

EXPERIMENTAL STUDY AND HYSTERESIS MODELING OF REINFORCED
CONCRETE MEMBERS UNDER PURE CYCLIC TORSIONAL LOADING

by

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to my dear son, Emir Efe

and my love, Osman..

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ABSTRACT

EXPERIMENTAL STUDY AND HYSTERESIS MODELING OF REINFORCED CONCRETE MEMBERS UNDER PURE CYCLIC TORSIONAL LOADING

The issue of torsion in reinforced concrete has been ignored for a long time, but the studies carried out in this thesis have revealed the importance of torsion. This study was achieved to better understand the behavior of reinforced concrete elements under torsion in terms of torsional moment carrying capacity, failure mode, crack formation, energy dissipation capacity and ductility capacity. After literature review, parameters that are effective on the torsional behavior of reinforced concrete elements have been determined. An experimental study was conducted on, real-size reinforced concrete elements having different reinforcement ratios and concrete classes. The test setup was designed in such a way that when the test elements were subjected to torsional moments, longitudinal elongation would not be prevented and the torsional moments will be applied reversibly. Furthermore, the longitudinal elongation of some test elements, as well as torsional moments and angles of rotation, were measured during the experiments. After testing of specimens, the behavior was analyzed using the results obtained from the experiments, and as a result, it was found that the lateral reinforcement ratio is the parameter that most affects the torsional capacity, as expected. In addition, a suitable mathematical model was developed using the experimental results. This model was also compared with the experimental results in the literature and its agreement was verified.

Keywords: Reinforced concrete, Reversible cyclic torsion, Torsional strength, Rotation angle, Failure mode, Ductility, Torsional rigidity, Hysteresis model.

ÖZET

BETONARME ELEMANLARIN TERSİNİR TEKRARLI YALIN BURULMA YÜKLERİ ALTINDA DAVRANIŞLARININ DENEYSEL OLARAK İNCELENMESİ VE ÇEVİRİMSSEL MODELLENMESİ

Betonarmede burulma konusu uzun süre ihmal edilmiş ancak bu tezde yapılan çalışmalar burulmanın önemini ortaya koymuştur. Bu çalışma betonarme elemanların burulma momentleri altındaki davranışını, burulma momenti taşıma kapasitesini, göçme modunu, çatlak oluşumunu, enerji kapasitesini ve süneklik kapasitesini daha iyi anlayabilmek amacıyla yapılmıştır. Literatür taraması yapıldıktan sonra betonarme elemanların burulma davranışında etkili olan parametreler belirlenmiştir. Bu doğrultuda gerçek boyutlu farklı donatı oranı ve beton sınıfına sahip betonarme elemanlar üretilmiştir. Deney düzeneği, test elemanları burulma momentlerine maruz kaldığında boyuna uzamalara engel olmayacak ve burulma momentlerini tersinir tekrarlı olarak uygulayacak şekilde tasarlanmıştır. Ayrıca bazı deney elemanlarının deney sırasında burulma momentleri ve dönme açılarının yanı sıra boyuna uzamaları da ölçülmüştür. Tüm numuneler test edildikten sonra deneylerden elde edilen sonuçlar kullanılarak davranış iredelenmiş sonuç olarak etriye oranının burulma kapasitesine en çok etki eden parametre olduğu bulunmuştur. Ayrıca deney sonuçları kullanılarak uygun bir matematiksel davranış/çevrim model geliştirilmiştir. Oluşturulan bu model literatürde bulunan deney sonuçlarıyla da karşılaştırılarak tutarlılığı ispat edilmiştir.

Anahtar Kelimeler: Betonarme, Tersinir tekrarlı burulma, Burulma dayanımı, Dönme açısı, Göçme modu, Süneklik, Burulma rijitliği, Çevrimsel model.

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LIST OF SYMBOLS

A_c	Area of concrete cross-section
A_{cp}	Area enclosed by outside perimeter of concrete cross-section
A_l	Area of longitudinal reinforcement
A_t	Area of transverse reinforcement
a_o	Height of equivalent stress block
A_o	Area enclosed by the centerline of the shear flow
A_{oh}	Area enclosed by the centerline of the hoop steel bar
b_w	Width of rectangular cross-section
C	Geometrical parameter
d	Effective depth of rectangular cross-section
D	Diameter of circular cross-section
E_c	Modulus of elasticity of concrete
E_D	Dissipated energy
E_s	Modulus of elasticity of steel
E_{so}	Strain energy
f_{2max}	Maximum compressive strength of cracked concrete
f'_c	Specified compressive strength of concrete
f_{cr}	Cracking strength of the concrete
f_d	Compressive stress within the concrete struts
f_l, f_t	Stress in longitudinal and transverse reinforcing steel
f_{max}	Ultimate stress
f_r	Modulus of rupture of concrete
f_t	Concrete tensile strength / Principal tensile stress
f_{t-max}	Maximum principal tensile stress
f_y	Yield stress of the steel
G	Shear modulus
GJ	Torsional stiffness
h_e	Equivalent viscous damping coefficient

J	Polar moment of inertia
J_1	Torsional constant for a cross-section
J_h	Torsional constant for a hollow-section
K	St. Venant torsional constant
k_1	Coefficient for high strength concrete
K_t	Torsional stiffness
LP	Loading pattern coefficient
M_s	Torsional moment
n	Modular ratio
P_{cp}	Outside perimeter of concrete cross-section
P_h	Perimeter of the centerline of outermost closed stirrups
P_o	Perimeter measured from centerline of the shear flow
P_v	Tensile forces in reinforcing hoops
Q_x	Horizontal components of this dowel action
q_y	Shear flow at yield
Q_y	Vertical components of this dowel action
r	Radius
RF	Loop reduction coefficient
S	Compressive strength reduction factor for the concrete strut
s_l	Spacing of longitudinal reinforcement
s_t	Spacing of transverse reinforcement
T	Torque
T_{cr}	Cracking torque
t_d	Thickness of the shear flow zone
t_e	Effective thickness
T_e	Elastic failure torque
T_m, T_p	Maximum and peak torsion of single cycle
T_n	Nominal torsional moment
T_{t-max}	Theoretical nominal maximum torsion
T_u	Ultimate torsional moment
u_c	Perimeter of cross-section

u_o	Effective perimeter measured from centerline of the shear flow
v_t	Maximum torsional shearing stress
x	Shorter dimension of rectangular cross-section
x_o	Horizontal dimension of the centerline of closed stirrup
y	Longer dimension of rectangular cross-section
y_o	Vertical dimension of the centerline of closed stirrup
α_1, β_1	Equivalent rectangular stress block conversion coefficients
α_e	Elastic theory coefficient or Saint- Venant's coefficient
β	Constant that depends on the d/b
γ_{lt}	Shear strain
δ_{min}	Minimum shear flow thickness
ε_1	Average tensile strain perpendicular to the strut direction
ε_{2max}	Strain distribution over the strut thickness
$\varepsilon_{3l}, \varepsilon_{3t}$	The strain in the longitudinal/transverse reinforcement
ε_{co}	Strain corresponding with concrete compressive strength
ζ_{eq}	Damping ratio
Θ	Unit torsion angle / Angle of twist
λ	Modification factor for lightweight concrete
ν	Poisson's ratio
ξ_{shape}	Shape factor
ρ	Reinforcement ratio
ρ_{lo}, ρ_{to}	Minimum reinforcement ratios defined in ACI
τ	Shear stress
τ_{max}	Maximum shear stress
ϕ	Strength reduction factor
ϕ_y	Yield twist
ψ	Twist
ω	Frequency

LIST OF ACRONYMS/ABBREVIATIONS

ACI	American Concrete Institute
AS-3600	Australian Standard for Concrete Structures
BWBN	Bouc-Wen-Baber-Noori Model
CEB-FIP	International Federation for Prestressing Concrete
CFRP	Carbon Fiber Reinforced Polymer
CSA-14	Canadian Standards Association
DBL	Degrading Bilinear Model
EC2	EuroCode2
FEMA	Federal Emergency Management Agency
FRP	Fiber Reinforced Polymer
GFRP	Glass Fiber Reinforced Polymer
GSVATM	Generalized Softened Variable Angle Truss Model
HSC	High Strength Concrete
JSCE-07	Japan Society of Civil Engineers 2007
LVDT	Linear Variable Displacement Transducer
NSC	Normal Strength Concrete
PHM	Polygonal Hysteresis Models
PTP	Plastic Trend Parameter
RA-STM	Rotating-Angle Softened Truss Model
RC	Reinforced Concrete
SDM	Smooth Differential Models
SHM	Smooth Hysteresis Models
SMMT	Softened Membrane Model for Torsion
STM	Softened Truss Model
TBEC	Turkish Building Earthquake Code
UBC	Uniform Building Code
VATM	Variable Angle Truss Model

1. INTRODUCTION

1.1. Motivation

Curiosity about the behavior of structural members under somewhat left behind many analytical and experimental studies on the investigation of behavior under bending and shear. A possible reason why torsion is ignored is that in buildings thought to consisted of simple articulated elements, torsion can be eliminated by specially arranging the main elements that make up the structure. If torsion cannot be eliminated, as in spandrel beam slabs, the problem has either been neglected or eliminated by reducing the allowable shear stresses. Lack of sufficient information about torsional damage could be considered as another reason for this indifference. Because there were not enough studies on torsion, the researchers had limited idea about the level of damages on the structure due to the torsion.

In recent years, sophisticated analysis methods have become available to calculate the stresses into the members and deformations of monolithic structures, for example the edge members as slabs or shells. Moreover, a great deal of work has been made toward the solution of many problems related to combination of shear/bending and/or axial load in reinforced concrete elements. In recent years, the increasing tendency for different and interesting designs to go beyond the standard in modern structures has led to problems such as torsion, which are not encountered in regular structures. Safe structures cannot be constructed by neglecting any effects of torsion that may exist in them, as long as the investigation of torsion is considered as a secondary matter. Here with, the prerequisite of understanding the effects of torsional moments has become a critical issue.

Torsional moments in reinforced concrete structures rarely happen without other forces. Torsion is often present in combination with bending, shear, and axial forces. However, the structural element analysis under combined loads needs to be generalized

in order to understand the behavior of elements subjected to pure torsion. However, very few studies have been reported on the reinforced concrete members behavior under reversed cyclic pure torsion.

1.2. Objectives

The primary objective of this study was to develop a hysteresis model for reinforced concrete members under pure torsion which includes significant stiffness degradation. In addition to that, the following objectives were summarized:

- To examine the behavior of reinforced concrete members under pure torsion.
- To explore the effects of the longitudinal and transverse steel ratio concerning the torsional behavior both at the serviceability and the ultimate limit states.
- To understand the effect of the compressive strength of concrete on torsional behavior.
- To analytically estimate the ultimate torsional capacity of reinforced concrete members with different compressive strength of concrete, stirrup, and longitudinal reinforcement ratios.
- To compare and investigate the failure mode, strain of steel reinforcement, behavior of hysteresis loops, skeleton curve, energy dissipation capacity, ductility factor and stiffness, and torsional bearing capacity calculation of reinforced concrete structural members subjected to re-cyclic torsion.

1.3. Scope

The following scope was employed to achieve the stated objectives:

- Testing of full-scale reinforced concrete members under reversed cyclic pure torsional moments.
- Specimens with three different concrete compressive strengths as 15, 35 and 50 MPa were examined.

- Only the torsional effects were investigated.
- Elongation of the test specimens was allowed by a specially-designed test setup
- The load is applied as a concentric, meanly torsional moment effect at the center of the specimen section.
- A hysteretic model is proposed to reflect the behavior significantly under pure cyclic torsional loading.

1.4. Methodology

To achieve the above-mentioned objectives, the following methodology was used:

- Planning an experimental test program on 11 reinforced concrete members having different compressive strengths of concrete, addition to the different transverse and longitudinal reinforcement ratios under reversed cyclic torsional moments.
- Designing and producing test specimens suitable for specified parameters.
- Designing and producing a proper test setup to perform the tests.
- Developing an analytical model based on test results.

1.5. Thesis Outline

To achieve the outlined objectives, the thesis was divided into seven chapters. In the first chapter, the motivation and objectives of the study were explained briefly and the methodology used to achieve these objectives was summarized. In the last part of this chapter, the outline of the dissertation was described briefly.

Torsional behavior was introduced in the second chapter. All theories and models proposed for torsional behavior were investigated in this section. The Coulomb Theory, Navier's Torsion Theory, Saint-Venant's Torsion Theory, and Bredt's Torsion Theory were discussed. Also, elastic and plastic torsional behavior were investigated. Lastly, in this chapter, the behavior of plain/reinforced concrete members under pure torsion was reported briefly.

In the third chapter, the experimental and analytical studies were investigated. The Coulomb theory of torsion was known as the first torsional theory to explain the twisting behavior of materials. Saint Venant's Torsion Theory also named as classical torsion theory was defined. The history of torsion theories and models was described to understand the new hypothesis and derived equations from these fundamental theories and models. Test setups, specimen specifications or test parameters were the main body of this chapter. Studies surveyed on the plain/ reinforced/high-strength concrete members were also described.

The experimental part was the basis of this study and all the works done for this part were reported in the fourth Chapter. Pre-study steps were introduced at the beginning of this chapter. The design steps of test specimens were derived briefly. The materials used, the production of the test members, the test setup and the instrumentation used were also reported in detail. Finally, the test results were presented and conclusions were drawn.

An analytical model was developed in Chapter Five. This model could be applied to reinforced concrete members with different parameters to calculate the torsional carrying capacity.

Comparison of the results obtained from the experimental and analytical studies was evaluated in the Discussion Chapter. In addition, the applicability of the model was examined by using the experimental data in the literature. For this purpose, torque-twist backbone curves were drawn for monolithic experiments and the accuracy of this curve was verified. A model was then developed for specimens tested under re-cyclic torsion and the validity of the defined model was verified by comparison with experimental results.

All results of the experimental and analytical study were evaluated in the seventh chapter.

2. TORSIONAL BEHAVIOR

There are many forces acting on structural elements. When these forces exceed the strength level of the structure, can cause damage or collapse of the structure, and the causing force determines the failure mode. Forces acting on members can be effective alone or in combination with each other. These forces are: compression, tension, bending, shear and torsion as illustrated in Figure 2.1. For safe and economical solution, these forces should be understood first one by one.

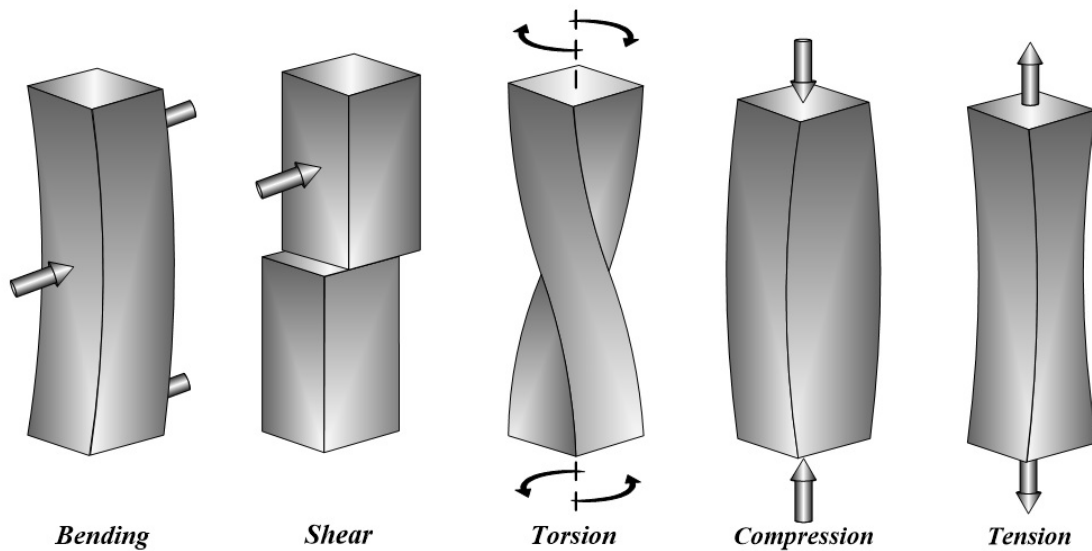


Figure 2.1. Forces acting on members.

Until 1970s years, torsion was neglected as the structures were built low-rise, symmetrical with straight facades. However, as of more and more architectural and aesthetics considerations increase in the construction industry, asymmetrical structures as well as multi-story structures were built, and thus, the issue of torsion has gained importance.

Torsion can be defined with simple terms for a circular shaft exposed to pure torsion means without any shear or bending. The basic torsion equation given with Equation (2.1) is derived from the geometrical property of cross-section of member

involved in the axis of the member that has a relation between the applied torque as

$$\frac{T}{J} = \frac{\tau}{r}, \quad (2.1)$$

where T is the torque, J is the polar moment of inertia whose SI unit is (m^4), r is the radius of circular shaft and τ is the resultant shearing stress. To derive this torsion equation few assumptions can be made as follows:

- Material of the member is homogeneous throughout the length and should follow Hooke's law,
- The material of the member should have shear stress proportional to shear strain,
- The circular section remains circular during torsion,
- Plane cross-section of the member before torsion remains plane after the torsional behaviour,
- Each point of the structure on the cross-section should rotate with the same twist angle,
- The stress of the material induced during torsion should not exceed the elastic limit.

Torsion, in the field of solid mechanics, is a twisting phenomenon of a rigid body subjected to a torsional moment. Another definition of torsion is the enforcing with a moment or a force to a structural element while holding one end or twisting it in one direction, and turning the other end in the opposite direction about a longitudinal axis.

Typical examples of beams may exist in the real life subjected to torsion are given in Figure 2.2. In order to analyze element-wise torsion, it is necessary to first understand the theories developed in this field.

Torsion may be occur with several ways in reinforced concrete members during the load application to a structural system. There are two types of torsion in structural members, equilibrium torsion and compatibility torsion, and therefore it is important for designers must be able to understand the differences between these two types of

torsions. The main difference can be defined as the equilibrium torsion is statically determinate torsion, the compatibility torsion is statically indeterminate torsion.

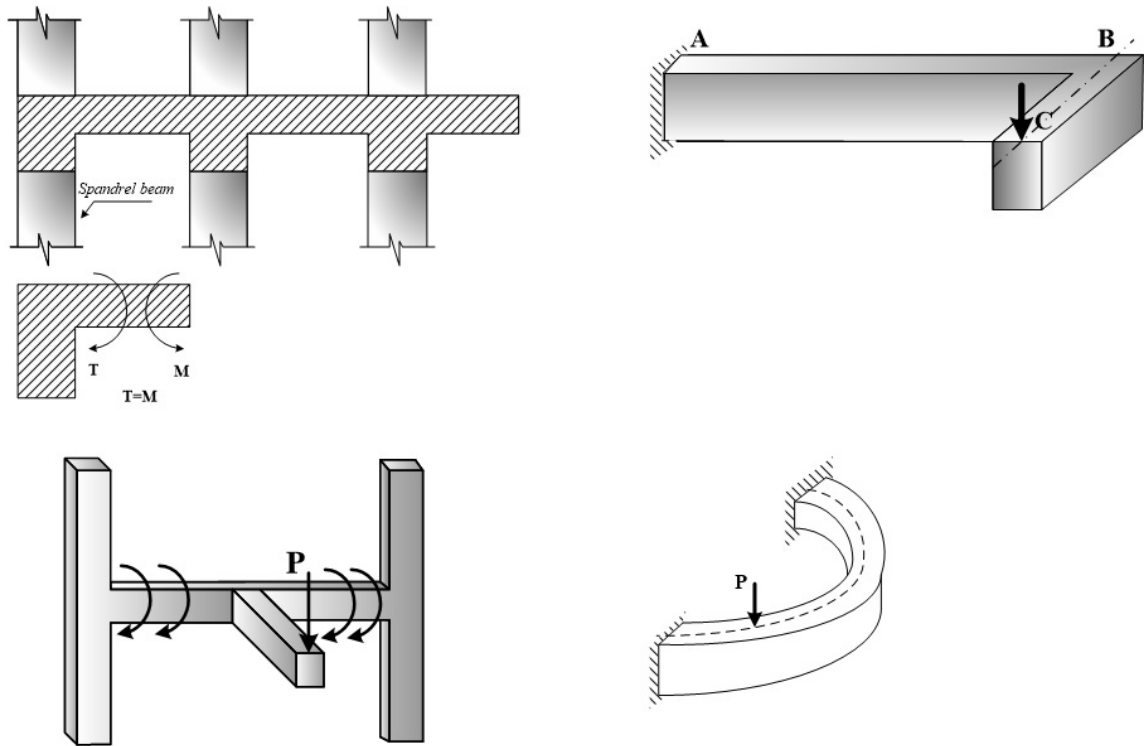


Figure 2.2. Typical examples of beams subjected to torsion

2.1. Equilibrium Torsion (Statically Determinate Torsion)

The torsional resistance needed to maintain the static equilibrium of the structure is called equilibrium torsion. It can also be defined as the torsion, which can only be determined by statics. In this case, if sufficient torsional resistance is not provided, the structure will become unstable and finally collapse. Since they have no alternative load path, external loads must resist torsional actions.

Structural members may be exposed to torsional moments, with a combination of other forces such as flexural force and/or shear force. As shown in Figure 2.3 (a), a cantilever beam is subjected to extreme torsion due to loading on transverse beams but there are also shear forces and bending moments developed due to the self weight of member as can be seen in the force diagrams (torsional moments, bending moments and

shear forces). The fixed-ended beam shown in Figure 2.3 (b) is exposed to significant torsional moments and additionally shear and bending as shown in the torsion, bending and shear diagrams below.

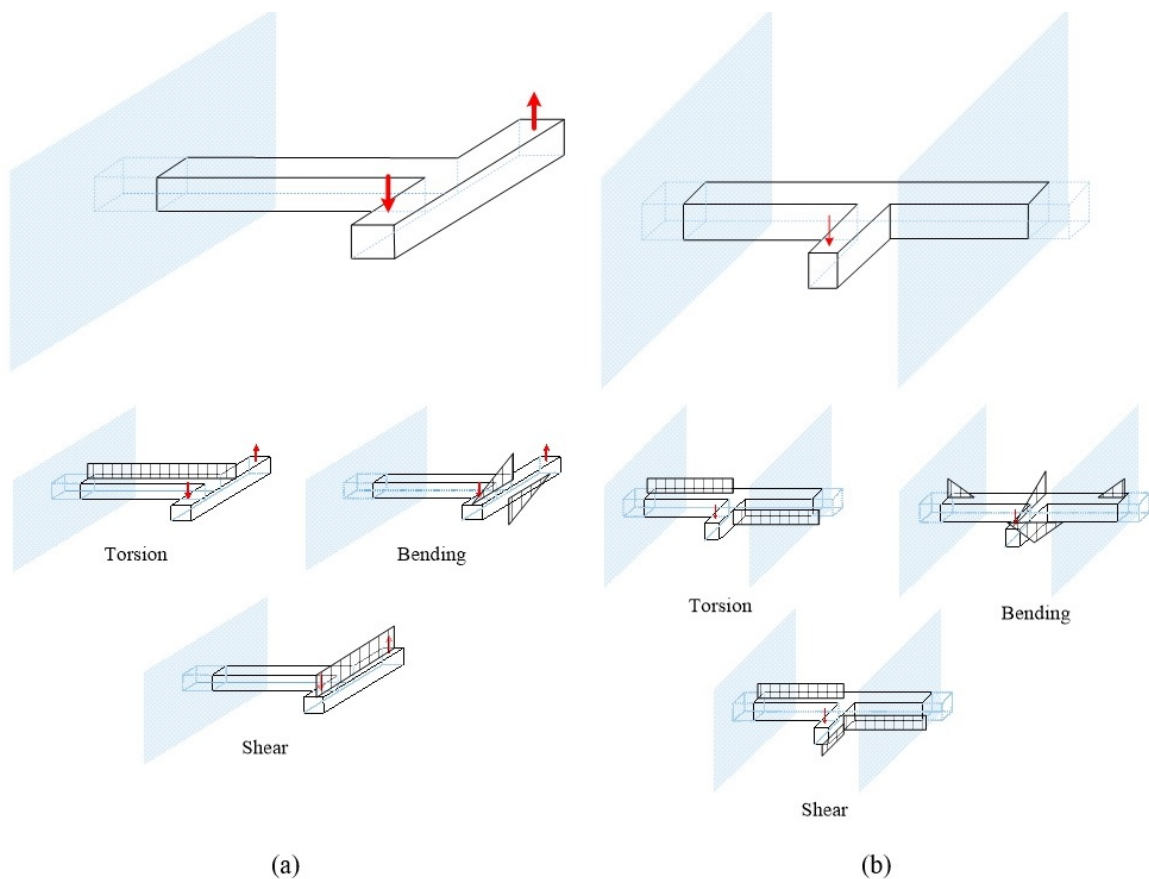


Figure 2.3. Structural members subjected to torsion.

In other words, in “*equilibrium torsion*”, the twisting moments can be determined sufficiently by using the equilibrium conditions of the structures and an eccentric loading (with respect to the shear center at any cross-section) induces the torsion.

In Figure 2.4, the example for the equilibrium torsional moment is illustrated. Such torsion is also induced in beams curved plan and subjected to gravity loads, and in beams where the transverse loads are eccentric with respect to the shear centre of the cross -section. In Figure 2.4(a), the cantilevered shelf roof self weight and loadings on the roof is shown. Due to these loading, the torsional moments were developed on the beam.

The freebody diagram of beam is shown in Figure 2.4(b) and the torsional moment internal force diagram of the beam were shown in Figure 2.4(c).

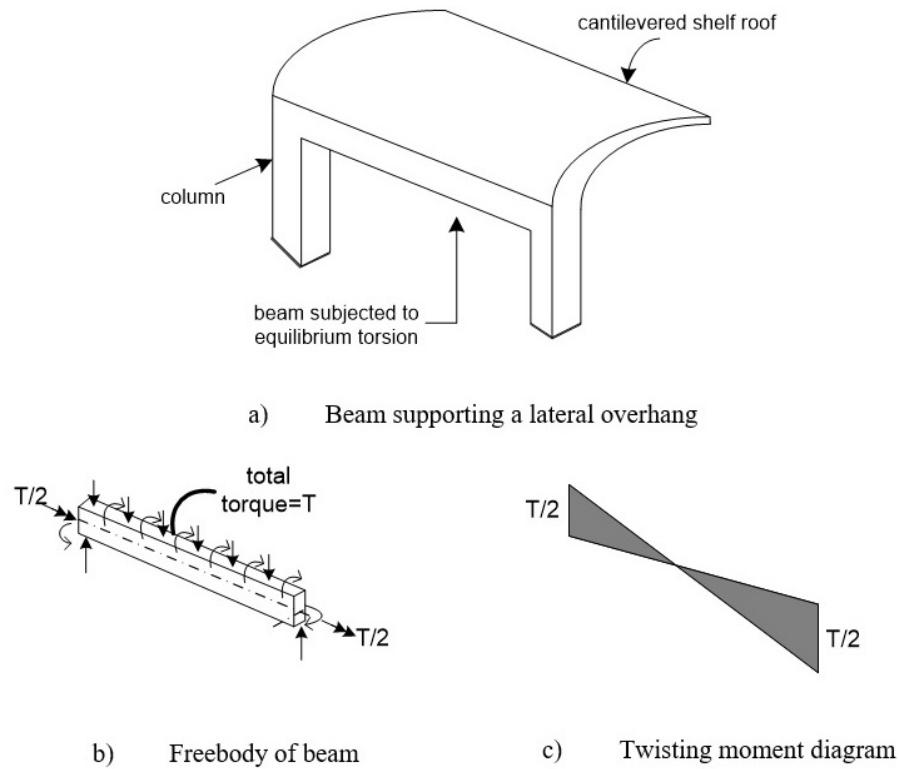


Figure 2.4. Example of equilibrium torsion

2.2. Compatibility Torsion (Statically Indeterminate Torsion)

Compatibility torsion develops when the member needs an angle of twist to maintain the deformation compatibility. Basically, in the case of compatibility torsion, torsion cannot be decided from statics and a twist (rotation) is necessary for deformational compatibility between connected members such as columns, slabs or spandrel beams. In systems where compatibility torsion is present, the failure mode is not torsional because the torsional moments are small enough to be neglected.

Spandrel beams, built integrally with the floor slab, which places at the building edges are subjected to torsional moments per unit length equal to the external end of the slab in addition to transverse loads. In a similar way, the outside ends of the floor beams take the torsional moments from the spandrel girders.

In “*compatibility torsion*”, it is assumed that the torsional moment remains constant in plastic joints formed by torsional cracking. This constant value will be equal to the cracking moment. Therefore, in cases where compatibility torsion is involved, there is no need to calculate the torsional moment. As shown in Figure 2.5, the torsion is formed owing to the need for the member to undergo an angle of twist to torsion occurs when a twisting angle is needed to maintain deformation compatibility of a structural member and this torsional moment depends on the member’s torsional stiffness.

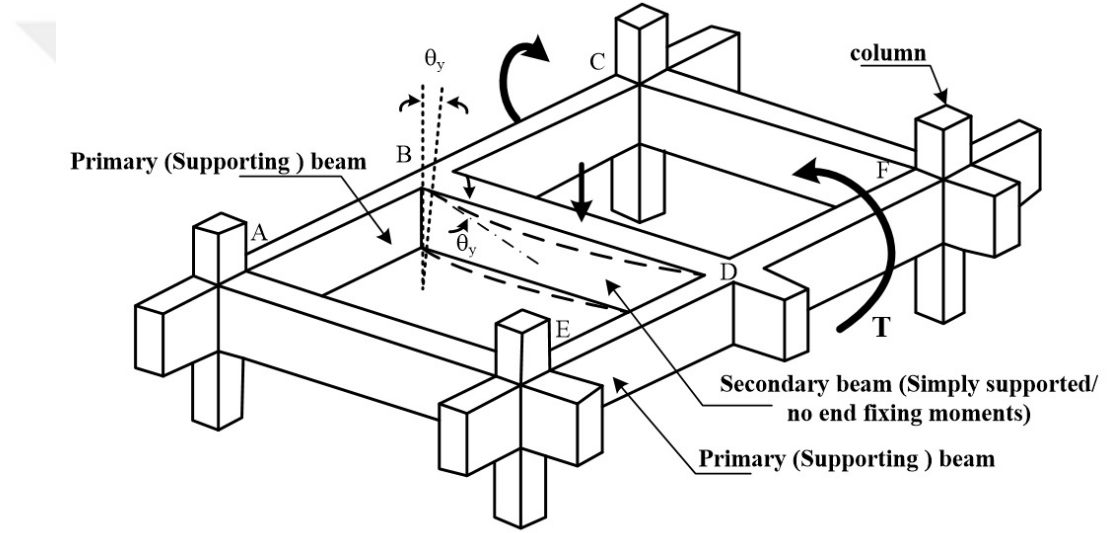


Figure 2.5. Example of compatibility torsion

For a statically indeterminate structure, the reduction of the torsional moment in a structural element can be formed from the internal forces redistribution and the maximum ultimate torsional moment $T_{u,comp}$ for the reinforced concrete members can be reduced as (according to ACI 318.11.5.2.2) [1]

$$T_u = \phi \cdot 0.33 \cdot \lambda \cdot \sqrt{f'_c} \left(\frac{A_{cp}^2}{P_{cp}} \right). \quad (2.2)$$

The terms in the given expression are; f'_c is the specified concrete compressive strength, A_{cp} and P_{cp} define the area and perimeter of the concrete cross-section, λ means the modification factor reflecting the lightweight concrete reduced mechanical properties and all type of concrete as relative to normal weight of the same compressive strength, ϕ indicates the reduction factor for the strength.

To summarize, the determinate and indeterminate behavior were clearly shown on structures with load at P at 90° to the plane of frame in Figure 2.6. For statically determinate torsion as shown in Figures 2.6 (a) and (b), the member needs to torsion based on the requirement of statics and the amount of torsion required to resist by the member is independent of the member stiffness. As the second case, for the statically indeterminate torsion as shown in Figures 2.6 (c) and (d), if it is possible to remove the statically indeterminacy there would be no torsion. For example, in Figure 2.6 (c), the torsion can be eliminated if the support at point A is removed. By the same way, in Figure 2.6 (d), the torsion can be eliminated if a flexural hinge is embedded at point B.

In cases of statically indeterminate torsion, the magnitude of torsional stiffness of member and interconnecting member stiffnesses are effective on the amount of torsion in an element.

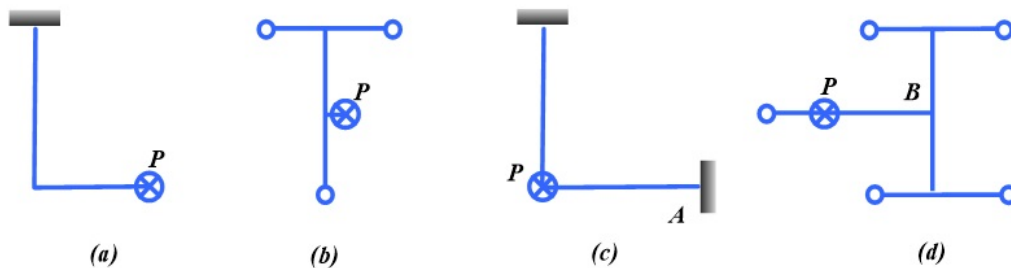


Figure 2.6. Examples for statically determinate torsion.

Separating the two types of torsion is become very important in terms of the safety of the structures. Table 2.1 briefly summarizes the differences between the two types of torsion. Equilibrium torsion cannot be released. Compatibility torsion can be released. If the design for compatibility torsion is chosen, the torsional stiffness needs to be modified by 0.5 which was suggested by some designers.

Without realizing these points, certain designers considers torsion for all member design and results in uneconomic designs. Some do not consider even equilibrium torsion and this results in unsafe designs. It is of high importance to understand these points for safe and economic designs.

Table 2.1. Comparison of the torsion types.

Cases	Equilibrium Torsion	Compatibility Torsion
<i>Additional Name</i>	Primary Torsion	Secondary Torsion
<i>Cause of Occurrence</i>	Occurs in cases where static balance can be maintained with torsional resistance.	Develops to maintain compatibility in deformation with the co-existent member
<i>Redistribution of Torsion</i>	<i>Cannot</i> be reduced by redistribution of moments	<i>Can</i> be reduced by internal forces redistribution
<i>Beam Analysis</i>	Combination of torsion flexure and shear	Combination of flexure and shear
<i>Statically Determinancy</i>	Statically <i>determinate</i> torsion	Statically <i>indeterminate</i> torsion
<i>Dependence on Stiffness</i>	Free from the torsional stiffness of the element	The torsional stiffness of the member is effective on the resulting twisting moment
<i>Consideration of Torsional Moment</i>	The element has to be designed according to the torsion and torsion <i>cannot</i> be ignored	For statically indeterminate structures, torsional moment <i>can</i> be ignored
<i>Example</i>	An edge beam supporting cantilever slab	An external (spandrel) beam in a frame

2.3. Torsion Theories

Well-known studies on torsional behavior started at the end of the 1700s. Firstly, researchers made basic definitions and assumptions to understand this complex behavior. Simply, they defined a prismatic member as an axis and a cross-section. So, a cylindrical bar can be defined as a straight axis and a constant cross-section. If a bar is twisted by torsional moments at the ends, it is said to be in pure torsion. In the beginning, researchers were interested in a circular shaft that is exposed to pure torsion in the field of solid mechanics. Until today, many mathematical models and theories derived and developed. Among torsion theories, well-known classical theories are Coulomb, Navier, Saint Venant and Bredt. In the studies conducted in the 20th century, researchers have realized that the three principles of material mechanics are highly effective on torsional behavior. Three principles, equilibrium, compatibility and materials stress-strain relationships, have been used to develop to skew bending and truss models [2–5].

2.3.1. Basic Torsion Theories

Basic torsion theories, such as Coulomb, Navier, Saint-Venant, and Bredt theory of torsion, formed the ground of torsion theories, and following studies and models were built on these simple theories [2–5].

2.3.1.1. The Coulomb Theory of Torsion. The first known hypothesis about the relationship between torque -twist for a circular bar was developed by Coulomb [2]. If we draw the outlines of the cross-section on the cylindrical surface of a bar, it can be seen that when the bar is twisted, the plane borders remain plane and the diameter lengths do not change and by using this information the deformation hypothesis for a circular bar subjected to torques can be formulated. Basically, the cross-section of a circular cylindrical bar subjected to torsional moments at the ends and can rotate without any distortion during the rotation remain plane. The deformation hypothesis developed by Irgens is the foundation of the Coulomb Theory of torsion for cylindrical bars [6]. To sum up, the main assumption of the mentioned theory is the plane sections normal to the axis of the member remain plane during torsion with no warping and via this hypothesis the correct torsion equation can be derived.

2.3.1.2. The Navier Theory of Torsion. Navier developed and applied the Coulomb Torsion theory for non-circular sections in 1826 and this theory later guided many theories explaining the reinforced concrete member behavior after cracking under torsional moments [3]. Researcher assumed also that the plane sections still remain plane during torsional moment. But, he got erroneous conclusions because of the assumption above.

During torsion, the centroidal polar moment of inertia of the cross-section and the rotation angle at the surface of the non-circular prismatic bar are inversely proportional to each other. Moreover, the maximum shearing stress develops at the farthest points from the cross section centroid. This approximation is in contradiction with the boundary conditions. These results are true for an elastic slender layer which corre-

sponds to a segment of the bar attached to rigid plates between two cross-sections [7]. Using continuum mechanics, B. Saint-Venant published the correct solution in 1855 [4].

2.3.1.3. Saint-Venant's Torsion Theory. According to Saint Venant's torsion theory (called as classical torsion) [4], a prismatic rod with length l is twisted by end torques (Figure 2.7) and the following assumptions can be done for this geometrical behavior:

- (i) Every cross section rotates as a rigid body @ each z .
- (ii) k is the constant defines rate of twist
- (iii) Cross-sections are free to warp ("warping" = extensional deformation in the direction of the axis about which the torque is applied) in the z -direction but the warping is the same for all cross-sections. (St. Venant Hypothesis).

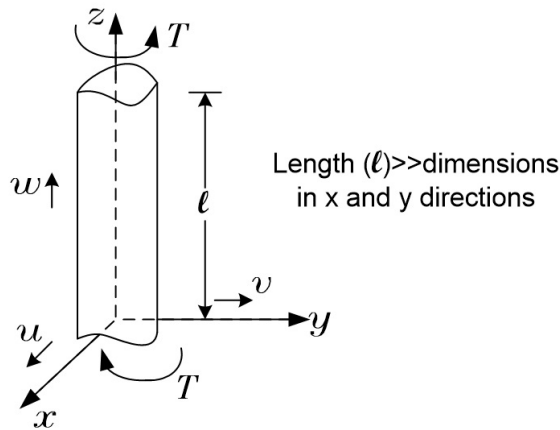


Figure 2.7. Example of a general long prismatic rod.

2.3.1.4. Bredt Torsion Theory. Derived equations for pure torsion of thin-walled closed section prismatic beams, can be called as Bredt's formulas, use the same procedure of asymptotic splitting method [5]. For example, the Saint-Venant formulation of a plain bar problem can be extended asymptotically. At this point, can started by verifying the assumptions derived for the shear stress distribution.

Additionally, the effect of the transverse force acting on the beam must be taken into account. It is needed to an adaptation of Bredt's formulas because the shear force

induces deformation in the cross-section. Hence, two definite formulations of the shear center can be obtained, as the kinematic and the energetic.

Bredt's first formula is given as

$$\tau_{max} = \frac{M_s}{2A\delta_{min}}, \quad (2.3)$$

here, M_s defines the torsional moment in cross-section, A states the enclosed area by the center line of shear flow zone and δ_{min} is the minimum shear flow thickness to calculate the deformation due to torsion can be defined as

$$\theta = \frac{M_s}{GJ}, \quad (2.4)$$

where θ is the unit torsion angle, G is the shear modulus of materials, J is the polar moment of inertia of cross-section and GJ is the torsional stiffness of member used in Equation (2.4).

The polar moment of inertia, J for circular section, determined from Equation (2.5), must be calculated to determine the stresses and strains in the cross-section, during torsion while using the theory of beams and can be expressed as

$$J = \frac{\pi}{2} (r_o^4 - r_i^4), \quad (2.5)$$

where r_o and r_i are the outer and inner radius of the circular hollow section, respectively, as shown in Figure 2.8;

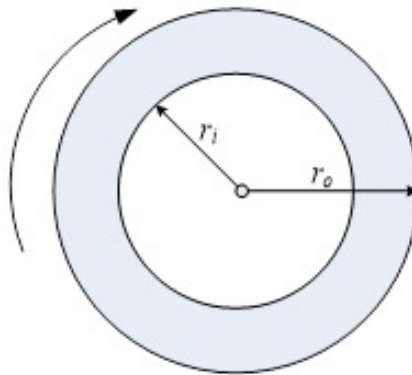


Figure 2.8. The polar moment of inertia.

2.3.2. Thin-Tube Theory

2.3.2.1. Two and Three-Dimensional Plane Truss Model. Initial theories for concrete members with shear cracks were formulated by using a two-dimensional (2D) plane truss model, in 1899. Later, in this model, reinforced concrete members were defined as a combination of concrete struts and reinforcement ties. The angle between the struts of concrete and longitudinal axis is assumed as 45 degrees, and it is accepted that the shear strength of the member is controlled by the yielding of stirrups. Finally, a three-dimensional (3-D) space truss model for reinforced concrete members under torsion was developed consisting of longitudinal and transverse steel for resist to tension and concrete struts for resist to compression.

2.3.2.2. Skew Bending and Space Truss Theories. After the definition of the plane truss model, researchers concentrated on the two theories; skew bending and space truss. Skew bending theory is based on the two failure modes and used only equilibrium equations. also, this theory assumes the yielding of reinforcements before the failure.

Space truss theory accepted that equilibrium equations can be used to calculate the slope of the compression diagonals at failure when the reinforcement yields. Studies conducted in the following years have shown that the results of skew-bending and space truss theories were in good agreement with each other.

2.3.2.3. Compression Field Theory. The main difference from the previous models is, compression field theory works out the torsional behavior up to the maximum torque.

2.3.2.4. Softened Truss Model. Softening truss model based on softening of the stress-strain behavior of the concrete was proposed in 1985. The difference from earlier models is, the model uses softening of concrete. Test results have been shown well in agreement with the compression field theory, which uses spalling of cover concrete.

Understanding the behavior of plain concrete subjected to torsion will be sensible because at the first stage of cracking, the contribution of the reinforcement is very small and negligible [8].

2.4. The Behavior of Plain Concrete Members Under Pure Torsion

The experimental tests on the behavior of pure concrete elements showed that a member with a convex cross-section collapse as soon as the formation of the initial crack. This failure is come true suddenly means develops without any warning and it is devastating. The torsional behavior up to failure can be described by small deformations and little evidence of distress in the concrete, until the first crack occurs and then the specimen suddenly collapses. The torque-twist curves of a pure concrete specimen behave nearly linear up to approximately 80 percent of the torsional capacity of the member (Figure 2.9). Sufficient inelastic stress redistribution occurs and it can cause the torque- twist curves to bend over slightly just before the failure.

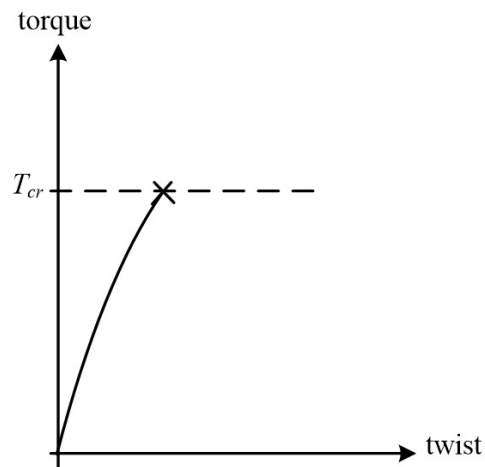


Figure 2.9. Typical torque-twist curve for plain concrete under pure torsion.

Basically, the formation of a flexural plastic hinge can be idealized as a bilinear elastic-plastic relation and which can be defined by moment redistribution and moment-curvature relation of under-reinforced members. By the way, a reinforced concrete member with sufficient torsional reinforcement shows a similar bilinear elastic-plastic

torque-twist relation (redistribution of torsional moments and the formation of a plastic hinge under torsion can occur).

In the case of compatibility torsion, the torsional stiffness decreases drastically and torsional cracking occurs. This reduction increases the angle of twist and that means the formation of a torsional hinge. Structures with members subjected to torsion can collapse where the torsional hinges occur. Some codes limit the maximum design torque as $0.67 T_{cr}$ to the reduction in torsional stiffness. In the given equation, the cracking torque, T_{cr} , of the spandrel beam represents a plastic torsional hinge.

Once a crack is formed, the torsional stiffness decreases drastically. At this stage, the estimation of the torsional stiffness gains importance. In the ordinary linear elastic analysis of framed structures, the torsional stiffness K_t can be defined as

$$K_t = \frac{T}{\theta} = \frac{GV}{L}. \quad (2.6)$$

In this expression, K_t states the member torsional stiffness and it can be described as the ratio of torsional moment T to the twist angle θ in the length L . G is the Shear Modulus may be taken as 0.4 times the modulus of elasticity of concrete E_c . Usually, GC defines the torsional rigidity and the geometrical parameter C may be taken as $0.5K$. Here, ‘St. Venant torsional constant’, K is the constant calculated for the plain concrete section given in Equation (2.7). Basically, for a rectangular section with dimension $b \times D$, with $b < D$ and

$$K_t = \beta b^3 D. \quad (2.7)$$

here, β is a constant taking values varying from 0.141 to 0.333 that depends on the D/b ratio, and calculated from

$$\beta = \left(1 - 0.63 \frac{b}{D}\right) \times \frac{1}{3}. \quad (2.8)$$

There is a 45 degrees angle between the principal stresses and centerline of the twist, and at any point of the element, the torsional shear stresses are equal in magnitude to both the principal compressive and tensile stresses. It can be expected that a pure concrete element under pure torsion can fail in tension. This is because, tensile stress which is equal to the ultimate tensile stress is produced by the applied torque. So, the

maximum shearing stress of pure concrete member is more than the maximum tensile stress under pure torsion.

The direct compressive and tensile stresses along with the diagonal directions develop under the state of pure shear, as shown in the Figure 2.10 (a) with the element A. Principal compressive and tensile stresses turn around the beam spirally and with an angle of 45 degrees to the orthogonal directions to the member axis.

Such a line thought to be a potential crack (it is the path affected by the tensile stress f_t acts) is highlighted in Figure 2.10 (a) and when the diagonal tensile stress equals the tensile strength of concrete such a crack would occur in a concrete member. The crack will rapidly move on from the outside of the member cross-section through to the inside due to the brittle characteristic of concrete. This case effectively eliminates the torsional resistance contributed primarily by the stresses in the outer fibers (which are the largest in magnitude and also have the largest lever arm). Therefore, in a pure concrete member, element, the section will fail suddenly after immediate damage because of the torsional cracking in the outer fibers. The torsional stress distribution of an element as in the example given is as shown in the Figure 2.10 (b). Some plasticity may occur as the failure mode is approached because greater stresses occur in the interior fibers of the member than predicted by the elastic theory. As explained in Figure 2.10 (c), it can be assumed that the material is rigid plastic which has a uniform stress distribution all over the cross-section.

The behavior of I-section concrete elements under torsion should be specifically evaluated. There is no extraordinary change in the behavior of convex members until cracking, but in the last stages before the failure, the flanges have been seen to resist nearly all of the applied torque.

A stress-strain curve of pure concrete behaves neither a completely elastic nor a completely plastic material. Conversely, in creating an analytical model to forecast the failure of a circular or a non-circular prismatic element, it can be assumed that the

material is either elastic or plastic. Before failure occurs, concrete can deform elastically to the extent required to impact a considerable redistribution of stresses. Nevertheless, under pure torsion, there is not sufficient inelastic strain capacity to allow a section to be completely plastic, and to have constant shearing stress across the entire section.

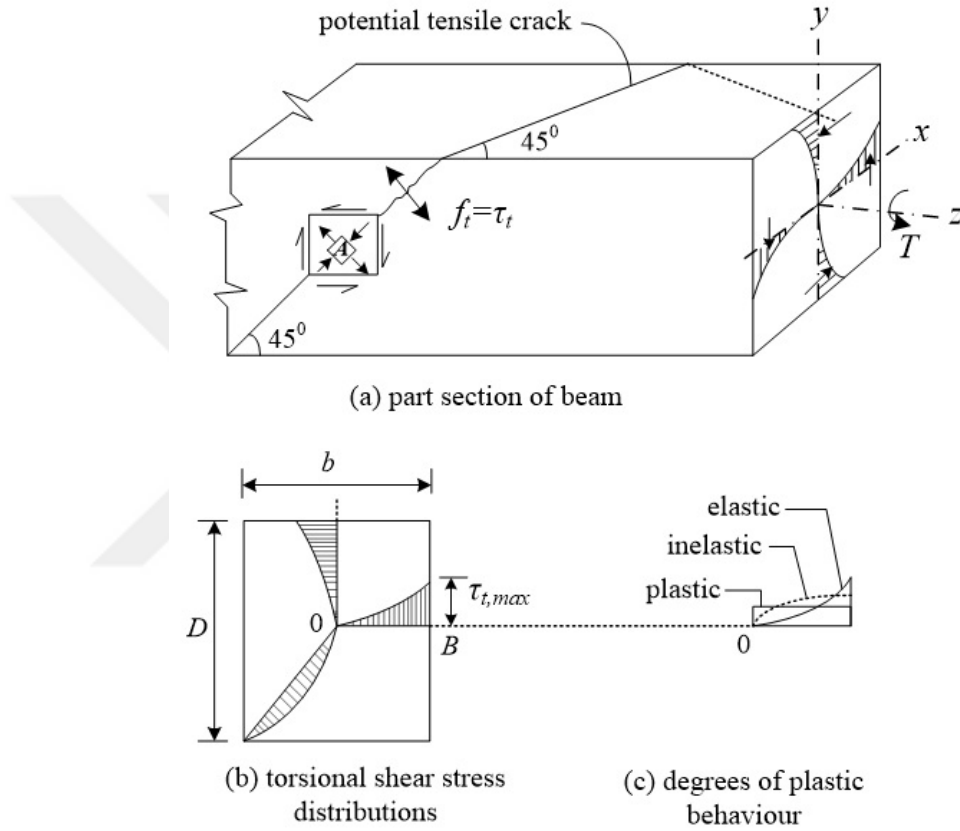


Figure 2.10. Torsional shear stresses in a rectangular section [8].

The ultimate torsional strength (resistance) can be calculated by using T_{cr} means the cracking torque of a pure concrete section. Generally, cracking torque can be calculated if the tensile strength of concrete equates to the theoretical nominal maximum torsional shear stress T_{t-max} (defines the resulting diagonal tension value). Different expressions for T_{cr} have been derived based on;

- (i) Elastic theory,
- (ii) Plastic theory,
- (iii) Skew bending theory.

Each of these different theories for T_{cr} should be experimentally correlated with a proper measure of the concrete's tensile strength to be used with it.

2.4.1. Elastic Theory

According to elastic theory, concrete is assumed as a homogeneous material, and the stress distribution is in accordance with Saint- Venant's theory [4]. Torsional failure may occur when the maximum principal tensile stress of a plain concrete element is equal to the concrete tensile strength. For the evaluation of torsional effects in plain concrete, the usual Saint-Venant theory can be used for the regular rectangular section of the concrete member. Therefore, in the middle of the long side of the element will be occurred the maximum torsional shearing stress v_t . Researchers like Navier, Saint Venant, Cauchy, and Bredt have also been interested in the torsional strength of reinforced concrete sections as well as the calculation of the torsional stress and torsional rigidity of homogeneous sections [3–5].

As illustrated in Figure 2.11, the shear stresses v_t across the cross-section resulted from the torsional moment T which is acting on a shaft with homogenous material.

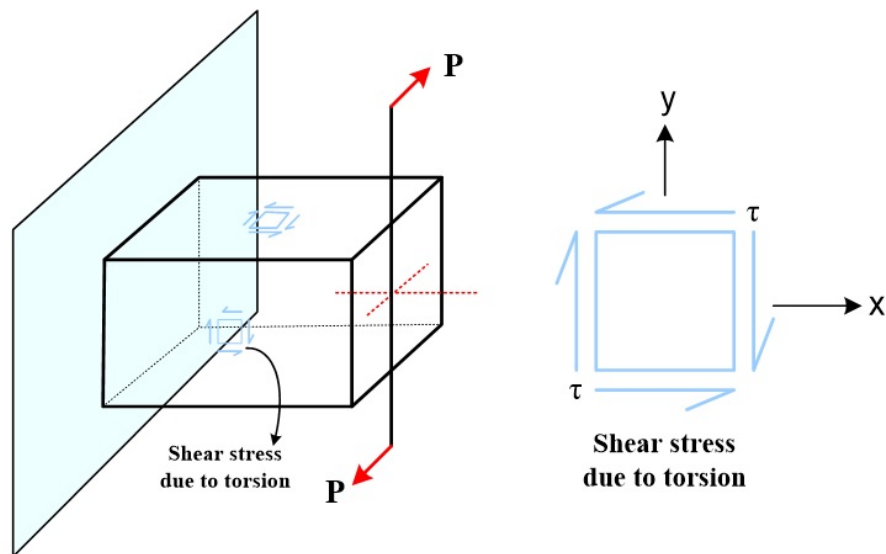


Figure 2.11. Shear stress due to torsion in homogenous section.

Derivation of the torsional shear stress distribution over a rectangular section of dimension x by y cannot be easily derived for a circular section. Derivation of the torsional shear stress distribution is difficult for a circular section when compared to a rectangular cross-section with dimensions x and y .

The non-circular cross-section bends under torsion where the plane sections remain plane, unlike the circular cross-sections. The maximum shear stress is formed at the farthest point from the center of the twist if the plane sections remain plane after twisting but this case is not valid for the rectangular cross sections. It can be defined by using the mathematical theory of elasticity, the maximum torsional shear stress v_t taking place parallel to line passing through the midpoint of the long side of the member (Equation (2.9)). The value of v_t is a function of the ratio of y to x which is the ratio of long to short sides, and

$$v_t = \frac{T_e}{\alpha_e x^2 y}, \quad (2.9)$$

where, elastic theory coefficient α is the Saint- Venant's coefficient and it is a function of the y/x ratio. And T_e torsional moment at section (elastic failure torque), y and x overall rectangular cross-section dimensions, ($x < y$).

2.4.2. Plastic Theory

Elements produced with ductile materials can yield due to the shear that occurs over the all area of a specific cross-section. Relatively it will be easier to calculate the plastic torque when the yielding happens over the whole section. On the other hand, perfectly plastic deformation occurs as a conclusion of shear stresses is nearly impossible due to the non-ductile nature of concrete especially in tension. Therefore, the proposed values by the membrane (fully elastic) and sand heap (fully plastic) analogies can help to determine the ultimate torsional strength of a plain concrete section.

The exterior fibers of a partially plastic section will be stressed up to the ultimate level while the center of the twist will be lower levels of stress and will exhibit elastic behavior. The contribution of overall resisting torsional moment is significantly more

than the less highly stressed elastic fibers near the twist center due to the outer fibers being far away from the center of the twist and more highly stressed. If the strain plasticity ratio is half, the ultimate torsional moment value for a circular cross-section can be approximately calculated from

$$T_u = \frac{\pi f D^3}{12^4}. \quad (2.10)$$

The ultimate torque for a fully plastic section is obtained from

$$T_u = \frac{\pi f D^3}{12} f_{max}, \quad (2.11)$$

where; f_{max} is the ultimate stress in material and D is the diameter [9].

For the non-circular section, the difference between the plasticity ratio of half and one is quite small. The nonlinear parts of the torque-twist curves in the region of the ultimate torque express the amount of inelastic strain in the concrete. For that reason, if it is assumed that the section was fully plastic at failure, it could be thought that the best results would be obtained by applying the plastic theory.

The maximum resisting torsional moment that is used in the equations should be equal to the maximum tensile stress of the concrete. Because, the pure concrete member will fail under tensile forces/stresses and the principal tensile stress equals the maximum torsional shearing stress at any point on the member. The ultimate torsional moment equation for a rectangular section is

$$T = \frac{b^2}{2} \left(d - \frac{1}{3}b \right) f_{max}, \quad (2.12)$$

where; T is the ultimate torque, b is the breadth, d is depth of the cross-section and f_{max} is ultimate stress.

2.4.3. Skew Bending Theory

The skew-bending theory states that when a pure concrete element is exposed to torsional moment, it can fail due to bending that occurs about a longitudinal axis parallel to the wider cross-sectional side with a 45-degrees angle. When the tensile stress which is formed due to flexure on the inclined side of the member reaches to

reduced rupture modulus of concrete then failure occurs. The derived equation is

$$T_u = \left(\frac{1}{3}\right) x^2 y (0.85 f_r), \quad (2.13)$$

where, x and y are the short and the long side dimension of the rectangular cross-section respectively and f_r modulus of rupture of concrete in Equation (2.13). The torsional stress factor as a constant takes place between the elastic theory and the plastic theory and its value is $1/3$.

The failure mode of the rectangular concrete members can be accurately calculated via the skew-bending theory but when applied to the flanged sections it should be assumed as an assembly of component rectangles.

According to the torsional shear stress, the strength of nominal torsional moment T_n of a rectangular pure concrete section may be expressed as

$$T_n = \alpha x^2 y [f_t(max)]. \quad (2.14)$$

In the case of pure torsion (same as pure shear), torsional stress v_t equals maximum principal tensile stress $f_t(max)$ and an elastic theory coefficient α is ranging from 0.208 to $1/3$. Additionally, y/x ratio varies from 1.0 to ∞ , and it is used to 0.50 in plastic theory. In 1968, Hsu expressed that when $f_t(max)$ is taken at about 5 to 6 $\sqrt{f'_c}$ and α at $1/3$ torsional cracking moment strength T_{cr} can be expressed as [10]

$$T_{cr} = \frac{1}{3} x^2 y \left(5 \text{ to } 6 \sqrt{f'_c} \right). \quad (2.15)$$

Contrary to popular belief, Hsu expressed that a rectangular cross-section under torsional moment does not cracked in a spiral form as expected from a circular cross-section.

As seen in the Figure 2.12, the rectangular section concrete beam subjected to torsion will cracked by bending with a 45-degrees angle and then collapse with the same angle and this is called the skew bending theory.

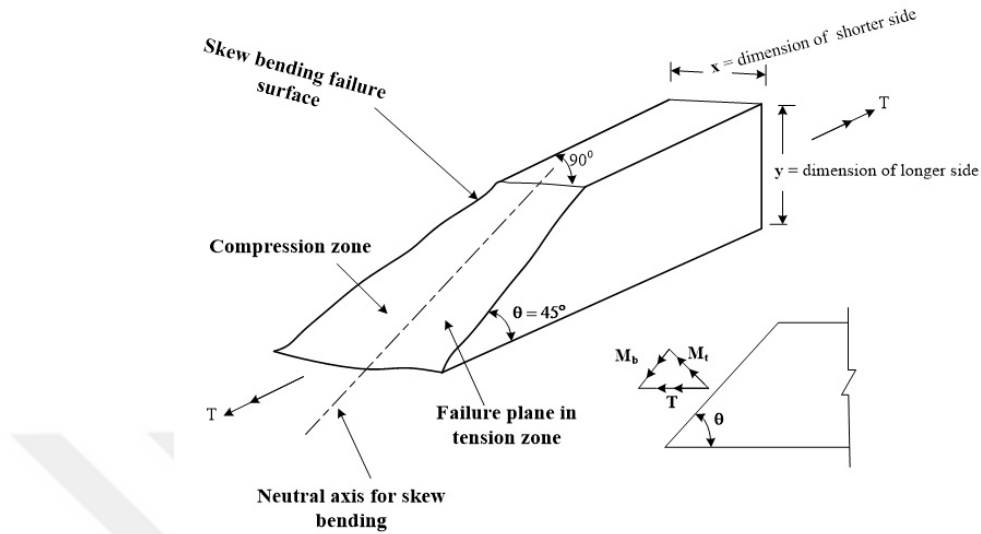


Figure 2.12. Failure of the rectangular beam without steel in torsion [11].

2.5. The Behavior of Reinforced Concrete Members Under Pure Torsion

Reinforced concrete elements subjected to torsional moments can be reinforced with only longitudinal bars or with both longitudinal and transverse reinforcement. The cracking torque may exceed about 15 percent of the ultimate torque in the case of using high amount of reinforcement. Since the members reinforced only longitudinal bars behave as plain concrete under torsion they do not have a significant importance and the torsional strength of members can be calculated as plain concrete members.

A reinforced concrete member needs to be made up of longitudinal bars and closed stirrups to resist torsion. The torque-twist curves of a few reinforced concrete beams with the same cross-sections and different percentages of longitudinal and transverse reinforcement are displayed in Figure 2.13 (ACI 445.1R-12) [12].

Each curve may be separated into two separate behavioral stages: the pre-cracking and post-cracking phases. The behavior is simply similar to the behavior of a plain concrete member in the pre-cracking stage. The elastic theory of Saint Venant can be used to assume the behavior at this point because the amount of rein-

forcement has very little effect on the torsional stiffness of the member. It is expected that the angle of twist increases immediately after cracking under constant torque, as seen in Figure 2.13. At this point, when the load is transferred to the reinforcement it will increase the stresses of the reinforcement unexpectedly and this case causes the establishment of a new configuration of the equilibrium.

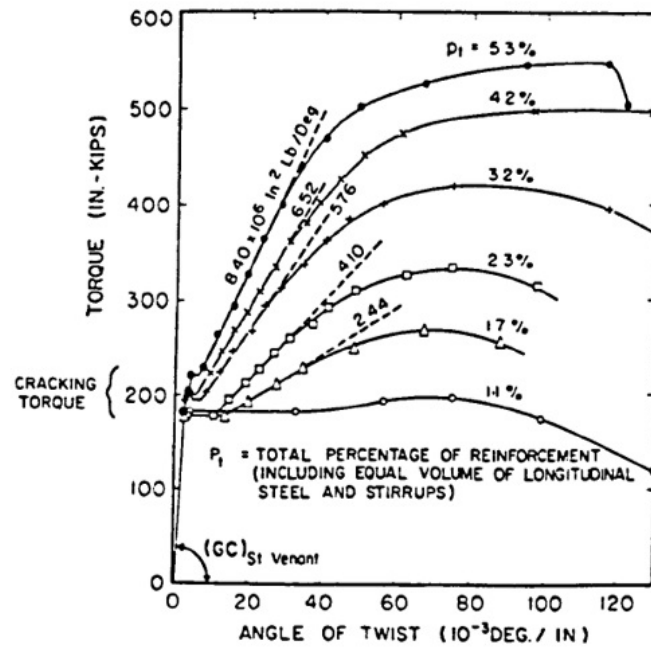


Figure 2.13. Effects of reinforcement ratio on torque-twist curve of beams [12].

Under torsion, the cracks on the reinforced concrete members were inclined at a 45-degree angle with the longitudinal axis. The cracks occurred at the same time on the wider and shorter faces of the members. After cracking longitudinal elongations occur as the applied torque increases, interestingly.

The elongation of the member causes the stretching of the longitudinal reinforcement. Stretching of the longitudinal steel is required to progress of tensile forces to maintain post-cracking equilibrium. If the longitudinal elongation of a beam is prevented, for example edge beams in reinforced concrete structures, a compression force will be produced in the beam. This self-generated compression will result in the induction of a concentrically pre-stress force, which will increase the torsional strength.

The torque-twist curve in Figure 2.13 rises with the increases of the twist under a constant torque and with a torsional stiffness that is much lower than it was before cracking. The percentage of reinforcement is strongly effective on the ultimate torsional strength and on the post-cracking torsional stiffness. The longitudinal and the transverse reinforcement can both yield at the ultimate level when small reinforcement percentages were used.

For an adequately reinforced concrete member, as in Figure 2.14 (a), the concrete crack pattern will be developed with the same or more than an unreinforced member's crack pattern at a similar torque value. The formation of the cracks is developed in a spiral pattern with θ angle as shown in Figure 2.14 (b). The torsional resistance of the concrete decreases at about half of the initial and the remaining is resisted by reinforcement.

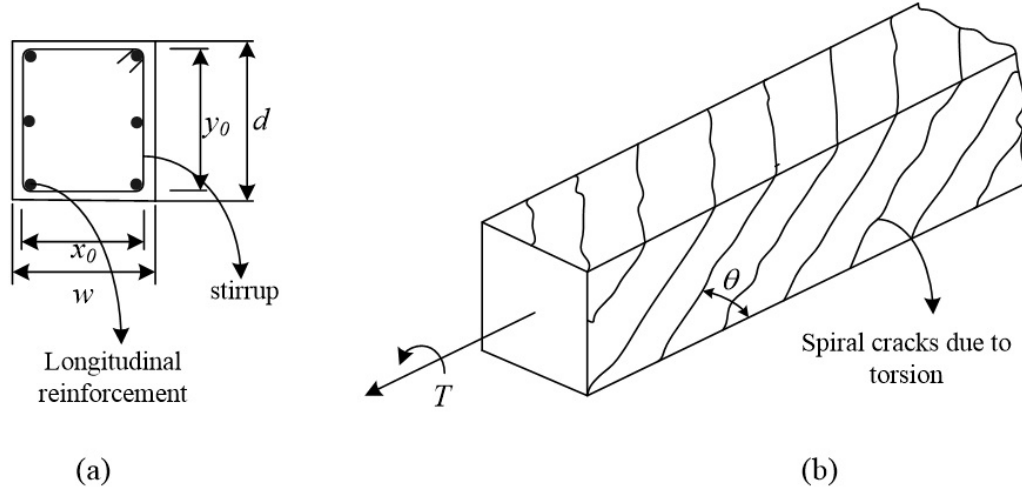


Figure 2.14. RC beams in torsion: (a) torsional reinforcement (b) torsional cracks.

The torque-twist curve is reflected the redistribution of internal forces as explained in Figure 2.15. This curve means that after the cracking torque, it shows continued twist at a constant torque until the redistribution of the internal forces from concrete to the reinforcement. The cover concrete outside the steel cracks and spalling of the concrete starts when the section reaches the ultimate torsional strength and the contribution of concrete will be progressively less to the torsional capacity of the

member. A few theories improved to calculation of the ultimate strength of reinforced concrete members with longitudinal and transverse steel. These theories can be divided into two categories such as the “space truss analogy” and the “skew bending theory”.

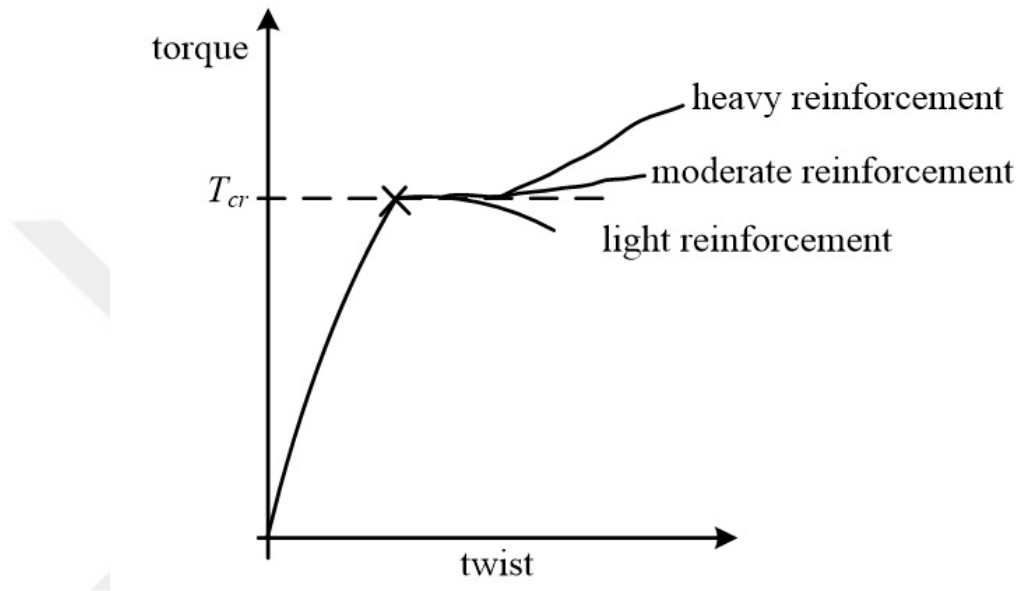


Figure 2.15. Torque-twist curves of RC members under pure torsion [8].

2.5.1. Space Truss Analogy

Flexural shear resistance can be defined better with the space truss analogy which is an advanced version of the plane truss analogy as shown in Figure 2.16. Under torsion, spiral concrete fibers can resist the parallel forces not perpendicular to the torsional cracks. The closed stirrups or ties resist tension forces which cause the cracks. As the last one, longitudinal reinforcement provides the tension chords.

All components of a reinforced concrete member resist to torsion and the member behaves like a space truss, this idea helps to analyze the torsional resistance of the member. Space-truss analogy uses the hollow tube to simplify the actual behavior. According to this analogy, the torsional strength is controlled by the transverse reinforcement strength independently from the strength of concrete. Although such a simplification helps understand behavior, it cannot accurately predict the large amount

of torsional strength that high-strength concretes have because it considers the torsional capacity of concrete as unimportant.

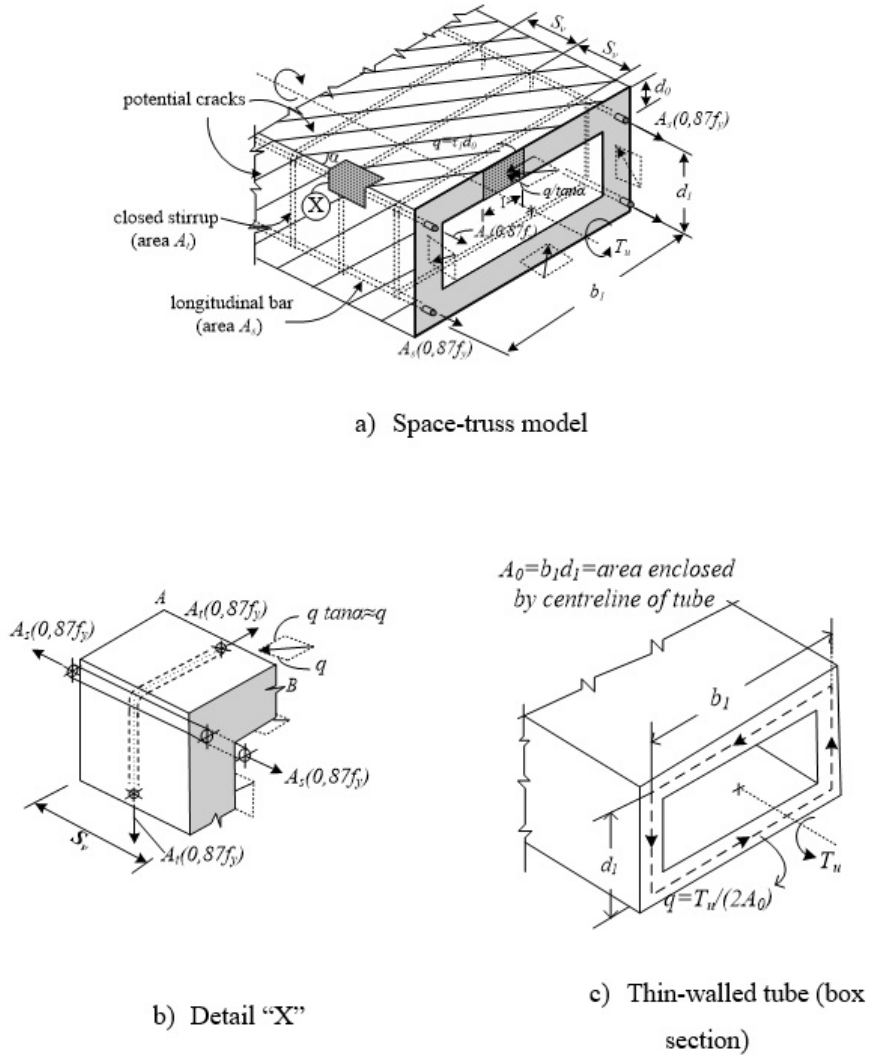


Figure 2.16. The idealized space-truss model [8].

The effective part of the beam consisting of concrete that covers the reinforcement cage and the transverse and longitudinal torsional reinforcement can be idealized using the space truss model. Therefore, in the post-cracking phase under torsional loading, the "thin-walled tube" concept becomes more effective. Working principle of the assumed truss consists of the concrete between the cracks as compression diagonals, the longitudinal corner steels as stringers, and the legs of closed stirrups as transverse ties.

2.5.2. Skew Bending Theory

The behavioral pattern of a section of a reinforced concrete element under torsion is different from that of plain concrete because of the reinforcement, both longitudinal and transverse placed into the member. During the torsional case, transverse reinforcement in the form of closed hoops shows a resistance similar to that against flexural shear. The behavior of the steel can be divided into two parts as the reinforcement resists to a little part of the total torsional moment before the cracking but after the cracking, it takes the large portion. In this case, the concrete contribution is nearly 40 percent of the total torsional strength for an unreinforced section. In line with all these assumptions, the defined failure mode according to this theory is the base of skew bending.

As another assumption of the theory, the failure portion is accepted to be a perpendicular plane to the wider side of the element and it is inclined at a 45-degree angle to the axis of the element as stated in Figure 2.17. Since the bending is also accepted as a failure mode, the compression zone with depth a at which the compressive stress can be considered uniform is taken into account as in any beam analysis. It is accepted as the tension side is not resist to tension because the concrete cracks consist on it. P_v expresses the tensile forces in reinforcing hoops and the longitudinal bars resist to shear which is occur across the cracked concrete by using the dowel action. Q_x and Q_y define the horizontal and vertical components of dowel action.

As long as the concrete transmits the shear force itself or stay as uncracked, the dowel action will not be seen. There is no longitudinal force that can exist on the tension side of the member. If the opposite happens, there has to be counterbalanced with a resistance force acting oppositely. The resultant force P in the tension zone should be directed upward (in the opposite direction to P in the compression zone) because, in the longitudinal direction of the tension side, there is no resultant compression or tension force that can exist under skew bending. It should not be forgotten that there is a potential failure zone can exist, opposite to that in Figure 2.17, where is the

3. LITERATURE REVIEW

3.1. History of Torsion Theories and Models

Early in the 20th century, engineers were concerned with torsion, realizing that torsion moments caused shear stresses, which increased the flexural shear stresses. Torsional shear stresses were usually very high due to torsional moments calculated using linear analyses. Engineers and researchers, who were aware of the brittle nature of diagonal tension failures, were alarmed by these high shear stresses. As a result of such observations, very strict design requirements for torsion were enforced in Europe. However, as time went by engineers and researchers realized that torsion rarely caused any serious problems in existing structures. As a result of these observations, design requirements were somewhat relaxed and research work deviated to other fields.

To estimate the torsional capacity of plain concrete members, researchers have developed several different theories, and today these theories form the fundamentals of many models. In 1787, the relationship between the torsional moment of an elastic circular shaft and the twisting angle was found by Coulomb [2]. Based on this definition, in 1826, Navier derived a new theoretical formula for calculating torsional strength of a non-circular elastic member [3]. As a more complex problem, pure torsion of non-circular elastic members and prediction of a precise expression for warping of the cross-section of the members required more developed mathematical tools such as the theory of elasticity and the Fourier series. To solve the problem, Saint Venant used the semi-inverse method three decades later [4].

A simple solution for a thin-walled element, and after for almost any cross-section, under pure torsion was proposed by Bredt in 1896. According to Bredt's theory, shear stress flows across the perimeter of the cross section as a result of the torque that the section is resisting. It is assumed that the core section of a solid member has any contribution to the torsional resistance. It is commonly accepted by the basic torsion

theories as reinforced concrete members are membrane elements in two dimensions that are subjected to both normal and shear stresses. Also assumes that the torque resisted by the section acts as flowing of the shear stresses around the perimeter of cross section [5]. Torsion design and shear design can be related to each other, since shear stresses will occur on the surfaces of a reinforced concrete element exposed to torsional moments. In 1899, Ritter proposed a simple model and called the classical truss analogy as shown in Figure 3.1.

