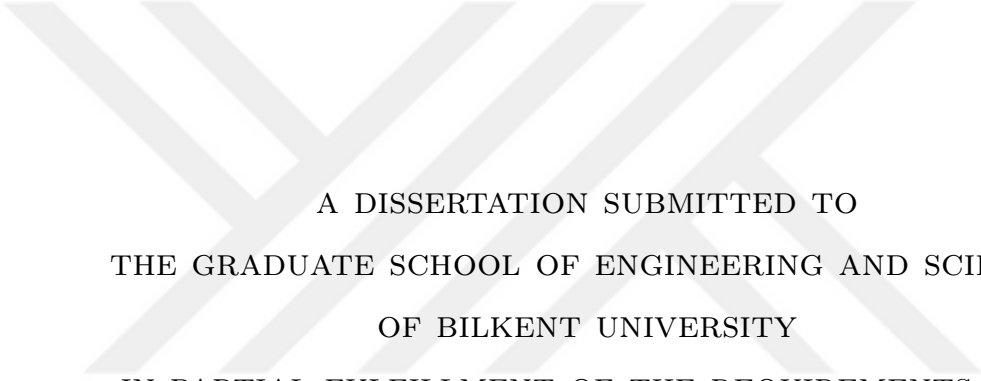


OPTIMIZING AIRLINE OPERATIONS UNDER UNCERTAINTY



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Özge Şafak Aydın
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June 2019

We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

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Ph.D. in Industrial Engineering

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Fluctuations in passenger demand, airport congestion, and high fuel costs are the main threats to airlines' profit, thereby need to be carefully addressed in airline scheduling problems. This study takes an advantage of aircraft cruise speed control in several scheduling problems to keep the cost of fuel manageable. We first generate a flight schedule by integrating strategic departure time decisions, tactical fleet and routing decisions and more operational flight timing decisions under stochastic demand and non-cruise times. Our model differs from the existing studies by including aircraft cruise speed decisions to compensate for increase in non-cruise time variations due to the airport congestion. To efficiently solve the problem, we provide a scenario group-wise decomposition algorithm. Then, we consider a new problem which aims to accommodate new flights into an existing flight schedule in a short time. We suggest some operational changes such as controlling the aircraft cruise speed, re-timing flight departures and swapping aircraft to open up time for new flights. However, nonlinear fuel cost function, and binary assignment and swapping decisions significantly increase the computational burden of solving scheduling problems. In this thesis, we propose strong mixed integer conic quadratic formulations. Finally, we extend the problem by including a strategic decision to lease an aircraft for introducing new flights. More importantly, we consider the effects of departure time decisions on the probability distribution of random demand. We propose a bounding method based on scenario group-wise decomposition for stochastic programs with decision dependent probabilities.

Keywords: aircraft cruise speed control, airline scheduling, decision dependent uncertainty, decomposition methods, mixed integer conic quadratic programming, stochastic programming.

ÖZET

BELİRSİZLİKLER ALTINDA HAVAYOLU OPERASYONLARINI EN İYİLEME

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Yolcu talebindeki dalgalanmalar, havaalanı yoğunluğu ve yüksek yakıt maliyetleri, havayollarının kazancı üzerindeki ana tehditlerdir, bu nedenle havayolu planlama problemlerinde dikkatlice ele alınması gerekir. Bu çalışma, yakıt maliyetini kontrol edilebilir şekilde tutmak için çeşitli çizelgeleme problemlerinde uçak seyir hızı kontrolünün avantajlarından yararlanır. Öncelikle rassal talep ve seyir dışı zamanlar altında stratejik kalkış zamanı kararlarını, taktiksel filo ve rota kararlarını ve daha operasyonel uçuş zamanlama kararlarını entegre ederek bir uçuş programı oluşturuyoruz. Modelimiz, havaalanı yoğunluğu nedeniyle seyir dışı zaman değişimlerindeki artışı telafi etmek için uçak seyir hızı kararlarını dahil ederek mevcut çalışmalardan farklıdır. Problemi etkin bir şekilde çözmek için senaryo grup bazında bir ayrıştırma algoritması sunuyoruz. Ardından, kısa bir sürede yeni uçuşları mevcut bir uçuş planına dahil etmeyi amaçlayan yeni bir problemi ele alıyoruz. Yeni uçuşlara yeterli zamanı açmak için, uçak seyir hızını kontrol etmek, uçuş saatlerini yeniden ayarlamak ve uçak değiş tokuşu gibi bazı operasyonel değişiklikler öneriyoruz. Bununla birlikte, doğrusal olmayan yakıt maliyeti fonksiyonu ve ikili tahsis ve uçak değiş tokuş kararları, çizelgeleme problemlerini çözmeyi önemli ölçüde zorlaştırmaktadır. Bu tezde, güçlü karma tamsayılı konik karesel formülasyonlar öneriyoruz. Son olarak, yeni uçuşlar için bir uçağın kiralanmasına yönelik stratejik bir karar ekleyerek bu problemi genişletiyoruz. Daha da önemlisi, uçuş kalkış zamanı kararlarının rassal talebin olasılık dağılımı üzerindeki etkilerini dikkate alıyoruz. Karar bağlı olasılıkları olan rassal programlar için senaryo grup bazında ayrıştırmaya dayanan bir sınırlama yöntemi önermekteyiz.

Anahtar sözcükler: uçak seyir hızı kontrolü, havayolu çizelgeleme, karara dayalı belirsizlik, ayrıştırma yöntemleri, karma tamsayılı konik karesel programlama, rassal programlama.

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Chapter 1

Introduction

At major airports, air traffic congestion may cause significant delays in flight departures and arrivals, in turn disrupting aircraft connections and passenger itineraries. According to the U.S. Department of Transportation (BTS [1]), around 18% of flights were delayed in 2016. Almost 29% of delays occurred due to the air carrier, 36% were due to the aircraft arriving late, and 31% were due to the National Aviation System. Only 4% of delays were due to heavy weather conditions and security issues. In 2010, FAA/Nextor [2] reported that the cost of all US air transportation delays in 2007 totalled \$31.2 billion. \$16.7 billion, or about half of the total cost was incurred from the extra cost of the passengers' longer travel times. The \$8.3 billion airline component consists of increased expenses for crew, fuel, and maintenance. Accordingly, and especially with consideration of a highly competitive market, it is crucial for airlines to manage their flights, aircraft and crews efficiently to minimize operational costs. Therefore, airlines are willing to seek additional advanced solution techniques that allow them to make decisions jointly, thus producing solutions which provide reliable performance despite the uncertainties involved in flight operations.

In the first part of this dissertation, we work on an airline scheduling problem in an uncertain environment. The problem of airline scheduling is disaggregated into different stages such as schedule design, fleet assignment, aircraft routing

and crew scheduling. Accordingly, a sequential planning approach has typically been employed. The first step, known as schedule design, is a strategic planning problem usually requiring reconciliation at least a year in advance of departures. Schedules of flights with predetermined origins, destination airports and flight departure and arrival times are generated by considering potential passenger demand, fleet and crew resources. Following completion of this first stage, airline engineers can begin the fleet assignment problem: here, aircraft types, each having a different seat capacity, are assigned to flight legs based on their equipment capabilities, availabilities and operational expenses. Following fleet assignment, a minimum cost route must be determined for each aircraft while satisfying the maintenance requirements.

There are four challenges arising in this study. First, airline scheduling problem itself is a complex problem with various subproblems. Solving each stage independently ignores the dependencies between stages so that the resulting schedule may not be feasible. On the other hand, solving an integrated model with hundreds of flights and aircraft could not be possible due to the increased computational burden.

Second, fluctuations in passenger demand bring an additional challenge in assigning the right type of aircraft to each leg. Realized demands may either exceed or fall short of the capacity of assigned aircraft type, resulting in loss of profit and customer goodwill.

Third, uncertainties in air traffic congestion make the problem significantly more difficult. These uncertainties are not included in any planning stages of a deterministic approach, despite the flight delays due to the air traffic congestions propagate through the network, ultimately affecting a large scale of aircraft routes and passenger itineraries. To minimize unexpected delays and ensure the passengers' connections, in some existing studies, safety times (i.e. slack) are inserted to flying times. However, if delays are not observed, slacks become useless. An aircraft stands idle at the airport. Therefore, this strategy decreases the utilization of an aircraft. An alternative way of ensuring on-time connections is to control the aircraft cruise speed. The speed of the aircraft can be increased

to compensate the increase in flight times due to air traffic congestion. In this thesis, we use the great advantage of cruise time controllability to handle the uncertainties in flight times.

Fourth and the last challenge is to consider the interplay between decisions in an uncertain environment. Let us consider a situation with high uncertainty in non-cruise times (taxi-in and out phases of a flight) due to air traffic congestion. On the one hand, airlines may choose to set a higher cruise speed to guarantee the minimum time requirements for passenger connections. Since speeding up the aircraft results in additional fuel burn and carbon emission costs, assigning a fuel efficient but smaller aircraft may be preferable. However, such an assignment may spill some of the passengers. Thus, passenger demand needs to be taken into account and a fuel efficient aircraft should only be assigned when the savings from fuel conservations compensate for the cost of spilled passengers. On the other hand, if a larger aircraft is used, it may be prohibitively costly to speed up the aircraft. In that case, carefully selecting departure times helps to reduce the costs caused by the late arrival of the aircraft. None of the approaches in the existing literature consider all these interplays between decisions while accounting for stochastic passenger demand and flight times. Therefore, the crucial contribution of our study is that we propose an integrated model which determines strategic departure time decisions, tactical fleetings and routing decisions and operational flight timing decisions while considering randomness in passenger demand and non-cruise times.

In this thesis, we also consider solution methods for proposed multi stage stochastic mixed integer nonlinear program (MINLP). For multi stage stochastic MINLP, there is no efficient solution method available, to the best of our knowledge. Difficulty of these problems is due to three main reasons. The first reason is the huge size of these problems. As the number of stages increases, scenario tree grows exponentially. Potentially large number of scenario-dependent variables and constraints arise in these problems. Second, nonlinearity brings an additional computational challenge. The third reason is the mixed integer nature of the decision variables.

Therefore, to handle the large number of scenarios, this thesis suggests a scenario group-wise decomposition (SGWD) algorithm to provide lower and upper bounds for the optimal value of the problem. The algorithm solves a number of group subproblems with reduced number of scenarios, and then uses their solutions to obtain an upper bound. However, solving group subproblems, which are still multi stage stochastic MINLPs may require significant computation times. Therefore, we also propose a cutting plane algorithm to efficiently solve each group subproblem. The performance of the cutting plane algorithm can be improved by utilizing different optimality cuts with different strengths.

Later, we address a new problem of accommodating new flights into an existing airline flight schedule. While operating a daily flight schedule, several events, such as tourism, business conferences or even a natural disaster, might necessitate introducing new flights on a particular day. In a relatively short time period, leasing or buying an aircraft just to serve these new flights may not be a feasible option. Therefore, an airline aims to accommodate new flights with minimum disruption on the existing schedule. In near real time, to accommodate these new flights into an existing flight schedule, the airline can only make some operational changes: either it can use the idle times, if any, in the existing schedule, or it can change the departure times of the existing flights, or it can increase the aircraft cruise speed to shorten the flight times. The airline can utilize any one of these alternatives or any combination of them to open up enough time to accommodate new flights. Increasing the cruise speed, however, has an adverse effect on the fuel burn, which in turn increases the fuel and carbon emission costs, i.e., the most significant component of an airline's operational costs. Since aircraft types have different fuel efficiencies, changing the aircraft assignments could be beneficial in reducing the fuel burn, and consequently decreasing the operational costs.

We propose two approaches to accommodate new flights into an existing schedule. The first approach carefully adjusts flight departure times and as well as aircraft cruise speed to allow the required time for operating new flights. Increasing the cruise speed of a flight directly reduces its block time, and thereby opening up space to accommodate new flights into the flight schedule. Although cruise time reduction provides a great opportunity to add new flights, increasing the

speed of an aircraft comes with significant additional cost of fuel burn and CO₂ emission. To keep the cost of fuel manageable, new flights may be assigned to a fuel-efficient, but smaller aircraft. However, such an assignment may spill some of the passengers due to the insufficient seat capacity, resulting in a loss of revenue. Therefore, in order to address this trade-off, we propose a second approach, which incorporates an explicit aircraft swapping mechanism together with cruise time controllability. Aircraft swapping provides a greater opportunity in reducing the fuel burn and capturing passenger demand of new flights. Through flight timing and assignment decisions, we trade-off the incremental fuel cost associated with the cruise time compression with the revenue from the passengers. Although the second approach may provide substantial improvements in the airline’s profit over the first one, the additional binary swapping decisions and the nonlinear fuel burn function make the optimization problem significantly more difficult to solve. Our aim is to provide a set of alternative schedules with increasing profit at a cost of additional computation time. A decision maker can interactively specify her preferences (or restrictions) and analyze their effect on the profit of airline when introducing new flights into an existing schedule.

Re-timing approach and aircraft swap opportunities have been widely used to minimize the delay propagation and ensure the passengers’ connections in the existing studies. However, in the context of introducing new flights, for the first time, we introduce these options of flight re-timing, aircraft cruise speed control and aircraft swapping to open up enough time for new flights in the schedule.

The binary assignment and swapping decisions, nonlinear (convex) cost terms for fuel burned and arrival tardiness of an aircraft increase the computational burden of solving this problem. Therefore, in this thesis, to efficiently handle the nonlinearity, we use convexification results from mixed integer conic quadratic optimization. We propose strong mixed integer conic quadratic (MICQ) formulation, and then improve this formulation by adding McCormick inequalities. These reformulations enable us to solve relatively large instances generated using a real data set of a major U.S. airline to optimality very efficiently.

Airbus group reported a global market forecast (GMF) 2018-2037 [3], which

shows a growing trend in air passenger traffic demand over the past 20 years. According to Airbus' GMF [3], air traffic will continue to grow at 4.4 % annually, requiring some 37,400 new passengers in 20 years. Therefore, in a highly competitive market, airlines would like to develop new routes or increase the frequencies of existing flights to meet more demand. In Chapter 5, we address a long term planning problem which aims to introduce new flights for the next season.

In this study, we introduce a strategic decision on leasing an aircraft to operate new flights. This option provides a great flexibility to meet passenger demand. The challenging part of leasing option is to determine the right type of aircraft with the right seat capacity before observing the passenger demand of new flights. Any larger aircraft than needed may presumably incur higher operational costs. On the other hand, smaller aircraft might spill some of the passengers, turning in loss of revenue. To overcome this trade off, we consider potential passenger demand scenarios for new flights.

Another important feature of this study is to consider the effect of departure time decisions on the probability distribution of random demand. For instance, it is more likely to observe high demand when the business flights or flights to coastal cities are introduced in the morning. On the other hand, when these flights are introduced around noon, there might be a high probability that these flights would not be preferred by many of passengers. Therefore, the departure times of new flights should be carefully determined to maximize an airline's profit.

If leasing an aircraft is costly or leased aircraft is not available, then we suggest to accommodate new flights into an existing flight schedule by shifting the current flights' arrival times and compressing the cruise times of flights. In this case, there arises another challenge to determine the most profitable aircraft that is available on the departure time of new flight. One departure time decision may provide a high potential to observe high demand. In this case, if a large aircraft is not available in the existing schedule, some of the revenue would be loss. On the other hand, another departure time decision might increase the probability of observing low demand. In this case, if a small fuel efficient aircraft is not available, then higher fuel and emission costs of new flights would be incurred. This study trades

off the fuel burn with the revenue under the departure time dependent demand uncertainties.

To model departure time dependent probability distribution of random demand, we generate alternative departure time vectors for new flights. We first formulate the problem as a standard two stage stochastic program for each departure time vector. The departure time vector which gives the maximum profit could be identified after solving each two stage stochastic program. However, this enumeration approach could be computationally expensive if the number of departure time vectors increases. Therefore, we present a unified model which chooses the most profitable departure time option among the alternatives. We formulate the unified model as a two stage stochastic mixed integer nonlinear program.

The proposed unified model with decision dependent uncertainty is significantly more difficult to solve than a standard two stage stochastic program, since decision dependent probability distributions cause nonlinearities in the objective function of the unified model. We handle these nonlinearities using the convexification results from the mixed integer conic quadratic programming. In order to solve the strengthened conic reformulations with large number of scenarios, we first devise a bounding method based on scenario group wise decomposition for the stochastic programs with decision dependent probabilities. Then, we develop an exact decomposition algorithm which solves a number of scenario group subproblems, and then evaluates the departure time decisions of each group subproblem on the original problem. After exploring the solutions, we cut them off from the feasible region of the original problem. The algorithm continues to explore new departure time alternatives until the termination criteria is satisfied.

The rest of the thesis is organized as follows. In Chapter 2, we present the related literature on airline scheduling, stochastic programming and its solution methods. In Chapter 3, we briefly introduce a stochastic airline scheduling problem and give a multi stage stochastic MINLP formulation. Then, we suggest a scenario group-wise decomposition algorithm to efficiently solve general multi stage stochastic MINLP. We also suggest a cutting plane algorithm improved

with lazy constraints callback feature of CPLEX to solve the group subproblems within the decomposition algorithm. Finally, we report computational results, comparing three stage stochastic programming approach with existing two stage stochastic implementations, and illustrating the high quality performances of the proposed algorithms using a sample schedule for a major U.S. airline. Despite the additional challenges of the third stage, computational experiments show that schedules obtained by the proposed three-stage stochastic programming formulation obtain 3% saving compared to the schedules of existing two stage stochastic programming formulations only considering the uncertainties in demand. Note that, 3% cost saving is important for airlines since this may translate into billions of dollars.

In Chapter 4, we define a new problem to accommodate new flights into an existing flight schedule. We suggest two novel approaches with different solution qualities and computational efforts. While the first one only uses the aircraft cruise speed control and flight re-timing options to open up space for new flights in the schedule, the second one incorporates an additional aircraft swapping mechanism to reduce the fuel burn. On the other hand, experiments show that the second one requires an average of 6800 seconds to solve the problem instances optimally. Therefore, in Chapter 4, we propose a strong conic quadratic formulation which efficiently handles the nonlinear fuel burn and arrival tardiness costs. The experiments, conducted using publicly available data of a U.S. airline, show the superiority of strong conic formulations over an alternative conic formulation. The alternative conic formulation times out for all test instances. On the other hand, the strengthened conic formulation with McCormick inequalities solves all the instances to optimality in reasonable compute times.

In Chapter 5, we extend the problem defined in Chapter 4 by considering the randomness in passenger demand. In addition to cruise time controllability and flight re-timing approaches proposed in Chapter 4, here we suggest to lease an aircraft to operate new flights. More importantly, different than the standard stochastic programming problem, we consider the departure time dependent probability distribution of random demand. We model decision dependency using two stage stochastic mixed integer nonlinear programming. We present strengthened

conic quadratic reformulations which handle the nonlinearities. In order to solve two stage stochastic MINLPs with decision dependent probabilities, we develop an exact solution algorithm based on scenario group wise decomposition.



Chapter 2

Literature Review

Airline scheduling problem is typically decomposed into several subproblems such as schedule design, fleet assignment, aircraft routing and crew scheduling. In this chapter, we present a brief survey of existing approaches in these subproblems. Then, we briefly present general two stage and multi stage stochastic programming formulations. We also give the existing literature on decomposition methods for multi stage stochastic programming problems. Finally, we give a survey on stochastic programming approaches to solve the subproblems of airline scheduling problem.

2.1 Airline Scheduling

The airline scheduling problem is a very large complex problem with hundreds of flights and fleets. Therefore, the problem of airline scheduling has been usually disaggregated into different stages and tackled sequentially, as explained by Barnhart and Cohn [4]. The sequential scheme is described as: schedule design, fleet assignment, aircraft maintenance routing and crew scheduling. The first step, known as schedule design, is a strategic planning problem. In this step, the markets and their frequencies to serve and how to schedule flights to meet these

frequencies are determined. Following completion of this first stage, airline engineers can begin to solve the fleet assignment problem: here, aircraft types, each having a different seat capacity, are assigned to scheduled flight legs. Following the fleet assignment, a minimum cost route must be determined for each aircraft while satisfying the maintenance requirements. Finally, crew scheduling problem assigns crews to each scheduled flight so that crew costs are minimized.

These subproblems include a large number of interrelated decisions and non-linear cost structures. For instance, the type of the aircraft assigned to one flight leg cannot be determined without determining the types of aircraft assigned to several other flight legs in the network, since the airline has a limited number of aircraft of each type. Modelling the interdependencies and nonlinearities of the subproblems turns into thousands of constraints and billions of decision variables in a very large scale program with integer variables. That is the reason why decision makers suffer from the tractability issues. To date, it has been impossible to construct and solve a single optimization model handling all stages. Therefore, a sequential planning approach which solves each subproblem independently has been employed. More recently, researchers have focused on integrating two or three subproblems to produce more cost effective solutions. However, even each subproblem involves significant challenges as discussed in the following subsections.

2.1.1 Schedule Design

The schedule design is a strategic planning problem usually requiring reconciliation at least six months to one year in advance of departures. In the schedule design, a feasible flight network is constructed by selecting a set of origin-destination (O-D) pairs, determining flight departure times and the operating frequencies. The schedule design defines the market share of an airline, thus it is one of the important stages which has a huge impact on the profitability of airlines.

There are several challenges arising in the schedule design step. First of all, the schedule design interacts with fleet assignment and crew scheduling stages.

However, including all linking constraints and interdependent decisions in a single model significantly increases the computational burden of solving the schedule design problem. Secondly, it is difficult to estimate the accurate passenger demand which is a critical input to construct a flight schedule. On the other hand, the flight departure times and the frequencies also affect the passengers' demand. Therefore, the interaction between the optimization decisions and the uncertain passenger demand makes the schedule design problem significantly more complicated. Third, average fares which are another crucial data for estimating revenue, are critically influenced by flight offerings of an airline and the flight schedule of competitors. Fares are determined by complex pricing strategies. Due to these difficulties, as stated by Barnhart and Cohn [4], in practice, flight schedules have been designed by limited optimization. Researchers have limited the complexity in schedule design problems and solved optimization models with the objective of maximizing the incremental profits of the new schedule subject to the maximum number of changes to the existing schedule.

Lohatepanont and Barnhart [5] simultaneously optimize the selection of flight legs and the assignment of aircraft types to these flight legs in their incremental optimization approach. In the experiments, they show significant increases in revenue of one major U.S. airline. Other researches in optimizing airline schedule design can be found in Armacost et al. [6] and Erdmann et al. [7].

Hai and Barnhart [8] introduce a dynamic airline scheduling approach that changes the schedule slightly by re-timing flight departures and re-fleeting during the booking period to reflect changing demand. Re-timing provides more cost effective passenger connection and re-fleeting opportunities. Re-timing approach has been also used to minimize the delay propagation in the entire airline flight network. Chiraphadnakul and Barnhart [9] adjust flight departure times to provide slacks across the connections so as to minimize delay propagation. Lan et al. [10] implement the re-timing approach to minimize misconnected passengers. Similarly, Ahmedbeygi et al. [11] propose a method to minimize the delay propagation by redistributing the existing slack in the flight schedule. Sohoni et al. [12] propose two stochastic models which incorporate block time uncertainty into the schedule development process.

2.1.2 Fleet Assignment

In the fleet assignment problem, aircraft types, each having a different seat capacity, are assigned to flight legs based on their equipment capabilities, availabilities and operational expenses. The fleet assignment decisions significantly affect the profit of airlines. On one hand, if a smaller aircraft than needed is assigned to a flight, some of passengers are spilled due to insufficient seat capacity of the aircraft, thereby leads to loss of revenue and customer goodwill. On the other hand, if a larger aircraft is assigned, higher operational costs would be incurred. When the large number of flights and fleet resources are considered, assigning the right type of aircraft to the right flight is a challenging problem. As a result, traditional approaches solve the fleet assignment problem in isolation from other airline scheduling stages under some assumptions such that flight schedules repeat daily and flight-leg demand is known.

Basic fleet assignment model (FAM) has been formulated as a mixed integer program using different network structures. Abara [13] is one of the first researchers who formulate the FAM using connection network in which the arcs represent all possible feasible flight connections. Later, Hane et. al. [14] formulate the problem as a multi-commodity network flow problem with side constraints using the time space network. Different than the connection network, here nodes represent the location (station) of flight departures and arrivals on the timeline. Arrival nodes are specified by considering the flight times and ground times. In the network, arcs connect departure and arrival nodes of a flight. Each aircraft type is a commodity. The problem is to flow each commodity with minimum cost while satisfying that each flight leg is assigned to exactly one aircraft type. The binary decisions represent whether to assign a fleet to a flight leg or not. Due to the large number of integer variables, many heuristic approaches have been developed to obtain good solutions in reasonable computation times. As stated by Barnhart and Cohn [4], commercial solvers adopted many of proposed techniques, allowing efficient solution to the large-scale problems. Daskin and Panayotopoulos [15] also formulate the fleet assignment problem as a mixed integer program and design heuristic solutions based on Lagrangian relaxation. Rushmeier and

Kontogiorgis [16] deal with solving similar model formulation with large-scale data set. They develop a heuristic approach which first solves an LP relaxation, and then its solution is rounded to get an initial solution that is given into a branch-and-bound process. Sherali et al. [17] provide an extensive review of various fleet assignment models and algorithms.

Integrated fleet assignment and schedule design: In a traditional sequential approach, following the completion of schedule design, the basic fleet assignment models discussed above are solved with given flight departure times. However, it is clear that the integration of schedule design with fleet assignment increases the airline revenue by allowing more profitable connections. Therefore, Rexing et al. [18] and Sherali et al. [19] allow the flight departure times to vary within certain time-windows to provide improved flight connections opportunities for the fleet assignment problem. Rexing et al. [18] make a copy of flight arc for each discretized departure time within the specified time interval on the time-space network. However, introducing multiple copies of each arc in the formulation increases the computational burden of solving the problem.

Fleet assignment with passenger considerations: A leg-based modelling approach assumes that the number of spilled passengers, their associated spill costs, demand and revenue can be computed per flight leg. However, Barnhart et. al. [20] show that there is no single scheme for allocating itinerary fares to flight legs that can always guarantee an optimal fleet assignment. They develop a path based fleet assignment model. Different than the previous studies, they include path variables that represent the assignment of passengers to itineraries. They aim to minimize the revenue loss due to spilled passengers on the paths. Later, Lohatepanont and Barnhart [5] integrate leg selection decisions with the path based fleet assignment model to provide more flexibility to minimize the cost. Jacobs et al. [21] propose a model that integrates the FAM model with the origin and destination revenue management model.

Re-fleeting and aircraft swapping mechanism: Based on the updated passenger demand during the booking period, re-fleeting adjusts the capacity by re-assigning aircraft types to flight legs to reduce the operational costs and increase

the revenues. Sherali et al. [22] develop a demand-driven re-fleeting model that dynamically re-assigns the aircraft in response to improved passenger demand forecast. They only allow aircraft re-assignment within the same aircraft family to keep the crew assignments unchanged. Jarrah et al. [23] also re-assign fleet types by limiting the number of changes on the original fleet assignment.

In addition to adjusting the capacity, most airlines make use of swap opportunities to build robust aircraft routings. Ageeva [24] adds a reward for each opportunity to swap aircraft in an aircraft routing model and encourage overlapping routes to have more swap opportunities in the case of an operational disruption. If two aircraft routings meet at more than one airport, aircraft can be swapped, and then returned to their original routings at a next meeting point. Therefore, if a flight is delayed, swapping the aircraft provides robustness by allowing a flight with high demand to be flown. On the other hand, Smith and Johnson [25] state that although setting a limit on the aircraft types operated at a station provides more re-fleeting opportunities, it significantly increases computation times required to solve the fleet assignment problem.

Aktürk et al. [26] use the idea of swapping aircraft between flights to reduce the effect of a disruption on the schedule. They provide approximately 30% cost savings compared to the delay propagation recovery approach. More recently, Arikan et al. [27] use both flight re-timing and aircraft swapping approaches to find minimum cost passenger and aircraft recovery plans. Based on an investigation of over 240,000 domestic routings of 13 major U.S. airlines, Lonzius and Lange [28] confirm the delay-reducing effect of swap opportunities. Wang and Regan [29] examine a dynamic yield management problem when the assigned capacities are subject to a swap.

2.1.3 Aircraft Maintenance Routing

Given the assignment of fleets to scheduled flight legs, the aircraft maintenance routing problem determines the minimum cost route for each flight such that each

flight leg is covered in exactly one aircraft routing while satisfying the maintenance requirement of each fleet.

Clarke et al. [30] present an aircraft rotation problem which determines the routes flown by each aircraft in a given fleet with an objective of maximizing the benefit derived from making specific connections. They model the problem as an asymmetric traveling salesman problem with side constraints. They use Lagrangian relaxation and subgradient optimization to solve the problem.

Desaulniers et al. [31] determine the sequence of flights (route) carried out by an aircraft with the departure and arrival times of these flights. The aim of the problem is to maximize the anticipated profits derived from the aircraft. They present two models: a set partitioning type formulation and a time constrained multi-commodity network flow formulation.

Haouari et al. [32] state that set partitioning formulations for the aircraft routing problem include exponentially many variables so that column generation or branch-and-price algorithms are required. Therefore, in order to reduce the computation times, they present a compact polynomial-sized representation for the aircraft routing problem, which is then linearized and lifted. Their experiments demonstrate the high quality solutions and significant savings in computational time.

In the existing literature, aircraft maintenance routing problem is usually integrated with the fleet assignment problem. Barnhart et al. [33] develop a flight string model where the strings represent the sequences of flights that can be assigned to aircraft. Each string begins and ends at the maintenance stations so that any assignment is feasible. This model allows to solve an integrated fleet assignment and aircraft maintenance routing problems. They propose a branch-and-price solution approach which uses a column generation procedure to solve the integrated problem. Gopalan and Talluri [34] also define lines of flight as a sequence of flights which can be operated during a day. After they construct the set of lines of flights, they use simple polynomial-time algorithms, which find a routing of aircraft whose routings satisfy the maintenance requirements. The details

of the types of maintenance checks required by Federal Aviation Administration regulations can be found in Gopalan and Talluri [34].

Haouari et al. [35] aim to investigate fast optimization based approximation algorithms for solving an integrated aircraft fleet and routing problem. They develop network flow-based heuristic approaches using the network flow techniques. Computational studies conducted on real data of TunisAir demonstrated that the proposed heuristic provides very near optimal solutions within the reasonable computation times.

Different than the daily flight schedule, Liang et al. [36] present a novel weekly aircraft maintenance routing problem. They propose a new network based mixed integer linear programming formulation. Since the LP relaxation of the formulation is very tight, they use a diving heuristic method to solve the problem efficiently.

More recently, Lan et al. [10] consider the impact of delays and cancellations in an aircraft maintenance routing problem. They propose a new approach to reduce delay propagation by intelligently routing aircraft. Then, they develop another approach which minimizes the number of passenger misconnections by re-timing the departure times of flights within a time window. Aloulou et al. [37] also propose a model which builds robust aircraft routes that are less vulnerable to disruptions through judiciously distributing slacks to critical connections. Toward this end, the model makes the assignment of aircraft to each leg and determines the departure times of flights while considering the time slot assignments at airports.

2.1.4 Crew Scheduling

The crew scheduling problem itself is usually solved sequentially. First a minimum cost of assignment of crews to itineraries is determined [4]. An itinerary is a round trip consisting of sequence of flights. In the second step, pairings are assembled into monthly work schedules, and assigned to individual crew members [4]. Since

the crew scheduling problem is not considered in this thesis, we will only give a brief literature. The details on the airline crew scheduling problem can be found in Barnhart et al. [38], Clarke and Smith [39].

Crew pairing problem has gained a great attention of researches. It is typically modelled as a set partitioning problem finding the minimum cost of crew-itinerary pairings such that each flight leg is included in exactly one pairing [4]. A column generation algorithm can be used to solve the set partitioning problem. Hoffman and Padberg [40] discuss an alternative branch-and-cut approach for solving large set partitioning problems to proven optimality, even when additional side constraints modelling labor requirements are incorporated.

Klabjan et al. [41] introduce a new approach for solving the crew scheduling problem that enumerates hundreds of millions random pairings. They first solve the linear programming relaxation and then select millions of columns with best reduced cost. They further reduce the number of columns by a heuristic. Lastly, they get an integer solution by enhancing the branching rule of the commercial solver. Their results show the superiority of the proposed approach over the approaches in the current practice.

In order to overcome the incompatible solutions, Cordeau et al. [42], Cohn and Barnhart [43] integrate airline crew scheduling problem with maintenance routing problem. Weide et al. [44] partially integrate aircraft routing and crew scheduling problems. They propose an iterative approach which starts from a minimum cost solution and produces a series of solutions that are increasingly robust to typical stochastic variability in airline operations.

2.2 Cruise Time Controllability

In the airline industry, there is an observation that cruise speed selections have a significant impact on the profit of an airline. Sherali et al. [17] state that airline optimization models are quite sensitive to fuel burn. Cook et al. [45]

discuss the option of flying faster to ensure the minimum time requirement for the connections of passengers and flying slower for conservation of fuel. In recent years, the aircraft speed control has been the subject of several research studies such as air traffic management (ATM), airline disruption management, aircraft recovery, and robust schedule design.

The joint work of FAA/Eurocontrol [46] emphasizes the importance of speed control for air traffic management to control the fuel burn and terminal congestion. While the aim of the proposed methodology in [46] is to save fuel by reducing cruise speed, once congestion in a terminal is determined, it is suggested to increase the speed of aircraft at the beginning of a rush period to avoid creating congestion and reduce the overall delay and fuel burn. Bertsimas et al. [47] control aircraft speed through adjustments in the time spent in each en route sector while deciding on an optimal combination of flow management actions including ground holding, rerouting and airborne holding.

In an overview of airline disruption management processes, Kohl et al. [48] discuss the ability to reduce passenger delay costs by accelerating the aircraft. Marla et al. [49] integrate disruption management with flight planning, which enables changes in the flight speed. Using a time-space network, they make multiple copies of flights representing different discrete departure times and cruise speeds. However, in the context of airline operations, this representation leads to a large number of copies of flights to be evaluated in the model. Arikan et al. [27] and Aktürk et al. [26] express cruise speed as a continuous variable and find an optimal trade-off between increased fuel cost and disruption costs such as delay and spilled passengers costs.

To manage disruptions in a less costly manner, airlines are also interested in building robust schedules. More recently, Duran et al. [50] use the idea of aircraft cruise speed control in response to the uncertainties in non-cruise times. Because of the congestion at major airports, non-cruise times including take-off, descent and taxi times may take longer than the expected. To absorb the increase in non-cruise times, earlier studies suggest to insert idle times so as to ensure the passenger connections. However, in these schedules, such an expensive aircraft

stands idle, if the non-cruise times are not realized long. Therefore, Duran et al. [50] simultaneously optimize aircraft speed and idle times in order to ensure the passengers' connections at a desired level. The experiments indicate that 60% of idle time costs can be reduced by only a 2% increase in fuel costs, turning in 12% savings in overall cost. These savings emphasize the effectiveness of aircraft cruise speed control strategy in schedule design. Later, Şafak et al. [51] use the aircraft cruise speed control option in an integrated robust schedule design and aircraft assignment problem. To ensure the passengers' connections, they also accelerate the aircraft to compensate the increase in non-cruise time variations. Different than the previous work, they suggest to assign a fuel efficient aircraft to reduce the fuel burn associated with accelerating the aircraft. However, the seat capacity of a fuel efficient aircraft could be less than others so that some of the passengers may be spilled. The experiments show that conservation of fuel can compensate the cost of spilled passengers so that 21% savings in overall cost is achieved. Then, Gürkan et al. [52] include aircraft cruise speed decisions in an integrated airline scheduling, fleet and routing problem.

Kang and Hansen [53] emphasize the importance of accurate flight fuel burn prediction to reduce the cost of an airline. They show how ensemble learning techniques can be used to improve flight trip fuel burn prediction. In their study, a novel discretionary fuel estimation approach is proposed to assist dispatchers with better discretionary fuel loading decisions.

2.3 Stochastic Programming

In many industries, the planning process is generally performed based on the expectation that operations will be maintained as planned. All data are assumed to be known with certainty a priori at the time of decision making. This approach implies that random parameters in the problem are represented by the expected values in the model. However, in practice, expected values may be too far from the realized values of the parameters, if the variability in the parameter is high. Therefore, resulting plan may perform poorly under certain realizations of the

random parameter. In order to provide solutions which can better withstand unexpected events, researchers recently have considered modelling the uncertainty in many areas of science and engineering, from medicine to manufacturing. Shapiro et al. [54] state that “although many ways have been proposed to model uncertain quantities, stochastic models have proved their flexibility and usefulness in diverse area of science”. An extensive discussion on the stochastic programming can be found in Shapiro et al. [54] and Birge and Louveaux [55].

In this thesis, we apply stochastic programming where potential scenarios for uncertain parameters are analyzed. In this chapter, we first address modelling issue of two stage stochastic programming and multi stage stochastic programming, respectively. Then, we give related literature on the airline scheduling applications of stochastic programming. We also provide recent decomposition methods to efficiently solve the stochastic programs. Finally, we extend the literature to decision dependent uncertainty in stochastic programming applications.

2.3.1 Two Stage Stochastic Programming

We demonstrate the general form of the two stage stochastic programming problems as:

$$\begin{aligned} \min \quad & c^\top x + \mathbb{E}_\xi Q(x, \xi) \\ \text{s.t.} \quad & Ax = b, \\ & x \in \mathbb{X}, \end{aligned}$$

where $Q(x, \xi)$ is the recourse function defined as the value of the optimal solution of the second stage problem:

$$\begin{aligned} \min \quad & d^\top y \\ \text{s.t.} \quad & Tx + Wy = h, \\ & y \in \mathbb{Y}. \end{aligned}$$

The vector of first stage decision variables x determines the actions which should be taken before the random parameters are known. $c \in \mathbb{R}^{n_1}$ and $b \in \mathbb{R}^{m_1}$

are known column vectors. $A \in \mathbb{R}^{m_1 \times n_1}$ is a known matrix and $\mathbb{X} = \mathbb{R}^{n_1 - k_1} \times \mathbb{Z}^{k_1}$. The feasible region of the first stage variables is defined by the set of constraints $Ax = b$ and by the set $\mathbb{X}$. The vector of second stage decision variables is denoted by y . Here, $\xi := (d, h, T, W)$ are the data of the second stage problem. Some or all elements of vector ξ might be random. The expectation operator at the first stage problem is taken with respect to the probability distribution of ξ . We assume ξ has a finite support Ω and each possible outcome $\omega \in \Omega$ occurs with probability $p(\omega)$. When W does not change according to realization of random vector ξ , the model is called as fixed recourse model.

Since ξ has a finite support Ω , two stage stochastic programming formulation to be written as the follows:

$$\begin{aligned}
\min \quad & c^\top x + \sum_{\omega \in \Omega} p(\omega) d^\top(\omega) y(\omega) \\
s.t. \quad & Ax = b, \\
& T(\omega)x + W(\omega)y(\omega) = h(\omega), \quad \omega \in \Omega \\
& x \in \mathbb{X} \\
& y(\omega) \in \mathbb{Y}, \quad \omega \in \Omega.
\end{aligned} \tag{2.1}$$

This formulation is called as extensive formulation.

2.3.2 Multi Stage Stochastic Programming

Multi stage stochastic programming can be viewed as the extension of the two stage stochastic programming to multiple stages. The uncertain data $\xi_1, \dots, \xi_T$ are observed gradually over time and decisions $x_1, \dots, x_T$ should be given based on these realizations. The decision process has the form of

$$\begin{aligned}
& \text{decision } (x_1) \rightsquigarrow \text{observation } (\xi_2) \rightsquigarrow \text{decision } (x_2) \rightsquigarrow \\
& \dots \rightsquigarrow \text{observation } (\xi_T) \rightsquigarrow \text{decision } (x_T).
\end{aligned}$$

The important phenomenon of multi stage stochastic programming is that the decision x_t is made based on the available information $\xi_{[t]} := (\xi_1, \dots, \xi_t)$ up to time t , but not on the future observations. This requirement is called as *non-anticipativity*. In order to show the algebraic representation of non-anticipativity constraints and derive an extensive formulation for the multi stage stochastic programming, we will use a simple linear programming problem with decision vectors $x_1, \dots, x_T$.

$$\begin{aligned}
\min \quad & c_1^\top x_1 + c_2^\top x_2 + c_3^\top x_3 + \dots + c_T^\top x_T \\
& A_1 x_1 = b_1, \\
& B_2 x_1 + A_2 x_2 = b_2, \\
& B_3 x_2 + A_3 x_3 = b_3, \\
& \dots \\
& B_T x_{T-1} + A_T x_T = b_T, \\
& x_1 \geq 0, x_2 \geq 0, \dots, x_T \geq 0,
\end{aligned}$$

The problem can be viewed as a multi period stochastic programming problem where c_1 , A_1 and b_1 are known, some entries of the cost vector x_t , matrices B_t and A_t , and right-hand side vectors b_t , $t = 2, \dots, T$ are random.

Our objective is the minimization of the expected total cost. Suppose that there are finitely many different scenarios denoted by Ω for the problem data. The probability associated with the scenario $\omega \in \Omega$ is $p(\omega)$ and the decisions for each scenario $\omega \in \Omega$ are represented by $x_1(\omega), x_2(\omega), \dots, x_T(\omega)$. Of course, it would not be appropriate, if we rewrite the problem as the following linear programming.

$$\begin{aligned}
\min \quad & \sum_{\omega \in \Omega} p(\omega) [c_1^\top x_1(\omega) + c_2^\top(\omega) x_2(\omega) + c_3^\top(\omega) x_3(\omega) + \dots + c_T^\top(\omega) x_T(\omega)] \\
& A_1 x_1(\omega) = b_1, \\
& B_2(\omega) x_1(\omega) + A_2(\omega) x_2(\omega) = b_2(\omega), \\
& B_3(\omega) x_2(\omega) + A_3(\omega) x_3(\omega) = b_3(\omega),
\end{aligned}$$

decomposition-based solution methods have been proposed. A widely used approach to solve two stage stochastic linear programming models is L-shaped method (see, [55] for details). It is a Benders' decomposition based method which correctly approximates the second stage objective functions by introducing optimality cuts to master problem. Recently, Bayram and Yaman [56] propose an exact algorithm based on Benders decomposition to solve a scenario-based two stage stochastic evacuation planning model. They use second order cone duality results to handle the nonlinear subproblems within a Benders' decomposition setting.

Introducing integer variables into stochastic programs brings an additional challenge to these such large-scale problems. Laporte and Louveaux [57] propose an integer L-shaped algorithm which requires pure binary first stage decision variables. Recently, Ahmed [58] develops an *evaluate-and-cut* procedure for two stage stochastic programming models with pure binary first stage decision variables. The algorithm is based on a scenario decomposition which iteratively evaluates the objective value for a set of binary first stage solutions, and then cut-off evaluated solutions with proposed *no good cuts*. That study shows the great advantage of parallel implementation of proposed scenario-based algorithm to solve the stochastic models.

While the multi stage stochastic program can reflect that the information gradually becomes available over the time, the large decision trees which grow exponentially as the number of stage increases, bring further complications. For solving linear multi stage stochastic programs, Rockafellar and Wets [59] propose a progressive hedging algorithm (PHA) which decomposes the stochastic model by scenarios. They provide theoretical convergence properties when all decision variables are continuous. For the discrete setting case, the PHA can be used as a heuristic by relaxing the integer variables, and then rounding the solution to achieve integrality (Listes and Dekker [60]). Then, Watson and Woodruff [61] introduce innovations for accelerating the convergence of PHA in stochastic mixed-integer programs. Gade et al. [62] present a method for computing lower bounds using the dual prices in PHA in order to assess the quality of the solutions generated by the algorithm. A detailed discussion on PHA can be found in

Ruszczynski [63].

