

DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**MECHANICAL AND DYNAMIC ANALYSIS OF
AN ALUMINUM - MINERAL COMPOSITE BEAM
AND PLATE**



by
Oğuzhan DAŞ

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İZMİR

MECHANICAL AND DYNAMIC ANALYSIS OF AN ALUMINUM - MINERAL COMPOSITE BEAM AND PLATE

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Program**

**by
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Ph.D. THESIS EXAMINATION RESULT FORM

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MECHANICAL AND DYNAMIC ANALYSIS OF AN ALUMINUM - MINERAL COMPOSITE BEAM AND PLATE

ABSTRACT

Frame structures are widely used in aerospace, maritime industries, construction, etc. These structures are subjected to static and dynamic loads, which may cause buckling, dynamic instability, or resonance. Hence, it is essential to determine the right material and understand the mechanic and dynamic properties of such structures. This thesis investigates the free vibration, buckling, and dynamic stability of isotropic and orthotropic curved beam and plate frame structures using the finite element method and beam or plate theory. The effects of the radius of curvature and stacking order of the single-bay and two-bay curved frame structures on the natural frequency, critical buckling load, and unstable regions are investigated. Similarly, the effects of the aspect ratio on the dynamic and buckling characteristics of the curved frame structures are examined. Besides, the effects of the thickness on the natural frequency and critical buckling load of the isotropic curved beam frame structure are investigated.

Euler – Bernoulli Beam Theory and Classical Plate Theory is employed with the Finite Element Method to obtain isotropic and orthotropic beam frame structures. Curved plate frame structures are modeled considering Kirchhoff – Love Plate Theory for isotropic frames, whereas the Classical Plate Theory and First-Order Shear Deformation Theory for orthotropic frames. Results are validated with those of ANSYS and SolidWorks.

Aluminum – potassium feldspar metal matrix composite materials are fabricated via the stir casting method. The mechanical properties of such materials are obtained by tensile testing. Free vibration, buckling, and dynamic stability analyses are performed considering a plate of such material.

Keywords: Curved frames, vibration, buckling, dynamic stability, metal matrix composites

ALÜMİNYUM-MİNERAL KOMPOZİT KİRİŞLERİN VE PLAKALARIN MEKANİK VE DİNAMİK ANALİZİ

ÖZ

Çerçeve yapılar havacılık, denizcilik, inşaat, vb. alanlarda sıkça kullanılmaktadırlar. Bu yapılar, burkulma, dinamik kararsızlık ve rezonans ile sonuçlanabilen statik ve dinamik yükler ile çeşitli uyarımlara maruz kalmaktadırlar. Bu nedenle bu yapıların mekanik ve dinamik özelliklerinin anlaşılması önem arz etmektedir. Bu tez kapsamında, Sonlu Elemanlar Yöntemi ve kiriş veya plaka teorileri kullanılarak izotropik ve ortotropik eğri kiriş ve plaka çerçevelerin serbest titreşim, burkulma ve dinamik stabilite analizleri araştırılmıştır. Eğrilik yarıçapının tek ve iki bölümlü eğri çerçeve yapılarının doğal frekans ve kritik burkulma yüküne olan etkileri incelenmiştir. Ayrıca, eğri plaka çerçeveler için en-boy oranının titreşim ve burkulma karakteristiklerine olan etkisi ve eğri kiriş çerçeveler için kalınlık etkisi incelenmiştir. Ek olarak, eğri kiriş ve plaka çerçeve yapılarının dinamik kararlılıkları da araştırılmıştır.

İzotropik ve ortotropik kirişler için Euler – Bernoulli Kiriş Teorisi ve Klasik Plaka Teorisi, Sonlu Elemanlar Yöntemi ile birleştirilerek modellenmiştir. Eğri plaka çerçevelerin modellenmesinde izotropik yapılar için Kirchhoff – Love Plaka Teorisi dikkate alınırken ortotropik yapılarda Klasik Plaka Teorisi ve Birinci Mertebe Kayma Deformasyon Teorisi kullanılmıştır. Elde edilen sonuçlar bilgisayar destekli çizim programları ile doğrulanmıştır.

Karıştırmalı döküm yöntemi ile alüminyum – potasyum feldspat metal matris kompozit malzemeler üretilmiştir. Üretilen malzemelerin mekanik ve malzeme özellikleri elde edilmiştir. Nümerik analiz yöntemi ile ilgili malzemeye sahip bir plaka modellenerek serbest titreşim, burkulma ve dinamik kararlılık analizleri yapılmıştır.

Anahtar Kelimeler: Eğri çerçeveler, titreşim, burkulma, dinamik kararlılık, metal matris kompozitler

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Curved frame structures, shown in Figures 1.1 and 1.2, have a massive role in many engineering fields like aerospace, naval, automotive, construction, etc. As the material technology develops, the design parameters and, as a result, the mechanical and dynamic characteristics of these structures changes. Therefore, the determination of the mechanic, and dynamic characteristics (e.g., natural frequency, critical buckling load, parametric resonance) of these structures is essential. Every structure should be operated under various static and dynamic conditions without any damage or failure. Hence, material selection, another critical aspect, becomes a significant issue during the design stage of such structures. Composite materials are used widely in engineering due to their high strength to weight ratio, corrosion, and fatigue resistance. These materials can be fabricated in many ways. Therefore, there are lots of composite types manufactured to satisfy the desired material properties. Laminated composite materials are one example of composite materials that comprise resin-infused fabrics whose fiber angles may vary. These composites are accepted as orthotropic materials whose modulus of elasticity and shear modulus differs with respect to the fiber angles. Metal matrix composite materials are another example of composite materials, which a metal and a reinforcement material, or materials are mixed together. The chemical composition and amount of the reinforcement material have a significant effect on the metal matrix composites' material and mechanical properties. Besides, the manufacturing parameters such as casting temperature, pre-heating, pressure, etc. also impact the properties of the final product. It is essential to determine these parameters accurately to obtain a useful composite material. It is still needed light, durable, productive, and affordable novel metal matrix composite materials to satisfy many requirements of the industry.



Figure 1.1 Arch Bridge (Garage, 2019)



Figure 1.2 Hangar (Pakforever, n.d.)

Free vibration analysis is one of the most critical dynamic analysis that is used to obtain natural frequency values and mode shapes of a system under different conditions. The significance of the natural frequency comes from the resonance phenomenon, which occurs when the frequency of a periodic force is equal to a natural frequency of a system. Such an incident may lead the structure to fail. Buckling analysis is performed to evaluate the critical buckling load that may harm the structure. Examining dynamic stability is also essential since parametric resonance may occur under a periodic load.

1.2 Literature Review

As mentioned in the previous section, curved frame structures play a significant role in many engineering fields. Even if there are lots of studies about free vibration analysis and buckling analysis in literature comprising beam, plate, curved plate, frame, and folded structures, a few studies are reported in the literature addressing curved plate frame or curved beam frame structures.

Vibration analysis of structures like beams, plates, frames, etc. has been a popular field of interest for a long time. Since it is not possible to mention all studies, some publications are discussed briefly as follows. Leontovich (1959) is the one who investigated the parabolic frames and arches. In this book, parabolic frames and arches are taken into account statically in various loading cases. Chidamparam & Leissa (1993) studied on the vibration behavior of planar curved beams, arches, and rings. They review many studies that comprise in-plane vibrations, out of plane vibrations, coupled vibrations, nonlinear vibrations, etc. Kang et al. (2003) worked on free vibration analysis of planar curved beams by utilizing wave propagation with general boundary conditions and supports. Zhu & Meguid (2004) developed a new curved beam element that satisfies inextensible bending mode conditions by using a three-node locking free curved beam element.

Akgöz & Civalek (2013) investigated the free vibration analysis of axially functionally graded tapered Bernoulli-Euler microbeams. They utilized the Rayleigh-Ritz method in order to obtain the solution of the free transverse vibration problem. Pradhan & Chakravery (2013) worked on the free vibration analysis of both Euler and Timoshenko functionally graded beams. They obtained the governing equations by using the Rayleigh-Ritz method. Eltaher et al. (2012) studied on the vibration analysis of Euler-Bernoulli nanobeams. They employ the finite element method in order to obtain numerical results. Bazoune & Khulief (1992) investigated the free vibration analysis of rotating tapered Timoshenko beams. They used a finite element beam that has four degrees of freedom including shear deformations.

Cleghorn et al. (1993) studied a finite element model for the vibration of rings with unsymmetrical cross-sections. They developed a finite element model for free, coupled in-plane and out of plane conditions. Fotouhi (2007) performed the dynamic analysis of flexible beams with large deflections. Cantilever beam and rotating flexible robot arms are taken into account as example structures. Ozturk (2011) studied on the vibration analysis of a pre-stressed laminated composite curved beam by employing the finite element method with a straight –beam element approach. The effects of orientation angle and vertical loading on the natural frequency are investigated for the first four modes. Carrera et al. (2012) used different theories based on the vibration modes and finite element analysis in order to investigate higher-order theories for free vibration analysis of beams. Sina et al. (2009) presented a new analytical method different from the first-order shear deformation theory for beams to analyze free vibration analysis of functionally graded beams. Marur & Kant (1996) carried out a study that was aimed to conduct free vibration analysis of fiber-reinforced composite beams by utilizing the Higher-Order Shear Deformation Theory (HSDT). They also investigated the dynamical behavior of sandwich composite beams in that work. Atlihan et al. (2009) utilized the Differential Quadrature Method (DQM) to perform free vibration analysis of a laminated composite beam. Ramtekkar (2009) used a 2-D plane stress mixed finite element model to perform a free vibration analysis of laminated beams with delaminations.

Jun & Hongxing (1996) investigated the free vibration behavior of axially loaded laminated composite beams by using Higher Order Shear Deformation Theory. They introduced the dynamic stiffness matrix method to solve both free vibration and buckling problems. Besides, several boundary conditions. Ozturk (2015) investigated the vibration analysis of a pre-stress laminated composite curved beam. In that study, a curved beam was modeled by utilizing the Finite Element Method with a straight beam element. In addition, to obtain the curved shape, the Reversion Method was used to obtain a non-linear deflection curve. Hodges et al. (1991) presented various methods to determine the natural frequencies and mode shapes of composite beams.

Vo et al. (2017) presented free vibration of axially loaded composite beams using a four-unknown shear and normal deformation theory by employing a two-node C1 beam element. Vo-Duy et al. (2018) performed a free vibration analysis of laminated functionally graded carbon nanotube-reinforced composite beams using the finite element method. Eken (2019) investigated the free vibration analysis of composite aircraft wings by considering them as thin-walled beams with NACA airfoil sections. Xiang et al. (2012) used an n th-order shear deformation theory in order to perform a free vibration analysis of isotropic plates. They solve the governing differential equations by using a meshless radial point collocation method. Burlayenko & Sadowski (2019) presented three-dimensional modeling for free vibration and static response of functionally graded (FGM) sandwich plates. They modelled those plates via ABAQUS by introducing a three-dimensional brick graded finite element. Ramu & Mohanty (2012) conducted a free vibration analysis of rectangular plates by using the Kirchhoff plate theory and the finite element method. Wang & Werely (2002) investigated the free in-plane vibration of rectangular plates with free and clamped boundary conditions. They presented an iteration scheme based on the Kantorovich-Krylov method in order to calculate the natural frequency values and corresponding mode shapes efficiently. Liew et al. (2004) used a meshfree method for the free vibration and buckling analysis of shear-deformable plates. They employed Mindlin/Reissner plate theory, or first-order shear deformation theory, and reproducing kernel approximation to construct displacement functions.

Liu et al. (2017) performed a free in-plane vibration analysis of plates by using differential quadrature hierarchical finite element method by transforming non-uniform rational B-splines. Bardell et al. (1996) calculated the in-plane frequencies of simply-supported, clamped, and free plates by employing the Rayleigh-Ritz method. Manna (2005) investigated free vibration analysis of isotropic rectangular plates with various thickness ratios, boundary conditions, and aspect ratios by using a high-order triangular element and the first-order shear deformation theory. Narita & Leissa (1992) presented an analytical approach to investigate the free vibration of cantilevered and symmetrically laminated rectangular plates by using the Ritz method.

Afshari & Widera (2000) developed a series of plates by considering the modified complementary energy principle formulations for free vibration analysis of composite plates. Pandit et al. (2007) investigated the free vibration analysis of composite rectangular plates by using a nine node isoparametric bending element with an effective mass lumping scheme. Besides simple rectangular plates, they also obtained natural frequency results not only for plates that have cutouts but also for plates on which distributed mass exists. Ngo-Cong et al. (2011) presented an effective Radial Basis Function for free vibration analysis of laminated composite plates via the First Order Shear Deformation Theory (FSDT). Ganesh et al. (2016) studied the free vibration analysis of laminated composite plates by using the finite element method and by considering several boundary conditions. El-Helloty (2016) examined parameters that affect the free vibration analysis of stiffened laminated composite plates. Belarbi et al. (2017) developed a higher-order layerwise finite element model for the free vibration analysis of multilayer sandwich plates. Fallah & Delzendeh (2018) utilized a Meshless Finite Volume (MFV) method for free vibration analysis of laminated composite plates. They tested MFV for different parameters such as plate thickness, stacking sequences, boundary conditions, and plate shapes. Cantin & Glough (1968) developed a curved cylindrical shell finite element for curved shells and proceeded a finite element analysis to find the displacement of a curved shell. Petyt & Deb Nath (1971) conducted a free vibration analysis for singly curved rectangular plates. Nosier et al. (1993) utilized Reddy's layerwise theory for free vibration analysis of laminated plates and compared their results with those evaluated from classical plate theory, first-order shear deformation theory, and third-order shear deformation theory.

Hu et al. (2002) investigated the vibration response of moderately thick composite plates having delamination using higher-order finite element. Pradyumna & Bandyopadhyay (2008) performed free vibration analysis of functionally graded curved panels by utilizing higher-order formulation. Marjanovic et al. (2017) used a dynamic stiffness element based on the first-order shear deformation theory and higher-order shear deformation theory to perform free vibration analysis of composite

plates. They investigated the effects of the boundary conditions and plate side to thickness and orthotropy ratios on the low and high modes of vibration.

Serdoun & Cherif (2016) investigated the free vibration analysis of composite thick rectangular plates using higher-order shear deformation theory. Chen et al. (2017) presented the free vibration analysis of composite truss core sandwich plates using third-order shear deformation theory and zig-zag theory. Demir et al. (2017) investigated free vibration analysis of annular sector plates using conical shell equations, first-order shear deformation theory, discrete singular convolution, and differential quadrature. Shankar & Mahato (2017) presented the vibration analysis of composite plates having delamination and/or damages using finite element method. Tornabene et al. (2018) investigated free vibration analysis of laminated plates and shells considering two different approaches, namely differential quadrature and integral quadrature, respectively. Abulanour et al. (2018) studied the free vibration analysis of simply supported functionally graded plates using a new shear deformation theory that comprises the stretching effect. Vidal et al. (2019) investigated the free vibration analysis of laminated composite plates by using variable separation approach. Yas & Chakravorty et al. (1996) presented a finite element analysis for free vibration analysis of doubly curved shells by using First-Order Shear Deformation Theory. Garmsiri (2010) investigated three-dimensional free vibration analysis of fiber-reinforced functionally graded cylindrical shells by using differential quadrature method (DQM). Ye et al. (2013) studied on free vibration analysis of laminated composite shallow shells with general elastic boundaries by using Rayleigh-Ritz procedure. Javed et al. (2016) studied on free vibration analysis of composite shells that have non-uniform thickness walls by using first-order shear deformation theory. Javed et al. (2016) utilized higher-order shear deformation theory to perform vibration analysis of antisymmetric angle-ply laminated plates for simply supported boundary conditions. Civalek (2017) investigated the vibration behavior of laminated composite truncated conical panels and annular sector panels with functionally graded materials (FGM) by using Love's shell theory and First Order Shear Deformation Theory. Governing differential equations of this study were solved via Differential Quadrature (DQ) and Discrete Singular Convolution (DSC) method. Chaubey & Kumar (2017)

conducted a free vibration finite element analysis of laminated composite cylindrical, spherical, hyperbolic, saddle, and elliptical shells with cutouts and concentrated mass by using Third Order Shear Deformation Theory (TSDT).

Javed et al. (2018) used higher-order shear deformation theory to evaluate the natural frequency values of the cross-ply laminated plates depending on the aspect ratio, side-to-thickness ratio, number of laminae, ply orientations, and stacking sequence. Hafizah & Viswanathan (2018) studied on the vibration of antisymmetric angle-ply composite annular plates by using first-order shear deformation theory and considering linear, exponential, and sinusoidal thickness variations.

The buckling characteristics of a structure have been investigated from researchers for a long time. Numerous studies about buckling analysis are presented in the literature. Some of those are mentioned as follows. Mehmood (2014) studied the vibration analysis of frame structure that is subjected to moving loads. Basci et al. (1979) presented an improved method for free vibration analysis of frame structures for various boundary conditions. Niyogi et al. (1999) investigated the free and forced vibration analysis of composite folded plate structures by using finite element analysis. They employed a nine-noded Lagrangian plate bending element which incorporates the rotary inertia and transverse shear deformation. Peng et al. (2006) presented a meshfree Galerkin method for free vibration analysis of stiffened and un-stiffened folded plates and plate structures based on the first-order shear deformation theory. Haldar & Sheikh (2005) performed the free vibration analysis of isotropic and composite folded plates by using a flexible shear element, which has 54 degrees of freedom. Lee et al. (2004) investigated the dynamic behavior of folded composite plates by using third-order plate theory and finite element analysis. Liu & Huang (1992) used the finite element – transfer matrix method in order to perform vibration analysis of folded plates. They considered one-fold and two-fold folded plates with various crank angles. Wittrick & Horsington (1984) and Peng (2015) dealt with flat folded plates with different methods and theories. Thinh et al. (2012) studied finite element bending and vibration analysis of multi-folding laminated composite plates

by using First-Order Shear Deformation Theory. Chan (1990) performed a buckling analysis of structures that are composed of tapered members.

Zhu et al. (2017) implemented Eringen's two-phase nonlocal model to perform buckling analysis of Euler-Bernoulli beams. Huynh et al. (2017) presented an isogeometric approach to analyze bending, buckling, and free vibration of functionally graded curved beams. They investigated Timoshenko beams with various curvatures by using non-uniform rational B-splines. Lee & Kim (2002) investigated the lateral buckling analysis of a laminated thin-walled channel-section composite beams using classical lamination theory. Goyal & Kapania (2007) presented a 21 degree of freedom element, which is based on the first-order shear deformation theory to perform buckling and vibration analysis of laminated composite beams. Zabihollah & Ganesan (2010) investigated the buckling behavior of tapered composite beams using higher-order finite element formulation. He & Yang (2014) studied buckling analysis of two-layer composite Timoshenko beams using the finite element method based on the higher-order beam theory. Wang et al. (2015) investigated the buckling analysis of laminated composite beams under different boundary conditions using the isogeometric finite element method based on non-uniform rational b-splines. Kahya (2016) studied finite element buckling analysis of laminated composite and sandwich beams using multilayered beam elements whose degrees of freedom vary with respect to the number of layers of the composite and sandwich beams. Ozdemir et al. (2018) carried out a buckling analysis of curvilinear stiffened plates by using a meshfree model. Bourada et al. (2016) investigated a new four variable refined plate theory to perform buckling analysis to isotropic plates and orthotropic plates. Apart from the higher-order shear deformation theories and first-order shear deformation theory, they introduced a new displacement field that incorporates undetermined integral terms. Reddy & Khdeir (1989) performed buckling and vibration analysis of cross-ply rectangular laminated composite plates considering classical plate theory, first-order shear deformation theory, and third-order shear deformation theory. Gal et al. (2006) presented buckling analysis of a composite panel using a new shell finite element. Singh & Chakrabarti (2012) investigated buckling analysis of laminated composite plates using C0 finite element model based on higher-order zig-zag theory.

Kasaeian et al. (2012) studied the inelastic local buckling analysis of the curved plates and curved plates having thickness-tapered sections using finite strip method. Barbero et al. (2014) presented the buckling analysis of folded laminated composite structure by introducing a mixed and quadrilateral 3D plate element. Huang et al. (2015) investigated the buckling analysis of grid stiffened composite panels, which have different stiffening arrangements, by presenting a unique finite element modelling technique. Hao et al. (2017) investigated the buckling analysis of composite variable stiffness panels based on isogeometric analysis and considering the Mindlin plate theory. Chikh et al. (2017) presented thermal buckling analysis of cross-ply laminated composite plates using a simplified higher-order shear deformation theory. Zghal et al. (2018) studied the buckling behavior of functionally graded materials and nanotubes reinforced composite flat and curved plates using double directors finite element shell model.

Structures are parametrically excited when they are subjected to dynamic loads. Dynamic instability may occur even the damping is existed due to the amplitudes of the response that grows exponentially. Hence it is significant for elastic structures. There are numerous studies on the dynamic stability of structures. A brief summarization of some of these studies are presented as follows. One of the well-known studies about the dynamic stability problem is Bolotin's approach (Bolotin, 1964), where the effect of an elastic foundation on the critical buckling loads and natural frequency values of a Bernoulli-Euler beam hinged from both ends is investigated. Tsai & Chen (2002) conducted dynamic stability analysis of the shape memory alloy wire reinforced composite beam using the finite element method and harmonic balance method. Yeh & Kuo (2004) investigated the dynamic stability of composite plates under parametric excitation considering the effects of the length, width, thickness, and fiber orientation of the beam on instability. Öztürk et al. (2006) examined the in-plane stability analysis of curved beams having non-uniform cross-section by using the finite element method. Machado & Cortínez (2009) studied the dynamic stability of periodically excited thin-walled composite beams using a seven degree of freedom shear deformable beam theory. Saravia et al. (2011) investigated the

dynamic stability of thin-walled rotating composite beams by employing the finite element method based on Bolotin's approach. Ke et al. (2013) presented the dynamic stability analysis of functionally graded nanocomposite beams that are reinforced with single-walled carbon nanotubes considering the Timoshenko beam theory. They used Bolotin's method to determine the unstable regions of the nanocomposite beams. Goren Kiral et al. (2015) performed a stability analysis for delaminated woven and laminated composite cantilever beams with different stacking sequences. Goren Kiral et al. (2009) investigated the stability of delaminated composite beams via finite element analysis for different stacking sequences. Chattopadhyay et al. (2000) used the higher-order shear deformation theory to perform a dynamic stability analysis of delaminated composite plates that are subjected to dynamic loads by using the finite element method. Oh et al. (2005) investigated the dynamic analysis of composite plates with multiple delaminations by using higher-order cubic zigzag theory by employing a four-node finite element. Dey & Singha (2006) studied the dynamic stability of laminated composite skew plates that are subjected to a dynamic in-plane load under simply supported boundary conditions by using Bolotin's method. Lee (2010) performed a dynamic stability analysis of laminated composite skew plates that have cutouts by using a higher-order shear deformation theory based finite element method. Fazilati (2017) performed stability analysis to variable stiffness laminated composite plates that have delaminations by using the finite strip method. Samukham et al. (2018) investigated dynamic stability analysis of variable angle tow composite with delamination placed around a cutout under simply supported boundary conditions. They used first-order shear deformation theory and finite element method to evaluate the governing equations of variable angle tow composite structure.

Composite materials are used widely due to their high strength to weight ratio, resistance to corrosion and fatigue. Laminated composites and metal matrix composites have significant importance among composite materials. Hence, many researchers studied about these two materials investigating their material properties, mechanical properties, manufacturing processes and challenges. There are countless studies about those materials, which is impossible to mention all of them. Therefore some researches are summarized and presented as follows.

Liu et al. (2006) investigated the mechanical performance of laminated composite materials having nanofibrous membranes. They determine the material and mechanical properties by using scanning electron microscopy (SEM), performing tensile, three-point bending, and short-beam shear tests. Zuo et al. (2006) manufactured $\text{Al}_2\text{O}_3/\text{Ni}$ laminated composites to investigate the mechanical properties of that material. Reis et al. (2006) researched the flexural behaviour of hybrid laminated composites. They performed static and fatigue tests to obtain the flexural strength and fatigue behavior of such material. Chen & Feng (2014) investigated the mechanical and thermal properties of epoxy/polyethyleneimine functionalized carbon fiber laminated composites. They unearthed the shear strength, thermal expansion coefficient, flexural strength, and flexural modulus of such composites. Abdellaoui et al. (2014) studied the fabrication, characterization, and modelling of woven jute fibres reinforced epoxy laminated composites. Xu et al. (2016) investigated the mechanical and electrical properties of laminated composites having continuous carbon nanotube film interleaves. Sharma et al. (2000) investigated the wear behaviour of feldspar particle reinforced magnesium alloy composites using pin-on-disc wear test. Singla et al. (2009) produced aluminium silicon carbide metal matrix composite using stir casting method and considering different weight fraction of silicon carbide. Rhee et al. (2015) investigated the mechanical properties of aluminium metal matrix composite considering different process conditions to give ideas about the optimum design of composite materials. Soltani et al. (2015) investigated the effects of the stir casting process parameters on aluminium silicon carbide metal matrix composites. Inegbenebor et al. (2016) manufactured silicon carbide particulate reinforced aluminium metal matrix composites using stir casting method. They conducted the manufacturing process through the dispersion of silicon carbide that effected the properties of metal metal matrix. Kumar & Birru (2017) investigated the mechanical properties and the microstructure of the aluminium metal matrix composites reinforced by bamboo leaf ash manufactured with stir casting method. Pandiyarajan et al. (2017) investigated the effects of zirconium dioxide and graphite on the tribological and mechanical properties of aluminium metal matrix composites using different weight percentages of reinforcement materials. Aybarc et al (2018) studied the effects of

different stirring methods on the mechanical properties of aluminium silicon carbide metal matrix composites. They employed mechanical, ultrasonic and hybrid (mechanical and ultrasonic) stirring during the production to determine the quality index and the tensile strength of the material, both experimental and numerical. Wang et al (2018) investigated the mechanical properties and the pore structure of porous aluminium titanate strontium feldspar mullite fiber composites. Gudimetla et al. (2019) developed a new metal aluminium matrix composite in which silicon and silicon balloons are used as reinforcement material, by using stir casting method and determine its tribological properties. Ye et al. (2019) investigated the particle size of silicon carbide on the mechanical properties and the failure of aluminium silicon carbide metal matrix composites under compression. Butola et al. (2019) investigated the mechanical properties of aluminum metal matrix composite reinforced by sugarcane, banana, and jute. They determined the ultimate tensile strength, yield strength, hardness, and percentage elongation.

1.3 Research Aims and Objectives

The aim of this research is determining the dynamic and mechanic characteristics of aluminum, aluminum – potassium (K) feldspar metal matrix composite, and various laminated composite curved beam and plate frame structures considering parameters such as the radius of curvature, aspect ratio, thickness, and boundary conditions of the structure. The objectives of this study can be summarized as follows:

- To perform finite element free vibration analysis on the aluminum and laminated composite curved beam and plate frame structures in order to evaluate the natural frequency values and mode shapes of the structures.
- To perform finite element buckling analysis on the aluminum and laminated composite curved beam and plate frame structures to calculate the critical buckling load values.

- To perform dynamic stability analysis on the laminated composite curved beam and plate frame structures to determine the unstable regions.
- To manufacture the aluminum – potassium feldspar metal matrix composite via stir casting method and obtain the material properties of such material.
- To perform finite element vibration, buckling, and dynamic stability analysis on a plate structure considered to be made by aluminum – potassium feldspar metal matrix composite material.

1.4 Thesis Outline

Chapter 2 presents the theoretical information about the free vibration, buckling, and dynamic stability analysis of curved frame structures, using the finite element method.

Chapter 3 gives the numerical results of the free vibration and buckling analysis of the isotropic and laminated composite curved frame structures considering various radius of curvatures and two thickness values. Besides, the results of the dynamic stability analysis of the laminated composite curved frame structures are presented. The effects of the stacking order of the laminated composite structures on the natural frequencies, critical buckling load, and unstable regions are investigated.

Chapter 4 presents the theoretical information about the free vibration, buckling, and dynamic stability analysis of the curved plate frame structures.

Chapter 5 gives the numerical results of the free vibration and buckling analysis of the isotropic and laminated composite curved thin plate frame structures considering various radius of curvatures and aspect ratios. Besides, the dynamic stability results of the laminated composite curved thin plate frame structures are presented. In addition to the curved thin plate structures, the free vibration analysis results of the laminated composite parabolic (curved) thick plate frame structures with different radius of

curvatures, aspect ratios, and two boundary conditions are presented. The effects of the stacking order of the laminated composite structures on the natural frequencies, critical buckling load, and unstable regions are investigated.

Chapter 6 presents the manufacturing process and material properties of the novel aluminum – potassium (K) feldspar metal matrix composite materials. The effect of the amount of the reinforcement on the material properties of such composites is presented. Besides, the free vibration, buckling, and dynamic stability analyses of the aluminum – K-feldspar metal matrix composite plate structure are performed, using the finite element method. The mechanical properties of aluminum – K-feldspar are compared with those of the A356 alloy plate and the required mechanical properties of car manufacturers to understand the effects of K-feldspar on such composites.

Chapter 7 draws general conclusions and presents the future work of this thesis.

CHAPTER 2

FINITE ELEMENT ANALYSIS OF CURVED FRAMES

2.1 Introduction

Numerous engineering structures take place in many fields, such as construction, automotive, naval industry, aerospace, etc. Curved frame structures, shown in Figure 2.1, have an important role in those industries. It is significant to determine the natural frequency values, critical buckling load, and the unstable regions of such structure in case of failure due to resonance, buckling, and parametric resonance. Therefore, this chapter presents the methods and relevant theories to perform vibration, buckling, and dynamic stability analysis of curved frame structures.

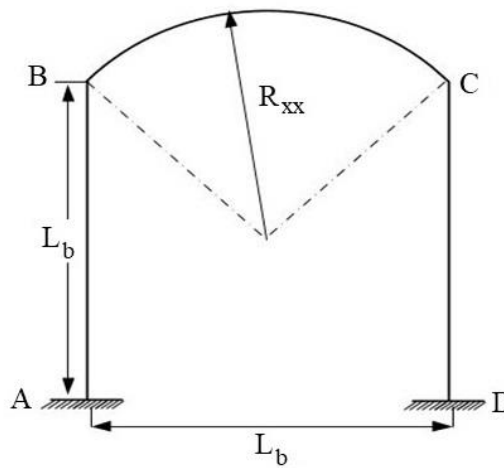


Figure 2.1 Curved (parabolic) frame structure

2.2 Finite Element Analysis of Curved Frame Structures

The finite element analysis of curved frame structures is conducted by using a two-noded frame element, shown in Figure 2.2, which has 3 degrees of freedom to satisfy the degrees of freedom of the curved frame structure. Since only thin frames are considered in this chapter, the stress and strain assumptions are made in accordance with the Euler-Bernoulli Beam Theory (Bauchau & Craig, 2009). However, for orthotropic materials, the Classical Plate Theory (Petyt, 2010) is utilized to obtain the

effective flexural modulus of elasticity to form the problem as Euler-Bernoulli Beam Theory. Equation 2.1 gives the displacement functions for the frame element.

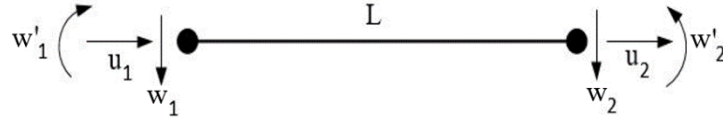


Figure 2.2 The frame element

$$\begin{aligned} u &= a_1 + a_2 x \\ w &= a_3 + a_4 x + a_5 x^2 + a_6 x^3 \\ w' &= \frac{dw}{dx} \end{aligned} \quad (2.1)$$

where u is the longitudinal displacement represented with a first-order polynomial equation while the bending displacement v , is defined as a cubic function in order to satisfy the rotation, w' .

The generalized displacement vector ($\mathbf{q}$) for the frame element is given in Equation 2.2.

$$\mathbf{q} = [u_1 \quad w_1 \quad w'_1 \quad u_2 \quad w_2 \quad w'_2] \quad (2.2)$$

The strain and kinetic energy equations (Petyt, 2010) are given in Equations 2.3 and 2.4, respectively.

$$U_e = \frac{1}{2} EI \int_0^L \frac{\partial^2 w}{\partial x^2} dx + \frac{1}{2} EA \int_0^L \frac{\partial u}{\partial x} dx \quad (2.3)$$

$$T_e = \frac{1}{2} \int_0^L \rho A (\dot{u}^2 + \dot{w}^2) dx \quad (2.4)$$

where E is the effective flexural modulus of elasticity, ρ denotes the density of the material, I and A is the second moment of inertia and the cross-sectional area of the frame element, respectively.

If the material is orthotropic, then Classical Plate Theory (CPT) should be employed to obtain the effective flexural modulus of elasticity. The moment equation for unit length is evaluated by considering plane strain and in-plane forces (Reddy, 2003), in CPT. Therefore, the moment equation is,

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (2.5)$$

and,

$$\begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = \begin{bmatrix} C_{11}^* & C_{12}^* & C_{16}^* \\ C_{12}^* & C_{22}^* & C_{26}^* \\ C_{16}^* & C_{26}^* & C_{66}^* \end{bmatrix} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \quad (2.6)$$

where M_{xx} , M_{yy} , and M_{xy} represent the moment components, C_{ij}^* denotes the inverse of the C_{ij} , which is the components of the bending stiffness matrix. C_{ij} components are given by

$$C_{ij} = \frac{1}{3} \sum_{k=1}^{NL} Q_{ij}^k (z_{k+1}^3 - z_k^3), \quad i, j = 1, 2, 6 \quad (2.7)$$

where NL is the number of laminates, z is the variable of coordinate of the kth laminate. Q_{ij} are the stiffness values given in Equation 2.8.

$$\begin{aligned}
Q_{11} &= q_{11}c^4 + 2(q_{12} + 2q_{66})s^2c^2 + q_{22}s^4 \\
Q_{12} &= (q_{11} + q_{22} - 4q_{66})s^2c^2 + q_{12}(s^4 + c^4) \\
Q_{22} &= q_{11}s^4 + 2(q_{12} + 2q_{66})s^2c^2 + q_{22}c^4 \\
Q_{16} &= (q_{11} - q_{12} - 2q_{66})sc^3 + (q_{12} - q_{22} + 2q_{66})s^3c \\
Q_{26} &= (q_{11} - q_{12} - 2q_{66})s^3c + (q_{12} - q_{22} + 2q_{66})sc^3 \\
Q_{66} &= (q_{11} + q_{22} - 2q_{12} - 2q_{66})s^2c^2 + q_{66}(s^4 + c^4)
\end{aligned} \tag{2.8}$$

where q_{ij} (i,j=1, 2, and 6) is given in Equation 2.9, c and s denote cosθ and sinθ, respectively. θ represents the fiber angle as shown in Figure 2.3.

$$\begin{aligned}
q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\
q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \\
q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\
q_{66} &= G_{12} \\
E_1\nu_{21} &= E_2\nu_{12}
\end{aligned} \tag{2.9}$$

where G₁₂ is the shear modulus with respect to x- and y- directions, ν₁₂ and ν₂₁ are the Poison ratio with respect to x-, y- and y-, x- directions, respectively. E₁ and E₂ denote the modulus of elasticity in x- and y- directions. (Petyt, 2010)

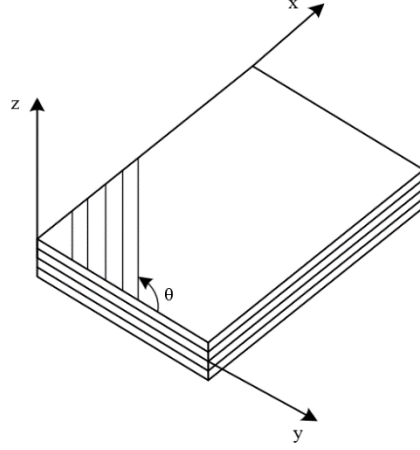


Figure 2.3 Laminated composite material

It is assumed that $M_{yy}=M_{xy}=0$ for laminated beam theory. Hence, Equation 2.5 becomes,

$$\frac{\partial^2 w}{\partial x^2} = C_{11}^* M_{xx} \quad (2.10)$$

Equation 2.10 can be written in the form that is well-known in Euler-Beam Theory as

$$M = hM_{xx}, \quad M_{xx} = E_{xx}I \frac{\partial^2 w}{\partial x^2}, \quad E_{xx} = E = \frac{12}{b^3 C_{11}^*} \quad (2.11)$$

where h and b are the thickness and width of the curved frame structure. If the material is isotropic, the effective modulus of elasticity is equal to the modulus of the elasticity of the isotropic material itself. Once the effective flexural modulus of elasticity is obtained, it can be substituted into Equations 2.3 and 2.4 can be written in matrix form as

$$\begin{aligned} U_e &= \frac{1}{2} \mathbf{q}^T \mathbf{k}_e \mathbf{q} \\ T_e &= \frac{1}{2} \mathbf{q}^T \mathbf{m}_e \mathbf{q} \end{aligned} \quad (2.12)$$

where $\mathbf{k}_e$ and $\mathbf{m}_e$ are the element elastic stiffness and element mass matrices given in Equations 2.13 and 2.19, respectively.

$$\mathbf{k}_e = \begin{bmatrix} \mathbf{k}_{be} & 0 \\ 0 & \mathbf{k}_{me} \end{bmatrix}_{6 \times 6} \quad (2.13)$$

where $\mathbf{k}_{be}$ and $\mathbf{k}_{me}$, given in Equation 2.14, are the flexural and membrane stiffness matrices, respectively.

$$\begin{aligned} \mathbf{k}_{be} &= \frac{1}{2} EI \int_0^L [\mathbf{N}_{vxx}]^T [\mathbf{N}_{vxx}] dx \\ \mathbf{k}_{me} &= \frac{1}{2} EA \int_0^L [\mathbf{N}_{ux}]^T [\mathbf{N}_{ux}] dx \end{aligned} \quad (2.14)$$

where

$$[\mathbf{N}_{vxx}]^T = \frac{d^2 \mathbf{N}_v}{dx^2}, \quad [\mathbf{N}_{ux}]^T = \frac{d \mathbf{N}_u}{dx} \quad (2.15)$$

where $\mathbf{N}_v$ and $\mathbf{N}_u$ are the shape function matrix representing the flexural and longitudinal displacements of the frame element, respectively. Therefore,

$$\mathbf{N}_v = \{w_k\}[\mathbf{V}]^{-1}, \quad \mathbf{N}_u = \{u_k\}[\mathbf{U}]^{-1} \quad (2.16)$$

where

$$\begin{aligned} [\mathbf{V}]^T &= [w(0) \ w'(0) \ w(x) \ w'(x)] \\ [\mathbf{U}]^T &= [u(0) \ u(x)] \end{aligned} \quad (2.17)$$

and,

$$\begin{aligned}\{w_k\}^T &= [1 \ x \ x^2 \ x^3] \\ \{u_k\}^T &= [1 \ x]\end{aligned}\tag{2.18}$$

Considering element mass matrix,

$$\mathbf{m}_e = \begin{bmatrix} \mathbf{m}_{be} & 0 \\ 0 & \mathbf{m}_{me} \end{bmatrix}_{6 \times 6}\tag{2.19}$$

where $\mathbf{m}_{be}$ and $\mathbf{m}_{me}$, given in Equation 2.20, are the element mass matrices for the flexural and membrane displacements of the frame element, respectively.

$$\begin{aligned}\mathbf{m}_{be} &= \frac{1}{2} \rho A \int_0^L \mathbf{N}_v^T \mathbf{N}_v dx \\ \mathbf{m}_{me} &= \frac{1}{2} \rho A \int_0^L \mathbf{N}_u^T \mathbf{N}_u dx\end{aligned}\tag{2.20}$$

The work done by the applied load (Goren Kiral et al., 2015) is given in Equation 2.21. The loading case of curved frames is shown in Figure 2.4(a). The distributed loading is first written in terms of single point load (P), as is shown in Figure 2.4(b). Afterward, the single point load is divided into two-point loads applied at the edges of the curved frame structure, as is shown in Figure 2.4(c). A similar application is performed for two-bay structure, as shown in Figures

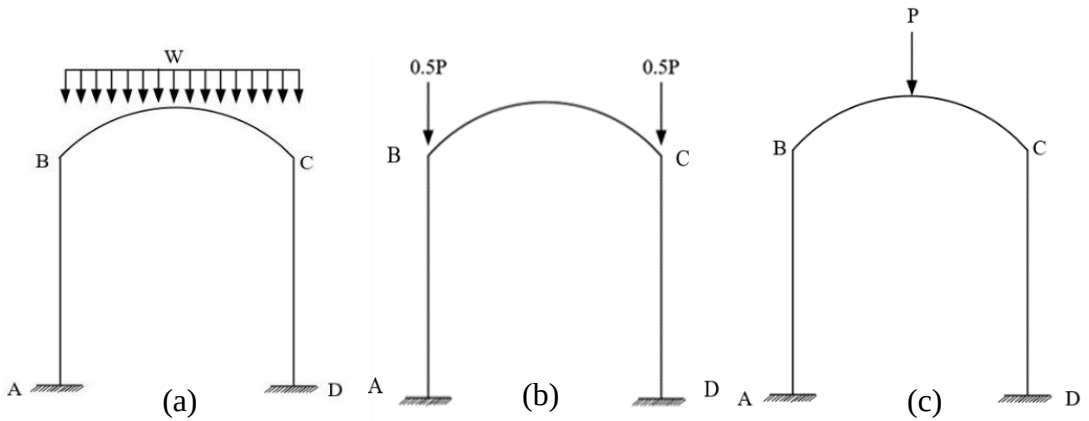


Figure 2.4 Distributed loading case (a), equivalent two-point loading (b), equivalent single-point loading (c) for the single-bay curved beam frame structure

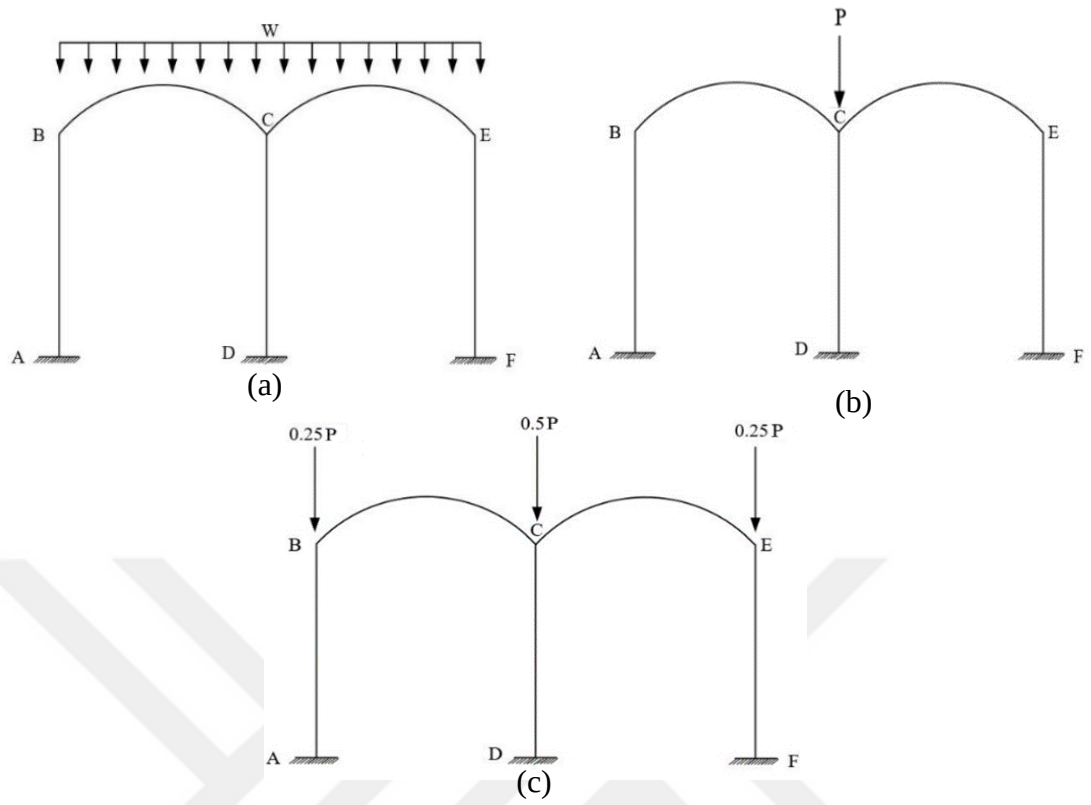


Figure 2.5 Distributed loading case (a), equivalent single-point loading (b), equivalent two-point loading (c) for the two-bay curved beam frame structure

$$V_e = \frac{1}{2} W \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \quad (2.21)$$

The distributed load (W) applied can be written in terms of point load (P) as is given in Equation 2.22.

$$P = W b_n L_b \quad (2.22)$$

where b_n is the number of bays. The matrix form of the work done is given in Equation 2.23.

$$V_e = \frac{1}{2} \mathbf{q}^T \mathbf{k}_{ge} \mathbf{q} \quad (2.23)$$

where $\mathbf{k}_{ge}$ is the element geometric stiffness matrix, which is given as

$$\mathbf{k}_{ge} = \begin{bmatrix} \mathbf{k}_{bge} & 0 \\ 0 & \mathbf{k}_{mge} \end{bmatrix}_{6 \times 6} \quad (2.24)$$

where $\mathbf{k}_{bge}$ and $\mathbf{k}_{mge}$ are the components of the element geometric stiffness matrix flexural and longitudinal. The longitudinal component, $\mathbf{k}_{mge}$, is set to zero since the equation of work done is a function, including flexural displacement only. Therefore,

$$\mathbf{k}_{bge} = \frac{1}{2} P \int_0^L [N_{vx}]^T [N_{vx}] dx \quad (2.25)$$

$$\mathbf{k}_{mge} = [0]_{2 \times 2}$$

where

$$[N_{vx}]^T = \frac{dN_v}{dx}, \quad (2.26)$$

Once all element matrices are obtained, they are needed to be transformed in terms of reference coordinates as it is given in Figure 2.6 by using Equation 2.27 since the curvature and the structure itself is composed of straight beams.

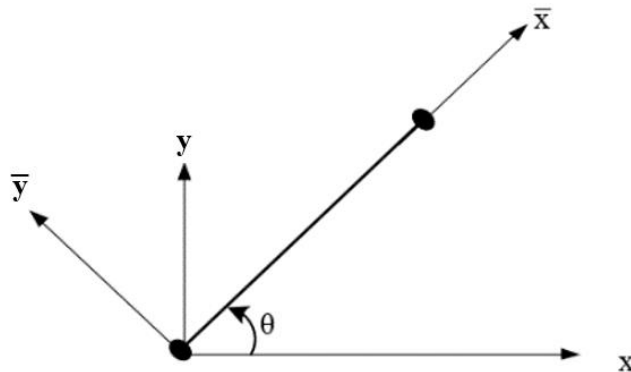


Figure 2.6 Transformation of local coordinates

$$\begin{aligned}
\bar{k}_e &= T^T k_e T \\
\bar{m}_e &= T^T m_e T \\
\bar{k}_{ge} &= T^T k_{ge} T
\end{aligned} \tag{2.27}$$

where $\mathbf{T}$ denotes the transformation matrix as is given in Equation 2.28.

$$\mathbf{T} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.28}$$

The curved frame structure's dynamic response can be obtained by using Lagrange's equation of motion, as is given in Equation 2.29.

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{K} + P_{cr}\mathbf{K}_g)\mathbf{q} = \mathbf{0} \tag{2.29}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{K}_g$ are the mass, elastic stiffness, and geometric stiffness matrix of the curved frame structure. P_{cr} is the critical buckling load of curved frame structure.

2.2.1 Free Vibration Analysis of Curved Frame Structures

Equation 2.30 gives the eigenvalue problem of the free vibration problem to obtain the natural frequencies of the structure.

$$(\mathbf{K} - \lambda\mathbf{M})\mathbf{q} = \mathbf{0} \tag{2.30}$$

2.2.2 Buckling Analysis of Curved Frame Structures

Equation 2.31 gives the eigenvalue buckling problem to obtain the critical buckling load of the structure.

$$(\mathbf{K} - P_{cr}\mathbf{K}_g)\mathbf{q} = \mathbf{0} \tag{2.31}$$

2.2.3 Dynamic Stability Analysis of Curved Frame Structures

Dynamic instability may occur by means of the parametric resonance due to periodic dynamic loadings. The main goal of the study is to determine the regions of stability and dynamic instability. Such a problem can be solved by considering it as an eigenvalue problem.

If the laminated composite curved frame is subjected to periodically distributed load, $W(t)$, instead of a static load, as shown in Figures 2.4(a) and 2.5(a). Such loading can be written in terms of point compressive periodic load as $P(t)=P+P_t\cos\lambda t$ instead of P as shown in Figures 2.4(b)-(c) and 2.5(b)-(c) where λ is the excitation frequency, P and P_t are the amplitudes of the static and time-dependent part of the periodic compressive load, respectively. Those two force components of the compressive load can be written in terms of the fraction of the static buckling load P_{cr} as, (Goren Kiral et al., 2015)

$$P(t) = \alpha P_{cr} + \beta P_{cr} \cos \lambda t, \quad \alpha = \frac{P}{P_{cr}}, \quad \beta = \frac{P_t}{P_{cr}} \quad (2.31)$$

where α and β are static and dynamic load factors. Substituting Equation 2.31 into the Lagrange's equation of motion given in Equation 2.29 gives,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} - P(t)\mathbf{K}_g\mathbf{q} = 0 \quad (2.32)$$

Substituting Equation 2.31 into Equation 2.32 gives,

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{K} - \alpha P_{cr}\mathbf{K}_{gst} - \beta P_{cr}\cos\lambda t\mathbf{K}_{gt})\mathbf{q} = 0 \quad (2.33)$$

$\mathbf{K}_{gst}$ and $\mathbf{K}_{gt}$ represent the influence of the static and time-dependent force components, P and P_t , respectively. Equation 2.33 reflects a system of a second-order differential equation, including periodic components of the Mathieu-Hill type.

Since $\mathbf{K}_{gst}$ and $\mathbf{K}_{gt}$ are identical when the static and time-varying components of the subjected compressive load are applied in the same manner, Equation 2.33 can be written as,

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{K} - P_{cr}(\alpha + \beta \cos \lambda t)\mathbf{K}_g)\mathbf{q} = 0 \quad (2.34)$$

The theory of linear equations, including periodic coefficients, indicate that the boundaries between unstable and stable solutions of Equation 2.34 are composed of periodic solutions where the periods $T=2\pi/\lambda$ and $2T= \pi/\lambda$. Bolotin (1964) indicated that considering $2T$ as period provides significant importance since the unstable regions' widths are usually larger than those solved with period T . Hence, performing a periodic solution considering the period as $2T$ on Equation 2.34 gives the eigenvalue problem of dynamic stability analysis as

$$\left[\mathbf{K} - P_{cr}(\alpha \mp 0.5\beta)\mathbf{K}_g - \frac{\lambda^2}{4}\mathbf{M} \right] \mathbf{q} = 0 \quad (2.35)$$

The dynamic stability analysis investigates the effect of α and β coefficients on the dynamic stability boundaries.