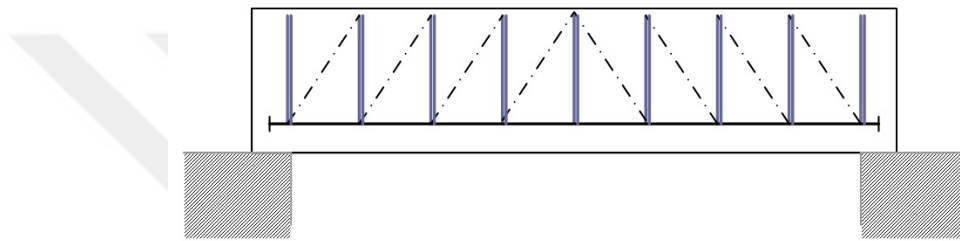
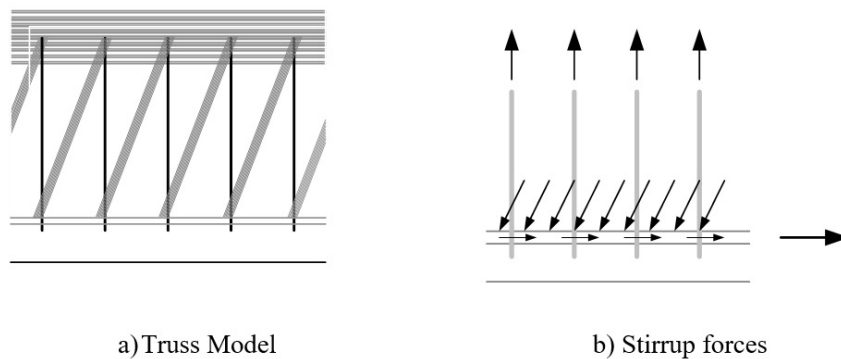


Figure 3.1. Ritter's original truss model [14].

A few years later, Mörsch studied on the same analogy [14, 15]. According to this analogy, a cracked reinforced concrete beam section acts like a truss. Under pure torsion, when the struts of the beam section affected from the shear forces, compression is resisted by concrete struts, and produced tension is resisted by the transverse reinforcement. as can be shown in Figure 3.2.



a) Truss Model

b) Stirrup forces

Figure 3.2. Mörsch's original truss model [15].

It is expressed by Prandtl that the deflection of a uniform elastic thin membrane member loaded by a uniform pressure is similar with the stress function within the

torsion, in 1903 [16]. After the membrane analogy for the plastic materials of Prandtl, the sand heap analogy was defined to estimate the fully plastic torque of solid sections. Nadai suggested a sand-heap analogy for a perfectly plastic material a corresponding analogy based on the shape of a sand-hill could be used for solid cylinders. Nadai presented this analogy first in 1923 at a meeting of the “German Society of Applied Mathematics and Mechanics” in Marburg, Germany. [17]. Sadowsky extended sand heap analogy derived by Nadai. Hence, it can be applied to plastic sections with holes [18].

After the definitions of the models, a calculation method was proposed by Rausch, in 1929. The torsional strength of the reinforced concrete members can be calculated by using this basic formula. He considered the reinforced concrete member section as a hollow section for simplification and later, experimental studies confirmed this assumption. Moreover, the formula overestimates the torsional strength of the cross-sections because of the incorrect determination of the centerline of the shear flow region [19].

The skew bending theory was proposed for the skew failure surface by Lessig. It is assumed that a reinforced concrete member under torsion will have a skewed failure surface according to theory. By using the equilibrium equations, Lessig proposed two skew failure modes; one has a compression zone at the top face of the member and the other one uses a compression zone along the side face [20]. On the other hand, initial skew bending theory was not able to determine three unknowns; the compression zone depth, the longitudinal reinforcement and the web steel area. In 1962, three equilibrium equations were proposed by Yudin to solve this problem [21]. These additional equations were; 1. “Equilibrium of moments about an axis through the centroid of the compression zone and parallel to the longitudinal axis of the beam”, 2. “Equilibrium of moments about an axis through the centroid of the compression zone and perpendicular to the longitudinal axis”, and 3. “Equilibrium of forces along the normal to the compression zone”.

At the Swiss Federal Institute of Technology in Zürich, Lampert and Thurlimann tested several series of specimens under the torsion and combined loading with bending and shear, and they suggested a model based on the space truss model to determine the combined effects of torsion and bending. When the failure of concrete is neglected for an inadequately reinforced member, failure can be calculated by the yielding of transverse and longitudinal reinforcement. With this assumption, the failure mode can be determined by the upper and lower bound theorems of the Theory of Plasticity. To approve this failure model, it is assumed an idealized elastic-plastic stress strain curve for the reinforcement, and equilibrium equations are formulated for an undeformed system [22, 23].

Additional studies were done by the researchers to understand the behavior of RC members subjected to torsional moment and combined with other forces. In 1972, Elfgren developed a calculation method to determine the capacity of reinforced concrete beams under combined shear and torsion by using the truss analogy. The first failure mechanism for combined torsion, bending and shear for beams where the steel yielded before the concrete failure due to compression defined by Elfgren. He derived an interaction equation to plot an interaction surface [24–28].

Another research group studying behavior under pure torsion was formed by Collins and Mitchell. They described the behavior of reinforced concrete beams subjected to pure torsion using equilibrium equations, geometry of deformations, and stress-strain relations between concrete and steel, and named this theoretical model as compression field theory [29–31].

Understanding the torsional theories, first linear models should be well defined. Studies on linear models started in the late 19th century. The first realistic theory for reinforced concrete members included Navier’s three principles of mechanics of materials: strain compatibility; stress equilibrium; and constitutive law of material. The Bernoulli compatibility truss model has been generalized the linear bending theory to cover the non-linear and linear theories for bending, axial load, and their combination.

Thus, four truss model contain the shear and torsion was defined as:

- “The equilibrium truss model”,
- “The Mohr compatibility truss model”,
- “The rotating angle softened truss model”,
- “The fixed angle softened truss model”.

Finally, a unified theory consisting of six rational models was developed, including Bernoulli’s strut-and-tie model and compatibility truss model. In addition, the rotating angle and the fixed angle softened truss model are also used to predict the nonlinear behavior. Although the model of the fixed angle softening truss is more complicated, this model also includes the “contribution of concrete”. Thus, the rotating angle softened truss model was used to more shear dominant structures such as; low-rise shear walls, framed shear walls, deep beams and shear transfer zones. In 1968, Hsu published a paper to explain these models and the application of the rotating angle softened-truss model to these shear dominant structures [32, 33].

To predict the post cracking torsional behavior, Hsu and Mo developed the Softened Truss Model (STM) in 1985 [34]. This model can predict very well all existing test results of the reinforced concrete beams under torsion up to the peak point. During torsion, softening of concrete increases the effective shear flow thickness. Jeng and Hsu proposed a new model called the Softened Membrane Model for Torsion (SMMT) [35]. This new model is capable of predicting the entire torque twist curve, including the pre-cracking and the post cracking levels by making the two modifications.

First, the pre-cracking stiffness and the strain at peak stress should each be increased by 45 percent in the tensile stress–strain relationship of concrete. Second, the Hsu/Zhu ratio for torsion is taken as 80 percent of the Hsu/Zhu ratio for shear. The analytical results obtained using the new model and the results of the experimental studies available in the literature were quite compatible [36].

Bernardo et.al. developed a new model to predict the ultimate behavior of the normal and high strength concrete beams under torsion. In addition, the researchers examined both plain and hollow beams. To take into consideration the softening and stiffening effect they work on σ - ε relationships (for concrete in compression and for steel in tension). They developed the new model on the variable angle truss model (VATM) instead of skew bending theory because of the better computing advantages. Researchers realize that the results were independent from the cross-section type (hollow or plain). Softening is more effective on the behavior of the HSC beams than the NSC beams. In addition, the researchers compared the existing codes using the model they developed. According to their results, although ACI is not accurate enough for the torsional strength of small section beams with high torsional reinforcement ratios. The other codes are more conservative for the NSC beams but not the HSC beams. Finally, when all the results were evaluated, they were suggested that the most realistic results can be obtained with the TORQUE_VATM model [37].

New models are constantly being developed for the behavior of reinforced concrete elements under torsion. One of them is the Generalized Softened Variable Angle Truss Model for (GSVATM) for reinforced concrete beams. Since the current model covers solid beams, it was developed for hollow reinforced concrete beams. The results obtained from the developed model were compared with the experimental results in the literature and it was observed that there was a very good agreement between the results [38].

Firstly, the relaxation methods for the elasto-plastic solution of an I-section were used by Christopherson [39]. On the other hand, analytical solutions for non-circular members subjected to pure torsion that have elasto-plastic material properties subjected to pure torsion have been derived only for prismatic bars with basic cross-sections. Analytical elasto-plastic models for overall cross-sections defined by Sokolovsky. After this improvement, it was possible to obtain an approximate solution combining the membrane and the sand heap analogies for different elasto-plastic cross-sections by Nadai [40,41].

3.2. Background of Plain Concrete Studies

Researchers were increasingly interested in the behavior under the torsion of structural members in the 1900s. Numerous analytical and experimental works were done to understand the behavior under the torsion of plain/reinforced concrete structural members.

The plain concrete rectangular members behave linearly under low torques and slowly become non-linear while the torque increases. The element bends about an axis parallel to the wider face with a 45-degrees angle to the longitudinal axis of the member. Failure is sudden and brittle. To estimate the torsional strength of pure concrete members three approaches have been improved and bases of these developed analogies are elastic theory, plastic theory, and skew bending theory.

According to the elastic theory, it is assumed that concrete is a homogenous material and the stress distribution becomes proper to Saint Venant's theory. When the concrete tensile strength, f_t , and the maximum principal tensile stress of the pure concrete member are equal to each other torsional failure occurs.

Outer fibers of a partially plastic section will be stressed up to the ultimate while fibers near the twisting center will be stressed not so much and their behavior will be elastic. Because of this case, the contribution of the outer fibers to the total resisting torque will be more than the less stressed elastic fibers near to the twisting center. Armstrong studied on the plasticity ratio of the non-circular sections and circular sections to explain the difference between the ratio of one-half and one, in 1956. As a result, it is proved that there is a quite small difference and a new equation was derived to calculate of ultimate torque of a fully plastic circular section [42].

Hsu worked on the torsional failure mechanism of rectangular beams due to the difficulties of applying the basic elastic and plastic theories to pure concrete elements. According to experimental tests, the first crack developed at the front face of the beam

with a 45-degrees angle to its longitudinal axis. As the torsional loading continued, the crack slightly widened and across to the top of the member. At the final stage of the test, cracks turned around the member spirally, and concrete crushing was observed. A similar failure mode with the plain flexural member took place. In 1968, Hsu improved an equation by using a bending mechanism for torsional failure to calculate the maximum torsional strength of plain concrete [32, 33].

The size effect is an important issue for all brittle failure types of concrete and reinforced concrete structures, also include torsion. In literature, there is quite less experimental data on the size effect in torsional behavior of concrete members than for shear. The known first studies were done by Humphreys, McMullen and Daniel etc. between 1957 and 1975 [43, 44].

Later, these data were collected by Bažant and Şener to understand the relation between the size effect and damage caused by distributed cracks. Recently, the results getting from fracture mechanics can be applied to torsional failure of unreinforced or longitudinally reinforced concrete beams without stirrups. They proved the existence of the size effect for concrete elements and confirmed the rules regarding the size effect with their experimental studies.

However, the range of data obtained from the experiments was so narrow and their distribution so large that the precise form of the size effect was difficult to define. To take the size effect into account, a simple and theoretically based equation was recommended. They concluded that the existence of the size effect was proved by using the test data on the torsional failure of the unreinforced and longitudinally reinforced concrete beams without stirrups, i.e., while the cross-section size increases the nominal stress at failure decreases.

In the following study, Bažant et al. performed a group experimental test to understand the size effect on the plain concrete and reinforced concrete members subjected to torsion (Figure 3.3). The experimental results are compatible with the pre-

vious study and confirmed the existence of a significant size effect. They also observed that the longitudinal reinforcing steel did not contribute significantly to the torsional strength [45–47].

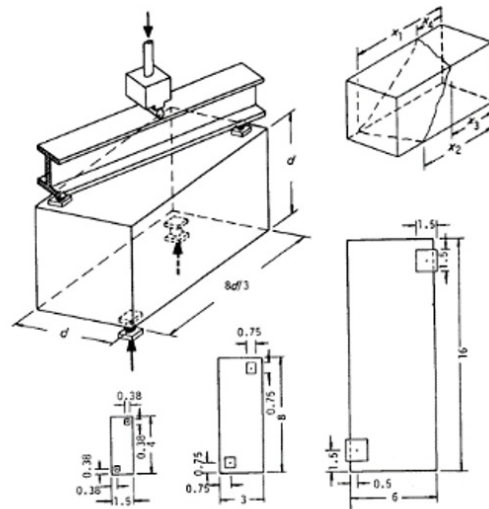


Fig. 1 Geometry of the specimens (dimensions in inches; 1 in. = 25.4 mm).

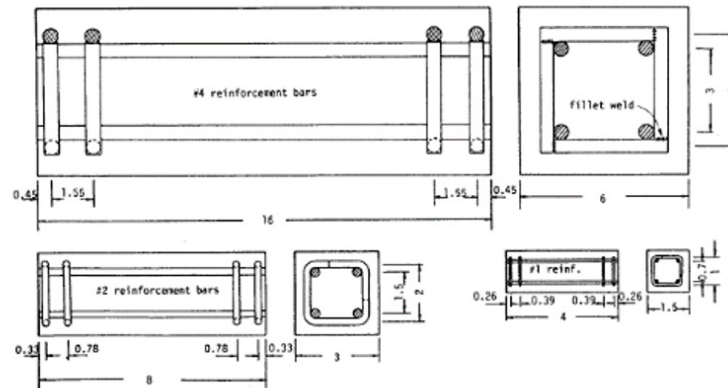


Fig. 2 Reinforcement of the specimens (dimensions in inches; 1 in. = 25.4 mm).

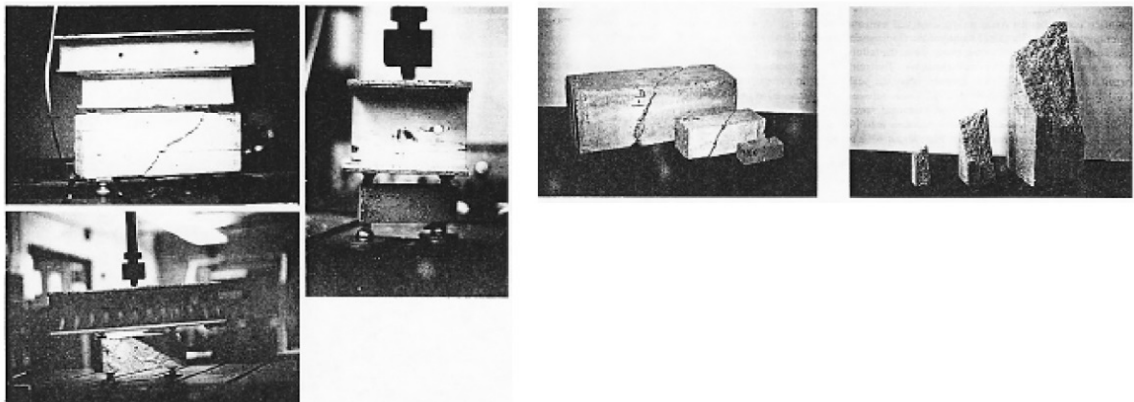


Figure 3.3. Typical torsional failure mode of micro-concrete beams [45–47].

In 1998, Subramaniam et al. conducted torsion tests to investigate the concrete shear response. It is difficult to obtain the full load-deformation response of unreinforced concrete members subjected to torsion due to test instabilities. The torsional tests of hollow concrete cylinders were carried out with a closed-loop test system. It was indicated that the snapback in the post-peak part of the torque-twist response curve of specimens was the reason for the test instability. It was observed during the test, that only one large crack propagated throughout the structure was observed to cause damage. A test signal that combination of torque and gauge rotation, was defined and shown to provide improved control by removing test instability. Concrete specimens were tested under torsion both controlled and uncontrolled conditions using exclusive test equipment. The test results were analyzed and reported in terms of the propagation of a single significant crack [48].

3.3. Background Research on Reinforced Concrete Studies

A basic formula developed by Rausch to forecast the torsional strength of reinforced concrete elements (1929). The torsional strength of reinforced concrete members is overestimated at an intolerable level by the classical formula of Rausch [19]. The centerline of the shear flow is determined incorrectly and this causes an error in results [12].

The space truss theory which was first defined by Rausch, calculates the theoretical torsional strength of reinforced concrete members under torsion. (1929) [19]. The cage that is composed of the reinforcement and the concrete cover surrounding the steel is the basis of the space truss analogy which calculates the torsional strength of a reinforced concrete section. A thin-walled section shown in Figure 3.4 is supposed to be a space truss. The longitudinal bars at the corners which are made up of diagonal concrete struts restrained by tension in the stirrup and longitudinal reinforcement, resist the tensile forces generated by the torsional moment, while the inclined spiral concrete strips between cracks resist the compressive forces. For reinforced concrete members under pure torsion, the diagonal cracks divide the concrete into struts af-

ter cracking. These compression diagonals consist of concrete between spiral cracks around the member with a constant angle, and resist torsion according to the space truss model.

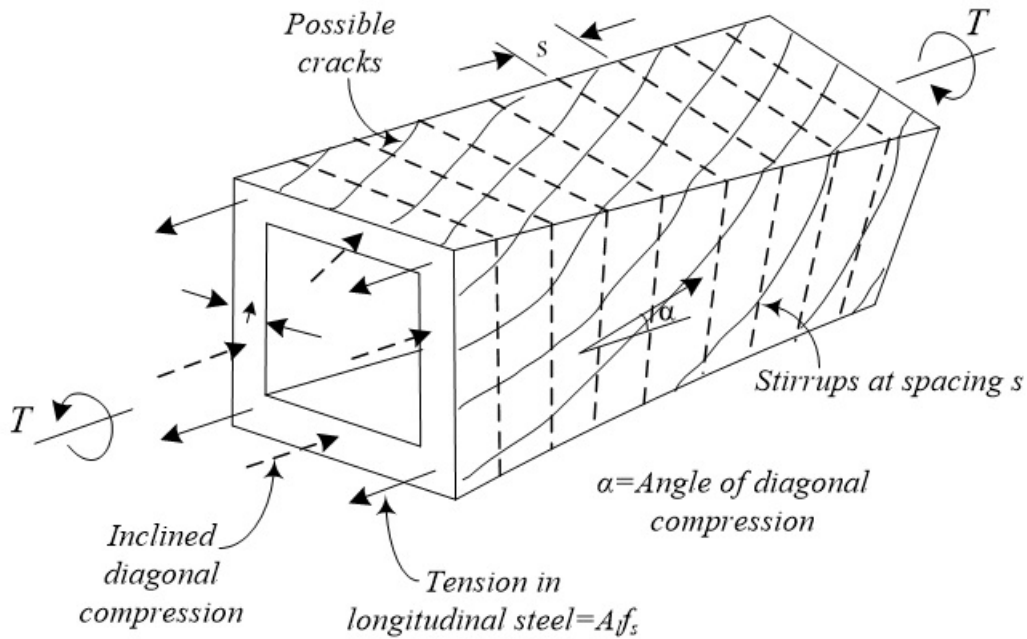


Figure 3.4. Forces on section in torsion (space truss analogy) [12].

Until 1961, insufficient studies had been done on the combined effects of bending, shearing and torsion in reinforced concrete beams. Kemp et.al. published a report on torsion about reinforced concrete members. The target point of the report was to investigate the experimental and analytical behavior of concrete beams under torsional moment and prove the current knowledge about the torsional behavior of reinforced concrete members. The report concerning the behavior of plain concrete, reinforced concrete and pre-stressed concrete members subjected to torsion [9].

Reinforced concrete structures exposed to torsion often arise from continuity between members. Various studies have been executed to improve the knowledge of the torsional behavior of Reinforced Concrete (RC) beams since the beginning of the last century.

There are several theories present for analysis of reinforced concrete members subjected to torsion. The two basic theories are:

- (i) “Skew bending theory”,
- (ii) “Space truss analogy”.

The skew bending theory was first introduced by Lessig in 1959 [20]. Later this theory was developed by many researchers such as Goode and Helmy in 1968, Hsu in 1968, and Rangan in 1977 [49–51]. The skew bending theory was applied to reinforced concrete beams subjected to torsion and bending. Nylander carried out the first tests on a combined loading includes shear, bending, and torsion (1945). The researcher realized that the torsional strength of the element was reduced by the bending moment when using only longitudinal reinforcement while ignoring the transverse steel [52]. ACI Committee accepted this theory and ACI Code of 1971 re-edited. The fundamental of the theory is bending about an axis parallel to the wider face of the section and inclined at about 45 degrees to the longitudinal axis of the beam causing failure of the rectangular section under torsion. The angle of the concrete struts was calculated by using a derivation of the compatibility equation defined by Collins [53, 54].

The effective thickness of the shear flow zone can be described as a tube wall and varies from zero to maximum to the side of the concrete member and it creates a powerful gradient of strain. In 1990, Hsu introduced a simple theoretical method based on the softened truss model theory to calculate the shear flow zone thickness (t_d). The centerline position of the shear flow zone directly depends on t_d . The calculation of shear flow thickness (t_d) in the torsion case is similar to the calculation of the compression zone depth in the bending case. In addition to the softened stress-strain relationship for concrete struts, the equilibrium and compatibility conditions are used in the advised method.

A precise torsional strength of the reinforced concrete members can be estimated by the calculation of t_d even if it is a tough procedure. The test results of the 61 beams

from the literature were compared with the results of the proposed model and it was observed that the conclusions were in good agreement with each other. In addition, a more simple t_d formula given with Equation (3.1) has been recommended to facilitate the torsional design of reinforced concrete elements [55];

$$t_d = \frac{4T_n}{A_c f'_c}. \quad (3.1)$$

Element size is also a very important parameter in the behavior of reinforced concrete beams subjected to torsion. However, this parameter is ignored in codes. In order to understand the importance of size effects on torsional behavior, Bazant et.al. performed a series of tests on reduced-scale beams in 1987. It is proved by the test results that the nominal stress was reduced when the beam size increased and this means that there was an evident size effect for both plain and reinforced concrete members. According to the test results, it is proved that longitudinal reinforcement has no significant contribution to the torsional strength [45, 46].

Kozonis tested six reinforced concrete beams under pure torsion and based on the experimental results examined methods to predict the response of the beams and proposed an equation for torsional capacity. He also repaired the specimens with the steel band bonding technique provided that external stirrups of suitable dimensions. Results showed that the repairing increases the torsional capacity of the beam compared to the reference specimen [56].

In 1997, a limit analysis method to estimate the torsional strength of RC beams was developed by Wang and Hsu. The modified constitutive law of the concrete Coulomb-Mohr failure criterion has accepted the relation between the post-yielding plastic flow and the failure surface of the member was used in the limit analysis method [13]. In this research, an ordinary equation for the ratio of energy dissipation along the yield line by the failure criterion as well as the upper bound method was used. It is possible to predict the ultimate torsional strength of reinforced concrete beams under pure torsional moment by the work equation based on the allowable failure mechanism at the ultimate level and the energy dissipation ratio of the member. The existing experimental results in the literature were compared with the theoretic-

cal torsional strengths and it was shown that the conclusions were found to be quite compatible [57].

Until the present day, many studies are done to understand the behavior of structural concrete members under torsional loadings and the effect of reinforcement on strength of members. One of these studies was done by Csikós and Hegedûs via comparing the test results of 15 reinforced and plain concrete beams. They compared the results of the different equations for cracking and ultimate moment with the test results. There is a good agreement between the test result and theoretical values [58].

Gunneswara, Rao and Seshu developed an analytical model for predicting the torque–twist response of steel fiber reinforced concrete members subjected to pure torsional loads considering the softening effect of concrete. At the same time, they tested 15 rectangular beams under pure torsion up to the ultimate torque. As a result of the study, they found that the addition of steel fibers in steel fiber reinforced concrete members improves the first cracking load and pre-cracking torsional stiffness of the member under pure torsional loading [59].

In 2007, Ameli, Ronagh and Dux tested twelve rectangular reinforced concrete beams under torsion. Six of the beams strengthened with Carbon Fiber Reinforced Polymer (CFRP) and the remaining six strengthened with Glass Fiber Reinforced Polymer (GFRP). As outcomes of the experimental study, the application of FRP can considerably improve the ultimate torque capacity of beams in addition to members behaving more ductile. Researchers completed a numerical study and performed analyses of the beams in the ANSYS finite element program. When the numerical results of the experiments and analyses were compared in terms of ultimate torque strength and the ultimate twist angle, it was concluded that they were in reasonable agreement. Although the torque-twist curve obtained from the analysis differs from the test observations at the post-cracking stage of concrete, the curve again catches up with the experimental results at around the ultimate torque [60].

An experimental study made by Chiu, Fang, Young and Shiau, on the thirteen full-sized beams produced with high and normal strength concrete and relatively insufficient amounts of torsional reinforcement. Researchers also examined the torsional strength and ductility levels of beams, the pattern of cracks in addition to the maximum crack widths at the ultimate torque level, and the reserve strength of the member after cracking. Their main parameters included the aspect ratio of the beam cross-section, the volumetric ratio of torsional reinforcements and the concrete compressive strength. This study proved that the total amount of torsional reinforcement added to the reinforcement ratio affected the reserve strength of specimens after cracking when compared with the relatively low amount of torsional reinforcement [61].

After the minimum torsional reinforcement ratio, researchers were interested in the maximum volume of steel. Because, torsional reinforcement ratio is very effective in preventing the failure of the concrete compressive struts before the steel yielding. To understand the compliance of the maximum reinforcement ratio recommended by the codes Hyunjin Ju et al conducted a parametric study to derive a theoretical maximum reinforcement ratio. Results showed that the ACI 318 gave the most reliable values for the maximum reinforcement ratio. CSA-14 estimated the acceptable ratio for the yielding of the transverse steel. The results for the maximum reinforcement ratio were taken from EC2 code was stayed on the unsafe side moderately. The proposed model suggested by the researchers could well predict the yielding of steel and the failure modes [62].

The study was done by Bernardo and Lopes is a comparative work to develop a model to draw a theoretical torque-twist curve. Researchers used the Theory of Elasticity and Skew-Bending Theory to predict the cracking torque of beams with plain sections. However, for hollow beams, Bredt's Thin-Tube Theory gives better results. To calculate the torsional stiffness, a reducing factor $K=0,7$ must be used to take care of the microcracking of the concrete. 3-D truss model was advised for the post cracking stage. Finally, the Variable Angle Truss Model was used to consider the softening effect and the calculations gave good results. The model developed by the

researchers was applied to the test results in the literature and the results were quite satisfactory [63].

Mullapudi and Ayoub investigated the interaction between the inelastic non-linear behavior under the bending, shear, axial force, and torsion of the reinforced concrete (RC) members with the seismic analysis of same members. A new concrete model was developed that uses the relation between the orthotropic constitutive behavior and the principal directions of the total strains. The basis of the model is the effects of softening and confining. There is a correlation between the experimental results and analytical results of the reinforced concrete elements to prove the validity of the model. According to the obtained conclusions from the study, under combined loading of bending and torsion, the energy dissipation capacity and ductility of the reinforced concrete elements were significantly reduced. [64].

Reinforced concrete members elongate axially under the cyclic torsional loading due to flexural yielding. Longitudinal elongation can reduce the deformation capacity and strength of the beam. The parameters that affect the elongation of the reinforced concrete members are investigated by performing a nonlinear truss model analysis. The primary effect that causes longitudinal elongation can be described as the plastic strain of the longitudinal reinforcement in the plastic hinge region. Researchers developed a model to evaluate member elongation and applied the proposed model to test specimens with various design parameters and loading conditions [65].

Many civil engineering structures subjected to torsion as significant loading conditions such as bridges, spandrel beams, and the reinforced core around the elevator shaft of buildings. During an earthquake, if the mass center of a slender building is eccentric, cyclic torsional loads are occurred on the reinforced concrete core. Usually, torsion is acted as a combination of shear and bending and for this reason, the required longitudinal and transverse reinforcement for torsional resistance is directly added to the shear reinforcement.

Today, there are a limited number of studies on the behavior of reinforced concrete structural elements under the influence of repeated cyclic torsional loading. Jacobsen, Hjorth-Hansen and Holand worked on the behavior of reinforced concrete box-members under cyclic torsional loading to explore the stiffness reduction and material energy dissipation. They tested six specimens; four in pure torsion and two with axial force in addition to the alternating torque. Their main parameters were the amount of reinforcement, the axial load (with/without) and the load history. They observed an increased energy dissipation with an increased twist level. Load cycling at the same twist level reduced the dissipated energy, but at a decreasing rate [66].

As an addition to the experimental studies, Belarbi and Greene defined a theoretical model for reinforced concrete members under cyclic torsion, in 2004. The improved model expanded upon the softened truss model by the addition of the tension-stiffened part of the torsional response. The bases of the proposed model are the bilinear relationship for concrete in tension and extra compatibility equations for the tension-stiffened zone. This case supplies a more developed prediction of the torsional behavior during either pre-cracking or post-cracking. As an experimental part of the study, one hollow reinforced concrete box girder was tested to understand the behavior of girders loaded in a combination of shear and torsion under seismic-like cyclic loading. At the end of the research, thanks to the proposed model, it will be possible to better predict the tension-stiffened region of the torsional response curve of elements with medium and high levels of reinforcement. The equations can be reduced by the suggested model using the softened truss analogy at the end of the tension-stiffened zone [67].

Although in reality cyclic pure torsion rarely affects the structures because they act on the structures together with other forces during seismic actions, pure torsion cyclic tests must be performed to understand the torsional behavior of reinforced concrete elements. An experimental investigation was done to understand reinforced concrete columns subjected to a loading that is a combination of cyclic uniaxial bending and cyclic torsion by Tirasit and Kawashima. Researchers produced seven reinforced concrete columnar specimens with the identical structural properties and tested under

different types of loading conditions such as; cyclic torsion, cyclic uniaxial bending, combined cyclic uniaxial bending-torsion with/without a constant axial loading. As a result of cyclic torsion tests, they observed that the torsional stiffness of the column deteriorates under significant torsion. When the test results of the specimen were exposed to torsion without axial force compared with the specimen under torsion with axial force, researchers examined that the effect of axial force on the hysteretical torsional behavior of members was not important according to the applied rotation increased past the point of maximum torsion [68].

The most explicit effects of torsion can be seen in the reinforced concrete bridge columns. Five reinforced concrete bridge columns with different cross-sectional shapes tested under reversed cyclic pure torsion. Test results proved that torsional stiffness is mainly affected by concrete strength and cross-section shape, but not by the amount of reinforcement. Moreover, the location and length of the torsional damage zone vary mainly depending on the cross-sectional shape, stirrup configuration, and aspect ratio [69].

As well known, concrete and steel work together to provide the torsional strength of a reinforced concrete member. However, the concrete contribution is negligible as the concrete cracks by acting brittle under torsion. An experimental and analytical study done by Khagehhosseini et.al. to investigate the torsional capacity of beams with only longitudinal steel without stirrup. After a series of analyses carried out using finite element analysis computation in ANSYS and compared the test results with the analysis result. Results were in good agreement with each other. As a result of tests, reinforced concrete beams reinforced with only longitudinal reinforcement exhibited better than expected torsional strength at large twist values [70].

To investigate the pure torsional behavior, longitudinal elongations should not be prevented and for this reason beams should not be axially restrained. Bernardo et al carried out a study to understand the effect of the axial restraint. According to the test results made by researchers, they developed a new model, named modified VATM, that

is convenient to foresee the ultimate strength of axially restrained reinforced concrete elements under torsion [71].

To fully comprehend the cyclic torsional behavior of hollow reinforced concrete beams an experimental program was carried out by Gajipara et.al. Twelve reinforced concrete hollow beams produced with different amounts of longitudinal and transverse steel and tested under pure cyclic torsion. As the interested test parameters were cracking torque, twist angle, energy dissipation capacity, crack pattern and failure mode. As a result, they observed that the longitudinal reinforcement is not effective at the pre-cracking stage but after cracking, all torsional moments resisted by the reinforcement. Also, they proved that the stirrup spacing is another important parameter to resist torsion. The torsional capacity of members is directly proportional to stirrup spacing and optimum spacing of lateral steel. Otherwise, the level of cracking torque is not affected by steel ratio and only it depends on the concrete grade. Longitudinal reinforcement is more effective in torsional behavior of reinforced concrete hollow beams at the post cracking levels [72].

Zhi et al investigated the torsional behavior of beams which were cast twice and reinforced with two-piece enclosed stirrups. They compared the experimental results of these specimens with the test results of beams produced by conventional methods. As a result, they found minor differences between the torsional behavior of composite beams reinforced with two-piece enclosed stirrups and those of monolithically cast beams reinforced with customary stirrups. Two -piece enclosed stirrups decreased torsional strengths of the reinforced concrete beams under torsion. Additionally, they observed that the second cast concrete on the upper part of specimens has relatively high strength compared to that of the first cast concrete at the lower part of the members [73].

The ductility value is one of the parameters of great importance in the analysis of reinforced concrete elements. When studied on the behavior of reinforced concrete elements under torsional moments, torsional ductility must be examined. For this

purpose, Teixeira and Bernardo developed a ductility index to analyze the torsional ductility of reinforced concrete beams. While creating this index, parameters such as concrete compressive strength, torsion reinforcement ratio and cross-section type of beam (plain or hollow) were used. As a result of the study, it was observed that the codes are quite restrictive in terms of torsional ductility. When we compare the reinforcement ratio with the concrete compressive strength, the steel ratio has a higher influence on the torsional ductility. Moreover, they realized that the concrete core has effectiveness on torsional ductility. For the similar reinforcement ratio, solid beams are more ductile than hollow beams [74].

Over reinforcement can cause the sudden torsional failure and to prevent this case, design codes limit the maximum steel ratio. To specify the accuracy of the limits given in design codes, 15 reinforced concrete beams with different ratios of reinforcement and different compressive strengths of concrete were tested. According to the experimental results, design codes ACI 318-14 and JSCE-07 gave more reasonable results, while design codes EC2-04 and CSA-14 overestimated the boundary between the two modes of torsional failure [75].

As well known, the torsional behavior of the reinforced concrete members basically depends on the steel and concrete. But there is not enough information about the cover concrete effect on the behavior. To understand the relation between the concrete cover and torsional behavior, nine reinforced concrete beams with different concrete cover depth were tested. It has been observed from the experiments that greater depth of concrete cover affects both torsional capacity and behavior of the beams. In addition, in the study, the criteria determined by the codes were examined and as a result, according to the ACI code, the larger concrete cover depth is the more secure, but the CSA and Eurocode provide more unsafe predictions [76].

An extensive database was searched and 202 rectangular reinforced concrete beam experiment data were used to improve the basic torsional equations. The researchers made a comparative analysis by examining the equations in the design codes used to

compute the torsional strength of reinforced concrete beams. The equations proposed by the researchers give slightly better values for the torsional strength of reinforced concrete members [77].

3.4. Background Research on High Strength Concrete Studies

Design of structures for resistance against seismic action of ground has seen a critical review in recent years, with the emphasis changing from “strength” to “performance”. Current seismic codes suggest a few design approaches to design the structures. Force-based and displacement-based design approaches are the most prominent of them. For example; Uniform Building Code (UBC) 2000, Federal Emergency Management Agency (FEMA) 1997 and Turkish Building Earthquake Code (TBEC) 2018, etc. performs the force-based demand-capacity analysis and also recognizes the importance of this methodology. Usually, an inelastic seismic response is indirectly and approximately defined with a factor that reduces the force demand and is estimated from a linear elastic response spectrum analysis which is implemented as a code design spectrum. Force-based design or in other words, performance-based design utilizes the mentioned force demand. After the determination of force demand, the demand capacity analysis runs for all of the members of the structure. To consider a structure to be safe and adequate, the elements of the structure can carry the force demands with the existing capacity. But in reality, although the seismic movements of the earth have a cyclic characteristic, reinforced concrete structural elements designed according to ultimate capacities often cannot maintain their initial stiffness and strength during the seismic response. Under these circumstances, the force-based design cannot determine the response phenomenon directly and this case causes the failures of the members and decreases the seismic performance of the structure.

Lopes et.al studied on the effects of ductility on the plastic hinge formation, using the results of their previous experiments. There are 4 different modes of failure based on the steel ratio of the reinforced concrete elements under pure torsion. If ordered from the least reinforcement ratio to the highest; brittle failure due to inadequate reinforcement,

ductile failure due to the balance case, brittle failure due to corner break-off due to over-reinforcement, and brittle failure due to inadequate strength of concrete. As an important failure mode, ductile failure occurs in a narrow range of reinforcement. In members with a ductile failure mode, plastic hinges form in regions where torsional rotation is concentrated, and the length of these regions is nearly equal to the beam height [78].

A new computational procedure has been developed for high-strength concrete hollow beams tested under pure torsion to foresee the torque-twist curve. Bredt's thin tube model gives the best results for the cracking torque (T_{cr}) value. Researchers defined new correcting factors of 0.85 that must be considered when computing the rupture modulus. The authors applied the model they developed with the studies in the literature and concluded that the theoretical results and the analysis results are in good agreement [79].

A study was conducted on the 16 hollow high strength (HSC) reinforced concrete specimens under torsion to discover the plastic behavior and twist capacity. Researchers used Plastic Trend Parameter (PTP), which is derived from the local plastic twist versus total twist curves of the tested specimens, to express the magnitude of the experimental twist capacity of the beams. Additionally, they search on the strength of concrete and the torsional steel ratio as parameters.

As a result, increasing concrete strength value decreases the PTP value and few important codes such as American, Canadian and European codes. Some important codes recommend the analysis over plastic twist capacity. Thus, the most appropriate maximum and minimum torsional reinforcement ratio can be imposed [80].

An experimental program consisting of 16 specimens was conducted by Lopes and Bernardo to investigate the failure mode and the analysis of the cracking development of hollow high-strength reinforced concrete members under pure torsional moment. Concrete compressive strength and the steel reinforcement ratio are the main param-

eters. As a result, they observed that the beams presented different kinds of behavior, failure mode, crack propagation under the torsional moments.

Moreover, researchers defined an uncommon form of failure (break off at the corners of beams) that can be observed for a particular range of steel ratios and that can prevent the beams from achieving the maximum levels of strength and ductility. Hence, it has been predicted that there is nonexistence of this kind of failure [81].

Today, codes used three main seismic design methods: performance based, displacement based and as the newest one is energy based seismic design method. Performance based seismic design method, as the most known method, preferred by the many codes for the design against destructive earthquakes. To develop this method a simple but effective method to predict the torsional behavior is needed and researchers improve very different methods. One of these methods is the 3D truss method which is proposed by Jung and Kwon. They first produce two specimens and test the beams under pure torsion. By using the optimization techniques and proposed 3D truss model, the torsional strength of the beams is evaluated. A 3-D truss model capable of predicting torsional behavior of reinforced concrete members is proposed [82].

3.5. Previous Research on Modeling of Hysteresis Behavior

Hysteresis modeling of reinforced concrete members under torsion should consider a constitutive relationship such as pinching effect and stiffness degradation. In addition, like all hysteretic models, the torsion hysteretic model should be mathematically applicable, generic and easily handled, and should be able to accurately predict behavior. In the literature, hysteresis models are divided into two main sections: 1- Polygonal Hysteresis Models (PHMs) and 2- Smooth Hysteresis Models (SHMs). The first hysteresis model considers the variation of stiffness during the cracking of member, strength, stiffness degradation and yielding levels. The second hysteresis model cares about the stiffness variation due to yielding and certain transitions during loading and unloading.

Sengupta and Li define the hysteretic models as given in Table 3.1 and as a result, the researchers proposed the BWBN model as the most suitable hysteretic model. The unique disadvantage of the proposed model is that the model requires a satisfactory solver due to complex mathematical expressions [83].

Table 3.1. Comparison of the hysteretic models.

Polygonal			Smooth	
Developer	Model	Disadvantages	Developer	Model
Veletsos et al.	The elasto-plastic model	Unable to represent realistic structural hysteresis	Bouc	smoothly varying hysteresis model for a single-degree-of-freedom (SDOF) system under forced vibration.
Clough et al.	The bilinear degrading stiffness model	Does not incorporate pinching effect	Baber and Wen	extended the model with the inclusion of stiffness and strength degradation.
Takeda	The trilinear primary curve model	Poor compared to the number of rules	Baber and Noori	extended the model with the inclusion pinching effects
Imbeault and Nielsen	In peak-oriented degrading bilinear (DBL) model	Unable to represent realistic structural hysteresis	Sengupta et al	modified original Bouc-Wen-Baber-Noori (BWBN) model by incorporating displacement-based stiffness ratio
Saidi and Sozen	In the bilinear primary curve of Q-hysteresis model	Impractically consider concrete section elastic until yielding while non-linearity initiates after appearance of cracks		
Otani	The bilinear primary curve of the hysteresis model	Impractically consider concrete section elastic until yielding while non-linearity initiates after appearance of cracks		
Ozcebe and Saatcioglu	The hysteresis shear model	Poor compared to the number of rules		
Dowell et al.	Pivot hysteresis model	Poor compared to the number of rules		
Sucuoglu and Erberik	The energy-based hysteresis model	Cannot perform satisfactorily at larger cycle numbers		
Ibarra et al.	The pinching hysteresis model	The contribution of four deterioration modes		

In order to create a good hysteretic model, a consistent back-bone curve is the envelope of the re-cyclic torque– twist behavior must first be drawn. The skeletal curve

to be created for this purpose should best express the monotonic behavior and linear and nonlinear regions should be well defined on the curve. The idealized torque-twist curve given in Figure 3.5 shows the key points of the envelope. Three points; cracking, yielding and the ultimate points, can be calculated by using the theories and analogies. Valipour and Foster recommended thin wall theory for the linear part and space truss analogy for the nonlinear part. When they compared the torque-twist curves obtained from the analysis method they developed with the experimental results in the literature, they obtained quite good results [84].

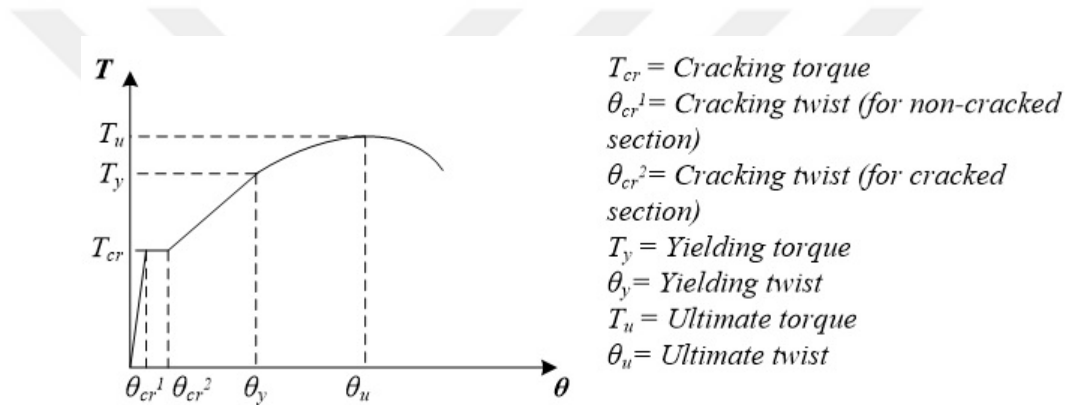


Figure 3.5. Idealized torque-twist envelope curve [84].

In 1987, Venkappa and Pandit worked on tests of the reinforced concrete beams subjected to cyclic torsion to investigate the deterioration of the torsional stiffness and strength under the repeated cyclic loading. Tests were done on nine normal-weight concrete and three light-weight concrete specimens. Their variables were the loading frequency and loading rate. In addition to the peak torque value. As a result of the study, it was seen that a peak torque lower than the static cracking torque must be repeated sufficiently for torsional cracking to occur. The pre-applied peak torque which is higher than the static cracking torque can be effective on the static torsional behavior of a beam. Specifically, a preloaded beam under cyclic torsion has substantially less torsional stiffness than the stiffness of a companion beam under raw loading [85].

As has been mentioned many times, torsion is a three-dimensional problem which involves the shear problem of 2D membrane elements as an addition to the buckling of

tube walls that causes bending. To model a torsional hysteresis, the hysteretic shear model should be understood. Ozcebe and Saatçioğlu proffered a hysteretic model to reflect the cyclic behavior of reinforced concrete members subjected to shear force and bending moments. Researchers used the primary curve as an envelope then, unloading and loading branches of the hysteretic model are established by using the huge number of test data. The proposed rules include the pinching and stiffness degradation of loops. Test results were compared with the results taken from the model and a good agreement was observed between the experimental and theoretical results [86].

A hysteresis can include stiffness degradation, pinching and deterioration of load. Generally, the pinching effect should be in consideration during the modeling of a torsional hysteresis. Pinching can be categorized into two types in the literature. The first one occurs, because the member encounters additional stiffness and the second type can occur, because of the differences in strengths during the loading in opposite directions. Here, stiffness degradation or load deterioration should be described in terms of hysterical energy. Mostaghel described an analytical model to describe the resistance-deformation relationship of the general hysteretic systems including the parameters such as stiffness degradation, pinching and deterioration of load. The suggested analytical model provided realistic descriptions for general hysteretic systems on the resistance-deformation behavior and the researcher confirmed this with examples [87].

To model the hysteresis of the reinforced concrete members under torsion, torque-twist curve of members must be stated correctly. For this purpose, the cracking torque and twist value should be calculated in the most accurate way. The softened membrane model for torsion (SMMT) was proposed to predict cracking torque and twist value. But the model is very complicated to apply. To simplify the model new empirical formulas derived by using SMMT were presented by Jeng. The formulas were approved by application to the 93 test results. Improved formulas proven by the test results are not only simple enough to be used but are also able to generate more realistic and accurate forecasts [88].

There are several hysteresis models in the literature, but most of these models ignore either pinching or stiffness degradation. Recently, a new model including these effects was proposed and the proposed model was tested for two circular and two square cross-section elements, and it was seen that the analysis results were very close to the experimental results. While developing this polygonal hysteresis model (PHM), researchers used the based softened truss model from mechanics to construct the primary curve of torsion. Researchers used the test results to derive the unloading and reloading rules. Finally, they applied the model to the studies in the literature and the analysis results were close to the test results [89].

4. EXPERIMENTAL STUDY

4.1. Introduction

To model the reinforced concrete members' behavior tested under reversed cyclic pure torsion, torque-twist hysteresis curve should be known. For this aim, 11 reinforced concrete members were designed according to the ACI 318R-11 "Building Code Requirements for Structural Concrete" and ACI 445.1R-12 "Report on Torsion in Structural Concrete". Three main parameters such as concrete strength, the spacing of the stirrup and longitudinal reinforcement ratio are examined [1, 12].

4.1.1. Test Specimens

The size effect is an important phenomenon due to the brittle behavior of concrete elements under loading such as torsion. It is observed that the deficiently reinforced or unreinforced concrete members were affected by the size effect under shear forces in the literature [45–47]. The size effect is a similar phenomenon for torsion as shear and can cause brittle failure. While the size effect is too little to be noticeable for the concrete structures with small sizes used in laboratory tests, it has a very strong effect on large structures in the field. In order to simulate the real behavior, full scale tests should be performed. The square cross-section dimensions of the members was 300 mm by 300 mm as shown in Figure 4.1. The overall length of the members was 2000 mm, and the 850 mm long middle-half of the members were designed as the test region. The outside of the test region of the specimen was reinforced heavily (the torsional reinforcement was placed in the transverse direction with 50 mm spacing) to prevent the primary failure at the end regions. Closed stirrups were prepared to be used as torsional reinforcement in the transverse direction.

A member under pure torsion behaves elastically with a torque-twist curve that is almost linear before the concrete cracks. The compression forces are carried by

the concrete and the tensile forces are resisted by the transverse and the longitudinal reinforcement at the post-cracking stage because the resistance mechanism changes after cracking. The tensile strains carried by the longitudinal reinforcement cause the elongation of members. The beam length almost linearly increases with the increased torque. A restraint on the longitudinal direction of a member acts as a compressive prestress force and increases the torsional strength of the element.

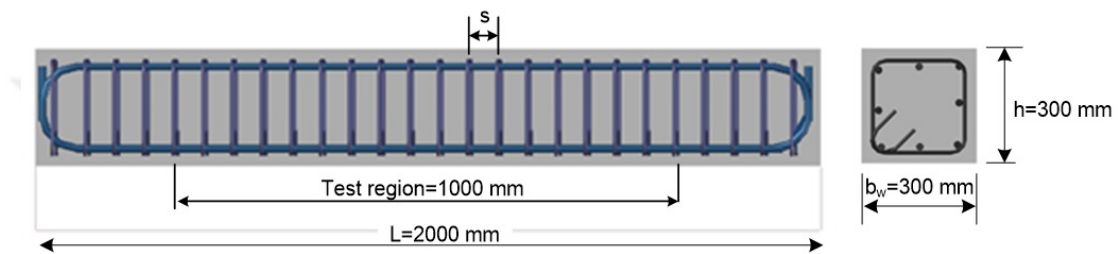


Figure 4.1. Test specimen.

4.1.1.1. Reduced Transverse Reinforcement. Observations have shown that most of the existing structures have insufficient transverse reinforcement. This inadequate transverse reinforcement detailing deteriorates the confinement effect which is necessary for ductility. In addition, studies have shown that transverse reinforcement is highly effective on torsional behavior. Specimens reinforced with only longitudinal steel behave similarly to unreinforced specimens in terms of torsional rigidity and strength, and finally, brittle failure happens after cracking. Structural elements with insufficient transverse reinforcement ratio exhibited a very weak behavior under torsional moments. To reflect this behavior, specimens were produced using 50 percent of the required amount of transverse reinforcement.

4.1.1.2. Reduced Longitudinal Reinforcement. There has not been enough study on the amount of insufficient longitudinal reinforcement. Since the relation between torsional behavior and longitudinal reinforcement has not been clearly examined, it was thought that it would be useful to examine the behavior of a structural member with insufficient length reinforcement exposed to pure torsional moments.

The ability of a member subjected to torsional moments to increase yield strength to resist shear flow depends on the amount of well-detailed transverse and longitudinal reinforcement. In order to obtain a good reinforcement detail, the interaction between the transverse and longitudinal reinforcement of the element must be considered.

4.1.1.3. Concrete Strength. Torsion is a 3D behavior which includes 2D shear. Pure concrete behaves very brittle under torsional moments because of the shear forces. Increasing concrete strength can be effective on the behavior of members exposed to pure torsion. To well understand the effect of concrete strength on torsional behavior, three different concrete types were studied as parameters. In addition, studies have shown that the concrete tensile stress is very effective on the torsional capacity of the member.

4.1.2. Torsion Design According to ACI

Tube-shaped cross-sections are the most effective section types that can be able to resist torsion proved by Saint-Venant's circulatory shear flow pattern.

The torsional behavior of a reinforced concrete member is a three-dimensional (3D) behavior as it includes both shear in the two-dimensional (2-D) wall member of a hollow reinforced concrete tube and the out-of-wall bending of the concrete struts. There are two simplifications defined in the ACI code. First; the bending on the concrete struts is neglected and transverse reinforcement can be determined by using Bredt's (1896) equilibrium equation given as [5]

$$q_y = \frac{T_u}{2A_o}, \quad (4.1)$$

here, q_y is the shear flow at yield (N/mm); T_u is the ultimate torsional moment (N.mm); and A_o is the lever arm area enclosed by the centerline of the shear flow (mm^2).

As second; it is assumed that the transverse and longitudinal reinforcement yield at the ultimate strength. It is sufficient to use three equilibrium equations to calculate

the reinforcement in any 2-D shear element. A simple equation for yield shear flow can be obtained by combining these three equations. The equations required for torsion analysis were given in detail and sequentially in Appendix A as

$$q_y = \sqrt{\left(\frac{A_t f_y}{s_t}\right) \left(\frac{A_l f_y}{s_l}\right)}, \quad (4.2)$$

where A_t , A_l are the area of transverse reinforcement and longitudinal reinforcement (mm^2), f_y is the yield stress of the steel (MPa), s_t , s_l are spacing of reinforcement (mm). Substituting the Equation(4.2) into the Equation (4.1) gives

$$T_u = 2A_o \sqrt{\left(\frac{A_t f_y}{s_t}\right) \left(\frac{A_l f_y}{s_l}\right)}. \quad (4.3)$$

If the moment arm of the shear flow sweeps around the axis of the twist with one full circle, the lever arm area A_o can be formed. To simplify the equation, the ACI codes provide a simple equation for calculating the moment arm area as

$$A_o = 0.85A_{oh}, \quad (4.4)$$

here, A_{oh} is the corresponding lever arm area (mm^2) to the centerline of the transverse reinforcements.

In line with the above-mentioned assumptions, the design steps proposed by ACI for elements subjected to pure torsion are given below;

(Step-1) Check if torsional effects are negligible

$$T_u < \phi \lambda \left(\frac{\sqrt{f'_c}}{12} \right) \left(\frac{A_{cp}^2}{P_{cp}} \right) \quad (f'_c \text{ in MPa}). \quad (4.5)$$

(Step-2) Check if the section dimensions of the cross-section are sufficient

$$\frac{T_u p_h}{1.7 A_{oh}^2} \leq \phi \left(\frac{5 \sqrt{f'_c}}{6} \right). \quad (4.6)$$

In case of pure torsion, taking the term as

$$\frac{v_c}{b_w d} = \frac{2 \lambda \sqrt{f'_c}}{12}. \quad (4.7)$$

Determine the dimensions of the cross-sections are adequate to each other, to calculate the torsional effects.

If torsional effects are disregarded for the rectangular cross-section, and the geometrical properties are given in Figure 4.2.

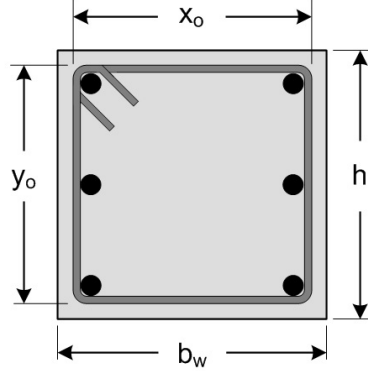


Figure 4.2. Reinforced concrete rectangular cross-section.

The perimeter of the outermost torsional reinforcement, p_h is determined from

$$p_h = 2(x_o + y_o), \quad (4.8)$$

where x_o , y_o are horizontal and vertical dimensions, respectively, of the centerline of the outermost closed transverse torsional reinforcement.

The strength reduction factor for shear and torsion can be accepted as equal to 0.75.

Modification factor of normal weight concrete = 1.0 and

$$\frac{v_c}{b_w d} = \frac{2\lambda\sqrt{f'_c}}{12} \quad (f'_c \text{ in MPa}), \quad (4.9)$$

concrete shear can be obtained as

$$v_c = \left(0.6\lambda\frac{\sqrt{f'_c}}{12} + 700(0.00689)\frac{v_u d}{M_u} \right) b_w d \quad (f'_c \text{ in MPa}), \quad (4.10)$$

and, the equilibrium should be satisfied. Therefore

$$\frac{T_u p_h}{1.7 A_{oh}^2} \leq \phi \left(\frac{v_c}{b_w d} + \frac{2\sqrt{f'_c}}{3} \right). \quad (4.11)$$

(Step-3) Calculate and check the transverse reinforcement details from

$$\frac{A_t}{s} \geq \frac{T_u}{\phi 2 f_{yt} A_o \cot \theta}, \quad (4.12)$$

the minimum transverse reinforcement can be determined from

$$\left(\frac{A_t}{s} \right)_{min} = \frac{0.75}{2} \frac{\sqrt{f'_c}}{12} \frac{b_w}{f_{yv}}. \quad (4.13)$$

(Step-4) Calculate and check the longitudinal reinforcement details from

$$A_l = \frac{A_t}{s} P_h \frac{f_{yt}}{f_{yl}} \theta, \quad (4.14)$$

the minimum longitudinal reinforcement can be determined from

$$A_{l,min} = \frac{5\sqrt{f'_c} A_{cp}}{12 f_{yl}} - \frac{A_t}{s} P_h \frac{f_{yt}}{f_{yl}}. \quad (4.15)$$

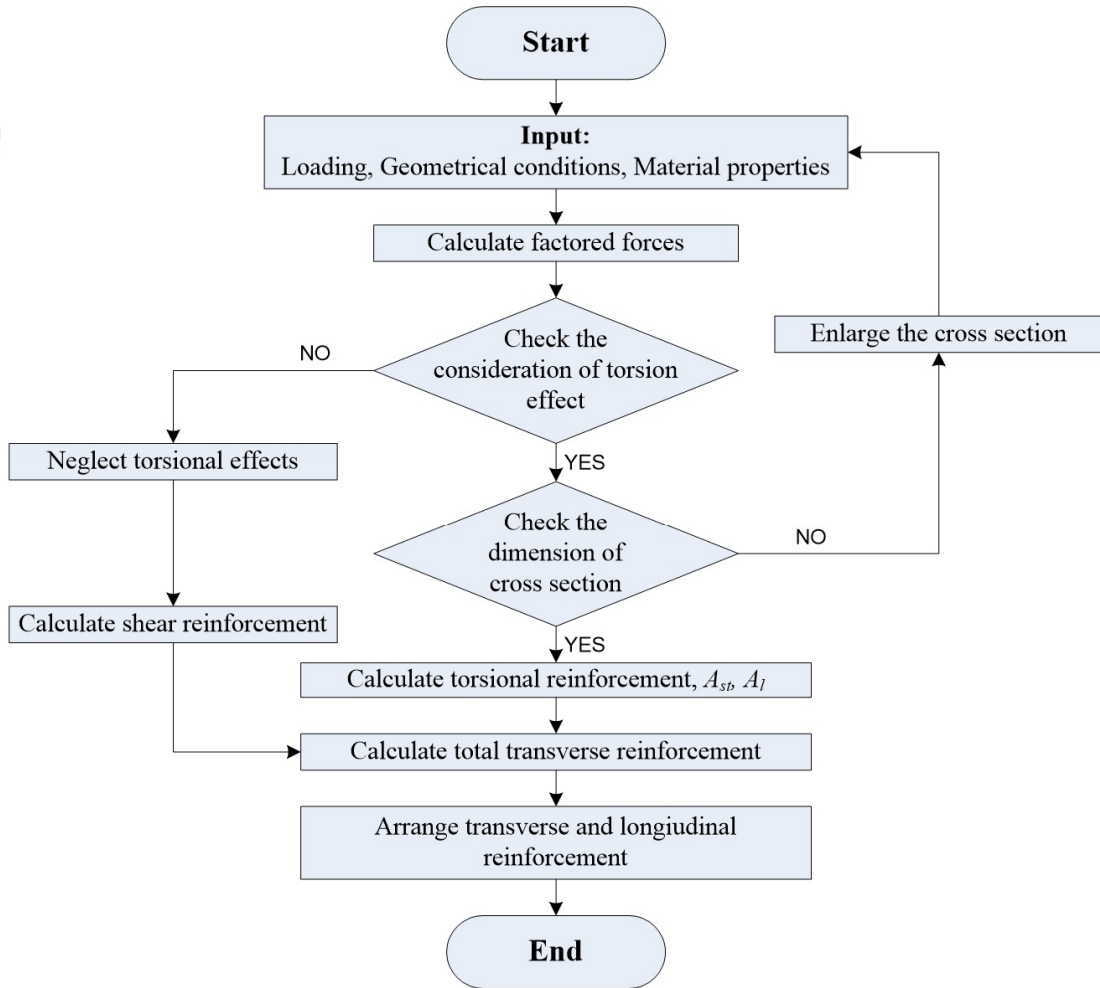


Figure 4.3. Design process flow chart for torsion effects [12].

Design procedure for the control specimen with 36 MPa concrete strength, according to the flow chart shown in Figure 4.3.

(Step-1) If torsional effects are disregarded for the shown cross-section: $f_c = 36$ MPa, $b_w = 300$ mm, $h = 300$ mm. Strength reduction and modification factor are

given before as 0.75 and 1.0 respectively. The perimeter is

$$p_{cp} = 2(b_w + h) = 1200 \text{ mm}, \quad (4.16)$$

and the concrete gross area can be calculated from

$$A_{cp} = b_w \times h = 90000 \text{ mm}^2, \quad (4.17)$$

the ultimate torsional strength can be determined from

$$T_u < \phi \lambda \left(\frac{\sqrt{f_c}}{12} \right) \left(\frac{A_{cp}^2}{P_{cp}} \right) = (0.75)(1.0) \left(\frac{\sqrt{36}}{12} \right) \left(\frac{90000^2}{1200} \right). \quad (4.18)$$

Therefore for $T_u = 26 \text{ kNm}$ and torsional check should be done by

$$\phi \lambda \left(\frac{\sqrt{f_c}}{12} \right) \left(\frac{A_{cp}^2}{P_{cp}} \right) = 2.11 \text{ kNm} < T_u = 26 \text{ kNm}. \quad (4.19)$$

The effects of torsion must be taken into account.

(Step-2) If the cross-section's dimensions are adequate: cross-section dimensions are $b_w = 300 \text{ mm}$, $h = 300 \text{ mm}$, in case of pure torsion, the perimeter is

$$p_h = 2(x_o + y_o) = 2(300 - (2 \times 50)) + (300 - (2 \times 50)) = 800 \text{ mm}, \quad (4.20)$$

the area enclosed by the stirrups is

$$A_{oh} = x_o \times y_o = 40000 \text{ mm}^2, \quad (4.21)$$

the ultimate torsional strength is obtained from

$$\frac{T_u p_h}{1.7 A_{oh}^2} = \phi \frac{5 \sqrt{f_c'}}{6}, \quad (4.22)$$

and ultimate torsional strength is found as

$$3.11 \text{ MPa} \leq 3.125 \text{ MPa}. \quad (4.23)$$

(Step-3) Cross section dimensions are adequate. It is required to calculate the enclosed of the transverse reinforcement for pure torsion from

$$\frac{A_t}{s} \geq \frac{T_u}{\phi 2 f_{yt} A_o \cot \theta}, \quad (4.24)$$

where, $f_{yt} = 420 \text{ MPa}$ (design strength of torsion transverse reinforcement) and

$$A_o = 0.85 A_{oh} = 34000 \text{ mm}^2, \quad (4.25)$$

if $\theta = 45$ degrees, the equation is become

$$\frac{A_t}{s} \geq 0.67 \text{ mm}^2/\text{mm}. \quad (4.26)$$

The minimum transverse reinforcement is computed according to Equation (4.27) taken from ACI 445.1R-12 [12]. In the case of pure torsion

$$\frac{A_t}{s} \geq \frac{0.75}{2} \frac{\sqrt{f'_c}}{12} \frac{b_w}{f_{yv}} = 0.11 \text{ mm}^2/\text{mm}. \quad (4.27)$$

But shall not be less than

$$\frac{A_t}{s} \geq \frac{0.35}{2} \frac{b_w}{f_{yv}} = 0.125 \text{ mm}^2/\text{mm}. \quad (4.28)$$

The minimum spacing of the stirrups will be

$$s_{max} = \min\left(\frac{p_h/8}{300 \text{ mm}}\right) = \min\left(\frac{135 \text{ mm}}{300 \text{ mm}}\right) \rightarrow s_{max} = 135 \text{ mm}. \quad (4.29)$$

For $A_t = 50.26 \text{ mm}^2$ and for $\phi 8$ reinforcement, the spacing is

$$s \leq 75.47 \text{ mm} \rightarrow \text{chosen} \rightarrow s = 75 \text{ mm}. \quad (4.30)$$

(Step-4) Calculation of the longitudinal reinforcement for torsion will be

$$A_l = \frac{A_t}{s} P_h \frac{f_{yt}}{f_{yl}} \theta = 719 \text{ mm}^2. \quad (4.31)$$

Although $7\phi 12$ (791.7 mm^2) longitudinal bars were sufficient for the torsional moment, due to the symmetric arrangement, $8\phi 12$ longitudinal bars with $A_l = 904.8 \text{ mm}^2$ were selected to be used for the control specimen.

4.2. Material Properties

The effects of concrete strength on torsional behavior were investigated in this research. Four specimens were produced with C36 (36 MPa) and four were produced with C17 (17 MPa) and the other four were produced with C50 (50 MPa) concrete type. Four cylindrical test samples were taken from each type of concrete during the casting. On the 28th day, one of the cylindrical samples of each concrete type was tested to determine the concrete compressive strength. The testing of samples and typical failure mode were shown in Figure 4.4.

Depending on the design of the specimens according to the code, the required area of reinforcing steel were determined. Several types of reinforcements were used in the cross-section. Diameters of $\phi 8$ and $\phi 10$ ribbed bars were used as transverse reinforcement, and $\phi 10$, $\phi 12$ and $\phi 14$ ribbed bars were used as longitudinal reinforcement, respectively.



Figure 4.4. Testing of concrete samples.

To determine the properties of the reinforcement, the direct tensile strength tests were carried out on three samples of each type of bar. Obtained yielding stresses were given in Table 4.1.

Table 4.1. Tension test results of samples.

Sample No:	Diameter (mm)	Yield Strength (MPa)
1	8	582.0
2	8	580.0
3	8	610.0
4	10	533.3
5	10	533.1
6	10	531.0
7	12	520.0
8	12	525.0
9	12	521.0
10	14	533.3
11	14	533.0
12	14	533.1

The compressive test results such as failure load, strength and average strength of samples were given in Table 4.2.

Table 4.2. Compression test results of samples.

Sample No:	Cross section area (cm^2)	Sample Age (day)	Failure Load (kN)	Strength (MPa)	Average (MPa)
1	176.7	28	290.0	16.41	16.88
2	176.7	28	306.6	17.35	
3	176.7	28	637.5	36.08	36.32
4	176.7	28	646.0	36.56	
5	176.7	28	849.1	48.05	46.43
6	176.7	28	897.7	50.80	

In Figure 4.5, the tensile testing of reinforcements, the failure mode and force-elongation curves obtained from the tests were illustrated for $\phi 8$, $\phi 10$, $\phi 12$, $\phi 14$ diameter of steels.

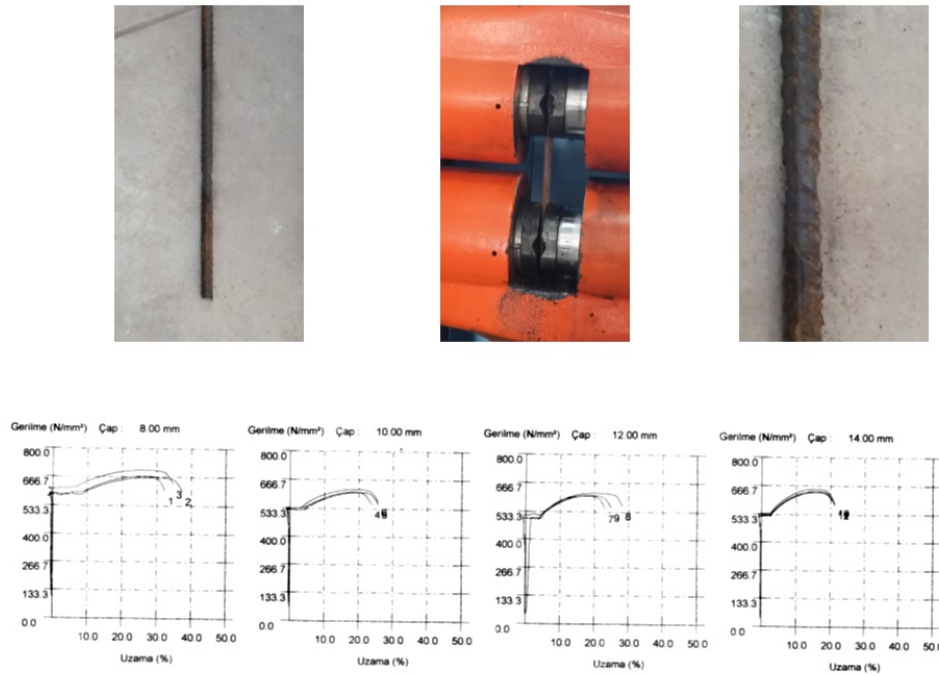


Figure 4.5. Testing of steel samples and results.

4.3. Test Parameters

This study investigates the behavior of reinforced concrete square cross-section members under pure torsion with different properties such as concrete compressive strength, diameter and spacing of transverse reinforcement, diameter and number of

longitudinal reinforcements. And transverse reinforcement ratios and longitudinal reinforcement ratios are calculated for the test specimens and given in Table 4.3.

The left side of the notations given in the *Specimen ID* column of the given table shows the concrete strength of the sample, and the right side shows the studied parameter as C = Control, T = Reduced transverse reinforcement, L = Reduced longitudinal reinforcement.

Table 4.3. Properties of specimens.

ID	#	f_c (MPa)	ρ_l	ρ_w	Trans.Reinf.	Long.Reinf.	Parameter
C36-C	2	36	0.0101	0.0056	$\phi 8/60$	$8\phi 12$	Control
C36-T	1	36	0.0101	0.0028	$\phi 8/120$	$8\phi 12$	Reduced ρ_w
C36-L	1	36	0.0050	0.0056	$\phi 8/60$	$4\phi 12$	Reduced ρ_l
C17-C	2	17	0.0069	0.0035	$\phi 8/95$	$8\phi 10$	Control
C17-T	1	17	0.0069	0.0022	$\phi 8/150$	$8\phi 10$	Reduced ρ_w
C17-L	1	17	0.0035	0.0035	$\phi 8/95$	$4\phi 10$	Reduced ρ_l
C50-C	2	50	0.0137	0.0045	$\phi 10/75$	$8\phi 14$	Control
C50-T	1	50	0.0137	0.0022	$\phi 10/150$	$8\phi 14$	Reduced ρ_w
C50-L	1	50	0.0068	0.0045	$\phi 10/75$	$4\phi 14$	Reduced ρ_l

4.4. Process of Production of the Test Specimens

Specimens were produced at the site. First, plywood molds were prepared according to the test specimens' dimensions as shown in Figure 4.6.



Figure 4.6. Producing of test specimens.

Then, reinforcement cages were produced. To measure the strain values during the testing, at least three strain gauges were installed on the longitudinal and transverse reinforcement in the test region. Before the installation, surface preparation was done.

The areas where the strain gauges would be glued were first sanded to obtain a smooth surface, and then these areas were wiped clean with alcohol to get the correct measurements. The strain gauges cables were soldered carefully to the strain gauges. And each cable was labeled. As the last step; strain gages were covered with plastic papers and plastic tubes to prevent damage during the casting of concrete (Figure 4.7).



Figure 4.7. Installation of strain gages.

Concrete was poured carefully after the reinforcement was placed into the plywood molds. In order to avoid gaps inside the concrete, the vibrators were used during the pouring of concrete as shown in Figure 4.8. Finally the surface of specimens and samples were smoothed and labeled.



Figure 4.8. Pouring concrete.

The specimens and samples were cured properly at the site. After 28 days, the concrete was set and all specimens and samples were transported carefully to the testing area of structural laboratory as seen in Figure 4.9.



Figure 4.9. Test specimens.

4.5. Test Procedure

In classical mechanics, longitudinal strains are produced in members subjected to pure torsion. However, for members made of a homogeneous and linearly elastic material, these strains are extremely small and are usually neglected. However, in reinforced concrete members due to cracking, the length of the beam has to increase and this increase has to be equal to the summation of the crack width along the member.

The elongation could produce significant strains in the longitudinal steel. Under these circumstances, the test setup for pure torsion should be free to the elongation of the member. If such elongations are prevented, axial compressive stresses will be generated as well as torsional stresses. Due to the longitudinal elongation and strains in the same direction, sufficient longitudinal steel should be provided in members subjected to torsion in addition to the transverse reinforcement.

The first test setup was produced via information taken from the literature as shown in Figure 4.10. The test setup had a loading beam over the specimen which was used for transferring the point load to the moment arms at both ends of the test specimen. Also, both ends of the specimen were free to longitudinal extensions thanks to the rollers inside the headpieces. However, during the test, the loading beam could not transfer the loads to the test specimen because of support conditions of the rigid element between loading frame and its moment arm. So that the loading frame was bent. Increasing the flexural capacity of the loading beam by changing the steel profile and welding additional steel plates for higher inertia, did not help to transfer the loads to the moment arms.

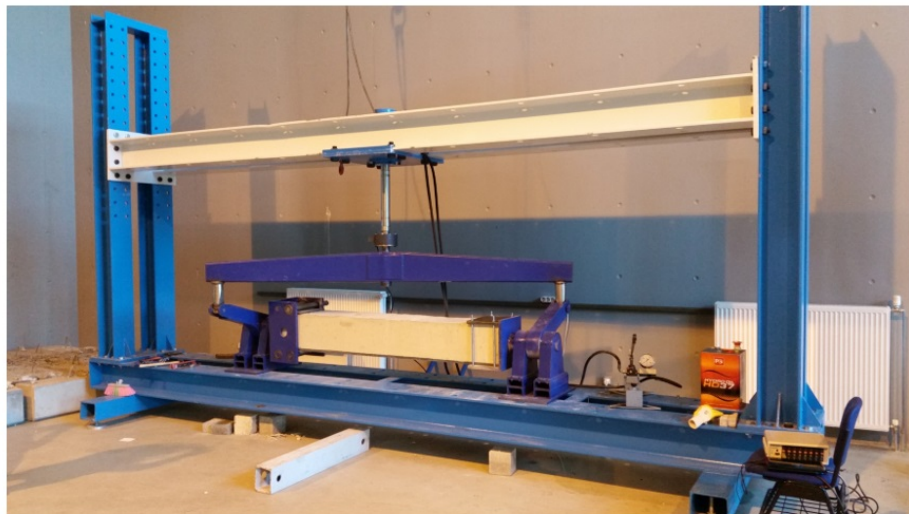


Figure 4.10. The first test setup.

Later, the rigid members were replaced with link elements, but in that case, also the point load was not transferred to the moment arms because of instability of loading

frame. Therefore, revision of the test setup did not look possible and as a result, the new test setup was designed and produced as seen in Figure 4.11.

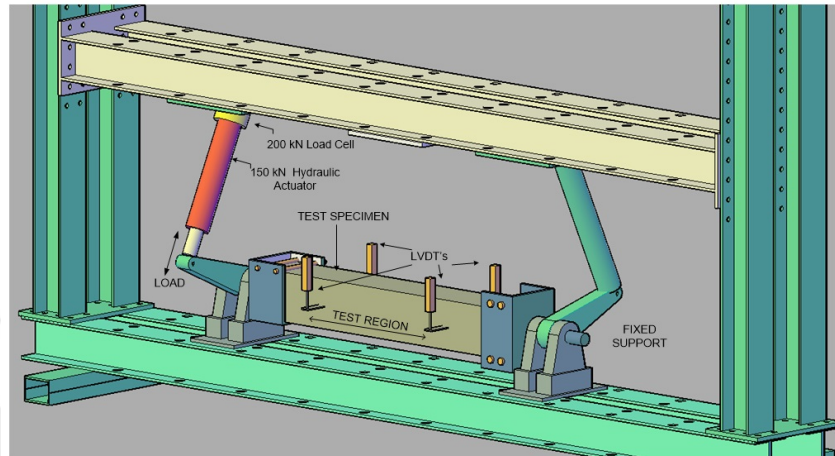


Figure 4.11. The sketch of test setup.

A detailed drawing of the new test setup and instrumentation was given in Figure 4.11. The torsional moment was given to the specimen from one end. The load was applied to the 530 mm long moment arm with a 300 kN capacity hydraulic actuator from one end. Also, the applied load level is measured by a 200 kN capacity load cell which is connected to the base of the hydraulic actuator as seen in Figure 4.12.

