DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

ACTUARIAL TECHNIQUES IN SOCIAL SECURITY INSURANCE

by
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İZMİR

ACTUARIAL TECHNIQUES
IN SOCIAL SECURITY INSURANCE

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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ACTUARIAL TECHNIQUES IN SOCIAL SECURITY INSURANCE"** completed by **BANU ÖZAKÇAN ÖZGÜREL** under supervision of **ASSOC. PROF. DR. GÜÇKAN YAPAR** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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ACTUARIAL TECHNIQUES IN SOCIAL SECURITY INSURANCE

ABSTRACT

Actuarial valuation is a special kind of financial calculation based upon some economical and demographic assumptions proposed for future. Purpose of this valuation is to compare actuarial present value of all future benefit payments which will be made to all members of the foundation and their dependants versus to actuarial present value of all future contributions into the fund by members and their employer. According to these assumptions at time *t*, sum of the actuarial present value of all future benefit payments and administrative expenses might be smaller than sum of the actuarial present value of all future contributions and current assets of the fund. Otherwise, all the assumptions must be revised and some adjustments must be made to increase actuarial value of future contributions and to decrease actuarial value of future benefits. The purpose of this thesis is actuarial valuation of a private fund which had been established by one of the Turkish Bank for their employees social security. This actuarial valuation will be made using aggregate and unit cost method on the evidence of current Foundation Voucher, member structure and economical indicators. In order to define the variables which will constitute the basis for the actuarial calculations, firstly the past activities of the Foundation have been analyzed and then the status has been determined according to the current Foundation Voucher purview. During the determination of the important variables such as interest, inflation and mortality, values matching the realities of Turkey and stochastic approximation have been chosen, and a flexible computer program, which can be adapted to changing conditions, has been prepared. In this thesis, all ready values for the future revenues and the expenditures of the Foundation with respect to end of 2006 fiscal year have been calculated (in accordance with the Foundation Voucher) under definite assumptions and the differential has been compared to the fund values in the balance sheets.

Keywords : Actuarial valuation, Pension system, Stochastic interest rate, Actuarial balance sheet.

SOSYAL GÜVENLİK SİGORTALARINDA AKTÜERYAL TEKNİKLER

ÖZ

Aktüeryal değerlendirme gelecek için öngörülmüş bazı ekonomik ve demografik varsayımlara dayanan özel bir finansal hesaplamadır. Bu değerlendirmenin amacı, gelecekte bir fonun üyeleri ve bağımlılarına yapılacak fayda ödemelerinin aktüeryal peşin değerinin bu fonun üyeleri ile işvereninin yaptığı katkıların aktüeryal peşin değeri ile karşılaştırılmasıdır. Bu varsayımlara göre herhangi bir t anında gelecekte yapılacak fayda ödemeleri ve idari masraflarının aktüeryal peşin değeri toplamı gelecekteki tüm katkıların aktüeryal peşin değeri ile fonun mevcut varlıklarının toplamından küçük olmalıdır. Aksi halde tüm varsayımlar gözden geçirilmeli ve gelecekteki katkılar arttırılmalı ve gelecekteki fayda ödemeleri düşürülmelidir. Bu tezin amacı, bir Türk bankası tarafından çalışanların sosyal güvenliği için kurulmuş özel bir fonun aktüeryal değerlendirmesinin yapılmasıdır. Bu aktüeryal değerlendirme vakfın mevcut senet hükümleri, üye yapısı ve ekonomik göstergelere göre toplam ve birim maliyet yöntemi kullanılarak yapılacaktır. Aktüeryal hesaplamalara baz teşkil edecek değişkenlerin belirlenebilmesi için öncelikle vakfın geçmiş dönemlerdeki faaliyetleri analiz edilmiş, daha sonra mevcut 2006 vakıf senedi hükümlerine göre durum tespiti yapılmıştır. Aktüeryal hesaplamalarda kullanılan sabit ve stokastik faiz oranı, enflasyon ve mortalite gibi önemli değişkenlerin belirlenmesinde Türkiye gerçeklerine uygun değerler seçilmekle birlikte, değişebilecek şartlara uyum sağlayan esnek bir bilgisayar programı hazırlanmıştır. Tezde vakfın gelecekteki tüm gelir ve giderlerinin, belirli varsayımlar altında, 2006 yılı itibariyle peşin değerleri hesaplanmış ve aradaki fark vakfın bilançolarında yer alan fon değerleriyle karşılaştırılmıştır.

Anahtar sözcükler : Aktüeryal değerlendirme, Emeklilik sistemleri, Stokastik faiz oranı, Aktüeryal Bilanço.

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CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW

1.1 Background and Motivation

Social security primarily refers to social welfare service concerned with social protection, or protection against socially recognized conditions, including poverty, old age, disability, unemployment and others. Social security may refer to social insurance, where people receive benefits or services in recognition of contributions to an insurance scheme. These services typically include provision for retirement pensions, disability insurance, survivor benefits and unemployment insurance.

Rising health care costs, the aging of populations, economic conditions and demographic bulges, all pose severe challenges to social insurance programs worldwide. Many countries face the unpleasant choice of reducing benefits or increasing taxes (contributions) to finance their social programs. The political issues faced by policy makers in many countries have limited their ability to enact social insurance reforms. Germany's pension reform, for example, only partially resolved projected in retirement income expenditures, but in doing so reduced the benefits for future retirees (Skipper, 1998).

The aging of the population is the most important challenge to social insurance programs worldwide. The irony of these programs is that their success has led to a healthier, longer-living population. As the length of time people spend in retirement increases, so does their need for retirement income and health care services. When these benefits are funded by active workers, the burden on workers increases. While the rationale for creating social insurance programs has not changed, the costs of maintaining them is likely to increase dramatically in the coming decades (Skipper, 1998).

The demographic pressure on social insurance programs is likely to produce a demand for change and innovation. New ways of funding and administering these programs will be developed and implemented. These changes are likely to create a number of opportunities and challenges for private insurers. New technologies that reduce economies of scale needed administer social insurance programs may allow some of those programs to be provided in the private sector. Conversely, new technologies that allow insurers to identify risks may create incentives in the private market to exclude segments of the population from coverage against the wishes of society. The result could be increased regulation in the private sectors or increased government provision of social insurance programs.

Actuarial valuation of social security or pension scheme is usual practical method. Its purpose is to look at the long-term position of a pension scheme. Using an appropriate basis and funding method, the funding level of the pension scheme can be calculated as well as the contribution rate necessary to satisfy the pension scheme's funding strategy over the following intervaluation period. Regular valuations allow the contributions to be adjusted in the light of a pension scheme's actual experience over the short term. At the same time, calculations can be made to asses the solvency of the pension scheme at the valuation date (Booth et all., 1999).

Actuarial valuation is based upon taking an interest assumption and comparing the difference between the present value of future payments to be done to members and their dependents, general management expenses and the present value of future collection of aids with the funds in hand. With the assumed interest rate, in case the sum of the expenses is more than the sum of revenues according to the actuarial calculations of revenues and expenses, the assumptions should be revised in order current ready money and realty values to be equal to this difference. In this point, interest rate is very important part in actuarial valuation.

This thesis aims at actuarial valuation of a private foundation's benefit liabilities provided by the Fund and SSK (Social Insurance Institution) in the framework of the aggregate cost method purview dated 31.12.2006 and on the evidence of new Foundation Voucher purview, member structure and economical indicators. All the benefits provided by the Fund and SSK will be evaluated according to the methods and assumptions of unit credit independently from the aggregate cost method.

In order to define the variables, which will constitute the basis for the actuarial calculations, first, the past activities of the Foundation have been analyzed and then the status has been determined according to the current 2006 Foundation Voucher purview.

During the determination of the important variables such as interest, inflation and mortality, values matching the realities of Turkey and stochastic approximation have been chosen, and a flexible computer program, which can be adapted to changing conditions, has been prepared.

In this thesis, all ready values for the future revenues and the expenditures of the Foundation with respect to 31.12.2006 have been calculated (in accordance with the Foundation Voucher) under definite assumptions and the differential has been compared to the fund values in the balance sheets.

In the analysis process, the revisions to be performed to eliminate the negative issues regarding the actuarial balance have been identified and some proposals for the Foundation to become more healthy and assured in fiscal and actuarial respects have been offered.

1.2 Brief Historical Perspective

We are interested in deterministic and stochastic interest rate in actuarial valuation. There has been a lot of work on stochastic interest rate and actuarial valuation.

Financial soundness and funding stability are two critical issues in financial management for public pension schemes. In recent years, with an increase in the percentage of population that comes under pension age in development countries, pension-related topics have taken on new significance, and much attention has been focused on the implementation of a better welfare program for the aging society. Hence the regular pension valuation carried out by pension actuaries becomes an important mechanism to monitor and maintain a plan's financial soundness. In performing a pension valuation, an appropriate stochastic model needs to be constructed to appropriately express the turnover pattern according to the current active workforce and project the corresponding future cash flows. Within the process of the valuation, the given actuarial assumptions are crucial in reporting the plan's annual cost on a sponsor's balance sheets (Anderson, 1992).

Dominicis et al. (1991) and Janssen and Manca (1997) have recently proposed a stochastic model to formulate the financial calculations in actuarial valuation. Projected cash flows are scrutinized through dynamic simulations. Extensive research has been conducted relating to the practical decision-making process adjusted by budgeting a plan's pension costs in optimizing its financial status. An extensive review of past pension cost analyses can be found in Shapiro (1985). Bowers et al. (1982) focus on the dynamics of pension funding. First, they discuss contribution rates consisting of the normal cost plus a generalized amortization method for unfunded supplemental present value. Then they examine aggregate cost funding methods for active members when consistent differences exist between assumed and actual returns. O'Brien (1986) proposes a stochastic-dynamic pension fund model via a stochastic differential equation using the fund ratio. O'Brien (1987) later establishes theoretical justifications for the optimal solution of a stochastic control problem in such a framework. He shows that the optimal solution of a stochastic control problem yields an affine function of the fund level and the present value of future benefits. Dufresne (1988, 1989) considers a funded pension plan and supposes that actuarial gains and losses are either directly amortized over a fixed number of years or indirectly spread over a moving term.

Haberman (1992, 1993, 1994), Gerrard and Haberman (1996), and Haberman and Wong (1997) extend this model and consider the return on the assets to be represented by the stochastic process. The expectations and the variability of fund and contribution levels are obtained. Haberman discusses the methods of funding that are optimal in the sense of the period for spreading surpluses and deficiencies by reducing the variability of fund or contributions. Cairns and Parker (1997) consider the stochastic behavior of the funding level through time and its relationship with the plan contribution rate. Given the rates of return to follow the AR(1) model, Cairns and Parker derive a recursive method for calculating the conditional distribution of the funding level. Owadally and Haberman (1999) discuss pension fund dynamics and the amortization of gains/losses due to random returns. They also investigate the evolution in time of the first and second moments of the pension fund and contribution levels.

Daykin et al. (1994) outline a practical simulation procedure in modeling pension dynamics. They also discuss the possibility of the valuation assumptions and the projection assumptions being different. Haberman and Sung (1994) propose two types of risk concerned with, respectively, the stability and security of funding: the contribution rate risk and the solvency risk. Haberman (1997) considers the contribution rate risk for defined benefit pension schemes and compares different approaches to funding through minimizing the variability in the present value of future contribution. In a recent study (Frees et al., 1997), a forecasting model was constructed to assist policymakers in projections of the Social Security system in the United States.

A quadratic loss function is discussed in Boulier et al. (1995, 1996) to find the optimal contribution rates and asset allocation in a continuous-time framework. In Cairns (1995, 1996), a continuous-time stochastic pension fund model under a more general loss function is proposed under two risky assets as well as randomness in the level of benefit outgoes. In his work, optimal and dynamic control strategies are compared given the quadratic loss function. Bacinello (1988) uses stochastic simulations to obtain the best estimates of plan dynamics. Recently, Chang (1999) combined stochastic simulations and dynamic optimization in deciding the optimal funding policy. He applies the theoretical results to the Taiwan Public Employees Retirement System (Tai-PERS) and presents the empirical findings.

In most of the insurance literature, the theory of life contingencies is developed in a deterministic way. This means that mortality happens according to a priori known mortality table and the interest rate is assumed to have a constant value. Nevertheless, the traditional theory of life contingencies implicitly deals with the stochastic nature of mortality and interest rates in those conservative assumptions are taken.

A first step forward was to consider the time until decrement as a random variable, while the interest rate was assumed to be constant. This approach is followed in Bowers et al. (1987). This (as one could call) “semi-stochastic” approach contains the traditional theory in that most actuarial functions can be considered as the expected values of certain stochastic functions.

It is only since about 1970 that there has been interest in actuarial models which consider both the time until death and the investment rate of return as random variable.

Boyle (1976) includes the stochastic nature of interest rates in assuming that the force of interest is generated by a white noise series that is forces of interest in the successive years are normally distributed and uncorrelated.

In the approach of Pollard (1971), the force of interest in a year is related to the force of interest in the preceding years by using an autoregressive process of order two.

Panjer and Bellhouse (1980) and Belhouse and Panjer (1981) develop a general theory including continuous and discrete models. The theory is further worked out for unconditional and conditional autoregressive processes of order one and two.

Historically, the theory of life contingencies has ignored explicit consideration of chance fluctuations in mortality, morbidity, interest, and expenses; but in practice, actuaries have implicitly considered chance fluctuations by using conservative assumptions for each of the factors entering a formula. In contrast, Hickman (1964), Pollard and Pollard (1969), Hickman and Gayda (1971), Taylor (1972), Boyle (1973), and Panjer (1978) have considered the role played by the time of decrement (death, disability, and so on) as a random variable in the calculation of actuarial functions. As a result, it is known that most actuarial functions are expected values of interest functions when time of decrement is considered as a random variable.

Giaccotto (1986) develops an algorithm for evaluating present value functions when interest rates are assumed to follow an ARIMA(p,0,q) or an ARIMA(p,1,q) process.

Dhaene (1989) develops for computing moments of insurance functions when interest rates are assumed to follow an autoregressive integrated moving average process. The goal of the Dhaene’s study is to state a methodology for computing, in an efficient manner, present value functions when the force of interest evolves according to an autoregressive integrated moving average process of order (p,d,q).

Stochastic interest rate scenarios use an assumed probability distribution for future interest rates along with simulation techniques to elicit single values of the rate for the next period.

1.3 Objectives and Scope of the Study

The basic objectives of this study are

i. to analyze the framework of recent social security system worldwide,
ii. to examine how the actuarial valuation could made,
iii. to look through necessary actuarial mathematics for actuarial valuation,
iv. to peruse structure of deterministic and stochastic interest rate,
v. to make actuarial valuation with selected assumptions on the foundation data set,

vi. to show results of actuarial valuation and the differences between actuarial valuation which is used in deterministic interest rate and stochastic interest rate,

vii. to make sensitive-analysis in order to compare the changing results of the parameters in actuarial valuation,

viii. to develop the computer program for carrying out the numerical computations involved in the actuarial valuation.

After this introductory chapter, the main concepts of social security insurance worldwide and in Turkey are presented Chapter two.

In Chapter three, methodologies of actuarial valuation and related topics will be given.

Chapter four contains some actuarial mathematics functions of actuarial valuation. The main concept of actuarial valuation and deterministic and stochastic interest rate in the literature are presented in Chapter four.

In Chapter five, the definitions of salary function, benefit function, actuarial liability, and pension mathematic functions will be given together with their numerical verifications and theoretical proofs.

Chapter six contains application of actuarial valuation. Introduction of the Foundation, demographic structure of the foundation, revenues of the Foundation, and assumptions are presented in Chapter six.

In Chapter seven, the summary of results and main conclusions of this research are presented.

CHAPTER TWO
SOCIAL INSURANCE

Almost all governments provide some forms of insurance for their citizens. Generally, they provide insurance against those risks that are not readily diversifiable, primarily catastrophic exposures, and against risks that are not otherwise adequately covered by the private insurance market. The latter category includes social insurance programs providing disability, unemployment, health, retirement, and survivor benefits.

Social security programs not only provide income security for individuals, but also they are important instruments of public policy that influence a country's macroeconomic health and redistribution of income to achieve social equity. These programs represent a significant percentage of the gross domestic product (GDP) of many countries.

2.1 Overview

Social security can be defined as reducing the income decreasing and expense increasing effects of social risks that occur beyond the wills and controls of individuals on themselves and on the people they are obliged to look after and the ability to guarantee a healthy life standard at the minimum level for individuals. Social security in a way is the guarantee of people's living with self-respect in their communities without needing others and of their personal freedom. People have had the need to protect themselves against certain risks throughout all ages of human history. In this section, social security is presented as fact that is as old as humanity and has constituted an essential part of social life throughout the history.

Social security systems play a very important role in society in preventing poverty and disparities in income distribution and in ensuring social piece. Today there is a social security system in all countries of the world. This situation shows that the existence of an institutional structure, the purpose of which is to ensure social security in the society, is a prerequisite for being an independent and modern country. It is possible to examine social security as a concept that is composed of three main mechanisms, namely social services, social benefits and social insurance.

Social services are the entire systematic and regular activities and programs protecting-preventing, curing-rehabilitating, changing-developing characteristics, with the aim of assisting people in eliminating their material and spiritual social deprivation arising from their own bodies and environmental conditions or beyond their control and meeting their needs, in becoming more self-sufficient and preventing their dependence on others, in strengthening their family relations, helping individuals, families, groups and societies to perform their social functions successfully, and ensuring the improvement and increase of living standards of people.

Social benefits are temporary or continuous, systematic and regular complimentary aids that are in kind – in cash, provided to people who are deprived from providing for themselves or for people they are obliged to look after even at a minimum level within local measures, based on the determination and control of the state of need, and with the aim of ensuring that they become self-sufficient. Social insurance, on the other hand, comprises of programs based on premiums, which protect individuals against social risks, the definition and scope of which change in time, where contribution and participation in the funds is obligatory.

While we are examining the institutional development of social security systems, we can take the social security system observed in countries, which experienced industrial revolution starting from the last quarter of the 19th century, as the starting point. On the following periods, the foundations of social security systems we observe in many countries today were established and the social security system came out of its structure that was composed of the contributions made, volunteer organizations and unions and started to transform into a structure where social insurances came forward. During the period following the Second World War, expenditures made by the state were increased in order to guarantee the continuity of social security systems and to ensure higher security standards. During the following years, as a result of people's demanding for better life standards and health services, the concept of social welfare state arose in Europe. During the period, which is also known as the welfare period which continued until 1975, duties and responsibilities of all states in the area of social security increased in general. During the same period, structures of social security systems implemented in different countries and responsibilities adopted by governments with regard to social security in these countries differentiated. For example, some countries such as Great Britain, Sweden, and Denmark adopted social security systems, which are generally financed by general taxes from government budgets and which cover the whole society, and countries Germany, France and Italy, on the other hand, organized their systems within the framework of social insurance. The most important country, which pioneered in the development of today's social security systems, was Germany, where Bismarck Model was developed. Great Britain, where Beveridge Model was implemented, followed Germany and then came the Continental Europe, North and South America and then Asian countries. The common characteristic in the systems adopted by the developing and developed countries today is the dominance of social insurance approach. After the second half of the 1970s social security systems in many developed countries were faced with serious financial crises as a result of changing social requirements and a new era where radical reforms were performed started.

As it was earlier, the reason for the existence of social security systems is to protect individuals against risks that affect their earnings and working abilities and to try to compensate for the damages caused in every country throughout the years. During the second half of the twentieth century, on the other hand, certain international standards, accepted by all social security systems, were created. The most important ones among these standards are; the "Universal Declaration of Human Rights" dated 1948, "Convention No 102 on the Minimum Norms of the Social Security" accepted in the 35th International Labor Conference in 1952, other conventions of the International Labor Organization (ILO) and the European Social Security Code, approved by Turkey in 1976. Compensations for social risks that are stipulated to be included within the social security system can be listed as sickness (both medical and financial support), unemployment, old-age, employment injury, occupational disease, maternity, invalidity, survivors and family benefits. The “care insurance” which was proposed in Germany for the care of old people and the disabled in their own homes can also be added to this list. In many developed countries, a major part or all of the population is being protected under social security systems against all these risks that are listed. Within the last 50 years, the periods for granting unemployment insurance have been extended, amount of salaries to be provided have been indexed to inflation, and free social security guarantees for the benefit of low income groups have been provided. These improvements achieved in social security services have at the same time increased the costs of social security systems.

The number of social insurance programs has grown significantly since 1940, with some 165 countries having some type of program today. Old-age programs and work injury programs are the most common types. The growth in the number and variety of social insurance programs reflects general worldwide economic growth since the early 1950s. Many countries are now restricting their social insurance programs as their rate of economic growth has slowed, but the growth in social insurance program costs generally has remained steady or even risen.

Social insurance programs are funded through three main sources: taxes collected through the workplace (payroll taxes), compulsory contributions, and general taxation. Payroll taxes are a percentage of covered wages or salaries earned by workers. They are usually levied on both the employer and the employee and paid to the government. Compulsory contributions need not be based on income, although they often are, and may be made to private entities that provide social insurance. Revenue from payroll taxes and compulsory contributions are almost always earmarked to fund the social insurance benefit. General taxation includes a variety of revenue sources from the government, none of which is earmarked for a specific use. Governments may contribute to a specific social insurance program from their general revenues.

As noted above, social insurance programs are often financed in a manner that redistributes income from one generation to another or from one income group to another. This redistribution is an important characteristic of social insurance programs in many countries, especially those providing retirement income security.

2.1.1 Types of Benefit Programs

Our examination of social insurance programs covers the following benefits;

- Retirement income
- Health
- Disability
- Survivor
- Occupational injury.

2.1.1.1 Retirement Income

Old age is a period of low or reduced income and of increased health risks. Support for the elderly has been a prime focus of social insurance. In OECD and many developing countries, retirement income programs provide coverage throughout the entire country. In some developing countries, coverage is limited to wage and salary workers in the capital city and perhaps in several provinces. While universal systems cover the entire population, certain conditions, such as citizenship and long-term national residence, are common. Most countries rely on voluntary private retirement pensions to supplement the public plans, but some countries make participation in private pensions mandatory. France, for example, has an employment-based public social security plan and requires all workers to participate in private employment-based pension plans. In 1981, Chile moved to a system in which most pension income comes from private pension funds. Finally, Sweden has a two-tiered public social security plan. The first tier is universal, providing a minimum set of benefits to all citizens financed through general tax revenue. In the second tier, benefits are based on contributions. Although participation in private pension plans is mandatory for collectively bargained plans only, the membership of most Swedes in unions makes private pension coverage nearly universal among workers. Many countries have mixed approaches like Sweden; the United Kingdom, Canada, and Japan also have two-tiered public programs.

Ordinarily, retirement income programs are funded from three sources; taxes paid by workers, taxes paid by employers, and a government contribution. Almost all pension programs under social insurance are financed from employer and employee contributions. About one-half of them derive their funds from all three sources. Contributions are usually related to earnings, with the amount determined by applying a percentage to salaries or wages up to a certain maximum. This percentage may be the same for both employer and employee, although in many cases the employer pays a larger share. The government's contribution may be derived from general revenues or, less frequently, from special earmarked or excise taxes (e.g., a tax on tobacco, gasoline, or alcoholic beverages). It may be used in different ways: to defray a portion off all expenditures, to make up deficits, or even to finance the entire program. For administrative purposes, many countries assess a single overall contribution rate covering several contingencies. Not only pensions, but other social security programs, such as sickness, work injury, unemployment, and survivor benefits may be financed from this contribution. Almost all OECD countries have pay-as-you-go retirement income security plans. Pay-as-you-go social insurance (often shortened as "paygo") means that benefits for current beneficiaries are paid from current worker contributions. In contrast, most private pension systems fund individual retirement benefits through contributions made over the employee's working life. Politically, the paygo system of financing retirement benefits is easiest to implement because it places the least burden on the initial generation of workers. Under a paygo system, the program's initial contributions cover the cost of providing benefits to current retirees only. Contributions are higher in the implementation period of a prefunded system because workers are funding not only their own retirement benefits but also those of current retirees. Worldwide, the number of the population between workers relative to the number of retirees has been declining. As a result, many countries are changing their public programs for retirement income to mitigate the impact of these programs on national budgets.

To receive old-age benefits, two requirements usually must be met: attainment of a specified age and completion of a specified period of contributions or covered employment. Another common requirement is total or substantial retirement from covered employment. Sometimes, eligibility is determined by resident status or citizenship. Old-age benefits generally become payable between ages 60 and 65. In some countries, length-of-service benefits are payable at any age after a certain period of employment, most commonly between 30 and 40 years. Many programs have the same pensionable age for women as for man. Other permit women to draw a full pension at an earlier age than men, despite their usually longer life expectancy. The differential is usually about five years. Internationally, however, the trend is toward equalizing the retirement age for men and women.

The retirement benefit in most countries is a wage-related, periodic payment. Some provide for a fixed amount unrelated to prior earnings. Formulas are used in determining the benefit amount. Some provide an amount equal to a percentage of average earnings, such as 35 or 50 percent, which is unchanged by length of coverage once the qualifying period is met. A more common practice is to provide a basic rate (e.g., 30 percent of average earnings) plus an increment of 1 or 2 percent of earnings either for each year of coverage or for each year in excess of a minimum number of years. Several countries have a weighted benefit formula which returns a larger percentage of earnings to lower-paid workers than to higher-paid workers. Most systems have developed some mechanism for limiting the size of the benefit. Many do this by establishing a ceiling on the earnings taken into account in the computation. Others establish a maximum cash amount or a maximum percentage of average earnings (e.g.,80 percent). Some systems combine two or more of these methods. Most systems add supplements to the benefit for a wife or children. The wife’s supplement may be 50 percent or more of the basic benefit, although many countries require the wife to have reached a specified age, to have children in her care, or to be disabled. Minimum benefits are often designed to maintain a minimum standard of living, but this is not always achieved. A maximum is often used to limit total benefits, including those of survivors, in the interest of the financial stability of the program. A maximum, reduces the effect large families have on benefits.

2.1.1.2 Health Benefit

After retirement income, the most important social insurance program worldwide is the provision of health care benefits. The majority of developed countries provide health care benefits through a social insurance mechanism. There are four fundamental reasons why a social insurance mechanism is employed to provide health benefits; (1) Adverse selection (2) Moral hazard (3) Externalities (4) Economies of scale (Skipper, 1998). Private health insurance markets are particularly vulnerable to adverse selection. Individuals have a great deal more information than insurers about their health status and possible need for health services. Information on individual health status is relatively expensive for insurers to procedure. As a result, most private health insurances throughout the world are supplemental to a social insurance program, tied to employment, or both. Individual policies are sold, but are generally much more expensive than group plans. Moral hazard affects private health insurance in several ways. First, insured individuals are more likely to seek care than uninsured individuals. Second, a principal-agent problem exists between health providers (agents) and patients (principals). Providers treating insured patients may have an incentive to recommend and perform more health services than they would in the absence of insurance. Finally, medical professionals may disagree on the beginning or end of a particular episode of illness and on the health services needed to alleviate the condition. Insurers may bear the risk of this uncertainty concerning the efficacy of care as well as the risk of needed care. Finally, it has been argued that the production of health care services has large economies of scale. As a result, it costs society less to produce these services using a public utility model instead of a competitive market model. The concept of economies of scale in health care may seem puzzling given the sheer number of physicians, hospitals, and other health care providers within most developed countries. However, the market for health care services is a local not a national one in all but the smallest countries. Thus, the markets for health care services encompass a relatively small geographic area. As a result, competition in all but the largest cities may not be sufficient for a workable market. In general, social insurance programs overcome the issues described above by compulsory participation, which removes the issue of adverse selection. Moral hazard remains an important issue. Most countries place implicit (but rarely implicit) limits on the services available to individuals. In many countries, the government also provides most or all health care services (e.g., United Kingdom) while in many others government providers compete with private providers in a strictly regulated market. These attempts to limit moral hazard are also intended to limit the growth of health care costs. They have not been very successful. Most countries have seen health care costs increase faster than national income. As a result, many countries are seeking ways to restructure both the financing and the delivery of health care services.

2.1.1.3 Disability Benefit

For insurance purposes, nonoccupational disability is generally classified in two ways: short term disability and long term disability. Short term disability insurance replaces income lost due to a temporary nonoccupational sickness or accident. Long term disability insurance provides income replacement for individuals who are permanently disabled. The risks of both types of disability have characteristics that make them ideal candidates for private insurance markets, and in the United States and some other countries, short term disability insurance is purchased primarily in the private market. These two disability benefits also have some characteristics of risks typically insured thorough a social insurance mechanism: for example, external benefits to consumption by the disabled. Adverse selection may also be a problem, particularly in economically depressed times. Moral hazard may be subjective. Finally, the presence of a social insurance disability benefit may encourage the taking of risks beneficial to society.

Generally, those eligible to participate in a country's retirement income social insurance program are also eligible to participate in the long term disability income program. In some instances, however, disability benefits may not be provided. Short term disability usually is tied to the country's health insurance program.

Long term disability income benefit funding ordinarily follows that of retirement income benefit funding. Thus, with wage-based plans, the worker may be assessed a percentage of covered wages; with universal plans, benefits usually are funded from general revenues. In most countries (e.g., Germany and France), short term disability benefits are funded through contributions for health insurance.

The principal requirements for receiving a disability benefits are loss of productive capacity and a minimum period of work or contributions. Usually, the full disability benefit is granted for a two-thirds loss of working capacity in the worker's customary occupation. In other case, however, this requirement may vary from one-third to one-half or even be as high as 100 percent. The qualifying period for a disability benefit is usually shorter than it is for an old-age benefit. Periods of three to five years of contributions of covered employment are most common. Entitlement to disability benefits usually is subject to minimum age (e.g., the teens) and maximum age (e.g., normal retirement age) limitations. Benefit qualification in some, particularly universal programs (e.g., Australia), is means tested.

Under most programs, provisions for persons who are permanently disabled due to nonoccupational causes are very similar to those for aged. The some basic formula used for old age usually applies to total disability-a cash amount frequently expressed as a percentage of average earnings. Increments and supplements for dependence are generally identical under the total disability and old-age programs. For the totally disabled, a constant-attendance supplement, most often 50 percent of the benefit, may be paid to those who need help on a daily basis. Partial disability benefits, if payable, are usually reduced, in terms of average earnings, according to a fixed scale. The system may also provide rehabilitation and training. Some countries provide higher benefits for worker's in arduous or dangerous employment. A waiting period of two to seven days is imposed under most short-term disability programs. Workers, ordinarily, may receive short term benefits for up to 26 weeks. In some instances, however, benefits may be drawn for considerably longer periods, or even for an unlimited duration. In most countries, the maximum period of benefit payment for short term disability marks the beginning of eligibility for long term disability coverage.

2.1.1.4 Survivor Benefit

Most social insurance programs provide for some type of payment to dependents of deceased workers. Most survivor benefits programs are patterned closely after the retirement income benefit program. Survivor benefit programs generally extend the coverage to the same population groups that receive retirement income benefits. Thus, with wage-based systems, only covered workers would be the target population, whereas a universal system would be extended to cover all qualifying persons. Funding for survivor benefits ordinarily follows that for retirement income benefits. Hence, with wage-based plans, payroll taxes usually provide the funding; under universal plans, general revenues usually provide the funding.

For survivors to be eligible for benefits, most programs require that deceased worker to be a pensioner at death or have completed a minimum period of covered employment or contribution. This period is often the same as that for the disability benefit. The surviving spouse and children must also often meet certain conditions, such as age requirements. Periodic benefits for survivor of covered persons or pensioners are provided under most systems, although a few pay only lump-sum benefits. Survivor benefits under most programs are a percentage of the benefit paid to the deceased at death, or the benefit to which the insured would have been entitled to if the individual had attained pensionable age or become disabled at that time. Survivor benefits are paid to sum categories of widows under nearly all programs. The amount of a widow's benefit, most frequently ranges from 50 to 75 percent of the deceased workers benefit; In some case (e.g., United States), it is 100 percent. In some countries, life time benefits are payable to every widow whose husband fulfills the necessary qualifying period. More commonly, the provision widows' benefits is confined, except possibly for a brief period, to those having young children in their care, those above a specified age, or those who are disabled. The age limits for children's benefits are often the same as for children's allowance through retirement income and disability income benefit programs. Many countries fix a somewhat higher limit for children who are attending school, undergoing an apprenticeship, or are incapacitated. In a larger number of countries, the age limit is removed for disabled orphans. Benefits are payable under a number of programs the certain widowers of insurers workers or pensioners. A widower usually must have been financially dependent on his wife and either disabled or old enough to receive an old-age benefit at her death. A widower's benefit is usually competed in the same way as a widow's benefit.

2.1.1.5 Occupational Injury Benefit

Occupationally injury benefit programs are the oldest and most widespread type of social insurance. They provide both income replacement and medical services. Most injuries covered under these programs are short term. As a result, these programs often promote the rehabilitation of workers and are designed to encourage returning to work. In developed countries, claims are associated with industrial injury and disease have decreased. This decrease has been countered by the growth in the number of recognized occupational diseases due to a greater understanding of the origins of certain diseases. Because work related injuries are generally not permanently disabling; a program's goal is to return the worker to work, and rehabilitation plays an important part in these benefits. Thus, benefits for medical services, including physical therapy and counseling, are typically provided.

Occupational injury systems come in two varieties: social insurance systems utilizing a public fund and requiring various forms of private or semiprivate arrangements. Many countries having occupational programs operate through a central public fund, which may or may not be a part of the general social insurance system. All employers subject to the program must pay contributions to the public carrier, which in turn pays the benefits. Countries that rely primarily on private arrangements, including Australia and the United States, require employers to insure their employees against the risk of employment injury. In some these countries, however, only private insurance is available. In the remainder, a public fund does exist, but employers are allowed the option of insuring with either a private or the public fund. The premiums charged by private insurance companies for protection against work-related injury usually vary according to past experience of work accidents in different under-takings or industries, and the cost of protection may vary widely. In some countries, notably Norway and Sweden, experience rating has been eliminated, and all employers within each country, regardless of type of business, contribute to the program at the same rate. In other instances, workers' compensation laws simply impose on employers a liability to pay direct compensation to injured workers or their survivors, without a requirement to carry insurance. Some employers covered under these laws may simply pay benefits from their own funds as injuries occur. Others voluntarily purchase private insurance to protect themselves against risk.

Work-injury programs commonly apply to wage and salary workers and exclude the self employed. The programs of some of the more highly industrialized nations cover practically all employees. However, many countries and a number of states in the United States either exclude all agricultural employees from coverage or cover only those work involves the operation of power-driven machinery. Some programs also exclude employees of small enterprises.

Work-injury benefits are financed primarily by compulsory employer contributions or payroll taxes, reflecting the traditional assumption that employers should be liable for payment of compensation when their employees suffer injuries on the job. Exceptions are found where certain elements of the work-injury program are meshed with one or more of the other branches of the social insurance system. In such case, financing often involves contributions from employees, employers, and the government. Another exception occurs in countries that provide medical treatment for work-connected illnesses under their ordinary public medical care programs. Contributions can vary widely across countries. The contribution rate in Germany, for example, is 1 percent of payroll while in Japan it is almost 15 percent for some workers.

Occupational injury programs provide disability income and medical benefits. Disability benefit may be subdivided into temporary disability, permanent total disability, and permanent partial disability benefits. No qualifying period of coverage or employment is ordinarily required for entitlement to benefits. The concept of work-related injury has gradually been liberalized in a number of countries to cover injuries occurring while commuting to and from work.

Temporary disability benefits are usually payable from the start of incapacity, although some programs require a waiting period of one to three days. Benefits normally continue for a limited period, such as 26 to 52 weeks, depending on the duration of incapacity; if incapacity lasts longer, the temporary disability benefit may be replaced by a permanent disability benefit. The temporary disability benefit is nearly always a percentage of the worker's average earnings during a period immediately before injury. Although benefits differ widely, they usually cover at least one-third to one-half of former earnings, subject to a maximum limit.

The second type of benefit is provided when permanent total disability occurs. Generally, it becomes payable immediately after the temporary disability benefit ceases, based on a medical evaluation that the worker's incapacity is both permanent and total. Under most programs, the permanent total disability benefit is usually payable for life, unless the worker's condition changes. A minority of programs pay only a single lump-sum grant equal to several years' wages. The permanent total disability benefit usually amounts to two-thirds to three-fourths of the worker's average earnings before injury, somewhat higher than it is for ordinary disability benefits. Supplements are often added for dependents and for pensioners requiring the constant attendance of another person, in which case benefits may exceed former earnings.

Still another form of cash work-injury benefit is that for permanent partial disability, payable when a worker loses partial working or earning capacity. It is usually equal to a portion of the full benefit corresponding to the percentage loss of capacity. Alternatively, permanent partial disability benefits may be paid in the form of a lump-sum grant. Aside from cash payments, medical and hospital care and rehabilitation services are provided to injured workers. Nearly always free, they may include a somewhat wider range of services than the general sickness program.

Most work-injury programs also provide benefits to survivors. These benefits are customarily payable to a widow, regardless of her age, until her death or remarriage; to a disabled widower; and to children below specified age limits. Survivors benefits are computed as percentage of the worker's average earnings immediately before death or of the benefit payable (or potentially payable) at death. Most systems also pay a funeral grant equivalent to a fixed sum or percentage of a worker's earnings.

Table 2.1 shows demographic and other statistics related to social security of some countries in the world (Sources of this table are United Nations). In selected countries, Monaco is the oldest country because of percentage of 65 or older persons. Therefore, dependency ratio 60.7% of Monaco is the highest rate. South Africa has the youngest population. Second young population is Turkey which has the lowest dependency ratio (8.8%). Andorra has the longest life expectancy at birth which is 80.6 years for male and 86.6 years for female. Average of life expectancy at birth is 72.8 years for male and 79.45 years for female in all selected countries. Average of retirement age is 63.8 years for male with standard deviation 1.87 and 61.5 years for female with standard deviation 3.5.

