CUKUROVA UNIVERSITY

INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

Adaptive Sliding Mode Control of MIMO Coupled Tank System

Özgür GEYLANİ

Computer Engineering Department

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MSc THESIS APPROVAL

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This Master's Thesis was evaluated by the following Jury Members on 22/06/2023 and was approved by unanimity of votes.

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Advisor: Asst. Prof. Dr. Barış ATA

Department of Computer Engineering

ABSTRACT

The aim of this thesis is to design an adaptive sliding mode controller for a multi input multi output quadruple tank system developed for liquid level control. Sliding mode control is a robust control approach designed to control complex nonlinear systems. The sliding mode control method is an efficient control technique in the presence of external disturbances and parametric uncertainties. This control method offers adaptability and can be combined with other control techniques to achieve enhanced outcomes. In this thesis, both the sliding mode control and adaptive sliding mode control methods are applied to the coupled tank system. The stability of the controllers is mathematically proven using the Lyapunov theorem. The implemantation of the adaptive sliding mode control method for the multi input multi output coupled tank system demonstrates the success of the designed controller in achieving more effective control performans for the system. In addition, the applicability of the proposed method for demanding complex systems such as coupled tanks has been verified. Although both methods achieved successful results, the proposed adaptive sliding mode control method offers better tracking performance compared to the conventional sliding mode method.

Keywords: Coupled Tank System, Sliding Mode Control, Adaptive Sliding Mode Control, Sliding Surface.

ÇUKUROVA ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

YÜKSEK LİSANS TEZİ

Çok Giriş Çok Çıkışlı Bağlantılı Tank Sisteminin Adaptif Kayan Mod Kontrolü

Özgür GEYLANİ

Danışman:Doktor Öğretim Üyesi Barış ATA

Bilgisayar Mühendisliği Anabilim Dalı

ÖZ

Bu tezin amacı, sıvı seviye kontrolü için geliştirilen çok girişli çok çıkışlı dörtlü tank sistemi için adaptif bir kayan kipli kontrolör tasarlamaktır. Kayan kipli kontrol, oldukça karmaşık doğrusal olmayan sistemleri kontrol etmek için tasarlanmış güçlü bir kontrol yaklaşımıdır. Kayan kipli kontrol yöntemi, dış etkenlerin ve parametrik belirsizliklerin varlığında etkili bir kontrol tekniğidir. Bu kontrol yöntemi adaptif olabileceği gibi diğer kontrol yöntemleri ile birlikte kullanılarak daha etkili sonuçlar alınabilir. Bu tezde, hem kayan kipli kontrol hem de uyarlamalı kayan kipli kontrol yöntemleri, bağlantılı tank sistemine uygulanmıştır. Kontrolörlerin kararlılığı Lyapunov teoremi ile kanıtlanmıştır. Adaptif kayan kipli kontrol yönteminin çok girişli çok çıkışlı birleştirilmiş tank sistemi için uyarlanması, daha efektif sonuçlar elde etmek için tasarlanan kontrolörün sistem üzerindeki başarısını göstermiştir. Ayrıca önerilen yöntemin bağlantılı tank gibi zorlu karmaşık sistemler için uygunluğu doğrulanmıştır. Her iki yöntem de başarılı sonuçlar elde etmesine rağmen, önerilen adaptif kayan kipli kontrolör geleneksel kayan kipli kontrol yöntemi ile karşılaştırıldığında daha iyi izleme performansı sunmuştur.

Anahtar Kelimeler: Bağlantılı Tank Sistemi, Kayan Kipli Kontrol, Adaptif Kayan Kipli Kontrol, Kayma Yüzeyi.

GENİŞLETİLMİŞ ÖZET

Bu tezde lineer olmayan çok değişkenli sistemlerin kontrolü için adaptif kayan kipli kontrol yöntemi kullanılarak bir kontrolör tasarlanmıştır. Lineer olmayan sistemlerin kontrolü zorlu ve karmaşık bir problemdir. Bu sistemler için uygun kontrol metodlarına karar vermek oldukça meşakkatli bir iştir. Son dönemlerde çok değişkenli sistemlerin kontrolü önem kazanmaya devam etmektedir. Bu tarz sistemler için çok sayıda kontrol metodu geliştirilmiştir ve hala hem var olan tekniklere yeni yaklaşımlar kazandırılmaya hem de literatüre yeni yaklaşımlar eklenmeye devam edilmektedir.

Bağlantılı tank sistemleri ünitesi mühendislik uygulamalarında yaygın bir şekilde kullanılan sistemlerdir. Bu sistemler başta içecek ve kimya sektörü olmak üzere bir çok endüstriyel alanda sıklıkla farklı amaçlar için kullanılmaktadır. Bağlantılı tank sistemi genellikle laboratuvar ortamlarında sıvı seviyelerini kontrol etmek için kullanılmaktadır. Bu sistemler yapısı ve kullanılan tank sayısı ile farklı çeşitlere sahiptir. Bağlantılı tank sistemleri tek girişli tek çıkışlı ve çok girişli çok çıkışlı yapıda kullanılabilmektedir. Bu tezde kullanılan bağlı tank sistemi çok girişli çok çıkışlı yapıdadır ve 4 tanka sahiptir. İki katlı bir sistem gibi görünen bu düzenekte altta iki adet ve üstte iki adet olmak üzere toplam dört tank konumlandırılmıştır. Alt alta konumlandırılan tanklar biribiri ile bir geçit sayesinde etkileşim halindedir. Aynı zamanda üst kattaki tanklar da birbirleri arasında bir geçit bulunduğu için etkileşim halindedir. Geçitler üzerinde manuel kapatma vanaları bulunmaktadır. Bu çalışmada vanalar açık kullanıldığı için kullanılan sistemin modeli çapraz bağlantılı tank sistemi olarak adlandırılmaktadır.

Sistem rezervuar içerisine daldırılmış iki pompa ile beslenir. Sıvı akışı pompadan tanklara doğrudur. Sıvı ilk olarak yukarıdaki tanklara dolar, borular aracılığıyla da alt tanklara akar. Amaç alt tanklardaki sıvı seviyelerinin istenen seviyeye gelmesi ve sonrasında sıvı seviyesinin korunmasıdır. Bu sürecin gerçekleşmesi için sıvı akışının sisteme uygun kontrolörler ile kontrol edilmesi gerekir.

Literatürde bağlantılı tank sistemlerinin kontrolü için bir çok farklı yöntem önerilmiştir. PID kontrol, zaman-optimal kontrol, kayan kipli kontrol ve doğrusal olmayan geri adımlamalı kontrol yöntemleri bağlantılı tank sistemlerinde sıvı seviyesini kontrol etmek için sıklıkla kullanılan tekniklerdendir.

Bu çalışmada kayan kipli kontrol adaptif bir yaklaşımla birlikte kullanılmıştır. Sistemin belirsiz parametreleri ve bazı doğrusal kabul edilen dinamikler sonuçta farklılıklara sebep olmaktadır. Kayan kipli kontrol metodu literatürde yapılan birçok çalışmadan da ıspatlandığı üzere sistem belirsizlikleri ve harici bozuculara karşı dayanıklıdır. Kayan kipli kontrol karmaşık ve lineer olmayan sistemlerin kontrolünde kullanılan etkinliği çok sefer kanıtlanmış bir metoddur. 1950lerin başında Sovyetler Birliği'nde Emelyanov tarafından önerilen ve Utkins ve Itkis gibi araştırmacılar tarafından detaylandırılan, o dönemden beri kullanılmaya başlanılan gün geçtikçe popülerleşen kayan kipli kontrol belirsizlik ve bozucu etkisine karşı dayanıklı olmasından dolayı sıklıkla tercih edilen doğrusal olmayan gürbüz bir kontrol yöntemidir. Kayan kipli kontrol yöntemi, doğrusal olmayan sistem, çok girişli çok çıkışlı sistemler, ayrık zamanlı modeller, büyük ölçekli ve sonsuz boyutlu sistemler ve stokastik sistemler dahil olmak üzere bir çok farklı sisteme uygulanmıştır.

Kayan kipli kontrol yaklaşımı, bir tür değişken yapı kontrolüdür. Temel olarak, lineer olmayan sistemin durumlarını durum uzayında kayma veya anahtarlama yüzeyi olarak adlandırılan belirli ve kullanıcı tarafından seçilen bir yüzeye sürmek ve bu yüzeyde tutmak için yüksek hızlı bir anahtarlama kontrol kanunu kullanır. Bu yüzeye anahtarlama yüzeyi denir, çünkü tesisin durum yörüngesi yüzeyin "üzerinde" ise, kontrol yolunun farklı bir kazancı ve yörünge yüzeyin "altına" düşmesi durumunda farklı bir kazancı vardır.

Gürbüz bir kontrolör tasarlarken belirsizlik aralığı belirlemek zorludur bu nedenle genellikle geniş aralıklar seçilir ve bu durum istenmeyen etkilere sebep olur. Bu etkiler yüksek kontrol sinyalleri üretilmesine ve çatırtı problemine sebebiyet verebilir.

Adaptif kayan kipli kontrol tekniği bu sorunun üstesinden gelmek için kullanılmıştır. Adaptif kayan kipli kontrol yaklaşımının amacı, sistemdeki belirsizliklerin üst sınırı hakkında herhangi bir ön bilgi yokluğunda denetleyici kazancını uyarlamalı olarak ayarlamaktır. Bu uyarlamalı kontrol kazancı mümkün olduğu kadar küçük olmalı, aynı zamanda bozulmaları ve sistem belirsizliklerini ortadan kaldırmak için yeterli olmalıdır. Uyarlama, değişen dış etkilere ve parametrelere göre sistemin kendini değiştirme durumu olarak tanımlanabilir. Uyarlama yaklaşımı gerçek zamanlı olarak gerçekleşir. Adaptif kayan kipli kontrol tekniğini kullanarak çeşitli kontrolörler tasaralanmış olsa da kontrol kazancının adaptasyon oranı hala bazı kısıtlamaları barındırır.

Bu çalışmadaki amaca istinaden geleneksel kayan kipli kontrol kullanımıyla elde edilen istenmeyen etkiler adaptif kontrol yaklaşımıyla en aza indirilmeye çalışılmıştır. İlk olarak sistemin tam matematiksel modeli hazırlanmıştır. Matematiksel modeli hazılanan sistemin simülasyon ortamında tasarımı gerçekleştirilmiştir. Öncelikle sisteme uygun geleneksel kayan kipli bir denetleyici tasarlanmış ve test edilmiştir. Sonrasında adaptif kayan kipli kontrol yöntemi kullanılarak önerilen kontrolör bağlantılı tank sistemi için tasarlanmıştır ve simüle edilmiştir. Hem geleneksel kayan kipli kontrol ve hem de adaptif kayan kipli kontrol simülasyon sonuçları sistemi kontrol etmede başarılı olmuştur. Her iki methodun etkinliği sayısal ve grafiksel olarak gözlemlenmiştir. Her iki methodun da kararlılığını ispatlamak için Lyapunov kararlılık teorisinden yararlanılmıştır. Lyapunov teoremi belirsizlik ve bozucular varlığında kontrolörlerin kararlılıkları için kullanılan bir çözümdür. Önerilen adaptif kayan kipli kontrol yöntemi ve geleneksel kayan kipli kontrol yöntemi yerleşme zamanı, aşım oranı ve kararlı hal hatası gibi performans ölçütleri bakımından karşılaştırılmıştır. Adaptif kayan kipli kontrol yöntemi geleneksel kayan kipli kontrol yöntemine göre daha iyi izleme performansı sunmuştur.

Sonuç olarak kayan kipli kontrol yöntemi altında adaptif bir yasa kullanılarak elde edilen kontrol sinyali daha etkili sonuçlar elde edilmesini sağlamıştır. Adaptif kayan kipli kontrol yöntemi ile kontrolörün tasarım aşamasında sistem parametrelerinin tam olarak bilinmesine gerek yoktur çünkü uyarlamalı bir yapı mevcuttur. Çatırtı problemi oldukça sönümlenmiştir ve kontrolörün harici bozulmalara karşı dayanıklılığı ispatlamıştır.

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SYMBOLS AND ABBREVIATIONS

SISO : Single Input Single Output
MIMO : Multi Input Multi Output
SMC : Sliding Mode Control
ASMC : Adaptive Sliding Mode Control
ISMC : Integral Sliding Mode Control
CCT : Cross Coupled Tank
QCCT : Quadruple Cross Coupled Tank
PI : Proportional Integral
PID : Proportional Integral Derivative
FOPI : Fractional Ordered Proportional Integral
BELBIC : Brain Emotional Logic Based Intelligent Controller
SOSMC : Second Order Sliding Mode Controller
VSC : Variable Structure Control
$\boldsymbol{s}$: sliding surface
$\boldsymbol{V}$: Lyapunov function
$\boldsymbol{u}$: Control vector
$\boldsymbol{e}$: Tracking error
T1 : Tank 1
T2 : Tank 2
T3 : Tank 3
T4 : Tank 4

1. INTRODUCTION

In modern control systems, the control of both linear and non-linear systems has become an increasingly significant. Among the various systems used for testing and implementing control strategies, coupled tank systems hold a notable position due to their non-linear structure. Control experiments have been carried out on these systems in the past and are still being developed today. Coupled tank systems are commonly used in various industries for different purposes (Vhora and Patel, 2016). These systems, which have become widespread in industrial environments where liquid distribution equipment is used, especially in the pharmaceutical industry, chemical industry, and food sectors can be used in quite different ways. In the coupled tanks systems, the number of tanks can be adjusted according to the purpose, allowing for the connection of 2, 3 or more tank parts. The number of inputs and outputs of the system, as well as the way the tanks are connected, varies accordingly, resulting in different system structures, such as single input single output (SISO) and multi input multi output (MIMO) systems.

The coupled tank setup is a chemical plant unit that can be configured in different ways offering varying system dynamics depending on the couplings in the mechanism. The flow direction of the liquid can be changed by means of these couplings, which can be operated manually.

The setup used in this study is the MIMO quadruple cross coupled tank (QCCT) model. The system consists of 4 tanks, with 2 tanks placed at the top and 2 tanks at the bottom. There are open valves between the upper tanks and the lower tanks, when these valves are open the liquid passage can be provided from here. At the same time, there is an open liquid passageway between the two upper tanks which connects them and allows liquid to flow between them. Therefore, the model is called the cross coupled tank (CCT) model. Two pumps are present in the system to carry out the liquid flow. The liquid flowing from these pumps to the tanks needs to be controlled in real-time (Feedback Instruments Ltd.).

