aligned} tb(L) &= \text{writhe}(p(L)) - \frac{1}{2}(\text{total cusps in } p(L)) \\ &= -3 - \frac{1}{2} \cdot 6 = -6 \quad \text{and} \\ rot(L) &= \frac{1}{2}(D - U) = \frac{1}{2}(4 - 2) = 1. \end{aligned}$$

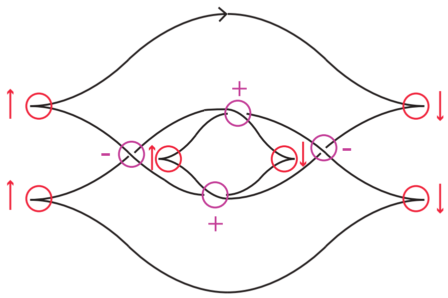

Figure 58: Crossings and cusps on Legendrian figure eight knot.

The writhe number of Legendrian figure eight knot is 0 in Figure 58. There are six cusps. Three of them are up cusps and the other three are down cusps.

$$\begin{aligned} tb(L) &= \text{writhe}(p(L)) - \frac{1}{2}(\text{total cusps in } p(L)) \\ &= 0 - \frac{1}{2}\cdot 6 = -3 \quad \text{and} \\ rot(L) &= \frac{1}{2}(D-U) = \frac{1}{2}(3-3) = 0. \end{aligned}$$

Example 4.2.9. Two different front projections of Legendrian unknot in Figure 59 are not Legendrian isotopic to each other.

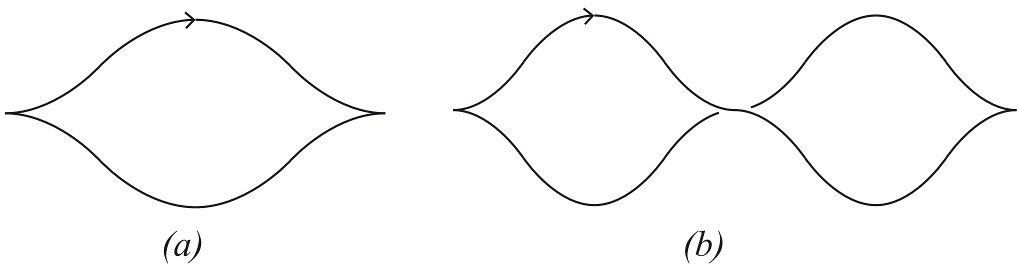

Figure 59: Front projections of non-isotopic Legendrian unknots.

The topological knot type of two Legendrian unknots is the unknot. However, their other classical invariants are different. In Figure 59(a), since the writhe number is zero and there are two cusps on the front projection, $tb(L_1) = -1$. In Figure 59(b), since the writhe number is -1 and there are two cusps on the front projection, $tb(L_2) = -2$. The rotation number of the first Legendrian unknot is $rot(L_1) = 0$, since it has one up cusp and one down cusp on the front projection. The second Legendrian unknot has $rot(L_2) = -1$, since it has two up cusps on the front projection. Hence, these two different front projections of the Legendrian unknot are not Legendrian isotopic.

Example 4.2.10. In Example 4.2.3, by using Legendrian Reidemeister moves it is shown that two Legendrian unknots in Figure 50 are Legendrian isotopic. In fact, two Legendrian unknots have the same invariants which are $tb = -2$ and $rot = 1$.

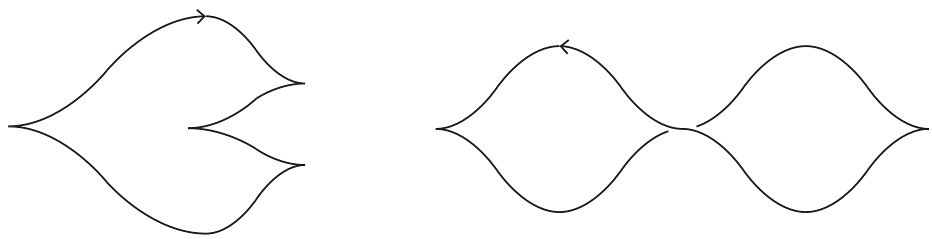

Figure 60: Legendrian isotopic unknots .