CHAPTER 3

NUMERICAL ANALYSIS OF CURVED FRAME STRUCTURES

3.1 Introduction

This section presents the free vibration, buckling, and dynamic stability analysis of single-bay and two-bay curved (parabolic) frame structures by using the finite element method. The finite element method given in section 2.2 is employed in order to perform free vibration, buckling, and dynamic stability analysis of the isotropic and laminated composite single-bay and two-bay curved frame structures as seen in Figure 2.1. The effects of the radius of curvature and thickness on the dynamic behavior of the structure are investigated. Computer code is developed to perform finite element analysis in MATLAB.

The numerical results are compared with the CAD model of the structure by using SolidWorks and ANSYS for various cases.

3.2 Numerical Results of the Finite Element Analysis of Isotropic Curved Frames

The finite element free vibration and buckling analyses are performed by considering aluminum whose material and geometric properties given in Table 3.1.

Table 3.1 Material and geometric properties of the isotropic curved frames

Property	Symbol	Quantity
Elastic Modulus	E	69 GPa
Density	ρ	2700 kg/m ³
Poisson Ratio	ν	0.33
Geometric Properties of the Structure		
Beam Length	L_b	1000 mm
Cross-section	h	$L_b/100$
	b	$L_b/100$

3.2.1 Free Vibration Analysis of Isotropic Curved Frames

In this section, the free vibration analysis of curved beam frames is presented. The first five natural frequency values of the curved beam frame structures are evaluated under fixed from two ends boundary conditions. The effect of the radius of curvature of single – bay and two – bay curved beam frame structure and the effect of the thickness of the single-bay curved beam frame structure on the natural frequency values is investigated.

3.2.1.1 Effect of the Radius of Curvature

The effect of radius of curvature (R_{xx}) on the natural frequency values of the single – bay and two – bay curved beam frames is presented. Figure 3.1 shows the four different radii of curvatures and straight frame structures investigated within this study.

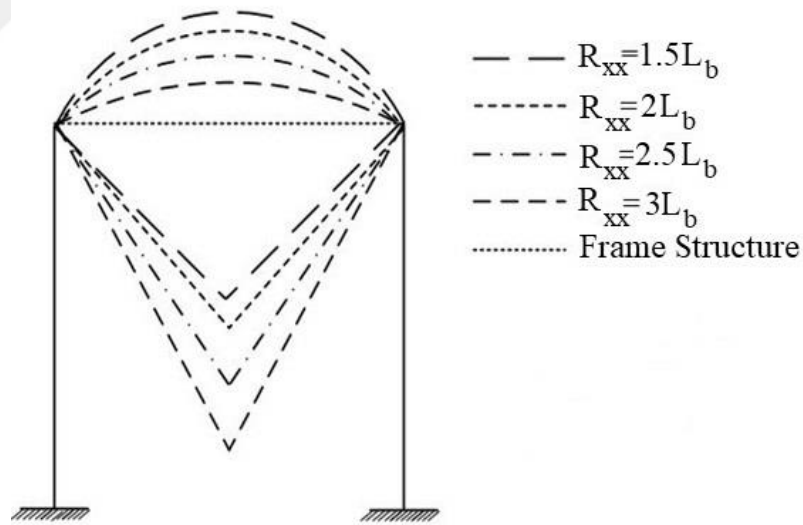


Figure 3.1 Single-bay curved beam frame structure with different curvatures

Comparative results of the free vibration analysis of the single-bay curved frame structure for various curvature values are given in Tables 3.2 and 3.3.

Table 3.2 Free vibration analysis results of the single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	SolidWorks	Present Study	Error (%)	SolidWorks	Present Study	Error (%)
λ_1	8.08	8.28	2.49%	8.13	8.29	1.86%
λ_2	30.49	31.87	4.49%	31.09	32.05	3.08%
λ_3	50.98	52.82	3.61%	51.56	53.02	2.82%
λ_4	61.34	60.22	1.84%	60.54	59.61	1.53%
λ_5	112.14	115.46	2.96%	113.54	115.66	1.87%

Table 3.3 Free vibration analysis results of the single-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	SolidWorks	Present Study	Error (%)	SolidWorks	Present Study	Error (%)
λ_1	8.16	8.29	1.61%	8.17	8.30	1.49%
λ_2	31.43	32.16	2.30%	31.64	32.23	1.87%
λ_3	51.88	53.14	2.41%	52.07	53.22	2.19%
λ_4	60.05	59.26	1.33%	59.68	59.03	1.09%
λ_5	114.23	115.79	1.37%	114.62	115.87	1.09%

The first five natural frequency values for four different curvatures are given in Tables 3.2 and 3.3. It is concluded that the present study is in very good agreement with SolidWorks results. As the curvature increases, the first three and the fifth natural frequency increase, whereas the fourth natural frequency decreases. All mode shapes are the same for all curvatures and given in Figure 3.2.

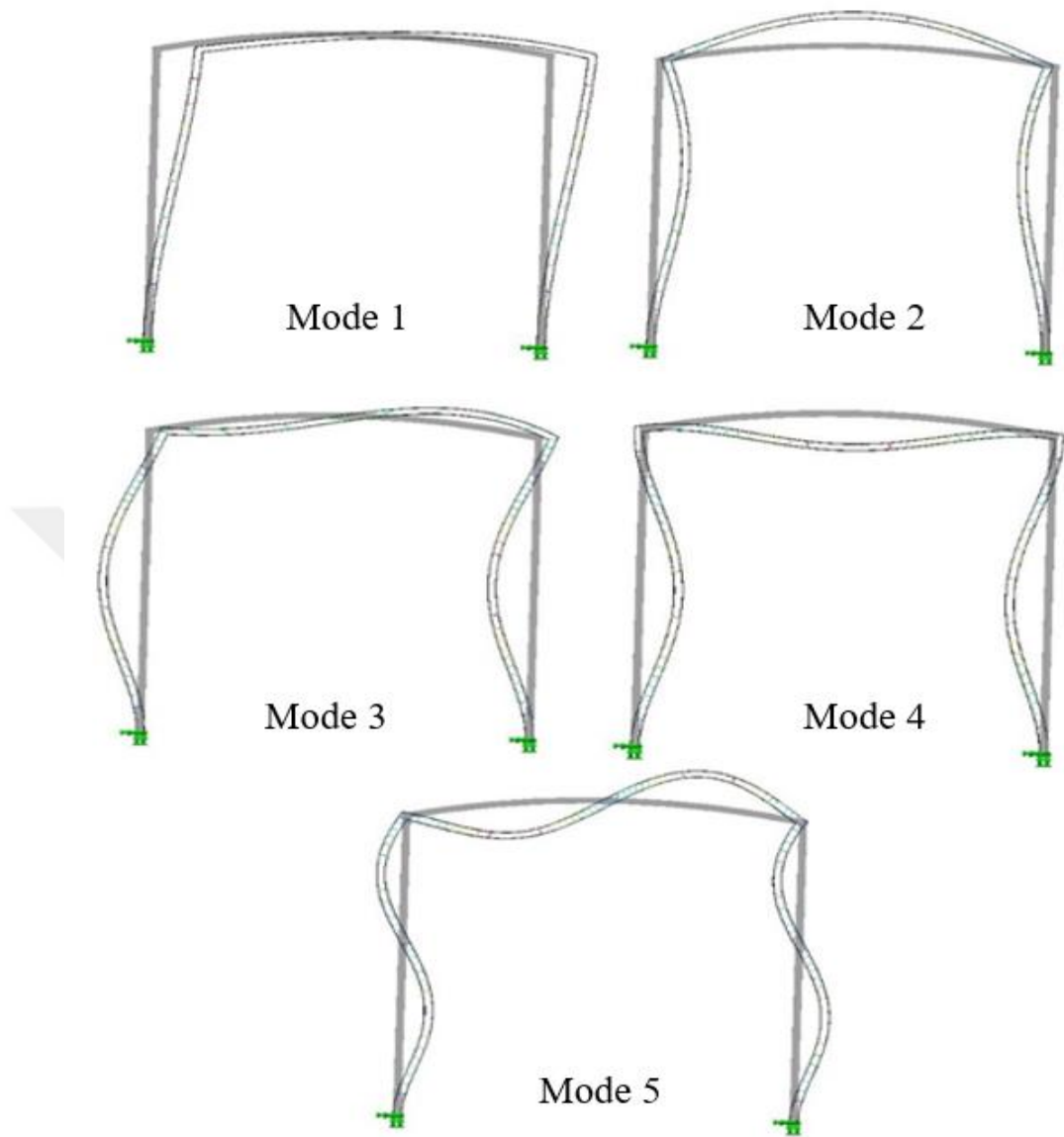


Figure 3.2 Mode shapes of the single-bay isotropic curved beam frame structure

Comparative results of the free vibration analysis of the straight frame structure ($R_{xx}=\infty$) for various curvature values are given in Table 3.4. It is seen that the first three and fifth natural frequencies are at the highest values when $R_{xx}=\infty$ (flat frame) as seen from Tables 3.2-3.5. In contrast, the fourth natural frequency is at the lowest value for the flat frame structure.

Table 3.4 Free vibration analysis results of the straight frame structure (N.F.: natural frequency values, SW: SolidWorks Results, PS: present study results, Err: error)

Natural Frequency (Hz)	Straight Frame Structure ($R_{xx}=\infty$)		
	SolidWorks	Present Study	Error (%)
λ_1	8.24	8.13	1.21%
λ_2	32.59	32.13	1.03%
λ_3	52.98	52.25	1.37%
λ_4	57.76	56.88	0.81%
λ_5	116.40	114.80	1.04%

Comparative results of free vibration analysis of the two-bay curved (parabolic) frame structure, shown in Figure 3.3, for four different curvature values are given in Tables 3.5 and 3.6.

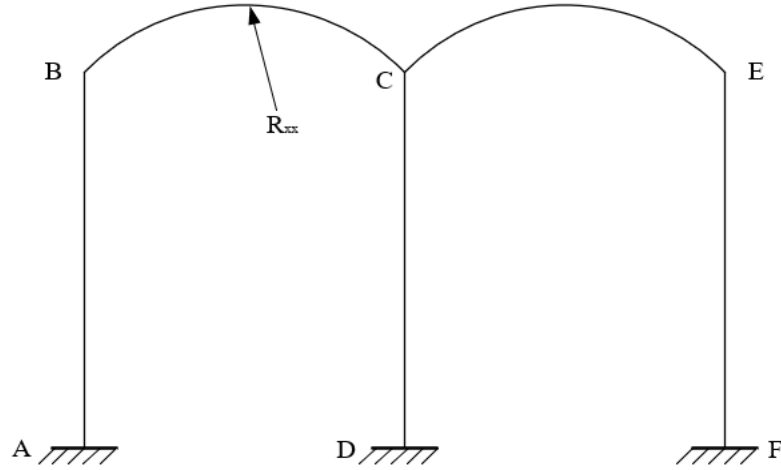


Figure 3.3 Two-bay curved beam frame structure

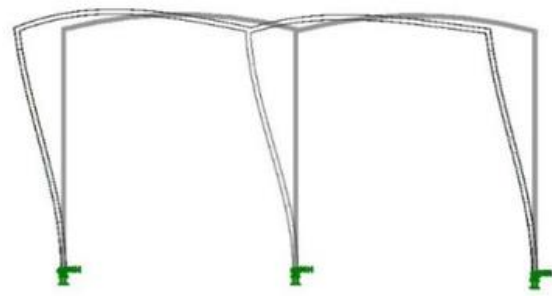
Table 3.5 Free vibration analysis results of the two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	SolidWorks	Present Study	Error (%)	SolidWorks	Present Study	Error (%)
λ_1	7.42	7.66	3.29%	7.47	7.67	2.71%
λ_2	29.48	31.01	5.19%	29.97	31.12	3.86%
λ_3	34.45	37.73	9.52%	35.74	38.26	7.04%
λ_4	51.13	53.29	4.22%	51.67	53.46	3.48%
λ_5	59.98	59.77	0.34%	59.37	59.30	0.12%

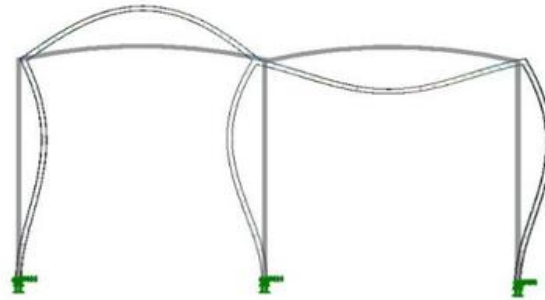
Table 3.6 Free vibration analysis results of the two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=2.5L_b$			$R_{xx}=3L_b$		
	SolidWorks	Present Study	Error (%)	SolidWorks	Present Study	Error (%)
λ_1	7.50	7.68	2.39%	7.51	7.68	2.29%
λ_2	30.23	31.19	3.20%	30.38	31.24	2.83%
λ_3	36.50	38.57	5.70%	36.97	38.78	4.90%
λ_4	51.93	53.57	3.15%	52.10	53.63	2.94%
λ_5	58.94	59.03	0.14%	58.65	58.85	0.34%

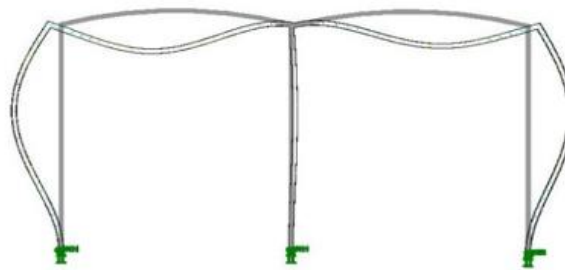
As seen in Tables 3.5 and 3.6, the agreement between the presented finite element models and SolidWorks results is close. Apart from the single-bay results, the fifth natural frequency value decreases as the curvature increases. This is because the fifth mode of the two-bay frame structure is similar to the fourth mode of the single-bay frame structure. The mode shapes of the two-bay curved beam frame structure are shown in Figure 3.4. Compared with the mode shapes of the single – bay frame structure, only the fifth mode of the two-bay curved frame structure is different because of the different geometries of those structures.



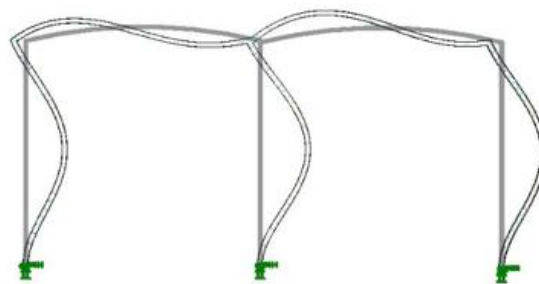
Mode 1



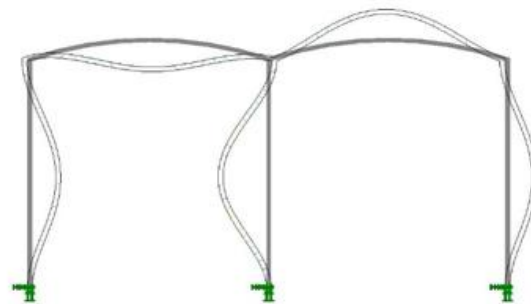
Mode 2



Mode 3



Mode 4



Mode 5

Figure 3.4 Mode shapes of two-bay curved beam frame

3.2.1.2 Effect of the Thickness

For the single-bay curved beam frame structure, the effect of the thickness (h) on dynamic behavior is investigated. Two different thickness parameters, $h_1=a/100$ and $h_2=a/50$, are selected. The curvature of the curved beam frame structure is set as $R_{xx}=2L_b$.

Table 3.7 shows the comparative results of the free vibration analysis of the single-bay curved beam frame structure with two different thickness parameters.

Table 3.7 Free vibration analysis results of the single-bay curved frame structure for $h_1=L_b/100$, $h_2=L_b/50$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$h_1=L_b/100$			$h_2=L_b/50$		
	SolidWorks	Present Study	Error (%)	SolidWorks	Present Study	Error (%)
λ_1	8.13	8.29	1.86%	16.21	16.57	2.21%
λ_2	31.09	32.05	3.08%	62.15	64.06	3.07%
λ_3	51.56	53.02	2.82%	102.59	106.03	3.35%
λ_4	60.54	59.61	1.53%	120.73	119.07	1.37%
λ_5	113.54	115.66	1.87%	226.16	230.81	2.06%

As seen from Table 3.7, two different thickness (h) parameter is compared each other in terms of natural frequency. The natural frequency values increase or decrease as the same as the thickness ratio as expected.

3.2.2 Buckling Analysis of Isotropic Curved Frames

Buckling analysis of curved beam frames is performed for the distributed loading case, as shown in Figure 2.4(a). For simplicity, the distributed load is first considered as a point load as seen in Figure 2.4(b) then the point load is divided into two-point loads that applied on the columns of the curved beam frame structure, as shown in Figure 2.4(c).

Similar to the single-bay curved beam structure, the distributed loading case is considered as point loads for the two-bay curved beam structure. The distributed load,

which is presented in Figure 2.5(a), is firstly redefined as point load, as is shown in Figure 2.5(b). Unlike the single-bay structure, three-point loads, as shown in Figure 2.5(c), are taken into account to represent the distributed loading condition for a two-bay curved beam frame structure.

3.2.2.1 Effect of the Radius of Curvature

The effect of the radius of curvature (R_{xx}) of the single-bay and two-bay curved beam frame structure on the critical buckling load is investigated. The same curvature values of section 3.2.2.1 are considered.

Table 3.8 presents the buckling analysis results of single-bay and two-bay curved beam frame structure for four different radii of curvature values.

Table 3.8 Buckling analysis results of the single-bay and two-bay curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame Critical Buckling Load (N)		
	SolidWorks	Present Study	Error (%)		SolidWorks	Present Study	Error (%)
$1.5L_b$	945.49	948.50	0.32%	$1.5L_b$	1429.1	1453.7	1.72%
$2L_b$	951.26	948.90	0.25%	$2L_b$	1439.8	1455.3	1.08%
$2.5L_b$	954.54	949.08	0.57%	$2.5L_b$	1437.2	1456.1	1.32%
$3L_b$	956.4	949.17	0.76%	$3L_b$	1448.3	1456.7	0.58%

As seen in Table 3.8, the results of the present study are in good agreement with SolidWorks results. The average error is 0.83%. For both single-bay and two-bay curved beam frame structure, the critical buckling load increases as the radius of curvature increases. However, this increment is relatively small since the change of the radius of curvature does not affect the geometric matrix considerably.

3.2.2.2 Effect of the Thickness

The effect of the thickness (h) on the critical buckling load is investigated for the single-bay curved ($R_{xx}=2L_b$) beam frame structure. Comparative results are given in Table 3.9.

Table 3.9 Buckling analysis results of the single-bay curved frame structure for $h=L_b/100$, $h=L_b/50$

Thickness (h)	Critical Buckling Load (N)		
	SolidWorks	Present Study	Error (%)
$L_b/100$	950.13	948.90	0.25%
$L_b/50$	15337	15180	1.02%

According to the results in Table 3.9, the critical buckling load increases as the thickness of the structure increases.

3.2.3 Conclusions

Section 3.2 covers free vibration and buckling analysis of the parabolic frame structure by using the finite element method. According to the results given in sections 3.2.2, and 3.2.3, it is concluded that

- The natural frequency values and critical buckling loads that are obtained via the finite element method by using a frame element are in very good agreement with SolidWorks results for both parabolic and straight frames.
- For the single-bay structure, all natural frequency values, except the fourth natural frequency value, increase as the radius of curvature increases. On the other hand, the fourth frequency value decreases.
- For the two-bay structure, all natural frequency values, except the fifth natural frequency value, increase as the radius of curvature increases. This situation is similar to the single-bay structure because the structural components behave in the same displacement characteristic in this mode.

- The critical buckling load increases when the radius of curvature increases for both single-bay and multi-bay parabolic frame structure.
- The equivalent loading approach that is used for straight frame structures in order to perform finite element analysis is also valid for parabolic frame structures under perpendicularly distributed loading.
- As is expected, the natural frequency and the critical buckling values increase as the thickness of the parabolic frame structure increase.

3.3 Numerical Results of the Finite Element Analysis of Laminated Composite Curved Frames

The finite element free vibration, buckling, dynamic stability analyses are performed by using the material and geometric properties given in Table 3.10 (Gay et al., 2003). Five different stacking orders are denoted C1, C2, C3, C4, and C5 for simplicity. Those stacking orders are presented as follows.

$$C1 = [0 \ 0 \ 0 \ 0], \ C2 = [90 \ 0 \ 0 \ 90], \ C3 = [0 \ 90 \ 90 \ 0], \\ C4 = [0 \ 45 \ -45 \ 0], \ \text{and} \ C5 = [0 \ 60 \ -60 \ 0]$$

where C2 and C3 are denoted as cross-ply laminates whereas C4 and C5 are called angle-ply laminates.

Table 3.10 Material properties and geometry of the structure of laminated composite curved frames

Property	Symbol	Quantity
Longitudinal Elasticity Modulus	E_x	45 GPa
Transverse Elasticity Modulus	E_y	12 GPa
Density	ρ	2080 kg/m ³
Poisson Ratio	ν	0.33
Geometric Properties of the Structure		
Beam Length	L_b	1000 mm
Cross-section	h	$L_b/100$
	b	$L_b/100$

3.3.1 Free Vibration Analysis of Laminated Composite Curved Frames

3.3.1.1 Effect of the Radius of Curvature

Finite element vibration analysis is performed for laminated composite curved beam structures considering four different radii of curvature (R_{xx}) values, as shown in Figure 3.1.

Tables 3.11 and 3.12 give the first five natural frequency values of the single-bay curved frame structure for C1 stacking order.

Table 3.11 Free vibration analysis results of C1 laminated composite single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.736	6.952	3.20%	6.777	6.986	3.09%
λ_2	25.400	26.427	4.04%	25.872	26.830	3.70%
λ_3	42.698	44.445	4.09%	43.120	44.737	3.75%
λ_4	51.068	50.324	1.46%	50.396	50.121	0.55%
λ_5	93.742	95.571	1.95%	94.782	96.738	2.06%

Table 3.12 Free vibration analysis results of C1 laminated composite single-bay laminated composite curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.797	7.002	3.02%	6.809	7.012	2.97%
λ_2	26.140	27.039	3.44%	26.311	27.164	3.24%
λ_3	43.357	44.890	3.54%	43.507	44.984	3.39%
λ_4	49.950	49.960	0.02%	49.636	49.837	0.40%
λ_5	95.344	97.305	2.06%	95.689	97.630	2.03%

According to the results given in Tables 3.11 and 3.12, the results of the present study are in close agreement with those obtained from ANSYS. The first three and the fifth natural frequency values increase slightly as the radius of curvature of the laminate composite curved frame structure increases. In contrast, the fourth natural frequency decreases as the radius of the curvature of the structure increases.

Tables 3.13 and 3.14 present the first five natural frequency values of single-bay curved frame structure for C2 stacking order.

Table 3.13 Free vibration analysis results of C2 laminated composite single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.042	4.171	3.20%	4.066	4.191	3.09%
λ_2	15.239	15.854	4.04%	15.522	16.097	3.70%
λ_3	25.616	26.664	4.09%	25.870	26.840	3.75%
λ_4	30.638	30.192	1.46%	30.235	30.070	0.55%
λ_5	56.240	57.337	1.95%	56.864	58.037	2.06%

Table 3.14 Free vibration analysis results of C2 laminated composite single-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.078	4.201	3.02%	4.085	4.207	2.97%
λ_2	15.682	16.222	3.44%	15.785	16.297	3.24%
λ_3	26.012	26.931	3.53%	26.102	26.987	3.39%
λ_4	29.967	29.973	0.02%	29.779	29.899	0.40%
λ_5	57.201	58.377	2.06%	57.408	58.572	2.03%

It is seen from Tables 3.13 and 3.14 that the results of the present study are in close agreement with ANSYS results. Similar to the change in natural frequency values of laminated composite curved frame structure with C1 stacking order, the first three and fifth natural frequency values increase, whereas the fourth natural frequency decreases as the curvature of the structure increases.

Tables 3.15 and 3.16 give the first five natural frequency results of the single-bay laminated composite curved frame structure having C3 stacking order.

Table 3.15 Free vibration analysis results of C3 laminated composite single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.435	6.640	3.20%	6.473	6.673	3.09%
λ_2	24.262	25.242	4.04%	24.712	25.628	3.71%
λ_3	40.784	42.453	4.09%	41.188	42.732	3.75%
λ_4	48.779	48.069	1.46%	48.138	47.875	0.55%
λ_5	89.541	91.288	1.95%	90.535	92.402	2.06%

Table 3.16 Free vibration analysis results of C3 laminated composite single-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.493	6.689	3.02%	6.504	6.697	2.97%
λ_2	24.968	25.827	3.44%	25.132	25.947	3.24%
λ_3	41.414	42.878	3.54%	41.557	42.967	3.39%
λ_4	47.711	47.721	0.02%	47.411	47.603	0.40%
λ_5	91.071	92.944	2.06%	91.401	93.254	2.03%

According to the results given in Tables 3.15 and 3.16, the results of the present study are close to ANSYS results. The first three and fifth natural frequency values increase, whereas the fourth natural frequency decreases as the radii of curvature of the frame structure increases.

Tables 3.17, 3.18, 3.19, and 3.20 give the first five natural frequency results of laminated composite curved frame structures for C4 and C5 stacking orders, respectively.

Table 3.17 Free vibration analysis results of C4 laminated composite single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.500	6.708	3.20%	6.539	6.741	3.09%
λ_2	24.509	25.499	4.04%	24.964	25.889	3.71%
λ_3	41.200	42.886	4.09%	41.607	43.168	3.75%
λ_4	49.276	48.559	1.46%	48.628	48.363	0.54%
λ_5	90.454	92.218	1.95%	91.457	93.344	2.06%

Table 3.18 Free vibration analysis results of C4 laminated composite single-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.559	6.757	3.02%	6.570	6.766	2.97%
λ_2	25.223	26.090	3.44%	25.388	26.211	3.24%
λ_3	41.836	43.315	3.54%	41.981	43.406	3.39%
λ_4	48.198	48.207	0.02%	47.895	48.088	0.40%
λ_5	91.999	93.891	2.06%	92.332	94.204	2.03%

Table 3.19 Free vibration analysis results of C5 laminated composite single-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.448	6.654	3.20%	6.487	6.687	3.09%
λ_2	24.313	25.295	4.04%	24.764	25.682	3.71%
λ_3	40.870	42.543	4.09%	41.274	42.823	3.75%
λ_4	48.882	48.170	1.46%	48.239	47.976	0.55%
λ_5	89.730	91.480	1.95%	90.725	92.597	2.06%

Table 3.20 Free vibration analysis results of C5 laminated composite single-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.506	6.703	3.02%	6.518	6.712	2.97%
λ_2	25.021	25.882	3.44%	25.185	26.002	3.24%
λ_3	41.501	42.969	3.54%	41.645	43.058	3.39%
λ_4	47.812	47.822	0.02%	47.511	47.704	0.41%
λ_5	91.262	93.140	2.06%	91.593	93.451	2.03%

It is seen from Tables 3.17, 3.18, 3.19, and 3.20 that the results of the present study are in close agreement with ANSYS results. For both laminated curved frame structures with C4 and C5 stacking orders, it is concluded that the first three and fifth natural frequency values increases when the radius of curvature of the laminated curved frame structure increase.

On the other hand, the fourth natural frequency decreases as the curvature of the structure increases. No matter which stacking order is, the mode shapes of the laminated composite single-bay curved frame structure are shown in Figure 3.5.

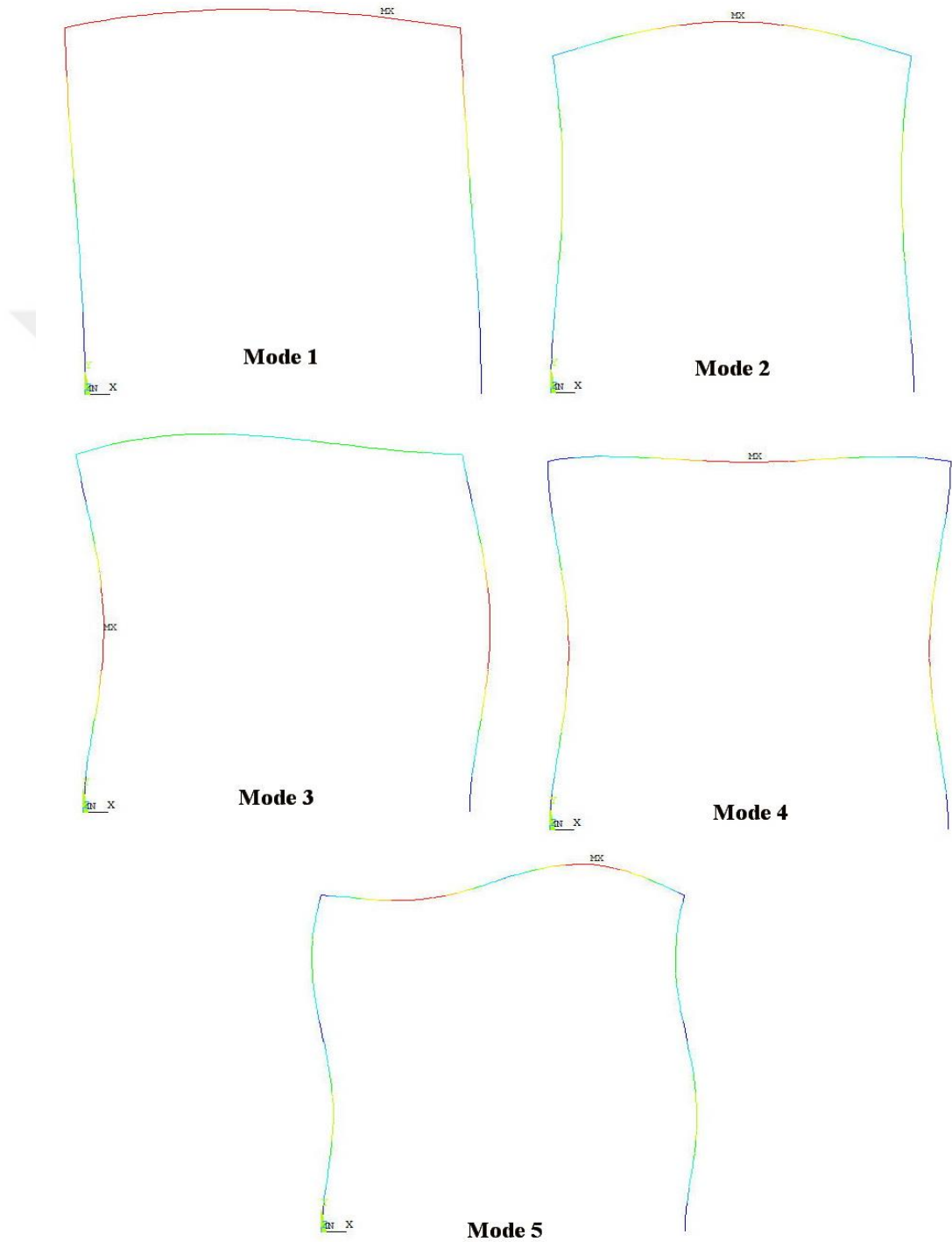


Figure 3.5 Mode shapes of the laminated composite single-bay curved beam frame

Tables 3.21 and 3.22 give the first five natural frequency results of the laminated curved two-bay frame structure as shown in Figure 3.3 considering C1 stacking order.

Table 3.21 Free vibration analysis results of C1 laminated composite two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.169	6.238	1.12%	6.197	6.276	1.27%
λ_2	25.823	24.875	3.81%	26.171	25.266	3.58%
λ_3	33.243	29.103	14.23%	33.773	30.157	11.99%
λ_4	41.510	43.160	3.82%	41.642	43.545	4.37%
λ_5	50.098	50.422	0.64%	50.073	49.865	0.42%

Table 3.22 Free vibration analysis results of C1 laminated composite two-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.210	6.296	1.37%	6.521	6.307	3.38%
λ_2	26.337	25.481	3.36%	26.505	25.616	3.47%
λ_3	34.001	30.768	10.51%	32.901	31.166	5.57%
λ_4	41.736	43.760	4.63%	45.499	43.895	3.65%
λ_5	49.935	49.503	0.87%	49.925	49.252	1.37%

According to the natural frequency results in Tables 3.21 and 3.22, the results of the present study are in good agreement with ANSYS results. The error rates are relatively high but diminish as the curvature of the laminated composite two-bay curved frame structure increases. The first four natural frequency values increase, while the fifth natural frequency value decreases as the curvature of the frame structure increases. Similar behaviors are seen for all stacking orders. The natural frequency results of the laminated composite two-bay curved frame structure with stacking orders C2, C3, C4, and C5 are given in Tables 3.23, 3.24, 3.25, 3.26, 3.27, 3.28, 3.29, and 3.30.

Table 3.23 Free vibration analysis results of C2 laminated composite two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.742	3.701	1.12%	3.765	3.718	1.27%
λ_2	14.923	15.492	3.81%	15.158	15.701	3.58%
λ_3	17.460	19.944	14.23%	18.093	20.262	11.99%
λ_4	25.894	24.904	3.82%	26.125	24.983	4.37%
λ_5	30.251	30.056	0.64%	29.916	30.041	0.42%

Table 3.24 Free vibration analysis results of C2 laminated composite two-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.777	3.725	1.37%	3.784	3.730	1.44%
λ_2	15.287	15.801	3.36%	15.368	15.857	3.18%
λ_3	18.459	20.399	10.51%	18.698	20.469	9.47%
λ_4	26.253	25.039	4.62%	26.335	25.080	4.77%
λ_5	29.699	29.958	0.87%	29.548	29.889	1.15%

Table 3.25 Free vibration analysis results of C3 laminated composite two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	5.959	5.892	1.12%	5.995	5.919	1.27%
λ_2	23.76	24.666	3.81%	24.133	24.998	3.58%
λ_3	27.799	31.753	14.22%	28.806	32.259	11.99%
λ_4	41.226	39.650	3.82%	41.594	39.776	4.37%
λ_5	48.163	47.853	0.64%	47.63	47.829	0.42%

Table 3.26 Free vibration analysis results of C3 laminated composite two-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.013	5.931	1.37%	6.024	6.228	3.38%
λ_2	24.339	25.157	3.36%	24.468	25.317	3.47%
λ_3	29.389	32.477	10.51%	29.769	31.427	5.57%
λ_4	41.798	39.865	4.62%	41.928	43.460	3.65%
λ_5	47.284	47.697	0.87%	47.044	47.688	1.37%

Table 3.27 Free vibration analysis results of C4 laminated composite two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	5.959	5.892	1.12%	5.995	5.919	1.27%
λ_2	23.76	24.666	3.81%	24.133	24.998	3.58%
λ_3	27.799	31.753	14.22%	28.806	32.259	11.99%
λ_4	41.226	39.650	3.82%	41.594	39.776	4.37%
λ_5	48.163	47.853	0.64%	47.63	47.829	0.42%

Table 3.28 Free vibration analysis results of C4 laminated composite two-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.075	5.992	1.37%	6.086	5.999	1.44%
λ_2	24.587	25.413	3.36%	24.717	25.504	3.18%
λ_3	29.689	32.808	10.51%	30.072	32.922	9.48%
λ_4	42.224	40.272	4.62%	42.355	40.337	4.76%
λ_5	47.766	48.183	0.87%	47.524	48.073	1.16%

Table 3.29 Free vibration analysis results of C5 laminated composite two-bay curved frame structure for $R_{xx}=1.5L_b$ and $R_{xx}=2L_b$

Natural Frequency (Hz)	$R_{xx}=1.5L_b$			$R_{xx}=2L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	5.971	5.905	1.12%	6.008	5.932	1.27%
λ_2	23.81	24.718	3.81%	24.184	25.051	3.59%
λ_3	27.858	31.820	14.22%	28.866	32.328	11.99%
λ_4	41.312	39.734	3.82%	41.682	39.860	4.37%
λ_5	48.264	47.954	0.64%	47.731	47.930	0.42%

Table 3.30 Free vibration analysis results of C5 laminated composite two-bay curved frame structure for $R_{xx}=2.5L_b$ and $R_{xx}=3L_b$

Natural Frequency (Hz)	$R_{xx}=2.5 L_b$			$R_{xx}=3 L_b$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.026	5.944	1.37%	6.037	5.951	1.44%
λ_2	24.390	25.210	3.36%	24.520	25.300	3.18%
λ_3	29.451	32.546	10.51%	29.832	32.658	9.47%
λ_4	41.886	39.950	4.62%	42.016	40.014	4.76%
λ_5	47.384	47.797	0.87%	47.143	47.688	1.16%

According to the results given in Tables 3.23-3.30, the results of the present study are in good agreement with ANSYS results. Similar to the single-bay frame structure regardless of the stacking order, the mode shapes of the laminated composite two-bay curved frame structure are shown in Figure 3.6.

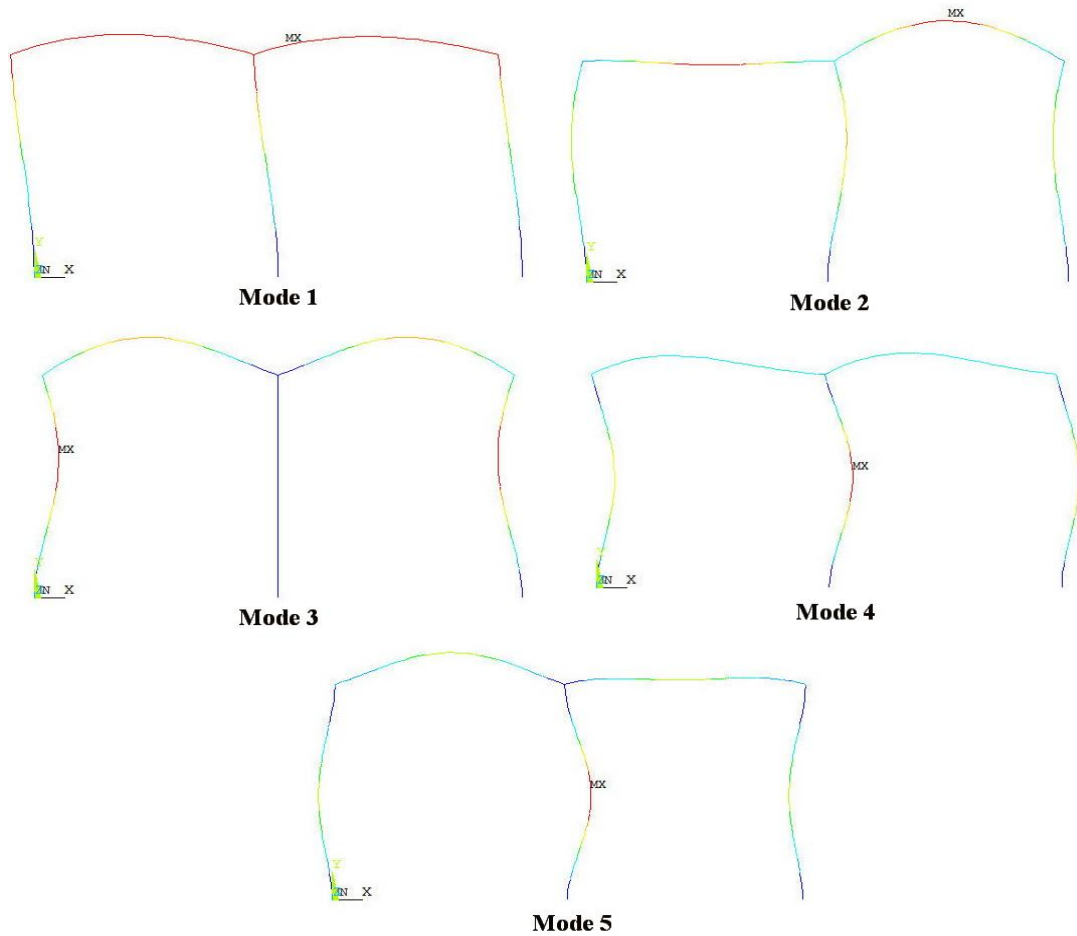


Figure 3.6 Mode shapes of the laminated composite two-bay curved beam frame

It is inferred from the results given in this section that the fiber angles of the first and the last laminate have the most impact on natural frequency values, whereas the effect of the middle laminates' fiber angles is quite small.

3.3.2 Buckling Analysis of Curved Frames

Buckling analysis of laminated composite single-bay and two-bay curved frames are conducted for the loading case shown in Figure 2.4(a). For simplicity, the distributed load is first considered as a point load as seen in Figure 2.4(b) then the point load is divided into two-point loads that applied on the columns of the curved beam frame structure, as shown in Figure 2.4(c). For the two-bay curved frame structure, the loading case is shown in Figure 2.5(a) is first considered as point load, as is shown in Figure 2.5(b). Afterward, three-point loads, as shown in Figure 2.5(c), are taken into account in order to represent the distributed loading condition for the two-bay curved beam frame structure.

3.3.2.1 Effect of the Radius of Curvature

The same curvature parameters, as in section 3.3.1, are considered for buckling analysis. Tables 3.31, 3.32, 3.33, 3.34, and 3.35 give the critical buckling load values for the different radius of curvatures (R_{xx}) of the single-bay and two-bay laminated composite curved frames with stacking orders C1, C2, C3, C4, and C5 respectively.

Table 3.31 Buckling analysis results of C1 single-bay and two-bay laminated composite curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
$1.5L_b$	566.350	543.860	4.14%	$1.5L_b$	799.980	829.750	3.59%
$2L_b$	567.800	546.550	3.89%	$2L_b$	801.560	834.700	3.97%
$2.5L_b$	568.450	547.900	3.75%	$2.5L_b$	802.210	837.400	4.20%
$3L_b$	568.800	548.690	3.67%	$3L_b$	875.860	839.070	4.38%

Table 3.32 Buckling analysis results of C2 single-bay and two-bay laminated composite curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5L _b	203.840	195.750	4.13%	1.5L _b	287.930	298.660	3.59%
2L _b	204.370	196.720	3.89%	2L _b	288.500	300.440	3.97%
2.5L _b	204.600	197.210	3.75%	2.5L _b	288.740	301.410	4.20%
3L _b	204.730	197.490	3.67%	3L _b	288.850	302.010	4.36%

Table 3.33 Buckling analysis results of C3 single-bay and two-bay laminated composite curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5L _b	516.720	496.200	4.14%	1.5L _b	729.870	757.050	3.59%
2L _b	518.040	498.660	3.89%	2L _b	731.320	761.570	3.97%
2.5L _b	518.640	499.890	3.75%	2.5L _b	731.910	764.020	4.20%
3L _b	518.960	500.610	3.67%	3L _b	799.120	765.550	4.39%

Table 3.34 Buckling analysis results of C4 single-bay and two-bay laminated composite curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame Pcr (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame Pcr (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5L _b	527.310	506.370	4.14%	1.5L _b	744.830	772.550	3.59%
2L _b	528.660	508.880	3.89%	2L _b	746.310	777.160	3.97%
2.5L _b	529.270	510.130	3.75%	2.5L _b	746.910	779.670	4.20%
3L _b	529.590	510.870	3.66%	3L _b	747.200	781.230	4.36%

Table 3.35 Buckling analysis results of C5 single-bay and two-bay laminated composite curved frame structure for $R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, and $R_{xx}=3L_b$

Radius of Curvature (R_{xx})	Single-Bay Curved Beam Frame P_{cr} (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Beam Frame P_{cr} (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
$1.5L_b$	518.900	498.300	4.13%	$1.5L_b$	732.960	760.240	3.59%
$2L_b$	520.230	500.760	3.89%	$2L_b$	734.410	764.770	3.97%
$2.5L_b$	520.830	502.000	3.75%	$2.5L_b$	735.000	767.240	4.20%
$3L_b$	521.150	502.720	3.67%	$3L_b$	735.290	768.780	4.36%

According to the results given in Tables 3.31-3.35, the results of the present study are in good agreement with ANSYS results. Regardless of the stacking order, the critical buckling load increases as the curvature of the laminated composite single-bay and two-bay curved frame structure increases.

It is inferred from the results given in this section, similar to the change in natural frequency values presented in the section 3.3.1, the fiber angles of the first and the last laminate have the most impact on the critical buckling load whereas the fiber angles of the middle laminates have the least effect on the critical buckling load of the laminated composite single-bay and two-bay curve frame structures.

3.3.3 Dynamic Stability Analysis of Laminated Composite Curved Frames

The dynamic stability of curved frames is performed within this section. The effects of the radius of curvature and the stacking order on the unstable region are investigated. Four different radii of curvatures and five stacking orders, as presented in section 3.3.1.1, are considered.

Figures 3.5 and 3.6 show the instability region of laminated composite curved frame structure with C1 stacking order and four different radii of curvatures for $\alpha=0$ and $\alpha=0.5$, respectively. It is inferred from Figures 3.7 and 3.8 that increasing the radius of curvature of the curved frame structure does not affect the instability region considerably.

Thus, the difference between the instability regions of different curvatures is so small that it can be neglected. This condition is observed for all other stacking orders and the two-bay curved frame structures. Besides, as the static load factor increase, the unstable region becomes narrow and moves towards origin.

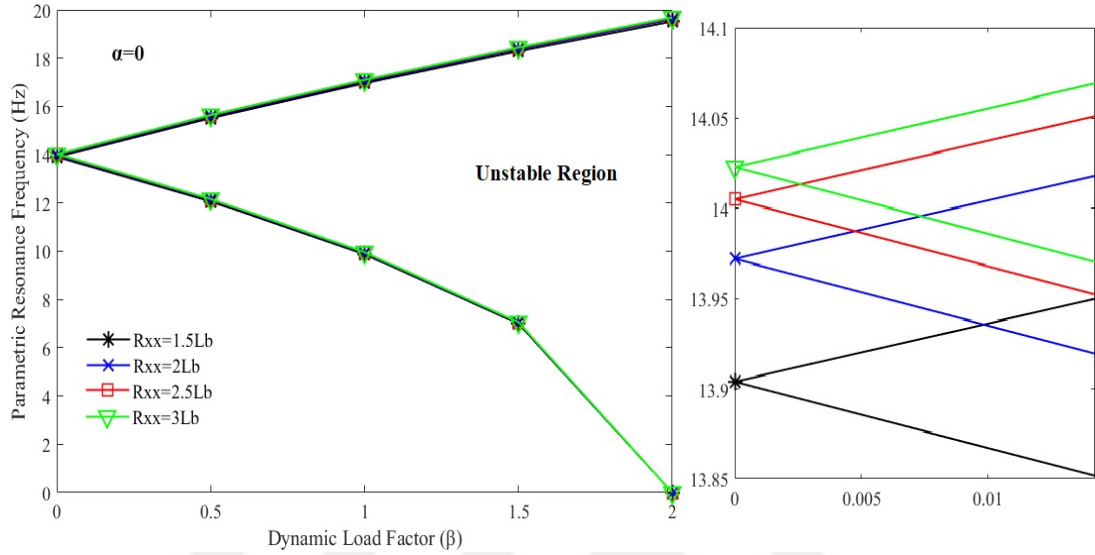


Figure 3.7 The effect of the radius of curvature on the dynamic instability regions for $\alpha=0$

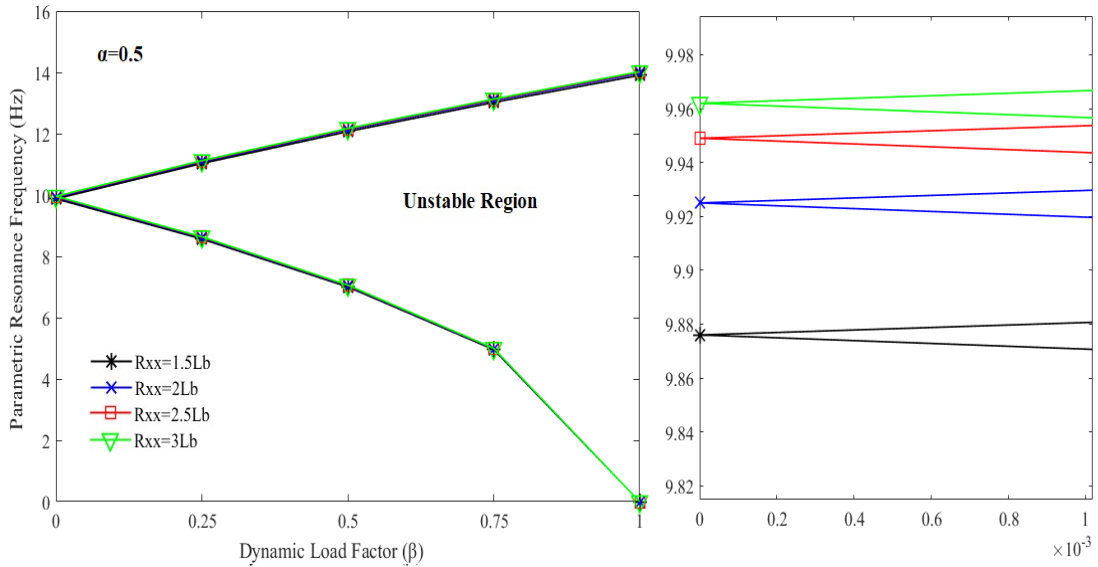


Figure 3.8 The effect of the radius of curvature on the dynamic instability regions for $\alpha=0.5$

Figures 3.9 and 3.10 show the dynamic instability region of laminated composite curved frame structures having different stacking orders.

It is concluded that the unstable region is almost the same for curved frame structures having C1, C3, C4, and C5 fiber angles. The distance of the unstable regions to the origin can be ordered from the closest to farthest as C2, C3, C5, C4, and C1, respectively. This is because even the stacking orders are different, the fundamental natural frequency values and the critical buckling load of the structures having C1, C3, C4, and C5 fiber angles do not change remarkably. The unstable region becomes narrow for the curved frame structure has C2 stacking order when compared with other stacking sequences. This is because the fiber angles of the first and the effects of the last layer, both fundamental natural frequency and critical buckling load significantly.

