ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

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APPLICATION OF BOOTSTRAP TECHNIQUES IN REGRESSION MODELS

DEPARTMENT OF STATISTICS

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ABSTRACT

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Due to the nature of multiple linear regression, the least squares estimation of regression parameters when explanatory variables are linearly related tends to be in the wrong sign and has large variance. To eliminate these kinds of abnormal effects, there are some useful techniques which reduce or totally eliminates this problem. Biased estimation methods are examples of these methods. In fact, this study aims to identify and observe the usefulness of these methods, particularly the bootstrap method. Furthermore, this study also aims to identify the best the k selection method while determining confidence interval levels. The advantage of using the bootstrap method is that this method enables the researcher to produce new data set from the existing one via resampling it.

On the other hand, k selection methods, which aim to minimize the mean square errors of the ridge estimators, have drawbacks when compared to the bootstrap method. The purpose of this study is to identify the existing literature on multiple linear regression via both ridge estimator and bootstrap, provide a comprehensive bootstrap study and simulations via considering the k selection method and its effects on confidence interval levels.

Key words: Bootstrap, Ridge estimator, Confidence intervals, Multicollinearity, Mean square error

ÖZ

YÜKSEK LİSANS TEZİ

BOOTSTRAP YÖNTEMİNİN REGRESYON MODELLERİNDE UYGULANMASI

Hüsniye ALTUNER

ÇUKUROVA ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ
İSTATİSTİK ANABİLİM DALI

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Çoklu doğrusal regresyon doğası gereğince, açıklayıcı değişkenler doğrusal ilişkili olduğunda regresyon parametrelerinin en küçük kareler tahmini yanlış işarette olma eğilimindedir ve büyük varyansa sahiptir. Bu tür anormal etkileri ortadan kaldırmak için, bu sorunu azaltan veya tamamen ortadan kaldıran bazı yararlı teknikler vardır. Önyargılı tahmin yöntemleri bu yöntemlerin örnekleridir. Aslında, bu çalışma, bu yöntemlerin, özellikle de bootstrap yönteminin yararlılığını tanımlamayı ve gözlemlemeyi amaçlamaktadır. Ayrıca, bu çalışma aynı zamanda güven aralığı seviyelerini belirlerken en iyi k seçim yöntemini belirlemeyi amaçlamaktadır. Bootstrap yöntemini kullanmanın avantajı, bu yöntemin araştırmacının yeniden örnekleme yoluyla mevcut olandan yeni bir veri kümesi oluşturmasına izin vermesidir.

Öte yandan, ridge tahmin edicilerinin ortalama kare hatalarını en aza indirmeyi amaçlayan k seçim metotları, bootstrap yöntemine kıyasla dezavantajlara sahiptir. Bu çalışmanın amacı, ridge tahmin edici ve bootstrap aracılığı ile çoklu doğrusal regresyon ile ilgili mevcut literatürü tanımlamak, k seçim yöntemini ve güven aralığı seviyeleri üzerindeki etkilerini göz önünde bulundurarak kapsamlı bootstrap ve simülasyonlar sağlamaktır.

Anahtar Kelimeler: Bootstrap, Ridge tahmin edici, Güven aralıkları, Çoklu bağlantı, Hata kareler ortalaması

EXPANDED SUMMARY

It is well known that working with data which are not familiar can be hard. This is simply because of the nature of the statistics. In this thesis, we try to give alternative and precise method to work with data sets which do not satisfy the regression model assumptions.

To begin with, Efron (1979), proposed a model which is called bootstrap method. Simply, this method is useful when the distribution of the data is unknown. Furthermore, the basis of the bootstrap method, which allows us to make inferences without any knowledge or assumptions about the distribution of the population, is to resampling to produce very large data sets from the current data set (Efron, 1979). As possible, the bootstrap method can be used in estimating the confidence intervals, hypothesis testing, and regression analysis. Furthermore, bootstrap method has properties that are limited in assumptions, easy to understand and use. This enables us to obtain reliable results when statistical methods and assumptions are inadequate.

Secondly, the Monte Carlo method is a method that consists of different types of methods but that perform the same operations.

These methods are quite important for many researchers since these techniques enable them to achieve true results for their studies. This study gives detailed explanation to above mentioned methods via considering real and computer based generated data set.

The main findings are as follows: According to the simulated data, it has been observed that k selection methods are dramatically affected from signal to noise ratio (SNR) and collinearity problem. Confidence intervals generated via k obtained by bootstrap method are obtained proposed by Delaney and Chatterjee (1986), and it has been observed that as SNR increases the length of the confidence intervals decrease. When the study is conducted via principal components regression (PCR), it is observed that PCR gives better results than least squares in terms of bootstrap confidence interval when multicollinearity exists.

As a result, the bootstrap k selection method proposed by Delaney and Chatterjee (1986) is a method that can compete with the existing ones in the literature in terms of providing smaller mean square error with increasing computer use popularity. In the case of low degree of collinearity and norm of true parameter vector with high SNR; the ridge confidence intervals are narrower than least square confidence interval which gives more reliable results. Furthermore, bootstrap k selection method results in narrower confident interval than some other k selection methods considered.

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CONTENTS **PAGE**

LIST OF TABLES **PAGE**

LIST OF FIGURES **PAGE**

LIST OF ABBREVIATIONS AND SYMBOLS

LS : Least Squares

PCR : Principal Components Regression

PCA : Principal Components Analysis

MSE : Mean Square Error

EMSE : Estimated Mean Square Error

RR : Reduction Rate

SNR : Signal-to-noise Ratio

VIF : Variance Inflation Factor

TV : Tolerance Value

Symbols

X : $n \times p$ matrix consisting of the explanatory variables

y : $n \times 1$ vector of response

β : $p \times 1$ unknown vector containing the regression coefficients

ε : $n \times 1$ error vector

$\hat{\beta}$: Least squares estimator

R^2 : Coefficient of determination

$\hat{\beta}_r$: Ridge estimator

$\hat{\beta}_{PC}$: Principal components estimator

k : Biasing parameter

ρ : The degree of collinearity

1. INTRODUCTION

The bootstrap method found by Bradley Efron in 1979 to find the standard error of any statistics has become a very important method used for statistical purposes. Methods which are similar to the bootstrap method can be listed as the Monte Carlo simulation introduced by Barnard in 1963, the method used by Hope in 1968 and the method used by Marriot in 1979. Efron revised the bootstrap method twice in 1981 and has been an important method that has survived to the present day. The basis of the bootstrap method, which allows us to make inferences without any knowledge or assumptions about the distribution of the population, is to resampling to produce very large data sets from the current data set (Efron, 1979). The bootstrap method which can be used in estimating the confidence intervals, hypothesis testing, and in regression analysis, has properties that are limited in assumptions, is easy to understand and use. The bootstrap methods are used to obtain reliable results when statistical methods and assumptions are inadequate. The bootstrap method helps us to create new data sets in different sizes by replacing the observations in the data set, which implies that the bootstrap method is a useful method.

On the other hand, Monte Carlo method, which is used to produce new data set, is similar to the bootstrap method. The Monte Carlo method is a mathematical technique applied to the computer, pointing to statistical sampling methods. Monte Carlo simulations can be used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. The main feature of this method and its distinguishing feature from the bootstrap method is that random numbers are used in the method.

We aim to use these two approaches in models of regression. Regression analysis is a statistical technique which measures the relationship between two or more variables. In this review, the multiple linear regression model will be discussed, describing the relationship between two or more explanatory variables that influence a variable by a linear method and are used to determine their results. In this regression model, which will be discussed, two situations will be considered. The first is that the explanatory variables are independent from each other. Under the assumption that the explanatory variables are independent from each other, the regression model is estimated by the least squares (LS) method. However, examples that infringe the assumption of independence between explanatory variables can be encountered. The linear dependence that exists between the explanatory variables is called multicollinearity. In the case of multicollinearity, the variances of LS estimates are large and the LS estimates can be quite far from their real values. Standard errors of estimators can be reduced by allowing a degree of bias. Therefore, when multicollinearity occurs, methods alternative to the LS method are used. Other alternative methods that will be used in this study are the ridge and PCR estimators.

Our purpose in this study is to use bootstrap methods in multiple linear regression model, where multicollinearity exists and to make inferences by comparing the different estimation methods or different selection methods for biasing parameter via a Monte Carlo simulation study and real data.

There will be very productive information that this study will add to the literature. As mentioned, bootstrap method will make an important contribution to comparative studies in the literature which takes account of Monte Carlo simulation. This study will be an important starting point for students, researchers and statisticians who will work on bootstrap and Monte Carlo in the future and will also provide an important ground for further study on these topics in the future.

The structure of this study is, introduction is given in Section 1, multiple linear regression that we consider is given in Section 2, ridge and PCR estimators are considered in Section 3 and Section 4, bootstrap method is considered in Section 5, Monte Carlo simulation method is considered in Section 6, a real data analysis is done in Section 7 and the study is finalized with conclusion presented by Section 8.

2. MULTIPLE LINEAR REGRESSION

The purpose of regression analysis is to create the best model that can predict the response variable from the explanatory variables or to determine which explanatory variables are affected by the response variable. Regression analysis has many uses in different disciplines. Including engineering, physical and chemical sciences, economics, psychology, and social sciences. Multiple linear regression is a mathematical method which uses more than one explanatory variable to predicts response variable. Multiple linear regression aims to observe the linear relationship between the explanatory variables and the response variable. Each value of the explanatory variable, which is denoted as x is related with a value of the response variable y.

The multiple linear regression model is as follows:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_q x_{iq} + \varepsilon_i \qquad \mathrm{i} = 1,2,\ldots,\mathrm{n} \quad (2.1)$$

where x_1, $x_2, \ldots,$ x_q are the explanatory variables, β_0, $\beta_1, \ldots,$ β_q are the regression coefficients and ε is the error term. The error terms have mean zero and unknown variance σ^2. In addition, the error terms are uncorrelated.

If we write the multiple linear regression in Eq. (1) the matrix form, we get

$$y = X\beta + \varepsilon \qquad (2.2)$$

where y is the $n \times 1$ vector of response, X is the $n \times (p = q + 1)$ matrix consisting of the explanatory variables, β is the $p \times 1$ vector containing the regression coefficients and ε is the $n \times 1$ vector of error terms distributed as $\mathrm{N}(0, \sigma^2 \mathrm{I})$ where σ^2 is the variance of each error terms and expected to be unidentified.

2.1. Assumptions of the Regression Model:

i. The regression model is linear in the coefficients and the error term

ii. The expected value of error terms is zero: $E(\varepsilon_i) = 0$.

iii. The variance of error terms is constant: $Var(\varepsilon_i) = \sigma^2$.

iv. Error terms are uncorrelated with each other: $Cov(\varepsilon_i, \varepsilon_j) = 0$.

v. There is no correlation between error terms and response variable: $Cov(\varepsilon, y) = 0$.

vi. The explanatory variables are assumed to be deterministic.

vii. For the hypothesis testing and confidence interval, the error terms can assumed to follow normal distribution.

The situation where one or more of these assumptions are not satisfy is called assumption distortions. Assumption distortions: "relationship" in the case of a relationship between error terms, "changing variance" if the variances of error terms are unequal, "non-normal" if errors are not normal, and "multicollinearity" if explanatory variables are dependent of each other.

β vector is unknown, the primary purpose of the regression analysis is to estimate β parameters by deducting the observed data.

2.1.1. Least - Squares Estimation of the Regression Coefficients

One of the most commonly used method for estimating the parameters in the model is the LS estimation method which allows to estimate the parameters by minimizing the sum of the squared error. The LS estimate of β is obtained by minimizing the objective function

$$S(\beta) = \sum_{i=1}^{n} {\varepsilon_i}^2 = \varepsilon'\varepsilon = (y - X\beta)'(y - X\beta) \tag{2.3}$$

$$= y'y - \beta'X'y - y'X\beta + 'X'X\beta$$

$$= y'y - 2\beta'X'y + \beta'X'X\beta$$

Differentiating Eq. (3) with respect to β, we get

$$\left.\frac{\partial S(\beta)}{\partial \beta}\right|_{\beta=\hat{\beta}} = -2X'y + 2X'X\hat{\beta} = 0$$

which simplifies to

$$X'X\hat{\beta} = X'y\,. \tag{2.4}$$

Hence, the LS estimator of β is obtained as

$$\hat{\beta} = (X'X)^{-1}X'y \tag{2.5}$$

given that the inverse of the $X'X$ matrix.

Estimation values corresponding to y obtained with this method are obtained as

$$\hat{y} = X\hat{\beta}\,. \tag{2.6}$$

It is also possible to write the $\hat{y}$ fitted values in terms of *y:*

$$\hat{y} = X(X'X)^{-1}X'y = Hy$$

where H is the $n \times n$ hat matrix.

The difference between the observed value y and the corresponding fitted value $\hat{y}$ is the residual ($e = y - \hat{y}$).

The properties of the LS Estimator are as follows:

- $\hat{\beta}$ is a linear combination of the observations of y:

$$\begin{aligned}\hat{\beta} &= (X'X)^{-1}X'y \\ &= \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{p1} & \cdots & C_{pn} \end{bmatrix} \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}\end{aligned}$$

- It is an unbiased estimator:

$$\begin{aligned}E(\hat{\beta}) &= E[(X'X)^{-1}X'y] = (X'X)^{-1}X'E(y) \\ &= (X'X)^{-1}X'E(X\beta + \varepsilon) = (X'X)^{-1}X'X\beta + E(\varepsilon) \\ &= (X'X)^{-1}X'X\beta = \beta \end{aligned} \tag{2.7}$$

- It has a minimum variance:

  $\hat{\beta}$'s covariance matrix is

$$\begin{aligned}Cov(\hat{\beta}) &= Cov[(X'X)^{-1}X'y] = (X'X)^{-1}X'Cov(y)X(X'X)^{-1} \\ &= \sigma^2(X'X)^{-1}\end{aligned} \tag{2.8}$$

Main diagonal elements of Eq. (8) are the variances of $\hat{\beta}_j's, j = 0,1,2,\ldots,p$ and off-diagonal elements give the covariances.

If the estimator is unbiased, but does not have the least variance, this does not mean the best. If the estimator has the least variance, this is the best estimate.

- $\hat{\beta}_j$ is the best linear unbiased (B.L.U.) estimator of β, refer to the Gauss-Markov theorem. The B.L.U.E properties implies that each $\hat{\beta}_j$, $j = 1,2,\ldots,p$ has the smallest variance among the class of all linear unbiased estimators of β_j.

2.2. Confidence Intervals on the Regression Coefficients

In regression, confidence intervals play a significant role. The confidence interval, is a kind of interval estimate in statistical science for a population parameter, which is an inferential statistical solution tool. There is a range of upper and lower limits that will cover the value of the parameter, and the value of the parameter is calculated from a single population. In this way, intervals of confidence indicate how accurate a prediction is.

2.2.1. Standard Normal Approximation Confidence Intervals

The confidence interval provides a measure of the reliability of our estimate of a statistic, whether it is the average or some other statistic that we calculate from our data.

$100(1-\alpha)\%$ approximate confidence interval for β_j is be given by

$$\hat{\beta}_j - z_{1-\alpha/2}\sqrt{\hat{\sigma}^2 C_{jj}} \ \le\ \beta_j \le\ \hat{\beta}_j + z_{1-\alpha/2}\sqrt{\hat{\sigma}^2 C_{jj}}$$

where C_{jj} is the jth diagonal element of the $(X'X)^{-1}$ matrix and $\hat{\sigma}^2$ is the estimate of the error variance. $z_{1-\alpha/2}$ represents the $1-\alpha/2$ percentile of the standard normal distribution.

$\hat{\sigma}^2$ estimation consists of the following steps:

As with the regression coefficient, we can derive $\hat{\sigma}^2$ from the residual sum of squares:

$$\begin{aligned} SS_{res} &= \left(y - X\hat{\beta}\right)'\left(y - X\hat{\beta}\right). \\ &= y'[I - X(X'X)^{-1}X']y \end{aligned}$$

The matrix $[I - X(X'X)^{-1}X']$ is symmetric idempotent. As a result, $\frac{SS_{res}}{\sigma^2}$ follows a χ^2distribution. The degrees of freedom comes from the rank of $[I - X(X'X)^{-1}X']$, which is the trace. Consequently, the degrees of freedom of the sum of residual squares is $n - p$, and the mean of residual squares is as follows:

$$MS_{res} = \frac{SS_{res}}{n-p} \ .$$

The expected value of MS_{res} is σ^2,

$$\begin{aligned} E(SS_{res}) &= E(y'[I - X(X'X)^{-1}X']y) \\ &= tr([I - X(X'X)^{-1}X']\sigma^2 I) + E(y)'[I - X(X'X)^{-1}X']E(y) \\ &= (n-p)\sigma^2 \end{aligned}$$

so, an unbiased estimator of σ^2 is given by

$$\hat{\sigma}^2 = \frac{MS_{res}}{n-p}.$$

2.2.2. t Approach Confidence Intervals

The only difference between the standard normal approximation confidence interval and the t approximation confidence interval is that the t distribution is used instead of the standard normal distribution. As a result, the t approximation confidence interval is as follows:

$$\hat{\beta}_{\mathrm{j}} - t_{1-\alpha/2,\mathrm{n-p}} \sqrt{\hat{\sigma}^2 \mathrm{C_{jj}}} \ \leq \ \beta_{\mathrm{j}} \leq \ \hat{\beta}_{\mathrm{j}} + t_{1-\alpha/2,\mathrm{n-p}} \sqrt{\hat{\sigma}^2 \mathrm{C_{jj}}}$$

where $t_{1-\alpha/2,\mathrm{n-p}}$ represents $1 - \alpha/2$ percentile of the t distribution with n-p degrees of freedom.

2.3. Multicollinearity

While explaining the multiple linear regression model, we mentioned many assumptions. One of these assumptions was the linear independency between explanatory variables with each other. In some cases, explanatory variables are significantly linearly related. In regression analysis, the presence of strong correlations between explanatory variables is called collinearity or multicollinearity and indicates the undesired situation (Orhunbilge, 2002). In these cases, inferences based on the regression model can be inaccurate. For example, if the regression coefficients are estimated incorrectly, the standard errors of the regression coefficients are exaggerated. The growth of the standard error will lead to negligible statistically significant regression coefficients resulting in a false result. In multicollinearity, variance and covariance of regression coefficients strengthen. As this occurs, there is a problem with multicollinearity. Multicollinearity can have many reasons (Montgomery et al., 2012). Some of these reasons are as follows:

1. Sampling techniques used: The problem of multicollinearity arises when the analyst samples only a subset of the independent set of variables, despite the fact that it is not.
2. Constraints in the model or population: Multicollinearity arising from physical constraints on the model or population are formed by preserving the actual relationship that exists in the population.
3. Determination of model: If the range of change of the explanatory variables are small, the problem of multicollinearity arises by adding a polynomial term to the regression model.
4. Overloaded model: Multicollinearity occurs when the number of variables is more than the number of observations. These types of models are more commonly encountered in areas such as behavioural sciences and medicine. In the case that the model is overloaded, some of the explanatory variables must be removed from the model.

If these points exist, that is, if the explanatory variables are related, the model may need to be omitted. In this case, the question in the minds will arise. What factors will be ignored?

Removing an incorrect attribute from the model would result in a mistake in design. On the other hand, we cannot use simple rules to incorporate and rule out claims in the model (Myers, 1990: Gujarati, 1995). In order to identify multicollinearity, there are some useful techniques to help us to identify the problem. These techniques are explained in the following section.

2.3.1. Multicollinearity Identification Techniques:

Multicollinearity identification techniques can be given as follows:

1.Examination of the Correlation Matrix

If the absolute value of the correlation coefficient between the explanatory variables is close to 1, while examining the $\mathrm{X}'\mathrm{X}$ matrix, it is stated that there is a multiple linear correlation between such explanatory variables. The determinant of the correlation matrix can be used to determine multicollinearity. It is stated that when the correlation matrix determinant is 1, there is no linear relationship between the explanatory variables, but when the correlation matrix determinant is 0, there is a linear correlation between the explanatory variables.

2.Variance Inflation Factors

The VIF tests the combined influence of the dependencies of the explanatory variables on the variance of that term on any term in the model. VIF values for explanatory variables are obtained by diagonal components of the $(X'X)^{-1}$. Generally, when any of the VIF value is above 10, it is said that there are multiple connections between explanatory variables.

VIF is also computed as follows:

$$\mathrm{VIF_j} = (1 - \mathrm{R_j}^2)^{-1} \tag{2.9}$$

where $\mathrm{R_j}^2$ is the coefficient of determination obtained when $\mathrm{x_j}$ is regressed on the remaining p−1 explanatory variables. If the explanatory variables do not have a linear relationship, then $\mathrm{R_j}^2$ is small.

3.Eigensystem Analysis of the Correlation Matrix

Eigenvalues $(\lambda_1, \lambda_2, \dots, \lambda_\mathrm{p})$ of the correlation matrix, can be used to measure the extent of multicollinearity in the data. If one or more eigenvalues of the correlation matrix is close to zero or equal to zero, it means that the model has multicollinearity problem.

4 Examination of the condition number of the $\mathrm{X'X}$ matrix.

Condition number is evaluated as follows:

$$\kappa = \begin{cases} \dfrac{\lambda_{\mathrm{max}}}{\lambda_{\mathrm{min}}} > 100, \text{there is serious problem with multicollinearity.} \\ \dfrac{\lambda_{\mathrm{max}}}{\lambda_{\mathrm{min}}} > 1000, \text{there is severe problem with multicollinearity.} \end{cases}$$

2.3.2. Multicollinearity Removal Techniques:

Many different techniques have been proposed for removing multicollinearity. Some of them are discussed as follows:

1. Additional data collection has been proposed as the best way to counter mu lticollinearity.
2. One or more explanatory variables can be removed from the model. However, this technique has its disadvantage. If we omit the wrong variable from the model, this may lead to misidentification of the model.

3. A further relation to two related variables can be taken as a single variable. In other words, related variables can be combined.
4. Less sensitive methods can be used instead of LS method. For example;

If there is multicollinearity in the linear regression model, then the LS estimator is again the best unbiased estimator. But its variance is enormous. So, the LS estimator will be unstable. To estimate regression coefficients without removing variables in the model, methods less sensitive to linearity than the LS method are often used. It has been concluded that using biased estimators to reduce variances is an appropriate approach for solving this problem. For this reason, the ridge estimator proposed by Hoerl and Kennard (1990) was used as the most common solution method for many years, where estimation variances were reduced by adding a small positive number to the diagonal elements of the correlation matrix. The ridge estimator is a complex function of the biasing parameter k, which is $k > 0$. For this reason, the various methods used to select the biasing parameter are often compared with complex equations. Considering the lack of a common opinion for selecting the biasing parameter, new explanatory variables have been proposed to address the problem of multiple internal relationships. For this reason, one of the alternative methods, PCR, in which vertical transformations are used instead of the original variables, can also be applied.

3. RIDGE REGRESSION

In the case of multicollinearity, the LS estimates are neutral, but as the variances inflate, the estimates can move away from their real values. Standardized regression coefficients calculated by multiple linear correlated data lose their stability (Darlington, 1978). The ridge regression technique helps to reduce the variance by adding a small biasing constant (k) to these estimates (Price, 1977). With the ridge regression technique, on the one hand, the variance of the estimates is reduced, and biased estimates are obtained in the ratio of this coefficient (k).

Diagonal of the correlation matrix in ridge regression analysis can be obtained by adding a small bias constant to the values, bias standardized regression coefficients are calculated. Many suggestions are provided to obtain ridge estimators of ridge regression coefficients. The ridge estimator is

$$\hat{\beta}_{R} = (X'X + kI)^{-1}X'y \qquad k > 0\,. \tag{3.1}$$

To simplify the equation, let's write it as

$$\hat{\beta}_{R} = (X'X + kI)^{-1}X'y = (X'X + kI)^{-1}(X'X)\hat{\beta} = Z\hat{\beta} \tag{3.2}$$

where $\hat{\beta}$ is the LS estimator. As we can see from here, the ridge estimator is a linear transformation of the LS estimator.

Hoerl and Kennard (1970), obtained the ridge estimator by following the path below.

Sum of error squares of the $\hat{\beta}_{R}$ estimator is,

$$\begin{aligned} SS_{Res,\hat{\beta}_{R}} &= \left(y - X\hat{\beta}_{R}\right)'\left(y - X\hat{\beta}_{R}\right) \qquad (3.3) \\ &= \left(y - X\hat{\beta}\right)'\left(y - X\hat{\beta}\right) + (\hat{\beta}_{R} - \hat{\beta})'X'X\left(\hat{\beta}_{R} - \hat{\beta}\right) \end{aligned}$$

where the term $(\hat{\beta}_R - \hat{\beta})'X'X(\hat{\beta}_R - \hat{\beta})$ is the square of its bias resulting from the use of the $\hat{\beta}_R$ estimator instead of the LS estimator. While the $X'X$ matrix is poorly conditional, the estimate value of $\hat{\beta}$ diverges from the actual value of parameter β. Therefore, the square of the distance of the $\hat{\beta}_R{}'\hat{\beta}_R$ estimator is required to be made minimum.

If the Lagranian method is evaluated as

$$\hat{\beta}'\hat{\beta} + \frac{1}{k}\left[\left(\hat{\beta} - \beta\right)'X'X\left(\hat{\beta} - \beta\right) - \psi\right], \tag{3.4}$$

we get the ridge estimator as given by Eq. (10)

The expected value of bias magnitude of this estimate is obtained with the following equation:

$$\begin{aligned} E\left(\hat{\beta}_R - \beta\right) &= (X'X + kI)^{-1}X'X\beta - \beta \\ &= [(I + k(X'X)^{-1})^{-1} - I]\beta \\ &= [I - k[I + (X'X)^{-1}k]^{-1}(X'X)^{-1} - I]\beta \\ &= [I - k[X'X + kI]^{-1} - I]\beta \\ &= -k(X'X + kI)^{-1}\beta \end{aligned} \tag{3.5}$$

and covariance matrix is

$$\begin{aligned} \mathrm{Var}\left(\hat{\beta}_R\right) &= \mathrm{Var}[(X'X + kI)^{-1}X'y] \\ &= (X'X + kI)^{-1}X'\mathrm{Var}(y)X(X'X + kI)^{-1} \\ &= \sigma^2(X'X + kI)^{-1}X'X(X'X + kI)^{-1} \\ &= \sigma^2[(X'X + kI)^{-1} - k(X'X + kI)^{-2}]. \end{aligned} \tag{3.6}$$

The MSE criterion is an important criterion for comparing the performance of ridge estimators. For a given value of k,

$$\begin{aligned}\mathrm{MSE}(\hat{\beta}_{\mathrm{R}}) &= \mathrm{E}\left[(\hat{\beta}_{\mathrm{R}}-\beta)(\hat{\beta}_{\mathrm{R}}-\beta)'\right] \qquad (3.7)\\ &= \mathrm{Var}(\hat{\beta}_{R}) + (\mathrm{E}(\hat{\beta}_{\mathrm{R}})-\beta)\,(\mathrm{E}(\hat{\beta}_{\mathrm{R}})-\beta)' \\ &= \sigma^2[(X'X+kI)^{-1} - k(X'X+kI)^{-2}] + k^2\beta(X'X+kI)^{-2}\beta'\end{aligned}$$

Hoerl and Kennard (1970) revealed the k parameter, which minimizes the $\mathrm{MSE}(\hat{\beta}_{\mathrm{R}})$.

To examine the optimum k constant, it profits from the graphs measured between bilateral normalized regression coefficients and k called skewed regression graph (Ridge Trace) (Hoerl and Kennard, 1970). Those graphs display skewed coefficients of regression as a function of k. The optimal k value is selected from the area where linear, skewed regression coefficients become stationary. In general, standardized coefficients of regression shift first with small k values very abnormally, and then become stationary. For this field, where the regression coefficients are stationary, the optimum k value is chosen as the smallest possible k value.

3.1. The Choice of Biasing Parameter

In this section, there are several k, biasing parameters ideas for obtaining estimators of ridge regression coefficients.

The orthogonal version of model (2) is

$$y = X^{*}\alpha + \varepsilon \qquad (3.8)$$

where $X^{*} = XS$ and $\alpha = S'\beta$. $X^{*\prime}X^{*} = S'X'XS = \Lambda = \mathrm{diag}(\lambda_1, \lambda_{2,\ldots,}\lambda_{\mathrm{p}})$ is a $\mathrm{p} \times \mathrm{p}$ diagonal matrix with the eigenvalues of $X'X$. S is a $\mathrm{p} \times \mathrm{p}$ orthogonal matrix, the columns of which are the eigenvectors aligned with $(\lambda_1, \lambda_{2,\dots,}\lambda_p)$. X^* is a new set of vertical explanatory variables and is called principal components. Assume that the explanatory variables are arranged in order of decreasing eigenvalues, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$.

The LS of $\widehat{\alpha}$ is as follows:

$$\widehat{\alpha} = (X^{*\prime}X^*)^{-1}X^{*\prime}\mathrm{y} = \Lambda^{-1}X^{*\prime}\mathrm{y} \tag{3.9}$$

and the covariance matrix of $\widehat{\alpha}$ is

$$\mathrm{Var}(\widehat{\alpha}) = \sigma^2(X^{*\prime}X^*)^{-1} = \sigma^2\Lambda^{-1} \tag{3.10}$$

In addition, the covariance matrix of the β is

$$\mathrm{Var}(\widehat{\beta}) = \mathrm{Var}(\mathrm{S}\widehat{\alpha}) = \mathrm{S}\Lambda^{-1}\mathrm{S}'\sigma^2 \tag{3.11}$$

The ridge estimator for α is

$$\widehat{\alpha}(\mathrm{k}) = (\mathrm{X}^{*\prime}\,\mathrm{X}^* + \mathrm{kI})^{-1}X^{*\prime}\,\mathrm{y}\,. \tag{3.12}$$

Hoerl and Kennard (1970) also gave the generalized ridge estimator which is defined as

$$\widehat{\alpha}(\mathrm{K}) = (\mathrm{X}^{*\prime}\mathrm{X}^* + \mathrm{K})^{-1}\mathrm{X}^{*\prime}\mathrm{y} \tag{3.13}$$

where $\mathrm{K} = \mathrm{diag}(\mathrm{k}_1, \mathrm{k}_2, \dots, \mathrm{k}_p)$ is the matrix of biasing parameters $\mathrm{k}_i > 0$.

The selection of the biasing parameter (k) affects the performance of the ridge estimator. In determining k here, we must choose k in such a way that it provides small, less biased, and stable regression coefficients. It is quite important that we choose the right k so we can get more accurate results. It is aimed to find a stable estimator that gives values smaller than the MSE value of the regression coefficients obtained by the LS method. It follows from Hoerl and Kennard (1970) that the values of $\mathrm{k_i}$ which minimize

$$\mathrm{MSE}\big(\widehat{\alpha}(\mathrm{K})\big) = \sigma^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k_i)^2} + \sum_{i=1}^{p} \frac{{k_i}^2 {\alpha_i}^2}{(\lambda_i + k_i)^2} \tag{3.14}$$

are obtained as

$$k_i = \frac{\sigma^2}{{\alpha_i}^2} \ \text{ for } i = 1, \dots, p\,.$$

The ideal $\mathrm{k_i}$ value relies entirely on the unknown σ^2 and α_i, and they must be estimated from the information observed. Hoerl and Kennard (1970) proposed that their respective unbiased estimators should substitute σ^2 and α_i which leads to

$$\widehat{k}_i \ = \frac{\widehat{\sigma}^2}{{\widehat{\alpha}_i}^2} \tag{3.15}$$

where $\widehat{\sigma}^2$ and $\widehat{\alpha}^2$ are unbiased estimators obtained by LS method which are respectively as $\hat{\sigma}^2 = \frac{(y-X\hat{\beta})'(y-X\hat{\beta})}{n-p}$ and ${\widehat{\alpha}_i}^2$is the ith element of the LS estimates of α.

Now we review some estimation methods of the biasing parameter which are as follows:

- Hoerl and Kennard (1970) Estimate:

Hoerl and Kennard (1970) suggested a choice of k given by $k = \hat{\sigma}^2 / \hat{\alpha}^2{}_{\max}$ where $\hat{\alpha}^2{}_{max}$ is the largest element of the LS estimate.

- Hoerl, Kennard and Baldwin (1975) Estimate:

Hoerl et al. (1975), by taking the harmonic mean of $\hat{k}_i$ in (24), proposed a distinct estimator of k as $k = \frac{p\hat{\sigma}^2}{\hat{\alpha}'\,\hat{\alpha}} = \frac{p\hat{\sigma}^2}{\hat{\beta}'\,\hat{\beta}}$

- Hocking, Speed and Lynn (1976) Estimate:

Hocking et al. (1976) suggest the following estimator for k

$$k = \frac{\hat{\sigma}^2 \sum_{i=1}^{p} (\lambda_i \hat{\alpha}_i)^2}{(\sum_{i=1}^{p} \lambda_i {\hat{\alpha}_i}^2)^2}$$

- Hoerl and Kennard Estimate (1976):

Hoerl and Kennard (1976) suggested a choice of k given by

$$\begin{gathered} k_0 = p\hat{\sigma}^2/\hat{\alpha}'\hat{\alpha} \\ k_1 = p\hat{\sigma}^2/\hat{\alpha}'(k_0)\hat{\alpha}(k_0) \\ k_2 = p\hat{\sigma}^2/\hat{\alpha}'(k_1)\hat{\alpha}(k_1) \\ . \\ . \\ . \\ k_i = p\hat{\sigma}^2/\hat{\alpha}'(k_{i-1})\hat{\alpha}(k_{i-1}) \end{gathered}$$

This iterated algorithm has been implemented by applying a 10^{-4} convergence criterion to successive k values and defaulting to the LS estimator if convergence is not obtained in 30 iterations (Hemmerle, 1975). If the k_i's are assumed to be equal to k, then the MSE function will be minimized when $\sum \lambda_i(k\gamma_i{}^2 - \widehat{\sigma^2})/(\lambda_i + k)^3$.

