T.C.
YOZGAT BOZOK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ
MATEMATİK ANABİLİM DALI

BAZI İNTEGRASYON ALGORİTMALARI İLE LPD MODELİNE OPTİK SOLİTONLAR

YASEMİN KORKMAZ BEŞCANLAR

YÜKSEK LİSANS TEZİ

Danışman: Prof. Dr. Abdullah SÖNMEZOĞLU

HAZİRAN – 2024

YOZGAT

T.C.
YOZGAT BOZOK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ
MATEMATİK ANABİLİM DALI

BAZI İNTEGRASYON ALGORİTMALARI İLE LPD MODELİNE OPTİK SOLİTONLAR

YASEMİN KORKMAZ BEŞCANLAR

YÜKSEK LİSANS TEZİ

Danışman: Prof. Dr. Abdullah SÖNMEZOĞLU

HAZİRAN –2024

YOZGAT

	YOZGAT BOZOK ÜNİVERSİTESİ **LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ** **LİSANSÜSTÜ TEZ ONAY FORMU**

T.C.

YOZGAT BOZOK ÜNİVERSİTESİ

LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

Enstitümüzün Matematik Anabilim Dalı Tezli Yüksek Lisans öğrencisi Yasemin KORKMAZ BEŞCANLAR' ın hazırladığı "**Bazı İntegrasyon Algoritmaları İle LPD Modeline Optik Solitonlar"** başlıklı tezi ile ilgili tez savunma sınavı, Lisansüstü Eğitim-Öğretim ve Sınav Yönetmeliği'nin ilgili maddeleri gereğince 26/06/2024 Çarşamba günü saat 11:00'da yapılmış, tezin onayına oy birliği ile karar verilmiştir.

Başkan **:** Prof. Dr. Akın Osman ATAGÜN

Jüri Üyesi **:** Prof. Dr. Abdullah SÖNMEZOĞLU

(Danışman)

Jüri Üyesi **:** Doç. Dr. Mehmet EKİCİ

ONAY:

Bu tezin kabulü, Enstitü Yönetim Kurulu'nun ……../……../….. tarih ve …… sayılı Enstitü Yönetim Kurulu Kararı ile onaylanmıştır.

……../……../……..

Prof. Dr. Ümit BUDAK

Lisansüstü Eğitim Enstitüsü Müdürü

TEZ BEYANI

Tez yazım kurallarına uygun olarak hazırlanan bu tezin yazılmasında bilimsel ahlak kurallarına uyulduğunu, başkalarının eserlerinden yararlanılması durumunda bilimsel normlara uygun olarak atıfta bulunulduğunu, tezin içerdiği yenilik ve sonuçların başka bir yerden alınmadığını, kullanılan verilerde herhangi bir tahrifat yapılmadığını, tezin herhangi bir kısmının bu üniversite veya başka bir üniversitedeki başka bir tez çalışması olarak sunulmadığını beyan eder, aksi bir durumda aleyhime doğabilecek tüm hak kayıplarını kabullendiğimi beyan ederim.

Yasemin KORKMAZ BEŞCANLAR

26/06/2024

ÖN SÖZ

Değerli görüş ve önerileriyle çalışmamı yönlendiren, özverili çalışma anlayışını örnek aldığım, ayrıca tez çalışmamın bu aşamaya gelmesinde göstermiş olduğu hoşgörü ve sabırdan dolayı çok değerli danışman hocam Prof. Dr. Abdullah SÖNMEZOĞLU'na ve çalışmamın araştırma kısmında bilgilerini ve yardımlarını esirgemeyen Doç. Dr. Mehmet EKİCİ'ye, ayrıca Prof. Dr. Akın Osman ATAGÜN'e ve Dr. Öğr. Üyesi Gökhan ÇELEBİ'ye desteklerinden dolayı teşekkürlerimi sunarım.

Bu günleri göremese de, beni hayatı boyunca destekleyen, bana güvenen, azmi ve çalışkanlığı ile bana ilham olan, kızı olmaktan gurur duyduğum rahmetli babam Hüseyin KORKMAZ'a her daim yanımda olan, bana güç veren, maddi ve manevi yardımları için minnet duyduğum canım annem Münire KORKMAZ'a, bu süreçteki yardım ve destekleri için sevgili eşim Soner BEŞCANLAR'a ve doğduğu andan itibaren hayatıma renk, enerji ve anlam kazandıran biricik oğlum Doğu Berk BEŞCANLAR'a teşekkür ederim.

Yasemin KORKMAZ BEŞCANLAR

26/06/2024

ÖZET

YÜKSEK LİSANS TEZİ

BAZI İNTEGRASYON ALGORİTMALARI İLE LPD MODELİNE OPTİK SOLİTONLAR

YASEMİN KORKMAZ BEŞCANLAR

YOZGAT BOZOK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ
MATEMATİK ANABİLİM DALI

TEZ DANIŞMANI: PROF. DR. ABDULLAH SÖNMEZOĞLU

Bu çalışma ilk olarak Kerr yasası nonlineerliği için Lakshmanan-Porsezian-Daniel denkleminin koyu ve tekil optik soliton çözümlerini iki entegrasyon şeması kullanarak elde etmektedir. Bunlar Jacobi'nin genişletilmiş eliptik fonksiyon yaklaşımı ve $exp(-\phi(\eta))$-genişleme yöntemidir. Bu solitonlar için kısıtlama koşulları olarak sunulan varoluş kriterleri de elde edilmiştir. Bir yan ürün olarak, periyodik tekil çözümler elde edilmiş ve bunlar da çalışmada listelenmiştir. Daha sonra vektör bağlantılı Lakshmanan-Porsezian-Daniel denklemini genişletilmiş Jacobi eliptik fonksiyon açılım şeması yardımıyla çift kırılımlı fiberlerde entegre etmektedir. Jacobi çift periyodik dalga çözümleri elde edilmiştir. Bu entegrasyon mekanizması ile elde edilen çözümler, sınırlayıcı durumda, karanlık solitonlara veya tekil solitonlara veya periyodik çözümlere yol açar.

2024, ix + 66 Sayfa

Anahtar Kelimeler: Optik Solitonlar, Lakshmanan-Porsezian-Daniel denklemi, Kerr yasası nonlineerliği, Genişletilmiş Jacobi fonksiyon.

ABSTRACT

MASTER THESIS

OPTİK SOLİTONS TO LPD MODEL BY SOME INTEGRATİON ALGORİTHMS

YASEMİN KORKMAZ BEŞCANLAR

YOZGAT BOZOK UNIVERSITY
SCHOOL OF GRADUATE STUDIES
DEPARTMENT OF MATHEMATICS

SUPERVISOR: PROF. DR. ABDULLAH SÖNMEZOĞLU

This study first obtains the dark and singular optical soliton solutions of the Lakshmanan-Porsezian-Daniel equation for Kerr's law nonlinearity by using two integration schemes. These are Jacobi's extended elliptic function approximation and exp(-ϕ(η))-expansion method. Existence criteria, which are presented as constraint conditions for these solitons, have also been obtained. As a by-product, periodic singular solutions were obtained and these are also listed in the study. It then integrates the vector-linked Lakshmanan-Porsezian-Daniel equation in birefringent fibers with the help of the extended Jacobi elliptic function expansion scheme. Jacobi double periodic wave solutions were obtained. Solutions obtained by this integration mechanism lead to dark solitons or singular solitons or periodic solutions in the limiting case.

2024, ix + 66 Page

Keywords: Optical solitons, Lakshmanan-Porsezian-Daniel equation, Kerr's law nonlinearity, Extended Jacobi function.

İÇİNDEKİLER

Sayfa

TEZ ONAY FORMU ii

TEZ BEYANI iii

ÖNSÖZ iv

ÖZET v

ABSTRACT vi

İÇİNDEKİLER vii

SİMGELER VE KISALTMALAR LİSTESİ viii

1.GİRİŞ 1

2. BAZI İNTEGRASYON ALGORİTMALARI İLE LPD MODELİNE OPTİK SOLİTONLAR 3

2.1. Ana Model 3

2.2. Matematiksel Analiz 3

2.3. Genişletilmiş Jacobi Fonksiyon Genişleme Şeması 8

2.4. $exp(-(\xi))$ açılım metodu 22

3. GENİŞLETİLMİŞ JACOBİ ELİPTİK FONKSİYON AÇILIM ŞEMASI İLE LAKSHMANAN-PORSEZİAN-DANİEL MODELİ İÇİN ÇİFT KIRILIMLI FİBERLERDE OPTİK SOLİTONLAR 35

3.1. Yönetim Modeli 35

3.2. Matematiksel Önbilgiler 35

3.3. Genişletilmiş Jacobi Eliptik Fonksiyon Açılım Şeması 44

4. SONUÇ 63

5. KAYNAKLAR 64

SİMGELER VE KISALTMALAR LİSTESİ

Bu çalışmada kullanılmış simgeler ve kısaltmalar, açıklamaları ile birlikte aşağıda sunulmuştur.

Simgeler	**Açıklamalar**
α	Alpha
β	Beta
θ	Theta
γ	Gamma
δ	Delta
ε	Epsilon
η	Eta
κ	Kappa
λ	Lambda
ξ	Xi
ρ	Rho
σ	Sigma
τ	Tau
ϕ	Phi
ω	Omega
$\lvert q \rvert$	q nun modülü
q^*	q nun eşleniği

Kısaltmalar	**Açıklamalar**
LPD	Lakshmanan-Porsezian-Daniel
STD	Uzaysal-zamansal dağılım

JEF	Jacobi Eliptik Fonksiyon
sn	Eliptik sinüs fonksiyonu
cn	Eliptik kosinüs fonksiyonu
dn	Üçüncü tip eliptik fonksiyon

1.GİRİŞ

Optik solitonlar, günümüz elektronik iletişim sisteminde devrim niteliğinde bir etki yaratan telekomünikasyon mühendisliğinin temel dokusunu oluşturmaktır. Facebook, Twitter, elektronik posta, internet, cep telefonu ve benzeri diğer iletişim araçları soliton yayılımı kavramı olmadan düşünülemez. Bu nedenle, bu soliton dinamiklerini bütünleşebilirlik sorunu, korunum yasaları, pertürbasyon şeması, stokastiklik ve diğerleri gibi çeşitli açılardan incelemek zorunludur (Abdou ve Elhanbaly, 2007; Bhrawy vd., 2013; Alqahtani vd., 2018; Bansal vd., 2018; Biswas vd., 2018; Guzman vd., 2017; Hubert vd., 2018; Jawad vd., 2018; Huiqun vd., 2007; Khan vd.,2013; Lakshmanan vd., 1988; Liu vd., 2016; Manafian vd., 2017; Ekici, 2018; Mirzazadeh vd., 2016 ; Roshid vd., 2014; Sonmezoglu, 2015; Taghizadeh vd., 2017). Bu çalışma, Lakshmanan-Porsezian-Daniel (LPD) denklemi olarak adlandırılan iyi bilinen bir modelin integrallenebilirlik yönünü incelemektedir. Bu model ilk olarak 1988 yılında Heisenburg spin zinciri bağlamında ortaya çıkmıştır (Lakshmanan vd., 1988). Daha sonra bu model fiber optik de dahil olmak üzere çeşitli bağlamlarda yaygın olarak çalışılmıştır. Bu, iki entegrasyon kullanılarak Kerr yasası nonlineerliği ile ele alınmıştır. Bu model için koyu ve tekil soliton çözümler elde edilmiş ve bu çözümlerin kısıt koşulları olarak sunulan varlık kriterleri de elde edilmiştir. Ayrıntıların tümü sonraki bölümlerde sıralanmıştır.

Optik solitonlar, doğrusal olmayan fiber optik alanındaki en önemli araştırma alanlarından biridir (Abdou ve Elhanbaly, 2007; Sonmezoglu, 2015). Bu bağlamda optik fiberler ve PCF üzerinden darbe iletiminin dinamiklerini yöneten Lakshmanan-Porsezian-Daniel (LPD) modeli son birkaç yılda oldukça popülerlik kazanmıştır. Bu modele solitonları ve soliter dalgaları ortaya çıkarmak için uygulanan birçok güçlü entegrasyon algoritması olduğunu belirtmek gerekir (Alqahtani vd., 2018; Bansal vd., 2018; Biswas vd., 2018; Guzman vd., 2017; Hubert vd., 2018; Jawad vd., 2018; Lakshmanan vd., 1988; Liu vd., 2016; Manafian vd., 2017; Ekici, 2018). Bu çalışma, çift kırılmayı tanımlayan vektör bileşenli LPD modeli ile polarizasyon modu dispersiyonu (PMD) fiberleri için soliton çözümlerinin çalışmasını ele alacaktır. Son zamanlarda, LPD'nin bu birleşik formu, belirlenmemiş katsayılar yöntemi, $\exp(-\phi(\xi))$-genişleme yaklaşımı ve genişletilmiş deneme denklemi şeması gibi birkaç güçlü entegrasyon algoritması kullanılarak çalışılmıştır (Arshed vd., 2018; Biswas vd.,2018; Guzmanvd., 2018). Ancak bu çalışmada, yönetim modeli için tam solitonları elde etmek için eski Jacobi eliptik fonksiyon (JEF) genişleme şeması benimsenecektir. Detaylar ilerleyen bölümlerde gösterilmektedir.

2. BAZI İNTEGRASYON ALGORİTMALARI İLE LPD MODELİNE OPTİK SOLİTONLAR

2.1. Ana Model

Bu çalışmada ele alınacak olan Kerr nonlineer ve uzaysal-zamansal dağılım (STD) içeren LPD modelinin boyutsuz formu (Alqahtani vd., 2018; Lakshmanan vd., 1988; Biswas vd., 2018; Guzman vd., 2017; Manafian vd., 2017) tarafından verilmiştir.

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2q = \sigma q_{xxxx} + \alpha(q_x)^2q^* + \beta|q_x|^2q + \gamma|q|^2q_{xx} + \lambda q^2q^*_{xx} + \delta|q|^4q \quad (2.1)$$

Bu sistemde bağımlı değişken $q(x,t)$ karmaşık değerli dalga fonksiyonunu temsil ederken bağımsız değişkenler sırasıyla uzayı ve zamanı temsil eden x ve t dir. Sol taraftaki ilk terim doğrusal olmayan dalganın zamansal evrimini temsil ederken, a katsayısı grup hızı dağılımı ve b , STD'dir. Sağ tarafta, σ dördüncü mertebeden dağılım ve δ, iki-foton emilimidir. Bir sonraki bölüm, (2.1) modelinin nonlineer dalga çözümlerini oluşturmak için temel mekanizmayı sağlamaktadır.

2.2. Matematiksel Analiz

(2.1) denkleminin integrallenebilirlik şemasına başlamak için, başlangıç hipotezi (Alqahtani vd., 2018; Biswas vd., 2018; Guzman vd., 2017) tarafından verilen hareket eden dalga argümanıdır.

$$q(x,t) = P(\xi)e^{i(-\kappa x+\omega t+\theta_0)} \quad (2.2)$$

Dalga değişkeni v solitonun hızı olmak üzere,

$$\xi = B(x - vt) \quad (2.3)$$

şeklindedir. Burada $P(\xi)$, dalga profilinin şeklini temsil eder ve faz bileşeni $\phi(x,t)$, aşağıdaki gibi tanımlanır;

$$\phi(x,t) = -\kappa x + \omega t + \theta_0 \quad (2.4)$$

Burada κ soliton frekansı, ω solitonun dalga sayısı, θ_0 faz sabitidir. (2.2), (2.3) ve (2.4) denklemleri kullanılarak (2.1) denkleminde bulunan q_t, q_{xx}, q_{xt}, $|q|^2$, q_{xxxx}, $(q_x)^2$, q^*, $|q_x|^2$, q^2, q^*_{xx}, $|q|^4$ terimleri aşağıdaki gibi hesaplanabilir:

$$q_t = -BvP'e^{i(-\kappa x+\omega t+\theta_0)} + i\omega Pe^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_t = (-BvP' + i\omega P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$iq_t = (-iBvP' - \omega P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_x = BP'e^{i(-\kappa x+\omega t+\theta_0)} - i\kappa Pe^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_x = (BP' - i\kappa P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xx} = (B^2P'' - i\kappa BP')e^{i(-\kappa x+\omega t+\theta_0)} + (BP' - i\kappa P)(-i\kappa)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xx} = (B^2P'' - i\kappa BP' - i\kappa BP' - \kappa^2 P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xx} = (B^2P'' - 2i\kappa BP' - \kappa^2 P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$aq_{xx} = a(B^2P'' - 2i\kappa BP' - \kappa^2 P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xt} = [B(-Bv)P'' - i\kappa(-Bv)P']e^{i(-\kappa x+\omega t+\theta_0)} + (BP' - i\kappa P)(i\omega)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xt} = (-B^2vP'' + i\kappa BvP' + i\omega BP' + \omega\kappa P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$bq_{xt} = b(-B^2vP'' + i\kappa BvP' + i\omega BP' + \omega\kappa P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$|q| = |P|\left|e^{i\varphi}\right|$$

$$|q| = |P|$$

$P > 0$ iken $|q| = P$

$$|q|^2 = P^2$$

$$c|q|^2q = cP^2Pe^{i(-\kappa x+\omega t+\theta_0)} = cP^3e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxx} = (B^3P''' - 2i\kappa B^2P'' - \kappa^2BP')e^{i(-\kappa x+\omega t+\theta_0)}$$
$$+(B^2P'' - 2i\kappa BP' - \kappa^2P)(-\dot{I}\kappa)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxx} = (B^3P''' - 2i\kappa B^2P'' - \kappa^2BP' - iB^2\kappa P'' - 2\kappa^2BP' + i\kappa^3P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxx} = (B^3P''' - 3i\kappa B^2P'' - 3\kappa^2BP' + i\kappa^3P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxxx} = \left(B^4P^{(4)} - 3i\kappa B^3P''' - 3\kappa^2B^2P'' + i\kappa^3BP'\right)e^{i(-\kappa x+\omega t+\theta_0)} + (B^3P''' - 3i\kappa B^2P''$$
$$-3\kappa^2BP' + i\kappa^3P)(-i\kappa)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxxx} = (B^4P^{(4)} - 3i\kappa B^3P''' - 3\kappa^2B^2P'' + i\kappa^3BP' - i\kappa B^3P''' - 3\kappa^2B^2P'' + 3i\kappa^3BP'$$
$$+i\kappa^4P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xxxx} = \left(B^4P^{(4)} - 4i\kappa B^3P''' - 6\kappa^2B^2P'' + 4i\kappa^3BP' + i\kappa^4P\right)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$\sigma q_{xxxx} = \sigma\left(B^4P^{(4)} - 4i\kappa B^3P''' - 6\kappa^2B^2P'' + 4i\kappa^3BP' + i\kappa^4P\right)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$(q_x)^2 = \left[(BP' - i\kappa P)e^{i(-\kappa x+\omega t+\theta_0)}\right]^2$$

$$(q_x)^2 = (B^2(P')^2 - 2iB\kappa P'P - \kappa^2P^2)e^{2i(-\kappa x+\omega t+\theta_0)}$$

$$q^* = Pe^{-i(-\kappa x+\omega t+\theta_0)}$$

$$\alpha(q_x)^2q^* = \alpha(B^2(P')^2 - 2iB\kappa P'P - \kappa^2P^2)e^{2i(-\kappa x+\omega t+\theta_0)}Pe^{-i(-\kappa x+\omega t+\theta_0)}$$

$$\alpha(q_x)^2q^* = \alpha(B^2(P')^2P - 2iB\kappa P'P^2 - \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$|q_x| = |BP' - i\kappa P|\left|e^{i(-\kappa x+\omega t+\theta_0)}\right|$$

$$|q_x| = \sqrt{B^2(P')^2 + \kappa^2P^2}$$

$$|q_x|^2 = B^2(P')^2 + \kappa^2P^2$$

$$\beta|q_x|^2q = \beta(B^2(P')^2 + \kappa^2P^2)Pe^{i(-\kappa x+\omega t+\theta_0)} = \beta(B^2(P')^2P + \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$\gamma|q|^2q_{xx} = \gamma P^2(B^2P'' - 2i\kappa BP' - \kappa^2P)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$\gamma|q|^2q_{xx} = \gamma(B^2P''P^2 - 2i\kappa BP'P^2 - \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$q_{xx}^* = (B^2P'' - \kappa^2P + 2i\kappa BP')e^{-i(-\kappa x+\omega t+\theta_0)}$$

$$q = Pe^{i(-\kappa x+\omega t+\theta_0)}$$

$$q^2 = P^2e^{2i(-\kappa x+\omega t+\theta_0)}$$

$$\lambda q^2q_{xx}^* = \lambda\left(P^2e^{2i(-\kappa x+\omega t+\theta_0)}\right)(B^2P'' - \kappa^2P + 2i\kappa BP')e^{-i(-\kappa x+\omega t+\theta_0)}$$

$$\lambda q^2q_{xx}^* = \lambda(B^2P''P^2 - \kappa^2P^3 + 2i\kappa BP'P^2)e^{i(-\kappa x+\omega t+\theta_0)}$$

$$|q| = P \quad \Rightarrow \quad |q|^4 = P^4$$

$$\delta|q|^4q = \delta P^4Pe^{i(-\kappa x+\omega t+\theta_0)} = \delta P^5e^{i(-\kappa x+\omega t+\theta_0)}$$

Bulunan sonuçlar,

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2q = \sigma q_{xxxx} + \alpha(q_x)^2q^* + \beta|q_x|^2q + \gamma|q|^2q_{xx} + \lambda q^2q_{xx}^* + \delta|q|^4q$$

denkleminde yerine yazılarak;

$$(-iBvP' - \omega P)e^{i(-\kappa x+\omega t+\theta_0)} + a(B^2P'' - \kappa^2P - 2i\kappa BP')e^{i(-\kappa x+\omega t+\theta_0)}$$

$$+b(-B^2vP'' + i\kappa BvP' + i\omega BP' + \omega\kappa P)e^{i(-\kappa x+\omega t+\theta_0)} + cP^3e^{i(-\kappa x+\omega t+\theta_0)}$$

$= \sigma\big(B^4P^{(4)} - 4i\kappa B^3P''' - 6\kappa^2B^2P'' + 4i\kappa^3BP' + i\kappa^4P\big)e^{i(-\kappa x+\omega t+\theta_0)}$

$+\alpha(B^2(P')^2P - 2iB\kappa P'P^2 - \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)} + \beta(B^2(P')^2P + \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)}$

$+\gamma(B^2P''P^2 - 2i\kappa BP'P^2 - \kappa^2P^3)e^{i(-\kappa x+\omega t+\theta_0)} + \lambda(B^2P''P^2 - \kappa^2P^3$

$+2i\kappa BP'P^2)e^{i(-\kappa x+\omega t+\theta_0)} + \delta P^5e^{i(-\kappa x+\omega t+\theta_0)}$

bulunur.

$(-iBvP' - \omega P + aB^2P'' - a\kappa^2P - 2ia\kappa BP' - bB^2vP'' + ib\kappa BvP' + ib\omega BP' + b\omega\kappa P$

$+cP^3)e^{i(-\kappa x+\omega t+\theta_0)} = (\sigma B^4P^{(4)} - 4i\sigma\kappa B^3P''' - 6\sigma\kappa^2B^2P'' + 4i\sigma\kappa^3BP' + i\sigma\kappa^4P$

$+\alpha B^2(P')^2P - 2i\alpha\kappa BP'P^2 - \alpha\kappa^2P^3 + \beta B^2(P')^2P + \beta\kappa^2P^3 + \gamma B^2P''P^2 - 2i\gamma\kappa BP'P^2$

$-\gamma\kappa^2P^3 + \lambda B^2P''P^2 - \lambda\kappa^2P^3 + 2i\lambda\kappa BP'P^2 + \delta P^5)e^{i(-\kappa x+\omega t+\theta_0)}$

denklemi düzenlendiğinde,

$(-iBv - 2ia\kappa B + ib\kappa Bv + ib\omega B)P' + (aB^2 - bB^2v)P'' + (-a\kappa^2 + b\omega\kappa - \omega P) + cP^3$

$= (\sigma B^4)P^{(4)} + (-4i\sigma\kappa B^3)P''' + (-6\sigma\kappa^2B^2)P'' + (4i\sigma\kappa^3B)P' + (\alpha B^2 + \beta B^2)(P')^2P$

$+(i\sigma\kappa^4)P + (-\alpha\kappa^2 + \beta\kappa^2 - \gamma\kappa^2 - \lambda\kappa^2)P^3 + (-2i\alpha\kappa B - 2i\gamma\kappa B + 2i\lambda\kappa B)P'P^2$

$+(\gamma B^2 + \lambda B^2)P''P^2 + \delta P^5$

elde edilir. Bu denklemdeki ifadeler eşitliğin bir tarafında toplanırsa,

$\sigma B^4P^{(4)} - 4i\sigma\kappa B^3P''' + B^2(-6\sigma\kappa^2 - a + bv)P'' + iB(4\sigma\kappa^3 + v + 2a\kappa - b\kappa v$

$-b\omega)P' + B^2(\alpha + \beta)(P')^2P + (i\sigma\kappa^4 + a\kappa^2 - b\omega\kappa + \omega)P + [(-\alpha + \beta - \gamma - \lambda)\kappa^2$

$-c]P^3 + 2i\kappa B\,(-\alpha - \gamma + \lambda)P'P^2 + B^2(\gamma + \lambda)P''P^2 + \delta P^5 = 0$

elde edilir. Bu denklem reel ve imajiner kısımlarına ayrılarak yazılırsa,

$\sigma B^4P^{(4)} + B^2(-6\sigma\kappa^2 - a + bv)P'' + B^2(\alpha + \beta)(P')^2P + (a\kappa^2 - b\omega\kappa + \omega)P$

$+[(-\alpha + \beta - \gamma - \lambda)\kappa^2 - c]P^3 + B^2(\gamma + \lambda)P''P^2 + \delta P^5 + i[-4\sigma\kappa B^3P'''$

$+B(4\sigma\kappa^3 + v + 2a\kappa - b\kappa v - b\omega P') + \sigma\kappa^4P + 2\kappa B(-\alpha - \gamma + \lambda)P'P^2] = 0$

elde edilir.

Bu denklemin reel kısmı,

$\sigma B^4P^{(4)} + B^2(-6\sigma\kappa^2 - a + bv)P'' + B^2(\alpha + \beta)(P')^2P + (a\kappa^2 - b\omega\kappa + \omega)P$

$$+[(-\alpha+\beta-\gamma-\lambda)\kappa^2-c]P^3+B^2(\gamma+\lambda)P''P^2+\delta P^5=0 \tag{2.5}$$

ve imajiner kısmı,

$$-4\sigma\kappa B^3P'''+B(4\sigma\kappa^3+v+2a\kappa-b\kappa v-b\omega)P'+\sigma\kappa^4P$$

$$+2\kappa B(-\alpha-\gamma+\lambda)P'P^2=0 \tag{2.6}$$

olarak bulunur. Denklemin imajiner kısmında, $\{P''', P', P, P'P^2\}$ kümesi lineer bağımsız olduğundan katsayılarının sıfıra eşit olması aşağıdaki kısıtlama koşullarını verir,

- $\sigma=0$ (2.7)
- $-\alpha-\gamma+\lambda=0$

  $\alpha=\lambda-\gamma$ (2.8)

ve sonuç olarak soliton hızı, söz konusu nonlineer olma durumundan bağımsız olarak, aşağıdaki eşitlikler ortaya çıkar,

- $v+2a\kappa-b\kappa v-b\omega=0$

  $v(1-b\kappa)=b\omega-2a\kappa$

  $v=\frac{b\omega-2a\kappa}{1-b\kappa}$ (2.9)

burada

$$b\kappa\neq 1 \tag{2.10}$$

dır.

Sonuç olarak (2.7) ve (2.8) kısıtlamaların her ikisi de (2.5) denkleminde yerine yazılarak, reel kısım

$$B^2(-a+bv)P''+B^2(\alpha+\beta)(P')^2P+(a\kappa^2-b\omega\kappa+\omega)P+[(-2\lambda+\beta)\kappa^2-c]P^3$$

$$+B^2(\gamma+\lambda)P''P^2+\delta P^5=0 \tag{2.11}$$

şeklinde elde edilir.

Elde edilen bu adi diferansiyel denklem, bundan sonraki alt bölümlerde, LPD modelinin parlak, koyu ve tekil soliton çözümlerini bulmak için genişletilmiş Jacobi eliptik fonksiyon genişleme yaklaşımı (Alqahtani vd., 2018; Bhrawy vd., 2013; Biswas vd., 2018; Ekici vd., 2017; Guzman vd., 2017) ve $exp(-\phi(\eta))$ genişleme şeması (Huiqun, 2007; Khan ve Akbar, 2013; Lakshmanan vd., 1988; Liu vd., 2016) ile analiz edilecektir.