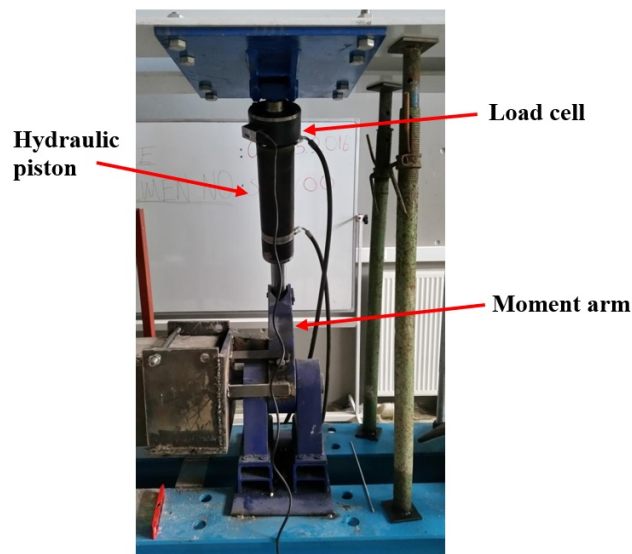


Figure 4.12. Twisted support.

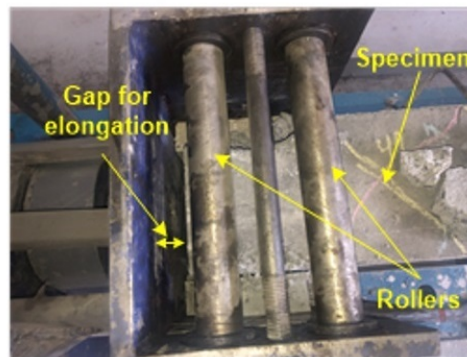
The actuator was pin connected to the loading frame. During the loading, when the specimen was twisted, actuator rotates about the pin connection. In order to measure the orientation of loading on moment arm, the rotation of head was also measured by LVDT. Therefore, the corrected torsional moment was calculated by using the perpendicular load on moment arm. The other end was called as fixed support which was restricted in rotational direction, but free in axial direction thanks to the roller inside the caps. Therefore, the specimen is become free to elongate in axial direction during torsional loading. The fixed support and roller for elongations were illustrated in Figure 4.13 (a) and (b). Also for giving a space to the specimens for elongation, a nearly 5 cm gap was left between the outer face of the specimen and the inner plate of the headpiece as seen in Figure 4.13(c).



a) Side view of the fixed headpiece



b) Inside of fixed headpiece



c) Details for the elongation on the fixed headpiece

Figure 4.13. Fixed supported headpiece.

Four Linear Variable Displacement Transducers (LVDT), which have 100 mm and 200 mm capacities, were attached to the front and back face of the member near the torsion critical region to measure the rotation of both ends of the test region. These rotation records were used to calculate the angle of twist as shown in Figure 4.14. One LVDT was used to measure the rotation angle of the headpiece for correction of the angle of the forces. Additionally, strain gauges fastened to the steel bars were used to measure the strains in the longitudinal and transverse steel bars in the test region. For the last two tests, two more LVDTs were used to measure the elongation in the specimen as shown in Figure 4.15.

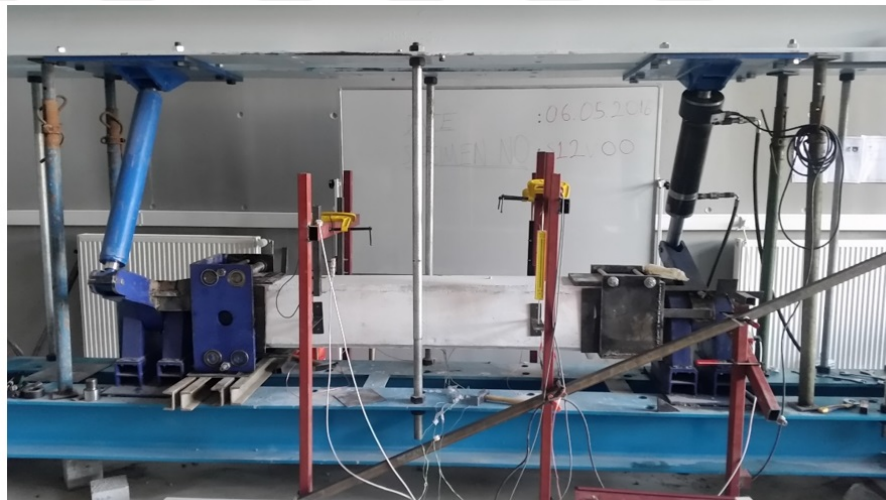


Figure 4.14. Test setup and instrumentation.

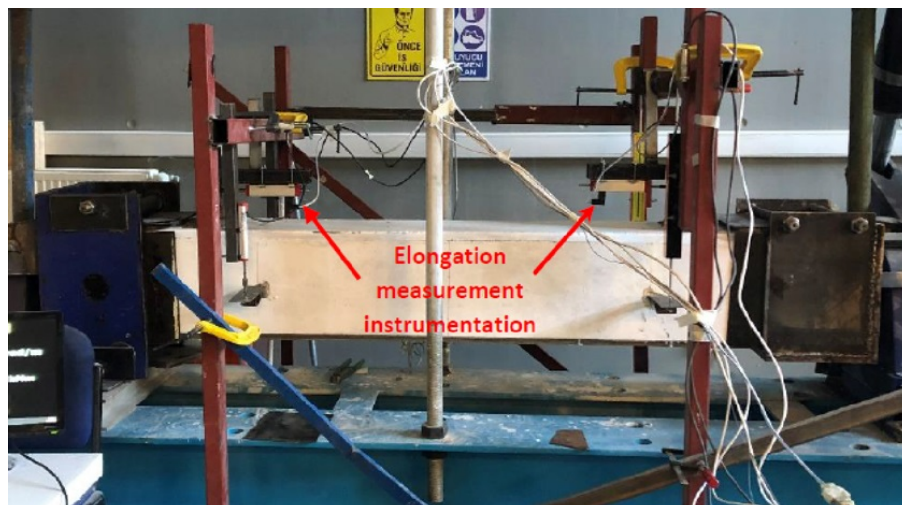


Figure 4.15. Instrumentation of elongation measurement.

4.6. Loading Protocol

By using a manually controlled hydraulic pump, reversed cyclic loading was applied from the free end of the member via the moment arm. As seen in Figure 4.16, the force (torque) controlled loading scheme was used up to the initial cracking level. A 500 kgf load increment is applied for each successive cycle in this part. After the initial cracking, the displacement (twist) controlled loading scheme was used throughout the tests.

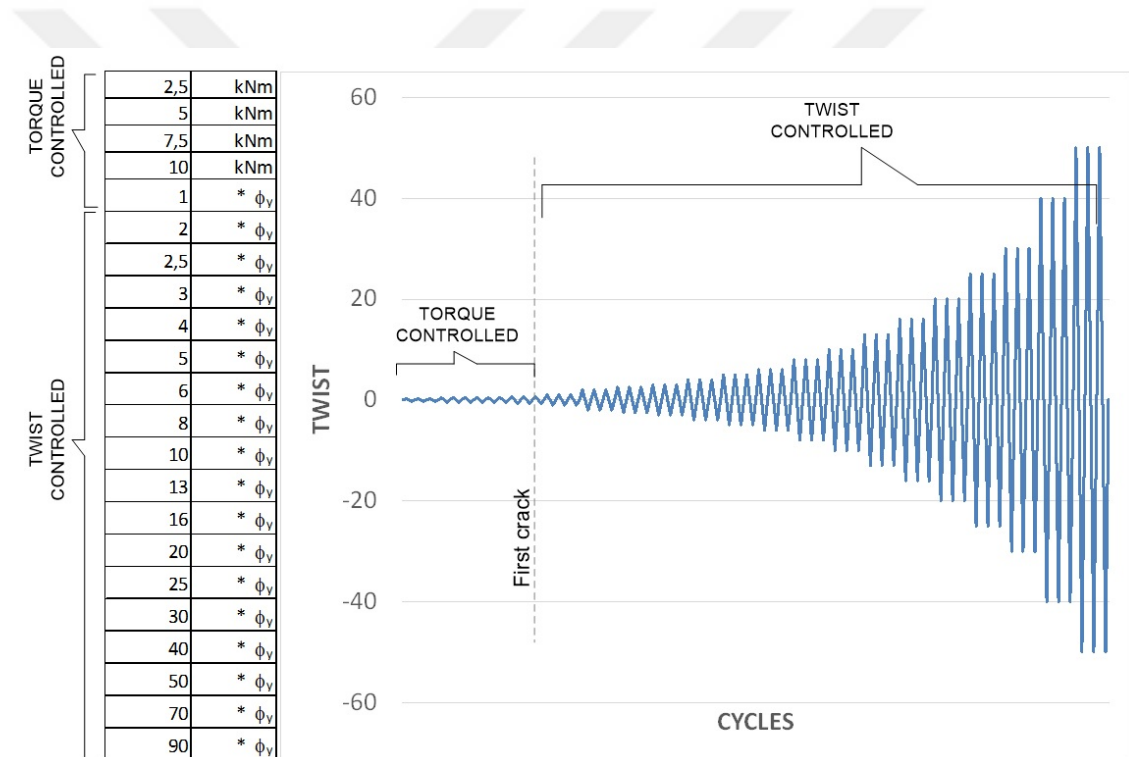


Figure 4.16. Loading protocol.

4.7. Test Results

The produced specimens were tested under pure reversed cyclic torsional moments after the curing period and transportation stage was completed. The test observations at certain twist level, crack propagation for each twist level, the failure modes and specimen behaviors at each loading level were explained in detail for each specimen with this section.

All details of the experimental setup, locations of measurement instrumentations (including elongation measurements), load cell location are shown in Figure 4.17. Additionally, the typical location of the strain-gauges on reinforcements were are shown clearly on the small drawing in Figure 4.17.

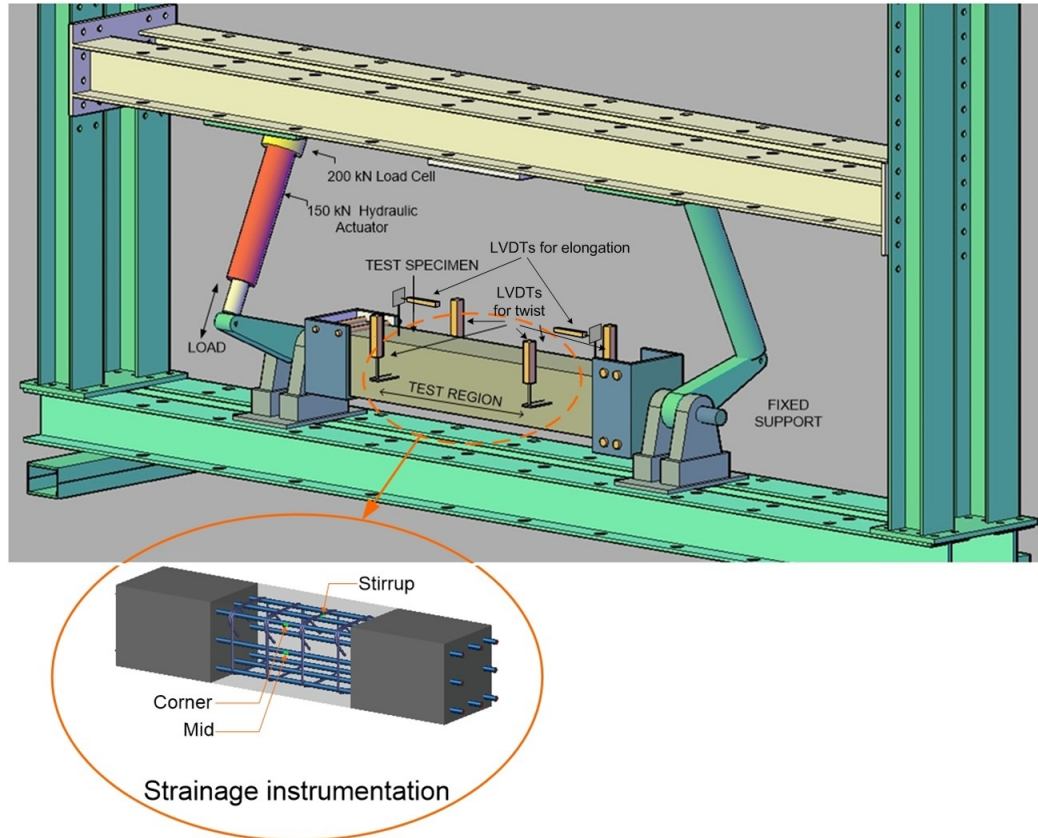


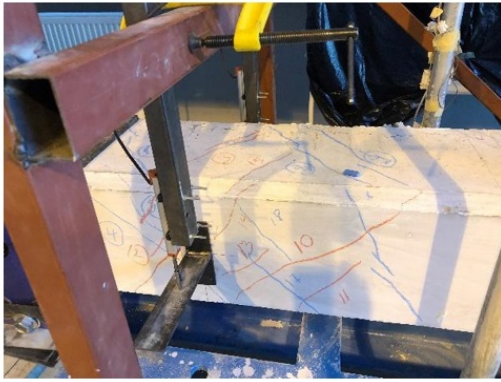
Figure 4.17. Test setup and strain gauge instrumentation.

4.7.1. Specimen #1: C36-C1

C36-C1 is the control specimen with 36 MPa concrete compressive strength. $8\phi 12$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 60 mm spacing are used in the test region. In the photographs in Figure 4.18 taken during the experiment, it can be seen that the cracks formed spirally around the specimen.

In the first cycle, the maximum twist was obtained as 0.13×10^{-3} rad/m for the 2.6 kNm moment and no damage was observed in this cycle.

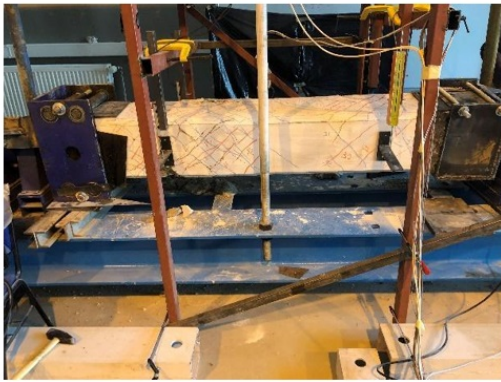
The first visible crack was observed in the 10th load cycle at 1.4×10^{-3} rad/m twist when the torsional moment was measured as 24.0 kNm. This point was accepted as the yield level of the specimen and called as ϕ_y . Up to this point test was carried on by torque controlled. After this cycle the test continued as twist controlled and each level called as in multiplies of ϕ_y . In successive cycles, diagonal torsional cracks were developed and propagated.



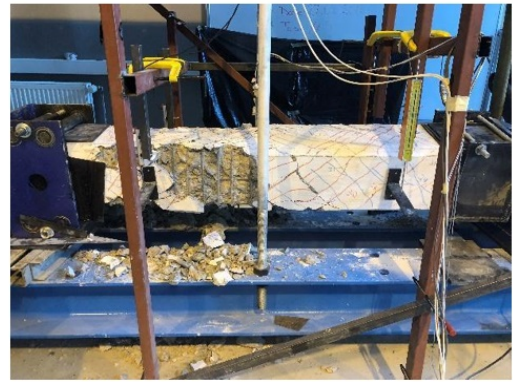
a) @ 24.65 kNm (moment) vs 5.36×10^{-3} rad/m (twist)



b) @ -28.9 kNm (moment) vs -13.4×10^{-3} rad/m (twist)



c) @ -28.1 kNm (moment) vs -33.5×10^{-3} rad/m (twist)



d) @ -25.7 kNm (moment) vs -67×10^{-3} rad/m (twist)

Figure 4.18. A few photos from the experiment of C36-C1.

At $2\phi_y$ twist level (3.5×10^{-3} rad/m twist), at moment level of 23.0 kNm, the specimen twisted suddenly, several cracks formed and moment capacity dropped down to 12.0 kNm. After this level up to $10\phi_y$ twist level many new cracks have formed and existing cracks were propagated and widened in the test region.

The member section acted as a space truss as shown in Figure 4.18. At the $10\phi_y$ twist level (-13.7×10^{-3} rad/m) the maximum torque was recorded as -28.9 kNm as indicated in Figure 4.18(b).

The concrete cover began to be crushed and fall off at the $25\phi_y$ twist level, when twist was 33.0×10^{-3} rad/m and moment was about 25.0 kNm. In addition, existing cracks were enlarged, elongated and spread over the entire surface.

The test ended at the 120.0×10^{-3} rad/m twist and 19.0 kNm moment during the $90\phi_y$ twist level, when the torque was decreased below 75 percent of the maximum torsional moment capacity. The ductile failure mode can be observed from the torque-twist curve.

As seen in Torque vs. Twist curves, the torsional stiffness was degraded for each cycle (Figure 4.19). Also, the area covered by each loop increased significantly which indicates the torsional energy dissipation capacity of the specimen.

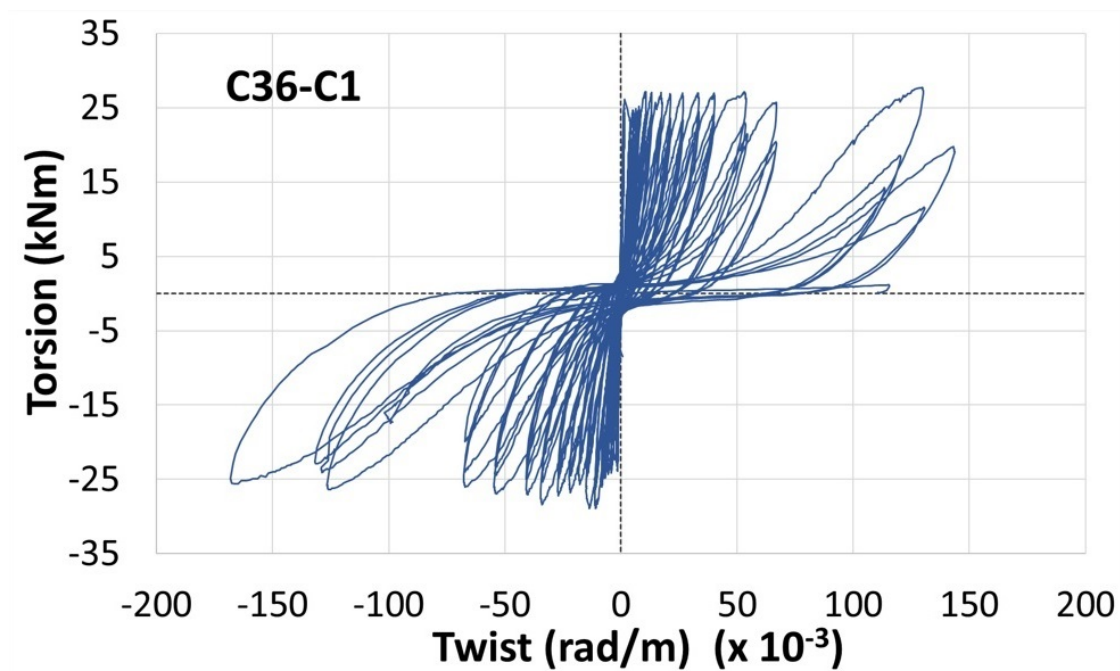


Figure 4.19. Torque vs. twist behavior of C36-C1.

Strain values obtained from strain-gauges installed on transverse and longitudinal reinforcements were indicated in Figures 4.20-4.22. As expected, the strain vs. torsional moment curves of the longitudinal reinforcement formed a butterfly shape due to cyclic torsional loading. During the twisting of the specimen either in clockwise direction or counter clockwise direction, the longitudinal reinforcement works always under tensile stresses.

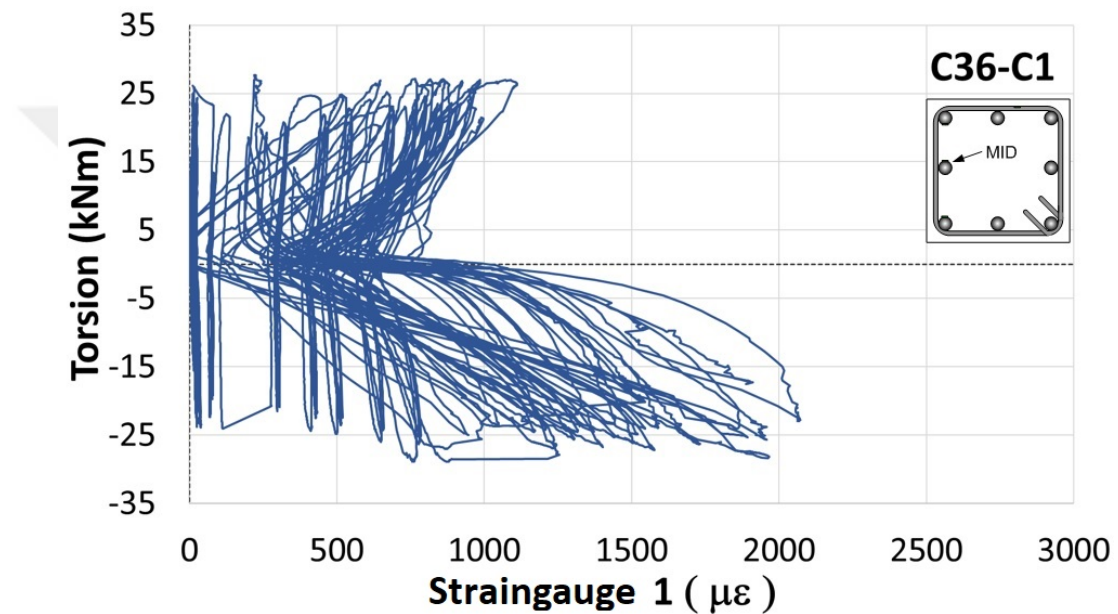


Figure 4.20. Strain curve of longitudinal reinforcement located at the mid.

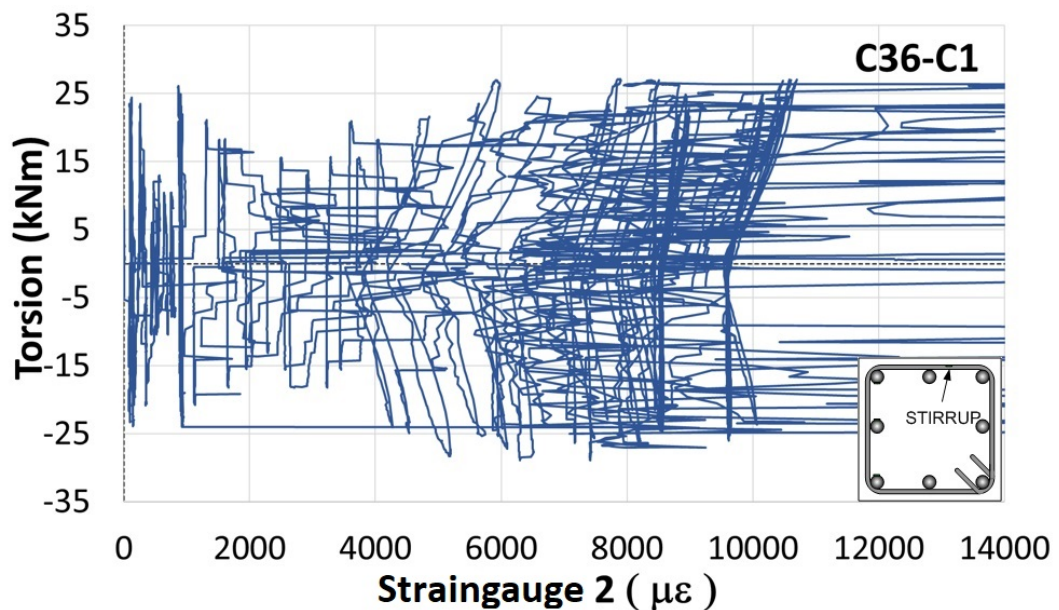


Figure 4.21. Strain curve of transverse reinforcement.

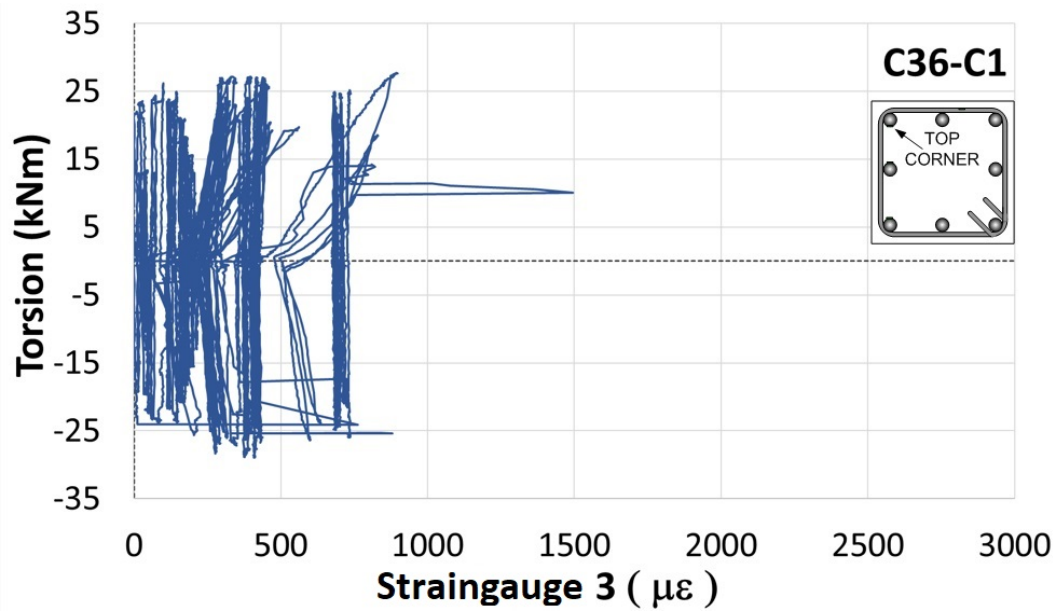


Figure 4.22. Strain curve of longitudinal steel located at the top corner.

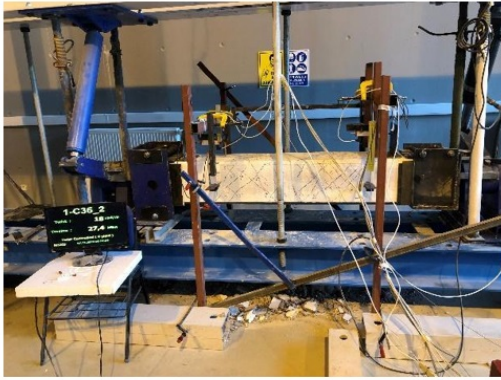
4.7.2. Specimen #2: C36-C2

C36-C2 is produced as a second control specimen which has the same specifications with the C36-C1.

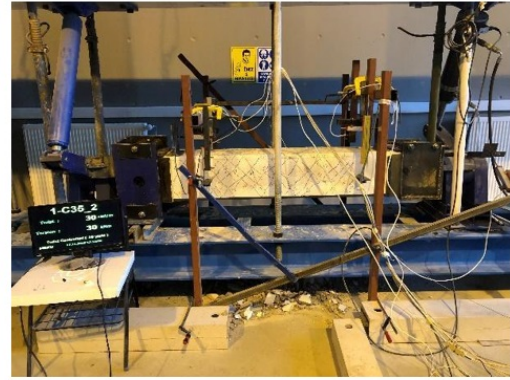
As expected, C36-C2 has a similar performance to C36-C1. The behavior of the specimen at different twist levels is shown in Figure 4.23. The first crack was formed at the 25.0 kNm moment and 1.6×10^{-3} rad/m twist level, and this cycle was accepted as the first ϕ_y twist level. Since the specimen twisted suddenly and more than planned at $2\phi_y$ twist level, the experiment was continued at $3\phi_y$ twist level, skipping intermediate rotation levels.

By increasing twist levels nearly 45-degrees angle cracks (as shear cracks) were formed and these cracks turned around the specimen like a space truss. Cracks occurred vertically to the tensile stresses on the three faces of the specimen. But the cracks on the last face formed because of the concrete crushing on a straight line [90]. The test was ended during the $40\phi_y$ twist levels because the plates adhered to the specimen surface for measuring fell along with the cover concrete. About 60 cracks were counted

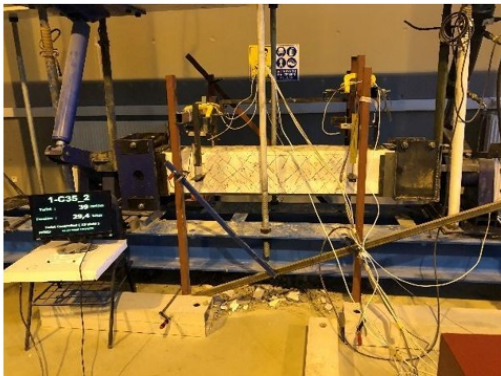
on the surface of the specimen within the test region which indicates that the behavior was quite ductile.



a) @ 27.4 kN.m (moment) vs 18.0×10^{-3} rad/m (twist)



b) @ 30.0 kN.m (moment) vs 30.0×10^{-3} rad/m (twist)



c) @ 29.4 kN.m (moment) vs 39.0×10^{-3} rad/m (twist)



d) @ 26.6 kN.m (moment) vs 120.0×10^{-3} rad/m (twist)

Figure 4.23. A few photos from the experiment of C36-C2.

Figure 4.24 gives the torque-twist behavior of the second control specimen. Cycles repeated twice because new instrumentations were implemented into the test setup to measure longitudinal elongation during the test. Figures 4.25-4.27 gives the strain values on the reinforcement during the test. C36-C2 was tested for measuring the longitudinal elongation under the torsional moments. According to elasticity theory, tensile stresses occur in the direction of the member axis subject to torsion, but these stresses and elongation are negligible since they are very small. On the other hand, after cracking formations due to these tensile stresses, axial deformations (elongation) became significantly large to be neglected.

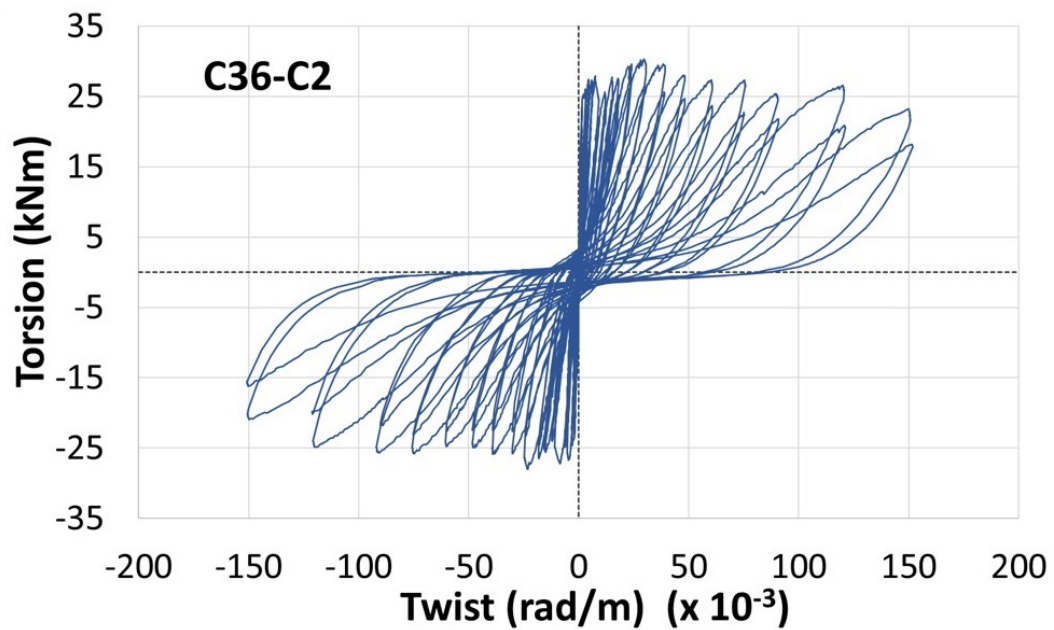


Figure 4.24. Torque vs. twist behavior of the C36-C2.

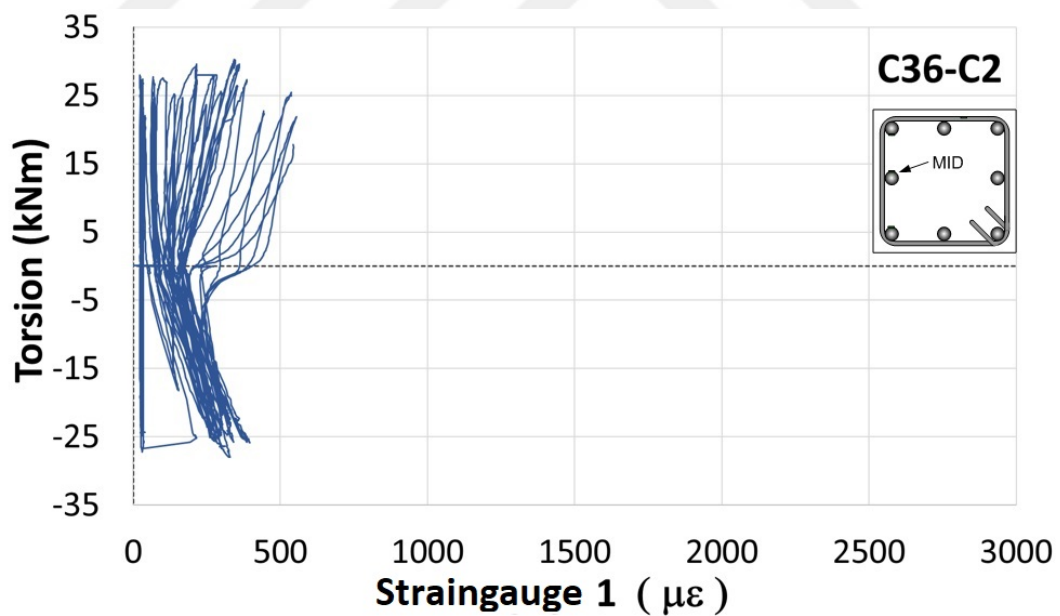


Figure 4.25. Strain curve of longitudinal reinforcement located at the mid.

Figures 4.28 and 4.31 shows the longitudinal elongation vs. twist and torsional moment respectively. This specimen elongated nearly Increasing torsional moments caused cracks to widen and thus elongation of the specimen. When the torsional moments were removed, the elongation decreased as the crack widths were closed. During the test, one of the plates implemented for the measure elongation ruptured

from the surface of the specimen and it was pasted again. To compare behavior and to see the similarity between the elongation behavior and the twist behavior of an element Torsional moment vs Member Strain and Twist curve is shown in Figure 4.30. Additionally, the Torsional moment vs Member strain and Rebar strain curve is shown in Figure 4.31.

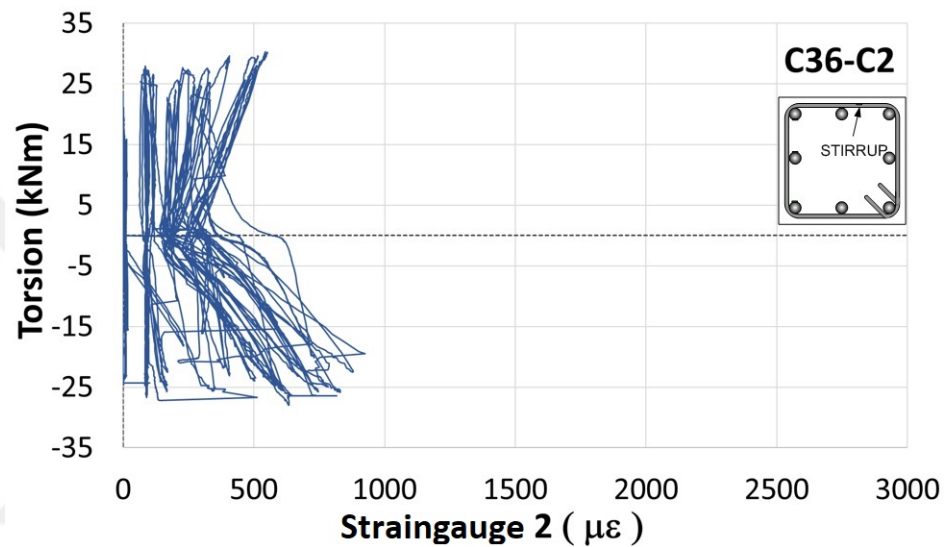


Figure 4.26. Strain curve of transverse reinforcement.

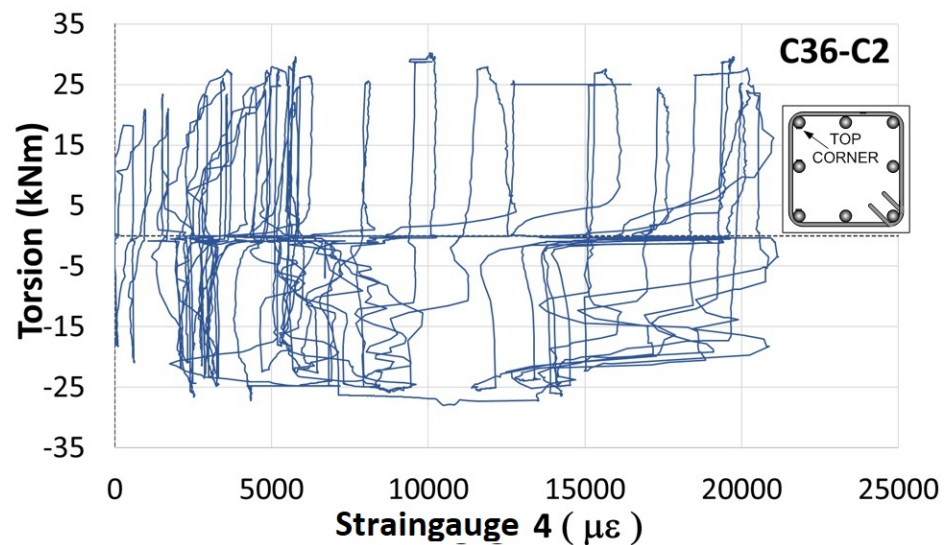


Figure 4.27. Strain curve of longitudinal steel located at the top corner.

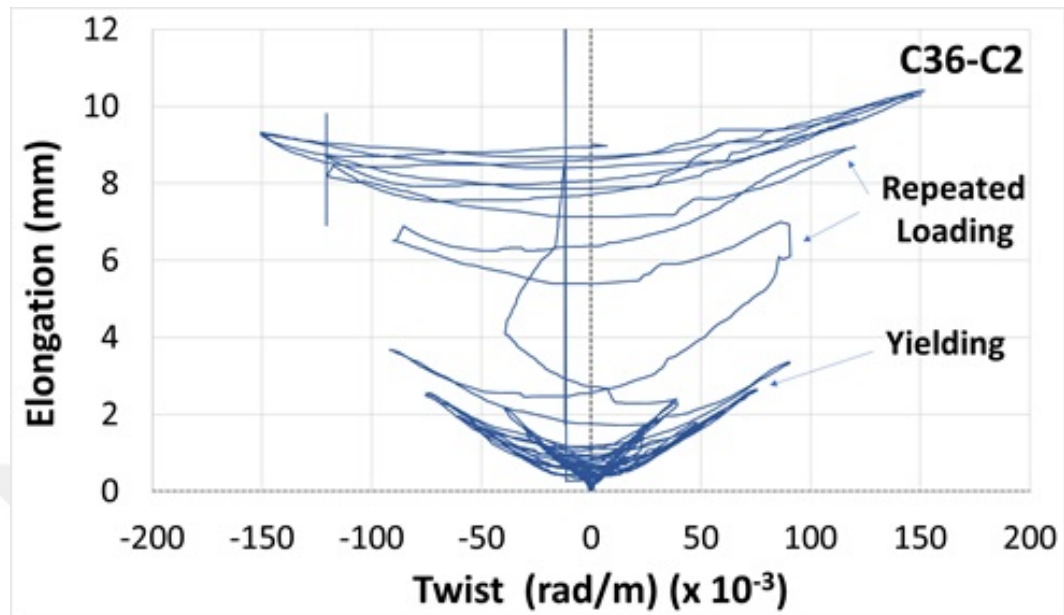


Figure 4.28. Longitudinal elongation vs. twist curve.

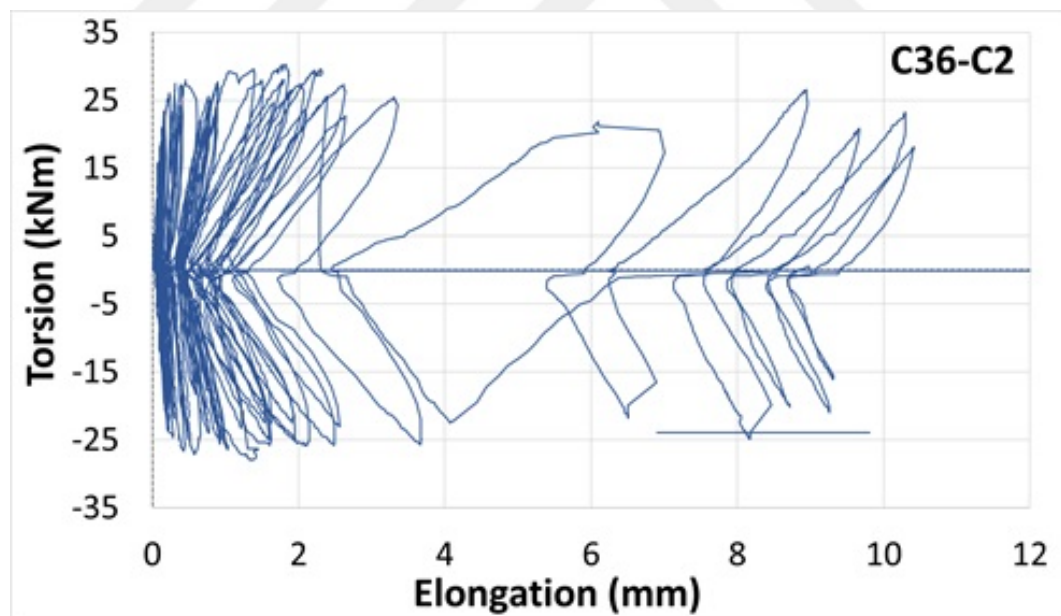


Figure 4.29. Torsional moment vs longitudinal elongation curve.

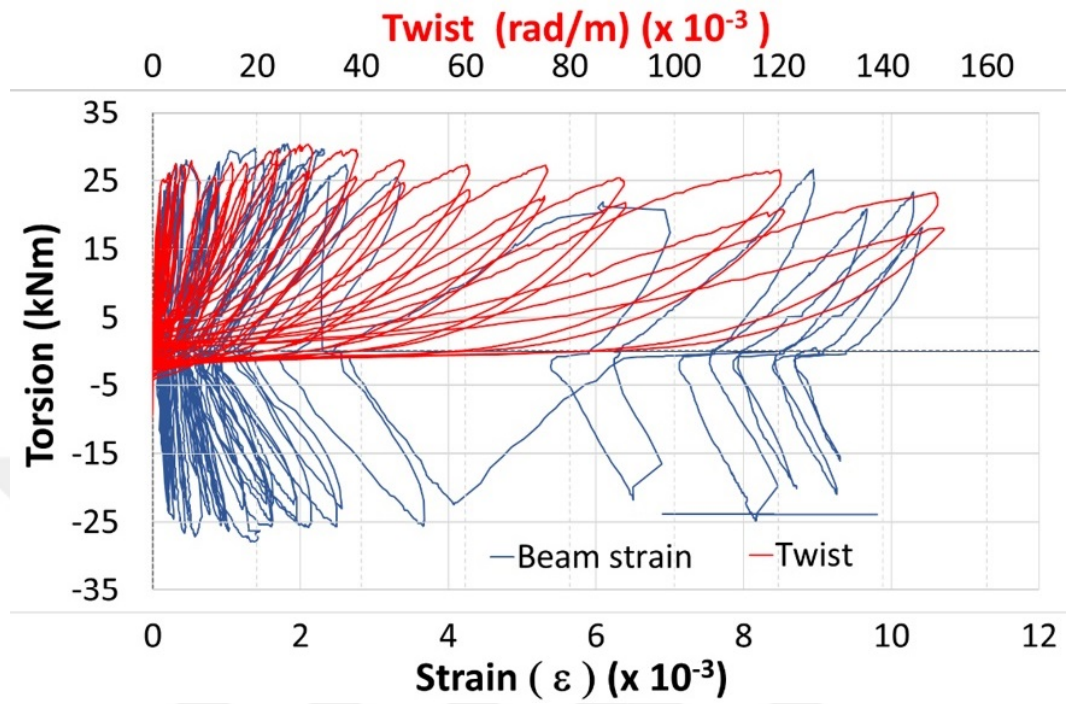


Figure 4.30. Torsional moment vs strain and twist curve.

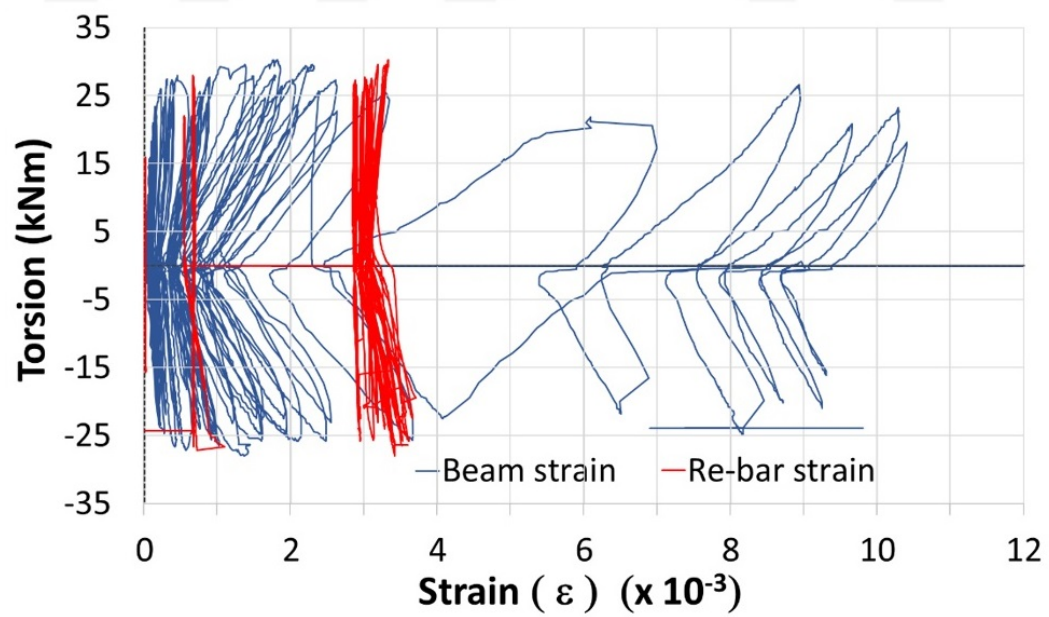


Figure 4.31. Torsional moment vs member strain and rebar strain curve.

4.7.3. Specimen #3: C36-T

C36-T has reduced transverse reinforcement ratio with 36 MPa concrete compressive strength. $8\phi 12$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 120 mm spacing are used in the test region at twice the range in the control specimens.



a) @ -24.0 kN.m (moment) vs - 7.5×10^{-3} rad/m (twist)



b) @ -24.2 kN.m (moment) vs - 30.0×10^{-3} rad/m (twist)



c) @ -22.8 kN.m (moment) vs - 39.0×10^{-3} rad/m (twist)



d) @ -8.7 kN.m (moment) vs - 210.0×10^{-3} rad/m (twist)

Figure 4.32. A few photos from the experiment of C36-T.

Up to the first cracking, test was done according to the torque-controlled loading pattern and no damage was observed. An initial crack was observed at the 26.9 kNm moment and 2.0×10^{-3} rad/m twist level. This twist level was taken as the ϕ_y and the test was continued as a twist-controlled loading pattern. After this level, diagonal torsional cracks propagated at each successive cycle.

The member section acted as a space truss as shown in Figure 4.32. The maximum torque was recorded as 28.5 kNm at 2.4×10^{-3} rad/m twist as indicated in Figure 4.33 at the ϕ_y twist level. The maximum torsional moment carrying capacity of this specimen is very close to the capacity of the first two samples, but the width of the cracks formed in this member was larger than the first two members. Also, the number of cracks on the member surface was less than the other two samples.

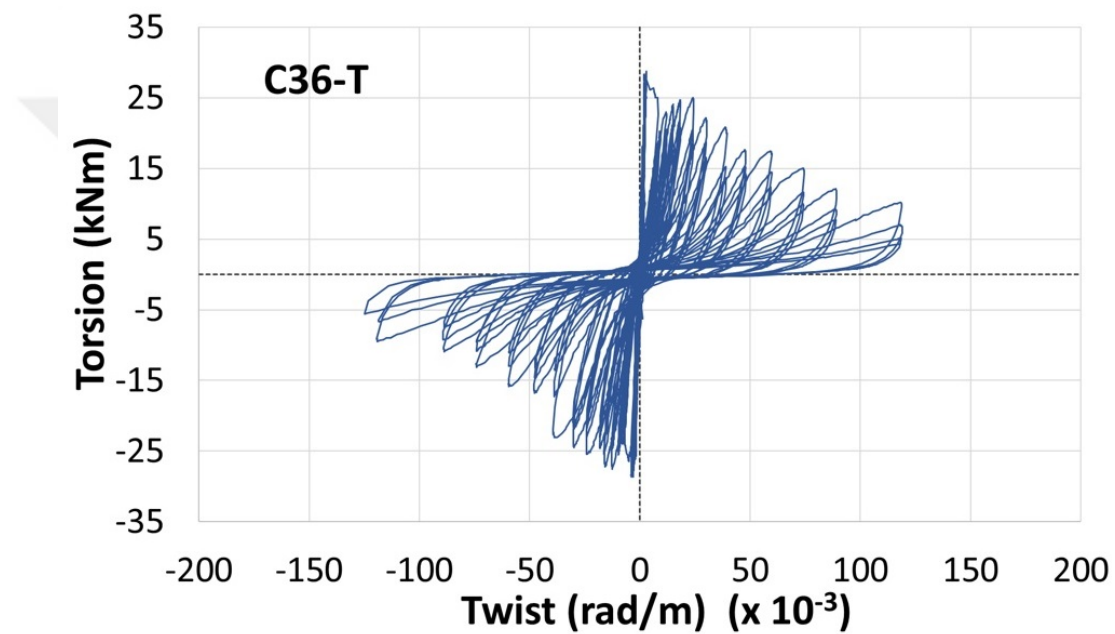


Figure 4.33. Torque vs. twist behavior of C36-T.

After this level, the cover concrete started to crush and existing cracks were widened. The test was ended at the 118.0×10^{-3} rad/m twist at the $13\phi_y$ twist level when the torque was decreased below 75 percent of the maximum torsional capacity. This specimen could be twisted up to larger twist angles with low moment forces due to insufficient stirrups. In the last cycles, large pieces of concrete were broken off from the specimen and fall of.

As seen from the Torque vs. Twist curves of C36-T specimen, torsional stiffness was degraded for successive twist level. Even in repeated cycles of same twist level, the stiffness degradation was also observed.

And also, the enclosed area covered by each loop increased significantly for increasing twist level, which indicates the torsional energy dissipation capacity of the specimen. Strain values for transverse and longitudinal reinforcements were indicated in Figures 4.34-4.36.

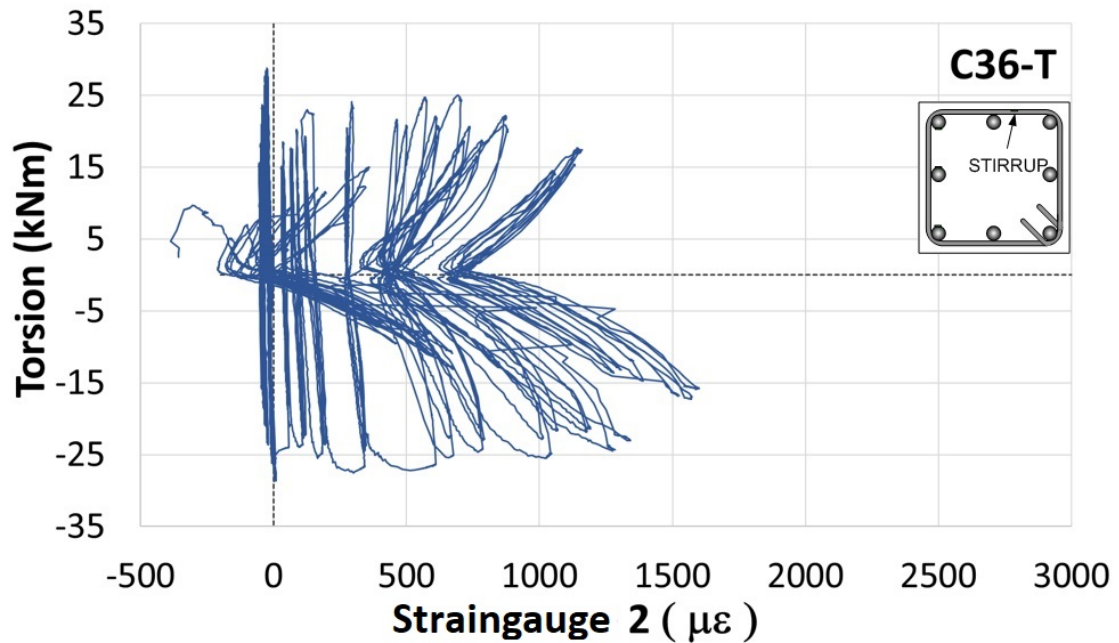


Figure 4.34. Strain curve of transverse reinforcement.

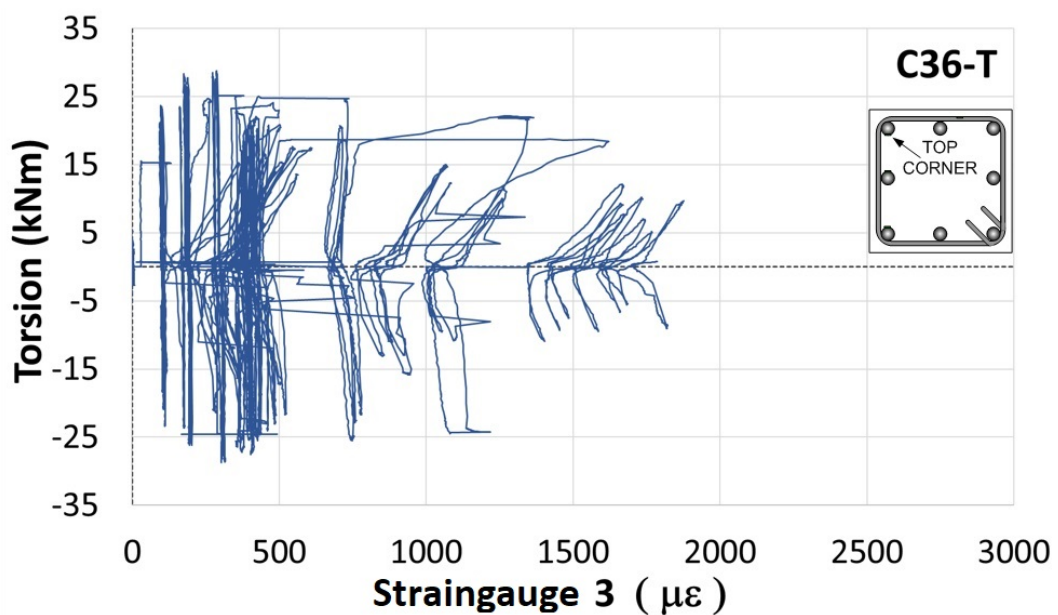


Figure 4.35. Strain curve of longitudinal steel located at the top corner.

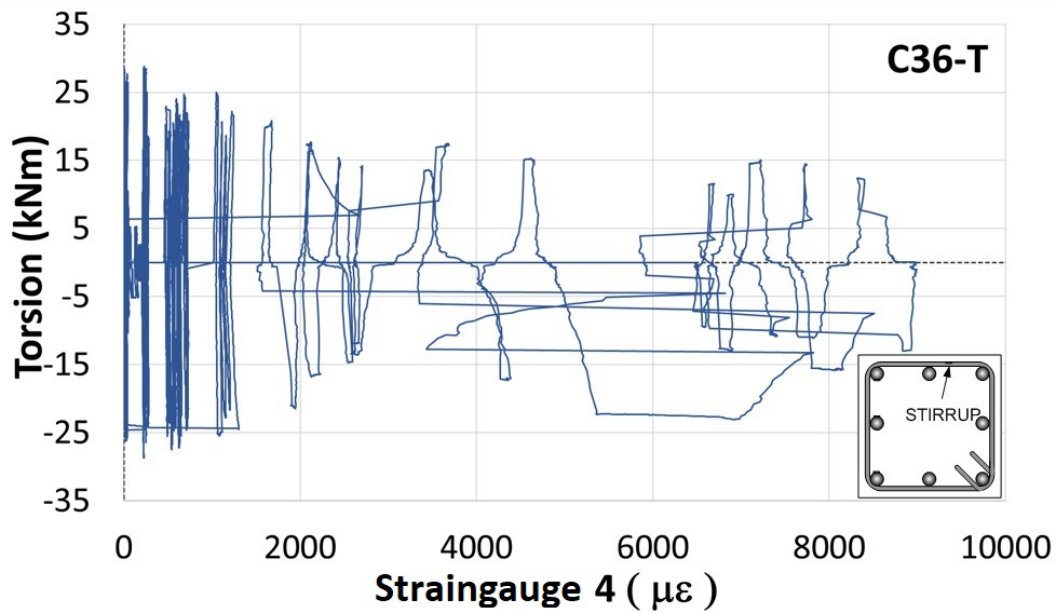


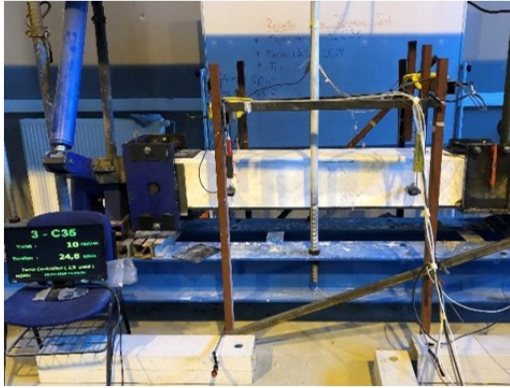
Figure 4.36. Strain curve of transverse reinforcement.

4.7.4. Specimen #4: C36-L

C36-L is the last specimen in the 36 MPa concrete compressive strength group with reduced longitudinal reinforcement ratio.

This specimen showed similar behavior with the previous specimens in the same concrete strength group. Cracking of the section started at the level of 26.0 kNm torsional moment with the 1.7×10^{-3} rad/m twist which is taken as the first ϕ_y twist level. Immediately after the initial crack, the torsional stiffness capacity and the torsional moment carrying capacity decreased. During this stage, many new cracks formed in the test region.

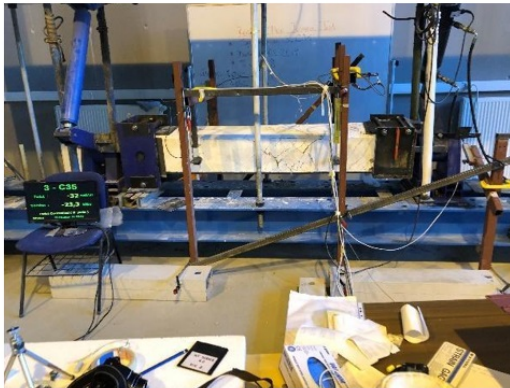
Reinforcement started to resist torsional moments due to the redistribution of internal forces in members at the 16.0×10^{-3} rad/m twist and torsional moment carrying capacity increased up to 26.5 kNm at the first ϕ_y . Also, new cracks formed and the existing crack width and length were increased at this twist level. The crack propagation of specimen at certain twist level were shown in Figure 4.37.



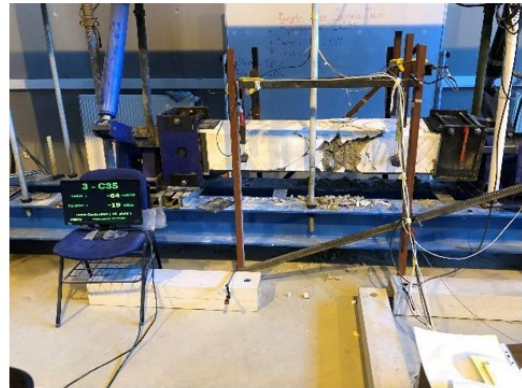
a) @ 24.8 kN.m (moment) vs 10.0×10^{-3} rad/m (twist)



b) @ -24.0 kN.m (moment) vs - 12.0×10^{-3} rad/m (twist)



c) @ -23.3 kN.m (moment) vs -32.0×10^{-3} rad/m (twist)



d) @ -19.0 kNm (moment) vs -64.0×10^{-3} rad/m (twist)

Figure 4.37. A few photos from the experiment of C36-L .

The cover concrete started to crush and to pour at $10\phi_y$ twist level when it was measured as 40.0×10^{-3} rad/m. The reduction of torsional moment capacity is observed for latter cycles. The test was ended up at $40\phi_y$ twist level when it was measured as 160.0×10^{-3} rad/m as seen in the torque-twist curve of the C36-L specimen in Figure 4.38.

The strain vs. torsional moment relations obtained from the strain gauges installed on transverse reinforcements and longitudinal reinforcements in test region were illustrated in Figures 4.39-4.41.

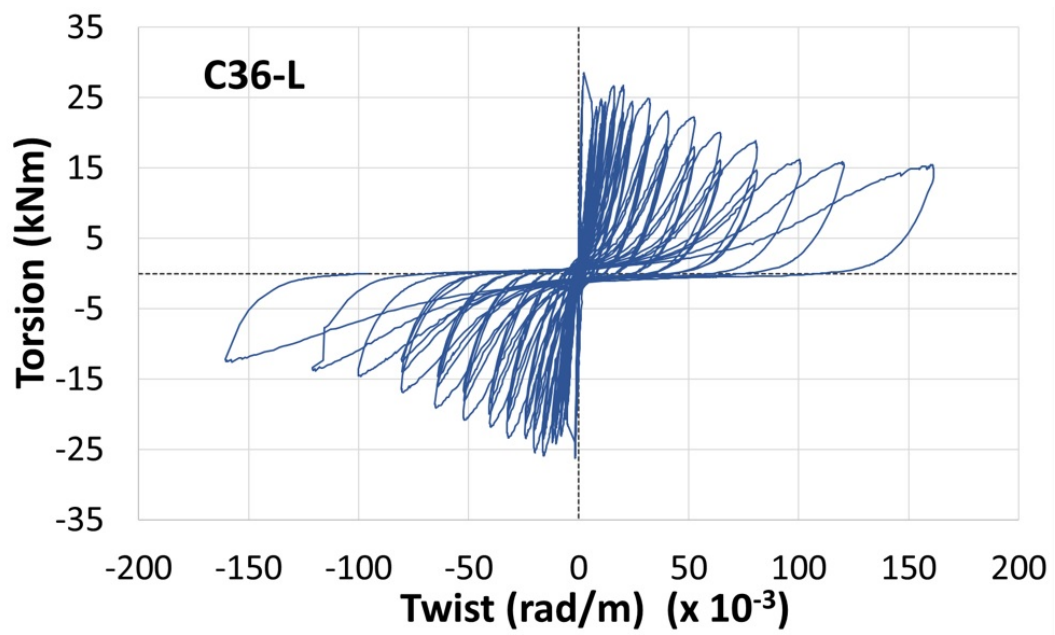


Figure 4.38. Torque vs. twist behavior of the C36-L.

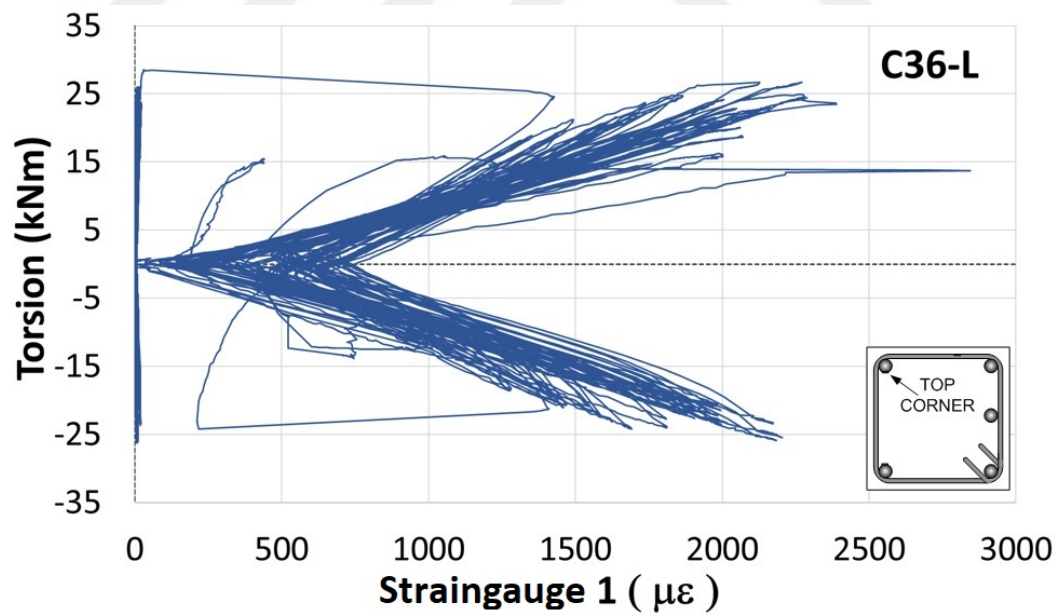


Figure 4.39. Strain curve of longitudinal reinforcement located at top corner.

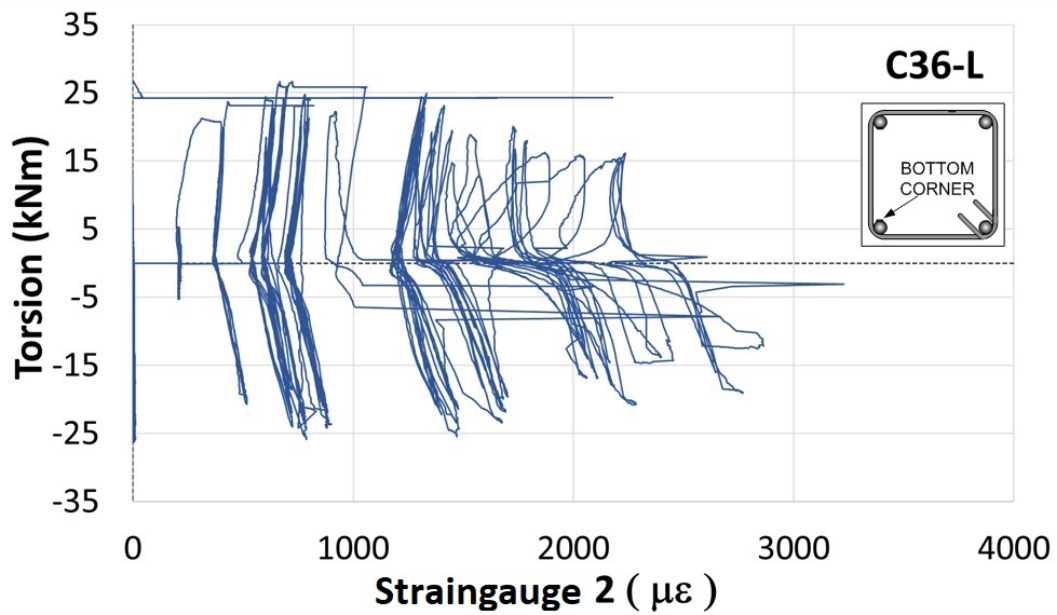


Figure 4.40. Strain curve of longitudinal steel located at bottom corner.

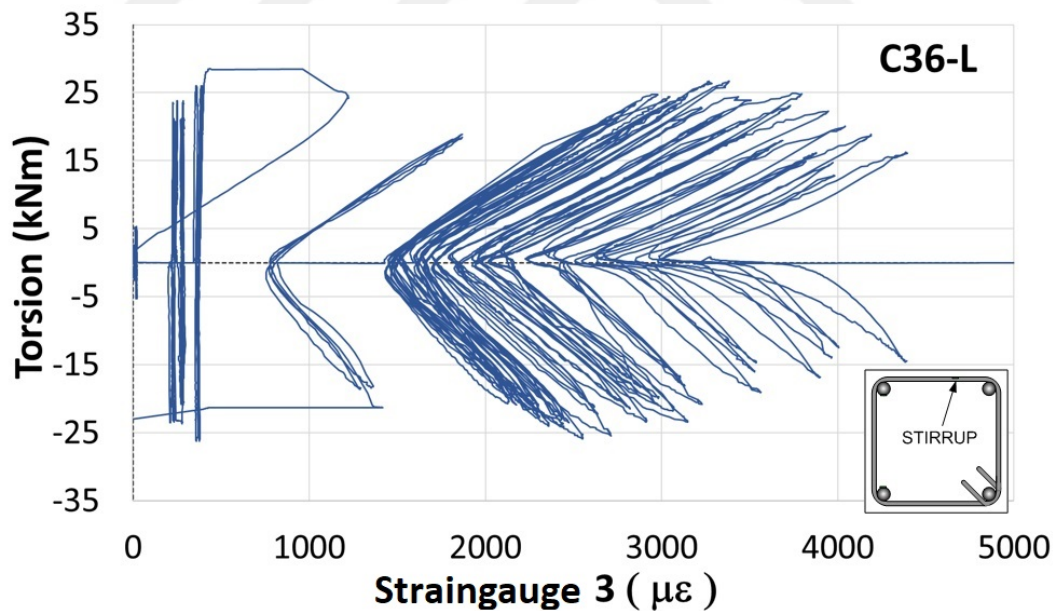


Figure 4.41. Strain curve of transverse reinforcement.

4.7.5. Specimen #5: C17-C1

C17-C1 is the control specimen of the weak specimen group. The concrete strength is 17 MPa. $8\phi 10$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 95 mm spacing are used in the test region.

An initial crack was observed at the seventh cycle when the torque was 15.4 kNm and the twist was 2.0×10^{-3} rad/m. After this level, diagonal torsional cracks propagated at each successive cycle. These cracks formed spirally around the beam and the member section acted as a space truss as shown in Figure 4.42.



a) @ 16.5 kN.m.(moment) vs 60.3×10^{-3} rad/m (twist)



b) @ 15.7 kN.m.(moment) vs 95.2×10^{-3} rad/m (twist)



c) @ 13.3 kN.m.(moment) vs 130.4×10^{-3} rad/m (twist)



d) @ 11.1 kN.m.(moment) vs 210.0×10^{-3} rad/m (twist)

Figure 4.42. A few photos from the experiment of C17-C1.

The maximum torque was recorded as 16.9 kNm at 52.0×10^{-3} rad/m twist level as indicated in Figure 4.43. After the maximum torque level, the concrete started to crush and the existing cracks widened. The test ended at the 151.0×10^{-3} rad/m twist level when the torque was decreased below 80 percent of the maximum torsional capacity. As seen in Torque vs. Twist curves, torsional stiffness was degraded for each

cycle. Failure behavior was ductile. In order to be sure that the new test setup was working properly, this weak specimen was selected and tested firstly according to the loading pattern with only one loop per each twist cycle.

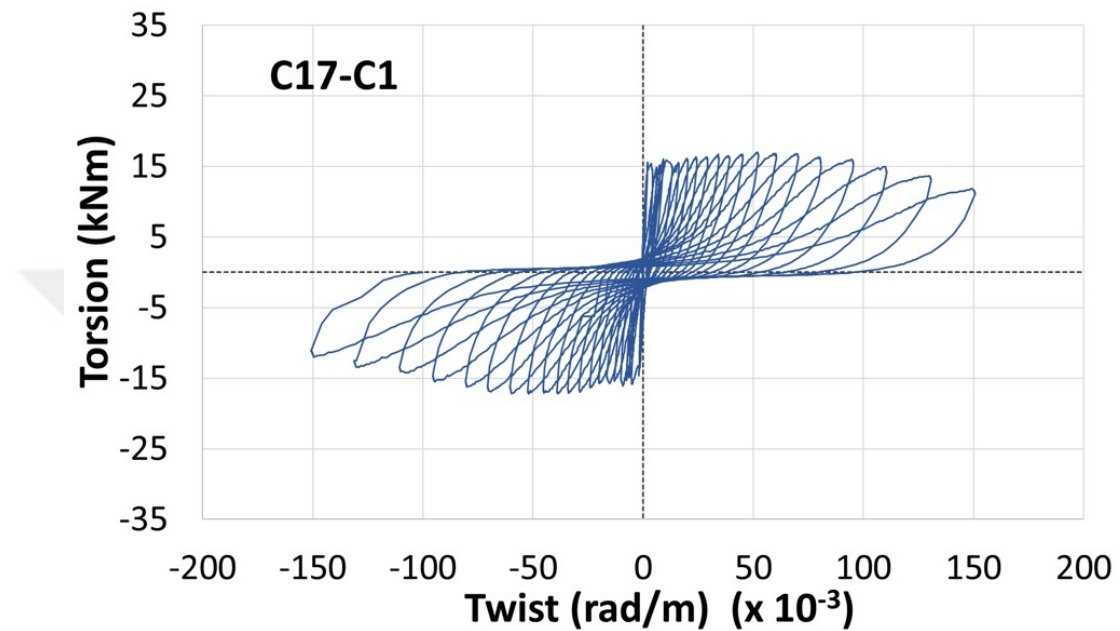


Figure 4.43. Torque vs. twist behavior of the C17-C1.

Strain values obtained from longitudinal and transverse reinforcements at the test region were given with Figures 4.44-4.47.

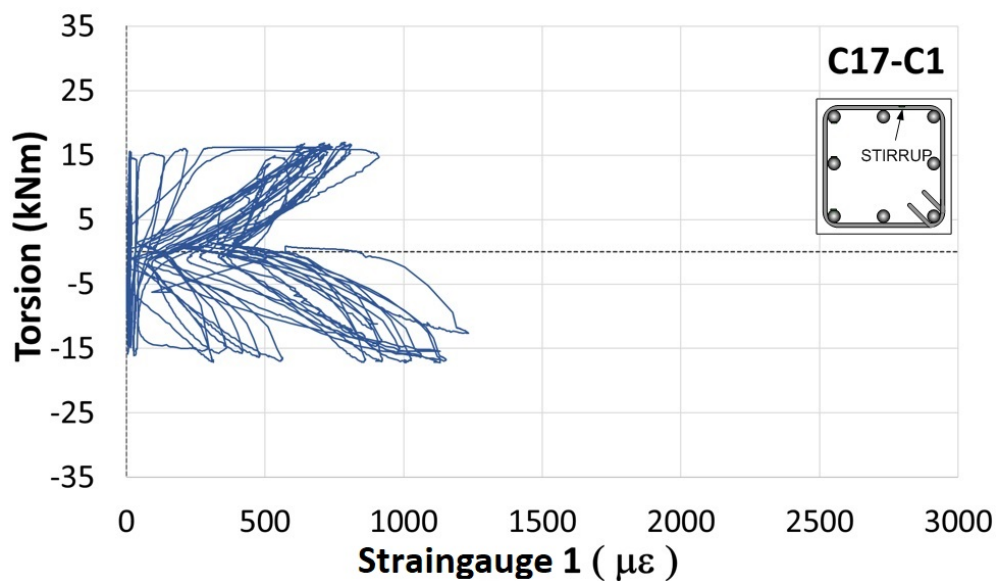


Figure 4.44. Strain curve of transverse reinforcement.