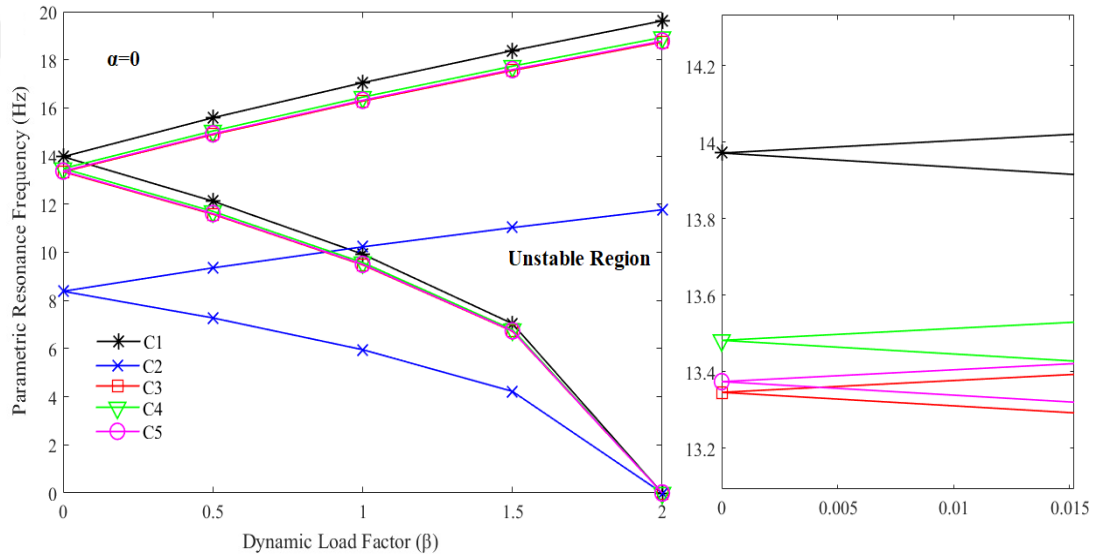


Figure 3.9 The dynamic instability regions of laminated composite curved frame structure with different stacking orders for $\alpha=0$

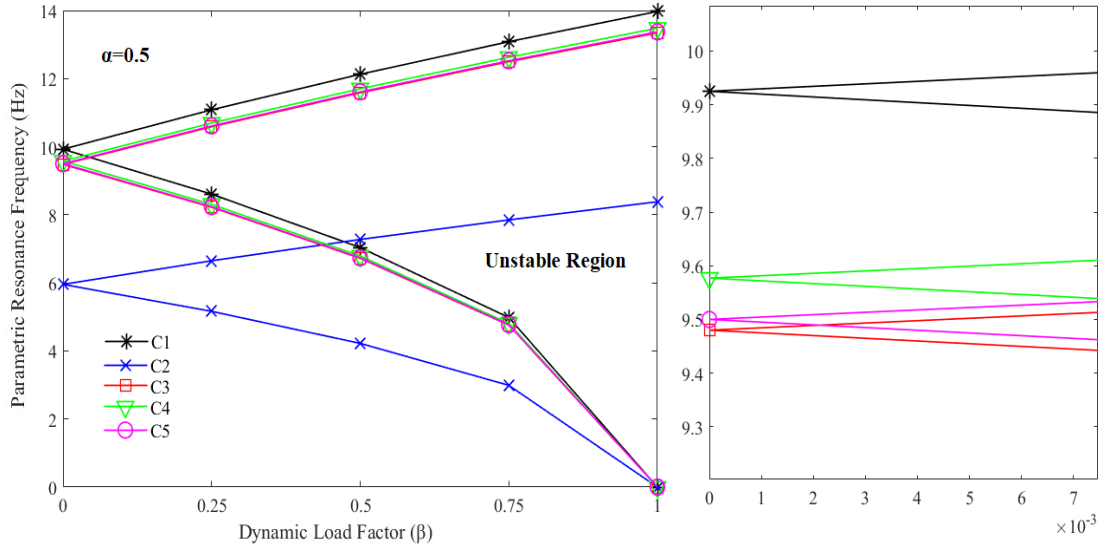


Figure 3.10 The dynamic instability regions of laminated composite curved frame structure with different stacking orders for $\alpha=0.5$

Figures 3.11 and 3.12 show the instability regions of laminated composite curved two-bay frame structures having different stacking orders.

It is seen that the unstable region becomes narrow as the static load factor increases. The unstable region of the laminated composite curved two-bay frame with C2 stacking order is narrow when compared with the curved frame structures that have C1, C3, C4, and C5 stacking order.

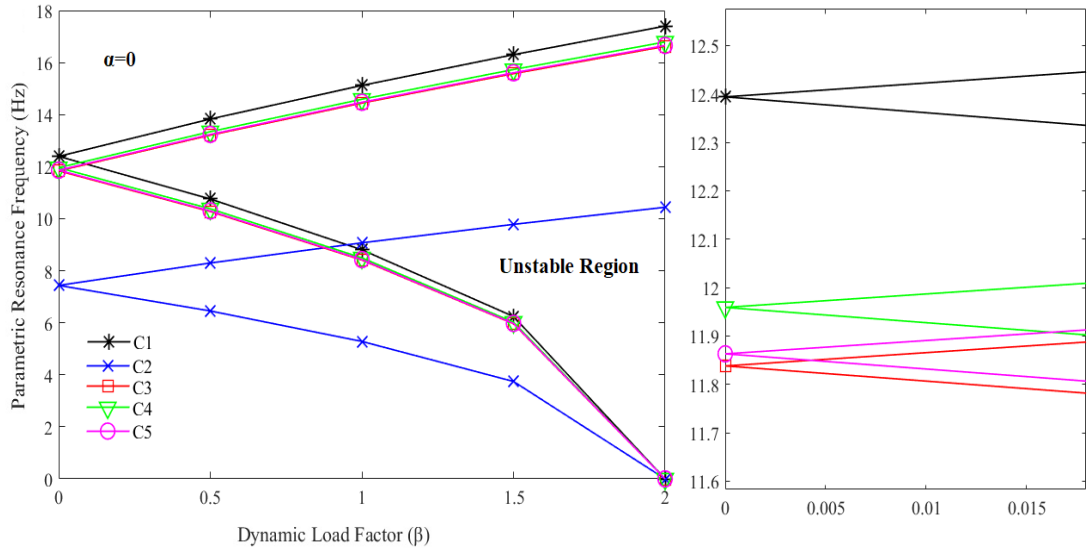


Figure 3.11 The dynamic instability regions of laminated composite curved two-bay frame structure with different stacking orders for $\alpha=0$

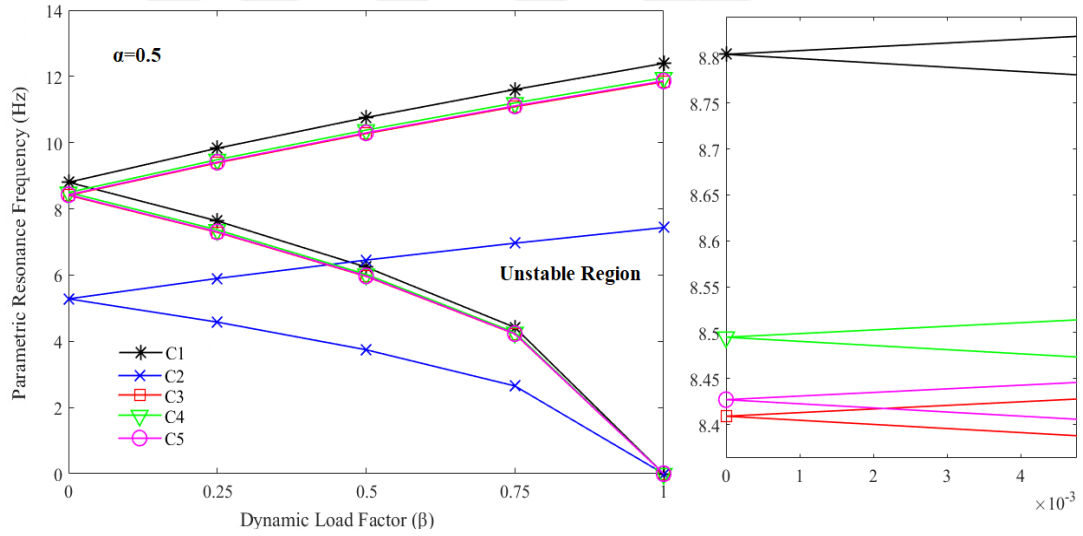


Figure 3.12 The dynamic instability regions of laminated composite curved two-bay frame structure with different stacking orders for $\alpha=0.5$

3.3.4 Conclusions

In this chapter, finite element analysis of isotropic and laminated composite curved frame structures is performed. In general, it is concluded that the radius of curvature of the curved frame structures does not affect the natural frequency values, critical buckling load values, and unstable regions significantly.

However, it should be noted that only four radii of curvature parameters ($R_{xx}=1.5L_b$, $R_{xx}=2L_b$, $R_{xx}=2.5L_b$, $R_{xx}=3L_b$) are investigated. Therefore, the results may change for curvature values smaller than $R_{xx}=1.5L_b$.

The stacking order of the laminated composite curved frame structures changes the natural frequency values, critical buckling load values, and unstable regions. However, these changes are significant only if the first and the last fiber angle of the laminated composite curved frame structure are different from zero. This condition only takes place when the curved frame structure is modeled with C2 stacking order. All other stacking orders have a negligible effect on the natural frequency values, critical buckling load values, and the unstable regions of the laminated composite curved frames. All interpretations made above are valid for two-bay curved frame structures.

CHAPTER 4

FINITE ELEMENT ANALYSIS OF CURVED PLATE FRAME STRUCTURES

4.1 Introduction

The curved plate frame structures, shown in Figure 4.1, are utilized in many engineering fields just as curved frame structures presented in Chapter 2. Likewise, it is essential to evaluate the natural frequency values, critical buckling load, and the dynamically unstable regions of curved plate frame structures in case of failure due to resonance, buckling, and parametric resonance. This chapter presents the methods and theories to perform vibration, buckling, and dynamic stability analysis of isotropic and laminated composite curved plate frame structures.

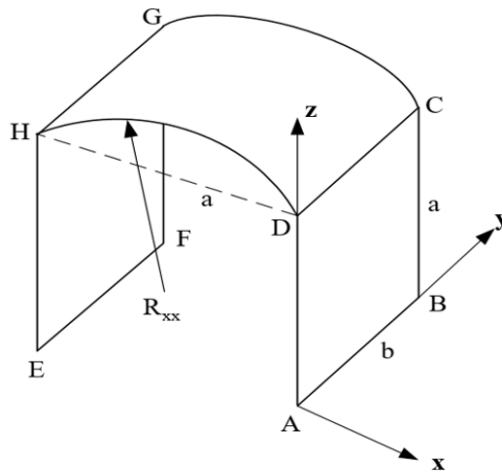


Figure 4.1 A curved plate frame structure

4.2 Finite Element Analysis of Curved Plate Frame Structures

Finite element analysis is performed to perform free vibration, buckling, and dynamic stability analyses of curved plate frames. Four node quadrilateral elements, shown in Figure 4.2, is utilized to model the curved plate frame structures.

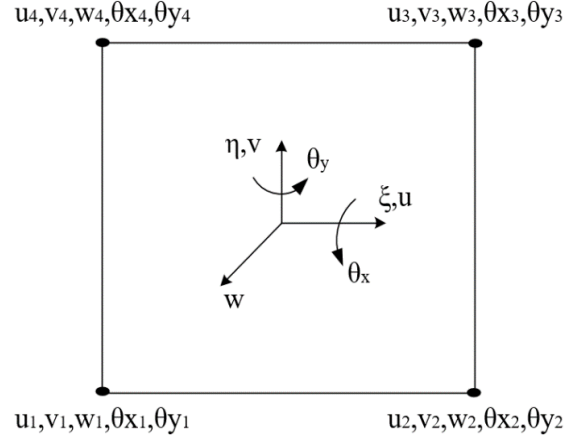


Figure 4.2 Four node quadrilateral element

Each node has five degrees of freedom (DOF) u , v , w , θ_x , and θ_y . Equation 4.1 gives displacement functions (Petyt, 2010).

$$\begin{aligned}
 u &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \\
 v &= \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy \\
 w &= \alpha_9 + \alpha_{10}x + \alpha_{11}y + \alpha_{12}x^2 + \alpha_{13}xy + \alpha_{14}y^2 + \alpha_{15}x^3 \\
 &\quad + \alpha_{16}x^2y + \alpha_{17}xy^2 + \alpha_{18}y^3 + \alpha_{19}x^3y + \alpha_{20}xy^3 \\
 \theta_x &= \frac{\partial w}{\partial y}, \quad \theta_y = \frac{\partial w}{\partial x}
 \end{aligned} \tag{4.1}$$

Equation 4.1 can be written in matrix form as

$$\begin{aligned}
 \{u\} &= \mathbf{N}_{mj}[u_j] \\
 \{v\} &= \mathbf{N}_{mj}[v_j] \\
 \begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} &= \mathbf{N}_{bj} \begin{Bmatrix} w_j \\ \theta_{xj} \\ \theta_{yj} \end{Bmatrix} \\
 j &= 1, \dots, 4
 \end{aligned} \tag{4.2}$$

Equation 4.3 represents the matrix forms of related displacement functions. $\mathbf{N}_m$ and $\mathbf{N}_b$ are the shape functions (Petyt, 2010) for in-plane and out of plane displacements, respectively.

$$\begin{aligned}
N_{mj} &= \left[\frac{1}{4} (1 + \xi_j \xi) (1 + \eta_j \eta) \right] \\
N_{bj} &= \frac{1}{8} \begin{bmatrix} (1 + \xi_j \xi) (1 + \eta_j \eta) (2 + \xi_j \xi + \eta_j \eta - \xi^2 - \eta^2) \\ b(1 + \xi_j \xi) (\eta_j + \eta) (\eta^2 - 1) \\ -a(\xi_j + \xi) (\xi^2 - 1) (1 + \eta_j \eta) \end{bmatrix} \\
j &= 1 \dots 4
\end{aligned} \tag{4.3}$$

where, a , b , ξ and η are the length, width and natural coordinates of the four node quadrilateral element. Equation 4.4 gives the strain energy of the finite element (U_e) (Petyt, 2010).

$$U_e = \frac{1}{2} \int_V \{\sigma\}^T \{\varepsilon\} dV \tag{4.4}$$

where $\{\sigma\}^T$ denotes the stress components matrix and $\{\varepsilon\}$ is the strain matrix. For thin and isotropic curved plate frame structures, Kirchhoff – Love Plate Theory (KLPT) is employed. To satisfy all stress and strain components of the curved thin plate structure, both in-plane and out of plane stress tensors $\{N_{ij}\}$ and $\{M_{ij}\}$ ($i, j = x, y$) are considered. The shear effect is negligible since the structure is thin. These stress tensor matrix can be written in terms of the material modulus matrix $[D_m]$. Hence, the constitutive relation between the stress tensor matrix and the material modulus matrix is given as

$$\{\sigma\}^T = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xc} \\ M_{yc} \\ M_{xy} \end{Bmatrix} = h[D_m]\{\gamma\} \tag{4.5}$$

where

$$[D_m] = \begin{bmatrix} D & 0 \\ 0 & \frac{h^2}{12} D \end{bmatrix} \tag{4.6}$$

where

$$[D] = \begin{bmatrix} E^* & E^*\vartheta & 0 \\ E^*\vartheta & E^* & 0 \\ 0 & 0 & G \end{bmatrix} \quad (4.7)$$

and,

$$E^* = \frac{E}{1 - \nu^2}, \quad G = \frac{E}{2(1 + \nu)} \quad (4.8)$$

where E , G and ν are the modulus of elasticity and shear modulus, and Poisson ratio of the material, respectively.

Substituting Equation 4.5 into Equation 4.4 gives

$$U_e = \frac{1}{2} \int_A (h\{\gamma\}^T [D_m] \{\gamma\}) dA \quad (4.9)$$

where h is the thickness of the structure of isotropic structure and,

$$\{\gamma\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial \xi} \\ \frac{\partial^2 w}{\partial \xi^2} \\ \frac{\partial^2 w}{\partial \eta^2} \\ \frac{\partial^2 w}{\partial \xi \partial \eta} \end{bmatrix} \quad (4.10)$$

If the material is orthotropic, the material will have two axes of symmetry, and the Classical Plate Theory has to be employed. Therefore, Equation 4.6 becomes as given in Equation 4.11.

$$[D_m] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \quad (4.11)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are the longitudinal, longitudinal-bending coupled, and bending stiffness matrices, which are given as

$$\begin{aligned} \mathbf{A} &= \sum_{k=1}^{NL} [Q_{bm}]^k (z_{k+1} - z_k) \\ \mathbf{B} &= \frac{1}{2} \sum_{k=1}^{NL} [Q_{bm}]^k (z_{k+1}^2 - z_k^2) \\ \mathbf{C} &= \frac{1}{3} \sum_{k=1}^{NL} [Q_{bm}]^k (z_{k+1}^3 - z_k^3) \end{aligned} \quad (4.12)$$

where

$$[Q_{bm}]^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}^k \quad (4.13)$$

where NL is the number of laminates, z is the variable of coordinate of the k^{th} laminate, and Q_{ij} ($i=1, 2$, and 6) are material coefficients given in the section 2.2. Equation 4.14 denotes the matrix form of Equation 4.9.

$$U_e = \frac{1}{2} \mathbf{q}_e^T \mathbf{k}_e \mathbf{q}_e \quad (4.14)$$

where

$$\mathbf{q}_e = [u_1 v_1 w_1 \theta_{x1} \theta_{y1} u_2 v_2 w_2 \theta_{x2} \theta_{y2} u_3 v_3 w_3 \theta_{x3} \theta_{y3} u_4 v_4 w_4 \theta_{x4} \theta_{y4}] \quad (4.15)$$

where $\mathbf{q}_e$ is the displacement vector of the four-node quadrilateral element. Considering Equations 4.9 and 4.14, the element elastic stiffness matrix, $\mathbf{k}_e$, is evaluated as

$$\mathbf{k}_e = \frac{1}{2} \int_A ([B_m]^T [D_m] [B_m]) dA \quad (4.16)$$

where

$$[B_m] = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix} = \sum_{i=1}^4 \begin{bmatrix} \frac{\partial N_{mi}}{\partial \xi} & 0 & 0 \\ 0 & \frac{\partial N_{mi}}{\partial \eta} & 0 \\ \frac{\partial N_{mi}}{\partial \eta} & \frac{\partial N_{mi}}{\partial \xi} & 0 \\ 0 & 0 & \mathbf{B}_{bi} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \end{bmatrix} \quad (4.17)$$

where

$$\mathbf{B}_{bi} = \begin{bmatrix} \frac{4}{a^2} \frac{\partial^2}{\partial \xi^2} \\ \frac{4}{b^2} \frac{\partial^2}{\partial \eta^2} \\ \frac{8}{ab} \frac{\partial^2}{\partial \xi \partial \eta} \end{bmatrix} N_{bi}^T(\xi, \eta), i = 1 \dots 4 \quad (4.18)$$

The kinetic energy, T_e , of the finite element is given in Equation 4.19 (Petyt, 2010).

$$T_e = \frac{1}{2} \rho h \int_A [(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)] dA \quad (4.19)$$

where ρ is the density of the material, and the point accent indicates the first derivative with respect to time. Equation 4.19 can be written in matrix form as

$$T_e = \frac{1}{2} \mathbf{q}_e^T \mathbf{m}_e \mathbf{q}_e \quad (4.20)$$

Hence the element mass matrix is obtained from

$$[\mathbf{m}_e] = \frac{1}{2} \int_A [\mathbf{N}]^T [\mathbf{IPV}] [\mathbf{N}] dA \quad (4.21)$$

where

$$[\mathbf{N}] = \sum_{i=1}^4 \begin{bmatrix} N_{mi} & 0 & 0 & 0 & 0 \\ 0 & N_{mi} & 0 & 0 & 0 \\ 0 & 0 & N_{bi}(1,1) & 0 & 0 \\ 0 & 0 & 0 & N_{bi}(2,1) & 0 \\ 0 & 0 & 0 & 0 & N_{bi}(3,1) \end{bmatrix} \quad (4.22)$$

and for thick structures,

$$[\mathbf{IPV}] = \sum_{i=1}^4 \begin{bmatrix} I_i & 0 & 0 & P_i & 0 \\ 0 & I_i & 0 & 0 & P_i \\ 0 & 0 & I_i & 0 & 0 \\ P_i & 0 & 0 & V_i & 0 \\ 0 & P_i & 0 & 0 & V_i \end{bmatrix} \quad (4.23)$$

where,

$$I_i = \sum_{i=1}^{NL} \rho \int_{z_i}^{z_{i+1}} dz, \quad P_i = \sum_{i=1}^{NL} \rho \int_{z_i}^{z_{i+1}} z dz, \quad V_i = \sum_{i=1}^{NL} \rho \int_{z_i}^{z_{i+1}} z^2 dz \quad (4.24)$$

For thin structures $[\mathbf{IPV}]$ matrix simplified into ρh . The matrix form of the work done by the applied load given in Equation 2.21 can be written as

$$V_e = \frac{1}{2} \mathbf{q}_e^T \mathbf{k}_{ge} \mathbf{q}_e \quad (4.25)$$

and,

$$k_{ge} = \frac{1}{2} \int_A \mathbf{G}^T \mathbf{H} \mathbf{G} dA \quad (4.26)$$

where

$$\mathbf{G} = \sum_{j=1}^4 \begin{bmatrix} \mathbf{N}_{mj,x} & \mathbf{N}_{bj,x}(1,1) \\ \mathbf{N}_{mj,y} & \mathbf{N}_{bj,y}(1,1) \end{bmatrix} \quad (4.27)$$

where $\mathbf{N}_{mj,x}$ and $\mathbf{N}_{bj,x}$ denotes the first derivatives of the shape functions with respect to x while $\mathbf{N}_{mj,y}$ and $\mathbf{N}_{bj,y}$ are the first derivatives of the shape functions with respect to y . H_{ij} ($i,j=x,y$) is the stress tensor matrix which is given as

$$\mathbf{H} = \begin{bmatrix} H_{xx} & H_{xy} \\ H_{xy} & H_{yy} \end{bmatrix} \quad (4.28)$$

The buckling cases of the single-bay and two-bay curved plate frame structure are demonstrated in Figures 4.3 and 4.4, respectively.

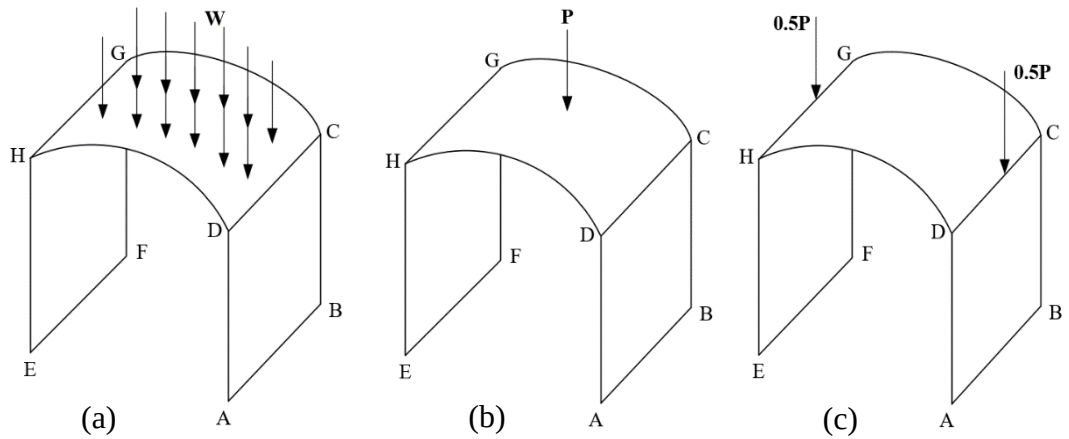


Figure 4.3 Loading case (a), equivalent single point loading (b), and equivalent loading for finite element analysis (c) of single-bay curved plate frame structure

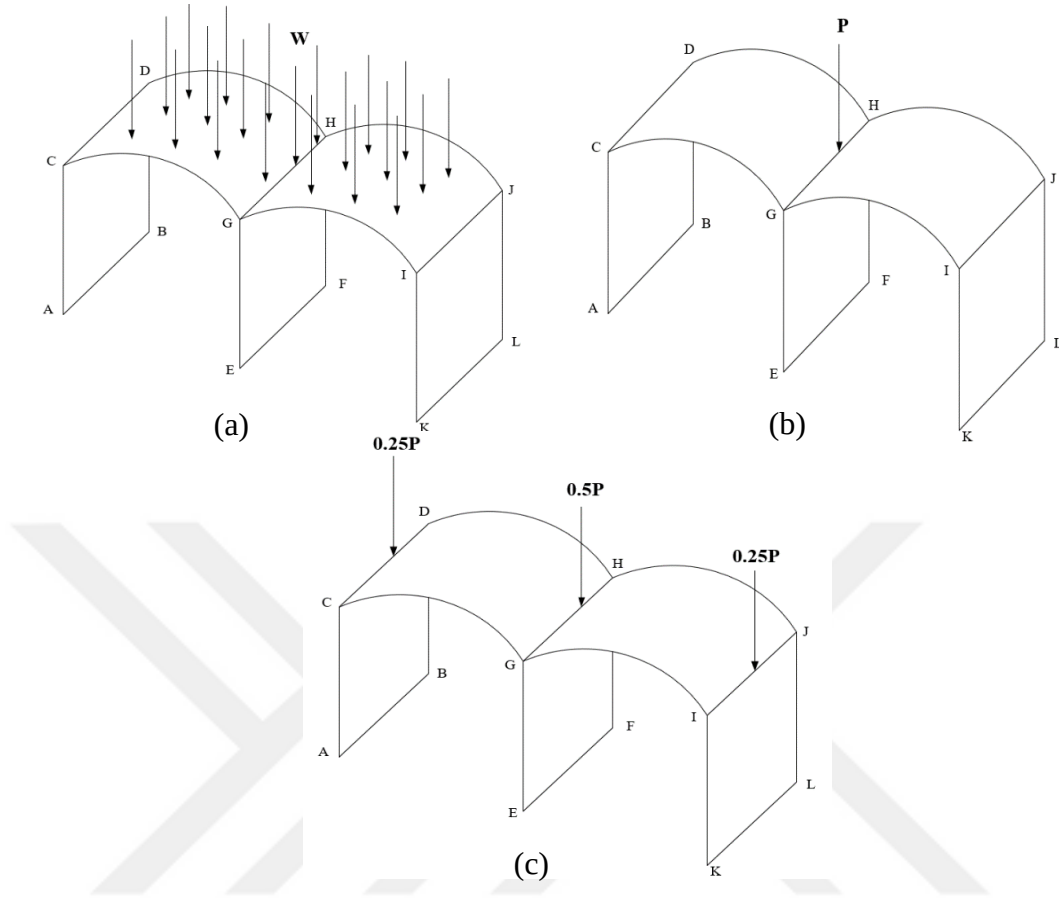


Figure 4.4 Loading case (a), equivalent single point loading (b), and equivalent loading for finite element analysis (c) of two-bay curved plate frame structure

In order to perform finite element analysis to calculate the critical buckling load, the loading conditions, shown in Figures 4.3(a) and 4.4(a), are first written in terms of single-point load (P), as shown in Figures 4.3(b) and 4.4(b). Afterward, the single-point load is divided into the two-point load for the single-bay frame structure and three-point load for two-bay frame structure, as presented in Figures 4.3(c) and 4.4(c), respectively. Since the loading case is unidirectional, the stress tensor in y -direction H_{yy} and the shear stress tensor H_{xy} is equal to zero. The stress tensor H_{xx} can be written in terms of single point load as

$$H_{xx} = P, H_{yy} = H_{xy} = [\quad] \quad (4.29)$$

The distributed load can be written in terms of the single load as

$$P = Wab \quad (4.30)$$

Since each node of the finite element has five degrees of freedom (DOF), the dimension of the element stiffness matrix, mass matrix, and the geometric matrix is obtained as 20x20. However, in order to satisfy all DOF of the curved frame structure, the drilling (θ_z) effect has to be considered.

Hence, the dimension of the element stiffness matrix, mass matrix, and the geometric matrix is expanded to 24x24 by implementing the drilling effect for each node of the finite element. The corresponding non-diagonal terms of θ_z are set to zero and the diagonal terms of θ_z are assumed 1000 times smaller of the minimal value of the element stiffness, mass, and geometric matrix.

Following the addition of θ_z , the element stiffness matrix, mass matrix, and geometric matrix are transformed, as seen in Figure 4.5, by using Equation 4.31 to model curved frame structure.

$$\bar{\mathbf{k}}_e = \mathbf{T}^T \mathbf{k}_e \mathbf{T}, \quad \bar{\mathbf{m}}_e = \mathbf{T}^T \mathbf{m}_e \mathbf{T}, \quad \bar{\mathbf{k}}_{ge} = \mathbf{T}^T \mathbf{k}_{ge} \mathbf{T} \quad (4.31)$$

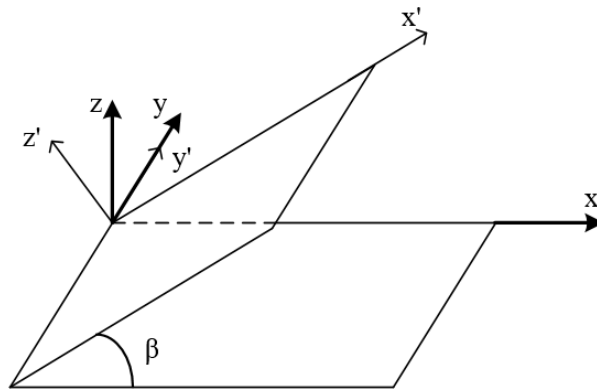


Figure 4.5 Transformation of local coordinates

where $\mathbf{T}$ is the transformation matrix which is given as

$$\mathbf{T} = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\beta) & 0 & \sin(\beta) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \quad (4.32)$$

The curvature of the structure is modeled by transforming the element stiffness matrix, mass matrix, and geometric matrix by using Equations 4.31 and 4.32.

If the structure length to thickness ratio (a/h) is bigger than twenty, the structure is considered as a thick structure (Steele & Balch, 2009). If so, some parameters such as the usage of finite element, displacement functions, and energy equations have to be considered different from the thin structures since the shear effect takes is not negligible in thick structures. Hence, instead of the Classical Plate Theory, the First-Order Shear Deformation Theory is employed.

In addition to the four-node quadrilateral element shown in Figure 4.2, the four-node cylindrical shell element, as shown in Figure 4.6, is used to perform finite element analysis of the laminated composite curved thick plate structure. The column plates of the structure are modeled via a four-node quadrilateral element, whereas the top of the frame structure is modeled with the four nodes cylindrical shell element.

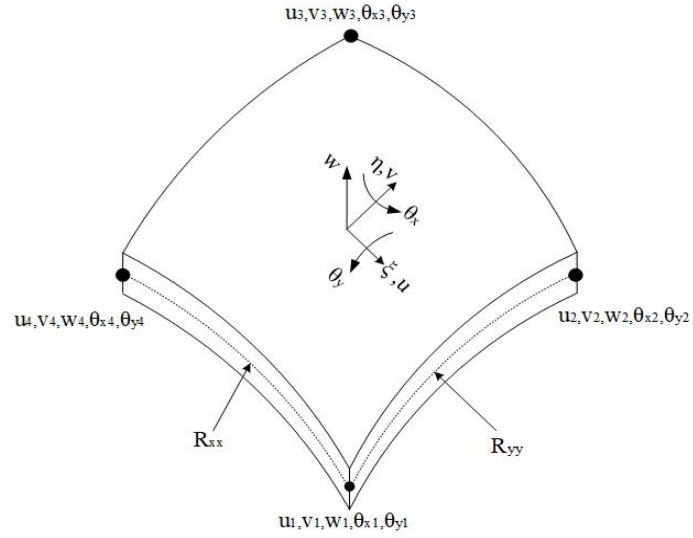


Figure 4.6 Cylindrical shell element

According to the First-Order Shear Deformation Theory, the displacement equations for laminated composite curved thick plate frame structure are

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\theta_y(x, y) \\ v(x, y, z) &= v_0(x, y) + z\theta_x(x, y) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (4.33)$$

where

$$\begin{aligned} u_0 &= \sum_{i=1}^4 u_{0i} N_{mi} & v_0 &= \sum_{i=1}^4 v_{0i} N_{mi} & w_0 &= \sum_{i=1}^4 w_{0i} N_{mi} \\ \theta_y &= \sum_{i=1}^4 \theta_{yi} N_{mi} & \theta_x &= \sum_{i=1}^4 \theta_{xi} N_{mi} \end{aligned} \quad (4.34)$$

The strain energy relation given in Equations 4.9 becomes

$$U_e = \frac{1}{2} \int_A (\{\delta\}^T [D_t] \{\delta\}) dA \quad (4.35)$$

where

$$\{\delta\} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \\ -\frac{\partial \theta_y}{\partial x} \\ \frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_x}{\partial x} - \frac{\partial \theta_y}{\partial y} \\ -\theta_y + \frac{\partial w}{\partial x} \\ \theta_x + \frac{\partial w}{\partial y} \end{Bmatrix} \quad (4.36)$$

and

$$[D_t] = \begin{bmatrix} \mathbf{A} & \mathbf{B} & 0 \\ \mathbf{B} & \mathbf{C} & 0 \\ 0 & 0 & \mu \mathbf{A}_s \end{bmatrix} \quad (4.37)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are given in Equation 4.12. $\mathbf{A}_s$ is the shear component of material coefficient matrix, $[D_t]$, representing shear stress and μ is the shear correction factor, which is needed to satisfy the variation of the shear stress and strains through thickness (Petyt, 2010). It is usually taken as 5/6 or $\pi^2/12$. $\mathbf{A}_s$ is given in Equation 4.38

$$\mathbf{A}_s = \sum_{k=1}^{NL} [Q_s]^k (z_{k+1} - z_k) \quad (4.38)$$

where $[Q_s]$ is

$$[Q_s]^k = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix}^k \quad (4.39)$$

where

$$\begin{aligned} Q_{44} &= q_{55}s^2 + q_{44}c^2 \\ Q_{45} &= q_{55}cs - q_{44}cs \\ Q_{55} &= q_{55}c^2 + q_{44}s^2 \end{aligned} \quad (4.40)$$

where c and s denote $\cos\theta$ and $\sin\theta$, respectively. θ represents the fiber angle and,

$$\begin{aligned} q_{44} &= G_{13} \\ q_{55} &= G_{23} \end{aligned} \quad (4.41)$$

where G_{13} and G_{23} are the shear modulus in x-, z- and x-, y- directions, respectively.

Substituting Equation 4.37 in Equation 4.35 and equalising to Equation 4.14 gives the element elastic stiffness matrix, $\mathbf{k}_e$, as

$$\mathbf{k}_e = \frac{1}{2} \int_A ([B_t]^T [D_m] [B_t]) dA \quad (4.42)$$

where

$$[B_t] \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \sum_{i=1}^4 \begin{bmatrix} \frac{\partial N_{mi}}{\partial x} & 0 & -\frac{w}{R_{xx}} & 0 & 0 \\ 0 & \frac{\partial N_{mi}}{\partial y} & -\frac{w}{R_{yy}} & 0 & 0 \\ \frac{\partial N_{mi}}{\partial y} & \frac{\partial N_{mi}}{\partial x} & -\frac{2w}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_{mi}}{\partial x} \\ 0 & 0 & 0 & \frac{\partial N_{mi}}{\partial y} & 0 \\ 0 & 0 & 0 & \frac{\partial N_{mi}}{\partial x} & -\frac{\partial N_{mi}}{\partial y} \\ 0 & 0 & \frac{\partial N_{mi}}{\partial x} & 0 & -N_{mi} \\ 0 & 0 & \frac{\partial N_{mi}}{\partial y} & N_{mi} & 0 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \end{bmatrix} \quad (4.43)$$

and,

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \end{bmatrix} - z \begin{bmatrix} -\frac{\partial \theta_y}{\partial x} \\ \frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_x}{\partial x} - \frac{\partial \theta_y}{\partial y} \end{bmatrix} \quad (4.44)$$

$$\begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} -\theta_y + \frac{\partial w}{\partial x} \\ \theta_x + \frac{\partial w}{\partial y} \end{bmatrix}$$

The evaluation of mass is the same as Equation 4.21. Note that to obtain the element mass matrix for thick structures, N_{bi} is replaced by N_{mi} in the $[\mathbf{N}]$ matrix given in Equation 4.22. The strain components ε_{xx} , ε_{yy} , and γ_{xy} have the curvature terms, $-w/R_{xx}$, $-w/R_{yy}$, and $-2w/R_{xy}$, respectively. However, y-axis dependent terms, w/R_{yy} and $2w/R_{xy}$, are neglected since the curvature only exists in x-direction. If all curvature components goes to infinity, the finite element become four-node quadrilateral plate element. The free vibration, buckling, and dynamic stability analysis are performed considering the same equations and methods given in section 2.2.

CHAPTER 5

NUMERICAL ANALYSIS OF CURVED PLATE FRAME STRUCTURES

5.1 Introduction

This chapter presents the numerical analysis of curved plate frame structures, shown in Figure 4.1, under fixed from all ends boundary conditions. The Finite Element Method (FEM) presented in chapter 4, is used to perform free vibration, buckling, and dynamic stability analysis of the curved frame structures. For this purpose, a computer code based on Kirchhoff-Love Plate Theory, Classical Plate Theory, and First-Order Shear Deformation Theory is developed and executed in MATLAB. The natural frequency and critical buckling load results of the present study are compared with the outcomes of ANSYS R18.2.

5.2 Numerical Results of Finite Element Analysis of Isotropic Curved Plate Frame Structures

The finite element analysis is performed to obtain the first ten natural frequencies and critical buckling loads of the single-bay and two-bay isotropic curved plate frames. The effects of the radius of curvature and the aspect ratio on the natural frequency and critical buckling values are investigated. All analyses are performed by using the material and geometric properties given in Table 5.1.

Table 5.1 Material and geometric properties of the isotropic curved plate frame

Property	Symbol	Quantity
Elastic Modulus	E	69 GPa
Density	ρ	2700 kg/m ³
Poisson Ratio	ν	0.33
Geometric Properties of the Structure		
Plate component length	a	1000 mm
Plate component width	b	1000 mm
The thickness of the structure	h	10 mm
The radius of curvature of the structure	R_{xx}	2a

5.2.1 Free Vibration Analysis of Isotropic Curved Plate Frame Structures

In this section, free vibration analysis of the curved plate frame fixed at all ends is investigated. Four different radii of curvature (R_{xx}) and aspect ratio (a/b) parameters are considered. The same parameters are taken into account for the two-bay structure.

Figure 5.1 shows the convergence analysis of used four-node quadrilateral elements for the curved plate frame structure ($R_{xx} = 2a$). The first ten natural frequency results of the present study with the number of 12x12, 18x18, 24x24, and 30x30 elements (meshes) are compared with the outcomes of ANSYS R18.2. The convergence analysis indicated that as the number of elements increases the present study gives better results and satisfactory convergence. Therefore, in this study, the number of 30x30 elements are used.

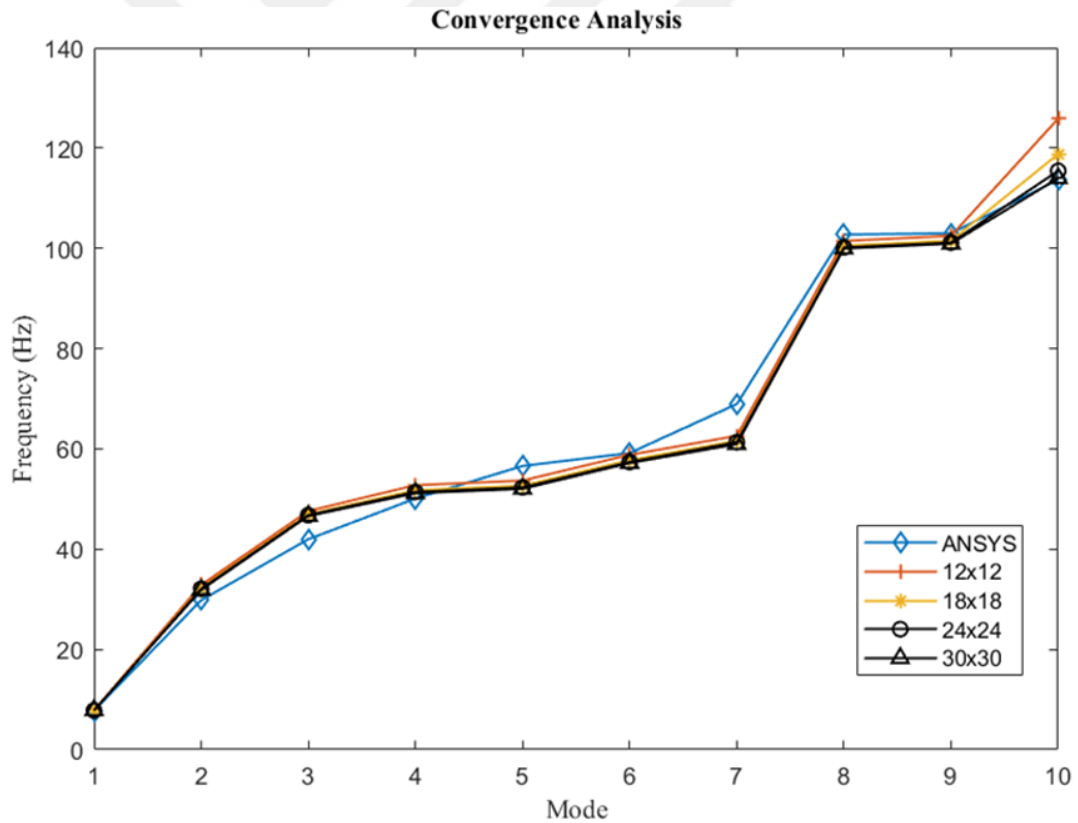


Figure 5.1 Convergence analysis of the quadrilateral element used in the analysis of the curved plate frame structure for $R_{xx} = 2a$.

5.2.1.1 Effect of the Radius of Curvature

The effect of the radius of curvature on the natural frequency values of the single-bay and two-bay curved plate frame structure is investigated. For this purpose, four different radii of curvature, as given in Figure 5.2, are considered. Tables 5.2 and 5.3 give the first ten natural frequency values. Figure 5.3 shows the structure's mode shapes having curvatures equal to $1.5a$, $2a$, $2.5a$, and $3a$, respectively.

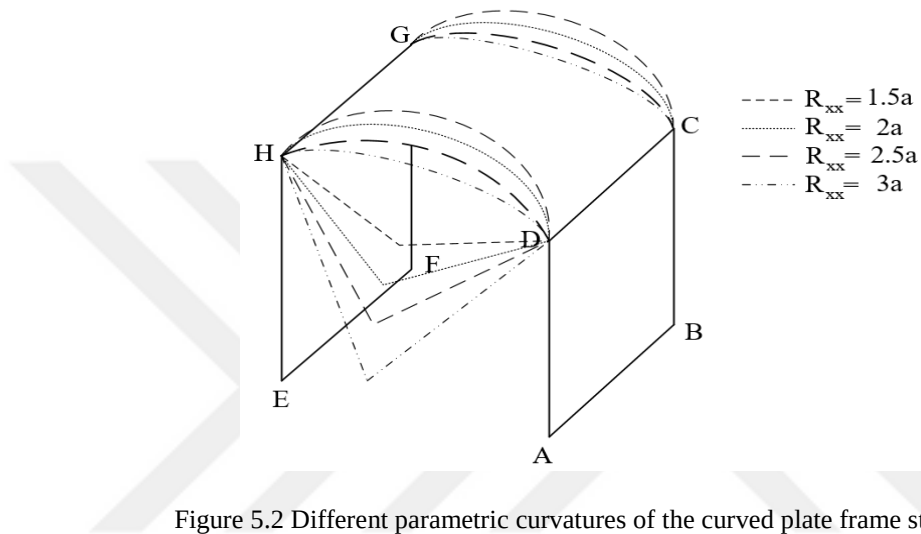


Figure 5.2 Different parametric curvatures of the curved plate frame structure

Table 5.2 The first ten natural frequency values of the curved plate structure for $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}= 2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.726	7.771	0.58%	7.773	7.836	0.81%
λ_2	29.270	31.814	8.69%	29.812	31.871	6.91%
λ_3	40.825	46.798	14.63%	41.955	46.597	11.06%
λ_4	49.575	49.885	0.63%	50.079	51.147	2.13%
λ_5	56.284	51.807	7.95%	56.574	52.024	8.04%
λ_6	59.997	56.743	5.42%	59.161	57.162	3.38%
λ_7	69.619	59.810	14.09%	68.928	60.932	11.60%
λ_8	102.880	99.315	3.47%	102.750	99.953	2.72%
λ_9	103.000	101.070	1.87%	102.980	100.930	1.99%
λ_{10}	112.490	112.990	0.44%	113.690	113.990	0.26%

Table 5.3 The first ten natural frequency values of the curved plate structure for $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.797	7.865	0.87%	7.811	7.868	0.74%
λ_2	30.107	31.846	5.78%	30.289	31.835	5.10%
λ_3	42.596	46.372	8.86%	43.005	46.234	7.51%
λ_4	50.354	51.512	2.30%	50.527	51.535	1.99%
λ_5	56.735	52.589	7.31%	56.838	53.082	6.61%
λ_6	58.612	57.272	2.29%	58.228	57.305	1.59%
λ_7	68.417	61.901	9.52%	68.038	62.503	8.14%
λ_8	102.650	100.170	2.42%	102.580	100.250	2.27%
λ_9	102.930	100.810	2.06%	102.860	100.720	2.08%
λ_{10}	114.310	114.310	0.00%	114.680	114.31	0.32%

According to the results given in Tables 5.2 and 5.3, the present study is in good agreement with ANSYS results for the natural frequency values of the structure for the different curvatures. The lowest error (0%) occurs at the tenth mode of $R_{xx}=2.5a$ (114.31 Hz ANSYS / 114.31 Hz Present Study), while the highest error (14.63%) occurs at the third mode of $R_{xx}=1.5a$ (40.825 Hz ANSYS / 46.798 Hz Present Study). The average error rate for all curvatures is 4.61%. As seen from Figure 5.3, higher error rates are evaluated at twisting and some bending modes. However, no matter which mode occurs, error rates diminish as the curvature of the structure increases. Hence, the evaluation of high error rates depends mostly on the characteristics of the curved frame structure's mode shape. The change in radius of curvature does not cause differences in the characteristics of the structure's mode shapes, as seen from Figure 5.3. However, some mode shapes shifted to another mode due to this change.

For instance, the seventh mode switched to the eighth mode when the radius of curvature changed from $1.5a$ to $2a$, as seen from Figure 5.3.

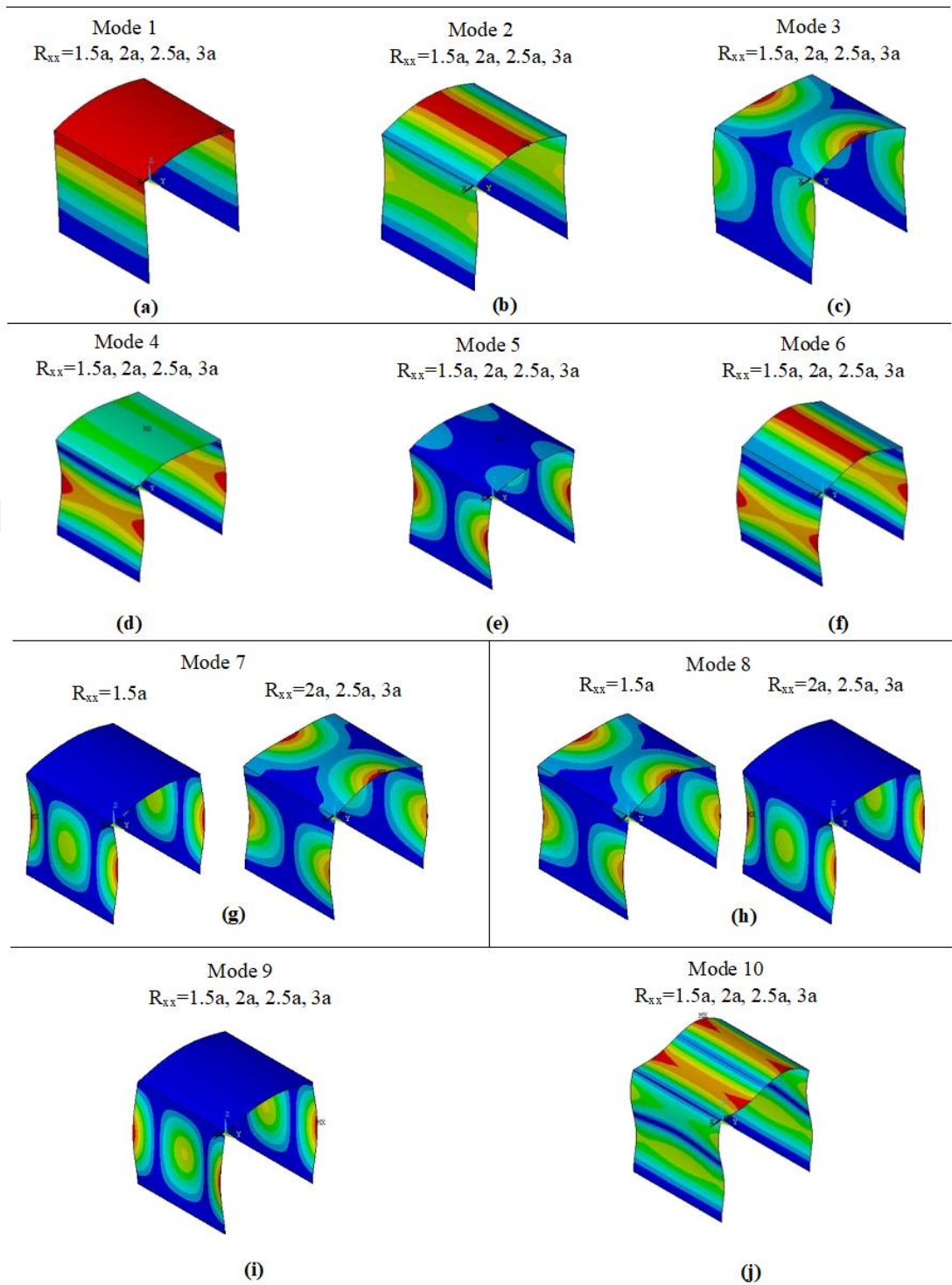


Figure 5.3 Mode shapes of single-bay curved plate frame with different radius of curvatures (R_{xx})

According to the natural frequency results of the present study, given in Tables 5.2 and 5.3, the difference between the first, second, and ninth natural frequency values of the curved frame structure with four different curvature is negligible.

The third natural frequency value decreases slightly as the curvature of the curved plate frame increases. The fifth, sixth, and eighth natural frequency values increase slightly as the curvature increases. However, as the curvature value increases, this increment gets smaller and becomes negligible at last. Apart from other modes, the increment in the fourth, seventh, and the tenth natural frequency values are relatively considerable (approximately 1 Hz) when the curvature changes from $R_{xx}=1.5a$ to $R_{xx}=2a$. Nevertheless, this increment changes similar to the fourth, fifth, sixth, and eight natural frequency values when the curvature value is bigger than $R_{xx}=2a$.

Figure 5.4 shows the isotropic two-bay curved plate frame structure. Tables 5.4 and 5.5 give the first ten natural frequency values of the two-bay structure with different radii of curvature. Figures 5.5 presents the mode shapes of the two-bay structure with curvatures 1.5, 2, 2.5, and 3, respectively.

