REPLENISHMENT AND TRANSPORTATION PRICING DECISIONS UNDER TWO INDEPENDENT CARRIERS WITH DIFFERENT MODES

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By

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ABSTRACT

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January, 2009

Freight transportation constitutes a substantial cost component in global economics and consumes significant amounts of energy. With the increasing oil prices, scarcity of resources and environmental concerns, efficient operation of transportation activities has become even more important than it was in the past. These raising concerns necessitate the companies which utilize this activity to plan for transportation more carefully and conservatively, and the ones that support it to appraise its value more cautiously.

In this thesis, we consider a setting consisting of a retailer, a truck-load carrier and a less-than-truck-load carrier. We model and solve the retailer's integrated inventory and transportation problem under the presence of two modes of carriers. Characterising the solution for the retailer's replenishment problem, we then analyze the transportation pricing decisions of the carriers. We show that substantial savings can be achieved by the retailer due to integrating inventory and transportation decisions. We also quantify the savings for the TL carrier and the LTL carrier through carefully determining their transportation price schedules.

Keywords: Transportation pricing, Truckload carrier, Less-than-truckload carrier, Transportation and inventory

ÖZET

BAĞIMSIZ İKİ FARKLI TAŞIMA ŞİRKETİ DURUMUNDA NAKLİYE HİZMETİNİN FİYATLANDIRILMASI VE İKMAL KARARLARI

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Eşya taşımacılığı global ekonomide önemli bir maliyet unsurunu teşkil etmekte ve önemli miktarda enerji tüketmektedir. Artan petrol fiyatlarına, kaynakların kıtlığına ve çevresel kaygılara bağlı olarak, nakliye faaliyetlerinin etkin bir şekilde kullanılması geçmişe nazaran çok daha önemli hale gelmiştir. Artan bu kaygılar, nakliye hizmeti alan şirketlerin daha dikkatli ve korunumlu planlama yapmasını, ve bu hizmeti verenlerin de daha ihtiyatlı bir şekilde fiyatlarını belirlemesini gerekli kılmıştır.

Bu tezde, bir perakendeci, bir kamyonyükü taşıyıcısı ve bir birimyük taşıyıcısını içeren bir ortam ele alınmıştır. Perakendecinin entegre envanter ve nakliye problemi iki taşıma şirketinin varlığı altında modellenmiş ve çözülmüştür. Perakendecinin ikmal probleminin çözümü karakterize ediltikten sonra taşıyıcıların nakliyet hizmetini fiyatlandırma kararları analiz edilmiştir. Envanter ve nakliye kararlarının birleştirilmesi sonucu perakendecinin önemli tasarruf elde edebileceğini görülmüştür. Ayrıca, dikkatli bir şekilde nakliye fiyat tarifesine karar verme yoluyla kamyonyükü taşıyıcısının ve birimyük taşıyıcısının muhtemel kazanımları sayısal olarak gösterilmiştir.

Anahtar Kelimeler: Nakliye hizmetinin fiyatlandırılması, Kamyonyükü taşıyıcısı, Birimyük taşıyıcısı, Nakliye ve envanter

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I devote this thesis and all my study to the memory of my dear father Prof. Dr. Şener Bingöl who passed away with a sudden heart attack in June, 2008. "I feel that you are with me and I promise you that I will do my best to be successful."

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

Freight transportation is a significant component of the global economy. United States (US) and European Union (EU) hold the first two places in the world for their freight transportation expenditures. Freight shipping costs have been over 10% of the US Gross Domestic Product (GDP) in 2007 [23]. In EU countries, volume of the freight transportation per ton/km is expected to increase by 78% from 2000 to 2020, while the EU GDP is expected to grow by 60% in the same period [13]. In Turkey, as an EU candidate, freight transportation by road is expected to increase by 148% from 2004 to 2020 and its usage among all transportation paths (road, air, sea) is expected to increase from 81.2% in 2004 to 84.3% in 2020 [16].

In light of the above statistics, minimization of shipping costs presents an important opportunity for companies to improve their profits. This requires an effort for all parties involved in the supply chain ranging from retailers to distributors to transporter companies. Integration of inventory replenishment and transportation decisions, aggregation of different products across suppliers or buyers, efficient routing decisions are some important tools to decrease transportation costs. Minimization of freight costs for the parties who utilize this function is a highly explored topic in recent supply chain studies. However, there is limited research suggesting operating policies for companies that undertake this functionality. With the increasing oil prices, uncertainty in the market and the raising competition among transporter companies, this topic has become well worth to study. The objectives of this thesis are to suggest methods for freight transporters to make their pricing decisions and to quantify the savings that can be achieved through these decisions.

With the above objectives in mind, we consider a system that consists of a retailer, a truckload (TL) carrier and a less than truckload (LTL) carrier. The retailer purchases the services of these carriers for inbound transportation and makes his/her replenishment decisions considering the related transportation costs and capacities. The TL carrier charges a fixed cost per truck regardless of whether the truck is fully or partially loaded. The LTL carrier charges in proportion to the number of units shipped. By loading as many quantities as possible in a truck, the retailer may get advantage of economies of scale inherent in fixed cost of each truck. However, this can happen to the extent that increased quantities do not raise the inventory related costs. If the quantity to be shipped does not justify the fixed cost of an additional truck, then the retailer uses the LTL carrier.

In the setting of the problem of interest, both the TL and the LTL carrier have their own costs. Besides the fixed costs associated with a transaction, the TL carrier incurs a cost for each truck utilized and the LTL carrier incurs a cost per unit shipped. The TL carrier and the LTL carrier announce their pricing schedules before the retailer decides on his/her replenishment decision.

In this thesis, we first model and solve the retailer's replenishment problem under the given pricing decisions of the TL and the LTL carrier. Due to the complex structure exhibited by the retailer's expected profit function, this analysis follows by investigating the structural properties of the profit function, which then leads to a characterization of the optimal solution. As it will be discussed in Chapter 3, the model and its solution can be used not only for the specific setting in this thesis but it also applies to a more general class of problems. In the later parts of the thesis, we study the pricing problems of the TL carrier and the LTL carrier assuming that they have full information about the retailer or his/her ordering behaviour.

More specifically, we first study the problem of the TL carrier to determine the price he/she will charge for a single truck with the objective of maximizing his/her own profits given the LTL carrier's pricing schedule. Secondly, for a similar objective for the LTL carrier, we study his/her problem to determine the price to be charged for the shipment of a single unit given the TL carrier's pricing schedule. The analysis for the two transportation pricing problems follows based on two different characterizations of the retailer's optimal response and structural properties of his/her objective function, which are discussed in Chapter 4.

In addition to an analytical investigation of the retailer's replenishment problem and the carriers' pricing problems, we illusrate over numerical examples the savings that can be achieved by the retailer as a result of jointly optimizing replenishment and transportation costs. We show that substantial savings can be achieved by the retailer in this setting due to the integrated model. We also illusrate the opportunity of savings for the TL carrier and the LTL carrier by carefully deciding on the value of their transportation services. We show that the TL carrier and the LTL carrier may increase their expected profits by percentages amounting to 27% and 20%, respectively. Another contribution of this thesis is that, we show the savings that can be achieved if the TL carrier and the LTL carrier coordinate. This can specifically be applicable to a situation where the truckload and the less-than-truck load transporation services are provided by the same company. Finally, we numerically show the impact of coordination between the retailer and one of the carriers.

The remainder of the thesis is organized as follows: In Section 2, a review of the related literature is provided. In Section 3, a detailed discussion of the problem definition and the mathematical models are presented. Section 4 is devoted to the analysis of the retailer's replenishment problem. In Section 5, the pricing problems of the TL and the LTL carriers are analyzed. The results of a numerical study for the transportation pricing problems are reported in Section 6. Finally, the conclusions of this study are summarized in Section 7 in relation to the future research directions.

Chapter 2

LITERATURE REVIEW

Integration of transportation and inventory decisions has recently attracted significant attention from both the academia and the industry. It has been shown that significant savings can be achieved if the decisions associated with these supply chain functions are coordinated. The research in this thesis is closely related to the group of studies (Aucamp [2], Lee [11], Toptal [22]) which model transportation costs and constraints within the replenishment problem. The underlying replenishment problem in the thesis is different from the ones in the existing research in the fact that, the setting of interest herein considers the existence of two modes of transportation. The closest to our setting is the one that is analyzed by Rieksts and Ventura [18], however, in their modeling, demand is deterministic and the planning horizon is infinite. Another matter that is of importance in this thesis is transportation pricing. To our best knowledge, there is no study in the literature on transportation pricing.

Integrated inventory and transportation models will be reviewed in section 2.1. In addition to the concept of coordinating inventory and transportation decisions for a single echelon, this thesis puts forward the idea of coordinating these functions between different parties in the supply chain. Therefore, a closely related area of the literature is coordination, which will be reviewed in Section 2.2.

2.1 Transportation Issues

Aucamp [2] extends the standard EOQ model to the case where freight costs are determined by the integer number of carloads required to fill the order. Proposed solution method to determine the optimal solution yields three candidate solutions and the best of three is found by picking the one with the least cost. Aucamp [2] finds out that the optimal solution is not far away from the standard EOQ. In fact, the optimal solution involves a shipment that either uses the same number of cars as EOQ, or one less car.

Carter and Ferrin [6] focus on the importance of including transportation costs in the optimal inventory-lot sizing decisions. The authors advocate the buyer to manage the inbound transportation. They give some company examples from the practice to support their decisions. They conclude that substantial reductions in the freight rate can be obtained even with small increases in order quantity. Also, when shipment charges are constant for any order quantity that falls between a weight break quantity and the next discount minimum quantity suggests that order quantities (inventories) could be decreased without increasing transportation costs.

Rieksts and Ventura [18] analyze theoretical inventory models with constant demand rate and two transportation modes. The transportation options are truckloads (TL) with fixed costs, less than truckloads (LTL) with a constant per unit, or using a combination of both modes simultaneously. They derive optimal algorithms for single stage models over both an infinite and a finite planning horizon. They find out that optimal order interval should be in the same range with the optimal order interval of classical EOQ model and optimal order interval can either be one of the breakpoints or one of the minimizers of the average total cost functions with transportation costs. Mendoza and Ventura [14] extend the work done by Rieksts and Ventura to the case where all units or incremental quantity discount structures exist. They derive the optimal algorithms for each quantity discount structure for single stage models over an infinite planning horizon.

Lee [11] proposes algorithms on the classical EOQ model with setup cost including a fixed cost and freight cost. There is a quantity discount in freight costs. Assumptions of the proposed model are the same as the EOQ model except for the setup structure. Lee [11] refers his proposed model as the general model. The author also talks about some special cases of the general model. It is assumed that in special case called Special Model 1, basic charge paid for the first P units is greater than the incremental charge paid for each P more units. In special case called Special Model 2, basic charge is assumed to be equal to the incremental charge. In this study, the author proposes two algorithms to find the optimal order quantity. First algorithm can be used to solve the general model. Second algorithm, which is proposed for Model 2, is more efficient. It compares three possible candidates for the optimal solution and selects the one with the least cost depending on some conditions.

The use of the actual transportation cost function, complete with ranges of over-declared shipments, requires a modification to the standard procedures for determining the optimal purchase order quantity. Russell and Krajewski [19] present an analytical procedure for finding the order quantity that minimizes total purchase costs which reflect both transportation economies and quantity discounts. They find out that the optimal purchase order quantity will be one of the four following possibilities: (1) the valid economic order quantity (EOQ), (2) a purchase price breakpoint in excess of valid EOQ, (3) a transportation rate breakpoint in excess of valid EOQ, (4) a modified EOQ which provides an over-declared shipment in excess of valid EOQ. They also develop an algorithm which systematically explores these four possibilities. Abad [1] extend the work done by Russell and Krajewski [19] to the case where there is a temporary price reduction (TPR) offered by the supplier. Abad considers the buyer's response to a TPR when the buyer is responsible for paying for freight. Abad models freight costs using the freight tariffs offered by public carriers in the practice. The author develops a search procedure for determining the optimal purchase lot size for the buyer in response to the TPR offered by the supplier.

Tersine and Barman [20] develop and demonstrate optimal inventory/transport decision algorithms when there are both unit and freight discounts. All units quantity/all weight freight, incremental quantity/incremental freight, all units quantity/incremental freight and incremental quantity/all-weight freight discount pairs are investigated. The authors also deal with single and no discount conditions in the paper.

2.2 Coordination Issues

Monahan [15] analyzes how a supplier can structure the terms of an optimal quantity discount schedule. Monahan [15] shows that supplier can increase his/her net profit by changing the ordering behavior of the buyer to increase his/her order size by a specific rate. It is shown that the optimal order increase rate is both dependent on the supplier's order processing/manufacturing setup cost and the buyer's fixed order processing cost. Lee and Rosenblatt [12] generalize Monahan's model to relax the implicit assumption of a lot-for-lot policy adopted by the supplier and to incorporate constraints imposed on the amount of discount that can be offered. Lee and Rosenblatt [12] also find out that lot-for-lot policy is optimal when the setup costs and holding costs of both the supplier and buyer are equal. Goyal [9] criticizes Lee and Rosenblatt's way to impose constraints on discount amount offered by the supplier. His assertion is that supplier may offer price discounts lower than the purchase price because high discounts on purchase prices may lead to greater economies of scale for the supplier [5, 9].

Banerjee [4] improves Monahan's quantity discount model by considering supplier as a manufacturer. This means that supplier will hold finished goods inventory until the order is shipped to the buyer. Banerjee shows that Monahan's model tends to overestimate the value of optimal order increase rate if the supplier is a manufacturer and claims that supplier will offer quantity discounts to the buyer to order smaller quantities which is not seen in practice a lot [4, 5]. Joglekar [10] criticizes Monahan's one-item, one-customer and one-supplier model and judges Monahan's implicit assumptions as unreasonable. Joglekar [10] shows that it is not economically beneficial for the supplier to match his/her production frequency with buyer's ordering frequency if the supplier's manufacturing setup costs are substantially larger than buyer's ordering costs. If these are of similar magnitude, Monahan's model does not have much practical significance even though Monahan's assumptions hold. Supplier can increase his/her profit more by considering a better production lot size policy instead of considering quantity discounts.

Weng and Wong [24] develop general models for the supplier's all-unit quantity discount policy. The models developed deal with four major issues: (a) one buyer or multiple buyers, (b) constant or price elastic demand, (c) the relationship between supplier's ordering or production policy and the buyer's ordering size, (d) supplier is either a manufacturer or a wholesaler. Objectives of these models are either the supplier's profit improvement or the supplier's increased profit share analysis. They also develop algorithms to find the optimal decision policies. This paper provides the supplier with both the optimal all-unit quantity discount policy and the optimal production (or ordering) policy.

Goyal [8] models the replenishment problems of the single buyer/ single vendor systems using independent (decentralized) and integrated (centralized) approaches under the following assumptions: Buyer demand is deterministic and constant, no stockouts are permitted, vendor is not a manufacturer, buyer and vendor replenishment lead times are zero and minimization of the variable cost is the criterion of optimality. Goyal [8] finds out that total cost of centralized policy can not be greater or equal to the total cost of decentralized policy. Another important result in the paper is that, in centralized policy, buyer will order less frequently but in larger sizes. Using these results, the author concludes that vendor and buyer can make some agreements to get the advantage of the centralized solution. Banerjee [3] also focuses on the replenishment decisions in a single-vendor, single-buyer and single-product environment under the following assumptions; vendor is a manufacturer, there is a replenishment lead time, cycle times of the vendor and the buyer are same, order quantity of vendor is an integer multiple of the order quantity of the buyer . This paper develops a joint economic lot size model for a special case where vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions. It is shown that a jointly optimal ordering policy, together with an appropriate price adjustment, can be beneficial for both parties or, at least does not place either at a disadvantage.

Pasternack [17] considers the pricing decision faced by a producer of a commodity with a short shelf or demand life in a newsboy setting under the following assumptions; retailer will place only one order with the manufacturer, retailer will sell the product until his/her inventory is depleted, or the shelf life is exhausted. Manufacturer's selection of the amount per unit charged to the retailer, the per unit credit for returned goods, and the percentage of purchased goods allowed to be returned are critical in achieving channel coordination and maximizing total profits. Pasternack [17] shows that the policy of a manufacturer allowing unlimited returns for full credit and the policy of a manufacturer allowing no returns are system suboptimal. On the other hand, a policy for unlimited returns at partial credit will be system optimal for appropriately chosen cost values. This policy can also achieve channel coordination in a multi-retailer environment.

Toptal and Çetinkaya [21] consider the coordination problem between a vendor and a buyer operating under generalized replenishment costs that include fixed costs as well as stepwise freight costs. They study the stochastic demand, single period setting where the buyer must decide on the order quantity to satisfy the random demand for a single item with a short product life cycle. They develop two models and derive the optimal order quantities both under decentralized and centralized settings. In the first model, they incorporate truck costs for the vendor only. In the second model, they incorporate truck costs both for the buyer and the vendor. Unlike the earlier research in this area, they prove that the vendor's expected profit is not increasing in buyer's order quantity. Their analysis show that the vendor can improve his/her profits by using an appropriate tariff schedule or a vendor managed delivery contract (VMD). Under VMD contract, vendor covers the buyer's transportation expenses. They show that a VMD arrangement potentially improves the centralized solution. Toptal [22] extend the work done by Toptal and Çetinkaya [21] by analyzing a single echelon replenishment problem under a general replenishment cost structure that includes stepwise freight costs and all-units quantity discounts. The author proves several useful properties of the expected profit function and utilizes these properties to develop a computational solution approach to find the optimal order quantity. The solution procedures developed in this paper can easily be applied to multi-echelon settings.

