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APPLICATION OF AHP TO MULTICRITERIA INVENTORY CLASSIFICATION

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**BY
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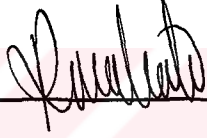
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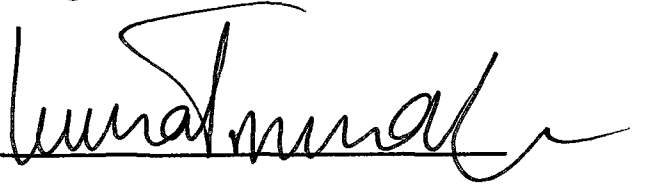
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ABSTRACT

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In this thesis, a new method based on the application of Analytic Hierarchy Process (AHP) to ABC inventory classification is investigated. The traditional ABC classification method utilizes only the unit price and the annual usage of inventory items. However, in some cases, the classification done using only these two criteria turns out to be insufficient. The method described in this thesis enables the integration of several criteria that can be organized in a hierarchy into ABC classification.

The method can be summarized as follows: A matrix is constructed by the pairwise comparison of criteria on the highest level. The elements of the eigen vector of this matrix represent the weights (priorities) of the criteria. If a criterion has subcriteria in the hierarchy, the weights computed in the similar manner for the subcriteria are multiplied by the weight of the criterion and inserted in its place. Repetition of these steps for all levels of the hierarchy, the weight of all criteria are determined.

Using the criteria weights determined by the AHP technique, the weighted score of each inventory item is computed. The items sorted by that weighted score are grouped in three classes: A, B, and C, as in the classical ABC classification.

This new method is applied to the classification of inventory items used in rock excavation jobs done using blasting by a construction company. The same inventory is also classified according to the classical ABC technique, and the results are compared.

Keywords: Inventory management, ABC classification, Analytic Hierarchy Process, multiple criteria



ÖZET

AHP TEKNİĞİNİN ÇOK KRİTERLİ ENVANTER SINIFLANDIRMASINA UYGULANMASI

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Bu çalışmada Analitik Hiyerarşi Süreci (AHS) tekniğinin ABC envanter sınıflandırmasına uygulanması esasına dayanan yeni metot incelenmiştir. Klasik ABC sınıflandırma metodu kriter olarak yalnızca birim fiyat ve yıllık ortalama kullanım değerlerini esas almaktadır. Ancak yalnızca bu iki kritere göre yapılan sınıflandırma bazı durumlarda yetersiz kalmaktadır. Buna karşılık ele alınan yeni metot bir hiyerarşi şeklinde organize edilebilmek şartıyla çok sayıda kriterin ABC sınıflandırmasında kullanılmasına olanak vermektedir.

Bu yeni metoda göre önce en üst düzeydeki kriterler ikişer ikişer karşılaştırılarak bir matris oluşturulur. AHS tekniğine göre bu matrisin eigen vektörünün elemanları kriterlerin ağırlık değerlerini verir. Hiyerarşinin alt düzeylerindeki kriterler için de benzer şekilde hesaplanan ağırlıklar üst düzeydeki vektörde karşılık düşen kriterin ağırlık değeri ile çarpılarak bu değer yerine konur. Bu adımlar hiyerarşinin tüm düzeyleri için tekrarlanarak bütün kriterlerin ağırlık değerleri bulunmuş olur.

AHS tekniđi yardımıyla bulunan bu kriter ağırlık değeri kullanarak her envanter kaleminin ağırlıklı değeri bulunur. Bu değere göre sıralanan kelemeler daha sonra klasik ABC sınıflandırmasında olduđu gibi A, B, ve C olmak üzere üç ayrı sınıfa yerleştirilir.

Bu yeni metot bir inşaat şirketi tarafından patlayıcı ile yapılan kaya hafriyatında kullanılan malzemelerin sınıflandırılmasına uygulanmıştır. Aynı malzemeler ayrıca klasik ABC tekniğine göre de sınıflandırılmış ve sonuçlar karşılaştırılmıştır.

Anahtar sözcükler: Envanter yönetimi, ABC sınıflandırması, Analitik Hiyerarşi Süreci, çoklu kriter



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1. INTRODUCTION

Inventory management represents one of the most important functions of production and operations management. Inventory managers try to maintain sufficient inventories to meet demand and achieve productivity, while at the same time to incur the lowest possible cost. In the literature, many models and approaches for planning and controlling inventories have been developed [11].

Management and control of inventories consisting of a large number of different items is usually done by classifying inventory items in three groups, called the ABC classification. In the traditional ABC classification, items are classified according to their total annual dollar usage. However, if criteria other than unit price and annual usage are important, managers must decide how to take them into account. Recently, it has been suggested that multiple criteria ABC classification can provide a more comprehensive managerial approach [8].

In the case of multiple criteria, a specific mechanical method is needed to reduce the classification to an ABC grouping. Flores, Olson and Dorai [7] have proposed the use of the Analytic Hierarchy Process (AHP) to reduce these multiple criteria to a univariate and consistent measure to consider multiple inventory management objectives.

1.1. A Review of the Related Literature

Some of the earliest formal work on classification was undertaken by Vilfredo Pareto in the late 1800's [13]. He claimed that a small percentage of the population of a country creates the majority of its output. Pareto thought that his 80-20 observations were generalizable but was never able to prove definitely that this distribution would hold over all applications.

In the late 1940's, H. Ford Dickie of General Electric Co. expanded on Pareto's concept and defined what he called the ABC inventory classification. Since then, classical ABC analysis has found many applications (e.g., [5]).

The classical ABC analysis is a very valuable tool for inventory management. However, there are many cases where the classical method is unsuitable. Therefore several extensions have been proposed and developed to overcome the difficulties arising in such cases.

In 1979, Peterson and Silver [9] recommend that items can be grouped according to whether they are slow (with a low demand, D , and high per unit cost, v) or fast moving items (with a high demand, D , and low per unit cost, v) in terms of average usage during lead time. Therefore there will not be any difficulties in the analysis of demand during a replenishment lead time.

In 1980, Sarai has extended the classical ABC analysis in two ways [17]. The first extension is the analysis of the items on the basis of their yearly chronological standard values. In this extended analysis, while annual dollar usage (value of the material consumption) is considered as a basis to classify items, relative numbers (cumulated ratio) are formed on the basis of chronological standards.

The other one is the refinement (precision) of the ABC analysis. In this case Sarai has considered some factors over the consumption value which is used in the classical ABC analysis. Those factors can be ranked as follows:

- Conditions of supply
- Conditions of consumption
- Storing conditions
- The link between goods (complementary and possibility for substitution)

According to these factors each class has some special criteria that reflect the features of these factors. Items whose characteristics fit these criteria are designed to one of the three classes. With this technique, besides the annual dollar usage, Sarai has considered other factors that are useful for controlling items efficiently. The refinement technique proposed by Sarai has been applied to the classification of construction items by Özaltın [12].

Classical ABC method may over-emphasize the importance of items that have high annual cost but are not as important to the firm. At the same time, focusing only upon one criterion may under-emphasize low annual cost items that are important. Therefore, use of annual usage only, in some cases, may lead the firm to mismanage its inventory assets.

In 1985, Flores and Whayberk [8] viewed the inventory classification technique from a different perspective by taking into account multiple criteria concept. They first proposed the use of joint criteria matrix to compare criteria pairwise [8]. They proposed the addition of several new criteria to the usual cost-volume criteria for ranking the importance of inventory items.

They pointed out that the importance of the criteria differs in each part of an organization. For example, engineering activities of a firm might deal with the obsolescence criterion. Because of the developments of high technology

items, some parts become obsolete, and they may not be used any more together with new items.

Similarly, one of the most important criteria that can influence the management of inventory in the purchasing area is the lead time. Both the length of lead time and its variability are important in maintaining an adequate supply of an item without excessive costs. The length of the lead time dictates the response time to a crisis. The variability of lead time determines the amount of safety stock, a firm has to supply to provide desired service level.

Substitutability is an important criterion for maintenance department. Substitution potential of items provide flexibility in response the problems by reducing the importance of them relative the less substitute items.

Repairability criterion also carries the similar advantages as the substitutability in terms of flexibility. Another possible criterion in the maintenance field is criticality. This factor is closely related to the idea of stock-out costs. A criticality index would be a method of classifying such items for management purposes.

The last criterion they proposed is commonality, which is a measure of the number of possible uses of a component. If an item can be used in many different ways, it might be meaningful to devote extra attention to that item.

After describing the criteria list, Flores and Whybark presented a multiple criteria approach to rank inventory items. However, they did not propose a specific methodology to integrate the utilization of several criteria, and only suggested a mechanical way to reduce the classification to ABC grouping.

Meanwhile, Saaty introduced the AHP in his book published in 1980 [16]. Saaty's AHP technique assumes that the criteria can be organized in a

hierarchy. The criteria are compared in a pairwise fashion to form a reciprocal square matrix. Then, according to the AHP technique, the eigenvector of the matrix gives the weights (priorities) of the criteria.

The AHP technique was developed in response to corporate and military contingency planning, decision making, the allocation of scarce resources, and to a need for political participation in negotiated agreements. Some applications of AHP include bank strategic planning [1], cost-benefit framework for highway projects[2], and allocation of livers for transplantation [4], and industrial bond rating [18].

Flores, Olson and Dorai [7], in 1992, extended the results of Flores and Whybark, and suggested the use of the Analytic Hierarchy Process (AHP) to integrate the use of several criteria and rank inventory items. They used the AHP technique to reduce these multiple criteria to a univariate and consistent measure to consider multiple inventory management objectives. The methodology used in this thesis is the one proposed by Flores, et. al. in 1992 [7].

1.2. Scope and Purpose of the Thesis

The purpose of the thesis is to determine the criteria that are applied for the classification of inventory items used in rock excavation jobs done by blasting, and apply the multicriteria ABC classification method proposed by Flores et. al in (Flores, 1992). The criteria used in the classification and their relative importance are the views of a civil engineer from the GÜRIŞ Construction and Engineering Co. Inc. These criteria will be put into a hierarchy first, and then using the AHP technique, these criteria hierarchy will be reduced into one level, and their relative priorities will be computed.

Using these criteria and their priorities, the inventory items used in rock excavation jobs involving explosives and blasting will be classified according to

the ABC classification. Since excavation is a part of most construction projects performed by GÜRIŞ, the company maintains an inventory of items used in such jobs. These items, equipment, explosives and consumables, are used continuously by GÜRIŞ. The reason behind choosing only this subset of all inventory is that this particular job requires a variety of items with different properties, which can help to illustrate the effect of multicriteria ABC inventory classification over the classical method.

In order to facilitate the AHP technique and the ABC classification, a computer program has been implemented. The program is called MCIC-AHP for MultiCriteria Inventory Classification using AHP technique. Using this program, it is possible to experiment with the effects of the differences in the relative importance of the criteria on the final classification of the inventory items.

1.3. Outline of the Thesis

The rest of the thesis consists of three main parts. The second chapter describes the classical ABC inventory classification technique. In that chapter some extensions of the ABC technique to handle multiple criteria will be discussed. The third chapter describes the Analytic Hierarchy Process and its application to the multiple criteria inventory classification. The fourth chapter contains an application and comparison of the methods described above to the inventory classification in rock excavation jobs involving explosives. The final chapter concludes with a summary and an overall evaluation of the methodology proposed by Flores et al. in [7].

Implementation details of the MCIC-AHP are given in the Appendix, and the way it is used is explained through examples.

2. ABC METHOD FOR INVENTORY CLASSIFICATION

The management of inventory of physical goods is accepted as a significant tool common to all enterprises. Inventories are idle goods or materials that are held for future use. Their importance to the firms stems from two reasons: financial and operational as explained below.

Inventory represents a major *financial* investment for any company and accounts for a large percentage of working capital. For many organizations, the expenses associated with financing and maintaining inventories are a substantial part of the cost of doing business. These costs can include transportation, warehousing, and capital costs.

On the other hand, from an *operational* perspective, inventories are essential to the successful operation of the organization. Although it is costly to carry inventories in terms of capital tied up, storage space used and insurance required, there are several reasons for keeping inventories. These include protection against variations in demand, maintaining smooth flow of production by decoupling function between stages of production, and lowering total material cost by taking advantage of quantity discounts. In addition, delayed

deliveries increase the risk of shortages. In order to prevent this risk, inventories are kept as “safety stocks.” Especially in countries where the inflation rate is high, inventories can be used as a protector from unexpected increases in material prices [3, 6].

Inventories which have a substantial effect on the cost and profit of a firm should be controlled and maintained very carefully. A company can realize substantial savings by employing a rational technique for inventory management.

One of the important issues in designing an inventory control system is the frequency of the assessment of stock on the hand. The other issues are the time and the size of the replenishment order.

Continuous monitoring of inventory levels is a costly process. Equal control effort for all items is not ordinarily justified, or even in some cases it would be incorrect. Therefore, the managers prefer grouping the inventory items in different classes, and applying different level monitoring to each group.

A universal technique for classifying and controlling inventories is the ABC analysis. It identifies and controls inventory items by classifying them into three categories. These categories are labeled A, B and C, respectively, leading to the term ABC analysis. Once the analysis is performed and the categories are determined, the argument is that attention should be concentrated on the “A category” items to maximize effectiveness. The “C category” items are given the least importance. According to Pareto’s famous 80-20 rule, about 20% of the inventory items comprise about 80% of the total annual dollar usage; these items are classified as Class A.

