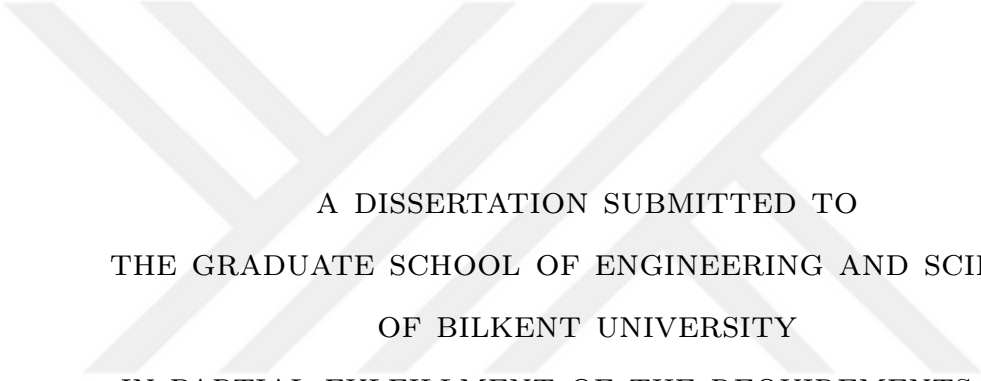


FAIR ALLOCATION OF IN-KIND DONATIONS IN POST-DISASTER PHASE



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By
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FAIR ALLOCATION OF IN-KIND DONATIONS IN POST-
DISASTER PHASE

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May 2024

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ABSTRACT

FAIR ALLOCATION OF IN-KIND DONATIONS IN POST-DISASTER PHASE

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Disaster response aims to address the immediate needs of the affected populations quickly in highly uncertain circumstances. In disaster relief supply chains, the demand comes from disaster victims (typically considered as internally displaced populations), while the supply mostly consists of in-kind donations. This dissertation focuses on finding a fair mechanism to distribute a scarce relief item among a set of demand points under supply uncertainty.

Primary concerns, restrictive elements, and unknown parameters change throughout the response phase, which substantially affects the structure of the underlying problems. Thus, the first part of this study provides a temporal classification of disaster response (e.g., into subphases) based on evolving features of demand and supply. As the next step, a donation management problem is structured considering the characteristics of a selected subphase.

We first focus on the deterministic donation management problem, which is formulated as a multi-criteria multi-period location-inventory problem with service distance constraints. A set of mobile facilities, called points of distribution (PoDs), is used to distribute the collected supply. In particular, two decisions are made for every period of the planning horizon: (i) where to locate a limited number of mobile PoDs and (ii) what quantity to deliver to each demand node from each PoD. We consider three criteria. The first two involve the so-called deprivation cost, which measures a population's "suffering" due to a shortage. The third objective is related to the total travel time. Two resulting vectorial optimization models are solved using the ε -constraint method, and the corresponding Pareto frontiers are obtained. Computational results are presented that result from applying the proposed methodological developments to an instance of the problem using real data as well as a generated one.

Finally, the stochastic counterpart of the problem is addressed with the aim of minimizing a deprivation cost-based objective. The uncertain supply parameters

are integrated into the model using a multi-stage stochastic programming (MSSP) approach. The MSSP model is tested on a real data set to assess and evaluate possible policies that can be adopted by decision-makers. Two matheuristic approaches are employed to handle the exponential growth of the scenario trees: a rolling horizon algorithm and a scenario tree reduction algorithm. A set of computational experiments is performed to evaluate the performance of the proposed methodologies. Overall, the results show that the proposed algorithms can better support the decision-making process when fairness is of relevance.

Keywords: Humanitarian logistics, donation management, disaster management, inventory management, fair resource allocation, deprivation cost, uncertainty, multi-stage stochastic programming.

ÖZET

AFET SONRASI DÖNEMDE AYNİ BAĞIŞLARIN ADİL DAĞITIMI

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Afet sonrası müdahale, belirsizliğin en çok karşılaşıldığı aşama olup afetzedelerin gördüğü zararın hızlıca en aza indirgenmesini amaçlamaktadır. Bu doğrultuda, ihtiyaçların ivedilikle, verimli ve adil bir şekilde karşılanması oldukça önemlidir. İnsani yardım lojistiğinde talep afetzedelerin ihtiyaçları iken, arz ise çoğunlukla bağışlardan oluşmaktadır. Bu tez kapsamında; müdahale aşamasında, bağışların afetzedelerin geçici olarak konakladığı çadırkentlere adil bir şekilde dağıtılması problemine odaklanılmıştır.

Müdahale aşamasında öncelikli amaçların, kısıtlayıcı etkenlerin ve arz/talep belirsizliklerinin değişim gösterdiği ve bu değişikliklerin bağış dağıtım problemlerinin yapısını etkilediği görülmüştür. Dağıtım problemlerinin doğru şekilde oluşturulabilmesi için, bu tezin ilk kısmında süreçler incelenip müdahale aşaması alt fazlara ayrılmıştır. Tüm fazlardaki problemler, karakteristik özelliklerine göre (örneğin ilgili amaçlar, dağıtılan ürün tipi, arz ve talep yapısı) tanımlanmış, sınıflandırılmış ve Yöneylem Araştırması literatürüne kazandırılmıştır. Daha sonra seçilen bir faz özelinde bağış yönetim problemi tanımlanmıştır. Bu çalışmanın sonraki kısımları, bu problemin modellenmesi ve çözülmesi için geliştirilen metotları içermektedir.

Tezin ikinci bölümünde bağış dağıtım probleminin deterministik versiyonu ele alınmıştır. Problem, kapsama kısıtlı çok-kriterli çok-periyotlu bir konumlandırma paylaşırma problemi olarak modellenmiştir. Bağışların dağıtımı için hareketli tesisler (dağıtım noktaları) kullanılmıştır. Planlama ufğunun her zaman periyodunda verilmesi gereken kararlar şu şekildedir: i) hareketli dağıtım noktalarının konumları, ii) talep noktalarının dağıtım noktalarından teslim alacağı bağış miktarı. Problemden üç amaç fonksiyonu ele alınmıştır. Bunlardan ikisi, adil bir dağıtımı sağlamak için, yoksunluk maliyeti bazlı amaçlardır. Yoksunluk maliyeti bir nüfusun kritik bir ürün eksikliğinde yaşadığı kaybı ifade eder. Sonuncu amaç

fonksiyonu ise toplam kat edilen mesafeyi enazlamayı amaçlamaktadır. Elde edilen iki vektör eniyileme modeli ε -kısıtı yöntemi ile çözülmüş ve ilgili Pareto eğrileri elde edilmiştir. Gerçek veri seti ve üretilmiş veri setinin kullanıldığı bilgisayarlı testlerin sonuçları paylaşılmıştır.

Tezin üçüncü kısmında problemin rassal versiyonuna odaklanılmış ve yoksunluk maliyeti bazlı bir amaç fonksiyonun en iyilenmesi amaçlanmıştır. Rassal parametreler modele çok-aşamalı rassal programlama (ÇARP) metodu kullanılarak entegre edilmiştir. ÇARP problemi gerçek bir veri seti üzerinde test edilmiştir. Üssel büyüyen senaryo ağaçları bu tip problemlerin çözüm sürelerini önemli ölçüde etkilemektedir. Daha kısa zamanlarda yüksek kalitede sonuçlar elde edebilmek için iki sezgisel metod geliştirilmiş; bu sayede daha küçük bir problem çözülerek orijinal problem için eniyiye yakınsayan çözümler elde edilmesi amaçlanmıştır. Sonuçlar, önerilen sezgisel yöntemlerin karar verme süreçlerini geliştirdiğini göstermektedir.

Anahtar sözcükler: İnsani lojistik, bağış yönetimi, afet yönetimi, envanter yönetimi, adil kaynak paylaşırma, yoksunluk maliyeti, belirsizlik, çok-aşamalı rassal programlama.

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Chapter 1

Introduction

Humanitarian logistics (HL) can be defined as the efficient and cost-effective planning, implementation, and control of the flow and storage of goods/materials and the related information from the point of origin to the point of destination to alleviate the suffering of vulnerable people [1]. As a relatively new domain, HL has drawn great attention from many Operations Research (OR) researchers in recent years concerning both development and disaster operations [2]. Disaster management (DM) is a sub-area of HL that focuses on developing mechanisms for better preparing for and responding to disasters. This covers a wide range of operations, including planning, capacity building, coordination, organization, prioritization, monitoring, and mobilization of resources in the pre- and post-disaster stages. This dissertation focuses on a DM problem emerging in a post-disaster scenario with the aim of delivering critical relief items to people in need while considering service equity.

Critical resources are most likely to be scarce in a disaster's aftermath. Multiple factors determine the level of scarcity, such as the scale of the disruption caused by the disaster, the level of preparedness, the proportion of pre-stocked supplies, and the socio-economic status of the affected regions. When faced with a low level of supply availability, the relief provided for the disaster victims may depend on voluntary donations. Islam (2013) states that in-kind donations present the fastest-growing segment of charitable giving as more donors have moved from

cash donations to donating in-kind [3]. The change in the donors' behavior can be explained by several reasons. First of all, cash donations have the risk of being used for purposes other than those intended by the donor. This possibility is eliminated when donors choose to donate gifts-in-kind so they can have a better idea of the resulting impact. Another reason is that both corporate and individual donors can use their available resources to donate in-kind without any acquiring cost. This provides a more sustainable and convenient alternative for donors.

The increase in the proportion of in-kind donations introduces a number of challenges to the post-disaster humanitarian supply chain, as explained below.

Uncertainty and unpredictability: Donations lead to a high level of uncertainty on the supply side of the distribution network, as the amount depends on the choice of volunteers. Donors' behavior depends on their motivation, background, socio-economic status, and financial means, among many other factors, making the donation choice hard to predict [4]. Although this adds to the complexity of a highly uncertain and chaotic supply chain, the methods and models should account for this uncertainty to better address the donation management challenge.

Logistical challenges: Compared to financial donations and vouchers, managing gifts-in-kind is more challenging. The relief aid needs to be stored in a physical facility and transported via a physical mode of transportation, which causes a number of logistical issues. First and foremost, the capacity of vehicles, facilities, and/or network edges is likely to be disrupted by the disaster and may fall short to carry all aid. Secondly, the collected donations must be processed before being delivered to the end user. This includes sorting, inspecting, packaging, and labeling [5]. These activities require processing facilities and a vast workforce, especially after large-scale disasters. Lastly, the products may become unusable during transportation and processing if their shelf life is not long enough. Therefore, additional storage conditions may be required if the relief item is somewhat perishable. Inventory and routing decisions should be made considering such limitations.

Donation pollution: As observed in previous disasters, in-kind donations may include unnecessary, excess, inappropriate, and/or unusable products. This creates donation pollution, which is a phenomenon used to describe the significant reduction in the effectiveness and efficiency of the humanitarian supply chain as a result of such items [6]. History shows many examples of this [7, 8, 9], one of which is the recent Kahramanmaraş earthquakes in Türkiye [10]. During the immediate response stage, a vast amount of large trucks carrying individual donation packages to the disaster area caused a traffic jam, blocking the emergency access routes and preventing other critical human, vehicle, and material resources from entering the zone. The distribution process for these donations was also challenging, as most of the donated items were left unattended. Not only was the donation amount excess, but there was also a significant proportion of inappropriate/unnecessary items. As a result, a majority of these donations could not be used by the people in need and accumulated as waste. Of course, the waste needs to be processed, increasing the burden on the humanitarian supply chain even more.

Multi-actor structure: There are many players in donation management systems, including governmental organizations (GOs) at national, regional, and local levels, non-governmental organizations (NGOs), individual and corporate donors, and workforce volunteers. Some of these agents act without communicating with each other, resulting in service overlaps, demand-supply mismatches, and an inequitable allocation of available resources. The issue amplifies when the disaster disrupts the information flow structure of the underlying network.

Conflicting objectives: Donation management activities in a post-disaster scenario have numerous and often conflicting objectives, such as minimizing response time, maximizing effectiveness, and fairness. Providing the right amount of material at the right time to all demand zones may not be feasible due to resource scarcity and time limitations. This causes a trade-off between equity- and efficiency-related objectives in humanitarian supply chains [11].

Motivated by the importance and challenge of managing the post-disaster relief chain in an efficient and equitable manner, this dissertation focuses on a donation management problem considering supply uncertainty. Before defining the problem setting, we conduct an in-depth analysis of the disaster response stage. We observe that the dynamics of the donation management problem (type of relief item, supply and demand structure, main objectives, etc.) drastically differ throughout disaster response. For instance, activities and priorities completely change one month after the disaster compared to one week after the disaster. At the beginning of the response phase, agility is sought, and decisions are made under uncertainties in a chaotic environment caused by the disaster, with the aim of reaching as many disaster victims as possible and meeting their critical needs as soon as possible. On the other hand, at the end of the response phase, the supply chain becomes more standardized in the sense of collection, storage, and distribution of relief items. Uncertainties mostly resolve, and the information flow is structured. Therefore, the first part of this thesis is dedicated to analyzing the temporal change in these dynamics and defining the corresponding problems from an operations research (OR) perspective. The response stage is divided into subphases based on main activities, uncertain problem parameters, distributed relief items, and primary objectives. Next, the donation management problem is defined for a selected subphase.

We consider the case where a set of demand nodes (e.g., shelter sites) must be supplied with some urgent commodity over a planning horizon. Based on the characteristics of this subphase, supply is treated as an uncertain parameter, and a nonperishable relief item with recurring demand (e.g., bottled water) is distributed. The supply for the item consists of donated goods and, thus, is irregular and time-dependent. We assume that supply information (i.e., amount of supply) reveals at the beginning of each time period upon processing (sorting and recording) the collected in-kind donations. This uncertainty poses a major challenge when it comes to distributing the available amount in a fair manner.

Upon the occurrence of a disaster, accessibility to the populations in need (or to the places where they are temporarily accommodated) may be a major issue. Therefore, supplying the demand nodes from the central warehouses where the

donations are collected is often not possible. Instead, mobile points of distribution (PoDs) such as trucks are utilized, carrying the relief aid and being positioned in such a way that they can be easily reached by the demand nodes. A coverage radius is defined, imposing that a mobile facility can only supply the demand nodes within that radius. The mobile PoDs are capacitated and typically exist in a limited number (e.g., due to a budgetary constraint).

The problem consists of determining where to set mobile PoDs in each period as well as the allocation amounts. As explained above, fairness is of major concern in such relief distribution problems as a main principle of humanitarian operations. A surrogate we consider for it is the so-called deprivation cost, which is a measure of the “suffering” caused by a shortage of a critical relief item. This is both quantity- and time-dependent, i.e., it is not only the deprivation quantity that matters but also the waiting time during which the demand node experiences shortage.

The rest of the thesis is organized as follows: Chapter 2 provides an overview of the related literature streams and our study’s contributions to the existing work. In Chapter 3, we define and analyze the temporal changes during the disaster response and relevant OR problems. After specifying a time frame, we define the deterministic version of the corresponding donation management problem in Chapter 4. The problem is formulated as a multi-objective, multi-period location allocation inventory problem. We first analyze the complexity of the model and show that even special cases of this problem fall into the category of NP-hard problems. Then, a case study is conducted to analyze the Pareto frontiers and the location and allocation schemes with respect to different objectives, where the model is solved using the ε -constraint method. Next, supply uncertainty is introduced to the problem in Chapter 5 through a multi-stage stochastic programming (MSSP) approach. The mathematical model is tested using instances built from real data available in the literature, using a set of scenarios with different supply availability levels. Two matheuristic algorithms are developed to tackle the exponentially growing scenario trees, utilizing rolling horizon and scenario reduction approaches, respectively. Computational tests are conducted on a set of benchmark instances with various sizes of scenario trees. The relevance of the

adopted methodology is evaluated based on measures such as the dynamic value of the stochastic solution (DVSS) and the expected value of perfect information (EVPI). Chapter 6 provides some conclusions drawn from the work done and possible directions for future research.



Chapter 2

Literature Review

The literature relevant to the problem tackled in this thesis can be classified into two streams: i) equity considerations in HL and iii) HL problems with uncertain information. Below, we review these streams and state the contributions of this study to the literature.

2.1 Equitable Relief Allocation¹

Social equity is an essential component of HL, especially for relief allocation operations where limited resources are available to address critical needs [11]. Other logistical constraints and uncertainties embedded in the relief management problems also contribute to the challenge of ensuring fairness. Motivated by this, researchers have shown increasing interest in integrating equity concerns into relief logistics problems in recent years. Yet, the number of studies with an explicit focus on equity measures/functions is still scarce. One first needs to quantify equity before attempting to ensure equitable distribution of resources. This is challenging, which explains why the HL literature lacks a common method applicable to all types of problems. In this section, we provide a detailed overview

¹This chapter is prepared as a working paper [12]. Available at SSRN: <https://ssrn.com/abstract=4396254>.

of fairness approaches utilized in the HL literature. To accomplish this, we review the literature based on the aspects specified in Figure 2.1.

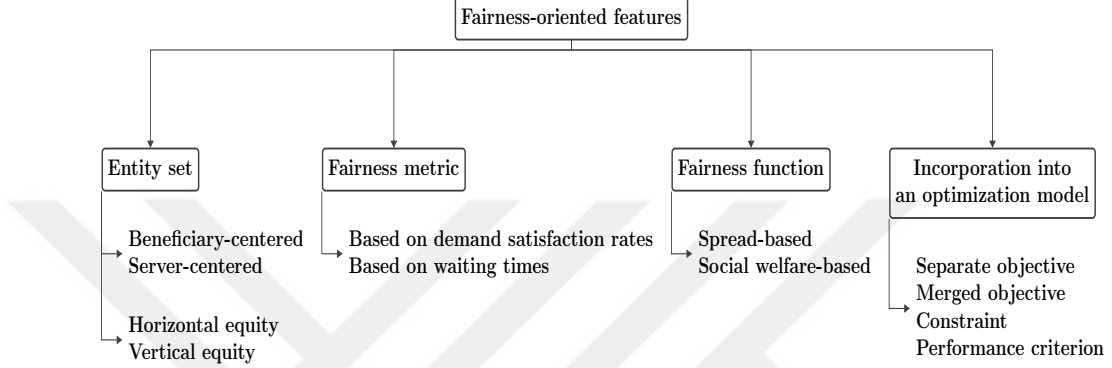


Figure 2.1: A classification framework for fairness-oriented approaches in HL.

The important aspects that will determine how equity will be incorporated are answers to the following questions: *what is distributed?* *to whom?* and *how it is (desired) to be distributed?* The first question identifies the metric [13], which is the good over which an equitable (or fair) allocation is sought. The second question refers to the notion of *entities*, the set of agencies for whom the decision-maker ought to uphold fairness. Finally, the evaluation of fairness based on the metric of choice relies on the *fairness function*. That is, the fairness function measures the level of (in)equity for a given vector of allocations. The following generic notation is used throughout this section so as to mathematically express the fairness functions: Given a fairness metric (that is the good (or the bad) to be distributed), let $x = (x_1, \dots, x_{|N|})$ be the *allocation vector* that shows the shares of entities in a set N . That is, each element x_i of this vector shows the allocated share of the selected metric to entity $i \in N$. A fairness (inequity) function, which will be denoted as $I(x)$, can be used to evaluate the (in)equity level of the allocation vector. The input of this function can take the form of a vector of (dis)utilities $u(x)$, determined by the allocation amounts, x . In other words, the inequity level can be measured based on x or $u(x)$.

2.1.1 Entity set

In the HL applications, the entities are typically the people in need, and as the underlying problem is a network optimization problem, they are the demand nodes. In disaster management problems, these nodes usually represent the populations in the affected areas or temporary settlements, such as shelter sites. On the other hand, in development logistics, the entity set (e.g., society) can include countries or different population groups with different needs and means [14].

Fairness can also be sought among the servers, e.g., the decision-maker may want to ensure a balanced workload across different rescue or response teams. Although such an approach is usually not a top priority in the HL operations since the main focus is to meet beneficiaries' needs, this situation alters when the disaster (or crisis) requires a massive workforce and the response phase expands to a longer time horizon, such as in the event of a pandemic. In such cases, a large amount of front-line workers are needed for a longer period of time in comparison to sudden disasters. Hence the workers should be equitably assigned to the required tasks, considering the effect of extensive workload on their physical and mental health [15, 16].

The definition of equity also includes the characteristics of the chosen entities. Since equal treatment of non-identical individuals may yield unfair, hence undesirable allocation results, one should be aware of the dissimilarities among the entities. In situations where these entities are identical, equal service levels are aimed, which is called *horizontal equity*. On the other hand, entities with different needs, means, and socio-economic statuses require an unequal but balanced treatment, i.e., *vertical equity* should be ensured [17, 18, 19]. The choice of differentiating the members of the entity set depends on the problem dynamics. Most of the reviewed studies that focus on disaster management problems assume an identical entity set as the information on population groups is challenging to obtain in the aftermath of a disaster, especially during the response phase.

2.1.2 Fairness metric

In the domain of HL, the equity metrics are determined based on the aims of humanitarian operations, which are reaching those in need, ensuring their safety, and meeting their needs as soon as possible. It has been observed that all of the articles reviewed in this section utilize demand coverage as the equity metric, although some of them focus on *distribution amounts* (how much the demand is covered) while others address *distribution times* (the duration of waiting for this demand coverage). That is, denoting the distribution amounts as $w \in \mathbb{R}^{|N|}$ and distribution times as $t \in \mathbb{R}^{|N|}$, it can be said that the allocation vector x is usually expressed as w , or t while the fairness function takes the form of $I(w)$ or $I(t)$.

The works that account for quantity-related metrics usually consider the satisfied proportion of demand (i.e., fill rate) or the amount of unmet demand. As a different approach, Çankaya et al. [20] and Ekici and Özener [21] study the equitable allocation of inventory and used the ratio of stock to demand as a measure of demand fulfillment. Time-related aspects of demand satisfaction can be defined in various ways, such as arrival time to a demand node, waiting time for a service, the number of facilities within a predetermined radius of a demand node, and visiting frequency.

Some of the related work presented in the literature includes a utility function $u(x)$ as an input to a social welfare or a spread-based function. After individual utilities, $u_i(x_i)$, are obtained based on delivery times or amounts, the corresponding allocation vector $u(x)$ is used as an input to the fairness function of choice within the model. The literature shows that in a vast number of studies, such utility functions are based on the amount or ratio of satisfied demand of beneficiaries. These are typically concave functions so as to encourage spread. If the allocation vector of concern (x) is something undesirable, such as unsatisfied demand or waiting time, then convex disutility functions are used, called loss functions. The summation of such functions over the entities can be specified as fairness functions, as the marginal cost of a unit loss increases with higher levels

of loss. Consider two entities with identical attributes other than their shortage levels. In such a case, the entity with the higher shortage rate will be prioritized due to the convex nature of the function of interest. Most of the reviewed work including such an approach adopts the following method: they utilize individual disutility functions which non-linearly increase with the disaster victims' waiting time for a relief product or a service while ensuring full demand satisfaction.

Another method used in the literature to ensure a fair division of the available resources is to use weights for the demand nodes to distinguish entities and ensure vertical equity. These weights suggest that the utility gained from receiving one unit of resource differs across the demand nodes. We refer to such functions as priority-based utility functions. The priority indices of populations can be based on poverty levels [22], risk associated with the area [23, 24, 25], urgency levels [26, 27, 28, 29], accessibility [30], and severity of the disaster damage [31, 32, 33].

A subcategory of (dis)utility functions addresses the need to account for the timing of delivery (t) as well as the allocated amounts (w), taking the form $u(w, t)$. Such approaches are relevant when the entities do not necessarily receive the goods at the same time, and the problem at hand involves decisions related to the allocation of time alongside the amount. In this work, we also adopt this approach as it is often the case for disaster response problems due to the low level of availability of resources and particular emphasis on agility.

The deprivation cost functions, introduced to the literature by Holguín-Veras et al. [34], are among the most frequently used methods (encountered in 21% of the reviewed works) when deficiencies caused by limited resources are defined in terms of both quantity and time. This approach attempts to quantify the welfare loss caused by lacking a vital good or service. Typically, these types of functions increase monotonously, are non-linear, and convex according to the deprivation time. The convexity over time occurs because this function must grow progressively as the deprivation period increases. At the same time, this function is likely to have hysteretic effects. For example, it is known that if individuals enter a hysteretic period after a certain period of time without water, the damage caused is irreversible.

Our review shows that there is a vast amount of approaches in the HL literature to select the fairness metric, which depends on the problem structure and characteristics of the entity set. We observe that fairness is usually sought in a single dimension, either in distribution amounts or in distribution times. The individual (dis)utility functions that account for both waiting time and amount distributed are not sufficiently studied, even though both aspects are significant in humanitarian settings. With that being said, it is also observed that this situation has been changing recently, especially after the notion of deprivation cost functions was introduced to the literature. Most (68%) of the reviewed studies that utilize deprivation cost functions were published after 2018, indicating that this concept has attracted increasing attention from OR researchers in recent years.

Upon forming the allocation vector x or $u(x)$, one needs to decide how to distribute this good or bad of interest, i.e., the form of the fairness function $I(\cdot)$. Next, we explain possible choices of fairness functions as well as the methods to incorporate them into the mathematical models.

2.1.3 Fairness functions

There is a vast amount of methods that can be implemented with regard to how to evaluate the fairness of an allocation vector x . This includes a variety of functions, which we categorize into two classes based on whether or not they encourage efficiency alongside fairness. While some of them measure only the spread of a distribution regardless of the overall efficiency, others consider both. The former class of functions is already defined in the literature as “dispersion type functions” or “inequity measures” [35, 36]. We name this category as *spread-based functions*. The second category is called *social welfare functions* as they account for the total amount distributed to the entity set (society) besides increasing the balance amongst them. In Figure 2.2, we present the list of fairness functions that fall into each category.

The main role of the spread-based fairness functions is to control the spread

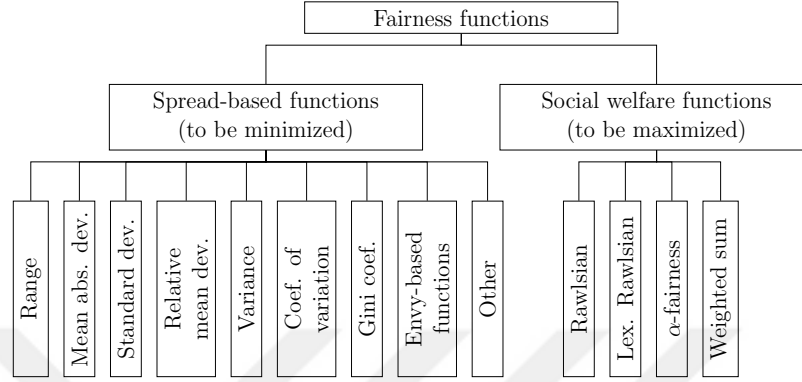


Figure 2.2: Frequently used fairness-based functions in the HL literature.

of the allocation to avoid highly inequitable results. Such functions are more frequently used as a performance criterion (to evaluate the solution vectors based on their equity level) compared to the other types of equity functions. This owes to the fact that such functions are indices yielding a score of (in)equity on a scale of 0 to 1. Moreover, such functions are frequently used within constraints in the reviewed mathematical models or merged with other objectives, as they fail to ensure efficiency on their own.

A social welfare function (SWF) quantifies the total welfare of the entities of concern, which are typically disaster victims or other beneficiaries in the humanitarian domain. Such functions embody equity in resource distribution in addition to the total resources enjoyed. SWFs include many egalitarian approaches, such as Rawlsian, weighted sum of concave (convex) (dis)utility functions, and other forms of functions in which the entities are prioritized based on a criterion. The majority of the reviewed work focuses on rather simple SWFs such as minimax, while more sophisticated methods are scarcely considered. Next, we present such approaches, including the well-known families of functions to encourage similar (balanced) allocation among the entities.

An example is the set of equitable aggregation functions [37], i.e., the functions that satisfy monotonicity, symmetry, and Pigou-Dalton principle of transfers. By their definition, these functions are Schur-concave functions. Two well-known

equitable aggregation function families incorporating welfare in optimization are Ordered Weighted Averaging (OWA) functions and α -fairness functions.

A social welfare function $I(\cdot)$ of the form $I(x) = \sum_{i \in N} w_i \vec{x}_i$ with $w \in \overleftarrow{\mathbb{R}}_+^m$ and $\sum_{i \in N} w_i = 1$, is called an OWA social welfare function [38]. These functions provide a welfare score for any allocation vector x by multiplying each element of its ordered permutation by a rank-based weight, giving higher weights to worse-off entities. Thanks to the rank-based nature of the aggregation, this form can encompass a class of social welfare functions that correspond to a wide range of attitudes to inequality. The two extreme equitable aggregation functions of Rawlsian and utilitarian are indeed OWA functions with weights $w_1 = 1, w_i = 0 \forall i \in N \setminus \{1\}$ and $w_i = \frac{1}{|N|} \forall i \in N$, respectively. Between these two extremes, each OWA function corresponds to a different degree of inequity-aversion.

Another equitable aggregation is the so-called α -fairness function, which has the following form:

$$I(x) = \begin{cases} \sum_{i \in N} \frac{(x_i^{(1-\alpha)} - 1)}{(1-\alpha)} & \text{if } \alpha \geq 0, \alpha \neq 1, \\ \sum_{i \in N} \log(x_i) & \text{if } \alpha = 1. \end{cases}$$

Similar to the OWA functions, α -fairness functions are capable of encompassing a class of social welfare functions between the two extremes of utilitarian ($\alpha=0$ case) and Rawlsian ($\alpha = \infty$ case) functions [39, 40]. The case when $\alpha = 1$ corresponds to the proportional fairness rule and yields the Nash bargaining solution [41]. Thanks to the parametric form of OWA and α -fairness functions, one can perform a sensitivity analysis and demonstrate how the suggested solutions alter for different degrees of inequity-aversion by systemically changing the inequity-related parameters of these aggregation functions, i.e., weight vector of the OWA aggregation and the α parameter in the α -fairness function.

The most common way to achieve a fair allocation among the reviewed literature is using one extreme form of OWA and α -fairness functions, namely the Rawlsian approach [42]. A vast amount of these studies maximize the minimum

proportion of satisfied demand or, analogously, minimize the maximum proportion of unmet demand among all demand nodes. In addition to the studies that use a minimax function in the objective, some use the Rawlsian approach in the form of a constraint and ensure that the minimum (resp. maximum) is above (resp. below) a predetermined threshold. A possible issue is that such an approach can yield an infeasible problem if the threshold value is not specified properly.

A generalized version of the maximin approach is called the lexicographic Rawlsian function, which orders the entities based on their allocation amount (for a good) in a non-decreasing manner. Then, the lexicographic Rawlsian function maximizes the allocation of entities starting from the worst-off entity, considering this sequence. This approach is able to distinguish the solutions that provide the same minimum value, whereas the Rawlsian function is indifferent among such solution vectors. Despite its advantage, it is noteworthy that only a single study utilizes this function, namely the study by Hakimifar et al. [43], among the reviewed works.

A possible way to utilize social welfare is to use a weighted sum of individual values given in the allocation vector. With such functions, the allocation vector consists of specially formed (dis)utility functions to encourage equity while optimizing the weighted aggregation within the underlying mathematical model. The functions that are summed are typically priority-based (dis)utility functions, deprivation cost, and other forms of penalty functions.

2.1.4 Incorporating fairness into optimization models

There are several ways to use a fairness function within an optimization model. Inequity function $I(.)$ can be minimized as a separate objective, or it can be integrated into a cost-related objective function (denoted as $f^C(.)$) by assigning a penalty value (p) per unit inequity. This constant can be interpreted as a measure of the importance placed on equity. Such an objective function is formed as follows: $\{\min f^C(x) + pI(x) | x \in \mathcal{X}\}$ where $\mathcal{X}$ denotes the feasible decision space.

The most challenging aspect of such a method is determining the penalty constant p . To do this, historical data and/or expert opinions can be utilized. However, assigning a constant monetary value to an inequity measure can lead to under or overestimating the actual suffering of entities due to the unfair allocation [34]. Despite this, we observe that this method is extensively used in the literature. For example, deprivation cost functions are usually minimized in the form of a social cost, i.e., combining logistics and deprivation cost values.

Another method is to limit the inequity level of the allocations using a hard constraint, which results in the following generic model: $\{\min f^C(x) | x \in \mathcal{X}, I(x) \leq \beta\}$. This requires specifying the parameter β similar to the previous case. Note that the hard constraint method is a version of the penalty model where $p = \infty$ for inequity levels higher than the threshold, which ensures fairness by enforcing infeasibility. The most frequently observed method is to enforce a minimum demand satisfaction level for all demand nodes as a way to balance the service levels.