Table 2.2 shows contribution rates for social security programs of some countries in the world (Source of this table is based on information in the web site. http://www.ssa.gov/policy/docs/progdesc/ssptw/index.html). Maximum contribution rate of insured person for old-age, disability, and survivors is Croatia. England has the highest contribution rate of employer for old-age, disability, and survivors. Average of contribution rate for all social security programs is 28% with standard deviation 1.83 for insured person, 53% with standard deviation 3.46 for employer.

Table 2.1 Demographic and other statistics related to social security for selected country, 2006

Country	Total populations (millions)	Percentage 65 or older	Dependency Ratio[a]	Life Expectancy at Birth (years)		Statutory Pensionable Age		Early Pensionable Age[b]		GDP per capita (US$)
				Men	Women	Men	Women	Men	Women	
Albania	3.1	8.3	54.5	71.7	77.4	65	60	c	c	4,584
Andorra	0.07	14	40.2	80.6	86.6	65	65	c	c	24,000
Austria	8.1	16.7	47.4	76.9	82.4	65	60	62.25	57.25	30,094
Belarus	9.7	14.7	42.6	63.1	74.5	60	55	c	c	6,052
Belgium	10.4	17.6	52.4	76.5	82.7	65	64	60	60	28,335
Bulgaria	7.7	16.8	43.8	69.8	76.3	63	58.5	c	c	7,731
Croatia	4.5	17.2	48.5	72.3	79.2	64	59	59	54	11,080
Cyprus	0.8	12.1	47	76.7	81.6	65	65	63	63	18,776
Czech Republic	10.2	14.2	40.4	73.1	79.4	61.5	55.66	58.5	52.66	16,357
Denmark	5.4	15	51	75.5	80.1	65	65	60	60	31,465
Estonia	1.3	16.5	46.1	67	78	63	59.5	60	56.5	13,539
Finland	5.2	15.9	49.7	76	82.4	65	65	62	62	27,619
France	60.4	16.6	53.3	76.6	83.5	60	60	c	c	27,677
Germany	82.6	18.8	49.4	76.4	82.1	65	65	63	63	27,756
Greece	11.1	18.2	48.1	76.1	81.3	65	60	60	55	19,954
Guernsey	0.06	17.8	48.8	77.4	83.5	65	65	c	c	40,000
Hungary	10	15.2	44.7	69.8	77.7	62	60	c	c	14,584
Iceland	0.3	11.8	51	79.5	83.2	67	67	c	c	31,243
Ireland	4.1	10.9	45.1	75.9	81.1	65	65	c	c	37,738
Isle of Man	0.07	17	52.2	75.1	82	65	60	c	c	28,500
Italy	58	20	51.5	77.5	83.6	65	60	c	c	27,119

GDP = gross domestic product.

a. Population aged 14 or younger plus population aged 65 or older, divided by population aged 15–64.
b. General early pensionable age only; excludes early pensionable ages for specific groups of employees.
c. The country has no early pensionable age, has one only for specific groups, or information is not available.
d. Statistics for Serbia also include data for Montenegro.
e. Regardless of age but subject to other conditions.

Table 2.1 Continue

Country	Total populations (millions)	Percentage 65 or older	Dependency Ratio[a]	Life Expectancy at Birth (years)		Statutory Pensionable Age		Early Pensionable Age[b]		GDP per capita (US$)
				Men	Women	Men	Women	Men	Women	
Jersey	0.09	15.7	49	76.8	82	65	65	63	63	40,000
Latvia	2.3	16.9	46.1	67.2	77.8	62	60.5	60	58	10,270
Liechtenstein	0.03	12.4	42.4	76.1	83.2	64	63	60	60	25,000
Lithuania	3.4	15.5	47.7	67.9	78.6	62.5	60	57.5	55	11,702
Luxembourg	0.4	13.8	48.5	75.9	82.2	65	65	60	60	62,298
Malta	0.4	13.5	45.1	76.6	81.3	61	60	c	c	17,633
Moldova	4.4	10.3	43.4	61.6	69.8	62	57	c	c	1,510
Monaco	0.03	22.6	60.7	75.8	83.7	65	65	60	60	27,000
Netherlands	16.2	14.1	47.7	76.3	81.6	65	65	c	c	29,371
Norway	4.6	15	52.9	77.8	82.5	67	67	c	c	37,670
Poland	38.5	12.9	41.2	71.2	79	65	60	c	c	11,379
Portugal	10.4	17.1	49	74.6	81.2	65	65	55	55	18,126
Romania	21.7	14.8	43.4	68.7	75.7	63	57.75	58	52.75	7,277
Russia	143	13.8	40.8	58.7	71.8	60	55	c	c	9,230
San Marino	0.03	17	51	78.2	85.5	65	65	c	c	34,600
Serbia[d]	10.5	14.1	47.9	71.7	76.4	63	58	c	c	5,000
Slovak Republic	5.4	11.8	39.8	71.1	78.7	62	62	e	e	13,494
Slovenia	2	15.6	42	73.5	80.7	61.5	55.33	c	c	19,150
Spain	43	16.5	44.5	76.5	83.8	65	65	c	c	22,391
Sweden	9	17.2	53.1	78.6	83	65	65	61	61	26,750
Switzerland	7.2	16	48.1	78.2	83.8	65	64	c	c	30,552
Turkey	67.8	5.69	8.8	66.2	70.9	60	58	60	55	5,062
Ukraine	46.4	16.1	44.7	60.7	72.5	60	55	c	c	5,491
United States	298.2	12.3	49	75.2	80.6	65.5	65.5	62	62	37,562
United Kingdom	59.6	16	51.2	76.7	81.2	65	60	c	c	27,147
South Africa	43.3	3.6	60.2	46.5	48.3	65	60	c	c	10,070

Table 2.2 Contribution rates for social security programs, 2006 (in percent)

Country	Old age, Disabilty, and Survivors			All Social Security Programs		
	Insured Person	Employer [a]	Total [a]	Insured Person [a]	Employer [a]	Total [a]
Albania[b]	8[c]	19.1	27.1[c]	9.5	30.2	39.7[d]
Andorra	2[c]	6[c]	8[c]	5	13	18
Austria[b]	10.25	12.55	22.8	17.2	25	42.2
Belarus	1[c]	10[c]	11[c]	1	11.2	12.2
Belgium[b]	7.5	8.86	16.36	13.07	24.77	37.84
Bulgaria[b]	8.05	14.95	23	12.425	23.475	35.9[d]
Croatia[b]	20	0	20	20	17.2	37.2[d]
Cyprus[b]	6.3[c]	6.3[c]	12.6[c]	6.3	6.3	12.6[d]
Czech Republic	6.5	21.5	28	12.5	37	47.5[d,a]
Denmark[b]	f	f	F	F	f	d,f
Estonia	2	20	22	3	33.5	36.5[d]
Finland	4.6	22.49	27.09	6.7	26.3	33[d]
France[b]	6.75	9.9	16.65	9.9	35.14	45.04
Germany[b]	9.75	9.75	19.5	20.55	20.98	41.53[d]
Greece[b]	6.67	13.33	20	11.55	22.1	33.65
Guernsey[b]	6[c]	5.5[c]	11.5[c]	6	5.5	11.5[d]
Hungary[b]	8.5[c]	18[c]	26.5[c]	13.5	32	45.5[d,g]
Iceland	4[c]	11.79[c]	15.79[c]	4	11.79	15.79[d]
Ireland[b]	4[c]	8.5[c]	12.5[c]	4	8.5	12.5[h]
Isle of Man[b]	10[c]	12.8[c]	22.8[c]	10	12.8	22.8[d]
Italy[b]	8.89	23.81	32.7	8.89	31.97	40.86
Jersey[b]	5.2[c]	5.3[c]	10.5[c]	6	6.5	12.5[d,l]
Latvia[b]	9[c]	24.09[c]	33.09[c]	9	24.09	33.09[d]
Liechtenstein[b]	10.55	10.55	21.1	12.3	13.65	25.95[d]
Lithuania	2.5	23.6	26.1	3	30.98	33.98[d]
Luxembourg[b]	8	8	16	14.05	13.66	27.71[d]
Malta[b]	10c	10[c]	20[c]	10	10	20
Moldova[b]	3[c]	26[c]	29[c]	3	26	29
Monaco[b]	6.15[c]	23.66[c]	29.81[c]	6.15	23.66	29.81[e]
Netherlands[b]	19.15	6.38	25.53	37.45	16.33	53.78[d,j]
Norway	7.8[c]	14.1[c]	21.9[c]	7.8	14.1	21.9[d]
Poland[b]	16.26	16.26	32.52	27.21	19.68	46.89[d]
Portugal	11[c]	23.75[c]	34.75[c]	11	23.75	34.75
Romania[b]	9.5[c]	20.5[c]	30[c]	17.5	33.25	50.75[d]
Russia[b]	0	20[c]	20[c]	0	26.2	26.2[l]
San Marino	1.9[c]	10[c]	11.9[c]	5.9	18.5	24.4
Serbia[b]	11[c]	11[c]	22[c]	17.9	17.9	35.8[d]
Slovak Republic[b]	7	17	24	13.4	30.2	43.6[d]
Slovenia	15.5[c]	8.85[c]	24.35[c]	22.1	16.1	38.2[d]
Spain[b]	4.7[c]	23.6[c]	28.3[c]	6.25	31.58	37.83[d]
Sweden[b]	7	11.91	18.91	7	23.43	30.43[d,k]
Switzerland[b]	11.9	11.9	23.8	14.72	13.19	27.91
Turkey	9	11	20	5	7.5	12.5
Ukraine[b]	3[c]	32.3[c]	35.3[c]	3.75	36.7	40.45
United States	6.2	6.2	12.4	7.65	8.45	16.1
United Kingdom[b]	11[c]	23.8[c]	12.8[c]	11	12.8	23.8[d]
South Africa	0	0	0	1	1	2

a. Includes Old Age, Disability, and Survivors; Sickness and Maternity; Work Injury; Unemployment; and Family Allowances. In some countries, the rate may not cover all of these programs. In some cases, only certain groups, such as wage earners, are represented. When the contribution rate varies, either the average or the lowest rate in the range is used.

b. Contributions are subject to a ceiling on some benefits.

c. Also includes the contribution rates for other programs.

d. Government pays the total cost of family allowance benefits.

e. Employers pay the total cost of work injury benefits.

f. Portion of set amount for Old Age, Disability, and Survivors. Central and local government and other types of contributions for the other programs.

g. Plus a flat-rate contribution by the employer for medical benefits.

h. Government pays most of the cost of family allowance benefits.

i. Government pays the total cost of unemployment benefits.

j. Plus a flat-rate contribution by the insured for medical benefit.

2.2 Social Security Organizations in Turkey

With the inspiration of social state concept, the Turkish Republic Constitution gives the constitutional and governmental guarantee for providing social security. It states in article 60 that: “Every state shall take required measurement and establish security organizations.”

In the light of the Constitution there are basically two social security systems. One of them is the non-contributory scheme, which provides means-tested pension to disabled and elderly people over 65 and is financed by general taxes. However, its amount is meaningless and not a remedy for any problem. The second one which draw their inspiration from the Bismarckian social insurance model like most European countries is based on statutory, occupational, earning related contributory scheme which provides employment injury and occupational disease, sickness and maternity as short term insurances, survivor (death), invalidity and old age insurances as long term by three main Social Security Organizations (SSO) according to employment. In other words, 7 out of 9 insurance branches stated in ILO no.102. Convention are provided by Turkish Social Security System to working population and their dependencies. Family insurance stated in that convention is not provided directly by SSO’s but by some other regulations.

There are three institutions for social security in Turkey. Firstly, Retirement Fund (RF) for public servants has been established in 6.8.1949. Benefits of RF are old-age, disability, survivors, and health. Total number of members for RF are 9.382.000. Of these members, 1.606.000 are retired, 2.413.000 are active, 5.363.000 are dependant. Secondly, Social Insurances Institutions (SII) for dependent employees has been established in 7.17.1964. Benefits of SII are old-age, disability, survivors, occupational diseases, maturity and health. Total number of members for SII are 45.730.000. Of these members, 4.493.000 are retired, 8.598.000 are active, 32.369.000 are dependant. Thirdly, Social Insurance Institution for tradesmen and craftsmen and other self employed (BAĞ-KUR) has been established. Benefits of BAĞ-KUR are old-age, disability, survivors, and health. Total number of members for BAĞ-KUR are 16.365.000. Of these members, 1.734.000 are retired, 3.375.000 are active, 11.256.000 are dependant. In Turkey, number of active workers are 14.5 million. But, 71.7 million people take advantages of RF, SSI and BAĞ-KUR.

Under the control of Ministry of Labour and Social Security some institutions such as banks, insurance companies, reassurance companies can set up pension funds for their employees under the condition that they should provide benefits not less than the level of SII.

2.2.1 Old Age, Disability and Survivors

Employees (including foreign nationals) aged 18 or older working under a service contract in the public or private sector. Special systems for civil servants; the self-employed; farmers; some categories of agricultural worker; and bank, insurance company, and stock exchange employees.

Contribution rate of insured person is 9% of monthly earnings. The minimum monthly earnings for contribution and benefit purposes are 444.15 liras. The maximum monthly earnings for contribution and benefit purposes are 2,887 liras (6.5 times minimum earnings). Contribution rate of employer is 11% of monthly payroll; 13% on behalf of employees in arduous employment.

In old-age pension, if first insured on or after September 8, 1999, age 60 (men) or age 58 (women) with 7,000 days of contributions or 25 years of insurance coverage with 4,500 days of contributions. If first insured before September 8, 1999, special conditions apply. Miners younger than age 50 who have worked underground permanently for at least 20 years and who have at least 5,000 days of paid contributions (worked alternately underground for at least 25 years and have at least 4,000 days of contributions) can ask to receive the old-age pension; miners aged 50 or older who have a minimum of 1,800 days of insured employment are subject to other conditions. Aged 50 or older and prematurely aged (and therefore unable to work until the full pensionable age), subject to other conditions. An insured person, of any age, whose disability began before starting insured employment and who has at least 15 years of insurance coverage including at least 3,600 days of paid contributions, is subject to the assessed degree of disability. Gainful employment must cease on retirement. (In certain cases, employment may be permitted while receiving the old-age pension. In such cases, a social security support contribution of 30% of earnings must be paid.) Deferred pension: There is no age limit for deferral.

For old-age benefit, if first insured on or after January 1, 2000, the pension is calculated using the insured's average annual indexed earnings, on the basis of 3.5% for each 360-day period of contributions up to 3,600 days, 2% for each 360-day period of the next 5,400 days, and 1.5% for each additional 360- day period. If first insured before January 1, 2000, special conditions apply. The minimum monthly pension is at least 35% of the lower limit of monthly earnings, or 400.80 liras. The maximum monthly pension is 878.68 liras.

Disability pension is the loss of 2/3 of working capacity with at least 1,800 days of contributions or insured for at least 5 years with an average of 180 days of paid contributions for each year of insurance. The disability pension is partially payable abroad under bilateral agreement.

The deceased met the contribution requirements for a disability pension or an old-age pension or was a pensioner at the time of death; was insured for at least 5 years and had paid contributions for an average of at least 180 days each year or for a total of 1,800 days. Eligible dependents include a spouse (the spouse's pension ceases on remarriage); children under age 18 (age 20 if in preuniversity education, age 25 if in university); a son aged 18 or older who is disabled and unemployed; an unmarried, widowed, or divorced daughter of any age who is without insured employment and is not receiving any social security benefits in her own right; and dependent parents.

The insured person was not eligible for a pension. A lump sum equal to total employee and employer contributions is split among survivors according to prescribed ratios. The survivor pension and survivor settlement are partially payable abroad under bilateral agreement.

For disability benefit, if first insured on or after January 1, 2000, the pension is calculated as 60% of the insured's average indexed earnings during the years before the onset of disability. If first insured before January 1, 2000, special conditions apply.

In survivor pension, if the deceased was first insured on or after January 1, 2000, the pension is calculated as 60% of the insured's average monthly earnings, plus 2% for each 360-day period of contributions beyond 8,100 days but not more than 9,000 days, plus 1.5% for each 360-day period of contributions beyond 9,000 days. If the deceased was first insured before January 1, 2000, special conditions apply. Eligible survivors include the spouse, orphans, and the deceased's parents. The minimum pension for one survivor is 335 liras; 365.57 liras for two survivors.

If the deceased was not eligible for a pension, a lump sum equal to total employee and employer contributions is split among survivors according to prescribed ratios. Eligible survivors include the spouse, orphans, and the deceased's parents.

2.2.2 Sickness and Maternity

Employees working under a service contract in the public and private sectors and their dependent family members have cash and medical benefits;. (Cash maternity benefits are provided only to an insured woman.) Pensioners and their dependents are covered for medical benefits. Special systems for civil servants, the self-employed, and some categories of agricultural worker.

Contribution rate of insured person is 5% of monthly earnings (sickness). The minimum monthly earnings for contribution and benefit purposes are 444.15 liras. The maximum monthly earnings for contribution and benefit purposes are 2,887 liras

(6.5 times minimum earnings).Contribution rate of employer is 1% of payroll (maternity).

Insured person has medical benefits in 90 days of contributions in the year before the date of the first diagnosis of illness. For the insured's dependents, the insured must have 120 days of contributions in the year before the date of the first diagnosis of illness. Eligible dependents include a spouse who does not work or receive any social security benefits; children under age 18 (age 20 if in preuniversity education, age 25 if in university); a son aged 18 or older who is disabled and unemployed; an unmarried, widowed, or divorced daughter of any age who is without insured employment and is not receiving any social security benefits in her own right; and dependent parents.

The sickness benefit for inpatient treatment is 1/2 of daily earnings. The benefit for outpatient treatment is 2/3 of daily earnings. The benefit is payable after a 2-day waiting period.

Incapacity for work is two-thirds of earnings, payable for up to 8 weeks before and 8 weeks after the expected date of childbirth. Pregnancy benefit is a lump sum of 50 liras (subject to the certification of pregnancy before the date of childbirth). Childbirth benefit is a lump sum of 56 liras for a birth without complications, 116 liras for a birth by forceps, or 220 liras for a birth by caesarian section. In all cases, the Social Insurance Institution must receive certification of the birth within 3 months after the birth. In cases in which medical services for pregnancy and childbirth cannot be provided directly through health facilities contracted by the Social Insurance Institution or government hospital, a fixed amount of monetary aid is provided according to the schedule in law. The fixed amount is increased for multiple births. Nursing grant is a lump sum of 50 liras for a live birth.

Medical services are usually provided directly to patients through the facilities of the Social Insurance Institution. Benefits include general and specialist care, hospitalization, laboratory services, medicines, maternity care, appliances, and transportation. Benefits are usually limited to 6 months; may be extended in special cases.

2.2.3 Work Injury

Employees working under a service contract in the public or private sector; applicants for apprenticeships, apprentices, and students; and convicted persons working in prison workshops.

There is no contribution of insured person. Contribution rata of employer is between 1.5% and 7% of payroll, according to the assessed degree of risk. The average contribution rate is 2.5% of payroll. The minimum monthly earnings for contribution and benefit purposes are 444.15 liras. The maximum monthly earnings for contribution and benefit purposes are 2,887 liras (6.5 times minimum earnings).

The Permanent disability pension is calculated on the basis of the insured's annual insurable earnings. Total disability is assessed as the loss of earning capacity as a result of a work accident or an occupational disease. For an assessed degree of disability of at least 10%, the pension is in proportion to the assessed degree of disability. For an assessed degree of disability of at least 10% but less than 25%, the pension may be paid as a lump sum. The minimum pension is at least 70% of the lower limit of monthly earnings for contribution and benefit purposes, with an assessed degree of disability of at least 25%. There is no maximum pension. Workers' medical benefits are medical treatment, surgery, hospitalization, medicines, appliances, and transportation. There is no limit to duration.

The minimum monthly survivor pension for one survivor is at least 80% (90% for two survivors) of 35% of the lower limit of monthly earnings for contribution and benefit purposes. There is no maximum pension. Eligible dependents include a spouse (the spouse pension ceases on remarriage); children under age 18 (age 20 if in preuniversity education, age 25 if in university); a son aged 18 or older who is disabled and unemployed; an unmarried, widowed, or divorced daughter of any age who is without insured employment and is not receiving any social security benefits in her own right; and dependent parents. If the total survivor pension awarded to the spouse and children is less than 70% of the insured’s annual earnings, the difference is paid to a dependent father and mother; if the total survivor pension awarded to the spouse and children is 70% or more of the insured’s annual earnings, no pension is paid for a dependent father and mother.

CHAPTER THREE
ACTUARIAL VALUATION

A pension scheme can be viewed as a series of cash flows – the flow of contributions and investment income into the scheme and the flow of benefit payments, and possibly expenses, out of the scheme.

Usually when a scheme is established there is little benefit outgo. The benefit outgo will gradually increase over the years until the income and outgo reach a balance. The scheme can then be said to be mature. If there are no new entrants, benefit outgo will eventually start to exceed income.

Most actuarial valuation computer systems will project the anticipated cash flows on a year by year basis. The cash flows are calculated on the basis of the various probabilities of mortality, retirement and withdrawal, and the economic assumptions of salary inflation and pension increases. The income and outgo in each year is then discounted, using the valuation rate of interest, back to the date of the valuation to produce capitalized (or present) values of the benefit outgo and contribution income. The valuation can therefore be viewed as a consolidation of future cash flows. The process can however disguise the timing of the income and the outgo. Merely presenting the capitalized values will not demonstrate the long term nature of the cash flows being considered.

There are a considerable number of different aspects to be considered in the valuation process. These are covered as follows;

- Overview of the valuation process and purposes of the valuation,
- Collecting and analyzing valuation data,
- Deciding on valuation assumptions,
- Valuing assets and liabilities,
- Initial valuation results and analysis of surplus and deficiency,
- Discontinuance / solvency valuation,
- Valuation results and report.

3.1 Valuation Process

The rate of interest assumed is a very important part of the valuation process because it is used to discount the future anticipated cash flows to produce the capitalized values of the future benefit payments and the contribution income.

The appropriate rate of interest to use for discounting is the assumed future return on the scheme's assets. The nature of this interest rate can be considered further by breaking down the investment return. The main factors affecting future cash flows can be divided into economic and demographic factors.

3.1.1 Economic Factors

The investment return is dependent on the assumed dividend growth. The cost of a defined benefit scheme is affected by investment returns. If returns are high this may reduce the contributions required to meet the benefits. The cost of a defined benefit final salary scheme is also affected by earnings inflation. High earnings inflation tends, over the long term, to be linked with high price inflation and also high investment returns. However, there will be periods when the rates of increase are out of step.

For example; few private UK company schemes guarantee pension increases in line with increases in the RPI (retail price index or consumer price index), however many will target to provide pension increases of a percentage of RPI. This target may then be taken into account in setting the valuation assumptions. The price inflation experience will then be a factor in the cash flows outgo.

The investment return is the assumed rate of return on the assets in the long term. Earnings and price inflation are the assumed increases affecting the liabilities in the long term. For viable funding of the liabilities, investment returns must exceed earnings/price inflation in the long term. The relationships between the economic factors are therefore very important in the long term; in fact more important than the absolute value of investment return and earnings/price inflation.

3.1.2 Demographic Factors

High mortality rates may either increase or decrease the benefit outgo depending on how the level of death benefits (lump sum and/or spouses'/dependant's pensions) compares with the benefits payable if the member was to survive.

If early retirement is allowed on favorable terms, and if ill-health retirements receive generous benefits then the number of such retirements will be a factor in the cash flow of the scheme. Cash flow is affected by early retirement pensions even if these are actuarially neutral; however the value of the cash flow may not be affected.

The affects here are fairly obvious. For example, a spouse's/dependent's pension is only payable if there is a spouse/dependent, and the younger the spouse/dependent, the longer the pension is likely to be paid and therefore the greater the cost. The increase in earnings due to increasing age/experience/promotion, will also affect the amount of benefit outgo.

Changes to benefits affect the cash flow. They occur because pension scheme design is not static; the requirements of employers and employees change over time, and are also needed to comply with legislation. In estimating a scheme's potential cash flows there are many uncertainties. Therefore it is reasonable that a degree of prudence is introduced when making the various assumptions. In deciding whether a valuation basis is prudent and reasonable all the assumptions have to be taken together. For example if a degree of prudence were introduced into each assumption, the actuarial basis when taken as a whole may be considered over cautions and produce an unrealistically high estimate of the cost of the benefits.

It is important to remember though, that the valuation basis does not affect the cost of the scheme-only the timing of the contributions. Also, in deciding on the assumptions to be adopted, the sensitivity of the resulting contribution rate to changes in the assumptions needs to be borne in mind.

3.2 Purposes of Valuation

Several of the valuation funding methods consider past service benefits separately from future service benefits. This is often a valuable feature because it allows the value of accrued benefits to be compared with the assets that have been built up. Also it allows the true cost of the benefits in respect of future service to be seen.

One of the main reasons for performing a valuation of a scheme is to assets how well the assets cover the liabilities. This process is necessary because it is not certain that a pension scheme will continue indefinitely. If a scheme is discounted, the assets it has at that time become important. If the scheme's assets are inadequate, members may not receive the benefits they expect.

The ratio Assets/Liabilities is known as the funding level. However, the phrase funding level is used in different ways. Assets can be valued in various different ways, but more importantly accrued liabilities can be calculated as

- the value of GMPs (Guaranteed Minimum Pension) and prior liabilities, (that is liabilities with priority over GMPs in the rules of a scheme in the event of the scheme winding up),
- the value of leaving service benefits,
- the value of minimum benefits promised on discontinuance,
- the value of accrued benefits allowing for future expected earnings increases.

So the phrase "funding level" needs to be carefully specified, but generally it would be taken to mean the ongoing funding level unless stated otherwise. A further ambiguity arises from the fact that each of these different funding levels may be calculated using different assumptions. The other main reason for valuing the liabilities and assets of a defined benefit scheme is to make recommendations about future contributions. To make contribution recommendations, values are required for

- benefit that will be paid (often split between past and future),
- assets,
- future contributions.

The values of these will depend on the method of funding adopted, the actuarial assumptions used and the method used to place a value on assets.

The pace of funding is one of the major decisions the trustees of the scheme need to agree with the employer at the outset of the scheme. This decision will be influenced by statutory and accounting requirements affecting the scheme and may change over the lifetime of the scheme.

The funding method, adopted for the valuation produces the financial information for the particular scheme which enables the trustees and employer to consider the pace of funding and the implications for company finances. There are a number of methods which fund at different speeds to meet the cost of benefits. Whichever method is chosen to meet the cost of benefits, a regular valuation is necessary to keep on course or to ascertain the consequences of a change in the funding method.

The shareholders of companies are interested in how much a company pension scheme costs. Since, as we have observed, there is a variety of speeds at which benefits may be paid for, the actual contributions to a pension scheme may in some circumstances be a poor guide to the ongoing cost of a scheme. The accounting profession after extensive discussion with the actuarial profession recognized this, and a valuation for the purpose of informing shareholders of the cost of a pension arrangement is a requirement both in the UK and USA. In many cases this valuation is the same as the ongoing valuation of a scheme, but in some cases the valuations differ for a number of reasons.

Before any benefit change is made the financial effect on the scheme and the employer should be investigated. This may be a simple and possibly approximate calculation based on the latest actuarial valuation. Alternatively if the benefit changes being considered are significant or the valuation was some time ago then a full actuarial valuation may be required.

One of the purposes of valuation is to be able to report to the members on the financial state of the pension scheme. This is important because such information will allow members to decide whether or not to remain in the scheme and also reassure them that their benefits are secure. There are also statutory requirements that schemes are regularly valued.

- Disclosure regulations require a valuation to be obtained from an actuary not more than three years and six months after the last one.
- The Inland Revenue will not approve a scheme unless it has a requirement for regular reviews in its trust deed.
- The Inland Revenue requires a scheme to be valued using the prescribed basis to establish if it is over funded in terms of the Surplus Regulations.

3.3 Valuation Data and Experience

In respect of the information needed by an actuary to perform an actuarial valuation of a pension scheme, including the analysis of the scheme's experience;

- Data requirements
  - Data relating to the operation of the scheme
  - Accounting data
  - Asset data
  - Membership data
    - Data required; the data required will vary with each scheme and it is not possible a comprehensive range of data requirements that will cover all schemes. However, as an example, the data that may be required from a typical contracted out final salary scheme at the valuation date is listed below:

- Active members both at this and the previous valuation
  - Name (or other identifier)
  - Date of birth
  - Sex
  - Category (for example works or staff)
  - Date joined company
  - Date joined scheme (and date pensionable service started, if not obvious from date joined company or date joined scheme)
  - Salary
  - Pensionable salary
  - Contributions paid
  - Spouse's details (or may make broad assumptions about marital status)
  - Details of any special benefit arrangements for a member
  - Date of leaving/death/retirement for members active at the previous valuation, who have since left this status.
- Deferred pensioners both at this and the previous valuation
  - Name (identifier)
  - Date of birth
  - Sex
  - Category (for example works or staff)
  - Pension due (split into components escalating or not)
  - Current level of pension if increases granted since date of leaving

- Date of joining this status
- Contingent pensions
- Contributions paid
- Spouse's details
- Date of leaving this status for deferred pensioner at the previous valuation who have since left status.

- Current pensioner's/spouse or dependent pensioners at this and the previous valuation
    - Name (identifier)
    - Date of birth
    - Sex
    - Category (if the pension is a dependent's or spouse's pension arising on the death of a member then these should be in a separate category and the date of death of the member should be recorded for reconciliation purposes)
    - Pension due (split into components escalating or not)
    - Pension at previous valuation (if increased in the period)
    - Pension before commutation
    - Date of exit from active status or deferred pensioners status
    - Contingent pensions
    - Date of commencement of pension (for guarantee period)
    - Date of leaving this status for pensioners at the previous valuation and status

- Data relating to the future operation of the scheme

- Data validation
    - General check
        - Membership reconciles with last valuation
        - No missing data
        - Average salaries and pension consistent with last time
        - Average past service consistent with last time
        - Contributions consistent with salaries and contribution rate
        - Employee and employer contributions consistent
        - Pensions paid in accounts tie in with pensions in data
        - Investment income consistent with level of assets
        - Value of assets consistent with investment manager's performance and contributions and outgo and last value of assets
    - Individual check
        - Data validation
        - Random spot check
- Summarized data
- Analysis of experience
    - The elements that might be analyzed
        - Investment return
        - General salary increase
        - Promotional salary increase
        - Pension increases
        - Mortality pre-retirement
        - Mortality post retirement
        - Withdrawals
        - Early retirement
        - Ill-health retirement
        - Late retirement
        - Proportion of pension commuted

        - Marital statistics (proportion to married, age differences)
- Experience analysis of salary increases
- Withdrawal experience
- Retirement experience.

3.4 Valuation Assumptions

Before the actuarial valuation or investigation can be made it is necessary to make assumptions about the economic and demographic factors which influence future cash flows from a pension scheme - the actuarial assumptions.

1. Economic assumption
    - d dividend yield
    - g dividend growth
    - i investment return
    - e earnings inflation
    - p price inflation
    - j state benefit inflation.
2. The demographic and other statistical assumptions
    - Decrement rates from which the active member's serviced table is constructed, which will include;
        - r_x probability of retiring on grounds other than ill health at age x last birthday
        - i_x probability of retiring on grounds ill-health at age x last birthday
        - d_x probability of deaths in service at age x last birthday
        - w_x probability of withdrawal at age x last birthday
    - Mortality rates of member's and spouses' after retirement
    - Other statistical elements
        - Family statistics
            - Proportions married amongst members dying in service, in deferment and in retirement

            - Proportions giving rise to a dependant's pension (includes spouses, common-law spouses and possible other financially dependent relatives)
            - Age difference between husband and wife (whether or not common-law spouses)
            - Rates of mortality of married male and female pensioners
            - Rates of mortality of widow(er)s of members who die in service or retirement
            - Rates of remarriage of widow(er)s
            - Number of children
        - The promotional salary scale before allowing for earnings inflation
        - The use of standard tables
        - New entrants.
3. Effect of choice of assumptions on actual cost.

3.5 Valuation of the Assets

The value of an investment for the purposes of an actuarial valuation lies in the stream of future income which it will bring. It is a simple matter to discount the future payments, at the valuation rate of interest, to arrive at their "present value" and thus an appropriate value for actuarial purposes. The merit of the method lies in the fact that assets are then valued in a manner consistent with the actuarial liabilities. In both cases a "discounted cash flow" procedure is used for future benefit payments and contributions and for future investment proceeds.

To value individual investment now would be a complex process. For example if we consider the future proceedings from a property owned by a pension scheme and leased to a tenant, an allowance needs to be made for rent reviews, expenses, periods where the property is vacant and possible resale expenses after the lease expires. Many investments require similar subjective assessments of the future outcomes. The full theoretical model of the expected future cash flows from the whole range of investments held by a typical pension scheme is now unmanageably complex, and so other methods are used.

The broad discounted income value method takes account of the actual scheme assets which are dividend into homogenous groups. A typical division by market value may be that adopted by the scheme's investment monitoring service as follows;

- Equities
- Overseas equities
- Conventional gilts
- Other fixed interest
- Index-linked gilts
- Property and property units
- Cash.

The following assumptions are required for the valuation of the scheme's assets:

- i: the long term average annual rate of return on new investment
- g: the average annual rate of growth in dividends for equities, and
- p: the long term annual rate of inflation.

These assumptions reflect the broad view of future economic conditions and are used in the calculation of discounted income values.

3.6 Analysis of Surplus and Deficiency

The actuary needs to be satisfied that the figures in a valuation are correct and that they give a sensible answer. The analysis of the results provides a semi-independent check on the new valuation results by reconciling this valuation's figures with the last valuation's results. It also looks at the reasons for any unexpected results and how the actual experience of the scheme has compared with the assumptions made.

The differences between the actual and expected results provide some measure of the suitability of the assumptions chosen for the valuation (and the financial stability of the funding method chosen). The analysis can also alert the actuary of potential future problems for the scheme. For example, there may be concern about the level of early retirement and the cost of such benefits.

Conventional wisdom is that the new valuation basis should be used for carrying out the analysis. Therefore, the previous valuation calculations will be reworked on the new assumptions. The reasons in favour of this approach are

- it provides a better check on the new valuation results,
- any changes in assumptions are likely to have taken into account the experience of the inter-valuation period, so actual and expected results should be closer together. There should therefore be fewer financially significant factors contributing to the surplus and deficiency, which may reduce the amount of work and level of accuracy required for the analysis,
- where the new assumptions differ from the experience, the financial effect of these differences will be easier to assets. This may affect the expected course of the required contribution rate, any short term advice and give a better idea of the level of prudence of the basis,
- the results of an analysis on the new basis may be useful in carrying out projections of what the experience of the scheme might be over the period to the next valuation.

In many cases, it may not be practical to rework the previous valuation on the new assumptions. In such cases, the analysis will be done on the old basis, by reworking this valuation on the old assumptions. It is assumed that the previous valuation results have been recalculated on the new assumptions. The valuation results should be set out in the form of a draft balance sheet showing the value of accrued benefits, the value of assets held in the fund and also the contribution rate needed to support future benefit accrual.

There is no single correct way in which the analysis should be done. In particular, because of the inter-relationship between various assumptions, there are second order terms, for example the difference between actual and expected investment return on mortality profit or loss. This can be counted either in the mortality term or in the investment term of the analysis. The choice on such matter is a matter of personal taste, convenience and practically as well as the order in which the various factors are investigated.

At all times when carrying out an analysis of surplus it is important to consider the “overlap” between different factors care should be taken not to double-count any aspects or to leave things out accidentally.

Although the analysis of surplus concentrates on the past service aspects of the balance sheet, some thought should also be given to any changes in the cost of future benefit accrual. In particular, the stability of pension costs under the chosen valuation method may depend on the stability of the age/sex structure of the membership. The validity of such assumptions for this particular scheme should be considered.

This preliminary calculation of the future service costs will, particularly if done using a Projected Unit Method, also give useful information about the cost of benefits which have accrued during the inter-valuation period. This information may be needed for analyzing the position at the end of the inter-valuation period in respect of accrued benefits at that date. The main part of the analysis looks at the inter-valuation period and the financial effect of events in that period and hence reconciling the figures at the beginning of the period with those at the end.

If the result of the valuation are very much as expected (after doing some rough calculations along the lines described) or are being done with some specific objective rather than as a basis for advice on a variety of matters, it may not be appropriate to delve much more deeply into the analysis of surplus. The analysis can be a costly and time-consuming piece of work and the figures cannot be reconciled exactly. The benefit of continuing and refining work on the analysis should be balanced against the costs involved.

At its most simplistic level, the previous valuation (on this time's basis) gave the net result of a surplus or a deficiency at the beginning of the period. Since then, the fund has been affected by some or all of the following factors:

- investment returns on money invested
- payment of contributions into the scheme by members and the employer
- transfer payments into and out of the scheme
- payment of other benefits, including lump sums and pensions on retirement and payment or non-payment of benefits due to deaths
- expenses
- salary increases on benefits earned at the previous valuation
- additional years of pension accrual
- benefit improvements, including pension increases
- members leaving service earlier than expected, either with deferred or early retirement benefits
- new members joining the scheme.

The assumptions to which the valuation results are most financially sensitive are usually the economic assumptions. It is therefore usual that the effect of differences between actual and expected investment return, salary increases, and pension and deferred pension increases are investigated first. These elements of the analysis often account for the largest individual items in an analysis and may render consideration of some of the minor items unnecessary.

Any difference between actual and expected investment return affects not only all the assets held at the beginning of the period (including any surplus), but also any net income (contributions minus outgo) during the inter-valuation period. The accounts will provide the "net new money" figure for this purpose and it may be appropriate to convert this figure to an assessed value at the assumed time of payment into the fund. However, for a mature fund, the effect of the notional initial value of net new money will not be significant. The expected value of the assets at the end of a year will be calculated as

$$\text{Initial calculated value} \times (1+i) + \text{net new money} \times (1+i)^{0.5} \qquad \text{(A)}$$

where i is the valuation rate of interest , t is 0.5 (6 months).

This formula should be adjusted appropriately for an inter-valuation period of longer than one year. It also assumes that income and outgo can reasonably be assumed to average out over the year. If there are any special circumstances (for example large capital injections or uneven payment of contributions or payments out such as large bulk transfer) the formula should be amended to improve the estimate. The result of this calculation is compared with the actual final calculated value (B). The investment profit is (B)-(A). This may be negative.