ABC classification provides a means of breaking down the tasks of dealing with suppliers and tracking the materials needed by the operation. Specific staff members can be assigned to deal with A items only, B items only,

or C items only; in this way purchasing department resources can be matched well to the importance of particular inventory items.

Different values of inventory items suggest that one should concentrate on higher valued items and be less concerned about lower valued items. On the other hand, even though an item may by itself be of low value, it is possible that the stock out cost could be substantial [3, 6].

2.1. Traditional ABC Classification

ABC inventory classification method is one universal inventory technique for any company and for any kind of manufacturing or service industry [9].

It has long been observed that in most companies a small fraction of the number of items in the inventory accounts for a disproportionately large fraction of purchasing expenditures. This small group of high-value items typically classified as A items. The A items usually account for 5 to 20 percent of the inventory.

In the traditional ABC inventory classification systems, two parameters for each item are used. The first parameter is the *average unit cost* and the other is the *annual demand*. For each item the parameter *annual dollar usage* is computed as the product of the average unit cost and the annual demand. The classification of inventory items is done on the basis of their annual dollar usage. In the ABC classification technique, the number of classes used in the classification is not important. For example, some companies may find it useful to use more than three classes. The important point is that control should provide a substantial value to the company.

In traditional ABC classification, the items are usually grouped as follows.

Class A: This group consists of a large annual dollar usage. The items with high stock out costs and those which comprise a large fraction of the total inventory fall into this class. The closest control might be reserved for raw materials that are used continuously in extremely high volume. For Class A items, periodic ordering, perhaps on a weakly basis, provides the necessary close surveillance over inventory levels. Plans and forecasts for Class A items should be evaluated frequently.

Class B: While this group consists of items of secondary importance with low demand and small unit value, they are still important and need intermediate control. The Class B items lay between the Class A and Class C items. These items should be monitored and controlled with periods larger than the periods of Class A items, e.g., biweekly or monthly. Stock out costs for Class B items should be moderate to low, and buffer stocks should provide adequate control for stock-outs.

Class C: This group consists of a small annual dollar usage but a relatively large percentage of total inventory items. Class C items account for the great bulk of inventory items, and can be controlled carefully but routinely. Attention should be much less frequent for these items, because the safety stock protection is deliberately set high.

Although the traditional ABC classification contains three categories, the number of classification categories can be increased depending upon the working conditions of a company.

2.2. Application of the ABC Analysis

Classical ABC analysis of an inventory is performed in the following steps:

- For each item, demand (typically one year) D , and the present value, v are determined.
- Total annual dollar usage (Dv) is calculated by multiplying D by v .
- All these items are ranked in descending orders in terms of annual dollar usage.
- The percentage of the annual dollar usage (Dv %) and the cumulative percentage of the annual dollar usage is calculated for every item [12].
- These items can be categorized as an A, B, C class with respect to cumulative percents according to the ratios shown in Table 2.1.

Table 2.1. Ratios of ABC.

Class	Percentage of items	Percentage of total annual usage
A	20	60
B	20	20
C	60	20

These percentages may change from one organization to another. The principle of separation is very important in management because it allows concentration of management efforts in the areas of highest payoff.

3. ANALYTIC HIERARCHY PROCESS

In this chapter, we describe the Analytic Hierarchy Process (AHP) which will be used in the application of multicriteria to the ABC classification.

AHP is a general theory of measurement. It deals with quantifiable or intangible criteria that have found rich applications in decision theory. The success of the theory is a consequence of its simplicity and robustness [19]. AHP is based on the principle that, to make decisions, experience and knowledge of people is at least as valuable as the data they use. This technique is proposed by Saaty, to provide a methodology for modeling unstructured problems in economic, social and management sciences.

There are four axioms of the theory.

Axiom 1: Reciprocal comparison. The intensity of the preferences of the decision maker must satisfy the reciprocal condition: If A is x times more preferred than B , then B is $1/x$ times more preferred than A .

Axiom 2: Homogeneity. The preference are represented by means of a bounded scale.

Axiom 3: Independence. In expressing preferences, criteria are assumed independent of the properties of the alternatives.

Axiom 4: Expectations. For the purpose of making a decision, the hierarchic structure is assumed to be complete.

It is important to satisfy these axioms in order to successfully apply the AHP technique to a decision making problem.

The next section explains the formation of a hierarchy and its evaluation. Section 3.2 describes how the weights (priorities) of elements are obtained [15]. Section 3.3 presents the methodology proposed by Flores et al. [7] for the application of AHP technique to the multicriteria ABC classification.

3.1. Hierarchic Design and its Evaluation

In making a decision, the most creative task is to choose the factors that are important for that decision. In the AHP technique, these factors are arranged in a hierarchical structure descending from an overall goal to the criteria, subcriteria and alternatives in successive levels [19].

Decision making applications of AHP are formed in two phases: hierarchic design and evaluation. The design of hierarchies requires the expertise and knowledge of the problem area. Two decision makers normally can construct two different hierarchies of the same problem; thus a hierarchy is not unique. On the other hand, even two people design the same hierarchy, their preferences can create different courses of action.

Criteria and subcriteria in a hierarchy serves two purposes. It provides an overall view of the complex relationships inherent in the situation and helps the decision maker assess whether the elements in each level are of the same order of magnitude [14].

Each element in a given level does not have to have subelements in the level below. In addition, a hierarchy should not be considered as a traditional decision tree. Each level may represent a different part of the problem. For example one level may represent social factors whereas another may represent economic factors.

The evaluation phase consists of pairwise comparisons. The elements in a level of the hierarchy are compared in relative terms. This comparison is made according to the importance or contribution of the criterion that is placed above the elements. This process of comparison yields a relative scale of measurement of the priorities or weights of the elements.

The comparisons are performed for the elements in a level with respect to all the elements in the level above. The weights of elements at the bottom level of the hierarchy are obtained by adding all the contributions of the elements in a level with respect to all the elements in the level above. This is known as the principle of hierarchic composition.

To make pairwise comparisons, a decision maker should answer these questions: when two criteria are compared in a given problem, it should be determined that which criteria should be preferred to the another. The results is a matrix of paired comparisons. This process is repeated for the all elements in the level above. This can be accomplished by using the principal right eigenvector of the matrix of paired comparisons [14, 19].

AHP is used to derive ratio scales from both discrete and continuous paired comparisons. These comparisons may be taken from actual measurements or from a priority scale. This scale reflects the relative strengths of preferences and feelings.

3.2. The Pairwise Comparison Matrix

The criteria of one level in a hierarchy are compared with each other one by one and the matrix is constructed.

Let us consider the criteria as $C_1, C_2, \dots, C_n$ of some level in a hierarchy. We wish to find their weights of influence, $w_1, w_2, \dots, w_n$ on some element in the next level.

The matrix represents our subjective judgments about the pairwise comparisons of criteria. The element a_{ij} indicates the relative strength of criterion C_i when compared to C_j . Therefore a_{ij} can be written as

$$a_{ij} = \frac{w_i}{w_j}, \text{ and } a_{ji} = \frac{1}{a_{ij}}, \quad i, j = 1, \dots, n.$$

$$A = \begin{matrix} & \begin{matrix} C_1 & C_j & C_n \end{matrix} \\ \begin{matrix} C_1 \\ C_i \\ C_n \end{matrix} & \begin{vmatrix} 1 & \frac{w_1}{w_j} & \frac{w_1}{w_n} \\ \frac{w_i}{w_1} & \frac{w_i}{w_j} & \frac{w_i}{w_n} \\ \frac{w_n}{w_1} & \frac{w_n}{w_j} & 1 \end{vmatrix} \end{matrix}$$

That is, the pairwise comparison matrix is a reciprocal square matrix whose diagonal is equal to 1. The size of the matrix is equal to the number of criteria.

3.2.1. Construction of the Matrix

Since the pairwise comparison is a subjective process, a priority scale is defined to compare two different criteria. This scale reflects the relative strengths of preferences and feelings. The preferences in a scale range from equally preferred to extremely preferred by giving numbers them from one through nine. The scales and their definitions are given in Table 3.1

Table 3.1. The fundamental scale.

Intensity of importance	Definition
1	Equal importance
3	Moderate importance of one over another
5	Essential or strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8 Intermediate values between the two adjacent judgments	

A pairwise comparison matrix has the feature that the main diagonal consists of 1's. In other words, each criterion is equally important when compared to itself. In addition, all other elements are reciprocal to the corresponding symmetric element. The rows represent the base factors. That is, if the row factor is less important than the column factor, then the value of the corresponding entry is the reciprocal of the scale given.

3.2.2. Obtaining the Weights (Priorities) of the Criteria

The next step consists of the computation of a vector of priorities from the constructed matrix. According to the AHP methodology, the eigenvector of the comparison matrix with the largest eigenvalue provides the priority ordering, and the eigenvalue is a measure of consistency of the judgment [16].

Computation of the exact Eigen vector of a matrix is complex and costly process. However, there are four approximate methods presented to compute the eigen vector of a matrix by Saaty [16].

- 1) Sum the elements in each row and normalize by dividing each sum by the total of all the sums, thus the results add up the unity.

- 2) Take the sum of the elements in each column and form the reciprocals of these sums, and divide each reciprocal by the sum of the reciprocals.
- 3) Divide the elements of each column by the sum of that column, and then add the elements in each resulting row, then divide this sum by the number of elements in the row.
- 4) Multiply the n elements in each row, and then take the n th root. Normalize the resulting numbers.

In our experiments we found that the 3rd method gives the best approximation, which is also observed by Saaty.

3.2.3. Consistency of the Matrix

If our judgment is perfect in all comparisons, then $a_{ik} = a_{ij} \cdot a_{jk}$ for all i, j, k and we call the matrix A *consistent*. Saaty has shown that if the diagonal of a matrix A consists of ones ($a_{ii} = 1$), and if A is consistent, then small variations of the a_{ij} keep the largest eigenvalue, $\lambda_{\max}$, close to n , and the remaining eigenvalues close to zero. Therefore, if A is the matrix of pairwise comparison values, in order to find the priority vector, the vector that satisfies $A\mathbf{w} = \lambda_{\max}\mathbf{w}$ must be found.

We multiply the matrix of comparisons on the right by the estimated eigenvector obtaining a new vector. If we divide the first component of this vector by the first component of the estimated eigenvector, the second component of the new vector by the second component of the estimated eigenvector, and so on, we obtain another vector. If we take the sum of the components of this vector, and divide by the number of components we get an approximation to the value of $\lambda_{\max}$ to use in estimating the consistency [16]. The

closer $\lambda_{\max}$ is to n (the number of criteria in the matrix) the more consistent is the result.

Deviations from consistency may be represented by

$$\frac{\lambda_{\max} - n}{n - 1}$$

which is called *Consistency Index* (C.I.). The consistency index of a randomly generated reciprocal matrix from 1 to 9, with reciprocals forced is called the *Random Index* (R.I.). At Oak Ridge National Laboratory, Dr. R. Uppuluri generated an average R.I. for matrices of order 1-15 using a sample size of 100. Average R.I. values for sizes 2 to 15 are given in Table 3.2. The R.I. values for sizes 1 and 2 are very close to 0.

Table 3.2. Average Ratio Index values.

n	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R.I	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

The ratio of C.I. to the average R.I. for the same size matrix is called the *consistency ratio* (C.R.). A consistency ratio of 0.10 or less is considered acceptable. Note that, since the average R.I. values are taken as zero for size less than three, the C.R. cannot be computed because of a division by zero error. If C.R. is more than 10% of R.I. then the matrix is considered as inconsistent. In this case, the matrix must be constructed again with different values.

These comparisons and computations establish the priorities of the criteria of one level of a hierarchy, with respect to one criterion of the one level above. If there are more than two levels, the various priority vectors can be combined into priority matrices, which yield one final priority vector for the bottom level. The element corresponding to a super-ordinate criterion (the one with subcriteria) in the eigenvector is replaced by the elements of the eigen

vector of the subordinate criterion. In this replacement, the elements of the eigenvector of the subordinate are multiplied by the element being replaced. Therefore, the sum of all elements of the eigenvector remains one.

For example, if the eigenvector at the highest level is $\langle w_1, w_2, \dots w_i, \dots w_k \rangle$ and the i th criterion has subcriteria $C_{i1}, C_{i2}, \dots C_{im}$, with eigenvector $\langle w_{i1}, w_{i2}, \dots w_{im} \rangle$, then the resulting eigenvector would be $\langle w_1, w_2, \dots, w_i * w_{i1}, w_i * w_{i2}, \dots w_i * w_{im}, \dots w_k \rangle$.

3.3. Application of AHP to Multicriteria ABC Classification

Flores et al. proposed a methodology to use AHP in multicriteria ABC inventory classification. It is based on classifying each item by a weighted value of the criteria utilized.

According to this methodology, the criteria hierarchy is constructed first. Then using the AHP technique the weights of each criterion is computed. The next step is to determine the values of each criterion for each inventory item. As the criteria have different units of measure, the measures have to be converted to a common 0-1 scale. Therefore, the values for each criteria are normalized using the following formula:

$$\frac{F_i - F_{\min}}{F_{\max} - F_{\min}},$$

where F_i is the i^{th} value of the criterion under transformation, $F_{\max}$ is the maximum value and $F_{\min}$ is the minimum value of the criterion under transformation.