4.2.3 Stabilizations

Stabilization is defined in $(\mathbb{R}^3, \xi_{std})$ and is used for constructing another Legendrian knot L that represents the same topological knot type. For a given strand of the Legendrian knot, the stabilization of the Legendrian knot can be achieved by deleting the strand and alternating it with the zigzags. Stabilization is named positive if down cusps are added and negative if the up cusps are added. $S_+(L)$ refers to positive stabilization and $S_-(L)$ refers to negative stabilization, [33]. Stabilizations are shown in Figure 61.

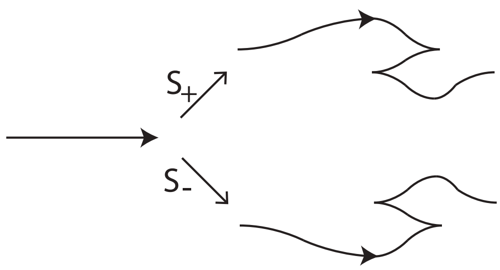

Figure 61: Positive and negative stabilization.

In the positive stabilization, two down cusps are added to the knot. In the negative stabilization, two up cusps are added to the knot. So, after this operation classical invariants will be changed as below:

$$tb(S_{\pm}(L)) = tb(L) - 1$$
$$r(S_{\pm}(L)) = r(L) \pm 1.$$

Stabilization is an operation which is done locally on a knot. A stabilization of a Legendrian knot can be moved to another region of the diagram over a cusp or a crossing thanks to Legendrian Reidemeister moves. Stabilization is a well-defined operation, since the operation is independent from the point that is applied. Since stabilization can be moved a region to another region on the knot diagram, the operation has the commutative property, that is, $S_+ \circ S_- = S_- \circ S_+$, [35].

Example 4.2.11. Figure 62 shows stabilizations of the given Legendrian unknot.

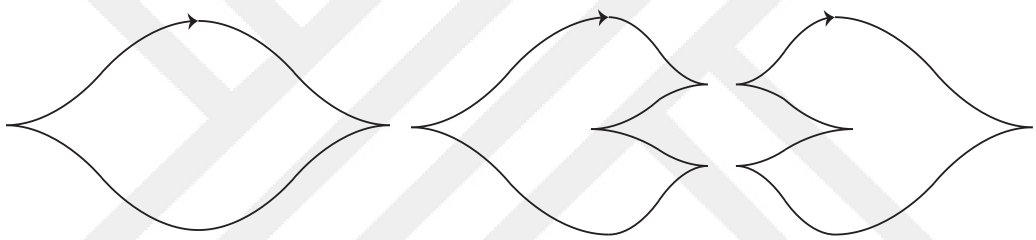

Figure 62: Legendrian unknot L, a positive and a negative stabilization of L.

Theorem 4.2.3. [36] *" Topologically isotopic Legendrian knots L and L' in* $(\mathbb{R}^3, \xi_{std})$ *will be Legendrian isotopic afterwards the application of the stabilization a few times on every one of them."*

4.3 Contact Surgery Along Legendrian Knots

The idea of contact surgery along Legendrian knots is similar to Dehn surgery along topological knots. The method of this surgery is that perform Dehn surgery along a Legendrian knot depending on contact framing and then extend the former contact structure to the resulting 3-manifold. The latter contact structure may differ from the former one. For more information see [5] and [37].

Take a Legendrian knot L in (M^3, ξ) and its tubular neighborhood vL of L. To execute Dehn surgery, remove vL from (M^3, ξ) and reglue solid torus along the boundary with its tight contact structure via diffeomorphism φ_L;

$$\varphi_L : \partial(S^1 \times D^2, \xi_1) \longrightarrow \partial((M^3, \xi) \setminus vL),\ [5],\ [38].$$

Contact α-surgery with the contact longitude λ_l can be presented as $\alpha = q\mu + p\lambda_l = \varphi_L(\{*\} \times D^2)$. So, $\alpha = \dfrac{p}{q}$ is called contact surgery coefficient. After contact surgery, a new contact 3-manifold $(M', \xi') = (M^3, \xi) \setminus vL \cup_{\varphi_L} (S^1 \times D^2, \xi_1)$ is constructed, [5]. Specifically, contact surgery with the contact framing (-1) is named as a *Legendrian surgery.*

Theorem 4.3.1. [5] *" All closed, oriented contact* 3*-manifolds is obtained via contact surgery with the contact framing* (± 1) *along a Legendrian knot in* (S^3, ξ_{std})*."*

Example 4.3.1. [37] Performing contact $(+1)$-surgery along the Legendrian unknot U with $tb(U) = -1$ and $rot(U) = 0$ corresponds performing Dehn surgery with the surface framing 0 along the topological unknot. This equivalent surgery results in the manifold $S^1 \times S^2$. As stated by Eliashberg, this 3-manifold yields the unique tight contact structure.