If the experience matched the assumptions during the inter-valuation period and if no remedial action was taken to reduce any surplus the last valuation, it would contribute

$$\text{Last time's surplus} \times (1+i)$$

towards this time surplus.

If there have been any benefits improvements in the inter-valuation period, the capital cost should be calculated, including those members who have benefited from the changes but left or retired during the inter-valuation period. The actual contribution rate paid may have varied considerably from the theoretical future service rate, particularly if the recommended rate had an adjustment to the "normal" cost as a result of there being a surplus or deficiency at the last valuation. The component of surplus from this source is calculated as

$$(\text{Actual contributions paid} - \text{expected contributions}) \times (1+i)^{0.5}$$

where expected contributions are estimated on the basis of actual salaries for the year but at the theoretical "future service cost" for benefits accruing during the inter-valuation period.

The formula will be adjusted as appropriate if regular payment of contributions throughout the year is not a reasonable assumption to make for the particular scheme. If there was a large surplus at the previous valuation, the contribution rate actually paid will usually be less than the amount needed to support future service benefit accrual. In such a case this formula will give a negative term, which is as expected if action was being taken to reduce the surplus.

If pensionable salaries have not increased exactly as expected, the salary related liabilities at the end of the period will be larger or smaller than expected. Also the contributions paid into the scheme and some of the benefits paid out will differ from the expected amounts.

Although the pensioner liabilities will technically consist of two groups:

- those who had retired at the last valuation, and
- those who retired during the year.

At the end of the period, the pensioner liabilities, (V*pen*), are based on actual pension increases granted. If the expected increases were higher than the actual increase, this will have contributed to the surplus, both in respect of the reserves needed at the end of the year being smaller and in respect of smaller payments during the year.

The effect on the surplus of the experience of these factors not being in line with the assumptions depends on how the value of the actual benefits paid out compares with the reserve held. Where the actual benefits cost less to provide than to reserve, for example withdrawals, a higher than expected number of withdrawals will lead to surplus being generated. The general term for this part of the analysis is therefore

$$(\text{actual decrement} - \text{expected decrement}) \times (\text{reserve held} - \text{cost of benefit given}).$$

Theoretically, this approach will be used at each individual age and summed, as the release of reserve will be age dependent. If a Projected Unit contribution rate is used for the assessment of the contribution paid, this will have made some implicit allowance for new entrants because of the underlying assumption of new entrants keeping the age/sex/salary distribution of the members stable. The effect of there being more or less than the expected number of new entrants can be seen by comparing the future service benefit accrual costs. If this approach is used, it will normally be the case that new entrants have had little financial effects. This may nevertheless be quantified by estimating the amount of additional cost/saving in cost over the period in terms of the difference in contribution income over this period, and rolling this up with investment return to the end of the period.

It has become increasingly common for the expenses of administering the scheme to be met out of the fund, rather than as a separate item of cost for the employer. An analysis of the actual amounts, compared with the expected amount should be carried out where needed. This will also check the reasoning of, and needed for, an allowance for expenses in the current valuation.

There may be other items of cost etc. which are scheme specific which have contributed to the surplus/deficiency. The list of items discussed above is not exhaustive and you should use your common sense and knowledge of the scheme in this respect.

3.7 Summary of Analysis

When carrying out the calculations, the actual experience should be compared with that assumed. Other items in the analysis should be the expected experience until the items has been analyzed, and the actual experience after it has been analyzed.

A summary should be made of actual versus expected results and the differences between the two. The net result of the calculations should be consistent with the preliminary valuation result. If it is not, the different elements of the analysis may give some indication as to which part of the membership data or benefit calculation should be reviewed.

In the light of the results of this analysis, one or more assumptions may be altered to give a better fit with experience, to provide a greater margin, or to allow for a contribution of any trend that has emerged.

CHAPTER FOUR
ACTUARIAL MATHEMATICS

This chapter is presenting the mathematical foundations for actuarial calculations in pension. In this chapter we will introduce a stochastic approach to complement the deterministic approach but also integrated annuity, life table, life insurance, and survival functions.

4.1 Interest Rate

The rate of interest assumed is a very important part of the valuation process because it is used to discount the future anticipated cash flows to produce the present values of the future benefit payments and the contribution income. Interest rate scenarios used in actuarial analysis are of two distinct types. Deterministic scenarios are determined a priori and are often used to "stress" a product's profitability in the event future interest rates are unfavorable. Scenarios of this type are sometimes prescribed by regulatory agencies to provide a test of sensitivity to interest rates that is common across products and companies.

Stochastic scenarios are scenarios that are created using an interest rate simulator of some sort. Interest rates, or periodic changes in rates, are assumed to follow a predefined probability distribution. A random number generator is then used to simulate annual rates from this distribution. These types of scenarios can be used either to assign values to interest sensitive products and cash flows or to assign probabilities to different levels of product probability. The development and study of interest rate generators that produce realistic interest rate scenarios and/or accurately reproduce interest sensitive values for financial products is an active research area in actuarial science. Regulators are beginning to require scenario testing of this sort as well.

4.1.1 Deterministic Interest Rate

Interest is in general may be described as money paid for the use of money which has been borrowed for a certain length of time. There are simple and compound interest. When only the original capital or indebtedness bears interest, it's called simple interest. When the interest payments on the capital as well as the original capital itself bear interest, it's called compound interest. In this section, we don't show simple interest. Because simple interest is ordinarily used only for short intervals of time.

Interest theory usually begins with the concept of the accumulation function, denoted $a(t)$, which gives the accumulated value, at time $t \geq 0$, of a unit of money invested at time $t=0$ (Kellison S.G., 1991). Under compound interest, the accumulation function has the exponential form,

$$a(t) = (1+i)^t \text{ for } t \geq 0, \tag{4.1}$$

where i is a parameter of the function.

In general, the original principal invested will not be one unit but will be some amount $k > 0$. It is defined as *amount function* $A(t)$ which gives the accumulated value at time $t > 0$ of an original investment of k. Then $A(t) = k.a(t)$ and $A(0) = k$.

The amount of interest earned during the n^{th} period from the date of investment denoted by I_n, then

$$I_n = A(n) - A(n-1) \text{ for integral } n \geq 1. \tag{4.2}$$

It should be noted that I_n involves the effect of interest over an interval of time, whereas $A(n)$ is an amount at a specific point in time.

The effective rate of interest is defined as the amount of money that one unit invested at the beginning of a period will earn during the period, where interest is paid at the end of the period. For the n^{th} time interval, which runs from *t=n-1* to *t=n*, the effective rate of interest is defined as

$$i_n = \frac{a(n)-a(n-1)}{a(n-1)} = \frac{(1+i)^n-(1+i)^{n-1}}{(1+i)^{n-1}} = i\,. \tag{4.3}$$

The compound interest discount factor which is the present value for one unit over one time interval is defined to be

$$v = (1+i)^{-1}\,. \tag{4.4}$$

It is important in many cases to be able to measure the intensity with which interest is operating at each moment of time, i.e., over infinitesimally small intervals of time. This measure of interest at individual moments of time is called the force of interest. In terms of calculus, the force of interest is the derivative of the accumulated value with respect to time expressed as a percentage of the accumulated value at time *t*

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{1}{a(t)}\frac{da(t)}{dt} = \frac{d}{dt}\ln a(t) \tag{4.5}$$

where $a'(t)$ is the first derivative of *a(t)* with respect to *t*. Under compound interest, becomes

$$\delta_t = \ln(1+i) = \delta \tag{4.6}$$

a constant function of time. Alternatively we can write

$$(1+i) = e^{\delta} \text{ and } v = e^{-\delta}\,. \tag{4.7}$$

It is also possible to define a force of discount analogous to formula (4.5). For this purpose, we use the discount function $a^{-1}(t)$ instead of the accumulation function *a(t).* The definition of the force of discount at time *t*, denoted by $\delta_t^{'}$, is given by

$$\delta_t' = -\frac{\frac{d}{dt}a^{-1}(t)}{a^{-1}(t)}. \tag{4.8}$$

The first type of variation considered is a continuously varying force of interest. The basic formula for use in problems involving a varying force of interest is

$$a(t) = e^{\int_0^t \delta_r \, dr}. \tag{4.9}$$

The second type of variation considered involves changes in the effective rate of interest over a period of time. This type of variation is probably the most commonly one encountered in practice. As before, let i_n denote the effective rate of interest during the n^{th} period from the date of investment. Then for integral $t \geq 1$, we have

$$a(t) = (1+i_1)(1+i_2)...(1+i_t) = \prod_{k=1}^{t}(1+i_k) \tag{4.10}$$

$$a^{-1}(t) = (1+i_1)^{-1}(1+i_2)^{-1}...(1+i_t)^{-1} = \prod_{k=1}^{t}(1+i_k)^{-1} = \prod_{k=1}^{t} v_k. \tag{4.11}$$

4.1.2 Stochastic Interest Rate

When calculating the actuarial present value for assets and liabilities in actuarial valuation, we will use mortality rates, inflation rate and interest rate. But we will assume that a single interest rate was valid throughout the life of the model, however long that might be. It can be risky to assume that interest rates will remain constant at today's rates. Indeed some insurance companies around the world have experienced bankruptcy as a result of pricing products at interest rates that proved to be too optimistic.

In this section it is considered the rate of interest directly as a random variable and then provides a basic introduction to several models with a stochastic basis which have important applications in practice.

Let the rate of interest during the t^{th} period, i.e., from time t-1 to t, be denoted by i_t for t = 1,2,...,n. The accumulated value of an investment of 1 at the end of n periods is

$$a(n) = (1+i_1)(1+i_2)...(1+i_n) = \prod_{t=1}^{n}(1+i_t)\,. \tag{4.12}$$

Now assume that the i_t's are independently and identically distributed with $E(i_t) = i$ and $Var(i_t) = s^2$, then the mean and variance of the accumulated value are given by

$$\begin{aligned} E[a(n)] &= E\left[\prod_{t=1}^{n}(1+i_t)\right] \\ &= \prod_{t=1}^{n}E(1+i_t) \\ &= (1+i)^n \end{aligned} \tag{4.13}$$

$$\begin{aligned} Var[a(n)] &= E\left[a^2(n)\right] - \{E[a(n)]\}^2 \\ &= E\left[a^2(n)\right] - (1+i)^{2n}\,. \end{aligned} \tag{4.14}$$

Assume that the i_t's have variance s^2. It can compute the second moment about the origin of $a(n)$

$$\begin{aligned} E\left[a^2(n)\right] &= E\left[\prod_{t=1}^{n}(1+i_t)^2\right] \\ &= \prod_{t=1}^{n}E(1+i_t)^2 \\ &= \prod_{t=1}^{n}E(1+2i_t+i_t^2) \\ &= (1+2i+i^2+s^2)^n\,. \end{aligned} \tag{4.15}$$

Thus, the variance of the accumulated value is given by

$$Var[a(n)] = (1 + 2i + i^2 + s^2)^n - (1 + i)^{2n} \tag{4.16a}$$

which can be written as

$$Var\left[a(n)\right]=(1+j)^n-(1+i)^{2n} \tag{4.16b}$$

where $j = 2\text{i} + i^2 + s^2$.

Parallel results to those above for accumulated values can also be developed for present values. However, we must be careful in the choice of interest rates to use, since in general

$$E\left[\frac{1}{1+i_t}\right] \neq \frac{1}{E\left[1+i_t\right]}$$

Thus, when working with present values it is defined i by

$$E[(1+i_t)^{-1}] = (1+i)^{-1}.$$

It must be stressed that this value of i is different than the value of i used above for accumulated values for which $E[i] = i$.

First result is to develop a formula for the mean of the present value of a single payment analogous to formula (4.13).

$$E[a^{-1}(n)] = (1+i_t)^{-n} \tag{4.17}$$

For the variance of the present value,

$$\begin{aligned} Var\left[a^{-1}(n)\right] &= E\left[a^{-2}(n)\right] - \left\{E\left[a^{-1}(n)\right]\right\}^2 \\ &= (1+k)^{-n} - (1+i)^{-2n} \end{aligned} \tag{4.18}$$

where $(1+k)^{-1} = E[(1+i_t)^{-2}]$.

Unfortunately, this is as far as it carried the development without knowing how i_t is distributed. The approach used above for finding the second moment for accumulated values will not work for present values. Evaluation of formula (4.18) will require computing the second moment based on a particular probability density function.

There is one important special case in which a usable result can be obtained analytically. Suppose that the random variable $\log_e(1+i_t)$ follows a normal distribution with mean μ and variance σ^2. Then the random variable $(1+i_t)$; follows a lognormal distribution with parameters μ and σ^2. The mean and variance of the lognormal distribution with parameters μ and σ^2 are given by

$$mean = e^{\mu+\sigma^2} \text{ and } variance = e^{2\mu+\sigma^2}(e^{\sigma^2}-1).$$

Now from formula (4.1),

$$\log_e a(n) = \sum_{t=1}^{n} \log_e(1+i_t). \tag{4.19}$$

The right-hand side is the sum of n independent normal random variables each with mean μ and variance σ^2. Thus, $\log_e a(n)$ is normal with mean $n\mu$ and variance $n\sigma^2$, so that *a(n)* is lognormal with parameters $n\mu$ and $n\sigma^2$. In this case we can directly make probability statements about the possible outcomes of *a(n)* and $a(n)^{-1}$, without resorting to simulation.

The use of the lognormal distribution is motivated by more than its convenience in evaluating the formulas developed in this section without the use of simulation. Empirical studies have shown that it provides a reasonably good model for variations in the rate of interest.

It is assumed that the rates of interest i_t in each successive period are independent. Historical experience suggests that it is more likely for rate to stay high or low for several successive periods than for it is for rates *t* bounce around randomly above and below some average rate. This seems even more plausible when it is considered the fact that the level of interest rates is tied to economic conditions and governmental policy. There are many different models which could be constructed to reflect dependence.

The primary models are *moving average* (MA) models, *autoregressive* (AR) models, and mixtures of the two. Experience has shown the AR models to be more successful in modeling interest rate movements than the MA models. This simple illustration actually involves an application of an *autoregressive process of order one* AR(1). Such a process makes the rate in one period dependent upon the rate in the prior period. A more sophisticated time series procedure is the *autoregressive process of order two* - AR(2).

Panjer and Bellhouse (1980) and Belhouse and Panjer (1981) develop a general theory including continuous and discrete models. The theory is further worked out for unconditional and conditional autoregressive processes of order one and two.

The goal of the Dhaene's (1989) study is to state a methodology for computing in an efficient manner present value functions when the force of interest evolves according to an autoregressive integrated moving average process of order (p,d,q). *Autoregressive Integrated Moving Average Processes* – ARIMA (p,d,q) in Dhaene (1989), assume that the stochastic model governing future forces of interest δ_t (t=1,2,…) belongs to the class of ARIMA (p,d,q) processes.

4.2 Annuity Functions

Finance theory is the field that deals with investment making decisions and the concept of the time value of money. The time value of money is based on the premise that an investor prefers to receive a payment of a fixed amount of money today, rather than an equal amount in the future, all else being equal. In other words, the present value of a certain amount a of money is greater than the present value of the right to receive the same amount of money at time t in the future. This is because the amount a could be deposited in an interest-bearing bank account (or otherwise invested) from now to time t and yield interest. The term *annuity* is used in finance theory to refer to any terminating stream of fixed payments over a specified period of time. This usage is most commonly seen in academic discussions of finance, usually in connection with the valuation of the stream of payments, taking into account time value of money are very important points for actuarial analysis.

An annuity for which the payments begin and end at fixed dates is called an annuity certain, evaluated at a constant rate of compound interest per payment period. The term *present value* implies the value *now*, at the present time, but according to the definition, it is the value at any earlier date of a sum of money due at a later time. The total amount received after a period of time is called the *accumulated value.*

A unit immediate annuity is one for which the unit payments are made at the ends of the respective payment periods. The present value of the annuity for a given interest rate *i*, denoted by $a_{\overline{n}|}$, is measured at time 0 and is given by

$$a_{\overline{n}|} = v + v^2 + \ldots + v^n = \frac{1-v^n}{i}. \tag{4.20}$$

The accumulated value of the annuity, denoted by $s_{\overline{n}|}$, is measured at time *n* and is given by

$$s_{\overline{n}|} = (1+i)^{n-1} + (1+i)^{n-2} + \ldots. + (1+i) + 1 = \frac{(1+i)^n - 1}{i}. \tag{4.21}$$

From equation (4.20) and (4.21) together, it is obtained

$$a_{\overline{n}|} = v^n . s_{\overline{n}|} \tag{4.22a}$$

$$s_{\overline{n}|} = (1+i)^n . a_{\overline{n}|}. \tag{4.22b}$$

In the limiting case, as $n \to \infty$, the notation of the unit immediate perpetuity with the present value is given by

$$a_{\overline{\infty}|} = v + v^2 + \ldots = \frac{1}{i}. \tag{4.23}$$

Now turning to the present value of an n period annuity-immediate with stochastic interest rate

$$a_{\overline{n|}} = (1+i_1)^{-1} + (1+i_1)^{-1}(1+i_2)^{-1} + ...(1+i_1)^{-1}(1+i_2)^{-1}...(1+i_n)^{-1}$$
$$= \sum_{t=1}^{n}\prod_{s=1}^{t}(1+i_s)^{-1}.$$

The mean of $a_{\overline{n|}}$ is the anticipated result,

$$E\left[a_{\overline{n|}}\right] = a_{\overline{n|}i}\,. \tag{4.24}$$

Finally, it is considered that the variance of $a_{\overline{n|}}$. Define m_1^a and m_2^a to be the first and second moments of $(1 + i_t)^{-1}$ about the origin, respectively, i.e.,

$$m_1^a = E\left[(1+i_t)^{-1}\right] = (1+i)^{-1} \tag{4.25}$$

$$m_2^a = E\left[(1+i_t)^{-2}\right] = (1+k)^{-1} \tag{4.26}$$

$$Var\left[a_{\overline{n|}}\right] = \frac{m_2^a + m_1^a}{m_2^a - m_1^a}a_{\overline{n|}k} - \frac{2m_2^a}{m_2^a - m_1^a}a_{\overline{n|}i} - (a_{\overline{n|}i})^2. \tag{4.27}$$

A unit annuity due is one for which the unit payments are made at the beginnings of the respective payment periods. The present value of the annuity, denoted by $\ddot{a}_{\overline{n|}}$, is measured at time 0 and is given by

$$\ddot{a}_{\overline{n|}} = 1 + v + v^2 + ... + v^{n-1} = \frac{1-v^n}{d}. \tag{4.28}$$

The accumulated value of the annuity, denoted by $\ddot{s}_{\overline{n|}}$, is measured at time n and is given by

$$\ddot{s}_{\overline{n}|} = (1+i)^n + (1+i)^{n-1} + ... + (1+i) = \frac{(1+i)^n - 1}{d}. \tag{4.29}$$

From equation (4.28) and (4.29) together, it is obtained

$$\ddot{a}_{\overline{n}|} = v^n.\ddot{s}_{\overline{n}|} \tag{4.30a}$$

$$\ddot{s}_{\overline{n}|} = (1+i)^n.\ddot{a}_{\overline{n}|}. \tag{4.30b}$$

In the limiting case, as $n \to \infty$, the notation of the unit immediate perpetuity with the present value is given by

$$\ddot{a}_{\overline{\infty}|} = 1 + v + v^2 + ... = \frac{1}{d}. \tag{4.31}$$

There are relations between immediate annuity and annuity due as follows,

$$\ddot{a}_{\overline{n}|} = (1+i).a_{\overline{n}|} \quad , \tag{4.32}$$

$$a_{\overline{n}|} = v.\ddot{a}_{\overline{n}|}, \tag{4.33}$$

$$\ddot{s}_{\overline{n}|} = (1+i).s_{\overline{n}|}, \tag{4.34}$$

$$s_{\overline{n}|} = v.\ddot{s}_{\overline{n}|}. \tag{4.35}$$

It extended the above analysis involving a single payment to a level annuity with stochastic interest rate. Consider an n period annuity-due. The accumulated value of this annuity is

$$\begin{aligned}\ddot{s}_{\overline{n}|} &= (1+i_n) + (1+i_n)(1+i_{n-1}) + ... + (1+i_n)(1+i_{n-1})...(1+i_1) \\ &= \sum_{t=1}^{n} \prod_{s=1}^{t} (1+i_{n-s+1}).\end{aligned}$$

The mean of $\ddot{s}_{\overline{n}|}$ can be obtained from previous equation as

$$E\left[\ddot{s}_{\overline{n}|}\right] = E\left[\sum_{t=1}^{n}\prod_{s=1}^{t}(1+i_{n-s+1})\right]$$
$$= \sum_{t=1}^{n}\prod_{s=1}^{t}E(1+i_{n-s+1})$$
$$= \sum_{t=1}^{n}(1+i)^{t}$$
$$= \ddot{s}_{\overline{n}|i}\,.$$

Define m_1^s and m_2^s to be the first and second moments of 1 + i_t about the origin, respectively, i.e.,

$$m_1^s = E[1+i_t] = 1+i \tag{4.36}$$

and

$$m_2^s = E[(1+i_t)^2] = 1+j \tag{4.37}$$

where j is defined in formula (4.16a). It is sought to find the variance of $\ddot{s}_{\overline{n}|}$.

$$Var\left[\ddot{s}_{\overline{n}|}\right] = \frac{m_2^s + m_1^s}{m_2^s - m_1^s}\ddot{s}_{\overline{n}|j} - \frac{2m_2^s}{m_2^s - m_1^s}\ddot{s}_{\overline{n}|i} - (\ddot{s}_{\overline{n}|i})^2\,. \tag{4.38}$$

In all the preceding types of annuities, the periodic payments have been equal. Situations sometimes occur in which the payments are unequal.

A formula is derived for the amount of an increasing annuity with the n successive payments 1,2,3,…,n-1,n due at the ends of the next n interest periods, when interest is at the rate i per period. The present value of this unit increasing annuity, denoted $(Ia)_{\overline{n}|}$, is measured at time 0 and is given by

$$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \ldots + nv^n = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}\,. \tag{4.39}$$

The accumulated value of the annuity, denoted by $(Is)_{\overline{n}|}$, is measured at time n and is given by

$$(Is)_{\overline{n}|} = (1+i)^{n-1} + 2(1+i)^{n-2} + ... + (n-1)(1+i) + n = \frac{\ddot{s}_{\overline{n}|} - n}{i}. \tag{4.40}$$

From equations (4.39) and (4.40) together it is clear that

$$(Ia)_{\overline{n}|} = v^n.(Is)_{\overline{n}|} \tag{4.41a}$$

and

$$(Is)_{\overline{n}|} = (1+i)^n.(Ia)_{\overline{n}|}. \tag{4.41b}$$

In the limiting case, as $n \to \infty$, the notation of the unit increasing immediate perpetuity, with the present value is given by

$$(Ia)_{\overline{\infty}|} = \frac{1}{id}. \tag{4.42}$$

Consider a decreasing annuity whose payments are $n, n-1, \ldots, 3, 2, 1$ due at the end of n successive interest periods, with interest rate i per period. The present value of this unit decreasing annuity, denoted $(Da)_{\overline{n}|}$, is measured at time 0 and is given by

$$(Da)_{\overline{n}|} = nv + (n-1)v^2 + ... + 2v^{n-1} + v^n = \frac{n - a_{\overline{n}|}}{i} \tag{4.43}$$

The accumulated value of the annuity, denoted by $(Ds)_{\overline{n}|}$, is measured at time n and is given by

$$(Ds)_{\overline{n}|} = n(1+i)^{n-1} + (n-1)(1+i)^{n-2} + ... + 2(1+i) + 1 = \frac{n(1+i)^{n-1} - \ddot{s}_{\overline{n}|}}{i}. \tag{4.44}$$

From equations (4.43) and (4.44) together it is clear that

$$(Da)_{\overline{n}|} = v^n.(Ds)_{\overline{n}|} \text{ and } (Ds)_{\overline{n}|} = (1+i)^n.(Da)_{\overline{n}|} \tag{4.45}$$

4.3 Survival Models

A survival model is a probabilistic model of a random variable that represents the time until the occurrence of an unpredictable event. For example, the life expectancy of a newborn baby, the future working life time of a machine until it fails.

The theory can be applied in a wide range of situations, in which the concept of "survival" may not be immediately obvious, for example:

- the time until a claim is made on an automobile insurance policy,
- the time until a patient in a coma recovers from the coma, given that he recovers,
- the time until a worker leaves employment.

Probabilities associated with these models play a central role in actuarial calculations such as pricing insurance contracts.

The most basic contingent payment model, in which a specified amount is paid if and only if a particular event occurs, is considered. Suppose that an amount P is to be paid in n years if a random event E occurs. Otherwise, if the complementary event occurs, then nothing is to be paid. At an effective annual rate of interest i, the random present value of the payment is

$$Z = \begin{cases} P.v^n & \text{if } E \text{ occurs} \\ 0 & \text{if } E' \text{ occurs} \end{cases}$$

where $v = (1+i)^{-1}$ is the one-year present value discount factor (Gauger, M.A., 2006).

The random present value of the payment, Z, is a discrete random variable. Its expected value is known as the actuarial present value of the payment, which incorporates the amount of the payment, the discount factor associated with the timing of the payment, and the probability of the payment being made

$$E[Z] = P.v^n.\Pr(E) + 0.\Pr(E')$$
$$= \underbrace{P}_{\text{Amount}}.\underbrace{v^n}_{\text{Discount}}.\underbrace{\Pr(E)}_{\text{Probability}}$$

There are three principal variables, all of which are measured in years;

- The random lifetime (i.e., time until death) of a newborn life is denoted X,
- The random future lifetime at age x, given that a newborn has survival to age x, is denoted $T(x)$, $T(x) = X - x | X > x$,
- The curtate future lifetime at age x, given that a newborn has survived to age x, is the complete number of years of future lifetime at age x and is denoted $K(x)$, $K(x) = [T(x)]$ (greatest integer).

The variables X and T are assumed to be continuous random variables, whereas K is obviously discrete. Notice that T is a function of X, and K is a function of T. So, the distributions of these three variables are closely related.

4.3.1 The Life Table – Discrete Survival Model

Life table is a discrete survival model commonly used in insurance applications. There are some notations.

l_x is defined as the number of lives expected to survive to age x from a group of l_0 newborn lives. A life table displays in a table format the values of l_x at ages x equal to $0,1,2,\ldots,\omega$, where ω is the first whole number at age which there are no remaining lives in the group.

d_x represents the number of lives among l_0 newborn lives that die in the age range $[x, x+1)$. It is computed as

$$d_x = l_x - l_{x+1} \tag{4.46}$$

${}_n p_x$ is denoted the probability that a life currently age *x* will survive *n* years. It is computed as

$$ {}_n p_x = \frac{l_{x+n}}{l_x} \tag{4.47} $$

It is standard convention to omit the *n* subscript when *n*=1, so the probability that a life currently age *x* will survive 1 year is

$$ p_x = \frac{l_{x+1}}{l_x} \tag{4.48} $$

${}_n q_x$ is described the probability that a life currently age *x* will die within *n* years. It is computed as

$$ {}_n q_x = 1 - {}_n p_x = 1 - \frac{l_{x+n}}{l_x} = \frac{l_x - l_{x+n}}{l_x} \tag{4.49} $$

It is standard convention to omit the *n* subscript when *n*=1, so the probability that a life currently age *x* will die 1 year is

$$ q_x = \frac{l_x - l_{x+1}}{l_x} \tag{4.50} $$

The probability that a life currently age *x* will survive for *m* years and die within the following *n* years is denoted ${}_{m|n} q_x$,

$$ {}_{m|n} q_x = \frac{l_{x+m} - l_{x+m+n}}{l_x} \tag{4.51} $$

Intuitively, ${}_{m|n} q_x$ is the probability that a life age *x* survives for *m* years, multiplied by the probability that a life age *m*+*n* dies within *n* years

$$ {}_{m|n} q_x = {}_m p_x \times {}_n q_{x+m} \tag{4.52} $$

It is standard convention to omit the n subscript when n=1, so the probability that a life currently age x will survive for m years and then die within 1 year is

$$ {}_{m|}q_x = \frac{l_{x+m} - l_{x+m+1}}{l_x} = \frac{d_{x+m}}{l_x} \tag{4.53} $$

4.3.2 Continuous Survival Models

Five different mathematical functions that can all be employed to specify the distribution of X are focused on the relations, the random lifetime (i.e., age at death) of a newborn life:

- the cumulative distribution function of X,
- the probability density function of X,
- the survival function,
- the life table function,
- the force of mortality.

The random lifetime (i.e., age at death) of a newborn life, is assumed to be continuous random variable.

In actuarial notation the probability that a newborn dies at or before age x is denoted by ${}_xq_0$. This is a new notation for the already familiar cumulative distribution function (CDF) $F_X(x)$, since

$$ {}_xq_0 = \Pr(X \le x) = F_X(x). \tag{4.54} $$

$F_X(x)$ is continuous and non-decreasing with $F_X(0) = 0$ and $F_X(w) = 1$ where w is the first age at which death is certain to have occurred for a newborn life.

The probability density function (PDF) is

$$ f_X(x) = F'_X(x) \tag{4.55} $$

wherever the derivative exists. The PDF $f_X(x)$ is not a probability itself. The probability that a newborn life dies between ages a and b is

$$P(a \le x \le b) = \int_a^b f_X(x)dx = F(b) - F(a) \tag{4.56}$$

In actuarial mathematics it is common to describe a survival model by giving the survival function rather that the density function or distribution function. The survival function denoted $s_X(x)$ and is defined as

$${}_x p_0 = s_X(x) = \Pr(X > x) \tag{4.57}$$

The survival function gives the probability that a newborn dies after age x. Properties of survival function are as follows;

- $s_X(x)$ is continuous and non-increasing with $s_X(0)=1$ and $s_X(\omega)=0$,
- $s_X(x) = 1 - F_X(x)$,
- $P(a \le x \le b) = \int_a^b f_X(x)dx = s_X(a) - s_X(b)$,
- $f_X(x) = -s'_X(x)$.

Deaths in an age group seem to be occurring at an increasing rate. Force of mortality (Failure rate - hazard rate) is denoted by μ_x. It is instantaneous measure of at age x, and it can be defined in several equivalent ways

$$\mu_x = \frac{f_X(x)}{s_X(x)} = -\frac{s_X'(x)}{s_X(x)} = -[\ln s_x(x)]' \tag{4.58}$$

The main properties of force of mortality are as follows;

- $s_X(x) = \exp\left(-\int_0^x \mu_y dy\right)$,
- μ_x is non-negative and piece-wise continuous where defined,
- $\int_0^\omega \mu_y dy = \infty$ in order that $s_X(\omega)=0$.

4.3.3 The Continuous Future Lifetime After Age x

Let the continuous random variable *X* again denote the random lifetime of a newborn. Now suppose that a newborn has survived to age *x*, that is, *X*>*x*. The future time lived after age *x* is *X-x*.

The conditional distribution of the time lived after age *x*, given survival to age *x*, is complete future lifetime; *T(x)=X-x*.

The continuous random variable *T(x)* is a survival model defined on the interval [0, ω-*x*]. It should be clear that the distribution of *T(x)* is closely related to the distribution of *X*. The relation between the distributions of *T(x)* and *X* is to calculate the survival function for *T(x)*, $s_{T(x)}(t) = s_T(t) = \Pr(T(x) > t)$. The probability of this event is simply ${}_t p_x$. So, we have

$$s_{T(x)}(t) = \Pr(T(x) > t) = {}_t p_x = \frac{l_{x+t}}{l_x} = \frac{s_X(x+t)}{s_X(x)} \text{ since } l_x = l_0 s_X(x).$$

Key results concerning the relation of the distribution of *X* and *T(x)* are as follows;

- $F_t(t) = {}_t q_x = 1 - s_T(t) = \dfrac{F_X(x+t) - F_X(x)}{s_X(x)}$,
- $f_t(t) = {}_t p_x \mu_{x+t} = \dfrac{f_X(x+t)}{s_X(x)}$.

4.3.4 The Curtate Future Lifetime After Age x

The curtate lifetime is a discrete random variable that is defined by $K(x) = [T(x)]$ i.e., the integer part (or greatest integer) of *T(x)*.

Since it is a function of *T(x)*, it is simple to calculate the probability function of *K*(*x*) from what it is known about *T(x)*. The possible values of *K(x)* are the numbers 0,1,2,…,ω-*x*-1.

The key observation is that if *K(x) = k* , then

$$k \le T(x) < k+1 .$$

This leads to the following formula for the probability function:

$$\Pr(K(x)=k) = \Pr(k \le T(x) < k+1) = {}_{k|}q_x = \frac{d_{x+k}}{l_x} \quad \text{for k = 0,1,2,...}\omega \text{ - x - 1.} \tag{4.59}$$

It is also useful to develop formulas for the cumulative distribution function and survival function of the curtate future lifetime.

Recall that for any random variable $F_X(x) = \Pr(X \le x)$, hence

$$\begin{aligned} F_K(k) &= \Pr(K(x) \le k) = \Pr(K(x)=0) + \Pr(K(x)=1) + ... + \Pr(K(x)=k) \\ &= \frac{l_x - l_{x+k+1}}{l_x} \\ &= {}_{\mathrm{k+1}}q_x \quad \text{for} \quad k = 0,1,...,\omega - x - 1. \end{aligned} \tag{4.60}$$

The survival function of the curtate future lifetimes is then easily derived as

$$S_K(k) = \Pr(K(x) > k) = 1 - F_K(k) = 1 - {}_{k+1}q_x = {}_{k+1}p_x . \tag{4.61}$$

4.3.5 Life Expectancy

Life expectancy for a newborn is the important life table function. The functions L_x and T_x are useful devices in the calculation of the life expectancy. They are defined in terms of the life table function, l_x , as follows;

- The function of L_x is calculated over just one of these one year periods

  $L_x = \int_x^{x+1} l_y dy$ which is the number of people-years lived by the survivors to age *x* during the next year,

- $T_x = \int_x^{\omega} l_y dy = L_x + L_{x+1} + ... + L_{\omega-1}$ which can be interpreted as the total people-years lived after age x by the survivors to age x. Beware of confusing T_x with $T(x)$, the random future lifetime of a single life age x.

There are two different life expectancy which are called complete and curtate life expectancy in life table.

The complete expected future life time at age x is denoted by $\overset{o}{e}_x$, and is defined as $\overset{o}{e}_x = E[T(x)]$. We can develop several methods to calculate this expected value

$$\overset{o}{e}_x = E[T(x)] = \int_0^{\omega-x} t\, f_{T(x)}(t)dt = \int_0^{\omega-x} s_T(t)dt = \frac{T_x}{l_x}.$$

The curtate expected future lifetime at age x is denoted by $e_x = E[K(x)]$, and is defined in terms of the curtate lifetime $K(x)$. So, e_x is the expected number of full years (the fractional part of the final year of life is not counted) lived by the life (x) after age x.

$$e_x = E[K(x)] = \sum_{k=0}^{\omega-x-1} k \Pr(K(x) = k) = \sum_{k=0}^{\omega-x-1} k\, {}_{k|}q_x = p_x + {}_2p_x + {}_3p_x + ... + {}_{\omega-x-1}p_x$$

4.4 Life Insurance

Life insurance can be defined in two different perspectives: from society's and from individual's. From society's point of view, life insurance is defined as a social device whereby individuals transfer the financial risks associated with loss of life to the group of individuals, and which involves the accumulation of funds by the group from these individuals to meet the uncertain financial losses associated with loss of life. There are two key elements for this definition. (i) for insurance to exist there must be transfer of the risk from the individual to the group; (ii) a sharing of losses by the group.

From individual's viewpoint, life insurance is defined as an agreement, which may be an insurance policy or certificate. The policy owner pays a stipulated consideration called as premium to the insurer, in return for which the insurer agrees to pay a defined amount of money if the person whose life is insured dies. The emphasis here is legal and financial (Black and Skipper, 1994).

In this section, the features of several standard life insurance policies are described. These policies provide benefits that are contingent on the survival of the policyholder for a certain period, or on the death of the policyholder within a certain period. Since the timing of the benefit payment is unknown at issue, the present value of the benefit payment is also unknown.

The random present value of benefits payable is a function of either the complete future lifetime variable, *T(x)*, or the curtate future lifetime *K(x)* (Gauger M.A., 2006). By the end of this section, we'll be able to

- calculate the expected present value of benefits under a life insurance policy,
- calculate the probability that the actual cost of benefits exceeds the expected amount,
- determine the probability that a certain fund of money will be sufficient to pay all benefits for a group of insured lives.

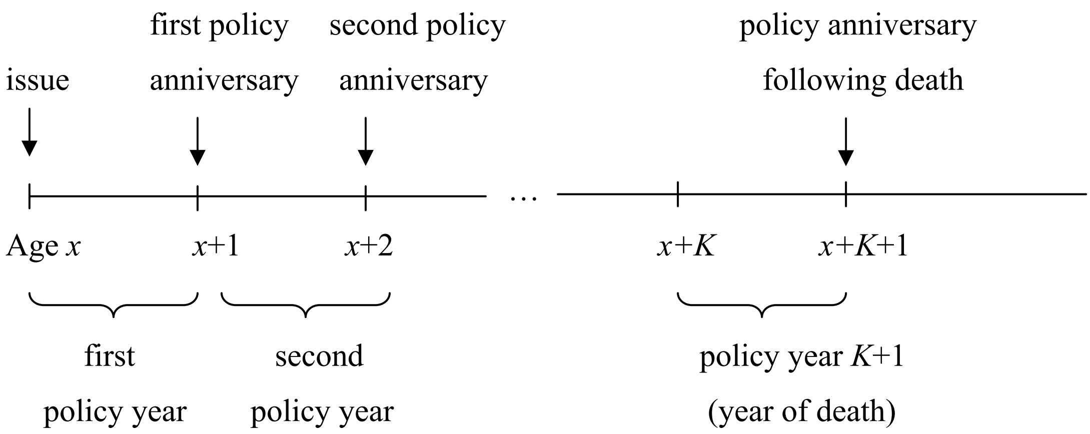
There are continuous and discrete models of life insurance. In this section, we'll interest in discrete models of life insurance. Before, we start to explain some of the terminology that we'll be using.