Using these normalized criteria values for each item and the criteria weights, the weighted scores of the inventory items are computed. The items are sorted in descending order according to their weighted score values. The ABC classification can be then performed by the cumulative weighted scores. The

A-B and B-C cut-off points are given as percentages of the total weighted score. The items from the top of the list, whose cumulative weighted scores form the A-B percent of the total weighted score are classified as Class A. The items from the top of the list, whose cumulative weighted scores form the B-C percent of the total weighted score, but not classified as Class A are classified as Class B. The remaining items are classified as Class C. That is, the cumulative weighted scores of the Class A items form the A-B percent of the total weighted score. The cumulative weighted scores of the Class B items form the B-C - A-B percent of the total weighted score.

Another way of classification is to use the desired sizes of each group. In this case the percentages of items in each class is given by the user. If, for example, the user wants to have 20% of all inventory in class A, then first 20% of all items with highest weighted scores are assigned to Class A. Class B and C items are classified in the same way.

4. AN APPLICATION: INVENTORY CLASSIFICATION IN ROCK EXCAVATION BY BLASTING

This chapter describes the application of multicriteria ABC classification technique described in the previous chapters to the classification of the inventory items used in rock excavation jobs using the blasting technique. The data used here are obtained from the records of the GÜRIŞ Construction and Engineering Co. Inc.

Rock excavation is one of the common tasks performed as a part of a large construction project. Therefore GÜRIŞ maintains an inventory of items used for excavation jobs. In this study we have worked on a small but complete portion of the items in the inventory. The portion covered here contains only the items used in the rock excavation done by blasting. The inventory includes items such as equipment, explosives and consumables.

The next section describes the criteria used in the ABC classification. The hierarchy of the criteria and the criteria weights obtained by the application of the AHP technique are also given. Section 4.1 presents the inventory items and their resulting ABC classification. Finally, Section 4.2 applies the classical

ABC classification technique to the same inventory and compares the results with its multicriteria classification.

4.1. Criteria used in the Classification

Discussions with the engineers of the GÜRIŞ Construction and Engineering Co. Inc. revealed that the main criteria to be utilized in the classification of inventory items used in rock excavation by blasting jobs are *unit price, lead time, annual usage (demand), criticality, order size requirements, stockability and commonality*, in decreasing order of importance. The criticality criterion can be further divided into four subelements as *scarcity, durability, substitutability, and repairability*, in decreasing order of importance. The resulting criteria hierarchy is shown in Fig. 4.1.

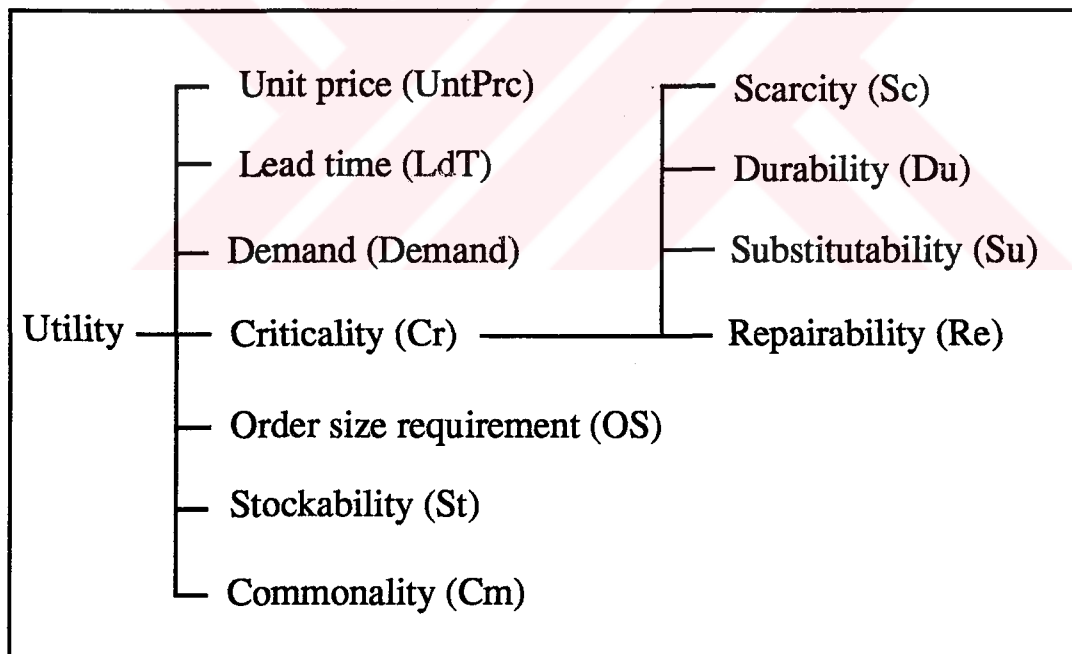


Fig. 4.1. Criteria hierarchy.

The most important criterion in the classification is the unit price of the items. Unit prices are going to be given in U.S. dollars.

The second most important criterion is the lead time. Since many of the items used in the rock excavation by blasting job are imported from abroad, the length of lead time and its variability become crucial factors. The lead time becomes even more important than the annual usage parameter used in the classical ABC analysis. The lead time is given in days.

Demand (annual usage) takes the third place in the classification. It represents the average consumption of a given item in a period of a year.

Criticality criterion is closely related to the idea of stock-out costs. The criticality of an item depends on four subcriteria: *scarcity*, *durability*, *substitutability* and *repairability*. These criteria will take on values in the range of 1 to 5.

Scarcity is a degree of the ease in acquiring an item. Scarcity can arise from the purchasing difficulties of an item. Scarcity is an important criterion for items that are imported from abroad. The value of 5 is assigned to scarce items.

Durability indicates the length of time that an item can stay in usable conditions. The existence of expired items in the stock can cause the stock-out costs for the company. For example, the explosive chemicals are not durable items. Usually, they have to be consumed in a few months of time. The durable items will take the value 1 while items with short usage periods will take the value 5.

Some items, when they are out of stock, can be substituted by similar items in the stock. In this case the stock-out cost of such items is less than the other items. The substitutability value of the items that have many substitutes in the inventory (e.g., brass handling rope) is 1, and for those that have no substitutes (e.g., seismic explosives) it is 5.

In case of stock-out, some items can be repaired and re-used. Closely related to the idea of substitutability for items is that of repairability. Repairable items provide a degree of flexibility similar to that of substitutability. The items that can be repaired quickly (e.g., containers and rock drills) will take the value 1 while the items that cannot be repaired (e.g., electrical and chemical items) will take the value 5.

There are three other criteria on the top level as described below. These criteria also take values in the range of 1 to 5.

For some items, a firm has to order a quantity which should not be less than a minimum order size required by suppliers or defined by a firm itself. This criterion can be called as order size requirement. The items with no order size requirements (e.g., anfo mixer and detonating box) takes the value 1, while items with large order size requirements (e.g., ignators and delay elements) take the value 5.

Stockability indicates the difficulty in storing an item. Items that require large space cause a difficulty in storage. The more importantly, for jobs involving blasting, the explosives cause dangers in storage areas. Therefore, explosives are given the value 5 for stockability criterion.

The final criterion used in this analysis is commonality. This is a measure of how many uses there are for an item. If an item is used in many different tasks, it might be important to devote extra attention to it. For example, trucks and screw drivers are items that are used commonly in many tasks; such items will take 5 as their commonality criterion.

4.2. Multicriteria ABC Classification

Multicriteria ABC classification of the inventory described in the previous sections of this chapter was done by the MCIC-AHP program

presented in the Appendix. Since the MCIC-AHP program requires that unit price and annual usage (demand) to be the first two criteria, we entered these two in the beginning.

According to the pairwise comparisons of the criteria made by an engineer of the company the following pairwise comparison matrix was constructed by the MCIC-AHP program.

	UntPrc	Demand	LdT	Cr	OS	St	Cm
UntPrc	1	3	2	4	6	7	9
Demand	1/3	1	1/2	1	4	2	7
LdT	1/2	2	1	3	5	6	8
Cr	1/4	1	1/3	1	3	5	6
OS	1/6	1/4	1/5	1/3	1	2	3
St	1/7	1/2	1/6	1/5	1/2	1	2
Cm	1/9	1/7	1/8	1/6	1/3	1/2	1

The Eigen vector of this matrix was computed as $\langle 0.358, 0.137, 0.246, 0.134, 0.057, 0.043, 0.025 \rangle$. Also, $\lambda_{\max} = 7.283$, $CI = 0.047$, and $CR = 0.036$. Relations between criteria are given by an engineer from the company as follows: $UntPrc \geq LdT \geq Demand \geq Cr \geq OS \geq St \geq Cm$ in order of importance to the company. After obtaining the weights of criteria, it is seen that the order of priorities are consistent with the above ranking. Since the matrix is consistent, we continued with the criticality criterion. The following matrix for the criticality was found in the similar way.

	Sc	Du	Su	Re
Sc	1	2	4	6
Du	1/2	1	3	5
Su	1/4	1/3	1	3
Re	1/6	1/5	1/3	1

The Eigen vector of this matrix was computed as <0.492, 0.309, 0.136, 0.063>. Also, $\lambda_{\max} = 4.080$, $CI = 0.027$, and $CR = 0.029$. This matrix is also consistent. The resulting weights (priority) of all criteria are as follows:

Index	Criterion	Priority
1	UntPrc	0.358
2	Demand	0.137
3	LdT	0.246
4	Sc	0.066
5	Du	0.041
6	Su	0.018
7	Re	0.008
8	OS	0.057
9	St	0.043
10	Cm	0.025

The scores of each item are computed as explained in the previous chapter. For example, the weighted score of the item “A.L.Truck” is calculated as

$$0.358 \frac{150000-1}{250000-1} + 0.137 \frac{2-1}{125000-1} + 0.246 \frac{90-1}{120-1} + 0.066 \frac{4-1}{5-1} + 0.041 \frac{2-1}{5-1} + 0.018 \frac{4-1}{5-1} + 0.008 \frac{2-1}{5-1} + 0.057 \frac{1-1}{5-1} + 0.043 \frac{4-1}{5-1} + 0.025 \frac{5-1}{5-1} = 0.5318$$

Two issues are important in determining the class boundaries. Firstly, we wanted to have the cut-off points in the large gaps between two consecutive scores, in order to have the items with similar score get the same class value. Secondly, we wanted to have about 20% of items in Class A, 20% in Class B and 60% in Class C. Considering these two issues we chose, as the class percentages, 20% for class A and 23.5% for class B. The resulting classification is given in Appendix I.

4.3. Classical ABC Classification and a Comparison

The same inventory is also classified using the classical ABC classification technique, and the results are compared. The items are sorted in decreasing order by their annual dollar usage. The multicriteria classification was resulted in twenty-three A class, twenty-seven B class, and sixty-five C class items. Therefore, in the classical ABC classification, we divided the inventory into the same size groups. The first twenty-three items were classified as Class A, the next twenty-seven items as Class B, and the remaining sixty-five items as Class C. The listing of the inventory classified by the multicriteria and classical ABC analysis is shown in Appendix I. By this comparison we wanted to see the number of items that are classified in different groups by the two method. The differences between the number of items in each class are shown in Table 4.1.

Fifteen items are classified as Class A, twelve items as Class B and fifty-six items as Class C by the both technique. The most important observation from this comparison is that the number of items that are classified as Class A by one method and Class C by the other method is very low, as expected. For example, there are no items that are classified as Class C by the multicriteria classification while Class A by the classical method. On the other hand, there exists only two items which are classified as Class A by the multicriteria classification and as Class C by the classical method. These two items, P.Grinder and Coupling, have relatively long lead times;100 and 90 days, respectively. Especially demand for P.Grinder is very low. Since the lead time criterion is more important than demand, this item was classified as Class A. In addition these two items are considered as critical in terms of scarcity and durability.

Eight items are classified as Class B by the multicriteria classification while as Class A by the classical ABC analysis. Five of these items have a

relatively short lead time. It should be remembered that the weight of lead time criterion constitute 26.4% of all criteria weights, and the second most important among all criteria. Two items, namely Jel.Dynam and Power.Dyn, have moderately high lead times. But their degree of scarcity is relatively low. This is due to the incorporation of the criticality criteria into the classification. One item, Anf.Mixer, has a relatively high lead time. However, this item does not have to be classified as class A since it is durable and not scarce.

On the other hand six items are classified as Class A by the multicriteria classification and as Class B by the classical method. These items have relatively long lead times and high scarcity values. These two criteria have high weights. Especially the item Nonel.Det is very critical in terms of all subelements of the criticality criterion. The items Bit, Rod and Shank have no substitutes and can not be repaired quickly.

Similarly, multicriteria technique classifies nine of the items as Class C, while the same items are classified as Class B by the classical analysis. Of the nine items that went down to Class C from B, two have moderately long lead times, but not high criticality ratings. Other seven items have short lead times and criticality values. Also note that Delay.El. item is classified as Class C while item C.Hose3/4 as class B; although both items have the same annual dollar usage value. Normally items with the same annual dollar usage value are classified as the same class. However, in this experiment we wanted to have the same number of items in class groups. If after the sorting operation these two items switched their position, they would have been classified in the same class by both techniques.