- Lawless and Wang (1976) Estimate:

  Lawless and Wang (1976) proposed that k be chosen as

$$k = {}^{p\hat{\sigma}^2}\Big/_{\sum_{i=1}^{p} \lambda_i \hat{\alpha}_i{}^2}$$

- Nomura (1988) Estimate:

  Nomura (1988) suggested that k be selected by

$$k = p\hat{\sigma}^2 / \left[\sum_{1}^{p} \hat{\alpha}_i{}^2 / \left(1 + \sqrt{1 + \lambda_i \hat{\alpha}_i{}^2 / \hat{\sigma}^2} \right) \right]$$

- Kibria (2003) Estimate:

  Kibria (2003) suggested some new estimators based on generalized ridge regression strategy, using the geometric mean of $\hat{k}_i$, which generates the following estimator:

$$k = \frac{\hat{\sigma}^2}{\left(\prod_1^p \hat{\alpha}_i{}^2\right)^{1/p}} \qquad i = 1,2, \dots, p$$

and using the median of $\hat{k}_i$, which produces the following estimator for $p \geq 3$

$$k = \text{median} \left\{ \frac{\hat{\sigma}^2}{\hat{\alpha}_i{}^2} \right\} \quad i = 1,2, \dots, p$$

- Khalaf and Shukur (2005) Estimate:

Khalaf and Shukur (2005) suggested a choice of k given by

$$k = \frac{\lambda_{\max}\hat{\sigma}^2}{\{\lambda_{\max}\hat{\alpha}^2{}_{\max} + (n - p)\hat{\sigma}^2\}}$$

- Alkhamisi, Khalaf and Shukur (2006) Estimate:

Alkhamisi et al. (2006) suggested a choice of biasing parameter given by

$$k = \max\left(\frac{\lambda_i\hat{\sigma}^2}{\lambda_i\hat{\alpha}_i{}^2 + (n - p)\hat{\sigma}^2}\right) \qquad i = 1,2,\ldots,p$$

- Alkhamisi and Shukur (2007) Estimate:

Alkhamisi and Shukur (2007) suggested the following estimators of biasing parameter

$$k = \max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i{}^2} + \frac{1}{\lambda_{\max}}\right)$$

$$k = \frac{1}{p}\sum_{1}^{p}\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i{}^2} + \frac{1}{\lambda_{\max}}\right)$$

$$k = \text{median}\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i{}^2} + \frac{1}{\lambda_{\max}}\right)$$

$$k = \frac{p\hat{\sigma}^2}{\sum_1^p \lambda_i\hat{\alpha}_i{}^2} + \frac{1}{\lambda_{\max}}$$

- Muniz and Kibria(2009) Estimate:

Muniz and Kibria (2009) suggests the following estimator for k

$$k = \left(\prod_{1}^{p}\sqrt{\hat{\alpha}_i{}^2/\hat{\sigma}^2}\right)^{1/p}$$

$$k = \left(\prod_{1}^{p}\sqrt{\hat{\sigma}^2/\hat{\alpha}_i{}^2}\right)^{1/p}$$

$$k = \text{median}\{\sqrt{\hat{\alpha}_i{}^2/\hat{\sigma}^2}\}$$

- Dorugade and Kashid (2010) Estimate:

Dorugade and Kashid (2010) proposed a fresh technique to determine the biasing parameter

$$k = \max\left(0, \frac{p\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}} - \frac{1}{n\left(\mathrm{VIF}_j\right)_{\max}}\right)$$

where VIF_j , j=1, 2,…,p is variance inflation factor of the jth explanatory variable.

- Allen’s (1974) PRESS:

Allen (1974) proposed a suitable selection of k by using the prediction error sum of squares error (PRESS). The PRESS criterion is given by

$$\mathrm{PRESS(k)} = \frac{1}{n}\sum e_i(k)^2/(1 - h_{ii}(k))^2$$

where $e_i(k)$ and $h_{ii}(k)$ refers to the ith ridge residual and the ith diagonal element of $H(k) = X(X'X + kI)^{-1}X'$.

The minimum of PRESS(k) provides Allen’s ordinary PRESS choice of k .

This method can be summarized as dropping one observation at a time while estimating the model and also predicting the left-out observation for each choice of k.

PRESS statistic is one of the methods which aims to select suitable k. The main idea here is that; the model can predict the left-out observation for each k via eliminating one observation per iteration. A mean square error of prediction (MSEP) needs to be calculated for each k, since the lowest MSEP is the optimal value for k.

- Generalized Cross-Validation of Wahba et al. (1975)

Wahba et al. (1975) advocated a method, called generalized cross-validation (GVC), which has some advantages over Allen's PRESS. This is also applicable when the number of parameters is not smaller and it does not require an estimate of the error variance. The GCV criterion is given by

$$\mathrm{GCV} = 1/\mathrm{n} \sum \mathrm{e_i(k)}^2 \,/ \sum (1 - \mathrm{h_{ii}(k)})^2$$

- Mallows (1973) Estimate:

Mallows (1973) proposed to find an extended k value in the state of ridge regression to minimize Cp statistics which is defined as

$$CP_k = \frac{\mathrm{SS_{Res}(k)}}{\hat{\sigma}^2} - \mathrm{n} + 2\mathrm{trH((k))}$$

- The Bootstrap Choice of the Biasing Parameter:

The following section will identify the bootstrap method which has been proposed by Delaney and Chatterjee (1986) in order to identify biasing parameters which can be used for ridge estimator. The main idea of biasing parameter, can be discussed as follows. This parameter is a combination of results obtained from cross-validation method and bootstrap. The parameter mentioned below gives the optimal formation which considers parameters based on MSEP. Each bootstrap sample gives computed parameter vector for each k. While conduction this estimation, there are some unchosen parameter according to the nature of the technique. However, when considering unchosen parameters, one needs to take the average of whole samples in order to achieve more specific values. This enables us to conduct simulations with more reliable determinants. The unchosen parameter can be described as the remainder observation from randomly selected observations. Therefore, in order to predict these unchosen parameters MSEP can be used. However, this can be a long process in order to get optimal MSEP for each value of k to be used. Hence, it requires repeated samples for each bootstrap and via taking average of these repeated samples will give the optimal value.

$$\mathrm{MSEP}(\mathrm{k_w})_\mathrm{j} = (\,\hat{\mathrm{Y}}_{\mathrm{L_j}}(\mathrm{k_w}) - \mathrm{Y_L})'(\,\hat{\mathrm{Y}}_{\mathrm{L_j}}(\mathrm{k_w}) - \mathrm{Y_L})/\,\mathrm{L_j} \qquad \mathrm{w} = 1,2,\dots,\mathrm{W}$$

where W values of k are used for each bootstrap sample and for the jth bootstrap sample, $\mathrm{k_w}$ equals k in MSEP which displays the number of unchosen observations,$\hat{\mathrm{Y}}_{\mathrm{L_j}}(\mathrm{k_w})$ is the prediction vector of unchosen observations. $\mathrm{L_j}$ indicates the number of unchosen observations. MSEP for each value of $\mathrm{k_w}$ used is obtained as follows:

$$\mathrm{MSEP}(\mathrm{k_w}) = {\sum_{\mathrm{j}=1}^{\mathrm{B}} \mathrm{MSEP}(\mathrm{k_w})_\mathrm{j}\,(\mathrm{L_j}\,)} \Big/ {\sum_{\mathrm{j}=1}^{\mathrm{B}} \mathrm{L_j}} \qquad \mathrm{w} = 1,2,\dots,\mathrm{W}$$

Based on the minimum MSEP, ideas of the bootstrap and cross-validation are combined in order to achieve optimal choice of biasing parameter. Estimated response values are computed in order to find MSEP for the unchosen members and this process is repeated a large number of times for every bootstrap sample. The value of k with minimum MSEP is the bootstrap optimal choice for the biasing parameter:

$$k = \min(\,\mathrm{MSEP}(\mathrm{k_w}))\,.$$

Table 3.1. The biasing parameters we use in the study

Research	*k* estimator	Notation
Hoerl et al. (1975)	$k = p\hat{\sigma}^2 / \hat{\alpha}' \hat{\alpha}$	k_2
Hocking et al. (1976)	$k = \hat{\sigma}^2 \sum_{i=1}^{p} (\lambda_i \hat{\alpha}_i)^2 / (\sum_{i=1}^{p} \lambda_i {\hat{\alpha}_i}^2)^2$	k_3
Lawless and Wang (1976)	$k = p\hat{\sigma}^2 / (\sum_{i=1}^{p} \lambda_i {\hat{\alpha}_i}^2)$	k_4
Kibria (2003)	$k = \hat{\sigma}^2 / \left(\prod_1^p {\hat{\alpha}_i}^2\right)^{1/p}$	k_5
Kibria (2003)	$k = \text{median}(\hat{\sigma}^2 / \hat{\alpha}^2{}_i)$	k_6
Khalaf and Shukur (2005)	$k = \lambda_{\max} \hat{\sigma}^2 / \{\lambda_{\max} \alpha^2{}_{\max} + (n - p)\hat{\sigma}^2\}$	k_7
Alkhamisi et al. (2006)	$k = \max(\lambda_i \hat{\sigma}^2 / \lambda_i {\hat{\alpha}_i}^2 + (n - p)\hat{\sigma}^2)$	k_8
Alkhamisi and Shukur (2007)	$k = \max(\hat{\sigma}^2 / {\hat{\alpha}_i}^2 + 1/\lambda_{\max})$	k_9
Alkhamisi and Shukur (2007)	$k = \frac{1}{p} \sum_1^p \left(\hat{\sigma}^2 / {\hat{\alpha}_i}^2 + 1/\lambda_{\max} \right)$	k_{10}
Alkhamisi and Shukur (2007)	$k = \text{median}(\hat{\sigma}^2 / {\hat{\alpha}_i}^2 + 1/\lambda_{\max})$	k_{11}
Alkhamisi and Shukur (2007)	$k = \frac{p\hat{\sigma}^2}{\sum_1^p \lambda_i {\hat{\alpha}_i}^2} + \frac{1}{\lambda_{\max}}$	k_{12}
Allen (1974)	$k = \min CV = 1/n \sum e_i(k)^2 / (1 - h_{ii}(k))^2$	k_{press}
Wahba et al. (1979)	$k = \min GCV = 1/n \sum e_i(k)^2 / \sum (1 - h_{ii}(k))^2$	k_{gvc}
Delaney and Chatterjee (1986)	$k = \min(MSEP(k_w))$	k_{boot}

4. PRINCIPAL COMPONENTS REGRESSION

First of all, let's explain the principal components analysis briefly via not considering the PCR analysis. Principal components analysis is the oldest and most well-known technique of multivariate analysis (Jolliffe, 2002). The technique, which was first used by Pearson (1901), was later independently developed by Hotelling (1933). Principal components analysis (PCA) is a size reduction process. The principal components are independent from each other. Thus, the dependency structure between variables is also eliminated (Joliffe, 2002). In addition, the PCA is one of the solutions proposed in the literature to the problem of multiple linear connection (For examples, see Chatterjee and Price, 1977; Hotelling 1933, 1936). In multivariate statistical analysis, p variables (properties) related to n objects are examined. It is difficult to analyse if many of these variables are interrelated and the number of variables is too large. In such cases, the main component analysis technique comes into play, explaining the variance-covariance structure of the set of variables through linear combinations of these variables, providing data reduction and interpretation. As a result of PCA, p new principal components that define the space with variable m. The disclosure of the information carried by m variable with less than m number is the main principal components. For this purpose, the first eigenvector, whose majority corresponds to the independent variables, and the percentage of the total change in the set of independent variables, is explained by the second eigenvector, and so on.

4.1. Principal Components Selection Methods

In the principal components analysis, one of the issues to consider is to settle on the number of essential principal components. For this reason, several techniques have been developed.

- The simplest of these is that when principal components are obtained from standard data, the number of eigenvalues of the $X'X$ matrix greater than 1 is taken as the number of important principal components.
- Principal components are considered important as the number of eigenvalues, so that the total variance explained by the eigenvalues should be at least 80%.

$$\sum_{j=1}^{p-m} \frac{\lambda_j}{\lambda_1 + \cdots + \lambda_p} \geq 80\%$$

- Scree plot drawn based on the eigenvalues of the principal components are used. In the graph, it shows the numerical values of eigenvalues on the y axis and the numbers of eigenvalues on the x axis. Eigenvalues with high acceleration, rapid declines in the graph give the number of important principal components.

PCR is a regression analysis technique based on PCA. PCR is based on the fact that each linear regression model can be re-explained, based on a set of unrelated (perpendicular) explanatory variables. It is known that using multiple estimation techniques instead of LS is the most appropriate approach in case of multiple connections (Albayrak, 2006). For the regression analysis of principal components, first of all response and explanatory variables are generalized by taking the differences from their averages and splitting them into standard deviations. In other words, all of the operations are solely relied on standardized results. Explanatory variables are converted to their basic components to perform PCR.

Therefore, the variance of $\hat{\beta}_j$ is a linear combination of reciprocal values of eigenvalues. Thus, one or more small eigenvalues are seen to distort the LS estimator.

To sum up, the eigenvalues are equal to one, then the explanatory variables are orthogonal, if the eigenvalues are equal to zero, it means that there is a linear relation between each explanatory variable. Based on this statement, when the determined eigenvalues are small, there exists strong multicollinearity.

The PCR approach combats multiple linearity using less than all the major components in the model. The components we will extract from the basic set of components are the main components that correspond to the eigenvalues of the explanatory variables that are approximately equal to zero.

We have previously assumed that eigenvalues are ordered in the order of $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$. Suppose that the last m of these eigenvalues are approximately equal to zero. Hence the principal components estimator is as follows:

$$\hat{\beta}_{PC} = S_{p-m} \Lambda^{-1}{}_{p-m} S_{p-m}{}' X'y \tag{4.1}$$

where $\Lambda_{p-m} = \mathrm{diag}(\lambda_1, \lambda_{2,\ldots,} \lambda_{p-m})$ is a diagonal matrix with the $(X'X)$'s eigenvalues. S_{p-m} is a $(p-m)\ x\ (p-m)$ orthogonal matrix, the columns of which are the eigenvectors aligned with $(\lambda_1, \lambda_{2,\ldots,} \lambda_{p-m})$.

5. BOOTSTRAP METHOD

Achieving statistical inferences based on long history, and therefore there are many different ways to make these inferences. One of these methods is the bootstrap method, which firstly proposed in 1979 and has the feature of being fully automatic. Bootstrap methods are the resampling methods which firstly proposed as an alternative to the Quenouille-Tukey jack-knife method in Efron's 1979 study (Efron, 1979). The basis of this method is to be resampling to produce very large datasets from the current dataset (Sacchi, 1998). This method, which is currently widely used in the field, enables to estimate statistics by averaging the estimates from a large number of small data sets. A significant bootstrap benefit is its usability. This is a simple way for complex estimators of distributed distribution parameters to obtain estimates of the standard errors and confidence intervals. Bootstrap is also a suitable way of controlling and testing result reliability. Whereas determining the true confidence interval is doubtful for most of the issues, bootstrap is asymptotically more reliable than the standard intervals obtained using sample variance and normality assumptions. Efron (1979) published significant research in the use of bootstrap function and statistical inference. This method can be used as a method to obtain accurate conclusions, especially in cases where established statistical methods and assumptions are inadequate. Sampling from large population, in applicable statistics, creates lack of time and energy. Therefore, it will generate new data sets of different sizes and quantities by adjusting the measurements in the data sets calculated by the bootstrap method. While considering samples in the cases where the distribution of is uncertain, the estimator's sampling distribution or test statistics are found depending on the obtained sample. In other words, new samples are derived by drawing the existing sample as a population and replacing it from whole sample. The bootstrap method can be inferred without knowing the population distribution or making any assumptions about the population distribution. Bootstrap methods can be used for parametric as well as non-parametric resampling. For non-parametric bootstrap methods, there are not any known conclusions regarding to the sample mass distribution, while the population distribution is known in the parametric bootstrap method. The use of the method for non-parametric bootstrap is as follows:

Assume that $\mathrm{x}_1, \mathrm{x}_{2,} \dots, \mathrm{x}_\mathrm{n}$ is a sample set which are independent from each other and comes from the same F distribution. F_n created by resampling, and $1/_\mathrm{n}$ is likely to be considered as the function of a population where only observed values can be observed $\mathrm{x}_1{}^*, \mathrm{x}_2{}^*, \dots, \mathrm{x}_\mathrm{n}{}^*$bootstrap sample each $\mathrm{x}_\mathrm{i}{}^*$random variable is created independently from the same F_n dispersed population.

To summarize the operation of the Bootstrap method,

Bootstrap samples are $\{\mathrm{x}_1, \mathrm{x}_{2,} \dots, \mathrm{x}_\mathrm{n}\}$ n-size random samples drawn from the n-size population. That is, bootstrap samples contain members of the original dataset, some members can be seen 0 times in bootstrap samples, others 2 times etc. n^n different samples from such a dataset are obtained. A discussed statistic can be computed from n^n samples and its distribution can be defined. The described process is called the bootstrap distribution. The bootstrap distribution cannot be generated but B is taken from n^n samples.

5.1. The Bootstrap algorithm is as follows:

The bootstrap procedure, which is graphically shown in Figure 1, can be explained with the following steps.

1. Select B independent $\mathrm{x}_1{}^*, \mathrm{x}_2{}^*, \dots, \mathrm{x}_\mathrm{B}{}^*$ (bootstrap samples), each of which consists of n size data values taken from the original data.
2. Evaluate the bootstrap replication corresponding to each bootstrap sample
$$\hat{\theta}^*(\mathrm{b}) = \mathrm{s}(\mathrm{x}^{*\mathrm{b}}) \qquad b = 1,2,3 \dots, B$$
3. Repeat steps 1-2 B times (i.e. b = 1, …, B)

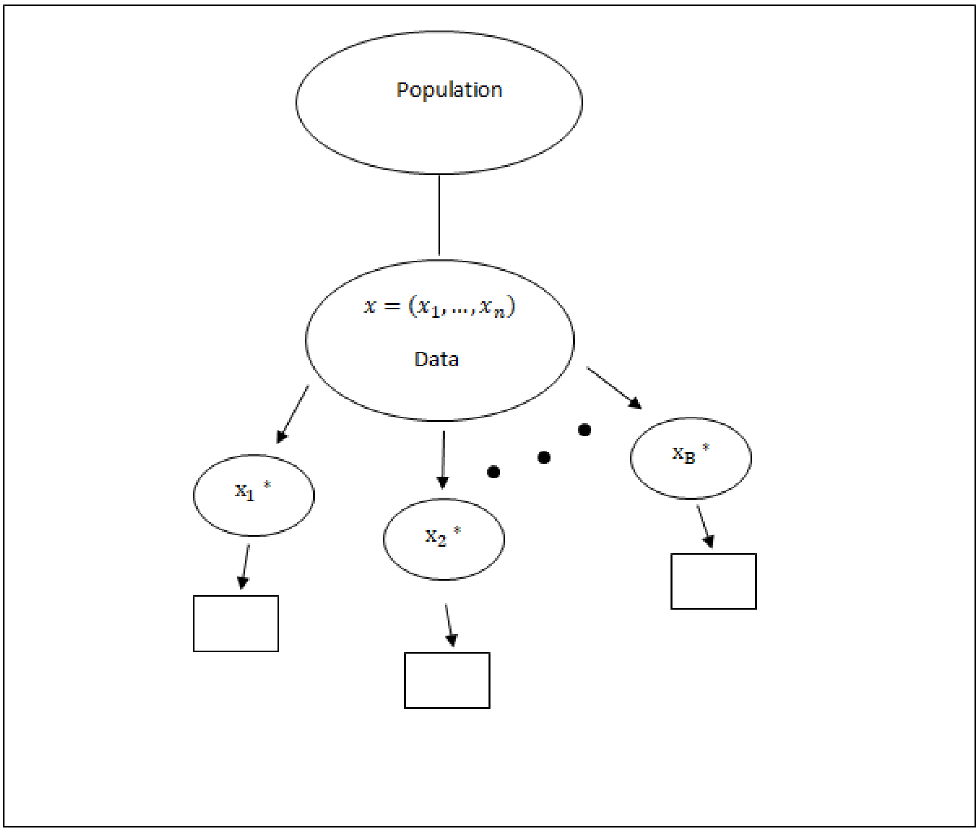

Figure 5.1. The Bootstrap Algorithm

To estimate any θ parameter, the sampling distribution is estimated by calculating the average and standard error of the θ parameter values from each bootstrap sample.

Efron and Tibshirani describe the bootstrap prediction of the standard error as consisting of the following steps:

1) Select B independent $x_1^*, x_2^*, \ldots, x_B^*$ (bootstrap samples), each of which consists of n size data values taken from the original data.
2) Evaluate the bootstrap replication corresponding to each bootstrap sample and calculate its average

$$\hat{\theta}^{*}(\mathrm{b}) = \mathrm{s}\left(\mathrm{X}^{*\mathrm{b}}\right) \qquad b = 1,2,3\ldots,B$$

$$\hat{\theta}^{*}(.) = \sum_{\mathrm{b}=1}^{\mathrm{B}} \hat{\theta}^{*}(\mathrm{b})/\mathrm{B}$$

Estimate the standard error by the sample standard deviation of B replications

$$\widehat{\mathrm{Se}}_{\mathrm{B}} = \left\{\sum_{\mathrm{b}=1}^{\mathrm{B}} \frac{\left[\hat{\theta}^{*}(\mathrm{b}) - \hat{\theta}^{*}(.)\right]^{2}}{\mathrm{B}-1}\right\}^{1/2} \qquad (4.2)$$

5.2. Bootstrap Confidence Intervals

In general, the confidence interval for any parameter gives more information than the point estimate of the θ parameter. It suggests various Bootstrap methods in establishing confidence intervals (Wehrens et al, 2000).

We can say that the most common confidence intervals for bootstrap are the percentile confidence interval for bootstrap and the confidence interval for bootstrap-t. In this study, bootstrap-t confidence interval and percentile of bootstrap confidence interval will be given in detail. The conditions for the effective use of bootstrap-t and percentile intervals for bootstrap have possibility of uncertainty. Unless the bootstrap distribution bias is low and the distribution is close to normal distribution, there should be similar agreement on the bootstrap-t and percentile confidence intervals. Like t intervals, percentile intervals do not neglect skewness. Therefore, percentile intervals are generally more reliable, provided the bias is low. Firstly, it is important to understand the percentile confidence interval for bootstrap method. The following section gives details regarding to the percentile confidence interval for bootstrap.

5.2.1. Percentile Confidence Interval for Bootstrap Method

As known, when creating a standard confidence interval point estimation and standard error of the parameter are needed. While creating a confidence interval with this method, there is no need for standard error of bootstrap parameter values. Just knowing the bootstrap distribution of any θ parameter will be sufficient to establish the bootstrap percent confidence interval. The operation of the method is as follows:

1. Evaluate the bootstrap replication $\hat{\theta}^*(\mathrm{b})$ corresponding to each bootstrap sample
2. The $\hat{\theta}^*(\mathrm{b})$values obtained are ordered from small to large. $\hat{\theta}^*(\mathrm{b1}) \leq \hat{\theta}^*(2) \ldots \leq \hat{\theta}^*(\mathrm{b})$
3. 100% $(1-\alpha)$ confidence level for the θ parameter can be obtained from (B-Bα)th and (αB)th $\hat{\theta}^*$ values . This forms $(\hat{\theta}^{*(\alpha)}, \hat{\theta}^{*(1-\alpha)})$

5.2.2. Bootstrap-t Confidence Interval

The bootstrap-t method is somewhat vague for correct use. If the deviation of the bootstrap distribution is small and the distribution is close to normal, the bootstrap-t confidence intervals are said to be reliable results. Intervals of confidence at the bootstrap disregard skew. This option, the bootstrap-t method, yields small satisfactory results, regardless of how large the deviations are. The bootstrap-t method was mentioned by Efron (1993).

The steps of the bootstrap-t method are as follows:

1. . We generate B bootstrap samples $\mathrm{x}^{*1}, \mathrm{x}^{*2} \ldots, \mathrm{x}^{*B}$ and for each we compute

$$\mathrm{Z}^*(\mathrm{b}) = \frac{\hat{\theta}^*(\mathrm{b}) - \hat{\theta}}{\widehat{se}^*(\mathrm{b})} \qquad \mathrm{b} = 1,2,\ldots,\mathrm{B} \tag{4.3}$$

where B is the number of bootstrap samples we have selected, $\hat{\theta}^*(\mathrm{b})$ is the value of $\hat{\theta}$ for the bootstrap sample and $\widehat{se}^*(\mathrm{b})$ is the estimated variance of $\hat{\theta}^*(\mathrm{b})$ for the bootstrap sample.

2. The α-th percentile of $\mathrm{Z}^*(\mathrm{b})$ is estimated by the value $\hat{\mathrm{t}}(\alpha)$ such that

$$\#\{\,\mathrm{Z}^*(\mathrm{b}) \leq \hat{\mathrm{t}}(\alpha)\,\}\,/\,\mathrm{B} = \alpha\mathrm{B} \qquad (4.4)$$

If we resolve the above equation, we first sort the Z^* values from small to large $\mathrm{Z}^*(1) \leq \mathrm{Z}^*(2) \leq \ldots \leq \mathrm{Z}^*(\mathrm{B})$. Here the two values of $\hat{\mathrm{t}}(\alpha)$ will be formed. $\hat{\mathrm{t}}(\alpha)$ is the (αB)th of the Z^* scores and $\hat{\mathrm{t}}(1-\alpha)$ is the $(\mathrm{B}-\mathrm{B}\alpha)$th of the Z^* scores.

3. $100\%\,(1-\alpha)$ confidence level for the $\hat{\theta}$ parameter can be obtained from $(\mathrm{B}-\mathrm{B}\alpha)$th and $(\alpha\mathrm{B})$th Z^*scores. These forms $(\hat{\theta}^*(\mathrm{b}) - \hat{\mathrm{t}}(1-\alpha)\widehat{Se}^*(\mathrm{b}), \hat{\theta}^*(\mathrm{b}) - \hat{\mathrm{t}}(\alpha).\widehat{Se}^*(\mathrm{b}))$.

Bootstrap confidence intervals, which are frequently used in literature, were mentioned above. But we did not use these methods in this thesis, since we will create confidence intervals for the ridge and PCR parameter.

Many methods have been published in the literature to calculate the bootstrap confidence interval of ridge regression parameters. Vinod (1987) reviewed four alternative methods to establish confidence intervals for ridge regression parameters.

1. The first method Obenchain (1977)

The simplest option, advocated by Obenchain (1977), is to use a confidence interval based on LS, even if it uses ridge regression to obtain a point estimate.

In this proposed confidence interval, point estimate uses the ridge estimator $\widehat{\beta}_{R} = (X'X + kI)^{-1}X'y$, and the LS unbiased estimate of the error variance of $\hat{\sigma}^2 = \frac{(y-X\hat{\beta})'(y-X\hat{\beta})}{n-p}$.

2. The approximate Bayes method,

Morris (1983) commented on Berger's ellipsoid, and pointed out that data analysts need the interval of each parameter to replace the familiar t interval, and further proposed an approximate empirical Bayesian confidence interval. The interval is centred on the ridge regression coefficient instead of the LS interval. The standard error is based on the diagonal of the$(X'X + kI)^{-1}$matrix. The approximate Bayesian 95% confidence interval based on

$$\widehat{\beta}_{R_j} - 1.96\left(V_{jj}{}^{B}\right)^{1/2} \leq \beta_{R_j} \leq \ \widehat{\beta}_{R_j} + 1.96\left(V_{jj}{}^{B}\right)^{1/2}$$

where $V_{jj}{}^{B}$ are the diagonal elements of the posterior covariance matrix

$$V^B = \sigma^2(X'X + kI)^{-1}$$

3. Stein's (1981) use of the unbiased estimate of the MSE (UMSE) of biased estimators

The UMSE confidence interval is as follows:

$$\hat{\alpha}^{lo}{}_j = \delta_j\hat{\alpha}_j - \sigma\lambda_j{}^{-1/2}(U^* + 2.0092S^*)^{1/2}$$

$$\hat{\alpha}^{up}{}_j = \delta_j\hat{\alpha}_j + \sigma\lambda_j{}^{-1/2}(U^* + 2.0092S^*)^{1/2}$$

where

$$U^* = \max[\delta^2, (\delta - 1)^2\sigma^{-2}\lambda_j\hat{\alpha}_j{}^2 + 2\delta - 1]$$

$$S^{*} = [\max\left(1{,}2(2\delta - 1)^{2}, 4\delta^{2} - 2 + 4(\delta - 1)^{2}\sigma^{-2}\lambda_{j}\hat{\alpha}_{j}{}^{2}\right)]^{1/2}$$

From these intervals for α, we generate conservative intervals for β, the main parameter of interest in a regression model, as follows.

$$b_{j}{}^{up} = \sum \max(s_{jl}\,\hat{\alpha}^{up}{}_{j}, s_{jl}\hat{\alpha}^{lo}{}_{j})$$

$$b_{j}{}^{lo} = \sum \min(s_{jl}\,\hat{\alpha}^{up}{}_{j}, s_{jl}\hat{\alpha}^{lo}{}_{j})$$

where S and α are notations used in Eq. (17). $\beta = S\alpha$ implies that $\beta_{j} = \sum s_{jl}\hat{\alpha}_{l}$ where s_{jl} is the element of S and δ is a shrinkage factor.

4. Confidence Interval for Bootstrap Method

3 types of bootstrap confidence intervals have been used here. The first percentile bootstrap confidence interval, the second is bootstrap reflected interval, and the third is bootstrap bias corrected interval.

We have already discussed the confidence interval of the percentile bootstrap.

The bootstrap reflected confidence interval in the second approach is based on the percentile bootstrap confidence interval. The confidence interval is

$$[2\hat{\beta} - \hat{\beta}_{R}{}^{*(1-\alpha)}, 2\hat{\beta} - \hat{\beta}_{R}{}^{*(\alpha)}]$$

(Efron and Tibshirani, 1993) proposed bias corrected confidence interval to overcome overload problems in percentile bootstrap confidence intervals. The estimated bootstrap correction factor $\hat{z}_{0}$ is that the ratio of the bootstrap estimate is smaller than the original parameter estimate

$$\hat{z}_{0} = \Phi^{-1}\left[CDF_{*}(\hat{\beta}_{R})\right] = \Phi^{-1}\left(\frac{\#\hat{\beta}_{R}{}^{*} < \hat{\beta}_{R}}{B}\right) \tag{4.5}$$

where Φ−1 is the inverse function of a standard normal cumulative distribution function.

The bias corrected percentile method leads to the following approximate confidence interval

$$[\, \mathrm{I}CDF_*\big(\Phi(2\hat{z}_0 - z_\alpha)\big), \mathrm{I}CDF_*\big(\Phi(2\hat{z}_0 + z_\alpha)\big) \,]$$

where $\mathrm{I}CDF_*$is the inverse CDF_* as before, and z_α is the upper a point for the N(0, 1) normal distribution function $\Phi(z_\alpha) = 1 - \alpha$

Crivelli et al. (1995) considered two confidence intervals for ridge regression parameters: one based on the standard normal distribution and the other on the Edgeworth expansion of the standard normal distributed statistic calculated by the percentile bootstrap concept. In this study, the confidence interval used for biasing parameter estimators, Crivelli et al. (1995) proposed, and in parallel, these confidence intervals were used within the PCR. In particular, Crivelli et al. (1995) created confidence intervals based on the ridge estimator using the standard normal approach, the student t approach, and the Edgeworth expansion of the standard normal distributed statistic. percentile is calculated by the concept of bootstrap (called Rayner's approach).

The Rayner's approximated confidence interval for $\hat{\beta}_{\mathrm{Rj}}$ is defined by Crivelli et al. (1995) as

$$[\hat{\beta}_{\mathrm{Rj}} - u_j \,, \hat{\beta}_{\mathrm{Rj}} + v_j \,]$$

where

$$u_j = \mathrm{S_{jj}}.\mathrm{z}_{1-\alpha/2} - \mathrm{S_{jj}}.\big(p^*\big(\mathrm{z}_{1-\alpha/2}\big) - (1-\alpha/2)\big)/\varphi(z/2),$$

$$v_j = \mathrm{S_{jj}}.\mathrm{z}_{1-\alpha/2} + \mathrm{S_{jj}}.\big(p^*\big(\mathrm{z}_{\alpha/2}\big) - (1-\alpha/2)\big)/\varphi(z/2)$$

and $\varphi(z)$ is the density of the standard normal distribution.