Carøe and Schultz [64] propose a branch-and-bound procedure that uses the bounds obtained by solving the Lagrangian dual problems in which the nonanticipativity constraints are dualized in a Lagrangian manner. They show that the proposed approach can be extended to multi stage stochastic programs with integer variables. Lulli and Sen [65] introduce a branch-and-price algorithm that relaxes the nonanticipativity constraints, thereby decomposes the problem by each scenario and applies column generation procedure.

More recently, Sandıkçı and Özaltın [66] propose a bounding method based on a scenario tree decomposition for mixed integer multi stage stochastic programs. This approach decomposes the problem into smaller group subproblems. Each group subproblem is a mixed integer multi stage stochastic program with the same number of stages as the original problem, but defined over a reduced number of scenarios. Therefore, each group subproblem is solvable with less computational effort. They show a weighted average of the optimal values of group subproblems gives a lower bound for the optimal value of the problem.

2.3.4 Stochastic Programming with Endogenous Uncertainty

A stochastic program with decision dependent uncertainty, called as *endogenous uncertainty*, is significantly more difficult to solve than the “standard” formulation of a stochastic program. In the standard scenario-based formulations, each random variable will be realized at a predetermined time and that scenarios are specified, giving a full set of random variable realizations. However, in the problems with *endogenous uncertainty*, the stochastic process is not assumed to be independent of the decisions.

The work on stochastic programs with endogenous uncertainty may be categorized into two groups with respect to the way optimization decisions affect the stochastic process. In the first type of problems, denoted as problems with

Decision-dependent Probabilities or Type 1, decisions directly influence the probability distribution of the random parameters. In these problems, a full set of scenarios are specified but the actions taken at a particular stage may alter the probability of occurrence of future events. However, in the second type of problems, called as problems with *Decision-dependent Information Structure* or Type 2, decisions can affect the times at which uncertain parameters are revealed.

Although there exists a numerous number of work dealing with exogenous uncertainty (see Birge and Louveaux [55] and Infanger [67] and references therein), the literature addressing endogenous uncertainty is sparse. The study by Jonsbraten et al. [68] is the first one addressing the class of problems where the time at which information becomes available is determined by the decisions in preceding stages. For instance, they concern stochastic production costs. Only after giving the decision on which product to produce, the product's cost is revealed. The authors generate possible decisions and then present an implicit enumeration algorithm for this class of problems.

Goel and Grossmann [69] address a real world example of the gas field development planning problem where the revelation of information depends on the decisions on which wells to be drilled and when. The uncertainties in the size and the quality of the reserves are only revealed when a facility is installed at that field. They model the problem as a disjunctive programming problem in which the nonanticipativity constraints are written based on the decision variables related to drilling. Then, they extend the problem by including both endogenous and exogenous uncertainty in the work of Goel and Grossmann [70]. In order to reduce the number of conditional nonanticipativity constraints, they develop some theoretical results. Although these makes great improvements to solve the problem, the problem size is still quite large. Therefore, they propose a branch and bound solution procedure based on Lagrangian duality.

Later, Solak et. al [71] consider R&D project portfolio optimization problem in which some uncertain parameters such as the returns are known only after making an investment. Solak [71] state that the general disjunctive programming

formulation and the solution suggested by Goel & Grossmann [70] does not contain a direct decomposition structure, which is commonly used to solve standard stochastic programming problems. Therefore, they aim to fill this gap by developing a formulation scheme that is amenable to scenario decomposition. They use the sample average approximation method in solving large scale problems with decision dependent uncertainty, where the sample problems are solved through Lagrangian relaxation and lower bounding heuristics.

In recent years, Gupta and Grossmann [72] also address the issue of a potentially large number of nonanticipativity constraints and present a new theoretical property that significantly reduces the problem size. However, since the reduced models are still large to be solved directly, they also propose solution strategies. Hooshmand and Mirhassani [73] works on a variant of single-vehicle routing problem with stochastic demands. The demand of a customer becomes known only when the customer is visited. They propose an approach to reduce the problem size.

In addition, stochastic problems with decision dependent uncertainty could be reformulated as stochastic programs with recourse. For example, Ntaimo et al. [74] consider the wildfire initial attacks. For each scenario, there are two possible outcomes for the cost incurred by each wildfire depending on whether the fire can be contained through an effective attack performed or not. Using the binary variables, they formulate the problem as a two stage stochastic mixed integer program.

Studies on the problems with decision-dependent probabilities have received little attention in the literature. In Chapter 5, we will work on the problem with this type of endogenous uncertainty.

Ahmed [75] made the first attempt to model the relationship between the probability distribution and the decision variable. He presents some examples in the context of network design, server selection and facility location.

Peeta et al. [76] introduce a network traversal problem where the investment

on each arc affects the probability of the arc being available for traversal after some disaster. They relax the integer investment variables and apply Taylor series expansion to derive a multilinear function. They show that using the first order terms of this function leads to a knapsack problem whose solution is a local optimum to the original problem. Escudero et al. [77] propose a multi stage stochastic model with both exogenous and endogenous uncertainty. In the model, the probabilities of each scenario group may be determined by decision variables from previous stages.

For some problems with decision-dependent probabilities, using a suitable transformation of the probability distribution, the decision dependency of distribution could be removed. This is called as push-in technique. For more detailed analysis on the method, please see Rubinstein and Shapiro [78].

2.3.5 Stochastic Programming in Airline Scheduling Problems

As competition increases among airlines, their profit margins are in threat. One of the significant reasons for low profit margins is the variability in passenger demand. Random demand fluctuations may result in either low average load factor or numerous number of spilled passengers who are turned away by the airlines due to insufficient seat capacity of the aircraft.

Traditional fleet assignment model assigns aircraft types to flight legs to match the seat capacity of the aircraft to the expected market demand. However, expected values may not reflect the actual demand realizations. As suggested by Yan et al. [79], passenger demand fluctuations have to be considered in modelling the scheduling problem. In recent years, a growing literature has focused on the fleet assignment problems with uncertain demand. Listes and Dekker [60] use a two stage stochastic programming model to determine an optimal airline fleet composition (i.e., number of aircraft contained in each fleet type) to provide a re-fleet assignment process. In the second stage, they assign aircraft to each leg under each demand

scenario by allowing swapping. One of the main drawbacks of stochastic programming is that the consideration of large number of scenarios simultaneously may lead to large decision trees significantly lengthening the computation time to solve the overall model. Therefore, for computational tractability, Listes and Dekker [60] use a scenario aggregation approach. Sherali and Zhu [80] analyze the several demand scenarios in their two stage stochastic fleet assignment model. In the first stage, they assign aircraft families having the same crew requirement with different aircraft types to flight legs. Given the assignment of such aircraft families, they subsequently assign aircraft types within the allotted family to each leg under each demand scenario. In their study, Sherali and Zhu [80] develop a Benders' decomposition-based solution framework. However, they were unable to optimize real problem instances for United Airlines within a 25-hour time limit. Cadaso and de Celis [81] propose a two stage stochastic programming formulation which updates base schedules in terms of timetable and fleet assignments while considering stochastic demand, and propose robust itineraries in order to ameliorate miss-connected passengers. However, they do not consider the integration of aircraft routing problem along with the fleeting decisions, which may provide more economical solutions and prevent some incompatibilities between the decisions.

Aside from passenger demand fluctuations, airport congestion is another crucial uncertainty involved in flight times. These uncertainties are not included in any planning stages of a deterministic approach, despite the disruptions that flight delays propagate through the network, ultimately affecting a large scale of aircraft routes and passenger itineraries. Therefore, Yen and Birge [82] consider the effect of random disruptions to develop a robust schedule which can better withstand delays. In their two stage stochastic integer programming formulation, in the first-stage, they determine pairings of a round-trip itinerary that a crew member might fly; in the second stage, they consider several disruption scenarios to determine actual departure and arrival times. They minimize the cost of flying with selected pairs and expected delay cost coming from crews switching planes due to the disruption. Sohoni et al. [12] propose two stochastic models which incorporate block time uncertainty into the schedule development process. More

recently, Dunbar et al. [83] incorporate delay scenarios within the aircraft routing and crew pairing problems. In order to minimize delay propagation, they adjust flight departure times, in turn providing more slack over critical connections and drawing excess slack from remaining connections.

2.4 Summary

Because of the tractability issue of solving an integrated airline scheduling problem, researchers have attempted to solve integration of two or three subproblems with some restrictive assumptions. Many of the existing studies in the literature generate a flight schedule by using deterministic passenger demand and flight times, although demand is highly uncertain at the planning stage and flying times are subject to air traffic congestion at the operation day. To the best of our knowledge, this is the first study which introduces a multi stage stochastic programming model to determine strategic departure time decisions, tactical fleeting and routing decisions and operational flight timing decisions while considering randomness in passenger demand and non-cruise times. Moreover, multi stage stochastic mixed integer nonlinear programming models lack efficient solution algorithms. In Chapter 3, we provide a scenario group-wise decomposition algorithm improved with a cutting plane algorithm.

In Chapter 4 and Chapter 5, we introduce a new problem to the literature. We first consider a deterministic problem of accommodating new flights into an existing airline flight schedule in Chapter 4, and then extend the problem by considering the randomness in passenger demand in Chapter 5. Although flight re-timing, aircraft swapping and cruise time controllability approaches have been widely used in the literature ([9]-[12], [24]-[29], [50] -[52]), the novelty in this study lies in the fact that they are implemented to allow for introducing new flights into the schedule.

A recent stream of research in stochastic programming includes the decision

dependent uncertainty ([68]-[76]). However, there has not been proposed a generally applicable efficient solution algorithm for stochastic programs with endogenous uncertainty. In Chapter 5, we propose an exact solution algorithm based on *evaluate-and-cut* procedure, where a number of group subproblems are solved to generate candidate solutions.



Chapter 3

Multi Stage Airline Scheduling Problem with Stochastic Passenger Demand and Non-cruise Times

In this chapter, we propose a three stage stochastic programming model which determines flight timing, fleet and routing decisions while considering the randomness of demand and non-cruise times. Our model differs from the existing two stage stochastic models by considering not only flight timing and potential passenger demand, but also expected operational expenses, such as fuel burn and carbon emission costs. We include aircraft cruise speed decisions to compensate for non-cruise time variability so as to satisfy the time requirements of the passenger connections. We handle nonlinear functions of fuel and emission costs associated with cruise speed adjustments by utilizing mixed integer second order cone programming. Because the three stage stochastic model leads to a large decision tree and can be very time-consuming to solve optimally, we suggest a scenario group-wise decomposition algorithm to obtain lower and upper bounds for the optimal value of the proposed model. The lower and upper bounds are obtained by solving a number of group subproblems, which are similar to proposed

multi stage stochastic model defined over a reduced number of scenarios. We suggest a cutting plane algorithm, along with improvements, to efficiently solve each group subproblem. In the numerical experiments, we provide a significant cost savings over two stage stochastic programming and deterministic approaches.

The organization of the chapter is as follows. In Section 3.1, we briefly describe the framework of the problem. We formulate a mixed integer three stage stochastic nonlinear programming model. Then, we reformulate proposed model using second order cone programming to handle nonlinearity. We also suggest valid inequalities to make the formulation stronger. We devote Section 3.2 to a scenario group-wise decomposition algorithm to find lower and upper bounds for the optimal value of the proposed model. Section 3.3 presents an improved cutting plane algorithm to solve group subproblems in bounding algorithms. In Section 3.4, we report computational results, comparing three stage stochastic programming approach with two stage stochastic implementations, and illustrating the high quality performances of the proposed algorithms using a sample schedule for a major U.S. airline. Finally, in Section 3.5, we conclude with remarks and outline possible research directions arising from this study.

The results of this chapter are published in Transportation Research Part B: Methodological [84].

3.1 Stochastic Airline Scheduling

In this section, we describe a mixed integer three stage stochastic nonlinear programming model that generates a flight schedule by considering the randomness of passenger demand and non-cruise times. The first stage variables determine the announced departure time for each flight, but unlike the classical deterministic approach, announced departure times are selected to reduce expected operational costs while considering the uncertainty that is represented by multiple potential demand and non-cruise time scenarios. After the departure times are announced, uncertain passenger demand data are realized. For each demand realization, the

second stage variables determine the fleet assignment and aircraft routing while considering the non-cruise time uncertainty. Finally, uncertain non-cruise time data are realized. The third stage decisions determine actual flight departure times and aircraft cruise speed for each scenario. The model takes the advantage of cruise speed control to compensate for the increase in non-cruise times due to the airport congestion, to be able to ensure the connection of passengers and aircraft itineraries. The decision process is provided in Figure 3.1.

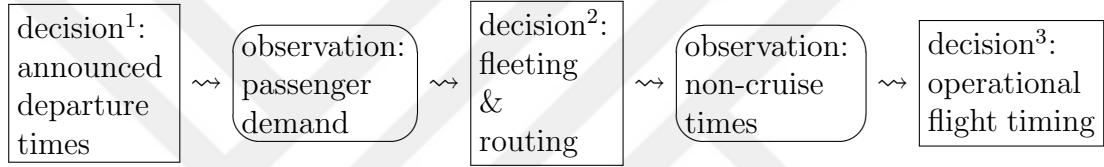


Figure 3.1: Decision process

The objective function minimizes the expected operational costs including delay cost, passenger misconnection cost, spilled passenger cost and fuel burn cost associated with aircraft cruise speed adjustments. The resulting three stage model assigns fleet types by considering not only passenger demand, as commonly done in the literature, but also expected operational expenses such fuel burn and carbon emission costs. The model can take the advantage of speeding up the aircraft to ensure the passengers' connections, which gives rise to a fuel burn cost. The fuel burn can be reduced through assigning a fuel efficient aircraft. However, assigning a fuel efficient, but smaller aircraft may result in spilled passengers, if the passenger demand realization yields above the seat capacity of assigned aircraft. In addition to aircraft cruise speed control, the model re-times the flight departures to satisfy the connections of passengers in response to the increase in non-cruise time variations. On the other hand, shifting the flight departures incurs an additional delay cost. Therefore, the model carefully determines the first stage announced departure times to minimize the expected costs. Because of the interplay between decisions, we propose a single model which integrates these three stage decisions using a stochastic programming where potential passenger demand and non-cruise time scenarios are included.

The notation used throughout this chapter is given below:

Sets:

Ω	set of scenarios
K	set of aircraft types
J	set of flights
P_i	set of flights which have passenger connections from flight $i \in J$
C_i	set of flights which have passenger connections to flight $i \in J$
U_i	set of flights that can be followed by flight $i \in J$
D_i	set of flights that can follow flight $i \in J$
C_E	set of flights $i \in J$ and $j \in J$ such that $i \neq j$ and flight i can connect flight j
J_k^e	set of flights that can be the last flight of aircraft type $k \in K$
J_k^s	set of flights that can be the first flight of aircraft type $k \in K$
$\bar{J}(\omega)$	set of scenarios that have the same demand realization as in scenario $\omega \in \Omega$, $\bar{J}(\omega) = \{\bar{\omega} \in \Omega : \mu_i(\omega) = \mu_i(\bar{\omega}), i \in J\}$

Parameters:

$\mu_i(\omega)$	demand realization for flight $i \in J$ under scenario $\omega \in \Omega$
$\eta_i(\omega)$	non-cruise time realization for flight $i \in J$ under scenario $\omega \in \Omega$
$p(\omega)$	probability of scenario $\omega \in \Omega$
ϕ	duration between consecutive departure time alternatives
T_i^{min}	earliest departure time unit of flight $i \in J$
T_i^{max}	latest departure time unit of flight $i \in J$
N_k	available number of aircraft type $k \in K$
τ_{ij}^k	turnaround time needed to prepare aircraft type $k \in K$ between flights $(i, j) \in C_E$
λ_{ij}	turntime needed to connect passengers between flights $i \in J, j \in P_i$
$[f_{ik}^\ell, f_{ik}^u]$	time window for the cruise time of flight $i \in J$ with an aircraft type $k \in K$
κ^k	number of seats in aircraft type $k \in K$

$pass_{ij}(\omega)$	number of passengers who have a connection from flight $i \in J$ to $j \in P_i$ under scenario $\omega \in \Omega$
c_{daily}^k	daily cost of using aircraft type $k \in K$
c_{sp}^i	opportunity cost of spilled passengers of flight $i \in J$
c_{fuel}	fuel price per kilogram (kg) of fuel burned by an aircraft
c_{CO_2}	cost of emission per kg of CO_2 emitted by an aircraft
c_{Idle}^k	unit idle time cost of aircraft type $k \in K$ in dollars per minute
$c_{del}^i(\omega)$	cost of delay of flight $i \in J$ under scenario $\omega \in \Omega$ in dollars per minute
c_{dist}^i	cost of disrupted passengers of flight $i \in J$ in dollars per passenger
M	a big number

Decision Variables:

x_i^1	announced departure time of flight $i \in J$ (first stage decision variable)
χ_i^1	discrete time unit in which announced departure time of flight i is scheduled, $i \in J$ (first stage decision variable)
$z_{ijk}^2(\omega)$	1 if both flights i and j are operated by aircraft type k and flight i is followed by flight j under scenario ω and 0 otherwise, $j \in J, i \in U_j, \omega \in \Omega, k \in K$ (second stage decision variable)
$y_{ik}^2(\omega)$	1 if flight i is the first flight operated by aircraft type k under scenario ω and 0 otherwise, $i \in J, k \in K, \omega \in \Omega$ (second stage decision variable)
$u_{ik}^2(\omega)$	1 if flight i is the last flight operated by aircraft type k under scenario ω and 0 otherwise, $i \in J, k \in K, \omega \in \Omega$ (second stage decision variable)
$d_i^3(\omega)$	actual departure time of flight i under scenario ω , $i \in J, \omega \in \Omega$ (third stage decision variable)
$a_i^3(\omega)$	actual arrival time of flight i under scenario ω , $i \in J, \omega \in \Omega$ (third stage decision variable)
$f_{ik}^3(\omega)$	cruise time of flight i operated by aircraft type k under scenario ω , $i \in J, k \in K, \omega \in \Omega$ (third stage decision variable)

$S_{ik}^3(\omega)$	idle time of aircraft type k after operating flight i under scenario ω , $i \in J, k \in K, \omega \in \Omega$ (third stage decision variable)
$del_i^3(\omega)$	delay amount of flight i under scenario ω , $i \in J, \omega \in \Omega$ (third stage decision variable)
$b_{ij}^3(\omega)$	1 if passengers in flight i miss flight j under scenario ω and 0 otherwise, $i \in J, j \in P_i, \omega \in \Omega$ (third stage decision variable)

3.1.1 Fuel and CO_2 Emission Cost Function

We use the idea of flying faster to compensate for the increase in non-cruise time variations to satisfy passenger connections or flying slower for conservation of fuel. To estimate fuel burn, we use the cruise stage fuel flow model developed by the Base of Aircraft Data (BADA) [85]. For all $i \in J, k \in K, \omega \in \Omega$, fuel burn (kg) as a function of cruise time $f_{ik}^3(\omega)$ (minutes) can be calculated as follows:

$$F_{ik}(f_{ik}^3(\omega)) = c_1^{ik} \frac{1}{f_{ik}^3(\omega)} + c_2^{ik} \frac{1}{(f_{ik}^3(\omega))^2} + c_3^{ik} (f_{ik}^3(\omega))^3 + c_4^{ik} (f_{ik}^3(\omega))^2. \quad (3.1)$$

The coefficients $c_s^{ik} > 0, s = 1, \dots, 4$, are expressed in terms of aircraft specific fuel consumption and drag as well as the mass of aircraft, air density and gravitational acceleration in Şafak et al. [51]. It is important to note that $F_{ik}(f_{ik}^3(\omega))$ is a convex function of $f_{ik}^3(\omega) > 0$.

Moreover, EUROCONTROL [86] states that each kg of fuel burn approximately produces 3.15 kg of CO_2 emission. Therefore, we can express fuel and CO_2 emission costs as a function of cruise time as follows:

$$C_{fuel}^{ik}(f_{ik}^3(\omega)) + C_{CO_2}^{ik}(f_{ik}^3(\omega)) = \begin{cases} (c_{fuel} + c_{CO_2}r) F_{ik}(f_{ik}^3(\omega)) & \text{if } m_{ik}^2(\omega) = 1 \\ 0 & \text{if } m_{ik}^2(\omega) = 0, \end{cases}$$

where c_{fuel} and c_{CO_2} are the unit prices (\$/kg) of fuel and CO_2 emission, respectively and r is CO_2 emission constant (e.g., $r = 3.15$). Here, $m_{ik}^2(\omega) := (y_{ik}^2(\omega) + \sum_{j \in U_i} z_{jik}^2(\omega))$, so that if aircraft type k is not assigned to flight i under scenario ω , then $C_{fuel}^{ik}(f_{ik}^3(\omega)) + C_{CO_2}^{ik}(f_{ik}^3(\omega)) = 0$.

3.1.2 Mathematical Model

We provide a three stage stochastic mixed integer nonlinear programming formulation below.

$$\begin{aligned}
\min \quad & \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} y_{ik}^2(\omega) c_{daily}^k \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} \max(0, \mu_i(\omega) - \kappa^k) \left(\sum_{j \in U_i} z_{jik}^2(\omega) + y_{ik}^2(\omega) \right) c_{sp}^i \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} (C_{fuel}^{ik}(f_{ik}^3(\omega)) + C_{CO_2}^{ik}(f_{ik}^3(\omega))) \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} S_{ik}^3(\omega) c_{Idle}^k \right) p(\omega) + \sum_{\omega \in \Omega} \left(\sum_{i \in J} c_{del}^i(\omega) del_i^3(\omega) \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{j \in P_i} c_{dist}^{ij} pass_{ij}(\omega) b_{ij}^3(\omega) \right) p(\omega) \tag{3.2}
\end{aligned}$$

subject to

$$T_i^{min} \leq \chi_i^1 \leq T_i^{max} \quad i \in J \tag{3.3}$$

$$x_i^1 = \phi \chi_i^1 \quad i \in J \tag{3.4}$$

$$\chi_i^1 \quad integer \quad i \in J \tag{3.5}$$

$$\sum_{k \in K} \left(\sum_{j \in U_i} z_{jik}^2(\omega) + y_{ik}^2(\omega) \right) = 1 \quad i \in J, \omega \in \Omega \tag{3.6}$$

$$\sum_{j \in U_i} z_{jik}^2(\omega) + y_{ik}^2(\omega) = 1 \quad i \in J, k \in K, \omega \in \Omega \tag{3.7}$$

$$\sum_{j \in D_i} z_{ijk}^2(\omega) + u_{ik}^2(\omega) = 1 \quad i \in J, k \in K, \omega \in \Omega \tag{3.8}$$

$$\sum_{i \in J} y_{ik}^2(\omega) \leq N_k \quad k \in K, \omega \in \Omega \tag{3.9}$$

$$y_{ik}^2(\omega) = 0 \quad i \in J \setminus J_k^s, k \in K, \omega \in \Omega \tag{3.10}$$

$$u_{ik}^2(\omega) = 0 \quad i \in J \setminus J_k^e, k \in K, \omega \in \Omega \tag{3.11}$$

$$z_{jik}^2(\omega) \in \{0, 1\} \quad i \in J, j \in U_i, k \in K, \omega \in \Omega \tag{3.12}$$

$$y_{ik}^2(\omega) \in \{0, 1\} \quad i \in J, k \in K, \omega \in \Omega \tag{3.13}$$

$$-x_i^1 + d_i^3(\omega) \geq 0 \quad i \in J, \omega \in \Omega \quad (3.14)$$

$$d_i^3(\omega) - x_i^1 - del_i^3(\omega) \leq 0 \quad i \in J, \omega \in \Omega \quad (3.15)$$

$$d_i^3(\omega) + \sum_{k \in K} f_{ik}^3(\omega) + \eta_i(\omega) = a_i^3(\omega) \quad i \in J, \omega \in \Omega \quad (3.16)$$

If $z_{ijk}^2(\omega) = 1$ then

$$a_i^3(\omega) + S_{ik}^3(\omega) + \tau_{ij}^k = d_j^3(\omega) \quad (i, j) \in C_E, k \in K, \omega \in \Omega \quad (3.17)$$

$$-f_{ik}^\ell \left(\sum_{j \in U_i} z_{jik}^2(\omega) + y_{ik}^2(\omega) \right) + f_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K, \omega \in \Omega \quad (3.18)$$

$$f_{ik}^u \left(\sum_{j \in U_i} z_{jik}^2(\omega) + y_{ik}^2(\omega) \right) - f_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K, \omega \in \Omega \quad (3.19)$$

$$d_j^3(\omega) \geq a_i^3(\omega) + \lambda_{ij} - Mb_{ij}^3(\omega) \quad i \in J, j \in P_i, \omega \in \Omega \quad (3.20)$$

$$S_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K, \omega \in \Omega \quad (3.21)$$

$$del_i^3(\omega) \geq 0 \quad i \in J, \omega \in \Omega \quad (3.22)$$

$$b_{ij}^3(\omega) \in \{0, 1\} \quad i \in J, j \in P_i, \omega \in \Omega \quad (3.23)$$

$$z_{jik}^2(\omega) = z_{jik}^2(\bar{\omega}) \quad i \in J, j \in U_i, \quad k \in K, \omega \in \Omega, \bar{\omega} \in \bar{J}(\omega) \quad (3.24)$$

$$y_{ik}^2(\omega) = y_{ik}^2(\bar{\omega}) \quad i \in J, j \in U_i, \quad k \in K, \omega \in \Omega, \bar{\omega} \in \bar{J}(\omega) \quad (3.25)$$

$$u_{ik}^2(\omega) = u_{ik}^2(\bar{\omega}) \quad i \in J, j \in U_i, \quad k \in K, \omega \in \Omega, \bar{\omega} \in \bar{J}(\omega) \quad (3.26)$$

The objective function (3.2) minimizes the expected operational costs of an airline. These interrelated costs are aircraft daily usage, spilled passengers, fuel and CO_2 emission, idle times, delay, and passenger misconnection costs. In our study, flight time is integer valued since airlines usually prefer the flight times to be assigned within a set of discrete time units for practical reasons. Therefore, the first stage constraints (3.3), (3.4) and (3.5) ensure the departure time of flights to be within a set of discrete time units which have already been determined by the airline. Constraints (3.6) - (3.9) are the second stage *fleeting and routing constraints* for each scenario. Constraint (3.6) assures that each scheduled flight

leg is covered by exactly one aircraft type. Constraints (3.7) and (3.8) maintain the route of an aircraft throughout the entire network. Constraint (3.9) limits the number of employed aircraft by N_k . Constraints (3.10) and (3.11) sustain maintenance policy which determines the first and last airports for each aircraft type. Constraints (3.12) and (3.13) define the domain of the second stage variables. Constraints (3.14) and (3.15) allow departure delays with respect to announced departure times of the first stage. Delays are penalized in the objective function (3.2). Constraint (3.16) determines the arrival time of each flight. The arrival time of a flight equals to sum of its departure time, cruise time and non-cruise time. Constraint (3.17) guarantees minimum aircraft turnaround time, needed between consecutive flights operated by the same aircraft, for each scenario. Constraints (3.18) and (3.19) apply cruise time upper and lower bounds for each flight. In constraint (3.20), if the time needed for the connection of passengers from flight i to any follow-on flight j is not satisfied under scenario ω , then passengers miss the connection with a penalty in the objective function (3.2). Constraints (3.21), (3.22) and (3.23) define the domain of the third stage variables. Constraints (3.24), (3.25) and (3.26) are the non-anticipativity constraints. At the second stage, fleeting decisions should have the same value for the scenarios which have the same history up to the second stage. In other words, fleeting decisions should depend on the realization of demands. Therefore, non-anticipativity constraints ensure that fleeting variables take equal values for the scenarios which share the same realization of demands.

Different than the existing two stage models, the proposed three stage stochastic model integrates fleet assignment and aircraft routing decisions along with operational flight timing decisions. Despite the additional complexity of the third stage, incorporating the cruise time decisions and considering potential non-cruise time scenarios result in improved solutions which can better withstand delays. In the computations, the great advantage of three stage stochastic programming approach can be clearly seen in the significant cost savings compared to a two stage approach. However, large number of scenarios, nonlinear fuel and carbon emission cost functions and binary fleet assignment decisions make the problem significantly more difficult to solve. To handle them, we propose valid inequalities

to strengthen the formulation and we utilize a mixed integer second order cone programming to handle nonlinearity. To deal with the large number of scenarios, we employ a scenario group-wise decomposition algorithm by solving a number of group subproblems. However, even the group subproblems, defined over a reduced number of scenarios, could not be solved to optimality by CPLEX, because of the large number of binary decisions and the second order conic inequalities. Therefore, we suggest a cutting plane algorithm to mitigate the computational effort to solve group subproblems. In the next section, we introduce valid inequalities.

3.1.3 Valid Inequalities

In order to mitigate the computational difficulty, we develop a set of valid inequalities which strengthen the formulation. Propositions 3.1, 3.2 and 3.3 present these valid inequalities.

Proposition 3.1. *Let v_i^l and v_j^l be a lower bound for $d_i^3(\omega)$ and $d_j^3(\omega)$, respectively, and $\max_{k \in K} f_{ik}^u$ be the maximum of upper bounds f_{ik}^u for all $i \in J, j \in P_i, k \in K, \omega \in \Omega$. Then, for all $i \in J, j \in P_i, \omega \in \Omega$, the inequality*

$$d_j^3(\omega) - v_i^l - \sum_{k \in K} f_{ik}^3(\omega) + (v_i^l + \max_{k \in K} f_{ik}^u + \eta_i(\omega) + \lambda_{ij} - v_j^l) b_{ij}^3(\omega) \geq \eta_i(\omega) + \lambda_{ij} \quad (3.27)$$

is a valid inequality for the polyhedron (3.3)-(3.26).

Proof. For any $i \in J, j \in P_i, \omega \in \Omega$, if $b_{ij}^3(\omega) = 0$ then for feasibility, from constraints (3.20) and (3.16), we need $d_j^3(\omega) \geq d_i^3(\omega) + \sum_{k \in K} f_{ik}^3(\omega) + \eta_i(\omega) + \lambda_{ij}$. Since $d_i^3(\omega) \geq v_i^l$, the inequality (3.27) is satisfied.

Otherwise, if $b_{ij}^3(\omega) = 1$, the inequality (3.27) becomes $d_j^3(\omega) - v_j^l - \sum_{k \in K} f_{ik}^3(\omega) + \max_{k \in K} f_{ik}^u \geq 0$. Note that $\max_{k \in K} f_{ik}^u$ is an upper bound for $\sum_{k \in K} f_{ik}^3(\omega)$, since from constraint (3.19), $f_{ik}^3(\omega)$ would take a positive value for only one $k \in K$. Since v_j^l is a lower bound for $d_j^3(\omega)$, the inequality (3.27) is again satisfied. $\square$

Proposition 3.2. Let v_i^l and v_j^l be a lower bound for $d_i^3(\omega)$ and $d_j^3(\omega)$, respectively, for all $(i, j) \in C_E, k \in K, \omega \in \Omega$. Then, for all $(i, j) \in C_E, k \in K, \omega \in \Omega$, the inequality

$$d_j^3(\omega) - v_i^l - f_{ik}^3(\omega) + (v_i^l + f_{ik}^u + \eta_i(\omega) + \tau_{ij}^k - v_j^l)(1 - z_{ijk}^2(\omega)) \geq \eta_i(\omega) + \tau_{ij}^k \quad (3.28)$$

is a valid inequality for the polyhedron (3.3)-(3.26).

Proof. For any $(i, j) \in C_E, k \in K, \omega \in \Omega$, if $z_{ijk}^2(\omega) = 1$ then for feasibility, from constraints (3.17) and (3.16), we need $d_j^3(\omega) - d_i^3(\omega) - f_{ik}^3(\omega) - S_{ik}^3(\omega) = \eta_i(\omega) + \tau_{ij}^k$. Since $d_i^3(\omega) \geq v_i^l$ and $S_{ik}^3(\omega) \geq 0$, the inequality (3.28) is satisfied.

Otherwise, if $z_{ijk}^2(\omega) = 0$, the inequality (3.28) becomes $d_j^3(\omega) - v_i^l - f_{ik}^3(\omega) + f_{ik}^u \geq 0$. Since v_j^l is a lower bound for $d_j^3(\omega)$ and f_{ik}^u is an upper bound for $f_{ik}^3(\omega)$, the inequality (3.28) is again satisfied. $\square$

Proposition 3.3. For all $(i, j) \in C_E, k \in K, \omega \in \Omega$, the inequality

$$z_{ijk}^2(\omega) \leq \sum_{n \in J} y_{nk}^2(\omega) \quad (3.29)$$

is a valid inequality for the polyhedron (3.3)-(3.26).

Proof. For any $k \in K, \omega \in \Omega$, if $\sum_{n \in J} y_{nk}^2(\omega) = 0$, by definition of variables $y_{nk}^2(\omega)$, it implies that under scenario $\omega \in \Omega$, aircraft type k is not used in the schedule. Then, we need an aircraft type k cannot be assigned to any consecutive flights $(i, j) \in C_E$ implying that $z_{ijk}^2(\omega) = 0$.

Otherwise, if $\sum_{n \in J} y_{nk}^2(\omega) = 1$, the valid inequality (3.29) becomes $z_{ijk}^2(\omega) \leq 1$. Since $z_{ijk}^2(\omega) \in \{0, 1\}$, the inequality (3.29) is trivially satisfied. $\square$

3.1.4 Conic Representation of the Fuel and CO_2 Cost Function

In the objective function, we have nonlinear cost terms due to controllable cruise times. To handle nonlinearity, we utilize a mixed integer second order cone programming. Let us redefine fuel and CO_2 emission cost functions as:

$$K(f_{ik}^3(\omega)) = \begin{cases} (c_{fuel} + c_{CO_2}r) \left(c_1^{ik} \frac{1}{f_{ik}^3(\omega)} + c_2^{ik} \frac{1}{(f_{ik}^3(\omega))^2} + c_3^{ik} (f_{ik}^3(\omega))^3 + c_4^{ik} (f_{ik}^3(\omega))^2 \right) & \text{if } m_{ik}^2(\omega) = 1 \\ 0 & \text{if } m_{ik}^2(\omega) = 0. \end{cases}$$

Note that $K(f_{ik}^3(\omega))$ is discontinuous and therefore its epigraph

$$E_F = \{(f_{ik}^3(\omega), t) \in \mathbb{R}^2 : K(f_{ik}^3(\omega)) \leq t\}$$

is nonconvex. In the next proposition, we describe how the convex hull of E_F is obtained. This proposition was previously proven in Şafak et al. [51], but for the sake of completeness, here we give the proof. To simplify the presentation, we drop the indices of the variables and parameters in the next proposition. A more detailed discussion can be found in Aktürk et al. [26], Aktürk et al. [87] and Günlük and Linderoth [88].

Proposition 3.4. *Let $q, \delta, \varphi, \vartheta \geq 0$. Then the convex hull of E_F can be expressed as*

$$t \geq (c_{fuel} + c_{CO_2}r)(c_1q + c_2\delta + c_3\varphi + c_4\vartheta) \quad (3.30)$$

$$m^2 \leq qf \quad (3.31)$$

$$m^4 \leq f^2\delta m \quad (3.32)$$

$$f^4 \leq m^2\varphi f \quad (3.33)$$

$$f^2 \leq \vartheta m \quad (3.34)$$

in the constraint set Moreover, each inequality (3.31) - (3.34) can be represented by conic quadratic inequalities..

Proof. According to Hiriart-Urruty and Lemaréchal [89], perspective of a convex function $k(f)$ is $mk(f/m)$. Observe that each of the nonlinear terms $\frac{1}{f}$, $\frac{1}{f^2}$, f^3

and f^2 is a convex function of $f \geq 0$. Therefore, epigraph of the perspective of each term can be written as, $\frac{m^2}{f} \leq q$, $\frac{m^3}{f^2} \leq \delta$, $\frac{f^3}{m^2} \leq \varphi$, $\frac{f^2}{m} \leq \vartheta$, respectively. Since $m, f \geq 0$, they can be written as stated in the proposition.

Observe that (3.31) and (3.34) are hyperbolic inequalities. The inequality (3.32) can be rewritten as the following two hyperbolic inequalities

$$m^2 \leq \pi f, \quad (3.35)$$

$$\pi^2 \leq \delta m, \quad (3.36)$$

where $\pi \geq 0$. Similarly, the inequality (3.33) can be restated as

$$f^2 \leq v m, \quad (3.37)$$

$$v^2 \leq \varphi f, \quad (3.38)$$

where $v \geq 0$. According to Ben-Tal and Nemirovski [90], the hypograph can be equivalently represented by hyperbolic inequalities of the form,

$$y^2 \leq u_1 u_2, \quad y, u_1, u_2 \geq 0$$

which can be represented by the second order conic inequality below

$$\|(2y, u_1 - u_2)\| \leq u_1 + u_2$$

that concludes the proof. $\square$

Below, we provide the mixed integer second order cone programming formulation of our model, called as MISOCP. For ease of representation, from now on, we substitute $\left(y_{ik}^2(\omega) + \sum_{j \in U_i} z_{jik}^2(\omega)\right)$ with $m_{ik}^2(\omega)$ for all $i \in J, k \in K, \omega \in \Omega$.

$$\begin{aligned} \min \quad & \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} y_{ik}^2(\omega) c_{daily}^k \right) p(\omega) \\ & + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} \max(0, \mu_i(\omega) - \kappa^k) m_{ik}^2(\omega) c_{sp}^i \right) p(\omega) \\ & + \sum_{\omega \in \Omega} \left(\sum_{k \in K} \sum_{i \in J} (c_{fuel} + c_{CO_2} r) (c_1^{ik} q_{ik}^3(\omega) + c_2^{ik} \delta_{ik}^3(\omega) + c_3^{ik} \varphi_{ik}^3(\omega) + c_4^{ik} \vartheta_{ik}^3(\omega)) \right) p(\omega) \end{aligned} \quad (\text{MISOCP})$$

$$\begin{aligned}
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} S_{ik}^3(\omega) c_{Idle}^k \right) p(\omega) + \sum_{\omega \in \Omega} \left(\sum_{i \in J} c_{del}^i(\omega) del_i^3(\omega) \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{j \in P_i} c_{distpass_{ij}}^i(\omega) b_{ij}^3(\omega) \right) p(\omega)
\end{aligned} \tag{3.39}$$

subject to

(3.3) – (3.26)

$$m_{ik}^2(\omega) = \left(y_{ik}^2(\omega) + \sum_{j \in U_i} z_{jik}^2(\omega) \right) \quad i \in J, k \in K, \omega \in \Omega \tag{3.40}$$

$$\| (2m_{ik}^2(\omega), q_{ik}^3(\omega) - f_{ik}^3(\omega)) \| \leq q_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.41}$$

$$\| (2f_{ik}^3(\omega), v_{ik}^3(\omega) - m_{ik}^2(\omega)) \| \leq v_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.42}$$

$$\| (2m_{ik}^2(\omega), \pi_{ik}^3(\omega) - f_{ik}^3(\omega)) \| \leq \pi_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.43}$$

$$\| (2\pi_{ik}^3(\omega), \delta_{ik}^3(\omega) - m_{ik}^2(\omega)) \| \leq \delta_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.44}$$

$$\| (2f_{ik}^3(\omega), v_{ik}^3(\omega) - m_{ik}^2(\omega)) \| \leq v_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.45}$$

$$\| (2v_{ik}^3(\omega), \varphi_{ik}^3(\omega) - f_{ik}^3(\omega)) \| \leq \varphi_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K, \omega \in \Omega \tag{3.46}$$

$$q_{ik}^3(\omega), \delta_{ik}^3(\omega), \varphi_{ik}^3(\omega), v_{ik}^3(\omega), \pi_{ik}^3(\omega), v_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K, \omega \in \Omega \tag{3.47}$$

Here, $\|\cdot\|$ denotes Euclidean norm. Note that, the objective function (3.39) is slightly different than the original objective function of the proposed model. The original objective (3.2) is represented by the new objective and conic constraints (3.41)-(3.46).

3.2 Bounding Algorithm by the Group Subproblem Approach

Although incorporating the third stage provides a greater flexibility to improve the solution quality, the three stage stochastic programming model may lead to large decision trees that could require significant computation time to solve the overall stochastic model. In order to mitigate the computational effort, in the previous section, we provide valid inequalities and handle the nonlinearity using a mixed integer second order cone programming. However, the overall three stage model cannot be solved for large number of scenarios. Therefore, in this section, in order to obtain lower and upper bounds for the optimal value of our problem, we suggest a scenario group-wise decomposition algorithm which is an extended version of the approach proposed by Sandıkçı and Özaltın [66] for multi stage integer stochastic optimization problems. This approach allows us to decompose the problem into group subproblems that are smaller in size and solvable with less computational effort. In Section 3.2.1, we describe the scenario group subproblem approach to obtain a lower bound. Then, in Section 3.2.2, we give our scenario group-wise decomposition algorithm to obtain lower and upper bounds for the optimal value of the proposed model.

3.2.1 Lower Bound from the Group Subproblems

In this section, our aim is to derive a lower bound by solving a number of group subproblems which are similar to our three stage problem, but smaller in size as discussed in Sandıkçı and Özaltın [66]. Each group subproblem contains the same number of stages as the original problem, but is defined over a reduced number of scenarios. The aim of this scenario group-wise decomposition is to mitigate computational effort. Let Ω_n be the scenarios considered in group subproblem $n \in N$, where N represents the set of group subproblems. To obtain a lower bound, we ensure that $\bigcup_{n \in N} \Omega_n = \Omega$. Moreover, we adjust the probabilities of the scenarios in each group subproblem to make sure that sum of the adjusted

probabilities equals to one. Let $p(\omega)$ be the probability of the scenario $\omega \in \Omega$ and $mc(\omega)$ count the number of group subproblems in which scenario ω appears. Then adjusted probability of scenario ω in group subproblem $n \in N$, represented by $\hat{p}_n(\omega)$, can be calculated as follows,

$$\hat{p}_n(\omega) = \frac{p(\omega) / mc(\omega)}{p(\Omega_n)}, \quad (3.48)$$

where $p(\Omega_n) = \sum_{\omega \in \Omega_n} p(\omega) / mc(\omega)$. Let $LB(N)$ denote the weighted average of the optimal values of group subproblems. It is defined as follows:

$$LB(N) = \sum_{n \in N} p(\Omega_n) obj(\Omega_n),$$

where $obj(\Omega_n)$ represents the optimal value of group subproblem $n \in N$.

The following proposition states that $LB(N)$ is a lower bound for the optimal value obj_{MISOCP} of the original problem.

Proposition 3.5. (*Proposition 1 in Sandıkçı and Özaltın [66].*) $LB(N) \leq obj_{MISOCP}$ for any set of group subproblems N satisfying $\bigcup_{n \in N} \Omega_n = \Omega$.

The steps of the lower bounding algorithm are given in Algorithm 1. The quality of the lower bound significantly depends on the choice of subsets Ω_n , $n \in N$. We are interested in finding subsets Ω_n , $n \in N$ that return a tighter lower bound. We therefore devise some grouping techniques and compare their performances in the experimental study provided in Section 3.4. Our scenario grouping strategies can be found in Appendix C.