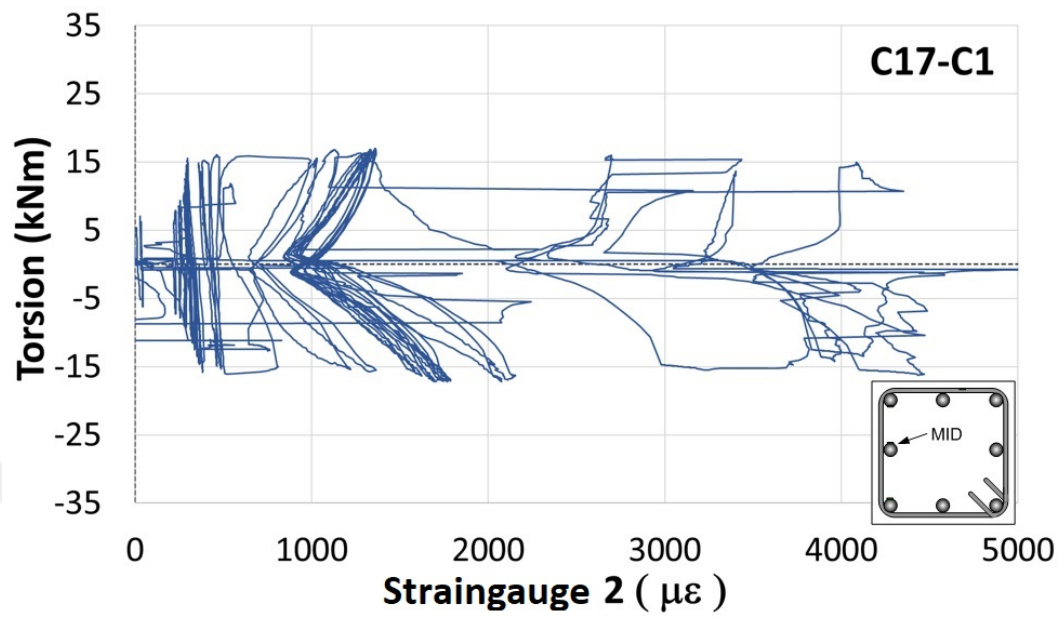


Figure 4.45. Strain curve of longitudinal reinforcement located at the mid.

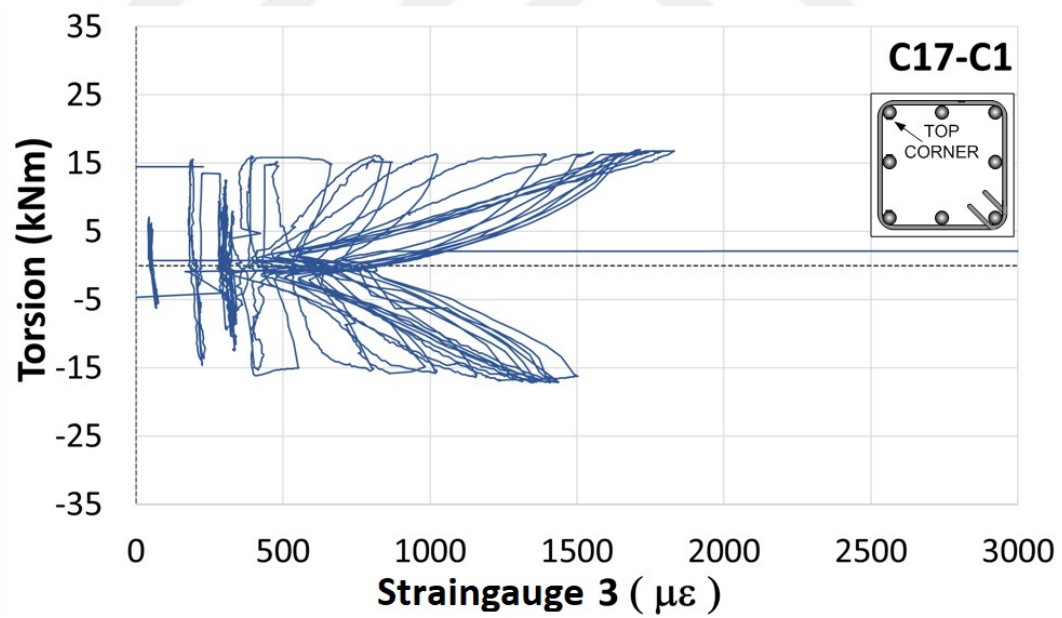


Figure 4.46. Strain curve of longitudinal steel located at the top corner.

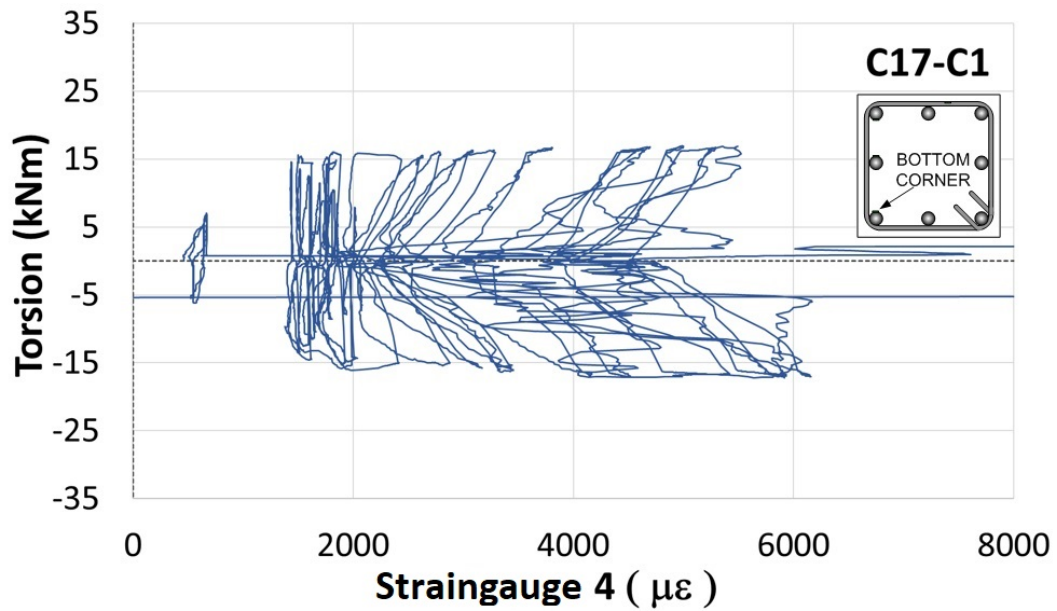


Figure 4.47. Strain curve of longitudinal reinforcement at the bottom corner.

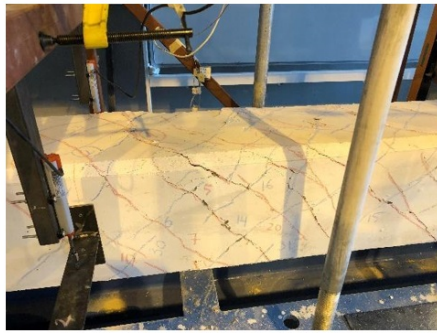
4.7.6. Specimen #6: C17-C2

C17-C2 is the second control specimen of the weak specimen group. All material properties and parameters are the same as with the C17-C1. During the first test (C17-C1) the loading was applied as one cycle per each twist level, but in the second one (C17-C2) three cycles were applied for each twist level.

This specimen behaved similarly to the C17-C1 as expected. An initial crack was observed at the 6th cycle when the torque was 13.8 kNm and the twist was 3.6×10^{-3} rad/m and the first ϕ_y twist level. However, starting from the $16\phi_y$ twist level, not many new cracks have formed, instead the existing cracks have widened and propagated. The member section acted as a space truss as shown in Figure 4.48.

After this level, diagonal torsional cracks propagated and turned around the specimen like a spiral. It was observed that most of the cracks were formed after the $5\phi_y$ twist level. The maximum torque was recorded as 17.7 kNm at 25.5×10^{-3} rad/m twist and $13\phi_y$ twist level as indicated in Figure 4.49. The concrete started to crush and existing cracks widened. The test was ended because the stroke was end at $120\phi_y$

twist level (203.0×10^{-3} rad/m) when the torque was decreased below 70 percent of the maximum torsional capacity. As seen in Torque vs. Twist curves, torsional stiffness was degraded for each cycle.



a) @ 16.9 kN.m.(moment) vs 57.0×10^{-3} rad/m (twist)



b) @ 15.6 kN.m.(moment) vs 95.2×10^{-3} rad/m (twist)



c) @ 13.3 kN.m.(moment) vs 133.0×10^{-3} rad/m (twist)



d) @ -10.3 kN.m.(moment) vs -228.0×10^{-3} rad/m (twist)

Figure 4.48. A few photos from the experiment of C17-C2.

The hysteresis loop shows the relationship between the torsional moment and twist angle which is an important basis for the seismic design of structures and structural members.

In Figure 4.49, at the initial stage of loading, there is a linear relationship before cracking between the torsional moment and the corresponding twist value. The fact that the area of the hysteresis loops is very small and narrow indicates that the test member is elastic. The slopes of the hysteresis loops are reduced because of the cracking and slippage between the reinforcement and the concrete of the test specimens while the torsional loads increase.

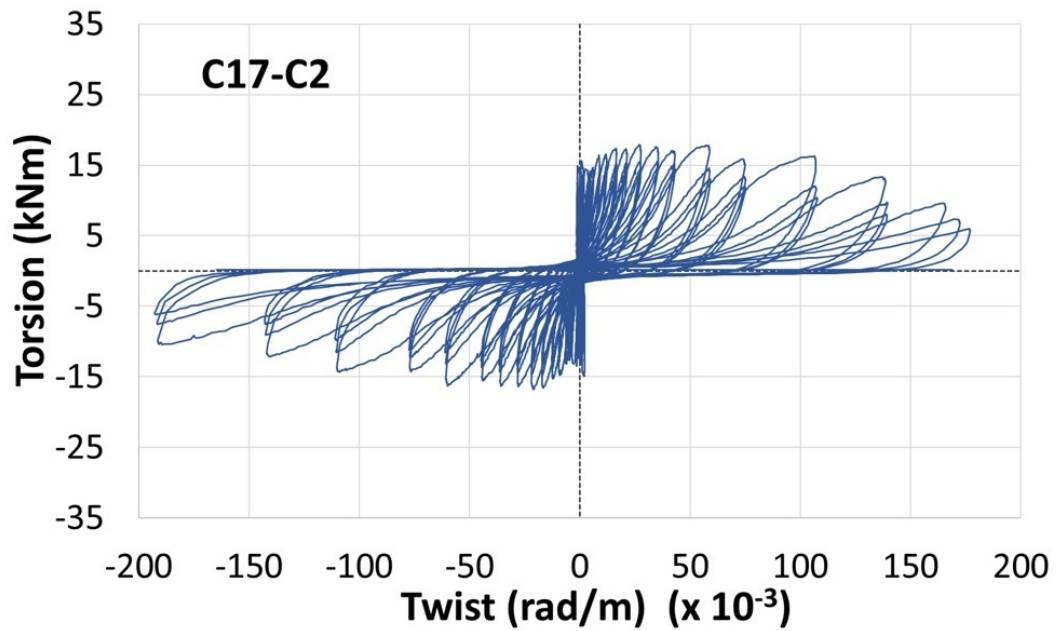


Figure 4.49. Torque vs. twist behavior of the C17-C2.

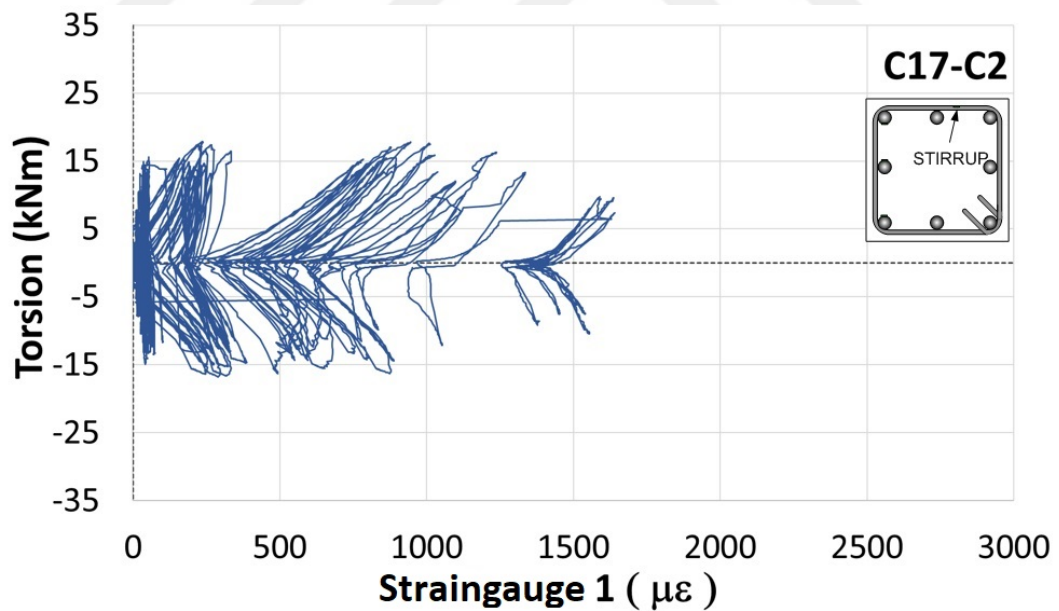


Figure 4.50. Strain curve of transverse reinforcement.

After the reinforcement of the test specimen yielded, the hysteresis loops gradually rotate towards the horizontal axis, and loops expanded with the increasing value of torsional loads. The transverse and longitudinal reinforcement strains in an element under reversed cyclic torsion are shown in Figures 4.50-4.53. Before the member cracked, it was found that the reinforcement steels had very low strains. After cracking,

all the reinforcement bars were in a state of tension. Strain curves of reinforcements performed a butterfly shape due to the re-cyclic torsional loading.

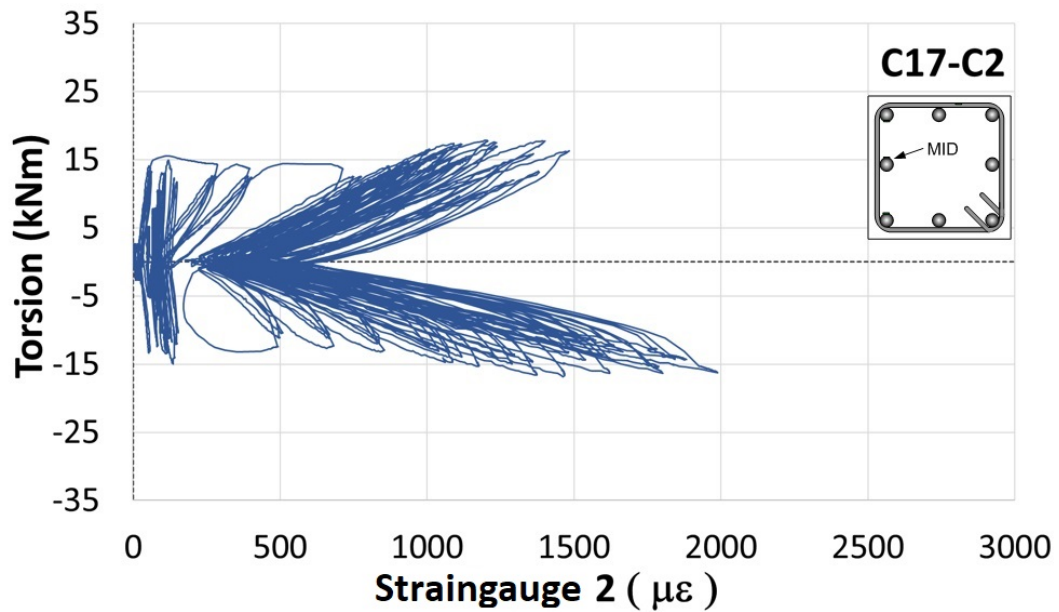


Figure 4.51. Strain curve of longitudinal reinforcement located at the mid.

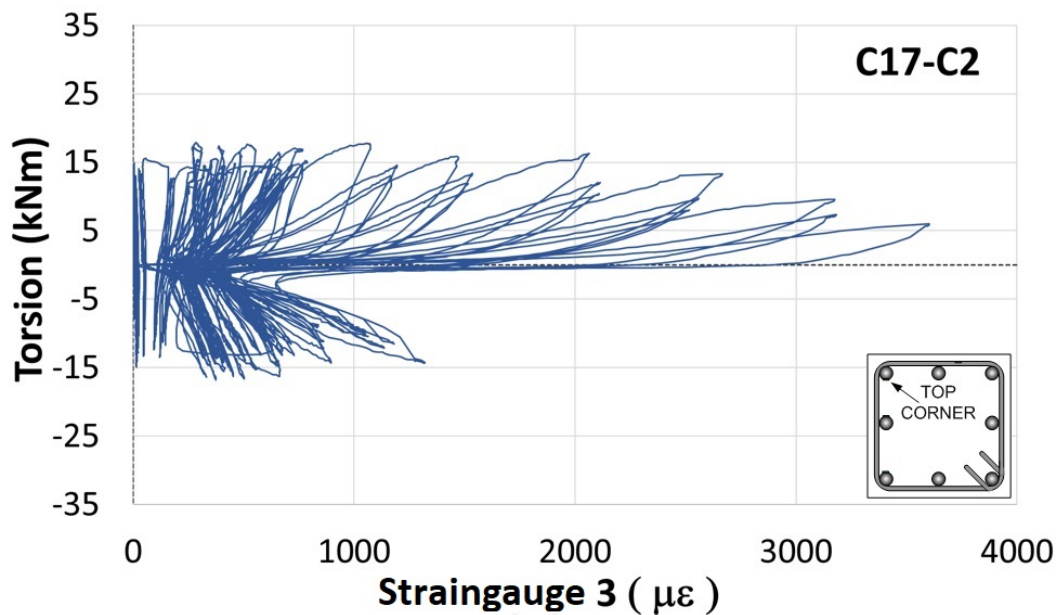


Figure 4.52. Strain curve of longitudinal steel located at the top corner.

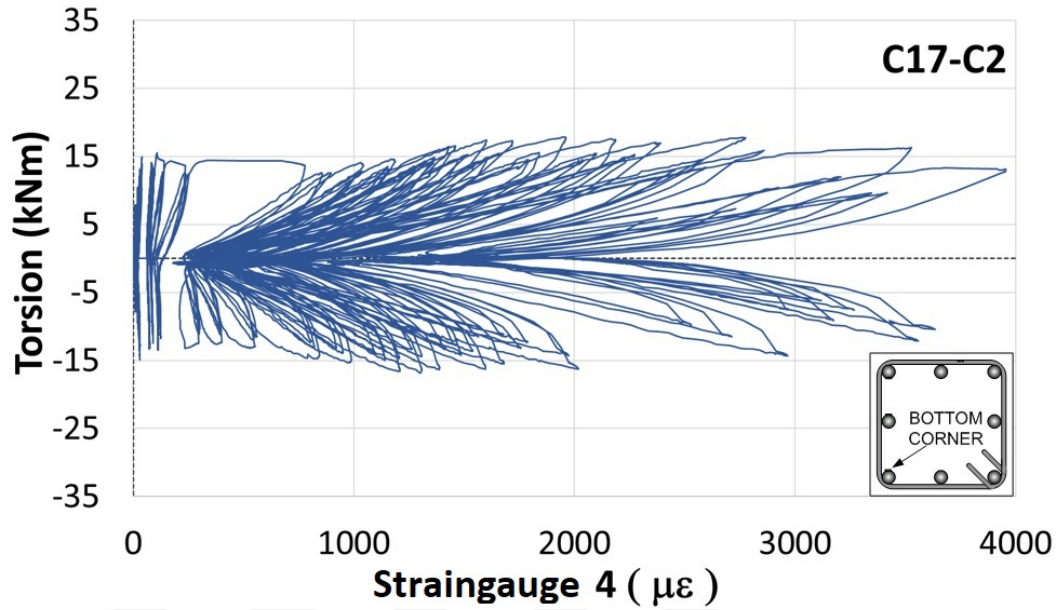


Figure 4.53. Strain curve of longitudinal reinforcement at the bottom corner.

4.7.7. Specimen #7: C17-T

C17-T has reduced transverse reinforcement with 17 MPa concrete compressive strength. $8\phi 10$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 150 mm spacing are used in the test region. The member was subjected to the same loading protocol with C17-C2 as three loops for each twist cycle. An initial crack was observed at 17.0 kNm. and the twist was 1.9×10^{-3} rad/m. After this level, diagonal torsional cracks were propagated at each ϕ_y twist level. Cracks formed outside the test region as shown in Figure 4.54. The formation of cracks outside the expected region prevented data collection. The sample became unstable in that place due to the condensation of the cracks at one point.

The maximum torque was recorded as -17.5 kNm. at -1.6×10^{-3} rad/m twist at $10\phi_y$ twist level in counter clockwise direction. The concrete started to crush and existing cracks widened.

The experiment was terminated at $30\phi_y$ twist level and at the 61.0×10^{-3} rad/m twist as indicated in Figure 4.55. Due to the failure location is formed outside the test

region, the head of the sample was crushed too much and rotation measuring plate was pulled down where the cracks were concentrated at this region as shown in Figure 4.55. The required data could not be collected because the sample collapsed from the head, out of the test region. However, the torque was decreased below 75 percent of the maximum torsional capacity.



a) @ -12.1 kN.m.(moment) vs -5×10^{-3} rad/m (twist)



b) @ 15.6 kN.m.(moment) vs 12×10^{-3} rad/m (twist)



c) @ -14.0 kN.m.(moment) vs -26.0×10^{-3} rad/m (twist)



d) @ -5.0 kN.m.(moment) vs -61.0×10^{-3} rad/m (twist)

Figure 4.54. A few photos from the experiment of C17-T.

Torsional moment and strain relationships for the reinforcements of the C17-T are given in Figures 4.56-4.59. As seen in Figure 4.56, there are not any strain records observed on the transverse reinforcement. The reason for this case is that the test ended early during 61.0×10^{-3} rad/m twist level, at which transverse reinforcements are not working effectively.

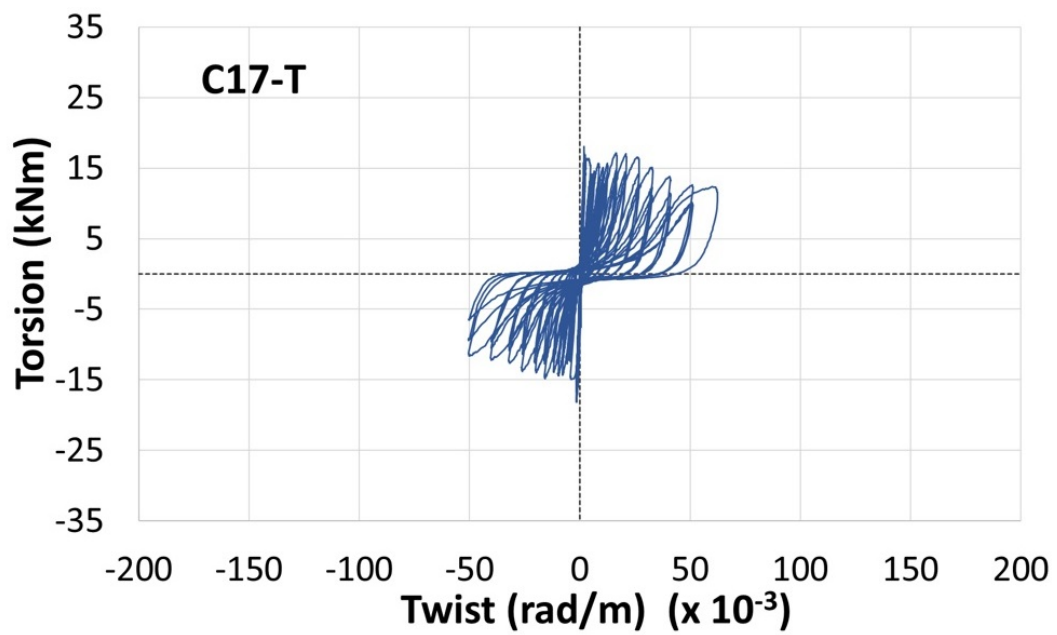


Figure 4.55. Torque vs. twist behavior of the C17-T.

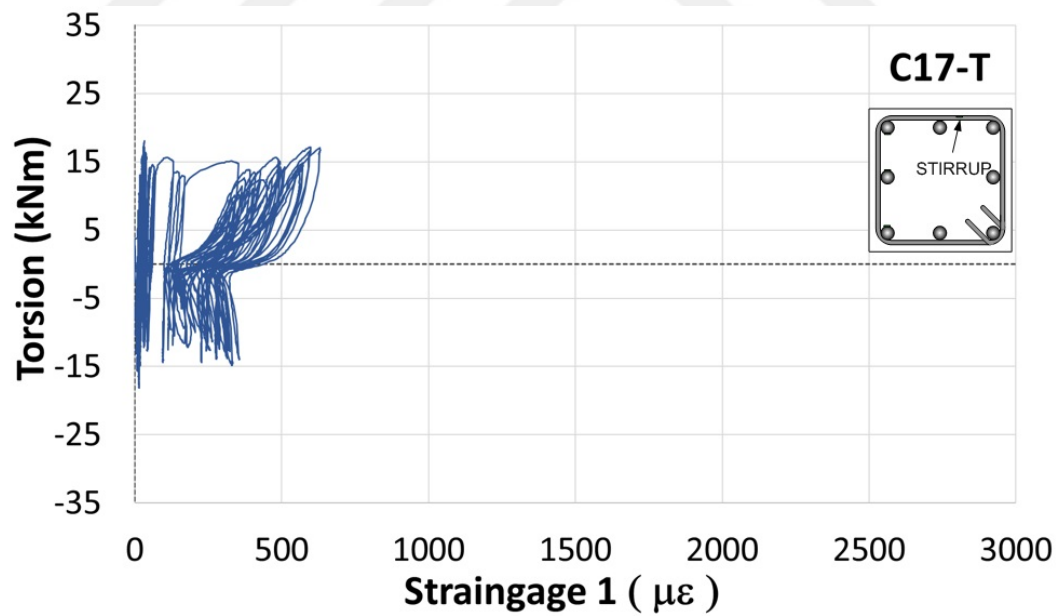


Figure 4.56. Strain curve of transverse reinforcement.

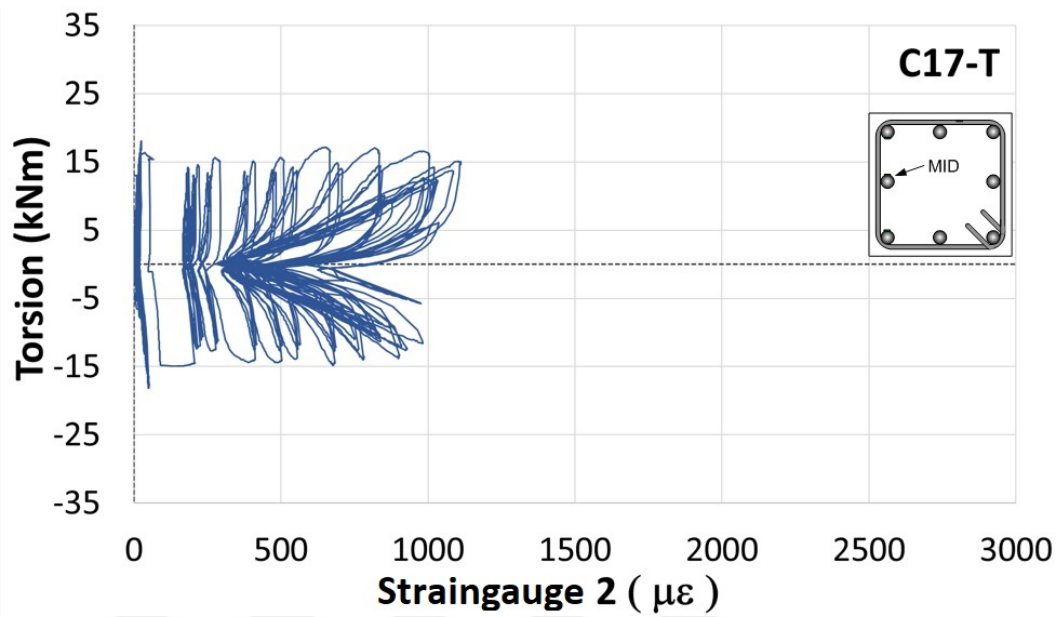


Figure 4.57. Strain curve of longitudinal reinforcement located at the mid.

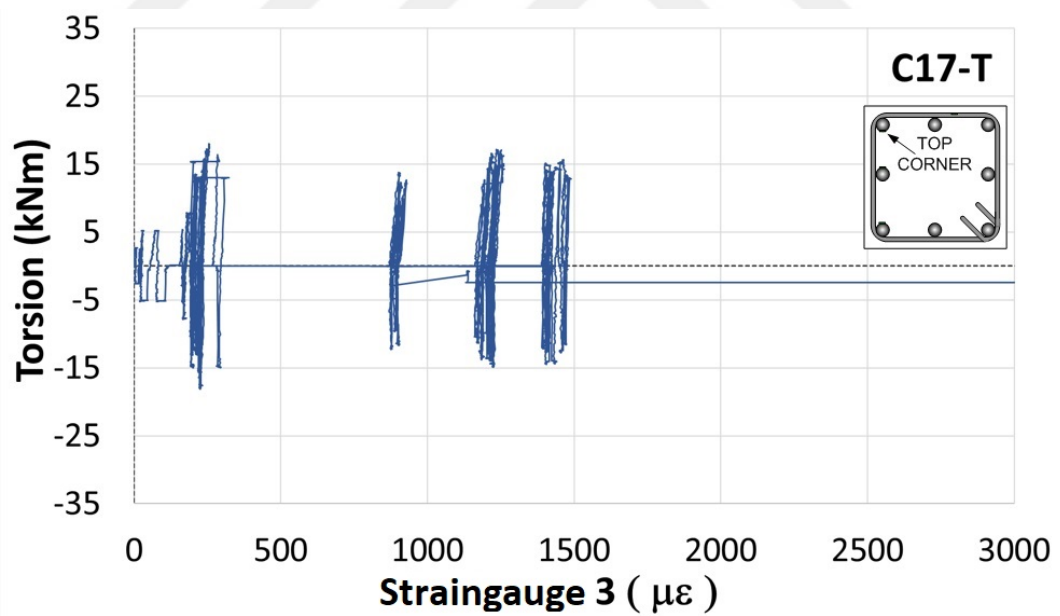


Figure 4.58. Strain curve of longitudinal steel located at the top corner.

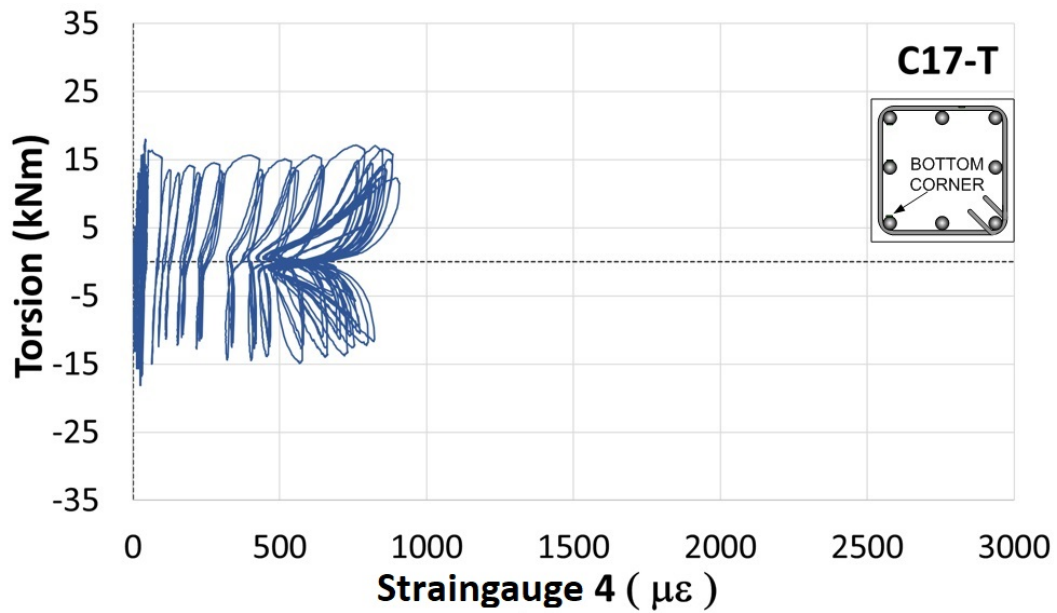


Figure 4.59. Strain curve of longitudinal reinforcement at the bottom corner.

4.7.8. Specimen #8: C17-L

The last specimen of the low strength concrete test group is the C17-L with reduced longitudinal reinforcement. From the designing of this specimen, $4\phi 10$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 95 mm spacing are used in the test region.

The worst test results were obtained from the testing of this specimen. During the testing of this specimen, an initial crack was observed at 2.0×10^{-3} rad/m twist level when the moment was 12.0 kNm. After this twist level, diagonal torsional cracks propagated at each successive cycles. Although this specimen has a weak concrete strength, the cracks developed out of the test is region unexpectedly as shown in Figure 4.60.

The failure occurred near the application point of load where the twist was not measured. The maximum torque was recorded as -15.5 kNm at 2.8×10^{-3} rad/m twist level as indicated in Figure 4.61 and Figures 4.62 to 4.64 shows the strain values of the reinforcements in the test region.



a) @ -14.7 kN.m (moment) vs -3.2×10^{-3} rad/m (twist)



b) @ -14.0 kN.m (moment) vs -5.3×10^{-3} rad/m (twist)



c) @ -14.6 kN.m (moment) vs -10.5×10^{-3} rad/m (twist)



d) @ -15.3 kN.m (moment) vs -12.6×10^{-3} rad/m (twist)

Figure 4.60. A few photos from the experiment of C17-L.

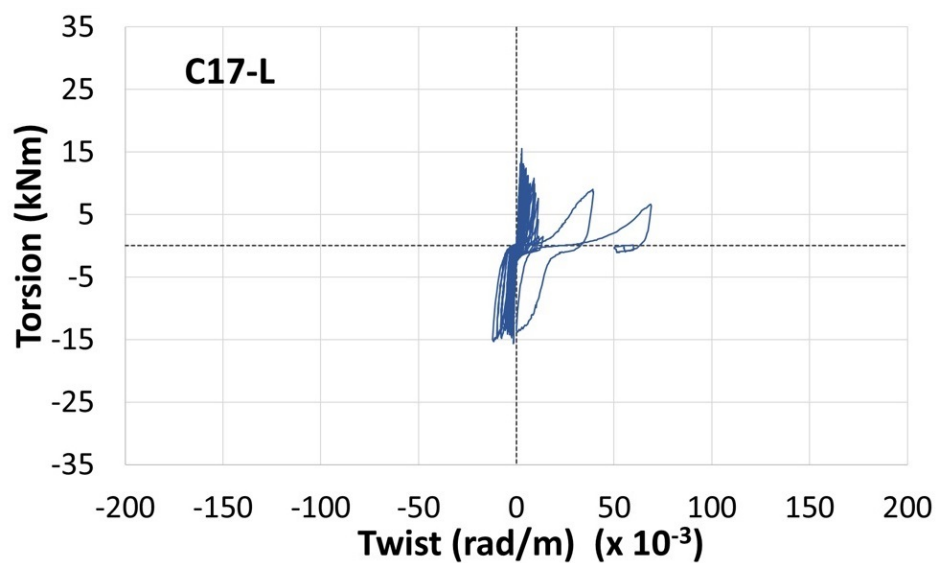


Figure 4.61. Torque vs. twist behavior of the C17-L.

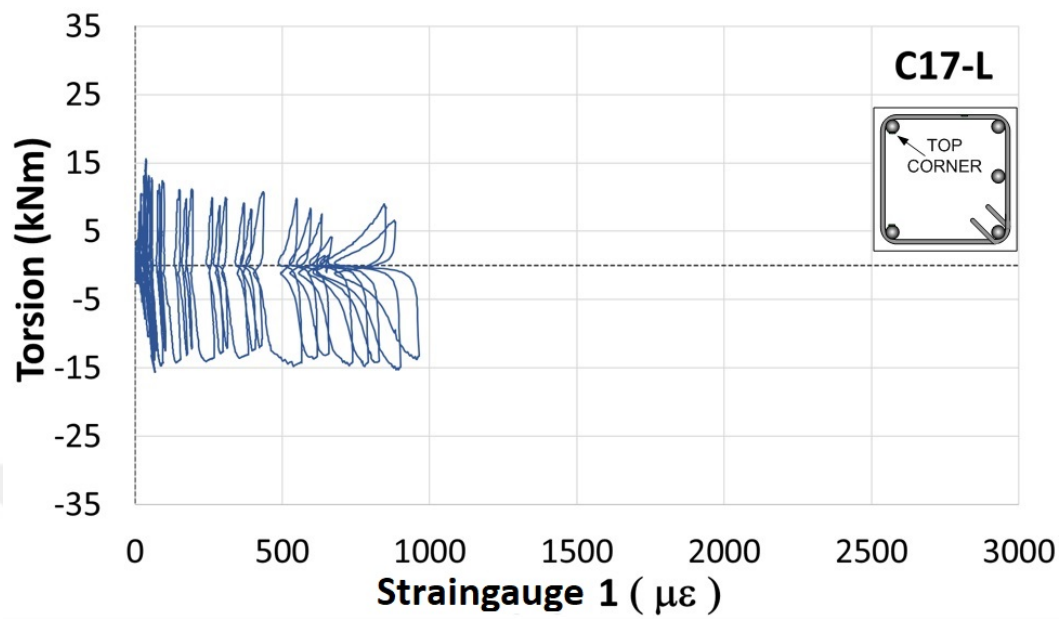


Figure 4.62. Strain curve of longitudinal steel located at the top corner.

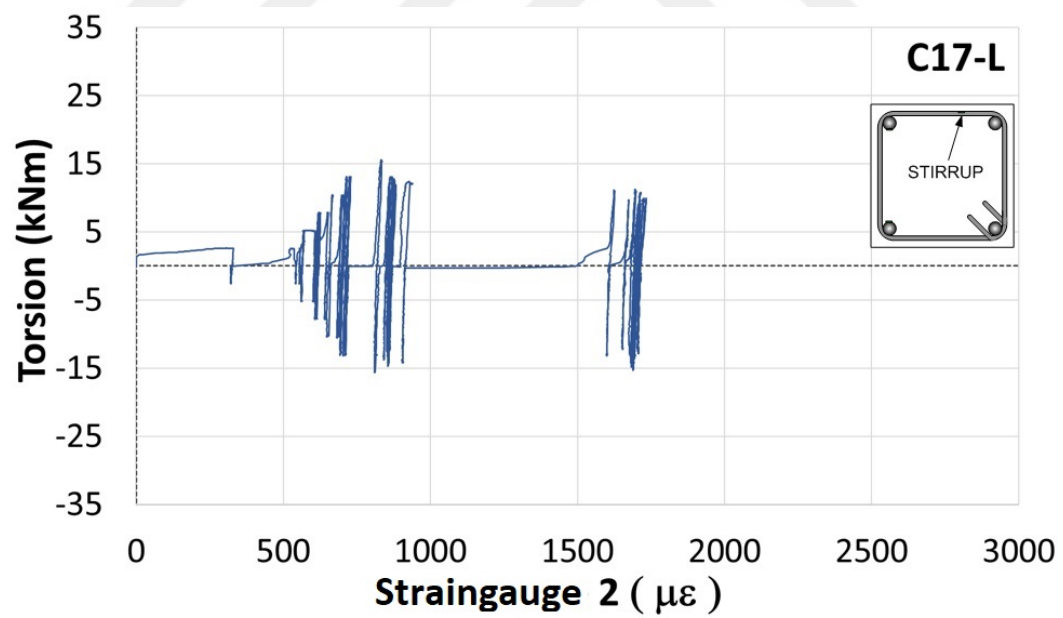


Figure 4.63. Strain curve of transverse reinforcement.

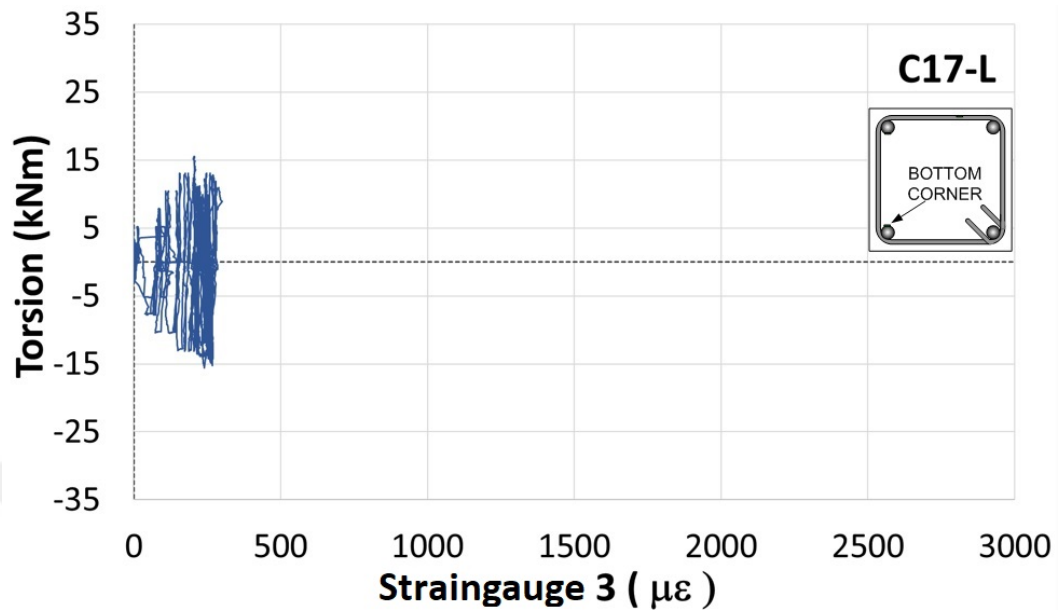


Figure 4.64. Strain curve of longitudinal steel at the bottom corner.

4.7.9. Specimen #9: C50-C

The longitudinal elongations were measured and recorded during the testing of the C50-C specimen. In order to measure axial elongation, the movement of points at the ends of the test region were recorded. Two more LVDTs and plates were implemented on the specimen to measure the longitudinal elongations.

This specimen is the control specimen of the high-strength concrete test group. $8\phi 14$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 75 mm spacing are used in the test region. This sample, which has the highest concrete strength and was produced with the capacity to meet the torsional moments, was not forced much under the incoming moments.

The yield twist level was reached with the torsional moment at 26.0 kNm and the twist at 2.0×10^{-3} rad/m. Likewise, the second crack occurred while turning it the other direction in this cycle. Since the behavior was quite brittle, the number of cracks increased rapidly. By the $5\phi_y$ twist level, the number of cracks had exceeded fifty. After the $5\phi_y$ twist level, at most 5 more cracks occurred. Experiment continued

with increasing crack widths and lengths. All the cracks spiraled around the sample in two opposite directions.

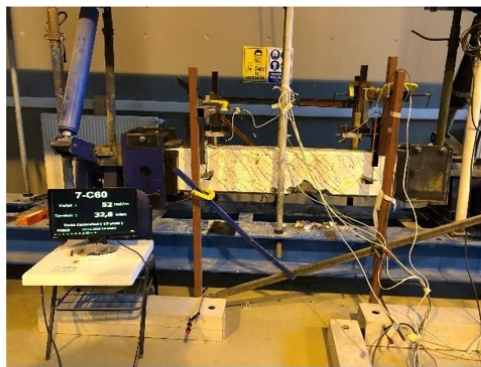
As can be seen in Figure 4.65, only cover concrete spalling was seen at $20\phi_y$ twist level, which can be considered the most advanced levels of the experiment.



a) @ 23.6 kN.m.(moment) vs 6.0×10^{-3} rad/m (twist)



b) @ 25.8 kN.m.(moment) vs 12.0×10^{-3} rad/m (twist)



c) @ 32.8 kN.m.(moment) vs 52.0×10^{-3} rad/m (twist)



d) @ 31.0 kN.m.(moment) vs 80.0×10^{-3} rad/m (twist)

Figure 4.65. A few photos from the experiment of C50-C.

The maximum torsional moment was 32.8 kNm when the twist was 32.0×10^{-3} rad/m at the $8\phi_y$ twist level. The widths of the cracks formed at the beginning were measured around 2 mm at these levels.

The core concrete started to crush at the $10\phi_y$ twist level and it is possible to see that the steel is working. The experiment had to be terminated because the stroke was

not enough at $40\phi_y$ twist level when the twist angle was 160.0×10^{-3} rad/m and the moment dropped at 17.0 kNm. Figure 4.66 gives the hysteretic behavior of this test specimen.

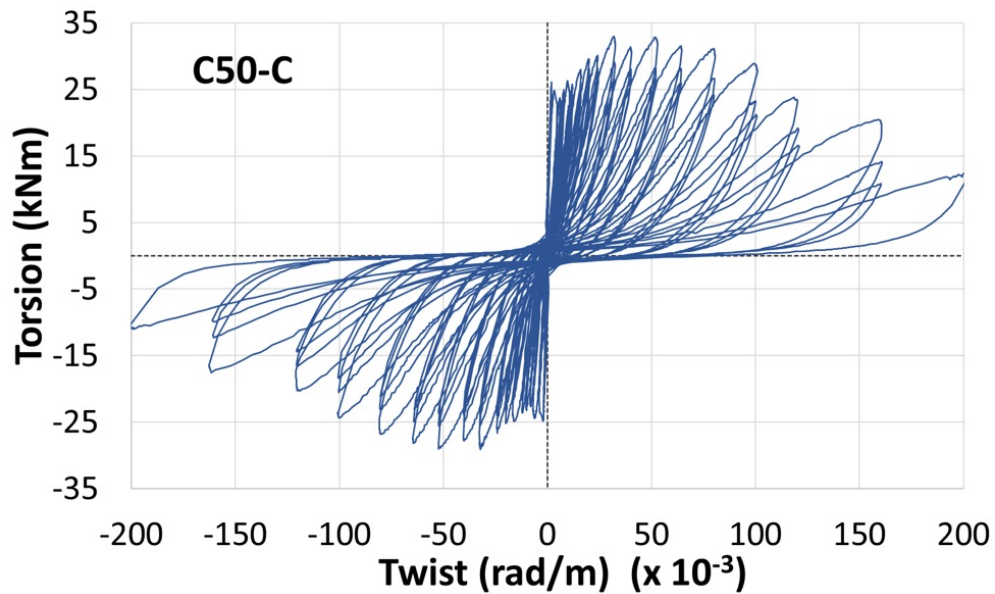


Figure 4.66. Torque vs. twist behavior of the C50-C.

Torsional moment and strain relationships for the reinforcements of the C50-C are given in Figures 4.67- 4.69.

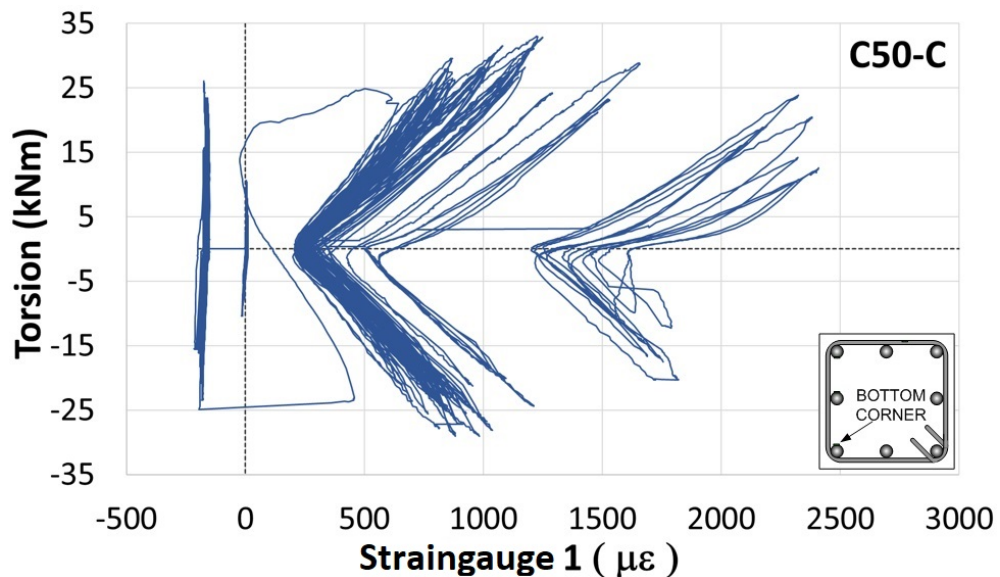


Figure 4.67. Strain curve of steel steel at the bottom corner.

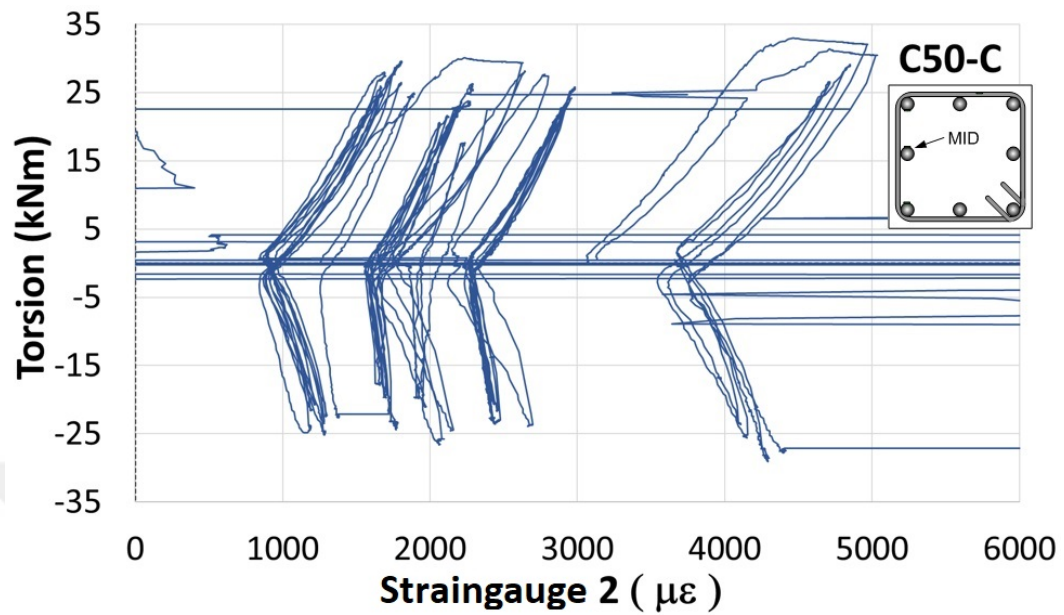


Figure 4.68. Strain curve of longitudinal reinforcement located at mid bar.

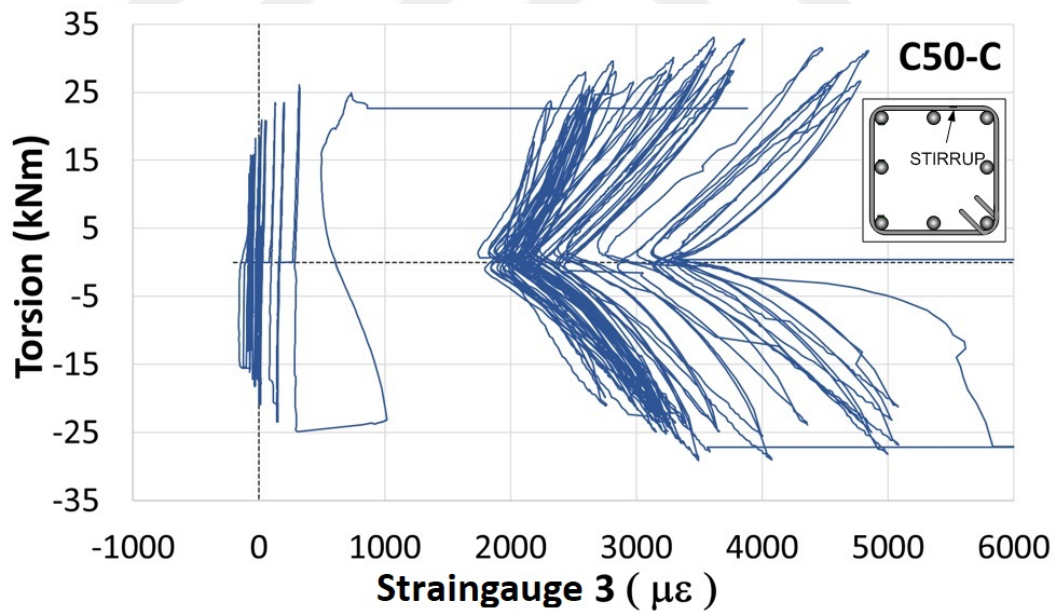


Figure 4.69. Strain curve of transverse reinforcement.

The similar elongation measurement made in specimen C36-C2 was also applied to this specimen. In Figures 4.70 and 4.71 the elongation vs twist and torsional moment relations were illustrated. It can be seen that the specimen elongates about 3 mm under torsional moments.

To compare behavior and to see the similarity between the elongation behavior and the twist behavior of an element Torsional moment vs Member Strain and Twist curve is shown in Figure 4.71.

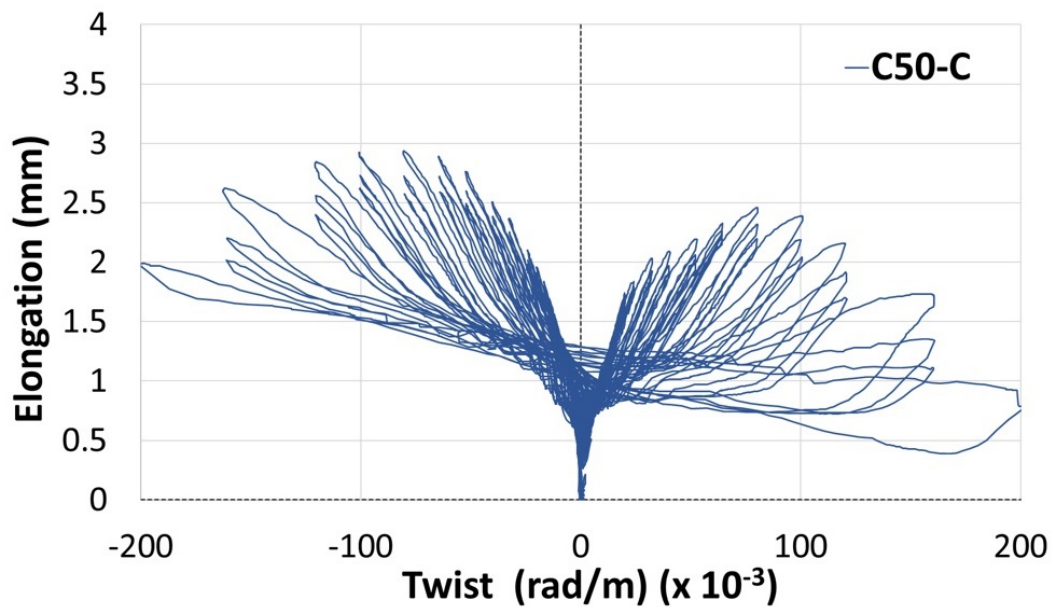


Figure 4.70. Longitudinal elongation vs. twist curve.

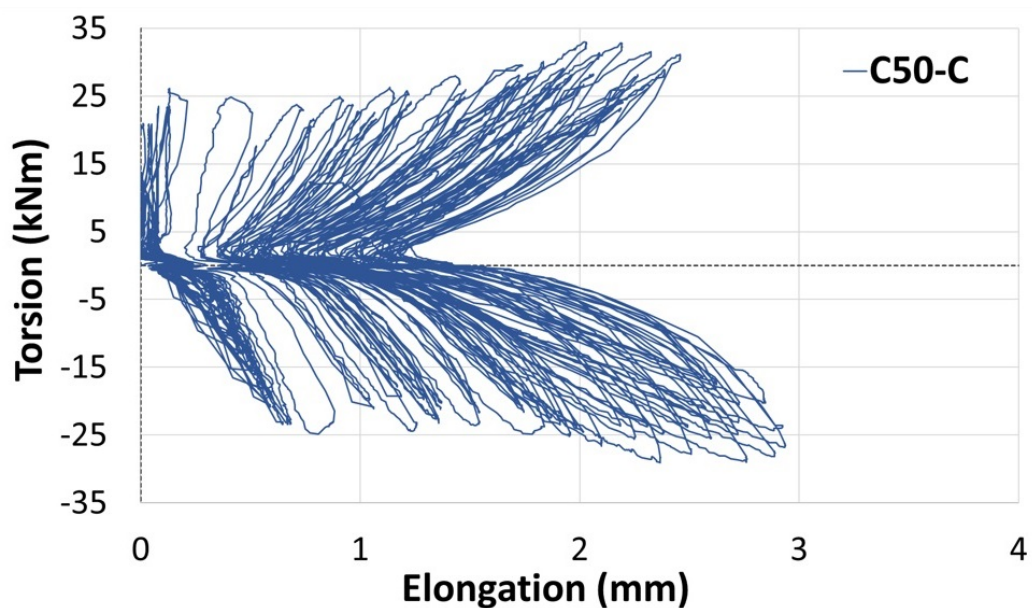


Figure 4.71. Torsional moment vs. longitudinal elongation curve.

Additionally, the strain obtained from straingauges and strain obtained from the elongations were compared in Figures 4.72 and 4.73. It can be seen clearly from the

figures that the both strain values indicates the similar behavior on strain vs. torsional moment relations.

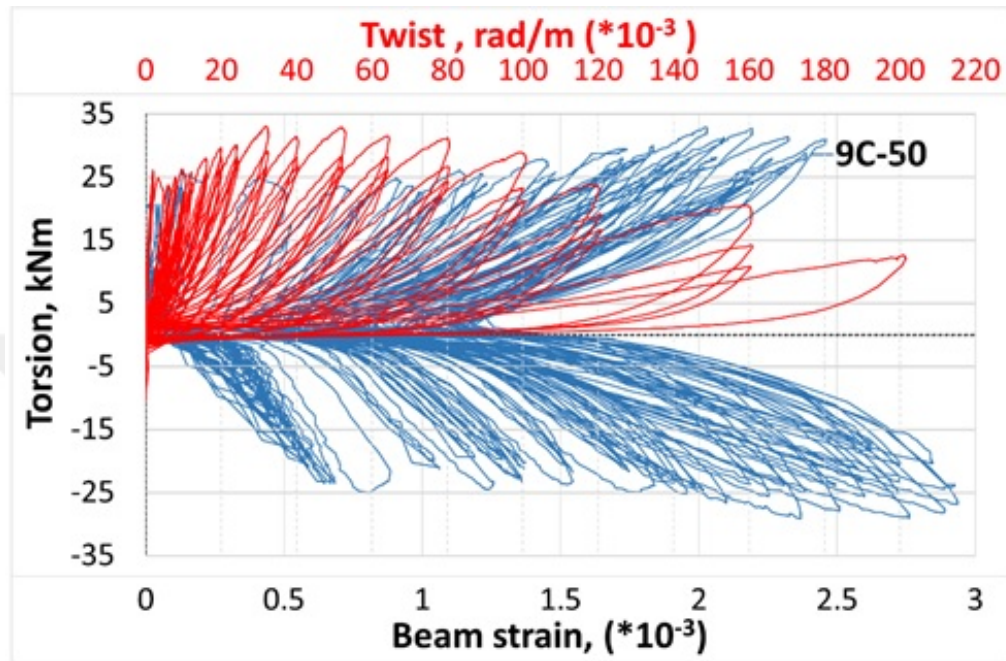


Figure 4.72. Torsional moment vs. strain and twist curve.

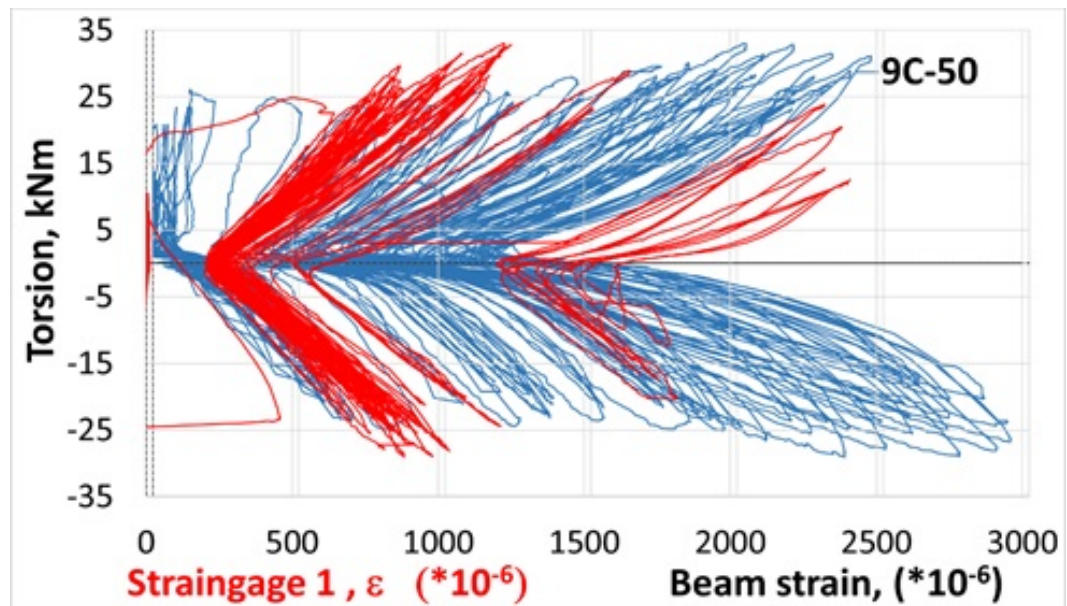


Figure 4.73. Torsional moment vs. member strain and rebar strain curve.

4.7.10. Specimen #10: C50-T

C50-T has reduced transverse reinforcement with 50 MPa concrete compressive strength. $8\phi 14$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 150 mm spacing are used in the test region.

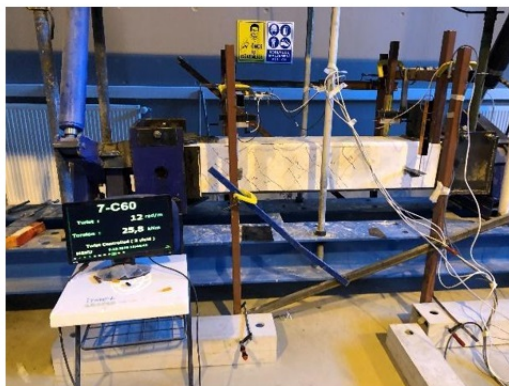
This specimen behaved brittle as the previous one under the cyclic torsional moments because of the high concrete compressive strength. The first crack was formed very close to the moving head of the specimen at the -2.2×10^{-3} rad/m twist angle and -23.3 kNm torsional moment. The member section acted as a space truss as shown in Figure 4.74.



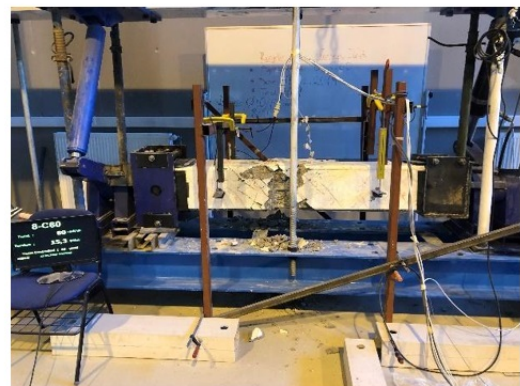
a) @ 26.0 kN.m.(moment) vs 3.1×10^{-3} rad/m (twist)



b) @ -23.9 kN.m.(moment) vs -20.0×10^{-3} rad/m (twist)



c) @ -21.6 kN.m.(moment) vs -50.0×10^{-3} rad/m (twist)



d) @ 15.3 kN.m.(moment) vs 80.0×10^{-3} rad/m (twist)

Figure 4.74. A few photos from the experiment of C50-T.

Many of the cracks were observed at the next loading cycle at the 26.0 kN.m moment ($2\phi_y$ twist level). Diagonal torsional cracks were propagated at each successive cycle.

After yielding, because of the redistribution of the internal forces, bearing capacity started to increase again up to the 26.0 kNm torsional moment at the $4\phi_y$ twist level. Also, this was the recorded maximum moment value at 20.0×10^{-3} rad/m twist angle as indicated in Figure 4.75.

The test was terminated at $6\phi_y$ twist level (127.0×10^{-3} rad/m) and at 13.0 kNm due to the stroke. However, the torque was already decreased below 75 percent of the maximum torsional capacity.

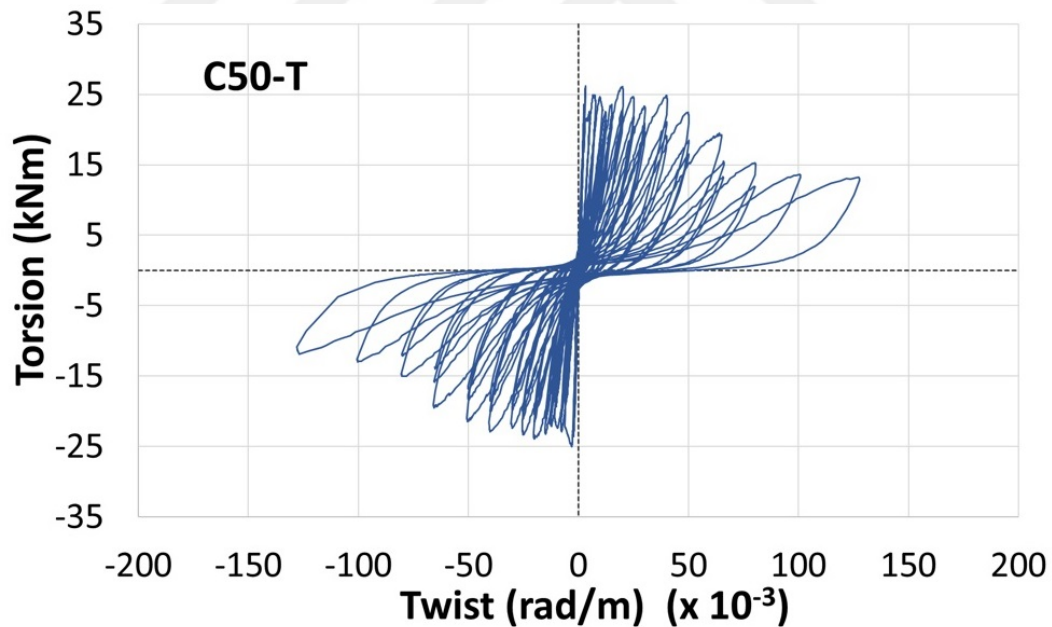


Figure 4.75. Torque vs. twist behavior of the C50-T.

Torsional moment vs. reinforcement strain relationships of the C50-T specimen are given in Figures 4.76- 4.78. The first two graphs shows the longitudinal reinforcement strain recorded from strain gauges, the last one indicates the transverse reinforcement strain.

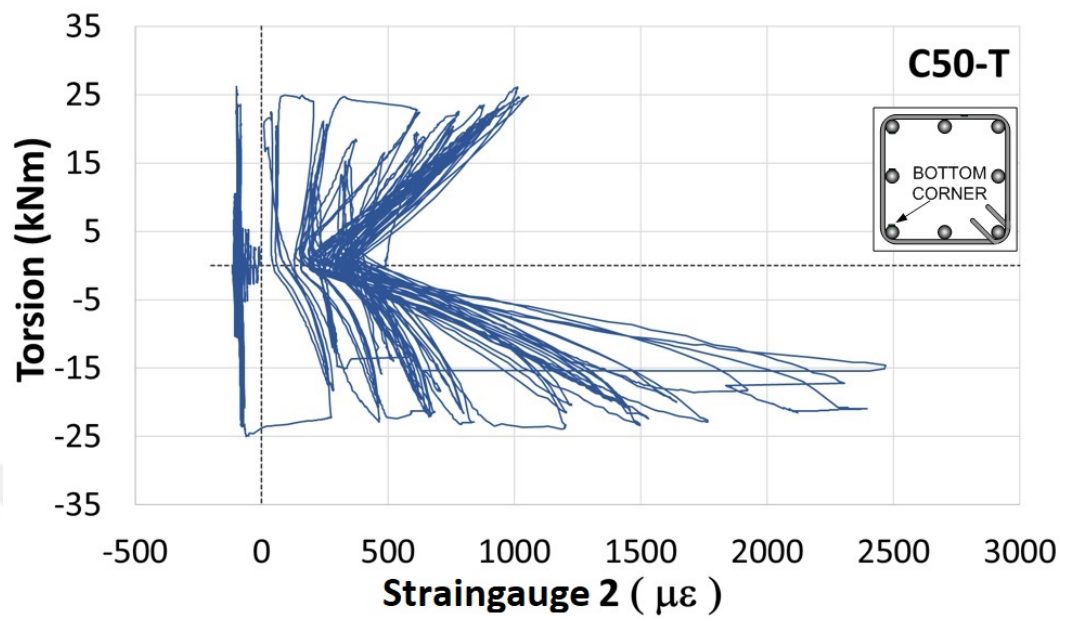


Figure 4.76. Strain curve of longitudinal steel is at the bottom corner.

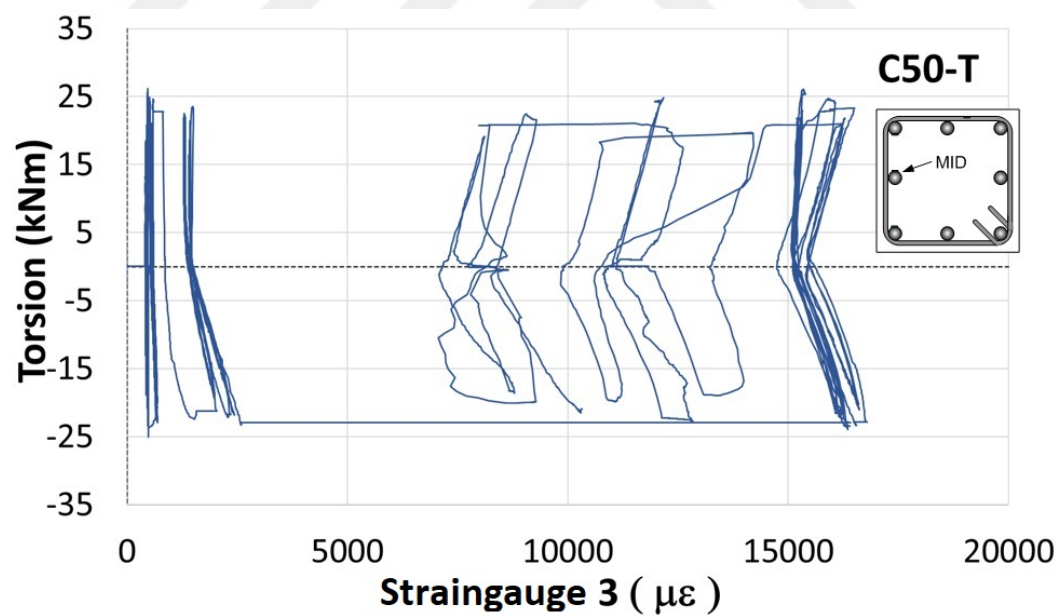


Figure 4.77. Strain curve of longitudinal steel located at the mid bar.

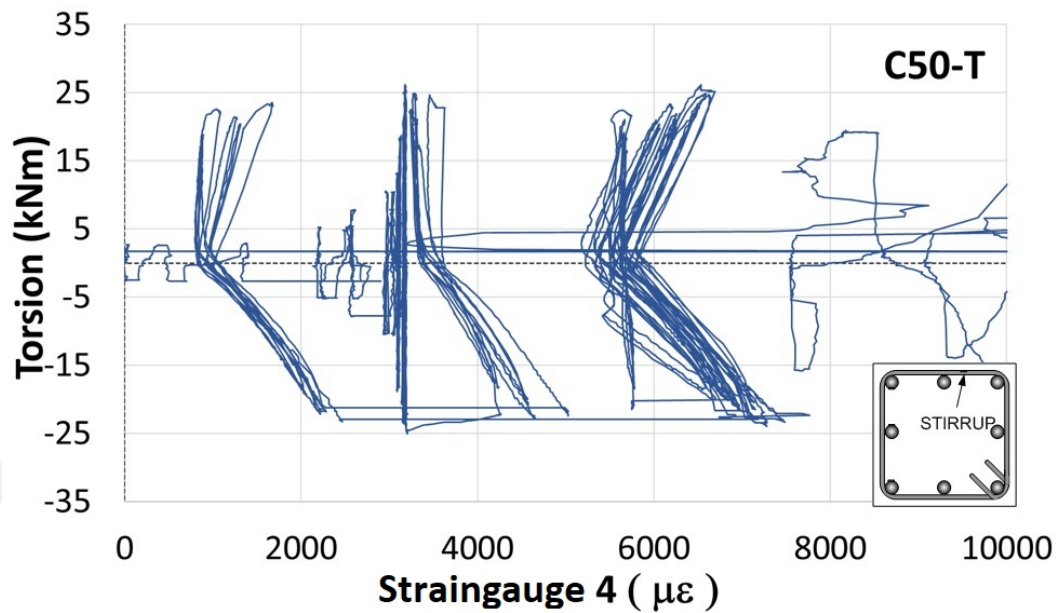


Figure 4.78. Strain curve of transverse reinforcement.

4.7.11. Specimen #11: C50-L

C50-L has reduced longitudinal reinforcement with 50 MPa concrete compressive strength. $8\phi 10$ bars are used as longitudinal reinforcement and 8 mm diameter stirrups with 150 mm spacing are used in the test region.

C50-L was the last specimen of the high-strength concrete group. The same brittle behavior was observed with the previous two high-strength specimens during the test. An initial crack was observed at 23.3 kNm and the twist was 2.2×10^{-3} rad/m. After this cycle, the specimen continued to twist and crack suddenly, despite the loading being stopped.

Many new cracks were formed. Diagonal torsional cracks were propagated at each successive cycle in the test region. The member section acted as a space truss as shown in Figure 4.79.

The maximum torque was recorded as -28.6 kNm at -26.3×10^{-3} rad/m twist level in counter clockwise direction as indicated in Figure 4.80. At 16 yield twist level,

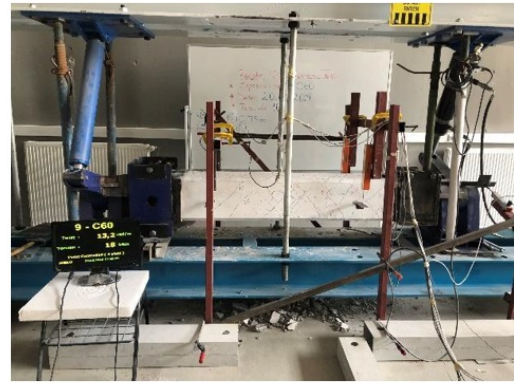
the concrete started to crush and existing cracks widened. The torsional ductility is very high for this specimen.

The test was terminated at the 50 yield twist level while 170.0×10^{-3} rad/m twist level because the stroke of the test setup was ended. However, the torque was decreased below 75 percent of the maximum torsional capacity.

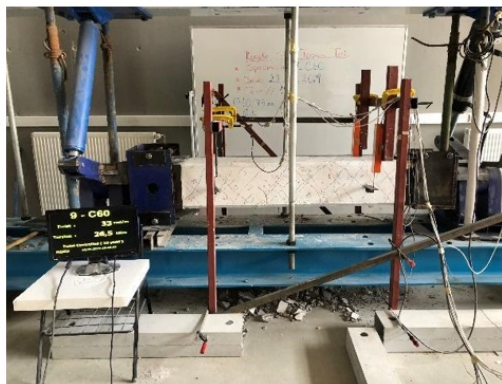
Torsional moment and strain relationships for the reinforcements of the C50-L are given in the Figures 4.81-4.83.