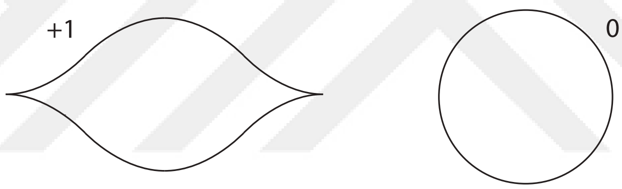

Figure 63: Contact $(+1)$-surgery along U.

Example 4.3.2. [37] [39] [40] Perform contact $(+1)$-surgery along a stabilized Legendrian unknot L which has $tb(L) = -2$, $rot(L) = 1$ and is given in Figure 64. It is equivalent to performing Dehn surgery with the surface framing (-1) applied to the topological unknot. This surgery results in the manifold S^3. According to [37], [39] and [40] contact surgery with the contact framing $(+1)$ applied to stabilized Legendrian unknot yields overtwisted contact structure on S^3.

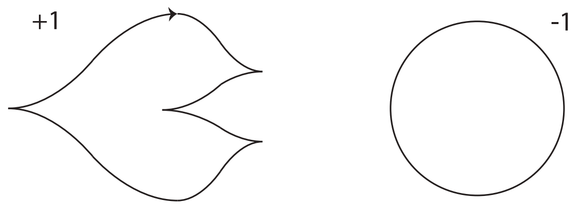

Figure 64: Contact $(+1)$-surgery along L.

4.3.1 Algorithm for Contact s-surgery

In [5] Ding and Geiges proved that all contact s-surgery with $s < 0$ or $s > 0$ can be achieved by a series of (-1)-surgeries or $(+1)$-surgeries respectively. The latter one is slightly differ from the former one. In [37] Ding, Geiges and Stipsicz gave the algorithm for these surgeries based on [5].

Algorithm for $s < 0$: Take Legendrian knot L in (M^3, ξ) for performing contact surgery with the contact framing s applied to the Legendrian knot. For integers $s_1, s_2, ..., s_n \leq -2$, compute s as a continued fraction which is given below;

$$s_1 + 1 - \cfrac{1}{s_2 - \cfrac{1}{... - \cfrac{1}{s_n}}}.$$

Think Legendrian knot L_1 as Legendrian knot L with $|s_1 + 2|$ extra zigzags like in Subsection 4.2.3. For the rest $k = 2, ..., n$ take Legendrian push off L_k of L_{k-1} with $|s_k + 2|$ extra zigzags. In that case, a contact surgery with the contact framing s applied to L equals to a series of contact surgeries with contact framings (-1) along $L_1, L_2, ..., L_n$.

Example 4.3.3. Apply the algorithm to the Legendrian unknot for contact $-\frac{7}{4}$-surgery. It is illustrated in Figure 65.

$$-\frac{7}{4} = -3 + 1 - \frac{1}{-4}$$

Since $s_1 = -3$, L_1 will be a Legendrian unknot with one extra zigzag. For $s_2 = -4$, L_2 will be Legendrian push off of L_1 with two extra zigzags. There are six possibilities of performing contact $\frac{-7}{4}$-surgery. One of the possibility is given in Figure 65.

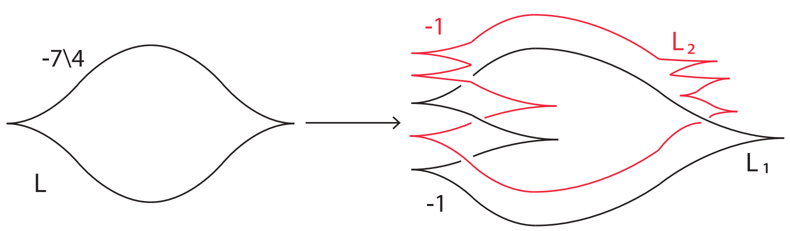

Figure 65: Contact $-\frac{7}{4}$-surgery along the Legendrian unknot.