When a time-related metric is utilized for the fairness function in this setting, the idea is usually to enforce a response time to cover entity demands. This approach is frequently used for facility location problems when the entities access the located facilities/servers by their own means. In the problems where vehicle routes are considered for delivery, the total route length is often limited with an upper bound. Such a constraint would serve more than one purpose: i) the workload of vehicles/teams is balanced by limiting the shift time, ii) all demand nodes are served in a timely manner. Most of the articles that utilize the deprivation cost concept consider such a time limit to indicate “the maximum time an individual can endure the deprivation”, also called “survivability time” [44]. A typical drawback of such constraints is that they may yield infeasibility, as satisfying a strict time or coverage limit is often not possible in post-disaster scenarios.

As another possibility, the fairness function can be used as a separate objective function, yielding a bi-objective problem in the form of $\{\min f^C(x), I(x) | x \in \mathcal{X}\}$. Such a setting does not require estimating a constant parameter; however, it

demands customized solution methodologies to obtain the exact or approximate Pareto front. There are other approaches for incorporating an inequity function into a mathematical model besides the multi-criteria models, hard constraints, and penalty approaches. Eisenhandler and Tzur [98], e.g., combine efficiency and equity-related functions in a single objective by multiplying them.

Overall, we observe that most of the literature tends to treat “equity” as a single-dimensional concept. However, this review reveals that it is, in fact, a multi-dimensional concept that requires the consideration of various factors such as the characteristics of the entity set, the length of the planning horizon, the demand satisfaction rates of affected populations, and the time passed until these demands are satisfied. In this study, we address this complex nature of equity. We summarize the works that are most similar to ours in Table 2.1, which includes the studies that consider deprivation cost functions within the context of humanitarian location-allocation problems.

Table 2.1: Literature review for humanitarian location-allocation models considering deprivation costs.

Reference	Multi-objective	Objective(s)	Multi-period	Uncertainty
Yushimito et al. (2012) [45]		TDC		D
Khayal et al. (2015) [46]		SC	✓	S
Pradhananga et al. (2016) [47]		SC		D
Loree and Aros-Vera. (2018) [48]		SC		D
Moreno et al. (2018) [49]	✓	LC, TDC	✓	S
Paul and Zhang (2019) [44]		SC		S
Paul and Wang (2019) [50]		SC		S
Cotes and Cantillo (2019) [51]		SC		D
Diehlmann et al. (2020) [52]		SC		D
Chang et al. (2021) [53]		SC	✓	S
Zhang et al. (2021) [54]		SC		S
Jamali et al. (2021) [55]	✓	LC, TDC		S
Yáñez Sandivari et al. (2023) [56]	✓	LC, TDC		S
<i>Current work</i>	<i>Chapter 4</i>	MaxDC, TDC, TD	✓	D
	<i>Chapter 5</i>	MaxDC	✓	S

TDC: total deprivation cost, MaxDC: maximum deprivation cost, LC: logistics cost, TD: total transportation distance, D: deterministic, S: stochastic

The main differences of this work from the existing studies are listed below:

- Ensuring fairness in multi-item, multi-period, and/or stochastic environments is an inherently challenging problem, and these extensions naturally arise in HL applications. Unlike most of the existing literature, we adopt a multi-period setting where the decisions are to be made over a planning horizon, considering supply uncertainty.
- Deprivation cost functions attempt to quantify human suffering when lacking access to critical needs, which is why they qualify as a strong match to address the equity challenges emerging in the humanitarian domain. Such functions are usually considered simultaneously with other cost elements such as facility setup, holding, assignment, and transportation costs. A common way to achieve this is to minimize the sum of logistics and deprivation costs (i.e., social cost) as a single objective function. This unified objective function is adopted by a substantial majority (80%) of the surveyed articles that take deprivation costs into account. Another frequently used approach, observed in 20% of these papers, is to include logistic costs as a separate objective function in a multi-objective setting besides the deprivation cost. Moreno et al. [49] test both approaches. As expected, the aggregated objective function is indifferent between a unit of logistics and deprivation cost, which can disregard the importance of the deprivation cost levels as opposed to logistics costs and prevent the decision-maker from observing the trade-off between these two criteria. Following this idea, in Chapter 4, we consider a deterministic multi-objective setting where the deprivation cost-related objectives and the total distance traveled to reach PoDs (i.e., efficiency) are minimized.
- Gutjahr and Fischer [57] argue that the utilitarian use of deprivation cost can still lead to arbitrarily inequitable solutions, which gives rise to the combined use of deprivation cost with other types of fairness functions. As an example of this approach, rather than solely minimizing the total deprivation cost, one may want to ensure that entities experience the deprivation cost in an equitable manner and use a spread-based function over

the deprivation cost vector. Following this idea, we focus on two deprivation cost-oriented objectives in Chapter 4: the total deprivation and the maximum deprivation cost. In Chapter 5, where the stochastic counterpart of the problem is tackled, the objective is determined as the minimization of the expected maximum deprivation cost experienced by the shelter sites. Table 2.1 shows that none of the similar works considers such an approach.

2.2 Uncertainty in Relief Allocation Problems²

The uncertain and dynamic nature of disasters presents significant challenges for humanitarian organizations in delivering aid effectively and efficiently. This has been the main motivation for the research efforts to address uncertainty in the HL problems over recent years. In the context of this dissertation, a review paper has been published as a comprehensive review of the research done on uncertainty in humanitarian contexts [58]. Although this study covers only the location problems, the findings and driven insights can be projected onto all relief distribution settings.

In this survey, 108 scientific papers published between the years 2007 and 2019 are reviewed based on: sources of uncertainty (demand, supply, and network connectivity), decisions that take place in the reviewed mathematical models in addition to location (pre-positioning, direct allocation, and routing), criteria used in these models (cost, equity, and reliability), methods to capture uncertainty in the mathematical models (stochastic programming, robust optimization, and chance constraints), and solution methodologies in general. A general modeling framework is also provided, considering the common features used in the reviewed literature. Recent trends, research gaps, and possible future directions are identified.

²The content of this chapter is published in Omega [58].

2.2.1 Sources of uncertainty

Uncertainty is intrinsic to many humanitarian problems since disasters are inherently unpredictable events in terms of their timing, magnitude, and the locations they affect. One possibility for hedging against uncertainty is to capture it in the parameters of the underlying optimization models. These parameters may appear on the demand side, on the supply side, or in the links in between [59]. Some examples include the location and amount of demand, the capacity of the suppliers, the connectivity of the network, the transportation times (which likely impact the response time), and the costs. We observe that out of 35 reviewed articles that were published in 2007–2013, 19 (54%) consider a single uncertainty source, while 12 (34%) capture two sources, and others (12%) consider all three sources.

One of the main challenges in humanitarian logistics is the difficulty in forecasting demand, which leads to a lack of reliable information about the locations and needs of affected communities prior to the disaster. As a result, most of the reviewed literature considering uncertain information focuses on demand-side uncertainty. They also often include methods to predict the demand, using geographic and demographic information and forecasts on the disaster scale, epicenter, and timing [60, 61]. The vulnerability levels of the potentially affected areas are also taken into account by some authors, such as Bozorgi-Amiri and Khorsi [62] and Salehi et al. [63].

Uncertainty in supply is possibly a major distinguishing feature of humanitarian supply chains, as the pre-positioned items and store facilities are likely to be located in close proximity to the potentially affected areas, which increases the risk of disruption. Despite this, supply uncertainty is studied less compared to demand-side uncertainty in the HL literature. It is worth noting that even fewer of the existing studies highlight that a key source of supply, donations, is also prone to uncertainty.

The status of the roads and bridges is also prone to damage in a disaster

scenario. It is not possible to determine beforehand the extent of a network disruption (if any) due to the unpredictability of disasters. Moreover, even after the event, the exact status of a transportation network may remain unknown due to communication issues. In the HL literature, this uncertainty is often reflected in the model parameters such as link capacities, transportation times, and/or transportation costs. Network disruption has been less studied compared to uncertainty in demand or supply. Nevertheless, this trend seems to be changing. In fact, we note that in the eight-year period 2007–2014, 20% of the reviewed articles (7 out of 35) consider this aspect, whereas, in the more recent years, the number rises to 36% (26 out of 73 articles).

2.2.2 Decisions involved

In humanitarian logistics, two major planning phases can be identified: preparedness (which includes preventive measures taken before a disaster occurs) and response (which includes the reaction to the consequences of the disaster). In the first phase, we find strategic decisions such as location decisions (common to all articles reviewed in this section) and inventory decisions. Inventory decisions are often implemented by pre-positioning relief items in the facilities prior to the disaster. In the second phase, we usually find operational distribution decisions that often appear in the form of allocation decisions (demand nodes to facilities) or routing decisions. Post-disaster inventory stored at distribution centers and demand nodes can also be regarded as operational decisions.

Resource allocation can be implemented via direct allocation, where the demand nodes are directly assigned to service facilities. A demand node can be supplied by more than one facility, i.e., multiple allocation is possible. In the reviewed literature, multiple allocation is used significantly more than single allocation. This can be explained by the specific type of problems we are covering: in an emergency situation, it makes sense to plan so that a disruption in a facility does not cause a full shortage at a demand point. In other words, multiple allocation is a way to increase reliability in humanitarian logistics. In some works, both

allocation patterns are considered. This holds, for instance, when a multi-layer network is considered. An alternative to direct allocation is to consider the sequence of service to the demand nodes. This is accomplished by means of vehicle routing decisions, which include the number and type of vehicles to use as well as the corresponding routes.

Although multi-period models are often used in the humanitarian context, using multiple stages to make location decisions, i.e., adopting a multi-stage facility location plan, is not frequently observed. If the setup cost of the facilities is not high compared to the operational cost, relocating can be used to perform deliveries more quickly. In relief distribution, temporary/mobile facilities with low setup costs can be used to meet the demand from different locations as much as possible. Such mobile facilities are re-located or moved in a multi-period setting in the studies by Eskandari-Khanghahi et al. [64], Kamyabniya et al. [65], Moreno et al. [49] and Vahdani et al. [66].

The studies that use mathematical models with relocation decisions are mostly motivated by the demand variability during the disaster response: the pre-positioned relief inventories in the existing facilities may fail to respond to the fluctuations of demand levels. Two examples of this approach can be observed in the papers by Chang et al. [53] and Zhang et al. [54]. The former work utilizes a rolling horizon approach to relocate facilities so as to respond to the demand fluctuation during hurricane relief operations, whereas in the latter, dynamic programming is adopted to decide on the location of a single depot, assuming a non-stationary stochastic demand.

Moving the relief inventory between static facilities is also studied in the literature as an alternative to location-relocation models [67]. Such models require a multi-depot setting. In some of the reviewed studies, mathematical models allow both facility and inventory relocation. Afshar and Haghani [26] investigate the dynamic location of temporary facilities (distribution centers and PoDs) while utilizing a set of permanent ones in a multi-modal relief supply chain to increase responsiveness. Their mathematical model also allows the transshipment of relief inventories across depots. Similarly, Khayal et al. [46] study a problem where

temporary mobile facilities are installed with the consideration of relief inventory relocation.

Location-relocation settings can also be adopted to improve the equity level of the service provided to demand nodes. This especially applies to cases where the resources are scarce and the total distance traveled (or time) to reach disaster victims is of relevance. Installing permanent depots may yield a high level of transportation cost for the demand nodes that are farther from these depots. If the service level is not ensured by hard constraints, inequity may arise in resource allocation, which conflicts with the main principles of humanitarian operations [11]. To prevent this from happening, one can improve coverage by increasing the number of supply facilities or moving these facilities in each period to ensure fair service among various regions. The latter provides a more cost-efficient approach since the same service level can be achieved with a smaller number of supply depots, assuming that the relocation cost is less than the cost of opening a new facility. Based on this idea, we consider the relocation of mobile points of distribution combined with inventory decisions to encourage equity among demand nodes.

2.2.3 Uncertainty paradigms

In recent years, various solution methodologies have been proposed to address the uncertainty faced during HL operations. The most frequently used ones are Stochastic Programming (SP) and Robust Optimization (RO).

The proper modeling framework depends on the available information regarding uncertainty and the type of constraints to be included in the model. A distinguishing feature concerns the knowledge on the probability distribution of the underlying random vector. If this information is not available (or it is irrelevant to take it into account), then RO is a good framework in many cases. In this method, an uncertainty set is assumed for each unknown parameter, and a solution is sought that is feasible no matter the value of the parameters. If such a distribution is known (e.g., can be estimated using historical data), then we can

resort to a SP model. In this case, we typically consider two- or multi-stage models. The here-and-now decisions (decisions to implement before any uncertainty is revealed) are defined as first-stage decisions. For the subsequent stages, a discrete set of scenarios is usually used to capture the uncertainty, and decisions are made that adapt to the occurring scenario. In the context of humanitarian facility location, the location and the inventory pre-positioning decisions are usually assumed as first-stage decisions. The allocation, routing, and demand satisfaction decisions have been mostly assumed as second-stage decisions. Most of the existing literature utilizes two-stage stochastic programming (TSSP), assuming that there is a single point in time where the uncertain information reveals completely. As we aim to search for fair allocation schemes for repetitively collected and processed in-kind donations, we use a solution methodology that allows us to formulate gradually revealing supply information over a planning horizon, namely multi-stage stochastic programming (MSSP). This method enables us to consider multiple stages of decision-making, where both location and allocation decisions are made at each stage based on the information available at that point in time. By considering the information updates, our purpose is to model the post-disaster donation management problem more in line with real-life situations [68]. Despite its advantages, MSSP is studied to a much smaller extent in the HL literature compared to TSSP. This can be explained by the increasing complexity of the models depending on the size of the scenario trees.

The works that address the post-disaster relief aid distribution problem via MSSP are reviewed in Table 2.2 along with this study. We remark that there is no common study in Tables 2.1 and 2.2. This indicates that, to the best of our knowledge, the deprivation cost concept has yet to be applied in a multi-stage stochastic framework for humanitarian location-allocation problems. This work aims to fill this gap by not only addressing the donation uncertainty via MSSP but also applying and evaluating multiple heuristic approaches to tackle large scenario trees.

To the best of our knowledge, there is no work combining the modeling perspectives explained below, which is a strong indicator of the novelty of the proposed problem as well as of the modeling framework (see also Tables 2.1 and 2.2):

Table 2.2: Review of the studies utilizing MSSP to capture uncertainty during the post-disaster relief aid distribution.

Reference	Decisions					Objective function			Source of uncertainty		
	FL		RA	I	VR	CC	EC		DS	SS	
	S	M					AB	TB		F/HR	VD
Rennemo et al. (2014) [69]	✓		✓		✓	✓	✓		✓	✓	
Zahiri et al. (2017) [70]	✓					✓			✓		
Escudero et al. (2018) [71]	✓		✓			✓			✓		
Hu et al. (2019) [72]	✓		✓	✓	✓	✓					✓
Cavagnini et al. (2022) [73]			✓	✓		✓			✓		
<i>Current work (Chapter 5)</i>		✓	✓	✓			✓	✓			✓

FL: facility location, S: static facilities, M: mobile facilities, RA: resource allocation, I: inventory, VR: vehicle routing, CC: cost-centered, EC: equity-centered, AB: amount-based, TB: timing-based, DS: demand-side, SS: supply-side, F/HR: fleet/human resources, VD: voluntary donations, NC: network connectivity

- The literature lacks a structured time framing for disaster response, which is characterized as highly dynamic. This constitutes great importance while defining the corresponding problems as the operations and dynamics substantially alter. In this dissertation, we aim to address this gap by introducing a detailed time-dependent classification framework for the response stage. Afterward, a donation management problem is defined for a selected subphase based on its features.
- In the HL literature, location-allocation models usually consider equity in terms of the distributed amounts. However, after a disaster, waiting time for the arrival of critical supplies greatly affects the suffering of the disaster victims. We capture time as a feature of relevance when measuring equity in a location-allocation model. This is accomplished through the concept of deprivation cost. Since minimizing the total deprivation cost may fail to ensure equity, the maximum deprivation cost among the demand nodes is minimized. We first investigate the deterministic variant of the problem, analyze its complexity, and provide insights on the trade-off between multiple objectives.

- Relocation decisions are made within a multi-period setting as new information becomes available, and the value of such relocation is analyzed in comparison to the traditional one-off settlement of supply facilities.
- We incorporate uncertain supply parameters (i.e., the amount of donations) into the relief distribution problem via multi-stage stochastic programming.
- We provide insights that would be very useful in practice by analyzing the location-relocation and allocation schemes that minimize alternative objectives for the stochastic problem, namely the maximum deprivation cost in the expected case and the expected maximum deprivation cost. These correspond to ex-ante and ex-post approaches, respectively.
- There is a major distinction between exact and approximate algorithms developed for solving stochastic problems in the humanitarian context. So as to take advantage of both, we design multiple matheuristic approaches. The results show that these algorithms considerably reduce the computational time while achieving high-quality solutions.
- One aspect that has been neglected to a large extent in the literature is the demonstration of the quantitative gain resulting from capturing uncertainty in the context of humanitarian logistics. Measures such as the value of the stochastic solution or the expected value of perfect information have not been considered in general. We analyze both measures to evaluate (i) how relevant it is to capture uncertainty and (ii) how relevant it is to consider a more elaborate model for capturing uncertainty compared to using a simplified (i.e., deterministic) one.

Chapter 3

Classification of Disaster Response Phases from an OR Perspective

3.1 Introduction

Disaster management problems can be categorized based on the timing of the operations relative to the disaster occurrence. Four stages are defined to address this aspect: Mitigation, preparedness, response, and recovery, which together form a *disaster management cycle* [74, 75]. Disaster response is the most dynamic time frame among these and spans a period from right after the disaster event until the daily operations of the humanitarian supply chain normalize. During this stage, the main objectives, operations, uncertainties, types of relief provided to disaster victims, and, consequently, the types of problems evolve significantly.

The existing literature defines *acute phase* as the first 72 hours during which agility is paramount, whereas cost-efficiency is prioritized in the first 100 days [76]. Some studies also suggest that the objectives gradually move from agility-based to cost-centered during the response [4, 77]. However, the response activities

should be further classified based on such changes so as to accurately capture the real-life dynamics while structuring the related problems. Motivated by this, we provide an event-based temporal classification framework for disaster response in this chapter, as depicted in Figure 3.1. The subphases typically overlap as the transitions between them do not happen instantaneously. Hence, the events specified in Figure 3.1 are not pinpoints that separate the phases completely. They rather specify the beginning of the transition periods. The attended HL activities are specified in Table 3.1, demonstrating how the operations evolve over time to address the changing needs of the affected populations.

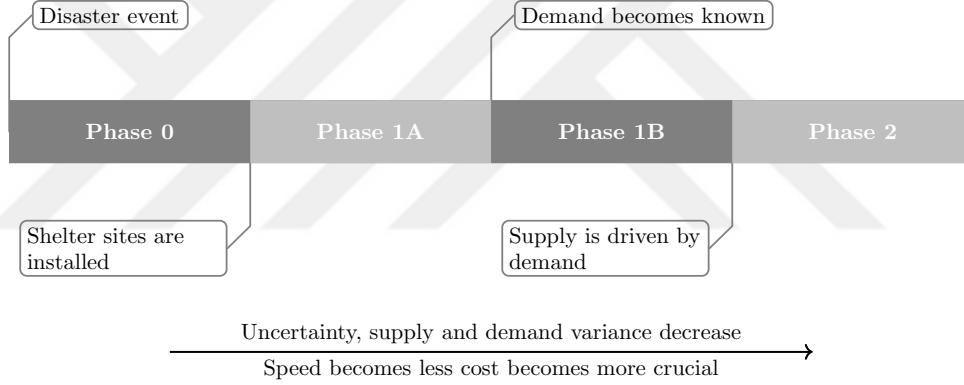


Figure 3.1: Timeline of disaster response subphases.

The timing of post-disaster operations can vary depending on numerous factors, such as the disaster’s magnitude, timing, scale, and the socio-economic status of the disaster-affected area. However, we construct this classification in a way that is free from such limitations, as it is based on important milestone events rather than the time passed after the disaster. The subphases are defined considering the earthquake action plans of the Türkiye government, yet they are applicable to i) other countries with a centralized disaster management structure and ii) other sudden-onset natural disasters where search and rescue operations, emergency relief, and temporary accommodation services are needed for the affected population.

The acute phase starts right after the disaster occurrence, which we refer to as *Phase 0*. The information flow is most limited during this phase, with the highest

Table 3.1: Relevant HL problems during disaster response.

HL problem	Phase 0	Phase 1	Phase 2
Damage/status assessment	✓		
Emergency access route design	✓		
Casualty transportation	✓		
Coordination center management	✓	✓	
Emergency response service management	✓	✓	
Mobile service management	✓	✓	✓
Workforce management	✓	✓	✓
Shelter site management		✓	✓
Donation management		✓	✓
Risk assessment and restoration			✓
Debris management			✓
Supplier selection			✓

level of uncertainty. The first task is to assess the damage. This can be performed via drone fleets or satellite images, which enhance the efficiency of the process without being disrupted by blocked roads/bridges. Based on this assessment, the routes to be used by emergency response teams (i.e., emergency access routes) can be determined. Coordination centers at regional and national levels are also set up during immediate response to facilitate a better communication channel among the authorities. The main focus is on emergency response services (e.g., search and rescue, safety, and emergency healthcare) and casualty transportation, with the aim of saving as many lives as possible as quickly as possible while managing the chaos. The temporary accommodation centers (e.g., shelter sites) are not established yet, and the affected groups start to travel to the pre-located gathering areas if they can. Following this, demand for relief aid starts to accumulate at these points. Mobile service facilities visit these locations to meet the most critical needs (food, water, tents, etc.) of the disaster victims. Such items are supplied mostly by the local and regional authorities as the national-level resources, including in-kind donations, have yet to arrive at the disaster area. However, the list of needed relief aid is announced prior to collecting donations to limit donation pollution to a minimum.

Phase 1A starts after temporary accommodation centers are established while

the emergency services still continue. Internally displaced people start to move to these centers, which act as demand points for the relief supply chain. Mobile service facilities start to serve these centers to maintain the basic needs until such services can be facilitated within the shelter sites. Demand cannot be known with certainty as the transition to shelter sites continues during this phase, which causes demand-side uncertainty in the underlying relief network. For ease of operations, the shelter sites can be assumed to be identical, or the demand can be estimated based on the size and disruption level of pre-assigned affected areas. The focus in this phase is to meet the critical needs of the disaster victims as quickly as possible. The relief aid consists of consumable items (items with recurring demand patterns), such as food and water, and non-consumable items (items with one-time demand patterns), such as tents, tent kits, kitchen kits, clothing, and blankets.

Phase 1B starts after the transition to shelter sites is completed. This milestone event specifies the time when the demand at these accommodation centers starts to stabilize. Due to this, the shelter sites are not considered identical, and the supply should be allocated proportional to the demand. The relief supply chain is redesigned during this phase, as the information flow is restructured and uncertainties surrounding the operations are mostly resolved. Yet, the supply remains uncertain as individual and corporate donations start to be collected and distributed to the disaster area. The relief includes mostly consumable items such as food and water. Non-perishable item groups can be stored both in demand zones and reliable facilities such as regional and local warehouses.

The transition to recovery begins with *Phase 2*. At this stage, the humanitarian supply chain starts to resemble a commercial one in terms of a demand-driven supply structure, standardization of operations, and prioritization of cost-centered objectives. In addition to seeking cost-minimizing solutions, the relief supply chain needs reliable and sustainable alternatives to meet the long-term needs of disaster victims. This includes establishing long-term contracts with third-party suppliers and utilizing government resources. As the public's response to the disaster diminishes, individual donations decrease in size and frequency, a phenomenon known as *donor fatigue*. Consequently, the supply becomes more stable

compared to previous stages.

During Phase 2, risk assessment and restoration activities start. This includes identifying uninhabitable residential and public buildings based on structural damage and reinforcement for those with minor and medium-level disruption. Waste and debris accumulated after the disaster also need to be handled, which is vital in normalizing life in the disaster zone.

Overall, we observe that the variability and uncertainty of both demand and supply fluctuate during disaster response, while the main focus moves from response time to cost and sustainability-related objectives. Below is a detailed discussion of the relief types and demand/supply structure during the subphases. Finally, we identify the OR problems faced in each subphase.

3.2 Characteristics of Response Subphases

3.2.1 Relief types

The usual characteristics of the relief needed in each subphase are explained in Table 3.2 based on item type, the main source of supply, inventory structure, and demand frequency.

The most urgent needs are prioritized due to the high level of agility and scarcity of resources, especially in the early phases of response, i.e., during Phase 0. These mostly consist of vehicles, equipment, and personnel for search and rescue activities. Moreover, the affected population needs to be provided with the basic relief items as soon as possible. As the national-level resources cannot be mobilized yet, local and regional governmental organizations (GOs) are usually responsible for this [78]. For example, in Türkiye, the Turkish Red Crescent provides warm food via mobile soup kitchens in gathering areas [79]. Inventory pre-positioned prior to the disaster can also be utilized to reduce the response time unless they become unusable (e.g., due to disaster disruption or spoilage).

Table 3.2: Characteristics of the relief items distributed during disaster response.

	Phase 0	Phase 1A	Phase 1B	Phase 2
Relief type	Food, water, tent, blanket, heater	Food, water, tent kits, kitchen kits, blankets, medical items, hygiene items, clothing	Food, water, clothing, medical items, hygiene items	Food, water, clothing, medical items, hygiene items, psychosocial needs
Relief source	Local GOs and pre-positioned items	Local and national GOs, donations	Mostly donations and national GOs	GOs and long-term contracts
Inventory	-	Facility inventories	Facility and shelter site inventories	Facility and shelter site inventories
Demand frequency	Consumable and non-consumable	Consumable and non-consumable	Mostly consumable	Consumable and non-consumable

Storing excess items is usually not applicable in Phase 0 as the demand nodes are typically gathering areas and the supply nodes are mobile facilities, with no installed storage units in either of them. The usual practice is to distribute the available resources periodically, especially for relief items with recurring demand patterns.

Donation collection and distribution activities start in Phase 1A. Shelter sites are the main demand locations during relief distribution, for which both mobile and static supply points are used for delivery. In addition to the most critical items, such as food and water, items such as tent kits, kitchen kits, and clothing are also distributed. Such relief goods can be classified as non-consumable items. Once the demand is met, the person can continue using the allocated resources for an extended period of time. Consumable relief items, such as medical and hygiene items, are also provided to maintain the lives of the disaster victims at the shelter sites. Non-perishable items can be stored at the static supply units and the shelter sites. For perishable items (medical items such as vaccines and serum and perishable food such as dairy products), periodic visits to shelter sites are required, as keeping storage of these can still be inapplicable.

Phase 1B starts after the recording process of shelter site residents is completed. As most of the demand for non-consumable items, such as tent kits and

kitchen kits, is already met during Phase 1A, perishable and non-perishable consumable relief aid is usually distributed. Donation volume and, consequently, donation pollution peak during this phase, which can cause bottlenecks in the humanitarian supply chain.

During Phase 2, the relief distribution activities focus on meeting the shelter site populations' long-term needs while periodically delivering critical products. The aim is to maintain and normalize the lives of disaster victims by providing extended psychosocial, educational, and healthcare support. The form of this support can vary based on the needs of different populations, such as the elderly, children, and disadvantaged groups. Regular visits to shelter sites for healthcare services and installing mobile schools for the children can be listed as examples of such practices. Relief is mostly provided by governmental organizations and third parties via long-term contacts. Donations, however, reduce in volume and frequency due to donor fatigue.

3.2.2 Supply structure

The type of relief needed affects the demand and supply structure during the disaster response. This shapes the underlying relief distribution problems and requires customized solution methodologies, as stochastic parameters and level of uncertainty alter over time. Thus, the specification of supply and demand features is of utmost importance while defining the subphases. In this section, we analyze the supply structure in each subphase as summarized in Table 3.3.

Supply in relief distribution problems consists of not only relief aid but also vehicles, equipment, and workforce to provide critical services. Since relief aid is mainly provided by the governmental organizations during Phase 0, supply volume is mostly stable compared to the future phases when the donation variability is introduced into the system. With that being said, the disaster may disrupt the available resources pre-positioned prior to the event or the network connections such as roads and bridges, preventing the immediate mobilization of resources

Table 3.3: Supply structure during disaster response.

	Phase 0	Phase 1A	Phase 1B	Phase 2
Supply variability	Mostly stable	Unstable, mostly sourced by donations	Unstable, mostly sourced by donations	Mostly stable
Supply uncertainty	Uncertain due to a possible disruption	Uncertain due to donations	Uncertain due to donations	Mostly known
Supply availability	Usually insufficient	Usually insufficient	Usually insufficient	Usually sufficient

during Phase 0. As a result of this, available supply falls short of meeting all demand. The extent of this disruption is unknown as it depends on the scale of the disaster, which introduces supply-side uncertainty to the humanitarian network. For instance, although the size of the ambulance fleet in the area is known, the usable proportion of them or the number of ambulances that can be mobilized from each base is uncertain in the aftermath of the disaster. Nonetheless, once this uncertainty is revealed, the number of ambulances in the area will be almost constant until the national-level resources arrive at the disaster zone.

Workforce is the most crucial resource during search and rescue, which partially relies on volunteers. The proportion of volunteers increases for larger-scale disasters, as the number of professional personnel will be insufficient to reach all affected regions. This results in supply variability and uncertainty as it depends on the choice of volunteers, similar to the donation procedure. The disaster likely affects the local emergency personnel, which limits the response capacity even more.

Supply variability grows as the donation activities start during Phase 1A. The accumulated amount of donations varies for each disaster zone following the days after the disaster due to the service overlaps of different NGOs and other agents. This demand-supply mismatch amplifies when faced with limited information flow. In addition to this, embedded uncertainties in donor behavior cause supply-side uncertainties in the relief supply chain in terms of size, type, and timing [80, 2]. Once the volume of incoming donations drops due to donation fatigue in

Phase 2, supply is mostly provided by GOs and third parties through them. This reduces supply variability as the authorities utilize a more structured information and communication network. As a result of the supply chain stabilization and long-term supplier contacts, supply availability is higher compared to the other phases.

3.2.3 Demand structure

Throughout disaster response, the affected populations (e.g., demand) accumulate at varying locations, such as gathering areas and shelter sites, based on the service provided and resources available. The demand structure also changes based on this demand movement and the ongoing uncertainties. In this section, we analyze the structure of demand in each subphase as summarized in Table 3.4.

Table 3.4: Demand structure during disaster response.

	Phase 0	Phase 1A	Phase 1B	Phase 2
Demand variability	Unstable	Unstable	Stable	Unstable
Demand uncertainty	Uncertain	Uncertain	Known	Known
Demand nodes	Population points and gathering areas	Gathering areas and shelter sites	Shelter sites	Shelter sites

Disasters affect population points depending on various factors such as their distance from the epicenter, vulnerability, and population density. These population point-specific attributes result in varying disruption levels in each location and, consequently, variable and uncertain demand values [81]. During Phase 0, where the main activity is specified as search and rescue, the population points and gathering areas are the main sources of demand. Demand in both groups can be considered as uncertain as the number of affected people is unknown to a great extent. There are a vast number of studies in the literature that estimate the demand based on geographical and demographic characteristics of the population areas [58].

During Phase 1A, affected populations start to move to temporary accommodation centers, such as shelter sites. As this transition continues, both gathering areas and shelter sites are regarded as demand points. Moreover, demand parameters are still unknown and time-dependent until the registration process for the shelter sites is completed. Demand at the shelter sites can be estimated during this phase or assumed to be identical for simplicity. The latter assumption implies anonymity, where *equality* must be sought among the entities so as to establish a fair allocation. Shelter site populations stabilize once the transition is finalized, i.e., when Phase 1B starts, reducing the demand variability and uncertainty at these locations. Now that the demand points are differentiated, the assumption of anonymity no longer holds. The service among these entities should be *balanced* instead of equalized. Registration records also reveal the demographics of the shelter sites. Thus, relief should be specified for population groups, including the ones with special needs. During Phase 2, the displaced population starts to seek permanent accommodation options, which result in a decrease in the shelter site populations.

3.3 Relevant OR Problems

The underlying networks in disaster management problems usually include a set of demand nodes and resources to be mobilized to serve these nodes. In that sense, disaster operations can be formulated as OR problems. In this section, we provide a detailed discussion of relevant OR problems faced during disaster response. We list such problems in Table 3.5 while Table A.1 in Appendix A provides a comprehensive modeling framework for each problem, including decisions, main objectives, services provided, possible limitations, and uncertain parameters. Overall, this section aims to reduce the gap between theory and practice in this field by integrating real-life dynamics and challenges into the problem definitions.

3.3.1 Phase 0

During Phase 0, main decisions include vehicle routing and team scheduling to mobilize the relevant teams for damage assessment and emergency response [84]. The activities should be performed quickly and efficiently under both supply and demand uncertainty. That is to say, service completion time or demand coverage given a time limit can be optimized. Fairness is another well-studied criterion during emergency response and usually competes with the other objectives.

Table 3.5: Relevant OR problems in disaster response.

