HUB LOCATION AND ROUTING PROBLEM

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By

Sinan Bayraktar

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

HUB LOCATION AND ROUTING PROBLEM

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Hubs are special facilities that consolidate and disseminate flows in many-to-many distribution systems. The hub location problem aims to find locations of hubs and allocate non-hub nodes directly to the hubs. However, this problem is necessary to extend when nodes do not have sufficient demand to justify direct connections between the non-hub nodes to the hubs since such direct connections increase the number of vehicles required and decrease the utilization of vehicles. Hence, it is necessary to construct local tours among the nodes allocated to the same hubs to generate economies of scale and to decrease vehicle costs. Nevertheless, forcing each non-hub node to be visited by a local tour is not the best way to design a many-to-many distribution system. Therefore, in this study two options for each non-hub node are given: (i) either it could be visited by a local tour or (ii) it could be directly connected to a hub without an economy of scale. We develop a mixed integer programming formulation and strengthen it with valid inequalities. We also develop three different Benders formulations as exact solution methods. In addition, we develop a hierarchical heuristic with two phases in order to solve large-sized problem instances. We test the performances of our solution methodologies on CAB and TR data sets.

Keywords: hub location, vehicle routing, Benders decomposition .

ÖZET

ADÜ YER SEÇİMİ VE ROTALAMA PROBLEMİ

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Ana dağıtım üsleri (ADÜ), çoklu dağıtım sistemlerinde noktalar arasında talebin toplandığı ve dağıtıldığı özel tesislerdir. ADÜ yer seçimi problemlerinde amaç ADÜ'lerin yerinin tespit edilip diğer talep noktalarını ADÜ'lere doğrudan atamaktır. Talebin doğrudan yapılan atamaya yetecek kadar yüksek olmadığı durumlarda doğrudan yapılan atamaların kullanılması gereken araç sayısını artırması ve bu araçların verimliliğini düşürmesi sebebiyle ADÜ yer seçimi probleminin genişletilmesi gerekmektedir. Bu yüzden aynı ADÜ'ye atanan talep noktaları arasında bölgesel turlar oluşturmak ölçek ekonomilerinden faydalanmak ve araç masraflarını düşürmek için gerekmektedir. Fakat, bütün talep noktalarına bölgesel turlar aracılığıyla gitmek çoklu dağıtım sistemleri için en iyi çözümü oluşturmamaktadır. Bu yüzden çalışmamızda her bir talep noktası oluşturulan bölgesel turlara ya da ölçek ekonomilerinden faydalanmadan doğrudan ADÜ'lere atanma ihtimali vardır. Bu problem için doğrusal karışık tamsayılı matematiksel model önerilmiştir ve geçerli eşitsizliklerle model kuvvetlendirilmiştir. Ayrıca Benders ayrıştırma yöntemi kullanılarak problem için kesin sonuçların bulunması amaçlanmıştır. Büyük ölçekli problemlerin çözülebilmesi için aşamalı sezgisel çözüm yöntemi geliştirilmiştir. Önerilen tüm model ve algoritmalar, literatürde kullanılan TR ve CAB data setleriyle test edilmiştir.

Anahtar sözcükler: ADÜ yer seçimi, rotalama, Benders ayrıştırma yöntemi.

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Chapter 1

Introduction

Hubs are facilities that are used as switching, transshipment and sorting points in many-to-many distribution systems. In such a network where traffic is collected from many origins to be distributed to many destinations, direct links between each origin-destination pair are not justified on the basis of cost. Hub network design is to replace large number of direct origin-destination links with fewer in-directed links. The construction of a hub network lowers the transportation cost since consolidation of flow on hubs generates economies of scale and reduces the number of links to ensure that each flow is routed to its destination. Design of such networks plays a very crucial role especially in the logistic systems thanks to its benefits for decreasing the total cost. This situation encourages academicians and practitioners to work on designing hub networks. This manifests itself in the location literature as the well-known hub location problem. Many variants of hub location problem have been studied and the main aim of these problems is to design a hub network by deciding on the location of hubs and the allocation of non-hub nodes to the hubs.

In many-to-many flow distribution systems, designing a hub network where allocation of non-hub nodes directly to the hubs increases the transportation cost when nodes do not have sufficient demand to justify direct connections with the hubs. In such cases, these direct connections increase the number of vehicles required and decrease the utilization of vehicle capacity. However, there could be cases where node without sufficient demand has to be directly connected to a hub node. To decrease the number of vehicles required and increase the utilization of vehicle capacity, nodes located in different origins are visited by the same vehicle to pick up traffic and send them to a terminal facility where flows are sorted and consolidated. The consolidated flows are then moved towards their destination through a network of terminals. Finally, the flows are deconsolidated and loaded into vehicles to deliver demands to nodes located in different destinations. Design of this network necessitates local tours established for pick-up and delivery task to decrease the transportation cost. In the literature, Nagy and Salhi [1] introduce hub location and routing problem. This problem decides the location of hubs and the allocation of non-hub nodes to the tours which start and end at the hub node while minimizing the total transportation cost.

The hub location and routing problem arises in many logistic applications. To exemplify, one important application of this problem is public postal services. Generally, in this application, flows should be sent from many origins to many destinations. Instead of directly connecting them, local pick-up tours starting and ending at a base are used to collect the flows from nodes in these tours. Afterwards, the flows are consolidated at this base and sent to another base where flows are sent through local delivery tours to the final destinations. The bases can be considered as hub nodes. This application necessitates to decide on the locations of hubs and the allocation of non-hub nodes to hubs and routing among the nodes allocated to the same hubs as in the hub location and routing problem.

In this study, two sets of nodes are given: the set of possible hub locations and the set of demand nodes. We are also given flows and distances between each origin-destination pair. We determine the location of hubs and the allocations of each non-hub node to either directly to a hub or to a local tour assigned to a hub. Each demand node can receive and send flow through a single hub node which is called single assignment in the location literature. The hub nodes are responsible for assembling flows from several origins that can come from directly from a non-hub node or a tour, re-routing these flows to other hub nodes where the flows are disassembled and delivered to again a tour or a non-hub node. We jointly decide on the location of hubs and the allocation of non-hub nodes to a hub or a tour assigned to a hub. Therefore, our problem is a combination of two well-known problems: the single assignment hub location problem and multi-depot vehicle routing problem.

The aim of our problem is to minimize the total cost of routing the traffic in the hub network, the tours and the direct links. The costs of sending flow through direct links between non-hub nodes and hub nodes and the costs of routing on the tours and the hub network are a function of the distance traversed and the flow sent through it. Moreover, we calculate fixed costs such as driver cost for the links in the network and these fixed costs are a function of distance traversed. This cost structure depicts the cost confronted in the real life. It is assumed that there is no capacity limit on the links between the hub nodes, however, each tour honors a predetermined capacity. Therefore, the flow that can be sent through any tour is limited. In the case where there is no limit on the local tours, a lot of nodes may be visited with a local tour and this situation increases the time spent in the tours. Moreover, increasing the visited nodes in the local tours will require huge vehicles which will increase the total cost. On the other hand, there is no limitation on the number of tours that are assigned to the same hub. This assumption relies on the fact that there will be hubs where traffic will be heavy due to geopolitical locations of these hubs which will increase the local tours required. Figure 1.1, illustrates a potential solution to our problem for an instance with 18 demand nodes and 4 hub nodes. The squared nodes depict the hub nodes: 1, 2, 3 and 4. The lines between any combination of the hub nodes represent the inter-hub complete network. Moreover, as it is seen in the figure, there are 4 local tours and 3 directly assigned non-hub nodes.

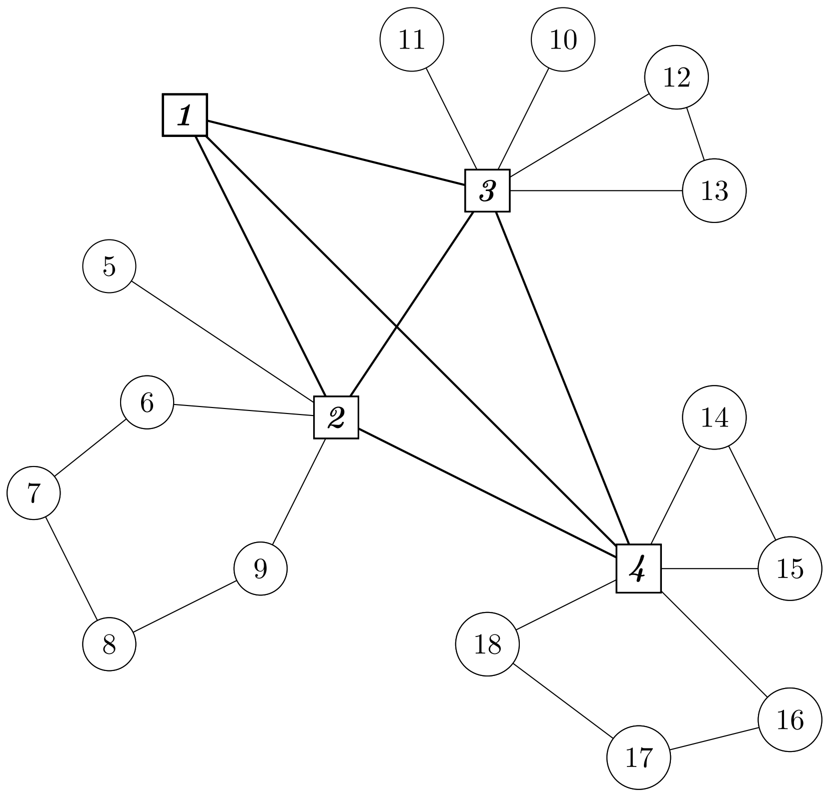

Figure 1.1: A feasible Solution

In this thesis, we develop a mixed integer mathematical model to our problem. The proposed model is then strengthened with valid inequalities. In addition to the mathematical formulation, we propose two more solution methodologies: Benders Decomposition algorithms and hierarchical heuristics. We propose three different Benders formulations: (i) we generate aggregated cuts at each iteration, as in the classical Benders procedure; (ii) we generate multiple cuts for each demand node and each hub node; (iii) we strengthen the first two Benders formulations with valid inequalities that eliminate subtours and the tours exceeding the maximum capacity in the iterations where Benders subproblem is infeasible. We also develop the iterative clustering-routing heuristic having two phases: clustering phase and routing phase. For each phase, mathematical models are developed. In the first phase, we decide on the location of hubs and assign non-hub nodes to the hubs. In the second phase, for each hub node, we allocate each non-hub node to either a local tour or directly to the hub and route flows among all the non-hub nodes assigned to the same tour. The heuristic provides upper bounds to our problem in reasonable CPU times. The solution methodologies are tested on the US Civil Aeronautics Board (CAB) and the Turkish Network (TR) data sets.

The outline of the thesis is as follows: Chapter 2 presents the hub location and the routing literature. In chapter 3, we formally represent the problem definition with the underlying assumptions and notations. In addition, we propose a mathematical mixed integer model and strengthen it with valid inequalities. Afterwards, as the solution methodologies, Chapter 4 gives the Benders Decomposition approach to our problem after pointing out the theory of the Benders Algorithm and Chapter 5 highlights the second solution approach to the problem, the Iterative Clustering-Routing Heuristic. In Chapter 6, computational experiments on our problem will be presented. Finally, a general discussion and future research related to our problem will be given in Chapter 7.

Chapter 2

Literature Review

In this chapter, the related literature is examined in two main sections titled as Hub Location Problems, and Hub Location and Routing Problem. In the first section, we point out various types of the hub location problems such as p-hub median problems, hub location with fixed costs, p-hub center problems and hub covering problems. In the second section entitled Hub Location and Routing Problem, we first give the literature of this problem which is the main scope of our study. Then, we illustrate the differences and similarities of our study. Meanwhile, throughout both sections, most studies are based on three assumptions: the hub network is complete, the traffic on hub-to-hub links is multiplied with a discount factor α and no direct nonhub-to-nonhub link is allowed. In our review, these three assumptions are satisfied unless otherwise is pointed out. Furthermore, readers could reach the detailed information through surveys by Campbell [2], O'Kelly and Miller [3], Campbell et al. [4] Alumur and Kara [5], Campbell and O'Kelly [6] and Farahani et al. [7].

2.1 Hub Location Problems

Hubs are particular facilities that serve as switching, transshipment and sorting points for transportation (air passenger, cargo, etc.) and telecommunication systems with many origins and destinations. To elaborate, hubs are centers where the flows are concentrated so as to take advantage of discount instead of serving each origin-destination pair directly. The Hub Location Problem is locating hub facilities and allocating other nodes to hubs in order to route traffic between origin and destination pairs. There are two types of hub networks - single allocation and multiple allocation. The difference between them is the allocation of the non-hub nodes to hubs. In the single allocation, each demand node can receive and send flow through a single hub node. On the other hand, in the multiple allocation, there is no restriction on the number of hubs that a non-hub node can receive and send flow through. In this section, we analyze hub location problems in four different subsections: p-hub median problem, hub location problem with fixed costs, p-hub center problem and hub covering problem. These hub location problems have analogous location versions such as p-median problem, facility location problem with fixed costs, p-center problem and covering problem.

2.1.1 p-Hub Median Problems

The p-hub median problem determines the location of p hubs and the allocation of each demand node to the hubs in order to serve the given set of flows between origin-destination pairs while minimizing the total transportation cost (time, distance, etc.).

The p-hub median problem is NP-Hard and Kara [8] demonstrates that even if the locations of the hubs are known, the allocation problem is still NP-hard.

O'Kelly [9] presents the first quadratic integer programming formulation for the single allocation p-hub median problem motivated by airline passenger networks. Campbell [10] introduces the first linear integer programming formulation. Skorin-Kapov et al. [11] propose a new mixed integer formulation whose linear relaxation is tighter than the formulation proposed by Campbell. Ernst and Krishnamoorthy [12] state a different linear integer programming formulation which gives the best computational time and requires fewer variables and constraints. However, in terms of required variables and constraints, Ebery [13] provides the best mathematical formulation.

Various heuristics are proposed for the single allocation p-hub median problem. The earlier heuristics are as follows: O'Kelly [9] proposes two heuristics based on the enumeration of p hub locations; Klincewicz [14], [15] develop an exchange heuristic, a GRASP (Greedy Randomized Search Procedure) and a tabu search heuristic; Skorin-Kapov [16] also provides a tabu search heuristic. O'Kelly [17] presents a lower bounding technique based on linerization of the quadratic objective function. Using the idea that the multiple allocation p-hub median problem provides a lower bound on the optimal solution of the single allocation p-hub median problem, Campbell [18] proposes two heuristics for the single allocation p-hub median problem. Later, Ernst and Krishnamoorthy [12] develop a simulated annealing heuristic. Pirkul and Schilling [19] propose a Langrangian relaxation method which finds lower and upper bounds in reasonable amount of CPU times.

In the multiple allocation p-hub median problem, each node can receive and send flow through more than one hub. Campbell [20] proposes the first linear integer formulation for the multiple allocation p-hub median problem. Another formulation in which some of the constraints are written in the aggregated form is stated by Skorin-Kapov [21]. LP relaxation of this formulation is tight and integral results are obtained for almost all instances using the CAB data set. Ernst and Krishnamoorthy [22] propose a new mathematical model which is based on the same notion they have suggested for the single allocation version of this problem.

Ernst and Krishnamoorthy [22] present two branch-and-bound algorithms for the multiple allocation p-hub median problem and they also obtain lower bounds by using LP relaxations. Recently, Benders decomposition based exact algorithms are proposed for the multiple allocation p-hub median problem. Camargo et al. [23] are the first ones to apply Benders decomposition. They propose three different Benders formulations. In the first one, they generate a single cut at each iteration as in the classical Benders procedure. Secondly, they propose multiple cuts for each origin-destination pair at each iteration. Their third formulation generates cuts with ϵ error margin. Contreras et al. [24] propose a Benders Decomposition algorithm where they generate cuts for each hub candidate to solve the uncapacitated multiple allocation p-hub median problem. They also construct pareto-optimal cuts to enhance the convergence of the algorithm. Camargo et al. [25] study this problem where the discount factor is defined as a piecewise-linear function. They propose Benders decomposition algorithm where they generate cuts for each origin-destination pair. Gelareh and Nickel [26] study on the uncapacitated multiple allocation hub location problem where the hub network is incomplete and the triangularity assumption does not hold. Benders formulation is proposed and the algorithm is tested on the AP data set. Moreover, Benders algorithm is also used for the capacitated version of the multiple allocation hub location problem. Rodríguez-Martín and Salazar-González [27] work on the capacitated hub location problem on an incomplete network and propose two Benders formulations: classical Benders formulation and nested two level algorithm based on Benders algorithm. Contreras et al. [28] study on the capacitated hub location problem and propose a Benders formulation where subproblem is a transportation problem that can be easily solved with a special algorithm.

Many variants of p-hub median problem have been studied in literature. Yaman et al. [29] work on star p-hub median problem with bounded path lengths. They select p hub nodes and connect them to a center hub, and then each non-hub node is assigned to a hub. The aim is to minimize the total cost subject to upper bounds on the path lengths. Yaman [30] introduces r-allocation p hub median problem where each node can be assigned to at most r hubs among selected p hubs. She proposes a mixed integer formulation and tests on AP, CAB and TR data sets. Perió et al. [31] propose a heuristic based on the GRASP methodology for r-allocation p hub median problem. Moreover, Martí et al. [32] propose scatter search algorithm for the uncapacitated r-allocation p hub median problem.

2.1.2 Hub Location Problem with Fixed Costs

In this version of hub location problem, there is a fixed cost for opening hub facilities, and therefore the number of hubs that should be open is a decision. O'Kelly [33] gives the first formulation as a quadratic integer program. This problem has uncapacitated and capacitated hub location versions with fixed costs in addition to single and multiple allocation versions. Campbell [10] introduces the linear integer programming formulations for multiple/single allocation uncapacitated/capacitated hub location problems. Abdinnour-Helm and Venkataramanan [34] propose a quadratic integer formulation based on the idea of multi-commodity flows in networks for the single allocation uncapacitated hub location problem. Aykin [35] presents the capacitated versions of the hub location problem with fixed costs. Ernst and Krishnamoorthy [36] present two new formulations modified versions of the previous mixed integer formulations to the p-hub median problem for the capacitated single allocation hub location problem. In addition, Ebery et al. [37] present a formulation based on the one proposed by Ernst and Krishnamoorthy for the multiple allocation p-hub median problem for the multiple allocation capacitated hub location problem.

Solution techniques for the uncapacitated multiple allocation version are as follows: Klincewicz [38] presents dual-ascent and dual adjustment with a branch-and-bound algorithm, Mayer and Wagner [39] develop a branch-and-bound method called the HubLocater, Canovas et al. [40] present a heuristic approach-a dual-ascent technique.

2.1.3 The p-hub Center Problem

The p-hub center problem is analogous to the p-center problem. The aim of this problem is to locate p hub nodes, and allocate each non-hub node to the hubs while minimizing the maximum cost between origin-destination pairs. The p-hub center problem has three various versions with different objectives such as minimization of maximum cost occurred for any origin-destination pair, minimization of the maximum cost on a single link that could provide movements of origin-to-hub, hub-to-hub and hub-to-destination and finally minimization of maximum cost of edge linking a hub and origin/destination. Campbell [10] proposes formulations for single and multiple allocation versions for all three types that we have mentioned above. Kara and Tansel [41] propose different formulations for the single allocation p-hub center problem and also give a combinatorial formulation and proof of NP-completeness of this problem. Ernst et al. [42] propose a new formulation which has more continuous variables compared to the formulation

Kara and Tansel [41] introduce. However, computational analysis on CAP and AP data sets indicates that Ernst et al. [42] formulation has better results in terms of CPU times. Ernst et al. [42] also study the multiple allocation p-hub center problem in the same paper and they present two new formulations.

Baumgartner [43] analyzes the polyhedral properties of the single allocation p-hub center problem and developed a branch-and-cut algorithm. Pamuk and Sepil [44] present a single relocation algorithm with tabu search. Later, Meyer [45] propose a two phase algorithm for the single allocation p-hub center problem and Gavriliouk [46] present heuristic procedures based on aggregation technique for both single and multiple allocation p-hub center problem.

Yaman et al. [29] introduce the star p-hub center problem where p hubs are chosen and connected to a center hub and each non-hub node is connected to a hub node. The aim is to minimize the longest path. Afterwards, Liang et al. [47] propose an approximation for the star p-hub center problem.