The calculation of $p^*(z_{\alpha/2})$ is based on the calculation of Z scores in the bootstrap-t confidence interval. Crivelli et al. (1995) constructed the Studentized statistic as

$$T_j^* = \frac{\hat{\beta}_{\mathrm{Rj}}^* - \hat{\beta}_{\mathrm{Rj}}}{S_{jj}^*} \tag{4.6}$$

where ${S_{jj}^*}^2 = \hat{V}(\hat{\beta}_{\mathrm{Rj}}^*)$ is the estimated variance of the bootstrapped ridge estimator, $\hat{\beta}_{\mathrm{Rj}}^*$. For each of B bootstrap samples drawn independently, the value t_j^* of the statistic T_j^* is calculated and $p^*(z_{\alpha/2})$ is computed from $\mathrm{p}^*(\mathrm{z}) = \#\{\mathrm{t}^* \ \leq \mathrm{z}\ \} \ / \ \mathrm{B}$.

5.3. Application of Bootstrap Method on Regression Models

As mentioned in the multiple regression analysis section, the regression model had assumptions. If all assumptions of the model satisfy, the model is suitable for data and the results will be reliable. But when there are problems with some assumptions, Efron and Tibshirani (1993) show that Bootstrap applies to general models by giving reasonable outputs, including the linearity of parameters. According to them, boot regression also applies to models with a mathematical form in addition to models without a mathematical solution.

5.3.1. Bootstrap Based On the Resampling Errors

In a regression model (2), there are two approaches to bootstrapping: bootstrapping pairs and bootstrapping residuals.

In the case of bootstrapping residuals, the interest of the model is that estimating the residuals via interacting them with each other. The way of computing this process is forming new singular matrix which consist from randomly selected residuals from each single estimation. The newly formed singular matrix will be used to estimate the new least square to minimize the error terms from the residuals. Each element of singular matrix has the probability of 1/n where n is the number of equations. The idea here is that the matrix can be formed from only one residual or n residuals since it is randomly selected. Therefore, the estimated least square equations with the singular matrix can perform better estimations.

The steps are follows;

1. Define the residual: $e_i = y_i - \hat{y}_i$
2. Bootstrap residual vector, $\hat{e}^*= \{e_1{}^*, e_2{}^*,.., e_n{}^*\}$, is created by selecting n-size random sample by replacing the n number of residual vector, $e = \{e_1, e_2, \ldots, e_n\}'$, obtained.
3. Create the bootstrap response vector: $y^*= X\hat{\beta} + e^*$
4. LS regression coefficients for each bootstrap sample is

   $\hat{\beta}^{*(b)}=(X'X)^{-1}y^*$ b=1, 2, …. , B ; B=bootstrap repeats

5. Bootstrap estimation of regression coefficients:

$$\hat{\beta}^* = \sum_{b=1}^{B} \hat{\beta}^{*(b)}/_{B} = \bar{\hat{\beta}}^*$$

6. Bootstrap regression model is obtained follows:

$$\hat{y} = X\hat{\beta}^* + \varepsilon$$

5.3.2. Bootstrap Based On The Resampling Observations

On the other hand, bootstrapping pairs, broadly speaking deals with interacting each observation. The idea here is that achieve optimum or ideal regression model which has been formed via using bootstrapped pairs. The newly formed least square equation is a product of randomly selected observations which are donated as X's. The procedure of selecting each observation is based on random selection method. This implies that each observation has probability of 1/n. Therefore, since the new formation is randomly selected, there is a probability which implies that the new formation can be formed from only one observation or totally from different observations. Since the idea is to minimize the possible drawbacks of first estimated equation, the second estimated equation with randomly selected observations gives better outcome. This is because the weighted average of randomly selected observations is used in the least square estimation.

In this type of bootstrap sampling there are n sample pairs (y_i, x_{ji}) that are considered as data to be resampled.

Let us assume that these sample pairs (y_i, x_{ji}) are independently and likewise drawn from the F distribution.

The original sample pairs of n numbers (y_i, x_{ji}) are sampled n times. (y_i^*, x_{ji}^*) is the bootstrap sample for i = 1,2, ..., n j = 1, 2, .., k

LS regression coefficients for each bootstrap sample is

$$\hat{\beta}^{*(b)} = (X^{*\prime} X^*)^{-1} y^*$$

Bootstrap estimation of regression coefficients;

$$\hat{\beta}^* = \sum_{b=1}^{B} \hat{\beta}^{*(b)} / _B = \bar{\hat{\beta}}^*$$

The obtained bootstrap regression model is:

$$\hat{y} = X\hat{\beta}^* + \varepsilon$$

Although there is not definitive conclusion about which method is better, Efron and Tibshirani (1998) mentioned that bootstrapping pairs is led sensitive to assumptions than bootstrapping residuals.

6. MONTE CARLO SIMULATION METHOD

Monte Carlo simulation method was introduced by Stanislaw Ulam in 1946 who as a physicist. Firstly, he was trying to come up with a method or a model to calculate the density of nuclear particles in order to know their energy level. The main idea of the Monte Carlo simulation method is to use random samples from the population. Ulam's proposed Monte Carlo method simply was selecting random variables from the population, and it had great achievement. Afterwards, Monte Carlo simulation method became a popular method to analyse huge population, since it gives reliable results. The use of Monte Carlo simulation enabled researchers to save their time while examining the huge data set which consists of variables such as normal, lognormal and uniform variables.

The simulation research structure is as follows:

1. By following McDonald and Galarneau (1975), explanatory variables were generated,

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{i(p+1)} \quad j = 2,3,\dots,p \qquad i = 1,2,3,\dots,n$$

where ρ is the degree of collinearity.

2. We computed parameter vector as,

$$\beta = (\|\beta\| / \sum_{j=1}^{p} u_j{}^2) u_j$$

which follows Delaney and Chatterjee (1986). Here, u_j is sampled from a uniform distribution on the interval [-1,1]. In this study, the $||\beta||$ values used are 2, 5, 10, 25.

3. The model generates the error terms ε_i from a normal distribution with zero expectation and σ^2 variance. The values of σ^2 are calculated as

$$\sigma^2 = \frac{||\beta||^2}{\mathrm{SNR}} = \frac{\beta'\beta}{\mathrm{SNR}}.$$

4. Ultimately the observations are derived from the following equation:

$$y_i = \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + \varepsilon_i \qquad i = 1,2,\ldots n$$

6.1. Ridge Confidence Interval Application

In this section, simulations were used to research the method of obtaining bootstrap-t confidence intervals based on ridge regression estimators. In this simulation study, sample size $n = 100$, number of explanatory variables $p = 4$, degree of collinearity $\rho = 0.99, 0.95, 0.90, 0.85,$ bootstrap sample $B = 100$ and number of simulations $MCN = 500$ were used as fixed factors. On the other hand, the actual variance of the error term that used as the variable σ^2 will vary depending on signal-to-noise ratio (SNR) and norm of the parameter vector ($||\beta||$). The values of SNR are considered as 4, 9, 25, 100, 200.

At the end of 4 steps in Section 6.1, ridge regression estimators and fourteen confidence intervals are computed for each model specification and for each replication based on the bootstrap-t method.

Appendix 1-Appendix 12 represent the outcome of the ridge confidence interval application. Furthermore, this section will also give detailed interpretation for each output.

The key conclusions obtained from Appendix 1-Appendix 12 are as follows:

1) It has been found that the confidence intervals for LS and ridge estimators for almost all values are getting smaller as SNR increases when ρ and $\beta'\beta$ remain at constant. However, k_3, k_5, k_6, k_9, k_{10} and k_{11} do not follow this pattern. Particularly, the confidence intervals for k_9, k_{10} and k_{11} values are either remaining constant or increasing. What is more, k_3 follows the pattern only at bootstrap-t confidence intervals.
2) The observation regarding to the confidence intervals for LS and Ridge estimators is that k values tend to increase as ρ increases and SNR, $\beta'\beta$ remain at constant. However, few samples fail to satisfy the latter argument. For example, as ρ changes from $\rho = 0.90$ to $\rho = 0.95$, the confidence interval of k_5, k_6, k_9, k_{10} and k_{11} decreases, and when ρ changes from $\rho = 0.95$ to $\rho = 0.99$, the confidence interval of k_3 decreases. But, on the other hand, for larger SNRs (such as 100 and 200), only k_9 and k_{11} follow the latter argument. This means that as the SNR increases, the value of k follows the general pattern. What is more, k_3 follows the pattern only at $\beta'\beta = 625$ and $\beta'\beta = 25$.
3) When SNR and ρ remain constant and $\beta'\beta$ values increase, confidence intervals for LS and Ridge estimators tend to increase for almost all k values. However, k_9 and k_{11} do not follow this pattern. In particular, when $\beta'\beta$ increases from $\beta'\beta = 25$ to $\beta'\beta = 100$ only k_9 and k_{11} become smaller for all confidence interval.
4) The confidence interval of the ridge estimate for all values is narrower than the confidence interval of LS. This finding is also similar to the study of Crivelli et al. (1995). In addition, for the standard normal confidence interval, the performance of k_{boot} is better than that of k_{press}, but when all three types of confidence intervals have narrow confidence intervals, the performance of k_{boot} is usually not better than k_{gvc}. When k_{boot} is

compared with other k values, it can be seen that it is usually better than some k values, such as k_4, k_7, k_8, k_{12} and worse than others, for example for all configurations, but in some $k$values Among them, the cases of k_5, k_6, k_9 , and k_{11} become the values of B and n, for example, k_2 , k_3 , and k_{10} . This shows that guided estimation can more accurately consider the lower and upper limits, so the results of k_{press}, k_4, k_7 and k_{12} can be seen better.

5) As a final and general observation, it has been found that bootstrap-t confidence intervals are the ones which have the most wider levels while on the other hand, standard normal approximation confidence intervals tend to have the smaller gaps among bootstrap-t, standard normal approximation and t approach confidence interval.

6.2. PCR Confidence Interval Application

Simulations were used in this section to research the method of obtaining bootstrap-t confidence intervals based on PCR estimators. In this simulation study, sample size $n = 100$, number of explanatory variables $p = 3$, degree of collinearity $\rho = 0.95, 0.90, 0.80,$ bootstrap sample $B = 100$ and number of simulations $MCN = 500$ were used as fixed factors. On the other hand, the actual variance of the error term that used as the variable σ^2 will vary depending on signal-to-noise ratio (SNR) and norm of the parameter vector (||β||). The values of SNR are considered as 4, 9, 25, 100, 200.

At the end of 4 steps which mentioned above, in Section 6.2, PCR estimators and two confidence intervals are computed for each model specification and for each replication based on the bootstrap-t method.

The method we use to determine the number of principal components $(p - m)$ in the study is based on the total variance criterion described by eigenvalues. Criterion is as follows:

$$\sum_{j=1}^{p-m} \frac{\lambda_j}{\lambda_1 + \cdots + \lambda_p} \geq 99\%$$

Tables from 6.1 to 6.12 represent the outcome of the PCR confidence intervals application.

The observed output from Tables 6.1-6.12 is that,

1) As $\beta'\beta$ decreases, ρ and SNR remains constant, confidence interval levels decrease for both PCR and LS for 3 types of confidence interval. When examining the difference between PCR and LS under the same conditions, it was observed that LS tended to react more than PCR.
2) For the 3 types of confidence interval, the confidence intervals for both LS and PCR decrease as SNR increases, while ρ and $\beta'\beta$ remains constant.
3) As ρ increases and SNR, $\beta'\beta$ remains constant, confidence intervals for LS for 3 types of confidence intervals increase. But the confidence length for PCR does not satisfy this trend.

Table 6.1. Bootstrap-t Confidence Intervals for LS and PCR estimators with $\beta'\beta = 625$

SNR	ρ	LS		PCR	
4	0.95	8.5741	25.0491	-23.3941	1.6050
		-16.4749		-24.9991	
	0.90	-21.1101	17.9248	-31.0000	1.6612
		-39.0349		-32.6612	
	0.80	8.3998	12.7226	38.6949	1.7455
		-4.3229		36.9494	
9	0.95	7.2676	16.7594	-0.2713	1.0588
		-9.4918		-1.3302	
	0.90	4.6926	11.9186	7.9633	1.1092
		-7.2260		6.8541	
	0.80	-17.1569	8.4699	12.3345	1.1689
		-25.6268		11.1656	
25	0.95	-3.7633	10.1365	15.7140	0.6501
		-13.8998		15.0638	
	0.90	0.2798	7.1605	9.8039	0.6585
		-6.8808		9.1454	
	0.80	5.5447	5.0217	-43.0413	0.6910
		0.5230		-43.7323	
100	0.95	-0.4026	5.0133	7.7277	0.3232
		-5.4159		7.4045	
	0.90	3.8186	3.5637	36.6355	0.3346
		0.2549		36.3009	
	0.80	2.8698	2.5495	-8.6812	0.3518
		0.3203		-9.0329	
200	0.95	-3.5292	3.5495	-9.9668	0.2270
		-7.0786		-10.1938	
	0.90	0.5381	2.5150	35.2148	0.2349
		-1.9769		34.9800	
	0.80	6.9131	1.7976	13.8394	0.2460
		5.1155		13.5934	

Table 6.2. Standard Normal Approximation Confidence Intervals for LS and PCR estimators with $\beta'\beta = 625$

SNR	ρ	LS		PCR	
4	0.95	9.8997	19.7635	0.8419	1.2865
		-9.8639		-0.4446	
	0.90	7.0217	14.0089	0.2897	1.3232
		-6.9872		-1.0335	
	0.80	4.9965	9.9609	0.7044	1.3968
		-4.9644		-0.6924	
9	0.95	6.5998	13.1757	0.4424	0.8577
		-6.5759		-0.4152	
	0.90	4.6811	9.3393	0.7534	0.8821
		-4.6582		-0.1287	
	0.80	3.3310	6.6406	0.4569	0.9312
		-3.3096		-0.4743	
25	0.95	3.9599	7.9054	0.4227	0.5146
		-3.9455		-0.0919	
	0.90	2.8087	5.6036	0.6570	0.5293
		-2.7949		0.1277	
	0.80	1.9986	3.9844	0.7804	0.5587
		-1.9858		0.2217	
100	0.95	1.9799	3.9527	0.3121	0.2573
		-1.9728		0.0548	
	0.90	1.4043	2.8018	-0.2472	0.2646
		-1.3974		-0.5118	
	0.80	0.9993	1.9922	-0.2026	0.2794
		-0.9929		-0.4819	
200	0.95	1.4000	2.7950	0.2488	0.1819
		-1.3950		0.0669	
	0.90	0.9930	1.9812	0.4262	0.1871
		-0.9881		0.2391	
	0.80	0.7066	1.4087	0.7414	0.1975
		-0.7021		0.5439	

Table 6.3. t Approach Confidence Intervals for LS and PCR estimators with $\beta'\beta = 625$

SNR	ρ	LS		PCR	
4	0.95	10.0257	20.0156	0.8501	1.3029
		-9.9899		-0.4529	
	0.90	7.1110	14.1876	0.2981	1.3401
		-7.0766		-1.0420	
	0.80	5.0600	10.0879	0.7133	1.4146
		-5.0279		-0.7013	
9	0.95	6.6838	13.3437	0.4479	0.8686
		-6.6599		-0.4207	
	0.90	4.7407	9.4584	0.7591	0.8934
		-4.7177		-0.1343	
	0.80	3.3734	6.7253	0.4628	0.9431
		-3.3519		-0.4802	
25	0.95	4.0103	8.0062	0.4260	0.5212
		-3.9960		-0.0952	
	0.90	2.8444	5.6751	0.6604	0.5360
		-2.8306		0.1243	
	0.80	2.0240	4.0352	0.7839	0.5658
		-2.0112		0.2181	
100	0.95	2.0051	4.0031	0.3137	0.2606
		-1.9980		0.0532	
	0.90	1.4222	2.8375	-0.2455	0.2680
		-1.4153		-0.5135	
	0.80	1.0120	2.0176	-0.2008	0.2829
		-1.0056		-0.4837	
200	0.95	1.4178	2.8306	0.2500	0.1843
		-1.4128		0.0657	
	0.90	1.0057	2.0064	0.4274	0.1895
		-1.0008		0.2379	
	0.80	0.7156	1.4266	0.7426	0.2001
		-0.7111		0.5426	

Table 6.4. Bootstrap-t Confidence Intervals for LS and PCR estimators with $\beta'\beta = 100$

SNR	ρ	LS		PCR	
4	0.95	5.7534	10.0313	6.1780	0.6410
		-4.2779		5.5369	
	0.90	5.9477	7.1697	9.0563	0.6635
		-1.2220		8.3928	
	0.80	-7.3193	5.1339	-14.2834	0.7047
		-12.4532		-14.9880	
9	0.95	9.1987	6.7472	3.5699	0.4325
		2.4515		3.1374	
	0.90	-4.3546	4.7278	0.1486	0.4405
		-9.0825		-0.2919	
	0.80	2.1256	3.3780	7.0347	0.4672
		-1.2524		6.5675	
25	0.95	-2.5161	4.0414	-4.9907	0.2598
		-6.5575		-5.2505	
	0.90	2.6219	2.8680	1.5766	0.2660
		-0.2461		1.3106	
	0.80	0.7840	2.0319	13.1470	0.2784
		-1.2479		12.8686	
100	0.95	2.3565	2.0169	0.8518	0.1292
		0.3396		0.7225	
	0.90	-0.4088	1.4251	-6.0972	0.1323
		-1.8339		-6.2295	
	0.80	0.1418	1.0200	3.2865	0.1398
		-0.8782		3.1467	
200	0.95	1.3344	1.4224	1.7090	0.0911
		-0.0879		1.6180	
	0.90	0.0106	1.0117	1.2789	0.0937
		-1.0011		1.1852	
	0.80	0.4110	0.7195	5.8285	0.0991
		-0.3085		5.7294	

Table 6.5. Standard Normal Approximation Confidence Intervals for LS and PCR estimators with $\beta'\beta = 100$

SNR	ρ	LS		PCR	
4	0.95	3.9599	7.9054	0.3338	0.5146
		-3.9455		-0.1808	
	0.90	2.8087	5.6036	0.2514	0.5293
		-2.7949		-0.2779	
	0.80	1.9986	3.9844	0.4780	0.5587
		-1.9858		-0.0807	
9	0.95	2.6399	5.2703	0.0736	0.3431
		-2.6304		-0.2695	
	0.90	1.8725	3.7357	0.1413	0.3529
		-1.8633		-0.2116	
	0.80	1.3324	2.6562	0.2269	0.3725
		-1.3238		-0.1456	
25	0.95	1.5839	3.1622	0.0133	0.2058
		-1.5782		-0.1925	
	0.90	1.1235	2.2414	0.2787	0.2117
		-1.1180		0.0670	
	0.80	0.7994	1.5937	0.1609	0.2235
		-0.7943		-0.0625	
100	0.95	0.7920	1.5811	0.0283	0.1029
		-0.7891		-0.0747	
	0.90	0.5617	1.1207	0.1867	0.1059
		-0.5590		0.0808	
	0.80	0.3997	0.7969	0.1909	0.1117
		-0.3972		0.0792	
200	0.95	0.5600	1.1180	0.1532	0.0728
		-0.5580		0.0804	
	0.90	0.3972	0.7925	0.1971	0.0749
		-0.3953		0.1222	
	0.80	0.2826	0.5635	0.2008	0.0790
		-0.2808		0.1218	

Table 6.6. t Approach Confidence Intervals for LS and PCR estimators with $\beta'\beta = 100$

SNR	ρ	LS		PCR	
4	0.95	4.0103	8.0062	0.3371	0.5212
		-3.9960		-0.1841	
	0.90	2.8444	5.6751	0.2547	0.5360
		-2.8306		-0.2813	
	0.80	2.0240	4.0352	0.4815	0.5658
		-2.0112		-0.0843	
9	0.95	2.6735	5.3375	0.0758	0.3474
		-2.6640		-0.2717	
	0.90	1.8963	3.7834	0.1435	0.3574
		-1.8871		-0.2138	
	0.80	1.3493	2.6901	0.2293	0.3772
		-1.3408		-0.1479	
25	0.95	1.6041	3.2025	0.0146	0.2085
		-1.5984		-0.1938	
	0.90	1.1378	2.2700	0.2801	0.2144
		-1.1323		0.0657	
	0.80	0.8096	1.6141	0.1624	0.2263
		-0.8045		-0.0640	
100	0.95	0.8021	1.6012	0.0289	0.1042
		-0.7992		-0.0753	
	0.90	0.5689	1.1350	0.1874	0.1072
		-0.5661		0.0802	
	0.80	0.4048	0.8070	0.1916	0.1132
		-0.4022		0.0785	
200	0.95	0.5671	1.1323	0.1536	0.0737
		-0.5651		0.0799	
	0.90	0.4023	0.8026	0.1975	0.0758
		-0.4003		0.1217	
	0.80	0.2862	0.5707	0.2013	0.0800
		-0.2844		0.1213	

Table 6.7. Bootstrap-t Confidence Intervals for LS and PCR estimators with $\beta'\beta = 25$

SNR	ρ	LS		PCR	
4	0.95	0.4752	5.0337	-1.0788	0.3221
		-4.5586		-1.4009	
	0.90	6.5077	3.5642	6.4313	0.3306
		2.9435		6.1006	
	0.80	2.1489	2.5375	4.3304	0.3491
		-0.3885		3.9812	
9	0.95	0.4889	3.3410	0.0711	0.2139
		-2.8520		-0.1428	
	0.90	-0.8235	2.3933	-3.9865	0.2204
		-3.2167		-4.2069	
	0.80	2.5065	1.6882	6.8887	0.2319
		0.8183		6.6568	
25	0.95	-1.9742	2.0164	-5.6797	0.1305
		-3.9905		-5.8102	
	0.90	-0.4202	1.4255	1.5122	0.1332
		-1.8457		1.3790	
	0.80	-1.4631	1.0115	-3.7454	0.1396
		-2.4746		-3.8850	
100	0.95	0.9258	1.0092	-0.4508	0.0649
		-0.0835		-0.5157	
	0.90	0.9990	0.7119	-0.9385	0.0661
		0.2871		-1.0046	
	0.80	0.5913	0.5109	-1.6046	0.0701
		0.0804		-1.6747	
200	0.95	0.4854	0.7134	2.3042	0.0454
		-0.2280		2.2588	
	0.90	0.4104	0.5044	3.2167	0.0469
		-0.0940		3.1698	
	0.80	-0.0056	0.3593	-1.8515	0.0496
		-0.3649		-1.9011	

Table 6.8. Standard Normal Approximation Confidence Intervals for LS and PCR estimators with $\beta'\beta = 25$

SNR	ρ	LS		PCR	
4	0.95	1.9799	3.9527	0.0795	0.2573
		-1.9728		-0.1778	
	0.90	1.4043	2.8018	0.1857	0.2646
		-1.3974		-0.0790	
	0.80	0.9993	1.9922	0.2278	0.2794
		-0.9929		-0.0516	
9	0.95	1.3199	2.6351	0.1210	0.1715
		-1.3152		-0.0505	
	0.90	0.9362	1.8678	0.0129	0.1764
		-0.9316		-0.1635	
	0.80	0.6662	1.3281	0.1675	0.1862
		-0.6619		-0.0187	
25	0.95	0.7920	1.5811	0.0439	0.1029
		-0.7891		-0.0590	
	0.90	0.5617	1.1207	0.1312	0.1059
		-0.5590		0.0253	
	0.80	0.3997	0.7969	0.1675	0.1117
		-0.3972		0.0558	
100	0.95	0.3960	0.7905	0.0178	0.0515
		-0.3946		-0.0337	
	0.90	0.2809	0.5604	-0.0508	0.0529
		-0.2795		-0.1038	
	0.80	0.1999	0.3984	-0.0596	0.0559
		-0.1986		-0.1155	
200	0.95	0.2800	0.5590	0.0455	0.0364
		-0.2790		0.0091	
	0.90	0.1986	0.3962	-0.0219	0.0374
		-0.1976		-0.0593	
	0.80	0.1413	0.2817	-0.0892	0.0395
		-0.1404		-0.1287	

Table 6.9. t Approach Confidence Intervals for LS and PCR estimators with $\beta'\beta = 25$

SNR	ρ	LS		PCR	
4	0.95	2.0051	4.0031	0.0812	0.2606
		-1.9980		-0.1794	
	0.90	1.4222	2.8375	0.1873	0.2680
		-1.4153		-0.0807	
	0.80	1.0120	2.0176	0.2296	0.2829
		-1.0056		-0.0533	
9	0.95	1.3367	2.6687	0.1221	0.1737
		-1.3320		-0.0516	
	0.90	0.9481	1.8917	0.0141	0.1787
		-0.9435		-0.1646	
	0.80	0.6747	1.3450	0.1687	0.1886
		-0.6704		-0.0199	
25	0.95	0.8021	1.6012	0.0446	0.1042
		-0.7992		-0.0597	
	0.90	0.5689	1.1350	0.1318	0.1072
		-0.5661		0.0246	
	0.80	0.4048	0.8070	0.1682	0.1132
		-0.4022		0.0551	
100	0.95	0.4010	0.8006	0.0181	0.0521
		-0.3996		-0.0340	
	0.90	0.2844	0.5675	-0.0505	0.0536
		-0.2831		-0.1041	
	0.80	0.2024	0.4035	-0.0593	0.0566
		-0.2011		-0.1158	
200	0.95	0.2836	0.5661	0.0457	0.0369
		-0.2826		0.0089	
	0.90	0.2011	0.4013	-0.0216	0.0379
		-0.2002		-0.0595	
	0.80	0.1431	0.2853	-0.0889	0.0400
		-0.1422		-0.1289	

Table 6.10. Bootstrap-t Confidence Intervals for LS and PCR estimators with $\beta'\beta = 4$

SNR	ρ	LS		PCR	
4	0.95	-0.4915	2.0059	-0.4432	0.1286
		-2.4975		-0.5718	
	0.90	-0.2227	1.4282	-0.4594	0.1332
		-1.6509		-0.5925	
	0.80	-1.7085	1.0150	-3.9067	0.1393
		-2.7235		-4.0461	
9	0.95	0.8666	1.3419	-1.2409	0.0857
		-0.4753		-1.3266	
	0.90	-0.6035	0.9486	-0.4779	0.0882
		-1.5522		-0.5662	
	0.80	0.7028	0.6754	0.5040	0.0932
		0.0273		0.4108	
25	0.95	0.5456	0.8093	1.0460	0.0514
		-0.2636		0.9946	
	0.90	-0.8741	0.5728	-2.9304	0.0530
		-1.4469		-2.9834	
	0.80	0.0728	0.4052	-1.3649	0.0561
		-0.3324		-1.4210	
100	0.95	-0.1280	0.4007	-1.7600	0.0256
		-0.5287		-1.7856	
	0.90	0.2809	0.2859	1.2224	0.0267
		-0.0050		1.1957	
	0.80	0.6129	0.2023	0.1855	0.0278
		0.4106		0.1577	
200	0.95	-0.2121	0.2832	1.8327	0.0182
		-0.4952		1.8146	
	0.90	0.2742	0.2005	2.0763	0.0186
		0.0737		2.0577	
	0.80	-0.1734	0.1446	-1.6202	0.0198
		-0.3180		-1.6400	

Table 6.11. Standard Normal Approximation Confidence Intervals for LS and PCR estimators with $\beta'\beta = 4$

SNR	ρ	LS		PCR	
4	0.95	0.7920	1.5811	0.0601	0.1029
		-0.7891		-0.0428	
	0.90	0.5617	1.1207	0.0868	0.1059
		-0.5590		-0.0191	
	0.80	0.3997	0.7969	0.0055	0.1117
		-0.3972		-0.1063	
9	0.95	0.5280	1.0540	0.0455	0.0686
		-0.5260		-0.0231	
	0.90	0.3745	0.7471	0.0469	0.0706
		-0.3726		-0.0237	
	0.80	0.2665	0.5312	0.0701	0.0745
		-0.2648		-0.0044	
25	0.95	0.3168	0.6324	0.0024	0.0412
		-0.3156		-0.0388	
	0.90	0.2247	0.4483	0.0191	0.0423
		-0.2236		-0.0233	
	0.80	0.1599	0.3187	0.0301	0.0447
		-0.1589		-0.0146	
100	0.95	0.1584	0.3162	-0.0087	0.0206
		-0.1578		-0.0293	
	0.90	0.1123	0.2241	0.0073	0.0212
		-0.1118		-0.0138	
	0.80	0.0799	0.1594	0.0165	0.0223
		-0.0794		-0.0059	
200	0.95	0.1120	0.2236	0.0068	0.0146
		-0.1116		-0.0078	
	0.90	0.0794	0.1585	0.0107	0.0150
		-0.0791		-0.0042	
	0.80	0.0565	0.1127	0.0526	0.0158
		-0.0562		0.0368	

Table 6.12. t Approach Confidence Intervals for LS and PCR estimators with $\beta'\beta = 4$

SNR	ρ	LS		PCR	
4	0.95	0.8021	1.6012	0.0608	0.1042
		-0.7992		-0.0435	
	0.90	0.5689	1.1350	0.0874	0.1072
		-0.5661		-0.0198	
	0.80	0.4048	0.8070	0.0062	0.1132
		-0.4022		-0.1070	
9	0.95	0.5347	1.0674	0.0460	0.0695
		-0.5328		-0.0235	
	0.90	0.3792	0.7566	0.0473	0.0715
		-0.3774		-0.0241	
	0.80	0.2699	0.5380	0.0706	0.0754
		-0.2681		-0.0049	
25	0.95	0.3208	0.6405	0.0026	0.0417
		-0.3197		-0.0391	
	0.90	0.2276	0.4540	0.0193	0.0429
		-0.2265		-0.0235	
	0.80	0.1619	0.3228	0.0304	0.0453
		-0.1609		-0.0149	
100	0.95	0.1604	0.3202	-0.0086	0.0208
		-0.1598		-0.0294	
	0.90	0.1138	0.2270	0.0075	0.0214
		-0.1132		-0.0140	
	0.80	0.0810	0.1614	0.0166	0.0226
		-0.0804		-0.0060	
200	0.95	0.1134	0.2265	0.0069	0.0147
		-0.1130		-0.0079	
	0.90	0.0805	0.1605	0.0108	0.0152
		-0.0801		-0.0043	
	0.80	0.0572	0.1141	0.0527	0.0160
		-0.0569		0.0367	

6.3. Comparing k Values According to MSE Criterion

In this section, our aim is to compare the bootstrap k selection method with the existing ones in the literature via Monte Carlo simulation study. In this simulation study, sample size $n = 50$, number of explanatory variables $p = 5$, degree of collinearity $\rho = 0.99, 0.95, 0.90, 0.85$, bootstrap sample, $B = 100$ and number of simulations $MCN = 500$ were used as fixed factors. On the other hand, the actual variance of the error term we use as the variable σ^2 will vary depending on signal-to-noise ratio (SNR) and norm of the parameter vector ($||\beta||$). The values of SNR are considered as 1,4, 9, 25, 400.

Repeat the experiment 500 times by generating new error terms and the estimated MSE (EMSE) is calculated to evaluate the estimators:

$$EMSE(\hat{\beta}) = \frac{1}{500}\sum_{r=1}^{500}(\hat{\beta}_{(r)} - \beta)'(\hat{\beta}_{(r)} - \beta)$$

where $\hat{\beta}_{(r)}$ is any estimator of β for the rth replication.