Liquid level control applications are commonly tested and implemented in laboratory environments using the coupled tanks systems. These systems belong to the class of multivariable systems. The control of multivariable systems, which are widely used today, has gained importance. Choosing the appropriate control approach for a given systems can indeed be a challenging task that requires careful consideration. Nevertheless, various controllers are applied and compared in practice. The primary purpose of using coupled tank systems is to bring the liquid level to the targeted level and use it as an automatic liquid distribution mechanism, which is frequently used in many industrial facilities. To achieve this, controllers are used to maintain the liquid level at the targeted level. For this reason, the liquid flowing towards the tank should be controlled with the most accurate methods. Fluid flow control is a challenging task and the controller design must be entirely compatible with the system.

Numerous remarkable control approaches have been developed, and are still being developed, using this coupled tank system, from the past to the present. In almost every study carried out in the literature, the aim is to achieve more stable and high quality results.

To control the liquid level of the coupled tank system with 2 identical tanks, the sliding mode controller (Alam et al., 2013), second order sliding mode controller (Khan et al., 2006), the backstepping controller (Pan et al., 2003), the proportional integral derivative (PID) controller (Roy et al., 2016), model predictive controller (Essahafi, 2014), fuzzy logic controller (Gupta and Verma, 2012) methods were used.

To control the liquid level of the MIMO structured coupled tank system with 4 identical tanks, sliding mode controller (Aksu and Coban, 2017), sliding mode PI controller with backstepping (Aksu and Coban, 2018), second order sliding mode controller, and brain emotional logic based intelligent controller (Debnanth and Mija, 2016) methods were used.

After Lyapunov and Poincare's approach, which brought a new idea to dynamic modeling studies in the 1950s, modern control techniques began to develop with a rising graphic. In an article published by Vadim Utkin in the 1970s, a relay-based control method was presented and this article formed the first step of the sliding mode theory (Ferrara, 2015). After these years, sliding mode theory has been used in many different fields and studies (Zhao et al., 2016; Fali et al., 2018).

The sliding mode controller is a popular controller widely used for controlling highly complex and non-linear systems. The Sliding Mode Control (SMC) approach is an easy to adjust and implement a control method that is insensitive to parametric uncertainties and external disturbances (Qureshi et al., 2018).

The SMC is a two-stage control method. The aim of the first stage is to drive the trajectory of the nonlinear system onto a surface called the sliding surface. The sliding surface is pre-designed by the user in accordance with the system. The purpose of the second stage is to ensure that the system trajectory, which is delivered to the sliding surface, remains here (Utkin, 1977).

The SMC method has both advantages and disadvantages. The most significant advantages are that the system dynamics can be adapted to the sliding surface, and the SMC is insensitive to system uncertainties. However, a notable disadvantage of the SMC is the chattering problem, which is still an active area of research (Utkin and Lee, 2006).

The chattering problem can have a detrimental effect on control stability, and therefore, different approaches have been developed to minimize or eliminate this problem. Several innovative approaches have been studied in the context of SMC such as decoupled backstepping sliding mode, high order sliding mode, integral sliding mode, and adaptive sliding mode controller (ASMC) (Ata and Coban, 2019; Utkin et al., 2020; Adhikary and Mahanta, 2013). One approach to overcome the chattering problem is to adapt switching gain. Adaptation refers to the ability of the system to adjust itself based on parametric changes within the system. The adaptation technique involves modifying the adaptation parameters based on the state of the applied system. In engineering applications, systems that can protect their own behavior against external influences and changes are known as adaptive systems. The adaptation technique that makes automatic adaptations works in real-time (Leyla et al., 2021).

By applying this technique to the sliding mode controller, both the chattering problem and the discontinuity issues can be reduced (Leyla et al., 2021). Moreover, the ASMC method provides more effective results in terms of performance compared to the conventional method (Huang et al., 2008).

The Lyapunov stability theorem is used to guarantee the system's stability. The Lyapunov stability theorem is a widely used method in the literature to demonstrate the stability of controllers. The Lyapunov stability theorem offers a robust solution for indicating the stability of the controller applied to a system in the presence of external disturbances and uncertainties.

The aim of this thesis is to design an adaptive sliding mode control law for a MIMO QCCT system. In order to analyze the suitability, efficiency, and performance of the proposed method, the conventional SMC technique has also been applied to the system for comparison purpose. The liquid level in the lower tanks is the part that is controlled on the system. In the installation consisting of four tanks, there are two tanks at the bottom and two tanks at the top. The objective is to maintain the liquid flowing into the two tanks at the bottom at the specified level. The complete mathematical model of the MIMO QCCT system has been examined and simulated. A law and an ASMC law are designed for a MIMO QCCT system. The stability of the system is guaranteed as both controllers are based on Lyapunov theory. The SMC and the ASMC methods were applied to the tank system through simulation tests. After analyzing the test results, both methods were compared in terms of performance. The results of the ASMC method and the conventional SMC are presented. The graphs obtained from the results and the table of the numerical results are included in the results and discussions section of the thesis for comparison purposes. Although both controllers yield robust results, the findings show that the proposed method provides more effective results against the conventional SMC method in terms of performance.

The structure of the thesis is as follows: In the 2nd chapter, preliminary works are discussed. In Chapter 3, the mathematical model for the MIMO QCCT system is presented and the SMC and the ASMC methods are explained theoretically. In Chapter 4, graphical results obtained from the tests are presented and various numerical analyses are provided.

2. PRELIMINARY WORK

The coupled tank system has been the subject of numerious studies conducted by researchers across the field. This chapter provides a foundation for the thesis by presenting the conceptual background related to the study.

Roy et al. carried out their experiments on the MIMO model of the two-tank setup. In general, in these setups, the liquid level in the first tank is maintained at a certain level, while the other tank follows the reference signal, which changes over time. Since some feed-forward controllers such as PID, PI, and 3DOF-PI could not maintain the liquid level in the first tank, the Fractional Ordered Proportional Integral (FOPI) controller was developed and tested to give better results than the PID family. The new controller is planned to use a reference field approach (Roy et al., 2016).

Pan et al., designed a backstepping method for precise liquid level control problem on coupled two tanks setup. First, a model-based control was proposed for liquid level control, and then an adaptive control was designed to alleviate the uncertainty in the orifice cross-sectional areas at the tank outputs. The proposed method has been tested experimentally and the performance results are compared in detail with the conventional PI control. With the Lyapunov theorem, it has been observed that the backstepping control approach gives a tracking response for trajectories up to the second order (Pan et al., 2003).

Gupta and Verma have worked on fuzzy logic controllers (FLCs). They designed FLC to control the liquid level of a coupled tank system (Gupta and Verma, 2012).

In the study by Alam et al., the conventional SMC technique and the integral SMC technique were applied to two identical tanks connected to each other. The system they use has a SISO structure. The experimental results obtained from the two control techniques were compared and it was observed that the chattering problem was reduced to a lesser extent in the experiment using the integral sliding mode control (ISMC) (Alam et al., 2013).

In the study of Khan et al., a second order sliding mode control (SOSMC) method was proposed in a twin tank setup with a MIMO structure that they used in the laboratory environment. The algorithm used introduces a new approach to sliding mode control and does not derive the sliding surface, in consequence terminating the need to create an observer or vertex detector. The algorithm produced a dynamic control signal and robust results were obtained and the proposed theory was confirmed. Results were also obtained by using PI control on the same mechanism, and the results of the quadratic sliding mode controller in terms of contextualization are presented together with the results of the PI controller (Khan and Spurgeon, 2006).

In 2017, Aksu and Coban applied the conventional sliding mode control method on the MIMO CCT mechanism, which is a non-linear and uncertain system. This mechanism, which consists of 4 tanks, is connected to each other through valves. The purpose here is to generate the signal that reaches the output tanks at the desired liquid level and ensures that it is maintained at the desired level. The method applied to this mechanism, which is in the cross structure, has been tested experimentally and has given quality tracking performance despite external disturbances. The durability of the method used has been proven (Aksu and Coban, 2017).

In their study in 2019, Aksu and Coban proposed an innovative approach to their previous work. They used the proportional-integral (PI) controller to avoid the steady-state error and combined this method with the backstepping technique of the sliding mode, which is insensitive to parameter and system uncertainties. This approach has been applied for the first time to this challenging and complicated system. During the experimental applications, this method was compared and analyzed with the results of sliding mode PI, backstepping sliding mode, and conventional SMC methods. After the analysis, it was observed that the proposed new approach achieved more effective results in many ways. The new approach offered both faster ascent and better tracking performance while reducing tracking error and overshoot rates (Aksu and Coban, 2019).

In the study by Datta et al., Decentralized Internal Model Control based Proportional Integral (IMC-PI) controllers are designed for MIMO 4-tank unit. In this unit there is a transition between the top two tanks, so the tanks behave interactively with each other. When this is the case, it is very difficult to control such processes. In this study, two IMC-PI controllers were designed to maintain the liquid level at the desired level. However, the interaction between the tanks prevented the controllers from achieving the desired level of success. After this experiment, a decoupler was placed between the tanks. Thus, the performance of the designed controllers in the presence and absence of the decoupler was compared. After the experiments, it was observed that the IMC-PI controller gave better results in maintaining the desired liquid level in the presence of the decoupler (Datta et al., 2015).

Debnath and Mija designed a SOSMC and Brain Emotional Logic Based Intelligent Controller(BELBIC) for a non-linear MIMO system with 4 equivalent tanks and presented a detailed performance review. SOSMC based on the super-twist algorithm is a method introduced to eliminate the chattering problem in the conventional SMC. The BELBIC, on the other hand, is a control technique based on emotional learning, a computational method based on the limbic system in the mammalian brain. Described as model-independent, the BELBIC learns from external sources, making it an intelligent controller. In the study, these two methodologies were designed for the liquid level controls of the two tanks located at the bottom of the 4-tank system. After the simulation, the effectiveness of both techniques was investigated comparatively. Both techniques were effective and yielded tremendous results, but the BELBIC outperformed the SOSMC in terms of performance (Debnath and Mija, 2016).

In the study of Mahapatro et al., an adaptive fuzzy sliding mode controller was designed for liquid level control of a MIMO coupled tank system. A fuzzy variable is placed on the sliding surface to minimize the chattering effect. To see the effectiveness of the proposed method, the results of the linear matrix inequality (LMI) based proportional integral (PI) controller are compared with the results. The proposed method showed superior control performance (Mahapatro et al., 2019).

3. MATERIAL AND METHOD

3.1. Coupled Tank System

Coupled tank systems are installations with common uses for industrial purposes in chemical plants (Pan et. al., 2005). The tanks are usually connected to each other by pipes, and the fluid flow can be adjusted by the couplings on the pipes. The liquid flowing into the tanks is provided by the pump or pumps (Feedback, 2006). Different algorithms can be developed, implemented on the coupled tanks and implemented on a real system to control the flow of the liquid and the liquid level in the tanks. The coupled tanks are available in different setups that serve a variety of purposes. These units can be used in single input single output (SISO) and multi input multi output (MIMO) structures (Aksu and Coban, 2017).

3.1.1. Single Input Single Output Coupled Tank System

SISO structured installations are usually consist of 2-tank system, with the tanks are connected to each other either upside down or side by side and being identical. In such installations, the first tank is fed by a pump and water flows to other tanks through the orifices between the tanks. A tank called the reservoir is located below the equipment, and the liquid fills this tank through the out orifices. Since these systems have a single output, the liquid fills the reservoir from this output, and the pump that feeds the first tank is submerged in the reservoir (Abbas et. al., 2012).

The aim is to maintain the liquid at a certain level, and to achieve this, the flow of liquid flowing from the pump needs to be controlled. The primary control objective can be defined as to managing the liquid level of the second tank (Malmborg and Eker, 1997; Almutairi and Zribi, 2006). The coupled tank systems are non-linear and complex, which makes them challenging to control. The tank systems shown in Figures 1 and 2 are examples of systems with the SISO structured systems.

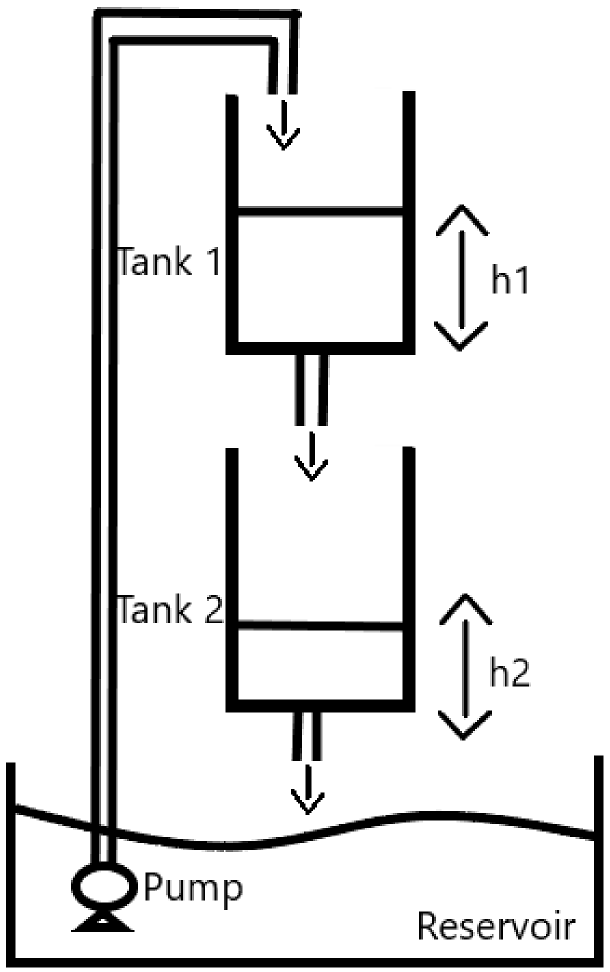

Figure 3.1. SISO Coupled Tank System

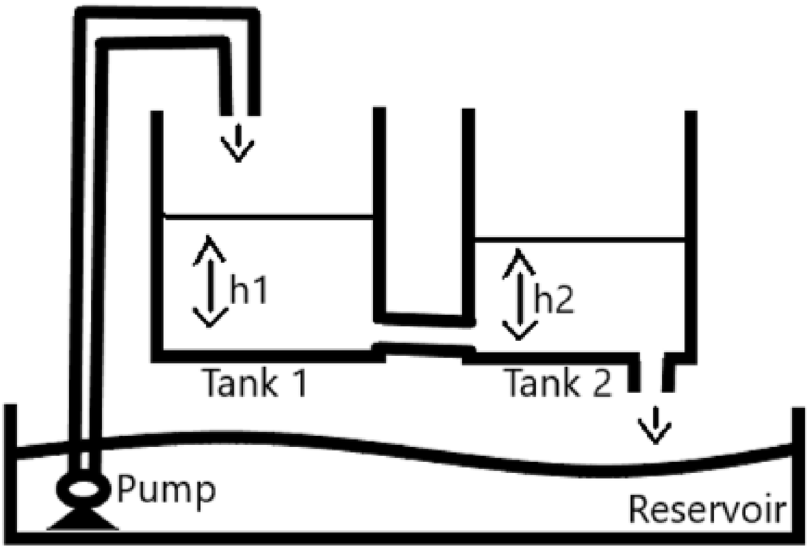

Figure 3.2. SISO Coupled Tank System

3.1.2. Multi Input Multi Output Coupled Tank System

The way the tanks are connected to each other can result in different system configurations. Installations with a MIMO structure have more than one input and one output in the system. In such installations, more than one pump feeds may be used to feed more than one tank. The orifices between the tanks allow for the flow of liquid. A tank called the reservoir is located under the equipment to collect the liquid flowing from the out orifices, and the pumps that feed the tanks are submerged in the reservoir (Feedback Instruments Ltd.).