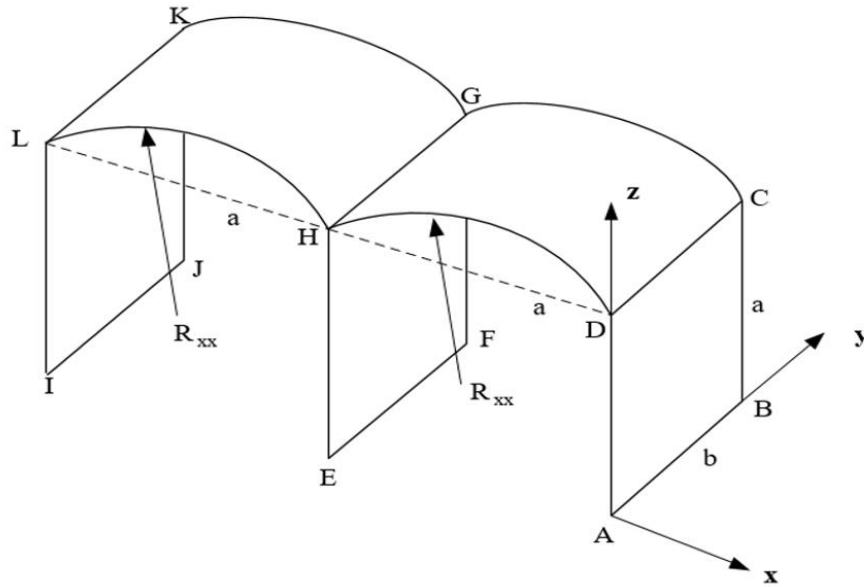


Figure 5.4 Two-bay curved plate frame structure

Table 5.4 The first ten natural frequency values of the curved plate structure for $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}= 2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.114	7.156	0.59%	7.160	7.237	1.07%
λ_2	28.240	30.341	7.44%	28.676	30.478	6.28%
λ_3	33.014	39.646	20.09%	34.231	39.935	16.66%
λ_4	40.178	46.400	15.49%	41.193	45.817	11.23%
λ_5	48.940	47.448	3.05%	49.392	49.265	0.26%
λ_6	54.910	50.917	7.27%	54.841	51.555	5.99%
λ_7	56.843	51.311	9.73%	56.828	52.266	8.03%
λ_8	57.487	56.683	1.40%	57.080	57.355	0.48%
λ_9	58.303	58.152	0.26%	57.894	58.161	0.46%
λ_{10}	67.502	60.329	10.63%	66.945	61.612	7.97%

Table 5.5 The first ten natural frequency values of the curved plate structure for $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}= 3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.182	7.272	1.25%	7.195	7.280	1.17%
λ_2	28.905	30.525	5.60%	29.041	30.561	5.23%
λ_3	34.921	39.813	14.01%	35.363	39.680	12.21%
λ_4	41.755	45.415	8.77%	42.108	45.224	7.40%
λ_5	49.636	50.710	2.16%	49.788	51.521	3.48%
λ_6	54.595	51.704	5.30%	54.264	51.809	4.52%
λ_7	56.404	53.051	5.94%	56.110	53.524	4.61%
λ_8	57.206	57.540	0.58%	57.121	57.475	0.62%
λ_9	57.473	58.132	1.15%	57.283	58.141	1.50%
λ_{10}	66.541	62.473	6.11%	66.245	62.988	4.92%

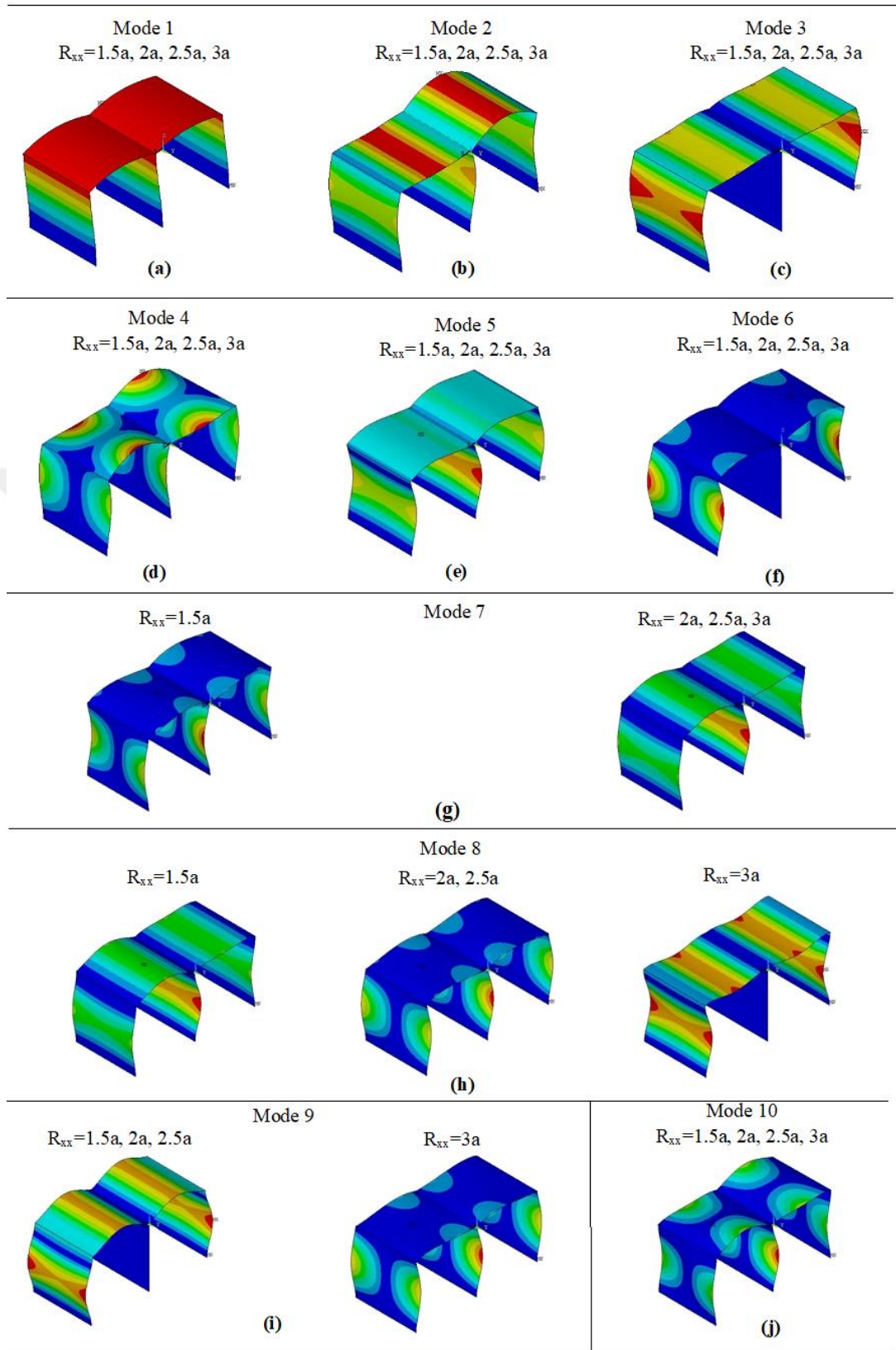


Figure 5.5 Mode shapes of two-bay curved plate frame with different radius of curvatures (R_{xx})

Analyses results from Tables 5.4 and 5.5 indicate that the results of the present study are in good agreement with ANSYS results in terms of the natural frequency values. The lowest error percentage (0.26%) occurs at the fifth mode of $R_{xx}=2a$ (49.392 Hz ANSYS / 49.265 Hz Present Study) and the ninth mode of $R_{xx}=1.5a$ (58.303 Hz ANSYS / 58.152 Hz Present Study) while the highest error percentage takes place at the third mode of $R_{xx}=1.5a$ (33.014 Hz ANSYS / 39.646 Hz Present Study). The average error rate for this case is 5.77. Similar to the single-bay structure, different types of bending and twisting (torsion) as seen from Figure 5.5, affect the accuracy of results of the natural frequencies. It seen from Figure 5.5 that the mode shapes of the seventh, eighth, and ninth modes differ with respect to radius of curvature. Two different modes are seen for the seventh and ninth mode while there are three different modes in the eighth mode when the radius of curvature changes.

As seen in Tables 5.4 and 5.5, the difference of first, second, and ninth natural frequency values among four different radii of curvature is negligible.

The third, sixth, and eighth natural frequencies increase slightly as the curvature shifts from $R_{xx}=1.5a$ to $R_{xx}=2a$. However, this change becomes negligible as the curvature increases from $R_{xx}=2a$ to $R_{xx}=3a$. The fourth natural frequency decreases slightly as the radii of curvature increases.

The increment of the fifth, seventh, and tenth natural frequency values with respect to increase in the curved plate frame structure is relatively considerable (between 0.5Hz and 1.8Hz).

5.2.1.2 Effect of the Aspect Ratio

The effect of aspect ratio (a/b) on the natural frequency values of the single-bay and two-bay curved plate frame structure is investigated. Four aspect ratio, 0.5, 1, 1.5, and 2, is considered. It should be noted that the length of the plate structure (a) is set as constant.

Tables 5.6 and 5.7 give the first ten natural frequency values and the critical buckling load. Figures 5.6 and 5.7 present the mode shapes of the structure with aspect ratios 0.5, 1, 1.5, and 2, respectively. It is seen from Figures 5.6 and 5.7 that the mode shapes of the first two modes remain the same while the third – tenth modes contain different types of bending or torsional mode shapes when the aspect ratio changes. The seventh mode has two different mode shapes whereas the eighth – tenth modes have three mode shapes as the aspect ratio differs. Besides, for each aspect ratio, four different mode shapes are seen in the fourth and fifth modes.

Table 5.6 The first ten natural frequency values of the curved plate structure for $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.808	7.836	0.36%	7.882	7.836	0.58%
λ_2	30.019	31.871	6.17%	29.836	31.871	6.82%
λ_3	33.358	36.175	8.44%	42.309	46.597	10.13%
λ_4	48.360	48.801	0.91%	49.564	51.147	3.19%
λ_5	50.359	51.147	1.56%	56.514	52.024	7.94%
λ_6	59.130	52.024	12.02%	58.880	57.162	2.92%
λ_7	60.089	54.098	9.97%	69.134	60.932	11.86%
λ_8	61.276	59.650	2.65%	102.560	99.953	2.54%
λ_9	61.802	60.977	1.33%	102.840	100.930	1.86%
λ_{10}	83.088	79.965	3.76%	109.240	113.990	4.35%

Table 5.7 The first ten natural frequency values of the curved plate structure for $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.739	7.836	1.26%	7.704	7.836	1.71%
λ_2	29.626	31.871	7.58%	29.461	31.871	8.18%
λ_3	49.832	51.147	2.64%	49.633	51.147	3.05%
λ_4	52.832	52.024	1.53%	58.756	52.024	11.46%
λ_5	58.950	59.749	1.36%	64.185	73.805	14.99%
λ_6	67.879	68.472	0.87%	80.811	81.297	0.60%
λ_7	79.103	70.828	10.46%	90.784	82.828	8.76%
λ_8	113.130	113.990	0.76%	112.590	113.990	1.24%
λ_9	144.280	137.790	4.50%	143.690	137.790	4.11%
λ_{10}	146.810	148.40	1.08%	168.030	156.090	7.11%

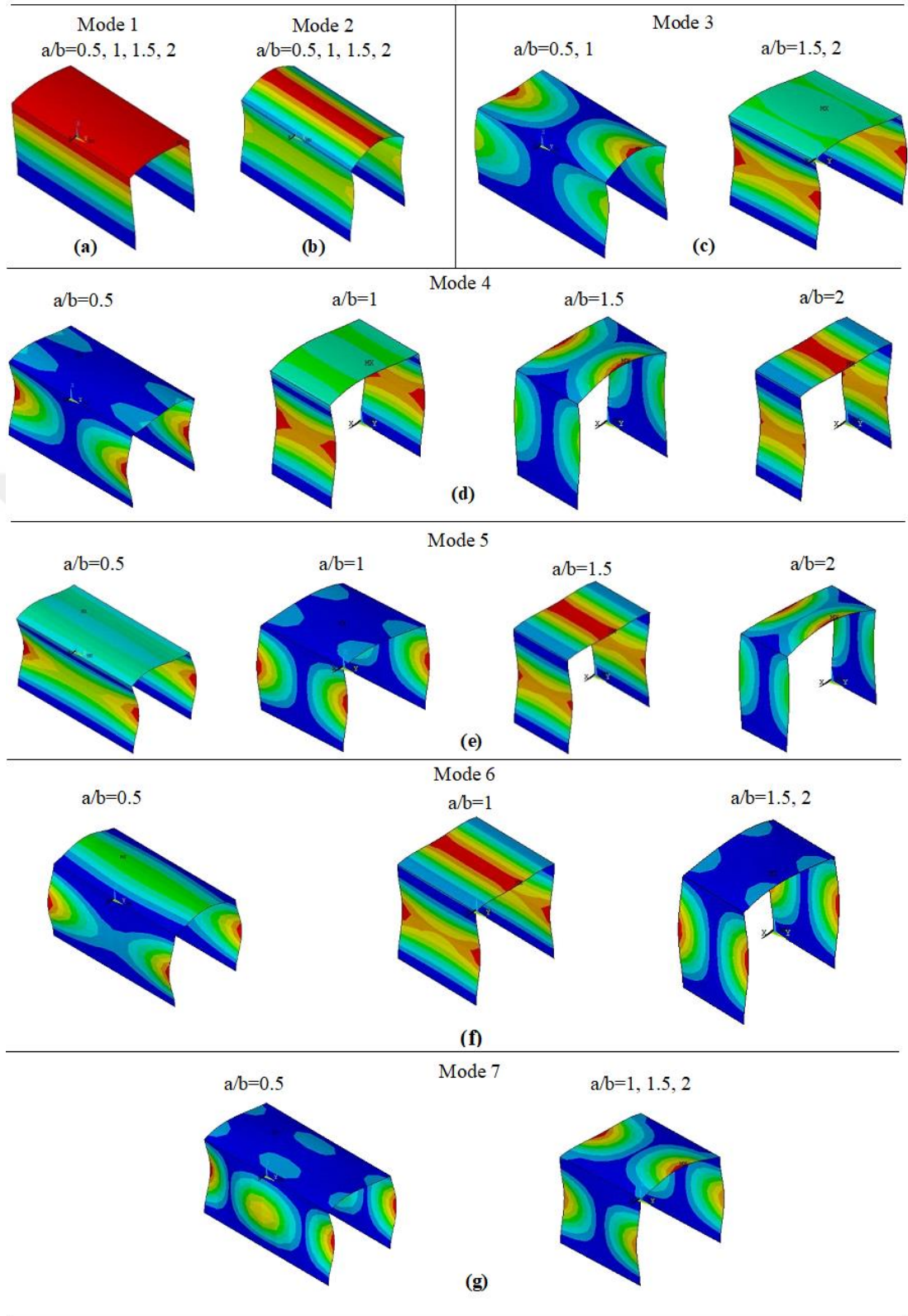


Figure 5.6 First seven mode shapes of single-bay curved plate frame with different aspect ratios (a/b)

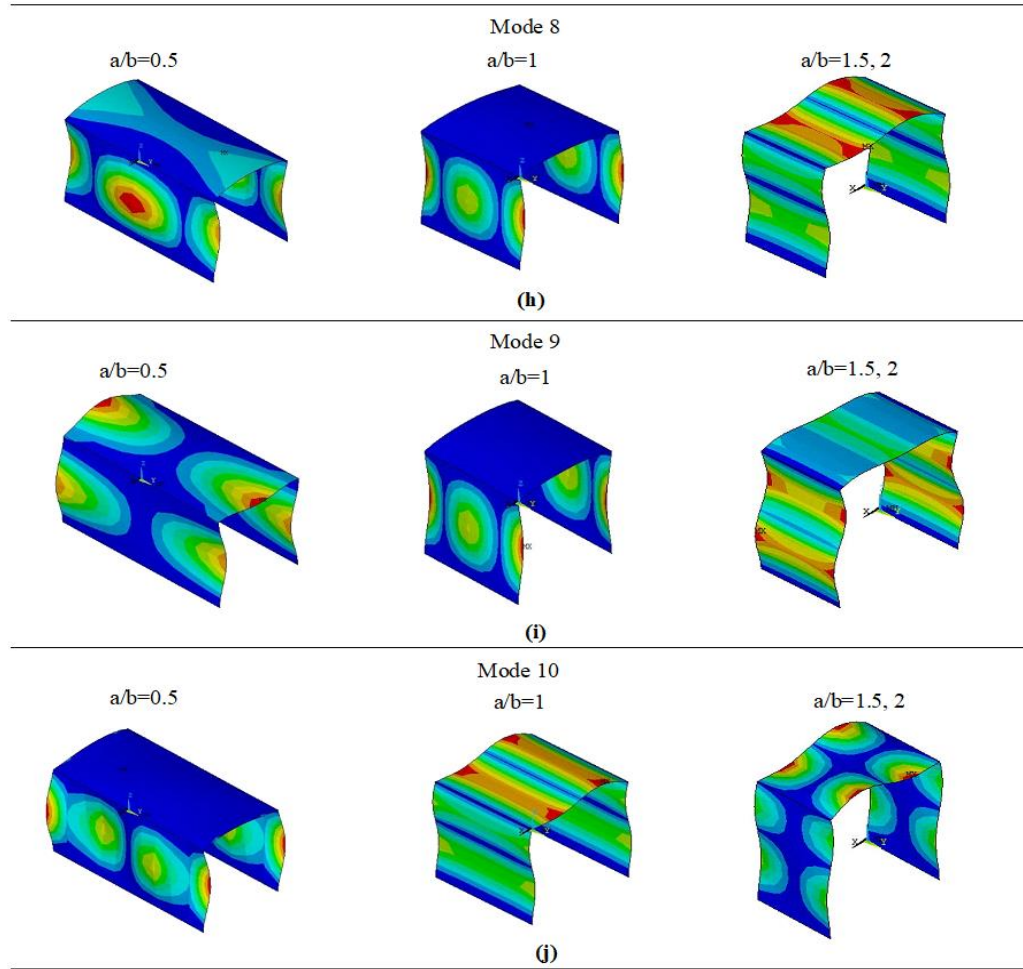


Figure 5.7 The eighth – tenth mode shapes of single-bay curved plate frame with different aspect ratios (a/b)

According to the results given in Tables 5.6 and 5.7, the present study is in good agreement with ANSYS results in terms of the natural frequency values of the structure when aspect ratio changes. The lowest error (0.36%) occurs at the first mode of $a/b=0.5$ (7.808 Hz ANSYS / 7.836 Hz Present Study), while the highest error (14.99%) occurs at the fifth mode of $a/b = 2$ (64.185 Hz ANSYS / 73.805 Hz Present Study). The average error rate for all aspect ratios is 4.82%. High error rates are obtained at different modes as the aspect ratio changes. However, those modes with corresponding high errors have the same mode shapes, which are the patterns of bending and twisting (torsion), as seen in Figure 5.6. The same interpretation can be made for the lower error rates, which take place at different modes but at the same mode shapes.

As seen in Tables 5.6 and 5.7, the first two natural frequencies change negligibly as the aspect ratio of the curved plate frame structure increases. The third, fourth, eighth, and ninth natural frequency values increase when the aspect ratio of the curve plate frame structure increases up to $a/b=1.5$. As the aspect ratio rises to $a/b=2$ from $a/b=1.5$, the difference in these natural frequency values are not considerable. The fifth, sixth, seventh, and tenth natural frequency values increase as the aspect ratio increases. Tables 5.8 and 5.9 present the first ten natural frequency values of the two-bay structure, shown in Figure 3.6, with different aspect ratios. Figures 5.8 and 5.9 give the mode shapes of the two-bay structure with aspect ratios, 0.5, 1, 1.5, and 2, respectively.

Table 5.8 The first ten natural frequency values of the two-bay curved plate structure for $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.193	7.237	0.61%	7.160	7.237	1.07%
λ_2	28.875	30.478	5.55%	28.676	30.478	6.28%
λ_3	32.400	34.760	7.28%	34.231	39.935	16.66%
λ_4	34.453	39.935	15.91%	41.193	45.817	11.23%
λ_5	45.697	49.132	7.52%	49.392	49.265	0.26%
λ_6	48.960	49.265	0.62%	54.841	51.555	5.99%
λ_7	49.684	49.901	0.44%	56.828	52.266	8.03%
λ_8	57.013	51.555	9.57%	57.080	57.355	0.48%
λ_9	57.922	52.266	9.76%	57.894	58.161	0.46%
λ_{10}	59.721	54.621	8.54%	66.945	61.612	7.97%

Table 5.9 The first ten natural frequency values of the two-bay curved plate structure for $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	7.127	7.237	1.55%	7.094	7.237	2.02%
λ_2	28.495	30.478	6.96%	28.328	30.478	7.59%
λ_3	34.038	39.935	17.32%	33.869	39.935	17.91%
λ_4	49.143	49.265	0.25%	48.940	49.265	0.66%
λ_5	52.202	51.555	1.24%	56.409	51.555	8.61%
λ_6	56.607	52.266	7.67%	57.467	52.266	9.05%
λ_7	57.673	59.787	3.67%	63.673	74.559	17.10%
λ_8	65.906	68.485	3.91%	78.106	81.212	3.98%
λ_9	68.276	69.395	1.64%	81.063	82.267	1.49%
λ_{10}	77.243	71.652	7.24%	89.134	83.579	6.23%

As inferred from Tables 5.8 and 5.9, the natural frequency results of the present study are in good agreement with ANSYS results for different aspect ratios. The lowest error rate (0.25%) is evaluated at the fourth node when the aspect ratio is $a/b=1.5$ (49.143 Hz ANSYS / 49.265 Hz Present Study), whereas the highest error occurs at the third mode of $a/b=2$ (33.869 Hz ANSYS / 39.935 Hz Present Study). The average error rate for this case is 6.26%. High error rates are seen in specific modes of bending and twisting (torsion) modes, as seen from Figures 5.8 and 5.9. It is seen from Figures 5.8 and 5.9 that only the first, second, and tenth mode shapes remain the same when the aspect ratio differs. The third – ninth modes have various mode shapes, including different types of bending and torsional modes as the aspect ratio changes. Two different mode shapes are seen in the third mode while there are three different mode shapes in the fourth, eighth, and ninth modes. The mode shapes of the fifth – seventh modes vary for each value of the aspect ratio.

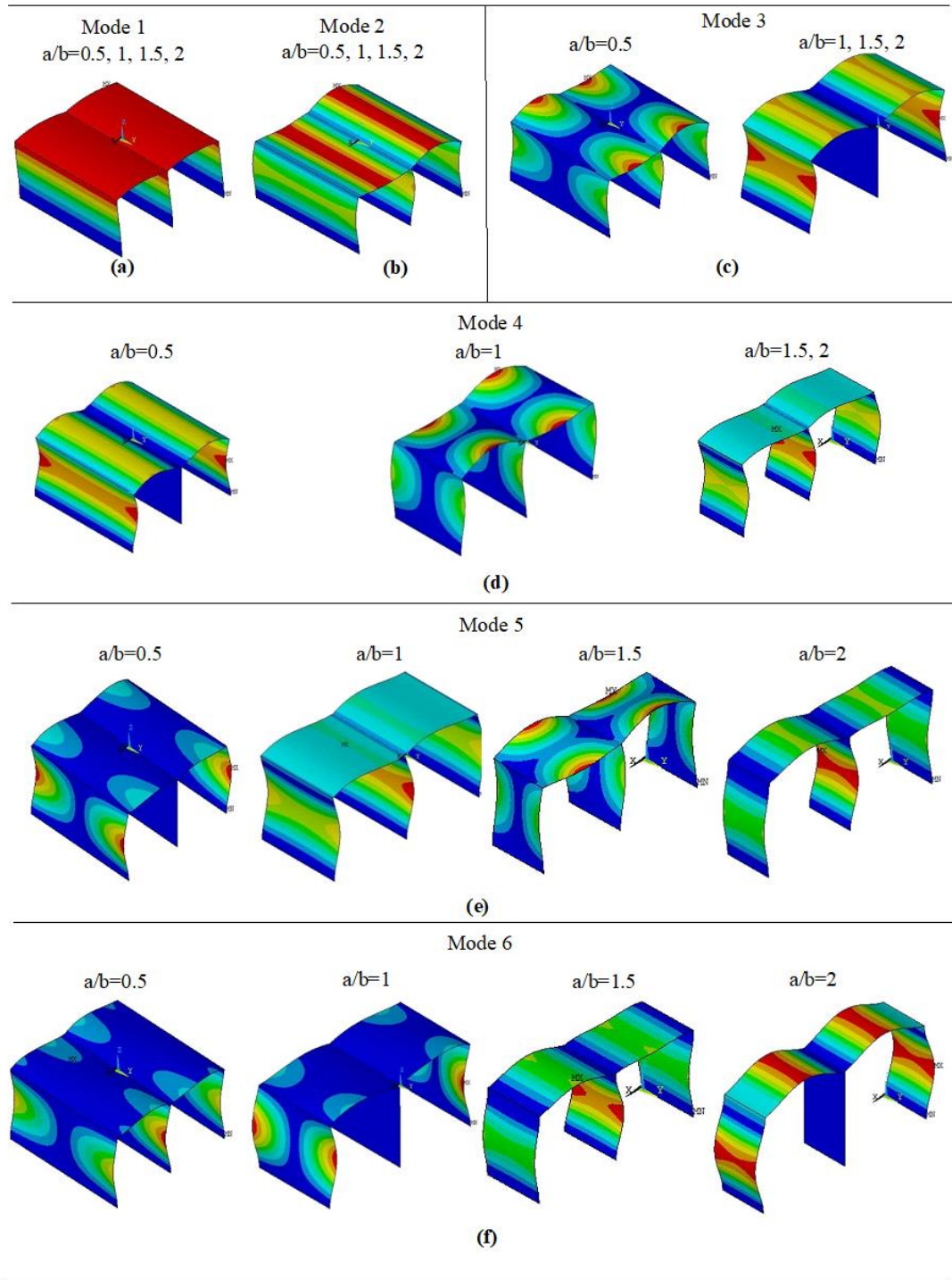


Figure 5.8 First six mode shapes of two-bay curved plate frame with different aspect ratios (a/b)

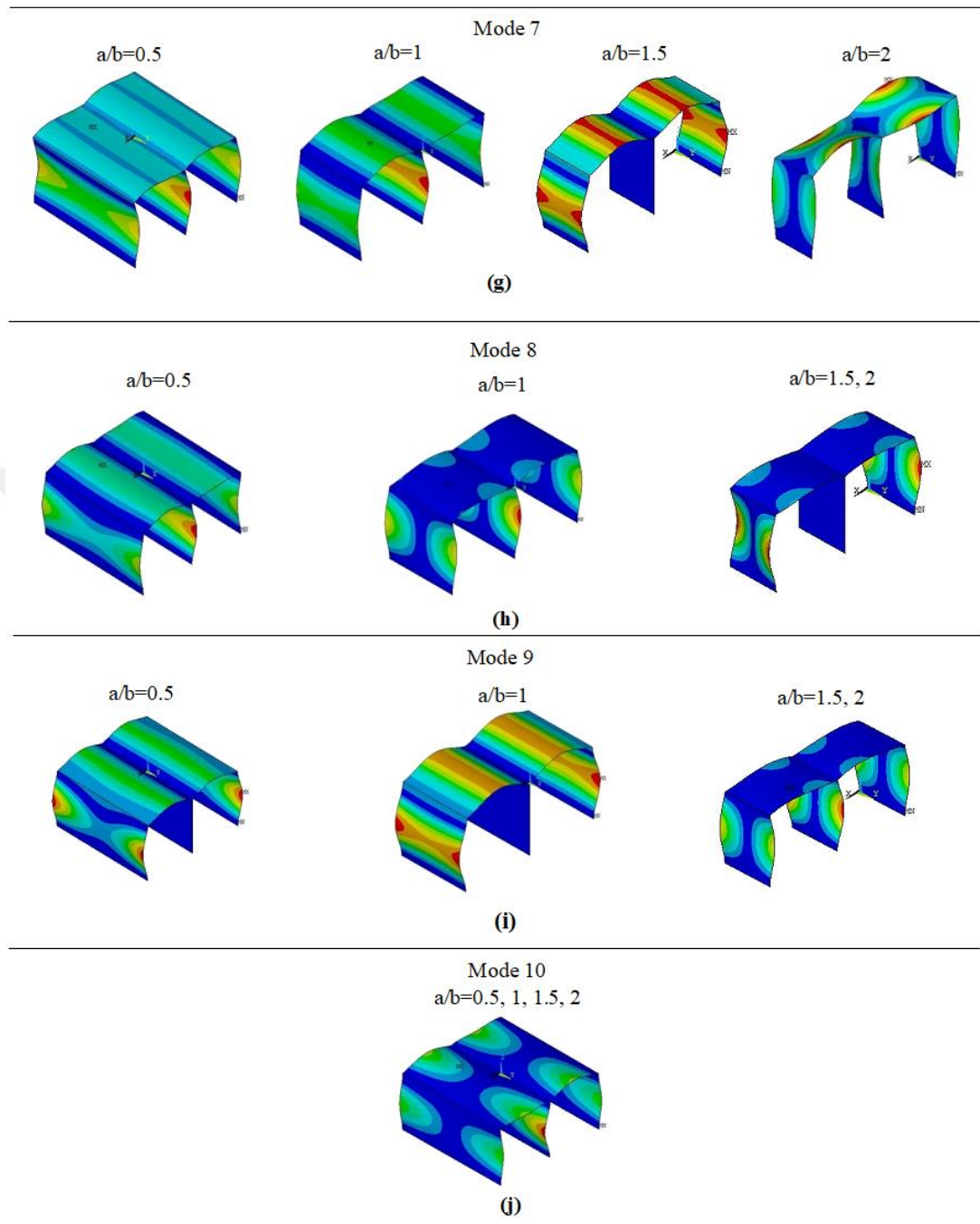


Figure 5.9 The seventh - ninth mode shapes of two-bay curved plate frame with different aspect ratios (a/b)

According to the results given in Tables 5.8 and 5.9, negligible changes occur for the first and second natural frequency values when the aspect ratio of the curved plate frame structure increases.

The third natural frequency increase as the aspect ratio shifts from $a/b=0.5$ to $a/b=1$. Nevertheless, it changes negligibly when the aspect ratio increases to $a/b=1.5$ and $a/b=2$, respectively. The fourth natural frequency increase as the aspect ratio increases from $a/b=0.5$ to $a/b=1.5$. However, this value remains almost the same as $a/b=1.5$ when the aspect ratio increases to $a/b=2$. The change of the fifth natural frequency value is negligible when the aspect ratio increases from $a/b=0.5$ to $a/b=1$. It increases as the aspect ratio increases from $a/b=1$ to $a/b=1.5$ and do not change considerably when the aspect ratio shifts from $a/b=1.5$ to $a/b=2$. The sixth natural frequency value increases as the aspect ratio of the curved plate frame increase from $a/b=0.5$ to $a/b=1.5$. Similar to the fifth natural frequency value, negligible difference occurs when the aspect ratio increases from $a/b=1.5$ to $a/b=2$. The seventh, eighth, ninth, and tenth natural frequency values increase as the aspect ratio increases.

5.2.2 Buckling Analysis of Isotropic Curved Plate Frames

5.2.2.1 Effect of the Radius of Curvature

Table 5.10 gives the critical buckling values of the single-bay and two-bay curved plate frame structures with four different radii of curvature.

Table 5.10 Buckling analysis results of the single-bay and two-bay curved frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
$1.5a$	92942.000	92635.000	0.33%	$1.5a$	143390.000	142690.000	0.49%
$2a$	93353.000	93798.000	0.45%	$2a$	144650.000	144470.000	0.12%
$2.5a$	93825.000	94313.000	0.52%	$2.5a$	145380.000	145210.000	0.12%
$3a$	94012.000	94574.000	0.60%	$3a$	145450.000	145590.000	0.10%

The present study is in a close agreement with ANSYS results for the critical buckling load. As seen in Table 5.10, the average error rate is 0.34%.

The critical buckling load increases as the radius of curvature of the curved plate frame structure increases. For the two-bay curved frame structure, the critical buckling load is evaluated approximately 1.5 times bigger than the critical buckling load of the single-bay structure for every radius of curvature values.

5.2.2.2 Effect of the Aspect Ratio

Table 5.11 gives the critical buckling values of the single-bay and two-bay curved plate frame structures with four different aspect ratio values.

Table 5.11 Buckling analysis results of the single-bay and two-bay curved frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=2$

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	188520.000	187680.000	0.49%	0.5	291660.000	289080.000	0.88%
1	93353.000	93798.000	0.45%	1	144650.000	144470.000	0.12%
1.5	61699.000	62506.000	1.29%	1.5	95553.000	96272.000	0.75%
2	45733.000	46857.000	2.44%	2	71170.000	72168.000	1.40%

As seen in Table 5.11, the present study is in a close agreement with ANSYS results in terms of evaluating the critical buckling load for different aspect ratios.

The average error rate is evaluated as 0.79%. It is concluded that the critical buckling load decreases as the aspect ratio of the two-bay curved plate frame structure increases. For the two-bay curved plate structure, the critical buckling load is 1.5 bigger than the critical buckling load of the single-bay curved plate structure for every aspect ratio values.

5.2.3 Conclusions

According to the convergence analysis results in Figure 5.1, the curved plate frame structure is represented effectively by combining Kirchhoff- Love Plate Theory and Finite Element Method. The error rates change with respect to the mode shape of the curved plate frame structure. Apart from some bending and twisting (torsional) modes where error rates are evaluated relatively high, the remained error rates are low and acceptable. Besides, the critical buckling load values for all cases are obtained accurately. This study's outcomes are important for the understanding of modeling for such structures that can be considered for further theoretical and experimental analyses.

The natural frequency values mostly increase as the curvature of the single-bay curved plate frame structure increases. However, this increment diminishes and becomes negligible, especially when the curvature of the frame structure is at its highest value. Hence, it is inferred that the natural frequencies remain nearly stable when the curvature of the frame structure is at its slightest value, which is, in this study, $R_{xx}=3a$. The natural frequency values of the two-bay curved plate frame structure behave in the same way as the single-bay curved plate frame structure. However, the increment is relatively high when compared with the single-bay curved frame structure. The natural frequency values do not remain stable at $R_{xx}=3a$ as it is for the single-bay curved frame structure. Hence, the curvature is not at its slightest value at $R_{xx}=3a$ for a two-bay structure. This means the curvature effect is still considerable at $R_{xx}=3a$ for the two-bay curved plate frame structure while it is negligible for the single-bay curved plate frame structure.

The increment or decrement in the aspect ratio of the single-bay curved plate frame structures has different effects on the natural frequency values of the curved plate frame structure depending on the value of the aspect ratio. The first and second natural frequency values do not affected considerably by this change. Likewise, the changes in the third, fourth, eighth, and ninth natural frequency values are negligible as the aspect ratio of the curved plate frame structure changes from $a/b=1.5$ to $a/b=2$.

Other natural frequency values increase as the aspect ratio increases. On the other hand, there are differences in terms of the change of the natural frequency values for the two-bay curved plate frame structure. The changes in the second and third natural frequency values are negligible as the aspect ratio of the curved plate frame structure changes from $a/b=1$ to $a/b=2$. Additionally, the fourth, fifth, and sixth natural frequency does not change significantly as the aspect ratio shifts from $a/b=1.5$ to $a/b=2$. Similar to the single-bay curved plate frame structure, the first natural frequency of the two-bay structure is not affected considerably by the change in aspect ratio. All other natural frequency values increase as the aspect ratio increases. The critical buckling load increases as the curvature of the single-bay and two-bay curved plate frame structure increases, whereas it decreases as the aspect ratio of these structures increases.

5.3 Numerical Results of Finite Element Analysis of Laminated Composite Curved Thin Plate Frames

The finite element analysis is performed to obtain the first ten natural frequencies critical buckling loads, and the instability regions of the single-bay and two-bay laminated curved thin plate frames. The effects of the radius of curvature and the aspect ratio on the natural frequency and critical buckling values are investigated. Besides, the effect of the radius of curvature of the curved plate frame structure on the unstable region is investigated.

All analyses are performed by using the material and geometric properties given in Table 5.12. Five different stacking orders are denoted C1, C2, C3, C4, and C5 for simplicity. The details about these fiber orientations are presented in section 3.3.

Table 5.12 Material properties and geometry of the structure of laminated composite curved frames

Property	Symbol	Quantity
Longitudinal Elasticity Modulus	E_x	45 GPa
Transverse Elasticity Modulus	E_y	12 GPa
Density	ρ	2080 kg/m ³
Poisson Ratio	ν	0.33
Geometric Properties of the Structure		
Plate component length	a	1000 mm
Plate component width	b	1000 mm
The thickness of the structure	h	10 mm

The number of 30x30 elements are used due to the convergence analysis results shown in Figure 5.1.

5.3.1 Free Vibration Analysis of Laminated Composite Curved Thin Plate Frames

In this section, the free vibration analysis of the laminated composite curved plate frame fixed at all ends is investigated. Four different radii of curvature (R_{xx}) and aspect ratio (a/b) parameters are considered. The same parameters are taken into account for the two-bay structure.

5.3.1.1 Effect of the Radius of Curvature

This chapter comprises the effect of the radius of curvature on the natural frequency values of the single-bay and two-bay laminated composite curved plate frame structure. For this purpose, four different radii, as presented in section 5.2.1.1, are considered. Tables 5.13 and 5.14 give the first ten natural frequency values of the single-bay laminated composite plate frame structure having C1 fiber orientation.

Table 5.13 The first ten natural frequency values of the laminated composite curved plate frame structure for C1 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.832	6.823	0.14%	6.875	6.880	0.07%
λ_2	25.935	27.914	7.63%	26.432	27.960	5.78%
λ_3	30.807	34.575	12.23%	31.634	34.545	9.20%
λ_4	43.880	43.776	0.24%	44.337	44.873	1.21%
λ_5	44.480	44.741	0.59%	44.731	45.060	0.74%
λ_6	52.828	45.445	13.98%	52.099	45.644	12.39%
λ_7	56.474	48.161	14.72%	55.947	49.210	12.04%
λ_8	64.082	63.458	0.97%	63.902	63.898	0.01%
λ_9	64.563	64.964	0.62%	64.516	64.798	0.44%
λ_{10}	98.642	99.178	0.54%	99.777	100.030	0.25%

Table 5.14 The first ten natural frequency values of the laminated composite curved plate frame structure for C1 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.896	6.905	0.13%	6.909	6.908	0.01%
λ_2	26.710	27.933	4.58%	26.885	27.920	3.85%
λ_3	32.094	34.450	7.34%	32.384	34.387	6.19%
λ_4	44.586	45.151	1.27%	44.743	45.180	0.98%
λ_5	44.869	45.177	0.69%	44.955	45.195	0.53%
λ_6	51.621	46.155	10.59%	51.288	46.589	9.16%
λ_7	55.547	50.051	9.89%	55.248	50.563	8.48%
λ_8	63.800	63.990	0.30%	63.744	63.984	0.38%
λ_9	64.412	64.705	0.45%	64.266	64.622	0.55%
λ_{10}	100.37	100.300	0.07%	100.74	100.290	0.45%

According to the natural frequency results in Tables 5.13 and 5.14, the outcomes of the present study are in good agreement with ANSYS results. The change in the first three natural frequency values with respect to the radius of curvature is negligible. The fourth and tenth natural frequency values slightly increase when the radius of curvature increases. However, this increment becomes negligible after $R_{xx}=2.5a$. The fifth, sixth, eighth, and ninth natural frequency values slightly increase with respect to the increment of the radius of curvature of the curved plate frame structure. Apart from the fourth and tenth natural frequency values, this increment becomes negligible after $R_{xx}=2a$. The sixth and seventh natural frequency values slightly increase as the radius of curvature increases.

Tables 5.15 and 5.16 give the first ten natural frequency results of laminated composite curved plate frame structures having C2 fiber orientation.

Table 5.15 The first ten natural frequency values of the laminated composite curved plate frame structure for C2 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.082	4.088	0.16%	4.116	4.114	0.05%
λ_2	16.698	15.518	7.60%	16.725	15.816	5.75%
λ_3	26.192	23.090	13.43%	26.390	23.735	11.19%
λ_4	26.564	26.266	1.13%	26.846	26.539	1.16%
λ_5	27.186	31.215	12.91%	27.308	31.209	12.50%
λ_6	31.373	31.647	0.87%	31.613	31.367	0.78%
λ_7	32.818	38.176	14.03%	33.378	37.784	11.66%
λ_8	59.356	59.089	0.45%	59.861	59.767	0.16%
λ_9	66.586	67.579	1.47%	66.908	67.517	0.90%
λ_{10}	67.436	67.584	0.22%	67.379	67.575	0.29%

Table 5.16 The first ten natural frequency values of the laminated composite curved plate frame structure for C2 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.131	4.126	0.10%	4.133	4.133	0.03%
λ_2	16.708	15.981	4.55%	16.700	16.086	3.82%
λ_3	26.234	24.101	8.85%	26.143	24.334	7.43%
λ_4	27.025	26.688	1.26%	27.035	26.782	0.94%
λ_5	27.617	30.922	10.69%	27.878	30.722	9.26%
λ_6	31.674	31.452	0.71%	31.692	31.505	0.59%
λ_7	33.887	37.497	9.63%	34.209	37.285	8.25%
λ_8	60.019	60.124	0.17%	60.009	60.339	0.55%
λ_9	67.029	67.465	0.65%	67.076	67.428	0.52%
λ_{10}	67.330	67.561	0.34%	67.293	67.540	0.37%

According to the natural frequency results given in Tables 5.15 and 5.16, the present study is in good agreement with ANSYS results. The first, second, third, and tenth natural frequencies do not change considerably as the radius of curvature of curved plate frame structure increases or decreases. Therefore, these changes are considered as negligible. The fourth, eighth, and ninth natural frequency values increase as the radius of curvature of curved plate frame structure increases.

However, this increment becomes negligible as the radius of the curvature of the frame structure reaches $R_{xx}=2.5a$. The fifth and seventh natural frequency values increase slightly with respect to the increment of the radius of curvature. The sixth natural frequency value increases as the radius of the curvature of the plate frame structure increases. Nevertheless, this increment becomes negligible as the radius of curvature of the curved plate frame structure reaches $R_{xx}=2a$.

Tables 5.17-5.22 give the first ten natural frequency results of a laminated composite curved plate frame structure with C3, C4, and C5 fiber orientation. Analysis results presented in Tables 5.17-5.22 indicate that the same interpretation done for C1 can be made for C3, C4, and C5.

Table 5.17 The first ten natural frequency values of the laminated composite curved plate frame structure for C3 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.519	6.503	0.24%	6.559	6.558	0.02%
λ_2	24.793	26.604	7.30%	25.269	26.648	5.46%
λ_3	29.879	33.564	12.33%	30.684	33.512	9.22%
λ_4	42.025	41.724	0.72%	42.462	42.771	0.73%
λ_5	42.973	43.103	0.30%	43.214	43.410	0.45%
λ_6	50.635	43.318	14.45%	49.933	43.506	12.87%
λ_7	54.454	46.290	14.99%	53.943	47.285	12.34%
λ_8	65.077	64.054	1.57%	64.866	64.492	0.58%
λ_9	65.268	65.463	0.30%	65.221	65.289	0.10%
λ_{10}	94.985	94.500	0.51%	96.074	95.317	0.79%

Table 5.18 The first ten natural frequency values of the laminated composite curved plate frame structure for C3 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.579	6.582	0.03%	6.591	6.585	0.10%
λ_2	25.534	26.623	4.26%	25.702	26.611	3.54%
λ_3	31.133	33.407	7.30%	31.417	33.340	6.12%
λ_4	42.700	43.064	0.85%	42.851	43.082	0.54%
λ_5	43.346	43.496	0.35%	43.429	43.523	0.22%
λ_6	49.472	43.989	11.08%	49.152	44.402	9.66%
λ_7	53.556	48.085	10.22%	53.267	48.575	8.81%
λ_8	64.719	64.596	0.19%	64.621	64.594	0.04%
λ_9	65.141	65.172	0.05%	65.032	65.084	0.08%
λ_{10}	96.648	95.577	1.11%	96.995	95.566	1.47%

Table 5.19 The first ten natural frequency values of the laminated composite curved plate frame structure for C4 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.600	6.585	0.24%	6.641	6.640	0.02%
λ_2	25.097	26.933	7.32%	25.578	26.976	5.47%
λ_3	30.438	34.223	12.44%	31.258	34.168	9.31%
λ_4	42.536	42.246	0.68%	42.978	43.303	0.76%
λ_5	43.687	43.833	0.33%	43.931	44.047	0.26%
λ_6	51.265	43.854	14.46%	50.554	44.144	12.68%
λ_7	55.296	47.022	14.96%	54.774	48.028	12.32%
λ_8	64.834	63.932	1.39%	64.635	64.390	0.38%
λ_9	65.127	65.396	0.41%	65.097	65.225	0.20%
λ_{10}	96.170	95.700	0.49%	97.270	96.522	0.77%

Table 5.20 The first ten natural frequency values of the laminated composite curved plate frame structure for C4 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.661	6.664	0.03%	6.673	6.667	0.10%
λ_2	25.846	26.950	4.27%	26.015	26.937	3.54%
λ_3	31.717	34.059	7.38%	32.006	33.988	6.19%
λ_4	43.219	43.594	0.87%	43.371	43.611	0.55%
λ_5	44.065	44.231	0.38%	44.149	44.259	0.25%
λ_6	50.088	44.542	11.07%	49.763	44.961	9.65%
λ_7	54.379	48.840	10.19%	54.085	49.337	8.78%
λ_8	64.506	64.501	0.01%	64.425	64.503	0.12%
λ_9	65.024	65.117	0.14%	64.921	65.038	0.18%
λ_{10}	97.849	96.782	1.09%	98.199	96.769	1.46%

Table 5.21 The first ten natural frequency values of the laminated composite curved plate frame structure for C5 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.549	6.534	0.24%	6.590	6.589	0.02%
λ_2	24.905	26.724	7.30%	25.382	26.768	5.46%
λ_3	30.185	33.938	12.43%	30.999	33.882	9.30%
λ_4	42.212	41.917	0.70%	42.65	42.966	0.74%
λ_5	43.336	43.478	0.33%	43.578	43.705	0.29%
λ_6	50.869	43.514	14.46%	50.163	43.787	12.71%
λ_7	54.856	46.647	14.96%	54.339	47.645	12.32%
λ_8	64.914	63.939	1.50%	64.706	64.393	0.48%
λ_9	65.134	65.381	0.38%	65.099	65.208	0.17%
λ_{10}	95.426	94.945	0.50%	96.518	95.762	0.78%

Table 5.22 The first ten natural frequency values of the laminated composite curved plate frame structure for C5 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.610	6.612	0.03%	6.622	6.615	0.10%
λ_2	25.648	26.742	4.27%	25.816	26.730	3.54%
λ_3	31.454	33.773	7.37%	31.74	33.703	6.18%
λ_4	42.890	43.258	0.86%	43.040	43.274	0.54%
λ_5	43.711	43.873	0.37%	43.795	43.901	0.24%
λ_6	49.701	44.193	11.08%	49.379	44.609	9.66%
λ_7	53.948	48.451	10.19%	53.657	48.943	8.79%
λ_8	64.565	64.504	0.09%	64.472	64.505	0.05%
λ_9	65.028	65.093	0.10%	64.927	65.009	0.13%
λ_{10}	97.093	96.020	1.11%	97.440	96.008	1.47%

According to the natural frequency results given in Tables 5.13 – 5.22, it is concluded that the effect of the radius of curvature of laminated composite curved plate frame structure on the first ten natural frequency values differs. Nevertheless, changing the radius of curvature has a slight or negligible impact on the first ten natural frequency values of laminated composite curved plate frame structures. Figure 5.10 shows the mode shapes of the single-bay curved plate frame structure having C1, C2, C3, C4, and C5 stacking order. It is seen from Figure 5.10 that the first seven and ninth modes remain the same when the radius of curvature and stacking order differs. However, the mode shapes eighth and tenth modes differ for C2 stacking order. It is seen that these mode shapes replace each other in such a way that the eighth mode shape of C2 becomes the tenth mode shape of C1, C3, C4 and C5.

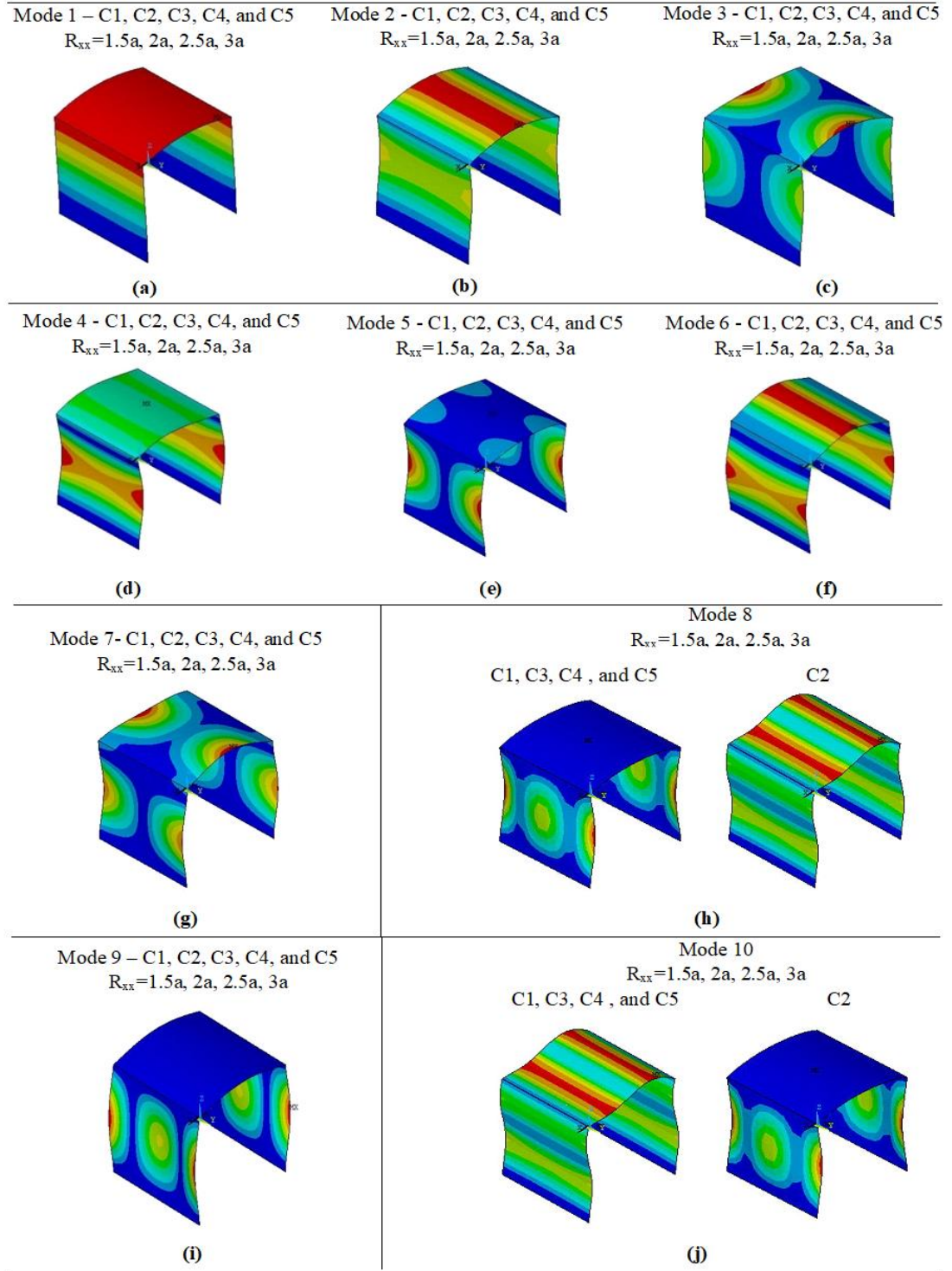


Figure 5.10 Mode shapes of the single-bay curved plate frame having C1, C3, C4, or C5 stacking orders with different radius of curvatures (R_{xx})

Tables 5.23 and 5.24 give the first ten natural frequency results of the two-bay laminated composite curved plate frame structure with C1 fiber orientation.