HL problem	Subphases	VR	FL	RA	TS	ND	C/P
Damage/status assessment	0	✓			✓		✓
Emergency access route design	0					✓	✓
Casualty transportation	0,1	✓					✓
Coordination center management	0,1		✓	✓	✓		
Emergency response service management	0,1	✓		✓	✓		✓
Mobile service management	0,1,2	✓	✓				✓
Workforce management	0,1,2				✓		
Shelter site management	1,2		✓	✓			
Donation management	1,2	✓	✓	✓			
Risk assessment and restoration	2	✓			✓	✓	✓
Debris management	2				✓		✓
Supplier selection	2			✓			

VR: vehicle routing, FL: facility location, RA: resource allocation, TS: team scheduling, ND: network design, C/P: clustering/prioritization

The humanitarian operations are mainly limited by the network link capacities as both the physical network and information network connections are disrupted by the disaster. For damage assessment, the use of unmanned aerial vehicles such as drone-like technologies can overcome the former when the roads and highways are blocked [85]. In this case, the problem casts as a distance-constrained vehicle routing problem due to the limited flight range of these vehicles. The roads that will be used by emergency response teams are identified upon the assessment of the damage. This can be considered as a network design problem where the aim is to maximize the reliability of the overall network (e.g., selecting open roads with minimal risk of further disruption).

Emergency response activities are usually modeled as team scheduling problems with incomplete information [86]. This can be studied in an online fashion since more information is collected as the teams continue their operations in the affected areas. One aspect that is less studied in the HL literature is the consideration of resting times for the personnel and task hand-overs, although it better reflects real-life disaster cases [87]. Another relevant problem emerges when the necessary equipment is also considered for search and rescue, which can be formulated as a joint scheduling problem assuming that the same equipment can be utilized by different teams. The operations of the first responders are usually limited by the size and capability of the teams. This plays a crucial role, especially for search and rescue activities, as such tasks require a specialized workforce depending on the type of collapse. Yet, professional teams are usually insufficient to cover all demand. This results in the utilization of a volunteer workforce with minimum to no training. This often yields a special type of scheduling problem, where the teams are assigned to tasks based on their competency levels. Another operational challenge during emergency response is the risk of aftershocks (in case of an earthquake) and secondary disasters [88, 89]. Kahramanmaraş earthquakes can be given as a recent example of this, after which over 30000 aftershocks are recorded [90].

Casualty transportation starts when people are rescued from the buildings. This includes the transportation of the patients to field hospitals for immediate medical intervention and, then, to facilities with higher capacities for a detailed examination and treatment. The latter is called patient referral, during which different modes of transportation can be utilized based on the severity level of the patients. Both types of problems are often constrained by the size of the fleet. In the HL literature, pick-up and distribution vehicle routing [91], capacity allocation [92], and prioritization (e.g., patient triage) [93] problems are often studied in the context of casualty management. Patient triage is usually coupled with functions to estimate the gradual decrease in survival probability with time as an interdisciplinary approach. Such functions add to the complexity of the problem, yet better estimate the time remaining for saving the patients [94].

3.3.2 Phase 1

Phase 1 encompasses operations such as facility management, relief distribution, personnel assignment, and scheduling to address the immediate needs of the disaster victims, such as safety, sheltering, medical care, and critical relief items. Emergency services and casualty transportation problems still continue at the beginning of Phase 1, i.e., Phase 1A. Similar to Phase 0, cost is typically not minimized. Rather, a fixed budget is considered as an operational constraint, which is dedicated to fixed and variable costs of facilities, vehicles, workforce, and item handling [95]. Predominantly considered objectives are efficiency and fairness. The former can appear as the distance to assigned facilities for location problems [91]. Moreover, a coverage radius can be set as a service requirement to ensure fair accessibility for demand nodes [96]. Relief distribution and personnel scheduling problems usually include objectives to maximize the covered demand and minimize waiting time for demand coverage. While the utilitarian use (i.e., sum) of these metrics can be considered as efficiency-centered, they may enhance equity when coupled with a fairness function such as minimax.

Relief distribution problems typically involve locating capacitated distribution centers and PoDs and distributing in-kind donations from these facilities. Such location problems can take the form of p-median (efficiency-based) and p-center (equity-based) problems. As the resources are collected (and distributed) over time, these problems can be formulated in a multi-period setting. In addition, considering multiple decision epochs enables inventory and resource pooling. Relief may be delivered via a fleet, e.g., during last-mile distribution. There are various types of routing problems in the HL literature, encompassing different constraints and objectives. If serving all demand nodes is not applicable due to capacity, relief distribution can be formulated as a selective routing problem [97]. Orienteering problems that aim to maximize the visited nodes are also quite common during this phase. Pick-up and distribution routing problems can be used if the vehicles simultaneously collect and distribute the relief from multiple depots to multiple demand locations [98].

Mobile service units are also located in Phase 1 to serve shelter sites. This is often merged with routing decisions to decide the order of demand nodes visited by each mobile unit and the time/resource devoted to these nodes [84]. Although such location-routing problems are challenging to solve, combining these decisions may significantly reduce transportation costs compared to the cases where they are attended sequentially.

3.3.3 Phase 2

In Phase 2, the focus shifts from agility to cost-minimization and sustainability. The problems faced during this phase are similar to those of recovery. To address the long-term needs of the affected population, the main activities are specified as procurement and contracting, restoration and risk assessment, waste management, and relief distribution. Contracting decisions can be modeled as supplier selection and budget allocation problems, where the capacity, lead time, and procurement cost of candidate suppliers are considered. Suppliers can also be selected in the preparation stage. Yet, determining the candidate supplier set can be challenging due to uncertainties embedded in the disaster event [99]. Supplier selection resembles a location problem due to the binary selection variables, fixed cost of contracts, and capacity limitations of the suppliers. Moreover, similar solution procedures can be adopted.

Restoration, risk assessment, and debris clearance are inherently team (and equipment) assignment problems. Here, demand nodes are prioritized based on their damage level, and the total cost of workforce and equipment is minimized. Such operations usually continue for an extended period of time, which slows down the normalization process of disaster regions. Thus, completion time should also be accounted for. The number of allocated teams or the proportion of restored structures in different disaster sites might be used as an equity measure in such problems [100].

During Phase 2, relief distribution and mobile services are periodically provided to the shelter sites, which can be formulated as periodic location or location routing problems. Owing to the similarities between the two, existing literature on maintaining and serving refugee camps can be used to model these operations. Management of both includes considering long-term needs and special population groups with the aim of designing a sustainable and reliable supply chain in a less chaotic and uncertain environment (compared to earlier phases of disaster response).

3.4 Concluding Remarks

Disaster response is traditionally divided into two time intervals: acute phase and early recovery. This is to distinguish the immediate response operations (such as search and rescue) rather than to address the evolution of problem features. This chapter provides a temporal classification framework for disaster response, which aims at attaining the latter. Unlike the existing literature, the transitions are defined by milestone events instead of the number of hours or days passed after the disaster occurrence. This enables us to address different disaster settings in which the length of operations is subject to change depending on the scale and type of the disaster.

Supply, demand, prioritized objectives, main activities, possible uncertainties, and limitations are defined and analyzed for each time bucket (i.e., subphase). Such information is crucial when planning for and responding to disasters to better manage the decision-making processes. We also discuss the relevant OR problems encountered in each subphase. By projecting the disaster management problems onto OR literature, we aim to build a comprehensive guideline for researchers in this field regarding which decisions, objectives, and constraints are relevant when formulating the corresponding mathematical models.

By building a foundation for our problem setting, this chapter also shapes the rest of this dissertation, where *allocation of in-kind donations to disaster victims during Phase 1B* is tackled. Donation accumulation and consequently the

intrinsic supply uncertainties peak during Phase 1B. This introduces a valuable methodological and conceptual challenge, which is why this subphase is addressed in this study.



Chapter 4

A Deterministic Bi-objective Location Allocation Inventory Problem for Post-disaster Donation Management

4.1 Introduction

This dissertation concerns the equitable and efficient allocation of in-kind donations in the aftermath of a disaster. The problem emerges after donation collection and distribution start (during Phase 1A) and remains relevant until the volume of in-kind gifts drops due to donor fatigue (during Phase 2). Phase 1B can be identified as the most challenging period in terms of managing the donated goods, as the donation level is much higher compared to other phases. Uncertainty on the supply side also grows due to the unpredictability of donor behavior. Moreover, supply availability does not proportionally increase with size since only a fragment of these items addresses the actual needs of the disaster-affected population. Due to these issues, the already stretched relief supply chain becomes more difficult to manage. Thus, this work focuses on Phase 1B, of

which the features are used to define the corresponding donation allocation problem. We tackle a simplified setting in this chapter, assuming that the available supply (i.e., the amount to be allocated) is known prior to the relief distribution. This assumption is to be relaxed in Chapter 5.

We consider a distribution network consisting of demand nodes (shelter sites) to be supplied over a planning horizon and candidate facility locations for delivery. Although we focus on only these two levels, they are part of a multi-echelon relief supply chain, as depicted in Figure 4.1. Both demand (disaster victims) and supply (in-kind donations) are typically mobilized in these settings. Disasters create demand in affected areas, which accumulates at shelter sites during Phase 1B. Supply is generated by a set of donors, including individuals, corporate organizations, and NGOs. The donated goods are collected and processed in a regional warehouse before being transported to local-level facilities such as distribution centers (DCs) and points of distribution (PoDs). The relief can be delivered to shelter sites by either locating DCs and routing vehicles from these facilities or locating PoDs to which the beneficiaries are expected to travel.

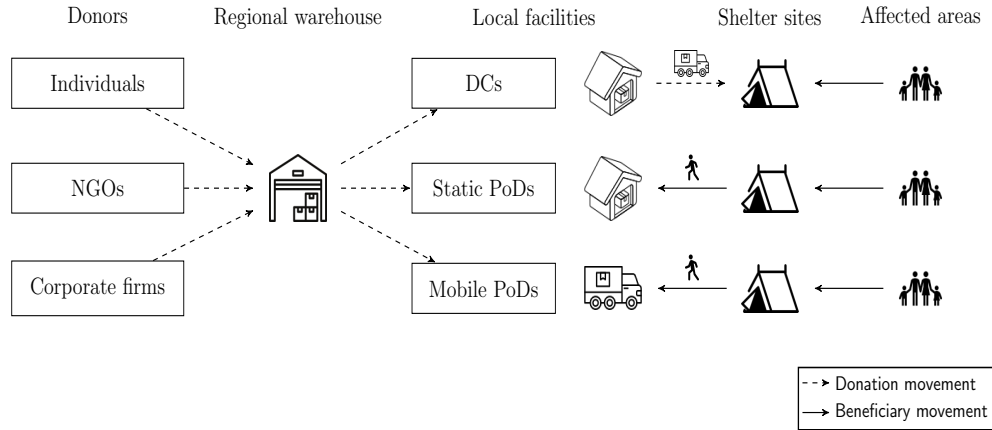


Figure 4.1: Design of the disaster relief supply chain.

Visiting shelter sites with vehicles might be challenging due to the chaotic environment at these locations and the possibility of looting. Moreover, delivery routes might be blocked due to disaster damage. Utilization of PoDs overcomes

these issues, yet it requires somehow accessible facilities for the convenience of disaster victims. This is usually done by limiting the service distance with a predetermined threshold, i.e., a coverage radius. However, available resources may not be sufficient to locate these facilities in a way that allows each demand node to have an assigned PoD within close proximity. A possible way to tackle this trade-off is to mobilize the PoDs. In this thesis, we adopt this approach: the PoDs are to be relocated in each time period so that shelter sites can receive service when a facility is located at a nearby site. We assume that the facilities are capacitated both in size and in number. They cannot store excess items in between periods due to security concerns.

Based on Phase 1B’s relief characteristics specified in Section 3.2, we consider the allocation of a consumable, non-perishable, and urgent relief commodity. Bottled water can be specified as one such item, used for both drinking and sanitization as a clean water source. Despite the accumulated plastic waste, bottled water is still preferred in disaster response. An example case is the 2004 Indian Ocean earthquake. Due to the tsunamis that followed the earthquake, the region (especially the coastal areas) suffered from immense damage to the water and sanitation system. Most non-coastal areas still had access to clean water, yet this option was inapplicable to shelter sites due to insufficient infrastructure. The disaster region relied on bottled water donations until the water distribution system was sustained to avoid water-borne diseases [101, 102].

We acknowledge that the relief supply is typically insufficient to meet the recurring demand for a relief item consumed at a high rate, such as bottled water. Thus, we expect shortages. Our aim in this work is to distribute this shortage as fairly as possible, considering the time aspect of resource allocation and the shortage amounts. We accomplish this through the use of a deprivation cost function. In each period, we penalize shortage amounts by the deprivation time, defined as the waiting time until a full-service level is provided. In other words, it is the number of consecutive time periods for which the demand node experiences a shortage. Figure 4.2 demonstrates a toy example explaining how the deprivation cost function is calculated over time given a shortage vector. As can be seen from the figure, we set the deprivation cost of each demand node in each

time period as the total amount of shortage accumulated until that point in time. When full demand satisfaction is observed, as in time period 5 in the example, the deprivation cost drops to zero. We focus on the accumulated deprivation cost so as to account for the hysteretic effects of lack of water. That is to say, if the demand is lost, the damage cannot be compensated in future time periods. Even if we observe a full demand coverage at some point, the suffering due to past shortages must be reflected in the objective function.

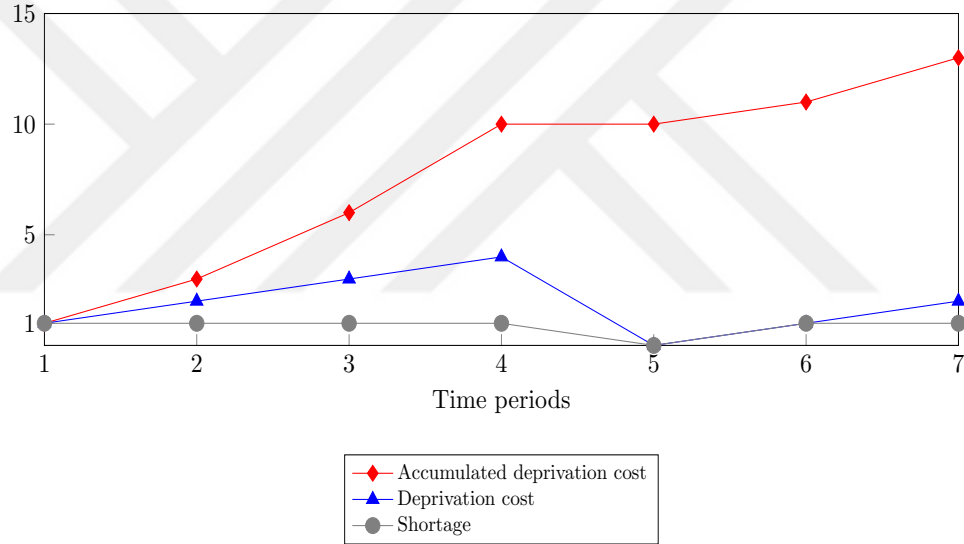


Figure 4.2: Form of the deprivation cost function.

4.2 Mathematical Formulation

In this section, we formulate the deterministic version of the donation management problem that takes place in Phase 1B. The aim is to allocate a critical good to disaster victims equitably and efficiently by locating mobile PoDs and distributing a supply of a single relief item to a set of demand nodes from these facilities over a planning horizon. To this end, assuming the supply of the distributed relief item in each period is a known amount, a deterministic multi-objective

mixed-integer programming problem (D–MOMIP) is proposed in this section. The following notation is considered while constructing the mathematical model:

Sets:

- I set of potential locations for the PoDs.
- J , set of shelter sites.
- T , set of time periods in the planning horizon.

Parameters:

- d_{jt} , demand of shelter site $j \in J$ in period $t \in T$.
- b_t , maximum number of PoDs that can be used in period $t \in T$.
- c_i , capacity of PoD $i \in I$.
- ℓ_{ij} , distance (time, road distance, etc.) between potential PoD location $i \in I$ and shelter site $j \in J$.
- γ , coverage radius for service.
- s_t , amount of supply in time period $t \in T$.
- α_j , weight associated with shelter site $j \in J$.

Since not all demand nodes are covered by all PoDs, for every demand node $j \in J$, we can compute the set of potential locations that cover j , I_j , as follows:

$$I_j = \{i \in I : \ell_{ij} \leq \gamma\}.$$

The problem can be formulated mathematically using the following sets of decision variables:

- $y_{it} = \begin{cases} 1, & \text{if PoD } i \in I \text{ is used in period } t \in T, \\ 0, & \text{otherwise.} \end{cases}$
- $x_{ijt} =$ amount delivered to shelter site $j \in J$ from PoD $i \in I$ in period $t \in T$.
- $v_{jt} =$ inventory at shelter site $j \in J$ at the end of period $t \in T$.

u_{jt} = shortage at shelter site $j \in J$ at the end of period $t \in T$ with respect to the demand in that period, d_{jt} . Additionally, we define a binary variable indicating whether shortage occurs at a node in each period:

$$w_{jt} = \begin{cases} 1, & \text{if } u_{jt} > 0, \\ 0, & \text{otherwise,} \end{cases} \quad j \in J, t \in T.$$

z_{jt} = deprivation cost at node $j \in J$ in period $t \in T$. This amount is affected by shortages both from period t and from previous periods.

As explained in Section 4.1, deprivation cost in each period for each demand node is calculated as the summation of shortage amounts in previous periods since the last time a full demand satisfaction is observed. If demand is fully satisfied at a time period, deprivation resets to zero. This calculation captures a non-linear increase in the accumulated cost value. In other words, deprivation cost for period t and shelter site j is calculated as follows:

$$z_{jt} = \begin{cases} z_{j(t-1)} + u_{jt}, & \text{if } w_{jt} = 1, \\ 0, & \text{otherwise} \end{cases} \quad j \in J, t \in T$$

where $z_{j0} = 0$ for all $j \in J$. Such a statement can be linearized by using a large enough value M with the use of an objective function that minimizes variables z :

$$z_{jt} \geq z_{j(t-1)} + u_{jt} - M(1 - w_{jt}) \quad j \in J, t \in T.$$

A possible value for M is determined as $\sum_{\tau=1}^{t-1} d_{j\tau}$ since for the case when w_{jt} and consequently u_{jt} are equal to zero, $z_{jt} \geq z_{j(t-1)} - \sum_{\tau=1}^{t-1} d_{j\tau}$ holds.

In order to establish a fair allocation in terms of allocation amounts and timing of distribution, a min-max objective function with deprivation cost values is utilized: maximum accumulated deprivation cost among all demand nodes is minimized. Such an objective may result in alternative optimal solutions; therefore, we use a secondary objective that minimizes the total deprivation cost. The variables z_{jt} can be weighted with parameters α_j in the deprivation cost-related objective functions, which further establishes fairness by capturing the priority of

demand nodes with respect to each other. Higher values of α_j imply that demand node j possesses a higher priority level.

On the other hand, since the facilities are PoDs, the distance traveled to receive the relief items is also a crucial concern. While setting a coverage radius ensures fairness in the accessibility of PoDs from shelter sites, an objective function is also utilized to ensure efficiency by minimizing the total distance traveled, weighted by the distribution amount. Although both distance and deprivation can be considered as cost elements, their ranges are quite different with respect to each other, and it is typically difficult to establish the true weight values in real life. Following this reasoning, using their weighted sum as a single objective is not preferred. Moreover, since demand satisfaction can be increased if one is willing to receive supply from a further PoD, the trade-off between total distance traveled and deprivation cost values can be observed intuitively. With the means of multi-objective optimization, the aim is to further analyze this trade-off and observe the strategies for minimizing these objectives.

Based on this discussion, three relevant objective functions are considered: the maximum deprivation cost ($f_1(y, x, v, u, w, z)$), the total deprivation cost ($f_2(y, x, v, u, w, z)$) and the total weighted distance traveled by the demand nodes ($f_3(y, x, v, u, w, z)$). For the sake of brevity, input variables will not be used throughout this chapter while denoting the objective functions. Using the introduced parameters, decision variables and objective functions; we can now formulate D-MOMIP:

$$\text{minimize } f_1(y, x, v, u, w, z) = \max_{j \in J} \left\{ \alpha_j \sum_{t \in T} z_{jt} \right\}, \quad (4.1)$$

$$\text{minimize } f_2(y, x, v, u, w, z) = \sum_{j \in J} \left\{ \alpha_j \sum_{t \in T} z_{jt} \right\}, \quad (4.2)$$

$$\text{minimize } f_3(y, x, v, u, w, z) = \sum_{j \in J} \sum_{i \in I} \ell_{ij} \left(\sum_{t \in T} x_{ijt} \right), \quad (4.3)$$

subject to:

$$\sum_{i \in I} y_{it} \leq b_t, \quad t \in T, \quad (4.4)$$

$$\sum_{j \in J: i \in I_j} x_{ijt} \leq c_i y_{it}, \quad i \in I, t \in T, \quad (4.5)$$

$$\sum_{j \in J} \sum_{i \in I_j} x_{ijt} = s_t, \quad t \in T, \quad (4.6)$$

$$v_{j(t-1)} + \sum_{i \in I_j} x_{ijt} - d_{jt} = v_{jt} - u_{jt}, \quad j \in J, t \in T, \quad (4.7)$$

$$u_{jt} \leq w_{jt} d_{jt}, \quad j \in J, t \in T, \quad (4.8)$$

$$w_{jt} \leq u_{jt}, \quad j \in J, t \in T, \quad (4.9)$$

$$v_{jt} \leq (1 - w_{jt}) \sum_{\tau=1}^t s_{\tau}, \quad j \in J, t \in T, \quad (4.10)$$

$$z_{j(t-1)} + u_{jt} - \sum_{\tau=1}^{t-1} d_{j\tau} (1 - w_{j\tau}) \leq z_{jt}, \quad j \in J, t \in T, \quad (4.11)$$

$$v_{j0} = 0, \quad j \in J, \quad (4.12)$$

$$z_{j0} = 0, \quad j \in J, \quad (4.13)$$

$$y_{it} \in \{0, 1\}, \quad i \in I, t \in T, \quad (4.14)$$

$$w_{jt} \in \{0, 1\}, \quad j \in J, t \in T, \quad (4.15)$$

$$x_{ijt} \geq 0, \quad i \in I, j \in J, t \in T, \quad (4.16)$$

$$u_{jt}, v_{jt}, z_{jt} \geq 0, \quad j \in J, t \in T. \quad (4.17)$$

The objective functions (4.1) and (4.2) account respectively for the maximum deprivation cost across all shelter sites and total deprivation cost whereas objective function (4.3) is the total weighted distance traveled by the beneficiaries to receive the assigned relief items. Inequalities (4.4) ensure the maximum number of PoDs operating in each period. Constraints (4.5) refer to the capacity of the PoDs. Inequalities (4.6) guarantee that in each time period, the total amount delivered to the demand nodes must be equal to the supply. The excess supply cannot be stored in mobile facilities, which is why this is the case. Equalities (4.7) ensure flow conservation at each demand node. Inequalities (4.8) and (4.9) define the binary indicator of shortage at a location in each period. Constraints (4.10) ensure that in each period, if a shortage occurs at a shelter site, then no amount is left in inventory. Note that the sum on the right side of this constraint defines an upper bound on the inventory amount in a node in each period in case the

inventory is positive. Expression (4.11) defines the deprivation amount at each demand node in each period. The initial inventory at the demand nodes and the initial missing amounts are set to zero by equalities (4.12) and (4.13). Finally, the domain of the decision variables is stated in (4.14)–(4.17).

In the above model, the non-linear objective function can be trivially linearized by replacing the entire objective function to be minimized by an auxiliary variable Z , which represents the maximum deprivation cost and thus, adding to the model the following constraint:

$$Z \geq \sum_{t \in T} z_{jt}, \quad j \in J. \quad (4.18)$$

Based on the preliminary analysis, it is observed that $f_1(\cdot)$ and $f_2(\cdot)$ do not imply a solid trade-off: they behave similarly due to their common tendency to increase with less demand satisfaction levels. Therefore, it makes more sense to utilize two bi-objective models, D-BOMIP1 and D-BOMIP2, which are constructed as follows:

$$\begin{aligned} D\text{-}BOMIP1 \quad & \text{minimize } f_1(\cdot) \\ & \text{minimize } f_3(\cdot) \\ & \text{subject to: } (4.4) - (4.18) \end{aligned}$$

$$\begin{aligned} D\text{-}BOMIP2 \quad & \text{minimize } f_2(\cdot) \\ & \text{minimize } f_3(\cdot) \\ & \text{subject to: } (4.4) - (4.17) \end{aligned}$$

Using these two mathematical models, one can i) analyze the trade-off between a total distance-related objective and an objective based on deprivation cost values and ii) observe the effect of utilizing a Rawlsian objective on location and allocation strategies as opposed to a utilitarian objective.

Before discussing the above multi-objective analysis, we analyze a few specific single-objective problems, namely, those emerging from some special cases.

4.3 Complexity Analysis

Similar to most discrete location problems, humanitarian facility location problems are typically NP-hard [103]. Thus, before attempting to construct the more complex stochastic counterpart, we analyze the complexity of our deterministic problem formulation in this section. To this end, the single-objective versions of the previously introduced multi-objective problem are investigated. These are the minimization of $f_1(\cdot)$ (maximum deprivation cost), $f_2(\cdot)$ (total deprivation cost), and $f_3(\cdot)$ (total distance traveled). We analyze these single-objective problems in the following subsections to elaborate on their computational complexities based on their relations to existing problems from the literature.

$$\begin{aligned} D\text{-}SOMIP1 \quad & \text{minimize } f_1(\cdot) \\ & \text{subject to: } (4.4) - (4.18) \end{aligned}$$

$$\begin{aligned} D\text{-}SOMIP2 \quad & \text{minimize } f_2(\cdot) \\ & \text{subject to: } (4.4) - (4.17) \end{aligned}$$

$$\begin{aligned} D\text{-}SOMIP3 \quad & \text{minimize } f_3(\cdot) \\ & \text{subject to: } (4.4) - (4.17) \end{aligned}$$

4.3.1 Minimization of the deprivation cost-related objectives

For this single-objective problem where we minimize the total deprivation cost, a special case with a single period is closely related to an existing problem from the logistics literature. Upon this observation, we provide a basis for the computational intractability of our single-objective problem D-SOMIP2.

For the static (single-period) problem, i.e., $|T| = 1$, the total deprivation cost reduces to the total weighted unmet demand. We make use of the notation

already introduced but ignore the time index t . The reduced model is formulated as follows:

$$\text{minimize} \quad \sum_{j \in J} \alpha_j u_j \quad (4.19)$$

$$\text{subject to} \quad \sum_{i \in I} y_i \leq b \quad (4.20)$$

$$\sum_{j \in J} \sum_{i \in I_j} x_{ij} = s \quad (4.21)$$

$$\sum_{i \in I_j} x_{ij} = d_j - u_j \quad \forall j \in J \quad (4.22)$$

$$\sum_{j \in J} x_{ij} \leq c_i y_i \quad \forall i \in I \quad (4.23)$$

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in J \quad (4.24)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (4.25)$$

For this special case with a single period, the constraint for determining the total amount distributed, (4.21), can be written as an inequality:

$$\sum_{j \in J} \sum_{i \in I_j} x_{ij} \leq s \quad (4.26)$$

The reason for this is the fact that the demand nodes do not have capacity levels to limit their assigned supply. In other words, the surplus (supply left undistributed) can be trivially sent to a demand node that can be reached from one of the opened facilities for this constraint to be held as equality. Note that this post-operation does not change the objective function value and can be performed as long as the original problem is feasible, i.e., there exists a subset of I which can carry all supply and reach at least one demand node.

The resulting problem is a *capacitated maximal covering problem (CMCP)* which falls into the class of NP-hard problems, when constraint (4.26) is set to be redundant by changing the value of the parameter s [104, 105]. Following this, we prove that the problem of minimizing the total deprivation cost is NP-hard even when the planning horizon consists of a single time period.

The problem of minimizing the maximum deprivation cost for a single time period is the minimax version of CMCP, which can be reformulated as a *capacitated p-center problem* [105]. This is considered NP-hard; thus, D-SOMIP1 can be classified as an NP-hard problem.

4.3.2 Minimization of the total transportation distance

Minimizing only the total weighted distance traveled to distribute the allocations yields a simpler version of the problem introduced in Section 4.2. In this case, decision variables other than y and x can be considered as auxiliary variables since they are irrelevant to the selected objective and thus can be omitted. The problem reduces to the following:

$$\begin{aligned} \text{minimize } f_3(y, x) &= \sum_{j \in J} \sum_{i \in I_j} \ell_{ij} \left(\sum_{t \in T} x_{ijt} \right), \\ \text{subject to: } \sum_{i \in I} y_{it} &\leq b_t, & t \in T, \end{aligned} \quad (4.4)$$

$$\sum_{j \in J: i \in I_j} x_{ijt} \leq c_i y_{it}, \quad i \in I, t \in T, \quad (4.5)$$

$$\sum_{j \in J} \sum_{i \in I_j} x_{ijt} = s_t, \quad t \in T, \quad (4.6)$$

$$y_{it} \in \{0, 1\}, \quad i \in I, t \in T, \quad (4.14)$$

$$x_{ijt} \geq 0, \quad i \in I, j \in J, t \in T. \quad (4.16)$$

This model can be fully decomposed into $|T|$ problems, one for each time period $t \in T$. Each such sub-problem, denoted by P_t , can be formulated mathematically as follows:

$$\begin{aligned} P_t \quad \text{minimize } f_{3t}(y, x) &= \sum_{j \in J} \sum_{i \in I_j} \ell_{ij} (x_{ijt}), \\ \text{subject to: } \sum_{i \in I} y_{it} &\leq b_t, \end{aligned} \quad (4.27)$$

$$\sum_{j \in J: i \in I_j} x_{ijt} \leq c_i y_{it}, \quad i \in I, \quad (4.28)$$

$$\sum_{j \in J} \sum_{i \in I_j} x_{ijt} = s_t, \quad (4.29)$$

$$y_{it} \in \{0, 1\}, \quad i \in I, \quad (4.30)$$

$$x_{ijt} \geq 0, \quad i \in I, j \in J. \quad (4.31)$$

Given that the optimal objective function value is f_{3t}^* for problem P^t , the optimal objective function value for the original problem D-SOMIP3 is $\sum_{t \in T} f_{3t}^*$.

Suppose that the set of facilities opened in an optimal solution to the above model P_t is I_t^* . Without loss of generality, we assume this is a minimal set of optimal facility locations. In other words, if a PoD is located, it must serve at least one demand point. If this is not the case, the facilities satisfying $\sum_{j \in J} x_{ijt} = 0$ can be simply removed from the set I_t^* . Note that neither does this operation harm feasibility nor does it change the objective function value.

Without loss of generality, let us reorder the facilities in set $I_t^* = \{1, \dots, m\}$ non-decreasingly in the values $\hat{\ell}_i = \min_{j \in J} \ell_{ij}$, i.e.

$$\hat{\ell}_1 < \hat{\ell}_2 < \dots < \hat{\ell}_m.$$

Due to the assumption that each opened facility is within reach of at least one demand node, we ensure that all values on this sorted set are less than or equal to the service distance threshold, i.e. $\hat{\ell}_i \leq \gamma, \forall i \in I_t^*$.

For the sake of simplicity, we further assume that none of the facilities has the same $\hat{\ell}$ value. If this is not the case, we rearrange the set I_t^* merging these facilities owing to the following logic: in the solution that minimizes the total distance traveled, decisions regarding the inventory, shortage, and deprivation amounts of the demand nodes are disregarded. Hence, it is safe to assume that two or more demand nodes can be combined as one unit as long as they are the same distance from the demand node closest to them. Thus, such nodes can be represented as a single demand node while minimizing the total transportation distance.

Proposition 4.3.1. *Given that the optimal solution to problem P_t has the minimal opened set of facilities I_t^* , a feasible allocation vector x is optimal only if it*

satisfies the following condition:

$$\sum_{j \notin J_i} x_{ijt} = 0 \quad \forall i \in I_t^*$$

where $J_i = \{j \in J | \ell_{ij} = \hat{\ell}_i\} \quad \forall i \in I_t^*$ i.e., demand nodes in set J_i are the closest demand nodes to the located facility i . This condition states that each opened facility only serves the demand points closest to them.

Proof. Suppose that there is an optimal allocation vector x^* for which this condition does not hold, i.e., $\exists i' \in I_t^*, j' \notin J_{i'}$ such that $x_{i'j't}^* = \epsilon_1 > 0$ (There is an open facility that serves a demand node that is not one of the closest options).

We want to show that reallocation of the positive amount ϵ_1 to a demand node $j^0 \in J_{i'}$ is feasible and the resulting objective function value is at least as good as that of x^* .

Consider an allocation vector, x^0 , such that:

- i) $x_{i'j't}^0 = 0$,
- ii) $x_{i'j^0t}^0 = x_{i'j^0t}^* + \epsilon_1$
- iii) $x_{i'jt}^0 = x_{i'jt}^* \quad \forall j \in J \setminus \{j', j^0\}$ and
- iv) $x_{ijt}^0 = x_{ijt}^* \quad \forall i \in I_t^* \setminus \{i'\}, \forall j \in J$.