The aim is to maintain the liquid at a certain level by controlling the flow of the liquid from the pumps. The flow of the liquid from the pumps should be controlled to achieve the desired conditions (Aksu and Coban,2017; Prusty et. al., 2016). The MIMO structured coupled tanks with complex structures are more difficult to control. The tank systems shown in Figures 3 and 4 are examples of systems with the MIMO structure.

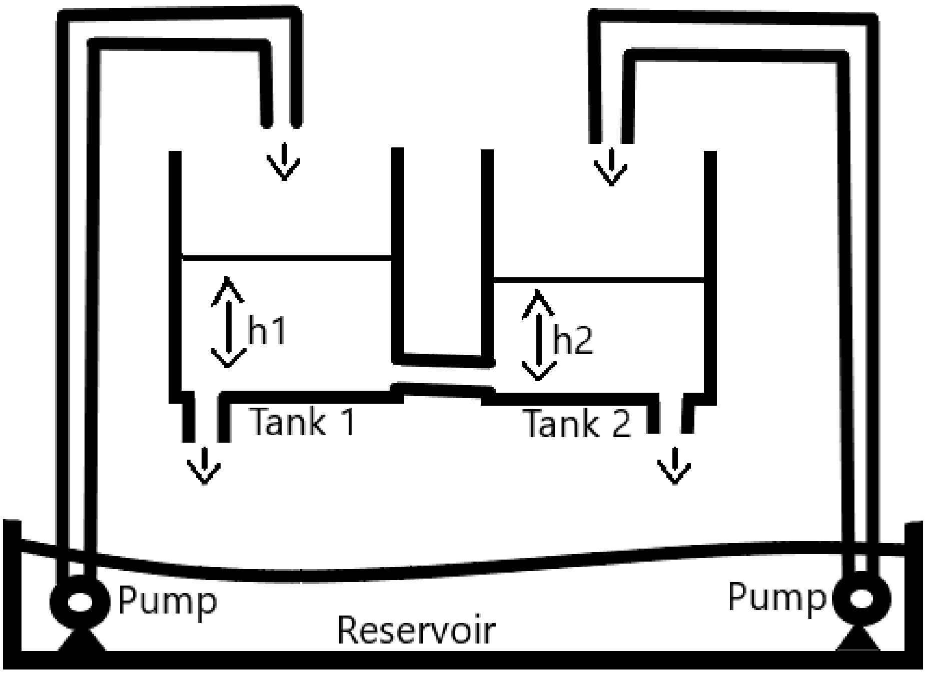

Figure 3.3. MIMO Coupled Tank System

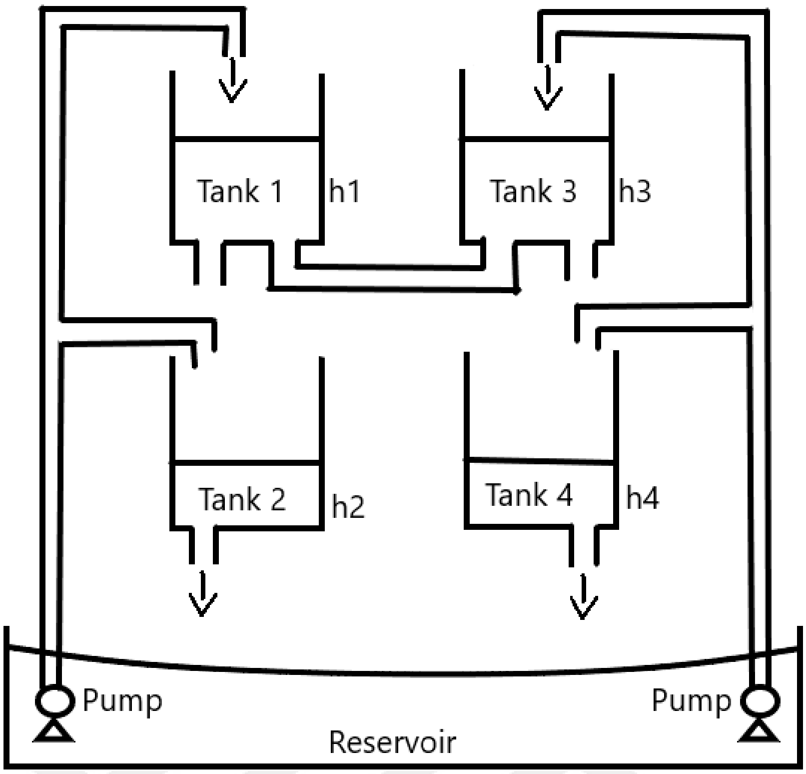

Figure 3.4. MIMO Coupled Tank System

Multi Input Multi Output Quadruple Cross Coupled Tank System

The quadruple cross-coupled tank (QCCT) system with a multi input and multi output structure, which is widely used in the industry, was utilized in this study. This system consists of four tanks, a reservoir, and two pumps submerged in the reservoir. T1 - T2 tanks and T3 - T4 tanks are interconnected by a pipe, and T1 and T3 tanks are also connected by a pipe with a valve on it. In this setup, the valve is opened to allow fluid passage between the T1 and T3 tanks. The system model used in the study is illustrated in Figure 5.

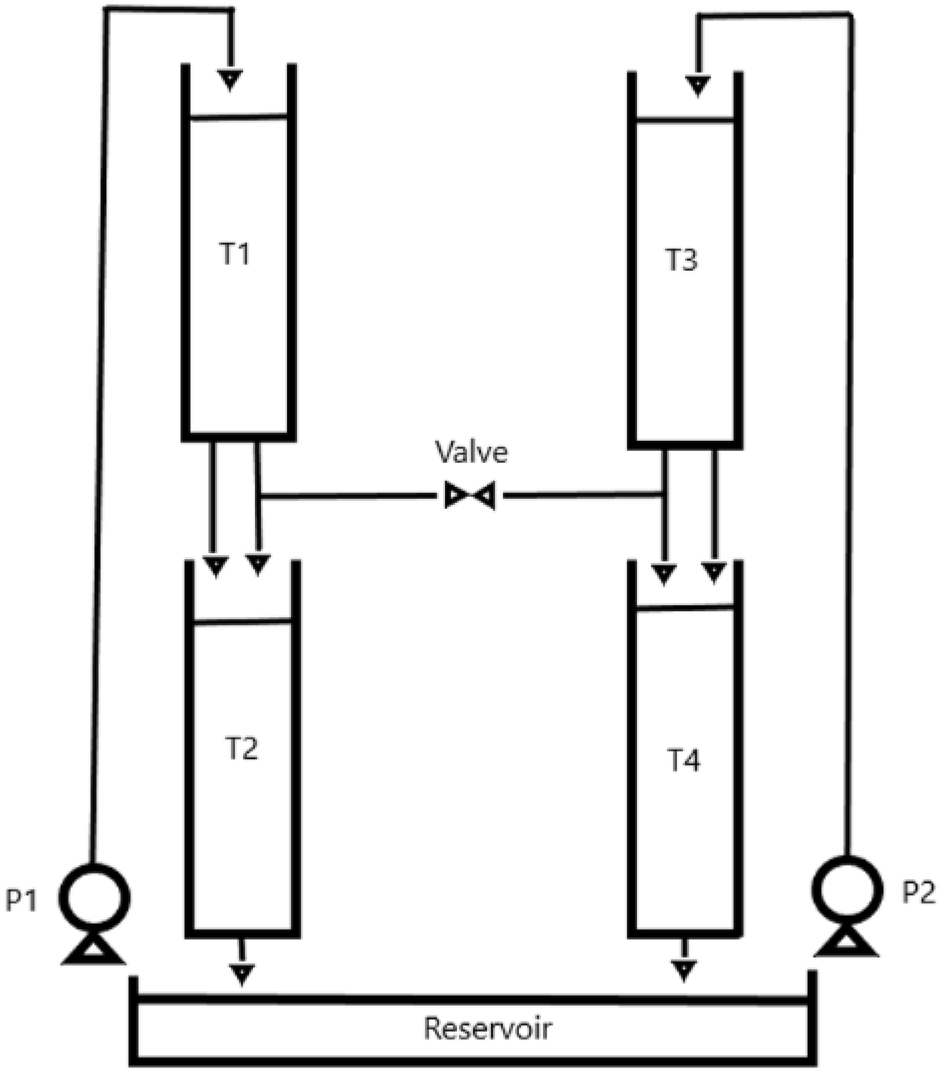

Figure 3.5. Structure of Quadruple Cross Coupled Tank System

When designing and adapting the definitions used by Almutairi and Zribi (2006) for the MIMO quadruple tank system, the following parameters can be defined: the inflow rate from T1 to T2 f_{i12} , the outflow rate of T2 f_{i2} , the inflow rate between T1 and T3 f_{i13}, the inflow rate from T3 to T4 f_{i34} and the outflow rate of T4 f_{i4} as follows (Almutairi and Zribi, 2006):

$$f_{i12} = a_{12}\sqrt{2g(l_1 - l_2)} \tag{3.1}$$

$$f_{i2} = a_2\sqrt{2g(l_2)} \tag{3.2}$$

$$f_{i13} = a_{13}\sqrt{2g(l_1 - l_3)} \tag{3.3}$$

$$f_{i34} = a_{34}\sqrt{2g(l_3 - l_4)} \tag{3.4}$$

$$f_{i4} = a_4\sqrt{2gl_4} \quad (3.5)$$

where l_1, l_2, l_3 and l_4 are the liquid level in T1, T2, T3, and T4, respectively; g is the acceleration due to gravity, a_{12} is the area of the pipe connecting T1 and T2, a_2 is the area of the T2 out, a_{13} is the area of the pipe connecting T1and T3, a_{34} is the area of the pipe connecting T3 and T4, and a_4 is the area of the T4 out.

Using Equations (3.1), (3.2), (3.3), (3.4) and (3.5), the flow equilibrium between all tanks can be written as (Almutairi and Zribi, 2006; Geylani and Ata, 2022):

$$A\frac{dl_1}{dt} = f_{ix} - f_{i12} - f_{i13} \quad (3.6)$$

$$A\frac{dl_2}{dt} = f_{i12} - f_{i2} \quad (3.7)$$

$$A\frac{dl_3}{dt} = f_{iy} - f_{i34} - f_{i13} \quad (3.8)$$

$$A\frac{dl_4}{dt} = f_{i34} - f_{i4} \quad (3.9)$$

where f_{ix} and f_{iy} are inflow rates and; A is the cross-sectional area of four tanks.

The dynamic equation of the coupled tank system can be written as (Almutairi and Zribi, 2006; Geylani and Ata, 2022):

$$\frac{dl_1}{dt} = -b_1\sqrt{l_1} - b_5 sign(l_1 - l_3)\sqrt{|(l_1 - l_3)|} + \frac{f_{ix}}{A} \quad (3.10)$$

$$\frac{dl_2}{dt} = b_1\sqrt{l_1} - b_2\sqrt{l_2} \quad (3.11)$$

$$\frac{dl_3}{dt} = -b_3\sqrt{l_3} - b_5 sign(l_3 - l_1)\sqrt{|(l_3 - l_1)|} + \frac{f_{iy}}{A} \quad (3.12)$$

$$\frac{dl_4}{dt} = b_3\sqrt{l_3} - b_4\sqrt{l_4} \quad (3.13)$$

where

$$b_1 = \frac{a_{12}\sqrt{2g}}{A}$$

$$b_2 = \frac{a_2\sqrt{2g}}{A}$$

$$b_3 = \frac{a_{34}\sqrt{2g}}{A} \tag{3.14}$$

$$b_4 = \frac{a_4\sqrt{2g}}{A}$$

$$b_5 = \frac{a_{13}\sqrt{2g}}{A}$$

and the sign function can be defined as

$$sign(l_1 - l_3) = \begin{cases} 1 & l_1 - l_3 > 0 \\ -1 & l_1 - l_3 < 0 \\ 0 & l_1 - l_3 = 0. \end{cases} \tag{3.15}$$

When the liquid levels reach balance, the derivatives of the liquid levels are equal to zero as below:

$$\frac{dl_1}{dt} = \dot{l_1} = 0$$

$$\frac{dl_2}{dt} = \dot{l_2} = 0 \tag{3.16}$$

$$\frac{dl_3}{dt} = \dot{l_3} = 0$$

$$\frac{dl_4}{dt} = \dot{l_4} = 0.$$

For this reason, the flow rates f_{ix} and f_{iy} can be rewritten as:

$$f_{ix} = A(b_1\sqrt{l_1} + b_5 sign(l_1 - l_3)\sqrt{|(l_1 - l_3)|}) \tag{3.17}$$

$$f_{iy} = A(b_3\sqrt{l_3} + b_5 sign(l_3 - l_1)\sqrt{|(l_3 - l_1)|}). \tag{3.18}$$

Because of the difference in input rate, when $l_1 \geq l_3$, the dynamic equation of the system should have $sign(l_1 > l_3) \geq 0$, therefore $\dot{l_1}$ and $\dot{l_3}$ can be rewritten as:

$$\dot{l_1} = -b_1\sqrt{l_1} - b_5\sqrt{|(l_1 - l_3)|} + \frac{f_{ix}}{A} \tag{3.19}$$

$$\dot{l_3} = -b_3\sqrt{l_3} + b_5\sqrt{|(l_3 - l_1)|} + \frac{f_{iy}}{A} \tag{3.20}$$

The flow rates in Equations (3.17) and (3.18) can be expressed as follows:

$$u_1 = f_{ix} \tag{3.21}$$

$$u_2 = f_{iy}. \tag{3.22}$$

The liquid levels of the T2 and T4 can be considered as the output of the system. The outputs can be defined as follows:

$$y_1 = l_2 \tag{3.23}$$

$$y_2 = l_4. \tag{3.24}$$

3.2. Design Method

3.2.1. Sliding Mode Control

SMC is a robust control technique that is frequently preferred in modern control systems that are insensitive to disturbances and uncertainties (Utkin, 1992). This technique, which can control linear and nonlinear systems, is a type of VSC method (Utkin, 1977).