It is assumed that a policyholder is age x when he/she buys the policy. In other words, the policy commences on a policyholder's birthday. The age at issue is called x, i.e., the age of the policyholder when the insurance policy is issued. It is always assumed that this term refers to an exact age.

For each life insurance model introduced, the random variable Z, which is the present value at the age of issue of the life insurance benefit payment will be studied. In discrete models, Z is a function of the curtate future lifetime $K(x)$, because it is assumed that any death benefit is paid at the end of the year of death.

Policy anniversaries occur at one-year intervals after the contract is issued. A policy tear runs from one policy anniversary to the next. The first policy year coincidence with the age interval $[x,x+1]$. The n^{th} policy year coincides with the age interval $[x+n\text{-}1,x+n)$.

In the discrete model, if the curtate future lifetime is $K(x)=K$, then the policyholder dies during the age interval $[x+k,\ x+K+1]$, which is during the $(K+1)^{\text{th}}$ policy.



Present values are calculated at a constant effective annual rate of interest is denoted i. the discount factor associated with this interest rate is denoted $v = 1/(1+i)$. The compound interest annuity notation is denoted as $a_{\overline{n}|} = \dfrac{1-v^n}{i}$.

Whole life insurance is a name given to a life insurance contract that pays a death when the policyholder dies, no matter when this may happen. For simplicity, it is assumed that the benefit payment is 1.

In a discrete life insurance model of whole life insurance, it is assumed that any death benefit is paid on the policy anniversary following death. This is equivalent to assuming that a payment of \$1 is made at a time $K(x)+1$ years after the contract is issued at age x. In practice, life insurance benefits are usually paid shortly after death, but this assumption serves the purpose of simplifying calculations when the survival model is a discrete life table.

Suppose that a whole life insurance paying 1 on the policy anniversary following death is issued on a life age x. The random present value of the benefit payment at issue is

$$Z = v^{K(x)+1} \text{ for } K(x) = 0,1,\ldots,w\text{-}x\text{-}1.$$

The variable Z is a function of the curtate lifetime variable $K=K(x)$, which has a probability function equal to

$$\Pr(K(x) = k) = {}_{k|}q_x = \frac{d_{x+k}}{l_x} \text{ for } k = 0,1,2,\ldots, w\text{-}x\text{-}1.$$

The expected value $E[Z]$ is known as the actuarial present value (APV) of the benefit, and we have

$$E[Z] = E\left[v^{K(x)+1}\right] = \sum_{k=0}^{\omega-x-1} v^{k+1}\,{}_{k|}q_x = \frac{1}{l_x}\sum_{k=0}^{\omega-x-1} v^{k+1} d_{x+k}\,.$$

When Z represents the random present value at age of issue x of a whole life insurance paying 1 on the policy anniversary following death, we can write $A_x = E[Z]$. This calculation illustrates the general principle that the actuarial present value is proportional to the amount of the benefit payment (known as the face value of the policy).

In this section, it is considered that other types of life insurance policies that a pay a fixed amount of benefit, either on death or survival to a certain age; term insurance, deferred insurance, pure endowment insurance, and endowment insurance.

Under the discrete model, the following notation based on a curtate future lifetime from issue of K (i.e., death occurs in the $(K+1)^{\text{th}}$ policy year) is used;

- The amount of the benefit paid is b_{K+1},
- The factor used to discount the benefit from the time of payment to issue is v_{K+1}.

Hence, the random present value of the benefit at issue is $Z = b_{K+1}\, v_{K+1}$ and the actuarial present value at issue is $E[Z]=E[b_{K+1}\, v_{K+1}]$.

Under an n year term insurance policy, a benefit is only paid if the death occurs within n years of issue. Hence, under the discrete model,

$$b_{K+1} = \begin{cases} 1 & K = 0,1,\dots,n-1 \\ 0 & K \geq n \end{cases}$$

$$v_{K+1} = v^{K+1}$$

$$\Rightarrow \; Z = \begin{cases} v^{K+1} & K = 0,1,2,\dots,n-1 \\ 0 & K \geq n \end{cases}.$$

The actuarial present value of an n year term insurance policy that pays a death benefit of 1 on the policy anniversary immediately following death is denoted by $A^{1}_{x:\overline{n}|}$. The actuarial present value is

$$A^{1}_{x:\overline{n}|} = E[Z] = \sum_{k=0}^{n-1} v^{k+1} \Pr(K = k) = \sum_{k=0}^{n-1} v^{k+1}\; {}_{k|}q_x \,.$$

Under an n year deferred insurance policy, a death is only paid if the death occurs more than n years after issues. Hence, under discrete model, we have

$$b_{K+1} = \begin{cases} 1 & K \leq n-1 \\ 0 & K \geq n \end{cases}$$

$$v_{K+1} = v^{K+1}$$

$$\Rightarrow \; Z = \begin{cases} 0 & K \leq n-1 \\ v^{K+1} & K \geq n \end{cases}.$$

The actuarial present value of an n year deferred insurance policy that pays a death benefit of 1 on the policy anniversary immediately following death is denoted by ${}_{n|}A_x$. The actuarial present value is

$$ {}_{n|}A_x = E[Z] = \sum_{k=0}^{\omega-x-1} v^{k+1} \Pr(K=k) = \sum_{k=0}^{\omega-x-1} v^{k+1} {}_{k|}q_x . $$

Under an n year pure endowment insurance policy, a benefit is paid n years after issue if and only if the policyholder is still alive at that time. The actuarial present value of an n year pure endowment insurance policy that pays a benefit of 1 at n years after issue if and only if the policyholder is still alive at time is denoted by $A_{x:\overset{1}{\overline{n|}}}$ or ${}_nE_x$. It is simple to see that the actuarial present value is

$$ A_{x:\overset{1}{\overline{n|}}} = v^n {}_np_x . $$

Under an n year endowment insurance policy, a benefit is paid at the earlier of

- the time of death, if the policyholder dies within n years of issue, and
- n years after issue, if the policyholder survives n years after issue.

Under the discrete model, any death benefit will be paid at the policy anniversary immediately following death. Hence, we have

$$ b_{K+1} = 1 \quad \text{for all } K \geq 0 $$

$$ v_{K+1} = \begin{cases} v^{K+1} & K \leq n-1 \\ v^n & K \geq n \end{cases} $$

$$ \Rightarrow \quad Z = \begin{cases} v^{K+1} & K \leq n-1 \\ v^n & K \geq n \end{cases} . $$

The actuarial present value of n year endowment insurance policy that pays a benefit of 1 after n years, or on the policy anniversary immediately following death (if earlier), is denoted by $A_{x:\overline{n|}}$. The actuarial present value is

$$A_{x:\overline{n}|} = E[Z] = \sum_{k=0}^{n-1} v^{k+1} \Pr(K = k) + \sum_{k=0}^{\omega-x-1} v^{n} \Pr(K = k) = \sum_{k=0}^{n-1} v^{k+1} {}_{k|}q_x + v^n {}_n p_x .$$

An n year endowment insurance policy is equivalent to a combination of an n year term insurance and an n year pure endowment insurance policy.

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}} .$$

Present values of life insurance are also calculated by another method, which is called commutations functions. Calculations of present values get easy with commutations functions. In order to save arithmetic a system of six symbol called commutations functions has been developed. These six symbols and their definitions are as follows

$$D_x = v^x l_x$$

$$N_x = D_x + D_{x+1} + D_{x+2} + \ldots + D_\omega$$

$$S_x = N_x + N_{x+1} + N_{x+2} + \ldots + N_\omega$$

$$C_x = v^{x+1} d_x$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \ldots + C_\omega$$

$$R_x = M_x + M_{x+1} + M_{x+2} + \ldots + M_\omega .$$

Given rate of interest and a mortality table, numerical values of these commutations symbols can be calculated for all values of x.

Actuarial present value of whole life insurance is

$$A_x = \frac{M_x}{D_x} .$$

Actuarial present value of n year term insurance is

$$A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x} .$$

Actuarial present value of n year deferred insurance is

$$ {}_{n|}A_x = \frac{M_{x+n}}{D_x} . $$

Actuarial present value of n year pure endowment insurance is

$$ A_{x:\overline{n}|}^{\;\;1} = \frac{D_{x+n}}{D_x} . $$

Actuarial present value of n year endowment insurance is

$$ A_{x:\overline{n}|} = \frac{M_{x+n} - M_x + D_{x+n}}{D_x} . $$

4.5 Life Annuities

In the preceding section, we studied payments contingent on death, as provided by various forms of life insurances. In this section, we study payments contingent on survival, as provided by various forms of life annuities. A *life annuity* is a series of payments made continuously or at equal intervals (such as months, quarters, years) while a given life survives. It may be temporary, that is, limited to a given term of years, or it may be payable for the whole of life. The payment intervals may commence immediately or, alternatively, the annuity may be deferred. Payments may be due at the beginnings of the payment intervals (annuities-due) or at the ends of such intervals (annuities-immediate).

Life annuities play a major role in life insurance operations. As we seen in the next section, life insurances are usually purchased be a life annuity of premiums rather than by a single premium. The amount payable at the time of claim may be converted through a settlement option into some form of life annuity for the beneficiary. Some types of life insurance carry this concept even further and, instead of featuring a lump sum payable on death, provide stated forms of income benefits. Thus, for example, there may be a monthly income payable to a surviving spouse or to a retired insured.

Annuities are even more central in pension systems. In fact, a retirement plan can be regarded as a system for purchasing deferred life annuities (payable during retirement) by some form of temporary annuity of contributions during active service. The temporary annuity may consist of varying contributions, and valuation of it may take into account not only interest and mortality, but other factors such as salary increases and the termination of participation for reasons other than death.

Life annuities also have a role in disability and workers' compensation insurances. In the case of disability insurance, termination of the annuity benefit by reason of recovery of the disabled insured may need to be considered. For surviving spouse benefits under workers' compensation, remarriage may terminate the annuity.

The random present value of benefits payable is a function of either the complete future lifetime variable, *T(x)*, or the curtate future lifetime *K(x)* (Gauger M.A., 2006). By the end of this section, we'll be able to

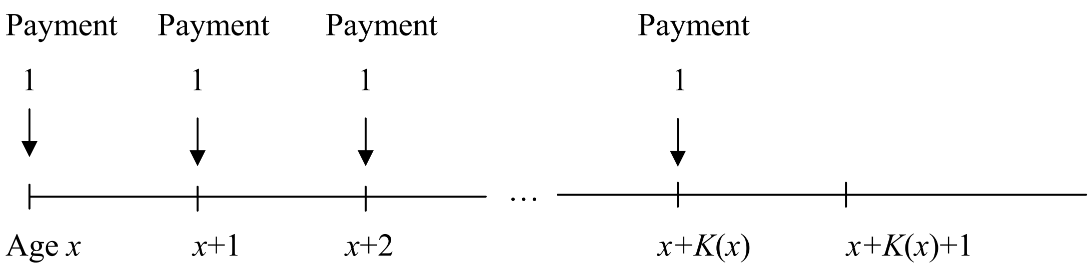
- calculate the expected present value of benefits under a life annuity policy,
- calculate the probability that the actual cost of benefits exceeds the expected amount,
- determine the probability that a certain fund of money will be sufficient to pay all the annuity benefits for a group of insured lives.

There are continuous and discrete models of life insurance. In this section, we'll interest in discrete models of life insurance. Before, we start to explain some of the terminology that we'll be using.

In a discrete life annuity model, payments are usually made at regular time intervals during the future lifetime of (x). The payments might be made annually, semiannually, or perhaps monthly. There are two main types of annuities, according to the timing of the payments:

- for an annuity due, the payments are made at the start of each period,
- for an annuity immediate, the payments are at the end of each period.

In the beginning, we will focus on a discrete model of a level whole life annuity due of 1 per year for a life (x). So, payments of 1 will be made at the start of each year, for as long as (x) is alive. The series of payments associated with this annuity are illustrated in a diagram below.



Recall that $T(x)$ lies somewhere between $K(x)$ and $K(x)$+1, so the annuity consists of $K(x)$+1 payments.

The random present value of this series of payments contingent on the survival of (x) is

$$Y = \ddot{a}_{\overline{K(x)+1}|} \text{ for } K(x)=0,1,2,\ldots,\omega - x - 1.$$

Y is a function of the curtate future lifetime, $K(x)=K$. It is also important to notice that Y is an increasing function of K. The longer that (x) survives, the more payments are made, and more payments result in a higher present value.

The actuarial present value (APV) of a whole life annuity due of 1 per year for (x) is written $\ddot{a}_x$. Hence, we have

$$\ddot{a}_x = E[Y] = E\left[\ddot{a}_{\overline{K(x)+1}|}\right] = \sum_{k=0}^{\omega-x-1} \ddot{a}_{\overline{k+1}|} \Pr(K(x)=k) = \sum_{k=0}^{\omega-x-1} \ddot{a}_{\overline{k+1}|}\, {}_{k|}q_x .$$

This method of calculating $E[Y]$ is known as the aggregate payment method, and it can be quite time consuming to evaluate, since we'll need to successively calculate annuity due values $\ddot{a}_{\overline{1}|}, \ddot{a}_{\overline{2}|}$, and so on.

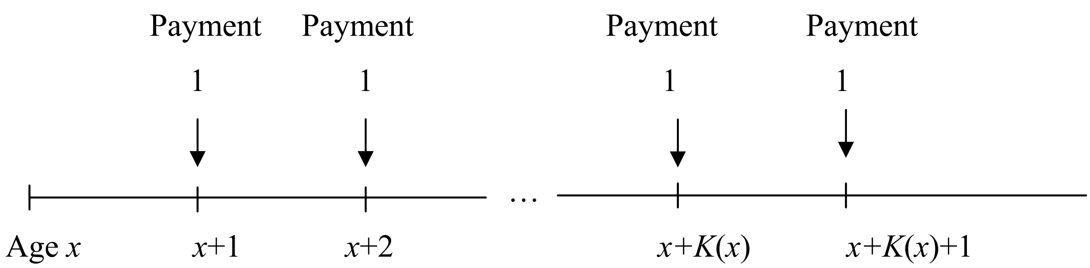
There is a alternative formula, known as the current payment formula, which uses the three factor method of writing an APV as a sum over possible payment times (n=0,1,2,…, $\omega - x - 1$) of a product of

- amount (1 for all payments),
- discount factor (v^n for a payment made n years after issue),
- probability of payment ($_np_x$ since the payment is made only if the life survives).

The current payment formula is derived as follows

$$\begin{aligned}\ddot{a}_x &= E[Y] = \sum_{k=0}^{\omega-x-1} \ddot{a}_{\overline{k+1}|}\ {}_{k|}q_x = \sum_{k=0}^{\omega-x-1}(1+v+\ldots+v^k)({}_kp_x - {}_{k+1}p_x) \\ &= 1 + vp_x + v^2\ {}_2p_x + \ldots + v^{\omega-x-1}\ {}_{\omega-x-1}p_x = \sum_{k=0}^{\omega-x-1} v^k\ {}_kp_x\end{aligned}.$$

It is surprisingly simple to extend this theory to a life annuity immediate. Under the discrete model of a level whole life annuity immediate of 1 per year for a life (x), payments of 1 will be made at the end of each year, for as long as (x) is alive. The series of payments associated with this annuity are illustrated in the diagram below.



Since there are $K(x)$ payments, the random present value of this life annuity immediate is $Y_1 = a_{\overline{K(x)}|}$ for $K(x)$ = 0,1,2,…, $\omega - x - 1$. Notice that the only difference between these payments and those under a life annuity due is that no payment is made at issue. Hence, we have $Y_1 = Y - 1$ where Y is the random present value of a life annuity due for (x). The actuarial present value of a whole life annuity immediate of 1 per year for (x) is written as a_x. Hence, we have

$$a_x = E[Y_1] = E[Y-1] = E[Y] - 1 = \ddot{a}_x - 1.$$

There are two additional types of life annuities: an n year temporary life annuity and n year deferred life annuity. These annuities will be assumed to be for 1 per year on a life currently age x.

For an n year temporary life annuity, payments are made only during the next n years and while (x) is surviving. In other words, payments cease on the earlier of:

- for death of the policyholder,
- the expiration of n years after the date of issue.

Under the discrete model, the random present value of an n year temporary life annuity due is a function of $K(x)$. It is given by

$$Y = \begin{cases} \ddot{a}_{\overline{K(x)+1}|} = \dfrac{1-v^{K(x)+1}}{d} & \text{if } 0 \le K(x) < n \\ \ddot{a}_{\overline{n}|} = \dfrac{1-v^{n}}{d} & \text{if } K(x) \ge n \end{cases}.$$

Actuarial present value is written as $\ddot{a}_{x:\overline{n}|}$. The actuarial present value is thus given by

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \, {}_k p_x \, .$$

For an n year deferred annuity, payments begin n years after issue and continue until the policyholder's death. It is possible to that no payments are made.

Under the discrete model, the random present value of an n year deferred life annuity due is a function of $K(x)$. It is given by

$$Y = \begin{cases} 0 & \text{if } K(x) < n-1 \\ \ddot{a}_{\overline{K(x)+1}|} - \ddot{a}_{\overline{n}|} & \text{if } K(x) \ge n \end{cases}.$$

The actuarial present value is written as ${}_{n|}\ddot{a}_x$. It is given by

$${}_{n|}\ddot{a}_x = E[Y] = \sum_{\mathrm{k=n}}^{\omega-x-1} v^k \, {}_k p_x \, .$$

Another type of annuity is known as the joint and survivor annuity. The term "joint" suggests that the payment amount is based on more than one status and the term "survivor" suggests that it pays as least some amount until the last status fails. For example, a 50 percent joint and survivor annuity pays one dollar annually while both statuses are alive (usually husband and wife, but not necessarily restricted to couples), and reduces to 50 cents after the death. Let x denote the age of the plan member, z the joint annuitant's age, and k the portion of the annual benefit paid to the survivor after the first death, regardless of who dies first. The 100k percent joint and survivor annuity may be represented as

$$ {}^{k}\ddot{a}_{xz} = \sum_{t=0}^{\infty} v^t \left[{}_t p_x^{(m)} \, {}_t p_z^{(m)} + k \, {}_t p_x^{(m)} (1 - {}_t p_z^{(m)}) + k \, {}_t p_z^{(m)} (1 - {}_t p_x^{(m)}) \right] $$

The first term inside the brackets represents a payment of \$1 if both x and z are alive at time t, the second term represents a payment of \$$k$ if only x is alive, while the third term represents a payment of \$$k$ if only z is alive.

A widely used variation of this annuity is known as a *contingent* joint and survivor annuity. Under this form, the annuity benefit is reduced only if the plan member is the first to die. The survivor's benefit might be any portion, with one-half and two-thirds representing choices usually available. A 100k percent contingent joint and survivor annuity may be expressed as

$$ {}^{k}\ddot{a}_{\overset{1}{x}z} = \sum_{t=0}^{\infty} v^t \left[{}_t p_x^{(m)} \, {}_t p_z^{(m)} + {}_t p_x^{(m)} (1 - {}_t p_z^{(m)}) + k \, {}_t p_z^{(m)} (1 - {}_t p_x^{(m)}) \right] $$

where the x subscript stipulates that, if x is the first to die, only k dollars are continued to z. The bracketed expression represents a payment of \$1 if both x and z are alive at time t, a payment of \$1 if x is alive and z is not alive, and a payment of \$$k$ if z is alive and x is not alive. This expression reduces to

$$ {}^{k}\ddot{a}_{\overset{1}{x}z} = \sum_{t=0}^{\infty} v^t \left[{}_t p_x^{(m)} + k \, {}_t p_z^{(m)} - k \, {}_t p_x^{(m)} \, {}_t p_z^{(m)} \right] $$

In this form, the bracketed term represents a payment of \$1 to x regardless of the whether or not z is alive, a payment of \$$k$ to z regardless of whether or not x is alive, and since this would result in a total payment of \$$(1+k)$ in the event both are alive in year t, \$$k$ is subtracted, both are alive at time t.

Present values of life annuities are also calculated by another method, which is called commutations functions. It is discussed earlier section.

Actuarial present values of whole life annuity immediate and due are

$$a_x = \frac{N_{x+1}}{D_x} \quad \text{and} \quad \ddot{a}_x = \frac{N_x}{D_x}.$$

Actuarial present values of n year temporary annuity immediate and due are

$$a_{x:\overline{n}|} = \frac{N_{x+1} - N_{x+n+1}}{D_x} \quad \text{and} \quad \ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x}.$$

Actuarial present values of n year deferred annuity immediate and due are

$$_{n|}a_x = \frac{N_{x+n+1}}{D_x} \quad \text{and} \quad _{n|}\ddot{a}_x = \frac{N_{x+n}}{D_x}.$$

4.6 Benefit Premiums

An insurance company that has sold a whole life insurance contract to a person at age x at issue of the contract knows how much will be needed to pay the eventual death benefit under the contract, but does not know when the death benefit cost will occur.

If the deposits will earn effective rate i per period, then we might determine P so that the accumulated fund at time k will provide exactly the needed amount X. Thus

$$P.\ddot{a}_{\overline{k}|i} = X.v_k\,.$$

This equation defines the periodic deposit by equating the present value of the funding scheme to the present value of the amount to be needed at time k, where each present value is taken at interest only.

With contingent payment models, including insurance contracts, the time at which amount X is needed is stochastic rather than fixed. The periodic funding payment is found by equating the actuarial present value (APV) of the funding scheme to the APV of the contingent payment. Funding payments determined in this manner are said to satisfy the equivalence principle.

In this section, we consider funding schemes for various types of contingent contracts, wherein the funding pattern is represented by a series of discrete annual funding payments.

Consider a discrete whole life insurance of b issued on a life (x). Suppose that P is the level annual benefit premium that would be paid at ages x,x+1, …, x+K(x), where K(x) is the curtate future lifetime of (x).

The random present value of the premium is $P\ddot{a}_{\overline{K(x)+1}|}$ so the actuarial present value is

$$E\left[P\ddot{a}_{\overline{K(x)+1}|}\right] = PE\left[\ddot{a}_{\overline{K(x)+1}|}\right] = P\ddot{a}_x .$$

Consider the whole life contingent payment model under which payment of X will be made at time k if the status of interest, which we denoted by (x), fails in the interval (k,k-1]. If this contingent payment is funded under the equivalence principle, then the periodic payment P is determined from

$$P.\ddot{a}_x = X.A_x$$

where $X.A_x$ is the actuarial present value (APV) of the contingent payment and $P.\ddot{a}_x$ is the APV of the funding scheme. Note that funding payments are made at the beginnings of each time interval including the beginning of the interval of failure.

To define the standard actuarial terminology and notation for contingent payment funding schemes, we consider the eventual contingent model, with time measured in years, the periodic funding payment is denoted by P_x. Thus we have

$$P_x.\ddot{a}_x = A_x \text{ or } P_x = \frac{A_x}{\ddot{a}_x}$$

from which P_x can be calculated from values A_x and $\ddot{a}_x$.

Consider other standard unit benefit plans of insurance such as n year term insurance and n year endowment insurance. For these insurances there would be no economic incentives for the policyholder to pay premiums for more than n years, since at that point no additional future benefit is possible. As a result, it is assumed that premiums are paid annually in advance for n years or until the death of the policyholder, whichever occurs first. So, in premium formulas, the present value of the premiums is calculated by $\ddot{a}_{x:\overline{n}|}$. Remember that all level annual benefit premium symbols for an insurance policy with a benefit of 1 start with a P. The level of annual benefit premium symbol for an n year endowment insurance issued on (x) is $P_{x:\overline{n}|}$. The level annual benefit premium symbol for an n year term insurance issued on (x) is $P^1_{x:\overline{n}|}$. Formulas of benefit premiums are as follows

$$P_x = \frac{A_x}{\ddot{a}_x} \qquad \text{and} \qquad P^1_{x:\overline{n}|} = \frac{A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

In discrete case, the level annual benefit premium for an n year pure endowment is written as

$$P_{x:\overset{1}{\overline{n}|}} = \frac{A_{x:\overset{1}{\overline{n}|}}}{\ddot{a}_{x:\overline{n}|}}.$$

The level annual benefit premium for endowment insurance can be split into the sum of the level annual benefit premium for term insurance and the level annual benefit premium for a pure endowment. In the discrete model, we have

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

4.7 Benefit Reserve

We introduce the very important concept of reserves under contingent contracts by returning to the simple funding arrangement recall that the periodic funding payment was determined such that the present value at time 0 (at interest only) of the funding scheme was equal to the present value at time 0 of the payment needed at time *k*.

In the insurance context, the contingent payment is called the benefi*t* under the insurance contract and the payments are called the benefit premiums. Therefore, the general idea of the t^{th} terminal benefit reserve, determined by the prospective method, under an insurance contract can be expressed as

$${}_tV = (AVP\ of\ future\ benefit) - (AVP\ of\ benefit\ premiums) \tag{4.62}$$

CHAPTER FIVE
PENSION MATHEMATICS

Each person has primarily responsibility for his or her own welfare. Personal thrift has played and probably always will play a major role in providing for old-age security. Government efforts for the most part have been directed toward providing a basic minimum benefit, as a matter of right, through various forms of social insurance. With assurance of a minimum benefit, the employee, through individual effort, supplemented by any benefits an employer may provide, can seek to raise his or her old-age income to in an adequate level. The ability to do this has been facilitated in recent years through federal income tax legislation that permits individuals to accumulate tax-favored savings for retirement purposes.

5.1 Pension Plan Benefits

The primary function of a pension plan is to provide income to employees in their retirement. Pension plans are not limited to providing retirement income, however, and all plans provide at least some of the following types of benefits: (1) vested termination benefits, (2) disability benefits, and (3) death benefits, the latter consisting of either a lump sum benefit or an annuity to a surviving spouse or other dependents (Winklevoss, 1993).

There are two basic types of pension plans: defined contribution (DC) and defined benefit (DB) (Winklevoss, 1993). A defined contribution plan provides an individual account for each participant. The benefits are based on the amount contributed and are also affected by income, expenses, gains and loses. A defined benefit plan promises the participant a specific monthly benefit at retirement and may state this as a certain amount. Monthly benefits could also be calculated through a formula that considers a participants salary and service. A participant is generally not required to make contributions in a private sector fund but most public sector funds require employee contributions. Unlike defined contribution plans, the participant is not required to make investment decisions. A defined benefit plan is sometimes referred to as a fully funded pension plan.

Advantages of defined benefit plans;

- Guaranteed retirement income security for workers,
- No investment risk to participants,
- Cost of living adjustments,
- Not dependant on the participant's ability to save,
- Tax deferred retirement savings medium.

Disadvantages of defined benefit plans;

- Difficult to understand by participant,
- Not beneficial to employees who leave before retirement.

Advantages of defined contribution plans;

- Tax deferred retirement savings medium,
- Participants have a certain degree of how much they choose to save,
- Can be funded through payroll deductions,
- Lump sum distributions may be eligible for special 10 year averaging,
- Participants can benefit from good investment results,
- Easily understood by participants.

Disadvantages of defined contribution plans;

- Difficult to build a fund for those who enter late in life,
- Participants bear investment risk.

5.1.1 Retirement Benefit

There are two categories of eligibility requirements related to retirement benefits, one setting the requirements for plan membership and the other specifying the requirements for retirement under the plan.

The most common type of benefit formula used in pension plans is the so-called unit benefit formula, which provides a unit of benefit for each year of credited service. There are three such formulas associated with defined benefit plans: (1) flat dollar, (2) career average, and (3) final average. The flat dollar benefit formula is the simplest of three, providing a dollar amount, such as $20, per month for each year of service rendered by the employee. The flat dollar amount is generally increased at periodic intervals by plan amendment, either to keep pace with the inflationary trends in the economy and/or in response to union negotiations.

The career average benefit formula provides a benefit defined in terms of some stipulated percentage of the employee's benefit accrual as 2 percent of each year's salary. It is permissible under current law to use a larger percentage for salary in excess of the average Social Security wage base than the percentage applied to salary under this level, an integration formula known as the step rate method.

The final average benefit formula is one providing a given percentage of the employee's final average (or highest average) salary per year of service. Since the benefits derived from this type of formula are based on the employee's salary near retirement, the percentage need not be as high as the career average formula percentage in order to provide equivalent benefits.

5.1.2 Vested Benefits

An employee has a vested benefit if its payment at retirement is no longer contingent upon remaining in the service of the employer. When an employee terminates employment with a vested benefit, the benefit amount generally becomes payable at the plan's normal retirement age; however, some plans permit payments to begin at an early retirement age, usually with an actuarial reduction for early payment.

In most cases, the benefit accruals used to determine vested benefits are those defined by application of the retirement benefit formula. The cost of providing benefits to vested terminating employees could be minimized by adopting a formula with disproportionate benefits provided at older ages and/or longer periods of service; however, the law has established guidelines to prevent such backloading.

In all cases, benefits attributable to employee contributions are fully and immediately vested. At the time of termination, employees may be entitled to a return of their contributions, usually with interest, instead of leaving them in the plan and receiving a deferred retirement benefit.

5.1.3 Disability Benefit

Two types of disability benefits are found in pension plans, one providing a deferred pension to disabled employees beginning at the plan's normal retirement age and continuing for life, and the second providing benefits that commence after a specified waiting period, such as 6 or 9 months, and continuing for life. The former type is generally found in combination with a long-term disability (LTD) benefit program operating outside the pension plan, with the LTD plan providing the desired level of benefits from the time of disability to the plan's normal retirement age.

The eligibility provisions for disability benefits vary widely among plans, but a minimum age or service requirement, or both, usually exists. The disability entitlement might coincide with eligibility for early retirement.

The most common method used to define disability benefits is simply to apply the retirement benefit formula to the employee's salary and years of service at the time of disability. Some plans use the total potential service of the employee up to the plan's normal retirement instead of actual service at the date of disability. This is almost invariably the case if the disability pension is the deferred type use in conjunction with an LTD program. In some instances, the amount of the disability benefit is a flat amount per month, irrespective of the employee's service or accrued benefit. The benefit assumed for the model pension plan is the unreduced accrued benefit commencing at the time of disability and payable for life.

5.1.4 Death Benefits

Death benefits may consist of a lump sum distribution, such as a flat dollar amount or, for active employees, some multiple of salary. Alternatively, the death benefit may take the form of an annuity payable to a surviving spouse.

Death benefit eligibility provisions are often related to the plan's eligibility for vesting. The benefit must be payable by the age the employee would have been eligible for early retirement, or it can be made available earlier. In either case, an actuarial reduction can be applied if the benefit commences prior to the employee's normal retirement age.

The model pension plan provides a surviving spouse benefit upon satisfying the requirements for vesting.

If a surviving spouse benefit is payable, the amount is generally some percentage of the deceased participant's accrued benefit. A common formula is 50 percent of the participant's attained age accruals, possibly reduced for early retirement or for the actuarial cost of the surviving spouse benefit itself. The benefit payable under the model pension plan is 50 percent of the employee's attained age accrued benefit without reduction.

5.2 Actuarial Assumptions

This section discusses the actuarial assumptions used to calculate pension costs and liabilities. These include various rates of decrement applicable to plan members, future salary estimates for plans with benefits linked to salary, and future interest returns on plan assets. In addition to a general discussion of actuarial assumptions, the specific assumptions used with the model pension plan are given.

5.2.1 Decrement Assumptions

Active plan participants are exposed to the contingencies of death, termination, disability, and retirement, whereas nonactive members exposed to death. These contingencies are dealt with in pension mathematics by rates of decrement. A rate of decrement refers to the proportion of participants leaving a particular status due to a given cause, under the assumption that there are no other decrements applicable. If such a rate is used in a single decrement environment (i.e., where there are in fact no other decrements applicable), it is also equal to the probability of decrement.

The rate of decrement in a multiple decrement environment (i.e., where more than one decrement is operating), is not equal to the probability of decrement. Active employees exist in a multiple decrement environment, being exposed to mortality, termination, disability, and retirement; hence, the rate of decrement is not equal to the probability of decrement because the other decrements prevent participants from being exposed to the contingency throughout the year (Özgürel, 2005).

A typical assumption for transforming a rate into a probability for a multiple-decrement environment is that all decrements occur on a uniform basis throughout the year, referred to as the uniform distribution of (UDD) assumption. With $q'^{(k)}$ denoted as the probability of decrement, the transformation of a rate into a probability in a double-decrement environment (*k=1,2*) under the UDD assumption is given by

$$q^{(1)} = q'^{(1)}\left[1-\frac{1}{2}q'^{(2)}\right].$$

The value of three decrement environment becomes

$$q^{(1)} = q'^{(1)}\left[1-\frac{1}{2}(q'^{(2)}+q'^{(3)})+\frac{1}{3}q'^{(2)}\,q'^{(3)}\right],$$

and four-decrement environment, we have

$$q^{(1)} = q'^{(1)}[1-\frac{1}{2}(q'^{(2)}+q'^{(3)}+q'^{(4)})+\frac{1}{3}(q'^{(2)}\,q'^{(3)}+q'^{(2)}\,q'^{(4)}+q'^{(3)}\,q'^{(4)})$$
$$-\frac{1}{4}(q'^{(2)}\,q'^{(3)}\,q'^{(4)})].$$

As noted previously, the prime symbol on $q'^{(k)}$ indicates a rate of decrement in a multiple-decrement environment, while $q^{(k)}$ denotes the corresponding probability of decrement. The following four rates will be discussed;

$q'^{(m)}$ = mortality rate	$q'^{(d)}$ = disability rate
$q'^{(t)}$ = termination rate	$q'^{(r)}$ = retirement rate.

5.2.2 Salary Assumption

If the plan's benefits are a function of salary, estimates of the employee's future salaries required. These estimates involve consideration of three factors: (1) salary increases due to merit, (2) increases due to labor's share of productivity gains, and (3) increases due to inflation.

The merit scale for a group of employees can be estimated by comparing the differences in salaries among employees at various ages and with various periods of service in a given year. A cross-sectional analysis of this type eliminates the effect of inflation and productivity increases. In many cases, a constant rate of increase at each age is used to approximate the age specific rates of a typical merit scale.

The second factor that affects the salaries of the entire group of employees is labor's share of productivity gains. This factor, which is difficult to estimate, may have diminished in importance over the years, and it varies among industries.

The third and most significant factor affecting an employee's future salary is inflation. This factor is more likely to be representing by a constant compound rate, unlike the merit component which generally increases salary at a decreasing rate with age. This need to be the case, however, and empirical trends that increase or decrease for a period of time to an ultimate level.

5.2.3 Interest Assumption

The interest assumption has a powerful effect of pension costs, since it is used to find the present value of financial obligations due to 20, 40, and even 60 years from the valuation date. Although it is common to find this assumption set at a constant compound rate, this is a special case of the more general assumption that would allow the rate of interest to vary over time. As with most actuarial assumptions, an element of subjectivity is involved in establishing the interest rate to be used in the valuation of pension costs and liabilities.

In any event, the interest assumption, like salary assumption, can be viewed as consisting of three components: (1) a risk-free rate of return, (2) a premium for investment risk, and (3) a premium for inflation.

The risk-free rate is one that would prevail for an investment completely secure as to principal and yield in an environment with no current or anticipated inflation. An estimate of this theoretical component would be the difference that varies widely from year to year, and is nearly zero over very long periods of time.

The second interest rate component is the investment risk inherent in the current and future portfolio of plan assets. A different investment risk, and hence risk premium, may be associated investments down only into several broad clasees for assignment of the risk premium.

A premium for the current and anticipated rate of inflation is the third interest rate component. This factor, it will be remembered, was present in the salary assumption also, and this sense the salary and interest assumptions have a common link.

5.3 Basic Actuarial Function

The purpose of this section is to introduce several actuarial functions used in the used in the development of pension mathematics. The composite survival function and interest function, perhaps the two most basic concepts in pension mathematics. Pension plan benefit functions are then represented.

5.3.1 Composite Survival Function

The composite survival function represents the probability that active plan participant survives in service for a given period, based on all of the decrement rates to which the employee is exposed. Whereas the probability of surviving one year in a multiple–decrement environment is equal to the complement of the rate of decrement, the probability of surviving one year in a multiple decrement environment is equal to the product of such complements for each applicable rate of decrement. The probability of an active participant aged x surviving one year is

$$p_x^{(T)} = (1 - q_x^{'(m)})(1 - q_x^{'(t)})(1 - q_x^{'(d)})(1 - q_x^{'(r)}) \tag{5.1a}$$

or equivalently

$$p_x^{(T)} = p_x^{'(m)}\ p_x^{'(t)}\ p_x^{'(d)}\ p_x^{'(r)} \tag{5.1b}$$

this same probability can be expressed in terms of multiple decrement probabilities

$$p_x^{(T)} = 1 - (q_x^{(m)} + q_x^{(t)} + q_x^{(d)} + q_x^{(r)})\ . \tag{5.2a}$$

A common approximation for multiple-decrement probabilities is illustrated for the mortality probability as follows

$$q_x^{(m)} \approx q_x^{'(m)}(1 - \frac{1}{2} q_x^{'(t)})(1 - \frac{1}{2} q_x^{'(d)})(1 - \frac{1}{2} q_x^{'(r)})\ . \tag{5.2b}$$

The probability of surviving in active service for n years is equal to the product of successive one-year composite survival probabilities

$$_n p_x^{(T)} = \prod_{t=0}^{n-1} p_{x+t}^{(T)}\ . \tag{5.2c}$$

5.3.2 Interest Function

The interest function is used to discount a future payment to the present time. It plays a crucial role in determining pension costs and, like the survival function of the previous chapter, it reduces such values. If i_t is the interest rate assumed for the t^{th} year, the present value of one dollar due in n years is given by

$$\frac{1}{(1+i_1)(1+i_2)...(1+i_n)} \tag{5.3}$$

and, if $i_1 = i_2 = ... = i_n$, we have

$$\frac{1}{(1+i)^n} = v^n \; . \tag{5.4}$$

Thus, v^n represents the present value of one dollar due in n years at an annual compound rate of interest equal to i. Details of interest function is previous chapter.