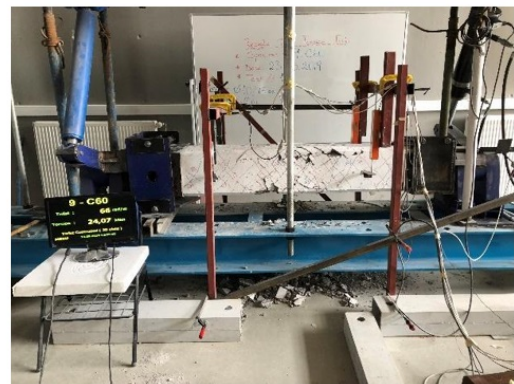
a) @ 20.5 kN.m.(moment) vs -4.9×10^{-3} rad/m (twist)



b) @ 18.0 kN.m.(moment) vs 13.2×10^{-3} rad/m (twist)



c) @ 26.5 kN.m.(moment) vs 33×10^{-3} rad/m (twist)



d) @ 24.1 kN.m.(moment) vs 66.0×10^{-3} rad/m (twist)

Figure 4.79. A few photos from the experiment of C50-L .

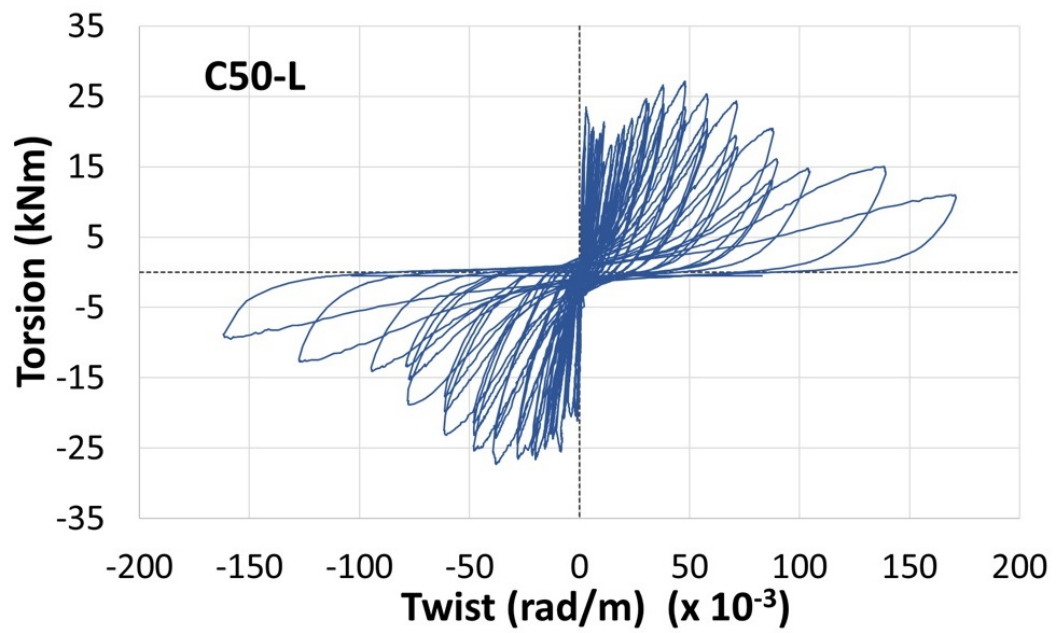


Figure 4.80. Torque vs. twist behavior of the C50-L.

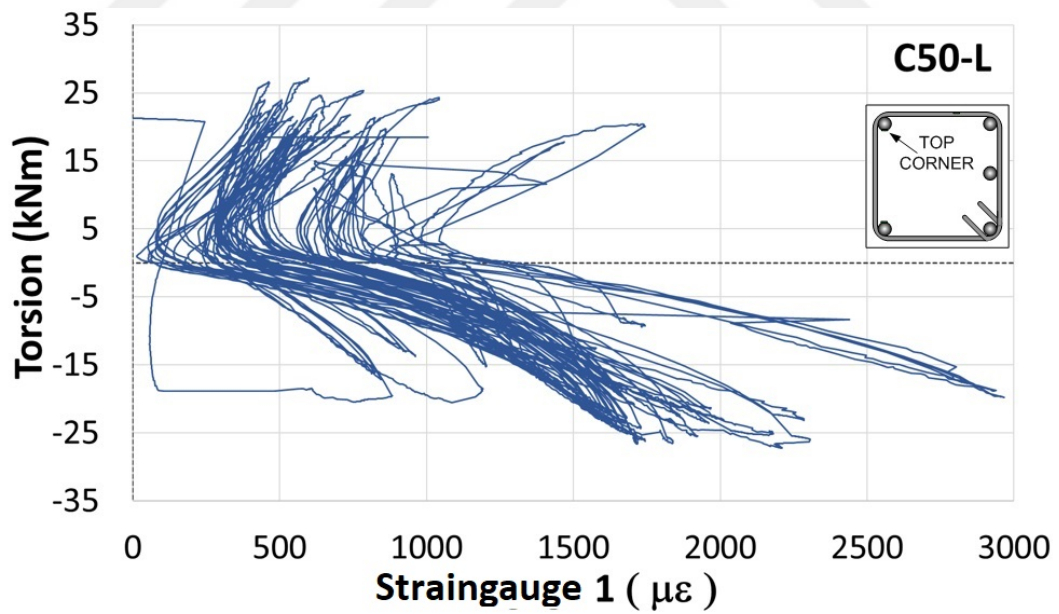


Figure 4.81. Strain curve of longitudinal steel located at the top corner.

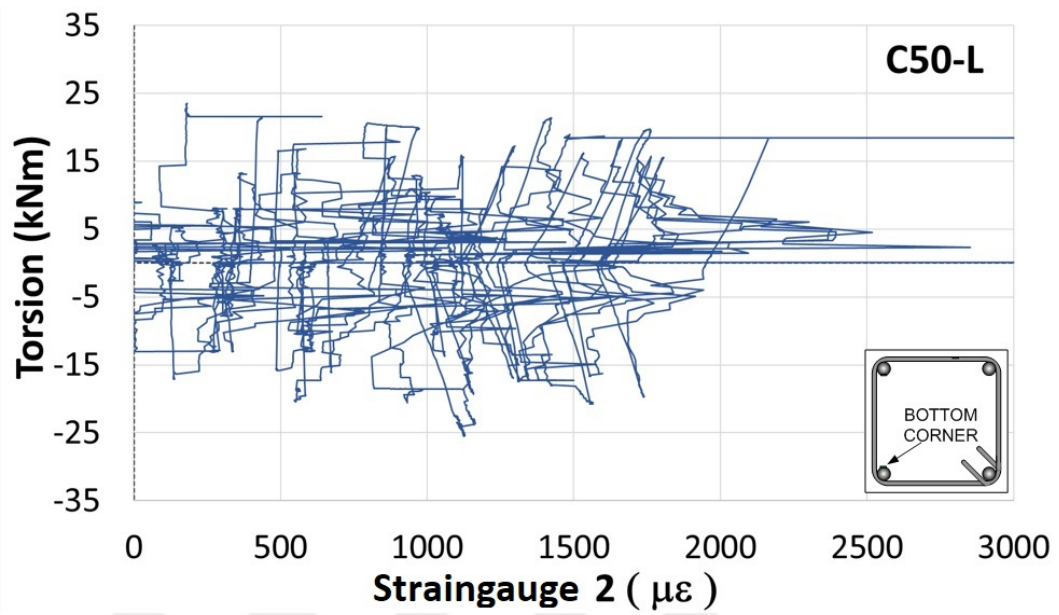


Figure 4.82. Strain curve of longitudinal steel at the bottom corner.

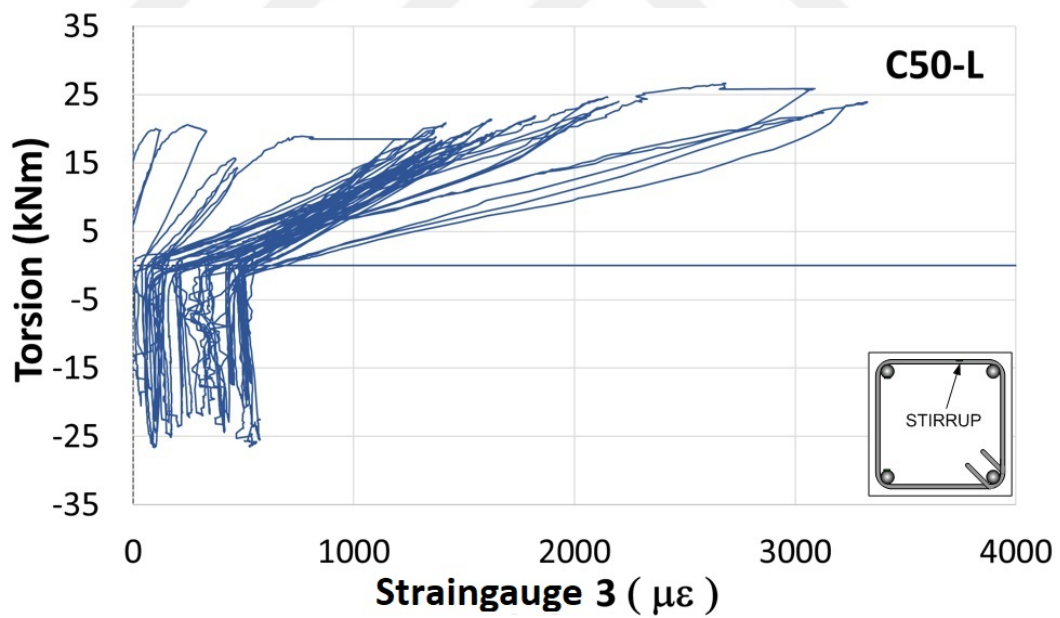


Figure 4.83. Strain curve of transverse reinforcement.

4.8. Discussion of Test Results

4.8.1. Backbone Curve

The curve formed by combining the peak points of the hysteresis loops is called as the backbone curve of members under reversed cyclic torsion. This curve can reflect the interrelationship between the peak torsional moment and the corresponding twist angle. The skeleton curve of all test specimen were obtained by connecting the peak points of first loop of each twist level were illustrated in Figures 4.84-4.94.

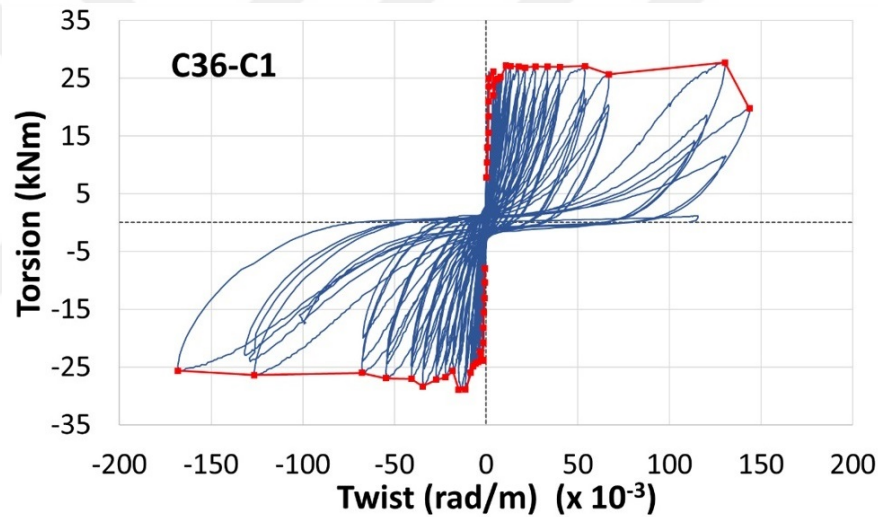


Figure 4.84. Skeleton curve of torque -twist hysteresis of C36-C1.

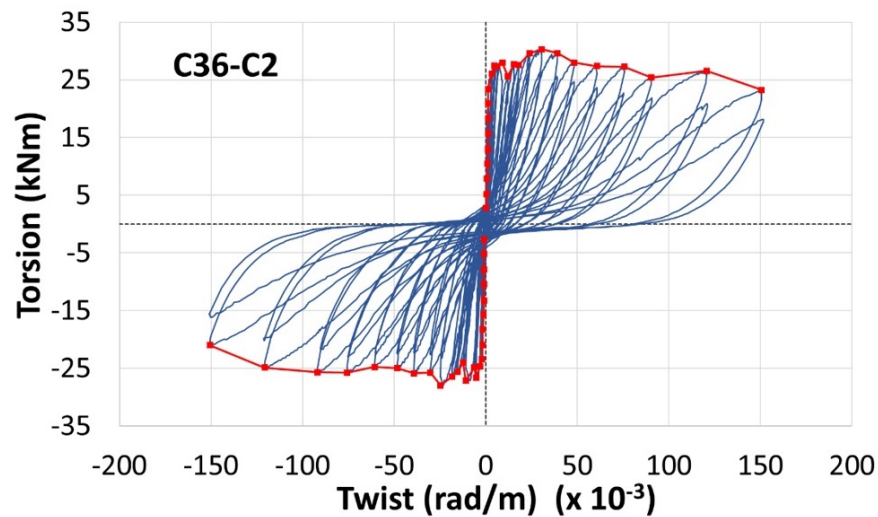


Figure 4.85. Skeleton curve of torque -twist hysteresis of C36-C2.

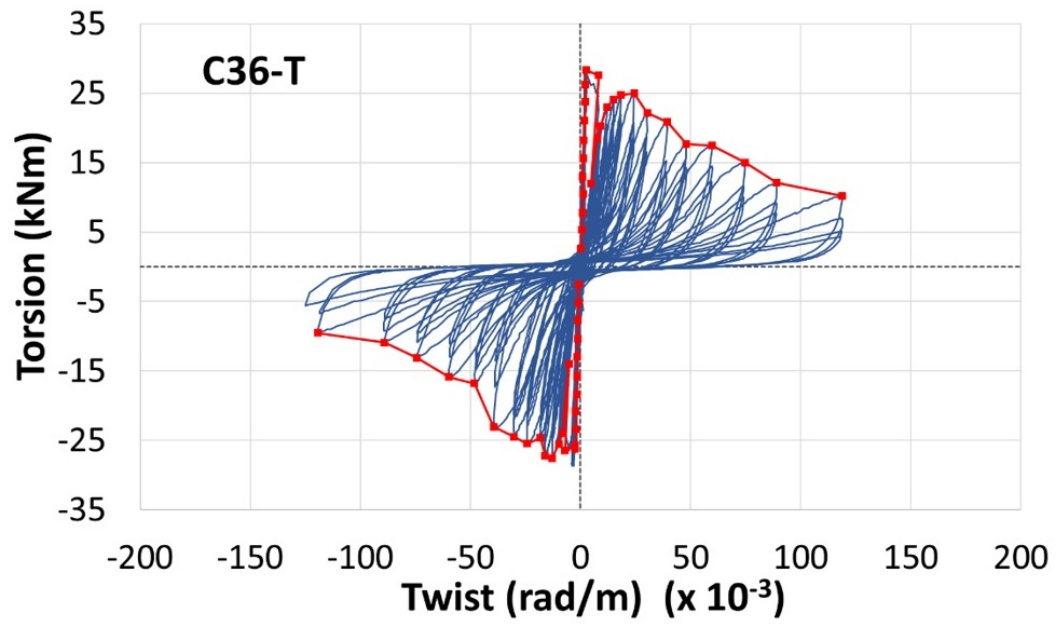


Figure 4.86. Skeleton curve of torque -twist hysteresis of C36-T.

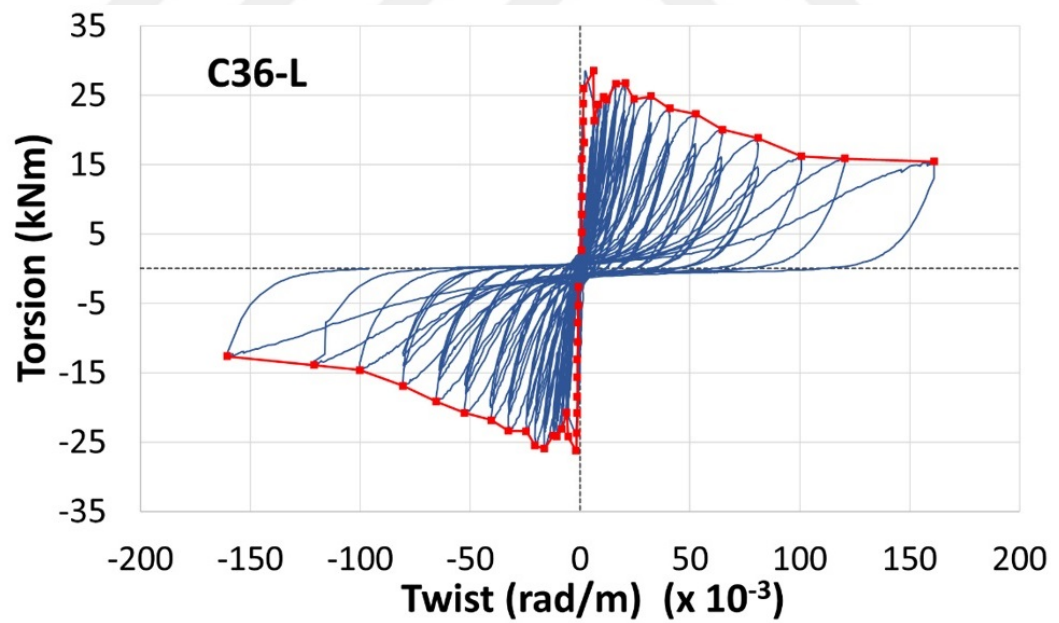


Figure 4.87. Skeleton curve of torque -twist hysteresis of C36-L.

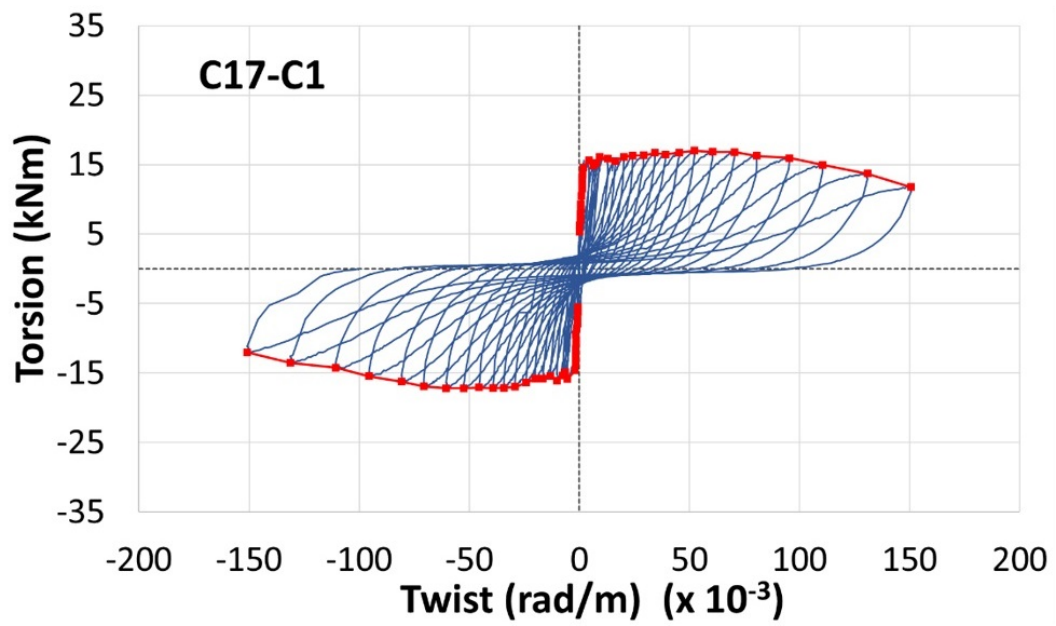


Figure 4.88. Skeleton curve of torque -twist hysteresis of C17-C1.

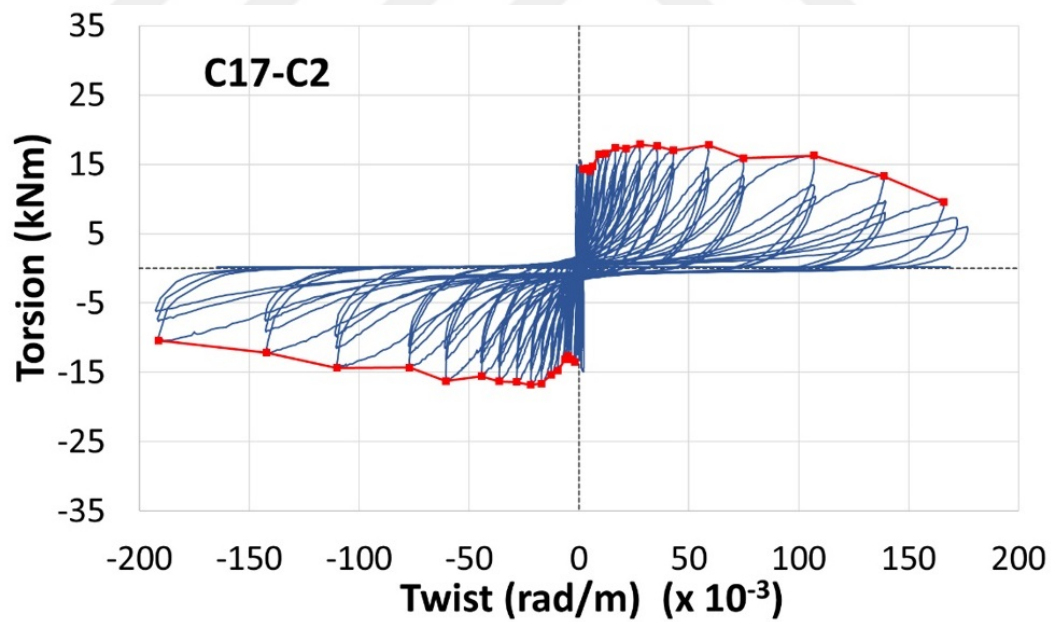


Figure 4.89. Skeleton curve of torque -twist hysteresis of C17-C2.

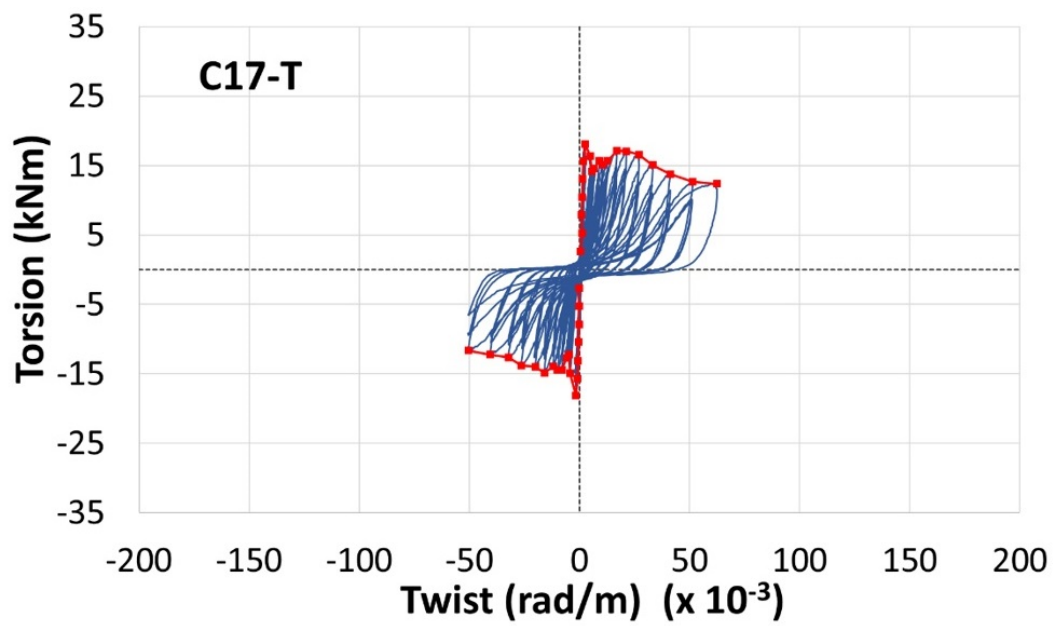


Figure 4.90. Skeleton curve of torque -twist hysteresis of C17-T.

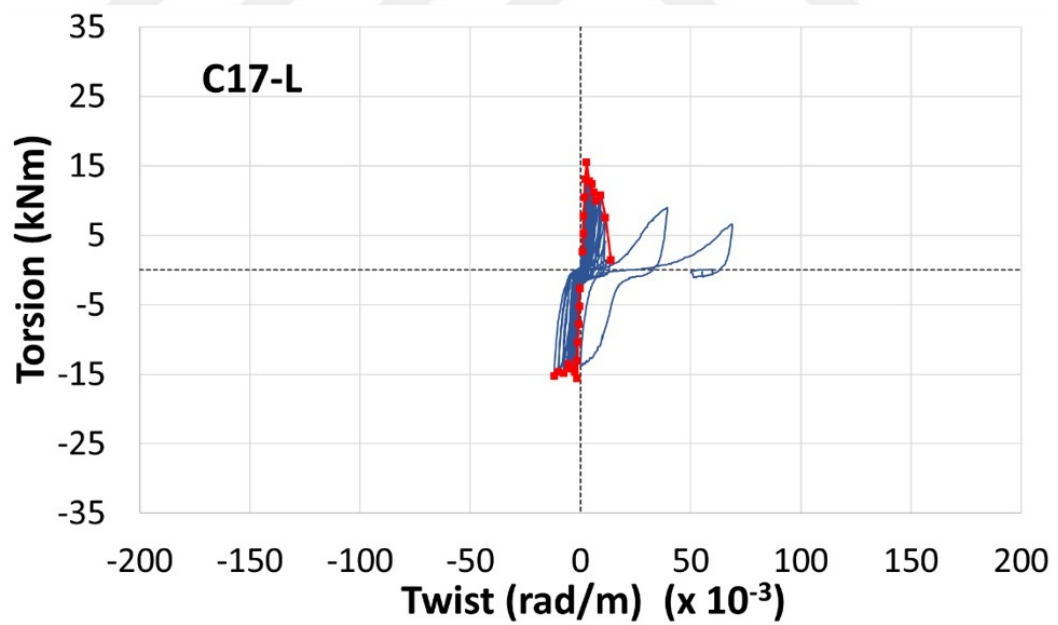


Figure 4.91. Skeleton curve of torque -twist hysteresis of C17-L.

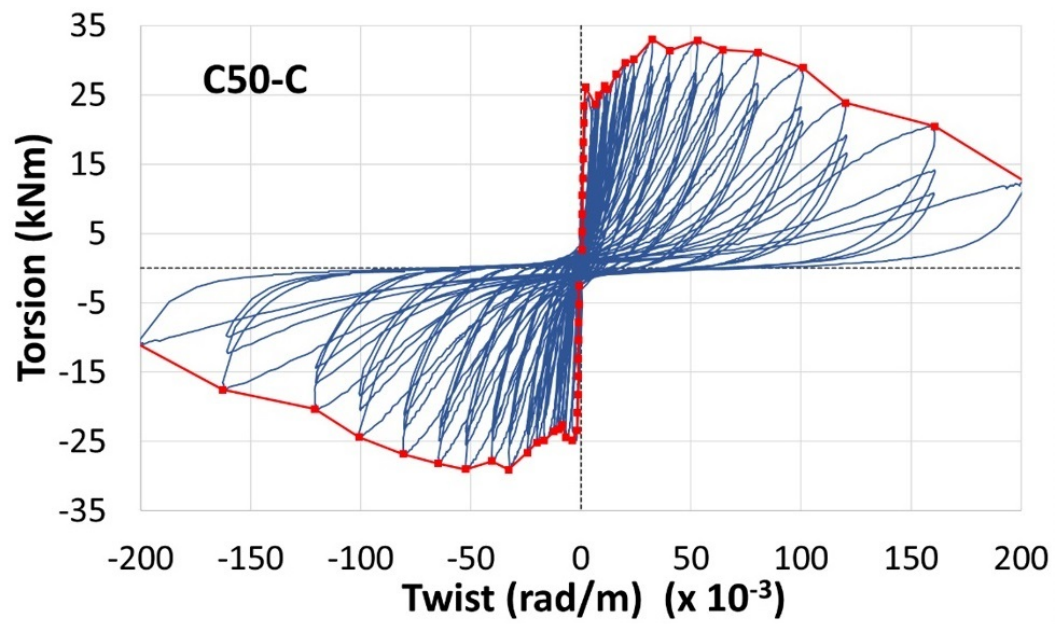


Figure 4.92. Skeleton curve of torque -twist hysteresis of C50-C.

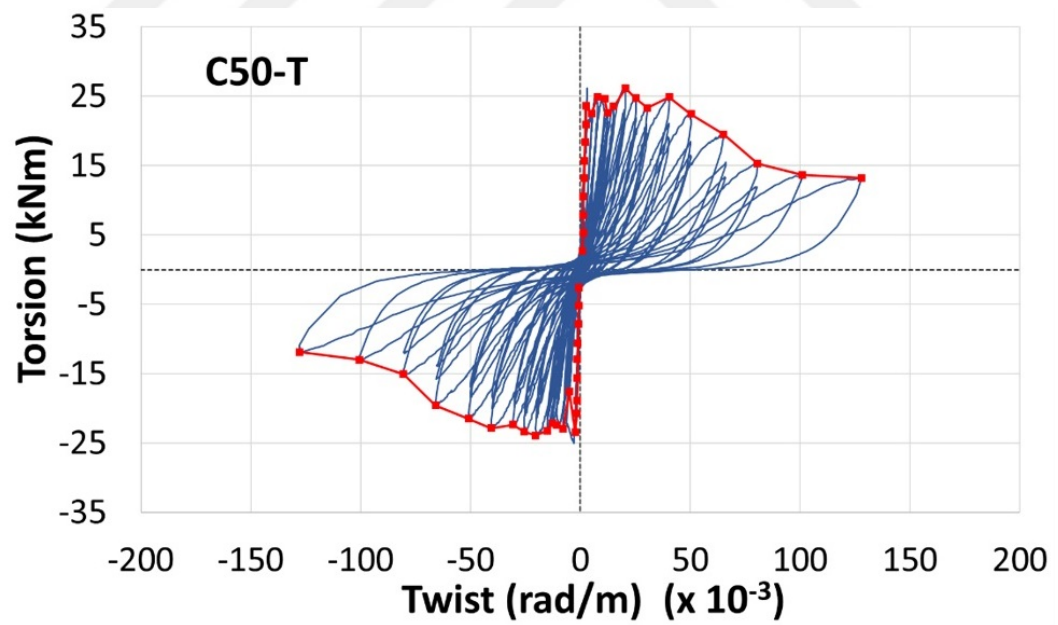


Figure 4.93. Skeleton curve of torque -twist hysteresis of C50-T.

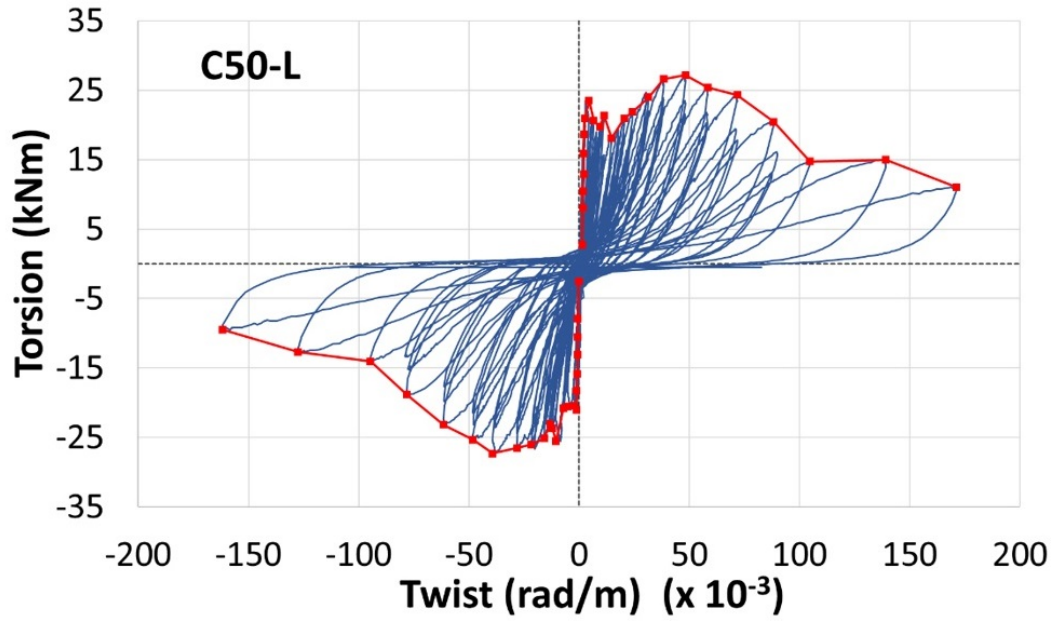


Figure 4.94. Skeleton curve of torque-twist hysteresis of C50-L.

The behavior is almost likely linear before cracking and members can be accepted in the elastic stage. Curves become nonlinear after the cracking and this means that the stiffness of the member decreases and at this level members in the elastic-plastic stage.

4.8.2. Damping Coefficient

The traditional approach to determining the equivalent viscous damping involves matching the energy consumption in the vibration hysteresis loop of real member to that of an equivalent viscous system. In the case of the real structure, the force-displacement relation is established through an experiment involving cyclic loading with displacement amplitude u_0 . This relation, which can be shaped as desired, is shown schematically in Figure 4.95. The dissipated energy in the real structure corresponds to E_D is obtained from enclosed area of the hysteresis loop. Equating the area and the dissipated energy obtained from viscous damping given by

$$4\pi\zeta_{eq}\frac{\omega}{\omega_n}E_{so} = E_D, \quad \text{or} \quad \zeta_{eq} = \frac{1}{4\pi}\frac{1}{\omega/\omega_n}\frac{E_D}{E_{so}}. \quad (4.32)$$

Here, $E_{so} = k_u^2/2$ gives the strain energy, and the stiffness k can be determined from the experiment.

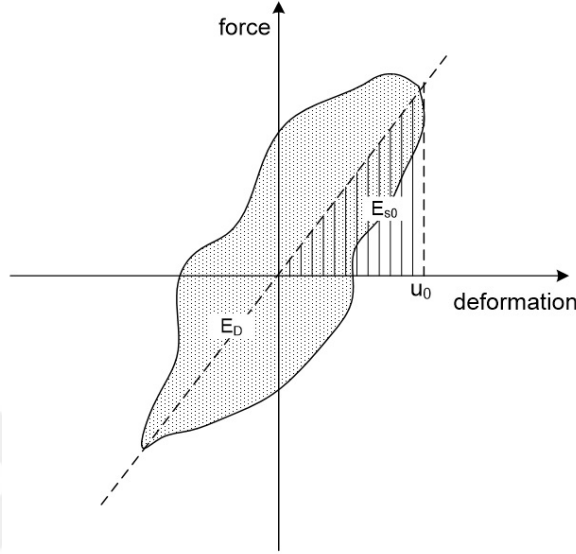


Figure 4.95. Dissipated energy in a hysteresis loop obtained from test.

The force-deformation curve of Figure 4.95 can be drawn by using the experimental data and hence, E_D conducted where the response of the system is most sensitive to damping ($\omega = \omega_n$). Thus, the damping coefficient will be

$$\zeta_{eq} = \frac{1}{4\pi} \frac{E_D}{E_{so}}. \quad (4.33)$$

It can be a satisfactory assumption but not be correct at any other exciting frequency that determining the damping ratio ζ_{eq} from a test at $\omega = \omega_n$ [91]. In 1984, a series of box columns were tested under the torsional moments by Jacobsen et. al. They evaluate the test results and calculate the equivalent viscous damping ratio from the dissipated energy to determine the relation between damping capacity and twisting angle. At the end of the study, the researchers observed that the damping ratio increased again before cycles at lower twist levels, and that after some cycles the damping ratio reached the lowest value for each twist level. The most important feature of the graphs is that the minimum values mentioned are the same irrespective of twist level and previous load history of the element [66]. In the experiments conducted in this dissertation, it was observed that while the energy dissipation value increased at

small rotation angles, the energy dissipation capacity remained almost constant as the rotation angle increased after the first crack occurred in the element.

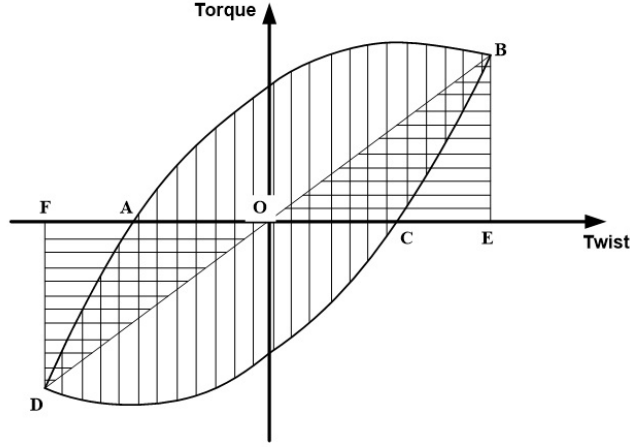


Figure 4.96. Definition of energy consumption.

Another damping ratio study was conducted by Wang et al [92]. The team examined the performance of recycled reinforced concrete beams with aggregates with and without fly ash under cyclic torsional moments. The area surrounded by the hysteresis loop indicates the seismic performance of members and the energy dissipation capacity of a beam under torsional loading. As the size of the enclosed area increases, the energy dissipation capacity of the element increases, and accordingly, its seismic behavior becomes better. Often, to measure the energy dissipation capacity of structural elements, the equivalent viscous damping coefficient h_e , is used, as given in Equation (4.33) and Figure 4.96. As a result of both the study done by Wang et al and the experiments conducted in this thesis, it was seen that the damping ratio of normal beams was 0.1 on average. The coefficient equivalent viscous damping, h_e can be calculated from

$$h_e = \frac{1}{2\pi} \frac{S_{ABC+CDA}}{S_{OBE+ODF}} \quad (4.34)$$

From Figure 4.96, $S(ABC+CDA)$ in the areas of regions ABC and CDA, describing the energy dissipation capacity of the member during the one cyclic torsional loading. In the same manner, $S(OBE+ODF)$ defines the areas of regions OBE and ODF, and states the energy dissipation capacity of the beam under torsional loading at the elastic level. As a result of their study, the researchers observed that h_e , value increased depending

on the cycle time just before the peak load and immediately after the beam yielded, and the viscous damping coefficient value increased faster accordingly [92].

In the light of given information above, the equivalent viscous damping coefficient was calculated for each member, and graphs were drawn according to the results as shown in Figures 4.97- 4.107.

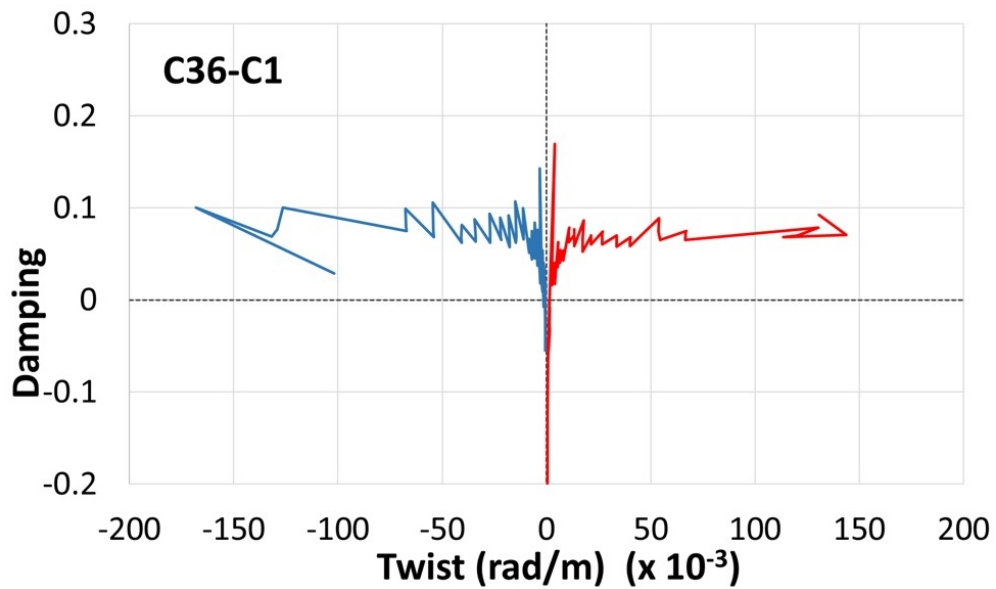


Figure 4.97. Viscous damping vs. twist for C36-C1.

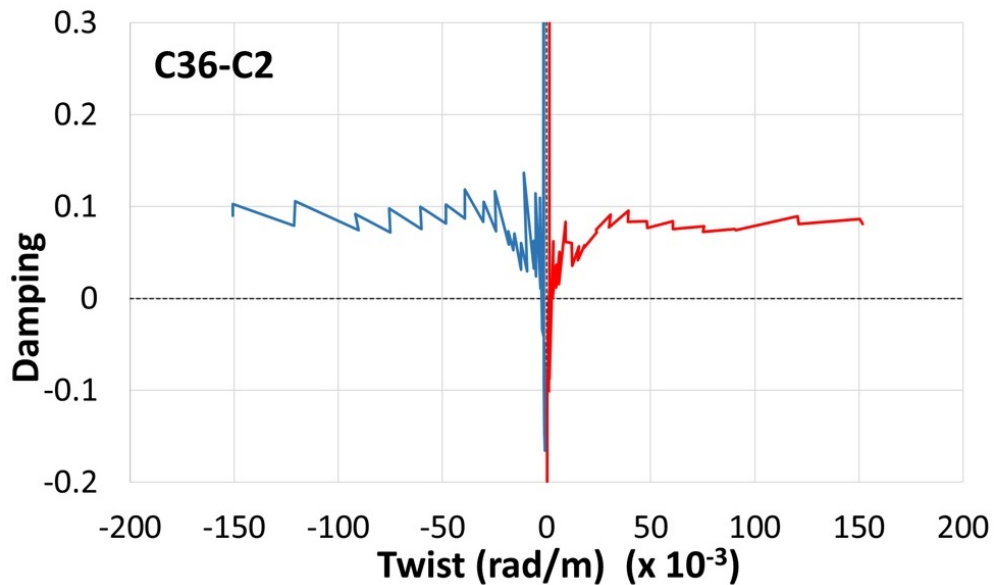


Figure 4.98. Viscous damping vs. twist for C36-C2.

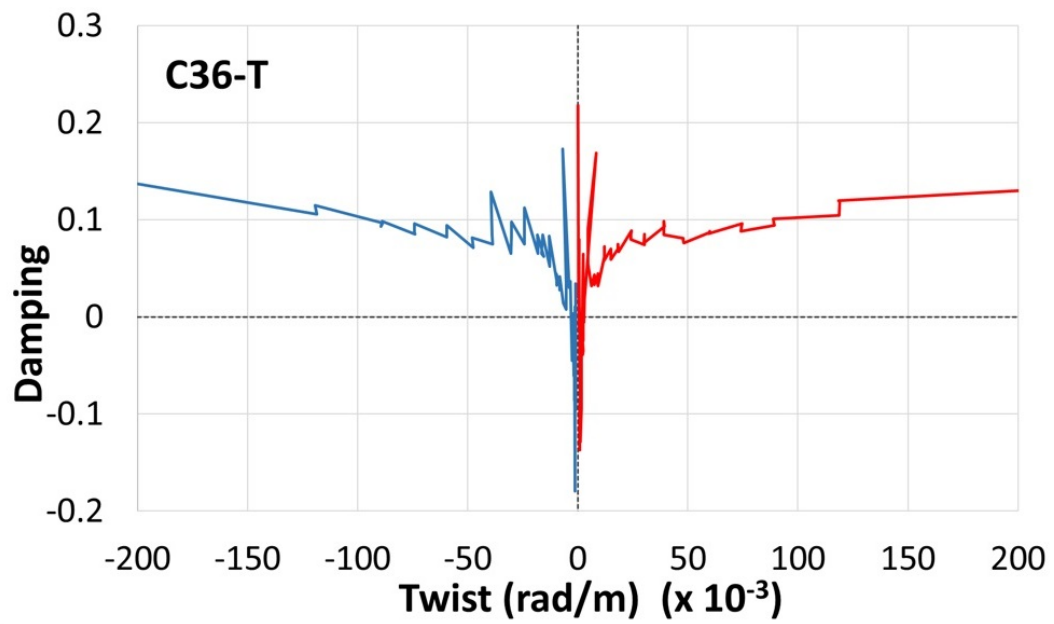


Figure 4.99. Viscous damping vs. twist for C36-T.

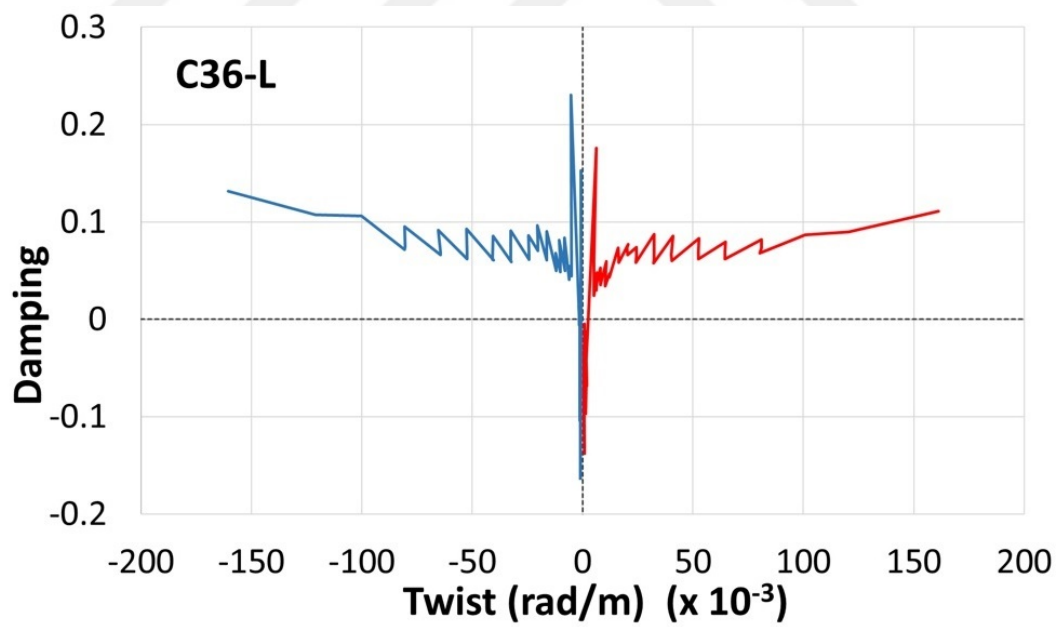


Figure 4.100. Viscous damping vs. twist for C36-L.

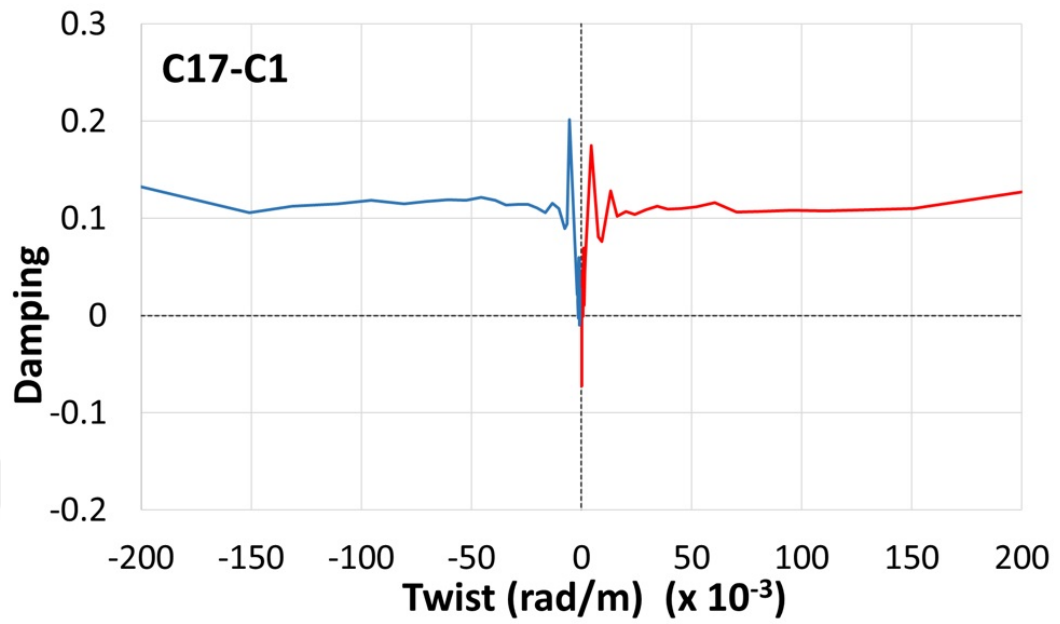


Figure 4.101. Viscous damping vs. twist for C17-C1.

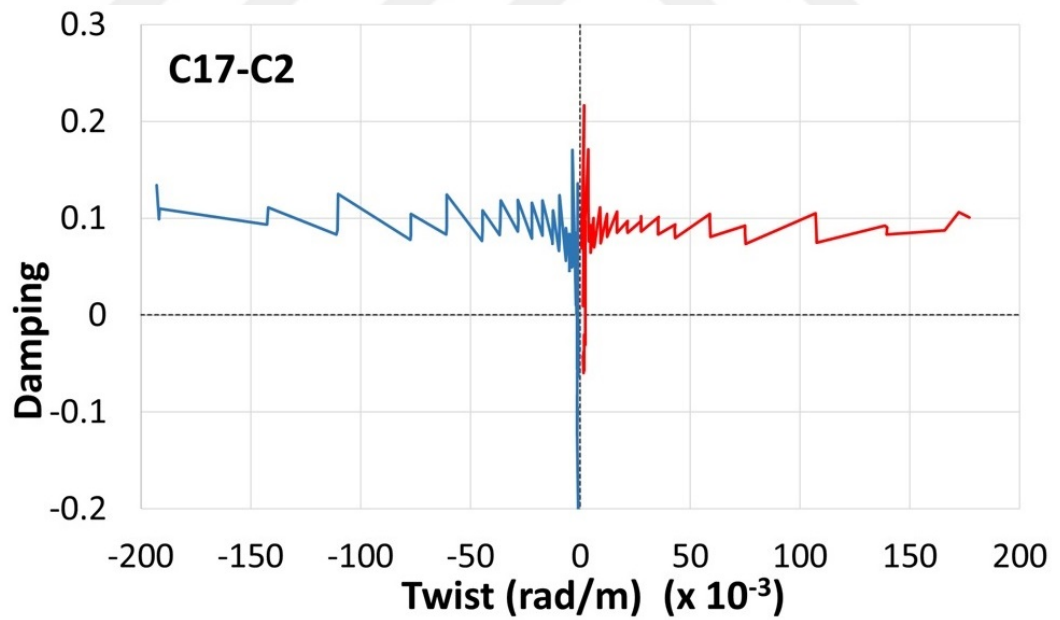


Figure 4.102. Viscous damping vs. twist for C17-C2.

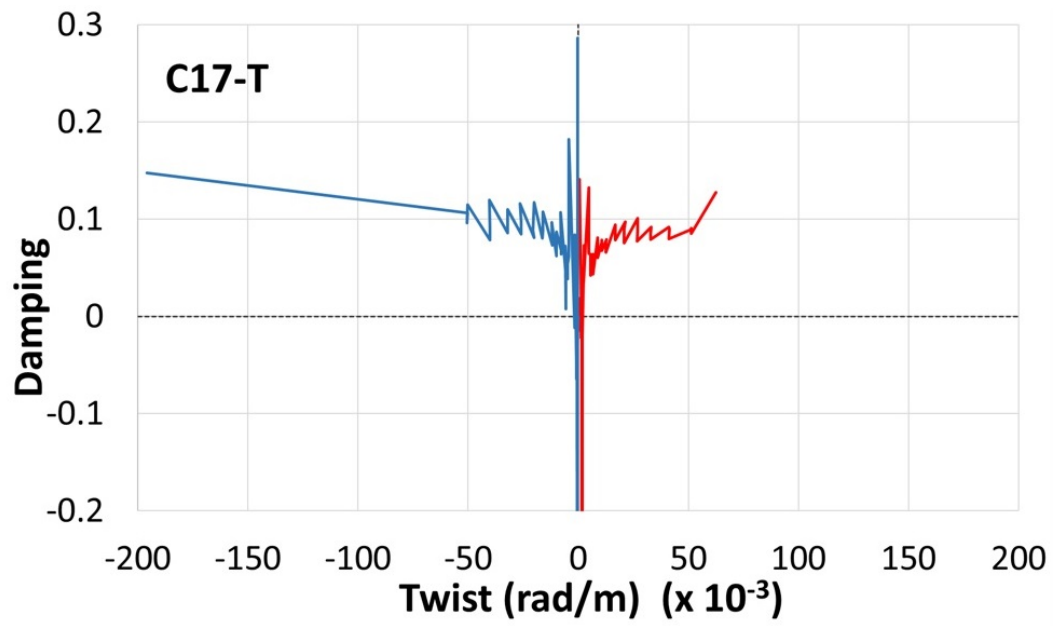


Figure 4.103. Viscous damping vs. twist for C17-T.

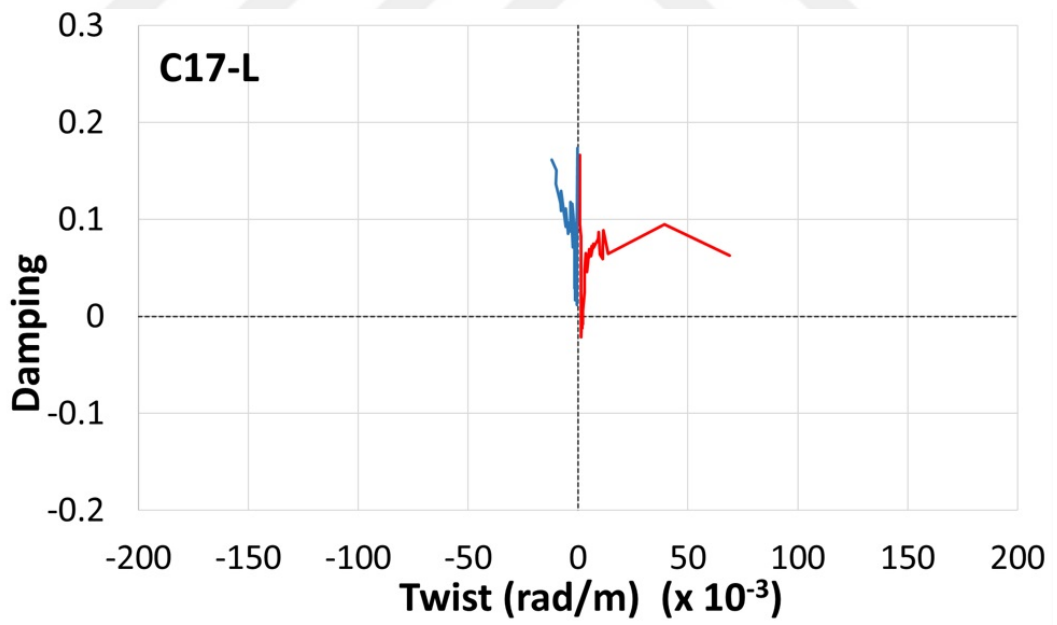


Figure 4.104. Viscous damping vs. twist for C17-L.

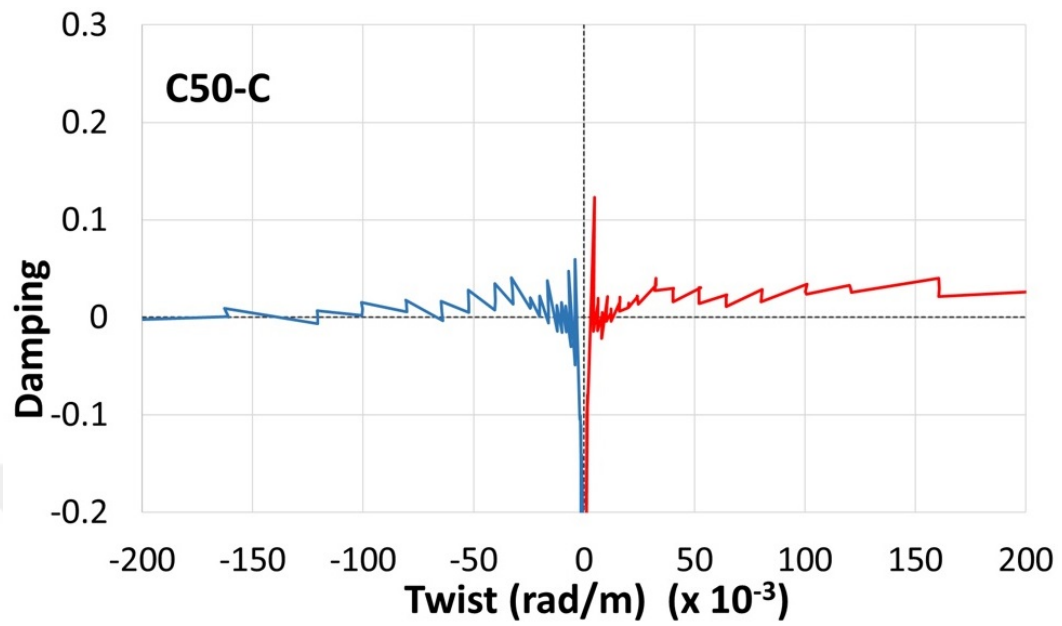


Figure 4.105. Viscous damping vs. twist for C50-C.

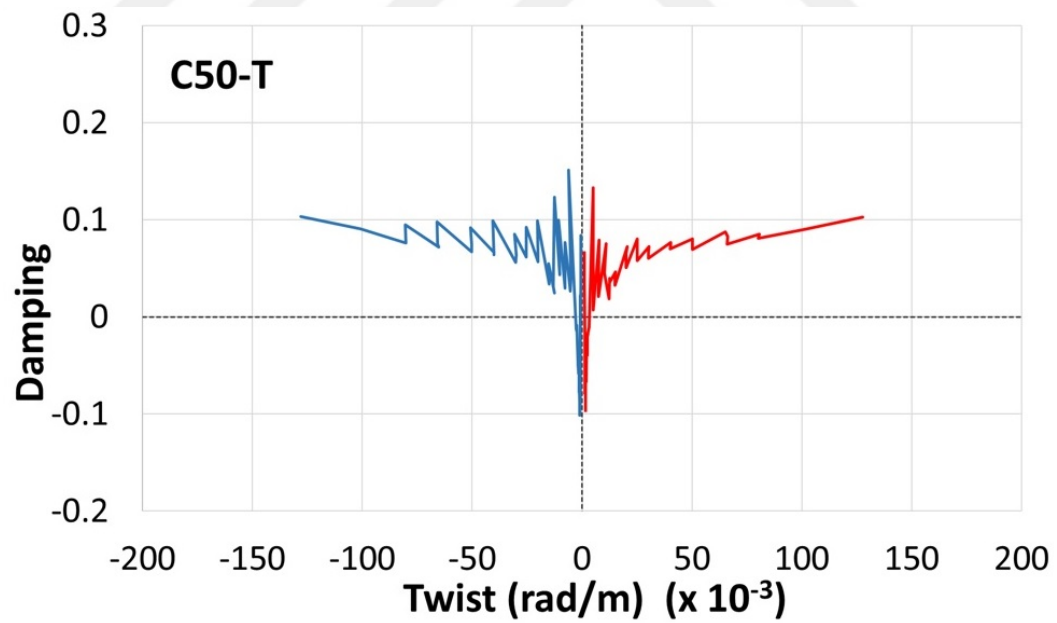


Figure 4.106. Viscous damping vs. twist for C50-T.

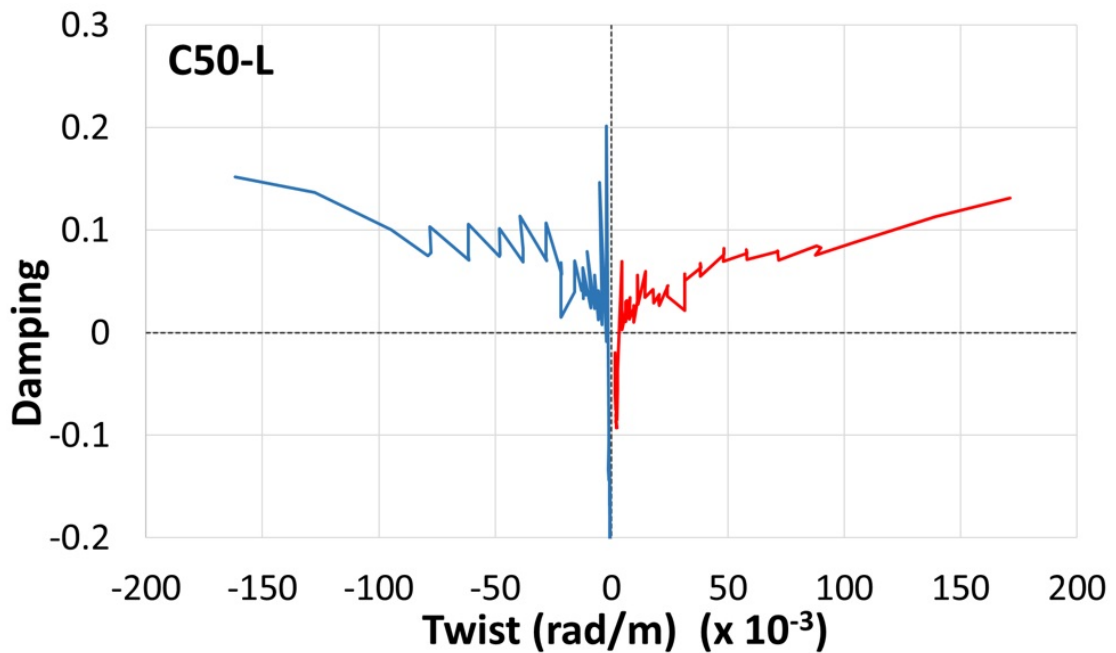


Figure 4.107. Viscous damping vs. twist for C50-L.

The equivalent viscous damping coefficient h_e is proportional to the energy dissipation capacity of reinforced concrete members. Higher h_e values mean better energy dissipation capacity. Figures 4.97 - 4.107 give the viscous damping capacity vs. twist graphs of the tested specimens.

Also, it is observed during the tests, that the viscous damping coefficient increased before the yield twist value but after the yielding of the beam, the value h_e increased much more rapidly. At the post-cracking stage, internal forces are redistributed into the members, and the internal shear forces are resisted by the reinforcement inside the cross-section.

4.8.3. Energy Dissipation Capacity

One of the most effective parameters in determining the behavior of reinforced concrete structures under seismic loads is the structure's ability to adequately dissipate energy. The energy dissipation of a reinforced concrete member during the cyclic loading takes place because the member behaves inelastic. Since a reinforced concrete

element consists of concrete and steel, the energy loss of the element is the total of the energy dissipated by the concrete and steel.

The energy dissipation capacity calculation depends on empirical formulas that do not give the real value, or the value can be defined with a complex nonlinear analysis. Simply put, the energy dissipation capacity, E_D , of a reinforced concrete beam under torsional loads can be approximately calculated (Equation (4.32)) depending on the area enclosed by each hysteresis loop (Figures 4.95), and the result also gives information about the seismic performance of the member. Namely, a larger loop area means higher energy dissipation capacity and better seismic behavior.

The yielding of reinforcement and cracking of concrete influence the energy dissipation capacity of members. In Figures 4.108 - 4.118, the cumulative energy dissipation capacities were given.

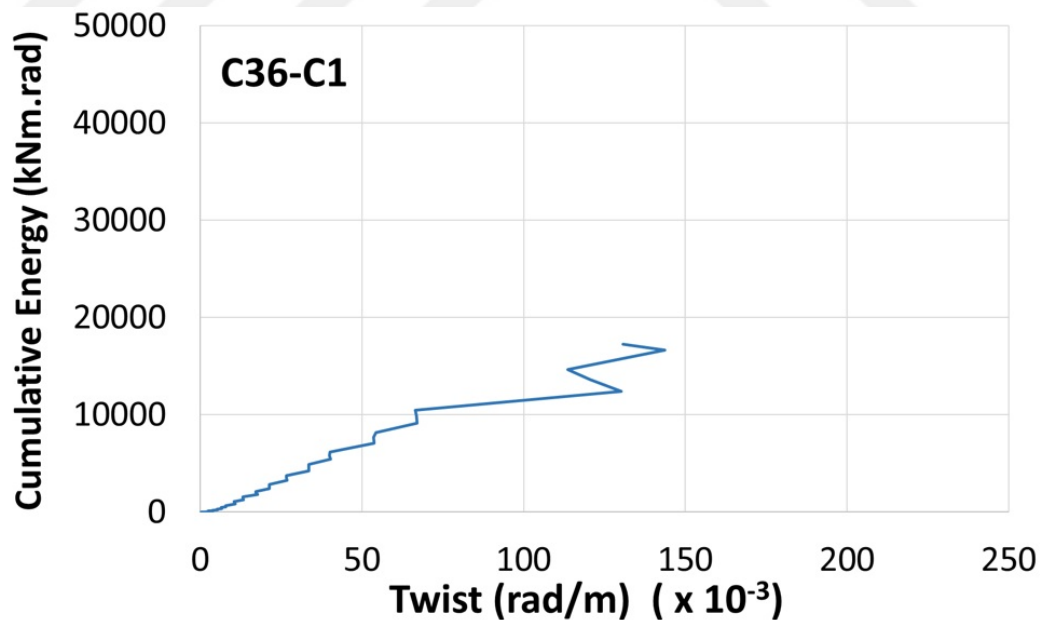


Figure 4.108. Cumulative energy vs. twist curve for C36-C1.

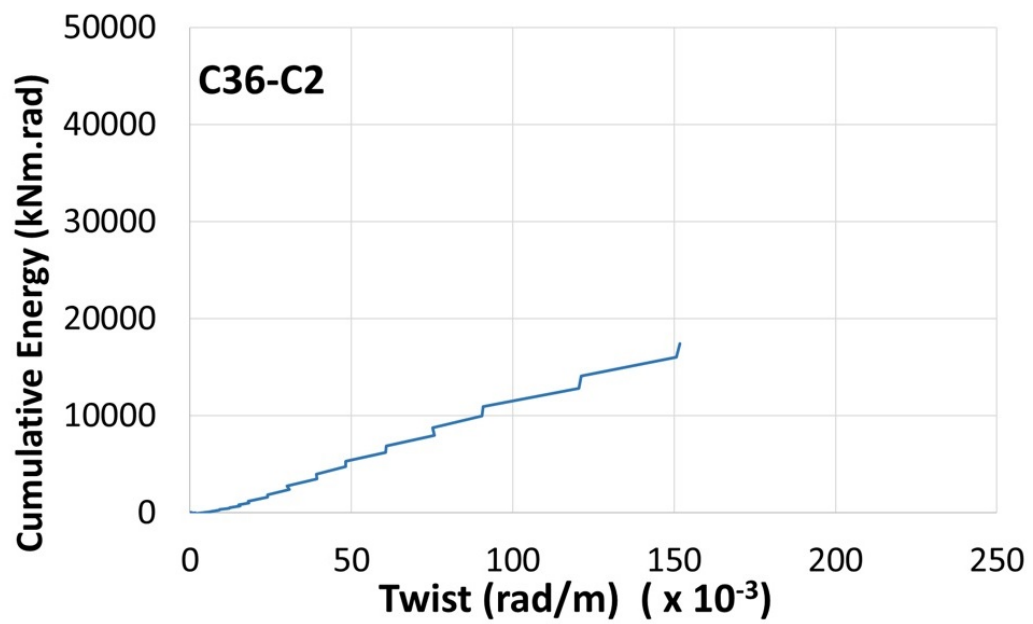


Figure 4.109. Cumulative energy vs. twist curve for C36-C2.

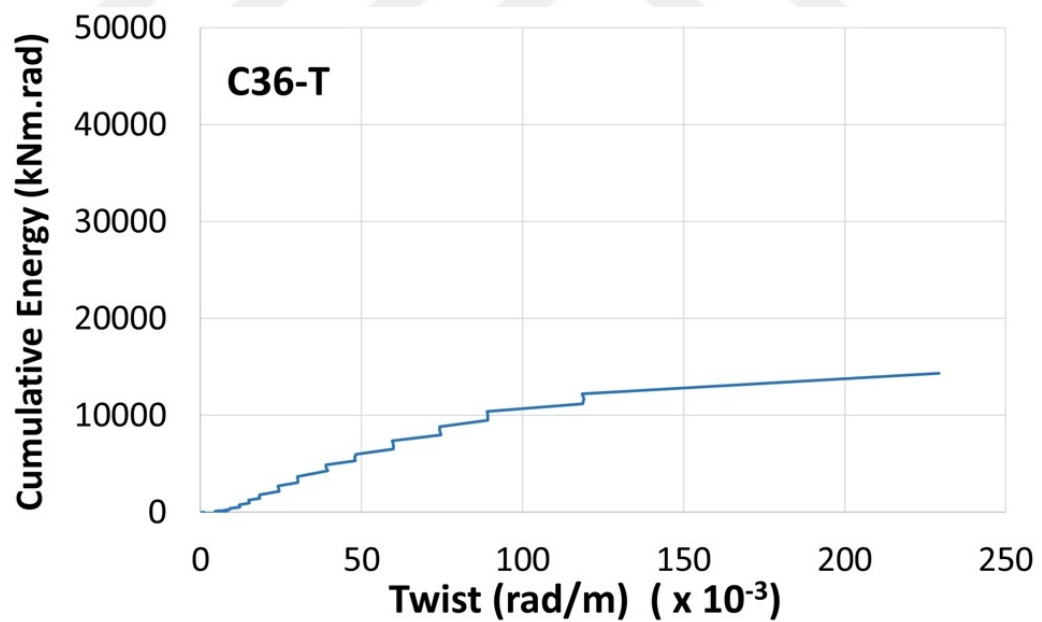


Figure 4.110. Cumulative energy vs. twist curve for C36-T.

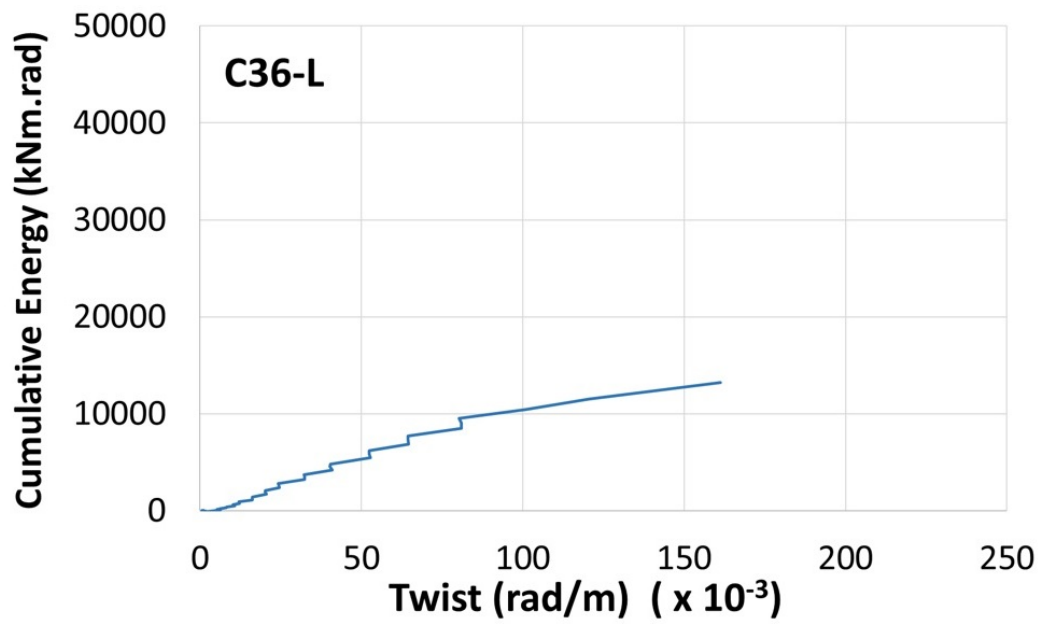


Figure 4.111. Cumulative energy vs. twist curve for C36-L.

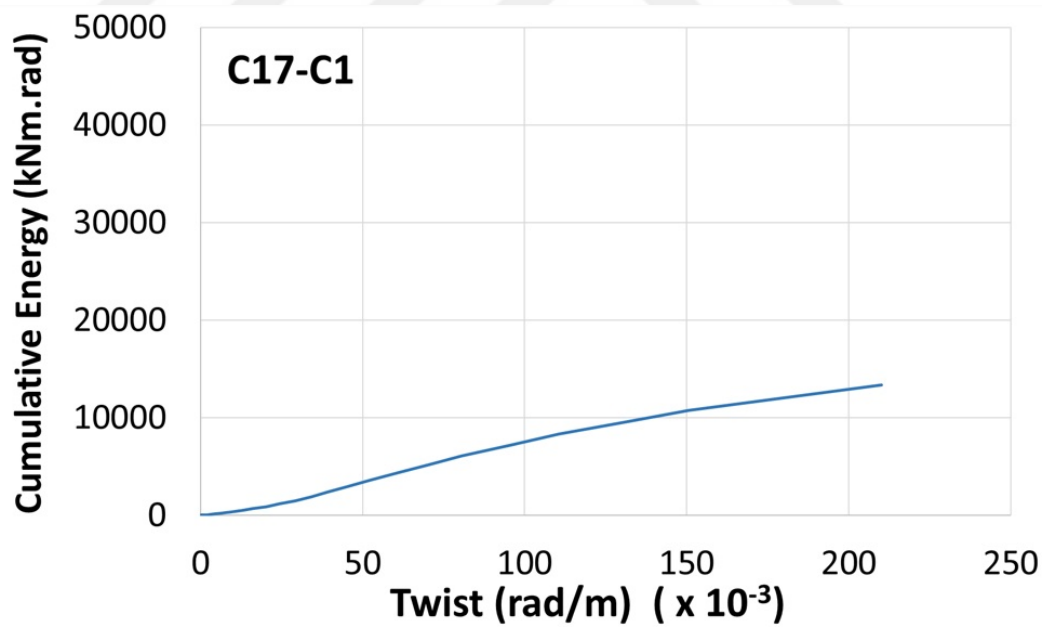


Figure 4.112. Cumulative energy vs. twist curve for C17-C1.

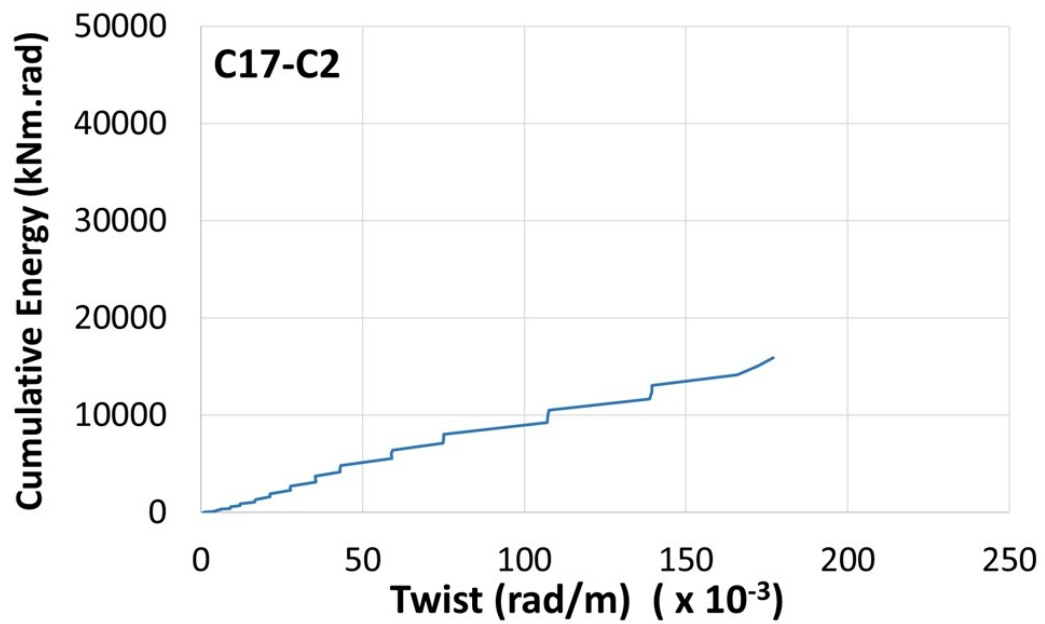


Figure 4.113. Cumulative energy vs. twist curve for C17-C2.

