DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

AN ANALYSIS OF CRITERIA INTERACTION TECHNIQUES IN MULTIPLE ATTRIBUTE DECISION MAKING AND A FUZZY TOPSIS BASED NEW MODEL PROPOSAL

by
İlker GÖLCÜK

December, 2013
İZMİR

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A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science in Industrial Engineering, Industrial Engineering Program

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December, 2013
İZMİR

M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"AN ANALYSIS OF CRITERIA INTERACTION TECHNIQUES IN MULTIPLE ATTRIBUTE DECISION MAKING AND A FUZZY TOPSIS BASED NEW MODEL PROPOSAL"** completed by **İLKER GÖLCÜK** under supervision of **PROF.DR. ADİL BAYKASOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Dr. Adil BAYKASOĞLU

Supervisor

Yrd.Doç.Dr. Derya EREN AKYOL

(Jury Member)

Doç. Dr. M. Evren TOYGAR

(Jury Member)

Prof.Dr. Ayşe OKUR

Director

Graduate School of Natural and Applied Sciences

ACKNOWLEDGMENTS

First of all, I would like to express my sincere gratitude to Prof. Adil BAYKASOĞLU, for his guidance, support and understanding. He not only served as my supervisor but also encouraged me throughout my graduate program. He always supported my decisions and gave me freedom in my research while helping me to overcome challenges and giving advices.

I would also like to express my thanks to all professors and colleagues in the Industrial Engineering Department of Dokuz Eylül University for their support, encouragement and tolerance.

Finally, I would like to express my special thanks to my parents, Aysel and Hüseyin GÖLCÜK for their love, patience and encouragement. Without their trust and moral support, this thesis would not have been possible.

İlker GÖLCÜK

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ABSTRACT

Increasing complexity of the very rapidly evolving business and technological environment entails making right decisions considering diversity of factors. For instance, decision makers should take into account multiple diverse aspects such as political, environmental, social, psychological, economic factors and their interactions, which are not always well defined nor well understood in depth. For that reason, Multi Criteria Decision Making (MCDM), which is one of the major disciplines in Operations Research/Management Science (OR/MS), supports decision makers with comprehensive collection of approaches to deal with complex, poorly-defined problems with multiple and interrelated criteria. MCDM continues to grow very swiftly to meet the contemporary needs of the decision situations, where restrictive assumptions of the traditional methods are inadequate to model. One of the most well-known assumptions of conventional MCDM approaches is that the criteria are assumed to be independent of each other.

The aim of this study is to shed light on the concept of criteria independence/dependence and propose a simple and effective method to deal with complex decision problems without the restrictive assumption of preferential independence. For this thought in line, we provide a comprehensive survey of the state of art methodologies used for modelling and solving MCDM problems with interdependent/interactive criteria. Initially, the independence assumptions of the classical decision theory are compiled from the multi attribute utility theory. Then focusing on the preferential independence concept, Analytical Network Process, Causal Maps, Decision-Making Trial and Evaluation Laboratory (DEMATEL), Fuzzy Cognitive Maps, Interpretive Structural Modelling, Bayesian Networks, System Dynamics, and Fuzzy Integral (specifically Choquet Integral) methods are explained and discussed. Literature survey of the pertinent methods is provided at the end of each chapter.

Due to the excessive number of alternative methods capable of modelling criteria interactions, characteristics of a sufficient MCDM method are determined as a complexity management strategy. In our regard, Fuzzy Cognitive Maps has much to offer in the field of MCDM due to its capabilities in qualitative modelling and simulating the dynamic systems. We propose a novel hybrid MCDM method by combining Hierarchical Fuzzy TOPSIS and Fuzzy Cognitive Maps for modelling interdependent, complex, and uncertain problems. The proposed method is illustrated in the strategy selection problem to demonstrate practicality and applicability of the method.

Keywords: Fuzzy cognitive maps, criteria interaction, fuzzy multiple attribute decision making

ÇOK KRİTERLİ KARAR VERME PROBLEMLERİNDE KRİTER ETKİLEŞİM TEKNİKLERİNİN ANALİZİ VE BULANIK TOPSIS TEMELLİ YENİ BİR MODEL ÖNERİSİ

ÖZ

Hızla gelişen iş ve teknoloji ortamındaki artan karmaşıklık, çeşitli faktörleri göz önünde bulundurarak doğru kararlar almayı zorunlu kılmaktadır. Örneğin, karar vericiler çoğu zaman iyi tanımlanmamış ya da yeterince anlaşılamamış politik, çevresel, sosyal, psikolojik ve ekonomik çok çeşitli faktörleri ve onların etkileşimlerini dikkate almak zorundadırlar. Bu sebeple, Yöneylem Araştırması ve Yönetim Biliminin önemli disiplinlerinden biri olan Çok Kriterli Karar Verme (ÇKKV), karar vericilere karmaşık, iyi tanımlanmamış, çok ve birbirine bağlı kriterlerden oluşan problemleri çözebilmeleri için geniş yaklaşımlar topluluğu sunmaktadır. ÇKKV hala geleneksel yöntemlerin kısıtlayıcı varsayımlarının yetersiz kaldığı karar durumlarındaki yeni ihtiyaçlara cevap verebilmek için hızla büyüyen bir alandır. Geleneksel ÇKKV yöntemlerinin en iyi bilinen varsayımlarından biri kriterlerin birbirinden bağımsız olduğu varsayımıdır.

Bu çalışmanın amacı kriterler arası bağımsızlık/bağımlılık kavramlarına ışık tutmak ve kısıtlayıcı tercihsel bağımsızlık varsayımına bağlı kalmayarak, karmaşık karar problemlerinin çözümü için basit ve etkili bir yöntem önermektir. Bu düşünceyle, birbirine bağlı kriterlerin olduğu karar problemlerini çözebilen en gelişmiş metodolojilerin kapsamlı bir literatür araştırması yapılmıştır. İlk olarak klasik karar teorisindeki bağımsızlık varsayımları, çok kriterli fayda teorisinden derlenmiştir. Ardından, tercihsel bağımlılık kavramına odaklanılarak, Analitik Ağ Süreci, Bilişsel Haritalar, DEMATEL, Bulanık Bilişsel Haritalar, Yorumlacıyıcı Yapısal Modelleme, Bayes Ağları, Sistem Dinamikleri ve Bulanık İntegral (özellikle Choquet integrali) metotları açıklanmış ve tartışılmıştır. Her bölümün sonunda ilgili metotlarla ilgili literatür araştırmasına yer verilmiştir.

Kriter etkileşimlerini modelleyecek çok fazla alternatif yöntem olması sebebiyle, bir karmaşıklık yönetimi stratejisi olarak, yeterli bir ÇKKV yönteminin sahip olması

gereken karakteristik özellikler belirlenmiştir. Bu bağlamda, Bulanık Bilişsel Haritaların kalitatif modelleme ve dinamik sistemlerin benzetimini yapabilme özellikleriyle ÇKKV alanına verebileceği çok fazla katkı vardır. Birbirine bağlı kriterlerden oluşan, karmaşık ve belirsiz problemlerin çözümü için Hiyerarşik Bulanık TOPSIS ve Bulanık Bilişsel Haritalardan oluşan orjinal bir hibrit ÇKKV yöntemi önerilmiştir. Önerilen yöntemin pratik ve uygulanabilirliğini göstermek amacıyla strateji seçimi problemi üzerinde örneklendirilmiştir.

Anahtar sözcükler: Bulanık bilişsel haritalar, kriter etkileşimi, bulanık çok kriterli karar verme

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CHAPTER ONE
INTRODUCTION

1.1 General Overview

Decision making is a ubiquitous human activity that occurs in life of every individual very often. People generally confront with decision situations in their social, personal or professional lives in which two or more alternatives are present. Generally, daily life problems are degraded into mono-criterion and outcomes are calculated intuitively. However, choosing the best alternative among set of options considering multiple, incommensurate and contradictory criteria exceeds cognitive limitations of decision makers and becomes a complicated issue. Making right decisions against complex problems yields significant influences on the future of individuals or groups.

Importance of making right decisions is not only restricted to personal decisions but also for business organizations operating under great pressure of competitive environment. Organizations encountering set of objectives should manage their diverse type of resources in a most efficient way to maintain their competitive advantage in the market. Hence, managers are decision makers, who have to make choices among set of distinct alternatives. Companies are exposed to the abundant amount of information, which transforms pretty much everything in a unique way. Because of the advancements in the information technology, reaching more information quickly brings with the increased number of alternatives that can be generated. Companies are obligated to consider market dynamics, social, economic and ecological factors, and actions of the competitors, as well as their daily basis operations. Therefore, making right decisions is a critical issue.

The multi criteria decision making is different from classical operations research instruments in the sense that there is no objectively optimal solution. Subjectivity is the inherent part of the multi criteria decisions; hence there is a discrepancy between fundamentals of the operations research and the subjective nature of decision making. According to Kuhn (1962), recognition of the ultimate right of a decision maker in the subjectivity of decisions is a sign of the appearance of a new paradigm of multi criteria decision-making. Actually, in decision making problems, objective constraints still exists which are mostly imposed by external environment (Pedrycz, Ekel, & Parreiras, 2011).

Subjectivity of decisions is mostly apparent in ill-structured problems. Decision making problems can be categorized as structured problems, semi-structured problems and unstructured problems (Larichev & Moshkovich, 1996; Lu et al., 2007). The structured problems can be described by the conventional mathematical models. The relationships between the problem components are expressed in symbols or numbers and they are numerical in nature. Standard solution techniques are successful to analyze possible outcomes.

In unstructured problems, the relationship between system components is not known, only descriptions of the problem components such as resources and characteristics are included. Conventional mathematical models cannot be used to solve this type of problems. The important features of unstructured problems are as follows (Larichev & Moshkovich, 1996) :

- The problem or some of the problem components are unique with which the decision maker did not deal before.
- There is an inherent uncertainty for assessment of alternative solutions
- The problem is mostly formulated in a verbal form, qualitative assessment is dominant.
- The alternatives are assessed according to only subjective judgment of decision makers.
- They are expert oriented, estimates are acquired from experts.

The semi-structured problems have both structured and unstructured components. They have mixed of qualitative and quantitative elements.

The field of industrial engineering is, for the most part, dominated by unstructured or semi-structured decision problems. Hence, problem structuring, specifically determination of the criteria and relationships among them is crucial topic for decision analysts. Clarification of interacting criteria, which is the subset of problem structuring problem, is of vital importance to make satisfying and right decisions. Especially, assuming decision criteria to be independent of each other under complex, uncertain and dynamic decision making environments lead to misleading and fallacious conclusions. Furthermore, a large number of psychological investigations demonstrate that decision makers, not being provided with additional analytical support, use simplified and, sometimes, contradictory decision rules (Pedrycz et al., 2011; Slovic, Fischhoff, & Lichtenstein, 1977). We thereby dig into the concept of criteria interaction and provide a hybrid technique to support decision makers with a easy to use and practical methodology in the scope of this thesis. In this section, we first give the basic definitions regarding multi criteria decision making.

1.1.1 Elements of Multi Criteria Decision Making

MCDM provides a systematized approach to model decision problems via the basic six elements as follows (Malczewski, 1999):

- Value: Something a person cares deeply about.
- Goal: Formulation of values in a given problem context
- Objective: Specification of goal in terms of the desired property of problem solution.
- Decision Maker: A single person, a group of people, or the whole organization responsible for making decisions
- Decision Alternatives: Feasible solutions to a decision problem.
- Criteria: Basis for evaluating decision alternatives. It may be used as attributes or objectives. An attribute measures the performance of an objective. An objective is a statement about the desired level of goal achievement.
- Outcomes: achievement of performance of each decision alternative

Potential actions or set of alternatives: The nature of decision making involves choice. It can be exercised if there are decision alternatives to choose from. It is the matter of testing whether or not these potential alternatives satisfy the basic decision problem in order to be admitted as feasible decision alternatives.

Potential action gives us the direction of the analysis problem on hand. It is a leading entity of decision analysis where the decision aiding is directed towards. The notion of potential refers to fact that whether the action is feasible or possible is not known a priori. The term alternative is a special case of potential actions, where the two distinct potential actions can in no way be conjointly put into operation (Roy, 2005).

In general, A symbolizes the set of potential actions in the decision analysis activity. This set is not fixed and might be augmented or degraded during the analysis process. It should be noted that decision analysis is a dynamic evaluation mechanism so that type and size of data can change likewise some aspects can be clarified during the process. Modification of the potential action set is also gives researchers some inspiration to dig into the problem structuring issues as well as changeable/dynamic spaces of decision domain in decision analysis.

The actions can be predetermined finite alternatives or sometimes denoted by Cartesian product of possible scores of criteria as in the Multi attribute utility theory. We denote the number of actions as:

$$A = \{a_1, a_2, \ldots a_m\} \tag{1.1}$$

Also every alternative is characterized by variables x_i, which denotes the possible scores of the scale X_i. As a result, set of alternatives A can be written as the subset of a Cartesian product $X = \prod_{i=1}^{n} X_i$, where n is the number of criteria (Roy, 2005). This sort of modeling approach is used in MAUT. A very important result of this representation is that the two actions having the same score for all x_i lose their identity and become indistinguishable.

Set of criteria: evaluation criteria represent measure for achieving those criteria. Some properties of criteria are: Exhaustiveness, cohesiveness, nonredundancy, understanding, and commitment (Roy, 1999).

Decision table: It represents the collection of criterion scores and thus provides the basis for the comparison of decision alternatives. A MADM problem can be depicted in a matrix format as:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & X_3 & X_4 \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & & x_{2n} \\ & & & \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array} \tag{1.2}$$

Where m alternatives and n attributes are denoted by A_i and X_i, respectively. The performance of ith alternative with respect to jth criteria is shown by x_{ij} (Chen & Hwang, 1992) .

Criterion scores: These scores represent achievements of decision alternatives on evaluation criteria.

Decision maker preferences: They are expressed in terms of weights. These weights express relative importance of evaluation criteria under consideration.

Aggregation functions: Sometimes called decision rule. It computes an overall assessment measure of each decision alternative by integrating decision maker's preferences with criterion scores.

Sensitivity analysis: It tests the stability of assessment measure of each decision alternative when weights and criterion scores are varied. The ranking of decision alternatives is said to be sensitive if small changes in the weighting or criterion scores produce significant changes in the order of ranked decision alternatives.

Final recommendation: The choice of the most appropriate decision alternatives.

1.1.2 Historical Perspective of Decision Making

Decision making is as old as the mankind. However it is very difficult to give the first recorded instance directly related to MCDM. The theoretical underpinnings mostly date back to 17th century, especially discussions about the St. Petersburg game. Bernstein (1996) defines St. Petersburg game as: "A game is played by flipping a fair coin until it comes up tails, and the total number of flips, n , determines the prize, which equals $\$2 \times n$. If the coin comes up heads the first time, it is flipped again, and so on. The problem arises: how much are you willing to pay for this game?"

The expected value theory states that $EV = \sum_{n=1}^{\infty} (1/2)^n \times 2^n$ where the expected value is infinity. However, this result was not convincing. The paradigm shifted when the first ideas of utility theory was published by Bernoulli (1738). According to utility theory, people do not choose the alternative with the best expected value but expected utility.

When the book of von Neumann and Morgenstern (1944) arrived, utility theory was concretized and new insights were opened for future developments of decision analysis. Today, most of the studies divide decision making approaches into two categories: multi attribute utility theory based methods and outranking based methods. ELECTRE (Roy, 1968) and PROMETHEE (Brans, Vincke, & Mareschal, 1986) are the widely used outranking approaches. Utility theory had its own drawbacks. First of all, constructing utility functions is costly. It requires an interactive procedure with decision maker and many questionnaires needed to be answered.

Additionally, utility functions had a lot of assumptions, such as preferential independence, which is the motivation of this thesis. Generally, the additive utility functions (Fishburn, 1970) were preferred due to its simplicity. Because of the fact that the utility functions were hard to elicit, other paradigms which did not base upon utility theory were arisen, such as outranking approaches.

On the other hand, regardless of multi criteria decision making, new advances were made in the measurement theory, like fuzzy measures (Choquet, 1954). In this doctorate thesis, Sugeno (1977) developed $\lambda-$measures which were very useful to overcome preferential independence assumption of utility theory.

Saaty (1972) laid the foundations of AHP, Hwang and Yoon (1981) developed TOPSIS method, Saaty (1996) proposed ANP method. Also the pioneering study of Zadeh (1965) completely changed the paradigm in decision analysis, fuzzy decision making approaches gained momentum.

The general picture of the development phases of different methods can be seen in Figure 1.1.

1.1.3 Process of Decision Making

In the literature of MCDM, decision components such as criteria, alternatives, goals and etc. assumed to be known a priori. In fact, the analyst or decision makers form the problem and it is a subset of a broader process of problem structuring issue. The process of decision making is illustrated in Figure 1.2 (Belton & Stewart, 2002).

A generic MCDM problem is initiated by the problem structuring phase. Problem structuring phase requires divergent thinking, and aims to manage the complexity which undoubtedly exists in decision problems

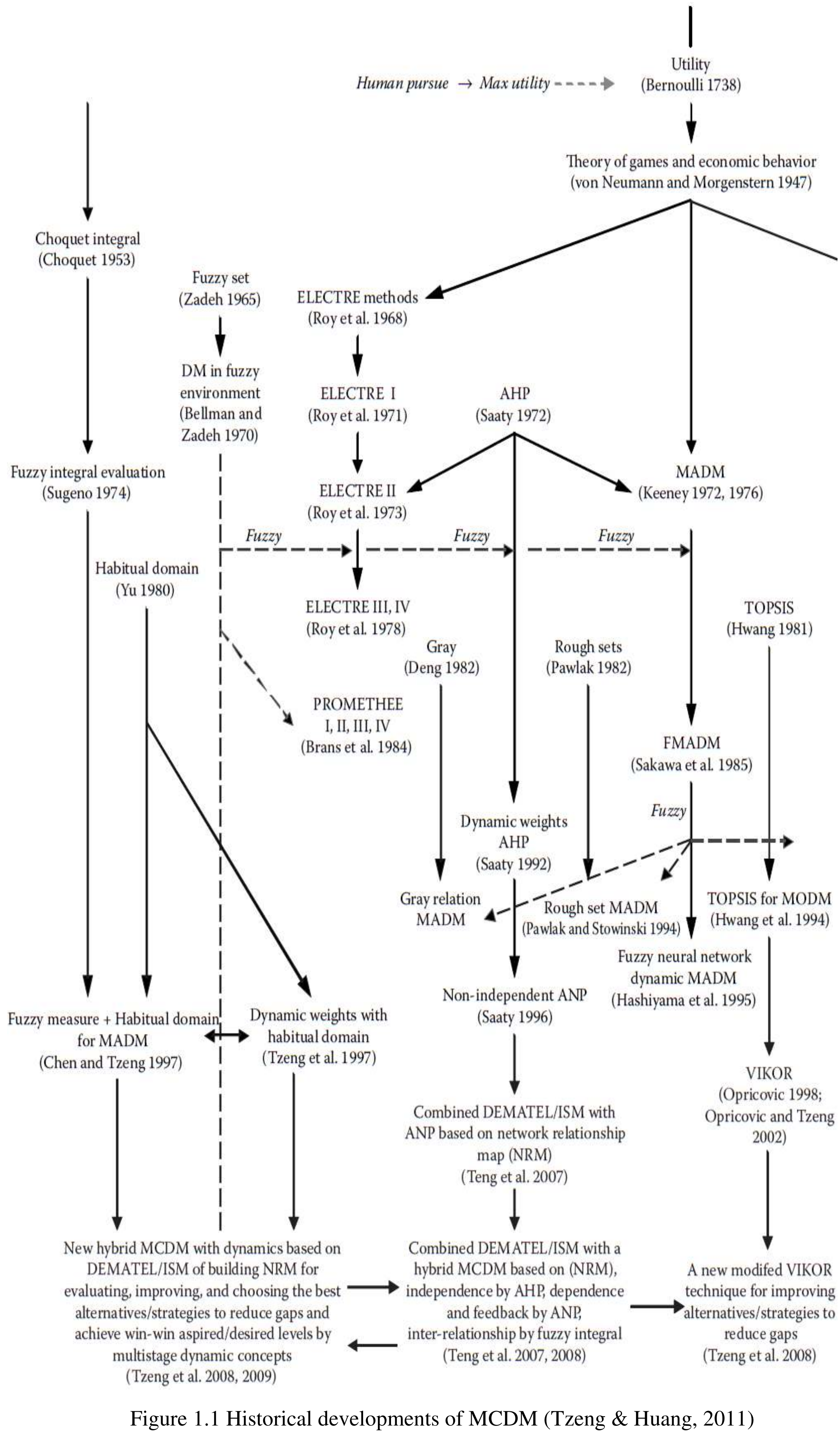


Figure 1.1 Historical developments of MCDM (Tzeng & Huang, 2011)

Problem structuring phase, actors central to the problem are identified, key issues, alternatives, criteria, goals and system boundaries are defined. In the model building phase decision matrix is filled up by eliciting performance scores and preferences of decision maker(s). Once the information is synthesized and initial results are obtained, challenge thinking is performed, consisting of sensitivity and robustness analysis and re-examination of the alternatives and problem.

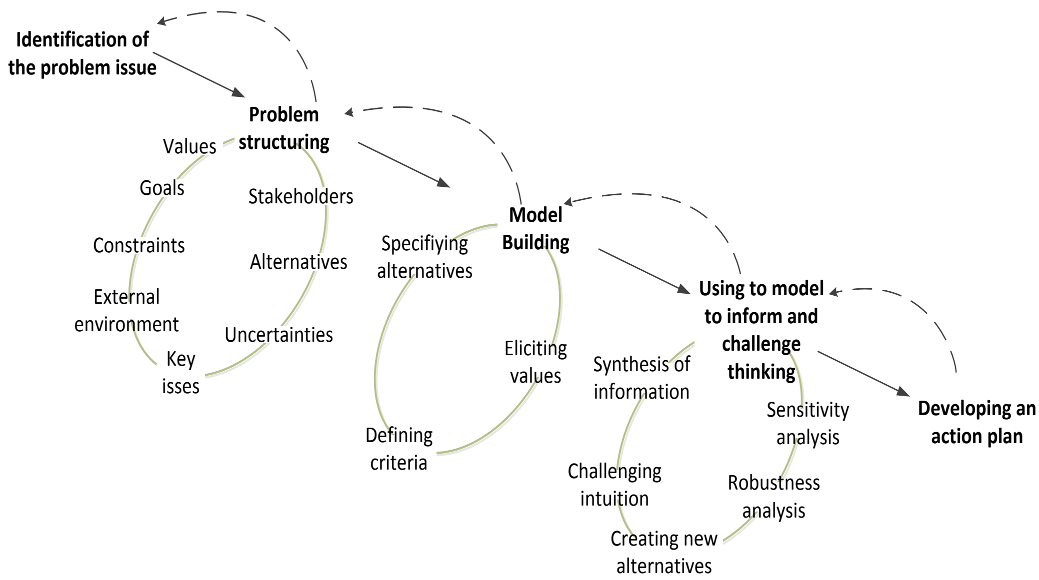


Figure 1.2 The process of MCDM (Belton & Stewart, 2002)

Additionally, a more general view is given by Clemen and Reilly (2001), who considered decision making as a process. The flowchart consisting of six phases is illustrated in figure 1.3. In the first phase, the decision maker identifies the decision situation and objectives. In this stage, alternatives and criteria are determined. Modeling phase is the vital feature of the process of decision making. In this stage mathematical or computer models have big impact on decision making. Decision tree or hierarchy like structures can be used to represent relationships between objectives and performance measures. Also uncertainty can be modeled based on the type of uncertainty, i.e. fuzzy or probabilistic. The subjective preferences of decision makers incorporating the uncertainties are mathematically represented. Finally, the best alternative is selected. Once the best alternative is selected, sensitivity analysis, which utilize what if questions is conducted. How small changes in the some aspects of the decision model affect final outcome is observed. The decision maker is allowed to return previous steps to make modifications if the decision is considered to be very sensitive.

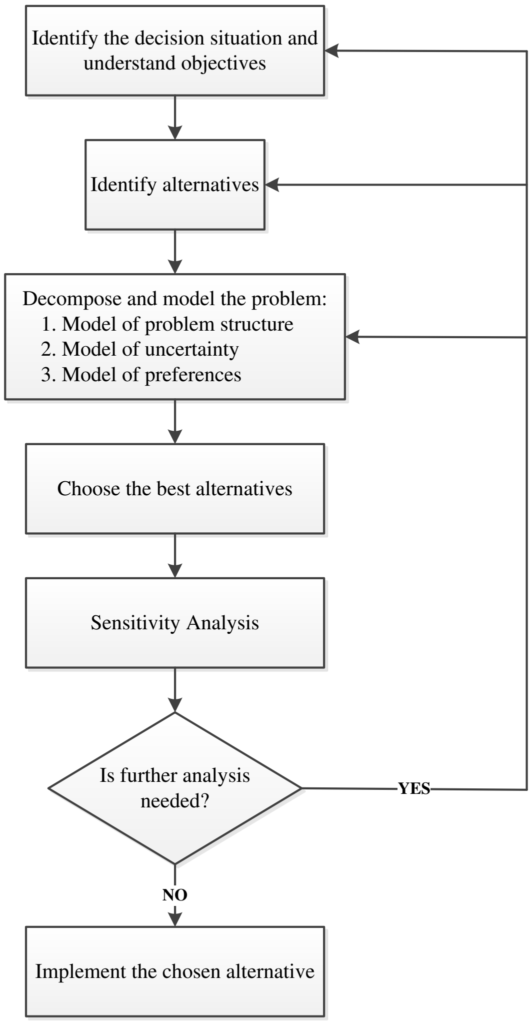


Figure 1.3 Flowchart of decision making process (Clemen & Reilly, 2001)

1.1.4 Classification of Problems and Methods

1.1.4.1 Classification of problems

Decision makers confront a plethora of different decision situations. There are many types of decision situations and their solution methodologies in the literature. Four main types of decision problems are identified by Roy (1981) as follows:

1. The choice problem. The goal is to select the single best option or reduce the group of options to a subset of equivalent or incomparable ‘good’ options. For example, selecting a person for a particular job and selecting an optional course at university examples of choice problems.
2. The sorting problem. Options are sorted into ordered and predefined groups, called categories. The aim is to then regroup the options with similar behaviours or characteristics for descriptive, organizational or predictive reasons. For instance, employees can be evaluated for classification into different categories such as ‘outperforming employees’, ‘average-performing employees’ and ‘weak-performing emplyees’. Based on these classifications, necessary measures can be taken. Sorting methods are useful for repetitive or automatic use. They can also be used as an initial screening to reduce the number of options to be considered in a subsequent step.
3. The ranking problem. Options are ordered from best to worst by means of scores or pairwise comparisons, etc. The order can be partial if incomparable options are considered, or complete. A typical example is the ranking of universities according to several criteria, such as teaching quality, research expertise and career opportunities.
4. The description problem. The goal is to describe options and their consequences. This is usually done in the first step to understand the characteristics of the decision problem.

In addition to these, Bana e Costa (1996) proposed elimination problem which is a particular branch of the sorting problem. Also, Keeney (1992c) mentioned “design problem”, aiming at identification or creating a new action to meet goals ans aspirations of the decision maker.

1.1.4.2 Classification of methods

Based upon Bell, Keeney, and Raiffa (1977), three different approaches can be raised in the field of decision analysis. These are normative, descriptive and prescriptive approaches. In the normative approach, rational choice is assumed and decision makers are expected to establish logical foundations for their decisions. It is optimality oriented and foundations dwell on management science and statistics. Traditional expected value model, the expected utility model, subjective expected utility model are examples of normative approach. Also, probability theory and Bayesian statistics have central roles in normative models.

The descriptive approaches are centered in the real behavior of decision makers and try to model the actual behavior mathematically. Models are compared to real life situations to evaluate the prediction quality. It is empirical research oriented and based on the idea that people are not always rational (Yoon & Hwang, 1995). Behavioral science and cognitive psychology are two important disciples feed descriptive approaches. A popular example of descriptive model is prospect theory model.

The prescriptive approach emphasizes on making better decisions utilizing normative approach, but also takes into account the irrational human judgment and limitations of the rational models in real life cases. It is the overlapping or intersection of the phenomenon that seeks out for optimal decision and that regards the human behavior for choosing an alternative from the physiological perspective. This model pursues the way that the tasks of the decision are as simple as possible and easy to understand by decision makers in order to hinder from the biases. Also, prescriptive approach strives to reduce the complexity of decision atmosphere and retains its manageable size.

On the other hand, more specific classification of basic methodologies for multicriteria problems are (Belton & Stewart, 2002):

1. Full aggregation approach (or American school). A score is evaluated for each criterion and these are then synthesized into a global score. This approach assumes compensable scores, which means that if a criterion has a bad score in terms of decision makers preference, good score of another

criterion can compensate this bad score. They are also known as value measurement models.

2. Outranking approach (or French school). A bad score may not be compensated for by a better score. The order of the options may be partial because the notion of incomparability is allowed. Two options may have the same score, but their behavior may be different and therefore incomparable.
3. Goal, aspiration or reference level approach. This approach defines a goal on each criterion, and then identifies the closest options to the ideal goal or reference level.

1.2 Motivation and Scope of the Study

MCDM provides distinct methodologies for solving particular class of problems. Unfortunately, most of the methods assume that the criteria are preferentially independent of each other, which is not realistic assumption in real life problems. In recent years, many methods appeared in the literature for modeling criteria interactions. However, the literature still lacks the overall and comprehensive view onto the criteria interactions in decision analysis. Also, there is no consensus on the basic definitions of the interaction concepts in the literature. As a result, further research is required to give detailed analysis on how specific methods consider interactions. If decision makers are informed about the available tools, then they can pick the best method which best suits their problem. The main objective of the thesis is twofold. They are

- To elucidate the concept of independence and provide the review of methods for modeling and solving criteria interactions in decision making. In the literature, methods dealing with criteria interactions are discussed very fragmentally. So a unified approach regarding the different modeling methodologies with comparative manner still lacks. This study is the initial step toward this direction.
- To propose a simple yet powerful method to handle interaction of criteria in a practical manner. Modeling interaction in decision making is costly. Models require too many survey questionnaires to be answered or call for numerous

parameters to be defined. A practical and intuitively easy decision making approach which takes into account criteria interactions is required. This thesis also tries to introduce a hybrid MCDM approach meeting the requirements of modeling complex, uncertain, dynamic decision environments.

To reach our objectives, we systematically analyzed papers in which interaction among criteria is considered and published in the highly reputable journals. The theoretical underpinnings of the decision analysis mostly rely on the utility theory (Bernoulli, 1738; von Neuman & Morgenstern, 1947). In utility theory, decision makers are assumed to have utility functions regarding each criterion, and also there is a global utility function aggregates each performance. Hence, most fundamental step in working with utility functions is the construction phase of the utility function. However, constructing utility function is a very difficult task. For that reason, additive utility function (Fishburn, 1970) is the most widely used utility function due to its practicality. The main drawback of the additive utility functions is that they come up with a set of assumptions which is not realistic for working with complex systems. Some of its assumptions are preferential independence, additive independence and so on.

It is quite interesting that even though the relatively recent MCDM methods do not have anything common with utility theory, i.e. outranking methods (Roy, 1990), basic notions of the utility theory are still used in the papers for referring to existence of dependence among criteria. The most widely used term is preferential independence. Although the utility theory is beyond the scope of this thesis, we had to revisit the basic terms pertaining to independence assumptions in the utility theory.

When the independence concepts in utility theory are introduced, we only focused on the preferential independence since other assumptions valid for methods making use of utility functions. However, most of the contemporary methods, such as Analytical Network Process or ELECTRE methods do not make use of utility function. However, from the ranking orders of the decision alternatives and the weights of the each criterion, preferential independence can be associated with the interpretation of the results.

In this thesis, different methodologies are brought together to answer how to model criteria interactions in multi attribute decision analysis. There are distinct methodologies used in this direction, where some of the methods directly take part in the original algorithm of the method, whereas others are hybridized with other decision analysis tools to obtain final priorities. We collected several methods and corresponding papers from the literature, which are the Analytical Network Process, Causal Maps, DEMATEL, Fuzzy Cognitive Maps, Interpretive Structural Modeling, Bayesian Networks, System Dynamics, and Choquet Integral. Then we selected Fuzzy Cognitive Maps for its desirable properties which will be mentioned in subsequent chapters to model and solve criteria dependencies in the context of hierarchical decision environments. The proposed model can also be interpreted from the problem structuring perspective by making use of hierarchical decomposition approach to structure criteria hierarchically to reduce complexity of the analysis while taking horizontal dependencies into account for more competent and realistic results.

1.3 Structure of the Thesis

The present thesis consists of thirteen chapters and organized as follows:

Chapter 2 lays the foundations of the independence concepts of decision analysis. For that purpose, preferential independence, utility independence, additive independence, difference independence and weak-difference independence concepts are explained with examples. The relationships between form of utility function and independence assumption are shown.

Chapter 3 presents Analytical Network Process method, which is the natural generalization of the Analytical Hierarchy Process. Main steps of the ANP are introduced and very detailed interpretation of the notion of dependency is analyzed. Literature analysis is used to show the popularity and high acceptance of ANP to model all sorts of dependency in decision making.

Chapter 4 presents causal (cognitive) maps -ancestors of the Fuzzy Cognitive Maps- for eliciting and modeling decision makers' causal beliefs and assertions. A brief literature review is given.

Chapter 5 is dedicated to DEMATEL method. The introduction of main steps of crisp/fuzzy DEMATEL method is given. Four different hybridization techniques of DEMATEL and ANP methods are thoroughly classified and explained with examples. Besides, literature review for different usage of DEMATEL method is given.

Chapter 6 presents Fuzzy Cognitive Maps. Brief definitions and mathematical foundations are given. Also a variant of Fuzzy Cognitive Maps which is called Fuzzy Decision Maps and its relationship with DEMATEL method is discussed. A literature review is given.

Chapter 7 gives the basics of Interpretive Structural Modeling. Binary matrix operations and structuring decision criteria are shown. Also partitioning criteria into different levels and MICMAC analysis are discussed. A brief literature review is given.

Chapter 8 presents Bayesian Networks. Bayesian probability theory and conditional probabilities are revised. The relationship between uncertain inference and causal dependency is discussed. Structural differences between causal thinking and Bayesian Networks representations are provided. A brief literature review is given.

Chapter 9 presents System Dynamics methodology. The place of system dynamics among the other simulation methodologies is discussed. Relationships between system behavior and model structure is given. The notion of causality, especially feedback causality is discussed. A brief literature review is given.

Chapter 10 is devoted to Fuzzy Integral methods. Basic definitions such as measures, integrals, fuzzy measures and fuzzy integrals are overviewed. Specifically Choquet integral, its calculation and interpretation is emphasized. Interpretation of both Choquet integral and k-additive Choquet integral are provided. Basics of Sugeno integral are also given. Interaction indices and their interpretations are discussed. Two important discussions; criticism of weighted average and need of Choquet integral, and relationship between Choquet integral and preferential independence/dependence are provided. A brief literature review is given.

Chapter 11 is dedicated to proposing a hybrid MCDM method by combining Hierarchical Fuzzy TOPSIS and Fuzzy Cognitive Maps. Since there is excessive number of decision analysis tools, selection of the most suitable method for the problem under consideration is a very challenging task. To support decision makers with a generic, powerful and simple method, a hybrid technique is developed. Initially, characteristics of a sufficient MCDM method must have are determined as hierarchical structure, criteria interactions and uncertainty. Hierarchical Fuzzy TOPSIS method is reinforced with the Fuzzy Cognitive Maps approach to model uncertainty in the form of fuzziness, taking criteria into account by Fuzzy Cognitive Maps in the hierarchically structured decision problems.

Chapter 12 presents an application study of the proposed method. A SWOT based strategy selection problem is studied as it is a hierarchically structured, strategic, and long term decision making problem involving uncertainties. Some desirable properties of Fuzzy Cognitive Mapping such as linguistic expressions of degree of interactions, and dynamic analysis via simulation are used to adjust criteria weightings. Proposed method is applied to the strategy selection problem of the department of industrial engineering at Dokuz Eylül University to illustrate the method.

Chapter 13 summarizes the thesis, reviews its contributions and suggests further research beyond the scope of this thesis.

CHAPTER TWO
BASIC INDEPENDENCE CONCEPTS

2.1 Introduction

Several assessment procedures and models are proposed for decision analysis in the literature. Multi attribute utility theory is one of the most prominent approaches in analyzing complex decision problems. Referring to historical development of the decision analysis, multi attribute utility theory is the milestone in the literature, where theoretical and axiomatic foundations of decision making are mostly formed by multi attribute utility theory.

Despite the name utility theory, its generalization as value models or value theory is more appropriate to describe these models. As stated by (Keeney, 1988), even though these terms are not very popular and seem to be new notions, they are not very different from the concept of objective function. In operations research or management science, objective functions are very often used, for example, to maximize profit or minimize cost. Actually, value models coincide with the objective functions in the operations research studies. Building a mathematical model for optimization problem is a similar process with forming a mathematical representation of preferences. Initially, decision maker should decide on the variables that will be used in the model and assess the model parameters using information available. In a value model, a number is calculated and it is called a value which is used for ranking of alternatives. A value model, just as in the optimization problems, can rank the alternatives depend on the maximization of desirability or minimization of undesirability.

A value model can be divided into two categories as utility functions and value functions. Utility functions refer to decision making under risk and deeply discussed by (Keeney & Raiffa, 1976; von Neuman & Morgenstern, 1947). On the other hand, measurable value functions refer to decision making context without concerning risks.

2.2 Basic Concepts and Definitions

Before beginning the independence concepts in preference theory, we give some basic and necessary definitions. Preference theory is mainly engaged with a binary preference relation $\succ$ on a choice set X, where X is generally set of alternatives. We can think of the two imaginary elements x, y, for instance, as two mobile phone alternatives where $x, y \in X$, X refers to set of all mobile phones. If customer prefers mobile phone x to mobile phone y, we write $x \succ y$, where the symbol $\succ$ represents strict preference. The absence of strict preference is symbolized as $\sim$, $x \sim y$ stands for not $x \succ y$ and not $y \succ x$.

Preference theory starts with the premise that the decision maker can assert his/her preferences over the alternative pairs without conflict; which means that decision maker does not strictly prefer first alternative to second alternative and second alternative to first alternative simultaneously. This property is called the preference asymmetry (Dyer, 2005). Preference asymmetry explains that $x \succ y$ and $y \succ x$ cannot be satisfied at the same time.

Another important definition is the negative transitivity. Negative transitivity means that in the situation of $x \succ y$, any other alternative, let's say z can be injected into the ordinal scale in such a way that it is less preferable than x and more preferable than y. This property can be represented as if $x \succ y$ is known, then z in X suits that $x \succ z$ or $z \succ y$ and the third alternative; both of them can be satisfied.

Essentially, we used negative transitivity and preference asymmetry to come up with the definition of a weak order. If a preference relation confirms the requirements of asymmetry and negative transitivity, then it is denoted as a weak order. Weak order is a vital property and it is indispensable part of preference ordering. Most of the definitions and properties in the field of preference theory presume that $\succ$ is a weak order. We give some properties of preference relations based on (Fishburn, 1970) in table assuming that strict preference is a weak order.

Table 2.1 Properties of preference relationships

Preference Relation	Property	Example
Strict Preference $(\succ)$	Transitive	if $x \succ y$ & $y \succ z$ then $x \succ z$
Indifference $(\sim)$	Transitive	if $x \sim y$ & $y \sim z$ then $x \sim z$
Indifference $(\sim)$	Reflexive	$\mathrm{x} \sim \mathrm{x}, \forall \mathrm{x} \in \mathrm{X}$
Indifference $(\sim)$	Symmetric	$(\mathrm{x} \sim y) \Rightarrow (y \sim x)$
Weak Preference $(\sim)$	Transitive	if $x \succeq y$ & $y \succeq z$ then $x \succeq z$
Weak Preference $(\sim)$	Complete	$x \succeq y$ or $y \succeq x$ holds.

Apart from the properties given in table 2.1, for each pair of x and y, one of the strict preferences or indifference relation should be satisfied.

2.3 Independence in Preference Functions for Certainty and Uncertainty

As we mentioned before, preference functions are backbone or kernel of the decision analysis methods in terms of the axiomatic foundations. A supportive argument can be given such that, in the early development stages of ELECTRE (Roy, 1990) and AHP (Saaty, 1980), as well as the other relatively new methods, people were tend to compare them with the multi attribute utility theory and its well-established theoretical basis during their primary introduction to the scientific community.

Unlike majority of the multi criteria decision making methods in the literature, multi attribute utility theory and multi attribute value theory have well-defined independence axioms. In our classification scheme, we categorize them under the preferential dependency title. In the literature, preference functions are divided into two categories, preference functions for certainty and for risky choice. Preference function for certainty takes our primary attention since we can build a bridge between its independence axioms with the AHP-like methods in the subsequent chapters of the thesis. Many methods in the literature assume preferential independence among criteria where the roots of the preferential independence term come from the preference function modeling under certainty. Indeed, we had to revisit the multi attribute utility theory and multi attribute value theory to trace back the origins of the important designations. Although they are not our main concern, we give independence axioms of preference functions for risky choice in order to clarify our categorization and to make every method fall into place.

In figure 2.1, the independence concepts are shown.

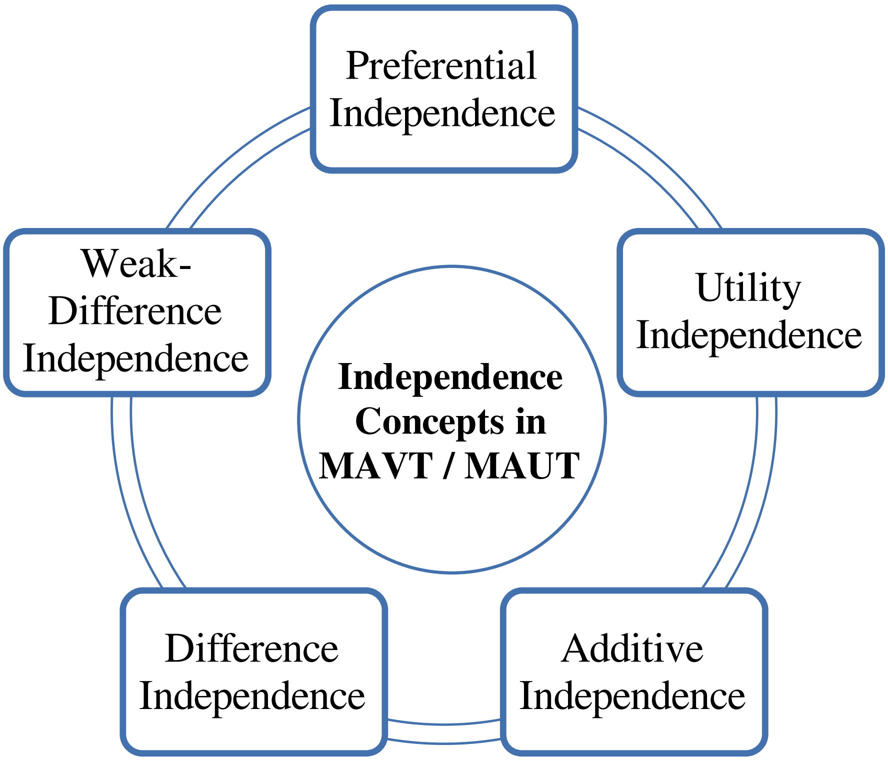


Figure 2.1 Preference independence concepts

We elaborate preferential independence, weak difference independence and difference independence underneath preference functions under certainty. On the other hand, utility independence and additive independence are the terms related to preference functions under uncertainty. Another important categorization provides a broader view such that preferential independence is the term regarding ordinal multiple attribute preference function for certainty. Weak-difference and difference independence concepts are related to measurable value functions under certainty. Finally utility independence and additive independence are linked to the cardinal multi attribute preference functions for the case of risk. Yet, we merely interested in the definitions of the concepts and evaluate them in terms of whether the uncertainty is involved in preference function. We begin with independence concepts in preference functions under certainty.

2.3.1 Independence in Preference Functions under Certainty

2.3.1.1 Preferential Independence

The condition for the attribute pairs X_1, X_2 to be preferentially independent of the other attributes is that consequences related to only changes or altering in the levels of interested attributes (X_1 and X_2) does not rely on the levels at which other attributes $X_3, \ldots, X_N$ are held fixed (Keeney, 1992a). Generalizing the definition, if preference independence condition is endured by all pairs of X_j attributes, then the mutual preference independence is satisfied.

More formal definition can be stated as follows; assume that subset of attribute indices are given by $J \in \{1, \ldots, n\}$ and X_j is defined as the subset of the attributes. Furthermore, $\bar{X}_j$ describes the complementary subset of the attributes. X_j is preferentially independent of $\bar{X}_j$ if (Dyer, 2005):

- $(\mathrm{y_j}, \bar{\mathrm{y}}_\mathrm{j}) \succeq (x_\mathrm{j}, \bar{\mathrm{y}}_\mathrm{j})$ for any $y_j, x_j \in X_j$ and $\bar{y}_j \in \bar{X}_j$ result in $(y_j, \bar{x}_j) \succeq (x_j, \bar{x}_j)$ for all $\bar{x}_j \in \bar{X}_j$.
- $X_1, X_2, \ldots, X_n$ are mutually preference independent if the set X_j of attributes is preferential independent of $\bar{X}_j$ for all $J \subseteq \{1, 2, \ldots, n\}$.

One of main implications of the preferential independence condition is that the indifference curves over X_1 and X_2 do not rely on the other attributes. In figure 2.2, we explain our assertion with an example (Keeney, 1992a) .

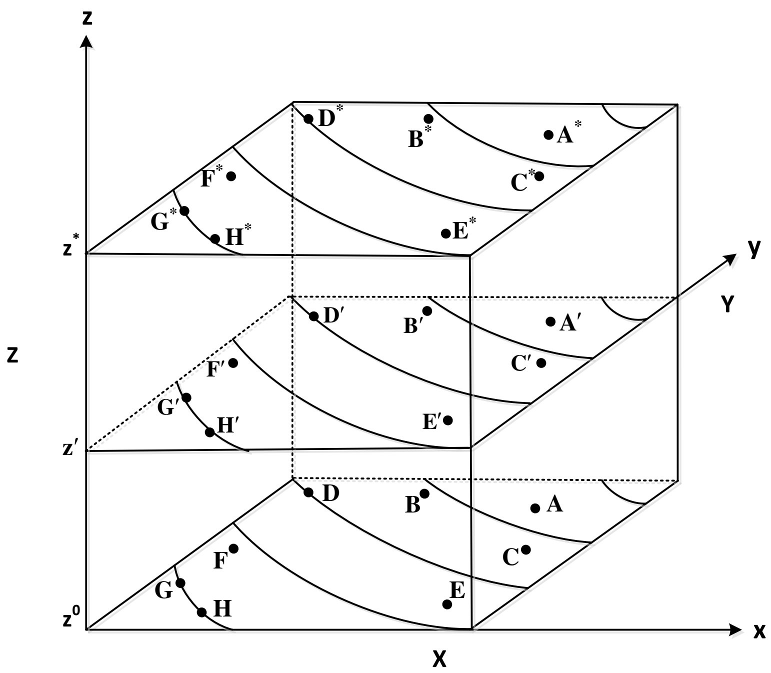


Figure 2.2 Representation of Preferential Independence (Keeney, 1992a)

In a nutshell, we express attributes by X, Y, Z with their levels x, y, z. Three planes can be seen in Figure 1. According to definition of preferential independence as explained above, if $\{X, Y\}$ is preferentially independent of Z, level of Z cannot affect the preference order of consequences in X, Y plane. Indifference curves represented in Figure 1 depicts the preferential independence situation where the $\{X, Y\}$ is independent of Z.

Let's take the bottom square of the indifference cube, where the level of Z is z^0. Preference order is $A, B, C, ..., F$ and G is indifferent to H in accordance with the indifference curve. When passed to the level z', we observe that the preference order is $A', B', C', ..., F'$ and G' is indifferent to H'. Accordingly, at the level of z^*, preference order is $A^*, B^*, C^*, ..., F^*$ and G^* is indifferent to H^*. Changes in the level of attribute Z do not alter the preference ordering due to the fact that attribute pair $\{X, Y\}$ is preferentially independent of Z. Indifference curve is only influenced by levels of X and Y.

To concretize the concept given above, we can introduce another example as selecting a mobile phone. This hypothetical example will be used to explain other methods in subsequent chapters and it is inspired from (Dyer, 2005). We have chosen this problem due to its inherent simplicity. Assume that we have three criteria to mobile phone selection. They are cost, CPU processing speed (we are interested in smartphones) and aesthetics. Suppose that decision maker's preferences between mobile phones only differ in cost and CPU speed. Elicited attribute levels and preferences are as $(\$180, 1\,\mathrm{GHz}, \mathrm{moderate}) \succ (\$195, 1.2\,\mathrm{GHz}, \mathrm{moderate})$ If the indifference curve between cost and CPU speed is not affected by the level of aesthetics, decision maker will pick $(\$180, 1\,\mathrm{GHz}, \mathrm{beautiful}) \succ$ $(\$195, 1.2\,\mathrm{GHz}, \mathrm{beautiful})$ and keeps the identical preference relation for the same aesthetics values.

Preferential independence is pragmatic to facilitate expressing preference orders only X, Y plane and convey this to other ones. We will discuss the relationships between preferential independence property and shape, type or the existence of the value/utility function later on in this chapter.

2.3.1.2 Weak-Difference Independence

Remember that weak-difference independence and difference independence are the topics of measurable multi attribute preference functions under certainty. Here the term measurable pertains to expression of strength of preference. Remembering the mobile phone selection example, we just ordered the decision maker's preferences, yet we did not quantify the strength of preference between pairs of consequences. In weak-difference and difference independence concepts, we mention about difference between consequences and quantify them.

Although we do not dig into the measurable preference functions, it is worth slightly mentioning them to be familiar with the underlying notation. Our problem is here represented by set of consequences, which is a product set $X = \prod_{i=1}^{n} X_i$.

Different components of X are represented by w, x, y, z. As an illustration of it, $z \in X$ is represented by $(z_1, \ldots, z_n)$. Suppose that $\succeq$ is a weak-order. Let $X = \{xy : \mathrm{x}, \mathrm{y} \in X\}$. Designation $xy \succeq wz$ is interpreted as preference difference between x and y is higher than preference difference between w and z.

Weak difference independence has a parallel function with utility independence concept of multi attribute utility theory. Main implication of weak-difference independence is to decide on the type of value function, whether it is multiplicative or any other non-additive formation. To give the formal definition of weak-difference independence, suppose that subset of attributes X_j is weak-difference independent of $\overline{X}_j$. If $(w_j, \overline{w}_j)(x_j, \overline{w}_j) \succeq (y_j, \overline{w}_j)(z_j, \overline{w}_j)$ then $(w_j, \overline{\mathrm{x}}_j)\ (x_j, \overline{x}_j) \succeq (y_j, \overline{x}_j)\ (z_j, \overline{\mathrm{x}}_j)$, $\forall \overline{\mathrm{x}}_\mathrm{j} \in \overline{\mathrm{X}}_\mathrm{j}$ is regarded by decision maker (Dyer, 2005). More clearly, preference differences depend merely on X_j, they are not dependent on the constant values of $\overline{X}_j$. Similar to the preferential independence, attributes are said to be mutually weak-difference independent if all subsets of attributes satisfy the weak-difference independence condition.

Even though we will go into details of relationships between independence axioms and type of value/utility functions, an important implication of weak-difference independence is worth mentioning. Weak difference independence leads to decomposition of measurable value function. Hence, we can capture multiplicative or additive preference function depending on the strength of preference.

To exemplify the weak- difference independence, let us take two attributes X and Y along with their levels x and y as seen in Figure 2.3 (Keeney, 1992a). We assume that we elicited from the decision maker that preference differences between $A-B$ and $B-C$ are equal.

Since attribute level of Y is y^0 for A, B and C, preference difference equalities should be hold for other two consequences

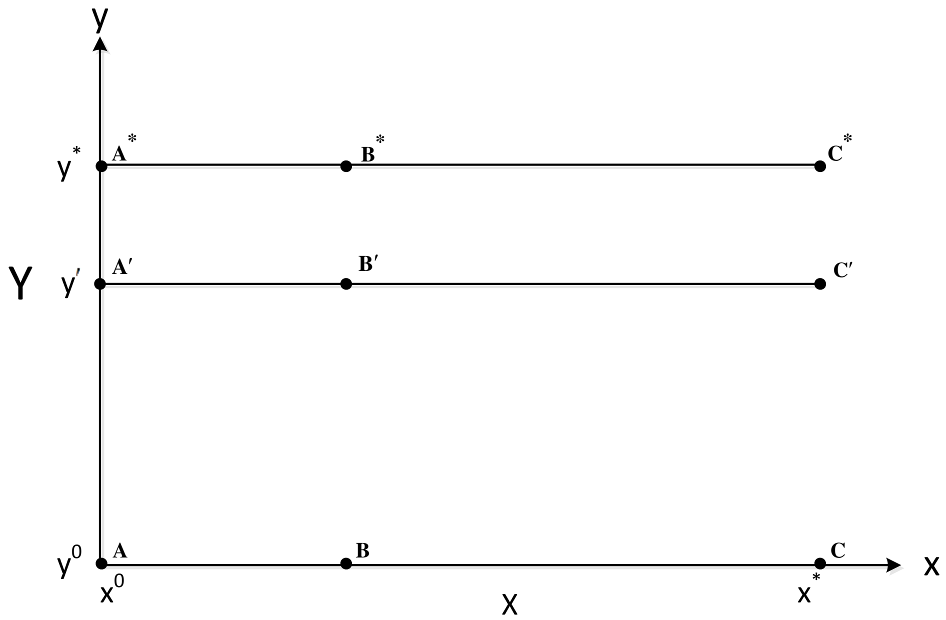


Figure 2.3 Representation of weak-difference and utility independence (Keeney, 1992a)

. If X is weak-difference independence of Y, then the preference difference between $A'-B'$ with $B'-C'$ and A^*-B^* with B^*-C^* should be the same. However the difference $A'-B'$ does not have to be equal to $B'-C'$. Furthermore, preference difference between $A-A'$ and $A'-A^*$ does not have to be equal. Another crucial point is that weak different independence does not have to be symmetrical. It means that X could be weak difference independence of Y, yet Y could not be weak difference independence of X.

Weak-difference independence can result in some confusion since it relies on an iterative process to elicit required information from the decision maker. To abandon this confusion, we turn back to our mobile phone selection example again.

Suppose that the decision maker prefers exchange of mobile phone $(\$180, 1.2\,\text{GHz}, \text{moderate})$ for the mobile phone $(\$195, 1.3\,\text{GHz}, \text{moderate})$ to the exchange of the mobile phone $(\$170, 1\,\text{GHz}, \text{moderate})$ for the mobile phone $(\$180, 1.2\,\text{GHz}, \text{moderate})$. If the first substitution over the second substitution does not rely on the same values of aesthetics, and if it is also valid for all other possible set of values of cost and CPU speed, then cost and CPU speed are weak difference independence of aesthetics.

2.3.1.3 Difference Independence

In essence, difference independence is the name of the circumstance that should be verified to use additive measurable preference function. As a formal definition, X_j is difference independent of $\bar{X}_j$ if: $(w_j, \bar{w}_j) \succeq (x_j, \bar{w}_j)$ for some $\bar{w}_j \in \bar{X}_j$, $(w_j, \bar{w}_j)(x_j, \bar{w}_j)$ indifferent to $(w_j, \bar{x}_j)(x_j, \bar{x}_j)$ for any $\bar{x}_j \in \bar{X}_j$.

Likewise the weak-difference independence, if all related subsets of the attributes satisfies the condition of difference independence, then the attributes are mutually different independent of their complementary attributes.

Consulting our regular example, suppose that the result of inquiry with decision maker is that exchanging or substituting a mobile phone represented by $(\$180, 1\,\text{GHz}, \text{moderate})$ for a mobile phone depicted by $(\$180, 1.2\,\text{GHz}, \text{moderate})$. Thereafter, another inquiry is provided with regarding substitution of $(\$195, 1\,\text{GHz}, \text{good})$ to $(\$195, 1.2\,\text{GHz}, \text{good})$. In fact, the inquiry is meant to be that does exchanging a mobile phone with 1 GHz for the other with 1.2 GHz be more favorable to decision maker when aesthetics attributes are moderate and costs are 180 dollars, or when costs are 195 dollars and aesthetics are nice? If the same value of cost and aesthetic attributes do not differ the importance of these substitutions, then CPU speed is difference independent of cost and aesthetics.

2.3.2 Independence Concepts in Preference Functions under Uncertainty

2.3.2.1 Utility Independence

Utility independence is a fundamental concept in building cardinal multi attribute preference functions under risky environment. Utility independence condition yields decomposing the utility function into simple assessment forms such as additive or multiplicative and brings great simplicity to analysis. Although our emphasis is on the preference functions under certainty, independence conditions related to utility functions would clarify the dependence condition.

Formally, X_j is utility independent of the other attributes denoted by $\bar{X}_j$, if the preference order for lotteries with dissimilar levels of X_j does not depend on the fixed levels of $\bar{X}_j$ attributes (Keeney, 1992a). If all related subsets of the attributes satisfy the condition of utility independence, then the attributes are mutually utility independent of their complementary attributes.

Utility independence has implications for lotteries as it undertakes uncertainties in the decision environment by incorporating probabilities into utility functions. Utility independence is, in essence, a similar concept to weak-difference independence; hence we revisit Figure 2.3 to give details.

Assume that in Figure 2.4, with probability of 0.5, the lottery B is indifferent to lottery A or C as seen at y^0 level. The condition for A to be utility independent of B is that this relation among A, B and C is conveyed to all other levels of Y. This means that fixing the probability as 0.5 for each lottery, B' is indifferent to lottery $A' - C'$ and B^* is indifferent to lottery $A^* - C^*$ conjointly.

Another example can be embodied as follows; suppose that the decision maker is facing with a choice situation to produce a new model of mobile phone. Attributes are cost, CPU speed and aesthetics again. Due to the immense competition in the market, competitive firms' strategy to launch a thinner and lighter brand new device force mobile phone producer to release equivalent products with slightly higher cost due to the composite materials used to make the phone thinner and with reduced CPU speed to focus on mobility and address almost all segments of customers. Our producer has two alternative products.

At present, the firm produces a mobile phone with the levels of $(\$170, 1.2\,\text{GHz}, \text{moderate})$. (Note that the alternative is described by the consequence space, which may result in confusion for those familiar with AHP-like methods notations. If we have two consequences for the equal values for each attribute level, then alternatives are said to be lose their identity).

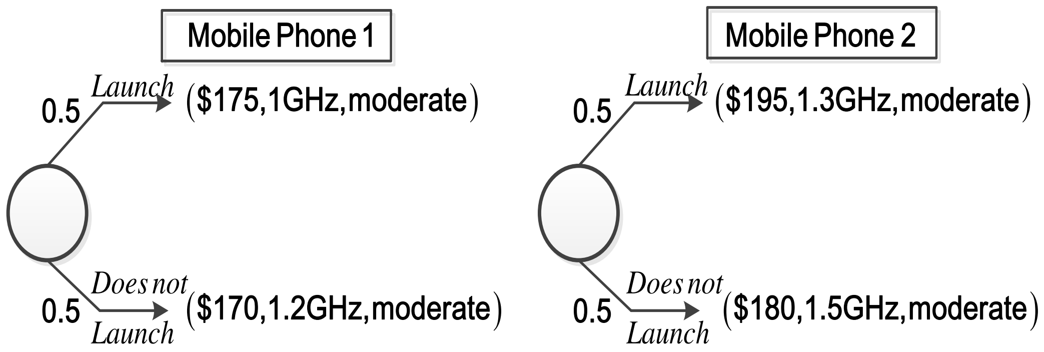


Figure 2.4 Two lotteries for new product launch

Keeping on the current situation, if the competitive firm launches a new product, our reaction is to release a product with $(\$175, 1\text{GHz}, \text{moderate})$ levels which raises the cost and reduce GHz level. Another alternative product in case of the situation that competitive firm launches a new product is to release a new mobile phone with levels of $(\$195, 1.3\text{GHz}, \text{moderate})$. In the second scenario if the competitive firm does not launch a new product, we produce a mobile phone with levels of $(\$180, 1.5\text{GHz}, \text{moderate})$. Decision maker approximates that competitive firm will launch a new product with probability of 0.5.

Accordingly, the decision maker confronts a choice situation between two lotteries as seen from Figure 2.4. Assume that the decision maker chooses mobile phone alternative 1 due to reduced cost and risk perception. If the decision maker's determination for lotteries does not rest on the same values of the aesthetics attribute, then cost and CPU speed are utility independent of aesthetics.

2.3.2.2 Additive Independence

Additive independence concept is highly related to decomposing Neumann-Morgenstern (von Neuman & Morgenstern, 1947) utility function into an additive form. Decision makers should satisfy the marginality condition to carry out an additive utility function in decision analysis. This marginality condition is exactly nothing different from the additive independence concept. The condition of the additive independence is that preference order for lotteries does not rely on the joint probability distributions yet they rely merely on their marginal probability distributions (Keeney, 1992a).

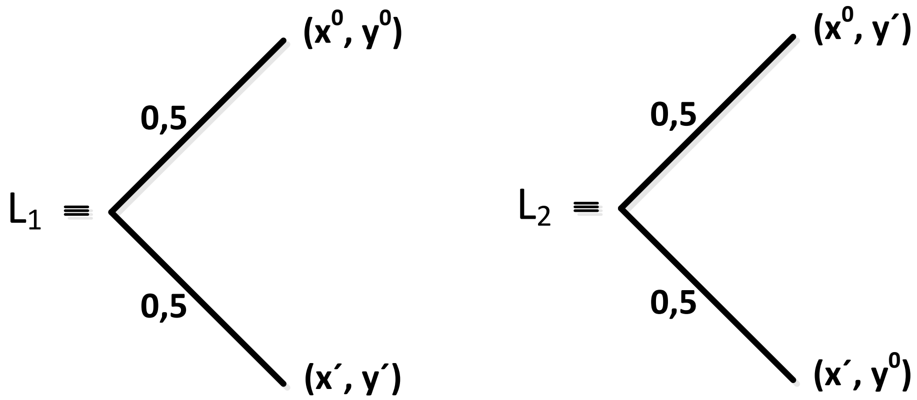


Figure 2.5 Representation of additive independence (Keeney, 1992a)

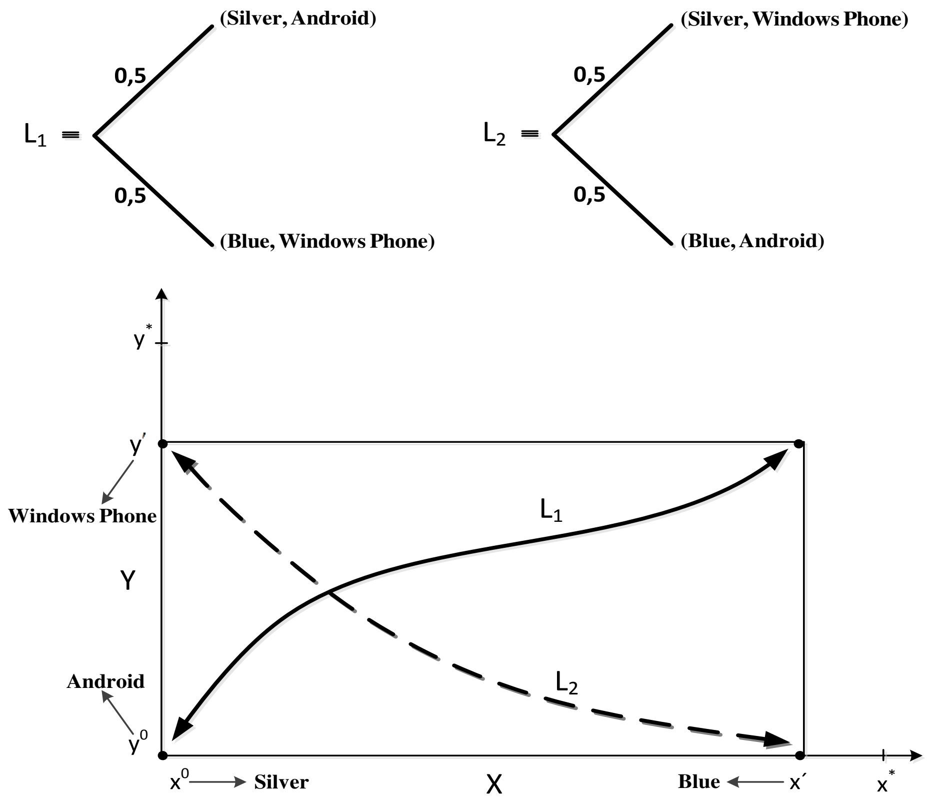


Figure 2.6 Additive independence for mobile phone selection, adopted from Keeney (1992a)

In Figure 2.5, we give the formal representation of additive independence. In Figure 2.6, we fitted our mobile phone selection example with the representation given by (Keeney, 1992a). In figure 2.5, we can say that lotteries have the same probabilities (0.5) for $x^0 - x'$ and $y^0 - y'$. It means that considering both of the lotteries, marginal probability distributions are identical for each attribute. Therefore, the condition for X and Y to be additive independent of each other is that the decision maker should be indifferent between L_1 and L_2 (Fishburn, 1965). In addition, expression of one attribute to be additive independent of the other is not meaningful. Additive independence is a bilateral situation, two attributes can be additive independent or not.

2.4 Independence Concepts and Type of Value Functions

Multi attribute utility theory and multi attribute value theory make the above mentioned independence assumptions for a very practical purpose: to construct a value/utility function. Without these assumptions, constructing utility/value functions are real headaches. Making such assumptions give decision makers an opportunity to decompose utility functions into multiplicative or additive forms, which facilitates evaluation of possible consequences conveniently.

In this section of the thesis, we present the summary of the relationship between type of preference function and the independence assumption made. The most widely used formulations in the literature are summarized in table 2.2.

2.5 Utility Functions and Analytic Hierarchy Process

Decision analysis literature witnessed great debate on the axiomatic differences between AHP and multi attribute utility theory. Lots of misinterpretations, corrections, discussions were reported in the highly cited scientific journals. Many authors in the literature exerted weaknesses and critics of the AHP; on the other hand some offended the AHP with respect to number of aspects. Examples of these discussions can be found in (Forman & Gass, 2001; Gass, 2005; Saaty, 1986, 1999a; Winkler, 1990). Most of the criticism was based on the fact that AHP did not conform the axioms of multi attribute utility theory, and exposed to transitivity and rank reversal (Forman & Gass, 2001).

Table 2.2 Independence assumptions and preference function formation

Criteria	Independence Assumption	Preference Function Formation
≥ 2	the attributes are utilityindependent of other attributes	$u(x_1,\ldots,x_N)=\sum_{i=1}^{N}k_i u_i(x_i)+\sum_{i=1}^{N}\sum_{j>i}^{N}k_{ij}u_i(x_i)u_j(x_j)$ $+\sum_{i=1}^{N}\sum_{j>i}^{N}\sum_{h>j}^{N}k_{ijh}u_i(x_i)u_j(x_j)u_h(x_h)$ $+\ldots+k_{1\ldots N}u_1(x_1)\ldots u_N(x_N)$
≥ 2	the attributes are additive independent	$u(x_1,\ldots,x_N)=\sum_{i=1}^{N}k_i u_i(x_i)$
≥ 3	The attributes are preferentially independent of other attributes	$u(x_1,\ldots,x_N)$ $=\sum_{i=1}^{N}k_i u_i(x_i)+k\sum_{i=1}^{N}\sum_{j>i}^{N}k_i k_j u_i(x_i)u_j(x_j)$ $+k^2\sum_{i=1}^{N}\sum_{j>i}^{N}\sum_{h>j}^{N}k_i k_j k_h u_i(x_i)u_j(x_j)u_h(x_h)+\cdots$ $+k^{N-1}k_1\ldots k_N u_1(x_1)\ldots u_N(x_N)$ If $k=0$; $u(x_1,\ldots,x_N)=\sum_{i=1}^{N}k_i u_i(x_i)$

On the other side, some authors like Salo and Hämäläinen (1997) criticized the debates in the literature with respect to the fact that different methods do not have to be seen as competitive or rivaling methodologies, adversely, they offended the idea that emphasize and discussion on similarities of methods can contribute to theory and development of methodologies. Authors mainly put forward the parallelism between AHP and MAVT and its conditions. Here we do not give details. Similar studies can be found in (Kamenetzky, 1982; Zahedi, 1987). The relationship between utility theory and AHP is still a controversial issue. Hence, we do not call independency of criteria in AHP as preferential independence, even though some authors do (Lin, Wang & Yu, 2008) . In the next chapter, we give detailed analysis of AHP/ANP.

CHAPTER THREE
ANALYTIC NETWORK PROCESS

3.1 Introduction

Analytical Network Process (ANP) is one of the most popular multiple criteria decision making methods. It is the generalization of Analytical Hierarchy Process (AHP), which can model only static and unidirectional interactions among decision elements (Sarkis, 1998). However, real life problems are very difficult to be modeled via hierarchical structures, since there are many interactions among the elements of different levels. Therefore, ANP is pragmatic tool to solve complex decision structures by utilizing the supermatrix formation. The supermatrix or influence matrix can be considered as the generalization of the AHP, since ANP offers much more flexibility for taking complex interactions among different elements into consideration (Saaty, 1996).

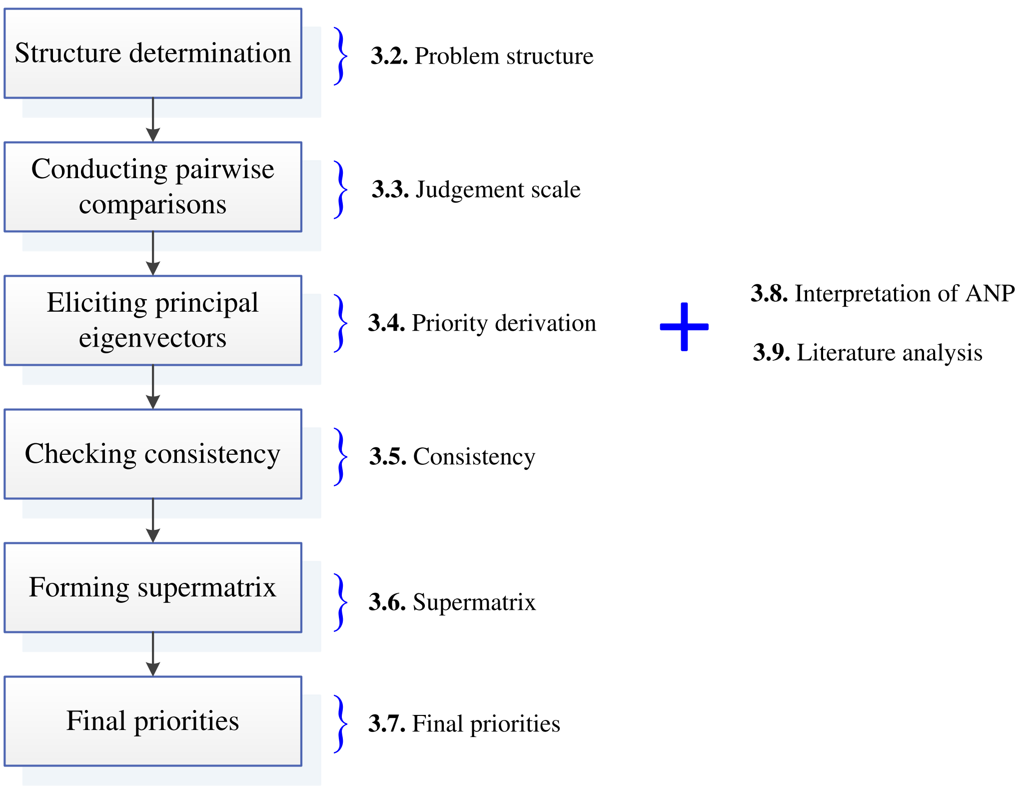


Figure 3.1 General steps of ANP model construction

The general steps of the ANP is shown in Figure 3.1. In this chapter, we also follow these steps to briefly explain basic concepts. Additionally, we give a detailed interpretation of ANP method in terms of modeling criteria dependencies. Finally we also depict the bibliographic analysis of published papers in order to demonstrate wide range of acceptance and applications of the method.