5.3.3 Salary Function

If a pension plan has benefits expressed in terms of salary, it is necessary to develop salary–related notation and procedures for estimating future salary. The current dollar salary for a participant age x is denoted by s_x, and S_x represents the *cumulative* salary from age y up to, but not including, age x. Thus, for $x>y$ we have

$$S_x = \sum_{t=y}^{x-1} s_t \; . \tag{5.5}$$

In order to estimate the dollar salary at age x, based on the employee's age-y salary, the following formula is used

$$s_x = s_y \frac{(SS)_x}{(SS)_y} \left[(1+I)(1+P)\right]^{(x-y)} \tag{5.6a}$$

where

s_y = Entry –age dollar salary,

$(SS)_x$ = Merit salary scale at age x,

I = Rate of inflation,

P = Rate of productivity reflected in the salary increases.

An age – y entrant's salary at age x can also be defined in terms of the age – z salary ($y<z<x$)

$$s_x = s_y \frac{(SS)_x}{(SS)_y} \left[(1+I)(1+P)\right]^{(x-z)} . \tag{5.6b}$$

If all of the salary increase assumptions were met from age y to age z, the employee's salary at age z would be equal to

$$s_z = s_y \frac{(SS)_z}{(SS)_y} \left[(1+I)(1+P)\right]^{(z-y)} . \tag{5.6c}$$

Substituting (5.6c) for s_z in (5.6b) reduces the latter to (5.6a), showing that s_x is identical, under the salary increase assumptions, whether derived from the entry age salary or the attained age salary.

The growth in a participant's future salary can increase pension cost estimates substantially. This is in contrast to the interest rate and decrement probabilities, both of which have a decreasing effect on pension cost estimates.

5.3.4 Benefit Function

The benefit function is used to determine the amount of benefits paid at retirement, vested termination, disablement, and death. This function, the interest function, and the survival function provide the basic components required to formulate pension costs. In this section consideration is given to the three most common types of benefit formulas used with defined benefit pension plans.

The symbol b_x denotes the annual benefit accrual during age x to age $x+1$ for an age y entrant, and is referred to as the benefit accrual function. The benefit accrual function can equal the formula accruals or, as discussed in this section, some other definition of accruals, such as a portion of the participant's projected retirement age benefit.

The accrued benefit, denoted by B_x, is equal to the sum of each attained age accrual up to, but not including, age x. This function is called the accrued benefit function and is defined for $x>y$ by $B_x = \sum_{t=y}^{x-1} b_t$.

5.3.5 Annuity Function

Annuities represent a combination of the survival and interest functions. Most annuities are based on the mortality-only survival function, and the materials in previous chapter reflect this emphasis.

5.4 Pension Liability Measure

A variety of liability measures are associated with pension plans, each one having a specified purpose. Some liabilities represents the financial obligations of the plan, either on a plan termination or ongoing basis, while others simply represent mathematical by products of various actuarial cost methods used for pension funding pension plans. Although the latter are not liabilities in the true sense of the word, they are referred to as *actuarial liabilities* to distinguish them from the term liability as used in the fields of finance and accounting. The accounting profession has promulgated several specific pension liability measures, and another set of liabilities is defined by federal statutes in determining minimum required and maximum tax deductible contributions. Finally, since none of these liabilities may represent what management believes to be the "true" long-term financial obligation of the plan, *economic liabilities* are sometimes used to evaluate a plan's funded status.

The purpose of this section is to present the fundamental mathematics of alternative liability measure.

5.4.1 Plan Termination Liability

The plan termination liability (PTL), sometimes referred to as the plan's *legal liability* is equal to the present value of all accrued benefits, both for active and retired employees. Assuming that the benefit is in the form of an annuity payable for the lifetime of the retiree, equation (5.7a) defines this liability for a participant age *x* prior to retirement, while (5.7b) is applicable after retirement

$$(PTL)_x = B_x \; {}_{r\text{-}x}p_x^{(m)} \; v_{r\text{-}x} \; \ddot{a}_r \text{ , for } x \leq r \tag{5.7a}$$

where

B_x = Accrued benefit as defined by the plan,

${}_{r-x}p_x^{(m)}$.= Probability of living from age *x* to age *r*,

v^{r-x} = Interest discount from age *x* to age *r*,

$\ddot{a}_r$ = Present value, at age *r*, of a life annuity;

$$(PTL)_x = B_r \, \ddot{a}_x \text{ for } x \geq r \tag{5.7b}$$

where

B_r = Retirement benefit payable for life,

$\ddot{a}_x$ = Present value, at age *x*, of a life annuity.

The $(PTL)_x$ function increase sharply with age prior to retirement, since the first three factors in (5.7a) increase with age, while the fourth term is constant. After retirement, the $(PTL)_x$ function decreases according to the annuity function, since the benefit function is constant.

The $(PTL)_x$ function for active employee utilizes the mortality survival function, whereas all of the other liability measures presented in this section involve the composite survival function, which includes decrements for termination and disability. The mortality survival function is appropriate for the $(PTL)_x$ function since only death would prevent the participant from receiving the accrued benefit at retirement if the plan were terminated. The participant's future employment status or disability status would have no bearing on the receipt of the accrued retirement benefit.

The mathematical definition of $(PTL)_x$ for retired participants is the same for all liability measures. Hence, the remainder of this section is devoted to alternative liability measures for active participants (i.e., all x's $\leq r$).

5.4.2 Plan Continuation Liability

The plan continuation liability for accrued benefits, sometimes referred to as the *ongoing liability* for accrued benefits, measures the financial obligation under the assumption that the plan will continue to exist. In this case, future employment and disability statuses are relevant. Equation (5.8a) defines this liability measure for an active participant at age x

$$^{AB}{}^{\mathrm{r}}(PCL)_x = B_x \;{}_{\mathrm{r\text{-}x}}p_x^{(T)}\; \mathrm{v}^{\mathrm{r\text{-}x}}\; \ddot{\mathrm{a}}_r \,. \tag{5.8a}$$

The AB prescript to the plan continuation liability symbol indicates that the liability is based on the accrued benefit as defined by the plan. The plan continuation liability also includes ancillary benefits; hence, the prescript r is used to denote that only retirement benefits are being evaluated at this point, a convention used hereafter until the liability associated with ancillary benefits is defined.

The plan continuation liability and the plan termination liability can be expressed in terms of each other

$$^{AB}{}^{\mathrm{r}}(PCL)_x = \frac{{}_{r-x}p_x^{(T)}}{{}_{r-x}p_x^{(m)}}\; {}^{r}(PTL)_x \tag{5.8b}$$

$$= \frac{{}_{r-x}p_x^{(m)}}{{}_{r-x}p_x^{(T)}}\; {}^{\mathrm{r}}(PCL)_x \,. \tag{5.8c}$$

Under identical actuarial assumptions, the $^{AB}{}^{\mathrm{r}}(PCL)_x$ function will be lower in value than the $(PTL)_x$ function until retirement, at which point they become equal. If the liability associated with termination, disability, and death benefits were to be included, however, the ${}^{AB}{}^{r}(PCL)_x$ may equal or exceed the $(PTL)_x$. If the plan had no disability or death benefits and the employee were fully vested, then the two liability values would be equal if the vested termination liability were included in the ${}^{AB}{}^{r}(PCL)_x$ function. In other words, in both cases only death prior to retirement would prevent the participant from receiving the accrued benefit.

As a practical matter, the actuarial assumptions used in evaluating these two liabilities are likely to be different. The interest rate used with the $(PTL)_x$ function, for example, might logically approximate the rate at which the plan sponsor could "sell" the liability to an insurance carrier, whereas the interest rate used for ${}^{AB}{}^{r}(PCL)_x$ is likely to represent the plan sponsor's expected long-run return on plan assets. Thus, it is difficult to predict the relative values of the plan termination and plan continuation liabilities.

5.4.3 Actuarial Liabilities

Several actuarial cost methods used with pension plans, and each method has an associated actuarial liability. In general terms, a cost method's actuarial liability is equal to the present value of benefits allocated to date, which can be expressed as follows (Winklevoss, H.E., 1993)

$$
{}^{r}(AL)_x = B'_x \, {}_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r \tag{5.9}
$$

where B'_x represents the benefits allocated under a given actuarial cost method, as discussed at a later point in this section. Observe that, if the benefit function is equal to the accrued benefit as defined by the plan, the expression is identical to the plan continuation liability given by (4.4a).

The actuarial liability of a given cost method may also be viewed as the portion of the participant's present value of future benefits (PVFB) allocated under the method.

The ${}^{r}(PVFB)_x$ function equals the present value of the participant's *total projected* retirement benefit

$$ {}^{r}(PVFB)_x = B_r \; {}_{r\text{-}x}p_x^{(T)} \; v^{r\text{-}x} \; \ddot{a}_r \, . \tag{5.10a}$$

The ${}^{r}(PVFB)_x$ function is the same as the ${}^{r}(AL)_x$ function evaluated with B_r instead of $B_x^{'}$. Since the actuarial liability represents the proportion of ${}^{r}(PVFB)_x$ allocated by the actuarial cost method being used, a generalized actuarial liability definition can be expressed in the following manner

$$ {}^{r}(AL)_x = k \; {}^{r}(PVFB)_x \tag{5.10b}$$

where k is a fraction dependent on each cost method and defined in subsequent section of this section.

There are three actuarial liability methods; (1) Accrued Benefit Method (Unit Credit), (2) Benefit Prorate Methods, and (3) Cost Prorate Methods. But we will use Unit Credit Method in this thesis, so we don't show other methods.

5.4.3.1 Accrued Benefit Method (Unit Credit)

Assuming that each employee is entitled to retire at age *y* with an annual pension (payable monthly) equal to *B*(*y*), a properly funded plan should have accumulated for each employee when he reaches age *y* an amount sufficient to fund his pension, i.e., an amount equal to $B(y)\ddot{a}_y^{(12)}$. This requirement is the first logical premise of the unit credit cost method (Anderson A.W., 1992).

Now the benefit *B*(*y*) does not arise suddenly at age at age *y*, but is earned or "accrued" in a more or less continuous fashion during the employee's active years of service. Thus, when the employee is hired, say at age *w*, his accrued benefit *B*(*w*) is exactly zero; at age *y* when he retires it is equal to its ultimate value *B*(*y*); and at any point in-between, at age x, it has some intermediate value $B(x)$, which we call his accrued benefit.

At any age x, earlier than y, the present value of employee j's accrued benefit is equal to $B^j(x)\ddot{a}^{(12)}_{y_j}\dfrac{D_{y_j}}{D_{x_j}}$. Note that the factor D_y / D_x is computed using a table of q_x's which represents probabilities of termination of employment before age y from all causes – not just from mortality, but also resignation, discharge, disablement, etc. This table of q's is called a service table – a term parallel to but more general than "mortality table".

So if we had assets on hand all times equal to $\sum_{A_t} B^j(x)\ddot{a}^{(12)}_{y_j}\dfrac{D_{y_j}}{D_{x_j}}$ than no matter what the distribution of ages was among the group A_t of active employees at time t, we should be assured of having sufficient funds for the employees to be the same age and all retire at once.

This observation is the source of the second premise of the unit-credit cost method, which distinguish it from all others: the ideal fund balance, or desired amount of assets, on hand at any given time t is equal to $\sum_{A_t} B^j(x)\ddot{a}^{(12)}_{y_j}\dfrac{D_{y_j}}{D_{x_j}}$, where A_t denotes the set of active employees at time t. This ideal fund balance is called the accrued liability

$$(Accrued\ Liability)_t = AL_t = \sum_{A_t} B^j(x)\ddot{a}^{(12)}_{y_j}\frac{D_{y_j}}{D_{x_j}}. \qquad (5.11)$$

In other words, under the unit credit cost method, the accrued liability is defined as the present value of accrued benefits. This definition distinguishes it from all other cost methods, and carries with it, by implication, a complete definition of what pension cost should be ascribed to any given year, as we shall now see.

Let us digress for a moment to remark our peculiar use of the word "liability" to denote a desired level of *assets*. This oddity, which has caused no end of confusion among accountants, arises from life-insurance terminology. In ordinary financial accounting, a business records each transaction twice – (roughly speaking) the sum of amounts actually owed to someone else. In life insurance accounting, by contrast, premiums received are not recorded on both sides of the ledger, but only as assets – liabilities being determined by a sort of inventory, the annual actuarial valuation. To a life insurance company, a "liability" is an actuarially determined amount which has first claim on the invested *assets* of the company. It is not, strictly speaking, an amount owed to anyone – although it will be if the reserve basis proves true – it is the amount of *assets* to be set aside for whatever the actual claims turn out to be. In the same way, the accrued liability of a pension plan represents a claim on plan assets.

From year to year the accrued liability changes, not only because the ages of the active participants increase, but also because the composition of the active groups itself changes. To keep things simple, we shall assume that there are no new entrants into plan; we shall put them in their own separate pension fund for the moment, recall tem later when we have need of them. Then, the active group can never grow but can only shrink during the year. Denote by T the set of all employees who terminate employment between time t and time t+1 and by R the set of employees during the year who reach age y, so we can write

$$A_{t+1} = A_t - T - R\,. \tag{5.12}$$

We now construct the following purely algebraic argument to show the relationship between the accrued liability at time t and the accrued liability at time t+1.

$$
\begin{aligned}
(Accrued\ Liability)_{t+1} = AL_{t+1} &= \sum_{A_{t+1}} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} \\
&= \sum_{A_t} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} - \sum_{T+R} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} \\
&= \sum_{\mathrm{A}_t} \left[B^j(x) + \Delta B^j\right]\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}}(1+i) + \sum_{A_{t+1}} q_x B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} \\
&\quad - \sum_{T+R} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}}
\end{aligned}
$$

where ΔB^j is the increase in j's accrued benefit during the year. This means that

$$
\begin{aligned}
AL_{t+1} &= \left[AL_t + \sum_{A_t} \Delta B^j \ddot{a}_y^{(12)} \frac{D_y}{D_x} \right](1+i) \\
&\quad - \left[\sum_{A_t} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} - \sum_{A_t} q_x B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}} \right]. \qquad (5.13) \\
&\quad - \sum_{R} B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}}
\end{aligned}
$$

Don't forget that D_y / D_x was computed using a table of q's which represent the probability at each age of withdrawing from the active group – not just the probability of dying. In other words, the D_x's are taken from our *service table*.

Now look at the second bracketed term of equation (4.7). If actual experience during the year is in direct accord with assumed experience, this term will be zero. That is easy to say, the expected release of liability on account of termination of employment before age y from all causes except retirement (the second summation) will exactly offset the actual amount of accrued liability released on account of employees who actually did terminate, i.e., who were members of the set T. Also, if actual experience is in accord with assumed, the ideal fund balance, AL_t will have grown to $AL_t(1+i)$ minus $\sum_R B^j(x+1)\ddot{a}_y^{(12)} \frac{D_y}{D_{x+1}}$ withdrawn for purchase of annuities.

Therefore, if the assumptions work out, an amount equal to

$$NC_t = \left[\sum_{A_t} \Delta B^j \ddot{a}_y^{(12)} \frac{D_y}{D_x} \right] = \sum_{A_t} NC_t^j \ . \tag{5.14}$$

will have to be added at the beginning of the year in order to raise the desired fund balance to its proper level at time t+1. This amount is called the *normal cost* of the plan, because to it is the costs of keeping the pension fund at the desired level if the assumptions work out and if fund assets equal the accrued liability – i.e., the cost under “normal” circumstances. This normal cost is the present value of the increase in accrued benefits between time t and time t+i, and is a single sum assumed to be paid at time t (in fact, the normal cost is never paid at time t because the valuation is not finished as of that date and by the time you get your data together and complete the calculations, you are many weeks at best into the next year).

The normal cost is not a proper reflection of the full cost of the plan except in this idea setting: i.e., except where the fund balance is exactly equal to the accrued liability and where the assumptions are exactly borne out in reality. In real life, (a) actual experience is not exactly in accord with assumptions during a given year, and (b) the fund balance is not equal to the accrued liability – either because when the plan was started, past service benefits were granted and the accrued liability started right off at some non-zero value, or because the plan has experienced good fortune (relative to assumptions) over the years so that there are assets in the fund in excess of the accrued liability (or bad experience has produced an accrued liability in excess of assets). Therefore, although the central component of the pension cost is the normal cost, there must also be adjustment in the cost to allow for these variations from the ideal.

Let us now assume that the fund balance is equal to F_t at time t, abandoning our previous assumption that the fund is exactly equal to AL_t. During the year between time t and t+1 the fund balance will increase by some amount (I) attributable to investment return and by contributions to fund (C), and will be diminished by amounts (P) withdrawn to “purchase” pensions

$$F_{t+1} = F_t + I + C - P \ . \tag{5.15}$$

The difference $AL_t - F_t$ between the accrued liability and the fund balance at time t is called the *unfunded accrued liability*. A negative unfunded accrued liability is often called a *surplus*, but we shall use the term "unfunded accrued liability" or simply "unfunded" to refer to this quantity whether it is positive or negative.

We now subtract equation (5.15) from equation (5.13) in order to derive a relationship between the unfunded accrued liability at time t and its value at time $t+1$

$$\begin{aligned}(\textit{Unfunded Accrued Liability})_{t+1} &\equiv UAL_{t+1} = AL_{t+1} - F_{t+1} \\ &= (AL_t + NC_t)(1+i) \\ &\quad - \left[\sum_T B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - \sum_{A_t} q_x B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}}\right] \\ &\quad - \sum_R B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - (F_t + I + C - P)\end{aligned}$$

or,

$$\begin{aligned}UAL_{t+1} &= UAL_t(1+i) - (I - iF_t) + \left[NC_t(1+i) - C\right] \\ &\quad - \left[\sum_T B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - \sum_{A_t} q_x B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}}\right]. \\ &\quad - \left[\sum_R B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - P\right]\end{aligned} \tag{5.16}$$

We should be able to say that all terms but the first in equation (4.10) would be zero if all assumptions worked out and if contributions actually were equal to the normal cost, but a bit of adjustment is needed to maneuver the equation into suitable form. Let I_c represent interest on the actual contributions as the assumed rate i from the date they are actually made to year-end. For example, if the contributions were made in a single deposit at the beginning of the year

$$I_C = iC$$

and if the contribution were made in a single sum half-way through the year

$$I_C = \left[(1+i)^{1/2} - 1\right]C,$$

etc. Define a similar term I_P for pension purchase. Then we can write

$$\begin{aligned} UAL_{t+1} &= UAL_t(1+i) - (I - iF_t - I_C + I_P) - \left[C + I_C - NC_t(1+i)\right] \\ &- \left[\sum_T B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - \sum_{A_t} q_x B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}}\right] \\ &- \left[\sum_R B^j(x+1)\ddot{a}_y^{(12)}\frac{D_y}{D_{x+1}} - P - I_P\right] \end{aligned} \quad . \quad (5.17)$$

Now look at equation (5.17) and note that if the actual rate of interest earned during the year were I, then the second term would equal zero, and that if the actual accrued liability released by those who actually terminated during the year before age would equal zero, and that if the actual accrued liability released by those who actually terminated during the year before age y worked out exactly as planned, then the fourth term would be exactly zero as well. Likewise, the fifth term would be zero if the amounts withdrawn for retirement were exactly those anticipated.

The unfunded measures the deviation of the actual fund balance F_t from its ideal value AL_t, and the sum of the second, fourth, and fifth terms represents the change in unfunded due to deviations of actual from expected *experience* (as opposed to amount of contributions). We have a name for the sum of these three terms: it is called the *actual* gain, and is defined as follows

$$Gain = (UAL_t + NC_t)(1+i) - C - I_C - UAL_{t+1}\,. \quad (5.18)$$

Of course, we could just as well have defined the gain as the sum of the second, fourth, and fifth terms of equation (5.17), but these terms are more difficult to compute. Historically, the gain has always been defined by (5.18). ("Gain and Loss Analysis", however, involves the direct computation of the components of the gain using terms similar to the second, fourth, and fifth terms of equation (5.17). A "loss" is just a negative gain).

Finally, looking at the third term of (5.17), you can see that the unfunded is not expected to decrease unless the actual contribution to the fund exceeds the normal cost with interest from the beginning of the year to the date of deposit. Any additional contribution, in excess of the normal cost and interest, goes to *amortize* the unfunded. The minimum funding standards prescribed by law in the U.S. and Canada, as well as the maximum limitation on deductible contributions in the U.S. prescribe bottom and top limits, respectively, on the amount which may be added to the normal cost each year to amortize the unfunded.

5.4.3.2 Aggregate Method

Unlike the individual methods, where the normal cost and actuarial liability are determined on a participant by participant basis and then summed for the entire plan, the aggregate method is based on group calculations (i.e., the present value of future benefits for all participants is divided by the present value of future payroll or future lifetimes).

The current liability for participants in pay status at age x, having earned a benefit of B_r, is given by

$$(CL)_x = B_r \ddot{a}_x \quad \text{if } x \geq r .$$

If the participant in question terminated employment through a disability, then the annuity factor in $(CL)_x$ should reflect disabled-life mortality. The current liability for non-active participants who terminated at age z with a vested benefit of B_z and who are currently age x with benefits payable at age r is given by

$$(CL)_x = B_z \, {}_{r-x}p_x^{(m)} v^{r-x} \ddot{a}_r \quad \text{if } z \leq x \leq r.$$

The current liability for active employees is not precisely defined by status or regulations; however, the calculation must be based on an ongoing rather than a terminating plan scenario. A reasonable interpretation would be to include all ancillary benefits; however, some experts believe that some benefits (e.g. disability and death benefits) need to be included in this calculation. With the broader interpretation, the current liability for active employees can be expressed as follows

$$\left(CL\right)_x = B_x \sum_{k=x}^{r} {}_{k-x}p_x^{(T)} v^{k-x} .(q_k^{(t)}\, {}^{v}F_k + q_k^{(d)}\, {}^{d}F_k + q_k^{(m)}\, {}^{s}F_k + q_k^{(r)}\, {}^{r}F_k \quad .$$

where each F_k function represents the value of the benefit payable at each decrement.

The unfunded current liability, which is used in determining the additional funding charge for underfunded plans, is defined as

$$\left(UCL\right)_t = \max\left\{\left(CL\right)_t - \left[\left(AV\right)_t - {}^{CB}\left(FSA\right)_t\right]\right.$$

where $(AV)_t$ is actuarial value of assets at beginning of year t and ${}^{CB}(FSA)_t$ is credit balance in the funding standard accounting at beginning of year t.

5.5 Accounting for Retiree Health Benefits

The funding and accounting for retiree health benefits is an important subject for many corporations. Many corporations provide health benefits and, sometimes, life insurance benefits to employees in retirement. Unlike pension plans, where advance funding and accrual accounting have been required for decades, retiree health benefits have been funded and accounted for on a pay-as-you-go basis by nearly all employers. The increase in the number and percentage of retired employees, plus the persistent increases in health care costs, ranging from 10 to 25 percent per year, has created a financial burden for many companies. The cost associated with recovering from decades of funding and accounting on a pay-as-you-go basis will create an even more substantial financial burden in future years.

5.5.1 Economic Liabilities and Costs

5.5.1.1 Health Benefits Cost Function

The starting point in formulating the economic liabilities and costs of retiree health benefits is to determine the total expected cost of such benefits at age x, which can be represented by the following

$${}^{Total}C_x = {}^{Hosp}C_x + {}^{Phy}C_x + {}^{Lab}C_x + {}^{Drgs}C_x + {}^{Other}C_x \tag{5.19}$$

where

$^{Total}C_x$ = Total expected health benefit costs at age x,

$^{Hosp}C_x$ = Expected hospital costs,

$^{Phy}C_x$ = Expected physician costs,

$^{Lab}C_x$ = Expected laboratory costs,

$^{Drgs}C_x$ = Expected prescription drug costs,

$^{Other}C_x$ = Expected other charges (e.g., nursing home costs).

It is assumed that the health benefits cost associated with any spouse or other dependent coverage is included (1) even though the equation is expressed in terms of an individual age (1) even though the equation is expressed in terms of an individual age x.

The employer's cost is reduced by: (1) payments made by the employee under the provisions of the plan (e.g., deductibles, co-payments, annual maximums, lifetime maximums, and/or employee contributions), (2) Medicare reimbursements, (3) Medicaid reimbursements, and (4) reimbursements through coordination with other insurance carriers. Thus, the employer's cost of health benefits is as follows

$$^{ER}C_x = {}^{Total}C_x - {}^{EE}C_x - {}^{MC}R_x - {}^{Other}R_x \tag{5.20}$$

where

$^{ER}C_x$ = Employer's expected health benefits cost for employee age x,

$^{EE}C_x$ = Employee's expected cost and/or contributions at age x,

$^{MC}R_x$ = Expected reimbursement from Medicare,

$^{Other}R_x$ =Expected reimbursement from other governmental programs and/or other private insurance programs.

There are three basic methods used to integrate Medicare reimbursement under a health care plan. The employer's cost for each method is given below.

Carve-Out Method:

$$^{ER}C_x = {}^{ER}(CP)_t\left[{}^{Total}C_x - D_t\right] - {}^{MC}R_x - {}^{Other}R_x$$

Exclusion Method:

$$^{ER}C_x = {}^{ER}(CP)_t\left[{}^{Total}C_x - D_t - {}^{MC}R_x\right] - {}^{Other}R_x$$

Coordination Method:

$$^{ER}C_x = Min\begin{Bmatrix} {}^{ER}(CP)_t[{}^{Total}C_x - D_t] \\ {}^{Total}C_x - {}^{MC}R_x \end{Bmatrix} - {}^{Other}R_x \tag{5.21}$$

where

D_t = Employee deductible in year t,

$^{ER}(CP)_t$ = Employer co-payment fraction.

Of these three methods, the carve-out method produces the lowest employer costs, while the coordination method produces the highest costs.

5.5.1.2 Economic Liabilities

Assuming that the health benefit cost function is defined and projected with the appropriate assumptions, determining the economic liability is a straightforward application of the actuarial mathematics presented previously. The economic liability for an employee in retirement at age x is given by (5.22), where benefits are adjusted with interest to the end of the year

$$^{HB}a_x = \sum_{t=0}^{\infty} {}^{ER}_{\ t}C_x \, {}_tp_x^{(m)} v^{t+1} \tag{5.22}$$

where

$^{ER}_{\ t}C_x$ = Employer's expected health benefits cost at age $x+t$ for a retiree currently age x.

The economic liability for an active employee is found by prorating the present value of future benefits to the age in question. This PVFB function for an employee age x is given by

$$^{HB}(PVFB)_x = \sum_{k=m}^{r''} {}_{k-x}^{\ \ r}p_x^{(T)} q_k^{(r)} v^{k-x} \, {}_{k-x|}^{HB}a_x \tag{5.23}$$

where

m = The greater or r' or x,

${}_{k-x|}^{HB}a_x$ = Deferred health benefits annuity, as defined in (5.19) at the point of retirement, with the deferral period reflecting increases in the employer's health benefits cost from age x to each specific retirement age, but not reflecting decrements and interest, which are explicit in (5.22).

The economic liability is equal to (5.22) with a salary proration factor included

$$^{HB}(EL)_x = \sum_{k=m}^{r''} {}_{k-x}^{\ \ r}p_x^{(T)} q_k^{(r)} v^{k-x} \frac{S_x}{S_k} \, {}_{k-x|}^{HB}a_x \ . \tag{5.24}$$

As noted previously in discussing the economic liability of pension benefits, the salary-based proration is a logical choice if benefits in retirement, whether pension or health, are viewed as deferred wages. Nevertheless, some employers may wish to define the economic liability as a service proration, in which case the salary fraction in (5.24) would be replaced by *(x-y)/(k-y)*.

CHAPTER SIX
APPLICATION

The purpose of this chapter is actuarial valuation of a private foundation which has been established by one of the Turkish Bank for its employees since 1957 according to conformity with the 20^{th} transitional provision of law No.506. The aim of this foundation is to provide beneficial owners benefit not less than the benefit provided by the Social Insurance Laws, in the event of the members retirement, disablement, death, ill health, maternity, occupational accidents and illnesses; and in the illnesses of the members spouses and children and parents which the member is liable to support, within the limits of current Foundation Voucher purview.

The ultimate cost of any retirement system can be represented by the formula:

System Costs=Assets - Liabilities
Assets=Investment Returns + Plan Contributions
Liabilities=Benefits to be Paid + Administrative Expenses.

Since none of these factors are known until the last benefit is paid, they must be measured and estimated by something called an Actuarial Valuation. The primary function of an Actuarial Valuation is to determine:

- the annual amount, the actuarially computed employer and employees rate that must be paid into the plan in order to pay for current and all future benefit costs,
- the ratio of fund assets to liabilities for benefits accrued to date.

A pension plan whose assets equal its liabilities is funded at 100% and is considered fully funded; any shortfall of assets is an unfunded liability, and a plan with an unfunded liability is considered underfunded. Underfunded does not mean that a plan is unable to pay the benefits for which it is presently obligated to pay or to meet it's current cash flow requirements. It simply means that in the unlikely event that all the plan's liabilities had to be settled today, it would be unable to meet them. Fortunately, a retirement system's obligations extend many years into the future so the plan has time to accrue the assets needed to ultimately meet all it's future obligations.

Calculating the contribution rate and the Actuarial Funding Ratio involves many variables together with financial, economic, and demographic assumptions. Over the short-term, many of these assumptions will be incorrect to one degree or another. Because no one knows what the future holds, an actuary uses a mixture of professional judgment, past experience and future expectations to estimate possible future economic outcomes. Assumptions used in the Actuarial Valuation should be chosen in a way that ensures:

1. the plan is funded in an orderly and stable way and
2. the plan's accumulated annual contributions and investment returns are adequate to provide participants with their promised benefit payouts by the end of funding period.

In order to define the variables which will constitute the basis for the actuarial calculations, first the past activities of the Foundation have been analyzed and then the status has been determined according to the current 2006 Foundation Voucher purview. During the determination of the important variables such as interest, inflation and mortality values matching the realities of Turkey have been chosen and also a flexible computer program which can be adapted to changing conditions has been prepared.

In this chapter, all ready values for the future revenues and the expenditures of the Foundation with respect to 2006 fiscal year, have been calculated (in accordance with the Foundation Voucher) under definite assumptions and the differential has been compared to the fund values in the balance sheets.

In the analysis process, the revisions to be performed to eliminate the negative issues regarding the actuarial balance have been identified and some proposal for the Foundation to become more healthy and assured in fiscal and actuarial respects.

6.1 Introduction

The aim of the foundation is to provide beneficial owners benefit not less than the benefit provided by the Social Insurance Laws, in the event of the members retirement, disablement, death, ill health, maternity, occupational accidents and illnesses; and in the illnesses of the members spouses and children and parents which the member is liable to support, within the limits of Foundation Voucher purview.

In accordance with this aim, the total number of members of the foundation are 14,327 at the end of 2006. Of these members, 7.560 are active, 5.781 are retired, 77 are disabled and the rest 909 are the dependent of deceased members. The member liabilities which receive medical benefit are not included in these values. The distribution of the foundation's members according to status and sex is represented in Table 6.1.

Table 6.1 Distribution of the foundation's according to status and sex

	Male	Female	Total
Active Member	3.979	3.581	7.560
Retired Member	3.453	2.328	5.781
Disabled	50	27	77
Widow	28	546	574
Orphan	118	215	333
Father	2	-	2
Total	**7.580**	**6.670**	**14327**

6.2 The Demographic Structure of The Foundation

6.2.1 Active (Employed) Members

Those who have been appointed to the staff of the bank are obliged to be affiliated with the Foundation. A written statement, determined by the management, is taken from these members at the beginning of the employment and affiliation to the foundation is performed. All the distributions of the current active members' age, sex, averages, and standard deviations in the gender groups are as presented in Table 6.2 and Figure 6.1.

Table 6.2 Number of active members according to age group and sex

Age	Male	Female	Total
<25	77	219	296
25-30	559	534	1.093
30-35	737	833	1.570
35-40	997	1.265	2.262
40-45	1.286	654	1.940
45-50	293	70	363
50-55	29	6	35
55-60	1	0	1
60>	0	0	0
Total	3.979	3.581	7.560
Average Age	37,4	35,1	36,3
Standard Deviation of Age	6,0	6,0	6,0

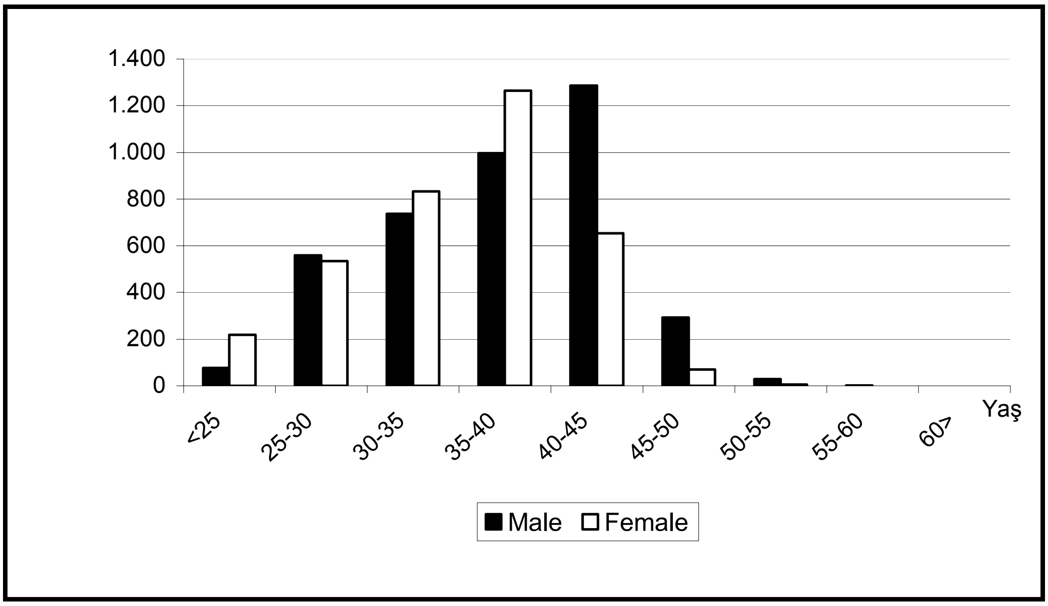


Figure 6.1 Distribution of age and sex of active members.

Considering the future service durations of the current active members, 94 members are continue to work although they gain right to retire. The number of the members who will have the right to retire will be 311 people in one year. The number of the active members to retire according to years is given in Table 6.3.

Table 6.3 Number of future retired members according to years

Year	***Cumulative***	***Frequency***
Retired	94	**94**
1	311	**217**
2	537	**226**
3	800	**263**
4	1.070	**270**
5	1.341	**271**
10	2.910	**1.569**
15	3.998	**1.088**
20	4.946	**948**
25	6.010	**1.064**
30	6.483	**473**
35	7.458	**975**
40	7.560	**102**

6.2.2 Passive (Retired) Members

Retirement age is 58 for women, 60 for men and at least 7.000 days or 58 for women, 60 for men and at least 25 years of insurance and members with at least 4.500 days of paid disablement, or old age insurance premium may retire at their will. The distribution of age and sex of all the passive members who has gained right to retire is as in Table 6.4 and Figure 6.2.

Table 6.4 Number of passive members according to age group and sex

Age	**Male**	**Female**	**Total**
<35	0	0	0
35-40	9	10	19
40-45	52	232	284
45-50	601	731	1.332
50-55	1.108	684	1.792
55-60	744	371	1.115
60-65	442	151	593
65-70	248	74	322
70>	249	75	324
Total	**3.453**	**2.328**	**5.781**
Average Age	**56,6**	**52,6**	**54,6**
Standard Deviation of Age	**8,0**	**7,8**	**8,0**

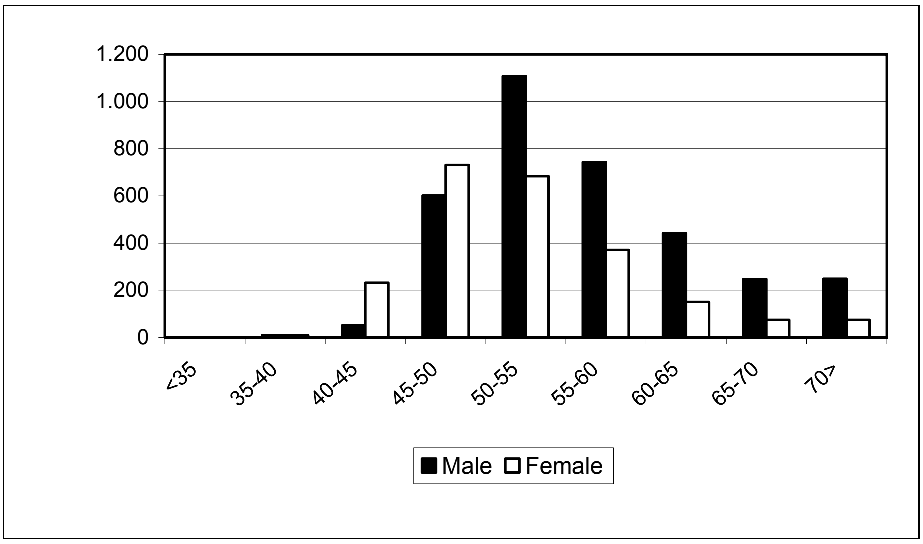


Figure 6.2 Distribution of age and sex of retired members.

6.2.3 Disabled Members

In order a member to be acknowledged as disabled, s/he should have lost at least 2/3 of the labour power or should be incapable of performing the duty due to incurable illnesses. The ones who lost 60% of their labour power due to occupational accidents or illnesses will be acknowledged disabled. The age and sex information and the age averages and standard deviations of the 77 disabled members are presented in Table 6.5.

Table 6.5 Number of disabled members according to age group and sex

Age	**Male**	**Female**	**Total**
<35	0	2	2
35-40	4	6	10
40-45	6	3	9
45-50	6	5	11
50-55	17	4	21
55-60	12	3	15
60-65	3	3	6
65-70	2	0	2
70>	0	1	1
Total	**50**	**27**	**77**
Average Age	**51,7**	**48,2**	**50,5**
Standart Deviation of Age	**7,8**	**7,7**	**7,8**

6.2.4 Dependent Members

The beneficial owners of the member who dies while receiving disablement or retirement salary or the member who has been insured for 900 days or 5 years total and dies, are paid widow-and-orphan's pension the next month following the date of death. The age, sex, age averages and standard deviations of the members who have gained the right to receive widows' pension are given in Table 6.6. The similar information on the members who have the right to receive orphan's pension is presented in Table 6.7.