Algorithm for $s > 0$: Take positive coprime integers p and q. For $s = \frac{p}{q} > 0$, take an integer $r > 0$ that fulfills the condition $q - rp < 0$ and fix $s' = \frac{p}{q - rp}$. Take r Legendrian push off $L_1, L_2, ..., L_r$ of a Legendrian knot L. Under these circumstances, a contact surgery with the contact framing s applied to L equals to a series of contact surgeries with contact framings $(+1)$ applied to $L, L_1, L_2, ..., L_{r-1}$ and a contact s'-surgery along L_r.

Example 4.3.4. Apply the algorithm to the Legendrian unknot for contact $\frac{1}{2}$-surgery. It is illustrated in Figure 66.

Since $s = \frac{p}{q} = \frac{1}{2}$, for $r = 3$ the condition $q - rp < 0$ is fulfilled. Contact surgeries with contact framings $(+1)$ applied to L, L_1, L_2 and a contact surgery with the framing (-1) applied to L_3 equal to the contact surgery with the contact framing $\frac{1}{2}$ along the Legendrian unknot.

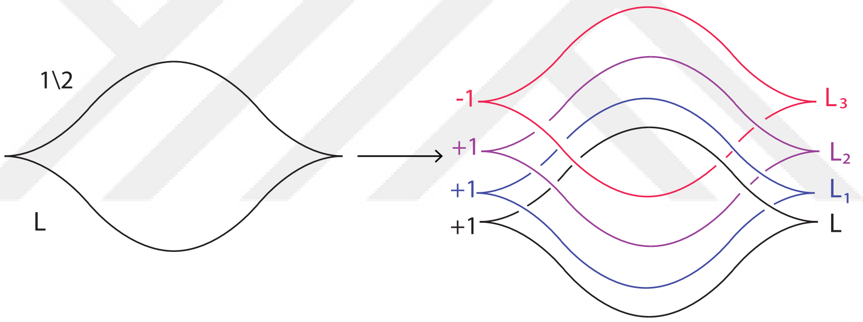

Figure 66: Contact $\frac{1}{2}$-surgery along the Legendrian unknot.

5 OPEN BOOK DECOMPOSITIONS AND CONTACT 3-MANIFOLDS

In this section, the relation between open book decompositions and contact structures will be analyzed. Contact structures are defined in Subsection 4.1. In here, positive contact structures will be used. A positive contact structure fulfills the conditions that are mentioned above with a slight difference, that is, $\alpha \wedge d\alpha > 0$. A definition will be given that relates open book decompositions to contact structures. For more information see [22] and [17].

In 2000, Giroux made a breakthrough in the field of contact topology by proving the following theorem. This theorem allows to study between contact structures and open book decompositions with inner peace.

Theorem 5.0.1. [8] *" Take a closed and oriented 3-manifold M^3. There exists a one-to-one correspondence between oriented contact structures on M^3 up to isotopy and open book decompositions of M^3 up to positive stabilization."*

Definition 5.0.1. [22] Take a contact 3-manifold (M^3, ξ) and an open book decomposition (S, K, p) of M^3. In other words, there exists a contact 1-form α for ξ wherever ξ is isotoped through contact structures. If this 1-form α is positive on the binding K and $d\alpha$ is also positive on the pages of (S, K, p), then (M^3, ξ) is *supported* by (S, K, p). It is also said that (S, K, p) is *compatible* with ξ on M^3.

Example 5.0.1. The abstract open book (D^2, id) supports (S^3, ξ_{std}).

The 3-sphere $S^3 = \{(k_1, k_2) \in \mathbb{C}^2 : |k_1|^2 + |k_2|^2 = 1\}$ and the standard tight contact structure $\xi_{std} = \ker(r_1^2 d\theta_1 + r_2^2 d\theta_2)$.

The binding $U = \{r_1 = 0\}$ and the fibration $p(r_1, \theta_1, r_2, \theta_2) = \theta_1$ gives an open book decomposition of S^3. For a fixed θ_1, $p^{-1}(\theta_1)$ is the interior of the Seifert surface for the unknot which is a disk. Its binding is the unknot and its pages are disks. Its monodromy map is identity. The abstract open book (D^2, id) supports (S^3, ξ_{std}): The base vector $\dfrac{\partial}{\partial \theta_2}$ of $\mathbb{C}^2$ is tangent to U and the contact form is $d\theta_2$ when restricted to the unknot. So, the binding U is transverse to ξ_{std}. The restricted contact form to a page is $r_2^2 d\theta_2$. Hence, $d(r_2^2 d\theta_2) = 2r_2 dr_2 \wedge d\theta_2$ is a volume form on the page, [17].