3.2 Problem structure

Structuring a decision problem is one of the most significant steps in the analysis. The AHP models only deal with the hierarchically structured problems, in which horizontal links are not allowed. On the other hand, the ANP considers problem as a network structure and overcomes the linearity assumption of AHP seen in Figure 3.2.

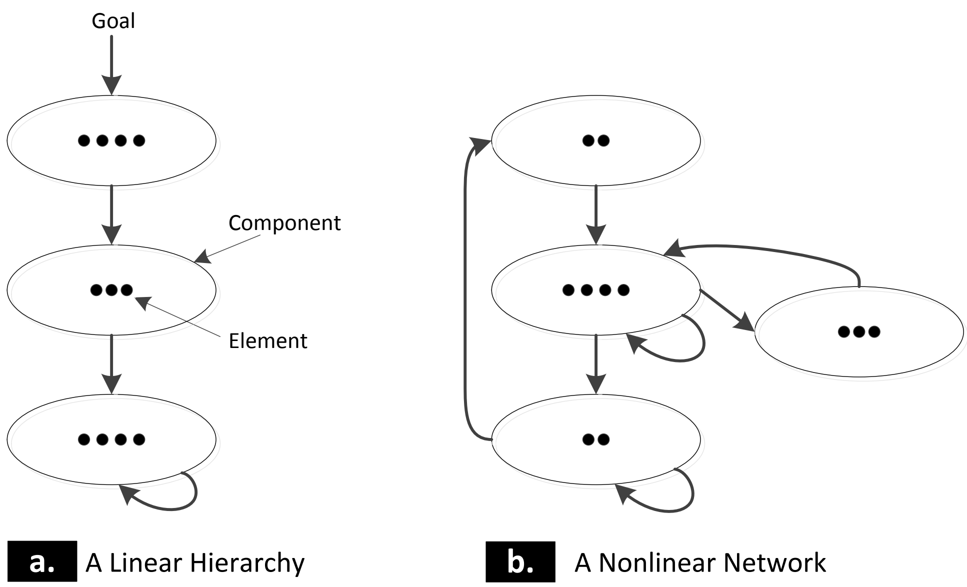
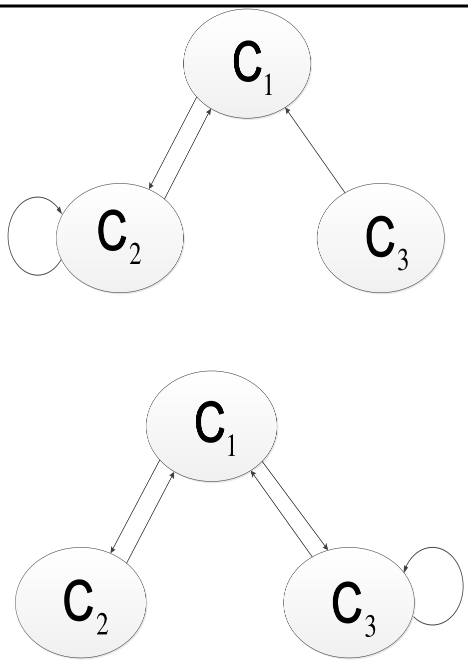


Figure 3.2 Linear hierarchy and nonlinear network

Depending on the criteria structure, the supermatrix can take several forms. Saaty (1996) mentioned different structures, such as hierarchy, holarchy, intarchy and so on.

In figure 3.3, we have shown two different structures and their corresponding supermatrices.



$$\rightarrow \quad \begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ W = \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \begin{bmatrix} 0 & W_{12} & W_{13} \\ W_{21} & W_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\rightarrow \quad \begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ W = \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \begin{bmatrix} 0 & W_{12} & W_{13} \\ W_{21} & 0 & 0 \\ W_{31} & 0 & W_{33} \end{bmatrix} \end{array}$$

Figure 3.3 Relationship between network structure and supermatrix

The W_{ij} values are the eigenvalues of the pairwise comparison matrix, which gives the influence degree of the element in the jth cluster on the other elements in the ith cluster. This process is done by pairwise comparison of the elements in the ith cluster with respect to elements in the jth cluster. The resultant eigenvectors are placed in the appropriate columns.

3.3 Judgment scale

The criteria and alternatives are weighted through pairwise comparison matrices using fundamental scale of Saaty given in table 3.1.

As seen from the table 3.1, ratio scales are used in pairwise comparison matrices of AHP/ANP. Verbal comparisons are the basis of judgment scale, since it is intuitively appealing and easier for decision maker. Verbal comparisons are more practical than using precise numbers. However, there some critiques over the ratio scale

Table 3.1 Saaty's Fundamental Scale

Value	**Definition**	**Explanation**
1	Equally important	Two decision elements have equal influence on the superior decision element
3	Moderately more important	One decision element has moderately more influence than the other
5	Strongly or essentially more important	One decision element has strongly more influence than the other
7	Very strong or demonstrated importance	One decision element has very strongly more influence than the other
9	Extremely more important	One decision element has extremely more influence than the other
2,4,6,8	Intermediate vales of judgment	
Reciprocals		If v is the judgment value when i is compared to j, then $1/v$ is the judgment value when j is compared to i

For example, Barzilai (2005) stated that absolute zero does not exist in the fundamental scale, so preferences cannot be modeled appropriately. Also F. J. Dodd and H. A. Donegan (1995) criticized the lack of absolute zero in the scale. An alternative approach to ratio scale is interval scales (Kainulainen et al., 2009). However, Saaty (1994) emphasizes that the ratio scales are necessary to aggregate incommensurate (i.e. not the same units) measurements. We give a small example to give difference between ratio scale and interval scale.

Example 3.1: Suppose that a basketball team is to select a pivot player among candidates. In the pre-selection phase, candidates are evaluated with respect to weight and height. We assume that greater values are better for two criteria.

Table 3.2 Weight and height values of the two players

	Weight [kg]	Height [meter]
Player A	80	2.00
Player B	90	1.90

Decision maker considers height three times as important as the weight. If two players are compared on a ratio scale:

$$\frac{Player\,B}{Player\,A} = 1 \times \frac{90}{80} + 3 \times \frac{1.90}{2.00} = 3.975 \tag{3.1}$$

If two players are compared on the interval scale:

$$Player\,B - Player\,A = 1 \times (90 - 80) + 3 \times (1.90 - 2.00) = 9.1 \tag{3.2}$$

Now suppose that we changed the scale of height from meter to centimeter. Calculation using the interval scale is:

$$Player\,B - Player\,A = 1 \times (90 - 80) + 3 \times (190 - 200) = -20 \tag{3.3}$$

Calculation using ratio scale:

$$\frac{Player\,B}{Player\,A} = 1 \times \frac{90}{80} + 3 \times \frac{190}{200} = 3.975 \tag{3.4}$$

Actually, interval scales are very sensitive to the unit of measurements. However, ratio scales are much more robust and reliable to aggregate and compare incommensurable units.

In addition to above raised issues, it is worth noting that fundamental scale is not the only option for measurement scale. In the literature, different scale types are proposed and some of them are summarized in table 3.3.

Choice of the best scale is a very debatable issue in the literature. We have to indicate that pairwise comparison matrices in AHP/ANP do not dictate linear or fundamental scale. Also it worth noting that in the literature, Saaty's fundamental scale is most widely used.

Table 3.3 Different scales for comparing alternatives (Ishizaka & Labib, 2009)

Scale Type	Values								
	Equall Important	**Weak**	**Moderate Importance**	**Moderate Plus**	**Strong Importance**	**Strong Plus**	**Verystrong Importance**	**Very very strong**	**Extreme Importance**
Linear (Saaty, 1977)	1	2	3	4	5	6	7	8	9
Power (Harker & Vargas, 1987b)	1	4	9	16	25	36	49	64	81
Geometric (Lootsma, 1989)	1	2	4	8	16	32	64	128	256
Logarithmic (Ishizaka, Balkenborg, & Kaplan, 2006)	1	1.58	2	2.32	2.58	2.81	3	3.17	3.32
Root square (Harker & Vargas, 1987b)	1	1.41	1.73	2	2.23	2.45	2.65	2.83	3
Asymptotical (F. Dodd & H. Donegan, 1995)	0	0.12	0.24	0.36	0.46	0.55	0.63	0.70	0.76
Inverse Linear (Ma & Zheng, 1991)	1	1.13	1.29	1.5	1.8	2.25	3	4.5	9
Balanced (Salo & Hämäläinen, 1997)	1	1.22	1.5	1.86	2.33	3	4	5.67	9

3.4 Priority derivation

3.4.1 Eigenvalue method

The eigenvalue method is proposed by Saaty (1980) in order to obtain the priority vector. Let us denote the priority vector by $\mathbf{p}$.

$$\mathbf{Ap} = n\mathbf{p} \tag{3.5}$$

where n is the number of criteria/alternative in the pairwise comparison matrix $\mathbf{A}$, and $\mathbf{p} = (p_1, p_2, \ldots, p_j, p_n)$.

Suppose that we have a fully consistent pairwise comparison matrix, where $a_{ij} = \frac{p_i}{p_j}$. Then the left hand side of the equation 3.5 is:

$$\frac{p_i}{p_1} p_1 + \frac{p_i}{p_2} p_2 + \cdots + \frac{p_i}{p_j} p_j + \cdots + \frac{p_i}{p_n} p_n = p_i + p_i + \cdots + p_i = np_i \qquad (3.6)$$

If the all rows of the comparison matrix is multiplied similarly, the priority vector becomes:

$$\begin{bmatrix} p_1/p_1 & p_1/p_2 & \cdots & p_1/p_n \\ p_2/p_1 & p_2/p_2 & \cdots & p_2/p_n \\ \cdots & \cdots & \cdots & \cdots \\ p_n/p_1 & p_n/p_2 & \cdots & p_n/p_n \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_n \end{bmatrix} = n \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_n \end{bmatrix} \qquad (3.7)$$

Example 3.2 (Ishizaka & Lusti, 2006) : Suppose that we have the consistency pairwise comparison matrix as seen in table 3.4.

Table 3.4 Pairwise comparison matrix of example 3.2

C_1	C_2	C_3	C_4
C_2	1	6	3
C_3	1/6	1	½
C_4	1/3	2	1

The equation 3.5 becomes:

$$\mathbf{Ap} = \begin{bmatrix} 1 & 6 & 3 \\ 1/6 & 1 & 1/2 \\ 1/3 & 2 & 1 \end{bmatrix} \mathbf{p} = 3\mathbf{p} \qquad (3.8)$$

The priority vector $\mathbf{p}$ is the solution of the following linear system:

$$\begin{aligned} &1 \times p_1 + 6 \times p_2 + 3 \times p_3 = 3 \times p_1 \\ &(1/6) \times p_1 + 1 \times p_2 + (1/2) \times p_3 = 3 \times p_1 \\ &(1/6) \times p_1 + 1 \times p_2 + (1/2) \times p_3 = 3 \times p_1 \end{aligned} \qquad (3.9)$$

The solution of the linear system is becomes:

However, pairwise matrices elicited from decision makers are not always consistent. In these inconsistency situations, $a_{ij} = p_i / p_j$ is no longer valid.

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0.667 \\ 0.111 \\ 0.222 \end{bmatrix} \tag{3.10}$$

Therefore, result of the multiplication operation in the left hand side of the formula x does not yield n times $\mathbf{p}$. So the n in the right hand side of the equation is replaced by the unknown λ. Thereby the equation takes the form;

$$\mathbf{Ap} = \lambda\mathbf{p} \tag{3.11}$$

which is the well-known eigenvalue problem in linear algebra. In this formula, λ value is called eigenvalue and the its associated eigenvector is $\mathbf{p}$. According to Perron's theorem, $\mathbf{A}$ has a unique positive eigenvalue. The non-trivial eigenvalue is called maximum eigenvalue which is denoted by $\lambda_{\max}$. If the matrix is fully consistent, $\lambda_{\max}$ equals to n. If the matrix is not perfectly consistent, $\lambda_{\max} - n$ is the measure of inconsistency. Note that the measure of inconsistency $\lambda_{\max} - n$ is used as the dividend in the consistency index formula.

In the literature, many methods have been proposed to calculate eigenvectors. The power method is a well-known algorithm for deriving principal eigenvector, which stems from the graph theory (Barzilai, 1997). Basic process of the power method is as follows:

1. The pairwise matrix is squared: $\mathbf{A}_{n+1} = \mathbf{A}_n\mathbf{A}_n$
2. Row sums are calculated and normalized.
3. Using the matrix $\mathbf{A}_{n+1}$, steps 1 and 2 are repeated.

4. Step 3 is repeated until the differece between these sums in two consecutive priorities calculations is smaller than a given stop criterion.

Actually, rasing matrix to powers is not peculiar to AHP/ANP. It is a property in graph theory, where raising matrix to powers provides different order transivities. Here we give the underlying mathematical basis of raising the matrix to powers in order to calculate the criteria weights. Raising matrix to powers is not peculiar to ANP, we mention about it also in DEMATEL or ISM though. We examplify the situation borrowed from Harker and Vargas (1987a).

Example 3.3 (Harker & Vargas, 1987a): Considering the criteria structure in Figure, we have three criteria represented by each node.

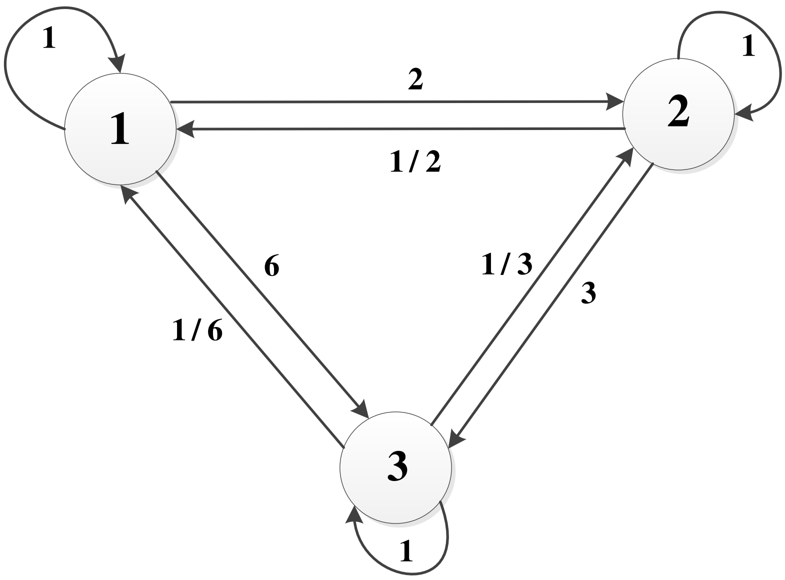


Figure 3.4 Graphical representation of pairwise comparison matrix

Influences among criteria can be seen in Figure 3.4 as well and depicted in the matrix form as:

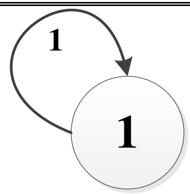
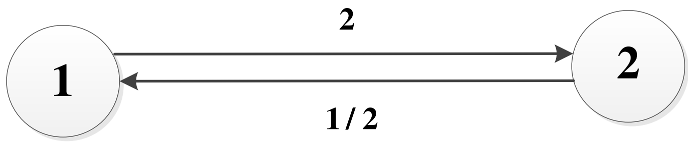
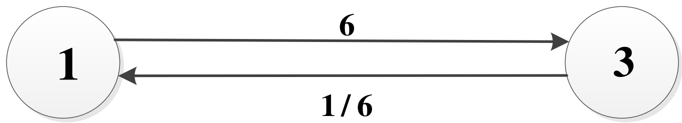
$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 6 \\ 1/2 & 1 & 3 \\ 1/6 & 1/3 & 1 \end{pmatrix} \tag{3.12}$$

Consequently, we calculated the princical eigenvalue as $\lambda_{\max} = 3$ and corresponding eigenvector $(w_1, w_2, w_3) = (0.6, 0.3, 0.1)$. We can reach the same values by a different analysis from the graph theory. Matrix A is directed and strongly connected graph represented in Figure 3.4 and elements are expressed by a_{ij}. Also second and third power of A is obtained as:

$$\mathbf{A}^2 = \begin{pmatrix} 3 & 6 & 18 \\ 1.5 & 3 & 9 \\ 0.5 & 1 & 3 \end{pmatrix} \qquad \mathbf{A}^3 = \begin{pmatrix} 9 & 18 & 54 \\ 4.5 & 9 & 27 \\ 1.5 & 3 & 9 \end{pmatrix} \tag{3.13}$$

In table 3.5, we have shown the paths for the lenght of two, begining from node 1 and ending in node 1. In table 3.6, the paths from node 1 to node 2 is shown with respect to path lenght of two.

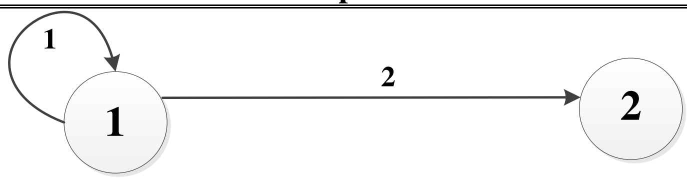
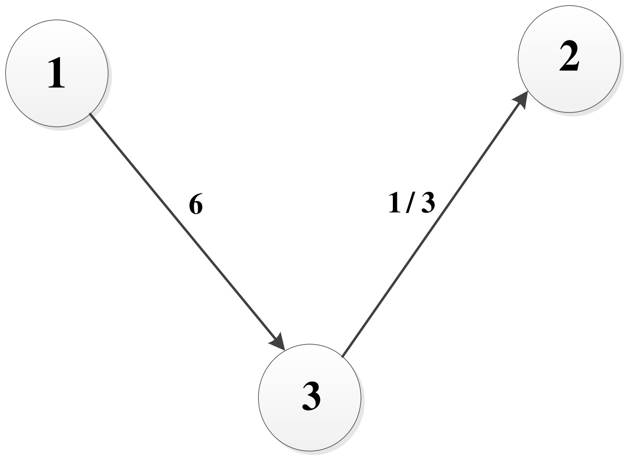
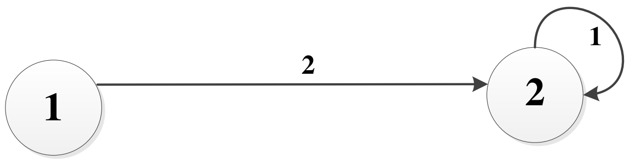
Table 3.5 Paths From Node 1 to Node 1

Path	**Graph**	**Intensity**
1-1-1		1
1-2-1		$2 \times (1/2) = 1$
1-3-1		$6 \times (1/6) = 1$

Actually, matrix A is a node-arc incidence matrix, which turns out to be that influence of node i on node j on paths of lenght k can be calcualted by raising A to kth power.

In the second power of the matrix A, $a_{11} = 3$ and $a_{12} = 6$. Summing the values in Table 3.5 which is node 1 to node 1 influences for the path lenght of 2, $1+1+1=3$ is found. In table 3.6, from node 1 to node 2, influences for the case of path lenght two is found as $2+2+2=6$. They indeed corresponds to the second power of the matrix. Raising the matrix to the kth power results in calculation of the total influence, posed from column criteria to the row criteria represented in the matrix form.

Table 3.6 Paths From Node 1 to Node 2

Path	**Graph**	**Intensity**
1-1-2		$1 \times (2) = 2$
1-3-2		$6 \times (1/3) = 2$
1-2-2		$1 \times (2) = 2$

$$\mathbf{A}^2 = \begin{pmatrix} \boldsymbol{3} & \boldsymbol{6} & 18 \\ 1.5 & 3 & 9 \\ 0.5 & 1 & 3 \end{pmatrix} \tag{3.14}$$

Besides, normalizing the second and third powers of the matrix A, we can obtain the identical criteria weights with eigenvalue method. Column sum of the second power of matrix A is $(5,10,30)$. If each column is normalized by $(5,10,30)$, the result is $(0.6, 0.3, 0.1)$ which coincides with the result we obtained by raising the matrix to powers.

3.5 Consistency

Consistency check is an important property of the weights derived from pairwise comparison matrices. A pairwise comparison matrix with elements a_{ij} is consistent if the following rules are satisfied:

- Transitivity:

$$a_{ij} = a_{ik}a_{kj} \tag{3.15}$$

where a_{ij} is the comparison of criterion i with criterion j. Transitivity rule assumes that if criterion A is twice as much important as criterion B, and criterion B is 5 times more important than C, then criterion A is 10 times more important than C.

- Reciprocity:

$$a_{ij} = \frac{1}{a_{ji}} \tag{3.16}$$

where i and j are any criteria or alternative. Reciprocity property assumes that if criterion A is twice as much important as criterion B, then the criterion B is half as important as criterion A.

Let p_i be the priority of the criterion i, If we know that the above two properties are satisfied, a totally consistent matrix can be written as:

$$A = \begin{bmatrix} p_1/p_1 & \cdots & p_1/p_j & \cdots & p_1/p_n \\ \cdots & 1 & \cdots & \cdots & \cdots \\ p_i/p_1 & \cdots & 1 & \cdots & p_i/p_n \\ \cdots & \cdots & \cdots & 1 & \cdots \\ p_n/p_1 & \cdots & p_n/p_j & \cdots & p_n/p_n \end{bmatrix} \tag{3.17}$$

The values of a_{ij} are given by the equation:

$$a_{ij} = \frac{p_i}{p_j} \tag{3.18}$$

In the literature, several methods are proposed for calculation consistency. Some of these methods are summarized in table 1. The most widely used method is the consistency index method (Saaty, 1977). Therefore we explain consistency index method here.

The consistency index method relies on the eigenvalue method. First of all, $\lambda_{\max}$ which is the maximal eigenvalue is calculated.

Consistency index is calculated based on the formula:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{3.19}$$

Table 3.7 Examples of the consistency calculation methods

Author	Method
(Crawford & Williams, 1985)	Difference between priorities and comparisons
(Ji & Jiang, 2003)	Transitivity rule based approach
(Peláez & Lamata, 2003)	Matrix determinant based appraoch
(Saaty, 1977)	Eigenvalue based method (Consistency index method)
(Salo & Hämäläinen, 1997)	Transitivity rule based approach
(Stein & Mizzi, 2007)	Normalized column of comparison matrix

Also the consistency ratio is given by:

$$CR = \frac{CI}{RI} \tag{3.20}$$

where RI is the random index.

Table 3.8 Random index developed by Saaty (1977)

n	3	4	5	6	7	8	9	10
RI	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

In the original paper, the random index is calculated by the average consistency index of the 500 randomly created matrices. In consistency index method, consistency ratio is considered acceptable if it is less than 10%. This means that inconsistency is found to be less than 10% percent of the 500 random matrices. Consistency index approach is criticized by some authors with respect to fact that it does not have a statistical basis (Salo & Hämäläinen, 1997).

3.6 Supermatrix

The influences among elements of each node on the other nodes in the network are gathered in a supermatix (Saaty, 2001). The general form of a supermatrix is shown in Figure 3.5. In figure 3.5, C_n denotes the nth cluster, e_{nm} denotes nth element in the mth cluster.

$$W = \begin{array}{cc} & \begin{array}{ccccc} C_1 & & C_k & & C_n \\ e_{11} \cdots e_{1m_1} \cdots & & e_{j1} \cdots e_{jm_j} \cdots & & e_{n1} \cdots e_{nm_n} \end{array} \\ \begin{array}{c} C_1 \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1m_1} \end{array} \\ \vdots \\ C_i \begin{array}{c} e_{i1} \\ e_{i2} \\ \vdots \\ e_{im_i} \end{array} \\ \vdots \\ C_n \begin{array}{c} e_{n1} \\ e_{n2} \\ \vdots \\ e_{nm_n} \end{array} \end{array} & \begin{bmatrix} W_{11} & \cdots & W_{1j} & \cdots & W_{1n} \\ \vdots & & \vdots & & \vdots \\ W_{i1} & \cdots & W_{ij} & \cdots & W_{in} \\ \vdots & & \vdots & & \vdots \\ W_{n1} & \cdots & W_{nj} & \cdots & W_{nn} \end{bmatrix} \end{array}$$

Figure 3.5 General form of a supermatrix

The goals, criteria and alternatives are place in the rows and columns of the supermatrix. The order of elements in the supermatrix is irrelevant. If there is no dependency between nodes, zero is entered. Any AHP model can be solved via supermatrix formation and results do not change. However, calculation with AHP is easier than calculations with the supermatix. AHP utilizes a weighted sum aggregation whereas ANP involves calculating square of the supermatrix many times. For the sake computational simplicity, AHP is recommended if there is no dependency. Positions of alternative and criteria with the fundamental regions are shown in Figure 3.6.

- Quadrant 1 stands for the influence of each criterion on other criteria. Eigenvector of influence on each criterion is placed in the first quadrant.
- Quadrant 2 stands for the influence of alternatives on criteria. Weight of the criteria with regard to each alternative is placed in the second quadrant.

Clusters and Nodes of the ANP Model		**Criteria**		**Alternatives**	
		C_1	C_2	A_1	A_2
Criteria	C_1	**Quadrant 1**		**Quadrant 2**	
	C_2				
Alternatives	A_1	**Quadrant 3**		**Quadrant 4**	
	A_2				

Figure 3.6 Fundamental regions in the supermatrix of ANP

- Quadrant 3 stands for the influence of each criterion on the alternatives. Local priority of alternative A_i with regard to criteria C_i is placed in the third quadrant.

- Quadrant 4 stands for the infuence of each alternative on other alternatives. Eigenvector of influence on each alternative is placed in the fourth quadrant.

The columns of the supermatrix must be equal to 1, which is the stochastic matrix necessary to use Markov Chain property. The matrix is raised to large powers to grasp the transmission of influence along all possible paths. For example, when we take the square of the matrix, direct influences are reflected. When taken cubic of the matrix, indirect influence of the second elements is considered and so on.

3.7 Final Priorities

Once the supermatrix is constructed, it is transformed into the weighted supermatrix by satisfying that all the columns are sum to unity. Next, the weighted supermatrix is raised to limiting powers to obtain global priorities using the equation:

$$\lim_{k \to \infty} \boldsymbol{W}^k \tag{3.21}$$

The black box of the equation x comes from the Markov Chain property, in which the system is subjected to transitions from one state to another without a memory mechanism. In other words, next state of the system is only influenced by the current state of the system. Let us give a small example of the transition matrix of the Markov Chain.

Example 3.4 (Kemeny, Snell, & Thompson, 1974): The weather of the land of Oz generally practices rainy or snowy conditions. The weather in the sequential days is never sunny. If the weather is currently sunny, it is very likely to be rainy or snowy next day. If it is rainy or snowy, it is also tend to maintain the same next day. Only half of the time the weather will be nice after a snowy or rainy day. The three states of the weather are rainy, nice and snowy is represented by a square matrix as equation 3.22. This matrix is called matrix of transition probabilities or the transition matrix. The Markov Chain provides interesting information, such that the evolution of the process after large number of steps.

$$\mathbf{P} = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} \end{array} \tag{3.22}$$

When we compute the matrix powers of the initial square matrix, we obtain the six days weather prediction. Finally the probabilities of the three types of weather after six days are 0.40 do rainy, 0.20 for nice and 0.40 for snowy weather conditions, regardless of where the chain started.

$$\mathbf{P}^1 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .500 & .250 & .250 \\ .500 & .000 & .500 \\ .250 & .250 & .500 \end{pmatrix} \end{array} \qquad \mathbf{P}^2 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .438 & .188 & .375 \\ .375 & .250 & .375 \\ .375 & .188 & .438 \end{pmatrix} \end{array}$$

$$\mathbf{P}^3 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .406 & .203 & .391 \\ .406 & .188 & .406 \\ .391 & .203 & .406 \end{pmatrix} \end{array} \qquad \mathbf{P}^4 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .402 & .199 & .398 \\ .398 & .203 & .398 \\ .398 & .199 & .402 \end{pmatrix} \end{array}$$

$$\mathbf{P}^5 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .400 & .200 & .399 \\ .400 & .199 & .400 \\ .399 & .200 & .400 \end{pmatrix} \end{array} \qquad \mathbf{P}^6 = \begin{array}{c} \\ Rain \\ Nice \\ Snow \end{array} \begin{array}{c} \begin{array}{ccc} Rain & Nice & Snow \end{array} \\ \begin{pmatrix} .400 & .200 & .400 \\ .400 & .200 & .400 \\ .400 & .200 & .400 \end{pmatrix} \end{array}$$

Figure 3.7 Powers of the Markov transition matrix

Therefore, we underlined the theoretical basis of the raising supermatrix to large powers for obtaining final states. However, in some situations, limiting supermatrix may not be the only one, which means that the supermatrix has the cyclicity effect.

Example 3.5 (Gürbüz, 2010) : Let a supermatrix has the form :

$$W = \begin{bmatrix} 0 & W_{12} & 0 \\ 0 & 0 & W_{23} \\ W_{31} & 0 & 0 \end{bmatrix} \tag{3.23}$$

When second and third powers of the supermatrix are calculated, following results are obtained.

$$W^2 = \begin{bmatrix} 0 & 0 & W_{12}W_{23} \\ W_{23}W_{31} & 0 & 0 \\ 0 & W_{31}W_{12} & 0 \end{bmatrix} \tag{3.24}$$

$$W^3 = \begin{bmatrix} W_{12}W_{23}W_{31} & 0 & 0 \\ 0 & W_{23}W_{31}W_{12} & 0 \\ 0 & 0 & W_{31}W_{12}W_{23} \end{bmatrix} \tag{3.25}$$

Generalizing the situation, it is observed that for arbitrary large powers, following matrices are obtained:

$$W^{3k} = \begin{bmatrix} (W_{12}W_{23}W_{31})^k & 0 & 0 \\ 0 & (W_{23}W_{31}W_{12})^k & 0 \\ 0 & 0 & (W_{31}W_{12}W_{23})^k \end{bmatrix} \tag{3.26}$$

$$W^{3k+1} = \begin{bmatrix} 0 & (W_{12}W_{23}W_{31})^k W_{12} & 0 \\ 0 & 0 & (W_{23}W_{31}W_{12})^k W_{23} \\ (W_{31}W_{12}W_{23})^k W_{31} & 0 & 0 \end{bmatrix} \tag{3.27}$$

$$W^{3k+2} = \begin{bmatrix} 0 & 0 & (\mathrm{W}_{12}\mathrm{W}_{23}\mathrm{W}_{31})^k\, \mathrm{W}_{12}\mathrm{W}_{23} \\ (\mathrm{W}_{23}\mathrm{W}_{31}\mathrm{W}_{12})^k\, \mathrm{W}_{23}\mathrm{W}_{31} & 0 & 0 \\ 0 & (\mathrm{W}_{31}\mathrm{W}_{12}\mathrm{W}_{23})^k\, \mathrm{W}_{31}\mathrm{W}_{12} & 0 \end{bmatrix} \tag{3.28}$$

Consequently, there are three cyclic forms that the supermatrix passes through and a single limiting supermatrix cannot be obtained.

If there are more than one limiting supermatrices, Cesaro sum is used to calculate the final priorities. The Cesaro sum is formulated as follows (Tzeng & Huang, 2011):

$$\lim_{k \to \infty} \left(\frac{1}{N} \right) \sum_{r=1}^{N} W_r^k \tag{3.29}$$

where $\boldsymbol{W}_r$ denotes the rth limiting priority. Casero sum is used to calculate the average influences of the limiting supermatrix. An example of Cesaro sum can be depicted in the example.

Example 3.6: Suppose that we have the structure of clusters as shown in Figure 3.8. The structure is known to produce different limiting supermatrixes depending on the power coefficient being either even or odd.

Assume that we conducted pairwise comparisons and acquired the unweighted supermatrix as seen in figure 3.9.

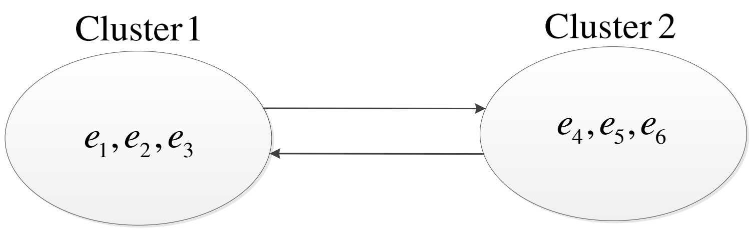


Figure 3.8 Network structure of example 3.6

$$
\begin{array}{c|ccc|ccc|}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\
\hline
e_1 & 0 & 0 & 0 & 0.250 & 0.243 & 0.216 \\
e_2 & 0 & 0 & 0 & 0.360 & 0.531 & 0.481 \\
e_3 & 0 & 0 & 0 & 0.390 & 0.226 & 0.303 \\
\hline
e_4 & 0.536 & 0.360 & 0.163 & 0 & 0 & 0 \\
e_5 & 0.102 & 0.240 & 0.412 & 0 & 0 & 0 \\
e_6 & 0.362 & 0.400 & 0.425 & 0 & 0 & 0 \\
\hline
\end{array}
$$

Figure 3.9 Unweighted supermatrix of example 3.6

Since all the columns sum to unity, it is a weighted supermatrix too. Finally, limiting power of weighted supermatrix is obtained to get the final weights. However, the limiting supermatrix when k is even obtained as:

	e_1	e_2	e_3	e_4	e_5	e_6
e_1	0.2346	0.2346	0.2346	0	0	0
e_2	0.4530	0.4530	0.4530	0	0	0
e_3	0.3124	0.3124	0.3124	0	0	0
e_4	0	0	0	0.3397	0.3397	0.3397
e_5	0	0	0	0.2614	0.2614	0.2614
e_6	0	0	0	0.3989	0.3989	0.3989

Figure 3.10 Weighted supermatrix of example 3.6 when k is even

Also when k is odd, the limiting supermatrix is as in figure 3.11.

	e_1	e_2	e_3	e_4	e_5	e_6
e_1	0	0	0	0.2346	0.2346	0.2346
e_2	0	0	0	0.4530	0.4530	0.4530
e_3	0	0	0	0.3124	0.3124	0.3124
e_4	0.3397	0.3397	0.3397	0	0	0
e_5	0.2614	0.2614	0.2614	0	0	0
e_6	0.3989	0.3989	0.3989	0	0	0

Figure 3.11 Weighted supermatrix of example 3.6 when k is odd

As seen from the limit supermatrix, the supermatrix has the cyclicity effect. In this case, the Cesaro sum is calculated by adding cycle matrices and dividing them by two (since we have two cycle matrices). Consequently, the limit supermatrix with Cesaro sum is found to be:

	e_1	e_2	e_3	e_4	e_5	e_6
e_1	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173
e_2	0.2265	0.2265	0.2265	0.2265	0.2265	0.2265
e_3	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562
e_4	0.1699	0.1699	0.1699	0.1699	0.1699	0.1699
e_5	0.1307	0.1307	0.1307	0.1307	0.1307	0.1307
e_6	0.1994	0.1994	0.1994	0.1994	0.1994	0.1994

Figure 3.12 Limit supermatrix of example 3.6 using Cesaro sum

3.8 Interpretation of ANP

Hierarchic and network systems are basic frameworks of the AHP/ANP for modeling unstructured problems. Network or a hierarchy is a useful tool to discern relationships among elements of the problem. What a hierarchy or network really represents will be discussed here.

Saaty (1987) distinguished two types of dependency as seen in Figure 3.13. Hierarchy or a network represents the type of functional dependency where a component of a system depends on the other component by directed arc. Besides, contextual dependency or semantic dependency is used interchangeably with functional dependency by Saaty.

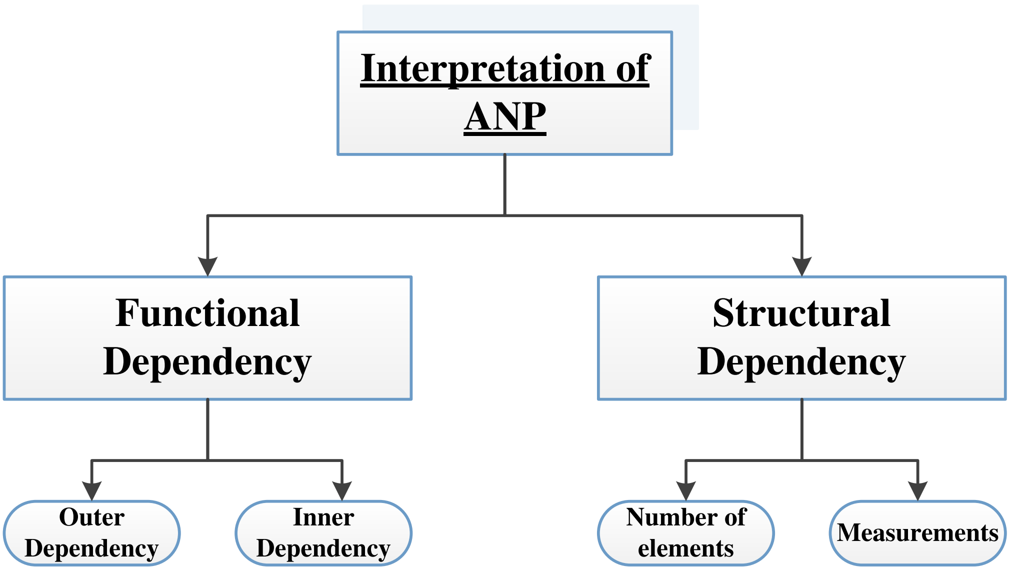


Figure 3.13 Dependency types in ANP

Futhermore, Saaty (1987) revealed his opinion regarding the functional dependency as: “A nonlinear network can be used to identify relationships among components using one’s own thoughts, relatively free of rules”. Therefore the dependency is highly contextual and its semantics rely on how decision maker interprets the problem, for that reason it is also deemed as contextual or semantic dependency.

On the other hand, structural dependency is the other term used in the context of AHP/AHP. Structural dependency is related to ranking of the alternatives, where the ranking influenced by the structural factors. We will discuss structural dependency later on in this chapter.

3.8.1 Functional Dependency

ANP is extended or generalized version of AHP (Saaty, 1980) with dependence and feedback structures. In other words, ANP overcomes the restriction of AHP in the sense that criteria do not have to form a linear hierarchical structure.

According to Axiom 3 of AHP (Saaty, 1986), Saaty defines three conditions that the hierarchy must conform (Wedley, Schoner, & Choo, 1993):

- The lower level is dependent upon every item of the immediate upper level
- Within each set of a level the items must be independent of each other
- The upper level does not depend upon one or more items of the lower level.

These specificities are known as principle of hierarchic composition of AHP. A hierarchy is a simple structure used to represent the simplest type of functional (contextual or semantic) dependence of one level or component of a system on another in a sequential manner (Saaty, 1987). However, many real life problems do not correspond with above mentioned requirements. Most of the time, decision problems involve interactions of various factors, with high-level factors occasionally depending on low-level factors (Lee & Kim, 2000;Saaty & Takizawa, 1986). A directional arc in the structure, for example, $A \rightarrow B$, indicates that B depends on A, functionally, contextually or semantically. Although giving the general meaning of the notion "dependency" in ANP is a very difficult task, we endeavor this way by raising two questions here.

- How does ANP consider interactions among criteria or alternatives?
- What does distinct ANP from other methods, specifically from those of causal models by means of handling interactions?

These questions are important because, in the following chapters, we present other methods to model criteria interactions in decision making, hence understanding the underlying logic and the point of view of the ANP is fundamental in this stage.

In ANP, decision making structure is treated as a network, constituting of source nodes, intermediate nodes and sink nodes. The relationships in a network represented by arcs, and the direction of arcs signify dependence (Saaty, 1996). Interactions are designated as outer and inner dependencies in ANP. Outer dependency is termed for interdependency between two nodes and represented by two-way arrow, whereas inner dependency is used for dependencies among elements in a node and depicted by a looped arc (Sarkis, 2003).

In this section, we give several examples in order to better explain the interpretation and the semantics of interaction concepts in ANP. Actually interpretation of the interaction phenomenon is not a trivial task, since interaction highly depends on the context of the study under investigation, Saaty (1987) also used the phase contextual dependency to describe the notion of interaction, along with the other terms in different papers. Keeping this fact in mind, examples used in this section should be interpreted in its own context, as the ANP is able to model numerous situations and the interaction phenomenon gains different meaning for different cases.

Using the terminology of the ANP, two types of dependency, namely, inner and outer dependency can be modeled in order to overcome the restriction of merely hierarchical problem representation of AHP. We first explain the inner dependency concept with examples; inner dependency among criteria cluster and among alternative cluster. Subsequently we give the meaning of the outer dependency with an example.

3.8.1.1 Inner dependency in the criteria cluster

Inner dependency is the dependency of the elements in the same cluster. Let us exemplify the inner dependency through a student evaluation problem. This problem is very popular example of the redundancy situation among criteria and widely used to describe the Choquet integral method (Grabisch, 1995).

Example 3.7 (Ishizaka & Nemery, 2013): A school has intended to enhance the educational level of the students. It is planned to reward the highest-achieving student in order to accomplish the strategic objectives. A plain methodology is requested to rank the students by the school administration. For the illustration purposes, Shakespeare and Newton are evaluated in terms of three classes, mathematics, physics and language. Newton is very successful in scientific topics, whereas Shakespeare excels in language courses.

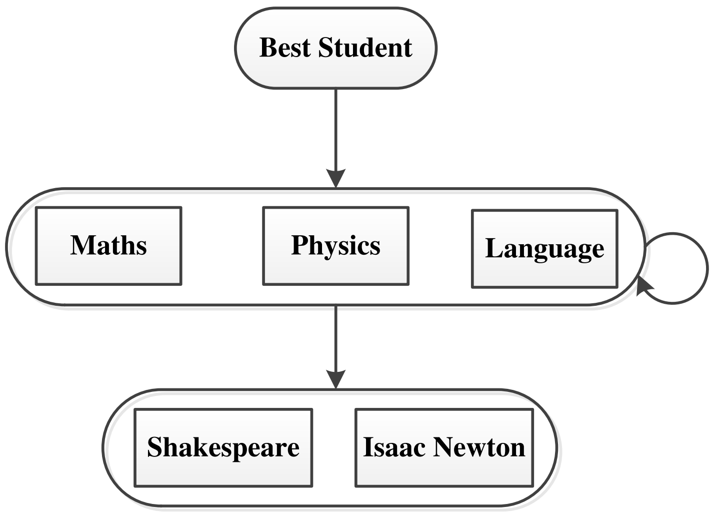


Figure 3.14 Inner dependency among criteria in example 3.7 (Ishizaka & Nemery, 2013)

It is known that there exists a correlation between mathematics and physics courses. Students successful at mathematics are usually successful at physics and vice versa. Hence inner dependency among criteria should be taken into account as seen in Figure 3.14. If this problem is modeled via traditional AHP method, the evaluation is said to be biased toward the scientifically oriented students; which, in our case, results in the fact that Newton would be rewarded. However, dependencies among criteria might change the overall priorities.

Dependencies among criteria are taken into consideration while filling the pairwise comparison matrices. Questions answered to get the pairwise comparison matrix as follows:

- The goal is to select the best student and you are evaluating the criteria against physics, which other criterion, language or mathematics would be more important and by how much? Because we know that physics and mathematics are correlated, higher importance (9 times more importance) is assigned to language criterion.
- The goal is to select the best student and you are evaluating the criteria against language, which other criterion, physics or mathematics would be more important and by how much? Because there is neither a correlation between language and mathematics nor between language and physics, equal weights are assigned to both criteria.
- The goal is to select the best student and you are evaluating the criteria against mathematics, which other criterion, language or physics would be more important and by how much? Because we know that there is a correlation between mathematics and physics, higher importance (9 times more importance) is assigned to language.

Assuming all criteria are 1 times more important than each other with respect to best student node (goal). Also Newton is 7 times more important than Shakespeare with respect to mathematics and physics, whereas Shakespeare is 7 times more important than Newton with respect to language. According to given information, the limiting priorities are calculated as:

Table 3.9 Result of the example 3.7 considering inner dependency

Node Name	Limiting priorities
Best student	0.0000
Newton	0.2598
Shakespeare	0.2401
Language	0.2368
Mathematics	0.1315
Physics	0.1315

As seen from the results in Table 3.9, priorities of alternatives are very close, and weights of scientific classes are lower than that of language course so as to hinder the double counting similar criteria. If the inner dependency among criteria is not taken into consideration, weights of mathematics and physics classes would be higher and the gap between limiting priorities of Newton and Shakespeare would be huge.

3.8.1.2 Inner dependency in the alternative cluster

Example 3.8: A student is to pick an application oriented, optional course for the new semester. The student prefers course A over course B. However, she learns that there is a third alternative course, which is taught by the same instructor of the course A. Then she worries about that fact that the instructor might not open the two courses simultaneously, she selects course B.

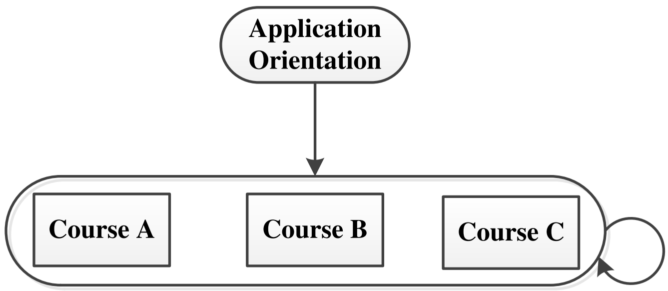


Figure 3.15 Example of inner dependency of example 3.8

This problem cannot be handled by AHP so the ANP is needed. In the problem structure, additional loop is included over the alternatives cluster in order to indicate that alternatives are not independent of each other. Apart from the pairwise comparison of alternatives with respect to application orientation criterion, alternatives are pairwise compared with respect to each other as well. These questions and answers would be as follows:

- The goal is to select your preferred optional course and you know that course A is in the evaluation cluster, which course, course B or course C, is more important and by how much? Because course A and course C are taught by the same lecturer, course B will be preferred (7 times more important).
- The goal is to select your preferred optional course and you know that course B is in the evaluation cluster, which course, course A or course C, is more important and by how much? Since the course B is neither correlated with course A nor with course C, they are equally preferred.
- The goal is to select your preferred optional course and you know that course C is in the evaluation cluster, which course, course A or course B, is more important and by how much? Because course C and course A are taught by the same lecturer, course B will be preferred (7 times more important).

If we solve the problem via Super decisions software, we obtain the final priorities as seen in Figure 3.16.

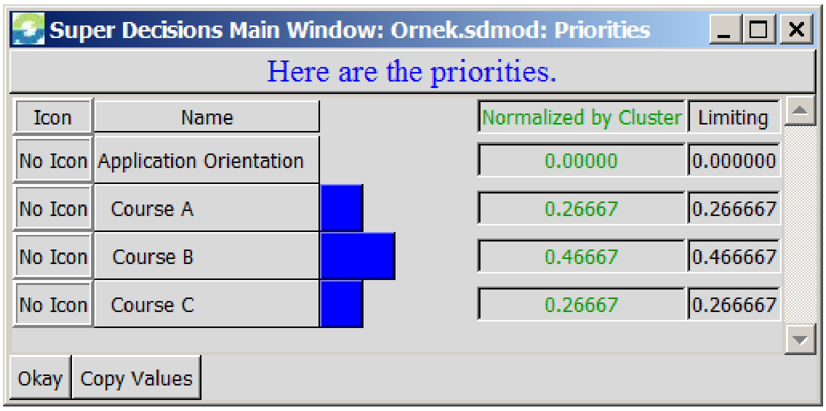


Figure 3.16 Result of the example 3.8 considering inner dependency

Note that in Example 3.8, individual alternative does not influence itself. For example, course A does not influence itself, so we did not question that if the course A is in the evaluation cluster, which course, course A or course B, is more important and by how much? However, in some situations, alternative might also be dependent on itself. In example 3.9, we illustrate this situation.

Example 3.9 (Saaty & Sodenkamp, 2010): Decision maker wants to know the most infuential industry for making electricity. These industries, which are electric, fuel and steel industries constituted in the alternative cluster. Inner dependency exists among these alternatives. Main inputs to make electricity are fuel and the steel. Steel is used to produce the turbines. Also electric industry supplies electricity to fuel and steel industries including itself.

Due to the material or energy flow among the electric, steel and fuel industries, a feedback loop is established in the cluster of alternatives.

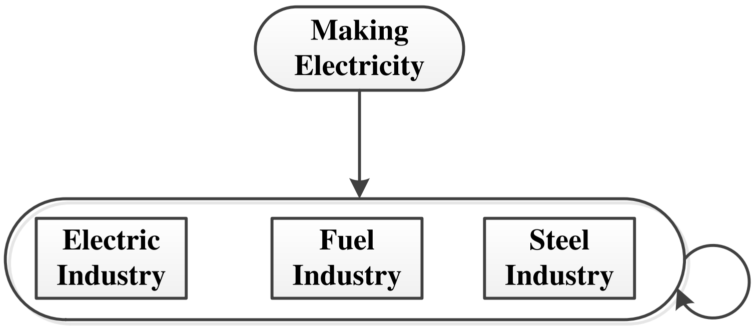


Figure 3.17 Inner dependency among different industries in example 3.9

As these industries depend on each other to maintain material or energy flow, we make pairwise comparisons to determine the individual intensities of each industry for the purpose of making electricity. We ask questions as follows:

- What does the electric industry depend on more to make electricity, itself or steel industry? Answer is steel industry.
- What does the electric industry depend on more to make electricity itself or the fuel industry? The answer is the fuel industry is much more important.
- What does the electric industry depend on more to make electricity, the steel or fuel industry? Fuel industry is more important. Indeed, the electric industry does not need its own electricity, but it needs fuel.

3.8.1.3 Outer dependency in the cluster

Outer dependency is the dependency between two clusters. In other words, an element is dependent on the elements of the other cluster. Feedback situation is also an outer dependency, for example, alternative cluster and criteria cluster can be dependent on each other simultaneously.

Example 3.10: Suppose that we evaluate alternative cars with respect to price, speed and comfort criteria as seen in Figure 3.18. As distinct from the conventional models, weight of criteria might change when evaluated with respect to alternatives.

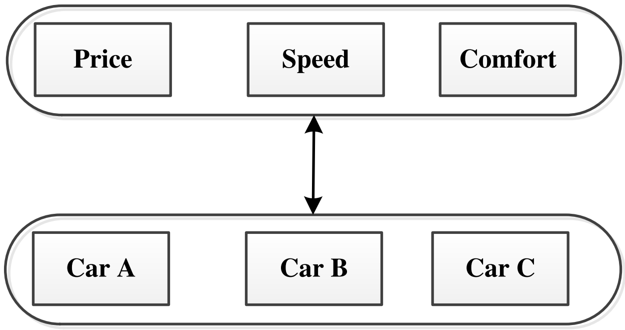


Figure 3.18 Example of outer dependency between alternative and criteria clusters

In AHP, weight of the criteria is changed with intuitive iterations. For example criteria are first pairwise compared with respect to criteria from the immediate upper level and the weights are obtained. However, when the alternatives are pairwise compared with respect to criteria, decision maker may conclude that the weight of the criteria should be changed. For example, decision maker can change the weight of the price criteria when she learns the price of a car. Different from the AHP, ANP does not necessarily require iterative judgments; decision maker can provide all sorts of dependencies at once.

We have given the pairwise comparison questions for comparing alternatives with respect to criteria. Here we only give the questions for pairwise comparison of criteria with respect to alternatives;

- If your goal is to buy a new car, and you know that car A is in the cluster of alternatives, which criterion, price or speed, is more important and by how much? The same question is repeated for the criteria pairs, price and comfort with speed and comfort criteria.
- If your goal is to buy a new car, and you know that car B is in the cluster of alternatives, which criterion, price or speed, is more important and by how much? The same question is repeated for the criteria pairs, price and comfort with speed and comfort criteria.
- If your goal is to buy a new car, and you know that car C is in the cluster of alternatives, which criterion, price or speed, is more important and by how much? The same question is repeated for the criteria pairs, price and comfort with speed and comfort criteria.

Hence, the weights of the criteria are determined by pairwise comparisons with respect to alternatives. Thinking of the imaginary cluster which is known to influence the decision criteria, pairwise comparisons would be conducted with respect to new cluster. Therefore, all the outer dependencies can be judged in one step. Once the weights of the criteria or alternatives are obtained, they are placed at the appropriate columns of the supermatrix.

As seen from the examples above, dependent relationships among criteria are quantified by pairwise comparisons in ANP. Actually, what ANP aims for is that it determines the priorities by taking the dependencies into account, where dependencies are defined in the predetermined network structure. In addition, means of chancing the importance of criteria is conducted by the influence mechanism; hence interpretation of influences found by pairwise comparisons is worth discussing. Saaty states that (Saaty & Sodenkamp, 2010):

"A network is concerned with all the influences from people and from nature that can affect an outcome. It is a model of continual chance because everything affects everything else and what we do now change the importance of the criteria that control the evolution of the outcome".

3.8.2 Structural Dependency

The notion of structural dependency is especially became an issue when the discussions on the rank reversal phenomenon of the AHP are raised. A well-known source of the rank reversal is that adding an alternative to the analysis, the rank of the alternatives is changed even though the performance values are not changed. Rank reversal is a pitfall of the AHP/ANP and gained many critiques. In recent years, robustness of a MCDM method is a hot research topic, which deals with how sensitive a specific decision making method to the changes in the parameters of the problem. From the AHP/ANP perspective, rank reversal is not a fallacy of the method, which is closely related to dependency issue. Actually main theme is that in most the situations, rank reversal occurs because the alternative added latter is not independent of the other alternatives. In other words, because there is dependency among alternatives, rank does not have to be preserved.

Although the structural dependency issue is seemed to be related with dependency among alternatives, the mechanism (i.e. pairwise comparisons) explaining the source of the occurrence of this situation is valid for criteria as well. Furthermore, criteria can also be dependent on the alternatives in ANP. Schmoldt et al. (2001) stated that "thus the measurement of the alternatives and their number which we call structural factors, always affect the importance of the criteria". Thus this situation is worth discussing.

The main aim of the decision analysis methods is generally rank the alternatives according to preferences of the decision maker. In AHP/ANP, pairwise comparisons are the main drivers of the priority elicitation. Using the pairwise comparison matrices, the final rankings of the alternatives depend on two important factors too.

They are:

- Quality of the alternatives, with which the alternative under consideration is compared,

- Number of alternatives, with which the alternative under consideration is compared (Saaty, 2006, 2009).

The quality of alternatives is based on the relative measurement or pairwise comparisons. Actually, the main source of structural dependence is that elements are compared with each other in the pairwise comparison matrix. Saaty (2009) stated that this approach itself makes elements dependent on each other in measurement. The other source of the structural dependence is number of alternatives. Especially, for the resource allocation problems, number of alternatives is an important factor that changes the final ranking, mostly due to the normalization.

We first explain the structural dependency due to the relative measurement of the elements. Then we elaborate how number of alternatives cause structural dependency.

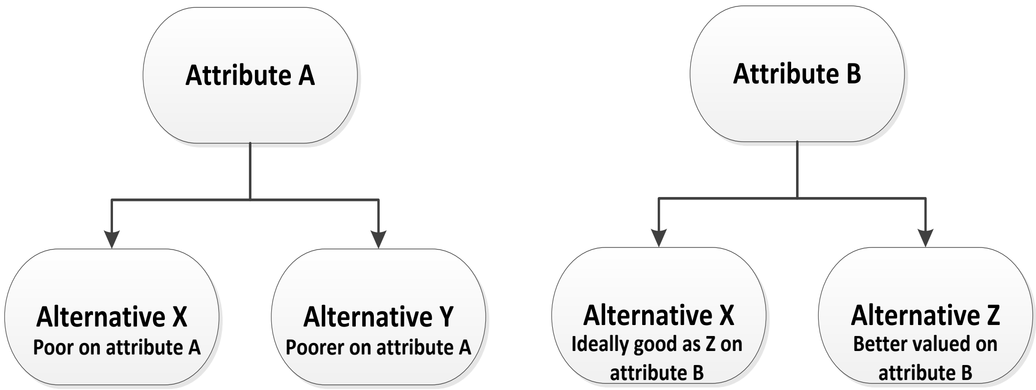
3.8.2.1 Structural dependence due to relative measurement

In AHP and ANP, two types of measurements are used to prioritize alternatives and criteria. They are relative measurement and absolute measurement. In relative measurement which is the mostly used method, the pair of alternatives or criteria is compared with respect to third alternative/criteria. On the other hand, absolute measurement a unique value is assigned to an alternative or criteria and performance of the other items are not taken into account. Absolute measurement provides unconditional independent situations that we will mention in rating method section.

In relative measurement, values or priorities derived through successive comparisons and each time an element is compared with the other. Depending on the element to which the other element is compared, different values are obtained. Naturally, obtained values from the pairwise comparisons are conditional (Saaty, 2006).

Similarly, when comparing alternatives in the pairwise comparisons, the derived values are dependent upon the quality of alternative with which the other alternative is compared. The quality of alternatives can be exemplified in a small example based on the ideas mentioned by Saaty (2006).

Example 3.11: Suppose that we are evaluating alternatives with respect to an attribute in traditional AHP/ANP framework. Suppose again we conduct pairwise comparison between alternatives with respect to attribute A. The alternative X is poor on attribute A. It is also known that the alternative Y is even poorer on attribute A than alternative X. Even though, two alternatives are poor with respect to alternative A, the result of the pairwise comparison matrix yields high priority for X and low priority (at least lower than priority of A) for Y.



Pairwise comparison ***w.r.t Attribute A***

$$\begin{array}{cc} & \begin{array}{cc} \boldsymbol{X} & \boldsymbol{Y} \end{array} \\ \begin{array}{c} \boldsymbol{X} \\ \boldsymbol{Y} \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array} = \begin{bmatrix} \text{High Priority} \\ \text{Low Priority} \end{bmatrix}$$

Pairwise comparison ***w.r.t Attribute B***

$$\begin{array}{cc} & \begin{array}{cc} \boldsymbol{X} & \boldsymbol{Z} \end{array} \\ \begin{array}{c} \boldsymbol{X} \\ \boldsymbol{Z} \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array} = \begin{bmatrix} \text{Low Priority} \\ \text{High Priority} \end{bmatrix}$$

Figure 3.19 Pairwise comparisons of example 3.11

Similarly, suppose that alternative X and alternative Z are compared with respect to attribute B. Attribute Z is known to have a very high performance on attribute B. Also alternative X is ideally good as alternative Z with respect to attribute B. Consequently, the pairwise comparison matrix results in a higher priority of alternative Z and lower priority of alternative X.

In the first part of the example, alternative X has a poor score yet obtained a high priority due to the fact that it is compared with an alternative with a poorer performance. On the other hand, in the second part of the example, alternative obtains a low priority because it is pairwise compared with a high performance alternative.

Therefore, the priorities of the alternatives depend on the quality of other alternatives with which it is compared (Saaty, 2006). Here quality (or measurement) means the performance of the alternative with respect to pertinent attribute.

3.8.2.2 Structural dependence due to number of alternatives

Another important structural factor is the number of alternatives. Number of alternatives has significant effect on the final priorities. The effect of the number of alternatives can be illustrated via the example originally proposed by (Corbin & Marley, 1974).

Example 3.12 (Corbin & Marley, 1974): A lady is shopping for hats in the mall. There are two alternatives denoted by A and B shown in Figure x. The lady's preference is as $A \succ B$.

Then, she realizes that there are many hats resembling hat A in the mall. The lady does not want to wear the same hat like the others so she changes her preference as $B \succ A$.

Actually, change of preference because of the knowledge that there exist many of the same alternative assumes there is dependence among alternatives (Saaty, 2006). To prevent this situation, decision maker can evaluate each alternative one by one, by not taking into account the others.

Then, she realizes that there are many hats resembling hat A in the mall. The lady does not want to wear the same hat like the others so she changes her preference as $B \succ A$.

Actually, change of preference because of the knowledge that there exist many of the same alternative assumes there is dependence among alternatives (Saaty, 2006). To prevent this situation, decision maker can evaluate each alternative one by one, by not taking into account the others. However, the decision whether number of alternatives is important or not is up to decision maker. For example, instead of selection of hats, if one is to select a computer, then the number of the same computers in the market would not affect the final ranking.

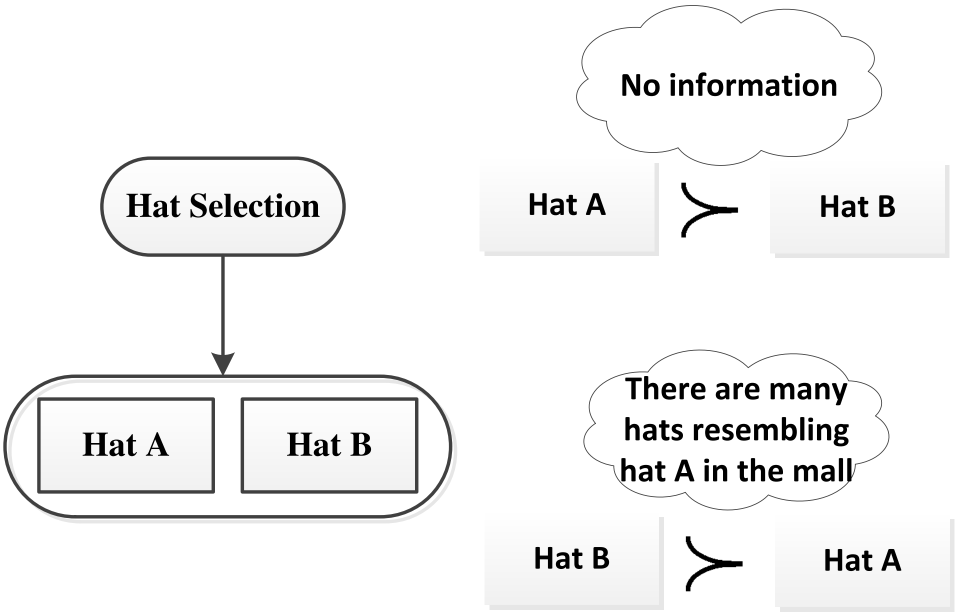


Figure 3.20 Change of preference in example 3.12

If the decision maker determines that the alternatives are totally independent, then rating method is suggested to use. The rating method is the well-known method not to take structural dependence into account.

3.8.3 Rating method and independence

Rating the alternatives requires the alternatives to be independent of each other. It means that the presence of an alternative has no influence on how the decision maker rates the other alternatives. This way of ranking also called absolute measurement or rating, which is a similar process with measurement with a physical device (Saaty, 2006).

Using rating method to rank alternatives, one needs to determine the intensity levels such as intensity levels of excellent, above average. These intensity levels are pairwise compared and the priorities are found after normalization, where the priorities are divided by the largest value among them. By this way, for example, the intensity of the excellent level becomes 1.00, other intensities becomes less than this value. Therefore, the alternatives are rated with respect to an ideal one.

Example 3.13 (Saaty, 2006): Suppose that we are choose the best city to live with respect to cultural, family, housing, jobs and transportation attributes. Priorities for the intensity levels of the cultural criterion can be found as in table 310.

Table 3.10 Priorities of the cultural criterion categories

	Extreme	Great	Significant	Moderate	Bad	**Derived Priorities**	**Idealized Priorities**
Extreme	1	5	6	8	9	0.569	**1.00**
Great	1/5	1	4	5	7	0.234	0.411
Significant	1/6	1/4	1	3	5	0.107	0.188
Moderate	1/8	1/5	1/3	1	4	0.060	0.106
Bad	1/9	1/7	1/5	1/4	1	0.030	0.052

The same operation is repeated for the other criterion categories. The resultant priorities are obtained as seen in Figure 3.21. The total score is calculated with the weighted sum.

Note that the pairwise comparisons are conducted for the attribute levels with respect to each attribute. Besides, the alternatives are rated according to attribute levels found by pairwise comparison. Hence the alternatives are not pairwise compared; their performances are rated separately, which does not assume dependence.

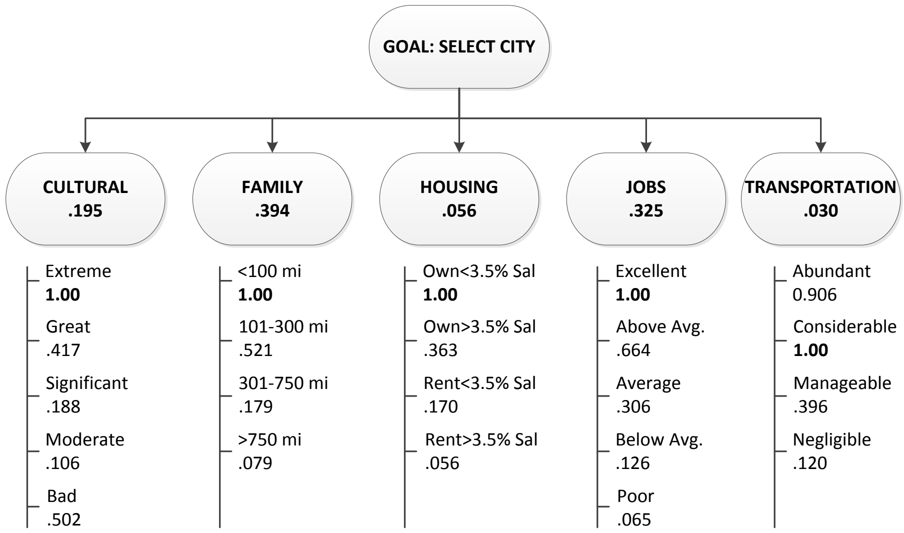


Figure 3.21 Choosing a city to live in ratings of example 3.13 (Saaty, 2006)

3.8.4 Relationships Between Causal Dependency and ANP

We have so far been discussed different interpretations of the notion of dependeny. As we have shown, these interpretations are fairly context-dependent and it is hard to give concrete definitions. However, AHP/ANP has a unique way of eliciting priorities by means of pairwise comparison matrices, which are slighly different from the models utilizing causal relationships. In this section, we discuss the the relationships causal inference and the ANP.

The notion of influence is essential in ANP (Saaty, 1999b). The main objective of the ANP's analytical procedure is to determine the synthesized influence of all the elements collectively (Saaty, 2005). Regardless of the mathematical properties on how the total influence in the network is calculated, we try to clarify the semantics of the influence concept in ANP. Despite the fact that the meaning of the influence or dependence has not explained in most of the applications in the literature, we agree with the interpretation that *influence on importance* is the key idea for the use of ANP/AHP (Khademi et al., 2012). Let us exemplify that;

Example 3.14 (Khademi et al., 2012): It is the fact that parents have substantial impact on the education and future career of their children. Assume that a group of social scientist inquiries parents' role in their children's education and for that purpose AHP model is constructed. We know that the AHP is the special case of the ANP, so hierarchical structure is enough for us to question the semantics of the influence, which flows from upper level through the lower levels.

Suppose that the role of the father and mother is compared with respect to only one criterion, which is the education of the children. The aim of the study is to prioritize and assign importance values to parents in the sense that how much importance they have on the education of the children. The AHP model is shown in Figure 3.22a.

In figure 3.22b, we compared causal influence with the influence concept of the original AHP structure.

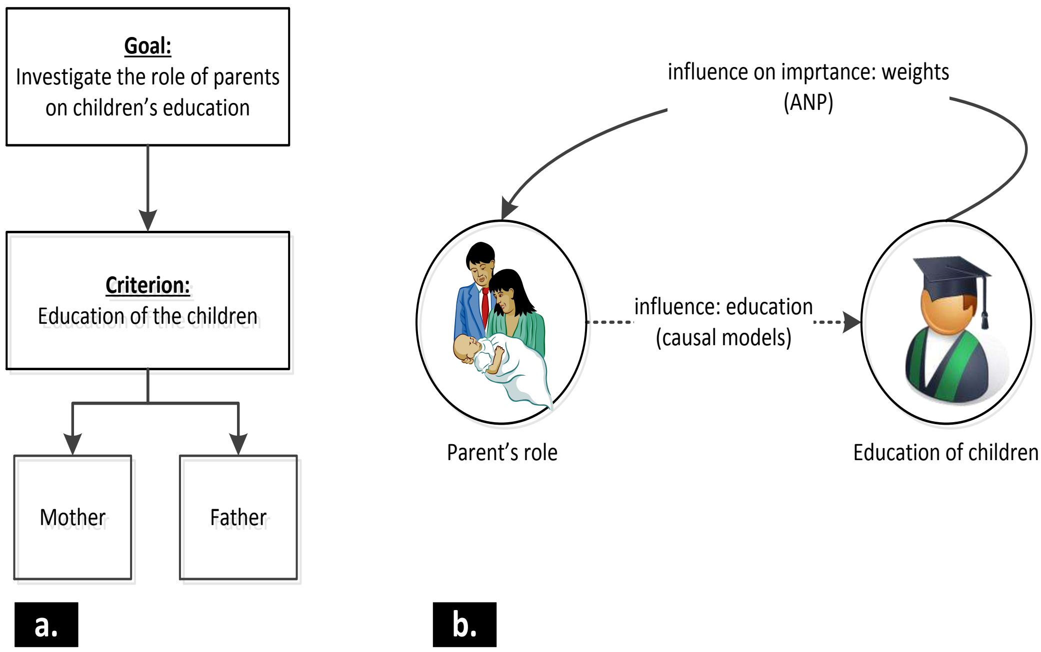


Figure 3.22 Influence on importance versus causal influence (Adapted from Khademi et al., 2012)

In this example, influence concept in the AHP method is considered as influence on importance. People intuitively think of the influence that parents directly affect their children's education. This approach coincides with causal thinking and is inherent part of causal models, which will be elaborated in following chapter. Contrary to the direct educational influence shown by dashed arrow in Figure 3.22b, considered AHP model receives influence on the importance of parents with respect to education of children. In other words, education of children has influence on the weights (importance) of the parents, where we expect that parents should have influence on childrens' education. Actually, influence on importance is moved through the direction of the arrow in structure, i.e. from upper levels to lever levels in AHP case (Khademi et al., 2012).

Consequently, in most of the studies related to ANP, models take the dependency into account on the basis of influence on importance concept, where criteria influence the importance of other criteria by pairwise comparisons. Since the way ANP structures the problem and the semantic of the influence concepts is not independent of the problem at hand, many other approaches are proposed in combination with ANP in order to structure the problem and obtain the degrees of influence among elements. One of the most important advancements in this direction is combining ANP and DEMATEL method, which will thoroughly be discussed in subsequent chapters.

3.9 Literature Analysis

In this section, we give some bibliographic information about the usage of ANP between 2003 and 2013. We have avoided conducting a review study, since there are already literature review studies regarding the theoretical and practical applications of ANP. Furthermore, we give very detailed literature review pertaining to applications of hybrid techniques with ANP in the subsequent chapters.

We have used SCOPUS online database to collect the information to depict the number of published work recently. We have found 634 articles published in international journals last 10 years. Comparing with other methodologies, ANP is far ahead of the other methods in terms of popularity and practical use.