Table 6.13. EMSE values of LS and ridge estimators with $\beta'\beta = 900$

SNR	ρ	ls	k_{boot}	k_{press}	k_{gvc}	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}
1	0.99	7286.3060	1800.6088	2280.0008	1606.3457	2352.1797	2054.9540	6116.3005	1400.8024	1774.1255	4041.7494	7027.2864	845.7570	1457.6767	845.7570	6015.2670
	0.95	1459.8001	822.3746	1017.2545	698.3393	745.2400	732.6441	1318.7173	659.6045	694.7881	1010.5935	1409.1261	676.7060	929.9270	676.7060	1314.4046
	0.90	731.7683	530.0630	605.6383	484.0733	498.2268	546.0325	694.3350	532.0173	521.6951	584.3536	706.4593	588.8102	605.4967	588.8102	693.1441
	0.85	489.2376	389.6338	429.9634	366.0012	380.8665	400.1089	483.7793	452.3855	433.6455	411.5889	472.3499	530.3187	473.9678	530.3187	483.1652
4	0.99	1821.5765	778.6258	787.6875	826.6831	846.9408	863.4396	1629.7561	709.4070	764.8264	1180.5252	1756.9656	765.6361	690.8121	765.6361	1602.7607
	0.95	364.9500	298.8219	289.0948	359.2209	321.8708	399.3895	353.0154	431.9346	384.3659	327.2247	352.5783	594.2095	464.0163	594.2095	351.9150
	0.90	182.9421	155.6372	155.9522	171.0035	164.8702	200.0925	180.2218	186.6835	173.8037	166.5930	177.6880	164.5944	175.0548	164.5944	179.9026
	0.85	122.3094	106.1521	108.9350	112.5458	110.9263	118.2178	121.5904	144.8332	131.2875	113.9137	118.5372	150.9232	123.3775	150.9232	121.4371
9	0.99	809.5896	527.2715	497.0788	570.7760	515.0701	547.3761	769.5830	525.0271	514.0080	610.1779	781.0434	753.1844	554.0354	753.1844	756.7160
	0.95	162.2000	148.9755	140.2099	153.0282	147.9999	291.0199	155.1437	234.3952	210.1200	149.5089	156.7722	313.3131	205.4799	313.3131	154.6844
	0.90	81.3076	81.4832	77.5986	81.6591	81.2003	80.0548	81.1700	105.8660	93.3449	79.0031	79.5385	78.3084	79.0402	78.3084	81.0198
	0.85	54.3597	56.7190	54.5559	56.2001	57.6579	58.4860	54.3039	82.2728	62.4665	54.1309	53.1812	64.5808	54.0027	64.5808	54.2349
25	0.99	291.4522	318.8087	294.4092	318.8983	274.7315	334.2213	288.8475	377.0767	342.5765	269.9778	281.6968	724.7892	425.2483	724.7892	284.3109
	0.95	58.3920	58.5620	57.9720	58.0947	58.1270	137.9964	57.2871	78.3677	64.1738	56.7825	57.0631	81.9124	55.4431	81.9124	57.1231
	0.90	29.2707	29.7034	29.3808	29.4204	29.5702	29.0375	29.2597	38.7478	30.1515	29.0265	28.5302	483.2159	269.0704	483.2159	29.2068
	0.85	19.5695	19.6368	19.5032	19.5171	19.5184	20.0138	19.4987	54.9232	26.2393	19.4240	19.2238	28.5399	19.7730	28.5399	19.4765
400	0.99	18.2158	18.4565	18.2582	18.2700	18.3513	18.1252	18.2117	22.9746	18.5906	18.1250	17.8626	730.1910	417.7649	730.1910	17.9656
	0.95	3.6495	3.6821	3.6573	3.6584	3.6680	3.6490	3.6494	3.7120	3.7450	3.6489	3.7228	42.9951	5.7381	42.9951	3.6405
	0.90	1.8294	1.8301	1.8257	1.8259	1.8260	1.8420	1.8284	1.8495	1.8348	1.8269	1.8293	4.1766	1.8922	4.1766	1.8256
	0.85	1.2231	1.2337	1.2231	1.2231	1.2237	1.2225	1.2231	1.2277	1.2267	1.2225	1.2247	1.8453	1.2383	1.8453	1.2218

Table 6.14. EMSE values of LS and ridge estimators with $\beta'\beta = 100$

SNR	ρ	ls	k_{boot}	k_{press}	k_{gvc}	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}
1	0.99	809.5896	196.5284	249.9228	175.5356	260.7812	214.6437	667.1548	158.6447	193.3421	449.4626	780.6777	94.1338	161.8693	94.1338	656.4308
	0.95	162.2000	93.7724	114.2431	82.6723	85.8884	85.5347	147.7385	75.3157	79.9585	113.4164	156.6514	78.2967	111.4214	78.2967	147.2516
	0.90	81.3076	59.2683	67.3872	54.7304	56.0386	59.2928	79.4529	59.1012	58.9536	64.6622	78.5791	58.9857	66.8143	58.9857	79.3091
	0.85	54.3597	42.8191	47.6514	37.0995	39.3267	42.2281	52.0237	50.5176	50.7107	45.7751	52.4699	62.2911	53.4151	62.2911	51.9615
4	0.99	202.3974	90.4316	92.8553	97.5248	97.4525	94.7611	174.9746	82.9648	88.2011	133.2713	195.2714	88.2347	82.8691	88.2347	172.2466
	0.95	40.5500	31.3076	31.0289	36.8159	33.9162	34.9848	40.1903	40.3605	36.2756	35.1713	39.1426	64.3158	49.4894	64.3158	40.0487
	0.90	20.3269	18.4207	17.7869	20.9788	20.0429	30.9934	19.9139	32.8018	29.1624	19.3518	19.7069	26.4829	21.0365	26.4829	19.8836
	0.85	13.5899	12.8332	12.4295	14.3650	14.0579	18.5352	13.5092	25.1367	24.9373	13.2540	13.1668	39.1690	24.0232	39.1690	13.4937
9	0.99	89.9544	60.9686	56.5741	66.7176	58.3677	72.5190	80.5797	64.6199	65.3291	69.8255	86.7546	84.4920	62.7524	84.4920	79.4605
	0.95	18.0222	17.4590	16.2660	18.4148	17.4920	17.1170	17.9786	22.5443	19.0074	17.0251	17.4152	60.3863	39.2056	60.3863	17.9151
	0.90	9.0342	8.9290	8.4684	8.9279	8.8535	15.7366	8.8959	11.1266	9.2341	8.6919	8.7776	12.6043	9.3751	12.6043	8.8821
	0.85	6.0400	6.2067	5.9827	6.2086	6.2770	9.0236	6.0122	11.0423	9.4530	5.9973	5.8812	22.7835	11.6604	22.7835	6.0059
25	0.99	32.3836	30.5420	29.3134	30.9859	28.0017	41.8189	31.1204	33.7781	29.9764	28.4430	31.3065	75.0398	38.8579	75.0398	30.6829
	0.95	6.4880	6.2543	6.2180	6.2224	6.1955	6.5942	6.4662	12.6509	13.2636	6.3054	6.2755	51.4384	28.5879	51.4384	6.4433
	0.90	3.2523	3.3058	3.2841	3.2893	3.3410	3.2415	3.2512	3.5481	3.5098	3.2398	3.2029	3.9083	3.2112	3.9083	3.2453
	0.85	2.1744	2.1884	2.1784	2.1804	2.1859	2.3028	2.1681	4.1396	2.2593	2.1596	2.1478	2.3146	2.1551	2.3146	2.1658
400	0.99	2.0240	2.0412	2.0287	2.0300	2.0390	2.0139	2.0235	2.5527	2.0656	2.0139	1.9847	81.1323	46.4183	81.1323	1.9962
	0.95	0.4055	0.4079	0.4060	0.4061	0.4071	0.4111	0.4055	0.4546	0.4111	0.4053	0.4053	4.1739	0.5942	4.1739	0.4045
	0.90	0.2033	0.2042	0.2032	0.2032	0.2032	0.2342	0.2032	0.2043	0.2036	0.2031	0.2032	0.4560	0.2103	0.4560	0.2029
	0.85	0.1359	0.1365	0.1358	0.1358	0.1358	0.1358	0.1359	0.1378	0.1367	0.1358	0.1366	0.1849	0.1367	0.1849	0.1336

Table 6.15. EMSE values of LS and ridge estimators with $\beta'\beta = 16$

SNR	ρ	ls	k_{boot}	k_{press}	k_{gvc}	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}
1	0.99	129.5343	31.4591	39.7519	28.0051	41.2994	36.3284	108.2469	24.5977	31.1026	71.4502	124.9232	14.8433	25.5375	14.8433	106.4660
	0.95	25.9520	14.1520	17.9850	11.7700	12.7338	12.9961	23.4102	11.3328	12.0816	17.6461	25.0398	12.5916	16.8696	12.5916	23.3328
	0.90	13.0092	9.3153	10.7462	8.1475	8.5137	9.0556	12.2055	8.9043	8.9266	10.1487	12.5610	10.4118	10.7573	10.4118	12.1849
	0.85	8.6976	6.9161	7.6461	6.3975	6.6801	7.0006	8.5847	7.5376	7.0104	7.2780	8.3959	10.6825	9.0705	10.6825	8.5739
4	0.99	32.3836	13.6957	14.0033	14.6966	15.0567	15.3500	28.9734	12.6117	13.5969	20.9871	31.2349	13.6113	12.2811	13.6113	28.4935
	0.95	6.4880	5.2163	5.0670	6.1905	5.5909	7.3275	6.2709	6.8106	6.3071	5.6926	6.2816	5.6171	5.7461	5.6171	6.2508
	0.90	3.2523	2.9059	2.8338	3.3292	3.1745	3.9735	3.2221	3.9416	3.5726	3.0456	3.1483	6.4264	4.4722	6.4264	3.2166
	0.85	2.1744	2.0272	1.9762	2.2194	2.1780	2.8100	2.1626	3.7358	3.1807	2.0757	2.1200	2.1418	2.1340	2.1418	2.1600
9	0.99	14.3208	9.7888	9.1225	10.7677	9.3963	11.2355	12.9429	10.3154	10.4653	11.1600	13.8128	13.5709	10.1258	13.5709	12.7562
	0.95	2.8692	2.7069	2.5362	2.7801	2.6855	4.1044	2.7962	3.2080	2.9173	2.6477	2.7775	4.2156	2.9954	4.2156	2.7871
	0.90	1.4383	1.4497	1.3748	1.4581	1.4497	1.4054	1.4367	1.8856	1.5302	1.4014	1.3926	9.0810	5.4825	9.0810	1.4341
	0.85	0.9616	0.9777	0.9447	0.9799	0.9871	0.9774	0.9606	1.0281	0.9970	0.9527	0.9430	0.9802	0.9479	0.9802	0.9594
25	0.99	5.1814	4.9839	4.7635	5.0514	4.5338	7.4752	4.9401	6.5748	6.2908	4.6695	5.0002	12.8191	7.6829	12.8191	4.8751
	0.95	1.0381	1.0619	1.0456	1.0483	1.0502	1.0212	1.0373	1.3948	1.0939	1.0205	1.0065	9.2018	5.3125	9.2018	1.0337
	0.90	0.5204	0.5348	0.5274	0.5283	0.5375	0.5220	0.5202	1.3941	0.5533	0.5197	0.5084	9.2725	5.3085	9.2725	0.5192
	0.85	0.3479	0.3532	0.3496	0.3499	0.3534	0.4220	0.3474	1.3789	1.4713	0.3465	0.3412	8.3764	4.5203	8.3764	0.3470
400	0.99	0.3238	0.3295	0.3279	0.3283	0.3345	0.3387	0.3238	0.5847	0.4619	0.3237	0.3209	12.1381	5.3546	12.1381	0.3203
	0.95	0.0649	0.0653	0.0650	0.0650	0.0652	0.0649	0.0649	0.0766	0.0658	0.0649	0.0648	0.7248	0.0984	0.7248	0.0647
	0.90	0.0325	0.0327	0.0325	0.0325	0.0325	0.0325	0.0325	0.0341	0.0326	0.0325	0.0363	4.1042	1.6586	4.1042	0.0328
	0.85	0.0217	0.0219	0.0217	0.0217	0.0217	0.0217	0.0217	0.0238	0.0291	0.0217	0.0226	0.0469	0.0226	0.0469	0.0217

Table 6.16. EMSE values of LS and ridge estimators with $\beta'\beta = 4$

SNR	ρ	ls	k_{boot}	k_{press}	k_{gcv}	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}
1	0.99	32.3836	7.9926	10.2106	7.1635	10.4309	8.7182	26.6073	6.2004	7.8495	17.7712	31.2328	3.8599	6.6747	3.8599	26.1788
	0.95	6.4880	3.7166	4.5421	3.1953	3.3620	3.3625	5.8067	3.0014	3.1243	4.5346	6.2615	3.3225	4.1262	3.3225	5.7882
	0.90	3.2523	2.3191	2.6855	2.0021	2.0876	2.2586	3.0696	2.2054	2.1983	2.5153	3.1405	2.4572	2.5974	2.4572	3.0644
	0.85	2.1744	1.7128	1.9064	1.4837	1.5648	1.6650	2.1166	1.8696	1.7772	1.7897	2.0986	2.4807	2.2000	2.4807	2.1140
4	0.99	8.0959	3.6422 95	3.7374	3.9089	3.9264	3.8537	6.8791	3.3251	3.5140	5.3248	7.8137	3.4270	3.1472	3.4270	6.7766
	0.95	1.6220	1.2568	1.2412	1.4726	1.3566	1.3994	1.6076	1.6144	1.4510	1.4069	1.5657	2.5726	1.9796	2.5756	1.6019
	0.90	0.8131	0.6849	0.6949	0.7638	0.7362	0.9025	0.7989	0.9949	0.9271	0.7446	0.7870	1.1827	0.9124	1.1827	0.7976
	0.85	0.5436	0.5054	0.4939	0.5588	0.5479	0.6426	0.5416	0.7567	0.6768	0.5210	0.5267	1.4926	0.9359	1.4926	0.5408
9	0.99	3.6342	2.4103	2.2584	2.6418	2.3340	2.7345	3.2552	2.4360	2.4431	2.7724	3.5078	3.3660	2.5111	3.3660	3.2082
	0.95	0.7281	0.7425	0.6790	0.7758	0.7344	0.7002	0.7267	1.0766	0.9694	0.6971	0.7072	0.8270	0.6854	0.8270	0.7241
	0.90	0.3650	0.3784	0.3580	0.3784	0.3805	0.7365	0.3610	0.6403	0.5255	0.3587	0.3560	0.4639	0.3728	0.4639	0.3605
	0.85	0.2440	0.2504	0.2411	0.2505	0.2527	0.3899	0.2430	0.3646	0.2555	0.2425	0.2375	1.2234	0.6444	1.2234	0.2428
25	0.99	1.2953	1.2764	1.2066	1.2926	1.1504	1.2520	1.2759	1.3510	1.2166	1.1530	1.2524	3.0540	1.6183	3.0540	1.2545
	0.95	0.2595	0.2668	0.2652	0.2661	0.2690	0.4042	0.2585	0.4265	0.3609	0.2572	0.2524	1.4412	0.7084	1.4412	0.2577
	0.90	0.1301	0.1330	0.1311	0.1313	0.1323	0.1292	0.1301	0.1558	0.1598	0.1292	0.1271	0.8122	0.3445	0.8122	0.1298
	0.85	0.0870	0.0881	0.0873	0.0873	0.0871	0.0977	0.0869	0.0895	0.0902	0.0866	0.0855	0.1552	0.0896	0.1552	0.0868
400	0.99	0.0810	0.0819	0.0811	0.0812	0.0819	0.0809	0.0809	0.1054	0.0866	0.0806	0.0796	2.9441	1.2169	2.9441	0.0798
	0.95	0.0162	0.0164	0.0162	0.0162	0.0163	0.0162	0.0162	0.0175	0.0178	0.0162	0.0162	0.1433	0.0222	0.1433	0.0162
	0.90	0.0081	0.0082	0.0081	0.0081	0.0081	0.0081	0.0081	0.0081	0.0081	0.0081	0.0081	0.0191	0.0084	0.0191	0.0081
	0.85	0.0054	0.0055	0.0054	0.0054	0.0054	0.0054	0.0054	0.0063	0.0056	0.0054	0.0054	0.0079	0.0055	0.0079	0.0054

The following results are obtained via using the following Reduction Rate equation. The RR equation is given below:

$$RR = \frac{EMSE(\tilde{\beta}) - EMSE(\hat{\beta}(k_{boot}))}{EMSE(\tilde{\beta})} \, .100$$

where $\tilde{\beta}$ is any estimator to be compared with k_{boot}. RR values help here us to evaluate the performance of k_{boot} over the competitors in percentage. The results are reported in Tables 6.13-6.16 and the comments are as follows:

As SNR and $\beta'\beta$ remains constant and ρ increases, it has been observed that k_{boot} loses its validity over LS, k_{press} and k_{gvc}. This means that, with smaller SNR k_{boot} performs better, however as SNR increases the validity of confidence intervals for all estimators do not vary among each other.

When $k_{boo□}$ is contrasted with k_9, k_{10}, k_{11} it is seen that the RR is enormous when SNR is huge regardless of what $\beta'\beta$ is. The RR level of k_{boot} over k_9, k_{10}, k_{11} changes from 99% to 5%.

The superiority of k_{boot} over LS is decreasing as ρ decreases and SNR has impact boot on this decreasement. As ρ decreases, the superiority percentage of k_{boot} over LS drops from 75% to 20% when SNR = 1 and from 55% to 5% when SNR = 4. As SNR increases, k_{boot} loses its superiority over LS.

While comparing the k_{boot} and k_{press}, the RR is expanding from 9% to 21% as ρ increases at SNR = 1. As SNR expands, the prevalence of k_{boot} over k_{gvc} becomes clear as opposed to the predominance of k_{boot} over k_{press}. This RR rate is somewhere in the range of 8% and 0.003% at SNR = 4,9. In any case, when SNR gets too enormous, k_{boot} loses its predominance over k_{press} and k_{gvc}.

According to the simulation study, it has been observed EMSE of some k values performed better than EMSE of LS estimators. Furthermore, k_{boot} tends to give more information than LS and k estimators while considering degree of collinearity.

7. REAL DATA ANALYSIS

In this section, we analyse the pollution data originally used by McDonald and Schwing (1973) to illustrate ridge regression. The data set consists of 15 explanatory variables, and the response variable is the total age-adjusted death rate of 60 metropolitan areas in the United States from 1959 to 1961.

After the explanatory variables are standardized by unit length scaling, the condition number, $\kappa = \frac{\lambda_{max}}{\lambda_{min}} = 930.5641$ indicates that there is serious collinearity. In addition, VIF values greater than 10, $VIF_{12} = 98.6399$ and $VIF_{13} = 104.9824$, indicate that the variables x_{12} and x_{13} contribute to collinearity, where x_{12} is the relative population potential of hydrocarbons, and x_13 is the relative pollution potential oxides of nitrogen. The simple correlation 0.9838 between the variables x_{12} and x_{13} also supports VIF results.

In order to perform ridge regression further, we should choose a regularization parameter. By examining the ridge trace, McDonald and Schwing (1973) determined that the k value was 0.2. Here we use 60 observations as the population of 1000 bootstrap samples. For each value of k selected in an interval from 0 to 0.2, with a step size of 0.001, the k values are randomly selected. 24082020 is used as the state number to generate the bootstrap sample. For these data, we report the results by Table 7.1, which presents several metrics for MSEP and selected extended metrics for the subset of values k considered. The average MSEP, 10% trimmed MSEP, median MSEP are presented by Table 7.1. The standard deviation and the interquartile range for the MSEP are also given in Table 7.1. The minimum MSEP is seen at $k = 0.1402$ for the average, $k = 0.0985$ for both the trimmed mean and median.

In addition, before drawing conclusions, we have to run another 1,000 bootstrap samples with status 26082020. Table 7.2 reports the results corresponding to this sample.

We propose an optimal choice of k because of the follwing reasons:

For the average MSEP, it has been observed that as the value of k increases, the average MSEP value decreases, reaching an optimal point. However, as the average MSEP reaches the optimal point of a certain k value, it starts to become larger as the optimal point is reached.

It can be seen from Table 7.1 that the optimal k value for a seed number of 24082020 is 0.1402. This is considered to be the best value of k because it corresponds to the average MSEP, trimmed average (10%), median and standard deviation. But, on the other hand, for the 26082020 seed, the best k value tends to be 0.1320, with the lowest MSEP, trimmed mean (10%), minimum median and standard deviation.

In general, it has been observed that, regardless to the seed number or to the regression, the optimal value for k follows same pattern for each iteration. This can be explained through "N-shape" or quadratic function. Simply for both regression as k values increases all correspondence values such as mean or standard deviation tends to decrease up to optimal point. When the K value goes further all other parameters tends to increase. Thus, they move far away from the optimal point.

Table 7.1. Bootstrap results for pollution data with seed number 24082020.

K	Average MSEP	Trimmed mean (10%)	Median	Standard deviation prediction	İnterquartile range
0	0.1929	0.1610	0.1533	0.3009	0.0786
0.0378	0.1844	0.1583	0.1458	0.1792	0.0743
0.0575	0.1825	0.1578	0.1481	0.2047	0.0744
0.0985	0.1799	0.1536	0.1451	0.1917	0.0743
0.1103	0.1789	0.1577	0.1468	0.1540	0.0758
0.1200	0.1787	0.1574	0.1487	0.1642	0.0753
0.1296	0.1774	0.1580	0.1495	0.1417	0.0741
0.1320	0.1772	0.1552	0.1456	0.1580	0.0703
0.1402	0.1728	0.1571	0.1481	0.1176	0.0766
0.1410	0.1809	0.1589	0.1488	0.1584	0.0759
0.1754	0.1824	0.1524	0.1458	0.2450	0.0747
0.1866	0.1874	0.1617	0.1519	0.1839	0.0808

Table 7.2. Bootstrap results for pollution data with seed number 26082020.

k	Average MSEP	Trimmed mean (10%)	Median	Standard deviation prediction	İnterquartile range
0	0.1863	0.1584	0.1468	0.2215	0.0724
0.0378	0.1815	0.1615	0.1484	0.1471	0.0784
0.0575	0.1806	0.1562	0.1454	0.1838	0.0734
0.0985	0.1792	0.1564	0.1465	0.1721	0.0749
0.1103	0.1765	0.1558	0.1488	0.1686	0.0756
0.1200	0.1762	0.1546	0.1448	0.1457	0.0728
0.1296	0.1744	0.1579	0.1476	0.1369	0.0828
0.1320	0.1679	0.1540	0.1476	0.1137	0.0695
0.1402	0.1796	0.1541	0.1461	0.2536	0.0677
0.1410	0.1811	0.1589	0.1495	0.1683	0.0815
0.1754	0.1853	0.1577	0.1504	0.2268	0.0777
0.1866	0.1890	0.1595	0.1520	0.1986	0.0787

8. CONCLUSIONS

The existing literature regarding to the choice of k is proposed and discussed many great researchers. The observation regarding to the performance of the ridge estimator is depended on k selection. Furthermore, this study extend the existing literature regarding to the k selection process via adding new methods such as $k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}$ and k_{12}.

According to the simulated data, it has been observed that k selection methods are dramatically affected from SNR and collinearity problem. Furthermore, one of the main finding is that, for every estimation there is a k which perform better than LS. When SNR is small enough and there is strong degree of collinearity, k_{boot} tends to perform better among others.

Furthermore, as for second scope of this study, confidence intervals generated via k_{boot} method analysed, and it has been observed that as SNR increases the confidence intervals decrease. Secondly, as ρ and $\beta'\beta$ decreases the confidence interval levels also decrease. Therefore, in the case of low ρ and $\beta'\beta$ with high SNR; the ridge confidence intervals are narrower than LS confidence interval which gives more reliable results.

Last but not least, PCR estimation also computed to analyses the strength of confidence level via considering k selection processes. The most interesting result for PCR estimation is that it performs in opposite to k_{boot}, while other determinants remain constant and ρ changes. When ρ is high, the PCR estimation is more favorable.

k selection methods can be seriously affected by SNR and strength of the collinearity. There are some cases where LS perform quite purely than some k's. Furthermore, when SNR is small enough and degree of collinearity is quite large, LS, k_7, k_8 and k_{12} perform weaker than k_{boot}. Although it loses its superiorty over

LS as SNR gets large, it performs significantly better than newly proposed k estimators in terms of EMSE criterion.

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Completed my primary and secondary education in Northern Cyprus. In 2014, I started my undergraduate education at Çukurova University, Department of Statistics. I finished my undergraduate education as the top student in the Faculty of Science and Letters, and immediately after that, I was accepted to the master's program in the same department at the Çukurova university in 2018. I started working as an Internal Auditor at Creditwest Bank in 2019. After working there for 10 months and I quit my job. In September 2020, I started to work at the TRNC Central Bank.

APPENDICES

APPENDIX-1: Bootstrap-t Confidence Intervals for LS and ridge estimators with $\beta'\beta = 625$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	-5.1275	55.0506	-4.3570	25.7046	-3.6470	25.6928	-3.8789	25.5515	-16.7416	24.9962	-51.2237	17.1166	-0.0754	49.3671	2.1931	13.9188	29.5280	15.0538	9.6290	36.6142	-4.2058	36.9149	-0.3011	12.3208	63.4740	27.4339	-0.3011	12.3208	-4.4975	51.2062
		-60.1781		-30.0616		-29.3397		-29.4304		-41.7378		-68.3402		-49.4425		-11.7256		14.4742		-26.9852		-41.1207		-12.6218		36.0401		-12.6218		-55.7037	
	0.95	24.4550	24.4290	27.6040	19.6004	27.0068	19.5513	27.0318	19.4961	30.4380	18.1404	26.6582	19.6486	24.3724	24.3375	21.1399	12.8951	25.7278	16.6214	26.5526	21.1557	24.4336	21.7044	23.2997	17.1098	23.0102	21.7702	23.2997	17.1098	24.3756	18.4400
		0.0259		8.0036		7.4555		7.5357		12.2976		7.0096		0.0349		8.2448		9.1063		5.3969		2.7292		6.1899		1.2400		6.1899		5.9356	
	0.90	-3.0922	17.3561	6.8486	15.4577	8.3398	15.4274	8.4561	15.4002	9.3773	14.6954	-13.4526	10.4720	-1.5721	17.1940	7.1579	10.2169	6.7894	14.4516	6.5844	15.8379	-2.2867	16.2632	8.4809	14.7189	2.2652	16.5779	8.4809	14.7189	8.5076	16.3919
		-20.4483		-8.6091		-7.0875		-6.9442		-5.3182		-23.9246		-18.7662		-3.0589		-7.6621		-9.2534		-18.5499		-6.2379		-14.3127		-6.2379		-7.8842	
	0.85	22.1652	14.2955	28.9772	13.2023	28.8424	13.1083	28.9141	13.0914	29.9803	12.8186	30.9048	11.8595	22.3614	14.2665	22.6161	10.0837	29.4693	11.5759	25.5224	13.7341	23.8545	13.6416	32.7885	11.7514	27.0136	13.5118	32.7885	11.7514	19.3507	13.2642
		7.8697		15.7749		15.7341		15.8226		17.1617		19.0453		8.0949		12.5325		17.8934		11.7883		10.2129		21.0371		13.5018		21.0371		6.0865	
9	0.99	2.9586	36.6732	8.8282	21.7824	10.4992	21.6759	10.3746	21.5296	12.9838	21.7213	-4.5015	17.3566	7.8831	34.7165	3.2543	15.7557	-3.8995	15.4306	14.0747	28.3210	1.1612	27.7468	-30.0425	6.0585	-40.7587	14.4949	-30.0425	6.0585	9.3204	26.3761
		-33.7146		-12.9542		-11.1768		-11.1550		-8.7375		-21.8581		-26.8335		-12.5014		-19.3301		-14.2463		-26.5857		-36.1010		-55.2536		-36.1010		-17.0557	
	0.95	12.4169	16.4204	2.2519	14.5980	2.3378	14.6178	2.1698	14.5824	0.4337	14.1946	3.5617	14.6482	12.3012	16.3997	1.9590	14.7655	-0.2430	13.5938	5.4197	15.2444	10.1149	15.3667	13.7710	12.2926	18.8199	15.0494	13.7710	12.2926	12.2038	15.9672
		-4.0035		-12.3461		-12.2801		-12.4126		-13.7609		-11.0865		-4.0985		-12.8065		-13.8369		-9.8247		-5.2518		1.4784		3.7704		1.4784		-3.7634	
	0.90	-12.0515	11.6862	-9.3385	11.0496	-9.6198	11.0688	-9.5854	11.0556	-9.0543	10.8241	-9.7843	10.9913	-12.0142	11.6789	-9.1217	10.0577	-8.9832	10.8670	-9.7432	11.1481	-11.6286	11.2926	-8.2539	10.7257	-10.6073	11.4279	-8.2539	10.7257	-2.9787	11.3318
		-23.7377		-20.3881		-20.6885		-20.6410		-19.8785		-20.7756		-23.6931		-19.1794		-19.8502		-20.8914		-22.9211		-18.9796		-22.0352		-18.9796		-14.3105	
	0.85	16.3701	9.5511	14.8997	9.1684	14.7598	9.1619	14.7313	9.1536	14.4726	9.0729	15.5889	9.3871	16.3613	9.5494	11.1947	7.8194	12.8719	7.2775	15.6157	9.3933	15.6644	9.2872	-36.1366	3.3131	-30.3472	5.3749	-36.1366	3.3131	15.3461	9.5206
		6.8190		5.7313		5.5979		5.5777		5.3997		6.2019		6.8119		3.3754		5.5944		6.2224		6.3772		-39.4497		-35.7221		-39.4497		5.8255	
25	0.99	6.4841	22.1632	2.9552	17.6512	2.8818	17.8046	2.8808	17.7286	2.6981	17.2281	1.9532	6.9266	2.1100	20.7616	5.2313	14.3506	2.7293	15.3352	4.3921	19.6318	5.3100	19.5936	6.1217	6.8935	2.7116	14.1485	6.1217	6.8935	2.0249	19.3800
		-15.6791		-14.6960		-14.9228		-14.8478		-14.5300		-4.9734		-18.6516		-9.1193		-12.6059		-15.2397		-14.2837		-0.7717		-11.4369		-0.7717		-17.3551	
	0.95	-3.7630	9.8687	-3.9606	9.4421	-3.7954	9.4525	-3.7909	9.4435	-3.7364	9.3272	-3.8809	9.7075	-3.7646	9.8670	3.1137	7.0190	4.8966	7.2676	-3.8800	9.7093	-3.8446	9.5804	8.1101	5.2541	-1.1784	7.5510	8.1101	5.2541	-3.1241	9.6980
		-13.6317		-13.4027		-13.2479		-13.2345		-13.0636		-13.5884		-13.6316		-3.9053		-2.3710		-13.5893		-13.4249		2.8561		-8.7294		2.8561		-12.8220	
	0.90	7.2371	6.9637	10.5396	6.8223	10.6156	6.8251	10.6854	6.8221	11.8615	6.7704	22.4592	6.1264	7.4370	6.9557	10.3120	6.2847	16.2515	6.5509	9.1786	6.8857	9.5006	6.8474	21.7448	6.1651	12.4454	6.7407	21.7448	6.1651	6.5205	6.8819
		0.2734		3.7172		3.7905		3.8633		5.0911		16.3328		0.4813		4.0273		9.7006		2.2930		2.6532		15.5797		5.7047		15.5797		-0.3614	
	0.85	2.9862	5.7219	0.2854	5.6308	0.5316	5.6365	0.4791	5.6346	-0.0151	5.6169	1.8230	5.6819	2.9740	5.7215	0.5241	5.6388	-2.7280	5.5161	1.8412	5.6825	1.2225	5.6465	-3.7237	5.4774	1.1551	5.6585	-3.7237	5.4774	1.9811	5.6602
		-2.7357		-5.3454		-5.1049		-5.1555		-5.6320		-3.8589		-2.7475		-5.1147		-8.2441		-3.8414		-4.4240		-9.2011		-4.5034		-9.2011		-3.6791	
100	0.99	3.8739	11.0181	11.1314	10.4405	10.9358	10.4447	11.0752	10.4319	12.6338	10.2784	-3.4070	5.5312	6.8183	10.7964	11.5444	9.3696	17.9506	9.3472	7.4068	10.7530	5.8217	10.6499	-10.4094	3.3334	17.4786	6.9073	-10.4094	3.3334	7.4164	10.9177
		-7.1442		0.6909		0.4911		0.6433		2.3553		-8.9382		-3.9781		2.1748		8.6034		-3.3461		-4.8282		-13.7428		10.5713		-13.7428		-3.5013	
	0.95	2.8186	4.9468	2.3645	4.8849	2.4258	4.8905	2.4169	4.8893	2.3297	4.8766	2.6915	4.9286	2.8173	4.9466	-6.7491	3.7515	-13.1785	3.0482	2.6919	4.9287	2.3450	4.8799	-6.5233	3.4499	-0.5982	4.4018	-6.5233	3.4499	2.4272	4.9169
		-2.1282		-2.5203		-2.4647		-2.4724		-2.5470		-2.2371		-2.1293		-10.5006		-16.2267		-2.2368		-2.5349		-9.9732		-4.9999		-9.9732		-2.4897	
	0.90	3.7402	3.5083	3.2435	3.4855	3.2720	3.4889	3.2618	3.4884	3.1399	3.4833	0.1675	3.3493	3.6950	3.5065	3.0254	3.3447	1.2913	3.4042	3.4898	3.4979	3.3963	3.4902	1.6073	3.4181	3.1806	3.4853	1.6073	3.4181	3.2687	3.4862
		0.2319		-0.2420		-0.2169		-0.2266		-0.3434		-3.1818		0.1886		-0.3193		-2.1129		-0.0081		-0.0939		-1.8108		-0.3047		-1.8108		-0.2175	
	0.85	3.4565	2.8542	4.5065	2.8404	4.1901	2.8439	4.2060	2.8437	4.4179	2.8407	4.1725	2.8442	3.4636	2.8541	4.1845	2.5534	4.4223	2.8407	4.1575	2.8444	4.5471	2.8397	6.9341	2.8048	4.3603	2.8417	6.9341	2.8048	3.8285	2.8484
		0.6023		1.6661		1.3462		1.3623		1.5772		1.3284		0.6095		1.6311		1.5816		1.3131		1.7074		4.1293		1.5186		4.1293		0.9801	
200	0.99	4.6565	7.7657	9.4663	7.5647	9.2811	7.5727	9.3707	7.5685	10.9290	7.4971	14.6171	5.5550	5.6079	7.7251	8.8026	7.6524	19.4804	7.0079	7.7616	7.6390	6.1757	7.6340	-14.6064	2.5346	22.3323	5.0889	-14.6064	2.5346	4.6814	7.7656
		-3.1092		1.9016		1.7084		1.8023		3.4318		9.0621		-2.1172		1.1502		12.4725		0.1225		-1.4583		-17.1411		17.2434		-17.1411		-3.0843	
	0.95	4.3457	3.4746	3.2901	3.4516	3.5844	3.4565	3.5686	3.4562	3.3174	3.4501	2.9982	3.4418	4.3325	3.4743	3.0955	3.3233	2.8086	3.4377	3.9494	3.4653	3.3689	3.4516	-7.0339	3.1710	0.9294	3.3931	-7.0339	3.1710	3.5985	3.4695
		0.8711		-0.1615		0.1279		0.1124		-0.1327		-0.4437		0.8582		-0.2278		-0.6290		0.4842		-0.0827		-10.2048		-2.4637		-10.2048		0.1289	
	0.90	-0.3090	2.4661	0.1108	2.4570	-0.0628	2.4602	-0.0574	2.4601	0.0570	2.4574	1.8527	2.4098	-0.2914	2.4657	-0.0629	2.4492	0.8002	2.4389	-0.1294	2.4619	-0.1510	2.4597	1.8817	2.4112	0.2802	2.4521	1.8817	2.4112	-0.2025	2.4573
		-2.7751		-2.3462		-2.5230		-2.5175		-2.4003		-0.5570		-2.7571		-2.5121		-1.6387		-2.5913		-2.6107		-0.5294		-2.1719		-0.5294		-2.6598	
	0.85	3.4027	2.0235	3.7851	2.0179	3.6324	2.0198	3.6372	2.0197	3.6989	2.0187	3.6696	2.0191	3.3853	2.0034	3.1048	2.0211	4.3728	2.0073	3.5150	2.0217	4.7686	2.0099	8.6170	1.9325	4.8829	1.9998	8.6170	1.9325	3.4756	2.0205
		1.3793		1.7672		1.6127		1.6176		1.6802		1.6505		1.3819		1.0837		2.3654		1.4933		2.7588		6.6845		2.8831		6.6845		1.4551	