Capacity and coverage radius requirements are obeyed as in constraints (4.5). All supply is distributed as stated imposed by equality (4.6). Finally, as in constraints (4.16), all allocation amounts are non-negative. Therefore, the allocation vector x^0 is feasible. The objective function value of such a vector x^0 would be:

$$f_{3t}(x^0) = \sum_{i \in I_t^*} \sum_{j \in J} \ell_{ij} x_{ijt}^0 = \sum_{i \in I_t^* \setminus \{i'\}} \sum_{j \in J} \ell_{ij} x_{ijt}^* + \sum_{j \in J \setminus \{j^0, j'\}} \ell_{i'j} x_{i'jt}^* + \hat{\ell}_{i'}(x_{i'j^0t}^* + \epsilon_1)$$

The objective function value of the solution x^* is given below.

$$f_{3t}(x^*) = \sum_{i \in I_t^*} \sum_{j \in J} \ell_{ij} x_{ijt}^* = \sum_{i \in I_t^* \setminus \{i'\}} \sum_{j \in J} \ell_{ij} x_{ijt}^* + \sum_{j \in J \setminus \{j^0, j'\}} \ell_{i'j} x_{i'jt}^* + \hat{\ell}_{i'}(x_{i'j^0t}^*) + \ell_{i'j'}(\epsilon_1)$$

Since $\hat{\ell}_{i'} < \ell_{i'j'}$, the objective function value resulting from the allocation vector x^0 is better, which contradicts with the optimality of vector x^* . Therefore, such a solution x^* cannot be optimal. $\square$

Considering the condition given in Proposition 4.3.1, the location-allocation problem for period t can be rewritten as follows:

$$\begin{aligned}
P_t \quad & \text{minimize} \quad f_3(x) = \sum_{i \in I} \hat{\ell}_i x_{it}, \\
& \text{subject to:} \quad \sum_{i \in I} y_{it} \leq b_t \\
& \quad \quad \quad \sum_{i \in I} x_{it} = s_t, \\
& \quad \quad \quad 0 \leq x_{it} \leq y_{it} c_i, \quad i \in I \\
& \quad \quad \quad y_{it} \in \{0, 1\},
\end{aligned}$$

where $x_{it} = \sum_{j \in J_i} x_{ijt}$. This problem is a variant of a *cardinality constrained knapsack problem (CCKP)* which is known to be NP-hard [106]. Therefore, we prove that the single-objective problem D-SOMIP3 is NP-hard since it can be decomposed into $|T|$ NP-hard problems.

Overall, we have shown that the two single-objective versions of the multi-objective problem D-MOMIP fall into the class of NP-hard problems. The complexity of the problem grows when the deprivation cost values are calculated in a multi-period setting.

4.4 Case Study

In this section, we solve the deterministic bi-objective problems D-BOMIP1 and D-BOMIP2 using a real data set adopted from the literature. First, we explain the solution methodology to address the bi-objective nature of the problems. Then, the problem data is constructed. We analyze the Pareto frontiers obtained after solving problems D-BOMIP1 and D-BOMIP2 and the trade-off between

the selected objectives, namely the efficiency- and deprivation-cost-related ones. Finally, we evaluate the value of mobility in this deterministic setting. That is, we compare the results of our models to those of the problem of locating static PoDs - without relocation decisions. In addition to the case study we explain in this section, we test our model with a large-size instance using a real data set from Kilis province. We explain the data and the resulting Pareto frontiers in Appendix D.

4.4.1 Solution methodology

We adopt a well-known procedure for finding all the Pareto solutions of a vectorial optimization problem, ε -constraint method [107], where a single objective model is considered using one of the objective functions and setting a threshold constraint for the other. Such a model would guarantee that we find a non-dominated solution in the weak sense. A second step is utilized to ensure obtaining a non-dominated solution, where the resulting optimal value is used as a constraint while the other objective is minimized. Next, we explain the procedure for D-BOMIP1, given that the same steps are to be applied also to D-BOMIP2.

For model D-BOMIP1, ε -constraint method requires solving the following models iteratively:

$$\begin{aligned} \min \quad & f_3(.) \\ \text{s.t.} \quad & f_1(.) \leq \varepsilon_1 \\ & (4.4) - (4.18), \end{aligned}$$

and

$$\begin{aligned} \min \quad & f_1(.) \\ \text{s.t.} \quad & f_3(.) \leq \varepsilon_2 \\ & (4.4) - (4.18), \end{aligned}$$

where ε_1 is reduced iteratively by a step size, and ε_2 is set as the optimal value of the first model at each iteration. The use of a Rawlsian objective as represented by function $f_1(.)$ may yield alternative optimal solutions. As we discussed above, among these solutions, the ones with less total deprivation cost value are more

desirable. Therefore, despite having observed that objective functions $f_1(.)$ and $f_2(.)$ are not much conflicting, we decided to post-process the Pareto solutions obtained by the ε -constraint method when applied to (D-BOMIP1) so that we keep those solutions that are the best with respect to total deprivation cost (among alternative solutions for $f_1(.)$).

Customarily, the ε -constraint method starts by finding the payoff table, i.e., the two most extreme points in the Pareto front. This allows finding the range of interest for both objective functions and, in particular, the range of interest to ε_1 . Then, we calculate a step size using the range of objective function $f_1(.)$ to define the value of ε_1 . Since ranges of objective functions are observed to be quite large, the algorithm performs a pre-determined number of iterations (N) so as to uniformly cover the true Pareto front. The step size is calculated by dividing the range for the objective $f_1(.)$ by the number of iterations. By doing so, at most N Pareto-optimal points are obtained after the algorithm runs.

Upon obtaining a Pareto front, we consider the objective function that was neglected until here: $f_2(.)$, namely the total weighted deprivation cost. To ensure that we end up with a non-dominated solution for the three-objective problem, we take each Pareto solution found and optimize the second objective restricted to not deteriorating the values obtained for the other two in this solution. The solution procedure is the same for D-BOMIP2, except for this post-process.

4.4.2 Problem data

The location-allocation-inventory problem is applied to a real-life data set of Kartal district, which is located in Istanbul, Türkiye, with approximately 475,000 inhabitants [108]. This data set is first introduced by Kılıcı et al. [109] and includes 20 sub-districts and 25 rallying points with available population information and node-to-node distance matrix. Of these 45 points, 37 are population points, and 8 are public service facilities such as schools and hospitals. For this case study, we choose 6 non-population points as potential mobile PoD locations after eliminating the closely-located ones, whereas 37 population points are considered

as the demand points. A planning horizon with 7 periods (e.g., 7 days, a full week) is adopted. Thus, models D-BOMIP1 and D-BOMIP2 are tested on a real network with 37 demand nodes, 6 potential depot locations, and 7 periods ($|J| = 37, |I| = 6, |T| = 7$) and the corresponding Pareto fronts are obtained.

The sets I and J , as well as the parameters ℓ_{ij} , are directly taken from the Kartal data set. Weights regarding the priority levels, α_j , are set to 1. The coverage radius, γ , is set as the 30th percentile of the distance values found in matrix $\ell = (\ell_{ij})$.

Other parameters are generated as follows. First, we retrieve the actual sizes of the populations (Pop_j). For generating the remaining data, we define three parameters: (i) demand to population ratio for each node for each period (θ_{jt}), (ii) total supply to total demand ratio (β_t), and (iii) the pre-determined percentage of depots that can be opened (λ_t) for each period. Table 4.1 shows the values set for these parameters.

Table 4.1: Preparation of demand and supply related parameters.

θ_{jt}	Demand to population ratio for demand node $j \in J$ in period $t \in T$	U(0.1, 0.3)
β_t	Total supply to total demand ratio in period $t \in T$	U(0.5, 0.7)
λ_t	Percentage of PoDs that can be opened in period $t \in T$	50%

The demand-to-population ratio was generated according to a uniform distribution in the interval (0.1, 0.3). This is in line with what was suggested by Kılçı et al. [109]. The total supply to total demand ratio was generated according to a uniform distribution in interval (0.5, 0.7) considering the scarcity of supply with respect to the demand.

Using the above information, a procedure was devised for computing d_{jt} , s_t , b_t , and c_i , for all $i \in I$, $j \in J$, $t \in T$. This procedure is detailed in Appendix B.

4.4.3 Results

Two bi-objective programming problems, D-BOMIP1 and D-BOMIP2, are solved with ϵ -constraint algorithm as detailed in Section 4.4.1 using the data set explained in Section 4.4.2. The algorithm is coded in Java 1.8.0 using CPLEX 12.71, and each model is given a time limit of 70 minutes. The resulting Pareto frontiers are plotted in Figures 4.3a and 4.3b. For model D-BOMIP1, 48 Pareto-optimal points are obtained. The CPU time required to construct the Pareto front is below 200 minutes. For model D-BOMIP2, we find 80 Pareto solutions in approximately 40 minutes. Note that the ranges for all three objectives are quite large. For this reason, all values were re-scaled to interval $[0, 100]$. For instance, if an objective function takes values from a range (a, b) a being the minimum, a point with objective function value c is now re-scaled as follows: $100 \times \frac{c-a}{b-a}$.

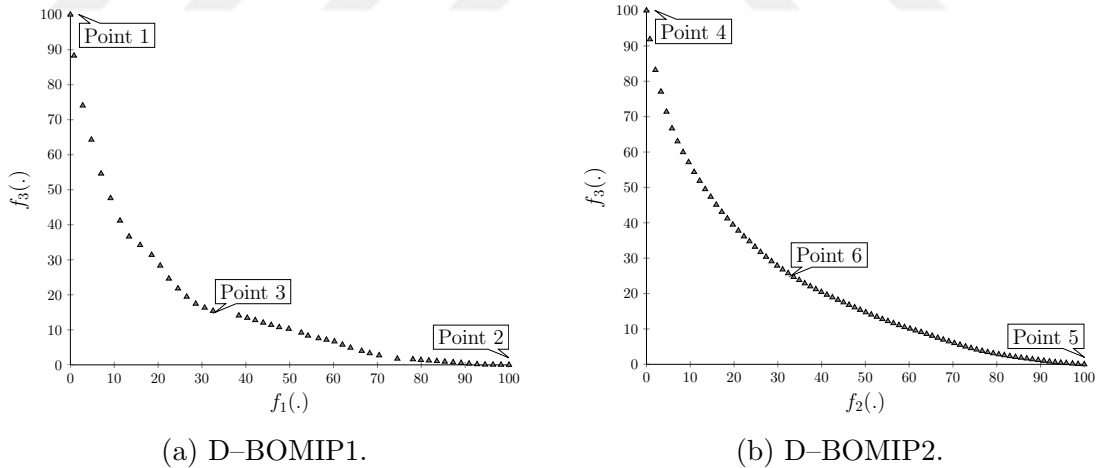


Figure 4.3: Pareto frontiers obtained for the bi-objective mathematical models.

Payoff tables for the two bi-objective problems are given in Table 4.2. These correspond to points 1 and 2 in Figure 4.3a for D-BOMIP1 and to points 4 and 5 in Figure 4.3b for D-BOMIP2. As explained before, the payoff table defines the range of interest for the objective functions. The tables indicate that optimizing the distance-related objective ($f_3(\cdot)$) significantly worsens the deprivation cost values. Yet, minimizing deprivation cost (especially with a utilitarian approach) does not cost that much increase in the efficiency objective, i.e., the total

distance. That is to say, deprivation costs can be substantially reduced without imposing much larger travel distances for beneficiaries. This owes to the coverage radius requirement in our problem, as we already ensure a maximum level for the distance-related objective via such a constraint. This is also observed in Pareto frontiers. For example, by reducing the maximum deprivation cost to around 70% of the worst value (the value given by Point 2), our loss regarding the total distance objective is marginal. This suggests that there are near-optimal solutions (for the distance-related objective) with a higher level of equity. Thus, instead of solely optimizing efficiency, equity should also be accounted for in these types of allocation problems. Another critical observation is that optimizing the total distance ($f_3(\cdot)$) costs much more for the minimax objective ($f_1(\cdot)$) than it does for the total deprivation cost ($f_2(\cdot)$). This indicates that its trade-off to the maximum deprivation cost is higher than to total deprivation cost.

Table 4.2: Payoff tables for bi-objective problems.

D-BOMIP1			D-BOMIP2		
Minimization of	Deviation from		Minimization of	Deviation from	
	$f_1^*(\cdot)$	$f_3^*(\cdot)$		$f_2^*(\cdot)$	$f_3^*(\cdot)$
$f_1(\cdot)$	-	209%	$f_2(\cdot)$	-	44%
$f_3(\cdot)$	1540%	-	$f_3(\cdot)$	675%	-

An important concept to our discussion is the so-called *the price of fairness* (PoF), which represents the loss on the utilitarian objective when the solution of the inequity-averse counterpart is applied [40]. Thus, for our donation allocation problem, it is calculated as follows:

$$PoF = \frac{f_2(x_1^*) - f_2^*(\cdot)}{f_2^*(\cdot)},$$

where x_1^* is the optimal solution that minimizes $f_1(\cdot)$. The value is computed as 9%. If the decision-maker is willing to pay this price, the equity over the deprivation cost values can be maximized instead of the total deprivation cost.

Next, considering the selected points in Figure 4.3, we analyze the corresponding solution vectors and the location and allocation policies favoring each objective function. Since these extreme points correspond to extreme cases where one of the objectives is almost completely neglected, on each Pareto front, a point towards the center of the curve (points 3 and 6) is also analyzed.

The solutions are depicted on the map of Kartal district for selected time periods of the planning horizon, as can be seen in Figures 4.4–4.8 (see Appendix C for solutions of all time periods). In these figures, demand nodes are represented by circles with sizes proportional to their demand, while the potential PoD locations are shown as rectangles. Location (y) and allocation (x) variables are used to analyze the solutions since the other variables regarding inventory (v), shortage (u), and deprivation cost (z) can be deduced using y and x values. Considering the allocation amounts, fill rates (percentage of satisfied demand) are visually represented on these maps as pie charts placed on each demand point. Diagonally patterned rectangles represent the open PoDs.

The solution for point 1 is given in Figure 4.4 for two consecutive time periods. It can be seen that various combinations of PoD locations are used to cover more demand points and minimize $f_1(.)$. In the 7 periods considered, 6 different location patterns were used. In periods 1, 3, 5, and 7, the upper part of the map is generally covered, while in periods 2, 4, and 6, demand points located on the lower side of the map are more covered. Due to the minimax objective, demand points that have not been visited lately are prioritized to limit the increase in their deprivation cost values. Partial demand satisfaction is observed frequently, which indicates that the strategy to minimize this Rawlsian function considers all demand nodes individually instead of relying solely on the total demand satisfaction level.

The solution corresponding to points 2 and 5, for which $f_3(.)$ is minimized without considering the other objectives, is intuitive: since all supply should be distributed to demand nodes, the PoDs that can carry this amount with their combined capacity are opened. Then, the supply assigned to each of these PoDs is allocated to the closest demand node. Regardless of the demand levels of nodes,

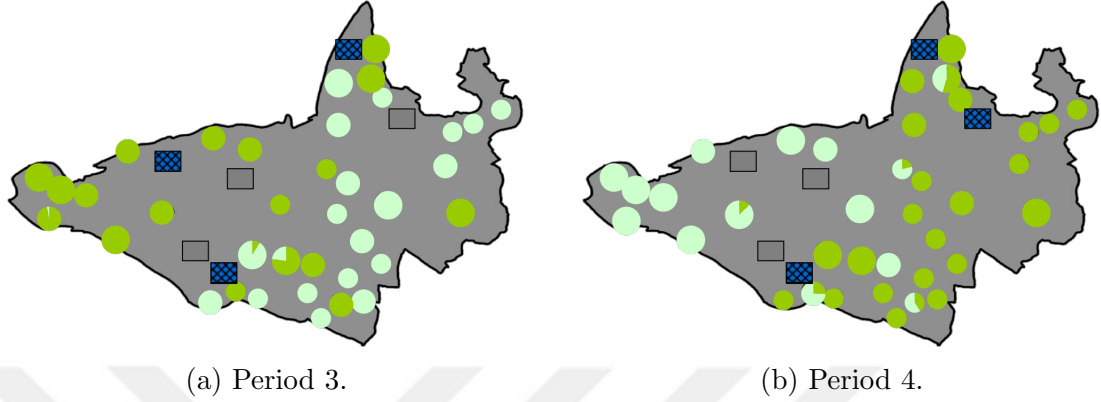


Figure 4.4: Operating PoDs and fill rates obtained by the solution corresponding to Pareto optimal point 1.

the model seeks to distribute all supply with minimum distance traveled. Hence, the same location and allocation decisions are made in each period. A visual representation of the operating PoDs and the fill rates for the first period is given in Figure 4.5. The exact location pattern with the same allocation structure is observed in the other time periods.

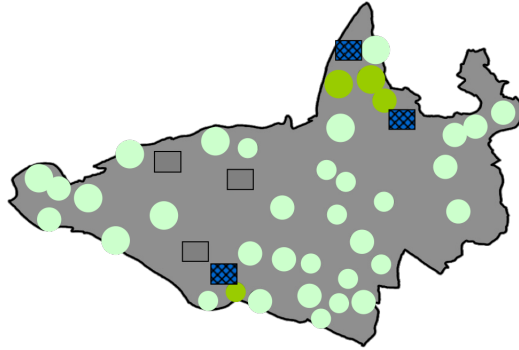


Figure 4.5: Depots and fill rates obtained by the solution corresponding to Pareto optimal points 2 and 5.

The solution corresponding to point 3 can be considered as a mixed strategy of the location-allocation policies mentioned above. The demand nodes closest to a PoD are provided a full demand satisfaction in each period, while other demand points are also visited to decrease their accumulated shortage. By doing so, the maximum deprivation cost is decreased. However, one can still observe some demand nodes that are not visited in any time period due to the distance limit imposed by the ε -constraint. Throughout the planning horizon, two different

location patterns are used as we can observe in Figure 4.6: in periods 1, 2, 4, 6 and 7, the same mobile PoDs are opened with allocation strategies similar to the solution corresponding to points 2 and 5 while more dispersed allocation schemes are adopted in periods 3 and 5 with a different location pattern. We also observe that optimizing the total deprivation cost does not necessarily ensure its fair allocation to demand nodes, as previously discussed by Gutjahr and Fischer [57]. This is demonstrated in Figure 4.7, where the accumulated deprivation cost levels are depicted for the solutions corresponding to Pareto optimal points 1 and 3. The Gini coefficient of the corresponding vectors are 0.33 and 0.03, respectively, when the total and the maximum deprivation cost are minimized.

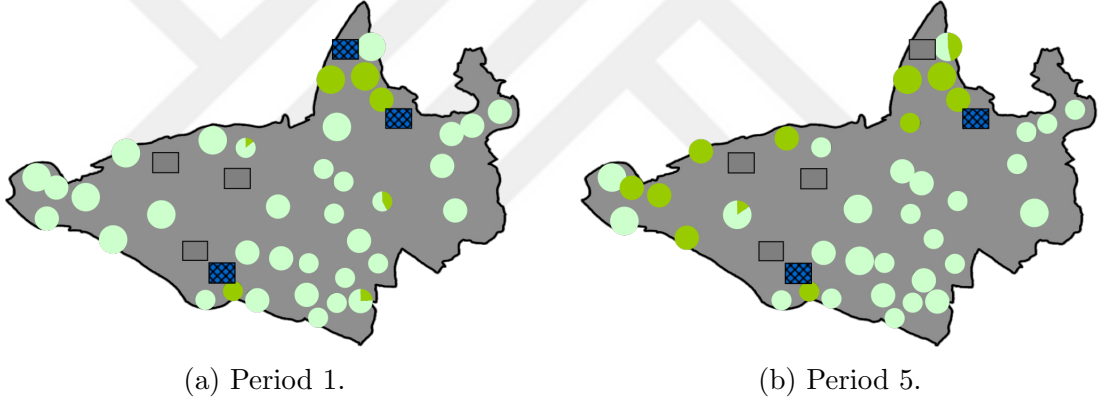


Figure 4.6: Operating PoDs and fill rates obtained by the solution corresponding to Pareto optimal point 3.

The solution corresponding to point 4, minimizing the utilitarian objective $f_2(\cdot)$ regarding deprivation costs demonstrates a similar policy to that of minimizing $f_1(\cdot)$: in each consecutive two periods, either the upper or the lower side of the map is more covered, while the location pattern changes throughout the planning horizon. Location and allocation schemes for such two periods are given in Figure 4.8. A remarkable difference is that now, partial demand satisfaction occurs less while binary fill rates are observed more frequently. The reason for this situation can be explained with an example: Suppose there are two entities sharing a single resource, and there are two allocation vectors $(1, 99)$ and $(50, 50)$. These two allocation vectors are the same in terms of a utilitarian objective since the total amount is the same. Therefore, decreasing deprivation cost for multiple demand nodes yields the same objective value as decreasing deprivation cost for

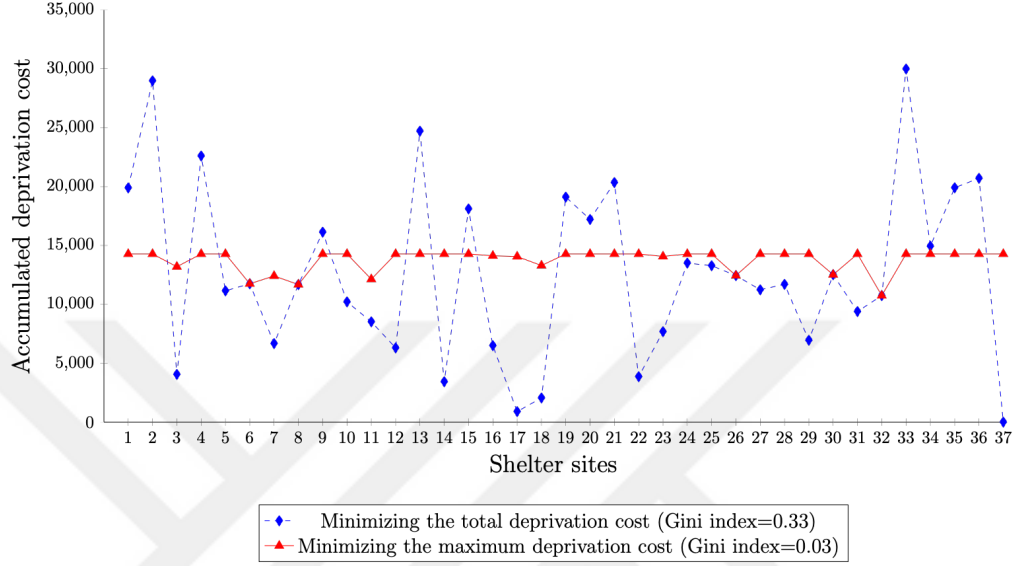


Figure 4.7: Accumulated deprivation cost levels when $f_1(\cdot)$ and $f_2(\cdot)$ are optimized.

a single node when the total amount decreased stays the same. Moreover, partial demand satisfaction becomes less attractive compared to the Rawlsian objective function $f_1(\cdot)$.

Similar to the solution represented by point 3, location and allocation decisions include different patterns in the solution corresponding to point 6, where both the total transportation distance and the total deprivation cost are considered. Binary fill rates are observed more often since a utilitarian objective is included in the model instead of minimizing the maximum deprivation cost among all demand nodes. A particular set of demand points (served points while minimizing the total distance objective, i.e., solution of point 5) are always provided with fill rate 1 in some periods (1 and 7). In these time periods, the solutions are identical to that of point 5 to reduce the total transportation distance. However, in other periods, other candidate locations are also used to serve the remaining demand nodes. The resulting location and allocation vectors for two periods are visualized in Figure 4.9. The utilization of various location allocation schemes indicates a mixed policy that aims to balance two objective functions.

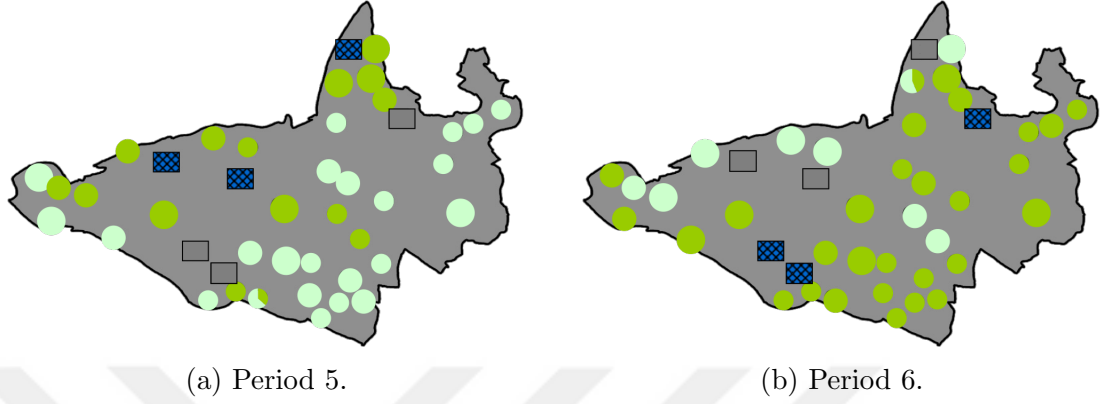


Figure 4.8: Operating PoDs and fill rates obtained by the solution corresponding to Pareto optimal point 4.

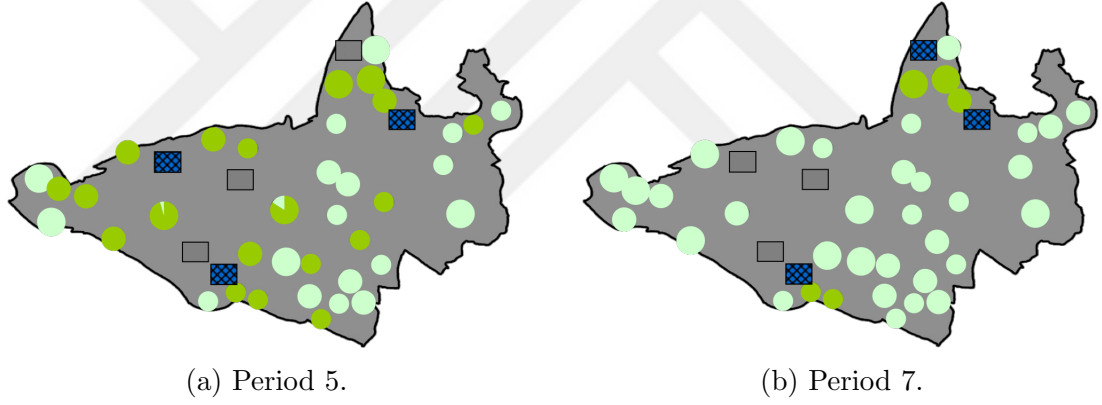


Figure 4.9: Operating PoDs and fill rates obtained by the solution corresponding to Pareto optimal point 6.

4.4.4 Value of mobility

Considering the increase in the deprivation cost as demand nodes are kept underserved for a long time, frequent deliveries to shelter sites become crucial. Under the imposed coverage radius, locating static supply facilities may lead to neglecting some demand nodes throughout the planning horizon. To this end, the proposed model allows supply facilities to change locations dynamically. In this section, we analyze the effect of a dynamic location/relocation of depots on deprivation cost-related objectives. If we neglect the time-dependency of the y -variables, we can remove the index t from those variables. In this case, the

problem defined in Section 4.2 is reduced to a static one where the supply facilities are located at the beginning of the planning horizon and always operate from there. We obtain the Pareto frontiers for D-BOMIP1 and D-BOMIP2 for both static and dynamic settlements, as depicted in Figure 4.10.

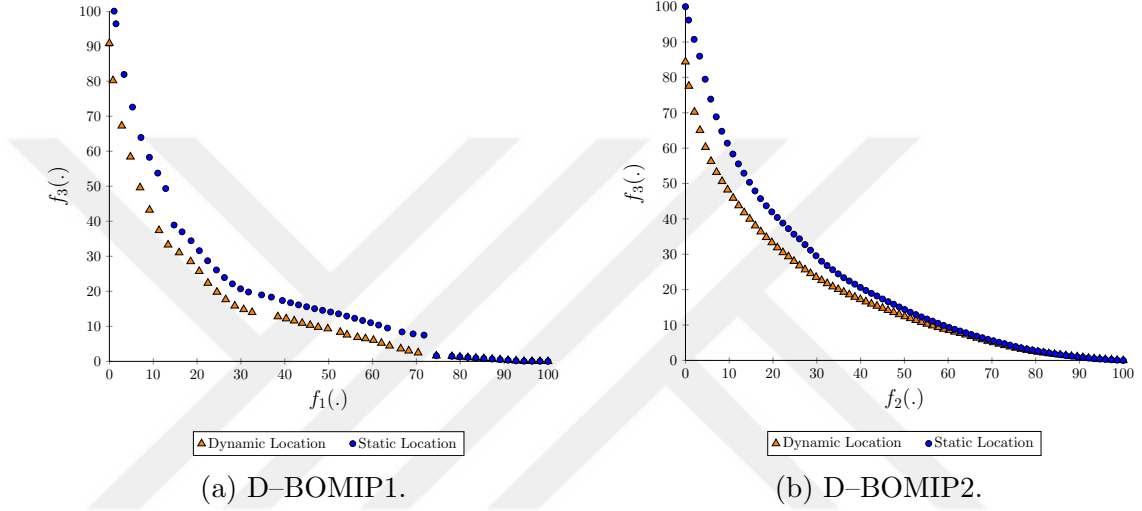


Figure 4.10: Pareto frontiers of the bi-objective mathematical models with dynamic and static location decisions.

When the facilities are not relocated, the minimum levels for $f_1(\cdot)$ and $f_2(\cdot)$ can still be achieved as observed in the above Pareto frontiers. However, utilizing dynamic location decisions leads to smaller total distance values compared to the use of static facilities. For any Pareto solution using the static model, there is a Pareto solution that is at least as good as it (with respect to both criteria) when the dynamic model is adopted. In both figures, we observe that when the value of the deprivation cost-related objective function is fixed, this value can be attained by traveling a shorter total distance in the dynamic location setting. In Figure 4.10, we also see that when we move towards the minimum distance edge, both static and dynamic settings result in the same solution, which is expected since the solution that minimizes $f_3(\cdot)$ has the same location pattern in each period. In other words, the difference between static and dynamic PoDs diminishes when near-optimal solutions for the distance-related objective are sought.

4.5 Concluding Remarks

In the humanitarian logistics domain, location-relocation models can provide a more advanced coping mechanism against the scarcity of resources and may increase the equity of service. Following this idea, we construct a deterministic multi-objective multi-period location-relocation-allocation model considering inventory decisions. We utilize the concept of deprivation cost in utilitarian and Rawlsian-type objective functions. We also consider an efficiency objective as a distance-related function of allocated amounts. A coverage radius is imposed as a service requirement to the demand nodes, and the total number of facilities positioned in each period is limited as a form of budget constraint. Upon observing the correlation between two of the three objective functions, namely the deprivation cost-centered ones, we consider two bi-objective models.

The models are applied to a real network of Kartal district. After acquiring the true Pareto fronts using ε -constraint method, we analyze the trade-offs between the objective functions. Results indicate a higher level of trade-off when we attempt to minimize the maximum deprivation cost alongside the total distance compared to the total deprivation cost. However, the improvement we gain for the equity objective (minimax deprivation cost) is significant when near-optimal solutions for the distance-related objective are accepted. This gives a valuable insight into the resource allocation problems in humanitarian settings: instead of optimizing only an efficiency objective and neglecting equity altogether, one should search for good-quality solutions (without sacrificing much from the optimal efficiency level) that are also equitable.

We observe that the solutions leading to the pay-off table show distinct location and allocation schemes. When the aim is minimizing the maximum deprivation cost among the demand nodes, PoDs are used in a more mobile pattern, i.e., the location setting is changed in every period. If a specific subset of the demand nodes is covered more in one period, others are compensated in future periods to the extent of supply availability. Additionally, partial demand coverage is frequently observed. In contrast, binary demand satisfaction is more encountered

while minimizing the total deprivation cost. We observe a trivial solution when the only concern is to minimize the total distance traveled: the closest demand nodes are allocated all of the supply. As expected, shortage and deprivation cost values are excessively high in this case. The solutions that attach importance to both objectives display mixed strategies regarding the location and allocation decisions.

We compare the mobile facility setting with traditional static supply facilities to analyze the value of relocation decisions. We observe that when using static PoDs, a higher total distance is traveled to obtain the same amount of deprivation cost. This proves that relocation decisions in our problem enable us to reduce transportation distance (or cost) for beneficiaries while covering the demand of shelter sites.

In addition to Kartal data, we apply our model to a large-size network from Kilis province. One critical observation is that, for large instances, finding the Pareto-optimal solutions becomes computationally exhausting as the maximum deprivation cost threshold gets smaller in the ε -constraint algorithm. This suggests that solving the model for the maximum deprivation cost objective is computationally more challenging (see Appendix D).