Ertogral et al. [7] analyze the single vendor-single buyer problem under equal-size shipment policy. Two new models that integrate the transportation cost explicitly are developed. The transportation cost is considered to be in an all-unit-discount format for the first model. The option of over declaring a shipment to exploit the transportation cost structure is explored in the second model. Two important conclusions can be derived from this paper. First, savings can be realized by explicitly integrating transportation cost into the supply chain under consideration, depending on the transportation contribution to the total cost. Second, production and inventory decisions are affected when transportation is considered explicitly in the model. Tables 2.1, 2.2 and 2.3 classify the papers reviewed in this thesis.

Table 2.1 Paper classifications according to the number of echelons, problem's time horizon, location and structure of the transportation cost

	Echelon	Time Horizon	Transportation Cost	Transportation Cost Structure
Abad (2006)	Single	Infinite	Buyer	LTL freight schedule
Aucamp (1982)	Single	Infinite	Buyer	Stepwise
Banerjee (1986)	Multi	Infinite	-	-
Banerjee (1986a)	Multi	Infinite	-	-
Ertogral et al. (2007)	Multi	Infinite	Buyer	All units discount
Goyal (1977)	Multi	Infinite	-	-
Goyal (1987)	Multi	Infinite	-	-
Joglekar (1988)	Multi	Infinite	-	-
Lee (1986)	Single	Infinite	Buyer	Quantity discount
Lee and Rosenblatt (1986)	Multi	Infinite	-	-
Mendoza and Ventura (2008)	Single	Infinite	Buyer	LTL and TL freight schedule
Monahan (1984)	Multi	Infinite	Vendor	All weight discount
Pasternack(1985)	Multi	Single Period	-	-
Rieksts and Ventura (2007)	Single	Finite & Infinite	Buyer	LTL and TL freight schedule
Russell and Krajewski (1991)	Single	Infinite	Buyer	LTL freight schedule
Tersine and Barman (1994)	Single	Infinite	Buyer	All weight / Incremental Weight / None
Toptal and Cetinkaya (2006)	Multi	Single Period	Buyer & Vendor	Stepwise
Toptal (2008)	Single	Single Period	Buyer	Stepwise
Weng and Wong (1993)	Multi	Infinite	-	-
Our study	Multi	Single Period	Buyer	Stepwise & Continuous

Table 2.2 Paper classifications according to the demand, wholesale price and unit cost structures

	Demand Structure	Wholesale Price	Unit cost
Abad (2006)	Constant & Deterministic	Temporary Price Reduction	-
Aucamp (1982)	Constant & Deterministic	Uniform	-
Banerjee (1986)	Constant & Deterministic	Uniform	Uniform
Banerjee (1986a)	Constant & Deterministic	Uniform	Uniform
Ertogral et al. (2007)	Constant & Deterministic	Uniform	Uniform
Goyal (1977)	Constant & Deterministic	Uniform	Uniform
Goyal (1987)	Constant & Deterministic	Uniform	Uniform
Joglekar (1988)	Constant & Deterministic	Uniform	Uniform
Lee (1986)	Constant & Deterministic	Uniform	-
Lee and Rosenblatt (1986)	Constant & Deterministic	Uniform	Uniform
Mendoza and Ventura (2008)	Constant & Deterministic	Uniform	-
Monahan (1984)	Constant & Deterministic	Uniform	Uniform
Pasternack(1985)	Stochastic	Uniform	Uniform
Rieksts and Ventura (2007)	Constant & Deterministic	Uniform	-
Russell and Krajewski (1991)	Constant & Deterministic	All units discount	-
Tersine and Barman (1994)	Constant & Deterministic	All units / Incremental unit / None	-
Toptal and Cetinkaya (2006)	Stochastic	Uniform	Uniform
Toptal (2008)	Stochastic	All units discount	All units discount
Weng and Wong (1993)	Uniform & Price Dependent Respectively	All units discount	Uniform
Our study	Stochastic	Uniform	Uniform

Table 2.3 Paper classifications according to the type of the vendor and decision variables

	Vendor is a Manufacturer	Ordering cost	Decision Variables
Abad (2006)	-	Constant	Q, q
Aucamp (1982)	-	Constant	Q
Banerjee (1986)	Yes	Constant	Q_p, Q_v, Q_J, d
Banerjee (1986a)	Yes	Constant	d_k(BE), K, Q_v, Q_J
Ertogral et al. (2007)	Yes	Constant	Q, n, q
Goyal (1977)	No	Constant	t_0
Goyal (1987)	No	Constant	K, k
Joglekar (1988)	Yes	Constant	S_2
Lee (1986)	-	Constant	Q
Lee and Rosenblatt (1986)	No	Constant	K, k
Mendoza and Ventura (2008)	-	Constant	Q, T
Monahan (1984)	No	Constant	d_k(BE), K
Pasternack(1985)	Yes	Constant	Q
Rieksts and Ventura (2007)	-	Constant	T
Russell and Krajewski (1991)	-	Constant	Q
Tersine and Barman (1994)	-	Constant	Q_s
Toptal and Cetinkaya (2006)	No	Constant	$Q_{c,1}$, $Q_{c,2}$, $Q_{d,1}$, $Q_{d,2}$
Toptal (2008)	No	Constant	Q
Weng and Wong (1993)	Yes & No	Constant	Q, d, n
Our study	-	Constant	$Q^*(R, s)$, R, s

Chapter 3

PROBLEM DEFINITION

In this study, we consider a system that includes a retailer and two carriers. The retailer operates in a Newsboy setting. That is, he/she makes a single replenishment decision at the beginning of a period during which he/she faces random demand. If the order quantity Q is greater than the demand, then excess items are salvaged at a $\$\, v/unit$ revenue. If it is less than the demand, then the retailer incurs a unit lost sale cost of \$$b$. The retailer pays for the transportation of incoming materials. There are two modes of transportation, a TL carrier and a LTL carrier. The TL carrier charges \$$R$ per each truck whether it is fully or partially loaded. Each truck has a capacity of carrying P units and the TL carrier has ample number of trucks available. The LTL carrier charges \$$s$ per unit where $R/P < s < R$. That is, for carrying a truck load of items, using the TL carrier is always less costly than using a LTL carrier. However, utilizing a truck has more costs when the quantity to be carried is less than R/s. More explicitly, the transportation cost of the retailer for replenishing Q units is given by

$$C(Q) = \min\left\{s(Q - iP) + iR, (i+1)R\right\} \text{ or}$$

$$C(Q) = \left\lfloor \frac{Q}{P} \right\rfloor R + \min\left\{R, s\left(Q - \left\lfloor \frac{Q}{P} \right\rfloor P\right)\right\},$$

where $iP \le Q < (i+1)P$ such that $i \in \{0\} \cup \mathbb{Z}^{+}$. Here, $\mathbb{Z}^{+}$ is the set of all positive integers. The above expression can be rewritten either as

$$C(Q) = \begin{cases} s(Q-iP)+iR & \text{if } iP \le Q < \frac{R}{s}+iP \\ & \\ (i+1)R & \text{if } \frac{R}{s}+iP \le Q < (i+1)P, \end{cases} \tag{1}$$

or as

$$C(Q) = \begin{cases} s\left(Q-\left\lfloor Q/P \right\rfloor P\right)+\left\lfloor Q/P \right\rfloor R & \text{if } \left\lfloor Q/P \right\rfloor P \le Q < R/s+\left\lfloor Q/P \right\rfloor P \\ & \\ \left\lceil Q/P \right\rceil R & \text{if } R/s+\left\lfloor Q/P \right\rfloor P \le Q < \left\lceil Q/P \right\rceil P, \end{cases}$$

For given values of R, P and s, retailer's expected profit for replenishing Q units, i.e. $H(Q)$, is given by $G(Q)-C(Q)$ where

$$G(Q) = (r-v)\mu-(c-v)Q+(r+b-v)\int_{Q}^{\infty}(Q-x)\,f(x)dx. \tag{2}$$

The TL carrier and the LTL carrier incur fixed setup costs as long as the retailer utilizes their services. Once the retailer decides on his/her replenishment quantity and how much of it will be carried by which carrier, the carriers are in charge of performing the required service. Namely, for long term business and customer satisfaction, neither the TL carrier nor the LTL carrier rejects an order even if the revenue does not cover the fixed costs implying that it is not a profitable transaction.

Before introducing TL carrier's and LTL carrier's expected profit functions, we next summarize the notation introduced so far and that will be used in the remaining parts of the text.

r: Retail price per unit.

b: Retailer's per unit lost sale cost.

v: Salvage value of an item unsold at the retailer.

c: Wholesale price per unit.

R: Per truck shipping price that the TL carrier charges.

$\overline{R}$: The TL carrier's fixed cost for utilizing one truck.

P: Capacity of a truck in number of units.

s: Per unit shipping price that the LTL carrier charges.

$\overline{s}$: The LTL carrier's cost for shipping one unit.

K_{TL}: Setup cost of the TL carrier.

K_{LTL}: Setup cost of the LTL carrier.

Q: Retailer's replenishment quantity.

$C(Q)$: Transportation cost of the retailer for replenishing Q units.

$G(Q)$: Retailer's expected profit function excluding the truck costs.

$H(Q)$: Retailer's expected profit function.

Q_1: Quantity transported using the TL carrier.

Q_2: Quantity transported using the LTL carrier.

$\Pi_{TL}(Q,R)$: TL carrier's profit if the retailer replenishes Q units.

$\Pi_{LTL}(Q,s)$: LTL carrier's profit if the retailer replenishes Q units.

X: Random variable showing demand in a single period.

$f(x)$: Probability density function of demand.

μ: Expected value of demand.

The TL carrier's profits $\Pi_{TL}(Q)$ in this system are given by

$$\Pi_{TL}(Q,R)=\begin{cases} i\times\left(R-\bar{R}\right)-K_{TL} & \text{if } iP\le Q<\frac{R}{s}+iP \text{ and } Q>0 \\ (i+1)\times\left(R-\bar{R}\right)-K_{TL} & \text{if } \frac{R}{s}+iP\le Q<(i+1)P, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

which can alternatively be written as

$$\Pi_{TL}(Q,R)=\begin{cases} \left\lfloor Q/P \right\rfloor\times\left(R-\bar{R}\right)-K_{TL} & \text{if } \left\lfloor Q/P \right\rfloor P\le Q<R/s+\left\lfloor Q/P \right\rfloor P \text{ and } Q>0 \\ \left\lceil Q/P \right\rceil\times\left(R-\bar{R}\right)-K_{TL} & \text{if } R/s+\left\lfloor Q/P \right\rfloor P\le Q<\left\lceil Q/P \right\rceil P, \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the LTL carrier's profits $\Pi_{LTL}(Q)$ are

$$\Pi_{LTL}(Q,s)=\begin{cases} (s-\bar{s})\times(Q-iP)-K_{LTL} & \text{if } iP<Q<\frac{R}{s}+iP \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

which can alternatively be represented as

$$\Pi_{LTL}(Q,s)=\begin{cases} (s-\bar{s})\times\left(Q-\left\lfloor Q/P \right\rfloor P\right)-K_{LTL} & \text{if } \left\lfloor Q/P \right\rfloor P<Q<R/s+\left\lfloor Q/P \right\rfloor P \\ 0 & \text{otherwise.} \end{cases}$$

Expression (1) implies that, if $iP \le Q < \frac{R}{s} + iP$, then $Q_1 = iP$ and $Q_2 = Q - iP$. Similarly, if $\frac{R}{s} + iP \le Q < (i+1)P$, then $Q_1 = (i+1)P$ and $Q_2 = 0$. Note also that, TL carrier does not get any profit from this transaction if $\left\lceil \frac{Q_1}{P} \right\rceil \le \frac{K_{TL}}{(R - \bar{R})}$ and the LTL carrier does not get any profit from this transaction if $Q_2 \le \frac{K_{LTL}}{(s - \bar{s})}$.

In order to write a more explicit expression for the retailer's expected profit function, we introduce the following two functions over $Q \ge 0$ and $j \in \{0\} \cup \mathbb{Z}^+$:

$$H_1(Q, j) = G(Q) - s(Q - jP) - jR \tag{5}$$

and

$$H_2(Q, j) = G(Q) - jR. \tag{6}$$

It turns out that we have

$$H(Q) = \begin{cases} H_1(Q,i) & \text{if } iP \le Q < \frac{R}{s} + iP \\ \\ H_2(Q,i+1) & \text{if } \frac{R}{s} + iP \le Q < (i+1)P. \end{cases} \tag{7}$$

Given the values of *R* and *s*, the retailer solves the following problem to decide on his/her replenishment quantity.

$$\begin{array}{ll} \max\limits_{Q} & H(Q) \\ \\ \text{s.t.} & Q \ge 0. \end{array}$$

Let the optimal solution of the above problem be $Q^*(R,s)$. In this thesis, we first analyze the structural properties of $H(Q)$ and propose a finite time exact solution procedure for finding $Q^*(R,s)$ given the values of R and s. Then we study the underlying transportation pricing problems that the TL and LTL carriers face. Namely, given the TL carrier's per truck shipping price R, we study the problem of the LTL carrier in deciding the value of s. We refer to this problem as *LTLP*. Then given the LTL carrier's per unit transportation charge, we study the problem of TL carrier in deciding the value of R, referred to as Problem *TLP*. Throughout this analysis, we assume that the two carriers know the retailer's inventory related costs and hence they predict the response of the retailer in terms of his/her order quantity, i.e., the value of $Q^*(R,s)$.

The pricing problems of the TL carrier as described above can be formulized as follows:

TLP:

$$\max_{R} \quad \Pi_{TL}\left(Q^*(R,s),R\right)$$

$$\text{s.t.} \quad R < sP, \quad R > s.$$

Similarly, the pricing problem of the LTL carrier is given by

LTLP:

$$\max_{s} \quad \Pi_{LTL}\left(Q^*(R,s),s\right)$$

$$\text{s.t.} \quad R/P < s < R.$$

In the next section, we study the retailer's replenishment problem to characterize his/her response to given values of R and s.

Chapter 4

RETAILER'S INTEGRATED INVENTORY REPLENISHMENT AND TRANSPORTATION PROBLEM

In this section, we propose an algorithm to find the maximizer of $H(Q)$. As seen in Figure 4.1, $H(Q)$ is a piecewise function. Therefore, our analysis will follow by proving some of its structural properties. Regarding Expression (2), we would like to note that the analysis in this chapter will use the fact that $G(Q)$ is a strictly concave function of Q $\left(\text{i.e., } \frac{d^2G(Q)}{dQ^2} < 0\right)$. Therefore, the analysis herein is not restricted to the Newsboy setting but it also applies to a more general class of problems where the production/inventory related expected profits of the retailer is strictly concave. The following lemma indicates the continuity of $H(Q)$ which forms the fundamental component of our analysis for finding the optimal solution of the replenishment problem.

Lemma 1 $H(Q)$ *is a continuous function of Q.*

Proof: Since $H_1(Q, j)$ and $H_2(Q, j)$ are continuous functions of *Q* for fixed *j*, we will prove that $H(Q)$ is continuous at the breakpoints which are *iP* and $\frac{R}{s}+iP,\ i\in\{0\}\cup\mathbb{Z}^+$. Let us first consider $Q=\frac{R}{s}+iP,\ i\in\{0\}\cup\mathbb{Z}^+$. Note that, we have

$$\lim_{Q\to\left(\frac{R}{s}+iP\right)^-} H(Q)=H_1\left(\frac{R}{s}+iP,i\right).$$

Using Expression (5), it follows that

$$\lim_{Q\to\left(\frac{R}{s}+iP\right)^-} H(Q)=G\left(\frac{R}{s}+iP\right)-s\left(\frac{R}{s}+iP\right)+(sP-R)i,$$

which leads to

$$\lim_{Q\to\left(\frac{R}{s}+iP\right)^-} H(Q)=G\left(\frac{R}{s}+iP\right)-R(i+1).$$

Expression (6) implies that the right hand side of above equality is $H_2\left(\frac{R}{s}+iP,i+1\right)$ which is equal to $H\left(\frac{R}{s}+iP\right)$.