Example 5.0.2. Take S as an annulus. The abstract open book (S, t_b) of the positive Hopf link supports (S^3, ξ_{std}).

The abstract open book (S, t_b) is the positive stabilization of (D^2, id). Its page, annulus, is achieved by taking union of the disk and a 1-handle and its monodromy is achieved by taking composition of id and a right-handed Dehn twist. By using Giroux's Theorem 5.0.1, the abstract open book (S, t_b) supports (S^3, ξ_{std}).

Example 5.0.3. Take S as an annulus. The abstract open book (S, t_b^{-1}) of the negative Hopf link supports (S^3, ξ_{ot}).

By Yilmaz, if the monodromy of an open book decomposition is of the form of a product of left-handed Dehn twists, then the open book is compatible with overtwisted contact structure, [41]. Since the monodromy of the open book is t_b^{-1}, then the abstract open book (S, t_b^{-1}) supports overtwisted contact structure on S^3.

The relation between abstract open book decompositions and contact surgeries can be understood via Theorem 5.0.2 down below.

Theorem 5.0.2. [22] *" Take an abstract open book decomposition (S, ψ) that is supporting (M^3, ξ) and a Legendrian knot L on the page of (S, ψ). To execute contact surgery with the contact framing (± 1) applied to the knot L is equal to adding a Dehn twist to the monodromy of the open book decomposition. Contact surgery with the contact framing $(+1)$ applied to a Legendrian knot on the page gives $(S, \psi \circ t_b^{-1})$. Contact surgery with the framing (-1) along a Legendrian knot on the page gives $(S, \psi \circ t_b^{+1})$. So, this new 3-manifold can be shown as below:*

$$(M^3, \xi)_{(L,\pm 1)} = (M^3_{(S,\psi \circ t_L^{\mp})}, L_{(S,\psi \circ t_L^{\mp})}).\text{"}$$

Theorem 5.0.3. [42] *" Take a 3-manifold M^3 which is closed and oriented. Every open book decomposition of M^3 supports a contact structure on M^3."*

Lemma 5.0.1. (Cancellation Lemma, [14]) *" Take a contact 3-manifold (M, ξ). A new 3-manifold (M', ξ') is formed by contact surgery with the contact framing (-1) applied to a Legendrian knot L in (M, ξ). After that, execute contact surgery with the framing $(+1)$ applied to a Legendrian push off of L in (M', ξ'), then the resulting manifold is contactomorphic to (M, ξ)."*

Proof. Take a Legendrian knot L in (M, ξ) and its Legendrian push off L'. One can always find an open book decomposition (S, ψ) supporting ξ and containing L and its

Legendrian push off L' on its page, [8], [43], [10]. To perform contact surgery with the contact framing (-1) applied to L gives a new contact 3-manifold (M', ξ') with $(S, \psi \circ t_L)$. Then performing contact surgery with the contact framing $(+1)$ applied to L' results in an open book decomposition $(S, \psi \circ t_L \circ t_{L'}^{-1})$. Hence, the former contact 3-manifold (M, ξ) is achieved, since $t_L \circ t_{L'}^{-1} = id$. $\square$

5.1 Invariants of Contact Structures from Open Book Decompositions

Contact structures have invariants from open book decompositions: the support genus and the binding number, that are described in [9].

Definition 5.1.1. [9] Take a contact 3-manifold (M^3, ξ). The minimal genus of a page of an abstract open book decomposition of M^3 is called the *support genus* of ξ.

$$sg(\xi) = \min\{\text{genus}(S) : (S, \psi) \text{ supports } \xi\}, \text{ [9]}.$$

Definition 5.1.2. [9] The minimal number of boundary components on the page of the open book decomposition that supports (M^3, ξ) and has the support genus is called the *binding number* of ξ on M^3.

$$bn(\xi) = \min\{|\partial S| : (S, \psi) \text{ supports } \xi \text{ with genus} = sg(\xi)\}, \text{ [9]}.$$

Example 5.1.1. In Example 5.0.1 the abstract open book (D^2, id) where the pages are disks supports (S^3, ξ_{std}). In Example 5.0.2 the abstract open book (S, t_b) where the pages are annuli supports (S^3, ξ_{std}). Let us examine the invariants of contact structures of these abstract open books which are compatible with ξ_{std} on S^3.