2.3. Genişletilmiş Jacobi Fonksiyon Genişleme Şeması

(2.11) denkleminin çözümünün sonlu bir seri olarak ifade edildiği varsayılırsa (Biswas vd., 2018):

$$P(\xi) = \sum_{i=0}^{N} \left(a_i sn^i \xi + \frac{b_i}{sn^i \xi} \right) \tag{2.12}$$

Burada $sn\xi$, k modülüne sahip Jacobi eliptik sinüs fonksiyonudur. $(0 < k < 1)$ ve a_0, a_i, b_i sabitleri $i = 1,2, \dots, N$ için belirlenecek sabitlerdir.

$(sn\xi)' = cn\xi dn\xi$

$(cn\xi)' = -sn\xi dn\xi$

$(dn\xi)' = -k^2 sn\xi cn\xi$

$cn\xi = \sqrt{1 - sn^2 \xi}$

$dn\xi = \sqrt{1 - k^2 sn^2 \xi}$

Burada

$F(k, \varphi) = \int_0^{\varphi} \frac{d\theta}{\sqrt{1 - k^2 sin^2 \theta}} \quad (0 < k < 1)$

birinci tip eliptik integraldir.

$$P(\xi) = \sum_{i=0}^{N} \left(a_i sn^i \xi + \frac{b_i}{sn^i \xi} \right)$$

fonksiyonunda;

$o(P) = N\,, \qquad o(P^2) = 2N\,, \qquad o(P^3) = 3N\,, \qquad o(P^4) = 4N\,, \qquad o(P^5) = 5N$

olarak bulunur. Böylece

$$P(\xi) = \sum_{i=0}^{N} \left(a_i sn^i \xi + \frac{b_i}{sn^i \xi} \right)$$

fonksiyonu

$$P(\xi) = \sum_{i=0}^{N} \left(a_i sn^i \xi + b_i sn^{-i} \xi \right)$$

olarak düzenlenebilir. $P(\xi)$ fonksiyonunun 1. mertebeden türevi alındığında,

$P'(\xi) = \sum_{i=0}^{N}\left(a_i i sn^{i-1}\xi cn\xi dn\xi + b_i(-i)sn^{-i-1}\xi cn\xi dn\xi\right)$

$$P'(\xi) = \sum_{i=0}^{N}\left(a_i i sn^{i-1}\xi - b_i i sn^{-i-1}\xi\right) cn\xi dn\xi$$

olarak bulunur.

$cn\xi = \sqrt{1 - sn^2\xi}$

$dn\xi = \sqrt{1 - k^2 sn^2\xi}$

eşitlikleri alınan türevde yerine yazılırsa;

$$P'(\xi) = \sum_{i=0}^{N}\left(a_i i sn^{i-1}\xi - b_i i sn^{-i-1}\xi\right)\sqrt{1 - sn^2\xi}\sqrt{1 - k^2 sn^2\xi}$$

$$= \sum_{i=0}^{N}\left(a_i i sn^{i-1}\xi - b_i i sn^{-i-1}\xi\right)\sqrt{1 - k^2 sn^2\xi - sn^2\xi + k^2 sn^4\xi}$$

$$= \sum_{i=0}^{N}\left(a_i i sn^{i-1}\xi - b_i i sn^{-i-1}\xi\right)\sqrt{1 - (k^2+1)sn^2\xi + k^2 sn^4\xi}$$

bulunur. Buradan;

$o(P') = N - 1 + 2 = N + 1$

olarak bulunur.

$P(\xi)$ fonksiyonunun 2. mertebeden türevi alınırsa,

$$P''(\xi) = \sum_{i=0}^{N}[a_i i(i-1)sn^{i-2}\xi cn\xi dn\xi$$

$$- b_i i(-i-1)sn^{-i-2}\xi cn\xi dn\xi]\sqrt{1 - (k^2+1)sn^2\xi + k^2 sn^4\xi}$$

$$+ \frac{[-2(k^2+1)sn\xi cn\xi dn\xi + 4k^2 sn^3\xi cn\xi dn\xi]}{2\sqrt{1 - (k^2+1)sn^2\xi + k^2 sn^4\xi}}$$

$$P''(\xi) = \sum_{i=0}^{N}\left[\left(a_i i(i-1)sn^{i-2}\xi + b_i i(i+1)sn^{-i-2}\xi\right)\sqrt{1 - (k^2+1)sn^2\xi + k^2 sn^4\xi}\right.$$

$$\left. + \frac{[-2(k^2+1)sn\xi + 4k^2 sn^3\xi]}{2\sqrt{1 - (k^2+1)sn^2\xi + k^2 sn^4\xi}}\right] cn\xi dn\xi$$

elde edilir. Benzer şekilde,

$cn\xi = \sqrt{1 - sn^2\xi}$ ve $dn\xi = \sqrt{1 - k^2 sn^2\xi}$

eşitlikleri yerine yazıldığında;

$$P''(\xi) = \sum_{i=0}^{N} \Bigg[\left(a_i i(i-1) sn^{i-2}\xi + b_i i(i+1) sn^{-i-2}\xi\right) \sqrt{1 - (k^2+1) sn^2\xi + k^2 sn^4\xi} + \frac{[-2(k^2+1) sn\xi + 4k^2 sn^3\xi]}{2\sqrt{1 - (k^2+1) sn^2\xi + k^2 sn^4\xi}} \Bigg] \sqrt{1 - sn^2\xi}\,\sqrt{1 - k^2 sn^2\xi}$$

elde edilir. Buradan

$o(P'') = N - 2 + 2 + 2 = N + 2$

elde edilir.

Böylece devam edilirse

$o(P''') = N + 3$

bulunur.

$o(P^2) = 2N\,,\quad o(P')^2 = 2N + 2\,,\quad o(PP'') = 2N + 2$

dır. Balans metoduna göre, (2.5) denkleminde P^2P'' ile P^5 in dengelenmesi sonucunda

$o(P^5) = o(P^2P'')$

eşitliği yazılabilir.

$o(P^5) = 5N$

$o(P^2P'') = 2N + N + 2 = 3N + 2$

eşitlikleri,

$o(P^5) = o(P^2P'')$

eşitliğinde yerine yazılırsa;

$$5N = 3N + 2 \tag{2.13}$$

$2N = 2$

$$N = 1 \tag{2.14}$$

elde edilir. Bulunan $N = 1$ değeri;

$$P(\xi)=\sum_{i=0}^{N}\left(a_i sn^i\xi+b_i sn^{-i}\xi\right)$$

fonksiyonunda yerine yazılırsa,

$$P(\xi)=\sum_{i=0}^{1}\left(a_i sn^i\xi+b_i sn^{-i}\xi\right)=a_0+a_1 sn\xi+b_0+b_1 sn^{-1}\xi$$

$$=a_0+a_1 sn\xi+b_1 sn^{-1}\xi \tag{2.15}$$

sonucu elde edilir.

$dn^2\xi=1-k^2sn^2\xi$

$cn^2\xi=1-sn^2\xi$

eşitliklerini kullanarak;

$$(cn\xi dn\xi)'=-sn\xi dn^2\xi-k^2sn\xi cn^2\xi=-[dn^2\xi+k^2cn^2\xi]sn\xi$$

$$=-[1-k^2sn^2\xi+k^2(1-sn^2\xi)]sn\xi$$

$$=-[1+k^2-2k^2sn^2\xi]sn\xi$$

bulunur.

(2.2) de verilen

$$q(x,t)=P(\xi)e^{i(-\kappa x+\omega t+\theta_0)}$$

denkleminde,

$$P(\xi)=a_0+a_1 sn\xi+b_1 sn^{-1}\xi$$

olarak elde edilmiş olur.

Bunun sonucu olarak, (2.11) denkleminde bulunan P'', $P(P')^2$, P^3, $P''P^2$ ve P^5 terimleri aşağıdaki gibi hesaplanabilir,

$$P'=(a_1cn\xi-b_1sn^{-2}\xi cn\xi)dn\xi=(a_1-b_1sn^{-2}\xi)cn\xi dn\xi$$

$$P''=2b_1sn^{-3}\xi.cn^2\xi dn^2\xi+(a_1-b_1sn^{-2}\xi)[-sn\xi dn\xi dn\xi-cn\xi k^2sn\xi dn\xi]$$

$$P''=2b_1sn^{-3}\xi cn^2\xi dn^2\xi-(a_1-b_1sn^{-2}\xi)[dn^2\xi+k^2cn^2\xi]sn\xi$$

$$P''=2b_1sn^{-3}\xi[(1-sn^2\xi)(1-k^2sn^2\xi)]$$

$$-(a_1-b_1sn^{-2}\xi)[1-k^2sn^2\xi+k^2(1-sn^2\xi)]sn\xi$$

$$P'' = 2b_1 sn^{-3}\xi[1-(1+k^2)sn^2\xi + k^2 sn^4\xi] - (a_1 - b_1 sn^{-2}\xi)[1+k^2-2k^2sn^2\xi]sn\xi$$

$$P'' = 2b_1 sn^{-3}\xi - 2b_1(1+k^2)sn^{-1}\xi + 2b_1k^2sn\xi - a_1(1+k^2)sn\xi + 2a_1k^2sn^3\xi$$
$$+b_1(1+k^2)sn^{-1}\xi - 2b_1k^2sn\xi$$

$$P'' = 2a_1k^2sn^3\xi \; - a_1(1+k^2)sn\xi - b_1(1+k^2)sn^{-1}\xi + 2b_1sn^{-3}\xi$$

$$P(\xi) = a_0 + a_1sn\xi + b_1sn^{-1}\xi$$

$$P^2 = (a_0 + a_1sn\xi)^2 + (b_1sn^{-1}\xi)^2 + 2(a_0 + a_1sn\xi)b_1sn^{-1}\xi$$

$$P^2 = {a_0}^2 + 2a_0a_1sn\xi + {a_1}^2sn^2\xi + {b_1}^2sn^{-2}\xi + 2a_0b_1sn^{-1}\xi + 2a_1b_1$$

$$P^2 = {a_0}^2 + 2a_1b_1 + {a_1}^2sn^2\xi + 2a_0a_1sn\xi + 2a_0b_1sn^{-1}\xi + {b_1}^2sn^{-2}\xi$$

$$P^3 = (a_0 + a_1sn\xi)^3 + 3(a_0 + a_1sn\xi)^2b_1sn^{-1}\xi + 3(a_0 + a_1sn\xi)(b_1sn^{-1}\xi)^2$$
$$+(b_1sn^{-1}\xi)^3$$

$$P^3 = {a_0}^3 + 3{a_0}^2a_1sn\xi + 3a_0{a_1}^2sn^2\xi + {a_1}^3sn^3\xi + 3{a_0}^2b_1sn^{-1}\xi + 6a_0a_1b_1$$
$$+3{a_1}^2b_1sn\xi + 3a_0{b_1}^2sn^{-2}\xi + 3a_1{b_1}^2sn^{-1}\xi + {b_1}^3sn^{-3}\xi$$

$$P^3 = {a_0}^3 + 6a_0a_1b_1 + {a_1}^3sn^3\xi + 3a_0{a_1}^2sn^2\xi + 3[{a_0}^2a_1 + {a_1}^2b_1]sn\xi$$
$$+3\left[{a_0}^2b_1 + a_1{b_1}^2\right]sn^{-1}\xi + 3a_0{b_1}^2sn^{-2}\xi + {b_1}^3sn^{-3}\xi$$

$$P^2P'' = \left[{a_0}^2 + 2a_1b_1 + {a_1}^2sn^2\xi + 2a_0a_1sn\xi + 2a_0b_1sn^{-1}\xi + {b_1}^2sn^{-2}\xi\right]$$
$$[2a_1k^2sn^3\xi - a_1(1+k^2)sn\xi - b_1(1+k^2)sn^{-1}\xi + 2b_1sn^{-3}\xi]$$

$$P^2P'' = 2{a_0}^2a_1k^2sn^3\xi - {a_0}^2a_1(1+k^2)sn\xi - {a_0}^2b_1(1+k^2)sn^{-1}\xi + 2{a_0}^2b_1sn^{-3}\xi$$
$$+4{a_1}^2b_1k^2sn^3\xi - 2{a_1}^2b_1(1+k^2)sn\xi - 2a_1{b_1}^2(1+k^2)sn^{-1}\xi + 4a_1{b_1}^2sn^{-3}\xi$$
$$+2{a_1}^3k^2sn^5\xi - {a_1}^3(1+k^2)sn^3\xi - {a_1}^2b_1(1+k^2)sn\xi + 2{a_1}^2b_1sn^{-1}\xi$$
$$+4a_0{a_1}^2k^2sn^4\xi - 2a_0{a_1}^2(1+k^2)sn^2\xi - 2a_0a_1b_1(1+k^2) + 4a_0a_1b_1sn^{-2}\xi$$
$$+4a_0a_1b_1k^2sn^2\xi - 2a_0a_1b_1(1+k^2) - 2a_0{b_1}^2(1+k^2)sn^{-2}\xi + 4a_0{b_1}^2sn^{-4}\xi$$
$$+2a_1{b_1}^2k^2sn\xi - a_1{b_1}^2(1+k^2)sn^{-1}\xi - {b_1}^3(1+k^2)sn^{-3}\xi + 2{b_1}^3sn^{-5}\xi$$

$$P^2P'' = -4a_0a_1b_1(1+k^2) + 2{a_1}^3k^2sn^5\xi + 4a_0{a_1}^2k^2sn^4\xi$$
$$+[2{a_0}^2a_1k^2 + 4{a_1}^2b_1k^2 - {a_1}^3(1+k^2)]sn^3\xi$$
$$+[-2a_0{a_1}^2(1+k^2) + 4a_0a_1b_1k^2]sn^2\xi$$

$$+\left[-{a_0}^2a_1(1+k^2)-2{a_1}^2b_1(1+k^2)-{a_1}^2b_1(1+k^2)+2a_1{b_1}^2k^2\right]sn\xi$$

$$+\left[-{a_0}^2b_1(1+k^2)-2a_1{b_1}^2(1+k^2)+2{a_1}^2b_1-a_1{b_1}^2(1+k^2)\right]sn^{-1}\xi$$

$$+\left[4a_0a_1b_1-2a_0{b_1}^2(1+k^2)\right]sn^{-2}\xi$$

$$+\left[2{a_0}^2b_1+4a_1{b_1}^2-{b_1}^3(1+k^2)\right]sn^{-3}\xi$$

$$+4a_0{b_1}^2sn^{-4}\xi+2{b_1}^3sn^{-5}\xi$$

$$P^2P''=-4a_0a_1b_1(1+k^2)+2{a_1}^3k^2sn^5\xi+4a_0{a_1}^2k^2sn^4\xi$$

$$+a_1[2{a_0}^2k^2+4a_1b_1k^2-{a_1}^2(1+k^2)]sn^3\xi$$

$$+2a_0a_1[-a_1(1+k^2)+2b_1k^2]sn^2\xi$$

$$+a_1[(-{a_0}^2-2a_1b_1-a_1b_1)(1+k^2)+2b_1k^2]sn\xi$$

$$+b_1[(-{a_0}^2-2a_1b_1-a_1b_1)(1+k^2)+2{a_1}^2]sn^{-1}\xi$$

$$+2a_0b_1[2a_1-b_1(1+k^2)]sn^{-2}\xi+b_1\left[2{a_0}^2+4a_1b_1-{b_1}^2(1+k^2)\right]sn^{-3}\xi$$

$$+4a_0{b_1}^2sn^{-4}\xi+2{b_1}^3sn^{-5}\xi$$

$$P^2P''=-4a_0a_1b_1(1+k^2)+2{a_1}^3k^2sn^5\xi+4a_0{a_1}^2k^2sn^4\xi$$

$$+a_1[2{a_0}^2k^2+4a_1b_1k^2-{a_1}^2(1+k^2)]sn^3\xi$$

$$+2a_0a_1[-a_1(1+k^2)+2b_1k^2]sn^2\xi$$

$$-a_1[({a_0}^2+3a_1b_1)(1+k^2)-2b_1k^2]sn\xi$$

$$-b_1[({a_0}^2+3a_1b_1)(1+k^2)-2{a_1}^2]sn^{-1}\xi$$

$$+2a_0b_1[2a_1-b_1(1+k^2)]sn^{-2}\xi+b_1\left[2{a_0}^2+4a_1b_1-{b_1}^2(1+k^2)\right]sn^{-3}\xi$$

$$+4a_0{b_1}^2sn^{-4}\xi+2{b_1}^3sn^{-5}\xi$$

$$(P')^2=(a_1-b_1sn^{-2}\xi)^2cn^2\xi dn^2\xi$$

$$(P')^2=(a_1-b_1sn^{-2}\xi)^2(1-sn^2\xi)(1-k^2sn^2\xi)$$

$$(P')^2=\left({a_1}^2-2a_1b_1sn^{-2}\xi+{b_1}^2sn^{-4}\xi\right)(1-k^2sn^2\xi-sn^2\xi+k^2sn^4\xi)$$

$$(P')^2=\left({a_1}^2-2a_1b_1sn^{-2}\xi+{b_1}^2sn^{-4}\xi\right)[1-(1+k^2)sn^2\xi+k^2sn^4\xi]$$

$$(P')^2={a_1}^2-{a_1}^2(1+k^2)sn^2\xi+{a_1}^2k^2sn^4\xi-2a_1b_1sn^{-2}\xi+2a_1b_1(1+k^2)$$

$$-2a_1b_1k^2sn^2\xi+{b_1}^2sn^{-4}\xi-{b_1}^2(1+k^2)sn^{-2}\xi+{b_1}^2k^2$$

$$(P')^2 = {a_1}^2 + 2a_1b_1(1+k^2) + {b_1}^2k^2 - [{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^2\xi + {a_1}^2k^2sn^4\xi$$
$$-\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-2}\xi + {b_1}^2sn^{-4}\xi$$

$$P(P')^2 = (a_0 + a_1sn\xi + b_1sn^{-1}\xi)\{{a_1}^2 + 2a_1b_1(1+k^2) + {b_1}^2k^2 + {a_1}^2k^2sn^4\xi$$
$$-[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^2\xi - \left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-2}\xi + {b_1}^2sn^{-4}\xi\}$$

$$P(P')^2 = a_0{a_1}^2 + 2a_0a_1b_1(1+k^2) + a_0{b_1}^2k^2 - a_0[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^2\xi$$
$$+a_0{a_1}^2k^2sn^4\xi - a_0\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-2}\xi + a_0{b_1}^2sn^{-4}\xi + {a_1}^3sn\xi$$
$$+2{a_1}^2b_1(1+k^2)sn\xi + a_1{b_1}^2k^2sn\xi - a_1[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^3\xi$$
$$+{a_1}^3k^2sn^5\xi - a_1\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-1}\xi + a_1{b_1}^2sn^{-3}\xi + {a_1}^2b_1sn^{-1}\xi$$
$$+2a_1{b_1}^2(1+k^2)sn^{-1}\xi + {b_1}^3k^2sn^{-1}\xi - b_1[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn\xi$$
$$+{a_1}^2b_1k^2sn^3\xi - b_1\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-3}\xi + {b_1}^3sn^{-5}\xi$$

$$P(P')^2 = a_0{a_1}^2 + 2a_0a_1b_1(1+k^2) + a_0{b_1}^2k^2 + {a_1}^3k^2sn^5\xi + a_0{a_1}^2k^2sn^4\xi$$
$$-a_1[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^3\xi + {a_1}^2b_1k^2sn^3\xi$$
$$-a_0[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^2\xi + {a_1}^3sn\xi + 2{a_1}^2b_1(1+k^2)sn\xi$$
$$+a_1{b_1}^2k^2sn\xi - b_1[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn\xi$$
$$-a_1\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-1}\xi + {a_1}^2b_1sn^{-1}\xi + 2a_1{b_1}^2(1+k^2)sn^{-1}\xi$$
$$+{b_1}^3k^2sn^{-1}\xi - a_0\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-2}\xi + a_1{b_1}^2sn^{-3}\xi$$
$$-b_1\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-3}\xi + a_0{b_1}^2sn^{-4}\xi + {b_1}^3sn^{-5}\xi$$

$$P(P')^2 = a_0{a_1}^2 + 2a_0a_1b_1(1+k^2) + a_0{b_1}^2k^2 + {a_1}^3k^2sn^5\xi + a_0{a_1}^2k^2sn^4\xi$$
$$-a_1[{a_1}^2(1+k^2) + a_1b_1k^2]sn^3\xi - a_0[{a_1}^2(1+k^2) + 2a_1b_1k^2]sn^2\xi$$
$$+a_1\left[{a_1}^2 + a_1b_1(1+k^2) - {b_1}^2k^2\right]sn\xi - b_1[{a_1}^2 - a_1b_1(1+k^2) - b_1k^2]sn^{-1}\xi$$
$$-a_0\left[2a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-2}\xi - b_1\left[a_1b_1 + {b_1}^2(1+k^2)\right]sn^{-3}\xi$$
$$+a_0{b_1}^2sn^{-4}\xi + {b_1}^3sn^{-5}\xi$$

$$P^5 = (a_0 + a_1sn\xi + b_1sn^{-1}\xi)^5$$

$$P^5 = (a_0 + a_1sn\xi)^5 + 5(a_0 + a_1sn\xi)^4b_1sn^{-1}\xi + 10(a_0 + a_1sn\xi)^3(b_1sn^{-1}\xi)^2$$

$+10(a_0+a_1 sn\xi)^2(b_1 sn^{-1}\xi)^3+5(a_0+a_1 sn\xi)(b_1 sn^{-1}\xi)^4+(b_1 sn^{-1}\xi)^5$

$P^5={a_0}^5+5{a_0}^4 a_1 sn\xi+10{a_0}^3{a_1}^2 sn^2\xi+10{a_0}^2{a_1}^3 sn^3\xi+5a_0{a_1}^4 sn^4\xi+{a_1}^5 sn^5\xi$

$+5[{a_0}^4+4{a_0}^3 a_1 sn\xi+6{a_0}^2{a_1}^2 sn^2\xi+4a_0{a_1}^3 sn^3\xi+{a_1}^4 sn^4\xi]b_1 sn^{-1}\xi$

$+10[{a_0}^3+3{a_0}^2 a_1 sn\xi+3a_0{a_1}^2 sn^2\xi+{a_1}^3 sn^3\xi]{b_1}^2 sn^{-2}\xi$

$+10[{a_0}^2+2a_0a_1 sn\xi+{a_1}^2 sn^2\xi]{b_1}^3 sn^{-3}\xi+5[a_0+a_1 sn\xi]{b_1}^4 sn^{-4}\xi+{b_1}^5 sn^{-5}\xi$

$P^5={a_0}^5+5{a_0}^4 a_1 sn\xi+10{a_0}^3{a_1}^2 sn^2\xi+10{a_0}^2{a_1}^3 sn^3\xi+5a_0{a_1}^4 sn^4\xi+{a_1}^5 sn^5\xi$

$+5{a_0}^4 b_1 sn^{-1}\xi+20{a_0}^3 a_1 b_1+30{a_0}^2{a_1}^2 b_1 sn\xi+20a_0{a_1}^3 b_1 sn^2\xi+5{a_1}^4 b_1 sn^3\xi$

$+10{a_0}^3{b_1}^2 sn^{-2}\xi+30{a_0}^2 a_1{b_1}^2 sn^{-1}\xi+30a_0{a_1}^2{b_1}^2+10{a_1}^3{b_1}^2 sn\xi$

$+10{a_0}^2{b_1}^3 sn^{-3}\xi+20a_0a_1{b_1}^3 sn^{-2}\xi+10{a_1}^2{b_1}^3 sn^{-1}\xi+5a_0{b_1}^4 sn^{-4}\xi$

$+5a_1{b_1}^4 sn^{-3}\xi+{b_1}^5 sn^{-5}\xi$

$P^5={a_0}^5+20{a_0}^3a_1b_1+30a_0{a_1}^2{b_1}^2+{a_1}^5 sn^5\xi+(10{a_0}^2{a_1}^3+5{a_1}^4b_1)sn^3\xi$

$+5a_0{a_1}^4 sn^4\xi+(10{a_0}^3{a_1}^2+20a_0{a_1}^3b_1)sn^2\xi$

$+\left(5{a_0}^4a_1+30{a_0}^2{a_1}^2b_1+10{a_1}^3{b_1}^2\right)sn\xi$

$+\left(5{a_0}^4b_1+30{a_0}^2a_1{b_1}^2+10{a_1}^2{b_1}^3\right)sn^{-1}\xi+\left(10{a_0}^3{b_1}^2+20a_0a_1{b_1}^3\right)sn^{-2}\xi$

$+\left(10{a_0}^2{b_1}^3+5a_1{b_1}^4\right)sn^{-3}\xi+5a_0{b_1}^4 sn^{-4}\xi+{b_1}^5 sn^{-5}\xi$

Elde edilen P'', $P(P')^2$, P^3, $P''P^2$ ve P^5 eşitlikleri daha önce adı geçen aşağıdaki (2.11) denkleminde

$B^2(-a+bv)P''+B^2(\alpha+\beta)(P')^2P+(a\kappa^2-b\omega\kappa+\omega)P+[(-2\lambda+\beta)\kappa^2-c]P^3$

$+B^2(\gamma+\lambda)P''P^2+\delta P^5=0$

yerine yazılırsa,

$B^2(-a+bv)\{2a_1k^2 sn^3\xi-a_1(1+k^2)sn\xi-b_1(1+k^2)sn^{-1}\xi+2b_1 sn^{-3}\xi\}$

$+B^2(\alpha+\beta)\{a_0{a_1}^2+2a_0a_1b_1(1+k^2)+a_0{b_1}^2k^2+{a_1}^3k^2 sn^5\xi+a_0{a_1}^2k^2 sn^4\xi$

$-a_1[{a_1}^2(1+k^2)+a_1b_1k^2]sn^3\xi-a_0[{a_1}^2(1+k^2)+2a_1b_1k^2]sn^2\xi$

$+a_1\left[{a_1}^2+a_1b_1(1+k^2)-{b_1}^2k^2\right]sn\xi-b_1[{a_1}^2-a_1b_1(1+k^2)-b_1k^2]sn^{-1}\xi$