The SMC uses a discontinuous control and applies a sliding function on a predefined sliding surface.

The conventional SMC is divided into two main stages; reaching the sliding surface and sliding on this surface. During the reaching stage, a suitable surface is selected by the user, and the system is shifted from the initial state to this surface. After reaching the surface, the sliding stage starts begins there the system is forced to stay on this surface and the system states are limited accordingly. Figure 3.6 shows the reaching and sliding stages; where e represents the tracking error of the system and $\dot{e}$ represents the derivative of the tracking error.

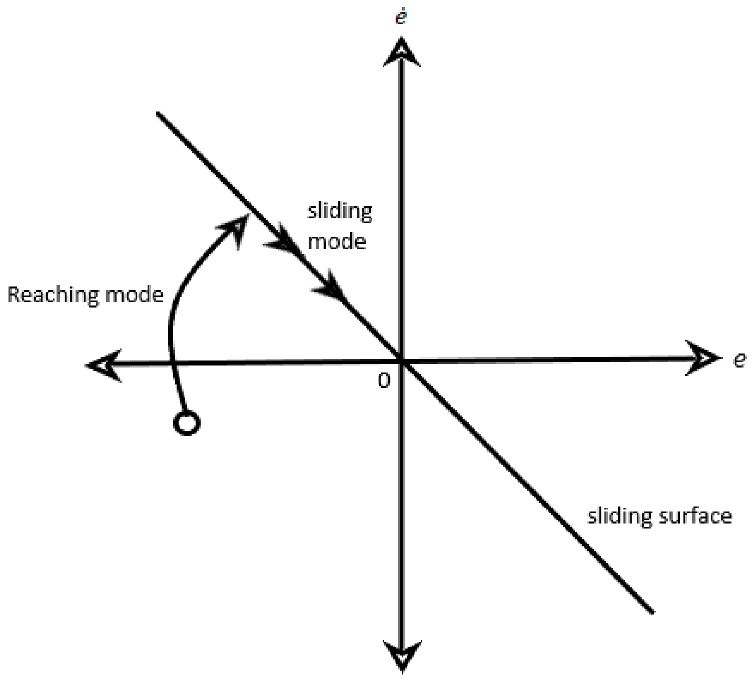

Figure 3.6. Reaching and sliding stage

The SMC includes two control laws; equivalent and switching control. The switching control is responsible for forcing the system trajectory to the predefined sliding surface and this is the most important function of the controller. Properly designing the switching control has a direct impact on the performance of the controller. The signum function $sign()$ is used in the literature when creating the switching control. The $sign()$ function can be defined as follows:

$$sign(s) = \begin{cases} -1 & for \quad s < 0 \\ 0 & for \quad s = 0 \\ 1 & for \quad s > 0 \end{cases}$$

and Figure 3.7 shows the $sign()$ function.

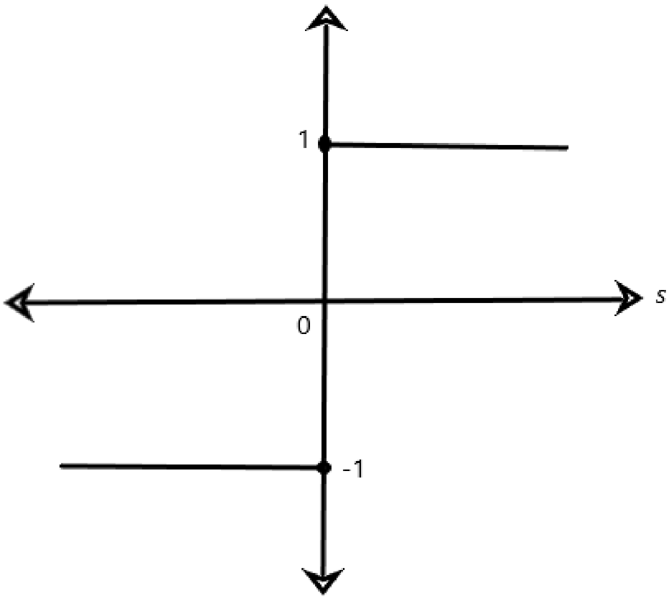

Figure 3.7. The signum function

Once the system reaches the sliding surface, the equivalent control is employed. Contrary to the advantages it offers, the SMC also has a significant disadvantage which is the chattering phenomenon. The chattering problem impacts the accuracy of the control technique. The chattering is an undesired oscillating phenomenon with a finite frequency and amplitude (Utkin and Lee, 2006).

Two main reasons can cause the chattering. The first reason is due to neglected system dynamics which means the dynamics with small time constants that are not modeled are generally ignored (Utkin and Lee, 2006). The second reason is the limited switching frequency. The SMC requires an infinite switching frequency, but in practice, the switching frequency is limited, leading to chattering (Young et al., 1999). Figure 3.8 illustrates the chattering phenomenon.

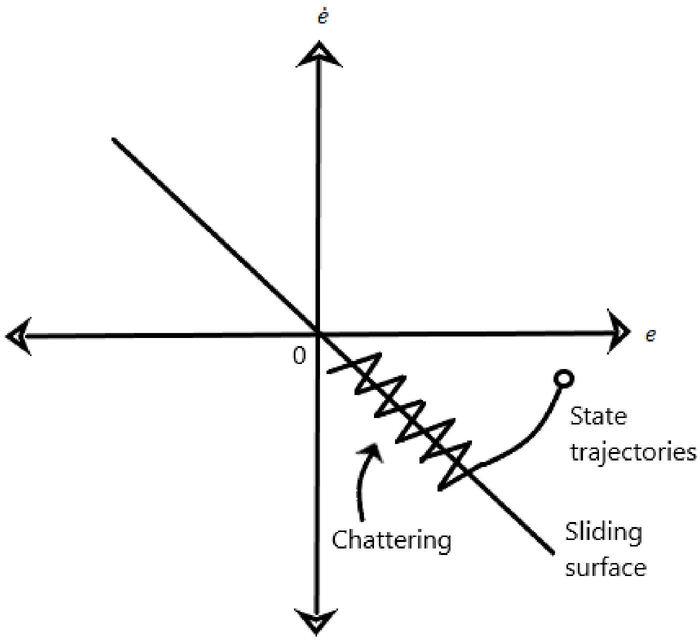

Figure 3.8. Chattering problem in the SMC

Various methods have been presented to avoid this zigzag movement on the sliding surface, which adversely affects the accuracy of the SMC method (Yasien and Kadhim, 2009). One of the most preferred techniques used to dampen the chattering effect is the boundary layer solution. (Slotine and Sastry, 1983; Burton and Zinober, 1986). The sign function $sign()$ in the switching control law is used with an approximation.

Figure 3.9 shows the boundary layer solution for the chattering phenomenon.

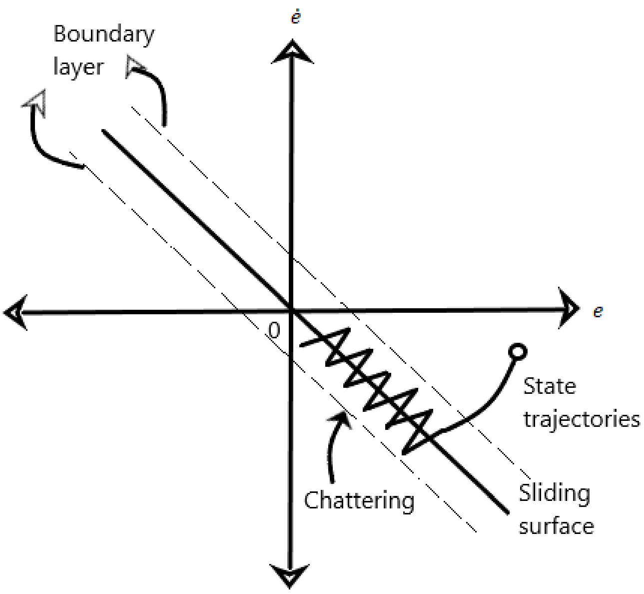

Figure 3.9. Boundary layer solution for the chattering phenomenon

A saturation function has finite upper and lower bounds. It is a response function that converts an input in the range (−∞, +∞) to the range (0, 1) and (−1, 1), respectively. In this solution, the saturation function is used instead of the signum function. The saturation function is defined as:

$$sat(s) = \begin{cases} s & for \quad |s| < 1 \\ sign(s) & for \quad |s| \geq 1 \end{cases}$$

and the saturation function is shown in Figure 3.10.

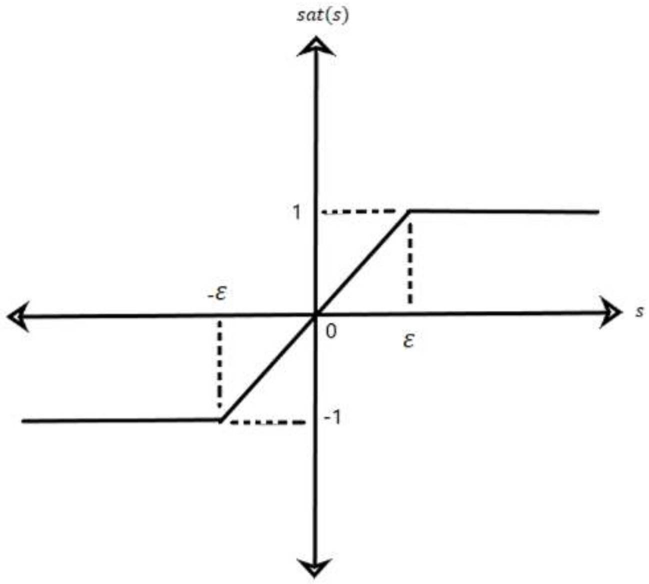

Figure 3.10. Saturation function

The single input single output nonlinear system is defined as (Shtessel et al., 2014)

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = f(x,t) + g(x,t)u(t) + \delta(t) \tag{3.25}$$

$$y(t) = x_1(t)$$

where $x_1(t)$ and $x_2(t)$ are state vectors, $f(x,t)$ is a nonlinear function, $g(x,t)$ is an uncertain function; $\delta(t)$ is the sum of uncertainty and disturbances, $u(t)$ is the control vector and $y(t)$ is the output.

When designing a sliding mode control, the sliding surface is defined as (Shtessel et al., 2014)

$$s(t) = \beta e(t) + \dot{e}(t) \tag{3.26}$$

where $e(t)$ is tracking error, $\dot{e}(t)$ derivative of tracking error and β is a positive constant.

The tracking error can be written as

$$e(t) = y(t) - y_d(t) \tag{3.27}$$

where $y_d(t)$ is the desired output.

The control law is described as

$$u(t) = u_{eq}(t) - u_{sw}(t) \tag{3.28}$$

where

$$u_{eq}(t) = \frac{1}{g(x,t)}(-f(x,t) - \beta\dot{e}(t)) \tag{3.29}$$

$$u_{sw}(t) = \frac{-ksign(s(t))}{g(x,t)}. \tag{3.30}$$

A Lyapunov function can be considered to examine the stability of the control system. This function can be selected as

$$V(s) = \frac{1}{2}s^2(t). \tag{3.31}$$

The derivative of the Lyapunov function $\dot{V}(s)$ can be writtten as

$$\dot{V}(s) = s(t)\dot{s}(t) \tag{3.32}$$

with

$$\begin{aligned} s(t) &= \beta e(t) + \dot{e}(t) \\ &= \beta(y(t) - y_d(t)) + (\dot{y}(t) - \dot{y}_d(t)) \end{aligned} \tag{3.33}$$

and

$$\begin{aligned} \dot{s}(t) &= \beta\dot{e}(t) + \ddot{e}(t) \\ &= \beta(\dot{y}(t) - \dot{y}_d(t)) + (\ddot{y}(t) - \ddot{y}_d(t)). \end{aligned} \tag{3.34}$$

Derivative of the Lyapunov function can be rearranged as

$$\dot{V}(s) = s(t)(\beta\dot{e}(t) + \ddot{y}(t) - \ddot{y}_d(t))$$
$$= s(t)(\beta\dot{e}(t) + \dot{x}_2(t) - \ddot{y}_d(t))$$
$$= s(t)(\beta\dot{e}(t) + f(x,t) + g(x,t)u(t) + \delta(t) - \ddot{y}_d(t))$$
$$= s(t)(\beta\dot{e}(t) + f(x,t) + g(x,t)(u_{eq}(t) + u_{sw}(t)) + \delta(t) - \ddot{y}_d(t)) \quad (3.35)$$
$$= s(t)(\beta\dot{e}(t) + f(x,t) + g(x,t)(\frac{1}{g(x,t)}(-f(x,t) - \beta\dot{e}(t) + \frac{-ksign(s(t))}{g(x,t)}) + \delta(t) - \ddot{y}_d(t))$$
$$= s(t)(-ksign(s(t)))$$
$$= -k|s(t)|$$
$$< 0.$$

.