Table 6.6 Number of widow members according to age group and sex

Age	Male	Female	Total
<40	2	21	23
40-45	0	34	34
45-50	2	52	54
50-55	2	71	73
55-60	5	81	86
60-65	7	65	72
65-70	5	68	73
70-75	3	73	76
75-80	2	44	46
>80	0	37	37
Total	**28**	**546**	**574**
Average Age	**61,2**	**61,4**	**61,3**
Standart Deviation of Age	**21,9**	**21,9**	**21,9**

Table 6.7 Number of orphan members according to age group and sex

Age	Male	Female	Total
0-18	**54**	**53**	**107**
18-25	**56**	**63**	**119**
25>	**8**	**99**	**107**
Total	**118**	**215**	**333**
Average Age	**44,5**	**47,8**	**47,7**
Standart Deviation of Age	**21,1**	**21,9**	**22,0**

If the sum of the salaries for the spouse and children is less than the salary of the member, the remainder is paid in equal shares to the parents who are documented to be supported by the member. The liabilities of the members and the number of the members benefiting from the medical benefit in this respect are given in Table 6.8. The total members are 32.000 people if we are to think the beneficiary owners and the members benefiting from the medical benefit together.

Table 6.8 Number of dependent members according to status

Active Member	Male	Female	Total
Spouse	2	1.839	1.841
Children	4.107	4.016	8.123
Mother/Father	82	168	250
Passive Member			
Spouse	30	3.149	3.179
Children	2.176	2.886	5.062
Mother/Father	79	264	343
Total	**13.814**	**18.037**	**31.851**

6.3 Revenues of The Foundation and Benefits Provided to The Members

6.3.1 Revenues

The necessary sources of income, for the Foundation to fulfill the liabilities to the members, are the insurance premiums, affiliation premiums, promotion differential and other revenues.

6.3.1.1 Premiums

Retirement (old age, disablement and death) and illness premiums of the proportions defined as employee share are collected every month from the salaries subject to the premium. Also, retirement (old age, disablement and death), occupational accidents and illnesses, illness and maternity premiums of the proportions defined as employer share. The percentage of monthly salaries paid as premiums by employer and employee according to benefits are given in Table 6.9.

Table 6.9 The percentage of monthly salaries paid as premiums by employer and employee according to benefits

	Employer	Employee	Total
Occupational Accident And Disease	1,5%	-	1,5%
Illness	6,0%	5,0%	11,0%
Maturity	1,0%	-	1,0%
Disabled, Old-Age, Death	11,0%	9,0%	20,0%
Total	**19,5%**	**14,0%**	**33,5%**

The premium to be collected from the members and the Bank is calculated according to the proportion of the sum of justified salary and other payments up to maximum salary. The amount exceeding this top limit is not subject to the premium.

6.3.1.2 Affiliation Premium and Promotion Differential

25% of the monthly salaries are collected as affiliation premium and %50 of wage increments are collected as promotion differential from the foundation's members without the maximum limitation. The affiliation premium is collected from the first full salary and the promotion differential is, from the first incremented salary. The same amount is paid by the Bank also. The retirement premiums, affiliation premiums, promotion differentials and the other retirement revenues (occupational accident dues, ex-officio retirement indemnifications) for the last 4 years are given in Table 6.10.

Table 6.10 Premium income for last 4 years (million YTL)

Premium and Income	***2006***	***2005***	***2004***	***2003***
Disabled, Old-age and Death Insurance	46.570	41.300	36.158	29.057
Health Insurance	25.658	22.754	19.607	15.739
Maternity Insurance	2.332	2.068	1.782	1.430
Occupational Diseases and Accident Insurance	3.499	3.102	2.673	2.146
Wage increments	907	986	458	920
Promotional	1.345	1.460	679	1.364
Affiliations	346	50	23	27
Other	558	521	38	395
Total	**81.219**	**72.244**	**61.423**	**51.082**

The revenues, as all kinds of revenues came by actuating the Foundation's funds, renting and bank deposits, and investment income, other then the retirement revenues are given in Table 6.11.

Table 6.11 Fund income for last 4 years (million YTL)

	2006	***2005***	***2004***	***2003***
Premium Income	81.219	72.244	61.423	51.082
Out of premium income	182.274	296.177	113.760	155.245
Total	**263.494**	**368.421**	**175.183**	**206.328**

6.3.2 Benefits

The following benefits under the terms and conditions defined in the Foundation Voucher are provided to the members of the foundations and their spouses and children and the parents which the member is liable to support:

- Retirement, Disablement and Death Benefits
    - Retirement salary
    - Disablement salary
    - Widow's pension
    - Orphan's pension
- Medical Benefits
    - Illness benefit
    - Illness benefit to spouses and children
    - Illness benefit to parents
    - Illness benefit to disabled and retired
- Maternity benefit
- Death and funeral expenses benefits
- Occupational accidents and illnesses benefits.

6.3.2.1 Retirement, Disablement and Death Benefits

In the calculation of the retirement, disablement salaries and widow-and-orphan's pensions, co-efficient based indicator system applies. Indicator and indicator determination tables are organized by the management as of 200 digits with 20 degrees each having 10 grades.

- **Organization of the Indicator Table**

The values used in the Indicator Table which came into effect in 31.03.2001 are taken as follows:

Maximum salary subject to premium	=	750 YTL
Minimum salary subject to premium	=	175 YTL
Co-efficient	=	18.240
Maximum Indicator	=	41.118
Minimum Indicator	=	9.595.

The difference between the maximum and the minimum indicator values is divided to the digit value in the indicator table minus 1, so the difference that should exist between the indicator values in every grade is found. Using this difference, the indicator values for each degree and grade are confirmed, thus the indicator table is made up.

(Maximum Indicator – Minimum Indicator) / (200-1) =(18.240-41118)/199=158.4.

In case, the in-effect maximum monthly salary is incremented, the new co-efficient with the new maximum salary is determined by the management.

- **Organization of Indicator Determination Table**

For the indicator determination tables corresponding to the salaries as of 01.04.2001;

a) The average of the maximum salaries in effect, in respect of the Foundation Voucher for 10 years past from each month’s last year; and the average of the minimum salaries in respect of No.506 Law in the same period of time are determined,

b) The average of the maximum salaries constitutes the maximum value of the indicator determination table and the average of the minimum salaries the minimum value,
c) The difference between the two average values is divided to the digits – 1 in the indicator determination table, so the difference that should exist between average salaries in every grade from the maximum salaries to the minimum salaries is found. Thus, using this difference, the annual average salaries for each degree and grade are determined and the indicator determination table is formed,
d) By starting from 01.04.2002, while re-organising the indicator determination tables to be used, the number of the years to be included in the calculation of the annual maximum and minimum salary averages is augmented each for 1 year. The 15 years salaries average used in the June 2006 dated indicator determination table is as follows:

Average of Maximum Salaries = 10.727,24
Average of Minimum Salaries = 1.994,19
(Average of Maximum Salaries – Average of Minimum Salaries) / 199 = 43,88.

- **Determination of The Co-efficient**

The co-efficient is calculated by dividing the current maximum salary to the maximum indicator value in the indicator determination table. Beginning from 01.04.2001, the co-efficient values are used according to the aforementioned article. The co-efficient value is 47,726 of June 2006.

- **Salary Subject to Allotment**

Regarding the salaries to be pensioned, the product of member's indicator value in the indicator table organized by the management, by the co-efficient is subject to salary calculation. This is called salary subject to allotment.

- **Salary Pensioning Rates**

Salary pensioning rate is the sum of member's retirement (old age, disablement, death) premiums, 3.5% per year in the first 10 year period, 2% per year in the following 15 year and 1.5% per year for following years. According to the Foundation Voucher, the salary pensioning rate cannot be less than 60% for members have the right for salary. Salary pensioning rates of all retired members and sex are presented in Table 6.12. The average of salary pensioning rates for male members is 62% and for female members, 55%.

Table 6.12 Average and standard deviation of salary pensioning rates of all retired members

Age	**Male**	**Female**
Average	61.9 %	54.7 %
Standart Deviation	2.0 %	2.2 %

- **Calculation of The Salary**

Following the Foundation Voucher purviews, for the calculation of the salaries, the average of the salaries subject to premium, for the last 10 year from the last retirement premium paid month, is calculated for the members demanding salary pensioning or who are ex-officio retired. In the calculation of the average, month up to ten year are taken in to account. The years to be added to the calculation of salaries subject to premium are increased by one for each year as of 01/04/2002. However, in case the period any member has covered paying the retirement premium is less than the years to be taken into consideration according to this clause, the salaries subject to premium in the premium paid period are calculated.

From the indicator determination table, the digit closer to the average of the salaries subject to premium shows the member's degree and grade. In case, any member's average salaries are below the lowest value in the table, the first grade of the lowest degree is acknowledged as the member's degree and grade. From the indicator table, the indicator value matching this degree and grade is determined and multiplied by the current co-efficient.

$$\text{Indicator} \times \text{Co-efficient} = \text{Salary subject to allotment}$$

The salary pensioning rate, which is calculated following the member's actual service period, is multiplied by the value of salary subject to allotment, so the retirement salary is determined.

$$\text{Retirement salary} = \text{Salary subject to allotment} \times \text{Salary pensioning rate}$$

According to the calculations detailed above, the calculation of current retired members' salaries' average and standard deviation are presented in Table 5.12.

Table 6.13 Average salaries for passive members (YTL)

	Male		Female	
	Mean	**StDev**	**Mean**	**StDev**
Retired	1.209	0.200	1.077	0.164
Disabled	1.102	0.182	1.133	0.196
Widow	701	0.167	778	0.222
Orphan	288	0.103	328	0.145

The disablement salary pensioning rate is 60% of the members' salaries subject to allotment; %70 for the ones' in need of care by others. The average and standard deviation of the current disabled members' salaries is given in Table 6.13.

50% of the determined retirement salary is paid to the widow. If there aren't any children receiving salary, this rate becomes 75%. If the widow remarries, the salary is discontinued. 25% of the determined retirement salary is paid to each child. The sum of the salaries to be paid to the beneficiary owners, spouses and children, cannot exceed the sum of the salaries paid to the insured. In order not to exceed this limit, some discounts are made from the salaries of the beneficiary owners. For male children to receive this salary, the beneficiary owner must be under 18 years old, if s/he is attending secondary education 20 years old, if s/he is at the higher education 25 years old. However, if the child is disabled, age class is not sought. For female children to have the right to receive orphan's pension, they should be single, widow or divorced and should not be employed. If the sum of the salaries to be pensioned to the spouse and the children is less than the salary of the member, the remainder is pensioned equally to the parents that have been proved to be supported by the member. Each share for the parents cannot exceed 25% of the member's salary. The averages and standard deviations of paid widow-and-orphan's pension according to the Foundation Voucher are given in Table 6.13.

The operation expenses of the foundation for the last 4 years are presented in Table 6.14, and the comparison of the revenues and expenses are given in Table 6.15.

Table 6.14 The operation expenses of the foundation for the last 4 years (million YTL)

	2006	*2005*	*2004*	*2003*
Pension	72.910	64.891	56.602	47.698
Disabled	978	859	780	673
Widow	4.852	4.130	3.471	2.897
Orphans	1.209	1.026	881	738
Benefit	5	5	2.659	10.358
Married payment for female orphans	51	50	60	13
Payment of abroad	44	33	23	19
Fellow pension	1.589	1.592	1.412	923
General Management	3.764	2.293	1.659	1.326
Member Health	15.309	14.422	11.902	11.751
Member Dependent Health	13.566	12.326	12.332	10.449
Maturity	29	1	12	8
Occupational Accident	28	13	9	8
Center of Health	316	246	235	118
Total	**114.655**	**101.894**	**92.042**	**86.987**

Table 6.15 The comparison of the revenues and expenses for last 4 years according to years

Increase of Outcome (%)	13%	11%	6%	-
Increase of Income (%)	-28%	110%	-15%	-
Increase of net term benefit	-44%	221%	-30%	-
Premium Income / Total of Outcomes(%)	71%	71%	67%	59%
Income / Outcome	230%	362%	190%	237%
Premium Income (2)	81.219	72.244	61.423	51.082
Out of Premium Income	182.274	296.177	113.760	155.245
Total of Incomes (3)	**263.494**	**368.421**	**175.183**	**206.328**
Net gain of term	**148.836**	**266.527**	**83.141**	**119.340**

6.3.2.2 Medical Benefits

The medical benefits, provided in cases of members' illnesses are shown below:

- Providing medical examination and treatment benefits,
- Providing medical prosthesis material,
- In required situations, sending other domestic facilities for examination and treatment,
- In cases when treatment is not possible in domestic facilities, sending abroad.

In order a member to utilize these benefits, s/he should have paid premium for at least 90 days in the previous year of the year the illness is discovered. The benefits provided in cases of illnesses continue until the recovery. However, these benefits are effective for the next 6 months period from the beginning of the illness. If the treatment continues, and if it is understood, with a medical commission report, that it is possible to prevent or reduce disablement this period will be extended. The benefits provided for the spouses, children and parents are subject to same conditions.

The benefits provided in coverage of maternity insurance are as follows:

- Providing pregnancy examination,
- Providing necessary medical benefits during delivery,
- Providing breastfeeding benefit money,
- For maternity reasons, sending to another domestic place.

In respect to years, the values pertaining to the medical benefit revenues, expenses and increase rates are presented in Table 6.16 and Figure 6.3.

Table 6.16 The values pertaining to the medical benefit revenues, expenses and increase rates

	2006	*2005*	*2004*	*2003*
Health Insurance Premium (million YTL)	**25.658**	**22.754**	**19.607**	**15.739**
Outcome of Member Health (million YTL)	**15.309**	14.422	11.902	11.751
Outcome of Dependent Health (million YTL)	**13.566**	12.326	2.332	10.449
Total of Outcome of Health (million YTL)	**28.875**	**26.748**	**24.235**	**22.200**
Average of Member Health Outcome (million YTL)	***2***	***1***	***1***	***1***
Income / Outcome	89%	85%	81%	71%
Average of Increasing of Annual Health Outcome	4%	9%	9%	-

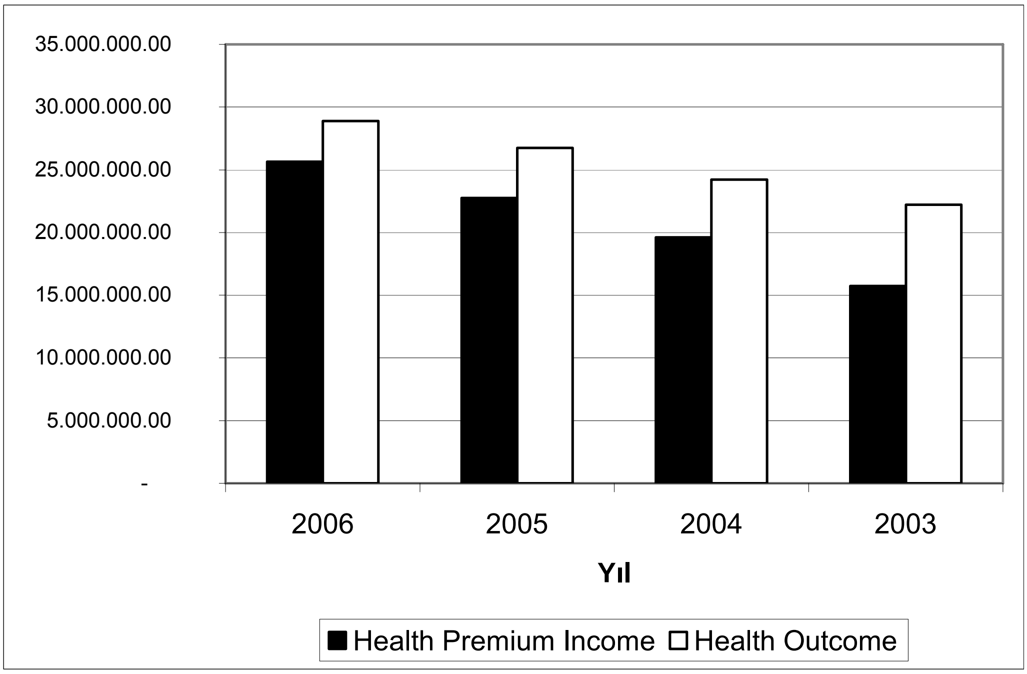


Figure 6.3 The medical benefit revenues and expenses.

6.4 Actuarial Valuation

Actuarial valuation is based upon taking an interest assumption and comparing the difference between the present value of future payments to be done to members and their dependents which the foundation is liable according to the foundation voucher purviews and general management expenses, and the present values of future collection of premiums with the funds in hand.

With the assumed interest rate, in case the sum of the expenses is more than the sum of revenues according to the actuarial calculations of the foundations revenues and expenses, the assumptions should be revised in order the foundations current ready money and realty values to be equal to this difference.

6.4.1 Variables

To calculate present value of the future aids and revenues, information such as members'

- Number,
- Status,
- Sex,
- Age,
- Salary subject to premium,
- Degree and grade,
- Salary pensioning rate,
- Past employment period,
- Fund membership period,
- Future service period etc.

should be obtained.

The statistical information about the entire active, passive, disabled, widow and orphans were collected in the previous section. In addition to this information, the statistical information such as fund membership, total service period and the future service period of the active members are presented in Tables 5.16. Number of days of member foundation, other foundation service and total service period for retired members are also presented in Table 6.17.

Table 6.17 Service time in days for active members and retired members

ACTIVE	Male		Female	
	Mean	StDev	Mean	StDev
Previous	4.574	2441,29	4.431	2124,9
Future	5.777	3721,36	5.324	3692,71
Member of Foundation	4.257	2222,86	4.289	2067,02
RETIRED				
Member of Foundation	7.476	1717,67	6.550	1399,71
Other foundation Service	943	1342,63	560	1139,11
Total Service Period	8.419	1168,24	7.109	854,89

The maximum salary subject to premium calculations as of 31.12.2006 is 3.451,50 YTL. and the minimum salary is 531 YTL. The average of salaries subject to premium for male members is 2.586 YTL and for the female members, 2.674 YTL. Averages and standard deviations of salary pensioning rates, rates of the salaries subject to premium, salaries subject to allotment and retirement salaries are calculated in respect to the principles defined in the foundation voucher and presented in Tables 6.18 for active members.

Table 6.18 Various average rates and values for active members

ACTIVE	Male		Female	
	Mean	StDev	Mean	StDev
Salary Pensioning Rate (%)	70.60	6.60	68.10	9.50
Salaries subject to premiums (YTL)	2.586	1.80	2.674	1.80
Salaries subject to allotment (YTL)	1553	451	1605	449
Pensions (YTL)	1097	184	1094	176

6.4.2 Assumptions

For performing the actuarial calculations, the next step to be taken is to make some consistent assumptions about the variables given below and to incorporate some of the valid values in practice;

- Mortality,
- Interest rate,
- Payroll,
- Maximum salary increment rate,
- Salary subject to allotment rate,
- Premium assessment,

- Salary subject to premium rate
- Employee's contribution rate
- Employer's contribution rate
- Salary pensioning rate

6.4.2.1 Mortality

The present values of the retirement salaries are calculated using the CSO (Commissioners' 1980 Standard Ordinary Mortality Table) 1980 Women and Men Mortality Tables (Appendix A). The reason for using CSO 1980 Table is that it gives an approximate value to the average life expectancy estimation made by Turkish Statistical Institute (TSI) in the beginning of 2000's based on population data. The estimation values of TSI and CSO 1980 according to sex are presented in Table 6.19.

Table 6.19 Average life expectancy of TSI and CSO 1980 according to sex

Year	1990 (TSI)	2000(TSI)	2005(TSI)	CSO 1980
Female	69.0	71.5	72.7	75.3
Male	64.4	66.9	68.0	70.3
Average	66.7	69.1	70.3	72.8

CSO 1980 Women and Men Mortality Table is used and recommended the table by private life insurance and retirement companies and Ministry of Labour and Social Security in the actuarial calculations of retirement insurances.

6.4.2.2 Real Interest Rate

The most important variable that will be used in the actuarial calculations is the determination of the technical (real) interest rate. As a result of the economical package's success, a rapid decrease in the real interest rates parallel to the inflation has become. But, when we look at the foundation's past funds management performance and current economical improvements, it is quite difficult to suggest an average real interest rate for long term. It is not so possible to anticipate the current

10% level real interest rate for a long term. In the following years, in current conditions it will be somewhat difficult to get these high real yields. It is meaningful to use low real interest rates in long term averages, but in short term, a graded decrease should be expected. Taking the structure of funds management and portfolio of the foundation into consideration, the obtained real funds income for last 4 years is given in Table 6.20.

Table 6.20 Pensions of active members (million YTL)

Year	*Active Total*	*Outcome-Income of Aim*	*Fund Income*	*Consumer Price Index*	*Real Income Fund*
2003	483.738	-35.904	-	-	
2004	593.252	-30.619	29.0%	9.3%	19.7%
2005	1.040.986	-29.649	80.5%	7.7%	72.8%
2006	1.190.259	-33.437	17.6%	9.65%	7.9%

In this study, in addition to the investigation of circumstances where the real interest rate is fixed, a stochastic investigation is performed. An actuarial valuation is also made under the assumption that the real interest rate is lognormal during the investigated period.

6.4.2.3 Dues and Salary Increments

A salary increment measure (s_x) has been constituted in order to reflect the dues and the real increments on the salaries into the actuarial calculations. Considering the rapid deteriorating assets/liability and actuarial balance of the foundation and the economic realities of the country, an increment below the inflation rate is expected in the retirement salaries and dues for a couple of years.

6.4.2.4 Other Members

By regaining a healthy accounting infrastructure, the revenues and expenses regarding current joint-retired members of the Foundation are given in Table 6.21 and the joint-retired members’ revenues are far more above the expenses. The information about the members who departed due to resignation or dismissal and the members who come from other institutions and become a member and the ones who pay the premium from outside as a voluntary member are given in Tables 6.22-6.24.

Table 6.21 Outcome and income of retired members based from other foundations

Foundation	Retired Income	Retired Outcome
T.C. EMEKLİ SANDIĞI	2.499	298
SSK	2.702	1.171
BAĞKUR	200	76
Other	215	43
Total	**5.618**	**1.589**

Table 6.22 Average of service period of withdrawal members

Number of Person	Average of Service Years
4.136	5.5

Table 6.23 Service period of other foundation of active members

Number of Person	Average of Service Years
2.457	2.1

Table 6.24 Voluntary members

	Male	Female	Total / Average
Number of Person	18	94	112
Average age	44	39	38
Ensurable Earnings	1.629	1.189	1.260

6.5 Actuarial Revenues and Expenses Calculations of The Foundation

After the identification of number of active and passive member numbers, the ages of these members, future service periods, employee and employer shares and salary pensioning rates, the present values of the foundations basic revenues and expenses entries are calculated according to 5% technical real interest rate separately for sex groups. In this study, the aim in choosing the real interest rate as 5% and lognormal distribution of interest rate are to see the effects of changes in the real interest rates in actuarial calculations.

Revenue:

The main revenue entry of the foundation, the employer's share along with the members' premium are calculated separately for men and women, taking into consideration each member's age, sex, future service period and payment will be monthly.

Expense:

The most important expenses of the foundation is the retirement salaries to be paid to the active members when they retire and the salaries of the passive members who are being paid already. Therefore, the present value of all the members' retirement salary is calculated separately for current active and passive members. Likewise the variables taken into consideration in calculating the revenue entries, the members' age, sex, past service period, membership period, future service period, salary pensioning rate and the payment will be monthly are considered and the total present value of the retirement salaries are calculated.

In this study according to calculated service times and average age of active members, average of salary pensioning rate is calculated. After this, present values of pensions are obtained using CSO 1980 Male and Female Table are given in Appendix A and B.

6.6 Other Revenues and Expenses

The other revenues and expenses entries such as retired expenses, medical expenses, general management expenses, other retired revenues, medical revenues, distinct from the foundation's actuarial modeled revenues and expenses entries which are the premium revenues and retirement salaries, are estimated by relating the previous years' revenues and expenses entries.

6.7 Actuarial Balance

6.7.1 Actuarial Valuation Methods

In this section, first the actuarial valuation of the foundation's benefit liabilities provided by the Fund is done according to the methods and assumptions Aggregate Cost Method and then the actuarial valuation of all the benefits provided by the Fund is done according to the methods and assumptions of Unit Cost Method. In this study, initially actuarial valuation is done under fixed interest rate assumption and then stochastic interest rate assumption.

6.7.1.1 Aggregate Cost Method (Fixed Interest Rate Assumption)

The basic assumptions to be used in the actuarial valuation of revenues and expenses according to aggregate cost method are as below;

- Real Interest Rate 5% (fixed),
- CSO 1980 Mortality Table,
- Annual Real Wage Increment 0%,
- Annual Real Salary Increment 0%,
- Medical Benefits, Annual medical expenses of SSK per head.

In the framework of these assumptions, all the operating revenues and expenses are calculated in accordance with foundation Voucher purviews. If Foundation Voucher purviews are used in calculating the general management expenses, 3% of the liabilities are reserved of expenses.

In the calculations according to Foundation Voucher purviews, average annual medical expense 2,015 YTL per member is used; this value is the average annual medical expense of the foundation per member as of the end of 2006.

The actuarial valuation of the Foundation is made in accordance with aggregate cost method, first according to the assumptions of transfer principles. The results pertaining to these calculations are re-made under same assumptions, only changing the real interest rate; the rates in which the foundation ensures the balance are determined. Calculations of each present value of assets and liabilities in actuarial balance sheet are given below.

i. *Present value of pension premium for active members (Liability of foundation)*: According to average age, past and future service time, and average age, present values of premiums which will be received at future are obtained and these present values are given in Table 6.25 (Appendix D,E,I). Present values of pensions for active members are calculated by ${}_{n|}\ddot{a}_x = \dfrac{N_{x+n}}{D_x}$. Pensions are paid every month; therefore, present values of pensions are converted "payable monthly deferred annuity" are used (Menge and Fisher, 1965).

$$ {}_{n|}\ddot{a}_x^{(m)} = {}_nE_x\left(\ddot{a}_{x+n} - \frac{m-1}{2m}\right). $$

Commutations functions used in calculations of present values are multiplied by salary scale. Calculated these values are multiplied by annually quantity of salary subject to allotment and calculated rate of monthly salary pensioning rate. Thus, present values of pensions for active members which will be paid at future are calculated. After this, these values are multiplied by total number of members.

Table 6.25 Present values of pensions for active members

	Average Age	Number of Members	Foundation Service Time	Average of Past Service Time (Year)	Average of Future Service Time (Year)	Salary Pensioning Rate (%)	${}_{n\|}\ddot{a}_x$	${}_{n\|}\ddot{a}_x^{(m)}$	Present Value of Total Pensions
Male	37	3979	12	13	16	71%	5,70	5,51	288.266.171
Female	35	3581	12	12	15	68%	7,54	6,77	318.420.922
Total									606.687.093

ii. *Present Value of Pensions for Retired Members (Liability of foundation)*: Salary pensioning rates of retired members according to average age are evaluated. Present values of pensions which will be paid at future are calculated. These present values are given in Table 6.26 (Appendix F,G,J). Present values of pensions for retired members are calculated by $\ddot{a}_x = \dfrac{N_x}{D_x}$. Pensions are paid every month; therefore, present values of pensions are converted "payable monthly whole life annuity" are used (Menge and Fisher, 1965).

$$\ddot{a}_x^{(m)} = \left(\ddot{a}_x - \frac{m-1}{2m} \right)$$

Commutations functions used in calculations of present values are multiplied by salary scale. Calculated these calculated values are multiplied by annually quantity of salary subject to allotment and calculated rate of monthly salary pensioning rate. Thus, present values of pensions for retired members which will be paid at future are calculated. After this, these values are multiplied by total number of members.

Table 6.26 Present values of pensions for retired members

	Average Age	Number of Members	Salary Pensioning Rate (%)	Number of Members	$\ddot{a}_x$	$\ddot{a}_x^{(m)}$	Present Value of Total Pensions (YTL)
Male	57	3453	62%	3453	12,3	11,9	591.743.496
Female	53	2328	55%	2328	14,7	14,2	428.477.081
Total							1.020.220.577

iii. *Present Value of Disability Pensions (Liability of Foundation)*: Total number of members is 14,327 people at the end of 2006. Of these members, 7.560 are active, 5.781 are retired, 77 are disabled. According to average age and salary liabilities of current disabled members are given in Table 6.27. Present values of disability pensions are calculated by

$$\sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)} v^{k-x}\, {}^{d}(TC)_k \qquad (6.1)$$

$${}^{d}(TC)_k = g_x^{(d)} B_x q_x^{(d)}\, {}_{w}^{d}p_{x+1}^{(m)} v^{w+1} \ddot{a}_{x+w}^{d}$$

$g_x^{(d)}$; grading function equal to the proportion of accrued benefit provided if disability occurs during age x,

$q_x^{(d)}$; probability of becoming disabled during age x,

w ; waiting period before disability benefits commence,

${}_{w}^{d}p_{x+1}^{(m)}$; probability that a disabled life age x lives w years,

$\ddot{a}_{x+w}^{d}$; life annuity based on disabled – life mortality.

$$\sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)} v^{k-x}\, {}^{d}(TC)_k = 21.745.616 \text{ YTL.}$$

Commutations functions used in calculations of present values are multiplied by number of disabled members and annual salary. The probability of disabled member is 0.08. This probability is obtained by SSK. Present values of disability pensions which will be paid disabled members is calculated by equation (6.1) (Appendix Q,R,S).

Table 6.27 Present values of disability pensions

Current	Count	Average Age	Present Value	Salary	Liability
Male	50	52	13,67	1.102	9.035.312
Female	27	48	15,74	1.133	5.778.014
Total	77				14.813.326

iv. *Present Value of Widows and Orphans Pensions (Liability of foundation)*: Total members of the foundation are 14,327 people at the end of 2006. Of these members, 7.560 are active, 5.781 are retired, 574 are widowed, and 333 are orphans. According to average age and salary liabilities of current widowed members are given in Table 6.28. Present values of widow pensions are calculated by

$$\sum_{k=x}^{r-1} {}_{k-x}p_x^{(T)} v^{k-x} \, {}^{s}(TC)_k \tag{6.2}$$

$${}^{s}(TC)_k = M \, g_x^{(s)} B_x \, q_x^{(m)} \, v \, \ddot{a}_{x+u+1}$$

M ; probability that the participant has a surviving spouse at death,

$g_x^{(s)}$; grading function equal to the proportion of accrued benefit provided to a surviving spouse if death occurs during age x,

$q_x^{(m)}$; probability of dying during age x,

u ; number of years that, when added to the participant's age, yield an assumed age for the surviving spouse,

$\ddot{a}_{x+u+1}$; life annuity based on the spouse's age at the death of the participant.

To calculate widow's liabilities, commutations functions used in calculations of present values are obtained that number of widow members multiplied by annual salary. There are additional assumptions in this section. These assumptions are given below;

- wife's age is smaller 3 years than husband's ages,
- widow members are paid widow pensions during 20 years.

Under these assumptions, present values of widow pensions which will be paid widow members are calculated by equation (6.2). Then, obtained present values of widow pensions are multiplied by number of widow members and salary. In this point, security loading is 10% and included calculations. General total of present values of widow pension is obtained that present values of current widow pension is summation of future widow pension (Appendix W,X,Y).

Table 6.28 Present values of widow pensions

Current	Count	Average Age	Present Value	Salary	Liability
Male	28	61	11,15	701	2.625.877
Female	546	61	12,68	778	64.704.130
Total					67.330.007

According to average age and salary, liabilities of current orphan members are given in Table 6.29 (Appendix T,U,V).

Table 6.29 Present values of orphan pension

Current	Count	Average Age	Time	Present Value	Salary	Liability
Male	118	18	7	6,74	288	2.747.639
Female	215	27	3	3,72	328	3.149.844
Total						5.897.484

v. *Present Value of Consociate Pensions (Liability of foundation)*: Average of consociate rate is 2.3%. This average rate is evaluated from last 4 years. Present value of consociate pensions is obtained that this rate multiplied by present values of retired pensions.

vi. *Present Value of Voluntary Pensions (Liability of foundation)*: Voluntary rate is 1.5%. This rate is evaluated that all of number of the voluntary members divided by all foundation members. Present value of voluntary pension is that present value of active pensions multiplied by voluntary rate.

vii. *Present Value of Withdrawal Pensions (Liability of foundation)*: Withdrawal rate is 12%. This rate is evaluated by 2005 foundation data. Present value of withdrawal pension is obtained that present value of active pensions multiplied by withdrawal rate.

viii. *Present value of other outcomes (Liability of foundation)*: Other outcomes are maturity, occupational decease, funeral of members, marriage of male orphans, abroad salary. Present value of other outcomes is calculated by rate of these outcomes and related rates.

ix. *Present Value of Outcome of Health (Liability of foundation)*: According to average age and annual average of expensive outcome of health of active and retired members are given in Table 6.29. Present values of outcomes of health for active members are calculated by ${}^{HB}a_x = \sum_{t=0}^{\infty} {}^{ER}_{t}C_{x\ t}p_x^{(m)} v^{t+1}$ where ${}^{ER}_{t}C_x$ is employer's expected health benefits cost at age $x + t$ for a retiree currently age x (Appendix L,M,N,O,P). Annual average of expensive is 2015 TL for active and retired members. Cost of outcome of health for active members is obtained that number of active members multiplied by annual average of expensive and present values of outcomes of health for active members.
Present value of outcomes of health for retired members is calculated by

$$ {}^{HB}(PVFB)_x = \sum_{k=m}^{r''} {}_{k-x}^{\ r}p_x^{(T)} q_k^{(r)} v^{k-x}\ {}_{k-x|}^{HB}a_x $$

Cost of outcome of health for retired members is obtained that number of retired members multiplied by annual average of expensive and present values of outcomes of health for retired members.

Table 6.29 Present values of outcomes of health

Current	**Male**	**Female**
Active		
Average of Age of Active Members	37	35
Number of Active Members	3979	3581
Annual Average of Expensive	2015	2015
Present Value	26,09	28,98
Cost	209.203.403	209.108.343
Total	418.311.749	
Passive		
Average of Age of Retired Members	57	53
Number of Retired Members	1305	2355
Annual Average of Expensive	2015	2015
Present Value	16,35	18,37
Cost	42.984.823	87.177.299
Total	130.162.122	
General Total	548.473.867	

x. *Present Value of General Management Outcomes (Liability of foundation)*: Present value of management outcomes is 3% of present values of all outcomes.

xi. *Assets on hand (Asset of foundation)*: Assets of foundation are 1.190.259.947 YTL. Assets of foundation are income of immovable goods, investment income etc.

xii. *Present Value of Pension Premium (Asset of foundation)*: According to average age and future service time present values of premiums which will be received at future are obtained and these present values are given in Table 6.30. Present values of premiums for active members are calculated by

$\ddot{a}_{x:\overline{n}|} = \dfrac{N_x - N_{x+n}}{D_x}$. Premiums are paid every month; therefore, present values of premiums are converted "payable monthly temporary life annuity" are used. (Menge and Fisher, 1965).

$$\ddot{a}^{(m)}_{x:\overline{n}|} = \left(\ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}\right).(1 - {}_nE_x)$$

Commutations functions used in calculations of present values are multiplied by salary scale. Obtained values are multiplied by salary subject to premium and contribution rate for employee. Thus, present values of premiums which will be paid by employee are calculated. Same calculations are used contribution rate for employer and present values of premiums which will be paid by employer are calculated. Summation of these present values multiplied by total number of members, so we obtained present values of total premiums.

Table 6.30 Present Values of premiums for active members

	Average Age	Number of Members	Average of Future Service Time (Year)	$\ddot{a}_{x:\overline{n}|}$	$\ddot{a}^{(m)}_{x:\overline{n}|}$	Present Value of Total Premium
Male	37	3.979	16	13,64	13,37	330.172.558
Female	35	3.581	15	13,50	13,26	304.707.826
Total						634.880.385

xiii. *Occupational decease premium (Asset of foundation)*: Present value of occupational decease premium is obtained that present value of pension premiums for active members is multiplied by average rate of occupational decease of 4 years. This rate is 7.5%.

xiv. *Maturity premium (Asset of foundation)*: Present value of maturity premium is obtained that present value of pension premiums for active members is multiplied by average rate of maturity of 4 years. This rate is 5%.

xv. *Voluntary premium (Asset of foundation)*: Present value of voluntary premium is obtained that present value of pension premiums for active members is multiplied by average rate of voluntary of 4 years. This rate is 5%.

xvi. *Consociate premium (Asset of foundation)*: Present value of consociate premium is obtained that present value of pension premiums for active members is multiplied by rate of consociate of 2005. This rate is 2.3%.

xvii. *Other conceits premium (Asset of foundation)*: Present value of conceits premium is obtained that present value of pension premiums for active members is multiplied by rate of other conceits of 2005. This rate is 34%.

xviii. *Other premiums (Asset of foundation)*: Other premiums are wage increments, promotional differential, affiliation premium, debt of members, and other premium incomes. Present value of other premiums is calculated by rate of these outcomes and related rates. Present value of other premium is obtained that present value of pension premiums for active members is multiplied by average rates of other premiums of 4 years. There are wage increments 2.2%, promotional differential 3.3%, affiliation premium 0.3%, debt of members 0.1%, and other premium income 0.3%.

xix. *Health premium (Asset of foundation)*: Present value of health premium is obtained that present value of pension premiums for active members is multiplied by average rate of health premium of 4 years. This rate is 54.6%.

Actuarial balance sheet, which is obtained using actuarial valuation under aggregate cost method, is given in Table 6.31. There are two columns which are called assets and liabilities in balance sheet.