$-a_0\left[2a_1b_1+{b_1}^2(1+k^2)\right]sn^{-2}\xi-b_1\left[a_1b_1+{b_1}^2(1+k^2)\right]sn^{-3}\xi+a_0{b_1}^2 sn^{-4}\xi$

$$+b_1{}^3 sn^{-5}\xi\} + (\alpha\kappa^2 - b\omega\kappa + \omega)(a_0 + a_1 sn\xi + b_1 sn^{-1}\xi)$$

$$+[(-2\lambda + \beta)\kappa^2 - c]\{a_0{}^3 + 6a_0a_1b_1 + a_1{}^3 sn^3\xi + 3a_0a_1{}^2 sn^2\xi$$

$$+3[a_0{}^2a_1 + a_1{}^2b_1]sn\xi + 3\left[a_0{}^2b_1 + a_1b_1{}^2\right]sn^{-1}\xi + 3a_0b_1{}^2 sn^{-2}\xi + +b_1{}^3 sn^{-3}\xi\}$$

$$+B^2(\gamma + \lambda)\{-4a_0a_1b_1(1 + k^2) + 2a_1{}^3k^2 sn^5\xi + 4a_0a_1{}^2k^2 sn^4\xi$$

$$+a_1[2a_0{}^2k^2 + 4a_1b_1k^2 - a_1{}^2(1 + k^2)]sn^3\xi + 2a_0a_1[-a_1(1 + k^2) + 2b_1k^2]sn^2\xi$$

$$-a_1[(a_0{}^2 + 3a_1b_1)(1 + k^2) - 2b_1k^2]sn\xi - b_1[(a_0{}^2 + 3a_1b_1)(1 + k^2) - 2a_1{}^2]sn^{-1}\xi$$

$$+2a_0b_1[2a_1 - b_1(1 + k^2)]sn^{-2}\xi + b_1\left[2a_0{}^2 + 4a_1b_1 - b_1{}^2(1 + k^2)\right]sn^{-3}\xi$$

$$+4a_0b_1{}^2 sn^{-4}\xi + 2b_1{}^3 sn^{-5}\xi\} + \delta\{a_0{}^5 + 20a_0{}^3a_1b_1 + 30a_0a_1{}^2b_1{}^2 + a_1{}^5 sn^5\xi$$

$$+5a_0a_1{}^4 sn^4\xi + (10a_0{}^2a_1{}^3 + 5a_1{}^4b_1)sn^3\xi + (10a_0{}^3a_1{}^2 + 20a_0a_1{}^3b_1)sn^2\xi$$

$$+\left(5a_0{}^4a_1 + 30a_0{}^2a_1{}^2b_1 + 10a_1{}^3b_1{}^2\right)sn\xi + \left(5a_0{}^4b_1 + 30a_0{}^2a_1b_1{}^2 + 10a_1{}^2b_1{}^3\right)sn^{-1}\xi$$

$$+\left(10a_0{}^3b_1{}^2 + 20a_0a_1b_1{}^3\right)sn^{-2}\xi + \left(10a_0{}^2b_1{}^3 + 5a_1b_1{}^4\right)sn^{-3}\xi + 5a_0b_1{}^4 sn^{-4}\xi$$

$$+b_1{}^5 sn^{-5}\xi = 0$$

ifadesi elde edilir. Bu ifade düzenlenirse,

$$-2a_1k^2aB^2 sn^3\xi + a_1(1 + k^2)aB^2 sn\xi + b_1(1 + k^2)aB^2 sn^{-1}\xi - 2b_1aB^2 sn^{-3}\xi$$

$$+2a_1k^2bvB^2 sn^3\xi - a_1(1 + k^2)bvB^2 sn\xi - b_1(1 + k^2)bvB^2 sn^{-1}\xi + 2b_1bvB^2 sn^{-3}\xi$$

$$+a_0a_1{}^2\alpha B^2 + 2a_0a_1b_1(1 + k^2)\alpha B^2 + a_0b_1{}^2k^2\alpha B^2 + a_1{}^3k^2\alpha B^2 sn^5\xi$$

$$+a_0a_1{}^2k^2\alpha B^2 sn^4\xi - a_1[a_1{}^2(1 + k^2) + a_1b_1k^2]\alpha B^2 sn^3\xi$$

$$-a_0[a_1{}^2(1 + k^2) + 2a_1b_1k^2]\alpha B^2 sn^2\xi + a_1\left[a_1{}^2 + a_1b_1(1 + k^2) - b_1{}^2k^2\right]\alpha B^2 sn\xi$$

$$-b_1[a_1{}^2 - a_1b_1(1 + k^2) - b_1k^2]\alpha B^2 sn^{-1}\xi - a_0\left[2a_1b_1 + b_1{}^2(1 + k^2)\right]\alpha B^2 sn^{-2}\xi$$

$$-b_1\left[a_1b_1 + b_1{}^2(1 + k^2)\right]\alpha B^2 sn^{-3}\xi + a_0b_1{}^2\alpha B^2 sn^{-4}\xi + b_1{}^3\alpha B^2 sn^{-5}\xi + a_0a_1{}^2\beta B^2$$

$$+2a_0a_1b_1(1 + k^2)\beta B^2 + a_0b_1{}^2k^2\beta B^2 + a_1{}^3k^2\beta B^2 sn^5\xi + a_0a_1{}^2k^2\beta B^2 sn^4\xi$$

$$-a_1[a_1{}^2(1 + k^2) + a_1b_1k^2]\beta B^2 sn^3\xi - a_0[a_1{}^2(1 + k^2) + 2a_1b_1k^2]\beta B^2 sn^2\xi$$

$$+a_1\left[a_1{}^2 + a_1b_1(1 + k^2) - b_1{}^2k^2\right]\beta B^2 sn\xi - b_1[a_1{}^2 - a_1b_1(1 + k^2) - b_1k^2]\beta B^2 sn^{-1}\xi$$

$$-a_0\left[2a_1b_1 + b_1{}^2(1 + k^2)\right]\beta B^2 sn^{-2}\xi - b_1\left[a_1b_1 + b_1{}^2(1 + k^2)\right]\beta B^2 sn^{-3}\xi$$

$$+a_0b_1{}^2\beta B^2 sn^{-4}\xi + b_1{}^3\beta B^2 sn^{-5}\xi + a_0\alpha\kappa^2 + a_1\alpha\kappa^2 sn\xi + b_1\alpha\kappa^2 sn^{-1}\xi - a_0b\omega\kappa$$

$-a_1 b\omega\kappa sn\xi - b_1 b\omega\kappa sn^{-1}\xi + a_0\omega \ + a_1\omega sn\xi + b_1\omega sn^{-1}\xi + a_0{}^3(-2\lambda+\beta)\kappa^2$

$+6a_0a_1b_1(-2\lambda+\beta)\kappa^2 + 3a_0a_1{}^2(-2\lambda+\beta)\kappa^2 sn^2\xi$

$+3[a_0{}^2a_1 + a_1{}^2b_1](-2\lambda+\beta)\kappa^2 sn\xi + 3\left[a_0{}^2b_1 + a_1b_1{}^2\right](-2\lambda+\beta)\kappa^2 sn^{-1}\xi$

$+3a_0b_1{}^2(-2\lambda+\beta)\kappa^2 sn^{-2}\xi + b_1{}^3(-2\lambda+\beta)\kappa^2 sn^{-3}\xi - a_0{}^3c - 6a_0a_1b_1c - a_1{}^3csn^3\xi$

$-3a_0a_1{}^2csn^2\xi - 3[a_0{}^2a_1 + a_1{}^2b_1]csn\xi - 3\left[a_0{}^2b_1 + a_1b_1{}^2\right]csn^{-1}\xi - 3a_0b_1{}^2csn^{-2}\xi$

$-b_1{}^3csn^{-3}\xi - 4a_0a_1b_1(1+k^2)\lambda B^2 + 2a_1{}^3k^2\lambda B^2 sn^5\xi + 4a_0a_1{}^2k^2\lambda B^2 sn^4\xi$

$+a_1[2a_0{}^2k^2 + 4a_1b_1k^2 - a_1{}^2(1+k^2)]\lambda B^2 sn^3\xi$

$+2a_0a_1[-a_1(1+k^2) + 2b_1k^2]\lambda B^2 sn^2\xi \ -a_1\left[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2b_1{}^2k^2\right]\lambda B^2 sn\xi$

$-b_1[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2a_1{}^2]\lambda B^2 sn^{-1}\xi + 2a_0b_1[2a_1 - b_1(1+k^2)]\lambda B^2 sn^{-2}\xi$

$+b_1\left[2a_0{}^2 + 4a_1b_1 - b_1{}^2(1+k^2)\right]\lambda B^2 sn^{-3}\xi + 4a_0b_1{}^2\lambda B^2 sn^{-4}\xi + 2b_1{}^3\lambda B^2 sn^{-5}\xi$

$-4a_0a_1b_1(1+k^2)\gamma B^2 + 2a_1{}^3k^2\gamma B^2 sn^5\xi + 4a_0a_1{}^2k^2\gamma B^2 sn^4\xi$

$+a_1\left[a_1{}^2 + a_1b_1(1+k^2) - b_1{}^2k^2\right]\alpha B^2 \ +a_1[2a_0{}^2k^2 + 4a_1b_1k^2 - a_1{}^2(1+k^2)]\gamma B^2 sn^3\xi$

$+2a_0a_1[-a_1(1+k^2) + 2b_1k^2]\gamma B^2 sn^2\xi \ -a_1\left[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2b_1{}^2k^2\right]\gamma B^2 sn\xi$

$-b_1[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2a_1{}^2]\gamma B^2 sn^{-1}\xi + 2a_0b_1[2a_1 - b_1(1+k^2)]\gamma B^2 sn^{-2}\xi$

$+b_1\left[2a_0{}^2 + 4a_1b_1 - b_1{}^2(1+k^2)\right]\gamma B^2 sn^{-3}\xi + 4a_0b_1{}^2\gamma B^2 sn^{-4}\xi + 2b_1{}^3\gamma B^2 sn^{-5}\xi$

$+a_0{}^5\delta + 20a_0{}^3a_1b_1\delta + 30a_0a_1{}^2b_1{}^2\delta + a_1{}^5\delta sn^5\xi + 5a_0a_1{}^4\delta sn^4$

$+(10a_0{}^2a_1{}^3 + 5a_1{}^4b_1)\delta sn^3\xi + (10a_0{}^3a_1{}^2 + 20a_0a_1{}^3b_1)\delta sn^2\xi$

$+\left(5a_0{}^4a_1 + 30a_0{}^2a_1{}^2b_1 + 10a_1{}^3b_1{}^2\right)\delta sn\xi$

$+\left(5a_0{}^4b_1 + 30a_0{}^2a_1b_1{}^2 + 10a_1{}^2b_1{}^3\right)\delta sn^{-1}\xi \ +\left(10a_0{}^3b_1{}^2 + 20a_0a_1b_1{}^3\right)\delta sn^{-2}\xi$

$+\left(10a_0{}^2b_1{}^3 + 5a_1b_1{}^4\right)\delta sn^{-3}\xi + 5a_0b_1{}^4\delta sn^{-4}\xi + b_1{}^5\delta sn^{-5}\xi = 0$

bulunur. Gerekli düzenlemeler yapılırsa,

$\{a_0a_1{}^2\alpha B^2 + a_0a_1{}^2\beta B^2 + 2a_0a_1b_1(1+k^2)\alpha B^2 + 2a_0a_1b_1(1+k^2)\beta B^2 \ +a_0b_1{}^2k^2\alpha B^2$

$+a_0b_1{}^2k^2\beta B^2 + a_0\alpha\kappa^2 - a_0b\omega\kappa + a_0\omega + a_0{}^3(-2\lambda+\beta)\kappa^2 + 6a_0a_1b_1(-2\lambda+\beta)\kappa^2$

$-a_0{}^3c - 6a_0a_1b_1c - 4a_0a_1b_1(1+k^2)\lambda B^2 - 4a_0a_1b_1(1+k^2)\gamma B^2 + a_0{}^5\delta$

$+20{a_0}^3a_1b_1\delta+30a_0{a_1}^2{b_1}^2\delta\}+\{{a_1}^3k^2\alpha B^2+{a_1}^3k^2\beta B^2+2{a_1}^3k^2\lambda B^2+2{a_1}^3k^2\gamma B^2$

$+{a_1}^5\delta\}sn^5\xi$

$+\{a_0{a_1}^2k^2\alpha B^2+a_0{a_1}^2k^2\beta B^2+4a_0{a_1}^2k^2\lambda B^2+4a_0{a_1}^2k^2\gamma B^2+5a_0{a_1}^4\delta\}sn^4\xi$

$+\{-2a_1k^2aB^2+2a_1k^2bvB^2-a_1[{a_1}^2(1+k^2)+a_1b_1k^2]\alpha B^2$

$-a_1[{a_1}^2(1+k^2)+a_1b_1k^2]\beta B^2+{a_1}^3(-2\lambda+\beta)\kappa^2-{a_1}^3c$

$+a_1[2{a_0}^2k^2+4a_1b_1k^2-{a_1}^2(1+k^2)]\lambda B^2$

$+a_1[2{a_0}^2k^2+4a_1b_1k^2-{a_1}^2(1+k^2)]\gamma B^2+(10{a_0}^2{a_1}^3+5{a_1}^4b_1)\delta\}sn^3\xi$

$+\{-a_0[{a_1}^2(1+k^2)+2a_1b_1k^2]\alpha B^2-a_0[{a_1}^2(1+k^2)+2a_1b_1k^2]\beta B^2$

$+3a_0{a_1}^2(-2\lambda+\beta)\kappa^2-3a_0{a_1}^2c+2a_0a_1[-a_1(1+k^2)+2b_1k^2]\lambda B^2$

$+2a_0a_1[-a_1(1+k^2)+2b_1k^2]\gamma B^2+(10{a_0}^3{a_1}^2+20a_0{a_1}^3b_1)\delta\}sn^2\xi$

$+\{a_1(1+k^2)aB^2-a_1(1+k^2)bvB^2+a_1a\kappa^2-a_1b\omega\kappa+a_1\omega$

$+3[{a_0}^2a_1+{a_1}^2b_1](-2\lambda+\beta)\kappa^2-3[{a_0}^2a_1+{a_1}^2b_1]c$

$-a_1\left[({a_0}^2+3a_1b_1)(1+k^2)-2{b_1}^2k^2\right]\lambda B^2$

$-a_1\left[({a_0}^2+3a_1b_1)(1+k^2)-2{b_1}^2k^2\right]\gamma B^2$

$+\left(5{a_0}^4a_1+30{a_0}^2{a_1}^2b_1+10{a_1}^3{b_1}^2\right)\delta\}sn\xi$

$+\{b_1(1+k^2)aB^2-b_1(1+k^2)bvB^2-b_1[{a_1}^2-a_1b_1(1+k^2)-b_1k^2]\alpha B^2$

$-b_1[{a_1}^2-a_1b_1(1+k^2)-b_1k^2]\beta B^2+b_1a\kappa^2-b_1b\omega\kappa+b_1\omega$

$+3\left[{a_0}^2b_1+a_1{b_1}^2\right](-2\lambda+\beta)\kappa^2-3\left[{a_0}^2b_1+a_1{b_1}^2\right]c$

$-b_1[({a_0}^2+3a_1b_1)(1+k^2)-2{a_1}^2]\lambda B^2-b_1[({a_0}^2+3a_1b_1)(1+k^2)-2{a_1}^2]\gamma B^2$

$+\left(5{a_0}^4b_1+30{a_0}^2a_1{b_1}^2+10{a_1}^2{b_1}^3\right)\delta\}sn^{-1}\xi$

$+\{-a_0\left[2a_1b_1+{b_1}^2(1+k^2)\right]\alpha B^2-a_0\left[2a_1b_1+{b_1}^2(1+k^2)\right]\beta B^2$

$+3a_0{b_1}^2(-2\lambda+\beta)\kappa^2-3a_0{b_1}^2c+2a_0b_1[2a_1-b_1(1+k^2)]\lambda B^2$

$+2a_0b_1[2a_1-b_1(1+k^2)]\gamma B^2+\left(10{a_0}^3{b_1}^2+20a_0a_1{b_1}^3\right)\delta\}sn^{-2}\xi$

$+\{-2b_1aB^2+2b_1bvB^2-b_1\left[a_1b_1+{b_1}^2(1+k^2)\right]\alpha B^2-b_1\left[a_1b_1+{b_1}^2(1+k^2)\right]\beta B^2$

$+{b_1}^3(-2\lambda+\beta)\kappa^2-{b_1}^3c+b_1\left[2{a_0}^2+4a_1b_1-{b_1}^2(1+k^2)\right]\lambda B^2$

$$+b_1\left[2a_0{}^2 + 4a_1b_1 - b_1{}^2(1+k^2)\right]\gamma B^2 + \left(10a_0{}^2b_1{}^3 + 5a_1b_1{}^4\right)\delta\}sn^{-3}\xi$$

$$+\{a_0b_1{}^2\alpha B^2 + a_0b_1{}^2\beta B^2 + 4a_0b_1{}^2\lambda B^2 + 4a_0b_1{}^2\gamma B^2 + 5a_0b_1{}^4\delta\}sn^{-4}\xi$$

$$+\{b_1{}^3\alpha B^2 + 2b_1{}^3\lambda B^2 + 2b_1{}^3\gamma B^2 + b_1{}^5\delta\}sn^{-5}\xi = 0$$

elde edilir. $\{sn^0\xi, sn^5\xi, sn^4\xi, sn^3\xi, sn^2\xi, sn\xi, sn^{-1}\xi, sn^{-2}\xi, sn^{-3}\xi, sn^{-4}\xi, sn^{-5}\xi\ \}$ kümesinin elemanları lineer bağımsız olduğundan katsayıları sıfıra eşittir. Buradan,

$$a_0a_1{}^2\alpha B^2 + a_0a_1{}^2\beta B^2 + 2a_0a_1b_1(1+k^2)\alpha B^2 + 2a_0a_1b_1(1+k^2)\beta B^2 + a_0b_1{}^2k^2\alpha B^2$$

$$+a_0b_1{}^2k^2\beta B^2 + a_0a\kappa^2 - a_0b\omega\kappa + a_0\omega + a_0{}^3(-2\lambda+\beta)\kappa^2 + 6a_0a_1b_1(-2\lambda+\beta)\kappa^2$$

$$-a_0{}^3c - 6a_0a_1b_1c - 4a_0a_1b_1(1+k^2)\lambda B^2 - 4a_0a_1b_1(1+k^2)\gamma B^2 + a_0{}^5\delta$$

$$+20a_0{}^3a_1b_1\delta + 30a_0a_1{}^2b_1{}^2\delta = 0$$

$$a_1{}^3k^2\alpha B^2 + a_1{}^3k^2\beta B^2 + 2a_1{}^3k^2\lambda B^2 + 2a_1{}^3k^2\gamma B^2 + a_1{}^5\delta = 0$$

$$a_0a_1{}^2k^2\alpha B^2 + a_0a_1{}^2k^2\beta B^2 + 4a_0a_1{}^2k^2\lambda B^2 + 4a_0a_1{}^2k^2\gamma B^2 + 5a_0a_1{}^4\delta = 0$$

$$-2a_1k^2aB^2 + 2a_1k^2bvB^2 - a_1[a_1{}^2(1+k^2) + a_1b_1k^2]\alpha B^2$$

$$-a_1[a_1{}^2(1+k^2) + a_1b_1k^2]\beta B^2 + a_1{}^3(-2\lambda+\beta)\kappa^2 - a_1{}^3c$$

$$+a_1[2a_0{}^2k^2 + 4a_1b_1k^2 - a_1{}^2(1+k^2)]\lambda B^2$$

$$+a_1[2a_0{}^2k^2 + 4a_1b_1k^2 - a_1{}^2(1+k^2)]\gamma B^2 + (10a_0{}^2a_1{}^3 + 5a_1{}^4b_1)\delta = 0$$

$$-a_0[a_1{}^2(1+k^2) + 2a_1b_1k^2]\alpha B^2 - a_0[a_1{}^2(1+k^2) + 2a_1b_1k^2]\beta B^2$$

$$+3a_0a_1{}^2(-2\lambda+\beta)\kappa^2 - 3a_0a_1{}^2c + 2a_0a_1[-a_1(1+k^2) + 2b_1k^2]\lambda B^2$$

$$+2a_0a_1[-a_1(1+k^2) + 2b_1k^2]\gamma B^2 + (10a_0{}^3a_1{}^2 + 20a_0a_1{}^3b_1)\delta = 0$$

$$a_1(1+k^2)aB^2 - a_1(1+k^2)bvB^2 + a_1a\kappa^2 - a_1b\omega\kappa + a_1\omega$$

$$+3[a_0{}^2a_1 + a_1{}^2b_1](-2\lambda+\beta)\kappa^2 - 3[a_0{}^2a_1 + a_1{}^2b_1]c$$

$$-a_1\left[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2b_1{}^2k^2\right]\lambda B^2 - a_1\left[(a_0{}^2 + 3a_1b_1)(1+k^2) - 2b_1{}^2k^2\right]\gamma B^2$$

$$+\left(5a_0{}^4a_1 + 30a_0{}^2a_1{}^2b_1 + 10a_1{}^3b_1{}^2\right)\delta = 0$$

$$b_1(1+k^2)aB^2 - b_1(1+k^2)bvB^2 - b_1[a_1{}^2 - a_1b_1(1+k^2) - b_1k^2]\alpha B^2$$

$$-b_1[a_1{}^2 - a_1b_1(1+k^2) - b_1k^2]\beta B^2 + b_1a\kappa^2 - b_1b\omega\kappa + b_1\omega$$

$$+3\left[a_0{}^2b_1 + a_1b_1{}^2\right](-2\lambda+\beta)\kappa^2 - 3\left[a_0{}^2b_1 + a_1b_1{}^2\right]c$$

$$-b_1[(a_0{}^2+3a_1b_1)(1+k^2)-2a_1{}^2]\lambda B^2-b_1[(a_0{}^2+3a_1b_1)(1+k^2)-2a_1{}^2]\gamma B^2$$

$$+\left(5a_0{}^4b_1+30a_0{}^2a_1b_1{}^2+10a_1{}^2b_1{}^3\right)\delta=0$$

$$-a_0\left[2a_1b_1+b_1{}^2(1+k^2)\right]\alpha B^2-a_0\left[2a_1b_1+b_1{}^2(1+k^2)\right]\beta B^2$$

$$+3a_0b_1{}^2(-2\lambda+\beta)\kappa^2-3a_0b_1{}^2c+2a_0b_1[2a_1-b_1(1+k^2)]\lambda B^2$$

$$+2a_0b_1[2a_1-b_1(1+k^2)]\gamma B^2+\left(10a_0{}^3b_1{}^2+20a_0a_1b_1{}^3\right)\delta=0$$

$$-2b_1aB^2+2b_1bvB^2-b_1\left[a_1b_1+b_1{}^2(1+k^2)\right]\alpha B^2-b_1\left[a_1b_1+b_1{}^2(1+k^2)\right]\beta B^2$$

$$+b_1{}^3(-2\lambda+\beta)\kappa^2-b_1{}^3c+b_1\left[2a_0{}^2+4a_1b_1-b_1{}^2(1+k^2)\right]\lambda B^2$$

$$+b_1\left[2a_0{}^2+4a_1b_1-b_1{}^2(1+k^2)\right]\gamma B^2+\left(10a_0{}^2b_1{}^3+5a_1b_1{}^4\right)\delta=0$$

$$a_0b_1{}^2\alpha B^2+a_0b_1{}^2\beta B^2+4a_0b_1{}^2\lambda B^2+4a_0b_1{}^2\gamma B^2+5a_0b_1{}^4\delta=0$$

$$b_1{}^3\alpha B^2+b_1{}^3\beta B^2+2b_1{}^3\lambda B^2+2b_1{}^3\gamma B^2+b_1{}^5\delta=0$$

eşitlikleri elde edilir.

Bu denklem sistemi çözülerek aşağıdaki çözüm setlerine ulaşılır (Biswas vd., 2018):

(i) Parametrelerin ilk seti aşağıdaki gibi verilir:

$$a_0=b_1=0\,,$$

$$a_1=\pm kB\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$\omega=\frac{\delta B^2(1+k^2)(a-bv)+a\delta\kappa^2-k^2B^4(\alpha+\beta)\left(\alpha+\beta+2(\gamma+\lambda)\right)}{\delta(b\kappa-1)} \tag{2.16}$$

$$c=\kappa^2(\beta-2\lambda)-B^2(1+k^2)(\alpha+\beta+\gamma+\lambda)+\frac{2\delta(a-bv)}{\alpha+\beta+2(\gamma+\lambda)}$$

(ii) Parametrelerin ikinci seti aşağıdaki gibi verilir:

$$a_0=a_1=0\,,$$

$$b_1=\pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$\omega=\frac{\delta B^2(1+k^2)(a-bv)+a\delta\kappa^2-k^2B^4(\alpha+\beta)\left(\alpha+\beta+2(\gamma+\lambda)\right)}{\delta(b\kappa-1)} \tag{2.17}$$

$$c=\kappa^2(\beta-2\lambda)-B^2(1+k^2)(\alpha+\beta+\gamma+\lambda)+\frac{2\delta(a-bv)}{\alpha+\beta+2(\gamma+\lambda)}$$

(iii) Parametrelerin üçüncü seti aşağıdaki gibi verilir:

$$a_0 = 0$$

$$a_1 = \pm kB\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$b_1 = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$\omega = \frac{\delta B^2\big(1+k(6+k)\big)(a-bv)+a\delta\kappa^2-4B^4k(1+m)^2(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)} \tag{2.18}$$

$$c = \frac{2\delta(a-bv)-\big(\alpha+\beta+2(\gamma+\lambda)\big)\big[B^2\big(1+m(6+m)\big)(\alpha+\beta+\gamma+\lambda)-\kappa^2(\beta-2\lambda)\big]}{\alpha+\beta+2(\gamma+\lambda)}$$

Bu sonuçlar yardımıyla, Jacobi eliptik fonksiyon çözümleri aşağıdaki gibi elde edilir:

$$q(x,t) = \pm kB\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\, sn\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x } exp\left[i\left\{-\kappa x+\left(\frac{\delta B^2\big(1+k^2\big)(a-bv)+a\delta\kappa^2-k^2B^4(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right] \tag{2.19}$$

$$q(x,t) = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\, ns\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x } exp\left[i\left\{-\kappa x+\left(\frac{\delta B^2\big(1+k^2\big)(a-bv)+a\delta\kappa^2-k^2B^4(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right] \tag{2.20}$$

$$q(x,t) = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left(ksn\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]+ns\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]\right)$$

$$\text{x } exp\left[i\left\{-\kappa x+\left(\frac{\delta B^2\big(1+k(6+k)\big)(a-bv)+a\delta\kappa^2-4B^4k(1+k)^2(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right] \tag{2.21}$$

Çözüm setleri, $\delta\big(\alpha+\beta+2(\gamma+\lambda)\big) < 0$ kısıtlama şartı sağlanması durumunda mevcut olacaktır.