It is important to note that items Gr.Robot and C.Hose3/4 are classified in different classes while they are classified as Class B by the traditional technique with the same annual dollar usage value. The reason is that, the lead time of Gr. Robot is ten times more than the lead time of C.Hose 3/4. In

addition, for Gr. Robot item, the unit price is relatively more than demand while there is an opposite relation for C Hose 3/4 item. If it is considered that unit price criterion is more important than demand, it is reasonable to assign Gr. Robot to Class A, while C.Hose 3/4 to Class C.

Seven items are classified as Class B by the multicriteria classification and as Class C by the traditional method. For three of these items, lead time and scarcity criteria are effective to move to the higher class. Other four items have high criticality values. Therefore this discrepancy is superficial, and occurs due to the random ordering of items with the same value during the sorting process.

Table 4.1. Comparison of Multicriteria and Classic ABC classification.

		Multicriteria ABC Classification			
Classic ABC Classification		A	B	C	Total
	A	15	8	0	23
	B	6	12	9	27
	C	2	7	56	65
	Total	23	27	65	115
Percentage		20	23.5	56.5	100

As a summary, there are two main reasons for the discrepancies between two classification techniques. Firstly, a long with unit price and annual usage, many other criteria are considered in the multicriteria classification. Secondly, the lead time criterion, which is not considered in the classical analysis, is more effective than the annual usage criterion.

The curves presented in Fig. 4.2 show the distribution of items in each class by the two methods. In multicriteria classification items that fall in Class A (42% of all total weighted cumulative score) constitute the 20% of all items. However, in classical ABC analysis, the Class A items constitute 98.8%, and the

Class B items constitute about 1% of all cumulative annual dollar usage. The item with the highest annual dollar usage, \$20,000,000, is emulsion explosive. The total annual dollar usage of the inventory is \$83,662,034. That is only the first item constitute the 23.9% of the total inventory. This shows that the classical ABC method with 80-20 rule would fail to classify the items in a meaningful way in this particular inventory.

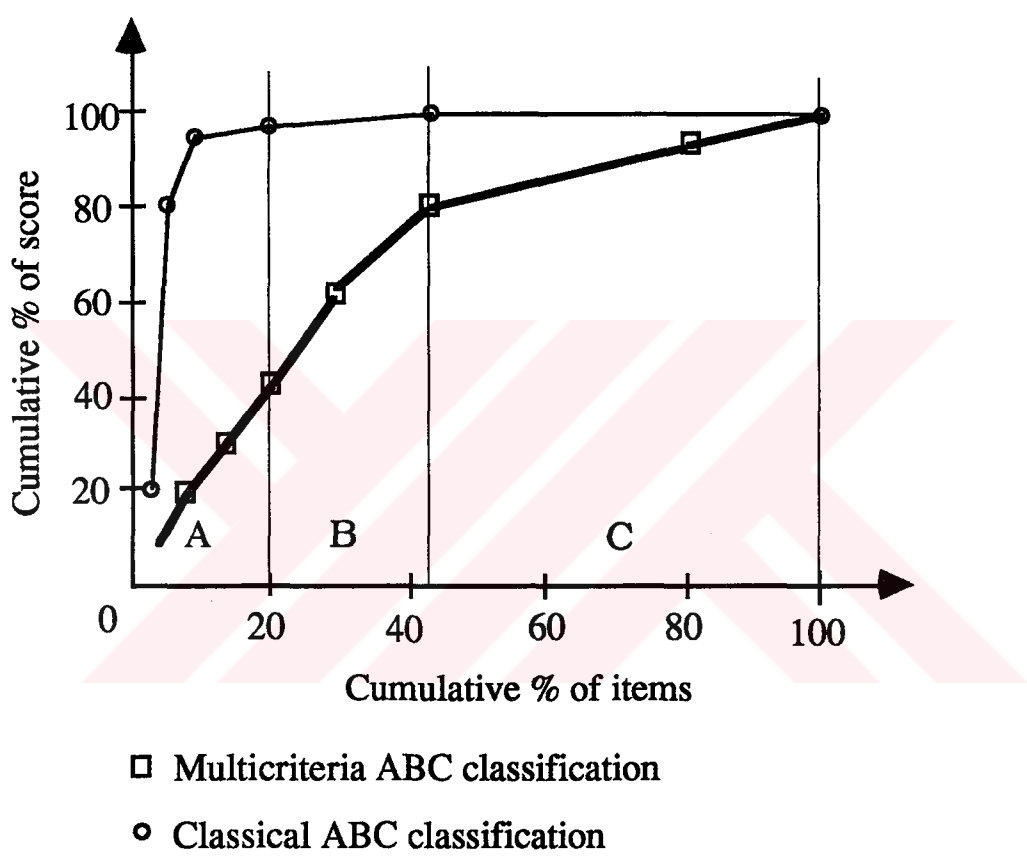


Fig. 4.2. Distribution of items in both methods.

The classifications obtained by the two methods were presented to a project manager from the GÜRIŞ Construction and Engineering Co. Inc., and he was asked to compare the results. He indicated that the classification done by the multicriteria ABC analysis was more accurate than the one obtained by the classical method.

4.4. Sensitivity Analysis

We altered some values in the original matrix given in section 4.2 to see whether the weights of all criteria are affected from this change. Firstly, one value which corresponds to the entry of unit price and criticality criterion is changed. In the original matrix, unit price’s relative importance to the criticality is between weakly and essentially more important, which is the value of 4. The value of 3 is assigned to this entry.

	UntPrc	Demand	LdT	Cr	OS	St	Cm
UntPrc	1	3	2	3	6	7	9
Demand	1/3	1	1/2	1	4	2	7
LdT	1/2	2	1	3	5	6	8
Cr	1/4	1	1/3	1	3	5	6
OS	1/6	1/4	1/5	1/3	1	2	3
St	1/7	1/2	1/6	1/5	1/2	1	2
Cm	1/9	1/7	1/8	1/6	1/3	1/2	1

The effect of this change onto the criteria weights are shown in the Weights1 column of the Table 4.2. The weights of the new matrix is close to the original one. However in the new case, the weight of criticality criterion is increased over the demand criterion, while these two criteria are given as equally important in the pairwise comparisons. The reason is that the relative importance of unit price with respect to criticality and demand criteria is the same. Therefore, the small changes in values can change the order of importance of criteria. This changes in the weights has resulted in the small changes in the weighted scores of the items, however, the classification of all items remained the same.

Secondly, the relative comparison values of one criterion with respect to other criteria are rearranged and the changes are given. For example the values

of demand, which is the third important criterion, are changed as 1/3, 2, 3, 3, 6 sequentially.

UntPrc	1	3	2	4	6	7	9
Demand	1/3	1	1/3	2	3	3	6
LdT	1/2	3	1	3	5	6	8
Cr	1/4	1/2	1/3	1	3	5	6
OS	1/6	1/3	1/5	1/3	1	2	3
St	1/7	1/3	1/6	1/5	1/2	1	2
Cm	1/9	1/6	1/8	1/6	1/3	1/2	1

According to this change, the weights of all criteria are shown in the weights2 column of Table 4.2. The striking observation in this example is that if the relative importance of demand with respect to any criteria is increased, the weights of criteria will decrease. For example, if the relative importance of demand with respect to stocability is inceased from 2 to 3, the weight of stocability decreases from 0.043 to 0.0040, while the weight of demand criterion increases from 0.137 to 0.139.

Table 4.2. Effects of changes in pairwise comparisons

Criterion	Original weight	Weights1	Weights2
UntPrc	0.358	0.346	0.354
Demand	0.137	0.137	0.139
LdT	0.246	0.250	0.260
Cr	0.134	0.140	0.124
OS	0.057	0.057	0.058
St	0.043	0.043	0.040
Cm	0.025	0.025	0.025

The changes in values did not change the order of importance of criteria. This changes in the weights has resulted in the small changes in the weighted scores of the items, however, the classification of all items remained the same, as in the previous case.

We have also experimented with the sensitivity of a change on a criteria value. The last item in the Class A group is the Coupling item. As the criterion to test the sensitivity, we chose the lead time criterion. We found out that in order to change the classification of Coupling item to Class B the lead time value must be dropped from 90 days to 84 days. In this experiment we kept the class percentages the same. For criteria with lower weights, the change in the value required to change the classification will be more.

In order to see the effect of changes in the low priority criteria, we have changed the scarcity value of the same item (Coupling) from 5 (maximum possible value) to 1 (minimum possible value). However this change in the scarcity value was not sufficient to move the Coupling item to Class B. As a conclusion, we can say that a small amount of change in the value of important criteria can change the classification of an item, while change in the value of low priority criteria do not have much effect in the classification.

5. SUMMARY AND CONCLUSION

In this thesis, a new method proposed by Flores et. al, which is based on the application of the Analytic Hierarchy Process (AHP) to ABC inventory classification, has been investigated. The traditional ABC classification method uses only the unit price and the annual usage of inventory items in ranking. However, in some cases, the classification done using only these two criteria turns out to be insufficient. On the other hand, the new method enables the integration of multicriteria into ABC classification. The requirement for the application of AHP is that the criteria must be organized in a hierarchical form.

The method first constructs a matrix by the pairwise comparison of criteria on the highest level. According to the AHP method, the elements of the eigen vector of this matrix represent the weights (priorities) of the criteria. If a criterion has subcriteria in the hierarchy, the weights computed in the similar manner for the subcriteria are multiplied by the weight of the criterion and inserted in its place. By repeating these steps for all levels of the hierarchy, the weight of all criteria are determined. Using the criteria weights determined by the AHP technique, the weighted score of each inventory item is computed. Then, the items sorted by that weighted score are grouped in three classes, A, B, and C, as in the classical ABC classification.

This new method is applied to the classification of inventory items used in rock excavation jobs done using blasting by GÜRIŞ, a construction company. The classification was done by a computer program called MCIC-AHP, which implements the this new method. The inventory contains 115 items including equipment, explosives and consumables. In the classification 10 criteria were used. According to an engineer from the company, unit price was the most important criteria, and lead time was the second most important criteria. Average annual usage (demand) came after lead time. That is lead time, which is not considered by the classical ABC method, was more important than annual usage. In general, we observed that the engineer rated the criteria which is not under control of the company higher than the others.

The same inventory is also classified according to the classical ABC technique, and the results are compared. The most striking observation was that if the classical ABC analysis were employed with 80-20 rule, only one item would be classified as Class A. The next item which has very similar properties had to be classified as Class B. This was due to the fact that explosives have very high price and very high annual usage, and constitute a very large portion of the total annual dollar usage. When the classical ABC analysis was performed using the number of items in each class as that of the multicriteria classification, the number of that changed class was 31 (27%). The resulting classifications were presented to the same engineer, he claimed that the multicriteria classification reflects more closely the way the items should be classified.

An analysis of the sensitivity of the multicriteria classification technique is conducted. In this analysis, we first changes the pairwise comparison value of two criteria by one point. The resulting values of the criteria weights are observed to remain the same. However, when we changed the importance of one criterion with respect to all other criteria in the pairwise comparisons, the order

of the importance has changed slightly. We have also experimented with the effect of change on a criteria value on the classification. About 4% change in the value of an important criterion (lead time) for the bottom most item in the Class A was sufficient to move it from class A to class B. Therefore, if the company can find another means of acquire that item, than it can be considered as Class B, instead of A. On the other hand, 80% change in the value of a low priority criterion (scarcity) for the same item was not sufficient to move to Class B. As a conclusion, a small amount of change in the value of important criteria can change the classification of an item, while change in the value of low priority criteria do not have much effect in the classification.



APPENDIX I

This appendix presents the inventory used in rock excavation jobs done by blasting. The inventory was classified first by the multicriteria ABC classification technique. The same inventory was then classified using the traditional ABC classification technique. Both classifications have been accomplished using the MCIC-AHP program. The output of the program displays the classification of the items, assigned by both techniques.