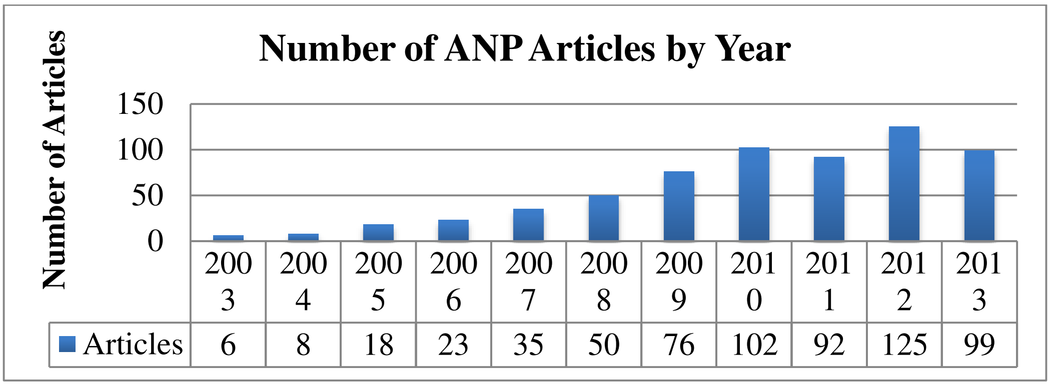


Figure 3.23 Number of journal articles applying ANP from 2003 to 2013

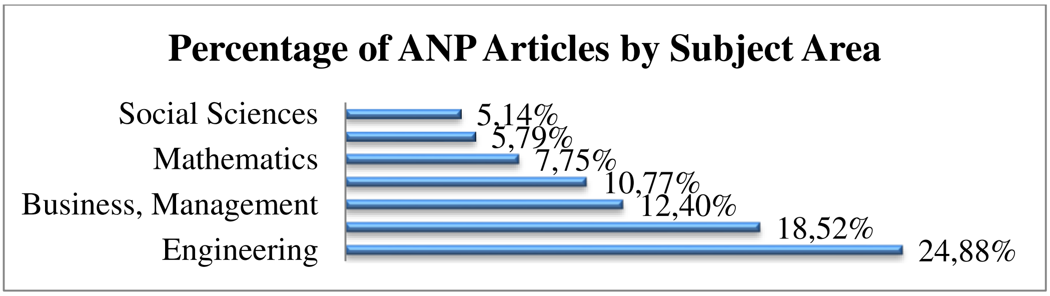


Figure 3.24 Percentage of journal articles applying ANP by subject

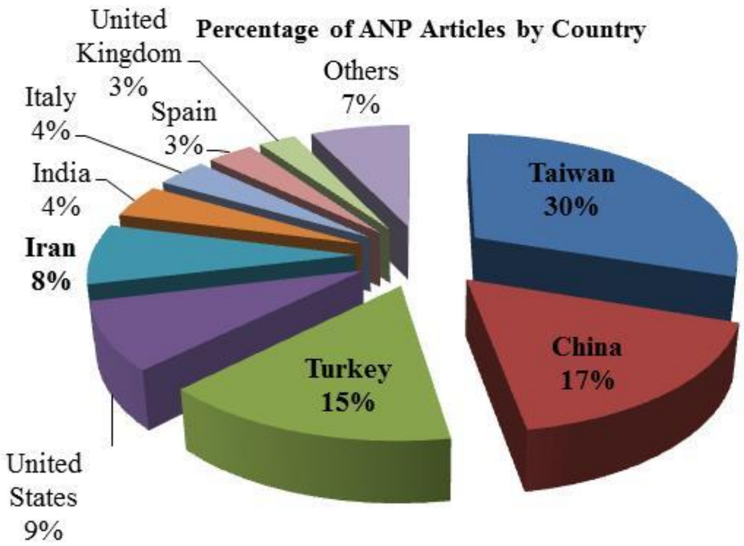


Figure 3.25 Percentage of ANP articles by country

There has been a great deal of papers applying ANP in the literature. Reviews concerning applications of ANP can be found in (Hülle, Kaspar, & Möller, 2013 ; Sipahi & Timor, 2010).

CHAPTER FOUR
CAUSAL MAPS

4.1 Introduction

From the historical point of view, organization science has given birth to the causal mapping approach (Narayanan, 2005). Researchers aimed at discerning the causal arguments of natural language used by managers in the organization. Axelrod (1976) was the first person used the term cognitive mapping to analyze the political elites' causal assertions.

The advent of cognitive mapping is attributed to Axelrod due to his book where he investigated the cognitive maps of political elites. He opened up a new area for the decision making with the idea that the causal beliefs and statements of a person were taken into consideration.

Axelrod, a political scientist involved in cognitive map elicitation, developed coding rules and obtained the cognitive maps of committee members of British Eastern Committee. After his study, several papers appeared in the literature under the name cognitive map (Narayanan, 2005). Actually, Axelrod's highly cited work Structure of Decision (Axelrod, 1976) is a result of the collecting and interpreting the cognitive map studies produced till his time.

According to Narayanan (2005), Axelrod's book supplied with some key methodological approaches as seen in Figure 4.1.

First of all, Axelrod provided with a nutritious definition of a cognitive map. Cognitive map is a way of depicting one's statements regarding a limited and specific domain such as a policy making problem. Cognitive mapping aims at grabbing the structure of the causal statements and it draws some conclusions based upon the structure.

Another important contribution of Axelrod was that he came up with a novel system that documents could be coded in a way that can be served as rules to capture the causal statements and generate causal maps from the document.

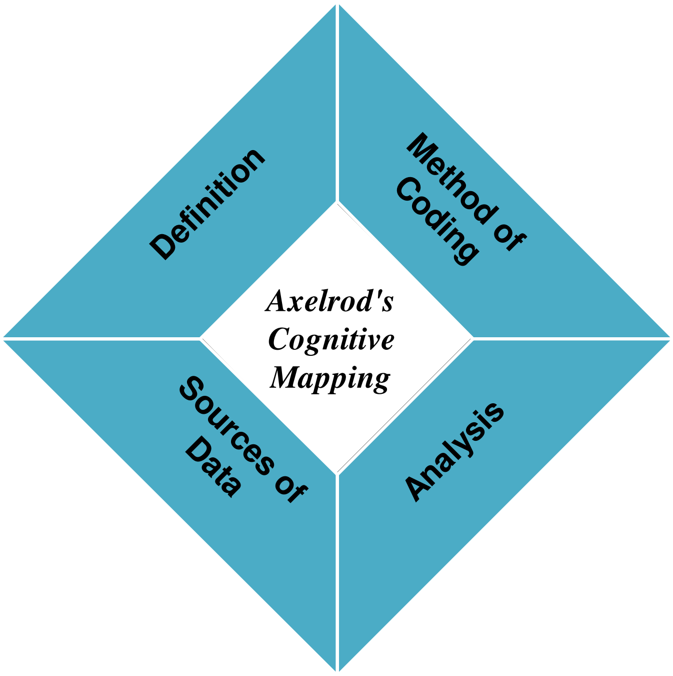


Figure 4.1 Axelrod's contributions to cognitive mapping

He also determined the sources of data that can be used to generate cognitive maps. They spanned from documents and questionnaires to interviews. Lastly, he mentioned the analysis of cognitive maps which are mostly known as the quantitative analysis as well as the methods like some statistical techniques and even simulation.

4.2 Causal Mapping Methodology

Causal mapping is a technique used to represent the cognition related to a specific domain. There are many tools that can be used to represent cognition such as repertory grid (Tan & Hunter, 2002), argument mapping (Huff, 1990), and Self-Q technique (Bougon, Weick, & Binkhorst, 1977). Causal mapping is the most widely known and applied method in the decision theory problems.

As there are many tools to model cognition such as repertory grid, argument mapping, cognitive maps can be seen as a sort of abstract or general term and does not evoke anything. Besides, cognitive maps and causal maps are used interchangeably in the literature. Actually, cognitive map is an umbrella term and causal map is a special type or subset of cognitive map. Since cognitive map strives to obtain the one's cognition about a specific situation, getting causal assertions and causal belief network related to specific domain is a part of the complex cognition of human mind. Causal maps refer to bunch of techniques to evaluate the structure and constituents of mental models (Fiol & Huff, 1992). Despite the minor difference between causal maps and cognitive maps, we use them interchangeably as in the majority of papers in the literature.

4.3 Types of Causal Maps

There exist many methods to be used in the context of mental model measuring. Mohammed, Klimoski, & Rentsch (2000) collected four methods as seen in Figure 4.2. They are interactively elicited causal maps, text based causal maps, multidimensional scaling, and pathfinder associative networks. We only discuss interactively elicited causal maps and text based causal maps since they involve alternative data collection techniques widely used in the literature.

4.3.1 Interactively Elicited Causal Maps

An interactively elicited causal map is a type of data collection technique that brings together the participants in order to cast the causal beliefs into cognitive structures related to a specific issue. It integrates the tacit knowledge of participant directly through interviews. This method has three important phases as follows (Armstrong, 2005) :

1. Sampling
2. Interview Protocol
3. Point of Redundency

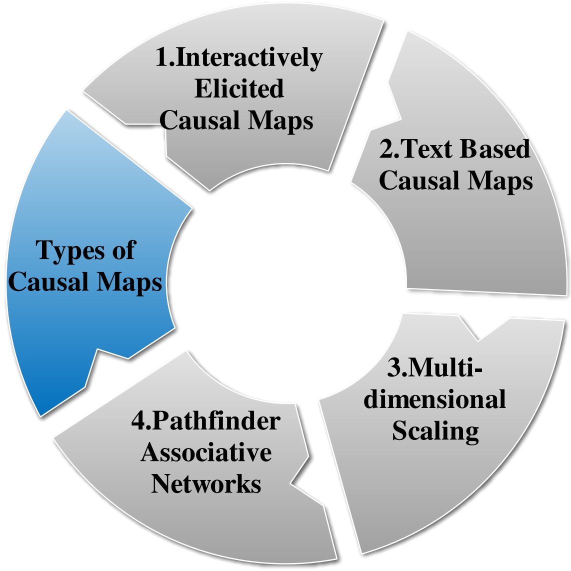


Figure 4.2 Methods for measuring mental models

Sampling phase is engaged in finding the appropriate expertise among uniformly distributed candidates. According to Axelrod (1976), sampling for expertise is started randomly, then each selected expert identifies the other expert in an interactive manner. This chain of identification proceeded by interviews, where each candidate is interviewed as soon as (s)he is selected.

Interview protocol is another important aspect of interactively elicited cognitive maps. Different data collection methods such as structured or semi-structured interviews can be used to obtain participants' thoughts regarding a specific domain. The interviews are then translated verbatim into a written form.

Moreover, there exists a stopping point for interview that additional information does not contribute to further concept elicitation. Actually this fact is highly correlated with sampling phase we discussed before. In causal map elicitation, each participants help identify unique concepts during the sequential process. In other words, each participant contributes to the total concepts captured. The point of redundancy can be considered as a point that determines the sufficiency of the sample.

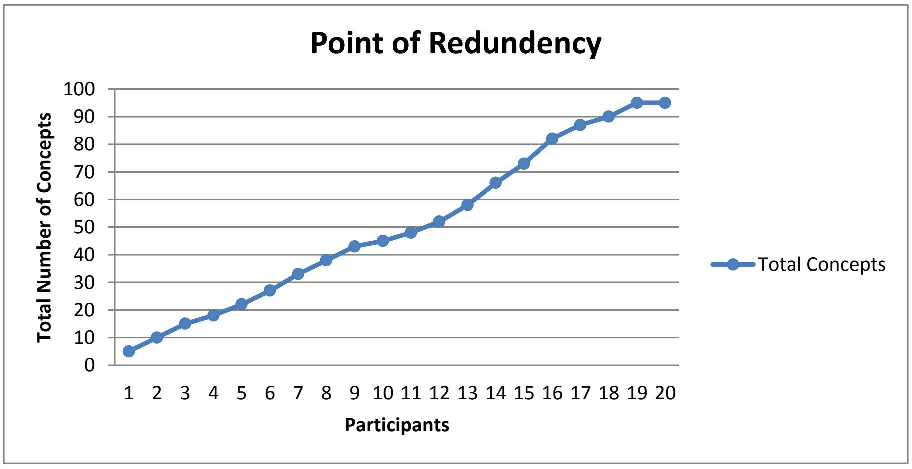


Figure 4.3 Point of redundancy is reached

In interactively elicited causal maps, interview transcript of the participant is analyzed and new concepts are distinguished. If additional concepts are not detected anymore, then it means that the point of redundancy is reached. In figure 4.3, an example of point of redundancy is shown. At the 18th participant 5 more new concepts are identified; however 19th and 20th participant could not provide new concepts. Hence the point of redundancy is reached at 18th point.

4.3.2 Text Based Causal Maps

Text based causal maps are another important type of cognitive maps we emphasize in this study. Text based causal maps differ from interactively elicited causal maps in terms of data sources. In text based causal maps, texts such as annual reports, written documentations, and legal decisions are used to establish cognitive maps. Relying on the documents and texts to capture causal beliefs of participants, it avoids biases of the interview (Axelrod, 1976).

One of major problem of text based causal maps is to determine the sample size of the documentation. Because of the abundance of data on hand, analyst might have difficulties to manage and interpret the information sources. According to Armstrong (2005), sampling strategies can be grouped into three categories as convenience, random and exhaustive sampling respectively. We summarize the key points of these methods analogous to (Armstrong, 2005).

In convenience sampling, there exist certain and available texts or statements to be analyzed by researcher. An example of convenience sampling would be that if the researchers seek for the impact of the new product, then he/she may refer to media releases regarding the new product. Moreover, the point of redundancy for which we mentioned earlier is only applicable to convenience sampling and carried out very similar manner with the steps mentioned in interactively elicited causal maps. Another sampling technique is random sampling, recommended to use when the data universe is very huge and cannot distinguished from each other easily. Additionally, due to the quite vast amount of data available, some decision rules are recommended to be constructed in order to limit and reduce the complexity. Such decision rules might be the time frame of the data source or number of data sources to be handled. The last sampling technique is exhaustive sampling. This sampling technique involves taking the entire sample universe into account.

4.4 Steps of Constructing Causal Maps

There are many causal mapping interpretations in the literature where the definitions of concepts may differ. We begin with giving the core concept definitions of causal mapping adopted from Armstrong (2005). Table 4.1 summarizes the basic terms and their definitions used in causal mapping studies.

According to Nelson et al. (2000), constructing causal maps require five steps.

Constructing causal maps involves;

- Identification of causal statements
- Constructing raw causal maps
- Developing a coding scheme
- Recasting raw causal maps into revealed causal maps

A detailed causal mapping process can be seen in Figure 4.4. We summarize the main steps of causal mapping construction in the subsequent paragraphs Besides we follow the example provided by (Nelson et al., 2000) to illustrate the steps.

Table 4.1 Causal mapping definitions (Armstrong, 2005)

Term	Definition
Causal Map	A network of causal links depicted in a matrix or diagram format.
Causal Statement	A phrase or sentence containing a cause/link/effect form.
Coding Scheme	A dictionary of terms and their definitions. It is a simplification of terms appeared in causal map.
Concept	A word or phrase that conveys the meaning of the participant's phase.
Construct	A word or phrase that conveys the meaning of the group of concepts.
Link	The relationship between two concepts.
Raw Causal Map	A causal map where the concepts are established by the raw data elicited from the participant.
Raw Causal Statement	A causal statement grabbed in the language of participant.
Revealed Causal Map	A causal map established via the causal assertions of participant to the extent that he/she prefers to reveal to the world.

4.4.1 Identification of Causal Statements

As we mentioned before, data sources might be interview transcript or text based documents like annual reports. According to Axelrod (1976), causal statements in the data sources are identified in the first step. Data source might involve causal statements such as "if-then", "so" or "because" in which these explicit causal statements are captured. On the other hand, some keywords might indicate the causal relationships as well. For instance "think", "know", "use" or "believe" can see as the implicit indicator of causal relationships (Armstrong, 2005).

Example of causal statements might be:

1. If a person has logical reasoning skills and experience in different environments, then I will consider him/her an expert.
2. Personal employee growth can lead to development of support personnel expertise.

In the first step, these kinds of statements are extracted from the interview transcript or documents available.

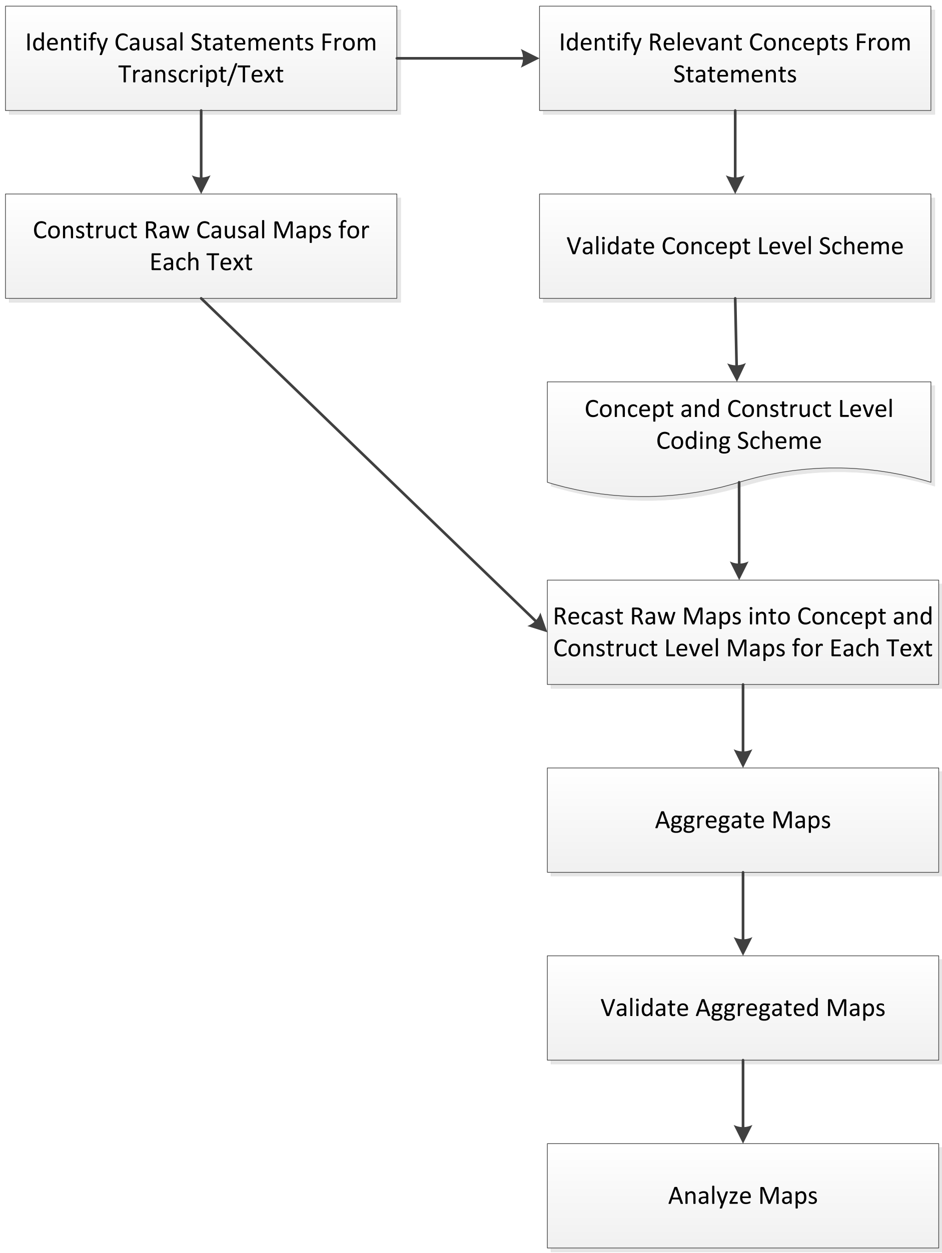


Figure 4.4 Detailed causal mapping process (Armstrong, 2005)

Axelrod (1976) provided some guidelines and tips in order to identify causal statements from the texts. Here we summarize the Axelrod's decision rules to capture causal statements as follows:

1. In some situations, cause effect relationships are not very clearly inferred from the sentence. So the analyst questions the text if the phase includes causal statements implicitly.
2. Participants' original expressions and language should be maintained.
3. Participants' causal statement should be noted in terms of number. If a causal relationship is mentioned more than once, then analyst should count and note number of times that causal relationships is stated.
4. Participant's viewpoint regarding assertions made by other participants should be paid close attention. If the participant surely agree with the assertion, then it should be coded as causal statement, otherwise, it should be disregarded.
5. Assertions made by participant should be established within a sentence. Linking whole paragraphs is not a pursued approach.

4.4.2 Constructing Raw Causal Maps

Identified causal statements are separated into cause and effect groups in second step of constructing causal maps. Divided cause and effect statements are utilized to form raw causal maps. Example of raw causal maps is given in table 4.2.

Table 4.2 Example of constructing raw causal map

Cause	Causal Connector	Effect
If a person has logical reasoning skills	If-then	Then I will consider him/her a support expert
If a person has diverse experience	If-then	Then I will consider him/her a support expert
Personal employee growth	Can lead to	Expertise of support personnel

In table 4.2, example of raw causal mapping is shown by separating explicitly mentioned cause and effect statements.

4.4.3 Developing a Coding Scheme

The coding scheme is aimed at reducing the cognitive load of developer and reader of causal maps (Armstrong, 2005). It transforms raw cause and effect statements into more generalized explicit ideas to enhance readability. Example of coding scheme is given in table 4.3.

Table 4.3 Example of a coding scheme

Raw Phase	Coded Concept
Person has logical reasoning skills	Cognitive abilities
Person has diverse experience	Diverse experience
Consider him/her a support expert expertise	Software operations support
Personal employee growth	Growth
Expertise of support personnel	Software operations support expertise

4.4.4 Recasting Raw Causal Maps into Revealed Causal Maps

In this step, coded concepts are inserted into suitable concept levels combined with directed arcs indicating the direction of link from cause to effect as in Figure 4.5.

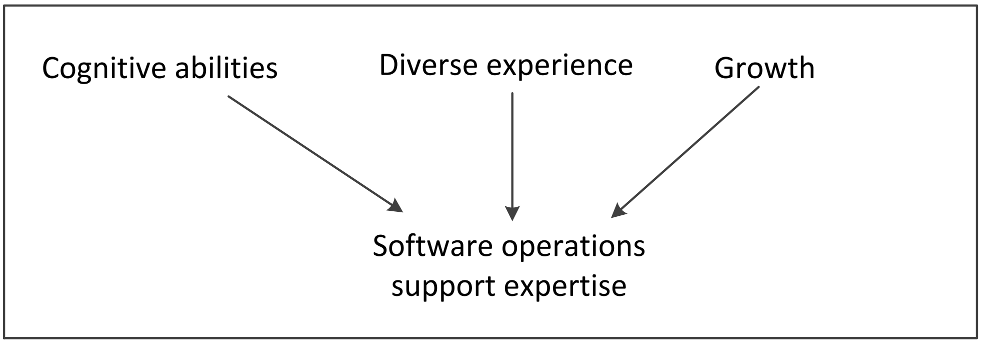


Figure 4.5 Example of concept level revealed causal map

4.4.5 Analysis of Causal Maps

The analysis of causal mapping reflects how the concepts and variables are organized in the map. Actually the analysis of causal maps lack advanced mechanisms, many of them are derived from the graph theory. Specifically, the theory comes from the social network analysis (Knoke & Kuklinski, 1982). Structural measures are related to number of concepts and analysis of linkages. We very briefly revise some the simplest structural measures:

- **Comprehensiveness**: It is assumed that when the number of concepts is increased in the map, then the cognition is increased too. The comprehensiveness is easily an overall number of concepts (Carley & Palmquist, 1992).
- **Density**: Density is a ratio of the connections among concepts and the number of all concepts in the map. (Carley & Palmquist, 1992).

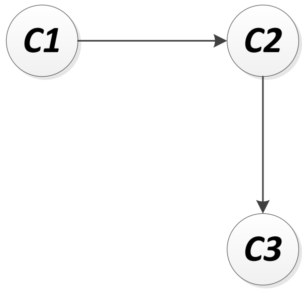


Figure 4.6 Sample of a density measure

Suppose that we have a causal map of an individual as in the Figure 4.6. The density measure is calculated as:

$$\text{Density}=\frac{\text{number of links}}{\text{number of concepts}} \tag{4.1}$$

$$Density = \frac{2}{3} = 0.66 \tag{4.2}$$

- **Centrality**: Centrality stands for the contribution of a variable into the cognitive map (Özesmi & Özesmi, 2004). It gives how connected the concepts with rest of the concepts and what the total strength of the relations.

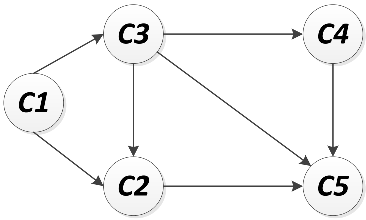


Figure 4.7 Sample of a centrality measure

Suppose that we analyze the cognitive map shown in Figure 4.7. The out degree and in-degree values are shown in table 4.4.

Table 4.4 In-degree and out-degree values of a cognitive map

	$C1$	$C2$	$C3$	$C4$	$C5$	Out-degree
$C1$	0	1	1	0	0	2
$C2$	0	0	0	0	1	1
$C3$	0	1	0	1	1	3
$C4$	0	0	0	0	1	1
$C5$	0	0	0	0	0	0
In-degree	0	2	1	1	3	

The centrality formula is given by (Armstrong, 2005) :

$$\text{Centrality}=\frac{\text{In-degree + Out-degree}}{\text{total number of linkages}} \tag{4.3}$$

Thus, for instance, centrality for $C3$ is given by:

$$Centrality\,of\ C3=\frac{1+2}{7}=\%43 \tag{4.4}$$

There are also many other measures used in analyzing cognitive maps. In subsequent sections, we will see the dynamic analysis of cognitive maps via Fuzzy Cognitive Maps.

4.5 Related Review

Suwignjo, Bititci, and Carrie (2000) developed a performance measurement system based on causal/cognitive map and AHP. Cognitive map was not only used to detect factors affecting performance but also used to capture the relationships between them. Then criteria in cognitive map transformed into the hierarchical tree diagram. Effects of each factor was characterized by its inherent effect, self-interaction effect, horizontal effects of other factors through interested factor and horizontal effects of interested factor through other factors within the same level. All these values were computed by AHP and then aggregated to reach the final weights. In a case study, total production cost per unit of a collaborator company was presented.

Montibeller et al. (2007) proposed a special type of causal map namely reasoning maps. The main aim of this method was to evaluate decision alternatives along chains of qualitative arguments. Reasoning maps were required two stage evaluations. First, the decision maker was constructed the means and ends chain which was consisted of attribute, consequence and value concepts. In the second stage, decision maker provided the strength of influence of each link within the user defined qualitative scale. The ultimate decision was made by calculating the partial and total effects. One of the obstacles of the method was that reasoning maps only performed static analysis and did not contain loops. A case study evaluating leadership capabilities of team members was provided. In another study by Montibeller and Belton (2009) discussed qualitative operators for reasoning maps such as maximum, minimum, mode, median, linear aggregation, weighted minimum with advantages and disadvantages.

Comes et al. (2011) developed a new framework for multi-criteria decision support which is called decision maps. Decision maps were based on the Directed Acyclic Graphs (DAG) facilitated by distributed scenario construction and assessment with MCDM techniques. Proposed method integrated scenario construction and evaluation of alternatives in a systematic way. Each decision map included a causal map and an attribute tree. Causal maps were used to process information at hand in a distributed approach. It makes possible to each expert, according to their local knowledge, use different uncertainty modeling techniques such as fuzzy, probabilistic or Bayesian, deterministic or limiting. On the other hand, attribute tree evaluated the causal map results with respect to decision maker’s preferences and goals. The proposed method was illustrated in the strategic emergency management example.

Ülengin et al. (2010) proposed a hybrid methodology combining cognitive maps, SEM, scenerio analysis and TOPSIS for analyzing transportation-environment relationships. Cognitive maps was used to study the relationhips between transportation system variables considering environment, society and energy perspectives. Quantification of causal relations was established by SEM. In the next step, scenario analysis was performed and different scenerios were generated. Finally, different scenerios were interpreted as consequence of different policies, hence the policy with the maximum performance was selected by TOPSIS method.

Ülengin et al. (2007) developed a transportation decision support system for transportation policy decisions in the case of Turkey. They used cognitive maps to analyze basic variables involved in transportation system. Causal relationships between variables were discovered by traditional econometric tehniques and ANN in conjunction with the expert judgements. Afterwards, causal model was transformed into the bayesian causal map. Finally, scenario analysis was performed based on the resulting map in order to analyze affect of socio economic and transport related variables onto future transportation demand.

CHAPTER FIVE
DEMATEL METHOD

5.1 Introduction

DEMATEL method was established at Battelle Memorial Institute of Geneva Research Center during the research of understanding and solving the intertwined real world problems such as population and hunger, environmental issues, and energy (Gabus & Fontela, 1973, 1976). DEMATEL method takes into account the subjective perceptions of individuals and captures the analyst's insight into the complex problem on hand. The aim of DEMATEL is to reveal direct/indirect causal relations (dependencies) among system variables (i.e. criteria). The final product of the analysis with DEMATEL method is a visual representation of the systems components in terms of their relationships and an individual map of the mind (Tzeng & Huang, 2011).

5.2 Steps of DEMATEL Method

Basic steps of the DEMATEL calculation is as follows (Lee et al., 2010):

Step 1: Definition of variables and establishment of measuring scale.

Brainstorming-like methods with the help of literature discussion are used to define variables influencing the complex system. Also a measuring scale is established in this step. Distinct measuring scales are used by different researchers in the literature. For example (Lin & Wu, 2008) used a measurement scale between 0 and 3 as no influence, low influence, high influence and very high influence.

Kim (2006) used a scale between 0-5 and (Huang, Shyu, & Tzeng, 2007) used a scale from no influence 0 to very high influence 10. Actually, the most commonly used scale is that of measurement values are between 0 and 5.

Table 5.1 Examples of measurement scales in DEMATEL

Reference	Measuremet Scale	Degree of Influence
(Lin & Wu, 2008)	3-point scale	No influence: 0 Low influence: 1 High influence:2 Very High influence:3
(Yang & Tzeng, 2011)	4-point scale	No influence: 0 Very low influence: 1 Low influence: 2 High influence:3 Very High influence:4
(Kim, 2006)	5-point scale	No influence: 0 Very low influence: 1 Low influence: 2 Normal influence: 3 High influence:4 Very High influence:5
(Huang et al., 2007)	10-point scale	No influence: 0 ⋮ Very high influence: 10

Step 2: Establish direct-relation matrix

Suppose that we have n system variables to be analyzed. According to expert opinions, a $n \times n$ matrix can be developed by comparing the effect of a variable on the other variable. It is assumed that the diagonal values of the pair-wise comparison matrix are all zero. Actually this property is hold in other causal modeling approaches such as fuzzy cognitive maps or Bayesian networks where the idea rooted from the philosophical standpoint implies that something cannot cause itself. We will not further discuss this point here though. Assuming the diagonal elements x_{ii} are zero, we have pair-wise comparison matrix, comprising of elements x_{ij} which represents the degree of x_i affecting x_j.

$$X = \begin{bmatrix} 0 & x_{12} & \cdots & x_{1n} \\ x_{21} & 0 & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & 0 \end{bmatrix} \tag{5.1}$$

Step 3: Calculate normalized direct-relation matrix

Once the direct-relation matrix is established, normalization is applied to direct-relation matrix subsequently. In the literature, two methods are very commonly used in order to normalize the direct-relation matrix. The first one is normalization by maximum sum of row or column vectors as follows (Tzeng, Chiang & Li, 2007) :

$$k = \mathrm{Min}\left[\frac{1}{\underset{1 \le i \le n}{\mathrm{Max}} \left(\sum_{j=1}^{n} x_{ij} \right)}, \frac{1}{\underset{1 \le j \le n}{\mathrm{Max}} \left(\sum_{i=1}^{n} x_{ij} \right)} \right] \tag{5.2}$$

where k is the normalization factor.

The second method commonly used in order to normalize the direct-relation matrix is based on the maximum sum of row vectors as provided below (Wu & Lee, 2007) :

$$k = \frac{1}{\underset{1 \le i \le n}{\mathrm{Max}} \left(\sum_{j=1}^{n} x_{ij} \right)} \tag{5.3}$$

In the scope of this study, we will use the first method for normalization. The normalized direct-relation matrix, denoted by N is calculated by multiplying k with direct-relation matrix X as follows:

$$N = k \times X \tag{5.4}$$

Step 4: Calculate direct/indirect relation matrix

An important mathematical property is given by (Papoulis & Pillai, 2002) and (Goodman, 1988) as follows:

$$\lim_{k\to\infty}\left(N^k\right)=0 \tag{5.5}$$

$$\lim_{k\to\infty}\left(I+N+N^2+\cdots+N^k\right)=(I-N)^{-1} \tag{5.6}$$

where 0 is a null or zero matrix and I is an identity matrix. As a matter of this fact, normalized direct-relation matrix N has characteristics of above equations and satisfies them. Therefore, the direct/indirect relation matrix, known as total relation matrix, can be calculated as follows (Huang et al., 2007):

$$T=\lim_{k\to\infty}\left(N+N^2+\cdots+N^k\right)=N\left(I-N\right)^{-1} \tag{5.7}$$

Proof of this formula can be given as:

$$T=N+N^2+\cdots+N^k \tag{5.8}$$

$$=N(\mathrm{I}+\mathrm{N}+\mathrm{N}^2+\cdots+N^{k-1}) \tag{5.9}$$

$$=N[(I+N+N^2+\cdots+N^{k-1})(I-N)](I-N)^{-1} \tag{5.10}$$

$$=N[I-N^k](I-N)^{-1} \tag{5.11}$$

$$=N(I-N)^{-1}, \tag{5.12}$$

where $\lim_{k\to\infty}N^k=[0]_{n\times n}$.

When taking the powers of the matrix, the indirect effects are continuously diminished. For example, as seen in Figure 5.1 (Tzeng & Huang, 2011), influence of variable a on variable h is smaller than the influence exerted on variable e, and also smaller than exerted on b.

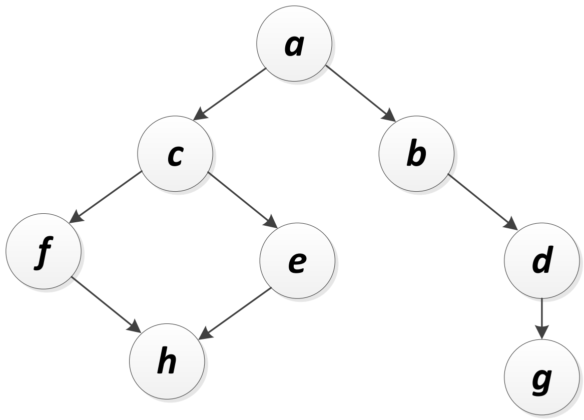


Figure 5.1 Example of a direct graph

The total relation matrix is the infinite series of direct and indirect effects of each factor. The final matrix (the total relation matrix) gives the ultimate structure after continuous process, where direct and indirect relationships are stabilized. Additionally, Lin and Wu (2008) provided with the calculation of indirect relation matrix, known as total-indirect relation matrix H as follows;

$$H = \lim_{k \to \infty}\left(N^2 + N^3 + \cdots + N^k\right) = N^2\left(I - N\right)^{-1} \tag{5.3}$$

Another very important feature of DEMATEL method is that the most affecting and the most affected factors can be visualized. After the total relation matrix T is calculated, the sum of row and the sum of column values can be obtained. Sum of row values of T is named prominence and formally expressed as:

$$D_i = \sum_{j=1}^{n} t_{ij} \ , \quad (i = 1, 2, \ldots, n) \tag{5.14}$$

Sum of the row values implies the sum of the degrees in which factor i exerts on factor j. In addition to sum of row values of T, the sum of column values of T can be calculated as:

$$R_j = \sum_{i=1}^{n} t_{ij}\ ,\quad (j = 1, 2, \ldots, n) \tag{5.15}$$

Similarly, sum of the column values implies the sum of the degrees in which factor j is affected by other factors i.

Furthermore, a visual causal diagram can be depicted by calculation of $(D+R)$ and $(D-R)$ values for each factor. The $(D+R)$ value is placed to x axis and called prominence. Prominence value indicates how important the factor is. On the other hand, the $(D-R)$ value is arranged in y axis, and called relation. The relation value is enabled factors to be grouped into cause and effect groups. If $(D-R)$ value is positive, then the factor is said to be part of the cause group. Contrarily, when $(D-R)$ is negative, the factor belongs to effect group. Thanks to visual representation of structural model, decision makers might gain invaluable insights into the problem while recognizing the cause and effect factors.

5.3 Fuzzy DEMATEL Method

There are different analytical procedures for the fuzzy DEMATEL calculations under group decision making setting. The two widely-used ones belong to Wu & Lee (2007). In this study, we adopted the method of Lin & Wu (2008) concisely described as follows:

Step 1: Form the decision maker committee and determine the objectives of decision. Obtaining relevant information, developing set of alternatives and discussing on pros and cons of each courses of action performed in this stage.

Step 2: Generate the evaluation criteria and develop the fuzzy linguistic scale. In this study, we adopted the linguistic scale developed by Li (1999) seen in table 5.2.

Step 3: Obtain the evaluations of decision-makers. In this stage, a committee of p decision makers provides pair-wise comparisons with linguistic terms in order to ascertain causal relationship between criteria.

Table 5.2 The correspondence of linguistic terms and linguistic values

Linguistic Terms	Linguistic Values
Very high influence (VH)	(0.75, 1.0, 1.0)
High influence (H)	(0.5, 0.75, 1.0)
Low influence (L)	(0.25, 0.5, 0.75)
Very low influence (VL)	(0, 0.25 0.5)
No influence (No)	(0, 0, 0.25)

Thereby, we have p fuzzy pair-wise matrices $\tilde{Z}^{\langle 1\rangle}, \tilde{Z}^{\langle 2\rangle}, \ldots, \tilde{Z}^{\langle p\rangle}$ consisting of evaluations with triangular fuzzy numbers. Here, $\tilde{Z}^{\langle k\rangle}$ is the initial direct-relation fuzzy matrix.

$$\tilde{Z}^{\langle k\rangle} = \begin{bmatrix} 0 & \tilde{z}_{12}^{\langle k\rangle} & \cdots & \tilde{z}_{1n}^{\langle k\rangle} \\ \tilde{z}_{21}^{\langle k\rangle} & 0 & \cdots & \tilde{z}_{2n}^{\langle k\rangle} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{n1}^{\langle k\rangle} & \tilde{z}_{n2}^{\langle k\rangle} & \cdots & 0 \end{bmatrix} \tag{5.16}$$

where $k = 1, 2, \ldots, p$ and $\tilde{z}_{ij}^{\langle k\rangle} = \left(l_{ij}^{\langle k\rangle}, m_{ij}^{\langle k\rangle}, u_{ij}^{k}\right)$.

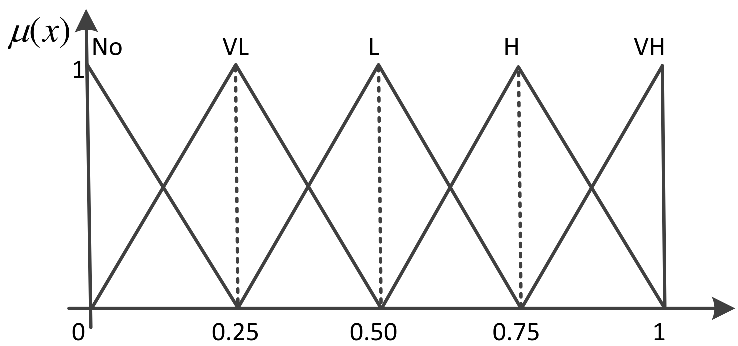


Figure 5.2 Triangular fuzzy numbers for linguistic variables

Step 4: Obtain the normalized direct-relation matrix. Let $\tilde{a}_i^{\langle k \rangle}$ be a triangular fuzzy number;

$$\tilde{a}_i^{\langle k \rangle} = \sum_{j=1}^{n} \tilde{z}_{ij}^{\langle k \rangle} = \left(\sum_{j=1}^{n} l_{ij}^{\langle k \rangle}, \sum_{j=1}^{n} m_{ij}^{\langle k \rangle}, \sum_{j=1}^{n} u_{ij}^{\langle k \rangle} \right) \tag{5.17}$$

And

$$r^{\langle k \rangle} = \max_{1 \le i \le n} \left(\sum_{j=1}^{n} u_{ij}^{\langle k \rangle} \right) \tag{5.18}$$

In order to obtain comparable scales, criteria scales are transformed by linear scale transformation. The normalized direct-relation fuzzy matrix of kth expert $\tilde{\mathrm{X}}^{\langle k \rangle}$ is given by

$$\tilde{X}^{\langle k \rangle} = \begin{bmatrix} \tilde{x}_{11}^{\langle k \rangle} & \tilde{x}_{12}^{\langle k \rangle} & \cdots & \tilde{x}_{1n}^{\langle k \rangle} \\ \tilde{x}_{21}^{\langle k \rangle} & \tilde{x}_{22}^{\langle k \rangle} & \cdots & \tilde{x}_{2n}^{\langle k \rangle} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1}^{\langle k \rangle} & \tilde{x}_{n2}^{\langle k \rangle} & \cdots & \tilde{x}_{nn}^{\langle k \rangle} \end{bmatrix} \tag{5.19}$$

where $k = 1, 2, \ldots, p$ and

$$\tilde{x}_{ij}^{\langle k \rangle} = \frac{\tilde{z}_{ij}^{\langle k \rangle}}{r^{\langle k \rangle}} = \left(\frac{l_{ij}^{\langle k \rangle}}{r^{\langle k \rangle}}, \frac{m_{ij}^{\langle k \rangle}}{r^{\langle k \rangle}}, \frac{u_{ij}^{\langle k \rangle}}{r^{\langle k \rangle}} \right) \tag{5.20}$$

It is assumed that for at least one i satisfies $\sum_{j=1}^{n} u_{ij}^{\langle k \rangle} < r^{\langle k \rangle}$. Thus average matrix $\tilde{\mathrm{X}}$ of $\tilde{\mathrm{X}}^{\langle 1 \rangle}, \tilde{\mathrm{X}}^{\langle 2 \rangle}, \ldots, \tilde{\mathrm{X}}^{\langle p \rangle}$ is calculated as;

$$\tilde{X} = \frac{\left(\tilde{X}^{\langle 1 \rangle} \oplus \tilde{X}^{\langle 2 \rangle} \oplus \cdots \oplus \tilde{X}^{\langle p \rangle} \right)}{p} \tag{5.21}$$

Also denoted by;

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nn} \end{bmatrix} \tag{5.22}$$

where $\tilde{x}_{ij} = \dfrac{\sum_{k=1}^{p} \tilde{x}_{ij}^{\langle k \rangle}}{p}$.

Note that the initial direct-relation matrices have not been aggregated when the required data is obtained from decision makers immediately after initial direct-relation fuzzy matrices are formed. Conversely, the normalized direct-relation fuzzy decision matrices are obtained for each decision maker, then arithmetic mean is utilized to aggregate normalized matrices. By doing so, it is argued that differences of individuals can be more apparent. This is actually essential information during the decision process.

Step 5: Construct the structural model and analyze. One of the assumptions underlying the computation of total-relation fuzzy matrix is that the convergence of $\lim_{w \to \infty} \tilde{X}^{w} = 0$. In order to compute the $\tilde{X}^{w}$, fuzzy multiplication formula, also a sort of approximation is used. Therefore, $\tilde{X}^{w}$ involves elements which are triangular fuzzy numbers. It is possible to define three crisp matrices separating the elements of $\tilde{x}_{ij} = \left(l_{ij}, m_{ij}, u_{ij} \right)$. Matrices with extracted elements from $\tilde{X}$ are as follows:

$$X_l = \begin{bmatrix} 0 & l_{12} & \cdots & l_{1n} \\ l_{21} & 0 & \cdots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & 0 \end{bmatrix} \tag{5.23}$$

$$X_m = \begin{bmatrix} 0 & m_{12} & \cdots & m_{1n} \\ m_{21} & 0 & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & 0 \end{bmatrix} \tag{5.24}$$

and

$$X_u = \begin{bmatrix} 0 & u_{12} & \cdots & u_{1n} \\ u_{21} & 0 & \cdots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & 0 \end{bmatrix} \tag{5.25}$$

Finally the total-relation fuzzy matrix $\tilde{T}$ is defined through

$$\tilde{T} = \tilde{X}(I - \tilde{X})^{-1} \tag{5.26}$$

$$\tilde{T} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{n1} & \tilde{t}_{n2} & \cdots & \tilde{t}_{nn} \end{bmatrix} \tag{5.27}$$

where $\tilde{t}_{ij} = (l'_{\mathrm{ij}}, m'_{\mathrm{ij}}, u'_{\mathrm{ij}})$ then

$$Matrix\left[l'_{ij}\right] = X_l(I - X_l)^{-1} \tag{5.28}$$

$$Matrix\left[m'_{ij}\right] = X_m(I - X_m)^{-1} \tag{5.29}$$

$$Matrix\left[u'_{ij}\right] = X_u(I - X_u)^{-1} \tag{5.30}$$

Once the total-relation fuzzy matrix $\tilde{T}$ is acquired, a causal diagram can be drawn pursuing the same procedure as in the crisp case.

The other well-known fuzzy DEMATEL calculation under group decision making setting belong to Wu & Lee (2007). The main characterization of their method is that defuzzification process has a central role, in other words, fuzzy numbers are transformed into crisp ones and calculation process continued with crisp scores. For the sake of simplicity, we abandon the procedural steps of the proposed approach, yet we only give the differentiated side of the approach. According to method, when decision makers' evaluations are obtained, assessment matrices are defuzzified using the CFCS method (Opricovic & Tzeng, 2003). Then arithmetic mean was used to acquire the initial direct-relation matrix. Since the relation matrix is included with crisp scores, remaining steps follows the traditional or non-fuzzy DEMATEL calculations.

5.4 DEMATEL with ANP

In recent years, there have been many applications of DEMATEL method in conjunction with ANP. As a powerful tool enables to model causal relationships, DEMATEL method applications increased and many different variants of hybrid techniques are proposed in the MCDM context. Therefore, selection and use of appropriate hybrid technique becomes a complicated problem. Under these circumstances, we try to shed light on the different variants of ANP and DEMATEL hybridization, thus researchers might find appropriate method for their analysis.

Even though, application variations are far too much, they can be divided into four general categories as;

- Network Relationship Map of ANP
- Inner Dependency in ANP
- Cluster Weighted ANP
- DEMATEL-Based ANP (DANP)

In Figure 5.3, DEMATEL method among the causal dependency models and the position of DEMATEL and ANP applications in our proposed framework have been shown.

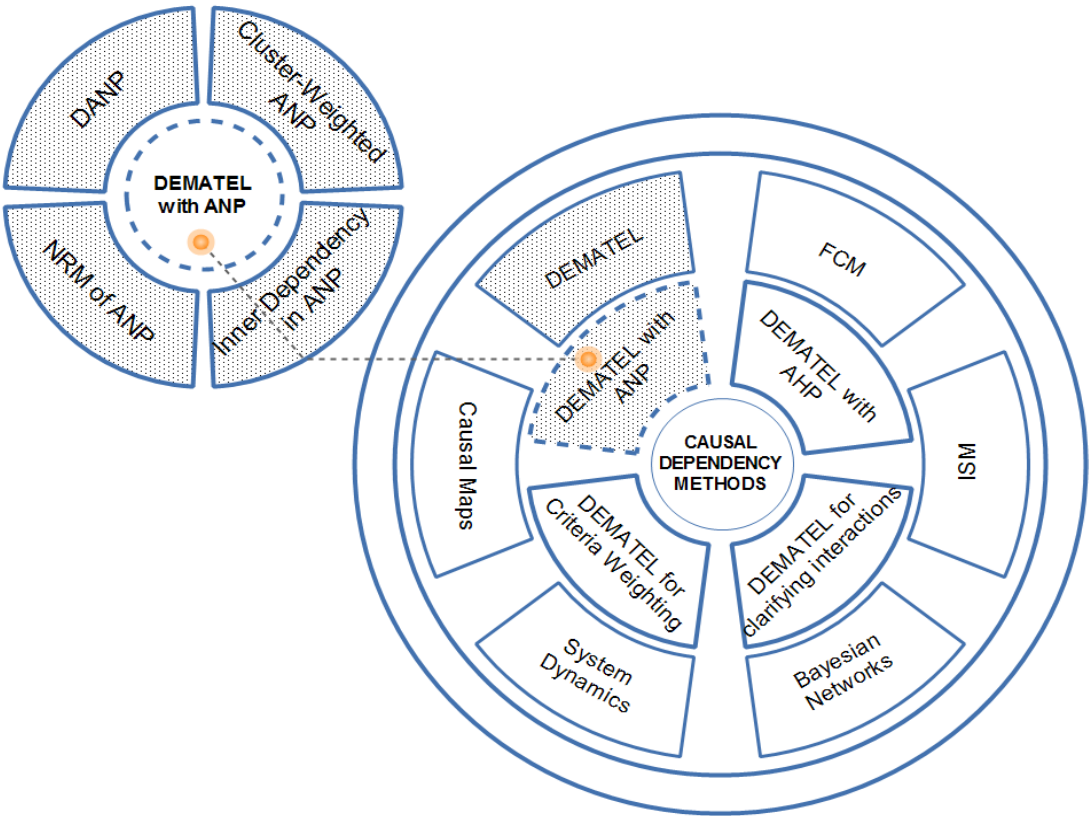


Figure 5.3 DEMATEL with ANP Applications

5.4.1 Network Relationship Map of ANP

In traditional AHP/ANP applications, it is assumed that the hierarchical or network structure of the decision problem is known a priori. However, this assumption does not comply with the real life situations where decision maker or analyst cannot form the problem structure easily. Specifically, utilizing ANP for complicated decision problems, the network structure is of vital importance in terms of accuracy of the model.

The ANP and AHP models quantify the influence between criteria based upon the pairwise comparisons. Additionally, pairwise comparisons are conducted with respect to problem structure. Here, we refer problem structure merely as relationships between criteria/clusters in the decision problem, since problem structuring is a much broader concept related to defining decision goals to criteria formalization, from identifying stakeholders to analyzing data requirements.

Because of the substantial role of the relationship between criteria and clusters in ANP, DEMATEL method is applied to depict the Network Relationship Map (NRM) of the decision problem. This type of usage of DEMATEL method with ANP constitutes the highest proportion (in terms of number of published articles) in comparison with other hybrid techniques of ANP. Studies implementing DEMATEL method prior to ANP intends to:

- Determining the relationships between decision criteria.
- Making traditional pairwise comparisons with respect to NRM by which DEMATEL method is used to construct.
- Utilizing some favorable features of DEMATEL such as visualizing cause and effect criteria, getting broader insight into relationships among criteria and clusters, and determining the most influential criteria and so on.

For the sake of increased understandability, we give examples and show the methodologies in a step by step approach, thus this study can be helpful for interested readers and especially for undergraduate students.

Example 5.1: Suppose that we have four clusters and twelve criteria in the decision problem. As we mentioned in the earlier chapter, let us assume that the decision maker collected criteria together into four distinct clusters based upon structural/functional dependency. For instance cluster 1 is named quality and quality-related criteria placed in cluster 1. Each cluster incorporates three criteria. The semi-completed structural model is shown in Figure 5.4. In order to carry out ANP method, one must know exactly whether the directed arrow indicated by question marks in Figure 5.4 exists or not. In traditional ANP approach, if the relationship between clusters is verified, the degree of relationship is assumed to be 1 inherently. It is actually nothing different than the assumption of equal importance of clusters. We will turn back to this situation later in subsequent sections. In ANP, influence of a criterion on a set of criteria is distributed by pairwise comparisons based on the NRM as seen in Figure 5.4. Hence, DEMATEL method resolves the problem of establishing network structure, in simple words; it helps to clarify the existence of directed arrows with question marks. Once the question marks in Figure 5.4 are replaced by solid arrows, pairwise comparisons might be conducted to quantify relationships.

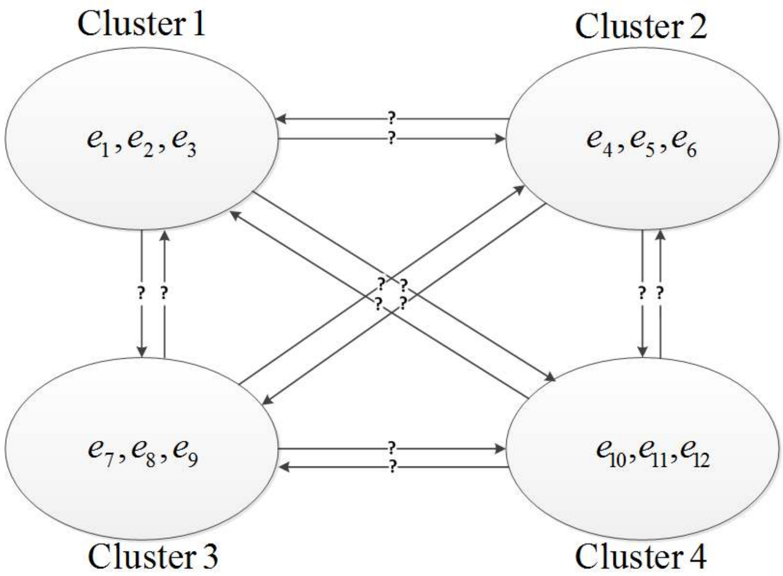


Figure 5.4 Network diagram of example 1

In figure 5.5, supermatrix representation regarding example 5.1 is provided. The partitions of the ANP structure can be seen as rectangles. The W_{ij} indicates the influence of cluster i on cluster j. In this example, influence of cluster i on cluster j characterized by 3×3 matrices which is the result of three pairwise comparison matrices in case of $W_{ij} \neq 0$.

In other words, if there is no arc from cluster 1 to cluster 4 in figure 1, all e_{ij} values in W_{14} seen in top-right of the supermatrix will be equal to zero

In order to determine the criteria structure, we assume that only one decision maker evaluates the assessment matrix and non-fuzzy scores are used for simplicity.

$$
\begin{array}{c|ccc|ccc|ccc|ccc|c}
\hline
 & e_{1,1} & e_{1,2} & e_{1,3} & e_{1,4} & e_{1,5} & e_{1,6} & e_{1,7} & e_{1,8} & e_{1,9} & e_{1,10} & e_{1,11} & e_{1,12} & \\
W_{1,1} \leftarrow & e_{2,1} & e_{2,2} & e_{2,3} & e_{2,4} & e_{2,5} & e_{2,6} & e_{2,7} & e_{2,8} & e_{2,9} & e_{2,10} & e_{2,11} & e_{2,12} & \rightarrow W_{1,4} \\
 & e_{3,1} & e_{3,2} & e_{3,3} & e_{3,4} & e_{3,5} & e_{3,6} & e_{3,7} & e_{3,8} & e_{3,9} & e_{3,10} & e_{3,11} & e_{3,12} & \\
\hline
 & e_{4,1} & e_{4,2} & e_{4,3} & e_{4,4} & e_{4,5} & e_{4,6} & e_{4,7} & e_{4,8} & e_{4,9} & e_{4,10} & e_{4,11} & e_{4,12} & \\
W_{2,1} \leftarrow & e_{5,1} & e_{5,2} & e_{5,3} & e_{5,4} & e_{5,5} & e_{5,6} & e_{5,7} & e_{5,8} & e_{5,9} & e_{5,10} & e_{5,11} & e_{5,12} & \rightarrow W_{2,4} \\
 & e_{6,1} & e_{6,2} & e_{6,3} & e_{6,4} & e_{6,5} & e_{6,6} & e_{6,7} & e_{6,8} & e_{6,9} & e_{6,10} & e_{6,11} & e_{6,12} & \\
\hline
 & e_{7,1} & e_{7,2} & e_{7,3} & e_{7,4} & e_{7,5} & e_{7,6} & e_{7,7} & e_{7,8} & e_{7,9} & e_{7,10} & e_{7,11} & e_{7,12} & \\
W_{3,1} \leftarrow & e_{8,1} & e_{8,2} & e_{8,3} & e_{8,4} & e_{8,5} & e_{8,6} & e_{8,7} & e_{8,8} & e_{8,9} & e_{8,10} & e_{8,11} & e_{8,12} & \rightarrow W_{3,4} \\
 & e_{9,1} & e_{9,2} & e_{9,3} & e_{9,4} & e_{9,5} & e_{9,6} & e_{9,7} & e_{9,8} & e_{9,9} & e_{9,10} & e_{9,11} & e_{9,12} & \\
\hline
 & e_{10,1} & e_{10,2} & e_{10,3} & e_{10,4} & e_{10,5} & e_{10,6} & e_{10,7} & e_{10,8} & e_{10,9} & e_{10,10} & e_{10,11} & e_{10,12} & \\
W_{4,1} \leftarrow & e_{11,1} & e_{11,2} & e_{11,3} & e_{11,4} & e_{11,5} & e_{11,6} & e_{11,7} & e_{11,8} & e_{11,9} & e_{11,10} & e_{11,11} & e_{11,12} & \rightarrow W_{4,4} \\
 & e_{12,1} & e_{12,2} & e_{12,3} & e_{12,4} & e_{12,5} & e_{12,6} & e_{12,7} & e_{12,8} & e_{12,9} & e_{12,10} & e_{12,11} & e_{12,12} & \\
\hline
\end{array}
$$

Figure 5.5 Supermatirx representation of example 5.1

Suppose that the direct relation matrix is formed as:

$$
X = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 2 & 0 & 4 & 1 \\ 1 & 3 & 0 & 2 \\ 3 & 3 & 2 & 0 \end{bmatrix} \tag{5.31}
$$

Following the calculation steps of DEMATEL method, the total influence matrix T is obtained as:

$$
T = \begin{bmatrix} 0.9910 & 1.4480 & 1.2851 & 1.2489 \\ 1.2187 & 1.3409 & 1.6109 & 1.1523 \\ 1.0739 & 1.5083 & 1.1719 & 1.1342 \\ 1.4721 & 1.7979 & 1.6290 & 1.1840 \end{bmatrix} \tag{5.32}
$$

Suppose that in order to filter the insignificant effects, $\alpha = 1.25$ value is set as a threshold and T_α is calculated:

$$T_{\alpha=1.25} = \begin{bmatrix} 0 & 1.4480 & 1.2851 & 0 \\ 0 & 1.3409 & 1.6109 & 0 \\ 0 & 1.5083 & 0 & 0 \\ 1.4721 & 1.7979 & 1.6290 & 0 \end{bmatrix} \tag{5.33}$$

So the resulting network structure is drawn as:

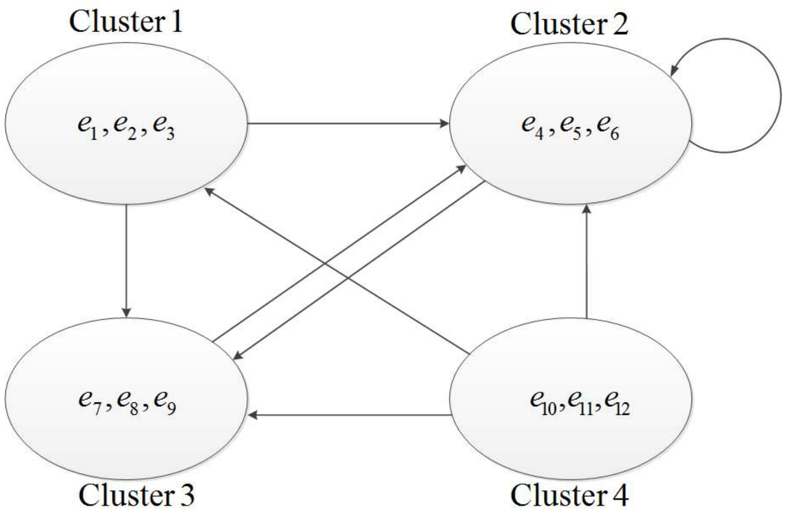


Figure 5.6 Resulting network diagram of Example 5.1 after DEMATEL

In the resulting supermatrix of ANP seen in Figure 5.7, 24 pairwise comparisons are required instead of 48 which account for the situation that all elements are compared. The values of elements in the supermatrix are obtained by traditional pairwise comparisons. What DEMATEL method made use of is that it determined the required pairwise comparison matrices by constructing the NRM. In the literature, studies utilizing DEMATEL with ANP for establishing NRM are sometimes perform extra analysis. For example, in addition to comparing clusters, criteria might be analyzed by DEMATEL method. However, this sort of studies does not use the comparison results of criteria in ANP, yet only managerial implications and insightful comments are provided.

$$
\begin{array}{c|ccc|ccc|ccc|ccc|c}
 & 0 & 0 & 0 & e_{1,4} & e_{1,5} & e_{1,6} & e_{1,7} & e_{1,8} & e_{1,9} & 0 & 0 & 0 & \\
W_{1,1} \leftarrow & 0 & 0 & 0 & e_{2,4} & e_{2,5} & e_{2,6} & e_{2,7} & e_{2,8} & e_{2,9} & 0 & 0 & 0 & \rightarrow W_{1,4} \\
 & 0 & 0 & 0 & e_{3,4} & e_{3,5} & e_{3,6} & e_{3,7} & e_{3,8} & e_{3,9} & 0 & 0 & 0 & \\
\hline
 & 0 & 0 & 0 & e_{4,4} & e_{4,5} & e_{4,6} & e_{4,7} & e_{4,8} & e_{4,9} & 0 & 0 & 0 & \\
W_{2,1} \leftarrow & 0 & 0 & 0 & e_{5,4} & e_{5,5} & e_{5,6} & e_{5,7} & e_{5,8} & e_{5,9} & 0 & 0 & 0 & \rightarrow W_{2,4} \\
 & 0 & 0 & 0 & e_{6,4} & e_{6,5} & e_{6,6} & e_{6,7} & e_{6,8} & e_{6,9} & 0 & 0 & 0 & \\
\hline
 & 0 & 0 & 0 & e_{7,4} & e_{7,5} & e_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & \\
W_{3,1} \leftarrow & 0 & 0 & 0 & e_{8,4} & e_{8,5} & e_{8,6} & 0 & 0 & 0 & 0 & 0 & 0 & \rightarrow W_{3,4} \\
 & 0 & 0 & 0 & e_{9,4} & e_{9,5} & e_{9,6} & 0 & 0 & 0 & 0 & 0 & 0 & \\
\hline
 & e_{10,1} & e_{10,2} & e_{10,3} & e_{10,4} & e_{10,5} & e_{10,6} & e_{10,7} & e_{10,8} & e_{10,9} & 0 & 0 & 0 & \\
W_{4,1} \leftarrow & e_{11,1} & e_{11,2} & e_{11,3} & e_{11,4} & e_{11,5} & e_{11,6} & e_{11,7} & e_{11,8} & e_{11,9} & 0 & 0 & 0 & \rightarrow W_{4,4} \\
 & e_{12,1} & e_{12,2} & e_{12,3} & e_{12,4} & e_{12,5} & e_{12,6} & e_{12,7} & e_{12,8} & e_{12,9} & 0 & 0 & 0 &
\end{array}
$$

Figure 5.7 Resulting supermatirx representation of Example 5.1 after DEMATEL

5.4.2 Inner Dependency of ANP

The ANP manages dependency as inner dependency, which comes to mean dependency within the cluster and outer dependency; dependency among different clusters in criteria structure. In almost all the DEMATEL and ANP hybridizations, DEMATEL serves as a means of providing NRM of the problem. This is the very basic capability included all the studies. Though, researchers developed other approaches to benefit from DEMATEL to compensate weaknesses of ANP or improve the modeling of the decision problem.

In ANP, inner dependency, outer dependency and self-feedback effects are all quantified by 9-scale pairwise comparisons. Nevertheless, ANP is a mathematically sound methodology in compared to other methods; especially in terms of its ability to handle all sorts of dependence and feedback, eliciting required pairwise comparison information from the decision makers is not trivial task. Yu & Tzeng (2006) emphasized the main shortcoming of ANP as:

- ANP requires too many pairwise comparisons which might be time consuming and difficult to obtain

- For particular situations, pairwise comparison questions might be meaningless or difficult to interpret.

Especially, in the inner dependency situation the pairwise comparison questions can be meaningless. Let us explain this concept on an example again.

Example 5.2: Let us concentrate on small portion of the above example and add a self-feedback effect to cluster 1 in order to better explain the approach. To demonstrate that the pairwise comparison questions might be vague and meaningless, let us entitle the decision criteria. Suppose that we are evaluating mobile phone alternatives. Cluster 1 is named costs, and cluster 2 is designated as ergonomics. Criteria in cluster 1 are purchasing cost, maintenance cost, accessory cost for e_1, e_2, e_3 respectively. On the other hand, criteria in cluster 2 are referred as appearance, durability, and usability for e_4, e_5, e_6 respectively. The network diagram of the problem is shown in Figure 5.8. Also supermatrix representation is given in Figure 5.9.

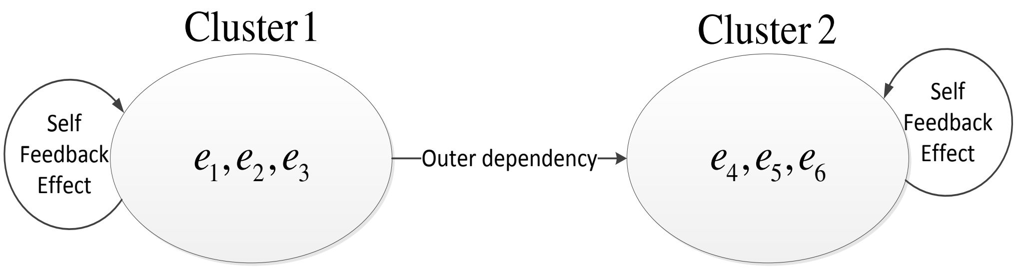


Figure 5.8 Network diagram of example 5.2

While conducting pairwise comparisons of outer dependency, decision makers are asked to, for example:

- For the criterion of purchasing cost;
    - How much more important the criterion of appearance than durability
    - How much more important the criterion of appearance than usability
    - How much more important the criterion of durability than usability
- For the criterion of maintenance cost;

- How much more important the criterion of appearance than durability
- How much more important the criterion of appearance than usability
- How much more important the criterion of durability than usability

- For the criterion of accessory cost;
  - How much more important the criterion of appearance than durability
  - How much more important the criterion of appearance than usability
  - How much more important the criterion of durability than usability

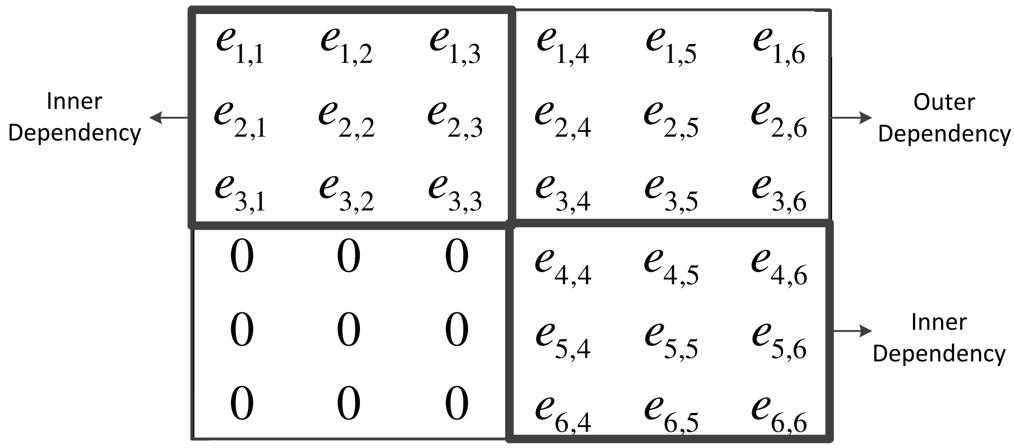


Figure 5.9 Supermatirx representation of example 5.2

Similarly, considering the inner dependencies, following pairwise comparison questions are asked for cluster 2:

- For the criterion of appearance;
  - How much more important the criterion of appearance than durability
  - How much more important the criterion of appearance than usability
  - How much more important the criterion of durability than usability
- For the criterion of durability;
  - How much more important the criterion of appearance than durability
  - How much more important the criterion of appearance than usability
  - How much more important the criterion of durability than usability
- For the criterion of usability;
  - How much more important the criterion of appearance than durability

- How much more important the criterion of appearance than usability
- How much more important the criterion of durability than usability

The first observation could be that the pairwise comparison questions for inner dependency are more challenging than those of outer dependencies. Furthermore, decision makers, in many situations, cannot estimate that how much the criterion of appearance have the influence on the importance of appearance, durability and usability respectively. The same question might be asked in another form as; thinking of the criterion of appearance, which criterion is more important: appearance or durability? This question is absolutely nonsense even for domain experts. Accordingly, a need for another method for treating with the inner dependency of supermatrix is obvious.

One solution approach to the above mentioned problem is to obtain inner dependency matrix from DEMATEL method, without Saaty's pairwise comparisons. The total-relation matrix obtained from DEMATEL method can be used as the inner dependency matrix in supermatrix.

Assume that the inner dependency matrix is obtained from DEMATEL method as;

$$T = \begin{bmatrix} t_{11} & \cdots & t_{1j} & \cdots & t_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1} & \cdots & t_{ij} & \cdots & t_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1} & \cdots & t_{nj} & \cdots & t_{nn} \end{bmatrix} \tag{5.34}$$

To be able to use this matrix for the place of inner dependency in supermatrix, it should be first normalized, and then transposed. The normalization is conducted as:

$$T = \begin{bmatrix} t_{11} & \cdots & t_{1j} & \cdots & t_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1} & \cdots & t_{ij} & \cdots & t_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1} & \cdots & t_{nj} & \cdots & t_{nn} \end{bmatrix} \begin{matrix} \rightarrow d_1 = \sum_{j=1}^{n} t_{1j} \\ \\ \rightarrow d_i = \sum_{j=1}^{n} t_{ij} \\ \\ \rightarrow d_n = \sum_{j=1}^{n} t_{nj} \end{matrix} \tag{5.35}$$

Each entry of the total relation matrix T is divided by sum values of each row as:

$$T = \begin{bmatrix} t_{11}/\mathrm{d}_1 & \cdots & t_{1j}/\mathrm{d}_1 & \cdots & t_{1n}/\mathrm{d}_1 \\ \vdots & & \vdots & & \vdots \\ t_{i1}/\mathrm{d}_i & \cdots & t_{ij}/\mathrm{d}_i & \cdots & t_{in}/\mathrm{d}_i \\ \vdots & & \vdots & & \vdots \\ t_{n1}/\mathrm{d}_n & \cdots & t_{nj}/\mathrm{d}_n & \cdots & t_{nn}/\mathrm{d}_n \end{bmatrix} \tag{5.36}$$

Finally transpose of the matrix is obtained;

$$T' = \begin{bmatrix} t_{11}/\mathrm{d}_1 & \cdots & t_{i1}/\mathrm{d}_i & \cdots & t_{n1}/\mathrm{d}_n \\ \vdots & & \vdots & & \vdots \\ t_{1j}/\mathrm{d}_1 & \cdots & t_{ij}/\mathrm{d}_i & \cdots & t_{nj}/\mathrm{d}_n \\ \vdots & & \vdots & & \vdots \\ t_{1n}/\mathrm{d}_1 & \cdots & t_{in}/\mathrm{d}_i & \cdots & t_{nn}/\mathrm{d}_n \end{bmatrix} \tag{5.37}$$

Once T' is obtained, it can be put into the appropriate place in supermatrix.

Suppose that the direct relation matrix for inner dependency of cost cluster is defined by:

$$X = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \tag{5.38}$$

The total relation matrix is obtained as:

$$T = \begin{bmatrix} 0.5484 & 0.6774 & 1.2581 \\ 0.9677 & 0.5484 & 1.1613 \\ 0.5806 & 0.6290 & 0.5968 \end{bmatrix} \tag{5.39}$$

Then normalization is applied to T as:

$$T = \begin{bmatrix} 0.2208 & 0.2727 & 0.5065 \\ 0.3614 & 0.2048 & 0.4337 \\ 0.3214 & 0.3482 & 0.3304 \end{bmatrix} \tag{5.40}$$

Finally transpose of the matrix T is calculated by:

$$T' = \begin{bmatrix} 0.2208 & 0.3614 & 0.3214 \\ 0.2727 & 0.2048 & 0.3482 \\ 0.5065 & 0.4337 & 0.3304 \end{bmatrix} \tag{5.41}$$

The total relation matrix is positioned into appropriate places in supermatrix as seen in Figure 5.10.