3.2.2 Upper Bound from the Group Subproblems

In this section, we again utilize the scenario group subproblems, defined in the previous section, to obtain an upper bound for the optimal value of the proposed model MISOCP. In order to proceed with scenario group-wise decomposition algorithm, it is useful to discuss the decision process and scenario tree in detail. In this study, we consider the random process (H, Z) , where H is the random

Algorithm 1 Lower bounding algorithm

Initialization: Given a scenario set Ω and corresponding probabilities $p(\omega), \omega \in \Omega$, define the sets $\Omega_n, n \in N$.

for each group subproblem $n \in N$ **do**

for each $\omega \in \Omega_n$ **do**

 Calculate $\hat{p}_n(\omega)$ as in equation (3.48).

 Solve the MISOCP model in Section 3.1.4 by considering the scenario set Ω_n and corresponding probabilities $\hat{p}_n(\omega), \omega \in \Omega_n$.

 Set $obj(\Omega_n)$ to the optimal value of the group subproblem n .

 Calculate $LB(N) = \sum_{n \in N} p(\Omega_n) \cdot obj(\Omega_n)$.

demand and Z is the random non-cruise time. The uncertain data H and Z are gradually revealed over time, first H is observed and then Z is observed. The first stage decisions are made before any uncertainty is revealed, the second stage decisions are made after the random demand H is observed and the third stage decisions are made after all uncertainty is resolved, in other words both H and Z are observed. Therefore, the scenario tree has three levels. At the first level, we have only one root node in which first stage decisions are made. Let the first stage decision variable $x^1 \in \mathbb{R}^{|J|}$ denote a vector of announced departure times $x_i^1, i \in J$. The value of the decision vector x^1 is determined without knowing the future observations (μ, η) of (H, Z) . The values of the second stage decisions depend on the revealed information of H . Therefore, at the second level of scenario tree, the number of nodes is equal to the number of different demand realizations μ of H . An arc connects each node of the second level with the root node. For each node of the second level (each node corresponds to a particular realization μ of H), we create nodes as the number of non-cruise time realizations η of Z . We connect them with the node μ by an arc. The values of the third stage decisions depend on the realization (μ, η) of (H, Z) .

Let $\hat{x}^1 \in \mathbb{R}^{|J|}$ denote a fixed value of x^1 . Solving the group subproblems proposes a number of first stage decisions $\hat{x}_n^1 \in \mathbb{R}^{|J|}$, each is a solution of a group subproblem $n \in N$. Each first stage decision $\hat{x}_n^1$ can be viewed as a candidate feasible first stage solution to the three stage stochastic model MISOCP. When we append one of these decisions to MISOCP, if the remaining problem is feasible,

then the optimal value of it gives an upper bound for the optimal value of MIS-COP. Note that for given $\hat{x}_n^1$, the remaining problem (scenario tree) decomposes in each demand realization μ of H . Under a demand realization μ with a fixed first stage decision $\hat{x}^1$, the problem turns into a two stage stochastic problem. We refer this constructed problem as *two stage SP*($\hat{x}^1, \mu$). Our aim is to solve the constructed *two stage SP*($\hat{x}_n^1, \mu$) models - one for each demand realization μ with a fixed first stage decision $\hat{x}_n^1$, $n \in N$ to obtain an upper bound for MIS-COP.

The *two stage SP*($\hat{x}_n^1, \mu$) has pure binary decisions in its first stage (i.e., second stage decisions $z_{ijk}^2(\omega)$, $j \in J, i \in U_j, \omega \in \Omega$; $y_{ik}^2(\omega)$, $u_{ik}^2(\omega)$, $i \in J, k \in K, \omega \in \Omega$ of the original three stage model), whereas the second stage (i.e., third stage of the original model) involves both continuous and binary decisions. To solve each *two stage SP*($\hat{x}_n^1, \mu$), we use the exact solution method suggested by Ahmed [58]. This approach is based on a simple scenario decomposition algorithm and uses the binary nature of the first stage variables. In that approach, the single scenario subproblems are solved. However, if the problem has a large number of scenarios, many single-scenario subproblems need to be solved and therefore, a large number of proposed candidate first stage decisions should be evaluated. For computational tractability, we suggest solving group subproblems in order to obtain fewer number of first stage decisions. Similarly, the lower bound obtained by solving group subproblems would be tighter than the lower bound of the simple scenario decomposition approach suggested by Ahmed [58]. We call the group subproblems obtained from the *reduced model two stage SP*($\hat{x}_n^1, \mu$) as *reduced group subproblems*. We also refer the proposed candidate first stage decisions of *reduced group subproblem* $r \in R$ as $\hat{z}_r^2(\hat{x}_n^1, \mu)$, $\hat{y}_r^2(\hat{x}_n^1, \mu)$, and $\hat{u}_r^2(\hat{x}_n^1, \mu)$. Afterwards, for each $r \in R$, we impose candidate $\hat{z}_r^2(\hat{x}_n^1, \mu)$, $\hat{y}_r^2(\hat{x}_n^1, \mu)$, $\hat{u}_r^2(\hat{x}_n^1, \mu)$ decisions upon the *two stage SP*($\hat{x}_n^1, \mu$). Then, *two stage SP*($\hat{x}_n^1, \mu$) decomposes in each realizations η of Z . Note that, each single scenario subproblem is a deterministic problem and can be referred as $detpr(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu), \eta)$. Given $\hat{z}_r^2(\hat{x}_n^1, \mu)$, $\hat{y}_r^2(\hat{x}_n^1, \mu)$, $\hat{u}_r^2(\hat{x}_n^1, \mu)$, the single scenario deterministic problems are solved to obtain an upper bound $UB^{red}(\hat{x}_n^1, \mu)$ for the reduced model *two stage SP*($\hat{x}_n^1, \mu$). The crucial point of the algorithm is to cut-off explored decisions $\hat{z}_r^2(\hat{x}_n^1, \mu)$ from the feasible set $X(\hat{x}_n^1, \mu)$ of the *two stage SP*($\hat{x}_n^1, \mu$). In order to accomplish this,

we use the idea of *no good cuts* suggested by Ahmed [58] and add the following no good cuts to each reduced group subproblem $r \in R$ of *two stage* $SP(\hat{x}_n^1, \mu)$:

$$\sum_{i,j,k:\hat{z}_{ijk}^2(\hat{x}_n^1, \mu)=1} (1 - z_{ijk}^2) + \sum_{i,j,k:\hat{z}_{ijk}^2(\hat{x}_n^1, \mu)=0} z_{ijk}^2 \geq 1, \quad \hat{z}^2(\hat{x}_n^1, \mu) \in \hat{S}, \quad (3.49)$$

where $\hat{S} := \bigcup_{r \in R} \{\hat{z}_r^2(\hat{x}_n^1, \mu)\}$, i.e. the set of evaluated candidate decisions. We do not need to add an additional cut to eliminate the rest of the decisions $(\hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu))$, since $\hat{z}_r^2(\hat{x}_n^1, \mu)$ determines both $\hat{y}_r^2(\hat{x}_n^1, \mu)$ and $\hat{u}_r^2(\hat{x}_n^1, \mu)$.

Algorithm 2 Scenario group-wise decomposition algorithm

Apply Lower Bounding Algorithm

for each candidate first stage decision $\hat{x}_n^1, n \in N$ **do**

for each demand realization μ **do**

 Initialize: **improvement** $\leftarrow$ **true**, $UB^{red}(\hat{x}_n^1, \mu) \leftarrow +\infty$, $LB^{red}(\hat{x}_n^1, \mu) \leftarrow -\infty$, $\hat{S} \leftarrow \emptyset$

while **improvement** **do**

 Set $\hat{X}(\hat{x}_n^1, \mu) \leftarrow X(\hat{x}_n^1, \mu) \setminus \hat{S}$

 Construct the set R of group subproblems.

 Apply Lower Bounding Algorithm with set R to solve *two stage* $SP(\hat{x}_n^1, \mu)$

 Set $LB^{red}(\hat{x}_n^1, \mu)$ to the $LB(R)$ of Lower Bounding Algorithm

 Set $\bar{S} \leftarrow \emptyset$

for each reduced group subproblem $r \in R$ **do**

 Set $\hat{S} \leftarrow \hat{S} \cup \{\hat{z}_r^2(\hat{x}_n^1, \mu)\}$

 Set $\bar{S} \leftarrow \bar{S} \cup \{(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu))\}$

for each $(\hat{z}^2, \hat{y}^2, \hat{u}^2) \in \bar{S}$ **do**

for each realization η of Z **do**

 Solve $detpr(\hat{z}^2, \hat{y}^2, \hat{u}^2, \eta)$

 Set $f(\hat{z}^2, \hat{y}^2, \hat{u}^2, \eta)$ to the optimal value of $detpr(\hat{z}^2, \hat{y}^2, \hat{u}^2, \eta)$

 Update $UB^{red}(\hat{x}_n^1, \mu) \leftarrow \min \left(UB^{red}(\hat{x}_n^1, \mu), \min_{(\hat{z}^2, \hat{y}^2, \hat{u}^2) \in \bar{S}} \left(\sum_{\eta \in Z} f(\hat{z}^2, \hat{y}^2, \hat{u}^2, \eta) \right) \right)$

if $LB^{red}(\hat{x}_n^1, \mu) > UB^{red}(\hat{x}_n^1, \mu)$ **then**

improvement $\leftarrow$ **false**

Set $UB \leftarrow \min_{n \in N} \left(\sum_{\mu} UB^{red}(\hat{x}_n^1, \mu) \right)$

We provide a scenario group-wise decomposition algorithm in Algorithm 2 to solve the proposed three stage stochastic model MISOCP. The algorithm starts with applying the *Lower Bounding Algorithm* to propose a lower bound and a set of candidate first stage decisions $\hat{x}_n^1, n \in N$ for MISOCP. For each $\hat{x}_n^1$,

$n \in N$ and demand realization $\mu \in H$, the aim of the algorithm is to solve the reduced model *two stage* $SP(\hat{x}_n^1, \mu)$ to obtain an upper bound UB for MIS-COP. In order to find an optimal solution to *two stage* $SP(\hat{x}_n^1, \mu)$, our algorithm uses a scenario group decomposition approach. The algorithm solves a number of *reduced group subproblems* to propose candidate solutions $\hat{z}_r^2(\hat{x}_n^1, \mu)$, $\hat{y}_r^2(\hat{x}_n^1, \mu)$, and $\hat{u}_r^2(\hat{x}_n^1, \mu)$, $r \in R$ to the reduced model. Furthermore, using the optimal values of the reduced group subproblems, a lower bound $LB^{red}(\hat{x}_n^1, \mu)$ for *two stage* $SP(\hat{x}_n^1, \mu)$ is calculated as described in Section 3.2.1. However, for given $\hat{x}_n^1$, if any *reduced group subproblem* is infeasible, then $LB^{red}(\hat{x}_n^1, \mu)$ is set to $+\infty$. We then iteratively append the candidate solutions $(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu))$, $r \in R$ to the reduced model *two stage* $SP(\hat{x}_n^1, \mu)$ and solve the model for the remaining decision variables, i.e. we solve single scenario *detpr* $(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu), \eta)$, $r \in R, \eta \in Z$ models. The best of the candidate solutions, the one giving minimum objective function value, provides an upper bound $UB^{red}(x_n^1, \mu)$ for *two stage* $SP(\hat{x}_n^1, \mu)$. Then, we cut-off each evaluated solution from the feasible region $X(\hat{x}_n^1, \mu)$ by adding no-good cuts (3.49) to each *reduced group subproblems*. The algorithm repeats the same steps until $LB^{red}(x_n^1, \mu)$ becomes greater than $UB^{red}(x_n^1, \mu)$. Since the feasible region of each reduced group subproblem gets strictly smaller after adding no-good cuts at each iteration, $LB^{red}(x_n^1, \mu)$ is nondecreasing. When these steps are completed for each demand realization μ , the algorithm computes an upper bound UB for MISOCOP.

The Algorithm 2 terminates in a finite number of iterations, since the number of feasible second stage solutions is finite and the number of candidate first stage decisions equals to the number of group subproblems, which is also finite. Moreover, under each demand realization μ and each fixed first stage decision x_n^1 , the algorithm detects an optimal solution for the reduced model *two stage* $SP(\hat{x}_n^1, \mu)$. This is the fact that we do not discard any solution without evaluation. Therefore, an optimal solution is either discarded after evaluation or is still in the feasible region in any iteration. If it is discarded, then $LB^{red}(x_n^1, \mu)$ becomes *greater* than $UB^{red}(x_n^1, \mu)$. The detailed proof can be found in Ahmed [58].

3.3 Cutting Plane Algorithm

The group subproblem approach provided in Section 3.2 has been recently developed to reduce the computational effort in solving multi stage stochastic programs. However, if group subproblems still involve a large number of decisions and constraints, then solving each group subproblem may require a significant computation time. In this section, our aim is to mitigate the computational effort by decomposing each group subproblem into problems smaller in size. We develop a cutting plane algorithm (see, Ruszczyński [91] and references therein for details) to solve group subproblems, which are mixed integer three stage stochastic models with a reduced number of scenarios.

Traditionally in stochastic programming literature, two stage stochastic problems with integer variables in the first stage and continuous decisions in the second stage are solved with the cutting plane (called *L-Shaped* for linear problems) method (see, [55] for details). We reformulate our proposed three stage model to have a structure which can be efficiently solved by the cutting plane method. Therefore, we integrate integer variables in the master program (MP) and continuous ones in the subproblems. In other words, we include original first and second stage decisions in the master program. Moreover, we include the binary decisions arising in the third stage of MISOCP in the master program as well. In the subproblems, we leave the original third stage continuous decisions together with second order conic inequalities. Then, we obtain the following formulation:

$$\begin{aligned}
\min \quad & \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} y_{ik}^2(\omega) c_{daily}^k \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} \max(0, \mu_i(\omega) - \kappa^k) m_{ik}^2(\omega) c_{sp}^i \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{j \in P_i} c_{dist}^i \text{pass}_{ij}(\omega) b_{ij}^3(\omega) \right) p(\omega) \\
& + \sum_{\omega \in \Omega} Z(x^1, m^2(\omega), z^2(\omega), b^3(\omega)) p(\omega) \\
\text{subject to} \quad & (3.3) - (3.13), (3.23) - (3.26), (3.40),
\end{aligned} \tag{MP}$$

where $x^1 \in \mathbb{R}^{|J|}$ and for $\omega \in \Omega$, $m^2(\omega) \in \{0, 1\}^{|J||K|}$, $z^2(\omega)$ is a vector of $z_j^2(\omega) \in \{0, 1\}^{|D_j||K|}$, $j \in J$, $b^3(\omega)$ is a vector of $b_i^3(\omega) \in \{0, 1\}^{|P_i|}$, $i \in J$, and $Z(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ is the optimal value of the following subproblem.

$$\begin{aligned} \min \sum_{i \in J} \sum_{k \in K} S_{ik}^3(\omega) c_{Idle}^k + \sum_{i \in J} c_{del}^i(\omega) del_i^3(\omega) \\ + \sum_{i \in J} \sum_{k \in K} (c_{fuel} + c_{CO_2r}) (c_1^{ik} q_{ik}^3(\omega) + c_2^{ik} \delta_{ik}^3(\omega) + c_3^{ik} \varphi_{ik}^3(\omega) + c_4^{ik} \vartheta_{ik}^3(\omega)) \end{aligned} \quad (3.50)$$

subject to

$$-x_i^1 + d_i^3(\omega) \geq 0 \quad i \in J \quad (3.51)$$

$$\begin{aligned} d_j^3(\omega) - d_i^3(\omega) - f_{ik}^3(\omega) - S_{ik}^3(\omega) \leq \tau_{ij}^k + \eta_i(\omega) \\ + M_1 (1 - z_{ijk}^2(\omega)) \quad (i, j) \in C_E, k \in K \end{aligned} \quad (3.52)$$

$$\begin{aligned} d_j^3(\omega) - d_i^3(\omega) - f_{ik}^3(\omega) - S_{ik}^3(\omega) \geq \tau_{ij}^k + \eta_i(\omega) \\ - M_1 (1 - z_{ijk}^2(\omega)) \quad (i, j) \in C_E, k \in K \end{aligned} \quad (3.53)$$

$$-f_{ik}^\ell m_{ik}^2(\omega) + f_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K \quad (3.54)$$

$$f_{ik}^u m_{ik}^2(\omega) - f_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K \quad (3.55)$$

$$d_i^3(\omega) - x_i^1 - del_i^3(\omega) \leq 0 \quad i \in J \quad (3.56)$$

$$d_j^3(\omega) - d_i^3(\omega) - \sum_{k \in K} f_{ik}^3(\omega) + M b_{ij}^3(\omega) \geq \eta_i(\omega) + \lambda_{ij} \quad i \in J, j \in P_i \quad (3.57)$$

$$S_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K \quad (3.58)$$

$$del_i^3(\omega) \geq 0 \quad i \in J \quad (3.59)$$

$$\|(2m_{ik}^2(\omega), q_{ik}^3(\omega) - f_{ik}^3(\omega))\| \leq q_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K \quad (3.60)$$

$$\|(2f_{ik}^3(\omega), \vartheta_{ik}^3(\omega) - m_{ik}^2(\omega))\| \leq \vartheta_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K \quad (3.61)$$

$$\|(2m_{ik}^2(\omega), \pi_{ik}^3(\omega) - f_{ik}^3(\omega))\| \leq \pi_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K \quad (3.62)$$

$$\|(2\pi_{ik}^3(\omega), \delta_{ik}^3(\omega) - m_{ik}^2(\omega))\| \leq \delta_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K \quad (3.63)$$

$$\|(2f_{ik}^3(\omega), v_{ik}^3(\omega) - m_{ik}^2(\omega))\| \leq v_{ik}^3(\omega) + m_{ik}^2(\omega) \quad i \in J, k \in K \quad (3.64)$$

$$\|(2v_{ik}^3(\omega), \varphi_{ik}^3(\omega) - f_{ik}^3(\omega))\| \leq \varphi_{ik}^3(\omega) + f_{ik}^3(\omega) \quad i \in J, k \in K \quad (3.65)$$

$$q_{ik}^3(\omega), \delta_{ik}^3(\omega), \varphi_{ik}^3(\omega), \vartheta_{ik}^3(\omega), \pi_{ik}^3(\omega), v_{ik}^3(\omega) \geq 0 \quad i \in J, k \in K, \quad (3.66)$$

where M_1 is a big number. Constraints (3.52) and (3.53) are utilized to linearize constraint (3.17). The problem (3.50) - (3.66) is a convex problem with second order conic inequalities (3.60)-(3.65) derived in Section 3.1.4. Note that there exists a point which makes problem (3.50) - (3.66) strictly feasible. Since it is also bounded, according to strong duality theorem for second order cone programming problems (see, [90] and [92]), we can express problem (3.50) - (3.66) in dual form by using second order cone duality. To the best of our knowledge, Bayram and Yaman [56] are also the first researchers who use second order cone duality to handle nonlinear subproblems within a cutting plane algorithm.

Let C be a second order (convex) cone. Since $C = \text{epi}(\|\cdot\|)$ is a convex cone, let's first compactly write the constraints (3.60) - (3.65) as:

$$\begin{aligned} \left(\begin{bmatrix} 2m_{ik}^2(\omega) \\ q_{ik}^3(\omega) - f_{ik}^3(\omega) \end{bmatrix}, q_{ik}^3(\omega) + f_{ik}^3(\omega) \right) &\in C_{ik}^1(\omega) \\ \left(\begin{bmatrix} 2f_{ik}^3(\omega) \\ \vartheta_{ik}^3(\omega) - m_{ik}^2(\omega) \end{bmatrix}, \vartheta_{ik}^3(\omega) + m_{ik}^2(\omega) \right) &\in C_{ik}^2(\omega) \\ &\vdots \\ \left(\begin{bmatrix} 2v_{ik}^3(\omega) \\ \varphi_{ik}^3(\omega) - f_{ik}^3(\omega) \end{bmatrix}, \varphi_{ik}^3(\omega) + f_{ik}^3(\omega) \right) &\in C_{ik}^6(\omega). \end{aligned} \quad (3.67)$$

In order to formulate the dual subproblem by using conic duality, we need to define the dual cones of the second order cones in equation (3.67).

Definition 3.1. *If C is a cone, then the set*

$$C^* = \{y \in \mathbb{R}^n : x^T y \geq 0, \quad \forall x \in C\}$$

is the dual cone of C .

Definition 3.2. *A cone C is called self-dual, if $C^* = C$.*

The second order cone is an example of self-dual cones. Therefore, for $i \in J, k \in K, \omega \in \Omega, s = 1, \dots, 6$, the corresponding dual cone $C_{ik}^{*s}(\omega)$ of each second order cone $C_{ik}^s(\omega)$ in equation (3.67) appears as a second order cone in the dual

problem of (3.50) - (3.66). The dual cones $C_{ik}^{*s}(\omega)$, $s = 1, \dots, 6$ with associated dual variables $\gamma_{ik}^{10}(\omega)$, $\gamma_{ik}^{11}(\omega)$, $\alpha_{ik}^1(\omega)$, $\gamma_{ik}^{12}(\omega)$, $\gamma_{ik}^{13}(\omega)$, $\alpha_{ik}^2(\omega)$, $\gamma_{ik}^{14}(\omega)$, $\gamma_{ik}^{15}(\omega)$, $\alpha_{ik}^3(\omega)$, $\gamma_{ik}^{16}(\omega)$, $\gamma_{ik}^{17}(\omega)$, $\alpha_{ik}^4(\omega)$, $\gamma_{ik}^{18}(\omega)$, $\gamma_{ik}^{19}(\omega)$, $\alpha_{ik}^5(\omega)$, $\gamma_{ik}^{20}(\omega)$, $\gamma_{ik}^{21}(\omega)$, $\alpha_{ik}^6(\omega)$ can be expressed as follows:

$$\begin{aligned} \left(\begin{bmatrix} \gamma_{ik}^{10}(\omega) \\ \gamma_{ik}^{11}(\omega) \end{bmatrix}, \alpha_{ik}^1(\omega) \right) &\in C_{ik}^{*1}(\omega) \\ \left(\begin{bmatrix} \gamma_{ik}^{12}(\omega) \\ \gamma_{ik}^{13}(\omega) \end{bmatrix}, \alpha_{ik}^2(\omega) \right) &\in C_{ik}^{*2}(\omega) \\ &\vdots \\ \left(\begin{bmatrix} \gamma_{ik}^{20}(\omega) \\ \gamma_{ik}^{21}(\omega) \end{bmatrix}, \alpha_{ik}^6(\omega) \right) &\in C_{ik}^{*6}(\omega). \end{aligned}$$

For $\omega \in \Omega, i \in J, j \in J, k \in K$, let $\gamma_i^1(\omega)$, $\gamma_{ijk}^2(\omega)$, $\gamma_{ijk}^3(\omega)$, $\gamma_{ik}^4(\omega)$, $\gamma_{ik}^5(\omega)$, $\gamma_i^6(\omega)$, $\gamma_{ij}^7(\omega)$ denote the dual variables associated with the constraints (3.51) - (3.57), respectively. Then, using the second order cone duality, the dual of problem (3.50) - (3.66) can be expressed as:

$$\begin{aligned} \max \quad & \sum_{i \in J} x_i^1 (\gamma_i^1(\omega) - \gamma_i^6(\omega)) \quad (DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))) \\ & + \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) - \gamma_{ijk}^2(\omega)) (\tau_{ij}^k + \eta_i(\omega)) \\ & - \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) + \gamma_{ijk}^2(\omega)) M_1 (1 - z_{ijk}^2(\omega)) \\ & + \sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (\gamma_{ik}^4(\omega) \ell_i^k - \gamma_{ik}^5(\omega) u_i^k + 2\gamma_{ik}^{10}(\omega) - \gamma_{ik}^{13}(\omega)) \\ & + \sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (-\alpha_{ik}^2(\omega) + 2\gamma_{ik}^{14}(\omega) - \gamma_{ik}^{17}(\omega)) \\ & + \sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (-\alpha_{ik}^4(\omega) - \gamma_{ik}^{19}(\omega) - \alpha_{ik}^5(\omega)) \\ & + \sum_{i \in J} \sum_{j \in P_i} \gamma_{ij}^7(\omega) (\eta_i(\omega) + \lambda_{ij} - Mb_{ij}^3(\omega)) \end{aligned} \quad (3.68)$$

subject to

$$\alpha_{ik}^1(\omega) - \gamma_{ik}^{11}(\omega) \leq (c_{fuel} + c_{CO_2}r) c_{ik}^1 \quad i \in J, k \in K \quad (3.69)$$

$$\alpha_{ik}^4(\omega) - \gamma_{ik}^{17}(\omega) \leq (c_{fuel} + c_{CO_2}r) c_{ik}^2 \quad i \in J, k \in K \quad (3.70)$$

$$\alpha_{ik}^6(\omega) - \gamma_{ik}^{21}(\omega) \leq (c_{fuel} + c_{CO_2}r) c_{ik}^3 \quad i \in J, k \in K \quad (3.71)$$

$$\alpha_{ik}^2(\omega) - \gamma_{ik}^{13}(\omega) \leq (c_{fuel} + c_{CO_2}r) c_{ik}^4 \quad i \in J, k \in K \quad (3.72)$$

$$\gamma_i^6(\omega) \leq c_{del}^i(\omega) \quad i \in J \quad (3.73)$$

$$\begin{aligned} & \gamma_i^1(\omega) + \sum_{j \in D_i} \sum_{k \in K} \gamma_{ijk}^2(\omega) - \sum_{j \in U_i} \sum_{k \in K} \gamma_{jik}^2(\omega) - \sum_{j \in D_i} \sum_{k \in K} \gamma_{ijk}^3(\omega) + \\ & \sum_{j \in U_i} \sum_{k \in K} \gamma_{jik}^3(\omega) - \gamma_i^6(\omega) - \sum_{j \in P_i} \gamma_{ij}^7 + \sum_{j \in C_i} \gamma_{ji}^7 \leq 0 \quad i \in J \end{aligned} \quad (3.74)$$

$$\alpha_{ik}^5(\omega) - 2\gamma_{ik}^{20}(\omega) - \gamma_{ik}^{19}(\omega) \leq 0 \quad i \in J, k \in K \quad (3.75)$$

$$\alpha_{ik}^3(\omega) - 2\gamma_{ik}^{16}(\omega) - \gamma_{ik}^{15}(\omega) \leq 0 \quad i \in J, k \in K \quad (3.76)$$

$$\sum_{j \in D_i} \gamma_{ijk}^2(\omega) - \sum_{j \in D_i} \gamma_{ijk}^3(\omega) \leq c_{idle}^k \quad i \in J, k \in K \quad (3.77)$$

$$\begin{aligned} & \alpha_{ik}^6(\omega) + \gamma_{ik}^{21}(\omega) - 2\gamma_{ik}^{18}(\omega) + \alpha_{ik}^3(\omega) + \gamma_{ik}^{15}(\omega) - 2\gamma_{ik}^{12}(\omega) + \alpha_{ik}^1(\omega) \\ & + \gamma_{ik}^{11}(\omega) - \sum_{j \in P_i} \gamma_{ij}^7(\omega) - \gamma_{ik}^5(\omega) + \gamma_{ik}^4(\omega) + \sum_{j \in D_i} \gamma_{ijk}^2(\omega) \\ & - \sum_{j \in D_i} \gamma_{ijk}^3(\omega) \leq 0 \quad i \in J, k \in K \end{aligned} \quad (3.78)$$

$$\|(\gamma_{ik}^{10}(\omega), \gamma_{ik}^{11}(\omega))\| \leq \alpha_{ik}^1(\omega) \quad i \in J, k \in K \quad (3.79)$$

$$\|(\gamma_{ik}^{12}(\omega), \gamma_{ik}^{13}(\omega))\| \leq \alpha_{ik}^2(\omega) \quad i \in J, k \in K \quad (3.80)$$

$$\|(\gamma_{ik}^{14}(\omega), \gamma_{ik}^{15}(\omega))\| \leq \alpha_{ik}^3(\omega) \quad i \in J, k \in K \quad (3.81)$$

$$\|(\gamma_{ik}^{16}(\omega), \gamma_{ik}^{17}(\omega))\| \leq \alpha_{ik}^4(\omega) \quad i \in J, k \in K \quad (3.82)$$

$$\|(\gamma_{ik}^{18}(\omega), \gamma_{ik}^{19}(\omega))\| \leq \alpha_{ik}^5(\omega) \quad i \in J, k \in K \quad (3.83)$$

$$\|(\gamma_{ik}^{20}(\omega), \gamma_{ik}^{21}(\omega))\| \leq \alpha_{ik}^6(\omega) \quad i \in J, k \in K \quad (3.84)$$

$$\gamma_i^1(\omega), \gamma_i^6(\omega) \geq 0 \quad i \in J \quad (3.85)$$

$$\gamma_{ijk}^2(\omega), \gamma_{ijk}^3(\omega) \geq 0 \quad (i, j) \in C_E, k \in K \quad (3.86)$$

$$\gamma_{ik}^4(\omega), \gamma_{ik}^5(\omega) \geq 0 \quad i \in J, k \in K \quad (3.87)$$

$$\gamma_{ij}^7(\omega) \geq 0 \quad i \in J, j \in P_i \quad (3.88)$$

$$\alpha_{ik}^1(\omega), \alpha_{ik}^2(\omega), \alpha_{ik}^3(\omega), \alpha_{ik}^4(\omega), \alpha_{ik}^5(\omega), \alpha_{ik}^6(\omega) \geq 0 \quad i \in J, k \in K \quad (3.89)$$

The feasible set of the dual subproblem (3.68)-(3.89) is nonempty and bounded. Furthermore, dual subproblem (3.68)-(3.89) and primal subproblem (3.50) - (3.66) are convex optimization problems and by strong duality, they attain the same optimal value.

It is important to note that feasible region of the dual formulation does not depend on the value of $(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$, which only affects the objective function. Therefore, instead of dealing with the primal subproblem, we need to solve the dual subproblem $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$.

Let us have an index set $\mathcal{T}$ of feasible solutions to MP. By an abuse of notation, for each $t \in \mathcal{T}$ and $\omega \in \Omega$, let $\alpha^t(\omega)$ and $\gamma^t(\omega)$ be an optimal solution to the dual subproblem for $(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$. By introducing auxiliary variables ν_ω , $\omega \in \Omega$ to approximate the third stage objective values, we can reformulate the proposed model as the following relaxed master program (RMP):

$$\begin{aligned}
\min \quad & \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} \max(0, \mu_i(\omega) - \kappa^k) m_{ik}^2(\omega) c_{sp}^i \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{k \in K} y_{ik}^2(\omega) c_{daily}^k \right) p(\omega) \\
& + \sum_{\omega \in \Omega} \left(\sum_{i \in J} \sum_{j \in P_i} c_{dist}^i pass_{ij}(\omega) b_{ij}^3(\omega) \right) p(\omega) \\
& + \sum_{\omega \in \Omega} p(\omega) \nu_\omega
\end{aligned} \tag{RMP}$$

subject to

$$\begin{aligned}
\nu_\omega \geq & \sum_{i \in J} x_i^1 (\gamma_i^{t1}(\omega) - \gamma_i^{t6}(\omega)) \\
& + \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^{t3}(\omega) - \gamma_{ijk}^{t2}(\omega)) (\tau_{ij}^k + \eta_i(\omega)) \\
& - \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^{t3}(\omega) + \gamma_{ijk}^{t2}(\omega)) (M_1 (1 - z_{ijk}^2(\omega))) \\
& + \sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (2\gamma_{ik}^{t14}(\omega) + 2\gamma_{ik}^{t10}(\omega) - \gamma_{ik}^{t13}(\omega) - \gamma_{ik}^{t17}(\omega) - \alpha_{ik}^{t4}(\omega)) \\
& + \sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (\gamma_{ik}^{t4}(\omega) \ell_i^k - \gamma_{ik}^{t5}(\omega) u_i^k - \gamma_{ik}^{t19}(\omega) - \alpha_{ik}^{t5}(\omega) - \alpha_{ik}^{t2}(\omega))
\end{aligned}$$

$$+ \sum_{i \in J} \sum_{j \in P_i} \gamma_{ij}^{t7}(\omega) (\eta_i(\omega) + \lambda_{ij} - M b_{ij}^3(\omega)) \quad \omega \in \Omega, t \in \mathcal{T} \quad (3.91)$$

(3.3) – (3.13), (3.23) – (3.26), (3.40)

In RMP, the exact calculation of the third stage objective value, that is $Z(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ is relaxed. The constraints (3.91) are called as optimality cuts. Note that, for each $t \in \mathcal{T}$ and $\omega \in \Omega$, $\alpha^t(\omega)$ and $\gamma^t(\omega)$ is a feasible solution to the dual subproblem for any feasible solution $(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ of MP. Therefore, for each $\omega \in \Omega$, the right hand side of optimality cuts provide a lower approximation of $Z(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$. This implies that the auxiliary variables ν_ω , $\omega \in \Omega$, together with the optimality cuts (3.91), denote appropriate approximations of the third stage objective values. We ensure that for each solution of RMP, every primal subproblem is feasible. Hence, we do not consider any feasibility cut.

In each iteration t of the algorithm, RMP is solved to optimality and an optimality cut for each $\omega \in \Omega$ is added. For an extensive discussion on the cutting plane method and the proofs for convergence, we refer to Ruszczyński [91] and references therein, and for L-shaped method, we refer to Van Slyke and Wets [93], Birge and Louveaux [55] and [94].

3.3.1 Improvements on Cutting Plane Algorithm

3.3.1.1 Defining Pareto-Optimal Cuts

In the cutting plane algorithm, convergence time often depends on the quality of the optimality cuts generated. If dual subproblem involves multi (alternative) optimal solutions, each multi optimal solution provides a different cut with different strength. Thus, an important question arises whether one could improve the performance of the cutting plane algorithm through defining the strong (pareto-optimal) cuts. Magnanti and Wong [95] call a cut pareto optimal if no

cut dominates it. Let $(x_0, m_0(\omega), z_0(\omega), b_0(\omega))$, $\omega \in \Omega$ be a core point in the relative interior of the convex hull of feasible first and second stage decision vectors and obj_ω be the optimal value of $DP(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$, $\omega \in \Omega$, where $(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$ is an optimal solution of RMP at iteration t . To generate a pareto-optimal cut, for each $\omega \in \Omega$, the following Magnanti-Wong problem is solved.

$$\begin{aligned}
\max \quad & \sum_{i \in J} x_{0i} (\gamma_i^1(\omega) - \gamma_i^6(\omega)) + \sum_{i \in J} \sum_{j \in P_i} \gamma_{ij}^7(\omega) (\eta_i(\omega) + \lambda_{ij} - Mb_{0ij}(\omega)) \\
& + \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) - \gamma_{ijk}^2(\omega)) (\tau_{ij}^k + \eta_i(\omega)) \\
& - \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) + \gamma_{ijk}^2(\omega)) M_1 (1 - z_{0ijk}(\omega)) \\
& + \sum_{i \in J} \sum_{k \in K} m_{0ik}(\omega) (\gamma_{ik}^4(\omega) \ell_i^k - \gamma_{ik}^5(\omega) u_i^k + 2\gamma_{ik}^{10}(\omega) - \gamma_{ik}^{13}(\omega) - \alpha_{ik}^2(\omega)) \\
& + \sum_{i \in J} \sum_{k \in K} m_{0ik}(\omega) (2\gamma_{ik}^{14}(\omega) - \gamma_{ik}^{17}(\omega) - \alpha_{ik}^4(\omega) - \gamma_{ik}^{19}(\omega) - \alpha_{ik}^5(\omega))
\end{aligned} \tag{3.92}$$

subject to

$$(3.69) - (3.89)$$

$$\begin{aligned}
obj_\omega = & \sum_{i \in J} x_i^{t1} (\gamma_i^1(\omega) - \gamma_i^6(\omega)) + \sum_{i \in J} \sum_{j \in P_i} \gamma_{ij}^7(\omega) (\eta_i(\omega) + \lambda_{ij} - Mb_{ij}^{t3}(\omega)) \\
& + \sum_{i \in J} \sum_{k \in K} m_{ik}^{t2}(\omega) (\gamma_{ik}^4(\omega) \ell_i^k - \gamma_{ik}^5(\omega) u_i^k + 2\gamma_{ik}^{10}(\omega) - \gamma_{ik}^{13}(\omega) - \alpha_{ik}^2(\omega)) \\
& + \sum_{i \in J} \sum_{k \in K} m_{ik}^{t2}(\omega) (2\gamma_{ik}^{14}(\omega) - \gamma_{ik}^{17}(\omega) - \alpha_{ik}^4(\omega) - \gamma_{ik}^{19}(\omega) - \alpha_{ik}^5(\omega)) \\
& + \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) - \gamma_{ijk}^2(\omega)) (\tau_{ij}^k + \eta_i(\omega)) \\
& - \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^3(\omega) + \gamma_{ijk}^2(\omega)) M_1 (1 - z_{ijk}^{t2}(\omega))
\end{aligned} \tag{3.93}$$

The Magnanti-Wong problem chooses the strongest cut associated with optimal dual variables chosen among the alternatives satisfying the constraint (3.93). However, finding a core point may not be an easy task. Papadakos [96] shows that $(x_0, m_0(\omega), z_0(\omega), b_0(\omega))$, $\omega \in \Omega$ does not have to be a core point. Any

point provides a valid optimality cut, since this only changes the objective function without altering the feasible region of Magnanti-Wong problem. Papadakos [96] suggests following equations to obtain approximate core points:

$$\begin{aligned}
x0^t &\leftarrow \frac{1}{2}x0^{t-1} + \frac{1}{2}x^t \\
m0^t(\omega) &\leftarrow \frac{1}{2}m0^{t-1}(\omega) + \frac{1}{2}m^t(\omega), \quad \omega \in \Omega \\
z0^t(\omega) &\leftarrow \frac{1}{2}z0^{t-1}(\omega) + \frac{1}{2}z^t(\omega), \quad \omega \in \Omega \\
b0^t(\omega) &\leftarrow \frac{1}{2}b0^{t-1}(\omega) + \frac{1}{2}b^t(\omega), \quad \omega \in \Omega,
\end{aligned} \tag{3.94}$$

where $(x^t, m^t(\omega), z^t(\omega), b^t(\omega))$, $\omega \in \Omega$ is an optimal solution of RMP at iteration t . In our computational experiments, for the first iteration, the point $(x0, m0(\omega), z0(\omega), b0(\omega))$, $\omega \in \Omega$ is set equal to an optimal solution of RMP, so that equation (3.94) gradually approaches to a core point.

However, a numerical unboundedness may occur due to equality constraint (3.93). Therefore, Papadakos [96] proposes an enhanced formulation by eliminating constraint (3.93) from the Magnanti-Wong problem. We denote improved cutting plane algorithm with this enhancement as CP-EMW. The main drawback of CP-EMW algorithm is to solve both dual subproblem $DP(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$ and the Magnanti-Wong problem for each scenario $\omega \in \Omega$ at every iteration t . To accelerate the convergence of the cutting plane method, Sherali and Lunday [97] propose an algorithm via the generation of maximal nondominated cuts. They use a preemptive priority multiple objective programming to solve the dual subproblems $DP(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$, $\omega \in \Omega$ optimally with the first priority while maximizing (3.92) among the alternative optimal solutions to the problem. Bayram and Yaman [56] implement this preemptive priority multiple objective program as the following weighted-sum problem:

$$\begin{aligned}
&\max \quad (3.68) - \xi(3.92) && (SL(x^1, m^2(\omega), z^2(\omega), b^3(\omega))) \\
&\text{subject to} \\
&\quad (3.69) - (3.89).
\end{aligned}$$

The new objective is the maximization of weighted sum of (3.68) and (3.92), where the weight vector is $(1, \xi)$. We take $\xi = 10^{-10}$. At iteration t , we update $(x0^t, m0^t(\omega), z0^t(\omega), b0^t(\omega))$, $\omega \in \Omega$ as described in (3.94) and replace $(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ in $SL(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ with $(x^{t1}, m^{t2}(\omega), z^{t2}(\omega), b^{t3}(\omega))$.

3.3.1.2 Defining Scenario Group Cuts

In our cutting plane algorithm, each subproblem is defined for each scenario. An optimality cut associated with each scenario can be added to RMP to accelerate convergence, thus resulting in a large number of cuts at each iteration of the cutting plane algorithm. However, this multi-cut version may perform worse due to time required to solve RMP with too many cuts. On the other hand, the single cut version, e.g. aggregating optimality cuts for each scenario, could be loose to approximate the expected third stage objective value, thus resulting in increased number of iterations to converge. To overcome these trade-offs, we use the idea of group subproblems in Section 3.2.1. We can add reduced number of cuts, one for each group of scenarios, to RMP. Compared to the single cut version, we could obtain a tighter bound to the third stage objective value. We create subsets $\Omega_g \subset \Omega$ of scenarios such that $\bigcup_{g \in G} \Omega_g = \Omega$, where G is the set of scenario groups. It is clear that quality of the cut depends on the scenario grouping. In order to strengthen the bound, we can apply scenario selection procedures in Appendix C. In scenario group cut strategy, the multi cuts (3.91) are replaced by the following group cuts:

$$\begin{aligned} \nu_g \geq & \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^{t3}(\omega) - \gamma_{ijk}^{t2}(\omega)) (\tau_{ij}^k + \eta_i(\omega)) \\ & - \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \left(\sum_{i \in J} \sum_{j \in D_i} \sum_{k \in K} (\gamma_{ijk}^{t3}(\omega) + \gamma_{ijk}^{t2}(\omega)) (M_1 (1 - z_{ijk}^2(\omega))) \right) \\ & + \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \left(\sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (\gamma_{ik}^{t4}(\omega) \ell_i^k - \gamma_{ik}^{t5}(\omega) u_i^k + 2\gamma_{ik}^{t10}(\omega)) \right) \\ & + \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \left(\sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (-\gamma_{ik}^{t13}(\omega) - \alpha_{ik}^{t2}(\omega) - \alpha_{ik}^{t4}(\omega) - \gamma_{ik}^{t17}(\omega)) \right) \end{aligned}$$

$$\begin{aligned}
& + \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \left(\sum_{i \in J} \sum_{k \in K} m_{ik}^2(\omega) (2\gamma_{ik}^{t14}(\omega) - \gamma_{ik}^{t19}(\omega) - \alpha_{ik}^{t5}(\omega)) \right) \\
& + \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \left(\sum_{i \in J} \sum_{j \in P_i} \gamma_{ij}^{t7}(\omega) (\eta_i(\omega) + \lambda_{ij} - Mb_{ij}^3(\omega)) \right) \\
& + \sum_{\omega \in \Omega_g} \hat{p}_g(\omega) \sum_{i \in J} x_i (\gamma_i^{t1}(\omega) - \gamma_i^{t6}(\omega)) \quad g \in G, t \in \mathcal{T} \tag{3.95}
\end{aligned}$$

We obtain group cuts in (3.95) by aggregating the optimality cuts for each scenario in the set Ω_g . To make sure that the sum of probabilities in each set Ω_g is one, we use adjusted probabilities $\hat{p}_g(\omega)$ as defined in Section 3.2.1. Then, we replace the third stage approximate cost in the objective of RMP by $\sum_{g \in G} \nu_g p(\Omega_g)$, where $p(\Omega_g) = \sum_{\omega \in \Omega_g} p(\omega)/mc(\omega)$.