Here, when the positive value from $s(t)$ in the absolute value is processed with the positive k value, $-k|s(t)|$ obtained as a result of solving the equation will give a negative value. According to the Lyapunov theorem, the fact that $\dot{V}(s)$ is less than zero, then the stability of the system is proven and it confirms the motion of the system orbit towards the sliding surface.

MIMO Sliding Mode Control

A MIMO system can be defined as follows

$$\dot{x}(t) = f(x,t) + g(x,t)u(t) \quad (3.36)$$

where $x(t) = [x_1(t)...x_a(t)]^T$ is the a-dimentional state vector; $f(x,t)$ is a nonlinear function; $g(x,t)$ is an uncertain function; $u(t)$ is the c-dimentional control vector (Benamor et al., 2011).

$$u(t) = [u_1(t)...u_c(t)]^T \quad (3.37)$$

In order to design a sliding mode controller for MIMO systems, the sliding surface can be defined as (Benamor et al., 2011)

$$s(t) = [s_1(t)...s_c(t)]^T \quad (3.38)$$

where

$s_i(t) = \sum_{j=0}^{n_i-1}(\beta_j)^i(e_i(t))^j$, for *i=1,...c.* (3.39)

Considering n_i is the notional degree of the error value $e_i(t)$, $(\beta_{n_i-1})^i = 1$ and $(e_i(t))^j$ is the j^{th} order derivative of the error value $e_i(t)$. The tracking error can be defined as

$$e_i(t) = y_i(t) - y_{id}(t) \text{ , for } i=1, \dots n_i - 1 \tag{3.40}$$

where $y_i(t)$ is the output, $y_{id}(t)$ is the desired output. The derivative of $s_i(t)$ can be written as

$$\dot{s}_i(t) = \sum_{j=0}^{n_i-1} (\beta_j)^i (\dot{e}_i(t) + \sum_{k=1}^{a} \dot{e}_i(t) \dot{x}_k(t)) \tag{3.41}$$

Substituting $\dot{x}_k(t)$ in Equation in (3.36), the Equation in (3.41) becomes:

$$\dot{s}_i(t) = \sum_{j=0}^{n_i-1} (\beta_j(t))^i (\dot{e}_i(t) + \sum_{k=1}^{a} \dot{e}_i(t) f_k + \sum_{t=1}^{c} \sum_{k=1}^{a} \dot{e}_i(t) g_{kt} u_t(t)) \tag{3.42}$$

which can be written as

$$\dot{s}_i(t) = m_i + \sum_{t=1}^{c} r_{it} u_t(t) \tag{3.43}$$

with

$$m_i = \sum_{j=0}^{n_i-1} (\beta_j)^i (\dot{s}_i(t) + \sum_{k=1}^{a} \dot{s}_i(t) f_i) \tag{3.44}$$

and

$$r_{ij} = \sum_{j=0}^{n_i-1} (\beta_j)^i (\sum_{k=1}^{a} \dot{s}_i(t) g_{kt}). \tag{3.45}$$

The derivative of the sliding surface can be written as

$$\dot{s}_i(t) = m + ru. \tag{3.46}$$

with:

$$m = \begin{bmatrix} m_1 \\ \vdots \\ m_c \end{bmatrix} \tag{3.47}$$

and

$$r = \begin{bmatrix} r_{11} & \cdots & r_{1c} \\ \vdots & \ddots & \vdots \\ r_{c1} & \cdots & r_{cc} \end{bmatrix}. \tag{3.48}$$

The control law can be defined as

$$u(t) = -r^{-1}(m + \begin{bmatrix} k_1 sign(s_1(t)) \\ \vdots \\ k_c sign(s_c(t)) \end{bmatrix}) \tag{3.49}$$

where r^{-1} is an invertible matrix, k_i is a positive constant.

To implement the sliding mode control to the cross coupled tank system, the sliding surface for this system can be written as (Aksu and Coban, 2017):

$$s_1(t) = \beta_1(l_2 - l_{2d}) + (\dot{l}_2 - \dot{l}_{2d}) \tag{3.50}$$

$$s_2(t) = \beta_2(l_4 - l_{4d}) + (\dot{l}_4 - \dot{l}_{4d}). \tag{3.51}$$

Also, m vector Equation in (3.46) can be defined as:

$$m_1 = \beta_1 b_1 \sqrt{2gl_1} - \beta_1 b_2 \sqrt{2gl_2} - \beta_1 \dot{l}_{2d} - b_1 b_1 g - b_1 b_5 g sign(l_1 - l_3) \frac{\sqrt{|l_1 - l_3|}}{\sqrt{l_1}} - b_1 b_2 g \frac{\sqrt{l_1}}{\sqrt{l_2}} + b_2 b_2 g - \ddot{l}_{2d} \tag{3.52}$$

and

$$m_2 = \beta_2 b_3 \sqrt{2gl_3} - \beta_2 b_4 \sqrt{2gl_4} - \beta_2 \dot{l}_{4d} - b_3 b_3 g - b_3 b_5 g sign(l_3 - l_1) \frac{\sqrt{|l_3 - l_1|}}{\sqrt{l_3}} - b_3 b_4 g \frac{\sqrt{l_3}}{\sqrt{l_4}} + b_4 b_4 g - \ddot{l}_{4d}. \tag{3.53}$$

Then, vector m can be given as

$$m = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}. \tag{3.54}$$

Similarly, r vector Equation in (3.46) can be defined as:

$$r_{11} = \frac{b_1\sqrt{2g}}{2A\sqrt{l_1}} \tag{3.55}$$

and

$$r_{22} = \frac{b_3\sqrt{2g}}{2A\sqrt{l_3}}. \tag{3.56}$$

Vector r can be given as

$$r = \begin{bmatrix} r_{11} & 0 \\ 0 & r_{22} \end{bmatrix}. \tag{3.57}$$

The control law for the conventional SMC method of the cross coupled tank system is defined as (Aksu and Coban, 2017):

$$u_1(t) = -{r_{11}}^{-1}(m_1 + k_1 sign(s_1(t))) \tag{3.58}$$

$$u_2(t) = -{r_{22}}^{-1}(m_2 + k_2 sign(s_2(t))). \tag{3.59}$$

Considering a Lyapunov function for the stability of the control system (Benamor et al., 2011):

$$V(s) = \frac{1}{2}(s_1^2(t) + s_2^2(t)) \tag{3.60}$$

and the derivative of the Lyapunov function $\dot{V}(s)$ can be writtten as

$$\begin{aligned} \dot{V}(s) &= s_1(t)\dot{s}_1(t) + s_2(t)\dot{s}_2(t) \\ &= s^T(t)(\dot{s}_1(t) + \dot{s}_2(t)). \end{aligned} \tag{3.61}$$

Using Equation (3.46)

$$\dot{V}(s) = s^T(t)(m + ru(t)), \tag{3.62}$$

replacing the control vector $u(t)$ by Equation (3.49) into Equation (3.62) yields

$$\begin{aligned} \dot{V}(s) &= s^T(t)(m + r(-r^{-1}(m + \begin{bmatrix} k_1 sign(s_1(t)) \\ k_2 sign(s_2(t)) \end{bmatrix}))) \\ &= -s^T(t)\begin{bmatrix} k_1 sign(s_1(t)) \\ k_2 sign(s_2(t)) \end{bmatrix} \\ &= -k_1|s_1(t)| < 0 \\ &= -k_2|s_2(t)| < 0. \end{aligned} \tag{3.63}$$

With the Lyapunov theorem, $\dot{V}(s)$ is calculated as negative definite since k_1 and k_2 are positive, in this case the system trajectory is tended to the sliding surface and will remain on this surface. According to the theorem, the stability of the control system has been proven.

3.2.2. Adaptive Sliding Mode Control

Adaptation law is one of the methods used to overcome the chattering, which negatively affects the stability of classical sliding mode control, and to improve the stability and performance of the system. The most challenging part of classical SMC design is adjusting the control gain to the upper limit of the uncertainties in the system. The adaptive sliding mode control method (ASMC) is one of the techniques frequently used to overcome this difficult step. In the ASMC method, the control gain is adjusted adaptively in real-time there is uncertainty in the system or a lack of upper-bound information. The adapted control gain should also be effective in removing system uncertainties and disturbances. Adaptation refers to the ability of a controller to adjust itself against the changes in system parameters and external influences. Although there are considerable studies with the ASMC method in the literature, there are still limitations regarding the rate of adaptation. Nevertheless, ASMC reduces both chattering and discontinuity by adapting the control gain based on the system used (Leyla et al., 2021).

The adaptive sliding mode controller, which is a dynamic controller, is designed to strengthen the performance and minimize the defects of the controller of the systems controlled by the conventional sliding mode controller.

In ASMC, the sliding surface can be defined as (Shtessel et al., 2014):

$$s_a(t) = \beta_a e_a(t) + \dot{e}_a(t) \tag{3.64}$$

where $e_a(t)$ is the tracking error, $\dot{e}_a(t)$ derivative of the tracking error, and β_a is a positive constant.

The tracking error and its derivative can be described as $e_a(t) = l_2 - l_{2d}$ and $\dot{e}_a(t) = \dot{l}_2 - \dot{l}_{2d}$, respectively for cross coupled tank system. To design an adaptive sliding mode controller for cross coupled tank system two different sliding surfaces can be defined using the tracking errors as follows (Aksu and Coban, 2017):

$$s_{a1}(t) = \beta_1(l_2 - l_{2d}) + (\dot{l}_2 - \dot{l}_{2d}) \tag{3.65}$$

$$s_{a2}(t) = \beta_2(l_4 - l_{4d}) + (\dot{l}_4 - \dot{l}_{4d}). \tag{3.66}$$

The adaptive control law is described by

$$u_a(t) = -r^{-1}(m + \begin{bmatrix} k_{a1} sign(s_{a1}(t)) \\ \vdots \\ k_{ac} sign(s_{ac}(t)) \end{bmatrix}). \quad (3.67)$$

Two control vectors are available for the cross coupled tank system $u_{a1}(t)$ and $u_{a2}(t)$,

$$u_{a1}(t) = -r_{11}{}^{-1}(m_1 + k_{a1} sign(s_{a1}(t))) \quad (3.68)$$

$$u_{a2}(t) = -r_{22}{}^{-1}(m_2 + k_{a2} sign(s_{a2}(t))) \quad (3.69)$$

where k_{a1} and k_{a2} are the adaptive term. k_{a1} and k_{a2} can be calculated according to the adaptation law as follows

$$k_{a1} = \varphi |s_{a1}(t)| sign(|s_{a1}(t)|) \quad (3.70)$$

$$k_{a2} = \varphi |s_{a2}(t)| sign(|s_{a2}(t)|) \quad (3.71)$$

where φ is positive constant. φ is used to adjust the adaptation rate k_{ai}. The adaptation law changes the gain k_{ai} according to the situation between the system trajectory and the sliding surface. The gain decreases when the trajectory is on the sliding surface (Huang et al., 2008).

The consistency of this adaptation law can be proved by the Lyapunov function. Considering the Lyapunov function as follows (Huang et al., 2008; Benamor et al., 2011):

$$V_a(s) = \frac{1}{2}(s_{a1}^2(t) + s_{a2}^2(t)) + \frac{1}{2}\lambda(\tilde{k}_{a1}^2 + \tilde{k}_{a2}^2) \quad (3.72)$$

where λ is a positive constant, used to avoid high control signals. $\tilde{k}_{ai}$ is the adaptation error and it can be defined as follow

$$\tilde{k}_{ai} = k_a - k_{am} \quad (3.73)$$

where k_{am} is the maximum value of k_a.

Presenting the derivative of the Equation (3.72) (Benamor et al., 2011)

$$\dot{V}_a(s) = s_{a1}(t)\dot{s}_{a1}(t) + s_{a2}(t)\dot{s}_{a2}(t) + \lambda(\tilde{k}_{a1}\dot{\tilde{k}}_{a1} + \tilde{k}_{a2}\dot{\tilde{k}}_{a2}), \quad (3.74)$$

$\dot{V}_a(s)$ can be written by simplifying

$$\dot{V}_a(s) = A + \lambda B \quad (3.75)$$

with:

$$A = s_{a1}(t)\dot{s}_{a1}(t) + s_{a2}(t)\dot{s}_{a2}(t) \quad (3.76)$$

and

$$B = \tilde{k}_{a1}\dot{\tilde{k}}_{a1} + \tilde{k}_{a2}\dot{\tilde{k}}_{a2}. \quad (3.77)$$

Equation (3.76) can be rewritten as

$$A = s_a^T(t)(\dot{s}_{a1}(t) + \dot{s}_{a2}(t)) \quad (3.78)$$

Substituting $\dot{s}(t)$ in Equation (3.46) into Equation (3.78) yields

$$A = s_a^T(t)(m + ru(t)) \quad (3.79)$$

and replacing the control vector $u(t)$ by Equation (3.49) into Equation (3.79) gives

$$A = s_a^T(t)(m + r(-r^{-1}(m + \begin{bmatrix} k_{a1}sign(s_{a1}(t)) \\ k_{a2}sign(s_{a2}(t)) \end{bmatrix})))$$

$$= -s_a^T(t)\begin{bmatrix} k_{a1}sign(s_{a1}(t)) \\ k_{a2}sign(s_{a2}(t)) \end{bmatrix} \quad (3.80)$$

$$< 0.$$

Similarly, Equation (3.77) can be rearranged as

$$B = \tilde{k}_{a1}\dot{\tilde{k}}_{a1} + \tilde{k}_{a2}\dot{\tilde{k}}_{a2}$$

$$= (k_{a1} - k_{am1})\dot{\tilde{k}}_{a1} + (k_{a2} - k_{am2})\,\dot{\tilde{k}}_{a2} \quad (3.81)$$

$$= (k_{a1} - k_{a1}|s_{a1}(t)|sign(|s_{a1}(t)|))\dot{\tilde{k}}_{a1} + (k_{a2} - k_{a2}|s_{a2}(t)|sign(|s_{a2}(t)|))\,\dot{\tilde{k}}_{a2}$$

$$< 0.$$

The fact that Equation (3.80) and Equation (3.81) are less than zero shows that $\dot{V}_a(s)$ is negative, so the stability of the system is proven according to the Lyapunov theory and the motion of the system orbit towards the sliding surface is confirmed.