Next, we will show that $H(Q)$ is continuous at $Q=iP,\ i\in\mathbb{Z}^+$. As *Q* approaches to *iP* from the below, the one sided limit of $H(Q)$ is given by,

$$\lim_{Q\to(iP)^-} H(Q)=H_2(iP,i).$$

Using Expression (6), we have

$$\lim_{Q \to (iP)^-} H(Q) = G(iP) - iR.$$

Rewriting the right-hand side of the above equality as $G(iP) - siP + i(sP - R)$, it can be observed from Expression (5) that $\lim_{Q \to (iP)^-} H(Q) = H_1(iP, i) = H(iP)$. ■

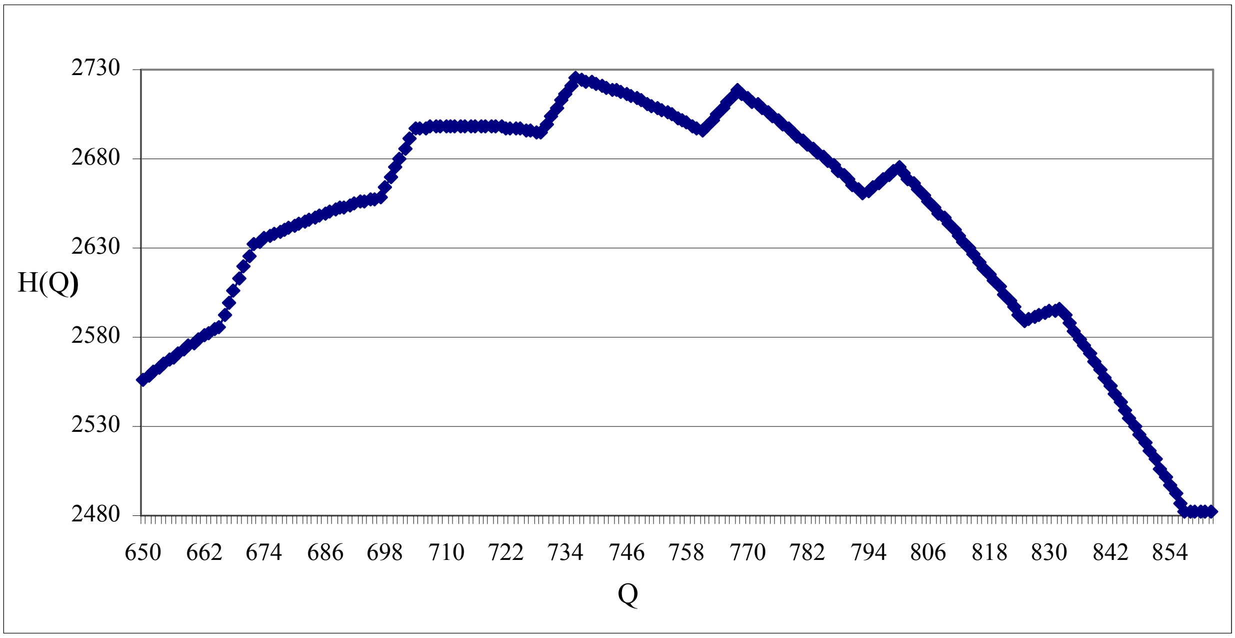

Figure 4.1 An illustration of the retailer's expected profit function

Since $G(Q)$ is a strictly concave function, it follows from Expressions (5) and (6) that $H_1(Q, j)$ and $H_2(Q, j)$ are strictly concave functions over Q for fixed j. Therefore, they have unique maximizers. Let q^* be the maximizer of $G(Q)$. It can be observed that q^* is also the unique maximizer of $H_2(Q, j)$, $\forall j$. Similarly, let z^* be the unique maximizer of

$H_1(Q, j), \forall j.$ It follows from Expression (5) that $G'(z^*) = s.$ The following lemma characterizes the ordinal relationship between q^* and z^*.

Lemma 2 *We have* $z^* < q^*.$

Proof: Since q^* is the unique maximizer of $G(Q),$ which is a strictly concave function of Q, we have $G'(Q) > 0 \ \ \forall Q$ s.t. $Q < q^*$ and $G'(Q) < 0 \ \ \forall Q$ s.t. $Q > q^*.$ By definition of z^*, we have $G'(z^*) = s.$ Since $s > 0,$ it follows that $z^* < q^*.$ ■

In the following analysis, we will prove some structural properties of $H(Q)$ to arrive at an algorithm that finds $Q^*(R, s).$ In this analysis, we say z^* is realizable if there exists $i \in \{0\} \cup \mathbb{Z}^+$ such that $iP \le Q < \frac{R}{s} + iP.$ Similarly, q^* is realizable if there exists $i \in \{0\} \cup \mathbb{Z}^+$ such that $\frac{R}{s} + iP \le Q < (i+1)P.$ In the next lemma, we show that z^* dominates all order quantities that are smaller than itself, therefore, in maximizing $H(Q),$ order quantities less than z^* should not be considered.

Lemma 3 *We have* $H(Q) < H(z^*), \ \forall Q < z^*.$

Proof: The proof will follow by considering the following two cases:

i) $H(z^*) = H_1(z^*, i)$ where $iP \le z^* < \frac{R}{s} + iP$ for some $i \in \{0\} \cup \mathbb{Z}^+.$ That is, z^* is realizable.

ii) $H(z^*) = H_2(z^*, i+1)$ where $\frac{R}{s} + iP \le z^* < (i+1)P$ for some $i \in \{0\} \cup \mathbb{Z}^+.$ That is, z^* is not realizable.

Case 1, z^* is realizable: First, we will show that $H(z^*) > H(Q),\ \forall Q$ s.t. $jP \le z^* < \frac{R}{s} + jP$ and $j \le i.$ To complete the proof for Case 1, secondly, we will show that $H(z^*) > H(Q),$ $\forall Q$ s.t. $\frac{R}{s} + jP \le z^* < (j+1)P$ and $j < i.$

Let us start with Q such that $jP \le Q < \frac{R}{s} + jP$ and $j \le i.$ Since z^* is the unique maximizer of $H_1(Q, j)$ over $Q,\ \forall j,$ we have

$$H_1(z^*, j) > H_1(Q, j), \quad jP \le Q < \frac{R}{s} + jP, \quad j \le i. \tag{8}$$

Using Expression (5), we further have

$$H_1(z^*, i) > H_1(z^*, j), \forall j < i. \tag{9}$$

It follows from Expressions (8) and (9) that

$$H_1(z^*, i) > H_1(Q, j), \quad jP \le Q < \frac{R}{s} + jP, \quad j \le i.$$

Recall from Expression (7) that we have $H(Q) = H_1(Q, j)$ for Q such that $jP \le Q < \frac{R}{s} + jP.$ Therefore, the above expression implies

$$H(Q) < H(z^*), \quad \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j \le i. \tag{10}$$

Now, let us consider Q such that $\frac{R}{s} + jP \le Q < (j+1)P$ and $j < i.$ Recall that $H_2(Q, j)$ is a strictly concave function of Q for fixed $j,$ and q^* is its unique maximizer. Since $q^* > z^* \ge iP,$ it follows that

$$H_2(Q, j) < H_2(jP, j), \quad \forall Q < jP \text{ and } j \leq i. \tag{11}$$

Note also that we have $H_1(jP, j) = H_2(jP, j)$. Combining this with the fact that $z^* \geq iP \geq jP$ further leads to

$$H_1(z^*, j) \geq H_1(jP, j) = H_2(jP, j), \ \forall j \leq i. \tag{12}$$

Using Expressions (11) and (12), we conclude that

$$H_1(z^*, j) > H_2(Q, j), \quad \forall Q < jP \text{ and } j \leq i.$$

The above expression implies that

$$H_1(z^*, j) > H_2(Q, j), \quad \forall Q \text{ s.t. } \frac{R}{s} + (j-1)P \leq Q < jP \text{ and } 1 \leq j \leq i,$$

which can be rewritten as

$$H_1(z^*, j) > H_2(Q, j), \quad \forall Q \text{ s.t. } \frac{R}{s} + jP \leq Q < (j+1)P \text{ and } 0 \leq j < i.$$

Recall from Expression (7) that, we have $H(Q) = H_2(Q, j)$ for Q such that $\frac{R}{s} + jP \leq Q < (j+1)P$. Therefore,

$$H(z^*) > H(Q), \quad \forall Q \text{ s.t. } \frac{R}{s} + jP \leq Q < (j+1)P \text{ and } 0 \leq j < i.$$

Combining Expression (10) with the above result, we conclude that $H(z^*) > H(Q)$, $\forall Q < z^*$ if $\exists\, i \in \{0\} \cup \mathbb{Z}^+$ such that $iP \leq Q < \frac{R}{s} + iP$.

Case 2, z^* is not realizable: The proof of this case is similar to that of Case 1. We will first show that $H(z^*) > H(Q)$, $\forall Q$ s.t. $jP \le Q < \frac{R}{s} + jP$ and $j \le i$. To complete the proof for Case 2, secondly, we will show that $H(z^*) > H(Q)$, $\forall Q$ s.t. $\frac{R}{s} + jP \le Q < (j+1)P$ and $Q < z^*$.

Let us start with Q such that $jP \le Q < \frac{R}{s} + jP$ and $j \le i$. The fact that $H(z^*) = H_2(z^*, i+1)$ implies

$$H_2(z^*, i+1) \ge H_1(z^*, i).$$

Since $H_1(z^*, i) > H_1(z^*, j)$, $\forall j < i$, and z^* is the unique maximizer of $H_1(Q, j)$ over Q, we have

$$H_1(z^*, i) > H_1(Q, j), \quad \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j \le i.$$

Therefore,

$$H_2(z^*, i+1) > H_1(Q, j), \ \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j \le i, \tag{13}$$

and hence,

$$H(z^*) > H(Q), \ \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j \le i. \tag{14}$$

Now, let us consider Q such that $\frac{R}{s} + jP \le Q < (j+1)P$ and $Q < z^*$. Since q^* is the unique maximizer of $H_2(Q, j)$ over Q and $z^* < q^*$, it follows that $H_2(z^*, i+1) > H_2(Q, i+1)$, and hence,

$$H\left(z^*\right) > H\left(Q\right),\ \forall Q \text{ s.t. } \frac{R}{s} + iP \le Q < z^*. \tag{15}$$

We have $q^* > z^* > iP$, therefore,

$$H_2\left(Q, j\right) < H_2\left(jP, j\right),\quad \forall Q < jP,\ 1 \le j \le i. \tag{16}$$

Expression (13) implies that

$$H_2\left(z^*, i+1\right) > H_1\left(jP, j\right),\ \forall j \le i. \tag{17}$$

Using the fact that $H_1\left(jP, j\right) = H_2\left(jP, j\right)$, Expressions (16) and (17) lead to

$$H_2\left(z^*, i+1\right) > H_2\left(Q, j\right),\ \forall Q \text{ such that } \frac{R}{s} + jP \le Q < \left(j+1\right)P \text{ and } j < i,$$

and hence,

$$H\left(z^*\right) > H\left(Q\right),\ \forall Q \text{ such that } \frac{R}{s} + jP \le Q < \left(j+1\right)P \text{ and } j < i. \tag{18}$$

Combining Expressions (15) and (18), we have

$$H\left(z^*\right) > H\left(Q\right),\ \forall Q \text{ such that } \frac{R}{s} + jP \le Q < \left(j+1\right)P \text{ and } Q < z^*. \tag{19}$$

■

In the next lemma, we show that q^* dominates all order quantities that are greater than itself, therefore, in maximizing $H\left(Q\right)$, order quantities greater than q^* should not be considered.

Lemma 4 *We have* $H(Q) < H(q^*),\ \forall Q > q^*$.

Proof: The proof will follow by considering the following two cases:

i) $H(q^*) = H_2(q^*, i+1)$ where $\frac{R}{s} + iP \le q^* < (i+1)P$ for some $i \in \{0\} \cup \mathbb{Z}^+$. That is, q^* is realizable.

ii) $H(q^*) = H_1(q^*, i)$ where $iP \le q^* < \frac{R}{s} + iP$ for some $i \in \{0\} \cup \mathbb{Z}^+$. That is, q^* is not realizable.

Case 1, q^* is realizable: First, we will show that $H(q^*) > H(Q),\ \forall Q$ s.t. $\frac{R}{s} + jP \le Q < (j+1)P,\ \ j \ge i$ and $Q > q^*$. Secondly, we will show that $H(q^*) > H(Q)$, $\forall Q$ s.t. $jP \le Q < \frac{R}{s} + jP,\ \ j > i$, and this will complete the proof for Case 1.

Let us start with Q such that $\frac{R}{s} + jP \le Q < (j+1)P,\ \ j \ge i$ and $Q > q^*$. Since q^* is the unique maximizer of $H_2(Q, j)$ over Q, $\forall j$, we have

$$H_2(q^*, j+1) > H_2(Q, j+1), \quad \frac{R}{s} + jP \le Q < (j+1)P, \quad j \ge i \text{ and } Q > q^*. \tag{20}$$

Using Expression (6), we further have

$$H_2(q^*, i+1) > H_2(q^*, j+1), \forall j > i. \tag{21}$$

It follows from Expressions (20) and (21) that

$$H_2(q^*, i+1) > H_2(Q, j+1), \quad \frac{R}{s} + jP \le Q < (j+1)P, \quad j \ge i \text{ and } Q > q^*.$$

Recall from Expression (7) that we have $H(Q) = H_2(Q, j+1)$ for Q such that $\frac{R}{s} + jP \le Q < (j+1)P.$ Therefore, the above expression implies

$$H(Q) < H(q^*), \qquad \forall Q \text{ s.t. } \frac{R}{s} + jP \le Q < (j+1)P, \quad j \ge i \text{ and } Q > q^*. \tag{22}$$

Now, let us consider Q such that $jP \le Q < \frac{R}{s} + jP$ and $j > i.$ Recall that $H_1(Q, j)$ is a strictly concave function of Q for fixed j, and z^* is its unique maximizer. Since $(i+1)P > q^* > z^*,$ it follows that

$$H_1(Q, j) < H_1(jP, j), \quad \forall Q > jP \text{ and } j > i. \tag{23}$$

Note also that we have $H_1(jP, j) = H_2(jP, j).$ Combining this with the fact that $jP \ge (i+1)P > q^*$ further leads to

$$H_2(q^*, j) > H_2(jP, j) = H_1(jP, j), \ \forall j > i. \tag{24}$$

Using Expressions (23) and (24), we conclude that

$$H_2(q^*, j) > H_1(Q, j), \quad \forall Q \ge jP \text{ and } j > i.$$

The above expression implies that

$$H_2(q^*, j) > H_1(Q, j), \quad \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j > i.$$

Recall from Expression (7) that, we have $H(Q) = H_1(Q, j)$ for Q such that $jP \le Q < \frac{R}{s} + jP.$ Therefore,

$$H(q^*) > H(Q), \quad \forall Q \text{ s.t. } jP \le Q < \frac{R}{s} + jP \text{ and } j > i.$$

Combining Expression (22) with the above result, we conclude that $H(q^*) > H(Q)$, $\forall Q > q^*$ if $\exists i \in \{0\} \cup \mathbb{Z}^+$ such that $\frac{R}{s} + iP \le q^* < (i+1)P$.

Case 2, q^* is not realizable: The proof of this case is similar to that of Case 1. We will first show that $H(q^*) > H(Q)$, $\forall Q$ s.t. $\frac{R}{s} + jP \le Q < (j+1)P$ and $j \ge i$. To complete the proof for Case 2, secondly, we will show that $H(q^*) > H(Q)$, $\forall Q$ s.t. $jP \le Q < \frac{R}{s} + jP$, $j \ge i$ and $Q > q^*$.

Let us start with Q such that $\frac{R}{s} + jP \le Q < (j+1)P$ and $j \ge i$. The fact that $H(q^*) = H_1(q^*, i)$ implies

$$H_1(q^*, i) > H_2(q^*, i+1).$$

Since $H_2(q^*, i+1) > H_2(q^*, j+1)$, $\forall j > i$, and q^* is the unique maximizer of $H_2(Q, j)$ over Q, we have

$$H_2(q^*, i+1) > H_2(Q, j+1), \quad \forall Q \text{ s.t. } \frac{R}{s} + jP \le Q < (j+1)P \text{ and } j \ge i.$$

Therefore,

$$H_1(q^*, i) > H_2(Q, j+1), \quad \forall Q \text{ s.t. } \frac{R}{s} + jP \le Q < (j+1)P \text{ and } j \ge i, \tag{25}$$

and hence,

$$H\left(q^*\right) > H\left(Q\right),\ \ \forall Q \text{ s.t. } \frac{R}{s} + jP \le Q < \left(j+1\right)P \text{ and } j \ge i. \qquad (26)$$

Now, let us consider Q such that $jP \le Q < \frac{R}{s} + jP$ and $Q > q^*$. Since z^* is the unique maximizer of $H_1\left(Q, j\right)$ over Q and $z^* < q^*$, it follows that $H_1\left(q^*, i\right) > H_1\left(Q, i\right)$, $\forall Q > q^*$, and hence,

$$H\left(q^*\right) > H\left(Q\right),\ \ \forall Q \text{ s.t. } q^* < Q < \frac{R}{s} + iP. \qquad (27)$$

We have $\left(i+1\right)P > q^* > z^*$, therefore,

$$H_1\left(Q, j\right) < H_1\left(jP, j\right),\ \ \forall Q > jP,\ \ j > i. \qquad (28)$$

Expression (25) implies that

$$H_1\left(q^*, i\right) > H_2\left(jP, j\right),\ \forall j > i. \qquad (29)$$

Using the fact that $H_1\left(jP, j\right) = H_2\left(jP, j\right)$, Expressions (28) and (29) lead to

$$H_1\left(q^*, i\right) > H_1\left(Q, j\right),\ \forall Q \text{ such that } jP \le Q < \frac{R}{s} + jP \text{ and } j > i,$$

and hence,

$$H\left(q^*\right) > H\left(Q\right),\ \forall Q \text{ such that } jP \le Q < \frac{R}{s} + jP \text{ and } j > i. \qquad (30)$$

Combining Expressions (27) and (30), we have

$$H\left(q^*\right) > H\left(Q\right),\ \forall Q \text{ such that } jP \le Q < \frac{R}{s} + jP \text{ and } Q > q^*. \qquad (31)$$

■

Lemma 3 and Lemma 4 jointly imply that we should only focus on Q such that $z^* \le Q \le q^*$ while optimizing $H(Q)$. In Lemma 5 and Lemma 6, by a similar analysis, we further eliminate some quantity values between z^* and q^*.

Lemma 5 *Let Q be an order quantity such that $z^* < Q < q^*$ and $jP \le Q < \frac{R}{s} + jP$, $j \in \{0\} \cup \mathbb{Z}^+$. Then, we have $H\left(\max\left\{z^*, \left\lfloor \frac{Q}{P} \right\rfloor P\right\}\right) > H(Q)$.*

Proof: Note that, if $\max\left\{z^*, \left\lfloor \frac{Q}{P} \right\rfloor P\right\} = z^*$, then $jP < z^* < Q < \frac{R}{s} + jP$. In this case, we have $H(z^*) = H_1(z^*, j)$ and $H(Q) = H_1(Q, j)$. Since z^* is the maximizer of $H_1(Q, j)$ over Q, it follows that $H_1(z^*, j) > H_1(Q, j)$, and hence, $H(z^*) > H(Q)$.