Table 5.23 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C1 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.333	6.271	0.98%	6.373	6.347	0.41%
λ_2	25.469	26.884	5.56%	25.881	27.096	4.69%
λ_3	29.863	33.305	11.53%	30.988	33.599	8.43%
λ_4	30.446	35.137	15.41%	31.196	35.488	13.76%
λ_5	44.018	42.085	4.39%	43.779	43.829	0.11%
λ_6	44.596	45.193	1.34%	45.015	45.772	1.68%
λ_7	45.945	45.360	1.27%	46.160	45.912	0.54%
λ_8	52.442	45.541	13.16%	51.830	46.459	10.36%
λ_9	53.193	46.598	12.40%	52.798	46.600	11.74%
λ_{10}	56.102	49.181	12.34%	55.653	50.405	9.43%

Table 5.24 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C1 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.393	6.380	0.22%	6.405	6.392	0.22%
λ_2	26.104	27.165	4.06%	26.242	27.113	3.32%
λ_3	31.605	33.692	6.60%	31.857	33.617	5.52%
λ_4	31.632	35.416	11.96%	32.046	35.270	10.06%
λ_5	43.387	45.038	3.81%	42.942	45.744	6.53%
λ_6	45.241	45.928	1.52%	45.383	45.786	0.89%
λ_7	46.273	45.986	0.62%	46.341	46.040	0.65%
λ_8	51.439	46.607	9.39%	51.170	46.642	8.85%
λ_9	52.404	47.132	10.06%	52.079	47.564	8.67%
λ_{10}	55.323	51.145	7.55%	55.079	51.619	6.28%

According to the results given in Tables 5.23 and 5.24, the present study is in good agreement with ANSYS results. The change in the fundamental natural frequency with respect to the change of the radius of curvature of two-bay laminated composite curved plate frame structure is negligible. The second, third, fourth, sixth, seventh, and eighth natural frequency values increase as the radius of curvature increases. However, this increment become negligible as the radius of curvature value reaches to $R_{xx}=2a$. The fifth and tenth natural frequency increase slightly as the radius of curvature increases. The change of the ninth frequency value is negligible when the value of the radius of

curvature shifts from $R_{xx}=1.5a$ to $R_{xx}=2a$. This change becomes more considerable for curvature values bigger than $R_{xx}=2a$.

Tables 5.25 and 5.26 present the first ten natural frequency values of laminated composite curved plate frame structure having C2 fiber orientation.

Table 5.25 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C2 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}=2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.789	3.752	0.99%	3.813	3.797	0.43%
λ_2	15.240	16.083	5.53%	15.486	16.209	4.67%
λ_3	17.875	21.027	17.63%	18.548	21.234	14.48%
λ_4	23.001	25.184	9.49%	23.587	25.869	9.67%
λ_5	26.695	25.827	3.25%	26.946	26.230	2.66%
λ_6	31.177	27.041	13.27%	31.044	27.384	11.79%
λ_7	31.411	27.245	13.26%	31.199	27.798	10.90%
λ_8	31.869	31.704	0.52%	31.633	32.082	1.42%
λ_9	32.046	32.513	1.46%	32.171	32.525	1.10%
λ_{10}	37.880	33.456	11.68%	37.550	34.176	8.99%

Table 5.26 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C2 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}=3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.825	3.817	0.24%	3.832	3.824	0.23%
λ_2	15.619	16.249	4.03%	15.701	16.217	3.29%
λ_3	18.932	21.189	11.92%	19.179	21.077	9.90%
λ_4	23.912	25.850	8.10%	24.116	25.739	6.73%
λ_5	27.082	26.955	0.47%	27.167	27.399	0.85%
λ_6	30.809	27.477	10.82%	30.648	27.548	10.11%
λ_7	31.127	28.202	9.40%	31.007	28.461	8.21%
λ_8	31.397	32.226	2.64%	31.201	32.236	3.32%
λ_9	32.237	32.532	0.92%	32.278	32.539	0.81%
λ_{10}	37.313	34.627	7.20%	37.141	34.921	5.98%

According to the results given in Tables 5.25 and 5.26, the present study is in good agreement with ANSYS results. As the radius of curvature of the laminated composite two-bay curved plate frame structure changes, the changes of the first, second, and ninth natural frequencies are negligible ($\Delta\lambda < 0.15$). The third, fourth, sixth, and eighth

natural frequency values increase slightly with respect to the increment of the radius of curvataure of the two-bay curved plate frame structure.

However, the increment of these natural frequency values become negligible for curvature values bigger than $R_{xx}=2a$. The fifth, seventh, and tenth natural frequency values increase slightly as the radius of curvature increases.

Tables 5.27 and 5.28 give the first ten natural frequency values of laminated composite two-bay curved plate frame structure with C3 fiber orientation.

Table 5.27 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C3 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}= 2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.037	5.978	0.98%	6.075	6.050	0.41%
λ_2	24.275	25.624	5.56%	24.668	25.826	4.69%
λ_3	28.461	32.359	13.70%	29.533	32.615	10.44%
λ_4	29.498	33.482	13.51%	30.227	33.819	11.88%
λ_5	42.228	40.107	5.02%	41.965	41.766	0.48%
λ_6	42.509	43.074	1.33%	42.909	43.630	1.68%
λ_7	44.232	43.410	1.86%	44.437	44.209	0.51%
λ_8	49.982	43.687	12.59%	49.398	44.281	10.36%
λ_9	50.689	44.873	11.47%	50.311	44.871	10.81%
λ_{10}	53.872	47.268	12.26%	53.439	48.434	9.37%

Table 5.28 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C3 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}= 3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.094	6.081	0.22%	6.105	6.092	0.21%
λ_2	24.881	25.891	4.06%	25.012	25.842	3.32%
λ_3	30.146	32.693	8.45%	30.541	32.613	6.78%
λ_4	30.624	33.751	10.21%	30.870	33.644	8.99%
λ_5	41.563	42.917	3.26%	41.121	43.635	6.11%
λ_6	43.125	43.779	1.52%	43.260	43.880	1.43%
λ_7	44.544	44.264	0.63%	44.610	44.010	1.35%
λ_8	49.025	44.878	8.46%	48.769	44.913	7.91%
λ_9	49.935	44.922	10.04%	49.624	45.334	8.65%
λ_{10}	53.121	49.139	7.50%	52.887	49.591	6.23%

According to the results given in Tables 5.27 and 5.28, the present study is in good agreement with ANSYS. The fundamental frequency does not change considerably as the radius of curvature of the laminated composite curved plate frame structure changes. The second, third, fourth, sixth, and seventh frequencies increase slightly as the radius of curvature increases.

However, the increment of these frequencies become negligible for curvature values bigger than $R_{xx}=2a$. The eighth frequency value increases as the curvature of the curved plate frame structure increases. Nevertheless, the increment becomes so small for the curvature values bigger than $R_{xx}=2.5a$. The change of the ninth frequency value is negligible when the value of the radius of curvature shifts from $R_{xx}=2a$ to $R_{xx}=2.5a$. This change becomes more considerable for curvature values bigger than $R_{xx}=2.5a$.

Tables 5.29 - 5.32 give the first ten natural frequency results of the two-bay laminated composite plate frame structure with C4 and C5, respectively.

Table 5.29 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C4 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}= 2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.111	6.053	0.96%	6.150	6.126	0.40%
λ_2	24.573	25.945	5.58%	24.970	26.148	4.72%
λ_3	28.812	33.001	14.54%	29.897	33.264	11.26%
λ_4	30.058	33.911	12.82%	30.802	34.248	11.19%
λ_5	43.029	40.625	5.59%	42.989	42.308	1.58%
λ_6	43.172	43.629	1.06%	43.432	44.181	1.73%
λ_7	44.955	43.959	2.21%	45.162	44.852	0.69%
λ_8	50.602	44.425	12.21%	50.011	44.967	10.09%
λ_9	51.321	45.629	11.09%	50.939	45.635	10.41%
λ_{10}	54.698	48.028	12.19%	54.257	49.210	9.30%

Table 5.30 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C4 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}= 3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.170	6.158	0.20%	6.181	6.169	0.20%
λ_2	25.185	26.213	4.08%	25.317	26.163	3.34%
λ_3	30.517	33.342	9.26%	30.916	33.259	7.58%
λ_4	31.207	34.177	9.52%	31.458	33.997	8.07%
λ_5	42.654	43.475	1.92%	42.257	44.196	4.59%
λ_6	43.651	44.332	1.56%	43.787	44.440	1.49%
λ_7	45.270	45.067	0.45%	45.336	44.882	1.00%
λ_8	49.633	45.502	8.32%	49.374	45.673	7.50%
λ_9	50.559	45.643	9.72%	50.244	45.919	8.61%
λ_{10}	53.934	49.925	7.43%	53.696	50.384	6.17%

Table 5.31 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C5 and $R_{xx}=1.5a$ and $R_{xx}=2a$

Natural Frequency (Hz)	$R_{xx}=1.5a$			$R_{xx}= 2a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.064	6.006	0.97%	6.103	6.079	0.40%
λ_2	24.385	25.742	5.57%	24.779	25.945	4.70%
λ_3	28.591	32.726	14.46%	29.667	32.983	11.18%
λ_4	29.808	33.642	12.86%	30.546	33.977	11.23%
λ_5	42.700	40.300	5.62%	42.539	41.967	1.34%
λ_6	42.746	43.280	1.25%	43.100	43.832	1.70%
λ_7	44.596	43.612	2.21%	44.802	44.493	0.69%
λ_8	50.212	44.062	12.25%	49.626	44.596	10.14%
λ_9	50.924	45.256	11.13%	50.544	45.260	10.45%
λ_{10}	54.267	47.640	12.21%	53.830	48.811	9.32%

Table 5.32 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C5 and $R_{xx}=2.5a$ and $R_{xx}=3a$

Natural Frequency (Hz)	$R_{xx}=2.5a$			$R_{xx}= 3a$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.122	6.110	0.21%	6.133	6.121	0.21%
λ_2	24.992	26.009	4.07%	25.123	25.959	3.33%
λ_3	30.282	33.061	9.18%	30.678	32.978	7.50%
λ_4	30.948	33.908	9.56%	31.196	33.729	8.12%
λ_5	42.183	43.125	2.23%	41.772	43.842	4.96%
λ_6	43.317	43.982	1.53%	43.453	44.087	1.46%
λ_7	44.910	44.682	0.51%	44.975	44.478	1.11%
λ_8	49.251	45.138	8.35%	48.993	45.300	7.54%
λ_9	50.167	45.268	9.76%	49.854	45.551	8.63%
λ_{10}	53.509	49.521	7.45%	53.273	49.976	6.19%

According to the results given in Tables 5.29 – 5.32, the present study is in good agreement with ANSYS results. The fundamental natural frequency values of laminated composite curved plate frame structure having stacking orders C4 and C5 do not change considerably as the radius of curvature of curved plate frame structure changes. The second, third, fourth, and sixth natural frequency values increase slightly as the curvature of the frame structure increases. However, the increment of those frequencies becomes negligible for curvatures bigger than $R_{xx}=2a$. The fifth natural frequency value increases slightly as the curvature of the laminated composite curved plate frame structure increases. The seventh frequency value increases as the curvature of the curved plate frame structure increases. Nevertheless, the increment becomes so small for the curvature values bigger than $R_{xx}=2.5a$. The change of the ninth frequency value is negligible when the value of the radius of curvature changes from $R_{xx}=2a$ to $R_{xx}=2.5a$. This change becomes more considerable for curvature values bigger than $R_{xx}=2.5a$.

It is inferred from Tables 5.23 – 5.32 that the same interpretation made for the single-bay laminated curved plate frame structure can be made for the two-bay laminated curved plate frame structure in terms of the effects of the radius of curvature and fiber orientation on the natural frequency values. Figure 5.11 shows the mode shapes of the two-bay laminated composite curved plate frame structure having different fiber orientations and radius of curvatures. It is seen from Figure 5.11 that the mode shapes of the first two and tenth modes remain the same when the radius of curvature and the stacking order changes. However, the mode shapes of the third – tenth modes differ as the radius of curvature changes. It is seen that the stacking sequence is more effective for two-bay structures when compared with the mode shapes of the single – bay structures. The diversity of the mode shapes of the two-bay structures is much more depended on the stacking order since there are unique modes for almost all stacking orders unlike the mode shapes of the single-bay structures.

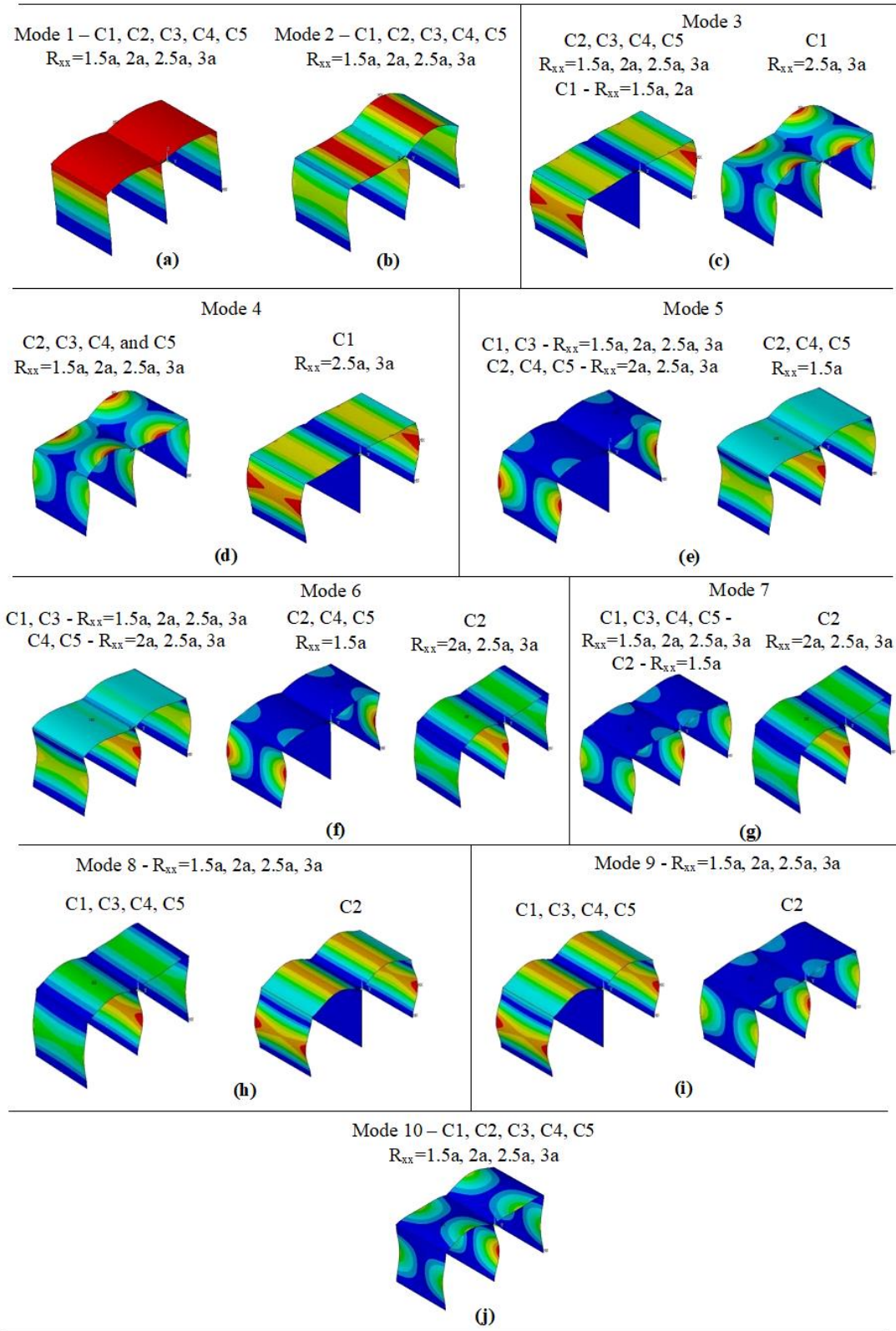


Figure 5.11 Mode shapes of the two-bay curved plate frame having C1, C3, C4, or C5 stacking orders with different radius of curvatures (R_{xx})

5.3.1.2 Effect of the Aspect Ratio

The effect of aspect ratio (a/b) on the natural frequency values of the single-bay and two-bay laminated composite curved plate frame structure is investigated. Four aspect ratios, 0.5, 1, 1.5, and 2, are considered. It should be noted that the length of the plate structure (a) is set as constant.

Tables 5.33 and 5.34 give the first ten natural frequency values of the single-bay laminated composite curved plate frame structure having C1 fiber orientation and different aspect ratios.

Table 5.33 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C1, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.887	6.885	0.03%	6.875	6.880	0.07%
λ_2	26.541	27.990	5.46%	26.432	27.960	5.78%
λ_3	27.855	29.762	6.85%	31.634	34.545	9.20%
λ_4	41.197	41.278	0.20%	44.337	44.873	1.21%
λ_5	44.546	44.891	0.77%	44.731	45.060	0.74%
λ_6	45.349	45.642	0.65%	52.099	45.644	12.39%
λ_7	45.515	45.953	0.96%	55.947	49.210	12.04%
λ_8	52.449	46.311	11.70%	63.902	63.898	0.01%
λ_9	53.195	47.179	11.31%	64.516	64.798	0.44%
λ_{10}	54.367	54.567	0.37%	99.777	100.030	0.25%

Table 5.34 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C1, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.877	6.869	0.12%	6.872	6.870	0.02%
λ_2	26.483	28.296	6.85%	26.457	27.904	5.47%
λ_3	37.012	41.282	11.54%	42.974	44.808	4.27%
λ_4	44.504	45.396	2.00%	44.470	45.580	2.50%
λ_5	50.281	46.182	8.15%	52.337	48.814	6.73%
λ_6	52.366	51.140	2.34%	56.893	57.295	0.71%
λ_7	60.680	54.349	10.43%	66.204	59.541	10.06%
λ_8	94.665	94.229	0.46%	100.660	99.876	0.78%
λ_9	94.813	95.065	0.27%	124.620	120.750	3.11%
λ_{10}	100.740	104.19	3.42%	128.210	125.470	2.14%

According to the results given in Tables 5.33 and 5.34, the present study is in good agreement with ANSYS results. As the aspect ratio increases, the changes in the first and second natural frequency values are negligible. The third, seventh, eighth, and ninth natural frequency values increase as the aspect ratio of the laminated composite curved plate frame structure increases. The fourth natural frequency increases considerably as the aspect ratio increases from 0.5 to 1. However, when the aspect ratio increases from 1 to 1.5, the change in the fourth natural frequency become negligible. As the aspect ratio shifts from 1.5 to 2, the increment of the fourth natural frequency becomes bigger but yet, not considerable. The fifth natural frequency increase slightly as the aspect ratio increases from 0.5 to 1. The increment of this frequency gets bigger as the aspect ratio increases from 1 to 2. The change in the sixth natural frequency is negligible when the aspect ratio increases from 0.5 to 1. Nevertheless, the increment of the sixth frequency becomes more considerable when the aspect ratio increases from 1 to 2. The tenth natural frequency first increases considerably as the aspect ratio increases from 0.5 to 1. Afterward, the increment becomes negligible when the aspect ratio shifts from 1 to 1.5. However, the increment of the tenth natural frequency gets bigger as the aspect ratio of the laminated composite curved plate frame structure increases from 1.5 to 2.

Tables 5.35 and 5.36 give the first ten natural frequencies of laminated composite curved plate frame structure having C2 fiber orientation and different aspect ratios.

Table 5.35 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C2, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.122	4.120	0.05%	4.114	4.116	0.05%
λ_2	15.887	16.749	5.43%	15.816	16.725	5.75%
λ_3	18.183	19.626	7.94%	23.735	26.390	11.19%
λ_4	26.058	26.115	0.22%	26.539	26.846	1.16%
λ_5	26.697	26.875	0.67%	31.209	27.308	12.50%
λ_6	31.409	27.337	12.96%	31.367	31.613	0.78%
λ_7	33.102	28.809	12.97%	37.784	33.378	11.66%
λ_8	34.294	34.302	0.02%	59.767	59.861	0.16%
λ_9	34.817	34.922	0.30%	67.517	66.908	0.90%
λ_{10}	52.727	51.313	2.68%	67.575	67.379	0.29%

Table 5.36 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C2, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	4.114	4.112	0.05%	4.109	4.108	0.04%
λ_2	15.844	16.705	5.43%	15.827	16.688	5.44%
λ_3	26.639	26.824	0.69%	26.619	26.807	0.71%
λ_4	30.620	27.286	10.89%	31.342	27.268	13.00%
λ_5	31.364	34.689	10.60%	37.686	43.435	15.26%
λ_6	38.643	38.858	0.56%	46.703	46.892	0.40%
λ_7	44.568	39.873	10.53%	52.070	47.613	8.56%
λ_8	60.329	59.798	0.88%	60.269	59.739	0.88%
λ_9	76.877	72.276	5.98%	76.812	72.215	5.98%
λ_{10}	82.223	81.927	0.36%	91.481	82.100	10.25%

According to the results given in Tables 5.35 and 5.36, the present study is in good agreement with ANSYS results. The change in the first and second natural frequency values are negligible as the aspect ratio of the single-bay laminated composite curved plate frame structure increases. The third natural frequency value increases until the aspect ratio increases to 1.5. The increment becomes negligible as the aspect ratio of the curved plate frame structure shifts from 1.5 to 2. The fourth natural frequency increase slightly as the aspect ratio increases from 0.5 to 1.5. This slight increment becomes negligible as the aspect ratio shifts from 1.5 to 2. The fifth natural frequency increases slightly just as the fourth frequency. Apart from the fourth frequency, the slight increment in the fifth natural frequency gets bigger as the aspect ratio shifts from 1.5 to 2. The sixth and seventh natural frequency values increase as the aspect ratio increases. The eighth natural frequency value increases until the aspect ratio increases from 0.5 to 1. The increment in this frequency gets smaller as the aspect ratio increases from 1 to 2. The ninth and tenth natural frequency values increase until the aspect ratio increases from 0.5 to 1.5. As the aspect ratio increases from 1.5 to 2, these frequencies increase slightly.

Tables 5.37 – 5.42 give the first ten natural frequencies of laminated composite curved plate frame structure with different aspect ratios and having C3, C4, and C5 fiber orientations, respectively.

Table 5.37 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C3, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.564	6.562	0.03%	6.559	6.558	0.02%
λ_2	25.295	26.675	5.46%	25.269	26.648	5.46%
λ_3	26.689	28.540	6.94%	30.684	33.512	9.22%
λ_4	39.392	39.479	0.22%	42.462	42.771	0.73%
λ_5	42.482	42.796	0.74%	43.214	43.410	0.45%
λ_6	44.217	43.529	1.56%	49.933	43.506	12.87%
λ_7	44.889	44.240	1.45%	53.943	47.285	12.34%
λ_8	49.983	44.600	10.77%	64.866	64.492	0.58%
λ_9	50.823	45.831	9.82%	65.221	65.289	0.10%
λ_{10}	54.408	54.302	0.19%	96.074	95.317	0.79%

Table 5.38 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C3, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.555	6.554	0.02%	6.551	6.550	0.02%
λ_2	25.243	26.622	5.46%	25.221	26.599	5.46%
λ_3	36.155	40.361	11.63%	42.208	42.717	1.21%
λ_4	42.427	42.742	0.74%	42.399	43.452	2.48%
λ_5	48.791	43.476	10.89%	49.881	48.117	3.54%
λ_6	49.907	49.106	1.60%	55.552	55.965	0.74%
λ_7	58.610	51.998	11.28%	64.268	57.990	9.77%
λ_8	95.998	95.245	0.78%	95.928	95.175	0.78%
λ_9	99.650	98.601	1.05%	120.780	115.090	4.71%
λ_{10}	99.699	99.355	0.35%	122.190	121.900	0.24%

Table 5.39 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C4, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.647	6.646	0.02%	6.641	6.640	0.02%
λ_2	25.613	27.012	5.46%	25.578	26.976	5.47%
λ_3	27.076	28.967	6.98%	31.258	34.168	9.31%
λ_4	39.964	40.066	0.26%	42.978	43.303	0.76%
λ_5	43.008	43.337	0.76%	43.931	44.047	0.26%
λ_6	44.723	44.080	1.44%	50.554	44.144	12.68%
λ_7	45.194	44.842	0.78%	54.774	48.028	12.32%
λ_8	50.618	45.153	10.80%	64.635	64.390	0.38%
λ_9	51.497	46.416	9.87%	65.097	65.225	0.20%
λ_{10}	54.450	54.511	0.11%	97.270	96.522	0.77%

Table 5.40 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C4, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.635	6.635	0.00%	6.629	6.629	0.00%
λ_2	25.544	26.942	5.47%	25.513	26.910	5.48%
λ_3	36.968	41.307	11.74%	42.889	43.227	0.79%
λ_4	42.930	43.262	0.77%	43.276	43.972	1.61%
λ_5	49.744	44.007	11.53%	50.483	49.373	2.20%
λ_6	50.518	50.083	0.86%	56.779	57.223	0.78%
λ_7	59.672	52.959	11.25%	65.612	59.219	9.74%
λ_8	96.832	95.946	0.91%	97.080	96.339	0.76%
λ_9	96.945	96.429	0.53%	123.140	116.500	5.39%
λ_{10}	97.221	96.760	0.47%	123.660	124.390	0.59%

Table 5.41 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C5, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.595	6.593	0.00%	6.590	6.589	0.02%
λ_2	25.414	26.801	5.46%	25.382	26.768	5.46%
λ_3	26.863	28.737	6.98%	30.999	33.882	9.30%
λ_4	39.647	39.744	0.24%	42.65	42.966	0.74%
λ_5	42.678	42.999	0.75%	43.578	43.705	0.29%
λ_6	44.466	43.735	1.64%	50.163	43.787	12.71%
λ_7	45.102	44.489	1.36%	54.339	47.645	12.32%
λ_8	50.224	44.879	10.64%	64.706	64.393	0.48%
λ_9	51.099	46.131	9.72%	65.099	65.208	0.17%
λ_{10}	54.430	54.408	0.04%	96.518	95.762	0.78%

Table 5.42 The first ten natural frequency values of the single-bay laminated composite curved plate frame structure for C5, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.584	6.584	0.02%	6.579	6.578	0.02%
λ_2	25.350	26.736	5.47%	25.322	26.707	5.47%
λ_3	36.645	40.947	11.74%	42.569	42.897	0.77%
λ_4	42.606	42.929	0.76%	42.877	43.636	1.77%
λ_5	49.330	43.668	11.48%	50.098	48.932	2.33%
λ_6	50.13	49.663	0.93%	56.289	56.725	0.77%
λ_7	59.178	52.523	11.25%	65.036	58.713	9.72%
λ_8	96.418	95.675	0.77%	96.340	95.590	0.78%
λ_9	97.841	96.858	1.00%	122.090	115.590	5.32%
λ_{10}	97.886	97.649	0.24%	122.720	123.330	0.50%

According to the results given in Tables 5.37-5.42, the present study is in good agreement with ANSYS results. The similar interpretation made for the curved plate structure having C1 fiber orientation can be made for these structures having C3, C4, and C5 fiber orientation with different aspect ratios. Only the fifth and eighth natural frequency values evaluated for C3, C4, and C5 change differently compared with those obtained for C1. The fifth natural frequency increases slightly as the aspect ratio increases from 0.5 to 1. When the aspect ratio shifts from 1 to 1.5, the change in the fifth natural frequency is negligible. This change becomes more considerable as the aspect ratio increases from 1.5 to 2. The eighth frequency increases until the aspect ratio value reaches to 1.5. As the aspect ratio increases from 1.5 to 2, the change in this frequency becomes negligible.

Figures 5.12-5.15 show the mode shapes of the single-bay laminated composite curved plate frame structure with different aspect ratios and C1, C2, C3, C4, and C5 fiber orientations, respectively. It is concluded that the fiber orientation and the aspect ratio of the single-bay laminated composite curved plate frame structure has a significant effect on the mode shapes. Except from the first and the second modes, all other modes behave differently as the aspect ratio and the fiber orientation changes. For structures having C1 and C3 stacking order, the first – third modes do not differ as the aspect ratio changes. On the other hand, there are two different modes in the fourth and seventh modes. Besides, for these stacking orders, three different modes are seen for the sixth, eighth, ninth, and tenth modes. For C2 stacking order, it is seen that the first two and seventh modes do not differ as the aspect ratio changes. Besides, two different mode shapes occur in the sixth and eighth modes while there are three different mode shapes in the fifth and ninth modes as the aspect ratio changes. Moreover, four various mode shapes take place in the fourth and tenth modes for each value of the aspect ratio. The mode shapes of C4 and C5 are identical and slightly differ from those of C1 and C3. Apart from C1 and C3, additional torsional and bending modes occur in the third and fourth modes of the structure having C4 and C5 stacking order.

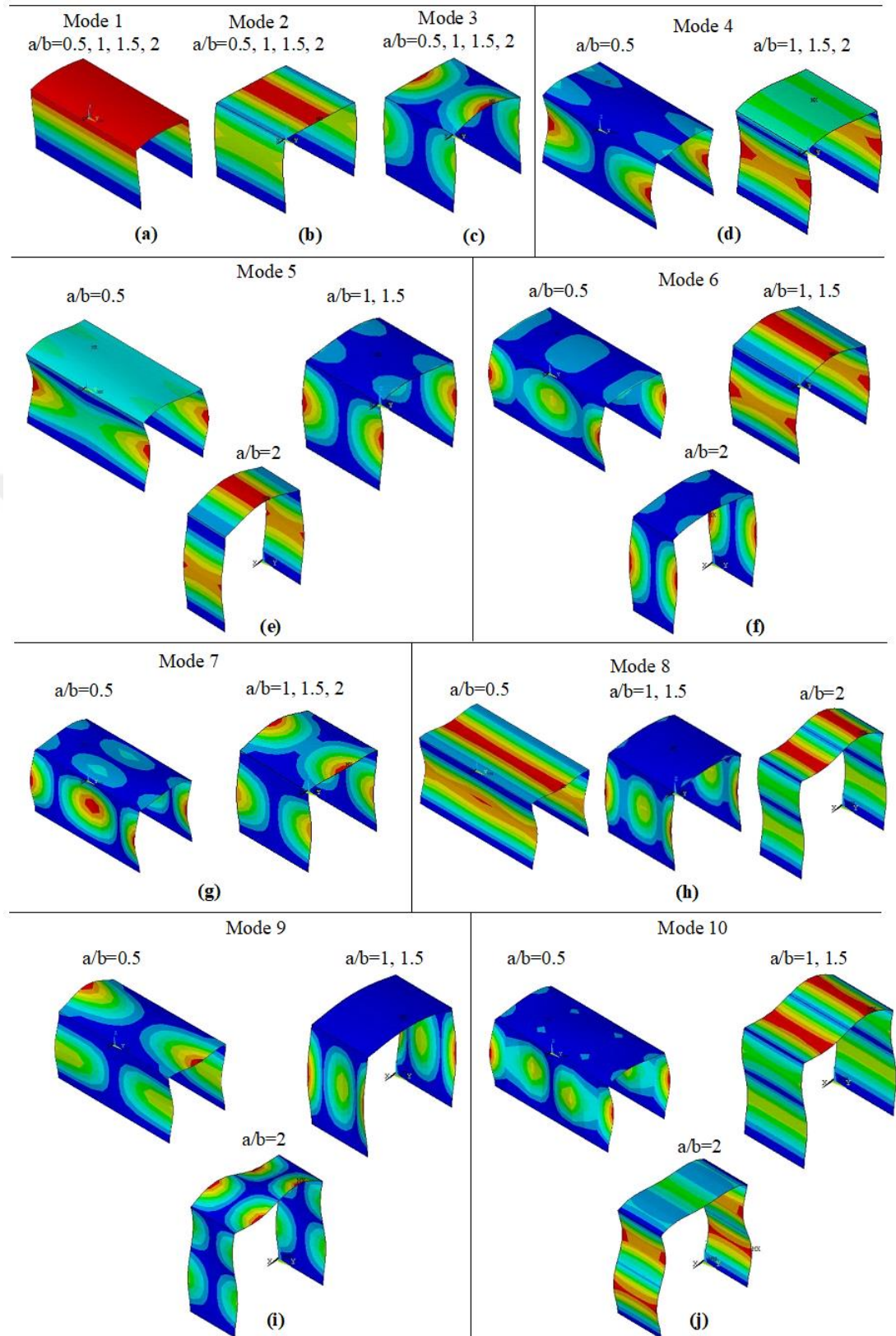


Figure 5.12 Mode shapes of the single-bay curved plate frame having C1 stacking order with different aspect of ratios (a/b)

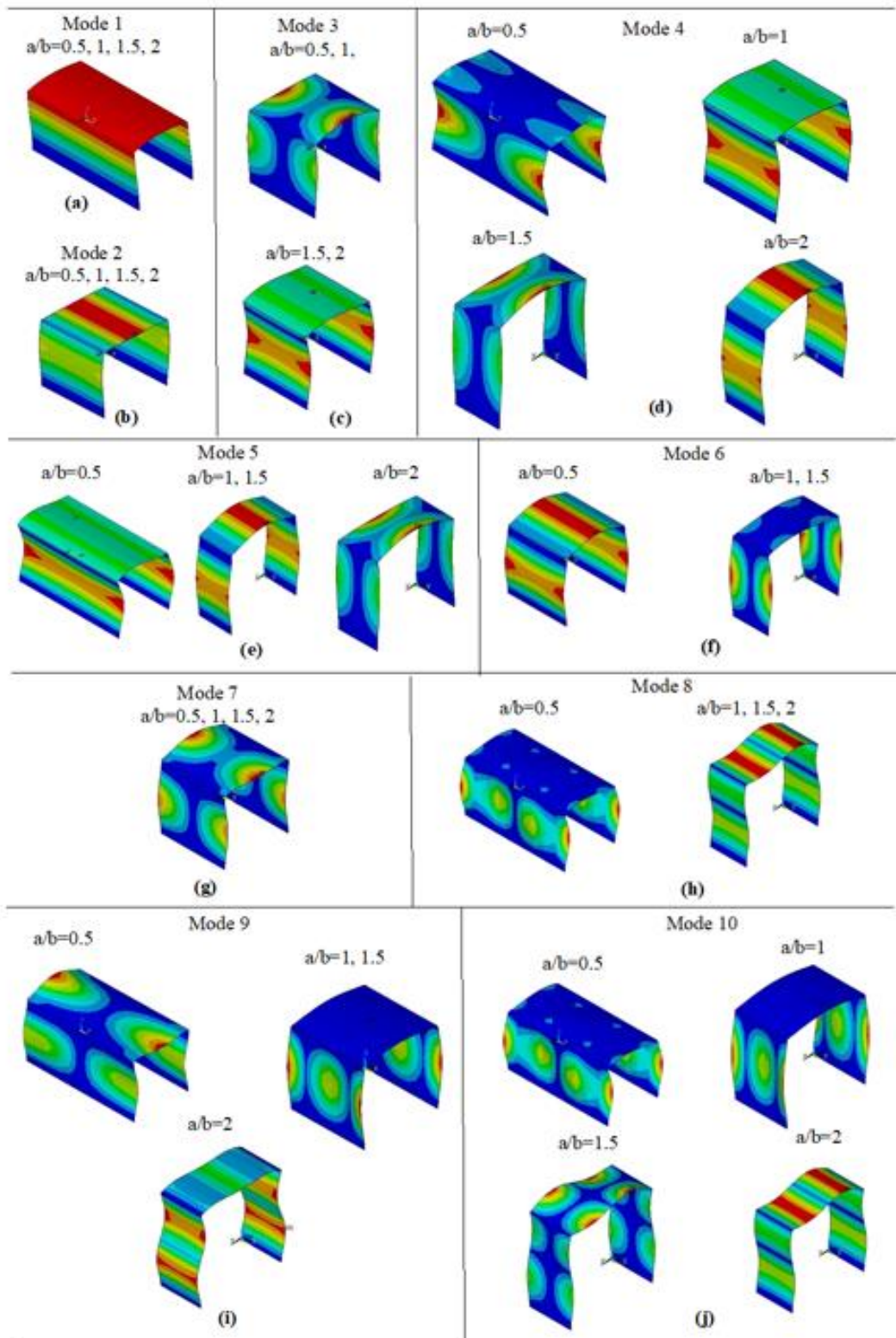


Figure 5.13 Mode shapes of the single-bay curved plate frame having C2 stacking order with different aspect of ratios (a/b)

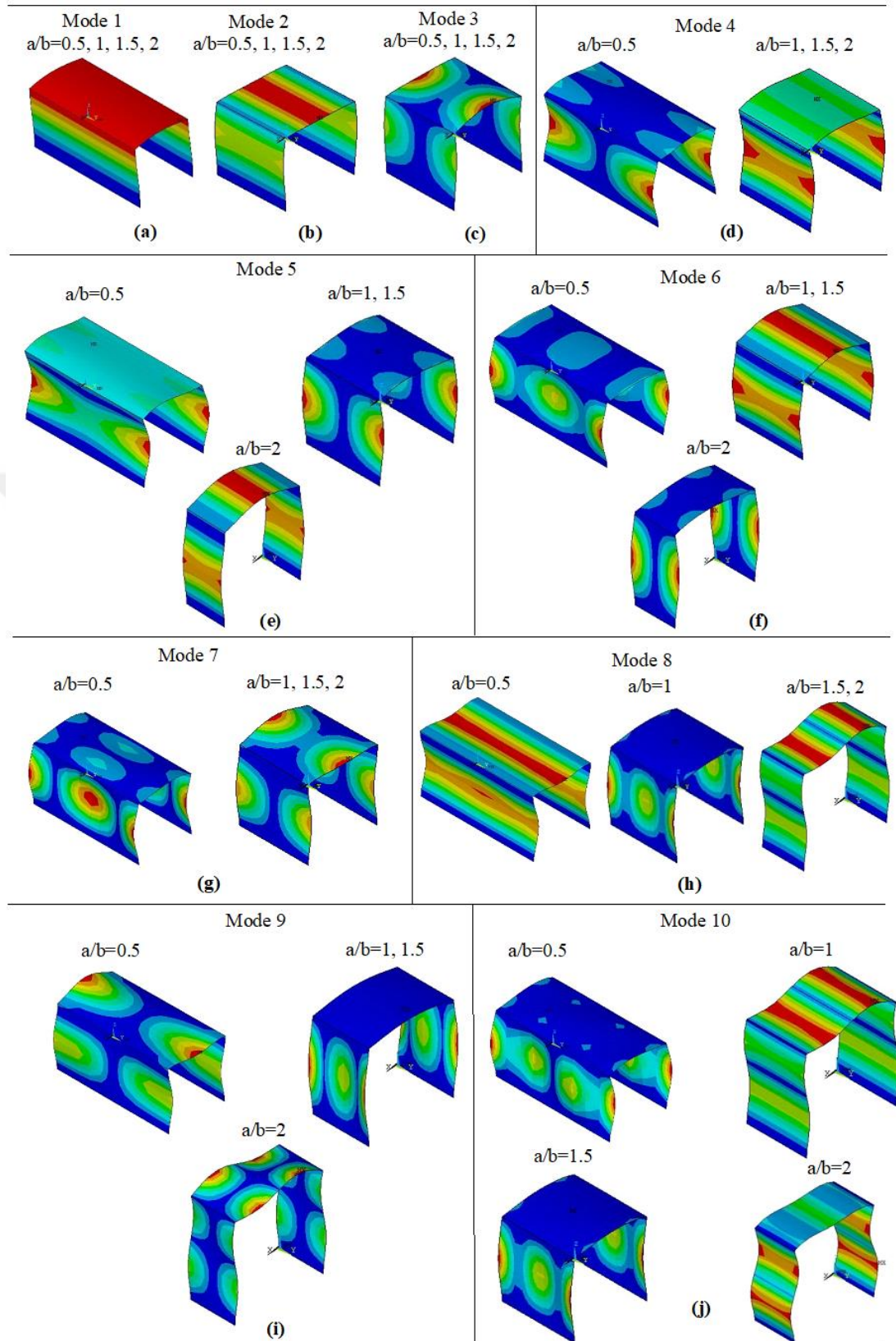


Figure 5.14 Mode shapes of the single-bay curved plate frame having C3 stacking order with different aspect of ratios (a/b)

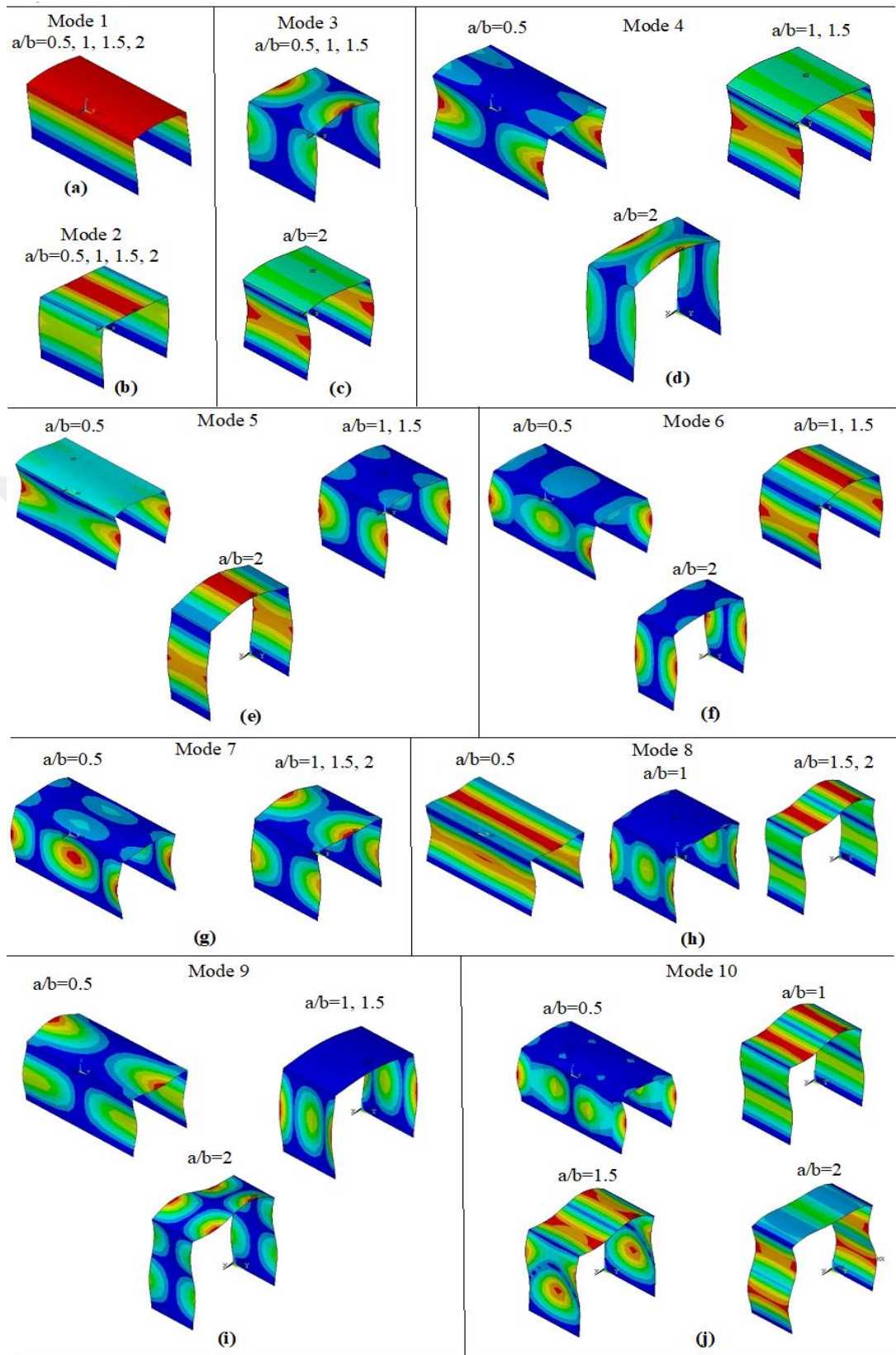


Figure 5.15 Mode shapes of the single-bay curved plate frame having C4 and C5 stacking order with different aspect of ratios (a/b)

Tables 5.43 and 5.44 give the first ten natural frequency results of the two-bay laminated composite curved plate frame structure with different aspect ratios and C1 fiber orientation.

Table 5.43 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C1, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.378	6.351	0.42%	6.373	6.347	0.41%
λ_2	25.909	27.123	4.69%	25.881	27.096	4.69%
λ_3	27.247	28.887	6.02%	30.988	33.599	8.43%
λ_4	31.020	35.519	14.51%	31.196	35.488	13.76%
λ_5	38.174	42.006	10.04%	43.779	43.829	0.11%
λ_6	42.581	42.804	0.52%	45.015	45.772	1.68%
λ_7	45.004	43.854	2.55%	46.160	45.912	0.54%
λ_8	45.580	45.790	0.46%	51.830	46.459	10.36%
λ_9	45.861	46.488	1.37%	52.798	46.600	11.74%
λ_{10}	48.371	46.631	3.60%	55.653	50.405	9.43%

Table 5.44 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C1, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.369	6.344	0.41%	6.365	6.340	0.40%
λ_2	25.854	27.070	4.70%	25.829	27.046	4.71%
λ_3	30.958	35.458	14.54%	30.932	35.432	14.55%
λ_4	36.628	40.089	9.45%	42.689	43.782	2.56%
λ_5	44.975	43.804	2.60%	44.941	45.708	1.71%
λ_6	49.436	45.737	7.48%	51.770	46.407	10.36%
λ_7	51.490	46.431	9.83%	52.740	47.428	10.07%
λ_8	51.799	51.383	0.80%	55.879	58.054	3.89%
λ_9	52.770	52.107	1.26%	58.032	58.842	1.40%
λ_{10}	60.211	55.040	8.59%	65.878	60.957	7.47%

According to the results given in Tables 5.43 and 5.44, the present study is in good agreement with ANSYS results. As the aspect ratio increases, the difference in the first and second natural frequencies are so small that they are negligible. The third natural frequency increases until the aspect ratio increases from 0.5 to 1.5. The change in the third natural frequency becomes negligible for the aspect ratio values bigger than 1.5.

The fourth and ninth natural frequency increase considerably as the aspect ratio increases from 1 to 2. The fifth and sixth natural frequency values first increase as the aspect ratio increases from 0.5 to 1, then the change in those frequencies becomes negligible as the aspect ratio shifts from 1 to 1.5. For the aspect ratio values bigger than 1.5, the increment of those frequencies increases considerably. The seventh natural frequency value increases until the aspect ratio increases from 0.5 to 1. For the aspect ratio values bigger than 1, this increment becomes slighter. The tenth natural frequency increases significantly as the aspect ratio of the curved plate frame structure increases.

Tables 5.45 and 5.46 give the first ten natural frequency values of the two-bay laminated composite curved plate frame structure having C2 stacking order and different aspect ratios.

Table 5.45 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C2, $a/b=0.5$ and $a/b=1$

<i>Natural Frequency (Hz)</i>	<i>a/b=0.5</i>			<i>a/b=1</i>		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.817	3.801	0.44%	3.813	3.797	0.43%
λ_2	15.509	16.231	4.65%	15.486	16.209	4.67%
λ_3	17.868	19.126	7.04%	18.548	21.234	14.48%
λ_4	18.573	21.259	14.46%	23.587	25.869	9.67%
λ_5	25.402	26.252	3.35%	26.946	26.230	2.66%
λ_6	26.846	26.638	0.78%	31.044	27.384	11.79%
λ_7	26.979	27.044	0.24%	31.199	27.798	10.90%
λ_8	31.069	27.413	11.77%	31.633	32.082	1.42%
λ_9	31.652	27.823	12.10%	32.171	32.525	1.10%
λ_{10}	32.747	29.504	9.90%	37.550	34.176	8.99%

Table 5.46 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C2, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	3.809	3.794	0.42%	3.806	3.791	0.41%
λ_2	15.467	16.190	4.68%	15.450	16.174	4.69%
λ_3	18.528	21.214	14.50%	18.511	21.197	14.51%
λ_4	26.921	26.214	2.63%	26.902	26.200	2.61%
λ_5	30.537	27.363	10.40%	31.000	27.346	11.79%
λ_6	31.021	27.781	10.45%	31.587	27.766	12.10%
λ_7	31.609	34.081	7.82%	37.698	42.727	13.34%
λ_8	38.295	39.301	2.63%	46.137	47.401	2.74%
λ_9	39.311	39.818	1.29%	47.339	48.012	1.42%
λ_{10}	44.205	40.773	7.76%	51.777	48.502	6.32%

It is seen from Tables 5.45 and 5.46 that the present study is in good agreement with ANSYS results. Similar to the two-bay curved plate structure having C1 stacking order, the changes in the first and second natural frequencies are negligible as the aspect ratio increases. The third natural frequency increases until the aspect ratio increases from 0.5 to 1. The increment of this frequency becomes negligible for the aspect ratio values bigger than 1. Similar to the third frequency, the fourth frequency increases as the aspect ratio increases. However, this increment becomes negligible for the aspect ratio values bigger than 1.5. As the aspect ratio increases from 0.5 to 1, the increment of the fifth natural frequency can be accepted as negligible. The increment becomes considerable when the aspect ratio of the two-bay curved plate frame structure increases from 1 to 1.5. However, it becomes negligible again as the aspect ratio increases from 1.5 to 2. The sixth natural frequency increases slightly and eventually becomes negligible as the aspect ratio increases. The seventh natural frequency increase slightly as the aspect ratio increases from 0.5 to 1, whereas the increment becomes more considerable as the aspect ratio shifts from 1 to 2. The eighth, ninth, and tenth natural frequency values increase as the aspect ratio of the two-bay laminated composite curved plate frame structure increases.

Tables 5.47 – 5.52 give the first ten natural frequency results of the two-bay laminated composite curved plate frame structures having different aspect ratios and C3, C4, and C5 fiber orientations, respectively.