4. RESULTS AND DISCUSSIONS

In this thesis, conventional SMC and ASMC methods are employed to design controllers for a cross coupled tank system. The adaptation law is incorporated in order to improve the performance of the classical method.

For both methods used, the same values were used as the system parameters of the system designed in the simulation. The results obtained after the simulation of the controllers designed using SMC and ASMC techniques were examined comparatively.

4.1. Simulation Result for Sliding Mode Control Method

The simulation setup and results are presented in this subsection for the conventional SMC.

4.1.1. Simulation Setup for Sliding Mode Control Method

The cross coupled tank system model described in Equation (3.10), Equation (3.11), Equation (3.12) and Equation (3.13) is simulated using MATLAB and Simulink software. The parameters of the tank model and the controller used in the simulations are given in Table 4.1 and Table 4.2, respectively. The controller described in Equation (3.58) and Equation (3.59) modeled to control the system in the simulation. Figure 4.1 shows the block diagram of the sliding mode controller that will control the cross coupled tank system. The reference value (R) is set as the desired liquid level. The simulation time for the simulation is selected as t=1000 sec.

Table 4.1. Parameters of Coupled Tank System (Feedback Instruments, 2006)

Symbols	Values	Meaning
l_1	25 cm	The initial liquid level in T1
l_2	0 cm	The initial liquid level in T2
l_3	25 cm	The initial liquid level in T3
l_4	0 cm	The initial liquid level in T4
l_{2d}, l_{4d}	15 cm	Desired liquid level for T2 and T4
g	981 cm/s^2	Gravitational acceleration
A	138.9 s^2	The cross-sectional area of the tanks
a_i	$50.2658 * 10^{-2}\ cm^2$	Area of the 2nd tank out

Table 4.2. Parameters of Sliding Mode Controller

Symbols	Values	Meaning
β_1	10	Positive constant for sliding surface 1
β_2	10	Positive constant for sliding surface 2
k_1	25	Positive constant for control vector 1
k_2	28	Positive constant for control vector 2

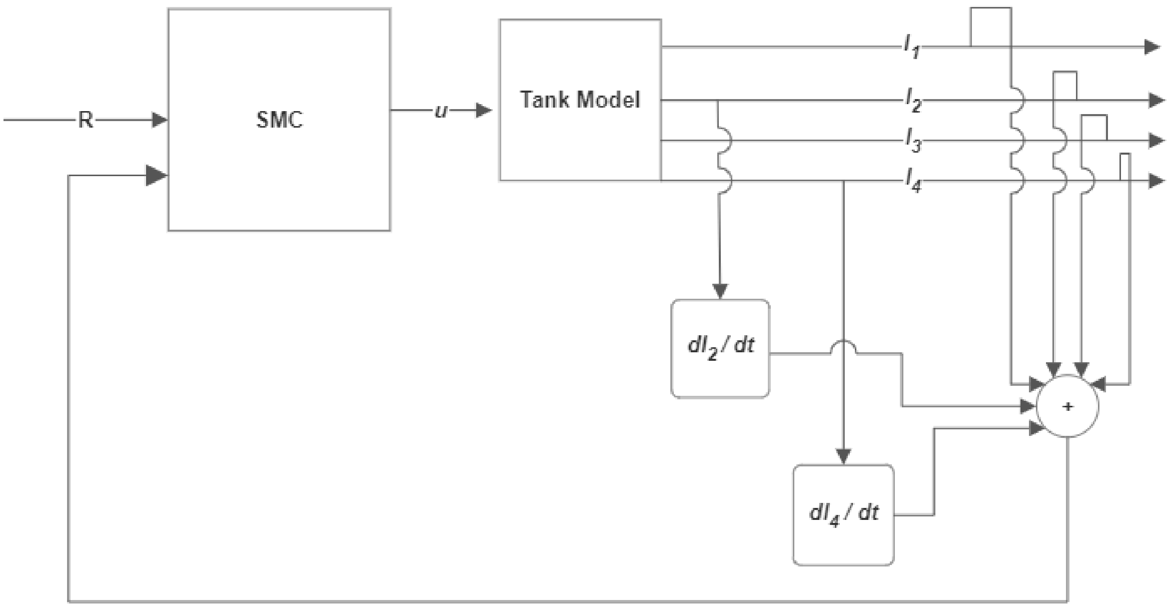

Figure 4.1. Block Diagram of the Sliding Mode Controller Simulation

4.1.2. The Sliding Mode Control Method Simulation Result

This simulation is designed to test the stability and performance of the sliding mode controller for a cross coupled tank system. The purpose of this simulation is to examine the effects of the classical sliding mode controller on the system used and compare the results obtained with the results of the proposed method versus the classical method.

In the simulation, the conventional SMC method is tested on the cross coupled tank system described in Equation (3.10), Equation (3.11), Equation (3.12) and Equation (3.13). The outputs from the system are the liquid levels of T2 and T4. The initial liquid level of T1 and T3 was set at 25 cm, and the initial liquid level of T2 and T4 was set at 0 cm. The intended liquid level target should be 15 cm for T2 and T4. The evolution of the liquid level in T2 and T4 is plotted in Figure 4.2. and Figure 4.3., respectively for the SMC method.

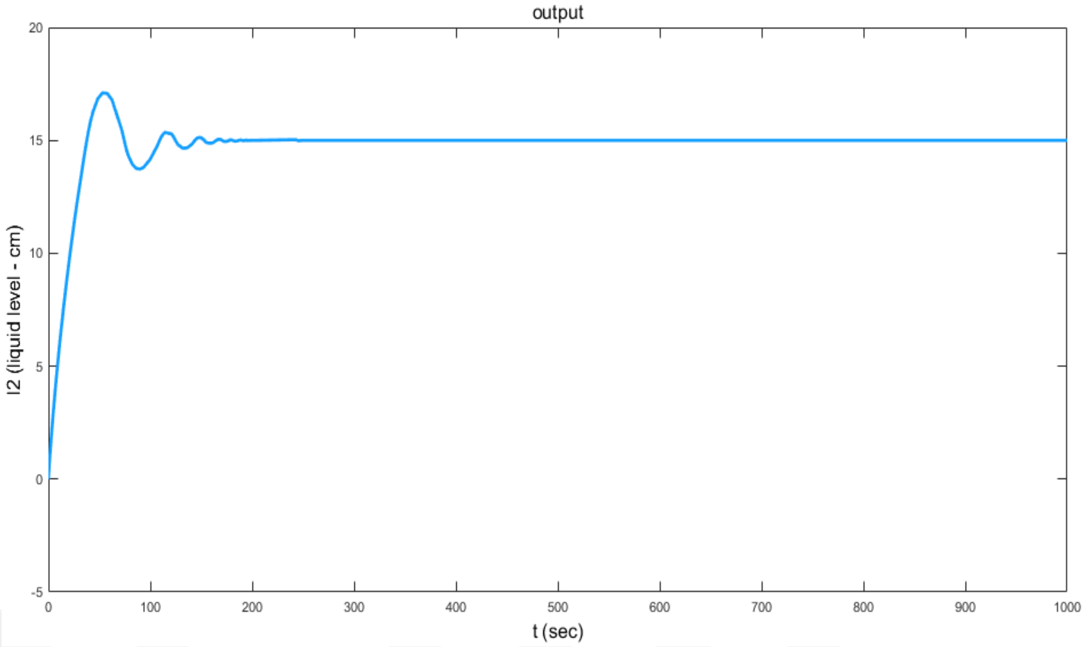

Figure 4.2. Evolution of the liquid level on T2 for SMC

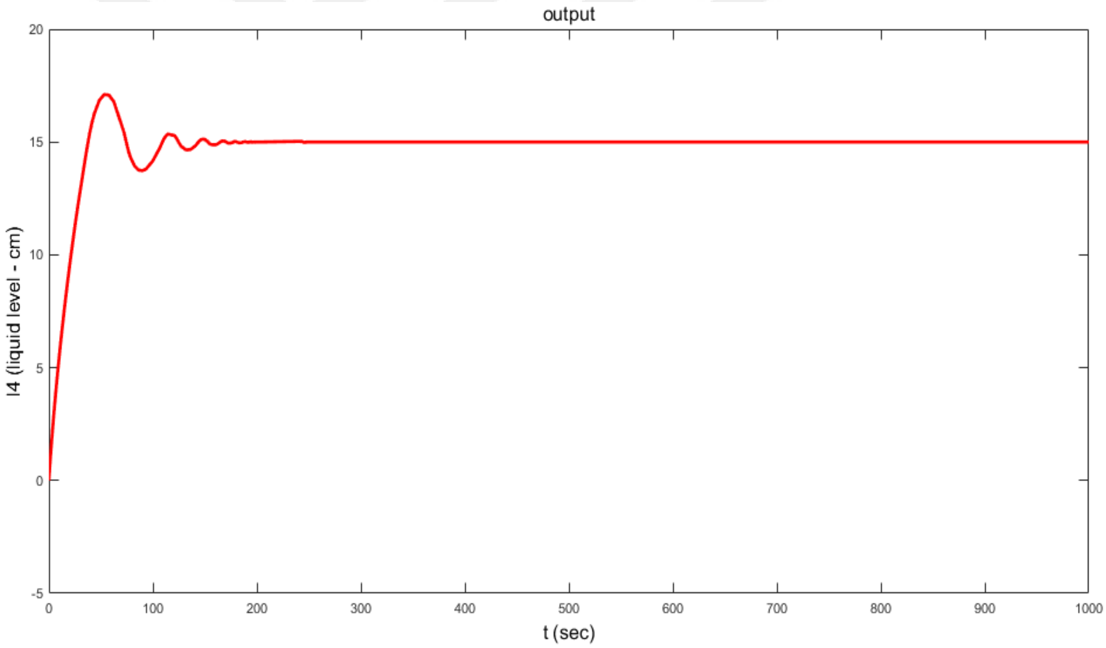

Figure 4.3. Evolution of the liquid level on T4 for SMC

In a finite time, the controller drives the system orbit to zero and tries to keep the orbital at the zero point. Errors e1(t) for T2 and e2(t) for T4 and their initial derivatives are shown for T2 and T4 in Figure 4.4 and Figure 4.5, respectively.

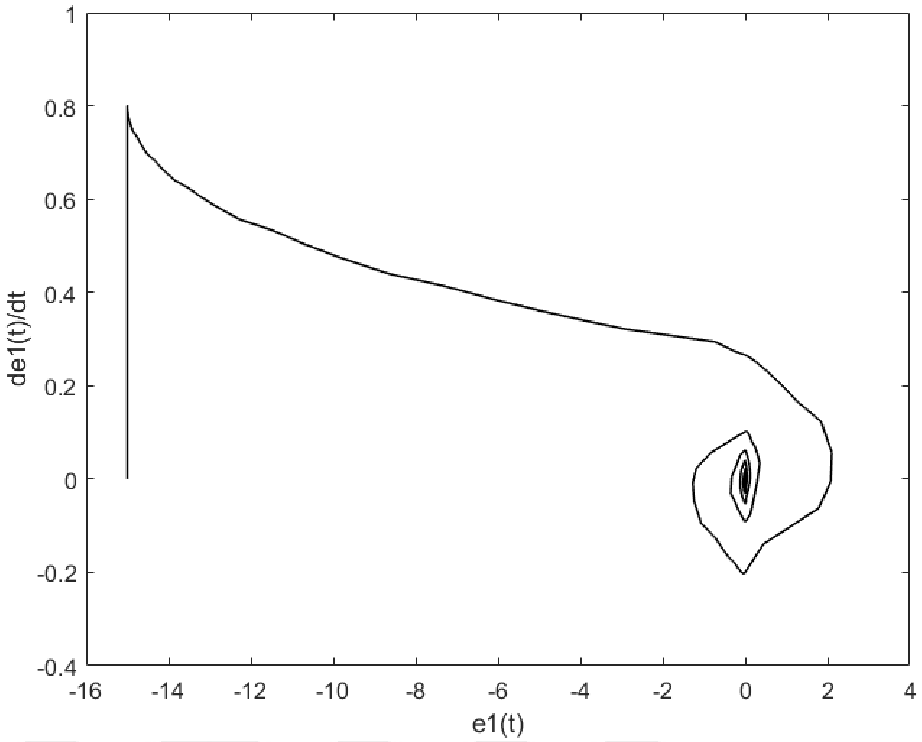

Figure 4.4. Motion of the system orbit for T2 with SMC

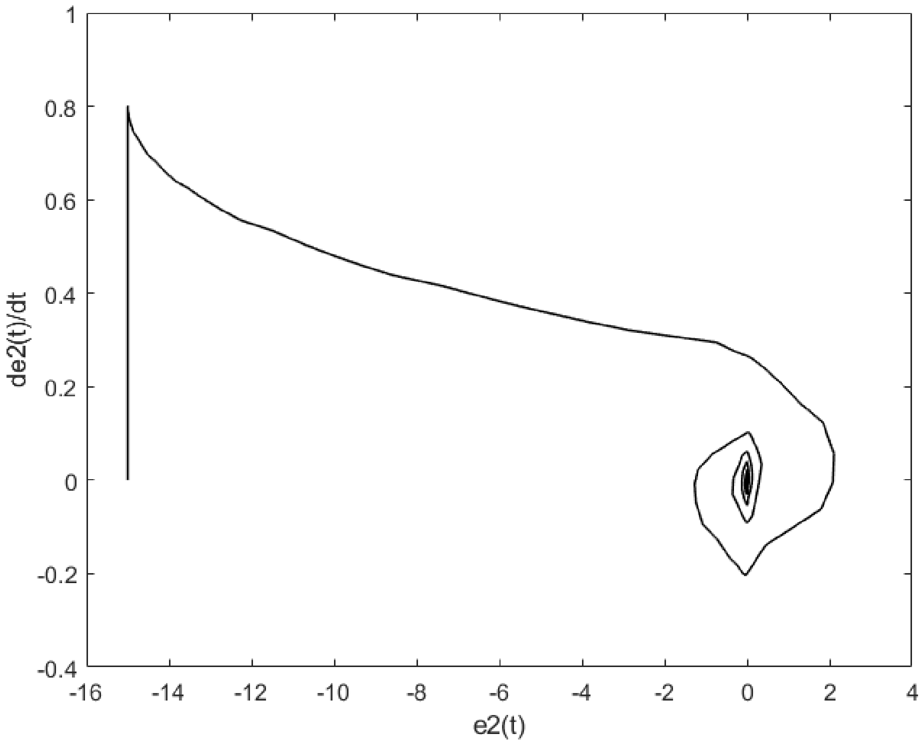

Figure 4.5. Motion of the system orbit for T4 with SMC

The sliding surfaces of the sliding mode controller depend on the error and its derivative, so the sliding surfaces are expected to approach zero. Considering the first sliding surface s1 and the second sliding surface s2, the time-dependent graphs of the sliding surfaces are shown in Figure 4.6 and Figure 4.7, respectively.