Chapter 5

A Multi-stage Stochastic Location Allocation Inventory Problem for Post-disaster Donation Management under Supply Uncertainty

5.1 Introduction

A significant proportion of relief supply is sourced by donations from institutions and individuals during disaster response. Thus, the amount of donations is unknown prior to relief distribution, which results in an exogenous uncertainty in the supply parameters. In this chapter, we address this supply uncertainty for the donation allocation problem that takes place in Phase 1B, where the aim is to ensure fairness among the demand nodes. The problem setting remains the same as the deterministic model (see Section 4.1). At the beginning of each time period, the donations are collected at a central depot (e.g., a regional warehouse) to be processed (sorted, inspected, and packaged). The availability of supplies

for this specific time period is unknown until these operations are finalized. As the possible PoD locations are typically parking lots and public buildings such as schools and hospitals, the involved third parties must be informed on the location of PoDs at the beginning of each time period. Thus, the location decisions take place before the donations are processed, i.e., prior to the supply realization. After this information is revealed, the relief supply is distributed to the demand nodes, i.e., shelter sites, via mobile units. The sequence of decisions with respect to the information arrival is depicted in Figure 5.1.

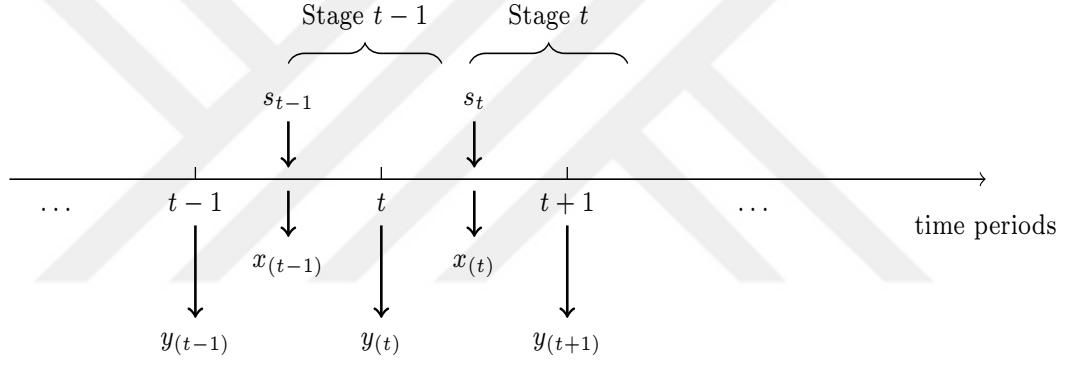


Figure 5.1: Timeline of events and arrival of new information during the planning horizon.¹

In this chapter, we formulate our problem using a multi-stage stochastic approach. Then, the model is applied to a real network of the Kartal district to analyze the resulting solution schemes. The problem we tackle in this chapter is computationally intractable; thus, we employ two matheuristic approaches to solve large-size instances. A set of computational tests is conducted to evaluate the performance of the proposed heuristics. Finally, we assess the relevance of our approach using two information measures we adopt from the literature: the expected value of perfect information and the dynamic value of stochastic solution.

¹ s_t denotes the supply amount for period t while location and allocation decisions are represented by $y_{(t)}$ and $x_{(t)}$, respectively.

5.2 Problem Formulation

The problem described in the previous section can be formulated and solved with multi-stage stochastic mixed-integer optimization. This is justified by the fact that some information is gradually revealed in each period, and the decisions are implemented sequentially [72, 110, 111].

The source of uncertainty in our problem is the supply amount available to be distributed in each time period. We assume that such uncertainty can be represented by a scenario tree, which describes the underlying discrete-time stochastic process or a sample drawn from it. An example of such a tree is depicted in Figure 5.2.

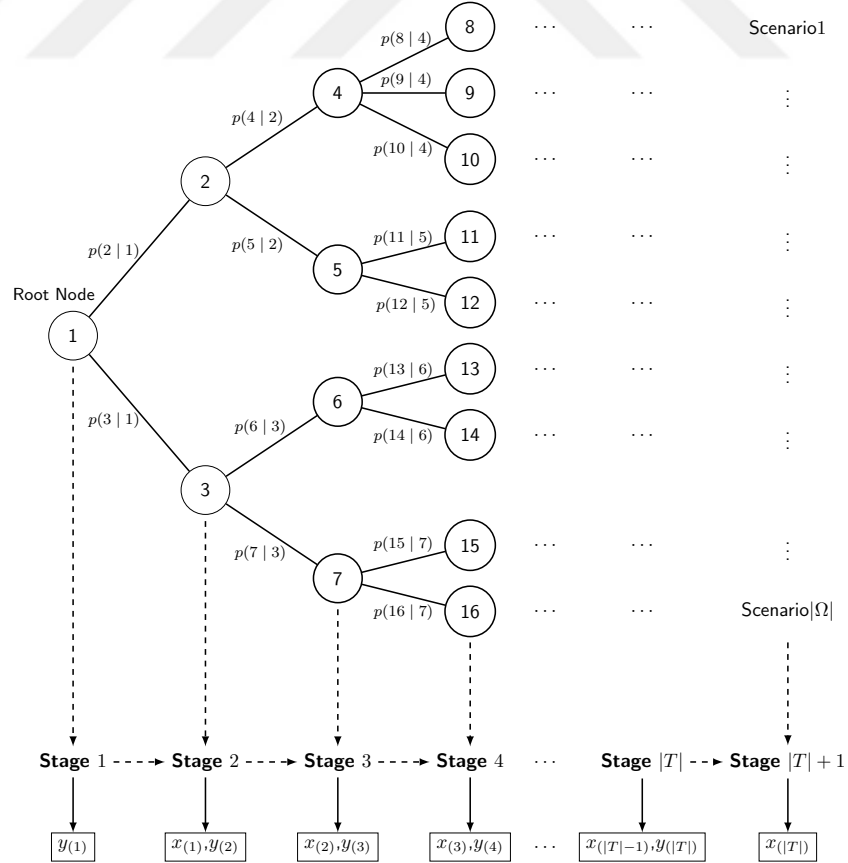


Figure 5.2: A sample scenario tree.

The nodes in a scenario tree define a set that we denote by $\mathcal{N}$. Each node $n \in \mathcal{N}$ results from a realization of the supply up to some time period and thus calls for decisions to be made—a stage in the decision-making process. This explains the different layers in the tree. Furthermore, each node n belongs to a specific stage, that we call $\sigma(n)$. Moreover, apart from the root node $n = 1$, each node n has a unique immediate parent, denoted as $\gamma(n)$. Owing to this, our formulation implicitly includes non-anticipativity property.

A path from the root node to a leaf node represents a scenario, i.e., a full realization of the supply throughout the planning horizon. Each node n in the tree induces a path, say $\mathcal{P}(n)$, which connects it to the root node. For instance, in Figure 5.2, $\mathcal{P}(8)$ is given by $1 \rightarrow 2 \rightarrow 4 \rightarrow 8$.

Each node n occurs with some given probability $\pi(n)$. Trivially, the total probability of nodes that belong to the same stage must be 1. Additionally, a probability value $p(n \mid \gamma(n))$ is assigned to each edge that connects a node n (apart from $n = 1$) to its predecessor $\gamma(n)$. This is the conditional probability of realizing node n given that its immediate predecessor is observed. Therefore, $\pi(n)$ can be written as $\prod_{m \in \mathcal{P}(n) \setminus \{1\}} p(m \mid \gamma(m))$ using the path information. For the sake of notation, the root node is assumed to be realized with probability 1.

Location decisions of the mobile supply units for the first period take place at the first stage of the scenario tree (beginning of the first period), under complete uncertainty. The information is partially revealed in each stage: the supply level of the corresponding period becomes known. After the supply amount of the first period is revealed, corresponding allocation decisions are to be made in the second stage—they are made based on how uncertainty is revealed. The location of PoDs in the second period also corresponds to a second-stage decision. In general, location decisions for period t are considered as *here-and-now* decisions, whereas the allocation of the realized supply is made as a *wait-and-see* decision with respect to period t . Note that due to the multi-stage structure of the problem, the location of the mobile PoDs for the next time period is also a recourse decision for the current period t .

The set of stages (layers in the scenario tree), say $\mathcal{M}$, is directly related to the set of time periods in the planning horizon, as the above reasoning makes clear. In particular, if T denotes the set of time periods in the planning horizon, then $|\mathcal{M}| = |T| + 1$.

We list all sets and parameters to better clarify the elements we use in our multi-stage stochastic programming problem (MSSP).

Sets:

- T , set of time periods in the planning horizon.
- I set of potential locations for the PoDs during the planning horizon.
- J , set of shelter sites.
- I_j , set of potential locations for the mobile PoDs that can supply shelter site $j \in J$.
- $\mathcal{M}$, set of stages in the scenario tree ($|\mathcal{M}| = |T| + 1$).
- $\mathcal{N}$, set of nodes in the scenario tree.
- $\mathcal{N}_m$, set of nodes in stage $m \in \mathcal{M}$.
- Ω , set of scenarios ($= \mathcal{N}_{|\mathcal{M}|}$).
- $\mathcal{P}(n)$, set of nodes in the path connecting nodes 1 to $n \in \mathcal{N}$ (inclusive).

Node-based parameters:

- $\sigma(n)$, stage to which node $n \in \mathcal{N}$ belongs.
- $\gamma(n)$, immediate predecessor of node $n \in \mathcal{N} \setminus \{1\}$ in the scenario tree.
- $\rho(n)$, period ending in node $n \in \mathcal{N} \setminus \{1\}$ ($= \sigma(n) - 1$).
- $\pi(n)$, probability that the sequence of events leading to node $n \in \mathcal{N}$ occurs.
- s_n , amount of supply at node $n \in \mathcal{N} \setminus \{1\}$.

Stage-based parameters:

- $d_{j\rho(n)}$, amount required by shelter site $j \in J$ during period $\rho(n)$, $n \in \mathcal{N} \setminus \{1\}$.
- $b_{\rho(n)}$, maximum number of PoDs that can be used in period $\rho(n)$, $n \in \mathcal{N} \setminus \{1\}$.

Static (time-invariant) parameters:

- c_i , capacity of a PoD operating at $i \in I$.
- ℓ_{ij} , distance (travel time, road distance, etc.) between potential PoD location $i \in I$ and shelter site $j \in J$.
- γ , coverage radius for shelter sites.
- ω_j , weight associated with shelter site $j \in J$.

The optimization model we propose makes use of the following sets of decision variables:

- $y_{in} = \begin{cases} 1, & \text{if a PoD is set operating at location } i \in I \text{ in period } \rho(n) \text{ if the} \\ & \text{sequence of events leads to node } n \in \mathcal{N} \setminus \{\Omega\}, \\ 0, & \text{otherwise.} \end{cases}$
- $x_{ijn} =$ amount delivered to shelter site $j \in J$ from a PoD operating at $i \in I_j$ at node $n \in \mathcal{N} \setminus \{1\}$.
- $v_{jn} =$ inventory at shelter site $j \in J$ at node $n \in \mathcal{N} \setminus \{1\}$.
- $u_{jn} =$ shortage at shelter site $j \in J$ at node $n \in \mathcal{N} \setminus \{1\}$ with respect to the demand in that location, $d_{j\rho(n)}$.
- $w_{jn} = \begin{cases} 1, & \text{if } u_{jn} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad j \in J, n \in \mathcal{N} \setminus \{1\}$
- $z_{jn} =$ deprivation cost at shelter site $j \in J$ at node $n \in \mathcal{N} \setminus \{1\}$.

The deprivation cost values are calculated as follows:

$$z_{jn} = \begin{cases} z_{j,\gamma(n)} + u_{jn}, & \text{if } w_{jn} = 1, \\ 0, & \text{otherwise,} \end{cases} \quad j \in J, n \in \mathcal{N} \setminus \{\{1\} \cup \mathcal{N}_2\}.$$

The above relation between the variables z , u and w can be ensured using the linear relations

$$z_{jn} \geq z_{j,\gamma(n)} + u_{jn} - M(1 - w_{jn}), \quad j \in J, n \in \mathcal{N} \setminus \{\{1\} \cup \mathcal{N}_2\},$$

as far as we seek to minimize somehow the values of z_{jn} . In these inequalities, M denotes a large enough constant that can be tuned by setting it equal to $\sum_{\tau \in \mathcal{P}(n) \setminus \{1, n\}} d_{j\rho(\tau)}$ because when w_{jn} (and consequently u_{jn}) are equal to zero,

$z_{jn} \geq z_{j,\gamma(n)} - \sum_{\tau \in \mathcal{P}(n) \setminus \{1,n\}} d_{j\rho(\tau)}$ holds. The deprivation cost for the first period (i.e., second stage) is trivially defined as the shortage that occurred in this time period, i.e. $z_{jn} = u_{jn}$ where $n \in \mathcal{N}_2$ for demand node $j \in J$.

We must first assess what equity means in this stochastic setting before setting the objective function of our model. The answer is subjective and depends on the risk-aversion level of the decision-maker. Two approaches are defined to address this: ex-post (EP) and ex-ante (EA). The former considers the expected equity, while the latter deals with the equity of the expected result. A detailed discussion on these and possible implications can be found in a recent study by Gutjahr [112].

Suppose a good is to be allocated to a set of entities (N) given a set of scenarios (S), i.e., possible realizations of future events. The allocation vector is given by x , where the amount $x(s, i)$ represents the share of entity $i \in N$ for a specific realization $s \in S$. Then, fairness is sought using a function of choice, $I(\cdot)$. This function returns a value indicating how equitable the allocation vector x is toward the entities. Now, the question is how to form the objective function: either $\mathbb{E}(I(x(s, i)))$ (EP approach) is or $I(\mathbb{E}(x(s, i)))$ (EA approach) is to be optimized. Mostajabdaveh et al. refer to these objectives as *change of equity* and *equity of changes*, respectively [61].

For our problem, the EP approach can be specified as minimizing *the expected maximum deprivation cost among demand nodes* and while the EA approach minimizes *the maximum deprivation cost among the demand nodes in the expected outcome*. Using the previously defined notation, the corresponding objective functions are formulated as follows:

$$\text{EP objective: minimize } \sum_{n \in \Omega} \left\{ \pi(n) \max_{j \in J} \left\{ \omega_j \sum_{\tau \in \mathcal{P}(n) \setminus \{1\}} z_{j\tau} \right\} \right\}, \quad (5.1)$$

$$\text{EA objective: minimize } \max_{j \in J} \left\{ \sum_{n \in \Omega} \pi(n) \left\{ \omega_j \sum_{\tau \in \mathcal{P}(n) \setminus \{1\}} z_{j\tau} \right\} \right\}. \quad (5.2)$$

In disaster management, EP policies seek to increase the expected balance

of the relief provided to affected populations following a disaster. However, if the decision-maker is willing to accept the solutions where the beneficiaries are equally likely to be underserved as perfectly equitable, EA is a good fit for the problem. Decision-makers are typically risk-averse in humanitarian settings; thus, the EP approach is preferred over EA [61]. We follow the same reasoning in this study. Therefore, the objective function (5.1) for the donation allocation problem adopts the EP approach and minimizes the expected maximum deprivation cost.

We can now present the mathematical model MSSP:

minimize (5.1)

$$\text{subject to } \sum_{i \in I} y_{in} \leq b_{\sigma(n)}, \quad n \in \mathcal{N} \setminus \Omega, \quad (5.3)$$

$$\sum_{j \in J: i \in I_j} x_{ijn} \leq c_i y_{i\gamma(n)}, \quad i \in I, n \in \mathcal{N} \setminus \{1\}, \quad (5.4)$$

$$\sum_{j \in J} \sum_{i \in I_j} x_{ijn} \leq s_n, \quad n \in \mathcal{N} \setminus \{1\}, \quad (5.5)$$

$$v_{j\gamma(n)} + \sum_{i \in I_j} x_{ijn} - d_{j\rho(n)} = v_{jn} - u_{jn}, \quad j \in J, n \in \mathcal{N} \setminus \{\{1\} \cup \mathcal{N}_2\}, \quad (5.6)$$

$$\sum_{i \in I_j} x_{ijn} - d_{j\rho(n)} = v_{jn} - u_{jn}, \quad j \in J, n \in \mathcal{N}_2, \quad (5.7)$$

$$u_{jn} \leq w_{jn} d_{j\rho(n)}, \quad j \in J, n \in \mathcal{N} \setminus \{1\}, \quad (5.8)$$

$$w_{jn} \leq u_{jn}, \quad j \in J, n \in \mathcal{N} \setminus \{1\}, \quad (5.9)$$

$$v_{jn} \leq (1 - w_{jn}) \sum_{\eta \in \mathcal{P}(n) \setminus \{1\}} s_\eta, \quad j \in J, n \in \mathcal{N} \setminus \{1\}, \quad (5.10)$$

$$z_{jn} \geq z_{j,\gamma(n)} + u_{jn} - \sum_{\tau \in \mathcal{P}(n) \setminus \{1, n\}} d_{j\rho(\tau)} (1 - w_{jn}), \quad j \in J, n \in \mathcal{N} \setminus \{\{1\} \cup \mathcal{N}_2\}, \quad (5.11)$$

$$z_{jn} = u_{jn}, \quad j \in J, n \in \mathcal{N}_2, \quad (5.12)$$

$$y_{in} \in \{0, 1\}, \quad i \in I, n \in \mathcal{N} \setminus \Omega, \quad (5.13)$$

$$w_{jn} \in \{0, 1\}, \quad j \in J, n \in \mathcal{N} \setminus \{1\}, \quad (5.14)$$

$$x_{ijn} \geq 0, \quad i \in I, j \in J, n \in \mathcal{N} \setminus \{1\}, \quad (5.15)$$

$$u_{jn}, v_{jn}, z_{jn} \geq 0, \quad j \in J, n \in \mathcal{N} \setminus \{1\}. \quad (5.16)$$

Constraint (5.3) limits the number of opened PoDs in each period as a budget. Facility capacities are ensured by Constraints (5.4). The total distributed amount cannot exceed the available supply, as Constraint (5.5) indicates. Constraints (5.6) and (5.7) are flow conservation constraints. Since the initial inventory levels are zero, (5.7) is separately written for the first period, corresponding to the scenario tree nodes belonging to the second stage. The logical relation between the binary variable w and shortage amounts is established through Constraints (5.8) and (5.9). Constraint (5.10) ensures that a shelter site cannot keep inventory if its demand is not fully satisfied within the same time period. In other words, either shortage or inventory levels must be zero for shelter sites. Deprivation cost values are calculated with Constraints (5.11) and (5.12). Since the initial deprivation cost levels are zero for all shelter sites, the deprivation cost is equal to the shortage for the first period. This is indicated by Constraint (5.12). Constraints (5.13)-(5.16) are domain constraints.

The following section is a case study in which we solve the donation management problem for a real network to analyze how the location and allocation policies evolve over multiple decision-making stages, given the uncertain nature of underlying parameters.

5.3 A Case Study

We apply the above-formulated MSSP to the real network of Kartal district. We prepare the deterministic data as explained in Section 4.4 using the same procedure we use for the deterministic problem, including 5 possible PoD locations and 37 demand nodes. We first explain the procedure for generating the uncertain parameters (i.e., the scenario tree). Then, we analyze the solutions with respect to a set of scenarios corresponding to different supply availability levels in each time period. The EA approach is also tested on the data set, which considers a less risk-averse decision-maker.

5.3.1 Problem data

The preparation of the stochastic part of the data set, i.e., node-based parameters, requires two steps: i) generation of a scenario tree, ii) generation of the information encapsulated by the nodes of the scenario tree (amount of donations). The procedures applied for both are explained in Appendix E.

The size of a scenario tree is determined by two factors: the number of decision-making stages and the number of nodes in each stage. These respectively set the horizontal and vertical length of the tree. We consider a setting with four stages, i.e., three time periods. The number of nodes (i.e., the number of possibilities) is a function of the degree of uncertainty. This can be dynamic; that is, the number of possibilities is subject to change across stages. The relief distribution problems typically have this feature as the supply and demand stabilize over time. Thus, for the case study, we assume that the number of possibilities (number of edges emanating from each node) is higher at the beginning of the planning horizon, encompassing a higher degree of uncertainty, as depicted in Figure 5.3. We use four equally likely possibilities for supply availability in the first time period (the second stage), ranging from extremely high to extremely low, to capture possible service duplications of agents in the donation supply chain. Next, we set the possible realizations in the following time periods depending on past events.

That is, in each node in stages 3 and 4, the supply amount either increases or decreases compared to that of its immediate parent (as denoted by high (H) and low (L) in the figure). As shown in the figure, this setting yields 29 nodes and 16 scenarios (leaf nodes). Note that the generated scenario tree is symmetric. Figure 5.4 shows the level of supply availability (supply to total demand ratio) for each scenario in the generated tree.

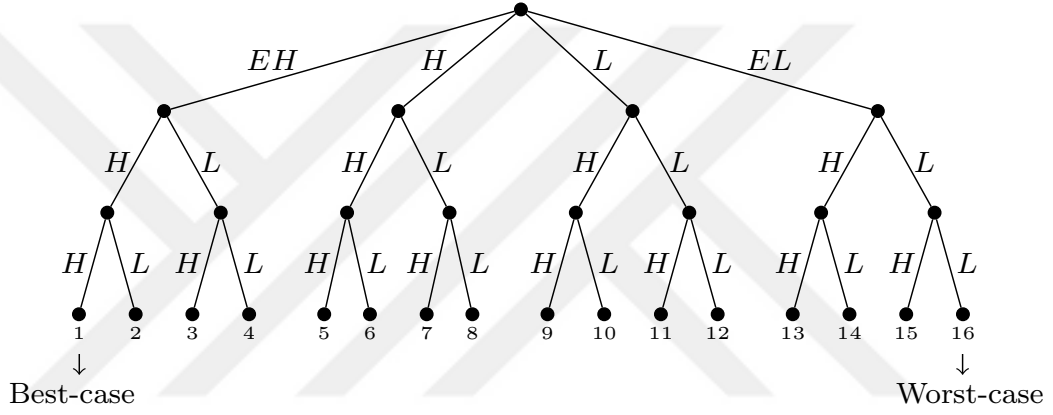


Figure 5.3: Generated scenario tree with decreasing degree of uncertainty.

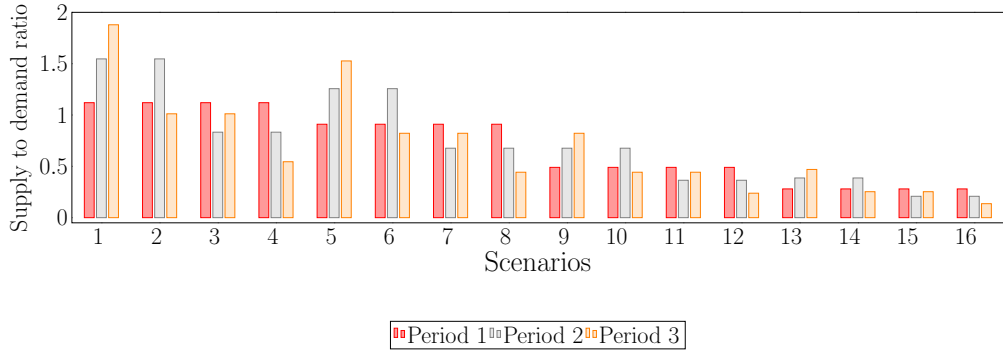


Figure 5.4: Supply to demand ratio across scenarios.

In the best-case scenario (scenario 1), the received donations are extremely large in quantity and increase even more in the following stages. Although this is a rather optimistic case, we may still observe inefficient or unfair distribution of the collected donations. This risk is posed by operational constraints such as

budget and limited capacity for operating facilities and workforce. The worst-case scenario (scenario 16) is that we start with an extremely low supply that continues to decrease throughout the planning horizon. This is characterized as the pessimistic case, where the main limitation is the scarcity of relief supplies. The decision-maker can face both possibilities and the ones in between, depending on numerous factors such as the accessibility of the disaster region, the extent of the damage, public reaction, and media coverage. Next, we provide some managerial insights as to how to deal with varying (and unknown priori) supply levels by analyzing the solutions corresponding to different scenarios.

5.3.2 Results

After solving the MSSP with the generated scenario trees, we extract the solutions for the nodes that belong to the best-case (scenario 1) and the worst-case (scenario 16), along with a more moderate scenario (scenario 7). The solutions for the selected scenarios are depicted on the map of Kartal district (see Figures F.1–F.4 in Appendix F).

For the best-case scenario, supply is sufficient to cover all demand at the beginning of the planning horizon. This represents a rapid accumulation of donations, which is possible due to service overlaps. Nonetheless, the availability of supply does not guarantee its accessibility in post-disaster scenarios. This is due to several reasons. First, disruptions in the underlying transportation network may hinder relief distribution processes. In addition, other resources, such as the capacity of supply facilities and budget, may limit the delivery of available supply. We observe the latter in the solutions. In the first period, we cannot meet all the demand of the shelter sites due to the inadequate number of PoDs (i.e., due to a limited budget), as shown in Figure 5.5. This situation changes for the second time period when the PoDs are relocated to compensate for the previously neglected demand points. Thus, the mobility of PoDs becomes crucial in such a setting as it enables the service providers to ensure spatial equity by covering different parts of the region in the course of the planning horizon. Full service

is achieved for the second and third time periods despite the limited number of PoDs. This resulted from the inventory of excess relief items at shelter sites.

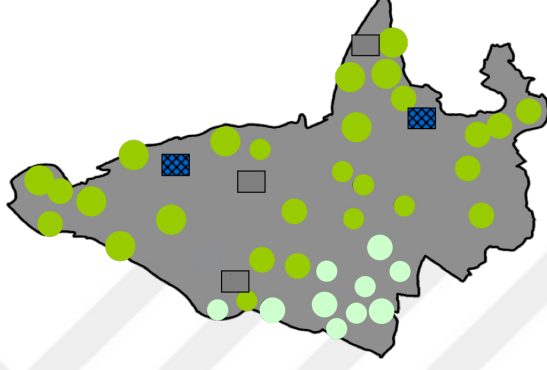
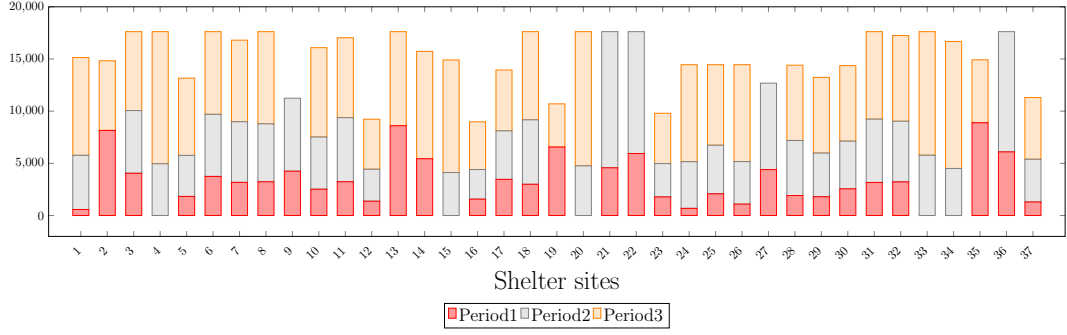
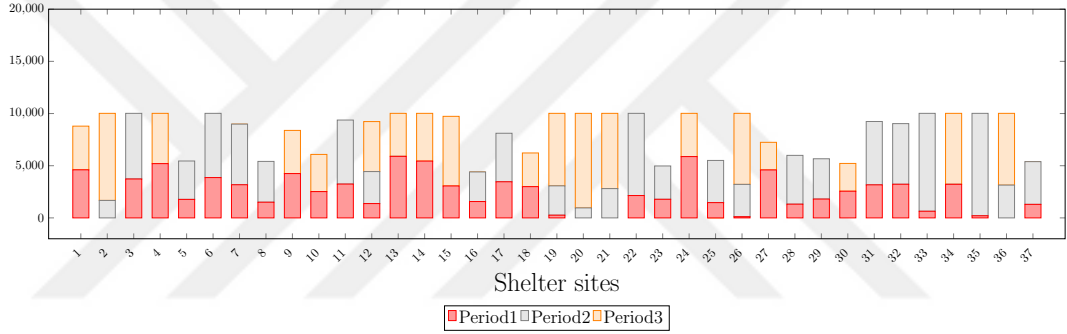


Figure 5.5: Operating PoDs and fill rates in the first time period for the best-case scenario.

Unlike the optimistic case, shortages and, consequently, deprivation costs are expected for more pessimistic scenarios. The deprivation cost values are shown in Figure 5.6 for the neutral and the worst-case scenarios. Trivially, the deprivation cost levels more drastically change for the latter case due to the higher scarcity of donations. In fact, there are some shelter sites that are not served during the planning horizon in the worst-case scenario. When similar supply levels are to be realized in consecutive time periods, as in the neutral scenario, we observe a tendency to prioritize demand nodes with longer deprivation time. This is because the shortage levels are expected to be balanced in future time periods, given that the supply availability will be more stable compared to more pessimistic scenarios. This shows how the adopted objective function accounts for the equitable allocation of service regarding the waiting time. Settlement of the mobile PoDs follows the same strategy. Different demand nodes are covered in turn in consecutive time periods, which prevents the inclined growth of deprivation cost by interrupting the waiting (i.e., deprivation) time of shelter sites. As the gap between supply and demand grows larger over time, it becomes difficult to maintain this consistent service to the shelter sites, and service frequency diminishes on average. This causes shelter sites to suffer from shortage in consecutive time periods, consequently resulting in the accumulation of deprivation cost levels.



(a) The worst-case scenario (scenario 16).



(b) A more moderate scenario (scenario 7).

Figure 5.6: Deprivation cost levels.

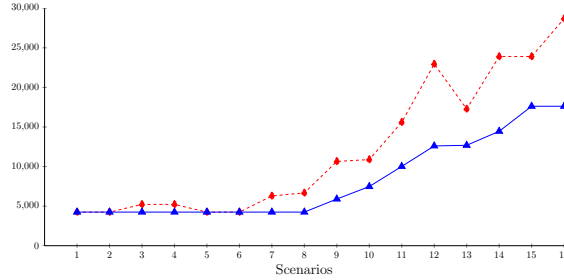
The solutions indicate that although dynamic prioritization of underserved shelter sites can achieve more equitable settings with more optimistic scenarios, more rigorous strategies are needed for pessimistic cases. This is simply because maintaining an equitable distribution of limited resources becomes exceptionally challenging when their availability significantly decreases over time.

5.3.2.1 Comparison with the ex-ante approach

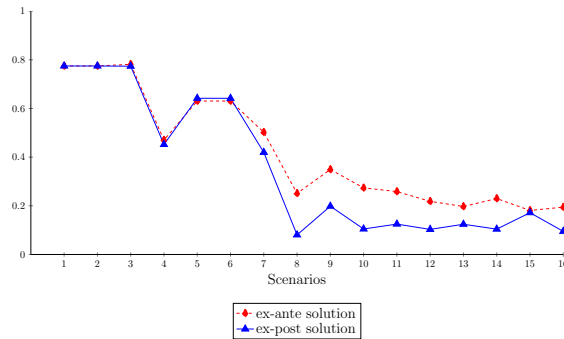
The risk-aversion behavior of decision-makers can alter the equity-based objectives and, consequently, the underlying mathematical models. As discussed earlier in this chapter, we consider a risk-averse decision-maker for the donation allocation problem, who tends to adopt an ex-post (EP) approach as opposed to

ex-ante (EA) policies. In this section, we address both possibilities for our multi-stage problem and compare the corresponding solutions. The resulting solution schemes for each case are depicted in Appendix F.

Figure 5.7 compares the fairness of accumulated deprivation costs using two different inequity functions: a minimax function (Figure 5.7a) and Gini coefficient (Figure 5.7b). The former measures the maximum suffering a demand node experiences. The latter measures the dispersion of these values and returns a value between 0 and 1, 0 indicating perfect equity. Both measures indicate that more equitable solutions are obtained in the pessimistic scenarios when the EP-based objective is minimized and that EA and EP approaches perform similarly for rather optimistic scenarios. This shows that EP-based policies can be preferred when the risk of underserving the demand nodes is high or, equivalently, the decision-maker seeks more robust equitable allocations.



(a) The maximum accumulated deprivation cost value.



(b) Gini coefficient of accumulated deprivation cost values.

Figure 5.7: Fairness evaluation of the EA- and EP-based solutions.

Figure 5.8 compares the expected accumulated deprivation cost levels when EA- and EP-based objectives are minimized. As expected, an EA-based objective function better controls the dispersion of these values. This is why it can be preferred for settings where i) the variability across possible future realizations is low and ii) the extreme scenarios are less likely to occur compared to others.

The selection of the appropriate objective also depends on the trade-off between EA and EP objectives. That is, instead of solely adopting either, more compromised solutions can be devised. To this end, Table 5.1 shows the deviation of the optimal value from each one when the other is optimized. Remarkably, we observe that adopting the EP approach works well regarding the EA-based objective (10% deviation from the optimal value). However, the cost of optimizing the EA-based objective is relatively high for the EP-based objective. This indicates that seeking fairness in the expected scenario significantly worsens the expected fairness.