Table 5.47 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C3, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.079	6.053	0.42%	6.075	6.050	0.41%
λ_2	24.693	25.850	4.69%	24.668	25.826	4.69%
λ_3	26.115	27.708	6.10%	29.533	32.615	10.44%
λ_4	29.561	33.847	14.50%	30.227	33.819	11.88%
λ_5	36.531	40.175	9.98%	41.965	41.766	0.48%
λ_6	40.709	40.931	0.55%	42.909	43.630	1.68%
λ_7	42.932	41.788	2.66%	44.437	44.209	0.51%
λ_8	44.512	43.654	1.93%	49.398	44.281	10.36%
λ_9	44.665	44.307	0.80%	50.311	44.871	10.81%
λ_{10}	47.266	45.246	4.27%	53.439	48.434	9.37%

Table 5.48 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C3, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.071	6.047	0.40%	6.067	6.043	0.40%
λ_2	24.644	25.803	4.70%	24.622	25.781	4.71%
λ_3	29.507	33.793	14.53%	29.484	33.770	14.54%
λ_4	35.816	39.307	9.75%	41.988	41.726	0.62%
λ_5	42.875	41.744	2.64%	42.847	43.577	1.70%
λ_6	47.652	43.600	8.50%	49.346	44.238	10.35%
λ_7	49.372	44.258	10.36%	50.260	46.825	6.83%
λ_8	49.942	49.862	0.16%	54.061	56.696	4.87%
λ_9	50.285	50.561	0.55%	56.636	57.460	1.46%
λ_{10}	58.183	53.256	8.47%	64.020	59.362	7.28%

Table 5.49 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C4, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.156	6.131	0.40%	6.150	6.126	0.40%
λ_2	25.003	26.179	4.71%	24.970	26.148	4.72%
λ_3	26.497	28.129	6.16%	29.897	33.264	11.26%
λ_4	29.934	34.284	14.53%	30.802	34.248	11.19%
λ_5	37.701	40.845	8.34%	42.989	42.308	1.58%
λ_6	41.280	41.556	0.67%	43.432	44.181	1.73%
λ_7	43.464	42.339	2.59%	45.162	44.852	0.69%
λ_8	44.972	44.216	1.68%	50.011	44.967	10.09%
λ_9	45.161	44.887	0.61%	50.939	45.635	10.41%
λ_{10}	47.737	45.830	3.99%	54.257	49.210	9.30%

Table 5.50 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C4, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.145	6.122	0.39%	6.140	6.117	0.38%
λ_2	24.938	26.118	4.73%	24.908	26.089	4.74%
λ_3	29.862	34.213	14.57%	29.830	34.182	14.59%
λ_4	36.632	40.232	9.83%	43.054	42.254	1.86%
λ_5	43.385	42.279	2.55%	43.345	44.106	1.76%
λ_6	48.829	44.141	9.60%	49.939	44.791	10.31%
λ_7	49.974	44.819	10.32%	50.869	48.028	5.58%
λ_8	50.902	50.854	0.10%	55.524	57.968	4.40%
λ_9	50.904	51.565	1.30%	57.870	58.747	1.52%
λ_{10}	59.226	54.254	8.39%	65.329	60.630	7.19%

Table 5.51 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C5, $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.108	6.083	0.41%	6.103	6.079	0.40%
λ_2	24.810	25.973	4.69%	24.779	25.945	4.70%
λ_3	26.289	27.904	6.14%	29.667	32.983	11.18%
λ_4	29.702	34.011	14.51%	30.546	33.977	11.23%
λ_5	37.254	40.498	8.71%	42.539	41.967	1.34%
λ_6	40.957	41.214	0.63%	43.100	43.832	1.70%
λ_7	43.131	41.996	2.63%	44.802	44.493	0.69%
λ_8	44.757	43.864	1.99%	49.626	44.596	10.14%
λ_9	44.921	44.525	0.88%	50.544	45.260	10.45%
λ_{10}	47.527	45.539	4.18%	53.830	48.811	9.32%

Table 5.52 The first ten natural frequency values of the two-bay laminated composite curved plate frame structure for C5, $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	6.098	6.074	0.40%	6.093	6.070	0.39%
λ_2	24.749	25.916	4.71%	24.721	25.889	4.73%
λ_3	29.634	33.945	14.55%	29.605	33.916	14.56%
λ_4	36.311	39.882	9.83%	42.663	41.918	1.75%
λ_5	43.057	41.941	2.59%	43.021	43.764	1.73%
λ_6	48.319	43.795	9.36%	49.559	44.437	10.33%
λ_7	49.592	44.463	10.34%	50.479	47.609	5.69%
λ_8	50.481	50.424	0.11%	54.909	57.462	4.65%
λ_9	50.512	51.129	1.22%	57.374	58.235	1.50%
λ_{10}	58.742	53.802	8.41%	64.773	60.108	7.20%

It is concluded from Tables 5.47 – 5.52 that the present study is in good agreement with ANSYS results. As the aspect ratio increases, the change in the first and second natural frequency values is negligible. The third natural frequency increases as the aspect ratio increases. However, this increment diminishes and becomes negligible when the aspect ratio reaches its biggest value. The fourth natural frequency increases when the aspect ratio of the two-bay laminated composite curved plate structure increases from 1 to 2. The fifth, sixth, and seventh natural frequency values increase as the aspect ratio increases from 0.5 to 1. However, these increments become negligible when the aspect ratio shifts from 1 to 1.5. Nevertheless, these frequencies increase considerably as the aspect ratio increases from 1.5 to 2. The eighth and ninth frequencies increase growingly as the aspect ratio increases. The tenth natural frequency increases as the aspect ratio increases.

Figures 5.16 – 5.20 show the mode shapes of the two-bay laminated composite curved plate frame structure having different aspect ratios and C1, C2, C3, C4, and C5 stacking order, respectively. It is concluded from those mode shapes that having an additional bay affects most of mode shapes significantly. Except for the first and the second modes, all modes change with respect to the aspect ratio and fiber orientation.

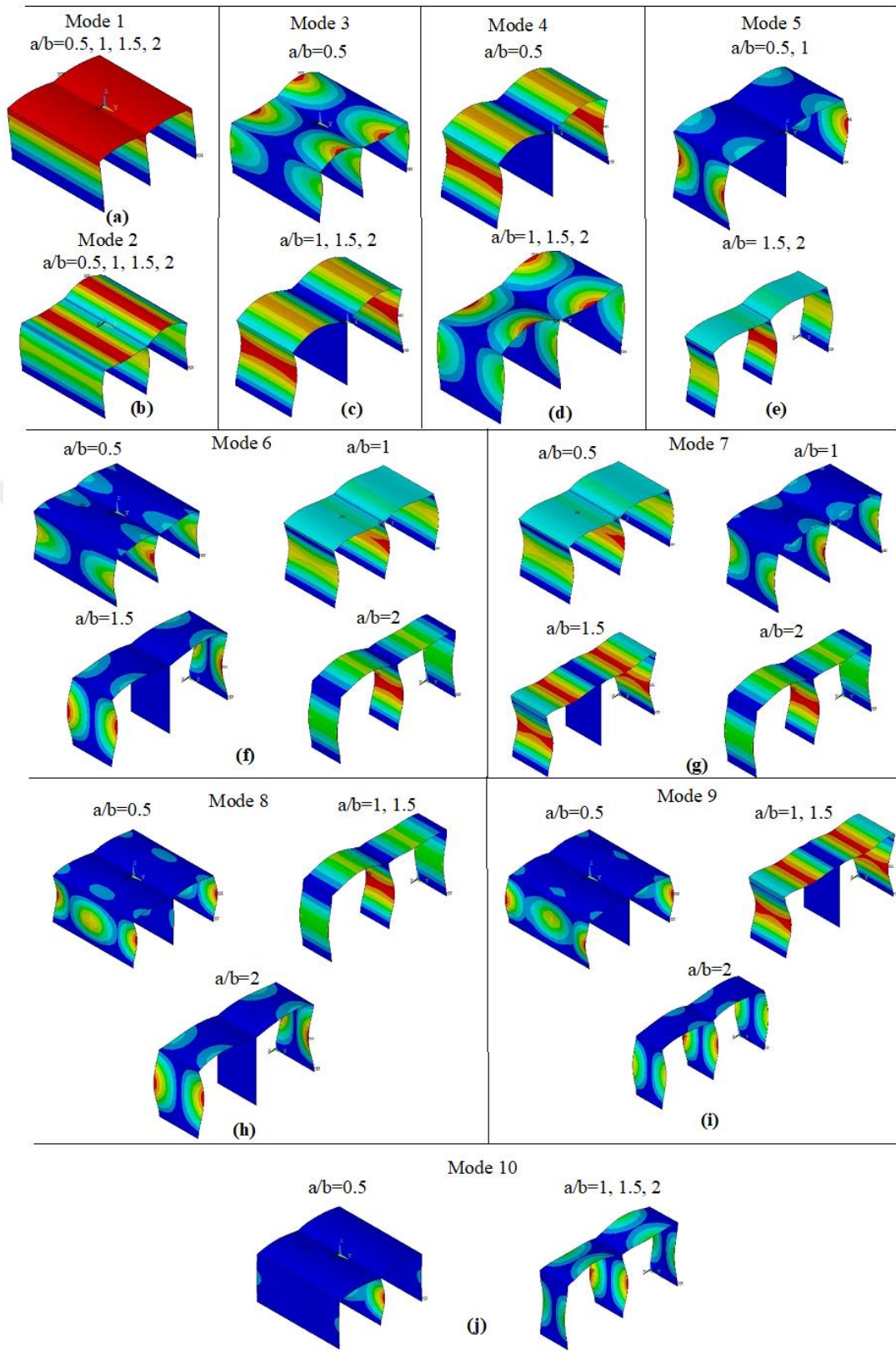


Figure 5.16 Mode shapes of the two-bay curved plate frame having C1 stacking order with different aspect of ratios (a/b)

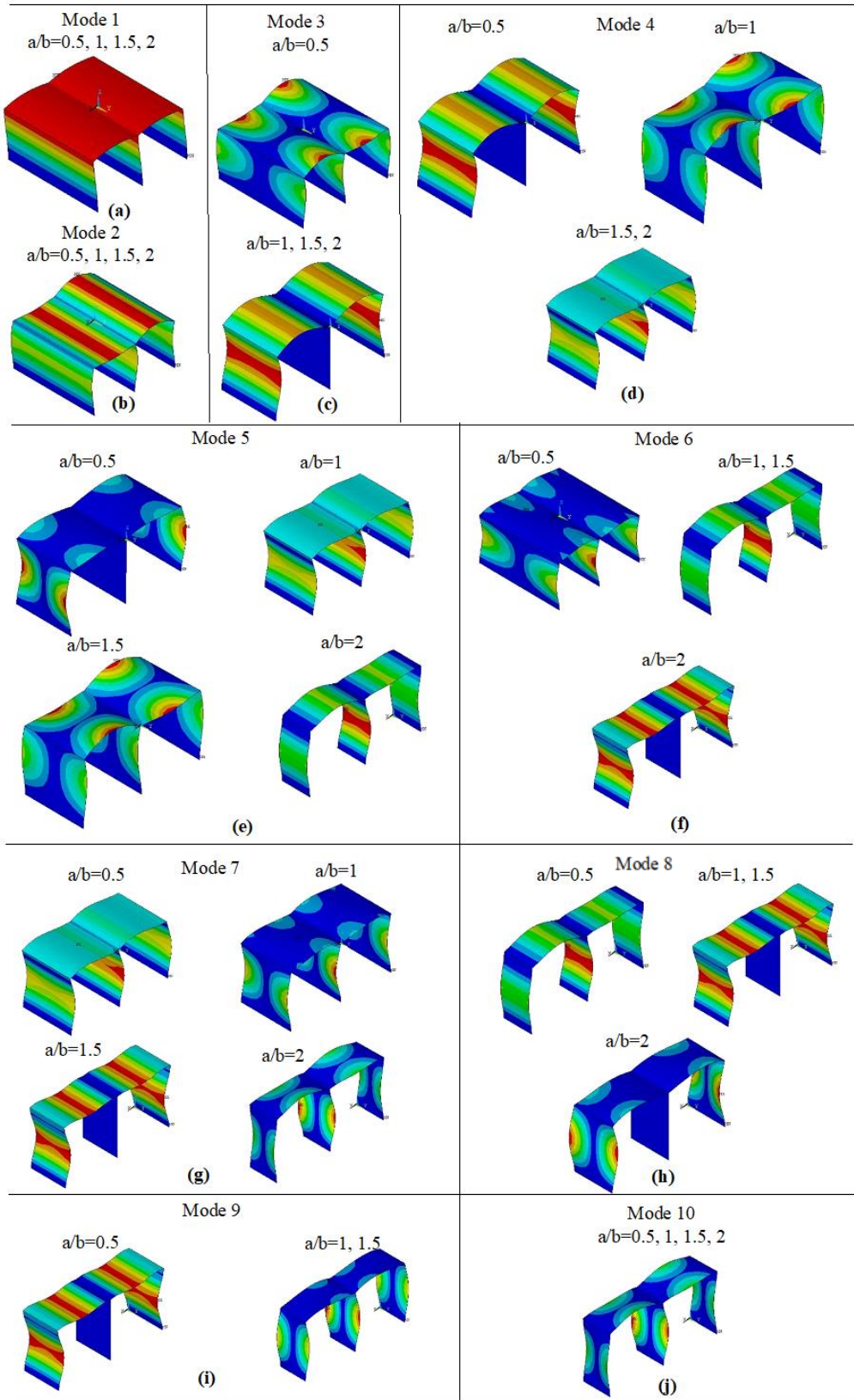


Figure 5.17 Mode shapes of the two-bay curved plate frame having C2 stacking order with different aspect of ratios (a/b)

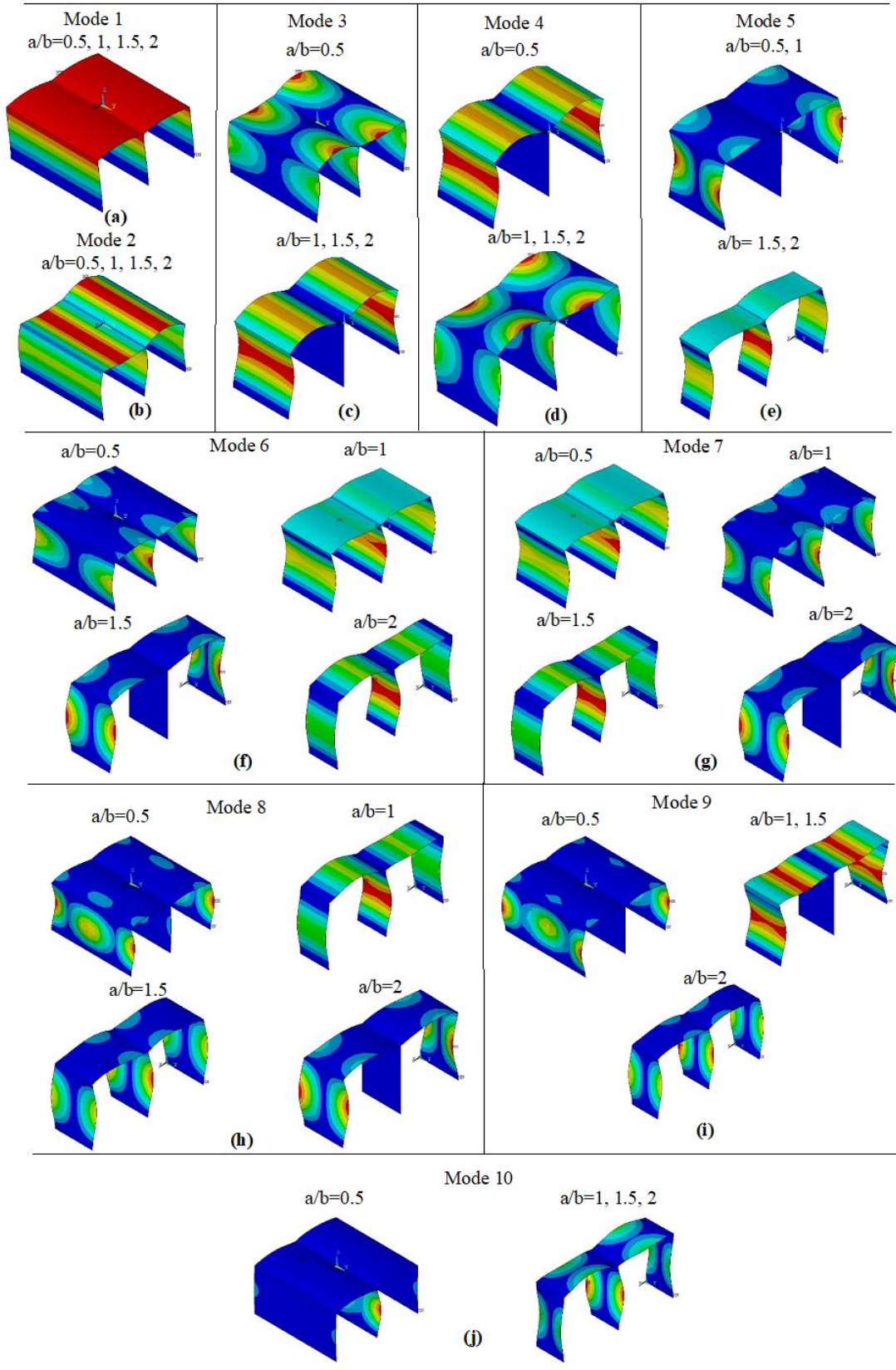


Figure 5.18 Mode shapes of the two-bay curved plate frame having C3 stacking order with different aspect of ratios (a/b)

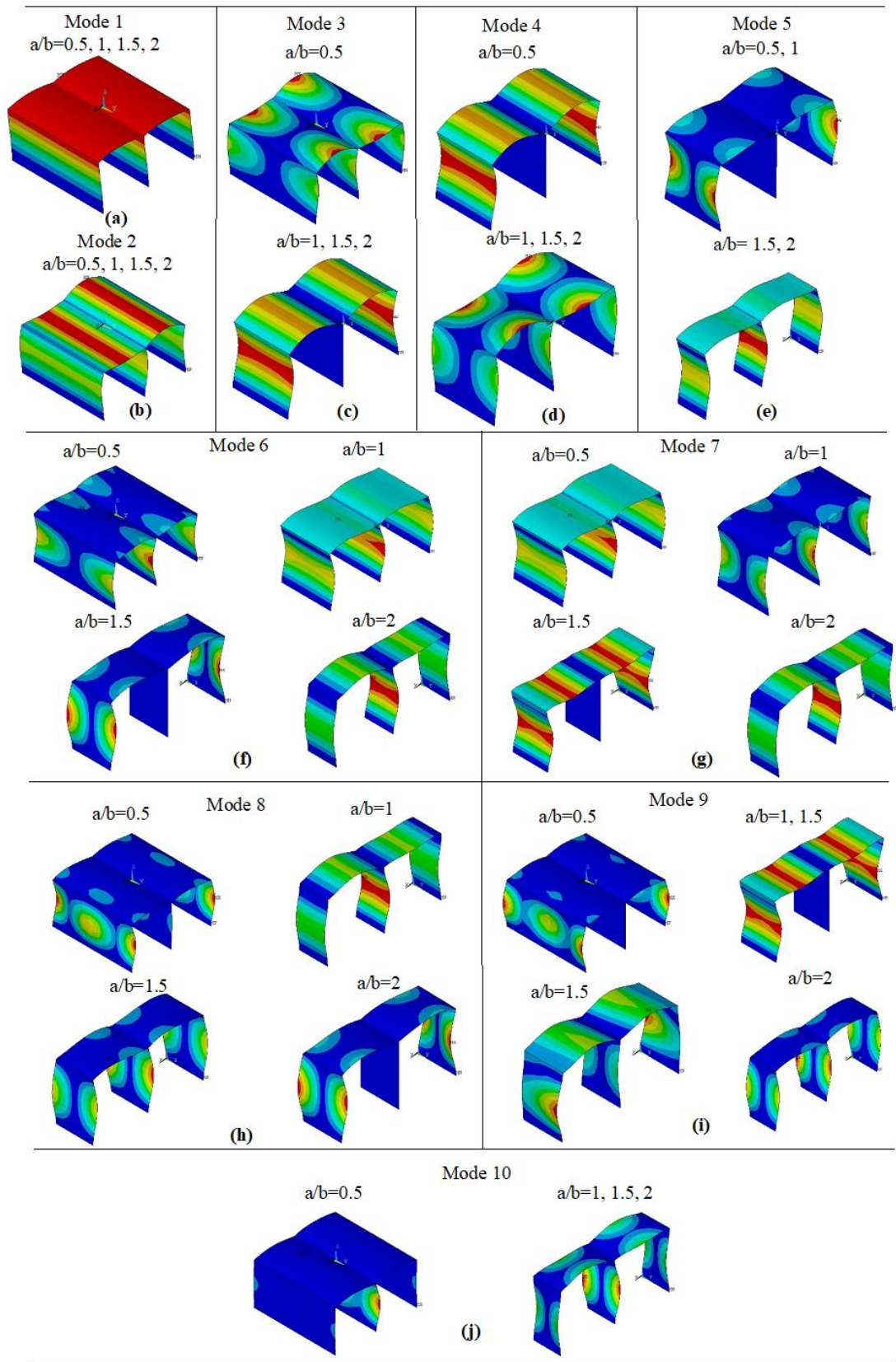


Figure 5.19 Mode shapes of the two-bay curved plate frame having C4 stacking order with different aspect of ratios (a/b)

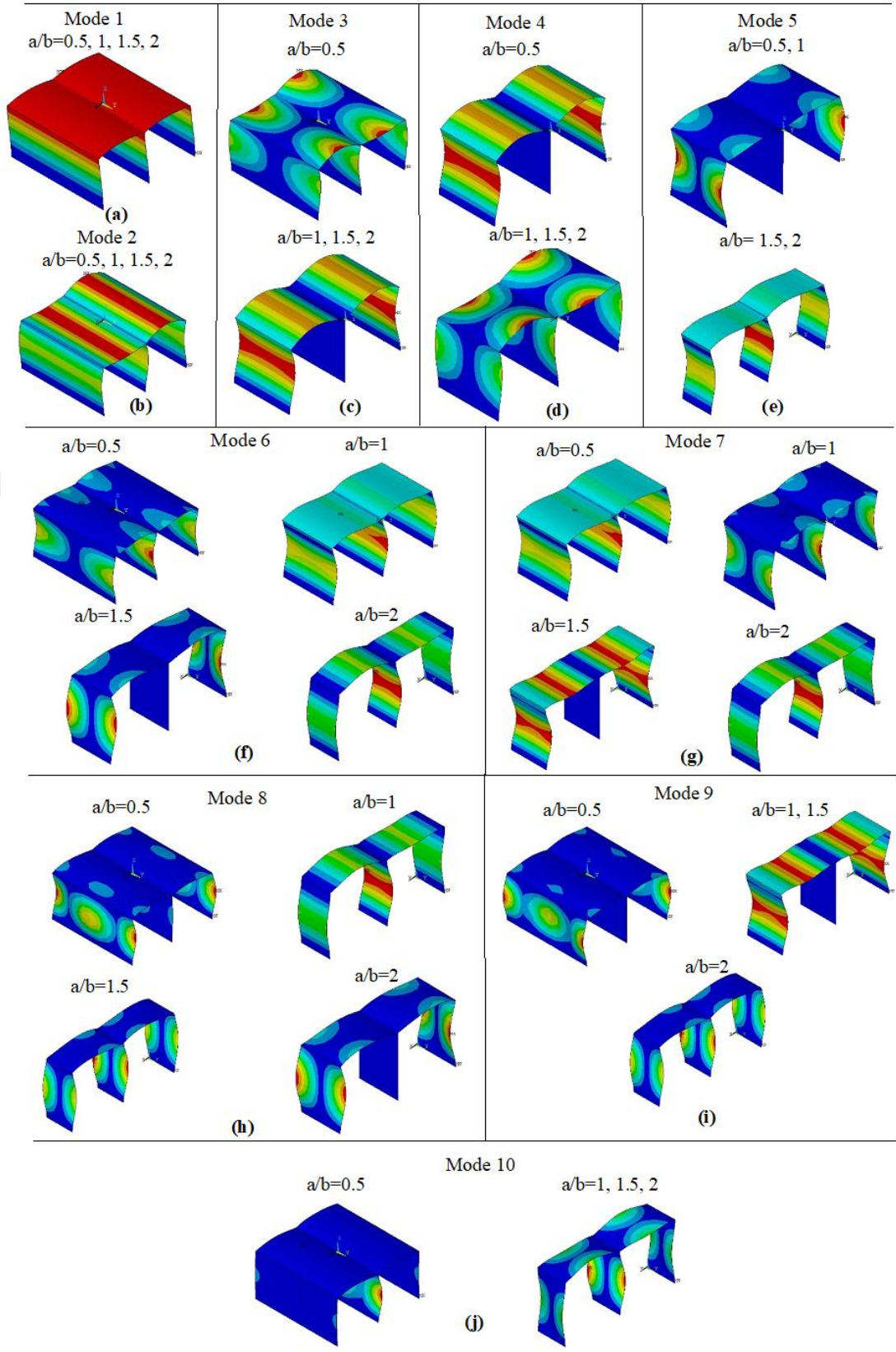


Figure 5.20 Mode shapes of the two-bay curved plate frame having C5 stacking order with different aspect of ratios (a/b)

5.3.2 Buckling Analysis of Laminated Composite Curved Thin Plate Frames

5.3.2.1 Effect of The Radius of Curvature

The effect of the radius of curvature (R_{xx}) on the critical buckling loads of the single-bay and two-bay laminated composite curved plate frame structure is investigated. Four different radii of curvature values $1.5a$, $2a$, $2.5a$, and $3a$, are considered.

Tables 5.53 – 5.57 give the critical buckling load of the single-bay and two-bay laminated composite curved plate frame structures having different radius of curvatures and C1, C2, C3, C4, and C5 fiber orientations, respectively.

Table 5.53 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$ and C1 stacking order

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
$1.5a$	54967.000	55974.000	1.80%	$1.5a$	84693.000	88281.000	4.06%
$2a$	55655.000	56315.000	1.17%	$2a$	85696.000	87091.000	1.60%
$2.5a$	55959.000	56483.000	0.93%	$2.5a$	86127.000	88404.000	2.58%
$3a$	56113.000	56580.000	0.83%	$3a$	86398.000	88388.000	2.25%

Table 5.54 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$ and C2 stacking order

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
$1.5a$	19677.000	20037.000	1.80%	$1.5a$	30320.000	31844.000	4.79%
$2a$	19923.000	20156.000	1.16%	$2a$	30679.000	31869.000	3.73%
$2.5a$	20031.000	20214.000	0.91%	$2.5a$	30832.000	31862.000	3.23%
$3a$	20086.000	20247.000	0.80%	$3a$	30928.000	31849.000	2.89%

Table 5.55 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$ and C3 stacking order

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5a	50851.000	49933.000	1.81%	1.5a	80564.000	76937.000	4.50%
2a	51161.000	50558.000	1.18%	2a	80647.000	77848.000	3.47%
2.5a	51314.000	50834.000	0.94%	2.5a	80644.000	78240.000	2.98%
3a	51403.000	50974.000	0.83%	3a	80620.000	78487.000	2.65%

Table 5.56 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$ and C4 stacking order

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5a	52118.000	51191.000	1.78%	1.5a	82566.000	80477.000	2.53%
2a	52434.000	51832.000	1.15%	2a	82586.000	80225.000	2.86%
2.5a	52589.000	52115.000	0.90%	2.5a	82582.000	79825.000	3.34%
3a	52679.000	52259.000	0.80%	3a	82487.000	78891.000	4.36%

Table 5.57 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $R_{xx}=1.5a$, $R_{xx}=2a$, $R_{xx}=2.5a$, and $R_{xx}=3a$ and C5 stacking order

Radius of Curvature (R_{xx})	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Radius of Curvature (R_{xx})	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
1.5a	51318.000	50401.000	1.79%	1.5a	81282.000	77665.000	4.45%
2a	51629.000	51032.000	1.16%	2a	81371.000	78585.000	3.42%
2.5a	51783.000	51311.000	0.91%	2.5a	81370.000	78980.000	2.94%
3a	51871.000	51452.000	0.81%	3a	81349.000	79228.000	2.61%

According to the critical buckling loads given in Tables 5.53-5.57, the present study is in close agreement with ANSYS results.

It is concluded that as the radius of curvature of the single-bay and two-bay laminated composite curved plate structure increases, the critical buckling load increases regardless of the stacking order.

5.3.2.2 Effect of The Aspect Ratio

The effect of aspect ratios (a/b) on the critical buckling loads of the single-bay and two-bay laminated composite curved plate frame structure is investigated. Four aspect ratio, 0.5, 1, 1.5, and 2, are considered. It should be noted that the length of the plate structure (a) is set as constant.

Tables 5.58 – 5.62 give the critical buckling load of the single-bay and two-bay laminated composite curved plate frame structures having different aspect ratios and C1, C2, C3, C4, and C5 fiber orientations, respectively.

Table 5.58 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=1.5$ and C1 stacking order

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	112820.000	111510.000	1.16%	0.5	172650.000	171690.000	0.56%
1	56315.000	55655.000	1.17%	1	87091.000	85696.000	1.60%
1.5	37479.000	37033.000	1.19%	1.5	59265.000	57027.000	3.78%
2	28062.000	27720.000	1.22%	2	44474.000	42687.000	4.02%

Table 5.59 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=1.5$ and C2 stacking order

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	40412.000	39941.000	1.17%	0.5	62541.000	61499.000	1.67%
1	20156.000	19923.000	1.16%	1	31869.000	30679.000	3.73%
1.5	13405.000	13250.000	1.16%	1.5	21257.000	20405.000	4.01%
2	10032.000	9916.000	1.16%	2	15923.000	15272.000	4.09%

Table 5.60 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=1.5$ and C3 stacking order

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	102480.000	101290.000	1.16%	0.5	150640.000	155960.000	3.53%
1	51161.000	50558.000	1.18%	1	80647.000	77848.000	3.47%
1.5	34054.000	33641.000	1.21%	1.5	53936.000	51803.000	3.95%
2	25500.000	25179.000	1.26%	2	40445.000	38775.000	4.13%

Table 5.61 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=1.5$ and C4 stacking order

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	105080.000	103890.000	1.13%	0.5	149380.000	159980.000	7.10%
1	52434.000	51832.000	1.15%	1	82582.000	79825.000	3.34%
1.5	34884.000	34475.000	1.17%	1.5	55242.000	53100.000	3.88%
2	26110.000	25793.000	1.21%	2	41413.000	39733.000	4.06%

Table 5.62 Buckling analysis results of the single-bay and two-bay laminated composite curved plate frame structure for $a/b=0.5$, $a/b=1$, $a/b=1.5$, and $a/b=1.5$ and C5 stacking order

Aspect Ratio (a/b)	Single-Bay Curved Plate Frame Critical Buckling Load (N)			Aspect Ratio (a/b)	Two-Bay Curved Plate Frame Critical Buckling Load (N)		
	ANSYS	Present Study	Error (%)		ANSYS	Present Study	Error (%)
0.5	103460.000	102270.000	1.15%	0.5	151380.000	157480.000	4.03%
1	51629.000	51032.000	1.16%	1	81371.000	78585.000	3.42%
1.5	34354.000	33946.000	1.19%	1.5	54415.000	52279.000	3.93%
2	25717.000	25399.000	1.24%	2	40792.000	39121.000	4.10%

According to the critical buckling loads given in Tables 5.58-5.62, the present study is in close agreement with ANSYS results. It is concluded that as the aspect ratio of the single-bay and two-bay laminated composite curved plate structure increases, the critical buckling load decreases regardless of the stacking order.

5.3.3 Dynamic Stability of Laminated Composite Curved Thin Plate Frames

The dynamic stability of the single-bay and two-bay laminated composite curved plate frames is performed within this section. The effects of the radius of curvature, aspect ratio, and the stacking order on the unstable region are investigated. The radius of curvature, aspect ratio, and the stacking orders are chosen, as presented in section 5.3.1.1.

Figures 5.21 and 5.22 show the instability region of laminated composite curved plate frame structure with C1 stacking order and four different radii of curvatures for $\alpha=0$ and $\alpha=0.5$, respectively. It is inferred that increasing the radius of curvature of the curved frame structure does not affect the instability region considerably. Thus, the difference between the instability regions of different curvatures is so small that it can be neglected. The unstable region moves towards the origin as the radius of curvature decreases. As the static load factor increase, the unstable region becomes narrow. These conditions are observed for all other stacking orders and the two-bay curved frame structures.

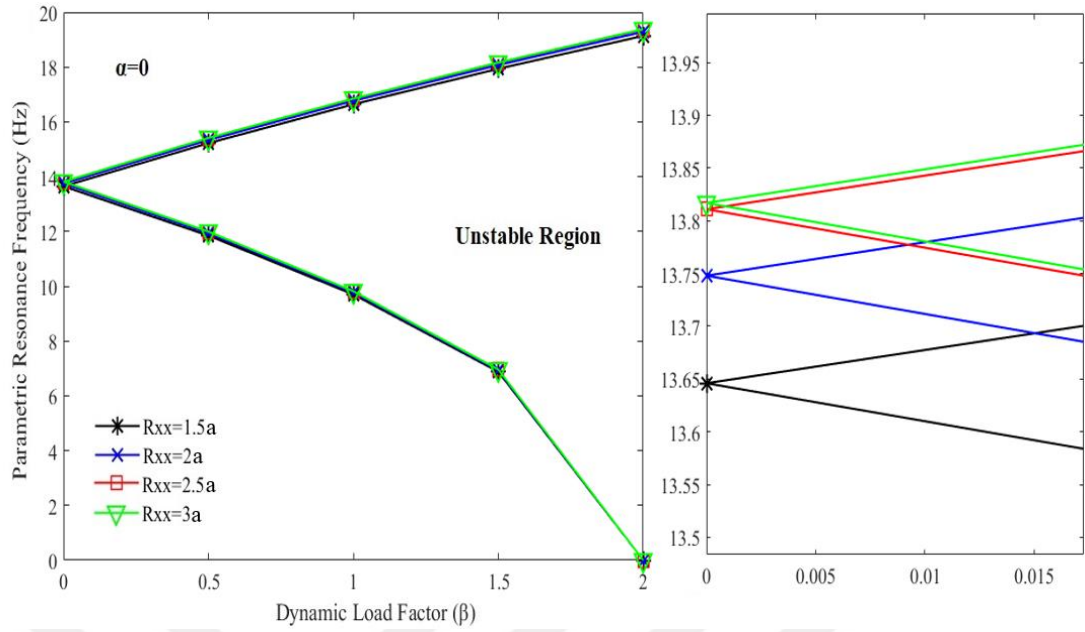


Figure 5.21 Unstable regions ($\alpha=0$) of the single-bay laminated composite curved plate frame structure having C1 stacking order and different radius of curvatures

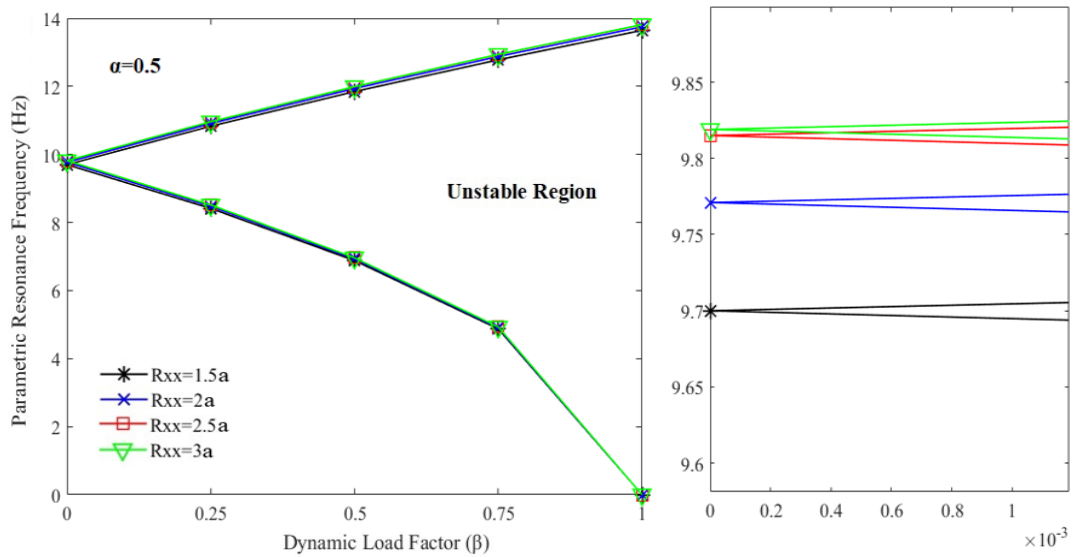


Figure 5.22 Unstable regions ($\alpha=0.5$) of the single-bay laminated composite curved plate frame structure having C1 stacking order and different radius of curvatures

Figures 5.23 and 5.24 show the instability region of laminated composite curved plate frame structure with C1 stacking order and four different aspect ratios for $\alpha=0$ and $\alpha=0.5$, respectively. It is inferred that just like increasing the radius of curvature, increasing the aspect ratio of the curved frame structure does not affect the instability region considerably. The unstable region moves towards the origin as the

aspect ratio increases. As the static load factor increase, the unstable region becomes narrow and moves towards the origin. These conditions are observed for all other stacking orders and the two-bay curved frame structures.

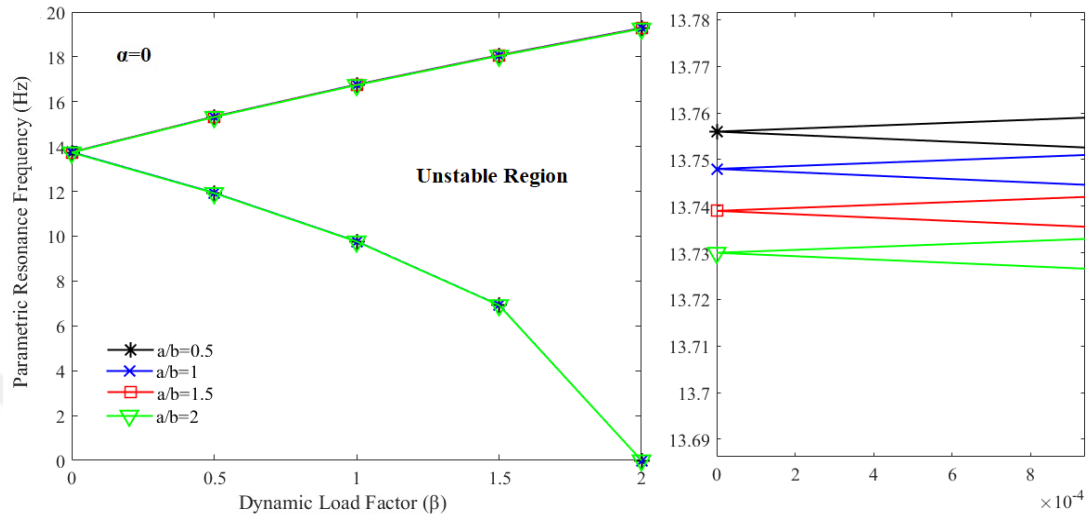


Figure 5.23 Unstable regions ($\alpha=0$) of the single-bay laminated composite curved plate frame structure having C1 stacking order and different aspect ratios

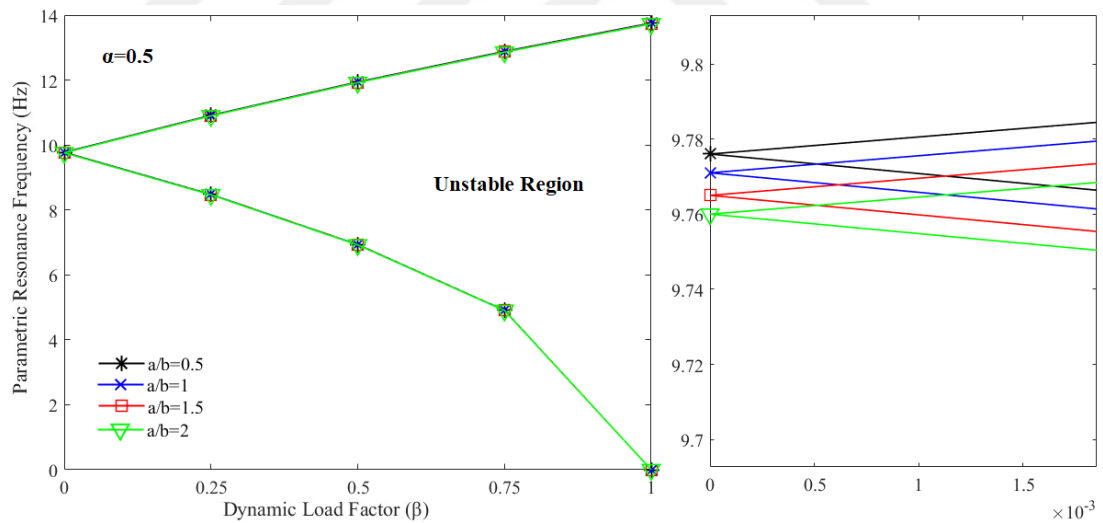


Figure 5.24 Unstable regions ($\alpha=0.5$) of the single-bay laminated composite curved plate frame structure having C1 stacking order and different aspect ratios

Figures 5.25 - 5.28 show the instability region of the single-bay and two-bay laminated composite curved plate frame structure, having different stacking orders for $\alpha=0$ and $\alpha=0.5$, respectively. It is concluded that the unstable region is almost the same for curved frame structures having C3, C4, and C5 stacking orders.

The distance of the unstable regions to the origin can be ordered from the closest to farthest as C2, C3, C5, C4, and C1, respectively. Such order occurs because even the stacking orders are different, the fundamental natural frequency values and the critical buckling load of the structures having C3, C4, and C5 fiber angles do not change remarkably. The unstable regions of the single-bay and two-bay laminated composite curved plate frame structure having C1 stacking order are similar to those structures having C3, C4, and C5 fiber angles. The area of the unstable regions of C1 is almost equal to those of C3, C4, and C5. On the other hand, the unstable region becomes narrow and moves towards the origin considerably for the curved frame structure has a C2 stacking order. This is due to the fiber angles of the first and the effects of the last layer, both fundamental natural frequency and critical buckling load significantly. Comparing single-bay and two-bay structures, the area of the unstable regions are the same. However, the unstable regions move towards origin when the structure has two bays. The unstable regions of all stacking orders move towards the origin as the static load factor increases.

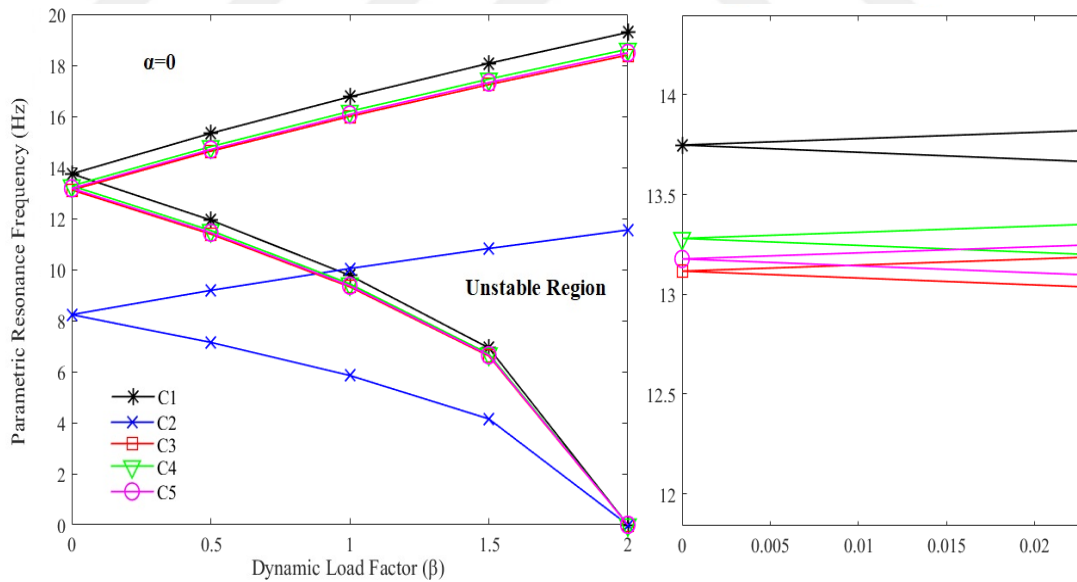


Figure 5.25 Unstable regions ($\alpha=0$) of the single-bay laminated composite curved plate frame structure having different stacking orders

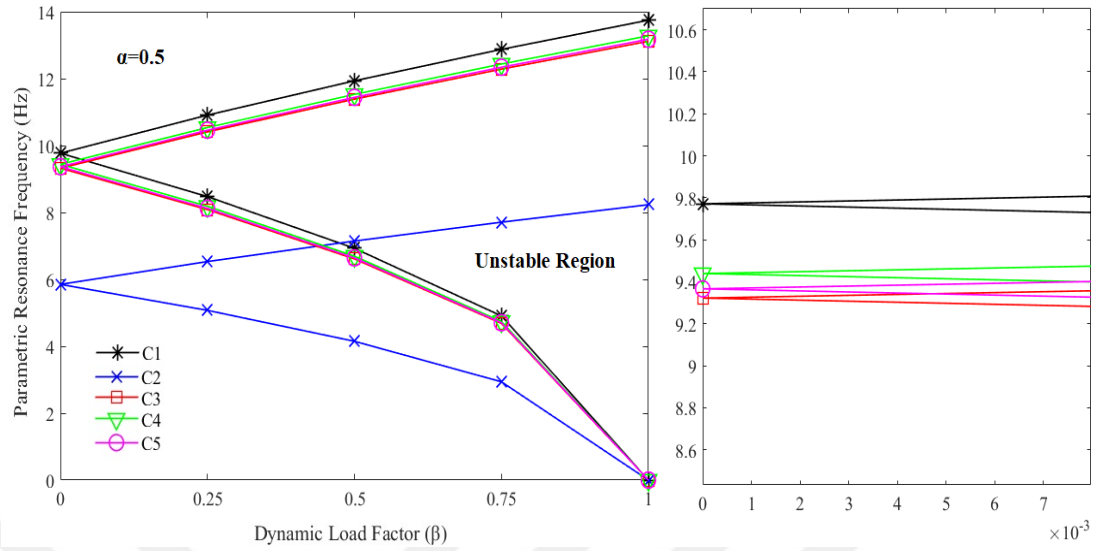


Figure 5.26 Unstable regions ($\alpha=0.5$) of the single-bay laminated composite curved plate frame structure having different stacking orders

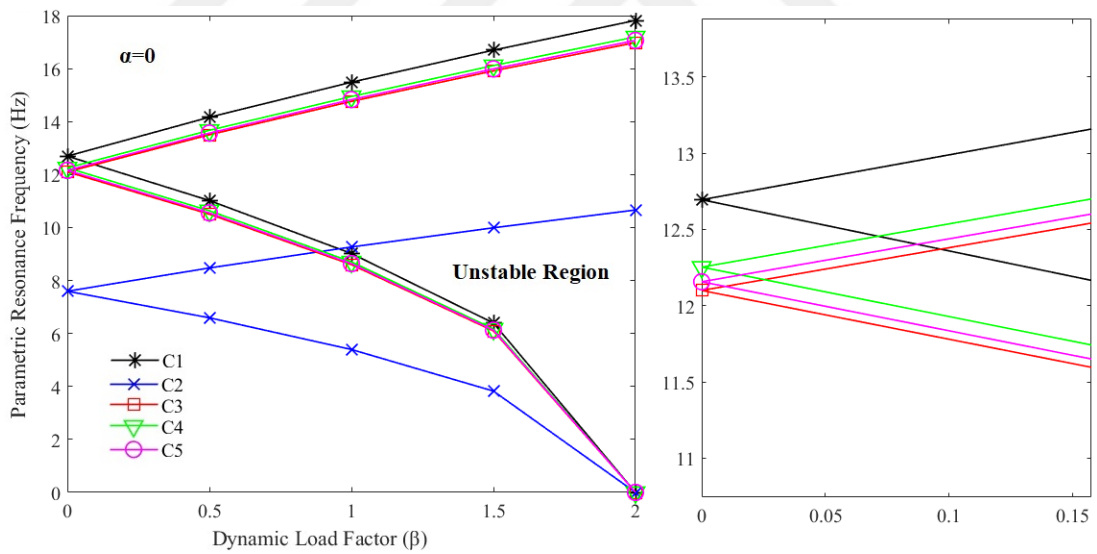


Figure 5.27 Unstable regions ($\alpha=0$) of the two-bay laminated composite curved plate frame structure having different stacking orders

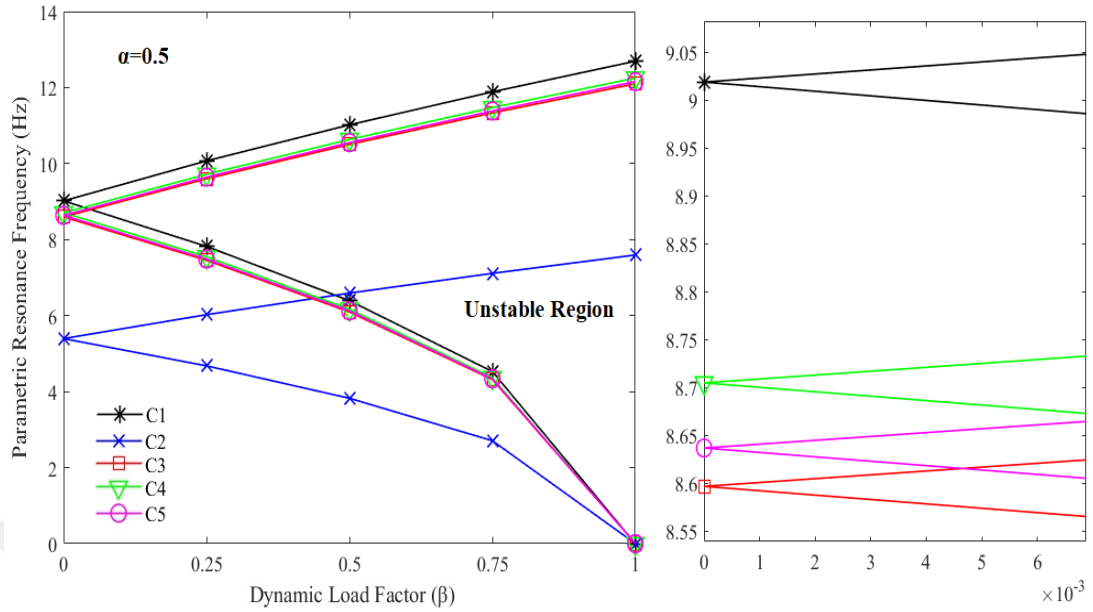


Figure 5.28 Unstable regions ($\alpha=0.5$) of the two-bay laminated composite curved plate frame structure having different stacking orders

5.3.4 Conclusions

It is concluded from the results given in Tables 5.13-5.62 and Figures 5.21 and 5.22 that the radius of curvature does not affect the first ten natural frequency values, the critical buckling loads, and unstable regions of the single-bay and two-bay laminated composite curved plate frame structures, considerably. According to Figures 5.10 and 5.11, the mode shapes change for some modes as the radius of curvature increases or decreases.

It is concluded from Tables 5.53-5.62 and Figures 5.23 and 5.24 that the aspect ratio has a considerable effect on the first ten natural frequencies and the critical buckling loads of the single-bay and two-bay laminated composite curved plate frame structures. However, the aspect ratio does not have an impact on unstable regions, significantly. This is due to the natural frequency, and the critical buckling load values increase or decrease in the same ratio. Figures 5.11-5.20 indicate that the mode shapes shift to each other as the aspect ratio changes.

It is inferred from the results that implementing C2 as the fiber orientation of the laminated composite curved plate frame structure affects considerably the first ten natural frequency values, critical buckling loads, and unstable regions of such structure. This is because of the maximum transverse displacement occurs at the first and last layer of the structure. Apart from the stacking orders C1, C3, C4, and C5, the fiber angles of the first and last layers of C2 is 90° , whereas it is 0° for other fiber orientations. Therefore, the natural frequency results of the curved plate frame structure having C1, C3, C4, or C5 are close to each other.