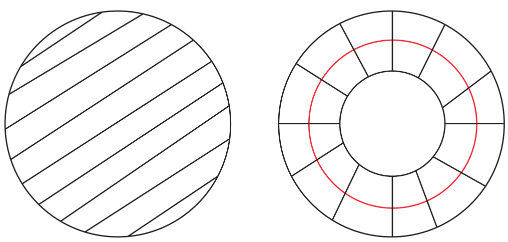

Figure 67: Disk and annulus.

In both examples, pages have zero genus, then genus of these open book decompositions is zero. This means that these open book decompositions are planar and both of them have support genus zero. The boundary components of the pages are one and two respectively. In this case, the minimal number of boundary components on the page of the open book which supports (S^3, ξ_{std}) with support genus zero is one. Therefore, the binding number of ξ_{std} on S^3 with planar pages is one, $bn(\xi_{std}) = 1$.

Lemma 5.1.1. [9] *" Assume (M^3, ξ) is a contact 3-manifold that is supported by an open book with genus zero.*

(i) If $bn(\xi) = 1$, then ξ is the standard tight contact structure on S^3.

(ii) Let $bn(\xi) = 2$. If ξ is tight, then ξ is the unique tight contact structure on $L(r, r-1)$ for some $r \geq 0$. Otherwise, if ξ is overtwisted, then ξ is overtwisted contact structure on $L(r, 1)$ for some $r > 0$."

Proof. (i) If the support genus is zero and the binding number is equal to 1, then the pages S of the open book decomposition are disks. It has been shown in Example 5.0.1 that (D^2, id) supports (S^3, ξ_{std}).

(ii) If the support genus is zero and the binding number is equal to 2, then the pages S of the open book are annular. Since its page is an annulus, its monodromy map is of the form of $t_b^{\pm r}$. Here, t_b stands for a right-handed Dehn twist about the core circle b of the page annulus and t_b^{-1} stands for a left-handed Dehn twist about the core circle b of the page annulus.

Assume the monodromy map ψ of the open book decomposition is $\psi = t_b^r$ where $r > 0$. Consider the open book decomposition of (S^3, ξ_{std}) where pages are annulus and the monodromy is equal to t_b which is given in Example 2.6.5. Since the monodromy is $\psi = t_b^r = t_b t_b^{r-1}$, the resulting 3-manifold can be seen as total $(r-1)$ Legendrian surgeries along $(r-1)$ Legendrian unknot with $tb = -1$. In Figure 68, Legendrian surgery picture is given.

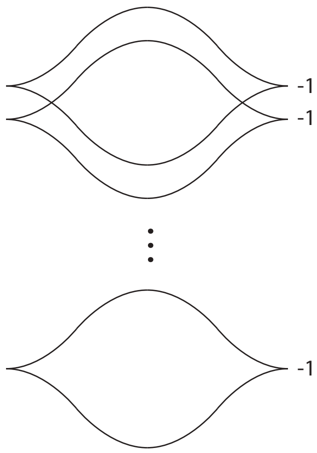

Figure 68: $(r-1)$ copies of Legendrian unknot.

From the algorithm 4.3.1 for contact s-surgery, Legendrian surgery along $(r-1)$ copies of Legendrian unknot equals to $-\dfrac{1}{(r-1)}$-contact surgery along the Legendrian unknot with $tb = -1$. By using the equation $\lambda_s = tb + \lambda_l$,

$$\lambda_s = -1 + \Big(-\frac{1}{r-1}\Big) \Rightarrow \lambda_s = -\frac{r}{r-1}.$$

Since the Seifert framing of the unknot is $-\dfrac{r}{r-1}$, the new 3-manifold is $L(r, r-1)$. In 2014, Wand proved that the tightness remains the same under Legendrian surgery in a tight contact structure. To obtain the new 3-manifold $L(r, r-1)$, we execute $(r-1)$ Legendrian surgeries. Therefore, this new contact 3-manifold is tight by Wand, [44].