If $\max\left\{z^*, \left\lfloor \frac{Q}{P} \right\rfloor P\right\} = \left\lfloor \frac{Q}{P} \right\rfloor P$, then $z^* < \left\lfloor \frac{Q}{P} \right\rfloor P$. Expression (7) implies that $H(Q) = H_1(Q, j)$ and $H\left(\left\lfloor \frac{Q}{P} \right\rfloor P\right) = H_1\left(\left\lfloor \frac{Q}{P} \right\rfloor P, j\right)$. It follows from the strict concavity of $H_1(Q, j)$ over Q and the fact that $\left\lfloor \frac{Q}{P} \right\rfloor P > z^*$, we have $H_1\left(\left\lfloor \frac{Q}{P} \right\rfloor P, j\right) > H_1(Q, j)$, which implies $H\left(\left\lfloor \frac{Q}{P} \right\rfloor P\right) > H(Q)$. ■

Lemma 6 *Let Q be an order quantity such that* $z^* < Q < q^*$ *and* $\frac{R}{s} + jP \le Q < (j+1)P$, $j \in \{0\} \cup \mathbb{Z}^+$. *Then, we have* $H\left(\min\left\{q^*, \left\lceil \frac{Q}{P} \right\rceil P\right\}\right) > H(Q)$.

Proof: If $\min\left\{q^*, \left\lceil \frac{Q}{P} \right\rceil P\right\} = q^*$, then $\frac{R}{s} + jP \le Q < q^* < (j+1)P$. In this case, we have $H(q^*) = H_2(q^*, j+1)$ and $H(Q) = H_2(Q, j+1)$. Since q^* is the maximizer of $H_2(Q, j+1)$ over Q, it follows that $H_2(q^*, j+1) > H_2(Q, j+1)$, and hence, $H(q^*) > H(Q)$.

If $\min\left\{q^*, \left\lceil \frac{Q}{P} \right\rceil P\right\} = \left\lceil \frac{Q}{P} \right\rceil P$, then $q^* > \left\lceil \frac{Q}{P} \right\rceil P$. It follows from the strict concavity of $H_2(Q, j)$ over Q and the fact that $\left\lceil \frac{Q}{P} \right\rceil P < q^*$, we have $H_2\left(\left\lceil \frac{Q}{P} \right\rceil P, j+1\right) > H_2(Q, j+1)$, which implies $H\left(\left\lceil \frac{Q}{P} \right\rceil P\right) > H(Q)$. ■

Lemma 5 and Lemma 6 imply that among the order quantities between z^* and q^*, we should only consider full truck loads. The following theorem provides a characterization of the retailer's optimal order quantity.

Theorem 1 *The order quantity which maximizes* $H(Q)$ *is given by*

$$Q^*(R,s) = arg\ max\ \left\{H(z^*), H(q^*), H(kP)\ s.t.\ k = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor, k \in \left\{\left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor\right\}\right\}.$$

Proof: The proof follows from Lemmas $(3), (4), (5)$ and (6).

The above theorem implies that the retailer does not use the LTL carrier if both z^* and q^* are full truck loads. The LTL carrier would generate some revenue from the business of the retailer only if the retailer orders z^* and z^* is realizable, or if the retailer orders q^* and q^* is not realizable. Because in all other cases, the retailer uses only the TL carrier. The following lemma further shows that in the second case (i.e., $Q^*(R,s) = q^*$ and q^* is not realizable), the quantity shipped using the LTL carrier is zero.

Lemma 7 *If there exists j,* $j \in \{0\} \cup \mathbb{Z}^+$ *such that* $jP \leq q^* < \frac{R}{s} + jP$, *that is q^* is not realizable, then the retailer orders q^* only if q^* is an integer number of full truck loads.*

Proof: Assume by the way of contradiction that the retailer orders q^* and q^* is not a full truckload. Since $jP < q^* < \frac{R}{s} + jP$, we have $H(q^*) = H_1(q^*, j)$. It follows from the strict concavity of $H_1(Q, j)$ and the fact that $z^* < q^*$, there exists $\overline{Q}$ such that $jP < \overline{Q} < q^* < \frac{R}{s} + jP$ and $H_1\left(\overline{Q}, \left\lfloor \frac{q^*}{P} \right\rfloor\right) > H_1\left(q^*, \left\lfloor \frac{q^*}{P} \right\rfloor\right)$. Note also that, we have $H\left(\overline{Q}\right) = H_1\left(\overline{Q}, \left\lfloor \frac{q^*}{P} \right\rfloor\right)$. Therefore, $H\left(\overline{Q}\right) > H(q^*)$, and hence, q^* cannot be retailer's optimal order quantity. ■

Lemma 8 *If* $\nexists\ j \in \{0\} \cup \mathbb{Z}^+$ *such that* $\frac{R}{s} + jP \leq z^* < (j+1)P$, *that is if z^* is not realizable, then it is not optimal.*

Proof: If z^* is not realizable, then we have $\frac{R}{s} + jP \leq z^* < (j+1)P$ for some $j \in \{0\} \cup \mathbb{Z}^+$. This implies that $H(z^*) = H_2(z^*, j+1)$. Since $z^* < q^*$ and q^* is the maximizer of $H_2(Q, j+1)$, it follows that there exists $\overline{Q}$ such that $z^* < \overline{Q} < (j+1)P$ and $H_2\left(\overline{Q}, j+1\right) > H_2(z^*, j+1)$. This further leads to $H\left(\overline{Q}\right) > H(z^*)$, and hence, z^* cannot be optimal. ■

The optimum order quantity of the retailer can also be characterized using the order quantity Q that maximizes the following function.

$$H_{TL}(Q) = G(Q) - \left\lceil \frac{Q}{P} \right\rceil R.$$

Let Q^*_{TL} be the maximizer of the above function. Note that, Q^*_{TL} is the optimal order quantity of the retailer if the TL carrier was the only transporter available. The value of Q^*_{TL} is derived by Toptal [22] and is given by the following expression.

$$Q^*_{TL} = \begin{cases} \arg\max\left\{H_{TL}(mP), H_{TL}((m+1)P)\right\} & \text{if } \mathbb{F} \neq \varnothing \\ \\ \arg\max\left\{H_{TL}(q^*), H_{TL}((l-1)P)\right\} & \text{if } \mathbb{F} = \varnothing \end{cases} \tag{32}$$

where $l = \left\lceil \frac{q^*}{P} \right\rceil$, $\mathbb{F} = \left\{k \in \{0,1,2,\ldots\} : G((k+1)P) - G(kP) \leq R,\ (k+1)P \leq q^*\right\}$ and $m = \min\{k \text{ s.t. } k \in \mathbb{F}\}$ when $\mathbb{F} \neq \varnothing$.

As it is implied from Expression (32), Q^*_{TL} may have two alternative values. If $\mathbb{F} \neq \varnothing$, it may be either mP or $(m+1)P$, or both of them. Similarly, if $\mathbb{F} = \varnothing$ then Q^*_{TL} may be either q^* or $\left(\left\lceil \frac{q^*}{P} \right\rceil - 1\right)P$, or both of them. In the following characterization for $Q^*(R,s)$, Q^*_{TL} refers to any one of these maximizers.

Proposition 1 *Given the values of Q^*_{TL}, z^* and q^*, we have the following:*

1. *If* $Q^*_{TL} < z^*$, *then* $Q^*(R,s) = z^*$.
2. *If* $z^* \le Q^*_{TL} \le q^*$, *then* $Q^*(R,s) = z^*$ *or* $Q^*(R,s) = Q^*_{TL}$.

Proof: We start the proof by pointing out that we have $H_{TL}(Q) = H_2\left(Q, \left\lceil \frac{Q}{P} \right\rceil\right)$. Observe also that, if $Q^*_{TL} < z^*$, then Q^*_{TL} is a full truck load and z^* is realizable. The first result follows due to Lemma 2 and Expression (32), and leads to $H\left(Q^*_{TL}\right) = H_1\left(Q^*_{TL}, \frac{Q^*_{TL}}{P}\right) = H_2\left(Q^*_{TL}, \frac{Q^*_{TL}}{P}\right) = H_{TL}\left(Q^*_{TL}\right)$. The second result follows, because, otherwise we would have $H(z^*) = H_2\left(z^*, \left\lceil \frac{z^*}{P} \right\rceil\right)$, but now Lemma 3 would imply that $H_2\left(z^*, \left\lceil \frac{z^*}{P} \right\rceil\right) > H_2\left(Q^*_{TL}, \frac{Q^*_{TL}}{P}\right)$, which contradicts with the fact that Q^*_{TL} is a maximizer of $H_{TL}(Q)$. Therefore, z^* is realizable and we have $H(z^*) = H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$. Recall from Theorem 1 that, $Q^*(R,s)$ can be z^* or q^* or any full truck load between those quantities. In the remaining part of the proof for the first case (i.e., $Q^*_{TL} < z^*$), we will first show that $H(z^*) > H(kP)$, for $k \in \left\{\left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor\right\}$. Then, we will show that $H(z^*) > H(q^*)$.

Let us start with kP values such that $k \in \left\{\left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor\right\}$. Note that, we have $H(kP) = H_1(kP,k) = H_2(kP,k) = H_{TL}(kP)$. Since Q^*_{TL} is a maximizer of $H_{TL}(Q)$, we have $H_{TL}\left(Q^*_{TL}\right) \ge H_{TL}(kP)$, which implies $H\left(Q^*_{TL}\right) \ge H(kP)$. It follows from Lemma 3 that $H(z^*) > H\left(Q^*_{TL}\right)$, hence, $H(z^*) > H(kP)$.

In order to show that $H(z^*) > H(q^*)$, we will consider two cases, that is q^* is realizable and q^* is not realizable. If q^* is realizable, then $H(q^*) = H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right) = H_{TL}(q^*)$. Since Q^*_{TL} is a maximizer of $H_{TL}(Q)$, we have $H_{TL}(Q^*_{TL}) \geq H_{TL}(q^*)$, and hence, $H(Q^*_{TL}) \geq H(q^*)$. Using Lemma 3, it follows that $H(z^*) > H(Q^*_{TL}) > H(q^*)$. If q^* is not realizable, then we either have $\left\lfloor \frac{z^*}{P} \right\rfloor P \leq z^* < q^* < \frac{R}{s} + \left\lfloor \frac{z^*}{P} \right\rfloor P$ or $\bar{k}P \leq q^* < \frac{R}{s} + \bar{k}P$ for $\bar{k} \in \{0\} \cup \mathbb{Z}^+$, $k > \left\lfloor \frac{z^*}{P} \right\rfloor$. In the first case, since z* is the maximizer of $H_1\left(Q, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$, we have $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) > H_1\left(q^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$, and hence, $H(z^*) > H(q^*)$. In the second case, we will use the earlier result in this proof that $H(z^*) > H(\bar{k}P)$ and the fact that $H_1(Q,k)$ is decreasing in the region where $Q \in \left[\bar{k}P, \frac{R}{s} + \bar{k}P\right)$. The latter implies that $H_1(\bar{k}P, k) > H_1(q^*, k)$, and therefore, $H(\bar{k}P) > H(q^*)$. Combining this with the fact that $H(z^*) > H(\bar{k}P)$, we further conclude that $H(z^*) > H(q^*)$. This completes the proof for the first part of the proposition, i.e., $Q^*_{TL} < z^*$.

Now, let us consider the second part of the proposition. Since Q^*_{TL} maximizes $H_{TL}(Q)$, it follows that $H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right) \geq H_2(kP, k)$, $\forall k$ such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor, k \in \{0\} \cup \mathbb{Z}^+$. We have $H(Q^*_{TL}) \geq H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$ and

$H(kP) = H_2(kP, k)$, therefore, $H\left(Q^*_{TL}\right) \geq H(kP)$. It follows from Theorem 1 that it suffices to consider z^* and Q^*_{TL} for finding a solution for $Q^*(R,s)$. ■

The following corollary indicates that if $z^* < Q^*_{TL} \leq q^*$ and z^* is not realizable, then the retailer should order the quantity that would maximize his/her expected profits if the TL carrier was the only transporter available.

Corollary 1 *If* $z^* < Q^*_{TL} \leq q^*$ and z^* *is not realizable,* $Q^*(R,s) = Q^*_{TL}$.

Proof: It follows from Proposition 1 that if $z^* < Q^*_{TL} \leq q^*$, then $Q^*(R,s)$ is either z^* or Q^*_{TL}. Since z^* is not realizable, we have $H(z^*) = H_2\left(z^*, \left\lceil \frac{z^*}{P} \right\rceil\right)$. Note also that, by definition, $H_{TL}(z^*) = H_2\left(z^*, \left\lceil \frac{z^*}{P} \right\rceil\right)$. Since Q^*_{TL} maximizes $H_{TL}(Q)$, we further have $H_{TL}\left(Q^*_{TL}\right) > H_{TL}(z^*)$, and hence, $H_{TL}\left(Q^*_{TL}\right) > H(z^*)$. Combining this with the fact that, $H\left(Q^*_{TL}\right) \geq H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right) = H_{TL}\left(Q^*_{TL}\right)$ leads to $H\left(Q^*_{TL}\right) > H(z^*)$. ■

Chapter 5

TRANSPORTATION PRICING PROBLEMS

In this chapter, we study the transportation pricing problems faced by the TL carrier and the LTL carrier. Recall that, we define the transportation pricing problem of the TL carrier as determining the value of the per truck price R that maximizes the TL carrier's expected profits under the optimal response of the retailer, for a given price schedule of the LTL carrier (i.e., unit shipping price s). A formulation of this problem is provided in Chapter 3 and is referred to as *Problem TLP*. Similarly, we define the transportation pricing problem of the LTL carrier as determining the value of the unit shipping price s to maximize the LTL carrier's expected profits under the optimal response of the retailer, for a given price schedule of the TL carrier (i.e., per truck price R). A formulation of this problem is presented in Chapter 3 and is referred to as *Problem LTLP*.

Solving the transportation pricing problems of the TL and the LTL carrier exhibits certain challenges primarily due to the piecewise structure of the retailer's expected profit function. As discussed in Chapter 4, there is no simple analytical expression for $Q^*(R,s)$. However, we have developed two different characterizations of the optimal solution through showing some structural properties of the objective function. In our following analysis for the transportation pricing problems, we will follow a similar approach. Based on our earlier results, we will further simplify the formulations of these problems, which will lead us to the optimal solutions. We first start with the TL carrier's pricing problem.

5.1 TL Carrier's Transportation Pricing Problem

In this section, we study the problem of the TL carrier in deciding the value of *R*. In our analysis, we solve *TLP* for the general demand distribution. We note that, for a specific demand distribution the solution may further be simplified. Two general constraints that we impose on the selection of R are: i) $R < sP$ and ii) $R \geq 0$.

The approach that we follow in solving *TLP* utilizes the first characterization of the retailer's optimal solution which we have derived in Chapter 4. This solution is stated in Theorem 1. It is implied by this theorem that $Q^*(R,s)$ can be z^*, or q^* or any full truck load between those quantities. Considering this implication, we divide all possible values of R (all real numbers such that $R < sP$ and $R \geq 0$) into three groups and proceed our analysis thereafter. More specifically, we optimize out of all feasible R values which lead to z^*. Secondly, we optimize over all feasible R values which lead to an optimal replenishment quantity of the retailer given by kP, where $k \in \left\{ \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor \right\}$ and is an integer value. Thirdly, we solve *TLP* over all feasible R values which lead to q^* as the optimal replenishment quantity of the retailer. Finally, among these solutions, we choose the R value which results in the maximum expected profits for the TL carrier. It is important to note that the reason why we use the first characterization for $Q^*(R,s)$ is that, the possible values for the optimal solution of the retailer (i.e., z^*, or q^* or any full truck load in between) is independent of the specific value of R for fixed s.

In what follows, we first present Proposition 2, Proposition 3 and Proposition 4 to identify the feasible R values which lead to each type of solution discussed above. Then, we formulate the three subproblems and discuss their solutions.

Proposition 2 *The optimal replenishment quantity of the retailer is given by q* if and only if we have* $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1\left(z^*,\left\lfloor \frac{z^*}{P}\right\rfloor\right)$ *and* $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1(kP,k)$ $\forall k$ *such that* $k=\left\lceil \frac{z^*}{P}\right\rceil,...,\left\lfloor \frac{q^*}{P}\right\rfloor$.

Proof: We first prove that if q^* is optimal then $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1\left(z^*,\left\lfloor \frac{z^*}{P}\right\rfloor\right)$ and $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1(kP,k)$ $\forall k$ such that $k=\left\lceil \frac{z^*}{P}\right\rceil,...,\left\lfloor \frac{q^*}{P}\right\rfloor$. The proof will be complete by showing that if $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1\left(z^*,\left\lfloor \frac{z^*}{P}\right\rfloor\right)$ and $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1(kP,k)$ $\forall k$ such that $k=\left\lceil \frac{z^*}{P}\right\rceil,...,\left\lfloor \frac{q^*}{P}\right\rfloor$, then q^* is optimal. Let us start with the first part of the proof.

Since q^* maximizes the retailer's expected profits, we have $H(q^*) \geq H_1\left(z^*,\left\lfloor \frac{z^*}{P}\right\rfloor\right)$ and $H(q^*) \geq H_1(kP,k)$ $\forall k$ such that $k=\left\lceil \frac{z^*}{P}\right\rceil,...,\left\lfloor \frac{q^*}{P}\right\rfloor$. If q^* is realizable, we further have $H(q^*) = H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right)$, which leads to $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1\left(z^*,\left\lfloor \frac{z^*}{P}\right\rfloor\right)$ and $H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right) \geq H_1(kP,k)$ $\forall k$ such that $k=\left\lceil \frac{z^*}{P}\right\rceil,...,\left\lfloor \frac{q^*}{P}\right\rfloor$. If q^* is not realizable while it is optimal, then Lemma 7 implies that it is given by an integer number of full truck loads. In this case, we again have $H(q^*) = H_2\left(q^*,\left\lceil \frac{q^*}{P}\right\rceil\right)$, and therefore, the result follows.