(2.19)-(2.21) denklemleri k modülü 1'e yaklaştığında koyu ve tekil soliton çözümler aşağıdaki gibi ortaya çıkar (Biswas vd., 2018) :

$$q(x,t) = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\, tanh\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x } exp\left[i\left\{-\kappa x+\left(\frac{2\delta B^2(a-bv)+a\delta\kappa^2-B^4(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right], \tag{2.22}$$

$$q(x,t) = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\, coth\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x } exp\left[i\left\{-\kappa x+\left(\frac{2\delta B^2(a-bv)+a\delta\kappa^2-B^4(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right], \qquad (2.23)$$

$$q(x,t)=\pm 2B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}coth2\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x}exp\left[i\left\{-\kappa x+\left(\frac{8\delta B^2(a-bv)+a\delta\kappa^2-16B^4(\alpha+\beta)\big(\alpha+\beta+2(\gamma+\lambda)\big)}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right] \qquad (2.24)$$

Bununla birlikte, (2.20) ve (2.21) denklemlerinde $k \to 0$ ise, periyodik çözümler aşağıdaki gibi türetilir:

$$q(x,t)=\pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}csc\left[B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]$$

$$\text{x}exp\left[i\left\{-\kappa x+\left(\frac{\delta B^2(a-bv)+a\delta\kappa^2}{\delta(b\kappa-1)}\right)t+\theta_0\right\}\right] \qquad (2.25)$$

2.4. $exp\big(-\Phi(\xi)\big)$ açılım metodu

Bu açılım ile başlamak için, (2.5) denkleminin çözümüne aşağıdaki başlangıç varsayımı alınır (Biswas vd., 2018):

$$P(x,t)=P(\xi)=\sum_{j=0}^{N}A_je^{-j\Phi(\xi)}=\sum_{j=0}^{N}A_j\big[exp\big(-\Phi(\xi)\big)\big]^j$$

Bu eşitlikte $\Phi(\xi)$,

$$\Phi'(\xi)=e^{-\Phi(\xi)}+\rho e^{\Phi(\xi)}+\tau$$

denklemini sağlamaktadır.

$$P(\xi)=\sum_{j=0}^{N}A_je^{-j\Phi(\xi)}$$

eşitliğinde;

$$o(P)=N$$

dır.

$$P(\xi)=\sum_{j=0}^{N}A_je^{-j\Phi(\xi)}$$

ifadesinin 1. mertebeden türevi alınırsa

$$P'(\xi) = \sum_{j=0}^{N} A_j(-j)e^{-j\Phi(\xi)}\Phi'(\xi)$$

$$P'(\xi) = \sum_{j=0}^{N} A_j(-j)e^{-j\Phi(\xi)}\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)$$

$$P'(\xi) = \sum_{j=0}^{N} A_j(-j)\left[e^{-j\Phi(\xi)-\Phi(\xi)} + \rho e^{-j\Phi(\xi)+\Phi(\xi)} + \tau e^{-j\Phi(\xi)}\right]$$

$$P'(\xi) = \sum_{j=0}^{N} A_j(-j)\left[e^{-(j+1)\Phi(\xi)} + \rho e^{-(j-1)\Phi(\xi)} + \tau e^{-j\Phi(\xi)}\right]$$

elde edilir. Buradan

$o(P') = N + 1$

olur.

$$P(\xi) = \sum_{j=0}^{N} A_j e^{-j\Phi(\xi)}$$

eşitliğinin 2. mertebeden türevi alınırsa;

$$P''(\xi) = \sum_{j=0}^{N} A_j(-j)\left[-(j+1)e^{-(j+1)\Phi(\xi)}\Phi'(\xi)\right] + A_j(-j)\rho\left[-(j-1)e^{-(j-1)\Phi(\xi)}\Phi'(\xi)\right]$$
$$+ A_j(-j)\tau\left[(-j)e^{-j\Phi(\xi)}\Phi'(\xi)\right]$$

$$P''(\xi) = \sum_{j=0}^{N} A_j(-j)\left\{\left[-(j+1)e^{-(j+1)\Phi(\xi)}\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)\right]\right.$$
$$+ \rho[-(j-1)]e^{-(j-1)\Phi(\xi)}\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)$$
$$\left. + \tau(-j)e^{-j\Phi(\xi)}\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)\right\}$$

$$P''(\xi) = \sum_{j=0}^{N} A_j(-j)\left[-(j+1)e^{-(j+1)\Phi(\xi)-\Phi(\xi)} - \rho(j+1)e^{-(j+1)\Phi(\xi)+\Phi(\xi)}\right.$$
$$- \tau(j+1)e^{-(j+1)\Phi(\xi)} - \rho(j-1)e^{-(j-1)\Phi(\xi)-\Phi(\xi)}$$
$$- \rho^2(j-1)e^{-(j-1)\Phi(\xi)+\Phi(\xi)} - \rho\tau(j-1)e^{-(j-1)\Phi(\xi)} + \tau(-j)e^{-j\Phi(\xi)-\Phi(\xi)}$$
$$\left. + \tau\rho(-j)e^{-j\Phi(\xi)+\Phi(\xi)} + \tau^2(-j)e^{-j\Phi(\xi)}\right]$$

$$P''(\xi) = \sum_{j=0}^{N} A_j(-j)\{[-(j+1)e^{-(j+2)\Phi(\xi)} - \rho(j+1)e^{-j\Phi(\xi)} - \tau(j+1)e^{-(j+1)\Phi(\xi)}$$
$$- \rho(j-1)e^{-j\Phi(\xi)} - \rho^2(j-1)e^{-(j-2)\Phi(\xi)} - \rho\tau(j-1)e^{-(j-1)\Phi(\xi)}$$
$$+ \tau(-j)e^{-(j+1)\Phi(\xi)} + \tau\rho(-j)e^{-(j-1)\Phi(\xi)} + \tau^2(-j)e^{-j\Phi(\xi)}]\}$$

elde edilir. Buradan;

$$o(P'') = N + 2$$

olur. Balans metoduna göre, (2.5) eşitliğinde P^2P'' ile P^5 in dengelenmesi sonucunda;

$$o(P^5) = o(P^2P'')$$

eşitliği yazılabilir.

$$o(P^5) = 5N$$

$$o(P^2P'') = 2N + N + 2 = 3N + 2$$

bulunur. Bu eşitlikler, $o(P^5) = o(P^2P'')$ eşitliğinde yerine yazılırsa,

$$5N = 3N + 2$$

$$2N = 2$$

$$N = 1$$

elde edilir. Bulunan $N = 1$ değeri

$$P(\xi) = \sum_{j=0}^{N} A_j e^{-j\Phi(\xi)}$$

eşitliğinde yerine yazılırsa,

$$P(\xi) = \sum_{j=0}^{1} A_j e^{-j\Phi(\xi)}$$

$$P(\xi) = A_0 + A_1 e^{-\Phi(\xi)} \qquad (2.26)$$

elde edilir.

$$P'(\xi) = -A_1 e^{-\Phi(\xi)}\Phi'(\xi)$$

$$P'(\xi) = -A_1 e^{-\Phi(\xi)}\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)$$

$$P'(\xi) = -A_1\left(e^{-2\Phi(\xi)} + \rho + \tau e^{-\Phi(\xi)}\right)$$

$$P''(\xi) = -A_1\left(-2e^{-2\Phi(\xi)}\Phi'(\xi) - \tau e^{-\Phi(\xi)}\Phi'(\xi)\right)$$

$$P''(\xi) = -A_1\left(-2e^{-2\Phi(\xi)} - \tau e^{-\Phi(\xi)}\right)\Phi'(\xi)$$

$$P''(\xi) = -A_1\left(-2e^{-2\Phi(\xi)} - \tau e^{-\Phi(\xi)}\right)\left(e^{-\Phi(\xi)} + \rho e^{\Phi(\xi)} + \tau\right)$$

$$P''(\xi) = -A_1\left(-2e^{-3\Phi(\xi)} - 2\rho e^{-\Phi(\xi)} - 2\tau e^{-2\Phi(\xi)} - \tau e^{-2\Phi(\xi)} - \tau\rho - \tau^2 e^{-\Phi(\xi)}\right)$$

$$P''(\xi) = -A_1\left[-2e^{-3\Phi(\xi)} - 3\tau e^{-2\Phi(\xi)} - (2\rho + \tau^2)e^{-\Phi(\xi)} - \tau\rho\right]$$

$$P''(\xi) = A_1\left[2e^{-3\Phi(\xi)} + 3\tau e^{-2\Phi(\xi)} + (2\rho + \tau^2)e^{-\Phi(\xi)} + \tau\rho\right]$$

$$P^2 = {A_0}^2 + 2A_0A_1e^{-\Phi(\xi)} + {A_1}^2e^{-2\Phi(\xi)}$$

$$P^3 = {A_0}^3 + 3{A_0}^2A_1e^{-\Phi(\xi)} + 3A_0{A_1}^2e^{-2\Phi(\xi)} + {A_1}^3e^{-3\Phi(\xi)}$$

$$P^5 = {A_0}^5 + 5{A_0}^4A_1e^{-\Phi(\xi)} + 10{A_0}^3{A_1}^2e^{-2\Phi(\xi)} + 10{A_0}^2{A_1}^3e^{-3\Phi(\xi)} + 5A_0{A_1}^4e^{-4\Phi(\xi)}$$
$$+{A_1}^5e^{-5\Phi(\xi)}$$

$$(P')^2 = {A_1}^2\left(e^{-2\Phi(\xi)} + \rho + \tau e^{-\Phi(\xi)}\right)^2$$

$$(P')^2 = {A_1}^2\left[\left(e^{-2\Phi(\xi)} + \rho\right)^2 + \left(\tau e^{-\Phi(\xi)}\right)^2 + 2\left(e^{-2\Phi(\xi)} + \rho\right)\left(\tau e^{-\Phi(\xi)}\right)\right]$$

$$(P')^2 = {A_1}^2\left(e^{-4\Phi(\xi)} + 2\rho e^{-2\Phi(\xi)} + \rho^2 + \tau^2e^{-2\Phi(\xi)} + 2\tau e^{-3\Phi(\xi)} + 2\rho\tau e^{-\Phi(\xi)}\right)$$

$$(P')^2 = {A_1}^2\left[e^{-4\Phi(\xi)} + 2\tau e^{-3\Phi(\xi)} + (2\rho + \tau^2)e^{-2\Phi(\xi)} + 2\rho\tau e^{-\Phi(\xi)} + \rho^2\right]$$

$$P(P')^2 = \left(A_0 + A_1e^{-\Phi(\xi)}\right){A_1}^2\left[e^{-4\Phi(\xi)} + 2\tau e^{-3\Phi(\xi)} + (2\rho + \tau^2)e^{-2\Phi(\xi)}\right.$$
$$\left.+2\rho\tau e^{-\Phi(\xi)} + \rho^2\right]$$

$$P(P')^2 = {A_1}^2\left[A_0e^{-4\Phi(\xi)} + 2A_0\tau e^{-3\Phi(\xi)} + A_0(2\rho + \tau^2)e^{-2\Phi(\xi)} + 2A_0\rho\tau e^{-\Phi(\xi)} + A_0\rho^2\right.$$
$$+A_1e^{-5\Phi(\xi)} + 2A_1\tau e^{-4\Phi(\xi)} + A_1(2\rho + \tau^2)e^{-3\Phi(\xi)} + 2A_1\rho\tau e^{-2\Phi(\xi)}$$
$$\left.+A_1\rho^2e^{-\Phi(\xi)}\right]$$

$$P(P')^2 = {A_1}^2\{A_1e^{-5\Phi(\xi)} + (A_0 + 2A_1\tau)e^{-4\Phi(\xi)} + [2A_0\tau + A_1(2\rho + \tau^2)]e^{-3\Phi(\xi)}$$
$$+[A_0(2\rho + \tau^2) + 2A_1\rho\tau]e^{-2\Phi(\xi)} + (2A_0\rho\tau + A_1\rho^2)e^{-\Phi(\xi)} + A_0\rho^2\}$$

$$P(P')^2 = {A_1}^3e^{-5\Phi(\xi)} + \left[{A_1}^2A_0 + 2{A_1}^3\tau\right]e^{-4\Phi(\xi)} + \left[2A_0{A_1}^2\tau + {A_1}^3(2\rho + \tau^2)\right]e^{-3\Phi(\xi)}$$
$$+\left[{A_1}^2A_0(2\rho + \tau^2) + 2{A_1}^3\rho\tau\right]e^{-2\Phi(\xi)} + \left[2A_0{A_1}^2\rho\tau + {A_1}^3\rho^2\right]e^{-\Phi(\xi)}$$
$$+A_0{A_1}^2\rho^2$$

$$P^2P'' = \left({A_0}^2 + 2A_0A_1e^{-\Phi(\xi)} + {A_1}^2e^{-2\Phi(\xi)}\right)A_1[2e^{-3\Phi(\xi)} + 3\tau e^{-2\Phi(\xi)}$$

$$+(2\rho + \tau^2)e^{-\Phi(\xi)} + \tau\rho]$$

$$P^2P'' = A_1[2{A_0}^2e^{-3\Phi(\xi)} + 3{A_0}^2\tau e^{-2\Phi(\xi)} + {A_0}^2(2\rho + \tau^2)e^{-\Phi(\xi)} + {A_0}^2\tau\rho$$

$$+4A_0A_1e^{-4\Phi(\xi)} + 6A_0A_1\tau e^{-3\Phi(\xi)} + 2A_0A_1(2\rho + \tau^2)e^{-2\Phi(\xi)} + 2A_0A_1\tau\rho e^{-\Phi(\xi)}$$

$$+2{A_1}^2e^{-5\Phi(\xi)} + 3{A_1}^2\tau e^{-4\Phi(\xi)} + {A_1}^2(2\rho + \tau^2)e^{-3\Phi(\xi)} + {A_1}^2\tau\rho e^{-2\Phi(\xi)}]$$

$$P^2P'' = A_1\{2{A_1}^2e^{-5\Phi(\xi)} + \left[4A_0A_1 + 3{A_1}^2\tau\right]e^{-4\Phi(\xi)}$$

$$+\left[2{A_0}^2 + 6A_0A_1\tau + {A_1}^2(2\rho + \tau^2)\right]e^{-3\Phi(\xi)}$$

$$+\left[3{A_0}^2\tau + 2A_0A_1(2\rho + \tau^2) + {A_1}^2\tau\rho\right]e^{-2\Phi(\xi)}$$

$$+\left[{A_0}^2(2\rho + \tau^2) + 2A_0A_1\tau\rho\right]e^{-\Phi(\xi)} + {A_0}^2\tau\rho\}$$

$$P^2P'' = 2{A_1}^3e^{-5\Phi(\xi)} + \left[4A_0{A_1}^2 + 3{A_1}^3\tau\right]e^{-4\Phi(\xi)}$$

$$+\left[2{A_0}^2A_1 + 6A_0{A_1}^2\tau + {A_1}^3(2\rho + \tau^2)\right]e^{-3\Phi(\xi)}$$

$$+\left[3{A_0}^2A_1\tau + 2A_0{A_1}^2(2\rho + \tau^2) + {A_1}^3\tau\rho\right]e^{-2\Phi(\xi)}$$

$$+\left[{A_0}^2A_1(2\rho + \tau^2) + 2A_0{A_1}^2\tau\rho\right]e^{-\Phi(\xi)} + {A_0}^2A_1\tau\rho$$

Elde edilen ifadeler

$$B^2(-a + bv)P'' + B^2(\alpha + \beta)(P')^2P + (a\kappa^2 - b\omega\kappa + \omega)P + [(-2\lambda + \beta)\kappa^2 - c]P^3$$

$$+B^2(\gamma + \lambda)P''P^2 + \delta P^5 = 0$$

denkleminde yerine yazılırsa,

$$B^2(-a + bv)A_1\left[2e^{-3\Phi(\xi)} + 3\tau e^{-2\Phi(\xi)} + (2\rho + \tau^2)e^{-\Phi(\xi)} + \tau\rho\right]$$

$$+B^2(\alpha + \beta)\left\{{A_1}^3e^{-5\Phi(\xi)} + \left[{A_1}^2A_0 + 2{A_1}^3\tau\right]e^{-4\Phi(\xi)}\right.$$

$$+\left[2A_0{A_1}^2\tau + {A_1}^3(2\rho + \tau^2)\right]e^{-3\Phi(\xi)} + \left[{A_1}^2A_0(2\rho + \tau^2) + 2{A_1}^3\rho\tau\right]e^{-2\Phi(\xi)}$$

$$\left.+\left[2A_0{A_1}^2\rho\tau + {A_1}^3\rho^2\right]e^{-\Phi(\xi)} + A_0{A_1}^2\rho^2\right\} + (a\kappa^2 - b\omega\kappa + \omega)\left(A_0 + A_1e^{-\Phi(\xi)}\right)$$

$$+[(-2\lambda + \beta)\kappa^2 - c]\left[{A_0}^3 + 3{A_0}^2A_1e^{-\Phi(\xi)} + 3A_0{A_1}^2e^{-2\Phi(\xi)} + {A_1}^3e^{-3\Phi(\xi)}\right]$$

$$+B^2(\gamma + \lambda)\left\{2{A_1}^3e^{-5\Phi(\xi)} + \left[4A_0{A_1}^2 + 3{A_1}^3\tau\right]e^{-4\Phi(\xi)}\right.$$

$$+\left[2{A_0}^2A_1 + 6A_0{A_1}^2\tau + {A_1}^3(2\rho + \tau^2)\right]e^{-3\Phi(\xi)}$$

$$+\left[3{A_0}^2A_1\tau+2A_0{A_1}^2(2\rho+\tau^2)+{A_1}^3\tau\rho\right]e^{-2\Phi(\xi)}$$

$$+\left[{A_0}^2A_1(2\rho+\tau^2)+2A_0{A_1}^2\tau\rho\right]e^{-\Phi(\xi)}+{A_0}^2A_1\tau\rho\}+\delta\left({A_0}^5+5{A_0}^4A_1e^{-\Phi(\xi)}\right.$$

$$\left.+10{A_0}^3{A_1}^2e^{-2\Phi(\xi)}+10{A_0}^2{A_1}^3e^{-3\Phi(\xi)}+5A_0{A_1}^4e^{-4\Phi(\xi)}+{A_1}^5e^{-5\Phi(\xi)}\right)=0$$

elde edilir.

$$-2A_1B^2ae^{-3\Phi(\xi)}-3A_1B^2a\tau e^{-2\Phi(\xi)}+A_1B^2a(2\rho+\tau^2)e^{-\Phi(\xi)}-A_1B^2a\tau\rho$$

$$+2A_1B^2bve^{-3\Phi(\xi)}+3A_1B^2bv\tau e^{-2\Phi(\xi)}+A_1B^2bv(2\rho+\tau^2)e^{-\Phi(\xi)}+A_1B^2bv\tau\rho$$

$$+{A_1}^3B^2\alpha e^{-5\Phi(\xi)}+\left[A_0{A_1}^2+2{A_1}^3\tau\right]B^2\alpha e^{-4\Phi(\xi)}$$

$$+\left[2A_0{A_1}^2\tau+{A_1}^3(2\rho+\tau^2)\right]B^2\alpha e^{-3\Phi(\xi)}+\left[A_0{A_1}^2(2\rho+\tau^2)+2{A_1}^3\rho\tau\right]B^2\alpha e^{-2\Phi(\xi)}$$

$$+\left[2A_0{A_1}^2\rho\tau+{A_1}^3\rho^2\right]B^2\alpha e^{-\Phi(\xi)}+A_0{A_1}^2\rho^2B^2\alpha+{A_1}^3B^2\beta e^{-5\Phi(\xi)}$$

$$+\left[A_0{A_1}^2+2{A_1}^3\tau\right]B^2\beta e^{-4\Phi(\xi)}+\left[2A_0{A_1}^2\tau+{A_1}^3(2\rho+\tau^2)\right]B^2\beta e^{-3\Phi(\xi)}$$

$$+\left[A_0{A_1}^2(2\rho+\tau^2)+2{A_1}^3\rho\tau\right]B^2\beta e^{-2\Phi(\xi)}+\left[2A_0{A_1}^2\rho\tau+{A_1}^3\rho^2\right]B^2\beta e^{-\Phi(\xi)}$$

$$+A_0{A_1}^2\rho^2B^2\beta+A_0a\kappa^2+A_1a\kappa^2e^{-\Phi(\xi)}-A_0b\omega\kappa-A_1b\omega\kappa e^{-\Phi(\xi)}+A_0\omega+A_1\omega e^{-\Phi(\xi)}$$

$$+{A_0}^3(-2\lambda+\beta)\kappa^2+3{A_0}^2A_1(-2\lambda+\beta)\kappa^2e^{-\Phi(\xi)}+3A_0{A_1}^2(-2\lambda+\beta)\kappa^2e^{-2\Phi(\xi)}$$

$$+{A_1}^3(-2\lambda+\beta)\kappa^2e^{-3\Phi(\xi)}-{A_0}^3c-3{A_0}^2A_1ce^{-\Phi(\xi)}-3A_0{A_1}^2ce^{-2\Phi(\xi)}-{A_1}^3ce^{-3\Phi(\xi)}$$

$$+2{A_1}^3B^2\gamma e^{-5\Phi(\xi)}+\left[4A_0{A_1}^2+3{A_1}^3\tau\right]B^2\gamma e^{-4\Phi(\xi)}$$

$$+\left[2{A_0}^2A_1+6A_0{A_1}^2\tau+{A_1}^3(2\rho+\tau^2)\right]B^2\gamma e^{-3\Phi(\xi)}$$

$$+\left[3{A_0}^2A_1\tau+2A_0{A_1}^2(2\rho+\tau^2)+{A_1}^3\tau\rho\right]B^2\gamma e^{-2\Phi(\xi)}$$

$$+\left[{A_0}^2A_1(2\rho+\tau^2)+2A_0{A_1}^2\tau\rho\right]B^2\gamma e^{-\Phi(\xi)}+{A_0}^2A_1\tau\rho B^2\gamma+2{A_1}^3B^2\lambda e^{-5\Phi(\xi)}$$

$$+\left[4A_0{A_1}^2+3{A_1}^3\tau\right]B^2\lambda e^{-4\Phi(\xi)}+\left[2{A_0}^2A_1+6A_0{A_1}^2\tau+{A_1}^3(2\rho+\tau^2)\right]B^2\lambda e^{-3\Phi(\xi)}$$

$$+\left[3{A_0}^2A_1\tau+2A_0{A_1}^2(2\rho+\tau^2)+{A_1}^3\tau\rho\right]B^2\lambda e^{-2\Phi(\xi)}$$

$$+\left[{A_0}^2A_1(2\rho+\tau^2)+2A_0{A_1}^2\tau\rho\right]B^2\lambda e^{-\Phi(\xi)}+{A_0}^2A_1\tau\rho B^2\lambda+{A_0}^5\delta+5{A_0}^4A_1\delta e^{-\Phi(\xi)}$$

$$+10{A_0}^3{A_1}^2\delta e^{-2\Phi(\xi)}+10{A_0}^2{A_1}^3\delta e^{-3\Phi(\xi)}+5A_0{A_1}^4\delta e^{-4\Phi(\xi)}+{A_1}^5\delta e^{-5\Phi(\xi)}=0$$

denklemi düzenlenirse,

$$\left\{{A_1}^3B^2\alpha+{A_1}^3B^2\beta+2{A_1}^3B^2\gamma+2{A_1}^3B^2\lambda+{A_1}^5\delta\right\}e^{-5\Phi(\xi)}+\left\{\left[A_0{A_1}^2+2{A_1}^3\tau\right]B^2\alpha\right.$$

$$+\left[A_0A_1^{\,2}+2A_1^{\,3}\tau\right]B^2\beta+\left[4A_0A_1^{\,2}+3A_1^{\,3}\tau\right]B^2\gamma+\left[4A_0A_1^{\,2}+3A_1^{\,3}\tau\right]B^2\lambda$$

$$+5A_0A_1^{\,4}\delta\}e^{-4\Phi(\xi)}+\{-2A_1B^2a+2A_1B^2bv+\left[2A_0A_1^{\,2}\tau+A_1^{\,3}(2\rho+\tau^2)\right]B^2\alpha$$

$$+\left[2A_0A_1^{\,2}\tau+A_1^{\,3}(2\rho+\tau^2)\right]B^2\beta+A_1^{\,3}(-2\lambda+\beta)\kappa^2-A_1^{\,3}c$$

$$+\left[2A_0^{\,2}A_1+6A_0A_1^{\,2}\tau+A_1^{\,3}(2\rho+\tau^2)\right]B^2\gamma+\left[2A_0^{\,2}A_1+6A_0A_1^{\,2}\tau+A_1^{\,3}(2\rho+\tau^2)\right]B^2\lambda$$

$$+10A_0^{\,2}A_1^{\,3}\delta\}e^{-3\Phi(\xi)}+\{-3A_1B^2a\tau+3A_1B^2bv\tau+\left[A_0A_1^{\,2}(2\rho+\tau^2)+2A_1^{\,3}\rho\tau\right]B^2\alpha$$

$$+\left[A_0A_1^{\,2}(2\rho+\tau^2)+2A_1^{\,3}\rho\tau\right]B^2\beta+3A_0A_1^{\,2}(-2\lambda+\beta)\kappa^2-3A_0A_1^{\,2}c$$

$$+\left[3A_0A_1^{\,2}\tau+2A_0A_1^{\,2}(2\rho+\tau^2)+A_1^{\,3}\rho\tau\right]B^2\gamma+3A_0A_1^{\,2}c$$

$$+\left[3A_0A_1^{\,2}\tau+2A_0A_1^{\,2}(2\rho+\tau^2)+A_1^{\,3}\rho\tau\right]B^2\lambda+10A_0^{\,3}A_1^{\,2}\delta\}e^{-2\Phi(\xi)}$$

$$+\{-A_1B^2a(2\rho+\tau^2)+A_1B^2bv(2\rho+\tau^2)+\left[2A_0A_1^{\,2}\rho\tau+A_1^{\,3}\rho^2\right]B^2\alpha$$

$$+\left[2A_0A_1^{\,2}\rho\tau+A_1^{\,3}\rho^2\right]B^2\beta+A_1a\kappa^2-A_1b\omega\kappa+A_1\omega+3A_0^{\,2}A_1(-2\lambda+\beta)\kappa^2$$

$$-3A_0^{\,2}A_1c+\left[A_0^{\,2}A_1(2\rho+\tau^2)+2A_0A_1^{\,2}\rho\tau\right]B^2\gamma+\left[A_0^{\,2}A_1(2\rho+\tau^2)+2A_0A_1^{\,2}\rho\tau\right]B^2\lambda$$

$$+5A_0A_1^{\,4}\delta\}e^{-\Phi(\xi)}+\{-A_1B^2a\rho\tau+A_1B^2bv\rho\tau+A_0A_1^{\,2}\rho^2B^2\alpha+A_0A_1^{\,2}\rho^2B^2\beta+A_0a\kappa$$

$$-A_0b\omega\kappa+A_0\omega+A_0^{\,3}(-2\lambda+\beta)\kappa^2-A_0^{\,3}c+A_0^{\,2}A_1\rho\tau B^2\gamma+A_0^{\,2}A_1\rho\tau B^2\lambda+A_0^{\,5}\delta\}=0$$

elde edilir.

$$\Phi'(\xi)=e^{-\Phi(\xi)}+\rho e^{\Phi(\xi)}+\tau$$

Riccati diferansiyel denkleminin bazı şartlar altında çözümü incelenirse,

$$\frac{d\Phi(\xi)}{d\xi}=e^{-\Phi(\xi)}+\rho e^{\Phi(\xi)}+\tau$$

$$\frac{d\Phi(\xi)}{e^{-\Phi(\xi)}+\rho e^{\Phi(\xi)}+\tau}=d\xi$$

$$\int\frac{d\Phi(\xi)}{e^{-\Phi(\xi)}+\rho e^{\Phi(\xi)}+\tau}=\int d\xi$$

$$\int\frac{e^{\Phi(\xi)}d\Phi(\xi)}{1+\rho e^{2\Phi(\xi)}+\tau e^{\Phi(\xi)}}=\xi+c$$

$$\frac{1}{\rho}\int\frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left[e^{2\Phi(\xi)}+\frac{\tau e^{\Phi(\xi)}}{\rho}+\frac{1}{\rho}\right]}=\xi+c$$

$$\frac{1}{\rho}\int \frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left(e^{\Phi(\xi)}+\frac{\tau}{2\rho}\right)^2+\frac{1}{\rho}-\frac{\tau^2}{4\rho^2}}=\xi+c$$

$$\frac{1}{\rho}\int \frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left(e^{\Phi(\xi)}+\frac{\tau}{2\rho}\right)^2+\frac{4\rho-\tau^2}{4\rho^2}}=\xi+c$$

i) $4\rho-\tau^2>0$ ise,

$$e^{\Phi(\xi)}+\frac{\tau}{2\rho}=\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}tgu$$

$$e^{\Phi(\xi)}d\Phi(\xi)=\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}(1+tg^2u)du=\frac{1}{\rho}\int\frac{\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}(1+tg^2u)du}{\frac{4\rho-\tau^2}{4\rho^2}tg^2u+\frac{4\rho-\tau^2}{4\rho^2}}$$

$$=\frac{1}{\rho\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}}\int du=\xi+c$$

$$\frac{2}{\sqrt{4\rho-\tau^2}}u=\xi+c$$

$$\frac{2}{\sqrt{4\rho-\tau^2}}arctg\left(\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}}\right)=\xi+c$$

$$arctg\left(\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}}\right)=\frac{(\xi+c)\sqrt{4\rho-\tau^2}}{2}$$

$$\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}}=tg\left[\frac{(\xi+c)\sqrt{4\rho-\tau^2}}{2}\right]$$

$$e^{\Phi(\xi)}=-\frac{\tau}{2\rho}+\sqrt{\frac{4\rho-\tau^2}{4\rho^2}}\ tg\left[\frac{(\xi+c)\sqrt{4\rho-\tau^2}}{2}\right]$$

$$\Phi(\xi) = ln\left[\frac{-\tau+\sqrt{4\rho-\tau^2}tg\left[\frac{(\xi+c)\sqrt{4\rho-\tau^2}}{2}\right]}{2\rho}\right] \tag{2.27}$$

bulunur.

ii) $4\rho - \tau^2 < 0$ ise,

$$\frac{1}{\rho}\int \frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left(e^{\Phi(\xi)}+\frac{\tau}{2\rho}\right)^2+\frac{4\rho-\tau^2}{4\rho^2}} = \xi + c$$

$$\frac{1}{\rho}\int \frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left(e^{\Phi(\xi)}+\frac{\tau}{2\rho}\right)^2-\left(\frac{\tau^2-4\rho}{4\rho^2}\right)} = \xi + c$$

$$e^{\Phi(\xi)}+\frac{\tau}{2\rho} = \sqrt{\frac{\tau^2-4\rho}{4\rho^2}}thu$$

$$e^{\Phi(\xi)}d\Phi(\xi) = \sqrt{\frac{\tau^2-4\rho}{4\rho^2}}(1-th^2u)du$$

$$\frac{1}{\rho}\int \frac{\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}(1-th^2u)du}{\frac{\tau^2-4\rho}{4\rho^2}th^2u-\left(\frac{\tau^2-4\rho}{4\rho^2}\right)} = \xi + c$$

$$-\frac{2}{\sqrt{\tau^2-4\rho}}\int du = \xi + c$$

$$-\frac{2}{\sqrt{\tau^2-4\rho}}u = \xi + c$$

$$-\frac{2}{\sqrt{\tau^2-4\rho}}\left[argth\left(\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}}\right)\right] = \xi + c$$

$$argth\left(\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}}\right)=-\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}$$

$$\frac{e^{\Phi(\xi)}+\frac{\tau}{2\rho}}{\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}}=th\left[-\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}\right]$$

$$e^{\Phi(\xi)}+\frac{\tau}{2\rho}=\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}\,th\left[-\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}\right]$$

$$e^{\Phi(\xi)}+\frac{\tau}{2\rho}=-\sqrt{\frac{\tau^2-4\rho}{4\rho^2}}\,th\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}$$

$$e^{\Phi(\xi)}=\frac{-\tau-\sqrt{\tau^2-4\rho}\,th\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}}{2\rho}$$

$$\Phi(\xi)=ln\left(\frac{-\tau-\sqrt{\tau^2-4\rho}\,th\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}}{2\rho}\right)$$

$$\Phi(\xi)=ln\left(-\frac{\tau+\sqrt{\tau^2-4\rho}\,th\frac{(\xi+c)\sqrt{\tau^2-4\rho}}{2}}{2\rho}\right) \tag{2.28}$$

bulunur.

iii) $4\rho-\tau^2=0,\ \rho\neq 0,\ \tau\neq 0$ ise,

$$\frac{1}{\rho}\int\frac{e^{\Phi(\xi)}d\Phi(\xi)}{\left(e^{\Phi(\xi)}+\frac{\tau}{2\rho}\right)^2}=\xi+c$$

$$e^{\Phi(\xi)}+\frac{\tau}{2\rho}=u$$

olsun.