Assume the monodromy ψ of the open book decomposition is $\psi = t_b^{-r}$ where $r > 0$. Consider the open book decomposition of (S^3, ξ_{std}) where pages are annulus and the monodromy is equal to t_b which is given in Example 2.6.5. Since the monodromy is $\psi = t_b^{-r} = t_b t_b^{-(r+1)}$, the resulting 3-manifold can be seen as total $(r+1)$ contact $(+1)$-surgeries along $(r+1)$ Legendrian unknot with $tb = -1$. In Figure 69, Legendrian surgery picture is given.

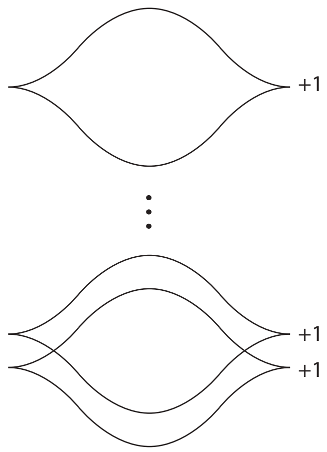

Figure 69: $(r+1)$ copies of Legendrian unknot.

From the algorithm 4.3.1 for contact s-surgery, contact $(+1)$-surgery along $(r+1)$ copies of Legendrian unknot equals to $\dfrac{1}{(r+1)}$-contact surgery along the Legendrian unknot with Thurston-Bennequin invariant $tb=-1$. By using the equation $\lambda_s = tb + \lambda_l$,

$$\lambda_s = -1 + \left(\frac{1}{r+1}\right) \Rightarrow \lambda_s = -\frac{r}{r+1}.$$

Since the Seifert framing of the unknot is $-\dfrac{r}{r+1}$, the new 3-manifold is $L(r, r+1)$. From the Theorem 2.7.1, $L(r, r+1)$ is equivalent to $L(r,1)$. In [41] Yilmaz show that if the monodromy of an open book decomposition is a product of left-handed Dehn twists, then the supporting contact structure is overtwisted. Thus, the contact structure on this new 3-manifold $L(r,1)$ is overtwisted by Yilmaz, [41].

Assume the monodromy ψ of the open book decomposition is $\psi = t_b^r$ where $r=0$. Consider the open book decomposition of (S^3, ξ_{std}) where pages are annulus and the monodromy is equal to t_b which is given in Example 2.6.5. Since the monodromy is $\psi = t_b t_b^{-1} = id$, the resulting 3-manifold can be seen as a contact surgery with the framing $(+1)$ applied to a Legendrian unknot with $tb=-1$. As it is given in Example 4.3.1 the resulting contact 3-manifold is $S^1 \times S^2$ and it is tight by [37]. □

6 CONCLUSION

Throughout history of contact topology, the real struggle is when one can get a tight or an overtwisted contact structure after contact surgeries. To understand these contact structures various techniques are improved. One of the most important improvement is made by Giroux. Giroux discovered a relation between open book decompositions and contact structures. The relation lets us better understanding on contact structures. Giroux's result allows to study contact 3-manifolds with regard to open book decompositions.

In this thesis, Alexander's theorem is examined. The proof which is given for this theorem is benefitted from the result of Wallace [3] and Lickorish [4], but the monodromy map of the resulting open book decomposition is unknown. For lighten up this monodromy issue Onaran's proof is analyzed. The proof of Onaran depends on the algorithm for planar open book decompositions and the pure braid representation of links, [13].

By using open book decompositions, surgery techniques are studied in detail and their useful applications are realized. As a matter of fact, proof of Cancellation Lemma of Ding and Geiges [14] is studied by using open book decompositions.

Moreover, in this thesis the invariants of contact structures from open book decompositions have been studied. Contact 3-manifolds which have support genus 0 and binding number 1, and contact 3-manifolds which have support genus 0 and binding number 2 are studied.

It is still unknown that which contact structures and contact 3-manifolds are obtained by fixing support genus 0 and binding number is greater than 4. Similarly, nothing is known for the case contact 3-manifolds with support genus 1 and binding number is greater than 1. In future work, the following listed open problems will be studied.

Open Problem 1: Classify all contact 3-manifolds which have support genus 0 and binding number is 5.

Open Problem 2: Classify all contact 3-manifolds which have support genus 1 and binding number is 2.

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