2.1.4 Hub Covering Problems

The hub covering problem could be investigated in two main types: hub set-covering problem and maximal hub-covering problem. The hub set-covering problem aims to locate hubs to cover all demands while minimizing the cost of opening hubs. On the other hand, the maximal hub-covering problem maximizes the demand covered with a certain number of opened hubs. These two kinds of problems are firstly presented by Campbell [10] and he gives three criteria for hub covering: (I) hubs k and l cover the origin-destination pair (i, j) if the cost of routing from i to j via k and l does not exceed a threshold; (II) the cost of links in the route from i to j via k and l does not exceed a threshold; (III) origin-to-hub and hub-to-destination links meet the specified values separately. Kara and Tansel [48] work on the single allocation hub set covering problem and their proposed linear model surpasses all of the other models. Wagner [49] improves the formulation of Kara and Tansel by aggregating some of the constraints and he also presents new formulations for both the single and the multiple allocation hub covering problems. Afterwards, Ernst et al. [50] propose a new formulation for the single allocation hub set-covering problem, which is based on the one proposed in Ernst et al. [42] for the p-hub center problem. Later, Ernst et al. [50] work on the multiple allocation of hub set-covering problems, present two new formulations and an implicit enumerative method for this problem. Hamacher and Mayer [51] compare various formulations of the hub covering problem and identify some facet-defining valid inequalities.

Peker et al. [52] introduce a new problem: $p-$hub maximal covering problem where they extend the definition of coverage and used partial coverage that changes with distance. They propose mixed integer models and test their results on the CAB and TR data sets.

Hub covering location perspective is used in different problems. Yıldız et al. [53] revisit the regenerator location problem with hub location perspective and they introduce flow-based compact formulations and cut formulation. They develop branch and cut algorithms based on the cut formulations.

2.2 Hub Location and Routing Problems

The hub location and routing problem is concerned with the locations of hub facilities, the allocations of non-hub nodes and the establishment of local tours among the nodes allocated to them. This problem arises when nodes do not have sufficient demand to justify direct connections with the hubs. In such cases, establishing the local tours after deciding on the hub locations and the allocations of non-hub nodes may result in sub optimal solutions. Therefore, location, allocation and establishment of local tours should be considered jointly in designing such network systems.

Nagy and Salhi [1] introduce the hub location and routing problem to the location literature. The authors propose a mathematical model where the objective function has two components: the first one is dependent on the distance traversed in the tours and between hubs and the other component is the fixed cost of opening hubs. They allow customers to be visited by two tours one for pick-up and the other for delivery while obeying capacity and distance constraints. They do not have a restriction on the number of hubs opened. However, they do not solve the problem exactly. Instead, they propose a hierarchical solution methodology to tackle with the problem.

Wasner and Zäpfel [54] extends the hub location and routing problem by allowing direct connections between non-hub nodes. They propose a mixed integer formulation which could not be solved to optimality due to the enormous number of constraints and variables. Therefore, they propose a heuristic method and they test their methodology on the Austria data in order to redesign of the Austrian postal service. The problem has a difficult aspect because Austria has a geographically complex region due to the Alps and approximation on the routing problem can result in extreme distortions of the problem.

Çetiner et al. [55] study multiple allocation version of hub location and routing problem for the Turkish postal delivery services. They assume that demand nodes are visited by uncapacitated vehicles that start and end their tours at the hub node. At most 450 km distance is allowed for each tour established. They propose an iterative heuristic as a solution methodology for this problem. In the first stage, they decide on the hub locations by using the formulation of Daskin [56] for the vertex p-center problem. In the second stage, the routing decision is made. The heuristic solution is tested on Turkish postal delivery system data.

The first exact solution method for the hub location and routing problem is proposed by Camargo et al. [57]. In this study, they develop a new mixed integer formulation that decides on the location of hubs and the allocations of tours to the hubs. There is no limit on the number of the hubs to be opened. In their study, multiple tours may be allocated to a hub. The cost of routing in the hub network is a function of the flows and the distance and the cycle costs are based on distance traversed. In addition, a fixed cost of opening a hub facility is included in the cost function. They define a set of possible arcs which can form a local tour-that decreases the number of variables dramatically. They also assume that there is a maximum time allowed for tours. The proposed mathematical model has 5 index variables. They propose Benders Decomposition algorithm as a solution methodology in order to obtain exact solution.

The second exact solution method for the hub location and routing problem is proposed by Martin et al. [58]. Their problem called Hub Cycle Location Problem (HCLP) aims to locate p hubs and assign each node to one of the hubs with a cycle while minimizing the total transportation cost which includes assigning nodes to hubs and routing flows in the network. To give details of the cost structure, the routing cost between hubs is a function of the distance and the flows. On the other hand, the cost incurred on the cycles depends on the distance traversed. Furthermore, there exist two discount factors α and β for the hub-to-hub traffic and cycles respectively. It is assumed that each cycle can include at most q nodes and one cycle could be assigned to one hub. To point out the solution methodology, they propose a mixed integer programming formulation strengthened by valid inequalities. They develop a branch-and-cut algorithm based on separation for the valid inequalities and test the algorithm on CAB and AP data sets. They solve instances up to 50 nodes. Figure (2.1) depicts a potential feasible solution for (HCLP) where $q \geq 5$ and $p = 4$

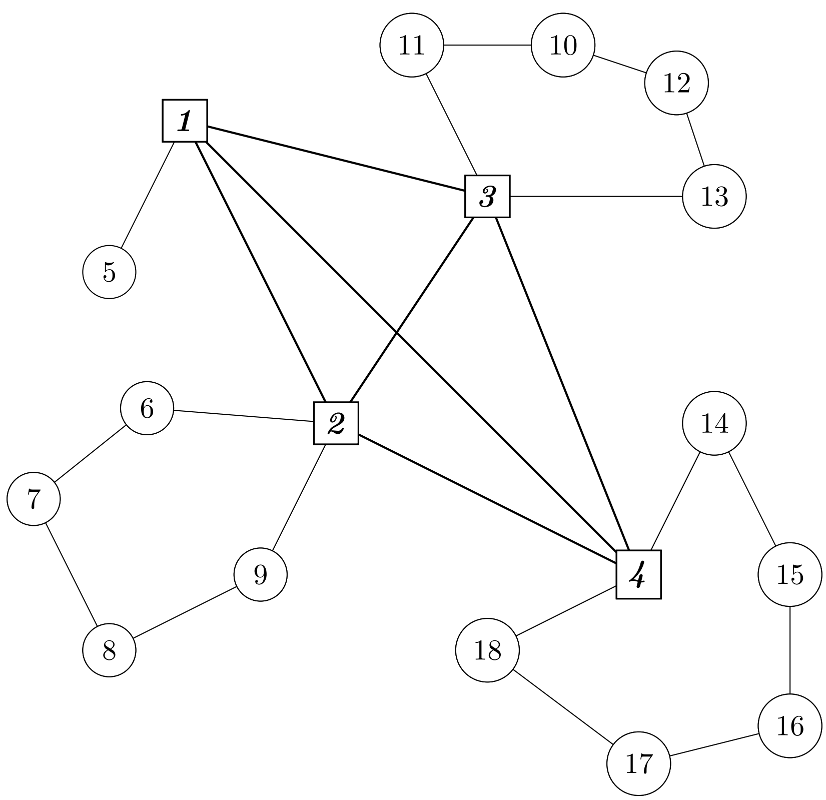

Figure 2.1: A HCLP feasible solution

In this study, we work on the single allocation hub location and routing problem. To the best of our knowledge, this is the third study in which an exact solution method for the hub location and routing problem is proposed. Compared to proposed exact solutions in the literate pointed above, our problem is generalized version of the hub location and routing problem. We have the following assumptions: (i) the set of nodes among which local tours can be established is the same as the set of demand node; (ii) there is no limitation on the number of local tours a hub node serves; (iii) there is maximum flow capacity on each local tour. In the existing studies, at least one of these assumptions is relaxed. Hence, our study contributes to the literature by proposing strong mixed integer formulation as an exact solution methodology to the generalized version of the hub location and routing problem. Furthermore, more realistic cost structure is considered in the objective function. Our objective is minimization of the following cost components: (i) the routing cost of flow sent from non-hub nodes to hub nodes with direct links, (ii) the routing cost of flow sent through local tours and hub network, (iii) the cost of the links to construct many-to-many distribution system. The first two component is function of the distance traversed and the flow carried. How much flow carried and the distance traversed by the flow affects the total transportation cost in many-to-many distribution system. Therefore, it is important to have a cost structure that includes this kind of cost terms. This is the first study incorporating the cost of the routing in the local tours as a function of both distance traversed and flow carried. In the literature, how much flows sent through local tours is not taken into consideration in the cost function. Besides, the third component pointed above can be considered as the fixed vehicle cost such as driver cost. That component plays also a crucial role to have a realistic cost structure.

The CAB data set is solved by the proposed mixed integer formulation. In addition to the mathematical model, we propose two more solution methodologies: Benders Decomposition algorithm and a hierarchical heuristic. We propose three different Benders formulations. In the first one, aggregated cuts are generated at each iteration. In the second one, we generate multiple cuts for each demand node and for each hub node. Finally, we strengthen the first two Benders formulations with valid inequalities that eliminate subtours and the tours exceed the maximum capacity in the iterations where Benders subproblem is infeasible. Our study is the second one which applies Benders Decomposition algorithm for single allocation type problems. We also develop the iterative clustering-routing heuristic in order to reach near optimal solution with reasonable CPU times. The proposed heuristic has two phases: the clustering phase and the routing phase. In each phase, we use mathematical model to make decisions. The proposed heuristic provides upper bounds to our problem.

Chapter 3

Problem Definition and Formulation

This chapter is divided into three sections: Problem Definition, Problem Formulation and Valid Inequalities. In Section 3.1, we state the problem definition along with the notations and assumptions. Then, our decision variables and a mixed integer mathematical model with a quadratic objective function will be represented in Section 3.2. The constraints of our proposed mathematical model will be explained in detail throughout this section. Finally, valid inequalities to strengthen our formulation will be stated in Section 3.3.

3.1 Problem Definition

We first introduce the notation. Let I be the set of demand nodes and J be the set of possible hub locations where $J \subseteq I$. There will be flows from any demand node to every other demand node and for all node pairs $i \in I$ and $j \in I$, w_{ij} represents the flow that should be sent from node i to node j. Let $O_i = \sum_{j \in I} w_{ij}$ be the total amount of flow emanating from node $i \in I$. We assume that the total amount of flow originating from node $i \in I$ is equal to the total amount of flow with destination node $i \in I$ (i.e., $O_i = \sum_{j \in I} w_{ij} = \sum_{j \in I} w_{ji}$). We denote the cost of routing a unit of flow from node $i \in I$ to $j \in I$ by c_{ij} and the cost of using arc (i, j) by g_{ij}. These cost parameters are dependent on the distance between each node pair. We assume that c_{ij} and g_{ij} are nonnegative, symmetric and satisfy the triangular inequality.

We select p hubs from the set J and construct a complete hub network with these p hubs. In the hub network, α factor is used to represent the economies of scale. In addition, we establish local tours and direct links with a hub. Therefore, each non-hub node has two options: (i) it could be directly connected to a hub; (ii) it could be visited through a local tour. We define a local tour as a route starting and ending at one of the hub nodes and each local tour should visit at least two non-hub nodes (i.e., direct connection between a non-hub node and a hub node is not considered as a local tour). The local tours are to increase the utilization of vehicles and decrease the number of vehicles required. Therefore, a factor β is used for local tours to represent economies of scale. In the direct connection case, the non-hub nodes are assigned directly to a hub. Consequently, we design a network where we construct a complete hub network with selected p hubs and each non-hub node is connected with the hub network by either a direct link or a local tour.

The connection rule described above within the hub network results in eight different scenarios for each origin-destination pair. If origin and destination nodes are assigned to the same hub node, the positions of the origin and destination can be as follows: (i) both origin and destination might be on the same tour;

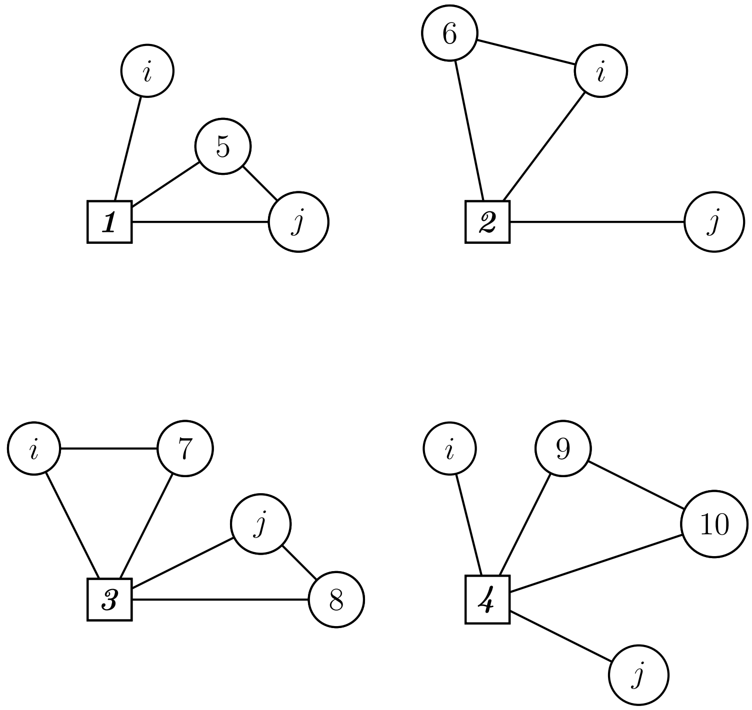

Figure 3.1: Possible Scenario for node pair (i, j) assigned to the same hub

(ii) both origin and destination might be directly assigned to the hub; (iii) origin might be on a cycle and destination might be directly assigned to the hub; (iv) origin might be directly assigned to the hub and destination might be on a cycle. In Figure 3.1, we give the possible scenarios for the origin-destination pair i and j that are assigned to same hub. The flow originating from node i that is assigned directly to hub node 1 will be sent firstly to its hub node then all the flows for delivering demand of node j including the flow emanating from node i will go through the local tour established. This is how the flow should be sent when the origin-destination pair i and j that are assigned to same hub and origin is directly assigned to the hub and destination is on the cycle as in the case (iv).

In the case where origin and destination nodes are not assigned to the same hub, routing the flow through the hub network is necessary so as to send the traffic between origin and destination. There are again four possibilities (i),(ii),(iii)

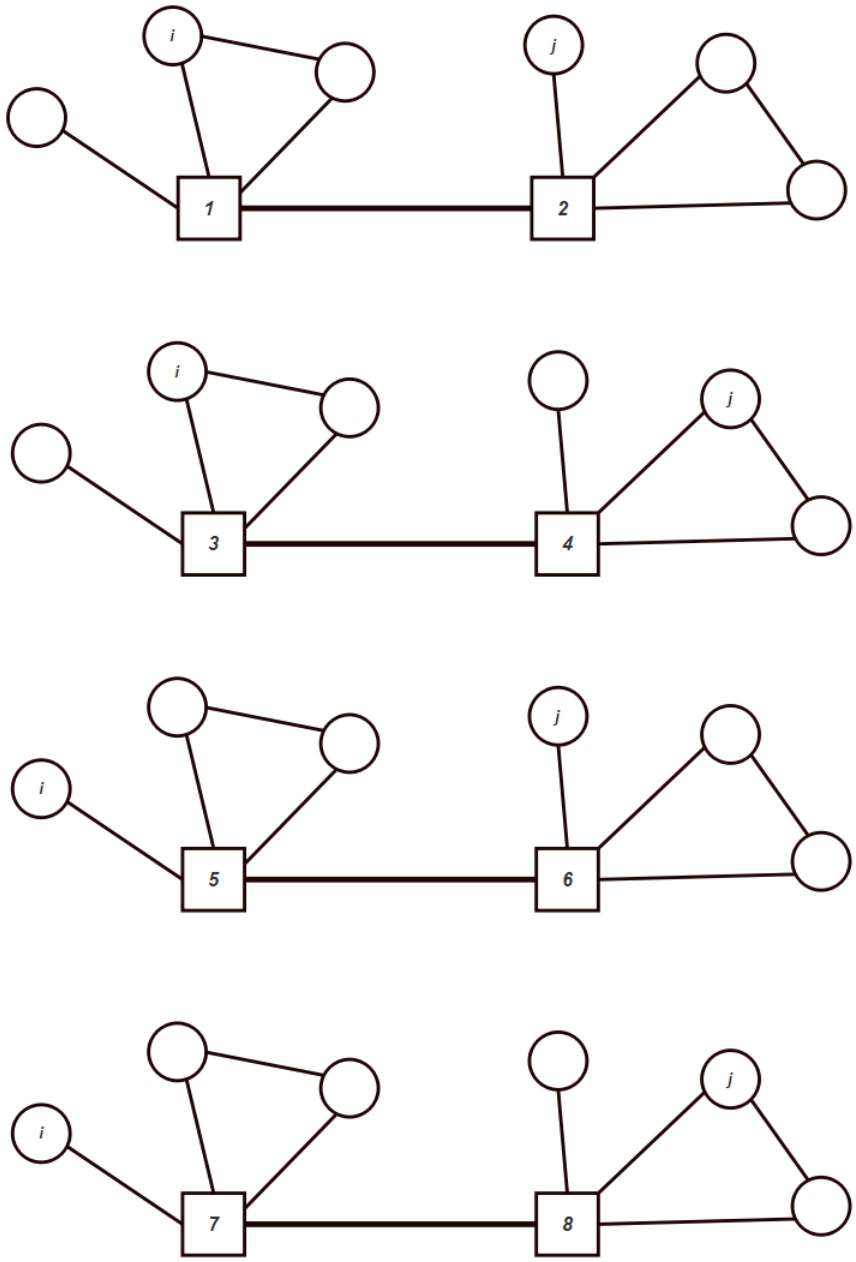

Figure 3.2: Possible Scenarios for node pair (i, j) assigned to different hubs

and (iv) for the locations of origin and destination nodes. In Figure 3.2, we give the possible scenarios for the origin-destination pair i and j that are assigned to different hubs. To exemplify how flows are sent between origin-destination pair i and j, the first scenario depicted in Figure 3.2 will be explained in detail as follows: the flow emanating from node i that is assigned to hub node 1 through a local tour will firstly complete its tour and come to the hub node 1. Then the flow will be sent to the hub node 2 through the hub network. Finally, all the flows for delivering demand of node j including the flow emanating from node i will be sent from the hub node 2 to node j.

To point out the specialities about the tours, each local tour should honor the capacity Q-that means a vehicle could carry a maximum Q units of flow. There is no restriction on the number of vehicles that can be initiated from a hub node. The nodes that are assigned to a local delivery tour will be travelled just in the reverse direction of the pick up tour. To explain it with an example, let $i_1 \in J$ be the hub node and i_2, i_3 and i_4 be the nodes assigned to the local tour that completes its tour on the hub i_1. Additionally, let $i_1 \rightarrow i_2 \rightarrow i_3 \rightarrow i_4 \rightarrow i_1$ be the tour P for the picking tour with the minimum cost (i.e., vehicle sent from hub node i_1 visit the nodes i_2, i_3 and i_4 in given order which is the way to reach minimum cost). Let $flow(i,j)$ be the amount of flow sent through arc (i,j). As it is seen in Figure 3.3 ,$flow(i_1,i_2) = 0$, $flow(i_2,i_3) = O_{i_2}$, $flow(i_3,i_4) = O_{i_2} + O_{i_3}$ and $flow(i_4,i_1) = O_{i_2} + O_{i_3} + O_{i_4}$. Then the routing cost in this local tour is $\sum_{(i,j)\in P} \beta c_{ij} flow(i,j)$. Then the tour $\overline{P}$ for delivery tour with the minimum cost will become $i_1 \rightarrow i_4 \rightarrow i_3 \rightarrow i_2 \rightarrow i_1$ since $\sum_{(i,j)\in P} \beta c_{ij} flow(i,j) = \sum_{(i,j)\in \overline{P}} \beta c_{ij} flow(i,j)$, P is the minimum cost local tour and the total amount of flow emanating from any node $i \in I$ is equal to the total amount of flow with destination node $i \in I$ as it is stated above. Therefore, we do not need to construct different tours for pick up and delivery tasks since we could establish any delivery tour by visiting the nodes in the reverse order of pick

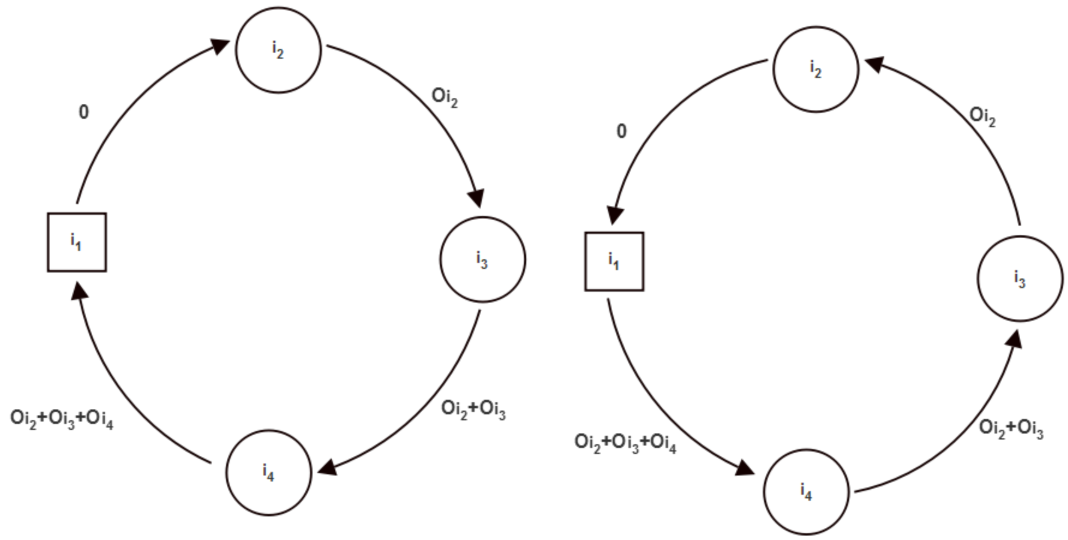

Figure 3.3: Local tour for pick up and delivery task

up tour. Consequently, establishing pick up tours is adequate to construct the local tours.