Here the symbols and their meanings are as follows:

UntPrc:	Unit Price
Demand:	Annual usage
LdT:	Lead time
Sc:	Scarcity
Du:	Durability
Su:	Substitutability
Re:	Reparability
OS:	Order Size requirement
St:	Stockability
Cm:	Commonality
WtScr:	Weighted Score for multicriteria classification
M_C:	Class value assigned by Multicriteria classification
Ann\$usg:	Annual Dollar Usage for traditional classification
T_C:	Class value assigned by Traditional classification

Item Name	UntPrc	Demand	LdT	Sc	Du	Su	Re	OS	St	Cm	WtScr	M_C	Ann\$usg	T_C
Eml.Explo	2000	10000	60	4	5	5	5	4	5	1	0.338	A	20000000	A
Cnt.Explo	4500	3000	90	4	5	5	5	4	5	1	0.396	A	13500000	A
Jel.Dynam	3500	3000	60	2	5	5	5	4	5	1	0.300	B	10500000	A
Power.Dyn	1500	7000	60	2	5	3	5	4	5	1	0.292	B	10500000	A
U.W.Explo	6000	1000	90	5	5	5	5	4	5	1	0.412	A	6000000	A
Liq.Explo	2000	3000	15	4	5	5	5	4	5	1	0.237	B	6000000	A
D.Agents	25000	200	120	5	5	5	5	4	5	1	0.501	A	5000000	A
Ignators	10000	500	100	5	5	5	5	5	4	1	0.442	A	5000000	A
Pls.Explo	15000	100	95	5	5	5	5	3	4	1	0.410	A	1500000	A
Seismic.E	5000	250	90	3	5	5	5	4	5	1	0.377	A	1250000	A
Rck.drill	250000	4	20	4	2	3	1	3	1	3	0.507	A	1000000	A
Det.Cert	4	100000	90	4	4	5	5	4	2	1	0.453	A	400000	A
A.L.truck	150000	2	90	4	2	4	2	1	4	5	0.531	A	300000	A
Wrk.boots	100	3000	30	2	3	4	4	3	2	2	0.165	B	300000	A
P.Compres	45000	6	95	4	2	5	2	1	1	2	0.345	A	270000	A
P.R.Drill	125000	2	90	2	1	5	2	1	4	3	0.444	A	250000	A
Elec.Det.	2	125000	10	4	5	5	5	4	3	1	0.336	A	250000	A
Delay.Det	3	75000	10	4	5	5	5	4	3	1	0.282	B	225000	A
Dest.Dack	2500	50	30	5	5	5	5	4	5	1	0.282	B	125000	A
R.Splitt	25000	4	90	5	2	3	3	2	4	2	0.362	A	100000	A
Prilled	300	300	90	4	5	2	5	4	4	2	0.369	A	90000	A
Anf.Mixer	30000	3	90	2	1	3	2	1	4	1	0.287	B	90000	A
Palm.Det.	2	50000	10	4	5	5	5	4	3	1	0.254	B	75000	A
Pwr.Pack	15000	4	95	5	2	3	3	2	3	2	0.347	A	60000	B
Fertilizr	100	600	10	2	4	2	5	3	4	2	0.146	C	60000	B
B.D.pumps	5000	10	90	4	2	5	2	2	2	2	0.302	B	50000	B
Anf.Load.	25000	2	90	2	1	3	2	2	5	1	0.305	B	50000	B
Bit	150	300	90	5	3	4	4	3	2	2	0.336	A	45000	B

R.Hammers	15000	3	30	3	3	4	2	2	1	4	0.183	B	45000	B
As.cutter	20000	2	15	4	2	5	2	2	2	2	0.169	B	40000	B
Nonel.Det	4	10000	90	5	5	5	5	4	2	1	0.381	A	40000	B
D.J.Hamm.	8000	5	90	4	3	4	2	2	2	4	0.325	B	40000	B
Shank	250	150	90	5	3	4	4	3	2	2	0.336	A	37500	B
PVC.Pipe	12	3000	7	5	3	3	1	2	1	3	0.138	C	36000	B
Rod	300	100	90	5	3	4	4	3	2	2	0.336	A	30000	B
LLwire25	3	10000	4	2	4	2	2	5	2	4	0.157	B	30000	B
Wrk.cloth	10	3000	35	2	2	4	4	4	1	2	0.169	B	30000	B
Deton.box	5000	6	60	4	1	5	4	1	2	1	0.213	B	30000	B
Alumin.Pw	4	5000	30	5	5	4	5	4	2	1	0.247	B	20000	B
D.H.Plug	5	4000	95	2	4	2	4	4	2	2	0.316	B	20000	B
C.Hammers	10000	2	30	2	2	4	2	1	1	4	0.135	C	20000	B
R.Breaker	2500	8	30	2	1	4	2	2	2	2	0.127	C	20000	B
S.Helmet	20	1000	7	2	3	4	4	3	2	3	0.122	C	20000	B
Jc.Hammer	3500	5	15	4	2	5	2	3	2	2	0.159	B	17500	B
S.R.drill	5000	3	30	4	1	4	3	4	2	2	0.194	B	15000	B
T.P.Cover	28	500	9	2	1	2	1	2	1	4	0.071	C	14000	B
B.H.rope	24	500	3	2	1	1	4	3	2	4	0.085	C	12000	B
L.P.G.Con	120	100	7	2	2	4	3	1	2	1	0.068	C	12000	B
Gr.Robot	10000	1	100	5	2	4	2	2	3	3	0.359	A	10000	B
C.Hose3/4	10	1000	10	2	2	5	4	4	2	4	0.143	C	10000	B
Delay.El.	2	5000	10	3	5	5	5	5	2	1	0.192	B	10000	C
Light.F.S	4500	2	30	4	2	5	4	1	1	2	0.156	B	9000	C
Int.Steel	150	60	7	2	1	4	4	3	1	1	0.077	C	9000	C
C.Hose.1	15	600	10	2	2	5	4	4	2	4	0.142	C	9000	C
Coupling	75	120	90	5	3	4	4	3	2	2	0.336	A	9000	C
Tape	2	4000	1	1	4	3	4	3	2	5	0.114	C	8000	C
U.W.Carr.	250	30	8	2	2	3	2	3	2	4	0.111	C	7500	C

F.T.I.St1	150	42	10	2	5	4	4	3	2	2	0.141	C	6300	C
N.P.I.St1	130	45	10	2	4	4	4	3	2	2	0.131	C	5850	C
Ohm.Meter	482	12	35	2	1	5	4	2	1	2	0.132	C	5784	C
LLwire	5	1000	4	2	4	2	2	4	2	4	0.133	C	5000	C
Siren	825	6	7	1	2	1	2	1	1	4	0.045	C	4950	C
L.F.I.St1	140	25	10	2	4	4	4	3	2	2	0.131	C	3500	C
Ear.Plug	10	300	30	1	4	2	3	3	1	2	0.134	C	3000	C
LLwire75	4	750	4	2	4	2	2	4	2	4	0.133	C	3000	C
Vib.Mon.S	700	3	60	4	1	5	3	1	2	1	0.205	B	2100	C
LLwire50	4	600	4	2	4	2	2	4	2	4	0.133	C	2100	C
G.compass	600	3	35	2	1	5	3	2	2	3	0.147	C	1800	C
Megaphone	230	5	7	1	3	1	1	1	1	5	0.058	C	1150	C
B.matress	250	4	8	2	1	4	1	1	1	2	0.051	C	1000	C
P.gloves	2	500	2	1	3	4	3	3	1	5	0.094	C	1000	C
FuelOil2	0	2000	3	2	4	3	5	5	4	4	0.179	B	900	C
Clw.Coup1	30	30	10	1	3	5	4	3	2	3	0.115	C	900	C
H.Nipple1	15	50	10	1	3	5	4	3	2	3	0.115	C	750	C
H.bucket	13	50	5	1	1	3	4	3	1	4	0.071	C	650	C
AirOil	2	300	3	2	4	2	5	4	4	1	0.139	C	600	C
P.Grinder	300	2	100	5	2	4	2	2	3	3	0.345	A	600	C
M.showel	20	30	2	2	2	3	3	3	2	4	0.100	C	600	C
C.drum	70	8	7	1	1	2	2	2	1	2	0.040	C	560	C
H.Calcul.	50	10	2	2	1	4	3	2	2	5	0.086	C	500	C
H.gasket	1	500	10	2	4	4	3	3	1	1	0.112	C	500	C
P.Lubric	40	12	90	4	2	4	4	2	2	4	0.307	B	480	C
Funnel	120	4	2	1	1	4	4	2	2	3	0.059	C	480	C
Draft.Set	150	3	7	2	1	3	3	2	2	3	0.080	C	450	C
Chisels	30	14	1	2	1	2	3	3	2	3	0.077	C	420	C
R.S.flags	4	100	3	2	3	5	3	3	2	2	0.109	C	400	C

H.Thread	10	40	10	1	3	5	4	3	2	5	0.127	C	400	C
C.Coupling	13	30	10	1	3	5	4	3	2	2	0.109	C	390	C
B.P.knife	18	20	2	1	3	2	3	3	1	5	0.085	C	360	C
W.Separat	50	7	90	4	2	4	4	2	2	2	0.295	B	350	C
S.whistle	7	50	2	1	1	3	3	3	2	5	0.079	C	350	C
Anfo.Sack	1	300	1	1	4	2	2	4	2	2	0.097	C	300	C
H.Nipple	10	30	8	2	2	4	3	5	1	2	0.122	C	300	C
W.Det.box	30	10	1	2	2	4	2	1	1	3	0.055	C	300	C
Timber.Pw	1	300	1	1	1	2	4	3	1	1	0.039	C	300	C
Explo.box	28	10	1	2	2	2	2	1	1	3	0.046	C	280	C
Tapp.rod	5	50	1	1	1	2	2	3	2	2	0.052	C	250	C
Cristilzd	1	500	7	4	5	2	5	4	2	2	0.176	B	250	C
G.hammer	50	5	30	2	1	5	3	2	2	4	0.142	C	250	C
Measure.T	15	14	2	2	2	4	2	1	2	4	0.074	C	210	C
Stemm.Mat	1	200	1	1	4	2	4	2	2	4	0.085	C	200	C
Ex.testers	100	2	30	3	1	4	4	1	2	1	0.123	C	200	C
Screw.Set	35	5	2	2	2	4	2	1	2	5	0.080	C	175	C
Stripper	15	10	30	2	2	2	4	3	1	3	0.138	C	150	C
Sc.driver	30	5	7	2	1	5	4	3	2	5	0.117	C	150	C
Nyl.cover	50	3	3	2	3	4	2	3	1	3	0.098	C	150	C
Container	3	50	1	1	4	2	1	1	1	4	0.054	C	150	C
Pad.Lock	5	30	3	1	2	3	3	3	2	4	0.085	C	150	C
Scissors	7	20	2	1	2	3	3	2	2	5	0.075	C	140	C
B.P.bobs	6	20	3	2	1	1	4	3	2	3	0.078	C	120	C
B.F.knife	22	5	4	1	2	2	3	3	2	4	0.083	C	110	C
P.crimper	5	15	2	2	3	4	3	3	1	4	0.104	C	75	C
W.stands	5	12	3	1	3	2	3	1	2	2	0.050	C	60	C
Notebook	10	5	3	1	2	1	4	3	2	4	0.078	C	50	C
P.lens.P.	10	4	1	2	3	4	3	3	2	3	0.106	C	40	C

APPENDIX II

The MCIC-AHP Program

MCIC-AHP (MultiCriteria Inventory Classifier using Analytic Hierarchy Process) is a computer program which can be used for ABC inventory classification. It is designed to handle multiple criteria used in the classification. The assumption is that the criteria used can be organized in a hierarchy. The program reduces this hierarchy into a single level and computes the priority weights for each criterion using the AHP technique as explained in Chapter 4. It can also be used to do traditional ABC classification. The program allows the comparison of both classification techniques.

The MCIC-AHP program is written in the Pascal (Turbo Pascal version 6.0) programming language, and runs on IBM PC compatible computers. In order to run on a wide range of computers, the program runs in the text mode, and, therefore, does not require any specific graphics hardware.

The flow-chart of the MCIC-AHP program is given in Figure A.1. The program is composed of two main subprograms. The first subprogram, called AHP, enables the user to enter a hierarchy of criteria along with their pairwise comparisons, and obtain a vector of criteria priorities (weights). The resulting priority vector can be saved into a text file for further processing; e.g., for ABC classification, printing, editing.

The second subprogram, called ABC, takes, as its input, a text file containing the inventory to be classified. The inventory file must be a text file, each line corresponding to an item. Two options are available: multicriteria classification, or traditional classification. One classification can be performed after the other, enabling the comparison of the effects of both techniques.

If multicriteria ABC classification option is selected, the program requires, as its input, a text file containing the criteria and their weights. This file is called criteria list file, and can be produced by the AHP subprogram. Using these criteria priorities the ABC subprogram classifies each item in the inventory file as A, B or C, and the resulting classified inventory can be saved in another text file for further processing.

If traditional ABC classification option is selected, the program assumes that the first two columns, following the item name column, of the inventory file contain the *unit price* and *annual usage*. The order of these two columns do not matter. The other criteria values of items are not considered.

In order to facilitate the creation and modification of inventory files or criteria list files, a multi purpose text editor, called EDIT, is provided. When the *editor* option is selected, the execution of the MCIC-AHP program is suspended, and the editor is invoked. The editor is menu driven, and can be used with a mouse, as well. When exited from the editor, the control returns back to the MCIC-AHP program.