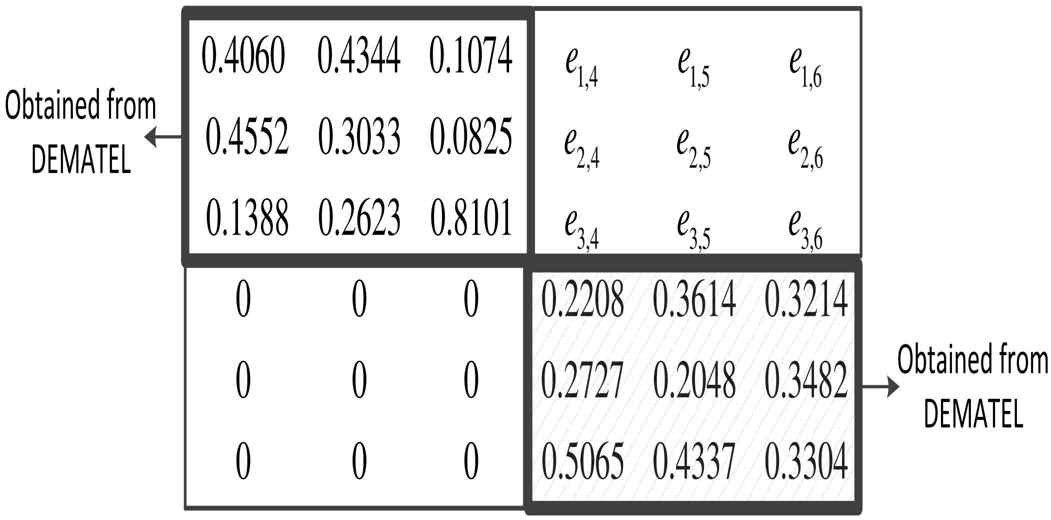


Figure 5.10 Resulting supermatirx representation of Example 5.2 after DEMATEL

Also we want to mention about different implementations exist in the literature for this type of usage. In the early applications of the DEMATEL method, some papers did not take the transpose of the matrix. However, in our study, we realized that the transpose of the matrix necessarily should be taken. We will further discuss this issue when DANP method will be representing. Furthermore, in the literature authors did not explain the reason why they have chosen a specific hybridization technique. Consequently, our understanding and comments are the primary source of the explaining the DEMATEL-ANP based calculations

5.4.3 Cluster Weighted ANP

Generalizing the hierarchical decomposition of AHP, the ANP method makes use of clusters and decision criteria which are referred as elements placed within the clusters. The relationship between elements is determined by pairwise comparisons. Then the corresponding eigenvalues are placed in the appropriate columns of the supermatrix. While relationships between elements are quantified in the ANP, those between clusters are not considered. The relationships between clusters are assumed to be the same.

In fact, each cluster has not the same priority for decision maker. The importance of the element not only affected by the influence emitted from other elements but also depends on the cluster that the element belongs to. Accordingly, the cluster weighted ANP approach exerts the influence degrees among clusters and use these values to weight the supermatrix. As we have shown earlier, the supermatrix takes the following form:

At the beginning of the cluster weighted ANP method, the total relation matrix is formed and then normalized. Suppose that we set a threshold to ignore the minor impacts having value smaller than α. When threshold is applied and the smaller values set to zero, the total influence matrix takes the form:as in 5.42.

$$
W = \begin{array}{cc}
 & \begin{array}{ccc} \overset{C_1}{e_{11}\cdots e_{1m_1}\cdots} & \overset{C_j}{e_{j1}\cdots e_{jm_j}\cdots} & \overset{C_n}{e_{n1}\cdots e_{nm_n}} \end{array} \\
\begin{array}{c}
C_1 \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1m_1} \end{array} \\
\vdots \\
C_i \begin{array}{c} e_{i1} \\ e_{i2} \\ \vdots \\ e_{im_i} \end{array} \\
\vdots \\
C_n \begin{array}{c} e_{n1} \\ e_{n2} \\ \vdots \\ e_{nm_n} \end{array}
\end{array}
&
\begin{bmatrix}
W_{11} & \cdots & W_{1j} & \cdots & W_{1n} \\
\vdots & & \vdots & & \vdots \\
W_{i1} & \cdots & W_{ij} & \cdots & W_{in} \\
\vdots & & \vdots & & \vdots \\
W_{n1} & \cdots & W_{nj} & \cdots & W_{nn}
\end{bmatrix}
\end{array}
$$

Figure 5.11 Supermatrix representation of ANP

$$
T_\alpha = \begin{bmatrix}
t_{11}^{\alpha} & \cdots & t_{1j}^{\alpha} & \cdots & t_{1n}^{\alpha} \\
\vdots & & \vdots & & \vdots \\
t_{i1}^{\alpha} & \cdots & t_{ij}^{\alpha} & \cdots & t_{in}^{\alpha} \\
\vdots & & \vdots & & \vdots \\
t_{n1}^{\alpha} & \cdots & t_{nj}^{\alpha} & \cdots & t_{nn}^{\alpha}
\end{bmatrix} \longrightarrow t_i = \sum_{j=1}^{n} t_{ij} \qquad (5.42)
$$

After normalization, the α -cut total influence matrix is expressed as:

$$
T_s = \begin{bmatrix}
t_{11}^{\alpha} / \mathrm{d}_1 & \cdots & t_{1j}^{\alpha} / \mathrm{d}_1 & \cdots & t_{1n}^{\alpha} / \mathrm{d}_1 \\
\vdots & & \vdots & & \vdots \\
t_{i1}^{\alpha} / \mathrm{d}_i & \cdots & t_{ij}^{\alpha} / \mathrm{d}_i & \cdots & t_{in}^{\alpha} / \mathrm{d}_i \\
\vdots & & \vdots & & \vdots \\
t_{n1}^{\alpha} / \mathrm{d}_n & \cdots & t_{nj}^{\alpha} / \mathrm{d}_n & \cdots & t_{nn}^{\alpha} / \mathrm{d}_n
\end{bmatrix} = \begin{bmatrix}
t_{11}^{s} & \cdots & t_{1j}^{s} & \cdots & t_{1n}^{s} \\
\vdots & & \vdots & & \vdots \\
t_{i1}^{s} & \cdots & t_{ij}^{s} & \cdots & t_{in}^{s} \\
\vdots & & \vdots & & \vdots \\
t_{n1}^{s} & \cdots & t_{nj}^{s} & \cdots & t_{nn}^{s}
\end{bmatrix} \qquad (5.43)
$$

Finally, the normalized total influence matrix is used to weight the unweighted supermatrix so that the weighted supermatrix W_w given as 5.44.

Once the weighted supermatrix is obtained, the limiting supermatrix can be calculated to reach overall priorities.

$$
\boldsymbol{W}_w = \begin{bmatrix} t_{11}^s \times \boldsymbol{W}_{11} & t_{21}^s \times \boldsymbol{W}_{12} & \cdots & \cdots & t_{n1}^s \times \boldsymbol{W}_{1n} \\ t_{12}^s \times \boldsymbol{W}_{21} & t_{22}^s \times \boldsymbol{W}_{22} & \vdots & & \vdots \\ \vdots & \vdots & \cdots & \cdots & t_{ni}^s \times \boldsymbol{W}_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ t_{1n}^s \times \boldsymbol{W}_{n1} & t_{2n}^s \times \boldsymbol{W}_{n2} & \cdots & \cdots & t_{nn}^s \times \boldsymbol{W}_{nn} \end{bmatrix} \tag{5.44}
$$

Example 5.3: Suppose that we have the total relation matrix as seen in equation 5.45. The circled elements are values higher than threshold value α.

$$
\boldsymbol{T} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \end{array} \tag{5.45}
$$

The unweighted supermatrix described by the above relation matrix is;

$$
\boldsymbol{W} = \begin{array}{c} \\ W_1 \\ W_2 \\ W_3 \end{array} \begin{array}{c} \begin{array}{ccc} W_1 & W_2 & W_3 \end{array} \\ \begin{bmatrix} W_{11} & 0 & 0 \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & 0 \end{bmatrix} \end{array} \tag{5.46}
$$

The NRM of the problem can be depicted as 5.12. Suppose that we have total relation matrix as in 5.47.

$$
T = \begin{bmatrix} 0.5469 & 0.9575 & 0.9649 \\ 0 & 0.5260 & 0.8020 \\ 0 & 0.6030 & 0 \end{bmatrix} \tag{5.47}
$$

In order to normalize the total relation matrix, each row should be divided by the value expressed in the formula 5.48.

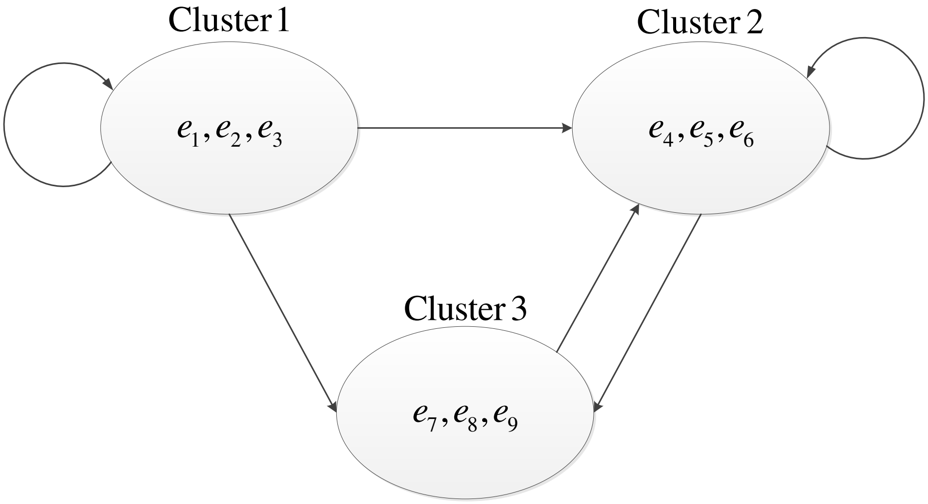


Figure 5.12 Network diagram of example 5.3

$$d_i = \sum_{j=1}^{n} t_{ij} \tag{5.48}$$

The equivalent d_i values are

$$\begin{array}{c} d_1 \\ d_2 \\ d_3 \end{array} \begin{bmatrix} 2.4693 \\ 1.3280 \\ 0.6030 \end{bmatrix} \tag{5.49}$$

Accordingly, the total relation matrix is normalized as

$$\begin{aligned} T_s &= \begin{bmatrix} \mathbf{0.5469}/2.4693 & \mathbf{0.9575}/2.4693 & \mathbf{0.9649}/2.4693 \\ 0 & \mathbf{0.5260}/1.3280 & \mathbf{0.8020}/1.3280 \\ 0 & \mathbf{0.6030}/0.6030 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0.2215} & \mathbf{0.3878} & \mathbf{0.3907} \\ 0 & \mathbf{0.3961} & \mathbf{0.6039} \\ 0 & \mathbf{1} & 0 \end{bmatrix} \end{aligned} \tag{5.50}$$

Finally, the supermatrix is multiplied by the influence between clusters. Calculations are given below:

$$W=\begin{bmatrix} 0.20 & 0.15 & 0.20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0.35 & 0.45 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.30 & 0.50 & 0.35 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.60 & 0.35 & 0.20 & 0.20 & 0.20 & 0.40 & 0.20 & 0.40 & 0.30 \\ 0.20 & 0.40 & 0.50 & 0.40 & 0.60 & 0.30 & 0.60 & 0.50 & 0.40 \\ 0.20 & 0.25 & 0.30 & 0.40 & 0.20 & 0.30 & 0.20 & 0.10 & 0.30 \\ 0.40 & 0.30 & 0.30 & 0.25 & 0.40 & 0.65 & 0 & 0 & 0 \\ 0.45 & 0.60 & 0.45 & 0.60 & 0.25 & 0.15 & 0 & 0 & 0 \\ 0.15 & 0.10 & 0.25 & 0.15 & 0.35 & 0.20 & 0 & 0 & 0 \end{bmatrix} \tag{5.51}$$

$$=\begin{bmatrix} 0.0443 & 0.0332 & 0.0443 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1108 & 0.0775 & 0.0997 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0664 & 0.1108 & 0.0775 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2327 & 0.1357 & 0.0776 & 0.0792 & 0.0792 & 0.1584 & 0.2000 & 0.4000 & 0.3000 \\ 0.0776 & 0.1551 & 0.1939 & 0.1584 & 0.2377 & 0.1188 & 0.6000 & 0.5000 & 0.4000 \\ 0.0776 & 0.0969 & 0.1163 & 0.1584 & 0.0792 & 0.1188 & 0.2000 & 0.1000 & 0.3000 \\ 0.1563 & 0.1172 & 0.1172 & 0.1510 & 0.2416 & 0.3925 & 0 & 0 & 0 \\ 0.1758 & 0.2344 & 0.1758 & 0.3623 & 0.1510 & 0.0906 & 0 & 0 & 0 \\ 0.0586 & 0.0391 & 0.0977 & 0.0906 & 0.2114 & 0.1208 & 0 & 0 & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix} \tag{5.52}$$

The weighted supermatrix is then raised to powers to calculate the final priorities as in the traditional ANP method.

5.4.4 DEMATEL based ANP (DANP)

We have seen different combinations of DEMATEL and ANP method so far. The DANP method gathers the advantages of other hybrid techniques we mentioned above. The DANP method proposed by Chen, Hsu & Tzeng (2011) by modifying the original ANP so as to reduce difficulty of ANP, yet preserving the mathematical power of the supermatrix at the same time.

As we already referred in previous sections, the survey questionnaire conducted in ANP is rather weird and it is indeed very difficult for decision makers to interpret and provide information. The DANP method combines four different approaches as seen in Figure 5.13.

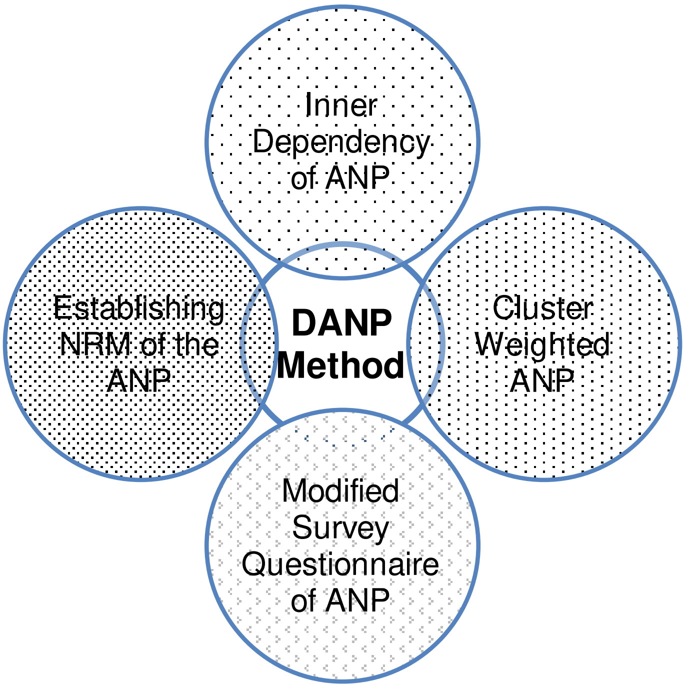


Figure 5.13 Components of DANP Method

Similar to other hybrid DEMATEL-ANP combinations, DANP method utilizes total influence matrix for clusters to establish NRM of the decision model. Based on the network relation map, the influential relationships are obtained. In traditional ANP method, the unweighted supermatrix is formed according to pairwise comparisons and ultimately the criteria weights which correspond to eigenvalues are placed to supermatrix. Due to the difficulty of pairwise comparisons questions and cognitive burden that the decision maker bears, DANP method modifies the pairwise comparison questions and abandons the Saaty's 9-scale matrix. Notwithstanding, DANP forms a comprehensive unweighted supermatrix by building initial direct influence matrix where pairwise comparisons not only conducted within clusters but for the whole system in compliance with the problem structure. In this respect, DANP method generalizes the modeling of inner dependency partitions by DEMATEL method. When the unweighted supermatrix is constructed, the total influence matrixes among clusters are used to weight the appropriate portions of the supermatrix in order to get the weighted supermatrix. Hence, DANP method also a general form of cluster-weighted ANP. Finally, DANP method avoids the tangled questions of 9-scale pairwise comparison survey and put the total relation matrix formation forward in lieu of it.

The basic steps of the DANP method can be given as follows (Chen, Hsu & Tzeng, 2011; Liu, Tzeng & Lee, 2012); in the first step of the evaluation, the NRM of the decision problem is developed by pairwise comparing criteria and dimensions separately . The total influence matrices denoted by T are separated into T_C and T_D for criteria and dimensions respectively. The total influence matrix T_C is given;

$$T_C = \begin{array}{cc} & \begin{array}{ccc} c_{11}\cdots c_{1m_1}\cdots & c_{j1}\cdots c_{jm_j}\cdots & c_{n1}\cdots c_{nm_n} \end{array} \\ \begin{array}{c} D_1 \begin{smallmatrix} c_{11}\\ c_{12}\\ \vdots \\ c_{1m_1} \end{smallmatrix} \\ \vdots \\ D_i \begin{smallmatrix} c_{i1}\\ c_{i2}\\ \vdots \\ c_{im_i} \end{smallmatrix} \\ \vdots \\ D_n \begin{smallmatrix} c_{n1}\\ c_{n2}\\ \vdots \\ c_{nm_n} \end{smallmatrix} \end{array} & \begin{bmatrix} T_c^{11} & \cdots & T_c^{1j} & \cdots & T_c^{1n} \\ \vdots & & \vdots & & \vdots \\ T_c^{i1} & \cdots & T_c^{ij} & \cdots & T_c^{in} \\ \vdots & & \vdots & & \vdots \\ T_c^{n1} & \cdots & T_c^{nj} & \cdots & T_c^{nn} \end{bmatrix} \end{array} \tag{5.53}$$

Then the total influence matrix T_C is normalized. The normalized T_C denoted by T_C^{α} expressed as;

$$T_C^{\alpha} = \begin{array}{cc} & \begin{array}{ccc} D_1 & D_j & D_n \\ c_{11}\cdots c_{1m_1}\cdots & c_{j1}\cdots c_{jm_j}\cdots & c_{n1}\cdots c_{nm_n} \end{array} \\ \begin{array}{c} D_1 \begin{smallmatrix} c_{11}\\ c_{12}\\ \vdots \\ c_{1m_1} \end{smallmatrix} \\ \vdots \\ D_i \begin{smallmatrix} c_{i1}\\ c_{i2}\\ \vdots \\ c_{im_i} \end{smallmatrix} \\ \vdots \\ D_n \begin{smallmatrix} c_{n1}\\ c_{n2}\\ \vdots \\ c_{nm_n} \end{smallmatrix} \end{array} & \begin{bmatrix} T_c^{\alpha 11} & \cdots & T_c^{\alpha 1j} & \cdots & T_c^{\alpha 1n} \\ \vdots & & \vdots & & \vdots \\ T_c^{\alpha i1} & \cdots & T_c^{\alpha ij} & \cdots & T_c^{\alpha in} \\ \vdots & & \vdots & & \vdots \\ T_c^{\alpha n1} & \cdots & T_c^{\alpha nj} & \cdots & T_c^{\alpha nn} \end{bmatrix} \end{array} \tag{5.54}$$

The normalization is conducted by the formula:

$$d_i^{11} = \sum_{j=1}^{m_1} t_{C^{ij}}^{11} \text{ , } i = 1,2,\ldots,\mathrm{m}_1 . \tag{5.55}$$

Formula above is repeated for each partition of the supermatrix. For the case of $T_C^{\alpha 11}$, the normalization procedure given by;

$$T_C^{\alpha 11} = \begin{bmatrix} t_{C^{11}}^{11}/d_1^{11} & \cdots & t_{C^{1j}}^{11}/d_1^{11} & \cdots & t_{C^{1m_1}}^{11}/d_1^{11} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{C^{i1}}^{11}/d_i^{11} & \cdots & t_{C^{ij}}^{11}/d_i^{11} & \cdots & t_{C^{im_1}}^{11}/d_i^{11} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{C^{m_1 1}}^{11}/d_{m_1}^{11} & \cdots & t_{C^{m_1 j}}^{11}/d_{m_1}^{11} & \cdots & t_{C^{m_1 m_1}}^{11}/d_{m_1}^{11} \end{bmatrix} = \begin{bmatrix} t_{C^{11}}^{\alpha 11} & \cdots & t_{C^{1j}}^{\alpha 11} & \cdots & t_{C^{1m_1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{C^{i1}}^{\alpha 11} & \cdots & t_{C^{ij}}^{\alpha 11} & \cdots & t_{C^{im_1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{C^{m_1 1}}^{\alpha 11} & \cdots & t_{C^{m_1 j}}^{\alpha 11} & \cdots & t_{C^{m_1 m_1}}^{\alpha 11} \end{bmatrix} \tag{5.56}$$

When the normalized total relation matrix is obtained, the next step is to form an unweighted supermatrix. A generic model of the supermatrix is specified as seen below where

$$W^{11} = \left(T_C^{\alpha 11}\right)'. \tag{5.57}$$

$$W = \left(T_C^{\alpha}\right)' = \begin{array}{cc} & \begin{array}{ccc} D_1 & D_i & D_n \\ c_{11}\cdots c_{1m_1}\cdots & c_{i1}\cdots c_{im_i}\cdots & c_{n1}\cdots c_{nm_n} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m_1} \\ \vdots \end{array} \\ D_j & \begin{array}{c} c_{j1} \\ c_{j2} \\ \vdots \\ c_{jm_j} \\ \vdots \end{array} \\ D_n & \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nm_n} \end{array} \end{array} & \begin{bmatrix} W^{11} & \cdots & W^{i1} & \cdots & W^{n1} \\ \vdots & & \vdots & & \vdots \\ W^{1j} & \cdots & W^{ij} & \cdots & W^{nj} \\ \vdots & & \vdots & & \vdots \\ W^{1n} & \cdots & W^{in} & \cdots & W^{nn} \end{bmatrix} \end{array} \tag{5.58}$$

In the supermatrix above, some W values would be zero. This indicates that the corresponding clusters/criteria has no influence. In other words, the criteria are independent of each other. Actually, dependent/independent character of criteria is determined at the beginning of the study. Especially, clusters are compared and the total relation matrix is acquired for the clusters, where the relationships are captured in cluster level. Furhermore, the total relation matrix is gathered for the whole system and unweighted supermatrix with positive and zero values are reached. By the way, it is suitable to explain one missing point here; some studies use criteria and cluster interchangably. From the AHP point of view, clusters might be seen as the criteria, and the elements inside the clusters are interpreted as sub-criteria. Nevertheless, we use cluster and criteria to depict the system with clusters hosting elements.

Recalling the unweighted supermatrix formation, for example, W^{11} can be expressed by following:

$$W^{11} = \left(T^{11}\right)' = \begin{array}{c} \\ c_{11} \\ \vdots \\ c_{i1} \\ \vdots \\ c_{m_1 1} \end{array} \begin{array}{c} \begin{array}{ccccc} c_{11} & \cdots & c_{i1} & \cdots & c_{m_1 1} \end{array} \\ \begin{bmatrix} t_{c^{11}}^{\alpha 11} & \cdots & t_{c^{i1}}^{\alpha 11} & \cdots & t_{c^{m_1 1}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{1j}}^{\alpha 11} & \cdots & t_{c^{ij}}^{\alpha 11} & \cdots & t_{c^{m_1 j}}^{\alpha 11} \\ \vdots & & \vdots & & \vdots \\ t_{c^{1m_1}}^{\alpha 11} & \cdots & t_{c^{im_1}}^{\alpha 11} & \cdots & t_{c^{m_1 m_1}}^{\alpha 11} \end{bmatrix} \end{array} \tag{5.59}$$

After the unweighted supermatrix is constructed, weighted supermatrix is needed to develop. Here the total relation matrix of clusters denoted by T_D given by;

$$T_D = \begin{bmatrix} t_D^{11} & \cdots & t_D^{1j} & \cdots & t_D^{1n} \\ \vdots & & \vdots & & \vdots \\ t_D^{i1} & \cdots & t_D^{ij} & \cdots & t_D^{in} \\ \vdots & & \vdots & & \vdots \\ t_D^{n1} & \cdots & t_D^{nj} & \cdots & t_D^{nn} \end{bmatrix} \tag{5.60}$$

The total relation matrix is normalized similar to relation matrix for criteria. For normalization below formula is applied;

$$d_i = \sum_{j=1}^{n} t^{ij}, \ i = 1, 2, \ldots, n \tag{5.61}$$

Consequently, the normalized total influence matrix for clusters is obtained as follows;

$$T_D^{\alpha}=\begin{bmatrix} t_D^{11}/d_1 & \cdots & t_D^{1j}/d_1 & \cdots & t_D^{1n}/\mathrm{d}_1 \\ \vdots & & \vdots & & \vdots \\ t_D^{i1}/d_i & \cdots & t_D^{ij}/d_i & \cdots & t_D^{in}/d_i \\ \vdots & & \vdots & & \vdots \\ t_D^{n1}/d_n & \cdots & t_D^{nj}/d_n & \cdots & t_D^{nn}/d_n \end{bmatrix}=\begin{bmatrix} t_D^{\alpha 11} & \cdots & t_D^{\alpha 1j} & \cdots & t_D^{\alpha 1n} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha i1} & \cdots & t_D^{\alpha ij} & \cdots & t_D^{\alpha in} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha n1} & \cdots & t_D^{\alpha nj} & \cdots & t_D^{\alpha nn} \end{bmatrix} \tag{5.62}$$

The final step of the DANP method is multiplication of normalized total influence matrix for clusters and the unweighted supermatrix as follows;

$$\boldsymbol{W}^{\alpha}=T_D^{\alpha}\boldsymbol{W}=\begin{bmatrix} t_D^{\alpha 11}\times\boldsymbol{W}^{11} & \cdots & t_D^{\alpha i1}\times\boldsymbol{W}^{i1} & \cdots & t_D^{\alpha n1}\times\boldsymbol{W}^{n1} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1j}\times\boldsymbol{W}^{1j} & \cdots & t_D^{\alpha ij}\times\boldsymbol{W}^{ij} & \cdots & t_D^{\alpha nj}\times\boldsymbol{W}^{nj} \\ \vdots & & \vdots & & \vdots \\ t_D^{\alpha 1n}\times\boldsymbol{W}^{1n} & \cdots & t_D^{\alpha in}\times\boldsymbol{W}^{in} & \cdots & t_D^{\alpha nn}\times\boldsymbol{W}^{nn} \end{bmatrix} \tag{5.63}$$

Example 5.4: We have given examples of other hybrid techniques in preview sections. For that reason, we very briefly introduce our example since DANP is just the combination of possible ideas that brings modeling capabilities of DEMATEL method together.

Assume that the problem involves two clusters and six criteria as shown in Figure 5.14.

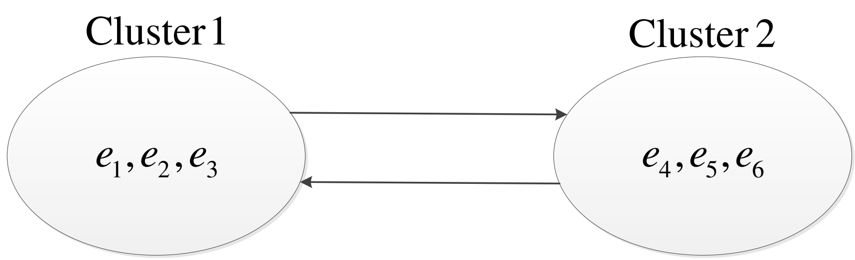


Figure 5.14 Network diagram of example 5.4

Suppose that the NRM belongs to our problem is obtained as follows:

$$T_D = \begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 1.8 & 0.7 \\ 3.2 & 1.4 \end{bmatrix} \end{array} \tag{5.64}$$

The unweighted supermatrix is seen in Figure 5.15. The main difference of DANP method is that the 4 partitions of the supermatrix (seen as rectangles) are obtained from the DEMATEL method.

	c_1	c_2	c_3	c_4	c_5	c_6
c_1	0.4410	0.1562	0.4268	0.3291	0.3114	0.6958
c_2	0.4903	0.3067	0.2164	0.3986	0.3476	0.0313
c_3	0.0687	0.5370	0.3568	0.2724	0.3409	0.2729
c_4	0.5558	0.4610	0.0959	0.0196	0.3218	0.0478
c_5	0.3848	0.0753	0.2851	0.4668	0.5378	0.1004
c_6	0.0593	0.4637	0.6190	0.5135	0.1405	0.5818

Obtained from DEMATEL

Obtained from DEMATEL

Figure 5.15 The unweighted supermatrix of Example 5.4

For example, once the total relation matrix is obtained, the normalization is applied. Then the matrix transpose of the normalized total relation matrix is attained. Ultimately, final total relation matrix is placed to appropriate positions in the supermatrix.

When the unweighted supermatrix is obtained, the cluster weighed ANP technique is applied to get all the columns sum to unity. The normalized total relation matrix for clusters is obtained as;

$$T_{D_s} = \begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 0.72 & 0.28 \\ 0.70 & 0.30 \end{bmatrix} \end{array} \tag{5.65}$$

The unweighted supermatrix is weighted by the influence degrees among clusters realized by DEMATEL method.

$$
\begin{array}{c|cccccc}
 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\
\hline
c_1 & 0.4410\times\mathbf{0.72} & 0.1562\times\mathbf{0.72} & 0.4268\times\mathbf{0.72} & 0.3291\times\mathbf{0.70} & 0.3114\times\mathbf{0.70} & 0.6958\times\mathbf{0.70} \\
c_2 & 0.4903\times\mathbf{0.72} & 0.3067\times\mathbf{0.72} & 0.2164\times\mathbf{0.72} & 0.3986\times\mathbf{0.70} & 0.3476\times\mathbf{0.70} & 0.0313\times\mathbf{0.70} \\
c_3 & 0.0687\times\mathbf{0.72} & 0.5370\times\mathbf{0.72} & 0.3568\times\mathbf{0.72} & 0.2724\times\mathbf{0.70} & 0.3409\times\mathbf{0.70} & 0.2729\times\mathbf{0.70} \\
c_4 & 0.5558\times\mathbf{0.28} & 0.4610\times\mathbf{0.28} & 0.0959\times\mathbf{0.28} & 0.0196\times\mathbf{0.30} & 0.3218\times\mathbf{0.30} & 0.0478\times\mathbf{0.30} \\
c_5 & 0.3848\times\mathbf{0.28} & 0.0753\times\mathbf{0.28} & 0.2851\times\mathbf{0.28} & 0.4668\times\mathbf{0.30} & 0.5378\times\mathbf{0.30} & 0.1004\times\mathbf{0.30} \\
c_6 & 0.0593\times\mathbf{0.28} & 0.4637\times\mathbf{0.28} & 0.6190\times\mathbf{0.28} & 0.5135\times\mathbf{0.30} & 0.1405\times\mathbf{0.30} & 0.5818\times\mathbf{0.30}
\end{array}
$$

Figure 5.16 Weighting of clusters in example 5.4

When the weighted supermatrix is raised to large powers, the steady-state criteria weights can be obtained as 5.67.

$$
\boldsymbol{W}_w = \begin{array}{r}
\\ \\ \\ \\ \\ \\ \textbf{Sum:}
\end{array}
\begin{bmatrix}
0.3175 & 0.1125 & 0.3073 & 0.2303 & 0.2180 & 0.4870 \\
0.3530 & 0.2209 & 0.1558 & 0.2790 & 0.2434 & 0.0219 \\
0.0495 & 0.3867 & 0.2569 & 0.1907 & 0.2387 & 0.1910 \\
0.1556 & 0.1291 & 0.0269 & 0.0059 & 0.0965 & 0.0143 \\
0.1078 & 0.0211 & 0.0798 & 0.1401 & 0.1613 & 0.0301 \\
0.0166 & 0.1298 & 0.1733 & 0.1541 & 0.0421 & 0.2555 \\
\mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1}
\end{bmatrix} \tag{5.66}
$$

$$
\boldsymbol{W}_f = \begin{bmatrix}
0.2733 & 0.2733 & 0.2733 & 0.2733 & 0.2733 & 0.4870 \\
0.2264 & 0.2264 & 0.2264 & 0.2264 & 0.2264 & 0.2264 \\
0.2146 & 0.2146 & 0.2146 & 0.2146 & 0.2146 & 0.2146 \\
0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 & 0.0875 \\
0.0801 & 0.0801 & 0.0801 & 0.0801 & 0.0801 & 0.0801 \\
0.1182 & 0.1182 & 0.1182 & 0.1182 & 0.1182 & 0.1182
\end{bmatrix} \tag{5.67}
$$

5.5 Review of DEMATEL Method

5.5.1 DEMATEL with ANP

5.5.1.1 DANP Method

Chen, Hsu and Tzeng (2011) introduced a DANP hybrid MCDM model for performance evaluation of hot spring hotels based on balanced score card, ANP and DEMATEL. First, the DEMATEL method used to draw a causal relationship map for hot spring hotel performance evaluation and then ANP was utilized to determine weights of criteria. Proposed method employed a modified DEMATEL survey questionnaire instead of that used in traditional ANP. Using DEMATEL method, total influence matrix for fifteen criteria and four perspectives were generated. Total influence matrix for criteria was facilitated to form unweighted supermatrix, while total influence matrix for perspectives utilized to form weighted supermatrix. In the last step, overall priorities with the limiting process method calculated to obtain the ANP weights.

Lee et al. (2011) studied the interactive relationships among equity investment factors combining DEMATEL and ANP methods. A hybrid method was developed by extending the study of (Yang et al., 2008). First, the DEMATEL method was used for clarifying the different degrees of impacts of the clusters. Then, total-influence matrix T was constructed considering all the criteria. After the normalization of total-influence matrix, it was incorporated into the unweighted supermatrix. Although the key interdependencies of clusters were obtained via DEMATEL, those between clusters were calculated by ANP. An empirical study was presented to prioritize equity investment factors in order to achieve the optimal investment strategy.

Wang and Tzeng (2012) proposed a hybrid MCDM model combining DEMATEL, ANP and VIKOR for ranking the priorities in brand marketing strategies. The DEMATEL method was not only used for constructing the IRM, but also providing input data for supermatrix formation in ANP. First, the initial influence matrix for criteria and clusters was formed. The total-influence matrix for criteria was obtained from DEMATEL and its transposed form was filled into the interdependence clusters in unweighted supermatrix. Then, normalized total-influence matrix for clusters was used to obtain weighted supermatrix by multiplying appropriate cells in unweighted supermatrix. Once the weighted supermatrix was formed, it was raised to a sufficiently large power until the supermatrix converged and became stable. Finally, the weights obtained from ANP used in VIKOR method to find a compromise ranking. An empirical case study was provided in electronic manufacturing industry in Taiwan to find the key success factors of brand marketing to create brand value.

Chen and Sun (2012) applied DANP method to find the best leisure sport and important factors that enhance senior citizens' participation on recreational sports. The DEMATEL method was used to obtain total influence matrix for criteria and clusters. Then the unweighted supermatrix was established incorporating the total influence matrix for criteria. Afterwards, unweighted supermatrix was normalized by the cluster weights obtained from DEMATEL method. Finally, weighted supermatrix was raised to sufficiently large powers to obtain the best alternative and criteria weights.

Tseng (2010) proposed fuzzy network balanced scorecard (FNBS) approach combining fuzzy set theory, DEMATEL and ANP. DEMATEL method served as a tool to obtain complex interrelationships among aspects and criteria by gathering the performance rating in the form of triangular fuzzy numbers. Then, these linguistic preferences were converted into crisp scores and DEMATEL calculations were made. Based on the DEMATEL calculations, cause and effect diagram of criteria was obtained and dependency matrix acquired from DEMATEL method was put into the supermatrix formation in ANP. The final results were achieved by converged supermatrix. Proposed method was applied in a private university of science and technology in Taiwan. Conducting a post-survey, benefits and drawbacks of the proposed method were given and the managerial implications were provided.

Liu, Tzeng, and Lee (2012) proposed a hybrid MCDM method with DANP and VIKOR methods to determine the optimal improvement plan for Taiwan tourism policy. DEMATEL method was not only used to draw a network relationship map (NRM) but also construct the unweighted supermatrix in ANP. Weights of criteria found by ANP was used with VIKOR method and gaps to the aspired level were discussed.

5.5.1.2 Inner Dependency in Supermatrix

Büyüközkan and Çifçi (2012) proposed a novel approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. The fuzzy DEMATEL and fuzzy ANP methods were used to determine the criteria weights. Since inner dependencies existed in the criteria structure, the total relationship matrix was derived from DEMATEL method and put into the appropriate columns in unweighted supermatrix. In the last step, using the weights obtained from ANP, fuzzy TOPSIS method was used to select the best alternative. Proposed method was implemented in Ford Motor Company in Turkey.

Büyüközkan and Öztürkcan (2010) proposed an integrated approach based on ANP and DEMATEL technique to help companies determine and prioritize critical Six Sigma projects. First, strategies, factors, sub-factors and project alternatives were determined. Strategies were divided into 3 categories as business excellence, revenue growth and productivity. Then, DEMATEL method was applied to analyze the relationship among strategies, factors and sub-factors. In the next step, the total relation matrices (inner dependencies), derived from DEMATEL technique was put into the appropriate columns in unweighted supermatrix where self-feedback loop exists in the structure. After weighted supermatrix obtained, the supermatrix was increased to sufficient large power until convergence occurs. Finally, the best project was selected.

Wu (2008) combined ANP and DEMATEL methods for choosing knowledge management (KM) strategies. Instead of making traditional pairwise comparisons for inner dependencies in unweighted supermatrix, inner dependency matrix was obtained from the DEMATEL method and deployed in unweighted supermatrix. In the paper, it was claimed that because the DEMATEL method could produce more valuable information for making decisions, it was used for the treatment of inner dependencies. When supermatrix was formed, it was raised to large powers to obtain steady-state matrix. A case study for evaluating and selecting a new KM strategy in Taiwanese company was provided to demonstrate the effectiveness of the proposed methodology.

Shen, Lin, and Tzeng (2011) proposed a hybrid MCDM method integrating fuzzy Delphi, DEMATEL and ANP to construct a technology selection model. First, the fuzzy Delphi method was applied to identify evaluation criteria for technology selection. Then, the DEMATEL method was used to build network relation map (NRM) among dimensions/criteria. Based on the NRM obtained from DEMATEL method, the ANP was employed to obtain the relative weights of the criteria. Interdependencies among criteria and dimensions were calculated by DEMATEL. After normalization of total relation matrix among criteria which was said to be the inner dependencies was put into the appropriate parts in unweighted supermatrix. Total relation matrix for dimensions/clusters gathered from DEMATEL was used to obtain weighted supermatrix. After supermatrix was formed, it was raised to large powers to obtain steady-state matrix. A case of OLED (Organic Light Emitting Diode) display technology selection where alternatives were generated by patent co-citation (PCA) method was proposed in order to verify the applicability of proposed method.

Tseng (2011a) proposed a framework based on fuzzy set theory, DEMATEL and ANP for measuring firm environmental knowledge management capability. From the literature review, 5 aspects and 22 criteria were determined. Group of experts made the linguistic assessments using triangular fuzzy numbers (TFN) for ANP and DEMATEL. Then, they were translated into crisp equivalences to make the matrix calculations. Using DEMATEL method, the inner dependency matrix with prominence and relation values was obtained. Then, the inner dependency matrix obtained from DEMATEL and the unweighted supermatrix in ANP were multiplied.

After normalization of the unweighted supermatrix, it was raised to sufficient powers to acquire steady-state matrix. Finally, success factors were determined and managerial implications were given.

Tseng (2009) combined DEMATEL and ANP methods for effective solution of municipal solid waste management (MSW) in Metro Manila region in Philippines. DEMATEL method was used to capture the cause and effect relationships among criteria. Unlike the traditional ANP, pairwise comparisons between criteria were not made, instead, the inner dependency matrix obtained from DEMATEL method was put into the unweighted supermatrix. Ultimately, limiting supermatrix was calculated and the best MSW management alternative was selected.

5.5.1.3 NRM of ANP

Ho et al. (2011) proposed a novel MCDM model with DEMATEL, ANP, and VIKOR for exploring portfolio selection based on CAMP (abbreviation of a formula, noting that expected return on a security is impacted by risk-free rate, expected market return, and beta of security). The DEMATEL method was used for obtaining NRM. Based on NRM obtained by DEMATEL method, ANP was carried out to find the weights of each criterion. Finally, the most suitable investment was identified by VIKOR. An empirical case using semiconductor portfolio as an example was displayed to illustrate the application of the proposed method. (Chen & Tzeng, 2011) Chen & Tzeng (2011) used the same methodology for selecting the aspired Intelligent Assessment Systems (IAS) for teaching materials. An empirical study of Mandarin Chinese teaching materials was provided.

Lin, Hsieh, and Tzeng (2010) proposed a model based on the DEMATEL, ANP and TOPSIS methods for evaluating vehicle telematics system (VTS). The DEMATEL method was used to construct the NRM of criteria. Based on the NRM obtained by DEMATEL method, ANP was applied to obtain the relative weights of dependent criteria. Finally, TOPSIS method was implemented to determine the most appropriate VTS product with respect to required utilities and services. As an empirical study, commercial VTSs from North America, Western Europe, Japan and Taiwan were investigated.

Kuo (2011) proposed a hybrid MCDM framework based on fuzzy DEMATEL method and a new fuzzy MCDM method comprised of additive/non-additive measures and TOPSIS for optimal location selection for an international distribution center. First, hierarchical/network structure was determined by fuzzy DEMATEL method. Then, fuzzy performance matrix was established. Next, the evaluation criteria weights were determined using AHP/ANP methods. Afterwards, aggregation fuzzy assessment calculations were made with respect to additive and non-additive measures. In the case of non-additive measurements, λ-measure and fuzzy integral were used. On the other hand, TOPSIS method was used for fuzzy evaluations under the condition of independent criteria. Finally, fuzzy synthetic performance of each alternative was calculated and alternatives were ranked. In the study, comparative study with some conventional methods was given and their ineffectiveness was argued.

Wu, Lin, and Chang (2011) developed a performance evaluation indices based on balanced scorecard (BSC), DEMATEL, ANP and VIKOR. DEMATEL method was not only used for capturing interrelationships among BSC perspectives and criteria but also construct the NRM. Based on the DEMATEL analysis, ANP was used to obtain the criteria weights. Finally, obtained criteria weights were used by VIKOR method to rank the alternatives. An empirical study was provided with the extension education centers of three Taiwanese universities.

Tsai and Chou (2009) proposed DEMATEL, ANP and ZOGP based method for selecting optimal management system under resource constraints. The balanced scorecard approach was adopted to define criteria for selection of management systems. DEMATEL method was used to capture the interrelationships among criteria and draw the NRM. Based on the NRM obtained by DEMATEL, ANP was carried out to determine the priorities for selecting management systems. Finally, the priority weights were used in ZOGP formulation which was used to select the alternatives without exceeding resource constraints. An illustrative example was provided to explain the steps of the proposed method.

Chen and Chen (2010b) constructed an innovation support system (ISS) for Taiwanese higher education institutes to evaluate innovation performance by DEMATEL, fuzzy analytic network process (FANP) and TOPSIS methods. DEMATEL method was used to obtain total influence matrix regarding dimensions and to construct the impact relation map (IRM). Based on the IRM obtained from DEMATEL, FANP pairwise comparisons were performed. Then, weights of criteria in the innovation support system were calculated. Finally, using criteria weights acquired from FANP, three alternatives, namely; research-intensive universities, teaching-intensive universities, and professional-intensive universities were ranked by TOPSIS method.

Tzeng and Huang (2011) proposed a novel MCDM framework for selecting a global manufacturing strategy by considering both aspects of global manufacturing and logistics. In the study, DEMATEL technique, ANP, VIKOR and GRA were used. Firstly, the interrelationships between criteria were captured and the NRM was constructed by DEMATEL method. Secondly, based on the NRM obtained from DEMATEL, the weights of each criterion versus the goal of the MCDM problem was derived by ANP. Then the priorities of global manufacturing and logistics system were ranked and selected by VIKOR method. Also the same calculation was made by TOPSIS so as to compare the results. Finally, in order to enhance the current global manufacturing and logistics system, possible manufacturing and logistics strategies were ranked for achieving aspired levels by GRA method.

Tsai and Hsu (2010) presented a hybrid model based on DEMATEL and ANP for selecting cost of quality (CoQ) model. DEMATEL method was used to obtain criteria structure incorporating cause and effect relationships. Based on the criteria structure acquired from DEMATEL method, ANP method was utilized to select the optimal CoQ model among ABC model, PAF model, process cost model and opportunity model.

Lo and Chen (2012) proposed a hybrid procedure for evaluating risk levels of information security under various security controls. The developed methodology applied the DEMATEL method, ANP and Fuzzy Linguistic Quantifiers-guided Maximum Entropy Order-Weighted-Averaging (FLQ-MEOWA) operator. The DEMATEL method was used to construct the interrelationships among security control areas and to obtain IRM. Based on the IRM derived from DEMATEL, the ANP method was applied to obtain the weights of criterion. In the last step, FLQ-OWA operator was used to aggregate impact values assessed by experts.

Liou et al. (2011) proposed a new hybrid multiple criteria decision-making model, combining the DEMATEL method, ANP and fuzzy preference programming for outsourcing service provider selection in airline company. The DEMATEL method was used to obtain the IRM, which captured the interrelationships among dimensions. Based on the IRM captured by the DEMATEL method, ANP method was applied to derive the weights of criteria. Since human judgments inherently contain imprecision, fuzzy preference programming was used for pairwise comparisons of elements in each level with respect to their relative importance toward their control criterion. Then, the weights obtained from fuzzy preference programming were put into corresponding positions in the unweighted supermatrix. In the next step, the limiting power of the un-weighted supermatrix was calculated until a steady state condition reached and criteria weights were obtained. Lastly, simple additive weighting (SAW) method was performed for selection of outsourcing partner/provider.

Tsai, Leu, et al. (2010) introduced a MCDM approach combining the DEMATEL method, ANP and zero-one-goal programming (ZOGP) to form an optimal project task sourcing portfolio in accordance with the goal and criteria of operations strategy and management. Using the DEMATEL method, total relation matrixes were constructed for perspectives and criterion to obtain IRM. Then, unweighted supermatrix was formed in accordance with the IRM obtained from DEMATEL method. Finally, the ANP weights combined with the ZOGP, taking into account the project budget and related resource constraints of the organization.

Chen and Chen (2010a) proposed a Pro-Performance Appraisal System (PPAS) for higher education in Taiwan using the DEMATEL method and fuzzy ANP. Using the DEMATEL method, the level of interrelationships between the dimensions of the evaluation system was determined. By setting a threshold value in total relation matrix, impact relation map (IRM) was obtained. The fuzzy ANP method was then carried out and the most important criteria were designated.

Hung (2011) proposed a DEMATEL, ANP and fuzzy goal programming based approach for divergent supply chain (DSC) forecasting and multi-objective production planning, incorporating the key factors of precise costing, managerial constraints, competitive advantage analysis and risk management. A case study of a consumer-oriented cell phone DSC forecasting and multi-objective production planning were presented. In terms of activity based costing approach, cost drivers at various levels were used to measure activity amounts and costs where the constraints of total budget were obtained. The five forces analysis was carried out to find the best competitive advantage for companies. The DEMATEL method was used to determine the interdependencies between five forces. Then, based upon the criteria structure derived from DEMATEL method, ANP method carried out in order to determine the criteria weights. Besides, value at risk was used to measure the potential largest loss. In the final step, fuzzy goal programming approach was applied so as to obtain the optimal product mix and the worst profit.

Tsai, Chou, and Lai (2010) and Tsai, Chou, and Leu (2011) proposed integrated model for evaluating websites in tourism and airline industries, combining DEMATEL, ANP and the modified VIKOR method. The DEMATEL method was used to capture the interrelationships among criteria and build a visual structural map. Then, ANP was used to obtain the relative weights of evaluation criteria. Next, the modified VIKOR method, which offers more rational formulas for computing gaps regarding the measure of closeness to an ideal alternative was used to rank the websites. Finally, weight-variance analysis (WVA) was suggested for improving website quality with limited resources.

Tsai, Hsu, et al. (2010) integrated the DEMATEL, AHP/ANP, ZOGP methods to find an optimal corporate social responsibility (CSR) programs and cost evaluation in the international tourist hotel. The DEMATEL method was used to detect complex relationships among the cost and differentiation advantage criteria. After the interdependencies were captured by DEMATEL, AHP or ANP method, depending on whether the criteria dependent or not, carried out to obtain criteria weights. Then, the AHP/ANP results were applied to ZOPG formulation. For each CSR program candidate, binary decision variables were used. After finding the optimal CSR program portfolio and optimal resource inputs, the ABC (Activity Based Costing) was applied to calculate CSR program's costs in order to acquire accurate cost information.

Lee and Tu (2011) proposed a hybrid MCDM combining DEMATEL, ANP and VIKOR methods to discuss the influence relationships and criterion weights among cash flow, weighted average cost of capital, tax shield and their sub-factors based on the Modigliani-Miller (MM) theorem. The DEMATEL method was used to establish a criteria structure. Based on the criteria structure obtained by DEMATEL, ANP was applied to derive criteria weights. Finally, VIKOR method was used to rank the alternatives. A case study, ranking of leading panel makers with respect to their company value, was proposed to show the applicability and effectiveness of the proposed model.

Padhi and Aggarwal (2011) investigated the revenue management (RM) problem of hotel industry, incorporating artificial neural network (ANN), DEMATEL, ANP and fuzzy goal programming approaches. First, real option values of hotel rooms (commodities) were forecasted and categorized by ANN methodology. Then, DEMATEL method was carried out to obtain network of criteria structure. Afterwards, ANP was used to acquire the dependent criteria weights. Finally, fuzzy goal programming approach was established in order to fix the optimal quota for each commodity where individual commodities contribute to the total profit of the hotel. A case study was provided using hotel-chain of mid-range hotels in non-metro cities to show the applicability of the proposed method.

Liou and Chuang (2010) used DEMATEL, ANP and VIKOR methods for selection of outsourcing providers. The DEMATEL method used to build criteria structure, addressing the dependent relationships among criteria. Based on the criteria structure obtained by DEMATEL, relative weights of criteria was determined using ANP method. The VIKOR method was then used to rank the alternatives. The Taiwanese airline case was examined to demonstrate the method.

Liou (2012) proposed a hybrid model combining DEMATEL, fuzzy preference programming (FPP) and ANP for selecting suitable partners for strategic alliances. DEMATEL method was used to construct the IRM for interrelated criteria. Then, based on the IRM derived from DEMATEL, ANP method was carried out. Pairwise comparisons between criteria were conducted using fuzzy preference programming which deals with the ambiguity and vagueness in human judgments by interval comparisons, and then the weight vectors were put into the unweighted supermatrix. Calculating the limiting supermatrix, the alternative with the largest value was selected. The method was demonstrated using data from a Taiwanese airline.

Tsai and Kuo (2011) integrated DEMATEL, ANP, and zero-one goal programming (ZOGP) methods for evaluating entrepreneurship policies and decision analysis for small and medium enterprises (SMEs) in Taiwan. The DEMATEL method was used to grasp the network structure considering the causal interrelations. Then ANP method was applied in order to obtain weights of alternatives. Finally, ZOGP was constructed to select the optimal alternatives and utilize resources without exceeding limited constraint.

Tzeng et al. (2007) proposed a hybrid MCDM model addressing the both dependent and independent criteria for evaluating intertwined effects in e-learning programs. Independent criteria were separated from dependent ones by factor analysis, whereas dependent criteria were treated by DEMATEL, fuzzy measures and fuzzy integral. When dependent criteria structure was obtained by DEMATEL method, fuzzy measures were employed to derive central criteria weights. The effectiveness of final affected elements was calculated using fuzzy integral approach.

Finally, the overall utility was determined by AHP method, combining independent and dependent criteria to obtain each e-learning program score.

Kuo and Liang (2011) proposed a novel hybrid decision-making model for selecting locations in a fuzzy environment, which deals with dependent and independent criteria at the same time. A new structural model, which combines fuzzy grey relational analysis and DEMATEL method, was introduced. For the purpose of structuring the criteria involved in MCDM problem, initial relation matrix was obtained by fuzzy grey relation calculations. After constructing the initial relation matrix, normalization matrix and total-relation matrix were calculated. On the basis of criteria structure, criteria weights were determined for dependent and independent criteria, using fuzzy ANP and fuzzy AHP, respectively. Then, based upon the fact that whether the criteria were dependent or not, addition measurement was used based on the concept of positive ideal and negative ideal solutions (TOPSIS), whereas non-additive measurement performed using Choquet integral. Finally, the closeness coefficients were calculated and the best alternative was selected from a set of feasible alternatives. A numerical example was proposed for the case of selecting an international distribution center in major Asian ports to show the effectiveness of the proposed model.

5.5.1.4 Weighting Clusters of ANP

Yang, Shieh, and Tzeng (2011) proposed a MCDM method, combining the modified VIKOR, DEMATEL and ANP for information security risk control assessment. Unlike the traditional method of normalizing unweighted supermatrix in ANP, which assumes that each cluster has the same weight, the DEMATEL method was applied to improve normalization process. So, different degrees of influence among clusters obtained using DEMATEL method was used for normalizing the unweighted supermatrix, which led to weighted supermatrix in ANP. When the criteria weights considering the interdependencies among criteria were obtained by DEMATEL and ANP, the modified VIKOR method was carried out. In modified VIKOR method, positive and negative ideal points were determined according to decision makers' aspired and tolerable levels, rather than maximum and minimum values of all criterion functions. Hence, the modified VIKOR method, which is called VIKORRUG, determined the compromise solution, with maximum group utility for the majority, and minimum individual maximal regret for the opponent. A case study, dealing with the information security controls of the Taiwanese government organization, was proposed to help IT managers diagnose the information security problem by emphasizing which control areas need to be improved by uncovering gaps in the control areas.

Yang and Tzeng (2011) proposed an integrated MCDM method, combined with DEMATEL and ANP. The DEMATEL method was served as the basis for normalization of supermatrix process in ANP, since using average method to obtain the weighted supermatrix deemed unrealistic in the context of non-equal cluster weights. In the study, the DEMATEL method was adopted to overcome the shortcomings of the traditional method in the supermatrix normalization process, in which influence degrees of criteria regarded as equal. When total relation matrix was acquired from the DEMATEL method, it is normalized and multiplied by the unweighted supermatrix to deal with interdependencies among criteria more accurately and realistically. A case study was proposed for selecting the appropriate vendor in a purchase project.

Chen and Chen (2011) explored the critical creativity criteria by DEMATEL and ANP methods. DEMATEL method was adopted to construct IRM. Based on the IRM derived by DEMATEL, ANP was utilized in order to obtain the final weights of each criterion. Once the unweighted supermatrix was formed, it was weighted by total influence matrix of dimensions extracted from DEMATEL method. Finally weighted supermatrix was raised to large powers and the critical criteria were indicated.

Chen, Lien, and Tzeng (2010) used hybrid MCDM model based on DEMATEL and ANP for evaluation of environmental watershed plans. DEMATEL method was used to construct the NRM. Then based on the NRM obtained by DEMATEL, unweighted supermatrix was formed involving Saaty’s 9-point scale. Then, multiplying unweighted supermatrix by each dimension weight obtained by DEMATEL method, the weighted supermatrix was acquired. Finally, limiting power of the weighted supermatrix was calculated. An empirical case study in the Pei-Keng brook in Taiwan was provided.

Wu, Chen, and Chen (2011) evaluated the Intellectual Capital (IC) in Taiwan’s higher education system by DEMATEL and ANP. DEMATEL method was used to develop Impact Relation Map (IRM). Based on the IRM, ANP method was exploited by using Saaty’s 1-9 measurement scale. After unweighted supermatrix was formed, weighted supermatrix was obtained by multiplying unweighted supermatrix with the influence matrix of dimensions acquired from DEMATEL. Finally, the most important criteria were determined and managerial insights were given**.**

5.5.2 DEMATEL with AHP

Chou, Sun, and Yen (2012) used the combination of fuzzy AHP and fuzzy DEMATEL methods in order to evaluate the criteria for human resources of science and technology (HRST). Fuzzy AHP method was used for weighting the each criterion and the DEMATEL method was used to establish the contextual relationship among those criteria. In the study, the weightings obtained from fuzzy AHP interpreted as the importance of criteria that might be improved for the short time period purposes. However, improvements in the effect-oriented criteria provided by DEMATEL were seen as an opportunity that could give a better performance for the long term strategies or goals. Hence, valuable managerial insights were given by the empirical study results.

Wu and Tsai (2011b) presented an integrated approach based on AHP and DEMATEL for evaluating the criteria of auto spare parts industry. Since AHP does not take into account the cause-effect relationship between criteria, the importance of criteria derived from it was considered to be a short term improvement opportunity. On the other hand, DEMATEL method was used to depict the cause and effect relationship between criteria and the results were interpreted as improvement chances in the long term perspective. A case study was conducted by survey incorporating seven local auto parts manufacturers in Taiwan.

Najmi and Makui (2011) proposed a hybrid model based on DEMATEL and AHP for measuring supply chain's performance. AHP method was used to derive criteria weights in accordance with the hierarchy. Then calculation of the interdependencies between criteria was made by DEMATEL method and the eigenvector of the total relation matrix was found. Next, the desirability index which considers interdependencies as a multiplier in the equation was calculated. Finally, the supply chain performance weighted index was established. A case study in Iranian automotive company was presented to compare the benchmark chain with the ideal chain.

5.5.3 DEMATEL for Weighting Criteria

Tseng (2009a) combined Fuzzy and Grey Theory with DEMATEL method for real estate agent service quality expectation ranking under uncertainty. First, criteria weights and criteria rating values for alternatives, described by grey numbers were obtained. Then inner dependency matrix was established using DEMATEL method. Then, the performance matrix including rating values and the inner dependency matrix obtained by DEMATEL method multiplied so as to establish grey-fuzzy DEMATEL decision matrix. After normalizing the obtained decision matrix, it was multiplied by criteria weights in order to establish the weighted normalized grey-fuzzy DEMATEL decision matrix. In the last step, ranking order of alternatives obtained by calculating grey possibility degrees.

Geng and Chu (2012) proposed a new importance-performance analysis (IPA) approach for customer satisfaction evaluation in supporting product-service system (PSS) design. The customer perception PSS quality attributes were identified through two-dimensional Kano questionnaire survey with linguistic variables, expressed in vague values. The criteria were separated into five Kano quality attributes. Then, DEMATEL method, using vague sets to express decision makers' evaluation, was performed to obtain cause-effect relationships among the derived attributes. Next, the importance of each attributes was obtained by taking into account the mutual influence relationships among them. The mutual influence of each attribute (D-R values) is added to modified importance of each attribute. The attributes with negative importance considered as much less important and ignored in IPA map. Finally, the IPA map was drawn and based on the quadrants that a criterion reside, criterion which needed to be improved were determined.

Dalalah, Hayajneh, and Batieha (2011) presented a hybrid model combining fuzzy DEMATEL and modified TOPSIS methods. Fuzzy DEMATEL was used for capturing the influential relationships and for weighting of criteria. In the graphical representation of the causal relationships, where axes represent prominence and effect of criteria, respectively, distance from the origin of a criteria calculated as the weight of a criteria. Then, modified fuzzy TOPSIS method model was implemented. In order to find the distances from ideal and anti-ideal points, distance based similarity assessment which is proposed by (Williams & Steele, 2002) was used. Application of developed method was presented for the supplier selection problem of a company in Jordan.

Tseng (2011b) proposed a hybrid MCDM method combining fuzzy TOPSIS and DEMATEL method using linguistic preferences for benchmarking hot spring hotels by evaluating the service quality expectations. DEMATEL method was used to capture the interrelationships among criteria and modify the criteria weights. Once the inner dependency matrix was obtained from DEMATEL method, the prominence and relation values were calculated. Based on the results of the DEMATEL calculations, the decision group evaluated the importance of criteria using triangular fuzzy numbers (TFN). Afterwards, the fuzzy decision matrix was constructed by incorporating modified criteria weights. Finally, fuzzy TOPSIS calculations were performed and the ranking alternatives were obtained.

Liaw et al. (2011) studied a reliability allocation problem using the maximal entropy ordered weighted averaging (ME-OWA)-based DEMATEL method. The ME-OWA method was used to derive ISPE (which are subsystem allocation factors, namely, intricacy, state of art, performance time, and environment) values. The DEMATEL method was used to discover the causal relationships between criteria and calculate the weights of factors. The R value in DEMATEL method was modified so as to represent the average severity of influence for each alternative. Then, the D-R value was used for the calculation of the weighting vector. Finally, a reasonable reliability rating was allocated into subsystems or components. The proposed method was applied to a fighter aircraft's digital flight control computer.

Hiete et al. (2011) investigated the cause and effect relationships between variables in a composite indicator for disaster resilience. Fuzzy DEMATEL method was extended to take into account trapezoidal membership functions. Along with the DEMATEL results, the weightings of sub-indicators were corrected by proposed equation in which dependency paradigm was taken into consideration. In a case study, Germany was assessed in terms of resilience at county level.

5.5.4 DEMATEL for Clarifying Interactions

Huang et al. (2007) developed an analytical framework for defining an innovation policy portfolio for Taiwan's SIP Mall industry using Delphi method, DEMATEL, Grey Relation Analysis (GRA) and Cluster Analysis. DEMATEL method served as a tool to deduct the key innovation requirements. Setting a threshold value in DEMATEL method, industry innovation requirements derived using Delphi were reduced.

Lin and Wu (2008) proposed a Fuzzy DEMATEL method to gather group ideas and analyze the cause-effect relationship of complex system under fuzzy environment. Involved criteria separated into cause and effect groups in order to help decision makers focus on those criteria that provide great influence. An empirical study using proposed method with triangular membership functions applied to R&D project selection of a Taiwanese company. The result showed the most important criteria in cause and effect groups, respectively.

Shieh, Wu, and Huang (2010), based on SERQUAL survey and DEMATEL method, identified the key success factors for hospital service quality. First, the survey was taken among patients by asking the importance of each criterion with a Likert-type 5 point scale. Then, seven major criteria were determined to be in a higher priority. Then, the DEMATEL method was constructed based on 19 experts. After total relation matrix was obtained, prominence and effect values were calculated. Based on the prominence and effect values, criteria which were considered to be great importance ranked.

Wu and Tsai (2011a) applied the DEMATEL method in auto spare parts industry in Taiwan. The survey was conducted in order to obtain causal relation matrices for seven dimensions and thirty criteria. Based on the purchasing managers' opinions, DEMATEL method computations were performed for both dimensions and criteria. In the study, digraphs of dimensions and criteria were provided. The traditional point of view that the most researchers rank the importance of criteria, based on the prominence values, was used and discussions regarding the causal relationships among criteria were given.

Jassbi, Mohamadnejad, and Nasrollahzadeh (2011) proposed a fuzzy DEMATEL methodology, for modeling cause and effect relationships of the strategy map in the Balanced Scorecard. Considering the real fuzzy data and group decision making judgments, developed method implemented. The prominence and effect groups were distinguished from each other, for the purpose of giving managerial insights. In addition to the proposed method, some valuable discussions were provided. The first issue raised to be discussed was that reversed relationships might exist among criteria, which account for that one object from upper perspective might produce negative value. Besides, necessity of managerial interpretations while calculating the weights for any relationship between objectives was emphasized.

Liou, Yen, and Tzeng (2008) suggested a method for building an effective safety management system for airlines, incorporated organization and management factors. The fuzzy DEMATEL method was applied to visualize the structural relations and to identify key factors in safety management system (SMS) for airlines. First, the fuzzy assessments of evaluators, the degree to which factor affects another one, were obtained. When calculated the fuzzy average matrix, it was deffuzified according to center of area method to determine the best non-fuzzy performance (BNP) value of fuzzy numbers due to practical considerations. Then traditional DEMATEL calculations were made and impact relations map was drawn. Finally, based on the prominence values, the most important factors were determined. Also, some discussions were provided with respect to IRM.

Lin and Tzeng (2009) used the DEMATEL method for a value-created system of science parks by establishing the value structures. In the study, industrial structures were established using DEMATEL by comparing various industrial clusters. Two well-known science parks in Taiwan were considered as an empirical study. The evaluation dimensions and key performance criteria of science parks were determined by DEMATEL method. The results might provide an insight for vendors and authorities of science (technology) parks.

Lee et al. (2010) used DEMATEL method with the theoretical extension of technology acceptance model (TAM2). Causal relationships and interaction levels between TAM2 variables were calculated using DEMATEL method. Hence, the core variables of TAM2 which carry the important aspects while promoting a new technology to resolve complicated problems were identified. A case study in Taiwanese plasma etching manufacturing industry was provided. In a similar study, the same authors (Lee et al., 2011) combined fuzzy DEMATEL and TAM2 methods. A questionnaire was designed to capture the ambiguity and vagueness in human judgments without which adhering to single or definite numbers so the real perception of respondents was expressed. The application of the proposed method in Taiwan optronics manufacturing industry was provided.

Tsai & Cheng (2011) analyzed the key performance indicators (KPIs) for e-commerce and internet marketing of elderly products combining the Balanced Scorecard (BSC) and DEMATEL methods. DEMATEL method was used for identifying the level of importance of BSC constructs, and for revealing the interdependencies among them, which gives valuable managerial insights to achieve functions of prevention, continuous improvement and innovation.

Hu et al. (2011) combined gap analysis, multiple regression analysis, importance-performance analysis (IPA) and the DEMATEL method to improve the order-winner criteria. The DEMATEL method was used to solve the influence of cause-effect relationships of quality characteristics. A case study was proposed in Taiwan's network communication equipment manufacturing industry to state that the proposed model could help correct the core problems of company and improve quality control therefore strengthen to win and be compatible for orders.

Liou et al. (2008) proposed a fuzzy DEMATEL method for building an effective safety management system (SMS) in airlines. A group of experts was formed from Taiwan Civil Aviation Administration (CAA), Aviation Safety Council (ASC) and industry practitioners to assess the questionnaire with fuzzy linguistic variables. Then, fuzzy DEMATEL method was carried out for visualizing the SMS for airlines and identifying the key factors.

Wu (2012) presented a structural evaluation methodology to determine the key performance indicators (KPI) in the strategy map of the balanced scorecard (BSC) for banking institutions. First, the most important evaluation indicators of banking performance were determined in regard to BSC perspectives. Then, DEMATEL method was applied to pinpoint the causal relationships between KPIs and the key factors that help performance improvement were determined. Hence, the most essential performance indicators helped constructing a strategy map, by which management could better allocate resources that need improvement. An empirical study was provided to give some insights that managers could apply the proposed methodology.

Wang et al. (2012) proposed a model to identify the divisions in the project organization who were responsible for the poor performance in the design project.

The satisfied importance analysis (SIA) was used to evaluate the degree of importance and satisfaction of each division. Then, a cause and effect impact relations map was constructed for each division by DEMATEL method. Afterwards, SIA and DEMATEL methods were integrated to analyze and identify the key divisions. Once the key divisions were determined, the improvement solutions and top managements actions were generated for those sections. Proposed model applied to high-tech facility design project in Taiwan to demonstrate the strengths of the model.

Cheng et al. (2012) proposed a method based on importance-performance and gap analysis (IPGA) and DEMATEL methods for enhancing service quality improvement strategies of fine-dining restaurants. Using IPGA analysis, advantages, disadvantages and improvement priority of restaurant service quality were determined by measuring the service quality of fine-dining restaurants. Since the IPGA analysis did not take the dependencies into account, the DEMATEL method was utilized to discover the causal relationships between service quality dimensions. Finally, the IPGA and DEMATEL methods were combined to discover the improvement directions of quality characteristics. The study was applied to eight fine-dining restaurants in Taipei City of Taiwan.

Fu, Zhu, and Sarkis (2011) evaluated green supplier development programs (GSDPs) using formalized grey-based DEMATEL methodology. Gathering grey input data from management, DEMATEL method was carried out by transforming the grey data into crisp data. After a series of DEMATEL calculation steps, a final causal relationship diagram with associated analysis was achieved. In the study, GSDP relationships to each other and the importance of GSDP for the organization was emphasized. An application in telecommunications provider organization was provided.

Wu, Chen, and Shieh (2010) examined the performance criteria of employment service outreach program personnel using DEMATEL method. Constructing the hierarchical structure of criteria used in performance evaluation, traditional

DEMATEL calculations were performed. Then, based on the prominence and effect values, importance of criteria was prioritized. Proposed method was applied for evaluation of outreach personnel with real data, where respondents were selected from the in Bureau of Employment and Vocational Training, Council of Labor Affairs of Executive Yuan in Taiwan.

Hsu (2012) proposed a model based on factor analysis and DEMATEL for evaluating criteria for blog design and an empirical study was conducted. . First, exploratory factor analysis (EFA) and principal component analysis were adopted in order to simplify and categorize factors. Questionnaire used in the study was answered by undergraduate students from National Chiao Tung University who are experienced in blogging. Then, DEMATEL method, consulting to the experts, was carried out so as to gain insight into the causal relationships between factors and degrees of influence. Therefore, the criteria that largely influence the other criteria were determined as the most important.

Wu, Lan, and Lee (2011) explored the decisive factors affecting and organization's software as a service (SaaS) adoption using DEMATEL method. Presuming that perceived risk (PR) would act as barriers to the SaaS adoption while perceived benefits (PB) would serve as driving forces, PR and PB were handled by DEMATEL approach separately to obtain the respective cause and effect factors. In addition, the results of cause effect analysis were combined into PB-PR matrix, which offered visible insights. By proposed method, the root causes that hindered the SaaS adoption were determined, also provided with clues to develop more effective marketing strategies in order to enhance growth of the SaaS business. A case study was provided with the company wishing to use SaaS as a way to perform sourcing stratagies in terms of customer relationships management and human capital management.

Seyedhosseini et al. (2011) employed balanced scorecard (BSC) and DEMATEL methods for extracting leanness criteria. Using Delphi method and nominal group technique, lean perspectives and objectives were identified. Then, going through the literature review, criteria for performance measurements were determined. Afterwards, cause and effect relationships between objectives and their priorities were discovered by DEMATEL method. Hence, lean strategy map was constructed. Ultimately, intensity of influence of objective on each other, final measurements and priorities were determined.

Liaw et al. (2011) investigated the most influential factors in selecting supply chain management suppliers using fuzzy DEMATEL method. A fuzzy DEMATEL questionnaire was designed and sent to seventeen professional purchasing personnel in the electronic industry. Then, the calculation of fuzzy DEMATEL method with triangular fuzzy numbers was performed. Finally, the strategy map and causal diagram was drawn. Therefore, the crucial factors were determined which would give managers valuable insights to focus on those with the greatest influence on other factors.

Rahman and Subramanian (2011) proposed a framework for end-of-life (EOL) computer recycling operations in reverse supply chains. The DEMATEL method was used for identifying the most important factors in EOL computer recycling operations as well as drawing the cognition map to visualize the interrelations between factors. An empirical study was provided with the two Melbourne-based companies engaged in recycling operations.

Wu and Lee (2007) questioned how to enrich global managers' competencies by the fuzzy DEMATEL method. Fuzzy DEMATEL method was used for segmenting the eight intelligence quotients (IQs) into meaningful portions and the IQs within the cause group were regarded as the critical criteria. An empirical study was proposed in Taiwanese high tech company.

Li and Tzeng (2009b) used DEMATEL method for discovering the key services to attract Semiconductor-Intellectual-Property (SIP) users and providers to an SIP Mall. A questionnaire was sent to companies from the "2003 Semiconductor Industry Yearbook" and based on the traditional DEMATEL calculations, prominence and effect groups were determined. Factors had the higher prominence and effect values than the average was considered as the key factors. Also, some insightful comments were made with respect to the prominence and effect values, to better manage the essential factors for having an attractive SIP Mall.