Now, let us assume that $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)\geq H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)$ and $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)\geq H_1\left(kP,k\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$. Note that we have $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)=H_{TL}\left(q^*\right)$ and $H_1\left(kP,k\right)=H_2\left(kP,k\right)=H_{TL}\left(kP\right)$. This leads to $H_{TL}\left(q^*\right)\geq H_{TL}\left(kP\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$. Using this result, Expression (32) implies that $Q^*_{TL}=q^*$, and hence, $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)\geq H_2\left(z^*,\left\lceil\frac{z^*}{P}\right\rceil\right)$. Combining this with the fact that $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)\geq H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)$ further leads to $H\left(q^*\right)\geq H\left(z^*\right)$. Note also that $H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)\geq H_1\left(kP,k\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$ implies $H\left(q^*\right)\geq H\left(kP\right)$. Therefore, q^* maximizes the retailer's expected profit function. ■

Proposition 3 *z^* is optimal if and only if we have $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)$ and $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_1\left(kP,k\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$.*

Proof: We first prove that if z^* is optimal then $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)$ and $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_1\left(kP,k\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$. The proof will be complete by showing that if $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_2\left(q^*,\left\lceil\frac{q^*}{P}\right\rceil\right)$ and $H_1\left(z^*,\left\lfloor\frac{z^*}{P}\right\rfloor\right)\geq H_1\left(kP,k\right)$ $\forall k$ such that $k=\left\lceil\frac{z^*}{P}\right\rceil,...,\left\lfloor\frac{q^*}{P}\right\rfloor$, then z^* is optimal. Let us start with the first part of the proof.

If z^* is optimal, then we have from Lemma 8 that z^* is realizable, and therefore, $H(z^*) = H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$. The optimality of z^* further implies that $H(z^*) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ and $H(z^*) \geq H_1(kP, k)$ $\forall k$ such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$. Combining these two results lead to $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ and $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_1(kP, k)$ $\forall k$ such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$.

Now, let us assume that $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ and $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_1(kP, k)$ $\forall k$ such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$. If $Q^*_{TL} < z^*$, then Proposition 1 implies that z^* maximizes the retailer's expected profits. If $Q^*_{TL} \geq z^*$, we have from Expression (32) that Q^*_{TL} is given by q^* or kP such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$. Since $H(kP) = H_1(kP, k)$ and $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_1(kP, k)$ $\forall k$ such that $k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$, we have $H(z^*) \geq H(kP)$. If q^* is not realizable, Lemma 7 implies that it cannot be optimal, therefore, z^* maximizes the retailer's expected profits. If q^* is realizable, then $H(q^*) = H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$. Combining this with the fact that $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ leads to $H(z^*) \geq H(q^*)$, and hence z^* is optimal. ■

Proposition 4 *The optimal replenishment quantity of the retailer is given by kP, for some* $k \in \mathbb{Z}^+$ *and* $\left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor$ *if and only if* $H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H_1(kP,k) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$.

Proof: We first prove that if kP is optimal then $H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H_1(kP,k) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$. The proof will be complete by showing that if $H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H_1(kP,k) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$, then kP is optimal. Let us start with the first part of the proof.

Recall that we have $H(kP) = H_1(kP,k)$. Since kP is optimal, it follows that $H(kP) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H(kP) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H(kP) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$. This in turn implies that $H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H_1(kP,k) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$.

Now, let us assume that $H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H_1(kP,k) \ge H_1(jP,j)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, ..., \left\lfloor \frac{q^*}{P} \right\rfloor$. Since $H_1(kP,k) = H(kP)$ and $H_1(jP,j) = H(jP)$, it follows that we have $H(kP) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$, $H(kP) \geq H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ and $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$. If z^* is realizable, then $H(kP) \geq H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ implies that $H(kP) \geq H(z^*)$. In this case, if q^* is also realizable, then $H(kP) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ implies $H(kP) \geq H(q^*)$, and hence, kP is optimal. If q^* is not realizable, then by Lemma 7, q^* would be optimal only if it is an integer multiple of a full truck load. Even in this case, $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$ implies that $H(kP) \geq H(q^*)$, therefore, kP is also optimal. If q^* is not realizable and is not an integer multiple of a full truck load, then it cannot be optimal, therefore, $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$ together with $H(kP) \geq H(z^*)$ guarantees that kP is optimal. If z^* is not realizable, we know from Lemma 8 that it cannot be optimal. In this case, if q^* is realizable, then $H(kP) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ implies that $H(kP) \geq H(q^*)$. Combining with the fact that $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$, we conclude that kP is optimal. If q^* is not realizable, then by Lemma 7, q^* would be optimal only if it is an integer multiple of a full truck load. Even in this case, $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$ implies that $H(kP) \geq H(q^*)$, therefore, kP is also optimal. If q^* is not realizable and is not an integer multiple of a full truck load, then it cannot be optimal. Therefore, $H(kP) \geq H(jP)$ $\forall j = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor$ guarantees that kP is optimal. ■

Before we characterize the solution to *TLP* by using Proposition 2, Proposition 3 and Proposition 4, let us define some new notation. Let $R^*(q^*)$ be the value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to q^* as the optimal replenishment quantity of the retailer. Similarly, let $R^*(z^*)$ be the value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to z^* as the optimal replenishment quantity of the retailer. Finally, let $R^*(kP)$ be the value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to kP as the optimal replenishment quantity of the retailer, k ranging from $\left\lceil \frac{z^*}{P} \right\rceil$ to $\left\lfloor \frac{q^*}{P} \right\rfloor$.

Note that we have by Proposition 2 that $R^*(q^*)$ is the solution the following model:

*Problem P*1:

$$\max_{R} \quad \Pi_{TL}(q^*, R)$$

$$\text{s.t.} \quad H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right) \geq H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right),$$

$$H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right) \geq H_1(kP, k), \quad k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor.$$

$$R < sP,$$

$$R \geq 0.$$

Similarly, we have from Proposition 3 that $R^*(z^*)$ is the solution to the following model:

*Problem P*2:

$$\max_{R} \quad \Pi_{TL}(z^*, R)$$

$$\text{s.t.} \quad H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right),$$

$$H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_1(kP, k), \quad k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor.$$

$$R < sP,$$

$$R \geq 0.$$

Finally, we have from Proposition 4 that for any integer value of k such that $\left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor$, $R^*(kP)$ is the solution to the following model:

*Problem P*3:

$$\max_{R} \quad \Pi_{TL}(kP, R)$$

s.t.

$$H_1(kP,k) \ge H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right),$$

$$H_1(kP,k) \ge H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right),$$

$$H_1(kP,k) \ge H_1(jP, j), \qquad j = \left\lceil \frac{z^*}{P} \right\rceil, \dots, \left\lfloor \frac{q^*}{P} \right\rfloor.$$

$$R < sP,$$

$$R \ge 0.$$

The following corollary characterizes the optimal solution of problem *TLP*.

Corollary 2 *The solution to TLP is given by the value of the per truck cost R among* $R^*(q^*)$, $R^*(z^*)$ *and* $R^*(kP)$ $\forall k$ *such that* $\left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor$, *which gives the maximum expected profits for the TL carrier. In other words, the solution to TLP is the second argument of the function which attains the following at its computed value*

$$\max\left\{\Pi_{TL}(q^*, R^*(q^*)), \Pi_{TL}(z^*, R^*(z^*)), \Pi_{TL}(kP, R^*(kP)) \,\forall k \text{ s.t.} \left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor\right\}.$$

In what follows, we provide simplified expressions for obtaining $R^*(q^*)$, $R^*(z^*)$ and $R^*(kP)$ $\forall k$ such that $\left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor$ as solutions to the mathematical formulations presented earlier in the chapter. These results are presented next in Lemma 9, Lemma 10 and Lemma 11. The proofs of these lemmas are in the Appendix.

Lemma 9 *The value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to q* as the optimal replenishment quantity of the retailer, that is $R^*(q^*)$, is given by*

$$\min\left\{\frac{G(q^*)-G(z^*)+sz^*-sP\left\lfloor \frac{z^*}{P}\right\rfloor}{\left\lceil \frac{q^*}{P}\right\rceil-\left\lfloor \frac{z^*}{P}\right\rfloor}, sP-\varepsilon, \frac{G(q^*)-G(kP)}{\left\lceil \frac{q^*}{P}\right\rceil-k}, \forall k \ s.t. \left\lceil \frac{z^*}{P}\right\rceil \le k \le \left\lfloor \frac{q^*}{P}\right\rfloor\right\}$$

where ε is a positive real number and $kP \neq q^*$ for the integer values of k that are in consideration.

Lemma 10 *The value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to z* as the optimal replenishment quantity of the retailer, that is $R^*(z^*)$, is given by $sP-\varepsilon$ where ε is a positive real number.*

Lemma 11 *The value of R which maximizes the TL carrier's expected profits among all feasible R values that lead to kP $\left(k \text{ is an integer such that } \left\lceil \frac{z^*}{P}\right\rceil \le k \le \left\lfloor \frac{q^*}{P}\right\rfloor\right)$ as the optimal replenishment quantity of the retailer, that is $R^*(kP)$, is given by*

$$\min\left\{\frac{G(kP)-G(z^*)+sz^*-sP\left\lfloor \frac{z^*}{P}\right\rfloor}{k-\left\lfloor \frac{z^*}{P}\right\rfloor}, sP-\varepsilon, \frac{G(kP)-G(jP)}{k-j}, \forall j \ s.t. \left\lceil \frac{z^*}{P}\right\rceil \le j \le k-1\right\}$$

where ε is a positive real number.

5.2 LTL Carrier's Transportation Pricing Problem

In this section, we study the problem of the LTL carrier in deciding the value of *s*. This problem is referred to as *Problem LTLP* in Chapter 3. Two general constraints that we impose on the selection of *s* are: i) $R < sP$ and ii) $s \geq 0$. In what follows next, based on our analysis in Chapter 4, we will prove a proposition which will later be used to simplify the formulation presented in Chapter 3 for *Problem LTLP*. The following proposition identifies the condition for *z** to be optimal.

Proposition 5 *z* is optimal if and only if* $H_1\left(z^*,\left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL},\left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$.

Proof: We will first proof that if $H_1\left(z^*,\left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL},\left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$, then *z** is optimal. The proof will be complete by showing that if *z** is optimal, then $H_1\left(z^*,\left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL},\left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$.

Let us start with the first part. Recall that we have $H_2\left(z^*,\left\lceil \frac{z^*}{P} \right\rceil\right) = H_{TL}(z^*)$ and $H_2\left(Q^*_{TL},\left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right) = H_{TL}\left(Q^*_{TL}\right)$. Combining this with the fact that $H_{TL}\left(Q^*_{TL}\right) \geq H_{TL}(z^*)$ leads to $H_2\left(Q^*_{TL},\left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right) \geq H_2\left(z^*,\left\lceil \frac{z^*}{P} \right\rceil\right)$, and hence, $H_1\left(z^*,\left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(z^*,\left\lceil \frac{z^*}{P} \right\rceil\right)$. This further implies that $H(z^*) = H_1\left(z^*,\left\lfloor \frac{z^*}{P} \right\rfloor\right)$. It follows from Expression (32) that Q^*_{TL}

is either a multiple of a full truck load $\left(\text{i.e., } Q^*_{TL} = kP,\ k \in \{0,1,2,...\}\right)$, or not, in which case it is equal to q^*. In the first case, we have $H\left(Q^*_{TL}\right) = H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right) = H_1\left(Q^*_{TL}, \left\lfloor \frac{Q^*_{TL}}{P} \right\rfloor\right)$. Since $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$ and $H(z^*) = H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$, it follows that, $H(z^*) \geq H\left(Q^*_{TL}\right)$. Recall from Proposition 1 that z^* and Q^*_{TL} are two possible solutions for $Q^*(R,s)$. As $H(z^*) \geq H\left(Q^*_{TL}\right)$, it turns out that $Q^*(R,s) = z^*$. In the second case $\left(i.e., \nexists\, k \in \{0,1,2,...\} \text{ such that } Q^*_{TL} = kP\right)$, assume that z^* is not optimal. Then, it follows from Lemma 7 that q^* is realizable, which in turn implies that $H(q^*) = H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$. Because $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(q^*, \left\lceil \frac{q^*}{P} \right\rceil\right)$ and $H(z^*) = H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$, this leads to $H(z^*) \geq H(q^*)$. However, this contradicts with the assumption that z^* is not optimal.

Now, let us consider the second part of the proof, i.e., z^* is optimal. Assume that $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) < H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$. This implies that z^* is not realizable, because otherwise, we would have $H(z^*) = H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right)$ which in turn leads to $H(z^*) < H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$, however this contradicts with the optimality of z^*. Yet, we have from Lemma 8 that, if z^* is not realizable then it is not optimal. Therefore, if z^* is optimal then we should have $H_1\left(z^*, \left\lfloor \frac{z^*}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right)$. ■

We know from Theorem 1 and Lemma 7 that the retailer does business with the LTL carrier only if he/she orders z^*. For all other quantities, the quantity shipped using the LTL carrier is 0. Therefore, the value s that maximizes the LTL carrier's expected profits should be looked for among all feasible values which lead to z^* as the optimal solution of the retailer. Proposition 5 suggests a necessary and sufficient condition for z^* to be optimal using the second characterization of the solution for the retailer's replenishment problem. Notice that the second characterization is based on Q^*_{TL}, which does not depend on the decision variable s.

Problem P4:

$$\max_{s} \quad \Pi_{LTL}\left(z^*(s), s\right)$$

s.t.

$$H_1\left(z^*(s), \left\lfloor \frac{z^*(s)}{P} \right\rfloor\right) \geq H_2\left(Q^*_{TL}, \left\lceil \frac{Q^*_{TL}}{P} \right\rceil\right),$$

$$R/P < s < R.$$

Our contribution in analyzing the LTL carrier's transportation problem is to come up with alternative formulations. Solving the above problem for general demand distribution, as we did it for the TL carrier, exhibits certain challenges. These challenges are mainly due to the complexity of the retailer's replenishment problem and the fact that z^* depends on the decision variable s.

Chapter 6

NUMERICAL RESULTS

In this chapter, we present some numerical results for the setting of interest in this thesis. The objectives of this numerical study are as follows:

i) To illustrate an application of the solution for the retailer's optimization problem,

ii) To show the implication of jointly considering inventory and transportation costs in the same problem for the retailer,

iii) To quantify the savings for the TL carrier that can be achieved through carefully deciding on the value of the shipping price per truck, i.e., R ,

iv) To quantify the savings for the LTL carrier that can be achieved through carefully deciding on the value of the shipping price per unit, i.e., s,

v) To show the opportunity for savings that could originate from a coordination of transportation pricing decisions between a TL carrier and an LTL carrier.

vi) To show the opportunity for savings that could originate from a coordination of transportation pricing decisions between a TL carrier and a retailer.

6.1 An Application of the Proposed Solution for the Retailer's Optimization Problem

In this section, we illustrate an application of Proposition 1 over a number of numerical instances (i.e., Example 1, Example 2, Example 3 and Example 4).

Example 1: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 16, r = 32, v = 11, b = 14, P = 250, R = 180, s = 2, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 30Q - 7000$. Therefore, $z^* = 800$ and $q^* = 857.14$. The retailer's expected profit function is given by

$$H(Q) = \begin{cases} -0.0175Q^2 + 28Q - 7000 + 320i & \text{if } \quad 250i \le Q < 250i + 90 \\ -0.0175Q^2 + 30Q - 7180 - 180i & \text{if } \quad 250i + 90 \le Q < 250(i+1). \end{cases}$$

Figure 6.1 shows a plot of the above function. The maximizer of $H_{TL}(Q) = -0.0175Q^2 + 30Q - 7000 - \left\lceil \dfrac{Q}{250} \right\rceil 180$, i.e., Q^*_{TL}, is equal to 857.14. Since $z^* < Q^*_{TL}$, it follows from Proposition 1 that, $Q^*(R,s)$ is either z^* or Q^*_{TL}. We have $H(800) = 5160$ and $H(857.14) = 5137.14$ Therefore, $Q^*(R,s) = z^* = 800.$

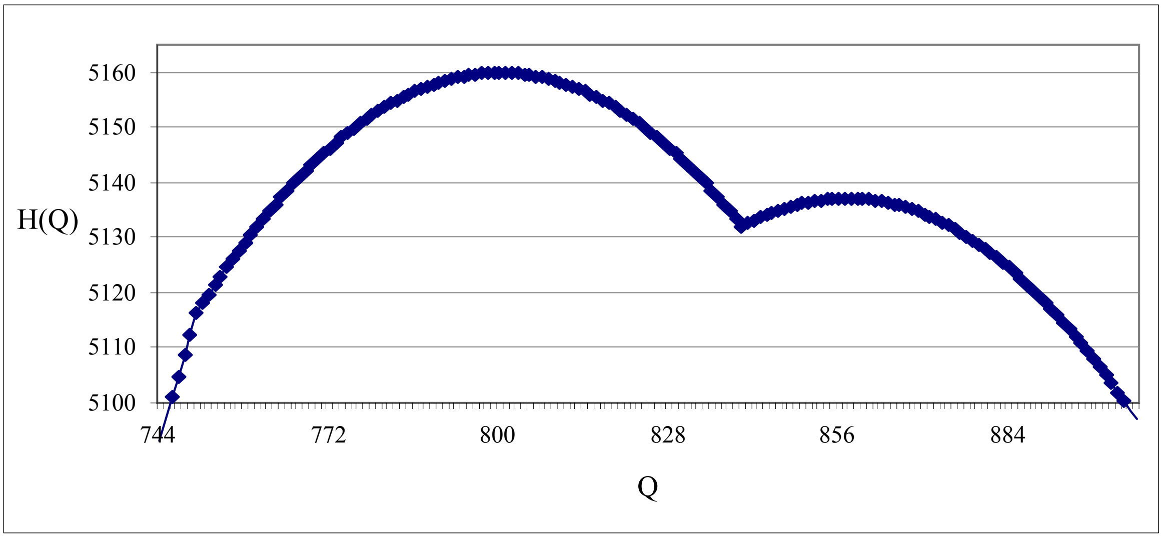

Figure 6.1 Plot of the retailer's expected profit function in Example 1

Example 2: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 18, r = 32, v = 11, b = 14, P = 90, R = 210, s = 5, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 28Q - 7000$. Therefore, $z^* = 657$ and $q^* = 800$. The retailer's expected profit function is given by

$$H(Q) = \begin{cases} -0.0175Q^2 + 23Q - 7000 + 240i & \text{if } \; 90i \le Q < 90i + 42 \\ -0.0175Q^2 + 28Q - 7210 - 210i & \text{if } \; 90i + 42 \; \le Q < 90(i+1). \end{cases}$$

Figure 6.2 shows a plot of the above function. The maximizer of $H_{TL}(Q) = -0.0175Q^2 + 28Q - 7000 - \left\lceil \frac{Q}{90} \right\rceil 210$, i.e., Q^*_{TL}, is equal to 720. Since $z^* < Q^*_{TL}$, it follows from Proposition 1 that, $Q^*(R,s)$ is either z^* or Q^*_{TL} .We have $H(657) = 2237.14$ and $H(720) = 2408$. Therefore, $Q^*(R,s) = Q^*_{TL} = 720$.