The program can be executed by typing MCIC-AHP at the MSDOS prompt as

```
C : \>MCIC-AHP
```

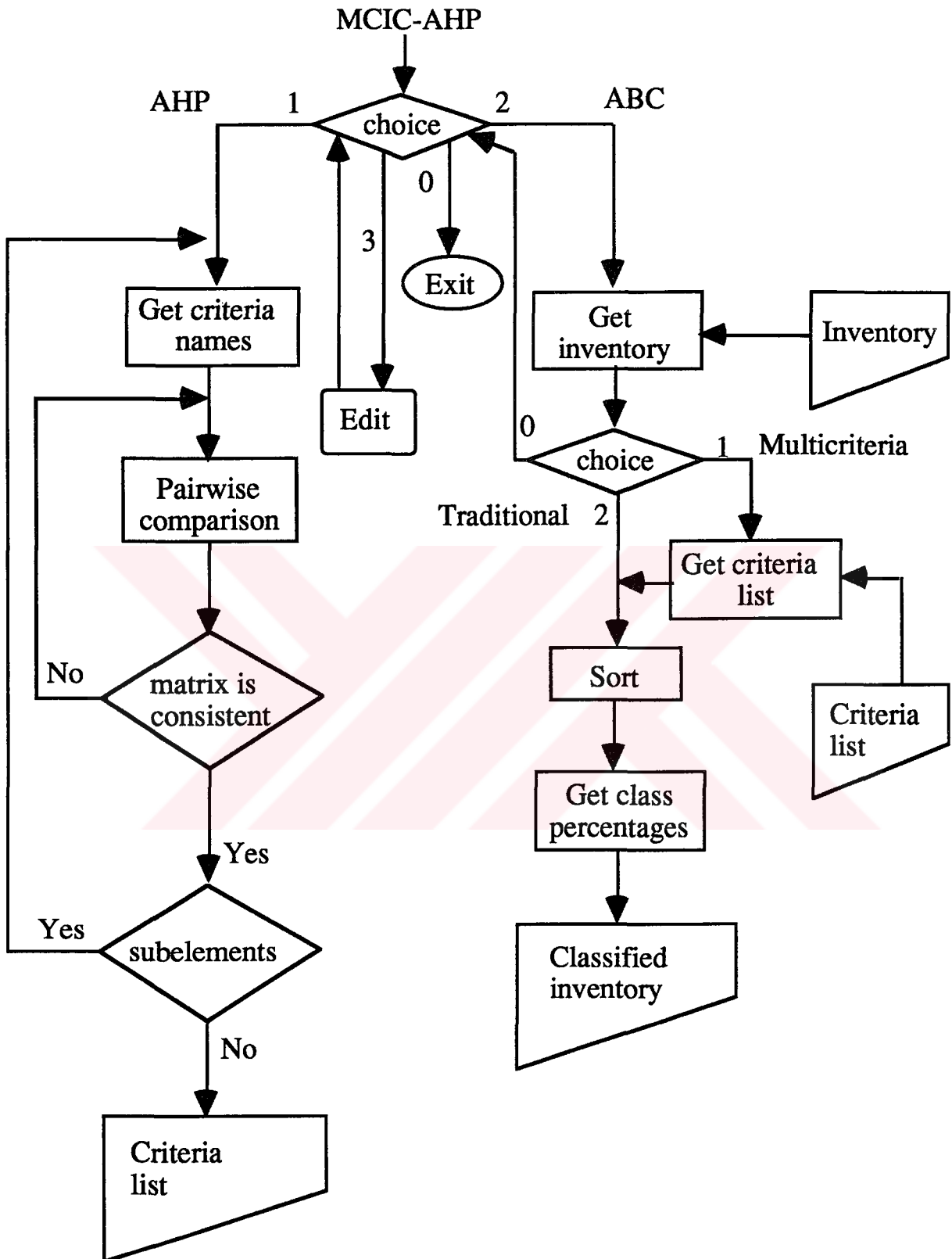


Fig. A.1. Flow-chart of MCIC-AHP.

The program starts with the screen presenting the main options as shown in Fig. A.2. Responding with 1 activates the AHP subprogram, while 2 activates the ABC classification subprogram. The editor can be invoked by the choice of 3. In order to exit the program, the user should type 0.

```
M C I C - A H P
MultiCriteria Inventory Classification using
Analytic Hierarchy Process
By H. Altay Güvenir & Nuray Güvenir

1: AHP
  Using this option you can create a file containing
  a list of criteria and their weights.
2: ABC Classification
  Using this option you can classify an inventory by
  multicriteria ABC classification,
  traditional ABC classification, or
  both.
3: Editor
0: Exit

What is your choice? (1/2/3/0):_
```

Fig. A.2. The main menu of the MCIC-AHP program.

The AHP subprogram first asks for the criteria names. It reads one name at a time, and starting from 1 indexes them sequentially, as shown in Fig. A.3. At the first time, only the criteria at the highest level are entered. The AHP subprogram allows the integration of subelements of a criterion after processing the highest level. Criteria names must not contain space character. When all the names are entered, the user simply presses the ENTER key at the next index number.


```
Index: Criterion name
1: Av-u-cst
2: An-$-usg
3: Criticality
4: LeadTime
5: -
```

Fig. A.3. AHP first reads the criteria names.

Having read all the criteria names, the AHP subprogram forms a square matrix whose each row and each column corresponds one criterion in the given order. The diagonal of the matrix is set to 1. In order to complete the matrix, the user is asked to compare each pair of criterion as shown in Fig. A.4. The pairwise comparison is done on the scale of 1 through 9. The meaning of the values on that scale are shown in the screen to guide the user. The default value is set to 1, meaning that the two criteria are equivalently important. In order to change this value, the user can use the up and down arrow keys. Pressing the up arrow key increases the importance of the first criterion by one, while pressing the down arrow key increases the importance of the second criterion. The changes in the values are reflected under the scale column, where the values are reciprocal of each other. If, for example, the Average Unit Cost is four times more important than the Annual Dollar Usage, then the user should press the up arrow key four times, as in Fig. A.4. Since the Average Unit Cost criterion has index 1, and the Annual Dollar Usage criterion has the index 2, the entry (1,2) of the matrix gets the value 4, while (2,1) gets the value $1/4 = 0.25$.

Criteria	Scale
=====	=====
Av-u-cst	4.000 Δ
An-\$-usg	0.250 V

Press up arrow Δ or dowd arrow V

Equal importance:1
Moderate importance:3
Essential or strong importance:5
Very strong importance:7
Extreme importance:9
Intermediate values between adjacent judgements: 2, 4, 6, 8

Fig. A.4. Pairwise comparison.

After all the pairwise comparisons are entered into the matrix, the AHP subprogram computes its eigen vector using the third approximate method given in page 19 of [16]. The approximate method is as follows.

Divide the elements of each column by the sum of that column (i.e., normalize the column) and then add the elements in each resulting row and divide this sum by the number of elements in the row.

A long with the matrix just entered, its eigen vector, L_{max} (λ_{max}), CI (Consistency Index) and CR (Consistency Ratio) are displayed on the screen (see Fig A.5). If the CR value is less then or equal to 0.1 then the matrix is consistent, otherwise it is inconsistent. If the matrix is found to be inconsistent, the pairwise comparisons are asked from the user again, and the process is repeated.

```

The matrix is:
  1.000   4.000   3.000   3.000
  0.250   1.000   2.000   2.000
  0.333   0.500   1.000   2.000
  0.333   0.500   0.500   1.000

Eigen vector is:
  0.506   0.214   0.165   0.115

Lmax: 4.185  CI: 0.062  CR: 0.068

The matrix is consistent.
Press ENTER to continue !_

```

Fig. A.5. A consistent matrix, and its relevant values.

After obtaining a consistent matrix, the AHP program list the criteria and their priorities (corresponding eigen vector entries) on the screen, and ask if any of these criteria has further subelements, as shown in Fig. A.6.

```

      Index: Criterion name      Priority
      1: Av-u-cst              0.506
      2: An-$-usg              0.214
      3: Criticality            0.165
      4: LeadTime               0.115

Does any of these criteria have subelements? (y/n): Y
What is the index of the criterion: 3

```

Fig. A.6. The criteria list.

Let us assume that the Criticality criterion is composed of three subelements: Impact, Scarcity, and Substitutes. In that case the user should type “y” to the first question in Fig. A.6, and “3” to the second question since the Criticality criterion has the index 3. The AHP subprogram will continue in a similar manner as shown in Fig. A.3, and read the names of these criterion. It will ask for pairwise comparison of the new criteria and form a new matrix and compute its eigen vector a long with other relevant parameters in the same way as before. If the new matrix is consistent it will replace the Criticality criterion in the original criteria list. The priority weights of the new criteria will be multiplied by the weight of the Criticality criterion.

If no other criteria have further substitutes the resulting criteria list can be saved into a text file as shown in Fig. A.7. This file can be printed on any printer and edited by any text editor. By pressing the ENTER key the AHP subprogram completes, and the main menu reappears on the screen in Fig. A.2.

```
Index: Criterion name      Priority
    1: Av-u-cst           0.506
    2: An-$-usg           0.214
    3: Impact             0.015
    4: Scarcity            0.040
    5: Subst              0.111
    6: LeadTime           0.115

Does any of these criteria have subelements? (y/n): N

Name of the file to save the criteria list: TEST1.CRL
Criteria list is saved into TEST1.CRL
Press ENTER to continue !_

```

Fig. A.7. The criteria list is saved into file TEST1.CRL

The next step is usually the classification of an inventory. The inventory file should be created (or organized) in a way that the values are in the order given in the criteria list file. The creation or modification of the inventory file can be done by the editor provided.

The second subprogram of the MCIC-AHP, called ABC, can be used to classify the inventory items which are stored in a text file. There are two input files for the ABC subprogram. They are criteria list file and inventory file; both of them are to be in the text format. The inventory file should be organized as follows. The file should contain one line for each inventory item. Each line should start with the name of the item followed by the numerical values of the criteria for that item. The item name and the values should be separated by one or more spaces. Most importantly the number of values must match the number of criteria that exist in the criteria file. The inventory file can be created by any text editor, or more conveniently, by printing to a file from any spread sheet program (e.g., LOTUS). Contents of example criteria list file and inventory file of five items are shown in Fig. A.8 and Fig. A.9, respectively.

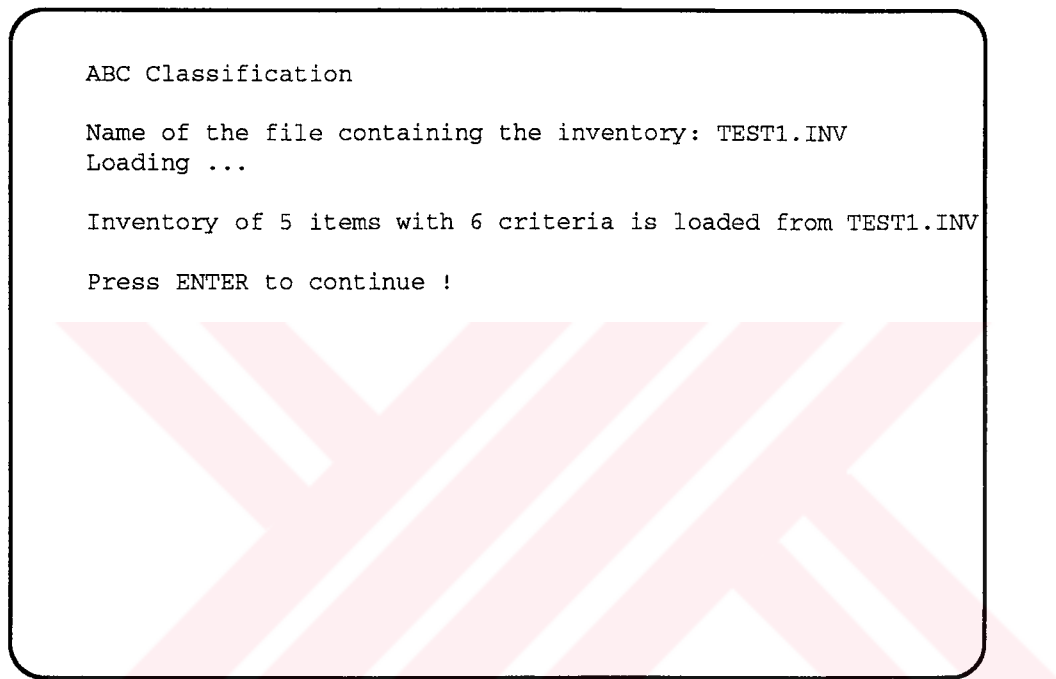
Index	Criterion name	Priority
1	Av-u-cst	0.506
2	An-\$-usg	0.214
3	Impact	0.015
4	Scarcity	0.040
5	Subst	0.111
6	LeadTime	0.115

Fig. A.8. The contents of a criteria file, TEST1.CRL

item1	550	2	1	2	1	1
item2	375	10	9	9	3	7
item3	560	55	4	10	2	15
item4	680	6	7	7	4	30
item5	700	2	3	4	5	60

Fig. A.9. The contents of an inventory file, TEST1.INV

The ABC subprogram starts by requesting from the user the name of the file containing the inventory. The inventory file is loaded, and the number of items in the file and the number of values for each item are displayed on the screen as in Fig. A.10. Then two options are available to the user (Fig. A.11). The first option is for multicriteria ABC classification, while the second is for the traditional one.



```
ABC Classification

Name of the file containing the inventory: TEST1.INV
Loading ...

Inventory of 5 items with 6 criteria is loaded from TEST1.INV

Press ENTER to continue !
```

Fig. A.10. ABC Classification subprogram.

If the multicriteria classification is selected, the name of the file containing the criteria list is asked. After the criteria list is loaded from the file, the program computes the weighted score for each item as explained in Chapter 3. Then the items are sorted in the descending order by their weighted score.

The items are classified according to the desired proportions of each class. The user is asked to give the percentages of the items in each class as shown in Fig. A.12. The program computes the number of items in each class according to the percentages given. The program then display the results on the screen asks for confirmation to go ahead with the classification.

```

What type of classification do you want to perform on TEST1.INV
  1: multicriteria ABC classification
  2: traditional ABC classification
  0: exit
Your choice [1/2/0]: 1

Name of the file containing the criteria list: TEST1.CRL
Criteria list is load from TEST1.CRL

Press ENTER to continue !

```

Fig. A.11. Two types of classification. ara possible.

```

Now we need the desired distribution of items in each class.

What is the desired percentage of items in class A? : 20
What is the desired percentage of items in class B? : 25

Distribution of items in each class will be as follows:
  1 items in class A (20.0%)
  1 items in class B (25.0%)
  3 items in class C (55.0%)
-----
  5 items in Total

Do you want to proceed with these percentages? [y/n]: _

```

Fig. A.12. Distribution of items in each class.