3.3.1.3 Improved Cutting Plane Algorithm with Lazy Constraints

Classical cutting plane algorithm involves sequentially solving RMP and subproblems. An optimality cut is added to RMP before it is re-optimized. One of the drawbacks of the classical cutting plane algorithm is that optimally solving RMP with too many optimality cuts may require a significant computational effort. Besides, whenever an optimality cut is added, a new search tree on RMP is constructed, thereby resulting in revisiting candidate solutions that had been already eliminated earlier. Therefore, we utilize the recent development that processes cutting plane algorithm on a single search tree by using a lazy constraint callback feature of CPLEX. In contrast to the classical cutting plane algorithm, we solve subproblems and generate optimality cuts whenever a new candidate incumbent on RMP is proposed by lazy constraint callback feature. Lazy constraints are initially put into a pool, they are not initially a part of the active model. As solutions are generated, the solver checks whether any lazy constraint is violated or not. Then, violated lazy constraints are added to active constraints set. This is a great advantage to reduce the computation time per iteration as the number of optimality cuts generated increases significantly. More detailed information on lazy constraints can be found in Rubin [98].

Algorithm 3 Improved cutting plane algorithm

Invoke CPLEX to solve RMP and initiate branch-and-cut procedure
while CPLEX determines the optimality of the current incumbent or user defined termination criteria is satisfied **do**
 Identify a candidate incumbent $\bar{x}^1, \bar{m}^2(\omega), \bar{z}^2(\omega), \bar{b}^3(\omega), \bar{\nu}_\omega, \omega \in \Omega$
 for each $\omega \in \Omega$ **do**
 Solve dual subproblem $DP(\bar{x}^1, \bar{m}^2(\omega), \bar{z}^2(\omega), \bar{b}^3(\omega))$ to optimality and let obj_ω denote the optimal value
 if $\bar{\nu}_\omega < obj_\omega - \epsilon$ **then**
 Add corresponding optimality cuts of the form (3.91) associated with the candidate incumbent to RMP as lazy constraints
 else
 Accept the candidate incumbent as the current incumbent with objective value of RMP

We define our improved cutting plane algorithm in Algorithm 3. We invoke CPLEX to solve RMP using the branch-and-cut procedure. In our implementation, whenever CPLEX identifies a candidate incumbent in the search tree, the lazy constraint callback routine is invoked. The goal of the callback routine is to identify the optimality cuts violated by the candidate incumbent and introduce the optimality cuts as lazy constraints or to approve the candidate incumbent as the current incumbent. Let $(\bar{x}^1, \bar{m}^2(\omega), \bar{z}^2(\omega), \bar{b}^3(\omega))$ be the candidate incumbent to RMP. In the callback routine, to generate the optimality cuts, the dual subproblem $DP(\bar{x}^1, \bar{m}^2(\omega), \bar{z}^2(\omega), \bar{b}^3(\omega))$ for each $\omega \in \Omega$ is solved to optimality. If the difference between the optimal value of any subproblem and the current optimal value of ν_ω , that is $\bar{\nu}_\omega$, is greater than ϵ , then we append the optimality cuts (3.91) associated with the candidate incumbent to RMP as lazy constraints. Here, ϵ is a very small number, which is used in order to handle the numerical precision in computations. Once all lazy constraints appended are satisfied in RMP, we accept the candidate incumbent as the current incumbent. The algorithm continues until the CPLEX determines the optimality of the current incumbent or any user defined termination criteria is satisfied. In the experimental study, we set a time limit as a termination criteria.

3.4 Computational Results

In this study, we propose a novel three stage stochastic programming model, which generates a schedule by incorporating operational flight timing decisions along with the fleet assignment and aircraft routing problem. Compared to the existing two stage stochastic fleet assignment problems which only consider passenger demand uncertainty, we additionally incorporate operational flight timing decisions and non-cruise time uncertainty in the third stage of our proposed model. In the experimental study, we first analyze the cost-wise benefits of including the third stage. On the other hand, the third stage brings an additional complexity together with a larger decision tree and nonlinear functions of fuel and emission costs. Therefore, we then analyze the impact of valid inequalities derived in Section 3.1.3 to efficiently handle the additional complexity of the third stage. Moreover, we test the ability of our scenario group-wise decomposition algorithm to handle the large number of scenarios arising in our three stage stochastic programming model. Finally, we analyze the effect of improvements in Section 3.3.1 on the convergence of our cutting plane algorithm.

We use three different schedules generated from the work of Aktürk et al. [26]. The flight information is extracted from database “Airline On-Time Performance Data,” provided by the Bureau of Transportation Statistics of the U.S. Department of Transportation, BTS [99]. We perform the numerical experiments in Sections 3.4.1 and 3.4.2 on a OS X Yosemite computer with 8 GB memory and 2.6 GHz Intel Core i5 processor. Then, we complete experiments on larger problem instances in Section 3.4.3 using a workstation with a 3.60 GHz Intel R Xeon R CPU E5-1650 and 32 GB main memory. All experiments are carried out by implementing proposed MISOCP model, scenario group-wise decomposition and cutting plane algorithms in JAVA programming language with a connection to commercial solver IBM ILOG CPLEX Optimization Studio 12.7.1. In order to analyze the effects of problem parameters, we conduct a 2^k full-factorial experimental design. We provide levels of experimental factors in Table 3.1.

Base Spill Cost represents the opportunity cost per each spilled passengers due

Factor Description	Levels	
	Low	High
Base Spill Cost (\$/passenger)	60	200
c_{fuel} (\$/kg)	0.6	1.2

Table 3.1: Factor values

to under capacity of assigned aircraft. We adjust the cost of spilled passengers for each flight using airport congestion coefficients, e.g. favoring the populated markets, as follows:

$$Csp_i = BaseSpillCost e_{O_i} e_{Dn_i}, \quad i \in J, \quad (3.96)$$

where e_{O_i} and e_{Dn_i} represent the congestion coefficients for origin and destination airports of flight $i \in J$ as discussed in Şafak et al. [51].

c_{fuel} is the price of jet fuel per kilogram (kg) of fuel burned by an aircraft. According to IATA fuel price monitor [100], the price of fuel per kg is fluctuating between \$0.6 and \$1.2 between years 2008 - 2017. In this study, we consider fuel prices as \$0.6/kg and \$1.2/kg for lower and higher settings, respectively.

Delay cost represents the cost of passengers' goodwill due to the flight delays. We calculate at-gate delay cost as a function of passenger demand, as follows:

$$c_{del}^i(\omega) = \beta \mu_i(\omega), \quad i \in J, \omega \in \Omega. \quad (3.97)$$

In the experimental design, we set β to \$0.08 per minute of delay per passenger. Eurocontrol [101] reports that delay costs 0.18 €/minute per passenger for the base case scenario by considering not only flight delays but also passengers disrupted due to misconnection. Since we penalize the misconnections in a separate cost term in the objective function, we set β to a value relatively smaller than 0.18 €. In the computations, we incur \$50 per passenger, when the passenger misses the next flight in his/her itinerary.

We include aircraft daily usage cost by adjusting the payment of purchasing a new aircraft on a daily basis under the assumption that it is regularly used over twenty years. We incorporate additional costs of insurance, hangar rental, taxes, etc., into daily usage cost parameter. We express the fuel burn and carbon

emission costs as in equation (3.1.1). We calculate the coefficients c_s^{ik} , $i \in J, k \in K, s = 1, \dots, 4$ using the corresponding values of fuel burn related parameters as well as the mass and surface in Table 3.2, as expressed in Şafak et al. [51]. We consider five different aircraft types and list the fuel burn related parameters, corresponding maximum range cruise (MRC) speed, seat capacity, idle time and daily usage cost parameters in Table 3.2.

Aircraft type	B737 500	MD 83	A320 111	A320 212	B767 300
Capacity (κ^k)	122	148	172	180	218
Mass (kgs)	50000	61200	62000	64000	135000
Surface (m^2)	105.4	118	122.4	122.6	283.3
$C_{D0,CR}$	0.018	0.0211	0.024	0.024	0.021
$C_{D2,CR}$	0.055	0.0468	0.0375	0.0375	0.049
Cf_1	0.46	0.7462	0.94	0.94	0.763
Cf_2	300	638.59	50000	100000	1430
cf_{CR}	1.079	0.9505	1.095	1.06	1.0347
MRC speed (km/h)	859.2	867.6	855.15	868.79	876.70
Base Turntime (min) (τ_{base}^k)	36	26	28	30	40
Idle Time Cost (\$)/min (c_{Idle}^k)	140	142	136	144	147
Daily Usage Cost (\$) (c_{daily}^k)	25000	27300	28250	29000	58550

Table 3.2: Aircraft parameters

We estimate the aircraft turnaround time, τ_{ij}^k , needed for an aircraft at an airport using the expression:

$$\tau_{ij}^k = \tau_{base}^k e_{Dn_i} \quad k \in K, (i, j) \in C_E, \quad (3.98)$$

where e_{Dn_i} represents the congestion level of destination airport for flight $i \in J$. Therefore, turntime of an aircraft visiting a congested airport takes longer.

In this study, we select the announced departure times from a time interval which has already been determined by an airline. We extracted departure time information from database “Airline On-Time Performance Data,” provided by BTS [99]. We determine the lower departure time for a flight by subtracting ten minutes from the departure time reported in the database [99]. Similarly, for the upper departure times, we added ten minutes to the departure times reported in the database [99]. Thus, we select an announced departure time for a flight from the twenty minutes time interval determined based on the history of flight departures. We only allow a flight to depart from one of the five discrete points

within this time interval. Duration between two consecutive points is taken as five minutes.

We generate a network of passenger connections using the flight departure information of sample schedule used in the computations. We only allow passenger connections between two flights $i \in J$ and $j \in J$, if the departure time of flight j in a sample schedule is within 45 minutes or 180 minutes of the arrival time of flight i and destination airport of flight i is same as the origin airport of the flight j . We take passenger connection times, λ_{ij} , $i \in J, j \in P_i$ uniformly between 25 and 40 minutes. We calculate upper bound f_{ik}^u , $i \in J, k \in K$ for cruise times by dividing the cruise length of flight i with MRC speed of aircraft type k . Max range cruise (MRC) speed is the minimizer of fuel burn of an aircraft. Although it is ideal to fly at MRC speed, we may prefer to accelerate the aircraft to satisfy the minimum time requirements of passenger connections. In our model, we allow compression on cruise times by 15% of f_{ik}^u .

3.4.1 Numerical Example

We provide a numerical example to show the great advantage of incorporating cruise time decisions, passenger demand and non-cruise time uncertainties within an integrated schedule design, fleet assignment and aircraft routing problem. We use a small sample schedule with 9 flights operated by two aircraft. We extract flight information from the schedule provided by Şafak et al. [51]. We consider randomness for passenger demands for two round way trips {Chicago O'Hare International Airport (ORD)-Dallas/Fort Worth International Airport (DFW)} and {ORD-LaGuardia Airport (LGA)}. The demand realization of a trip (ORD-DFW) in the morning would be different than the demand realization of a flight at noon of the same trip (ORD-DFW). Therefore, for each trip, we segment the passenger demand into three different levels according to the flight departure times of the sample schedule. This segmentation based on flight departure times is explained in detail in Appendix A. We assume that the demand levels on all segments belonging to same trip are positively correlated, taking either some

“high” or “low” level of demand simultaneously. On the other hand, demands of different round trips are independent. Therefore, such a demand pattern would yield a total of $2^2 = 4$ demand realizations. In addition, we select two key cities (ORD, DFW), e.g., hubs in a hub-and-spoke network. We consider four data points for ORD airport and two points for DFW airport to generate non-cruise time scenarios. Similarly, we assume that “taxi-out”, “departure delay”, “taxi-in” and “arrival delay”, which are the components of non-cruise time of a flight, are positively correlated for the same airport. On the other hand, non-cruise times at different airports are independent. Thus, this pattern yields in $4 \cdot 2 = 8$ non-cruise time realizations. In conclusion, total number of scenarios, which is characterized by multiple demand and non-cruise time realizations, would yield $4 \cdot 8 = 32$ scenarios. More detailed information on scenario generation procedure along with the realization values can be found in Appendices A and B.

In this example, we initially start with analyzing what might be the impact of including uncertain passenger demand and non-cruise times in our proposed model. First, we solve the proposed MISOCP model by replacing the passenger demand and non-cruise time data with their expected values. We call it as Deterministic Approach. Then, we solve the proposed three stage stochastic MISCOP model. It is important to note that we use the same demand and non-cruise time data whose mean values are equal to the values used in the deterministic approach. In Table 3.3, we compare expected costs for the deterministic and three stage stochastic approach. We calculate the expected cost of the Deterministic Approach by imposing the first and second stage decisions upon the MISOCP model and optimizing the third stage decisions. In the first column of Deterministic Approach, we consider cruise time as a decision variable as we propose. To analyze the impact of cruise time controllability, in the second column, we take the cruise time as a fixed parameter. For each flight $i \in J$, we set the cruise time to f_{ik}^u , $k \in K$ which is the ideal duration when an aircraft type k flies at a speed with minimum fuel burn (MRC speed).

Considering the uncertainty of multiple demand and non-cruise time scenarios may lead to large decisions trees, potentially resulting in significant computation time. Despite the complexity of considering many scenarios, the three stage

Factor		Deterministic				Three Stage
Spill (\$/pass.)	c_{fuel} (\$/kg)	Speed Control		No Speed Control		Stochastic
		Cost	Incr.(%)	Cost	Incr.(%)	Cost
60	0.6	137,736	0.70	138,312	1.12	136,776
	1.2	180,588	0.65	180,957	0.86	179,417
200	0.6	140,268	2.55	140,844	2.97	136,776
	1.2	183,120	1.92	183,488	2.13	179,670
Avg:			1.46		1.77	

Table 3.3: Comparison of expected costs for the deterministic and three stage stochastic approach

stochastic approach leads to an average of 1.77% in cost saving over the deterministic approach. More importantly, cost savings increase for high spill cost setting, since the deterministic approaches do not consider the fluctuations in passenger demand during the planning process. The cost saving could reach to 2.97%, when the deterministic model does not utilize the advantage of cruise time controllability to compensate for the non-cruise time uncertainty.

In the second experiment, we extend the deterministic approach by putting 30 minutes buffer time into the flight durations. In the current practical solutions of airlines, instead of accelerating the aircraft, airlines may add an additional 30 minutes to announced flight duration to absorb the non-cruise time uncertainty. In this experiment, we want to see the difference between the deterministic approach with buffer times and our three stage stochastic approach with controllable cruise times. In Table 3.4, we compare the resulting schedules.

Factor		Deterministic with Buffer Time		Three Stage
Spill (\$/pass.)	c_{fuel} (\$/kg)	No Speed Control		Stochastic
		Cost	Incr.(%)	Cost
60	0.6	138,731	1.43	136,776
	1.2	181,375	1.09	179,417
200	0.6	141,262	3.28	136,776
	1.2	183,907	2.36	179,670
Avg:			2.04	

Table 3.4: Comparison of expected costs for the deterministic approach with buffer time and three stage stochastic approach

Although nonlinear function of fuel burn together with many scenarios make the three stage stochastic approach difficult to solve, it pays off since the average cost saving reaches to 2.04%. If the fuel price is low, the three stage stochastic

approach takes the advantage of cruise time controllability more, then the cost saving increases. It is important to mention, the current deterministic approaches put buffer times without considering the potential scenarios so that some buffer times may be unnecessary for some realizations of non-cruise times, which gives rise to an additional idle time cost. Therefore, it is more cost efficient to determine the amount of idle times and cruise time compressions simultaneously for each scenario as in the three stage stochastic approach. Furthermore, adding an additional 30 minutes of buffer time in flight durations adversely affects the aircraft utilization, and could reduce the number of flights that can be assigned to a single aircraft.

In the third experiment, we analyze the impact of incorporating third stage decisions and non-cruise time uncertainty. For this analysis, we compare our proposed model with a two stage stochastic model. The two stage model corresponds to the first two stages of our proposed model. i.e., two stage model considers first and second stage decisions and the demand uncertainty, but third level operational flight timing decisions as well as the non-cruise time uncertainty are not taken into account. We calculate the expected cost for the two stage approach by using optimal solution of two stage model as an input to the original three stage MISOCP model and then optimizing the third stage decisions for each scenario under two settings; one calculates the expected cost by considering cruise times as a third stage decision and the other takes the cruise times as fixed parameters.

Factor		Two Stage Stochastic				Three Stage
Spill (\$/pass.)	c_{fuel} (\$/kg)	Speed Control Cost	Incr.(%)	No Speed Control Cost	Incr.(%)	Stochastic Cost
60	0.6	139,779	2.20	140,618	2.81	136,776
	1.2	184,599	2.89	185,172	3.21	179,417
200	0.6	139,779	2.20	140,618	2.81	136,776
	1.2	184,599	2.74	185,172	3.06	179,670
Avg			2.51		2.97	

Table 3.5: Comparison of expected costs for two stage and three stage stochastic approach

Table 3.5 shows that by incorporating the third stage, the three stage approach provides an average of 2.97% in cost saving over the two stage stochastic approach. Even if the fuel price is high, cost saving could reach to 3.21%. Since

the existing two stage stochastic approaches for solving the fleet assignment problem do not take the cruise time decisions and non-cruise time uncertainty into consideration, they fail to adjust the fleet decisions to reflect the increase in fuel price. However, fuel expenses account for around 30% of the total operating cost. In 2008, the share for fuel costs reached 50% due to the sharp increase in the fuel prices [102]. Thus, 3.21% cost saving is significantly important for airlines since this may translate into billions of dollars.

3.4.2 Analysis on the Lower Bounds

In this section, we use a sample schedule generated from the study [26]. The schedule has 15 flights operated by 4 different aircraft. We use the same scenario set as in the numerical example in Section 3.4.1. In this experiment, we compare the lower bounds achieved by solving group subproblems with the lower bounds achieved by the cutting plane (CP) algorithm and CPLEX solver. To be fair in comparison, we set the time limit of CP algorithm and CPLEX solver as much as the time consumed to solve the group subproblems. In Table 3.6, we report the relative gaps between the lower bound achieved by each approach and the optimal value of the problem. As it is stated, the quality of the lower bound clearly depends on the choice of the group subproblems. For simple case, we start with combining two scenarios in each group as provided in Procedure P1. For each group subproblem, two scenarios are selected in a way that they do not have common demand or non-cruise time realizations. This diversity gives a better representation of overall MISOCP model, thus has a potential to improve the lower bound compared to any scenario selection procedure with common demand or non-cruise time realizations. Then, to increase the variety of scenarios, we implement Procedure P2 in which we extend the group size to four by randomly combining two scenario groups of Procedure P1. Solving the group subproblems larger in size brings an additional complexity, which in turn leads to higher CPU times. Despite the longer CPU times, increasing the number of scenarios in each group leads to smaller optimality gaps, which can be attributed entirely to tighter lower bounds.

Factor		LB Algorithm				Cutting Plane Algorithm				CPLEX	
Spill (\$/pass.)	c_{fuel} (\$/kg)	Procedure P1		Procedure P2		Group Cut		Multi Cut		Solver	
		Gap (%)	Cpu (sec)	Gap (%)	Cpu (sec)	Gap (%)	Cpu (sec)	Gap (%)	Cpu (sec)	Gap (%)	Cpu (sec)
60	0.6	0.27	337	0.15	3170	3.04	3170	0.75	3170	2.91	3170
	1.2	0.49	396	0.35	6695	3.05	6695	1.37	6695	3.92	6695
200	0.6	0.50	109	0.42	226	3.83	226	2.85	226	2.06	226
	1.2	0.45	143	0.36	390	4.24	390	2.01	390	1.77	390
Avg:		0.43		0.32		3.54		1.74		2.66	

Table 3.6: Comparison of strength of lower bounds

In Table 3.6, we also observe that the optimality gap between the optimal value and the lower bound at termination is higher for both CP algorithm and CPLEX solver. For group subproblem approach, the optimality gap is less than 0.5% for all instances. On the other hand, group cut version of CP algorithm yields an average of 3.5% optimality gap and the optimality gap for CPLEX solver reaches to 4% at termination when fuel price is high. Moreover, CPLEX solver cannot provide an incumbent (feasible) solution for some instances within the time limit even though this experiment is designed with small instances. This experiment clearly demonstrates the effectiveness of the group subproblem approach to obtain a lower bound for the proposed MISOCP model.

We take ten replications to observe if randomly combining scenario groups in Procedure P2 has any impact on the lower bound values. For each replication, we conduct a 2^2 full-factorial experimental design. In Table 3.7, for each replication, we report the minimum, average and maximum relative gaps with the lower bound and the optimal value. We could conclude that there is no statistically significant randomization effect on the lower bound values.

To see the effect of each factor on the computation time of the LB algorithm with Procedure P2, we report minimum, average and maximum values of CPU's in seconds over five replications in Table 3.8. Increase in fuel price makes the fleet assignment decisions more crucial. As expected, CPU time to deal with the nonlinear fuel and emission cost functions increases. If the base spill cost increases, then the solution has a tendency to closely match the seat capacity of aircraft and potential passenger demands with a less consideration of the fuel

Replications	Optimality Gap(%)		
	Min	Avg	Max
1	0.16	0.33	0.40
2	0.14	0.31	0.38
3	0.19	0.34	0.42
4	0.14	0.30	0.40
5	0.19	0.33	0.41
6	0.15	0.32	0.42
7	0.15	0.32	0.42
8	0.15	0.31	0.37
9	0.15	0.32	0.40
10	0.14	0.31	0.40

Table 3.7: Comparison for different replications

efficiency of the aircraft. Thus, solution time decreases.

Factor	Level	CPU Time (sec)		
		Min	Avg	Max
c_{fuel} (\$/kg)	0.6	102	1384	7886
	1.2	117	3111	8585
Spill (\$/passenger)	60	291	4276	8585
	200	102	219	694

Table 3.8: CPU time analysis of the LB algorithm

3.4.3 Analysis on a Schedule with 48 Flights

In this section, we select the first 48 flights of the sample schedule provided in [51]. There are 13 aircraft with 5 different types. We consider the same scenario set as in the previous section. The corresponding three stage stochastic MISOCP formulation has 188,080 rows, 258,839 columns and 924,860 non-zeroes after pre-solve is completed. This is a large problem even with 32,584 binary decisions and 70,880 quadratic constraints.

In this experiment, we compare the scenario group-wise decomposition algorithm in Section 3.2.2 with cutting plane algorithm in Section 3.3. We implement different improvements suggested in Section 3.3 within the cutting plane algorithm. We test the impact of each valid inequality derived in Section 3.1.3 separately. We summarize the proposed algorithms below:

Scenario group-wise decomposition algorithm (SGWD): We implement the scenario group-wise decomposition algorithm (Algorithm 2). First, Algorithm 1 is implemented to obtain a lower bound by solving group subproblems. We apply the scenario grouping Procedure P2. We solve each group subproblems using our cutting plane algorithm (Algorithm 3). Instead of solving each group subproblem to optimality, we set a time limit to 10 minutes and consider the lower bounds at the termination for each group subproblem. To obtain an upper bound, we evaluate randomly selected two candidate first stage decisions $\hat{x}_n^1, n \in N$ within the Algorithm 2. For each given $\hat{x}_n^1$, we solve reduced group subproblems obtained from the reduced model *two stage* $SP(\hat{x}_n^1, \mu)$ using the cutting plane algorithm (Algorithm 3) within four minutes of time limit. Then, single scenario $detpr(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu), \eta)$ models are solved to optimality using CPLEX solver with an option to solve a quadratically constrained mixed integer program. In the first column of Table 3.9, we report the relative gap between the objective function value of the best incumbent solution and the lower bound derived within the scenario group-wise decomposition along with CPU hours.

Cutting plane algorithm (CP): We solve three stage MISOCP model by implementing cutting plane algorithm (Algorithm 3). We use the optimality cuts derived in (3.91) by solving the dual subproblem $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$. We test both scenario group cut and multi-cut strategies. Scenarios are grouped using Procedure P1. For consistency, we set the time limit of the cutting plane algorithm to the time consumed by scenario group-wise decomposition algorithm. In the second column of Table 3.9, we report the relative gaps between the lower and upper bounds provided by the cutting plane algorithm.

Cutting plane algorithm with the Magnanti-Wong enhancement (CP-EMW): In this version of cutting plane algorithm (Algorithm 3), as suggested by Papadakos [96], we solve an enhanced Magnanti-Wong auxiliary problem which is obtained by eliminating constraint (3.93) from the Magnanti-Wong problem. At each iteration, we add two types of optimality cuts (3.91), one is derived by solving the original dual subproblem $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ and the other is derived by solving the enhanced Magnanti-Wong problem. We update the points

$(x0^t, m0^t(\omega), z0^t(\omega), b0^t(\omega))$ in enhanced Magnanti-Wong problem as in equation (3.94) to approximate core points. We use multi-cut strategy.

Cutting plane algorithm with preemptive priority multiple objective dual subproblem (CP-SL): In this version of cutting plane algorithm, we solve the weighted-sum problem $SL(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ instead of solving $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$. We give the first priority to the objective of $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ (3.68) and consider the objective of Magnanti-Wong problem (3.92) with a weight ξ . We take $\xi = 10^{-10}$. We update the points $(x0^t, m0^t(\omega), z0^t(\omega), b0^t(\omega))$ as in equation (3.94) to approximate core points. We use multi-cut strategy.

Factor			SGWD	CP		CP-EMW		CP-SL		CPU(hr)
Spill	C_{fuel}	Valid		Group	Multi	Group	Multi	Group	Multi	
(\$/pass.)	(\$/kg)	Ineq.	Gap(%)	Cut	Cut	Cut	Cut	Cut	Cut	
			Gap(%)	Gap(%)	Gap(%)	Gap(%)	Gap(%)	Gap(%)	Gap(%)	
60	0.6	No	5.23	5.81	6.55	7.07	6.09	6.47	6.09	6.4
		(3.27)	5.30	6.77	5.92	6.71	6.32	6.73	6.06	6.5
		(3.28)	5.32	6.55	6.31	6.94	6.40	6.36	6.30	6.4
		(3.29)	5.10	6.52	6.72	6.61	6.64	6.01	6.45	6.5
	1.2	No	5.86	7.30	7.20	6.87	7.56	7.41	7.42	6.4
		(3.27)	5.72	7.31	7.42	8.01	7.48	7.47	6.92	6.4
		(3.28)	5.84	7.34	7.30	7.60	7.43	7.22	7.56	6.4
		(3.29)	5.73	7.84	7.22	7.31	7.35	7.24	6.66	6.4
	200	No	5.14	6.45	6.70	6.62	6.76	6.06	6.25	6.4
		(3.27)	5.15	6.87	7.25	6.84	6.51	6.33	6.53	6.4
		(3.28)	5.26	6.40	6.66	6.89	6.91	6.23	6.21	6.4
		(3.29)	5.31	6.61	6.31	7.18	7.41	6.58	6.37	6.5
	1.2	No	5.98	7.21	7.58	7.31	7.47	7.11	6.84	6.4
		(3.27)	5.92	7.13	6.85	7.03	7.80	7.87	7.13	6.4
		(3.28)	5.97	7.70	7.61	7.13	7.34	7.00	6.92	6.4
		(3.29)	5.95	7.33	7.45	7.35	7.68	7.21	6.82	6.4
Avg:			5.55	6.95	6.94	7.09	7.07	6.83	6.66	6.4

Table 3.9: Comparison on relative gaps

The results in Table 3.9 indicate that our scenario group-wise decomposition algorithm performs the best over all versions of the cutting plane algorithm for each problem instance. While the scenario group-wise decomposition algorithm provides an average of 5.55% optimality gap, cutting plane algorithm yields an optimality gap around 6.94% on the average. It is also clear that the performance of the cutting plane algorithm could be improved by the optimality cuts provided

in CP-SL. Furthermore, the results show that including either valid inequality (3.27) or (3.29) decreases the optimality gap for many instances.

3.4.4 Analysis on a Schedule with 102 Flights

For this computational study, we utilize the sample schedule provided in [51]. We extract the flight information of the first 102 flights of the schedule. These flights are operated by 26 aircraft with five different types. We again perform 2^2 full-factorial experimental design.

In this section, we use the scenario group-wise decomposition algorithm in Section 3.2.2 to solve the instances. We obtain a lower bound by solving group subproblems. We employ scenario grouping Procedure P2 so that each group subproblem has four scenarios. We set the time limit for solving each group subproblem to 15 minutes using the cutting plane algorithm (Algorithm 3). To obtain an upper bound, we evaluate randomly selected two candidate first stage decisions $\hat{x}_n^1, n \in N$ within the Algorithm 2. For each given $\hat{x}_n^1$, we solve reduced group subproblems obtained from the reduced model *two stage* $SP(\hat{x}_n^1, \mu)$ within a time limit of six minutes using the cutting plane algorithm (Algorithm 3). Then, we solve the single scenario models $detpr(\hat{z}_r^2(\hat{x}_n^1, \mu), \hat{y}_r^2(\hat{x}_n^1, \mu), \hat{u}_r^2(\hat{x}_n^1, \mu), \eta)$ to optimality using CPLEX solver. Within the scenario group-wise decomposition algorithm, we employ different versions of cutting plane algorithm for solving group subproblems and reduced group subproblems obtained from the reduced models *two stage* $SP(\hat{x}_n^1, \mu)$. The details are provided below.

Scenario group-wise decomposition algorithm with simple cutting plane (CP): In this version, we use the optimality cut (3.91) derived from solving the original dual subproblem $DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ within the cutting plane algorithm (Algorithm 3).

Scenario group-wise decomposition algorithm with CP-SL: This version differs from the previous one by solving the weighted-sum problem $SL(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ instead of solving the original dual subproblem

$DP(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$ within the cutting plane algorithm (Algorithm 3). We derive the optimality cut (3.91) associated with the optimal solutions of $SL(x^1, m^2(\omega), z^2(\omega), b^3(\omega))$. We take weight vector as $(1, \xi)$, where $\xi = 10^{-10}$.

In both version, we use the multi cut strategy for the optimality cuts. In Table 3.10, we report the relative gaps between the lower and upper bounds provided by the scenario group-wise decomposition algorithm. The results indicate that on the average, *SGWD* with *CP-SL* can improve the solutions by slightly increasing the CPU times.

Factor			Scenario Group-Wise Decomposition Alg. with			
Spill (\$/pass.)	C_{fuel} (\$/kg)	Valid Ineq.	CP		CP-SL	
			Gap(%)	CPU(hr)	Gap(%)	CPU(hr)
60	0.6	No	6.58	17.7	6.67	18.2
		(3.27)	6.68	18.9	6.68	20.7
		(3.28)	6.75	18.9	6.54	19.2
		(3.29)	6.59	17.2	6.69	19.3
	1.2	No	7.72	17.8	7.66	18.5
		(3.27)	7.57	19.3	7.65	18.4
		(3.28)	7.77	19.2	7.71	18.9
		(3.29)	7.87	19.4	7.70	19.1
	200	No	6.71	20.2	6.66	19.2
		(3.27)	6.52	18.8	6.56	19.3
		(3.28)	6.67	19.3	6.55	20.5
		(3.29)	6.97	18.8	6.70	18.8
	1.2	No	7.66	21.2	7.67	20.3
		(3.27)	7.69	19.7	7.78	18.7
		(3.28)	7.86	20.3	7.69	20.1
		(3.29)	7.59	19.1	7.89	20.1
	Avg:		7.20	19.1	7.17	19.3

Table 3.10: Comparison on relative gaps

3.5 Conclusion

We propose a novel three stage stochastic programming model which determines strategic departure time decisions, tactical fleeting and routing decisions and the more operational flight timing decisions while examining potential demand and non-cruise time scenarios. The model utilizes the great advantage of cruise time

controllability to compensate for non-cruise time uncertainty. To achieve a minimum cost schedule, the model assigns fleet types through consideration of not only passenger demand, as commonly done in the literature, but also operational expenses such as fuel burn and carbon emission costs. Thus, the major contribution of this work is to incorporate operational flight timing decisions and different levels of uncertainty into fleet assignment and aircraft routing decisions. In turn, as indicated by our computational results, the resulting three stage stochastic model yields substantial savings compared to the deterministic and two stage stochastic programming approaches.

In this study, we propose different techniques to handle the additional complexity of the third stage decisions in airline scheduling. We provide valid inequalities to strengthen the formulation. We suggest a scenario group-wise decomposition algorithm to handle the large number of scenarios. The algorithm separately solves group subproblems similar to the original three stage model, but are defined over a reduced number of scenarios. Even the group subproblems can be very time-consuming to solve because of large number of decisions and constraints. Therefore, we implement cutting plane algorithm to efficiently solve each group subproblem. We handle the nonlinear subproblems using the second order cone duality. It is finally worth mentioning that the proposed method can be generalized to solve three stage stochastic programming models, which include pure binary decisions in their second stages.

There are some future research directions arising from this study. We have demonstrated the importance of scenario selection procedures, but it is still open to evaluate the impact of various selection procedures. Moreover, we currently provide pareto optimality and scenario group cuts to improve the performance of cutting plane algorithm. Analysing the effect of different cut strategies is of another future interest. Finally, given a novel three stage stochastic programming approach to airline scheduling problem, one can take advantages of parallel computing technology within the framework of the scenario group-wise decomposition algorithm so that an increased number of scenarios could be explored.

Chapter 4

Accommodating New Flights into an Existing Airline Flight Schedule

In this chapter, we present two novel approaches to alter a flight network for introducing new flights while maximizing airline's profit. A key feature of the first approach is to adjust the aircraft cruise speed to compensate for the block times of the new flights, trading off flying time and fuel burn. In the second approach, we introduce aircraft swapping as an additional mechanism to provide a greater flexibility in reducing the incremental fuel cost and adjusting the capacity. The nonlinear fuel-burn function and the binary aircraft swap and assignment decisions complicate the optimization problem significantly. We propose strong mixed-integer conic quadratic formulations to overcome the computational difficulties. The reformulations enable solving instances with 300 flights from a major U.S. airline optimally within reasonable compute times.

The organization of the chapter is as follows: In Section 4.1, we briefly describe the framework of the problem and then present a numerical example illustrating the benefits of cruise time controllability and the proposed aircraft swapping mechanism. Section 4.2 introduces the mixed-integer nonlinear programming

formulations for the two proposed approaches. In Section 4.3 we present stronger reformulations of the models to improve their solvability. We computationally test the proposed mathematical models using a real-world data of a major U.S. airline in Section 4.4 and conclude with a few final remarks in Section 4.5.

The results of this chapter are published in Transportation Research Part C: Emerging Technologies [103].

4.1 Problem Definition

In this section, we briefly describe the problem setting. Consider a set of new flight pairs (i.e., consecutive flights, specifically a flight from the hub to a new demand point and its return flight to hub) to be accommodated into the existing flight schedule in near real time. An airline needs to accommodate new flights into an existing flight schedule without excessively disrupting the existing schedule. Therefore, we only shift the departure times of existing flights within the intervals already determined by the airline. Moreover, any arrival tardiness of the existing flights due to inserting new flights is penalized in the objective. In hub-and-spoke networks, connecting passengers represent a non-negligible percentage of the total number of passengers. Therefore, we also respect the connection of passengers at the hub airport while optimizing the departure and cruise times of flights. There are two cases for a new flight: (1) If the new flight is assigned to a larger aircraft, then this assignment may capture more passengers, providing a greater revenue, but compressing the cruise times of flights to accommodate new flights may increase the fuel cost significantly; (2) if the new flight is assigned to a smaller fuel efficient aircraft to reduce the fuel burn, then there may be an additional cost of spilled passengers with a decrease in revenue. To increase the profit for an airline, we introduce a second model which additionally includes aircraft swapping decisions.

In the following sections, we first define the fuel burn as a function of cruise time and the penalty function for arrival tardiness of flights, and then provide

a numerical example to show how to utilize the cruise time controllability and aircraft swapping to open-up enough space for the block times of new flights.

4.1.1 Fuel and CO₂ Emission Cost Function

One of the main contributions of this study is to increase the aircraft cruise speed to compensate for the time required to operate new flights. However, we need to consider the adverse effect of increasing cruise speed on fuel and carbon emissions costs. To estimate the fuel burn, we use the cruise stage fuel flow model developed by the Base of Aircraft Data (BADA) project of EUROCONTROL [85]. This model has been widely used in the literature. The fuel burn (kg) of a flight as a function of its cruise time f (min) and its aircraft type t can be calculated as follows:

$$F^t(f) = \alpha^t \frac{1}{f} + \beta^t \frac{1}{f^2} + \gamma^t f^3 + \nu^t f^2.$$

The coefficients $\alpha^t, \beta^t, \gamma^t, \nu^t > 0$ are expressed in terms of aircraft specific fuel consumption coefficients as well as the mass of the aircraft, air density and gravitational acceleration as provided in Şafak et al.[51]. It is important to note that F^t is a convex function whenever $f > 0$. The minimizer of the fuel consumption function F^t is represented by u^t , which is the ideal cruise time when an aircraft flies at the most fuel-efficient speed, referred to Maximum Range Cruise (MRC) speed. Although the fuel burn is minimized at MRC speed, airlines may set higher cruise speed due to the scheduling constraints.

EUROCONTROL [86] states that each kg of fuel burn approximately produces 3.15 kg of CO₂ emission. Therefore, we can express fuel and CO₂ emission costs as a function of cruise time as follows:

$$c^t(f) = c_o F^t(f) \tag{4.1}$$

where c_o is the total cost of fuel and CO₂ emitted by an aircraft per kg of fuel burned. Note that $c_o = c_{fuel} + 3.15c_{CO_2}$, where c_{fuel} is the fuel price per kilogram (kg) of fuel burned by an aircraft and c_{CO_2} is the cost of emission per kg of CO₂ emitted by an aircraft.

4.1.2 Penalty Function for Arrival Tardiness

In this paper, we aim to accommodate new flights with minimal disruptions on the existing flight schedule. In this quest, we penalize the deviation from the original arrival times for the existing flights. Hoffman and Ball [104] suggest to use a nonlinear delay cost function to better reflect the reality since flight delay costs tend to grow with time at a greater rate than linear rate. Moreover, EUROCONTROL [101] performs a detailed investigation of airline cost functions and reports that the power curve provides a good fit to passenger costs as a function of delay duration. Therefore, we penalize the arrival tardiness with a convex increasing function of tardiness b as

$$P(b) = \rho b^\zeta \quad (4.2)$$

where $\rho \geq 0$. As in Aktürk et al. [26], we let $\zeta = 1.5$ in our computational experiments. It is important to note that the US costs might differ from the ones provided in the EUROCONTROL [101].

Arrival tardiness can be reduced by compressing the cruise time of flights with an additional fuel burn cost. Therefore, the assignment of fuel efficient aircraft to new flights becomes more critical in order to reduce both delay and fuel burn costs.

4.1.3 Numerical Example

In this section, first we will provide a numerical example to show how the cruise time controllability can be utilized to accommodate new flights into an existing airline flight schedule. Then, we will extend the example to show how aircraft swapping together with the cruise speed control can be used to achieve a more profitable schedule.

We give a sample schedule for two aircraft in Table 4.1. Tail numbers of the aircraft and the flight numbers along with the origin and destination airports, planned departure times in local ORD time, planned block times and demand for

the flights are listed in the table. Each aircraft visits ORD at least once in a day. Let us now introduce new round-trip flights from ORD to MSP and back.

Table 4.1: Original schedule.

	Tail #	Flight #	From	To	Plan. Dep.	Plan. Dur.	Plan. Arr.	Demand
Existing flights	N53442	1586	ORD	MCO	08:00	03:04	11:04	200
		633	MCO	ORD	12:00	03:07	15:07	180
		451	ORD	IAH	18:10	03:08	21:18	190
		584	IAH	ORD	22:30	02:50	01:20	186
Existing flights	N45425	527	ORD	IAH	08:45	03:02	11:47	151
		521	IAH	ORD	12:32	03:03	15:35	154
		623	ORD	MCO	17:00	03:02	20:02	160
		679	MCO	ORD	21:10	03:10	00:20	163
New flights		1842	ORD	MSP	13:15	01:40	14:55	183
		430	MSP	ORD	16:10	01:45	17:55	168

Figure 4.1 gives the time-space network representation of the original schedule. The red and blue arcs in Figure 4.1 represent routes for aircraft N53442 and N45425 respectively. The flight arcs originate from the departure airport at the planned departure time and end at the destination airport after the planned block time. Ground arcs represent the aircraft turnaround times needed to prepare the aircraft for the next flight.

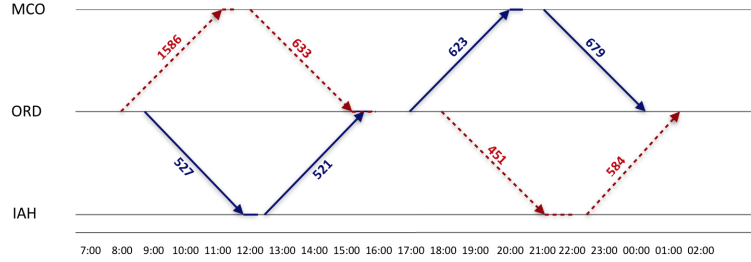


Figure 4.1: Original time-space network.

In this example, we assume that aircraft N53442 is a Boeing 767-300 and N45425 is an Airbus 320-212. The number of seats of the aircraft are 218 and 180, respectively. In the original schedule we assume that the aircraft fly at the most fuel efficient speed (MRC speed) and estimate the fuel burn rates as 87 kg/min and 40 kg/min for aircraft N53442 and N45425, respectively. The fuel burn rate is calculated using the fuel flow model of BADA as mentioned above. We assume that for each flight, non-cruise stages take 30 min. Then, cruise stages take 30 min less than flight block times given in Table 4.1. Assuming $c_{fuel} = 1.2$

\$/kg and $c_{CO_2} = 0.2$ \$/kg, the total cruise stage fuel and carbon cost for the original schedule is \$100,593.

Let us assume that an airline wants to operate two new flights 1842 and 430. In order to open up sufficient time to accommodate these new flights, one approach is to compress the cruise times and adjust the departure times simultaneously. We refer to this approach with cruise time controllability as CTC. If the airline wants to meet the passenger demand of 183 for the new flight 1842, the only way to do so is to assign the new flights to aircraft N53442. These new flights can be placed between flights 633 and 451. The necessary block times for the new flights can be made available by left-shifting flights 1586 and 633 and right-shifting flights 451 and 584. However, we assume that the airline wishes to keep such a new schedule close to the original one. Thus, we only allow departure times to deviate at most 90 min from the planned departure times in the original schedule. In addition, if a flight arrives 15 min later than the original planned arrival times, we penalize the tardiness with a nonlinear function in (4.2). We let $\rho = 5$ and $\zeta = 1.5$ in the penalty function. Because of these scheduling limitations, the new flights cannot be accommodated by only shifting the flight departure times. We also need to compress the cruise times of the existing flights 633, 451 and 584 by 17, 17 and 3 min, respectively. Besides, the cruise times of new flights 1842 and 430 are compressed by 8 min to satisfy the scheduling restrictions. However, flight 451 arrives 37 min later than the original planned arrival time, thus 22 min of this tardiness are penalized by \$552. In Figure 4.2, we give the time-space network representation of resulting schedule. In this figure, there might be two arcs originating from the same point. The shorter arc represents the flights with compressed cruise times.