The total transportation cost consists of four components: (1) routing cost of flow sent from non-hub nodes to hub nodes with direct links; (2) routing cost of flow sent through local tours; (3) routing cost of traffic in the hub network; (4) fixed cost of links that are used to construct a many-to-many distribution network system. In the first three components, costs are a function of the distance traversed and the traffic. Using this cost structure makes the problem realistic, however, it increases the number of variables and constraints used in the mathematical formulation. To the best of our knowledge, this is the first study that considers the routing cost in the local tours as a function of both the distance traversed and the traffic on these tours. Nevertheless, the components (1), (2) and (3) are not enough to depict the cost confronted in the real life without the component (4) because these three components do not take into consideration the vehicle fixed cost. Hence, the aim of the component (4) is to calculate the fixed cost such as driver cost which is a function of the distance traversed.

We use a β discount factor for the component (2) as local tours increase the utilization of vehicle capacity and decrease the number of vehicles required. We use another discount factor α for the component(3) since in the hub network, the utility of the vehicles is high due to the fact that hubs are consolidation points.

Consequently, our study determines jointly the location of p hubs and the allocations of each non-hub node to either directly to a hub or a tour with capacity Q that completes its tour on a hub while minimizing the total transportation cost. To reach our goal, we design a complete hub network among p hub nodes and connect each non-hub node with the hub network by either a direct link or a local tour that starts and ends its tour at one of the hub nodes. Hence, our study is based on deciding on the locations of p hubs, allocations of non-hub nodes and routing among the nodes that are allocated to the same local tour.

3.2 Problem Formulation

In this part, we first define the decision variables and then propose the mixed integer model with quadratic cost function to our problem. Finally, we linearize the cost function in this section.

Decision Variables

x_{ij} : 1 if node i is assigned to hub j and 0 otherwise

y_{ijk} : 1 if node i precedes node j at the route that completes its tour on hub k and 0 otherwise

f^i_{jl} : flow that originates at node i and travels from hub j to hub l

r^k_{ij} : flow that travels from node i to node j in the route that completes its tour on hub k

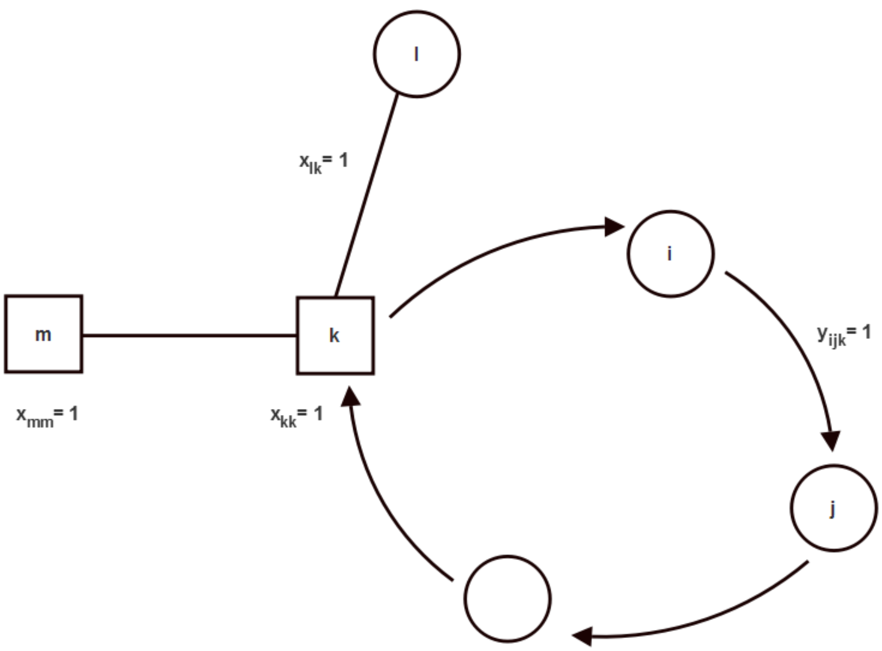

Figure 3.4: Representation of the variables

The model is as follows:

$$\min \sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}} \alpha c_{jl} f^i_{jl} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} 2\beta c_{ij} r^k_{ij} + \sum_{i\in I}\sum_{j\in J} 2O_i c_{ij} x_{ij}$$
$$+ \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} g_{ij} y_{ijk} + \sum_{i\in I}\sum_{j\in J} g_{ij} x_{ij} + \sum_{j\in J:j<k}\sum_{k\in J} g_{jk} x_{jj} x_{kk} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}}\sum_{k\in J} y_{ijk} \geq 1 \qquad \forall i \in I \tag{3.2}$$

$$\sum_{i\in I\setminus\{j\}} y_{ijk} - \sum_{i\in I\setminus\{j\}} y_{jik} = 0 \qquad \forall j \in I, k \in J \tag{3.3}$$

$$y_{ikk} + y_{kik} \leq 1 \qquad \forall k \in J, i \in I : i \neq k \tag{3.4}$$

$$y_{ijk} \leq x_{kk} \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{3.5}$$

$$x_{ij} \leq x_{jj} \qquad \forall i \in I, j \in J \tag{3.6}$$

$$\sum_{j\in J} x_{jj} = p \tag{3.7}$$

$$\sum_{l\in J\setminus\{j\}} (f^i_{jl} - f^i_{lj}) = \sum_{m\in I} w_{im}(\sum_{k\in I\setminus\{i\}} y_{ikj}) - \sum_{m\in I\setminus\{j\}} w_{im}(\sum_{k\in I\setminus\{m\}} y_{mkj})$$
$$+ \sum_{m\in I} w_{im}(x_{ij} - x_{mj}) \quad \forall i \in I, j \in J : i \neq j \tag{3.8}$$

$$\sum_{l\in J\setminus\{j\}} (f^j_{jl} - f^j_{lj}) = \sum_{m\in I} w_{jm}\Big(x_{jj} - x_{mj} - \sum_{k\in I\setminus\{m\}} y_{mkj}\Big) \quad \forall j \in J \tag{3.9}$$

$$\sum_{j\in J\setminus\{i\}} (r^k_{ij} - r^k_{ji}) = O_i \sum_{m\in I\setminus\{i\}} y_{imk} \qquad \forall i \in I, k \in J : i \neq k \tag{3.10}$$

$$0 \leq r^k_{ij} \leq Q y_{ijk} \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{3.11}$$

$$f^i_{jl} \geq 0 \qquad \forall i \in I, j \in J, l \in J \setminus \{j\} \tag{3.12}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{3.13}$$

$$y_{ijk} \in \{0,1\} \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{3.14}$$

The objective function (3.1) minimizes the total transportation cost that consists of six terms : (1) the routing cost of flow sent in the hub network - the cost is multiplied by the discount factor α, (2) the routing cost of flow sent through the local tours - the cost is multiplied by the discount factor β, (3) the routing cost of flow sent directly from single assigned non-hub nodes to hub nodes, (4) the fixed cost of travelling the local tours which is a function of distance traversed, (5) the fixed cost of travelling from single assigned non-hub nodes to hub nodes which is a function of distance traversed and (6) the fixed cost of travelling through the hub network. In addition, the second and third terms are multiplied by two in order to calculate the cost of both delivery and pick up.

Constraints (3.2) ensure that each node in the set I will be assigned directly to a hub or assigned to a tour that completes its tour on a hub. To elaborate, if node i is not a hub node, i.e $x_{ii} = 0$, then this node is either directly assigned to a hub node or a local tour: $\sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}} \sum_{k\in J} y_{ijk} = 1$. In the case where node i is a hub node, i.e $x_{ii} = 1$, there are two possibilities: (i) There is at least one local tour assigned to the hub node i, then $\sum_{j\in I\setminus\{i\}} y_{iji} \geq 1$, which leads $\sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}} \sum_{k\in J} y_{ijk} \geq 1$. (ii) There is no local tour assigned to the hub node i, then $\sum_{j\in I\setminus\{i\}} y_{iji} = 0$, which leads $\sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}} \sum_{k\in J} y_{ijk} = 1$.

Constraints (3.3) impose that the number of incoming arcs to any node i is equal to the number of outgoing arcs from any node i that are assigned to a tour that completes its tour on hub k.

Constraints (3.4) ensure that there is no local tour with just one node.

Constraints (3.5) and (3.6) impose that if a node is not chosen as a hub node, any demand node which is either a part of a local tour or single cannot be assigned to this node.

Constraints (3.7) ensure that p nodes should be chosen as hub locations.

Constraints (3.8) and (3.9) are flow balance constraints for the hub network. If node j is not a hub node, then the right sides of both constraints will be zero which means there cannot be flow sent through the hub network that touches node j. In the case of constraints (3.8) where node i is not a hub node, but node j is a hub node i.e $x_{ii} = 0$, $x_{jj} = 1$, then node i is either directly assigned to a hub node or a local tour as stated above i.e, $\sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}} \sum_{k\in J} y_{ijk} = 1$. If node i is assigned to the hub node j then $x_{ij} + \sum_{k\in I\setminus\{i\}} y_{ikj} = 1$. Then the total flow emanating from node i will be $\sum_{m\in I} w_{im}(\sum_{k\in I\setminus\{i\}} y_{ikj}) + \sum_{m\in I} w_{im}x_{ij}$. Some of this flow will not go through the hub network but will be sent to nodes either individually or by a local tour to the hub j which is calculated by $\sum_{m\in I\setminus\{j\}} w_{im}(\sum_{k\in I\setminus\{m\}} y_{mkj}) + \sum_{m\in I} w_{im}x_{mj}$. Therefore, the flow emanating from node i and going through the hub network will be the total flow emanating from node i minus the flow sent to nodes either individually or by a local tour to the hub j, i.e,$\sum_{m\in I} w_{im}(\sum_{k\in I\setminus\{i\}} y_{ikj}) - \sum_{m\in I\setminus\{j\}} w_{im}(\sum_{k\in I\setminus\{m\}} y_{mkj}) + \sum_{m\in I} w_{im}(x_{ij} - x_{mj})$. If node i is not assigned to the hub node j then $x_{ij} + \sum_{k\in I\setminus\{i\}} y_{ikj} = 0$ that ensures that the flow originating node i cannot be sent from node j. In the case of constraints (3.9) where node j is a hub node i.e $x_{jj} = 1$, the total flow emanating from node j will be $\sum_{m\in I} w_{jm}x_{jj}$, however, some of this flow will not go through the hub network but will be sent to nodes either individually or by a local tour to the hub j which is calculated by $\sum_{m\in I} w_{jm}\Big(x_{mj} + \sum_{k\in I\setminus\{m\}} y_{mkj}\Big)$. Hence, the flow emanating from node j and going through the hub network will be the total flow emanating from node j minus the flow sent to the nodes either individually or by a local tour to hub j, i.e, $\sum_{m\in I} w_{jm}\Big(x_{jj} - x_{mj} - \sum_{k\in I\setminus\{m\}} y_{mkj}\Big)$.

Constraints (3.10) are flow balance for the local tours. The total outgoing flow minus the total incoming flow from non-hub node i will be equal to its demand.

Constraints (3.11) ensure that the capacity on tours is not exceeded.

Constraints (3.12), (3.13) and (3.14) are the variable restrictions.

The objective function of above problem can be easily linearized by defining a new variable $z_{jk} = x_{jj}x_{kk}$ and adding new constraints as follows:

$$z_{jk} \geq x_{jj} + x_{kk} - 1 \qquad \forall j \in J, k \in J : j < k \tag{3.15}$$

$$z_{jk} \leq x_{jj} \qquad \forall j \in J, k \in J : j < k \tag{3.16}$$

$$z_{jk} \leq x_{kk} \qquad \forall j \in J, k \in J : j < k \tag{3.17}$$

3.3 Valid Inequalities

In this section, we propose some inequalities to strengthen our proposed mixed integer programming formulation.

Lemma 1. The following constraints are valid for (3.2)-(3.17)

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in i \in I : O_i \geq Q \tag{3.18}$$

$$y_{ijk} = 0, y_{jik} = 0 \qquad \forall i \in I, j \in I, k \in J : i \neq j, O_i + O_j > Q \tag{3.19}$$

$$\sum_{j \in J} x_{ij} + \sum_{j \in I \setminus \{i\}} \sum_{k \in J \setminus \{i\}} y_{ijk} \leq 1 \qquad \forall i \in I \tag{3.20}$$

$$\sum_{m \in I \setminus \{k\}} y_{mkk} \geq \sum_{m \in I \setminus \{i\}} y_{imk} \qquad \forall i \in I, k \in J \tag{3.21}$$

$$\sum_{m \in I \setminus \{k\}} y_{kmk} \geq \sum_{m \in I \setminus \{i\}} y_{imk} \qquad \forall i \in I, k \in J \tag{3.22}$$

$$y_{ikk} + y_{kik} \leq 1 - \sum_{j \in J} x_{ij} \qquad \forall k \in J, i \in I : i \neq k \tag{3.23}$$

The constraints (3.18) are valid since any node which has demand greater than or equal to capacity Q cannot be a part of a local tour. Constraints (3.19) aim to eliminate local tours among the customers with total demand exceeding the capacity Q. Constraints (3.20) are valid because they force each node except a hub node to have one incoming arc and one outgoing arc as desired. Constraints (3.21) and (3.22) ensure that if a node is assigned to the hub k, there should be incoming and outgoing arc(s) to the hub k. Finally, constraints (3.23) strengthen the constraints (3.4) because if a non-hub node is assigned directly to a hub then this node cannot be a part of a local tour.

Chapter 4

Benders Decomposition

In this chapter, we first give the information about the theory of Benders Decomposition Algorithm to point out how to construct master problem, subproblems and generate feasibility and optimality cuts. Afterwards, we explain how the Benders Decomposition Algorithm is applied to our problem: adding aggregated cuts to the master problem, multiple cuts to the master problem and adding the inequalities that are obtained by checking the subtours or local tours that exceed their capacity.

4.1 Benders Decomposition Methodology

The Benders decomposition method (Benders, 1962) is based on partitioning procedure for solving the mixed integer linear and mixed integer non-linear programs. The main idea of the Benders Decomposition algorithm is the reformulation of the original problem by projecting out a set of variables in order to reach a problem with fewer variables. To explain the procedure briefly, firstly variables are decomposed as complicating and non-complicating variables. Afterwards, complicating variables are fixed by solving the master problem (MP) that is relaxation of the original problem and for these fixed values of variables the remaining Benders subproblem (SP) become easier to solve. Solution of this easier problem enables us to generate Benders cuts (BC) for the master problem.

To explain the idea of the Benders algorithm, let consider the following problem (4.1):

$$\min\{cu + g\nu : Au = b, D\nu = d + Bu, u \in \{0,1\}^n, \nu \geq 0\} \tag{4.1}$$

where u is the vector of binary decision variables and ν is the vector of continuous decision variables, A,B,D are the matrices, c,g are vectors of the parameters.

Suppose that the problem (4.1) will be easier to solve, perhaps, due to the structure of the parameters or matrices when we fix u variables. Then the corresponding Benders subproblem (SP) for the problem (4.1) will become as follows:

$$z_{P1}(u) = min\{g\nu : D\nu = d + Bu, \nu \geq 0\} \tag{4.2}$$

Taking the dual of problem (4.1) , we obtain:

$$z_{D1}(u) = max\{w(d + Bu) : wD \leq g\} \tag{4.3}$$

Before examining the problems above, let us first define the following useful sets:

$$K_1 = \{u : Au = b, u \in \{0,1\}^n\}$$

$$K_2 = \{u \in K_1 : \exists \nu \geq 0 \quad s.t \quad D\nu = d + Bu\}$$

Some specialities for the problem indicated above as follows:

1. Problem (4.2) is linear for any given value of $u \in K_1$.

2. If Problem (4.2) is unbounded for any value of $u \in K_1$ then the original problem (4.1) will also be unbounded.

3. The feasible region of Problem (4.3) is independent of u. Therefore, dual problem(4.3) is preferred in the Benders formulation. Let π^j be extreme rays for $j = 1, .., n$ and w^p be extreme points for $j = 1, .., m$ for the feasible region of Problem (4.3).

4. If the feasible region of Problem (4.3) is empty, then either Problem (4.2) is unbounded for some $u \in K_1$ that means the original problem (4.1) is also unbounded or the feasible region of Problem (4.2) is empty for all $u \in K_1$ that means the original problem (4.1) is infeasible.

5. If the feasible region of Problem (4.2) is empty for some $\bar{u} \in K_1$, then $u = \bar{u}$ is not a feasible for the original problem (4.1). Therefore, the following problems will be evaluated in order to cut this kind of solution.

$$z_{P2}(u) = min\{e^T w^- + e^T w^+ : D\nu + w^+ - w^- = d + Bu, \nu \geq 0, w^+ \geq 0, w^- \geq 0\} \tag{4.4}$$

$$z_{D2}(u) = max\{\pi(d + Bu) : D^T\pi \leq 0, -e \leq \pi \leq e\} \tag{4.5}$$

   i) If Problem (4.4) has a positive optimal value for a given $\bar{u}$, by the strong duality theorem Problem (4.5) has also a positive optimal value i.e, $\pi(d+B\bar{u}) > 0$ where $\bar{\pi}$ is the optimal solution for the problem (4.5). However, this means that the problem (4.3) is unbounded. To eliminate this solution, we add $\bar{\pi}(d + B\bar{u}) \leq 0$ feasibility cut to the system.

ii) If the optimal value of Problem (4.4) is zero for a given $\bar{u}$, then Problem (4.2) is feasible for $\bar{u}$.

This approach enables us to obtain the following equalities :

$$
\begin{aligned}
& min\{cu + g\nu : Au = b, D\nu = d + Bu, u \in \{0,1\}^n, \nu \geq 0\} \\
& = min_{u\in K_1}\{cu + min\{g\nu : D\nu = d + Bu, \nu \geq 0\}\} \\
& = min_{u\in K_2}\{cu + max\{w(d + Bu) : wD \leq g\}\} \\
& = min_{u\in K_2}\{cu + max_{p=1,..,m}\{w^p(d + Bu)\}\}
\end{aligned}
$$

Set $\eta = max_{p=1,..,m}\{w^p(d + Bu)\}$

Then master problem is obtained as follows:

$$
\begin{aligned}
\min\ & cu + \eta \\
\text{s.t.}\ & \eta \geq w^p(d + Bu) \qquad \forall p = 1,..,m \\
& \pi^j(d + Bu) \leq 0 \qquad \forall j = 1,..,n \\
& Au = b \\
& u \in \{0,1\}^n
\end{aligned}
$$

Next, we provide the Benders decomposition algorithm:

Benders Decomposition Algorithm

Initialization
Let $k \leftarrow 0$
Set $\eta \leftarrow -\infty$
do
Solve the master problem
$k \leftarrow k+1$
Let (u^k, η_k) be the optimal solution of master problem.
if Problem(4.2) for u^k has a finite optimal objective value **then**
Solve Problem (4.3) for u^k and let w^k be the optimal value
if $w^k(d+Bu^k) > \eta_k$ **then**
add the optimality cut $(\eta \geq w^k(d+Bu))$
else the optimal solution is obtained **break**
end if
else Solve Problem (4.5) for u^k and let π^k be the optimal value
add the feasibility cut $(\pi^k(d+Bu) \leq 0)$ to cut u^k
end if
while(true)

4.2 Applying Benders Decomposition Algorithm to Our Problem

From now on, we will focus on how we apply Benders decomposition to our problem.