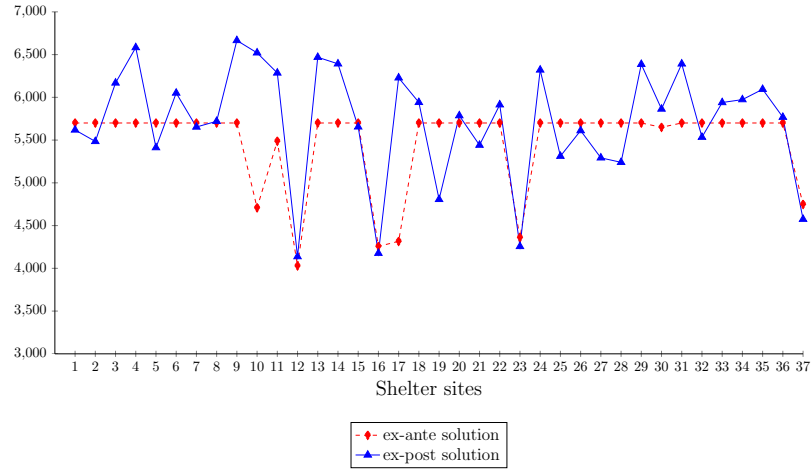


Figure 5.8: Expected accumulated deprivation cost levels.

We also analyze the so-called price of fairness, i.e., the cost of ensuring fairness in terms of the total value (the expected total deprivation cost, denoted by $f_{TDC}(\cdot)$). The value is notably higher when the EP objective is favored. The EA objective controls the maximum value of a vector of expected deprivation cost levels. Thus, it also contributes to minimizing the total amount. This explains the marginal increase in the total deprivation cost when an EA policy is

of relevance.

Table 5.1: Payoff tables for EA and EP objectives.

Minimization of	Deviation from		
	$f_{EA}^*(.)$	$f_{EP}^*(.)$	$f_{TDC}^*(.)$
$f_{EA}(.)$	-	150%	4%
$f_{EP}(.)$	10%	-	60%

Overall, the EP and EA approaches have their pros and cons and can be preferred based on the problem context and the underlying stochastic process. The EA approach can be preferred if the decision-maker aims to control the efficiency (total expected allocation) rather than solely opting for fairness (expected balance of allocations) under uncertainty. It is also a good fit for the stochastic problems where the degree of uncertainty is low, i.e., possible realizations are similar. On the other hand, the EP approach is more suitable to ensure fairness given highly uncertain data as it increases the level of robustness in inequity against more extreme scenarios.

5.4 Matheuristic Algorithms

Multi-stage stochastic programming models are challenging to solve due to the fast growth of the scenario tree depending on the number of stages, i.e., length of the planning horizon [113]. Larger scenario trees can better represent the complexity of real-life uncertainties; hence, the adopted methodology should mainly account for the size of the scenario tree. To achieve this, we employ two matheuristic approaches: a two-stage rolling horizon approach and a scenario reduction procedure. Both methods include solving a sequence of simpler problems with the aim of producing good-quality solutions for the original scenario tree in a shorter time frame. Next, we explain both methods and compare their performances for scenario trees of different sizes.

5.4.1 A two-stage rolling horizon algorithm

One possible way to make decisions in multiple stages is to consider a series of two-stage problems in a rolling horizon framework [114, 115, 116]. That is, the decisions are made sequentially, considering only two stages in each step. The first stage includes the here-and-now decisions (HN), i.e., the decisions for the current time in the planning horizon, while the second stage is defined by the wait-and-see decisions (WS), i.e., the decisions for possible future realizations. We adopt an anticipatory approach for this heuristic and preserve the multi-period structure of the MSSP. That is, we consider the expected values for future stages instead of fully disregarding them. This yields a less myopic approximation of the original scenario tree [117]. At the beginning of each step, the decisions for the past stages are fixed as they are already realized. The procedure for this heuristic approach is depicted in Figure 5.9.

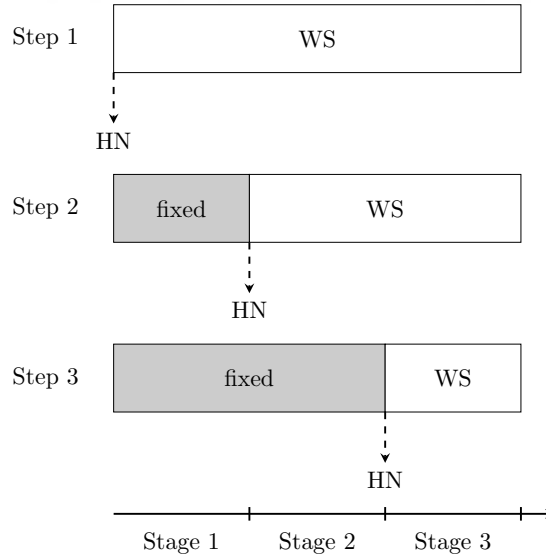


Figure 5.9: Overview of the two-stage rolling horizon algorithm.

Figure 5.10 shows the steps of the heuristic for a four-stage problem. The first step is to solve a two-stage problem under complete uncertainty. The possible realizations are considered for the next stage, while the nodes of the other stages are represented by the expected values. Afterward, we fix the decisions we make for the first stage and observe the new information. Since we fix the root node

variables, the tree becomes decomposable over the nodes of the current stage. We solve $|\mathcal{N}_m|$ two-stage problems in each stage $m \in \mathcal{M}$. The process continues until uncertainty is fully resolved and the objective value is calculated by the expected optimal value of the problems solved in the last stage.

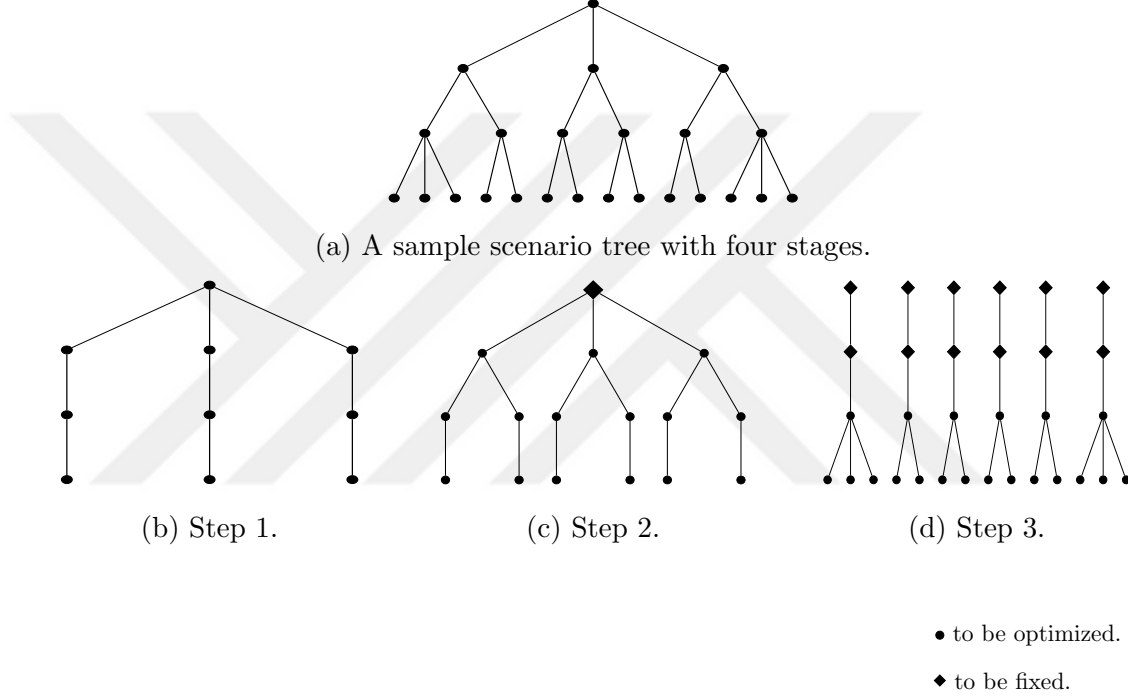


Figure 5.10: Two-stage problems to approximate a sample four-stage scenario tree.

5.4.2 A scenario reduction algorithm

A common way of handling the computational challenge induced by MSSP is to utilize reduced scenario trees, which are generated to approximate the original scenario trees. We develop a solution framework following this idea. This framework involves three main steps: First, we create a representative reduced scenario tree by eliminating some nodes from the original tree. We then solve the corresponding reduced problem. Finally, we obtain a solution to the original problem using the solution obtained for the reduced one. Figure 5.11 explains these steps and the methods we employ to tackle them. Below, we explain each step in detail.

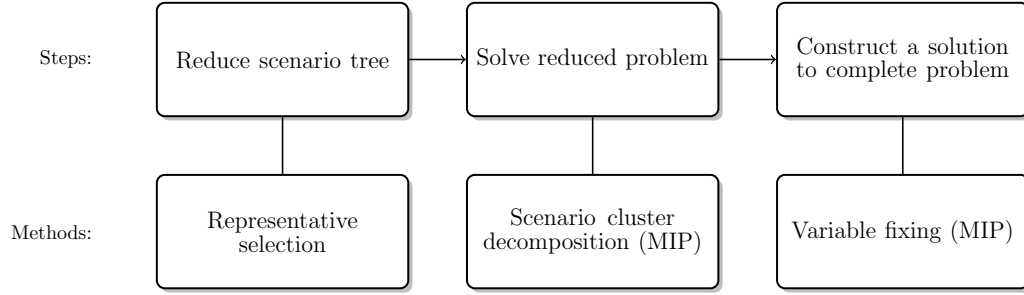


Figure 5.11: Steps of the scenario reduction algorithm.

Scenario reduction: We first select the nodes (scenarios) that will be preserved in the reduced tree in a way that they adequately represent the original tree. The methods used for this selection are mainly categorized into two groups based on whether they consider the similarity of the stochastic information included in the scenarios [118] or of the objective function values of the corresponding deterministic problems [119]. We choose the former approach and select the scenarios based on the similarity of the encapsulated events, i.e., the realization of supply parameters.

When constructing the scenario tree, we preserve one member of each sibling set within the leaf nodes and eliminate the others as they are well-represented by the chosen sibling (see Figure 5.12). This is because these nodes share the same ancestors, i.e., the corresponding scenarios have the same events except for the last stage.

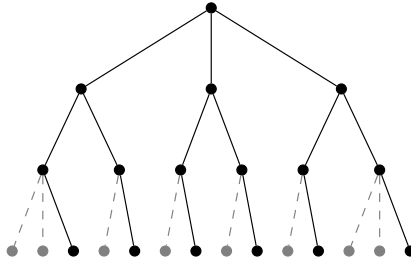


Figure 5.12: Scenario selection procedure.

This strategy is simple, yet effective in generating a highly representative reduced set of scenarios as each leaf node (scenario) is represented by either itself or one of its siblings. Note that selecting only one member per sibling set also

helps us control the size of the reduced tree. Based on our preliminary results, we choose the most pessimistic node, i.e., the one with the least supply availability, in each sibling set of the last stage. We recall that the number of sibling leaf sets in a tree is given by $|\mathcal{N}_{|\mathcal{M}|-1}|$, i.e., the number of nodes in the previous stage. Thus, the original tree is reduced by $|\mathcal{N}_{|\mathcal{M}|}| - |\mathcal{N}_{|\mathcal{M}|-1}|$ nodes.

Solving the reduced problem: After the reduction step, we obtain a reduced tree that has the same nodes up to stage $|\mathcal{M}| - 1$ as the original tree and one member of each sibling set for the leaf nodes. We utilize a decomposition idea to be able to solve this reduced problem in a reasonable time frame. That is, the reduced tree, e.g., the one given in Figure 5.12, is decomposed into a set of clusters as shown in Figure 5.13 by utilizing explicit non-anticipativity constraints for the first-stage decision variables [120, 121]. Here, we note that the scenario tree involves only the binary location variables (y_1) (see Figure 5.13) in the first-stage. When the set of possible location combinations has a reasonable size, one can enumerate all possibilities of the first-stage decision variables. For each enumeration, the decisions for the first-stage variables are fixed, and the problem corresponding to each cluster is solved accordingly. Thus, the exact solution of the reduced problem is obtained when the problem of each cluster is solved for $|\mathcal{Y}_1|$ many times, where $\mathcal{Y}_1$ denotes the set of all feasible location combinations, i.e., $y_1 \in \mathcal{Y}_1$. The number of such enumerations is as follows: $|\mathcal{Y}_1| = \binom{I}{b_1}$, where I is the set of possible locations and b_1 is the number of PoDs that can be operated in the first time period.

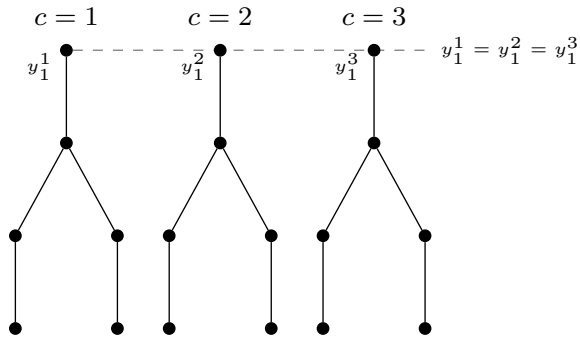


Figure 5.13: Scenario cluster decomposition for a reduced tree.²

²First stage location decisions are denoted by y_1^c for each cluster c in the figure.

Finding a solution for the complete scenario tree: After solving the reduced problem, we obtain a solution that tells the decision-maker what to do in the scenarios included in the reduced scenario tree. However, the decisions corresponding to the eliminated nodes are not yet determined. Such nodes are represented by their chosen sibling in the reduced tree. Hence, we mimic the decisions we made for the chosen sibling so as to find a complete solution. For this, we first retrieve the solutions corresponding to the chosen siblings, fix the variables for all leaf nodes, including both selected and eliminated ones, and then solve the mathematical model corresponding to the complete tree.

The only decisions made in the leaf nodes are the allocation decisions of the last period. Thus, we need to determine the values for the corresponding allocation variables to fix the solutions for the eliminated nodes. Directly equalizing the allocation amount of the eliminated node to that of its chosen sibling can yield infeasibility. The reason is that the supply in the eliminated node may be insufficient to repeat the same allocation values of the selected sibling. For this, we fix the allocation proportions instead of fixing the allocation amounts directly. The allocation decisions are now fixed for all leaf nodes, indicating which PoDs will be operated to perform these deliveries. Using this information, we fix the location decisions for stage $|\mathcal{M}| - 1$, i.e., for the last time period. After fixing all variables for the last time period, we solve the complete problem.

5.5 Computational Tests

We test both procedures explained in the previous section on a set of generated instances and analyze their performances compared to that of the MSSP. First, we generate synthetic data consisting of both deterministic (the underlying network and demand information) and stochastic (the scenario tree) parts. So as to properly observe the performance of the proposed heuristics with respect to the scenario tree size, we use a fixed network consisting of 10 demand nodes and 5 possible locations for the mobile facilities. We consider varying lengths for the planning horizon, including small (4 stages), medium (5 or 6 stages), or large (7

stages) scenario trees. These possibilities together yield four distinct cases, for each 10 instances are generated and solved via CPLEX 12.71. In each instance, we randomly decide the number of children of each node, except the leaf nodes. That is, each realization may yield 2 or 3 possibilities for the following stage, resulting in a random vertical growth across instances. Next, we compare the solution qualities and computational times. The detailed results for each case are given in Appendix H.

5.5.1 Results

For each instance, we first solve the MSSP applied on the complete tree given a time limit of 1 hour. The results are shared in Table 5.2. As expected, the scenario trees exponentially grow as we increase the number of stages in the decision-making process. Consequently, given the time limit, the optimality gap of the solutions given by the MSSP increases. We also apply the matheuristic algorithms to the same instance set. The results show that both approaches outperform the mathematical model when the decisions are made considering a longer planning horizon.

Table 5.2: Computational experiments for MSSP.

Case #				z_{MSSP}^*			%gap		
	$ \mathcal{M} $	avg. $ \mathcal{N} $	avg. $ \Omega $	avg	min	max	avg	min	max
1	4	26	16	8.75	8.42	9.00	4.32	4.00	6.09
2	5	66	40	12.12	11.68	12.40	12.52	10.35	15.50
3	6	165	100	16.84	16.16	17.46	21.85	17.83	26.76
4	7	414	249	109.93	33.96	182.58	74.60	39.25	91.54

Table 5.3 provides an overview of the results in terms of both solution quality and computational time when matheuristics are applied. We consider the deviation from the best solution the model can find in 1 hour so as to evaluate the performance of the matheuristic methods. Denoting this solution as z_{MSSP}^* , we

compute the deviation as follows:

$$\%dev = \frac{z_H - z_{MSSP}^*}{z_{MSSP}^*} \times 100,$$

where z_H is the objective obtained by the corresponding heuristic algorithm.

This table shows that, indeed, our MSSP problem can be approximated by either i) a smaller scenario tree or i) a set of two-stage problems. The former is the scenario reduction approach, where we disregard some nodes in the last stage and make the decisions according to the preserved ones. This enables us to omit one stage of the scenario tree. In each test case, we were able to reduce the original size of the scenario tree by approximately 35% on average. As indicated in Table 5.3, this reduction does not cost significant positive deviations from the objective of MSSP, while reducing the solution times by 93% on average for small and medium-sized scenario trees. Yet, for large scenario trees, solving even the reduced problems becomes computationally exhausting. On the other hand, the rolling horizon approach requires solving only two-stage problems regardless of the length of the planning horizon. Thus, it works faster for all cases tested and better estimates the original problem for large instances. When the planning horizon consists of 7 stages, the average optimality gap observed for z_{MSSP}^* is 75%, which can be reduced to 5% by solving a sequence of two-stage problems.

Table 5.3: Computational experiments for matheuristic algorithms.

Case #	$ \mathcal{M} $	Scenario reduction heuristic						Rolling horizon heuristic					
		%dev			%CPU imp.			%dev			%CPU imp.		
		avg	min	max	avg	min	max	avg	min	max	avg	min	max
1	4	4.28	3.32	5.33	99.44	96.31	99.96	2.68	1.42	4.11	98.26	89.89	99.84
2	5	0.54	-0.73	2.09	97.67	95.41	99.76	2.24	0.21	4.78	99.57	98.16	99.86
3	6	-1.42	-9.76	2.44	85.79	76.91	89.33	-1.12	-6.28	4.20	98.83	96.61	99.65
4	7	-66.13	-83.97	-27.62	74.09	66.51	81.05	-70.28	-88.45	-36.41	97.57	96.35	98.58

5.6 Relevance of the Adopted Methodology

When considering a problem such as the one presented in Section 5.2, one may ask whether it is necessary to have a more complex model, namely one embedding uncertainty. In this section, we address this question. We use two information measures for this: the expected value of the perfect information (EVPI) and the dynamic value of the stochastic solution (DVSS). The former measures how much a decision-maker would be willing to pay to get access to perfect information about future demand realization(s). Low values mean that the results do not drastically change when the stochasticity of the problem parameters is considered; thus, the value of capturing them is insignificant. The second measure, DVSS, evaluates whether or not solving a series of deterministic expected value problems (the expectations are updated as new information reveals) can give good enough solutions for the stochastic problem. Here, low values indicate better approximations, and that the added computational burden due to uncertainty is not justified.

The EVPI is defined by the difference between the results of two settings. In the first one, the decisions take place before the realization of the uncertainty. In the second case, the future events are known and allows the decision-maker to observe the random parameters before making the decisions. Thus, the difference between them indicates the value of this observation. In stochastic programming, the former is characterized by the optimal value of the stochastic problem, which we denote as z_{MSSP}^* . For the latter case, we solve a deterministic single scenario problem for all possible scenarios as if we have perfect information on each possible realization. We denote the single scenario problem corresponding to scenario ω as SS^ω , $\omega \in \Omega$. Figure 5.14 depicts the set of single scenario problems for a sample scenario tree with three stages.

Denoting the optimal objective value of a single-scenario problem corresponding to scenario ω by V_{SS^ω} , the value of the wait-and-see solution for a minimization

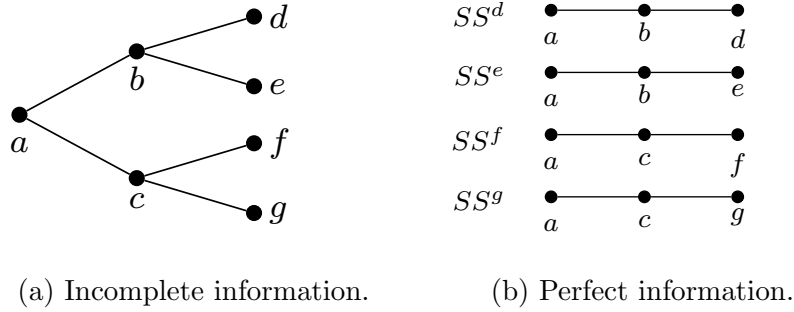


Figure 5.14: A comparison of the settings with incomplete and perfect information structure for a sample three-stage problem.

setting is calculated as follows:

$$WS = \sum_{\omega \in \Omega} \pi(\omega) V_{SS^\omega},$$

and we have,

$$EVPI = z_{MSSP}^* - WS,$$

or, in relative terms:

$$\%EVPI = \frac{EVPI}{WS} \times 100.$$

For two-stage stochastic problems, VSS is a frequently used to measure the difference between the stochastic solution and the expected result of using the solution of the expected value problem. Let us explain the latter term. Expected value problem for each node $n \in \mathcal{N}$ of the scenario tree (let us denote this problem as EV^n) is a deterministic problem, where only the expected values are considered for the uncertain parameters of future stages. The expected result of using this approach is denoted as EEV and computed as follows: $\sum_{n \in \Omega} \pi(n) EV^n$.

A small example with two stages is shown in Figure 5.15. First, the expected value problem is solved for root node a . This problem, denoted as EV^a , considers the expected values in each stage. An average scenario defined by scenarios b and c , denoted as $A(b, c)$, is used. The solution for the decisions associated with node a is obtained as the result. Next, the expected value problems for the second stage (i.e., for nodes b and c) are solved by fixing the first-stage variables. The objective values obtained by these expected value problems are denoted as

V_{EV^b} and V_{EV^c} , respectively. EEV is the expectation of these two values, i.e., $EEV = \pi(b)V_{EV^b} + \pi(c)V_{EV^c}$. Finally, VSS is calculated as follows:

$$VSS = EEV - z_{TSSP}^*,$$

where z_{TSSP}^* is the optimal objective value of the two-stage stochastic problem.

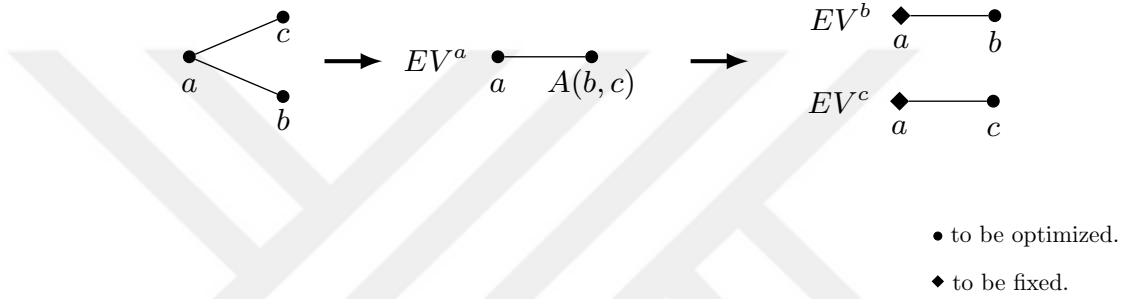


Figure 5.15: Expected value problems for a sample two-stage scenario tree.

Although VSS is often used for two-stage problems, several authors such as Escudero et al. [113] and Ziegler [122] have generalized it to multi-stage decision processes. EEV is dynamically calculated for multiple stages in this setting, thus referred to as $EDEV$ (expected result of applying the dynamic expected value solution). Following this, VSS is redefined for multi-stage problems as *the dynamic value of the stochastic solution* ($DVSS$). The associated expected value problems are shown in Figure 5.16 for a sample scenario tree with three stages (e.g., the one depicted in Figure 5.14a).

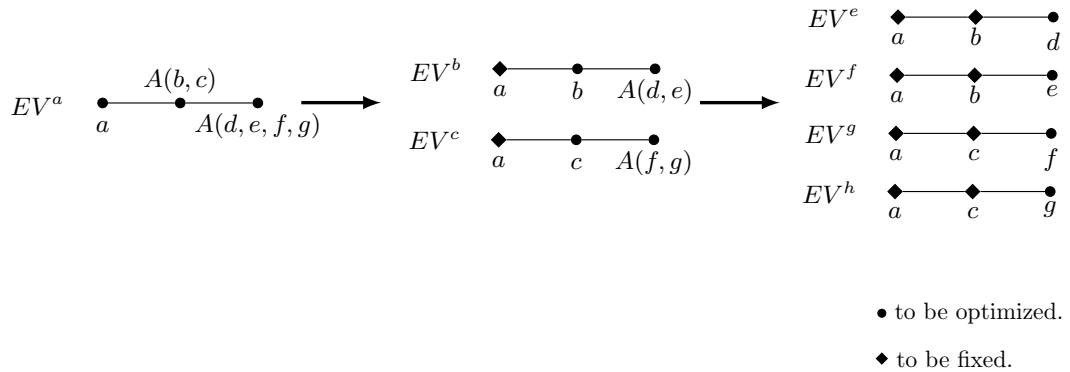


Figure 5.16: Expected value problems for a sample three-stage scenario tree.

Dynamic calculation of EDEV yields a vector of values; $EDEV_m$ represents this expected result for stage m and is calculated as follows:

$$EDEV_m = \sum_{n \in \mathcal{N}_m} \pi(n) V_{EV^n}$$

where $m \in \mathcal{M}$. The optimal solution to EV problems in stage $m+1$ is also feasible for the EV problems of the previous stage m . Thus, $EDEV_{m+1} \geq EDEV_m$ for all $m \in \mathcal{M} \setminus \Omega$.

Finally, DVSS is computed:

$$DVSS = EDEV_{|\mathcal{M}|} - z_{MSSP}^*,$$

and

$$\%DVSS = \frac{DVSS}{z_{MSSP}^*} \times 100,$$

We conduct computational experiments to measure these values for our problem. The average EVPI and DVSS are shown for a set of cases with different scenario tree sizes in Table 5.4 (see Table H.5 and Appendix I for details). This table shows that access to perfect information on future events significantly alters the results even for shorter planning horizons. Moreover, making sequential decisions based on the expectations of the future supply realizations costs 14% on average. This implies that the uncertainty in our problem setting is relevant, and the resulting problem is not well approximated by solving a series of deterministic problems. Thus, our attempt to incorporate uncertainty into the donation allocation problem is fully justified.

Table 5.4: Overview of EVPI and DVSS for test instances.

Case #	$ \mathcal{M} $	%EVPI			%DVSS		
		avg	min	max	avg	min	max
1	4	20.40	18.86	21.13	14.64	11.98	16.70
2	5	21.86	17.44	24.04	12.46	9.86	15.82
3	6	28.64	22.72	37.55	13.88	7.77	17.91

5.7 Concluding Remarks

In this chapter, we incorporate uncertain supply parameters into the problem of allocating a single type of durable and frequently demanded relief item to a set of shelter sites in the aftermath of a disaster. We represent the uncertain information resulting from the donation unpredictability using a discrete set of scenarios. Then, a multi-stage stochastic programming (MSSP) approach is adopted to formulate the problem given the set of possible realizations.

The mathematical model, obtained with the MSSP method, is applied to a real network from the Kartal district of Istanbul, Türkiye. For this study, we test two possible equity-based objectives a decision-maker might be willing to employ: the equity of the expected case (ex-ante approach) and the expected equity (ex-post approach). The solutions indicate that the latter is preferable when i) the degree of uncertainty is high and ii) robustness towards the pessimistic scenarios is needed. These align with a post-disaster scenario, which makes the ex-post approach a good fit for the problem at hand.

Given the exponential growth of scenario trees, MSSP problems can quickly become unsolvable. To address this issue, we employ and test two heuristic approaches, namely a scenario reduction method and a rolling horizon algorithm. In the scenario reduction method, we represent the scenario tree with a reduced one by systematically eliminating some of the possibilities. In the second approach, we solve a set of two-stage problems as new information becomes available instead of making all wait-and-see decisions at once. The results show that for large instances, both methods produce better solutions than the mathematical model (given a time limit) while significantly reducing the computational time. Moreover, the rolling horizon method includes smaller subproblems and, thus, better tackles the large scenarios trees.

We perform additional experiments to analyze the EVPI and DVSS for our problem. These evaluate whether it is important to include stochastic parameters in the problem and, if so, how well these can be approximated by solving

dynamically updated expected value problems. Our numerical tests indicate that uncertainty is relevant, yet the corresponding expected value problems fall short of capturing it. Thus, the decisions should be made considering the possible realizations of future events.



Chapter 6

Conclusion and Future Work

Similar to other logistics operations, humanitarian logistics (HL) aims to provide the right products at the right time and at the right place. Nonetheless, these operations are quite different from those of the commercial sector in terms of the activities, limiting constraints, and objectives prioritized. Disaster management, defined as the branch of HL that specifically focuses on improving disaster preparedness and response processes, is inherently more challenging, considering the chaos and uncertainties that arise in the aftermath of disasters. One typical challenge is to address the immediate needs of internally displaced people urgently and equitably with highly limited resources, which is the focus of this study. Motivated by the intrinsic challenges of this problem, this work aims to help decision-makers make more informed decisions by i) accurately quantifying equity for a critical relief item in a post-disaster scenario and ii) developing methods to ensure equitable allocation of a scarce resource that is also uncertain in quantity.

The first step in approaching a relief distribution problem, or any other disaster management problem for that matter, is to identify the timing relative to the disaster event. This matters as the post-disaster environment and the needs of the affected population evolve in time, affecting the limitations and criteria of the relevant HL operations. Moreover, the affected population (i.e., demand) is in

movement after a disaster as they are displaced, and the points they accumulate change over time. The supply side of the underlying network is also dynamic due to the possible disruptions. The relief sources (e.g., suppliers) alter in addition to this: new sources may become available in time. For example, in-kind donations cannot be mobilized in the immediate response phase but become the main supply source over time. Acknowledging this dynamic nature is specifically important during disaster response, where the problem features may change even within hours. Thus, in Chapter 3, we develop a temporal classification of the disaster response based on the aforementioned features. We also project this classification onto OR literature to provide a comprehensive guideline for the researchers in this field regarding the type of objectives, constraints, and decisions (potentially) used to formulate the disaster response problems. Then, we specify a time frame for our relief distribution problem using this framework. By doing so, we aim to mirror real-life dynamics while addressing a relevant post-disaster problem.

We specify the demand nodes of the relief distribution network as the temporary accommodation points, e.g., shelter sites, while the supply of a non-perishable consumable relief item, e.g., bottled water, is considered. The main source of relief is the in-kind donations. Thus, the amount available is considered as an exogenous uncertain parameter due to several factors, such as the unpredictability of donor behavior, donation pollution, and service overlaps of the related organizations. This information only reveals upon processing the collected donations. The problem, consequently the proposed solution methodology and insights driven from them, are applicable to a wide range of settings where i) supply for the allocated critical item is sourced with voluntary donations, yielding uncertain availability, ii) the disaster type is sudden onset, thus, the timing of the deliveries is crucial to help the affected populations and save lives.

We consider a finite planning horizon divided into discrete time periods. Within the scope of this work, routing is not considered since a milk-run type delivery may not be possible because of the chaotic environment after a disaster. Therefore, the supply units act as points of distribution (PoD) to which beneficiaries from the demand nodes (possibly managers of shelter sites) are expected to arrive and receive their assigned donation packages. Hence, rather than means

of the distribution, accessibility is considered: the demand nodes can receive service only from the supply facilities that are close enough to them. We also allow mobilization of the supply facilities, i.e., PoDs can be relocated throughout the planning horizon to increase demand coverage.

Full demand coverage is typically infeasible during disaster response (e.g., due to insufficient supply and fast-growing demand for critical items), which results in inevitable shortages. Thus, equity is considered predominantly in such cases. If the temporal effects are disregarded, this would be equivalent to balancing demand fulfillment levels. However, the increase in disutility based on waiting time should not be neglected for urgent needs. That is, a fair service should be provided to beneficiaries regarding the waiting times and allocation amounts. A specific objective, a deprivation cost function, is adopted for this purpose. Both the deprived amount (shortage levels) and the deprivation time (delay of services) are taken into consideration to capture the multi-dimensional nature of equity.