Compressing the cruise times of the flights incurs additional costs of fuel burn and CO₂ emission. The total fuel burn and CO₂ emission costs of the existing flights increases to \$101,286. Moreover, for new flights, the total fuel burn and CO₂ emission cost is \$16,044. Therefore, the fuel and emission cost increment compared to the original schedule is \$16,737 calculated as $\$16,737 = \$101,286 + \$16,044 - \$100,593$. To reduce the fuel burn by reassigning the aircraft, we also

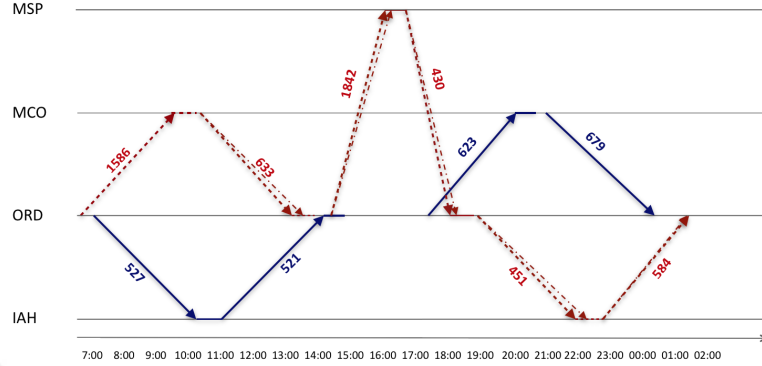


Figure 4.2: Time-space network with CTC.

propose an aircraft swapping mechanism together with the cruise time controllability, referred to as CTC-AS.

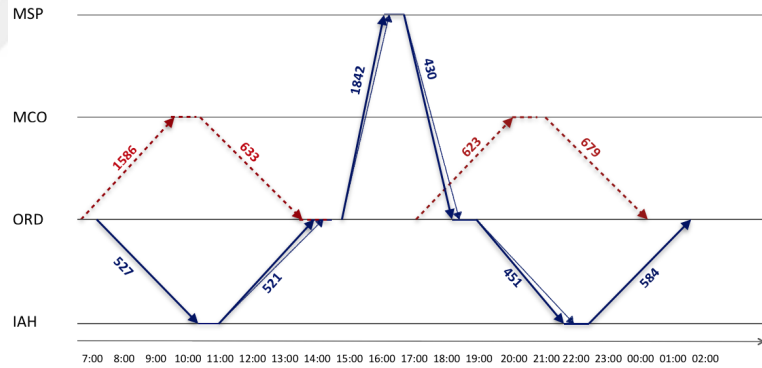


Figure 4.3: Time-space network with CTC-AS.

In the CTC-AS approach, we swap the aircraft of flights 451 and 623 before departure. In the new schedule, aircraft N53442 operates flights 1586, 633, 623, and 679, whereas aircraft N45425 operates 527, 521, 451, and 584. We provide the time-space network representation of the resulting schedule in Figure 4.3. To reduce the fuel expenses, the new flights are assigned to the fuel efficient aircraft N45425. However 10, 6 and 3 passengers of flights 451, 584 and 1842, respectively are spilled due to the low capacity of the aircraft N45425, thus resulting in a cost of \$2007. Similarly, passengers' revenue obtained from ticket sales of new flights are reduced to \$49,548 from \$49,923, since 3 passengers of new flights are not served due to low seat capacity. An additional \$500 cost of swapping is incurred in the new schedule. On the other hand, the savings from the fuel burn and CO₂

emission costs may compensate for these additional cost of spilled passengers, revenue loss and cost of swapping. Indeed, the fuel expenses and CO₂ cost of the new flights are reduced to \$8118, almost half of the fuel expenses of the CTC approach. Even though the total fuel burn and CO₂ emission cost of existing flights slightly increases to \$101,590, the fuel and emission cost increment significantly decreases from \$16,737 to \$9115. Penalization cost for arrival tardiness of flight 451 also decreases to \$115. Therefore, resulting schedule improves the airline's profit from \$30,234 to \$35,412.

We give the operational cost components and revenues of two schedules achieved by CTC and CTC-AS approaches in Table 4.2. Airline's profit is calculated as follows:

$$\begin{aligned} \text{Profit} = & \text{Revenue} - (\text{Fuel \& Emiss. Cost Increment}) - \text{Spilled Cost} \\ & - \text{Penalty Cost} - \text{Swap Cost} - (\text{Crew \& Service Costs}). \end{aligned}$$

Table 4.2: Cost comparison.

	Fuel & Emiss. Cost Increment (\$)	Spilled Pass. Cost (\$)	Deviation Penalty (\$)	Swap Cost (\$)
CTC	16,044	0	552.0	0
CTC-AS	8,118	2,007	115.0	500
	Crew & Service Cost (\$)	Passengers Revenue (\$)	Profit (\$)	
CTC	4,400	49,923	30,234	
CTC-AS	4,400	49,548	35,412	

4.2 Mathematical Formulations

In this section, we present the mathematical formulations of the two approaches described in the previous section. We start with the simpler CTC model that adjusts the departure times and controls the cruise time, and then extend it to CTC-AS by incorporating aircraft swapping as well.

4.2.1 Formulation with Cruise Time Controllability

We first give a list of sets, parameters and decision variables used in the model.

Sets:

E	set of existing flights in the schedule
E_O	set of existing outbound flights from the hub
E_I	set of existing inbound flights arriving to the hub
N	set of new flights
N_O	set of new outbound flights from the hub to a new demand point
N_I	set of new inbound flights from a new demand point to the hub
T	set of aircraft types
C_E	set of pairs of existing consecutive flights of the same aircraft, (i, j) , $i \in E, j \in E$
C_N	set of pairs of new consecutive flights of the same aircraft, (i, j) , $i \in N_O, j \in N_I$
U_i	set of flights that can follow flight i , $i \in E \cup N_I$
G_i	set of existing outbound flights which have passenger connections from flight $i \in E_I$

Parameters:

χ_i^t	1 if aircraft type t is originally assigned to existing flight i , and 0 o.w., $i \in E, t \in T$,
$[\ell_i^t, u_i^t]$	time window for the cruise time of flight i with aircraft type t , $i \in E \cup N, t \in T$
$[d_i^\ell, d_i^u]$	time window for the departure of flight i , $i \in E \cup N$
η_i	non-cruise time of flight i , $i \in E \cup N$
τ_i^t	turnaround time needed to prepare aircraft type t for the connection after flight i , $i \in E \cup N, t \in T$
λ_{ij}	time needed for the connection of passengers from flight i to flight j , $i \in E_I, j \in G_i$

κ^t	number of seats of an aircraft type t , $t \in T$
μ_i	number of passenger demand of each new flight i , $i \in N$
$t(i)$	aircraft type assignment of existing flight i , $i \in E$
π_i	ticket price of new flight i , $i \in N$
σ_i	cost of spilled passengers of new flight i , $i \in N$
ϕ	crew and serving cost for new flights
a_i^o	original arrival time of flight i , $i \in E$

Decision variables:

f_i^t	cruise time of flight i with aircraft type t , $i \in E \cup N, t \in T$
d_i	departure times of flight i , $i \in E \cup N$
a_i	arrival time of flight i to its destination, $i \in E \cup N$
b_i	deviation from the original arrival time of flight i , $i \in E$
z_i^t	1 if aircraft type t is assigned to flight i , and 0 o.w., $i \in N, t \in T$
y_{ij}	1 if flight i is followed by flight j , and 0 o.w., $i \in E \cup N_I, j \in U_i$

In the new schedule, an existing flight $i \in E$ can be followed by a new outbound flight $j \in N_O$. Similarly, each new inbound flight $i \in N_I$ can be followed by an existing flight $j \in E$.

For each $i \in E \cup N, t \in T$, we redefine the fuel and CO₂ emission cost function as

$$c_i^t(f_i^t) = \begin{cases} c_o \left(\alpha_i^t \frac{1}{f_i^t} + \beta_i^t \frac{1}{(f_i^t)^2} + \gamma_i^t (f_i^t)^3 + \nu_i^t (f_i^t)^2 \right) & \text{if } z_i^t = 1 \\ 0 & \text{if } z_i^t = 0, \end{cases}$$

so that if aircraft type t is not assigned to flight i , then $c_i^t(f_i^t) = 0$.

Using the notation above, we now provide a mathematical model of the problem (CTC):

$$\begin{aligned} \max \quad & \sum_{t \in T} \sum_{i \in N} \pi_i (\min(\mu_i, \kappa^t)) z_i^t - \sum_{i \in E} \left(c_i^{t(i)}(f_i^{t(i)}) - c_i^{t(i)}(u_i^{t(i)}) \right) \\ & - \sum_{t \in T} \sum_{i \in N} c_i^t(f_i^t) - \sum_{t \in T} \sum_{i \in N} \sigma_i \max(0, \mu_i - \kappa^t) z_i^t - \sum_{i \in E} \rho b_i^\zeta - \phi \end{aligned}$$

s.t.

$$\sum_{i \in E_I} y_{in} \leq 1, \quad n \in N_O \quad (4.3)$$

$$\sum_{i \in E_O} y_{ni} \leq 1, \quad n \in N_I \quad (4.4)$$

$$\sum_{i \in E_I} y_{in} + \sum_{i \in E_O} y_{mi} \geq 1, \quad (n, m) \in C_N \quad (4.5)$$

$$\sum_{n \in N_O} y_{in} \leq 1, \quad i \in E_I \quad (4.6)$$

$$\sum_{m \in N_I} y_{mi} \leq 1, \quad i \in E_O \quad (4.7)$$

$$y_{in} = y_{mj}, \quad (i, j) \in C_E, (n, m) \in C_N \quad (4.8)$$

$$|z_n^t - \chi_i^t| \leq (1 - y_{in}), \quad i \in E_I, n \in N_O, t \in T \quad (4.9)$$

$$|z_n^t - \chi_i^t| \leq (1 - y_{ni}), \quad i \in E_O, n \in N_I, t \in T \quad (4.10)$$

$$z_n^t = z_m^t, \quad (n, m) \in C_N, t \in T \quad (4.11)$$

$$\sum_{t \in T} z_n^t = 1, \quad n \in N_O \quad (4.12)$$

$$d_i + f_i^{t(i)} + \eta_i = a_i, \quad i \in E \quad (4.13)$$

$$d_n + \sum_{t \in T} f_n^t + \eta_n = a_n, \quad n \in N \quad (4.14)$$

$$a_i + \lambda_{ij} \leq d_j, \quad i \in E_I, j \in G_i \quad (4.15)$$

$$a_n + \sum_{t \in T} \tau_n^t z_n^t \leq d_m, \quad (n, m) \in C_N \quad (4.16)$$

$$\text{If } y_{in} = 1 \text{ then } a_i + \tau_i^{t(i)} \leq d_n, \quad i \in E_I, n \in N_O \quad (4.17)$$

$$\text{If } y_{ni} = 1, \text{ then } a_n + \tau_n^{t(i)} \leq d_i, \quad i \in E_O, n \in N_I \quad (4.18)$$

$$\text{If } \sum_{n \in N_O} y_{in} = 0, \text{ then } a_i + \tau_i^{t(i)} \leq d_j, \quad (i, j) \in C_E \quad (4.19)$$

$$a_i - (a_i^0 + 15) \leq b_i, \quad i \in E \quad (4.20)$$

$$\rho_i^{t(i)} \leq f_i^{t(i)} \leq u_i^{t(i)}, \quad i \in E \quad (4.21)$$

$$\ell_i^t z_i^t \leq f_i^t \leq u_i^t z_i^t, \quad i \in N, t \in T \quad (4.22)$$

$$d_i^\ell \leq d_i \leq d_i^u, \quad i \in E \cup N \quad (4.23)$$

$$z_i^t \in \{0, 1\}, \quad i \in N, t \in T \quad (4.24)$$

$$y_{ij} \in \{0, 1\}, \quad i \in E \cup N_I, j \in U_i \quad (4.25)$$

$$b_i \geq 0, \quad i \in E. \quad (4.26)$$

For given aircraft routes, we aim to generate a new flight schedule to introduce new flights with the goal of maximizing airline's profit. The first term of the objective is the revenue from ticket sales. The number of tickets sold can be determined as the minimum of passenger demand and seat capacity of the assigned aircraft. The remaining terms in the objective function represent the operational costs. The second term is the incremental cost of fuel burn associated with speeding up the aircraft of existing flights. Similarly, the third term represents the total cost of fuel burn and carbon emissions for new flights. The fourth term is the cost of spilled passengers due to insufficient seat capacity of assigned aircraft to new flights. The fifth term is a penalty for arrival tardiness of the existing flights. Finally, the sixth term is the total costs of crew and service for new flights including landing fees, ground handling fees, insurance fees, onboard services costs, etc.

Constraint (4.3) ensures that new outbound flight n follows at most one existing flight i arriving to the hub airport. Similarly, constraint (4.4) guarantees that new inbound flight n is followed at most one existing flight i departing from the hub airport. Constraint (4.5) assures that new flight pair (n, m) is covered by an aircraft route. Constraints (4.6)–(4.7) ensure that an existing flight does not follow or is immediately followed by two different new flights. Constraint (4.8) keeps the sequence of existing flights as in the original schedule. If a new flight pair (n, m) is operated between an existing flight pair (i, j) , then the model ensures that $y_{in} = y_{mj} = 1$. Otherwise, $y_{in} = y_{mj} = 0$ for $(i, j) \in C_E$.

Constraints (4.9)–(4.12), determine the aircraft type assignment to a new flight pair (n, m) . If $y_{in} = 1$ or $y_{ni} = 1$, then the corresponding aircraft of existing flight $i \in E$ is assigned to new flight pair (n, m) . In the schedule, we only allow new flight n to depart from the hub and be immediately followed by a return flight m so that we make the same fleet assignment to flights n and m in constraint (4.11). A new flight is assigned to one fleet type in constraint (4.12).

Constraints (4.13)–(4.14) define the arrival time of the flights to their destination airport. Constraints (4.15) ensure the minimum time requirement for the passenger connections. Similarly, constraints (4.16)–(4.19) maintain the precedence relations among the flights assigned to the same aircraft in the new schedule. For a new flight pair (n, m) , constraint (4.16) guarantees the minimum time requirement for aircraft connections between flights n and m . If an existing flight i is followed by the new flight n , then constraint (4.17) ensures that flight n does not depart before the arrival time of flight i plus its turnaround time. If a new flight n is followed by an existing flight i , then constraint (4.18) enables incoming aircraft of flight n to catch flight i . On the other hand, if no new flight is scheduled between existing flights i and j , then constraint (4.19) keeps the minimum aircraft turnaround time between flights i and j as in the original schedule. Constraints (4.20) determine the arrival tardiness for new flights. In this study, we do not penalize the first 15 min of tardiness as it is a common notion in practice.

Constraints (4.21)–(4.22) apply cruise time upper and lower bounds for each flight, respectively. Constraint (4.23) defines the time intervals for the departures of both existing and new flights. For instance, due to time sensitivity of the business trips, one can allow departures in the morning within certain time intervals, which have already been determined by the airline. The rest of the constraints (4.24)–(4.26) define the domain of decision variables.

An important feature of the proposed mathematical formulation is that the problem is formulated without keeping track of individual aircraft. The decision variable y denotes on which flights are operated before/after the new flights. Since we also keep the sequence of existing flights operated by the same aircraft as it was in the sample schedule, following the index of y decision determines the route of each aircraft. In the model, the aircraft tail of existing flights are given and not changed. This is a great advantage so that the proposed model determines the aircraft tail assignment with less computational effort. Note that the aircraft type information is also necessary for the cost calculations in the objective function.

The proposed formulation is a mixed-integer optimization model with nonlinear cost terms in the objective function and logical constraints (4.17), (4.18), and

(4.19). In Section 4.3, we present the logical constraints mathematically using the Big-M method and McCormick inequalities, respectively. These reformulations enable us to solve relatively large instances to optimality very efficiently.

4.2.2 Formulation with Cruise Time Controllability and Aircraft Swapping

In this section, we additionally include an option of swapping aircraft in the model. Although the ability of swapping the aircraft provides a greater flexibility to make time spaces for new flights with an increased profit, several challenges arise. First, additional binary aircraft assignment decisions for the existing flights are required. Second, there exists a trade-off between the fuel burn and number of spilled passengers of existing flights, since swapping the aircraft of a flight with a more fuel efficient but smaller aircraft not only decreases the fuel burn but may also spill some of the passengers. Third, cruise time decisions of the existing flights depend on the aircraft assignments due to fuel burn as a function of aircraft type.

In order to include an option of aircraft swapping, we first define additional sets and parameters, and redefine the decision variables.

Sets & Parameters:

R	set of aircraft routes in the original schedule
E_r	set of flights in each aircraft route r , $r \in R$
$S(i)$	set of possible flights whose aircraft can be swapped with the aircraft of flight i , $i \in E$
$p(i)$	predecessor of flight i , $i \in E$
σ_i	cost of spilled passengers of flight i , $i \in E \cup N$
ψ	cost of swapping an aircraft
$t(p(i))$	aircraft type assignment of the predecessor of existing flight i , $i \in E$

Decision variables:

z_i^t	1 if aircraft type t is assigned to flight i and 0 o.w., $i \in E \cup N, t \in T$
s_{ij}	1 if the aircraft of flight i and flight j are swapped at their origin and 0 o.w., $i \in E, j \in S(i)$

Then, the mathematical formulation that includes the option of the aircraft swapping (CTC-AS) is stated as follows:

$$\begin{aligned}
& \max \sum_{t \in T} \sum_{i \in N} \pi_i \min(\mu_i, \kappa^t) z_i^t - \sum_{i \in E} \left(\sum_{t \in T} c_i^t(f_i^t) - c_i^{t(i)}(u_i^{t(i)}) \right) \\
& - \sum_{t \in T} \sum_{i \in N} c_i^t(f_i^t) - \sum_{t \in T} \sum_{i \in E \cup N} \sigma_i \max(0, \mu_i - \kappa^t) z_i^t \\
& - \sum_{i \in E} \rho b_i^\zeta - \sum_{i \in E} \sum_{j \in S(i)} (\psi/2) s_{ij} - \phi
\end{aligned}$$

s.t. Constraints (4.3) – (4.7)

$$|z_n^t - z_i^t| \leq (1 - y_{in}), \quad i \in E_I, n \in N_O, t \in T \quad (4.27)$$

$$|z_n^t - z_i^t| \leq (1 - y_{ni}), \quad i \in E_O, n \in N_I, t \in T \quad (4.28)$$

$$z_n^t = z_m^t, \quad (n, m) \in C_N, t \in T \quad (4.29)$$

$$\sum_{t \in T} z_n^t = 1, \quad n \in N_O \quad (4.30)$$

$$|z_{p(i)}^t - z_i^t| \leq \sum_{j \in S(i)} s_{ij}, \quad t \in T, i \in E \quad (4.31)$$

$$z_j^{t(p(i))} \geq s_{ij}, \quad i \in E, j \in S(i) \quad (4.32)$$

$$\sum_{i \in E_r} \sum_{j \in S(i)} s_{ij} \leq 1, \quad r \in R \quad (4.33)$$

$$s_{ij} = s_{ji}, \quad i \in E, j \in S(i) \quad (4.34)$$

$$|y_{in} - y_{mj}| \leq \sum_{k \in S(j)} s_{jk}, \quad (i, j) \in C_E, (n, m) \in C_N \quad (4.35)$$

$$|y_{in} - y_{mk}| \leq 1 - s_{jk}, \quad k \in S(j), (i, j) \in C_E, (n, m) \in C_N \quad (4.36)$$

$$d_i + \sum_{t \in T} f_i^t + \eta_i = a_i, \quad i \in E \cup N \quad (4.37)$$

$$a_n + \sum_{t \in T} \tau_n^t z_n^t \leq d_m, \quad (n, m) \in C_N \quad (4.38)$$

$$\text{If } y_{in} = 1, \text{ then } a_i + \sum_{t \in T} \tau_i^t z_i^t \leq d_n, \quad i \in E_I, n \in N_O \quad (4.39)$$

$$\text{If } y_{ni} = 1, \text{ then } a_n + \sum_{t \in T} \tau_n^t z_n^t \leq d_i, \quad i \in E_O, n \in N_I \quad (4.40)$$

$$\text{If } \sum_{k \in S(j)} s_{jk} = 0, \text{ then } a_i + \sum_{t \in T} \tau_i^t z_i^t \leq d_j, \quad (i, j) \in C_E \quad (4.41)$$

$$\text{If } s_{jk} = 1, \text{ then } a_i + \sum_{t \in T} \tau_i^t z_i^t \leq d_k, \quad k \in S(j), (i, j) \in C_E \quad (4.42)$$

$$a_i + \lambda_{ij} \leq d_j, \quad i \in E_I, j \in G_i \quad (4.43)$$

$$a_i - (a_i^o + 15) \leq b_i, \quad i \in E \quad (4.44)$$

$$\ell_i^t z_i^t \leq f_i^t, \leq u_i^t z_i^t, \quad i \in E \cup N, t \in T \quad (4.45)$$

$$d_i^\ell \leq d_i \leq d_i^u, \quad i \in E \cup N \quad (4.46)$$

$$z_i^t \in \{0, 1\}, \quad i \in E \cup N, t \in T \quad (4.47)$$

$$y_{in} \in \{0, 1\}, \quad i \in E \cup N_I, n \in U_i \quad (4.48)$$

$$s_{ij} \in \{0, 1\}, \quad i \in E, j \in S(i) \quad (4.49)$$

$$b_i \geq 0, \quad i \in E. \quad (4.50)$$

The objective function of the second model is slightly different than the objective of the first model. If the aircraft of a flight is swapped with a smaller aircraft, then some of the passengers of the subsequent flights might be spilled. Therefore, we include an additional cost term for spilled passengers of the existing flights. We also add a new swap cost term to cover the cost of changes caused by swapping the aircraft. The rest of the objective terms are same as the first model. Despite the additional cost of spilled passengers and swapped aircraft, the CTC-AS introduces potential for greater profit by reducing the fuel burn.

We use the same constraints (4.3)–(4.7) of the first formulation to accommodate new flight pairs in an aircraft route. However, the aircraft type assignment constraints (4.27)–(4.30) are slightly different. If new flight n follows an existing flight i , then aircraft type assignments of flights i and its immediate successor n will be same per constraint (4.27). Similarly, if new flight n is followed by an

existing flight i , then constraint (4.28) assigns the same aircraft type to flights n and i . Constraint (4.29) ensures that the aircraft type assignments of the consecutive new flights are same. Constraint (4.30) assigns exactly one aircraft type to each new flight pair.

Constraints (4.31)–(4.32) relate aircraft swap decisions to assignment decisions. If aircraft of flight i is not swapped with another one before its departure, then the aircraft type assignment of flight i and its predecessor flight $p(i)$ will be the same. In other words, if there is no swap before the departure of flight i , then $s_{ij} = 0$ for all flights $j \in S(i)$. Therefore, $z_i^t = z_{p(i)}^t$ for each aircraft type t per constraint (4.31). Otherwise, i.e., $s_{ij} = 1$, then the aircraft type assignment of flight j and the predecessor flight $p(i)$ of flight i will be the same. That is, flight j is taken over by the aircraft of the predecessor of i , in the original schedule. Aircraft type assignment of flight j is modeled in constraint (4.32). Constraint (4.33) limits the number of swaps on an aircraft path. Constraint (4.34) guarantees the symmetry of swap decisions between flights.

If an aircraft is not swapped with another, we keep the same sequence of flights in an aircraft route as in the original schedule. If the aircraft of flight pair (i, j) is not swapped, a new flight pair (n, m) may be accommodated between flights i and j . That is, if the aircraft of flight j is not swapped before its departure, i.e., $\sum_{k \in S(j)} s_{jk} = 0$, then constraint (4.35) ensures that $y_{in} = y_{mj}$ for new flight pair (n, m) as in the constraint (4.8) of the first formulation. Otherwise, if the aircraft of flight j is swapped with an aircraft of any flight k , i.e., $s_{jk} = 1$, then constraint (4.36) guarantees that $y_{in} = y_{mk}$ for new flight pair (n, m) .

Constraint (4.37) defines the arrival time of flights to their destination airport. To model flight departure times and cruise times, we need to make sure that the departure of the successor of flight i is later than the arrival time of flight i plus its aircraft turn time. We first define the successor flight of i in the new schedule. There are three cases for the precedence relations of flight i . Case 1: the new flight n follows flight i , i.e., $y_{in} = 1$, in which case constraint (4.39) enables incoming aircraft of flight i to catch new flight n . Case 2: no swap is made after flight i (i.e., $\sum_{k \in S(j)} s_{jk} = 0$), in which case flight i is followed by either a new

flight or its successor. If it is followed by a new outbound flight, then the inbound flight of new trip will follow the successor of flight i as well. Therefore, in both situations, departure time of the successor of flight i will be later than the arrival time of flight i plus its turnaround time as it is guaranteed by constraint (4.41). Case 3: Aircraft of flight i is swapped with aircraft of flight k , in which case flight k follows flight i in the new schedule. Therefore, constraint (4.42) guarantees the minimum aircraft turn time between flight i and k . We ensure the minimum time requirements for the aircraft connections of new flight pair with constraint (4.38) as well as the aircraft connections between the new flight and its successor with constraint (4.40). Similarly, we guarantee the minimum connection time for passengers with constraints (4.43). Constraints (4.44) determine the deviation from the original arrival times for existing flights.

The mathematical formulation for the CTC-AS approach also provides aircraft tail assignment. Following the index of decisions y and x determines the sequence of flights operated by the same aircraft. Then, aircraft tail information of the existing flights helps to identify the tail assignment for each route developed. The logical constraints (4.39)-(4.42) are also represented using the BigM and McCormick inequalities in Section 4.3.

The mathematical formulations above include nonlinear (convex) fuel and CO₂ emission cost and penalty cost terms in the objective function. Using the convexification results from mixed-integer conic quadratic optimization, we efficiently handle the nonlinearity. The reformulation of nonlinear fuel and emission cost function with conic quadratic inequalities was previously given in Chapter 3, but for the sake of completeness, here we redefine the fuel and emission cost function using the notation provided in this chapter, and then give the convex hull of its epigraph.

$$c(f) = \begin{cases} c_o(\alpha \frac{1}{f} + \beta \frac{1}{f^2} + \gamma f^3 + \nu f^2) & \text{if } z = 1 \\ 0 & \text{if } z = 0. \end{cases}$$

The convex hull of the set E_F can be expressed as

$$t \geq c_o(\alpha p + \beta q + \gamma r + \nu h) \tag{4.51}$$

$$z^2 \leq pf \quad (4.52)$$

$$z^4 \leq f^2 qz \quad (4.53)$$

$$f^4 \leq z^2 rf \quad (4.54)$$

$$f^2 \leq hz \quad (4.55)$$

Moreover, each inequality (4.52)–(4.55) can be represented by conic quadratic inequalities as provided in the Proposition 3.4 in Chapter 3.

The next proposition 4.1 also shows conic quadratic representation of the non-linear penalty function.

Proposition 4.1. *The epigraph of penalty function, $E_P = \{(b, g) \in \mathbb{R}_+^2 : b^{1.5} \leq g\}$ is conic quadratic representable.*

Proof. Since $b^{1.5}$ is a convex function for $b \geq 0$, its epigraph E_P is a convex set. Let us restate $b^{1.5} \leq g$ as

$$b^4 \leq g^2 b. \quad (4.56)$$

Observe that (4.56) can be rewritten as two hyperbolic inequalities

$$b^2 \leq xg \quad (4.57)$$

$$x^2 \leq b1 \quad (4.58)$$

where $x \geq 0$. According to Ben-Tal and Nemirovski [90], hyperbolic inequality (4.57) can be represented by conic quadratic inequality below

$$\|(2b, x - g)\| \leq x + g.$$

Similarly, hyperbolic inequality (4.58) can be represented by the conic quadratic inequality. That concludes the proof. $\square$

4.3 Stronger Reformulations

The nonlinear fuel and emission costs, binary aircraft assignment and swapping decisions and the logical “if-then” constraints in the models of the preceding section increase the computational burden of solving the problem significantly. In

order to solve the problem with less computational effort, in this section we give alternative, stronger reformulations. We first introduce a simple linearization of the logical constraints using the well known Big-M method with carefully computed constants. Then we improve this formulation using McCormick estimators.

4.3.1 Reformulations with Big-M Constraints

Formulations with logical constraints by means of conditional “if-then” statements can be numerically more robust than the Big-M formulations if the Big-M formulations use large constants to express the constraints linearly. Solvers may exploit the explicit conditional statements to improve the preprocessing and branching algorithms. Details on logical constraints can be found in [105]. However, for our formulations, we are able to carefully tighten the Big-M constants using the implied upper and lower bounds on the variables, leading to more effective formulations. In the following, we will present linear reformulations of the logical constraints of CTC and CTC-AS with the corresponding Big-M constraints.

The formulation CTC involves logical constraints (4.17), (4.18), and (4.19). We introduce below three linear constraints to replace these logical constraints, respectively.

Proposition 4.2. *For $i \in E_I$, $n \in N_O$, inequality*

$$d_n - d_i - f_i^{t(i)} \geq \tau_i^{t(i)} + \eta_i - \delta_{in}^1 (1 - y_{in}), \quad (4.59)$$

where $\delta_{in}^1 := \max(d_i^u + u_i^{t(i)} + \tau_i^{t(i)} + \eta_i - d_n^\ell, 0)$, is equivalent to (4.17).

Proof. For any $i \in E_I$, $n \in N_O$, if $y_{in} = 1$, then constraint (4.59) is same as the (4.17). Otherwise, (4.59) reduces to the redundant inequality $d_i^u - d_i + u_i^{t(i)} - f_i^{t(i)} + d_n - d_n^\ell \geq 0$ since $u_i^{t(i)}$ is an upper bound for $f_i^{t(i)}$, d_n^ℓ is an lower bound for d_n , and d_i^u is an upper bound for d_i . $\square$

The rest of the inequalities are stated without proof as they are similar to the one above.

Proposition 4.3. *For all $i \in E_O, n \in N_I$, inequality*

$$d_i - d_n - \sum_{t \in T} f_n^t \geq \tau_n^{t(i)} + \eta_n - \delta_{ni}^2 (1 - y_{ni}), \quad (4.60)$$

where $\delta_{ni}^2 := \max \left(d_n^u + \max_{t \in T} u_n^t + \tau_n^{t(i)} + \eta_n - d_i^\ell, 0 \right)$, is equivalent to (4.18).

Proposition 4.4. *For $(i, j) \in C_E$, inequality*

$$d_j - d_i - f_i^{t(i)} \geq \tau_i^{t(i)} + \eta_i - \delta_{ij}^3 \left(\sum_{n \in N_O} y_{in} \right), \quad (4.61)$$

where $\delta_{ij}^3 := \max \left(d_i^u + u_i^{t(i)} + \tau_i^{t(i)} + \eta_i - d_j^\ell, 0 \right)$, is equivalent to (4.19).

The logical constraints (4.39) - (4.42) of formulation CTC-AS are replaced with the following linear inequalities, respectively.

Proposition 4.5. *For $i \in E_I, n \in N_O$, inequality*

$$d_n - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{in}^4 (1 - y_{in}), \quad (4.62)$$

where $\delta_{in}^4 := \max \left(d_i^u + \max_{t \in T} u_i^t + \max_{t \in T} \tau_i^t + \eta_i - d_n^\ell, 0 \right)$, is equivalent to (4.39).

Proposition 4.6. *For $i \in E_O, n \in N_I$, inequality*

$$d_i - d_n - \sum_{t \in T} f_n^t \geq \sum_{t \in T} \tau_n^t z_n^t + \eta_n - \delta_{ni}^5 (1 - y_{ni}), \quad (4.63)$$

where $\delta_{ni}^5 := \max \left(d_n^u + \max_{t \in T} u_n^t + \max_{t \in T} \tau_n^t + \eta_n - d_i^\ell, 0 \right)$, is equivalent to (4.40).

Proposition 4.7. *For $(i, j) \in C_E$, inequality*

$$d_j - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{ij}^6 \left(\sum_{k \in S(j)} s_{jk} \right), \quad (4.64)$$

where $\delta_{ij}^6 := \max \left(d_i^u + \max_{t \in T} u_i^t + \max_{t \in T} \tau_i^t + \eta_i - d_j^\ell, 0 \right)$, is equivalent to (4.41).

Proposition 4.8. For $(i, j) \in C_E, k \in S(j)$, inequality

$$d_k - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{ik}^7 (1 - s_{jk}), \quad (4.65)$$

where $\delta_{ik}^7 := \max(d_i^u + \max_{t \in T} u_i^t + \max_{t \in T} \tau_i^t + \eta_i - d_k^\ell, 0)$, is equivalent to (4.42).

We provide the reformulation with Big-M and the hyperbolic inequalities, which can be written as conic quadratic inequalities, below.

$$\begin{aligned} \max \quad & \sum_{t \in T} \sum_{i \in N} \pi_i \min(\mu_i, \kappa^t) z_i^t - \sum_{i \in E} \rho g_i - \phi \\ & - \sum_{i \in E} \left(\sum_{t \in T} c_o (\alpha_i^t p_i^t + \beta_i^t q_i^t + \gamma_i^t r_i^t + \nu_i^t h_i^t) - c_i^{t(i)} (u_i^{t(i)}) \right) \\ & - \sum_{t \in T} \sum_{i \in N} c_o (\alpha_i^t p_i^t + \beta_i^t q_i^t + \gamma_i^t r_i^t + \nu_i^t h_i^t) \\ & - \sum_{t \in T} \sum_{i \in E \cup N} \sigma_i \max(0, \mu_i - \kappa^t) z_i^t - \sum_{i \in E} \sum_{j \in S(i)} (\psi/2) s_{ij} \end{aligned}$$

s.t. (4.3) – (4.7), (4.27) – (4.38), (4.43) – (4.50)

$$(z_i^t)^2 \leq p_i^t f_i^t, \quad i \in E \cup N, t \in T \quad (4.66)$$

$$(z_i^t)^4 \leq (f_i^t)^2 q_i^t z_i^t, \quad i \in E \cup N, t \in T \quad (4.67)$$

$$(f_i^t)^4 \leq (z_i^t)^2 r_i^t f_i^t, \quad i \in E \cup N, t \in T \quad (4.68)$$

$$(f_i^t)^2 \leq h_i^t z_i^t, \quad i \in E \cup N, t \in T \quad (4.69)$$

$$b_i^4 \leq g_i^2 b_i, \quad i \in E \quad (4.70)$$

$$d_n - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{in}^4 (1 - y_{in}), \quad i \in E_I, n \in N_O \quad (4.71)$$

$$d_i - d_n - \sum_{t \in T} f_n^t \geq \sum_{t \in T} \tau_n^t z_n^t + \eta_n - \delta_{ni}^5 (1 - y_{ni}), \quad i \in E_O, n \in N_I \quad (4.72)$$

$$d_j - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{ij}^6 \left(\sum_{k \in S(j)} s_{jk} \right), \quad (i, j) \in C_E \quad (4.73)$$

$$d_k - d_i - \sum_{t \in T} f_i^t \geq \sum_{t \in T} \tau_i^t z_i^t + \eta_i - \delta_{ik}^7 (1 - s_{jk}), \quad (i, j) \in C_E, k \in S(j). \quad (4.74)$$

Objective function is slightly different than the original objective function of the proposed model for CTC-AS in Section 4.2.2. The original objective is

represented by the new objective and constraints (4.66) - (4.70), which can be restated by conic quadratic inequalities. Moreover, logical constraints (4.39) - (4.42) are represented using the Big-M inequalities (4.71) - (4.74). The remaining constraints are same as the original constraints of the proposed model for CTC-AS.

As an alternative to the Big-M method, Mannino et. al. [106] use the Path&Cycle Algorithm. In their experiments, the Big-M formulation is slowed down due to the weak bounds on optimality compared to the Path&Cycle formulation. As another alternative to the Big-M formulation, we introduce stronger McCormick estimators in the next section.

4.3.2 Improved Reformulation with McCormick Inequalities

In this section, we improve and strengthen the formulations by using McCormick estimators to represent the logical constraints. We will demonstrate the construction of McCormick inequalities only for constraints (4.41).

Let us define auxiliary variables $v_i = d_i + \sum_{t \in T} f_i^t + \eta_i + \sum_{t \in T} \tau_i^t z_i^t$. We can state inequality (4.41) as

$$d_j \geq v_i \left(1 - \sum_{k \in S(j)} s_{jk} \right), \quad (i, j) \in C_E.$$

The problem formulation can be strengthened using linear inequalities based on McCormick estimators for the bilinear terms $w_{ij} = v_i(1 - \sum_{k \in S(j)} s_{jk})$, $(i, j) \in C_E$. To do so, note the valid upper and lower bounds on v_i :

$$\begin{aligned} v_i^u &:= d_i^u + \max_{t \in T} u_i^t + \max_{t \in T} \tau_i^t + \eta_i \geq v_i \\ v_i^\ell &:= d_i^\ell + \min_{t \in T} \ell_i^t + \min_{t \in T} \tau_i^t + \eta_i \leq v_i. \end{aligned}$$

Using these bounds on v_i , $i \in E$, the following McCormick inequalities [107] are

valid for each bilinear term $\omega_{ij} = v_i \left(1 - \sum_{k \in S(j)} s_{jk}\right)$:

$$\omega_{ij} \leq v_i^u \left(1 - \sum_{k \in S(j)} s_{jk}\right) \quad (i, j) \in C_E \quad (4.75)$$

$$\omega_{ij} \geq v_i^\ell \left(1 - \sum_{k \in S(j)} s_{jk}\right) \quad (i, j) \in C_E \quad (4.76)$$

$$\omega_{ij} \leq v_i - v_i^\ell \sum_{k \in S(j)} s_{jk} \quad (i, j) \in C_E \quad (4.77)$$

$$\omega_{ij} \geq v_i - v_i^u \sum_{k \in S(j)} s_{jk} \quad (i, j) \in C_E. \quad (4.78)$$

Therefore, constraints (4.41) can be replaced with the following constraints

$$d_j \geq \omega_{ij} \quad (4.75) - (4.78).$$

Observe that constraints (4.77) do not need to be added. If $\sum_{k \in S(j)} s_{jk} = 1$, then constraints (4.77) and (4.78) become redundant. On the other hand, if $\sum_{k \in S(j)} s_{jk} = 0$, then constraints (4.77) and (4.78) ensure that $d_j \geq \omega_{ij} = v_i$. If constraints (4.77) are not included in the model, then we obtain $d_j \geq \omega_{ij} \geq v_i$. By this way, we still ensure that flight j cannot depart before the arrival time of flight i plus its turnaround time. Therefore, the optimal solution is same as the original one.

Similarly, logical constraints (4.39), (4.40), and (4.42) can be replaced by the stronger McCormick inequalities. We provide the improved reformulation with the McCormick inequalities and hyperbolic inequalities, which can be written as conic quadratic inequalities, below.

$$\begin{aligned} \max \quad & \sum_{t \in T} \sum_{i \in N} \pi_i \min(\mu_i, \kappa^t) z_i^t - \sum_{i \in E} \rho g_i - \phi \\ & - \sum_{i \in E} \left(\sum_{t \in T} c_o (\alpha_i^t p_i^t + \beta_i^t q_i^t + \gamma_i^t r_i^t + \nu_i^t h_i^t) - c_i^{t(i)} (u_i^{t(i)}) \right) \\ & - \sum_{t \in T} \sum_{i \in N} c_o (\alpha_i^t p_i^t + \beta_i^t q_i^t + \gamma_i^t r_i^t + \nu_i^t h_i^t) \\ & - \sum_{t \in T} \sum_{i \in E \cup N} \sigma_i \max(0, \mu_i - \kappa^t) z_i^t - \sum_{i \in E} \sum_{j \in S(i)} (\psi/2) s_{ij} \end{aligned}$$

s.t.

$$(4.3) - (4.7), (4.27) - (4.38), (4.43) - (4.50)$$

$$(z_i^t)^2 \leq p_i^t f_i^t, \quad i \in E \cup N, t \in T \quad (4.79)$$

$$(z_i^t)^4 \leq (f_i^t)^2 q_i^t z_i^t, \quad i \in E \cup N, t \in T \quad (4.80)$$

$$(f_i^t)^4 \leq (z_i^t)^2 r_i^t f_i^t, \quad i \in E \cup N, t \in T \quad (4.81)$$

$$(f_i^t)^2 \leq h_i^t z_i^t, \quad i \in E \cup N, t \in T \quad (4.82)$$

$$b_i^4 \leq g_i^2 b_i, \quad i \in E \quad (4.83)$$

$$d_n \geq \omega_{in}^1, \quad i \in E_I, n \in N_O \quad (4.84)$$

$$\omega_{in}^1 \leq v_i^u y_{in}, \quad i \in E_I, n \in N_O \quad (4.85)$$

$$\omega_{in}^1 \geq v_i^\ell y_{in}, \quad i \in E_I, n \in N_O \quad (4.86)$$

$$\omega_{in}^1 \leq v_i - v_i^\ell (1 - y_{in}), \quad i \in E_I, n \in N_O \quad (4.87)$$

$$\omega_{in}^1 \geq v_i - v_i^u (1 - y_{in}), \quad i \in E_I, n \in N_O \quad (4.88)$$

$$d_i \geq \omega_{ni}^2, \quad i \in E_I, n \in N_O \quad (4.89)$$

$$\omega_{ni}^2 \leq v_n^u y_{ni}, \quad i \in E_O, n \in N_I \quad (4.90)$$

$$\omega_{ni}^2 \geq v_n^\ell y_{ni}, \quad i \in E_O, n \in N_I \quad (4.91)$$

$$\omega_{ni}^2 \leq v_n - v_n^\ell (1 - y_{ni}), \quad i \in E_O, n \in N_I \quad (4.92)$$

$$\omega_{ni}^2 \geq v_n - v_n^u (1 - y_{ni}), \quad i \in E_O, n \in N_I \quad (4.93)$$

$$d_j \geq \omega_{ij}^3, \quad (i, j) \in C_E \quad (4.94)$$

$$\omega_{ij}^3 \leq v_i^u \left(1 - \sum_{k \in S(j)} s_{jk}\right), \quad (i, j) \in C_E \quad (4.95)$$

$$\omega_{ij}^3 \geq v_i^\ell \left(1 - \sum_{k \in S(j)} s_{jk}\right), \quad (i, j) \in C_E \quad (4.96)$$

$$\omega_{ij}^3 \leq v_i - v_i^\ell \sum_{k \in S(j)} s_{jk}, \quad (i, j) \in C_E \quad (4.97)$$

$$\omega_{ij}^3 \geq v_i - v_i^u \sum_{k \in S(j)} s_{jk}, \quad (i, j) \in C_E \quad (4.98)$$

$$d_k \geq \omega_{ik}^4, \quad k \in S(j), (i, j) \in C_E \quad (4.99)$$

$$\omega_{ik}^4 \leq v_i^u s_{jk}, \quad k \in S(j), (i, j) \in C_E \quad (4.100)$$

$$\omega_{ik}^4 \geq v_i^\ell s_{jk}, \quad k \in S(j), (i, j) \in C_E \quad (4.101)$$

$$\omega_{ik}^4 \leq v_i - v_i^\ell (1 - s_{jk}), \quad k \in S(j), (i, j) \in C_E \quad (4.102)$$

$$\omega_{ik}^4 \geq v_i - v_i^u (1 - s_{jk}), \quad k \in S(j), (i, j) \in C_E \quad (4.103)$$

$$v_i = d_i + \sum_{t \in T} f_i^t + \eta_i + \sum_{t \in T} \tau_i^t z_i^t, \quad i \in E \cup N. \quad (4.104)$$

4.4 Computational Study

In this section, we first test and compare the performance of two approaches, CTC and CTC-AS, proposed in the paper in terms of their effectiveness to improve the airline's profitability computationally through a full-factor experimental design. Then we test and compare the effectiveness of the stronger reformulations described in Section 4.3 in solving the computationally intensive approach CTC-AS with aircraft swapping.