Firstly, recall the mathematical formulation of our problem stated in Chapter 3:

$$
\begin{aligned}
\min\ & \sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}} \alpha c_{jl} f^i_{jl} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} 2\beta c_{ij} r^k_{ij} + \sum_{i\in I}\sum_{j\in J} 2O_i c_{ij} x_{ij} \\
& + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} g_{ij} y_{ijk} + \sum_{i\in I}\sum_{j\in J} g_{ij} x_{ij} + \sum_{j\in J: j<k}\sum_{k\in J} g_{jk} z_{jk} \\
\text{s.t.}\ & \sum_{j\in J} x_{ij} + \sum_{j\in I\setminus\{i\}}\sum_{k\in J} y_{ijk} \geq 1 \qquad \forall i\in I \\
& \sum_{i\in I\setminus\{j\}} y_{ijk} - \sum_{i\in I\setminus\{j\}} y_{jik} = 0 \qquad \forall j\in I, k\in J \\
& y_{ikk} + y_{kik} \leq 1 \qquad \forall k\in J, i\in I : i\neq k \\
& y_{ijk} \leq x_{kk} \qquad \forall i\in I, j\in I, k\in J : i\neq j \\
& x_{ij} \leq x_{jj} \qquad \forall i\in I, j\in J \\
& \sum_{j\in J} x_{jj} = p \\
& \sum_{l\in J\setminus\{j\}} (f^i_{jl} - f^i_{lj}) = \sum_{m\in I} w_{im} (\sum_{k\in I\setminus\{i\}} y_{ikj}) - \sum_{m\in I\setminus\{j\}} w_{im} (\sum_{k\in I\setminus\{m\}} y_{mkj}) \\
& \qquad\qquad + \sum_{m\in I} w_{im}(x_{ij} - x_{mj}) \quad \forall i\in I, j\in J : i\neq j \\
& \sum_{l\in J\setminus\{j\}} (f^j_{jl} - f^j_{lj}) = \sum_{m\in I} w_{jm} \Big(x_{jj} - x_{mj} - \sum_{k\in I\setminus\{m\}} y_{mkj} \Big) \quad \forall j\in J \\
& \sum_{j\in J\setminus\{i\}} (r^k_{ij} - r^k_{ji}) = O_i \sum_{m\in I\setminus\{i\}} y_{imk} \qquad \forall i\in I, k\in J : i\neq k \\
& 0 \leq r^k_{ij} \leq Q y_{ijk} \qquad \forall i\in I, j\in I, k\in J : i\neq j \\
& f^i_{jl} \geq 0 \qquad \forall i\in I, j\in J, l\in J\setminus\{j\} \\
& x_{ij} \in \{0,1\} \qquad \forall i\in I, j\in J \\
& y_{ijk} \in \{0,1\} \qquad \forall i\in I, j\in I, k\in J : i\neq j \\
& z_{jk} \geq x_{jj} + x_{kk} - 1 \qquad \forall j\in J, k\in J : j<k \\
& z_{jk} \leq x_{jj} \qquad \forall j\in J, k\in J : j<k \\
& z_{jk} \leq x_{kk} \qquad \forall j\in J, k\in J : j<k
\end{aligned}
$$

In the formulation, x, y and z variables correspond to u variable, and f,r variables correspond to ν variable in the Benders formulation provided in Section 4.1. In the classical Benders decomposition algorithm, the master problem is solved to the optimality at each iteration. In our implementations, branch-and-cut framework is used and we separate Benders cuts each time an integer solution is found. Furthermore, LazyConstraintCallback class provided by CPLEX is used to implement Benders Decomposition algorithm in the branch-and-cut framework.

Before starting to explain how to apply Benders Decomposition to our problem, let us define the following useful set and functions:

$\mathcal{X} = \{(x, y) : (3.2) - (3.7), (3.12) - (3.17)\}$

$$\boxed{A_{ij} = \sum_{m \in I} w_{im}(x_{ij} - x_{mj} + \sum_{k \in I \setminus \{i\}} y_{ikj}) - \sum_{m \in I \setminus \{j\}} w_{im}(\sum_{k \in I \setminus \{m\}} y_{mkj}) \quad \forall i \in I, j \in J : i \neq j}$$

$$\boxed{A_{ij} = \sum_{m \in I} w_{im}(x_{ij} - x_{mj} - \sum_{k \in I \setminus \{m\}} y_{mkj}) \qquad \forall i \in I, j \in J : i = j}$$

$$\boxed{B_{ik} = O_i \sum_{m \in I \setminus \{i\}} y_{imk} \qquad \forall i \in I, k \in J : i \neq k}$$

$$\boxed{C_{ijk} = Q y_{ijk} \qquad \forall i \in I, j \in I, k \in J : i \neq j}$$

The functions A,B and C will get the values A^s,B^s and C^s respectively by embedding (x^s, y^s) that is obtained from the solution of the master problem at the s^{th} stage.

Three Benders formulations are proposed. In the first formulation, Benders subproblem can be decomposed into two due to its structure when x, y variables are fixed. These two subproblems enable us to generate aggregated cuts to our master problem. Second formulation is based on partitioning Benders subproblem into small-sized problems. These problems enable us to generate multiple cuts for each demand node $i \in I$ and for each hub node obtained from solution of master problem. In the last formulation, we generate cuts that aim to eliminate subtours and local tours that exceed the capacity along with optimality cuts generated in the first and second formulations. In the following subsections, these formulations are explained in detail.

4.2.1 Aggregated Cuts to Master Problem

The following Benders subproblem at each stage s is obtained when x, y variables are fixed.

(Primal Problem)

$$\min \sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}} \alpha c_{jl} f^i_{jl} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} 2\beta c_{ij} r^k_{ij}$$

$$\text{s.t.} \sum_{l\in J\setminus\{j\}} (f^i_{jl} - f^i_{lj}) = A^s_{ij} \qquad \forall i \in I, j \in J$$

$$\sum_{j\in J\setminus\{i\}} (r^k_{ij} - r^k_{ji}) = B^s_{ik} \qquad \forall i \in I, k \in J : i \neq k$$

$$0 \leq r^k_{ij} \leq C^s_{ijk} \qquad \forall i \in I, j \in I, k \in J : i \neq j$$

$$f^i_{jl} \geq 0 \qquad \forall i \in I, j \in J, l \in J \setminus \{j\}$$

The primal problem can be decomposed into two due to its structure as follows:

$$\textbf{(P1)} \quad \min \sum_{i \in I} \sum_{j \in J} \sum_{l \in J \setminus \{j\}} \alpha c_{jl} f_{jl}^i \tag{4.6}$$

$$\text{s.t.} \sum_{l \in J \setminus \{j\}} (f_{jl}^i - f_{lj}^i) = A_{ij}^s \qquad \forall i \in I, j \in J \tag{4.7}$$

$$f_{jl}^i \geq 0 \qquad \forall i \in I, j \in J, l \in J \setminus \{j\} \tag{4.8}$$

Problem **(P1)** aims to find the flows after projecting (x, y) into the minimum cost hub network problem. This problem is always feasible because $\sum_{i \in I, j \in J} A_{ij}^s = 0$ and our hub network is complete. Therefore, we always obtain optimality cuts from the solution of this problem.

$$\textbf{(P2)} \quad \min \sum_{i \in I \setminus \{j\}} \sum_{j \in I} \sum_{k \in J} 2\beta c_{ij} r_{ij}^k \tag{4.9}$$

$$\text{s.t.} \sum_{j \in J \setminus \{i\}} (r_{ij}^k - r_{ji}^k) = B_{ik}^s \qquad \forall i \in I, k \in J : i \neq k \tag{4.10}$$

$$0 \leq r_{ij}^k \leq C_{ijk}^s \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{4.11}$$

Problem **(P2)** aims to find flows in the local tours established. This problem could be feasible or infeasible. The infeasibility is caused by subtours or by exceeding the capacity.

The duals of Problem **(P1)** and **(P2)** are Problem **(D1)** and **(D2)** respectively as follows:

$$\textbf{(D1)} \quad \max \sum_{i \in I} \sum_{j \in J} A_{ij}^s \pi_{ij} \tag{4.12}$$

$$\text{s.t.} \ \pi_{ij} - \pi_{il} \leq \alpha c_{jl} \qquad \forall i \in I, j \in J, l \in J \setminus \{j\} \tag{4.13}$$

where dual variable π is associated with constraints (4.7)

$$\textbf{(D2)} \quad \max \sum_{i \in I} \sum_{k \in J \setminus \{i\}} B^s_{ik} \phi_{ik} - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} \sum_{k \in J} C^s_{ijk} \psi_{ijk} \tag{4.14}$$

$$\text{s.t. } \phi_{ik} - \phi_{jk} - \psi_{ijk} \leq 2\beta c_{ij} \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{4.15}$$

$$\phi_{kk} = 0 \quad \forall k \in J \tag{4.16}$$

$$\psi_{ijk} \geq 0 \qquad \forall i \in I, j \in I, k \in J : i \neq j \tag{4.17}$$

where dual variables ϕ and ψ are associated with constraints (4.10) and (4.11) respectively.

The master problem becomes:

$$\min \sum_{i \in I} \sum_{j \in J} 2O_i c_{ij} x_{ij} + \sum_{i \in I \setminus \{j\}} \sum_{j \in I} \sum_{k \in J} g_{ij} y_{ijk} + \sum_{i \in I} \sum_{j \in J} g_{ij} x_{ij} + \sum_{j \in J : j < k} \sum_{k \in J} g_{jk} z_{jk} + \eta + \gamma \tag{4.18}$$

$$\text{s.t. } \eta \geq \sum_{i \in I} \sum_{j \in J} A_{ij} \pi_{ij} \quad \forall \pi \in S_1 \tag{4.19}$$

$$\gamma \geq \sum_{i \in I} \sum_{k \in J \setminus \{i\}} B_{ik} \phi_{ik} - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} \sum_{k \in J} C_{ijk} \psi_{ijk} \quad \forall (\phi, \psi) \in T_1 \tag{4.20}$$

$$\sum_{i \in I} \sum_{k \in J \setminus \{i\}} B_{ik} \phi_{ik} - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} \sum_{k \in J} C_{ijk} \psi_{ijk} \leq 0 \quad \forall (\phi, \psi) \in T_2 \tag{4.21}$$

$$(x, y) \in \mathcal{X} \tag{4.22}$$

where S_1 is the set of extreme points of Problem **(D1)** and T_1, T_2 are the sets of extreme points and extreme rays (ϕ,ψ) of Problem **(D2)** respectively.

4.2.2 Multiple Cuts to the Master Problem

We evaluate the following Benders subproblems at each stage s for each node i in the set I in order to obtain multiple cut to the master problem:

$$\textbf{(P1 } i\textbf{)} \quad \min \sum_{j \in J} \sum_{l \in J \setminus \{j\}} \alpha c_{jl} f_{jl}^{i} \tag{4.23}$$

$$\text{s.t.} \quad \sum_{l \in J \setminus \{j\}} (f_{jl}^{i} - f_{lj}^{i}) = A_{ij}^{s} \qquad \forall j \in J \tag{4.24}$$

$$f_{jl}^{i} \geq 0 \qquad \forall j \in J, l \in J \setminus \{j\} \tag{4.25}$$

Moreover, at each iteration we evaluate the following subproblems for each hub node k where $k \in K = \{k \in J : x_{kk}^{s} = 1\}$ to construct multiple cuts:

$$\textbf{(P2 } k\textbf{)} \quad \min \sum_{i \in I \setminus \{j\}} \sum_{j \in I} 2\beta c_{ij} r_{ij}^{k} \tag{4.26}$$

$$\text{s.t.} \quad \sum_{j \in J \setminus \{i\}} (r_{ij}^{k} - r_{ji}^{k}) = B_{ik}^{s} \qquad \forall i \in I : i \neq k \tag{4.27}$$

$$0 \leq r_{ij}^{k} \leq C_{ijk}^{s} \qquad \forall i \in I, j \in I : i \neq j \tag{4.28}$$

$$\tag{4.29}$$

Duals of the above problems are given below:

$$\textbf{(D1 } i\textbf{)} \quad \max \sum_{j \in J} A^s_{ij} \pi_j \tag{4.30}$$

$$\text{s.t. } \pi_j - \pi_l \leq \alpha c_{jl} \qquad j \in J, l \in J \setminus \{j\} \tag{4.31}$$

$$\textbf{(D2 } k\textbf{)} \quad \max \sum_{i \in I \setminus \{k\}} B^s_{ik} \phi_i - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} C^s_{ijk} \psi_{ij} \tag{4.32}$$

$$\text{s.t. } \phi_i - \phi_j - \psi_{ij} \leq 2\beta c_{ij} \qquad \forall i \in I, j \in I : i \neq j \tag{4.33}$$

$$\phi_k = 0 \tag{4.34}$$

$$\psi_{ij} \geq 0 \qquad \forall i \in I, j \in I : i \neq j \tag{4.35}$$

The master problem becomes:

$$\min \sum_{i \in I} \sum_{j \in J} 2O_i c_{ij} x_{ij} + \sum_{i \in I \setminus \{j\}} \sum_{j \in I} \sum_{k \in J} g_{ij} y_{ijk} + \sum_{i \in I} \sum_{j \in J} g_{ij} x_{ij} + \sum_{j \in J : j < k} \sum_{k \in J} g_{jk} z_{jk}$$
$$+ \sum_{i \in I} \eta_i + \sum_{i \in K} \gamma_k \tag{4.36}$$

$$\text{s.t. } \eta_i \geq \sum_{j \in J} A_{ij} \pi^i_j \quad \forall (\pi_i) \in S^i_1, \forall i \in I \tag{4.37}$$

$$\gamma_k \geq \sum_{i \in I \setminus \{k\}} B_{ik} \phi^k_i - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} C_{ijk} \psi^k_{ij} \quad \forall (\phi^k, \psi^k) \in T^k_1, \forall k \in K \tag{4.38}$$

$$\sum_{i \in I \setminus \{k\}} B_{ik} \phi^k_i - \sum_{i \in I} \sum_{j \in I \setminus \{i\}} C_{ijk} \psi^k_{ij} \leq 0 \quad \forall (\phi^k, \psi^k) \in T^k_2, \forall k \in K \tag{4.39}$$

$$(x, y) \in \mathcal{X} \tag{4.40}$$

where S^i_1 is the set of extreme points (π^i) of Problem **(D1** i**)** for each $i \in I$ and T^k_1, T^k_2 are the sets of extreme points and extreme rays (ϕ^k,ψ^k) of Problem **(D2** k**)** for each $k \in K$ respectively.

4.2.3 Benders Decomposition with Special Cuts

As stated before, in our implementations, branch-and-cut framework is used and we devise separation routines for Benders cuts each time an integer solution is found. When an integer solution is found from the master problem, this solution is embedded into our subproblems which could be either feasible or infeasible for fixed x and y variables. In the case where subproblem is infeasible, we add feasibility cuts to the master problem. However, knowing why infeasibility of the subproblems stems from enables us to generate special feasibility cuts to the master problem. As mentioned before, infeasibility is brought about either by subtours or by not satisfying the capacity constraint. Therefore, we strengthen our Benders formulation with valid inequalities which eliminate infeasibility and generate cuts with these valid inequalities to the master problem by detecting the subtours and the local tours that exceed the capacity.

We first find all the tours that we have obtained by solving the master problem in each iteration of Benders Decomposition Algorithm. The algorithm used to find all the tours is given below in Table (4.1). In this algorithm, initially all nodes are marked as 0 and set of hub nodes will removed from the list and their marks will remain 0 throughout algorithm. Afterwards, when the first non-hub node marked as 0 is detected, all the nodes that are assigned to the same local tour with it will be found and be given the same mark 1. Then, the second non-hub nodes marked 0 will be found and again all the nodes that are assigned to the same local tour with it will be found and be given the same mark 2. This will goes on until there is no non-hub node with mark 0 and the number of different marks given is equal to the number of local tours established in the solution of the master problem.

Detect Tours

Initialization
Let $K = \{i \in I : \sum_{j \in J} x_{ij} = 1\}$ be the set singled assigned nodes
Set $mark(i) = 0 \quad \forall i \in I$
Set $component = 0$
for $i \in I$ **do**
if $(mark(i) = 0 \quad and \quad i \notin K)$ **then**
component ++;
SM(i,component);
 	
SM(i,component)
for $k \in \{k \in J : x_{kk} = 1\}$ **do**
for $j \in I$ **do**
if$((y_{ijk} = 1 \quad \|\| \quad y_{jik} = 1), mark(j) = 0 \quad and \quad j \notin K)$ **then**
SM(j,component)
end if
end for
end for

Table 4.1: Tour Detection Algorithm

For each local tour found in the algorithm one of the following outcomes occurs:(i) it is not assigned to a hub node, (ii) it exceeds its capacity and (iii) it is feasible to our problem. Let S_l be the set of arcs that are obtained by the first two outcomes $l = 1, .., n$. In these cases, we add the following cut to the master problem:

$$\sum_{(i,j) \in S_l} y_{ijk} \leq (|S_l| - r(S_l))x_{kk} \quad l = 1, .., n \quad where \quad r(S_l) = \lceil \sum_{i \in S_l} O_i / Q \rceil$$

These cuts eliminate infeasible solutions to our problem. If we encounter the third outcome for all the tours established in the solution of master problem, then we add the optimality cuts to our problem that are generated by the solutions of the subproblems in the subsections (4.2.1) and (4.2.2). Then the master problems will become as follows:

(Master Problem 1)

$$\min \sum_{i\in I}\sum_{j\in J} 2O_i c_{ij} x_{ij} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} g_{ij} y_{ijk} + \sum_{i\in I}\sum_{j\in J} g_{ij} x_{ij} + \sum_{j\in J: j<k}\sum_{k\in J} g_{jk} z_{jk}$$

$$+ \eta + \gamma \tag{4.41}$$

$$\text{s.t. } \eta \geq \sum_{i\in I}\sum_{j\in J} A_{ij}\pi_{ij} \quad \forall \pi \in S_1 \tag{4.42}$$

$$\gamma \geq \sum_{i\in I}\sum_{k\in J\setminus\{i\}} B_{ik}\phi_{ik} - \sum_{i\in I}\sum_{j\in I\setminus\{i\}}\sum_{k\in I} C_{ijk}\psi_{ijk} \quad \forall(\phi,\psi) \in T_1 \tag{4.43}$$

$$\sum_{i\in I}\sum_{k\in J\setminus\{i\}} B_{ik}\phi_{ik} - \sum_{i\in I}\sum_{j\in I\setminus\{i\}}\sum_{k\in J} C_{ijk}\psi_{ijk} \leq 0 \quad \forall(\phi,\psi) \in T_2 \tag{4.44}$$

$$\sum_{(i,j)\in S} y_{ijk} \leq (|S| - r(S))x_{kk} \quad \forall S : |S| \geq 2 \tag{4.45}$$

$$(x, y) \in \mathcal{X} \tag{4.46}$$

where S_1 is the set of extreme points of Problem **(D1)** and T_1, T_2 are the sets of extreme points and extreme rays (ϕ,ψ) of Problem **(D2)** respectively as in the master problem in Subsection (4.2.1) and $r(S) = \lceil \sum_{i\in S} O_i/Q \rceil$

(Master Problem 2)

$$\min \sum_{i\in I}\sum_{j\in J} 2O_i c_{ij} x_{ij} + \sum_{i\in I\setminus\{j\}}\sum_{j\in I}\sum_{k\in J} g_{ij} y_{ijk} + \sum_{i\in I}\sum_{j\in J} g_{ij} x_{ij} + \sum_{j\in J: j<k}\sum_{k\in J} g_{jk} z_{jk}$$

$$+ \sum_{i\in I}\eta_i + \sum_{i\in K}\gamma_k \tag{4.47}$$

$$\text{s.t. } \eta_i \geq \sum_{j\in J} A_{ij}\pi_j^i \quad \forall(\pi_i) \in S_1^i, \forall i \in I \tag{4.48}$$

$$\gamma_k \geq \sum_{i\in I\setminus\{k\}} B_{ik}\phi_i^k - \sum_{i\in I}\sum_{j\in I\setminus\{i\}} C_{ijk}\psi_{ij}^k \quad \forall(\phi^k, \psi^k) \in T_1^k, \forall k \in K \tag{4.49}$$

$$\sum_{i\in I\setminus\{k\}} B_{ik}\phi_i^k - \sum_{i\in I}\sum_{j\in I\setminus\{i\}} C_{ijk}\psi_{ij}^k \leq 0 \quad \forall(\phi^k, \psi^k) \in T_2^k, \forall k \in K \tag{4.50}$$

$$\sum_{(i,j)\in S} y_{ijk} \leq (|S| - r(S))x_{kk} \quad \forall S : |S| \geq 2 \tag{4.51}$$

$$(x, y) \in \chi \tag{4.52}$$

where S_1^i is the set of extreme points (π^i) of Problem **(D1** i**)** for each $i \in I$ and T_1^k, T_2^k are the sets of extreme points and extreme rays (ϕ^k,ψ^k) of Problem **(D2** k**)** for each $k \in K$ respectively as in the master problem in Subsection (4.2.2) and $r(S) = \lceil \sum_{i\in S} O_i/Q \rceil$

Chapter 5

Iterative Clustering-Routing Heuristic

In this chapter, we propose a heuristic algorithm to our problem. The heuristic algorithm has two phases: the clustering phase and the routing phase. Mixed integer programming is used for each phase. The solution of the clustering phase is used in the routing phase of the heuristic algorithm. In the clustering phase, we decide on the locations of p hubs and the allocations of non-hub nodes to the hubs. In the routing phase, we have p subproblems where the hub nodes and the non-hub nodes assigned to each hub are the inputs obtained from the clustering phase. For each hub node, determining whether a non-hub node is assigned directly to the hub or a local tour that completes its tour at the hub is the aim of the routing phase. In the following two sections, the clustering phase and the routing phase will be explained in more detail.