5.4 Numerical Results of the Finite Element Analysis of Laminated Composite Curved Thick Plate Frames

This section presents the free vibration finite element analysis of laminated parabolic thick plate frames (LCPTPF), which includes two square flat plates and one singly curved plate perpendicular to those flat plates. Finite element analysis based on First-Order Shear Deformation Theory (FSDT) presented in chapter 4, is performed to obtain the first ten natural frequencies of LCPTPF.

The natural frequency results of free vibration analysis of LCPTPF are calculated for several parameters: curvature, aspect ratio, boundary conditions, stacking orders, and multi-bay structure. For comparison, the same structure is modeled and simulated via ANSYS.

5.4.1 Free Vibration Analysis of Laminated Composite Curved Thick Plate Frames

In this section, free vibration analysis of the single-bay and two-bay laminated composite parabolic (curved) thick plate frame structures is investigated by changing the radii of curvature, aspect ratio and boundary conditions of the curved frame structure. Stacking sequences are set as $[0^\circ/0^\circ/0^\circ/0^\circ]$, $[90^\circ/0^\circ/0^\circ/90^\circ]$, $[0^\circ/90^\circ/90^\circ/0^\circ]$, $[0^\circ/45^\circ/-45^\circ/0^\circ]$ and $[0^\circ/60^\circ/-60^\circ/0^\circ]$ for all cases. These stacking sequences are denoted as C1, C2, C3, C4 and C5 in order to simplify.

Material properties are given in Table 5.63 (Reddy, 2003). The density is assumed as 1250 kg/m^3 . Natural frequency values are obtained in hertz (Hz) for each case mentioned below.

Table 5.63 Parametric material properties and geometry of the structure of laminated composite curved frames

Property	Symbol	Quantity
Transverse Elasticity Modulus	E_y	1
Longitudinal Elasticity Modulus	E_x	$40E_y$
Shear Modulus in xy- direction	G_{xy}	$0.6E_y$
Shear Modus in xz- direction	G_{xz}	$0.6E_y$
Shear Modus in yz- direction	G_{yz}	$0.5E_y$
Density	ρ	1250 kg/m^3
Poisson Ratio	ν	0.25
Geometric Properties of the Structure		
Single-Bay Structure Length (a), Width (b)	a	800 mm
	b	800 mm
Structure Thickness	h	a/10

5.3.1.1 Effect of The Radius of Curvature

Four different radii of curvature parameters depended on thickness (h) are introduced. The radius of curvatures (R_{xx}) are ranged gradually from $500h$ to $10h$. The structure is considered as fixed from both ends. Finite element analysis is performed with a mesh size of 30×30 . For comparison of the accuracy, the same structure is modeled using ANSYS. The natural frequencies obtained from the analysis of the structure are given in Tables 5.64-5.73.

Table 5.64 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C1 and $R_{xx}=500h$ and $R_{xx}=250h$

Natural Frequency (Hz)	$R_{xx}=500h$			$R_{xx}=250h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	78.880	75.132	4.75%	78.953	75.133	4.84%
λ_2	135.480	135.790	0.23%	135.490	135.790	0.22%
λ_3	148.130	154.960	4.61%	148.270	154.960	4.51%
λ_4	270.980	276.700	2.11%	271.410	276.670	1.94%
λ_5	276.170	278.660	0.90%	276.470	278.630	0.78%
λ_6	309.090	314.120	1.63%	309.870	314.140	1.38%
λ_7	311.790	323.760	3.84%	312.250	323.800	3.70%
λ_8	339.150	326.280	3.79%	340.140	325.870	4.20%
λ_9	345.980	327.280	5.40%	347.960	327.720	5.82%
λ_{10}	347.960	354.150	1.78%	349.830	354.090	1.22%

Table 5.65 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C1 and $R_{xx}=100h$ and $R_{xx}=10h$

Natural Frequency (Hz)	$R_{xx}=100h$			$R_{xx}=10h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	78.840	75.139	4.69%	77.010	75.797	1.58%
λ_2	135.390	135.780	0.29%	131.420	134.650	2.46%
λ_3	148.070	154.960	4.65%	145.530	155.090	6.57%
λ_4	269.210	276.430	2.68%	243.970	259.020	6.17%
λ_5	274.260	278.410	1.51%	245.240	261.150	6.49%
λ_6	308.970	314.250	1.71%	304.760	314.720	3.27%
λ_7	310.340	324.040	4.41%	305.180	320.680	5.08%
λ_8	338.970	324.620	4.23%	335.420	341.280	1.75%
λ_9	349.420	329.140	5.80%	351.820	347.090	1.34%
λ_{10}	351.360	353.690	0.66%	369.630	350.460	5.19%

Tables 5.64 and 5.65 shows the first ten natural frequencies for C1 in accordance with the change of radii of curvature. It is inferred that the changes are negligible in the natural frequency values as the curvature increases. However, as the curvature decreases from 100h to 10h, the fourth, fifth, seventh, and tenth natural frequencies decrease while the eighth and ninth natural frequencies increase.

Table 5.66 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C2 and $R_{xx}=500h$ and $R_{xx}=250h$

Natural Frequency (Hz)	$R_{xx}=500h$			$R_{xx}=250h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	36.540	34.342	6.02%	36.530	34.343	5.99%
λ_2	131.630	132.140	0.39%	131.560	132.140	0.44%
λ_3	133.810	144.920	8.30%	133.790	144.920	8.32%
λ_4	135.700	150.330	10.78%	135.500	150.330	10.94%
λ_5	152.570	168.370	10.36%	152.360	168.370	10.51%
λ_6	201.800	202.940	0.56%	201.240	202.950	0.85%
λ_7	204.030	215.610	5.68%	204.520	215.620	5.43%
λ_8	204.310	247.470	21.12%	204.520	247.470	21.00%
λ_9	213.330	254.460	19.28%	213.860	254.460	18.98%
λ_{10}	384.350	403.825	5.07%	384.310	403.820	5.08%

Table 5.67 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C2 and $R_{xx}=100h$ and $R_{xx}=10h$

Natural Frequency (Hz)	$R_{xx}=100h$			$R_{xx}=10h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	36.510	34.356	5.90%	35.430	35.524	0.27%
λ_2	131.290	132.090	0.61%	118.720	128.380	8.14%
λ_3	133.730	144.920	8.37%	122.300	144.930	18.50%
λ_4	134.880	150.300	11.43%	131.320	148.050	12.74%
λ_5	151.770	168.370	10.94%	145.220	168.650	16.13%
λ_6	201.340	203.040	0.84%	193.200	211.500	9.47%
λ_7	204.010	215.700	5.73%	198.340	222.600	12.23%
λ_8	205.990	247.470	20.14%	225.620	248.780	10.27%
λ_9	215.440	254.460	18.11%	234.820	254.470	8.37%
λ_{10}	384.140	403.810	5.12%	369.120	403.290	9.26%

Tables 5.66 and 5.67 present the first ten natural frequency values of C2. The changes in the first ten natural frequencies are negligible until the curvature decreases to 100h. As the radius of curvature shifts to 10h from 100h, the second and fourth natural frequencies decrease while the first, sixth, seventh, eighth, and tenth natural frequencies increase.

Table 5.68 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C3 and $R_{xx}=500h$ and $R_{xx}=250h$

Natural Frequency (Hz)	$R_{xx}=500h$			$R_{xx}=250h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.420	70.874	4.76%	74.600	70.874	4.99%
λ_2	131.750	132.180	0.33%	131.830	132.170	0.26%
λ_3	143.880	152.830	6.22%	144.300	152.830	5.91%
λ_4	254.750	260.300	2.18%	256.520	260.260	1.46%
λ_5	260.280	264.120	1.48%	261.700	264.080	0.91%
λ_6	299.100	307.290	2.74%	301.170	307.320	2.04%
λ_7	326.390	311.040	4.70%	330.000	310.980	5.76%
λ_8	328.660	316.260	3.77%	331.940	316.350	4.70%
λ_9	343.050	364.310	6.20%	346.260	364.310	5.21%
λ_{10}	370.330	379.790	2.55%	373.080	379.830	1.81%

Table 5.69 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C3 and $R_{xx}=100h$ and $R_{xx}=10h$

Natural Frequency (Hz)	$R_{xx}=100h$			$R_{xx}=10h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.370	70.880	4.69%	72.610	71.281	1.83%
λ_2	131.620	132.160	0.41%	127.300	130.960	2.88%
λ_3	143.810	152.830	6.27%	141.090	152.950	8.41%
λ_4	253.170	259.990	2.69%	230.050	239.240	3.99%
λ_5	258.600	263.830	2.02%	232.470	244.230	5.06%
λ_6	298.950	307.550	2.88%	294.900	304.520	3.26%
λ_7	329.470	310.620	5.72%	333.410	323.320	3.03%
λ_8	331.750	316.880	4.48%	353.200	334.130	5.40%
λ_9	342.620	364.310	6.33%	364.380	364.530	0.04%
λ_{10}	369.670	380.170	2.84%	377.750	394.530	4.44%

The first ten natural frequency values of C3 are given in Tables 5.68 and 5.69. The second, fourth, fifth, and sixth natural frequency values decrease while the seventh, eighth, and tenth frequencies increase when R_{xx} decreases from 100h to 10h.

Table 5.70 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C4 and $R_{xx}=500h$ and $R_{xx}=250h$

Natural Frequency (Hz)	$R_{xx}=500h$			$R_{xx}= 250h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.670	71.606	4.10%	74.860	71.606	4.35%
λ_2	255.590	264.130	3.34%	257.370	264.100	2.61%
λ_3	258.090	273.470	5.96%	258.600	273.380	5.72%
λ_4	271.880	281.830	3.66%	273.000	281.780	3.22%
λ_5	294.710	289.150	1.89%	295.450	289.240	2.10%
λ_6	305.310	312.680	2.41%	306.640	312.710	1.98%
λ_7	327.180	321.100	1.86%	330.800	321.120	2.93%
λ_8	332.930	328.910	1.21%	336.140	329.120	2.09%
λ_9	343.430	371.030	8.04%	346.620	371.040	7.05%
λ_{10}	354.230	375.700	6.06%	356.910	375.750	5.28%

Table 5.71 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C4 and $R_{xx}=100h$ and $R_{xx}=10h$

Natural Frequency (Hz)	$R_{xx}=100h$			$R_{xx}= 10h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.620	71.611	4.03%	72.850	72.107	1.02%
λ_2	254.020	263.840	3.87%	218.020	241.900	10.95%
λ_3	255.290	272.800	6.86%	230.760	244.280	5.86%
λ_4	271.420	281.510	3.72%	263.860	279.810	6.04%
λ_5	294.880	289.790	1.73%	290.220	297.700	2.58%
λ_6	306.220	312.940	2.19%	328.070	328.950	0.27%
λ_7	330.300	321.300	2.72%	333.860	341.060	2.16%
λ_8	336.180	330.550	1.67%	343.560	359.850	4.74%
λ_9	343.060	371.040	8.16%	363.610	371.470	2.16%
λ_{10}	355.410	376.130	5.83%	365.610	384.740	5.23%

Table 5.72 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C5 and $R_{xx}=500h$ and $R_{xx}=250h$

Natural Frequency (Hz)	$R_{xx}=500h$			$R_{xx}= 250h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.855	71.175	4.92%	75.040	71.176	5.15%
λ_2	245.860	258.760	5.25%	245.650	258.670	5.30%
λ_3	249.350	260.310	4.40%	250.140	260.310	4.07%
λ_4	256.530	262.040	2.15%	258.330	262.010	1.42%
λ_5	269.350	270.460	0.41%	270.650	270.500	0.06%
λ_6	300.110	309.850	3.25%	301.190	309.880	2.89%
λ_7	329.610	316.930	3.85%	333.290	316.950	4.90%
λ_8	333.070	329.750	1.00%	336.240	329.950	1.87%
λ_9	345.060	367.510	6.51%	347.620	367.510	5.72%
λ_{10}	346.010	369.460	6.78%	349.230	369.540	5.82%

Table 5.73 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C5 and $R_{xx}=100h$ and $R_{xx}=10h$

Natural Frequency (Hz)	$R_{xx}=100h$			$R_{xx}=10h$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	74.804	71.180	4.84%	73.020	71.612	1.93%
λ_2	243.410	258.110	6.04%	210.040	231.280	10.11%
λ_3	248.960	260.320	4.56%	231.740	241.180	4.07%
λ_4	254.950	261.750	2.67%	241.790	260.420	7.71%
λ_5	269.720	270.770	0.39%	266.870	275.700	3.31%
λ_6	301.040	310.110	3.01%	324.240	325.950	0.53%
λ_7	332.720	317.110	4.69%	336.270	332.540	1.11%
λ_8	336.120	331.320	1.43%	342.480	356.710	4.15%
λ_9	345.620	367.520	6.34%	351.500	367.780	4.63%
λ_{10}	346.040	370.100	6.95%	367.880	392.190	6.61%

According to Tables 5.70-5.73, the change trends of the natural frequency values of C4 and C5 are the same as other fiber orientations. Outcomes of the present study are in good agreement with ANSYS results. It is seen from the results given in this section, the first five natural frequencies of all structures, regardless of their stacking sequence, decrease as the radius of curvature decreases from 100h to 10h. Other natural frequency values behave differently with respect to the stacking sequence. The last five natural frequency values increase for C4 and C5 when curvature decreases.

Although there are some differences between ANSYS results, the present work shows a good agreement with ANSYS results, especially when the radius of curvature increases. This is because the selected finite element's representation ability for the curved part of the structure is very good for slightly curved structures. The stacking sequence has also affected the increment and the values of natural frequencies. Besides, it is concluded that the representation of the present study changes with respect to the fiber angle of each layer. Furthermore, it is observed that the first and last layer's fiber angle is more effective than those of the middle layer on the ability of representation of this study.

5.3.1.2 Effect of The Aspect Ratio

The effect of aspect ratio on the free vibration response of LCPTPF is investigated in this section. Four different aspect ratios varying only with respect to the structure height (a) are taken into account. The boundary condition is selected as fixed from all ends. The dimensions of the structure are set as $a=0.8$ m to 1.6 m with 0.4 m increments, $a_c=b=0.8$ m, $R_{xx}=10h$ and $b/h=10$. For comparison, the same structure is modeled via ANSYS. The natural frequency results for this case are given in Tables 5.74-10.

Table 5.74 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C1 and $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	163.270	163.390	0.07%	77.010	75.797	1.58%
λ_2	202.680	196.810	2.90%	131.420	134.650	2.46%
λ_3	261.620	251.930	3.70%	145.530	155.090	6.57%
λ_4	305.220	314.270	2.97%	243.970	259.020	6.17%
λ_5	315.270	321.390	1.94%	245.240	261.150	6.49%
λ_6	374.370	382.380	2.14%	304.760	314.720	3.27%
λ_7	494.600	480.020	2.95%	305.180	320.68	5.08%
λ_8	516.390	518.440	0.40%	335.420	341.280	1.75%
λ_9	561.380	540.670	3.69%	351.820	347.090	1.34%
λ_{10}	595.490	594.040	0.24%	369.630	350.460	5.19%

Table 5.75 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C1 and $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	44.540	43.930	1.37%	28.890	28.758	0.46%
λ_2	96.300	98.270	2.05%	73.770	76.060	3.10%
λ_3	99.530	110.100	10.62%	74.690	85.393	14.33%
λ_4	161.990	180.230	11.26%	109.120	121.650	11.48%
λ_5	166.030	181.380	9.25%	113.900	124.950	9.70%
λ_6	190.970	194.920	2.07%	130.350	132.280	1.48%
λ_7	212.710	210.030	1.26%	142.690	142.840	0.11%
λ_8	217.210	225.060	3.61%	171.540	176.440	2.86%
λ_9	224.820	231.380	2.92%	174.920	179.240	2.47%
λ_{10}	276.370	275.550	0.30%	223.170	231.700	3.82%

According to results given in Tables 5.74 and 5.75, the natural frequency values of C1 changes significantly with respect to aspect ratio. As the aspect ratio rises, all natural frequencies declines. As it is expected, the error values decrease when the aspect ratio has smaller values.

Table 5.76 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C2 and $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	86.940	88.850	2.19%	35.430	35.524	0.27%
λ_2	163.190	174.390	6.86%	118.720	128.380	8.14%
λ_3	174.770	189.760	8.58%	122.300	144.930	18.50%
λ_4	225.790	228.080	1.01%	131.320	148.050	12.74%
λ_5	226.460	250.280	10.52%	145.220	168.650	16.13%
λ_6	374.570	467.650	24.85%	193.200	211.500	9.47%
λ_7	388.560	472.170	21.52%	198.340	222.600	12.23%
λ_8	459.970	504.850	9.76%	225.620	248.780	10.27%
λ_9	494.710	578.410	16.92%	234.820	254.470	8.37%
λ_{10}	502.960	600.430	19.38%	369.120	403.290	9.26%

Table 5.77 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C2 and $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	19.200	19.780	3.02%	12.030	12.981	7.91%
λ_2	75.450	93.220	23.55%	48.170	64.740	34.40%
λ_3	85.290	97.904	14.79%	59.380	69.430	16.93%
λ_4	86.010	99.013	15.12%	61.460	71.235	15.90%
λ_5	94.830	113.720	19.92%	66.370	73.430	10.64%
λ_6	105.850	113.910	7.61%	69.730	77.276	10.82%
λ_7	118.650	127.960	7.85%	83.150	93.890	12.92%
λ_8	160.110	180.890	12.98%	119.810	150.600	25.70%
λ_9	171.220	192.440	12.39%	135.040	166.590	23.36%
λ_{10}	241.930	279.050	15.34%	156.730	186.780	19.17%

As seen from the results which are given in Tables 5.76 and 5.77, the aspect ratio has the same effect on C2 as C1. The natural frequency values of C2 are smaller than that of C1. It is seen that, the natural frequency values of C2 decrease as the aspect ratio decreases. The error rates obtained for C2 are relatively higher than those of C1.

Table 5.78 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C3 and $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	154.700	154.320	0.25%	72.610	71.280	1.83%
λ_2	193.350	199.100	2.97%	127.300	130.960	2.88%
λ_3	246.650	256.710	4.08%	141.090	152.950	8.41%
λ_4	296.400	282.980	4.53%	230.050	239.240	3.99%
λ_5	306.100	297.970	2.66%	232.470	244.230	5.06%
λ_6	460.760	456.860	0.85%	294.900	304.520	3.26%
λ_7	482.070	496.090	2.91%	333.410	323.320	3.03%
λ_8	599.910	627.320	4.57%	353.200	334.130	5.40%
λ_9	601.960	648.520	7.73%	364.380	364.530	0.04%
λ_{10}	608.080	656.850	8.02%	377.750	394.530	4.44%

Table 5.79 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C3 and $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	41.890	41.220	1.60%	27.130	26.965	0.61%
λ_2	91.550	93.917	2.59%	68.920	71.208	3.32%
λ_3	94.880	108.880	14.76%	69.500	85.019	22.33%
λ_4	152.810	169.340	10.82%	102.810	114.630	11.50%
λ_5	157.270	171.860	9.28%	107.930	118.650	9.93%
λ_6	182.510	186.190	2.02%	124.290	126.390	1.69%
λ_7	201.050	198.160	1.44%	134.540	134.600	0.04%
λ_8	262.650	260.100	0.97%	211.140	220.570	4.47%
λ_9	284.680	274.400	3.61%	222.520	233.920	5.12%
λ_{10}	295.930	306.830	3.68%	230.940	237.980	3.05%

Tables 5.78 and 5.79 show the first ten natural frequency values of C3. It is concluded that the interpretation made for C1 can be made for C3 since the natural frequency values of C3 are changing in the same way as that of C1. The natural frequency values of C3 are greater due to the stacking sequence when compared with C1. Again, the same trend as C1 in terms of error rate can be seen for C3.

Table 5.80 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C4 and $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	155.090	156.750	1.07%	72.850	72.107	1.02%
λ_2	278.780	282.440	1.31%	218.020	241.900	10.95%
λ_3	297.350	289.400	2.67%	230.760	244.280	5.86%
λ_4	516.800	514.180	0.51%	263.860	279.810	6.04%
λ_5	529.040	562.270	6.28%	290.220	297.700	2.58%
λ_6	552.150	579.230	4.90%	328.070	328.950	0.27%
λ_7	601.540	664.270	10.43%	333.860	341.060	2.16%
λ_8	609.990	675.020	10.66%	343.560	359.850	4.74%
λ_9	644.960	707.820	9.75%	363.610	371.470	2.16%
λ_{10}	702.180	720.650	2.63%	365.610	384.740	5.23%

Table 5.81 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C4 and $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	42.050	41.619	1.02%	27.240	27.202	0.14%
λ_2	153.260	166.140	8.40%	100.880	112.250	11.27%
λ_3	154.090	172.050	11.66%	103.140	116.190	12.65%
λ_4	154.290	174.280	12.96%	109.180	119.920	9.84%
λ_5	174.810	182.490	4.39%	117.280	125.000	6.58%
λ_6	201.520	201.010	0.25%	134.920	136.260	0.99%
λ_7	211.820	214.080	1.07%	146.680	150.580	2.66%
λ_8	234.260	255.460	9.05%	186.100	205.290	10.31%
λ_9	239.080	264.180	10.50%	186.630	208.070	11.49%
λ_{10}	285.770	280.240	1.94%	231.690	242.710	4.76%

Table 5.82 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C5 and $a/b=0.5$ and $a/b=1$

Natural Frequency (Hz)	$a/b=0.5$			$a/b=1$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	156.020	155.410	0.39%	73.017	71.612	1.92%
λ_2	275.690	276.030	0.12%	210.040	231.280	10.11%
λ_3	299.040	285.570	4.50%	231.720	241.180	4.08%
λ_4	465.570	492.500	5.78%	241.790	260.420	7.71%
λ_5	471.470	516.020	9.45%	266.870	275.700	3.31%
λ_6	513.220	539.000	5.02%	324.240	325.950	0.53%
λ_7	605.830	655.000	8.12%	336.270	332.540	1.11%
λ_8	610.400	661.130	8.31%	342.480	356.710	4.15%
λ_9	647.090	701.920	8.47%	351.500	367.780	4.63%
λ_{10}	661.340	713.220	7.84%	367.880	392.190	6.61%

Table 5.83 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C5 and $a/b=1.5$ and $a/b=2$

Natural Frequency (Hz)	$a/b=1.5$			$a/b=2$		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	42.060	41.365	1.65%	27.200	27.047	0.56%
λ_2	144.130	157.790	9.48%	95.350	108.190	13.47%
λ_3	146.460	159.190	8.69%	103.280	110.390	6.88%
λ_4	153.700	170.500	10.93%	103.730	115.310	11.16%
λ_5	164.790	178.590	8.37%	111.400	122.700	10.14%
λ_6	202.480	199.400	1.52%	135.310	135.290	0.01%
λ_7	203.440	205.440	0.98%	140.060	144.220	2.97%
λ_8	233.860	266.050	13.76%	186.940	222.690	19.12%
λ_9	237.070	276.600	16.67%	187.680	226.610	20.74%
λ_{10}	286.950	279.000	2.77%	232.470	239.810	3.16%

The first ten natural frequency values of C4 and C5 are given in Tables 5.80-5.83. It is seen that the fundamental frequencies of C4 and C5 are almost equal. The aspect ratio has the same effect as the other structures with different stacking sequences. The natural frequency values of C4 and C5 decreases as the aspect ratio increases.

The error rates of the present study decrease as the value of the aspect ratio decreases. For all structures with different stacking sequences, it is concluded from the results given in this section that the increment of the aspect ratio affects the natural frequency values, significantly. As the aspect ratio increases, the natural frequency values of those structures decreases. This is because when the aspect ratio increase, the stiffness matrix of the structure decrease.

5.3.1.3 Effect of the Boundary Conditions

In this section, LCPTPF structures fixed from two sides (CC) and simply supported (SS) boundary conditions are investigated for C1, C2, C3, C4, and C5 stacking sequences. For SS boundary condition, it is considered that only rotations are permitted. Structure dimensions are set as $a=a_c=b=0.8$ m, $R_{xx}/h=10$ and $b/h=10$.

All mode shapes have been defined according to general deflection tendencies to form (i.e., Bn: Bending; Ln: Longitudinal; Tr: Torsion) and presented in Figures 5.28-5.32.

Tables 5.84 -5.88 give the first ten natural frequencies of laminated composite parabolic thick plate frame structures under fixed and simply supported boundary conditions for C1, C2, C3, C4, and C5 stacking orders, respectively.

Table 5.84 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C1 under simply supported and fixed boundary conditions

Natural Frequency (Hz)	SS			CC		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	39.760	37.130	6.59%	77.010	75.797	1.58%
λ_2	131.260	134.610	2.55%	131.420	134.650	2.46%
λ_3	135.380	145.790	7.69%	145.530	155.090	6.57%
λ_4	213.650	235.460	10.21%	243.970	259.020	6.17%
λ_5	218.470	238.830	9.32%	245.240	261.150	6.49%
λ_6	272.260	279.810	2.77%	304.760	314.720	3.27%
λ_7	277.370	291.740	5.18%	305.180	320.680	5.08%
λ_8	298.510	302.170	1.23%	335.420	341.280	1.75%
λ_9	312.940	309.420	1.12%	351.820	347.090	1.34%
λ_{10}	349.860	326.990	6.54%	369.630	350.460	5.19%

Table 5.85 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C2 under simply supported and fixed boundary conditions

Natural Frequency (Hz)	SS			CC		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	16.950	15.760	7.02%	35.430	35.520	0.27%
λ_2	97.450	124.870	28.14%	118.720	128.380	8.14%
λ_3	109.260	125.690	15.04%	122.300	144.930	18.50%
λ_4	119.790	132.880	10.93%	131.320	148.050	12.74%
λ_5	133.180	151.360	13.65%	145.220	168.650	16.13%
λ_6	153.900	159.560	3.68%	193.200	211.500	9.47%
λ_7	171.130	179.200	4.72%	198.340	222.600	12.23%
λ_8	199.930	217.370	8.72%	225.620	248.780	10.27%
λ_9	212.360	223.370	5.18%	234.820	254.470	8.37%
λ_{10}	357.550	389.040	8.81%	369.120	403.290	9.26%

Free vibration analysis results for C1 and C2 are given in Tables 5.84 and 5.85. The natural frequency values are higher for that of fixed boundary conditions. The error rates of the C2 are relatively higher than those of C1.

Table 5.86 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C3 under simply supported and fixed boundary conditions

Natural Frequency (Hz)	SS			CC		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	37.360	34.610	7.34%	72.610	71.280	1.83%
λ_2	127.110	130.930	3.01%	127.300	130.960	2.88%
λ_3	131.250	144.330	9.97%	141.090	152.950	8.41%
λ_4	201.140	218.180	8.47%	230.050	239.240	3.99%
λ_5	206.810	223.990	8.31%	232.470	244.230	5.06%
λ_6	261.860	268.960	2.71%	294.900	304.520	3.26%
λ_7	295.550	284.620	3.70%	333.410	323.320	3.03%
λ_8	332.400	310.350	6.63%	353.200	334.130	5.40%
λ_9	341.960	335.570	1.87%	364.380	364.530	0.04%
λ_{10}	353.170	368.750	4.41%	377.750	394.530	4.44%

Table 5.86 shows the first ten natural frequency values of C3 for simply supported and fixed from two ends boundary conditions. The same interpretation made for C1 is valid for C3 stacking sequence.

Table 5.87 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C4 under simply supported and fixed boundary conditions

Natural Frequency (Hz)	SS			CC		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	37.500	34.930	6.85%	72.850	72.100	1.02%
λ_2	199.680	221.890	11.12%	218.020	241.900	10.95%
λ_3	201.640	228.140	13.14%	230.760	244.280	5.86%
λ_4	230.390	250.090	8.55%	263.860	279.810	6.04%
λ_5	265.630	277.110	4.32%	290.220	297.700	2.58%
λ_6	291.810	288.710	1.06%	328.070	328.950	0.27%
λ_7	295.690	312.080	5.54%	333.860	341.060	2.16%
λ_8	310.490	328.340	5.75%	343.560	359.850	4.74%
λ_9	341.680	341.550	0.04%	363.610	371.470	2.16%
λ_{10}	347.760	361.830	4.05%	365.610	384.740	5.23%

The first ten natural frequency values for simply supported and fixed from all ends boundary conditions for C4 and C5 are given in Tables 5.87 and 5.88, respectively.

Table 5.88 The first ten natural frequency values of the laminated composite curved thick plate frame structure for C5 under simply supported and fixed boundary conditions

Natural Frequency (Hz)	SS			CC		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	37.500	34.710	7.44%	73.020	71.610	1.93%
λ_2	194.550	219.490	12.82%	210.040	231.280	10.11%
λ_3	202.140	220.880	9.27%	231.740	241.180	4.07%
λ_4	213.730	235.120	10.01%	241.790	260.420	7.71%
λ_5	241.530	257.190	6.48%	266.870	275.700	3.31%
λ_6	284.970	286.310	0.47%	324.240	325.950	0.53%
λ_7	297.230	299.780	0.86%	336.270	332.540	1.11%
λ_8	306.490	327.810	6.96%	342.480	356.710	4.15%
λ_9	326.220	338.240	3.68%	351.500	367.780	4.63%
λ_{10}	344.970	367.220	6.45%	367.880	392.190	6.61%

As seen in Tables 5.87 and 5.88, changes in natural frequency for P5 and P4 are similar to C1, C2, and C3. Error rates change with respect to the mode of the structure for relevant boundary conditions, as seen in Figures 5.28-5.32.

For fixed from all ends boundary condition, it is seen from Figures 5.29-5.33 that for all stacking orders, the first mode is the coupling of bending and longitudinal vibration. Two different vibration mode is observed for the second mode. C1 and C3 have the same vibration mode, which is the coupling of bending and longitudinal vibration. On the other hand, C2, C4, and C5 vibrate in torsional and bending coupled mode. For the third mode, while C1 and C3 vibrate in the torsional vibration mode, C2, C4, and C5 vibrate in the bending mode. The fourth mode is the symmetry of the third mode. C1 and C3 vibrate in the bending mode, whereas C2, C4, and C5 vibrate in the torsional mode. For the fifth mode, C1, C2, C3, C4, and C5 vibrate in the coupling of bending and torsional modes.

The sixth and the seventh and eighth modes are the same for C1 and C2, which are the bending, torsional, and bending vibration, respectively. C3 behaves differently from all other stacking sequences after the fifth mode.

C4 and C5 behave the same for all modes. This is because even they have different fiber angles, they have the same angle pattern.

It is seen from Figures 5.29-5.33 that for simply supported boundary conditions, the same mode of the fixed boundary condition is present for all stacking sequences. For the second mode, there are three different vibration modes. C1 and C3 vibrate in the coupling of bending and longitudinal mode, whereas C4 and C5 vibrate in the coupling of torsion and bending mode. C2 is in the pure bending vibration in this mode. For the third mode, C1 and C3 are in the torsional vibration mode. C4 and C5 are in the bending mode. C2 vibrates in the coupling of torsional and bending mode. The fourth and the fifth modes have the same behavior as of the fixed boundary condition for all stacking orders.

Different bending modes occur at the sixth modes for each stacking sequence. Except for C4 and C5, all stacking orders have different mode shapes for the seventh, eighth, ninth, and tenth modes.

It is concluded that no matter which boundary condition is applied, the first mode is the coupling of bending and longitudinal vibration regardless of the stacking order. For the other modes, the angle and the angle pattern change the vibration mode of the structure. In addition, except for the first and the sixth modes, the boundary condition also changes the mode of the structure as expected.

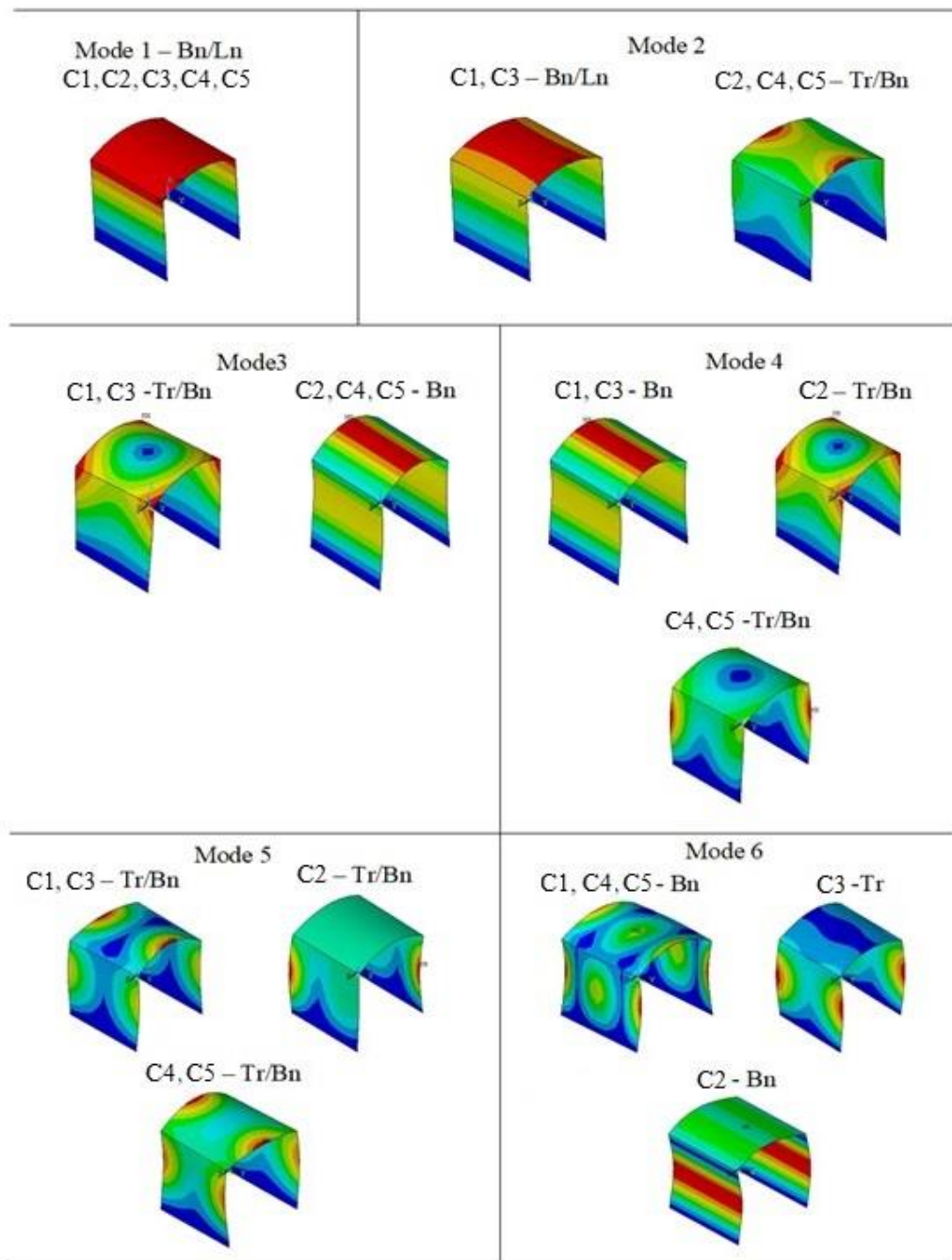


Figure 5.29 The first six modes of laminated composite curved thick plate frames under fixed from two sides boundary conditions

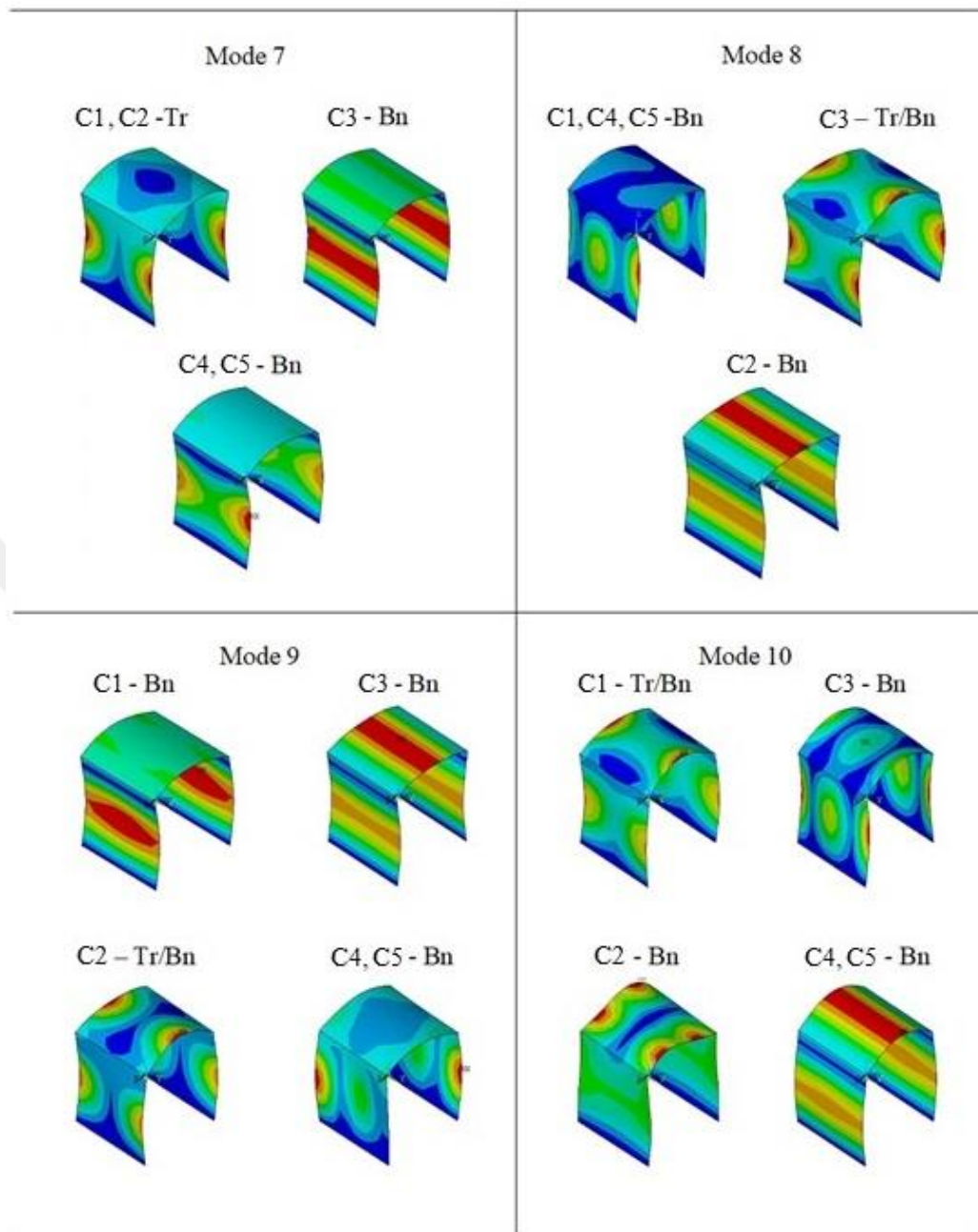


Figure 5.30 The seventh - tenth modes of laminated composite curved thick plate frames under fixed from two sides boundary conditions

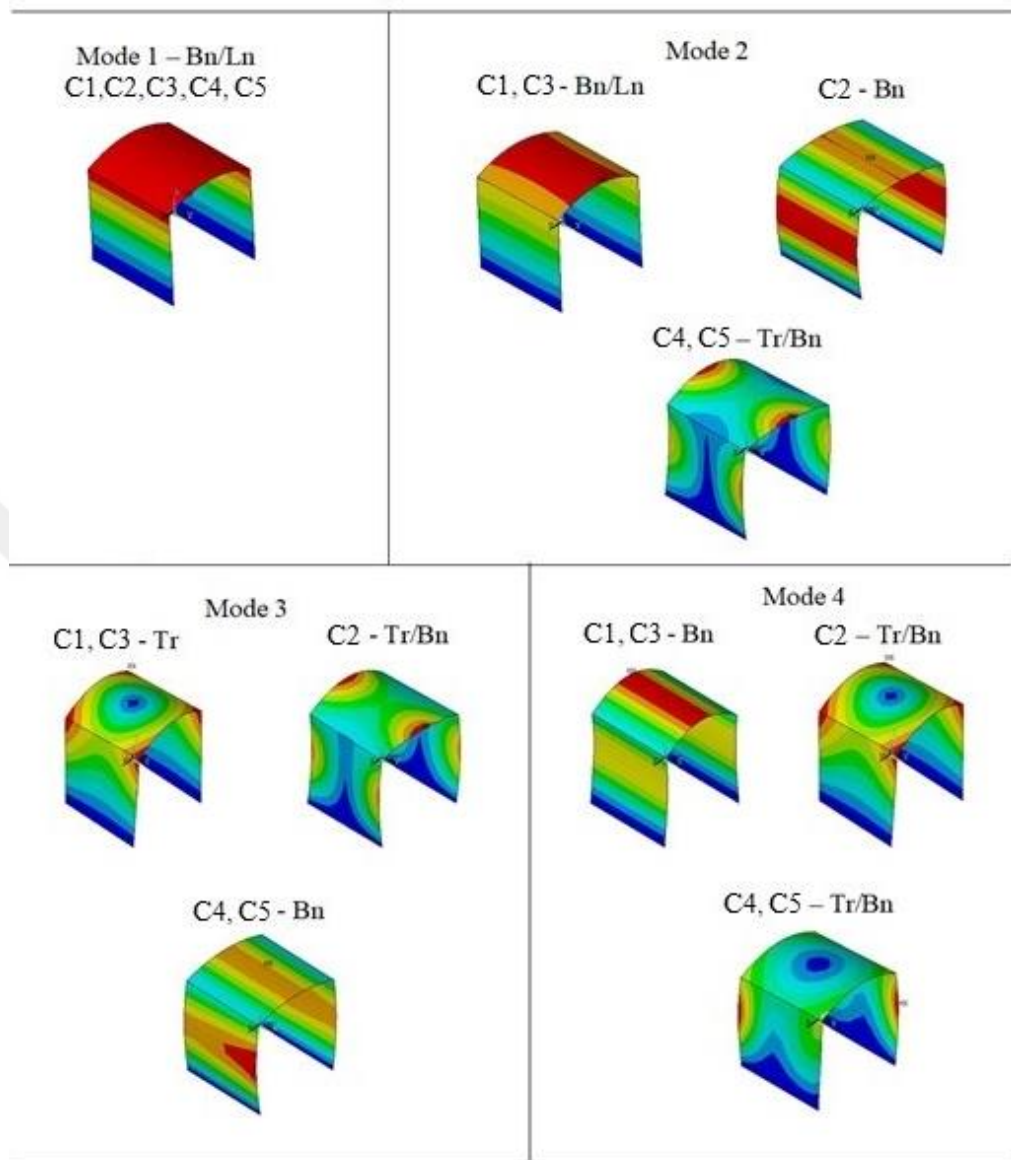


Figure 5.31 The first four modes of laminated composite curved thick plate frames under simply supported boundary conditions

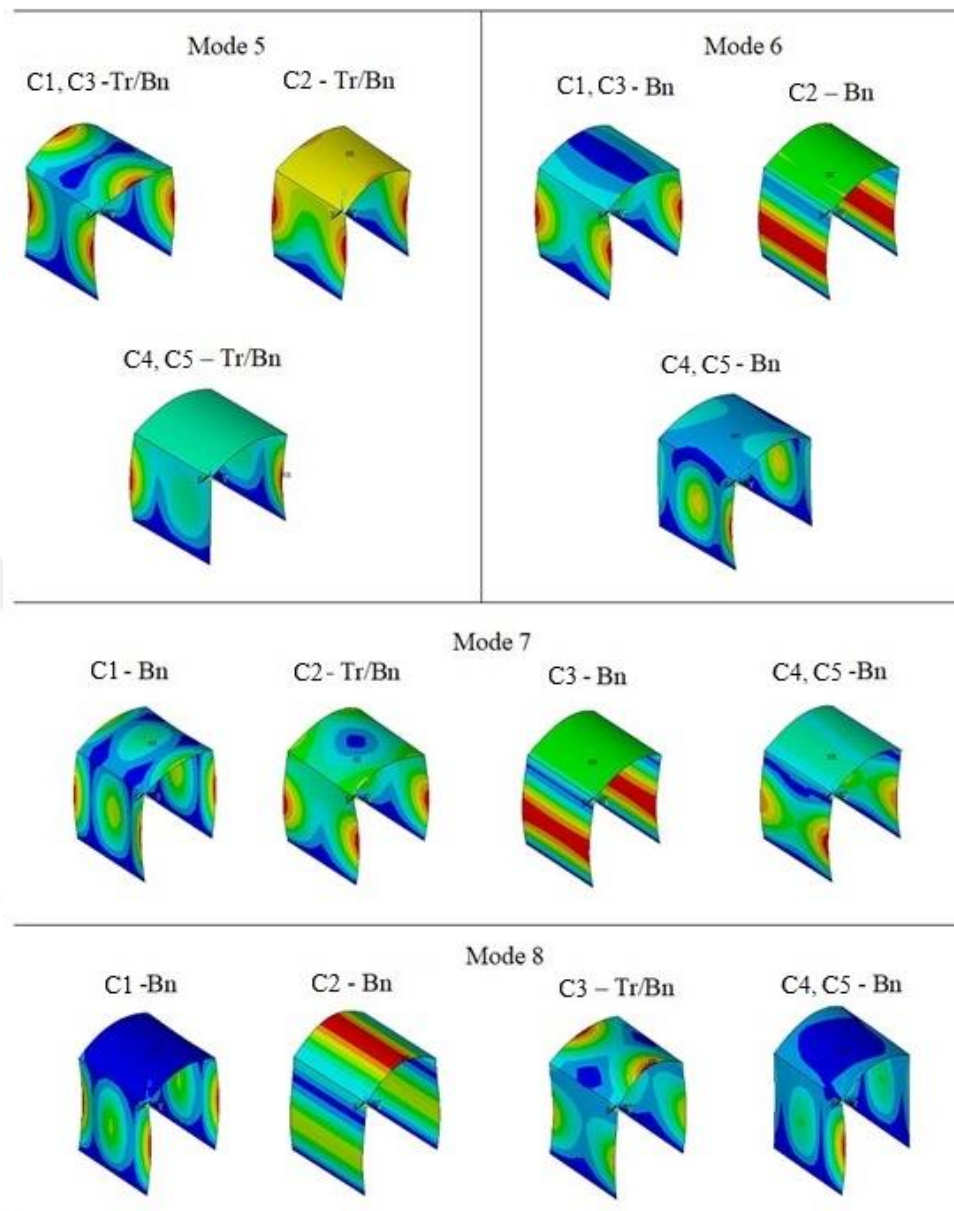


Figure 5.32 The fifth - eighth modes of laminated composite curved thick plate frames under simply supported boundary conditions

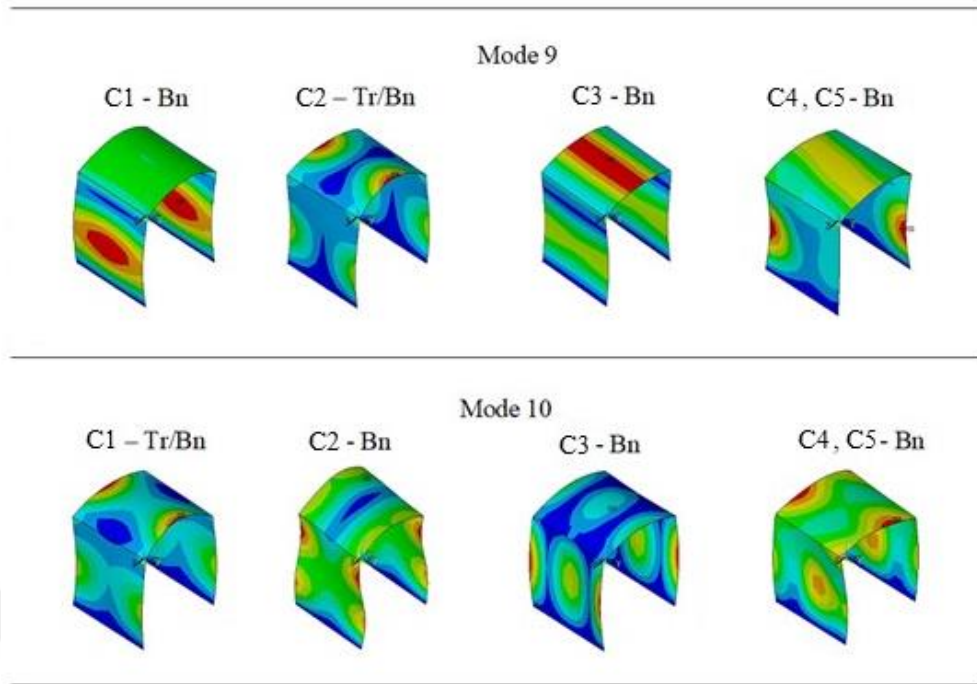


Figure 5.33 The ninth and tenth modes of laminated composite curved thick plate frames under simply supported boundary conditions

To understand the difference between the single-bay and two-bay structure, free vibration analysis is performed for two-bay structures considering fixed boundary condition. Tables 5.89-5.93 give the first ten natural frequency values of the two-bay laminated composite curved thick plate frame structures under fixed boundary conditions.

Table 5.89 The first ten natural frequency values of the C1 and C2 two-bay laminated composite curved thick plate frame structure under fixed boundary conditions

Natural Frequency (Hz)	C1			C2		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	71.53	73.345	2.54%	32.96	37.906	15.01%
λ_2	118.49	122.88	3.70%	108.95	118.6	8.86%
λ_3	138.64	145.76	5.14%	122.93	139.01	13.08%
λ_4	221.64	239.08	7.87%	124.46	168.06	35.03%
λ_5	240.55	263.72	9.63%	129.38	181.46	40.25%
λ_6	241.46	276.89	14.67%	140.6	191.93	36.51%
λ_7	247.44	298.93	20.81%	149.43	198.02	32.52%
λ_8	299.93	316.5	5.52%	196.83	228.18	15.93%
λ_9	302.96	328.01	8.27%	197.34	233.71	18.43%
λ_{10}	312.52	335.09	7.22%	219.84	242.45	10.28%

Table 5.90 The first ten natural frequency values of the C3 and C4 two-bay laminated composite curved thick plate frame structure under fixed boundary conditions

Natural Frequency (Hz)	C3			C4		
	ANSYS	Present Study	Error (%)	ANSYS	Present Study	Error (%)
λ_1	67.49	69.107	2.40%	67.71	70.031	3.43%
λ_2	114.59	119.77	4.52%	208.56	244.37	17.17%
λ_3	134.38	142.49	6.04%	226.66	251.32	10.88%
λ_4	212.19	224.42	5.76%	228.05	274.68	20.45%
λ_5	227.26	237.85	4.66%	234.26	276.39	17.98%
λ_6	228.57	255.96	11.98%	276.91	292.05	5.47%
λ_7	233.58	266.27	14.00%	283.62	298.14	5.12%
λ_8	291.16	305.96	5.08%	328.11	328.79	0.21%
λ_9	292.53	313.62	7.21%	335.49	337.81	0.69%
λ_{10}	335.24	322.91	3.68%	337.6	350.17	3.72%

Table 5.91 The first ten natural frequency values of the C5 two-bay laminated composite curved thick plate frame structure under fixed boundary conditions

Natural Frequency (Hz)	C5		
	ANSYS	Present Study	Error (%)
λ_1	67.87	69.499	2.40%
λ_2	193.52	229.42	18.55%
λ_3	220.49	240.19	8.93%
λ_4	229.19	257.51	12.36%
λ_5	235.24	269.14	14.41%
λ_6	256.43	276.86	7.97%
λ_7	265.39	283.38	6.78%
λ_8	323.34	325.66	0.72%
λ_9	334.73	333.13	0.48%
λ_{10}	338.39	340.8	0.71%

It is seen from Tables 5.89 – 5.91 that the natural frequency values of single-bay structure are greater than the natural frequency values of the two-bay structure. For the most modes, the present study is in a good agreement with ANSYS results.