The list of the classified inventory items are displayed on the screen in the sorted order, the items of class A being on top as shown in Fig. A.13. If the

number of items is more than 20 the program display only 20 items at a time, allowing the user check the classification of each item.

The ABC Classification program then enables the user to save the classified inventory in another text file for further processing, such as reclassification, printing, editing, or reading into a spread sheet program.

Item Name	Av-u-cst	An-\$-usg	Impact	Scarcity	Subst	LeadTime	WtScr	M_C	Ann\$usg	T_C
item5	700	2	3	4	5	60	0.746	A	0	
item4	680	6	7	7	4	30	0.667	B	0	
item3	560	55	4	10	2	15	0.603	C	0	
item1	550	2	1	2	1	1	0.272	C	0	
item2	375	10	9	9	3	7	0.149	C	0	

Press ENTER to continue!

Name of the file to save the classified inventory: TEST1.CIN
Classified inventory is saved in file TEST1.CIN
Press ENTER to continue !_

Fig. A.13. The classified inventory using multicriteria classification.

When returned from the multicriteria ABC classification option, the ABC subprogram enables the user to apply the traditional classification, as well. The program allocates four extra fields for each inventory item. They are weighted score (WtScr), class for multicriteria classification (M_C), annual dollar usage (Ann\$usg), and class for traditional classification (T_C).

If the user then select the 2nd option, the program computes the annual dollar usage of each item using the first two values, following the inventory name in the inventory file. The rest of the classification is performed in the same way as the multicriteria classification.

Item Name	Av-u-cst	An-\$-usg	Impact	Scarcity	Subst	LeadTime	WtScr	M_C	Ann\$usg	T_C
item3	560	55	4	10	2	15	0.603	C	30800	A
item4	680	6	7	7	4	30	0.667	B	4080	B
item2	375	10	9	9	3	7	0.149	C	3750	C
item5	700	2	3	4	5	60	0.746	A	1400	C
item1	550	2	1	2	1	1	0.272	C	1100	C

Press ENTER to continue!

Name of the file to save the classified inventory: TEST1.CIN

Classified inventory is saved in file TEST1.CIN

Press ENTER to continue !_

Fig. A.14. The classified inventory using traditional classification.

The traditional ABC classification of the example TEST1.INV file is shown in Fig. A.14. In this classification the same percentage values have been used as in the multicriteria classification. However, the user can chose a different set of percentages. Both of the class values are displayed to enable the comparison of the two classification techniques.

The program listing

```
program MCIC_AHP;
{$M $C000,0,0}

Uses
    Crt, Dos;

Const
    MaxSize = 12;
    MaxInv  = 140;

Type
    Strng    = string[20];
    Matrix   = array[1..MaxSize, 1..MaxSize] of Real;
    Vector   = array[1..MaxSize] of Real;
    StrArray = array[1..MaxSize] of Strng;
    Inventory= record
        name:    strng;
        values: Vector;
        MC_score: real;
        MC_class: char;
        TR_score: real;
        TR_class: char;
    end;
    InvArray = array[1..MaxInv] of Inventory;

Var
    selection: integer;

function upCaseStr (str: string): string;
var i: integer;
begin
    upCaseStr := str;
    for I:= 1 to length (str) do upCaseStr[i] := upCase(str[i]);
end;

procedure showMatrix (n: integer; var M: Matrix);
var r, c: integer;
begin
    for r := 1 to n do begin
        for c:= 1 to n do write (M[r,c]:7:3, ' ');
        writeln;
    end; {for r}
end; {showMatrix}

procedure showVector (n: integer; var V: Vector);
var
    r: integer;
begin
    for r := 1 to n do write (V[r]:7:3, ' ');
    writeln;
end; {showVector}

procedure getCriteriaList (var n: integer; var CL: StrArray);
begin
    ClrScr; writeln; writeln;
    Writeln ('Index: Criterion name':40);
    n := 0;
    repeat
        n := n + 1;
        write (' ':19, n:5, ' '); readln(CL[n]);
    until CL[n] = '';
    n := n -1;
end; {getCriteriaList}

procedure showCriteriaList (n: integer; var CL: StrArray; var PL: Vector);
var
```

```

    i: integer;
begin
    ClrScr; writeln; writeln;
    writeln ('Index: Criterion name          Priority':48);
    for i := 1 to n do
        writeln(' ':13, i:5, ' ', CL[i], ' ':20-length(CL[i]), PL[i]:5:3);
end; {showCriteriaList}

procedure saveCriteriaList (n: integer; var CL: StrArray; var PL: Vector;
                           var fp: text);
var
    i: integer;
begin
    writeln (fp, 'Index Criterion name          Priority':46);
    for i := 1 to n do
        writeln(fp, i:17, ' ', CL[i], ' ':20-length(CL[i]), PL[i]:5:3);
    close (fp);
end;

procedure getName (var f: text; var name: string);
var c: char;
    i: integer;
begin
    i := 0; name := '';
    read (f, c);
    while c in [#32,#10,#13] do read (f, c);
    if not eoln (f) then
        repeat
            i := i+1; name[i] := c; name[0] := chr(i); read (f, c);
        until c = #32;
end; {getName}

procedure loadCriteriaList (var n: integer; var CL: StrArray; var PL: Vector;
                           var f: text);
{Read into the priority list from the criteria list file}
var
    line: string;
    c: char;
begin
    readln (f); {skip the first line containing the header}
    while not eof (f) do begin
        read (f, n);
        read (f, c); {skip the space}
        getName (f, CL[n]);
        readln (f, PL[n]);
    end; {while}
    close(f);
end;

procedure loadInventory (var Csize, Isize: integer; var min, max: Vector;
                        var IA: InvArray; var f: text);
var
    i: integer;
    name: string;
begin
    repeat
        {Skip the blank lines}
        getName (f, IA[1].name);
    until IA[1].name <> '';
    {get the first line, and determine the number of criteria}
    Csize := 0;
    while not eoln (f) do begin
        Csize := Csize +1;
        read (f, IA[1].values[Csize]);
        min[Csize] := IA[1].values[Csize]; max[Csize] := min[Csize];
    end;
    Isize := 2;
    writeln ('Loading ...');
    while not eof (f) do begin
        getName (f, IA[Isize].name);
        if IA[Isize].name <> '' then begin {test is to skip the blank lines}

```

```

        for i:= 1 to Csize do begin
            read (f, IA[Isize].values[i]);
            if IA[Isize].values[i] < min[i] then
                min[i] := IA[Isize].values[i]
            else if IA[Isize].values[i] > max[i] then
                max[i] := IA[Isize].values[i]
            end; {for i to Csize}
            Isize := Isize + 1;
        end; {if then}
    end; {while}
    Isize := Isize-1;
    close (f);
    for i := 1 to Isize do begin {initialize}
        IA[i].MC_score := 0; IA[i].MC_class := ' ';
        IA[i].TR_score := 0; IA[i].TR_class := ' ';
    end; {for}
end; {loadInventory}

procedure showInventory (Csize, Isize: integer;
                        var CL: StrArray; var IA: InvArray);
var
    i, j, k, left : integer;
begin
    i := 0;
    while i < Isize do begin
        ClrScr;
        write ('Item Name');
        for k := 1 to Csize do write (' ', CL[k]);
        writeln (' WtScr M_C Ann$usg T_C');
        left := Isize - i;
        if left > 20 then left := 20;
        for j := 1 to left do begin
            i := i + 1;
            write (IA[i].name:9);
            for k := 1 to Csize do
                write (' ', IA[i].values[k]:length(CL[k]):0);
            writeln (IA[i].MC_score:6:3, ' ', IA[i].MC_class:2, ' ',
                    IA[i].TR_score:8:0, ' ', IA[i].TR_class:2);
        end; {for}
        write ('Press ENTER to continue !'); readln;
    end; {while}
end; {showInventory}

procedure saveInventory (Csize, Isize: integer;
                        var CL: StrArray; var IA: InvArray; var fp: text);
var
    i, k: integer;
begin
    for i := 1 to Isize do begin
        write (fp, IA[i].name:9);
        for k := 1 to Csize do
            write (fp, ' ', IA[i].values[k]:length(CL[k]):0);
        writeln (fp, IA[i].MC_score:6:3, ' ', IA[i].MC_class:2, ' ',
                IA[i].TR_score:8:0, ' ', IA[i].TR_class:2);
    end; {for i}
    close (fp);
end; {saveInventory}

procedure getEntry (var CL: StrArray; i, j: integer; var Entry: Real);
{get the pairwise comparison of two criteria i and j,
 which is the (i,j) entry above the diagonal in the matrix}
var
    c: char;
begin
    Entry := 1;
    ClrScr; writeln; writeln; writeln;
    writeln ('Criteria          Scale');
    writeln ('=====');
    writeln (CL[i], ' ':27-length(CL[i]), Entry:5:3, ' ', #30);
    writeln (CL[j], ' ':27-length(CL[j]), (1/Entry):5:3, ' ', #31);

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writeln;
writeln ('Press up arrow ', #30, ' or down arrow ', #31, ' ');
writeln; writeln;
writeln('Equal importance: ..... 1');
writeln('Moderate importance: .....3');
writeln('Essential or strong importance: .....5');
writeln('Very strong importance: .....7');
writeln('Extreme importance: .....9');
writeln('Intermediate values between adjacent judgements: 2, 4, 6, 8');
gotoXY(34,6);
repeat
  c := readkey; if c = #0 then c := readkey;
  case c of
    #72: begin
      gotoXY(1,4);
      if Entry >= 1 then Entry := Entry + 1
      else Entry := Entry / (1 - Entry);
    end;
    #80: begin
      gotoXY(34,7);
      if Entry > 1 then Entry := Entry - 1
      else Entry := Entry / (1 + Entry);
    end;
  end; {Case}
  if Entry < (1/9) then Entry := 1/9;
  if Entry > 9 then Entry := 9;
  gotoXY(1,6);
  writeln (CL[i], ' ':27-length(CL[i]), Entry:5:3, ' ', #30);
  writeln (CL[j], ' ':27-length(CL[j]), (1/Entry):5:3, ' ', #31);
  if c = #72 then gotoXY(34,6)
  else if c = #80 then gotoXY(34,7);
until c = #13;
end;

procedure Normalize (n: integer; var V: Vector);
var
  i: integer;
  total: real;
begin
  total := 0;
  for i := 1 to n do
    total := total + V[i];
  for i := 1 to n do
    V[i] := V[i] / total;
end; {Normalize}

procedure Method3 (n: integer; M: Matrix; var EigenVector: Vector);
{This is 3rd approximation method for Eigen vector in Saaty's book}
var
  r, c: integer; {r: row, c: column}
  total: real;
begin
  for c := 1 to n do begin
    total := 0.0;
    for r := 1 to n do
      total := total + M[r,c];
    for r := 1 to n do
      M[r,c] := M[r,c] / total;
    end; {for c}
    for r := 1 to n do begin
      total := 0.0;
      for c := 1 to n do
        total := total + M[r,c];
      EigenVector[r] := total / n;
    end; {for r}
  end; {Method 3}

function LambdaMax(n: integer; M: Matrix; EigenVector: Vector): real;
var
  r, c: integer;

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    total: real;
    Vector1: Vector;
begin
    for r := 1 to n do begin
        total := 0.0;
        for c := 1 to n do
            total := total + M[r,c]*EigenVector[c];
        Vector1[r] := total / EigenVector[r];
    end; {for r}
    total := 0.0;
    for r := 1 to n do
        total := total + Vector1[r];
    LambdaMax := total / n;
end; {LambdaMax}

function CI (n: integer; LambdaMax: real): real;
begin
    CI := (LambdaMax - n) / (n - 1);
end; {CI}

function RI (n:integer): Real;
begin
    case n of
        1, 2: RI := 0.0;
        3: RI := 0.58;
        4: RI := 0.9;
        5: RI := 1.12;
        6: RI := 1.24;
        7: RI := 1.32;
        8: RI := 1.41;
        9: RI := 1.45;
        10: RI := 1.49;
        11: RI := 1.51;
        12: RI := 1.48;
        13: RI := 1.56;
        14: RI := 1.57;
        15: RI := 1.59;
    end; {case}
end; {RI}

function CR (n: integer; CIval: real): real;
begin
    CR := CIval / RI(n);
end; {CR}

procedure makeCriteriaList (var n: integer;
                           var CL: StrArray; var PL: Vector);
var
    M: Matrix;
    r, c: integer;
    Lmax, ciVal, crVal: real;
begin
    repeat
        getCriteriaList (n, CL);
        if n <= 2 then begin
            write ('You must have at least three criteria !!!'); readln;
        end;
    until n >2 ;
    for r:= 1 to n do writeln(CL[r]);
    repeat
        for r := 1 to n do begin
            M[r,r] := 1;
            for c := r+1 to n do begin
                getEntry(CL, r, c, M[r,c]);
                M[c,r] := 1 / M[r,c];
            end; {for c}
        end; {for r}
        ClrScr; writeln; writeln; writeln;
        writeln('The matrix is:');
        showMatrix(n, M);
    end;
end;