Table 6.31 Actuarial balance sheet for aggregate cost method

ASSETS			**LIABILITIES**
Assets (31.12.2006)	**1.190.259.947 TL**	**Active Members**	**606.687.093 TL**
		Active Male Member Pension	288.266.171 TL
Present Value of Pension Premium	**634.880.385 TL**	***Active Female Member Pension***	318.420.922 TL
Male Members Pension Premium	330.172.558 TL		
Female Member Pension Premium	304.707.826 TL	**Passive Members**	**1.020.220.577 TL**
		Male Pension	591.743.496 TL
		Female Pension	428.477.081 TL
Occupational Diseases Premium	**47.311.265 TL**		
Maturity Premium	**31.539.569 TL**	**Disability Pension**	**21.745.616 TL**
Voluntary Premium	**3.384.709 TL**	**Widows and Orphans Pension**	**116.404.151 TL**
Consociate Pensions	**64.629.669 TL**	**Consociate Pensions**	**23.126.820 TL**
Other Conceits	**16.500.158 TL**	**Voluntary Pension**	**8.822.724 TL**
Other	**39.047.631 TL**	**Withdrawal pensions**	**73.020.730 TL**
Health Premium	**346.929.920 TL**	**Other Outcomes (Maturity, Occupational Decease, …)**	**8.679.757 TL**
		Outcome of Health	**548.473.867 TL**
		General Management Outcome (3%)	**72.815.440 TL**
		Actuarial Surplus	**-125.513.524 TL**
TOTAL	**2.374.483.252 TL**	**TOTAL**	**2.374.483.252 TL**

6.7.1.2 Unit Credit Method (Fixed Interest Rate Assumption)

The basic difference between Unit Credit Method and Aggregate Cost Method of the actuarial valuation is the actuarial calculation of the benefits provided to the employees. In this method, the actuarial present value of the retirement salaries of any active member is the actuarial present value of the realized salary considering the past service period of the same member. Therefore, the contributions the member will make after the valuation date are not taken into consideration. Since the benefits provided to the retired and dependent members are realized totally, the calculations for these members will be the same as the results of the calculations made in respect of aggregate cost method. The main problem in unit credit method calculations is to figure the realized portion of the medical expenses of active members because, the active members are not paid the realized portion in losses with the past service period taken into consideration. Hence, in the actuarial calculations of medical expenses of active members, the rate obtained from the realized retirement salaries are used. In calculations made with respect to unit credit method principles, while the demographic assumptions (CSO Mortality Table - Appendix A,B) used in aggregate cost method are used, in the fiscal assumptions the following rates are used considering the current economical improvements. Fiscal assumptions are given in Table 6.31.

Table 6.31 Fiscal Assumptions for Unit Credit Method

Assumptions	***12.31.2006***
Inflation	5.00%
Increasing of General Wage	5.00%
Increasing Rate of Promotional Differential	2.00%
Increasing of Pension	5.00%
Discount Rate	11.50%
Increasing Rate of Health Cost	6.00%
Increasing Rate of Dependent with Old-age in Health Cost	1.26%

Calculations of liability of retired members, disability pensions, salary of orphans and widows, consociate pensions, voluntary pensions are same with aggregate cost method. There is different calculation for accrued liability of active members. Accrued liabilities of active members are calculated by are given in Table 6.32. Commutations functions used in calculations of liabilities are multiplied by annual salary and number of active members.

Table 6.32 Accrued liabilities of active members

	Average Age	Number of Members	Average of Monthly Salary (YTL)	$\ddot{a}_r^{(m)}$	Present Value of Total Pensions
Male	37	3979	627	1.75	170.667.639
Female	35	3581	636	2.7	189.098.249
Total					359.765.888

According to assumptions, actuarial balance sheet, which is obtained using actuarial valuation under unit credit method, is given in Table 6.33.

Table 6.33 Actuarial balance sheet for unit credit method

TOTAL LIABILITIES OF FOUNDATION	
Accrued Liability of Active Members	**359.765.888 TL**
Active Male Members Pensions	170.667.639 TL
Active Female Members Pensions	189.098.249 TL
Liability of Retired Members	**1.020.220.577 TL**
Retired Male Pensions	591.743.496 TL
Retired Female Pensions	428.477.081 TL
Disability Pensions	**21.745.616 TL**
Salaries of Orphans and Widows	**116.404.151 TL**
Consociate Pensions	**15.017.723 TL**
Voluntary Pensions	**5.438.015 TL**
Outcome of Health	**284.500.781 TL**
Active Member Accrued Health Outcome	120.496.508 TL
Passive Member Health Outcome	164.004.273 TL
General Management Outcome (3%)	**53.589.882 TL**
Total Liability	**1.861.664.910 TL**
Assets (31.12.2006)	**1.190.259.947 TL**
Actuarial Surplus	**-671.404.963 TL**

6.7.1.3 Aggregate Cost Method (Stochastic Interest Rate)

Stochastic interest rate scenarios use an assumed probability distribution for future interest rates along with simulation techniques to elicit single values of the rate for the next period. We will focus here on different behavioral features of scenarios produced by lognormal distribution. Under this model the interest rate in period *n* is given by $i_n = e^x - 1$ where *x* is drawn from a normal distribution with mean μ and standard deviation σ. Here μ is force of interest corresponding to the current annual interest rate and σ represents appropriate deviations from that rate. Although this model is in some ways superior to the normal distribution model, it is also assumes independence between rates in successive periods.

We begin with two different models for generating future interest rates. Firstly, we calculated assets and liabilities of foundation for constant μ and different σ under aggregate cost method. Then, same calculations are made for different μ and constant σ. Assets and liabilities of foundation are obtained μ=6 and σ=1,2, and 3 under aggregate cost method are given in Table 6.34.

Table 6.34 Asset and Liability of Foundation under aggregate cost method (Constant mean)

	ASSET	LIABILITY
μ =6 and σ =1	2.792.381.050 YTL	4.286.956.208 YTL
μ =6 and σ =2	2.794.843.839 YTL	4.295.349.329 YTL
μ =6 and σ =3	2.794.843.839 YTL	4.327.200.427 YTL

Assets and liabilities of foundation are obtained σ=2 and μ=4,5, and 6 under aggregate cost method are given in Table 6.35.

Table 6.35 Asset and Liability of Foundation under aggregate cost method (Constant standard deviation)

	ASSET	LIABILITY
μ =4 and σ =2	2.878.337.603 YTL	4.813.208.264 YTL
μ =5 and σ =2	2.839.538.250 YTL	4.539.073.274 YTL
μ =6 and σ =2	2.794.843.839 YTL	4.295.349.329 YTL

Assets and liabilities of foundation are obtained μ=6 and σ=1,2, and 3 under unit credit method are given in Table 6.36.

Table 6.36 Asset and Liability of Foundation under unit credit method (Constant mean)

	ASSET	LIABILITY
μ =6 and σ =1	1.190.259.947 YTL	2.397.433.414 YTL
μ =6 and σ =2	1.190.259.947 YTL	2.405.587.954 YTL
μ =6 and σ =3	1.190.259.947 YTL	2.399787.851 YTL

Assets and liabilities of foundation are obtained σ=2 and μ=4,5, and 6 under unit credit method are given in Table 6.37.

Table 6.36 Asset and Liability of Foundation under unit credit method (Constant standard deviation)

	ASSET	LIABILITY
μ =4 and σ =2	1.190.259.947 YTL	2.526.756.952 YTL
μ =5 and σ =2	1.190.259.947 YTL	2.464.281.074 YTL
μ =6 and σ =2	1.190.259.947 YTL	2.405.587.954 YTL

CHAPTER SEVEN
CONCLUSION

Social security refers to social insurance, where people receive benefits in recognition of contributions. These benefits generally include provision for retirement, disability, death and health. All these and other benefits might be presented by social or private insurance system. But, whoever the provider is, the system must be under the control. However, many factors affect the harmony of the system and some of these factors are out of control. Increasing health cost, the aging of populations, economic and demographic conditions all pose severe challenges to social insurance programs worldwide. Many countries face the unpleasant choice of reducing benefits or increasing contributions to finance their social programs.

Every social or private retirement system is a living mechanism therefore it must be delivered healthy and grow up healthy up to its death. The only way of keeping the system healthy is actuarial valuation and this valuation must be repeated and implemented regularly. Whenever an unhealthy status is observed, the necessary precautions must be taken immediately. These regular valuations allow the contributions and benefits to be adjusted and asses the solvency.

A pension scheme can be viewed as a series of cash flows and quantity of this cash flows depends upon contributions, investment income, benefit payments and expenses. The valuation can therefore be viewed as a consolidation of future cash flows. The cash flows can be calculated on the basis of the various probabilities for future demographic and economical contingencies. Professionally prepared actuarial valuation computer program can project the anticipated cash flows on a year by year basis. Furthermore, the main factors affecting future cash flows can be tested using this program.

To point out the importance of actuarial valuation process in social insurance mechanism one of the private social insurance has been chosen and its fiscal and actuarial structure analyzed. The fiscal and actuarial structure has been affected negatively due to the economical crises and rapid deteriorating of active/passive balance parallel to the Bank's employment policy. However, the negative actuarial balance of the foundation has been positively affected with the successful initial public offering of the institution nestling the foundation and the negative issues that could be encountered in the near future are avoided.

The actuarial balance of the foundation, both in the valuation regarding the assumptions of aggregate cost method and in the valuation corresponding to unit credit method, has no actuarial deficit. The actuarial balances produce 125 millions YTL surplus in the valuation regarding the aggregate cost method, and 671 millions YTL surplus in the valuation corresponding to the unit credit method.

REFERENCES

ACTEX, Pensions, *Core Reading*, 1996.

Anderson, A.W.(1992). *Pension Mathematics for Actuaries* (2nd Edition). Actex Pub., USA.

Bacinello, A. R. (1988). A Stochastic Simulation Procedure for Pension Scheme. *Insurance: Mathematics and Economics*, (*7)*: 153–161.

Belhouse, D.R., & Panjer, H.H. (1981). Stochastic Modeling of Interest Rates with Application to Life Contingencies – Part II. *Journal of Risk and Insurance*, *(47),* pp. 628-637.

Black, K., & Skipper H.D. (1994). *Life Insurance* (12th Edition). Prentice Hall, London.

Booth, P., Chadburn, R., Cooper, D., & James, D. (1999). *Modern Actuarial Theory and Practice*. Chapmann & Hall, USA.

Boulier, J. F., Trussant E., & Florens D. (1995). A Dynamic Model for Pension Funds Management. *Proceedings of the 5th AFIR International Colloquium*, (*1)*: 361–384.

Boulier, J. F., Trussant E., & Florens D. (1996). Optimizing Investment and Contribution Policies of a Defined Benefit Pension Fund. *Proceedings of the 6th AFIR International Colloquium*, (*1)*: 593–607.

Bowers, N. L., Gerber H. U., Hickman J. C.,. Jones D. A, & Nesbittt C. J. (1982). Notes on the Dynamics of Pension Funding. *Insurance: Mathematics and Economics*, (*1)*: 261–270.

Box G.E.P. & Jenkins G.M. (1976). *Time series Analysis* (2nd Edition). Holden Day, San Francisco.

Boyle P.P. (1973). Note on the Variance of a Widow's Pension. *Journal of the Institute of Actuaries*, *(99)*, 103-107.

Boyle P.P. (1976). Rates of return as random variables. *Journal of Risk and Insurance*, *(XLIII),* no.4, 693-713.

Cairns A. J. G. (1995). Pension Funding in a Stochastic Environment: The Role of Objectives in Selecting an Asset-Allocation Strategy. *Proceedings of the 5th AFIR International Colloquium*, (*1)*: 429–453.

Cairns A. J. G. (1996). Continuous-Time Stochastic Pension Funding Modeling *Proceedings of the 6th AFIR International Colloquium*, (*1)*: 609–624.

Cairns, A. J., & Parker G. (1997). Stochastic Pension Fund Modeling. *Insurance: Mathematics and Economics*, (*21)*: 43–79.

Chang, S. C. (1999). Optimal Pension Funding Through Dynamic Simulations: The Case of Taiwan Public Employees Retirement System. *Insurance: Mathematics and Economics*, (*24)*: 187–199.

Cunningham, R., Herzog, T., & London, R. (2005). Models for Quantifying Risk. Actex Pub., USA.

Daykin C.D., Bernstein G.D., & Coutts S.M. (1987). Assessing the Solvency and Financial Strength of a General Insurance Company. *Journal of Institute of Actuaries, (114)*, pp. 227 – 310 (with discussion).

Daykin C.D., Pentikainen T. & Pesonen M. (1994). Assessing the Solvency and financial Strength of a General Insurance Company. *Journal of Institute of Actuaries*, *(114)*, pp. 227-310 (with discussion).

Dominicis R. D., Manca L., & Granata L. (1991). The Dynamics of Pension Funds in a Stochastic Environment. *Scandinavian Actuarial Journal*, (*2)*: 118–128.

Dufresne D. (1986). The dynamics of Pension Funding. PhD. Thesis, City University, London.

Dufresne D. (1988). Moments of Pension Fund Contributions and Fund Levels when Rates of Return are Random. *Journal of Institute of Actuaries*, *(115)*, pp. 535-544.

Dufresne D. (1989). Stability of Pension Systems when Rates of Return are Random. *Insurance: Mathematics and Economics*, *(8)*, pp. 71-76.

Dufresne D. (1990). The distribution of a Perpetuity , with Applications to Risk Theory and Pension Funding. *Scandinavian Actuarial Journal*, 37-79.

Frees E., Kung Y. C., Rosenberg M., Young V., & Lai S. W. (1997). Forecasting Social Security Actuarial Assumptions. *North American Actuarial Journal*, (*1)*: 49–82.

Gauger M., A. (2006). Actuarial Models, BPP Professional Education Inc., USA.

Gerrard R.J. & Haberman S. (1996). Stability of Pension Systems with Gains / Loses are Amortized and Rates of Return of Autoregressive. *Insurance: Mathematics and Economics*, *(18)*, pp. 59-72.

Giaccotto C. (1986). Stochastic modeling of interest rates: actuarial vs. equilibrium approach. *Journal of Risk and Insurance*, *(LIII)* no.3, 435-453.

Haberman S. (1992). Pension Funding Time Delays: A Stochastic Approach. *Insurance: Mathematics and Economics*, *(11)*, pp. 179-189.

Haberman S. (1993a). Pension Funding Time Delays and Autoregressive Rates of Investment Return. *Insurance: Mathematics and Economics*, *(13)*, pp. 45-56.

Haberman S. (1993b). Pension Funding: The Effect of Changing the Frequency of Valuations. *Insurance: Mathematics and Economics*, *(13)*, pp. 263-270.

Haberman S. (1994b). Autoregressive Rates of Return and the Variability of Pension Contributions and Fund Levels for a Defined Benefit Pension Scheme. *Insurance: Mathematics and Economics*, *(14)*, pp. 219-240.

Haberman S. (1994b). Pension Funding Modeling and Stochastic Investment Returns. *Presented to Joint Meeting of Staple Inn Actuarial Society and Royal Statistical Society of General Applications Section.*

Haberman S. (1997). Stochastic Investment Returns and Contribution Rate Risk in a Defined Benefit Pension Scheme. *Insurance: Mathematics and Economics*, (*19)*: 127–139.

Haberman S., & Sung J. H. (1994). Dynamic Approaches to Pension Funding. *Insurance: Mathematics and Economics*, (*15)*: 151–162.

Haberman S., & Wong L. Y. (1997). Moving Average Rates of Return and the Variability of Pension Contributions and Fund Levels for a Defined Benefit Pension Scheme. *Insurance: Mathematics and Economics*,(*20)*: 115–135.

Hickman J.C. (1964). A Statistical Approach to Premiums and Reserves in Multiple Decrement Theory. *Transactions of the Society of Actuaries*, *(XVI)*, 1-16.

Hickman J.C. & Gayda (1971). Discussion on the Paper ‘Actuarial Functions as Expected Values’. *Transactions of the Society of Actuaries*, *(XXIII)*, 143-149.

Kellison S.G. (1991). *The Theory of Interest* (2nd Edition). *Irwin/McGraw Hill*, USA.

Loades D.H. (1992). Instability in Pension Funding. *Transactions of the 24th International Congress of Actuaries*, *(2)*, pp. 137-154.

Menge W. O. & Fischer C. H. (1965). The Mathematics of Life Insurance. The MacMillan Company, USA.

McCutcheon J.J. & Scott W.F. (1986). An Introduction to the Mathematics of Finance. Heinemann, London.

O'Brien T. (1986). A Stochastic-Dynamic Approach to Pension Funding. *Insurance: Mathematics and Economics*, (*5)*: 141–146.

O'Brien T. (1987). A Two-Parameter Family of Pension Contribution Functions and Stochastic Optimization. *Insurance: Mathematics and Economics*, (*6)*: 129–134.

Owadally M.I. & Haberman S. (1999). Pension Fund Dynamics and Gains / Losses due to Random Rates of Investment Return. *North American Actuarial Journal*.

Özgürel B. (2005). 2005 Yılı için Çoklu Azalım Tablosu Öngörümlemesi. *İstatistik Araştırma Dergisi, (3)*: 63-76.

Panjer H.H. (1978). Stochastic Foundation of Life Contingencies. Presented at Canadian Institute of Actuaries, London, Canada.

Panjer H.H. & Bellhouse D.R. (1980). Stochastic modeling of interest rates with applications to life contingencies. *Journal of Risk and Insurance*, *(XLVII),* no.1, 91-110.

Parmenter M.M. (1999). Theory of Interest and Life Contingencies, with Application: A Problem Solving Approach. (3rd Edition). ACTEX Publication.

Pollard A.H. & Pollard J.H. (1969). A Stochastic Approach to Actuarial Functions. *Journal of the Institute of Actuaries*, *(95)*, 79-113.

Pollard J.H. (1971). On fluctuating interest rates. *Bulletin van de Koninklije Vereniginh van Belgische Actuarissen*, *(66)*, 68-97.

Shapiro A. F. (1985). Contributions to the Evolution of Pension Cost Analysis. *Journal of Risk and Insurance*, *(52)*: 81–99.

Skipper H.D. (1998). International Risk and Insurance. Irwin McGraw-Hill, USA.

Taylor G.C. (1972). On Calculation of Moments of the Value of a Life Assurance Liability. *Journal of the Institute of Actuaries*, *(98)*, 157-164.

Trowbridge C.L. (1952). Fundamentals of Pension Funding. *Transactions of the Society of Actuaries*, *(4)*, pp. 17-43.

Wilkie A.D. (1986). A Stochastic Model for Actuarial Use. *Transactions of the Faculty of Actuaries*, *(39)*, pp. 341-403.

Winklevoss H.E.(1993). *Pension Mathematics with Numerical Illustrations*. (2nd Edition). USA.

APPENDICES

APPENDIX A 1980 CSO Male Mortality and Commutations Table at Interest Rate 5%

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
0	0.00418	0.99582	1.000.000	4.180	1.000.000	19.862.632	3.981	54.160	19.863
1	0.00107	0.99893	995.820	1.066	948.400	18.862.632	966	50.179	19.889
2	0.00099	0.99901	994.754	985	902.272	17.914.232	851	49.213	19.855
3	0.00098	0.99902	993.770	974	858.456	17.011.961	801	48.362	19.817
4	0.00095	0.99905	992.796	943	816.776	16.153.505	739	47.561	19.777
5	0.00090	0.99910	991.853	893	777.142	15.336.730	666	46.822	19.735
6	0.00086	0.99914	990.960	852	739.470	14.559.587	606	46.156	19.689
7	0.00080	0.99920	990.108	792	703.651	13.820.118	536	45.550	19.641
8	0.00076	0.99924	989.316	752	669.608	13.116.466	485	45.014	19.588
9	0.00074	0.99926	988.564	732	637.237	12.446.859	449	44.529	19.533
10	0.00073	0.99927	987.832	721	606.443	11.809.622	422	44.080	19.474
11	0.00077	0.99923	987.111	760	577.143	11.203.178	423	43.659	19.411
12	0.00085	0.99915	986.351	838	549.237	10.626.035	445	43.235	19.347
13	0.00099	0.99901	985.513	976	522.638	10.076.798	493	42.791	19.281
14	0.00115	0.99885	984.537	1.132	497.258	9.554.159	545	42.298	19.214
15	0.00133	0.99867	983.405	1.308	473.035	9.056.901	599	41.753	19.146
16	0.00151	0.99849	982.097	1.483	449.910	8.583.867	647	41.154	19.079
17	0.00167	0.99833	980.614	1.638	427.839	8.133.957	680	40.507	19.012
18	0.00178	0.99822	978.976	1.743	406.785	7.706.118	690	39.827	18.944
19	0.00186	0.99814	977.234	1.818	386.725	7.299.334	685	39.137	18.875
20	0.00190	0.99810	975.416	1.853	367.624	6.912.609	665	38.452	18.803
21	0.00191	0.99809	973.563	1.860	349.453	6.544.985	636	37.787	18.729
22	0.00189	0.99811	971.703	1.837	332.177	6.195.532	598	37.151	18.651
23	0.00186	0.99814	969.867	1.804	315.761	5.863.355	559	36.553	18.569
24	0.00182	0.99818	968.063	1.762	300.165	5.547.595	520	35.994	18.482
25	0.00177	0.99823	966.301	1.710	285.351	5.247.430	481	35.474	18.389
26	0.00173	0.99827	964.591	1.669	271.282	4.962.078	447	34.993	18.291
27	0.00171	0.99829	962.922	1.647	257.917	4.690.796	420	34.546	18.187
28	0.00170	0.99830	961.275	1.634	245.215	4.432.879	397	34.126	18.078
29	0.00171	0.99829	959.641	1.641	233.141	4.187.664	380	33.729	17.962
30	0.00173	0.99827	958.000	1.657	221.660	3.954.523	365	33.349	17.841
31	0.00178	0.99822	956.343	1.702	210.739	3.732.863	357	32.984	17.713
32	0.00183	0.99817	954.640	1.747	200.347	3.522.124	349	32.627	17.580
33	0.00191	0.99809	952.893	1.820	190.457	3.321.777	346	32.277	17.441
34	0.00200	0.99800	951.073	1.902	181.041	3.131.320	345	31.931	17.296
35	0.00211	0.99789	949.171	2.003	172.076	2.950.279	346	31.586	17.145
36	0.00224	0.99776	947.168	2.122	163.536	2.778.203	349	31.240	16.988
37	0.00240	0.99760	945.047	2.268	155.399	2.614.667	355	30.891	16.825
38	0.00258	0.99742	942.779	2.432	147.644	2.459.268	363	30.536	16.657
39	0.00279	0.99721	940.346	2.624	140.251	2.311.624	373	30.173	16.482
40	0.00302	0.99698	937.723	2.832	133.199	2.171.373	383	29.801	16.302
41	0.00329	0.99671	934.891	3.076	126.474	2.038.174	396	29.418	16.115
42	0.00356	0.99644	931.815	3.317	120.055	1.911.700	407	29.021	15.924

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
43	0.00387	0.99613	928.498	3.593	113.931	1.791.645	420	28.614	15.726
44	0.00419	0.99581	924.905	3.875	108.086	1.677.715	431	28.194	15.522
45	0.00455	0.99545	921.029	4.191	102.507	1.569.629	444	27.763	15.312
46	0.00492	0.99508	916.838	4.511	97.182	1.467.122	455	27.319	15.097
47	0.00532	0.99468	912.328	4.854	92.099	1.369.940	467	26.864	14.875
48	0.00574	0.99426	907.474	5.209	87.246	1.277.841	477	26.397	14.646
49	0.00621	0.99379	902.265	5.603	82.615	1.190.595	489	25.920	14.411
50	0.00671	0.99329	896.662	6.017	78.192	1.107.980	500	25.431	14.170
51	0.00730	0.99270	890.645	6.502	73.969	1.029.787	514	24.932	13.922
52	0.00796	0.99204	884.144	7.038	69.933	955.818	530	24.417	13.668
53	0.00871	0.99129	877.106	7.640	66.072	885.886	548	23.887	13.408
54	0.00956	0.99044	869.466	8.312	62.378	819.813	568	23.339	13.143
55	0.01047	0.98953	861.154	9.016	58.840	757.436	587	22.771	12.873
56	0.01146	0.98854	852.138	9.766	55.451	698.596	605	22.184	12.598
57	0.01249	0.98751	842.372	10.521	52.205	643.145	621	21.579	12.320
58	0.01359	0.98641	831.851	11.305	49.098	590.940	635	20.958	12.036
59	0.01477	0.98523	820.546	12.119	46.125	541.842	649	20.323	11.747
60	0.01608	0.98392	808.427	13.000	43.280	495.717	663	19.674	11.454
61	0.01754	0.98246	795.427	13.952	40.556	452.437	677	19.011	11.156
62	0.01919	0.98081	781.476	14.997	37.947	411.881	694	18.334	10.854
63	0.02106	0.97894	766.479	16.142	35.447	373.934	711	17.640	10.549
64	0.02314	0.97686	750.337	17.363	33.048	338.488	728	16.929	10.242
65	0.02542	0.97458	732.974	18.632	30.746	305.440	744	16.201	9.934
66	0.02785	0.97215	714.342	19.894	28.537	274.694	757	15.457	9.626
67	0.03044	0.96956	694.448	21.139	26.421	246.157	766	14.700	9.317
68	0.03319	0.96681	673.309	22.347	24.397	219.735	771	13.934	9.007
69	0.03617	0.96383	650.962	23.545	22.464	195.338	774	13.163	8.695
70	0.03951	0.96049	627.416	24.789	20.621	172.874	776	12.389	8.383
71	0.04330	0.95670	602.627	26.094	18.863	152.253	778	11.613	8.072
72	0.04765	0.95235	576.533	27.472	17.187	133.390	780	10.835	7.761
73	0.05264	0.94736	549.061	28.903	15.588	116.203	781	10.055	7.454
74	0.05819	0.94181	520.159	30.268	14.065	100.615	779	9.273	7.154
75	0.06419	0.93581	489.891	31.446	12.615	86.550	771	8.494	6.861
76	0.07053	0.92947	458.445	32.334	11.243	73.935	755	7.723	6.576
77	0.07712	0.92288	426.111	32.862	9.953	62.691	731	6.968	6.299
78	0.08390	0.91610	393.249	32.994	8.748	52.739	699	6.237	6.029
79	0.09105	0.90895	360.255	32.801	7.632	43.991	662	5.538	5.764
80	0.09884	0.90116	327.454	32.366	6.607	36.358	622	4.876	5.503
81	0.10748	0.89252	295.089	31.716	5.670	29.751	580	4.254	5.247
82	0.11725	0.88275	263.372	30.880	4.820	24.081	538	3.673	4.996
83	0.12826	0.87174	232.492	29.819	4.052	19.261	495	3.135	4.753
84	0.14025	0.85975	202.673	28.425	3.364	15.209	449	2.640	4.521
85	0.15295	0.84705	174.248	26.651	2.755	11.844	401	2.191	4.300
86	0.16609	0.83391	147.597	24.514	2.222	9.090	352	1.789	4.090
87	0.17955	0.82045	123.082	22.099	1.765	6.867	302	1.438	3.891
88	0.19327	0.80673	100.983	19.517	1.379	5.102	254	1.136	3.700
89	0.20729	0.79271	81.466	16.887	1.060	3.723	209	882	3.514
90	0.22177	0.77823	64.579	14.322	800	2.664	169	673	3.330
91	0.23698	0.76302	50.257	11.910	593	1.864	134	504	3.144

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
92	0.25345	0.74655	38.347	9.719	431	1.271	104	370	2.950
93	0.27211	0.72789	28.628	7.790	306	840	79	266	2.743
94	0.29590	0.70410	20.838	6.166	212	534	60	187	2.514
95	0.32996	0.67004	14.672	4.841	142	321	45	127	2.258
96	0.38455	0.61545	9.831	3.780	91	179	33	82	1.971
97	0.48020	0.51980	6.050	2.905	53	88	24	49	1.656
98	0.65798	0.34202	3.145	2.069	26	35	17	25	1.326
99	1.00000	0.00000	1.076	1.076	9	9	8	8	1.000

APPENDIX B 1980 CSO Female Mortality and Commutations Table at Interest Rate 5%

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
0	0.00289	0.99711	1.000.000	2.890	1.000.000	20.097.284	2.752	42.986	20.097
1	0.00087	0.99913	997.110	867	949.629	19.097.284	787	40.234	20.110
2	0.00081	0.99919	996.243	807	903.621	18.147.656	697	39.447	20.083
3	0.00079	0.99921	995.436	786	859.895	17.244.034	647	38.750	20.054
4	0.00077	0.99923	994.649	766	818.300	16.384.140	600	38.103	20.022
5	0.00076	0.99924	993.883	755	778.734	15.565.839	564	37.503	19.989
6	0.00073	0.99927	993.128	725	741.087	14.787.106	515	36.939	19.953
7	0.00072	0.99928	992.403	715	705.282	14.046.019	484	36.424	19.915
8	0.00070	0.99930	991.688	694	671.214	13.340.736	447	35.941	19.876
9	0.00069	0.99931	990.994	684	638.804	12.669.523	420	35.493	19.833
10	0.00068	0.99932	990.310	673	607.965	12.030.719	394	35.073	19.789
11	0.00069	0.99931	989.637	683	578.620	11.422.754	380	34.680	19.741
12	0.00072	0.99928	988.954	712	550.687	10.844.134	378	34.299	19.692
13	0.00075	0.99925	988.242	741	524.086	10.293.447	374	33.922	19.641
14	0.00080	0.99920	987.501	790	498.755	9.769.361	380	33.547	19.587
15	0.00085	0.99915	986.711	839	474.625	9.270.606	384	33.167	19.532
16	0.00090	0.99910	985.872	887	451.639	8.795.981	387	32.783	19.476
17	0.00095	0.99905	984.985	936	429.746	8.344.342	389	32.396	19.417
18	0.00098	0.99902	984.049	964	408.893	7.914.596	382	32.007	19.356
19	0.00102	0.99898	983.085	1.003	389.040	7.505.703	378	31.626	19.293
20	0.00105	0.99895	982.082	1.031	370.136	7.116.663	370	31.248	19.227
21	0.00107	0.99893	981.051	1.050	352.141	6.746.527	359	30.878	19.159
22	0.00109	0.99891	980.001	1.068	335.013	6.394.386	348	30.519	19.087
23	0.00111	0.99889	978.933	1.087	318.712	6.059.373	337	30.171	19.012
24	0.00114	0.99886	977.846	1.115	303.199	5.740.660	329	29.834	18.934
25	0.00116	0.99884	976.732	1.133	288.432	5.437.462	319	29.505	18.852
26	0.00119	0.99881	975.599	1.161	274.378	5.149.030	311	29.186	18.766
27	0.00122	0.99878	974.438	1.189	261.001	4.874.652	303	28.875	18.677
28	0.00126	0.99874	973.249	1.226	248.270	4.613.650	298	28.572	18.583
29	0.00130	0.99870	972.023	1.264	236.149	4.365.381	292	28.274	18.486
30	0.00135	0.99865	970.759	1.311	224.612	4.129.232	289	27.982	18.384
31	0.00140	0.99860	969.448	1.357	213.627	3.904.620	285	27.693	18.278
32	0.00145	0.99855	968.091	1.404	203.170	3.690.993	281	27.408	18.167
33	0.00150	0.99850	966.687	1.450	193.214	3.487.823	276	27.127	18.052
34	0.00158	0.99842	965.237	1.525	183.738	3.294.609	276	26.851	17.931
35	0.00165	0.99835	963.712	1.590	174.712	3.110.871	275	26.575	17.806
36	0.00176	0.99824	962.122	1.693	166.118	2.936.160	278	26.300	17.675
37	0.00189	0.99811	960.429	1.815	157.929	2.770.042	284	26.022	17.540
38	0.00204	0.99796	958.614	1.956	150.124	2.612.113	292	25.738	17.400
39	0.00222	0.99778	956.658	2.124	142.684	2.461.989	302	25.446	17.255
40	0.00242	0.99758	954.534	2.310	135.587	2.319.306	312	25.144	17.106
41	0.00264	0.99736	952.224	2.514	128.818	2.183.718	324	24.832	16.952
42	0.00287	0.99713	949.710	2.726	122.360	2.054.900	334	24.508	16.794
43	0.00309	0.99691	946.985	2.926	116.199	1.932.539	342	24.174	16.631
44	0.00332	0.99668	944.059	3.134	110.324	1.816.340	349	23.832	16.464

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
45	0.00356	0.99644	940.924	3.350	104.722	1.706.016	355	23.483	16.291
46	0.00380	0.99620	937.575	3.563	99.380	1.601.295	360	23.128	16.113
47	0.00405	0.99595	934.012	3.783	94.288	1.501.915	364	22.768	15.929
48	0.00433	0.99567	930.229	4.028	89.434	1.407.627	369	22.404	15.739
49	0.00463	0.99537	926.201	4.288	84.807	1.318.193	374	22.036	15.544
50	0.00496	0.99504	921.913	4.573	80.394	1.233.386	380	21.662	15.342
51	0.00531	0.99469	917.340	4.871	76.186	1.152.992	385	21.282	15.134
52	0.00570	0.99430	912.469	5.201	72.173	1.076.806	392	20.897	14.920
53	0.00615	0.99385	907.268	5.580	68.344	1.004.633	400	20.505	14.700
54	0.00661	0.99339	901.688	5.960	64.690	936.288	407	20.104	14.474
55	0.00709	0.99291	895.728	6.351	61.202	871.599	413	19.697	14.241
56	0.00757	0.99243	889.378	6.733	57.874	810.397	417	19.284	14.003
57	0.00803	0.99197	882.645	7.088	54.701	752.523	418	18.867	13.757
58	0.00847	0.99153	875.557	7.416	51.678	697.822	417	18.448	13.503
59	0.00894	0.99106	868.141	7.761	48.800	646.144	415	18.031	13.241
60	0.00947	0.99053	860.380	8.148	46.061	597.344	415	17.616	12.969
61	0.01013	0.98987	852.232	8.633	43.452	551.283	419	17.201	12.687
62	0.01096	0.98904	843.599	9.246	40.964	507.831	428	16.781	12.397
63	0.01202	0.98798	834.353	10.029	38.586	466.867	442	16.354	12.100
64	0.01325	0.98675	824.324	10.922	36.306	428.281	458	15.912	11.796
65	0.01459	0.98541	813.402	11.868	34.119	391.975	474	15.454	11.488
66	0.01600	0.98400	801.535	12.825	32.021	357.856	488	14.980	11.176
67	0.01743	0.98257	788.710	13.747	30.008	325.835	498	14.492	10.858
68	0.01884	0.98116	774.963	14.600	28.081	295.827	504	13.994	10.535
69	0.02036	0.97964	760.363	15.481	26.240	267.746	509	13.490	10.204
70	0.02211	0.97789	744.882	16.469	24.481	241.507	516	12.981	9.865
71	0.02423	0.97577	728.412	17.649	22.800	217.025	526	12.466	9.519
72	0.02687	0.97313	710.763	19.098	21.188	194.225	542	11.939	9.167
73	0.03011	0.96989	691.665	20.826	19.637	173.037	563	11.397	8.812
74	0.03393	0.96607	670.839	22.762	18.139	153.400	586	10.834	8.457
75	0.03824	0.96176	648.077	24.782	16.689	135.261	608	10.248	8.105
76	0.04297	0.95703	623.295	26.783	15.286	118.572	626	9.640	7.757
77	0.04804	0.95196	596.512	28.656	13.933	103.286	637	9.015	7.413
78	0.05345	0.94655	567.855	30.352	12.632	89.353	643	8.377	7.074
79	0.05935	0.94065	537.503	31.901	11.387	76.721	644	7.734	6.737
80	0.06599	0.93401	505.602	33.365	10.202	65.333	641	7.090	6.404
81	0.07360	0.92640	472.238	34.757	9.075	55.132	636	6.449	6.075
82	0.08240	0.91760	437.481	36.048	8.006	46.057	628	5.813	5.753
83	0.09253	0.90747	401.433	37.145	6.997	38.051	617	5.185	5.438
84	0.10381	0.89619	364.288	37.817	6.047	31.054	598	4.568	5.135
85	0.11610	0.88390	326.471	37.903	5.161	25.007	571	3.970	4.845
86	0.12929	0.87071	288.568	37.309	4.345	19.846	535	3.400	4.568
87	0.14332	0.85668	251.259	36.010	3.603	15.501	492	2.865	4.302
88	0.15818	0.84182	215.249	34.048	2.940	11.898	443	2.373	4.048
89	0.17394	0.82606	181.201	31.518	2.357	8.958	390	1.930	3.801
90	0.19075	0.80925	149.683	28.552	1.854	6.602	337	1.540	3.561
91	0.20887	0.79113	121.131	25.301	1.429	4.747	284	1.203	3.322
92	0.22881	0.77119	95.830	21.927	1.077	3.319	235	919	3.082
93	0.25151	0.74849	73.903	18.587	791	2.242	189	684	2.835

Age	q_x	p_x	l_x	d_x	D_x	N_x	C_x	M_x	$\ddot{a}_x$
94	0.27931	0.72069	55.316	15.450	564	1.451	150	495	2.574
95	0.31732	0.68268	39.866	12.650	387	887	117	345	2.293
96	0.37574	0.62426	27.215	10.226	252	500	90	228	1.989
97	0.47497	0.52503	16.989	8.069	150	249	68	138	1.664
98	0.65585	0.34415	8.920	5.850	75	99	47	70	1.328
99	1.00000	0.00000	3.070	3.070	25	25	23	23	1.000

APPENDIX C   Input Page in Microsoft Excel

	A	B
1	**Interest Rate**	5.00%
2	**Salary Increasing Rate**	0.00%
3	**Premium Increasing Rate**	0.00%

	A	B
1	**Total Number of Active Members**	7.560
2	Total Number of Active Male Members	3.979
3	Total Number of Active Female Members	3.581
4	**Total Number of Passive Members**	5.781
5	Total Number of Passive Male Members	3.453
6	Total Number of Passive Female Members	2.328
7	**Total Number of Disabled Members**	77
8	Total Number of Disabled Male Members	50
9	Total Number of Disabled Female Members	27
10	**Total Number of Widow Members**	574
11	Total Number of Widowed Members	28
12	Total Number of Widower Members	546
13	**Total Number of Orphan Members**	333
14	Total Number of Orphan Male Members	118
15	Total Number of Orphan Female Members	215