We first formulate the deterministic version of the above-explained problem in Chapter 4 as a multi-objective, multi-period, location allocation inventory problem. The total deprivation cost, the maximum deprivation cost, and the total transportation distance are considered as the main criteria. The first two objectives appear to behave similarly. Therefore, we formulate two bi-objective problems. In these, we minimize the total distance along with the maximum and the total deprivation cost objectives, respectively. The resulting models are applied to a real data set to analyze i) the trade-off between the selected objectives, ii) solutions favoring each objective and the compromising ones. The findings yield some significant insights:

- Although fairness and efficiency are typically conflicting objectives, the former can be significantly improved when the optimality of the latter is marginally sacrificed. Thus, instead of solely optimizing efficiency-related objectives, equity should also be accounted for. Optimistically, there may be alternate solutions with higher equity levels, e.g., the cost of considering equity (to some extent) might be negligible.

- The results show that optimizing the total deprivation cost does not directly ensure equity in terms of the accumulated deprivation cost levels, as previously discussed by Gutjahr and Fischer (2018) [57]. This justifies the use of the maximin objective for the stochastic problem as opposed to adopting a utilitarian approach.
- The solutions indicate a specific policy while minimizing the deprivation cost-based objectives: the demand nodes that have been waiting the most are usually prioritized. Moreover, the different location schemes are used in consecutive time periods so that such nodes are covered. This implies a queueing-type strategy to increase the fairness of allocations.
- Mobilization of the distribution facilities provides operational flexibility to our problem and reduces the total transportation distance for fixed values of deprivation cost objectives.

In Chapter 5, uncertain supply parameters are integrated into the problem using a node-based multi-stage stochastic programming (MSSP) approach. Including uncertainty is a natural extension of a simple deterministic disaster management problem. Yet, the question of how to define equity in stochastic environments remains: either expected equity or equity of expected allocation can be considered. These represent the ex-ante and ex-post approaches, respectively. We observe that ex-post policies better suit our donation allocation problem, as they enable a more robust solution towards the risk of inequitable service due to supply scarcity.

The mathematical model, obtained with the MSSP method, is applied to a case study based on real data from the Kartal district. The results suggest different location and allocation patterns depending on how the supply levels fluctuate over time. We observe that demand nodes are often prioritized based on the waiting time and visited periodically, especially when the supply is scarce but stable across the time periods. Even though such strategies may perform well in some specific scenarios, they may fail to ensure equity for the whole scenario tree as the decisions are made before the uncertainties resolve. This highlights

the need for more sophisticated methodologies to address the uncertainty in the supply parameters over a planning horizon.

Multi-stage stochastic programming models are challenging to solve due to the fast growth of the scenario tree depending on the number of stages, i.e., length of the planning horizon [113]. Moreover, the solution methodologies should account for the size of these scenario trees to better capture the complexity of real-life uncertainties. To this end, we develop two matheuristic algorithms to tackle the problems with longer planning horizons, i.e., larger scenario trees. We solve simplified versions of the scenario tree in both procedures, aiming to obtain a good-quality solution for the original problem in shorter time frames.

First, we solve a sequence of two-stage stochastic programming problems in a rolling horizon fashion as an approximation for the multi-stage problem. The first stage decisions are the ones related to the current stage (here-and-now), and for the next stage, we make the second stage decisions (wait-and-see). The rest of the stages are specified by the expected value of supply realizations corresponding to them. Thus, variables associated with these stages are also considered as the second stage decisions. In the second method, we select a set of nodes for the last stage of the scenario tree, assume that they will be realized with probability 1, and eliminate other possibilities. Then, we solve this reduced problem. If the selected nodes are actually realized, we apply the decisions we made for the reduced tree. If not, we mimic these decisions for the realized nodes using a customized procedure. As a result, decisions for the last time period are fixed in the complete problem, and the remaining variables are optimized. The test results show that both heuristic approaches outperform the MSSP regarding the solution time and solution quality. The improvement is more significant when the rolling horizon approach is adopted, as it is affected less by the original size of the underlying scenario tree. We also analyze some information measures, namely the EVPI and the DVSS (as a multi-stage adaptation of well-known VSS), to see if the value added to the problem justifies the computational burden. These indicate that incorporating uncertainty in a multi-stage stochastic setting into our problem is, indeed, relevant.

The domain of equitable distribution of in-kind donations presents a promising avenue for future research. First of all, easy-to-apply and easy-to-communicate location and allocation policies can be developed to be used by donation management practitioners. These policies should address the uncertainty in supply availability while aiming to ensure an efficient and fair allocation.

Another research direction would be to extend our mathematical models accounting for the disaster response phases other than Phase 1B. This would yield problems with different uncertainty settings involving strategic and operational decisions other than the ones we considered in this work. For example, the donation allocation problem for Phase 1A encompasses both demand- and supply-side uncertainty as a result of the ongoing demand movement. The sequence of deliveries might also be relevant in this phase due to a higher sense of agility. Thus, the corresponding problem also embodies a valuable methodological challenge: incorporating vehicle routes in a multi-stage stochastic environment.

Our study only considers the last-mile distribution of in-kind donations to the disaster victims. Addressing the logistics problems at upper levels of the relief supply chain would be of great value by providing a holistic approach to the decision-making processes during disaster response.

A possible extension to our work is considering multiple objectives under uncertainty, which is still very much unexplored in the literature, especially for multi-stage problems. This would provide valuable insights as to how uncertainty alters the price of fairness and how the decision-makers can interpret the trade-offs between efficiency- and fairness-related objectives under stochastic information.

Lastly, the assumption that the probability distribution of supply parameters is known can be relaxed. Then, the problem can be addressed via robust optimization techniques. One drawback is that the solutions may be overly conservative. Using an approach that combines both, such as distributionally robust optimization, more compromising solutions can be devised for the donation allocation problem.

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Appendix A

Modelling frameworks adopted for disaster logistics problems

Table A.1: Problem structures during disaster response operations.

		Decisions					Service type		Objectives		Uncertainties		Limitations														
Subphase	Problem type	Location Routing	Resource allocation	Team formation	Team scheduling	Clustering/prioritization	Network design	Relief items	Workforce	Equipment	Fleet	Accommodation	Response time	Coverage	Accessibility	Fairness	Reliability	Cost	Link capacities	Supply type	Supply amount	Demand locations	Demand amount	Number of facilities	Size of fleet	Capacity	Competency levels
Phase 0	Damage/status assessment	+		+	+				+	+				+								+	+		+	+	
	Emergency access route design					+	+							+	+	+		+			+				+		
	Coordination center management	+	+	+			+	+						+	+	+					+	+	+	+		+	
	Emergency response service management		+	+	+	+	+		+	+	+		+	+	+	+	+		+	+	+	+	+		+	+	
	Casualty transportation	+				+			+	+	+		+	+	+		+		+		+	+		+	+	+	
	Mobile service management	+	+			+		+		+				+	+	+		+				+	+	+	+	+	
Phase 1	Shelter site management		+									+		+	+	+	+		+		+	+	+	+		+	
	Donation management	+	+	+				+	+	+				+	+	+				+	+		+	+	+	+	
	Workforce management for critical facilities			+	+			+						+	+				+	+					+	+	
	Coordination center management		+	+			+	+						+	+									+	+		
	Emergency response service management		+	+	+	+	+		+	+	+		+	+	+		+		+	+				+	+	+	
	Mobile service management	+	+			+		+		+				+	+	+								+	+	+	
Phase 2	Shelter site management		+									+						+					+	+			
	Mobile service management	+	+			+		+		+				+	+	+		+						+	+	+	
	Donation management	+	+				+	+	+					+	+	+		+		+	+		+	+	+	+	
	Risk assessment and restoration	+	+	+	+	+		+	+	+	+		+		+	+								+	+	+	
	Debris management		+	+	+			+	+	+	+		+		+			+						+	+		
	Workforce management for critical facilities	+	+	+				+	+					+	+	+		+		+	+				+	+	
	Supplier selection		+					+	+					+	+	+		+							+		

Appendix B

Data generation procedure for Kartal case study

Algorithm 1 Generating supply and demand related data.

```
1: for  $j \in J, t \in T$  do  
2:    $\theta_{jt} \sim U(0.1, 0.3)$   
3:    $d_{jt} = \theta_{jt} \times Pop_j$   
4: end for  
5: for  $t \in T$  do  
6:    $\beta_t \sim U(0.5, 0.7)$   
7:    $s_t = \beta_t \times \sum_{j \in J} d_{jt}$   
8:    $\lambda_t \leftarrow 50\%$   
9:    $p_t = \lfloor |I| \times \lambda_t \rfloor$   
10: end for  
11: for  $i \in I$  do  
12:    $c_i = \max_{t \in T} \{ \frac{s_t}{p_t} \}$   
13: end for
```

Appendix C

Visualization of selected Pareto-optimal solutions for Kartal case study

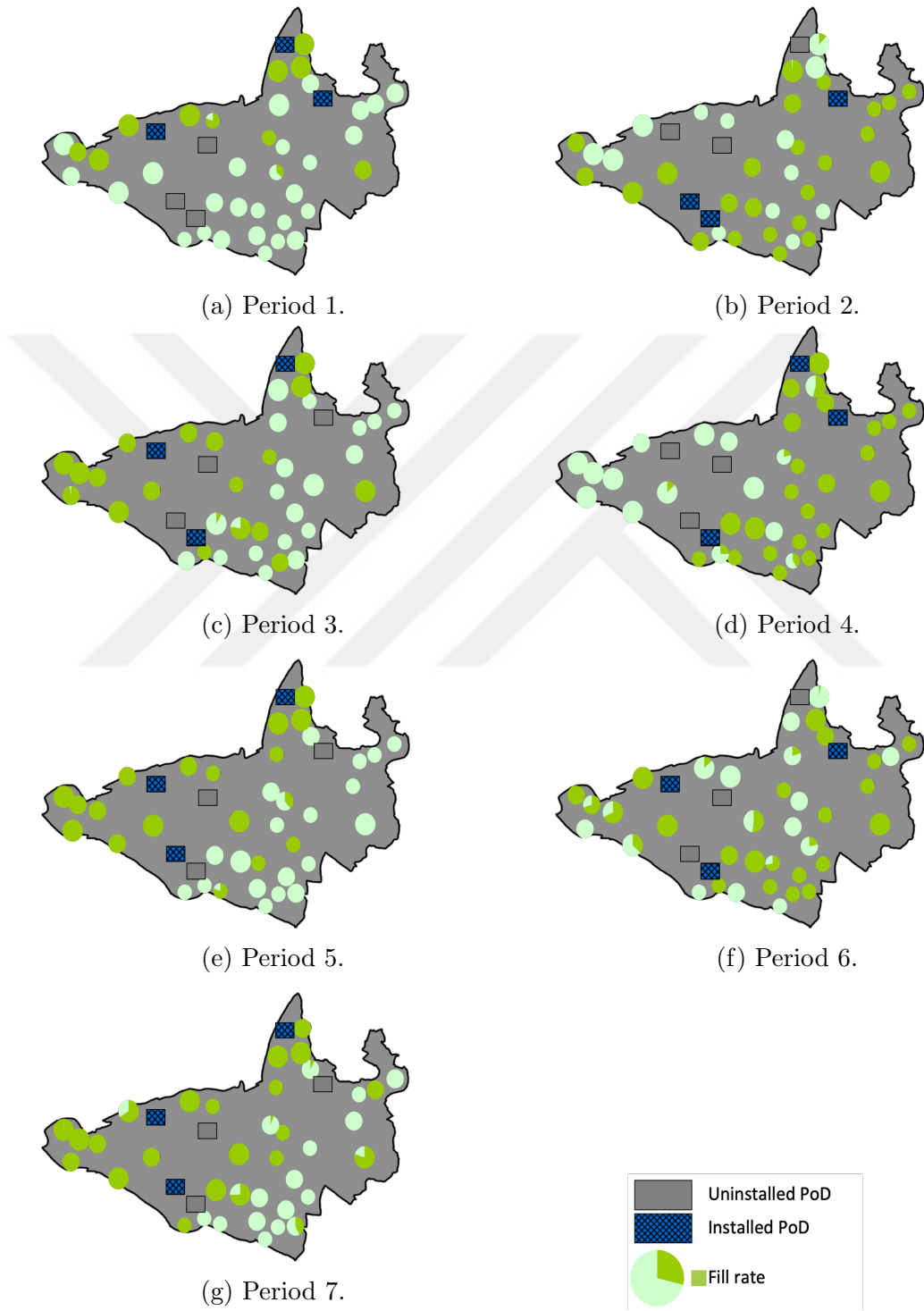


Figure C.1: Operating PoDs and fill rates obtained by the solution corresponding to Pareto-optimal point 1 to minimize the maximum deprivation cost, using three PoDs.

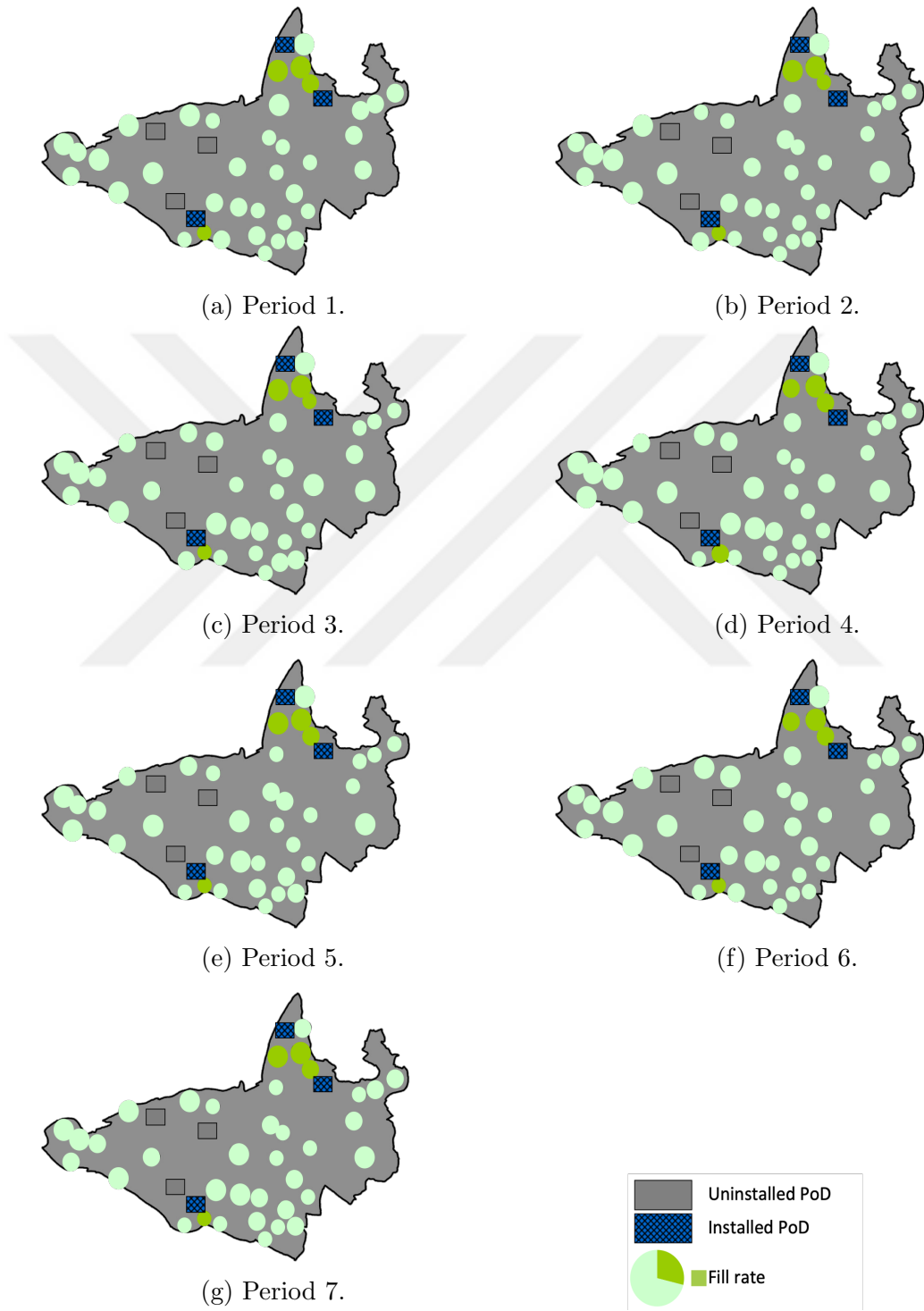


Figure C.2: Operating PoDs and fill rates obtained by the solution corresponding to Pareto-optimal points 2 and 5 to minimize the total transportation distance, using three PoDs.

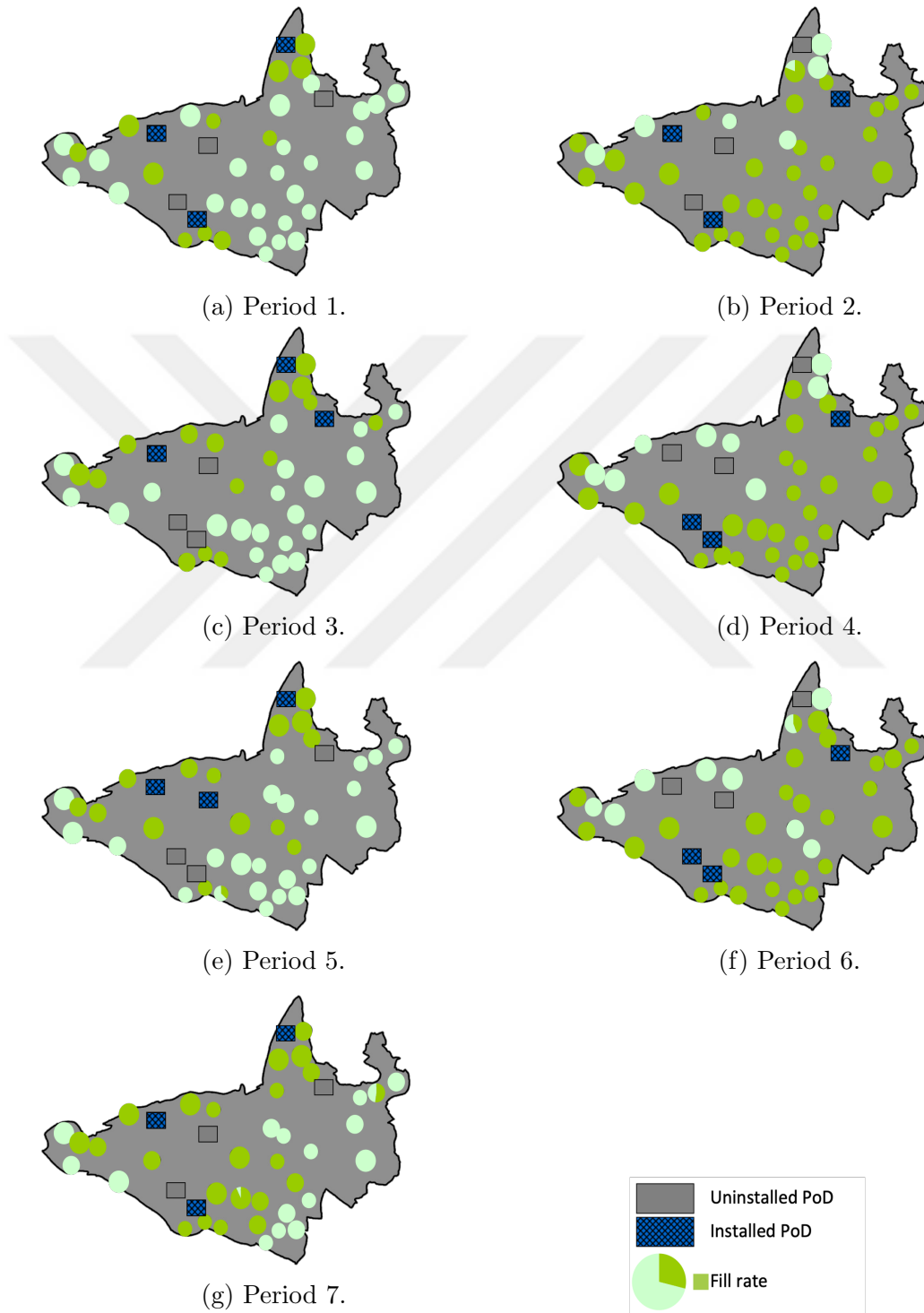


Figure C.3: Operating PoDs and fill rates obtained by the solution corresponding to Pareto-optimal point 4 to minimize the total deprivation cost, using three PoDs.

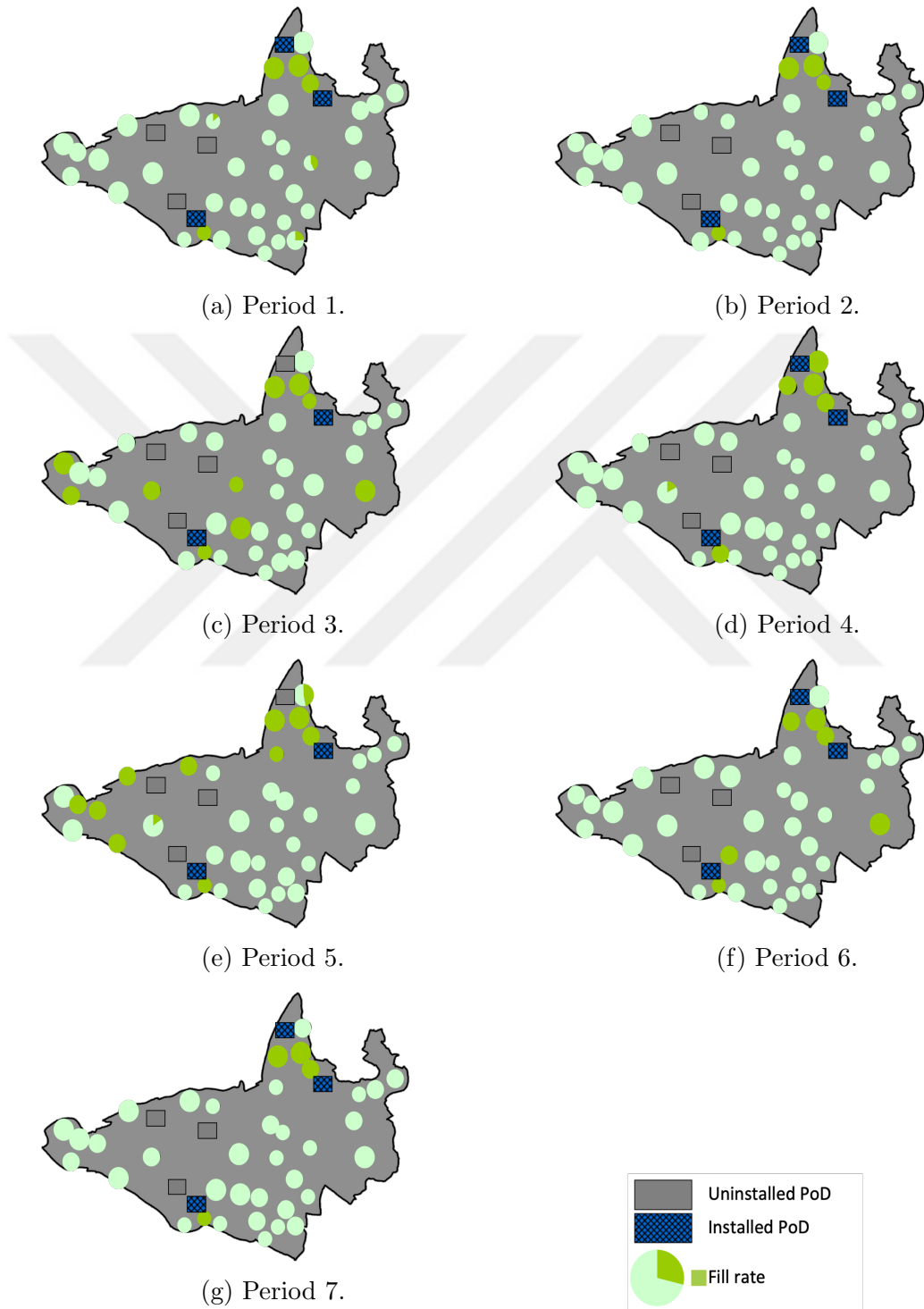


Figure C.4: Operating PoDs and fill rates obtained by the solution corresponding to Pareto-optimal point 3 to balance the maximum deprivation cost and the total transportation distance, using three PoDs.

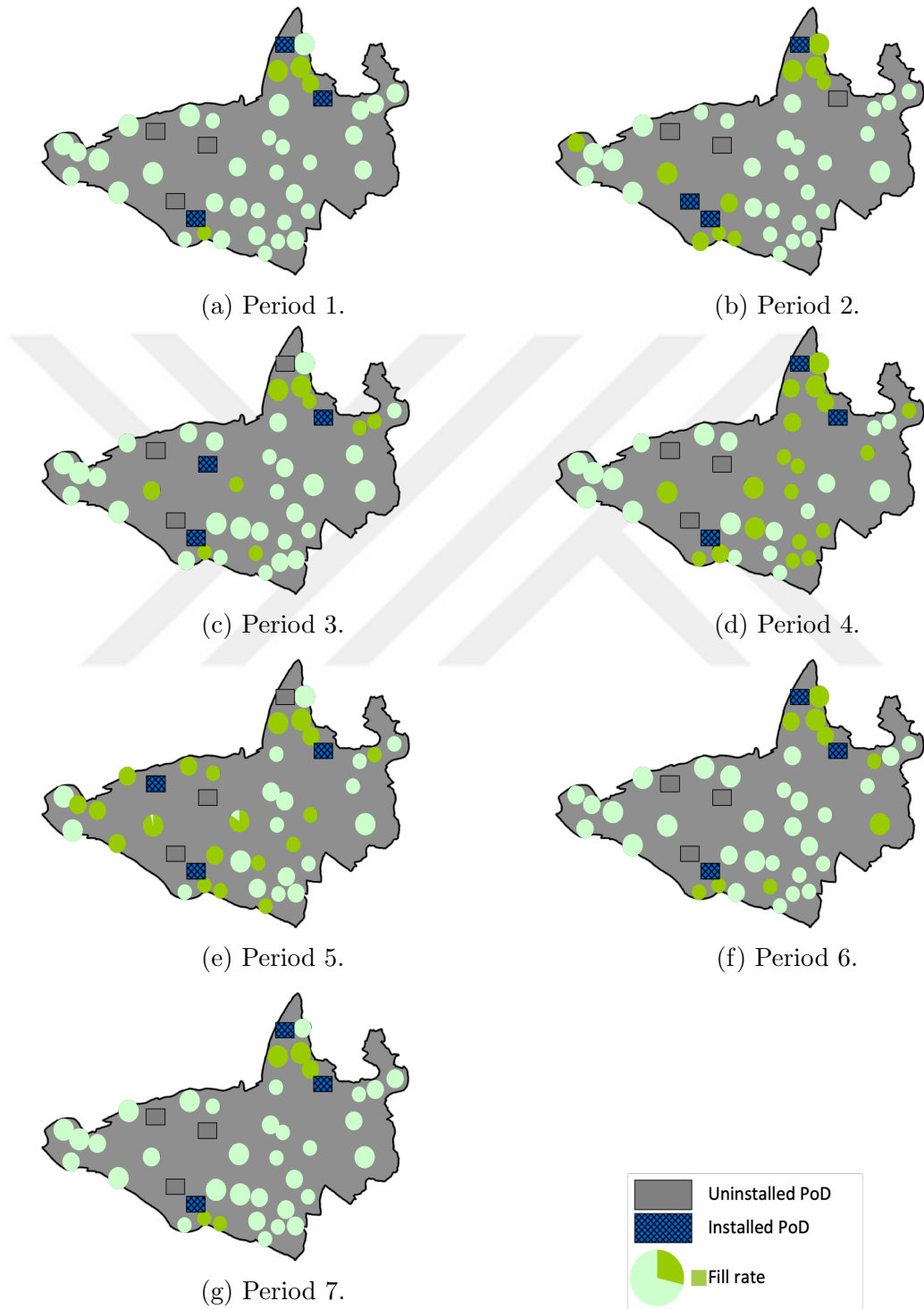


Figure C.5: Operating PoDs and fill rates obtained by the solution corresponding to Pareto-optimal point 6 to balance the total deprivation cost and the total transportation distance, using three PoDs.

Appendix D

Kilis case study

The mathematical model D-BOMIP1 is applied to a real-life data set of Kilis province, which is located in southern Türkiye [123]. This dataset includes 19 potential service points (hospitals, schools, etc.) together with 187 (109 villages and 78 neighborhoods) population points where Syrian refugees inhabit. Due to the sparsity of the network, most of the population points are not reachable from potential service points. In this study, we use some of the population points as the potential PoD locations in addition to 19 service points. By doing so, 130 population points can be covered if a PoD is located on a nearby site. Similar to the Kartal case study, 7 periods (e.g., 7 days) are considered. The rest of the data set is generated following the same procedure in the Kartal case study, and the ε -constraint algorithm is coded in Java 1.8.0 using CPLEX 12.71.

The resulting Pareto front is shown in Figure D.1. 85 Pareto optimal points are found in approximately 35 hours (shown as circles in Figure D.1). Starting with the point that corresponds to total distance minimization, 77 Pareto-optimal points are obtained in approximately 6 hours, meaning that 82% of the total time is spent on finding the remaining 8 points (10%). As the maximum deprivation cost threshold gets smaller in the ε -constraint algorithm, finding the Pareto optimal points becomes computationally exhausting.

Due to this computational challenge, 7 additional points are obtained with a 3.5% optimality gap (shown as triangular points in Figure D.1) in approximately 4.5 hours and included in the Pareto front.

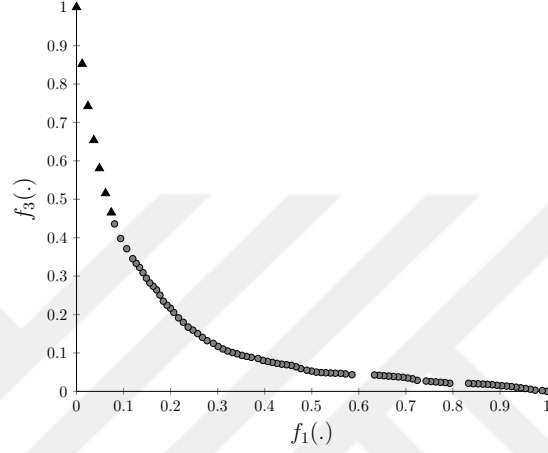


Figure D.1: Pareto frontier obtained for D-BOMIP1 for Kilis case study. ¹

¹3.5% optimality gap is allowed only for triangular points.

Appendix E

Scenario tree generation procedure

$MaxEdge$ and $MinEdge$ are, respectively, the upper and lower bounds for the number of possible realizations after a node in the scenario tree, α is the magnitude of change in the supply value in each of these realizations, s_1 is the base value for supply for which the possible realizations for the first time period are calculated.