```

```

Method3 (n, M, PL);
writeln;
writeln('Eigen vector is: ');
showVector(n, PL);
Lmax := LambdaMax(n, M, PL);
ciVal := CI(n, Lmax);
crVal := CR(n, ciVal);
writeln;
writeln('Lmax:', Lmax:6:3, ' CI:', ciVal:6:3, ' CR:', crVal:6:3);
writeln;
if crVal <= 0.1 then writeln ('The matrix is consistent.')
else begin writeln ('The matrix is INCONSISTENT!');
      writeln ('We have to form the matrix again');
    end;
  write ('Press ENTER to continue !'); readln;
until crVal <= 0.1;
showCriteriaList(n, CL, PL);
end; {MakeCriteriaList}

procedure mergeCriteriaList(var size, SubSize, index: integer;
                           var CL, SCL: StrArray; var PL, SPL: Vector);
var
  i, shift: integer;
  Priority: Real;
begin
  shift := SubSize - 1;
  for i := size downto index+1 do begin
    CL[i + shift] := CL[i];
    PL[i + shift] := PL[i];
  end; {for}
  Priority := PL[index];
  for i := 1 to subSize do begin
    CL[index + i -1] := SCL[i];
    PL[index + i -1] := Priority * SPL[i];
  end; {for}
  size := size + shift;
end; {mergeCriteriaList}

procedure main_menu (var choice: integer);
begin
  repeat
    clrScr;
    writeln; writeln; writeln;
    writeln ('      M C I C - A H P');
    writeln ('      MultiCriteria Inventory Classification using');
    writeln ('      Analytic Hierarchy Process');
    writeln ('      By H. Altay GÄvenir & Nuray GÄvenir');
    writeln; writeln;
    writeln ('      1: AHP');
    writeln ('      Using this option you can create a file containing');
    writeln ('      a list of criteria and their weights.');
```

X

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    writeln ('      2: ABC Classification');
    writeln ('      Using this option you can classify an inventory by');
    writeln ('      multicriteria ABC classification,');
    writeln ('      traditional ABC classification, or');
    writeln ('      both.');
```

X

```

    writeln ('      3: Editor');
    writeln ('      0: Exit');
    writeln; writeln;
    write ('      What is your choice? (1/2/3/0): ');
    readln (choice);
  until choice in [0,1,2,3];
  clrScr;
end; {main_menu}

procedure getFileR (message: string; var fp: text; var fname: string);
var
  IOcode: integer;
begin
  {I-}
```

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repeat
  writeln; write ('Name of the file containing ', message, ': ');
  readln (fname);
  assign (fp, fname); reset (fp); IOCode := IOResult;
  if IOCode <> 0 then
    writeln ('File ', upCaseStr(fname), ' does not exist!');
until (IOCode = 0) and (fname <> ''); {$I+}
end; {getFile}

procedure getFileW (message: string; var fp: text; var fname: string);
begin
  writeln; write ('  Name of the file to save ', message, ': ');
  readln (fname);
  assign (fp, fname); rewrite (fp);
end; {getFile}

procedure AHP;
var
  M: Matrix;
  size, subSize, index, r, c : Integer;
  CL, SCL: StrArray;
  PL, SPL: Vector;
  fname, response: string;
  fp: text;
begin
  makeCriteriaList(size, CL, PL);
  repeat
    writeln;
    write ('  Does any of these criteria have subelements? (y/n): ');
    readln (response);
    if response[1] in ['y','Y'] then begin
      write ('  What is the index of the criterion: '); readln(index);
      if (index > 0) and (index <= size) then begin
        ClrScr;
        makeCriteriaList(subSize, SCL, SPL);
        mergeCriteriaList(size, subSize, index, CL, SCL, PL, SPL);
        showCriteriaList(size, CL, PL);
      end {if valid index}
      else writeln ('Invalid index!!!');
    end; {if response = yes}
  until not (response[1] in ['y','Y']);

  getFileW ('the criteria list', fp, fname);
  if fname <> '' then begin
    saveCriteriaList (size, CL, PL, fp);
    writeln ('  Criteria list is saved into ', upCaseStr(fname));
    write ('  Press ENTER to continue !'); readln;
  end; {save criteria list}
end; {AHP}

procedure MC_computeScores (Csize, Isize: integer; var min, max, PL: vector;
                           var IA: InvArray);
var
  i, c: integer;
  score: real;
begin
  for i := 1 to Isize do begin
    score := 0;
    for c := 1 to Csize do
      score := score + PL[c] * (IA[i].values[c] - min[c]) /
        (max[c] - min[c]);
    IA[i].MC_score := score;
  end; {for}
end; {MC_computeScores}

procedure TR_computeScore (Isize: integer; var IA: InvArray);
var
  i: integer;
begin
  for i := 1 to Isize do

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    IA[i].TR_score := IA[i].values[1] * IA[i].values[2];
end; {TR_computeScores}

procedure swap (var IA: InvArray; i, j: integer);
var
    tmp: Inventory;
begin
    tmp := IA[i];
    IA[i] := IA[j];
    IA[j] := tmp;
end; {swap}

procedure sortInv (Isize: integer; var IA: InvArray; kind: integer);
{Using Bubble sort algorithm}
var
    i, j: integer;
begin
    for i := 1 to Isize-1 do
        for j := i+1 to Isize do
            case kind of
                1: if IA[i].MC_score < IA[j].MC_score then swap (IA, i, j);
                2: if IA[i].TR_score < IA[j].TR_score then swap (IA, i, j);
            end; {case}
        end;
    end; {sortInv}

procedure Classify (Isize: integer; var IA: InvArray; kind: integer);
{for MC_ABC kind = 1; for TR_ABC kind = 2}
var
    Aper, Bper: real;
    Acnt, Bcnt: integer;
    i: integer;
    response: char;
begin
    repeat
        ClrScr;
        writeln; writeln;
        writeln ('Now we need the desired distribution of items in each class. ');
        writeln;
        write ('What is the desired percentage of items in class A? : ');
        readln (Aper); Acnt := round (Aper * Isize / 100);
        write ('What is the desired percentage of items in class B? : ');
        readln (Bper); Bcnt := round (Bper * Isize / 100); writeln;
        writeln;
        writeln ('Distribution of items in each class will be as follows:');
        writeln (Acnt:10, ' items in class A (', Aper:4:1, '%)');
        writeln (Bcnt:10, ' items in class B (', Bper:4:1, '%)');
        writeln (Isize-(Acnt+Bcnt):10, ' items in class C (',
            (100-Aper-Bper):4:1, '%)');
        writeln ('-----');
        writeln (Isize:10, ' items in Total');
        writeln;
        write ('Do you want to proceed with these percentages? [y/n]: ');
        readln (response);
    until response in ['y','Y'];
    i := 1;
    while (i <= Acnt) do begin
        if kind = 1 then IA[i].MC_class := 'A'
        else if kind = 2 then IA[i].TR_class := 'A';
        i := i+1;
    end; {while for class A}
    while (i <= Acnt+Bcnt) do begin
        if kind = 1 then IA[i].MC_class := 'B'
        else if kind = 2 then IA[i].TR_class := 'B';
        i := i+1;
    end; {while for class B}
    while i <= Isize do begin
        if kind = 1 then IA[i].MC_class := 'C'
        else if kind = 2 then IA[i].TR_class := 'C';
        i := i + 1;
    end; {while}
end;

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end; {Classify}

procedure MC_ABC (Csize, Isize: integer; var CL: StrArray;
                 var PL, min, max: Vector; var IA: InvArray);
var
  Csize_CF: integer;
  fp: text;
  fname: string;
begin
  getFileR ('the criteria list', fp, fname);
  loadCriteriaList (Csize_CF, CL, PL, fp);
  if Csize_CF <> Csize then
    writeln ('Number of criteria do not match !!!')
  else begin
    writeln ('Criteria list is loaded from ', upCaseStr(fname));
    writeln;
    write ('Press ENTER to continue !'); readln;
    MC_computeScores (Csize, Isize, min, max, PL, IA);
    sortInv (Isize, IA, 1);
    Classify (Isize, IA, 1);
  end; {else}
end; {MC_ABC}

procedure TR_ABC (Isize: integer; var IA: InvArray);
begin
  TR_computeScore (Isize, IA);
  sortInv (Isize, IA, 2);
  Classify (Isize, IA, 2);
end; {TR_ABC}

procedure ABC;
var
  i, choice: integer;
  Csize, Isize: integer; {Csize: criteria size, Isize: inventory size}
  IA: InvArray;
  CL: StrArray;
  PL, min, max: Vector;
  fp: text;
  invFname, cinFname: string;
begin
  ClrScr; writeln; writeln ('ABC Classification');
  getFileR ('the inventory', fp, invFname);
  loadInventory (Csize, Isize, min, max, IA, fp); writeln;
  writeln ('Inventory of ', Isize, ' items with ', Csize,
    ' criteria is loaded from ', upCaseStr(invFname));
  writeln;
  write ('Press ENTER to continue !'); readln;
  writeln; writeln;
  CL[1] := 'UntPrc'; {Default name of the first criterion: Unit Price}
  CL[2] := 'AnnUsg'; {Default name of the second criterion: Demand}
  for i := 3 to Csize do CL[i] := 'Cr'+Chr(i+48);
  repeat
    ClrScr; writeln; writeln;
    writeln ('What type of classification do you want to perform on ',
      upCaseStr(invFname), ' ?');
    writeln (' 1: multicriteria ABC classification');
    writeln (' 2: traditional ABC classification');
    writeln (' 0: exit');
    write ('Your choice [1/2/0]: '); readln (choice);
    if choice = 1 then MC_ABC (Csize, Isize, CL, PL, min, max, IA)
    else if choice = 2 then TR_ABC (Isize, IA);
    if choice in [1,2] then begin
      ShowInventory (Csize, Isize, CL, IA);
      getFileW ('the classified inventory', fp, cinFname);
      if cinFname <> '' then begin
        saveInventory (Csize, Isize, CL, IA, fp);
        writeln ('Classified inventory is saved in file ',
          upCaseStr(cinFname));
        write ('Press ENTER to continue !'); readln;
      end; {Save classified inventory file}
    end;
  until choice = 0;
end;

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        end; {if choice <> Exit}
    until choice = 0;
end; {ABC}

procedure EDITOR;
begin
    SwapVectors;
    Exec ('c:\command.com','/c edit');
    SwapVectors;
end; {EDITOR}

begin
    repeat
        main_menu (selection);
        case selection of
            1: AHP;
            2: ABC;
            3: EDITOR;
        end; {case}
        clrScr;
    until selection = 0;
end.

```



BIBLIOGRAPHY

- [1] A. Arbel and Y. E. Orgler, "An Application of the AHP to Bank Strategic Planning: The Mergers and Acquisitions Process," *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 27-37, 1990.
- [2] I. J. Azis, "Analytic Hierarchy Process in the Benefit-Cost Framework: A Post-Evaluation of the Trans-Sumatra Highway Project," *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 38-48, 1990.
- [3] E. S. Buffa and R. K. Sarin, *Modern Production / Operations Management*, John Wiley & Sons, New York, 1987.
- [4] D. R. Cook, S. Staschak and W. T. Green, "Equitable Allocation of livers for Orthotopic Transplantation: An Application of the Analytic Hierarchy Process," *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 49-56, 1990.
- [5] M. Duru, "ABC Inventory Classification: An Application: Özdemirler," MBA thesis, Faculty of Business Administration, Bilkent University, Jan. 1989
- [6] J. R. Evans, D. R. Anderson, D. J. Sweeney and T. A. Williams, *Applied Production & Operations Management*, West Publishing Co. St. Paul, 1978.
- [7] B. E. Flores, D. L. Olson and V. K. Dorai, "Management of Multicriteria Inventory Classification," *Mathl. Comput. Modeling* Vol. 16, No. 12, pp. 71-82, 1992.

- [8] B. E. Flores and D. C. Whybark, "Multiple Criteria ABC Analysis" *International Journal of Operations and Production Management* Vol. 6, No. 3, pp. 38-46, 1986.
- [9] J. H. Greene, *Production and Inventory Control Handbook*, McGraw-Hill, New York, 1987.
- [10] R. L. Janson, *Handbook of Inventory Management*, Englewood Cliffs, NJ: Prentice-Hall, 1987.
- [11] M. R. Leenders, H. E. Fearon, W. B. England, *Purchasing and Materials Management*, Irwin Inc., 1989.
- [12] A. Özaltın, "An Inventory Management System for Özaltın Construction and Trading Company," MBA thesis, Faculty of Business Administration, Bilkent University, Feb. 1992.
- [13] V. Pareto, *Manual of Political Economy*, English Translation, A. M. Kelly, New York, 1971.
- [14] T. L. Saaty, "How to make a Decision: The Analytic Hierarchy Process," *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 9-26, 1990.
- [15] T. L. Saaty, *Mathematical Methods of Operations Research*, New York, McGraw-Hill, 1988.
- [16] T. L. Saaty, *The Analytic Hierarchy Process*, McGraw-Hill, New York, 1980.
- [17] J. Sarai, "Practical Approach to the ABC Analysis, Its Extension and Application," Proc. First Int. Symp. on Inventories, Budapest, Hungary, pp. 255-261, 1980,

- [18] V. Srinivasan and P. J. Bolster, , “An Industrial Bond Rating Model based on the Analytic Hierarchy Process,” *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 105-119, 1990.
- [19] L. G. Vargas, “An Overview of the Analytic Hierarchy Process and its Applications,” *European Journal of Operational Research*, Vol. 48, No. 1, North-Holland, pp. 2-8, 1990.