5.1 Clustering Phase

This phase aims to find the locations of p hub nodes and the allocations of non-hub nodes to the hubs by using the mathematical model of *p-hub median problem with single assignment*. We use the formulation of Ernst and Krishnamoorthy (1996) with a slight modification in the objective function (i.e.,adding cost of using arcs to the objective function). We use the formulation of Ernst and Krishnamoorthy as it requires fewer variables and constraints compared to the other formulations developed in the literature.

Decision Variables

x_{ij} : 1 if node i is assigned to hub j and 0 otherwise

f^i_{jl} : flow that originates at node i travels from hub j to hub l

The mathematical model is indicated below:

$$\min \sum_{i\in I}\sum_{j\in J}\sum_{l\in J\setminus\{j\}} \alpha c_{jl} f^i_{jl} + \sum_{i\in I}\sum_{j\in J} 2O_i c_{ij} x_{ij} + \sum_{i\in I}\sum_{j\in J} g_{ij} x_{ij} + \sum_{j\in J:j<k}\sum_{k\in J} g_{jk} z_{jk} \tag{5.1}$$

$$\text{s.t.} \sum_{j\in J} x_{ij} = 1 \qquad \forall i \in I \tag{5.2}$$

$$x_{ij} \le x_{jj} \qquad \forall i \in I, j \in J \tag{5.3}$$

$$\sum_{j\in J} x_{jj} = p \tag{5.4}$$

$$\sum_{l\in J\setminus\{j\}} (f^i_{jl} - f^i_{lj}) = \sum_{m\in I} w_{im}(x_{ij} - x_{mj}) \qquad \forall i \in I, j \in J \tag{5.5}$$

$$f^i_{jl} \ge 0 \qquad \forall i \in I, j \in J, l \in J \setminus \{j\} \tag{5.6}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{5.7}$$

$$(3.15), (3.16), (3.17)$$

The outputs of this model are the locations of p hub nodes and the assignments of non-hub nodes to a hub node. When hub nodes and the assignments of non-hub nodes are decided, we obtain p sub problems which will be solved in the routing phase.

5.2 Routing Phase

The routing phase is the second step of our heuristic algorithm in which the results of the first model is used. Let x^* be the solution of the previous problem and $J^* = \{j \in J : x^*_{jj} = 1\}$ be the set of hub locations. Define $S_j = \{i \in I : x^*_{ij} = 1\}$ for each hub node $j \in J^*$. In this phase, for each $j \in J^*$, a mathematical model is solved in order to allocate the nodes in the set S_j to either a tour that completes its tour at hub j or directly to hub j. This mathematical model is based on the idea of our proposed model in Chapter 3. This time, we know the locations of hubs and the non-hub nodes assigned to each hub node. Therefore, for each hub node j, we decide on local tours and direct connections for the non-hub nodes assigned to this hub.

The decision variables required are:

w_i : 1 if node i is assigned to hub j and 0 otherwise

y_{im} : 1 if node i precedes node m and 0 otherwise

r_{im} : flow travelling from node i to node m

The following mixed integer formulation for each $j \in J^*$ is used to complete the routing phase of the heuristic:

$$\mathbf{P}(S_j) \quad \min \sum_{i\in S_j} 2O_i c_{ij} w_i + \sum_{i\in S_j} g_{ij} w_i + \sum_{i\in S_j\setminus\{m\}} \sum_{m\in S_j} 2\beta c_{im} r_{im} + \sum_{i\in S_j\setminus\{m\}} \sum_{m\in S_j} g_{im} y_{im} \tag{5.8}$$

$$\text{s.t. } w_i + \sum_{m\in S_j\setminus\{i\}} y_{im} = 1 \qquad \forall i \in S_j : i \neq j \tag{5.9}$$

$$w_j + \sum_{m\in S_j\setminus\{j\}} y_{jm} \geq 1 \tag{5.10}$$

$$y_{im} + y_{mi} \leq 1 \qquad \forall i \in S_j, m \in S_j : i < m \tag{5.11}$$

$$\sum_{i\in S_j\setminus\{m\}} y_{im} - \sum_{i\in S_j\setminus\{m\}} y_{mi} = 0 \qquad \forall m \in S_j \tag{5.12}$$

$$\sum_{m\in S_j\setminus\{i\}} (r_{im} - r_{mi}) = O_i \sum_{k\in S_j\setminus\{i\}} y_{ik} \qquad \forall i \in S_j : i \neq j \tag{5.13}$$

$$0 \leq r_{im} \leq Q y_{im} \qquad \forall i \in S_j, m \in S_j : i \neq m \tag{5.14}$$

$$y_{im} \in \{0,1\} \qquad \forall i \in S_j, m \in S_j : i \neq m \tag{5.15}$$

$$w_i \in \{0,1\} \qquad \forall i \in S_j \tag{5.16}$$

The objective function (5.8) minimizes the total transportation cost (i.e., routing cost of flow sent through local tours and sent from single assigned non-hub nodes plus arc-adding cost). Constraints (5.9) and constraints (5.10) ensure that each non-hub node is assigned either directly to the hub j or a local tour. Constraints (5.11) impose that each local tour has at least two nodes other than hub node. Constraints (5.12) and Constraints (5.13) are the flow balance constraints for the traffic on local tours. Constraints (5.14) are the capacity constraints to guarantee that each local tour honors the capacity Q. Constraints (5.15) and Constraints (5.16) are domain restrictions.

Chapter 6

Computational Study

In this study, we design a many-to-many distribution system where we construct a complete hub network with selected p hubs and each non-hub node is connected to the hub network by either direct link or a local tour. Our problem aims to determine jointly the location of p hubs and allocations of each non-hub node to either directly to a hub or to a tour while minimizing the total transportation cost. We propose three solution methodologies for our problem: mathematical formulation, Benders formulations and a heuristic algorithm. Although our mathematical model provides optimal solutions, when the dimensions increase, the time spent to solve the problem becomes too much. Therefore, we propose different solution methods for our problem. To solve the problem exactly, we propose different Benders Decomposition algorithms. However, it takes even more time to solve the problem compared to the mathematical model. To find near optimal solution for the large-scale problem instances in reasonable CPU times, we have developed the iterative clustering-routing heuristic.

In this chapter, we conduct computational studies to test our solution methodologies mentioned above. We code the solution methodologies in Java and ran it on 4XAMD Opreton Interlagos 16C 6282SE 2.6G 16M 6400MT computer. We use CPLEX 12.6 as mixed integer linear programming solver. In addition, we deactivate some of the default cuts used in CPLEX 12.6 since these default cuts increase CPU times according to our analysis on the preliminary computational studies. These cuts are flow path cuts, mixed integer rounding cuts and gomory fractional cuts.

6.1 Data

The performances of our solution methodologies are tested on two data sets commonly used in the hub location literature: CAB and TR. Given parameters in these data sets are the flows and the distances between each origin destination pair. In the data sets, properties of the flows provided are as follows: (i) there is flow between any demand node to every other demand node, (ii) total emanating flow originating from a demand node i is equal to the total amount of flow with destination node i. These two properties are desired properties for our problem settings. In addition, we use the scaled version of the given flow values as it is done in literature. The flows are scaled as follows: $w_{ij}/\sum_{i\in I,j\in I} w_{ij}$ where w_{ij} represents the flow that should be sent from node i to node j and I is the set of nodes. The distances provided are non-negative, symmetric and satisfy the triangle inequality. These properties of distance matrices are also suitable for our problem setting.

We generate instances with 20, 25 and 81 demand nodes. The maps in Figures 6.1 and 6.2 show the locations of demand nodes. We assume that each demand node is a candidate hub where CAB data set is used. However, in the case where TR data set is used, the set of possible hub locations is not the same with the demand set. We used the set of possible hub locations commonly used in the literature. The cardinality of this set is 22 and the locations of these hubs are circled in Figure 6.3.

Figure 6.1: The locations of demand nodes in the CAB data set

The parameter settings are as follows:

For all instances, the sets of discount factors $\alpha = \{0.4, 0.6, 0.8\}$ and $\beta = \{0.4, 0.6, 0.8, 1\}$ where $\alpha \leq \beta$. This relationship between discount factors relies on the perception that carriers used in the hub network causes less cost compared to the ones assigned to local tours, i.e., we assume that a larger carrier is used in the hub-to-hub link, which decreases the number of carriers required for the hub-to-hub links. Moreover, for the discount factor we obtain the following:

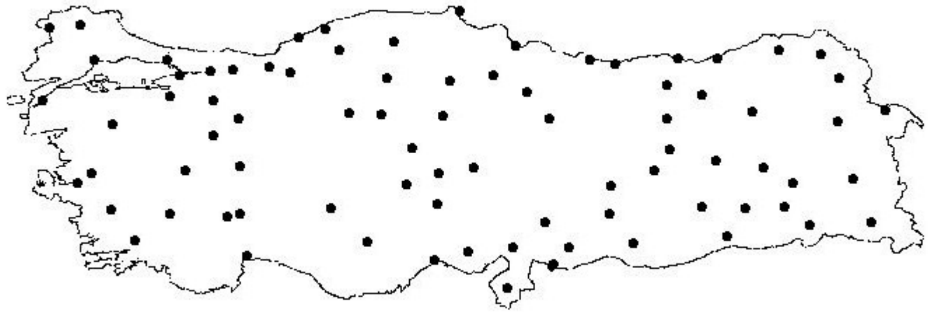

Figure 6.2: The locations of demand nodes in the TR data set

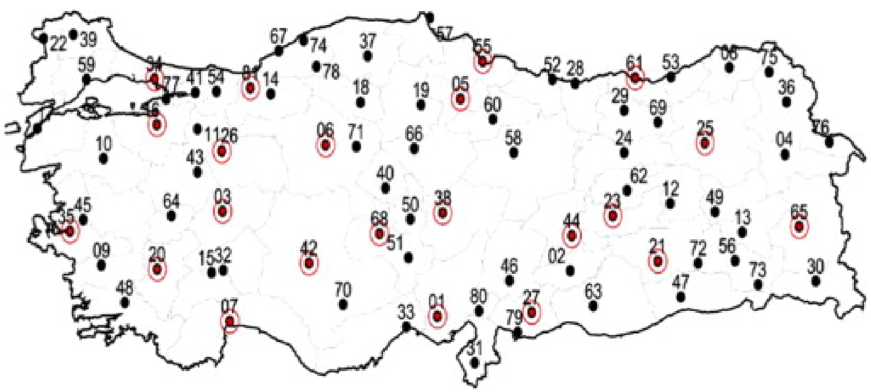

Figure 6.3: Possible hub locations in the TR data set

Remark If $(\alpha, \beta) \in \{((\alpha, \beta) : 0 \leq \alpha \leq 1, \beta = 1)\}$, all the demand nodes are assigned directly to a hub.

Proof Let $i \in I$ and $j \in J$. Let P be the path from the demand node i to the hub j. $\sum_{a \in P} 2\beta O_i c_a \geq 2O_i c_{ij}$ due to triangle inequality, and all other cost incurred in the network will be same. Therefore, in such a parameter setting, all the nodes will be assigned directly to a hub (i.e., there is no local tour established). $\square$

We conducted experiments with $p = \{2, 3, 4, 5\}$ and capacities $Q_1 = 2\sum_{i \in I} O_i/n, Q_2 = 5\sum_{i \in I} O_i/n$ and $Q_3 = 10\sum_{i \in I} O_i/n$ where n is the number of demand nodes.

6.2 Computational Experiments

Before pointing out the computational experiments conducted, sketch of one of the optimal solutions for the CAB instance is provided to depict the possible solution to our problem. Figure 6.4 indicates the optimal solution for the parameters n=25, p=4, $Q = Q_1$, α=0.8, β=0.8. In the optimal solution, $1, 4, 12$ and 17 are chosen as the hub nodes and $3, 5$ and 19 are directly connected to the hubs $17, 4$ and 12 respectively. Local tours are established among other nodes. As it can be seen from the figure, the local tour among the demand nodes $12, 22, 23$ are assigned to the hub 12, the local tours $1-10-16-1$, $1-13-7-1$ and $1-14-24-1$ are initiated from the hub 1, the local tours $4-6-9-4, 4-7-21-4$ and $4-8-11-4$ are initiated from the hub 4, and $17-20-18-17, 17-25-2-17$ are the tours initiated from the hub 17. Sketch of the heuristic solution of the problem under the settings above is depicted in Figure 6.5. In the heuristic solution, the hub nodes are chosen as $1, 4, 12$ and 18. The number of direct connections with the hub nodes increased compared to the optimal solution provided. Local tours and direct connections that are assigned to the hub node $1, 4$ and 12 are the same with the optimal solution. However, the hub node 17 in the optimal solutions is not a hub node in the heuristic solutions. Instead, 18 is chosen as the hub node. $3, 17$ and 20 are directly connected to the hub node 18 and the local tour $18 - 2 - 25 - 18$ are established and connected with the hub node 18. Recall that the first phase of the heuristic where the locations of the hubs are determined does not take routing into consideration. This situation reveals the importance of joint decisions - locating the hubs and routing the flows in the network.

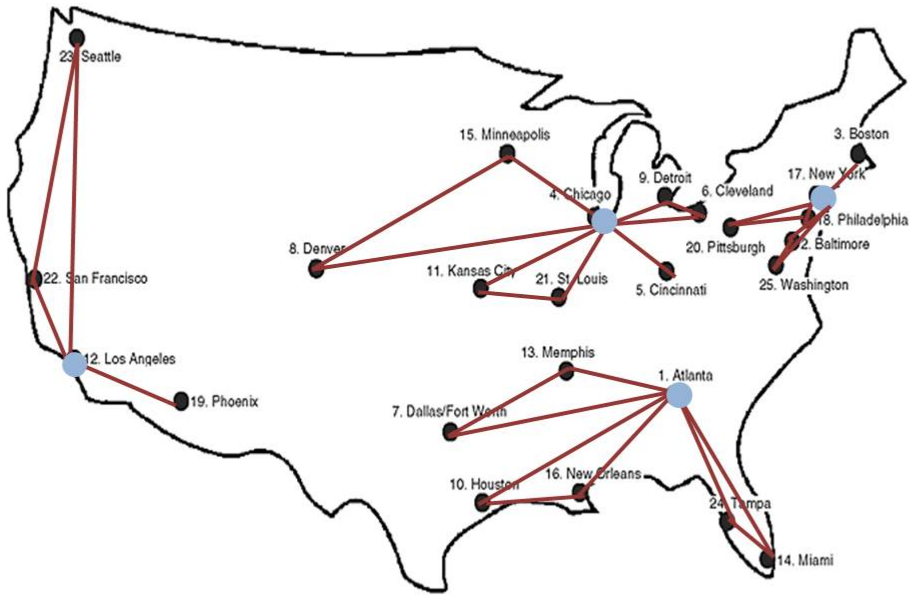

Figure 6.4: Optimal Solution for CAB25 with p=4, Q=0.08, α=0.8, β=0.8

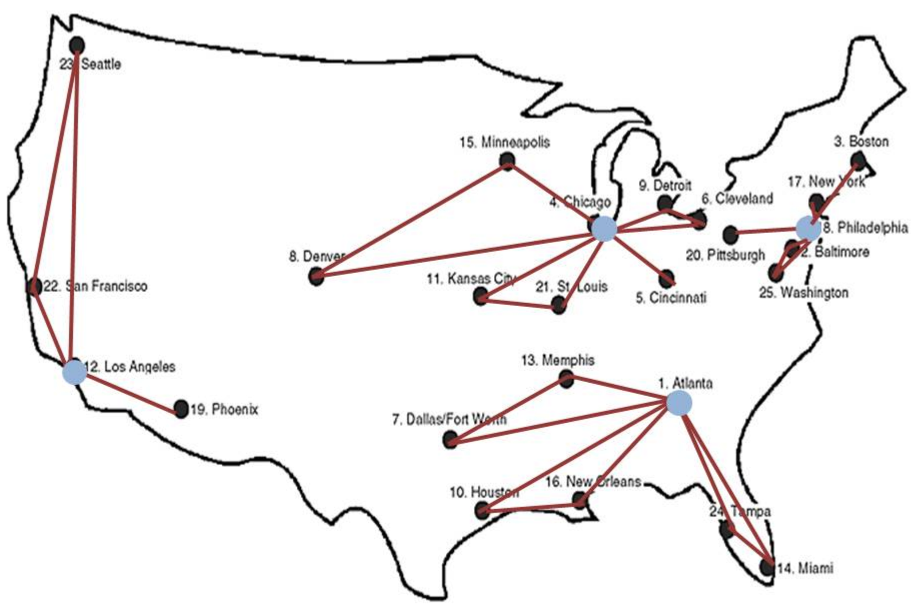

Figure 6.5: Heuristic Solution for CAB25 with p=4, Q=0.08, α=0.8, β=0.8

We first report the performances of our solution methodologies on the CAB data set. We start with the results of the mathematical model. Tables A.1-A.6 in Appendix depict the results of the mathematical model on the CAB data sets for different parameter settings. Tables A.1-A.3 give the result for 20 demand nodes with different capacity values. On the other hand, Tables A.4-A.6 give the results for 25 demand nodes with again different capacity values. In these tables, the first three columns are dedicated to the parameters p, α and β. For each instance, $time, gap, objvalue, \% - cycle, hubs$ and $node$ report solution time, the gap obtain at the end of the solution time, transportation cost, the ratio of the number of demand nodes assigned to local tour to all the demand nodes considering only the non-hub nodes, which is calculated as $\% - cycle = 100 * (n - \sum_{i \in I, j \in J: i \neq j} x_{ij})/(n-p)$, the locations of hub nodes and the number of nodes exploited in the branch-and-bound tree respectively. We impose time limit of five hours for each run. When the limit is exceeded, we report the best solution. Although we impose time limits of five hours, as it can be observed from these tables we solve almost all the instances optimally for the CAB data set with the parameter settings above within two hours.

When we look at the CPU times of the mathematical model in Tables A.1-A.6, we can see that decreasing β - discount factor for local tours- increases the CPU times. The motivation of the factor β is that local tours increase the utilization of the vehicles and decrease the number of vehicles required. Hence, when the factor β decreases, the routing cost of flow through local tours will also decrease and establishing local tours requires trying more combinations compared to the direct connections. That situation probably increases the CPU times of the mathematical model. Another important parameter affecting the CPU times of the model is the capacity of the local tours. Increasing the capacity on the local tours results in high CPU times since it increases the number of possible routes that can be constructed among the local tours.

A closer look at Tables A.1-A.6 reveals that increasing the value of the factor α, in other words, reducing the effect of hubs in the inter-hub connections, causes the high CPU times. When the factor α increases, the hub nodes in the inter-hub network will be probably closer to each other. This situation will probably increase the importance of local tours and more local tours will be tried to be established. Moreover, amount of load on vehicles will be close to the capacity of local tours. This situation increases the integrality gap due to the numbers of possible local tours that can be constructed.

Another important question is whether there is an effect of the change in β on the locations of hubs. The results of Tables A.1-A.6 reveal that some of the instances are affected by the value of the discount factor β. To exemplify, in Table A.3, the locations of the hubs change when the factor β changed for fixed $p = 5$ and $\alpha = 0.4$. This situation proves how decisions of establishing local tours is important.

In addition to the effects of parameters pointed above, p has also greater impact on the CPU times. As is apparent from Tables A.1-A.6, CPU times increase when we increase the number of locations to be opened.