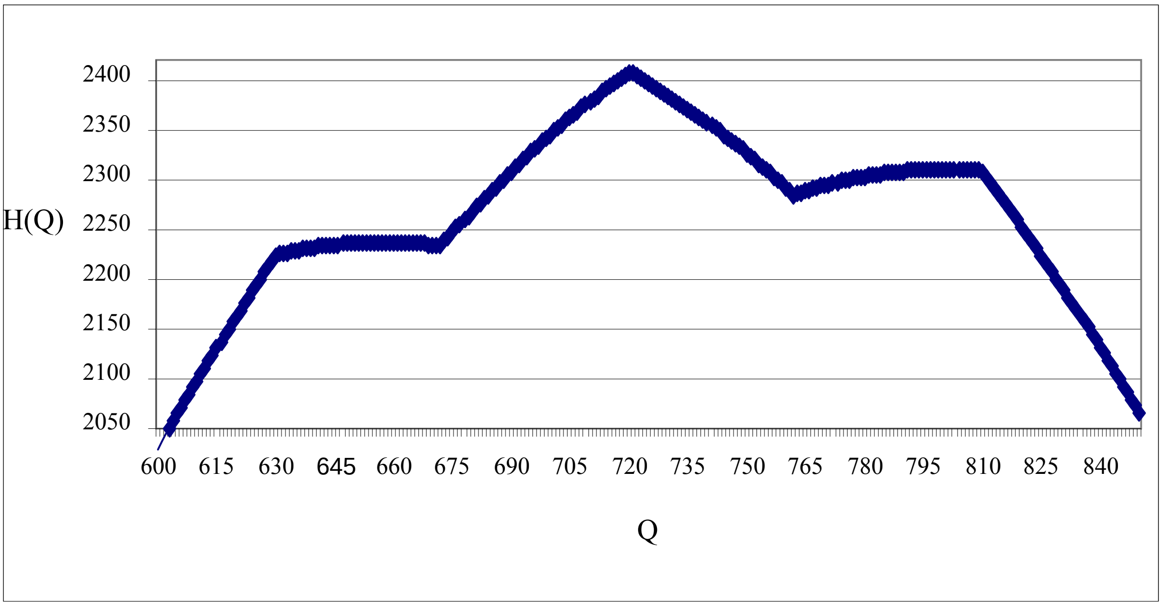

Figure 6.2 Plot of the retailer's expected profit function in Example 2

Example 3: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 18, r = 32, v = 11, b = 14, P = 75, R = 50, s = 1, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 28Q - 7000$. Therefore, $z^* = 771.43$ and $q^* = 800$. The retailer's expected profit function is given by

$$H(Q) = \begin{cases} -0.0175Q^2 + 27Q - 7000 + 25i & \text{if} \quad 75i \le Q < 75i + 50 \\ -0.0175Q^2 + 28Q - 7050 - 50i & \text{if} \quad 75i + 50 \le Q < 75(i+1). \end{cases}$$

Figure 6.3 shows a plot of the above function. The maximizer of $H_{TL}(Q) = -0.0175Q^2 + 28Q - 7000 - \left\lceil \frac{Q}{75} \right\rceil 50$, i.e., Q^*_{TL}, is equal to 750. Since $Q^*_{TL} < z^*$, it follows from Proposition 1 that, $Q^*(R,s) = z^* = 771.43$.

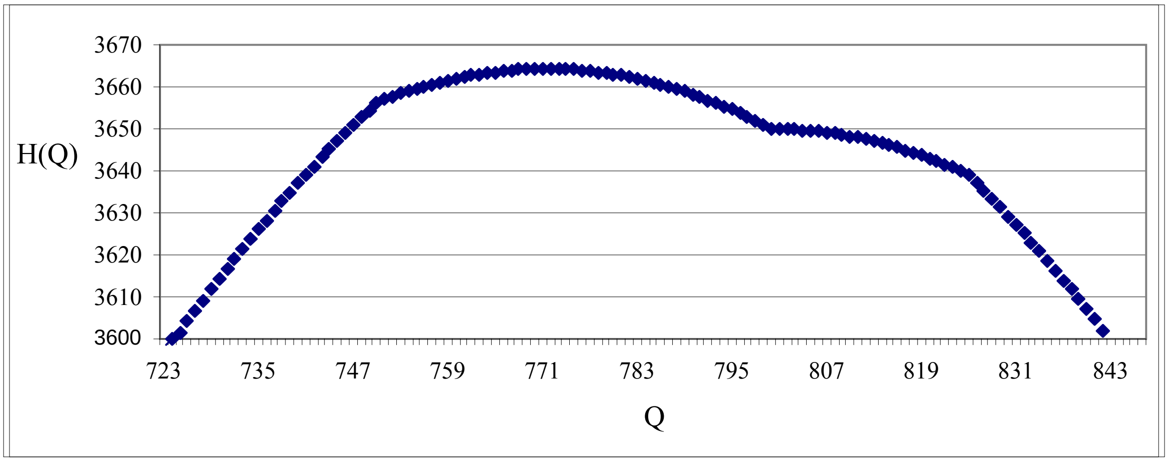

Figure 6.3 Plot of the retailer's expected profit function in Example 3

Example 4: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 14, r = 32, v = 11, b = 14, P = 250, R = 200, s = 2, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 32Q - 7000$. Therefore, $z^* = 857.14$ and $q^* = 914.29$. The retailer's expected profit function is given by

$$H(Q) = \begin{cases} -0.0175Q^2 + 30Q - 7000 + 300i & \text{if } \; 250i \le Q < 250i + 100 \\ -0.0175Q^2 + 32Q - 7200 - 200i & \text{if } \; 250i + 100 \le Q < 250(i+1). \end{cases}$$

Figure 6.4 shows a plot of the above function. The maximizer of $H_{TL}(Q) = -0.0175Q^2 + 32Q - 7000 - \left\lceil \frac{Q}{250} \right\rceil 200$, i.e., Q^*_{TL}, is equal to 914.29. Since Q^*_{TL} = q^*, it follows from Proposition 1 that $Q^*(R,s)$ is either z^* or Q^*_{TL}. We have

$3 \times 250 + 100 < z^* = 857.14 < (3+1) \times 250$, therefore, z^* is not realizable. Corollary 1 implies that in this case z^* cannot be optimal, and hence, $Q^*(R,s) = Q^*_{TL} = 914.29$.

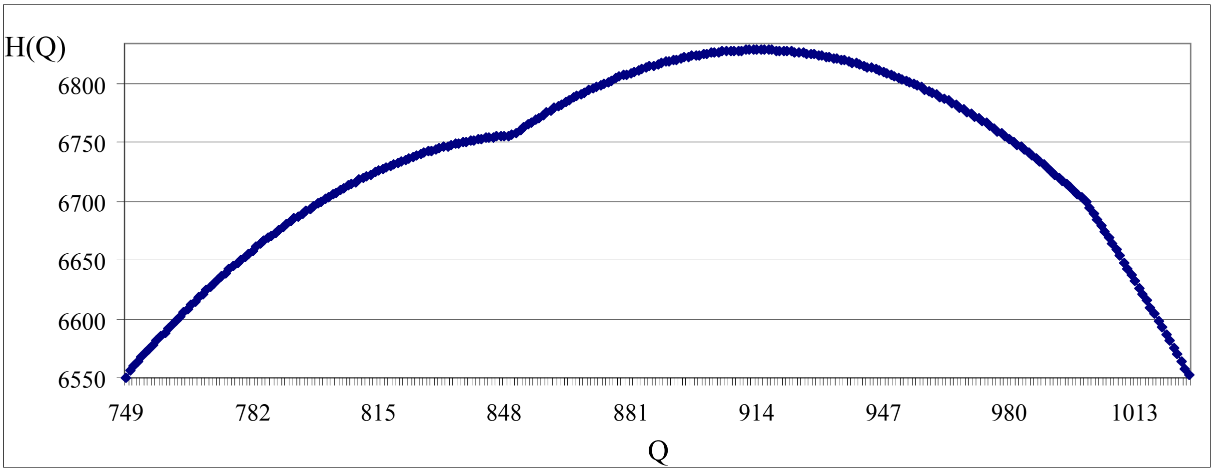

Figure 6.4 Plot of the retailer's expected profit function in Example 4

6.2 The Impact of Jointly Considering Inventory and Transportation Costs

Recent supply chain studies promote the joint modelling and optimization of inventory and transportation costs. One of the premises of this thesis lies in the fact that substantial savings can be achieved by the retailer through this consideration. With this in mind, in Chapter 4 of this thesis, we analyzed the retailer's problem in detail taking into account replenishment and transportation costs simultaneously. The purpose of this section is to quantify the savings that can be achieved through this joint modelling in comparison to a hierarchical approach.

In the hierarchical approach, we assume that the retailer makes his/her replenishment decisions using a model which does not explicitly incorporate transportation costs. In this model, transportation costs are not jointly minimized with the replenishment costs. However, the retailer has an idea about the minimum and the maximum shipping cost per unit that would result from any order quantity. Therefore, he/she makes the decision regarding the order quantity based on this rough idea of transportation costs. In order to quantify the savings from joint modelling, we compare the retailer's corresponding optimum expected profits with those resulting from the hierarchical approach under the consideration of minimum and maximum shipping cost per unit. The retailer's minimum shipping cost per unit is R/P, which is realized when he/she purchases a quantity which is an integer multiple of a full truck load and utilizes only the TL carrier. The retailer's minimum shipping cost per unit is s, which is realized when he/she utilizes only the LTL carrier. In the hierarchical approach, firstly, the retailer solves his/her replenishment problem both by incorporating the minimum per unit transportation cost R/P and the maximum per unit transportation cost s. After that, he/she sets his/her optimal order quantity as the quantity which gives the maximum profit under these two considerations. More explicitly, let $P_1(Q)$ be the expected profit function of the retailer under the minimum transportation cost R/P and define $Q_f *$ as the maximizer of this function. Similarly, let $P_2(Q)$ be the expected profit function of the retailer under the minimum transportation cost s and define $Q_g *$ as the maximizer of this function. That is,

$$P_1(Q) = G(Q) - \frac{R}{P}Q,$$

$$P_2(Q) = G(Q) - sQ,$$

$$\overline{Q} = \arg\max\left(P_1\left(Q_f *\right), P_2\left(Q_g *\right)\right)$$

More specifically, we consider the following measure to show the improvement in the retailer's expected profits as a result of jointly considering inventory and transportation costs:

$$\frac{H\left(Q*(R,s)\right) - H\left(\overline{Q}\right)}{H\left(\overline{Q}\right)} \times 100\%.$$

The following two examples show that the savings due to joint modelling of inventory and transportation costs are in fact very significant in the problem setting of interest.

Example 5: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 19, r = 32, v = 11, b = 14, P = 150, R = 400, s = 8, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 27Q - 7000$, $P_1(Q) = -0.0175Q^2 + 24.33Q - 7000$, $P_2(Q) = -0.0175Q^2 + 19Q - 7000$, $q^* = 771.43$, $Q_f^* = 695.14$ and $Q_g^* = 542.85$. $P_1(695.14) = 1456.413$ and $P_2(542.85) = -1842.86$ so $\overline{Q} = 695.14$. The retailer's expected profit function is given by

$$H(Q) = \begin{cases} -0.0175Q^2 + 19Q - 7000 + 800i & \text{if } \; 150i \le Q < 150i + 50 \\ -0.0175Q^2 + 27Q - 7400 - 400i & \text{if } \; 150i + 50 \le Q < 150(i+1). \end{cases}$$

It can be shown that $Q^*(R,s) = 750$ and the retailer's expected profits at $\overline{Q}$ and $Q^*(R,s)$ are equal to 1312.43 and 1406.25, respectively. Therefore, the percentage improvement that can be achieved due to joint optimization of inventory and transportation costs over hierarchical decision making is 7.14 % ((1406.25-1312.43)/1312.43×100%).

Example 6: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 18, r = 32, v = 11, b = 14, P = 70, R = 280, s = 7, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 28Q - 7000$, $P_1(Q) = -0.0175Q^2 + 24Q - 7000$, $P_2(Q) = -0.0175Q^2 + 21Q - 7000$, $q^* = 800$, $Q_f^* = 685.71$ and $Q_g^* = 600$. $P_1(685.71) = 1228.571$ and $P_2(600) = -700$ so $\overline{Q} = 685.71$. The retailer's expected profit function is given by

$$H(Q)=\begin{cases}-0.0175Q^2+21Q-7000+210i & \text{if } \ 70i\le Q<70i+40\\ \\ -0.0175Q^2+28Q-7280-280i & \text{if } \ 70i+40\ \le Q<70(i+1).\end{cases}$$

It can be shown that $Q^*(R,s)=700$ and the retailer's expected profits at $\overline{Q}$ and $Q^*(R,s)$ are equal to 1171.41 and 1225, respectively. Therefore, the percentage improvement that can be achieved due to joint optimization of inventory and transportation costs over hierarchical decision making is 4.57 % ((1225-1171.41)/1171.41×100%).

6.3 A Quantification of Savings for the TL Carrier due to Transportation Pricing

In Chapter 5, an analysis of the TL carrier's transportation pricing problem is provided. In this section, we use the result presented in Corollary 2 of Chapter 5 to quantify the savings the TL carrier can achieve through carefully deciding the value of the shipping price per truck, i.e., R. For this purpose, we use the following examples.

Example 7: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 19, r = 32, v = 11, b = 14, P = 150, $\overline{R}$ = 0, s = 8, $\bar{s}$ = 0.1, $K_{TL}=100$, $K_{LTL}=10$, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q)=-0.0175Q^2+27Q-7000$, $z^*=542.86$ and $q^*=771.43.$ Using Lemma 9, Lemma 10 and Lemma 11, we conclude that $R^*(q^*)=8,$ $R^*(z^*)=1200-\varepsilon,$ $R^*(4\times150)=1142.86$ and $R^*(5\times150)=506.25$. We further have

$\Pi_{TL}(771.43, 8)=-52,$

$\Pi_{TL}(542.86, 1200-\varepsilon)=3500-\varepsilon,$

$\Pi_{TL}(600, 1142.86) = 4471.44,$ and

$\Pi_{TL}(750, 506.25) = 2431.25.$

Since $\Pi_{TL}(600, 1142.86)$ gives the maximum expected profit for the TL carrier, Corollary 2 implies that the optimum value of R should be 1142.86.

In Table 6.1, for various values of R, we report the retailer's optimal order quantity, the TL carrier's, the LTL carrier's and the retailer's corresponding expected profits. In contrary to the common belief that the higher the TL carrier sets his/her price within the feasible range, the better his/her expected profits are, the expected profits of the TL carrier at R = 1142.86 are more than those at R = 1199. In fact, the percentage difference in expected profits for the TL carrier by carefully determining the value of R, instead of setting it to the highest price, amounts to 27.86%.

Table 6.1 Expected profits of the parties resulting from the retailer's optimal replenishment decision for various values of R

R	$Q^*(R,s)$	Q^*_{TL}	$\prod_{TL}(Q^*(R,s),R)$	$H(Q^*(R,s))$	$\prod_{LTL}(Q^*(R,s),s)$
8	771.43	771.43	-52	3366.2857	0
100	750	750	400	2906.25	0
200	750	750	900	2406.25	0
300	750	750	1400	1906.25	0
400	750	750	1900	1406.25	0
500	750	750	2400	906.25	0
506.25	750	750	2431.25	875	0
600	600	600	2300	500	0
700	600	600	2700	100	0
800	600	600	3100	-300	0
900	600	600	3500	-700	0
1000	600	600	3900	-1100	0
1100	600	600	4300	-1500	0
1142.86	600	600	4471.44	-1671.44	0
1199	542.86	450	3497	-1839.857	723.594

Example 8: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 18, r = 32, v = 11, b = 14, P = 70, $K_{TL} = 50$, $K_{LTL} = 10$, $\overline{R} = 5$, $s = 3$, $\overline{s} = 0.3$ and X = Uniform (0,1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 28Q - 7000$, $z^* = 714.2857$ and $q^* = 800$. Using Lemma 9, Lemma 10 and Lemma 11, we conclude that $R^*(q^*) = 15.75$, $R^*(z^*) = 210 - \varepsilon$ and $R^*(7 \times 70) = 155.6785$. We further have

$\Pi_{TL}(800, 15.75) = 79,$

$\Pi_{TL}(714.2857, 210 - \varepsilon) = 1995 - \varepsilon,$

$\Pi_{TL}(770, 155.6785) = 1637.4635.$

Since $\Pi_{TL}(714.2857, 210 - \varepsilon)$ gives the maximum expected profit for the TL carrier, Corollary 2 implies that the optimum value of R should be $210 - \varepsilon$.