$$e^{\Phi(\xi)}d\Phi(\xi)=du$$

olur.

$$\frac{1}{\rho}\int \frac{du}{u^2} = \xi + c$$

$$-\frac{1}{\rho u} = \xi + c \quad \Rightarrow u = -\frac{1}{\rho(\xi + c)}$$

$$e^{\Phi(\xi)} + \frac{\tau}{2\rho} = -\frac{1}{\rho(\xi + c)}$$

$$e^{\Phi(\xi)} = -\frac{\tau}{2\rho} - \frac{1}{\rho(\xi + c)}$$

$$\Phi(\xi) = ln\left[-\frac{\tau}{2\rho} - \frac{1}{\rho(\xi + c)}\right]$$

$4\rho = \tau^2$ olduğundan,

$$\Phi(\xi) = ln\left[-\frac{2}{\tau} - \frac{4}{\tau^2(\xi+c)}\right] = ln\left[-\frac{2\tau(\xi+c)-4}{\tau^2(\xi+c)}\right] = ln\left\{\frac{-2[\tau(\xi+c)+2]}{\tau^2(\xi+c)}\right\} \tag{2.29}$$

bulunur.

iv) $\rho = 0,\ \tau = 0,\ \tau^2 - 4\rho = 0$ ise,

$$\int \frac{e^{\Phi(\xi)} d\Phi(\xi)}{1 + \rho e^{2\Phi(\xi)} + \tau e^{\Phi(\xi)}} = \xi + c$$

$$\int e^{\Phi(\xi)} d\Phi(\xi) = \xi + c$$

$$e^{\Phi(\xi)} = \xi + c$$

$$\Phi(\xi) = ln(\xi + c) \tag{2.30}$$

bulunur.

v) $\rho = 0,\ \tau \neq 0,\ \tau^2 - 4\rho > 0$ ise,

$$\int \frac{e^{\Phi(\xi)} d\Phi(\xi)}{1 + \rho e^{2\Phi(\xi)} + \tau e^{\Phi(\xi)}} = \xi + c$$

$$\int \frac{e^{\Phi(\xi)} d\Phi(\xi)}{1 + \tau e^{\Phi(\xi)}} = \xi + c$$

$$1 + \tau e^{\Phi(\xi)} = u$$

olsun.

$$\tau e^{\Phi(\xi)} d\Phi(\xi) = du$$

olur.

$$\int \frac{du}{\tau u} = \xi + c$$

$$lnu = \tau(\xi + c)$$

$$u = e^{\tau(\xi+c)}$$

$$1 + \tau e^{\Phi(\xi)} = e^{\tau(\xi+c)}$$

$$\tau e^{\Phi(\xi)} = e^{\tau(\xi+c)} - 1$$

$$e^{\Phi(\xi)} = \frac{e^{\tau(\xi+c)} - 1}{\tau}$$

$$\Phi(\xi) = ln\left[\frac{e^{\tau(\xi+c)}-1}{\tau}\right] = -ln\left[\frac{\tau}{e^{\tau(\xi+c)}-1}\right] \qquad (2.31)$$

bulunur.

$$P(\xi) = A_0 + A_1 e^{-\Phi(\xi)}$$

ifadesinin indirgenmiş (2.11) denkleminde yerine yazılması, $exp(-\Phi(\xi))$'nin her kuvvetinin katsayılarının düzenlenmesi ve elde edilen sistemin çözülmesiyle aşağıdaki çözümler elde edilir:

$$A_0 = \pm\frac{\tau B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$A_1 = \pm B\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}$$

$$\omega = \frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)} \qquad (2.32)$$

$$c = \frac{4\delta(a-bv)+(\alpha+\beta+2(\gamma+\lambda))[2\kappa^2(\beta-2\lambda)+B^2(\alpha+\beta+\gamma+\lambda)(4\rho-\tau^2)]}{2(\alpha+\beta+2(\gamma+\lambda))}$$

Burada ρ ve τ keyfi sabitlerdir. (2.32) denklemini (2.26) denkleminde yerine koyarak çözüm formülü (2.26) denklemi aşağıdaki gibi yeniden yazılabilir:

$$P(\xi) = \pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\{\tau + 2e^{-\Phi(\xi)}\} \qquad (2.33)$$

Bu sonuçlar ışığında hareket eden dalga çözümleri şu şekilde elde edilir:

$$q(x,t)=\pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left\{\tau-\frac{4\rho}{\sqrt{\tau^2-4\rho}\tanh\left[\frac{\sqrt{\tau^2-4\rho}}{2}\left(B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)+\xi\right)\right]+\tau}\right\}$$

$$\mathrm{x}e^{i\left\{-\kappa x+\left(\frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)}\right)t+\theta_0\right\}} \tag{2.34}$$

$$q(x,t)=\pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left\{\tau+\frac{4\rho}{\sqrt{4\rho-\tau^2}\tan\left[\frac{\sqrt{4\rho-\tau^2}}{2}\left(B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)+\xi\right)\right]-\tau}\right\}$$

$$\mathrm{x}e^{i\left\{-\kappa x+\left(\frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)}\right)t+\theta_0\right\}} \tag{2.35}$$

$$q(x,t)=\pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left\{\tau+\frac{2\tau}{exp\left[\tau\left(B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)+\xi\right)\right]-1}\right\}$$

$$\mathrm{x}e^{i\left\{-\kappa x+\left(\frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)}\right)t+\theta_0\right\}} \tag{2.36}$$

$$q(x,t)=\pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left\{\frac{2\tau}{\tau\left(B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)+\xi\right)+2}\right\}$$

$$\mathrm{x}e^{i\left\{-\kappa x+\left(\frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)}\right)t+\theta_0\right\}} \tag{2.37}$$

$$q(x,t)=\pm\frac{B}{2}\sqrt{-\frac{\alpha+\beta+2(\gamma+\lambda)}{\delta}}\left\{\tau+\frac{2}{B\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)+\xi}\right\}$$

$$\mathrm{x}e^{i\left\{-\kappa x+\left(\frac{8a\delta\left(2\kappa^2+B^2(\tau^2-4\rho)\right)+B^2(4\rho-\tau^2)[8bv\delta-B^2(\alpha+\beta)(\alpha+\beta+2(\gamma+\lambda))(4\rho-\tau^2)]}{16\delta(b\kappa-1)}\right)t+\theta_0\right\}} \tag{2.38}$$

Bu solitonların aşağıdaki kısıtlamalar için var olduğunu gözlemlemek önemlidir

$\delta\big(\alpha+\beta+2(\gamma+\lambda)\big)<0$

3. GENİŞLETİLMİŞ JACOBİ ELİPTİK FONKSİYON AÇILIM ŞEMASI İLE LAKSHMANAN-PORSEZİAN-DANİEL MODELİ İÇİN ÇİFT KIRILIMLI FİBERLERDE OPTİK SOLİTONLAR

3.1. Yönetim Modeli

Kerr Yasası nonlineerliğe sahip LPD denklemi boyutsuz yapısıyla aşağıdaki gibi verilmiştir (Ekici, 2018):

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2q = \sigma q_{xxxx} + \alpha(q_x)^2q^* + \beta|q_x|^2q + \gamma|q|^2q_{xx} + \lambda q^2q_{xx}^* \\ +\delta|q|^4q \qquad (3.1)$$

(3.1) denkleminde a ve b katsayıları sırasıyla grup hızı dağılımını ve uzaysal-zamansal dağılımı ifade etmektedir. Daha sonra, c Kerr yasası nonlineerlik katsayısıdır ve σ dördüncü dereceden dağılımdır. δ ise iki foton emilimini açıklar. Geri kalan terimler diğer dispersif fenomen biçimlerinden kaynaklanmaktadır (Bansal vd., 2018). Solitonlar, dağılma ve nonlineerlik etkiler arasındaki hassas bir dengenin sonucudur. Bu nedenle, çift kırılımlı fiberler için model LPD'nin çift vektör formuna yol açan iki bileşene ayrılır. 4WM'nin etkilerini göz ardı ederek bu birleşik sistem (Arshed vd., 2018; Biswas vd.,2018; Guzmanvd., 2018) aşağıdaki şekli alır:

$$iq_t + a_1q_{xx} + b_1q_{xt} + (c_1|q|^2 + d_1|r|^2)q = \sigma_1q_{xxxx} + (\alpha_1q_x^2 + \beta_1r_x^2)q^* \\ +(\gamma_1|q_x|^2 + \delta_1|r_x|^2)q + (\lambda_1|q|^2 + \theta_1|r|^2)q_{xx} + (\xi_1q^2 + \eta_1r^2)q_{xx}^* \\ +(f_1|q|^4 + g_1|q|^2|r|^2 + h_1|r|^4)q \qquad (3.2)$$

$$ir_t + a_2r_{xx} + b_2r_{xt} + (c_2|r|^2 + d_2|q|^2)r = \sigma_2r_{xxxx} + (\alpha_2r_x^2 + \beta_2q_x^2)r^* \\ +(\gamma_2|r_x|^2 + \delta_2|q_x|^2)r + (\lambda_2|r|^2 + \theta_2|q|^2)r_{xx} + (\xi_2r^2 + \eta_2q^2)r_{xx}^* \\ +(f_2|r|^4 + g_2|r|^2|q|^2 + h_2|q|^4)r \qquad (3.3)$$

(3.2) ve (3.3) denklemlerinden $j = 1,2$ için c_j ve f_j self-faz modülasyonunu açıklarken d_j, g_j ve h_j katsayıları çapraz faz modülasyonel etkiden gelir.

3.2. Matematiksel Önbilgiler

Bu birleşik sistemin entegrasyonuna başlamak için hipotez,

$$q(x,t) = U_1(\tau)e^{i\phi} \qquad (3.4)$$

$$r(x,t) = U_2(\tau)e^{i\phi} \qquad (3.5)$$

şeklinde seçilir. Burada $l=1,2$ için $U_l(\tau)$ soliton genlik bileşenini temsil eder. v soliton hızı olmak üzere,

$$\tau = x - vt \tag{3.6}$$

faz kısmı ϕ şu şekilde tanımlanır,

$$\phi = -\kappa x + \omega t + \theta \tag{3.7}$$

Burada κ soliton frekansını, ω soliton dalga sayısını ve θ faz sabitini gösterir.

(3.4) ve (3.5) denklemlerini (3.2) ve (3.3) denklemlerinde yerine koyarak, reel ve imajiner kısımlara ayırarak sırasıyla reel ve imajiner kısımlar aşağıdaki gibi oluşur,

$$q_t = -vU_l' e^{i(-\kappa x+\omega t+\theta)} + U_l i\omega e^{i(-\kappa x+\omega t+\theta)} = (-vU_l' + i\omega U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_x = U_l' e^{i(-\kappa x+\omega t+\theta)} - i\kappa U_l e^{i(-\kappa x+\omega t+\theta)} = (U_l' - i\kappa U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xx} = (U_l'' - i\kappa U_l')e^{i(-\kappa x+\omega t+\theta)} + (U_l' - i\kappa U_l)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xx} = (U_l'' - i\kappa U_l' - i\kappa U_l' - \kappa^2 U_l)e^{i(-\kappa x+\omega t+\theta)} = (U_l'' - 2i\kappa U_l' - \kappa^2 U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xt} = [-vU_l'' - i\kappa(-v)U_l']e^{i(-\kappa x+\omega t+\theta)} + (U_l' - i\kappa U_l)(i\omega)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xt} = (-vU_l'' + i\kappa v U_l' + i\omega U_l' + \kappa\omega U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxx} = (U_l''' - 2i\kappa U_l'' - \kappa^2 U_l')e^{i(-\kappa x+\omega t+\theta)} + (U_l'' - 2i\kappa U_l' - \kappa^2 U_l)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxx} = (U_l''' - 2i\kappa U_l'' - \kappa^2 U_l' - i\kappa U_l'' - 2\kappa^2 U_l' + i\kappa^3 U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxx} = (U_l''' - 3i\kappa U_l'' - 3\kappa^2 U_l' + i\kappa^3 U_l)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxxx} = \left(U_l^{(4)} - 3i\kappa U_l''' - 3\kappa^2 U_l'' + i\kappa^3 U_l'\right)e^{i(-\kappa x+\omega t+\theta)}$$
$$+(U_l''' - 3i\kappa U_l'' - 3\kappa^2 U_l' + i\kappa^3 U_l)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxxx} = \left(U_l^{(4)} - 3i\kappa U_l''' - 3\kappa^2 U_l'' + i\kappa^3 U_l' - i\kappa U_l''' - 3\kappa^2 U_l'' + 3i\kappa^3 U_l'\right.$$
$$\left.+\kappa^4 U_l\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$q_{xxxx} = \left(U_l^{(4)} - 4i\kappa U_l''' - 6\kappa^2 U_l'' + 4i\kappa^3 U_l' + \kappa^4 U_l\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$(q_x)^2 = [(U_l')^2 - 2i\kappa U_l' U_l - \kappa^2 U_l^2]e^{2i(-\kappa x+\omega t+\theta)}$$

$$|q| = |U_l|\left|e^{i(-\kappa x+\omega t+\theta)}\right|$$

$$|q| = |U_l|, \qquad U_l > 0 \text{ iken } |q| = U_l$$

$$|q|^2 = U_l^2$$

$$|q|^4 = U_l^4$$

$$|q_x| = |U_l' - i\kappa U_l|\left|e^{i(-\kappa x+\omega t+\theta)}\right|$$

$$|q_x| = \sqrt{(U_l')^2 + \kappa^2 U_l^2}$$

$$|q_x|^2 = (U_l')^2 + \kappa^2 U_l^2$$

$$q_{xx}^* = (U_l'' - \kappa^2 U_l + 2i\kappa U_l')e^{-i(-\kappa x+\omega t+\theta)}$$

$$q^2 = U_l^2 e^{2i(-\kappa x+\omega t+\theta)}$$

$$q^* = U_l(\tau)e^{-i(-\kappa x+\omega t+\theta)}$$

$$r_t = -vU_{\bar{l}}' e^{i(-\kappa x+\omega t+\theta)} + U_{\bar{l}} i\omega e^{i(-\kappa x+\omega t+\theta)}$$

$$r_t = \left(= -vU_{\bar{l}}' + i\omega U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_x = U_{\bar{l}}' e^{i(-\kappa x+\omega t+\theta)} - i\kappa U_{\bar{l}} e^{i(-\kappa x+\omega t+\theta)} = \left(U_{\bar{l}}' - i\kappa U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xx} = \left(U_{\bar{l}}'' - i\kappa U_{\bar{l}}'\right)e^{i(-\kappa x+\omega t+\theta)} + \left(U_{\bar{l}}' - i\kappa U_{\bar{l}}\right)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xx} = \left(U_{\bar{l}}'' - i\kappa U_{\bar{l}}' - i\kappa U_{\bar{l}}' - \kappa^2 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)} = \left(U_{\bar{l}}'' - 2i\kappa U_{\bar{l}}' - \kappa^2 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xt} = \left(-vU_{\bar{l}}'' + iv\kappa U_{\bar{l}}'\right)e^{i(-\kappa x+\omega t+\theta)} + \left(U_{\bar{l}}' - i\kappa U_{\bar{l}}\right)(i\omega)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xt} = \left(-vU_{\bar{l}}'' + iv\kappa U_{\bar{l}}' + i\omega U_{\bar{l}}' + \kappa\omega U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xt} = \left[-vU_{\bar{l}}'' + i(v\kappa + \omega)U_{\bar{l}}' + \kappa\omega U_{\bar{l}}\right]e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxx} = \left(U_{\bar{l}}''' - 2i\kappa U_{\bar{l}}'' - \kappa^2 U_{\bar{l}}'\right)e^{i(-\kappa x+\omega t+\theta)} + \left(U_{\bar{l}}'' - 2i\kappa U_{\bar{l}}' - \kappa^2 U_{\bar{l}}\right)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxx} = \left(U_{\bar{l}}''' - 2i\kappa U_{\bar{l}}'' - \kappa^2 U_{\bar{l}}' - i\kappa U_{\bar{l}}'' - 2\kappa^2 U_{\bar{l}}' + i\kappa^3 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxx} = \left(U_{\bar{l}}''' - 3i\kappa U_{\bar{l}}'' - 3\kappa^2 U_{\bar{l}}' + i\kappa^3 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxxx} = \left(U_{\bar{l}}^{(4)} - 3i\kappa U_{\bar{l}}''' - 3\kappa^2 U_{\bar{l}}'' + i\kappa^3 U_{\bar{l}}'\right)e^{i(-\kappa x+\omega t+\theta)}$$
$$+\left(U_{\bar{l}}''' - 3i\kappa U_{\bar{l}}'' - 3\kappa^2 U_{\bar{l}}' + i\kappa^3 U_{\bar{l}}\right)(-i\kappa)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxxx} = \left(U_{\bar{l}}^{(4)} - 3i\kappa U_{\bar{l}}''' - 3\kappa^2 U_{\bar{l}}'' + i\kappa^3 U_{\bar{l}}' - i\kappa U_{\bar{l}}''' - 3\kappa^2 U_{\bar{l}}'' + 3i\kappa^3 U_{\bar{l}}'\right.$$
$$\left.+\kappa^4 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$r_{xxxx} = \left(U_{\bar{l}}^{(4)} - 4i\kappa U_{\bar{l}}''' - 6\kappa^2 U_{\bar{l}}'' + 4i\kappa^3 U_{\bar{l}}' + \kappa^4 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$(r_x)^2 = \left[\left(U_{\bar{l}}'\right)^2 - 2i\kappa U_{\bar{l}}U_{\bar{l}}' - \kappa^2 U_{\bar{l}}^2\right] e^{2i(-\kappa x+\omega t+\theta)}$

$|r| = |U_{\bar{l}}|\left|e^{i(-\kappa x+\omega t+\theta)}\right|$

$|r| = |U_{\bar{l}}|\,,\quad U_{\bar{l}} > 0$ iken $|r| = U_{\bar{l}}$

$|r|^2 = U_{\bar{l}}^2$

$|r|^4 = U_{\bar{l}}^4$

$r^* = U_{\bar{l}}e^{-i(-\kappa x+\omega t+\theta)}$

$|r_x| = \left|U_{\bar{l}}' - i\kappa U_{\bar{l}}\right|$

$|r_x|^2 = \left(U_{\bar{l}}'\right)^2 + \kappa^2 U_{\bar{l}}^2$

$r_{xx}^* = \left(U_{\bar{l}}'' - \kappa^2 U_{\bar{l}} + 2i\kappa U_{\bar{l}}'\right)e^{-i(-\kappa x+\omega t+\theta)}$

$r^2 = U_{\bar{l}}^2 e^{2i(-\kappa x+\omega t+\theta)}$

Elde edilen q_t , q_{xx} , q_{xt} , $|q|^2$, $|r|^2$, q_{xxxx} , q_x^2 , r_x^2 , q^* , $|q_x|^2$, $|r_x|^2$, q_{xx} , q^2 , r^2 , q_{xx}^* , $|q|^4$, $|r|^4$ ifadeleri (3.2) denkleminde yerine yazılırsa,

$i(-vU_l' + i\omega U_l)e^{i(-\kappa x+\omega t+\theta)} + a_l(U_l'' - 2i\kappa U_l' - \kappa^2 U_l)e^{i(-\kappa x+\omega t+\theta)}$

$+b_l(-vU_l'' + i\kappa v U_l' + i\omega U_l' + \kappa\omega U_l)e^{i(-\kappa x+\omega t+\theta)} + \left(c_l U_l^2 + d_l U_{\bar{l}}^2\right)U_l e^{i(-\kappa x+\omega t+\theta)}$

$= \sigma_l\left(U_l^{(4)} - 4i\kappa U_l''' - 6\kappa^2 U_l'' + 4i\kappa^3 U_l' + \kappa^4 U_l\right)e^{i(-\kappa x+\omega t+\theta)}$

$+\{\alpha_l[(U_l')^2 - 2i\kappa U_l' U_l - \kappa^2 U_l^2]e^{2i(-\kappa x+\omega t+\theta)}$

$+\beta_l\left[\left(U_{\bar{l}}'\right)^2 - 2i\kappa U_{\bar{l}}U_{\bar{l}}' - \kappa^2 U_{\bar{l}}^2\right] e^{2i(-\kappa x+\omega t+\theta)}\}U_l e^{-i(-\kappa x+\omega t+\theta)}$

$+\left\{\gamma_l[(U_l')^2 + \kappa^2 U_l^2] + \delta_l\left[\left(U_{\bar{l}}'\right)^2 + \kappa^2 U_{\bar{l}}^2\right]\right\}U_l e^{i(-\kappa x+\omega t+\theta)}$

$+\left(\lambda_l U_l^2 + \theta_l U_{\bar{l}}^2\right)(U_l'' - 2i\kappa U_l' - \kappa^2 U_l)e^{i(-\kappa x+\omega t+\theta)}$

$+\left(\xi_l U_l^2 e^{2i(-\kappa x+\omega t+\theta)} + \eta_l U_{\bar{l}}^2 e^{2i(-\kappa x+\omega t+\theta)}\right)(U_l'' - \kappa^2 U_l + 2i\kappa U_l')e^{-i(-\kappa x+\omega t+\theta)}$

$+\left(f_l U_l^4 + g_l U_l^2 U_{\bar{l}}^2 + h_l U_{\bar{l}}^4\right)U_l e^{i(-\kappa x+\omega t+\theta)}$

elde edilir. Bu ifade düzenlenirse,

$(-ivU_l' - \omega U_l + a_l U_l'' - 2ia_l\kappa U_l' - a_l\kappa^2 U_l - b_l v U_l'' + ib_l\kappa v U_l' + ib_l\omega U_l' + b_l\kappa\omega U_l$

$+c_l U_l^3 + d_l U_l U_{\bar{l}}^2)e^{i(-\kappa x+\omega t+\theta)}$

$$= [\sigma_l U_l^{(4)} - 4i\sigma_l \kappa U_l''' - 6\sigma_l \kappa^2 U_l'' + 4i\sigma_l \kappa^3 U_l' + \sigma_l \kappa^4 U_l + \alpha_l U_l (U_l')^2 - 2i\alpha_l \kappa U_l^2 U_l'$$

$$-\alpha_l \kappa^2 U_l^3 + \beta_l U_l \left(U_{\bar{l}}'\right)^2 - 2i\beta_l \kappa U_l U_{\bar{l}} U_{\bar{l}}' - \beta_l \kappa^2 U_l U_{\bar{l}}^2 + \gamma_l U_l (U_l')^2 + \gamma_l \kappa^2 U_l^3$$

$$+\delta_l U_l \left(U_{\bar{l}}'\right)^2 + \delta_l \kappa^2 U_l\, U_{\bar{l}}^2 + \lambda_l U_l^2 U_l'' - 2i\lambda_l \kappa U_l^2 U_l' - \lambda_l \kappa^2 U_l^3 + \theta_l U_{\bar{l}}^2 U_l'' - 2i\theta_l \kappa U_{\bar{l}}^2 U_l'$$

$$-\theta_l \kappa^2 U_l U_{\bar{l}}^2 + \xi_l U_l^2 U_l'' - \xi_l \kappa^2 U_l^3 + 2i\xi_l \kappa U_l^2 U_l' + \eta_l U_{\bar{l}}^2 U_l'' - \eta_l \kappa^2 U_l U_{\bar{l}}^2 + 2i\eta_l \kappa U_{\bar{l}}^2 U_l'$$

$$+f_l U_l^5 + g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4 \big] e^{i(-\kappa x + \omega t + \theta)}$$

bulunur. Buradan $e^{i(-\kappa x+\omega t+\theta)}$ nin çarpanları birbirine eşitlenerek

$$-ivU_l' - \omega U_l + a_l U_l'' - 2ia_l \kappa U_l' - a_l \kappa^2 U_l - b_l v U_l'' + ib_l \kappa v U_l' + ib_l \omega U_l' + b_l \kappa \omega U_l$$

$$+c_l U_l^3 + d_l U_l U_{\bar{l}}^2 = \sigma_l U_l^{(4)} - 4i\sigma_l \kappa U_l''' - 6\sigma_l \kappa^2 U_l'' + 4i\sigma_l \kappa^3 U_l' + \sigma_l \kappa^4 U_l + \alpha_l U_l (U_l')^2$$

$$-2i\alpha_l \kappa U_l^2 U_l' - \alpha_l \kappa^2 U_l^3 + \beta_l U_l \left(U_{\bar{l}}'\right)^2 - 2i\beta_l \kappa U_l U_{\bar{l}} U_{\bar{l}}' - \beta_l \kappa^2 U_l U_{\bar{l}}^2 + \gamma_l U_l (U_l')^2 + \gamma_l \kappa^2 U_l^3$$

$$+\delta_l U_l \left(U_{\bar{l}}'\right)^2 + \delta_l \kappa^2 U_l U_{\bar{l}}^2 + \lambda_l U_l^2 U_l'' - 2i\lambda_l \kappa U_l^2 U_l' - \lambda_l \kappa^2 U_l^3 + \theta_l U_{\bar{l}}^2 U_l'' - 2i\theta_l \kappa U_{\bar{l}}^2 U_l'$$

$$-\theta_l \kappa^2 U_l U_{\bar{l}}^2 + \xi_l U_l^2 U_l'' - \xi_l \kappa^2 U_l^3 + 2i\xi_l \kappa U_l^2 U_l' + \eta_l U_{\bar{l}}^2 U_l'' - \eta_l \kappa^2 U_l U_{\bar{l}}^2 + 2i\eta_l \kappa U_{\bar{l}}^2 U_l'$$

$$+f_l U_l^5 + g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4$$

elde edilir. Bu ifadeler düzenlenirse,

$$\sigma_l U_l^{(4)} - 4i\sigma_l \kappa U_l''' - 6\sigma_l \kappa^2 U_l'' + 4i\sigma_l \kappa^3 U_l' + \sigma_l \kappa^4 U_l + \alpha_l U_l (U_l')^2 - 2i\alpha_l \kappa U_l^2 U_l'$$

$$-\alpha_l \kappa^2 U_l^3 + \beta_l U_l \left(U_{\bar{l}}'\right)^2 - 2i\beta_l \kappa U_l U_{\bar{l}} U_{\bar{l}}' - \beta_l \kappa^2 U_l U_{\bar{l}}^2 + \gamma_l U_l (U_l')^2 + \gamma_l \kappa^2 U_l^3 + \delta_l U_l \left(U_{\bar{l}}'\right)^2$$

$$+\delta_l \kappa^2 U_l U_{\bar{l}}^2 + \lambda_l U_l^2 U_l'' - 2i\lambda_l \kappa U_l^2 U_l' - \lambda_l \kappa^2 U_l^3 + \theta_l U_{\bar{l}}^2 U_l'' - 2i\theta_l U_{\bar{l}}^2 U_l' - \theta_l \kappa^2 U_l U_{\bar{l}}^2$$

$$+\xi_l U_l^2 U_l'' - \xi_l \kappa^2 U_l^3 + 2i\xi_l \kappa U_l^2 U_l' + \eta_l U_{\bar{l}}^2 U_l'' - \eta_l \kappa^2 U_l U_{\bar{l}}^2 + 2i\eta_l \kappa U_{\bar{l}}^2 U_l' + f_l U_l^5$$

$$+g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4 + ivU_l' + \omega U_l - a_l U_l'' + 2ia_l \kappa U_l' + a_l \kappa^2 U_l + b_l v U_l'' - ib_l \kappa v U_l'$$

$$-ib_l \omega U_l' - b_l \kappa \omega U_l - c_l U_l^3 - d_l U_l U_{\bar{l}}^2 = 0$$

bulunur.

$$(\sigma_l \kappa^4 + \omega + a_l \kappa^2 - b_l \kappa \omega) U_l + (-\alpha_l \kappa^2 + \gamma_l \kappa^2 - \lambda_l \kappa^2 - \xi_l \kappa^2 - c_l) U_l^3 + f_l U_l^5$$

$$+(-\beta_l \kappa^2 + \delta_l \kappa^2 - \theta_l \kappa^2 - \eta_l \kappa^2 - d_l) U_l U_{\bar{l}}^2 + g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4 + (\alpha_l + \beta_l) U_l \left(U_{\bar{l}}'\right)^2$$

$$+(\beta_l + \delta_l) U_l \left(U_{\bar{l}}'\right)^2 + (-6\sigma_l \kappa^2 - a_l + b_l v) U_l'' + (\lambda_l + \xi_l) U_l^2 U_l'' + (\theta_l + \eta_l) U_{\bar{l}}^2 U_l''$$

$$+\sigma_l U_l^{(4)} + i(-4\sigma_l\kappa U_l''' + 4\sigma_l\kappa^3 U_l' - 2\alpha_l\kappa U_l^2 U_l' - 2\beta_l\kappa U_l U_{\bar{l}} U_{\bar{l}}' - 2\lambda_l\kappa U_l^2 U_l' - 2\theta_l\kappa U_{\bar{l}}^2 U_l'$$

$$+2\xi_l\kappa U_l^2 U_l' + 2\eta_l\kappa U_{\bar{l}}^2 U_l' + vU_l' + 2a_l\kappa U_l' - b_l\kappa v U_l' - b_l\omega U_l') = 0$$

Elde edilen bu ifade düzenlenirse,

$$(\sigma_l\kappa^4 + \omega + a_l\kappa^2 - b_l\kappa\omega)U_l - [c_l + \kappa^2(\alpha_l - \gamma_l + \lambda_l + \xi_l)]U_l^3 + f_l U_l^5$$

$$-[d_l + \kappa^2(\beta_l - \delta_l + \theta_l + \eta_l)]U_l U_{\bar{l}}^2 + g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4 + (\alpha_l + \beta_l)U_l\left(U_{\bar{l}}'\right)^2$$

$$+(\beta_l + \delta_l)U_l\left(U_{\bar{l}}'\right)^2 - (6\sigma_l\kappa^2 + a_l - b_l v)U_l'' + (\lambda_l + \xi_l)U_l^2 U_l'' + (\theta_l + \eta_l)U_{\bar{l}}^2 U_l'' + \sigma_l U_l^{(4)}$$

$$+i\{[4\sigma_l\kappa^3 + v + 2a_l\kappa - b_l(v\kappa + \omega)]U_l' - 2\kappa(\alpha_l + \lambda_l - \xi_l)U_l^2 U_l' + 2\kappa(\eta_l - \theta_l)U_{\bar{l}}^2 U_l'$$

$$-2\beta_l\kappa U_l U_{\bar{l}} U_{\bar{l}}' - 4\sigma_l\kappa U_l'''\} = 0$$

Buradan reel kısım,

$$(\sigma_l\kappa^4 + \omega + a_l\kappa^2 - b_l\kappa\omega)U_l - [c_l + \kappa^2(\alpha_l - \gamma_l + \lambda_l + \xi_l)]U_l^3 + f_l U_l^5$$

$$-[d_l + \kappa^2(\beta_l - \delta_l + \theta_l + \eta_l)]U_l U_{\bar{l}}^2 + g_l U_l^3 U_{\bar{l}}^2 + h_l U_l U_{\bar{l}}^4 + (\alpha_l + \beta_l)U_l\left(U_{\bar{l}}'\right)^2$$

$$+(\beta_l + \delta_l)U_l\left(U_{\bar{l}}'\right)^2 - (6\sigma_l\kappa^2 + a_l - b_l v)U_l'' + (\lambda_l + \xi_l)U_l^2 U_l'' + (\theta_l + \eta_l)U_{\bar{l}}^2 U_l''$$

$$+\sigma_l U_l^{(4)} = 0 \qquad (3.8)$$

ve imajiner kısım,

$$[v + 2a_l\kappa - b_l(v\kappa + \omega) + 4\sigma_l\kappa^3]U_l' - 2\kappa(\alpha_l + \lambda_l - \xi_l)U_l^2 U_l' + 2\kappa(\eta_l - \theta_l)U_{\bar{l}}^2 U_l'$$

$$-2\beta_l\kappa U_l U_{\bar{l}} U_{\bar{l}}' - 4\sigma_l\kappa U_l''' = 0 \qquad (3.9)$$

olarak bulunur.