In the experimental study, we test performance of MICQ reformulations with Big-M constraints and McCormick inequalities, respectively. All experiments are performed on a workstation with a 3.60 GHz Intel R Xeon R CPU E5-1650 and 32 GB main memory. The mixed-integer conic quadratic reformulations are implemented using JAVA programming language with a connection to IBM ILOG CPLEX Optimization Studio 12.7.1.

We use a sample schedule extracted from the database "Airline On-Time Performance Data," provided by the Bureau of Transportation Statistics of the US Department of Transportation, BTS [99], and query the planned departure and arrival times of all United Airlines (UA) domestic flights for the date of 03/01/2018 from the database. In our computations, departure time intervals $[d^\ell, d^u]$ are determined by adding/subtracting ninety min to planned departure times of the sample schedule. The flight block times in the sample schedule are calculated by taking the difference between the scheduled arrival and departure times. Then, we assume that 30 min of a flight block time is non-cruise time (η) and remaining is cruise time (u). In experiments, we compress the cruise times at most 15%, hence $\ell = 0.85u$. Moreover, in the proposed models, we keep each aircraft route, i.e., sequence of flights as provided in the sample schedule.

In this study, new flights are connected to the existing schedule at hub airport ORD. Therefore, in the sample schedule, we remove the route of aircraft that does not visit ORD airport for this particular day. This is reasonable, since the flights of aircraft which does not visit ORD will not be affected by introducing new flights. The resulting sample schedule includes 300 flights operated by 81 aircraft.

4.4.1 Description of the Data for the Experimental Study

In order to analyze the effects of problem parameters on the airline's profit, we conduct a 2^k full-factorial experimental design. The experimental factors and their levels are given in Table 4.3.

Table 4.3: Factor values.

Factor Description	Levels	
	Low	High
c_{fuel} (\$/kg)	0.6	1.2
σ_b (\$/passenger)	60	200
ψ (\$/swap)	500	1000

The fuel prices for lower and higher settings, respectively, are estimates based on the history of fuel prices obtained from IATA fuel price monitor [100], which shows a fluctuation between \$0.6/kg and \$1.2/kg during years 2008 – 2018. In the table σ_b is a base value for the opportunity cost for each of the spilled passengers due to the insufficient seat capacity of the aircraft. Şafak et al. [51] express the cost of spilled passenger for each flight using airport congestion coefficients, e.g., favoring the populated markets as follows:

$$\sigma_i = \sigma_b(e_{O_i})(e_{D_i}), \quad i \in E \cup N, \quad (4.105)$$

where e_{O_i} and e_{D_i} represent the congestion coefficients for the origin and destination airports of flight $i \in E \cup N$. These coefficient values are provided in Şafak et al. [51]. ψ is the cost of changes caused by swapping the aircraft of the flights. For low and high values of swap cost, we have used \$500 (proposed by Marla et al. [49]) and \$1000, respectively.

We consider six aircraft types and list the fuel burn related parameters, the corresponding maximum range cruise (MRC) speed, and the seat capacity in Table 4.4. The coefficients of the fuel burn function, $\alpha_i^t, \beta_i^t, \gamma_i^t, \nu_i^t$, are calculated as specified in Şafak et al. [51] using the corresponding values of fuel burn related parameters in Table 4.4.

Table 4.4: Aircraft parameters.

Aircraft type	B727 228	B737 500	MD 83	A320 111	A320 212	B767 300
Capacity	134	122	148	172	180	218
Mass (kgs)	74000	50000	61200	62000	64000	135000
Surface(m^2)	157.9	105.4	118	122.4	122.6	283.3
$C_{D0,CR}$	0.018	0.018	0.0211	0.024	0.024	0.021
$C_{D2,CR}$	0.06	0.055	0.0468	0.0375	0.0375	0.049
C_{f1}	0.53178	0.46	0.7462	0.94	0.94	0.763
C_{f2}	276.72	300	638.59	50000	100000	1430
cf_{CR}	0.954	1.079	0.9505	1.095	1.06	1.0347
MRC speed (km/h)	867.6	859.2	867.6	855.15	868.79	876.70
τ_b^t (min)	32	36	26	28	30	40

For a flight i , the aircraft turnaround time (τ_i^t) needed to prepare the aircraft for the next flight is estimated using the expression

$$\tau_i^t = \tau_b^t e_{D_i} \quad t \in T, \quad (4.106)$$

where τ_b^t is a base value for aircraft turnaround time. Therefore, turnaround time of an aircraft visiting a congested airport will take longer. The calculated aircraft turnaround times match with the aircraft turnaround times given in Arıkan et al. [108].

The passenger demand for existing flights are generated uniformly between 110 and 134, 110 and 122, 110 and 148, 150 and 172, 160 and 180, 160 and 218, for aircraft types B727 228, B737 500, MD 83, A320 111, A320 212 and B767 300, respectively. Without loss of generality, we assume that the original aircraft assignments meet all passenger demand. Under this experimental setting, we can analyze the performance of aircraft swapping while trading-off between the cost of fuel burn and cost of spilled passengers.

4.4.2 Performance Analysis of CTC and CTC-AS

While aircraft swapping in addition to re-timing departures and cruise speed control provides a greater flexibility to accommodate the new flights, it is of interest to study the incremental increase in the airlines profit due to the heavy computational burden of solving CTC-AS.

In this experimental study, we use 300 flights operated by 81 aircraft of the sample schedule. We consider adding round-way trips $\{\text{ORD-IAH}\}$, $\{\text{ORD-BOS}\}$ and $\{\text{ORD-MSP}\}$ so that there are six new flights to be added into the new schedule. The demand for the new flights are generated uniformly between 120 and 200. The estimated demand ranges are defined based on the seat capacity of the aircraft types commonly used to operate these trips. The fares for the new flights are generated uniformly between USD 120 and USD 350, based on an analysis of the ticket prices of the flights in these trips for United Airlines. According to Eurocontrol's report [109] on dynamic cost indexing, total crew cost per block hour varies between \$280 and \$800. In this study, we let the crew cost for each new flight as \$400/hr. Using an expected block time for each new flight, the total crew cost for six new flights is calculated as \$6500. For each new flight, we also consider \$1500 service cost including landing fees, ground handling fees, insurance fees, onboard services costs, etc.

We design a 2^3 experimental study with two levels for each experimental factor. For each combination of the factor levels, we solve 10 randomly generated instances with the approaches CTC and CTC-AS, respectively. In both formulations, we replace logical constraints with McCormick inequalities and handle the nonlinear fuel and emission costs using mixed integer conic quadratic programming described in Section 4.2.2. Then, each instance is approximately solved less than sixty seconds by the CTC approach. On the other hand, with formulation CTC-AS, the average CPU time is 6,100 seconds due to increased number of conic constraints with binary assignment decisions and binary swapping decisions. Despite the additional computational complexity, the CTC-AS approach provides

substantial profit improvement over the simpler cruise time controllability approach CTC, calculated as

$$\text{Profit improvement (\%)} = 100 \times \frac{\text{Optval (CTC-AS)} - \text{Optval (CTC)}}{\text{Optval (CTC)}}.$$

Table 4.5 summarizes the results for 80 instances. Observe that the profit improvement significantly increases as the fuel price increases. As the fuel expenses are a major cost component of airlines, this cost component becomes more important with higher fuel price. In order to reduce fuel burn, CTC-AS has an advantage of reassigning flights to more fuel efficient aircraft. For high fuel price, the profit improvement can reach to 131% over CTC. On the other hand, if the spill cost is high, then the profit improvement decreases. Similarly, the profit improvement decreases as the swapping cost parameter increases. Because, swapping the aircraft of a flight with a smaller aircraft may spill some of the passengers, and such swaps incur additional spilled passenger cost for CTC-AS approach. CTC-AS approximately yields a 53% improvement in airline’s profit compared to CTC for the factor values analyzed in this study.

Table 4.5: Profit improvement of CTC-AS over CTC.

		Profit Impr. (%)		
		min	avg	max
c_{fuel}	Low	8	25	44
	High	42	81	131
σ_b	Low	25	66	131
	High	8	39	107
ψ	Low	10	55	131
	High	8	51	125
avg			53	

4.4.2.1 What-if Analysis on the Number of Aircraft Swaps

CTC-AS approach utilizes the aircraft swapping mechanism to reduce the fuel burn. On the other hand, if the aircraft of a flight is swapped with a smaller aircraft, then some of the passengers of the subsequent flights might be spilled. Therefore, CTC-AS approach trades-off the cost of fuel burn with the cost of

spilled passengers. To see the effect of the number of swaps, we restrict the maximum number of swaps with the following constraint:

$$\sum_{i \in J} \sum_{j \in AS(i)} x_{ij} \leq \text{max_swap} \quad (4.107)$$

A schedule planner can specify and modify the maximum number of swaps and analyze the influence of the number of swaps on the airline's profit. In Figure 4.4, we provide the efficient frontier for a problem instance with 300 flights and 81 aircraft solved with different levels of factors in Table 4.6. If the fuel price is high, then the airline's profit significantly increases as the number of swaps increases. Since the fuel cost is the major cost component of airlines, fuel burn has a greater influence on airline's profit in this case. The total fuel burn can be reduced by reassigning the fuel efficient aircraft to longer trips and using the less efficient aircraft for shorter trips. On the other hand, if the fuel price is low and the spill parameter is high, the profit is slightly improved as the number of swaps increases and the profit remains constant after seven swaps. In this case, it is not preferred to spill passengers due to the high spill cost. As shown in Figure 4.5, the percentage of spilled passengers is the lowest for factor combinations 3 and 7 as expected. The diminishing rate of return in the profit increase allows the airline to limit the number of swaps to a few to gain a large benefit with a small number of swaps.

Table 4.6: Combination of factor levels.

Factor combination	Levels of		
	ψ	σ_b	c_{fuel}
1	Low	Low	Low
2	Low	Low	High
3	Low	High	Low
4	Low	High	High
5	High	Low	Low
6	High	Low	High
7	High	High	Low
8	High	High	High

The computational results clearly demonstrate that the benefit of CTC-AS over CTC and highlights the need to address the computational difficulty for solving the CTC-AS model. In the next section, we will test the performance of reformulations of CTC-AS.

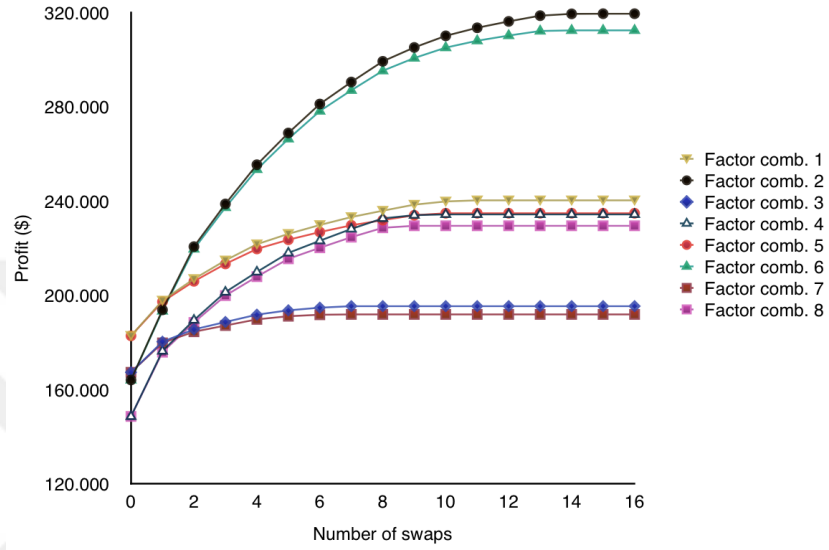


Figure 4.4: Efficient frontier of aircraft swaps.

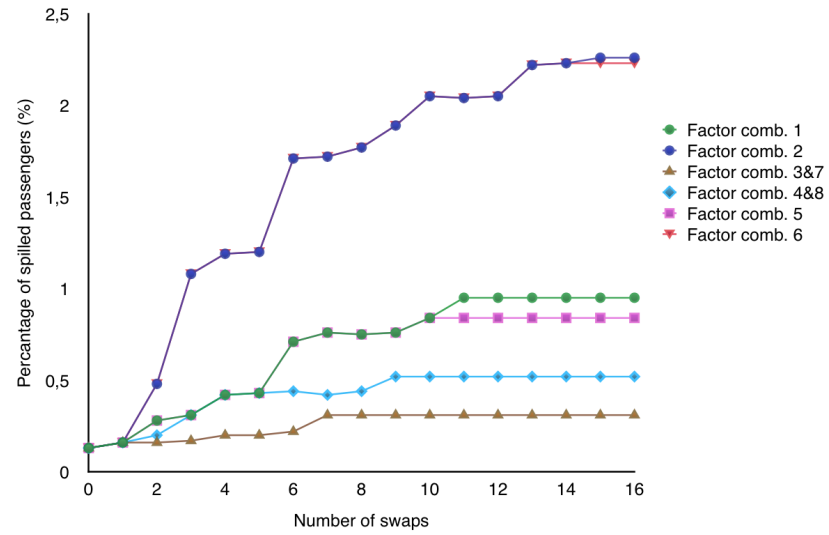


Figure 4.5: The effect of swap decisions on spilled passengers.

4.4.2.2 What-if Analysis for Aircraft Dependent Swap Costs.

An airline may consider its passenger's satisfaction when the aircraft of a passenger's flight is swapped. While passengers would be satisfied when the aircraft of their flights is swapped with a larger one, in other case, passengers would not feel comfortable with smaller aircraft. Therefore, in this section, we consider the aircraft dependent swapping cost parameters provided in Table 4.7. If aircraft type (t) in the first row is swapped with another aircraft type (t') in one of the columns, we incur an additional penalty cost $\Phi^{tit'}$ to original swap cost ψ for this swap.

Table 4.7: Aircraft dependent swap cost.

Aircraft type	B727 228	B737 500	MD 83	A320 111	A320 212	B767 300
B727 228	0	0	0	0	0	0
B737 500	0	0	0	0	0	0
MD 83	100	100	0	0	0	0
A320 111	150	150	100	0	0	0
A320 212	150	150	100	0	0	0
B767 300	200	200	150	100	100	0

We redefine the cost of swap (ϕ_{ij}) between the aircraft of flights i and j and calculate as:

$$\phi_{ij} = \psi + \Phi^{t(i),t(j)}.$$

ψ is still considered as \$500 and \$1000 for low and high values, respectively. For each combination of fuel price, base spill cost and swap cost levels, we solve ten instances. Then, we report the minimum, average and maximum profit improvements of CTC-AS approach over the CTC approach in Table 4.8.

Table 4.8: Profit improvement of CTS-AS over CTC under aircraft dependent swap costs.

		Profit Impr. (%)		
		min	avg	max
c_{fuel}	Low	8	24	43
	High	40	78	127
σ_b	Low	24	64	127
	High	8	38	103
ψ	Low	10	53	127
	High	8	49	121
avg			51	

As expected, if there is an additional aircraft dependent swap cost, then the profit improvement slightly decreases compared to the single swap cost case provided in Table 4.8. On the other hand, CTC-AS approach with aircraft type dependent swap cost still significantly increases the profit compared to the CTC approach, if the fuel price increases. Because, CTC-AS approach has an advantage of reducing the fuel burn by reassigning the fuel efficient aircraft among subset of flights. However, Table 4.9 shows that these reassignments spill more passengers as the fuel price increases. If the spill cost parameter increases, then the number of swaps decreases so that the number of spilled passengers decreases. The results indicate that an average of 0.9% of passengers are spilled while maximizing the airline profit to capture the additional demand of new flights.

Table 4.9: Percentage of the spilled passengers

		Percentage of spilled pax. (%)		
		min	avg	max
c_{fuel}	Low	0.2	0.6	1.3
	High	0.3	1.2	2.3
σ_b	Low	0.7	1.4	2.3
	High	0.2	0.4	0.7
ψ	Low	0.2	0.9	2.3
	High	0.2	0.9	2.3
avg			0.9	

4.4.3 Analysis of the Reformulations

The original formulation in Section 4.2.2 has nonlinear fuel burn and carbon emission functions in the objective. To efficiently handle them, we propose a mixed-integer conic quadratic reformulation. Here, we compare two alternative mixed-integer conic quadratic reformulations referred to as MICQ1 and MICQ2. The second one MICQ2 is formulated using the strengthened inequalities as shown in Table 4.10. It is clear that inequalities of MICQ2 are valid for MICQ1 since for $z \in \{0, 1\}$ they reduce to inequalities of MICQ1. Recall that the additional constraints $\ell z \leq f \leq uz$, which force f to zero whenever z is zero.

With each of these formulations we compare two different ways of forcing the

logical constraints. First, “MICQ1/2+BigM”, corresponds to the mixed integer conic quadratic reformulation, where the logical constraints are replaced with the Big-M constraints (4.71)-(4.74). Second, “MICQ1/2+MC”, replaces the logical constraints with the McCormick inequalities (4.84)–(4.103) described in Section 4.3.2. We do not need to add inequalities (4.87), (4.92), (4.97) and (4.102) as discussed in Section 4.3.2, therefore we do not include them in the experiments.

We perform a 2^3 experimental design with two levels of experimental factors in Table 4.3. For each of these eight factor combinations, we generate 10 instances, resulting in a total of 80 instances. A time limit of 18000 seconds is used for each runtime of each instance.

Table 4.10: Alternative conic formulations.

	MICQ1	MICQ2
	$z^2 \leq pf$	$z^2 \leq pf$
Hyperbolic	$z^4 \leq f^2 q1$	$z^4 \leq f^2 qz$
inequalities	$f^4 \leq 1^2 rf$	$f^4 \leq z^2 rf$
	$f^2 \leq h1$	$f^2 \leq hz$

The computational performance results with the alternative formulations are summarized in Tables 4.11–4.12. Table 4.11 displays the results with the conic formulation MICQ1, whereas Table 4.12 presents the improved results with the strengthened conic formulation MICQ2. For each fuel price, base spill cost, and swap cost parameters, the first column “# nodes” reports the average number of nodes explored in the branch-and-bound algorithm. The second column “time” reports the average CPU time (in seconds) for the instances that could be solved to optimality within the time limit with the number of such instances in the parenthesis. The symbol (-) indicates that none of the instances could be solved to optimality within the time limit. The third column “gap” reports the average percentage optimality gap between the best bound at termination and the integer objective with the number of such instances that could not be solved to optimality in parenthesis.

Table 4.11 shows that none of the instances could be solved to optimality using MICQ1 with either Big-M or McCormick inequalities within the time limit. On the other hand, the computational performances of strong conic formulations

MICQ2+BigM/MC are significantly better. The formulation MICQ2+BigM solves the most of instances to optimality within the time limit as indicated in Table 4.12. Even, for each instance that could not be solved by formulation MICQ1+BigM to optimality, the stronger formulation MICQ2+BigM achieves a lower optimality gap. When the McCormick valid inequalities are added to the strong conic formulation MICQ2, the computational performance further improves. The McCormick inequalities help to solve all instances faster within the time limit. For some instances, the strong conic programming formulation MICQ2 with McCormick estimators is solved more than twice as fast than the best alternative.

Table 4.11: Comparison of conic formulations.

			MICQ1+Big-M				MICQ1+MC			
ψ	σ_b	c_{fuel}	#	nodes	cpu (sec)	gap (%)	#	nodes	cpu (sec)	gap (%)
500	60	0.6	55726	-	-	11.69(10)	35416	-	-	7.78(10)
		1.2	51729	-	-	13.71(10)	33419	-	-	9.38(10)
	200	0.6	101801	-	-	7.09(10)	62260	-	-	4.83(10)
		1.2	70944	-	-	8.55(10)	39552	-	-	4.74(10)
1000	60	0.6	56412	-	-	11.99(10)	36445	-	-	7.68(10)
		1.2	56506	-	-	13.45(10)	35584	-	-	9.01(10)
	200	0.6	102414	-	-	7.00(10)	55232	-	-	4.32(10)
		1.2	71287	-	-	7.74(10)	37564	-	-	5.25(10)

Table 4.12: Comparison of the strengthened conic formulations.

			MICQ2+Big-M				MICQ2+MC			
ψ	σ_b	c_{fuel}	#	nodes	cpu (sec)	gap (%)	#	nodes	cpu (sec)	gap (%)
500	60	0.6	13511	12603(9)	0.07(1)	0.07(1)	3538	8206(10)	0(0)	0(0)
		1.2	25209	12988(6)	1.46(4)	1.46(4)	6677	8688(10)	0(0)	0(0)
	200	0.6	19908	5798(9)	3.48(1)	3.48(1)	7640	3436(10)	0(0)	0(0)
		1.2	12981	7304(9)	2.35(1)	2.35(1)	3783	3033(10)	0(0)	0(0)
1000	60	0.6	9900	10601(9)	1.00(1)	1.00(1)	3641	8074(10)	0(0)	0(0)
		1.2	17262	12930(8)	0.47(2)	0.47(2)	6161	8803(10)	0(0)	0(0)
	200	0.6	13969	5085(9)	3.06(1)	3.06(1)	6560	3380(10)	0(0)	0(0)
		1.2	10958	7043(9)	0.08(1)	0.08(1)	8935	5608(10)	0(0)	0(0)

In Table 4.13, we summarize the results for 80 instances. The conic formulation MICQ1 including BigM inequalities could not solve any instances to optimality within the time limit. If we replace BigM inequalities with McCormick inequalities in formulation MICQ1, then the average optimality gap is decreased from 10.15% to 6.61%. If we use strong conic formulation MICQ2 with BigM inequalities, we can further decrease the average optimality gap to 1.40% over 12 instances that are not solved to optimality. When we reformulate the logical

constraints with McCormick inequalities in strong conic formulation MICQ2, the computational performance is the best. All instances are solved to optimality with a dramatic reduction in number of nodes explored.

Table 4.13: Average performances of conic formulations.

	# nodes	CPU (sec)	Gap (%)
MICQ2+MC	5867	6153(80)	0(0)
MICQ2+Big-M	15462	9294(68)	1.40(12)
MICQ1+MC	41934	-	6.61(80)
MICQ1+Big-M	70852	-	10.15(80)

Figure 4.6 analyzes the average number of nodes explored in the branch-and-bound algorithm for each reformulation. For each factor combination, conic formulation MICQ1 with Big-M constraints explores a large number of nodes within the time limit. By adding McCormick inequalities to MICQ1, the average number of nodes is reduced by half, but it is still more than three times required by the conic formulations MICQ2. With strengthened inequalities of conic formulation MICQ2+Big-M, a large number of the nodes are fathomed and the optimal solutions are found for many instances within the time limit. Further, conic formulation MICQ2 can be improved and strengthened by adding McCormick inequalities, which leads to further significant reduction in the number of nodes.

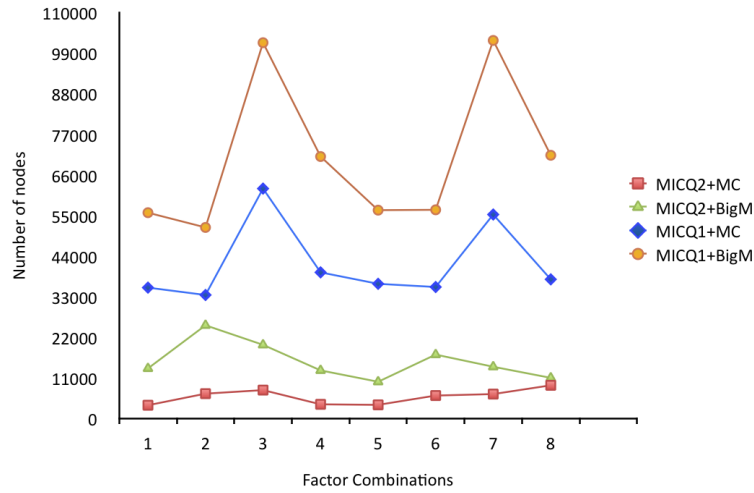


Figure 4.6: Analysis on number of nodes.

4.5 Conclusion

We propose two approaches to accommodate new flights into an existing flight schedule of a particular day. Both of the approaches make use of re-timing of the flight departure times and cruise time controllability to reduce the block times of the existing flights, thereby making time to operate the new flights in the schedule. The second approach additionally takes the advantage of flexibility offered by aircraft swapping among flights. The second approach provides substantial cost savings in fuel burn by reassigning flights to fuel-efficient aircraft. However, the nonlinear fuel and emission costs together with additional binary swapping and assignment decisions significantly complicate the problem. To overcome the computational difficulty, we present strong conic quadratic reformulations. The experiments show the superiority of strong conic formulations over an alternative conic formulation. The alternative conic formulation times out for all test instances. On the other hand, the Big-M reformulation of logical constraints with strengthened conic inequalities can solve most of test instances to optimality. For only 12 instances over 80, Big-M reformulation provides an average of 1.40% optimality gap at termination. As an alternative to Big-M method, when we add McCormick inequalities to the strong conic formulations, all test instances can be solved to optimality with a dramatic reduction in the number of nodes.

In conclusion, we provide two alternative approaches as CTC and CTC-AS with different quality of results and computational difficulties to an airline. While CTC-AS provides an average of 53% profit improvement over CTC, the average CPU time to solve the CTC-AS to optimality by a strong conic reformulation together with McCormick inequalities is 6,100 seconds. On the other hand, CTC approach can be easily solved within sixty seconds.

This study may lead to several potential research directions. The computational advantages of the strong conic quadratic models may pave the way for researchers to integrate the consideration of crew itineraries while determining the flight departures. Another extension of this study would be addressing a strategic planning problem that aims at introducing new flights to new demand

points for the next season. Many potential demand scenarios considering the competitor's flights could be analyzed. Moreover, leasing an aircraft to hedge for the demand uncertainties may be an additional mechanism to introduce new flights.



Chapter 5

Introducing New Flights under Departure Time Dependent Demand Uncertainty

This study aims to introduce new flights by considering the randomness in passenger demand. To open up enough time for new flights, we not only make operational changes such as shifting the departure times of current flights and adjusting the aircraft cruise speed, but also introduce a more strategic decision to lease an aircraft. We take the advantage of leasing option to assign new flights to the right type of aircraft with the right seat capacity and fuel burn rate. The crucial contribution of this study is to consider the effect of departure times of new flights on the probability distribution of random demand. We formulate the problem as a two stage stochastic mixed integer nonlinear program with decision dependent probabilities. Then, we represent the nonlinear fuel burn function with strengthened conic quadratic inequalities and reformulate the bilinear terms with strong McCormick inequalities. To efficiently solve the reformulation, we suggest an exact decomposition algorithm which depends on an evaluate-and-cut procedure. The algorithm generates candidate solutions by solving a number of group subproblems, and then cuts off the evaluated solutions.

5.1 Problem Definition

We propose a two stage stochastic model which generates a new flight schedule to introduce new flights with stochastic passenger demand. As pointed out by Barnhart and Cohn [4], due to complexity and problem size issues, researchers have focused on producing a new schedule by applying a limited number of changes to the existing schedule. Similarly, the aim of this study is to maximize the incremental profit of new schedule while keeping the strategic and tactical decisions of existing flights unchanged. We only allow changes in operational flight times of existing flights.

Different than the previous study in Chapter 4, here we consider a long planning horizon and accommodate new flights into an existing flight schedule by considering the randomness of passenger demand. Before observing the passenger demand, in the first stage, planned departure times of new flights are determined. The important phenomena is that departure times of new flights affect the passenger demand. For instance, it is more likely to observe high demand when new business trips are introduced in the morning, while it is expected to be less when they are operated around noon. Therefore, departure time decisions may change the probabilities of the realizations of random demand. Another strategic decision to lease an aircraft is also made in the first stage. It is critical to determine the right type of aircraft to lease, since each aircraft type has different seat capacity, fuel consumption rate and cost to lease. Before observing the passenger demand, it is a more challenging task to determine the right type of aircraft. In addition, an aircraft family is assigned to each new flight in the first stage. Each aircraft family requires the same cockpit design and crew requirement, but the seat capacities of aircraft types within the same family may differ ([80]).

Then, uncertain passenger demand data are realized. Given the departure time dependent probabilities, the second stage problem maximizes the expected profit of operating new flights through an aircraft type assignment to new flight legs. For each realization of passenger demand, the second stage variables make aircraft assignment to each new flight with an aircraft type from the corresponding aircraft

family that is assigned to it in the first stage. Consequently, the crew schedules are not disturbed, since crew members are usually qualified to fly aircraft types within a single family. The second stage variables introduce two alternatives to schedule new flights: (1) a new flight can be accommodated into the existing flight schedule and operated using an existing aircraft; (2) a new flight can be operated by a leased aircraft, if it is leased in the first stage. In the first case, sufficient time for new flights in the schedule can only be opened up by changing the departure times of current flights and by compressing the cruise times. However, increasing the speed of an aircraft brings an additional cost of fuel burn and CO₂ emission. To keep the cost of fuel manageable, assigning new flights to smaller fuel efficient aircraft may be preferable, despite the additional costs of spilled passengers. However, a fuel efficient aircraft may not be available on the departure time of a new flight. In this case, assigning a larger aircraft presumably incurs higher operational costs. Therefore, the departure times of new flights should be carefully selected.

From the modelling perspective, the “standard” scenario-based formulation of a stochastic program assumes that each random variable will be realized at a predetermined time and that a full set of random variable realizations are specified. In this formulation, decisions do not influence the distribution of the random parameters. The problem is significantly more difficult to solve when the distribution of random parameters depend on decisions made by the decision maker. Similarly, in this study, the departure times made for new flights alter the probability distribution of their passenger demand. That is the decision maker may change the probabilities of the scenarios by making one possibility more likely than the other. For instance, when an airline decides to introduce new business flights in the morning, it is more likely to observe high demand compared to the case when these flights are introduced around noon. Note that in this study, the outcome (realization) of each scenario is assumed to be known, only the probabilities of scenarios depend on the departure time decisions given in the first stage.

To model the departure time dependent probabilities, we first create alternative departure time vectors and index each alternative by $j \in J$, where J is the indexed

set. Let N be the set of new flights and K be the set of time zones in which a new flight might depart. We denote departure time vectors by q_{jnk} 1 if the new flight $n \in N$ departs within the time zone $k \in K$ at alternative $j \in J$, and 0 otherwise. As we discuss, each alternative j of departure time vectors affects the probability distribution of random demand. We then denote these probabilities attached to each scenario ω of Ω with $p_{j\omega}$ for each alternative j . The aim of the problem is to identify the best alternative (j^*) so that the expected profit is maximum.

Sets:

Ω	set of scenarios
J	indexed set for alternative departure time vectors
E	set of existing flights in the schedule
E_O	set of existing outbound flights originating from hub
E_I	set of existing inbound flights arriving to hub
N	set of new flights
N_O	set of new outbound flights from hub to new demand point
N_I	set of new inbound flights from new demand point to hub
H	set of aircraft families
T_h	set of aircraft types in an aircraft family h , $h \in H$
T	set of aircraft types in the entire fleet, where $T = \cup_{h \in H} T_h$
C_E	set of existing flight pairs including flights i and j such that $i \neq j$ and flight i can connect flight j , $i \in E, j \in E$
C_N	set of new flight pairs including flights i and j such that $i \neq j$ and flight i can connect flight j , $i \in N_O, j \in N_I$
K	set of time zones
U_i	set of flights that can follow flight i , $i \in E \cup N_I$
G_i	set of existing outbound flights which have passenger connections from flight i , $i \in E_I$

Parameters:

q_{jnk}	1 if the new flight n departs within the time zone k for the alternative j , and 0 otherwise, $n \in N, k \in K, j \in J$
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$\mu_{i\omega}$	demand realization for new flight i under scenario ω , $i \in N, \omega \in \Omega$
$p_{j\omega}$	probability of scenario ω for the alternative j , $j \in J, \omega \in \Omega$
χ_{it}	1 if aircraft type t is already assigned to existing flight i , and 0 otherwise, $t \in T, i \in E$
$[f_{it}^\ell, f_{it}^u]$	time window for cruise time of flight i with an aircraft type t , $i \in E \cup N, t \in T$
$\bar{x}_{nk}$	desired departure time for new flight n under time zone k , $n \in N, k \in K$
$[d_i^\ell, d_i^u]$	time window for the departure of existing flight i , $i \in E$
η_i	non-cruise time of flight i , $i \in E \cup N$
τ_{it}	turnaround time needed to prepare aircraft type t for the connection after flight i , $i \in E \cup N, t \in T$
λ_{ij}	time needed for the connection of passengers from flight i to flight j , $i \in E_I, j \in G_i$
κ_t	number of seats of an aircraft type t , $t \in T$
t_i	original aircraft type assigned to existing flight i , $i \in E$
δ_i	fare of new flight i , $i \in N$
θ_t	cost of leasing an aircraft with type t , $t \in T$
σ_i	cost of each spilled passenger of new flight i , $i \in N$
ϕ	crew and serving costs for new flights
a_i^o	original arrival time of flight i , $i \in E$
γ	allowable deviation from the planned departure time

First stage decision variables:

π_{nmh}	1 aircraft family h is assigned to new flight pair (n, m) , and 0 otherwise, $h \in H, (n, m) \in C_N$
x_n	planned departure time of new flight $n \in N$
r_t	1 if an aircraft with type t is leased, and 0 otherwise, $t \in T$

Second stage decision variables:

$\ell_{nm\omega}$	1 if new flight pair (n, m) is operated by a leased aircraft under scenario ω , 0 otherwise, $(n, m) \in C_N, \omega \in \Omega$
$v_{nm\omega}$	1 if new flight pair (n, m) is the first flight pair operated by the leased aircraft under scenario ω , and 0 otherwise, $(n, m) \in C_N, \omega \in \Omega$
$u_{nm\omega}$	1 if new flight pair (n, m) is the last flight pair operated by the leased aircraft under scenario ω , and 0 otherwise, $(n, m) \in C_N, \omega \in \Omega$
$s_{nmfg\omega}$	1 if both new flight pairs (n, m) and (f, g) are operated by a leased aircraft and flight pair (n, m) is followed by other flight pair (f, g) under scenario ω , and 0 otherwise, $(n, m) \in C_N, (f, g) \in C_N, \omega \in \Omega$
$y_{ij\omega}$	1 if flight i is followed by flight j under scenario ω , and 0 otherwise, $i \in E \cup N_I, j \in U_i, \omega \in \Omega$
$z_{it\omega}$	1 if aircraft type t is assigned to flight i under scenario ω , and 0 otherwise, $i \in E \cup N, t \in T, \omega \in \Omega$
$f_{it\omega}$	cruise time of flight i with aircraft type t under scenario ω , $i \in E \cup N, t \in T, \omega \in \Omega$
$d_{i\omega}$	actual departure time of flight i under scenario ω , $i \in E \cup N, \omega \in \Omega$
$a_{i\omega}$	actual arrival time of flight i under scenario ω , $i \in E \cup N, \omega \in \Omega$

We first give a mathematical formulation for each alternative j of departure time vectors $\bar{q}_j := (q_{jnk})_{n \in N, k \in K}$. Then, we will give a unified mathematical model which determines the optimal alternative j^* among all departure time vectors while maximizing the expected profit of an airline.

For each departure time vector $\bar{q}_j$, the problem can be formulated as a standard two stage stochastic program.

$$\begin{aligned}
\max \quad & \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N} (\min(\mu_{i\omega}, \kappa_t)) z_{it\omega} \delta_i \right) p_{j\omega} - \sum_{t \in T} r_t \theta_t \quad (F(\bar{q}_j)) \\
& - \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N \cup E} c_{it}(f_{it\omega}) \right) p_{j\omega} + \sum_{i \in E} c_{it_i}(f_{it_i}^u)
\end{aligned}$$

$$- \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N} \max(0, \mu_{i\omega} - \kappa_t) z_{it\omega} \sigma_i \right) p_{j\omega} - \phi \quad (5.1)$$

subject to

$$x_n = \sum_{k \in K} \bar{x}_{nk} q_{jnk}, \quad n \in N \quad (5.2)$$

$$\sum_{h \in H} \pi_{nmh} = 1, \quad (n, m) \in C_N \quad (5.3)$$

$$\sum_{t \in T} r_t \leq 1 \quad (5.4)$$

$$\sum_{t \in T_h} z_{nt\omega} = \pi_{nmh}, \quad (n, m) \in C_N, h \in H, \omega \in \Omega \quad (5.5)$$

$$\sum_{i \in E_I} y_{in\omega} + \sum_{i \in E_O} y_{mi\omega} + \ell_{nm\omega} \geq 1, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.6)$$

$$\ell_{nm\omega} + \sum_{i \in E_I} y_{in\omega} \leq 1, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.7)$$

$$\ell_{nm\omega} + \sum_{i \in E_O} y_{mi\omega} \leq 1, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.8)$$

$$\sum_{n \in N_O} y_{in\omega} \leq 1, \quad i \in E_I, \omega \in \Omega \quad (5.9)$$

$$\sum_{n \in N_I} y_{ni\omega} \leq 1, \quad i \in E_O, \omega \in \Omega \quad (5.10)$$

$$y_{in\omega} = y_{mj\omega}, \quad (i, j) \in C_E, (n, m) \in C_N, \omega \in \Omega \quad (5.11)$$

$$|z_{nt\omega} - \chi_{it}| \leq (1 - y_{in\omega}), \quad i \in E_I, n \in N_O, t \in T, \omega \in \Omega \quad (5.12)$$

$$|z_{nt\omega} - \chi_{it}| \leq (1 - y_{ni\omega}), \quad i \in E_O, n \in N_I, t \in T, \omega \in \Omega \quad (5.13)$$

$$|z_{nt\omega} - r_t| \leq (1 - \ell_{nm\omega}), \quad (n, m) \in C_N, t \in T, \omega \in \Omega \quad (5.14)$$

$$z_{nt\omega} = z_{mt\omega}, \quad (n, m) \in C_N, t \in T, \omega \in \Omega \quad (5.15)$$

$$v_{nm\omega} + \sum_{(f,g) \in C_N} s_{fgnm\omega} = \ell_{nm\omega}, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.16)$$

$$u_{nm\omega} + \sum_{(f,g) \in C_N} s_{nmfg\omega} = \ell_{nm\omega}, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.17)$$

$$\sum_{(n,m) \in C_N} v_{nm\omega} \leq 1, \quad \omega \in \Omega \quad (5.18)$$

$$d_{i\omega} + \sum_{t \in T} f_{it\omega} + \eta_i = a_{i\omega}, \quad i \in E \cup N, \omega \in \Omega \quad (5.19)$$

If $y_{in\omega} = 1$, then

$$a_{i\omega} + \tau_{it_i} \leq d_{n\omega}, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.20)$$

If $y_{ni\omega} = 1$, then

$$a_{n\omega} + \tau_{nt_i} \leq d_{i\omega}, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.21)$$

If $s_{nmfg\omega} = 1$, then

$$a_{m\omega} + \sum_{t \in T} \tau_{mt} z_{mt} \leq d_{f\omega}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.22)$$

$$a_{i\omega} + \tau_{it_i} \leq d_{j\omega}, \quad (i, j) \in C_E, \omega \in \Omega \quad (5.23)$$

$$a_{n\omega} + \sum_{t \in T} \tau_{nt} z_{nt\omega} \leq d_{m\omega}, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.24)$$

$$a_{i\omega} + \lambda_{ij} \leq d_{j\omega}, \quad i \in E_I, j \in G_i, \omega \in \Omega \quad (5.25)$$

$$d_i^\ell \leq d_{i\omega} \leq d_i^u, \quad i \in E, \omega \in \Omega \quad (5.26)$$

$$x_n - \gamma_n \leq d_{n\omega} \leq x_n + \gamma_n, \quad n \in N, \omega \in \Omega \quad (5.27)$$

$$f_{it}^\ell z_{it\omega} \leq f_{it\omega} \leq f_{it}^u z_{it\omega}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.28)$$

$$z_{it\omega} = \chi_{it}, \quad i \in E, t \in T, \omega \in \Omega \quad (5.29)$$

$$\pi_{nmh} \in \{0, 1\}, \quad (n, m) \in C_N, h \in H \quad (5.30)$$

$$r_t \in \{0, 1\}, \quad t \in T \quad (5.31)$$

$$z_{it\omega} \in \{0, 1\}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.32)$$

$$y_{ij\omega} \in \{0, 1\}, \quad i \in E \cup N_I, j \in U_i, \omega \in \Omega \quad (5.33)$$

$$v_{nm\omega} \in \{0, 1\}, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.34)$$

$$u_{nm\omega} \in \{0, 1\}, \quad (n, m) \in C_N, \omega \in \Omega \quad (5.35)$$

$$s_{nmfg\omega} \in \{0, 1\}, \quad (n, m) \in C_N, (f, g) \in C_N, \omega \in \Omega \quad (5.36)$$

$$\ell_{nm\omega} \in \{0, 1\}, \quad (n, m) \in C_N, \omega \in \Omega. \quad (5.37)$$

The goal of adding new flights to existing flight schedule is to maximize the expected profit of an airline. The first term of the objective function calculates the revenue from accepted number of passengers. The number of tickets sold can be determined as the minimum of passenger demand realization and the seat capacity of assigned aircraft. The second term is the total cost of leasing an aircraft. The third term is the expected total cost of fuel burned and CO₂ emitted by aircraft of both new and existing flights. Since we only consider the

incremental fuel and emission costs incurred by accelerating the aircraft to open up time for new flights, we add the fuel and emission costs of existing flights under the ideal cruise times in the fourth term. The fifth term is the expected total cost of spilled passengers due to the insufficient seat capacity of assigned aircraft. Finally, the sixth term in the objective function is the total costs of crew and service for new flights including landing fees, ground handling fees, insurance fees, onboard services costs, etc.

Constraint (5.2) determines the planned departure times for new flights for given departure time vector $(\bar{q}_j)$. Constraint (5.3) assigns exactly one aircraft family to each of new flights. Constraint (5.4) determines the type of aircraft to lease.

Based on the realization of passenger demand, Constraint (5.5) assures that each new flight leg is covered by exactly one aircraft type from the corresponding family that is assigned to it. Constraint (5.6) ensures that new flight is either assigned to a leased aircraft or accommodated into the existing flight schedule. However, a new flight cannot be operated by a leased aircraft or an existing aircraft at the same time through Constraints (5.7) and (5.8). Constraints (5.9)-(5.10) ensure that an existing flight can follow or be followed by at most one new flight. If a new flight pair (n, m) is accommodated between an existing flight pair (i, j) , then the model ensures that $y_{in} = y_{mj} = 1$ in Constraint (5.11). In other words, existing flight i is followed by new flight n and new flight m is followed by the existing flight j .

Constraints (5.12) and (5.15) relate aircraft type assignment decisions (z_{itw}) of new flights with the scheduling variables (y_{ijw}, ℓ_{nmw}) . Constraint (5.12) and (5.13) ensures that if either y_{inw} or y_{niw} is 1, then corresponding aircraft of the existing flight i operates new flight pair (n, m) . Similarly, Constraint (5.14) assigns the type of leased aircraft to new flight pair if ℓ_{nmw} is 1. In the schedule, we only allow that new flight n departs from the hub and is immediately followed by its return flight m so that we make the same aircraft assignment to flights n and m in Constraint (5.15).