APPENDIX-2: Standard Normal Approximation Confidence Intervals for LS and ridge estimators with with $\beta'\beta = 625$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	21.7754	43.5104	12.6453	25.2433	12.6646	25.2816	12.5955	25.1431	12.2211	24.3949	8.4992	16.9451	20.1779	40.3140	7.7839	15.5221	7.8134	15.5821	16.4560	32.8670	21.5806	43.1208	6.7803	13.5038	13.1004	26.1465	6.7803	13.5038	20.1273	40.2127
		-21.7351		-12.5980		-12.6171		-12.5476		-12.1738		-8.4459		-20.1361		-7.7382		-7.7687		-16.4111		-21.5402		-6.7235		-13.0462		-6.7235		-20.0854	
	0.95	9.7631	19.4850	8.5013	16.9082	8.5175	16.9404	8.4755	16.8556	8.0573	16.0044	8.4604	16.8307	9.7410	19.4402	7.0690	13.9874	7.5413	14.9512	8.9307	17.7863	9.6806	19.3163	7.7015	15.2762	9.0743	18.0795	7.7015	15.2762	9.7359	19.4298
		-9.7219		-8.4069		-8.4229		-8.3800		-7.9470		-8.3703		-9.6992		-6.9184		-7.4100		-8.8556		-9.6357		-7.5747		-9.0051		-7.5747		-9.6938	
	0.90	6.9231	13.8042	6.4395	12.8467	6.4512	12.8700	6.4242	12.8162	6.2278	12.4268	4.8365	9.6641	6.8836	13.7258	5.7769	11.5297	6.1507	12.2748	6.5401	13.0457	6.8659	13.6910	6.2321	12.4342	6.7307	13.4226	6.2321	12.4342	6.8818	13.7220
		-6.8811		-6.4071		-6.4187		-6.3920		-6.1990		-4.8276		-6.8421		-5.7528		-6.1241		-6.5056		-6.8251		-6.2021		-6.6920		-6.2021		-6.8403	
	0.85	5.6686	11.2942	5.3990	10.7492	5.4086	10.7684	5.3714	10.6936	5.2998	10.5491	5.0294	9.9976	5.6616	11.2802	4.5348	9.0032	4.9487	9.8364	5.5327	11.0198	5.6218	11.1998	4.9990	9.9440	5.4760	10.9059	4.9990	9.9440	5.6606	11.2781
		-5.6256		-5.3502		-5.3598		-5.3222		-5.2492		-4.9683		-5.6185		-4.4684		-4.8877		-5.4871		-5.5779		-4.9450		-5.4299		-4.9450		-5.6175	
9	0.99	14.5169	29.0070	10.1222	20.1805	10.1408	20.2175	10.0472	20.0299	10.1187	20.1736	8.1363	16.1873	13.9795	27.9281	7.4934	14.8990	7.6731	15.2590	12.2673	24.4897	14.3855	28.7428	3.4444	6.7623	7.1930	14.2897	3.4444	6.7623	13.9435	27.8559
		-14.4901		-10.0583		-10.0767		-9.9827		-10.0549		-8.0510		-13.9487		-7.4056		-7.5858		-12.2224		-14.3573		-3.3179		-7.0967		-3.3179		-13.9124	
	0.95	6.5087	12.9900	6.0329	12.0680	6.0440	12.0902	6.0305	12.0633	5.9235	11.8561	6.0440	12.0885	6.5037	12.9802	5.3633	10.7703	5.7440	11.5078	6.2083	12.4080	6.4516	12.8795	5.3427	10.7298	6.1492	12.2924	5.3427	10.7298	6.5001	12.9732
		-6.4813		-6.0351		-6.0462		-6.0328		-5.9326		-6.0445		-6.4765		-5.4069		-5.7638		-6.1997		-6.4279		-5.3871		-6.1432		-5.3871		-6.4732	
	0.90	4.6154	9.2028	4.4526	8.8910	4.4607	8.9072	4.4549	8.8954	4.3939	8.7787	4.4382	8.8630	4.6136	9.1993	4.3026	8.6044	4.4052	8.8003	4.4791	8.9417	4.5799	9.1350	4.3674	8.7282	4.5506	9.0788	4.3674	8.7282	4.6123	9.1968
		-4.5874		-4.4384		-4.4465		-4.4406		-4.3848		-4.4249		-4.5857		-4.3018		-4.3951		-4.4627		-4.5550		-4.3608		-4.5282		-4.3608		-4.5845	
	0.85	3.7790	7.5295	3.6910	7.3430	3.6977	7.3565	3.6880	7.3367	3.6692	7.2968	3.7419	7.4509	3.7786	7.5286	3.2729	6.4555	3.1994	6.2995	3.7434	7.4539	3.7470	7.4616	1.7085	3.1945	2.4662	4.7703	1.7085	3.1945	3.7780	7.5272
		-3.7504		-3.6520		-3.6587		-3.6487		-3.6276		-3.7090		-3.7500		-3.1826		-3.1001		-3.7106		-3.7145		-1.4860		-2.3041		-1.4860		-3.7492	
25	0.99	8.7101	17.4042	7.5244	15.0077	7.5392	15.0371	7.5555	15.0705	7.4203	14.7967	3.5194	6.9037	8.3595	16.6951	6.3074	12.5462	6.8152	13.5747	8.0827	16.1359	8.6403	17.2629	3.7394	7.3481	6.4999	12.9325	3.7394	7.3481	8.3383	16.6523
		-8.6940		-7.4833		-7.4980		-7.5150		-7.3764		-3.3844		-8.3356		-6.2388		-6.7596		-8.0531		-8.6226		-3.6088		-6.4326		-3.6088		-8.3140	
	0.95	3.9052	7.7940	3.8068	7.5873	3.8138	7.6012	3.8078	7.5894	3.7805	7.5321	3.8688	7.7175	3.9049	7.7932	3.3878	6.7075	3.2217	6.3582	3.8692	7.7184	3.8722	7.7245	2.5124	4.8811	3.2468	6.4192	2.5124	4.8811	3.9029	7.7891
		-3.8888		-3.7805		-3.7874		-3.7816		-3.7516		-3.8487		-3.8884		-3.3196		-3.1365		-3.8492		-3.8523		-2.3687		-3.1724		-2.3687		-3.8862	
	0.90	2.7692	5.5217	2.7350	5.4534	2.7400	5.4633	2.7351	5.4535	2.7225	5.4283	2.5569	5.0965	2.7673	5.5178	2.6880	5.3592	2.6676	5.3182	2.7505	5.4843	2.7475	5.4781	2.5669	5.1168	2.7151	5.4133	2.5669	5.1168	2.7666	5.5163
		-2.7524		-2.7183		-2.7232		-2.7184		-2.7058		-2.5396		-2.7505		-2.6711		-2.6506		-2.7338		-2.7307		-2.5499		-2.6982		-2.5499		-2.7498	
	0.85	2.2674	4.5177	2.2430	4.4738	2.2470	4.4819	2.2441	4.4759	2.2394	4.4673	2.2568	4.4986	2.2673	4.5175	2.2244	4.4405	2.2119	4.4180	2.2570	4.4989	2.2515	4.4892	2.2011	4.3989	2.2505	4.4873	2.2011	4.3989	2.2668	4.5166
		-2.2503		-2.2308		-2.2349		-2.2317		-2.2279		-2.2418		-2.2502		-2.2161		-2.2062		-2.2419		-2.2377		-2.1977		-2.2368		-2.1977		-2.2498	
100	0.99	4.3551	8.7021	4.2163	8.4208	4.2240	8.4362	4.2142	8.4166	4.1762	8.3395	2.6374	5.2187	4.3020	8.5945	3.8612	7.7008	3.9291	7.8380	4.2922	8.5747	4.3192	8.6294	1.8291	3.5781	3.2043	6.3679	1.8291	3.5781	4.2908	8.5719
		-4.3470		-4.2045		-4.2122		-4.2024		-4.1633		-2.5813		-4.2925		-3.8396		-3.9089		-4.2825		-4.3102		-1.7490		-3.1635		-1.7490		-4.2810	
	0.95	1.9526	3.8970	1.9367	3.8679	1.9403	3.8749	1.9379	3.8700	1.9346	3.8640	1.9480	3.8885	1.9526	3.8969	1.5394	3.1408	1.3355	2.7677	1.9480	3.8885	1.9342	3.8633	1.4947	3.0592	1.8000	3.6179	1.4947	3.0592	1.9514	3.8948
		-1.9444		-1.9312		-1.9347		-1.9321		-1.9294		-1.9405		-1.9443		-1.6014		-1.4322		-1.9405		-1.9291		-1.5645		-1.8179		-1.5645		-1.9434	
	0.90	1.3846	2.7608	1.3787	2.7502	1.3812	2.7552	1.3795	2.7515	1.3781	2.7491	1.3425	2.6845	1.3841	2.7600	1.3669	2.7289	1.3573	2.7115	1.3819	2.7559	1.3812	2.7546	1.3610	2.7181	1.3786	2.7500	1.3610	2.7181	1.3837	2.7592
		-1.3762		-1.3714		-1.3739		-1.3720		-1.3709		-1.3420		-1.3758		-1.3619		-1.3541		-1.3740		-1.3734		-1.3571		-1.3714		-1.3571		-1.3755	
	0.85	1.1337	2.2588	1.1311	2.2523	1.1331	2.2564	1.1317	2.2539	1.1311	2.2524	1.1318	2.2541	1.1337	2.2588	1.1310	2.2521	1.1311	2.2524	1.1318	2.2542	1.1308	2.2516	1.1242	2.2352	1.1313	2.2529	1.1242	2.2352	1.1335	2.2584
		-1.1251		-1.1212		-1.1232		-1.1222		-1.1213		-1.1223		-1.1251		-1.1211		-1.1213		-1.1223		-1.1208		-1.1110		-1.1216		-1.1110		-1.1248	
200	0.99	3.0795	6.1533	3.0302	6.0545	3.0357	6.0655	3.0317	6.0575	3.0142	6.0223	2.4504	4.8909	3.0700	6.1342	2.9342	5.8619	2.8900	5.7734	3.0489	6.0920	3.0648	6.1239	1.3580	2.6973	2.3323	4.6541	1.3580	2.6973	3.0618	6.1179
		-3.0738		-3.0243		-3.0298		-3.0258		-3.0081		-2.4405		-3.0642		-2.9277		-2.8834		-3.0431		-3.0591		-1.3393		-2.3219		-1.3393		-3.0561	
	0.95	1.3807	2.7556	1.3744	2.7445	1.3769	2.7495	1.3757	2.7467	1.3740	2.7438	1.3717	2.7398	1.3806	2.7554	1.3409	2.6855	1.3706	2.7378	1.3782	2.7511	1.3744	2.7446	1.2944	2.6040	1.3582	2.7160	1.2944	2.6040	1.3798	2.7540
		-1.3749		-1.3701		-1.3726		-1.3711		-1.3698		-1.3681		-1.3748		-1.3446		-1.3672		-1.3730		-1.3701		-1.3096		-1.3579		-1.3096		-1.3742	
	0.90	0.9791	1.9522	0.9766	1.9479	0.9784	1.9514	0.9775	1.9494	0.9767	1.9480	0.9638	1.9251	0.9790	1.9520	0.9745	1.9441	0.9717	1.9392	0.9779	1.9502	0.9780	1.9504	0.9642	1.9258	0.9753	1.9455	0.9642	1.9258	0.9787	1.9515
		-0.9731		-0.9712		-0.9730		-0.9719		-0.9713		-0.9613		-0.9730		-0.9696		-0.9674		-0.9722		-0.9723		-0.9616		-0.9702		-0.9616		-0.9728	
	0.85	0.8017	1.5972	0.8003	1.5946	0.8017	1.5975	0.8007	1.5955	0.8005	1.5950	0.8006	1.5952	0.8016	1.5972	0.7989	1.5919	0.7977	1.5896	0.8012	1.5964	0.7963	1.5870	0.7789	1.5534	0.7958	1.5860	0.7789	1.5534	0.8015	1.5969
		-0.7956		-0.7943		-0.7958		-0.7947		-0.7945		-0.7946		-0.7956		-0.7930		-0.7919		-0.7952		-0.7907		-0.7745		-0.7902		-0.7745		-0.7954	

APPENDIX-3: t Approach Confidence Intervals for LS and ridge estimators with $\beta'\beta = 625$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	22.0495	44.0588	12.8043	25.5614	12.8051	25.5625	12.7539	25.4600	12.3748	24.7023	8.6060	17.2005	20.4320	40.8221	7.8817	15.7177	7.9116	15.7784	16.6631	33.2812	21.8523	43.6642	6.8654	13.6739	13.2651	26.4760	6.8654	13.6739	20.3806	40.7194
		-22.0093		-12.7571		-12.7573		-12.7060		-12.3275		-8.5946		-20.3901		-7.8360		-7.8668		-16.6181		-21.8119		-6.8086		-13.2109		-6.8086		-20.3388	
	0.95	9.8859	19.7306	8.6078	17.1212	8.5971	17.0996	8.5817	17.0680	8.1582	16.2061	8.5665	16.9329	9.8635	19.6851	7.1571	14.1637	7.6355	15.1396	9.0428	18.0104	9.8023	19.5597	7.7978	15.4687	9.1883	18.3073	7.7978	15.4687	9.8583	19.6746
		-9.8447		-8.5134		-8.5024		-8.4862		-8.0479		-8.3665		-9.8216		-7.0066		-7.5042		-8.9676		-9.7574		-7.6709		-9.1190		-7.6709		-9.8163	
	0.90	7.0101	13.9781	6.5205	13.0086	6.5123	12.9922	6.5050	12.9777	6.3061	12.5834	4.8974	9.4855	6.9701	13.8987	5.8496	11.6750	6.2280	12.4295	6.6223	13.2101	6.9522	13.8636	6.3104	12.5909	6.8152	13.5918	6.3104	12.5909	6.9682	13.8950
		-6.9680		-6.4881		-6.4799		-6.4727		-6.2773		-4.5881		-6.9286		-5.8254		-6.2014		-6.5878		-6.9114		-6.2804		-6.7765		-6.2804		-6.9267	
	0.85	5.7397	11.4365	5.4667	10.8847	5.4431	10.8371	5.4388	10.8284	5.3663	10.6820	5.0924	11.0849	5.7327	11.4223	4.5915	9.1167	5.0107	9.9604	5.6022	11.1587	5.6924	11.3409	5.0617	10.0693	5.5447	11.0434	5.0617	10.0693	5.7317	11.4202
		-5.6968		-5.4180		-5.3940		-5.3896		-5.3157		-5.9926		-5.6896		-4.5251		-4.9496		-5.5566		-5.6485		-5.0076		-5.4987		-5.0076		-5.6885	
9	0.99	14.6997	29.3725	10.2494	20.4348	10.2235	20.3828	10.1735	20.2823	10.2458	20.4278	8.2383	16.4087	14.1554	28.2801	7.5873	15.0868	7.7693	15.4513	12.4216	24.7983	14.5666	29.1050	3.4870	6.8475	7.2831	14.4698	3.4870	6.8475	14.1190	28.2069
		-14.6728		-10.1855		-10.1594		-10.1089		-10.1821		-8.1705		-14.1247		-7.4995		-7.6820		-12.3767		-14.5384		-3.3606		-7.1867		-3.3606		-14.0879	
	0.95	6.5906	13.1537	6.1089	12.2200	6.1164	12.2344	6.1066	12.2153	5.9982	12.0055	6.1201	13.1360	6.5854	13.1437	5.4312	10.9060	5.8165	11.6529	6.2865	12.5644	6.5327	13.0418	5.4103	10.8650	6.2266	12.4473	5.4103	10.8650	6.5818	13.1367
		-6.5631		-6.1111		-6.1180		-6.1088		-6.0073		-7.0159		-6.5583		-5.4748		-5.8363		-6.2779		-6.5091		-5.4547		-6.2206		-5.4547		-6.5549	
	0.90	4.6734	9.3188	4.5086	9.0030	4.5144	9.0142	4.5109	9.0075	4.4492	8.8893	4.4940	8.6975	4.6716	9.3153	4.3568	8.7128	4.4606	8.9112	4.5354	9.0544	4.6375	9.2501	4.4224	8.8382	4.6078	9.1932	4.4224	8.8382	4.6702	9.3127
		-4.6454		-4.4944		-4.4998		-4.4966		-4.4401		-4.2035		-4.6437		-4.3560		-4.4505		-4.5190		-4.6126		-4.4158		-4.5854		-4.4158		-4.6425	
	0.85	3.8265	7.6244	3.7372	7.4355	3.7362	7.4333	3.7343	7.4292	3.7152	7.3887	3.7889	6.9917	3.8261	7.6235	3.3136	6.5368	3.2391	6.3789	3.7903	7.5479	3.7940	7.5556	1.7287	3.2348	2.4962	4.8304	1.7287	3.2348	3.8254	7.6221
		-3.7979		-3.6983		-3.6971		-3.6949		-3.6736		-3.2029		-3.7974		-3.2232		-3.1398		-3.7575		-3.7615		-1.5061		-2.3342		-1.5061		-3.7967	
25	0.99	8.8198	17.6235	7.6190	15.1968	7.6724	15.3049	7.6504	15.2604	7.5135	14.9831	6.5629	12.8911	8.4647	16.9055	6.3865	12.7043	6.9007	13.7458	8.1844	16.3392	8.7491	17.4804	3.7857	7.4407	6.5814	13.0955	3.7857	7.4407	8.4433	16.8621
		-8.8037		-7.5779		-7.6324		-7.6100		-7.4696		-6.3282		-8.4408		-6.3178		-6.8451		-8.1548		-8.7314		-3.6551		-6.5140		-3.6551		-8.4189	
	0.95	3.9543	7.8922	3.8546	7.6829	3.8577	7.6895	3.8556	7.6851	3.8279	7.6270	3.9174	7.6529	3.9540	7.8914	3.4301	6.7920	3.2618	6.4383	3.9178	7.8156	3.9209	7.8219	2.5431	4.9426	3.2873	6.5001	2.5431	4.9426	3.9520	7.8872
		-3.9379		-3.8283		-3.8318		-3.8295		-3.7991		-3.7355		-3.9375		-3.3619		-3.1765		-3.8978		-3.9010		-2.3995		-3.2129		-2.3995		-3.9353	
	0.90	2.8040	5.5913	2.7694	5.5221	2.7702	5.5237	2.7695	5.5222	2.7567	5.4967	2.5890	5.3545	2.8021	5.5874	2.7218	5.4267	2.7011	5.3852	2.7851	5.5534	2.7820	5.5472	2.5992	5.1813	2.7492	5.4815	2.5992	5.1813	2.8013	5.5859
		-2.7872		-2.7527		-2.7535		-2.7527		-2.7400		-2.7656		-2.7853		-2.7049		-2.6841		-2.7683		-2.7652		-2.5821		-2.7323		-2.5821		-2.7845	
	0.85	2.2959	4.5746	2.2711	4.5301	2.2728	4.5332	2.2723	4.5323	2.2675	4.5236	2.3851	4.7098	2.2958	4.5744	2.2524	4.4965	2.2397	4.4737	2.2853	4.5556	2.2798	4.5458	2.2288	4.4543	2.2788	4.5439	2.2288	4.4543	2.2953	4.5736
		-2.2787		-2.2590		-2.2603		-2.2599		-2.2561		-2.3246		-2.2786		-2.2441		-2.2340		-2.2703		-2.2660		-2.2255		-2.2651		-2.2255		-2.2783	
100	0.99	4.4099	8.8118	4.2694	8.5270	4.2705	8.5292	4.2673	8.5227	4.2287	8.4446	3.6703	7.5968	4.3561	8.7028	3.9097	7.7978	3.9785	7.9368	4.3462	8.6827	4.3736	8.7381	1.8516	3.6232	3.2445	6.4481	1.8516	3.6232	4.3448	8.6799
		-4.4019		-4.2576		-4.2587		-4.2554		-4.2159		-3.9266		-4.3467		-3.8881		-3.9583		-4.3365		-4.3645		-1.7716		-3.2036		-1.7716		-4.3351	
	0.95	1.9772	3.9461	1.9611	3.9167	1.9626	3.9193	1.9623	3.9187	1.9589	3.9127	1.9725	3.9355	1.9771	3.9460	1.5592	3.1804	1.3529	2.8025	1.9725	3.9375	1.9585	3.9119	1.5140	3.0977	1.8228	3.6635	1.5140	3.0977	1.9760	3.9439
		-1.9689		-1.9555		-1.9567		-1.9565		-1.9537		-1.9630		-1.9689		-1.6212		-1.4496		-1.9650		-1.9534		-1.5838		-1.8407		-1.5838		-1.9679	
	0.90	1.4020	2.7956	1.3961	2.7848	1.3969	2.7864	1.3968	2.7862	1.3954	2.7837	1.3594	2.2959	1.4015	2.7947	1.3841	2.7633	1.3744	2.7456	1.3993	2.7907	1.3985	2.7893	1.3781	2.7524	1.3960	2.7847	1.3781	2.7524	1.4011	2.7940
		-1.3936		-1.3888		-1.3895		-1.3894		-1.3883		-0.9365		-1.3932		-1.3791		-1.3712		-1.3914		-1.3908		-1.3743		-1.3887		-1.3743		-1.3929	
	0.85	1.1479	2.2873	1.1453	2.2807	1.1459	2.2824	1.1459	2.2823	1.1453	2.2808	1.0460	2.2021	1.1479	2.2873	1.1452	2.2805	1.1453	2.2808	1.1460	2.2826	1.1450	2.2800	1.1383	2.2634	1.1455	2.2813	1.1383	2.2634	1.1478	2.2868
		-1.1394		-1.1354		-1.1364		-1.1364		-1.1355		-1.1561		-1.1393		-1.1353		-1.1355		-1.1365		-1.1350		-1.1251		-1.1358		-1.1251		-1.1391	
200	0.99	3.1183	6.2309	3.0684	6.1308	3.0710	6.1360	3.0699	6.1339	3.0521	6.0982	2.4812	5.6755	3.1086	6.2115	2.9711	5.9358	2.9264	5.8461	3.0873	6.1688	3.1034	6.2011	1.3750	2.7313	2.3616	4.7128	1.3750	2.7313	3.1004	6.1950
		-3.1126		-3.0624		-3.0650		-3.0640		-3.0461		-3.1943		-3.1029		-2.9646		-2.9198		-3.0815		-3.0976		-1.3563		-2.3512		-1.3563		-3.0946	
	0.95	1.3981	2.7903	1.3917	2.7791	1.3931	2.7815	1.3930	2.7813	1.3913	2.7784	1.3890	2.3635	1.3980	2.7902	1.3578	2.7193	1.3878	2.7723	1.3955	2.7858	1.3917	2.7791	1.3108	2.6368	1.3753	2.7503	1.3108	2.6368	1.3971	2.7887
		-1.3923		-1.3874		-1.3885		-1.3884		-1.3871		-0.9745		-1.3922		-1.3615		-1.3845		-1.3903		-1.3874		-1.3260		-1.3750		-1.3260		-1.3915	
	0.90	0.9914	1.9768	0.9889	1.9724	0.9898	1.9740	0.9898	1.9739	0.9890	1.9726	0.9759	1.4565	0.9913	1.9766	0.9868	1.9686	0.9839	1.9636	0.9902	1.9748	0.9903	1.9750	0.9763	1.9500	0.9876	1.9700	0.9763	1.9500	0.9910	1.9761
		-0.9854		-0.9835		-0.9842		-0.9842		-0.9836		-0.4806		-0.9853		-0.9818		-0.9797		-0.9845		-0.9846		-0.9737		-0.9825		-0.9737		-0.9851	
	0.85	0.8117	1.6174	0.8103	1.6147	0.8108	1.6156	0.8108	1.6156	0.8105	1.6151	0.8107	1.1386	0.8117	1.6174	0.8089	1.6120	0.8077	1.6096	0.8113	1.6165	0.8063	1.6070	0.7887	1.5730	0.8058	1.6060	0.7887	1.5730	0.8116	1.6170
		-0.8057		-0.8044		-0.8048		-0.8048		-0.8045		-0.3280		-0.8056		-0.8031		-0.8019		-0.8052		-0.8007		-0.7843		-0.8002		-0.7843		-0.8055	

APPENDIX-4: Bootstrap-t Confidence Intervals for LS and ridge estimators with $\beta'\beta = 100$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	14.8710 -7.1034	21.9744	4.7492 -5.6368	10.3861	4.3593 -5.9382	10.2975	4.2808 -5.9589	10.2397	10.6840 0.7945	9.8895	3.5095 -8.5437	12.0532	13.1648 -7.8343	20.9991	9.4494 3.7870	5.6624	2.5316 -3.4255	5.9572	9.4776 -5.1422	14.6197	7.7663 -7.0659	14.8322	6.8662 3.0127	3.8535	6.9541 -2.0753	9.0294	6.8662 3.0127	3.8535	12.0647 -9.1106	21.1753
	0.95	10.9134 1.0409	9.8726	17.8206 9.9433	7.8773	17.5831 9.7386	7.8445	17.6201 9.7973	7.8228	19.3384 12.0110	7.3274	18.0237 10.4734	7.5503	11.1332 1.3070	9.8262	12.4488 5.6894	6.7594	20.9399 14.6979	6.2420	15.6452 7.0428	8.6024	11.7545 3.0097	8.7448	14.3669 8.7154	5.6515	17.6971 9.8025	7.8946	14.3669 8.7154	5.6515	11.1714 2.1446	9.0268
	0.90	14.5952 7.5665	7.0287	10.6899 4.4593	6.2306	10.5801 4.3958	6.1843	10.5360 4.3622	6.1738	10.0114 4.0142	5.9972	9.3665 3.3875	5.9790	14.5074 7.4915	7.0159	5.9720 0.4984	5.4737	9.4278 3.6174	5.8105	11.9285 5.4170	6.5115	13.8515 7.2790	6.5725	11.1083 4.8572	6.2511	13.4485 6.6354	6.8131	11.1083 4.8572	6.2511	11.9867 4.9982	6.9886
	0.85	7.7539 2.0390	5.7149	9.5210 4.2268	5.2942	9.6226 4.3450	5.2776	9.6477 4.3770	5.2706	9.8494 4.7506	5.0988	7.7681 3.4965	4.2717	7.8909 2.1961	5.6947	8.8374 4.3083	4.5291	10.4287 5.4963	4.9324	9.3395 3.9630	5.3764	7.8455 2.3806	5.4649	9.5271 4.2985	5.2286	8.4478 2.8646	5.5832	9.5271 4.2985	5.2286	7.9031 2.6864	5.2167
9	0.99	8.3549 -6.3597	14.7146	-0.0718 -8.6551	8.5833	-0.3378 -8.9258	8.5880	-0.3566 -8.8886	8.5320	-1.2917 -9.9589	8.6672	-7.4192 -15.9169	8.4976	7.4529 -6.8459	14.2988	-0.2241 -6.5344	6.3104	-3.6426 -9.8222	6.1795	-0.1467 -11.4501	11.3034	4.8288 -6.1982	11.0271	3.3289 0.8094	2.5195	3.3855 -2.5434	5.9289	3.3289 0.8094	2.5195	7.1835 -4.7624	11.9459
	0.95	5.8306 -0.7050	6.5356	3.8875 -1.8941	5.7816	3.8824 -1.8760	5.7584	3.8477 -1.8960	5.7438	3.6388 -2.0262	5.6650	4.9808 -1.2044	6.1853	5.8208 -0.7110	6.5318	2.0424 -2.2327	4.2752	1.1611 -3.4019	4.5631	4.9728 -1.2292	6.2020	5.4882 -0.6124	6.1005	1.4311 -3.3917	4.8228	4.1858 -1.7745	5.9603	1.4311 -3.3917	4.8228	5.3295 -1.1233	6.4528
	0.90	2.3220 -2.3387	4.6607	2.8427 -1.5308	4.3735	2.7385 -1.6312	4.3697	2.7442 -1.6195	4.3637	2.7871 -1.5362	4.3233	2.7228 -1.7085	4.4313	2.3271 -2.3312	4.6583	3.5358 -0.4022	3.9380	2.8642 -1.4398	4.3040	2.6159 -1.8602	4.4760	2.4207 -2.0644	4.4851	3.1055 -1.0700	4.1755	2.6492 -1.8780	4.5272	3.1055 -1.0700	4.1755	2.5475 -1.8371	4.3846
	0.85	2.5359 -1.2792	3.8151	1.3816 -2.2816	3.6632	1.4213 -2.2353	3.6567	1.3974 -2.2559	3.6533	1.2119 -2.4159	3.6278	2.1615 -1.5983	3.7598	2.5319 -1.2826	3.8145	1.1216 -1.5143	2.6359	-4.9482 -7.7982	2.8500	2.1728 -1.5888	3.7616	2.0855 -1.6261	3.7116	-4.3566 -7.2470	2.8904	0.2083 -3.2847	3.4930	-4.3566 -7.2470	2.8904	2.4735 -1.3286	3.8020
25	0.99	7.4944 -1.3283	8.8227	2.3896 -4.7336	7.1232	2.7334 -4.3791	7.1125	2.6907 -4.3917	7.0823	2.5589 -4.3229	6.8818	3.5757 -3.1477	6.7234	7.4479 -1.3616	8.8095	2.0893 -2.9617	5.0510	-0.1093 -5.6506	5.5412	4.5554 -3.2811	7.8365	5.6742 -2.1775	7.8517	-1.0250 -3.4048	2.3798	-4.2901 -9.3936	5.1035	-1.0250 -3.4048	2.3798	6.3043 -2.4060	8.7103
	0.95	1.0047 -2.9275	3.9322	2.0751 -1.7063	3.7814	2.1969 -1.5838	3.7808	2.2194 -1.5581	3.7775	2.5914 -1.1262	3.7177	2.1244 -0.7732	2.8976	1.1391 -2.7777	3.9168	2.8524 -0.2469	3.0993	4.0613 0.9687	3.0927	1.8186 -2.0180	3.8366	1.2811 -2.5565	3.8377	3.4497 -0.0940	3.5437	1.9076 -1.9193	3.8269	3.4497 -0.0940	3.5437	2.1712 -1.6380	3.8092
	0.90	-1.2180 -4.0007	2.7827	-0.8477 -3.5676	2.7199	-0.9239 -3.6450	2.7212	-0.9181 -3.6379	2.7198	-0.8550 -3.5603	2.7053	-0.5864 -3.2181	2.6317	-1.2104 -3.9917	2.7813	-0.9116 -3.4521	2.5405	-0.6729 -3.3394	2.6664	-1.0436 -3.7911	2.7476	-1.1317 -3.8732	2.7415	-0.7908 -3.4819	2.6910	-1.0998 -3.8590	2.7592	-0.7908 -3.4819	2.6910	-1.2040 -3.9237	2.7198
	0.85	-2.2724 -4.5732	2.3008	-1.3356 -3.6020	2.2664	-1.3262 -3.5948	2.2687	-1.3057 -3.5736	2.2680	-1.0264 -3.2846	2.2582	-1.3397 -3.6090	2.2693	-2.2629 -4.5634	2.3005	1.2607 -0.8547	2.1155	1.4686 -0.6914	2.1600	-1.7470 -4.0300	2.2831	-1.3786 -3.6471	2.2685	4.1908 2.1518	2.0390	-0.2125 -2.4400	2.2275	4.1908 2.1518	2.0390	-1.2412 -3.5155	2.2743
100	0.99	-1.2038 -5.5916	4.3878	1.7764 -2.3850	4.1614	1.7222 -2.4398	4.1619	1.7775 -2.3798	4.1573	2.4842 -1.6072	4.0914	0.6129 -3.6433	4.2562	-1.1836 -5.5700	4.3864	3.6013 0.4083	3.1930	4.5752 0.8740	3.7012	0.5964 -3.6613	4.2578	-0.4298 -4.6761	4.2463	-4.9855 -6.3898	1.4043	5.3174 2.4710	2.8464	-4.9855 -6.3898	1.4043	1.6300 -2.5603	4.1904
	0.95	-0.5054 -2.4688	1.9634	-0.7551 -2.6970	1.9419	-0.7209 -2.6644	1.9435	-0.7252 -2.6683	1.9431	-0.7985 -2.7344	1.9359	-1.8474 -3.6323	1.7849	-0.5607 -2.5190	1.9583	-0.6754 -2.2270	1.5516	-1.1420 -3.0406	1.8987	-0.6220 -2.5748	1.9528	-0.5966 -2.5453	1.9487	-1.8602 -3.6615	1.8013	-0.9636 -2.8837	1.9201	-1.8602 -3.6615	1.8013	-0.5802 -2.5325	1.9524
	0.90	0.4705 -0.9207	1.3913	0.1799 -1.2017	1.3816	0.2014 -1.1821	1.3835	0.1954 -1.1879	1.3833	0.1317 -1.2497	1.3814	-0.0086 -1.3857	1.3772	0.4659 -0.9252	1.3911	0.1989 -1.1880	1.3869	-0.2420 -1.6124	1.3704	0.3521 -1.0357	1.3878	0.0578 -1.3227	1.3805	-1.2949 -2.6311	1.3363	-0.0213 -1.3983	1.3771	-1.2949 -2.6311	1.3363	0.1937 -1.1881	1.3818
	0.85	-0.5016 -1.6445	1.1428	-0.8182 -1.9555	1.1373	-0.7118 -1.8507	1.1388	-0.7164 -1.8552	1.1387	-0.7836 -1.9210	1.1375	-0.8805 -2.0161	1.1357	-0.5052 -1.6479	1.1428	-0.7907 -1.9311	1.1404	-1.0069 -2.1401	1.1331	-0.6027 -1.7436	1.1409	-1.3962 -2.5278	1.1315	-0.6261 -1.0679	0.4418	-1.9342 -2.6277	0.6935	-0.6261 -1.0679	0.4418	-0.5215 -1.6565	1.1350
200	0.99	-0.2349 -3.3656	3.1307	1.2888 -1.7562	3.0451	1.2694 -1.7788	3.0482	1.3027 -1.7437	3.0464	1.7180 -1.3036	3.0216	5.7587 3.2678	2.4909	-0.0731 -3.1952	3.1221	2.2255 -0.0223	2.2478	3.3733 0.4630	2.9103	0.5465 -2.5428	3.0893	0.4357 -2.6337	3.0694	-5.9868 -6.9904	1.0036	5.4673 3.4346	2.0328	-5.9868 -6.9904	1.0036	1.2311 -1.8146	3.0457
	0.95	0.7727 -0.6174	1.3901	0.9048 -0.4757	1.3806	0.8605 -0.5218	1.3824	0.8623 -0.5199	1.3822	0.8839 -0.4964	1.3803	0.8010 -0.5866	1.3876	0.7730 -0.6171	1.3901	0.9735 0.1556	0.8179	3.4290 2.6006	0.8284	0.8009 -0.5867	1.3876	1.0107 -0.3646	1.3753	3.2906 2.4237	0.8670	2.8439 1.6786	1.1653	3.2906 2.4237	0.8670	0.7873 -0.5965	1.3837
	0.90	-0.0859 -1.0790	0.9931	-0.1925 -1.1817	0.9892	-0.1640 -1.1543	0.9903	-0.1655 -1.1557	0.9902	-0.1829 -1.1724	0.9896	-0.1707 -1.1607	0.9901	-0.0867 -1.0798	0.9931	-0.2637 -1.2054	0.9417	-0.1913 -1.1805	0.9893	-0.1372 -1.1284	0.9912	-0.1342 -1.1245	0.9903	-0.6843 -1.6550	0.9706	-0.2442 -1.2316	0.9874	-0.6843 -1.6550	0.9706	0.3125 -0.6770	0.9895
	0.85	-0.8854 -1.6932	0.8078	-0.9060 -1.7117	0.8057	-0.8758 -1.6822	0.8064	-0.8757 -1.6821	0.8063	-0.8726 -1.6785	0.8059	-0.8820 -1.6893	0.8073	-0.8854 -1.6932	0.8078	-0.8691 -1.6318	0.7627	-0.7668 -1.5604	0.7936	-0.8820 -1.6893	0.8073	-0.8329 -1.6359	0.8030	-0.6485 -1.4303	0.7817	-0.8350 -1.6361	0.8011	-0.6485 -1.4303	0.7817	-0.8839 -1.6897	0.8058