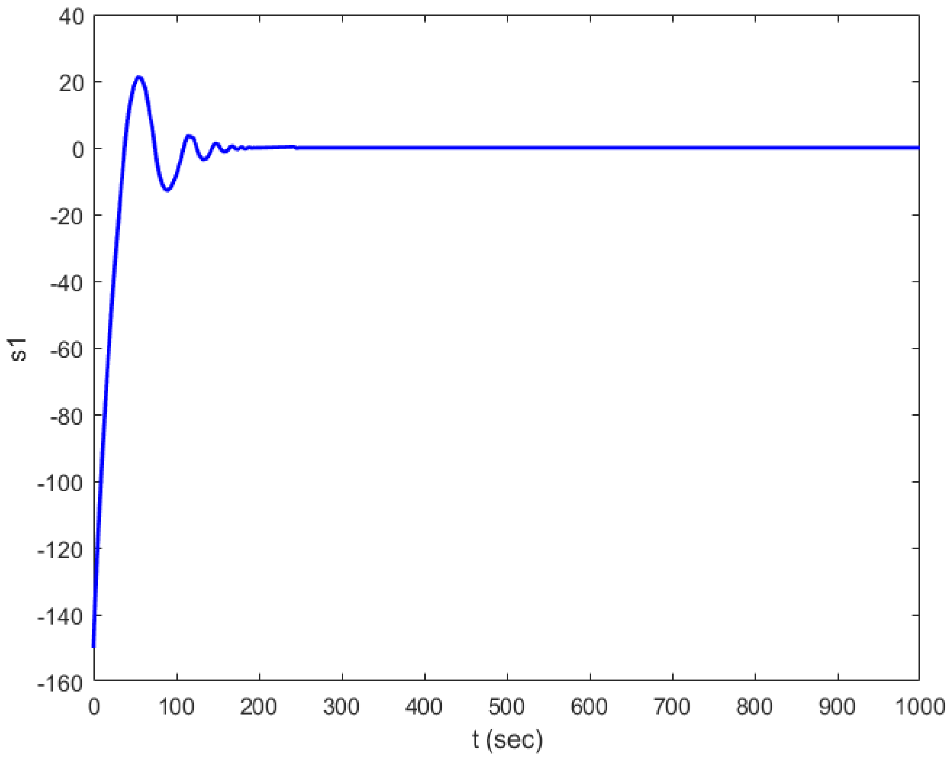

Figure 4.6. First sliding surface for SMC

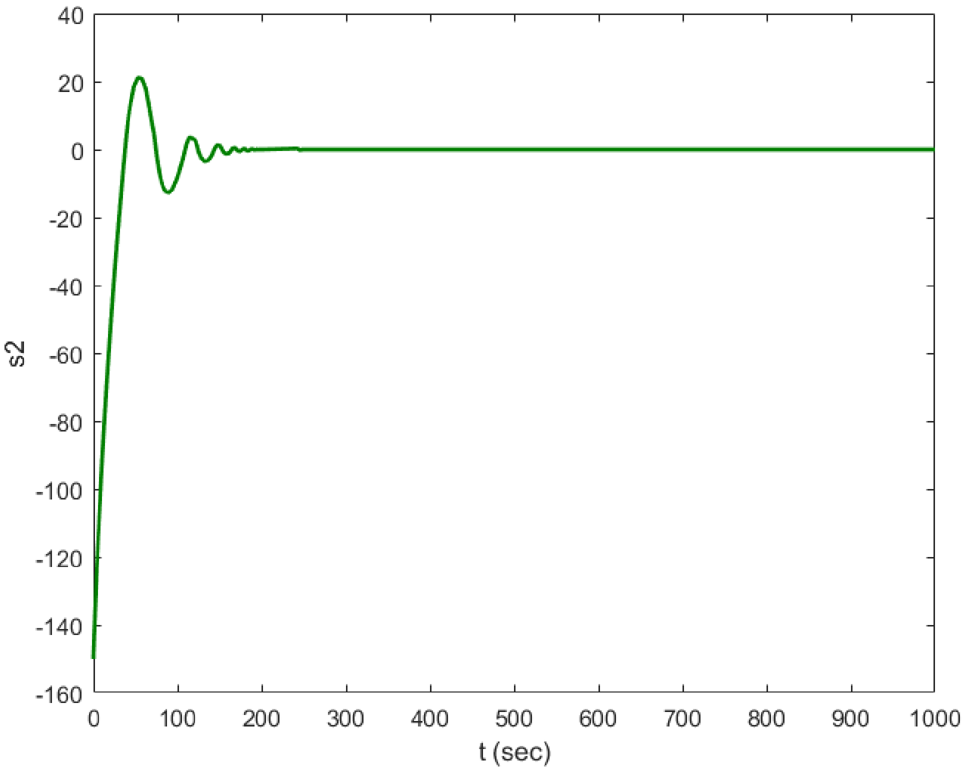

Figure 4.7. Second sliding surface for SMC

4.2. Simulation Result for Adaptive Sliding Mode Control Method

The simulation setup and results are presented in this subsection for ASMC.

4.2.1. Simulation Setup for Adaptive Sliding Mode Control Method

The parameters of the tank model used in the simulation were used identical to the parameters of the SMC method explained in the previous title. The values of the parameters used for the controller parameters are given in the Table 4.3. The tank system and the adaptive controller modeled for the system are simulated in the simulation program. Figure 4.8 shows the block diagram of the adaptive sliding mode controller that will control the cross coupled tank system. The simulation time for the simulation was set as t=1000 sec.

Table 4.3. Parameters of Adaptive Sliding Mode Controller

Symbols	Values	Meaning
β_1	10	Positive constant for sliding surface
β_2	10	Positive constant for sliding surface

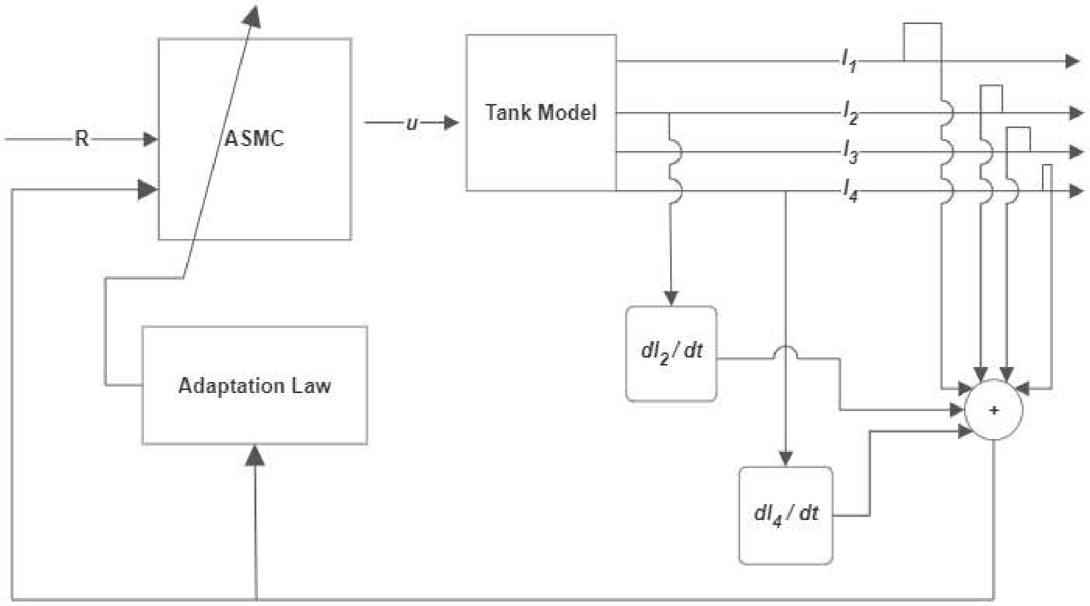

Figure 4.8. Block Diagram of the Adaptive Sliding Mode Controller Simulation

4.2.2. The Adaptive Sliding Mode Control Method Simulation Result

This simulation is designed for the proposed adaptation law to improve the performance of the conventional sliding mode control method for a cross coupled tank system. The purpose of this simulation is to examine the performance effects of the adaptive sliding mode controller on the system used.

In this simulation, the ASMC method is tested on the cross coupled tank system described in Equation (3.10), Equation (3.11), Equation (3.12) and Equation (3.13). The evolution of the liquid level in T2 and T4 is plotted in Figure 4.9 and Figure 4.10 for the ASMC method.

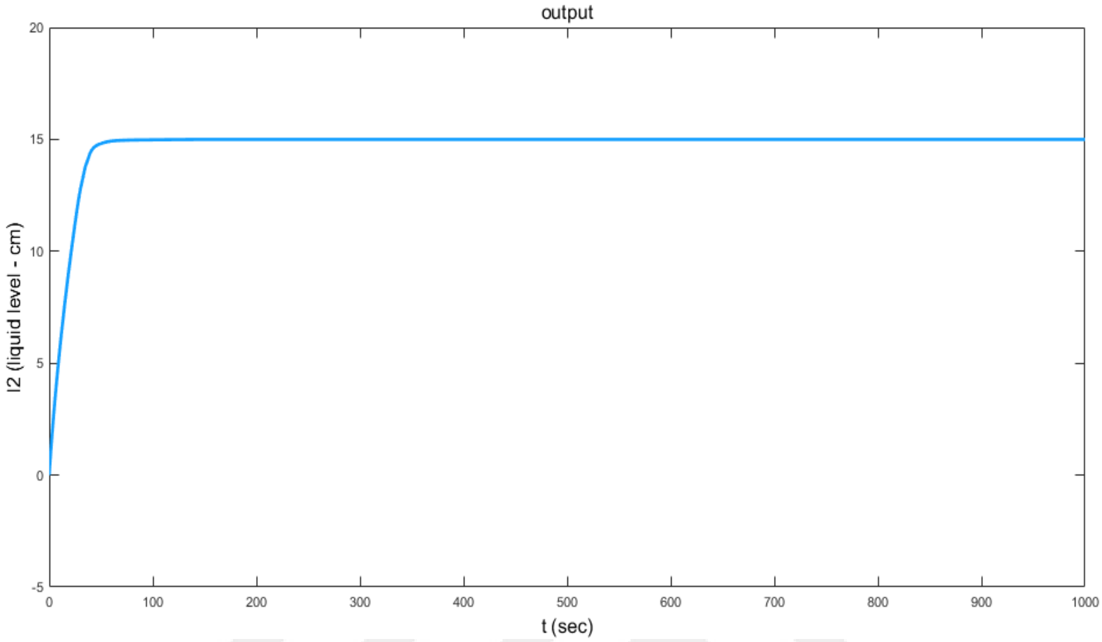

Figure 4.9. Evolution of the liquid level on T2 for ASMC

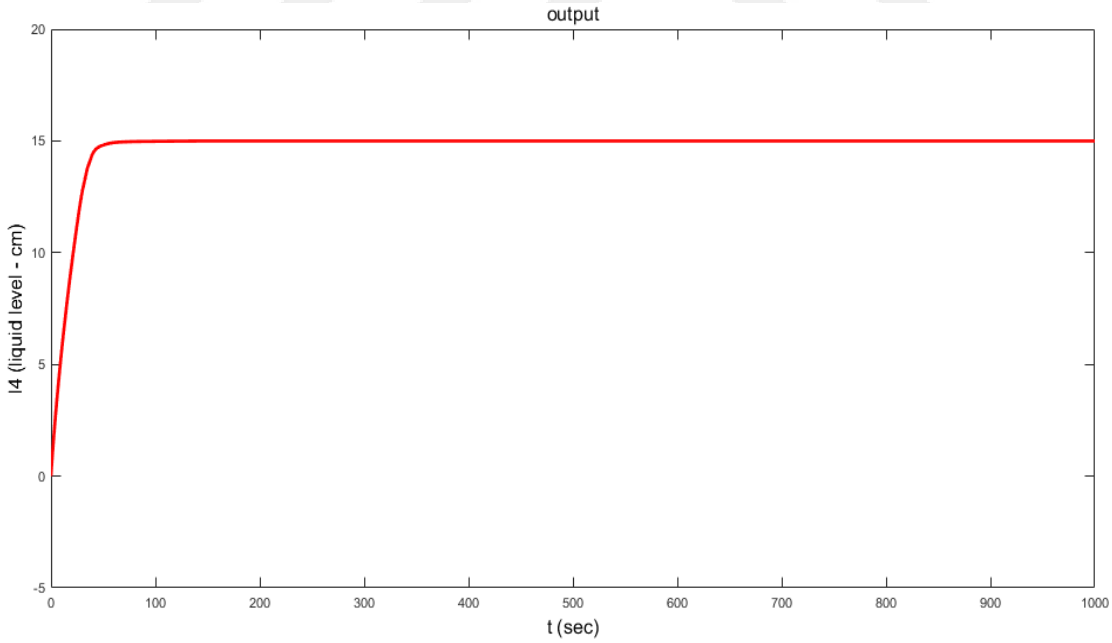

Figure 4.10. Evolution of the liquid level on T4 for ASMC

Errors for output of tanks and their derivatives are shown in Figure 4.11 and Figure 4.12, respectively. The system trajectory is forced to zero and is tried to be kept there by the controller.