Table 6.2 Expected profits of the parties resulting from the retailer's optimal replenishment decision for various values of R

R	$Q^*(R,s)$	Q^*_{TL}	$\prod_{TL}(Q^*(R,s),R)$	$H(Q^*(R,s))$	$\prod_{LTL}(Q^*(R,s),s)$
10	800	800	10	4080	0
15.75	800	800	79	3995.25	0
20	770	770	115	3964.25	0
30	770	770	225	3854.25	0
40	770	770	335	3744.25	0
50	770	770	445	3634.25	0
100	770	770	995	3084.25	0
150	770	770	1545	2534.25	0
155.6785	770	770	1607.4635	2471.7865	0
200	714.2857	700	1900	2028.571	28.571
209	714.2857	700	1990	1938.571	28.571

In Table 6.2, for various values of R, we report the retailer's optimal order quantity, the TL carrier's , the LTL carrier's and the retailer's corresponding expected profits. As it is seen in Table 6.2, for the given value of s the TL carrier can increase his/her profits by a factor of two, if she/he changes the per truck cost R from 100 to 209. This again illustrates the extent of savings that can be achieved by the TL carrier through carefully deciding on the value of per truck cost.

6.4 A Quantification of Savings for the LTL Carrier due to Transportation Pricing

The purpose of this section is to quantify the savings the LTL carrier can achieve through carefully deciding the value of the per unit shipping price, i.e., s. For this purpose, we use the following examples.

Example 9: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 16, r = 32, v = 11, b = 14, P = 250, $\overline{R}$ = 0, $\bar{s}$ = 1, K_{TL} = 100, K_{LTL} = 30, X = Uniform (0, 1000).

Table 6.3 Expected profits of the parties resulting from the retailer's optimal replenishment decision for various values of s and R = 120

s	$Q^*(R,s)$	z^*	Q_2	$\prod_{LTL}(Q^*(R,s),s)$	$\prod_{TL}(Q^*(R,s),R)$	$H(Q^*(R,s))$
1.2	822.857	822.857	72.857	-15.429	330	5389.1429
1.37	818	818	68	-4.84	330	5377.1700
1.371	857.143	817.971	0	0	450	5377.1429
1.4	857.143	817.143	0	0	450	5377.1429
1.6	857.143	811.429	0	0	450	5377.1429
1.8	857.143	805.714	0	0	450	5377.1429
2	857.143	800	0	0	450	5377.1429
4	857.143	742.857	0	0	450	5377.1429
6	857.143	685.714	0	0	450	5377.1429
12	857.143	514.286	0	0	450	5377.1429
16	857.143	400	0	0	450	5377.1429
20	857.143	285.714	0	0	450	5377.1429

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 30Q - 7000$, $q^* = 857.143$ and $Q^*_{TL} = 857.143$. The retailer's optimal order quantity and the expected profits of all parties given $R = 120$, are summarized in Table 6.3 for varying values of s.

For $R = 120$, there is no value of s that would put the LTL carrier in a profitable position. Any s value that is greater than or equal to 1.371 will have the retailer order from the TL carrier and yield zero expected profits for the LTL carrier. Therefore, in this case, there are alternative optimal values.

Table 6.4 Expected profits of the parties resulting from the retailer's optimal replenishment decision for various values of s and R = 180

s	$Q^*(R,s)$	z^*	Q_2	$\prod_{LTL}(Q^*(R,s),s)$	$\prod_{TL}(Q^*(R,s),R)$	$H(Q^*(R,s))$
1.2	822.857	822.857	72.857	-15.429	510	5209.1429
1.4	817.143	817.143	67.143	-3.143	510	5195.1429
1.6	811.429	811.429	61.429	6.857	510	5182.2857
1.8	805.714	805.714	55.714	14.571	510	5170.5714
2	800	800	50	20	510	5160
2.2	794.286	794.286	44.286	23.1432	510	5150.5714
2.36	789.714	789.714	39.714	24.011	510	5143.8514
2.37	789.429	789.429	39.429	24.0177	510	5143.4557
2.38	789.143	789.143	39.143	24.0173	510	5143.0629
2.4	788.571	788.571	38.571	23.9994	510	5142.2857
2.41	788.286	788.286	38.286	23.9833	510	5141.9014
2.54	784.571	784.571	34.571	23.2393	510	5137.1657
2.55	784.286	784.286	0	0	690	5137.1429
2.6	857.143	782.857	0	0	690	5137.1429
2.8	857.143	771.143	0	0	690	5137.1429
8	857.143	628.571	0	0	690	5137.1429
16	857.143	400	0	0	690	5137.1429
20	857.143	285.714	0	0	690	5137.1429

Now, let us consider the scenario where $R = 180$. Under the given parameters, we have $G(Q) = -0.0175Q^2 + 30Q - 7000$, $q^* = 857.143$ and $Q^*_{TL} = 857.143$. The retailer's optimal order quantity and the expected profits of all parties are summarized in Table 6.4 for varying values of s.

As it can be seen from Table 6.4, the optimal value of s that maximizes the LTL carrier's expected profits under the optimal response of the retailer given R = 180, is 2.37. As the value of s gets smaller, the quantity that is shipped using the LTL carrier, i.e., Q_2, increases. However, the expected profits of the LTL carrier decrease due to the decreasing marginal revenue from the shipment of a single unit. Table 6.4 also shows the impact of carefully deciding the value of s on the LTL carrier's expected profits. For example, instead of setting the value of s to 2, the LTL carrier can increase his/her expected profits by 20% by setting the value of s to 2.37. Another observation is that if $s \geq 2.55$, then the retailer does not use the services of the LTL carrier.

Now, let us consider the case where $R = 300$. Under the given parameters, we have $G(Q) = -0.0175Q^2 + 30Q - 7000$, $q^* = 857.143$ and $Q^*_{TL} = 750$. The retailer's optimal order quantity and the expected profits of all parties are summarized in Table 6.5 for varying values of s.

The optimal value of s that maximizes the LTL carrier's expected profits under the optimal response of the retailer given R = 300, is 2.37. Similar to our observations for R = 180, it can be observed that as the value of s gets smaller, the quantity that is shipped using the LTL carrier, i.e., Q_2, increases. However, the expected profits of the LTL carrier decrease due to the decreasing marginal revenue from the shipment of a single unit. In case of R = 300, the retailer does not use the services of the LTL carrier if $s \geq 3.75$.

When the results in Table 6.3, Table 6.4 and Table 6.5 are compared, it can be observed that as the value of per truck cost R increases, the expected profit of the TL carrier increases and those of the retailer decreases. The LTL carrier, on the other hand, has an increased chance of improving his/her expected profits. However, as R gets larger, after a certain point on, the optimal s value and the LTL carriers' expected profits at the optimal solution do not change. It can also be observed that the TL carrier's expected profits are nondecreasing in s.

Table 6.5 Expected profits of the parties resulting from the retailer's optimal replenishment decision for various values of s and R = 300

s	$Q^*(R,s)$	z^*	Q_2	$\prod_{LTL}(Q^*(R,s),s)$	$\prod_{TL}(Q^*(R,s),R)$	$H(Q^*(R,s))$
1.4	817.143	817.143	67.143	-3.143	870	4835.1429
1.6	811.429	811.429	61.429	6.857	870	4822.2857
1.8	805.714	805.714	55.714	14.571	870	4810.5714
2	800	800	50	20	870	4800
2.2	794.286	794.286	44.286	23.1432	870	4790.5714
2.36	789.714	789.714	39.714	24.011	870	4783.8514
2.37	789.429	789.429	39.429	24.0177	870	4783.4557
2.4	788.571	788.571	38.571	23.9994	870	4782.2857
2.41	788.286	788.286	38.286	23.9833	870	4781.9014
2.6	782.857	782.857	32.857	22.5712	870	4775.1429
2.8	777.143	777.143	27.143	18.8574	870	4769.1429
3	771.429	771.429	21.429	12.858	870	4764.2857
3.2	765.714	765.714	15.714	4.5708	870	4760.5714
3.4	760	760	10	-6	870	4758
3.6	754.286	754.286	4.286	-18.8564	870	4756.5714
3.72	750.857	750.857	0.857	-27.669	870	4756.2629
3.73	750.571	750.571	0.571	-28.4411	870	4756.2557
3.74	750.286	750.286	0.286	-29.2164	870	4756.2514
3.75	750	750	0	0	870	4756.25
3.8	750	748.571	0	0	870	4756.25
4	750	742.857	0	0	870	4756.25
6	750	685.714	0	0	870	4756.25
8	750	628.571	0	0	870	4756.25
10	750	571.429	0	0	870	4756.25
12	750	514.286	0	0	870	4756.25
16	750	400	0	0	870	4756.25
18	750	342.857	0	0	870	4756.25
20	750	285.714	0	0	870	4756.25

Figure 6.5 provides plots of the LTL carrier's expected profits under the optimal response of the retailer with respect to *s* for the three chosen values of *R*. It can be observed from the figure that for a given value of *R*, within the region where the retailer does business with the LTL carrier (i.e., ships some positive quantity), the LTL carrier's expected profits are strictly concave with respect to *s*. This strictly concave function as long as the retailer utilizes the services of the LTL carrier, has a unique maximizer. The specific value of *R* does not have an effect on the value of the unique maximizer, however, it only determines how soon the retailer stops using the services of the LTL carrier before the value of *s* is increased drastically. In Example 9, the value of the unique maximizer is 2.37. When R = 120, the retailer utilizes the LTL carrier only at small values of *s*. However, the resulting revenue for the LTL carrier at these *s* values does not justify his/her fixed costs. This further implies that the LTL carrier cannot compete with the TL carrier at R = 120. On the other hand, at R = 180 and R = 300, the maximizing value of the LTL carrier's expected profits, i.e., s = 2.37, is still a good option for the retailer in comparison to what the TL carrier charges. Therefore, the LTL carrier competes with the TL carrier and further maximizes his/her expected profits. It is important to note that, the structural characteristic of the LTL carrier's expected profit function herein is specific to uniform demand distribution and may not apply to other demand distributions.

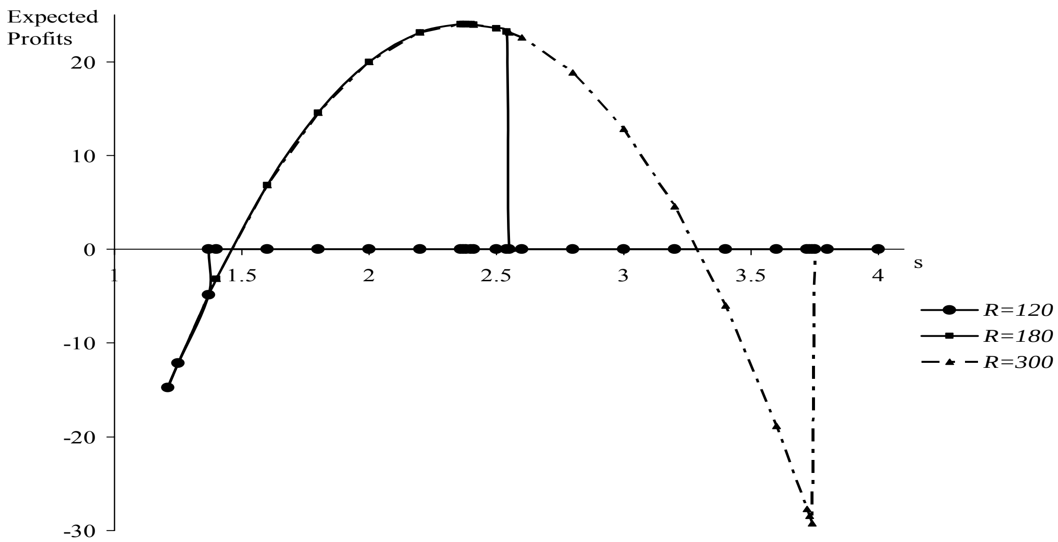

Figure 6.5 Plot of the LTL carrier's expected profits with respect to *s* for varying values of *R*

6.5 Coordination of Transportation Pricing Decisions Between a TL Carrier and an LTL Carrier

In earlier sections of this chapter, we have illustrated over numerical examples the savings that could be achieved by the TL carrier/LTL carrier due to transportation pricing decisions under the retailer's optimal response to a given transportation price schedule of the LTL carrier/TL carrier. In this section, we investigate the opportunity of savings that can be achieved if the TL carrier and the LTL carrier coordinate their transportation pricing decisions knowing the best response of the retailer. Example 10 will be used to support this suggestion.

In the following example setting, we will show that for the given value of s, there may be found a value of R which does not necessarily maximize the TL carrier's expected profits under the best response of the retailer, however it increases the sum of TL carrier's and LTL carrier's expected profits. Furthermore, this value of R is such that, the savings of the LTL carrier on top of what his/her expected profits would be under the TL carrier's first choice, exceeds the TL carrier's loss due to choosing this value. Therefore, by an appropriate agreement between the TL and the LTL carrier, the LTL carrier may be better off while compensating the TL carrier for his/her losses.

Example 10: Consider a setting with a retailer, a TL carrier and an LTL carrier with the following parameters: c = 19, r = 32, v = 11, b = 14, P = 150, $\overline{R}$ = 0, s = 2, $\bar{s}$ = 0.1, $K_{TL} = 100$, $K_{LTL} = 10$, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 27Q - 7000$, $z^* = 714.29$ and $q^* = 771.43$. It can be shown that the optimal value of R for the TL carrier without any coordination with the LTL carrier is approximately 277. At this value of R, the retailer orders 750 units with resulting expected profits as 2021.25. The TL carrier's and the LTL carrier's expected profits at R = 277 are 1285 and 0, respectively. If the TL carrier sets the value of R as 299 instead, the retailer's order size is 714.29 with resulting expected profits as 2732.57.

The TL carrier's and the LTL carrier's expected profits are 1096 and 207.15, respectively. If R = 299 instead of R = 277, the TL carrier's expected profits decrease by 189, however, the LTL carrier's expected profits and the retailer's expected profits increase by 207.15 and 711.32. Therefore, as shown in this example, there is an incentive for the retailer and the LTL carrier if the TL carrier increases the price he/she charges for each truck. Furthermore, the systemwide savings exceed the TL carrier's expected losses.

6.6 Coordination of Transportation Pricing Decisions Between a Retailer and a TL Carrier

In this section, we investigate the opportunity of coordination that can be achieved between a retailer and a TL carrier. Example 11 will be used to illustrate this situation.

Example 11: Consider a setting with a retailer and a TL carrier with the following parameters: c = 18, r = 32, v = 11, b = 14, P = 70, $\overline{R}$ = 0, $\overline{s} = 0.3$, $K_{TL} = 50$, $K_{LTL} = 10$, X = Uniform (0, 1000).

Under the given parameters, we have $G(Q) = -0.0175Q^2 + 28Q - 7000$ and $q^* = 800$. It can be shown that the TL carrier reaches the optimal profit value at R = 450, in which case his/her expected profits will be 4000 whereas the retailer's expected profits will be –355.75. However, in this case, the retailer may choose to exit the business since he/she is having some loss. If the TL carrier sets R = 400, then the retailer's and the TL carrier's expected profits will be 94.25 and 3550, respectively. A better solution may be realized when the TL carrier agrees to set his/her per truck cost to 203.75. In this case, without any agreement between the parties, the resulting expected profits will be 1987.5 for both the retailer and the TL carrier. However, the total expected profits amount to 3975, which is greater than those at R = 400 (i.e., 3544.25). The retailer can use his/her savings from an R value of 203.75 to compensate the TL carrier's loss. This would lead to 3550 as the TL carrier's expected profits and 425 as the retailer's expected profits.

Chapter 7

CONCLUSIONS

This thesis analyzes the replenishment and transportation pricing problems in a setting consisting of a retailer and two carriers (one truck-load carrier and one less-than-truckload carrier). In this setting, the retailer makes a single replenishment decision at the beginning of the selling period during which she/he faces random demand. The inbound replenishment quantity can be shipped utilizing both the TL carrier and the LTL carrier. The costs due to inbound transportation are incurred by the retailer. The TL carrier charges a fixed cost for each truck whether it is fully or partially loaded. The LTL carrier, on the other hand, charges in proportion to the number of units shipped. That is, a unit shipping price is charged by the LTL carrier. Under this transportation cost structure, the replenishment problem is defined as determining the retailer's optimal order quantity given the price schedules of the carriers. The transportation problem for the TL carrier is defined as finding the value of the per truck shipping price that maximizes the TL carrier's expected profits under the retailer's optimal response for given per unit shipping price of the LTL carrier. Similarly, the transportation problem for the LTL carrier is defined as determining the value of the per unit shipping price that maximizes the LTL carrier's expected profits under the retailer's optimal response for a fixed value of TL carrier's per truck shipping price.

The expected profit function of the retailer in the corresponding replenishment problem is piecewise, therefore, it cannot be maximized using the classical differentiation techniques. Chapter 4 of the thesis provides two characterizations of the solution for the retailer's expected profit function. The results in this chapter are based on a detailed analysis of the structural properties of the retailer's expected profit function. Part of the retailer's expected profit function that belongs to production/inventory related profits in the single period stochastic replenishment problem, namely the Newsboy Problem, is strictly concave. This is one property that is critical to the analysis in Chapter 4 and the succeeding chapters. Regarding the retailer's expected profit function excluding the transportation costs, the analysis in Chapter 4 utilizes the fact that this part of the function is strictly concave. Therefore, the analysis in Chapter 4 is generic and can be used in other problem settings where a supply chain entity that has strictly concave production/inventory related expected profits faces a transportation cost structure as in this thesis.