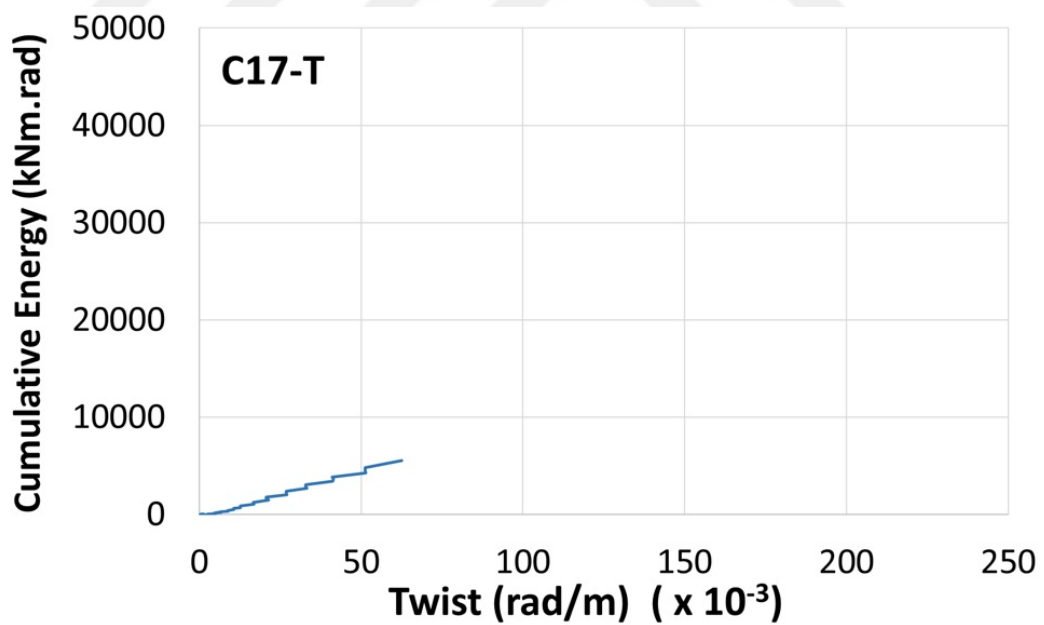


Figure 4.114. Cumulative energy vs. twist curve for C17-T.

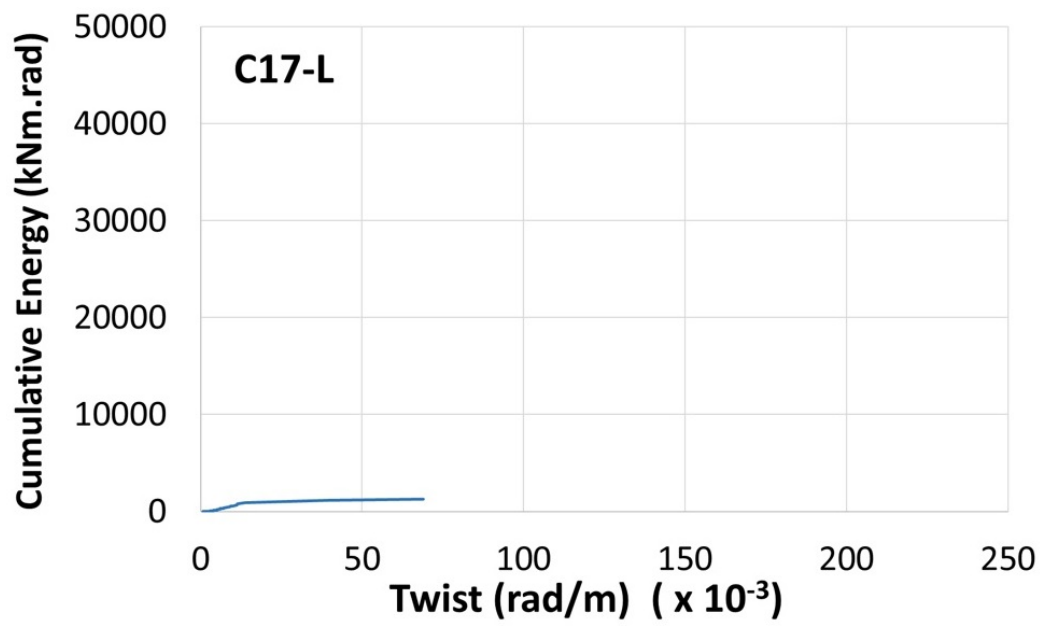


Figure 4.115. Cumulative energy vs. twist curve for C17-L.

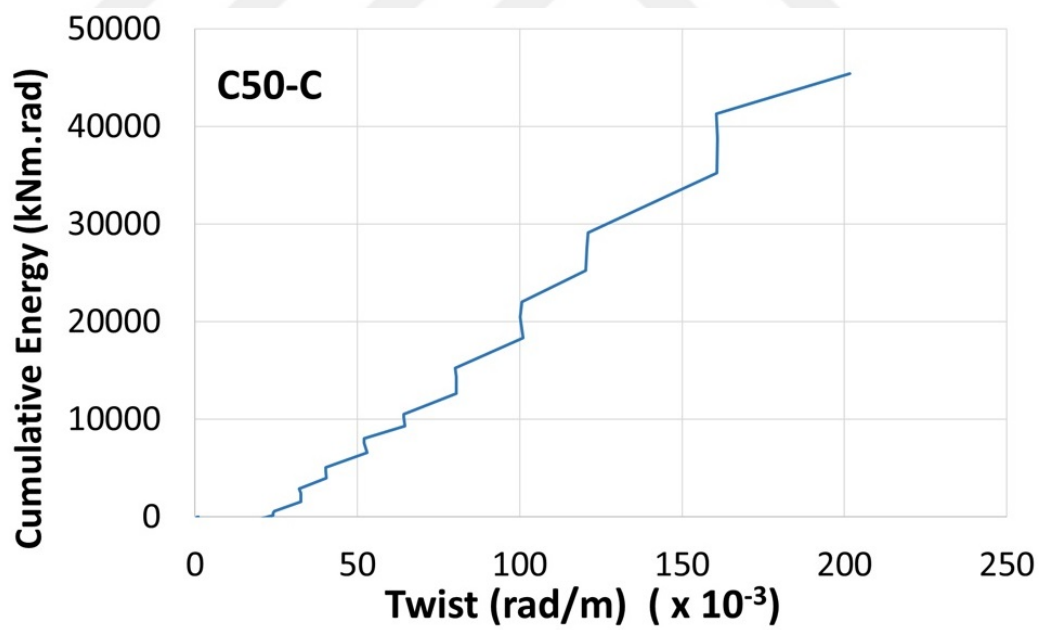


Figure 4.116. Cumulative energy vs. twist curve for C50-C.

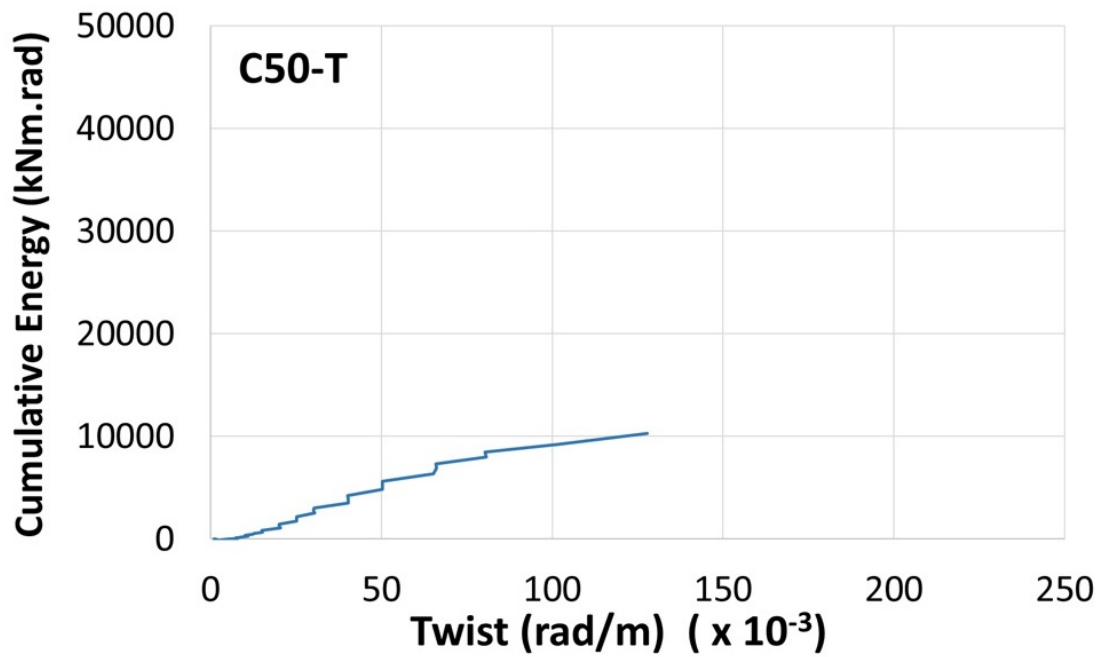


Figure 4.117. Cumulative energy vs. twist curve for C50-T.

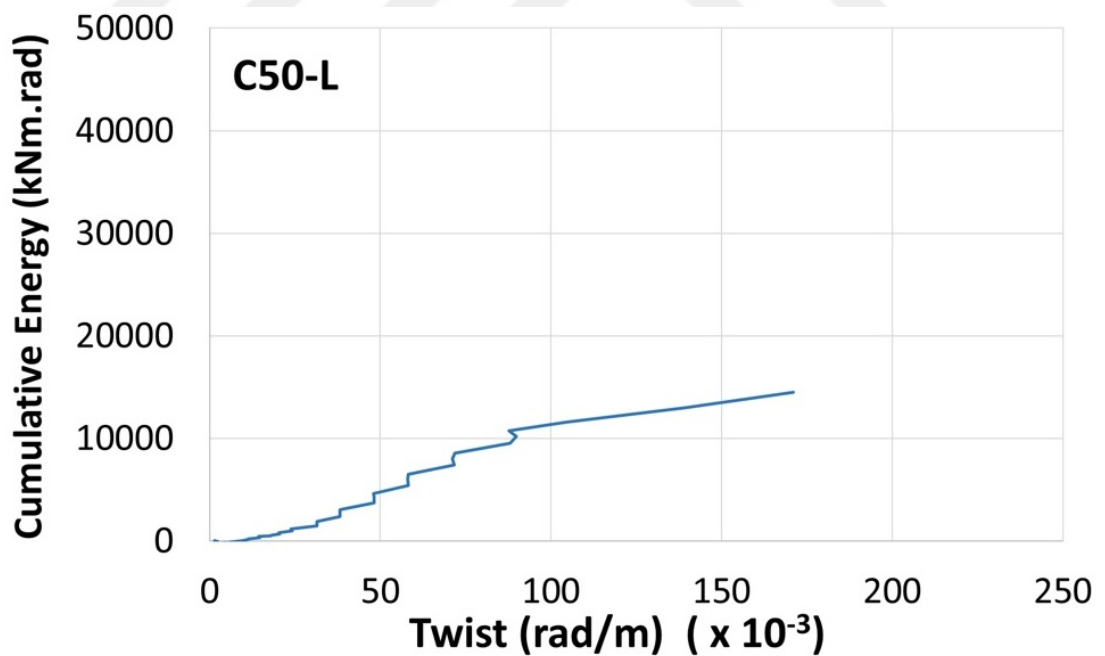


Figure 4.118. Cumulative energy vs. twist curve for C50-L.

4.8.4. Stiffness Degradation

After the yielding of reinforcement, the stiffness of hysteresis cycles decreases because of the new crack formation in the beam. The secant stiffness which is the

slope of hysteresis loops measured from origin to the peak point of each cycle was used to evaluate the stiffness degradation as indicated in Figure 4.119. This relationship between torque-twist can be determined from

$$G_j = \frac{\sum_{i=1}^n |T_{ij}|}{\sum_{i=1}^n |\theta_{ij}|}, \quad (4.35)$$

where, G_j is the stiffness of the member at the j^{th} level of cyclic torsional loading. T_{ij} is the torsional moment of the i times cycle at the j^{th} level and the θ_{ij} is the torsional curvature of the i times cycle at the j^{th} level of loading.

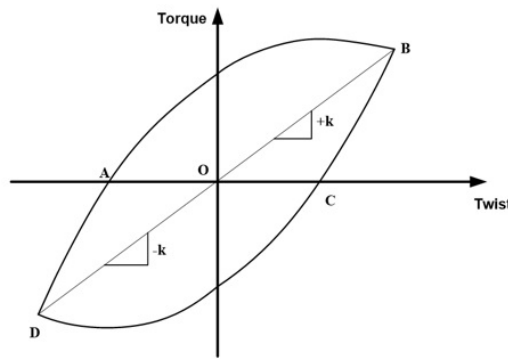


Figure 4.119. Stiffness calculation.

In accordance with the rule for torsional stiffness, curves of all specimens were drawn for each tested member and given in Figure 4.120 - 4.130.

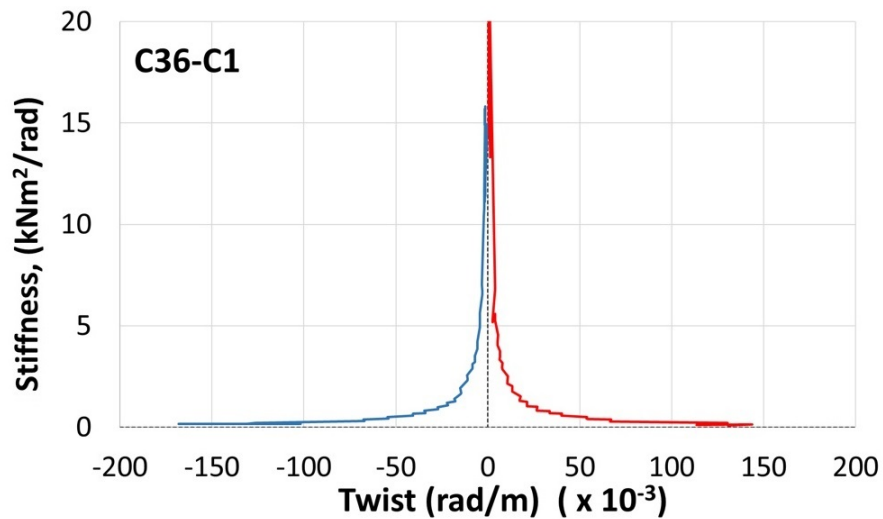


Figure 4.120. Torsional stiffness vs. twist curve for C36-C1.

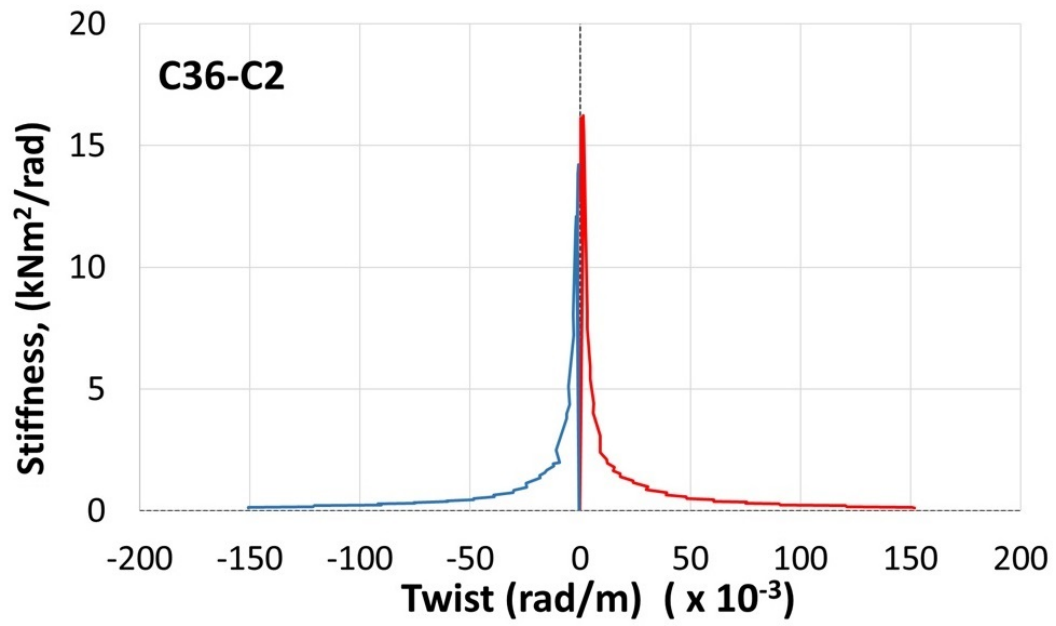


Figure 4.121. Torsional stiffness vs. twist curve for C36-C2.

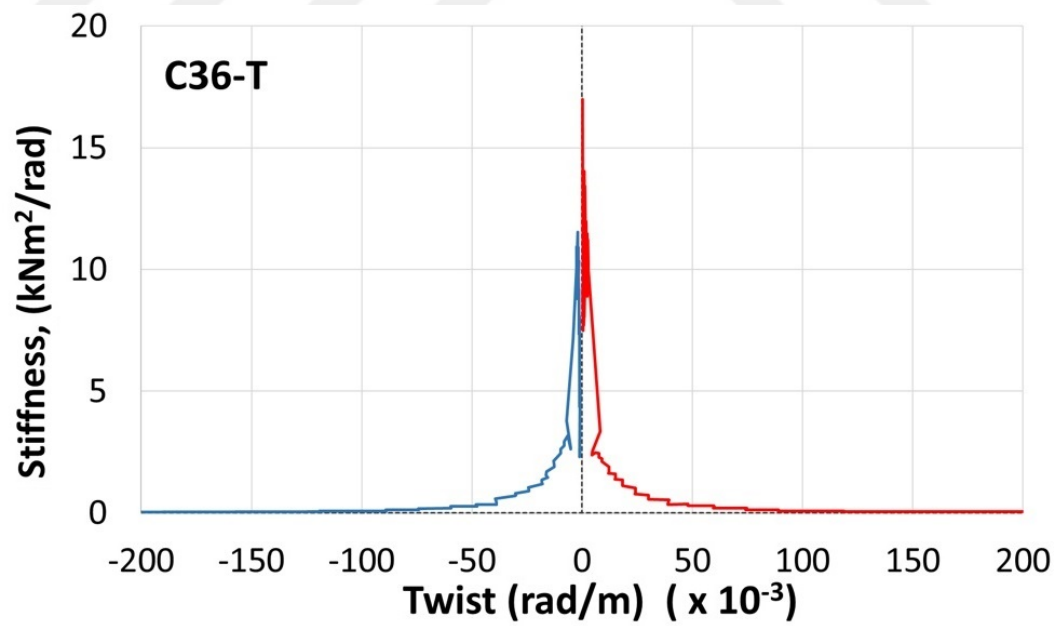


Figure 4.122. Torsional stiffness vs. twist curve for C36-T.

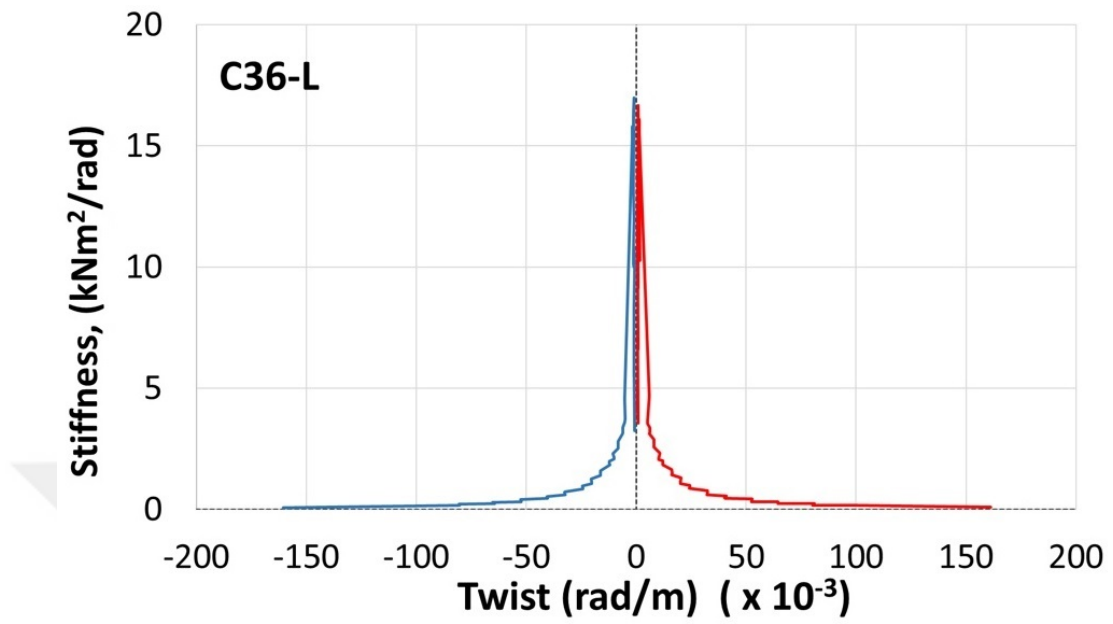


Figure 4.123. Torsional stiffness vs. twist curve for C36-L.

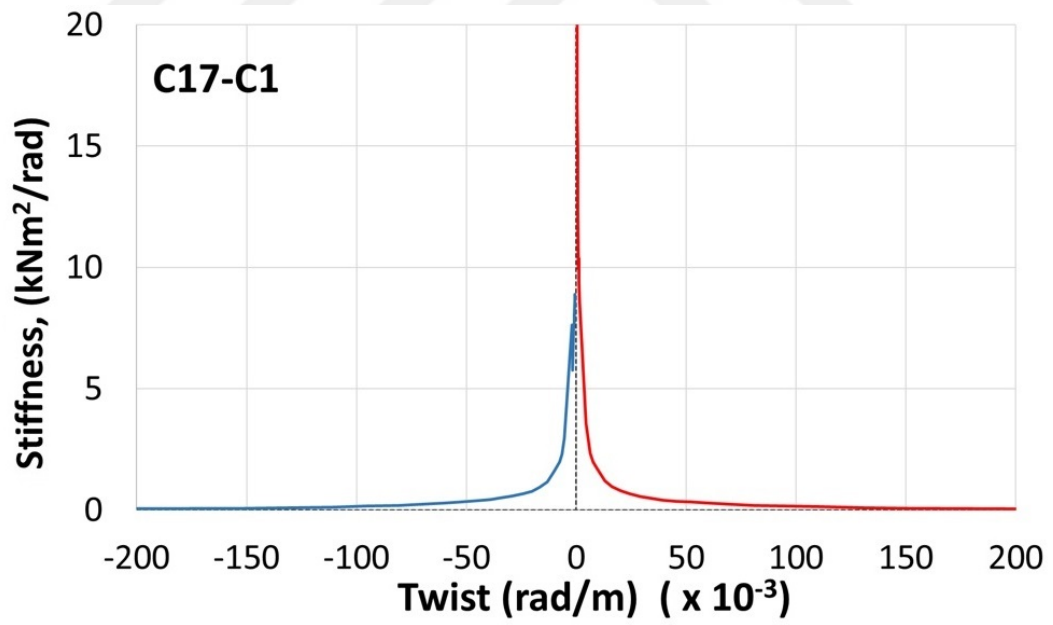


Figure 4.124. Torsional stiffness vs. twist curve for C17-C1.

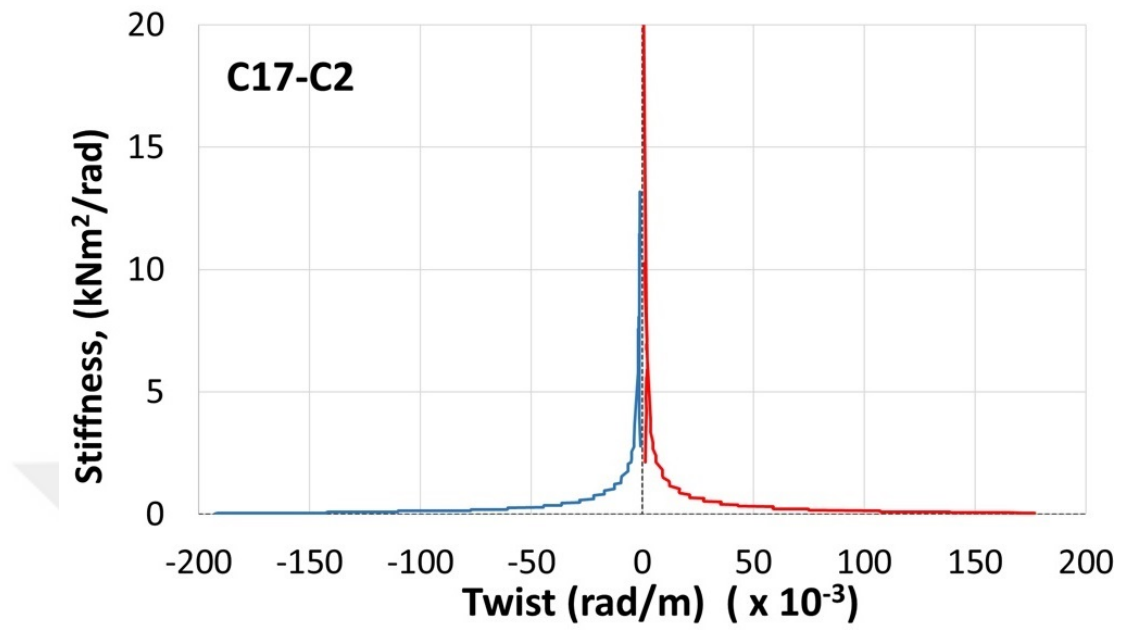


Figure 4.125. Torsional stiffness vs. twist curve for C17-C2.

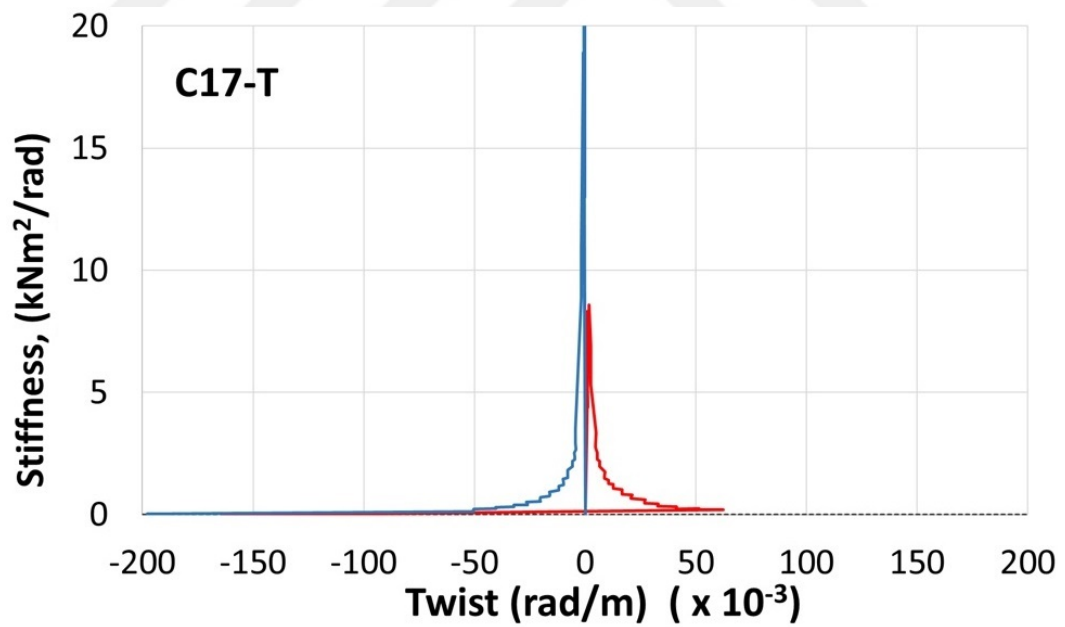


Figure 4.126. Torsional stiffness vs. twist curve for C17-T.

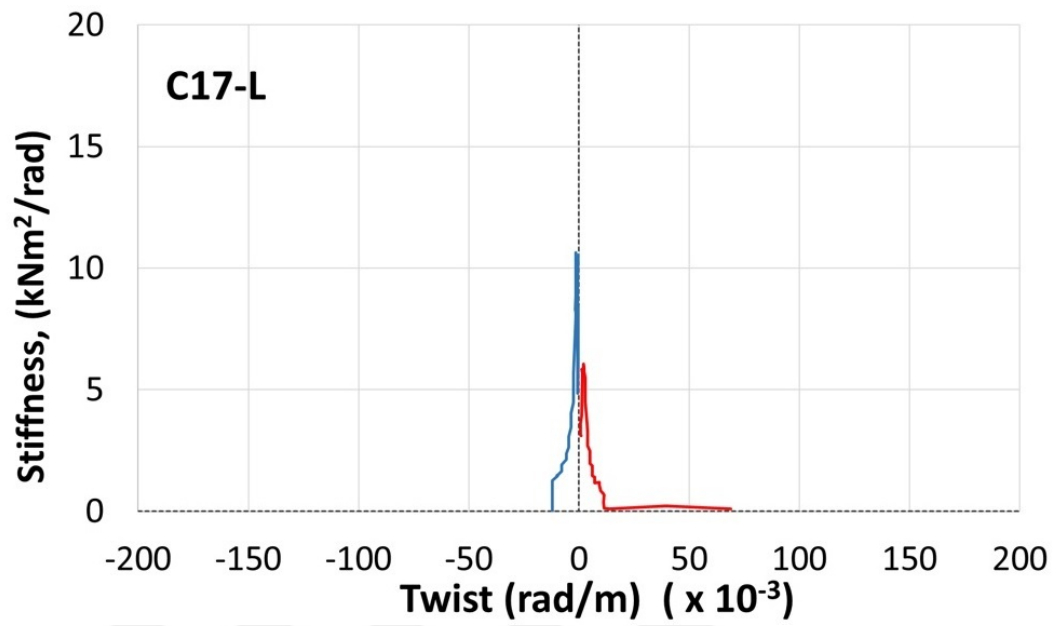


Figure 4.127. Torsional stiffness vs. twist curve for C17-L.

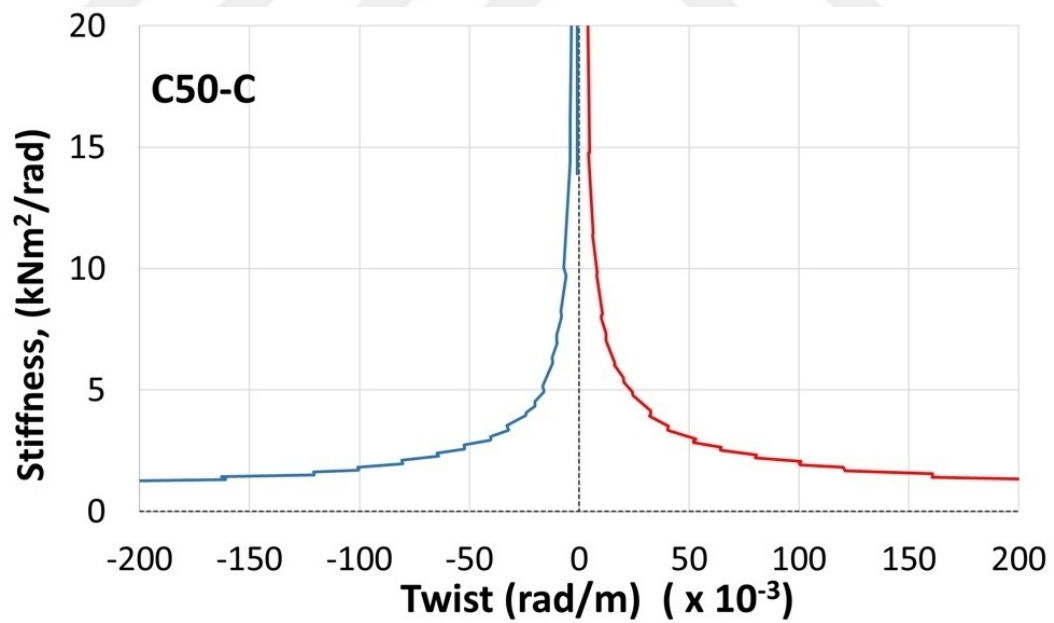


Figure 4.128. Torsional stiffness vs. twist curve for C50-C.

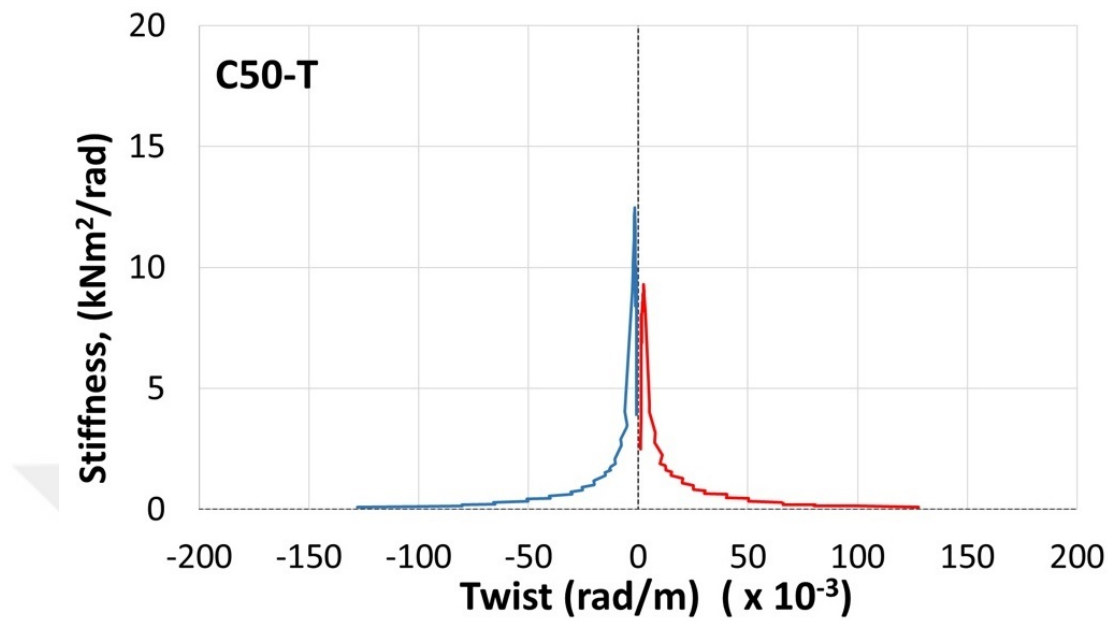


Figure 4.129. Torsional stiffness vs. twist curve for C50-T.

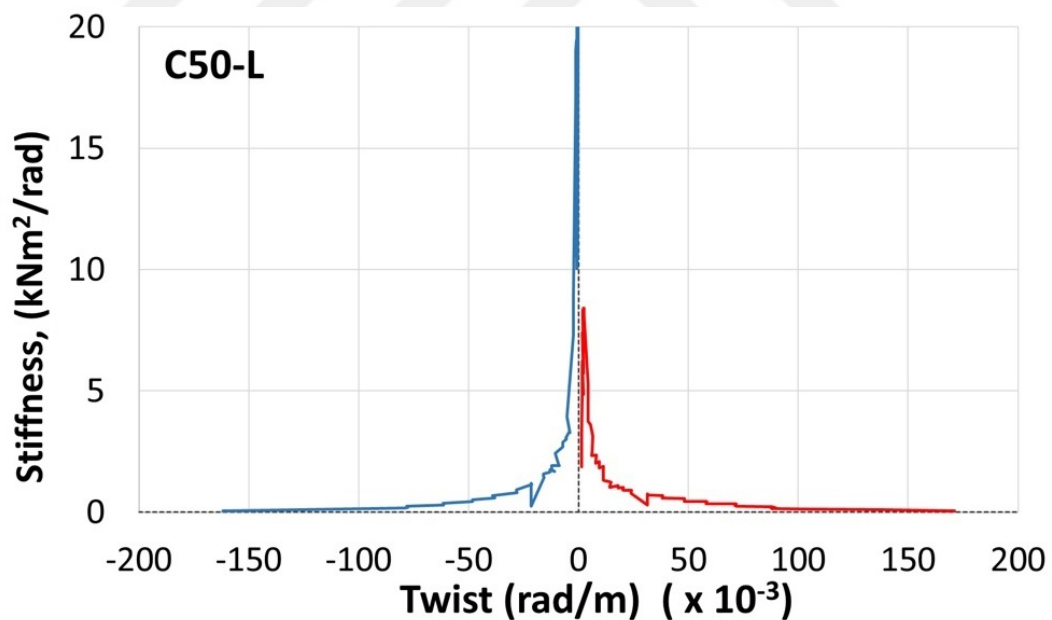


Figure 4.130. Torsional stiffness vs. twist curve for C50-L.

4.9. Comparison of Test Results

C36-C1, C36-C2, C17-C1, C17-C2, and C50-C are the control specimens of each different concrete compressive strength group. Figure 4.131 was drawn to compare the backbone curves of all control specimens. It can be observed from the curve, C36-C1

has a better bearing capacity due to higher compressive strength. The reason for the difference between the C17-C1 and C17-C2 is the application of the loading pattern, but the backbone curves are similar. One cycle for each twist level was applied for C17-C1 whereas three cycles for each twist level were applied for C17-C2. The behavior of the specimen with high concrete strength is more brittle than the Specimen C36 group.

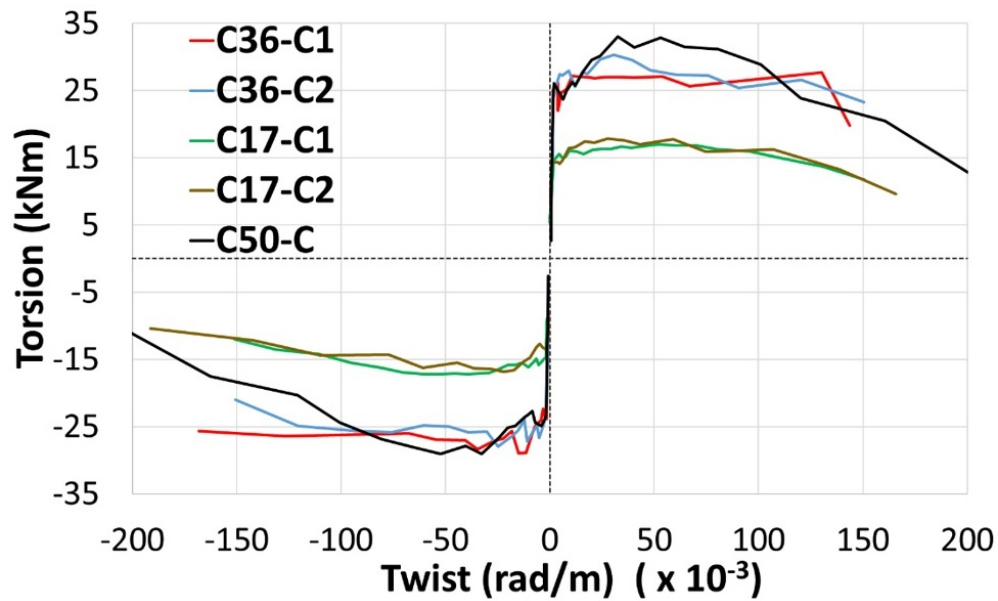


Figure 4.131. Comparison of backbone curves for control specimens

Figure 4.132 compares the backbone curves of all specimens inside the 36 MPa concrete compressive strength group. C36-C1 and C36-C2 are the control specimens, C36-T has inadequate transverse reinforcement and C36-L has insufficient longitudinal reinforcement. Up to the yielding, all specimens behave nearly similarly in the elastic stage.

Less bearing capacity was observed after the peak torsional moment point when compared to the other two specimens. In addition, the curve proves that transverse reinforcement is more effective than longitudinal reinforcement for better ductility.

Test results of the weak concrete compressive strength group are compared with the curve shown in Figure 4.133. C17-C1 and C17-C2 were the control specimens with similar backbone curves because they had the same specifications except for the

application of the loading pattern during the test. C17-T has the worst bearing capacity and ductility according to other specimens because a failure occurred outside of the test region and it was not possible to take enough measurements.

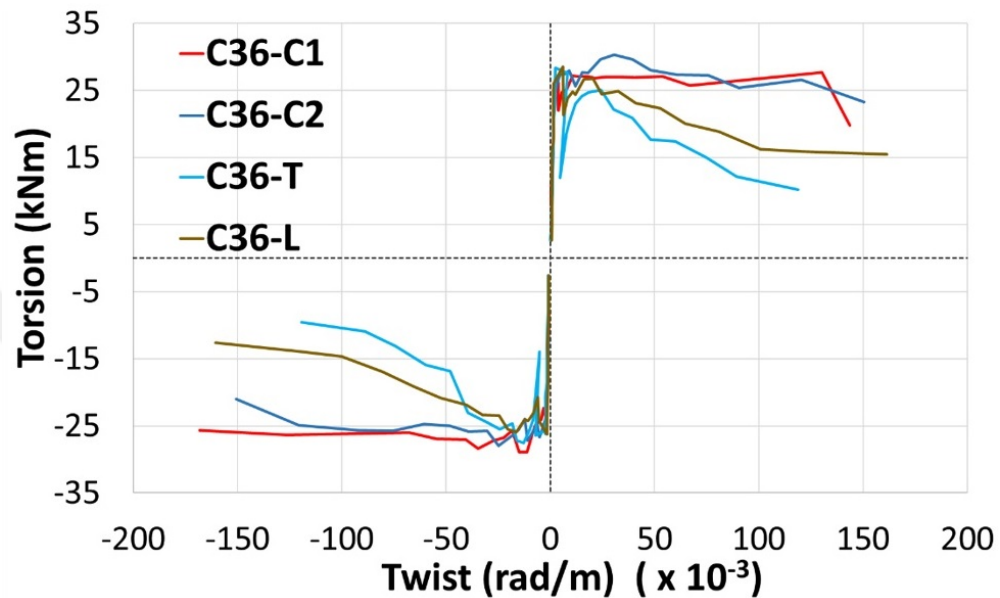


Figure 4.132. Comparison of backbone curves for C36 group specimens.

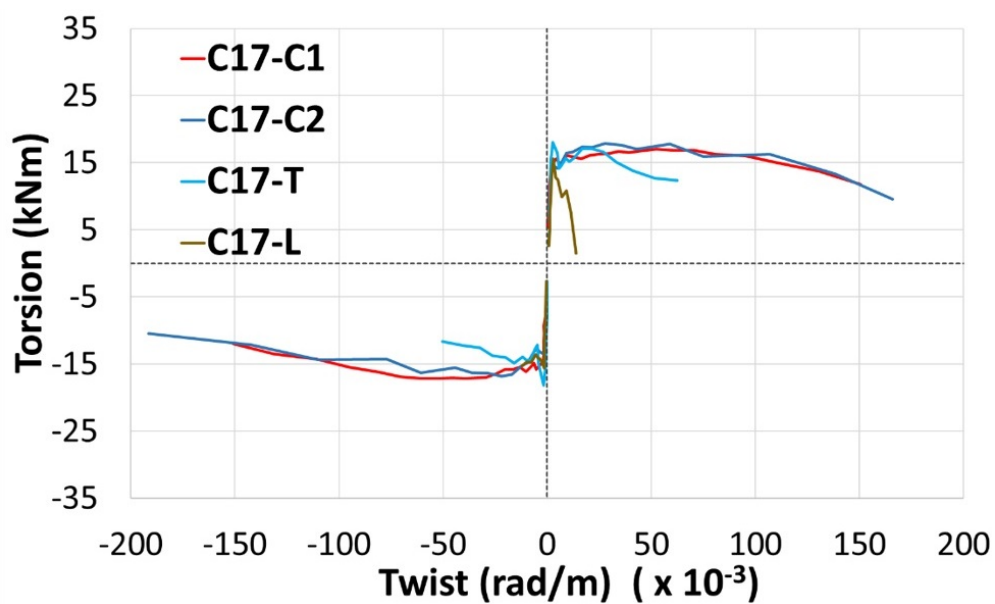


Figure 4.133. Comparison of backbone curves for C17 group specimens.

From the tested specimens' results given in Figure 4.134, it can be observed clearly from the comparison of backbone curves of the C50-T and C50-L that the transverse

reinforcement is more effective than the longitudinal reinforcement for better bearing capacity. C50-C was the control specimen that had the best torsional capacity.

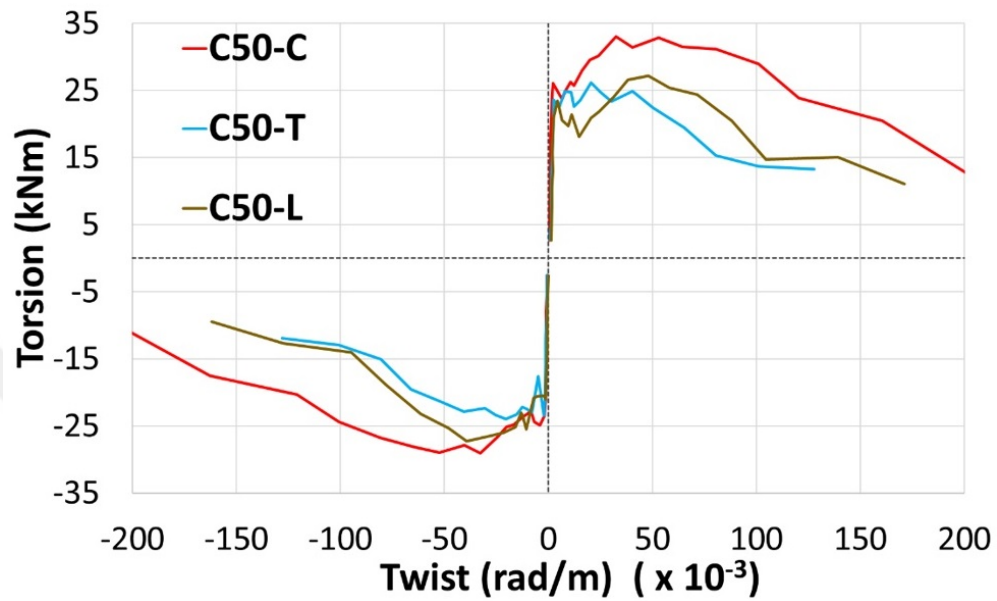


Figure 4.134. Comparison of backbone curves for C50 group specimens.

Figure 4.135 is comparing the cumulative energy capacities of all control specimens. C50-C has the best cumulative energy capacity. The curve of the C17-C1 is smoother because one cycle is done for each twist level.

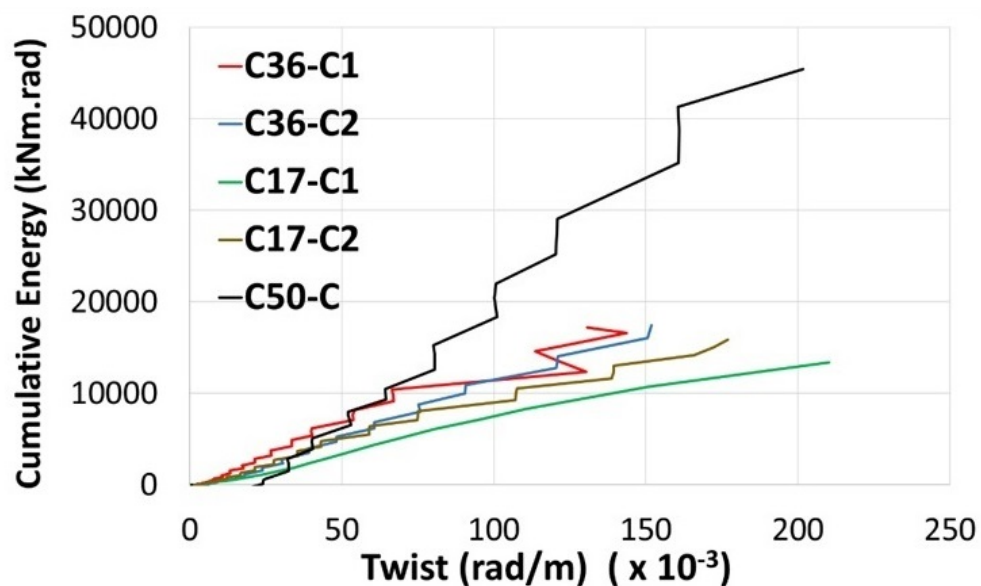


Figure 4.135. Comparison of energy vs. twist curves of control specimens.

Comparison of Energy dissipation capacities were given in Figures 4.136 - 4.138 for C36, C17 and C50 group specimens, respectively.

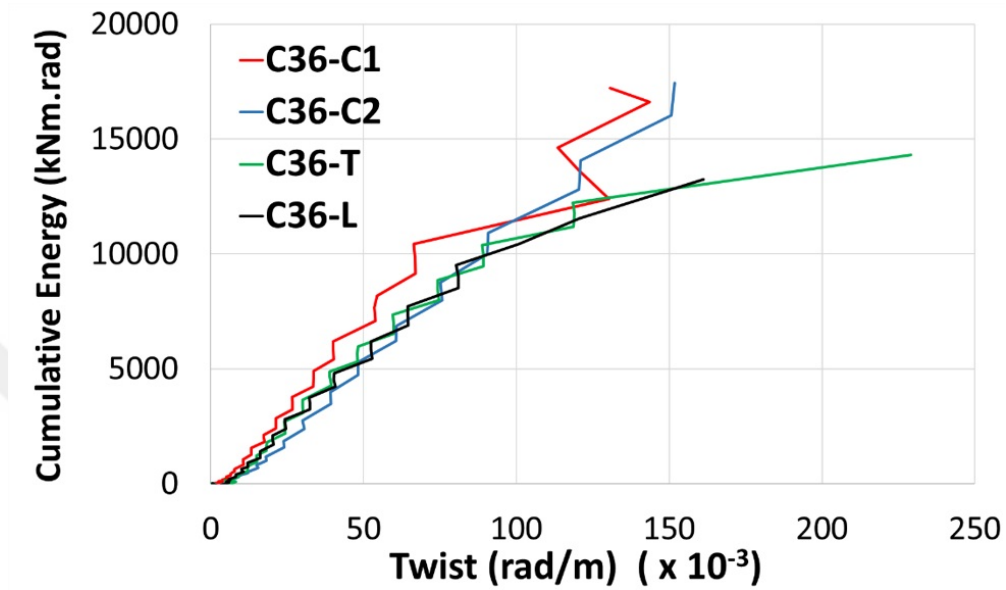


Figure 4.136. Comparison of cumulative energy vs. twist curves of C36 group.

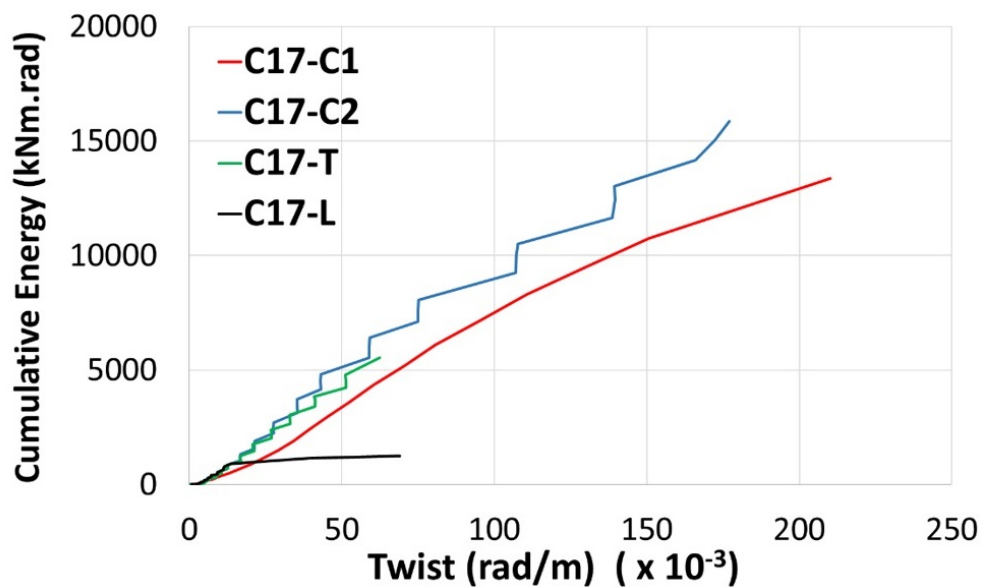


Figure 4.137. Comparison of cumulative energy vs. twist curves of C17 group.

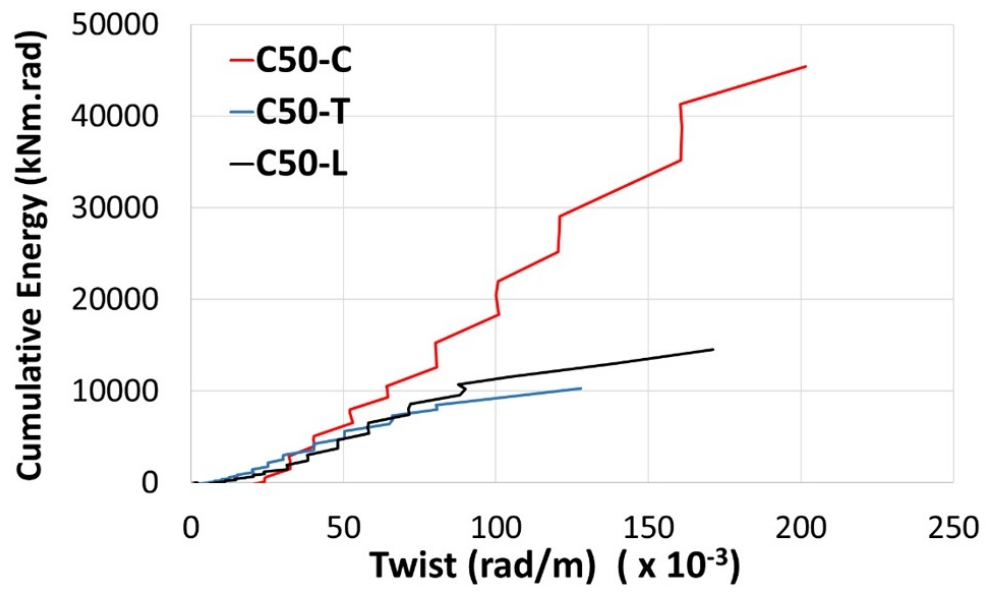


Figure 4.138. Comparison of cumulative energy vs. twist curves of C50 group.

5. ANALYTICAL MODELING

5.1. Introduction

A reinforced concrete element can undergo four fundamental force types: normal force (axial load), flexural load (bending), shear force and torsional moments. Initially, these forces can be designed and analyzed by using fundamental basic principles of mechanics of materials defined in single unified theory. These principles are:

- the condition of stress equilibrium,
- the condition of strain compatibility, and
- the constitutive laws governing concrete and steel.

The theory defines 6 analytical models depending on principles of mechanics of materials. These models can give more rational results for the four basic types of loadings: normal, flexural, shear (2D) and torsion (3D) [93]. These models are called as:

- (i) strut-and-tie model,
- (ii) equilibrium truss model (plasticity),
- (iii) compatibility truss model (Bernoulli),
- (iv) compatibility truss model (Mohr),
- (v) softened truss model, and
- (vi) softened membrane model.

Shear force causes a shear deformation for 2-D membrane element in the wall of tube so that is a 2-D problem. However, the torsional moment causes in addition to the shear deformations, bending of concrete struts, warping of tube walls, and equilibrium and compatibility of the whole 3-D member. So that, it is a 3-D problem and more complicated.

The concrete strain varies from 0 to a maximum level at the edge causing a gradient of strain in the effective thickness of the tube wall which is defined by the shear flow region [55]. In addition to softened concrete stress-strain curve obtained from RA-STM (Rotating-Angle Softened Truss Model), and the two equations for member coming from compatibility (twisting angle and curvature), are used to estimate torsional behavior between the cracking and maximum point [34]. The model showed a good agreement with test results found in the literature. Also, it indicated a reason for the model developed by Rausch which overestimates the ultimate torque. Essentially, concrete softening increases the effective thickness of the shear flow and reduces the lever arm area. Thus, torsional resistance is reduced [93].

The SMM (Softened Membrane Model), developed for shear behavior, was started to use for torsional behavior [35]. Bending effect and poisson effect were considered in this model. The bending effect consist of constitutive relationship of concrete in tension and bending of 2D elements in the principal tension direction. Therefore, before the cracking the response of the member is become predictable. On the other hand, due to the Poisson effect of the 2-D elements in the principal compression direction, the response is become predictable after the cracking [94]. However, the Poisson effect, needs to be diluted by 20 percent to consider the changes of strain due to the bending of the concrete struts. So that, entire torque-twist curve can be predicted by this model.

Structural members have characteristic features in terms of their hysteretic behavior. Having enough knowledge on the hysteretic behavior is crucial to determine the behavior of reinforced concrete system under seismic loading. For this aim, the different hysteretic models have been developed up to the present day in order to estimate the behavior of reinforced concrete members under cyclic loading. There are several researchers such as Takeda, Clough, Jirsa, Feghali, Sozen and Nielsen, Saiidi, Stone and Taylor, proposed hysteresis models for reinforced concrete members. The most common one is the Takeda's Hysteresis Model.

Hysteresis modeling approaches which are adopted by different researchers in the past, consists of mainly two types; i) Polygonal Hysteresis Models (PHMs) and ii) Smooth Differential Models (SDM). In the former one piecewise linear or nonlinear functions are used to define the paths. However in latter one empirical laws derived from experimental observations were used to define the path.

5.2. Analysis of Pure Torsion

Concrete strength is high in compression but very low in tension. In designing of concrete, the cracks in the tensile region can be prevented by reinforcing with high strength material in tension, such as steel. In order to calculate the cracking torque level, The Thin Wall Tube theory can be used for the pre-cracking stage, and the for calculating the cracking twist level, the Saint Venant's theory can be used. For the post-cracking stages, RA-STM (Rotating Angle Softened Truss Model), which uses the material models, can be used. The term, STM (Softened Truss Model), comes from properties of reinforced concrete such as nonlinear behavior and softening of reinforced concrete based on the biaxial constitutive laws.

5.2.1. Primary Curve

The primary curve (can be called as the backbone curve of the hysteresis) represents the torque-twist curve of a RC member under monotonic loading. It means that the primary curve provides a convenient means of defining strength boundaries for the purpose of modeling [84]. The primary curve or theoretical torque-twist relation for a RC section consists of four branches. Namely; the first branch shown with the OA line shows the first stage which is "pre-cracking behavior" and point A indicates the cracking point; line AB shows the cracking stage; curve BC shows the post-cracking behavior without yielding of steel bars and as a last portion, curve CD cracked state until failure with yielding of reinforcing steels as shown in Figure 5.1. Space truss analogy can be used to obtain the primary torque-twist curve because the model includes the reinforcement ratio.

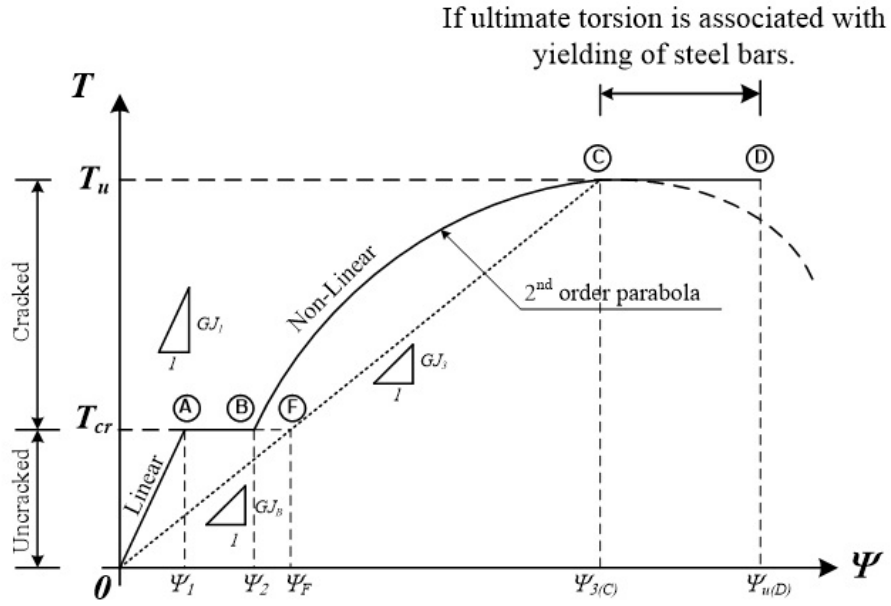


Figure 5.1. Idealized torque- twist relations for a member subjected to pure torsion [84].

5.2.1.1. Pre-cracking Stage. Linear part of the curve can be obtained by using the thin-walled tube theory.

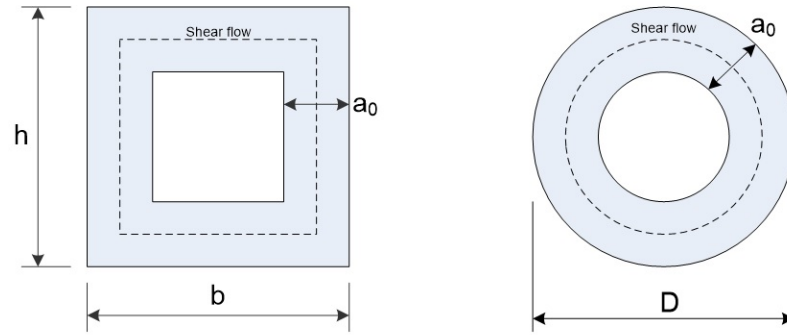


Figure 5.2. Shear flow thickness for rectangular and circular equivalent hollow section cross-section.

Basically, the cracking torsion, T_{cr} , can be expressed as

$$T_{cr} = T_A = T_B = [1 + (n - 1)\rho] \frac{A_c^2}{u_c} f_{cr} f_t k_1, \quad (5.1)$$

where, A_c is the gross area, u_c is the circumferences of the cross-section and f_{cr} , which is the strength of the concrete at cracking, can be taken as the direct tensile strength of

concrete, f_t . In Equation (5.1), the cracking torque is directly depending on reinforcing steel, where ρ represents the total volumetric steel ratio, n represents the modular ratio which is obtained from $n = E_s/E_c$. The gross area of section is calculated as

$$A_c = \begin{cases} b.h & \text{(a) rectangular section} \\ \frac{\pi D^2}{4} & \text{(b) circular section,} \end{cases} \quad (5.2)$$

perimeter of the cross-section is calculated from

$$u_c = \begin{cases} 2(b + h) & \text{(a) rectangular section} \\ \pi D & \text{(b) circular section,} \end{cases} \quad (5.3)$$

k_1 is the coefficient for high strength concrete defined by Bernardo and Lopes. Experimental studies showed that the T_{cr} must be multiplied by $k_1=0.85$ for high strength concrete, when $f_c \geq 50$ MPa [79]

$$k_1 = \begin{cases} 0.85 & f_c \geq 50 \text{ MPa} \\ 1 & f_c < 50 \text{ MPa.} \end{cases} \quad (5.4)$$

Cracking twist value at point A , shown as ψ_1 can be obtained by using Equation (5.5) via elasticity theory of torsion which is derived by Saint-Venant as

$$\psi_A = \psi_1 = \frac{T_{cr}}{G_c J_1}, \quad (5.5)$$

here, the concrete shear modulus, G_c is taken as approximately 40 percent of modulus of elasticity of concrete and obtained from $G_c = E_c/2(1 + \nu)$ and ν is the Poisson's ratio.

Poisson's ratio determined from member strain ratio in lateral direction to longitudinal direction. This ratio ranges from 0.1 for high-strength concrete to 0.2 for weaker mixes. In engineering practice, it is typically assumed as 0.15 for strength-focused design and 0.2 for considerations related to serviceability [94].

J_1 is the torsional constant for a cross-section. Torsional constant for rectangular and circular sections are calculated as

$$J_1 \cong \begin{cases} bh^3 \left[\frac{1}{3} - 0.21 \frac{h}{b} \left(1 - \frac{h^4}{12b^4} \right) \right], & h \geq b & \text{(a) rectangular section} \\ \frac{1}{2} \pi r^4 & & \text{(b) circular section.} \end{cases} \quad (5.6)$$

Here, b and h are the dimensions of rectangular cross-section, and d is the diameter of the circular cross-section. Equation of torsional constant J for the different shaped section is given in Appendix B.

Calculated theoretical cracking torque and twist values by using the equations given in Equation (5.1) and Equation (5.4) are compared with experimentally obtained cracking torque and twist values. As can be seen from Table 5.1, analytically obtained and experimentally obtained cracking torsional moment and twist values are in a good agreement.

Table 5.1. Experimental and theoretical torque-twist values.

Specimen	$T_{cr,exp}$ (kNm)	$\Phi_{cr,exp}$ (10^{-3} rad/m)	$T_{cr,theo}$ (kNm)	$\Phi_{cr,theo}$ (10^{-3} rad/m)	$\frac{T_{cr,theo}}{T_{cr,exp}}$	$\frac{\Phi_{cr,theo}}{\Phi_{cr,exp}}$
C36-C1	24.0	1.40	26.6	1.85	1.11	1.32
C36-C2	24.9	1.58	26.6	1.85	1.07	1.17
C36-T	26.9	2.00	26.2	1.82	0.97	0.91
C36-L	26.0	1.70	25.9	1.80	0.99	1.06
C17-C1	15.4	2.00	18.3	1.76	1.19	0.88
C17-C2	14.4	2.00	18.3	1.76	1.27	0.88
C17-T	17.0	1.95	18.1	1.75	1.07	0.89
C17-L	12.0	2.00	17.8	1.71	1.48	0.86
C50-C	25.6	2.04	26.1	1.72	1.02	0.85
C50-T	25.0	2.24	25.84	1.71	1.03	0.76
C50-L	23.3	2.20	25.2	1.67	1.08	0.76

5.2.1.2. Post-cracking Stage. At this stage, after full torsional cracking, the section will begin to behave like a space truss. Namely, stiffness controlled by reinforcement in the longitudinal and transverse direction. The torsion is counteracted by a truss-like mechanism.

The compressive stresses are carried by diagonal concrete struts and the tensile stresses carried by the longitudinal and transverse reinforcements. Importantly, there is no shear transfer along the surface of the crack in this scenario, and this condition is referred to as the "space truss analogy". The circulating shear stresses formed at the edges of cross-section is called as shear flow q , and the effective thickness (t_e) shown in Figure 5.3 is variable thickness depending on equilibrium and compatibility

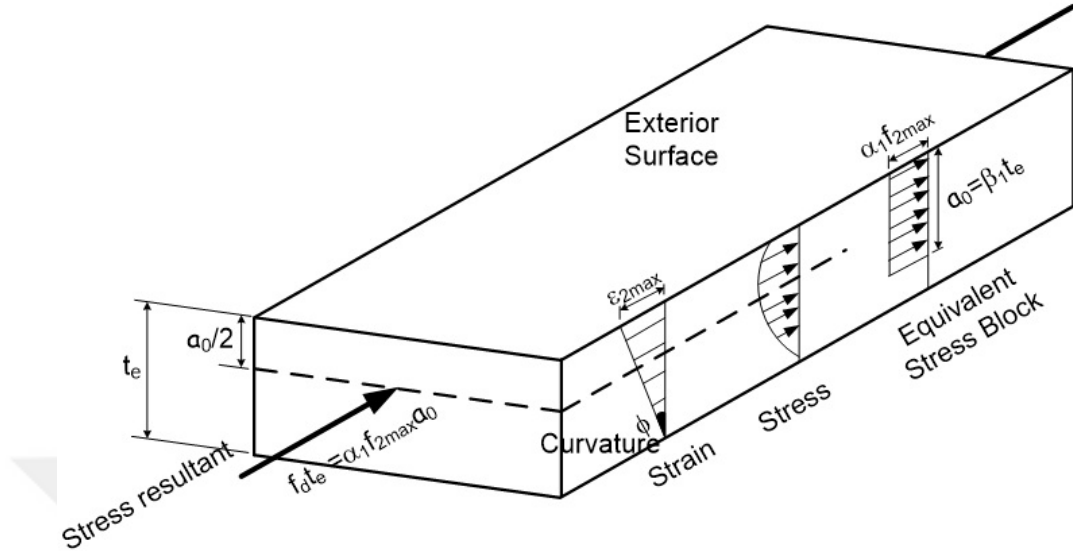


Figure 5.5. Description of a concrete strut and corresponding rectangular stress block [84].

The cross-sectional properties of equivalent hollow section and definition of equivalent stress block are defined in Figure 5.4 and Figure 5.5 and area is expressed as

$$A_o = \begin{cases} (b - a_o)(h - a_o) & \text{(a) rectangular section} \\ \frac{\pi}{4}(D - a_o)^2 & \text{(b) circular section} \\ A_c - 0.5u_c a_o + a_o^2 \xi_{shape} & \text{(c) all sections,} \end{cases} \quad (5.7)$$

effective perimeter, u_o is calculated from

$$u_o = \begin{cases} 2[(b - a_o) + (h - a_o)] = 2(b + h - 2a_o) & \text{(a) rectangular section} \\ \pi(D - a_o) & \text{(b) rectangular section} \\ u_c - 4a_o \xi_{shape} & \text{(c) all sections,} \end{cases} \quad (5.8)$$

where, ξ_{shape} is shape factor, and obtained by

$$\xi_{shape} = \begin{cases} 1 & \text{(a) rectangular section} \\ \pi/4 & \text{(b) circular section.} \end{cases} \quad (5.9)$$

The height of wall thickness of the equivalent rectangular compressive stress block, which is defined and illustrated in Figure 5.5, is calculated by using β_1 factor.

The height of the equivalent rectangular stress block, (a_o) is a factor of effective wall thickness (t_e) and calculated from

$$a_o = \beta_1 t_e, \quad (5.10)$$

where, β_1 factor is taken from AS-3600 and calculated from

$$\beta_1 = \begin{cases} 0.85 & f_c \leq 27 \text{ MPa} \\ 0.85 - 0.007(f_c - 27) & f_c > 27 \text{ MPa}. \end{cases} \quad (5.11)$$

For the unit dimension member on the walls of the equivalent hollow section given in Figure 5.3, the lateral (horizontal) force equilibrium can be written as

$$f_l = \frac{qs_l}{A_l} \cot \theta, \quad (5.12)$$

the transverse (vertical) force equilibrium can be written as

$$f_t = \frac{qs}{A_t} \tan \theta, \quad (5.13)$$

also, the transverse force can be defined as

$$f_t = \frac{q}{t_e} \left(\frac{1}{\sin \theta \cos \theta} \right), \quad (5.14)$$

where, q is the shear flow, f_d denotes the compressive stress of the concrete struts, f_l and f_t are indicates the stresses on reinforcing steel in transverse and longitudinal direction, respectively, A_l and A_t are longitudinal reinforcement area, and the area of one leg of a transverse reinforcement, respectively. s and s_l are the average spacing of transverse and longitudinal reinforcements, respectively. θ is the slope of the compressive struts with respect to the longitudinal axis of the member.

Sectional and Material Properties: The torsional behavior of reinforced concrete member is 3D problem. In addition to 2D shear problem of membrane elements in the tube wall, it consists equilibrium and compatibility of the whole 3-D member. The Mohr compatibility truss model employs equation sets for analyzing the response of two-dimensional RC elements when subjected to external membrane stresses.

Three equations used for maintaining equilibrium, three of them used for ensuring compatibility and three of them used for describing the constitutive law of the materials. The constitutive relations are obtained from Hooke's law for steel and also for concrete

(Figure 5.6), the behavior of the 2-D element is predicted as linear. Consequently, the Mohr compatibility truss model offers a framework for linear analysis. For nonlinear analyses of reinforced concrete members subjected to torsion, it is more advantageous to employ Navier's three principles of the mechanics of materials.

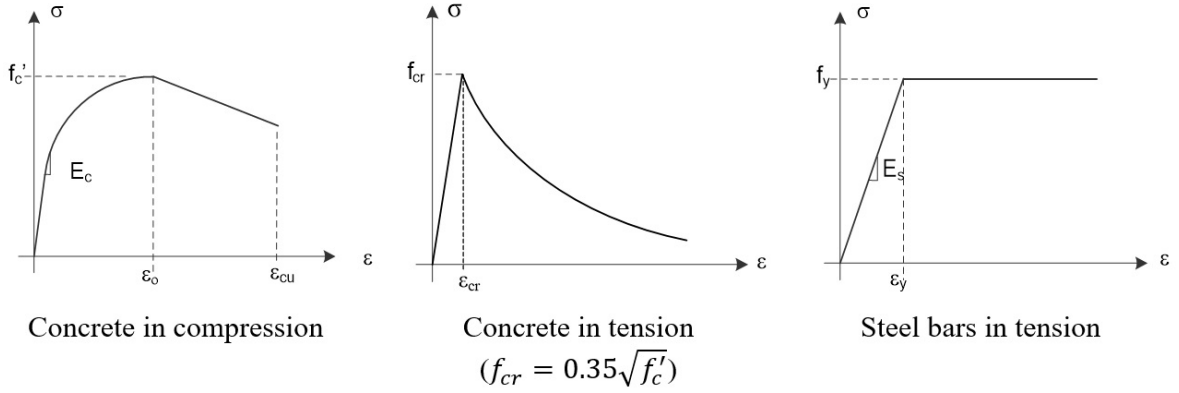


Figure 5.6. Stress-strain relationships for the materials.

To accurately represent the compressive and tensile strength of concrete struts, it's necessary to employ stress-strain ($\sigma - \varepsilon$) relationships that consider the softening effect in concrete under compression stresses and the stiffening effect in concrete under tension stresses.

Equilibrium equations: In Figure 5.2, a reinforced concrete member under T torsion is shown. This external torsion, T , causes an internal torsion due to the shear flow q formed along the circumference of the cross-section. The torsional moment can be obtained by using thin-walled tube theory from Equation (5.15) for the equivalent hollow section given in Figure 5.3 as

$$T = 2A_oq, \quad (5.15)$$

where the region occupied by shear flow, q , is called as the shear flow zone, that has a wall thickness of t_d and effective wall thickness of a_o and torsion will be

$$T = 2A_o k_1 \sqrt{\frac{A_l f_l}{u_o} \frac{A_t f_t}{s}}. \quad (5.16)$$

By substituting q from Equation (5.15) into Equation (5.16) to Equation (5.18), the slope can be calculated as

$$\tan \theta = \sqrt{\frac{A_t f_t}{s} \frac{u_o}{A_l f_l}}. \quad (5.17)$$

The constant for equivalent hollow section, J_h , is obtained from

$$J_h = 4t_e \left(\frac{A_o^2}{u_o} \right). \quad (5.18)$$

5.2.1.3. Cracking Branch AB. In Figure 5.1, the point A shows the first torsional cracking, point F indicates the fully cracked section. The torsional stiffness of torque-twist curve on the section corresponding to point F can be determined by using the Collins and Lampert [95] post-cracking approach for pure torsion, which is based on Equations (5.15) and (5.16). This indicates that the truss torsional stiffness value is approximately equal to the torsional stiffness of a tube member with the identical dimensions and an effective wall thickness

$$t_e = \sqrt{\frac{A_t A_l}{s u_h}}. \quad (5.19)$$

The torsional stiffness, GJ_F , which depends on the effective enclosed area of the stirrups, can be obtained by substituting Equation (5.18) into Equation (5.19) and

$$GJ_F = 4G_s \left(\frac{A_h^2}{u_h} \right) \sqrt{\frac{A_t A_l}{s u_h}}, \quad (5.20)$$

where, A_h and u_h are the enclosed area and perimeter of stirrup centerline shown in Figure 5.3. G_s is the shear modulus of steel. The twist value at the point F , can be calculated by obtained torsional stiffness as

$$\psi_F = \frac{T_{cr}}{GJ_F}. \quad (5.21)$$

The twist at point B can be determined from Equation (5.22), which is an intuitive approach, proposed by CEB-FIP model code 1990 [96]. In this approach, tension stiffening is considered in torsion to obtain the twist value as

$$\psi_B = \lambda \psi_A + (1 - \lambda) \psi_F, \quad (5.22)$$

where, λ is calibration based on experiments with different reinforcement ratio

$$\lambda = 0.6 - \frac{0.2}{e^{1000\rho}}. \quad (5.23)$$

5.2.1.4. Post Cracking Branch BC. The equivalent stress block coefficients, shown in Figure 5.4 (α_1 and β_1), can be found from adopted stress-strain behavior of concrete. The final stress is obtained from

$$f_d t_e = \alpha_1 f_{2max} a_o, \quad (5.24)$$

where, $\alpha_1 = 0.9$, f_{2max} is the maximum strength of cracked concrete in compression and

$$f_{2max} = S f_c, \quad (5.25)$$

here, S is the softening factor for strength of concrete obtained from

$$0.4 \leq S = \frac{1}{0.8 + 170\varepsilon_1} \leq 1.0, \quad (5.26)$$

and, f_c is the strength of concrete in compression and

$$\frac{A_l f_l}{u_o} + \frac{A_t f_t}{s} = \alpha_1 f_{2max} a_o = \alpha_1 f_{2max} \beta_1 t_e. \quad (5.27)$$

In a structural member under torsion, the twisting angle leads warping of member's wall, and leads to bending of concrete struts. The relations between the twisting angle, ψ , and the curvature of the concrete struts, ϕ , is defined by

$$\phi = \psi \sin 2\theta, \quad (5.28)$$

the twisting angle is calculated for point C from

$$\psi = \psi_c = \psi_{3(c)} = \frac{\varepsilon_{2max}}{t_e \sin 2\theta}, \quad (5.29)$$

the strain at outer of the shear flow zone calculated from

$$\varepsilon_{2max} = 0.13 S \varepsilon_{co}, \quad (5.30)$$

where, ε_{co} is the concrete strain corresponding to the strength of concrete in compression direction.