APPENDIX-5: Standard Normal Approximation Confidence Intervals for LS and ridge estimators with $\beta'\beta = 100$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	8.7101 -8.6940	17.4042	5.1101 -5.0812	10.1913	5.1223 -5.0933	10.2156	5.0608 -5.0315	10.0923	4.8462 -4.8171	9.6633	5.5220 -5.4934	11.0154	8.4230 -8.4061	16.8291	3.0411 -3.0070	6.0480	3.1078 -3.0746	6.1824	6.5705 -6.5472	13.1177	8.6313 -8.6149	17.2463	2.1581 -2.1193	4.2774	4.4289 -4.4000	8.8290	2.1581 -2.1193	4.2774	8.4011 -8.3840	16.7852
	0.95	3.9052 -3.8888	7.7940	3.3941 -3.3558	6.7498	3.4004 -3.3620	6.7625	3.3807 -3.3418	6.7226	3.2293 -3.1850	6.4144	3.2730 -3.2321	6.5051	3.8943 -3.8775	7.7718	2.6239 -2.5547	5.1785	2.8570 -2.7984	5.6554	3.5905 -3.5609	7.1514	3.8714 -3.8534	7.7249	2.6310 -2.5634	5.1943	3.3466 -3.3086	6.6552	2.6310 -2.5634	5.1943	3.8923 -3.8754	7.7676
	0.90	2.7692 -2.7524	5.5217	2.5690 -2.5548	5.1238	2.5736 -2.5594	5.1330	2.5540 -2.5400	5.0940	2.5057 -2.4923	4.9980	2.4965 -2.4815	4.9780	2.7662 -2.7494	5.5156	2.2760 -2.2663	4.5423	2.4524 -2.4392	4.8916	2.6417 -2.6266	5.2683	2.7474 -2.7309	5.4784	2.5733 -2.5596	5.1329	2.7171 -2.7010	5.4181	2.5733 -2.5596	5.1329	2.7655 -2.7487	5.5141
	0.85	2.2674 -2.2503	4.5177	2.1618 -2.1494	4.3112	2.1657 -2.1533	4.3190	2.1559 -2.1438	4.2997	2.1103 -2.1001	4.2105	1.8669 -1.8656	3.7325	2.2625 -2.2455	4.5081	2.0361 -2.0285	4.0646	2.0629 -2.0549	4.1178	2.1832 -2.1699	4.3531	2.2499 -2.2335	4.4834	2.1448 -2.1329	4.2777	2.2352 -2.2195	4.4547	2.1448 -2.1329	4.2777	2.2621 -2.2451	4.5072
9	0.99	5.8068 -5.7960	11.6028	3.9778 -3.9784	7.9562	3.9866 -3.9871	7.9737	3.9624 -3.9630	7.9254	4.0162 -4.0166	8.0328	3.8403 -3.8430	7.6833	5.6947 -5.6847	11.3794	2.8629 -2.8704	5.7333	3.0288 -3.0360	6.0648	4.8847 -4.8795	9.7642	5.7539 -5.7435	11.4974	1.3853 -1.4058	2.7911	2.8918 -2.9033	5.7951	1.3853 -1.4058	2.7911	5.6797 -5.6698	11.3495
	0.95	2.6035 -2.5925	5.1960	2.4082 -2.4038	4.8120	2.4125 -2.4080	4.8206	2.3991 -2.3949	4.7940	2.3776 -2.3741	4.7517	2.5161 -2.5078	5.0239	2.6026 -2.5916	5.1942	2.0115 -2.0224	4.0339	2.0310 -2.0417	4.0727	2.5204 -2.5121	5.0325	2.5809 -2.5707	5.1516	2.1204 -2.1275	4.2479	2.4544 -2.4491	4.9035	2.1204 -2.1275	4.2479	2.6012 -2.5903	5.1914
	0.90	1.8462 -1.8349	3.6811	1.7754 -1.7654	3.5407	1.7786 -1.7686	3.5471	1.7730 -1.7630	3.5360	1.7627 -1.7529	3.5156	1.7899 -1.7794	3.5694	1.8456 -1.8344	3.6800	1.6853 -1.6772	3.3625	1.7577 -1.7481	3.5057	1.8012 -1.7907	3.5918	1.8318 -1.8208	3.6526	1.7237 -1.7149	3.4386	1.8137 -1.8031	3.6168	1.7237 -1.7149	3.4386	1.8451 -1.8339	3.6790
	0.85	1.5116 -1.5002	3.0118	1.4721 -1.4656	2.9376	1.4747 -1.4682	2.9429	1.4697 -1.4634	2.9331	1.4629 -1.4575	2.9204	1.4975 -1.4878	2.9852	1.5115 -1.5000	3.0115	1.2527 -1.2743	2.5269	1.2263 -1.2520	2.4783	1.4979 -1.4882	2.9861	1.4978 -1.4881	2.9860	1.2374 -1.2617	2.4991	1.4235 -1.4237	2.8472	1.2374 -1.2617	2.4991	1.5112 -1.4998	3.0109
25	0.99	3.4841 -3.4776	6.9617	3.0387 -3.0258	6.0645	3.0444 -3.0315	6.0759	3.0300 -3.0170	6.0470	2.9756 -2.9616	5.9372	3.2077 -3.1969	6.4045	3.4810 -3.4745	6.9554	2.4998 -2.4803	4.9801	2.5165 -2.4973	5.0138	3.2392 -3.2289	6.4681	3.4539 -3.4469	6.9008	1.3129 -1.2741	2.5870	2.4005 -2.3779	4.7784	1.3129 -1.2741	2.5870	3.4718 -3.4651	6.9369
	0.95	1.5621 -1.5555	3.1176	1.5241 -1.5197	3.0438	1.5269 -1.5225	3.0494	1.5233 -1.5190	3.0423	1.5080 -1.5045	3.0125	1.2668 -1.2759	2.5427	1.5583 -1.5519	3.1102	1.3609 -1.3653	2.7263	1.3198 -1.3254	2.6451	1.5383 -1.5331	3.0715	1.5530 -1.5470	3.1000	1.4623 -1.4615	2.9239	1.5359 -1.5309	3.0668	1.4623 -1.4615	2.9239	1.5574 -1.5511	3.1085
	0.90	1.1077 -1.1010	2.2087	1.0922 -1.0859	2.1781	1.0942 -1.0879	2.1821	1.0923 -1.0860	2.1783	1.0887 -1.0825	2.1712	1.0699 -1.0645	2.1344	1.1074 -1.1006	2.2080	1.0835 -1.0775	2.1610	1.0789 -1.0731	2.1520	1.0991 -1.0927	2.1918	1.1030 -1.0964	2.1994	1.0851 -1.0791	2.1643	1.1020 -1.0954	2.1974	1.0851 -1.0791	2.1643	1.1070 -1.1003	2.2074
	0.85	0.9070 -0.9001	1.8071	0.9001 -0.8907	1.7907	0.9017 -0.8923	1.7940	0.9004 -0.8911	1.7914	0.8984 -0.8884	1.7867	0.9006 -0.8914	1.7921	0.9069 -0.9000	1.8069	0.8786 -0.8614	1.7400	0.8779 -0.8604	1.7383	0.9034 -0.8952	1.7986	0.9009 -0.8918	1.7927	0.8513 -0.8241	1.6755	0.8921 -0.8797	1.7718	0.8513 -0.8241	1.6755	0.9068 -0.8998	1.8066
100	0.99	1.7420 -1.7388	3.4808	1.6861 -1.6830	3.3691	1.6892 -1.6861	3.3753	1.6854 -1.6823	3.3677	1.6687 -1.6657	3.3343	1.7100 -1.7069	3.4169	1.7417 -1.7385	3.4802	1.6094 -1.6067	3.2161	1.5607 -1.5583	3.1190	1.7104 -1.7073	3.4177	1.7291 -1.7259	3.4550	0.7520 -0.7499	1.5019	1.3084 -1.3062	2.6145	0.7520 -0.7499	1.5019	1.7371 -1.7339	3.4709
	0.95	0.7810 -0.7778	1.5588	0.7761 -0.7723	1.5485	0.7776 -0.7737	1.5513	0.7764 -0.7726	1.5490	0.7748 -0.7708	1.5456	0.7382 -0.7305	1.4686	0.7799 -0.7765	1.5564	0.7507 -0.7440	1.4947	0.7661 -0.7612	1.5274	0.7786 -0.7751	1.5537	0.7793 -0.7758	1.5550	0.7430 -0.7355	1.4785	0.7712 -0.7668	1.5379	0.7430 -0.7355	1.4785	0.7795 -0.7760	1.5555
	0.90	0.5538 -0.5505	1.1043	0.5515 -0.5482	1.0997	0.5525 -0.5492	1.1017	0.5519 -0.5486	1.1005	0.5514 -0.5482	1.0996	0.5504 -0.5472	1.0976	0.5538 -0.5505	1.1043	0.5476 -0.5445	1.0921	0.5488 -0.5456	1.0943	0.5530 -0.5497	1.1027	0.5510 -0.5477	1.0987	0.5402 -0.5374	1.0777	0.5504 -0.5472	1.0975	0.5402 -0.5374	1.0777	0.5537 -0.5503	1.1040
	0.85	0.4535 -0.4501	0.9035	0.4522 -0.4487	0.9009	0.4530 -0.4495	0.9025	0.4525 -0.4491	0.9016	0.4522 -0.4488	0.9010	0.4518 -0.4483	0.9001	0.4535 -0.4500	0.9035	0.4456 -0.4419	0.8875	0.4512 -0.4477	0.8989	0.4530 -0.4496	0.9026	0.4495 -0.4459	0.8954	0.2144 -0.2082	0.4227	0.3074 -0.3023	0.6097	0.2144 -0.2082	0.4227	0.4534 -0.4499	0.9033
200	0.99	1.2318 -1.2295	2.4613	1.2117 -1.2088	2.4205	1.2139 -1.2110	2.4249	1.2120 -1.2091	2.4212	1.2061 -1.2030	2.4092	1.0618 -1.0541	2.1159	1.2298 -1.2275	2.4572	1.1869 -1.1832	2.3702	1.1791 -1.1751	2.3542	1.2222 -1.2196	2.4417	1.2233 -1.2208	2.4441	0.5465 -0.5216	1.0681	0.9328 -0.9207	1.8535	0.5465 -0.5216	1.0681	1.2266 -1.2241	2.4507
	0.95	0.5523 -0.5500	1.1022	0.5503 -0.5473	1.0976	0.5513 -0.5483	1.0996	0.5506 -0.5478	1.0984	0.5502 -0.5473	1.0975	0.5518 -0.5493	1.1010	0.5523 -0.5499	1.1022	0.4798 -0.4539	0.9337	0.4044 -0.3544	0.7588	0.5518 -0.5493	1.1010	0.5481 -0.5444	1.0925	0.4159 -0.3701	0.7860	0.4972 -0.4774	0.9746	0.4159 -0.3701	0.7860	0.5520 -0.5496	1.1016
	0.90	0.3916 -0.3893	0.7809	0.3907 -0.3884	0.7790	0.3914 -0.3891	0.7805	0.3909 -0.3886	0.7795	0.3908 -0.3884	0.7792	0.3909 -0.3886	0.7795	0.3916 -0.3892	0.7809	0.3905 -0.3882	0.7786	0.3907 -0.3884	0.7791	0.3912 -0.3888	0.7800	0.3912 -0.3889	0.7801	0.3862 -0.3841	0.7702	0.3902 -0.3879	0.7782	0.3862 -0.3841	0.7702	0.3915 -0.3891	0.7807
	0.85	0.3207 -0.3182	0.6389	0.3204 -0.3175	0.6379	0.3209 -0.3181	0.6390	0.3205 -0.3177	0.6382	0.3204 -0.3176	0.6380	0.3206 -0.3181	0.6386	0.3207 -0.3182	0.6389	0.3197 -0.3159	0.6355	0.3187 -0.3135	0.6321	0.3206 -0.3181	0.6386	0.3196 -0.3158	0.6354	0.3169 -0.3094	0.6264	0.3197 -0.3160	0.6357	0.3169 -0.3094	0.6264	0.3206 -0.3181	0.6388

APPENDIX-6: t Approach Confidence Intervals for LS and ridge estimators with $\beta'\beta = 100$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	8.8198 -8.8037	17.6235	5.1743 -5.1454	10.3197	5.1454 -5.1162	10.2616	5.1244 -5.0951	10.2195	4.9071 -4.8780	9.7851	5.5914 -9.7267	15.3182	8.5291 -8.5121	17.0412	3.0792 -3.0451	6.1243	3.1468 -3.1135	6.2603	6.6532 -6.6298	13.2830	8.7400 -8.7236	17.4636	2.1851 -2.1462	4.3313	4.4846 -4.4557	8.9402	2.1851 -2.1462	4.3313	8.5069 -8.4898	16.9967
	0.95	3.9543 -3.9379	7.8922	3.4366 -3.3983	6.8349	3.4291 -3.3905	6.8196	3.4231 -3.3842	6.8073	3.2697 -3.2255	6.4952	3.3140 -4.2921	7.6061	3.9433 -3.9264	7.8697	2.6565 -2.5873	5.2438	2.8926 -2.8341	5.7267	3.6356 -3.6059	7.2415	3.9201 -3.9021	7.8222	2.6637 -2.5961	5.2598	3.3885 -3.3505	6.7390	2.6637 -2.5961	5.2598	3.9412 -3.9243	7.8655
	0.90	2.8040 -2.7872	5.5913	2.6013 -2.5871	5.1884	2.5889 -2.5749	5.1638	2.5861 -2.5721	5.1582	2.5372 -2.5238	5.0609	2.1279 -3.2244	5.3523	2.8010 -2.7842	5.5851	2.3046 -2.2949	4.5995	2.4832 -2.4701	4.9533	2.6749 -2.6598	5.3347	2.7820 -2.7654	5.5474	2.6057 -2.5919	5.1976	2.7512 -2.7352	5.4864	2.6057 -2.5919	5.1976	2.8002 -2.7834	5.5836
	0.85	2.2959 -2.2787	4.5746	2.1890 -2.1765	4.3655	2.1849 -2.1726	4.3575	2.1830 -2.1709	4.3539	2.1369 -2.1266	4.2635	1.8904 -1.8215	3.7119	2.2909 -2.2739	4.5649	2.0617 -2.0541	4.1158	2.0889 -2.0808	4.1697	2.2107 -2.1973	4.4080	2.2781 -2.2618	4.5399	2.1717 -2.1599	4.3316	2.2633 -2.2476	4.5109	2.1717 -2.1599	4.3316	2.2905 -2.2735	4.5640
9	0.99	5.8799 -5.8691	11.7490	4.0280 -4.0285	8.0565	4.0315 -4.0320	8.0635	4.0123 -4.0129	8.0252	4.0668 -4.0672	8.1340	3.8887 -7.7254	11.6141	5.7664 -5.7564	11.5228	2.8990 -2.9066	5.8056	3.0670 -3.0742	6.1412	4.9462 -4.9410	9.8872	5.8263 -5.8160	11.6423	1.4029 -1.4234	2.8262	2.9283 -2.9398	5.8681	1.4029 -1.4234	2.8262	5.7512 -5.7413	11.4926
	0.95	2.6362 -2.6253	5.2615	2.4385 -2.4341	4.8726	2.4333 -2.4290	4.8624	2.4293 -2.4251	4.8544	2.4075 -2.4041	4.8116	2.3477 -2.8766	5.2243	2.6353 -2.6244	5.2597	2.0369 -2.0478	4.0847	2.0567 -2.0673	4.1240	2.5521 -2.5438	5.0959	2.6133 -2.6032	5.2165	2.1471 -2.1543	4.3014	2.4853 -2.4800	4.9653	2.1471 -2.1543	4.3014	2.6339 -2.6230	5.2569
	0.90	1.8694 -1.8581	3.7275	1.7977 -1.7877	3.5854	1.7968 -1.7868	3.5836	1.7953 -1.7853	3.5806	1.7848 -1.7750	3.5599	1.8124 -1.6508	3.4633	1.8688 -1.8576	3.7263	1.7065 -1.6984	3.4048	1.7797 -1.7702	3.5499	1.8238 -1.8133	3.6371	1.8548 -1.8438	3.6986	1.7454 -1.7365	3.4819	1.8365 -1.8259	3.6624	1.7454 -1.7365	3.4819	1.8683 -1.8571	3.7253
	0.85	1.5306 -1.5191	3.0497	1.4906 -1.4841	2.9747	1.4891 -1.4827	2.9717	1.4882 -1.4819	2.9701	1.4813 -1.4759	2.9572	1.5163 -1.1541	2.6703	1.5304 -1.5190	3.0495	1.2686 -1.2902	2.5588	1.2419 -1.2676	2.5095	1.5167 -1.5070	3.0237	1.5167 -1.5070	3.0236	1.2532 -1.2774	2.5306	1.4414 -1.4417	2.8831	1.2532 -1.2774	2.5306	1.5301 -1.5187	3.0489
25	0.99	3.5279 -3.5215	7.0494	3.0769 -3.0640	6.1409	3.0769 -3.0640	6.1409	3.0681 -3.0551	6.1232	3.0130 -2.9990	6.0120	3.2480 -3.3338	6.5818	3.5248 -3.5183	7.0431	2.5312 -2.5117	5.0428	2.5480 -2.5289	5.0770	3.2799 -3.2697	6.5496	3.4973 -3.4904	6.9877	1.3292 -1.2904	2.6196	2.4307 -2.4080	4.8386	1.3292 -1.2904	2.6196	3.5155 -3.5088	7.0243
	0.95	1.5817 -1.5752	3.1569	1.5433 -1.5389	3.0822	1.5434 -1.5389	3.0823	1.5425 -1.5381	3.0807	1.5270 -1.5235	3.0505	1.2828 -0.8531	2.1359	1.5779 -1.5715	3.1494	1.3781 -1.3825	2.7606	1.3364 -1.3421	2.6785	1.5577 -1.5525	3.1102	1.5725 -1.5665	3.1390	1.4807 -1.4800	2.9607	1.5552 -1.5502	3.1054	1.4807 -1.4800	2.9607	1.5770 -1.5707	3.1477
	0.90	1.1216 -1.1149	2.2365	1.1059 -1.0997	2.2056	1.1064 -1.1001	2.2065	1.1060 -1.0998	2.2058	1.1024 -1.0962	2.1986	1.0833 -0.5895	1.6728	1.1213 -1.1145	2.2358	1.0971 -1.0911	2.1882	1.0925 -1.0867	2.1791	1.1129 -1.1065	2.2194	1.1169 -1.1103	2.2272	1.0988 -1.0928	2.1915	1.1158 -1.1093	2.2251	1.0988 -1.0928	2.1915	1.1210 -1.1142	2.2352
	0.85	0.9184 -0.9115	1.8298	0.9113 -0.9019	1.8133	0.9118 -0.9025	1.8143	0.9116 -0.9023	1.8140	0.9096 -0.8996	1.8092	0.9119 -0.4131	1.3250	0.9183 -0.9114	1.8297	0.8896 -0.8724	1.7620	0.8888 -0.8714	1.7602	0.9147 -0.9066	1.8213	0.9122 -0.9031	1.8153	0.8619 -0.8347	1.6966	0.9032 -0.8909	1.7941	0.8619 -0.8347	1.6966	0.9181 -0.9112	1.8294
100	0.99	1.7640 -1.7607	3.5247	1.7073 -1.7042	3.4116	1.7078 -1.7047	3.4125	1.7066 -1.7035	3.4101	1.6897 -1.6867	3.3764	1.7316 -1.5165	3.2480	1.7636 -1.7604	3.5240	1.6297 -1.6270	3.2566	1.5804 -1.5780	3.1583	1.7320 -1.7288	3.4608	1.7508 -1.7477	3.4985	0.7615 -0.7594	1.5209	1.3248 -1.3227	2.6475	0.7615 -0.7594	1.5209	1.7589 -1.7557	3.5147
	0.95	0.7909 -0.7876	1.5784	0.7859 -0.7821	1.5680	0.7863 -0.7825	1.5688	0.7862 -0.7824	1.5686	0.7845 -0.7805	1.5650	0.7474 -0.2769	1.0243	0.7897 -0.7863	1.5760	0.7601 -0.7534	1.5135	0.7758 -0.7708	1.5466	0.7884 -0.7849	1.5733	0.7891 -0.7856	1.5746	0.7523 -0.7448	1.4972	0.7808 -0.7765	1.5573	0.7523 -0.7448	1.4972	0.7893 -0.7858	1.5751
	0.90	0.5608 -0.5574	1.1183	0.5584 -0.5552	1.1136	0.5589 -0.5556	1.1145	0.5588 -0.5556	1.1144	0.5584 -0.5551	1.1135	0.5573 -0.1551	0.7124	0.5608 -0.5574	1.1182	0.5545 -0.5514	1.1058	0.5556 -0.5525	1.1081	0.5600 -0.5566	1.1166	0.5579 -0.5546	1.1125	0.5470 -0.5442	1.0912	0.5573 -0.5541	1.1114	0.5470 -0.5442	1.0912	0.5606 -0.5573	1.1179
	0.85	0.4592 -0.4557	0.9149	0.4578 -0.4544	0.9122	0.4582 -0.4548	0.9130	0.4582 -0.4547	0.9130	0.4579 -0.4544	0.9123	0.4575 -0.1036	0.5611	0.4592 -0.4557	0.9149	0.4512 -0.4475	0.8987	0.4569 -0.4534	0.9102	0.4587 -0.4553	0.9140	0.4551 -0.4516	0.9067	0.2171 -0.2109	0.4280	0.3112 -0.3061	0.6174	0.2171 -0.2109	0.4280	0.4591 -0.4556	0.9147
200	0.99	1.2473 -1.2450	2.4923	1.2270 -1.2240	2.4510	1.2277 -1.2248	2.4526	1.2273 -1.2244	2.4517	1.2213 -1.2182	2.4395	1.0752 -0.5844	1.6595	1.2453 -1.2429	2.4882	1.2019 -1.1982	2.4000	1.1939 -1.1899	2.3838	1.2375 -1.2350	2.4725	1.2387 -1.2362	2.4749	0.5533 -0.5283	1.0816	0.9444 -0.9324	1.8768	0.5533 -0.5283	1.0816	1.2420 -1.2396	2.4816
	0.95	0.5592 -0.5569	1.1161	0.5572 -0.5542	1.1115	0.5576 -0.5548	1.1124	0.5576 -0.5547	1.1123	0.5572 -0.5542	1.1113	0.5587 -0.1564	0.7152	0.5592 -0.5569	1.1161	0.4857 -0.4598	0.9455	0.4092 -0.3592	0.7683	0.5587 -0.5562	1.1149	0.5550 -0.5513	1.1063	0.4208 -0.3751	0.7959	0.5033 -0.4836	0.9869	0.4208 -0.3751	0.7959	0.5590 -0.5566	1.1155
	0.90	0.3966 -0.3942	0.7907	0.3956 -0.3933	0.7889	0.3959 -0.3935	0.7894	0.3958 -0.3935	0.7893	0.3957 -0.3934	0.7890	0.3958 -0.0779	0.4737	0.3965 -0.3942	0.7907	0.3954 -0.3931	0.7884	0.3956 -0.3933	0.7889	0.3961 -0.3937	0.7898	0.3961 -0.3938	0.7899	0.3910 -0.3889	0.7800	0.3952 -0.3928	0.7880	0.3910 -0.3889	0.7800	0.3964 -0.3941	0.7905
	0.85	0.3247 -0.3223	0.6469	0.3244 -0.3215	0.6459	0.3245 -0.3218	0.6463	0.3245 -0.3218	0.6463	0.3244 -0.3216	0.6460	0.3246 -0.0518	0.3764	0.3247 -0.3223	0.6469	0.3237 -0.3199	0.6435	0.3226 -0.3175	0.6401	0.3246 -0.3221	0.6467	0.3236 -0.3198	0.6434	0.3209 -0.3134	0.6343	0.3237 -0.3200	0.6437	0.3209 -0.3134	0.6343	0.3247 -0.3222	0.6468