Zhou, Huang, and Zhang (2011) figured out the critical success factors (CSF) of emergency management by fuzzy DEMATEL method. Importance of each factor and cause-effect relationships were acquired by fuzzy DEMATEL method. Then, five CSFs were determined with respect to cause-effect relationship diagram.

Fan, Suo, and Feng (2012) proposed a 2-tuple fuzzy linguistic representation based DEMATEL method to identify IT outsourcing risk factors. First, the initial relation matrix was transformed into the 2-tuple form. Then, group direct relation matrix was formed by aggregating matrices in the 2-tuple form. Next, normalized direct relation matrix, indirect relation matrix and total relation matrix were constructed. Afterwards, prominence and relation values of risk factors were calculated. Importance of risk factors was determined by prominence values while classification of risk factors was determined based on relations. A case study was implemented in a Chinese resource company involved in investment, trade and logistic service of mineral resources. After calculations of the proposed method were completed, insightful comments with suggestions regarding how to reduce the influence of different risks were provided.

Wu (2012) carried out fuzzy DEMATEL method in order to segment the critical factors for successful knowledge management (KM) implementations. Fuzzy DEMATEL method used to unveil the causal relationships among factors and distinguish them into cause and effect groups. Factors in the cause group were considered to be the critical factors since they are difficult to move. A case study was implemented in a Taiwanese company involved in broadband wireless networking solutions.

Jeng and Tzeng (2012) studied the effect of social influence toward medical professionals behavioral intention when a new Clinical Decision Support System (CDSS) was introduced. Fuzzy DEMATEL method was used to explore the causal relationships among the significant Unified Theory of Acceptance and Use of Technology (UTAUT) variables. In the study, it was found that the social influence does not matter in the behavioral intention to use CDSS for medical professionals.

Tan and Tan (2012) investigated the factors for successful transformation from single purpose smart-card-based e-micropayment scheme to multi-purpose scheme. DEMATEL method was used to discover the interrelationships between cause and effect of criteria and obtain the structural model visually. Based on the literature review and interviews with business experts, eleven factors were classified into non-participant-related and participant-related factors. Then, DEMATEL questionnaire was filled up by general users. After traditional DEMATEL calculations, structural maps of the participant related and non-participant related factors were drawn. Finally, managerial implications and insightful comments were made.

Lin et al. (2011) investigated the case of an Integrated Circuit (IC) design company in order to discover the core competences of semiconductor industry. Seven core competences were determined and intertwined effects between them were explored by utilizing DEMATEL method. After traditional DEMATEL calculations, cause and effect groups were determined. Hence criteria in the cause group were considered as core competences. Drawing the impact relation map, managerial insights and recommendations for improvement of core competences were provided.

Hsu et al. (2011) applied the DEMATEL method to discover the influential factors of carbon management in green supply chain for improving the performance of suppliers regarding carbon management. Using DEMATEL method, causal relationships among criteria were captured and the most influential factors were determined. Managerial insights and some suggestions for supplier selection were provided.

Lin (2011) evaluated green supply chain management (GSCM) practices using fuzzy DEMATEL method. Considering practices, performances and external pressures, eight influential factors were examined and their structural model was constructed. According to findings, the most influential factors were determined and managerial implications were given.

Tseng (2009b) proposed an extension of DEMATEL method (EDEMATEL method) for assessing service quality perceptions. Unlike considering only one group's perceptions in traditional DEMATEL method, the perceptions of both employee and customer on service quality were integrated into one cause and effect model. A linguistic scale and its corresponding triangular fuzzy numbers were used to handle ambiguity and vagueness in human judgments. Calculating the proposed method, causal diagram was drawn and the most influential factors were determined.

Li and Tzeng (2009a) proposed a method to determine the threshold value for the DEMATEL by the maximum de-entropy algorithm. Since taking all the intertwined relationships into account was costly and impractical, appropriate threshold value identification was of great importance. A real case study was implemented to find the interrelationships between the services of a Semiconductor Intellectual Property Mall. Furthermore, results of the proposed method compared with the traditional method, and showed that the proposed method was favorable and helpful to select a consistent threshold value.

Tseng (2010) analyzed environmental practices in knowledge management capability (EKMC) using fuzzy DEMATEL approach. A cause and effect model was developed and the most influential factors were determined. A case study was conducted in an integrated healthcare services provider company.

Tseng & Lin (2009) developed a cause and effect model of municipal solid waste management in Metro Manila using fuzzy DEMATEL method. Calculating the constructed model, criteria were separated into cause and effect groups. Furthermore, MSW management criteria were prioritized so that authorities could focus the more important criteria in the decision making process.

Chang and Cheng (2009) proposed an algorithm to evaluate the orderings of risk for failure problems by fuzzy ordered weighted averaging (OWA) and DEMATEL. In the study, the OWA operator was utilized to calculate ordered weights of severity, occurrence, and detection rather than using traditional RPN numbers. OWA weights were calculated based on the maximum entropy (Fuller & Majlender, 2001) with different α values. Then aggregated value was calculated. Afterwards, DEMATEL procedure was performed. Priority for assessing failure risk was ranked by D-R values of DEMATEL.

Suo, Feng, and Fan (2011) proposed an extension of the DEMATEL method for trapezoidal fuzzy numbers in uncertain environment. First, linguistic judgments obtained from decision makers were transformed into trapezoidal fuzzy numbers. Then these numbers were aggregated by operations of trapezoidal fuzzy numbers. Next, uncertain overall-relation matrix was established. Afterwards, overall intensities of influencing and influenced correlations with prominence and relation values were determined. Finally, centroid method was used to defuzzify the results and causal diagram was drawn. An illustrative example on risk factor identification of a R&D alliance was proposed.

Fekri, Aliahmadi, and Fathian (2008) identified cause and effect factors for the agile New Product Development (NPD) process by fuzzy DEMATEL method. Using explanatory factor analysis, six critical success factors were extracted among 32 main items regarding agile NPD projects in Iranian companies. Then, fuzzy DEMATEL method was utilized to discover the cause and effect relationships between factors. Finally, the most important and the critical factors were determined. The critical factors were considered to be the ones affected mostly by other factors.

Dytczak and Ginda (2009) investigated the building repair policy choice criteria by DEMATEL method and zero unitarisation method (ZUM). Criteria were assessed in terms of economical merits and social merits influence. A sensitivity analysis was made to assign each attribute to three classes, namely, the key attributes, the medium attributes and the small importance attributes.

CHAPTER SIX
FUZZY COGNITIVE MAPS

6.1 Introduction

Modeling and analysis of dynamic systems has long been examined using different techniques (Campello & Amaral, 2003). Despite the growing interest towards the dynamic systems, critical difficulties are inherent in the modeling phase. From the knowledge representation perspective, decision makers are able to use two modeling options in order to model dynamic systems; namely quantitative and qualitative methods (Aguilar, 2005).

The first group includes quantitative techniques such as operational research methods (i.e. mathematical programming), statistical approaches (i.e. statistical regression) and system dynamics. Despite the appealing properties of these methods, significant exertion and sophisticated knowledge from the analyst in which the required data is generally beyond the domain of interest is needed. For instance, methods based upon mathematical programming require set of difference or differential equations. On the other hand, the system dynamics approach demands identification of stock/flow elements and necessitates numerical data in order to express the rate of change in stock level in time when the flow is occurred. Furthermore, as stated by (Pedrycz, Lam, & Rocha, 1995) , quantitative approaches might not provide satisfactory results due to nonlinearities, unknown physical structure and systemic feedbacks.

The second group of methods helps overcome the difficulties of quantitative models. The second group consists of, namely, qualitative approaches where this group of models strives to describe the given system qualitatively (Pedrycz et al., 1995). In this type of studies, descriptors of the system variables are given much broader terms, rather than strict values. As an example, the terms negative, zero or positive would be system descriptors. In the qualitative models, two class of approaches come into prominence; fuzzy models (Pedrycz, 1996; Zadeh, 1965) and neural networks (Kosko, 1992). Fuzzy Cognitive Maps is the combination of these two classes of approaches.

The Fuzzy Cognitive Maps grounds on the cognitive maps introduced by (Axelrod, 1976). We have already mentioned about causal/cognitive mapping in earlier sections. Although the cognitive mapping approach is the cornerstone in analysis of social systems, its inadequacy for modeling and describing much more complex systems is realized. Main reason of the insufficiency of cognitive maps is related to limited representation of the relations among system variables.

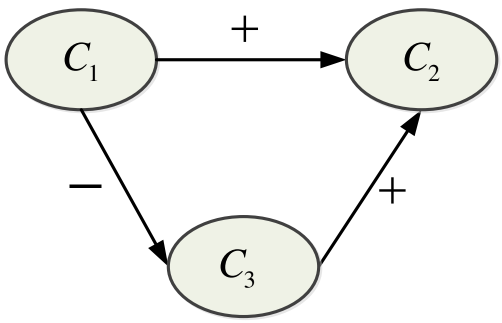


Figure 6.1a A simple example of cognitive map

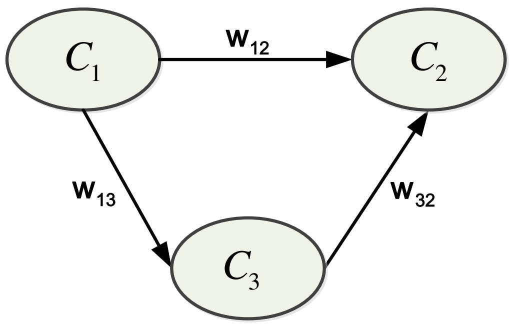


Figure 6.1b A simple example of fuzzy cognitive map

After ten years of the Axelrod's pioneering study, Fuzzy Cognitive Maps is introduced by (Kosko, 1986) in order to overcome the restrictions of conventional cognitive maps. In Fuzzy Cognitive Maps, causal relations among system variables are not only described by sign, but also degrees of causal relations or weights are incorporated with the links. Comparison of a simple Cognitive Map and Fuzzy Cognitive Maps is seen in Figure 6.1. These links between system variables can be described by fuzzy terms such as medium, weak, very strong, strong and etc. Fuzzy terms used to depict relationship between system variables actually quantify how much a concept causes the linked concept. The weight values among the system variables are stored in the connection matrix and usually they are normalized into $[-1,+1]$ interval. It is important to note that the Fuzzy Cognitive Maps implies fuzziness in the concepts and weight matrix by linguistic variables, being an Artificial Neural Network is the other feature that the Fuzzy Cognitive Maps possess.

The Fuzzy Cognitive Maps encapsulates the power of Artificial Neural Networks (ANNs) with an additional advantage: the conventional ANNs behave like "black boxes", neurons and connections among neurons do not have specific meanings; on the contrary, in the Fuzzy Cognitive Maps, concepts and connections have precise meanings (Tsadiras, 2008). An example of a Fuzzy Cognitive Maps (Maruyama, 1963; Tsadiras, 2008) in the civil engineering field is given.

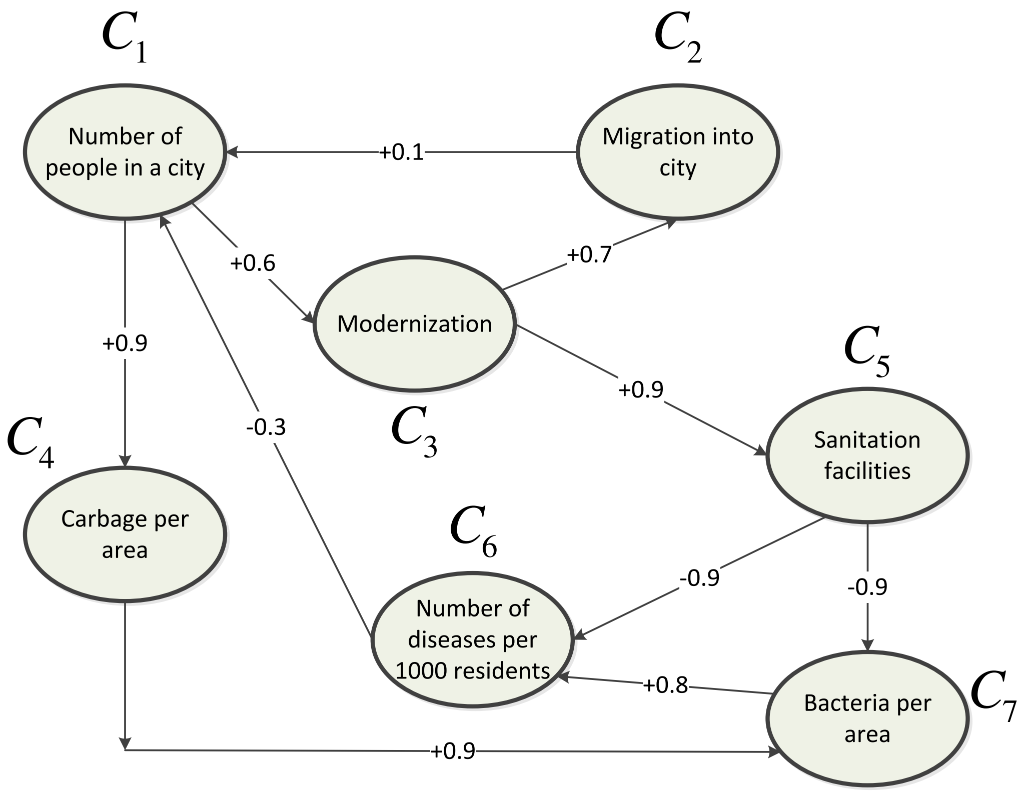


Figure 6.2 Example of a Fuzzy Cognitive Map (Maruyama, 1963; Tsadiras, 2008)

In the example above, the concepts take the value in the interval $[-1,+1]$. The causal relationships are represented by the links between concepts. For instance, the first concept, which is the population of the city, causes increase of the concept 4 which stands for carbage per area. Similary, the carbage per capita leads to increase in bacteria per capita. The linguistic terms are quantified and placed into the arcs denoting influence degrees with sign. As can be seen in the example, interpretation of the Fuzzy Cognitive Maps is very clear which makes it very attractive to be used in MCDM. The formal definitions and the mathematical underpinning of the Fuzzy Cognitive Maps is given below.

6.2 Mathematical Foundations

6.2.1 Formal Representation

A Fuzzy Cognitive maps is 4-tuple $(N, \boldsymbol{E}, \boldsymbol{C}, f)$ (Kosko, 1986) where:

1. $N = \{N_1, N_2, \ldots, N_n\}$ is the set of n concepts which are the nodes in the graph.
2. $\boldsymbol{E} : (N_i, N_j) \rightarrow e_{ij}$ is a function which associate e_{ij} to a pair of concepts (N_i, N_j) where the e_{ij} stands for weight of the arc from N_i to N_j (N_i, N_j). If $i = j$, e_{ij} is equal to zero. Hence $\boldsymbol{E}(N \times N) = (e_{ij}) \in K^{n \times n}$
3. $\boldsymbol{C} : N_i \rightarrow C_i$ is a function that it associates each concept N_i with sequence of its activation degrees such as for $t \in \boldsymbol{N}$, $C_i(t) \in L$ given its activation degree at the moment t. $\boldsymbol{C}(0) \in L^n$ indicates the initial vector and specifies initial values of all concept nodes and $\boldsymbol{C}(t) \in L^n$ is a state vector at certain iteration t.
4. $f : \boldsymbol{R} \rightarrow L$ is a transformation function, which includes recurring relationship on $t \geq 0$ between $\boldsymbol{C}(t+1)$ and $\boldsymbol{C}(t)$. The state updating equation is (Xirogiannis, Stefanou, & Glykas, 2004):

$$\forall i \in \{1, \ldots, n\}, \qquad C_i^{(t+1)} = f\left(C_i^{(t)} + \sum_{\substack{j=1 \\ j \neq i}}^{n} e_{ji} C_j^{(t)} \right) \tag{6.1}$$

Each concept value in the state vector is calculated for successive iterations based on the formula above. System dynamic simulation is conducted by performing the iterations, and state of the system is observed. The state vector provides the current value of each node/variable in a particular iteration.

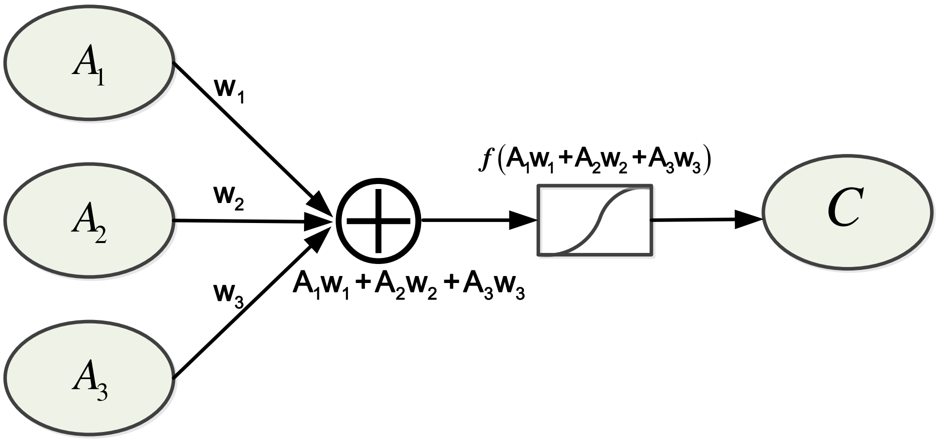


Figure 6.3 Inference scheme on Fuzzy Cognitive Maps (Carvalho, 2013)

On the other hand, the transformation function f is used to bind the weighted sum in the range $[0,1]$. Figure 4 (Carvalho, 2013) shows the inference scheme on Fuzzy Cognitive Maps.

6.2.2 Transformation Functions

The causal effects of the concepts determined the value of the other concept in the next iteration by integrating transformation function on the weighted sum. There are many transformation functions in the literature used in Fuzzy Cognitive Maps studies. Selection of a transformation function is a part of design of the Fuzzy Cognitive Maps. The most frequently used transfer functions can be summarized as (Bueno & Salmeron, 2009; Tsadiras, 2008) :

- The bivalent function

$$f(x) = \begin{cases} 0, & if\ x \leq 0 \\ 1, & if\ x > 0 \end{cases} \tag{6.2}$$

  Bivalent transformation function allows developing binary Fuzzy Cognitive Maps where binary variables used to indicate whether the concept is activated or not.

- The trivalent function

$$f(x) = \begin{cases} -1, & if\ x < 0 \\ 0, & if\ x = 0 \\ +1, & x > 0 \end{cases} \tag{6.3}$$

In trivalent function, three states of Fuzzy Cognitive Maps are developed. If the activation level of concept is 1, then it means that the concept is increasing. On the contrary, activation value of -1 implies that the concept is decreasing. The 0 value of concept stands for the stable state of the concept.

- The unipolar sigmoid (logistic) function

$$f(x) = \frac{1}{1 + e^{-\lambda x}} \tag{6.4}$$

where parameter $\lambda > 0$ determines the slope of the function, which is used to obtain the proper shape. The output of the unipolar sigmoid function lies within $[0,1]$.

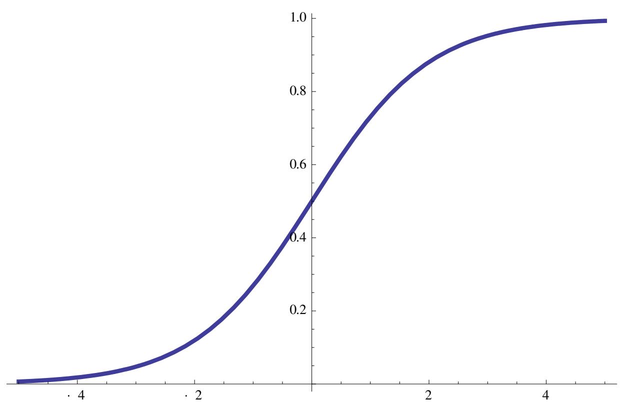


Figure 6.4 Shape of the sigmoid function

- The hyperbolic tangent function

$$f(x) = \tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \tag{6.5}$$

The hyperbolic tangent function is suitable when concept values can take negative values and their values lie within $[-1,1]$.

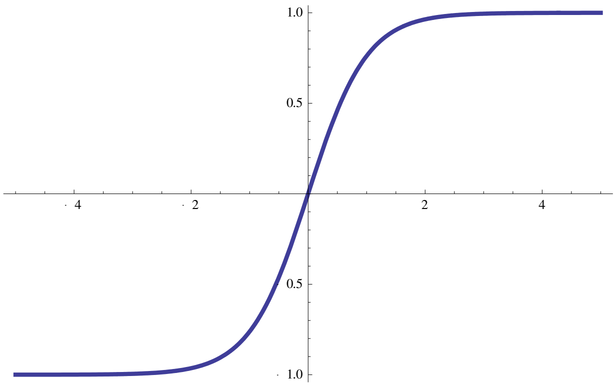


Figure 6.5 Shape of the hyperbolic tangent function

6.2.3 System Behavior

When one of the transformation functions is applied to interacting concepts, the Fuzzy Cognitive Maps is expected to converge one the following states (Kosko, 1997)

- A fixed point is reached, which means that the state vector stabilizes and remains unchanged for sequential iterations.
- A limit cycle is reached with the state vector falls in a loop of repeating states indefinitely.
- A chaotic behavior is occurred where the state vector keeps changing with iterations and no repeating states are observed.

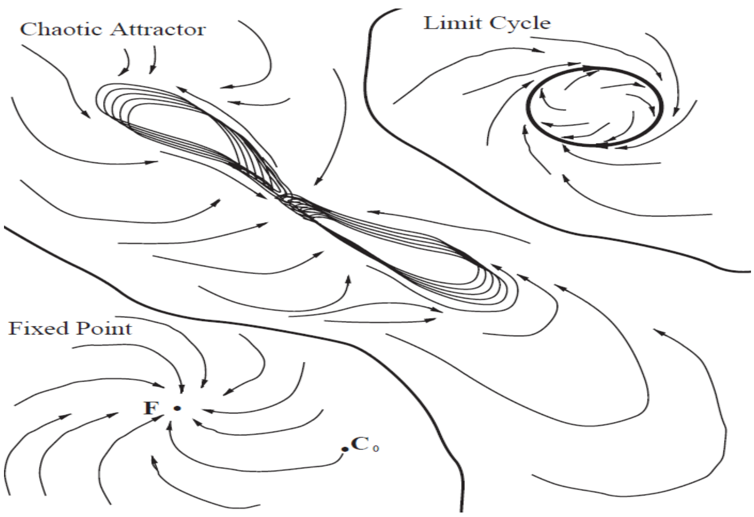


Figure 6.6 Possible situations of state vector (Kosko, 1997)

According to Tsadiras (2008), if bivalent or trivalent transformation functions are used, then a fix point situation (also known as hidden pattern, fixed-point attractor) or limit cycle situations are observed. If a continuous function is selected such as sigmoid function or tangent hyperbolic, then chaotic behavior can appear in addition to fixed point or limit cycle.

6.3 Construction of Fuzzy Cognitive Maps

There are many methods to construct Fuzzy Cognitive Maps in the literature. The method used highly depends on the nature of the study under consideration. Generally, two mainstream approaches exist to model Fuzzy Cognitive Maps: expert-based methods and computational methods.

In expert-based methods, the domain knowledge of expert is the key element of developing Fuzzy Cognitive Maps. For the sake of increased credibility of the model, group of experts, rather than individualistic perspective, are gathered together and different ideas are fused. As a very generic framework, expert based Fuzzy Cognitive Maps follows three steps (Khan & Quaddus, 2004):

- Significant concepts are determined.
- Causal relationships among the identified concepts are captured.
- The strength of causal relationships is estimated.

Maybe the real challenge in construction of the Fuzzy Cognitive Maps is to articulate the strengths of influence among concepts. Especially, for the large models consisting of hundreds of concepts, determination of influence degrees demands extra effort. Nevertheless influential relationships require strength of relationship and the sign. Accordingly, subjective interpretation of decision maker is inevitable. A common approach to determine relationships between concepts can be given as (Khan & Quaddus, 2004):

- Decide on the sign of each relationship

- Assign linguistic terms to each relationship, for example, strong, weak, very strong.
- Map linguistic terms into numeric values, for example, a linguistic variable medium corresponds to 0.5 and 0.75 means strong.

Linguistic expressions make assigning the degrees of causality easier for decision makers because experts tend to evade specification of exact numerical values. To find numerical values of the degrees of causality, MCDM techniques might be helpful.

Other family of methods to construct Fuzzy Cognitive Maps is computational methods, which involve a learning algorithm and available data to support forming of the map. Actually, computational models are very important especially when expert knowledge is not adequate or available. Automatic construction of Fuzzy Cognitive Maps is a hot research topic; results might have great impacts on MCDM methods in terms of problem structuring and hierarchy/network construction directly from data. We only focus on the expert based side of the Fuzzy Cognitive Maps in the scope of this thesis; hence, computational methods will be our primary concern in the future studies.

6.4 Fuzzy Decision Maps

6.4.1 Fuzzy Cognitive Maps versus Fuzzy Decision Maps

Fuzzy Decision Maps is a different interpretation of Fuzzy Cognitive Maps in order to model dependence and feedback among criteria in MCDM problems (Yu & Tzeng, 2006). The main difference between Fuzzy Decision Maps and the Fuzzy Cognitive Maps is that in Fuzzy Decision Maps, output of the analysis returns a $n \times n$ matrix where n denotes the number of criteria. However in Fuzzy Cognitive Maps, the state vector contains concept values and has dimensions of $1 \times n$.

A Fuzzy Decision Maps is 4-tuple $(N, \boldsymbol{E}, \boldsymbol{C}, f)$ where $N = \{N_1, N_2, \ldots, N_n\}$ are concepts, E is the connection matrix, C is the state matrix, $C^{(0)}$ is the initial matrix, $C^{(t)}$ is the state matrix after t iterations, and f is the threshold function. The threshold function can be bivalent, trivalent, sigmoid, hyperbolic tangent etc.

The updating equation is given by (Yu & Tzeng, 2006) :

$$\boldsymbol{C}^{(t+1)} = f(\boldsymbol{C}^{(t)}\boldsymbol{E}), \ \ \boldsymbol{C}^{(0)} = I_{n\times n} \tag{6.6}$$

Where $I_{n\times n}$ denotes the identity matrix.

Let us give an example to illustrate the operations conducted in Fuzzy Decision Maps.

Example 6.1: Suppose that we criteria structure shown in Figure 6.7. The mapping of linguistic terms into numerical values is indicated in the figure as well.

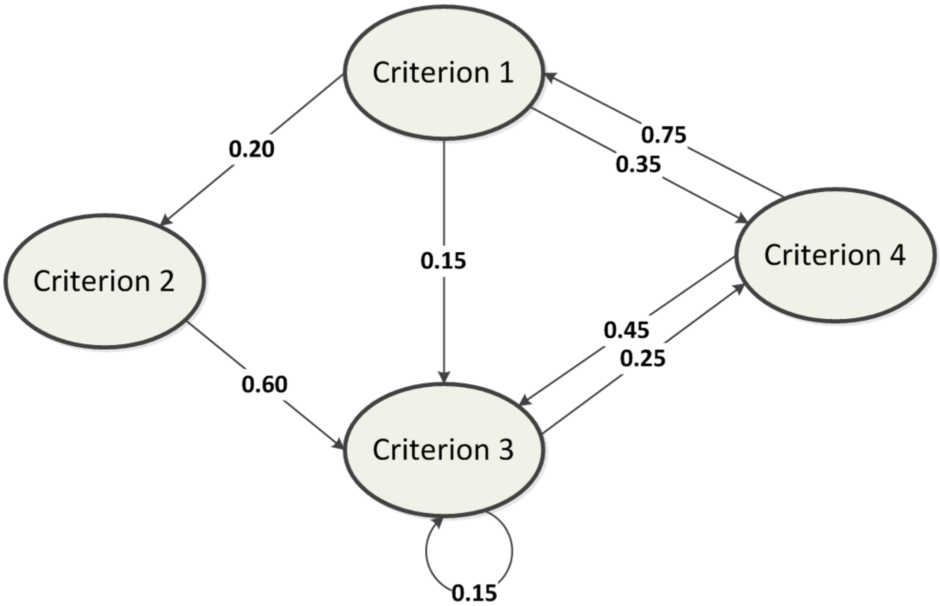


Figure 6.7 The initial criteria structure

The initial connection matrix is as follows:

$$
\boldsymbol{E} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}
\begin{array}{c}
\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\
\begin{bmatrix} 0 & 0.20 & 0.15 & 0.35 \\ 0 & 0 & 0.60 & 0 \\ 0 & 0 & 0.15 & 0.25 \\ 0.75 & 0 & 0.45 & 0 \end{bmatrix}
\end{array} \tag{6.7}
$$

Applying the updating equation with hyperbolic tangent function, the resulting state matrix happens to be :

$$
\boldsymbol{E} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}
\begin{array}{c}
\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\
\begin{bmatrix} 0.4024 & 0.2733 & 0.6188 & 0.5687 \\ 0.1696 & 0.0339 & 0.6923 & 0.2283 \\ 0.3228 & 0.0645 & 0.4586 & 0.4463 \\ 0.7914 & 0.1570 & 0.7488 & 0.4335 \end{bmatrix}
\end{array} \tag{6.8}
$$

The resulting criteria structure is shown in Figure 6.8. The dashed arrows emphasize the additional interactions captured by Fuzzy Decision Maps.

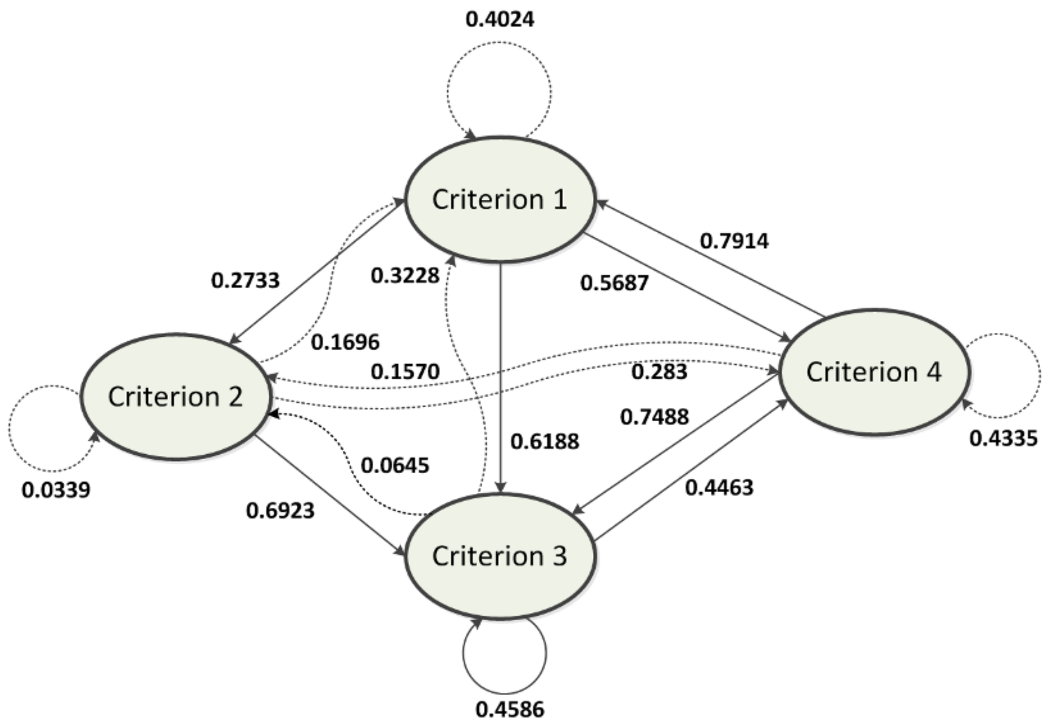


Figure 6.8 Resulting criteria structure after Fuzzy Decision Maps

Similarly, the resulting state matrix with the linear threshold function is as follows:

$$
\boldsymbol{E} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0.5903 & 0.3181 & 0.9218 & 0.7871 \\ 0.2426 & 0.0485 & 0.9542 & 0.3235 \\ 0.4043 & 0.0809 & 0.5903 & 0.5391 \\ 1.3749 & 0.2749 & 1.4070 & 0.8329 \end{bmatrix} \end{array} \tag{6.9}
$$

6.4.2 Fuzzy Decision Maps versus DEMATEL

DEMATEL method is the special case of the Fuzzy Decision Maps where the threshold function is linear $(f(x) = x)$. For mathematical proof, interested readers can refer to (Tzeng et al., 2009). An interesting question would be that why do decision makers use Fuzzy Decision Maps instead of DEMATEL method or which method should be selected? Actually these questions require some discussion and some remarks can be made as follows.

Because the Fuzzy Decision Maps generalizes the DEMATEL method, all the capabilities of DEMATEL method is preserved, and also additional features are attained. DEMATEL method falls behind Fuzzy Decision Maps with following respects (Tzeng et al., 2009):

- DEMATEL method assumes that criteria are linearly interactive because the underlying threshold function is linear in DEMATEL method. However, Fuzzy Decision Maps allows utilizing different threshold functions.
- In DEMATEL method, the diagonal elements of the initial relation matrix are assumed to be zero. Fuzzy Decision Maps permits to manage criteria with self-feedback loops which mean that the diagonal elements do not have to be zero.
- DEMATEL method obeys the rules of discrete time Markov property where the criteria pass different states with dynamical linearly. However, Fuzzy Decision Maps permits dynamical non-linearity in transition of states.

Although some methods seem to superior than the others in some respects, ease of use and practicality is also very important for selection of the modeling method. Decision makers are free to evaluate and select the best method which best fits their problem.

6.5 Related Review

Yu and Tzeng (2006) employed eigenvalue method, fuzzy cognitive maps and weighting equation to MCDM problems with dependence and feedback effects. In the study, ANP method was criticized as to assumption that network structure of criterion was known in advance and excessive pairwise comparison questions which, sometimes, could be difficult to answer. Fuzzy cognitive maps (fuzzy decision maps) was used to handle the problems described above. In the study, fuzzy cognitive map was used to obtain the connection matrix which indicates the degree of causal relationships between criteria. The local weights of criteria were extracted by pairwise comparisons and eigenvalue method. In the last step, proposed weighting equation combined connection matrix and the local weights to reach the final weights. The pure-linear and the hyperbolic-tangent functions were tried out as activation functions in fuzzy cognitive maps. Results from different activation functions and ANP were compared. The proposed method was found to be more practical and convenient approach in comparison with ANP/AHP.

Tzeng et al. (2009) demonstrated that the fuzzy decision maps (FDM) which was comprised of the eigenvalue method, the fuzzy cognitive maps and the weighting equation was the generalization of the DEMATEL method. It was mathematically proved that the DEMATEL was the specific case of FDM where the threshold function was linear. In the study, two numerical examples were provided.

Akgun, Kandakoglu, and Ozok (2010) proposed fuzzy integrated vulnerability assessment model (FIVAM) integrating fuzzy set theory, Simple Multi-Attribute Rating Technique (SMART) and fuzzy cognitive maps in a group decision making setting. Fuzzy SMART method was used to find out the weight of vulnerability criteria. FCM was used to simulate the system vulnerability behavior based on the interdependencies among the system functions. Finally, considering the results of FCM simulation, the system function and system vulnerabilities were recalculated and the most critical functions and components were ranked. Proposed method was illustrated by a hypothetical example.

Xiao, Chen, and Li (2012) introduced FCM and soft fuzzy set model to solve supplier selection problem considering operational risks. First, the importance among criteria was compared and the local weight vector was obtained. Then, FCM was constructed and degree of influence among criteria was gathered from the experts. Next, FCM was trained by Particle Swarm Optimization algorithm to obtain the steady-state matrix. Afterwards, global criteria weights were obtained from weighting equation proposed by (Yu & Tzeng, 2006). Finally, weighted performance ratings of each criterion with respect to alternatives were evaluated by fuzzy soft sets and the most desirable suppliers were ranked.

CHAPTER SEVEN
INTERPRETIVE STRUCTURAL MODELING

7.1 Introduction

A preliminary condition to make better decisions in complex environments requires identification and understanding of the system. System theory has given birth to many methodologies in this direction where the structural models constitute the greatest part. The interpretive structural modeling (ISM) is developed by (Warfield, 1973a, 1973b, 1974a, 1974c) in order to understand complex systems or situations. ISM is a computer-assisted methodology used to capture the fundamental relationships among system elements. The mathematical foundations of the ISM rely on graph theory, group decision making, discrete mathematics, computer assistance and social sciences (Warfield, 1974a, 1974c).

The output of the ISM is a directed graph indicating the contextual relationships among system variables. It is a convenient tool to model unclear, unstructured, poorly defined mental models so as to transform the system into understandable and visualized model to be used for various analyses. The ISM consists of nodes and links where nodes represent the system variables; while links show the direction of the dependency. However, the interpretation of links is not clear in ISM method. In the literature, ISM is considered as a tool to model contextual relationships (Kannan, Pokharel, & Sasi, 2009; Sushil, 2012) which, in turn, means that the meaning of the relationship is dependent upon the context and should be evaluated around the semantic sphere of the involved system. That is why the ISM involves the term interpretive, which refers to fact that relationship between variables needs the decision makers' interpretation. For instance, (Lin & Yeh, 2013) stated that they have chosen the phrase "lead to" as the meaning of the linkages between system variables used in their study. According to (Sushil, 2012), the directed link in the ISM accounts for 'will help achieve'. For instance if there exist a directed arrow from the target X to target Y, then the interpretation would be that X will help attain the target Y. Actually, this interpretation is very similar to that of DEMATEL method which invokes causal relationships. Furthermore, we argue that in the context of MCDM, the applications of ISM method coincides with the causal relations modeling. A supporting argument of our assertion would be that there are many studies employing cause and effect relationships (Thakkar et al., 2007) in order to decompose the system into smaller ones to be able to depict the criteria structure and get a deeper inside about driving and dependent criteria. Furthermore, (Chang, Hu, & Hong, 2012) stated that they used ISM in order to illustrate the causal structure of factors. Many other examples can be found in the literature in which the causality or causal dependency is selected as a sort of contextual relationship.

Aside from the term interpretive, the other term structural refers to the graphical representation of the system variables by which the system structure is extracted from the complex system. For that reason it is also a modeling technique providing with the digraph of the system with many interrelated variables. Also ISM is first design to be used by group of experts, though it can also be used individually. The main steps of the ISM methodology are as follows (Faisal, Banwet, & Shankar, 2006):

- All the variables influencing the system are listed. These variables might be different depending on the system under consideration.
- The contextual relationships are determined among the variables by analyzing the pairs of variables.
- A structural self-interaction matrix (SSIM) is established. SSIM is the pairwise relationships among system variables.
- Reachability matrix is formed from the SSIM and transitivity property is inspected. The transitivity is an important assumption of ISM method.
- The reachability matrix is divided into different levels.
- A directed graph is drawn in compliance with the reachability matrix and the transitive links are deleted.
- The digraph is transformed into ISM and nodes are named with the statements.
- Possible inconsistencies are checked in the conceptual model and necessary fixes are made.

7.2 Components of Interpretive Structural Modeling

7.2.1 Structural Self-Interaction Matrix (SSIM)

The ISM methodology generally requires the opinions of the group of experts. Assuming that the contextual relationship is determined, the group of experts is questioned regarding the relationships for each variable and the associated directions of these relationships. Four symbols are used to describe the relationships among variables. Also the names variable and enabler are used interchangeably in ISM methodology. The symbols used to describe relationships among enablers i and j as follows (Sushil, 2012) :

- V: for the relation from enabler i to enabler j (not for both directions)
- A: for the relation from enabler j to enabler i (not for both directions)
- X: for the relation from enabler i to enabler j and from j to i .
- O: there is no relation between the enablers.

For the SSIM, each pairwise interaction is determined with the symbols above by filling the questionnaire form.

7.2.2 Reachability matrix

The SSIM is configured into a binary matrix by transforming the entries of the SSIM into zeros and ones. Hence the initial reachability matrix A is obtained. Four situations can emerge as follows (Chang et al., 2012):

- When the entry (i, j) is V, the entry (i, j) of initial reachability matrix takes the value of 1, and the entry (j,i) becomes 0.
- When the entry (i, j) is A, the entry (i, j) of initial reachability matrix takes the value of 0, and the entry (j,i) becomes 1.
- When the entry (i, j) is X, the entry (i, j) of initial reachability matrix takes the value of 1, and the entry (j,i) becomes 1 too.

- When the entry (i, j) is O, the entry (i, j) of initial reachability matrix takes the value of 0, and the entry (j, i) becomes 0 too.

Then the reachability matrix T is calculated as (Tzeng & Huang, 2011b):

$$T = (A + I) \tag{7.1}$$

$$T^* = T^k = T^{k+1},\ k > 1 \tag{7.2}$$

where the identity or unit matrix is represented as I, A is the initial relation matrix, power is denoted by k, and T^* is the ultimate reachability matrix under the boolean arithmetic. The boolean addition and multiplication operations can be seen in Table 7.1 (Tzeng & Huang, 2011b):

Table 7.1 Boolean multiplication and addition operations

Binary Addition	Binary Multiplication
$1+1=1$	$1\times1=1$
$1+0=1$	$1\times0=0$
$0+1=1$	$0\times1=0$
$0+0=0$	$0\times0=0$

7.2.3 Level Partitions

Once the reachability matrix is obtained, the reachability and antecedent sets for each enabler are calculated as the first step of extracting the diagraph (Warfield, 1974b, 1974c). The reachability set includes the variable itself and the other variables which are influenced by the variable under consideration. On the other hand, the antecedent set contains the variable itself and the other variables which have influence on the variable under consideration (Faisal et al., 2006).

When the reachability and antecedent sets are derived, the intersection of these sets is calculated. The enablers having the same reachability and intersection sets occupy the top most position in the hierarchy. The top level elements can be reached from the elements in other levels, but they are not able to reach the other levels. Once the top level enablers are identified, assuming we have more than one enabler at the top level, they are excluded from the other elements. The enablers with the same reachability and intersection sets are searched within the remaining variables and the process repeats itself iteratively for finding next levels.

We give a very basic example show the main steps of the ISM method.

Example 7.1 (Yang et al., 2008): Suppose that the enablers of our system are the family members namely: father, mother, son, daughter, and cat denoted by $F, M, S, D,$ and C respectively. The contextual relationship is not specified in the original article so we leave it to readers' imagination. Many different relationships can be considered according to given variables. We skip the first step and directly give the binary relation matrix as follows:

$$T = \begin{array}{c} \\ F \\ M \\ S \\ D \\ C \end{array} \begin{array}{c} \begin{array}{ccccc} F & M & S & D & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{7.3}$$

The corresponding relationship graph is shown in Figure 7.1.

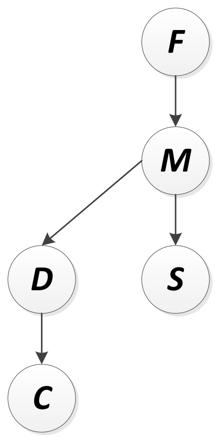


Figure 7.1 The relationship among variables in example 7.1

The initial reachability matrix is obtained as:

$$T = A + I = \begin{array}{c} \\ F \\ M \\ S \\ D \\ C \end{array}\begin{array}{c} \begin{array}{ccccc} F & M & S & D & C \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{7.4}$$

Thereafter, the limiting power of initial reachability matrix is calculated.

$$T^2 = \begin{array}{c} \\ F \\ M \\ S \\ D \\ C \end{array}\begin{array}{c} \begin{array}{ccccc} F & M & S & D & C \end{array} \\ \begin{bmatrix} 1 & 1 & 1^* & 1^* & 0 \\ 0 & 1 & 1 & 1 & 1^* \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{7.5}$$

$$T^3 = \begin{array}{c} \\ F \\ M \\ S \\ D \\ C \end{array}\begin{array}{c} \begin{array}{ccccc} F & M & S & D & C \end{array} \\ \begin{bmatrix} 1 & 1 & 1^* & 1^* & 1^{**} \\ 0 & 1 & 1 & 1 & 1^* \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{7.6}$$

It is found out that $T^3 = T^4$ so the $k = 3$ satisfies the stopping criteria. Based on the final reachability matrix, the reachability, antecedent and priority sets are calculated as given in table 1. Unlike the original article, we solve the problem via a step by step approach in order to illustrate the all iterative steps clearly.

Once the reachability and antecedent sets are determined, the intersection elements of two sets are calculated. In our situation, two candidate elements can be placed in the first level. The enabler C has been chosen heuristically, and assigned to level 1.

Table 7.2 Determination of the first level of the hierarchy

e_i	$R(t_i)$	$A(t_i)$	$R(t_i) \cap A(t_i)$	Level
F	F, M, S, D, C	F	F	
M	M, S, D, C	F, M	M	
S	S	F, M, S	S	
D	D, C	F, M, D	D	
C	$\boldsymbol{C}$	F, M, D, C	$\boldsymbol{C}$	**1**

In the second iteration, the enabler C is removed from the table. The next element for whom the intersection of reachability and antecedent sets overlaps with the reachability set is assigned to second level. As is seen in table 3, the enabler D is placed into level 2.

Table 7.3 Determination of the second level of the hierarchy

e_i	$R(t_i)$	$A(t_i)$	$R(t_i) \cap A(t_i)$	Level
F	F, M, S, D	F	F	
M	M, S, D	F, M	M	
S	S	F, M, S	S	
D	$\boldsymbol{D}$	F, M, D	$\boldsymbol{D}$	**2**
C	$-$	F, M, D	-	

This iterative process is continued for determining the third, fourth and fifth level of elements in the hierarchy. The related calculations are shown in table 4, 5 and 6.

Table 7.4 Determination of the third level of the hierarchy

e_i	$R(t_i)$	$A(t_i)$	$R(t_i) \cap A(t_i)$	Level
F	F, M, S	F	F	
M	M, S	F, M	M	
S	$\boldsymbol{S}$	F, M, S	$\boldsymbol{S}$	**3**

D	**-**	F,M	**-**	**2**
C	–	F,M	-	**1**

Table 7.5 Determination of the fourth level of the hierarchy

e_i	$R(t_i)$	$A(t_i)$	$R(t_i) \cap A(t_i)$	Level
F	F,M	F	F	
M	$\boldsymbol{M}$	F,M	$\boldsymbol{M}$	**4**
S	**-**	F,M	–	**3**
D	**-**	F,M	**-**	**2**
C	–	F,M	-	**1**

Table 7.6 Determination of the fifth level of the hierarchy

e_i	$R(t_i)$	$A(t_i)$	$R(t_i) \cap A(t_i)$	Level
F	$\boldsymbol{F}$	F	$\boldsymbol{F}$	**5**
M	**-**	F	–	**4**
S	**-**	F	–	**3**
D	**-**	F	**-**	**2**
C	–	F	-	**1**

As a result, the hierarchical structure belonging to example 7.1 is drawn as seen in Figure 7.2.

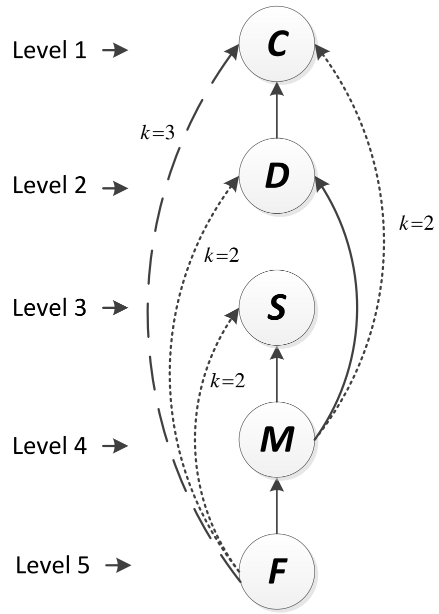


Figure 7.2 Resulting hierarchical structure of the elements in example 7.1

7.3 MICMAC Analysis

The Cross-Impact Matrix Multiplication Applied to Classification (MICMAC) analysis (Mandal & Deshmukh, 1994) is used to understand the dependencies among the system variables under consideration. Especially, MICMAC analysis is a pragmatic tool to analyze the driver power and dependence power of the variables.

In the reachability matrix, 1 and 0 values in the columns imply dependence whereas values of rows indicate the driver power. In table 7.7, the dependence and driver power values related to example 7.1 are shown.

Table 7.7 Driver and dependence powers of example 1

	F	M	S	D	C	**Driver Power**
F	1	1	1	1	1	**5**
M	0	1	1	1	1	**4**
S	0	0	1	0	0	**1**
D	0	0	0	1	1	**2**
C	0	0	0	0	1	**1**
Dependence	**1**	**2**	**3**	**3**	**4**	

MICCMAC analysis separates the variables into 4 clusters namely, autonomous, dependent, linkage and independent. In figure 7.3, an example of driver power and dependence diagram can be seen. Explanations of the four clusters can be given by (Chang et al., 2012):

- Autonomous factors which have weak dependence and weak driving powers. Factors fall into this cluster are said to be relatively disconnected from other elements since they have few links with other variables. Generally they have low impact on the system under consideration.
- Dependent factors weak driver power and strong dependence power. These factors are highly dependent upon the other factors. Also they are weak drivers so they are generally place at the top of the digraph of ISM.

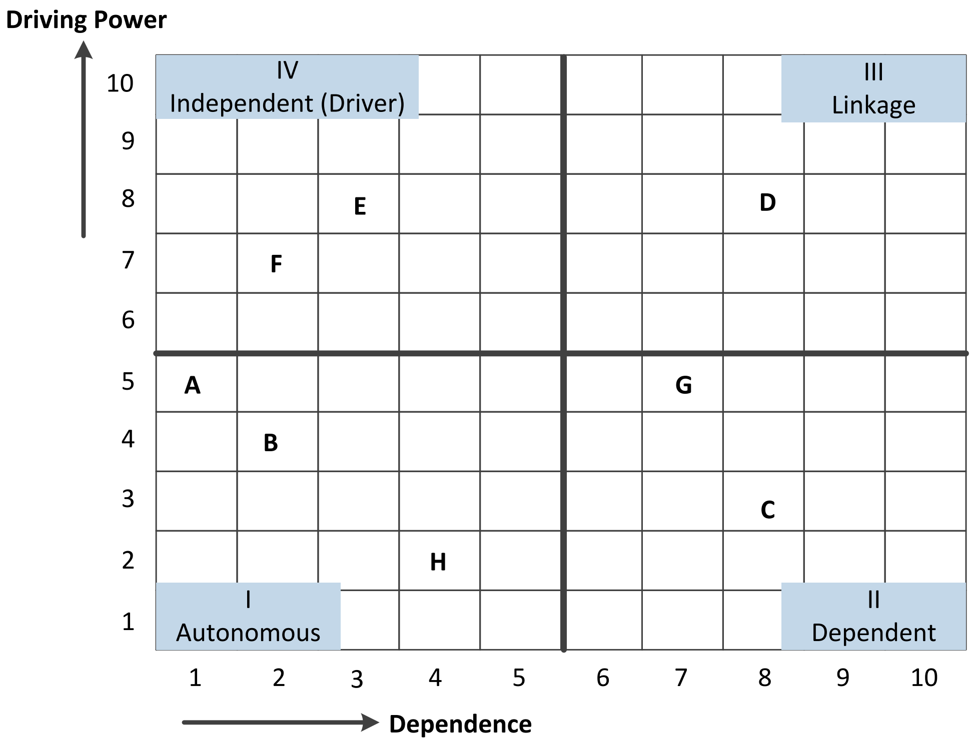


Figure 7.3 Cluster of variables in MICMAC analysis

- Linkage factors are strong driver power and strong dependence power. These factors are commonly unstable due to the fact that they can influence and be influenced by other factors easily. Furthermore, linkage factors usually have feedback effects.

- Independent factors strong driver power but weak dependence. These factors have high impact on the other criteria. Independent factors and linkage factors considered as key variables. Decision makers should pay extra attention on independent factors.

7.4 Related Review

Lee, Wang, and Lin (2010) proposed an evaluation framework of technology transfer of new equipment in high tech industry combining Fuzzy Delphi Method (FDM), Interpretive Structural Modeling (ISM) and Fuzzy ANP. The most critical factors were determined by group of experts using FDM. After evaluation criteria were identified, ISM was employed to capture the interrelationships among criteria and sub-criteria separately. Based on the results of ISM, FANP was applied to compare the equipment suppliers in terms of technology transfer performances. A case study was conducted regarding TFT-LCD industry in Taiwan.

Chen and Wu (2010) investigated automobile-distributer partnership by ISM, AHP and ANP. ISM was used to develop three-stage hierarchical/network structure on the basis of relationships between twenty criteria. This structure was composed of partnership selection, partnership establishment and partnership maintenance. Afterwards relative importance of criteria was found by AHP/ANP regarding whether criteria was independent or not. Finally, the most investment-worthy aspects for manufacturer-distributer partnership were determined. In addition, optimum distributers were identified by the proposed model and compared with the current practices. It was reported that proposed was found highly consistent.

Kannan et al. (2009) developed a hybrid approach using ISM and fuzzy TOPSIS for third party reverse logistic provider (3PRLP). ISM was used to establish the contextual relationships between criteria and form the basis of evaluation hierarchy. When ISM calculations were completed, MICMAC analysis was performed and criteria were separated into four distinct clusters with respect to their driving and dependence power. Result of MICMAC analysis was visualized, and helped decision makers with assigning linguistic values to each criteria to achieve criteria importance weights. Finally fuzzy TOPSIS procedure was applied to select the best 3PRLP.

Chang et al. (2012) analyzed critical agility factors when launching a new product into mass production by ISM and ANP. Causal interdependencies between agility factors were captured by ISM method. Then, based on the interdependency structure obtained from the result of ISM, ANP method was applied to acquire importance weights of agility criteria. Finally, visualized ISM analysis and importance weights of factors discussed and managerial insights were given.

CHAPTER EIGHT
BAYESIAN NETWORKS

8.1 Introduction

Bayesian networks are well-established tools in modeling large body of knowledge under uncertainty. They are computerized tools that try to approach human reasoning mechanisms by computers. Indeed, manipulating immense body of knowledge and ascertaining the relationships among model entities or objects is not a trivial task. Ever since the booming of information technology and widespread use of the computers, researchers attempt to generate some programming concepts that are capable of handling the decision problems and, the most importantly, deduce some inferences, as the way people do. Bayesian networks are great tools to reason about the decision situation even faster than humans.

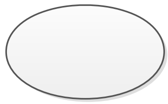
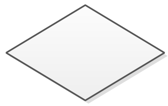
According to (Jensen & Nielsen, 2007), a Bayesian network consists of:

- Variables with finite set of mutually exclusive levels
- Acyclic directed graph
- Conditional probability tables for each variable with its parents.

In decision problems, Bayesian networks can be expanded with decision options and utility functions. These augmented Bayesian networks are designated as influence diagrams (Kjaerulff & Madsen, 2008). Vertex symbols of influence diagrams under discrete case are depicted in table 8.1. In the rest of the thesis, we will not focus on the influence diagrams, rather we will discuss the Bayesian networks to give broader sense of understanding.

As we quite a bit often repeat in this thesis; causality is the indispensable body of human reasoning mechanism. Bayesian networks are causal representations of decision entities, which might be criteria and their levels or states and courses of actions.

Table 8.1 Vertex symbols in influence diagrams

Node Name	Node Symbol
Chance Node	
Decision Node	
Utility Node	

Another vital property of Bayesian networks is the notion of probabilistic causality. Causality is a very generic term, which is also philosophically conflicting that researchers do not reach a consensus regarding its meaning in different contexts. Since the term causality is not a crisp term, its modeling might cover different methodologies. In Bayesian networks, we will suppose that causal influence will obey the rules of probability theory. We will elaborate this concept later.

Another merit of Bayesian networks is the graphical representation of decision problem so it made very easy to handle relationships among decision entities with their structural levels. Also graphical representations provide with the exploitation of several inference algorithms. These algorithms are used to obtain probability distribution or structure of the Bayesian network. However, inference algorithms are completely out of the context of this thesis.

From now on, will we explain the key components of Bayesian networks as probabilistic inference, calculation of probabilities in a chain, graphical representation properties, variables and vertices etc. in a very brief manner.

8.2 Probability Theory

8.2.1 Basic Definitions

There are many alternative approaches to model uncertainty in the literature. The most prominent and strong methods which are also alternatives against Bayesian probability are Dempster Shafer Theory (Dempster, 1968; Shafer, 1976) and fuzzy set theory (Zadeh, 1965).

In probability theory, source of uncertainty is the occurrence of the event, yet occurrence is well defined. However, if the occurrence itself is ill-defined and contains ambiguities, then probability theory is not an appropriate choice. Contrary to classical probability theory, Dempster Shafer Theory assigns probabilities, which is namely basic probability assignments, for a collection of events rather than to the single events (Bender, 1996). The other form is the fuzzy set theory which is discussed already. However, Bayesian networks have very strong mathematical foundation and have great variety of application areas. Despite the fuzzy set theory and Dempster Shafer Theory, Bayesian probability is still prominent and dominant method. Here we give some basic fundamentals of probability theory. We define sample space as possible outcomes of an experiment. Experiment is a theoretical term to denote any achieved result that is uncertain. Throwing a die is an example of the experiment. We denote sample space with S. For the die throwing experiment, we have $S = \{1,2,3,4,5,6\}$. An event is defined as subset of a sample space. For instance, getting four or higher value when throwing a die event is a subset $\{4,5,6,7\} \subseteq \{1,2,3,4,5,6\}$.

As we will deeply discuss when we are explaining the Choquet Integral and nonadditive measures; probability is just one of the measures used in the measure theory. Measure theory has been taken great attention recent years, where generalized measure theory also deals with upper and lower probabilities. In either situation, whether it is a probability measure or generalized probability measure, we are measuring the degree of uncertainty regarding an event, and $P(\mathrm{A})$ is assigned for each $A \subseteq S$. We mention three key axioms of classical probability theory where the Bayesian networks are also rely upon these (Doob, 1996):

- Axiom 1: $P(S) = 1$. If we are absolutely sure that we will observe the event S, we assign the probability of 1.

- Axiom 2: For all $A \subseteq S$, $P(A) \geq 0$ does hold. Any event has a probability value that is not negative.
- Axiom 3: Supposing $A \subseteq S$, $B \subseteq S$ and $A \bigcap B = \varnothing$; $P(A \bigcup B) = P(A) + P(B)$.

Based on the axioms of probability theory, conditional probabilities are also characterizing the Bayesian Networks. Conditional probabilities mean that the statement of a probability is conditioned on the other statement. This coincides with the structure that "given a specific event or events, probability of the considered event is p". As a definition, conditional probability for two events that conditional probability for A given B is (Berger, 1985);

$$P(A \mid B) = \frac{P(A \bigcap B)}{P(\mathrm{B})} \tag{8.1}$$

More generally, condition on multiple events can be defined as (Berger, 1985);

$$P(A \mid B \bigcap C) = \frac{P(A \bigcap B \bigcap C)}{P(B \bigcap C)} \tag{8.2}$$

Based on the conditional probability definitions above, Bayes' rule can be stated that

$$P(A \mid B) = \frac{P(B \mid A, C) P(A \mid C)}{P(B \mid C)} \tag{8.3}$$

Furthermore, it is also worth mentioning conditional dependence and independence concepts. Two event are said to be independent if (Jensen & Nielsen, 2007);

$$P(A \mid B) = P(A) \tag{8.4}$$

For the case of the conditional independency, if A and B are conditionally independent given the event C (Jensen & Nielsen, 2007),

$$P(A \mid B \cap C) = P(A \mid C) \tag{8.5}$$

In the subsequent title, we give example of probability calculation in Bayesian Network.

8.2.2 Calculation of Probabilities

In the above basic definitions, we mentioned about probabilities of simple events. However, in Bayesian networks we also talk about collection of sample spaces namely variables. Here variables are very similar to attribute levels in MAUT. For instance sample space A might be represented as $(a_1, a_2, \ldots, a_n)$. For each state, uncertainty of a variable to be in the specific state is represented by a probability distribution $P(A) = (x_1, \ldots x_n)$ where $\sum_{i=1}^{n} x_i = x_1 + \cdots + x_n$ and for all x_i ; $x_i \geq 0$.

Consequently, we have conditional probabilities not only for events, but also for variables. With n variables of A and m variables of B, conditional probability $P(A \mid B)$ have $n \times m$ probabilities (Jensen & Nielsen, 2007).

Example 8.1 (Jensen & Nielsen, 2007) : Suppose that we have a conditional probability of $P(\mathrm{A} \mid \mathrm{B})$ shown in table. Each event consists of elements $a_i \in A$ and $b_j \in B$.

Table 8.2 An example of conditional probability table

	b1	b2	b3
a1	0,2	0,4	0,7
a2	0,8	0,6	0,3

Notice that sum of the probabilities of each column is equal to 1 due to the axiom 1. $P(\mathrm{A} \mid \mathrm{B})$ means that for each state of B , total of A must be 1. In addition, suppose that we have $p(B) = (0.3, 0.5, 0.2)$. To calculate the joint probability distribution, which implies that the probability of collective or joint outcomes for distinct experiments, we implement the equation $P(A|B)P(B) = P(A \cap B)$ known as fundamental rule (Jensen & Nielsen, 2007).

Therefore, we have

$$P(A,B) \quad = \quad \begin{array}{c|ccc} & b1 & b2 & b3 \\ a1 & 0.2 \times 0.3 = 0.06 & 0.4 \times 0.5 = 0.20 & 0.7 \times 0.2 = 0.14 \\ a2 & 0.8 \times 0.3 = 0.24 & 0.6 \times 0.5 = 0.30 & 0.3 \times 0.2 = 0.06 \end{array} \tag{8.6}$$

8.3 Causality in Bayes Networks

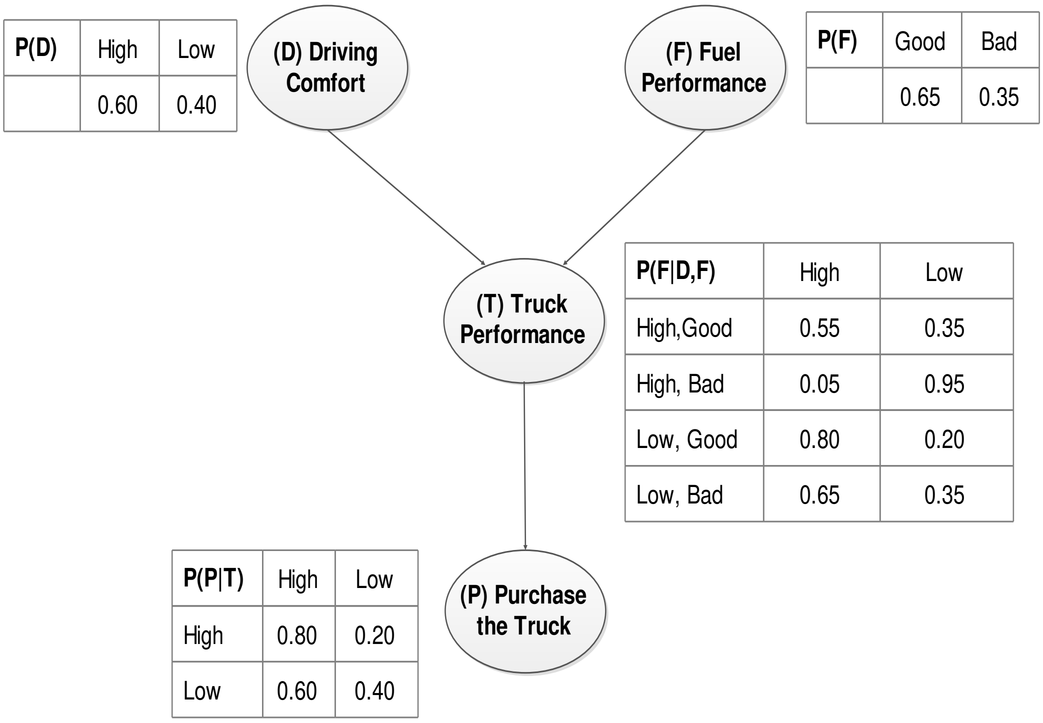
In the literature, there are mainly two different approaches to build Bayesian networks. The first one is based on the data that these models facilitate the probabilistic methods to construct Bayesian network from data (Heckerman, 1997). The other method which is our main interest is the knowledge-based approach, where the domain expert provides with the causal knowledge in order to establish the Bayesian network. It is crucial to point out that we do not generalize the fact that Bayesian network methods are always causal methods. Bayesian networks have very different implementations in distinct application areas, so they do not always possess causal inference. Yet in the context of this thesis, we bring together the Bayesian networks applications in decision making framework, where decision maker provides causal knowledge regarding decision variables. Actually, if Bayesian networks are used in the decision analysis context (evaluation of alternatives or an alternative), they very often exploit causal knowledge of domain expert to model causal dependencies among decision variables.

Bayesian network is very suitable tool to model expert's uncertain and incomplete knowledge. As we mentioned before, both quantitative and qualitative representations constitute the net. Directed acyclic graph representation is considered as quantitative aspect, while conditional independence relations and probability tables are seen as the quantitative side of the method. The way decision variables influence any other is based purely on the graphical structure of the problem and Bayesian probability calculations. Consequently, uncertain inference and dependence/independence among decision variables are intertwined concepts. We will explain causality, independency and their relationship with the conditional probability tables in proceeding paragraphs.

We prefer explaining conditional independency concepts by example rather than giving concrete definitions. We pursue the explanation of (Nadkarni & Shenoy, 2004) for conditional independency. Example is inspired from truck selection problem presented by Baykasoğlu et al. (2013) and Nadkarni & Shenoy (2004).

Example 8.2: Suppose that we evaluate the decision of purchasing second hand truck for the logistic company. For the sake of simplicity, we only consider driving comfort and fuel performance that influence the truck performance. Based on the truck performance, we make the purchasing decision. Variables have set of values assumed to be mutually exclusive and exhaustive. Driving comfort variable has two levels, high and low, and fuel performance has good and bad levels. Also truck performance has high and low levels and purchase the truck has levels of yes and no.

Truck performance is said to have two parent nodes as driving comfort and fuel performance. Also purchase the truck has one parent as truck performance. Parent and child relationships can be easily read from the graph. Direction of the arc clarifies the relationship such that, i.e. arrow pointing from A to B means A is the parent of the B. Moreover, conditional probability tables are shown in Figure 8.1. Tables $P(D), P(F), P(T \mid D, F)$ and $P(P \mid T)$ are defined. As we mentioned before, calculation by probability tables rely on determining the joint probability distributions.



P(D)	High	Low
	0.60	0.40

P(F)	Good	Bad
	0.65	0.35

P(F|D,F)	High	Low
High,Good	0.55	0.35
High, Bad	0.05	0.95
Low, Good	0.80	0.20
Low, Bad	0.65	0.35

P(P|T)	High	Low
High	0.80	0.20
Low	0.60	0.40

Figure 8.1 Bayesian Network of truck selection example

We calculate the joint probability distribution as;

$$P(D,F,T,P) = P(D) \otimes P(F) \otimes P(T \mid D,F) \otimes P(P \mid \mathrm{T}) \tag{8.7}$$

According to rule of total probability we get ($\otimes$ means point wise multiplication);

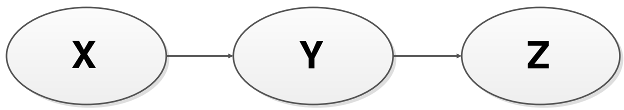
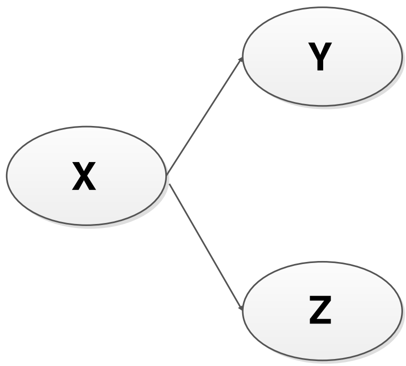
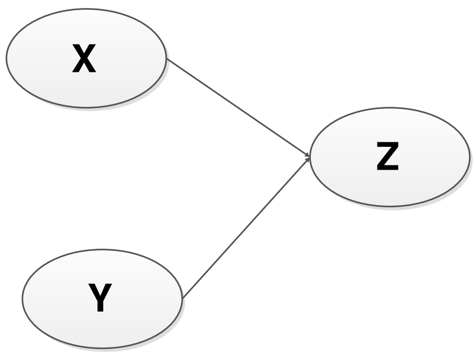
$$P(D,F,T,P) = P(D) \otimes P(F \mid D) \otimes P(T \mid D,F) \otimes P(P \mid \mathrm{D,F,T}) \tag{8.8}$$

From the equations above, we conclude that $P(F) = P(F \mid D)$ which is interpreted as fuel performance is independent of driving comfort. Additionally, $P(P \mid T) = P(P \mid D,F,T)$ refers to Purchase the Truck is conditionally independent of Driving Comfort and Fuel Performance given Truck Performance.

These independence conditions can be seen directly from the graph. Graphical representation, indeed, enhances the understandability of the model and makes it easy to infer the conditional independencies. Following the directed arc, sequence of variables can be seen easily. In our example, D,F,T,P is a sequence. Absence of an arc from D to F points out that D is independent of F. Absence of an arc from D to P indicates that P is conditionally independent of D and F given T.

Although we presented the conditional independency in Bayesian networks, it is not the same term with causality. According to (Nadkarni & Shenoy, 2001, 2004), direct causality concept is exploited to discernment of conditional independence.

Table 8.3 Notion of causality and its relationship with conditional independence

Situation	Graphical Presentation of Relationship
• X causes Y and Y causes Z (Causal interpretation) • Y is a mediation node • If true state of Y is known, then more knowledge about X does not influence Z. • Hence Z is conditionally independent of X given Y. (Conditional independence)	
• X causes Y and also X causes Z. (Causal interpretation) • If true state of X is known, then more knowledge about Y does not influence Z. • So Y is conditionally independent of Z given X. (Conditional independence)	
• X and Y are independently causes Z. (Causal interpretation) • If true state of Z is known, X and Y are not said to be independent • Suppose that state of Z is true and X is false, then Y becomes very likely true. Hence Y is not conditionally independent of X given Z.	

In table 8.3, we explained the relationship between direct causality and its effect on the conditional independence situations. Another important point in causality is the situations of indirect causal relations. In figure 8.2, the difference between causal modeling and Bayesian representation. In Bayesian representation, suppose that the decision maker has the complete knowledge about the truck performance, and then fuel performance does not influence the purchase the truck node. Consequently, determining the direct and indirect influences and their interpretation are of vital importance in modeling. Ill-defined representations might increase the complexity of the model, which, especially, is very costly in Bayesian networks.

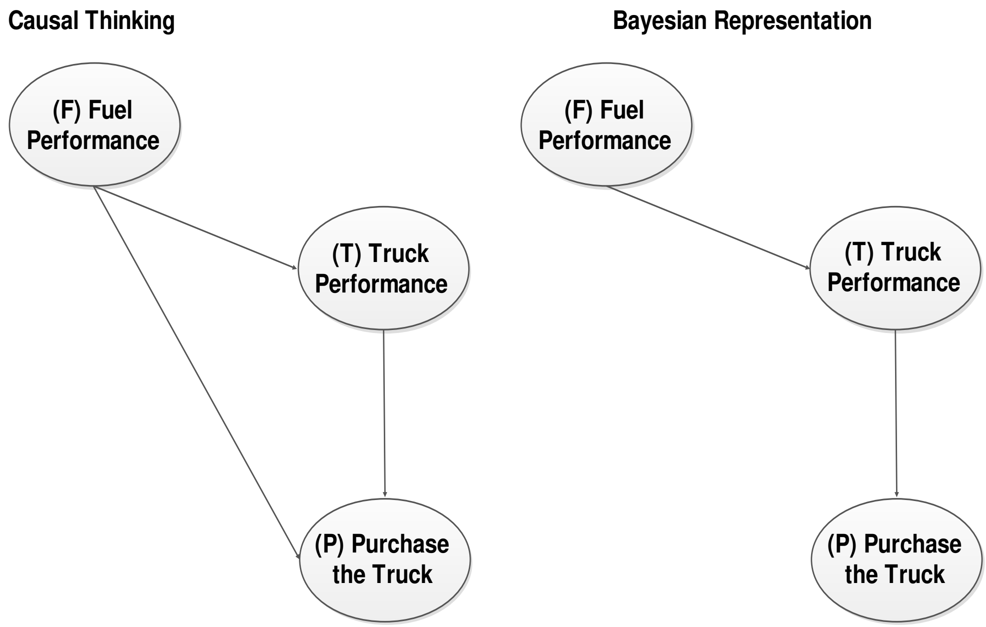


Figure 8.2 Comparison of causal thinking and Bayesian representation

Lastly, Bayesian networks do not permit the circular relations. The most common approach to resolve the problem is separating the nodes according to time frame. Circular or feedback situations, generally, underlies the past and future of the variable. For instance, variable influences the other variable at time t, then affected variable influence the affecting variable at time t+1. These kinds of situations lead to the circular structure. In figure 8.3, we depicted the above mentioned situation.