The strain of the steel in longitudinal direction and transverse direction can be calculated from

$$\begin{cases} \varepsilon_{3l} = \left(\frac{\alpha_1 \beta_1 A_o^2 f_{2max}}{T_u u_o \cot \theta} - \eta \right) \varepsilon_{2max} & (a) \\ \varepsilon_{3t} = \left(\frac{\alpha_1 \beta_1 A_o^2 f_{2max}}{T_u u_o \tan \theta} - \eta \right) \varepsilon_{2max} & (b), \end{cases} \quad (5.31)$$

where η is the average of η_l and η_t described in Figure 5.3. η_l and η_t are ratio of

distances of steel location from the hollow region over effective wall thicknesses. These ratios are calculated from

$$\left\{ \begin{array}{ll} \eta = \frac{\eta_l + \eta_t}{2} & (a) \\ \eta_l = 1 - \frac{d_l}{t_e} & (b) \\ \eta_t = 1 - \frac{d_t}{t_e} & (c), \end{array} \right. \quad (5.32)$$

here, d_l and d_t are the concrete cover from the center point of longitudinal reinforcement and transverse reinforcement, and calculated from

$$\left\{ \begin{array}{ll} d_l = d' + \phi_w + \frac{\phi_l}{2} & (a) \\ d_t = d' + \frac{\phi_w}{2} & (b), \end{array} \right. \quad (5.33)$$

Finally, the torque value for point C is equal to the ultimate torque value will be

$$T_C = T_u. \quad (5.34)$$

5.2.1.5. Degrading Part CD. Torsional stiffness in the degrading part is directly depending on the transverse and longitudinal reinforcement. For the control specimens generally, we did not observe degradation, however, for inadequate transverse or inadequate longitudinal reinforcement there is degradation. From the experimental backbone curves, the ratio of ultimate twist over the twist level at C point is obtained as 6. Therefore, the relation of twist value is found as

$$\psi_D = 6\psi_C, \quad (5.35)$$

and also from the test results, the torsion values are decreased when the reinforcement ratio are decreased. Therefore, the relation for torsion values are obtained as

$$T_D = T_C \frac{\rho_l}{\rho_{lo}} \left(\frac{\rho_t}{\rho_{to}} \right)^2 k_1, \quad (5.36)$$

where, ρ_{lo} and ρ_{to} are minimum longitudinal reinforcement ratio and minimum transverse reinforcement ratio, respectively. These ratios are defined in ACI code and used in control specimens. k_1 is the coefficient for high strength concrete defined by Bernardo and Lopes [78].

5.2.2. Procedure for Iterative Solution

Summary of the analysis is given with the flowchart in the Figure 5.7, and step by step process was explained below:

- Iteration 1- Iteration for t_e : f_l , f_t and S values are assumed initially. The equilibrium equation defined in Equation (5.27) should be satisfied by changing t_e . In the equation, a_o and u_o are updated by using the new t_e , according to Equation (5.8) and Equation (5.11).
- Iteration 2: Iteration for f_l and f_t : By using the t_e obtained from 1st iteration, the reinforcement stresses, f_l , and f_t are updated from

$$\begin{cases} f_l = \varepsilon_{3l} E_s < f_{yl} & \text{(a)} \\ f_t = \varepsilon_{3t} E_s < f_{yt} & \text{(b)}, \end{cases} \quad (5.37)$$

here, ε_{3l} and ε_{3t} are calculated from Equation (5.31), which includes updated values of A_o , u_o , T_u , θ and η depending on t_e , f_l , and f_t .

- Iteration 3: Iteration for S : By using the strain values obtained from 2nd iteration, ε_1 is updated from Equation (5.38) and S is calculated from

$$\varepsilon_1 = \varepsilon_{3t}. \quad (5.38)$$

These three steps are repeated until all iteration solutions are satisfied. In other words, when the calculated t_e , f_l , f_t and S values are almost equal to the assumed values at the beginning, the iteration can be stopped and the values are obtained. And finally, t_e , f_l , f_t and S values are used to calculate torque and twist value of point C .

In Figure 5.7, the flow chart for prediction of backbone curve is given. First of all depending on material properties and geometrical properties, torque and twist values were calculated for point A and B, and for point B. Iterative process should be done for obtaining the values for point C and the experimental values are used to find the values for point D.

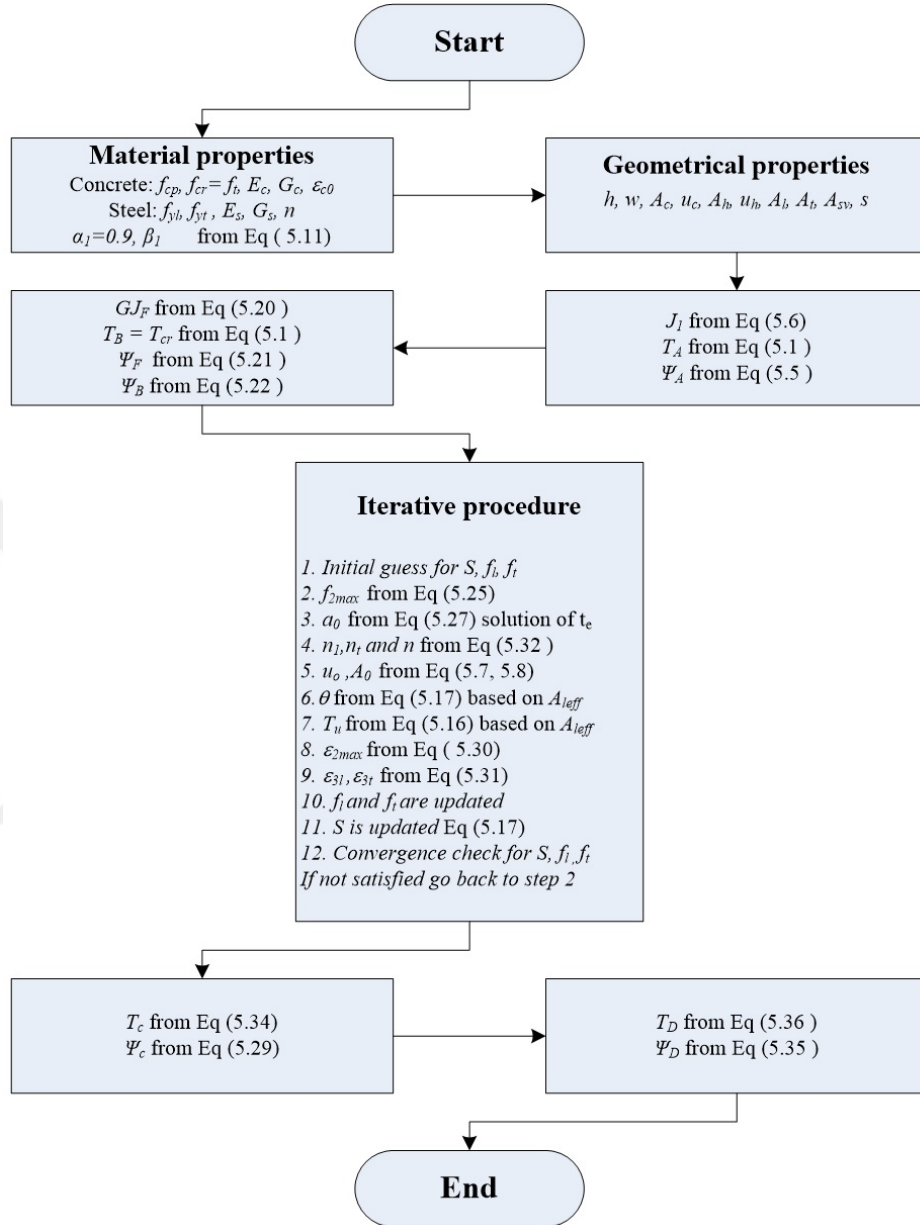


Figure 5.7. Flowchart for modeling of backbone curve.

5.3. Hysteresis Modeling Description

The proposed model comprises two components: the initial segment deals with the primary torsional moment-twist correlation during monotonic loading, while the subsequent segment addresses the unloading and reloading aspects within hysteresis loops during cyclic loading

The branches of loading and unloading of the hysteresis model are identified by analyzing of experimental data. A comprehensive experimental study was conducted to establish the rules for these branches. Specific parameters such as concrete strength, reinforcement ratio, and torsional ductility ratio that have a significant impact on the behavior of the system were identified.

5.3.1. Unloading Branches

To define unloading branches of hysteretic loading, two slopes on the primary curve are utilized as indicators. The initial slope, k_1 , is measured from the line slope drawn from the origin to the cracking point. Likewise, the slope of the current loop (i^{th} loop slope), is measured from the line slope drawn from the origin to the target point on the backbone curve. These slopes are shown on Figure 5.8.

The cracking torque, denoted as T_{cr} , is determined using the Valipour's equation [84]. On the other hand, the torque value, T_1 , where the curvature of the loop is changed, is obtained based on the Saint-Venant equation.

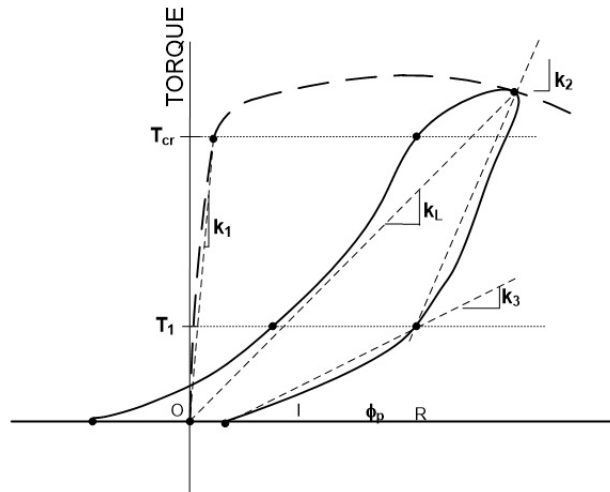


Figure 5.8. Slopes of torque twist curves used in unloading branches.

The unloading slopes in the torque-twist relationship are influenced by the twist and torsional moment levels reached at the start of unloading. Analysis of experimental

data reveals that when unloading initiates below the cracking torque and the cracking torque has not been surpassed, the unloading behavior follows a nearly straight line from the torque reversal point. This straight line exhibits a slope of k_1 . A linear variation is proposed as a function of torsional ductility [97].

Experimental observations have further revealed that when the cracking torque is surpassed, the unloading curve undergoes a change in slope near the T_1 torque level. The portion of the curve below the cracking torque exhibits a relatively flatter slope compared to the portion above it. The statistical analysis performed on the experimental data has yielded insights into the correlation between the torsional ductility and the slope of unloading branches. For the portion of the curve above the T_1 , the analysis has indicated that the correlation between the torsional ductility and slope of unloading branches can be defined by a logarithmic function as illustrated in Figure 5.9.

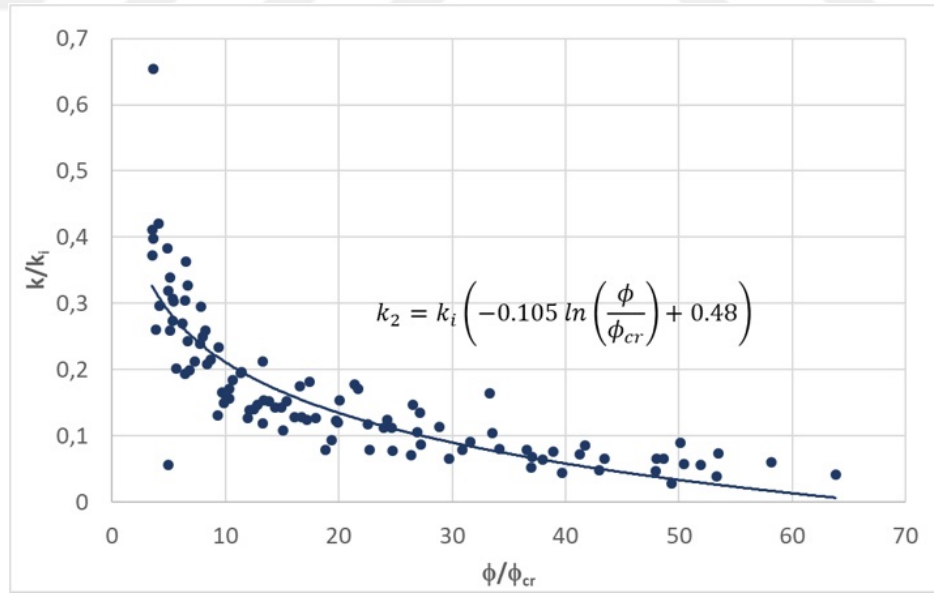


Figure 5.9. Variation slope of unloading part, k_2 for above T_1 .

However, for the portion of the hysteresis curve below the T_1 , a nearly constant relationship is observed with the hysteresis loop slope. Therefore, in Figure 5.10, the variation of the k/k_1 ratio is observed to range between 0.6 and 1.0. In order to simplify the analysis and ensure consistency, a constant value of 0.8 is selected and used

in analytical study for the k/k_1 ratio. The unloading branches rules can be defined as given in below: The example of proposed single loop is shown in Figure 5.11.

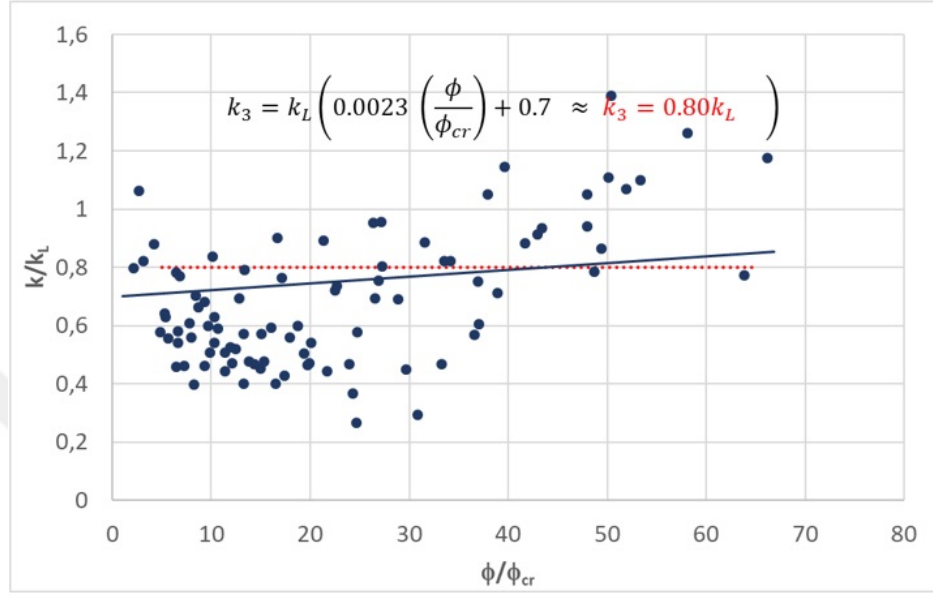


Figure 5.10. Variation slope of unloading part, k_3 for below T_1 .

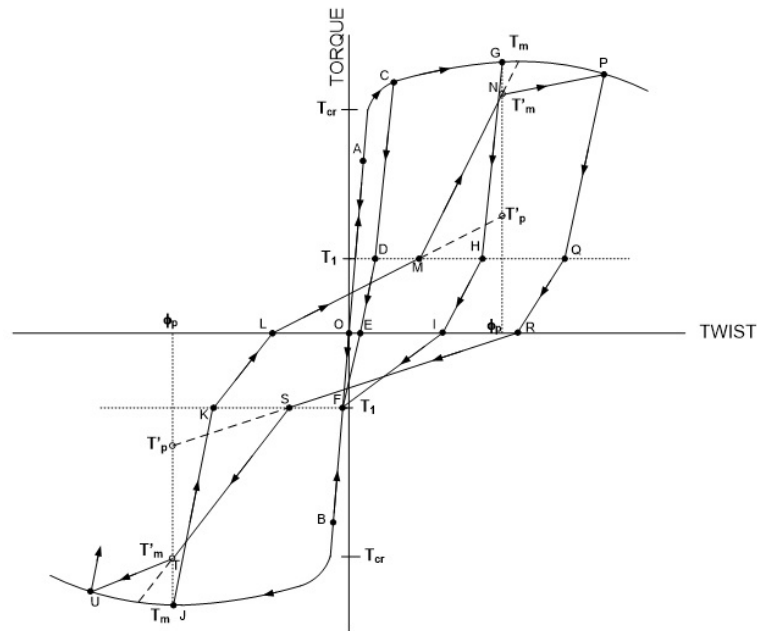


Figure 5.11. Single loop of proposed hysteretic model.

- The unloading behavior follows the k_1 slope if the torque at the beginning of unloading is lower than the cracking torque T_{cr} and T_{cr} has not been surpassed previously in either direction.

- However, if the cracking torque T_{cr} has been exceeded at least once in one direction, the unloading slope above the T_1 level (Figure 5.9) is determined by

$$k = k_1 \left(-0.105 \ln \left(\frac{\phi}{\phi_{cr}} \right) + 0.48 \right). \quad (5.39)$$

However, the unloading stiffness below the T_1 (Figure 5.10) is determined by

$$k = 0.80k_L, \quad (5.40)$$

where, k_1 is initial stiffness, k_L is the slope of the loop starting from origin towards the target point on the primary curve, ϕ_{cr} is twist at cracking, ϕ is twist where unloading starts.

5.3.2. Loading and Reloading Branches

The loading and reloading branches in the hysteresis response of the system follow a specific pattern. During the initial loading phase, the response of the system follows the backbone curve until the point where unloading begins. When the loading direction is reversed, the initial cycle is directed towards T_1 in the opposite direction. Once the T_1 is reached, the response traces the rules defined by using experimental observations for loading, unloading, and reloading branches.

Reinforced concrete exhibits a characteristic behavior known as stiffness degradation under inelastic twist cycles. This means that increasing the amount of twist cycles or increasing the magnitude of inelastic twist values, the member behaves progressively softer during reloading. In addition, the torque-twist hysteretic relationship typically displays a phenomenon called pinching of hysteresis loops. This pinching occurs due to the sliding motion in the cracks that were developed during the loading of previous cycles.

As the cracks in the member close, it begins to exhibit a stiffer response, resulting in a steeper slope of the reloading branches. To capture this behavior, two slopes are defined for reloading branches of the hysteretic behavior. These slopes represent the

changing stiffness of the member during reloading, reflecting the pinching phenomenon and the subsequent closure of cracks.

Reloading slopes are established using T'_p/T_p and T'_m/T_p ratios for the portions below and above the T_1 , respectively. T'_p , T'_m and T_p are peak and maximum point of the previous loop, respectively and the prime indicates factored points as shown in Figure 5.11. Variations of these parameters versus stiffness degradation, are illustrated in Figure 5.12 and Figure 5.13.

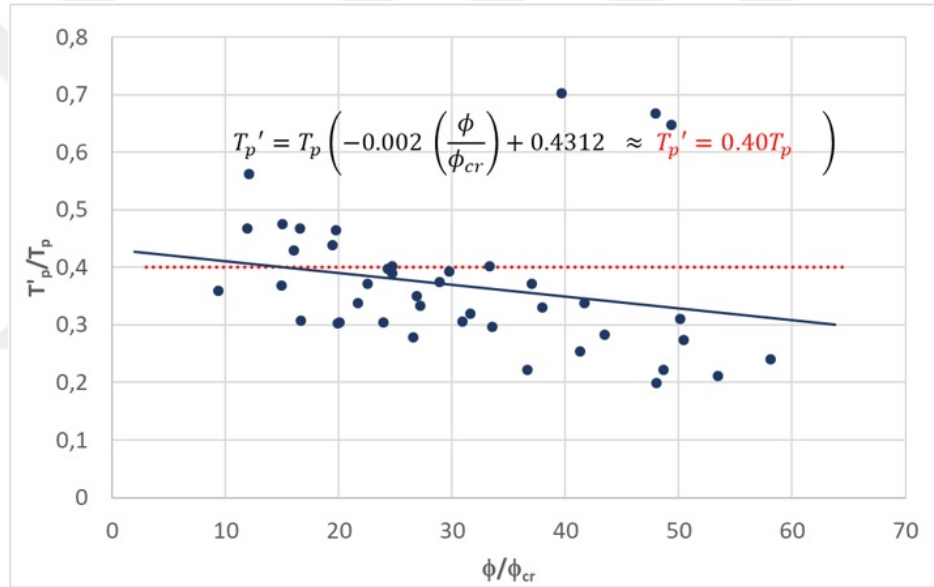


Figure 5.12. Stiffness degradation vs. ductility at reloading for below T_1 .

In Figure 5.12, the variation of the T'_p/T_p ratio is observed to range between 0.2 and 0.6. Therefore, it is assumed that T'_p is equal to 40 percent of T_p for simplification of analysis.

Similarly, in Figure 5.13, the variation of the T'_m/T_p ratio is observed to range between 0.8 and 1.0. In order to simplify the analysis a constant value of 0.4 and 0.9 is selected for the T'_p/T_p and T'_m/T_p ratio respectively.

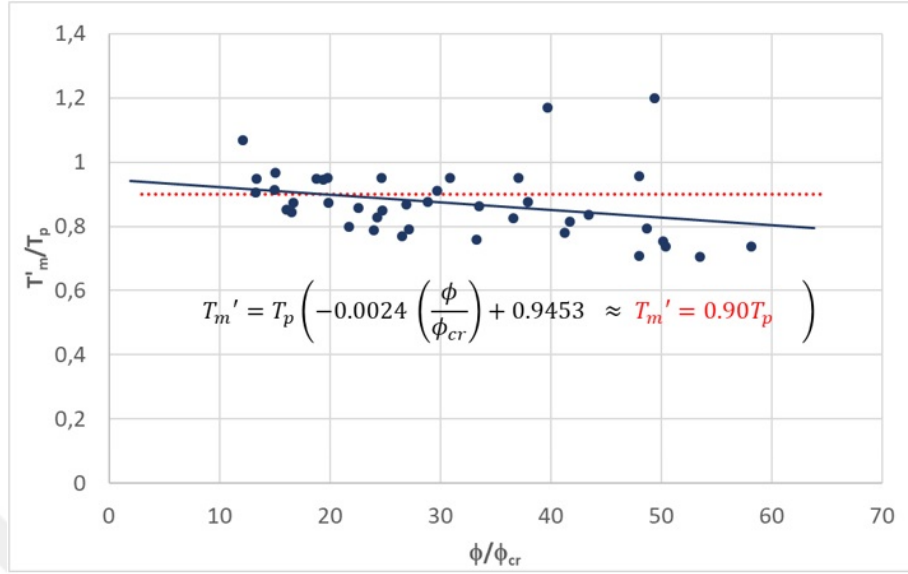


Figure 5.13. Stiffness degradation vs. ductility at reloading for above T_1 .

For loading and reloading stages, the rules for hysteresis were defined as below. The typical indication of hysteresis loops segment is shown in Figure 5.11.

- The loading and reloading stages, which is less than T_{cr} , trace the backbone curves.
- If the member is not subjected the torque greater than T_{cr} before in one direction, the loading in that direction is directed towards achieving the cracking torque even if the member has reached T_{cr} in the opposite direction (e.g., EF , IF).
- If the loading or reloading is greater than T_{cr} ;

- (i) Reloading until to the T_1 , will continue linearly passing through the 40 percent of the peak point (ϕ_p, T'_p) (e.g., LM , RS)

$$T'_p = 0.4T_p, \quad (5.41)$$

- (ii) Reloading beyond T_1 will trace a straight line up to the point (ϕ_p, T'_m) (e.g., MN , ST);
- (iii) Beyond the T'_m , loading trace a straight line to the target point on the primary curve (e.g., NP , TU)

$$T'_m = 0.9T_p. \quad (5.42)$$

Experimental results indicate that maximum torsion values decrease for successive loops at the same twist level due to stiffness degradation. To quantify this reduction in maximum torsion, a reduction coefficient RF is introduced. Figure 5.14 presents the reduction factors of the specimens.

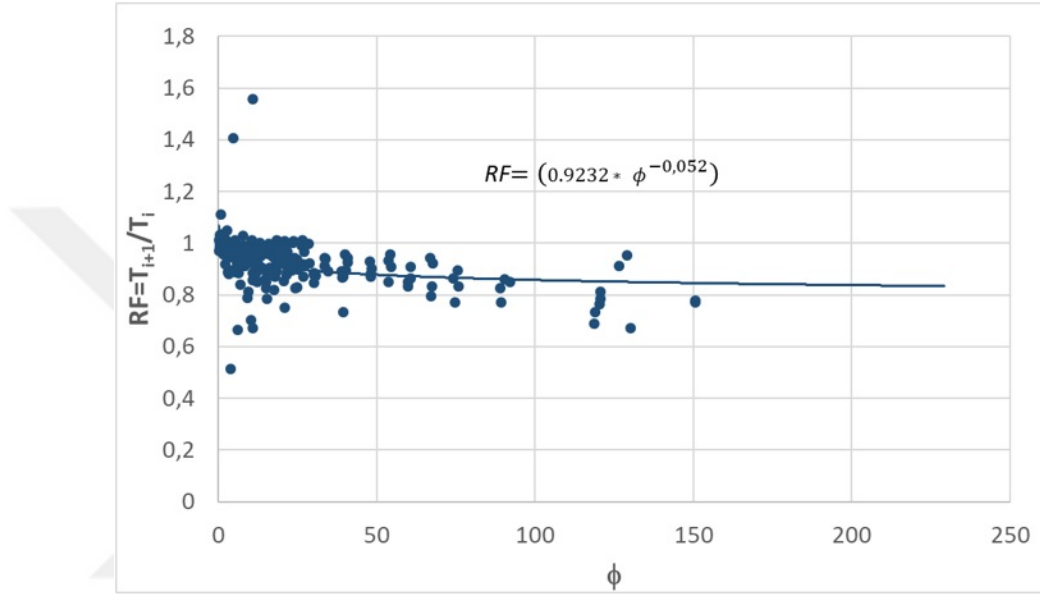


Figure 5.14. Torsion reduction factor of successive loops.

The ratio of torsional strengths among the successive cycles of single twist level i called as RF and determined by

$$RF = (0.9232 \times LP \times \phi^{-0.052})^{n-1}, \quad (5.43)$$

where, n is the number of successive loops for each twist value. LP is the loading pattern coefficient for the twist-controlled loading system. LP is the ratio of the twist applied to the specimen to the cracking twist level (ie: $1.5 \phi_{cr}$). If a single loop is done for each twist level, the RF coefficient becomes 1, that means there is no decrease in stiffness for successive loops. The maximum torsion, T_m , for each successive cycle are updated depending on the number of loops determined by

$$T_m = T_m \times RF. \quad (5.44)$$

6. DISCUSSION

6.1. Comparison of Analytical and Experimental Backbone Curves

Theoretical backbone curves and experimental backbone curves were drawn for the all tested specimens as shown in Figures 6.1 - 6.11. It was observed that the results were quite consistent with each other.

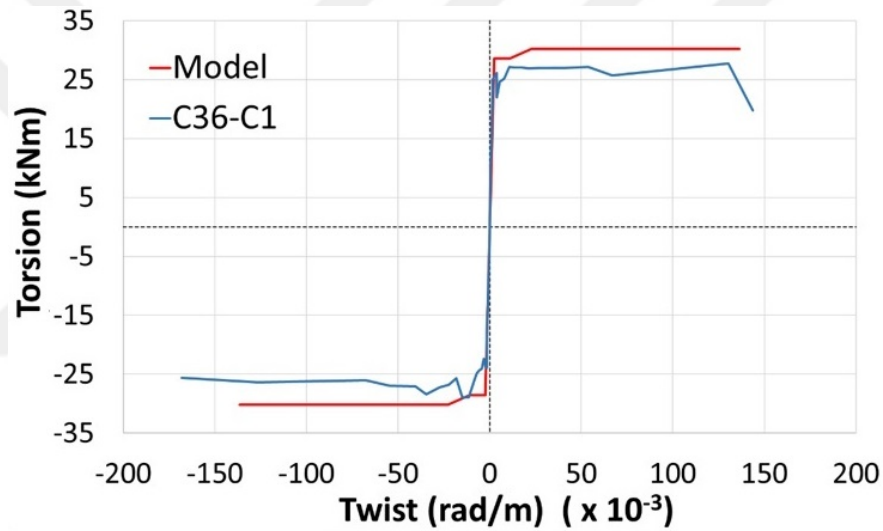


Figure 6.1. Comparison of experimental - theoretical backbone for C36-C1.

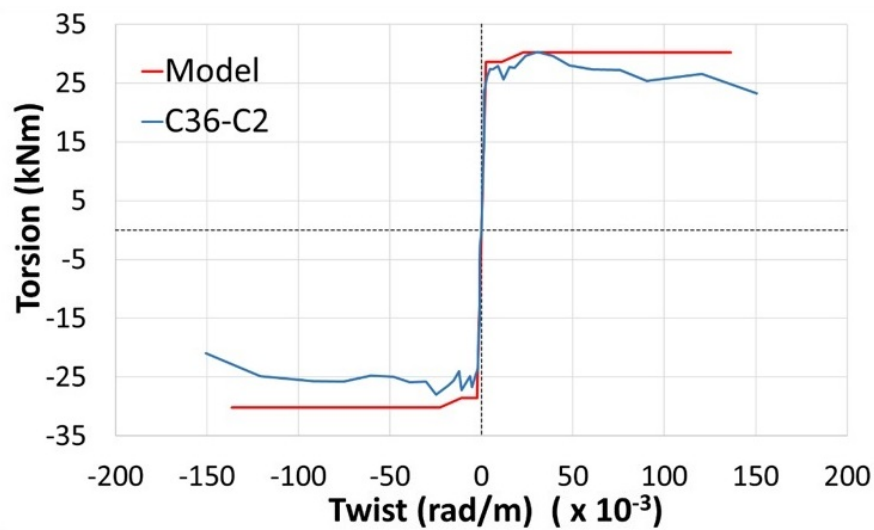


Figure 6.2. Comparison of experimental - theoretical backbone for C36-C2.

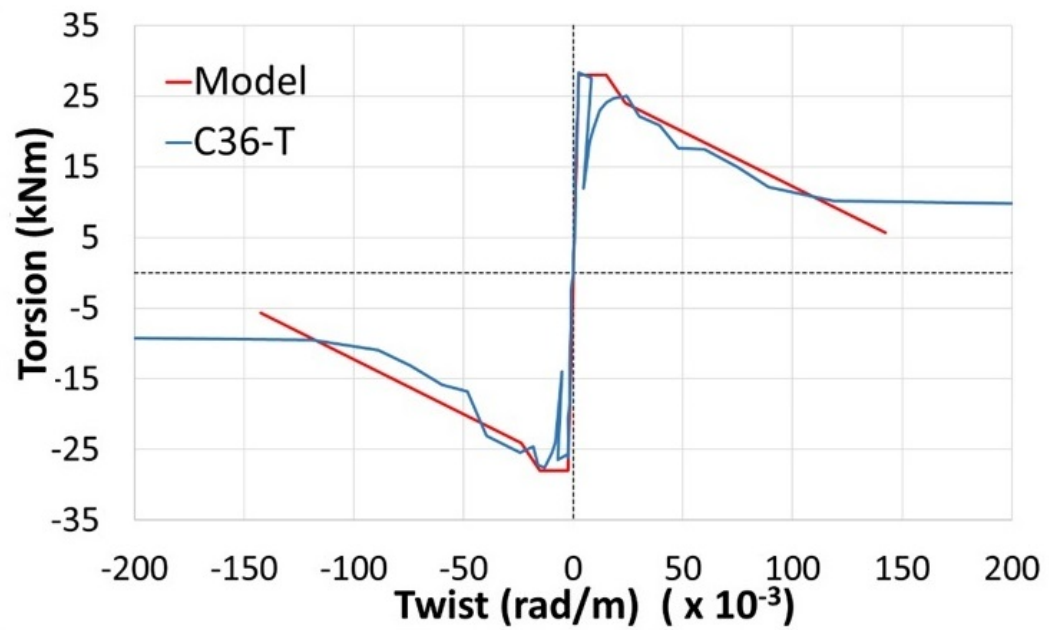


Figure 6.3. Comparison of experimental - theoretical backbone for C36-T.

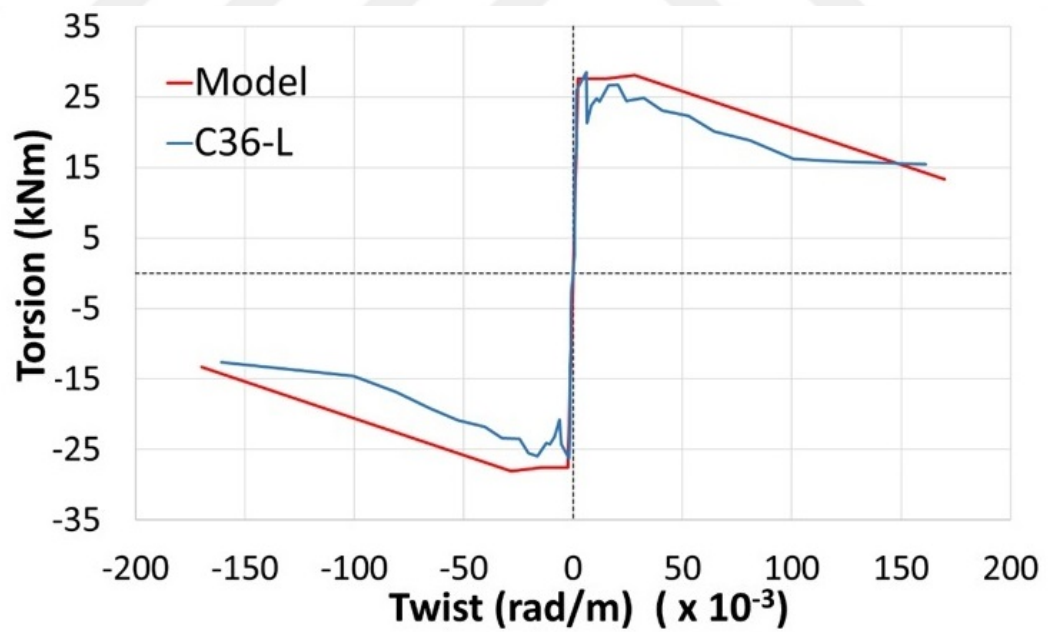


Figure 6.4. Comparison of experimental - theoretical backbone for C36-L.

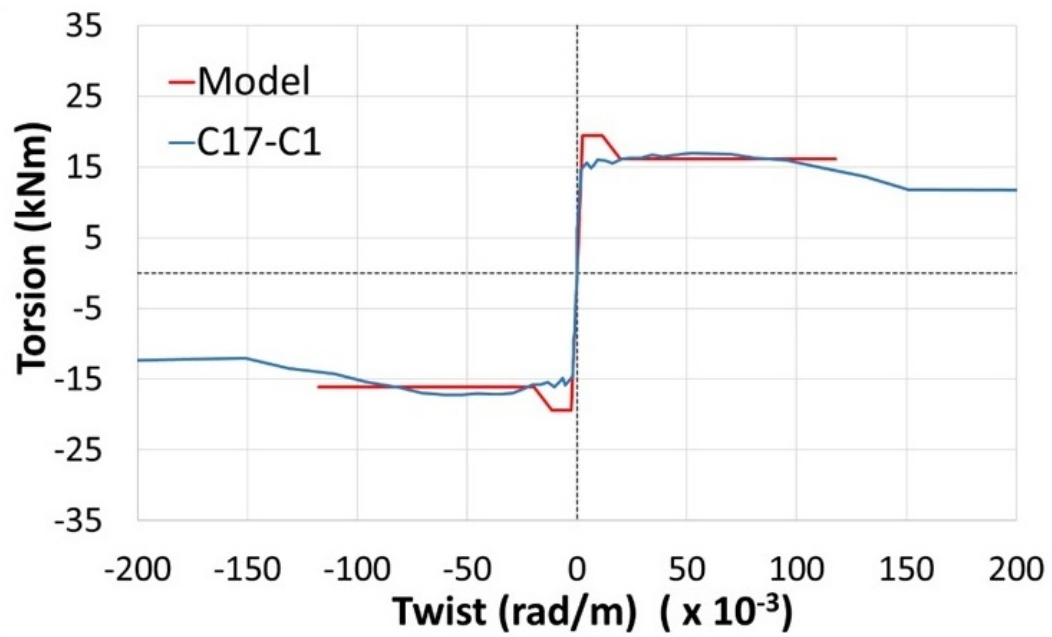


Figure 6.5. Comparison of experimental - theoretical backbone for C17-C1.

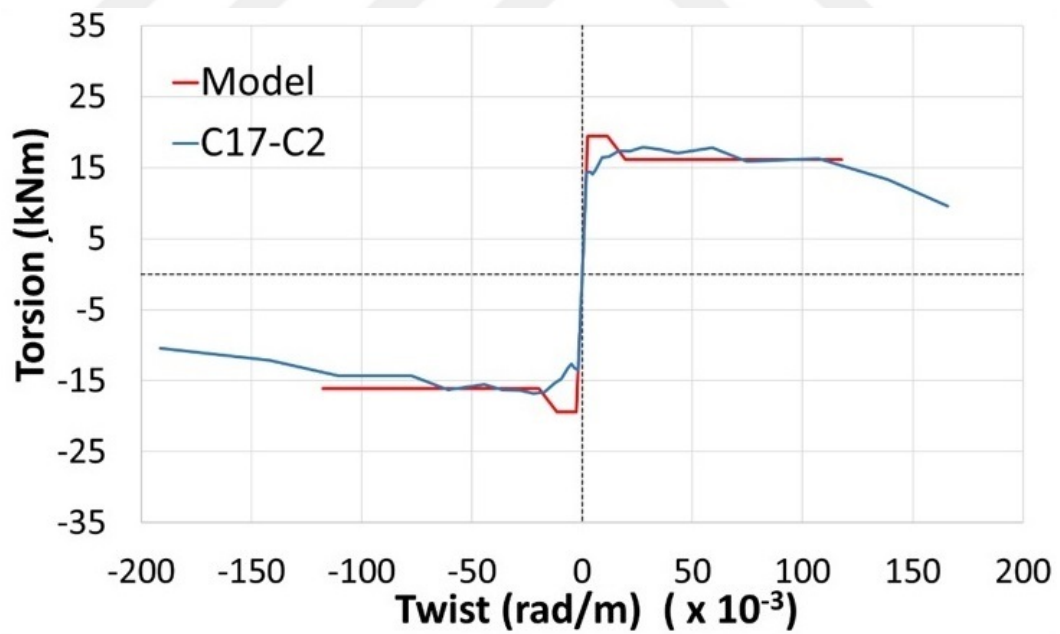


Figure 6.6. Comparison of experimental - theoretical backbone for C17-C2.

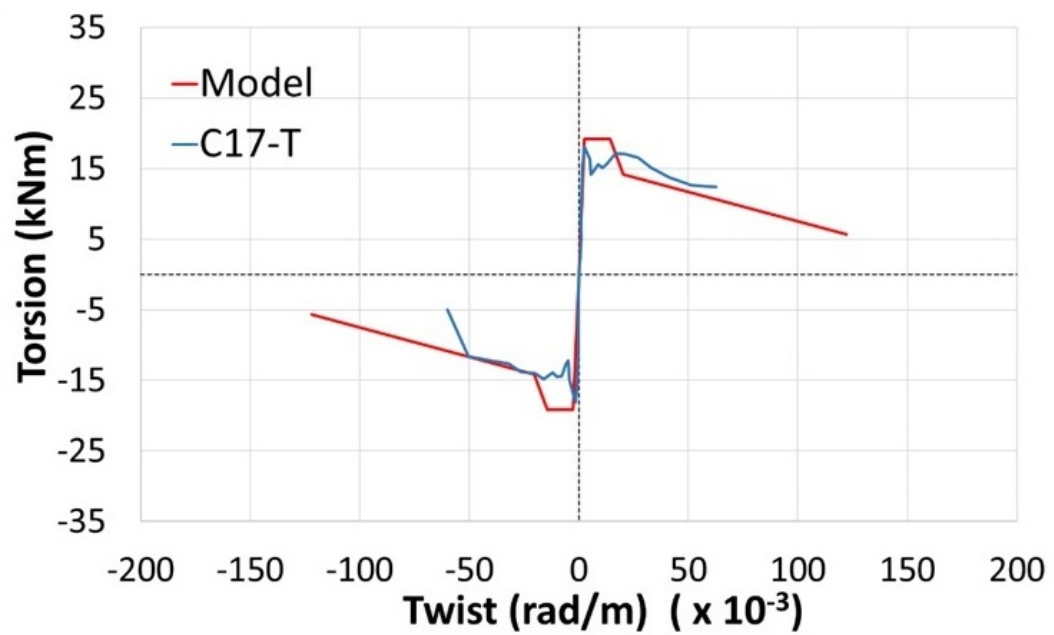


Figure 6.7. Comparison of experimental - theoretical backbone for C17-T.

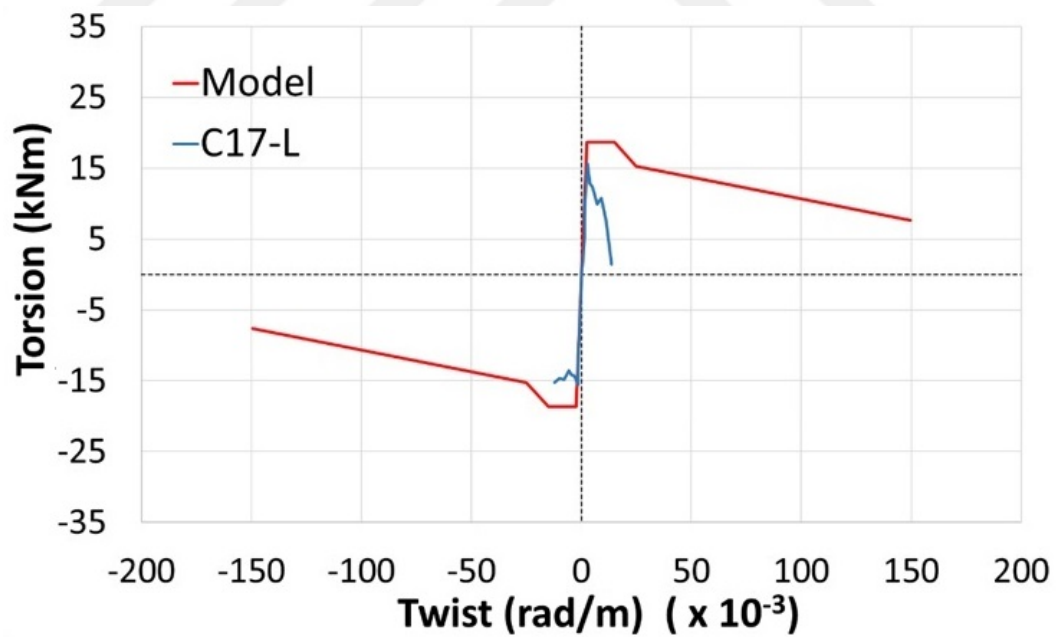


Figure 6.8. Comparison of experimental - theoretical backbone for C17-L.

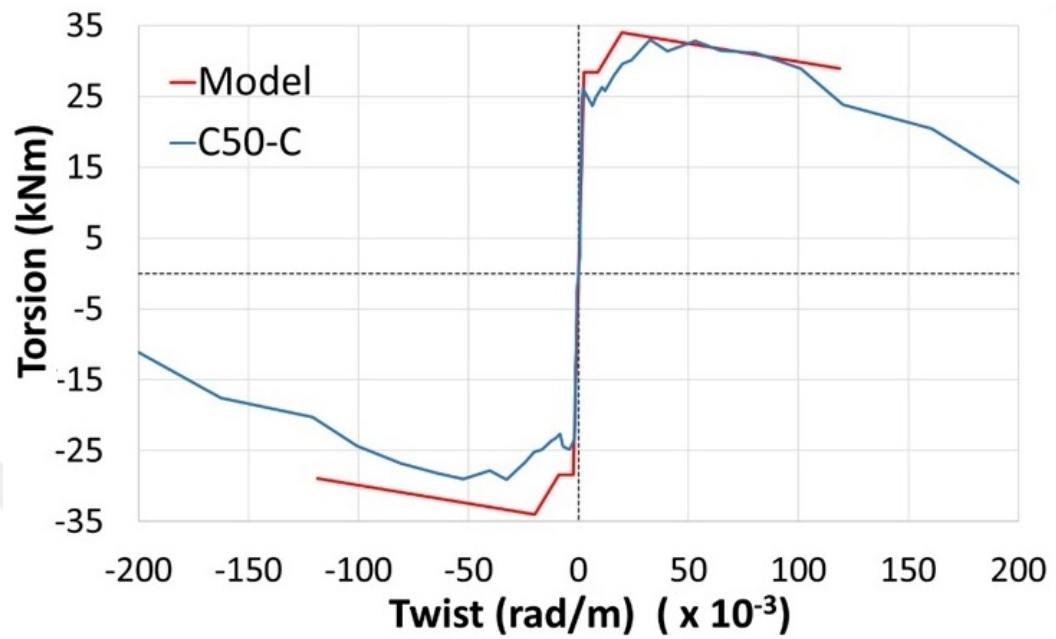


Figure 6.9. Comparison of experimental - theoretical backbone for C50-C.

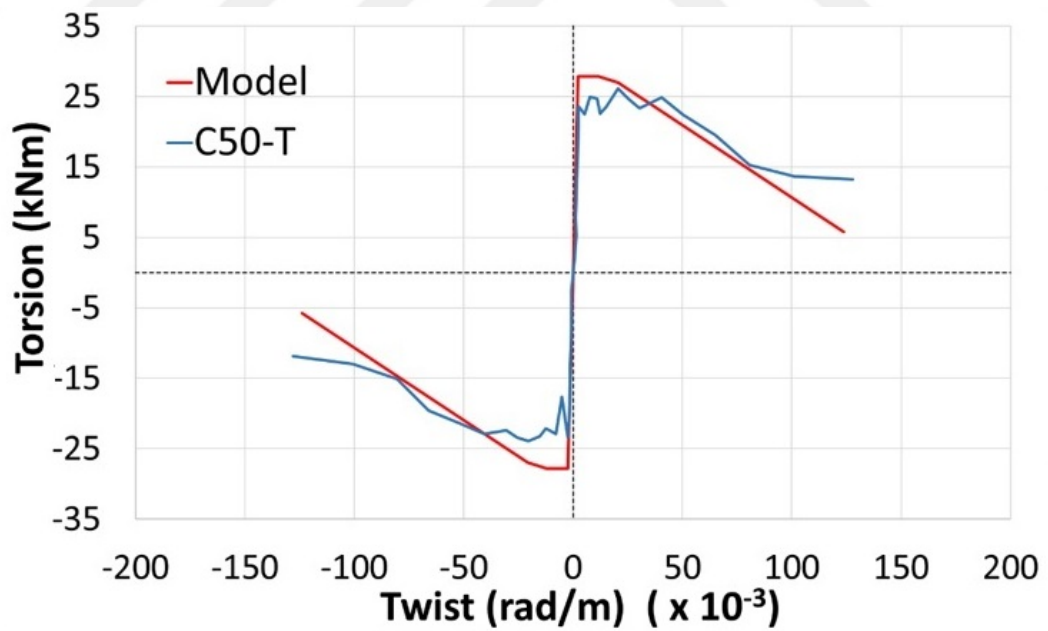


Figure 6.10. Comparison of experimental - theoretical backbone for C50-T.

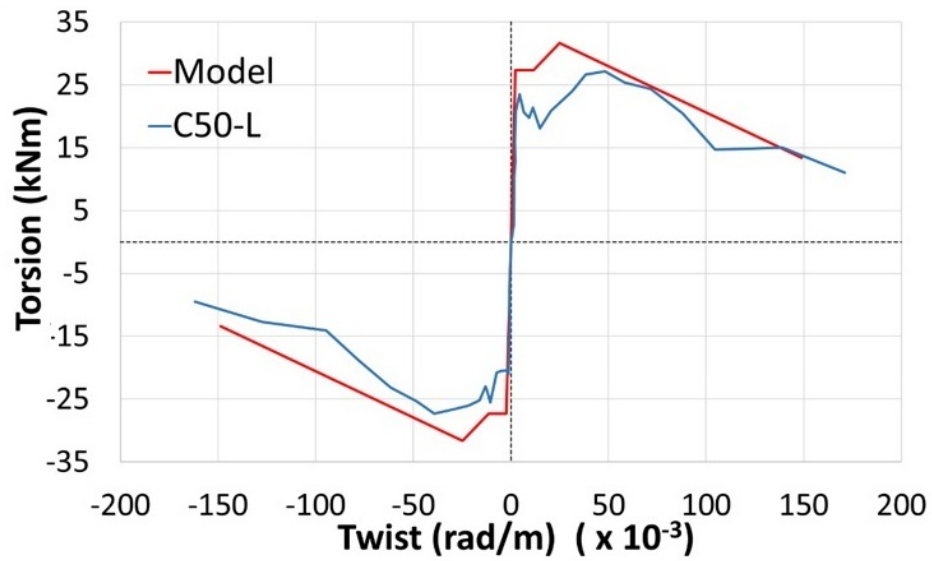


Figure 6.11. Comparison of experimental - theoretical backbone for C50-L.

6.2. Comparison of Analytical and Experimental Hysteresis Loops

Comparison of the analysis and test results are shown with the Figures 6.12 - 6.22. Also comparison of hysteresis loops are given in Appendix C.

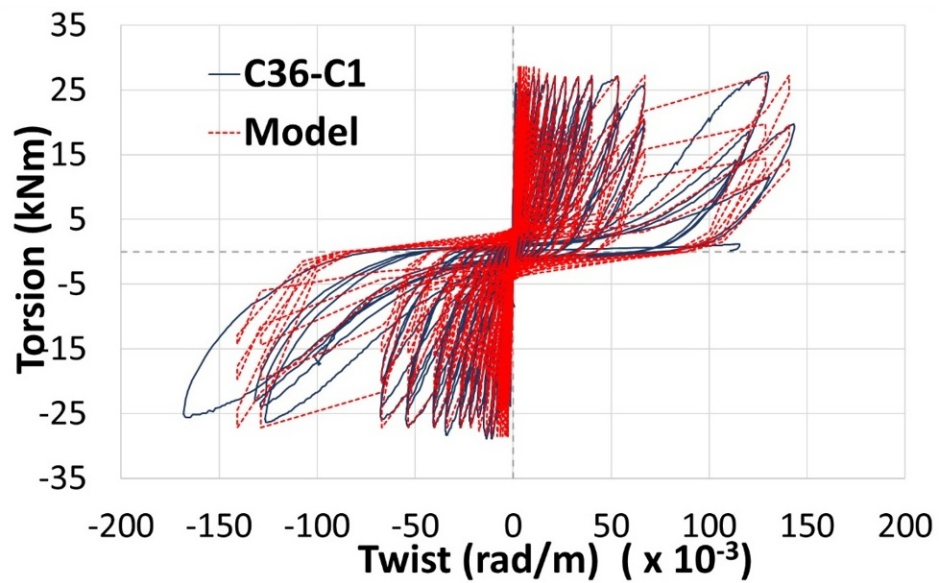


Figure 6.12. Experimental and theoretical torque-twist hysteresis of C36-C1.

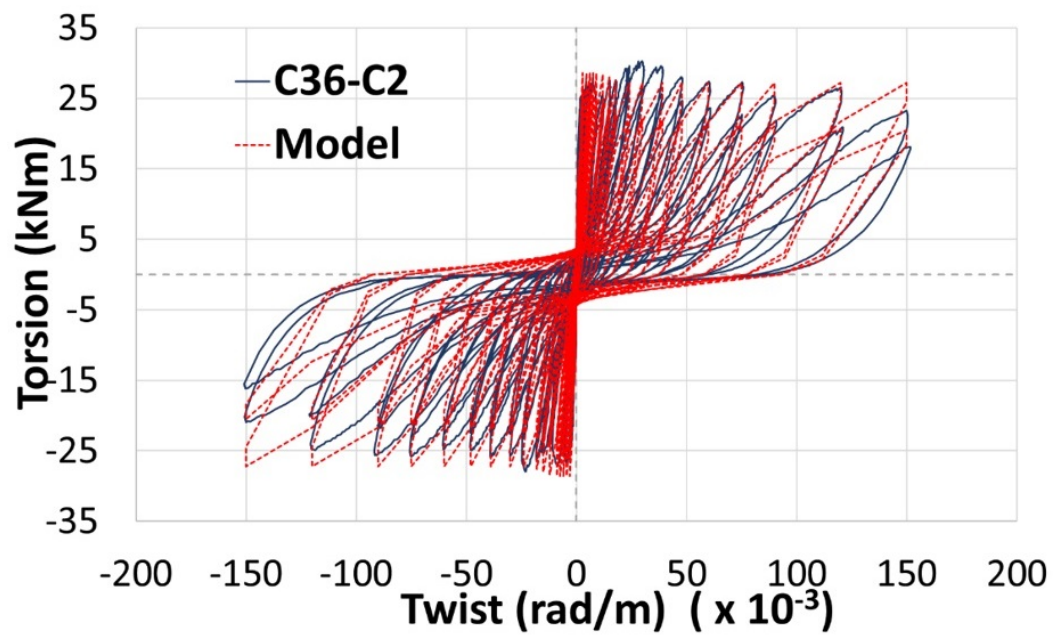


Figure 6.13. Experimental and theoretical torque-twist hysteresis of C36-C2.

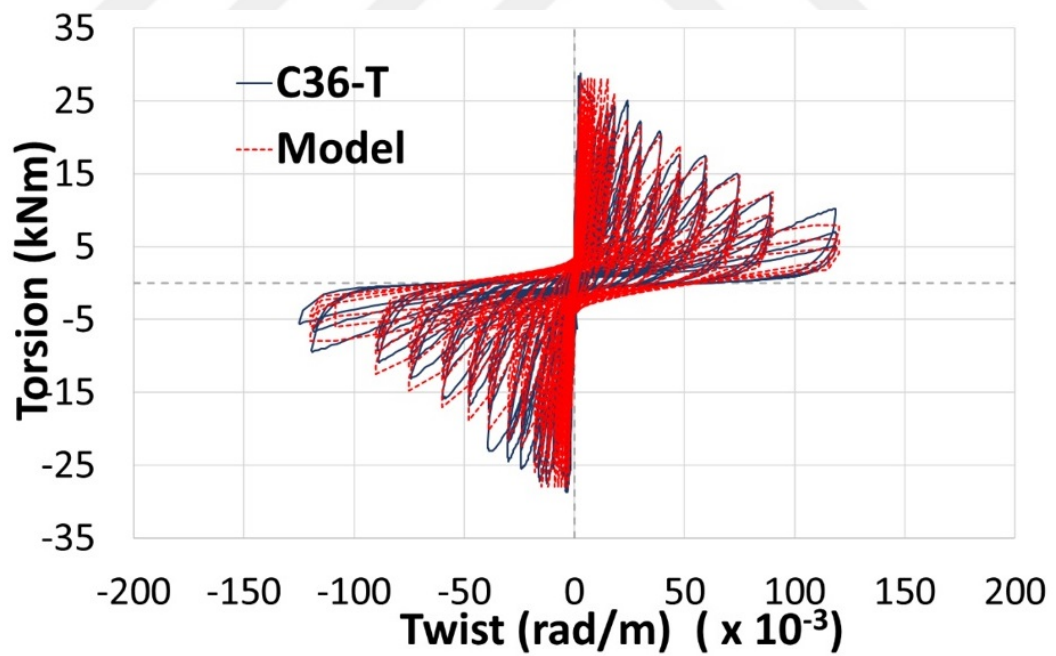


Figure 6.14. Experimental and theoretical torque-twist hysteresis of C36-T.

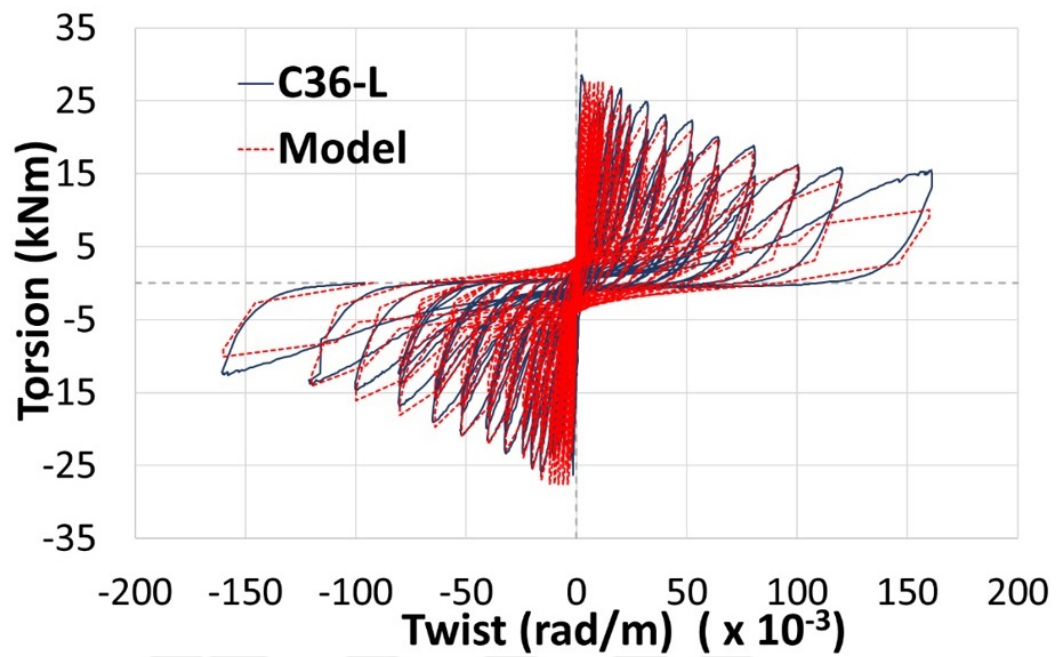


Figure 6.15. Experimental and theoretical torque-twist hysteresis of C36-L.

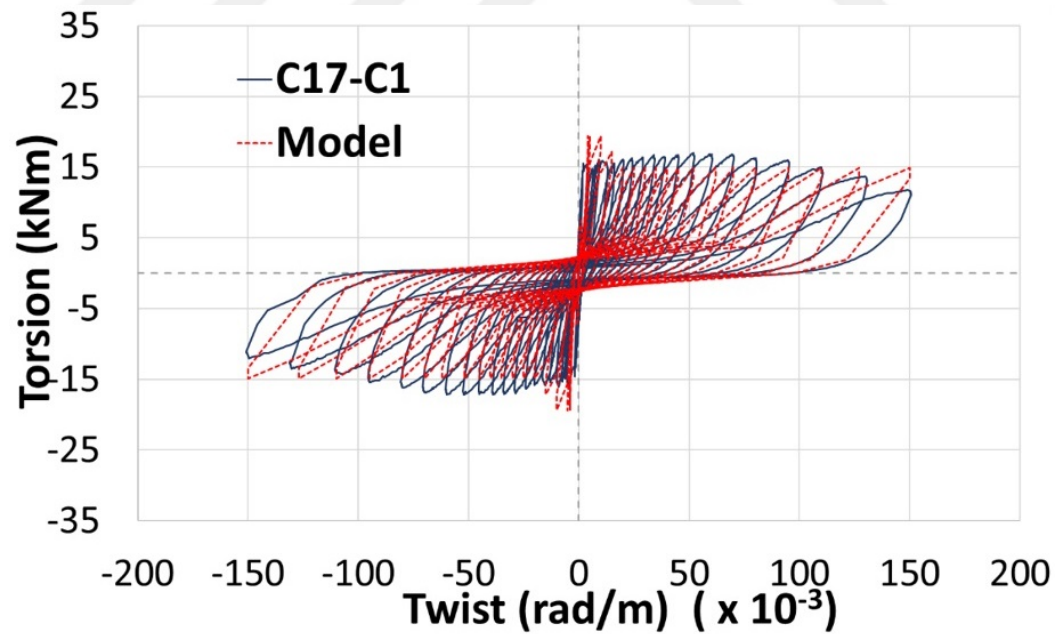


Figure 6.16. Experimental and theoretical torque-twist hysteresis of C17-C1.

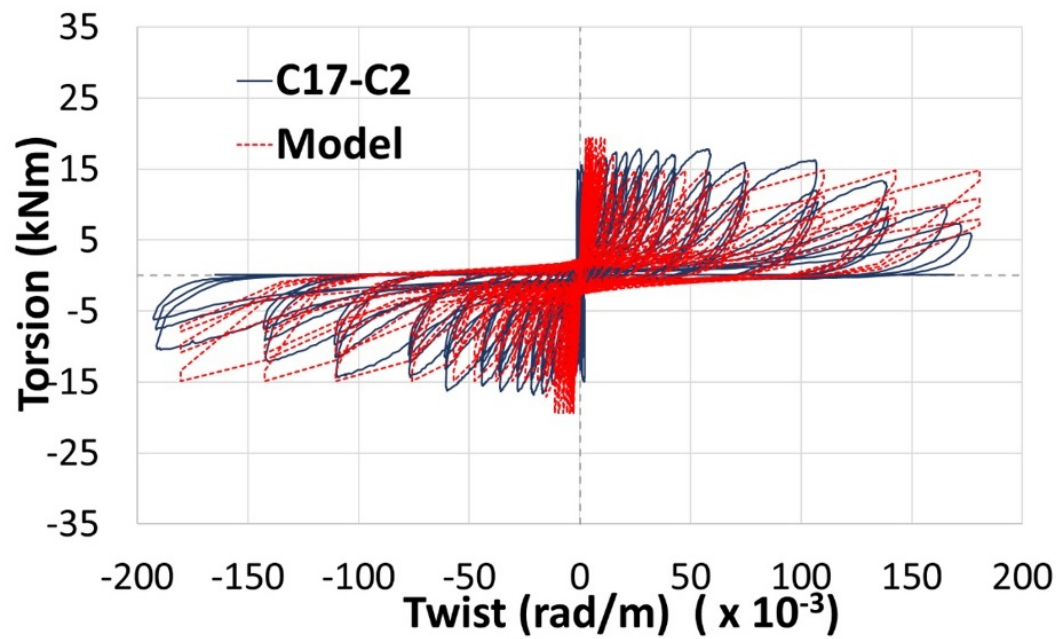


Figure 6.17. Experimental and theoretical torque-twist hysteresis of C17-C2.

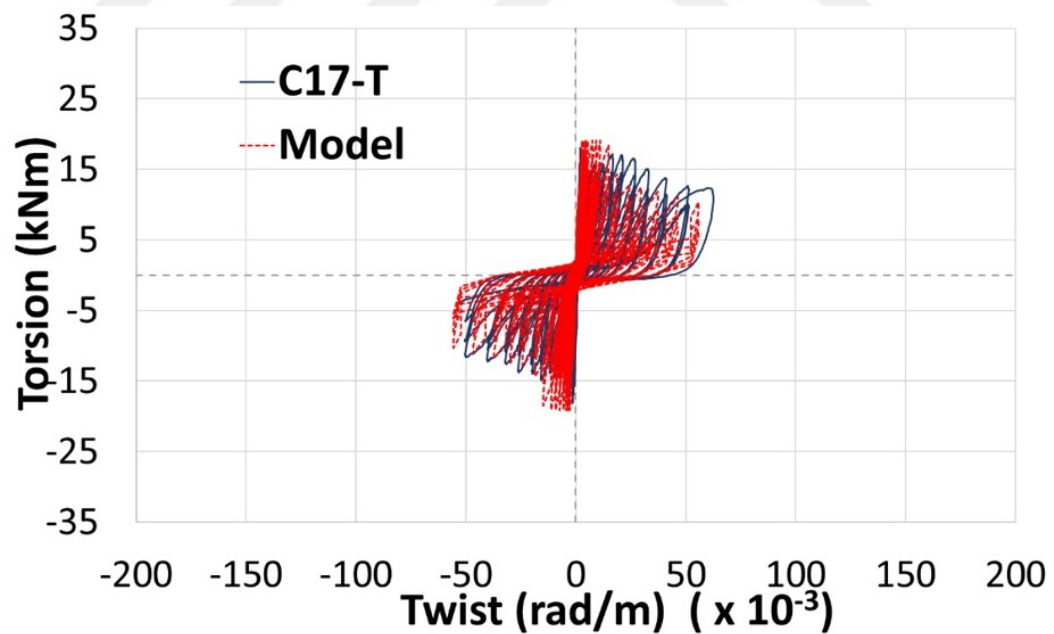


Figure 6.18. Experimental and theoretical torque-twist hysteresis of C17-T.

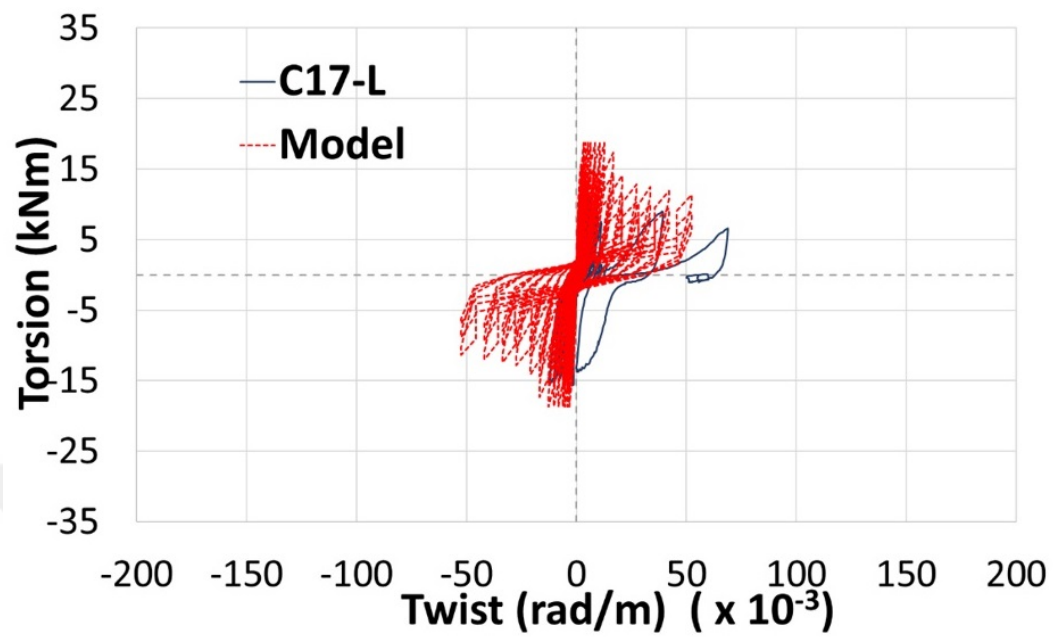


Figure 6.19. Experimental and theoretical torque-twist hysteresis of C17-L.

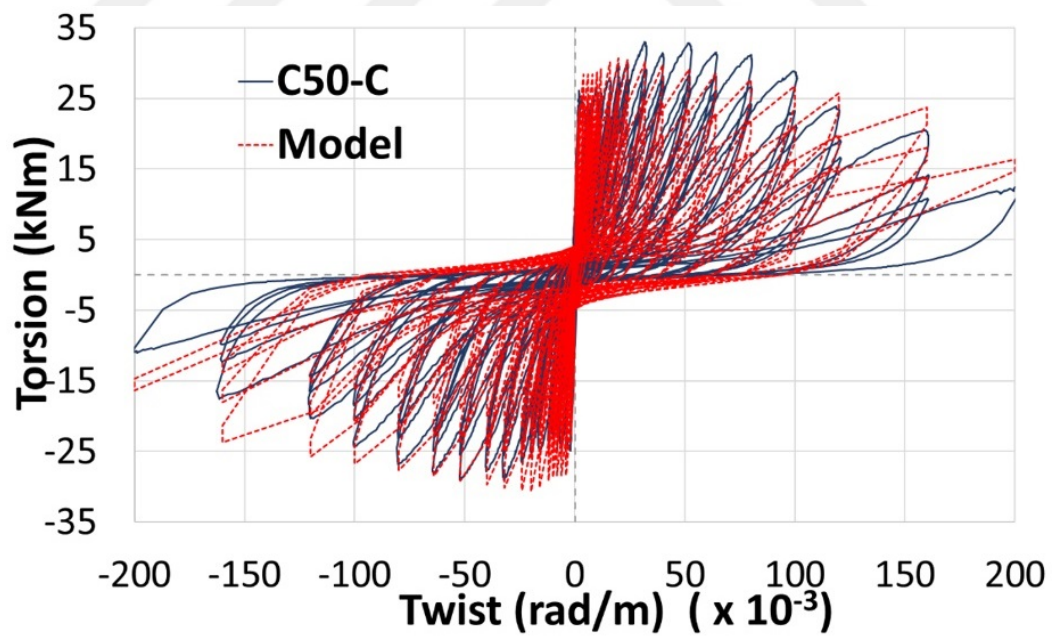


Figure 6.20. Experimental and theoretical torque-twist hysteresis of C50-C.

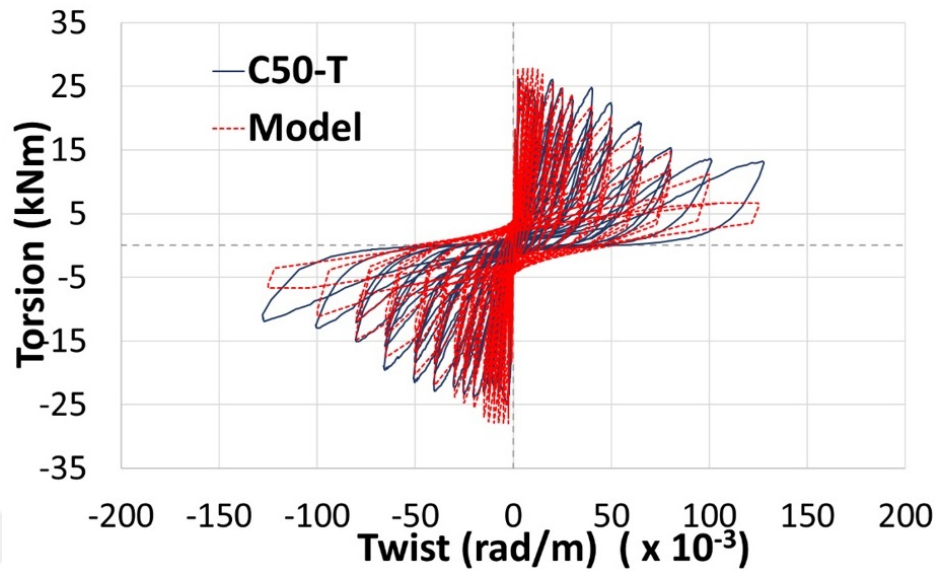


Figure 6.21. Experimental and theoretical torque-twist hysteresis of C50-T.

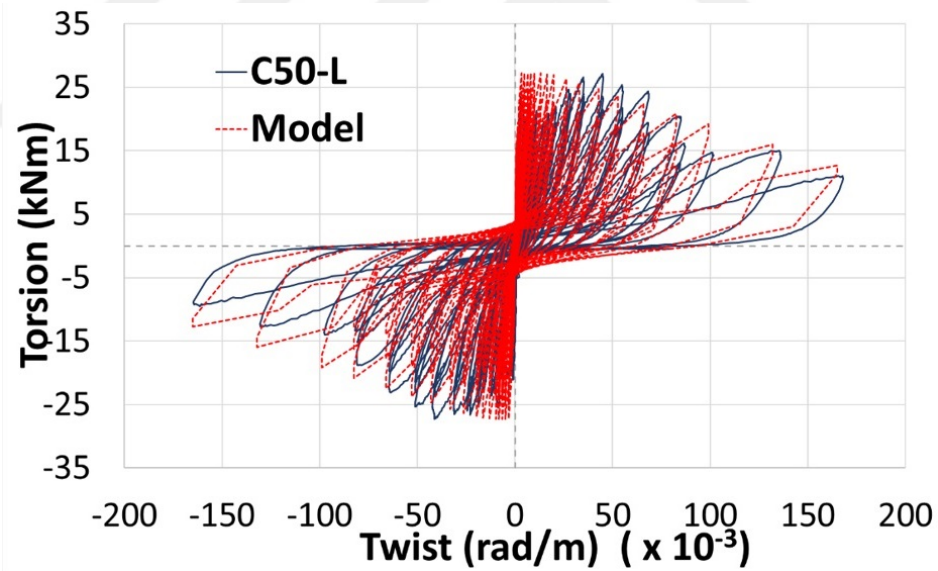


Figure 6.22. Experimental and theoretical torque-twist hysteresis of C50-L.

The cumulative energy dissipation capacities calculated from the area enclosed by experimental hysteresis curves and theoretical hysteresis curves are compared in Table 6.1. As seen from the table, the predicted cumulative energy dissipation values are in good agreement with experimental cumulative energy dissipation values. The failure C17-T specimen is developed out of the test region, so that the prediction for that specimen is not correct.

Table 6.1. Experimental and theoretical energy dissipation.

Specimen	Φ (10^{-3} rad/m)	$(Energy)_{exp}$ (Joule)	$(Energy)_{theo}$ (Joule)	$\frac{(Energy)_{theo}}{(Energy)_{exp}}$
C36-C1	140.7	16616.63	20004.41	1.20
C36-C2	150.0	17423.71	22983.28	1.32
C36-T	120.0	12230.05	14497.80	1.19
C36-L	160.0	13248.27	14486.07	1.09
C17-C1	150.0	10752.55	9900.47	0.92
C17-C2	180.5	15871.29	18829.61	1.19
*C17-T	55.8	5541.02	5389.82	0.97
C17-L	52.5	1247.02	5024.55	4.03
C50-C	136.0	35817.48	38397.30	1.07
C50-T	125.0	10273.22	12534.32	1.22
C50-L	165.0	14514.73	17845.55	1.23

*Experimental failure

Also, the comparison of experimental and analytical loop energy graphs are given in Appendix D and the comparison of experimental and analytical stiffness degradation curves are given in Appendix E.

6.3. Model Verification

In order to verify the model, several experiments from the literature is going to be analyzed. The tested members had different strength of concrete, aspect ratio and percentage of transverse reinforcement.

Okay and Engin studied the behavior of fiber reinforced square beams under pure torsional loading. Two of the specimens, called as the control specimens, were produced without fibers. The comparison of torsion-twist relations obtained from the experimental study (for specimens L08F00V0 and L12F00V0) and theoretical model defined above are given in Figure 6.23 and Figure 6.24. The result showed that the experimentally obtained and analytically obtained torsion twist behaviors of were in good agreement.