Algorithm 2 Generating the scenario tree.

```

1:  $\mathcal{N} \leftarrow \{1\}$ 
2:  $\mathcal{N}_1 \leftarrow \{1\}$ 
3:  $NbEdge_1 \sim U(MaxEdge, MinEdge)$ 
4:  $NodeCount \leftarrow 1$ 
5:  $\mathcal{P}_{1,1} \leftarrow 1$ 
6: for  $m \in \mathcal{M} \setminus \{1\}$  do
7:    $\mathcal{N} \leftarrow \mathcal{N} \cup \{NodeCount + 1, \dots, NodeCount + \sum_{\nu \in \mathcal{N}_{m-1}} NbEdge_\nu\}$ 
8:    $\mathcal{N}_m \leftarrow \{NodeCount + 1, \dots, NodeCount + \sum_{\nu \in \mathcal{N}_{m-1}} NbEdge_\nu\}$ 
9:   for  $n \in \mathcal{N}_m$  do
10:    if  $m < |\mathcal{M}|$  then
11:       $NbEdge_n \sim U(MaxEdge, MinEdge)$ 
12:    end if
13:     $\pi(n) \leftarrow \frac{1}{|\mathcal{N}_m|}$ 
14:     $\sigma(n) \leftarrow m$ 
15:     $\rho(n) \leftarrow m - 1$ 
16:     $NodeCount \leftarrow NodeCount + 1$ 
17:  end for
18: end for
19: for  $n \in \mathcal{N} \setminus \{1\}$  do
20:    $\mathcal{P}_{n,\sigma(n)} \leftarrow n$ 
21:   for  $n_1 \in \mathcal{N}_{\sigma(n)-1}$  do
22:     $cnt \leftarrow \mathcal{N}_{\sigma(n)-1}(|\mathcal{N}_{\sigma(n)-1}|)$ 
23:    if  $\sigma(n) \geq 3$  then
24:      for  $n_2 \in \{\mathcal{N}_{\sigma(n)-1}(1), \dots, n_1 - 1\}$  do
25:         $cnt \leftarrow cnt + NbEdge_{n_2}$ 
26:      end for
27:    end if
28:    if  $cnt \leq n \leq cnt + NbEdge_{n_1}$  then
29:       $\mathcal{P}_{n,\sigma(n)-1} \leftarrow n_1$ 
30:       $\gamma(n) \leftarrow n_1$ 
31:      for  $m_1 \in \{1, \dots, \sigma(n) - 2\}$  do
32:         $\mathcal{P}_{n,m_1} \leftarrow \mathcal{P}_{n_1,m_1}$ 
33:      end for
34:    end if
35:  end for
36: end for

```

Algorithm 3 Generating the stochastic information.

```

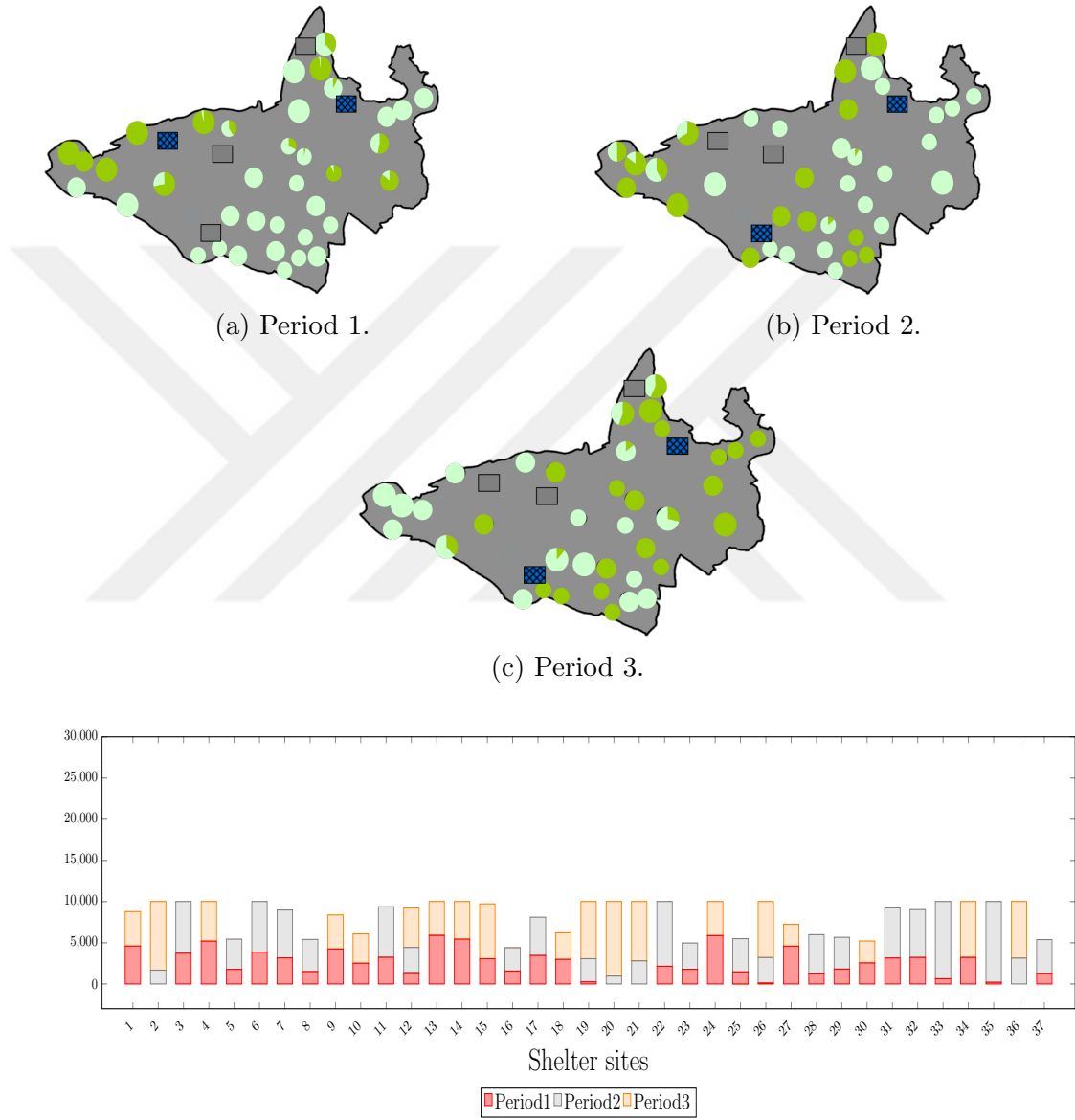
1: for  $n \in \mathcal{N} \setminus \mathcal{N}_{|\mathcal{M}|}$  do
2:   for  $n_1 \in \{1, \dots, NbEdge_n\}$  do
3:     if  $NbEdge_n$  is odd then
4:        $a = \frac{NbEdge_n - 1}{2}$ 
5:       if  $n \leq a$  then
6:          $s_{n_1} \leftarrow s_n \times (1 + an\alpha)$ 
7:       else if  $n = a + 1$  then
8:          $s_{n_1} \leftarrow s_n$ 
9:       else
10:         $s_{n_1} \leftarrow s_n \times (1 - a(a + 1 - n)\alpha)$ 
11:      end if
12:    else
13:       $a = \frac{NbEdge_n}{2}$ 
14:      if  $n \leq a$  then
15:         $s_{n_1} \leftarrow s_n \times (1 + an\alpha)$ 
16:      else
17:         $s_{n_1} \leftarrow s_n \times (1 - a(a + 1 - n)\alpha)$ 
18:      end if
19:    end if
20:  end for
21: end for

```

Appendix F

Visualization of selected solutions to MSSP for Kartal case study

In Figures F.1- F.4, demand nodes, i.e., shelter sites, are represented by circles with sizes proportional to their demand, while the potential PoD locations are shown as rectangles. Considering the allocation amounts, fill rates (percentage of satisfied demand) are visually represented on these maps as pie charts placed on each demand point. The operated PoDs are represented by diagonally patterned rectangles.



(d) Accumulated deprivation cost levels during the planning horizon.

Figure F.1: Operating PoDs, fill rates and accumulated deprivation cost levels for a neutral scenario based on EP solution, using two PoDs.

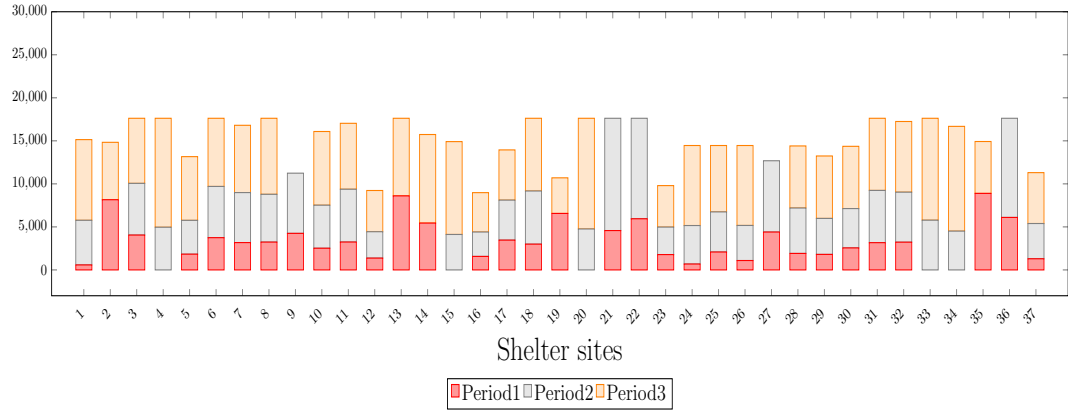
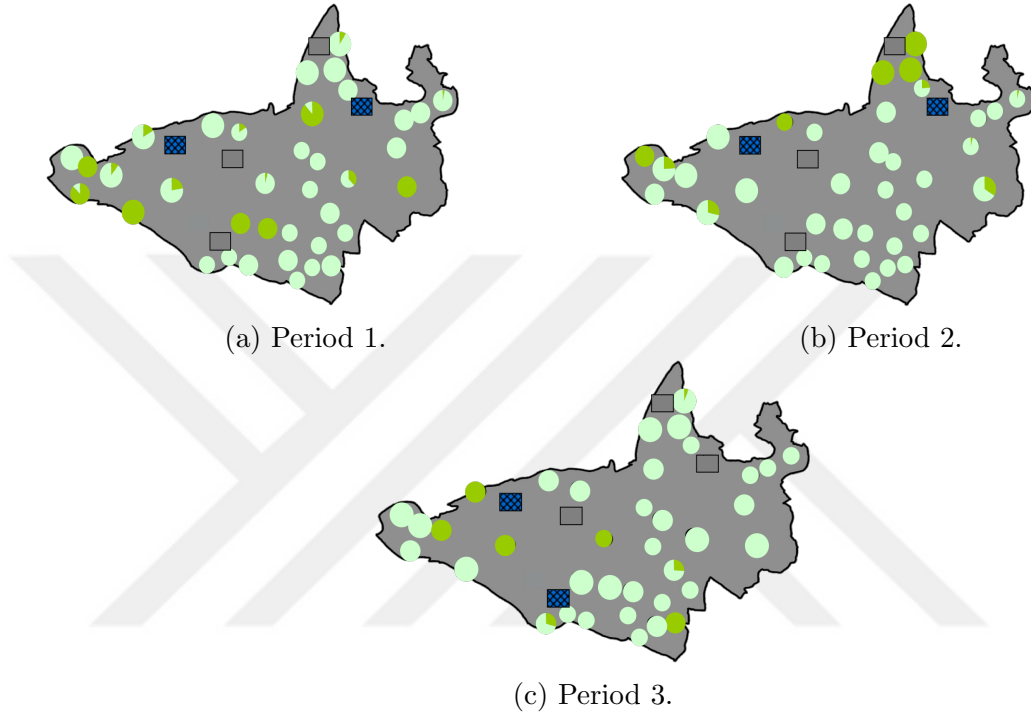
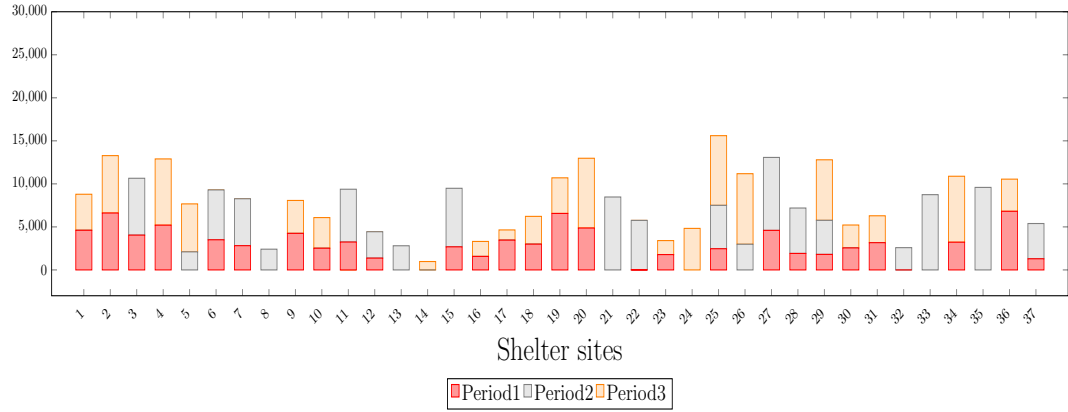
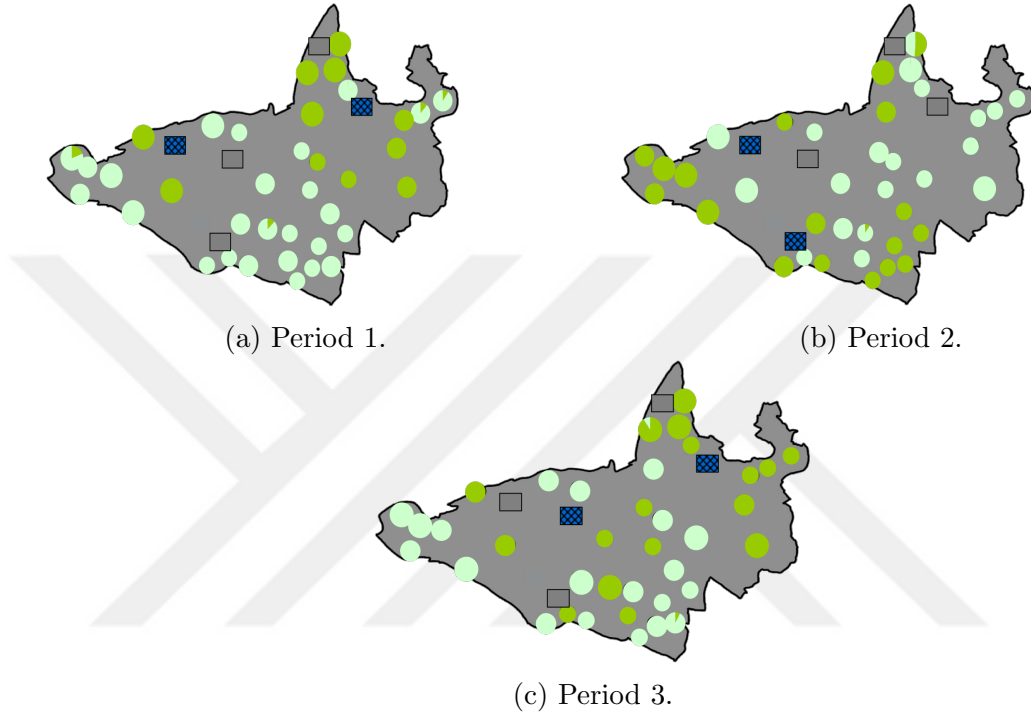


Figure F.2: Operating PoDs, fill rates and accumulated deprivation cost levels for the worst case scenario based on EP solution, using two PoDs.



(d) Accumulated deprivation cost levels during the planning horizon.

Figure F.3: Operating PoDs, fill rates and accumulated deprivation cost levels for a neutral scenario based on EA solution, using two PoDs.

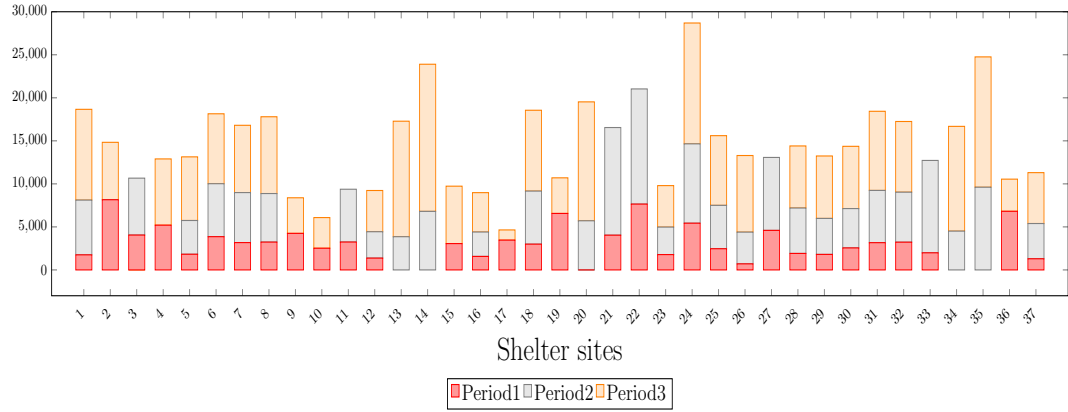
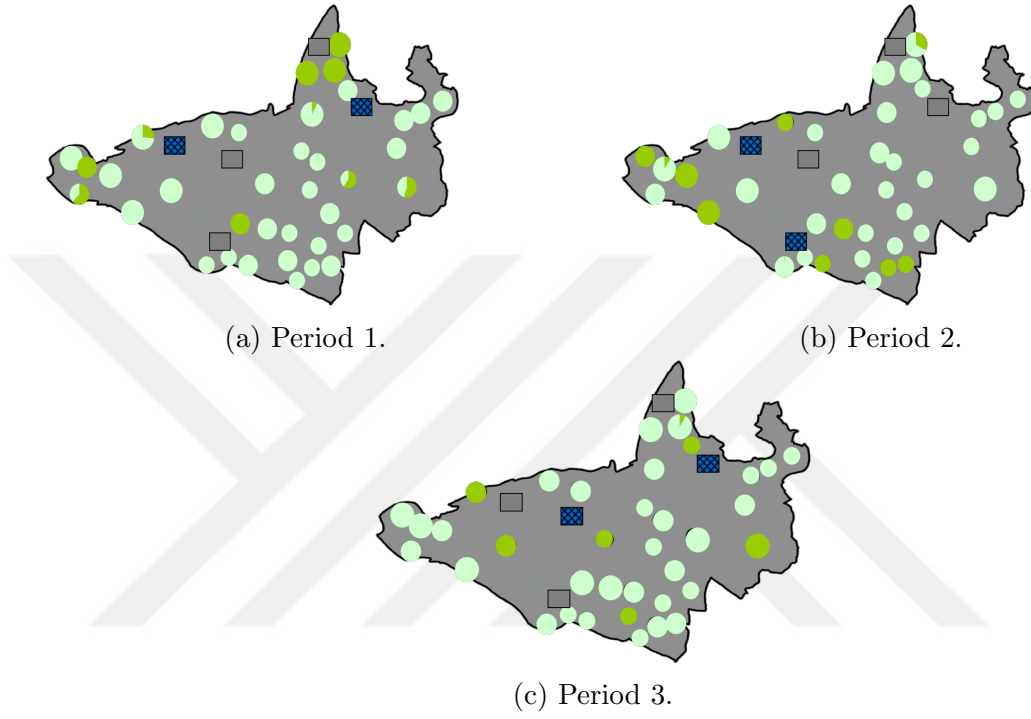


Figure F.4: Operating PoDs, fill rates and accumulated deprivation cost levels for the worst case scenario based on EA solution, using two PoDs.

Appendix G

Scenario reduction based heuristic algorithm

Algorithm 4 Heuristic algorithm based on scenario reduction and clustering.

```

1:  $\Omega^R \leftarrow \emptyset, \mathcal{N}^R \leftarrow \mathcal{N}, \pi^R \leftarrow \pi$ 
2:  $NbClusters = NbChildren_1, NbChildren^R \leftarrow NbChildren$ 
3: for  $n \in \mathcal{N}_{|\mathcal{M}-1|}$  do
4:    $\bar{m} \leftarrow \{m \in \Omega : \gamma(m) = n, \gamma(l) = n, s_m \leq s_l, \forall l \in \Omega\}$ 
5:    $SelectedChild_n \leftarrow \bar{m}$ 
6:    $R(\bar{m}) \leftarrow \bar{m}$ 
7:    $\Omega^R \leftarrow \Omega^R \cup \{\bar{m}\}$ 
8:    $NbChildren_n^R = 1$ 
9: end for
10: for  $n \in \Omega$  do
11:   if  $n \in \Omega_R$  then
12:      $\pi^R(\bar{m}) \leftarrow \pi(\gamma(\bar{m}))$ 
13:   else
14:      $\mathcal{N}^R \leftarrow \mathcal{N}^R \setminus \{n\}$ 
15:      $\pi^R(\bar{m}) \leftarrow 0$ 
16:      $R(n) \leftarrow SelectedChild_{\gamma(n)}$ 
17:   end if
18: end for
19: for  $c \in \{1, \dots, NbClusters\}$  do
20:    $\mathcal{N}^{R,c} \leftarrow \mathcal{N}^{R,c} \cup \{1\}$ 
21:    $\mathcal{N}_1^{R,c} \leftarrow \mathcal{N}_1^{R,c} \cup \{1\}$ 
22:    $\pi_c^R(c+1) \leftarrow 1$ 
23:    $NbChildren_1^{R,c} = 1$ 
24:   for  $n \in \mathcal{N} \setminus \{1\}$  do
25:     if  $\mathcal{P}_{n,2} = (c+1)$  then
26:        $\mathcal{N}^{R,c} \leftarrow \mathcal{N}^{R,c} \cup \{n\}$ 
27:        $\mathcal{N}_{\sigma(n)}^{R,c} \leftarrow \mathcal{N}_{\sigma(n)}^{R,c} \cup \{n\}$ 
28:        $\pi_c^R(n) \leftarrow \frac{\pi_c^R(\gamma(c))}{NbChildren_{\gamma(n)}^{R,c}}$ 
29:     end if
30:   end for
31: end for
32: for  $k \in \{1, \dots, \binom{|I|}{f_1}\}$  do
33:   for  $c \in \{1, \dots, NbClusters\}$  do
34:     Solve  $RP(c, k) : \min_{x, y \in \mathcal{X}_c^R} f^R(x, y)$  such that  $y_{i1} = 1, \forall i \in I^k$ 
35:     Return  $x^{R*}, y^{R*}$ 
36:     Solve  $CP(c, k) : \min_{x, y \in \mathcal{X}_c} f^R(x, y)$  such that  $y_1^m = 1, \forall m \in I^k, \frac{x_{ijn}}{s_n} =$ 
37:        $\frac{x_{ijR(n)}^{R*}}{s_{R(n)}}, y_{i\gamma(n)} = y_{i\gamma(R(n))}^{R*}, \forall n \in \Omega$ 
38:     Return  $x^{C*}(c, k), z^{C*}(c, k)$ 
39:   end for
40:    $z^*(k) = \sum_{c=1}^{NbClusters} \pi(c+1) z^{C*}(c, k)$ 
41:    $I^* \leftarrow \arg \min_{k \in \{1, \dots, \binom{|I|}{f_1}\}} z^*(k)$ 
42:    $z_H^* \leftarrow \min_{k \in \{1, \dots, \binom{|I|}{f_1}\}} z^*(k)$ 
43: end for

```

Appendix H

Computational experiments

Table H.1: Numerical results for 4 stages, $|\mathcal{M}| = 4$, $\alpha = 30\%$.

Instance #	MSSP				Scenario reduction heuristic			Rolling horizon heuristic	
	$ \mathcal{N} $	z_{MSSP}^*	CPU time (sec)	%gap	$ \mathcal{N} _{RT}$	z_{SR}	CPU time (sec)	z_{RH}	CPU time (sec)
1	25	8.76	773.39	4.00	20	9.05	1.17	8.93	4.23
2	36	8.42	>1 hr	6.09	15	8.87	1.69	8.54	9.25
3	20	8.95	199.89	4.00	20	9.34	1.05	9.23	3.40
4	21	8.99	268.10	4.00	13	9.34	1.09	9.36	5.16
5	20	9.00	297.47	4.00	15	9.35	1.08	9.23	3.37
6	33	8.49	1898.36	4.00	16	8.90	1.32	8.73	5.47
7	26	8.79	767.16	4.00	22	9.15	1.20	8.97	3.54
8	29	8.58	>1 hr	5.14	13	8.90	1.50	8.87	5.87
9	21	8.75	21.10	4.00	13	9.21	0.78	9.06	2.13
10	25	8.80	643.51	4.00	13	9.16	1.19	8.96	5.28

Table H.2: Numerical results for 5 stages, $|\mathcal{M}| = 5$, $\alpha = 30\%$.

Instance #	MSSP				Scenario reduction heuristic			Rolling horizon heuristic	
	$ \mathcal{N} $	z_{MSSP}^*	CPU time (sec)	%gap	$ \mathcal{N} _{RT}$	z_{SR}	CPU time (sec)	z_{RH}	CPU time (sec)
1	63	11.98	>1 hr	10.51	40	12.23	8.68	12.41	7.93
2	93	11.68	>1 hr	15.50	59	11.64	135.03	11.81	66.15
3	48	12.37	>1 hr	11.59	32	12.47	111.56	12.59	8.20
4	52	12.40	>1 hr	11.14	34	12.31	45.67	12.71	9.39
5	49	12.39	>1 hr	12.89	32	12.36	135.45	12.65	7.47
6	90	11.69	>1 hr	12.44	54	11.84	12.09	11.99	7.61
7	71	12.24	>1 hr	13.40	43	12.40	165.06	12.55	22.68
8	72	11.86	>1 hr	13.89	46	11.84	47.14	12.01	10.53
9	51	12.33	>1 hr	10.35	34	12.49	14.62	12.92	5.09
10	67	12.28	>1 hr	13.51	41	12.31	163.84	12.31	8.93

Table H.3: Numerical results for 6 stages, $|\mathcal{M}| = 6$, $\alpha = 30\%$.

Instance #	MSSP				Scenario reduction heuristic			Rolling horizon heuristic	
	$ \mathcal{N} $	z_{MSSP}^*	CPU time (sec)	%gap	$ \mathcal{N} _{RT}$	z_{SR}	CPU time (sec)	z_{RH}	CPU time (sec)
1	156	16.72	>1 hr	18.87	101	16.13	589.81	16.55	13.11
2	236	16.82	>1 hr	25.38	150	16.20	831.32	16.53	122.19
3	114	17.29	>1 hr	21.41	76	16.48	408.67	17.38	16.93
4	134	16.96	>1 hr	20.51	83	17.34	399.81	16.73	12.78
5	124	16.93	>1 hr	20.63	78	17.11	412.57	17.38	17.65
6	230	17.46	>1 hr	26.76	147	15.75	626.96	16.36	23.36
7	182	17.17	>1 hr	25.77	116	16.90	488.69	16.14	59.26
8	181	16.39	>1 hr	22.09	115	16.55	512.18	16.04	82.59
9	125	16.46	>1 hr	17.83	81	16.86	384.26	17.15	39.82
10	172	16.16	>1 hr	19.26	109	16.53	462.41	16.16	33.12

Table H.4: Numerical results for 7 stages, $|\mathcal{M}| = 7$, $\alpha = 30\%$.

Instance #	MSSP				Scenario reduction heuristic			Rolling horizon heuristic	
	$ \mathcal{N} $	z_{MSSP}^*	CPU time (sec)	%gap	$ \mathcal{N} _{RT}$	z_{SR}	CPU time (sec)	z_{RH}	CPU time (sec)
1	383	56.19	>1 hr	71.27	249	22.64	1203.90	20.40	50.99
2	594	176.32	>1 hr	91.54	379	28.26	1205.78	20.36	131.42
3	279	140.26	>1 hr	87.87	180	23.63	758.48	21.45	65.14
4	341	85.39	>1 hr	78.84	216	25.13	792.69	21.65	80.59
5	316	33.96	>1 hr	50.16	199	24.58	682.36	21.59	69.68
6	571	56.82	>1 hr	73.51	370	25.78	1079.98	28.05	102.07
7	470	182.58	>1 hr	91.32	293	53.22	800.79	36.41	129.20
8	446	53.98	>1 hr	71.20	290	25.74	1200.90	32.23	67.40
9	311	131.56	>1 hr	39.25	199	23.68	800.48	21.76	60.74
10	431	182.22	>1 hr	91.05	277	42.80	800.66	21.84	118.44

Table H.5: Numerical results for EVPI and DVSS.

$ \mathcal{M} $	Instance #	$ \mathcal{N} $	z^*	WS	%EVPI	EDEV	%DVSS
3	1	25	8.76	7.23	21.13	10.06	14.85
	2	36	8.42	7.01	20.14	9.73	15.51
	3	20	8.95	7.44	20.23	10.30	15.09
	4	21	8.99	7.45	20.63	10.49	16.69
	5	20	9.00	7.44	20.96	10.38	15.36
	6	33	8.49	7.05	20.43	9.68	14.05
	7	26	8.79	7.29	20.51	9.84	11.98
	8	29	8.58	7.10	20.81	9.84	14.63
	9	21	8.75	7.36	18.86	10.14	15.82
	10	25	8.80	7.31	20.36	9.89	12.42
4	1	63	11.98	9.86	21.50	13.68	14.25
	2	93	11.68	9.48	23.30	12.88	10.28
	3	48	12.37	10.15	21.86	14.33	15.82
	4	52	12.40	10.15	22.15	13.76	11.01
	5	49	12.39	10.14	22.17	14.17	14.41
	6	90	11.69	9.61	21.53	13.08	11.93
	7	71	12.24	9.87	24.04	13.49	10.21
	8	72	11.86	9.63	23.14	13.63	14.88
	9	51	12.33	10.15	21.45	13.80	11.97
	10	67	12.28	10.46	17.44	13.50	9.86
5	1	156	16.13	13.05	28.11	18.84	16.80
	2	236	16.20	12.53	34.32	17.77	9.64
	3	114	16.48	13.46	28.41	19.12	16.01
	4	134	16.73	13.40	26.55	18.82	12.47
	5	124	16.93	13.45	25.85	18.81	11.12
	6	230	15.75	12.69	37.55	18.57	17.91
	7	182	16.14	13.04	31.72	18.83	16.67
	8	181	16.04	12.80	28.06	18.54	15.56
	9	125	16.46	13.41	22.72	18.90	14.85
	10	172	16.16	13.12	23.13	17.41	7.77

Appendix I

Dynamic value of the stochastic solution

Additional notation is defined for calculating DVSS for the formulation provided in Chapter 5:

Ω^n , set of scenarios induced by node $n \in \mathcal{N}$ or, in other words, that include the node.

Γ^n , set of all successors of node $n \in \mathcal{N}$.

Γ_1^n , set of immediate successors of node $n \in \mathcal{N}$.

In our problem, the parameters capturing uncertainty are s_n , $n \in \mathcal{N} \setminus \{1\}$. A possible use of the expected value solution consists of solving a model, say, EV^n in every node of the scenario tree. This should be done sequentially for all stages. In a specific stage and for every node in that stage we consider the subtree rooted at that node and compute the corresponding conditional expected solution considering the future supply realizations.

In the first step, we solve problem EV^1 (i.e., for the root node) denoting its optimal value by V_{EV^1} and save the optimal values of the first-stage variables

$\{\bar{\mathbf{y}}^1\}$. This is accomplished as follows:

Problem EV^1 is a single scenario problem rooted at node 1, i.e., we consider a single node in every stage. For each stage $m \in \mathcal{M} \setminus \{1\}$ we consider the following data defining the unique scenario in that stage, which represents the expected supply:

$$\bar{s}_m = \sum_{n \in \mathcal{N} | \sigma(n)=m} \pi(n) s_n;$$

After solving problem EV_1 we obtain a solution that we denote by $\{\bar{\mathbf{y}}_m^1\}_{m \in \mathcal{M} \setminus \Omega}$; $\{\bar{\mathbf{x}}_m^1\}_{m \in \mathcal{M} \setminus \{1\}}$; $\{\bar{\mathbf{v}}_m^1\}_{m \in \mathcal{M} \setminus \{1\}}$; $\{\bar{\mathbf{w}}_m^1\}_{m \in \mathcal{M} \setminus \{1\}}$; $\{\bar{\mathbf{z}}_m^1\}_{m \in \mathcal{M} \setminus \{1\}}$.

For the next stages, we fix the first-stage solution according to the values $\{\bar{\mathbf{y}}_1\}$.

For every node $n \in \mathcal{N}$ belonging to stage m (i.e., such that $\sigma(n) = m$), we solve problem EV^n and denote its optimal value by V_{EV^n} . This problem corresponds to a single scenario (deterministic) problem that has the decisions corresponding to the previous stages fixed according to the computations already done. A single node (representing the average scenario) is considered in every remaining stage. In other words, for every remaining stage $\kappa = \sigma(n) + 1, \dots, |\mathcal{M}|$ we define a single (sub)scenario until the end using the following parameters, representing the conditional expectation of the future supply values:

$$\bar{s}_\kappa = \sum_{\nu \in \Gamma^n | \sigma(\nu)=\kappa} \frac{\pi(\nu)}{\pi(n)} s_\nu;$$

After solving problem EV^n using the above data we obtain a solution that we denote by $\{\bar{\mathbf{y}}_\kappa^n\}_{\kappa=\sigma(n), \dots, |\mathcal{M}|-1}$; $\{\bar{\mathbf{x}}_\kappa^n\}_{\kappa=\sigma(n), \dots, |\mathcal{M}|}$; $\{\bar{\mathbf{v}}_\kappa^n\}_{\kappa=\sigma(n), \dots, |\mathcal{M}|}$; $\{\bar{\mathbf{w}}_\kappa^n\}_{\kappa=\sigma(n), \dots, |\mathcal{M}|}$;

$$\{\bar{\mathbf{z}}_\kappa^n\}_{\kappa=\sigma(n), \dots, |\mathcal{M}|}.$$

When considering problem EV^ν for a node ν that is a successor of n (i.e. $\nu \in \Gamma_1^n$) we fix the solution corresponding to node n .

The expected value problems for the last stage, $m = |\mathcal{M}|$, are considered at the

beginning of period T . The decisions that remain to be made are the allocation (x) and the resulting expected deprivation cost in the last period.

We define the expected outcome in stage m from using the above dynamic solution of the average scenario, and we denote it by $EDEV_m$ ($m \in \mathcal{M}$). That is,

$$EDEV_m = \sum_{n \in \mathcal{N} \mid \sigma(n)=m} \pi(n) V_{EV^n}, \quad m \in \mathcal{M}.$$

Finally, we compute the dynamic value of the stochastic solution (or its relative value) as follows:

$$DVSS = EDEV_{|\mathcal{M}|} - z_{MSSP}^*$$

or

$$\%DVSS = \frac{DVSS}{z_{MSSP}^*} \times 100,$$

where z_{MSSP}^* is the optimal value of the multi-stage stochastic model.