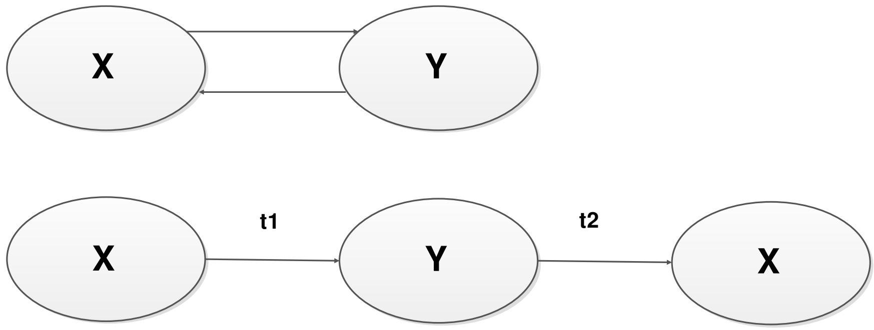


Figure 8.3 Decomposing variables into time frames

8.4 Advantages and Disadvantages

There are many advantages of Bayesian networks. Some of them are:

- Mathematically sound and relies on well-established mathematical axioms.
- Close connection with expected utility calculation that the most decision analyst are familiar with.
- Data-base models facilitate the automatic building of the model.
- Based on causal relationships that is easy to understand.
- Efficient.

Also there are some requirements of the method.

- Variables and nodes should be well defined.
- Causal relations should be determined.
- Conditional probabilities are needed.
- If influence diagram is utilized, utility and decision nodes should be incorporated.
- Uncertainty should be inherent, at least some of the variables involved with uncertainty.

8.5 Related Review

Ferreira and Borenstein (2012) proposed a novel approach integrating influence diagram and fuzzy logic to support supplier selection problem. Influence diagram was an extension of Bayesian network for decision models. State of each node in the influence diagram was presented by linguistic fuzzy variables. In the model, first, influence diagram was constructed. Then performance rating of each node and the weights of criteria were determined by linguistic variables. The supplier performance and rating of each criteria in the influence diagram were calculated by the algorithm proposed by Shachter (1986). A case study for selecting potential vegetal oil supplying alternatives in biodiesel industry was provided. Furthermore, computational implementation including different modules and databases was developed so as to store the historical data and support learning and responding dynamical changes.

Fenton and Neil (2001) discussed the relation between Bayesian nets and MCDA comprehensively. It was pointed out that BBNs was a powerful technique used to capture causal factors and predict the effects of changes to causal factors had influence on the decision making attributes. In addition, giving different examples, relationship between causal dependence and uncertain inference was explained. It was claimed that uncertainty in various factors were result of causal dependency where it was not possible to know the exact state of the factor due to its dependence on other factors so that uncertain inference by BBNs was required. A framework was proposed to combine the MCDA methods and BBNs. Factors with causal dependency and uncertainty were handled by BBNs first and then chosen MCDA method was applied.

Dogan and Aydin (2011) combined Bayesian networks and total cost of ownership in supplier selection analysis. BNs was used to handle uncertainty and capture the interdependencies between supplier selection and different cost items in the process. TCO was applied to obtain the true cost of acquisition of a product accurately. Illustrative case study was provided to select the best supplier and sensitivity analysis with different scenarios was carried out to see the effects of factors on total cost.

Dogan (2012) proposed similar logic for facility location problem of an international manufacturing plant. Qualitative and quantitative criteria and their probabilistic cause-effect relations modeled by BN. TCO was used to measure all the costs that were taken into account in selecting facility location. In the case study, facility with the minimum total cost was selected.

Yang, Bonsall, and Wang (2009) developed a hybrid uncertain multiple attribute decision making technique for effective safety management. In the study, BN and MAUT were combined where BN was used to obtain the attribute values represented by posterior probabilities. Non-additive utility function expressed by linguistic variables was developed and incorporated into a novel fuzzy TOPSIS method with entropy measures. Entropy method was used to acquire criteria weights by changing the predefined relative weights of the decision attributes. Proposed model illustrated in a container transportation delay problem and risk control options were ranked.

Sedki and Delcroix (2012) proposed an influence diagram based model for MCDM. Influence diagram which was one of the Probabilistic Graphical Models (PGM) extended the classical Bayesian Networks by defining chance, decision and utility nodes. Proposed model was able to consider attributes, alternatives, decision maker's preferences and internal or external factors. Interdependencies among decision variables were modeled by probabilistic influences in the influence diagram. In the model importance of each alternative, quality index of each alternative with respect to each criterion and general satisfaction of each alternative were computed. Proposed model was explained and illustrated in transportation problem used by Fenton and Neil (2001) previously.

CHAPTER NINE
SYSTEM DYNAMICS

9.1 Introduction

Companies are operating under very dynamic and uncertain environment. Increasing unemployment, political conflicts, global warming, environmental pressures and new legislations, high inflation rates, democracy and human rights dilemmas, rise of terrorism in global extent, education and healthcare problems, great poverty and economic unbalance are all surrounding and influencing us in a vigorous way. The problems we face all require dynamic and long term perspectives in order to manage them. We consider this sort of long term decision making problems as dynamic since the situations chance over time. The decisions should be always monitored and traced during the time. System dynamics, hence, is a very suitable tool to address long term, strategic, uncertain and dynamic decision making problems.

System dynamics is established under the industrial dynamics name by (Forrester, 1961). In the study, well known business problems were handled. This study was followed by urban dynamics approach, a dynamic approach related to development of the urban areas (Forrester, 1969). Consequently, methodology gained broader sense of domain and designated as system dynamics. System dynamics deals with the socio economic, and usually complex and interrelated world problems. It is different from the dynamic systems simulation methodology in the sense that problem in question is not a complex mechanical or other technical problems like mechatronics or automotive industry; yet socio-economic and business related. We focus on the terminology of system dynamics related to world affairs and business dynamics and briefly explain the main concepts.

In Figure 9.1, we have shown the status of the system dynamics methodology among other simulation tools due to (Borshchev & Filippov, 2004). System dynamics offers macro level view and high abstraction.

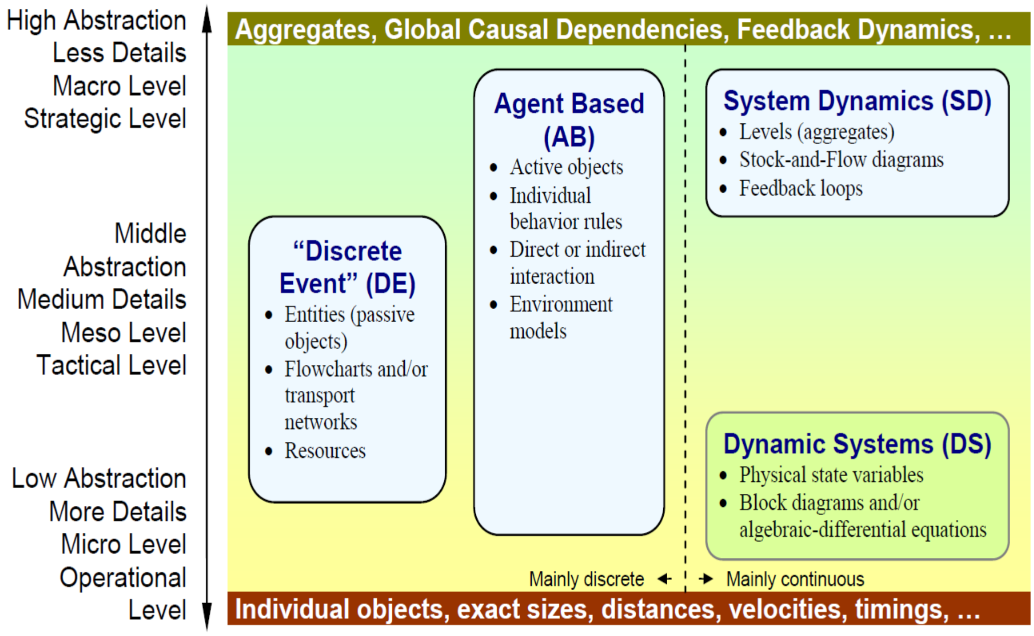


Figure 9.1 system dynamics among simulation paradigms (Borshchev & Filippov, 2004)

It does not focus on the details yet analyze the situation in the strategic level. Also dynamic systems and system dynamics are not the same, while they are very prone to confusion for the reason of name similarities. Dynamic systems are usually related to mechanical or biological systems that they provide with a very low level abstraction with great details. They are usually modeled via Matlab Simulink software, whereas Anylogic, Vensim and Stella are market leaders for modeling and analysis of system dynamcis methodology.

9.2 Dynamic Feedback Structures

The core concept of the system dynamics methodology is the feedback structures. In the system dynamic models, variables are subjected to undergo changes while the time passes. In real life, indeed, systems are not static and different patterns of behaviors are observed. World population is exponentially increasing during the past millennium, stock prices are waving, and price of oil per gallon is dramatically changing. Even in our body, many alterations might be observed in a day. So the question is how the system dynamics methodology handles the dynamic systems and what does it aim for?

System dynamics method looks for the causes of the dynamism in the internal structure of the system (Barlas, 2002). It is also known as endogenous perspective. From the system dynamics point of view, systemic feedbacks are generated due to the internal interactions of the system components and variables. These interactions are actually causal relations and we will mention about it later.

In some situations, a system can be influenced by the external factors dramatically. However, this type of situations are inevitably might become to happen, and most of the times managers or decision maker have to admit the results of this circumstances. Suppose that a disastrous earthquake or some sort of very strong natural disaster has happened and the material flow from the suppliers has disrupted in the supply chain. This external factor is such very powerful and system dynamics has nothing to do with this kind of cases.

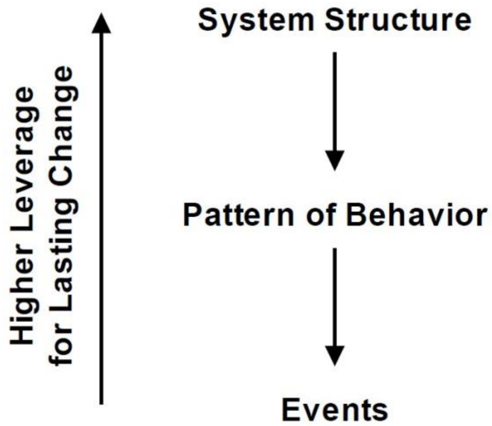


Figure 9.2 Event oriantation versus system structure perspectives

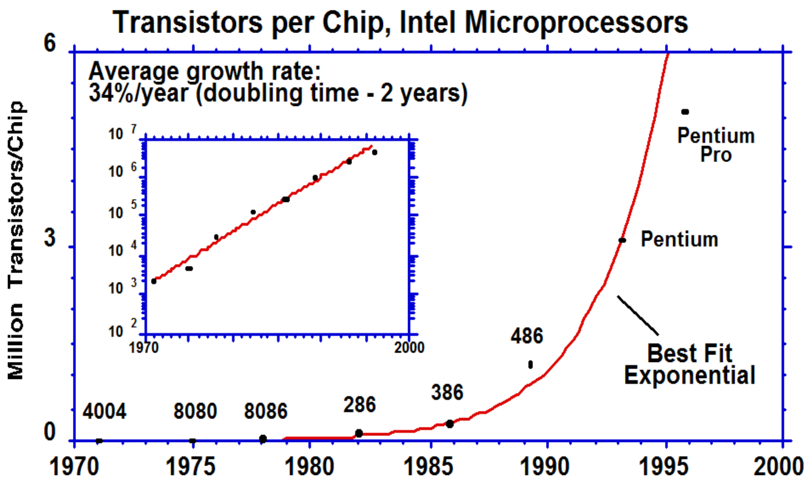
Philosophy of the system dynamics methodology is that the system itself, and its components within the internal relationships, is responsible for the system behavior. Feedback term is highly correlated with this view. Thus, managers or decision makers focus on the system itself, analyze the relationships among variables, depict the behavior and draw the conclusions. Also, stated by Barlas (2002), people have many problems in daily life and we are eager to make excuses and blame out factors instead of ourselves. In a philosophical standpoint, system dynamics improves the system as it is, strives to make it resistant to changes in outside the system. System dynamics seeks for some space in the system to managerial initiative; i.e. in the earthquake example, there is no space for decision makers to change the system, since earthquake does not result in the structure of the system.

As system dynamics focus on the system itself, the term system can be very broad and lead to misunderstanding. Level of abstraction is the key element to manage complexity of the model. In system dynamics, the system is not modeled as it is; components are selected based on the relevancy to the problem on hand.

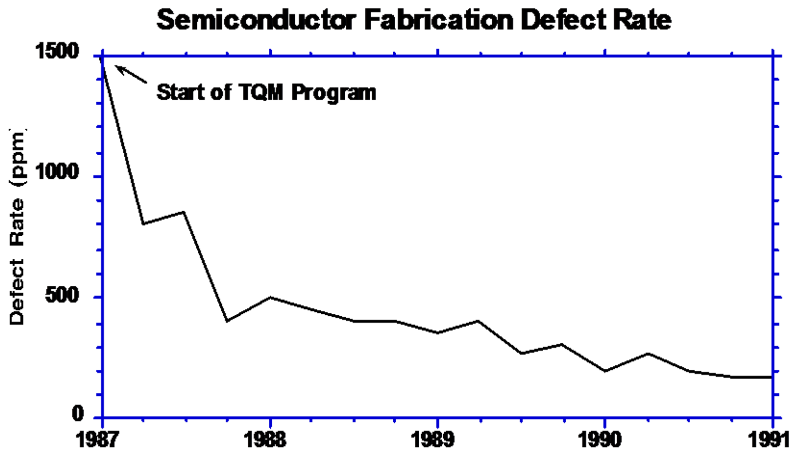
9.3 Model Structure and System Behavior

System dynamic method implicitly defines structure as relationships among the system variables. Selected variables, their influences on other variables determined by equations are all account for the structure. As we mentioned before, system structure determines the system behavior and decision makers intend to find the causes of the behavior in the system to derive conclusions. Some very common behaviors in system dynamic models reported as exponential growth, goal seeking, s-shaped growth, oscillation, growth with overshoot and overshoot and collapse (Sterman, 2000). In table 9.1, some examples of system behavior from real life are shown.

Table 9.1 Real life examples of system behaviors

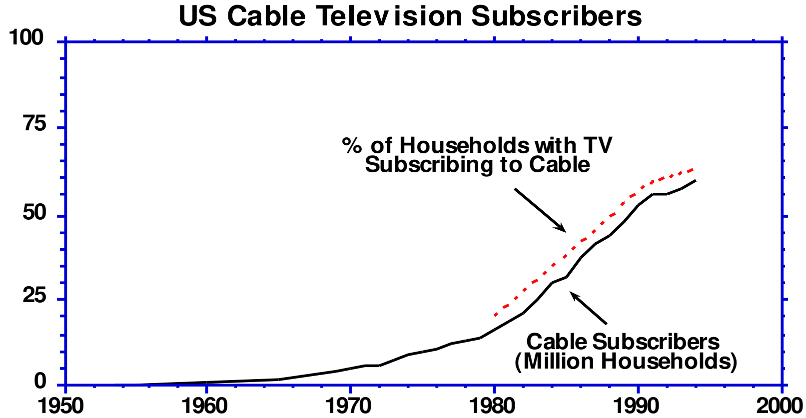
System Behavior	**Real Life Example**
Exponential Growth (Joglekar, 1996)	

Goal-seeking behavior (Sterman, Repenning, & F., 1997)

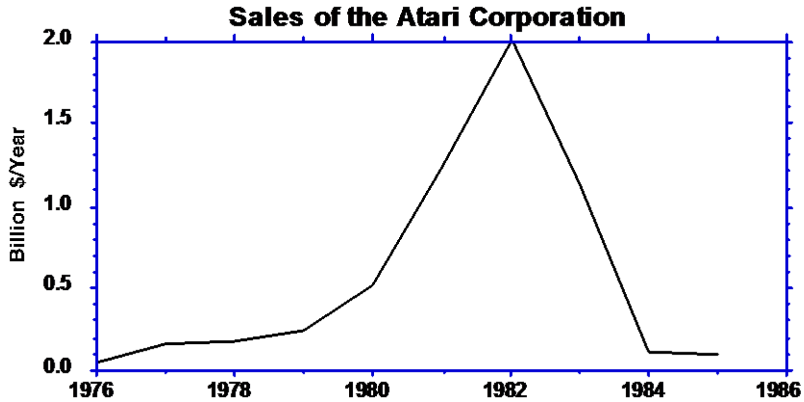


Oscillation behavior Source: US Bureau of Economic Analysis

S-Shaped Growth (Kurian, 1994)



Overshoot and Collapse (Paich & Sterman, 1993)



In order to better explain the relationship between system structure and system behavior, let us give a very common example from the literature. Suppose that we are investigating the population dynamics of a species in an isolated island under predetermined assumptions (Sterman, 2000).

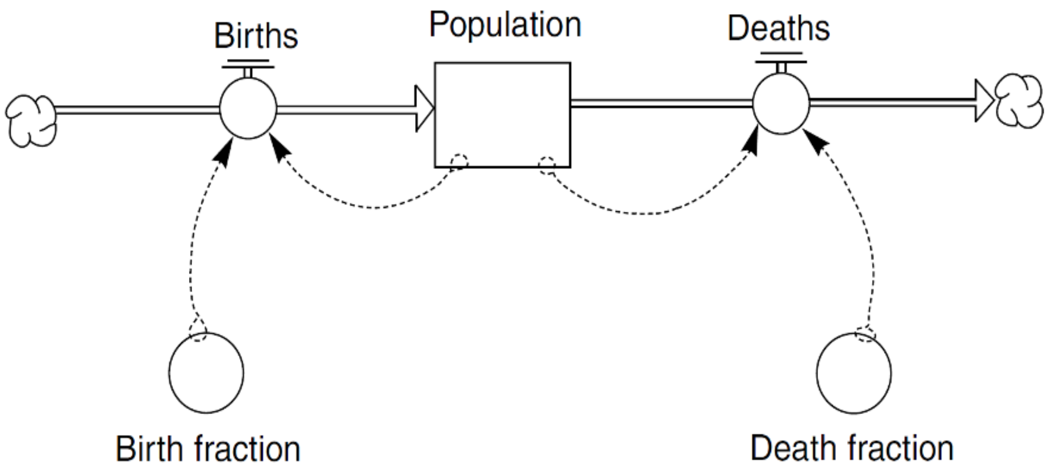


Figure 9.3 Example of population dynamics (Sterman, 2000)

The problem is depicted in Figure 9.3. For the sake of reducing complexity, we explain concepts very briefly within the example. Population is model as a stock and births with deaths are flows. Birth fraction and death fraction are assumed to be constants are called auxiliary variables. We will show that this system correspond to exponential growth behavior.

$\frac{dx}{dt} = births - deaths$ can be read from the figure easily. Suppose that b is the birth fraction per year and f is death fraction per year. We assumed that these fractions are constant. Then we have $\frac{dx}{dt} = bx - fx$. This equation corresponds to

$$x(t) = x(0) + \int_0^t (bx - fx)dt \tag{9.1}$$

Putting a constant k instead of $b - f$, equation becomes

$$\frac{dx}{dt} = (b - f)x = kx \tag{9.2}$$

$$\frac{dx}{x} = kdt \tag{9.3}$$

Integrating both sides of the equation yields,

$$\int \frac{1}{x} dx = \int kdt \tag{9.4}$$

So we have $\ln x + C_1 = kt + C_2$. Then $x = e^{kt + C_3}$ which equals $e^{kt} e^{C_3}$. Substituting e^{C_3} by another constant C, we have $x = Ce^{kt}$. Ultimately, we reach the final solution that

$$x(t) = x(0)e^{kt} \tag{9.5}$$

which is an exponential growth.

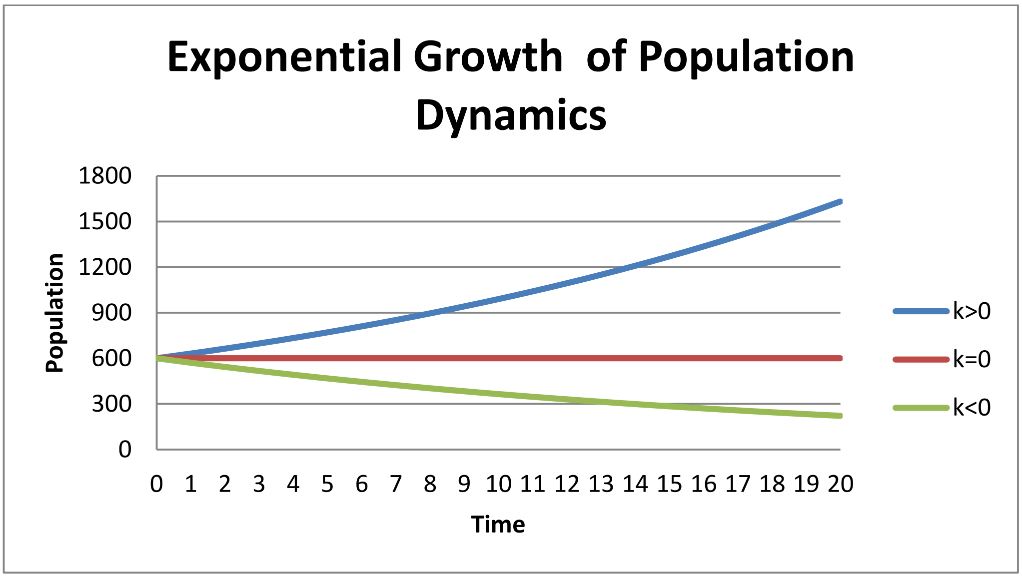


Figure 9.4 Exponential growth of population dynamics

Remember that k means difference between birth faction and death fraction. If birth fraction is higher than death fraction, then exponential growth of population is observed. In figure 9.4, we have shown the population dynamics as means of a chart, assuming that initial population is 600. The difference between birth and death fractions are assumed to be 0.05, 0 and -0.05 respectively. Population dynamics can be clearly seen from the chart that when the birth faction is bigger, equal or smaller than the death fraction situations.

9.4 Causality and Feedback

Ignoring the modeling details of system dynamics, we focus on the concept of causality and feedback structure which yield a distinct philosophy to model interaction. System dynamics models constitute of causal relations among variables, not solely as a means of correlations in statistics. Secondly, circular causality or feedback causality is in the core of the method. And third, dynamic pattern orientation is emphasized compared to event orientation. Lastly, structure of the system is seen as the cause of the dynamic behavior (Barlas, 2002).

System dynamics models aspire to understand the causes of the dynamic pattern of the system then it manage and improve it appropriately. It is long-term oriented approach to depict the system behavior in time. In a short run, correlational models might be more convenient approach to model non-causal relations. However, system dynamics considers the causal relationships in the long run of general picture and analyze system behavior. Suppose that variable A has causal influence on variable B. It means that increase in A causes B to decline, or increase in A causes B to increase too. In the population dynamics example above, if death rate is increased, then population is declined.

Feedback causality is another important feature of system dynamics. feedback is defined as transmission and return of information (Richardson & L., 1981). Feedback loops are closed paths of sequence of causes and effects. Causal loop diagrams are central and substantial tools to depict the feedback causality. They grab the beliefs regarding the causes of dynamics, capture the cognitive models of decision maker and depict the vital feedbacks that are crucial for the problem.

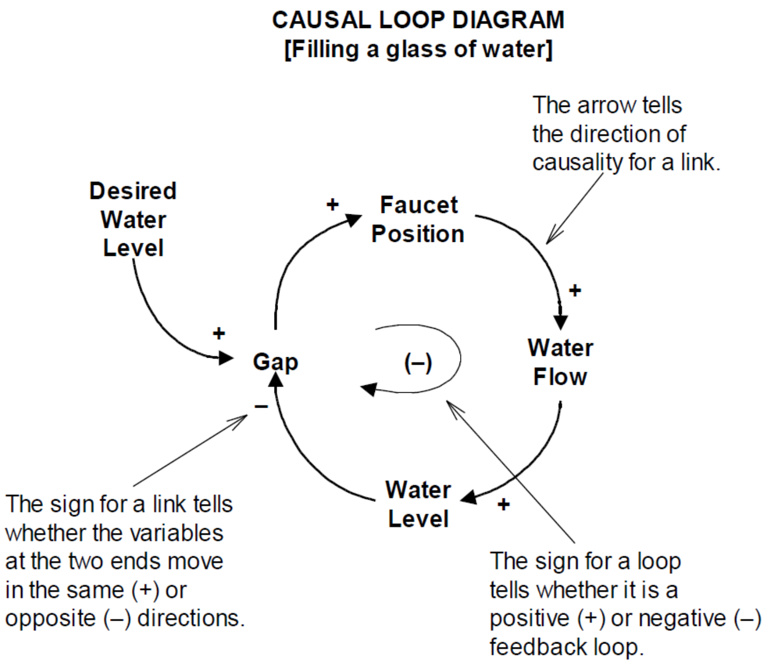


Figure 9.5 Example of a causal loop diagram (Kirkwood, 1998)

Also note that in some models, cause and effects do not close back on themselves so that this sort of models are named open loop thinking (Kirkwood, 1998). However, open loop thinking is not as strong as the feedback loops in means of depicting the full range of influences.

Before giving an example of causal loop diagram, it is wise to introduce very basic components of system dynamics models: stock and flows. Stocks are generally represents the state of the system in which our decisions are mostly based upon. Also they accumulate in time, which is an important clue to distinguish it from flows. On the other hand, flows are functions of the stocks and other state variables and parameters (Sterman, 2000). Some examples of stock and flows can be seen in table 9.2.

In Figure 9.5, we provided a simple example of causal loop diagram, filling a glass of water, in order to explain the underlying logic of system dynamics approach. In the causal loop diagram in Figure 4, faucet position is the beginning point of the model. If the faucet position is escalated, the flow of water naturally becomes escalated too. In a similar logic, if water flows faster, water level in glass rises. Therefore sign of these relationships is positive.

Table 9.2 Discrimination of stocks and flows in different fields (Sterman, 2000)

Field	Stocks	Flows
Mathematics, engineering	Integrals, states, state variables, stocks	Derivatives, rates of change, flows
Chemistry	Reactants and reaction products	Reaction rates
Manufacturing	Buffers, inventories	Throughput
Economics	Levels	Rates
Accounting	Stocks, balance sheet items	Flows, cash flow or income statement items
Biology, physiology	Compartments	Diffusion rates, flows
Medicine, epidemiology	Prevalence, reservoirs	Incidence, infection, mortality rates

If a causal link is positive where the direction of the influence is from X to Y, positive sign means that:

- X adds to Y, or
- A change occurred in X yields a change in Y in same direction.

Continuing the chain of the causal loop diagram in Figure 9.5, next element is the gap which refers to the difference between desired water level and current water level. By this definition, if the water level increases, then the gap is decreased. Hence the sign of the relationship is negative. . If a causal link is negative where the direction of the influence is from X to Y, negative sign means that:

- X subtracts from Y, or
- A change occurred in X yields a change in Y in opposite direction.

Finally, the higher value of the gap leads to rising of faucet position, since we desire to make glass filled. Accordingly, there is a positive sign between gap and faucet position. Furthermore, the complete loop has also a sign which is calculated by number of positive and negative signs as follows (Kirkwood, 1998)

- A positive feedback loop, denoted by positive sign in the parentheses, encompasses even number of negative causal links.
- A negative feedback loop, denoted by negative sign in the parentheses, encompasses odd number of negative causal links.

The sign of the feedback loop is the one component of the system structure, actually sign of the feedback loop determines the pattern of behavior of the whole system. We do not give any further details about specific feedback loops and resulting system behaviors, since we have already mentioned earlier as briefly summarized in table 9.1.

9.5 Further Discussion on Causality in System Dynamics Models

System dynamics models, specifically causal loop diagrams represent causal relationships between variables. We believe that this point deserves some further discussion which is significant to understand type of interactions the system dynamics is able to model.

System dynamics models are expected to mimic the real system so that the results of the model will be concordant with the actual system under consideration. The system behavior is not the repetition of the historical events; it usually comes up with the novel responses. As we already emphasized, the pattern of behavior is emerged from the structure of the system. Hence, causal influences, or what we interpreted to be causal, determine the behavior of the system.

A misleading interpretation of the causal influences might bring about modeling with correlations. Actually, correlation does not describe the system; it is only the result when the simulation model is run. Correlation depends on the historical facts, so trying different strategies or policies in the system dynamics model is not possible with correlational relationships, since correlation among variables are not valid for the new situations.

Suppose that it can infer from the historical data that the ice cream sales are positively correlated with the murder rate. Should we use this relationship in system dynamics model as in the Figure 9.6a?

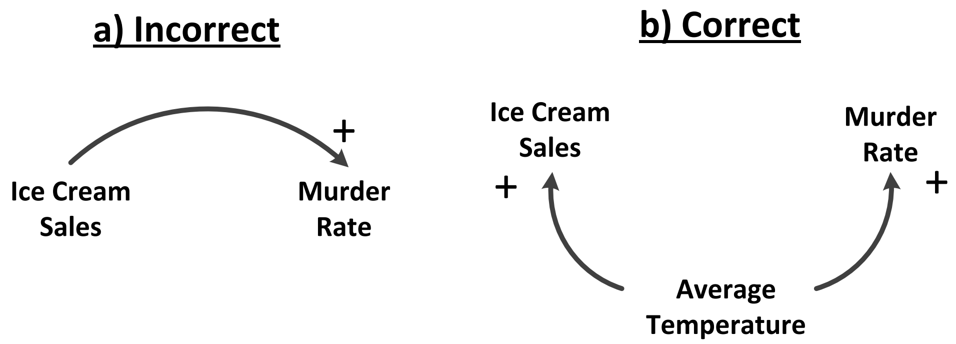


Figure 9.6 Correct and incorrect causal relationship (Sterman, 2000)

The answer is exactly no. If the ice cream sales decrease dramatically, murder rate will not be influenced by this fact. The accurate modeling approach is given in Figure 9.6b, the rise of average temperature leads to increase in ice cream sales and murder rates; they both fall in the winter due to the decrease in the average temperature and rise in the summer. Actually, modelers should keep in mind that statistical indicators are not very strong in modeling with system dynamics; causality and the modeler's interpretation is in the center while constructing the causal loop diagrams.

9.6 Related Review

Tesfamariam and Lindberg (2005) proposed an aggregate analysis of manufacturing system configurations by System Dynamics and ANP. System Dynamics which captures the causal relationships of the factors and dynamic behavior of the system was used to acquire the system performance, while ANP was provided with weights of lower level performance criteria. It was indicated that both the system dynamics and ANP made use of Causal Loop Diagrams (CLD), dependencies and feedback loops obtained from CLD in system dynamics model facilitates and simplifies to establish an accurate ANP model. An illustrative hypothetical case study comparing job shop, cellular shop and the dedicated lines was presented. The performances of alternatives were determined by aggregate simulation study and then these alternatives were pairwise compared to gather the relative performances. The desirability index for each alternative was defined which was the multiplication of relative performance of an alternative and weights of criteria obtained by ANP.

Brans et al. (1998) proposed System Dynamics and PROMETHEE methodologies in order to control complex and socio economic structures. Since socio-economic systems are complex and even hyper complex, necessity of providing decision makers with the future assessments was emphasized. In that sense, System Dynamics came into the front with abilities to integrate decision maker's mental representations with causal loop diagrams and to help understand the influence between system elements with the lapse of time. In the proposed methodology, System Dynamics was used to model the dynamic system and simulate the scenarios and strategic options. Different scenarios were meant to be the change in the SD model structure or setting of the system variables. When different scenarios were simulated, multi criteria evaluation table was established. System variables in the SD model were considered as criteria and their final value at horizon T with respect to strategy was used in the multi criteria evaluation table. It was also recommended that criteria that take the patterns in the SD model into account such as time to reach a steady state could be included. Finally, PROMETHEE method was applied to select the best strategy.

Santos, Belton, and Howick (2002) proposed performance measurement framework by integrating multi criteria decision analysis and system dynamics. The proposed method was illustrated in hypothetical application in health care resources management Performance measurement system was considered as a continuous loop consisting of design, measure, analyze and improve phases. In the design phase, it was indicated that capturing the holistic view of the system and the relationships between measures had vital importance for effective performance management system. In order to determine the performance measures accurately causal loop diagram (system dynamics) was constructed which had ability to model cause and effect, feedback loops and delays. In the measurement phase, causal loop structure was moved to a tree structure and hierarchical, weighted additive value function with visual interactive sensitivity analysis (VISA) was applied to arrive at an overall performance indicator. In the analysis and improvement phases, causes of the poor performance were determined and alternative courses of actions were simulated by system dynamics. Then the best alternative was selected by the previously mentioned MCDM method.

CHAPTER TEN
FUZZY INTEGRAL

10.1 Basic Definitions

10.1.1 Measures and Integrals

Let X be a nonempty set, $\mathbf{C}$ be a nonempty class of subsets of X, and $\mu : \mathbf{C} \to [0, \infty]$ be a nonnegative, extended real valued set function defined on $\mathbf{C}$.

Definition 10.1 (Wang & Klir, 2009) : A set A in $\mathbf{C}$ is called the null set iff $\mu(A) = 0$.

Definition 10.2 (Murofushi & Sugeno, 2000): μ is additive iff

$$\mu(A \cup B) = \mu(A) + \mu(B) \qquad (10.1)$$

Whenever

$$A \in \mathbf{C},\ B \in \mathbf{C},\ A \cup B \in \mathbf{C}, \text{and } A \cap B = \varnothing, \qquad (10.2)$$

Definition 10.3 (Murofushi & Sugeno, 2000): μ is monotone iff

$$\mu(A) \le \mu(B) \qquad (10.3)$$

whenever

$$A \in \mathbf{C},\ B \in \mathbf{C},\ A \subset B \qquad (10.4)$$

Definition 10.4 (Wang & Klir, 2009): μ is finitely additive iff

$$\mu\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mu(A_i) \qquad (10.5)$$

for any disjoint and finite class $\{A_1, A_2, \ldots A_n\}$ of sets in $\mathbf{C}$ whose union is also in $\mathbf{C}$.

Definition 10.5 (Wang & Klir, 2009): μ is countably additive iff

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i) \tag{10.6}$$

for any disjoint sequence $\{A_n\}$ of sets in $\mathbf{C}$ whose union is also in $\mathbf{C}$.

Definition 10.6 (Murofushi & Sugeno, 2000)**:** A non-negative additive (or countably additive) set function defined on 2^X is a measure on X.

Definition 10.7 (Murofushi & Sugeno, 2000) Let μ be a measure on X and f a function on X. The integral $\int f(x)dm(x)$ of with respect to μ is defined as:

$$\int f(x)d\mu(x) = \int fd\mu = \sum_{x\in X} f(x)\mu(\{\mathrm{x}\}) \tag{10.7}$$

The graphical representation of integral can be seen in Figure 10.1, which satisfies

$$\int fd\mu = C_1 + C_2 + C_3 + C_4 + C_5 \tag{10.8}$$

where $C_i = f(x_i)\mu(\{\mathrm{x_i}\})$, with $i = 1,\ldots,5$.

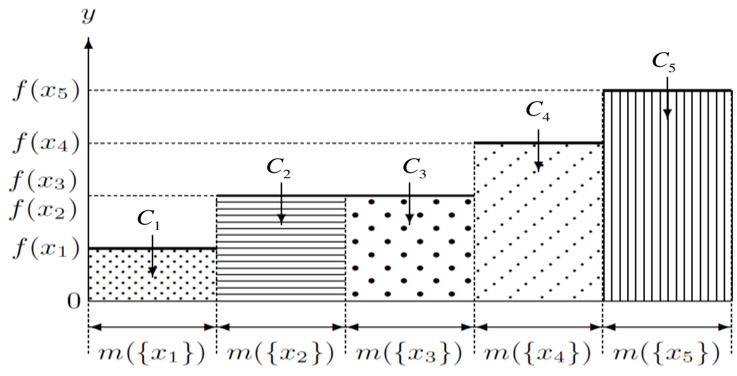


Figure 10.1 Graphical representation of integral

10.1.2 Fuzzy Measures

Definition 10.8 (Sugeno, 1977): A fuzzy measure μ on X is a function $\mu : \wp(X) \to [0,1]$, satisfying following axioms:

- $\mu(\varnothing) = 0$
- $A \subset B \subset X$ implies $\mu(A) \le \mu(B)$.

Also for modeling importance of criteria, usually $\mu(X) = 1$ is assumed, it is not compulsory in general though. A fuzzy measure μ is said to be:

- Additive if $\mu(A \cup B) = \mu(A) + \mu(B)$ when $A \cap B = \varnothing$
- Super additive if $\mu(A \cup B) \ge \mu(A) + \mu(B)$ when $A \cap B = \varnothing$
- Sub additive if $\mu(A \cup B) \le \mu(A) + \mu(B)$ when $A \cap B = \varnothing$

Among many fuzzy measures, λ-fuzzy measure (Sugeno, 1974) is the most widely used in the literature for modeling multiple attributes due to its simplicity. Satisfying the following properties (Sugeno, 1974) :

$$\forall A, B \in \wp(X), A \cap B = \varnothing \tag{10.9}$$

$$\mu_\lambda(A \cup B) = \mu_\lambda(A) + \mu_\lambda(B) + \lambda \mu_\lambda(A) \mu_\lambda(B), \text{ for } -1 \le \lambda < \infty \tag{10.10}$$

Fuzzy measure can be calculated based on the fuzzy densities $\mu_i = \mu_\lambda(\{x_i\})$ as follows (Sugeno, 1977):

$$\mu_\lambda(\{x_1, x_2, \ldots, x_n\}) = \sum_{i=1}^{n} \mu_i + \lambda \sum_{i_1=1}^{n-1} \sum_{i_2=i1+1}^{n} \mu_{i_1} \mu_{i_2} + \cdots + \lambda^{n-1} \mu_1 \mu_2 \cdots \mu_n \tag{10.11}$$

$$= \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1 + \lambda \mu_i) - 1 \right|, \text{ for } -1 \le \lambda < \infty \tag{10.12}$$

For criteria A and B, three cases can be observed as follows (Tzeng & Huang, 2011):

- If $\lambda > 0$, then $\mu_\lambda(A \cup B) > \mu_\lambda(A) + \mu_\lambda(B)$, which indicates multiplicative effect (complementarity, super-additive, conjunctive behavior)
- If $\lambda < 0$, then $\mu_\lambda(A \cup B) < \mu_\lambda(A) + \mu_\lambda(B)$, which indicates substitutive effect (redundancy, sub-additive)
- If $\lambda = 0$, then $\mu_\lambda(A \cup B) = \mu_\lambda(A) + \mu_\lambda(B)$, which indicates additive effect. In other words, there is no interaction.

Example 10.1 (Sugeno, 1977): Suppose that identical products are produced in a manufacturing plant. A group of workers in a workshop are responsible for production and we are dealing with the performance of workers. Set of all workers given by X and groups may work in different ways: in divided work or joint work. It is supposed that all groups work with their maximum efficiency.

Number of products produced by group A in one hour is denoted by $\mu(A)$. The fuzzy measure μ can also be non-additive. Suppose that A and B are different group of workers and productivity of coupled group is represented by $A \cup B$. If the groups work separately, equality becomes $\mu(A \cup B) = \mu(A) + \mu(B)$. However, this equation is not always satisfied, as the groups interact with each other. If the effective cooperation among the workers of two groups is established, $\mu(A \cup B) > \mu(A) + \mu(B)$ is reached. On the other hand, due to the several reasons such as limited space or inadequate number of equipment, working together might lead to $\mu(A \cup B) < \mu(A) + \mu(B)$.

Also note that without monotonicity condition, productivity may be diminished due to the conflicts between two groups and $\mu(A \cup B) < \mu(A)$ does hold. However, the assumption of working in the most efficient way satisfied the inequalities $\mu(A \cup B) \geq \mu(A)$ and $\mu(A \cup B) \geq \mu(B)$ that working of coupled groups leads to the performance at least as good as the performance which is obtained by only one of the groups.

10.1.3 Interpretation of Fuzzy Measures

In the literature, there are many interpretations of fuzzy measures differing mainly in application area and the context of the studies. The interpretations generally are tightly related to type of data and the specific fuzzy measures used in studies. We give the main interpretations of fuzzy measures relying on (Torra & Narukawa, 2007b) as in the Figure 10.2.

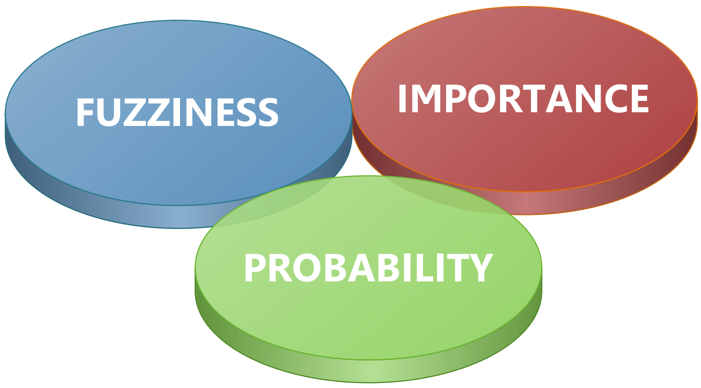


Figure 10.2 Different interpretations of fuzzy measures

1. **Fuzziness**: A fuzzy measure μ for a set A, $\mu(A)$ come to mean the grade that the element of set X belongs to the set A. Here, fuzziness is a sort of uncertainty, differ from the probability distributions, which indicates the degree to which element belongs to set by pursuing the membership function interpretation of fuzzy sets. For example, suppose that X refers to the set of information sources. Fuzzy measures are determined to modeling some aspects regarding these information sources supplying information. $\mu(A)$ is the degree that accurate information is provided by the information source that belongs to A.
2. **Importance**: $\mu(A)$ stands for the importance or weights used as an input to the aggregation function. Interpreting fuzzy measures as importance is very convenient when modeling criteria or experts. For example, when there are

no information sources available, then the importance equals to 0. On the other hand, the maximum importance is acquired when all the sources are considered and the maximum importance is equal to 1.

3. **Probability**: Especially belief and plausibility measures are interpreted as the probability as well as the some other types of fuzzy measures. For example, when the μ is additive, it coincides with the probability distribution on the information sources. In this case, $\mu(A)$ is interpreted as the probability of the information sources of A which are correct. On the other hand, when the μ is non-additive, then we talk about generalized probabilities on the information sources.

10.2 Choquet Integral

10.2.1 Definition of Choquet Integral

Definition 10.9 (Choquet, 1954): Let μ be a fuzzy measure on X. The elements of X are denoted by $x_1, x_2, \ldots, x_n$. The discrete Choquet integral of a function $f : X \to \mathbb{R}^+$ with respect to μ is defined by

$$C_\mu(f) = \sum_{i=1}^{n} f(x_{(i)}) - f(x_{(i-1)})\mu(A_{(i)}) \qquad (10.13)$$

where $.(i)$ implies that the indices have been permuted so that $0 \le f(x_{(1)}) \le \cdots \le f(x_{(n)})$ and $A_{(i)} = \{x_{(i)}, \ldots, x_{(n)}\}$ and $f(x_{(0)}) = 0$.

where $C_i = (a_i - a_{i-1})\mu(\{x \mid f(x) \ge a_i\})$ with $i = 1, \ldots, 4$ and

$$C_\mu(f) = \int f dm = C_1 + C_2 + C_3 + C_4 \qquad (10.14)$$

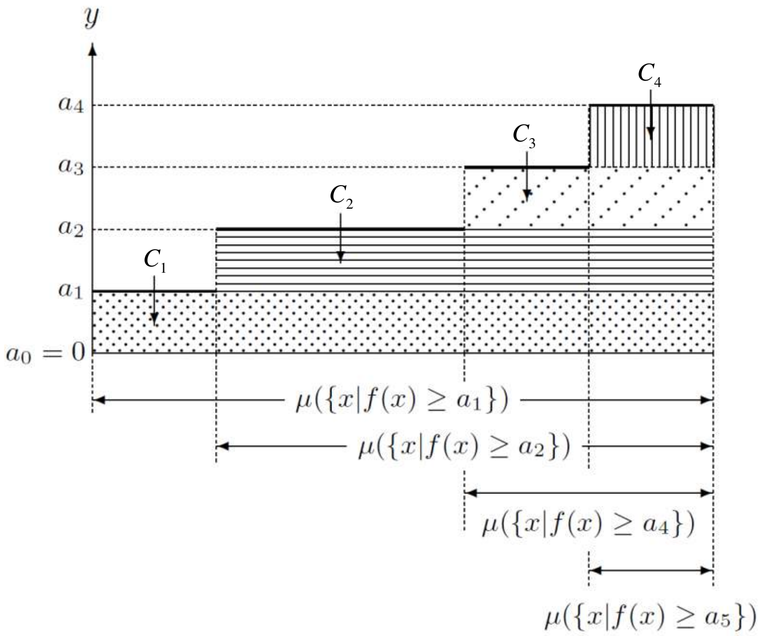


Figure 10.3 Graphical representation of Choquet integral (Murofushi & Sugeno, 2000)

10.2.2 Calculation Algorithm of Choquet Integral

The calculation of Choquet integral in an algorithmic form is as follows (Beliakov, Pradera, & Calvo, 2007b) :

1. Sort the items of $((x_1,1),(x_2,2),\ldots,(x_n,n))$ in increasing order according to the first item of each pair. Therefore, the items of $((x_{(1)},i_1),(x_{(2)},i_2),\ldots,(x_{(n)},i_n))$ are obtained, hence $x_{(j)} = xi_j$ and $x_{(j)} \leq x_{(j+1)}$ for all i. Let also $x_{(0)} = 0$
2. Let $A = \{1,\ldots,n\}$ and $C = 0$
3. For $j = 1,\ldots,n$ do
    a. $C = C + [x_{(j)} - x_{(j-1)}]\mu(A)$
    b. $A = A\,|\,\{i_j\}$
4. Return C

Example 10.2 (Beliakov et al., 2007b): Suppose that $n = 3$, related fuzzy measures μ be given and $x = (0.8, 0.1, 0.6)$

1. The items $((0.8,1),(0.1,2),(0.6,3))$ are considered. Sorting this vector as in the first step of the algorithm leads to $((0.1,2),(0.6,3),(0.8,1))$
2. $A=\{1,2,3\}$ and $C=0$.
3. Inside the loop, following calculations are made;
   a. $C=0+[0.1-0]\mu(\{1,2,3\})=0.1\times 1=0.1;$
   b. $A=\{1,2,3\}\,|\,\{2\}=\{1,3\};$
   a. $C=0.1+[0.6-0.1]\mu(\{1,3\})=0.1+0.5\mu(\{1,3\});$
   b. $A=\{1,3\}\,|\,\{3\}=\{1\};$
   a. $C=[0.1+0.5\mu(\{1,3\})]+[0.8-0.6]\mu(\{1\})$
4. $C_\mu(x)=0.1+0.5\mu(\{1,3\})+0.2\mu(\{1\})$

10.2.3 Interpretation of Choquet Integral

Example 10.3 (Grabisch, 1995): A high school students are to be evaluated by the director in terms of their grades in mathematics, literature and physics. It is assumed that the school is scientifically oriented and importance of mathematics and physics is higher than that of literature. The director of the high school assigns weights of 3,3, and 2 for mathematics, physics and literature respectively. The performance matrix is and aggregation results with weighted mean are shown in Table 10.1.

Table 10.1 Performance table for evaluating students

Student	Mathematics	Physics	Literature	Global evaluation (weighted mean)
A	18	16	10	$A=\dfrac{18\times 3+16\times 3+10\times 2}{3+3+2}=15.25$
B	10	12	18	$B=\dfrac{10\times 3+12\times 3+18\times 2}{3+3+2}=12.75$
C	14	15	15	$C=\dfrac{14\times 3+15\times 3+15\times 2}{3+3+2}=14.62$

However, the director is not pleased with the obtained ranking. According to director, student C is more preferable than student A due to the fact that student C is good at scientific classes and also well equilibrated with the literature, whereas student A is bad at literature. The problem here is that there is no weighting vector that makes student C more preferable than student A. Consequently, Choquet integral is selected as a modeling tool. Director decides that:

1. The importance of scientific classes is higher (mathematics and physics).
2. Mathematics and physics are similar classes so students good at both of them should not be favored in the aggregation.
3. It is not very common situation that the students are good at both mathematics (physics) and literature.

As a result, fuzzy measure representation of above mentioned features are given as follows:

1. $\mu(\{Mathematics\}) = \mu(\{Physics\}) = 0.45$ and $\mu(\{Literature\}) = 0.3$
2. $\mu(\{Mathematics, Physics\}) = 0.5 < \mu(\{Mathematics\}) + \mu(\{Physics\})$ (redundancy)
3. $\mu(\{Mathematics, Literature\}) = \mu(\{Physics, Literature\}) = 0.9 > 0.45 + 0.3$ (complementarity)

Table 10.2 Choquet integral aggregation for student evaluation

Student	Global Evaluation (Choquet integral)
A	$A = 10\times\mu(\{Mathematics, Physics, Literature\}) + (16-10)\times\mu(\{Mathematics, Physics\}) + (18-16)\times\mu(\{Mathematics\}) = 10\times1 + 6\times0.5 + 2\times0.45 = 13.9$
B	$B = 10\times\mu(\{Mathematics, Physics, Literature\}) + (12-10)\times\mu(\{Physics, Literature\}) + (18-12)\times\mu(\{Literature\}) = 10\times1 + 2\times0.9 + 6\times0.30 = 13.6$
C	$C = 14\times\mu(\{Mathematics, Physics, Literature\}) + (15-14)\times\mu(\{Physics, Literature\}) + (15-15)\times\mu(\{Literature\}) = 14\times1 + 1\times0.9 + 0\times0.30 = 14.9$

The results of the Choquet integral calculation brings the results summarized in table 10.2. Students are rankled as $C \succ A \succ B$ which corresponds to the preferences of director. Because the school is scientifically oriented, student B has the smallest total value. On the other hand, student C being good at both literature and science classes led it to be ranked first, which is more appropriate compared to student A who is bad at literature.

10.3 k-additive Choquet Integral

10.3.1 Definitions of k-additive Choquet Integral

Despite the strong mathematical underpinnings of fuzzy measures in the use of multiple criteria decision making, reported real world applications in the top scientific journals are scarce. Main reason of rare applications of fuzzy measures is it's intrigued and difficult to understand mathematical formalism.

According to (Grabisch, 1997); two reasons can be counted to explain small number of practical applications of fuzzy measures.

- If there are n criteria in the decision problem, 2^n positive coefficients should be determined. Off course, monotonicity conditions should be satisfied. So the biggest challenge the decision maker confronts is to make a choice between selecting an additive measure like probability measure to reduce complexity with n coefficients, on the other hand, accepting the difficulty of defining 2^n coefficients for the sake of a richer analysis.
- Coefficients of fuzzy measures rely on the strength of coalitions among criteria. In most real life situations, decision maker is able to assign importance degrees to singletons of criteria. However, when it comes to fuzzy measures, decision maker is expected to assign importance degrees to the couple of criteria. Satisfying monotonicity conditions and distinguishing the importance of coalition, for example, difference between importance of $\mu\{x_1, x_2\}$ and $\mu\{x_1, x_2, \mathrm{x}_3\}$ is not a trivial task.

One of the well-known techniques proposed in order to manage the above mentioned obstacles is using k-order additive fuzzy measures which is introduced by (Grabisch, 1997). For simplicity, we also use the term k -additive fuzzy measures.

Definition 10.10 (Grabisch, 1997): A capacity of order k (k - monotone) is a monotone measure μ such that

$$\mu\left(\bigcup_{j=1}^{k} A_j\right) \geq \sum_{\substack{K\subseteq X_k \\ K\neq\varnothing}} (-1)^{|K|+1} \mu\left(\bigcap_{j\in K} A_j\right) \tag{10.15}$$

holds for all family of subsets $A_1,\ldots,A_k$.

k -additive capacity only permits the interaction of k number of criteria. For example 1 -additive fuzzy measures do not allow to model interaction. Albeit, 2 - additive fuzzy measures enable to model interaction up to between 2 -criteria. This property holds for other values of k .

The greatest advantage of using k -additive measures is that it requires $\sum_{i=1}^{k}\binom{n}{i}$ coefficients. Remember that fuzzy measures need 2^n coefficients to be defined. As $\mu(\varnothing)=0$ and $\mu(X)=1$ are known, fuzzy measures necessitate to determine 2^n-2 coefficients. Since decision makers seek for the balance between richness of the analysis and the cognitive burden they are forced to bear, 2 -additive fuzzy measures take special attention by research community. For its practicality and satisfying the best compromise between complexity and richness, particular interest focused on Choquet integral with respect to 2 -additive fuzzy measures.

Grabisch (1997) defined Choquet integral for μ which is 2 -additive fuzzy measure, and f is a real-valued function on N . Choquet integral with respect to 2 - additive fuzzy measure can be expressed as:

$$C_\mu(f) = \sum_{i,j \in N | I_{ij} > 0} (f_i \wedge f_j) I_{ij} + \sum_{i,j \in N | I_{ij} < 0} (f_i \vee f_j) \left| I_{ij} \right| + \sum_{i \in N} f_i \left[\phi_i - \frac{1}{2} \sum_{j \neq i} \left| I_{ij} \right| \right] \qquad (10.16)$$

where ϕ_i represents Shapley value of μ, and I_{ij} is the interaction index between criteria i and j.

Besides, the alternative representation of the Choquet integral with respect to 2-additive capacities is expressed as follows (Grabisch & Labreuche, 2010):

$$C_\mu(f) = \sum_{i=1}^{n} \phi_i f_i - \sum_{\{i,j\} \subseteq N} \frac{I_{ij}}{2} \left| f_i - f_j \right| \qquad (10.17)$$

10.3.2 Möbius Transformation and k-additive Fuzzy Measure

Möbius transformation representation allow us to check the k-monotonocity of fuzzy measure. Futhermore, Möbius transformation is pragmatic tool to determine whether the fuzzy measure is a Choquet capacity or not. Choquet capacities satisfy the following criteria (Klir, 2005);

1. ${}^{\mu}m(\varnothing) = 0$
2. $\sum_{A \in P(X)} {}^{\mu}m(A) = 1$
3. $\sum_{\{x\} \subseteq B \subseteq A} {}^{\mu}m(B) \geq 0$, $\forall A \in P(X)$ and $x \in A$

Additionally, Möbius transformation of fuzzy measure should satisfy the following properties to be k-order additive measure. To be k-order additive measure;

4. μ is a Choquet capacity if ${}^{\mu}m(A) \geq 0$, where $2 \leq |A| \leq k$
5. μ is a Choquet capacity of order ∞ if and only if ${}^{\mu}m(A) \geq 0$, $\forall A \in P(X)$

6. μ is probability measure if and only if ${}^{\mu}m(A) > 0$ when $|A| = 1$ and ${}^{\mu}m(A) = 0$ otherwise.

7. μ is a Choquet capacity of order $k(k \geq 2)$ if and only if

$$\sum_{C \subseteq B \subseteq A} {}^{\mu}m(B) \geq 0 \tag{10.18}$$

for $\forall A \in P(X)$ and $\forall C \in P(X)$ such that $2 \leq |C| \leq k$

Example 10.4:

Table 10.3 Fuzzy measures of example 10.4

x_1	x_2	x_3	$\mu(A)$	${}^{\mu}m(\text{A})$
0	0	0	0	0
1	0	0	0.55	0.55
0	1	0	0.25	0.25
0	0	1	0.35	0.35
1	1	0	0.55	-0.25
1	0	1	0.90	0
0	1	1	0.55	-0.05
1	1	1	1	0.15

In the example 10.4, $\mu(A)$, also represented by both Hasse diagram in figure 10.4, is not a 2-order fuzzy measure for two reasons. First, ${}^{\mu}m_A(\{\text{x}_1, \text{x}_2\})$ and ${}^{\mu}m_A(\{\text{x}_2, \text{x}_3\})$ are not positive so the fourth property above is violated. Also, it does not satisfy the these equalities;

- $\mu(\{\text{x}_1\} \cup \{\text{x}_2\}) < \mu(\{\text{x}_1\}) + \mu(\{\text{x}_2\})$ which becomes $0.55 < 0.55 + 0.25$
- $\mu(\{\text{x}_2\} \cup \{\text{x}_3\}) < \mu(\{\text{x}_2\}) + \mu(\{\text{x}_3\})$ which becomes $0.55 < 0.25 + 0.35$

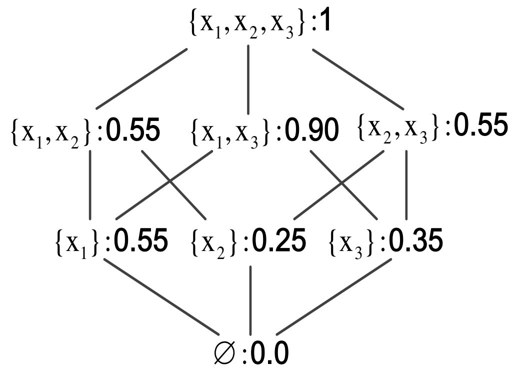


Figure 10.4 Hasse diagram of example 10.4

10.3.3 Interpretation of 2-additive Choquet integral

As we discussed earlier, there are many different types of fuzzy measures introduced in the literature. Maybe the most commonly used fuzzy measure is λ-measure proposed by (Sugeno, 1974, 1977). Apart from the λ-measure, k-additive measures (Grabisch, 1997), k-tolerant and k-intolerant measures (Marichal, 2004), and p-symmetrical measures (Miranda, Grabisch, & Gil, 2002) are important fuzzy measures especially suitable for multi criteria decision making contexts.

In the case of k-additive fuzzy measures, the way Choquet aggregation operates brings about important merits of the method. In the Choquet integral formula above, aggregation function can be divided into three categories or parts according to sign of the interaction indices which is defined in the interval $[-1,+1]$ as follows (Büyüközkan & Ruan, 2010):

- A positive I_{ij} stands for conjunctive behavior, in other words, complementarity or super-additive situation, which means that simultaneous satisfaction of both of the criteria have substantial effect whereas satisfaction of only one criterion does not affect the overall performance.
- A negative I_{ij} stands for disjunctive behavior, in other words, redundancy, substitutive or sub-additive situation, which means that satisfaction of one of the criteria is sufficient for the overall performance.

- The zero I_{ij} stands for no interaction situation. Aggregation corresponds to weighted means.

Interaction index and Shapley value have great influence in the 2-additive Choquet integral aggregation. Dilating on 2-additive Choquet aggregation while keeping in mind the properties of interaction indices with respect to its sign, following remarkable points are observed;

- When $I_{ij} > 0$, positive interaction occurs. Consequently, interaction leads to conjunctive aggregation. In this case, according to aggregation formula, if the difference between the performance scores of an alternative with respect to criterion i and j ($|f_i - f_j|$) increase, then the overall value of aggregation is decreased due to fact that interaction phenomenon imposes a penalty on aggregation. More rigorously, aggregation is penalized by $I_{ij}/2$ since positive interaction is an undesirable situation that the correlated criteria are double counted in the aggregation (we do not want to double count the value of physics and mathematics classes so aggregation result is decreased by the factor $I_{ij}/2$).
- When $I_{ij} < 0$, negative interaction occurs and disjunctive behavior happens. If difference between f_i and f_j ($|f_i - f_j|$) increases, interaction phenomenon compensates it by escalating the overall aggregation value. For example, suppose that $f_i < f_j$; low score of f on the ith criterion is compensated to a degree $I_{ij}/2$ by a high evaluation on j_{th} criterion (Grabisch & Labreuche, 2010).
- When $I_{ij} = 0$, no interaction occurs, Shapley value corresponds to the weight of criteria and aggregation is actually weighted mean.

10.4 Sugeno Integral

Sugeno integral is one of the aggregation functions used also for aggregating preferences in decision making. Sugeno integral is the generalization of the aggregation functions such as medians, weighted minimum and weighted maximum (Beliakov, Pradera, & Calvo, 2007a).A survey paper regarding Sugeno integral and its applications in MCDM is given by (Dubois et al., 2001).

Definition 10.11 (Sugeno, 1974): Let μ be a fuzzy measure on $(X, 2^X)$. Sugeno integral $S_\mu(f)$ of a function $f : X \to [0,1]$ with respect to μ is defined by

$$S_\mu(f) = \max_{j=1,\ldots,n} \min\{f(x_{s(j)}), \mu(A_{s(j)})\} \tag{10.19}$$

where indices of $f(x_{s(i)})$ have been permuted so that

$$0 \le f(x_{s(1)}) \le \cdots \le f(x_{s(n)}) \le 1,\ A_{s(i)} = \{x_{s(i)}, \ldots, x_{s(n)}\}, A_{s(n+1)} = \varnothing \tag{10.20}$$

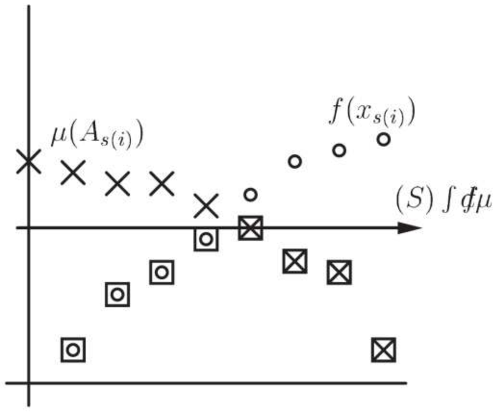


Figure 10.5 Graphical interpratation of Sugeno integral (Torra & Narukawa, 2007a)

In Figure 10.5, the values $f(x_{s(i)})$ are shown by dots, the measures $\mu(A_{s(i)})$ are denoted by crosses and the values $\min(f(x_{s(i)}), \mu(A_{s(i)}))$ are represented by squares. It can be inferred from the graphical representation that the Sugeno integral chooses the importance that overcomes certain thresholds. The integral seeks for a tradeoff between importance of the sets and the members of the sets have assigned (Torra & Narukawa, 2007a).

Another property of the Sugeno integral is that it calculates the length of the square with maximum area for the continuous monotone function as seen in Figure 10.6. Actually, Sugeno integral is the generalization of weighted maximum aggregation operator. The only difference is that in the Sugeno integral, fuzzy measure is used instead of weights, where the each value of the function is weighted by the measure supporting a value equal or larger than $f(x_{s(i)})$.

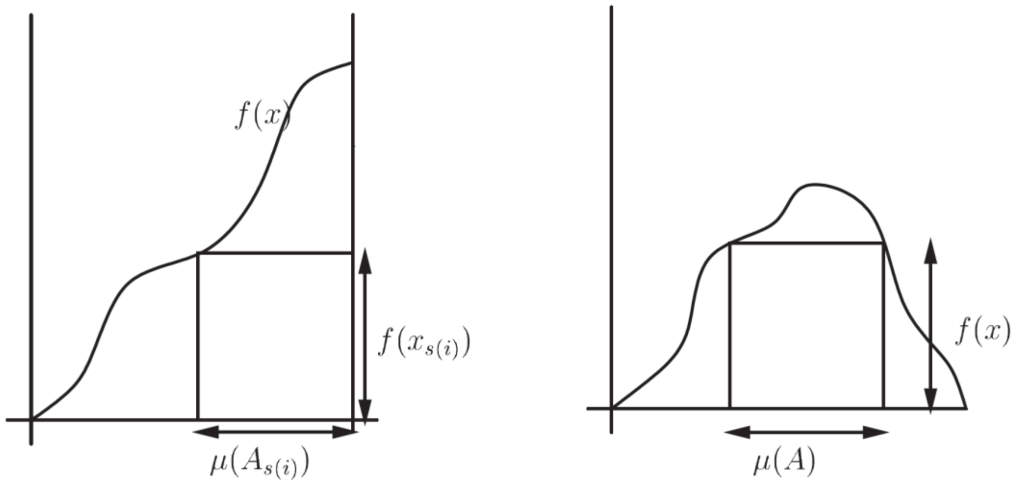


Figure 10.6 Calculation of lenght of square with maximum area by Sugeno integral (Torra & Narukawa, 2007a)

We illustrate the Sugeno integral by the following example:

Example 10.5 (Torra & Narukawa, 2006): Suppose that a group of experts evaluate the reliability of three machines. Experts are denoted by $X = \{x_1, x_2, \ldots, x_n\}$. Further, the reliability of a machine is expressed by $f(x_i)$ which correspond to reliability of a machine according to expert x_i. Also reliability of group of experts is determined. Reliability of experts are given by fuzzy measures as: $\mu(\{x_1\}) = 0.2$, $\mu(\{x_2\}) = 0.3$, $\mu(\{x_3\}) = 0.4$. When the experts are together, the group reliabilities are $\mu(\{x_1, x_2\}) = 0.3$, $\mu(\{x_1, x_3\}) = 0.4$, $\mu(\{x_2, x_3\}) = 0.8$. As a result of the boundary conditions, we know that $\mu(\emptyset) = 0$ and $\mu(\{x_1, x_2, x_3\}) = 1$. The evaluations of experts regarding the reliability of machines are given by: $f(x_1) = 0.3$, $f(x_2) = 0.7$, $f(x_3) = 0.6$ as in the table 10.4.

Table 10.4 Reliability evaluations of three experts

Set	x_1	x_2	x_3
f	0.3	0.7	0.6

Reliability degrees of machines μ_f are calculated as seen in table 10.5.

Table 10.5 Reliability degrees of machines

Set	x_1	x_2	x_3
μ_f	1	0.3	0.8

The reliability degree is defined by $\mu_f(x_i) = \mu(\{x \mid f(x) \geq f(x_i)\})$. For instance, in case of machine 1:

$$\begin{aligned} \mu_f(x_1) &= \mu(\{x \mid f(x) \geq f(x_1)\}) = \mu(\{x \mid f(x) \geq 0.3\}) \\ &= \mu(\{x_1, x_2, x_3\}) = 1 \end{aligned} \tag{10.21}$$

The reliability degree of machine 2 is calculated by:

$$\begin{aligned} \mu_f(x_2) &= \mu(\{x \mid f(x) \geq f(x_2)\}) = \mu(\{x \mid f(x) \geq 0.7\}) \\ &= \mu(\{x_2\}) = 0.3 \end{aligned} \tag{10.22}$$

The reliability degree of machine 3 is calculated by:

$$\begin{aligned} \mu_f(x_3) &= \mu(\{x \mid f(x) \geq f(x_3)\}) = \mu(\{x \mid f(x) \geq 0.6\}) \\ &= \mu(\{x_2, \mathrm{x}_3\}) = 0.8 \end{aligned} \tag{10.23}$$

Table 10.6 Calculation of machine reliability by Sugeno integral

Set	x_1	x_2	x_3
f	0.3	0.7	0.6
μ_f	1	0.3	0.8
$f(x_i) \wedge \mu_f(x_i)$	0.3	0.3	0.6
$\max\limits_{i=1,2,3} f(x_i) \wedge \mu_f(x_i)$	0.6		

10.5 Interaction Indices

When modeling with multiple attributes, it is often the case that there are positive or negative interactions among the attributes. If the attributes are aggregated by means of conventional methods such as weighted means, then, for example, the scores of some attributes might be double counted. On the other hand, an attribute might not be very important in terms of its contribution to total score, conversely it might become more important in a coalition with the other attributes.

Utilizing fuzzy measure to model coalitions of attributes, importance of an attribute or interaction degrees among the attributes can be needed. Contributions of the attribute in various coalitions are required to be calculated.

Definition 10.12 (Shapley value) (Beliakov et al., 2007b): Let μ be a fuzzy measure. The Shapley index for every $i \in N$ is

$$\phi(i) = \sum_{A \subseteq N \setminus \{i\}} \frac{(n-|A|-1)!\,|A|!}{n!} [\mu(A \cup \{i\}) - \mu(A)] \tag{10.24}$$

The Shapley value is vector $\phi(v) = (\phi(1), \ldots, \phi(n))$. The Shapley value is the average of the contribution of each criterion in all coalitions.

Example 10.6:

Table 10.7 Fuzzy measures of example 10.6

x_1	x_2	x_3	$\mu(A)$
0	0	0	0
1	0	0	0.30
0	1	0	0.20
0	0	1	0.25
1	1	0	0.35
1	0	1	0.40
0	1	1	0.45
1	1	1	1

Let us calculate the Shapley value for $\{x_1\}$ in a step by step approach. We have four situations according to different coalitions involving $\{x_1\}:\{x_1\},\{x_1,x_2\},\{x_1,x_3\}$,and $\{x_1,x_2,x_3\}$. We calculate the Shapley value for $\{x_1\}$ based on the coalitions of the $\{x_1\}$ as $\phi(x_1)_1, \phi(x_1)_2, \phi(x_1)_3$, $\phi(x_1)_4$.

When only x_1 is individually in the coalition, $A=\{x_2,x_3\}$ and $|A|=2$, Shapley value is calculated as follows:

$$\begin{aligned}\phi(x_1)_1 &= \frac{(3-|0|-1)!|0|!}{3!}\times[\mu_1-\varnothing] \\ &= \frac{2}{6}\times 0.30 = 0{,}10\end{aligned} \tag{10.25}$$

When x_1,x_2 is in the coalition, $A=\{x_3\}$ and $|A|=1$, Shapley value is calculated as follows:

$$\begin{aligned}\phi(x_1)_2 &= \frac{(3-|1|-1)!|1|!}{3!}\times[\mu_{12}-\mu_2] \\ &= \frac{1}{6}\times(0.35-0.20) = 0.025\end{aligned} \tag{10.26}$$

When x_1,x_3 is in the coalition, $A=\{x_2\}$ and $|A|=1$, Shapley value is calculated as follows:

$$\begin{aligned}\phi(x_1)_3 &= \frac{(3-|1|-1)!|1|!}{3!}\times[\mu_{13}-\mu_3] \\ &= \frac{1}{6}\times(0.40-0.25) = 0.025\end{aligned} \tag{10.27}$$

When x_1,x_2,x_3 is in the coalition, $A=\{\varnothing\}$ and $|A|=0$, Shapley value is calculated as follows:

$$\phi(x_1)_4 = \frac{(3-|0|-1)!|0|!}{3!} \times [\mu_{123} - \mu_{23}]$$
$$= \frac{2}{6} \times (1-0.45) = 0.1833 \qquad (10.28)$$

Finally, Shapley value for the first attribute $\phi(x_1)$ is as follows:

$$\begin{aligned} \phi(x_1) &= \phi(x_1)_1 + \phi(x_1)_2 + \phi(x_1)_3 + \phi(x_1)_4 \\ &= 0.10 + 0.025 + 0.025 + 0.1833 \\ &= 0.3333 \end{aligned} \qquad (10.29)$$

The same calculation is done for the rest of the attributes and Shapley values are found as in the table 10.8.