How much discount is made for local tours plays a very crucial role. To observe this relationship, we define the ratio of the number of demand nodes assigned to local tour to all the demand nodes considering only the non-hub nodes as $\% - cycle$ that is stated in Tables A.1-A.6. We construct Tables 6.1 and 6.2 which are the summaries of Tables A.1-A.6 in order to understand easily the effect of the discount factor on the number of nodes assigned to local tours. In these tables, for each α, β and Q values, minimum,

Table 6.1: %-cycle results for CAB20

Parameters			**%-cycle**		
Q	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**minimum**	**average**	**maximum**
Q_1	0.8	0.8	62.50	72.16	88.89
		1	0.00	0.00	0.00
	0.6	0.6	87.50	90.88	94.44
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
	0.4	0.4	93.33	98.33	100.00
		0.6	88.24	92.44	94.44
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
Q_2	0.8	0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
	0.6	0.6	93.33	96.86	100.00
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
	0.4	0.4	100.00	100.00	100.00
		0.6	88.24	91.05	93.75
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
Q_3	0.8	0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
	0.6	0.6	93.33	96.86	100.00
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00
	0.4	0.4	100.00	100.00	100.00
		0.6	93.33	95.30	100.00
		0.8	66.67	78.23	88.89
		1	0.00	0.00	0.00

Table 6.2: %-cycle results for CAB25

Parameters			**%-cycle**		
Q	α	β	**minimum**	**average**	**maximum**
Q_1	0.8	0.8	76.19	79.07	81.82
		1	0.00	0.00	0.00
	0.6	0.6	85.71	88.39	90.91
		0.8	78.26	83.72	90.91
		1	0.00	0.00	0.00
	0.4	0.4	86.96	91.91	95.45
		0.6	85.71	88.39	90.91
		0.8	78.26	83.72	90.91
		1	0.00	0.00	0.00
Q_2	0.8	0.8	85.00	88.18	95.65
		1	0.00	0.00	0.00
	0.6	0.6	90.00	94.04	100.00
		0.8	80.00	84.71	91.30
		1	0.00	0.00	0.00
	0.4	0.4	95.24	98.81	100.00
		0.6	90.00	94.09	95.65
		0.8	80.00	85.80	95.65
		1	0.00	0.00	0.00
Q_3	0.8	0.8	90.00	92.85	100.00
		1	0.00	0.00	0.00
	0.6	0.6	90.00	95.17	100.00
		0.8	81.82	88.24	95.65
		1	0.00	0.00	0.00
	0.4	0.4	95.45	98.86	100.00
		0.6	90.48	94.15	100.00
		0.8	80.00	89.21	100.00
		1	0.00	0.00	0.00

average and maximum of the $\% - cycles$ is calculated considering all possible p values. The first observation is the one stated in the remark above, that each demand node is directly assigned to a hub when $\beta = 1$. However, the case $\beta = 1$ is not realistic when nodes do not have sufficient demand to justify direct connection with hubs. When nodes do not have sufficient demand, establishing local tours decreases the number of vehicles required and increases utilization of vehicles. That generates economies of scale and therefore β should be less than one. However, the case $\beta = 1$ gives us opportunity to observe the results of *p-hub median problem* with our cost structure. The second observation that can be deduced from the tables is that as β increases, the value of $\% - cycle$ decreases for all instances. Finally, as capacity of local tours increases, the value of $\% - cycle$ increases as it is expected. The results of the optimal solutions to our problem indicate how the realistic problem settings are taken in the cost structure.

Tables A.7-A.9 in Appendix depict the results of the mathematical model on the TR data sets for different parameter settings. The meaning of each column in Tables A.7-A.9 is the same with Tables A.7-A.9. We impose two hours time limit for each instance. When the limit is exceeded, we report the best solution. As it can be observed in Tables A.7-A.9, we cannot obtain optimal solutions for the large-scale problem instances generated from TR data set. This situation leads us to develop Benders decomposition and the iterative clustering-routing heuristic in order to be able to solve large-scale instances.

We first report Benders Decomposition Algorithm for large-scale instances. Recall that we formulate three different Benders formulations by (i) adding aggregated cuts to the master problem, (ii) adding multiple cuts to the master problem and (iii) adding valid inequalities that are obtained by checking the subtours or local tours that exceed their capacity to the Benders formulations pointed above. Our aim with this algorithm is to obtain exact solutions in reasonable CPU times. However, the CPU times of the algorithm with all of three approaches stated above are so high. Even the best one where we add aggregated cuts to the master problem takes too much time compared the mathematical model we have proposed in our problem settings. Unfortunately, the Benders algorithm does not work well on our problem. Hence, we only report the results of the Benders formulation that is obtained by adding aggregated cuts to the master problem. In the other formulations, generating too many cuts during the subproblems and adding them into the master problems increase CPU times dramatically. Even though adding cuts are expected to speed up the convergence of the problem, generating many cuts from the subproblems increases the solution times of both subproblems and master problems. Therefore, CPU times in these formulations are a lot higher compared to the mathematical model and aggregated version of Benders formulation.

In Table 6.3, we compare the mathematical model with the Benders formulation where aggregated cuts are added to the master problem. The parameter settings are as follows: $p = 2, Q = Q_1$. The comparisons are made under different discount factors α and β. We report the gap between the objective value of the Benders formulation and the objective value of the mathematical model. When we look closer into Table 6.3, it is observed that mathematical models solve all the instances optimally with CPU times less than six minutes. Unfortunately, the Benders Algorithm solves the problem with a lot higher CPU times. Consequently, although different Benders formulations are developed, these algorithms do not work well on our problem.

Table 6.3: Comparisions of Mathematical Model with the Benders Formulation

Parameters		**Benders Formulation**			**Mathematical Model**		**Comparison**
α	β	**time**	**objvalue**	**# of cuts**	**time**	**objvalue**	**gap**
0.8	0.8	3615.16	998.23	667	251.24	1033.84	3.57
	1	3612.29	1181.41	242	107.78	1181.41	0.00
0.6	0.6	3629.43	739.49	1669	165.35	791.09	6.98
	0.8	3620.56	947.00	961	142.30	970.36	2.47
	1	37.11	1117.93	96	32.64	1117.93	0.00
0.4	0.4	3604.19	511.64	376	119.56	537.87	5.13
	0.6	3600.70	695.58	1637	122.25	727.62	4.61
	0.8	3628.04	892.68	956	35.55	906.88	1.59
	1	36.53	1054.45	65	24.02	1054.45	0.00

Finally, we report the performances of the iterative clustering-routing heuristic on the CAB and TR data sets. Tables A.10-A.18 in Appendix show the results of the heuristic algorithm for different parameter settings. Tables A.10-A.15 give the results on the CAB data set. On the other hand, Tables A.15-A.18 give the results on TR data set. In these tables, the first three columns are dedicated to the parameters p, α and β. For each instance, the results of the mathematical model, results of the heuristic algorithm and their comparisons are reported. To compare the results, the gap between the objective value of the heuristics and the objective value of the mathematical model is calculated. The proposed mathematical model solves almost all instances to optimality in the CAB data set. Therefore, gaps in Tables A.10-A.15 are generally optimality gaps. In the cases where optimal solution cannot be found in the given time limit heuristic results are compared with the best solution found so far. Since the TR data sets cannot be solved optimally, gaps in Tables A.16-A.18 are calculated with the best solution found at the end of time limit.

The most important measure whether our heuristic algorithm works well or not is how close the objective function values of the heuristic algorithm are to the objective function values of mathematical model. Namely, whether we obtain near optimal solution or not determines the quality of the heuristic algorithm. The average gap of 216 instances in the CAB data set is 1.29% and the average of 108 instances in the TR data set is 0.58%. The gap results on the CAB data sets show that near optimal solutions are found with the proposed heuristic algorithm. Besides, the gap results on TR data set are less than one on average since the proposed mathematical model cannot solve instances generated from this data set.

To observe the results on gaps easily, Tables 6.4, 6.5 and Tables 6.6 provide the summaries of Tables A.10-A.18. Decreasing the value of the discount factor

β increases gap since decrease in β results in establishing more local tours to benefit from the economies of scale. Increase in the number of local tours probably affects on the locations of the hub nodes and the allocations of non-hub nodes to hub nodes. Since these two decisions are made in the clustering phase of our algorithm without routing decision, decrease in β increases gap.

Another important measure for heuristic algorithms is whether reasonable CPU times are spent in the solution procedure or not. A closer look into Tables A.10-A.18 indicates that the instances generated from CAB and TR data sets are solved in less than 60 seconds and 20 minutes respectively. These results indicate the efficiency of the proposed heuristic algorithm.

Table 6.4: Heuristic gaps for CAB20

Parameters		**gap**		
$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**minimum**	**average**	**maximum**
0.8	0.8	0.00	2.00	3.64
	1.0	0.00	0.00	0.00
0.6	0.6	0.00	4.59	8.02
	0.8	0.00	0.19	0.64
	1.0	0.00	0.00	0.00
0.4	0.4	0.73	6.15	10.36
	0.6	0.00	1.59	3.52
	0.8	0.00	0.28	0.74
	1.0	0.00	0.00	0.00

Table 6.5: Heuristic gaps for CAB25

Parameters		**gap**		
$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**minimum**	**average**	**maximum**
0.8	0.8	0.00	0.95	1.93
	1	0.00	0.00	0.00
0.6	0.6	0.00	2.24	5.51
	0.8	0.00	0.66	1.81
	1	0.00	0.00	0.00
0.4	0.4	0.33	2.98	6.48
	0.6	0.00	1.35	3.33
	0.8	0.00	0.33	1.24
	1	0.00	0.00	0.00

Table 6.6: Heuristic gaps for TR

Parameters		**gap**		
$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**minimum**	**average**	**maximum**
0.8	0.8	0.37	0.78	1.04
	1	0.36	0.73	0.98
0.6	0.6	0.36	0.99	1.19
	0.8	0.43	0.88	1.06
	1	0.45	0.82	0.99
0.4	0.4	0.40	0.81	1.47
	0.6	0.37	0.91	1.19
	0.8	0.39	0.83	1.06
	1	0.38	0.90	0.99

Chapter 7

Conclusion

In this thesis, we have studied the hub location and routing problem. This problem is closely related to the single allocation hub location problem and multi-depot vehicle routing problem, which are known as difficult problems. Direct connections between non-hub nodes to hub nodes are not justified when nodes do not have sufficient demands. Such direct connections increase the number of vehicles required and decrease the utilizations of vehicles. Hence, it is necessary to construct local tours among the nodes allocated to the same hubs when nodes do not have sufficient demands. However, forcing each hub node to be visited by a local tour is not the best solution to design many-to-many distribution system. Therefore, we give two options for each non-hub node: (i) it could be visited by a local tour or (ii) it could be assigned directly to a hub. Moreover, in the real-life applications, the cost has a lot of components so it is important to find a solution by taking into account a realistic cost structure even if it increases the complexity of the problem. Consequently, our study aims to design a network system by justifying with real world examples.

First, we develop a mixed integer mathematical model to our problem. The proposed model is then strengthened with valid inequalities. We conduct comprehensive computational studies on both CAB and Turkish network data set. In the computational studies, we observe the effect of the capacity of local tours, number of hubs and discount factors on the locations of hubs, CPU times, number of nodes allocated to the local tours and most importantly on the global transportation cost.

Secondly, we develop three different Benders formulations to our problem:(i)we generate aggregated cuts at each iteration, as in the classical Benders procedure; (ii) we generate multiple cuts for each demand node; (iii) we strengthen the first two Benders formulations with valid inequalities that eliminate subtours and the tours exceed the maximum capacity in the iterations where Benders subproblem is infeasible. We code these algorithms on Java and we used CPLEX 12.6 as a mixed integer solver. Although we approach with different Benders formulations to our problem, the CPU times unfortunately are higher compared to the mathematical model we proposed.

In addition to these solution methods, we propose the iterative clustering-routing heuristic to solve larger instances. The proposed heuristic approach has two phases: the clustering phase and the routing phase. Both of the phases are solved by mathematical models. In the first phase, we decide on the locations of hubs and the allocations of non-hub nodes to the hubs. In the clustering phase, we decide on the allocations of each non-hub to either a local tour or a hub and routing among the nodes assigned to the same tour. The average gap of 216 instances in the CAB data set is 1.29%. In addition, the instances generated from CAB and TR data sets are solved in less than 60 seconds and 20 minutes respectively. These results show the efficiency of the proposed heuristic approach.

As possible future directions for our research, the problem can be studied under the stochastic demand and two stage stochastic programming formulation can be developed. As in our hierarchical heuristic, in the first stage, the locations of hubs can be determined and then in the second stage routing on the local tours and direct connection decisions can be made. Moreover, capacities on the hubs and hub-to-hub flows can be added into the model. Also, different vehicle types can be imposed to the model and this enables to apply different capacities for the hub-to-hub links, local tours and direct links between non-hub nodes and hub nodes. The number of vehicles initiated from a hub node can be limited. In the cost point of view, instead of using discount factor β for local tours, the number of vehicles sent from each hub node can be a decision variable and the cost can be calculated based on the number of vehicles initiated from hub nodes.

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Appendix A

Appendix

Table A.1: Results of Mathematical Model for CAB20 with $Q = Q_1$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	251.24	0.0	1033.84	88.89	12;20	16069
		1	107.78	0.0	1181.41	0.00	12;20	19031
	0.6	0.6	165.35	0.0	791.09	94.44	12;20	34555
		0.8	142.30	0.0	970.36	88.89	12;20	18072
		1	32.64	0.0	1117.93	0.00	12;20	1832
	0.4	0.4	119.56	0.0	537.87	100.00	12;20	18845
		0.6	122.25	0.0	727.62	94.44	12;20	20157
		0.8	35.55	0.0	906.88	88.89	12;20	8084
		1	24.02	0.0	1054.45	0.00	12;20	863
3	0.8	0.8	1039.13	0.0	984.60	70.59	4;12;25	198989
		1	147.92	0.0	1102.81	0.00	2;4;12	26612
	0.6	0.6	1281.36	0.0	757.55	88.24	4;12;17	292687
		0.8	376.37	0.0	893.73	82.35	4;12;17	84703
		1	25.99	0.0	984.25	0.00	2;4;12	1474
	0.4	0.4	830.35	0.0	514.60	100.00	4;12;17	201243
		0.6	212.19	0.0	658.69	88.24	4;12;17	40595
		0.8	96.78	0.0	770.50	82.35	4;12;17	14359
		1	15.01	0.0	861.02	0.00	4;12;18	32
4	0.8	0.8	1400.97	0.0	944.53	62.50	1;4;12;17	373907
		1	124.79	0.0	1020.84	0.00	1;4;12;18	16992
	0.6	0.6	3614.57	0.0	730.80	87.50	1;4;12;17	836156
		0.8	900.29	0.0	838.11	75.00	1;4;12;17	250551
		1	38.63	0.0	883.72	0.00	1;4;12;17	2577
	0.4	0.4	2018.82	0.0	495.86	100.00	1;4;12;17	524719
		0.6	363.78	0.0	609.64	93.75	1;4;12;17	86874
		0.8	188.86	0.0	691.63	75.00	1;4;12;17	52720
		1	22.44	0.0	741.66	0.00	1;4;12;17	0
5	0.8	0.8	1087.75	0.0	909.42	66.67	1;4;6;12;17	320720
		1	128.71	0.0	961.77	0.00	1;4;7;12;18	23629
	0.6	0.6	2181.97	0.0	702.00	93.33	1;4;6;12;17	568833
		0.8	507.60	0.0	783.95	66.67	1;4;7;12;17	157602
		1	31.35	0.0	812.47	0.00	4;7;12;14;17	1429
	0.4	0.4	3207.89	0.0	477.92	93.33	1;4;12;17;20	843885
		0.6	528.30	0.0	564.70	93.33	1;4;7;12;17	88025
		0.8	150.22	0.0	624.10	66.67	4;7;12;14;17	23589
		1	12.53	0.0	649.97	0.00	4;7;12;14;17	0

Table A.2: Results of Mathematical Model for CAB20 with $Q = Q_2$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	4642.61	0.0	1023.46	88.89	12;20	970870
		1	164.13	0.0	1181.41	0.00	12;20	26584
	0.6	0.6	2912.69	0.0	777.23	100.00	12;20	574572
		0.8	1138.90	0.0	961.39	88.89	12;20	106864
		1	30.89	0.0	1117.93	0.00	12;20	389
	0.4	0.4	2140.21	0.0	523.71	100.00	12;20	492800
		0.6	1599.10	0.0	721.42	88.89	12;20	229182
		0.8	953.59	0.0	897.91	88.89	12;20	74515
		1	32.66	0.0	1054.45	0.00	12;20	533
3	0.8	0.8	4041.90	0.0	973.34	82.35	4;12;25	635110
		1	454.79	0.0	1102.81	0.00	2;4;12	90332
	0.6	0.6	7208.52	1.2	745.97	94.12	4;12;17	850008
		0.8	3003.24	0.0	893.00	82.35	4;12;17	413345
		1	57.20	0.0	984.25	0.00	2;4;12	16002
	0.4	0.4	7209.94	2.8	503.17	100.00	4;12;17	608368
		0.6	2792.34	0.0	658.14	88.24	4;12;17	475419
		0.8	209.49	0.0	769.77	82.35	4;12;17	47402
		1	21.83	0.0	861.02	0.00	4;12;18	0
4	0.8	0.8	7165.15	0.0	941.56	75.00	1;4;12;17	1602943
		1	247.47	0.0	1020.84	0.00	1;4;12;18	35310
	0.6	0.6	7207.60	1.7	722.41	100.00	1;4;12;17	1397653
		0.8	5014.74	0.0	838.11	75.00	1;4;12;17	757226
		1	38.12	0.0	883.72	0.00	1;4;12;17	2950
	0.4	0.4	7212.18	3.4	487.23	100.00	1;4;12;17	1288725
		0.6	4253.12	0.0	609.64	93.75	1;4;12;17	799333
		0.8	795.00	0.0	691.63	75.00	1;4;12;17	97545
		1	25.01	0.0	741.66	0.00	1;4;12;17	122
5	0.8	0.8	6278.30	0.0	909.40	66.67	1;4;6;12;17	938440
		1	152.88	0.0	961.77	0.00	1;4;7;12;18	17722
	0.6	0.6	7209.75	3.3	703.93	93.33	1;4;6;12;17	1364563
		0.8	2257.70	0.0	783.95	66.67	1;4;7;12;17	344625
		1	43.03	0.0	812.47	0.00	4;7;12;14;17	2754
	0.4	0.4	7217.58	2.6	476.14	100.00	1;4;12;17;20	1367560
		0.6	1360.10	0.0	564.70	93.33	1;4;7;12;17	220213
		0.8	473.55	0.0	624.10	66.67	4;7;12;14;17	75817
		1	18.13	0.0	649.97	0.00	4;7;12;14;17	0

Table A.3: Results of Mathematical Model for CAB20 with $Q = Q_3$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	1921.82	0.0	1011.91	88.89	12;20	301591
		1	129.73	0.0	1181.41	0.00	12;20	22236
	0.6	0.6	1582.98	0.0	768.53	100.00	12;20	221206
		0.8	1329.68	0.0	961.39	88.89	12;20	134201
		1	30.40	0.0	1117.93	0.00	12;20	478
	0.4	0.4	2492.88	0.0	517.86	100.00	12;20	246938
		0.6	1287.98	0.0	719.53	100.00	12;20	293424
		0.8	528.72	0.0	897.91	88.89	12;20	60989
		1	27.81	0.0	1054.45	0.00	12;20	0
3	0.8	0.8	7200.17	0.8	971.01	82.35	4;12;25	642503
		1	609.82	0.0	1102.81	0.00	2;4;12	126147
	0.6	0.6	6892.08	0.0	738.91	94.12	4;12;17	744860
		0.8	4143.84	0.0	892.51	82.35	4;12;17	388524
		1	95.26	0.0	984.25	0.00	2;4;12	11006
	0.4	0.4	5991.26	0.0	498.33	100.00	4;12;17	650511
		0.6	2574.60	0.0	655.13	94.12	4;12;17	300047
		0.8	315.18	0.0	769.77	82.35	4;12;17	56881
		1	24.67	0.0	861.02	0.00	4;12;18	0
4	0.8	0.8	7213.71	1.8	937.55	75.00	1;4;12;17	475885
		1	179.53	0.0	1020.84	0.00	1;4;12;18	28125
	0.6	0.6	7146.55	0.0	714.17	100.00	1;4;12;17	1329925
		0.8	3205.60	0.0	838.11	75.00	1;4;12;17	815169
		1	34.19	0.0	883.72	0.00	1;4;12;17	403
	0.4	0.4	7210.25	1.8	481.58	100.00	1;4;12;17	518844
		0.6	1850.91	0.0	607.97	93.75	1;4;12;17	386368
		0.8	761.96	0.0	691.63	75.00	1;4;12;17	83583
		1	28.36	0.0	741.66	0.00	1;4;12;17	168
5	0.8	0.8	7030.92	0.0	909.40	66.67	1;4;6;12;17	952832
		1	168.43	0.0	961.77	0.00	1;4;7;12;18	20291
	0.6	0.6	7208.84	1.6	695.56	93.33	1;4;6;12;17	632693
		0.8	1140.98	0.0	783.95	66.67	1;4;7;12;17	270175
		1	40.62	0.0	812.47	0.00	4;7;12;14;17	2533
	0.4	0.4	7216.23	2.6	470.38	100.00	1;4;12;17;20	543302
		0.6	1695.46	0.0	562.41	93.33	1;4;7;12;17	185969
		0.8	294.66	0.0	624.10	66.67	4;7;12;14;17	67132
		1	18.52	0.0	649.97	0.00	4;7;12;14;17	0