Constraints (5.16)-(5.18) define the route of a leased aircraft. Constraint (5.16) ensures that if a new flight pair (n, m) is assigned to the leased aircraft, then flight pair (n, m) is either the first flight pair of leased aircraft or follows another new flight pair. Similarly, Constraint (5.17) guarantees that new flight pair (n, m) is either the last flight pair of the leased aircraft or followed by another new flight pair. In Constraint (5.18), we limit the number of routes of leased aircraft by one.

Constraint (5.19) defines the arrival time of the flights to their destination airport. Constraints (5.20)-(5.24) satisfy the precedence relations among the flights that are assigned to the same aircraft in new schedule. If an existing flight i is followed by a new flight n , then Constraint (5.20) guarantees that flight n cannot depart before the arrival of flight i plus the turnaround time needed to prepare the aircraft for the next flight. Similarly, Constraint (5.21) ensures the minimum time requirement for the aircraft connection between new flight n and existing flight i , if new flight n is followed by an existing flight i . If leased aircraft consecutively operates new flight pairs (n, m) and (f, g) , then Constraint (5.22) enables incoming aircraft of flight m to catch flight f . Constraint (5.23) keeps similar precedence relations between existing flight pair (i, j) as in the original schedule. Lastly, Constraint (5.24) ensures that for each new flight pair (n, m) , an aircraft first operates new outbound flight n , and then operates its inbound flight. Constraint (5.25) ensures minimum time requirement for the connection of passengers at hub airport.

In Constraint (5.26), existing flights can only be allowed to depart within the given time interval. Similarly, Constraint (5.27) allows the actual flight departure times of new flights to deviate from the planned ones by γ_i amount due to the scheduling limitations of airlines and airports. Constraint (5.28) apply cruise time lower and upper limits. In Constraint (5.29), for existing flights, we assign the same type of the aircraft as in the original schedule. Constraints (5.30) - (5.37) define the domain of the variables.

We introduce a new binary variable e_j which takes value of 1 if the departure time vector $\bar{q}_j$ is selected in the solution, and 0 otherwise. Then, the problem

which considers all alternatives of departure time vectors $\bar{q}_j$ for $j \in J$ can be formulated as a two stage stochastic nonlinear mixed integer program.

$$\begin{aligned}
\max \quad & \sum_{j \in J} \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N} (\min(\mu_{i\omega}, \kappa_t)) z_{it\omega} \delta_i \right) p_{j\omega} e_j - \sum_{t \in T} r_t \theta_t \quad (\text{UF}) \\
& - \sum_{j \in J} \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N \cup E} c_{it}(f_{it\omega}) \right) p_{j\omega} e_j + \sum_{i \in E} c_{it_i}(f_{it_i}^u) \\
& - \sum_{j \in J} \sum_{\omega \in \Omega} \left(\sum_{t \in T} \sum_{i \in N} \max(0, \mu_{i\omega} - \kappa_t) z_{it\omega} \sigma_i \right) p_{j\omega} e_j - \phi \quad (5.38)
\end{aligned}$$

$$\text{subject to} \quad (5.3) - (5.37) \quad (5.39)$$

$$\sum_{j \in J} e_j = 1 \quad (5.40)$$

$$x_n = \sum_{j \in J} \sum_{k \in K} \bar{x}_{nk} q_{jnk} e_j, \quad n \in N \quad (5.41)$$

$$e_j \in \{0, 1\}, \quad j \in J. \quad (5.42)$$

This unified model identifies the best alternative of departure time vectors $\bar{q}_j$ which maximizes the expected profit of an airline. Constraint (5.40) ensures that exactly one of the alternative departure time vectors $\bar{q}_j$ is selected. Constraint (5.41) is slightly different than the Constraint (5.2). Based on the selected vector $\bar{q}_j$, Constraint (5.41) determines the planned departure times of new flights. The rest of the constraints are same as Constraints (5.3) - (5.37) of the problem $F(\bar{q}_j), j \in J$.

5.2 Stronger Reformulations

The unified formulation UF of the previous section involves nonlinearities in the objective function (5.38) due to multiplication of second stage decisions with the binary variable e_j . Moreover, binary aircraft type assignment decisions, leasing option and the logical constraints bring additional challenges to solve the problem. In this section, we first introduce McCormick inequalities to linearize the

logical constraints and the product of two binary decisions in the objective function. Then, we suggest alternative reformulations to handle the product of binary variable e_j with the nonlinear fuel and emission cost function $c_{it}(f_{it\omega})$.

5.2.1 Reformulation of Logical Constraints

The well known method to linearize the logical constraints is to use the Big-M constant. However, if the reformulation uses large constants to express the constraints linearly, the Big-M formulation could be slowed down due to weak bounds on optimality. An alternative way to present linear reformulations of logical constraints is to use strong McCormick estimators. In Section 4.5, we test the performance of both Big-M method and McCormick inequalities. The results indicate the superiority of McCormick estimators over the Big-M constants. Therefore, in this section, we choose to reformulate the logical constraints with McCormick inequalities. We will demonstrate the construction of McCormick inequalities only for constraints (5.22) in Proposition 5.1.

Proposition 5.1. *For $\rho_{nmfg\omega}^1 \geq 0$, $(f, g) \in C_N$, $(n, m) \in C_N$, $\omega \in \Omega$, inequalities*

$$\rho_{nmfg\omega}^1 \leq b_m^u s_{nmfg\omega} \quad (5.43)$$

$$\rho_{nmfg\omega}^1 \geq b_m^\ell s_{nmfg\omega} \quad (5.44)$$

$$\rho_{nmfg\omega}^1 \leq b_{m\omega} - b_m^\ell (1 - s_{nmfg\omega}) \quad (5.45)$$

$$\rho_{nmfg\omega}^1 \geq b_{m\omega} - b_m^u (1 - s_{nmfg\omega}) \quad (5.46)$$

$$d_{f\omega} \geq \rho_{nmfg\omega}^1, \quad (5.47)$$

where $b_m := a_{m\omega} + \sum_{t \in T} \tau_{mt} z_{mt\omega}$, $b_m^u := \max_{k \in K} x_{mk}^u + \gamma_m + \max_{t \in T} f_{mt}^u + \max_{t \in T} \tau_{mt} \geq b_{m\omega}$ and $b_m^\ell := \min_{k \in K} x_{mk}^\ell - \gamma_m + \min_{t \in T} f_{mt}^\ell + \min_{t \in T} \tau_{mt} \leq b_{m\omega}$ equivalently represent the Constraints (5.22).

Proof. The inequality (5.22) can be restated as

$$d_{f\omega} \geq b_{m\omega} s_{nmfg\omega}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega. \quad (5.48)$$

Then, the linear inequalities (5.43)-(5.47) based on McCormick estimators for the bilinear terms $\rho_{nmfg\omega}^1 = b_{m\omega}s_{nmfg\omega}$ equivalently represent the Constraints (5.22).

If $s_{nmfg\omega} = 0$, then Constraints (5.43) and (5.44) ensure that $\rho_{nmfg\omega}^1 = 0$. In this case, Constraints (5.47) become redundant. If $s_{nmfg\omega} = 1$, then Constraints (5.45) and (5.46) ensure that $\rho_{nmfg\omega}^1 = b_{m\omega}$, where $b_{m\omega} = a_{m\omega} + \sum_{t \in T} \tau_{mt}z_{mt\omega}$. Therefore, Constraints (5.47) become same as Constraints (5.22).

□

Similarly logical constraints (5.20) and (5.21) can be replaced by the stronger McCormick inequalities.

Observe that constraints (5.45) can be deleted. If $s_{nmfg\omega} = 0$, then constraints (5.43) and (5.44) ensure that $\rho_{nmfg\omega}^1 = 0$. Therefore, there is no need to add constraints (5.45). If $s_{nmfg\omega} = 1$, then constraints (5.46) and (5.47) guarantee that $\rho_{nmfg\omega}^1 \geq a_{m\omega} + \sum_{t \in T} \tau_{mt}z_{mt\omega}$. Since $d_{f\omega} \geq \rho_{nmfg\omega}^1$, we ensure that flight f cannot depart before the arrival time of flight m plus its turnaround time. This is the situation desired by the logical constraints (5.22).

5.2.2 Reformulation of Product of Two Binary Variables

In Section 5.1, we first give a two stage stochastic program for each alternative of the departure time vector $\bar{q}_j$. Then, by introducing a binary variable e_j , we present a unified model which selects an optimal alternative of the departure time vectors. However, due to product of binary variable e_j with the binary assignment decisions $z_{it\omega}$, computational burden of solving the problem may significantly increase. Therefore, we reformulate this nonlinear term using the following Proposition 5.2.

Proposition 5.2. *For $\beta_{ijt\omega} \geq 0$, $j \in J, i \in N, t \in T, \omega \in \Omega$, inequalities*

$$\beta_{ijt\omega} \leq e_j \tag{5.49}$$

$$\beta_{ijt\omega} \leq z_{it\omega} \tag{5.50}$$

$$\beta_{ijt\omega} \geq e_j + z_{it\omega} - 1 \quad (5.51)$$

$$\beta_{ijt\omega} \in \{0, 1\} \quad (5.52)$$

are valid for each term $\beta_{ijt\omega} = e_j z_{it\omega}$.

These are the special case of McCormick inequalities introduced in Section 5.2.1. If we take upper bounds 1 and lower bounds 0 for the binary variables, and then constructing the McCormick inequalities as defined in Section 5.2.1, we obtain the inequalities (5.49)-(5.52).

5.2.3 Reformulation of Product of a Binary Variable with Nonlinear Fuel and Emission Cost Function

The nonlinear functions of fuel and emission costs complicate two stage stochastic program significantly. Moreover, in the unified model UF, the multiplication of binary variable e_j with the nonlinear fuel and emission cost function brings an additional challenge to solve the problem. To efficiently handle these nonlinearities, we propose two alternative reformulations.

In Chapter 4, we propose strengthened conic reformulations for the nonlinear fuel burn function. The computations indicate that while the proposed strengthened conic reformulations are able to solve relatively large scale problems within reasonable times, an alternative conic formulation could not solve any of these instances within the same time limit. Therefore, in this chapter, we use the strengthened conic quadratic inequalities as proposed in Chapter 4. For the sake of completeness, here we redefine the fuel and emission cost function using the notation provided in this chapter. To simplify the presentation, we drop the indices of variables and parameters as follows:

$$c(f) = \begin{cases} c_o(c_1 \frac{1}{f} + c_2 \frac{1}{(f)^2} + c_3(f)^3 + c_4(f)^2) & \text{if } z = 1 \\ 0 & \text{if } z = 0, \end{cases}$$

where coefficients $c_j > 0$, $j=1, \dots, 4$, are expressed in terms of aircraft specific fuel

consumption and drag as well as the mass of aircraft, air density, gravitational acceleration.

To handle the nonlinear term $c(f)e$ in the unified model UF, we will first use the convexification results from the mixed integer conic quadratic optimization. Let us redefine the cruise time decisions as $f_{jit\omega}$ the cruise time of flight $i \in E \cup N$ with aircraft type $t \in T$ for alternative $j \in J$ under scenario $\omega \in \Omega$ and apply lower and upper bounds on the cruise time decisions by the following constraint:

$$f^\ell ze \leq f \leq f^u ze, \quad (5.53)$$

so that the decision f is controlled by the product ze . Note that the linearization of the product $\beta = ze$ is given in Section 5.2.2. Then, we can represent the nonlinear term $k(f) = c(f)e$ as follow:

$$k(f) = \begin{cases} c_o(c_1 \frac{1}{f} + c_2 \frac{1}{(f)^2} + c_3(f)^3 + c_4(f)^2) & \text{if } \beta = 1 \\ 0 & \text{if } \beta = 0. \end{cases}$$

The function $k(f)$ with the indicator variable β is discontinuous and its epigraph $E_k = \{(\beta, f, t) \in \{0, 1\} \times \mathbb{R}_+^2 : k(f) \leq t, f^\ell \beta \leq f \leq f^u \beta\}$ is nonconvex. The Proposition 5.3 describes the convex hull of E_k . The convexification of convex functions with indicators are discussed in detail in Aktürk et al. [87] and Günlük and Linderoth [88].

Proposition 5.3. *The convex hull of the set E_k can be expressed as*

$$t \geq c_o(c_1 \Gamma + c_2 \Theta + c_3 \Phi + c_4 \Psi) \quad (5.54)$$

$$(\beta)^2 \leq \Gamma f \quad (5.55)$$

$$(\beta)^4 \leq (f)^2 \Theta \beta \quad (5.56)$$

$$(f)^4 \leq (\beta)^2 \Phi f \quad (5.57)$$

$$(f)^2 \leq \Psi \beta. \quad (5.58)$$

Each inequality (5.55)–(5.58) can be represented by conic quadratic inequalities as shown in Section 3.1.4.

Proof is given in Section 3.1.4.

For the unified model UF, we give the conic reformulation below.

$$\begin{aligned}
\max \quad & \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N} \min(\mu_{i\omega}, \kappa_t) \delta_i \beta_{ijtw} p_{j\omega} - \sum_{t \in T} r_t \theta_t \quad (\text{MICQ1}) \\
& - \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N \cup E} c_o (c_1^{it} \Gamma_{ijtw} + c_2^{it} \Theta_{ijtw} + c_3^{it} \Phi_{ijtw} + c_4^{it} \Psi_{ijtw}) p_{j\omega} + \sum_{i \in E} c_{it_i} (f_{it_i}^u) \\
& - \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N} \max(0, \mu_{i\omega} - \kappa_t) \sigma_i \beta_{ijtw} p_{j\omega} - \phi \quad (5.59)
\end{aligned}$$

$$\text{s.t. (5.3) - (5.18), (5.23) - (5.27), (5.29) - (5.37) (5.40) - (5.42)}$$

$$d_{i\omega} + \sum_{j \in J} \sum_{t \in T} f_{j\text{it}\omega} + \eta_i = a_{i\omega}, \quad i \in E \cup N, \omega \in \Omega \quad (5.60)$$

$$f_{it}^\ell \beta_{ijtw} \leq f_{j\text{it}\omega} \leq f_{it}^u \beta_{ijtw}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.61)$$

$$(\beta_{ijtw})^2 \leq \Gamma_{ijtw} f_{j\text{it}\omega}, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.62)$$

$$(\beta_{ijtw})^4 \leq (f_{j\text{it}\omega})^2 \Theta_{ijtw} \beta_{ijtw}, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.63)$$

$$(f_{j\text{it}\omega})^4 \leq (\beta_{ijtw})^2 \Phi_{ijtw} f_{j\text{it}\omega}, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.64)$$

$$(f_{j\text{it}\omega})^2 \leq \Psi_{ijtw} \beta_{ijtw}, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.65)$$

$$\beta_{ijtw} \leq e_j, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.66)$$

$$\beta_{ijtw} \leq z_{it\omega}, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.67)$$

$$\beta_{ijtw} \geq e_j + z_{it\omega} - 1, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.68)$$

$$\beta_{ijtw} \in \{0, 1\}, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.69)$$

$$\rho_{in\omega}^2 \leq b_i^u y_{in\omega}, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.70)$$

$$\rho_{in\omega}^2 \geq b_i^\ell y_{in\omega}, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.71)$$

$$\rho_{in\omega}^2 \leq b_{i\omega} - b_i^\ell (1 - y_{in\omega}), \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.72)$$

$$\rho_{in\omega}^2 \geq b_{i\omega} - b_i^u (1 - y_{in\omega}), \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.73)$$

$$d_{n\omega} \geq \rho_{in\omega}^2, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.74)$$

$$\rho_{ni\omega}^3 \leq b_n^u y_{ni\omega}, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.75)$$

$$\rho_{ni\omega}^3 \geq b_n^\ell y_{ni\omega}, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.76)$$

$$\rho_{ni\omega}^3 \leq b_{n\omega} - b_n^\ell (1 - y_{ni\omega}), \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.77)$$

$$\rho_{ni\omega}^3 \geq b_{n\omega} - b_n^u (1 - y_{ni\omega}), \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.78)$$

$$d_{i\omega} \geq \rho_{ni\omega}^3, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.79)$$

$$\rho_{nmfg\omega}^1 \leq b_m^u s_{nmfg\omega}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.80)$$

$$\rho_{nmfg\omega}^1 \geq b_m^\ell s_{nmfg\omega}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.81)$$

$$\rho_{nmfg\omega}^1 \leq b_{m\omega} - b_m^\ell (1 - s_{nmfg\omega}), \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.82)$$

$$\rho_{nmfg\omega}^1 \geq b_{m\omega} - b_m^u (1 - s_{nmfg\omega}), \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.83)$$

$$d_{f\omega} \geq \rho_{nmfg\omega}^1, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.84)$$

$$b_{i\omega} = a_{i\omega} + \sum_{t \in T} \tau_i^t z_{it\omega}, \quad i \in E \cup N, \omega \in \Omega \quad (5.85)$$

The objective function (5.59) is slightly different than the original objective function of the proposed model UF. The original nonlinear objective (5.38) is represented by the new linear objective, conic constraints (5.62)- (5.65) and the linear constraints (5.66)-(5.69). The logical constraints (5.20)-(5.22) are replaced with the strong McCormick inequalities (5.70)-(5.85). The remaining constraints are the same as the original constraints of the proposed model UF.

Although the conic reformulation MICQ1 has a linear objective function, additional conic quadratic inequalities may bring a heavy computational burden as the number of alternative departure time decisions ($|J|$) increases. Therefore, we also introduce an alternative conic reformulation, called as MICQ2, with less number of conic quadratic inequalities. In the following two steps, we will construct the conic reformulation MICQ2.

First, we represent the nonlinear fuel and emission cost function $c(f)$ using mixed integer conic quadratic programming as described in Chapter 3 and 4. The convex hull of epigraph $E_c = \{(z, f, t) \in \{0, 1\} \times \mathbb{R}_+^2 : c(f) \leq t, f^\ell z \leq f \leq f^u z\}$ for the nonlinear fuel and emission cost function $c(f)$ is given in Proposition 5.4.

Proposition 5.4. *The convex hull of the set E_c can be expressed as*

$$t \geq c_o(c_1\zeta + c_2\vartheta + c_3\varphi + c_4\nu) \quad (5.86)$$

$$(z)^2 \leq \zeta f \quad (5.87)$$

$$(z)^4 \leq (f)^2 \vartheta z \quad (5.88)$$

$$(f)^4 \leq (z)^2 \varphi f \quad (5.89)$$

$$(f)^2 \leq \nu z. \quad (5.90)$$

Then, the nonlinear fuel and emission cost function $c(f)$ in the objective function (5.38) of unified model can be replaced with a linear term $c_o(c_1\zeta + c_2\vartheta + c_3\varphi + c_4\nu)$ by adding the hyperbolic inequalities (5.87)-(5.90) to the constraint set. Each inequality (5.87)-(5.90) can be represented by conic quadratic inequalities as described in Chapter 3. However, in the objective function of reformulation, there still exist nonlinearities due to product of binary variable e with the decisions $\zeta, \vartheta, \varphi, \nu$. Therefore, to efficiently handle these nonlinearities, in the second step, we will use McCormick estimators for each bilinear term $\psi^1 = \zeta e$, $\psi^2 = \vartheta e$, $\psi^3 = \varphi e$, $\psi^4 = \nu e$. In Proposition 5.5, we will only construct McCormick inequalities for the bilinear term $\psi^1 = \zeta e$. The rest of them could be constructed similarly.

Proposition 5.5. *The linear inequalities based on McCormick estimators*

$$\begin{aligned}\psi^1 &\leq \zeta^u e \\ \psi^1 &\geq \zeta^\ell e \\ \psi^1 &\leq \zeta - \zeta^\ell(1 - e) \\ \psi^1 &\geq \zeta - \zeta^u(1 - e),\end{aligned}$$

where $\zeta^u = \frac{1}{f^\ell}$, $\zeta^\ell = 0$, are valid for bilinear term $\psi = \zeta e$.

We provide an improved conic quadratic reformulation MICQ2 for the unified model UF below.

$$\begin{aligned}\max \quad & \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N} \min(\mu_{i\omega}, \kappa_t) \delta_i \beta_{ijtw} p_{j\omega} - \sum_{t \in T} r_t \theta_t \quad (\text{MICQ2}) \\ & - \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N \cup E} c_o (c_1^{it} \psi_{jit\omega}^1 + c_2^{it} \psi_{jit\omega}^2 + c_3^{it} \psi_{jit\omega}^3 + c_4^{it} \psi_{jit\omega}^4) p_{j\omega} + \sum_{i \in E} c_{it_i}(f_{it_i}^u) \\ & - \sum_{j \in J} \sum_{\omega \in \Omega} \sum_{t \in T} \sum_{i \in N} \max(0, \mu_{i\omega} - \kappa_t) \sigma_i \beta_{ijtw} p_{j\omega} - \phi \quad (5.91)\end{aligned}$$

$$\text{s.t. (5.3) - (5.19), (5.23) - (5.37), (5.40) - (5.42)}$$

$$(z_{it\omega})^2 \leq \zeta_{it\omega} f_{it\omega}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.92)$$

$$(z_{itw})^4 \leq (f_{itw})^2 \vartheta_{itw} z_{itw}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.93)$$

$$(f_{itw})^4 \leq (z_{itw})^2 \varphi_{itw} f_{itw}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.94)$$

$$(f_{itw})^2 \leq \nu_{itw} z_{itw}, \quad i \in E \cup N, t \in T, \omega \in \Omega \quad (5.95)$$

$$\beta_{ijtw} \leq e_j, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.96)$$

$$\beta_{ijtw} \leq z_{itw}, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.97)$$

$$\beta_{ijtw} \geq e_j + z_{itw} - 1, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.98)$$

$$\beta_{ijtw} \in \{0, 1\}, \quad j \in J, i \in N, t \in T, \omega \in \Omega \quad (5.99)$$

$$\rho_{inw}^2 \leq b_i^u y_{inw}, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.100)$$

$$\rho_{inw}^2 \geq b_i^\ell y_{inw}, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.101)$$

$$\rho_{inw}^2 \leq b_{iw} - b_i^\ell (1 - y_{inw}), \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.102)$$

$$\rho_{inw}^2 \geq b_{iw} - b_i^u (1 - y_{inw}), \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.103)$$

$$d_{nw} \geq \rho_{inw}^2, \quad i \in E_I, n \in N_O, \omega \in \Omega \quad (5.104)$$

$$\rho_{niw}^3 \leq b_n^u y_{niw}, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.105)$$

$$\rho_{niw}^3 \geq b_n^\ell y_{niw}, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.106)$$

$$\rho_{niw}^3 \leq b_{nw} - b_n^\ell (1 - y_{niw}), \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.107)$$

$$\rho_{niw}^3 \geq b_{nw} - b_n^u (1 - y_{niw}), \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.108)$$

$$d_{iw} \geq \rho_{niw}^3, \quad i \in E_O, n \in N_I, \omega \in \Omega \quad (5.109)$$

$$\rho_{nmfgw}^1 \leq b_m^u s_{nmfgw}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.110)$$

$$\rho_{nmfgw}^1 \geq b_m^\ell s_{nmfgw}, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.111)$$

$$\rho_{nmfgw}^1 \leq b_{mw} - b_m^\ell (1 - s_{nmfgw}), \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.112)$$

$$\rho_{nmfgw}^1 \geq b_{mw} - b_m^u (1 - s_{nmfgw}), \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.113)$$

$$d_{fw} \geq \rho_{nmfgw}^1, \quad (f, g) \in C_N, (n, m) \in C_N, \omega \in \Omega \quad (5.114)$$

$$b_{iw} = a_{iw} + \sum_{t \in T} \tau_i^t z_{itw}, \quad i \in E \cup N, \omega \in \Omega \quad (5.115)$$

$$\psi_{jitw}^1 \leq \zeta_{it}^u e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.116)$$

$$\psi_{jitw}^1 \geq \zeta_{it}^\ell e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.117)$$

$$\psi_{jitw}^1 \leq \zeta_{itw} - \zeta_{it}^\ell (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.118)$$

$$\psi_{jitw}^1 \geq \zeta_{itw} - \zeta_{it}^u (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.119)$$

$$\psi_{jitw}^2 \leq \vartheta_{it}^u e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.120)$$

$$\psi_{jitw}^2 \geq \vartheta_{it}^\ell e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.121)$$

$$\psi_{jitw}^2 \leq \vartheta_{itw} - \vartheta_{it}^\ell (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.122)$$

$$\psi_{jitw}^2 \geq \vartheta_{itw} - \vartheta_{it}^u (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.123)$$

$$\psi_{jitw}^3 \leq \varphi_{it}^u e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.124)$$

$$\psi_{jitw}^3 \geq \varphi_{it}^\ell e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.125)$$

$$\psi_{jitw}^3 \leq \varphi_{itw} - \varphi_{it}^\ell (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.126)$$

$$\psi_{jitw}^3 \geq \varphi_{itw} - \varphi_{it}^u (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.127)$$

$$\psi_{jitw}^4 \leq \nu_{it}^u e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.128)$$

$$\psi_{jitw}^4 \geq \nu_{it}^\ell e_j, \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.129)$$

$$\psi_{jitw}^4 \leq \nu_{itw} - \nu_{it}^\ell (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.130)$$

$$\psi_{jitw}^4 \geq \nu_{itw} - \nu_{it}^u (1 - e_j), \quad i \in E \cup N, t \in T, j \in J, \omega \in \Omega \quad (5.131)$$

In the improved conic quadratic reformulation MICQ2, the nonlinear objective function (5.38) of the unified model (UF) is represented by the linear objective function (5.91), hyperbolic inequalities (5.92)-(5.95) and linear inequalities (5.96)-(5.99), (5.116)-(5.131). The inequalities (5.96)-(5.99) linearize the product of binary variables z and e . The logical constraints in the unified model are replaced with the strong McCormick inequalities (5.100)-(5.115). Constraints (5.96)-(5.115) are common for both conic quadratic reformulations. Different than the conic quadratic reformulation MICQ1, there exists less number of hyperbolic inequalities (5.92)-(5.95) in MICQ2. On the other hand, there are additional McCormick inequalities (5.116)-(5.131), one for each bilinear term ζe , ϑe , φe , and νe . In the experimental study, we compare the performance of two conic quadratic reformulations.

5.3 Decomposition Algorithm

The logical constraints and the nonlinearities in the unified model bring computational challenges to solve the problem. To present the logical constraints

mathematically, in the previous section, we propose strong McCormick inequalities. Moreover, we introduce alternative reformulations to efficiently handle the nonlinearities in the unified model due to the product of a binary variable and the nonlinear fuel burn function. However, even the strong conic quadratic reformulations may not solve the problem for large number of scenarios in reasonable compute times. Therefore, in this section, we propose an exact algorithm to solve proposed two stage stochastic programming formulation with decision dependent probabilities. The algorithm depends on *evaluate-and-cut* procedure where a number of group subproblems are solved to produce candidate solutions. Each candidate solution is iteratively evaluated, and then cut off from the feasible region of the problem.

In recent years, the group subproblem approach proposed by Sandıkçı et al. [110] has been widely used to obtain a bound for the optimal value of the mixed integer two stage stochastic programming formulations. Using the scenario tree decomposition, they construct group subproblems that are similar to original mixed integer two stage stochastic programs, but defined over reduced number of scenarios. However, this approach is not applicable to solve the stochastic programs with decision dependent probabilities, since it does not consider the influence of departure time decisions on the probability distribution of random process. In this study, we extend the group subproblem approach to provide a valid upper bound for the proposed maximization problem with decision dependent uncertainty.

5.3.1 Upper Bound from the Group Subproblems

The aim is to derive subproblems similar to proposed two stage stochastic program with endogenous uncertainty. Therefore, each subproblem smaller in size could be solved with less computational effort. As discussed in Sandıkçı et al. [110], one can write a mathematical program similar to unified model for only a subset $\Omega_n \subseteq \Omega$ of scenarios. Before giving an extensive formulation for group subproblems, we give a closed formulation for proposed unified two stage stochastic

program with decision dependent uncertainty.

$$\begin{aligned} \max \quad & c^\top x + \sum_{j \in J} \sum_{\omega \in \Omega} d_{j\omega}^\top y_\omega p_{j\omega} e_j \\ \text{s.t.} \quad & Ax = b \end{aligned} \tag{5.132}$$

$$\sum_{j \in J} e_j = 1 \tag{5.133}$$

$$Tx + Wy_\omega = h, \quad \omega \in \Omega \tag{5.134}$$

$$y_\omega \in \mathbb{Y}_\omega, \quad \omega \in \Omega \tag{5.135}$$

$$e_j \in \{0, 1\}, \quad j \in J \tag{5.136}$$

$$x \in \mathbb{X}, \tag{5.137}$$

where $x \in \mathbb{X}$ is the first stage decision vector representing the collection of decisions π_{nmh} , r_t and x_n of the unified model UF. Thus, the first term of the objective function represents the second term of the unified model. In the second stage, a finite number of random events $\omega \in \Omega$ are realized. Each possible realization $\omega \in \Omega$ occurs with probability $p_{j\omega}$ for $j \in J$. As defined in the previous section, the probabilities depend on the first stage decisions $e_j \in \{0, 1\}$. The second stage decision vector $y_\omega \in \mathbb{Y}_\omega$ is the collection of the second stage decisions of the unified model UF. Therefore, the equation (5.134) corresponds to the collection of constraints (5.5)-(5.29) in the unified model UF. Similarly, constraints (5.132) represent the collection of the constraints (5.3), (5.4) and (5.41) in the unified model UF.

Note that the first term $c^\top x$ of the original problem can be replaced with the $\sum_{j \in J} \sum_{\omega \in \Omega} c^\top x p_{j\omega} e_j$, since $\sum_{j \in J} \sum_{\omega \in \Omega} p_{j\omega} e_j = 1$ and x does not depend on the realization of scenario ω .

Let Ω_n be a subset of scenarios such that $\bigcup_{n \in N} \Omega_n = \Omega$, where N is the indexed set. We also define adjusted probabilities, denoted by $\hat{p}_{j\omega}$ as

$$\hat{p}_{j\omega} = p_{j\omega} / mc_\omega, \tag{5.138}$$

where mc_ω counts the number of subsets Ω_n in which scenario ω appears. It can be defined as $mc_\omega = \sum_{n \in N} 1_{\Omega_n}(\omega)$, where $1_{\Omega_n}(\omega) = 1$ if $\omega \in \Omega_n$, and 0 otherwise.

Definition 5.1. The group subproblem for a block of scenario indices $\Omega_n \subseteq \Omega$ is

$$\begin{aligned}
\text{Obj}(UF(\Omega_n)) = \max \quad & \sum_{j \in J} \sum_{\omega \in \Omega_n} c^\top x \hat{p}_{j\omega} e_j + \sum_{j \in J} \sum_{\omega \in \Omega_n} d_\omega^\top y_\omega \hat{p}_{j\omega} e_j \\
\text{s.t.} \quad & Ax = b \\
& \sum_{j \in J} e_j = 1 \\
& Tx + Wy_\omega = h, \quad \omega \in \Omega_n \\
& y_\omega \in \mathbb{Y}_\omega, \quad \omega \in \Omega_n \\
& e_j \in \{0, 1\}, \quad j \in J.
\end{aligned}$$

Solving a number of group subproblems provides an upper bound for the optimal value of the original problem. Let $UB(N)$ denote the weighted average of the optimal values of group subproblems. It is defined as follows:

$$UB(N) = \sum_{n \in N} \text{Obj}(UF(\Omega_n)).$$

The following proposition states that $UB(N)$ is an upper bound for the optimal value ($\text{Obj}(UF)$) of the original problem.

Proposition 5.6. $UB(N) \geq \text{Obj}(UF)$ for any indexed set N satisfying $\bigcup_{n \in N} \Omega_n = \Omega$.

Proof. Let $x^*, (e_j^*)_{j \in J}, (y_\omega^*)_{\omega \in \Omega}$ be an optimal solution to the original problem UF . Substituting this solution into the objective function of the group subproblem $n \in N$, we find

$$\text{Obj}(UF(\Omega_n)) \geq \sum_{j \in J} \sum_{\omega \in \Omega_n} c^\top x^* \hat{p}_{j\omega} e_j^* + \sum_{j \in J} \sum_{\omega \in \Omega_n} d_\omega^\top y_\omega^* \hat{p}_{j\omega} e_j^*. \quad (5.139)$$

Summing over $n \in N$, we get

$$\sum_{n \in N} \text{Obj}(UF(\Omega_n)) \geq \sum_{n \in N} \sum_{j \in J} \sum_{\omega \in \Omega_n} c^\top x^* \hat{p}_{j\omega} e_j^* + \sum_{n \in N} \sum_{j \in J} \sum_{\omega \in \Omega_n} d_\omega^\top y_\omega^* \hat{p}_{j\omega} e_j^* \quad (5.140)$$

$$\begin{aligned}
&= \sum_{n \in N} \sum_{j \in J} \sum_{\omega \in \Omega_n} c^\top x^* \frac{p_{j\omega}}{mc_\omega} e_j^* + \sum_{n \in N} \sum_{j \in J} \sum_{\omega \in \Omega_n} d_\omega^\top y_\omega^* \frac{p_{j\omega}}{mc_\omega} e_j^* \\
&\quad (5.141)
\end{aligned}$$

$$= \sum_{j \in J} \sum_{\omega \in \Omega} c^\top x^* p_{j\omega} e_j^* + \sum_{j \in J} \sum_{\omega \in \Omega} d_\omega^\top y_\omega^* p_{j\omega} e_j^* \quad (5.142)$$

$$= c^\top x^* + \sum_{j \in J} \sum_{\omega \in \Omega} d_\omega^\top y_\omega^* p_{j\omega} e_j^* \quad (5.143)$$

$$= \text{Obj}(UF), \quad (5.144)$$

where equality (5.142) follows because $\bigcup_{n \in N} \Omega_n = \Omega$ and the number of Ω_n 's, $n \in N$ including the scenario ω is mc_ω . Equality (5.143) also follows since $\sum_{j \in J} \sum_{\omega \in \Omega} p_{j\omega} e_j^* = 1$.

□

Algorithm 4 Upper bounding algorithm

Initialization: Given a scenario set Ω and corresponding probabilities $p_{j\omega}, \omega \in \Omega, j \in J$, define the sets $\Omega_n, n \in N$.

for each group subproblem $n \in N$ **do**

 Solve the model UF with scenario set Ω_n .

 Set $\text{Obj}(UF(\Omega_n))$ to the the optimal value of the group subproblem n .

Calculate $UB(N) = \sum_{n \in N} \text{Obj}(UF(\Omega_n))$.

We provide the steps of **Upper bounding algorithm** in Algorithm 4. The quality of the upper bound significantly depends on the choice of subsets $\Omega_n, n \in N$. When the number of scenarios increases in each group subproblem, the algorithm provides a tighter upper bound. On the other hand, solving each group subproblem with increased number of scenarios may increase the computation times of the algorithm. Our scenario grouping strategies can be found in Appendix C.

5.3.2 Decomposition Algorithm

In this section, we provide an exact solution algorithm which decomposes the problem into group subproblems, and then evaluates their first stage solutions in the original problem. The crucial point of the algorithm is to cut off the explored solutions using 0-1 nature of the first stage decisions. The algorithm iteratively proposes candidate solutions, explores and then cuts them off until the

termination criteria is satisfied. The details of the **Decomposition algorithm** can be found in Algorithm 5.

The Algorithm 5 starts with solving group subproblems to provide an upper bound for the optimal value of the unified model. Let $\hat{e}(g) \in \mathbb{B}^{|J|}$ denote the optimal value of $(e_j)_{j \in J}$ in the solution of the group subproblem $g \in G$. Each $\hat{e}(g)$ can be viewed as a candidate first stage solution to the unified model UF. The idea here is to evaluate the candidate solutions $\hat{e}(g)$, $g \in G$ by iteratively imposing them into the unified model UF. The model UF with given $\hat{e}(g)$ turns into the model $F(\bar{q}_{j^*}(g))$, since $\hat{e}_j^*(g) = 1$ indicates that alternative j^* of departure time vectors is selected in the solution. Let us denote the selected vector by $\bar{q}_{j^*}(g)$. In order to solve $F(\bar{q}_{j^*})$, we apply **Lower bounding algorithm** in Algorithm 6. For fixed decision $\bar{q}_{j^*}(g)$, if the problem $F(\bar{q}_{j^*}(g))$ is feasible, then the optimal value of the problem $F(\bar{q}_{j^*}(g))$ gives a lower bound for the original unified model. We set the lower bound LB to the maximum of optimal values of $F(\bar{q}_{j^*}(g))$, one for each $\hat{e}(g) \in \bar{S}$, where $\bar{S}$ is the set of proposed candidate decisions. The important point is to cut off these explored solutions $\hat{e}(g)$, $g \in G$ from the feasible region $\hat{X}$ of the unified model. In order to accomplish this, we use 0-1 nature of the decisions $(e_j)_{j \in J}$ and add the following no good cuts suggested by Ahmed [58] to each group subproblem $g \in G$:

$$\sum_{j: \hat{e}_j(g)=1} (1 - e_j) + \sum_{j: \hat{e}_j(g)=0} e_j \geq 1, \quad \hat{e}(g) \in \hat{S}, \quad (5.145)$$

where $\hat{S} := \bigcup_{g \in G} \{\hat{e}(g)\}$, i.e. the set of evaluated candidate decisions.

Then, the algorithm proceeds with similar steps. Group subproblems are solved to propose new candidate solutions $\hat{e}(g)$. If the upper bound UB obtained by group subproblem approach is greater than or equal to lower bound LB , the algorithm continues to evaluate new candidate solutions $\hat{e}(g)$ to obtain a tighter lower bound, and then similarly cuts off explored solutions by adding no good cuts (5.146). If the upper bound UB is less than the lower bound LB , the algorithm terminates. When the algorithm terminates, lower bound LB is the optimal value of the unified model. Because in any iteration of the algorithm, the optimal solution is either evaluated or cut off from the feasible region $\hat{X}$. If

it is cut off, LB corresponds the optimal value of the unified model UF, since the optimal solution is already explored and recorded as LB . If it is still in the feasible region $\hat{X}$, then the upper bound UB would be greater than the lower bound LB . In this case, the algorithm proceeds to identify the optimal solution and evaluate it. The algorithm terminates in a finite number of steps, since the feasible set $\hat{X}$ is finite. The proof can be found in Ahmed [58].

Algorithm 5 Decomposition algorithm

```

Initialize: improvement  $\leftarrow$  true,  $UB \leftarrow +\infty$ ,  $LB \leftarrow -\infty$ ,  $\hat{S} \leftarrow \emptyset$ 
while improvement do
  Set  $\hat{X} \leftarrow X \setminus \hat{S}$ 
  Construct the set  $G$  of group subproblems
  Apply Upper Bounding Algorithm with set  $G$ 
  Set  $UB \leftarrow UB(G)$ 
  Set  $\bar{S} \leftarrow \emptyset$ 
  if  $UB \geq LB$  then
    for each group subproblem  $g \in G$  do
      Set  $\hat{e}(g)$  to the optimal value of  $(e_j)_{j \in J}$  in the solution of the group
      subproblem  $g$ 
      Set  $\hat{S} \leftarrow \hat{S} \cup \{\hat{e}(g)\}$ 
      Set  $\bar{S} \leftarrow \bar{S} \cup \{\hat{e}(g)\}$ 
      for each  $\hat{e}(g) \in \bar{S}$  do
        Find the  $j^*$  s.t.  $\hat{e}_j(g) = 1$ ,  $j \in J$ 
        Apply Lower Bounding Algorithm to optimally solve  $F(\bar{q}_{j^*})$ 
        Update  $LB \leftarrow$  return value of Lower Bounding Algorithm
    else
      improvement  $\leftarrow$  false

```

In each iteration of the Algorithm 5, we optimally solve the problem $F(\bar{q}_{j^*})$ using the exact solution algorithm in Algorithm 6. Notice that the problem $F(\bar{q}_{j^*})$ is a two stage stochastic programming problem with pure binary first stage decisions $(r_t)_{t \in T}$ and $(\pi_{nmh})_{(nm) \in C_N, h \in H}$. Therefore, we apply similar *evaluate-and-cut* procedure to obtain an optimal solution to the problem $F(\bar{q}_{j^*})$. In Algorithm 5, we again solve many group subproblems to propose candidate solutions $\{\hat{r}_s, \hat{\pi}_s\}$ for each group subproblem $s \in G^{rd}(\hat{e}_g)$. When we append the candidate solution $\{\hat{r}_s, \hat{\pi}_s\}$ to the problem $F(\bar{q}_{j^*})$, then the problem is scenario-wise decomposed. In order to mitigate computational difficulty, we iteratively solve scenario subproblems with given decisions $(\hat{e}_g, \hat{r}_s, \hat{\pi}_s)$, one for each scenario $\omega \in \Omega$.

We call each subproblem with given decisions $(\hat{e}_g, \hat{r}_s, \hat{\pi}_s)$ as $\detpr(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$. The optimal value of the problem with fixed decisions $(\hat{e}_g, \hat{r}_s, \hat{\pi}_s)$ can be calculated as $\sum_{\omega \in \Omega} f(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$. If the lower bound LB^{rd} found so far is less than $\sum_{\omega \in \Omega} f(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$, we set the lower bound LB^{rd} to this summation. Since the proposed solutions are evaluated and their objective values are recorded, the solutions $\{\hat{r}_s, \hat{\pi}_s\}$ can be cut off from the feasible region of the problem $F(\bar{q}_{j*})$ by adding the following *no-good cuts*:

$$\sum_{t: \hat{r}_t(s)=1} (1-r_t) + \sum_{nmh: \hat{\pi}_{nmh}(s)=1} (1-\pi_{nmh}) + \sum_{t: \hat{r}_t(s)=0} r_t + \sum_{nmh: \hat{\pi}_{nmh}(s)=0} \pi_{nmh} \geq 1, \quad \{\hat{r}_s, \hat{\pi}_s\} \in \hat{S}^{rd}, \quad (5.146)$$

where $\hat{S}^{rd} := \bigcup_{s \in G^{rd}(\hat{e}_g)} \{\hat{r}_s, \hat{\pi}_s\}$, i.e. the set of evaluated candidate decisions. The algorithm terminates when the upper bound UB^{rd} from the group subproblem approach is less than the lower bound LB found so far.

Algorithm 6 Lower bounding algorithm

Initialize: Given problem set of $F(\bar{q}_{j*})$, $\hat{e}(g)$ and LB , $\text{improvement}^{rd} \leftarrow \text{true}$, $UB^{rd} \leftarrow +\infty$, $LB^{rd} \leftarrow -\infty$, $\hat{S}^{rd} \leftarrow \emptyset$

while improvement^{rd} **do**

Set $\hat{X}^{rd}(\hat{e}_g) \leftarrow X^{rd}(\hat{e}_g) \setminus \hat{S}^{rd}$

Construct the set $G^{rd}(\hat{e}_g)$ of reduced group subproblems

Apply **Upper Bounding Algorithm** with set $G^{rd}(\hat{e}_g)$

Set $UB^{rd} \leftarrow UB(G^{rd}(\hat{e}_g))$

Set $\bar{S}^{rd} \leftarrow \emptyset$

if $UB^{rd} \geq LB$ **then**

for each reduced group subproblem $s \in G^{rd}(\hat{e}_g)$ **do**

Set $\hat{r}_s$ to the solution $(r_t)_{t \in T}$ of the group subproblem s

Set $\hat{\pi}_s$ to the solution $(\pi_{nmh})_{(nm) \in C_N, h \in H}$ of the group subproblem s

Set $\hat{S}^{rd} \leftarrow \hat{S}^{rd} \cup \{\hat{r}_s, \hat{\pi}_s\}$

Set $\bar{S}^{rd} \leftarrow \bar{S}^{rd} \cup \{\hat{r}_s, \hat{\pi}_s\}$

for each $\{\hat{r}_s, \hat{\pi}_s\} \in \bar{S}^{rd}$ **do**

for each $\omega \in \Omega$ **do**

Solve $\detpr(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$

Set $f(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$ to the optimal value of $\detpr(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega)$

Update $LB^{rd} \leftarrow \max(LB^{rd}, \sum_{\omega \in \Omega} f(\hat{e}_g, \hat{r}_s, \hat{\pi}_s, \omega))$

Update $LB \leftarrow \max(LB, LB^{rd})$

else

$\text{improvement}^{rd} \leftarrow \text{false}$

return LB

5.4 Computational Results

In this study, we propose a novel two stage stochastic programming formulation which introduces new flights to an existing airline flight schedule by considering the randomness in passenger demand. The important feature of the formulation is to reflect the changes in the probability distribution of random demand according to the departure time decisions given for new flights. However, this dependency causes nonlinearities in the objective function that significantly makes the problem computationally difficult to solve. Therefore, to mitigate computational effort, we propose alternative mixed integer strengthened conic quadratic reformulations, called as MICQ1 and MICQ2. Since the reformulations can be very time consuming to optimally solve the problem with large number of scenarios, we suggest a decomposition algorithm that provides the optimal value of the proposed model.