Table 10.8 Shapley indices of example 10.6

$\phi(1)$	$\phi(2)$	$\phi(3)$
0.3333	0.3083	0.3583

Definition 10.13 (Interaction index) (Grabisch, 1997): Let v be a fuzzy measure. The interaction index for every pair $i, j \in N$ is

$$I_{ij} = \sum_{A \subseteq N|\{i,j\}} \frac{(n-|A|-2)!|A|!}{(n-1)!} [\mu(A \cup \{i,j\}) - \mu(A \cup \{i\} - \mu(A \cup \{j\} + \mu(A)] \qquad (10.30)$$

where $I_{ij} \in [0,1]$. The interaction index indicates complementarity situation when $I_{ij} > 0$ or redundancy situation when $I_{ij} < 0$.

The results of interaction index is calculated by Kappalab package (Grabisch, Kojadinovic, & Meyer, 2009) for the GNU R statistical system (R Development Core Team, 2005) as follows:

Table 10.9 Interaction indices of example 10.6

I_{ij}	x_1	x_2	x_3
x_1	-	0.125	0.125
x_2	0.125	-	0.275
x_3	0.125	0.275	-

10.6 Further Discussion of Choquet Integral

10.6.1 Criticism of Weighted Average and Need of Choquet Integral

Weighted arithmetic mean is one of the most popular aggregation operator used in MCDM methods. However, how well it can depict the preferences of the decision maker is questionable. Let us give an example to illustrate our assertion. The example is adapted from (Keeney, 1992b).

Example 10.7: Suppose that we have three alternatives and which are evaluated with respect to two criteria. Alternatives are denoted by a, b and c respectively.

Table 10.10 Decision matrix of example 10.7

	c1	**c2**
a	0,4	0,4
b	0	1
c	1	0

$$\begin{aligned} &u_1(a) = 0.4, \quad u_1(b) = 0, \quad u_1(c) = 1, \\ &u_2(a) = 0.4, \quad u_2(b) = 1, \quad u_2(c) = 0, \end{aligned} \tag{10.31}$$

where utility scores are between [0,1]. Also assume that decision maker states his/her preference as $a \succ b \sim c$. Here we try to find the criteria weights that the weighted sum aggregation operator reflects the preference of decision maker. Suppose that decision maker gives the preferences as $b \sim c$ and $\mathrm{a} \succ \mathrm{b}$.

$$\begin{aligned} &b \sim c \iff w_1 = w_2, \\ &\mathrm{a} \succ \mathrm{b} \iff 0.4(w_1 + w_2) > w_2 \end{aligned} \tag{30.32}$$

which means that $0.8w_2 > w_2$, so it is contradiction.

Actually, we infer that the decision maker favors the alternatives which have well balanced scores on two criteria. Decision maker prefers alternative a to alternative b as b has relatively unbalanced scores as 0 and 1. Thus, weighted arithmetic method does not comply with the decision makers preferences. Practical solution to this problem could be specifying the weights for the group of criteria, rather than for each single criterion.

A weight w_{12} corresponds to the score that the decision maker determine for the alternative which is fulfilled on both of the criteria. Since decision maker prefers the alternative that satisfies both of the criterion, maximum value of 1 can be assigned for w_{12}. This means that the importance of satisfaction of both of the criteria is very important for decision maker. When it comes to w_1 and w_2, we know that decision maker prefers the alternative which satisfy both of the criteria simultaneously and he/she is not pleased with the satisfaction of only one criterion. Thereby, assignment $w_1 = w_2 = 0.3$ can be made. Since we assign the weights for the group of criteria, w_1 and w_2 do not necessarily yields $w_1 + w_2 = 1$.

Our calculation procedure follows these steps:

- Calculation for $U(a)$: Alternative a has the same scores for both of the criterion which was described by w_{12}. When we use w_{12},
  - $U(a)$ becomes $U(a) = 0.4 \times \mathrm{w}_{12} = 0.4 \times 1 = 0.4$
- Calculation for $U(\mathrm{b})$: Weight of the criteria was represented by w_1 and w_2,
  - $U(\mathrm{b})$ becomes $U(\mathrm{b}) = 0 \times w_1 + 1 \times w_2 = 1 \times w_2 = 1 \times 0.3 = 0.3$
- Calculation for $U(\mathrm{c})$: Weight of the criteria was represented by w_1 and w_2,
  - $U(\mathrm{c})$ becomes $U(\mathrm{c}) = 1 \times w_1 + 0 \times w_2 = 1 \times w_1 = 1 \times 0.3 = 0.3$

Consequently, preference order $a \succ b \sim c$ is satisfied. This example demonstrates that different preference orders can be realized if one can determine suitable criterion weights, i.e., in our example, w_1, w_2, w_{12}.

Continuing with the same example, Assume that we have one more alternative which is denoted by d.

Table 10.11 Decision matrix of example 10.7 with additional alternative

	c1	**c2**
a	0,4	0,4
b	0	1
c	1	0
d	0,3	0,7

Also, decision maker's preferences are $a \succ d,\ d \succ b,\ d \succ c$. To solve this problem, we create two artificial alternatives d_1 and d_2 which constitute the alternative d.

Table 10.12 Decision matrix of example 10.7 with dummy alternatives

	c1	**c2**
a	0,4	0,4
b	0	1
c	1	0
d1	0,3	0,3
d2	0	0,4

Calculations for alternative d as follows:

- Calculation for $U(\mathrm{d}_1)$: Alternative d_1 has the same scores for both of the criterion which was described by w_{12}. When we use w_{12},
  - $U(\mathrm{d}_1)$ becomes $U(a) = 0.3 \times \mathrm{w}_{12} = 0.3 \times 1 = 0.3$

- Calculation for $U(\mathrm{d}_2)$: Weight of the criteria was represented by w_1 and w_2,
    - $U(\mathrm{d}_2)$ becomes $U(\mathrm{d}_2) = 0 \times w_1 + 0.4 \times w_2 = 0.4 \times w_2 = 0.4 \times 0.3 = 0.12$
- Calculation for $U(\mathrm{d})$: $U(\mathrm{d}) = U(\mathrm{d}_1) + U(\mathrm{d}_2) = 0,30 + 0,12 = 0,32$

Consequently, we get the result as $a \succ d \succ b \sim c.$ The way we calculated the total performances of alternatives were indeed Choquet integral calculation.

10.6.2 Choquet Integral and Preferential Independence

Preferential independence is an important assumption that should be satisfied in order to make use of additive utility function. Furthermore, if an aggregation operator is associative and strictly increasing, then it implies mutual preferential independence (Dubois & Prade, 1985; Grabisch, 1995). Therefore, preferential independence assumption takes place very often in decision analysis problems. The most comprehensive study revealing the relationship between Choquet integral and independence concepts of multi attribute utility theory that we revisited in earlier chapters provided by Sugeno and Murofushi (2000). In their study, Choquet integral representation of well-defined independence concepts such as preferential independence, utility independence and difference independence are provided. As they stated in the concluding remarks, the preferential independence assumption, which is the weakest one considering other independence concepts, can be avoided in a Choquet integral model.

To illustrate how preferential independence concept is not necessarily assumed in Choquet integral, we give example of student evaluation problem based on Marichal (2000).

Suppose that we evaluate four students denoted by a, b, c and, with respect to their performance on statistics, probability and algebra courses. Each criterion has two levels: good and bad. We assume that the first level corresponds to good score, and the second level represents bad score. For instance, statistics criterion represented by X and levels of it are represented by x_1 and x_2, where x_1 is preferred to x_2. The performance matrix regarding our example is provided in Table 10.13.

Table 10.13 Performance table of student evaluation problem with numerical values

Student	Statistics (X) (x_1=19, x_2=11)	Probability (Y) (y_1=18, y_2=15)	Algebra (Z) (z_1=18, z_2=15)
A	19	15	18
B	19	18	15
C	11	15	18
D	11	18	15

For the compatibility of the notation used in the multi attribute utility theory chapter, we give the performance matrix as in the form of Table 10.14.

Table 10.14 Performance table of student evaluation problem with their attribute levels

Student	Statistics (X)	Probability (Y)	Algebra (Z)
A	x_1	y_2	z_1
B	x_1	y_1	z_2
C	x_2	y_2	z_1
D	x_2	y_1	z_2

The decision maker is expected to rank the students. From the performance table of the problem, two ranking orders are easily derived as:

1. $A \succ C$: Because of the fact that $(x_1, y_2, z_1) \succ (x_2, y_2, z_1)$.
2. $B \succ D$: Because of the fact that $(x_1, y_1, z_2) \succ (x_2, y_1, z_2)$.

The remaining rankings are not obviously obtained from the performance table since associated preference profiles are intertwined. It is assumed that the decision maker defined a rule such that:

- **Rule 1**: If a student is good at statistics, than the student has a better performance in the algebra course is preferred to the student with a better performance of probability course. By this rule, third ranking order is derived as:

3. $A \succ B$: Because of the fact that $(x_1, y_2, z_1) \succ (x_1, y_1, z_2)$.

The other decision rule is the reciprocal of the earlier one such that:

- **Rule 2**: If a student is not good at statistics, than the student has a better performance in the probability course is preferred to the student with a better performance of algebra course.

Consequently, this rule leads to the forth ranking order as:

4. $D \succ C$: Because of the fact that $(x_2, y_1, z_2) \succ (x_2, y_2, z_1)$.

Let us take the last ranking order. Also recalling the multi attribute utility theory chapter, the condition for probability and algebra to be preferentially independent of statistics is that

$$(\mathbf{x_2}, y_1, z_2) \succ (\mathbf{x_2}, y_2, z_1) \Leftrightarrow (\mathbf{x_1}, y_1, z_2) \succ (\mathbf{x_1}, y_2, z_1) \tag{10.33}$$

where we only changed the level of the x (x_2 is used in the left side of the ranking order, while at the right side x_1 is used) and level of y and z are endured. Actually, the condition of preferential independency is violated due to the first rule, which implies that the right side of the ranking order is not satisfied. If student is good at statistics, then the student with better performance in algebra (z_1) is preferred rather than a student with good performance of probability (y_1). As a result, $(\mathbf{x_1}, y_2, z_1) \succ (\mathbf{x_1}, y_1, z_2)$ should be satisfied in the case of $(\mathbf{x_2}, y_1, z_2) \succ (\mathbf{x_2}, y_2, z_1)$. Ultimately, probability and algebra is not preferentially independent of statistics. Since the defined rules are examples of redundancy among attributes, preferential independence situation is not necessarily assumed in Choquet integral models.

10.7 Related Review

Tzeng et al. (2005) proposed hierarchical model with Choquet integral for evaluating enterprise intranet web sites. In the study, an algorithm was developed to calculate λ values in λ-measures when fuzzy densities were gathered from the decision maker. A case study was provided in which the hierarchical λ-fuzzy measure based Choquet integral was used to evaluate enterprise intranet web sites and results were compared with the AHP evaluation.

Yang et al. (2008) integrated ISM, AHP and Choquet integral for vendor selection problem. First, ISM method was used to map out the relationships between criteria. Then, based on the map obtained from ISM, fuzzy analytic hierarchy process (FAHP) was employed to compute the relative weights of criteria. Non-additive fuzzy integral was used to find the fuzzy synthetic performance of each criterion. Finally, best vendor was determined by multiplying the fuzzy synthetic utilities with the fuzzy weights obtained from FAHP. Also some suggestions were given about improving the sub criteria.

Tseng, Chiang, and Lan (2009) proposed a hierarchical framework for selecting optimal supplier in supply chain management strategy (SCMS) using ANP and Choquet integral (non-additive fuzzy integral). ANP was used to calculate the weights of all attributes. Based on the values obtained by ANP, λ-fuzzy measures were identified and required equations were solved. Final aggregation and determining best alternatives were conducted by Choquet integral procedure. Ultimate results when λ=0 and λ=1 were provided.

Büyüközkan, Feyzioğlu, and Ersoy (2009) evaluated fourth party logistics (4PL) operating models by 2-additive Choquet Integral. First, hierarchical model of decision criteria for 4PL operating model selection was constructed. Then criteria interactions with negative and positive values were clarified with the inquiry of experts. It was assumed that criteria in the same hierarchical level were interacting and these interactions. Finally alternatives were evaluated by 2-additive Choquet integral. Case study was conducted in a local 3PL aiming at expand its operations.

Buyukozkan et al. (2010) also used a 2-additive Choquet integral based approach for software development risk assessment. In addition, elucidation phases were included in order to inform decision makers about the different steps of the decision making process. Proposed model was implemented in a Turkish software company and results compared with weighted arithmetic means (WAM) and TOPSIS methods.

Yazgan, Boran, and Goztepe (2009) used ANP with Choquet integral for selection of dispatching rules in Flexible Manufacturing System (FMS). Benefit, Opportunity, Cost and Risk (BOCR) analyses was incorporated with the ANP procedure. In addition to ANP calculations, priorities of alternatives and weights of main criteria were determined by λ-fuzzy measure based Choquet integral. Dispatching rules were ranked in terms of multiplicative and additive analysis and the results were compared.

Feng, Wu, and Chia (2010) developed a hybrid fuzzy integral decision making model for selecting manufacturing center. First, common factors were extracted by factors analysis to reduce the attribute space. Then ISM method was used to structuring criteria relationship. Afterwards, Markov chain method was employed to find steady state status of criteria relationships. Next, λ-fuzzy measures were calculated and performance score of each criterion was obtained by Choquet integral. In the last step, simple additive weighted method was used to aggregate the synthetic scores for each facility location.

Demirel, Demirel, and Kahraman (2010) developed a hierarchical evaluation model based on generalized Choquet integral for warehouse location selection. Importance of criteria and coalitions were represented in terms of intervals rather than real numbers as in the standard Choquet integral. An eight step methodology was applied on the grounds of the study (Tsai & Lu, 2006). A real case study was proposed in warehouse selection problem of a Turkish logistic firm.

Magoč and Modave (2011) used 2-additive Choquet integral to model the dependencies among assets in portfolio selection. In the study, a two stage algorithm was developed. In the first step decision maker was given the Shapley values of each criterion and interaction indices were approximated. Choquet integral aggregation was employed to select top n assets. In the second stage, optimal distribution of wealth was found among the top n assets utilizing genetic algorithm. Proposed method was tested on data from Shanghai stock market between 2000 and 2007. It was reported that the proposed model was outperformed the benchmark portfolios in most of the tests.

Tan, Wu, and Ma (2011) considered interaction among linguistic preference of decision makers in the group decision making setting. Hence, linguistic ordered geometric averaging operator and extended Choquet integral operators were developed for multiplicative linguistic preference relations and additive linguistic preference relations. A hypothetical case study concerning supplier selection was provided as a case study to illustrate the proposed methods.

Dhouib and Elloumi (2011) introduced a new multi criteria approach combining 2-tuple linguistic representation, 2-additive Choquet integral and PROMETHEE II method for end-of-life (EOL) product strategy evaluation. In PROMETHEE method, linguistic approach was used rather than preference functions which were taken part in the traditional PROMETHEE. Besides, both quantitative and qualitative deviations were taken into account in the model. Due to the interaction phenomenon, 2-additive fuzzy measures were used to calculate the Shapley index. Then, calculated Shapley values were used as the criteria weightings in PROMETHEE method. Finally, alternative strategies were ranked according to net flows. In the illustrative example, personal computers were used as a product of EOL strategy and proposed method applied with sensitivity analysis.

Gürbüz, Alptekin, and Işıklar (2012) proposed a hybrid method for ERP selection problem using ANP, Choquet Integral and Measuring Attractiveness by a Categorical Based Evaluation Technique) MACBETH. Criteria and sub-criteria were evaluated based on the interaction types which were the existence of interdependency, conjunctive/disjunctive behavior or both of them. ANP was used to calculate priorities of interdependent criteria. Furthermore, conjunctive and disjunctive behaviors between criteria were handled by MACBETH and Choquet integral with 2-additive measures. In the study, four ERP alternatives were ranked according the proposed method and results were compared with the solution where interactions were ignored. It was reported that the proposed method was able to provide more accurate results.

CHAPTER ELEVEN
PROPOSED METHOD

11.1 Motivation

The consciousness regarding the long term impact of good quality decisions on the overall organizational performance has sparked off increased attention to MCDM methods. In the literature, there are many methods and tools to help decision makers make better evaluations. However, practitioners in real life do not recognize all the methods in the literature and cannot estimate the benefits of a particular method specific to their problem. Generally, decisions are made based on ad hoc way, and choice of the method happens to be based on familiarity to a specific approach such as AHP.

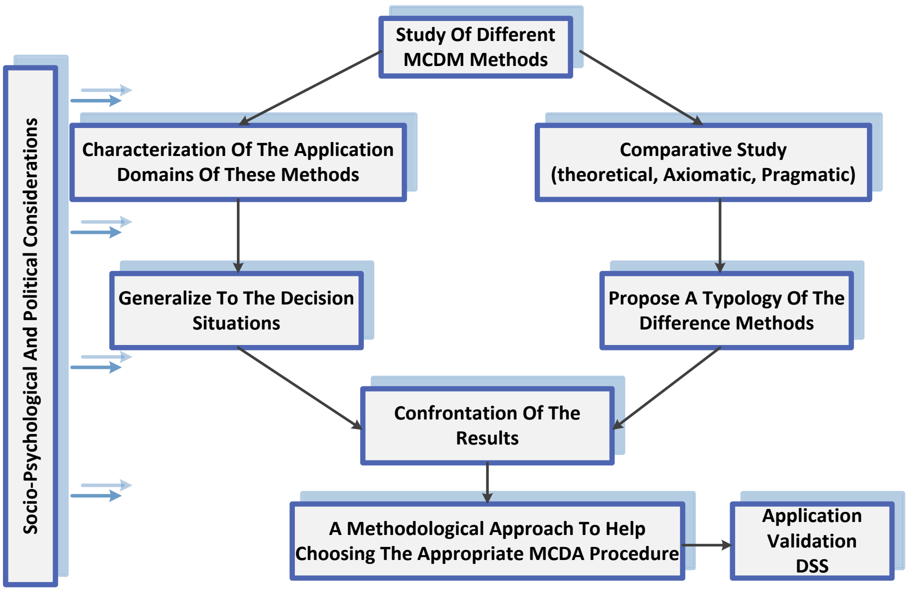


Figure 11.1 Framework to choose appropriate MCDM model (Guitouni & Martel, 1998)

In the literature, MCDM methods have very distinct features and hundreds of methods are available. Selection of a MCDM method is a challenging task and no simple answer exists. One of the attempts to help decision makers select an appropriate method is provided in Figure 11.1 (Guitouni & Martel, 1998). Although the proposed methodology is relevant for most of the situations, trying out different

MCDM methods is improbable in today's highly agile production and service systems decisions.

Furthermore, it is very difficult to find an expert, who has knowledge of all the decision analysis methods with many years of experience in both theory and practice, recognizes your problem and advises you concerning potential analysis tools. This difficulty sometimes leads to the fact that firms abandon scientific methods of decision making, rather they analyze and solve problems by the ways they are accustomed to, which are mostly intuitionistic.

In this thesis, one of our aims is to propose a simple hybrid decision making technique, which is intuitively easy to understand, very generic that suits the needs to solve most of the real life problems, and has strong features, which make it superior to traditional methods.

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11.2 Characteristics of a Sufficient MCDM Method

In the development of the MCDM as a research field, methods gradually emerged for specific needs. Incapability of a method with respect to some aspects gave rise to establishment of different approaches. For example, ANP was developed to overcome the restrictions of AHP, which cannot take dependence and feedback situations among criteria into account. While new features are added to MCDM methods, the complexity of the method increases too. However, practicality is of vital importance for a MCDM method to be able to gain high acceptance among practitioners.

Among many MCDM methods in the literature, there is no approach that satisfies all the needs of decision situations. Hence, decision makers confront a trade-off situation between selecting a method with some desirable properties at the expense of abandoning others' strong feature.

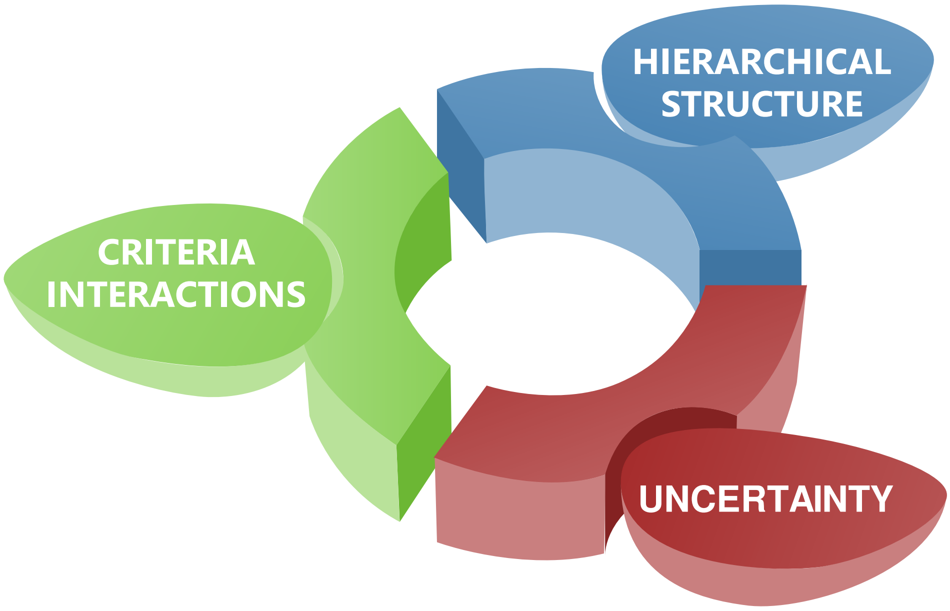

Figure 11.2 Three pillars of a sufficient MCDM method

As a result, expected features of the MCDM method which are crucial for generic decision making situations should be determined. Maybe, tens of evaluation criteria can be counted regarding which features a sufficient MCDM method should provide. In our regard, there are three main characteristics that a MCDM method absolutely must have:

- Uncertainty
- Hierarchical Structure
- Criteria Interactions

We elaborate these concepts below.

11.2.1 Uncertainty

There are many methods to model uncertainty in decision problems. Probabilistic approaches, fuzzy logic, Dempster-Shafer theory, possibility theory are some examples of options. In this study, we use fuzzy logic for uncertainty handling tool, since its practicality and flexibility. Decision making problems often require experts' subjective evaluations. Fuzziness should be incorporated into decision making since problems in real life involves different sorts of uncertainties, of which fuzzy logic is able to successfully manage. There are different sources of the imprecision in decision making process such as (Chen & Hwang, 1992)

- **Unquantifiable information:** Decision making problems usually contain intangible and tangible criteria together. Some information can be easily obtained such as price of a car or amount of money in the bank. However, when it comes to intangibles, providing exact information is impossible. For instance, criteria such as safety, aesthetics, or comfort cannot be quantified, and related information is often given by linguistic terms like good, high, low, etc.
- **Incomplete information:** The information obtained from the decision maker cannot be exact; it can only be an approximation. For instance, speed of an object can be measured by an equipment as "about 110 kmph", yet not "surely 110 kmph".
- **Non-obtainable information**: Sometimes, no information is available or obtaining required information is extremely costly. For these types of situations, fuzzy logic provides promising results by expert's approximations.
- **Partial ignorance**: Since the evaluations of the decision maker rely on the partial information and the part of the fact, partial ignorance is encountered frequently.

In classical MCDM methods, the importance degrees of criteria and the performance scores of alternatives are assumed to be known precisely. However, practical constraints of the real world hinder the use of crisp values. The problems faced in practice take place in such an environment that goals, constraints and consequences of alternatives are not precise (Bellman & Zadeh, 1970). Furthermore, the ambiguities, uncertainties and vagueness inherent in decision makers' evaluations necessitate the usage of fuzzy set theory. We believe that a sufficient MCDM method should be fuzzy, in other words, fuzzy MCDM method is essential to model today's complex decision making problems.

11.2.2 Hierarchical Structure

A typical MCDM problem solution procedure starts with problem structuring. Obviously, problem is disaggregated into smaller parts since experts can more easily handle the smaller portions of the problem. It is indeed extremely difficult to manage the entire problem at once. Another benefit of problem structuring is that the decision makers can better understand the problem and improve their information processing capabilities with these smaller portions of the problem (Aschenbrenner, Jaus, & Villani, 1980).

Most of the problem structures in the literature are geometric in nature. Geometric structures are not only triangles, circles, but they are also vertices and edges connecting them via paths and cycles (Saaty & Shih, 2009). In most studies, vertices stand for the attributes or system variables and the lines or arcs mean the perceived influence associating them. Hence geometric structures have attractive features, which escalate our abstract cognition of influences among elements and their interactions.

Among many geometric structures, hierarchical structure is most widely used. Hierarchical structure is very suitable for human thinking process, where our brains utilize hierarchies as a powerful tool to classify or order information in order to understand the complex world surrounding us. In decision problems, influences among criteria can be very complex; in this case, network of influences is taken into account. Nevertheless, network structures contain hierarchies, in other words, collection of hierarchies constitute networks (Saaty & Shih, 2009). Hence hierarchies are not as complex as networks, they are stratified systems used to organize all sorts of information and even intangibles such as one's beliefs and values.

Although there are "general purpose" approaches to help structuring problems, in this thesis, we adopted qualitative top-down hierarchical decomposition (Saaty, 1990). In the top-down hierarchical decomposition approach, the overall objective is determined and placed at the top of the hierarchy. The top element of the hierarchy is decomposed into sub levels at successive iterations. In order to decompose the system into sub-levels, questions like “which sub-goals must be satisfied to fulfill this objective?” guided the descending process from the top (Maier & Stix, 2013). This kind of decomposition approach is very useful and intuitive; and, because of the fact that most of the practitioners are familiar with AHP approach, they do not have difficulties to interpret and apply the methodology.

In this thesis, we reinforced the top-down hierarchical decomposition approach with horizontal causal influences so as to model criteria interactions. Since the criteria structure and the interaction phenomenon are highly interrelated, we explain further concepts in the next section.

11.2.3 Criteria Interactions

Theoretical foundations of the decision analysis mostly rely on MAUT. In MAUT, criteria are assumed to be preferentially independent (Fishburn, 1970; Grabisch, 1995). Actually, preferential independence concept ignores the dependence and feedback situations among criteria (Yu & Tzeng, 2006). Furthermore, interactive criteria cannot be modeled under the preferential independence assumption. In the scope of thesis, we have shown many techniques for modeling interactions such as:

- Structural/functional dependencies: ANP is the generalization of the AHP technique and used to model all sorts of dependency in the network structure. In ANP, relationships are quantified via the weights elicited from pairwise comparisons which require decision maker intervention and his/her cognitive effort.
- Causal dependencies: In this group of methods, causality is the driving logic of the analysis. This group consist of:
    - Causal/Cognitive Maps
    - DEMATEL
    - Fuzzy Cognitive Maps

    - Interpretive Structural Modeling
    - Bayesian Networks
    - System Dynamics
- Criteria interactivity: Apart from the dependency concepts, criteria can be interactive and they can be modeled via:
    - Choquet Integral and Sugeno integral with modeling capabilities of
        - Complementarity situation (super-additive, conjunctive behavior etc.)
        - Redundancy situation (sub-additive, disjunctive behavior etc.)
        - Independent situation

In our regard, a sufficient MCDM method is expected to take criteria interactions into account. We have selected Fuzzy Cognitive Maps for modeling interactions for following reasons:

- In Fuzzy Cognitive Maps, criteria are represented by concepts and causal influences are represented by links connecting them, which provide intuitively easy interpretation of all modeling components. Also causal relationship is very close to human thinking and it is quite easy to elicit information from decision makers.
- Fuzzy Cognitive Maps make use of concept values and influence degrees among concepts using fuzzy logic. Linguistic values are much easier to obtain than that of exact crisp values.
- Fuzzy Cognitive Maps has ability to model positive and negative causal influences simultaneously.
- Fuzzy Cognitive Maps has flexibility to exploit different threshold functions which help capture system behavior in the long term. Besides, dynamic nonlinearity can be visualized by time.
- Fuzzy Cognitive Maps also allow static analysis without conducting simulation experiments.

- Semi-automated or automated construction of Fuzzy Cognitive Maps is possible. In this case, minimum or no expert intervention is required to form Fuzzy Cognitive Maps.
- Utilizing different learning algorithms, influence degrees among concepts can be learned.
- There are many variants of Fuzzy Cognitive Mapping such as Rule-Based Fuzzy Cognitive Maps (Carvalho & Tome, 2001), Fuzzy Cognitive Networks (Kottas, Boutalis, & Christodoulou, 2007), and Evolutionary Fuzzy Cognitive Maps (Mateou & Andreou, 2008). An opportunity to select the most appropriate Fuzzy Cognitive Maps extension is possible.
- Updating equation of the Fuzzy Cognitive Maps can be modified according to nature of the problem under consideration. Different updating equations are possible, i.e. different aspects can be incorporated into model such as time delay weights, conditional weights and non-linear membership functions (Hagiwara, 1992).

As a result Fuzzy Cognitive Maps is highly flexible, easy to implement and practical. We integrated Fuzzy Cognitive Maps for modeling horizontal dependencies in the hierarchies.

We also suggest a generic hierarchical structure which is extended version of structure mentioned by Wedley et al. (1993) by means of horizantal links. This structure has been recently used by Baykasoğlu et al. (2013). Suggested generic hierarchical structure has following properties:

- The lower level is dependent upon every item of the immediate upper level. For example, in Figure 11.3, all sub-attributes in level 2 are dependent upon the main attribute in level 1.
- The upper level does not depend upon one or more items of the lower level. For example, In Figure 11.3, the main attribute in level 1 is independent of all the sub-attributes of level 2.

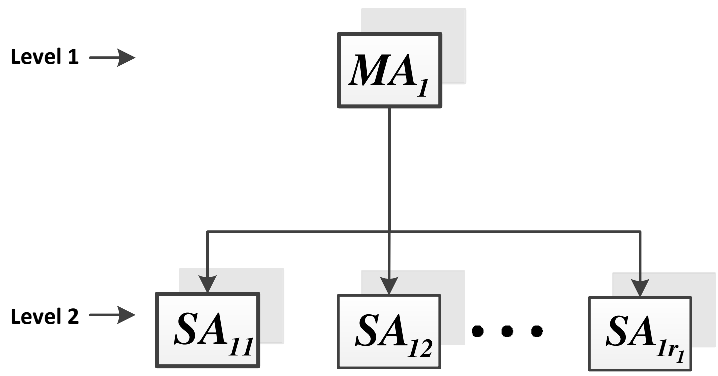

Figure 11.3 Dependencies between upper and lower level criteria

- Each attribute belongs to only one attribute of the level immediately above. In other words, an attribute belongs to ith level of the hierarchy is a sub-attribute of only one of the attribute of the $(i-1)-$th level. This type of structure is called partitioned structure (Corrente, Greco, & Słowiński, 2013). We also call the group of attributes influenced by the same attribute of the immediately above level as partition.

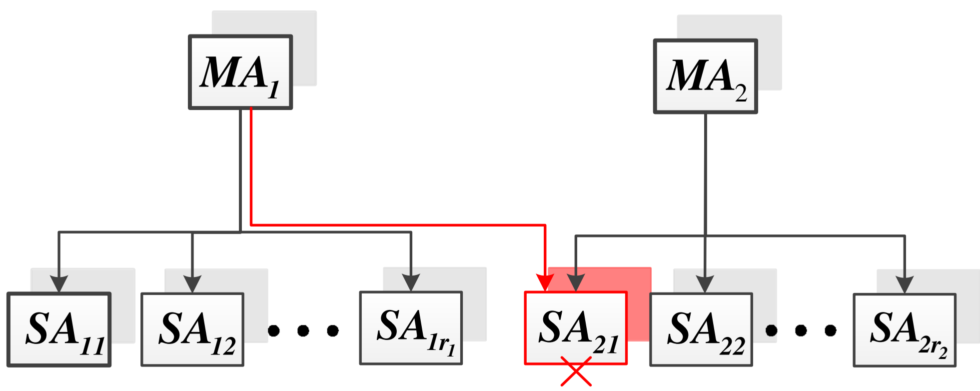

Figure 11.4 Example of a non-partitioned hierarchical structure

For example in Figure 11.4 an example of a non-partitioned hierarchical structrure is given. The necessary condition to be a partitioned structure is violated due to the fact that the sub-attribute, SA_{21}, belongs to both MA_1 and MA_2 main attributes from the first level.

- The same level of attributes whose immediate upper level attribute is also the same can be dependent of each other. In other words, interaction may exist among the attributes of each partitions. Different partitions are assumed to be independent.

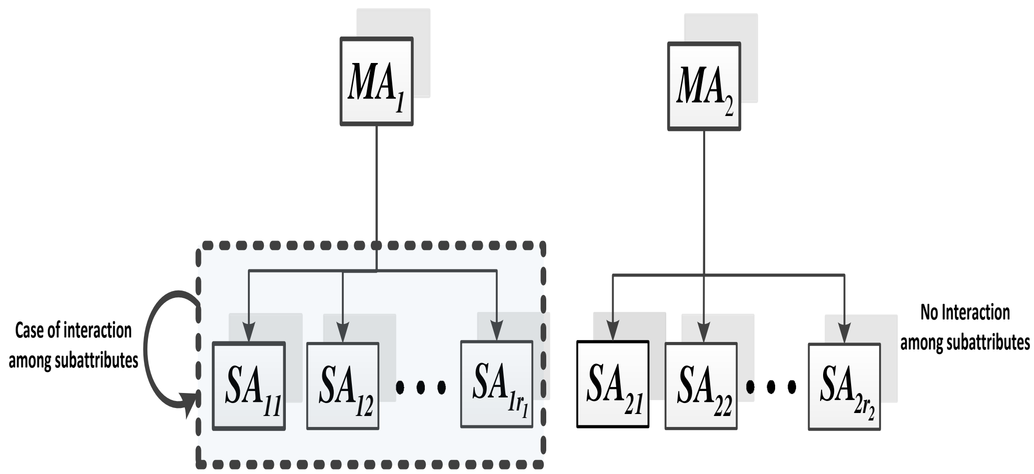

Figure 11.5 Example of hierarchy of attributes with/without interaction

In figure 11.5, interaction among the same level of attributes is shown. Note that this representation allows modeling the attributes which can be either dependent or independent.

In figure 11.6, traditional hierarchy, top-down hierarchical decomposition with horizontal interactions, and network structure are shown. We argue that traditional hierarchy is a very simple form, suitable only for modeling uncomplicated problems. AHP is the most widely applied method by utilizing traditional hierarchical structure. However, real life problems are rarely easily done, much complicated problems are encountered and network structures are used such as ANP.

We criticize the fact that decision makers, in many situations, are stuck between traditional hierarchical decomposition and the network structures. If problem is highly complex and interdependencies exist, the ANP seems to be the only choice. Although the ANP is a strong technique capable of modeling all sorts of interdependencies, the cost of switching from AHP to ANP is immense. We raise the question: is network structure the only choice for modeling interactions, specifically dependence and feedback situations?

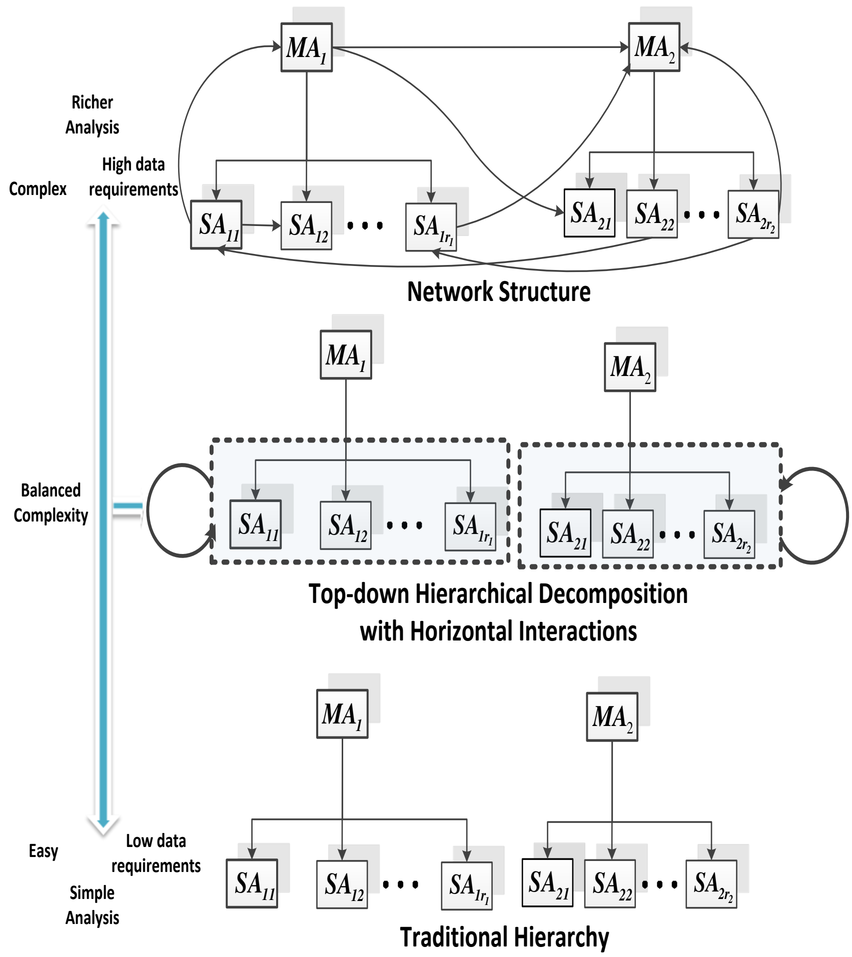

Figure 11.6 Balancing complexity of the problem structure

We argue that modeling with ANP is unnecessary for many situations. Even for relatively simple problems, tens of pairwise comparisons are required, of which the major pair-wise comparison questions are semantically pointless.

Figure 11.8 shows our suggested problem structuring scheme. Let's take the first level into consideration; interactions among the attributes of the first level are used to determine the weight of the main attributes. In the second level, interaction among the attributes of each partition was calculated to obtain the weight of the sub-attributes. This process is continued for sub-sub attribute levels. Actually, going through the lower levels of the hierarchy, interaction phenomenon is only considered among attributes which belong to same partition. Through the lower levels in the hierarchy, interaction among the attributes in the upper level is implicitly reflected onto the attributes of lower levels by means of multiplication, which in turn reduce the required data regarding interactions. This process is depicted in Figure 11.7

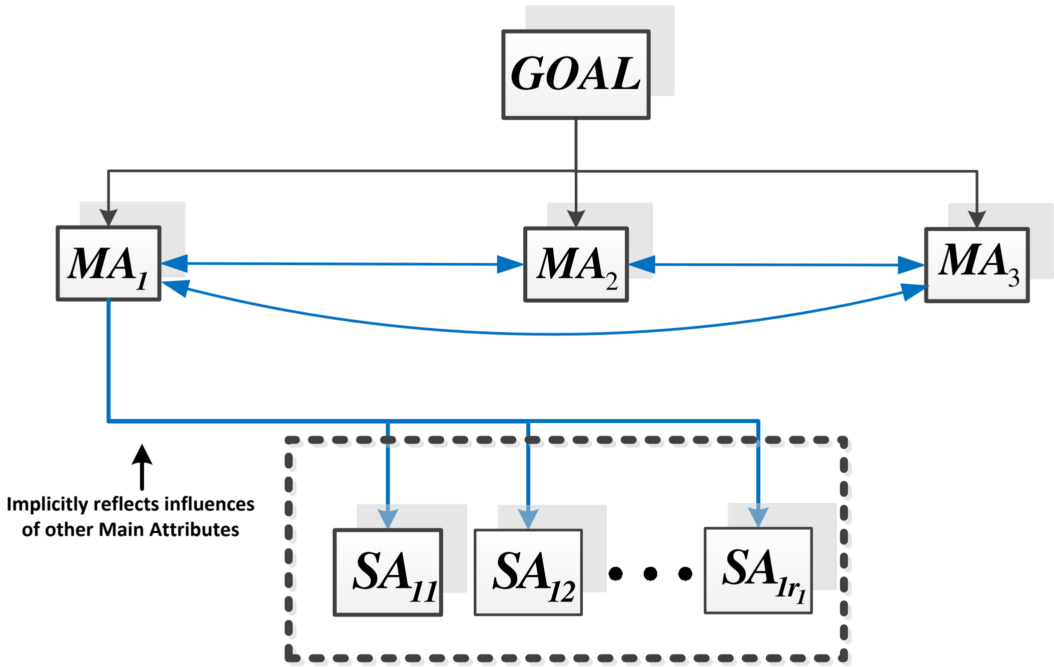

Figure 11.7 Implicit flow of influence in the hierarchy

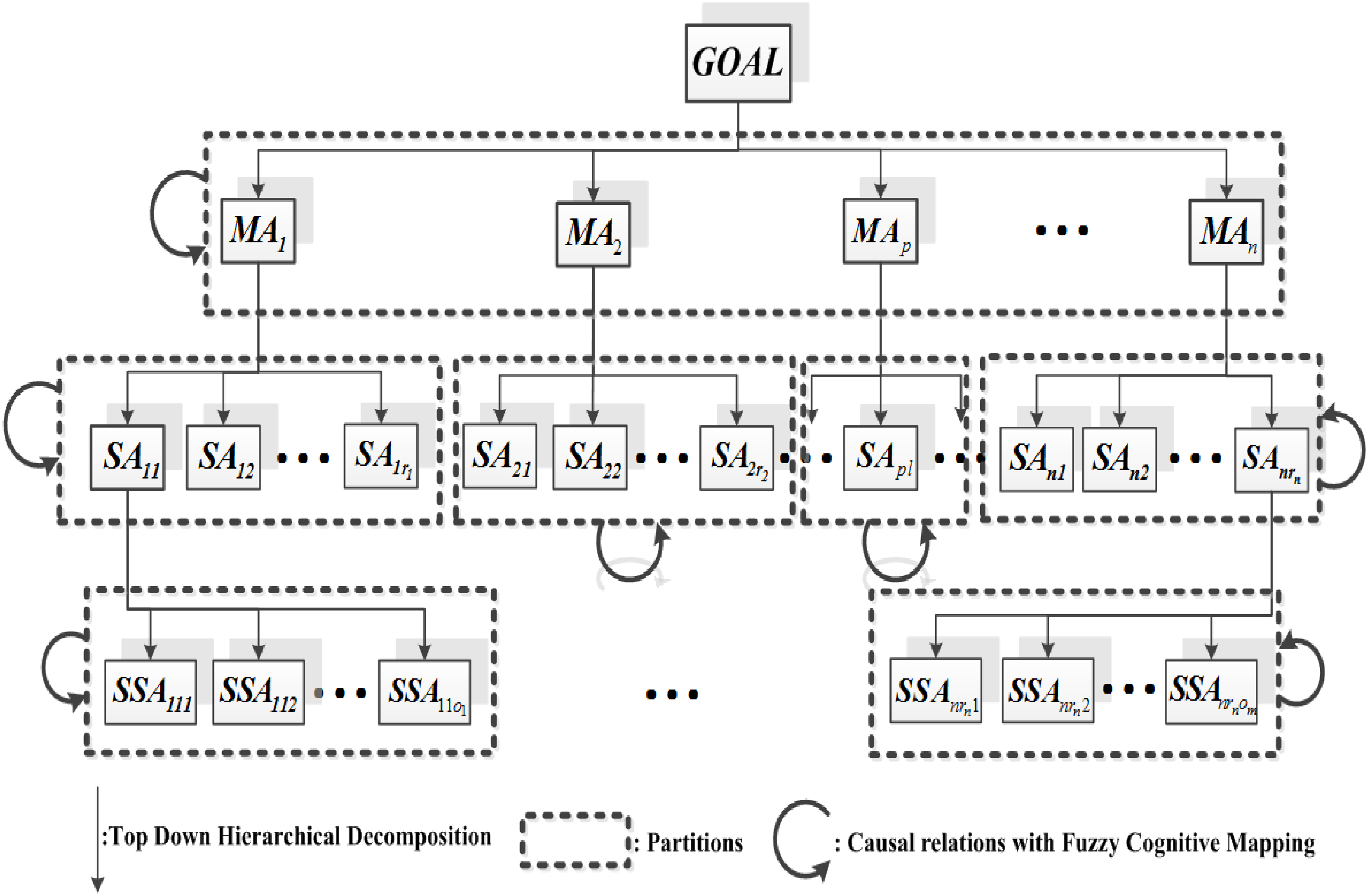

Figure 11.8 The complete evaluation scheme with top-down hierarchical decomposition approach and interacting attributes

11.3 Fuzzy Hierarchical TOPSIS and Fuzzy Cognitive Maps Integration

We proposed a hybrid MCDM technique combining Fuzzy Hierarchical TOPSIS and Fuzzy Cognitive Maps in order to evaluate hierarchically structured decision problems, which incorporates fuzziness and criteria interactions into analysis. The methodology is well suited for hierarchical structures as seen in Figure 11.8. We seek for a balance between practicality and richness of the model, the top-down hierarchical decomposition approach of the hierarchical fuzzy TOPSIS with modeling horizontal causal relationships via Fuzzy Cognitive Maps is the most convenient approaches. Hence, proposed novel hybrid hierarchical fuzzy decision making technique conform characteristics of a sufficient MCDM method mentioned earlier. Flowchart of the proposed method is given in Figure 11.10.

The hierarchical fuzzy TOPSIS method is the extension of the classical TOPSIS method proposed by (Kahraman et al., 2007). The hierarchical fuzzy TOPSIS method is composed of two significant parts: acquiring the criteria weights from the hierarchical structures and fuzzy TOPSIS algorithm implementation to rank the alternatives. We evaluate the two approaches separately.

In the MCDM literature, there is no doubt that the AHP is the most widely used decision making method. The path-breaking article of Saaty (1977) enabled prioritize the alternatives in the hierarchical structures by utilizing the principal eigenvectors. The AHP outperforms other methods by taking the hierarchical structures into account, as hierarchy is intuitive and more human-centric that facilitates the analysis of complex problems. Although TOPSIS method (Chen & Hwang, 1992) became one of the prominent methods in MCDM studies with its practicality and ease of use, it does not take hierarchy into account. To manage this problem, researchers hybridize the AHP and TOPSIS methods in many studies. (Dağdeviren, Yavuz, & Kılınç, 2009; Işıklar & Büyüközkan, 2007; Lin et al., 2008; Önüt & Soner, 2008) However, integrating notion of hierarchical decomposition logic into TOPSIS algorithm without using AHP method would add great value to TOPSIS. Fortunately, (Kahraman et al., 2007) incorporated a hierarchy with fuzzy TOPSIS as expressed below. We give the basic steps of the hierarchical fuzzy TOPSIS and Fuzzy

Cognitive Maps combination in two parts. The first part consists of obtaining overall attribute weights, and the second part is dedicated to fuzzy TOPSIS algorithm in order to rank the alternatives.

11.3.1 Obtaining Attribute Weights from the Hierarchical Structure

Let us assume that we have the criteria hierarchy as seen in Figure 11.9. We use the same notations as in the (Kahraman et al., 2007), n attributes, m sub-attributes, o sub-sub attributes, k alternatives and s decision makers. It is assumed that there are r_i sub attributes belonging to each main attributes. There are m sub-attributes which are expressed by $\sum_{i=1}^{n} r_i$. Similarly, total number of sub-sub attributes belong to sub-attributes is given by $\sum_{i=1}^{m} o_i$.

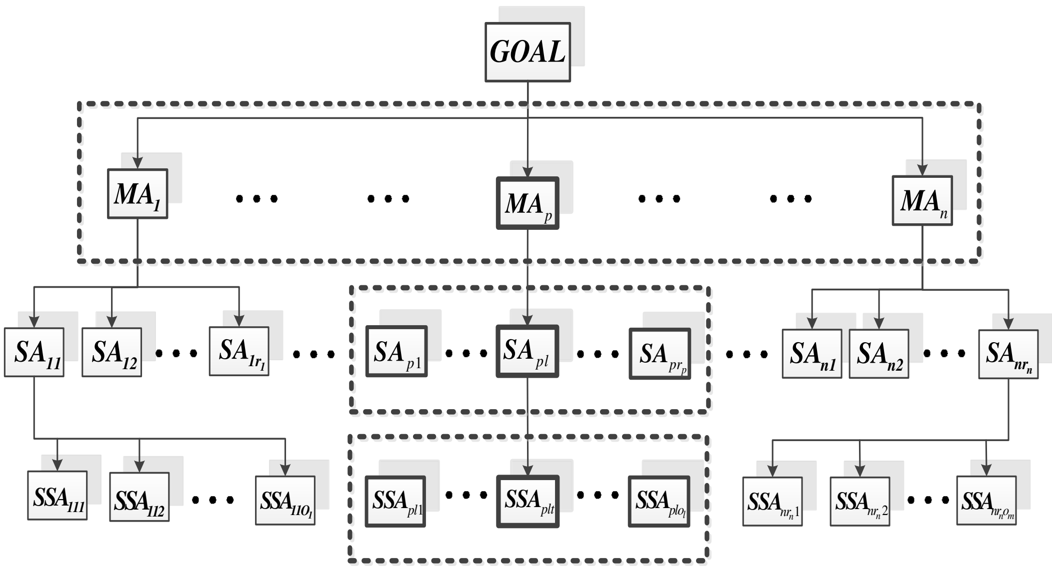

Figure 11.9 Criteria hierarchy of TOPSIS

11.3.1.1 Obtain Criteria Weights for Main Attributes

In the hierarchical fuzzy TOPSIS evaluation, matrices for main attributes, sub-attributes, sub-sub attributes and performance matrices are required. Following the top-down decomposition approach, the first matrix is for the main attributes, which is denoted by I_{MA}.

In the matrix $\tilde{I}_{MA}$, the weights of main attributes with respect to goal is given. The $\tilde{w}_p$ values are arithmetic mean of the group of decision makers which is calculated as:

$$\tilde{w}_p = \frac{\sum_i^s \tilde{w}_{pi}}{s}, \quad p = 1, 2, \ldots, n \tag{11.1}$$

where $\tilde{w}_{pi}$ is the fuzzy weight score of the ith decision maker for the pth main attribute with respect to goal.

Then, fuzzy weights are used as initial states of the Fuzzy Cognitive Maps concepts in order to take interactions among main attributes into account. Centroid method is used to defuzzify the fuzzy weights of main attributes.Centroid method is one of the defuzzzification method gained widespread acceptance and applications due to its simplicity. It utilizes the concept of center of gravity (CoG) of the membership function where membership function is used for weighting the average. In the literature, the crisp equivalent, so called Best Non-fuzzy Performance (BNP), of a triangular fuzzy number $\tilde{N} = (l, m, r)$ is given by CoG method as (Opricovic & Tzeng, 2003):

$$BNP = l + [(m - l) + (r - l)] / 3 \tag{11.2}$$

$$nw_p = \frac{w_p}{\sum_{i=1}^n w_i}, \tag{11.3}$$

where nw_p is the normalized crisp weight vector of pth main attribute. Then normalized crisp weight vectors are regarded as the initial concept values of the Fuzzy Cognitive Maps.

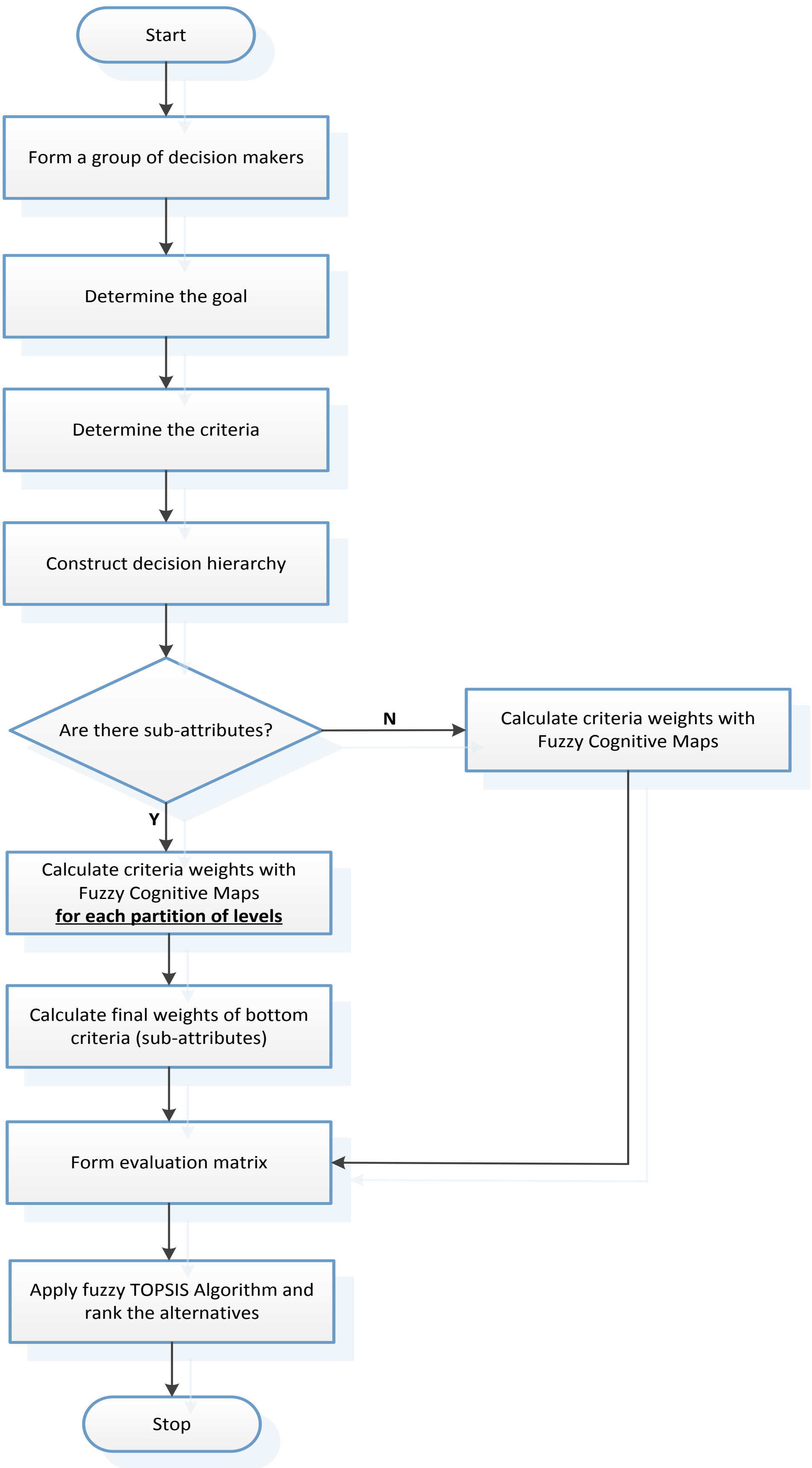

Figure 11.10 Flowchart of the proposed hybrid model

$$\begin{bmatrix} C_{MA_1}^{t=0} \\ C_{MA_2}^{t=0} \\ \vdots \\ C_{MA_p}^{t=0} \\ \vdots \\ C_{MA_n}^{t=0} \end{bmatrix} = \begin{bmatrix} nw_1 \\ nw_2 \\ \vdots \\ nw_p \\ \vdots \\ nw_n \end{bmatrix} \tag{11.4}$$

where $C_{MA_i}^{t=0}$ is the concept value of the ith main attribute when $t = 0$. Next, causal influences among main attributes are obtained via linguistic terms. These linguistic variables are mapped into the numerical crisp values and consequently aggregated crisp connection matrix is denoted by e_{ji}^{MA}. To capture the long term influences among main attributes, Fuzzy Cognitive Map is simulated with an appropriate threshold function as:

$$C_{MA_i}^{t+1} = f(\mathrm{C}_{MA_i}^{t} + \sum_{\substack{j=1 \\ j \neq i}}^{n} e_{ji}^{MA} \times C_{MA_j}^{t}) \tag{11.5}$$

where $C_{MA_i}^{t}$ is the concept value of the ith main attribute at time t. The steady state concept values are normalized hence the final steady state weights of main attributes are obtained. Final crisp criteria weights are shown as:

$$\mathrm{I}_{\mathrm{MA}} = \begin{array}{c} \\ MA_1 \\ MA_2 \\ \vdots \\ MA_P \\ \vdots \\ MA_n \end{array} \begin{array}{c} Goal \\ \begin{bmatrix} w_1 \\ w_2 \\ \\ w_p \\ \\ w_n \end{bmatrix} \end{array}, \quad \text{where} \quad \sum_{i=1}^{n} w_i = 1 \tag{11.6}$$

11.3.1.2 Obtain Criteria Weights for Sub Attributes

The next matrix $\tilde{I}_{SA}$ stands for the weights of sub-attributes with respect to main attributes. The values $\tilde{w}_{pl}$ is the arithmetic mean of the weights of decision makers calculated by:

$$\tilde{w}_{pl} = \frac{\sum_{i}^{s} \tilde{w}_{pli}}{s} \tag{11.7}$$

where $\tilde{w}_{pl}$ is the fuzzy weight score of the ith decision maker for the lth sub-attribute with respect to pth main attribute.

Similarly, fuzzy weights for sub attributes are defuzzified by centroid method. Then normalization is given by:

$$nw_{pl} = \frac{w_{pl}}{\sum_{i=1}^{r_p} w_{pi}}, \tag{11.8}$$

where nw_{pl} is the normalized weight of the lth sub-attribute with respect to pth main attribute. Then normalized crisp weight vectors are regarded as the initial concept values of the Fuzzy Cognitive Maps.

$$\begin{bmatrix} C_{SAp_1}^{t=0} \\ C_{SAp_2}^{t=0} \\ \vdots \\ C_{SAp_l}^{t=0} \\ \vdots \\ C_{SAp_{r_p}}^{t=0} \end{bmatrix} = \begin{bmatrix} nw_{p1} \\ nw_{p2} \\ \vdots \\ nw_{pl} \\ \vdots \\ nw_{pr_p} \end{bmatrix} \tag{11.9}$$

where $C_{SAp_l}^{t=0}$ is the concept value of the lth sub attribute with respect to pth main attribute when $t = 0$. Number of sub attributes under the pth main attribute is denoted by r_p. Then connection matrix $e_{ji}^{SA_p}$ which denotes the crisp influence degrees among sub-attributes under pth main attribute. The Fuzzy Cognitive Map calculation is conducted as:

$$C_{SAp_i}^{t+1} = f(\mathrm{C}_{SAp_i}^{t} + \sum_{\substack{j=1 \\ j \neq i}}^{r_p} e_{ji}^{SA_p} \times C_{SAp_j}^{t}) \tag{11.10}$$

where $C_{SAp_i}^{t}$ is the concept value of the ith sub attribute under the pth main attribute at time t.The steady state concept values are normalized hence the final steady state weights of main attributes are obtained. Final crisp criteria weights are shown as:

$$I_{SA} = \begin{array}{c} \\ \\ SA_{11} \\ SA_{12} \\ \vdots \\ SA_{1r_1} \\ SA_{21} \\ SA_{22} \\ \vdots \\ SA_{2r_2} \\ \vdots \\ SA_{pl} \\ \vdots \\ SA_{n1} \\ SA_{n2} \\ \vdots \\ SA_{nr_n} \end{array} \begin{array}{c} \begin{array}{ccccc} w_1 & w_2 & \cdots & w_p & \cdots & w_n \\ MA_1 & MA_2 & \cdots & MA_p & \cdots & MA_n \end{array} \\ \begin{bmatrix} w_{11} & 0 & \cdots & 0 & \cdots & 0 \\ w_{12} & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ w_{1r_1} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & w_{21} & \cdots & 0 & \cdots & 0 \\ 0 & w_{22} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & w_{2r_2} & \cdots & 0 & \cdots & 0 \\ \vdots & 0 & & \vdots & & \vdots \\ 0 & 0 & \cdots & w_{pl} & \cdots & 0 \\ \vdots & \vdots & & & & 0 \\ 0 & 0 & \cdots & 0 & \cdots & w_{n1} \\ 0 & 0 & \cdots & 0 & \cdots & w_{n2} \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & w_{nr_n} \end{bmatrix} \end{array} \tag{11.11}$$

11.3.1.3 Obtain Criteria Weights for Sub-Sub Attributes

Another matrix is $\tilde{I}_{SSA}$ which corresponds to weights of the sub-sub attributes with respect to sub-attributes. Also note that sub attributes do not have to contain sub-sub attributes. In this case, alternatives are compared in terms of sub-attributes. However, we follow the hierarchy given in figure 11.9 to explain how to derive weights of the sub-sub attributes.

The values $\tilde{w}_{plt}$ is the arithmetic mean of the weights of decision makers calculated by:

$$\tilde{w}_{plt} = \frac{\sum_{i}^{s} \tilde{w}_{plti}}{s}, \quad 1 \le t \le o_l \tag{11.12}$$

where $\tilde{w}_{plti}$ is the fuzzy weight score of the ith decision maker for the tth sub-sub attribute with respect to lth sub-attribute. Initially, in accordance with the weight calculations of main attributes and sub-attributes, fuzzy weights for sub-sub attributes are defuzzified by centroid method. Then normalization is given by:

$$nw_{plt} = \frac{w_{plt}}{\sum_{i=1}^{o_l} w_{pli}}, \tag{11.13}$$

where nw_{plt} is the normalized weight of the tth sub-sub attribute with respect to lth sub-attribute. Then normalized crisp weight vectors are regarded as the initial concept values of the Fuzzy Cognitive Maps, given in equation 11.14.

$C_{SSAp_{lt}}^{t=0}$ is the concept value of the tth sub-sub attribute with respect to lth sub-attribute when $t = 0$. Number of sub-sub attributes under the plth sub attribute is denoted by o_l.

$$\begin{bmatrix} C_{SSApl_1}^{t=0} \\ C_{SSApl_2}^{t=0} \\ \vdots \\ C_{SSApl_t}^{t=0} \\ \vdots \\ C_{SSApl_{o_l}}^{t=0} \end{bmatrix} = \begin{bmatrix} nw_{pl1} \\ nw_{pl2} \\ \vdots \\ nw_{plt} \\ \vdots \\ nw_{plo_l} \end{bmatrix} \tag{11.14}$$

Then connection matrix $e_{ji}^{SSA_{pl}}$ which denotes the crisp influence degrees among sub-sub attributes under plth sub attribute. The Fuzzy Cognitive Map calculation is conducted as:

$$C_{SSApl_i}^{t+1} = f(\mathrm{C}_{SSApl_i}^{t} + \sum_{\substack{j=1 \\ j \neq i}}^{o_l} e_{ji}^{SSA_{pl}} \times C_{SSApl_j}^{t}) \tag{11.15}$$

where $C_{SSApl_i}^{t}$ is the concept value of the ith sub-sub attribute under the plth sub attribute at time t.

The steady state concept values are normalized hence the final steady state weights of main attributes are obtained. Final crisp criteria weights are shown in Equation 11.16

$$
\begin{array}{rl}
 & \begin{array}{cccccc} w_{11} & w_{12} & \cdots & w_{pl} & \cdots & w_{nm} \\ SA_{11} & SA_{12} & \cdots & SA_{pl} & \cdots & SA_{nm} \end{array} \\
I_{SSA} = \begin{array}{c} SSA_{111} \\ SSA_{112} \\ \vdots \\ SSA_{11o_1} \\ SSA_{121} \\ SSA_{122} \\ \vdots \\ SSA_{12o_2} \\ \vdots \\ SSA_{plt} \\ \vdots \\ SSA_{nr_n1} \\ SSA_{nr_n2} \\ \vdots \\ SA_{nr_no_m} \end{array} &
\begin{bmatrix}
w_{111} & 0 & \cdots & 0 & \cdots & 0 \\
w_{122} & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & & \vdots \\
w_{11o_1} & 0 & \cdots & 0 & \cdots & 0 \\
0 & w_{121} & \cdots & 0 & \cdots & 0 \\
0 & w_{122} & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & & \vdots \\
0 & w_{12o_2} & \cdots & 0 & \cdots & 0 \\
\vdots & 0 & & \vdots & & \vdots \\
0 & 0 & \cdots & w_{plt} & \cdots & 0 \\
\vdots & \vdots & & & & 0 \\
0 & 0 & \cdots & 0 & \cdots & w_{nr_n1} \\
0 & 0 & \cdots & 0 & \cdots & w_{nr_n2} \\
\vdots & \vdots & & \vdots & & \vdots \\
0 & 0 & \cdots & 0 & \cdots & w_{nr_no_m}
\end{bmatrix}
\end{array}
\qquad (11.16)
$$

11.3.1.4 Obtain Overall Criteria Weights

Finally, the overall attribute weights are calculated. For the sub-attribute w_{pl}, the overall weight W_{pl} is calculated as

$$W_{pl} = \sum_{j=1}^{n} w_p w_{pj} \qquad (11.17)$$

It is known that $w_{pj} = 0$ in the case of $j \neq l$, hence the overall weight W_{pl} is calculated as

$$W_{pl} = w_p w_{pl} \qquad (11.18)$$

In a similar manner, the overall sub-sub attribute weight W_{plt} is calculated as

$$\tilde{W}_{plt} = \sum_{j=1}^{m} \tilde{w}_p \tilde{w}_{pl} \tilde{w}_{plj} \tag{11.19}$$

Because $w_{plj} = 0$ for $j \neq t$, final weights are obtained as

$$W_{plt} = w_p w_{pl} w_{plt} = W_{pl} w_{plt} \tag{11.20}$$

11.3.2 Fuzzy TOPSIS Calculations

When the criteria weights are determined in the hierarchical fuzzy TOPSIS, the fuzzy TOPSIS calculations are made in order to reach the final ranking of alternatives. In the literature, many different fuzzy TOPSIS implementations exist. These differences generally emerge from following aspects:

- Fuzzy numbers used in the study such as trapezoidal or triangular fuzzy numbers
- Linguistic variables for importance of criteria and rating of alternatives
- Group or individual decision making
- Aggregation functions used in the group decision making setting
- The normalization formula used in the TOPSIS algorithm (vector normalization, linear normalization etc.)
- Determination of fuzzy positive ideal solution and fuzzy negative ideal solution.
- Fuzzy distance method used in the calculation algorithm.

Among many different implementations of the fuzzy TOPSIS method, we applied a highly accepted method with more than a thousand citations developed by (Chen, 2000). Since our aim is to develop an easy and practical approach, we choose Chen's method. The fuzzy TOPSIS calculation steps are as follows (Chen, 2000):

Step 1: Construct the problem and determine the linguistic variables.

Step 2: A group of experts involving s persons provided the weights of criteria and rating of alternatives. The aggregated ratings and importance weights are given by

$$\tilde{x}_{ij} = \frac{1}{s}[\tilde{x}_{ij}^1 \oplus \tilde{x}_{ij}^2 \oplus \cdots \oplus \tilde{x}_{ij}^s] \quad (11.21)$$

$$\tilde{w}_j = \frac{1}{s}[\tilde{\mathrm{w}}_j^1 \oplus \tilde{w}_j^2 \oplus \cdots \oplus \tilde{w}_j^s] \quad (11.22)$$

where $\tilde{x}_{ij}^s$ and $\tilde{w}_j^s$ represents the performance rating and criteria weighting provided by sth decision maker. However, we obtained criteria weights using equations 11.18 and 11.19. The fuzzy evaluation matrix is obtained as:

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} X_1 & \cdots & X_j & \cdots & X_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1j} & \cdots & \tilde{x}_{1n} \\ \vdots & & \vdots & & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \cdots & x_{in} \\ \vdots & & \vdots & & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array} \quad (11.23)$$

Also the aggregated criteria weights are as follows:

$$W = \left[\tilde{w}_1, \tilde{\mathrm{w}}_2, \ldots, \tilde{\mathrm{w}}_{\mathrm{n}}\right] \quad (11.24)$$

where $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

Step 3: Normalize the fuzzy evaluation matrix. The normalized fuzzy decision matrix is denoted by $\tilde{\boldsymbol{R}} = \left[\tilde{r}_{ij}\right]_{m \times n}$, where benefit criteria is represented by B and cost criteria is given by C, the normalization formula is:

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), \quad j \in B \quad (11.25)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right), \; j \in C \tag{11.26}$$

$$c_j^* = \max_i c_{ij}, \; if \; j \in B \tag{11.27}$$

$$a_j^- = \min_i a_{ij}, \; if \; j \in C \tag{11.28}$$

After normalization with the above formulas, the fuzzy elements in the fuzzy evaluation matrix take the values between $[0,1]$.

Step 4: Calculate the weighted normalized decision matrix. The weighted normalized fuzzy decision matrix $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$ by the formula:

$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j, \; i = 1,2,\ldots,m, \text{ and } j = 1,2,\ldots,n \tag{11.29}$$

Step 5: Determine the positive and negative ideal solutions. Since the elements of the evaluation matrix are mapped into $[0,1]$, in a very practical manner, the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solutions (FNIS) can be given as:

$$A^* = \left(\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*\right), \tag{11.30}$$

$$A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right), \tag{11.31}$$

Where $\tilde{v}_j^* = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0)$ and $j = 1,2,\ldots,n$

Step 6: Calculate distance of each alternative from FPIS and FNIS. The distances are calculated according to formulas:

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*) \ , \ i = 1,2,\ldots,m \tag{11.32}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-) \ , \ i = 1,2,\ldots,m \tag{11.33}$$

The distance between two triangular fuzzy numbers $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]} \tag{11.34}$$

Step 7: Compute the closeness coefficients of each alternative and rank the alternatives. Closeness coefficient is calculated by:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \ i = 1,2,\ldots,m \tag{11.35}$$

If the CC_i value of the ith alternative is closer to 1, then the alternative is said to be closer to FPIS and, farther from the FNIS.

CHAPTER TWELVE
APPLICATION

12.1 Motivation

Higher education systems undergo profound transformations due to globalization. As a great contributor to nation's competitiveness, governments are forced to adapt their higher education systems to the needs of the present. Especially in the era of knowledge societies, universities have active role in the development of economy (Coccia, 2008). Ongoing turmoil in different regions of the world, particularly in the Middle East, sluggish economic growth with increased unemployment, and competitive pressures are all triggered the array of revisions to the current higher education systems of nations. For instance, in Europe, a need to form some assessment tools for higher education is realized at European Union and nation-wide levels (Murias, Miguel, & Rodríguez, 2008). In Spain and Portuguese, studies regarding faculty or academic staff evaluation are presently in progress (Bana e Costa & Oliveira, 2012).

Turkey has recently recognized that the only way to be competitive is to generate new policies for the higher education system that fosters innovation, high-tech talent and sustainable development. According to Mizikaci (2006), Turkey has the state-center model in higher education according to classification of Olsen (2007). The Council of Higher Education (YÖK), holding administrative control, coordinates most of the aspects of the higher education system. In state-centered model, universities are considered as instruments in order to reach pre-determined national targets (Dobbins, Knill, & Vögtle, 2011). Turkish higher education system is still anchored in state-centered regularities; however the general paradigm has also been changing.

Currently, the total number of universities has been reached 175 in 81 different cities in Turkey. Also the shift toward much more innovative and entrepreneurial universities is observed. Unlike the historical peculiarities of higher education system, new universities are more global, have greater autonomy in terms of crucial aspects of governance. Despite the promising developments in the higher education system, debate on the categorization of universities is still unsolved. Some argue that universities should be divided into two categories: research-intensive universities and the teaching-intensive universities. This sort of categorization brings about the different resource allocation policies based upon the university category. Although there are some developments in this direction, they are still in their infancy. In parallel to progress in the top management of higher education, universities, individually, should arrive at a comprehensive portray of policy developments. Furthermore, departments are expected to come up with multifaceted policies, span from research directions to relationships with society.

The industrial engineering department of Dokuz Eylül University with its forty years of academic background is among the top departments in Turkey. As a reactive organization due to its strong academic ties, industrial engineering department focused on the domestic exigencies by taking external pressures into account, and decided to reshape its current statue with the changing paradigms. The department is currently making several reforms, such as making changes in the curriculum, establishing new laboratories to strengthen physical infrastructure, and reinforce the academic staff to better adapt itself for highly competitive real world. Also it has long been realized that the success heavily relies on the department’s ability to create its own strategies and act according to these strategies in a participatory manner. Preparatory efforts to develop renewed strategic plan motivated us to study strategy selection problem with our proposed method.