Table A.4: Results of Mathematical Model for CAB25 with $Q = Q_1$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	8420.98	0.0	1196.95	78.26	12;20	584837
		1	358.33	0.0	1309.89	0.00	12;20	13337
	0.6	0.6	2677.05	0.0	962.07	86.96	12;20	269016
		0.8	783.47	0.0	1104.07	78.26	12;20	93514
		1	58.33	0.0	1217.01	0.00	12;20	1118
	0.4	0.4	574.69	0.0	716.14	86.96	12;20	47169
		0.6	956.90	0.0	869.19	86.96	12;20	57910
		0.8	285.50	0.0	1010.53	78.26	12;20	22567
		1	56.42	0.0	1116.98	0.00	12;20	3289
3	0.8	0.8	18002.74	0.5	1091.92	81.82	4;12;25	1287963
		1	574.02	0.0	1174.23	0.00	2;4;12	51516
	0.6	0.6	3215.45	0.0	835.81	90.91	4;12;17	271018
		0.8	1723.32	0.0	959.74	90.91	4;12;17	155464
		1	302.03	0.0	1048.97	0.00	2;4;12	22715
	0.4	0.4	2909.31	0.0	574.39	95.45	4;12;17	232717
		0.6	1497.61	0.0	699.51	90.91	4;12;17	115032
		0.8	747.07	0.0	823.43	90.91	4;12;17	71780
		1	72.14	0.0	917.38	0.00	4;12;18	8695
4	0.8	0.8	18014.13	2.6	1038.31	76.19	1;4;12;17	1045047
		1	490.41	0.0	1104.13	0.00	1;4;12;18	32407
	0.6	0.6	11592.90	0.0	794.21	85.71	1;4;12;17	886392
		0.8	1597.87	0.0	886.82	85.71	1;4;12;17	105336
		1	157.35	0.0	956.00	0.00	1;4;12;17	14065
	0.4	0.4	17234.15	0.0	541.58	95.24	1;4;12;17	1158764
		0.6	824.13	0.0	642.52	85.71	1;4;12;17	50983
		0.8	436.76	0.0	735.13	85.71	1;4;12;17	24762
		1	61.92	0.0	804.31	0.00	1;4;12;17	145
5	0.8	0.8	18002.21	0.8	1003.87	80.00	1;4;6;12;17	1487264
		1	545.15	0.0	1053.19	0.00	1;4;7;12;18	23669
	0.6	0.6	18009.67	2.2	770.15	90.00	1;4;6;12;17	1347701
		0.8	2454.21	0.0	850.86	80.00	1;4;7;12;17	236996
		1	194.96	0.0	897.73	0.00	4;7;12;14;17	16289
	0.4	0.4	18016.53	3.4	529.08	90.00	1;4;12;17;20	1203638
		0.6	1827.35	0.0	616.11	90.00	1;4;7;12;17	145411
		0.8	519.47	0.0	686.73	80.00	4;7;12;14;17	37185
		1	49.46	0.0	728.83	0.00	4;7;12;14;17	257

Table A.5: Results of Mathematical Model for CAB25 with $Q = Q_2$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	18015.51	3.0	1145.61	95.65	12;20	1398530
		1	626.52	0.0	1309.89	0.00	12;20	19262
	0.6	0.6	18016.71	2.6	866.50	100.00	12;20	1123555
		0.8	18000.37	0.1	1065.58	91.30	12;20	2801159
		1	198.95	0.0	1217.01	0.00	12;20	11453
	0.4	0.4	18019.76	4.4	593.61	100.00	12;20	1260617
		0.6	4325.63	0.0	785.91	95.65	12;20	724192
		0.8	4539.62	0.0	972.91	95.65	12;20	632673
		1	103.05	0.0	1116.98	0.00	12;20	11340
3	0.8	0.8	18037.06	2.3	1076.55	86.36	4;12;25	836602
		1	699.67	0.0	1174.23	0.00	2;4;12	54475
	0.6	0.6	18028.73	4.9	817.74	90.91	4;12;17	174866
		0.8	18011.32	1.0	952.65	81.82	4;12;17	981239
		1	522.82	0.0	1048.97	0.00	2;4;12	29583
	0.4	0.4	18031.82	5.5	557.28	100.00	4;12;17	201516
		0.6	18000.28	0.4	692.87	95.45	4;12;17	1129101
		0.8	11811.74	0.0	821.14	81.82	4;12;17	828816
		1	168.00	0.0	917.38	0.00	4;12;18	12381
4	0.8	0.8	18016.98	2.5	1028.32	85.71	1;4;12;17	879435
		1	754.07	0.0	1104.13	0.00	1;4;12;18	30363
	0.6	0.6	18023.85	1.0	778.86	95.24	1;4;12;17	986856
		0.8	5766.41	0.0	886.82	85.71	1;4;12;17	506747
		1	324.21	0.0	956.00	0.00	1;4;12;17	13524
	0.4	0.4	18030.61	3.0	529.31	95.24	1;4;12;25	780277
		0.6	6752.81	0.0	641.92	95.24	1;4;12;25	538829
		0.8	2789.87	0.0	735.13	85.71	1;4;12;17	120110
		1	85.00	0.0	804.31	0.00	1;4;12;17	509
5	0.8	0.8	18023.48	4.7	1002.75	85.00	1;4;6;12;17	310863
		1	771.82	0.0	1053.19	0.00	1;4;7;12;18	30773
	0.6	0.6	18031.82	3.4	762.70	90.00	1;4;6;12;25	982529
		0.8	16437.86	0.0	850.86	80.00	1;4;7;12;17	1100298
		1	254.69	0.0	897.73	0.00	4;7;12;14;17	14715
	0.4	0.4	18045.91	4.8	524.20	100.00	1;2;4;7;8	931972
		0.6	18012.33	1.6	615.46	90.00	1;4;6;12;17	626137
		0.8	1899.85	0.0	686.73	80.00	4;7;12;14;17	149807
		1	70.93	0.0	728.83	0.00	4;7;12;14;17	1241

Table A.6: Results of Mathematical Model for CAB25 with $Q = Q_3$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	18025.03	2.7	1137.23	100.00	12;20	386329
		1	2278.52	0.0	1309.89	0.00	12;20	63559
	0.6	0.6	18016.95	1.9	858.66	100.00	12;20	1173959
		0.8	4895.30	0.0	1054.44	95.65	12;20	286946
		1	782.27	0.0	1217.01	0.00	12;20	10867
	0.4	0.4	18044.86	2.6	582.68	100.00	12;20	477753
		0.6	9441.53	0.0	775.29	95.65	12;20	715027
		0.8	1894.60	0.0	961.00	100.00	12;20	108399
		1	641.41	0.0	1116.98	0.00	12;20	11279
3	0.8	0.8	18032.11	2.7	1065.64	90.91	4;12;25	231914
		1	3044.90	0.0	1174.23	0.00	2;4;12	114149
	0.6	0.6	18027.91	2.0	807.29	95.45	4;12;17	710806
		0.8	15504.38	0.0	945.02	81.82	4;12;17	555417
		1	1911.04	0.0	1048.97	0.00	2;4;12	77576
	0.4	0.4	18027.00	3.4	550.95	95.45	4;12;17	230761
		0.6	9245.05	0.0	685.82	95.45	4;12;17	409151
		0.8	4594.54	0.0	815.35	86.36	4;12;17	299798
		1	723.08	0.0	917.38	0.00	4;12;18	30909
4	0.8	0.8	18026.58	3.3	1030.42	90.48	1;4;12;17	284966
		1	2332.06	0.0	1104.13	0.00	1;4;12;18	85241
	0.6	0.6	18030.41	4.8	785.72	95.24	1;4;12;17	274179
		0.8	10430.26	0.0	881.46	90.48	1;4;12;17	250508
		1	716.53	0.0	956.00	0.00	1;4;12;17	12702
	0.4	0.4	18029.79	5.1	534.87	100.00	1;4;12;17	249256
		0.6	3247.30	0.0	633.94	90.48	1;4;12;17	136428
		0.8	2821.65	0.0	734.91	90.48	1;4;12;17	102838
		1	521.82	0.0	804.31	0.00	1;4;12;17	12772
5	0.8	0.8	18027.56	3.0	994.07	90.00	1;4;6;12;17	309076
		1	1239.23	0.0	1053.19	0.00	1;4;7;12;18	35179
	0.6	0.6	18029.45	3.4	755.23	90.00	1;4;6;12;17	279345
		0.8	17745.85	0.0	845.76	85.00	1;4;7;12;17	538551
		1	614.98	0.0	897.73	0.00	4;7;12;14;17	18843
	0.4	0.4	18028.94	5.4	520.05	100.00	1;4;12;17;20	257742
		0.6	10198.89	0.0	607.68	95.00	1;4;7;12;17	392554
		0.8	3870.59	0.0	686.73	80.00	4;7;12;14;17	152810
		1	171.91	0.0	728.83	0.00	4;7;12;14;17	5553

Table A.7: Results of Mathematical Model for TR with $Q = Q_1$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	7210.23	7.46	637.34	81.01	26;38	12672
		1	7210.86	100.00	1138.65	0.00	6;7	0
	0.6	0.6	7209.40	5.78	516.63	89.87	26;38	14329
		0.8	7211.17	7.90	609.59	82.28	26;38	10852
		1	7209.07	3.01	639.88	27.85	6;44	14532
	0.4	0.4	7208.70	56.50	865.35	0.00	6;7	64
		0.6	7211.53	4.98	482.34	89.87	26;38	4871
		0.8	7267.08	2.76	544.42	87.34	26;38	16324
		1	7213.55	2.91	602.93	17.72	6;44	17813
3	0.8	0.8	7211.77	7.78	601.21	67.95	6;26;38	9768
		1	7218.05	5.29	640.89	16.67	1;6;23	4657
	0.6	0.6	7211.24	6.26	480.28	87.18	3;34;68	4320
		0.8	7208.58	100.00	1241.00	0.00	6;7;35	0
		1	7214.75	4.95	594.50	30.77	3;34;38	8465
	0.4	0.4	7210.17	62.14	896.42	7.69	6;7;35	30
		0.6	7216.58	8.68	449.90	85.90	3;34;68	11374
		0.8	7209.40	10.61	524.82	78.21	5;26;27	9112
		1	7210.50	4.27	538.75	23.08	6;16;44	4674
4	0.8	0.8	7211.32	7.52	575.69	70.13	3;6;38;81	11986
		1	7212.05	6.20	612.43	25.97	1;3;6;23	4416
	0.6	0.6	7215.53	10.00	477.99	80.52	3;5;34;68	9234
		0.8	7211.78	29.60	686.12	5.19	6;7;34;35	6
		1	7213.66	5.96	554.54	20.78	3;6;27;34	8138
	0.4	0.4	7211.04	8.82	351.59	88.31	3;6;34;38	9168
		0.6	7210.66	8.73	415.96	84.42	3;5;27;34	855
		0.8	7290.88	6.46	456.49	75.32	3;5;27;34	11752
		1	7211.11	2.36	486.18	31.17	3;5;27;34	12604
5	0.8	0.8	7213.28	9.56	570.79	69.74	1;3;6;34;38	34
		1	7214.43	5.90	585.70	21.05	1;3;6;23;34	2718
	0.6	0.6	7234.38	7.19	447.03	78.95	6;26;34;35;38	14314
		0.8	7211.46	7.57	498.61	48.68	1;3;6;23;34	10805
		1	7212.47	3.73	511.29	23.68	1;3;6;23;34	11704
	0.4	0.4	7209.72	100.00	993.64	0.00	6;7;34;35;42	0
		0.6	7210.49	36.59	565.50	21.05	6;7;34;35;42	44
		0.8	7210.55	5.59	425.64	46.05	1;3;6;23;34	19480
		1	7209.90	3.98	441.94	15.79	1;6;23;34;35	10423

Table A.8: Results of Mathematical Model for TR with $Q = Q_2$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	7211.96	48.47	1064.19	8.86	34;35	0
		1	7210.83	56.02	1456.58	0.00	34;35	0
	0.6	0.6	7211.33	19.76	529.75	97.47	6;34	4552
		0.8	7211.37	6.03	561.21	96.20	26;38	9915
		1	7210.46	100.00	1327.59	0.00	34;35	0
	0.4	0.4	7210.71	100.00	1198.60	0.00	34;35	0
		0.6	7210.59	100.00	1198.60	0.00	34;35	0
		0.8	7210.40	13.27	570.61	94.94	16;38	0
		1	7208.51	16.63	694.30	39.24	6;21	7238
3	0.8	0.8	7211.59	44.57	939.55	12.82	34;35;42	0
		1	7211.38	31.95	881.97	14.10	34;35;42	9
	0.6	0.6	7215.73	13.90	461.56	92.31	6;34;38	981
		0.8	7212.71	30.25	692.31	62.82	34;35;42	38
		1	7211.88	27.18	768.17	89.74	6;16;44	23
	0.4	0.4	7209.44	25.56	361.37	98.72	6;34;38	0
		0.6	7210.94	18.78	439.64	94.87	6;34;44	438
		0.8	7210.70	48.62	850.37	17.95	34;35;42	0
		1	7216.41	11.08	574.35	62.82	6;34;44	30
4	0.8	0.8	7209.90	52.83	1060.07	24.68	16;23;61;65	0
		1	7213.21	36.77	901.28	44.16	16;23;61;65	8
	0.6	0.6	7209.93	69.25	1234.84	19.48	16;23;61;65	0
		0.8	7209.11	16.43	541.97	88.31	3;23;34;38	14
		1	7212.46	16.39	617.58	71.43	3;6;34;44	41
	0.4	0.4	7210.36	77.66	1150.23	19.48	16;23;61;65	0
		0.6	7209.93	59.87	823.51	45.45	16;23;61;65	3
		0.8	7209.92	50.42	801.53	42.86	16;23;61;65	0
		1	7210.47	15.85	544.69	63.64	1;6;16;23	22
5	0.8	0.8	7209.77	28.05	676.84	80.26	1;3;5;6;34	0
		1	7208.86	21.00	692.03	40.79	1;3;5;34;81	0
	0.6	0.6	7211.95	34.57	563.74	90.79	1;3;5;6;34	0
		0.8	7209.06	33.50	649.21	57.89	1;3;5;34;81	0
		1	7211.38	12.23	555.82	57.89	1;3;5;23;34	17
	0.4	0.4	7213.27	37.52	400.25	92.11	1;3;5;6;34	1
		0.6	7211.62	19.19	386.44	96.05	1;3;6;23;34	17
		0.8	7209.32	35.38	571.89	85.53	1;3;6;23;34	0
		1	7211.81	20.60	530.25	61.84	1;3;5;21;34	14

Table A.9: Results of Mathematical Model for TR with $Q = Q_3$

p	α	β	time	gap (%)	objvalue	%-cycle	hubs	node
2	0.8	0.8	7211.38	53.51	1168.87	32.91	34;65	23
		1	7210.83	100.00	1740.11	0.00	34;65	0
	0.6	0.6	7211.46	100.00	1602.52	0.00	34;65	0
		0.8	7210.27	62.39	1385.58	21.52	34;65	0
		1	7211.92	26.11	826.85	96.20	6;44	5860
	0.4	0.4	7212.25	20.31	354.80	100.00	6;34	3580
		0.6	7210.24	100.00	1464.94	0.00	34;65	0
		0.8	7210.06	65.44	1417.36	0.00	34;65	8148
		1	7209.51	24.06	758.74	0.00	6;26	6942
3	0.8	0.8	7211.08	59.85	1286.78	19.23	34;35;65	0
		1	7210.51	53.79	1294.11	14.10	34;35;65	9
	0.6	0.6	7211.03	66.55	1173.00	28.21	34;35;65	1
		0.8	7210.79	60.61	1214.84	19.23	34;35;65	0
		1	7210.53	48.02	1072.71	20.51	34;35;65	30
	0.4	0.4	7209.38	74.73	1048.32	29.49	34;35;65	0
		0.6	7210.14	100.00	1364.95	0.00	34;35;65	0
		0.8	7212.92	24.00	569.73	97.44	6;34;44	15
		1	7209.26	56.52	1168.69	3.85	34;35;65	0
4	0.8	0.8	7211.18	100.00	1597.22	0.00	34;35;38;65	0
		1	7209.60	49.75	1128.60	35.06	34;35;38;65	0
	0.6	0.6	7209.81	100.00	1424.19	0.00	34;35;38;65	0
		0.8	7208.93	53.42	963.39	61.04	34;35;38;65	0
		1	7212.85	50.03	1029.11	42.86	34;35;38;65	0
	0.4	0.4	7209.74	61.74	662.87	77.92	34;35;38;65	0
		0.6	7213.66	55.35	730.41	63.64	34;35;38;65	0
		0.8	7212.20	55.99	892.53	48.05	34;35;38;65	0
		1	7211.01	49.06	896.56	37.66	34;35;38;65	0
5	0.8	0.8	7214.30	50.33	974.27	44.74	34;35;38;42;65	0
		1	7210.11	46.29	1014.70	27.63	34;35;38;42;65	0
	0.6	0.6	7211.70	56.28	838.59	53.95	34;35;38;42;65	0
		0.8	7208.75	48.39	830.84	51.32	34;35;38;42;65	0
		1	7208.41	44.68	877.86	35.53	34;35;38;42;65	0
	0.4	0.4	7212.93	60.53	627.70	76.32	34;35;38;42;65	0
		0.6	7210.95	55.65	697.97	57.89	34;35;38;42;65	0
		0.8	7209.24	55.16	817.89	35.53	34;35;38;42;65	0
		1	7209.37	62.70	1121.45	0.00	34;35;38;42;65	0

Table A.10: Results of Heuristic for CAB20 with $Q = Q_1$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	251.24	0.0	1033.84	1.62	1033.84	0.00
		1	107.78	0.0	1181.41	1.31	1181.41	0.00
	0.6	0.6	165.35	0.0	791.09	1.49	791.09	0.00
		0.8	142.30	0.0	970.36	1.44	970.36	0.00
		1	32.64	0.0	1117.93	1.08	1117.93	0.00
	0.4	0.4	119.56	0.0	537.87	1.04	541.80	0.73
		0.6	122.25	0.0	727.62	1.09	727.62	0.00
		0.8	35.55	0.0	906.88	1.12	906.88	0.00
		1	24.02	0.0	1054.45	0.74	1054.45	0.00
3	0.8	0.8	1039.13	0.0	984.60	5.36	1007.04	2.28
		1	147.92	0.0	1102.81	5.21	1102.81	0.00
	0.6	0.6	1281.36	0.0	757.55	1.84	781.92	3.22
		0.8	376.37	0.0	893.73	1.70	893.73	0.00
		1	25.99	0.0	984.25	1.68	984.25	0.00
	0.4	0.4	830.35	0.0	514.60	0.93	540.32	5.00
		0.6	212.19	0.0	658.69	0.95	658.69	0.00
		0.8	96.78	0.0	770.50	0.86	770.50	0.00
		1	15.01	0.0	861.02	0.78	861.02	0.00
4	0.8	0.8	1400.97	0.0	944.53	6.18	970.69	2.77
		1	124.79	0.0	1020.84	6.17	1020.84	0.00
	0.6	0.6	3614.57	0.0	730.80	1.49	771.44	5.56
		0.8	900.29	0.0	838.11	1.48	838.83	0.09
		1	38.63	0.0	883.72	1.51	883.72	0.00
	0.4	0.4	2018.82	0.0	495.86	1.40	525.64	6.01
		0.6	363.78	0.0	609.64	1.25	629.38	3.24
		0.8	188.86	0.0	691.63	1.12	696.77	0.74
		1	22.44	0.0	741.66	0.98	741.66	0.00
5	0.8	0.8	1087.75	0.0	909.42	5.88	921.28	1.30
		1	128.71	0.0	961.77	5.65	961.77	0.00
	0.6	0.6	2181.97	0.0	702.00	1.97	742.11	5.71
		0.8	507.60	0.0	783.95	1.92	789.00	0.64
		1	31.35	0.0	812.47	1.99	812.47	0.00
	0.4	0.4	3207.89	0.0	477.92	1.04	519.09	8.61
		0.6	528.30	0.0	564.70	1.00	579.61	2.64
		0.8	150.22	0.0	624.10	0.98	626.49	0.38
		1	12.53	0.0	649.97	0.89	649.97	0.00

Table A.11: Results of Heuristic for CAB20 with $Q = Q_2$

Parameters			Mathematical Model			Heuristic		Comparison
p	**α**	**β**	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	4642.61	0.0	1023.46	1.75	1024.87	0.14
		1	164.13	0.0	1181.41	1.31	1181.41	0.00
	0.6	0.6	2912.69	0.0	777.23	1.49	784.90	0.99
		0.8	1138.90	0.0	961.39	1.46	961.39	0.00
		1	30.89	0.0	1117.93	1.12	1117.93	0.00
	0.4	0.4	2140.21	0.0	523.71	1.23	531.58	1.50
		0.6	1599.10	0.0	721.42	1.10	721.42	0.00
		0.8	953.59	0.0	897.91	1.12	897.91	0.00
		1	32.66	0.0	1054.45	0.75	1054.45	0.00
3	0.8	0.8	4041.90	0.0	973.34	5.27	1006.31	3.39
		1	454.79	0.0	1102.81	5.16	1102.81	0.00
	0.6	0.6	7208.52	1.2	745.97	1.77	781.37	4.75
		0.8	3003.24	0.0	893.00	1.76	893.00	0.00
		1	57.20	0.0	984.25	1.65	984.25	0.00
	0.4	0.4	7209.94	2.8	503.17	0.88	533.11	5.95
		0.6	2792.34	0.0	658.14	0.84	658.14	0.00
		0.8	209.49	0.0	769.77	0.92	769.77	0.00
		1	21.83	0.0	861.02	0.71	861.02	0.00
4	0.8	0.8	7165.15	0.0	941.56	6.22	970.69	3.09
		1	247.47	0.0	1020.84	5.78	1020.84	0.00
	0.6	0.6	7207.60	1.7	722.41	1.52	771.44	6.79
		0.8	5014.74	0.0	838.11	1.47	838.83	0.09
		1	38.12	0.0	883.72	1.35	883.72	0.00
	0.4	0.4	7212.18	3.4	487.23	1.11	525.64	7.88
		0.6	4253.12	0.0	609.64	1.09	629.38	3.24
		0.8	795.00	0.0	691.63	1.06	696.77	0.74
		1	25.01	0.0	741.66	0.99	741.66	0.00
5	0.8	0.8	6278.30	0.0	909.40	5.63	921.28	1.31
		1	152.88	0.0	961.77	5.69	961.77	0.00
	0.6	0.6	7209.75	3.3	703.93	2.02	742.11	5.42
		0.8	2257.70	0.0	783.95	1.97	789.00	0.64
		1	43.03	0.0	812.47	1.83	812.47	0.00
	0.4	0.4	7217.58	2.6	476.14	1.04	519.09	9.02
		0.6	1360.10	0.0	564.70	1.06	579.61	2.64
		0.8	473.55	0.0	624.10	0.96	626.49	0.38
		1	18.13	0.0	649.97	0.91	649.97	0.00