5.4.2 Conclusions

In this study, finite element free vibration analysis of LCPTPF structure is investigated by using FSDT for various stacking orders. Parameters such as radii of curvature, aspect ratio, boundary conditions are taken into account to find out the ability of the representation of the present study. Additionally, free vibration analysis

is performed for the multi-bay structure of LCPTPF. Based on these analyses results, the following conclusions can be drawn as:

- It is concluded that the natural frequency values of such a structure depend on not only the stacking sequence but also the radii of curvature, aspect ratio, and boundary conditions. However, the stacking order and the aspect ratio have the most impact on the natural frequency values for the LCPTPF structure.
- In terms of stacking order, for all cases, fiber angles of the first and the last layer effects on the natural frequency values significantly.
- The radius of curvature has a slight effect on the natural frequency values of LCPTPF structure
- Regardless of stacking order, for all LCPTPF structures, the natural frequency values decrease considerably as the aspect ratio increases.
- The natural frequency values decrease as the boundary condition of the structure is set as simply supported. It is also concluded that the boundary condition affected the natural frequency values of all structures similar regardless of their stacking sequence.
- According to results for multi-bay structure, the natural frequency values are higher when compared with single-bay structures.
- Except for C2, in general, the present study is in good agreement with ANSYS results. This is because the first and the last layer of the laminated composite structure affects the shear characteristics of the structure. Considering the representation of FSDT depends strongly on the shear correction factor, the natural frequency results of the present study for LCPTPF with C2 stacking order shows a moderate agreement with ANSYS results as it is expected. Nevertheless, the use of FSDT for free vibration analysis of the LCPTPF structure gives accurate results.

- The error rates decrease when R_{xx} increases.
- The error rates decrease when the aspect ratio gets higher values.
- According to the free vibration analysis results, the error rates change with respect to boundary conditions because mode shapes are different for each boundary condition and stacking order.
- For multi-bay LCPTPF structures, the natural frequency results indicate that this study is in good agreement with ANSYS results for most natural frequency values.
- Although the error rates increase for some natural frequency values, the present study is in good agreement with ANSYS results for most natural frequency values.
- Mode shapes of this structure are depended on not only the boundary conditions but also the stacking order. Besides, the angle pattern affects the vibration mode considerably.
- Considering the mean error of the natural frequency results of LCPTPF, it is concluded that the combination of four-node quadrilateral and 4-node cylindrical shell elements satisfies the model of the structure.
- It can be concluded that using the novel approach, which is the combination of 4 – node quadrilateral element and 4 – node cylindrical shell element, for modeling LCPTPF to perform free vibration analysis of LCPTPF structure using finite element analysis is effective.
- The mathematical model of laminated composite parabolic thick plate frames is essential for future analyses (i.e., dynamic stability analysis) that could not be performed via computer-aided design programs such as ANSYS, is evaluated.

CHAPTER 6

MECHANICAL PROPERTIES OF ALUMINUM – POTASSIUM FELDSPAR METAL MATRIX COMPOSITES

6.1 Introduction

Feldspars (KAlSi_3O_8 - $\text{NaAlSi}_3\text{O}_8$ - $\text{CaAl}_2\text{Si}_2\text{O}_8$) are a group of rock-forming tectosilicate minerals that make up about 50% of the Earth's continental crust by weight (Deer et al., 1991). The feldspars are classified chemically as members of the ternary system $\text{NaAlSi}_3\text{O}_8$ (albite, Ab) - KAlSi_3O_8 (K-feldspar, Or) - $\text{CaAl}_2\text{Si}_2\text{O}_8$ (anorthite, An). The composition between $\text{NaAlSi}_3\text{O}_8$ and KAlSi_3O_8 is referred to as alkali feldspar and those between $\text{NaAlSi}_3\text{O}_8$ and $\text{CaAl}_2\text{Si}_2\text{O}_8$ as plagioclase feldspar. The difference between those two groups comes from the bonding ion between the silica tetrahedral and aluminum tetrahedral. As can be inferred, the bonding ion is K for K-feldspar, whereas it is Na or Ca for plagioclase feldspar. The alkali feldspars are as follows; orthoclase (monoclinic) KAlSi_3O_8 , sanidine (monoclinic) $(\text{K},\text{Na})\text{AlSi}_3\text{O}_8$, microcline (triclinic) KAlSi_3O_8 , anorthoclase (triclinic) $(\text{Na},\text{K})\text{AlSi}_3\text{O}_8$. Sanidine is stable at the highest temperatures, whereas the microcline is stable at the lowest temperatures. The physical properties and appearance of the feldspar group are quite similar to each other. K-feldspars are generally used as decorative building stones. By volume, however, the most critical use of K-feldspar is as aggregate in concrete and asphalt. They are also used to manufacture glass, enamel, and scouring powder due to its mechanical properties (i.e., high hardness, durability, resistance to chemical corrosion). Particularly, microcline is essential in the manufacture of ceramics and ceramic glazes as an assisting material to lower the melting temperature of the ceramic mixture. Powdered microcline is mixed with clay and quartz minerals and then fired to form a wide range of porcelain and ceramic products (inclusive of electric insulators, sanitaryware, pottery, tableware, and tile). (Iqbal & Lee, 2000; Olupot et al., 2006; Su et al., 2014). In addition, the density of the feldspars is approximately 2.6 g/cm^3 . Due to their mechanical properties, it is estimated that they will be used in many other applications or materials in the future.

Feldspars can be utilized in metal matrix composite materials due to their mechanical properties. Besides, considering that feldspars constitute almost 50% of the Earth's continental crust by weight, such material will be cost-effective, which makes it more preferable. According to the literature, there are few studies considering metal matrix composites, including feldspar mineral (Sharma et al., 2000).

This chapter comprises the manufacturing and mechanical properties of aluminum-K-feldspar. The manufacturing process is performed by considering 0.25%, 0.5%, 0.75%, and 1.5% weight fraction (wt.) of K-feldspar. Figure 6.1 shows a brief flowchart of the entire work.

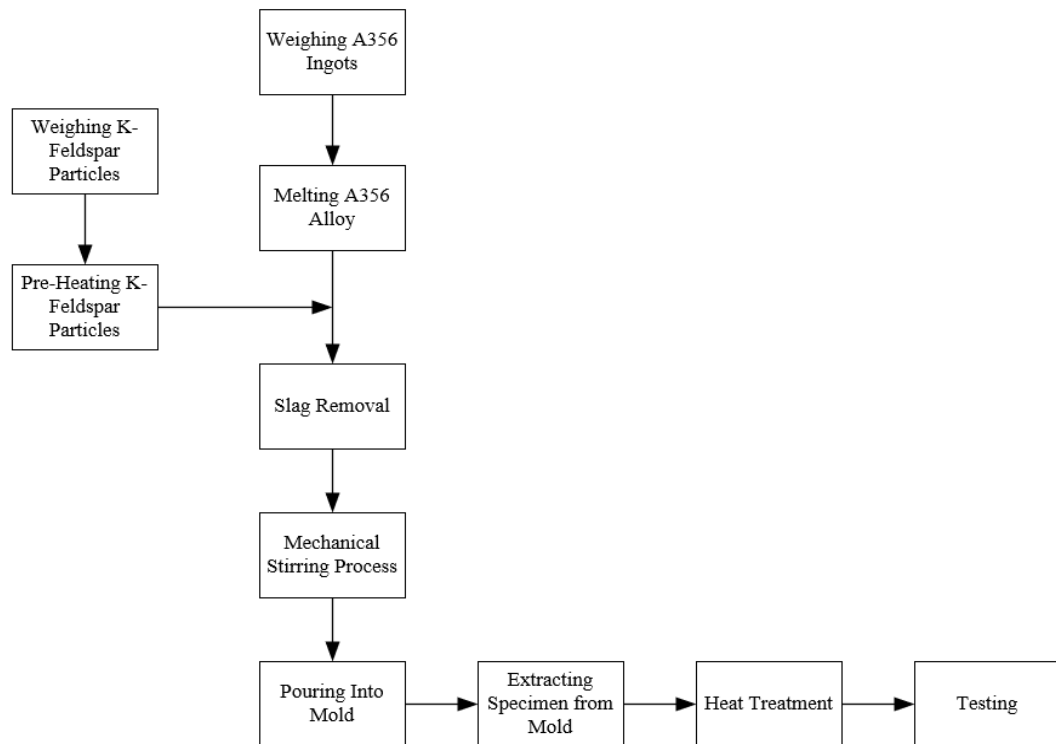


Figure 6.1 The flowchart of the process

6.2 Manufacturing Process of Aluminum – K Feldspar Metal Matrix Composites

To obtain matrix material, an amount of 4 kg A356, whose chemical composition is given in Table 6.1, was melted at 750 °C in a resistant furnace. K-feldspar, whose physical and chemical properties are given in Table 6.2, was utilized as reinforcement

material. A mechanical stirrer was used to create a vortex during the production of aluminum–K-feldspar metal matrix composite samples by stir casting. Figure 6.2 shows the schematic diagram of the stir casting method.

Table 6.1 Chemical composition of A356 alloy

Alloy/Element	Si	Fe	Cu	Mn	Mg	Zn	Ti	B	Al
A356	6.83	0.1	0.003	0.004	0.28	0.05	0.107	0.0016	balanced

Table 6.2 Technical data of K-feldspar mineral

	Particle Size				Other Properties					
	Sieve Residue %	Malvern (Mastersizer 2000)			Humidity (%)	Hardness (Mohs)	Density (g/cm³)			
	+0.075 mm	D/10	D/50	D/97						
Typ.	1.0	4.8µm	22.1µm	86.0µm	0.2	6	2.60			
Chemical Composition										
	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃	TiO ₂	CaO	MgO	Na ₂ O	K ₂ O	P ₂ O ₅	L.O.I
Typ.	69.70	16.50	0.13	0.04	0.40	0.07	2.20	10.6	0.06	0.30

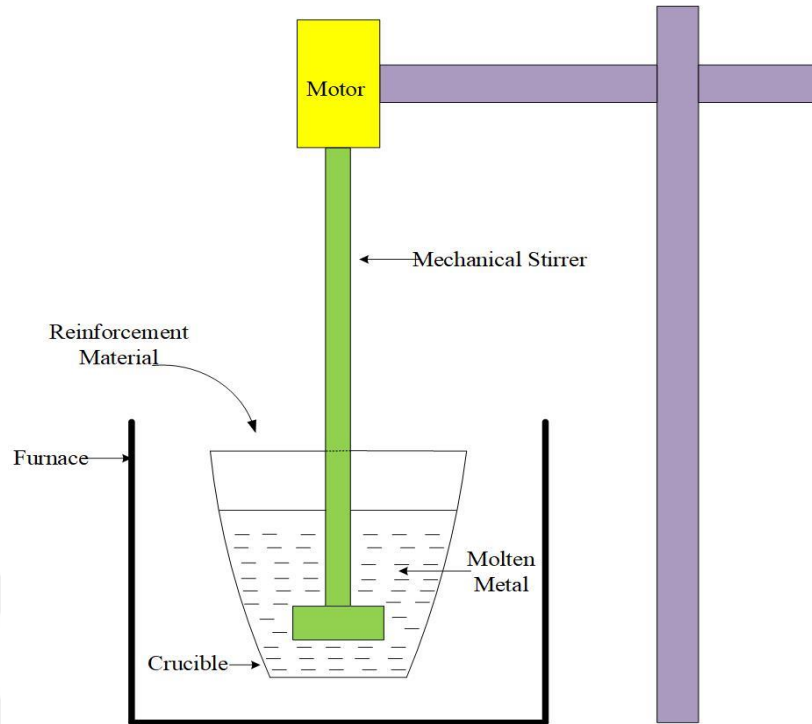


Figure 6.2 The schematic diagram of stir casting method

K-feldspar particles were weighed to be 0.25%, 0.5%, 0.75%, and 1.5% wt. The reinforcement particles were preheated to 750 °C for 30 minutes in another furnace to improve wettability by obtaining an oxidized surface. After removing the dross from the surface of melted A356, the preheated K-feldspar was added into the vortex existed in the melt due to the simultaneous mechanical stirring, which was applied for 2 minutes at 420 rpm by Optimum B20 – 400V. The melt was stirred and poured into mold preheated to 320 °C as shown in Figure 6.3.



Figure 6.3 Preheated mold (Aybarc et al., 2018)

Although the manufacturing process of aluminum – K feldspar metal matrix composite material is successfully completed, the unmixed K feldspar is observed for 1.5% wt., as seen in Figure 6.4. Such an incident may occur due to the saturation limit of the mixture, which consists of 4 kg A356 aluminum. Hence, the mechanical properties of aluminum – K-feldspar metal matrix composite having 1.5% wt. is excluded from the comparative results.



Figure 6.4 Unmixed K feldspar occurrence (Personal archive, 2020)

6.3 Mechanical Properties of Aluminum – K Feldspar Metal Matrix Composites

Ten samples were produced for each amount of reinforcement, then T6 heat treatment process (solutionized for 4 hours at 540°C, then quenched in water at 80°C, artificially aged for 6 hours at 155°C) was applied to all samples. The mechanical properties (i.e., modulus of elasticity, yield strength, tensile strength, elongation) of aluminum – K feldspar metal matrix composites were obtained via the tensile testing considering DIN EN 10002 -1 by Zwick Z100 model tensile testing machine. The tensile test samples, shown in Figure 6.5, were prepared in accordance with DIN EN ISO 6892-1. The tensile tests were conducted by 5 mm/min tensile speed and 5N/mm² preloading with 10mm/min preloading speed.

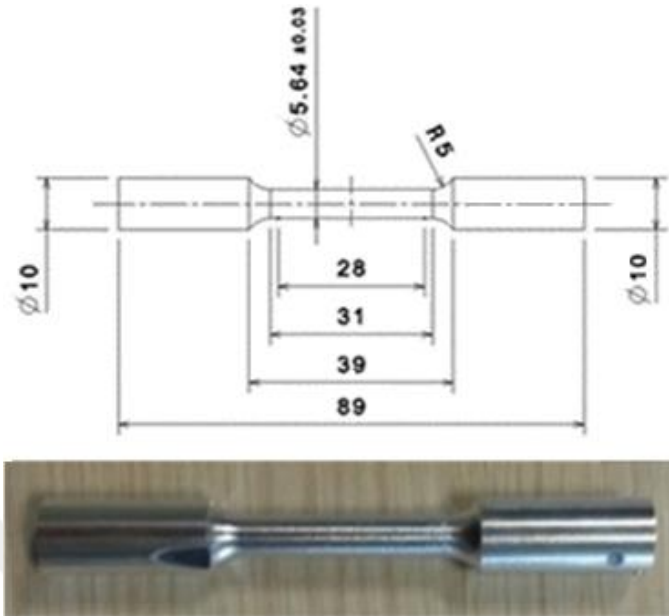


Figure 6.5 Tensile test specimen (Aybarc et al., 2018)

Figures 6.6 – 6.10 present the boxplots of modulus of elasticity, yield strength, tensile strength, elongation, and the quality index of aluminum – K- feldspar metal matrix composite materials.

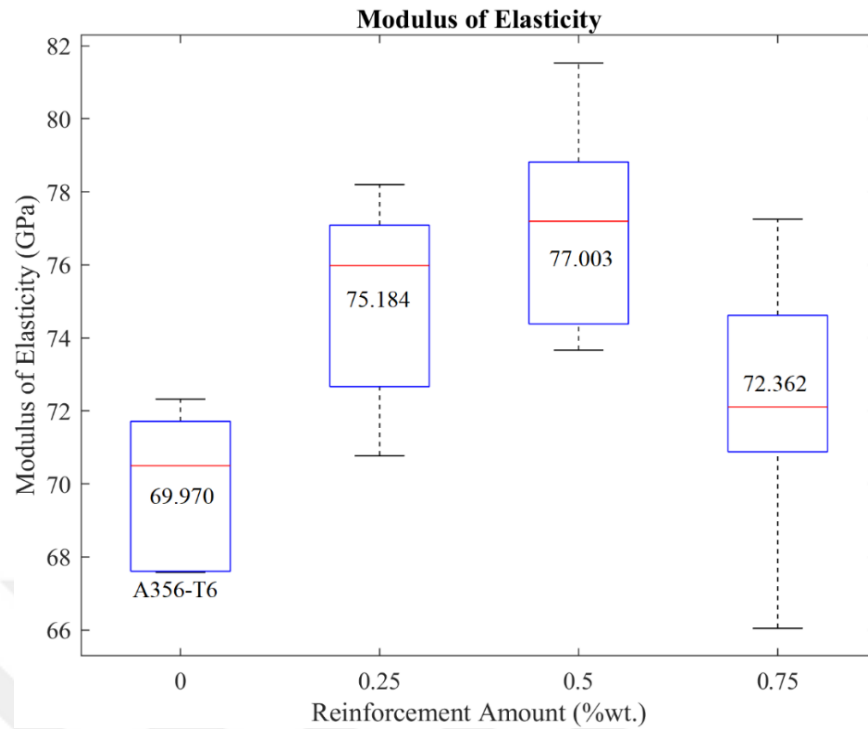


Figure 6.6 Modulus of elasticity values of aluminum – K feldspar metal matrix composites having different amounts of reinforcement

As seen from Figure 6.6, the maximum modulus of elasticity was obtained as 77.003 GPa, whereas the minimum value is 72.362 GPa. It is concluded that the addition of K-feldspar made the material stiffer than A356 in the elastic region since the modulus of elasticity values of aluminum – K-feldspar metal matrix composites were evaluated higher than the elasticity modulus of A356-T6.

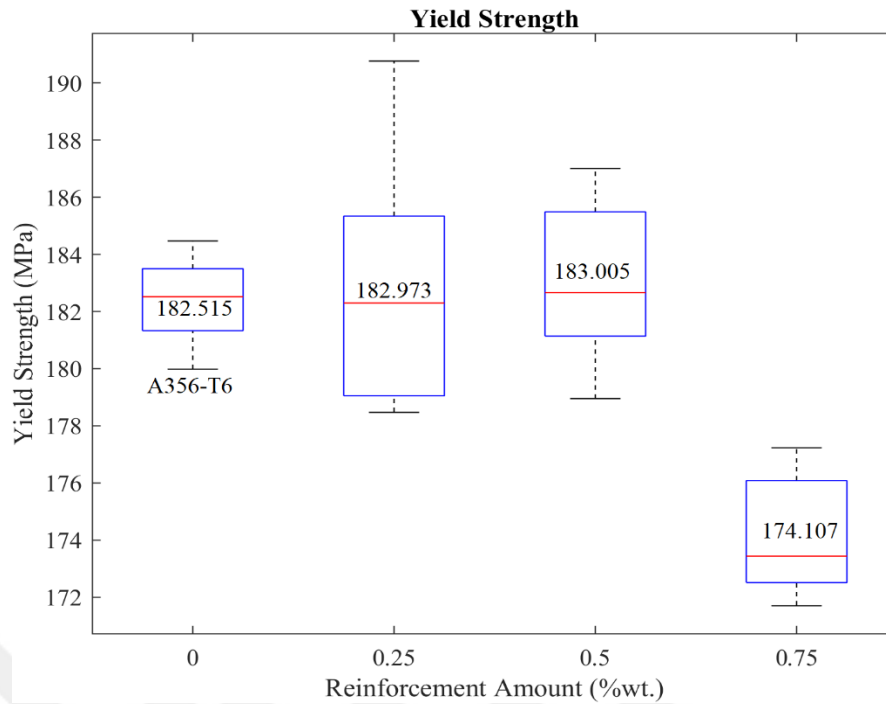


Figure 6.7 Yield stress values of aluminum – K feldspar metal matrix composites having different amounts of reinforcement

According to the results given in Figure 6.7, the yield strength values were ranged between 174.107-183.005 MPa. Comparing with the yield strength range of A356, it was seen that the yield strength values of aluminum – K-feldspar metal matrix composite materials having 0.25% and 0.50% wt. additives were slightly higher than A356-T6. On the other hand, the yield strength of the composite consisted of 0.75% wt. K-feldspar was smaller than A356-T6.

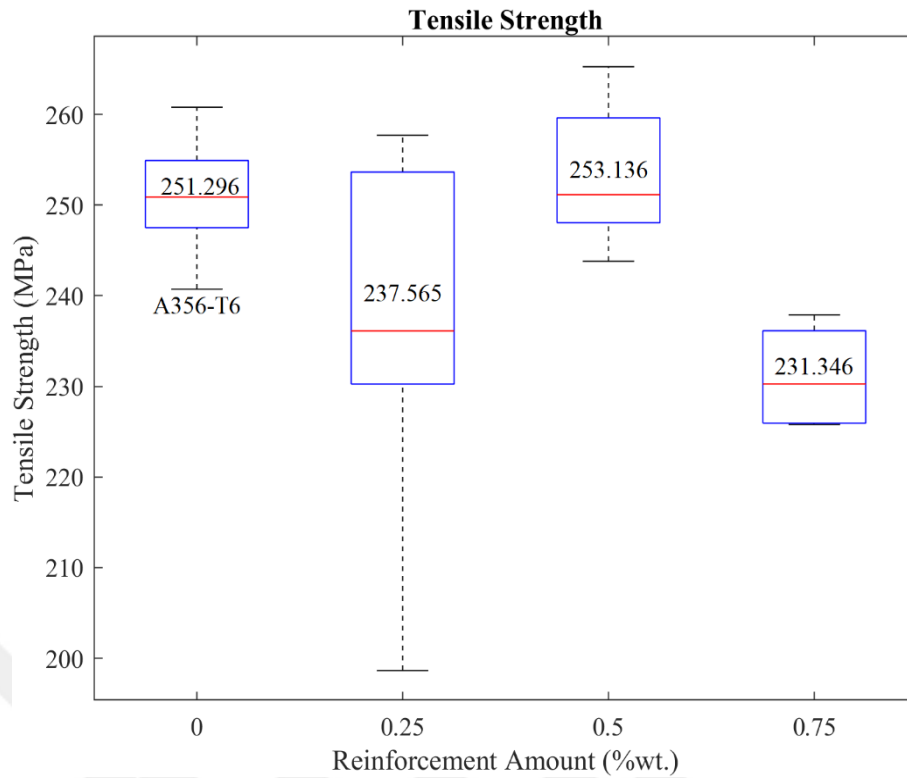


Figure 6.8 Tensile stress values of aluminum – K feldspar metal matrix composites having different amounts of reinforcement

As seen from Figure 6.8, the tensile strength values of aluminum – K-feldspar metal matrix composite materials were evaluated between 231.346– 253.136 MPa. Apart from the yield strength, the tensile strength of the aluminum – K-feldspar metal matrix composite material having 0.50% wt. additive was obtained higher than A356-T6.

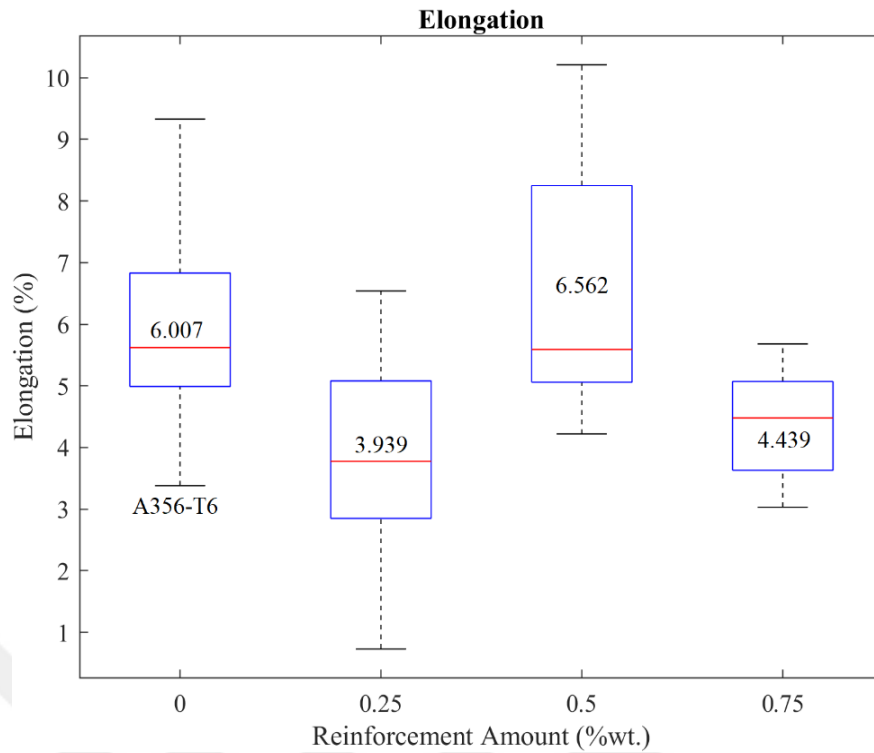


Figure 6.9 Elongation values of aluminum – K feldspar metal matrix composites having different amounts of reinforcement

According to Figure 6.9, the elongation values of aluminum – K-feldspar metal matrix composite materials were ranged between 3.939 – 6.562%. Similar to tensile strength results, only the elongation percentage of aluminum – K-feldspar composite having 0.50% reinforcement material surpassed the elongation of A356-T6.

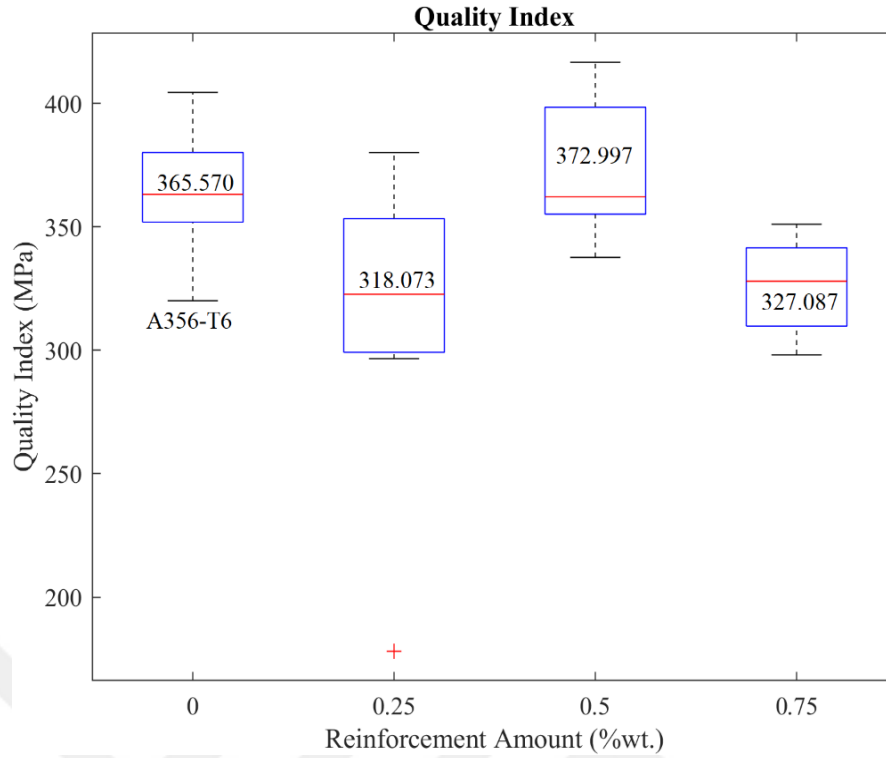


Figure 6.10 Quality index values of aluminum – K feldspar metal matrix composites having different amounts of reinforcement

The quality index is known as the durability of construction components depending on the plasticity and strength of the material. The quality index can be evaluated by using Equation 6.1 (Czekaj et al., 2016).

$$Q = UTS + c \log(\text{elongation}) \quad (6.1)$$

where UTS denotes the ultimate tensile strength, c is a constant which is 150MPa for A356 alloy (Aybarc et al., 2018).

As seen in Figure 6.10, the quality index values of aluminum – K-feldspar metal matrix composite materials were obtained between 318.073 – 372.997 MPa. Hence, it was concluded that aluminum – K-feldspar metal matrix composite having 0.50%wt. had better quality index value when compared with A356-T6 material.

The density measurements of aluminum – K-feldspar metal matrix composite materials having 0.25%, 0.50%, and 0.75% wt. reinforcement are presented in Table 6.3.

Table 6.3 The density and hardness values of A356 and aluminum – K-feldspar metal matrix composites

Amount of additive (wt.)	Density (g/cm ³)	Hardness (HB)
0% (A356 – T6)	2.600	81.0
0.25%	2.634	78.1
0.50%	2.599	84.2
0.75%	2.630	75

According to density measurements given in Table 6.3, aluminum – K-feldspar metal matrix composites almost had the same density as A356-T6 material. It was evaluated from Table 6.3 that the average density value of the composite is 2.621 g/cm³. According to Table 6.3, only the hardness value of the composite having 0.50% wt. of reinforcement material surpassed those of A356 – T6 material.

It is concluded that the addition of 0.50% wt. of K-feldspar has superior mechanical properties when compared with the matrix material (A356-T6). Besides, considering the required mechanical properties in automotive manufacturing industry (Kaba et al., 2018) the mechanical properties of aluminum – K-feldspar metal matrix composites having 0.25%, 0.50%, and 0.75% wt. reinforcement are suitable to be used in this field.

6.4 Finite Element Analysis of Aluminium – K Feldspar Metal Matrix Composites

Finite element analysis of aluminum – K feldspar metal matrix composite plates is performed via a computer code developed in MATLAB considering finite element analysis equations and theories given in chapter four. The plates are theoretically considered to be fabricated with W1, W2, and W3 (0.25%, 0.50%, and 0.75% wt.) aluminum – K feldspar metal matrix composite materials. Additionally, a plate with A356 T6 alloy is modeled to compare the finite element analysis results. The finite element analysis is performed considering cantilever boundary conditions as seen in Figure 6.11(a) and the loading case, shown in Figure 6.11(b) for buckling and dynamic

stability analysis. Table 6.4 gives information about the geometric structure of aluminum – K feldspar metal matrix composite plates shown in Figure 6.11(a). The modulus of elasticity varies with respect to the amount of the reinforcement material, as seen in Figure 6.6.

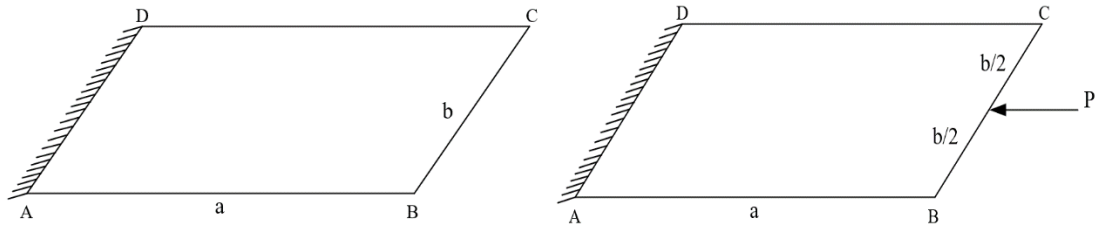


Figure 6.11 Theoretical plate structure (a), loading case of the theoretical plate structure (b)

Table 6.4 The geometric structure of aluminum – K feldspar metal matrix composite plates

Property	Symbol	Quantity
Plate width	a	1000 mm
Plate length	b	1000 mm
Structure Thickness	h	a/100

6.3.1 Free Vibration Analysis of the Aluminium – K Feldspar Metal Matrix Composite Plate

Table 6.5 gives the first ten natural frequency values of aluminum – K feldspar metal matrix composite plates having different amounts of reinforcement material.

Table 6.5 The first ten natural frequency values of aluminum – K-feldspar metal matrix composite plates and A356 alloy plate

Natural Frequency (Hz)	W1	W2	W3	A356
λ_1	9.139	9.312	8.974	8.875
λ_2	21.803	22.215	21.410	21.174
λ_3	57.316	58.400	56.283	55.663
λ_4	69.404	70.717	68.154	67.403
λ_5	80.727	82.254	79.273	78.400
λ_6	137.106	139.699	134.636	133.154
λ_7	160.816	163.857	157.919	156.180
λ_8	170.913	174.146	167.834	165.987
λ_9	183.593	187.065	180.285	178.300
λ_{10}	230.899	235.266	226.739	224.243

In order to interpret the first ten natural frequency values of aluminum – K feldspar metal matrix composite plates having four different amount of reinforcement material, it is important to remember the modulus of elasticity values of these materials. Therefore, the modulus of elasticity values of the material in W1, W2, and W3 (0.25%, 0.50%, and 0.75 % wt.) conditions are 75.184 GPa, 77.003 GPa, and 72.362 GPa, respectively. The modulus of elasticity of A356 is considered as 69.970 GPa.

According to the natural frequency results given in Table 6.5, all natural frequency values of all composites are higher than those of A356, as expected.

6.3.2 Buckling Analysis of an Aluminium – K Feldspar Metal Matrix Composite Plate

Buckling analysis is performed considering the loading case shown in Figure 6.11(b). Table 6.6 gives the critical buckling load values of aluminum – K feldspar metal matrix composite plates having different amounts of reinforcement material and the critical buckling load of A356 alloy plate for comparison.

Table 6.6 The critical buckling loads of aluminum – K-feldspar metal matrix composite plates and A356 alloy plate

<i>Critical Buckling Load (N)</i>	W1	W2	W3	A356
P _{cr}	17345	17645	16694	16142

As seen in Table 6.6, the critical buckling load values of W1, W2, and W3 are higher than those of A356 as expected. The highest critical buckling load is evaluated for W2, which has the highest modulus of elasticity value, 77.003 GPa.

6.3.3 Dynamic Stability of Aluminium – K Feldspar Metal Matrix Composite Plates

Dynamic stability analysis of plate structures are performed considering the loading case, shown in Figure 6.11(b). Figures 6.12 and 6.13 shows the unstable regions of A356 alloy plate and aluminum – K feldspar metal matrix composite plates having W1, W2, and W3 conditions.

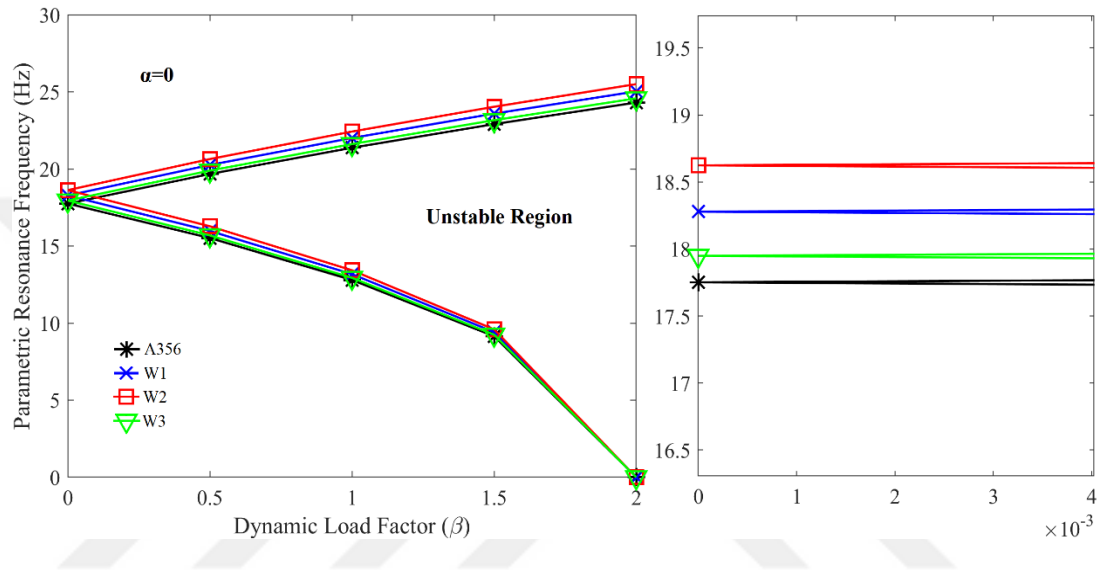


Figure 6.12 Unstable regions ($\alpha=0$) of A356 alloy plate and aluminum – K feldspar metal matrix composite plates having W1, W2, and W3 amount of reinforcement

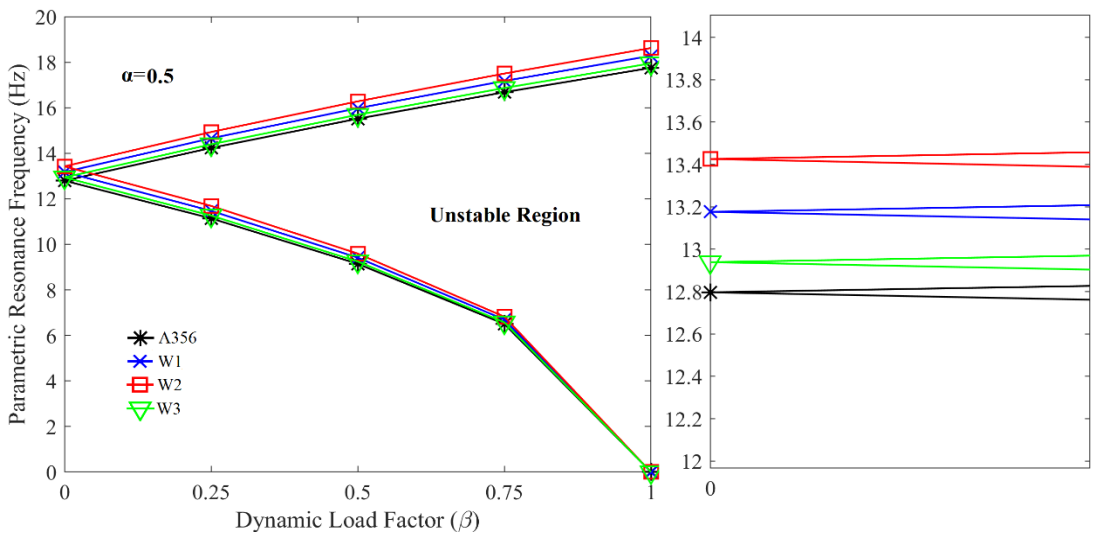


Figure 6.13 Unstable regions ($\alpha=0.5$) of A356 alloy plate and aluminum – K feldspar metal matrix composite plates having W1, W2, and W3 amount of reinforcement

According to the results given in Figures 6.11 and 6.12, as the modulus of elasticity decreases, the unstable region moves towards the origin. Hence, W2 has the highest parametric resonance frequency values. The unstable regions do not change considerably as the reinforcement amount changes. However, these regions become narrow as the static load increases. The distance of the unstable regions to the origin can be ordered from the farthest to closest considering the amount of reinforcement material as W2 (0.50%), W1 (0.25%), W3 (0.75%), and A356 (0%), respectively.

6.5 Conclusions

In this chapter, the manufacturing and mechanical properties of the novel aluminum – K feldspar metal matrix composite materials are investigated. In addition, the first ten natural frequency values, critical buckling loads, and unstable regions of plate, shown in Figure 6.11(a) is considered to be made by these materials are obtained theoretically. The conclusions of this chapter are drawn as follows.

- According to the mechanical properties, especially the high ultimate tensile strength and yield strength, of the novel material, it is concluded that a good wettability was achieved. Besides, the manufacturing parameters, i.e., the casting and preheating temperatures, the particle size of the reinforcement material, manufacturing under vacuumed environment stirring time, and stirring speed, are sufficient to produce such material. However, the optimization of such properties by increasing the number of experiments may improve the mechanical properties of aluminum-K-feldspar metal matrix composite materials.
- It is concluded that K-feldspar mineral can be utilized as a reinforcement material to fabricate aluminum metal matrix composite materials.
- The measured density values of aluminum – K-feldspar metal matrix composite material having 0.5% wt. additive indicate that the composite is slightly lighter (0.005 g/cm^3) than A356 – T6 material.

- Aluminum – K-feldspar metal matrix composites having 0.50% wt. reinforcement material had superior mechanical properties than the matrix material, A356-T6. Besides, using K-feldspar as a reinforcement material for aluminum metal matrix composites satisfies the required mechanical properties of a material in the automotive industry (Kaba et al., 2018)
- Considering almost 50% of Earth's ground contains feldspar mineral and the economic benefits of the stir casting method, it is estimated that using aluminum – K-feldspar metal matrix composite materials will be a cost-effective material for many structures.
- Due to its density, which is close to those of A356 alloy, utilizing K-feldspar with aluminum will make the final product slightly lighter, but not heavier.
- The natural frequency values are higher for all types of aluminum – K-feldspar metal matrix composite plates than the A356 plate as expected.
- The critical buckling load of the plates made by aluminum – K-feldspar metal matrix composite material is higher than the A356 alloy plate.
- The unstable regions of aluminum – K-feldspar metal matrix composite plates are neither wider nor narrower than A356 alloy plates. However, the distance of the unstable regions to the origin can be ordered from the farthest to closest considering the amount of reinforcement material as W2 (0.50%), W1 (0.25%), W3 (0.75%), and A356 (0%), respectively.
- It is estimated that optimizing the manufacturing parameters such as the amount and particle size of K-feldspar, pre-heating temperature and duration of K-feldspar, casting temperature and duration, the duration and speed of stirring, casting environment, and other applications that may improve the wettability of K-feldspar (e.g., encapsulation with aluminum foil).

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Overview of the Thesis

Curved frame structures are used widely in many engineering applications such as construction, naval industry, automotive, etc. Hence, it is essential to determine their mechanical behaviors under several circumstances. Besides, the material used in such structures or more simple structures such as plates, sheets, and beams is another essential factor that should be determined since it can both affect the structure positively or negatively.

It is crucial to obtain the natural frequency values of such structures due to the resonance phenomenon that may lead the structures to damage or failure. Another significant event, buckling, may occur once the load subjected to the structures is equal to the critical buckling load. The unstable regions of a structure should be determined. Because, if a structure is under parametric resonance, a failure may occur. Hence, those three fundamental concepts are examined for curved frame structures by using finite element analysis and considering beam and plate theories.

The accurate material selection for such structures is another key factor for structure. Choosing the right material will improve the mechanical properties of the structure, making the structure useful for wider applications. Hence, many researchers are manufacturing new materials or improving existing materials in many ways. Within the scope of this thesis, a novel aluminum – K-feldspar metal matrix composite is manufactured via the stir casting method. The effect of the amount of the reinforcement material on mechanical properties (i.e., modulus of elasticity, yield strength, tensile strength, elongation, and quality index) of this novel material is investigated via tensile test by considering DIN EN 10002 -1 standard using tensile test samples prepared under DIN EN ISO 6892-1 standard. The same phenomenons as those of curved frame structures investigated in this thesis are examined for aluminum – K-feldspar metal matrix composite plates theoretically.

7.2 General Conclusions of the Finite Element Analysis of Curved Frame Structures

The first research topic focused on this thesis is the investigation of finite element vibration, buckling, and dynamic stability analysis of thin isotropic and composite curved beam/plate structures considering the corresponding theories presented in chapter two and chapter four. Some computer codes are developed via MATLAB to perform finite element analysis to these structures. According to the finite element analysis results of these structures, the following conclusions are drawn:

In chapter two, Laminated Beam Theory and Euler-Bernoulli Beam Theory is presented as the main tool of the finite element method. A two-noded frame element is utilized for finite element analysis. The modeling stage of curved beam frame structures and eigenvalue equations to solve free vibration, buckling, and dynamic stability analysis are presented.

In chapter three, the first five natural frequency values and critical buckling loads are evaluated by considering the differences in the radius of curvature and thickness of single-bay and two-bay isotropic curved frame structures. On the other hand, the effect of radius of curvature and stacking order on the first five natural frequency, critical buckling loads, and unstable regions are examined for single-bay and two-bay laminated composite curved frame structures.

In chapter four, Kirchhoff- Love plate theory, Classical Plate Theory, and First-Order Shear Deformation Theory are presented as the fundamental theories for finite element analysis. Thin isotropic and laminated composite curved plate frame structures and thick laminated composite curved plate frames are modeled by using a four-node quadrilateral plate element and four-node cylindrical shell element (for thick plate only).

In chapter five, the first ten natural frequency values and critical buckling loads are evaluated by considering the differences in the radius of curvature and aspect-ratio of

single-bay and two-bay isotropic and laminated thick composite curved plate frame structures. On the other hand, the effect of radius of curvature, aspect ratio, and stacking order on the first ten natural frequency, critical buckling loads, and unstable regions are examined for single-bay and two-bay laminated thin composite curved plate frame structures.

It has been concluded that the presented models represent the curved frame structures effectively. The results are satisfying when compared with those of ANSYS software.

For both isotropic and composite single-bay and two-bay curved frame structures, the radius of curvature affects the natural frequency values. However, the effect is quite small. The difference in radius of curvature does changes some of the mode shapes of the curved frame structure, whether it is isotropic or laminated. The increment in the radius of curvature of the curved frame structures resulted in an increment in the critical buckling load. Hence, a slightly curved frame structure can handle bigger loads than curved frame structures. The unstable regions of the frame structures do not affected considerably as the radius of curvature of these structures changes. The decrement in curvature moves the unstable region slightly towards the origin.

For isotropic curved beam frame structure, increment in thickness resulted in higher natural frequency values as the resistance of the structure increases due to the increment in the moment of the inertia that affects the stiffness matrix of the structure. Similarly, the critical buckling load increases because of the same reason.

The stacking order affects the natural frequency values of the laminated composite curved beam/plate frame structure significantly. Among all stacking orders, C2 ([90 0 0 90]) has the highest effect on the natural frequency values. Apart from all other stacking orders whose fiber angles of the first and last layer are 0o, the fiber angles of the first and the last layer of C2 is 90o, which makes the structure less resistant to flexural displacement due to the change in the modulus of elasticity. The ranking of

the stacking orders from the highest to lowest in terms of the first five natural frequency values of curved frame structures is C1, C4, C5, C3, and C2, respectively. Hence, it can be concluded that as the fiber angles of the middle layer increase, the natural frequency values of the curved frame structure decrease. Although the fiber orientation affected the natural frequency values, the mode shapes are the same regardless of which fiber orientation the curved beam frame structure has. On the other hand, the mode shapes vary significantly for laminated composite curved plate frame structures because the modulus of elasticity is taken into account as a two-axis instead of a single axis. The stacking order has the same impact on the critical buckling load as of the natural frequency values of laminated composite curved frame structures. The fiber orientation affects the unstable regions of the laminated composite curved frame structure considerably. The distance of the unstable regions to the origin can be ordered from the closest to farthest as C2, C3, C5, C4, and C1, respectively. The areas of each unstable region are almost identical to each other.

The two-bay curved frame structures have higher natural frequency values and critical buckling load values. It is concluded that the unstable regions of the single-bay structure are closer to the origin when compared with those of the two-bay structure. However, all the above conclusions are valid for not only the single-bay curved frame structures but also two-bay curved frame structures.

For both isotropic and composite single-bay and two-bay curved frame structures, the radius of curvature affects the first five natural frequency values. However, the effect is quite small. The difference in radius of curvature does not change the mode shapes of the curved frame structure, whether it is isotropic or laminated. The increment in the radius of curvature of the curved frame structures resulted in an increment in the critical buckling load. Hence, a slightly curved frame structure can handle bigger loads than curved frame structures. The unstable regions of laminated composite curved frame structures do not affected as the radius of curvature of these structures changes. The decrement in curvature moves the unstable region slightly towards the origin.

7.3 General Conclusions of Aluminum – K-Feldspar Metal Matrix Composite Materials

In chapter six, the manufacturing and mechanical properties of novel aluminum – K-feldspar metal matrix composite materials are investigated. The manufacturing process is conducted via the stir casting method by using A356 alloy and four different amounts of K-feldspar. The mechanical properties (i.e., modulus of elasticity, tensile strength, yield strength, elongation, and quality index) of such materials are unearthed for each material by performing the tensile test. The finite element free vibration, buckling, and dynamic stability analysis of aluminum – K-feldspar metal matrix composite plates are performed using the material's mechanical properties. The results of these analyses are compared with those of A356 alloy plates.

It is concluded that using K-feldspar for casting aluminum metal matrix composite is possible for a certain ratio of matrix – reinforcement material. According to the mechanical properties of the material, the best result is obtained for W2, where 20-grams of K-feldspar is utilized into 4 kilograms of A356 alloy. The amount of 1.5% wt. K-feldspar did not mix with the molten metal and remained on the surface. According to the mechanical properties of aluminum – K-feldspar metal matrix composite materials, it is estimated that such materials can be utilized in many applications due to these mechanical properties, the abundance, and the cost of K-feldspar. Using W2 (0.50% wt.) of K-feldspar made the novel material slightly lighter and slightly harder than A356 alloy. On the other hand, using W1 (0.25%) and W3 (0.75%) of K-feldspar made the metal matrix composite slightly heavier and slightly softer than A356. The yield and tensile strength of aluminum – K-feldspar metal matrix composite materials are quite suitable for many engineering applications.

The results of the finite element analysis of aluminum – K-feldspar metal matrix composite plates indicate that the natural frequency values of plates with aluminum – K-feldspar metal matrix composite materials are higher than those of A356 alloy plates. Similarly, the critical buckling loads are higher for plates made by aluminum – K-feldspar metal matrix composites. This means that the structures made by aluminum

– K-feldspar metal matrix composite materials can handle more load than those made by A356 alloy. The areas of unstable regions of aluminum metal matrix composite plates are identical with those of A356 alloy plates. However, the distance of the unstable regions to the origin can be ordered from the farthest to closest considering the amount of reinforcement material as W2 (0.50%), W1 (0.25%), W3 (0.75%), and A356 (0%), respectively.

7.4 The Future Work of Curved Frame Structures

Future research about curved frame structures are given as follows:

- Static analysis can be performed on curved frame structures by using the methods presented in this thesis
- The proposed modeling method can be used to analyze the effects of some mechanical phenomenons such as crack and delamination.
- For vibration problems of curved frame structures, including damping and/or force, the proposed finite element method can be considered.

7.5 The Future Work of Aluminum – K-Feldspar Metal Matrix Composite Materials

Future research about aluminum – K-feldspar metal matrix composite materials are given as follows:

- For industrial purposes, aluminum – K-feldspar metal matrix composite materials can be utilized.
- K-feldspar can be considered as an important reinforcement material for other potential metal matrix composite materials.

- The casting parameters such as casting temperature, pre-heating temperature, vacuumed environment casting, stirring time, stirring speed, particle size, particle amount, etc. can be changed and optimized for aluminum – K-feldspar metal matrix composite materials.
- It is known that K-feldspar has good behavior in wear resistance (Sharma et al., 2000). Hence, K-feldspar can be considered for metal matrix composites manufactured to obtain high wear resistance.



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