Elde edilen $r_t, r_{xx}, r_{xt}, |r|^2, |q|^2, r_{xxxx}$, $(r_x)^2, (q_x)^2, |r_x|^2, |q_x|^2, |r|^2, |q|^2, r^2, q^2, r_{xx}^*,$ $|r|^4, |q|^4$

ifadeleri (3.3) denkleminde yerine yazılırsa,

$$i\left(-vU_{\bar{l}}' + i\omega U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)} + a_{\bar{l}}\left(U_{\bar{l}}'' - 2i\kappa U_{\bar{l}}' - \kappa^2 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$+b_{\bar{l}}\left(-vU_{\bar{l}}'' + i\kappa v U_{\bar{l}}' + i\omega U_{\bar{l}}' + \kappa\omega U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)} + \left(c_{\bar{l}}U_{\bar{l}}^2 + d_{\bar{l}}U_l^2\right)U_{\bar{l}}e^{i(-\kappa x+\omega t+\theta)}$$

$$= \sigma_{\bar{l}}\left(U_{\bar{l}}^{(4)} - 4i\kappa U_{\bar{l}}''' - 6\kappa^2 U_{\bar{l}}'' + 4i\kappa^3 U_{\bar{l}}' + \kappa^4 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$+\{\alpha_{\bar{l}}\left[\left(U_{\bar{l}}'\right)^2 - 2i\kappa U_{\bar{l}}U_{\bar{l}}' - \kappa^2 U_{\bar{l}}^2\right]e^{2i(-\kappa x+\omega t+\theta)}$$

$$+\beta_{\bar{l}}[(U_l')^2 - 2i\kappa U_l U_l' - \kappa^2 U_l^2]e^{2i(-\kappa x+\omega t+\theta)}\}U_{\bar{l}}e^{-i(-\kappa x+\omega t+\theta)}$$

$$+\left\{\gamma_{\bar{l}}\left[\left(U_{\bar{l}}'\right)^2 + \kappa^2 U_{\bar{l}}^2\right] + \delta_{\bar{l}}[(U_l')^2 + \kappa^2 U_l^2]\right\}U_{\bar{l}}e^{i(-\kappa x+\omega t+\theta)}$$

$$+\left(\lambda_{\bar{l}}U_{\bar{l}}^2 + \theta_{\bar{l}}U_l^2\right)\left(U_{\bar{l}}'' - 2i\kappa U_{\bar{l}}' - \kappa^2 U_{\bar{l}}\right)e^{i(-\kappa x+\omega t+\theta)}$$

$$+\left(\xi_{\bar{l}}U_{\bar{l}}^2 e^{2i(-\kappa x+\omega t+\theta)} + \eta_{\bar{l}}U_l^2 e^{2i(-\kappa x+\omega t+\theta)}\right)\left(U_{\bar{l}}'' - \kappa^2 U_{\bar{l}} + 2i\kappa U_{\bar{l}}'\right)e^{-i(-\kappa x+\omega t+\theta)}$$

$$+\left(f_{\bar{l}}U_{\bar{l}}^4 + g_{\bar{l}}U_{\bar{l}}^2U_l^2 + h_{\bar{l}}U_l^4\right)U_{\bar{l}}e^{i(-\kappa x+\omega t+\theta)}$$

elde edilir. Bu ifade düzenlenirse,

$$[-i\nu U_{\bar{l}}' - \omega U_{\bar{l}} + a_{\bar{l}}U_{\bar{l}}'' - 2ia_{\bar{l}}\kappa U_{\bar{l}}' - a_{\bar{l}}\kappa^2 U_{\bar{l}} - b_{\bar{l}}\nu U_{\bar{l}}'' + ib_{\bar{l}}(\nu\kappa + \omega)U_{\bar{l}}' + b_{\bar{l}}\kappa\omega U_{\bar{l}}$$

$$+c_{\bar{l}}U_{\bar{l}}^3 + d_{\bar{l}}U_l^2U_{\bar{l}}]e^{i(-\kappa x+\omega t+\theta)} = [\sigma_{\bar{l}}U_{\bar{l}}^{(4)} - 4i\sigma_{\bar{l}}\kappa U_{\bar{l}}''' - 6\sigma_{\bar{l}}\kappa^2 U_{\bar{l}}'' + 4i\sigma_{\bar{l}}\kappa^3 U_{\bar{l}}' + \sigma_{\bar{l}}\kappa^4 U_{\bar{l}}$$

$$+\alpha_{\bar{l}}U_{\bar{l}}\left(U_{\bar{l}}'\right)^2 - 2i\alpha_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' - \alpha_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \beta_{\bar{l}}U_{\bar{l}}(U_l')^2 - 2i\beta_{\bar{l}}\kappa U_l U_{\bar{l}}U_l' - \beta_{\bar{l}}\kappa^2 U_l^2 U_{\bar{l}}$$

$$+\gamma_{\bar{l}}U_{\bar{l}}(U_l')^2 + \gamma_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \delta_{\bar{l}}U_{\bar{l}}(U_l')^2 + \delta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \lambda_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - 2i\lambda_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' - \lambda_{\bar{l}}\kappa^2 U_{\bar{l}}^3$$

$$+\theta_{\bar{l}}U_l^2U_{\bar{l}}'' - 2i\theta_{\bar{l}}\kappa U_l^2U_{\bar{l}}' - \theta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \xi_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - \xi_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + 2i\xi_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' + \eta_{\bar{l}}U_l^2U_{\bar{l}}''$$

$$-\eta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + 2i\eta_{\bar{l}}\kappa U_l^2U_{\bar{l}}' + f_{\bar{l}}U_{\bar{l}}^5 + g_{\bar{l}}U_{\bar{l}}^3U_l^2 + h_{\bar{l}}U_l^4U_{\bar{l}}]e^{i(-\kappa x+\omega t+\theta)}$$

bulunur. Buradan $e^{i(-\kappa x+\omega t+\theta)}$ nin çarpanları birbirine eşitlenerek

$$[-i\nu U_{\bar{l}}' - \omega U_{\bar{l}} + a_{\bar{l}}U_{\bar{l}}'' - 2ia_{\bar{l}}\kappa U_{\bar{l}}' - a_{\bar{l}}\kappa^2 U_{\bar{l}} - b_{\bar{l}}\nu U_{\bar{l}}'' + ib_{\bar{l}}(\nu\kappa + \omega)U_{\bar{l}}' + b_{\bar{l}}\kappa\omega U_{\bar{l}}$$

$$+c_{\bar{l}}U_{\bar{l}}^3 + d_{\bar{l}}U_l^2U_{\bar{l}}] = [\sigma_{\bar{l}}U_{\bar{l}}^{(4)} - 4i\sigma_{\bar{l}}\kappa U_{\bar{l}}''' - 6\sigma_{\bar{l}}\kappa^2 U_{\bar{l}}'' + 4i\sigma_{\bar{l}}\kappa^3 U_{\bar{l}}' + \sigma_{\bar{l}}\kappa^4 U_{\bar{l}} + \alpha_{\bar{l}}U_{\bar{l}}\left(U_{\bar{l}}'\right)^2$$

$$-2i\alpha_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' - \alpha_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \beta_{\bar{l}}U_{\bar{l}}(U_l')^2 - 2i\beta_{\bar{l}}\kappa U_l U_{\bar{l}}U_l' - \beta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \gamma_{\bar{l}}U_{\bar{l}}(U_l')^2 + \gamma_{\bar{l}}\kappa^2 U_{\bar{l}}^3$$

$$+\delta_{\bar{l}}U_{\bar{l}}(U_l')^2 + \delta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \lambda_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - 2i\lambda_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' - \lambda_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \theta_{\bar{l}}U_l^2U_{\bar{l}}'' - 2i\theta_{\bar{l}}\kappa U_l^2U_{\bar{l}}'$$

$$-\theta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \xi_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - \xi_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + 2i\xi_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' + \eta_{\bar{l}}U_l^2U_{\bar{l}}'' - \eta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + 2i\eta_{\bar{l}}\kappa U_l^2U_{\bar{l}}'$$

$$+f_{\bar{l}}U_{\bar{l}}^5 + g_{\bar{l}}U_{\bar{l}}^3U_l^2 + h_{\bar{l}}U_l^4U_{\bar{l}}]$$

elde edilir. Bu ifade düzenlenirse,

$$\sigma_{\bar{l}}U_{\bar{l}}^{(4)} - 4i\sigma_{\bar{l}}\kappa U_{\bar{l}}''' - 6\sigma_{\bar{l}}\kappa^2 U_{\bar{l}}'' + 4i\sigma_{\bar{l}}\kappa^3 U_{\bar{l}}' + \sigma_{\bar{l}}\kappa^4 U_{\bar{l}} + \alpha_{\bar{l}}U_{\bar{l}}\left(U_{\bar{l}}'\right)^2 - 2i\alpha_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}'$$

$$-\alpha_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \beta_{\bar{l}}U_{\bar{l}}(U_l')^2 - 2i\beta_{\bar{l}}\kappa U_l U_{\bar{l}}U_l' - \beta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \gamma_{\bar{l}}U_{\bar{l}}(U_l')^2 + \gamma_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \delta_{\bar{l}}U_{\bar{l}}(U_l')^2$$

$$+\delta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + \lambda_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - 2i\lambda_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' - \lambda_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + \theta_{\bar{l}}U_l^2U_{\bar{l}}'' - 2i\theta_{\bar{l}}\kappa U_l^2U_{\bar{l}}' - \theta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}}$$

$$+\xi_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}'' - \xi_{\bar{l}}\kappa^2 U_{\bar{l}}^3 + 2i\xi_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}' + \eta_{\bar{l}}U_l^2U_{\bar{l}}'' - \eta_{\bar{l}}\kappa^2 U_l^2U_{\bar{l}} + 2i\eta_{\bar{l}}\kappa U_l^2U_{\bar{l}}' + f_{\bar{l}}U_{\bar{l}}^5$$

$$+g_{\bar{l}}U_{\bar{l}}^3U_l^2+h_{\bar{l}}U_l^4U_{\bar{l}}+ivU_{\bar{l}}'+\omega U_{\bar{l}}-a_{\bar{l}}U_{\bar{l}}''+2ia_{\bar{l}}\kappa U_{\bar{l}}'+a_{\bar{l}}\kappa^2U_{\bar{l}}+b_{\bar{l}}vU_{\bar{l}}''$$

$$-ib_{\bar{l}}(v\kappa+\omega)U_{\bar{l}}'-b_{\bar{l}}\kappa\omega U_{\bar{l}}-c_{\bar{l}}U_{\bar{l}}^3-d_{\bar{l}}U_l^2U_{\bar{l}}=0$$

elde edilir.

$$\sigma_{\bar{l}}U_{\bar{l}}^{(4)}-6\sigma_{\bar{l}}\kappa^2U_{\bar{l}}''+\sigma_{\bar{l}}\kappa^4U_{\bar{l}}+\alpha_{\bar{l}}U_{\bar{l}}\left(U_{\bar{l}}'\right)^2-\alpha_{\bar{l}}\kappa^2U_{\bar{l}}^3+\beta_{\bar{l}}U_{\bar{l}}(U_l')^2-\beta_{\bar{l}}\kappa^2U_l^2U_{\bar{l}}$$

$$+\gamma_{\bar{l}}U_{\bar{l}}\left(U_{\bar{l}}'\right)^2+\gamma_{\bar{l}}\kappa^2U_{\bar{l}}^3+\delta_{\bar{l}}U_{\bar{l}}(U_l')^2+\delta_{\bar{l}}\kappa^2U_l^2U_{\bar{l}}+\lambda_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}''-\lambda_{\bar{l}}\kappa^2U_{\bar{l}}^3+\theta_{\bar{l}}U_l^2U_{\bar{l}}''$$

$$-\theta_{\bar{l}}\kappa^2U_l^2U_{\bar{l}}+\xi_{\bar{l}}U_{\bar{l}}^2U_{\bar{l}}''-\xi_{\bar{l}}\kappa^2U_{\bar{l}}^3+\eta_{\bar{l}}U_l^2U_{\bar{l}}''-\eta_{\bar{l}}\kappa^2U_l^2U_{\bar{l}}+f_{\bar{l}}U_{\bar{l}}^5+g_{\bar{l}}U_{\bar{l}}^3U_l^2+h_{\bar{l}}U_l^4U_{\bar{l}}$$

$$+\omega U_{\bar{l}}-a_{\bar{l}}U_{\bar{l}}''+a_{\bar{l}}\kappa^2U_{\bar{l}}+b_{\bar{l}}vU_{\bar{l}}''-b_{\bar{l}}\kappa\omega U_{\bar{l}}-c_{\bar{l}}U_{\bar{l}}^3-d_{\bar{l}}U_l^2U_{\bar{l}}+i\left[-4\sigma_{\bar{l}}\kappa U_{\bar{l}}'''+4\sigma_{\bar{l}}\kappa^3U_{\bar{l}}'\right.$$

$$-2\alpha_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}'-2\beta_{\bar{l}}\kappa U_lU_{\bar{l}}U_l'-2\lambda_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}'-2\theta_{\bar{l}}\kappa U_l^2U_{\bar{l}}'+2\xi_{\bar{l}}\kappa U_{\bar{l}}^2U_{\bar{l}}'+2\eta_{\bar{l}}\kappa U_l^2U_{\bar{l}}'+vU_{\bar{l}}'$$

$$\left.+2a_{\bar{l}}\kappa U_{\bar{l}}'-b_{\bar{l}}(v\kappa+\omega)U_{\bar{l}}'\right]=0$$

Bu ifade düzenlenirse,

$$(\sigma_{\bar{l}}\kappa^4+\omega+a_{\bar{l}}\kappa^2-b_{\bar{l}}\kappa\omega)U_{\bar{l}}-[c_{\bar{l}}+\kappa^2(\alpha_{\bar{l}}-\gamma_{\bar{l}}+\lambda_{\bar{l}}+\xi_{\bar{l}})]U_{\bar{l}}^3+f_{\bar{l}}U_{\bar{l}}^5$$

$$-[d_{\bar{l}}+\kappa^2(\beta_{\bar{l}}-\delta_{\bar{l}}+\theta_{\bar{l}}+\eta_{\bar{l}})]U_l^2U_{\bar{l}}+g_{\bar{l}}U_{\bar{l}}^3U_l^2+h_{\bar{l}}U_l^4U_{\bar{l}}+(\alpha_{\bar{l}}+\gamma_{\bar{l}})U_{\bar{l}}\left(U_{\bar{l}}'\right)^2$$

$$+(\beta_{\bar{l}}+\delta_{\bar{l}})U_{\bar{l}}(U_l')^2-(6\sigma_{\bar{l}}\kappa^2-a_{\bar{l}}+b_{\bar{l}}v)U_{\bar{l}}''+(\lambda_{\bar{l}}+\xi_{\bar{l}})U_{\bar{l}}^2U_{\bar{l}}''+(\theta_{\bar{l}}++\eta_{\bar{l}})U_l^2U_{\bar{l}}''$$

$$+\sigma_{\bar{l}}U_{\bar{l}}^{(4)}+i\left\{[(4\sigma_{\bar{l}}\kappa^3+v+2a_{\bar{l}}\kappa)-b_{\bar{l}}(v\kappa+\omega)]U_{\bar{l}}'-2\kappa(\alpha_{\bar{l}}+\lambda_{\bar{l}}-\xi_{\bar{l}})U_{\bar{l}}^2U_{\bar{l}}'\right.$$

$$\left.+2\kappa(\eta_{\bar{l}}-\theta_{\bar{l}})U_l^2U_{\bar{l}}'-2\beta_{\bar{l}}\kappa U_lU_{\bar{l}}U_l'-4\sigma_{\bar{l}}\kappa U_{\bar{l}}'''\right\}=0$$

bulunur. Buradan reel kısım,

$$(\omega+a_{\bar{l}}\kappa^2-b_{\bar{l}}\kappa\omega+\sigma_{\bar{l}}\kappa^4+)U_{\bar{l}}-[c_{\bar{l}}+\kappa^2(\alpha_{\bar{l}}-\gamma_{\bar{l}}+\lambda_{\bar{l}}+\xi_{\bar{l}})]U_{\bar{l}}^3+f_{\bar{l}}U_{\bar{l}}^5$$

$$-[d_{\bar{l}}+\kappa^2(\beta_{\bar{l}}-\delta_{\bar{l}}+\eta_{\bar{l}}+\theta_{\bar{l}})]U_l^2U_{\bar{l}}+g_{\bar{l}}U_{\bar{l}}^3U_l^2+h_{\bar{l}}U_l^4U_{\bar{l}}+(\alpha_{\bar{l}}+\gamma_{\bar{l}})U_{\bar{l}}\left(U_{\bar{l}}'\right)^2$$

$$+(\beta_{\bar{l}}+\delta_{\bar{l}})U_{\bar{l}}(U_l')^2-(a_{\bar{l}}\mp b_{\bar{l}}v+6\sigma_{\bar{l}}\kappa^2)U_{\bar{l}}''+(\lambda_{\bar{l}}+\xi_{\bar{l}})U_{\bar{l}}^2U_{\bar{l}}''+(\eta_{\bar{l}}+\theta_{\bar{l}})U_l^2U_{\bar{l}}''$$

$$+\sigma_{\bar{l}}U_{\bar{l}}^{(4)}=0 \tag{3.10}$$

ve imajiner kısım,

$$(v+2a_{\bar{l}}\kappa)-b_{\bar{l}}(v\kappa+\omega)+4\sigma_{\bar{l}}\kappa^3]U_{\bar{l}}'-2\kappa(\alpha_{\bar{l}}+\lambda_{\bar{l}}-\xi_{\bar{l}})U_{\bar{l}}^2U_{\bar{l}}'+2\kappa(\eta_{\bar{l}}-\theta_{\bar{l}})U_l^2U_{\bar{l}}'$$

$$-2\beta_{\bar{l}}\kappa U_lU_{\bar{l}}U_l'-4\sigma_{\bar{l}}\kappa U_{\bar{l}}'''=0 \tag{3.11}$$

olarak bulunur.

Burada $l = 1,2$ ve $\bar{l} = 3 - l$ dir. Dengeleme ilkesi yardımıyla,

$$U_{\bar{l}} = U_l \tag{3.12}$$

sonucunu verir ve daha sonra (3.8) ve (3.9) denklemleri, (3.12) eşitliği yardımıyla sırasıyla aşağıdaki (3.13) ve (3.15) denklemlerine dönüşür.

$$(\omega + a_l\kappa^2 - b_l\kappa\omega + \kappa^4\sigma_l)U_l - [c_l + d_l + \kappa^2(\mathcal{H}_l + \mathcal{R}_l)]U_l^3 + \mathcal{J}_lU_l^5 + \mathcal{L}_lU_l(U_l')^2$$

$$-(a_l - b_lv + 6\kappa^2\sigma_l)U_l'' + \mathcal{R}_lU_l^2U_l'' + \sigma_lU_l^{(4)} = 0 \tag{3.13}$$

Burada;

$$\mathcal{J}_l = f_l + g_l + h_l$$

$$\mathcal{H}_l = \alpha_l + \beta_l - \gamma_l - \delta_l$$

$$\mathcal{L}_l = \alpha_l + \beta_l + \gamma_l + \delta_l$$

$$\mathcal{R}_l = \eta_l + \theta_l + \lambda_l + \xi_l \tag{3.14}$$

ve

$$[v + 2a_l\kappa - b_l(v\kappa + \omega) + 4\sigma_l\kappa^3]U_l' - 2\kappa(\alpha_l + \beta_l - \eta_l + \theta_l + \lambda_l - \xi_l)U_l^2U_l'$$

$$-4\sigma_l\kappa U_l''' = 0 \tag{3.15}$$

Böylece (3.15) denkleminin 3. mertebeden terimi,

$$\sigma_l = 0 \tag{3.16}$$

Bu, LPD sistemi için çözümlerin dördüncü dereceden dağılımın ortadan kalkmasıyla var olmasını gerektirir. (3.15) denkleminin diğer lineer bağımsız fonksiyonları,

$$\eta_l + \xi_l = \alpha_l + \beta_l + \theta_l + \lambda_l \tag{3.17}$$

kısıtlamasını sağlar ve soliton hızı v aşağıdaki gibi bulunur. Gerçekten U_l' ve $U_l^2U_l'$ lineer bağımsız olduğundan katsayıları sıfıra eşittir.

- $v + 2a_l\kappa - b_l(v\kappa + \omega) = 0$

  $v + 2a_l\kappa - b_lv\kappa - b_l\omega = 0$

  $v(1 - b_l\kappa) = b_l\omega - 2a_l\kappa$

  $$v = \frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\ , \ \ b_l\kappa \neq 0 \tag{3.18}$$

(3.18) eşitliğindeki soliton hızının iki değerinin eşitlenmesi ile yani $l = 1,2$ için;

$$\frac{b_1\omega - 2a_1\kappa}{1 - b_1\kappa} = \frac{b_2\omega - 2a_2\kappa}{1 - b_2\kappa}$$

$$(1 - b_1\kappa)(b_2\omega - 2a_2\kappa) = (1 - b_2\kappa)(b_1\omega - 2a_1\kappa) \tag{3.19}$$

sonucuna varılır.

Sonuç olarak (3.13) denklemi,

$$(\omega + a_l\kappa^2 - b_l\kappa\omega)U_l - [c_l + d_l + \kappa^2(\mathcal{H}_l + \mathcal{R}_l)]U_l^3 + \mathcal{J}_lU_l^5 + \mathcal{L}_lU_l(U_l')^2$$

$$-(a_l - b_lv)U_l'' + \mathcal{R}_lU_l^2U_l'' = 0 \tag{3.20}$$

denklemine indirgenir.