Table A.12: Results of Heuristic for CAB20 with $Q = Q_3$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	1921.82	0.0	1011.91	1.72	1024.87	1.28
		1	129.73	0.0	1181.41	1.32	1181.41	0.00
	0.6	0.6	1582.98	0.0	768.53	1.47	784.90	2.13
		0.8	1329.68	0.0	961.39	1.33	961.39	0.00
		1	30.40	0.0	1117.93	1.10	1117.93	0.00
	0.4	0.4	2492.88	0.0	517.86	1.03	531.58	2.65
		0.6	1287.98	0.0	719.53	1.03	721.42	0.26
		0.8	528.72	0.0	897.91	0.97	897.91	0.00
		1	27.81	0.0	1054.45	0.74	1054.45	0.00
3	0.8	0.8	7200.17	0.8	971.01	5.31	1006.31	3.64
		1	609.82	0.0	1102.81	4.97	1102.81	0.00
	0.6	0.6	6892.08	0.0	738.91	1.97	781.37	5.75
		0.8	4143.84	0.0	892.51	1.76	893.00	0.06
		1	95.26	0.0	984.25	1.61	984.25	0.00
	0.4	0.4	5991.26	0.0	498.33	0.92	533.11	6.98
		0.6	2574.60	0.0	655.13	0.96	658.14	0.46
		0.8	315.18	0.0	769.77	0.95	769.77	0.00
		1	24.67	0.0	861.02	0.70	861.02	0.00
4	0.8	0.8	7213.71	1.8	937.55	6.03	970.69	3.53
		1	179.53	0.0	1020.84	5.87	1020.84	0.00
	0.6	0.6	7146.55	0.0	714.17	1.68	771.44	8.02
		0.8	3205.60	0.0	838.11	1.45	838.83	0.09
		1	34.19	0.0	883.72	1.32	883.72	0.00
	0.4	0.4	7210.25	1.8	481.58	1.09	525.64	9.15
		0.6	1850.91	0.0	607.97	1.11	629.38	3.52
		0.8	761.96	0.0	691.63	1.07	696.77	0.74
		1	28.36	0.0	741.66	1.00	741.66	0.00
5	0.8	0.8	7030.92	0.0	909.40	5.95	921.28	1.31
		1	168.43	0.0	961.77	5.61	961.77	0.00
	0.6	0.6	7208.84	1.6	695.56	2.14	742.11	6.69
		0.8	1140.98	0.0	783.95	1.95	789.00	0.64
		1	40.62	0.0	812.47	1.99	812.47	0.00
	0.4	0.4	7216.23	2.6	470.38	1.04	519.09	10.36
		0.6	1695.46	0.0	562.41	1.13	579.61	3.06
		0.8	294.66	0.0	624.10	0.99	626.49	0.38
		1	18.52	0.0	649.97	0.92	649.97	0.00

Table A.13: Results of Heuristic for CAB25 with $Q = Q_1$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	8420.98	0.0	1196.95	2.89	1196.95	0.00
		1	358.33	0.0	1309.89	2.57	1309.89	0.00
	0.6	0.6	2677.05	0.0	962.07	2.45	962.07	0.00
		0.8	783.47	0.0	1104.07	1.95	1104.07	0.00
		1	58.33	0.0	1217.01	1.65	1217.01	0.00
	0.4	0.4	574.69	0.0	716.14	1.91	718.47	0.33
		0.6	956.90	0.0	869.19	1.69	869.91	0.08
		0.8	285.50	0.0	1010.53	1.50	1010.53	0.00
		1	56.42	0.0	1116.98	1.17	1116.98	0.00
3	0.8	0.8	18002.74	0.5	1091.92	12.86	1102.37	0.96
		1	574.02	0.0	1174.23	13.11	1174.23	0.00
	0.6	0.6	3215.45	0.0	835.81	2.85	862.89	3.24
		0.8	1723.32	0.0	959.74	3.04	977.10	1.81
		1	302.03	0.0	1048.97	2.67	1048.97	0.00
	0.4	0.4	2909.31	0.0	574.39	2.09	598.10	4.13
		0.6	1497.61	0.0	699.51	2.05	714.60	2.16
		0.8	747.07	0.0	823.43	2.12	828.34	0.60
		1	72.14	0.0	917.38	1.79	917.38	0.00
4	0.8	0.8	18014.13	2.6	1038.31	12.83	1040.74	0.23
		1	490.41	0.0	1104.13	12.74	1104.13	0.00
	0.6	0.6	11592.90	0.0	794.21	4.22	794.21	0.00
		0.8	1597.87	0.0	886.82	4.36	886.82	0.00
		1	157.35	0.0	956.00	3.95	956.00	0.00
	0.4	0.4	17234.15	0.0	541.58	2.12	549.92	1.54
		0.6	824.13	0.0	642.52	1.92	642.52	0.00
		0.8	436.76	0.0	735.13	1.87	735.13	0.00
		1	61.92	0.0	804.31	1.76	804.31	0.00
5	0.8	0.8	18002.21	0.8	1003.87	14.45	1012.51	0.86
		1	545.15	0.0	1053.19	13.83	1053.19	0.00
	0.6	0.6	18009.67	2.2	770.15	5.54	796.80	3.46
		0.8	2454.21	0.0	850.86	4.96	856.52	0.66
		1	194.96	0.0	897.73	4.83	897.73	0.00
	0.4	0.4	18016.53	3.4	529.08	2.25	560.98	6.03
		0.6	1827.35	0.0	616.11	2.46	627.91	1.91
		0.8	519.47	0.0	686.73	2.12	687.62	0.13
		1	49.46	0.0	728.83	1.96	728.83	0.00

Table A.14: Results of Heuristic for CAB25 with $Q = Q_2$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	18015.51	3.0	1145.61	3.02	1159.23	1.19
		1	626.52	0.0	1309.89	2.49	1309.89	0.00
	0.6	0.6	18016.71	2.6	866.50	2.11	879.48	1.50
		0.8	18000.37	0.1	1065.58	2.11	1066.35	0.07
		1	198.95	0.0	1217.01	1.62	1217.01	0.00
	0.4	0.4	18019.76	4.4	593.61	1.82	598.79	0.87
		0.6	4325.63	0.0	785.91	1.79	787.71	0.23
		0.8	4539.62	0.0	972.91	1.44	972.91	0.00
		1	103.05	0.0	1116.98	1.17	1116.98	0.00
3	0.8	0.8	18037.06	2.3	1076.55	13.29	1082.21	0.53
		1	699.67	0.0	1174.23	13.02	1174.23	0.00
	0.6	0.6	18028.73	4.9	817.74	3.01	825.63	0.97
		0.8	18011.32	1.0	952.65	3.04	956.95	0.45
		1	522.82	0.0	1048.97	2.71	1048.97	0.00
	0.4	0.4	18031.82	5.5	557.28	2.25	566.34	1.62
		0.6	18000.28	0.4	692.87	2.13	701.21	1.20
		0.8	11811.74	0.0	821.14	2.10	825.35	0.51
		1	168.00	0.0	917.38	1.82	917.38	0.00
4	0.8	0.8	18016.98	2.5	1028.32	13.01	1038.60	1.00
		1	754.07	0.0	1104.13	12.71	1104.13	0.00
	0.6	0.6	18023.85	1.0	778.86	4.61	794.21	1.97
		0.8	5766.41	0.0	886.82	4.32	886.82	0.00
		1	324.21	0.0	956.00	4.21	956.00	0.00
	0.4	0.4	18030.61	3.0	529.31	2.18	541.58	2.32
		0.6	6752.81	0.0	641.92	2.09	642.52	0.09
		0.8	2789.87	0.0	735.13	1.99	735.13	0.00
		1	85.00	0.0	804.31	2.14	804.31	0.00
5	0.8	0.8	18023.48	4.7	1002.75	14.11	1010.37	0.76
		1	771.82	0.0	1053.19	14.15	1053.19	0.00
	0.6	0.6	18031.82	3.4	762.70	5.13	796.80	4.47
		0.8	16437.86	0.0	850.86	5.23	856.52	0.66
		1	254.69	0.0	897.73	5.60	897.73	0.00
	0.4	0.4	18045.91	4.8	524.20	2.21	553.72	5.63
		0.6	18012.33	1.6	615.46	2.26	627.91	2.02
		0.8	1899.85	0.0	686.73	2.20	687.62	0.13
		1	70.93	0.0	728.83	2.13	728.83	0.00

Table A.15: Results of Heuristic for CAB25 with $Q = Q_3$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	**α**	**β**	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	18025.03	2.7	1137.23	3.02	1159.23	1.93
		1	2278.52	0.0	1309.89	2.49	1309.89	0.00
	0.6	0.6	18016.95	1.9	858.66	2.11	879.48	2.42
		0.8	4895.30	0.0	1054.44	2.11	1066.35	1.13
		1	782.27	0.0	1217.01	1.62	1217.01	0.00
	0.4	0.4	18044.86	2.6	582.68	1.82	598.79	2.77
		0.6	9441.53	0.0	775.29	1.79	787.71	1.60
		0.8	1894.60	0.0	961.00	1.44	972.91	1.24
		1	641.41	0.0	1116.98	1.17	1116.98	0.00
3	0.8	0.8	18032.11	2.7	1065.64	13.29	1082.21	1.56
		1	3044.90	0.0	1174.23	13.02	1174.23	0.00
	0.6	0.6	18027.91	2.0	807.29	3.01	825.63	2.27
		0.8	15504.38	0.0	945.02	3.04	956.95	1.26
		1	1911.04	0.0	1048.97	2.71	1048.97	0.00
	0.4	0.4	18027.00	3.4	550.95	2.25	566.34	2.79
		0.6	9245.05	0.0	685.82	2.13	701.21	2.24
		0.8	4594.54	0.0	815.35	2.10	825.35	1.23
		1	723.08	0.0	917.38	1.82	917.38	0.00
4	0.8	0.8	18026.58	3.3	1030.42	13.01	1038.60	0.79
		1	2332.06	0.0	1104.13	12.71	1104.13	0.00
	0.6	0.6	18030.41	4.8	785.72	4.61	794.21	1.08
		0.8	10430.26	0.0	881.46	4.32	886.82	0.61
		1	716.53	0.0	956.00	4.21	956.00	0.00
	0.4	0.4	18029.79	5.1	534.87	2.18	541.58	1.25
		0.6	3247.30	0.0	633.94	2.09	642.52	1.35
		0.8	2821.65	0.0	734.91	1.99	735.13	0.03
		1	521.82	0.0	804.31	2.14	804.31	0.00
5	0.8	0.8	18027.56	3.0	994.07	14.11	1010.37	1.64
		1	1239.23	0.0	1053.19	14.15	1053.19	0.00
	0.6	0.6	18029.45	3.4	755.23	5.13	796.80	5.51
		0.8	17745.85	0.0	845.76	5.23	856.52	1.27
		1	614.98	0.0	897.73	5.60	897.73	0.00
	0.4	0.4	18028.94	5.4	520.05	2.21	553.72	6.48
		0.6	10198.89	0.0	607.68	2.26	627.91	3.33
		0.8	3870.59	0.0	686.73	2.20	687.62	0.13
		1	171.91	0.0	728.83	2.13	728.83	0.00

Table A.16: Results of Heuristic for TR with $Q = Q_1$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	7210.23	7.46	637.34	43.48	645.64	1.01
		1	7210.86	100.00	1138.65	43.43	647.19	0.57
	0.6	0.6	7209.40	5.78	516.63	24.54	612.20	1.18
		0.8	7211.17	7.90	609.59	25.37	615.63	1.01
		1	7209.07	3.01	639.88	24.35	617.18	0.96
	0.4	0.4	7208.70	56.50	865.35	15.86	571.25	0.66
		0.6	7211.53	4.98	482.34	14.36	574.74	1.19
		0.8	7267.08	2.76	544.42	14.07	578.17	1.06
		1	7213.55	2.91	602.93	12.79	579.72	0.96
3	0.8	0.8	7211.77	7.78	601.21	87.57	624.75	1.04
		1	7218.05	5.29	640.89	87.12	625.53	0.98
	0.6	0.6	7211.24	6.26	480.28	62.72	570.56	1.19
		0.8	7208.58	100.00	1241.00	64.42	574.00	0.46
		1	7214.75	4.95	594.50	60.57	575.55	0.97
	0.4	0.4	7210.17	62.14	896.42	55.25	525.90	0.59
		0.6	7216.58	8.68	449.90	52.86	529.58	1.18
		0.8	7209.40	10.61	524.82	52.31	531.00	1.01
		1	7210.50	4.27	538.75	52.26	531.78	0.99
4	0.8	0.8	7211.32	7.52	575.69	69.48	597.24	1.04
		1	7212.05	6.20	612.43	70.90	598.02	0.98
	0.6	0.6	7215.53	10.00	477.99	58.97	539.23	1.13
		0.8	7211.78	29.60	686.12	59.87	543.39	0.79
		1	7213.66	5.96	554.54	58.28	547.06	0.99
	0.4	0.4	7211.04	8.82	351.59	37.96	468.52	1.33
		0.6	7210.66	8.73	415.96	38.94	472.00	1.13
		0.8	7290.88	6.46	456.49	38.73	475.49	1.04
		1	7211.11	2.36	486.18	38.61	478.05	0.98
5	0.8	0.8	7213.28	9.56	570.79	64.71	566.67	0.99
		1	7214.43	5.90	585.70	64.00	567.45	0.97
	0.6	0.6	7234.38	7.19	447.03	41.76	498.52	1.12
		0.8	7211.46	7.57	498.61	41.74	499.29	1.00
		1	7212.47	3.73	511.29	41.02	500.07	0.98
	0.4	0.4	7209.72	100.00	993.64	39.86	428.58	0.43
		0.6	7210.49	36.59	565.50	39.37	429.35	0.76
		0.8	7210.55	5.59	425.64	39.29	430.13	1.01
		1	7209.90	3.98	441.94	38.87	430.91	0.98

Table A.17: Results of Heuristic for TR with $Q = Q_2$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	7211.96	48.47	1064.19	50.86	627.35	0.59
		1	7210.83	56.02	1456.58	45.14	632.61	0.43
	0.6	0.6	7211.33	19.76	529.75	30.12	589.75	1.11
		0.8	7211.37	6.03	561.21	29.82	597.34	1.06
		1	7210.46	100.00	1327.59	26.66	602.60	0.45
	0.4	0.4	7210.71	100.00	1198.60	25.18	544.50	0.45
		0.6	7210.59	100.00	1198.60	26.19	552.29	0.46
		0.8	7210.40	13.27	570.61	20.43	559.87	0.98
		1	7208.51	16.63	694.30	15.41	565.14	0.81
3	0.8	0.8	7211.59	44.57	939.55	88.15	624.39	0.66
		1	7211.38	31.95	881.97	87.39	625.53	0.71
	0.6	0.6	7215.73	13.90	461.56	70.60	548.12	1.19
		0.8	7212.71	30.25	692.31	72.56	555.70	0.80
		1	7211.88	27.18	768.17	67.52	560.97	0.73
	0.4	0.4	7209.44	25.56	361.37	67.08	515.73	1.43
		0.6	7210.94	18.78	439.64	57.62	523.32	1.19
		0.8	7210.70	48.62	850.37	55.24	527.51	0.62
		1	7216.41	11.08	574.35	52.54	530.39	0.92
4	0.8	0.8	7209.90	52.83	1060.07	71.54	596.88	0.56
		1	7213.21	36.77	901.28	74.72	598.02	0.66
	0.6	0.6	7209.93	69.25	1234.84	63.03	529.30	0.43
		0.8	7209.11	16.43	541.97	61.49	536.84	0.99
		1	7212.46	16.39	617.58	60.11	543.28	0.88
	0.4	0.4	7210.36	77.66	1150.23	40.87	455.65	0.40
		0.6	7209.93	59.87	823.51	41.77	463.40	0.56
		0.8	7209.92	50.42	801.53	40.75	469.99	0.59
		1	7210.47	15.85	544.69	38.11	475.11	0.87
5	0.8	0.8	7209.77	28.05	676.84	63.45	566.32	0.84
		1	7208.86	21.00	692.03	63.30	567.45	0.82
	0.6	0.6	7211.95	34.57	563.74	42.07	496.89	0.88
		0.8	7209.06	33.50	649.21	40.99	498.94	0.77
		1	7211.38	12.23	555.82	41.47	500.07	0.90
	0.4	0.4	7213.27	37.52	400.25	39.42	424.70	1.06
		0.6	7211.62	19.19	386.44	40.25	427.73	1.11
		0.8	7209.32	35.38	571.89	39.43	429.78	0.75
		1	7211.81	20.60	530.25	42.22	430.91	0.81

Table A.18: Results of Heuristic for TR with $Q = Q_3$

Parameters			**Mathematical Model**			**Heuristic**		**Comparison**
p	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**time**	**gap (%)**	**objvalue**	**time**	**objvalue**	**gap**
2	0.8	0.8	7211.38	53.51	1168.87	60.39	620.17	0.53
		1	7210.83	100.00	1740.11	50.58	628.59	0.36
	0.6	0.6	7211.46	100.00	1602.52	212.05	578.15	0.36
		0.8	7210.27	62.39	1385.58	43.48	590.15	0.43
		1	7211.92	26.11	826.85	48.19	598.58	0.72
	0.4	0.4	7212.25	20.31	354.80	937.02	522.46	1.47
		0.6	7210.24	100.00	1464.94	191.21	540.68	0.37
		0.8	7210.06	65.44	1417.36	31.02	552.69	0.39
		1	7209.51	24.06	758.74	21.76	561.11	0.74
3	0.8	0.8	7211.08	59.85	1286.78	85.96	624.39	0.49
		1	7210.51	53.79	1294.11	87.68	625.53	0.48
	0.6	0.6	7211.03	66.55	1173.00	276.73	536.51	0.46
		0.8	7210.79	60.61	1214.84	81.73	548.52	0.45
		1	7210.53	48.02	1072.71	79.89	556.94	0.52
	0.4	0.4	7209.38	74.73	1048.32	182.86	509.35	0.49
		0.6	7210.14	100.00	1364.95	74.53	521.67	0.38
		0.8	7212.92	24.00	569.73	56.35	527.51	0.93
		1	7209.26	56.52	1168.69	52.69	530.39	0.45
4	0.8	0.8	7211.18	100.00	1597.22	69.97	596.88	0.37
		1	7209.60	49.75	1128.60	70.47	598.02	0.53
	0.6	0.6	7209.81	100.00	1424.19	78.46	528.04	0.37
		0.8	7208.93	53.42	963.39	62.31	536.84	0.56
		1	7212.85	50.03	1029.11	62.44	543.28	0.53
	0.4	0.4	7209.74	61.74	662.87	47.18	455.09	0.69
		0.6	7213.66	55.35	730.41	39.88	463.40	0.63
		0.8	7212.20	55.99	892.53	40.16	469.99	0.53
		1	7211.01	49.06	896.56	38.85	475.11	0.53
5	0.8	0.8	7214.30	50.33	974.27	62.92	566.32	0.58
		1	7210.11	46.29	1014.70	61.60	567.45	0.56
	0.6	0.6	7211.70	56.28	838.59	41.85	496.89	0.59
		0.8	7208.75	48.39	830.84	41.46	498.94	0.60
		1	7208.41	44.68	877.86	41.31	500.07	0.57
	0.4	0.4	7212.93	60.53	627.70	40.53	424.70	0.68
		0.6	7210.95	55.65	697.97	39.46	427.73	0.61
		0.8	7209.24	55.16	817.89	39.39	429.78	0.53
		1	7209.37	62.70	1121.45	39.89	430.91	0.38