12.2 Problem Statement

12.2.1 SWOT Factors

SWOT analysis is considered as a part of the contemporary strategic planning process of industrial engineering department of Dokuz Eylül University. Although we regarded the industrial engineering department, this study can be generalized to the faculty evaluation, since the criteria taken into consideration are also well suits to the case of the SWOT analysis of the faculty. Studies related to faculty evaluation with SWOT analysis.

Table 12.1 Studies related to higher education with SWOT analysis

Author	Topic
(Hargis et al., 2013)	The role of mobile learning devices in the national college system in framework
(Zhang et al., 2013)	Evaluation of current development status of a university
(Lanning et al., 2012)	Evaluation of the revised curriculum at the Virginia Commonwealth University School of Dentistry
(Henzi et al., 2007)	Analyzing students perspective about the US and Canadian dental school curriculums by developing Curriculum SWOT (C-SWOT).
(Al Hijji, 2012)	Strategic analysis of academic libraries in Oman
(Kurmanov, Zhumanova, & Kirichok, 2013)	Analysis of business education of Kazakhstan with SWOT and PEST analysis.
(Hromovyk & Horilyk, 2013)	Current situational analysis of pharmaceutical education of Ukraine
(Hamidi & Delbahari, 2011)	Strategy formulation for Islamic Azad University using SWOT analysis
(Kahveci & Meads, 2008)	Evaluation of development of health technology assessment program of Turkey

Table 12.2 Studies related to higher education with SWOT analysis

Author	Topic
(Zavadskas, Turskis, & Tamosaitiene, 2011)	SWOT based strategy selection for construction enterprises
(Ekmekcioglu, Kutlu, & Kahraman, 2011)	SWOT based nuclear power plant site selection with fuzzy AHP and fuzzy TOPSIS
(Chen, 2011)	Supplier selection and evaluation with SWOT, DEA and TOPSIS.
(C. Lin et al., 2008)	Developing SWOT based group decision support system to aid process of core knowledge selection

(Yuksel & Dagdeviren, 2007)	Strategy selection in a textile firm with SWOT and ANP.
(Pesonen et al., 2001)	Evaluation of resource management strategies at Finnish Forest and Park service with AHP in SWOT analysis (A'WOT).
(Kangas et al., 2003)	Assessing management strategies of a forestland estate with A'WOT and Stochastic Multiple Criteria Acceptability Analysis with Ordinal Criteria (SMAA-O).

The preliminary SWOT analysis is given in Figure 12.1.

Figure 12.1 The preliminary SWOT analysis

12.2.2 Problem Hierarchy

As we actually mentioned comprehensively in earlier chapters, our suggested hierarchical structure is much more easy and practical comparing with the ANP. In the top down decomposition approach, similar criteria consist of homogenous groups under specific upper level criteria. Also when number of levels increase, criteria at the bottom levels are highly dissimilar from each other. In our opinion, identifying interactions among the bottom level of hierarchy is very difficult task. Let us take two main criteria as an example: threats and opportunities criteria. We know that threats and opportunities affect each other in the long run. However, when it comes to sub and sub-sub criteria, it is really very challenging to state how a specific threat factor influence another opportunity related sub-sub attribute. Since criteria are specialized through to down of the hierarchy, interactions are based on indirect relations. However, decision makers can more easily capture the direct influences. Furthermore, eliciting interaction degrees among sub-sub criteria requires excessive cognitive effort; in which decision makers avoid allocate time to fill obscure survey questionnaire. The best solution to this problem is to elicit interaction degrees at the higher levels among non-homogeneous criteria, and at the lower levels influences only among the same partition of criteria are considered. During calculation of the overall criterion weights, top down multiplication reflects the implicit influences among non-homogeneous criteria into the lower level criteria. Problem hierarchy is given in Figure 12.2.

12.3 Solution

A group decision making setting involving three evaluators is established to solve our problem. First of all, the linguistic scales are determined. For the relative importance of attribute weights, linguistic scale as shown in table 12.3 is constructed. Linguistic scale for causal relationships is formed as in the table 12.4. The membership functions of the linguistic variables regarding the causal influences are seen in Figure 12.3. The other linguistic scale is used to the rate the alternatives with respect to attributes as seen in table 12.5.

Table 12.3 Linguistic variables for relative importance weights of attributes

Linguistic variable	Triangular fuzzy number
Very low (VL)	(0,0,0.1)
Low (L)	(0,0.1,0.3)
Medium low (ML)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
Medium high (MH)	(0.5,0.7,0.9)
High (H)	(0.7,0.9,1)
Very High (VH)	(0.9,1,1)

Table 12.4 Linguistic variables for causal relationships among attributes

Linguistic variable	Triangular fuzzy number
Very very low (VVL)	(0,0.1,0.2)
Very low (VL)	(0.1,0.2,0.35)
Low (L)	(0.2,0.35,0.5)
Medium (M)	(0.35,0.5,0.65)
High (H)	(0.5,0.65,0.8)
Very high (VH)	(0.65,0.8,0.9)
Very very high (VVH)	(0.8,0.9,1)

Table 12.5 Linguistic variables for rating of alternatives

Linguistic variable	Triangular fuzzy number
Very poor (VP)	(0,0,1)
Poor (P)	(0,1,3)
Medium poor (MP)	(1,3,5)
Fair (F)	(3,5,7)
Medium good (MG)	(5,7,9)
Good (G)	(7,9,10)
Very good (VG)	(9,10,10)

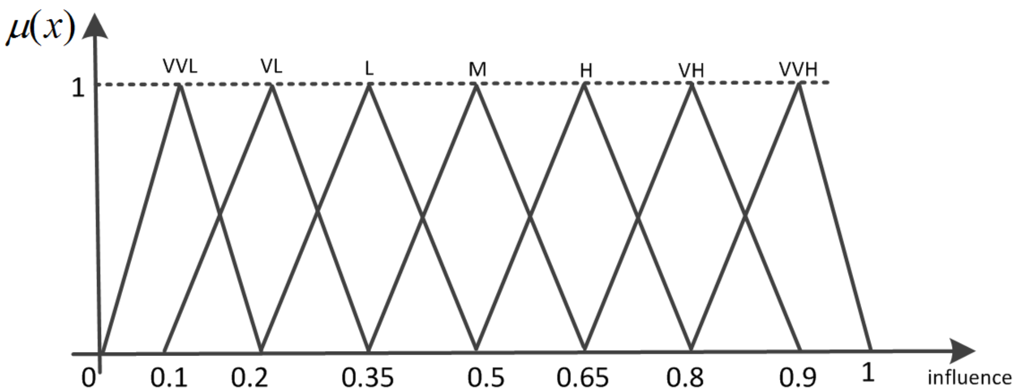

Figure 12.2 Membership functions of linguistic variables for causal relationships

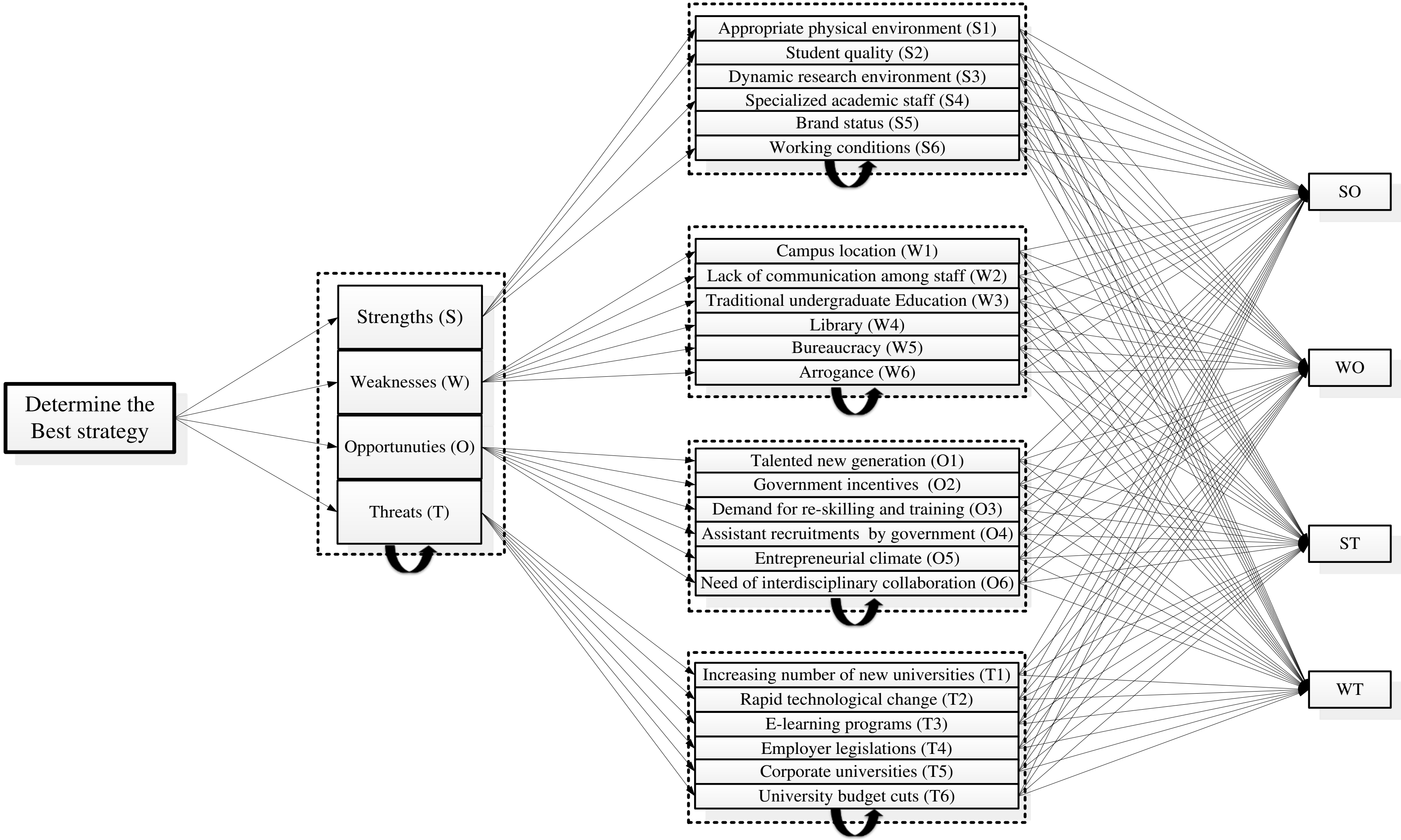

Figure 12.3 Hierarchical evaluation framework

In order to solve the problem by hybrid hierarchical fuzzy TOPSIS and Fuzzy Cognitive Maps, we first determine the overall attribute weights and then calculate the best strategy using fuzzy TOPSIS algorithm. Initially, we calculate the attribute weights for main attributes.

12.3.1 Calculating Weights of the Main Attributes

Initially, the relative importance weights of each main attributes, S, W, O, and T are obtained from three decision makers as seen in table 4.

Table 12.6 The relative importance weights of the main attributes by three decision makers

	D1	**D2**	**D3**
S	H	H	H
W	ML	L	M
O	H	VH	MH
T	H	VH	ML

Then, the degrees of dependency among each main attributes are obtained.

Table 12.7 Dependency degrees among main attributes obtained from the first decision maker

D1	**S**	**W**	**O**	**T**
S	-	VVL	L	
W	L	-		H
O			-	VL
T		H	M	-

Table 12.8 Dependency degrees among main attributes obtained from the second decision maker

D2	**S**	**W**	**O**	**T**
S	-	VL	H	
W	VL	-		M
O			-	VL
T		VH	VH	-

Table 12.9 Dependency degrees among main attributes obtained from the third decision maker

D3	**S**	**W**	**O**	**T**
S	-	M	L	
W	VVL	-		H
O			-	M
T		VH	H	-

When the linguistic assessments are obtained from three decision makers, aggregated fuzzy evaluations for relative importance of weights and causal dependency degrees are acquired by equation 11.1. Aggregated relative importance weights of main attributes is seen in Table 12.10.

Table 12.10 Aggregated relative importance weights of main attributes

	Aggregate Weights	**Defuzzified Weights**	**Normalized Weights**
S	(0.70, 0.90, 1)	0.87	0.32
W	(0.13, 0.30, 0.50)	0.31	0.11
O	(0.70, 0.87, 0.97)	0.84	0.31
T	(0.57, 0.73, 0.83)	0.71	0.26

Similarly, aggregated dependency degrees among main attributes are calculated as table 12.11.

Table 12.11 Aggregated dependency degrees among main attributes

	S	**W**	**O**	**T**
S	(0, 0, 0)	(0.15, 0.27, 0.40)	(0.30, 0.45, 0.60)	(0, 0, 0)
W	(0.10, 0.22, 0.35)	(0, 0, 0)	(0, 0, 0)	(0.45, 0.60, 0.75)
O	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.18, 0.30, 0.45)
T	(0, 0, 0)	(0.60, 0.75, 0.87)	(0.50, 0.65, 0.78)	(0, 0, 0)

Also, aggregated dependency degrees among main attributes are defuzzified to be used in Fuzzy Cognitive Maps as seen in table 12.12.

Table 12.12 Aggregated defuzzified dependency degrees among main attributes

	S	**W**	**O**	**T**
S	0	0.27	0.45	0
W	0.22	0	0	0.60
O	0	0	0	0.31
T	0	0.74	0.64	0

The underlying assumption of normalized weights seen in table 8 is that the main attributes are independent of each other. However, this assumption is not valid in our situation where attributes influence each other in the long term as seen in Figure 12.4. To be able to capture the long term influences, normalized weights are considered as the initial values of the concepts in Fuzzy Cognitive Maps according to equation 11.4.

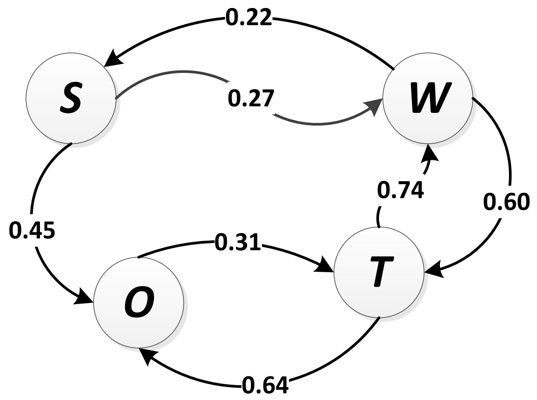

Figure 12.4 Influıence degrees among S,W,O,T attributes

The aggregated defuzzified dependency degrees among main attributes are considered as connection matrix of the Fuzzy Cognitive Maps. Employing equation 11.5, dynamic behavior of main attributes is simulated via Fuzzy Cognitive Maps.

Table 12.13 Concept values of main attributes for each iteration

	Main Attributes			
Iteration	S	W	O	T
0	0.320	0.110	0.310	0.260
1	0.331	0.370	0.551	0.399
2	0.391	0.638	0.742	0.659
3	0.486	0.843	0.872	0.854
4	0.586	0.923	0.927	0.926
5	0.658	0.943	0.945	0.943
6	0.699	0.949	0.951	0.947
7	0.720	0.951	0.954	0.948
8	0.730	0.951	0.955	0.948
9	0.735	0.952	0.955	0.948
10	0.737	0.952	0.956	0.948
11	0.738	0.952	0.956	0.948
12	0.739	0.952	0.956	0.948

Table 12.14 Final weights of main attributes

	Steady State Concepts	Normalized Weights
S	0.739	0.21
W	0.952	0.26
O	0.956	0.27

T	0.948	0.26

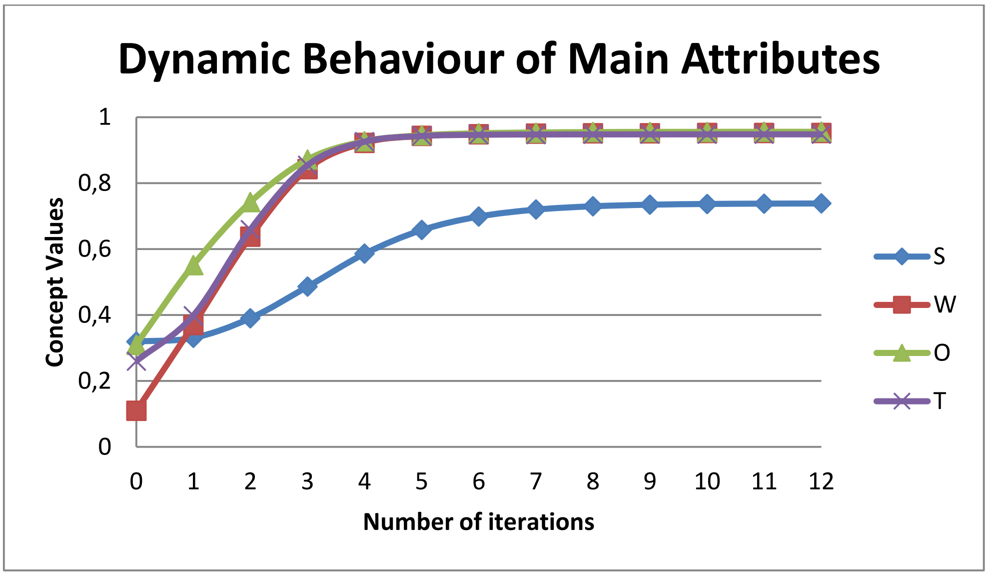

Figure 12.5 Dynamic behaviour of main attributes

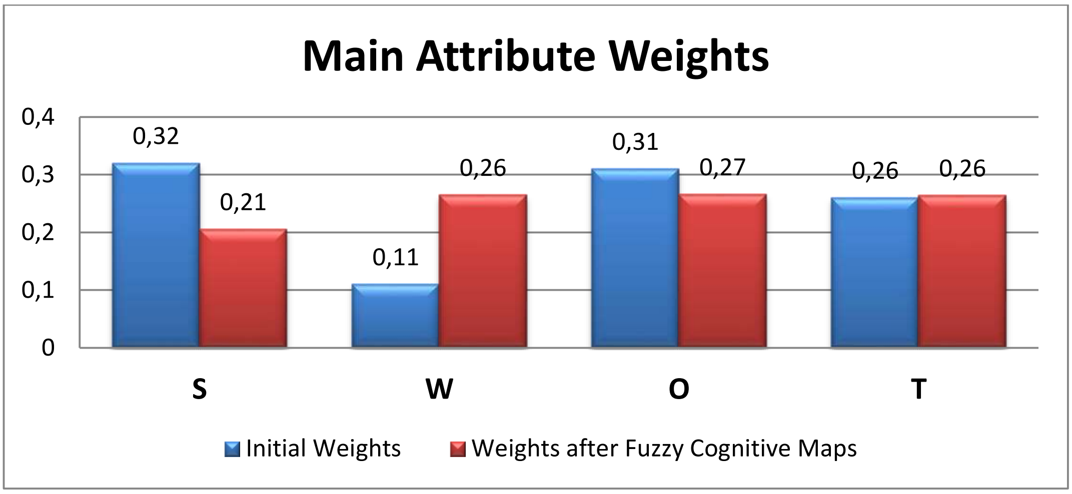

Figure 12.6 Weights of main attributes before/after simulation

12.3.2 Calculating Weights of the Sub Attributes

12.3.2.1 Calculating weights of sub attributes under strengths main attribute

The relative importance weights of the sub attribute strengths are obtained as seen in table 13.

Table 12.15 The relative importance weights of the sub attribute: strenghts by three DMs

	D_1	D_2	D_3
S_1	ML	ML	M
S_2	MH	M	MH
S_3	VH	M	VH
S_4	H	MH	H
S_5	H	MH	VH
S_6	VH	M	H

Then the degrees of dependency among each strength sub attributes are obtained as table 12.16-12.18.

Table 12.16 Dependency degrees among strenght sub attribute obtained from the first DM

D1	S1	S2	S3	S4	S5	S6
S1	-	L	VL		M	M
S2		-	H		L	H
S3		VH	-	H	M	H
S4			H	-	VH	H
S5					-	
S6		L	VH		M	-

Table 12.17 Dependency degrees among strenght sub attribute obtained from the second DM

D_2	S_1	S_2	S_3	S_4	S_5	S_6
S_1	-	M	L		H	H
S_2		-	M		VL	M
S_3		H	-	M	VH	L
S_4			H	-	VVH	H
S_5					-	
S_6		VL	H		H	-

Table 12.18 Dependency degrees among strenght sub attribute obtained from the third DM

D_3	S_1	S_2	S_3	S_4	S_5	S_6
S_1	-	H	VL		H	H
S_2		-	M		VL	L
S_3		VH	-	M	M	L
S_4			H	-	VH	H
S_5					-	
S_6		VL	H		M	-

Then the aggregated fuzzy evaluations are calculated.

Table 12.19 Aggregated relative importance weights of strenght sub attributes

	Aggregate Weights	Defuzzified Weights	Normalized Weights
S1	(0.17, 0.37, 0.57)	0.37	0.09
S2	(0.43, 0.63, 0.83)	0.63	0.15
S3	(0.70, 0.83, 0.90)	0.81	0.19
S4	(0.63, 0.83, 0.97)	0.81	0.19
S5	(0.70, 0.87, 0.97)	0.84	0.20
S6	(0.63, 0.80, 0.90)	0.78	0.18

Similarly, aggregated dependency degrees among strength sub attributes are calculated as table 12.20.

Table 12.20 Aggregated dependency degrees among strenght sub attributes

	S1	S2	S3	S4	S5	S6
S1	(0, 0 ,0)	(0.35, 0.50, 0.65)	(0.13, 0.25, 0.40)	(0, 0 ,0)	(0.45, 0.60, 0.75)	(0.45, 0.60, 0.75)
S2	(0, 0 ,0)	(0, 0 ,0)	(0.40, 0.55, 0.70)	(0, 0 ,0)	(0.13, 0.25, 0.40)	(0.35, 0.50, 0.65)
S3	(0, 0 ,0)	(0.60, 0.75, 0.87)	(0, 0 ,0)	(0.40, 0.55, 0.70)	(0.45, 0.60, 0.73)	(0.30, 0.45, 0.60)
S4	(0, 0 ,0)	(0, 0 ,0)	(0.50, 0.65, 0.80)	(0, 0 ,0)	(0.70, 0.83, 0.93)	(0.50, 0.65, 0.80)
S5	(0, 0 ,0)	(0, 0 ,0)	(0, 0 ,0)	(0, 0 ,0)	(0, 0 ,0)	(0, 0 ,0)
S6	(0, 0 ,0)	(0.13, 0.25, 0.40)	(0.55, 0.70, 0.83)	(0, 0 ,0)	(0.40, 0.55, 0.70)	(0, 0 ,0)

The aggregated dependency degrees among strength sub attributes are defuzzified to be used in Fuzzy Cognitive Maps as seen in table 12.21.

Table 12.21 Aggregated defuzzified dependency among strenght sub attributes

	S1	S2	S3	S4	S5	S6
S1	0	0.50	0.26	0	0.60	0.60
S2	0	0	0.55	0	0.26	0.50
S3	0	0.74	0	0.55	0.59	0.45
S4	0	0	0.65	0	0.82	0.65
S5	0	0	0	0	0	0
S6	0	0.26	0.69	0	0.55	0

The normalized weights of the strength sub attributes are used as the initial values of the concepts in Fuzzy Cognitive Maps. Also the aggregated defuzzified dependency degrees among main attributes are regarded as connection matrix of the

Fuzzy Cognitive Maps. Consequently, employing equation 11.10, dynamic behavior of sub attributes under strength main attribute is simulated through Fuzzy Cognitive Maps.

Table 12.22 Concept values of strenght sub attributes for each iteration

	Sub-attributes of Strenght Main Attribute					
Iteration	S1	S2	S3	S4	S5	S6
0	0.090	0.150	0.190	0.190	0.200	0.180
1	0.090	0.365	0.496	0.286	0.578	0.476
2	0.090	0.716	0.844	0.507	0.908	0.808
3	0.089	0.921	0.973	0.749	0.987	0.959
4	0.089	0.959	0.990	0.858	0.995	0.984
5	0.089	0.963	0.992	0.886	0.996	0.987
6	0.089	0.964	0.993	0.892	0.996	0.988
7	0.088	0.964	0.993	0.893	0.996	0.988
8	0.088	0.964	0.993	0.894	0.996	0.988

Table 12.23 Final weights of strenght sub attributes

	Steady State Concepts	Normalized Weights
S1	0.088	0.02
S2	0.964	0.20
S3	0.993	0.20
S4	0.894	0.18
S5	0.996	0.20
S6	0.988	0.20

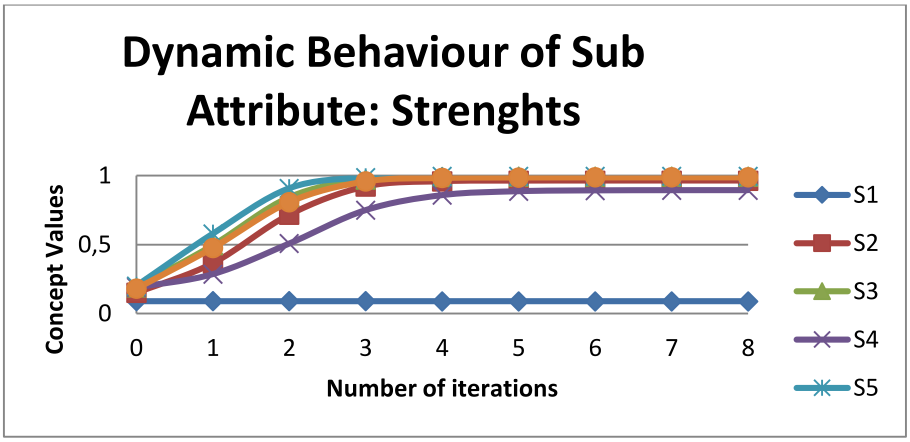

Figure 12.7 Dynamic behaviour of strenght sub attributes

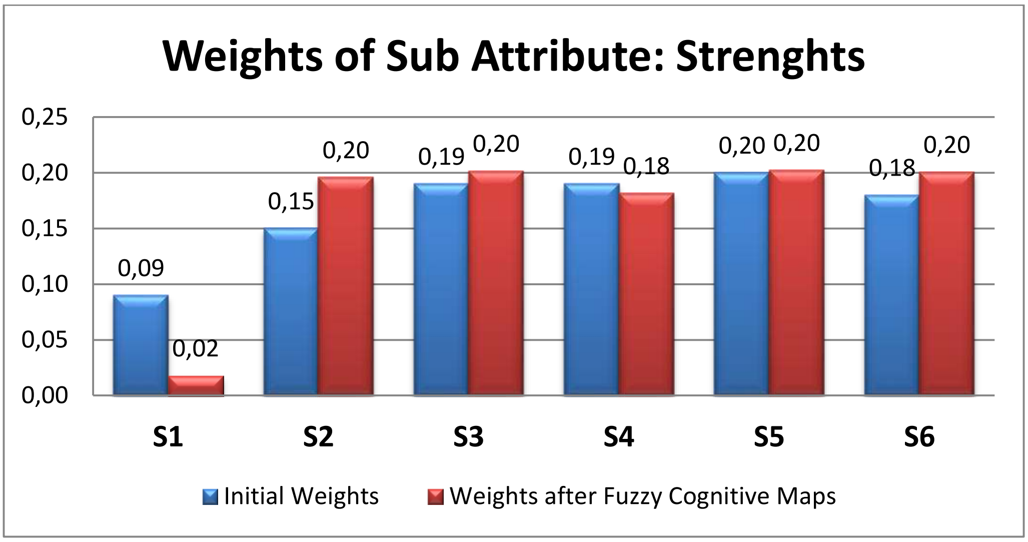

Figure 12.8 Weights of strenght sub attributes before/after simulation

12.3.2.2 Calculating weights of sub attributes under weakness main attribute

The relative importance weights of the sub attributes weaknesses are obtained as seen in table 12.24.

Table 12.24 The relative importance weights of the sub attribute: weaknesses by three DM

	D_1	**D_2**	**D_3**
W_1	VH	H	H
W_2	MH	M	ML
W_3	L	M	M
W_4	ML	L	MH
W_5	H	M	MH
W_6	M	L	MH

Degrees of dependency among each strength sub attributes are obtained as table 12.25-12.27.

Table 12.25 Dependency degrees among weaknesses sub attribute obtained from the first DM

D_1	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**W_6**
W_1	-			H	VL	
W_2		-			M	H
W_3		L	-			L
W_4				-		
W_5		M	L		-	L
W_6		H	L		H	-

Table 12.26 Dependency degrees among weaknesses sub attribute obtained from the second DM

D_2	W_1	W_2	W_3	W_4	W_5	W_6
W_1	-			VH	M	
W_2		-			H	M
W_3		M	-			VL
W_4				-		
W_5		H	VL		-	VL
W_6		M	VL		M	-

Table 12.27 Dependency degrees among weaknesses sub attribute obtained from the third DM

D_3	W_1	W_2	W_3	W_4	W_5	W_6
W_1	-			VH	VL	
W_2		-			VH	L
W_3		M	-			L
W_4				-		
W_5		M	VL		-	M
W_6		H	VVL		L	-

Then the aggregated fuzzy evaluations are calculated.

Table 12.28 Aggregated relative importance weights of weaknesses sub attributes

	Aggregate Weights	Defuzzified Weights	Normalized Weights
W1	(0.77, 0.93, 1)	0.90	0.27
W2	(0.30, 0.50, 0.70)	0.50	0.15
W3	(0.20, 0.37, 0.57)	0.38	0.11
W4	(0.20, 0.37, 0.57)	0.38	0.11
W5	(0.50, 0.70, 0.87)	0.69	0.21
W6	(0.27, 0.43, 0.63)	0.44	0.14

Similarly, aggregated dependency degrees among weaknesses sub attributes are calculated as table 12.29.

Table 12.29 Aggregated dependency degrees among weaknesses sub attributes

	W1	W2	W3	W4	W5	W6
W1	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.60, 0.75, 0.87)	(0.18, 0.30, 0.45)	(0, 0, 0)
W2	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.50, 0.65, 0.78)	(0.35, 0.50, 0.65)
W3	(0, 0, 0)	(0.30, 0.45, 0.60)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.17, 0.30, 0.45)
W4	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
W5	(0, 0, 0)	(0.40, 0.55, 0.70)	(0.13, 0.25, 0.40)	(0, 0, 0)	(0, 0, 0)	(0.22, 0.35, 0.50)

W6	(0, 0, 0)	(0.45, 0.60, 0.75)	(0.10, 0.22, 0.35)	(0, 0, 0)	(0.35, 0.50, 0.65)	(0, 0, 0)

The aggregated dependency degrees among weaknesses sub attributes are defuzzified to be used in Fuzzy Cognitive Maps as seen in table 12.30.

Table 12.30 Aggregated defuzzified dependency among weaknesses sub attributes

	W1	**W2**	**W3**	**W4**	**W5**	**W6**
W1	0.00	0.00	0.00	0.74	0.31	0.00
W2	0.00	0.00	0.00	0.00	0.64	0.50
W3	0.00	0.45	0.00	0.00	0.00	0.31
W4	0.00	0.00	0.00	0.00	0.00	0.00
W5	0.00	0.55	0.26	0.00	0.00	0.36
W6	0.00	0.60	0.22	0.00	0.50	0.00

The normalized decision weights of the weaknesses sub attributes are used as the initial weights of the concepts, while aggregated defuzzified dependency degrees among main attributes are considered as connection matrix of Fuzzy Cognitive Maps. Therefore, employing the equation 11.10, dynamic behavior of sub attributes under weaknesses main attribute is simulated by Fuzzy Cognitive Maps.

Table 12.31 Concept values of weakness sub attributes for each iteration

	Sub-attributes of Weaknesses Main Attribute					
Iteration	W1	W2	W3	W4	W5	W6
0	0.270	0.150	0.110	0.110	0.210	0.140
1	0.264	0.379	0.193	0.300	0.430	0.314
2	0.258	0.712	0.357	0.458	0.722	0.616
3	0.252	0.927	0.592	0.571	0.916	0.872
4	0.247	0.977	0.771	0.640	0.966	0.952
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
64	0.133	0.986	0.871	0.612	0.972	0.969
65	0.132	0.986	0.871	0.611	0.972	0.969
66	0.131	0.986	0.871	0.610	0.972	0.969

Table 12.32 Final weights of weaknesses sub attributes

	Steady State Concepts	**Normalized Weights**
W1	0.131	0.03
W2	0.986	0.22
W3	0.871	0.19

W4	0.610	0.13
W5	0.972	0.21
W6	0.969	0.21

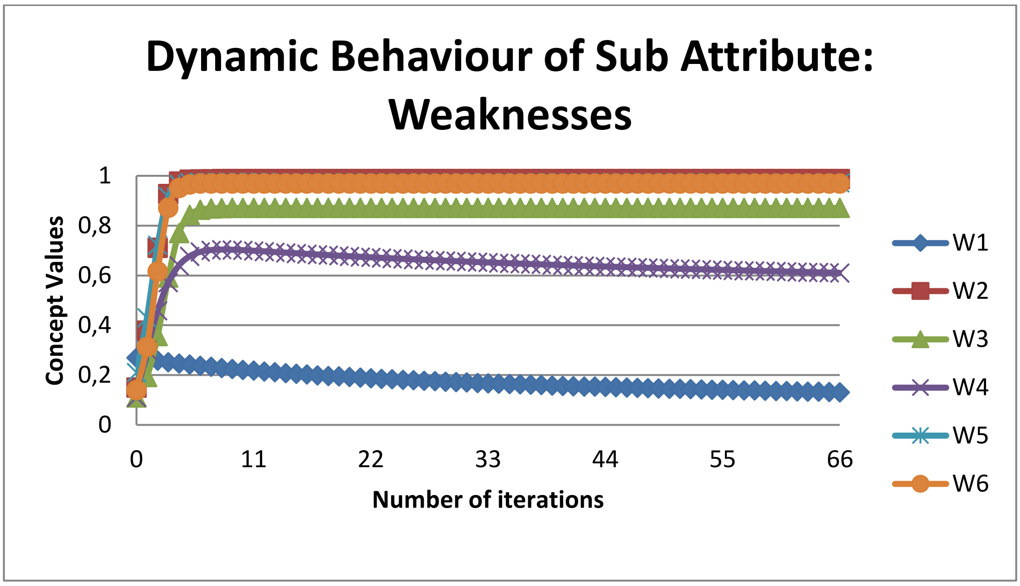

Figure 12.9 Dynamic behaviour of weakness sub attributes

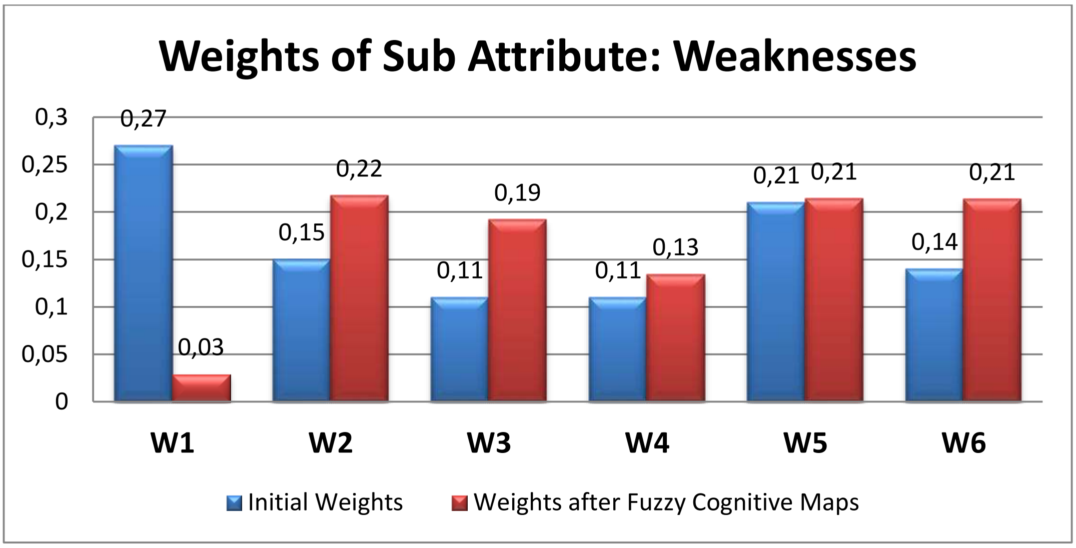

Figure 12.10 Weights of weakness sub attributes before/after simulation

12.3.2.3 Calculating weights of sub attributes under opportunities main attribute

The relative importance weights of the sub attributes opportunities are obtained as seen in table 12.33.

Table 12.33 The relative importance weights of the sub attribute: opportunuties by three DM

	D1	**D2**	**D3**
O1	L	ML	M
O2	VH	H	VH
O3	H	VH	L
O4	M	MH	H
O5	VH	H	H
O6	MH	ML	VH

Degrees of dependency among each opportunities sub attributes are obtained as table 12.34-12.36.

Table 12.34 Dependency degrees among opportunities sub attribute obtained from the first DM

D1	**O1**	**O2**	**O3**	**O4**	**O5**	**O6**
O1	-					
O2		-		VH	VH	H
O3	M	M	-			
O4		L		-	VL	
O5	L	M			-	VH
O6		M	H		L	-

Table 12.35 Dependency degrees among opportunities sub attribute obtained from the second DM

D_2	**O_1**	**O_2**	**O_3**	**O_4**	**O_5**	**O_6**
O_1	-					
O_2		-		VVH	H	M
O_3	L	H	-			
O_4		M		-	M	
O_5	VL	H			-	VVH
O_6		H	L		VH	-

Table 12.36 Dependency degrees among opportunities sub attribute obtained from the third DM

D_3	**O_1**	**O_2**	**O_3**	**O_4**	**O_5**	**O_6**
O_1	-					
O_2		-		VVH	M	H
O_3	M	H	-			
O_4		L		-	H	
O_5	VVL	L			-	VVH
O_6		H	L		L	-

Then the aggregated fuzzy evaluations are calculated.

Table 12.37 Aggregated relative importance weights of opportunuties sub attributes

	Aggregate Weights	Defuzzified Weights	Normalized Weights
O1	(0.13, 0.30, 0.50)	0.31	0.08
O2	(0.83, 0.97, 1)	0.93	0.23
O3	(0.53, 0.67, 0.77)	0.66	0.16
O4	(0.50, 0.70, 0.87)	0.69	0.17
O5	(0.77, 0.93, 1)	0.90	0.22
O6	(0.50, 0.67, 0.80)	0.66	0.16

Table 12.38 Aggregated dependency degrees among opportunuties sub attributes

	O1	O2	O3	O4	O5	O6
O1	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
O2	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.75, 0.87, 0.97)	(0.50, 0.65, 0.78)	(0.45, 0.60, 0.75)
O3	(0.30, 0.45, 0.60)	(0.45, 0.60, 0.75)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
O4	(0, 0, 0)	(0.25, 0.40, 0.55)	(0, 0, 0)	(0, 0, 0)	(0.32, 0.45, 0.60)	(0, 0, 0)
O5	(0.10, 0.22, 0.35)	(0.35, 0.50, 0.65)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.75, 0.87, 0.97)
O6	(0, 0, 0)	(0.45, 0.60, 0.75)	(0.30, 0.45, 0.60)	(0, 0, 0)	(0.35, 0.50, 0.63)	(0, 0, 0)

Table 12.39 Aggregated defuzzified dependency among opportunuties sub attributes

	O1	O2	O3	O4	O5	O6
O1	0.00	0.00	0.00	0.00	0.00	0.00
O2	0.00	0.00	0.00	0.86	0.64	0.60
O3	0.45	0.60	0.00	0.00	0.00	0.00
O4	0.00	0.40	0.00	0.00	0.46	0.00
O5	0.22	0.50	0.00	0.00	0.00	0.86
O6	0.00	0.60	0.45	0.00	0.49	0.00

The normalized decision weights of the opportunities sub attributes are used as the initial weights of the concepts, while aggregated defuzzified dependency degrees among main attributes are considered as connection matrix of Fuzzy Cognitive Maps. Therefore, employing the equation 7.10, dynamic behavior of sub attributes under opportunities main attribute is simulated by Fuzzy Cognitive Maps.

Table 12.40 Concept values of opportunities sub attributes for each iteration

Sub-attributes of Opportunities Main Attribute						
Iteration	O1	O2	O3	O4	O5	O6
0	0.08	0.23	0.16	0.17	0.22	0.16
1	0.198	0.537	0.228	0.352	0.481	0.452
2	0.385	0.868	0.406	0.672	0.836	0.830
3	0.636	0.980	0.653	0.889	0.971	0.969
4	0.816	0.993	0.796	0.939	0.986	0.983
5	0.883	0.994	0.845	0.946	0.988	0.985
6	0.902	0.995	0.859	0.947	0.988	0.985
7	0.906	0.995	0.862	0.947	0.988	0.985
8	0.907	0.995	0.863	0.947	0.988	0.985
9	0.907	0.995	0.863	0.947	0.988	0.985

Table 12.41 Final weights of opportunities sub attributes

	Steady State Concepts	Normalized Weights
O1	0.907	0.16
O2	0.995	0.17
O3	0.863	0.15
O4	0.947	0.17
O5	0.988	0.17
O6	0.985	0.17

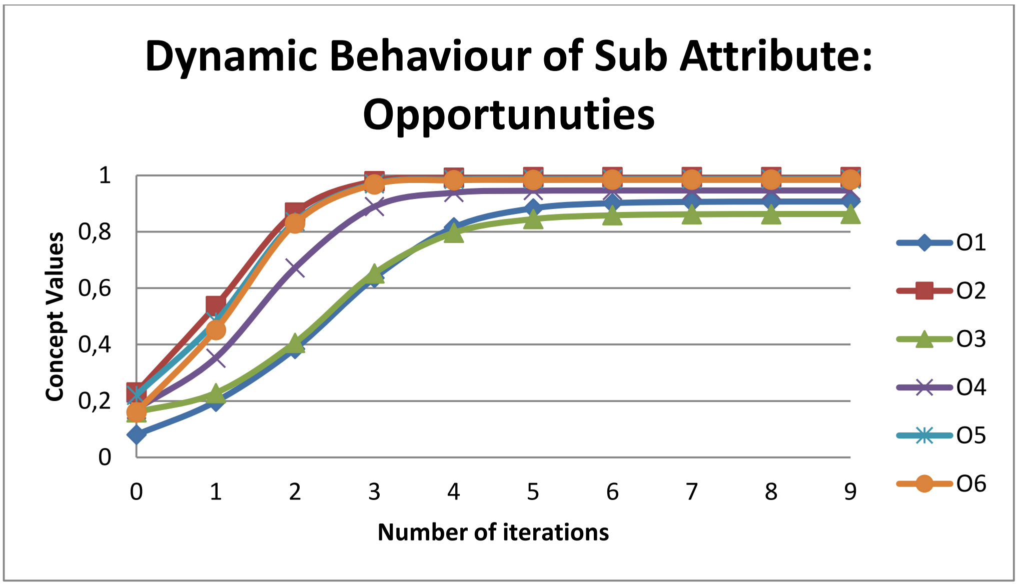

Figure 12.11 Dynamic behaviour of opportunities sub attributes

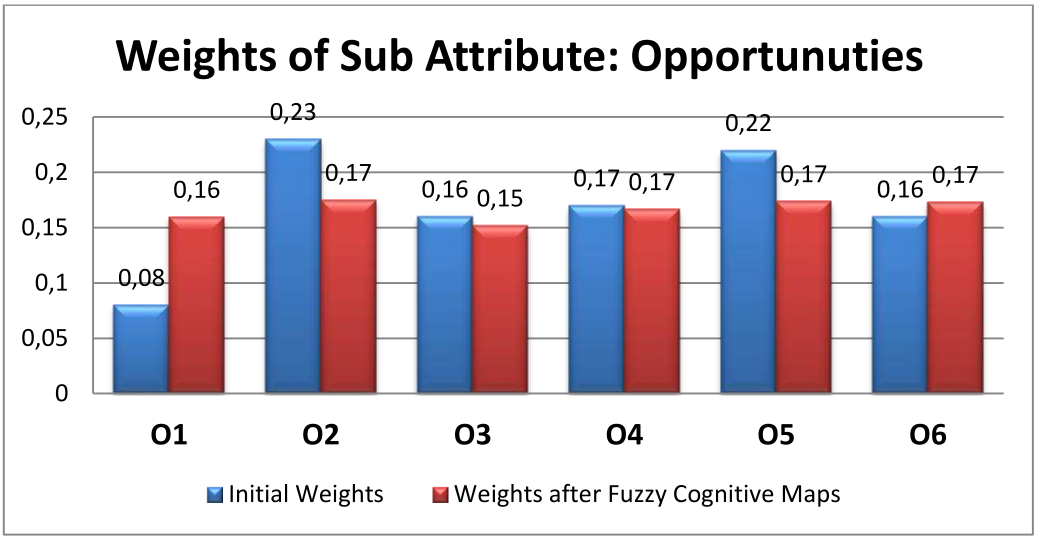

Figure 12.12 Weights of opportunities sub attributes before/after simulation

12.3.2.3 Calculating weights of sub attributes under threats main attribute

Table 12.42 The relative importance weights of the sub attribute: treats by three decision makers

	D_1	D_2	D_3
T_1	VH	H	MH
T_2	H	H	H
T_3	H	ML	M
T_4	VL	L	L
T_5	VH	H	MH
T_6	L	VL	L

Table 12.43 Dependency degrees among treats sub attribute obtained from the first decision maker

D_1	T_1	T_2	T_3	T_4	T_5	T_6
T_1	-		H	M	VH	L
T_2		-	H			M
T_3		L	-			VVL
T_4				-	M	
T_5	L		L		-	M
T_6			VVL		L	-

Table 12.44 Dependency degrees among treats sub attribute obtained from the second decision maker

D_2	T_1	T_2	T_3	T_4	T_5	T_6
T_1	-		VH	H	M	M
T_2		-	M			H
T_3		M	-			L
T_4				-	L	
T_5	L		M		-	L
T_6			VVL		VVL	-

Table 12.45 Dependency degrees among treats sub attribute obtained from the third DM

D_3	T_1	T_2	T_3	T_4	T_5	T_6
T_1	-		VH	VH	M	VVL
T_2		-	L			L
T_3		VL	-			VVL
T_4				-	L	
T_5	M		VVL		-	H
T_6			VVL		M	-

Table 12.46 Aggregated relative importance weights of treats sub attributes

	Aggregate Weights	Defuzzified Weights	Normalized Weights
T1	(0.70, 0.87, 0.97)	0.84	0.26
T2	(0.70, 0.90, 1)	0.87	0.26
T3	(0.37, 0.57, 0.73)	0.56	0.17
T4	(0, 0.07, 0.23)	0.10	0.03
T5	(0.70, 0.87, 0.97)	0.84	0.26
T6	(0, 0.07, 0.23)	0.10	0.03

Table 0.47 Aggregated dependency degrees among treats sub attributes

	T1	T2	T3	T4	T5	T6
T1	(0, 0, 0)	(0, 0, 0)	(0.60, 0.75, 0.87)	(0.50, 0.65, 0.78)	(0.45, 0.60, 0.73)	(0.18, 0.32, 0.45)
T2	(0, 0, 0)	(0, 0, 0)	(0.35, 0.50, 0.65)	(0, 0, 0)	(0, 0, 0)	(0.35, 0.50, 0.65)
T3	(0, 0, 0)	(0.22, 0.35, 0.50)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.07, 0.18, 0.30)
T4	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.25, 0.40, 0.55)	(0, 0, 0)
T5	(0.25, 0.40, 0.55)	(0, 0, 0)	(0.18, 0.32, 0.45)	(0, 0, 0)	(0, 0, 0)	(0.35, 0.50, 0.65)
T6	(0, 0, 0)	(0, 0, 0)	(0, 0.10, 0.20)	(0, 0, 0)	(0.18, 0.32, 0.45)	(0, 0, 0)

Table 12.48 Aggregated defuzzified dependency among treats sub attributes

	T1	T2	T3	T4	T5	T6
T1	0.00	0.00	0.74	0.64	0.59	0.32
T2	0.00	0.00	0.50	0.00	0.00	0.50
T3	0.00	0.36	0.00	0.00	0.00	0.18
T4	0.00	0.00	0.00	0.00	0.40	0.00
T5	0.40	0.00	0.32	0.00	0.00	0.50
T6	0.00	0.00	0.10	0.00	0.32	0.00

The normalized decision weights of the threats sub attributes are used as the initial weights of the concepts, while aggregated defuzzified dependency degrees among main attributes are considered as connection matrix of Fuzzy Cognitive Maps. Therefore, employing the equation 11.10, dynamic behavior of sub attributes under threats main attribute is simulated by Fuzzy Cognitive Maps.

Table 12.49 Concept values of threats sub attributes for each iteration

	Sub-attributes of Threats Main Attribute					
Iteration	T1	T2	T3	T4	T5	T6
0	0.26	0.26	0.17	0.03	0.26	0.03
1	0.349	0.311	0.522	0.194	0.409	0.383
2	0.472	0.461	0.802	0.394	0.673	0.739
3	0.630	0.635	0.932	0.602	0.873	0.922
4	0.753	0.749	0.970	0.764	0.945	0.967
5	0.811	0.800	0.980	0.847	0.964	0.977
6	0.833	0.819	0.983	0.878	0.970	0.980
7	0.840	0.825	0.984	0.888	0.972	0.981
8	0.842	0.827	0.985	0.891	0.972	0.981
9	0.843	0.828	0.985	0.892	0.973	0.981

Table 12.50 Final weights of threats sub attributes

	Steady State Concepts	**Normalized Weights**
T1	0.843	0.15
T2	0.828	0.15
T3	0.985	0.18
T4	0.892	0.16
T5	0.973	0.18
T6	0.981	0.18

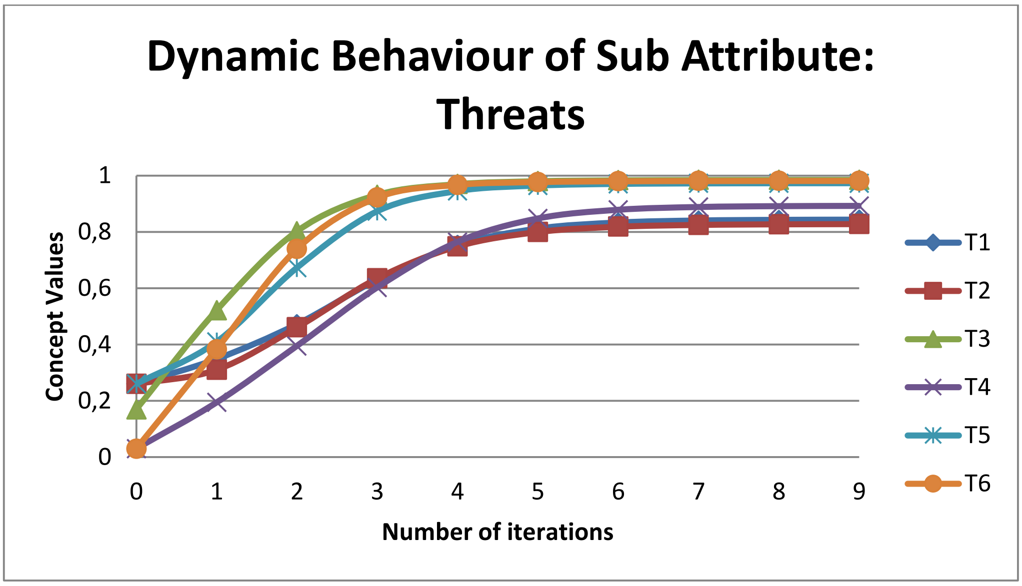

Figure 12.13 Dynamic behaviour of threats sub attributes

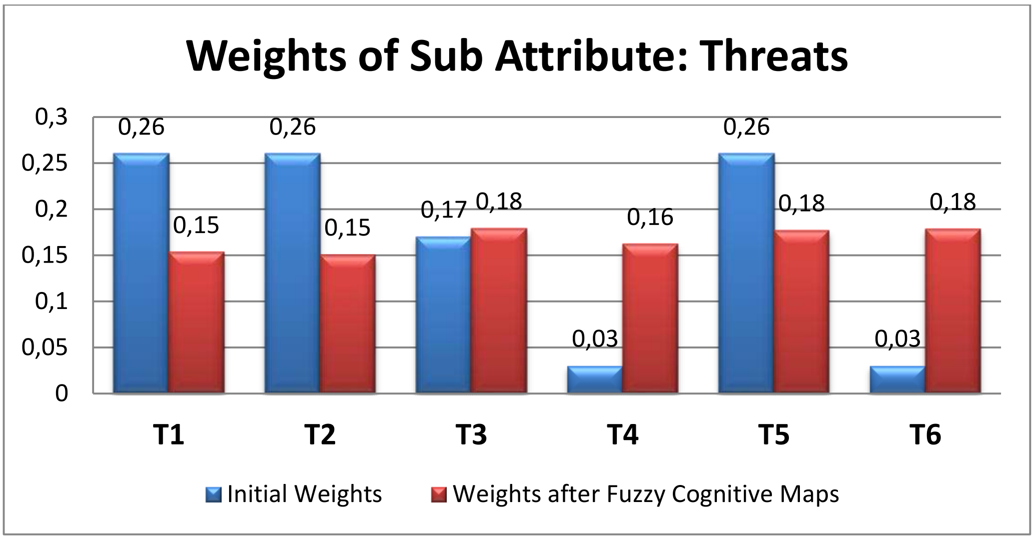

Figure 12.14 Weights of threats sub attributes before/after simulation

12.3.3 Calculating Overall Attribute Weights

The overall attribute weights are calculated according to equation 11.20.

Table 12.51 Overall priorities of factors and sub-factors

SWOT factors	Factor Weights	SWOT sub-factors	Sub-factor Weights	Overall Weights
Strengths	0.210	Appropriate physical environment (S1)	0.020	0.004
		Student quality (S2)	0.200	0.042
		Dynamic research environment (S3)	0.200	0.042
		Specialized academic staff (S4)	0.180	0.038
		Brand status (S5)	0.200	0.042
		Working conditions (S6)	0.200	0.042
Weaknesses	0.260	Campus location (W1)	0.030	0.008
		Lack of communication among staff (W2)	0.220	0.057
		Traditional undergraduate education (W3)	0.190	0.049
		Library (W4)	0.130	0.034
		Bureaucracy (W5)	0.210	0.055
		Arrogance (W6)	0.210	0.055
Opportunities	0.270	Talented new generation (O1)	0.160	0.043
		Government incentives (O2)	0.170	0.046
		Demand for re-skilling and training (O3)	0.150	0.041
		Assistant recruitments by government (O4)	0.170	0.046
		Entrepreneurial climate (O5)	0.170	0.046
		Need of interdisciplinary collaboration (O6)	0.170	0.046
Threats	0.260	Increasing number of new universities (T1)	0.150	0.039
		Rapid technological change (T2)	0.150	0.039
		E-learning programs (T3)	0.180	0.047
		Employer legislations (T4)	0.160	0.042
		Corporate universities (T5)	0.180	0.047
		University budget cuts (T6)	0.180	0.047

12.3.4 Calculating TOPSIS Algorithm

12.3.4.1 Obtain Performance Values from Decision Makers

The fuzzy rating values of each strategy with respect to attributes are obtained as in the table 12.52.

Table 12.52 Fuzzy rating values of alternatives with respect to attributes obtained from DMs

	SO			WO			ST			WT		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
S_1	G	VG	MG	G	G	VG	VG	G	G	P	MP	F
S_2	VG	G	MG	G	VG	G	G	MG	VG	F	MG	G
S_3	MG	P	F	P	F	MP	MP	F	F	F	G	MG
S_4	F	MG	G	P	MG	F	P	MP	F	P	P	F
S_5	G	F	MP	P	F	MG	MP	F	P	G	VG	F
S_6	P	VP	G	G	VG	G	G	VG	MG	G	F	F
W_1	P	MP	P	VG	G	MG	G	MG	F	P	F	VG
W_2	G	VG	MG	G	G	MG	VG	G	F	P	G	F
W_3	VG	VG	G	VG	G	F	G	F	MG	P	MP	G
W_4	F	MP	G	MG	F	G	P	MP	F	P	MP	F
W_5	F	MP	P	G	VG	F	G	F	VG	P	F	MP
W_6	G	MG	MG	F	MP	F	MP	P	P	MP	F	MG
O_1	VG	G	VG	G	P	VP	F	MG	MG	G	MG	F
O_2	G	MG	F	VG	G	MG	F	MP	F	G	F	MG
O_3	G	MG	MG	F	F	G	G	G	F	VG	G	F
O_4	P	MP	F	F	G	MG	MP	MP	P	F	MP	P
O_5	G	VP	P	P	G	MG	G	G	MG	F	MG	MG
O_6	VG	G	MG	VG	G	F	G	VG	MG	MG	G	G
T_1	MG	G	MP	G	VG	G	MP	F	P	F	G	F
T_2	G	MG	VG	G	VG	G	F	MP	MG	G	F	VP
T_3	P	MP	F	P	MP	F	VG	G	VG	P	G	G
T_4	F	MG	F	VG	G	MG	G	F	MG	P	F	VG
T_5	F	MG	G	G	VG	G	MG	VG	G	VG	G	G
T_6	P	MP	F	P	VP	MP	MP	VG	G	G	F	MG

12.3.4.2 Aggregate Performance Matrices

The rating values of performance matrix are aggregated using equation 11.22. The aggregated fuzzy decision matrix is given in table 12.53.

Table 12.53 Aggregated fuzzy decision matrix

	SO	WO	ST	WT
S1	(7, 8.67, 9.67)	(7.67, 9.33, 10)	(7.67, 9.33, 10)	(1.33, 3, 5)
S2	(7, 8.67, 9.67)	(7.67, 9.33, 10)	(7, 8.67, 9.67)	(5, 7, 8.67)
S3	(2.67, 4.33, 6.33)	(1.33, 3, 5)	(2.33, 4.33, 6.33)	(5, 7, 8.67)
S4	(5, 7, 8.67)	(2.67, 4.33, 6.33)	(1.33, 3, 5)	(1, 2.33, 4.33)
S5	(3.67, 5.67, 7.33)	(2.67, 4.33, 6.33)	(1.33, 3, 5)	(6.33, 8, 9)
S6	(2.33, 3.33, 4.67)	(7.67, 9.33, 10)	(7, 8.67, 9.67)	(4.33, 6.33, 8)
W1	(0.33, 1.67, 3.67)	(7, 8.67, 9.67)	(5, 7, 8.67)	(4, 5.33, 6.67)
W2	(7, 8.67, 9.67)	(6.33, 8.33, 9.67)	(6.33, 8, 9)	(3.33, 5, 6.67)
W3	(8.33, 9.67, 10)	(6.33, 8, 9)	(5, 7, 8.67)	(2.67, 4.33, 6)
W4	(3.67, 5.67, 7.33)	(5, 7, 8.67)	(1.33, 3, 5)	(1.33, 3, 5)
W5	(1.33, 3, 5)	(6.33, 8, 9)	(6.33, 8, 9)	(1.33, 3, 5)

W6	(5.67, 7.67, 9.33)	(2.33, 4.33, 6.33)	(0.33, 1.67, 3.67)	(3, 5, 7)
O1	(8.33, 9.67, 10)	(2.33, 3.33, 4.67)	(4.33, 6.33, 8.33)	(5, 7, 8.67)
O2	(5, 7, 8.67)	(7, 8.67, 9.67)	(2.33, 4.33, 6.33)	(5, 7, 8.67)
O3	(5.67, 7.67, 9.33)	(4.33, 6.33, 8)	(5.67, 7.67, 9)	(6.33, 8, 9)
O4	(1.33, 3, 5)	(5, 7, 8.67)	(0.67, 2.33, 4.33)	(1.33, 3, 5)
O5	(2.33, 3.33, 4.67)	(4, 5.67, 7.33)	(6.33, 8.33, 9.67)	(4.33, 6.33, 8.33)
O6	(7, 8.67, 9.67)	(6.33, 8, 9)	(7, 8.67, 9.67)	(6.33, 8.33, 9.67)
T1	(4.33, 6.33, 8)	(7.67, 9.33,10)	(1.33, 3,5)	(4.33, 6.33, 8)
T2	(7, 8.67, 9.67)	(7.67, 9.33, 10)	(3, 5,7)	(3.33, 4.67, 6)
T3	(1.33, 3, 5)	(1.33, 3, 5)	(8.33, 9.67, 10)	(4.67, 6.33, 7.67)
T4	(3.67, 5.67, 7.67)	(7, 8.67, 9.67)	(5, 7, 8.67)	(4, 5.33, 6.67)
T5	(5, 7, 8.67)	(7.67, 9.33, 10)	(7, 8.67, 9.67)	(7.67, 9.33, 10)
T6	(1.33, 3, 5)	(0.33, 1.33, 3)	(5.67, 7.33, 8.33)	(5, 7, 8.67)

12.3.4.3 Normalize Fuzzy Performance Matrix

The fuzzy decision matrix is normalized according to equations 11..25-11.28. The normalized fuzzy decision matrix is given in table 12.54.

Table 12.54 Normalized fuzzy decision matrix

	SO	WO	ST	WT
S1	(0.700, 0.867, 0.967)	(0.767, 0.933, 1.000)	(0.767, 0.933, 1.000)	(0.133, 0.300, 0.500)
S2	(0.700, 0.867, 0.967)	(0.767, 0.933, 1.000)	(0.700, 0.867, 0.967)	(0.500, 0.700, 0.867)
S3	(0.308, 0.500, 0.731)	(0.154, 0.346, 0.577)	(0.269, 0.500, 0.731)	(0.577, 0.808, 1.000)
S4	(0.577, 0.808, 1.000)	(0.308, 0.500, 0.731)	(0.154, 0.346, 0.577)	(0.115, 0.269, 0.500)
S5	(0.407, 0.630, 0.815)	(0.296, 0.481, 0.704)	(0.148, 0.333, 0.556)	(0.704, 0.889, 1.000)
S6	(0.233, 0.333, 0.467)	(0.767, 0.933, 1.000)	(0.700, 0.867, 0.967)	(0.433, 0.633, 0.800)
W1	(0.034, 0.172, 0.379)	(0.724, 0.897, 1.000)	(0.517, 0.724, 0.897)	(0.414, 0.552, 0.690)
W2	(0.724, 0.897, 1.000)	(0.655, 0.862, 1.000)	(0.655, 0.828, 0.931)	(0.345, 0.517, 0.690)
W3	(0.833, 0.967, 1.000)	(0.633, 0.800, 0.900)	(0.500, 0.700, 0.867)	(0.267, 0.433, 0.600)
W4	(0.423, 0.654, 0.846)	(0.577, 0.808, 1.000)	(0.154, 0.346, 0.577)	(0.154, 0.346, 0.577)
W5	(0.148, 0.333, 0.556)	(0.704, 0.889, 1.000)	(0.704, 0.889, 1.000)	(0.148, 0.333, 0.556)
W6	(0.607, 0.821, 1.000)	(0.250, 0.464, 0.679)	(0.036, 0.179, 0.393)	(0.321, 0.536, 0.750)

O1	(0.833, 0.967, 1.000)	(0.233, 0.333, 0.467)	(0.433, 0.633, 0.833)	(0.500, 0.700, 0.867)
O2	(0.517, 0.724, 0.897)	(0.724, 0.897, 1.000)	(0.241, 0.448, 0.655)	(0.517, 0.724, 0.897)
O3	(0.607, 0.821, 1.000)	(0.464, 0.679, 0.857)	(0.607, 0.821, 0.964)	(0.679, 0.857, 0.964)
O4	(0.154, 0.346, 0.577)	(0.577, 0.808, 1.000)	(0.077, 0.269, 0.500)	(0.154, 0.346, 0.577)
O5	(0.241, 0.345, 0.483)	(0.414, 0.586, 0.759)	(0.655, 0.862, 1.000)	(0.448, 0.655, 0.862)
O6	(0.724, 0.897, 1.000)	(0.655, 0.828, 0.931)	(0.724, 0.897, 1.000)	(0.655, 0.862, 1.000)
T1	(0.433, 0.633, 0.800)	(0.767, 0.933, 1.000)	(0.133, 0.300, 0.500)	(0.433, 0.633, 0.800)
T2	(0.700, 0.867, 0.967)	(0.767, 0.933, 1.000)	(0.300, 0.500, 0.700)	(0.333, 0.467, 0.600)
T3	(0.133, 0.300, 0.500)	(0.133, 0.300, 0.500)	(0.833, 0.967, 1.000)	(0.467, 0.633, 0.767)
T4	(0.379, 0.586, 0.793)	(0.724, 0.897, 1.000)	(0.517, 0.724, 0.897)	(0.414, 0.552, 0.690)
T5	(0.500, 0.700, 0.867)	(0.767, 0.933, 1.000)	(0.700, 0.867, 0.967)	(0.767, 0.933, 1.000)
T6	(0.154, 0.346, 0.577)	(0.038, 0.154, 0.346)	(0.654, 0.846, 0.962)	(0.577, 0.808, 1.000)

12.3.4.4 Weight Normalized Fuzzy Decision Matrix

The normalized fuzzy decision matrix is weighted by the weights as seen in table 12.51 using equation 11.29. The weighted normalized fuzzy decision matrix is seen as table 12.55.

Table 12.55 Weighted normalized fuzzy decision matrix

	SO	WO	ST	WT
S1	(0.003, 0.003, 0.004)	(0.003, 0.004, 0.004)	(0.003, 0.004, 0.004)	(0.001, 0.001, 0.002)
S2	(0.029, 0.036, 0.041)	(0.032, 0.039, 0.042)	(0.029, 0.036, 0.041)	(0.021, 0.029, 0.036)
S3	(0.013, 0.021, 0.031)	(0.006, 0.015, 0.024)	(0.011, 0.021, 0.031)	(0.024, 0.034, 0.042)
S4	(0.022, 0.031, 0.038)	(0.012, 0.019, 0.028)	(0.006, 0.013, 0.022)	(0.004, 0.010, 0.019)
S5	(0.017, 0.026, 0.034)	(0.012, 0.020, 0.030)	(0.006, 0.014, 0.023)	(0.030, 0.037, 0.042)
S6	(0.010, 0.014, 0.020)	(0.032, 0.039, 0.042)	(0.029, 0.036, 0.041)	(0.018, 0.027, 0.034)

W1	(0.000, 0.001, 0.003)	(0.006, 0.007, 0.008)	(0.004, 0.006, 0.007)	(0.003, 0.004, 0.006)
W2	(0.041, 0.051, 0.057)	(0.037, 0.049, 0.057)	(0.037, 0.047, 0.053)	(0.020, 0.029, 0.039)
W3	(0.041, 0.047, 0.049)	(0.031, 0.039, 0.044)	(0.025, 0.034, 0.042)	(0.013, 0.021, 0.029)
W4	(0.014, 0.022, 0.029)	(0.020, 0.027, 0.034)	(0.005, 0.012, 0.020)	(0.005, 0.012, 0.020)
W5	(0.008, 0.018, 0.031)	(0.039, 0.049, 0.055)	(0.039, 0.049, 0.055)	(0.008, 0.018, 0.031)
W6	(0.033, 0.045, 0.055)	(0.014, 0.026, 0.037)	(0.002, 0.010, 0.022)	(0.018, 0.029, 0.041)
O1	(0.036, 0.042, 0.043)	(0.010, 0.014, 0.020)	(0.019, 0.027, 0.036)	(0.022, 0.030, 0.037)
O2	(0.024, 0.033, 0.041)	(0.033, 0.041, 0.046)	(0.011, 0.021, 0.030)	(0.024, 0.033, 0.041)
O3	(0.025, 0.034, 0.041)	(0.019, 0.028, 0.035)	(0.025, 0.034, 0.040)	(0.028, 0.035, 0.040)
O4	(0.007, 0.016, 0.027)	(0.027, 0.037, 0.046)	(0.004, 0.012, 0.023)	(0.007, 0.016, 0.027)
O5	(0.011, 0.016, 0.022)	(0.019, 0.027, 0.035)	(0.030, 0.040, 0.046)	(0.021, 0.030, 0.040)
O6	(0.033, 0.041, 0.046)	(0.030, 0.038, 0.043)	(0.033, 0.041, 0.046)	(0.030, 0.040, 0.046)
T1	(0.017, 0.025, 0.031)	(0.030, 0.036, 0.039)	(0.005, 0.012, 0.020)	(0.017, 0.025, 0.031)
T2	(0.027, 0.034, 0.038)	(0.030, 0.036, 0.039)	(0.012, 0.02, 0.027)	(0.013, 0.018, 0.023)
T3	(0.006, 0.014, 0.024)	(0.006, 0.014, 0.024)	(0.039, 0.045, 0.047)	(0.022, 0.030, 0.036)
T4	(0.016, 0.025, 0.033)	(0.030, 0.038, 0.042)	(0.022, 0.030, 0.038)	(0.017, 0.023, 0.029)
T5	(0.024, 0.033, 0.041)	(0.036, 0.044, 0.047)	(0.033, 0.041, 0.045)	(0.036, 0.044, 0.047)
T6	(0.007, 0.016, 0.027)	(0.002, 0.007, 0.016)	(0.031, 0.040, 0.045)	(0.027, 0.038, 0.047)

12.3.4.5 Calculate distances from ideal and anti-ideal solutions

The distances d_i^* and d_i^- of each strategies are calculated based on the equations 11.32-11.33 where $\tilde{v}_j^* = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0)$, $j = 1,2,\ldots 24$. The distances from FPIS and FNIS are given in table 12.56.

Table 12.56 Distances of each alternative from positive and negative ideal solutions

	d_i^*	d_i^-
SO	23.362	0.657
WO	23.318	0.699
ST	23.365	0.657
WT	23.391	0.629

12.3.4.6 Calculate the Closeness Coefficients

The closeness coefficient is calculated using equation 11.35. The closeness coefficients are given in table 12.57.

Table 12.57 Closeness coefficients of each alternatives

Strategies	CC_i
SO	0.0274
WO	0.0291
ST	0.0273
WT	0.0262

According to closeness coefficients, alternative strategies are ranked as: $WO \succ SO \succ ST \succ WT$.

CHAPTER THIRTEEN
CONCLUSIONS

In this thesis, several methods are overviewed for modeling and solving interactions among multiple criteria. We first, give the basic independence axioms of the utility theory. Then we anayzed many methods for modeling interactions among criteria. Furhtermore, we give the characteristics of a sufficient MCDM method should have, and proposed a method based on Fuzzy Cognitive Maps and Fuzzy TOPSIS, which deals with interactions among criteria..

There are also other methods, where we did not mentioned in the scope of this thesis. For future studies, following methods will be incorporated:The use of decision rules in decision problems. Specifically Dominance-Based Rough Set approach (Greco, Matarazzo, & Slowinski, 2001) is able to deal with modeling criteria interactions and approaximate preference relations for sorting and rankig problems. In the distance based methods, Mahalanobis distance, which is able to measure the correlations between criteria where the relations are defined by covariance matrix can be used instead of Euclidean distance (Antuchevičiene, Zavadskas, & Zakarevičius, 2010). In ELECTRE type methods, updating concordance index for mutual strengthening, mutual weakening and antagonistic effects can be studied further (Figueira, Greco, & Roy, 2009). A recent review of Graph Theoretical-Matrix Permanent Approach to Decision Making (GT-MP-DM) method is provided by Baykasoglu (2012). Possible hyrib techniques with GT-MP-DM can be used. Structural Equation Modeling (SEM) can be used in conjunction with Fuzzy Cognitive Maps and DEMATEL, especially for justification of interactions.

Main difficulty is that the methods cannot pricisely categorised since they are mostly contextual. The second difficulty is the excessive number of published papers from diversity of distinct disciplines. However, this study is a big step toward reaching the ultimate picture of whole methods for modeling interactions. A concise survey paper with classification of methods for modeling interactions will be published in the near future.

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