In computational study, we first analyze the effects of departure time decisions on the profit of an airline. Then, we analyze the benefits of including stochastic passenger demand on the profit of an airline. We test the ability of decomposition algorithm to handle large number of scenarios. Moreover, we compare the performance of alternative conic reformulations MICQ1 and MICQ2 within the decomposition algorithm.

5.4.1 Description of Data for Experimental Study

In this study, we introduce round-way trips $\{\text{ORD-IAH}\}$, $\{\text{ORD-DEN}\}$ and $\{\text{ORD-MSP}\}$. It is assumed that new outbound flight which departs from the ORD airport is immediately followed by its return flight to ORD airport. In this way, an aircraft does not need to fly with empty seats leading to deadhead cost. In constructed schedule, the new outbound and inbound flights of a pair are either operated by an aircraft that is leased or added into an existing flight schedule. We use a sample schedule extracted from the database “Airline On-Time Performance Data,” provided by the Bureau of Transportation Statistics of

the US Department of Transportation, BTS [99], and query the planned departure and arrival times of all United Airlines (UA) domestic flights for the date of 03/01/2018 from the database. Then, we remove the route of aircraft that does not visit ORD airport for this particular day, since the new flights are connected to existing flights at ORD airport. This is reasonable, since the route of an aircraft which does not visit ORD will not be affected by introducing new flights. The resulting sample schedule includes 300 flights operated by 81 aircraft.

In the experiments, departure time intervals $[d_i^\ell, d_i^u]$ for existing flights are computed by adding/subtracting ninety minutes to planned departure times of the sample schedule. The flight block times in the sample schedule are calculated by taking the difference between the scheduled arrival and departure times. Then, we assume that 30 minutes of a flight block time is non-cruise time (η_i) and remaining is cruise time upper bound (f_{it}^u) of the aircraft that is operated in the sample schedule. In experiments, we compress the cruise times at most 15%, hence $f_{it}^\ell = 0.85f_{it}^u$.

For the new flights, we generate alternative departure time vectors and these vectors define the probabilities of scenarios. In the experiments, for each new flight pair, we consider two alternative time zones as the most preferred and the alternative one. We assume that both outbound and inbound flights of the same trip depart in the same time zone. Therefore, such a pattern would yield a total of $2^3 = 8$ alternative departure time vectors. In Table 5.5, we list all departure time vectors that we consider in this study. Then, in Table 5.6, for each departure time vector, we specify the departure times of new flights under each time zone. Time zone 1 refers to the most preferred one and time zone 2 is relatively less preferred one.

In the optimal solution, if the decision vector 3 is selected, then the planned departure times of flight ORD-IAH and IAH-ORD become 7:00am and 11:00am, respectively. Similarly, departure times of flights ORD-DEN and DEN-ORD become 3:30 pm and 7:30 pm, respectively. However, to provide a feasible as well as more profitable schedule, the model may deviate the planned departure times by 30 min (γ) in the second stage.

Table 5.5: List of departure time vectors.

Dep. Time	New flight pairs		
Vector	{ORD-IAH}	{ORD-DEN}	{ORD-MSP}
1	zone 1	zone 1	zone 1
2	zone 1	zone 1	zone 2
3	zone 1	zone 2	zone 1
4	zone 2	zone 1	zone 1
5	zone 1	zone 2	zone 2
6	zone 2	zone 2	zone 1
7	zone 2	zone 1	zone 2
8	zone 2	zone 2	zone 2

Table 5.6: Departure times of new flights.

Flights	Departure time of new flights under	
	time zone 1	time zone 2
ORD-IAH	7:00 am	3:30 pm
IAH-ORD	11:00 am	7:30 pm
ORD-DEN	7:00 am	3:30 pm
DEN-ORD	11:00 am	7:30 pm
ORD-MSP	7:00 am	5:00 pm
MSP-ORD	9:30 am	7:30 pm

In this study, we assume that the demand levels for each new flight is either “high” or “low” and demands of flights are independent. Therefore, such a demand pattern would yield a total of $2^6 = 64$ demand scenarios. We generate the demand realizations uniformly between 120-160 and 160-200 for low and high setting, respectively. We assumed that time zone 1 specifies the desired departure times of an airline. In other words, an airline would expect to see high demand realizations with higher probability if the new flights depart in the time zone 1. Therefore, for each new flight, we assign the probability of observing high demand if the flight departs in the time zone 1 as 0.7, otherwise 0.3. Similarly, we assign the probability of observing low demand for each new flight, if it departs in the time zone 2 as 0.7, otherwise 0.3. For instance, for departure time vector 2, the probability of the scenario (ω^*) which observes high demand for all new flights can be calculated as follows:

$$p(2, \omega^*) = 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3 \times 0.3 = 0.021609.$$

In order to analyze the effect of problem parameters on the profit of an airline, we conduct a 2^k full-factorial experimental design. The experimental factors and

their levels are given in Table 4.3.

Table 5.3: Factor values.

Factor Description	Levels	
	Low	High
c_{fuel} (\$/kg)	0.6	1.2
σ_b (\$/passenger)	60	200

The fuel prices for lower and higher settings, respectively, are estimates based on the history of fuel prices obtained from IATA fuel price monitor [100], which shows a fluctuation between \$0.6/kg and \$1.2/kg during years 2008 – 2018. In Table 5.3, σ_b is a base value for the opportunity cost for each of the spilled passengers due to the insufficient seat capacity of the aircraft. Şafak et al. [51] express the cost of spilled passenger for each flight using airport congestion coefficients, e.g., favoring the populated markets as follows:

$$\sigma_i = \sigma_b(e_{O_i})(e_{D_i}), \quad i \in E \cup N, \quad (5.147)$$

where e_{O_i} and e_{D_i} represent the congestion coefficients for the origin and destination airports of flight $i \in N$, respectively. These coefficient values are provided in Şafak et al. [51].

We consider six different aircraft types and list the fuel burn related parameters, the corresponding maximum range cruise (MRC) speed, and the seat capacity in Table 5.4. The coefficients of the fuel burn function (5.2.3), c_1, c_2, c_3, c_4 , are calculated as specified in Şafak et al. [51] using the corresponding values of fuel burn related parameters in Table 5.4.

Table 5.4: Aircraft parameters.

Aircraft type	B727 228	B737 500	MD 83	A320 111	A320 212	B767 300
Capacity	134	122	148	172	180	218
Mass (kgs)	74000	50000	61200	62000	64000	135000
Surface(m^2)	157.9	105.4	118	122.4	122.6	283.3
$C_{D0,CR}$	0.018	0.018	0.0211	0.024	0.024	0.021
$C_{D2,CR}$	0.06	0.055	0.0468	0.0375	0.0375	0.049
Cf_1	0.53178	0.46	0.7462	0.94	0.94	0.763
Cf_2	276.72	300	638.59	50000	100000	1430
cf_{CR}	0.954	1.079	0.9505	1.095	1.06	1.0347
MRC speed (km/h)	867.6	859.2	867.6	855.15	868.79	876.70
τ_b^t (min)	32	36	26	28	30	40
θ_t (\$)	5,250	5,000	5,500	6,250	7,000	8,000

For a flight i , the aircraft turnaround time (τ_i^t) needed to prepare the aircraft for the next flight is estimated using the expression

$$\tau_i^t = \tau_b^t e_{D_i} \quad t \in T, \quad (5.148)$$

where τ_b^t is a base value for aircraft turnaround time. Therefore, turnaround time of an aircraft visiting a congested airport will take longer. The calculated aircraft turnaround times match with the aircraft turnaround times given in Arıkan et al. [108].

In the experiment, we generate the fares for new flights uniformly between \$120 and \$350, based on an analysis of the ticket prices of the flights in these trips for United Airlines. According to Eurocontrol's report [109] on dynamic cost indexing, total crew cost per block hour varies between \$280 and \$800. Therefore, we let the crew cost for each new flight as \$400/hr. Based on the expected block time of each new flight, the total crew cost for six new flights is calculated as \$6500. For each new flight, we also add \$1500 service cost including landing fees, ground handling fees, insurance fees, onboard services costs, etc.

5.4.2 Value of Decision Dependent Uncertainty

One of the main contributions of our study is to consider alternative departure times for each new flight. Since, the departure time decisions given in the first stage alter the probability distributions of random demand, identifying the most profitable departure time vector is a challenging problem. One departure time vector may provide higher probability of observing high demand, thereby providing an opportunity to capture more demand. On the other hand, because of the scheduling limitations defined by the departure time vector, the resulting schedule may achieve a lower profit due to fuel and emission costs incurred by accelerating the aircraft to open up space in the existing schedule. Due to trade-off between the revenue and the fuel burn, the selection of the best departure time vector is not a straightforward task. To identify the best vector, in Section 5.1, we provide a unified model UF. However, the model UF involves nonlinearities in the objective function due to decision dependent probabilities. Therefore, it is of

interest to study the benefit of considering alternative departure time vectors over the profit of an airline before dealing with heavy computational burden of solving stochastic programming formulation with decision dependent probabilities.

We design a 2^3 experimental study with two levels for each experimental factor. For each combination of factor levels, we solve problem $F(\bar{q}_j)$ and call its optimal value as $\text{Optval}(F(\bar{q}_j))$ for each departure time vector, $j \in J$. Then, we report the deviation of the objective function values from the optimal value of the unified model UF, called as $\text{Optval}(\text{UF})$. The deviation can be calculated as

$$\text{Profit reduction (\%)} = 100 \times \frac{\text{Optval}(\text{UF}) - \text{Optval}(F(\bar{q}_j))}{\text{Optval}(\text{UF})}.$$

Table 5.5 clearly shows the importance of considering alternative departure time vectors. Each departure time vector provides a different opportunity to operate new flights. For instance, departure time vector 1 operates all new flights in the most preferred time zone of an airline. Therefore, the probability of observing high demand would be higher, enabling the airline to increase the revenue by selling more tickets. In the optimal solution of departure time vector 1, the flight pair {ORD-DEN} is assigned to the B767 300 aircraft with the largest seat capacity to meet more passenger demand. On the other hand, B767 300 aircraft is the least fuel efficient aircraft with the fuel burn rate defined as 87 kg/min. In the solution, new flight ORD-DEN follows an existing flight already operated by B767 300 aircraft. Therefore, to operate the new flight ORD-DEN around 7:00 am, the existing flight has to arrive ORD airport before 7:00 am. Moreover, turnaround time should be ensured between the arrival time of existing flight and the departure time of flight ORD-DEN. In order to satisfy these scheduling limitations, B767 300 aircraft has to be accelerated to arrive ORD airport earlier, thus bringing an additional fuel burn cost, which is the significant cost component of airline operational costs.

In the optimal solution of departure time vector 1, leased aircraft can only operate the flight pair {ORD-IAH}, since the planned departure times of new flights determined by the decision vector 1 overlap. Although the type of leased aircraft is A320 212 with the fuel burn rate defined as 40 kg/min, the departure

time vector 1 could not take the advantage of leasing a fuel efficient aircraft.

If the fuel price is low, then the optimal solution coincides with the solution of departure time vector 1. However, if the fuel price increases, the expected profit with the departure time vector 1 is approximately 2.9% less than the optimal value. When the fuel price is high, the fuel consumed by B767 300 aircraft becomes significantly important. In order to reduce the fuel burn, the original model UF assigns both flight pairs {ORD-DEN} and {ORD-IAH} to the fuel efficient leased aircraft A320 212 by choosing the departure time vector 3 in the optimal solution. In other words, the model UF assigns new flight pair {ORD-DEN} to the least preferred zone 2 in order to increase the utilization of fuel efficient aircraft. On the other hand, with this assignment, model UF decreases expected revenue from new flight pair {ORD-DEN} due to the low probability of observing high demand in time zone 2. In addition, model UF incurs an additional expected cost of spilled passengers due to insufficient seat capacity of the A320 212 aircraft. In Figure 5.1, it is clearly seen that the fuel burn reduction of the optimal solution compared to the departure time vector 1 can compensate the additional expected cost of spilled passengers and the reduction in the revenue, when fuel price is high.

Table 5.5: Value of decision dependent uncertainty.

		Deviation (%) from the optimal value of decision vectors							
σ_b	c_{fuel}	1	2	3	4	5	6	7	8
60	0.6	0.0	3.2	0.6	1.3	6.6	7.0	7.4	16.7
	1.2	2.8	6.3	0.0	0.8	6.3	6.7	7.2	17.7
200	0.6	0.0	2.4	0.3	1.5	8.6	9.5	9.5	23.8
	1.2	3.3	5.9	0.0	1.3	8.1	9.1	10.0	25.3

Table 5.5 also indicates that compared to the optimal solution of UF model, the departure time vector alternatives 5, 6 and 7 decrease the profit of an airline by 8% on the average. These vectors reduce the expected revenue from ticket sales, since they operate two new flight pairs in the least preferred times when the probability of observing high demand is lower. When the departure time vector 7 is selected, the reduction in the profit of an airline may reach to 25.3% for high setting of experimental factors. There are two main reasons of this significant

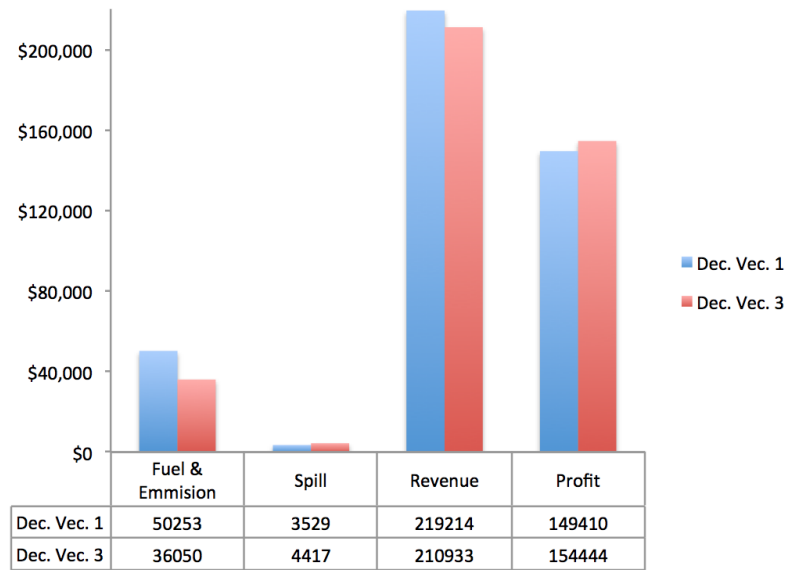


Figure 5.1: The comparison of cost components and the profit.

reduction. One is the overlapping departure time of new flights so that leased aircraft can only operate one new flight pair. The remaining new flights can only be accommodated into the existing flight schedule by shifting the current flights' departure times and compressing the cruise times which may incur additional fuel and emission costs significantly. The second reason of the reduction is that the vector 7 defines the departure times of all new flights in the least preferred time zone 2 so that the probability of observing high demand is lower for each new flight. This leads to a lower expected revenue.

In conclusion, this experiment clearly shows the need of unified model where all departure time vectors are considered to accommodate new flights. Because of the trade off between the revenue and fuel burn in the problem, finding the optimal solution is not always trivial as choosing the vector 1 which maximizes the probability of observing high demand. As it is seen in the experiment, the profit of an airline with vector 1 could be 3.3% worse than the optimal value. In the context of airline operations, 3.3% reduction in the profit may cause millions of dollar loss.

5.4.3 Decision Dependent Value of Stochastic Solution

In the airline industry, estimation of the passenger demand during the planning stage is one of the most challenging and crucial tasks. Overestimating demand may result in assigning a larger aircraft than needed. However, the larger aircraft would presumably incur a higher operational cost. On the other hand, in a highly competitive market, underestimating demand may cause reduction in the market share of an airline. Therefore, one of the main contributions of our study is to capture the variations in passenger demand using a two stage stochastic programming approach.

To analyze the impact of uncertain passenger demand on the profit of an airline, we compare our stochastic model called as model UF with a deterministic approach in which the passenger demand data are replaced with their expected value. Since each departure time vector provides a different probability distribution for random demand, for each departure time vector, we separately calculate the expected demand using the associated probabilities of corresponding departure time vector. Then, we formulate the deterministic approach by replacing the demand values in the UF model with computed expected demand values. The deterministic approach also chooses the best departure time vector. We take the optimal first stage solution of the deterministic approach and then impose upon the model UF to calculate the expected profit for the deterministic approach. In Table 5.6, we report the improvement in the profit of an airline for the UF model compared to the deterministic approach. The improvement can be calculated as

$$\text{Profit improvement (\%)} = 100 \times \frac{\text{Optval (UF)} - \text{Optval (Deterministic Approach)}}{\text{Optval (Deterministic Approach)}}.$$

Considering many potential demand scenarios may lead to large decisions trees, potentially resulting in significant computation time. Despite the complexity of model UF, Table 5.6 indicates that model UF achieves an average of 7.2% improvement in the profit of an airline over a deterministic approach. As the cost for each spilled passenger increases, the profit improvement reaches to 9.7%, since the deterministic approach fails to adjust the decisions according to the fluctuations in passenger demand.

Table 5.6: Decision dependent value of stochastic solution.

σ_b	c_{fuel}	Profit (\$) of		
		Deterministic Appr.	UF	Improvement
60	0.6	167,350	175,761	5.0%
	1.2	150,706	157,536	4.5%
200	0.6	157,909	173,290	9.7%
	1.2	141,265	154,444	9.3%

5.4.4 What-if Analysis for Leasing Option

In the proposed study, we introduce a strategic decision of leasing an aircraft. This option provides an advantage of reducing the fuel burn and capturing more demand by a flexibility of leasing an aircraft with the right type. In order to analyze the benefits of leasing option, we solve the unified model by closing leasing option and then compare its results with the optimal value of the original model UF. We conduct a 2³ experimental study with two levels for each experimental factor. For each combination of the factor levels, we generate five instances with different ticket price values, resulting in a total of 20 instances. We solve each instance by closing leasing option, and then calculate the reduction in the profit of an airline as

$$\text{Profit reduction (\%)} = 100 \times \frac{\text{Optval (UF)} - \text{Optval (UF w/o lease)}}{\text{Optval (UF)}}.$$

Table 5.7 summarizes the results for 20 instances. Observe that the reduction in the profit significantly increases as the fuel price increases. In the model without leasing option, the sufficient times for new flights in the existing schedule could be only opened up by compressing the cruise times of flights in addition to re-timing of departure times. However, the fuel efficient aircraft may not be available at the time of the departures of new flights. In this case, accelerating an aircraft could be extremely costly when the fuel price is high. As the fuel expenses are a major cost component of airlines, the expected profit significantly reduces.

Similarly, as the cost for each spilled passenger increases, the reduction in the profit significantly increases. Due to the scheduling limitations, the right type of

aircraft with the right seat capacity may not be available at the departure times of new flights. Therefore, new flights could be operated with an aircraft which has smaller seat capacity than needed, thus resulting in additional spilled cost and revenue loss. In addition, the model without leasing option may choose a departure time vector which gives a higher probability of observing low demand, thus giving a reduction in the expected revenue. The reason of choosing less profitable departure time vector is based on the availability of existing aircraft and the trade off between the fuel burn and revenue.

In summary, the experiment clearly shows that leasing option provides a greater flexibility to assign new flights to the right type of aircraft, thus achieving a greater profit. When leasing option is not considered, the profit reduces by 6.5% on the average. If the fuel price and cost for each spilled passenger increase, then the reduction reaches to 15.1%. For a high fuel price case, the unified model with leasing option assigns two new flight pairs to the fuel efficient aircraft that is leased by departing one of the new flight pairs in the time zone 1 and another in the time zone 2. Since the probability of observing low demand in the time zone 2 is high, the revenue of the second flight pair could be decreased. On the other hand, the utilization of the fuel efficient leased aircraft is increased, thereby operating cost of new flight pairs is significantly reduced. This is a great advantage of a leasing option to increase the profit of an airline.

Table 5.7: Profit reduction of UF w/o lease over UF.

		Profit Impr. (%)		
		min	avg	max
c_{fuel}	Low	2.4	4.8	8.4
	High	5.1	8.2	15.1
σ_b	Low	2.4	4.4	6.2
	High	5.2	8.6	15.1
avg		6.5		

5.4.5 Analysis of the Reformulations

In the unified model, there exist nonlinearities due to multiplication of binary decisions with the nonlinear fuel and emission cost function. In order to handle

these nonlinearities, we propose alternative conic formulations called as MICQ1 and MICQ2. In this section, we will test performances of alternative conic reformulations within the decomposition algorithm 5 provided to solve the stochastic programs with decision dependent uncertainty. If we formulate the group subproblems as MICQ1 in Algorithm 5, we will call it as *Decomposition algorithm with MICQ1*. If we use MICQ2, we refer it as *Decomposition algorithm with MICQ2*. Remember that the group subproblem approach provides an upper bound for our maximization problem. Therefore, in order to mitigate the computational effort of solving group subproblems, we relax binary decisions except the decision e_j in each group subproblem, since the relaxation still gives a valid upper bound. The reason of keeping e_j 's as binary decisions is that they are later imposed upon the unified model UF to obtain a lower bound.

We again design a 2^3 experimental study with two levels for each experimental factor. For each combination of the factor levels, we solve four instances using the decomposition algorithm with alternative conic reformulations. Table 5.8 shows the superiority of MICQ2 over the MICQ1. The average CPU time for the decomposition algorithm with MICQ1 exceeds 24 hours to solve the problem instance. Remember that MICQ2 has less number of conic quadratic inequalities, although it has additional variables and McCormick inequalities compared to MICQ1. The reduction in the number of conic quadratic inequalities enables us to solve the problem instances 8 times faster. Average CPU time for the decomposition algorithm with MICQ2 is 3.6 hours. The significant reduction in CPU times is promising to solve larger problem instances. This is a great advantage to attempt to solve the larger problem instances. Thus, in the next section, we will consider the problem instances with an increased number of departure time vectors.

Table 5.8: Comparison of conic formulations.

		Decomposition Alg. with					
		MICQ1			MICQ2		
		CPU (hr)			CPU (hr)		
σ_b	c_{fuel}	min	avg	max	min	avg	max
60	0.6	28.5	36.7	41.3	3.4	3.5	3.6
	1.2	14.5	20.7	23.8	3.5	3.8	4.1
200	0.6	25.8	30.1	35.0	3.3	3.5	3.6
	1.2	14.3	18.8	21.1	3.5	3.6	3.8

5.4.6 Analysis on Increased Number of Departure Time Vectors

In the experiments above, we consider two different time zones for each new flight pair. However, a schedule planner may want to consider more alternative departure times that would potentially yield more profitable schedule. In order to see the impact of the number of departure time vectors on the profit of an airline, we design a new 2^3 experimental study.

In new experiment, for new flight pairs {ORD-DEN} and {ORD-MSP}, we consider an extra time zone. In new time zone, flight ORD-DEN can depart at 5:30 am and the departure time for new flight DEN-ORD can be announced as 9:00 am. Similarly, the departure times for new flights ORD-MSP and MSP-ORD can be announced as 6:00 am and 8:00 am, respectively. We assume that probability of observing high demand is 0.65 if the flights are determined to depart at new time zone. In summary, in new experiments, we consider three time zones for new flight pairs {ORD-DEN} and {ORD-MSP} and two time zones for round trip {ORD-IAH}, resulting in 18 departure time vectors. In Table 5.9, we compare the profit of the schedules with different number of departure time vectors. As expected, increased number of departure time vectors provides alternative solutions, potentially increasing the profit of an airline. In these problem instances, the profit increases by 2.5% on average.

In Table 5.10, we report the CPU time requirements for solving the instances with 18 departure time vectors. We solve the instances using the decomposition algorithm with MICQ2. The results show the high performance of the algorithm.

Table 5.9: Comparison of the schedules with different number of departure time vectors.

σ_b	c_{fuel}	Profit (\$) with		
		18 vectors	8 vectors	Improvement
60	0.6	179,429	175,761	2.1%
	1.2	162,268	157,536	3.0%
200	0.6	177,477	173,290	2.4%
	1.2	158,628	154,444	2.7%

For a schedule planning problem which is usually solved once in advance of departures, an average of 11.7 CPU hours can be reasonable.

Table 5.10: CPU time analysis.

σ_b	c_{fuel}	CPU (hr)
60	0.6	11.9
	1.2	11.7
200	0.6	11.9
	1.2	11.4

5.5 Conclusion

This study introduces new flights into an existing flight schedule by considering the randomness in passenger demand. One could increase the probability of observing high demand, when the departure time decisions for new flights are made at their desired times. The sufficient time for new flights in the existing schedule could be opened up by re-timing the departures and compressing the cruise times. The study also takes an advantage of the flexibility offered by leasing an aircraft to operate new flights. The goal of the decision maker is to maximize the profit of an airline by trading off the fuel and emission costs incurred to operate new flights with the revenue. The problem is modelled as a two stage stochastic mixed integer nonlinear program in which the departure times given in the first stage alter the probability distribution of random demand and the second stage makes the aircraft assignment to new flights based on the availabilities of aircraft within the interval of departure times of new flights.

We reformulate the nonlinearities in the objective using mixed integer conic

quadratic programming and McCormick estimators. Then, we devise an exact solution algorithm based on scenario group wise decomposition to handle the large number of scenarios of the two stage stochastic MINLPs with decision dependent probabilities. In the experiments, the decomposition algorithm can solve instances with 300 flights that are generated using publicly available data of one of U.S. major airline within 3.5 hours CPU times. The experiments also indicates that the proposed stochastic formulation yields an improvement of 7.2%, on the average, in the profit of an airline over a deterministic approach.

This study leads to several potential research directions. One can take advantages of parallel computing technology within the scenario group-wise decomposition algorithm so that the number of scenarios explored could be increased. Another extension of this study would be addressing a strategic accept or reject decision to operate new flights. It is important to note that the passenger demand is only revealed when new flights are accepted to be operated.

Chapter 6

Conclusion

6.1 Summary

In this thesis, we consider several schedule planning problems arising in the airline industry. In Chapter 3, we address an airline scheduling problem which determines strategic planning departure times decisions, tactical fleet and routing decisions and more operational flight timing decisions by considering the randomness in passenger demand and non-cruise times. This study compensates the increase in non-cruise time variations by accelerating the aircraft, thus incurring an additional fuel and emission cost. In order to reduce the fuel burn, we consider the fuel efficiency of the aircraft while making the fleet assignment decisions. Assigning a fuel efficient but smaller aircraft could be less costly when the reduction in the fuel burn compensates the increase in the spilled costs due to the insufficient seat capacity. Therefore, in this study, we trade off between the fuel and emission costs with the cost of spilled passengers in an uncertain environment.

We model the problem as a three stage stochastic mixed integer nonlinear program. It is an extremely challenging problem with hundreds of binary variables and nonlinear fuel and emission costs. To mitigate the computational effort, we devise a scenario group-wise decomposition algorithm to provide a lower and

upper bound for the optimal value of the problem. We further improve the performance of the algorithm by solving each group subproblem with a cutting plane algorithm. We handle the nonlinear subproblems using second order cone duality within the cutting plane algorithm.

In Chapter 4, we introduce a novel problem on accommodating new flights into an existing airline flight schedule. This study suggests two alternatives to introduce new flights. The first approach changes the departure times of existing flights or increases the aircraft cruise speed to shorten the flight times. However, this approach trades off the fuel cost with the spilled passenger costs of new flights as discussed in the earlier study. Therefore, in order to address this trade-off, we introduce a second approach which considers an aircraft swapping alternative together with the aircraft cruise speed control approach. Although these strategies have been previously used in the literature, here for the first time they are implemented to allow for introducing new flights into the schedule.

The additional binary swapping and assignment decisions of the second approach together with the nonlinear fuel and emission costs make the problem significantly more difficult to solve. Therefore, we start with handling the nonlinear functions of fuel and emission cost by proposing a strong mixed integer conic quadratic (MICQ) formulation. However, formulations still involve logical constraints to ensure the precedence relations of aircraft. We first introduce a simple linearization of the logical constraints using the well known Big-M method. Then we improve and strengthen the original MICQ formulation using McCormick estimators. We test the performance of the reformulations on real data set of United Airlines including 300 domestic flights and 81 aircraft. In the experiment, we compare the strong MICQ reformulation with an alternative MICQ reformulation. Alternative MICQ reformulation times out for all instances. When we use the strengthened MICQ reformulation with Big-M method, then most of the instances could be solved to optimality within the time limit. When we add McCormick inequalities to the strengthened MICQ reformulation, then the results are significantly better. All instances could be solved optimally even faster than the best alternative reformulation.

In Chapter 5, we extend the problem defined in Chapter 4 by including the randomness of passenger demand for new flights. We also introduce a flexibility offered by leasing an aircraft to operate new flights. In this study, the probability of observing high demand can be increased by allowing the new flights to depart at their desired times. We model the problem as a two stage stochastic mixed integer nonlinear program where the first stage determines the departure time of new flights and the aircraft that is leased. In the second stage, new flights are either assigned to an existing fleet resource by re-timing the flight departures and compressing the cruise times or operated by a leased aircraft.

In Chapter 5, we extend the group subproblem approach defined in Chapter 3 to solve the two stage stochastic programs with decision dependent probabilities. Then, we use these findings in an exact solution algorithm which proposes candidate solutions by solving group subproblems and then cuts-off the explored solutions using the 0-1 nature of the first stage solutions. The nonlinearities in the group subproblems due to multiplication of binary variable with the nonlinear fuel and emission costs are handled by mixed integer conic quadratic programming and McCormick estimators.

6.2 Future Remarks

Sherali et al. [17] emphasize the sensitivity of airline optimization decisions to fuel burn. Therefore, aircraft cruise speed control which has a direct impact on controlling the fuel burn should have been given necessary importance in order to increase airline's profit. A slot allocation problem that has not yet taken the advantage of cruise time controllability can be addressed. When an airport in the US is about to be subject to inclement weather that reduces the number of usable runways, the Federal Aviation Administration (FAA) institutes a ground delay program (GDP). Flights that were to arrive at the affected airport during the heavy weather are issued delayed departure times at their origin. In addition to GDP, the aircraft cruise speed control strategy to handle a short period of air traffic congestion can be suggested. On the other hand, airborne

delay might be costly due to the high fuel prices. To address this, an additional mechanism, namely aircraft re-routing, can be considered to handle the large number of excess demand. Furthermore, to keep the fuel cost manageable in response to the fluctuations in fuel prices, A stochastic model where many of potential fuel prices are analyzed can be provided. A stochastic model which schedules the arrivals of flights using aircraft cruise speed control and re-routing strategies within GDP can be very interesting for practitioners and researchers.

Another research area in which the advantage of aircraft cruise speed can be taken is airport slot exchange mechanism. Within collaborative decision making, FAA considers the allocation of arrival slots to airlines which gives permission for an airline to either reassign any of its flights within its own part of the landing schedule, or trade them with other airlines. For instance, after slots are allocated, a flight delay results in a situation of newly vacated landing slot. On one hand, airline can take the advantage of accelerating the aircraft of its subsequent flight to shift its arrival into its own vacant slot and then, continue to accelerate the aircraft of subsequent flights as necessary to shift them into earlier slots and reassign the delayed flight to a later slot. If this strategy is not feasible, then airline can trade the vacant slot for a slot belonging to another airline. However, new slot obtained by trading still may not be feasible for the flights of the aircraft. In this case, airline again takes the advantage of cruise time controllability to assign either the delayed flight or any of its flights to the slot gained by trading. A nonlinear mathematical model with cruise time controllability to evaluate possible benefits that can be obtained from slot trade mechanism could be developed. In this work, mixed integer conic quadratic programming techniques to improve and strengthen the formulations can be further explored.

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Appendix A

Generation of Potential Demand Realizations

The fleet assignment problem is typically modeled by considering expected demand values based on historical data and forecasts. However, because of the stochastic nature of passenger demand, certain realizations may result in either spilled passengers, who cannot be accommodated due to under capacity, or unsold seats and presumably higher operational costs. On the other hand, as the competition increases and the profit margins of airlines get in threat, they have to produce solutions that perform well relative to this uncertainty in demand. Hence, we explicitly consider several potential demand scenarios in making fleet-ing and routing decisions.

We segment passenger demand volume with respect to the flight departure times. Passengers may not want to waste their valuable time on a flight during business hours, e.g., business travellers would prefer to depart early in the morning and going back late at night. Hence, in this segmentation, we consider different demand volumes for different departure time intervals to better reflect reality. We segment three different levels of passenger demand by the flight departure times as follows:

- 1st segment: 6:00am - 8:59am and after 5:00pm
2nd segment: 9:00am - 11:59am and 3:00pm-4:59pm
3rd segment: 12:00 noon - 2:59pm.

In this manner, we refer time interval (6:00 - 8:59am) and time after 5pm as peak times which are mostly preferred by passengers. After 9am, passenger demand slightly decreases and departures at noon are unfavourable. Afterwards, passenger demand increases again and reaches a pick point after 5pm.

Moreover, we consider stochastic passenger demand for three different origin-destination points, ORD-DFW and ORD-LGA . To estimate potential passenger demand realizations, we analyze the seat capacities of assigned aircraft to these flights for a major U.S. airline. We also analyze passenger demand data of [111] which are constructed by extracting 10% of the number of passengers provided by airline origin and destination survey (DB1B) [112]. By combining these analyses, we generate two levels of demand realizations for two origin-destination points, ORD-DFW and ORD-LGA as in Table A.1.

	ORD - DFW		ORD - LGA	
	Realization		Realization	
	Low (L)	High (H)	Low (L)	High (H)
1 st Seg.	161	200	121	140
2 nd Seg.	121	160	101	120
3 rd Seg.	81	120	81	100
Probability	0.5	0.5	0.4	0.6

Table A.1: Demand realizations & associated probabilities

It is important to clarify that, under low realization for ORD-DFW flight, if the departure time is within 6am - 08:59am or after 5pm, we would consider passenger demand as 161. If departure time is within 9am - 11:59am or 3pm-4:59pm, then realization would be 121.

Appendix B

Generation of Potential Non-cruise Time Realizations

In this study, we separate flight block times into two components as cruise and non-cruise times. In non-cruise times, we include departure and arrival delays as well as the taxi-in and taxi-out times, which are all highly uncertain due to airport congestions. To handle these uncertainties, we analyze potential non-cruise time scenarios, and then set a cruise speed for each non-cruise time realization. Cruise time for each flight is a decision variable. We may set a higher cruise speed to compensate for variations in non-cruise time so as to ensure passenger connections on the subsequent flights and set maximum range cruise (MRC) speed for conservation of fuel.

In this study, we consider uncertain non-cruise times for flights departing from and arriving to Chicago O’Hare International Airport (ORD) and Dallas/Fort Worth International Airport (DFW), which are used as hub airports of major U.S. airlines. To generate potential non-cruise time scenarios, we extracted flight information from the database “Airline On-Time Statistic,” provided by the Bureau of Transportation Statistics of the US Department of Transportation, BTS [113] We queried the “departure delays,” “taxi-in times,” “taxi-out times” and “arrival delays” of the flights of a commercial airline for three months. We then

clustered data with respect to the observed values queried and generated a value to represent each cluster. Representative value for each cluster is generated by taking weighted average of all observed values in a cluster, where the weights correspond to observation frequencies over three months. We assign probabilities for each representative value by taking proportion of total observation frequencies in a cluster to the number of all observations in the data so that the sum of the probabilities, each one is associated with one cluster, is equal to one.

In the experimental study, we consider four realizations for departure delay, taxi-out time, taxi-in time and arrival delay for ORD airport. In other words, we make four clusters and get one representative value for each of the clusters. These representative values can be viewed as potential realizations. We provide these four realizations and their associated probabilities in Table B.1. It is important to note that we assume that the realization levels for all departure delay, taxi-out, arrival delay and taxi-in times at the same airport are positively correlated, taking some “lowest”, “low”, “high” or “highest” frequency level simultaneously. Because, these components of non-cruise times are exposed to the same weather condition and airport congestion.

	Realization with			
	highest frequency (HH)	high frequency (H)	low frequency (L)	lowest frequency (LL)
Dep. delay (min)	0	10	30	50
Taxi-out (min)	15	25	10	35
Arr. delay (min)	0	10	30	50
Taxi-in (min)	15	25	10	35
Probability	0.65	0.20	0.1	0.05

Table B.1: Non-cruise time scenarios for ORD

For DFW airport, we consider two different realizations. Table B.2 shows realizations and associated probabilities for DFW airport.

To clarify how we generate a non-cruise time of a flight, let’s consider a flight departing from Chicago O’Hare International Airport (ORD) and arriving to Dallas/Fort Worth International Airport (DFW). Under a scenario ω^* involving realization with low frequency for ORD and realization with high frequency for DFW, observed values of departure delay, taxi-out, arrival delay, taxi-in times are

	Realization with	
	high frequency (H)	low frequency (L)
Dep. delay (min)	5	40
Taxi-out (min)	15	35
Arr. delay (min)	5	40
Taxi-in (min)	15	35
Probability	0.85	0.15

Table B.2: Non-cruise time scenarios for DFW

equal to 30, 10, 30 and 10 for ORD and 5, 15, 5 and 15 for DFW, respectively. Then, non-cruise time of this flight is calculated as follows:

$$\begin{aligned}
\eta_i(\omega^*) &= \text{dep. delay}_{ORD}^L + \text{taxi-out}_{ORD}^L + \text{taxi-in}_{DFW}^H + \text{arr. delay}_{DFW}^H \\
&= 30 + 10 + 15 + 5 = 60.
\end{aligned}
\tag{B.1}$$

The associated probability for scenario ω^* can be calculated as follows:

$$\begin{aligned}
p(\omega^*) &= (prob_{ORD}^L) \cdot (prob_{DFW}^H) \\
&= (0.1) \cdot (0.85) = 0.085.
\end{aligned}
\tag{B.2}$$

Appendix C

Scenario Generation and Selection Procedures

We use the following notation through the scenario generation procedure.

- J' : set of flights that have random demand realizations
- $D(j)$: set of demand realizations for each flight $j \in J'$, indexed by $d_1(j), d_2(j), \dots, d_{|D(j)|}(j)$
- B' : set of airports that have random non-cruise times
- $A(b)$: set of arrival delay, departure delay, taxi-out and taxi-in time realizations for each airport $b \in B'$, indexed by $a_1(b), a_2(b) \dots, a_{|A(b)|}(b)$

Let demand and non-cruise time realizations for each set $D(j)$, $j \in J'$ and $A(b)$, $b \in B'$, respectively, be in increasing order with respect to indices. Then, the following pseudocode generates the scenarios.

Since, the realizations are in increasing order with respect to indices, the first scenario generated (ω_1) represents lowest demand realizations for each flight $j \in J'$ and non-cruise time (delay) realizations with lowest frequency for each airport $b \in B'$. Similarly, the last scenario generated represents the highest realizations.

Algorithm 7 Scenario generation procedure

```

Initialization:  $index \leftarrow 0$ ;
for  $i = 1$  to  $|D(1)|$  do
  for  $j = 1$  to  $|D(2)|$  do
    .
    .
    for  $k = 1$  to  $|D(|J'|)|$  do
      for  $l = 1$  to  $|A(1)|$  do
        for  $m = 1$  to  $|A(2)|$  do
          .
          .
          for  $r = 1$  to  $|A(|B'|)|$  do
             $index \leftarrow index + 1$ ;
             $\omega_{index} \leftarrow (d_i(1), d_j(2), \dots, d_k(|J'|), a_l(1), a_m(2), \dots, a_r(|B'|))$ ;

```

Let Ω_n denote the set of scenarios in group subproblem $n \in N$. We generate subsets $\Omega_n \subset \Omega$ of scenarios such that $\bigcup_{n \in N} \Omega_n = \Omega$. In Procedure P1, we consider two scenarios in each set Ω_n for all $n \in N$. We put the first and last scenarios $(\omega_1, \omega_{|\Omega|})$ in the first set Ω_1 , then put the second first and last scenarios $(\omega_2, \omega_{|\Omega|-1})$ in the second set Ω_2 . We repeat the same selection procedure to construct the rest of group subproblems. We summarize the steps to construct Ω_n , $n \in N$ in Procedure P1. Scenario sets Ω_n form a complete partition of scenarios Ω . With this selection procedure, we ensure that Ω_n has no common demand or non-cruise time realizations among two scenarios, i.e., each group subproblem with Ω_n has no deterministic demand or non-cruise time. Therefore, considering the uncertain demand and non-cruise times in each group subproblem may better reflect the original stochastic problem. Procedure P1 is a simple way of obtaining this variety in each Ω_n as a result of generating scenarios in an increasing order of indices. Note that the number of demand realizations we consider for each flight $i \in J'$ is an even number, similarly, the number of non-cruise time realizations for each airport $b \in B'$ is even. This is another reason that Procedure P1 provides stochastic passenger demand and non-cruise time in each group subproblem.

Another approach to strengthen the lower bound is to increase the size of the subsets Ω_n , $n \in N$. We provide another scenario selection procedure, called as Procedure P2, which includes four scenarios in each group subproblem. We

Algorithm 8 Combine first-last scenarios procedure (P1)

Initialization: Given set of scenarios Ω , $n \leftarrow 0$;
for $i = 1$ **to** $|\Omega|/2$ **do**
 $n \leftarrow n + 1$;
 $\Omega_n \leftarrow \{\omega_i, \omega_{|\Omega|-i+1}\}$;

first call Procedure P1 to construct group subproblems of size two. Then, in each iteration of Procedure P2, we randomly combine two sets Ω_n and construct a group subproblem with $\hat{\Omega}_n$ including four scenarios. Note that new sets of scenarios $\hat{\Omega}_n$ form the complete partition of scenarios Ω . The detailed steps are given in Algorithm 9.

Algorithm 9 Combine randomly chosen group subproblems from first-last scenarios procedure (P2)

Initialization: Given set of scenarios Ω ;
Call Procedure P1 to construct $\Omega_n, n \in N$;
Randomly order numbers $\{1, 2, \dots, |\Omega|/2\}$ represented by $random_i, i = 1, \dots, |\Omega|/2$, respectively
 $n \leftarrow 0$
for $i = 1$ **to** $|\Omega|/2$ **by** 2 **do**
 $n \leftarrow n + 1$;
 $\hat{\Omega}_n \leftarrow \{\Omega_{random_i}, \Omega_{random_{i+1}}\}$;