	F	H
1		**Average Age**
2	Active Male	37
3	Active Female	35
4	Passive Male	57
5	Passive Female	53
6	Disabled Male	52
7	Disabled Female	48
8	Widowed	61
9	Widower	61
10	Orphan Male	18
11	Orphan Female	27

	F	I	J
1		**Future Service Time (year)**	**Past Service Time (year)**
2	Active Male	16	13
3	Active Female	15	12
4	Passive Male		21
5	Passive Female		18

	F	K	L	M	N	O
1		**Salary Pension in Rate (%)**	**Salary Subject to Allotment (YTL)**	**Retirement Salary (YTL)**	**Salary Subject to Premium (YTL)**	**Average of Health Expensive (YTL)**
2	Active Male	71%	1.553	1.097	2.586	2.015
3	Active Female	68%	1.605	1.094	2.674	2.015
4	Passive Male	62%		1.204		2.015
5	Passive Female	55%		1.077		2.015

	F	N
1		**Salary (YTL)**
6	Disabled Male	1.102
7	Disabled Female	1.133
8	Widowed	701
9	Widower	778
10	Orphan Male	288
11	Orphan Female	328

APPENDIX D - Calculation of Pensions for Active Male Members
Male – Active Sheet in Microsoft Excel

	B	C		B	C
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	53	401.812.35	**52**	102	0.00
4	54	379.345.30	**53**	103	0.00
5	55	357.827.39	**54**	104	0.00
6	56	337.219.94	**55**	105	0.00
7	57	317.481.34	**56**	106	0.00
8	58	298.586.66	**57**	107	0.00
9	59	280.503.68	**58**	108	0.00
10	60	263.200.61	**59**	109	0.00
11	61	246.636.52	**60**	110	0.00
12	62	230.771.92	**61**	111	0.00
13	63	215.565.15	**62**	112	0.00
14	64	200.976.52	**63**	113	0.00
15	65	186.977.07	**64**	114	0.00
16	66	173.546.78	**65**	115	0.00
17	67	160.679.52	**66**	116	0.00
18	68	148.369.94	**67**	117	0.00
19	69	136.614.80	**68**	118	0.00
20	70	125.403.28	**69**	119	0.00
21	71	114.712.95	**70**	120	0.00
22	72	104.519.88	**71**	121	0.00
23	73	94.799.54	**72**	122	0.00
24	74	85.532.65	**73**	123	0.00
25	75	76.719.53	**74**	124	0.00
26	76	68.376.10	**75**	125	0.00
27	77	60.527.18	**76**	126	0.00
28	78	53.199.35	**77**	127	0.00
29	79	46.415.17	**78**	128	0.00
30	80	40.180.06	**79**	129	0.00
31	81	34.484.44	**80**	130	0.00
32	82	29.312.43	**81**		5.387.379.44
33	83	24.643.38	**82**		**5.700648**
34	84	20.459.64			
35	85	16.752.55			
36	86	13.514.52			
37	87	10.733.23			
38	88	8.386.74			
39	89	6.443.65			
40	90	4.864.71			
41	91	3.605.59			
42	92	2.620.13			
43	93	1.862.91			
44	94	1.291.42			
45	95	865.99			
46	96	552.62			
47	97	323.91			
48	98	160.35			
49	99	0.00			
50	100	0.00			
51	101	0.00			

APPENDIX E - Calculation of Pensions for Active Female Members
Female – Active Sheet in Microsoft Excel

	B	C		B	C
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	50	467.794.56	**53**	100	0.00
4	51	443.455.88	**54**	101	0.00
5	52	420.244.13	**55**	102	0.00
6	53	398.107.27	**56**	103	0.00
7	54	376.988.63	**57**	104	0.00
8	55	356.828.71	**58**	105	0.00
9	56	337.590.55	**59**	106	0.00
10	57	319.235.27	**60**	107	0.00
11	58	301.732.05	**61**	108	0.00
12	59	285.056.33	**62**	109	0.00
13	60	269.182.76	**63**	110	0.00
14	61	254.072.64	**64**	111	0.00
15	62	239.682.45	**65**	112	0.00
16	63	225.956.63	**66**	113	0.00
17	64	212.838.24	**67**	114	0.00
18	65	200.266.59	**68**	115	0.00
19	66	188.202.91	**69**	116	0.00
20	67	176.625.75	**70**	117	0.00
21	68	165.523.56	**71**	118	0.00
22	69	154.893.79	**72**	119	0.00
23	70	144.738.66	**73**	120	0.00
24	71	135.039.79	**74**	121	0.00
25	72	125.765.77	**75**	122	0.00
26	73	116.874.73	**76**	123	0.00
27	74	108.318.39	**77**	124	0.00
28	75	100.054.21	**78**	125	0.00
29	76	92.056.54	**79**	126	0.00
30	77	84.320.29	**80**	127	0.00
31	78	76.854.33	**81**		7.271.037.00
32	79	69.678.33	**82**		**7.54482**
33	80	62.813.36			
34	81	56.271.79			
35	82	50.055.64			
36	83	44.163.37			
37	84	38.594.58			
38	85	33.355.64			
39	86	28.469.52			
40	87	23.965.91			
41	88	19.873.67			
42	89	16.214.65			
43	90	12.999.82			
44	91	10.227.27			
45	92	7.882.30			
46	93	5.938.98			
47	94	4.361.98			
48	95	3.109.43			
49	96	2.134.22			
50	97	1.387.61			
51	98	824.98			
52	99	412.51			

APPENDIX F - Calculation of Pensions for Passive Male Members
Male – Retirement Sheet in Microsoft Excel

	E	F		E	F
2	Age	Dx	2	Age	Dx
3	57	842.372.50	52	106	0.00
4	58	792.239.30	53	107	0.00
5	59	744.259.78	54	108	0.00
6	60	698.349.58	55	109	0.00
7	61	654.400.12	56	110	0.00
8	62	612.306.61	57	111	0.00
9	63	571.958.52	58	112	0.00
10	64	533.250.54	59	113	0.00
11	65	496.105.83	60	114	0.00
12	66	460.471.26	61	115	0.00
13	67	426.330.61	62	116	0.00
14	68	393.669.62	63	117	0.00
15	69	362.479.74	64	118	0.00
16	70	332.732.24	65	119	0.00
17	71	304.367.61	66	120	0.00
18	72	277.322.37	67	121	0.00
19	73	251.531.39	68	122	0.00
20	74	226.943.60	69	123	0.00
21	75	203.559.76	70	124	0.00
22	76	181.422.15	71	125	0.00
23	77	160.596.62	72	126	0.00
24	78	141.153.72	73	127	0.00
25	79	123.153.26	74	128	0.00
26	80	106.609.67	75	129	0.00
27	81	91.497.50	76	130	0.00
28	82	77.774.62	77	131	0.00
29	83	65.386.23	78	132	0.00
30	84	54.285.52	79	133	0.00
31	85	44.449.50	80	134	0.00
32	86	35.858.04	81		10.377.648.79
33	87	28.478.46	82		**12.3195**
34	88	22.252.53			
35	89	17.096.93			
36	90	12.907.53			
37	91	9.566.69			
38	92	6.951.98			
39	93	4.942.86			
40	94	3.426.53			
41	95	2.297.73			
42	96	1.466.26			
43	97	859.44			
44	98	425.46			
45	99	138.59			
46	100	0.00			
47	101	0.00			
48	102	0.00			
49	103	0.00			
50	104	0.00			
51	105	0.00			

APPENDIX G – Calculation of Pensions for Passive Female Members
Female – Retirement Sheet in Microsoft Excel

	E	F		E	F
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	53	907.268.09	**52**	102	0.00
4	54	858.750.85	**53**	103	0.00
5	55	812.451.91	**54**	104	0.00
6	56	768.277.74	**55**	105	0.00
7	57	726.154.17	**56**	106	0.00
8	58	686.022.05	**57**	107	0.00
9	59	647.820.42	**58**	108	0.00
10	60	611.456.10	**59**	109	0.00
11	61	576.824.39	**60**	110	0.00
12	62	543.791.58	**61**	111	0.00
13	63	512.220.60	**62**	112	0.00
14	64	481.965.43	**63**	113	0.00
15	65	452.932.75	**64**	114	0.00
16	66	425.070.92	**65**	115	0.00
17	67	398.352.17	**66**	116	0.00
18	68	372.770.38	**67**	117	0.00
19	69	348.330.84	**68**	118	0.00
20	70	324.989.36	**69**	119	0.00
21	71	302.670.33	**70**	120	0.00
22	72	281.272.98	**71**	121	0.00
23	73	260.681.11	**72**	122	0.00
24	74	240.792.39	**73**	123	0.00
25	75	221.545.05	**74**	124	0.00
26	76	202.926.83	**75**	125	0.00
27	77	184.959.10	**76**	126	0.00
28	78	167.689.21	**77**	127	0.00
29	79	151.167.83	**78**	128	0.00
30	80	135.424.78	**79**	129	0.00
31	81	120.464.86	**80**	130	0.00
32	82	106.284.42	**81**		13.336.449.80
33	83	92.882.46	**82**		**14.6996**
34	84	80.274.33			
35	85	68.515.29			
36	86	57.676.82			
37	87	47.828.37			
38	88	39.022.48			
39	89	31.285.62			
40	90	24.613.15			
41	91	18.969.70			
42	92	14.292.86			
43	93	10.497.63			
44	94	7.483.21			
45	95	5.136.26			
46	96	3.339.45			
47	97	1.985.41			
48	98	992.76			
49	99	325.39			
50	100	0.00			
51	101	0.00			

APPENDIX H - Calculation of Premiums for Active Male Members
Male – Premium Sheet in Microsoft Excel

	E	F		E	F
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	37	945.046.81	**53**	87	0.00
4	38	897.884.48	**54**	88	0.00
5	39	852.921.84	**55**	89	0.00
6	40	810.040.18	**56**	90	0.00
7	41	769.137.01	**57**	91	0.00
8	42	730.101.47	**58**	92	0.00
9	43	692.859.35	**59**	93	0.00
10	44	657.312.36	**60**	94	0.00
11	45	623.388.78	**61**	95	0.00
12	46	591.002.25	**62**	96	0.00
13	47	560.090.02	**63**	97	0.00
14	48	530.581.28	**64**	98	0.00
15	49	502.414.99	**65**	99	0.00
16	50	475.519.04	**66**	100	0.00
17	51	449.836.49	**67**	101	0.00
18	52	425.288.27	**68**	102	0.00
19	53	401.812.35	**69**	103	0.00
20	54	379.345.30	**70**	104	0.00
21	55	357.827.39	**71**	105	0.00
22	56	337.219.94	**72**	106	0.00
23	57	317.481.34	**73**	107	0.00
24	58	298.586.66	**74**	108	0.00
25	59	280.503.68	**75**	109	0.00
26	60	0.00	**76**	110	0.00
27	61	0.00	**77**	111	0.00
28	62	0.00	**78**	112	0.00
29	63	0.00	**79**	113	0.00
30	64	0.00	**80**	114	0.00
31	65	0.00	**81**		12.886.201.30
32	66	0.00	**82**		**13.6355**
33	67	0.00			
34	68	0.00			
35	69	0.00			
36	70	0.00			
37	71	0.00			
38	72	0.00			
39	73	0.00			
40	74	0.00			
41	75	0.00			
42	76	0.00			
43	77	0.00			
44	78	0.00			
45	79	0.00			
46	80	0.00			
47	81	0.00			
48	82	0.00			
49	83	0.00			
50	84	0.00			
51	85	0.00			
52	86	0.00			

APPENDIX I - Present Values of Pensions for Active Members

Active Sheet in Microsoft Excel

Male

	A	B	C	D	E	F
4	Average Age	Number of Members	Future Service Time	${}_{n\|}\ddot{a}_x$	${}_{n\|}\ddot{a}_x^{(m)}$	PresentValue of Total Pension
15	Total	**=INPUT!G2**	**=INPUT!H2**	**=INPUT!I2**	**='MALE-ACTIVE'!C82**	**=VLOOKUP(C15+D15;'CSO 80 male'!$A$5:$E$104;4)/ VLOOKUP(C15;'CSO 80 male'!$A$5:$E$104;4) *VLOOKUP(D15;SCALE!$A$1:$G$103;7) *('MALE-RETIREMENT'!$U$166 -(($I$1-1)/(2*$I$1)))**

Female

	A	B	C	D	E	F
21	Average Age	Number of Members	Future Service Time	${}_{n\|}\ddot{a}_x$	${}_{n\|}\ddot{a}_x^{(m)}$	PresentValue of Total Pension
32	Total	**=INPUT!G3**	**=INPUT!H3**	**=INPUT!I3**	**='FEMALE-ACTIVE'!C82**	**=VLOOKUP(C32+D32;'CSO 80 female'!$A$5:$E$104;4)/ VLOOKUP(C32;'CSO 80 female'!$A$5:$E$104;4) *VLOOKUP(D32;SCALE!$A$1:$G$103;7) *('FEMALE-RETIREMENT'!$R$166 -(($I$1-1)/(2*$I$1)))**

APPENDIX J - Present Values of Pensions for Passive Members

Retirement Sheet in Microsoft Excel

Male

	A	B	C	D	E
4	Average Age	Number of Members	$\ddot{a}_x$	$\ddot{a}_x^{(m)}$	PresentValue of Total Pension
15	Total	=INPUT!G4	=INPUT!H4	='MALE-RETIREMENT'!F82	=D15-(($I$1-1)/(2*$I$1))

Female

	A	B	C	D	E	F
21	Average Age	Number of Members	Future Service Time	${}_{n\vert}\ddot{a}_x$	${}_{n\vert}\ddot{a}_x^{(m)}$	PresentValue of Total Pension
32	Total	=INPUT!G5	=INPUT!H5	='FEMALE-RETIREMENT'!F82	=D32-(($I$1-1) /(2*$I$1))	=B32*E32*12*INPUT!M5

APPENDIX K - Present Values of Premiums for Active Members Premium Sheet in Microsoft Excel

Male

	A	B	C	D	E
4	Average Age	Number of Members	$\ddot{a}_x$	$\ddot{a}_x^{(m)}$	PresentValue of Total Pension
15	Total	=INPUT!G2	=INPUT!H2	=INPUT!I2	='MALE-PREMIUM'!F82

Female

	A	B	C	D	E	F
19	Average Age	Number of Members	Future Service Time	${}_{n\|}\ddot{a}_x$	${}_{n\|}\ddot{a}_x^{(m)}$	PresentValue of Total Pension
30	Total	=INPUT!G3	=INPUT!H3	=INPUT!I3	='FEMALE-PREMIUM'!F82	=E30-(($L$2-1)/(2*$L$2))*(1-VLOOKUP(C30+D30;'CSO 80 male'!$A$5:$E$104;4)/VLOOKUP(C30;'CSO 80 male'!$A$5:$E$104;4)*VLOOKUP(D30;SCALE!$A$1:$G$103;7))

APPENDIX L - Calculation of Health Expensive for Male Active Members Health Sheet in Microsoft Excel

	A	B		A	B
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	37	945.046.81	**53**	87	47.053.48
4	38	924.821.01	**54**	88	37.869.69
5	39	904.864.78	**55**	89	29.968.70
6	40	885.152.78	**56**	90	23.303.98
7	41	865.670.48	**57**	91	17.790.41
8	42	846.387.71	**58**	92	13.315.88
9	43	827.310.29	**59**	93	9.751.62
10	44	808.411.30	**60**	94	6.962.90
11	45	789.690.26	**61**	95	4.809.20
12	46	771.123.89	**62**	96	3.160.98
13	47	752.714.15	**63**	97	1.908.37
14	48	734.448.58	**64**	98	973.07
15	49	716.323.64	**65**	99	326.47
16	50	698.315.75	**66**	100	0.00
17	51	680.418.05	**67**	101	0.00
18	52	662.585.26	**68**	102	0.00
19	53	644.790.87	**69**	103	0.00
20	54	626.999.99	**70**	104	0.00
21	55	609.177.18	**71**	105	0.00
22	56	591.317.21	**72**	106	0.00
23	57	573.406.61	**73**	107	0.00
24	58	555.459.14	**74**	108	0.00
25	59	537.474.06	**75**	109	0.00
26	60	519.449.18	**76**	110	0.00
27	61	501.361.27	**77**	111	0.00
28	62	483.185.15	**78**	112	0.00
29	63	464.885.92	**79**	113	0.00
30	64	446.426.94	**80**	114	0.00
31	65	427.790.02	**81**		24.658.864.65
32	66	408.974.35	**82**		**26.0927**
33	67	390.011.37			
34	68	370.936.77			
35	69	351.794.42			
36	70	332.611.54			
37	71	313.384.91			
38	72	294.104.58			
39	73	274.755.44			
40	74	255.334.36			
41	75	235.895.95			
42	76	216.548.96			
43	77	197.441.94			
44	78	178.744.45			
45	79	160.628.78			
46	80	143.222.51			
47	81	126.607.99			
48	82	110.847.78			
49	83	95.987.05			
50	84	82.081.93			
51	85	69.225.75			
52	86	57.520.76			

APPENDIX M - Calculation of Health Expensive for Female Active Members Health Sheet in Microsoft Excel

	C	D		C	D
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	35	963.712.35	**53**	85	124.807.69
4	36	943.796.09	**54**	86	108.216.23
5	37	924.189.58	**55**	87	92.430.19
6	38	904.872.52	**56**	88	77.674.85
7	39	885.826.07	**57**	89	64.142.75
8	40	867.024.12	**58**	90	51.976.51
9	41	848.451.14	**59**	91	41.260.81
10	42	830.092.92	**60**	92	32.020.90
11	43	811.944.64	**61**	93	24.223.83
12	44	794.017.91	**62**	94	17.785.94
13	45	776.307.83	**63**	95	12.573.99
14	46	758.810.00	**64**	96	8.420.51
15	47	741.527.92	**65**	97	5.156.46
16	48	724.457.59	**66**	98	2.655.73
17	49	707.581.25	**67**	99	896.56
18	50	690.889.81	**68**	100	0.00
19	51	674.368.47	**69**	101	0.00
20	52	658.010.66	**70**	102	0.00
21	53	641.797.91	**71**	103	0.00
22	54	625.701.31	**72**	104	0.00
23	55	609.726.08	**73**	105	0.00
24	56	593.871.64	**74**	106	0.00
25	57	578.149.82	**75**	107	0.00
26	58	562.583.33	**76**	108	0.00
27	59	547.193.14	**77**	109	0.00
28	60	531.971.68	**78**	110	0.00
29	61	516.897.08	**79**	111	0.00
30	62	501.914.99	**80**	112	0.00
31	63	486.958.49	**81**		27.927.972.79
32	64	471.941.34	**82**		**28.9796**
33	65	456.817.87			
34	66	441.578.56			
35	67	426.236.86			
36	68	410.830.26			
37	69	395.412.31			
38	70	379.983.40			
39	71	364.504.21			
40	72	348.897.56			
41	73	333.055.59			
42	74	316.874.38			
43	75	300.291.92			
44	76	283.307.64			
45	77	265.969.46			
46	78	248.369.57			
47	79	230.616.24			
48	80	212.797.18			
49	81	194.968.89			
50	82	177.178.81			
51	83	159.482.53			
52	84	141.968.93			

APPENDIX N - Calculation of Health Expensive for Male Passive Members Health Sheet in Microsoft Excel

	E	F		E	F
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	57	842.372.50	**53**	107	0.00
4	58	816.006.48	**54**	108	0.00
5	59	789.585.20	**55**	109	0.00
6	60	763.105.44	**56**	110	0.00
7	61	736.533.10	**57**	111	0.00
8	62	709.831.18	**58**	112	0.00
9	63	682.948.38	**59**	113	0.00
10	64	655.830.91	**60**	114	0.00
11	65	628.452.03	**61**	115	0.00
12	66	600.810.55	**62**	116	0.00
13	67	572.952.69	**63**	117	0.00
14	68	544.930.82	**64**	118	0.00
15	69	516.809.44	**65**	119	0.00
16	70	488.628.51	**66**	120	0.00
17	71	460.383.31	**67**	121	0.00
18	72	432.059.22	**68**	122	0.00
19	73	403.634.04	**69**	123	0.00
20	74	375.103.19	**70**	124	0.00
21	75	346.546.87	**71**	125	0.00
22	76	318.124.84	**72**	126	0.00
23	77	290.055.35	**73**	127	0.00
24	78	262.587.50	**74**	128	0.00
25	79	235.974.38	**75**	129	0.00
26	80	210.403.41	**76**	130	0.00
27	81	185.995.57	**77**	131	0.00
28	82	162.842.77	**78**	132	0.00
29	83	141.011.37	**79**	133	0.00
30	84	120.583.82	**80**	134	0.00
31	85	101.697.24	**81**		13.769.994.18
32	86	84.501.83	**82**		**16.3467**
33	87	69.124.70			
34	88	55.633.10			
35	89	44.026.02			
36	90	34.235.11			
37	91	26.135.30			
38	92	19.561.92			
39	93	14.325.78			
40	94	10.228.97			
41	95	7.065.03			
42	96	4.643.69			
43	97	2.803.52			
44	98	1.429.51			
45	99	479.61			
46	100	0.00			
47	101	0.00			
48	102	0.00			
49	103	0.00			
50	104	0.00			
51	105	0.00			
52	106	0.00			

APPENDIX O – Calculation of Health Expensive for Female Passive Members Health Sheet in Microsoft Excel

	E	F		E	F
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	53	877.105.98	**53**	103	0.00
4	54	852.905.12	**54**	104	0.00
5	55	828.660.84	**55**	105	0.00
6	56	804.366.01	**56**	106	0.00
7	57	780.002.30	**57**	107	0.00
8	58	755.588.45	**58**	108	0.00
9	59	731.123.43	**59**	109	0.00
10	60	706.604.27	**60**	110	0.00
11	61	681.999.36	**61**	111	0.00
12	62	657.274.48	**62**	112	0.00
13	63	632.382.12	**63**	113	0.00
14	64	607.272.46	**64**	114	0.00
15	65	581.920.74	**65**	115	0.00
16	66	556.325.87	**66**	116	0.00
17	67	530.530.63	**67**	117	0.00
18	68	504.583.54	**68**	118	0.00
19	69	478.544.29	**69**	119	0.00
20	70	452.449.91	**70**	120	0.00
21	71	426.296.02	**71**	121	0.00
22	72	400.069.07	**72**	122	0.00
23	73	373.748.52	**73**	123	0.00
24	74	347.330.13	**74**	124	0.00
25	75	320.888.15	**75**	125	0.00
26	76	294.570.52	**76**	126	0.00
27	77	268.579.33	**77**	127	0.00
28	78	243.145.23	**78**	128	0.00
29	79	218.502.57	**79**	129	0.00
30	80	194.824.91	**80**	130	0.00
31	81	172.224.25	**81**		16.113.486.38
32	82	150.785.71	**82**		**18.3712**
33	83	130.570.73			
34	84	111.655.66			
35	85	94.167.46			
36	86	78.245.22			
37	87	64.006.63			
38	88	51.513.96			
39	89	40.766.28			
40	90	31.700.30			
41	91	24.200.22			
42	92	18.113.53			
43	93	13.265.08			
44	94	9.471.61			
45	95	6.541.93			
46	96	4.299.86			
47	97	2.595.94			
48	98	1.323.67			
49	99	444.10			
50	100	0.00			
51	101	0.00			
52	102	0.00			

APPENDIX P - Present Values of Health for Active and Passive Members Health Sheet in Microsoft Excel

	M	N	O
1	**Foundation**	**Male**	**Female**
2	**Active Age**	=ACTIVE!C15	=ACTIVE!C32
3	**Active Number**	=ACTIVE!B15	=ACTIVE!B32
4	**Annual Average**	=INPUT!O2	=INPUT!O3
5	**Cost**	=$N$4*$N$3*$B$82	=$O$4*$O$3*$D$82
6	**TOTAL**	=N5+O5	
7			
8		Male	Female
9	**Passive Age**	=RETIREMENT!C15	=RETIREMENT!C32
10	**Passive Number**	=RETIREMENT!B15+DISABILITY!L2	=RETIREMENT!B32+DISABILITY!L3
11	**Annual Average**	=INPUT!O4	=INPUT!O5
12	**Cost**	=$N$10*$N$11*$F$82	=$O$10*$O$11*$H$82
13	**TOTAL**	=N12+O12	
14			
15	**General Total**	=N6+N13	

APPENDIX Q - Calculation of Disability Benefits for Male Members
Disability Sheet in Microsoft Excel

	S	T		S	T
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	52	884.143.76	**53**	102	0.00
4	53	835.339.02	**54**	103	0.00
5	54	788.631.64	**55**	104	0.00
6	55	743.897.45	**56**	105	0.00
7	56	701.056.04	**57**	106	0.00
8	57	660.020.89	**58**	107	0.00
9	58	620.740.22	**59**	108	0.00
10	59	583.147.01	**60**	109	0.00
11	60	547.175.17	**61**	110	0.00
12	61	512.739.61	**62**	111	0.00
13	62	479.758.25	**63**	112	0.00
14	63	448.144.46	**64**	113	0.00
15	64	417.815.75	**65**	114	0.00
16	65	388.711.90	**66**	115	0.00
17	66	360.791.28	**67**	116	0.00
18	67	334.041.19	**68**	117	0.00
19	68	308.450.45	**69**	118	0.00
20	69	284.012.36	**70**	119	0.00
21	70	260.704.41	**71**	120	0.00
22	71	238.479.98	**72**	121	0.00
23	72	217.289.33	**73**	122	0.00
24	73	197.081.42	**74**	123	0.00
25	74	177.816.25	**75**	124	0.00
26	75	159.494.40	**76**	125	0.00
27	76	142.149.00	**77**	126	0.00
28	77	125.831.65	**78**	127	0.00
29	78	110.597.63	**79**	128	0.00
30	79	96.493.80	**80**	129	0.00
31	80	83.531.47	**81**		12.084.227.29
32	81	71.690.68	**82**		**13.667718**
33	82	60.938.45			
34	83	51.231.82			
35	84	42.534.12			
36	85	34.827.34			
37	86	28.095.72			
38	87	22.313.62			
39	88	17.435.44			
40	89	13.395.89			
41	90	10.113.39			
42	91	7.495.76			
43	92	5.447.06			
44	93	3.872.86			
45	94	2.684.78			
46	95	1.800.33			
47	96	1.148.85			
48	97	673.39			
49	98	333.36			
50	99	108.59			
51	100	0.00			
52	101	0.00			

APPENDIX R - Calculation of Disability Benefits for Female Members
Disability Sheet in Microsoft Excel

	U	V		U	V
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	48	930.229.13	**52**	97	1.555.62
4	49	882.096.42	**53**	98	777.86
5	50	836.202.20	**54**	99	254.95
6	51	792.432.99	**55**	100	0.00
7	52	750.690.64	**56**	101	0.00
8	53	710.868.29	**57**	102	0.00
9	54	672.853.76	**58**	103	0.00
10	55	636.577.33	**59**	104	0.00
11	56	601.965.71	**60**	105	0.00
12	57	568.960.79	**61**	106	0.00
13	58	537.516.22	**62**	107	0.00
14	59	507.584.25	**63**	108	0.00
15	60	479.091.85	**64**	109	0.00
16	61	451.957.00	**65**	110	0.00
17	62	426.074.93	**66**	111	0.00
18	63	401.338.24	**67**	112	0.00
19	64	377.632.53	**68**	113	0.00
20	65	354.884.66	**69**	114	0.00
21	66	333.054.19	**70**	115	0.00
22	67	312.119.35	**71**	116	0.00
23	68	292.075.34	**72**	117	0.00
24	69	272.926.33	**73**	118	0.00
25	70	254.637.67	**74**	119	0.00
26	71	237.150.12	**75**	120	0.00
27	72	220.384.74	**76**	121	0.00
28	73	204.250.47	**77**	122	0.00
29	74	188.667.14	**78**	123	0.00
30	75	173.586.34	**79**	124	0.00
31	76	158.998.48	**80**	125	0.00
32	77	144.920.30	**81**		14.641.108.77
33	78	131.388.88	**82**		**15.73925**
34	79	118.443.95			
35	80	106.108.86			
36	81	94.387.37			
37	82	83.276.62			
38	83	72.775.84			
39	84	62.897.04			
40	85	53.683.52			
41	86	45.191.30			
42	87	37.474.78			
43	88	30.575.14			
44	89	24.513.11			
45	90	19.285.04			
46	91	14.863.26			
47	92	11.198.83			
48	93	8.225.17			
49	94	5.863.29			
50	95	4.024.39			
51	96	2.616.55			

APPENDIX S - Present Values of Disability Benefits
Disability Sheet in Microsoft Excel

	K	L	M	N	O	P
1	**Current**	**Count**	**Age**	**Present Value**	**Salary**	**Liability**
2	**Disability Male**	=INPUT!G6	=INPUT!H6	=T82	=INPUT!M6	=N2*L2*(12*O2)
3	**Disability Female**	=INPUT!G7	=INPUT!H7	=V82	=INPUT!M7	=N3*L3*(12*O3)
4	**Total**					=SUM(P2:P3)

APPENDIX T - Calculation of Male Orphan Benefits

Orphans Sheet in Microsoft Excel

	T	U		T	U
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	18	978.976.23	**51**	66	0.00
4	19	930.698.72	**52**	67	0.00
5	20	884.731.07	**53**	68	0.00
6	21	841.000.07	**54**	69	0.00
7	22	799.422.63	**55**	70	0.00
8	23	759.915.93	**56**	71	0.00
9	24	722.383.32	**57**	72	0.00
10	25	686.731.98	**58**	73	0.00
11	26	0.00	**59**	74	0.00
12	27	0.00	**60**	75	0.00
13	28	0.00	**61**	76	0.00
14	29	0.00	**62**	77	0.00
15	30	0.00	**63**	78	0.00
16	31	0.00	**64**	79	0.00
17	32	0.00	**65**	80	0.00
18	33	0.00	**66**	81	0.00
19	34	0.00	**67**	82	0.00
20	35	0.00	**68**	83	0.00
21	36	0.00	**69**	84	0.00
22	37	0.00	**70**	85	0.00
23	38	0.00	**71**	86	0.00
24	39	0.00	**72**	87	0.00
25	40	0.00	**73**	88	0.00
26	41	0.00	**74**	89	0.00
27	42	0.00	**75**	90	0.00
28	43	0.00	**76**	91	0.00
29	44	0.00	**77**	92	0.00
30	45	0.00	**78**	93	0.00
31	46	0.00	**79**	94	0.00
32	47	0.00	**80**	95	0.00
33	48	0.00	**81**		6.603.859.95
34	49	0.00	**82**		**6.745680**
35	50	0.00			
36	51	0.00			
37	52	0.00			
38	53	0.00			
39	54	0.00			
40	55	0.00			
41	56	0.00			
42	57	0.00			
43	58	0.00			
44	59	0.00			
45	60	0.00			
46	61	0.00			
47	62	0.00			
48	63	0.00			
49	64	0.00			
50	65	0.00			

APPENDIX U - Calculation of Female Orphan Benefits

Orphans Sheet in Microsoft Excel

	V	W		V	W
2	**Age**	**Dx**	**2**	**Age**	**Dx**
3	27	974.437.68	**51**	75	0.00
4	28	926.903.68	**52**	76	0.00
5	29	881.653.12	**53**	77	0.00
6	30	838.578.07	**54**	78	0.00
7	31	0.00	**55**	79	0.00
8	32	0.00	**56**	80	0.00
9	33	0.00	**57**	81	0.00
10	34	0.00	**58**	82	0.00
11	35	0.00	**59**	83	0.00
12	36	0.00	**60**	84	0.00
13	37	0.00	**61**	85	0.00
14	38	0.00	**62**	86	0.00
15	39	0.00	**63**	87	0.00
16	40	0.00	**64**	88	0.00
17	41	0.00	**65**	89	0.00
18	42	0.00	**66**	90	0.00
19	43	0.00	**67**	91	0.00
20	44	0.00	**68**	92	0.00
21	45	0.00	**69**	93	0.00
22	46	0.00	**70**	94	0.00
23	47	0.00	**71**	95	0.00
24	48	0.00	**72**	96	0.00
25	49	0.00	**73**	97	0.00
26	50	0.00	**74**	98	0.00
27	51	0.00	**75**	99	0.00
28	52	0.00	**76**	100	0.00
29	53	0.00	**77**	101	0.00
30	54	0.00	**78**	102	0.00
31	55	0.00	**79**	103	0.00
32	56	0.00	**80**	104	0.00
33	57	0.00	**81**		3.621.572.55
34	58	0.00	**82**		**3.71658**
35	59	0.00			
36	60	0.00			
37	61	0.00			
38	62	0.00			
39	63	0.00			
40	64	0.00			
41	65	0.00			
42	66	0.00			
43	67	0.00			
44	68	0.00			
45	69	0.00			
46	70	0.00			
47	71	0.00			
48	72	0.00			
49	73	0.00			
50	74	0.00			

APPENDIX V - Present Values of Orphan Benefits
Orphan Sheet in Microsoft Excel

	K	L	M	N	O	P	Q
	Current	**Count**	**Age**	**Time**	**Present Value**	**Salary**	**Liability**
1	**Orphan Male**	=INPUT!G10	=INPUT!H10	=25-M2	=U82	=INPUT!N10	=O2*L2*(12*P2)
2	**Orphan Female**	=INPUT!G11	=INPUT!H11	=30-M3	=W82	=INPUT!N11	=O3*L3*(12*P3)
3	**Total**						=SUM(Q2:Q3)

APPENDIX W – Calculation of Male Widow Benefits
Widow Sheet in Microsoft Excel

	S	T		S	T
2	**Age**	**D_x**	**2**	**Age**	**D_x**
3	61	795.427.43	**49**	107	0.00
4	62	744.262.51	**50**	108	0.00
5	63	695.219.15	**51**	109	0.00
6	64	648.169.37	**52**	110	0.00
7	65	603.019.74	**53**	111	0.00
8	66	559.705.70	**54**	112	0.00
9	67	518.207.52	**55**	113	0.00
10	68	478.507.89	**56**	114	0.00
11	69	440.596.39	**57**	115	0.00
12	70	404.438.11	**58**	116	0.00
13	71	369.960.73	**59**	117	0.00
14	72	337.087.07	**60**	118	0.00
15	73	305.737.98	**61**	119	0.00
16	74	275.851.36	**62**	120	0.00
17	75	247.428.16	**63**	121	0.00
18	76	220.519.76	**64**	122	0.00
19	77	195.206.19	**65**	123	0.00
20	78	171.573.23	**66**	124	0.00
21	79	149.693.56	**67**	125	0.00
22	80	129.584.72	**68**	126	0.00
23	81	111.215.78	**69**	127	0.00
24	82	94.535.53	**70**	128	0.00
25	83	79.477.37	**71**	129	0.00
26	84	65.984.38	**72**	130	0.00
27	85	54.028.64	**73**	131	0.00
28	86	43.585.68	**74**	132	0.00
29	87	34.615.75	**75**	133	0.00
30	88	27.048.08	**76**	134	0.00
31	89	20.781.43	**77**	135	0.00
32	90	15.689.19	**78**	136	0.00
33	91	11.628.38	**79**	137	0.00
34	92	8.450.18	**80**	138	0.00
35	93	6.008.07	**81**		8.873.715.41
36	94	4.164.97	**82**		**11.155908**
37	95	2.792.91			
38	96	1.782.25			
39	97	1.044.65			
40	98	517.15			
41	99	168.45			
42	100	0.00			
43	101	0.00			
44	102	0.00			
45	103	0.00			
46	104	0.00			
47	105	0.00			
48	106	0.00			

APPENDIX X – Calculation of Female Widow Benefits
Widow Sheet in Microsoft Excel

	U	V		U	V
2	**Age**	**D_x**	**2**	**Age**	**D_x**
3	61	852.232.34	**48**	106	0.00
4	62	803.427.83	**49**	107	0.00
5	63	756.783.11	**50**	108	0.00
6	64	712.082.45	**51**	109	0.00
7	65	669.187.96	**52**	110	0.00
8	66	628.023.34	**53**	111	0.00
9	67	588.547.59	**54**	112	0.00
10	68	550.751.62	**55**	113	0.00
11	69	514.643.30	**56**	114	0.00
12	70	480.157.29	**57**	115	0.00
13	71	447.181.92	**58**	116	0.00
14	72	415.568.29	**59**	117	0.00
15	73	385.144.73	**60**	118	0.00
16	74	355.760.02	**61**	119	0.00
17	75	327.322.94	**62**	120	0.00
18	76	299.815.34	**63**	121	0.00
19	77	273.268.84	**64**	122	0.00
20	78	247.753.33	**65**	123	0.00
21	79	223.343.73	**66**	124	0.00
22	80	200.084.08	**67**	125	0.00
23	81	177.981.46	**68**	126	0.00
24	82	157.030.50	**69**	127	0.00
25	83	137.229.70	**70**	128	0.00
26	84	118.601.75	**71**	129	0.00
27	85	101.228.29	**72**	130	0.00
28	86	85.214.93	**73**	131	0.00
29	87	70.664.28	**74**	132	0.00
30	88	57.653.98	**75**	133	0.00
31	89	46.223.12	**76**	134	0.00
32	90	36.364.83	**77**	135	0.00
33	91	28.026.89	**78**	136	0.00
34	92	21.117.06	**79**	137	0.00
35	93	15.509.78	**80**	138	0.00
36	94	11.056.11	**81**		10.812.386.07
37	95	7.588.60	**82**		**12.68713**
38	96	4.933.89			
39	97	2.933.36			
40	98	1.466.76			
41	99	480.75			
42	100	0.00			
43	101	0.00			
44	102	0.00			
45	103	0.00			
46	104	0.00			
47	105	0.00			

APPENDIX Y - Present Values of Widow Benefits
Widow Sheet in Microsoft Excel

Current	Count	Age	Coefficient	Salary	Liability
Widow Male	=INPUT!G8	=INPUT!H8	=T82	=INPUT!M8	=N2*L2*(12*O2)
Widow Female	=INPUT!G9	=INPUT!H9	=V82	=INPUT!M9	=N3*L3*(12*O3)
TOTAL					=SUM(P2:P3)