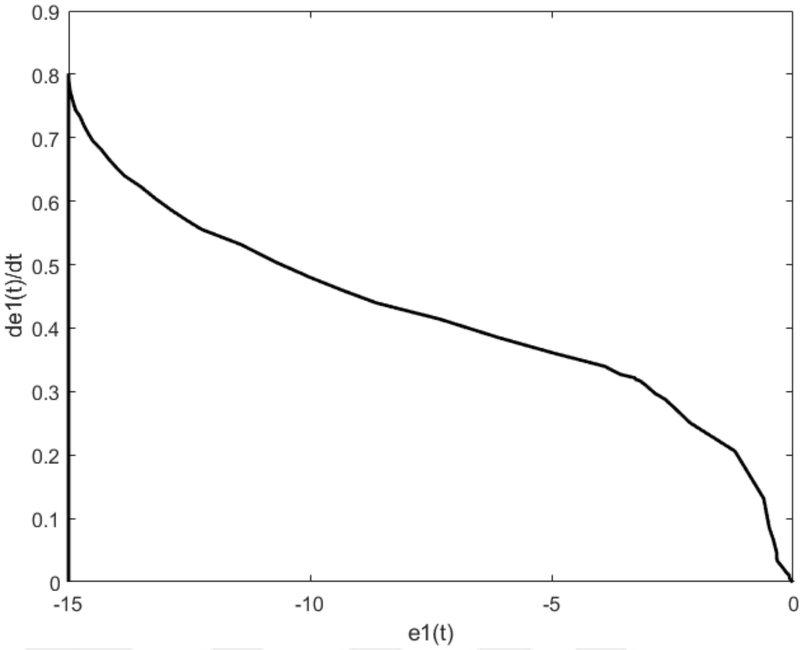

Figure 4.11. Motion of the system orbit for T2 with ASMC

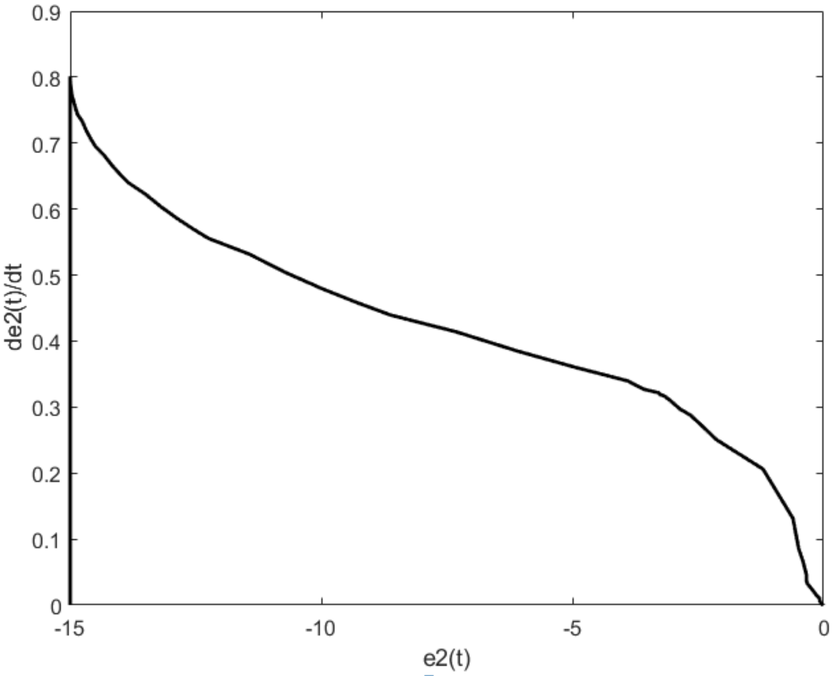

Figure 4.12. Motion of the system orbit for T4 with ASMC

The sliding surfaces, which depend on the error and the derivative of the error, approach zero. Considering the first sliding surface s_1 and the second sliding surface s_2, the time-dependent graphs of the sliding surfaces are shown in Figure 4.13 and Figure 4.14, respectively.

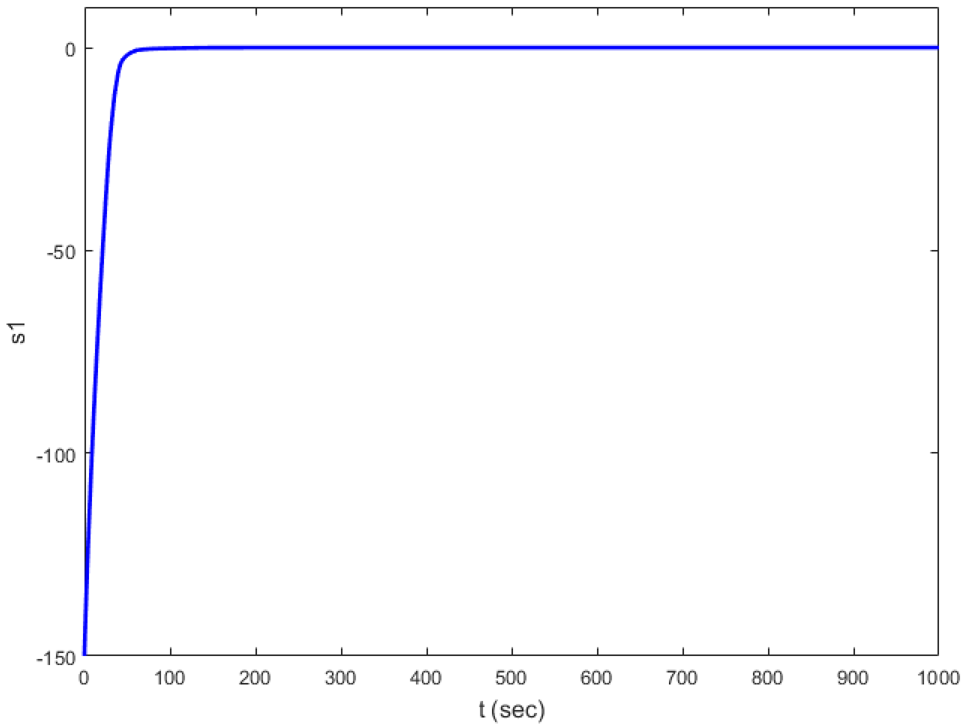

Figure 4.13. First sliding surface for ASMC

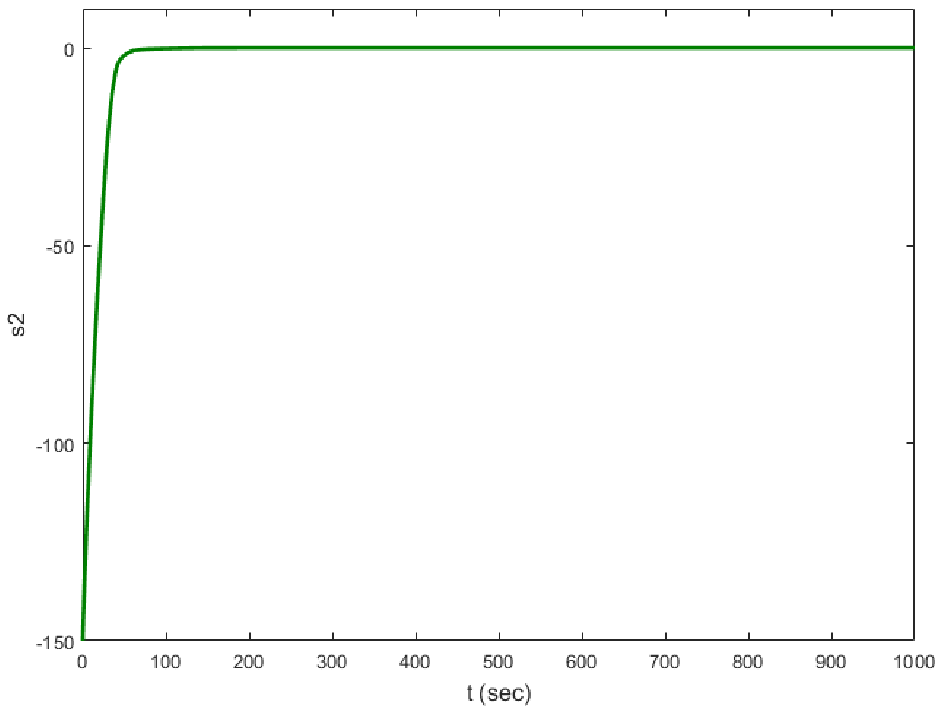

Figure 4.14. Second sliding surface for ASMC

4.3. Comparison Results for Simulation

Both SMC and ASMC methods were applied to the cross coupled tank system described in Equation (3.10), Equation (3.11), Equation (3.12) and Equation (3.13). The evolution of the liquid level in T2 and T4 are shown in Figure 4.15 and Figure 4.16.

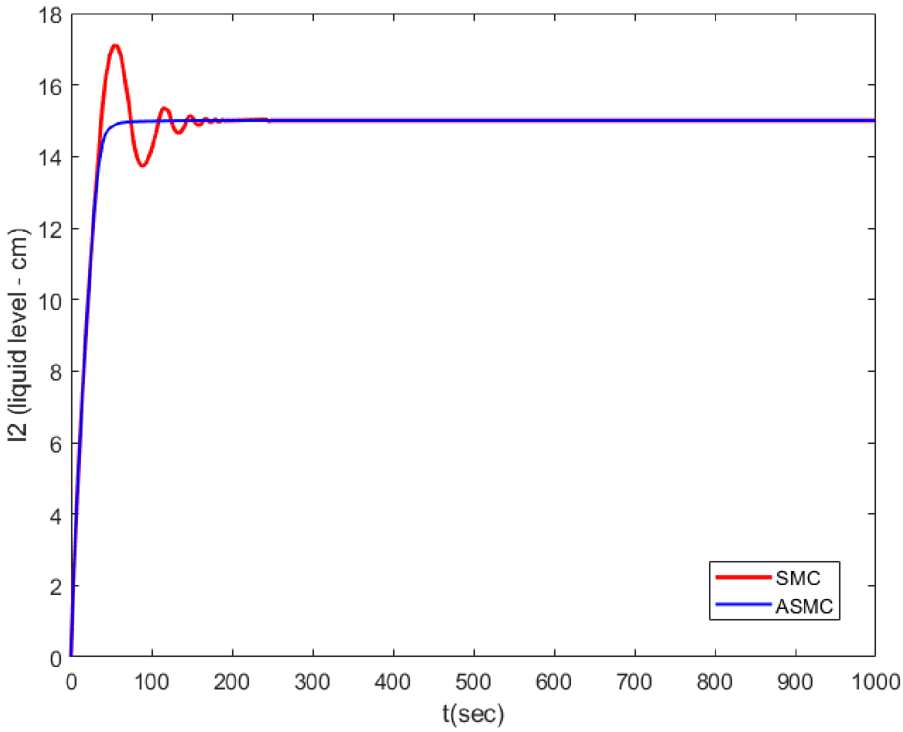

Figure 4.15. Evolutions of the liquid levels on T2

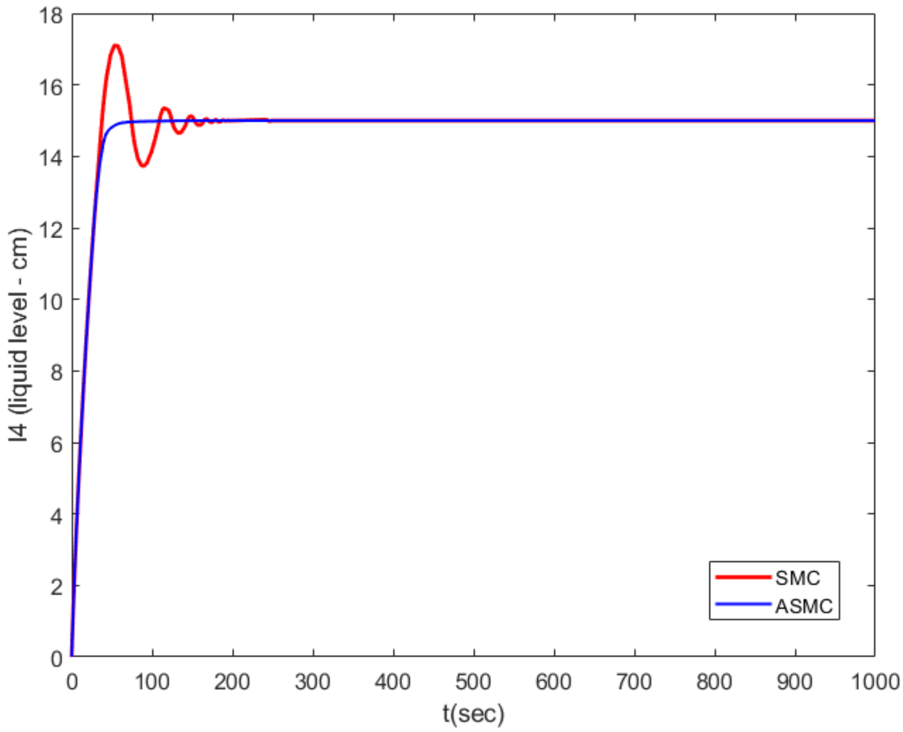

Figure 4.16. Evolutions of the liquid levels on T4

Table 4.4. Performance Results

	Settling Time	Overshoot	Steady State Error
SMC	141.0819	14.0662	0.3241
ASMC	51.8341	0	0.0177

Both methods are successfully managed to control the liquid level at the desired level. The liquid level states of T2 obtained from the conventional SMC method and the ASMC method are presented in Figure 4.15. The liquid level states of T4 obtained from the conventional SMC method and the ASMC method are shown in Figure 4.16. It should be clearly understood from the Figures 4.15 and 4.16 that the proposed method has reached the desired level faster than the conventional method and has achieved a more effective result in terms of performance. The performance results of the tested controllers are shown in Table 4.4. The settling time of the controller designed using the ASMC method is shorter than the controller designed using the SMC method. The overshoot value obtained when testing the conventional sliding mode controller was not obtained in the adaptive sliding mode controller test. The steady state error of the controller designed using the ASMC method resulted in a more minimized value compared to the conventional controller.

5. CONCLUSIONS

In this study, it is aimed to control the liquid level of a MIMO cross coupled tank system. The controller designed using the adaptive sliding mode control method was applied to a coupled tank system. The system was simulated to examine the performance and effectiveness of the controller obtained using the adaptation law for a MIMO cross coupled tank system. The simulation results show that the proposed controller has managed to control the liquid level for the coupled tank system.

To compare the performance of the proposed controller, the coupled tank system is tested with the control signal obtained using the conventional sliding mode control method.. The comparison of the two methods showed that the ASMC method offers a more efficient control law in terms of both time and performance when compared to the conventional SMC law. In addition, the simulation results prove that the proposed controller produces a more accurate control signal.

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