APPENDIX-7: Bootstrap-t Confidence Intervals for LS and ridge estimators with $\beta'\beta = 25$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	5.1114 -5.9276	11.0390	-0.3748 -5.5260	5.1512	-0.2486 -5.4326	5.1840	-0.2470 -5.4010	5.1540	-0.3099 -5.3526	5.0428	0.9961 -2.5569	3.5530	5.1976 -4.6820	9.8796	-0.1303 -5.0439	4.9136	-1.1668 -4.7737	3.6069	0.7804 -6.4591	7.2395	2.6159 -4.7843	7.4002	-2.8575 -4.8200	1.9625	-4.6799 -9.3131	4.6332	-2.8575 -4.8200	1.9625	5.1039 -3.2506	8.3545
	0.95	5.9647 1.0190	4.9456	2.9482 -0.9801	3.9284	2.7037 -1.1992	3.9029	2.6829 -1.2100	3.8929	2.5118 -1.1964	3.7082	2.7980 -0.9408	3.7388	5.8632 0.9395	4.9237	2.0335 -1.8024	3.8359	2.2572 -1.1351	3.3923	3.6077 -0.6875	4.2953	5.1695 0.7972	4.3723	2.4773 -1.2561	3.7333	4.3493 -0.2019	4.5512	2.4773 -1.2561	3.7333	5.8427 1.5113	4.3315
	0.90	1.2364 -2.2654	3.5019	2.6198 -0.5000	3.1198	2.7143 -0.3982	3.1125	2.7264 -0.3805	3.1069	2.8894 -0.0716	2.9610	0.7981 -1.6093	2.4073	1.3606 -2.1212	3.4817	2.3243 0.0859	2.2384	2.7523 -0.0171	2.7694	2.5437 -0.6628	3.2065	1.3795 -1.9028	3.2823	2.8258 -0.2470	3.0728	1.8846 -1.4963	3.3809	2.8258 -0.2470	3.0728	1.3705 -2.0462	3.4168
	0.85	0.4168 -2.4336	2.8504	2.7063 0.0802	2.6261	2.6448 0.0214	2.6234	2.6749 0.0549	2.6200	3.0674 0.5201	2.5474	3.2969 1.0543	2.2426	0.6380 -2.1948	2.8327	2.1135 0.2099	1.9035	-0.0709 -1.9909	1.9200	1.5453 -1.2003	2.7455	0.8378 -1.8776	2.7154	-6.1652 -7.6532	1.4880	-4.3332 -6.4572	2.1241	-6.1652 -7.6532	1.4880	2.6490 0.0195	2.6295
9	0.99	3.7753 -3.5643	7.3396	1.2925 -2.9966	4.2891	1.1737 -3.1565	4.3302	1.1210 -3.1797	4.3007	0.0036 -4.2980	4.3016	-10.5541 -12.9563	2.4022	3.5636 -3.1154	6.6790	0.5268 -1.3301	1.8570	-6.2937 -8.1178	1.8241	3.5207 -2.3115	5.8321	2.5988 -2.9066	5.5053	-4.1569 -6.3128	2.1559	1.5351 -2.9787	4.5138	-4.1569 -6.3128	2.1559	3.6023 -2.8563	6.4586
	0.95	4.7984 1.5045	3.2938	2.6747 -0.2728	2.9475	2.8082 -0.1338	2.9420	2.7837 -0.1514	2.9351	2.4490 -0.3937	2.8427	5.8575 4.0614	1.7961	4.4081 1.1627	3.2455	2.8066 0.9978	1.8088	7.9432 6.0439	1.8993	3.4181 0.3290	3.0890	4.2712 1.1761	3.0950	2.3228 -0.5309	2.8537	3.8668 0.6962	3.1705	2.3228 -0.5309	2.8537	2.9852 -0.1467	3.1320
	0.90	-1.6261 -3.9541	2.3280	-0.7341 -2.9233	2.1892	-0.7662 -2.9568	2.1907	-0.7492 -2.9370	2.1878	-0.5614 -2.7183	2.1569	-1.2886 -3.5637	2.2751	-1.6224 -3.9499	2.3275	0.4760 -0.7358	1.2118	-0.7500 -1.8948	1.1447	-1.2978 -3.5746	2.2768	-1.4415 -3.6818	2.2403	-19.4459 -20.2209	0.7750	-17.9361 -19.2211	1.2850	-19.4459 -20.2209	0.7750	-1.6047 -3.9125	2.3079
	0.85	1.0088 -0.8816	1.8904	0.1243 -1.6938	1.8181	0.2076 -1.6143	1.8219	0.1925 -1.6279	1.8204	-0.0436 -1.8398	1.7962	0.1364 -1.6792	1.8156	0.9986 -0.8911	1.8896	0.1037 -1.7367	1.8404	-0.0691 -1.8629	1.7937	0.3060 -1.5257	1.8316	0.6636 -1.1777	1.8414	-0.8933 -2.5359	1.6426	0.2043 -1.6151	1.8194	-0.8933 -2.5359	1.6426	0.8905 -0.9743	1.8648
25	0.99	1.1119 -3.2937	4.4056	1.9601 -1.5755	3.5356	2.0886 -1.4567	3.5452	2.0895 -1.4409	3.5304	2.1753 -1.2716	3.4468	-2.8812 -5.1749	2.2937	1.2380 -3.0475	4.2855	1.7183 -1.2798	2.9981	0.8524 -2.0955	2.9480	1.8882 -2.0338	3.9220	1.4406 -2.4712	3.9117	-1.3128 -2.6903	1.3775	1.5722 -1.2487	2.8209	-1.3128 -2.6903	1.3775	1.7899 -1.7529	3.5428
	0.95	2.2025 0.2356	1.9669	3.1456 1.2567	1.8888	3.1455 1.2550	1.8906	3.1632 1.2743	1.8889	3.4602 1.6000	1.8601	3.7370 1.9199	1.8171	2.2235 0.2582	1.9653	3.0754 1.7038	1.3716	3.5188 1.6685	1.8502	3.0958 1.1988	1.8970	2.5076 0.5929	1.9147	4.6026 2.9586	1.6440	3.3373 1.4640	1.8733	4.6026 2.9586	1.6440	2.2488 0.3039	1.9448
	0.90	2.4202 1.0225	1.3977	1.6006 0.2320	1.3686	1.6626 0.2929	1.3698	1.6467 0.2776	1.3691	1.3955 0.0363	1.3592	-1.2459 -2.4498	1.2039	2.3499 0.9546	1.3952	0.5978 -0.6877	1.2854	0.8150 -0.5193	1.3343	1.9644 0.5832	1.3812	1.9218 0.5472	1.3746	-0.2393 -1.5214	1.2822	1.5731 0.2065	1.3667	-0.2393 -1.5214	1.2822	1.5313 0.1582	1.3731
	0.85	-0.0730 -1.2123	1.1393	0.0906 -1.0334	1.1240	0.0559 -1.0686	1.1244	0.0588 -1.0653	1.1241	0.1112 -1.0071	1.1182	0.0046 -1.1257	1.1303	-0.0722 -1.2114	1.1392	0.0558 -0.9081	0.9638	0.2434 -0.8598	1.1033	0.0033 -1.1272	1.1305	0.0125 -1.1141	1.1267	0.2210 -0.8848	1.1059	0.0000 -1.1307	1.1307	0.2210 -0.8848	1.1059	-0.0691 -1.1932	1.1241
100	0.99	0.6346 -1.5642	2.1988	1.6062 -0.4811	2.0873	1.5907 -0.4998	2.0906	1.6080 -0.4802	2.0883	1.8807 -0.1695	2.0502	1.3510 -0.7732	2.1242	0.6429 -1.5552	2.1980	1.7380 0.9267	0.8113	-3.9590 -4.7977	0.8387	1.2784 -0.8532	2.1316	0.8640 -1.2664	2.1304	-2.9139 -3.6249	0.7110	2.0636 0.6293	1.4344	-2.9139 -3.6249	0.7110	1.6489 -0.5229	2.1718
	0.95	0.3379 -0.6480	0.9859	0.3809 -0.5920	0.9729	0.3656 -0.6087	0.9743	0.3662 -0.6079	0.9740	0.3710 -0.6010	0.9720	0.3486 -0.6326	0.9812	0.3380 -0.6478	0.9859	0.3582 -0.5385	0.8967	0.4234 -0.5320	0.9553	0.3485 -0.6327	0.9812	0.3612 -0.6134	0.9746	0.6249 -0.2614	0.8863	0.4072 -0.5516	0.9588	0.6249 -0.2614	0.8863	0.3602 -0.6155	0.9757
	0.90	-0.0402 -0.7349	0.6947	0.0229 -0.6675	0.6904	0.0192 -0.6720	0.6912	0.0205 -0.6706	0.6912	0.0446 -0.6451	0.6897	-0.0019 -0.6944	0.6925	-0.0399 -0.7345	0.6946	0.0190 -0.6229	0.6419	0.0921 -0.5946	0.6866	-0.0021 -0.6946	0.6925	0.0456 -0.6443	0.6900	0.3127 -0.3582	0.6709	0.0634 -0.6251	0.6886	0.3127 -0.3582	0.6709	-0.0282 -0.7219	0.6937
	0.85	0.0127 -0.5584	0.5711	-0.0961 -0.6645	0.5683	-0.0683 -0.6375	0.5692	-0.0702 -0.6393	0.5691	-0.0989 -0.6673	0.5684	-0.0666 -0.6358	0.5692	0.0119 -0.5592	0.5711	-0.1682 -0.7277	0.5594	-0.0975 -0.6659	0.5685	-0.0529 -0.6225	0.5695	-0.0853 -0.6539	0.5685	-0.3419 -0.9045	0.5626	-0.0781 -0.6470	0.5689	-0.3419 -0.9045	0.5626	0.0045 -0.5643	0.5688
200	0.99	0.6428 -0.9095	1.5523	0.2106 -1.3059	1.5165	0.1731 -1.3425	1.5156	0.1620 -1.3529	1.5149	-0.0286 -1.5269	1.4983	-0.5282 -1.9644	1.4362	0.6261 -0.9249	1.5510	0.2663 -0.9633	1.2296	-0.9089 -2.3041	1.3951	0.3967 -1.1371	1.5338	0.4976 -1.0315	1.5291	3.7751 3.2698	0.5053	-0.4818 -1.4974	1.0156	3.7751 3.2698	0.5053	0.3794 -1.1681	1.5475
	0.95	0.5097 -0.1891	0.6988	0.7636 0.0693	0.6943	0.7001 0.0050	0.6951	0.7041 0.0091	0.6951	0.7640 0.0701	0.6939	2.5612 1.9086	0.6525	0.5338 -0.1645	0.6984	0.4796 -0.1469	0.6265	2.5331 1.8803	0.6528	0.5822 -0.1153	0.6974	1.0314 0.3399	0.6916	4.4168 3.9261	0.4907	3.2971 2.6735	0.6236	4.4168 3.9261	0.4907	0.5669 0.0050	0.5620
	0.90	-0.0957 -0.5890	0.4934	-0.0443 -0.5358	0.4915	-0.0679 -0.5601	0.4922	-0.0674 -0.5596	0.4921	-0.0552 -0.5468	0.4916	-0.0598 -0.5516	0.4918	-0.0953 -0.5887	0.4933	-0.0678 -0.5408	0.4730	-0.0495 -0.5409	0.4914	-0.0724 -0.5647	0.4924	-0.0762 -0.5682	0.4920	0.1595 -0.3227	0.4822	-0.0311 -0.5216	0.4905	0.1595 -0.3227	0.4822	-0.0903 -0.5813	0.4910
	0.85	1.0889 0.6840	0.4049	1.0974 0.6936	0.4037	1.0959 0.6916	0.4042	1.0962 0.6919	0.4042	1.0990 0.6950	0.4040	1.1162 0.7137	0.4025	1.0891 0.6842	0.4049	1.0946 0.6974	0.3973	1.1019 0.6982	0.4037	1.0939 0.6895	0.4044	1.0926 0.6885	0.4041	1.1372 0.7365	0.4007	1.1006 0.6968	0.4039	1.1372 0.7365	0.4007	1.0905 0.6875	0.4030

APPENDIX-8: Standard Normal Approximation Confidence Intervals for LS and ridge estimators with $\beta'\beta = 25$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	4.3551 -4.3470	8.7021	2.5271 -2.5337	5.0608	2.5313 -2.5379	5.0693	2.5286 -2.5351	5.0637	2.4546 -2.4608	4.9154	1.7364 -1.7505	3.4869	4.0278 -4.0222	8.0500	1.7586 -1.7706	3.5293	1.8237 -1.8345	3.6582	3.2607 -3.2606	6.5214	4.3154 -4.3077	8.6231	1.0840 -1.1041	2.1881	2.2588 -2.2686	4.5274	1.0840 -1.1041	2.1881	4.0176 -4.0122	8.0297
	0.95	1.9526 -1.9444	3.8970	1.6854 -1.6792	3.3646	1.6883 -1.6821	3.3704	1.6762 -1.6701	3.3463	1.6195 -1.6138	3.2333	1.6137 -1.6061	3.2198	1.9474 -1.9392	3.8866	1.4101 -1.4065	2.8165	1.5146 -1.5098	3.0244	1.7880 -1.7809	3.5689	1.9360 -1.9279	3.8639	1.6241 -1.6192	3.2434	1.8542 -1.8470	3.7013	1.6241 -1.6192	3.2434	1.9464 -1.9381	3.8845
	0.90	1.3846 -1.3762	2.7608	1.2878 -1.2832	2.5710	1.2901 -1.2856	2.5757	1.2846 -1.2802	2.5648	1.2442 -1.2412	2.4854	1.0674 -1.0697	2.1371	1.3798 -1.3715	2.7513	1.1560 -1.1562	2.3123	1.1837 -1.1827	2.3665	1.3107 -1.3052	2.6160	1.3734 -1.3655	2.7389	1.2748 -1.2707	2.5455	1.3550 -1.3478	2.7028	1.2748 -1.2707	2.5455	1.3794 -1.3712	2.7506
	0.85	1.1337 -1.1251	2.2588	1.0794 -1.0684	2.1477	1.0813 -1.0703	2.1516	1.0780 -1.0669	2.1449	1.0594 -1.0475	2.1069	0.9717 -0.9557	1.9275	1.1296 -1.1208	2.2504	0.8890 -0.8697	1.7586	0.8649 -0.8441	1.7090	1.1089 -1.0992	2.2081	1.1239 -1.1149	2.2387	0.7027 -0.6743	1.3771	0.9173 -0.8982	1.8155	0.7027 -0.6743	1.3771	1.1294 -1.1206	2.2500
9	0.99	2.9033 -2.8980	5.8013	1.9873 -1.9889	3.9762	1.9902 -1.9918	3.9819	1.9927 -1.9942	3.9870	1.9932 -1.9946	3.9878	1.1774 -1.1863	2.3637	2.7207 -2.7167	5.4374	1.0636 -1.0727	2.1364	0.9491 -0.9593	1.9084	2.4906 -2.4881	4.9788	2.8785 -2.8734	5.7519	1.1583 -1.1676	2.3259	2.0910 -2.0925	4.1835	1.1583 -1.1676	2.3259	2.7138 -2.7099	5.4237
	0.95	1.3017 -1.2962	2.5980	1.2167 -1.2084	2.4251	1.2190 -1.2107	2.4296	1.2136 -1.2052	2.4189	1.1896 -1.1804	2.3699	0.8476 -0.8284	1.6760	1.2903 -1.2845	2.5748	0.9374 -0.9208	1.8582	0.8731 -0.8558	1.7288	1.2527 -1.2456	2.4983	1.2923 -1.2865	2.5787	1.1930 -1.1838	2.3769	1.2726 -1.2661	2.5387	1.1930 -1.1838	2.3769	1.2897 -1.2838	2.5735
	0.90	0.9231 -0.9175	1.8405	0.8910 -0.8809	1.7720	0.8927 -0.8826	1.7752	0.8909 -0.8808	1.7718	0.8836 -0.8725	1.7561	0.9112 -0.9039	1.8150	0.9229 -0.9173	1.8403	0.6537 -0.6111	1.2648	0.5652 -0.5128	1.0780	0.9116 -0.9043	1.8159	0.9153 -0.9086	1.8239	0.4064 -0.3465	0.7529	0.5940 -0.5529	1.1469	0.4064 -0.3465	0.7529	0.9227 -0.9171	1.8398
	0.85	0.7558 -0.7501	1.5059	0.7387 -0.7315	1.4703	0.7401 -0.7329	1.4730	0.7394 -0.7322	1.4716	0.7335 -0.7259	1.4595	0.7382 -0.7310	1.4691	0.7556 -0.7499	1.5055	0.7304 -0.7225	1.4530	0.7329 -0.7252	1.4582	0.7420 -0.7351	1.4772	0.7500 -0.7437	1.4937	0.6943 -0.6835	1.3778	0.7390 -0.7319	1.4710	0.6943 -0.6835	1.3778	0.7555 -0.7497	1.5052
25	0.99	1.7420 -1.7388	3.4808	1.5085 -1.5097	3.0182	1.5111 -1.5123	3.0234	1.5072 -1.5084	3.0156	1.4840 -1.4858	2.9698	1.0507 -1.0619	2.1126	1.7111 -1.7085	3.4196	1.3142 -1.3193	2.6334	1.3205 -1.3253	2.6457	1.6185 -1.6176	3.2361	1.7279 -1.7250	3.4529	0.7273 -0.7450	1.4723	1.2916 -1.2977	2.5893	0.7273 -0.7450	1.4723	1.7066 -1.7041	3.4107
	0.95	0.7810 -0.7778	1.5588	0.7615 -0.7590	1.5205	0.7629 -0.7604	1.5233	0.7617 -0.7591	1.5208	0.7543 -0.7521	1.5064	0.7430 -0.7411	1.4842	0.7807 -0.7774	1.5581	0.7513 -0.7492	1.5005	0.7518 -0.7496	1.5014	0.7637 -0.7611	1.5248	0.7747 -0.7717	1.5464	0.6957 -0.6957	1.3915	0.7576 -0.7553	1.5129	0.6957 -0.6957	1.3915	0.7802 -0.7770	1.5572
	0.90	0.5538 -0.5505	1.1043	0.5466 -0.5436	1.0902	0.5476 -0.5446	1.0922	0.5468 -0.5438	1.0906	0.5443 -0.5415	1.0858	0.5029 -0.5026	1.0055	0.5532 -0.5499	1.1032	0.5334 -0.5311	1.0645	0.5379 -0.5355	1.0735	0.5498 -0.5467	1.0965	0.5496 -0.5465	1.0962	0.5244 -0.5228	1.0471	0.5461 -0.5432	1.0894	0.5244 -0.5228	1.0471	0.5531 -0.5498	1.1029
	0.85	0.4535 -0.4501	0.9035	0.4504 -0.4458	0.8962	0.4512 -0.4466	0.8978	0.4504 -0.4458	0.8962	0.4492 -0.4441	0.8934	0.4517 -0.4475	0.8992	0.4535 -0.4500	0.9035	0.4474 -0.4415	0.8889	0.4462 -0.4398	0.8860	0.4517 -0.4476	0.8993	0.4515 -0.4473	0.8988	0.4467 -0.4406	0.8873	0.4518 -0.4477	0.8994	0.4467 -0.4406	0.8873	0.4534 -0.4499	0.9033
100	0.99	0.8710 -0.8694	1.7404	0.8433 -0.8422	1.6854	0.8448 -0.8437	1.6885	0.8437 -0.8426	1.6864	0.8341 -0.8331	1.6672	0.8527 -0.8515	1.7042	0.8708 -0.8692	1.7400	0.6036 -0.6068	1.2104	0.4013 -0.4075	0.8088	0.8546 -0.8533	1.7079	0.8652 -0.8637	1.7289	0.3755 -0.3823	0.7578	0.6551 -0.6574	1.3125	0.3755 -0.3823	0.7578	0.8685 -0.8669	1.7354
	0.95	0.3905 -0.3889	0.7794	0.3876 -0.3856	0.7731	0.3883 -0.3863	0.7745	0.3878 -0.3859	0.7737	0.3874 -0.3854	0.7727	0.3895 -0.3877	0.7772	0.3905 -0.3889	0.7794	0.3830 -0.3804	0.7634	0.3835 -0.3811	0.7646	0.3895 -0.3877	0.7772	0.3883 -0.3864	0.7748	0.3670 -0.3626	0.7296	0.3843 -0.3820	0.7663	0.3670 -0.3626	0.7296	0.3903 -0.3886	0.7790
	0.90	0.2769 -0.2752	0.5522	0.2757 -0.2744	0.5501	0.2762 -0.2749	0.5511	0.2760 -0.2745	0.5505	0.2756 -0.2743	0.5498	0.2763 -0.2748	0.5511	0.2769 -0.2752	0.5522	0.2752 -0.2740	0.5492	0.2747 -0.2736	0.5483	0.2763 -0.2748	0.5512	0.2755 -0.2742	0.5498	0.2703 -0.2704	0.5407	0.2752 -0.2740	0.5493	0.2703 -0.2704	0.5407	0.2768 -0.2752	0.5520
	0.85	0.2267 -0.2250	0.4518	0.2259 -0.2246	0.4504	0.2263 -0.2250	0.4513	0.2261 -0.2247	0.4508	0.2259 -0.2246	0.4505	0.2261 -0.2247	0.4509	0.2267 -0.2250	0.4518	0.2254 -0.2244	0.4498	0.2259 -0.2246	0.4505	0.2262 -0.2248	0.4510	0.2260 -0.2246	0.4506	0.2240 -0.2237	0.4477	0.2261 -0.2247	0.4507	0.2240 -0.2237	0.4477	0.2267 -0.2250	0.4517
200	0.99	0.6159 -0.6148	1.2307	0.6075 -0.6059	1.2133	0.6086 -0.6070	1.2155	0.6071 -0.6055	1.2127	0.6032 -0.6014	1.2046	0.5880 -0.5853	1.1733	0.6156 -0.6145	1.2301	0.5914 -0.5889	1.1804	0.5777 -0.5745	1.1522	0.6116 -0.6102	1.2218	0.6135 -0.6122	1.2257	0.2798 -0.2604	0.5402	0.4703 -0.4610	0.9314	0.2798 -0.2604	0.5402	0.6140 -0.6128	1.2268
	0.95	0.2761 -0.2750	0.5511	0.2751 -0.2739	0.5490	0.2756 -0.2744	0.5500	0.2753 -0.2740	0.5493	0.2750 -0.2738	0.5488	0.2653 -0.2632	0.5284	0.2760 -0.2749	0.5509	0.2705 -0.2689	0.5394	0.2653 -0.2632	0.5285	0.2758 -0.2746	0.5505	0.2739 -0.2725	0.5464	0.2199 -0.2140	0.4340	0.2575 -0.2548	0.5123	0.2199 -0.2140	0.4340	0.2759 -0.2747	0.5506
	0.90	0.1958 -0.1946	0.3904	0.1955 -0.1941	0.3895	0.1958 -0.1944	0.3903	0.1956 -0.1943	0.3899	0.1955 -0.1941	0.3896	0.1956 -0.1942	0.3897	0.1958 -0.1946	0.3904	0.1955 -0.1940	0.3895	0.1955 -0.1940	0.3895	0.1956 -0.1943	0.3900	0.1957 -0.1944	0.3900	0.1939 -0.1912	0.3851	0.1953 -0.1938	0.3891	0.1939 -0.1912	0.3851	0.1958 -0.1946	0.3903
	0.85	0.1603 -0.1591	0.3194	0.1602 -0.1587	0.3189	0.1604 -0.1590	0.3195	0.1602 -0.1589	0.3191	0.1602 -0.1588	0.3190	0.1600 -0.1583	0.3183	0.1603 -0.1591	0.3194	0.1602 -0.1587	0.3189	0.1601 -0.1587	0.3189	0.1603 -0.1590	0.3192	0.1603 -0.1590	0.3192	0.1597 -0.1577	0.3174	0.1602 -0.1588	0.3189	0.1597 -0.1577	0.3174	0.1603 -0.1591	0.3194

APPENDIX-9: t Approach Confidence Intervals for LS and ridge estimators with $\beta'\beta = 25$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	4.4099 -4.4019	8.8118	2.5590 -2.5656	5.1246	2.5714 -2.5778	5.1492	2.5605 -2.5670	5.1275	2.4856 -2.4918	4.9774	1.7584 -2.2857	4.0441	4.0785 -4.0730	8.1514	1.7809 -1.7929	3.5737	1.8467 -1.8576	3.7043	3.3018 -3.3017	6.6035	4.3697 -4.3620	8.7317	1.0977 -1.1179	2.2156	2.2874 -2.2971	4.5845	1.0977 -1.1179	2.2156	4.0682 -4.0627	8.1309
	0.95	1.9772 -1.9689	3.9461	1.7066 -1.7004	3.4070	1.7002 -1.6940	3.3941	1.6973 -1.6911	3.3884	1.6399 -1.6342	3.2741	1.6340 -1.3784	3.0124	1.9719 -1.9637	3.9356	1.4278 -1.4242	2.8520	1.5337 -1.5288	3.0625	1.8105 -1.8034	3.6138	1.9604 -1.9523	3.9126	1.6446 -1.6397	3.2842	1.8776 -1.8704	3.7479	1.6446 -1.6397	3.2842	1.9708 -1.9626	3.9335
	0.90	1.4020 -1.3936	2.7956	1.3040 -1.2994	2.6034	1.3023 -1.2978	2.6001	1.3008 -1.2963	2.5971	1.2598 -1.2569	2.5167	1.0809 -0.6093	1.6902	1.3971 -1.3889	2.7860	1.1706 -1.1708	2.3414	1.1986 -1.1977	2.3963	1.3272 -1.3217	2.6489	1.3907 -1.3827	2.7734	1.2909 -1.2867	2.5776	1.3721 -1.3648	2.7369	1.2909 -1.2867	2.5776	1.3967 -1.3885	2.7852
	0.85	1.1479 -1.1394	2.2873	1.0929 -1.0819	2.1748	1.0923 -1.0813	2.1736	1.0915 -1.0804	2.1719	1.0726 -1.0608	2.1334	0.9839 -0.4785	1.4624	1.1438 -1.1350	2.2788	0.9000 -0.8807	1.7808	0.8756 -0.8549	1.7305	1.1228 -1.1131	2.2359	1.1380 -1.1290	2.2670	0.7114 -0.6830	1.3944	0.9287 -0.9096	1.8383	0.7114 -0.6830	1.3944	1.1436 -1.1348	2.2784
9	0.99	2.9399 -2.9345	5.8744	2.0123 -2.0139	4.0263	2.0280 -2.0294	4.0574	2.0179 -2.0194	4.0372	2.0183 -2.0197	4.0380	1.1923 -1.0250	2.2174	2.7549 -2.7510	5.5059	1.0771 -1.0862	2.1633	0.9611 -0.9713	1.9324	2.5220 -2.5195	5.0415	2.9148 -2.9096	5.8244	1.1729 -1.1823	2.3552	2.1174 -2.1189	4.2362	1.1729 -1.1823	2.3552	2.7480 -2.7441	5.4920
	0.95	1.3181 -1.3126	2.6307	1.2320 -1.2237	2.4557	1.2307 -1.2224	2.4531	1.2289 -1.2205	2.4494	1.2045 -1.1953	2.3998	0.8582 -0.3817	1.2399	1.3066 -1.3007	2.6073	0.9491 -0.9325	1.8816	0.8840 -0.8667	1.7506	1.2684 -1.2613	2.5297	1.3085 -1.3027	2.6112	1.2080 -1.1988	2.4068	1.2886 -1.2821	2.5707	1.2080 -1.1988	2.4068	1.3059 -1.3000	2.6059
	0.90	0.9347 -0.9291	1.8637	0.9022 -0.8921	1.7943	0.9028 -0.8928	1.7956	0.9021 -0.8920	1.7941	0.8947 -0.8836	1.7783	0.9226 -0.4247	1.3473	0.9345 -0.9289	1.8635	0.6617 -0.6191	1.2808	0.5720 -0.5196	1.0916	0.9230 -0.9158	1.8388	0.9268 -0.9201	1.8468	0.4112 -0.3512	0.7624	0.6012 -0.5601	1.1614	0.4112 -0.3512	0.7624	0.9343 -0.9286	1.8629
	0.85	0.7653 -0.7596	1.5248	0.7480 -0.7408	1.4888	0.7490 -0.7419	1.4909	0.7486 -0.7415	1.4902	0.7427 -0.7351	1.4779	0.7474 -0.2770	1.0244	0.7651 -0.7594	1.5245	0.7396 -0.7317	1.4713	0.7421 -0.7344	1.4766	0.7514 -0.7444	1.4958	0.7594 -0.7531	1.5125	0.7030 -0.6922	1.3952	0.7483 -0.7412	1.4895	0.7030 -0.6922	1.3952	0.7650 -0.7592	1.5242
25	0.99	1.7640 -1.7607	3.5247	1.5275 -1.5287	3.0562	1.5306 -1.5317	3.0624	1.5262 -1.5274	3.0536	1.5027 -1.5045	3.0072	1.0640 -0.6885	1.7525	1.7326 -1.7301	3.4627	1.3308 -1.3358	2.6666	1.3371 -1.3420	2.6791	1.6389 -1.6380	3.2769	1.7496 -1.7467	3.4964	0.7365 -0.7543	1.4909	1.3079 -1.3140	2.6220	0.7365 -0.7543	1.4909	1.7281 -1.7256	3.4536
	0.95	0.7909 -0.7876	1.5784	0.7711 -0.7686	1.5397	0.7717 -0.7691	1.5408	0.7712 -0.7687	1.5400	0.7638 -0.7616	1.5254	0.7524 -0.2854	1.0378	0.7905 -0.7872	1.5777	0.7608 -0.7586	1.5194	0.7612 -0.7591	1.5203	0.7733 -0.7707	1.5440	0.7845 -0.7814	1.5659	0.7045 -0.7045	1.4090	0.7672 -0.7648	1.5320	0.7045 -0.7045	1.4090	0.7901 -0.7868	1.5768
	0.90	0.5608 -0.5574	1.1183	0.5535 -0.5505	1.1040	0.5538 -0.5509	1.1047	0.5537 -0.5507	1.1044	0.5511 -0.5483	1.0995	0.5092 -0.1312	0.6405	0.5602 -0.5569	1.1171	0.5401 -0.5378	1.0779	0.5447 -0.5423	1.0870	0.5567 -0.5536	1.1103	0.5566 -0.5534	1.1100	0.5310 -0.5294	1.0603	0.5530 -0.5501	1.1031	0.5310 -0.5294	1.0603	0.5600 -0.5567	1.1168
	0.85	0.4592 -0.4557	0.9149	0.4561 -0.4514	0.9075	0.4561 -0.4515	0.9077	0.4561 -0.4514	0.9075	0.4549 -0.4498	0.9046	0.4573 -0.1031	0.5604	0.4592 -0.4557	0.9149	0.4530 -0.4471	0.9001	0.4518 -0.4454	0.8972	0.4574 -0.4532	0.9106	0.4572 -0.4530	0.9101	0.4523 -0.4462	0.8985	0.4574 -0.4533	0.9108	0.4523 -0.4462	0.8985	0.4591 -0.4556	0.9147
100	0.99	0.8820 -0.8804	1.7624	0.8539 -0.8528	1.7067	0.8549 -0.8538	1.7088	0.8544 -0.8532	1.7076	0.8446 -0.8436	1.6882	0.8635 -0.3770	1.2405	0.8818 -0.8802	1.7620	0.6112 -0.6145	1.2257	0.4064 -0.4126	0.8190	0.8653 -0.8640	1.7294	0.8761 -0.8746	1.7507	0.3803 -0.3870	0.7673	0.6634 -0.6657	1.3291	0.3803 -0.3870	0.7673	0.8794 -0.8779	1.7573
	0.95	0.3954 -0.3938	0.7892	0.3924 -0.3904	0.7829	0.3928 -0.3908	0.7836	0.3927 -0.3907	0.7834	0.3922 -0.3902	0.7825	0.3944 -0.0777	0.4720	0.3954 -0.3938	0.7892	0.3878 -0.3853	0.7730	0.3883 -0.3859	0.7742	0.3944 -0.3926	0.7870	0.3932 -0.3913	0.7846	0.3716 -0.3672	0.7388	0.3892 -0.3868	0.7760	0.3716 -0.3672	0.7388	0.3952 -0.3936	0.7888
	0.90	0.2804 -0.2787	0.5591	0.2792 -0.2779	0.5571	0.2794 -0.2780	0.5575	0.2794 -0.2780	0.5574	0.2790 -0.2777	0.5567	0.2798 -0.0388	0.3186	0.2804 -0.2787	0.5591	0.2786 -0.2774	0.5561	0.2782 -0.2771	0.5552	0.2798 -0.2783	0.5581	0.2790 -0.2777	0.5567	0.2737 -0.2738	0.5475	0.2787 -0.2775	0.5562	0.2737 -0.2738	0.5475	0.2803 -0.2787	0.5590
	0.85	0.2296 -0.2279	0.4575	0.2287 -0.2274	0.4561	0.2290 -0.2276	0.4565	0.2290 -0.2275	0.4565	0.2287 -0.2274	0.4562	0.2290 -0.0257	0.2547	0.2296 -0.2279	0.4575	0.2283 -0.2272	0.4554	0.2287 -0.2274	0.4562	0.2291 -0.2276	0.4567	0.2288 -0.2275	0.4563	0.2268 -0.2265	0.4533	0.2289 -0.2275	0.4564	0.2268 -0.2265	0.4533	0.2295 -0.2278	0.4574
200	0.99	0.6237 -0.6225	1.2462	0.6151 -0.6135	1.2286	0.6150 -0.6134	1.2283	0.6148 -0.6132	1.2280	0.6108 -0.6090	1.2198	0.5954 -0.1776	0.7730	0.6234 -0.6222	1.2456	0.5989 -0.5964	1.1953	0.5850 -0.5818	1.1668	0.6193 -0.6179	1.2372	0.6212 -0.6199	1.2412	0.2832 -0.2638	0.5470	0.4762 -0.4669	0.9431	0.2832 -0.2638	0.5470	0.6217 -0.6205	1.2422
	0.95	0.2796 -0.2785	0.5581	0.2786 -0.2773	0.5559	0.2788 -0.2775	0.5563	0.2787 -0.2775	0.5562	0.2785 -0.2772	0.5557	0.2686 -0.0352	0.3038	0.2795 -0.2783	0.5578	0.2739 -0.2723	0.5462	0.2686 -0.2665	0.5351	0.2793 -0.2781	0.5574	0.2773 -0.2759	0.5532	0.2227 -0.2168	0.4394	0.2607 -0.2580	0.5188	0.2227 -0.2168	0.4394	0.2794 -0.2782	0.5575
	0.90	0.1983 -0.1971	0.3954	0.1979 -0.1965	0.3945	0.1981 -0.1967	0.3948	0.1981 -0.1967	0.3948	0.1980 -0.1965	0.3945	0.1980 -0.0191	0.2171	0.1983 -0.1971	0.3954	0.1979 -0.1965	0.3944	0.1979 -0.1965	0.3944	0.1981 -0.1968	0.3949	0.1981 -0.1968	0.3949	0.1963 -0.1936	0.3899	0.1978 -0.1962	0.3940	0.1963 -0.1936	0.3899	0.1982 -0.1970	0.3952
	0.85	0.1623 -0.1611	0.3235	0.1622 -0.1607	0.3229	0.1622 -0.1609	0.3231	0.1622 -0.1609	0.3231	0.1622 -0.1608	0.3230	0.1620 -0.0124	0.1743	0.1623 -0.1611	0.3235	0.1622 -0.1608	0.3229	0.1622 -0.1607	0.3229	0.1623 -0.1610	0.3232	0.1623 -0.1610	0.3233	0.1617 -0.1597	0.3214	0.1622 -0.1608	0.3230	0.1617 -0.1597	0.3214	0.1623 -0.1611	0.3234