It is important to emphasize that, in comparison to the traditional systems where inventory replenishment decisions are made first followed by transportation decisions, the replenishment problem herein considers the inventory and transportation costs jointly. In recent supply chain research, it has been shown that substantial savings can be achieved due to such integrated models. However, the Newsboy Problem with two modes of transportation has not been studied in the literature.

Although the problem of maximizing the objective function as stated in Expression (7) corresponds to the replenishment problem of the retailer in this thesis, an important application area of the solution can be in outsourcing decisions. The trucking cost structure of the TL carrier is also known as the multiple set-up cost structure in the literature. It can also be used to model the production setup costs of a manufacturer who incurs a fixed setup cost for operating a capacitated machine which has the capability of processing several items at a time. The unit shipping price that the LTL carrier charges may be considered as the cost per unit if the manufacturer decides to outsource the items rather than manufacture inhouse. As a result of solving the problem with the objective of maximizing Expression (7), the manufacturer can end up with one of the following decisions in addition to determining the replenishment quantity: manufacture all items inhouse, outsource all items from a third party or a combination of both.

Transportation pricing is an unexplored area in supply chain management. The existing research on integrated inventory and transportation decisions assume that the transportation price schedules are already given. This thesis shows that the TL carrier and the LTL carrier may increase their expected profits substantially if they carefully make their transportation pricing decisions. The theoretical challenges behind solving the transportation pricing problems as defined in this thesis are as follows: First, the carrier in the corresponding transportation pricing problem has to come up with the optimal response of the retailer as a function of both his/her and the competing carrier's price schedule. However, due to the structural complexity of the retailer's expected profit function, the solution to his/her replenishment problem can only be characterized by a finite number of quantities that dominate other solutions (of infinite size), rather than a simple analytical expression. Furthermore, the retailer's expected profit function is piecewise and the decision variables in the transportation pricing problems not only affect the function value at a fixed quantity but they also affect the range of quantities where each function is defined. A contribution of this thesis is to propose alternative mathematical models for transportation pricing problems of the TL carrier and the LTL carrier using the detailed analysis of the retailer's optimization problem. Using these alternative models, the TL carrier's transportation pricing problem is solved exactly.

The opportunity of savings due to making transportation pricing decisions carefully is illustrated over several numerical examples both for the TL carrier and the LTL carrier. Furthermore, it is illustrated that the carriers, by coordinating their transportation pricing decisions, can increase the total supply chain profits. Also, it is shown that there may be a coordination possibility between the retailer and the TL carrier. These observations lead to several future research directions involving scenarios where two of the parties cooperate against the third, or all of the parties coordinate their decisions. The contractual agreements within this context constitute some topics of interest worthy of investigation.

The setting investigated in this thesis is a stylistic one that concerns two competing carriers that are in business with the same retailer. The objectives of this thesis can also be extended to other settings involving different characteristics of demand, selling period, number of replenishment opportunities, modes of carrier, transportation price structures, etc.

BIBLIOGRAPHY

[1] Abad, P., “Buyer’s response to a temporary price reduction incorporating freight costs,” to appear in *European Journal of Operational Research*.

[2] Aucamp, D., “Nonlinear freight costs in the EOQ problem,” *European Journal of Operational Research*, 9, (1982), 61-63.

[3] Banerjee, A., “A joint economical lot size model for purchaser and vendor,” *Decision Sciences*, 16 (1986a), 292-311.

[4] Banerjee, A., “On a quantity discount pricing model to increase vendor profits,” *Management Science*, 32, 11 (1986), 1513-1517.

[5] Benton, W.C., and Park, S., “A classification of literature on determining the lot size under quantity discounts,” *European Journal of Operational Research*, 92, 2 (1996), 219-238.

[6] Carter, J.R. and Ferrin, B.G., “Transportation costs and inventory management: Why transportation costs matter?,” *Production and Inventory Management Journal*, 37, 3 (1996), 58-62.

[7] Ertogral, K., Darwish, M. and Ben-Daya, M., “Production and shipment lot sizing in a vendor-buyer supply chain with transportation cost,” *European Journal of Operational Research*, 176, (2007), 1592-1606.

[8] Goyal, S.K., “An integrated inventory model for a single supplier-single customer problem,” *International Journal of Production Research*, 15, 1 (1977), 107-111.

[9] Goyal, S.K., "Comment on: A generalized quantity discount pricing model to increase supplier's profit," *Management Science,* 33, 2 (1987), 1635-1636.

[10] Joglekar, P.N., "Comments on a quantity discount pricing model to increase vendor profits," *Management Science,* 34, 11 (1988), 1391-1400.

[11] Lee, C.Y., "The economic order quantity for freight discount costs," *IIE Transactions*, 18, (1986), 318-320.

[12] Lee, H.L., and Rosenblatt, J., "A generalized quantity discount pricing model to increase supplier's profits," *Management Science,* 33, 9 (1986), 1167-1185.

[13] Logistical Magazine, **"**Küreselleşme ve Taşımacılık Sorunları," *Logisticsclub.com*, 2006, 1 July 2008, <http://www.logisticsclub.com/modules.php?name=News&file=print&sid=144/>.

[14] Mendoza, A. and Ventura, J.A., "Incorporating quantity discounts to the EOQ model with transportation costs," to appear in *International Journal of Production Economics*.

[15] Monahan, J.P., "A quantity pricing model to increase vendor profits," *Management Science*, 30, 6 (1984), 720-726.

[16] Mueller, R., "Technical Assistance to Transportation Infrastructure Needs Assessment for Turkey," *Republic of Turkey Ministry of Transport*, 2007, 1 July 2008, <www.ubak.gov.tr/op/doc/TINA_Turkey-Final_Report.pdf/>.

[17] Pasternack, B.A., "Optimal pricing and return policies for perishable commodities," *Marketing Science*, 4, 2 (1985), 166-176.

[18] Rieksts, B.Q. and Ventura, J.A., "Optimal inventory policies with two modes of freight transportation," *European Journal of Operational Research*, 186, (2008), 576-585.

[19] Russell, R.M. and Krajewski, L.J., “Optimal purchase and transportation cost lot sizing for a single item,” *Decision Sciences*, 22, (1991), 940-952.

[20] Tersine, R. and Barman, S., “Optimal lot sizes for unit and shipping discount situations,” *IIE Transactions*, 26, 2 (1994), 97-101.

[21] Toptal, A. and Çetinkaya S., “Contractual agreements for coordination and vendor managed delivery under explicit transportation considerations,” *Naval Research Logistics*, 53, (2006), 1-21.

[22] Toptal, A., “Replenishment decisions under an all-units quantity discount schedule and stepwise freight costs,” *European Journal of Operational Research*, (2008), doi:10.1016/j.ejor.2008.09.037.

[23] “US Freight Shipping Costs Beyond 10% of GDP,” *FreightClick.com*, 2008, 1 July 2008, <http://www.freightclick.com/blog/freight-usa-2008-06-26-us-freight-shipping-costs-beyond-10-of-gdp.html/>.

[24] Weng, Z.K. and Wong, R.T., “General models for the supplier’s all-unit quantity discount policy,” *Naval Research Logistics*, 40, (1993), 971-991.

APPENDIX

A.1 Proof of Lemma 9

In proving the lemma, we will use the mathematical formulation in *Problem P*1 and Lemma 7. We know from Proposition 2 that under the constraints of *Problem P1*, q^* is optimal. It follows from Lemma 7 that, q^* is either realizable, or is an integer number full truck loads. In both cases, the TL carrier's objective function will be given by $\Pi_{TL}(q^*, R) = \left\lceil \frac{q^*}{P} \right\rceil (R - \bar{R}) - K_{TL}$. If we execute the constraints in *Problem P*1, it reduces to

$$\max \quad \left\lceil \frac{q^*}{P} \right\rceil (R - \bar{R}) - K_{TL}$$

s.t.

$$R \times \left(\left\lceil \frac{q^*}{P} \right\rceil - \left\lfloor \frac{z^*}{P} \right\rfloor \right) \leq G(q^*) - G(z^*) + sz^* - sP \left\lfloor \frac{z^*}{P} \right\rfloor,$$

$$R \times \left(\left\lceil \frac{q^*}{P} \right\rceil - k \right) \leq G(q^*) - G(kP), \qquad k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor.$$

$$R < sP,$$

$$R \geq 0.$$

Note that, the objective function of the TL carrier is increasing in the value of R. Therefore, the largest value in the feasible region, as characterized by the constraints of the above formulation, should give us $R^*(q^*)$. We have from Lemma 2 that $z^* < q^*$. This in turn implies that $\left\lceil \frac{q^*}{P} \right\rceil > \left\lfloor \frac{z^*}{P} \right\rfloor$, and hence, the left side of the first constraint is positive. Since q^* is the unique maximizer of $G(Q)$, we further have $G(q^*) > G(z^*)$. Combining this with the fact that $sz^* \geq sP \left\lfloor \frac{z^*}{P} \right\rfloor$, we conclude that the right side of the first constraint is also positive. Therefore, the first constraint yields the following upper bound on the solution:

$$R \le \frac{G(q^*) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{\left\lceil \frac{q^*}{P} \right\rceil - \left\lfloor \frac{z^*}{P} \right\rfloor}.$$

Now, let us consider the second constraint. Since $G(Q)$ is a strictly concave function with a unique maximizer q^* and $kP \le q^*,$ it follows that $G(q^*) \ge G(kP)$ for any integer k such that $\left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor$. The fact that $kP \le q^*,$ further yields to $\left\lceil \frac{q^*}{P} \right\rceil \ge k$. If for some integer k we have $kP = q^*$, then the second constraint is trivially satisfied. Otherwise, it implies the following upper bound

$$R \le \frac{G(q^*) - G(kP)}{\left\lceil \frac{q^*}{P} \right\rceil - k}.$$

Finally, we have $R < sP$ as stated in the third constraint. Therefore, the largest value of R in the feasible region is given by

$$\min\left\{ \frac{G(q^*) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{\left\lceil \frac{q^*}{P} \right\rceil - \left\lfloor \frac{z^*}{P} \right\rfloor}, sP - \varepsilon, \frac{G(q^*) - G(kP)}{\left\lceil \frac{q^*}{P} \right\rceil - k}, \forall k \text{ } s.t. \left\lceil \frac{z^*}{P} \right\rceil \le k \le \left\lfloor \frac{q^*}{P} \right\rfloor \right\}$$

where ε is a positive real number and $kP \ne q^*$ for the integer values of k that are in consideration.

■

A.2 Proof of Lemma 10

In proving the lemma, we will use the mathematical formulation in *Problem P*2 and Lemma 8. We know from Proposition 3 that under the constraints of *Problem P2*, z^* is optimal. It follows from Lemma 8 that z^* is therefore realizable. The TL carrier's objective function is then given by $\prod_{TL}(z^*, R) = \left\lfloor \frac{z^*}{P} \right\rfloor (R - \bar{R}) - K_{TL}$. If we execute the constraints in *Problem P*2, it reduces to

$$\max \quad \left\lfloor \frac{z^*}{P} \right\rfloor (R - \bar{R}) - K_{TL}$$

s.t.

$$R \times \left(\left\lfloor \frac{z^*}{P} \right\rfloor - \left\lceil \frac{q^*}{P} \right\rceil \right) \le G(z^*) - G(q^*) - sz^* + sP \left\lfloor \frac{z^*}{P} \right\rfloor,$$

$$R \times \left(\left\lfloor \frac{z^*}{P} \right\rfloor - k \right) \le G(z^*) - G(kP) - sz^* + sP \left\lfloor \frac{z^*}{P} \right\rfloor,$$

$$k = \left\lceil \frac{z^*}{P} \right\rceil, \ldots, \left\lfloor \frac{q^*}{P} \right\rfloor.$$

$$R < sP,$$
$$R \ge 0.$$

Note that, the objective function of the TL carrier is increasing in the value of *R*. Therefore, the largest value in the feasible region, as characterized by the constraints of the above formulation, should give us $R^*(z^*)$. Since $\left\lceil \frac{q^*}{P} \right\rceil > \left\lfloor \frac{z^*}{P} \right\rfloor$, the left side of the first constraint is negative. As it has been shown in the proof of Lemma 9, the right side of the first constraint is also negative. Therefore, the first constraint yields the following lower bound on the solution:

$$R \geq \frac{G(q^*) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{\left\lceil \frac{q^*}{P} \right\rceil - \left\lfloor \frac{z^*}{P} \right\rfloor}.$$

Now, let us consider the second constraint. Since $G(Q)$ is a strictly concave function with a unique maximizer q^* and $\left\lceil \frac{z^*}{P} \right\rceil \leq k \leq \left\lfloor \frac{q^*}{P} \right\rfloor$, it follows that we have $G(z^*) \leq G(kP)$ for any integer k within the specified range. Combining this with the fact that $sz^* \geq sP\left\lfloor \frac{z^*}{P} \right\rfloor$, we conclude that the right side of the second inequality is not positive. Since $\left\lfloor \frac{z^*}{P} \right\rfloor \geq k$, the left side of this constraint is not positive either. Note also that, if for some integer k we have $kP = z^*$, then the second constraint is trivially satisfied. Otherwise, it implies the following lower bound:

$$R \geq \frac{G(kP) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{k - \left\lfloor \frac{z^*}{P} \right\rfloor}.$$

If any of the following lower bounds exceeds the upper bound given by the third constraint, that is $R < sP$, then there is no feasible R value that leads to z^* as the optimal replenishment quantity of the retailer. Otherwise, we should choose the largest value of R in the feasible region, which is given by $sP - \varepsilon$ where ε is a positive real number. ■

A.3 Proof of Lemma 11

In proving the lemma, we will use the mathematical formulation in *Problem P*3. We know from Proposition 4 that under the constraints of *Problem P3*, *kP* is optimal. The TL carrier's objective function is then given by $\Pi_{TL}(kP,R)=k\left(R-\bar{R}\right)-K_{TL}$. If we execute the constraints in *Problem P*3, it reduces to

$$\max \quad k\left(R-\bar{R}\right)-K_{TL}$$

s.t.

$$R\times\left(k-\left\lceil\frac{q^*}{P}\right\rceil\right)\le G(kP)-G(q^*),$$

$$R\times\left(k-\left\lfloor\frac{z^*}{P}\right\rfloor\right)\le G(kP)-G(z^*)+sz^*-sP\left\lfloor\frac{z^*}{P}\right\rfloor,$$

$$R\times(k-j)\le G(kP)-G(jP), \qquad j=(k+1),\ldots,\left\lfloor\frac{q^*}{P}\right\rfloor.$$

$$R\times(k-j)\le G(kP)-G(jP), \qquad j=\left\lceil\frac{z^*}{P}\right\rceil,\ldots,(k-1).$$

$$R<sP,$$
$$R\ge 0.$$

The objective function of the TL carrier is increasing in the value of *R*. Therefore, the largest value in the feasible region, as characterized by the constraints of the above formulation, should give us $R^*(kP)$. Since k is an integer such that $\left\lceil\frac{z^*}{P}\right\rceil\le k\le\left\lfloor\frac{q^*}{P}\right\rfloor$, we have $k\le\left\lfloor\frac{q^*}{P}\right\rfloor$. Therefore, the left side of the first constraint is not positive. Combining $k\le\left\lfloor\frac{q^*}{P}\right\rfloor$ with the fact that $G(Q)$ is a strictly concave function with a unique maximizer q^*, we conclude that the right side of the first constraint is also not positive. If $kP = q^*$, the first constraint is trivially satisfied. Otherwise, it implies the following lower bound:

$$R \geq \frac{G(q^*) - G(kP)}{\left\lceil \frac{q^*}{P} \right\rceil - k}.$$

Now, let us consider the second constraint. Since *k* is an integer such that $\left\lceil \frac{z^*}{P} \right\rceil \leq k \leq \left\lfloor \frac{q^*}{P} \right\rfloor$, we have $k \geq \left\lfloor \frac{z^*}{P} \right\rfloor$. Therefore, the left side of the second constraint is nonnegative. Since q^* is the unique maximizer of the strictly concave function $G(Q)$ and $k \geq \left\lfloor \frac{z^*}{P} \right\rfloor$, we further have $G(kP) \leq G(z^*)$. Combining this with the fact that $sz^* \geq sP\left\lfloor \frac{z^*}{P} \right\rfloor$, we conclude that the right side of the second constraint is also nonnegative. If $kP = z^*$, the second constraint is trivially satisfied. Otherwise, it yields the following upper bound on the solution:

$$R \leq \frac{G(kP) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{k - \left\lfloor \frac{z^*}{P} \right\rfloor}.$$

It can also be observed that the third constraint implies the following lower bound for $j = (k+1),\ldots,\left\lfloor \frac{q^*}{P} \right\rfloor$.

$$R \geq \frac{G(jP) - G(kP)}{j - k}.$$

Finally, the fourth constraint implies the following upper bound $j = \left\lceil \frac{z^*}{P} \right\rceil,\ldots,(k-1)$.

$$R \leq \frac{G(kP) - G(jP)}{k - j}.$$

If any of the lower bounds exceeds the upper bounds including the one provided by the fifth constraint, then there is no feasible R value that leads to kP as the maximizer of the TL carrier's expected profits. Otherwise, we choose the largest value of R in the feasible region, which is given by

$$\min\left\{\frac{G(kP) - G(z^*) + sz^* - sP\left\lfloor \frac{z^*}{P} \right\rfloor}{k - \left\lfloor \frac{z^*}{P} \right\rfloor}, \lim_{\varepsilon \to 0}(sP - \varepsilon), \frac{G(kP) - G(jP)}{k - j}, \forall j \ s.t. \left\lceil \frac{z^*}{P} \right\rceil \leq j \leq k - 1\right\}$$

.

where ε is a positive real number. ■