3.3. Genişletilmiş Jacobi Eliptik Fonksiyon Açılım Şeması

Bu bölümün odak noktası, genişletilmiş JEF açılım şeması ile vektör bileşenli LPD modeline tam solitonların elde edilmesi olacaktır. Jacobi açılım şeması (Abdou vd.,2007; Abdou vd., 2018; Bhrawy vd., 2013; Ekici vd., 2017; Huiqun vd., 2007; Sonmezoglu, 2015). (3.20) denkleminin $sn\tau$ Jacobi fonksiyonu cinsinden aşağıdaki gibi bir çözüm yapısına sahip olduğunu varsayılırsa,

$$U_l = \sum_{j=0}^{M} \varepsilon_j^{(l)} sn^j\tau + \sum_{j=1}^{M} \xi_j^{(l)} sn^{-j}\tau \tag{3.21}$$

Burada $j = 1,2, \dots M$ için $\varepsilon_0^{(l)}, \varepsilon_j^{(l)}, \xi_j^{(l)}$ sabitlerdir. Şimdi eşitlik (3.21) denkleminde M parametresini tespit etmek için $U_l^2U_l''$ ile U_l^5 balans edilerek $M = 1$ elde edilir. Böylece uygulanan algoritma sonlu açılımın kullanılmasına izin verir.

$$U_l = \varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau + \xi_1^{(l)} sn^{-1}\tau \tag{3.22}$$

(3.22) denklemini (3.20) denkleminde yerine yazarak,

$$(U_l)^3 = \left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau + \xi_1^{(l)} sn^{-1}\tau\right)^3$$

$$(U_l)^3 = \left[\left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau\right)^3 + 3\left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau\right)^2 \xi_1^{(l)} sn^{-1}\tau\right.$$

$$\left. +3\left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau\right)\left(\xi_1^{(l)} sn^{-1}\tau\right)^2 + \left(\xi_1^{(l)} sn^{-1}\tau\right)^3\right]$$

$$(U_l)^3 = \left(\varepsilon_0^{(l)}\right)^3 + 3\left(\varepsilon_0^{(l)}\right)^2 \varepsilon_1^{(l)} sn\tau + 3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)} sn\tau\right)^2 + \left(\varepsilon_1^{(l)} sn\tau\right)^3 + 3\left(\varepsilon_0^{(l)}\right)^2 \xi_1^{(l)} sn^{-1}\tau$$

$$+6\varepsilon_0^{(l)}\varepsilon_1^{(l)}sn\tau\xi_1^{(l)}sn^{-1}\tau+3\left(\varepsilon_1^{(l)}sn\tau\right)^2\xi_1^{(l)}sn^{-1}\tau+3\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$+3\varepsilon_1^{(l)}sn\tau\left(\xi_1^{(l)}\right)^2sn^{-2}\tau+\left(\xi_1^{(l)}\right)^3sn^{-3}\tau$$

$$(U_l)^3=\left(\varepsilon_0^{(l)}\right)^3+3\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau+3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau+\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+3\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau$$

$$+6\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+3\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau+3\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau+3\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau$$

$$+\left(\xi_1^{(l)}\right)^3sn^{-3}$$

$$(U_l)^3=\left(\varepsilon_0^{(l)}\right)^3+6\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+3\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau+3\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau+3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau$$

$$+\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+3\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau+3\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau+3\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$+\left(\xi_1^{(l)}\right)^3sn^{-3}$$

$$(U_l)^3=\left(\varepsilon_0^{(l)}\right)^3+6\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+3\left[\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}\right]sn\tau+3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau$$

$$+\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+3\left[\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2\right]sn^{-1}\tau+3\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$+\left(\xi_1^{(l)}\right)^3sn^{-3}\tau$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau+\xi_1^{(l)}sn^{-1}\tau\right)^5$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)^5+5\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)^4\xi_1^{(l)}sn^{-1}\tau$$

$$+10\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)^3\left(\xi_1^{(l)}sn^{-1}\tau\right)^2+10\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)^2\left(\xi_1^{(l)}sn^{-1}\tau\right)^3$$

$$+5\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)\left(\xi_1^{(l)}sn^{-1}\tau\right)^4+\left(\xi_1^{(l)}sn^{-1}\tau\right)^5$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}\right)^5+5\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}sn\tau+10\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}sn\tau\right)^2+10\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}sn\tau\right)^3$$

$$+5\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}sn\tau\right)^4+\left(\varepsilon_1^{(l)}sn\tau\right)^5+5\left[\left(\varepsilon_0^{(l)}\right)^4+4\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}sn\tau\right.$$

$$\left.+6\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}sn\tau\right)^2+4\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}sn\tau\right)^3+\left(\varepsilon_1^{(l)}sn\tau\right)^4\right]\xi_1^{(l)}sn^{-1}\tau$$

$$+10\left[\left(\varepsilon_0^{(l)}\right)^3+3\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau+3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}sn\tau\right)^2+\left(\varepsilon_1^{(l)}sn\tau\right)^3\right]\left(\xi_1^{(l)}sn^{-1}\tau\right)^2$$

$$+10\left[\left(\varepsilon_0^{(l)}\right)^2+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}sn\tau+\left(\varepsilon_1^{(l)}sn\tau\right)^2\right]\left(\xi_1^{(l)}sn^{-1}\tau\right)^3$$

$$+5\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau\right)\left(\xi_1^{(l)}sn^{-1}\tau\right)^4+\left(\xi_1^{(l)}sn^{-1}\tau\right)^5$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}\right)^5+5\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}sn\tau+10\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2sn^2\tau+10\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3sn^3\tau$$

$$+5\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4sn^4\tau+\left(\varepsilon_1^{(l)}\right)^5sn^5\tau+5\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}sn^{-1}\tau+20\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$+30\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau+20\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}sn^2\tau+5\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}sn^3\tau$$

$$+10\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn^{-2}\tau+30\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau+30\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2$$

$$+10\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn\tau+10\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-3}\tau+20\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3sn^{-2}\tau$$

$$+10\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-1}\tau+5\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-4}\tau+5\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-3}\tau$$

$$+\left(\xi_1^{(l)}\right)^5sn^{-5}\tau$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}\right)^5+20\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}+30\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2+5\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}sn\tau$$

$$+30\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau+10\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn\tau+10\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2sn^2\tau$$

$$+20\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}sn^2\tau+10\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+5\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}sn^3\tau$$

$$+5\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4sn^4\tau+\left(\varepsilon_1^{(l)}\right)^5sn^5\tau+5\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}sn^{-1}\tau$$

$$+30\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau+10\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-1}\tau+10\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$+20\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3sn^{-2}\tau+10\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-3}\tau+5\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-3}\tau$$

$$+5\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-4}\tau+\left(\xi_1^{(l)}\right)^5sn^{-5}\tau$$

$$(U_l)^5=\left(\varepsilon_0^{(l)}\right)^5+20\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}+30\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2$$

$$+5\left[\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}+6\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+2\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2\right]sn\tau$$

$$+10\left[\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2+2\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}\right]sn^2\tau+5[2\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3$$

$$+\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}]sn^3\tau+5\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4sn^4\tau+\left(\varepsilon_1^{(l)}\right)^5sn^5\tau$$

$$+5\left[\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}+6\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+2\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3\right]sn^{-1}\tau$$

$$+10\left[\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3\right]sn^{-2}\tau$$

$$+5[2\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4\right]sn^{-3}\tau+5\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-4}\tau+\left(\xi_1^{(l)}\right)^5sn^{-5}\tau$$

$$U_l'=\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau+\xi_1^{(l)}sn^{-1}\tau\right)'$$

$$U_l'=\varepsilon_1^{(l)}cn\tau dn\tau-\xi_1^{(l)}sn^{-2}\tau cn\tau dn\tau$$

$$U_l'=\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)cn\tau dn\tau$$

$$U_l''=\left(2\xi_1^{(l)}sn^{-3}\tau cn\tau dn\tau\right)cn\tau dn\tau$$

$$+\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)[-sn\tau dn\tau dn\tau+cn\tau(-k^2)sn\tau cn\tau]$$

$$U_l''=2\xi_1^{(l)}sn^{-3}\tau cn^2\tau dn^2\tau+\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)(-sn\tau dn^2\tau-k^2sn\tau cn^2\tau)$$

$$U_l''=2\xi_1^{(l)}sn^{-3}\tau(1-sn^2\tau)(1-k^2sn^2\tau)+\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)(-dn^2\tau-k^2cn^2\tau)sn\tau$$

$$U_l''=2\xi_1^{(l)}sn^{-3}\tau(1-k^2sn^2\tau-sn^2\tau+k^2sn^4\tau)$$

$$+\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)[-(1-k^2sn^2\tau)-k^2(1-sn^2\tau)]sn\tau$$

$$U_l''=2\xi_1^{(l)}sn^{-3}\tau[1-(1+k^2)sn^2\tau+k^2sn^4\tau]$$

$$+\left(\varepsilon_1^{(l)}-\xi_1^{(l)}sn^{-2}\tau\right)[-(1+k^2-2k^2sn^2\tau)sn\tau]$$

$$U_l''=2\xi_1^{(l)}sn^{-3}\tau-2\xi_1^{(l)}(1+k^2)sn^{-1}\tau+2\xi_1^{(l)}k^2sn\tau-\varepsilon_1^{(l)}(1+k^2)sn\tau+2\varepsilon_1^{(l)}k^2sn^3\tau$$

$$+\xi_1^{(l)}(1+k^2)sn^{-1}\tau-2\xi_1^{(l)}k^2sn\tau$$

$$U_l''=-\varepsilon_1^{(l)}(1+k^2)sn\tau+2\varepsilon_1^{(l)}k^2sn^3\tau-\xi_1^{(l)}(1+k^2)sn^{-1}\tau+2\xi_1^{(l)}sn^{-3}\tau$$

$$(U_l')^2 = \left[\left(\varepsilon_1^{(l)} - \xi_1^{(l)} sn^{-2}\tau\right) cn\tau dn\tau\right]^2$$

$$(U_l')^2 = \left[\left(\varepsilon_1^{(l)} - \xi_1^{(l)} sn^{-2}\tau\right)\right]^2 cn^2\tau dn^2\tau$$

$$(U_l')^2 = \left[\left(\varepsilon_1^{(l)}\right)^2 - 2\varepsilon_1^{(l)}\xi_1^{(l)} sn^{-2}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau\right](1 - sn^2\tau)(1 - k^2 sn^2\tau)$$

$$(U_l')^2 = \left[\left(\varepsilon_1^{(l)}\right)^2 - 2\varepsilon_1^{(l)}\xi_1^{(l)} sn^{-2}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau\right](1 - k^2 sn^2\tau - sn^2\tau + k^2 sn^4\tau)$$

$$(U_l')^2 = \left[\left(\varepsilon_1^{(l)}\right)^2 - 2\varepsilon_1^{(l)}\xi_1^{(l)} sn^{-2}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau\right][1 - (1 + k^2) sn^2\tau + k^2 sn^4\tau]$$

$$\begin{aligned}(U_l')^2 = {} & \left(\varepsilon_1^{(l)}\right)^2 - \left(\varepsilon_1^{(l)}\right)^2 (1 + k^2) sn^2\tau + \left(\varepsilon_1^{(l)}\right)^2 k^2 sn^4\tau - 2\varepsilon_1^{(l)}\xi_1^{(l)} sn^{-2}\tau \\ & + 2\varepsilon_1^{(l)}\xi_1^{(l)}(1 + k^2) - 2\varepsilon_1^{(l)}\xi_1^{(l)} k^2 sn^2\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau - \left(\xi_1^{(l)}\right)^2 (1 + k^2) sn^{-2}\tau \\ & + \left(\xi_1^{(l)}\right)^2 k^2\end{aligned}$$

$$\begin{aligned}(U_l')^2 = {} & \left(\varepsilon_1^{(l)}\right)^2 + 2\varepsilon_1^{(l)}\xi_1^{(l)}(1 + k^2) + \left(\xi_1^{(l)}\right)^2 k^2 - \left[\left(\varepsilon_1^{(l)}\right)^2 (1 + k^2) + 2\varepsilon_1^{(l)}\xi_1^{(l)} k^2\right] sn^2\tau \\ & + \left(\varepsilon_1^{(l)}\right)^2 k^2 sn^4\tau - \left[2\varepsilon_1^{(l)}\xi_1^{(l)} + \left(\xi_1^{(l)}\right)^2 (1 + k^2)\right] sn^{-2}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau\end{aligned}$$

$$\begin{aligned}(U_l)(U_l')^2 = {} & \left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)} sn\tau + \xi_1^{(l)} sn^{-1}\tau\right)\Big\{\left(\varepsilon_1^{(l)}\right)^2 + 2\varepsilon_1^{(l)}\xi_1^{(l)}(1 + k^2) + \left(\xi_1^{(l)}\right)^2 k^2 \\ & - \left[\left(\varepsilon_1^{(l)}\right)^2 (1 + k^2) + 2\varepsilon_1^{(l)}\xi_1^{(l)} k^2\right] sn^2\tau + \left(\varepsilon_1^{(l)}\right)^2 k^2 sn^4\tau \\ & - \left[2\varepsilon_1^{(l)}\xi_1^{(l)} + \left(\xi_1^{(l)}\right)^2 (1 + k^2)\right] sn^{-2}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-4}\tau\Big\}\end{aligned}$$

$$\begin{aligned}(U_l)(U_l')^2 = {} & \varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2 + 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1 + k^2) + \varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 k^2 \\ & - \varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2 (1 + k^2) sn^2\tau - 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)} k^2 sn^2\tau + \varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2 k^2 sn^4\tau \\ & - 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)} sn^{-2}\tau - \varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 (1 + k^2) sn^{-2}\tau + \varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 sn^{-4}\tau \\ & + \left(\varepsilon_1^{(l)}\right)^3 sn\tau + 2\left(\varepsilon_1^{(l)}\right)^2 \xi_1^{(l)}(1 + k^2) sn\tau + \varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2 k^2 sn\tau \\ & - \left(\varepsilon_1^{(l)}\right)^3 (1 + k^2) sn^3\tau + 2\left(\varepsilon_1^{(l)}\right)^2 \xi_1^{(l)} k^2 sn^3\tau + \left(\varepsilon_1^{(l)}\right)^3 k^2 sn^5\tau\end{aligned}$$

$$-2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau-\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau$$

$$+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau+2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau+\left(\xi_1^{(l)}\right)^3k^2sn^{-1}\tau$$

$$-\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau$$

$$-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau-\left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau+\left(\xi_1^{(l)}\right)^3sn^{-5}\tau$$

$$(U_l)(U_l')^2=\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2k^2+\left(\varepsilon_1^{(l)}\right)^3sn\tau$$

$$+2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau-\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau$$

$$-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau-\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)sn^2\tau-2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2sn^2\tau$$

$$-\left(\varepsilon_1^{(l)}\right)^3(1+k^2)sn^3\tau\ +2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau$$

$$+\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau+\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau-2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau$$

$$-\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau$$

$$+2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau+\left(\xi_1^{(l)}\right)^3k^2sn^{-1}\tau-2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}sn^{-2}\tau$$

$$-\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-2}\tau+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau$$

$$-\left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau+\left(\xi_1^{(l)}\right)^3sn^{-5}\tau$$

$$(U_l)(U_l')^2=\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2k^2$$

$$+\left[\left(\varepsilon_1^{(l)}\right)^3+2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2-\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)\right.$$

$$\left.-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2\right]sn\tau-\left[\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2\right]sn^2\tau$$

$$-\left[\left(\varepsilon_1^{(l)}\right)^3(1+k^2)-2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2-\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2\right]sn^3\tau$$

$$+\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau+\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau-\left[2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)\right.$$

$$-\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)} - 2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2) - \left(\xi_1^{(l)}\right)^3 k^2]sn^{-1}\tau - \left[2\varepsilon_0^{(l)}\ \varepsilon_1^{(l)}\xi_1^{(l)}\right.$$

$$+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)]sn^{-2}\tau + \left[\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2 - 2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2\right.$$

$$-\left(\xi_1^{(l)}\right)^3(1+k^2)]sn^{-3}\tau + \varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 sn^{-4}\tau + \left(\xi_1^{(l)}\right)^3 sn^{-5}\tau$$

$$(U_l)(U_l')^2 = \varepsilon_0^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2 + 2\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) + \left(\xi_1^{(l)}\right)^2 k^2\right]$$

$$+\varepsilon_1^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2 + \varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) - \left(\xi_1^{(l)}\right)^2 k^2\right]sn\tau$$

$$-\varepsilon_0^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2(1+k^2) + 2\varepsilon_1^{(l)}\xi_1^{(l)}k^2\right]sn^2\tau$$

$$-\left(\varepsilon_1^{(l)}\right)^2\left[\varepsilon_1^{(l)}(1+k^2) - 3\xi_1^{(l)}k^2\right]sn^3\tau + \varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2 k^2 sn^4\tau$$

$$+\left(\varepsilon_1^{(l)}\right)^3 k^2 sn^5\tau$$

$$-\left[\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)} - \varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2) - \left(\xi_1^{(l)}\right)^3 k^2\right]sn^{-1}\tau$$

$$-\varepsilon_0^{(l)}\xi_1^{(l)}\left[2\varepsilon_1^{(l)} + \xi_1^{(l)}(1+k^2)\right]sn^{-2}\tau$$

$$-\left[\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2 + \left(\xi_1^{(l)}\right)^3(1+k^2)\right]sn^{-3}\tau$$

$$+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 sn^{-4}\tau + \left(\xi_1^{(l)}\right)^3 sn^{-5}\tau$$

$$U_l^2 = \left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)}sn\tau + \xi_1^{(l)}sn^{-1}\tau\right)^2$$

$$U_l^2 = \left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)}sn\tau\right)^2 + 2\left(\varepsilon_0^{(l)} + \varepsilon_1^{(l)}sn\tau\right)\xi_1^{(l)}sn^{-1}\tau + \left(\xi_1^{(l)}sn^{-1}\tau\right)^2$$

$$U_l^2 = \left(\varepsilon_0^{(l)}\right)^2 + 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}sn\tau + \left(\varepsilon_1^{(l)}\right)^2 sn^2\tau + 2\varepsilon_0^{(l)}\xi_1^{(l)}sn^{-1}\tau + 2\varepsilon_1^{(l)}\xi_1^{(l)} + \left(\xi_1^{(l)}\right)^2 sn^{-2}\tau$$

$$U_l^2 = \left(\varepsilon_0^{(l)}\right)^2 + 2\varepsilon_1^{(l)}\xi_1^{(l)} + 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}sn\tau + \left(\varepsilon_1^{(l)}\right)^2 sn^2\tau + 2\varepsilon_0^{(l)}\xi_1^{(l)}sn^{-1}\tau + \left(\xi_1^{(l)}\right)^2 sn^{-2}\tau$$

$$U_l^2 U_l'' = \left[\left(\varepsilon_0^{(l)}\right)^2 + 2\varepsilon_1^{(l)}\xi_1^{(l)} + 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}sn\tau + \left(\varepsilon_1^{(l)}\right)^2 sn^2\tau + 2\varepsilon_0^{(l)}\xi_1^{(l)}sn^{-1}\tau\right.$$

$$+\left(\xi_1^{(l)}\right)^2 sn^{-2}\tau]\ [-\varepsilon_1^{(l)}(1+k^2)sn\tau + 2\varepsilon_1^{(l)}k^2 sn^3\tau - \xi_1^{(l)}(1+k^2)sn^{-1}\tau$$

$$+2\xi_1^{(l)}sn^{-3}\tau]$$

$$U_l^2U_l'' = -\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}(1+k^2)sn\tau + 2\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}k^2sn^3\tau - \left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn^{-1}\tau$$

$$+2\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-3}\tau - 2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau + 4\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau$$

$$-2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau + 4\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau - 2\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)sn^2\tau$$

$$+4\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau - 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) + 4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}sn^{-2}\tau$$

$$-\left(\varepsilon_1^{(l)}\right)^3(1+k^2)sn^3\tau + 2\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau - \left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau$$

$$+2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau - 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) + 4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2sn^2\tau$$

$$-2\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-2}\tau + 4\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau - \varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau$$

$$+2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau - \left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau + 2\left(\xi_1^{(l)}\right)^3sn^{-5}\tau$$

$$U_l^2U_l'' = -2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) - 2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) - \left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}(1+k^2)sn\tau$$

$$-2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau - \left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau + 2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau$$

$$-2\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)sn^2\tau + 4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2sn^2\tau + 2\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}k^2sn^3\tau$$

$$+4\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau - \left(\varepsilon_1^{(l)}\right)^3(1+k^2)sn^3\tau + 4\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau$$

$$+2\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau - \left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn^{-1}\tau - 2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau$$

$$+2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau - \varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau + 4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}sn^{-2}\tau$$

$$-2\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-2}\tau + 2\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-3}\tau + 4\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau$$

$$-\left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau + 4\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau + 2\left(\xi_1^{(l)}\right)^3sn^{-5}\tau$$

$$U_l^2U_l'' = -4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)$$

$$-\varepsilon_1^{(l)}\left[\left(\varepsilon_0^{(l)}\right)^2(1+k^2) + 3\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2) - 2\left(\xi_1^{(l)}\right)^2k^2\right]sn\tau$$

$$-2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left[\varepsilon_1^{(l)}(1+k^2) - 2\xi_1^{(l)}k^2\right]sn^2\tau$$

$$+\varepsilon_1^{(l)}\left[2\left(\varepsilon_0^{(l)}\right)^2k^2+4\varepsilon_1^{(l)}\xi_1^{(l)}k^2-\left(\varepsilon_1^{(l)}\right)^2(1+k^2)]sn^3\tau\right.$$

$$+4\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau+2\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau$$

$$-\xi_1^{(l)}\left[\left(\varepsilon_0^{(l)}\right)^2(1+k^2)+3\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)-2\left(\varepsilon_1^{(l)}\right)^2\right]sn^{-1}\tau$$

$$+2\varepsilon_0^{(l)}\xi_1^{(l)}\left[2\varepsilon_1^{(l)}-\xi_1^{(l)}(1+k^2)\right]sn^{-2}\tau+\xi_1^{(l)}\left[2\left(\varepsilon_0^{(l)}\right)^2+4\varepsilon_1^{(l)}\xi_1^{(l)}\right.$$

$$\left.-\left(\xi_1^{(l)}\right)^2(1+k^2)\right]sn^{-3}\tau+4\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau+2\left(\xi_1^{(l)}\right)^3sn^{-5}\tau$$

Elde edilen bu ifadeler daha önce adı geçen (3.20) denkleminde

$$(\omega+a_l\kappa^2-b_l\kappa\omega)U_l-[c_l+d_l+\kappa^2(\mathcal{H}_l+\mathcal{R}_l)]U_l^3+\mathcal{J}_lU_l^5+\mathcal{L}_lU_l(U_l')^2$$

$$-(a_l-b_lv)U_l''+\mathcal{R}_lU_l^2U_l''=0$$

yerine yazılırsa,

$$(\omega+a_l\kappa^2-b_l\kappa\omega)\left(\varepsilon_0^{(l)}+\varepsilon_1^{(l)}sn\tau+\xi_1^{(l)}sn^{-1}\tau\right)$$

$$-[c_l+d_l+\kappa^2(\mathcal{H}_l+\mathcal{R}_l)]\left\{\left(\varepsilon_0^{(l)}\right)^3+6\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+3\left[\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}+\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}\right]sn\tau\right.$$

$$+3\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau+\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+3\left[\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2\right]sn^{-1}\tau$$

$$\left.+3\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau+\left(\xi_1^{(l)}\right)^3sn^{-3}\right\}+\mathcal{J}_l\left\{\left(\varepsilon_0^{(l)}\right)^5+20\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}\right.$$

$$+30\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2+5\left[\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}+6\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2+2\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2\right]sn\tau$$

$$+10\left[\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}+2\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}\right]sn^2\tau+5\left[2\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3+\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}\right]sn^3\tau$$

$$+5\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4sn^4\tau+\left(\varepsilon_1^{(l)}\right)^5sn^5\tau+5\left[\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}+6\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2\right.$$

$$\left.+2\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3\right]sn^{-1}\tau+10\left[\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2+2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3\right]sn^{-2}\tau$$

$$\left.+5\left[2\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3+\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4\right]sn^{-3}\tau+5\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-4}\tau+\left(\xi_1^{(l)}\right)^5sn^{-5}\tau\right\}$$

$$+\mathcal{L}_l\left\{\varepsilon_0^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2+2\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\left(\xi_1^{(l)}\right)^2k^2\right]\right.$$

$$+\varepsilon_1^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2+\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)-\left(\xi_1^{(l)}\right)^2k^2\right]sn\tau$$

$$-\varepsilon_0^{(l)}\left[\left(\varepsilon_1^{(l)}\right)^2(1+k^2)+2\varepsilon_1^{(l)}\xi_1^{(l)}k^2\right]sn^2\tau$$

$$-\left(\varepsilon_1^{(l)}\right)^2\left[\varepsilon_1^{(l)}(1+k^2)-3\xi_1^{(l)}k^2\right]sn^3\tau+\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau+\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau$$

$$-\left[\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}-\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)-\left(\xi_1^{(l)}\right)^3k^2\right]sn^{-1}\tau$$

$$-\varepsilon_0^{(l)}\xi_1^{(l)}\left[2\varepsilon_1^{(l)}+\xi_1^{(l)}(1+k^2)\right]sn^{-2}\tau-\left[\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+\left(\xi_1^{(l)}\right)^3(1+k^2)\right]sn^{-3}\tau$$

$$+\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau+\left(\xi_1^{(l)}\right)^3sn^{-5}\tau\}-(a_l-b_l\nu)\{-\varepsilon_1^{(l)}(1+k^2)sn\tau+2\varepsilon_1^{(l)}k^2sn^3\tau$$

$$-\xi_1^{(l)}(1+k^2)sn^{-1}\tau+2\xi_1^{(l)}sn^{-3}\tau\}+\mathcal{R}_l\{-4\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)$$

$$-\varepsilon_1^{(l)}\left[\left(\varepsilon_0^{(l)}\right)^2(1+k^2)+3\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)-2\left(\xi_1^{(l)}\right)^2k^2\right]sn\tau$$

$$-2\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left[\varepsilon_1^{(l)}(1+k^2)-2\xi_1^{(l)}k^2\right]sn^2\tau$$

$$+\varepsilon_1^{(l)}\left[2\left(\varepsilon_0^{(l)}\right)^2k^2+4\varepsilon_1^{(l)}\xi_1^{(l)}k^2-\left(\varepsilon_1^{(l)}\right)^2(1+k^2)\right]sn^3\tau+4\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau$$

$$+2\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau-\xi_1^{(l)}\left[\left(\varepsilon_0^{(l)}\right)^2(1+k^2)+3\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)-2\left(\varepsilon_1^{(l)}\right)^2\right]sn^{-1}\tau$$

$$+2\varepsilon_0^{(l)}\xi_1^{(l)}\left[2\varepsilon_1^{(l)}-\xi_1^{(l)}(1+k^2)\right]sn^{-2}\tau$$

$$+\xi_1^{(l)}\left[2\left(\varepsilon_0^{(l)}\right)^2+4\varepsilon_1^{(l)}\xi_1^{(l)}-\left(\xi_1^{(l)}\right)^2(1+k^2)\right]sn^{-3}\tau+4\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau$$

$$+2\left(\xi_1^{(l)}\right)^3sn^{-5}\tau\}=0$$

elde edilir. Bu denklem sistemi çözülerek,

$$\omega\varepsilon_0^{(l)}+\omega\varepsilon_1^{(l)}sn\tau+\omega\xi_1^{(l)}sn^{-1}\tau+a_l\kappa^2\varepsilon_0^{(l)}+a_l\kappa^2\varepsilon_1^{(l)}sn\tau+a_l\kappa^2\xi_1^{(l)}sn^{-1}\tau-b_l\kappa\omega\varepsilon_0^{(l)}$$

$$-b_l\kappa\omega\varepsilon_1^{(l)}sn\tau-b_l\kappa\omega\xi_1^{(l)}sn^{-1}\tau-c_l\left(\varepsilon_0^{(l)}\right)^3-6c_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-3c_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau$$

$$-3c_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau-3c_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau-c_l\left(\varepsilon_1^{(l)}\right)^3sn^3\tau-3c_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau$$

$$-3c_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau-3c_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau-c_l\left(\xi_1^{(l)}\right)^3sn^{-3}\tau-d_l\left(\varepsilon_0^{(l)}\right)^3$$

$$-6d_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-3d_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau-3d_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau-3d_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau$$

$$-d_l\left(\varepsilon_1^{(l)}\right)^3sn^3\tau-3d_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau-3d_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau-3d_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$-d_l\left(\xi_1^{(l)}\right)^3sn^{-3}\tau-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^3-6\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}sn\tau-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2sn^2\tau-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^3sn^3\tau$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-2}\tau-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\xi_1^{(l)}\right)^3sn^{-3}\tau+\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^5$$

$$+20\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}+30\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}sn\tau$$

$$+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn\tau+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn\tau+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2sn^2\tau$$

$$+20\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}sn^2\tau+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3sn^3\tau+5\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}sn^3\tau$$

$$+5\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4sn^4\tau+\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^5sn^5\tau+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}sn^{-1}\tau$$

$$+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-1}\tau+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-1}\tau+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2sn^{-2}\tau$$

$$+20\mathcal{J}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3sn^{-2}\tau+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3sn^{-3}\tau+5\mathcal{J}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-3}\tau$$

$$+5\mathcal{J}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4sn^{-4}\tau+\mathcal{J}_l\left(\xi_1^{(l)}\right)^5sn^{-5}\tau+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)$$

$$+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2k^2+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3sn\tau+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau$$

$$-\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)sn^2\tau-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2sn^2\tau-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)sn^3\tau$$

$$+3\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau$$

$$+\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3k^2sn^{-1}\tau-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}sn^{-2}\tau$$

$$-\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-2}\tau-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau-\mathcal{L}_l\left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau$$

$$+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2 sn^{-4}\tau+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3 sn^{-5}\tau+a_l\varepsilon_1^{(l)}(1+k^2)sn\tau-2a_l\varepsilon_1^{(l)}k^2sn^3\tau$$

$$+a_l\xi_1^{(l)}(1+k^2)sn^{-1}\tau-2a_l\xi_1^{(l)}sn^{-3}\tau-b_l v\varepsilon_1^{(l)}(1+k^2)sn\tau+2b_l v\varepsilon_1^{(l)}k^2sn^3\tau$$

$$-b_l v\xi_1^{(l)}(1+k^2)sn^{-1}\tau+2b_l v\xi_1^{(l)}sn^{-3}\tau-4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)$$

$$-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}(1+k^2)sn\tau-3\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn\tau+2\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2sn\tau$$

$$-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)sn^2\tau+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2sn^2\tau+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}k^2sn^3\tau$$

$$+4\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2sn^3\tau-\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)sn^3\tau+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2sn^4\tau$$

$$+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3k^2sn^5\tau-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}(1+k^2)sn^{-1}\tau-3\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-1}\tau$$

$$+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}sn^{-1}\tau+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}sn^{-2}\tau-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)sn^{-2}\tau$$

$$+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}sn^{-3}\tau+4\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-3}\tau-\mathcal{R}_l\left(\xi_1^{(l)}\right)^3(1+k^2)sn^{-3}\tau$$

$$+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2sn^{-4}\tau+2\mathcal{R}_l\left(\xi_1^{(l)}\right)^3sn^{-5}\tau=0$$

bulunur. Bu eşitlik düzenlendiğinde,

$$\omega\varepsilon_0^{(l)}+a_l\kappa^2\varepsilon_0^{(l)}-b_l\kappa\omega\varepsilon_0^{(l)}-c_l\left(\varepsilon_0^{(l)}\right)^3-6c_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-d_l\left(\varepsilon_0^{(l)}\right)^3-6d_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^3-6\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^5+20\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$+30\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2k^2$$

$$-4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\Big\{\omega\varepsilon_1^{(l)}+a_l\kappa^2\varepsilon_1^{(l)}-b_l\kappa\omega\varepsilon_1^{(l)}-3c_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}$$

$$-3d_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}-3d_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}$$

$$+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3$$

$$+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2+a_l\varepsilon_1^{(l)}(1+k^2)-b_l v\varepsilon_1^{(l)}(1+k^2)$$

$$-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}(1+k^2)-3\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)+2\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2\}sn\tau$$

$$+\Big\{-3c_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2-3d_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2$$

$$+20\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}-\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2$$

$$-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2\}sn^2\tau$$

$$+\Big\{-c_l\left(\varepsilon_1^{(l)}\right)^3-d_l\left(\varepsilon_1^{(l)}\right)^3-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^3+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3$$

$$+5\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)+3\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2-2a_l\varepsilon_1^{(l)}k^2+2b_lv\varepsilon_1^{(l)}k^2$$

$$+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}k^2+4\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2-\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)\}sn^3\tau$$

$$+\Big\{5\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2\}sn^4\tau$$

$$+\Big\{\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^5+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3k^2+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3k^2\}sn^5\tau+\Big\{\omega\xi_1^{(l)}+a_l\kappa^2\xi_1^{(l)}-b_l\kappa\omega\xi_1^{(l)}$$

$$-3c_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}-3c_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-3d_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}-3d_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}$$

$$+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)$$

$$+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3k^2+a_l\xi_1^{(l)}(1+k^2)-b_lv\xi_1^{(l)}(1+k^2)-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}(1+k^2)$$

$$-3\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}\Big\}sn^{-1}\tau+\Big\{-3c_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2-3d_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2+20\mathcal{J}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$-\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)\}sn^{-2}\tau$$

$$+\Big\{-c_l\left(\xi_1^{(l)}\right)^3-d_l\left(\xi_1^{(l)}\right)^3-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\xi_1^{(l)}\right)^3+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3+5\mathcal{J}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4$$

$$-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-\mathcal{L}_l\left(\xi_1^{(l)}\right)^3(1+k^2)-2a_l\xi_1^{(l)}+2b_lv\xi_1^{(l)}+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}$$

$$+4\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-\mathcal{R}_l\left(\xi_1^{(l)}\right)^3(1+k^2)\}sn^{-3}\tau+\Big\{5\mathcal{J}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2$$

$$+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2\Big\}sn^{-4}\tau+\Big\{\mathcal{J}_l\left(\xi_1^{(l)}\right)^5+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3+2\,\mathcal{R}_l\left(\xi_1^{(l)}\right)^3\Big\}sn^{-5}\tau=0$$

elde edilir. $\{sn\tau, sn^2\tau, sn^3\tau, sn^4\tau, sn^5\tau, sn^{-1}\tau, sn^{-2}\tau, sn^{-3}\tau, sn^{-4}\tau, sn^{-5}\tau\}$ kümesinin elemanları lineer bağımsız olduğundan katsayıları sıfıra eşittir.