APPENDIX-10: Bootstrap-t Confidence Intervals for LS and ridge estimators with $\beta'\beta = 4$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	-1.0693 -5.4824	4.4131	0.1088 -1.9571	2.0659	0.0756 -2.0073	2.0830	0.0644 -2.0066	2.0710	0.8695 -1.1549	2.0244	2.1366 0.2935	1.8431	-0.1602 -4.1149	3.9547	0.8674 -0.5591	1.4264	1.0487 -0.3984	1.4470	-0.0021 -2.9031	2.9010	-0.1876 -3.1513	2.9637	-0.7166 -1.5065	0.7899	-0.5081 -2.3686	1.8605	-0.7166 -1.5065	0.7899	-0.1642 -3.7870	3.6227
	0.95	-0.0950 -2.0646	1.9696	0.3802 -1.1848	1.5650	0.4444 -1.1165	1.5608	0.4474 -1.1092	1.5566	0.7424 -0.7149	1.4573	0.2055 -1.5569	1.7624	-0.0928 -2.0599	1.9671	0.3466 -0.4709	0.8175	0.1880 -0.6058	0.7939	0.1295 -1.6507	1.7802	0.0026 -1.7382	1.7408	-7.0571 -7.7079	0.6508	-8.6318 -9.7268	1.0951	-7.0571 -7.7079	0.6508	-0.0931 -1.7091	1.6160
	0.90	0.1471 -1.2464	1.3934	0.6469 -0.5936	1.2405	0.5977 -0.6425	1.2403	0.6028 -0.6352	1.2380	0.5576 -0.6228	1.1805	0.8822 -0.2341	1.1163	0.1711 -1.2186	1.3897	0.5034 -0.5882	1.0916	0.6816 -0.4707	1.1523	0.5100 -0.7662	1.2762	0.1986 -1.1071	1.3057	0.4808 -0.7385	1.2192	0.3168 -1.0273	1.3441	0.4808 -0.7385	1.2192	0.3746 -0.9177	1.2923
	0.85	1.4275 0.2881	1.1394	0.4310 -0.6170	1.0480	0.4158 -0.6315	1.0473	0.4036 -0.6423	1.0460	0.2254 -0.7942	1.0196	-0.2038 -1.0833	0.8795	1.3577 0.2235	1.1343	0.2945 -0.2031	0.4976	1.1174 0.4000	0.7174	0.9651 -0.1366	1.1018	1.1704 0.0860	1.0844	2.8679 2.5755	0.2923	1.9141 1.3831	0.5310	2.8679 2.5755	0.2923	1.3528 0.2827	1.0702
9	0.99	1.9915 -0.9466	2.9380	1.4190 -0.3158	1.7348	1.5125 -0.2265	1.7390	1.5063 -0.2210	1.7273	1.5476 -0.1938	1.7414	3.2304 1.6957	1.5347	2.1236 -0.7032	2.8268	1.0450 -0.7941	1.8391	0.8219 -0.4237	1.2456	2.0432 -0.2158	2.2590	1.4138 -0.8019	2.2157	-0.9244 -1.3846	0.4602	-1.0885 -2.1938	1.1053	-0.9244 -1.3846	0.4602	2.1403 -0.3593	2.4996
	0.95	0.0495 -1.2713	1.3208	0.1001 -1.0731	1.1732	0.2065 -0.9628	1.1693	0.2071 -0.9592	1.1663	0.2088 -0.9280	1.1367	-0.4855 -1.6174	1.1319	0.1078 -1.1897	1.2975	0.1608 -0.8159	0.9767	0.4504 -0.5014	0.9519	0.1639 -1.0775	1.2414	-0.0068 -1.2433	1.2365	0.2067 -0.9395	1.1462	0.1300 -1.1420	1.2720	0.2067 -0.9395	1.1462	0.1097 -1.0670	1.1767
	0.90	0.1018 -0.8315	0.9333	-0.1166 -0.9946	0.8781	-0.1309 -1.0073	0.8763	-0.1354 -1.0106	0.8752	-0.1692 -1.0352	0.8661	-0.1226 -0.9881	0.8655	0.0992 -0.8334	0.9326	-0.2175 -1.0796	0.8622	-0.2236 -1.0757	0.8521	-0.0492 -0.9458	0.8967	0.0362 -0.8639	0.9000	-0.1368 -1.0119	0.8751	0.0355 -0.8825	0.9180	-0.1368 -1.0119	0.8751	-0.1979 -1.0764	0.8785
	0.85	0.0845 -0.6765	0.7611	-0.2271 -0.9571	0.7300	-0.2550 -0.9863	0.7314	-0.2619 -0.9926	0.7307	-0.3410 -1.0645	0.7235	-0.0419 -0.7922	0.7502	0.0832 -0.6778	0.7610	-0.1775 -0.4857	0.3082	-0.0052 -0.4016	0.3963	-0.0383 -0.7888	0.7506	-0.0442 -0.7840	0.7398	5.6400 5.3461	0.2940	3.8974 3.4345	0.4629	5.6400 5.3461	0.2940	-0.2803 -1.0256	0.7453
25	0.99	0.9938 -0.7629	1.7566	-0.2279 -1.6482	1.4203	-0.2340 -1.6432	1.4091	-0.2459 -1.6493	1.4033	-0.3075 -1.6888	1.3813	0.0990 -1.4414	1.5404	0.9802 -0.7738	1.7540	-0.4205 -1.6328	1.2123	-0.5097 -1.7542	1.2445	0.1836 -1.3801	1.5638	0.7090 -0.8560	1.5650	-0.0747 -0.3647	0.2900	0.3520 -0.3420	0.6939	-0.0747 -0.3647	0.2900	0.8364 -0.9080	1.7444
	0.95	0.3477 -0.4387	0.7864	0.2364 -0.5191	0.7555	0.2208 -0.5358	0.7566	0.2186 -0.5373	0.7560	0.1819 -0.5620	0.7439	0.1803 -0.5664	0.7467	0.3453 -0.4406	0.7860	0.2071 -0.4911	0.6983	0.1822 -0.5614	0.7435	0.2445 -0.5180	0.7625	0.3115 -0.4553	0.7668	0.1077 -0.5964	0.7041	0.2486 -0.5154	0.7640	0.1077 -0.5964	0.7041	0.3195 -0.4576	0.7771
	0.90	-0.0681 -0.6299	0.5618	-0.3459 -0.8960	0.5501	-0.3722 -0.9222	0.5499	-0.3785 -0.9282	0.5497	-0.4589 -1.0053	0.5464	-0.4148 -0.9633	0.5485	-0.0717 -0.6334	0.5617	-0.4896 -1.0130	0.5234	-0.4644 -1.0104	0.5460	-0.3775 -0.9275	0.5500	-0.2205 -0.7735	0.5530	-0.7898 -1.3221	0.5322	-0.2730 -0.8271	0.5541	-0.7898 -1.3221	0.5322	-0.0289 -0.5813	0.5525
	0.85	0.1494 -0.3091	0.4584	-0.1146 -0.5662	0.4516	-0.0921 -0.5441	0.4520	-0.0973 -0.5492	0.4519	-0.1675 -0.6175	0.4500	-0.4830 -0.9239	0.4410	0.1432 -0.3150	0.4583	-0.6743 -1.0616	0.3873	-0.9130 -1.3405	0.4275	0.0503 -0.4055	0.4558	-0.1090 -0.5607	0.4517	-0.1322 -0.4323	0.3002	-0.3563 -0.7461	0.3898	-0.1322 -0.4323	0.3002	0.0369 -0.4213	0.4582
100	0.99	-0.3046 -1.1823	0.8778	-0.6818 -1.5149	0.8331	-0.6718 -1.5057	0.8339	-0.6787 -1.5116	0.8329	-0.7732 -1.5918	0.8185	0.5114 -0.0221	0.5335	-0.3807 -1.2470	0.8664	-0.4612 -1.2642	0.8030	-0.4726 -1.0114	0.5388	-0.4641 -1.3246	0.8605	-0.3668 -1.2182	0.8514	1.1377 0.8525	0.2852	-0.7671 -1.3411	0.5740	1.1377 0.8525	0.2852	-0.7176 -1.5578	0.8402
	0.95	0.3608 -0.0338	0.3946	0.4009 0.0112	0.3897	0.3885 -0.0016	0.3901	0.3891 -0.0009	0.3900	0.3944 0.0053	0.3891	0.4010 0.0130	0.3881	0.3613 -0.0333	0.3946	0.3856 0.0194	0.3662	0.3937 0.0044	0.3893	0.3815 -0.0097	0.3911	0.3763 -0.0147	0.3910	0.5352 0.1749	0.3603	0.4171 0.0317	0.3854	0.5352 0.1749	0.3603	0.3636 -0.0302	0.3937
	0.90	-0.0618 -0.3434	0.2816	-0.0189 -0.2987	0.2797	-0.0302 -0.3104	0.2802	-0.0295 -0.3097	0.2801	-0.0186 -0.2982	0.2796	0.1374 -0.1350	0.2724	-0.0602 -0.3117	0.2516	-0.0300 -0.2767	0.2467	0.0109 -0.2674	0.2783	-0.0379 -0.3184	0.2805	-0.0417 -0.3219	0.2802	0.0876 -0.1871	0.2747	-0.0232 -0.3031	0.2799	0.0876 -0.1871	0.2747	-0.0590 -0.3401	0.2812
	0.85	0.2425 0.0136	0.2288	0.3192 0.0915	0.2277	0.2991 0.0711	0.2280	0.3004 0.0724	0.2280	0.3191 0.0914	0.2277	0.4358 0.2097	0.2261	0.1941 0.0153	0.1788	0.2972 0.1776	0.1196	0.4099 0.1835	0.2264	0.2688 0.0403	0.2284	0.4423 0.2154	0.2269	1.2524 1.0397	0.2126	0.5422 0.3177	0.2245	1.2524 1.0397	0.2126	0.2493 0.0713	0.1780
200	0.99	0.0621 -0.5609	0.6230	0.2957 -0.3094	0.6051	0.2981 -0.3080	0.6061	0.3038 -0.3019	0.6057	0.3594 -0.2419	0.6013	0.6880 0.1199	0.5681	0.0707 -0.5517	0.6224	0.6240 0.0276	0.5964	0.5479 -0.0365	0.5844	0.1905 -0.4236	0.6141	0.1953 -0.4142	0.6095	-1.1552 -1.3195	0.1643	-0.0407 -0.3947	0.3540	-1.1552 -1.3195	0.1643	0.2985 -0.3150	0.6135
	0.95	0.5108 0.2320	0.2787	0.5923 0.3154	0.2768	0.5818 0.3046	0.2772	0.5835 0.3063	0.2771	0.6023 0.3256	0.2767	0.5585 0.2808	0.2777	0.5112 0.2325	0.2787	0.5802 0.3404	0.2398	0.6302 0.3541	0.2761	0.5583 0.2807	0.2777	0.6088 0.3321	0.2767	1.4876 1.2343	0.2534	0.8190 0.5472	0.2719	1.4876 1.2343	0.2534	0.5731 0.2960	0.2771
	0.90	0.4221 0.2241	0.1979	0.3640 0.1668	0.1972	0.3831 0.1856	0.1975	0.3823 0.1848	0.1974	0.3664 0.1692	0.1972	0.0524 -0.1406	0.1930	0.4190 0.2211	0.1979	0.3216 0.1255	0.1961	0.3273 0.1305	0.1967	0.3986 0.2010	0.1976	0.3336 0.1366	0.1970	-0.0159 -0.2080	0.1921	0.3036 0.1071	0.1965	-0.0159 -0.2080	0.1921	0.3833 0.1860	0.1973
	0.85	0.0903 -0.0711	0.1615	0.1134 -0.0476	0.1610	0.1031 -0.0581	0.1612	0.1034 -0.0578	0.1612	0.1086 -0.0525	0.1611	0.1012 -0.0600	0.1612	0.0904 -0.0710	0.1615	0.1030 -0.0497	0.1527	0.1103 -0.0507	0.1610	0.1012 -0.0600	0.1612	0.1037 -0.0574	0.1611	0.1764 0.0168	0.1597	0.1121 -0.0489	0.1610	0.1764 0.0168	0.1597	0.0928 -0.0582	0.1510

APPENDIX-11: Standard Normal Approximation Confidence Intervals for LS and ridge estimators with $\beta'\beta = 4$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	1.7420 -1.7388	3.4808	1.0099 -1.0126	2.0225	1.0116 -1.0143	2.0259	1.0114 -1.0140	2.0255	0.9819 -0.9843	1.9662	0.7946 -0.7002	1.4948	1.6111 -1.6089	3.2200	0.7035 -0.7082	1.4117	0.7295 -0.7338	1.4633	1.3043 -1.3042	2.6085	1.7261 -1.7231	3.4492	0.4336 -0.4416	0.8752	0.9035 -0.9074	1.8110	0.4336 -0.4416	0.8752	1.6070 -1.6049	3.2119
	0.95	0.7810 -0.7778	1.5588	0.6767 -0.6693	1.3460	0.6780 -0.6706	1.3486	0.6745 -0.6670	1.3415	0.6439 -0.6354	1.2793	0.7294 -0.7242	1.4536	0.7805 -0.7772	1.5576	0.4146 -0.3967	0.8113	0.3965 -0.3783	0.7748	0.7343 -0.7293	1.4637	0.7741 -0.7705	1.5447	0.3279 -0.3105	0.6383	0.4959 -0.4837	0.9796	0.3279 -0.3105	0.6383	0.7801 -0.7767	1.5568
	0.90	0.5538 -0.5505	1.1043	0.5142 -0.5130	1.0272	0.5151 -0.5139	1.0290	0.5137 -0.5125	1.0262	0.4975 -0.4971	0.9946	0.4764 -0.4768	0.9532	0.5529 -0.5496	1.1025	0.4552 -0.4572	0.9124	0.4885 -0.4885	0.9771	0.5240 -0.5222	1.0461	0.5493 -0.5462	1.0955	0.5084 -0.5074	1.0158	0.5416 -0.5389	1.0804	0.5084 -0.5074	1.0158	0.5528 -0.5495	1.1022
	0.85	0.4535 -0.4501	0.9035	0.4304 -0.4276	0.8580	0.4312 -0.4283	0.8596	0.4299 -0.4271	0.8570	0.4229 -0.4202	0.8432	0.3820 -0.3803	0.7623	0.4522 -0.4488	0.9011	0.3342 -0.3344	0.6686	0.3237 -0.3246	0.6483	0.4442 -0.4410	0.8852	0.4493 -0.4460	0.8953	0.1448 -0.1486	0.2934	0.2438 -0.2454	0.4893	0.1448 -0.1486	0.2934	0.4521 -0.4487	0.9009
9	0.99	1.1613 -1.1591	2.3204	0.8054 -0.8001	1.6055	0.8070 -0.8016	1.6086	0.8040 -0.7986	1.6025	0.8091 -0.8038	1.6129	0.6831 -0.7007	1.3838	1.1312 -1.1288	2.2599	0.6014 -0.5941	1.1955	0.6174 -0.6103	1.2278	0.9772 -0.9735	1.9507	1.1508 -1.1485	2.2993	0.2615 -0.2508	0.5123	0.5483 -0.5401	1.0884	0.2615 -0.2508	0.5123	1.1282 -1.1258	2.2540
	0.95	0.5207 -0.5185	1.0391	0.4833 -0.4821	0.9655	0.4842 -0.4830	0.9673	0.4818 -0.4806	0.9624	0.4739 -0.4729	0.9468	0.3823 -0.3456	0.7279	0.5151 -0.5130	1.0281	0.4072 -0.4081	0.8152	0.4164 -0.4169	0.8333	0.5013 -0.4996	1.0008	0.5168 -0.5147	1.0315	0.4766 -0.4756	0.9522	0.5089 -0.5070	1.0159	0.4766 -0.4756	0.9522	0.5148 -0.5128	1.0276
	0.90	0.3692 -0.3670	0.7362	0.3556 -0.3534	0.7090	0.3562 -0.3540	0.7103	0.3550 -0.3528	0.7077	0.3527 -0.3505	0.7032	0.3524 -0.3502	0.7026	0.3690 -0.3668	0.7358	0.3493 -0.3472	0.6965	0.3491 -0.3469	0.6959	0.3603 -0.3581	0.7185	0.3667 -0.3645	0.7312	0.3549 -0.3528	0.7077	0.3655 -0.3633	0.7288	0.3549 -0.3528	0.7077	0.3689 -0.3667	0.7356
	0.85	0.3023 -0.3000	0.6023	0.2940 -0.2932	0.5871	0.2945 -0.2937	0.5882	0.2942 -0.2934	0.5876	0.2922 -0.2918	0.5840	0.2994 -0.2977	0.5971	0.3023 -0.3000	0.6023	0.2154 -0.2285	0.4439	0.1748 -0.1938	0.3686	0.2995 -0.2978	0.5973	0.2993 -0.2976	0.5969	0.1293 -0.1518	0.2811	0.1969 -0.2103	0.4072	0.1293 -0.1518	0.2811	0.3022 -0.2999	0.6022
25	0.99	0.6968 -0.6955	1.3923	0.6073 -0.6070	1.2143	0.6083 -0.6081	1.2164	0.6030 -0.6028	1.2058	0.5968 -0.5967	1.1935	0.6412 -0.6406	1.2818	0.6962 -0.6949	1.3911	0.5168 -0.5176	1.0344	0.5530 -0.5533	1.1063	0.6478 -0.6472	1.2950	0.6904 -0.6892	1.3797	0.1593 -0.1650	0.3243	0.3411 -0.3443	0.6855	0.1593 -0.1650	0.3243	0.6943 -0.6931	1.3874
	0.95	0.3124 -0.3111	0.6235	0.3052 -0.3032	0.6084	0.3058 -0.3037	0.6095	0.3054 -0.3033	0.6087	0.3025 -0.3002	0.6027	0.3031 -0.3009	0.6040	0.3123 -0.3110	0.6233	0.2934 -0.2902	0.5836	0.3024 -0.3001	0.6025	0.3069 -0.3050	0.6119	0.3106 -0.3091	0.6197	0.2928 -0.2895	0.5823	0.3073 -0.3054	0.6127	0.2928 -0.2895	0.5823	0.3122 -0.3108	0.6230
	0.90	0.2215 -0.2202	0.4417	0.2186 -0.2175	0.4361	0.2190 -0.2179	0.4369	0.2185 -0.2174	0.4359	0.2177 -0.2167	0.4343	0.2182 -0.2172	0.4354	0.2215 -0.2202	0.4417	0.2176 -0.2166	0.4342	0.2176 -0.2166	0.4342	0.2186 -0.2175	0.4361	0.2201 -0.2189	0.4390	0.2140 -0.2134	0.4274	0.2196 -0.2185	0.4381	0.2140 -0.2134	0.4274	0.2214 -0.2201	0.4416
	0.85	0.1814 -0.1800	0.3614	0.1796 -0.1786	0.3581	0.1799 -0.1789	0.3588	0.1796 -0.1786	0.3583	0.1791 -0.1782	0.3574	0.1767 -0.1763	0.3530	0.1814 -0.1800	0.3613	0.1742 -0.1743	0.3485	0.1729 -0.1734	0.3463	0.1807 -0.1795	0.3602	0.1796 -0.1786	0.3583	0.1290 -0.1381	0.2671	0.1600 -0.1628	0.3229	0.1290 -0.1381	0.2671	0.1813 -0.1800	0.3613
100	0.99	0.3484 -0.3478	0.6962	0.3376 -0.3366	0.6743	0.3382 -0.3373	0.6755	0.3376 -0.3367	0.6743	0.3340 -0.3330	0.6670	0.2448 -0.2409	0.4857	0.3457 -0.3450	0.6907	0.2634 -0.2602	0.5236	0.2480 -0.2444	0.4924	0.3443 -0.3435	0.6879	0.3466 -0.3459	0.6926	0.1557 -0.1492	0.3049	0.2648 -0.2616	0.5264	0.1557 -0.1492	0.3049	0.3448 -0.3440	0.6888
	0.95	0.1562 -0.1556	0.3118	0.1550 -0.1544	0.3094	0.1553 -0.1546	0.3100	0.1551 -0.1544	0.3096	0.1549 -0.1542	0.3091	0.1547 -0.1540	0.3086	0.1562 -0.1555	0.3117	0.1545 -0.1538	0.3082	0.1549 -0.1543	0.3092	0.1554 -0.1547	0.3101	0.1556 -0.1550	0.3106	0.1479 -0.1470	0.2949	0.1540 -0.1533	0.3074	0.1479 -0.1470	0.2949	0.1561 -0.1555	0.3116
	0.90	0.1108 -0.1101	0.2209	0.1104 -0.1096	0.2200	0.1106 -0.1098	0.2204	0.1105 -0.1097	0.2202	0.1104 -0.1095	0.2199	0.1089 -0.1075	0.2165	0.1108 -0.1101	0.2208	0.1102 -0.1093	0.2195	0.1101 -0.1092	0.2193	0.1106 -0.1098	0.2204	0.1106 -0.1099	0.2205	0.1094 -0.1082	0.2176	0.1104 -0.1096	0.2200	0.1094 -0.1082	0.2176	0.1107 -0.1100	0.2208
	0.85	0.0907 -0.0900	0.1807	0.0905 -0.0897	0.1802	0.0906 -0.0899	0.1805	0.0905 -0.0898	0.1803	0.0905 -0.0897	0.1802	0.0902 -0.0892	0.1794	0.0907 -0.0900	0.1807	0.0903 -0.0894	0.1796	0.0902 -0.0893	0.1796	0.0906 -0.0899	0.1805	0.0902 -0.0892	0.1794	0.0875 -0.0853	0.1728	0.0899 -0.0888	0.1786	0.0875 -0.0853	0.1728	0.0907 -0.0900	0.1807
200	0.99	0.2464 -0.2459	0.4923	0.2422 -0.2415	0.4836	0.2426 -0.2419	0.4845	0.2423 -0.2416	0.4839	0.2413 -0.2405	0.4818	0.2330 -0.2318	0.4648	0.2462 -0.2458	0.4920	0.2353 -0.2342	0.4694	0.2372 -0.2362	0.4733	0.2443 -0.2437	0.4880	0.2443 -0.2437	0.4879	0.0939 -0.0846	0.1785	0.1683 -0.1634	0.3317	0.0939 -0.0846	0.1785	0.2456 -0.2451	0.4907
	0.95	0.1105 -0.1100	0.2204	0.1101 -0.1095	0.2195	0.1103 -0.1097	0.2199	0.1101 -0.1096	0.2197	0.1100 -0.1095	0.2195	0.1102 -0.1097	0.2200	0.1105 -0.1100	0.2204	0.1100 -0.1094	0.2193	0.1099 -0.1093	0.2192	0.1102 -0.1097	0.2200	0.1100 -0.1094	0.2194	0.1048 -0.1031	0.2079	0.1090 -0.1082	0.2172	0.1048 -0.1031	0.2079	0.1104 -0.1099	0.2203
	0.90	0.0783 -0.0779	0.1562	0.0781 -0.0777	0.1558	0.0783 -0.0778	0.1561	0.0782 -0.0777	0.1559	0.0782 -0.0777	0.1558	0.0771 -0.0767	0.1538	0.0783 -0.0778	0.1562	0.0780 -0.0776	0.1556	0.0780 -0.0776	0.1556	0.0783 -0.0778	0.1560	0.0781 -0.0776	0.1556	0.0769 -0.0765	0.1534	0.0780 -0.0775	0.1555	0.0769 -0.0765	0.1534	0.0783 -0.0778	0.1561
	0.85	0.0641 -0.0636	0.1278	0.0641 -0.0635	0.1276	0.0642 -0.0636	0.1278	0.0641 -0.0636	0.1276	0.0641 -0.0635	0.1276	0.0641 -0.0636	0.1277	0.0641 -0.0636	0.1278	0.0641 -0.0635	0.1276	0.0641 -0.0635	0.1276	0.0641 -0.0636	0.1277	0.0641 -0.0635	0.1276	0.0639 -0.0630	0.1269	0.0641 -0.0635	0.1276	0.0639 -0.0630	0.1269	0.0641 -0.0636	0.1278

APPENDIX-12: t Approach Confidence Intervals for LS and ridge estimators with $\beta'\beta = 4$

SNR	ρ	LS		k_{boot}		k_{press}		k_{gvc}		k_2		k_3		k_4		k_5		k_6		k_7		k_8		k_9		k_{10}		k_{11}		k_{12}	
4	0.99	1.7640	3.5247	1.0227	2.0480	1.0286	2.0597	1.0242	2.0510	0.9942	1.9910	0.7034	1.0708	1.6314	3.2606	0.7124	1.4295	0.7387	1.4817	1.3207	2.6414	1.7479	3.4927	0.4391	0.8863	0.9150	1.8338	0.4391	0.8863	1.6273	3.2524
		-1.7607		-1.0253		-1.0311		-1.0268		-0.9967		-0.3674		-1.6292		-0.7171		-0.7430		-1.3207		-1.7448		-0.4472		-0.9188		-0.4472		-1.6251	
	0.95	0.7909	1.5784	0.6852	1.3629	0.6841	1.3608	0.6830	1.3584	0.6520	1.2954	0.7386	1.0110	0.7903	1.5773	0.4197	0.8215	0.4014	0.7845	0.7436	1.4821	0.7839	1.5641	0.3319	0.6464	0.5020	0.9920	0.3319	0.6464	0.7899	1.5764
		-0.7876		-0.6777		-0.6767		-0.6754		-0.6434		-0.2724		-0.7870		-0.4018		-0.3832		-0.7385		-0.7803		-0.3145		-0.4899		-0.3145		-0.7865	
	0.90	0.5608	1.1183	0.5207	1.0401	0.5208	1.0403	0.5202	1.0391	0.5037	1.0071	0.4824	0.6020	0.5599	1.1164	0.4610	0.9239	0.4947	0.9894	0.5305	1.0593	0.5562	1.1093	0.5148	1.0286	0.5484	1.0940	0.5148	1.0286	0.5597	1.1161
		-0.5574		-0.5195		-0.5195		-0.5190		-0.5034		-0.1195		-0.5565		-0.4629		-0.4947		-0.5288		-0.5531		-0.5138		-0.5457		-0.5138		-0.5564	
	0.85	0.4592	0.9149	0.4358	0.8688	0.4357	0.8685	0.4353	0.8678	0.4282	0.8538	0.3868	0.4618	0.4579	0.9124	0.3384	0.6771	0.3278	0.6564	0.4498	0.8964	0.4550	0.9066	0.1466	0.2971	0.2469	0.4954	0.1466	0.2971	0.4578	0.9122
		-0.4557		-0.4330		-0.4328		-0.4325		-0.4256		-0.0750		-0.4545		-0.3386		-0.3287		-0.4466		-0.4516		-0.1504		-0.2485		-0.1504		-0.4544	
9	0.99	1.1759	2.3497	0.8156	1.6258	0.8180	1.6307	0.8140	1.6227	0.8193	1.6333	0.7162	1.0204	1.1454	2.2884	0.6089	1.2106	0.6252	1.2432	0.9895	1.9752	1.1653	2.3283	0.2647	0.5188	0.5552	1.1021	0.2647	0.5188	1.1424	2.2824
		-1.1738		-0.8102		-0.8127		-0.8087		-0.8140		-0.3042		-1.1430		-0.6017		-0.6181		-0.9857		-1.1630		-0.2541		-0.5470		-0.2541		-1.1400	
	0.95	0.5272	1.0522	0.4894	0.9776	0.4886	0.9761	0.4878	0.9745	0.4798	0.9587	0.3663	0.4233	0.5216	1.0411	0.4123	0.8255	0.4217	0.8438	0.5076	1.0134	0.5233	1.0445	0.4826	0.9642	0.5153	1.0287	0.4826	0.9642	0.5213	1.0405
		-0.5250		-0.4882		-0.4874		-0.4867		-0.4789		-0.0570		-0.5195		-0.4132		-0.4221		-0.5059		-0.5212		-0.4816		-0.5134		-0.4816		-0.5193	
	0.90	0.3739	0.7455	0.3601	0.7179	0.3597	0.7173	0.3594	0.7167	0.3571	0.7120	0.3569	0.4199	0.3737	0.7451	0.3537	0.7053	0.3534	0.7047	0.3649	0.7275	0.3713	0.7404	0.3594	0.7166	0.3701	0.7380	0.3594	0.7166	0.3736	0.7449
		-0.3716		-0.3579		-0.3575		-0.3572		-0.3549		-0.0630		-0.3714		-0.3516		-0.3513		-0.3627		-0.3691		-0.3572		-0.3679		-0.3572		-0.3713	
	0.85	0.3061	0.6099	0.2977	0.5945	0.2981	0.5953	0.2979	0.5950	0.2959	0.5914	0.3032	0.3487	0.3061	0.6099	0.2182	0.4495	0.1771	0.3733	0.3033	0.6048	0.3031	0.6044	0.1311	0.2846	0.1995	0.4123	0.1311	0.2846	0.3060	0.6097
		-0.3038		-0.2969		-0.2972		-0.2971		-0.2954		-0.0455		-0.3038		-0.2313		-0.1961		-0.3015		-0.3013		-0.1535		-0.2128		-0.1535		-0.3037	
25	0.99	0.7056	1.4099	0.6149	1.2296	0.6123	1.2244	0.6106	1.2210	0.6043	1.2085	0.6492	0.8628	0.7050	1.4086	0.5233	1.0475	0.5600	1.1202	0.6560	1.3113	0.6991	1.3970	0.1613	0.3284	0.3454	0.6941	0.1613	0.3284	0.7031	1.4049
		-0.7043		-0.6147		-0.6121		-0.6104		-0.6042		-0.2136		-0.7037		-0.5242		-0.5603		-0.6553		-0.6979		-0.1671		-0.3487		-0.1671		-0.7018	
	0.95	0.3163	0.6314	0.3090	0.6161	0.3093	0.6167	0.3092	0.6164	0.3063	0.6103	0.3069	0.3532	0.3163	0.6312	0.2971	0.5909	0.3062	0.6100	0.3108	0.6196	0.3145	0.6275	0.2965	0.5897	0.3111	0.6204	0.2965	0.5897	0.3161	0.6308
		-0.3150		-0.3070		-0.3073		-0.3072		-0.3040		-0.0463		-0.3149		-0.2939		-0.3039		-0.3089		-0.3130		-0.2932		-0.3093		-0.2932		-0.3147	
	0.90	0.2243	0.4473	0.2213	0.4416	0.2213	0.4416	0.2212	0.4414	0.2204	0.4398	0.2209	0.2451	0.2243	0.4472	0.2203	0.4396	0.2203	0.4396	0.2213	0.4416	0.2228	0.4445	0.2167	0.4328	0.2224	0.4436	0.2167	0.4328	0.2242	0.4471
		-0.2230		-0.2203		-0.2203		-0.2202		-0.2194		-0.0241		-0.2229		-0.2193		-0.2193		-0.2203		-0.2216		-0.2161		-0.2212		-0.2161		-0.2229	
	0.85	0.1837	0.3660	0.1818	0.3627	0.1819	0.3629	0.1819	0.3628	0.1814	0.3619	0.1789	0.1949	0.1836	0.3659	0.1764	0.3529	0.1751	0.3506	0.1830	0.3647	0.1819	0.3628	0.1307	0.2705	0.1621	0.3269	0.1307	0.2705	0.1836	0.3658
		-0.1823		-0.1808		-0.1809		-0.1809		-0.1805		-0.0160		-0.1823		-0.1765		-0.1755		-0.1817		-0.1809		-0.1397		-0.1649		-0.1397		-0.1822	
100	0.99	0.3528	0.7049	0.3419	0.6828	0.3421	0.6832	0.3419	0.6828	0.3382	0.6754	0.2478	0.2788	0.3500	0.6994	0.2667	0.5302	0.2512	0.4986	0.3486	0.6965	0.3510	0.7013	0.1577	0.3088	0.2681	0.5330	0.1577	0.3088	0.3491	0.6975
		-0.3521		-0.3409		-0.3411		-0.3409		-0.3372		-0.0309		-0.3493		-0.2635		-0.2475		-0.3479		-0.3503		-0.1511		-0.2649		-0.1511		-0.3484	
	0.95	0.1582	0.3157	0.1570	0.3133	0.1571	0.3135	0.1571	0.3135	0.1568	0.3130	0.1566	0.1686	0.1582	0.3157	0.1564	0.3121	0.1569	0.3131	0.1573	0.3140	0.1576	0.3145	0.1497	0.2986	0.1560	0.3112	0.1497	0.2986	0.1581	0.3155
		-0.1575		-0.1563		-0.1564		-0.1564		-0.1562		-0.0120		-0.1575		-0.1557		-0.1562		-0.1567		-0.1569		-0.1489		-0.1553		-0.1489		-0.1574	
	0.90	0.1122	0.2237	0.1118	0.2228	0.1119	0.2230	0.1119	0.2229	0.1118	0.2227	0.1103	0.1157	0.1121	0.2236	0.1116	0.2223	0.1115	0.2221	0.1119	0.2231	0.1120	0.2232	0.1108	0.2203	0.1118	0.2228	0.1108	0.2203	0.1121	0.2236
		-0.1115		-0.1110		-0.1111		-0.1111		-0.1109		-0.0054		-0.1115		-0.1107		-0.1106		-0.1112		-0.1112		-0.1096		-0.1110		-0.1096		-0.1114	
	0.85	0.0918	0.1830	0.0916	0.1825	0.0917	0.1826	0.0917	0.1826	0.0916	0.1825	0.0913	0.0950	0.0918	0.1830	0.0914	0.1819	0.0914	0.1818	0.0918	0.1828	0.0913	0.1816	0.0886	0.1749	0.0910	0.1809	0.0886	0.1749	0.0918	0.1829
		-0.0911		-0.0908		-0.0909		-0.0909		-0.0908		-0.0037		-0.0911		-0.0905		-0.0905		-0.0910		-0.0904		-0.0864		-0.0899		-0.0864		-0.0911	
200	0.99	0.2495	0.4985	0.2452	0.4897	0.2454	0.4902	0.2454	0.4900	0.2443	0.4879	0.2360	0.2634	0.2493	0.4982	0.2382	0.4753	0.2401	0.4793	0.2474	0.4941	0.2473	0.4941	0.0950	0.1808	0.1704	0.3359	0.0950	0.1808	0.2487	0.4969
		-0.2490		-0.2445		-0.2448		-0.2447		-0.2436		-0.0275		-0.2489		-0.2371		-0.2392		-0.2468		-0.2468		-0.0857		-0.1655		-0.0857		-0.2482	
	0.95	0.1118	0.2232	0.1114	0.2223	0.1115	0.2225	0.1115	0.2225	0.1114	0.2223	0.1116	0.1177	0.1118	0.2232	0.1113	0.2221	0.1113	0.2220	0.1116	0.2227	0.1114	0.2222	0.1061	0.2105	0.1104	0.2199	0.1061	0.2105	0.1118	0.2231
		-0.1114		-0.1109		-0.1110		-0.1110		-0.1109		-0.0060		-0.1114		-0.1108		-0.1107		-0.1111		-0.1108		-0.1044		-0.1096		-0.1044		-0.1113	
	0.90	0.0793	0.1581	0.0791	0.1578	0.0792	0.1579	0.0792	0.1579	0.0791	0.1578	0.0781	0.0810	0.0793	0.1581	0.0790	0.1575	0.0790	0.1576	0.0792	0.1580	0.0790	0.1576	0.0779	0.1553	0.0790	0.1574	0.0779	0.1553	0.0793	0.1581
		-0.0788		-0.0787		-0.0787		-0.0787		-0.0787		-0.0028		-0.0788		-0.0785		-0.0785		-0.0788		-0.0786		-0.0774		-0.0785		-0.0774		-0.0788	
	0.85	0.0649	0.1294	0.0649	0.1292	0.0649	0.1293	0.0649	0.1293	0.0649	0.1292	0.0649	0.0668	0.0649	0.1294	0.0649	0.1292	0.0649	0.1292	0.0649	0.1293	0.0649	0.1293	0.0647	0.1285	0.0649	0.1292	0.0647	0.1285	0.0649	0.1294
		-0.0645		-0.0643		-0.0644		-0.0644		-0.0643		-0.0019		-0.0645		-0.0643		-0.0643		-0.0644		-0.0644		-0.0638		-0.0643		-0.0638		-0.0644	