$$\omega\varepsilon_0^{(l)}+a_l\kappa^2\varepsilon_0^{(l)}-b_l\kappa\omega\varepsilon_0^{(l)}-c_l\left(\varepsilon_0^{(l)}\right)^3-6c_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-d_l\left(\varepsilon_0^{(l)}\right)^3-6d_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^3-6\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}+\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^5+20\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$+30\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^2+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2k^2$$

$$-4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}(1+k^2)=0$$

$$\omega\varepsilon_1^{(l)}+a_l\kappa^2\varepsilon_1^{(l)}-b_l\kappa\omega\varepsilon_1^{(l)}-3c_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}-3d_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}-3d_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}$$

$$-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\varepsilon_1^{(l)}$$

$$+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)$$

$$-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2+a_l\varepsilon_1^{(l)}(1+k^2)-b_lv\varepsilon_1^{(l)}(1+k^2)-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}(1+k^2)$$

$$-3\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}(1+k^2)+2\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2k^2=0$$

$$-3c_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2-3d_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\varepsilon_1^{(l)}\right)^2$$

$$+20\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^3\xi_1^{(l)}-\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2$$

$$-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2(1+k^2)+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}k^2=0$$

$$-c_l\left(\varepsilon_1^{(l)}\right)^3-d_l\left(\varepsilon_1^{(l)}\right)^3-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_1^{(l)}\right)^3+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\varepsilon_1^{(l)}\right)^3+5\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^4\xi_1^{(l)}$$

$$-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)+3\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2-2a_l\varepsilon_1^{(l)}k^2+2b_lv\varepsilon_1^{(l)}k^2+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}k^2$$

$$+4\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}k^2-\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3(1+k^2)=0,$$

$$5\mathcal{J}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^4+\mathcal{L}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\varepsilon_1^{(l)}\right)^2k^2=0$$

$$\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^5+\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^3k^2+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^3k^2=0$$

$$\omega\xi_1^{(l)}+a_l\kappa^2\xi_1^{(l)}-b_l\kappa\omega\xi_1^{(l)}-3c_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}-3c_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-3d_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}$$

$$-3d_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+5\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^4\xi_1^{(l)}$$

$$+30\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_1^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3-\mathcal{L}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}+\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)$$

$$+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3k^2+a_l\xi_1^{(l)}(1+k^2)-b_lv\xi_1^{(l)}(1+k^2)-\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}(1+k^2)$$

$$-3\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)+2\mathcal{R}_l\left(\varepsilon_1^{(l)}\right)^2\xi_1^{(l)}=0$$

$$-3c_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2-3d_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2-3\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^3\left(\xi_1^{(l)}\right)^2$$

$$+20\mathcal{J}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^3-2\mathcal{L}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}-\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)+4\mathcal{R}_l\varepsilon_0^{(l)}\varepsilon_1^{(l)}\xi_1^{(l)}$$

$$-2\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2(1+k^2)=0$$

$$-c_l\left(\xi_1^{(l)}\right)^3-d_l\left(\xi_1^{(l)}\right)^3-\kappa^2(\mathcal{H}_l+\mathcal{R}_l)\left(\xi_1^{(l)}\right)^3+10\mathcal{J}_l\left(\varepsilon_0^{(l)}\right)^2\left(\xi_1^{(l)}\right)^3+5\mathcal{J}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^4$$

$$-\mathcal{L}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-\mathcal{L}_l\left(\xi_1^{(l)}\right)^3(1+k^2)-2a_l\xi_1^{(l)}+2b_lv\xi_1^{(l)}+2\mathcal{R}_l\left(\varepsilon_0^{(l)}\right)^2\xi_1^{(l)}$$

$$+4\mathcal{R}_l\varepsilon_1^{(l)}\left(\xi_1^{(l)}\right)^2-\mathcal{R}_l\left(\xi_1^{(l)}\right)^3(1+k^2)=0$$

$$5\mathcal{J}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^4+\mathcal{L}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2+4\mathcal{R}_l\varepsilon_0^{(l)}\left(\xi_1^{(l)}\right)^2=0$$

$$\mathcal{J}_l\left(\xi_1^{(l)}\right)^5+\mathcal{L}_l\left(\xi_1^{(l)}\right)^3+2\mathcal{R}_l\left(\xi_1^{(l)}\right)^3=0$$

Bu sistem çözülerek aşağıdaki parametreler elde edilir.

Set-1

$$\varepsilon_0^{(l)}=0$$

$$\varepsilon_1^{(l)}=\pm m\sqrt{-\frac{\mathcal{L}_l+2\mathcal{R}_l}{\mathcal{J}_l}}$$

$$\xi_1^{(l)}=0$$

$$\omega = \frac{-m^2\mathcal{L}_l^2+\mathcal{J}_l\left(a_l(1+m^2+\kappa^2)-b_l v(1+m^2)\right)-2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa-1)}$$

$$c_l = \frac{2\mathcal{J}_l(a_l-b_l v)-(\mathcal{L}_l+2\mathcal{R}_l)\left(d_l+\kappa^2(\mathcal{H}_l+\mathcal{R}_l)+(1+m^2)(\mathcal{L}_l+\mathcal{R}_l)\right)}{\mathcal{L}_l+2\mathcal{R}_l} \tag{3.23}$$

Set-2

$$\varepsilon_0^{(l)} = 0$$

$$\varepsilon_1^{(l)} = 0$$

$$\xi_1^{(l)} = \pm\sqrt{-\frac{\mathcal{L}_l+2\mathcal{R}_l}{\mathcal{J}_l}}$$

$$\omega = \frac{-m^2\mathcal{L}_l^2+\mathcal{J}_l\left(a_l(1+m^2+\kappa^2)-b_l v(1+m^2)\right)-2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa-1)}$$

$$c_l = \frac{2\mathcal{J}_l(a_l-b_l v)-(\mathcal{L}_l+2\mathcal{R}_l)\left(d_l+\kappa^2(\mathcal{H}_l+\mathcal{R}_l)+(1+m^2)(\mathcal{L}_l+\mathcal{R}_l)\right)}{\mathcal{L}_l+2\mathcal{R}_l} \tag{3.24}$$

Set-3

$$\varepsilon_0^{(l)} = 0$$

$$\varepsilon_1^{(l)} = \pm m\sqrt{-\frac{\mathcal{L}_l+2\mathcal{R}_l}{\mathcal{J}_l}}$$

$$\xi_1^{(l)} = \pm\sqrt{-\frac{\mathcal{L}_l+2\mathcal{R}_l}{\mathcal{J}_l}}$$

$$\omega = \frac{4m(m-1)^2\mathcal{L}_l^2+\mathcal{J}_l\left(a_l\left(1+\kappa^2+m(m-6)\right)-b_l v\left(1+m(m-6)\right)\right)+8m(m-1)^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa-1)}$$

$$c_l = -d_l + \frac{2\mathcal{J}_l(a_l-b_l v)-(\mathcal{L}_l+2\mathcal{R}_l)\left(\kappa^2(\mathcal{H}_l+\mathcal{R}_l)+\left(1+m(m-6)\right)(\mathcal{L}_l+\mathcal{R}_l)\right)}{\mathcal{L}_l+2\mathcal{R}_l} \tag{3.25}$$

Sonuç olarak (3.23)-(3.25) çözüm setleri yardımıyla (3.2) ve (3.3) LPD sistemine JEF çözümler aşağıdaki gibi listelenir,

$$q(x,t) = \pm m\sqrt{-\frac{\mathcal{L}_1+2\mathcal{R}_1}{\mathcal{J}_1}}\,sn\left[x-\left\{\frac{b_1\omega-2a_1\kappa}{1-b_1\kappa}\right\}t\right]$$

$$\times\, exp\left[i\left\{-\kappa x+\left(\frac{-m^2\mathcal{L}_l^2+\mathcal{J}_l\left(a_l(1+m^2+\kappa^2)-b_l v(1+m^2)\right)-2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.26}$$

$$r(x,t) = \pm m\sqrt{-\frac{\mathcal{L}_2 + 2\mathcal{R}_2}{\mathcal{J}_2}}\, sn\left[x - \left\{\frac{b_2\omega - 2a_2\kappa}{1 - b_2\kappa}\right\}t\right]$$

$$\times\, exp\left[i\left\{-\kappa x + \left(\frac{-m^2\mathcal{L}_l^2 + \mathcal{J}_l\left(a_l(1+m^2+\kappa^2) - b_l v(1+m^2)\right) - 2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa - 1)}\right)t + \theta\right\}\right] \quad (3.27)$$

$$q(x,t) = \pm\sqrt{-\frac{\mathcal{L}_1 + 2\mathcal{R}_1}{\mathcal{J}_1}}\, ns\left[x - \left\{\frac{b_1\omega - 2a_1\kappa}{1 - b_1\kappa}\right\}t\right]$$

$$\times\, exp\left[i\left\{-\kappa x + \left(\frac{-m^2\mathcal{L}_l^2 + \mathcal{J}_l\left(a_l(1+m^2+\kappa^2) - b_l v(1+m^2)\right) - 2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa - 1)}\right)t + \theta\right\}\right] \quad (3.28)$$

$$r(x,t) = \pm\sqrt{-\frac{\mathcal{L}_2 + 2\mathcal{R}_2}{\mathcal{J}_2}}\, ns\left[x - \left\{\frac{b_2\omega - 2a_2\kappa}{1 - b_2\kappa}\right\}t\right]$$

$$\times\, exp\left[i\left\{-\kappa x + \left(\frac{-m^2\mathcal{L}_l^2 + \mathcal{J}_l\left(a_l(1+m^2+\kappa^2) - b_l v(1+m^2)\right) - 2m^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa - 1)}\right)t + \theta\right\}\right] \quad (3.29)$$

$$q(x,t) = \pm\sqrt{-\frac{\mathcal{L}_1 + 2\mathcal{R}_1}{\mathcal{J}_1}}\left(m\, sn\left[x - \left\{\frac{b_1\omega - 2a_1\kappa}{1 - b_1\kappa}\right\}t\right] - ns\left[x - \left\{\frac{b_1\omega - 2a_1\kappa}{1 - b_1\kappa}\right\}t\right]\right)$$

$$\times\, exp\left[i\left\{-\kappa x + \left(\frac{4m(m-1)^2\mathcal{L}_l^2 + \mathcal{J}_l\left(a_l(\kappa^2 + \bar{m}) - b_l v\bar{m}\right) + 8m(m-1)^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa - 1)}\right)t + \theta\right\}\right] \quad (3.30)$$

$$r(x,t) = \pm\sqrt{-\frac{\mathcal{L}_2 + 2\mathcal{R}_2}{\mathcal{J}_2}}\left(m\, sn\left[x - \left\{\frac{b_2\omega - 2a_2\kappa}{1 - b_2\kappa}\right\}t\right] - ns\left[x - \left\{\frac{b_2\omega - 2a_2\kappa}{1 - b_2\kappa}\right\}t\right]\right)$$

$$\times\, exp\left[i\left\{-\kappa x + \left(\frac{4m(m-1)^2\mathcal{L}_l^2 + \mathcal{J}_l\left(a_l(\kappa^2 + \bar{m}) - b_l v\bar{m}\right) + 8m(m-1)^2\mathcal{L}_l\mathcal{R}_l}{\mathcal{J}_l(b_l\kappa - 1)}\right)t + \theta\right\}\right] \quad (3.31)$$

Burada $\bar{m} = 1 + m(m - 6)$ dır. Burada bu çözümlerin var olması için

$$\mathcal{J}_1(\mathcal{L}_1 + 2\mathcal{R}_1) < 0 \quad (3.32)$$

ve

$$\mathcal{J}_2(\mathcal{L}_2 + 2\mathcal{R}_2) < 0 \quad (3.33)$$

eşitsizliklerinin sağlanmasının gerekli olduğu bahsedilmelidir.

$$F(m,\phi)=\int_0^{\phi}\frac{d\theta}{\sqrt{1-m^2sin^2\theta}}$$

Jacobi eliptik fonksiyonlar $m\to 1$ olduğunda aşağıda verildiği gibi hiperbolik fonksiyonlara indirgenir

$\Rightarrow\ sn\xi\to th\xi$

$\Rightarrow\ cn\xi\to sech\xi$

$\Rightarrow\ dn\xi\to sech\xi$

m modülü 1'e yaklaştığında (3.26)-(3.31) çözümleri aşağıdaki koyu solitonlara,

$$q(x,t)=\pm\sqrt{-\frac{\mathcal{L}_1+2\mathcal{R}_1}{\mathcal{J}_1}}tanh\left[x-\left\{\frac{b_1\omega-2a_1\kappa}{1-b_1\kappa}\right\}t\right]$$
$$\times exp\left[i\left\{-\kappa x+\left(\frac{-\mathcal{L}_l^2+\mathcal{J}_1(a_l(2+\kappa^2)-2b_lv)-2\mathcal{L}_1\mathcal{R}_1}{\mathcal{J}_l(b_1\kappa-1)}\right)t+\theta\right\}\right] \tag{3.34}$$

$$r(x,t)=\pm\sqrt{-\frac{\mathcal{L}_2+2\mathcal{R}_2}{\mathcal{J}_2}}tanh\left[x-\left\{\frac{b_2\omega-2a_2\kappa}{1-b_2\kappa}\right\}t\right]$$
$$\times exp\left[i\left\{-\kappa x+\left(\frac{-\mathcal{L}_l^2+\mathcal{J}_1(a_l(2+\kappa^2)-2b_lv)-2\mathcal{L}_1\mathcal{R}_1}{\mathcal{J}_l(b_1\kappa-1)}\right)t+\theta\right\}\right] \tag{3.35}$$

ve aşağıdaki singüler solitonlara

$$q(x,t)=\pm\sqrt{-\frac{\mathcal{L}_1+2\mathcal{R}_1}{\mathcal{J}_1}}coth\left[x-\left\{\frac{b_1\omega-2a_1\kappa}{1-b_1\kappa}\right\}t\right]$$
$$\times exp\left[i\left\{-\kappa x+\left(\frac{-\mathcal{L}_l^2+\mathcal{J}_1(a_l(2+\kappa^2)-2b_lv)-2\mathcal{L}_1\mathcal{R}_1}{\mathcal{J}_l(b_1\kappa-1)}\right)t+\theta\right\}\right] \tag{3.36}$$

$$r(x,t)=\pm\sqrt{-\frac{\mathcal{L}_2+2\mathcal{R}_2}{\mathcal{J}_2}}coth\left[x-\left\{\frac{b_2\omega-2a_2\kappa}{1-b_2\kappa}\right\}t\right]$$
$$\times exp\left[i\left\{-\kappa x+\left(\frac{-\mathcal{L}_l^2+\mathcal{J}_1(a_l(2+\kappa^2)-2b_lv)-2\mathcal{L}_1\mathcal{R}_1}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.37}$$

$$q(x,t)=\pm 2\sqrt{-\frac{\mathcal{L}_1+2\mathcal{R}_1}{\mathcal{J}_1}}csch2\left[x-\left\{\frac{b_1\omega-2a_1\kappa}{1-b_1\kappa}\right\}t\right]$$
$$\times exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{J}_1(a_l(\kappa^2-4)+4b_lv)}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.38}$$

$$r(x,t)=\pm 2\sqrt{-\frac{\mathcal{L}_2+2\mathcal{R}_2}{\mathcal{J}_2}}csch2\left[x-\left\{\frac{b_2\omega-2a_2\kappa}{1-b_2\kappa}\right\}t\right]$$

$$\times\, exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{J}_1\big(a_l(\kappa^2-4)+4b_l v\big)}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.39}$$

dönüşür.

Jacobi eliptik fonksiyonlar, $m \to 0$ olduğunda aşağıda verildiği gibi trigonometrik fonksiyonlara indirgenir,

$sn\xi \to sin\xi\,,\quad cn\xi \to cos\xi\,,\quad dn\xi \to 1$

Bununla beraber $m \to 0$ ise (3.28)-(3.31) çözümleri aşağıdaki periyodik singüler çözümlere indirgenir

$$q(x,t)=\pm\sqrt{-\frac{\mathcal{L}_1+2\mathcal{R}_1}{\mathcal{J}_1}}\,csc\left[x-\left\{\frac{b_1\omega-2a_1\kappa}{1-b_1\kappa}\right\}t\right]$$

$$\text{x}\;\; exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{J}_1\big(a_l(1+\kappa^2)-b_l v\big)}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.40}$$

$$r(x,t)=\pm\sqrt{-\frac{\mathcal{L}_2+2\mathcal{R}_2}{\mathcal{J}_2}}\,csc\left[x-\left\{\frac{b_2\omega-2a_2\kappa}{1-b_2\kappa}\right\}t\right]$$

$$\text{x}\;\; exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{J}_1\big(a_l(1+\kappa^2)-b_l v\big)}{\mathcal{J}_l(b_l\kappa-1)}\right)t+\theta\right\}\right] \tag{3.41}$$

Uyarı: Modele diğer Jacobi katlı periyodik dalga çözümleri elde etmek için $sn\xi$ ve $dn\xi$ Jacobi fonksiyonlarının açılımları aşağıdaki gibi göz önüne alınmalıdır:

$$U_l=\sum_{j=0}^{M}\varepsilon_j^{(l)}cn^j\tau+\sum_{j=1}^{M}\zeta_j^{(l)}cn^{-j}\tau \tag{3.42}$$

ve

$$U_l=\sum_{j=0}^{M}\varepsilon_j^{(l)}dn^j\tau+\sum_{j=1}^{M}\zeta_j^{(l)}dn^{-j}\tau \tag{3.43}$$

(3.42) ve (3.43) denklemleri yardımıyla (3.2) ve (3.3) denklem sistemi için üçgensel fonksiyon, düğümsel dalga, dnoidal dalga çözümlerini içeren non solitonik ve ek soliton çözümler elde edilir. Ancak bu açılımlar kolaylık için ihmal edilmiştir.

4. SONUÇ

Bu çalışma, iki entegrasyon şemasını kullanarak koyu ve tekil optik soliton çözümlerini güvence altına almıştır. Jacobi eliptik fonksiyon şeması ve $\exp(-\phi(\eta))$ genişleme yöntemi, bu soliton çözümlerini, kısıtlama koşulları olarak listelenen varlık kriterleriyle birlikte başarıyla elde etti. Bu çalışmanın sonuçları gerçekten de bu modelle daha fazla araştırma faaliyeti yürütmeyi teşvik etmektedir. Gelecekte, deterministik ve stokastik tipte pertürbasyon terimleri dahil edilecek ve bu, mevcut çalışmadaki çözümlerin genişletilmesi ve genelleştirilmiş versiyonu olarak hizmet edecek ek bir çözüm setine yol açacaktır. Ayrıca solitonlara ve diğer çözümlere ulaşmak için ek entegrasyon planları uygulanacaktır. Bunlar Kudryashov'un yöntemi, Lie simetri analizi ve diğerleridir. Bu sonuçlar ilerleyen süreçte kademeli olarak ve sırayla rapor edilecektir.

Bu çalışma, PMD fiberler için LPD modeline Jacobi eliptik fonksiyon çözümlerini ele almıştır. Bu çözümleri göstermek için bu çalışmada kullanılan matematiksel araç, güçlü genişletilmiş JEF genişleme yaklaşımıdır. Sınırlayıcı durumda, m modülü $m \to 1$ olduğunda, etkili çözümler fotonik çalışmalarında çok yararlı olan koyu ve singüler solitonlara yaklaşır. Ancak $m \to 0$ ise, çözümler periyodik singüler çözümlere dönüşür. Bu sonuçlar ışığında, benimsenen şemanın, yöneten model için ilginç ve anlamlı olan çözümlerin yaygınlığına neden olduğunu belirtmek gerekir.

5. KAYNAKLAR

Abdou, M. A., & Elhanbaly, A. (2007). Construction of periodic and solitary wave solutions by the extended Jacobi elliptic function expansion method. *Communications in Nonlinear Science and Numerical Simulation*, *12*(7), 1229-1241.

Abdou, M. A., Soliman, A. A., Biswas, A., Ekici, M., Zhou, Q., & Moshokoa, S. P. (2018). Dark-singular combo optical solitons with fractional complex Ginzburg–Landau equation. *Optik*, *171*, 463-467.

Alqahtani, R. T., Babatin, M. M., & Biswas, A. (2018). Bright optical solitons for Lakshmanan-Porsezian-Daniel model by semi-inverse variational principle. *Optik*, *154*, 109-114.

Arshed, S., Biswas, A., Majid, F. B., Zhou, Q., Moshokoa, S. P., & Belic, M. (2018). Optical solitons in birefringent fibers for Lakshmanan–Porsezian–Daniel model using exp (− ϕ (ξ))-expansion method. *Optik*, *170*, 555-560.

Bansal, A., Biswas, A., Triki, H., Zhou, Q., Moshokoa, S. P., & Belic, M. (2018). Optical solitons and group invariant solutions to Lakshmanan–Porsezian–Daniel model in optical fibers and PCF. *Optik*, *160*, 86-91.

Bhrawy, A. H., Abdelkawy, M. A., & Biswas, A. (2013). Cnoidal and snoidal wave solutions to coupled nonlinear wave equations by the extended Jacobi's elliptic function method. *Communications in Nonlinear Science and Numerical Simulation*, *18*(4), 915-925.

Biswas, A., Ekici, M., Sonmezoglu, A., & Babatin, M. M. (2018). Optical solitons with differential group delay and dual-dispersion for Lakshmanan–Porsezian–Daniel model by extended trial function method. *Optik*, *170*, 512-519.

Biswas, A., Ekici, M., Sonmezoglu, A., Triki, H., Majid, F. B., Zhou, Q. & Belic, M. (2018). Optical solitons with Lakshmanan–Porsezian–Daniel model using a couple of integration schemes. *Optik*, *158*, 705-711.

Biswas, A., Kara, A. H., Alqahtani, R. T., Ullah, M. Z., Triki, H., & Belic, M. (2018). Conservation laws for optical solitons of Lakshmanan–Porsezian–Daniel model. In *Proc. Rom. Acad. Ser. A* (Vol. 19, No. 1, pp. 39-44).

Biswas, A., Yıldırım, Y., Yaşar, E., & Alqahtani, R. T. (2018). Optical solitons for Lakshmanan–Porsezian–Daniel model with dual-dispersion by trial equation method. *Optik*, *168*, 432-439.

Biswas, A., Yildirim, Y., Yasar, E., Zhou, Q., Moshokoa, S. P., & Belic, M. (2018). Optical solitons for Lakshmanan–Porsezian–Daniel model by modified simple equation method. *Optik*, *160*, 24-32.

Ekici, M. (2018). Optical solitons in birefringent fibers for Lakshmanan–Porsezian–Daniel model by extended Jacobi's elliptic function expansion scheme. *Optik*, *172*, 651-656.

Ekici, M., Mirzazadeh, M., Sonmezoglu, A., Zhou, Q., Triki, H., Ullah, M. Z., ... & Biswas, A. (2017). Optical solitons in birefringent fibers with Kerr nonlinearity by exp-function method. *Optik*, *131*, 964-976.

Ekici, M., Zhou, Q., Sonmezoglu, A., Manafian, J., & Mirzazadeh, M. (2017). The analytical study of solitons to the nonlinear Schrödinger equation with resonant nonlinearity. *Optik*, *130*, 378-382.

Hubert, M. B., Betchewe, G., Justin, M., Doka, S. Y., Crepin, K. T., Biswas, A., ... & Belic, M. (2018). Optical solitons with Lakshmanan–Porsezian–Daniel model by modified extended direct algebraic method. *Optik*, *162*, 228-236.

Jawad, A. J. M., Abu-AlShaeer, M. J., Biswas, A., Zhou, Q., Moshokoa, S., & Belic, M. (2018). Optical solitons to Lakshmanan-Porsezian-Daniel model for three nonlinear forms. *Optik*, *160*, 197-202.

Khan, J.K. & Akbar, M.A. (2013). Application of $exp\big(-\phi(\xi)\big)$-expansion method to find the exact solutions of modified Benjamin–Bona–Mahony equation, WorldAppl. Sci. (10) 1373–1377.

Lakshmanan, M., Porsezian, K., & Daniel, M. (1988). Effect of discreteness on the continuum limit of the Heisenberg spin chain. *Physics Letters A*, *133*(9), 483-488.

Liu, W., Qiu, D. Q., Wu, Z. W., & He, J. S. (2016). Dynamical Behavior of Solution in Integrable Nonlocal Lakshmanan—Porsezian—Daniel Equation. *Communications in Theoretical Physics*, *65*(6), 671.

Manafian, J., Foroutan, M., & Guzali, A. (2017). Applications of the ETEM for obtaining optical soliton solutions for the Lakshmanan-Porsezian-Daniel model. *The European Physical Journal Plus*, *132*, 1-22.

Mirzazadeh, M., Ekici, M., Sonmezoglu, A., Ortakaya, S., Eslami, M., & Biswas, A. (2016). Soliton solutions to a few fractional nonlinear evolution equations in shallow water wave dynamics. *The European Physical Journal Plus*, *131*, 1-11.

Roshid, H. O., Kabir, M. R., Bhowmik, R. C., & Datta, B. K. (2014). Investigation of Solitary wave solutions for Vakhnenko-Parkes equation via exp-function and Exp (− ϕ (ξ))-expansion method. *SpringerPlus*, *3*, 1-10.

Sönmezoğlu, A. (2015). Exact solutions for some fractional differential equations. Advances in Mathematical Physics, 2015(1), 567842.

Taghizadeh, N., Zhou, Q., Ekici, M., & Mirzazadeh, M. (2017). Soliton solutions for Davydov solitons in α-helix proteins. *Superlattices and Microstructures*, *102*, 323-341.

Vega-Guzman, J., Alqahtani, R. T., Zhou, Q., Mahmood, M. F., Moshokoa, S. P., Ullah, M. Z., ... & Belic, M. (2017). Optical solitons for Lakshmanan–Porsezian–Daniel model with spatio-temporal dispersion using the method of undetermined coefficients. *Optik*, *144*, 115-123.

Vega-Guzman, J., Biswas, A., Mahmood, M. F., Zhou, Q., Moshokoa, S. P., & Belic, M. (2018). Optical solitons with polarization mode dispersion for Lakshmanan–Porsezian–Daniel model by the method of undetermined coefficients. *Optik*, *171*, 114-119.

Zhang, H. (2007). Extended Jacobi elliptic function expansion method and its applications. *Communications in Nonlinear Science and Numerical Simulation*, *12*(5), 627-635.