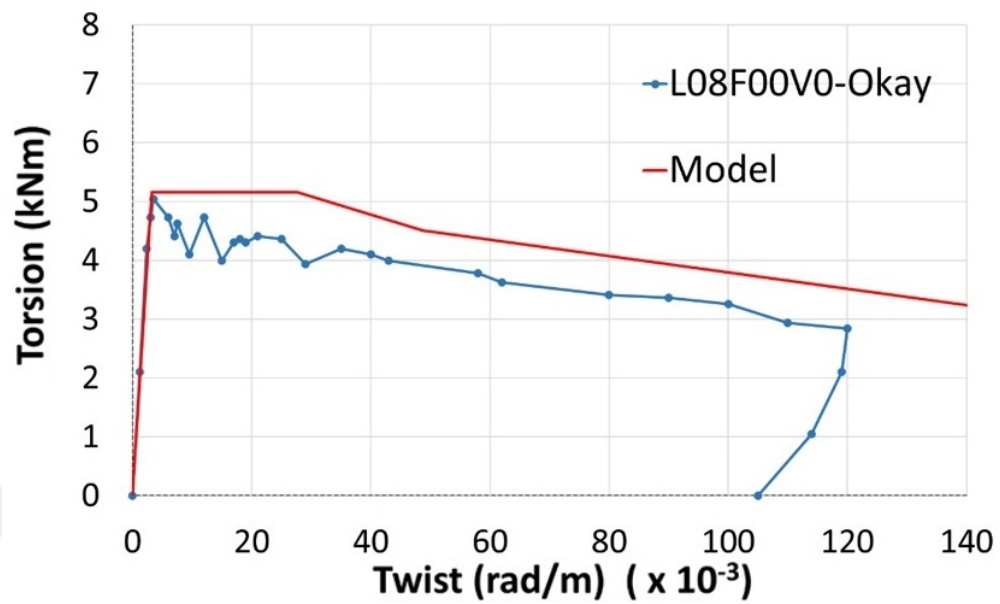


Figure 6.23. Predicted torque-twist curve of L08F00V0 [98].

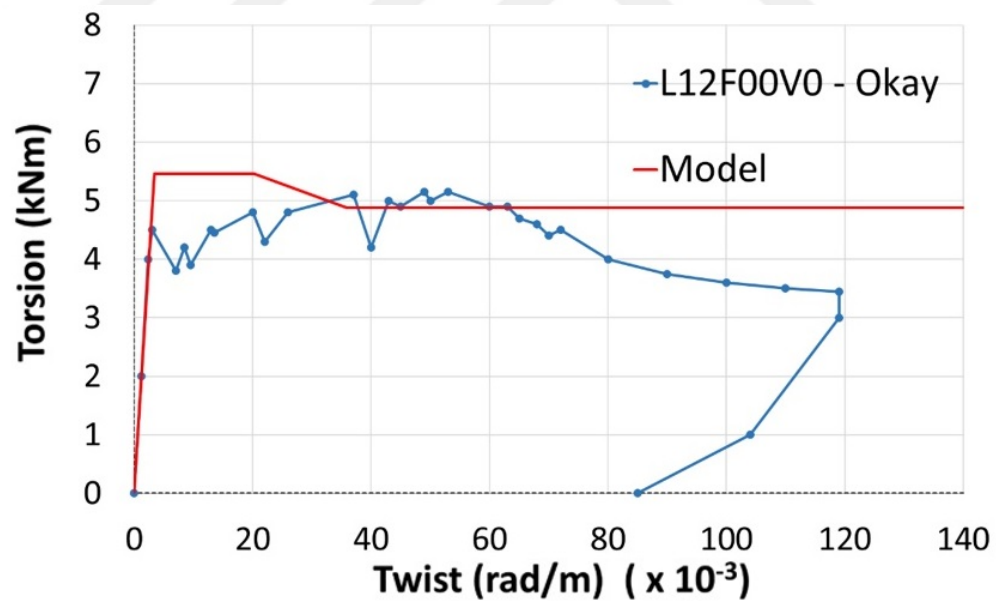


Figure 6.24. Predicted torque-twist curve of L12F00V0 [98].

Torsion of structural concrete was studied in 1968 [10] and Specimen G6 tested under pure torsion. This specimen was tested as monotonically and the torsion twist behavior of experiments and proposed model were drawn as shown in Figure 6.25. The cracking point and the behavior of curve is good predicted by analytical model.

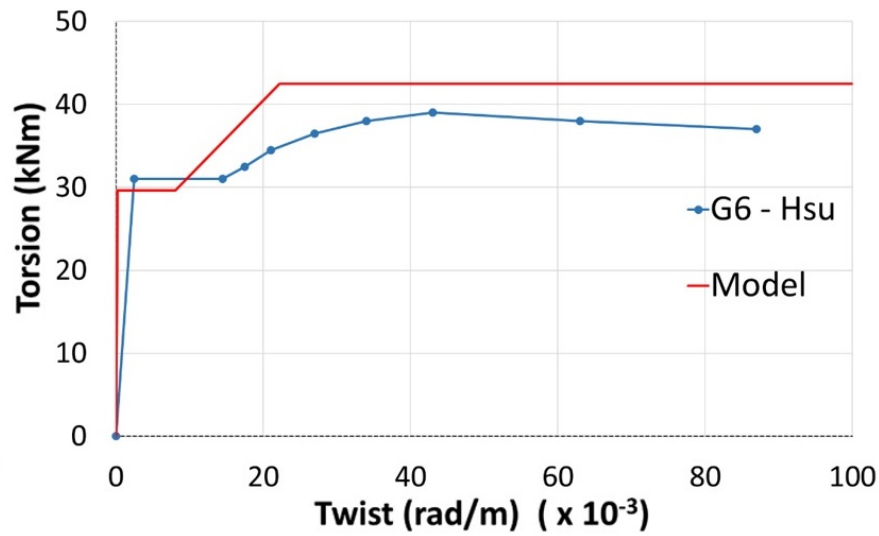


Figure 6.25. Predicted torque-twist curve of G6 [10].

One reinforced concrete specimen (Column P2) tested under the pure re-cyclic torsional moments by Tirasit and Kawashima [68, 99]. The experimental hysteresis taken from the literature is compared with predicted backbone curve and hysteresis in Figures 6.26 and 6.27 respectively. Torsional cracking calculated from the model is 57.9 kN.m and the twist value is 1.54×10^{-3} rad/m. These values have been obtained as 60.0 kN.m and 5.0×10^{-3} rad/m during the experiment.

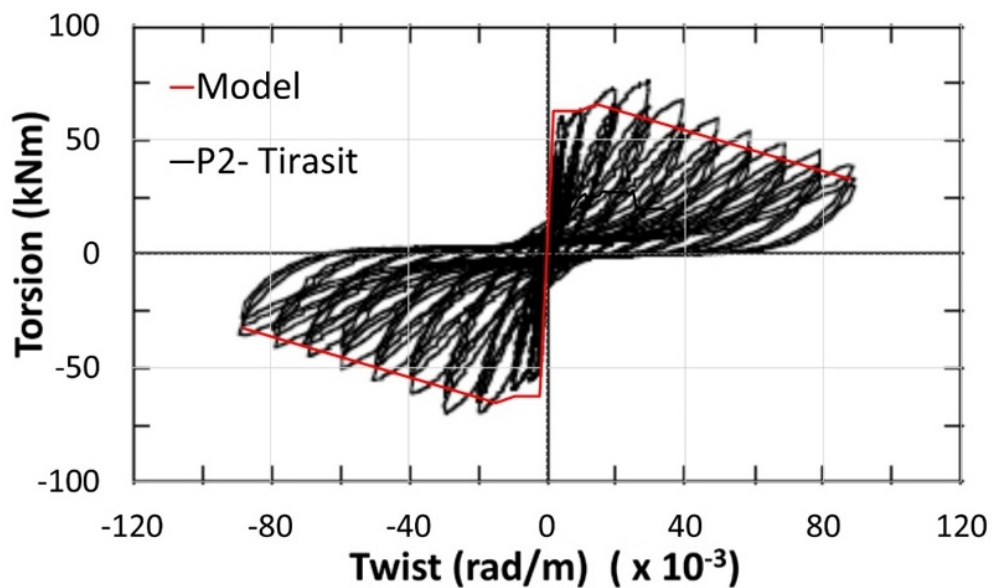


Figure 6.26. Predicted torque-twist curve of P2 [68].

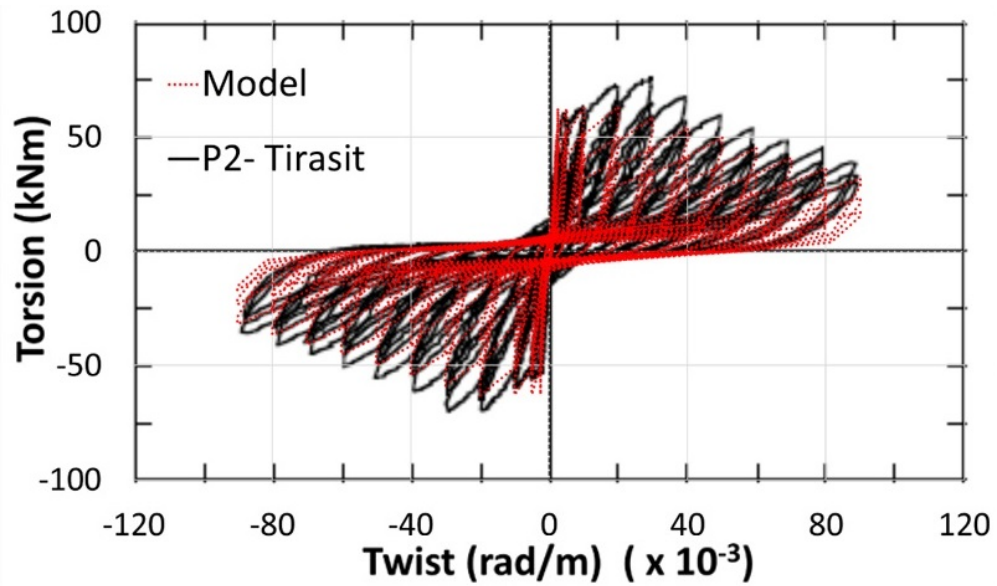


Figure 6.27. Predicted torque-twist hysteresis of P2 [68].

Wang et al. carried out a sets of experiments to examine the seismic response of RC beams produced with recycled aggregate subjected to cyclic torsion. And they named their natural aggregate samples NAC1 [100]. The test results of NAC1 were taken to compare the model backbone and hysteresis as shown in Figures 6.28 and 6.29.

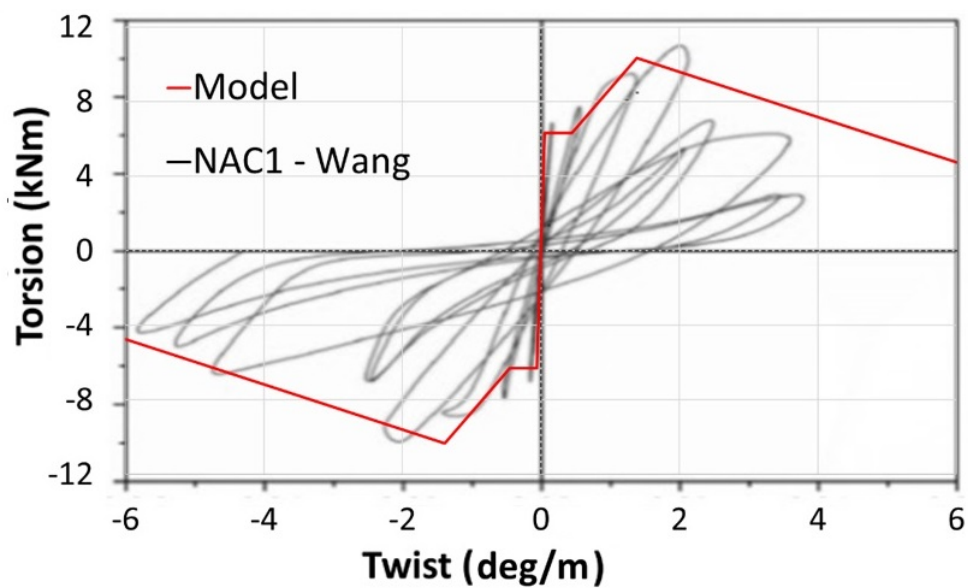


Figure 6.28. Predicted torque-twist curve of NAC1 [100].

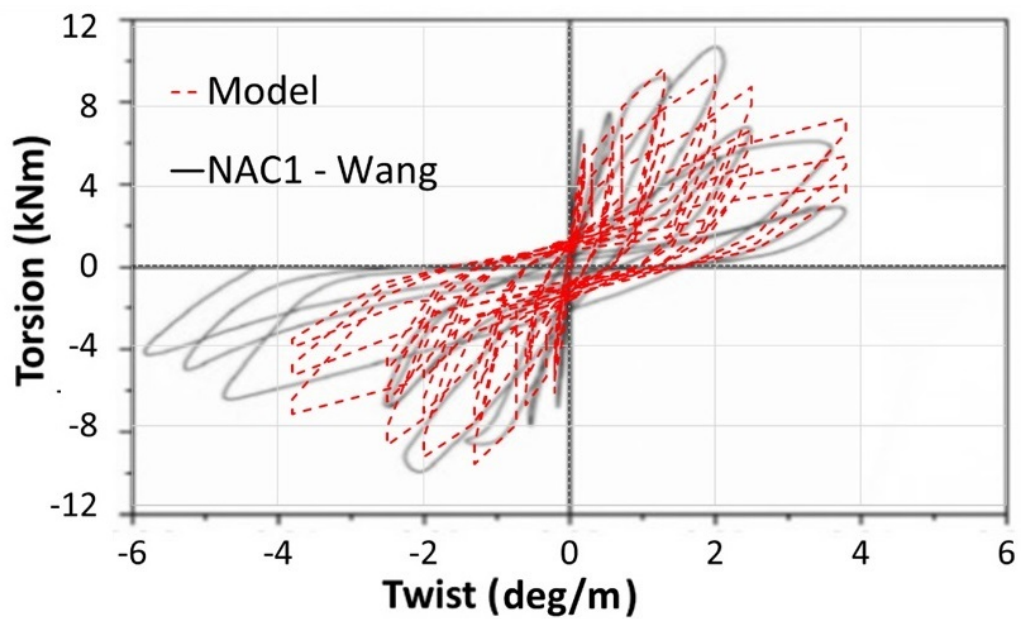


Figure 6.29. Predicted torque-twist hysteresis of NAC1 [100].

The developed model gives satisfactory results for different cross sections. Belarbi et al studied the torsional behavior of rectangular hollow box and circular members. The torsion twist backbone and hysteresis curves of 14.7 meter long hollow box girder [67] were given in Figures 6.30 and 6.31.

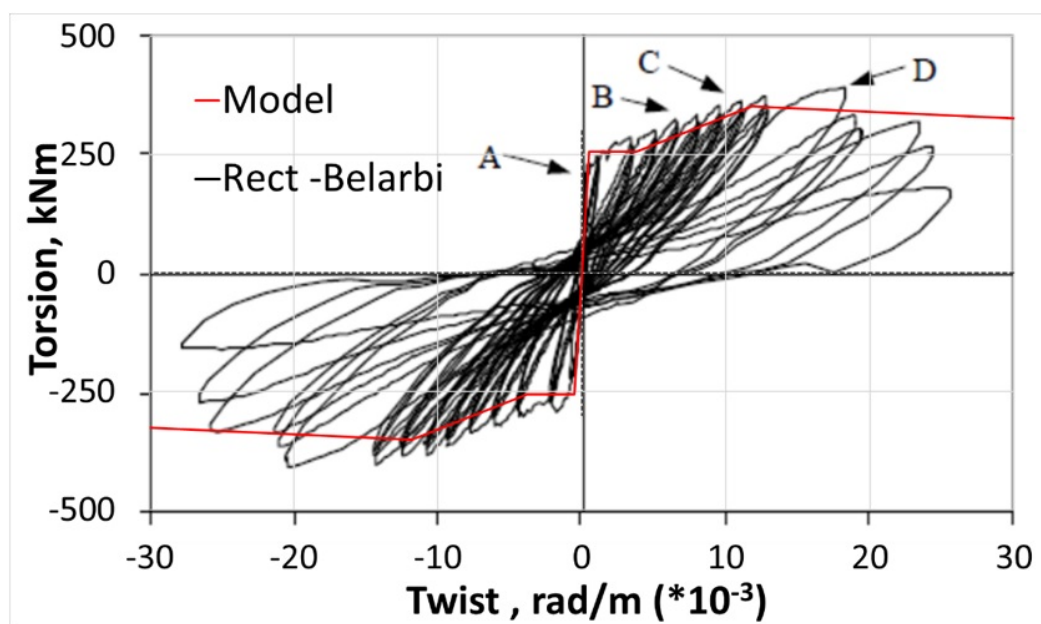


Figure 6.30. Predicted torque-twist curve of hollow section [67].

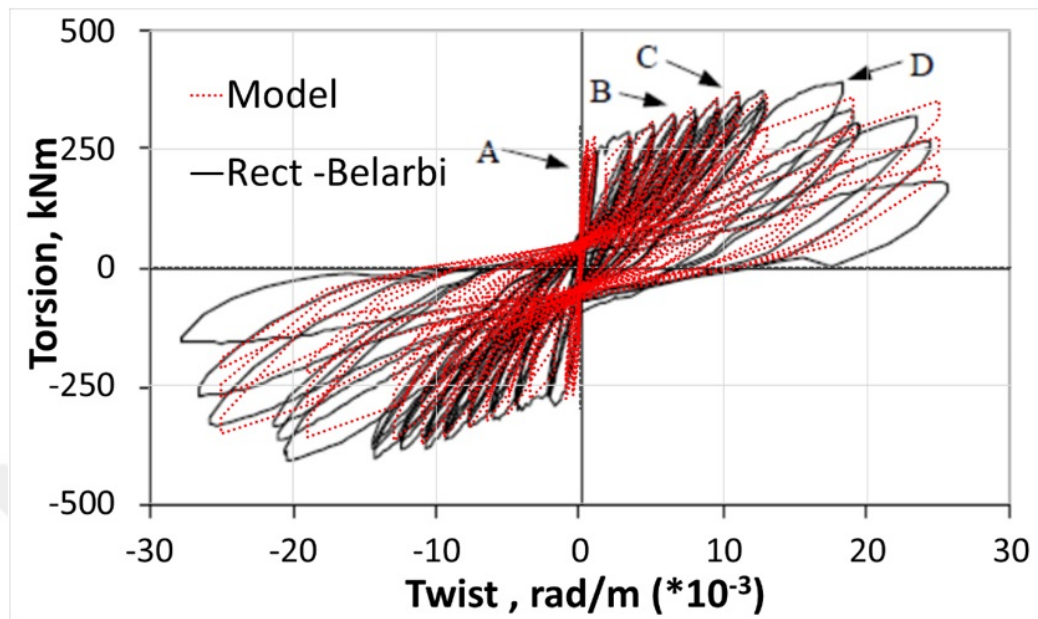


Figure 6.31. Predicted torque-twist hysteresis of hollow section [67].

The backbone curves and hysteresis loops obtained from the experiments of circular members with 0.73 percent and 1.32 percent spiral reinforcement ratio [101,102] were compared with the proposed model in Figures 6.32 to 6.34, respectively.

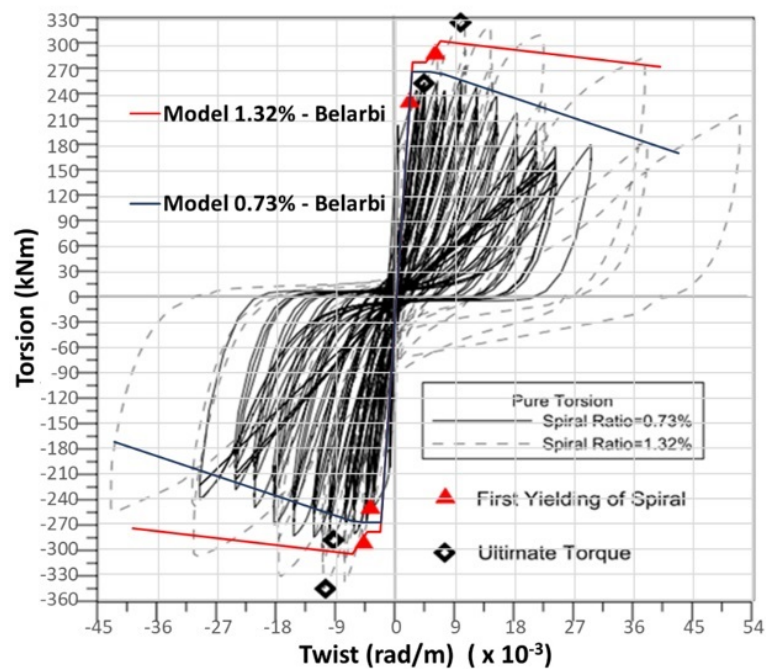


Figure 6.32. Predicted torque-twist curves of circular sections [101].

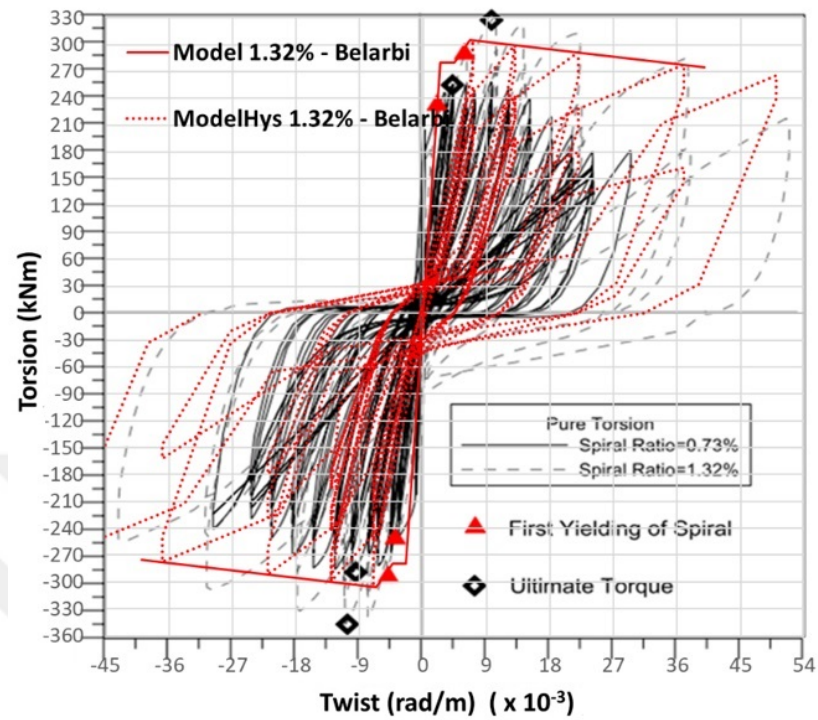


Figure 6.33. Predicted torque-twist hysteresis of circular sections (1.32 percent) [101].

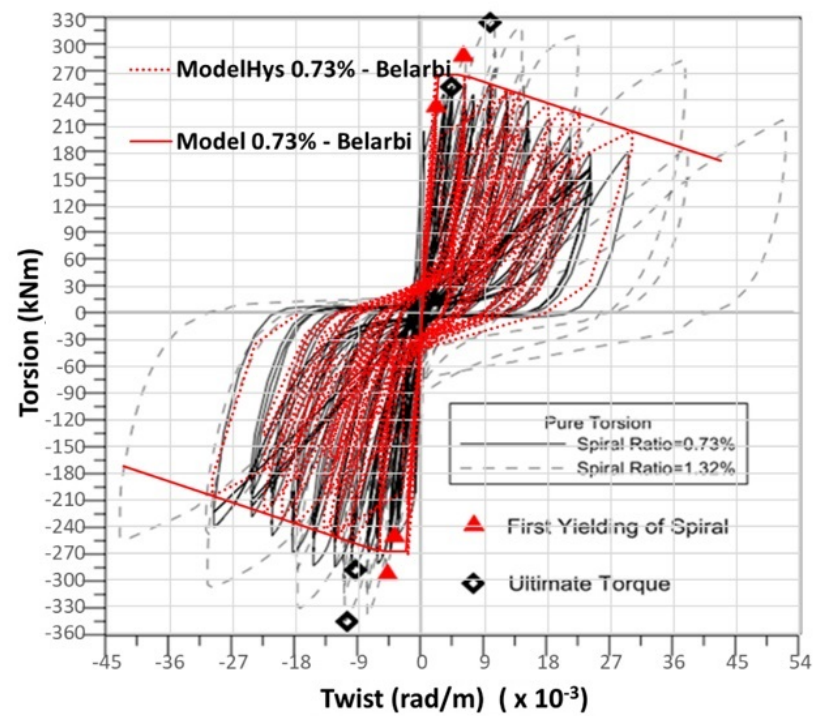


Figure 6.34. Predicted torque-twist hysteresis of circular sections (0.73 percent) [101].

7. CONCLUSION

An experimental series were conducted on 11 reinforced concrete members with different concrete strengths and reinforcement configuration, subjected to pure torsion, and focusing on the study of hysteresis and energy dissipation evolution. Based on the results, the following conclusions can be drawn:

- (i) The concrete strength and the reinforcement ratio have a significant influence on the total energy dissipation. The loops of the torque vs twist curve of the control specimens is fatter than the others, so that the energy dissipation capacities are higher in the control ones.
- (ii) When the concrete strength increases from 17 MPa to 36 MPa, and also from 36 MPa to 50 MPa, the ductility decreases about 20 percent, however energy dissipation increases about 20 percent. Hence, it can be said that increasing the concrete strength, reduces the ductility and increases energy dissipation capacity.
- (iii) Considering stirrup as the second parameter, when the transverse reinforcement ratio decreased by 50 percent, the energy dissipation decreased about 30 percent, and also the ductility decreased about 40 percent. Moreover, it can be concluded that, the reduction in ductility is more in low-strength concrete
- (iv) As the last parameter, when the longitudinal reinforcement ratio decreased by 50 percent, the energy dissipation decreased about 25 percent, and also the ductility decreased about 50 percent. Moreover, it can be concluded that, the reduction in ductility is more in low-strength concrete.
- (v) The failure of RC members under pure re-cyclic torsion was initiated by yielding of reinforcement, severe diagonal cracking of concrete, and concrete cover spalling and the failure was finalized by the formation of core concrete crushing near the middle of the member.
- (vi) Control specimens had the best performance under the re-cyclic torsional loading. The loss in the torsional moment and ductility capacity, due to the lack of stirrup was higher than the loss due to lack of longitudinal reinforcements.

- (vii) Before the cracking, the torsional stiffness is mainly affected by concrete strength but not by the amount of reinforcement. As the number of reinforcements increases, the number of cracks increases. The energy dissipation capacity increased within the higher twist levels. Successive loading at the same twist level decreases the energy dissipation capacity at a decreasing rate. The energy dissipation capacity approached a slightly descending asymptote for each twist level.
- (viii) Under pure re-cyclic torsional moment, longitudinal elongation develops in RC members. When elongations are prevented, in addition to torsional stresses, axial compressive stresses occur in the elements. Elongation has a significant effect on the seismic response of the structure and it can reduce the shear strength and deformation capacity of the members. It is found that the amount of elongation is related to the concrete compressive strength (f_c), of the specimen. The axial elongations were measured as 8.0 mm and 3.5 mm for Specimens C36-C and C50-C, respectively. This also showed the importance of proper test setup design.
- (ix) The strain-torsional moment curves of longitudinal reinforcement and stirrup performed a butterfly-shape due to the cyclic torsion loading.

The test results were investigated and an appropriate hysteresis model was developed using the test results. In this study, thin walled tube theory up to the cracking, space truss analogy for the post cracking stage and finally, the truss model analogy for the cracking branch was used for the proposed model. Rules of the hysteretic loops were developed to predict experimentally observed stiffness degradation, pinching and softening. The developed torque-twist hysteretic model was in agreement with the experimental results.

REFERENCES

1. ACI-Committee-318, “Building Code Requirements for Structural Concrete:(ACI 318-11) and Commentary (ACI 318R-11)”, *American Concrete Institute*, American Concrete Institute, Farmington Hills, 2011.
2. Coulomb, C. A., *Theoretical and Experimental Research on the Force of Torsion, and on the Elasticity of Metal Wires*, Mémoires de l’Académie Royale des Sciences, France, 1784.
3. Navier, C., *Résumé des Lecons Données à L’école des Ponts et Chausses sur L’application de la Mécanique à L’établissement des Constructions et des Machines*, Firmin Didot père et Fils, Paris, 1826.
4. Saint-Venant, A. J. C. B., *Memoire Sur La Torsion Des Prismes*, Mémoires Présentés Par Divers Savants, Paris, 1855.
5. Bredt, R., “Kritische Bemerkungen zur Drehungselastizitat”, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 40, No. 28, pp. 785–790, 1896.
6. Irgens, F., *Continuum Mechanics*, Springer, Berlin, 2008.
7. Timoshenko, S. and J. Goodier, *Theory of Elasticity*, McGraw-Hill, New York, 1951.
8. Pillai, S. U. and D. Menon, *Reinforced Concrete Design*, Second Edition, Tata McGraw Hill Publishing Company, New Jersey, 2003.
9. Kemp, E. L., M. A. Sozen and C. P. Siess, *Torsion in Reinforced Concrete- A Report on a Research Project*, University of Illinois, Urbana, 1961.
10. Hsu, T. T. C., “Torsion of Structural Concrete-Behavior of Reinforced Concrete

- Rectangular Members, In Torsion of Structural Concrete ACI SP-18", *American Concrete Institute*, Vol. 18, No. 10, pp. 261–306, 1968.
11. Wang, C. K. and C. G. Salmon, *Reinforced Concrete Design*, Fifth Edition, Addison-Wesley, Dallas, 1998.
 12. Reportedby-Joint-ACI-ASCE-Committee-445, *Report on Torsion in Structural Concrete*, Vol. 1, American Concrete Institute, Texas, 2013.
 13. Wang, W. and T. T. C. Hsu, "Limit Analysis of Reinforced Concrete Beams Subjected to Pure Torsion", *Journal of Structural Engineering, ASCE*, Vol. 123, No. 1, pp. 86–94, 1997.
 14. Ritter, W., "The Hennebique Design Method (Die Bauweise Hennebique)", *Schweizerische Bauzeitung*, Vol. 33, No. 7, pp. 59–61, 1899.
 15. Mörsch, E., *Concrete-Steel Construction: (Der Eisenbetonbau)*, Engineering News Publishing Company, Michigan, 1909.
 16. Prandtl, L., "Zur Torsion Von Prismatischen Stäben", *Zeitschrift für Physik*, Vol. 4, pp. 758–770, 1903.
 17. Nadai, A., "Der Beginn des Fließvorganges in Einem Tordierten Stab", *Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 3, No. 6, pp. 442–454, 1923.
 18. Sadowsky, M. A., "An Extension of the Sand-Heap Analogy in Plastic Torsion Applicable to Cross Sections Having One or More Holes", *Journal of Applied Mechanics*, Vol. 8, No. 4, pp. 166–168, 1941.
 19. Rausch, E., *Design of Reinforced Concrete in Torsion (Berechnung des Eisenbetons Gegen Verdrehung und Abscheren)*, Ph.D. Thesis, Berlin Technische Hochschule, 1929.

20. Lessig, N. N., *The Determination of the Load-Bearing Capacity of Reinforced Concrete Elements with Rectangular Sections Subjected to Flexure and Torsion Work, (Translation: Foreign Literature Study)*, Concrete and Reinforced Concrete Institute, PCA Research and Development Laboratories, Moscow, 1959.
21. Yudin, V. K., “Determination of the Load-Bearing Capacity of Reformed Concrete Elements of Rectangular Cross-section under Combined Torsion and Bending”, *Betoni Zhelkezobeton*, Vol. 6, No. 8, p. 265, 1962.
22. Lampert, P. and B. Thürlimann, “Torsionsversuche an Stahlbetonbalken (Torsion Tests of Reinforced Concrete Beams)”, *Bericht/Institut für Baustatik*, Vol. 6506, No. 2, 1968.
23. Lampert, P. and B. Thürlimann, “Torsion-Beuge-Versuche an Stahlbetonbalken (Torsion-Bending Tests on Reinforced Concrete Beams)”, *Bericht/Institut für Baustatik*, Vol. 6506, No. 3, 1969.
24. Elfgren, L. and I. Karlsson, *Tests on Rectangular Reinforced Concrete Beams in Combined Torsion, Bending and Shear*, Report 71:2, Division of Concrete Structures, Chalmers University of Technology, Göteborg, 1971.
25. Elfgren, L., *Reinforced Concrete Beams Loaded in Combined Torsion, Bending and Shear. A study of the Ultimate Load-Carrying Capacity*, Ph.D. Thesis, Chalmers University of Technology, 1972.
26. Elfgren, L., “Torsion-Bending-Shear in Concrete Beams : A Kinematic Model”, *Plasticity in Reinforced Concrete : Final Report*, Vol. 29 of *Reports of the Working Commissions- IABSE Reports*, pp. 111–118, Zürich, 1979.
27. Elfgren, L., I. Karlsson and A. Losberg, “Torsion-Bending-Shear Interaction for Concrete Beams”, *Journal of the Structural Division*, Vol. 100, No. 8, pp. 1657–1676, 1974.

28. Elfgren, L., I. Karlsson and A. Losberg, “Nodal Forces in the Analysis of the Ultimate Torsional Moment for Rectangular Beams”, *Magazine of Concrete Research*, Vol. 26, No. 86, pp. 21–28, 1974.
29. Collins, M. P. and D. Mitchell, “Discussion of Torsional-Balanced Steel in Concrete Beams”, *Journal of the Structural Division*, Vol. 104, No. 10, pp. 1684–1687, 1978.
30. Collins, M. P. and D. Mitchell, “Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams”, *PCI Journal*, Vol. 25, No. 5, pp. 32–100, 1980.
31. Collins, M. P. and D. Mitchell, *Prestressed Concrete Structures*, Prentice Hall, New Jersey, 1991.
32. Hsu, T. T. C., “Torsion of Structural Concrete, A Summary of Pure Torsion, In Torsion of Structural Concrete ACI SP-18”, *American Concrete Institute*, Vol. 18, No. 06, pp. 165–178, 1968.
33. Hsu, T. T. C., “Torsion of Structural Concrete- Plain Concrete Rectangular Sections, In Torsion of Structural Concrete ACI SP-18”, *American Concrete Institute*, Vol. 18, No. 08, pp. 203–238, 1968.
34. Hsu, T. T. C. and Y. Mo, “Softening of Concrete in Torsional Members-Theory and Tests”, *Journal of American Concrete Institute*, Vol. 82, No. 3, pp. 290–303, 1985.
35. Jeng, C. H. and T. T. C. Hsu, “A Softened Membrane Model for Torsion in Reinforced Concrete Members”, *Engineering Structures*, Vol. 31, No. 9, pp. 1944–1954, 2009.
36. Hsu, T. T. C. and R. R. H. Zhu, “Softened Membrane Model for Reinforced Concrete Elements in Shear”, *Structural Journal*, Vol. 99, No. 4, pp. 460–469, 2002.

37. Bernardo, L. F. A., J. M. A. Andrade and S. M. R. Lopes, “Softened Truss Model for Reinforced NSC and HSC Beams under Torsion: A Comparative Study”, *Engineering Structures*, Vol. 42, pp. 278–296, 2012.
38. Bernardo, L., “Generalized Softened Variable Angle Truss Model for RC Hollow Beams under Torsion”, *Materials*, Vol. 12, No. 13, p. 2209, 2019.
39. Christopherson, D. G., “A Theoretical Investigation of Plastic Torsion in an I-Beam”, *Journal of Applied Mechanics*, Vol. 7, No. 1, pp. 1–4, 2021.
40. Sokolovsky, W. W., “The Theory of Plasticity—An Outline of Work Done in Russia”, *Journal of Applied Mechanics*, Vol. 13, No. 1, pp. 1–10, 1946.
41. Nadai, A., *Theory of Flow and Fracture of Solids*, McGraw-Hill, New York, 1954.
42. Armstrong, S., “The Strength of Concrete Members in Combined Bending and Torsion”, *Symposium on Strength of Concrete Structures, Cement and Concrete Association*, London, pp. 101–122, 1956.
43. Humphreys, R., “Torsional Properties of Prestressed Concrete”, *The Structural Engineering*, Vol. 35, No. 6, pp. 213–224, 1957.
44. McMullen, A. E. and H. R. Daniel, “Torsional Strength of Longitudinally Reinforced Beams Containing an Opening”, *ACI Journal*, Vol. 72, No. 8, pp. 415–420, 1975.
45. Bazant, Z. P. and S. Sener, “Size Effect in Torsional Failure of Concrete Beams”, *Journal of Structural Engineering, ASCE*, Vol. 113, No. 10, pp. 2125–2130, 1987.
46. Bažant, Z. P., “Size Effect Tests of Torsional Failure of Plain and Reinforced Concrete Beams”, *Materials and Structures / Matériaux et Constructions*, Vol. 21, No. 6, pp. 425–430, 1988.

47. Kirane, K., K. Singh and Z. Bažant, “Size Effect in Torsional Strength of Plain and Reinforced Concrete”, *ACI Structural Journal*, Vol. 113, No. 6, pp. 1253–1262, 2016.
48. Subramaniam, K., J. Popovics and S. Shah, “Testing Concrete in Torsion: Instability Analysis and Experiments”, *Journal of Engineering Mechanics - ASCE*, Vol. 124, No. 11, pp. 1258–1268, 1998.
49. Goode, C. and M. Helmy, “Ultimate Strength of Reinforced Concrete Beams Subjected to Combined Torsion and Bending, In Torsion of Structural Concrete, ACI SP-18”, *American Concrete Institute*, Vol. 18, No. 13, pp. 361–393, 1968.
50. Hsu, T. T. C., “Ultimate Torque of Reinforced Rectangular Beams”, *Journal of the Structural Division*, Vol. 94, No. 2, pp. 485–510, 1968.
51. Rangan, B. V., R. F. Staley and A. S. Hall, “Behavior of Concrete Beams in Torsion and Bending”, *Journal of the Structural Division*, Vol. 103, No. 4, pp. 759–772, 1977.
52. Nylander, H., *Vridning och Vridningsinspänning vid Betongkonstruktioner: Torsion and Torsional Restraint by Concrete Structures Bull. D-19*, Statens Kommittee för Byggnadsforskning, Stockholm, 1945.
53. Collins, M. P., P. F. Walsh, F. E. Archer and A. S. Hall, “Reinforced Concrete Beams Subjected to Combined Torsion, Bending and Shear”, *University of New South Wales, UNICIV Report No. R-14*, p. 41, 1965.
54. Walsh, P. F., M. P. Collins, F. E. Archer and A. S. Hall, “The Ultimate Strength Design of Rectangular Reinforced Concrete Beams Subjected to Combined Torsion, Bending and Shear”, *Civil Engineering Transactions, The Institution of Engineers*, Vol. 8, No. 2, pp. 143–157, 1966.
55. Hsu, T. T. C., “Shear Flow Zone in Torsion of Reinforced Concrete”, *Journal of*

- Structural Engineering*, Vol. 116, No. 11, pp. 3206–3226, 1990.
56. Kozonis, D., *Strength Evaluation and Retrofit of Reinforced Concrete Beams Subjected to Pure Torsion*, Ph.D. Thesis, Rice University, 1997.
 57. Hsu, T. T. C., “Unified Approach to Shear Analysis and Design”, *Cement and Concrete Composites*, Vol. 20, No. 6, pp. 419–435, 1998.
 58. Csikos, A. and I. Hegedûs, “Torsion of Reinforced Concrete Beams”, *Technical University of Budapest, Department of Reinforced Concrete Structures*, 1998.
 59. Rao, T. D. G. and D. R. Seshu, “Analytical Model for the Torsional Response of Steel Fiber Reinforced Concrete Members under Pure Torsion”, *Cement and Concrete Composites*, Vol. 27, No. 4, pp. 493–501, 2005.
 60. Ameli, M., H. R. Ronagh and P. F. Dux, “Behavior of FRP Strengthened Reinforced Concrete Beams under Torsion”, *Journal of Composites for Construction*, Vol. 11, No. 2, pp. 192–200, 2007.
 61. Chiu, H.-J., I.-K. Fang, W.-T. Young and J.-K. Shiau, “Behavior of Reinforced Concrete Beams with Minimum Torsional Reinforcement”, *Engineering Structures*, Vol. 29, No. 9, pp. 2193–2205, 2007.
 62. Ju, H., D. Lee, J. R. Kim and K. S. Kim, “Maximum Torsional Reinforcement Ratio of Reinforced Concrete Beams”, *Structures*, Vol. 23, pp. 481–493, 2020.
 63. Bernardo, L. F. A. and S. M. R. Lopes, “Behaviour of Concrete Beams under Torsion: NSC Plain and Hollow Beams”, *Materials and Structures*, Vol. 41, pp. 1143–1167, 2008.
 64. Mullapudi, T. R. S. and A. Ayoub, “Coupled Model for Non Linear Analysis of Reinforced Concrete Members Subjected to 3D Loading”, *Earth and Space ASCE: Engineering, Science, Construction, and Operations in Challenging Envi-*

- ronments*, pp. 2679–2688, 2010.
65. Eom, T.-S. and H.-G. Park, “Elongation of Reinforced Concrete Members Subjected to Cyclic Loading”, *Journal of Structural Engineering*, Vol. 136, No. 9, pp. 1044–1054, 2010.
 66. Jakobsen, B., E. Hjorth-Hansen and I. Holamd, “Cyclic Torsion Tests of Concrete Box Columns”, *Journal of Structural Engineering*, Vol. 110, No. 4, pp. 803–822, 1984.
 67. Belarbi, A. and G. Greene, “Reinforced Concrete Box Girders under Cyclic Torsion”, *Proceedings of the 13th World Conference on Earthquake Engineering*, Paper 998, Vancouver, 2004.
 68. Tirasit, P. and K. Kawashima, “Seismic Performance of Square Reinforced Concrete Columns under Combined Cyclic Flexural and Torsional Loadings”, *Journal of Earthquake Engineering*, Vol. 11, No. 3, pp. 425–452, 2007.
 69. Belarbi, A. and Q. Li, “Ductility and Damage Characteristics of RC Bridge Columns under Cyclic Pure Torsion”, *Proceedings of the 15th World Conference on Earthquake Engineering*, Lisbon, 2012.
 70. Khagehhosseini, A., R. Porhosseini, R. Morshed and A. Eslami, “An Experimental and Numerical Investigation on the Effect of Longitudinal Reinforcements in Torsional Resistance of RC Beams”, *Structural Engineering and Mechanics*, Vol. 47, No. 2, pp. 247–263, 2013.
 71. Bernardo, L. F. A., C. S. B. Taborda and J. M. A. Andrade, “Ultimate Torsional Behaviour of Axially Restrained RC Beams”, *Computers and Concrete*, Vol. 16, pp. 67–97, 2015.
 72. Gajipara, J. M., P. V. Patel and S. D. Raiyani, “Behavior of RC Hollow Beam Under Pure Cyclic Torsion”, *IJRET: International Journal of Research in Engi-*

- neering and Technology*, Vol. 4, No. 25, pp. 407–412, 2015.
73. Zhang, Z., Q. Wang and F. Lin, “Experimental Investigation on Torsional Behavior of Composite Beams Reinforced with Two-piece Enclosed Stirrups”, *Modular and Offsite Construction (MOC) Summit Proceedings*, Shanghai, 2017.
 74. Teixeira, M. M. and L. F. A. Bernardo, “Ductility of RC beams Under Torsion”, *Engineering Structures*, Vol. 168, pp. 759–769, 2018.
 75. Lee, J. Y., K. H. Kim, S. H. Lee, C. Kim and M. H. Kim, “Maximum Torsional Reinforcement of Reinforced Concrete Beams Subjected to Pure Torsion”, *ACI Structural Journal*, Vol. 115, No. 3, pp. 749–760, 2018.
 76. Ibrahim, M. S., E. Gebreyouhannes, A. Muhdin and A. Gebre, “Effect of Concrete Cover on the Pure Torsional Behavior of Reinforced Concrete Beams”, *Engineering Structures*, Vol. 216, p. 110790, 2020.
 77. Bernardo, L. F. A., M. M. Teixeira, D. De Domenico and J. M. Gama, “Improved Equations for the Torsional Strength of Reinforced Concrete Beams for Codes of Practice Based on the Space Truss Analogy”, *Materials*, Vol. 15, No. 11, p. 3827, 2022.
 78. Lopes, S. M. R. and L. F. A. Bernardo, “Twist Behavior of High-Strength Concrete Hollow Beams-Formation of Plastic Hinges along the Length”, *Engineering Structures*, Vol. 31, No. 1, pp. 138–149, 2009.
 79. Bernardo, L. F. A. and S. M. R. Lopes, “Theoretical Behavior of HSC Sections Under Torsion”, *Engineering Structures*, Vol. 33, No. 12, pp. 3702–3714, 2011.
 80. Bernardo, L. F. A. and S. M. R. Lopes, “Plastic Analysis and Twist Capacity of High-Strength Concrete Hollow Beams under Pure Torsion”, *Engineering Structures*, Vol. 49, pp. 190–201, 2013.

81. Lopes, S. M. R. and L. F. A. Bernardo, “Cracking and Failure Mode in HSC Hollow Beams under Torsion”, *Construction and Building Materials*, Vol. 51, pp. 163–178, 2014.
82. Jung, W. Y. and M. Kwon, “Prediction of Nonlinear Torsional Behavior of High Strength RC Beams”, *International Journal of Civil and Environmental Engineering*, Vol. 8, No. 6, pp. 751–756, 2014.
83. Sengupta, P. and B. Li, “Hysteresis Modeling of Reinforced Concrete Structures: State of the Art”, *ACI Structural Journal*, Vol. 114, No. 1, pp. 25–38, 2017.
84. Valipour, H. R. and S. J. Foster, “Nonlinear Reinforced Concrete Frame Element with Torsion”, *Engineering Structures*, Vol. 32, No. 4, pp. 988–1002, 2010.
85. Venkappa, V. and G. S. Pandit, “Cyclic Torsion Tests on Reinforced Concrete Beams”, *Journal of Structural Engineering*, Vol. 113, No. 6, pp. 1329–1340, 1987.
86. Ozcebe, G. and M. Saatcioglu, “Hysteretic Shear Model for Reinforced Concrete Members”, *Journal of Structural Engineering*, Vol. 115, No. 1, pp. 132–148, 1989.
87. Mostaghel, N., “Analytical Description of Pinching, Degrading Hysteretic Systems”, *Journal of Engineering Mechanics*, Vol. 125, No. 2, pp. 216–224, 1999.
88. Jeng, C. H., “Simple Rational Formulas for Cracking Torque and Twist of Reinforced Concrete Members”, *ACI Structural Journal*, Vol. 107, No. 2, 2010.
89. Mondal, T. G., S. R. Kothamuthyala and S. S. Prakash, “Hysteresis Modelling of Reinforced Concrete Columns under Pure Cyclic Torsional Loading”, *Structural Engineering and Mechanics*, Vol. 64, No. 1, pp. 11–21, 2017.
90. Ersoy, U., *Betonarmede Burulma: Davranış Mukavemet Şartnameler Hesap İçin Öneriler*, Güven Kitabevi, Ankara, 1975.

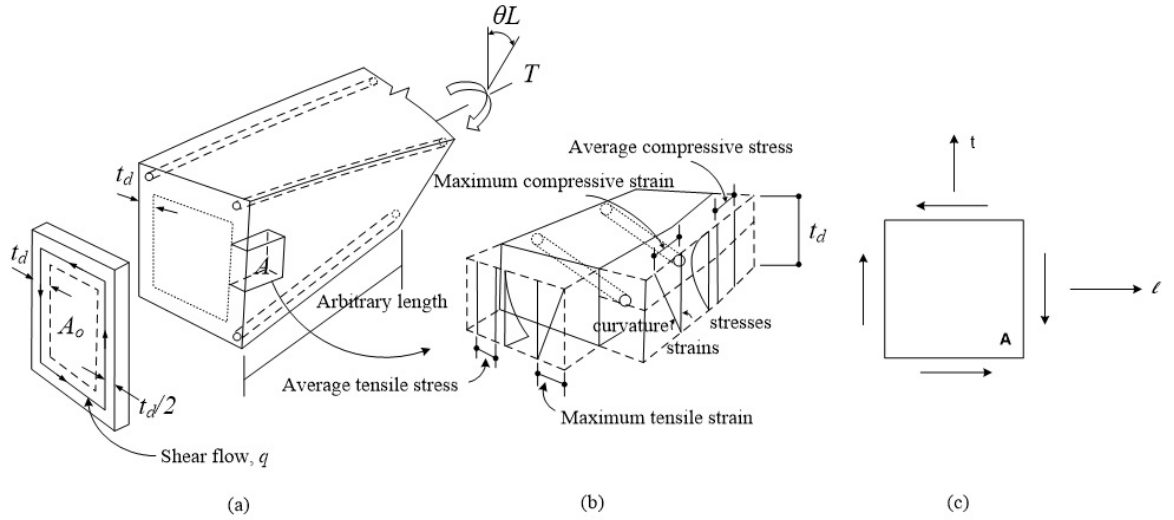
91. Chopra, A. K., *Dynamics of Structures*, Second Edition, University of California, Prentice Hall, Berkeley, 2001.
92. Wang, X., B. Liu and C. Zhang, “Cyclic Torsion Performance of Recycled Aggregate Concrete Beams with or without Fly Ash”, *Construction and Building Materials*, Vol. 132, pp. 529–542, 2017.
93. Hsu, T. T. C., *Unified Theory of Reinforced Concrete*, Taylor & Francis, Houston, 1992.
94. Zhu, R. R. H. and T. T. C. Hsu, “Poisson Effect in Reinforced Concrete Membrane Elements”, *Structural Journal*, Vol. 99, No. 5, pp. 631–640, 2002.
95. Collins, M. P. and P. Lampert, “Redistribution of Moments at Cracking-the Key to Simpler Torsion Design”, *ACI Special Publication*, Vol. 35, pp. 343–384, 1973.
96. Euro-International-Committee, *CEB-FIP Model Code 90 for Concrete Structures*, Thomas Telford Publishing, London, 1993.
97. Saatcioglu, M., “Modeling Hysteretic Force-Deformation Relationships for Reinforced Concrete Elements”, *ACI Special Publication*, Vol. 127, pp. 153–198, 1991.
98. Okay, F. and S. Engin, “Torsional Behavior of Steel Fiber Reinforced Concrete Beams”, *Construction and Building Materials*, Vol. 28, No. 1, pp. 269–275, 2012.
99. Tirasit, P. and K. Kawashima, “Combined Cyclic Bending-Torsional Loading Test of Reinforced Concrete Bridge Columns”, <https://www.pwri.go.jp/eng/ujnr/tc/g/pdf/21/21-8-4kawashima.pdf>, accessed on Oct 3, 2005.
100. Wang, X., B. Liu and C. Zhang, “Seismic Behavior of Recycled Aggregate Concrete Beams under Cyclic Torsion”, *Construction and Building Materials*, Vol. 129, pp. 193–203, 2016.

101. Belarbi, A., S. Prakash and Y. M. You, “Effect of Spiral Reinforcement on Flexural-Shear Torsional Seismic Behavior of Reinforced Concrete Circular Bridge Columns”, *Structural Engineering Mechanics*, Vol. 33, No. 2, pp. 137–158, 2009.
102. Prakash, S., A. Belarbi and Y. M. You, “Seismic Performance of Circular RC Columns Subjected to Axial Force, Bending, and Torsion with Low and Moderate Shear”, *Engineering Structures*, Vol. 32, No. 1, pp. 46–59, 2010.



APPENDIX A: ANALYSIS OF TORSION

The equations given for torsion analysis in this section were derived by Hsu using the rotating angle softened truss model (RA-STM). A 2-D member shown in Figure A.1 (a) subjected to a torsional moment at one end. Stresses and strains on element A in the shear flow zone taken from the member were shown in Figure A.1 (b) and (c) [93].



(a) R.C. element in shear flow zone (b) stress distribution on element A (c) shear stresses on element A

Figure A.1. A simple RC member under torsional moment.

A.1. Equilibrium Equations

Equilibrium equations derived from the equilibrium of a membrane element in shear flow zone, and Bredt's equilibrium equation is derived for the shear flow zone thickness.

A reinforced concrete beam under external torque, T , which is resisted by an internal torque causes a shear flow zone along the periphery of the member section as shown in Figure A.1. Shear flow, q , and as the variable can be calculated from

equilibrium equations, t_d is the shear flow zone effective thickness which is different from, h , the wall thickness of a hollow section and shear stress will be

$$\tau_{lt} = \frac{q}{t_d}, \quad (\text{A.1})$$

the longitudinal reinforcement ratio is

$$\rho_l = \frac{A_l}{p_o t_d}, \quad (\text{A.2})$$

the transverse reinforcement ratio is

$$\rho_t = \frac{A_t}{s t_d}, \quad (\text{A.3})$$

for the prestresses members, the longitudinal prestressed steel ratio is

$$\rho_{lp} = \frac{A_{lp}}{p_o t_d}, \quad (\text{A.4})$$

for the prestresses members, the transverse prestressed steel ratio is

$$\rho_{tp} = \frac{A_{tp}}{s t_d}, \quad (\text{A.5})$$

equilibrium equation in longitudinal direction is

$$\sigma_l - \rho_l f_l - \rho_{lp} f_{lp} = \sigma_d \alpha_r, \quad (\text{A.6})$$

equilibrium equation in transverse direction is

$$\sigma_t - \rho_t f_t - \rho_{tp} f_{tp} = \sigma_d \alpha_r, \quad (\text{A.7})$$

shear stress calculation is obtained from

$$\tau_{lt} = (-\sigma_d) \sin \alpha_r \cos \alpha_r, \quad (\text{A.8})$$

and, Bredt's equilibrium equation is

$$\tau_{lt} = \frac{T}{2A_o t_d}, \quad (\text{A.9})$$

here, ρ_l and ρ_t are the reinforcement ratios; f_l and f_t are the stresses, ρ_{lp} and ρ_{tp} are the prestressing reinforcement ratio; f_{lp} and f_{tp} are prestressing steel stresses in the longitudinal and transverse directions respectively. A_l and A_{lp} are the total area of reinforcement and prestressing steel, A_t and A_{tp} are the area of the transverse steel and prestressing steel strand, respectively. p_o is the perimeter of the shear flow zone and s is the spacing between the transverse steel. For the final equation, τ_{lt} is the shear stress and A_o is the cross-sectional area of the shear flow zone. These four equilibrium Equations (A.6) to (A.9), should be satisfied to maintain the equilibrium.

A.2. Compatibility Equations

The shear stress on 2-D Element A in shear flow region is shown in Figure A.1. Given three compatibility equations should be satisfied to prevent the out of plane deformation.

Element A is a 2-D element in the shear flow region and is subjected to shear stresses as shown in Figure A.1. Given three compatibility equations should be satisfied to prevent the out of plane deformation. Longitudinal strain will be

$$\varepsilon_l = \varepsilon_r \alpha_r + \varepsilon_d \alpha_d, \quad (\text{A.10})$$

the transverse strain equilibrium equation is

$$\varepsilon_r = \varepsilon_r \alpha_r + \varepsilon_d \alpha_d, \quad (\text{A.11})$$

the shear deformation is calculated from

$$\frac{\gamma_{lt}}{2} = (\varepsilon_r - \varepsilon_d) \sin \alpha_r \cos \alpha_r, \quad (\text{A.12})$$

where, θ is the angle of twist and γ_{lt} is the shear strain into the element A. ε_l and ε_r are the strains in the l and r directions respectively. Here, twist can be defined as

$$\theta = \frac{p_o}{2A_o} \gamma_{lt}. \quad (\text{A.13})$$

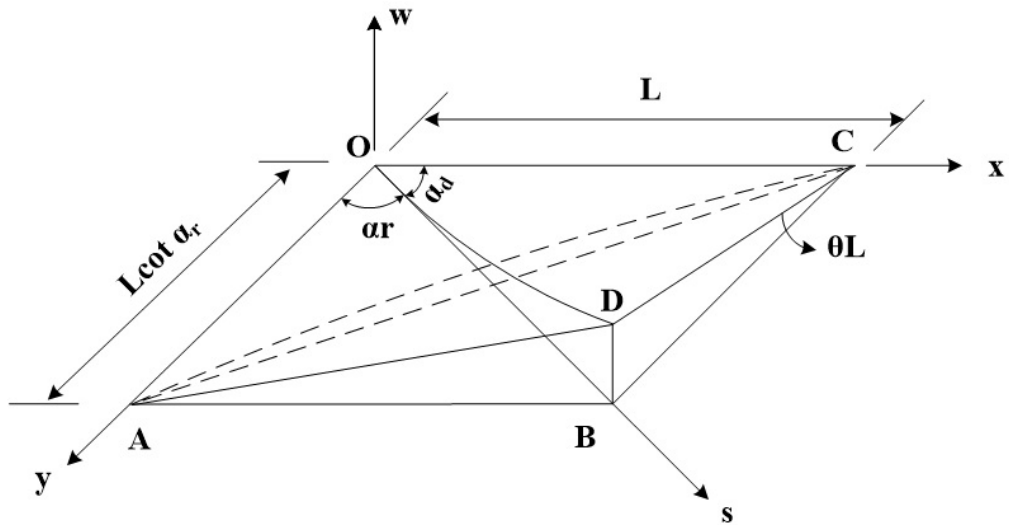


Figure A.2. Bending of a strut on a simple RC member subjected to torsion.

Twist angle, θ , can produce both warping in the wall surface of the member and bending of concrete struts. The bending curvature ψ of a bending strut shown in Figure A.2 is obtained from the Equation (A.17) for the analysis. Here, $OADC$ is a hyperbolic paraboloid surface and w , x , and y are the coordinate axes of this paraboloid element. The second derivative of the w with respect s gives the curvature

$$w = \theta xy, \quad (\text{A.14})$$

the first derivative of the w with respect to s is

$$\frac{dw}{ds} = \frac{\partial w}{\partial x} \frac{dx}{ds} + \frac{\partial w}{\partial y} \frac{dy}{ds} = (\theta_y) \sin \alpha_r + (\theta_x) \cos \alpha_r. \quad (\text{A.15})$$

And for the curvature, second derivative of the w with respect to s

$$\frac{d^2w}{ds^2} = \frac{\partial}{\partial x} \left(\frac{dw}{ds} \right) \frac{dx}{ds} + \frac{\partial}{\partial y} \left(\frac{dw}{ds} \right) \frac{dy}{ds} = (\theta \cos \alpha_r) \sin \alpha_r + (\theta \sin \alpha_r) \cos \alpha_r. \quad (\text{A.16})$$

Finally, the equation will be obtained as

$$\psi = \theta \sin 2\alpha_r. \quad (\text{A.17})$$

The strain and the stress variation for the unit width and h thickness of reinforced concrete struts was illustrated as seen In Figure A.3 (a). The height of the compression region measured from the neutral axis to the extreme fiber of concrete in compression region was called as shear flow zone (t_d).

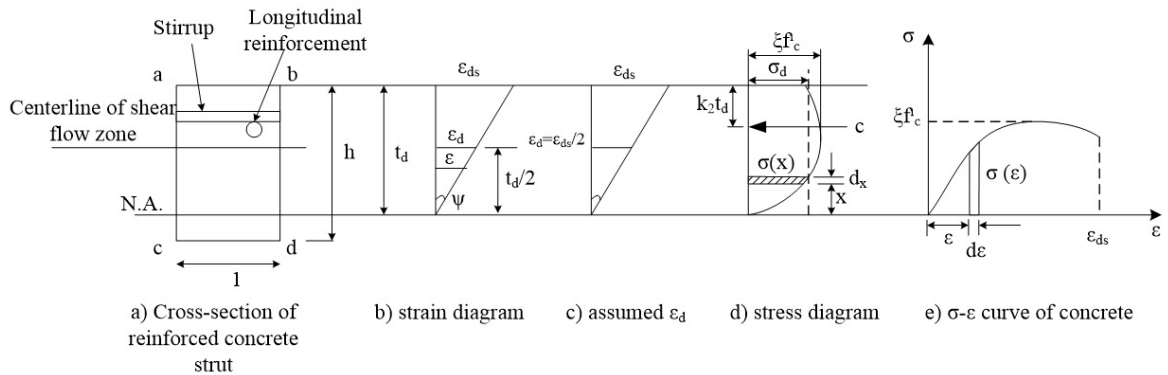


Figure A.3. Stress and strains of the concrete strut under torsion.

There is a linear relation between t_d , ψ (curvature) and ε_{ds} (strain for extreme fiber) and the shear flow zone effective thickness will be

$$t_d = \frac{\varepsilon_{ds}}{\psi}. \quad (\text{A.18})$$

And, the average strain ε_d will be defined as

$$\varepsilon_d = \frac{\varepsilon_{ds}}{2}. \quad (\text{A.19})$$

By using the softened compression block, peak stress σ_p can be defined as

$$\sigma_p = \zeta f'_c. \quad (\text{A.20})$$

A.3. Constitutive Law of Concrete in Compression

The average stress of the concrete strut σ_d was shown in Figure A.3 (d). The average stress can be calculated from

$$\sigma_d = k_1 \sigma_p = k_1 \zeta f'_c, \quad (\text{A.21})$$

here, k_1 is mean stress over peak stress ratio, and Equation (A.22) (a) gives the k_1 coefficient at the ascending branch as shown in Figure A.3 (e). For the descending branch, k_1 could be calculated as

$$k_1 = \begin{cases} \frac{\varepsilon_{ds}}{\zeta \varepsilon_o} \left(1 - \frac{1}{3} \frac{\varepsilon_{ds}}{\zeta \varepsilon_o}\right) & \frac{\varepsilon_{ds}}{\zeta \varepsilon_o} \leq 1 \quad (\text{a}) \\ \left[1 - \frac{\zeta^2}{(2-\zeta)^2}\right] \left(1 - \frac{1}{3} \frac{\zeta \varepsilon_o}{\varepsilon_{ds}}\right) + \frac{\zeta^2}{(2-\zeta)^2} \frac{\varepsilon_{ds}}{\zeta \varepsilon_o} \left(1 - \frac{1}{3} \frac{\varepsilon_{ds}}{\zeta \varepsilon_o}\right) & \frac{\varepsilon_{ds}}{\zeta \varepsilon_o} > 1 \quad (\text{b}), \end{cases} \quad (\text{A.22})$$

where, the concrete softening coefficient is

$$\zeta = \frac{0.9}{\sqrt{1 + 600 \varepsilon_r}}. \quad (\text{A.23})$$

A.4. Constitutive Law of Mild Steel

The longitudinal reinforcement stress is

$$\begin{cases} f_l = E_s \varepsilon_l & \varepsilon_l < \varepsilon_{ly} \quad (\text{a}) \\ f_l = f_{ly} & \varepsilon_l \geq \varepsilon_{ly} \quad (\text{b}), \end{cases} \quad (\text{A.24})$$

and the transverse reinforcement stress is

$$\begin{cases} f_t = E_s \varepsilon_l & \varepsilon_t < \varepsilon_{ty} \quad (\text{a}) \\ f_t = f_{ty} & \varepsilon_t \geq \varepsilon_{ty} \quad (\text{b}). \end{cases} \quad (\text{A.25})$$

APPENDIX B: TORSIONAL CONSTANTS

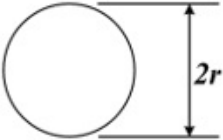
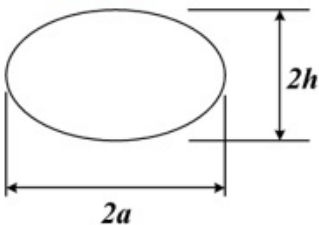
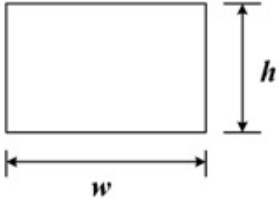
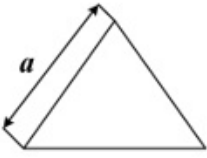
1. Solid Circular Section: 	$J = \frac{1}{2} \pi r^4$
2. Solid <u>Elliptical</u> Section: 	$J = \frac{\pi a^3 h^3}{a^2 + h^2}$
3. Solid Square Section 	$J = 2,25 h^4$
4. Solid Rectangular Section 	$J = wh^3 \left[\frac{1}{3} - 0,21 \frac{h}{w} - \left(1 - \frac{h^4}{12w^4} \right) \right]$
5. Solid Triangular Section 	$J = \frac{a^4 \sqrt{3}}{80}$

Figure B.1. Formulas for torsional constants.

APPENDIX C: COMPARISON OF HYSTERESIS LOOPS

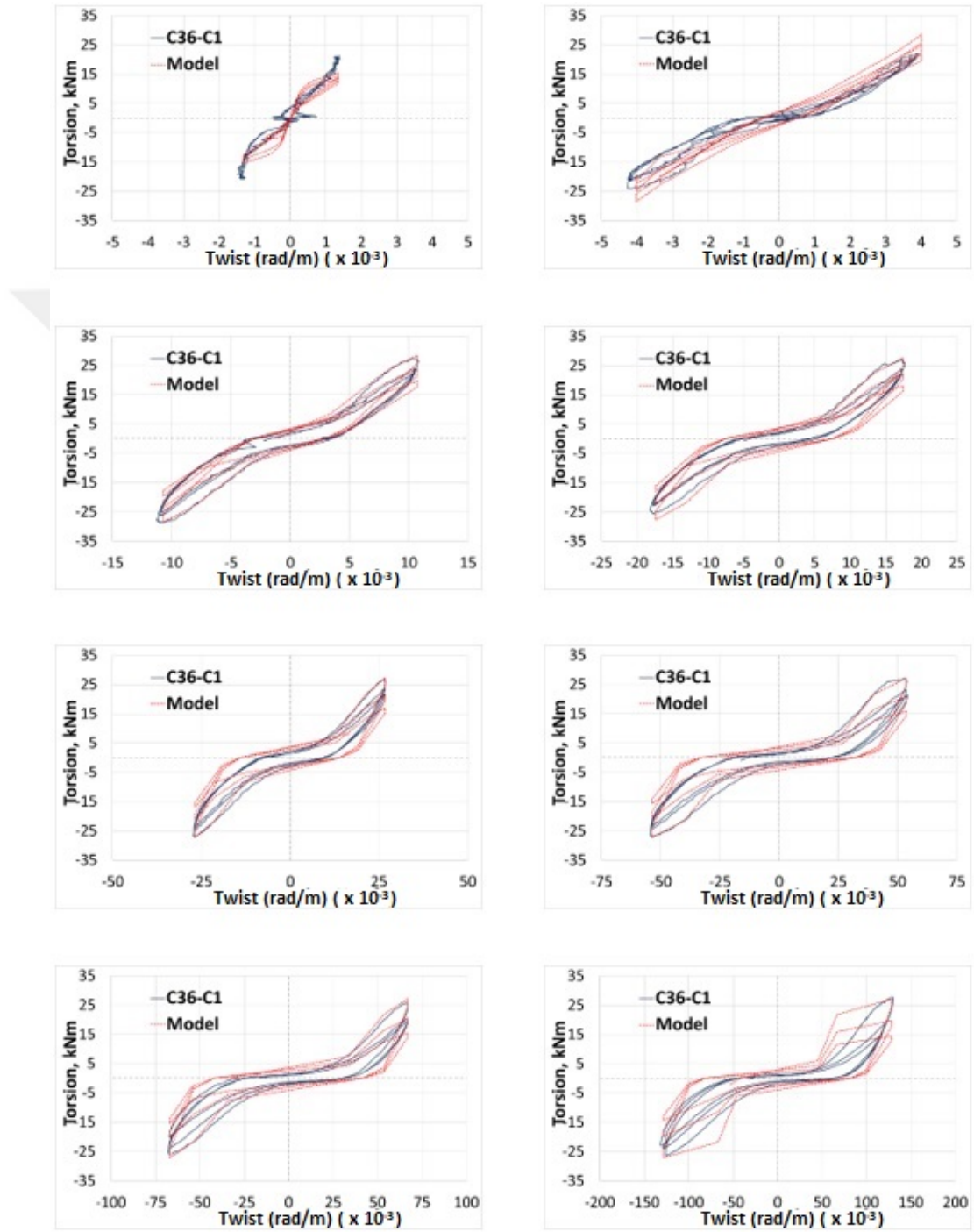


Figure C.1. Hysteresis loops for C36-C1.

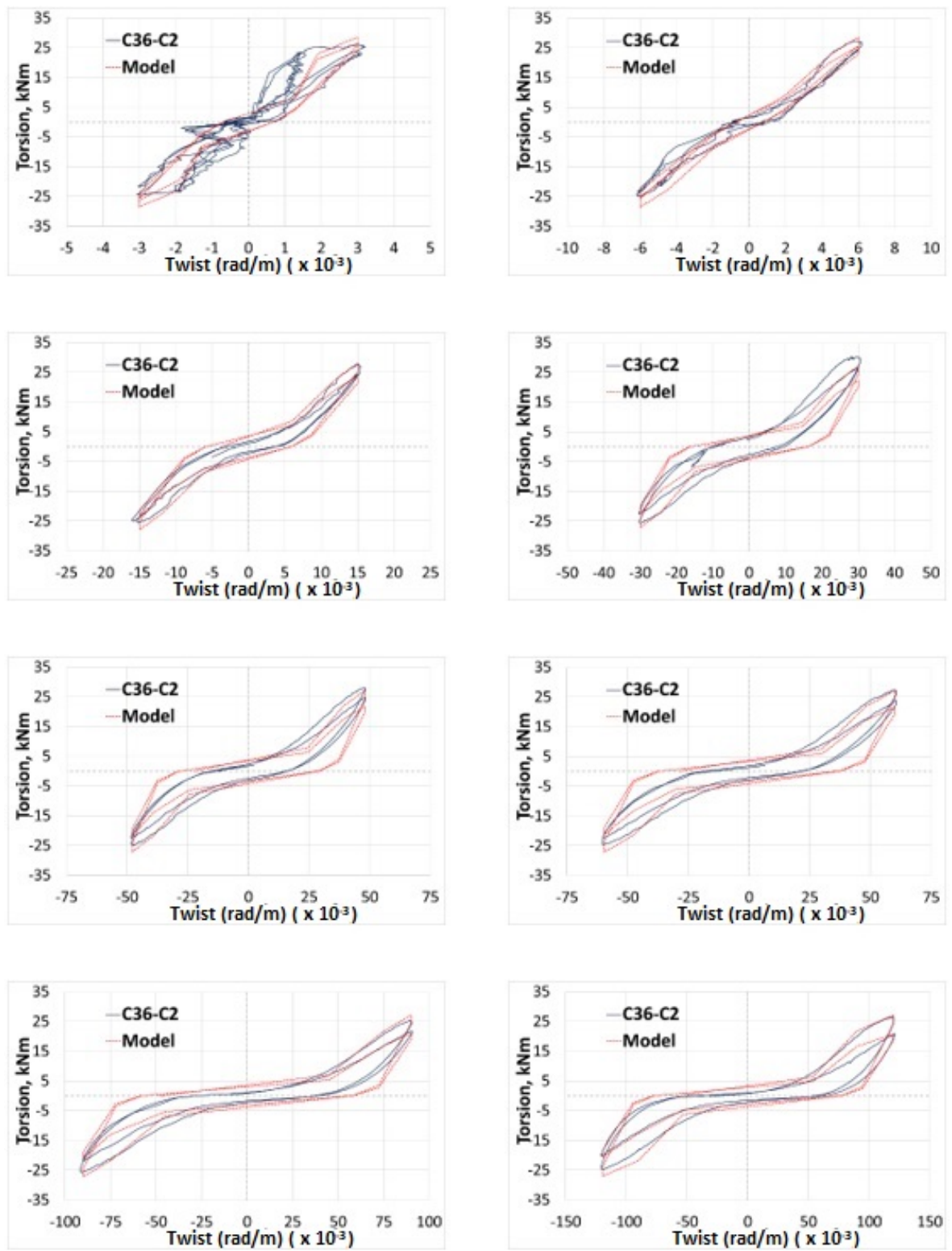


Figure C.2. Hysteresis loops for C36-C2.

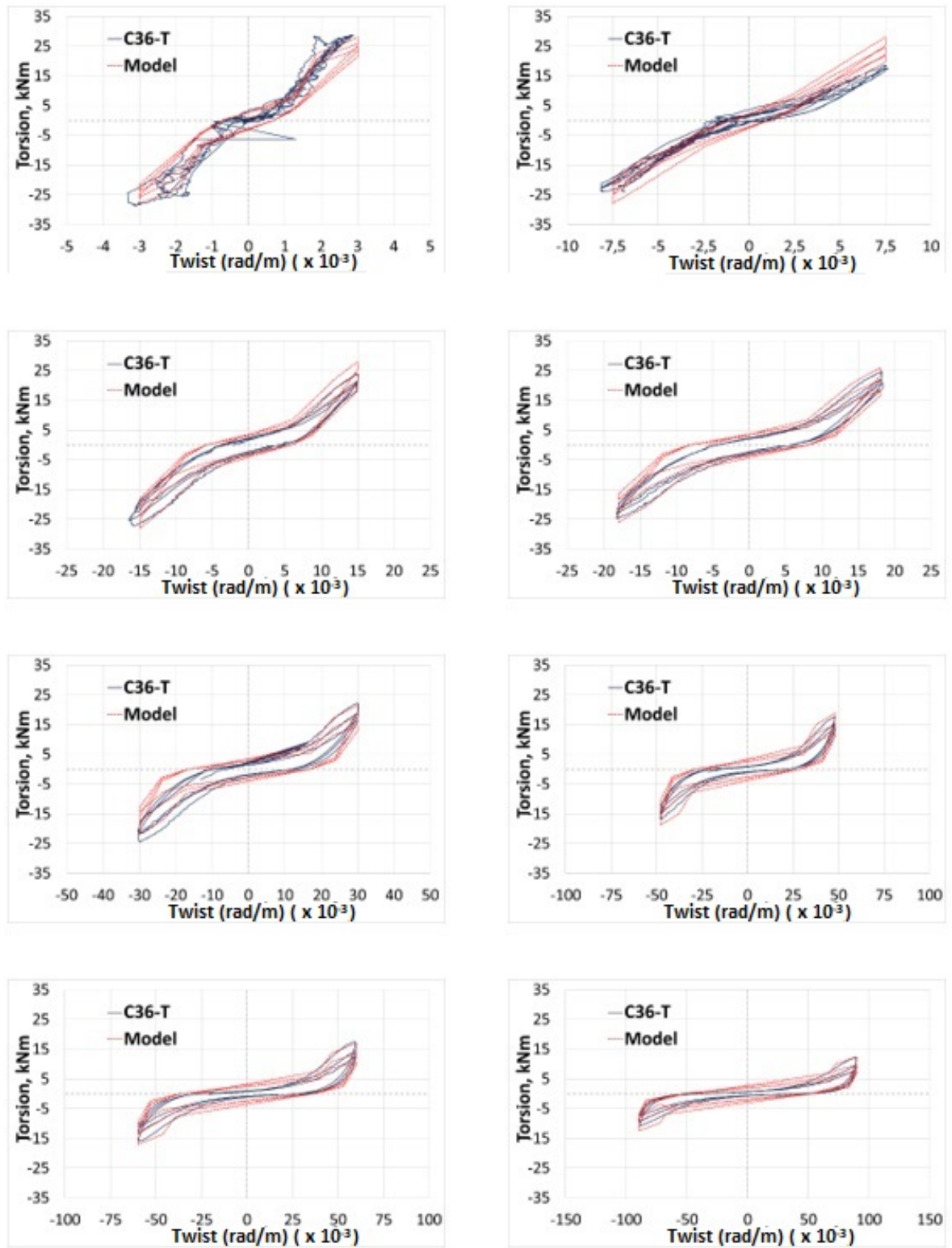


Figure C.3. Hysteresis loops for C36-T.

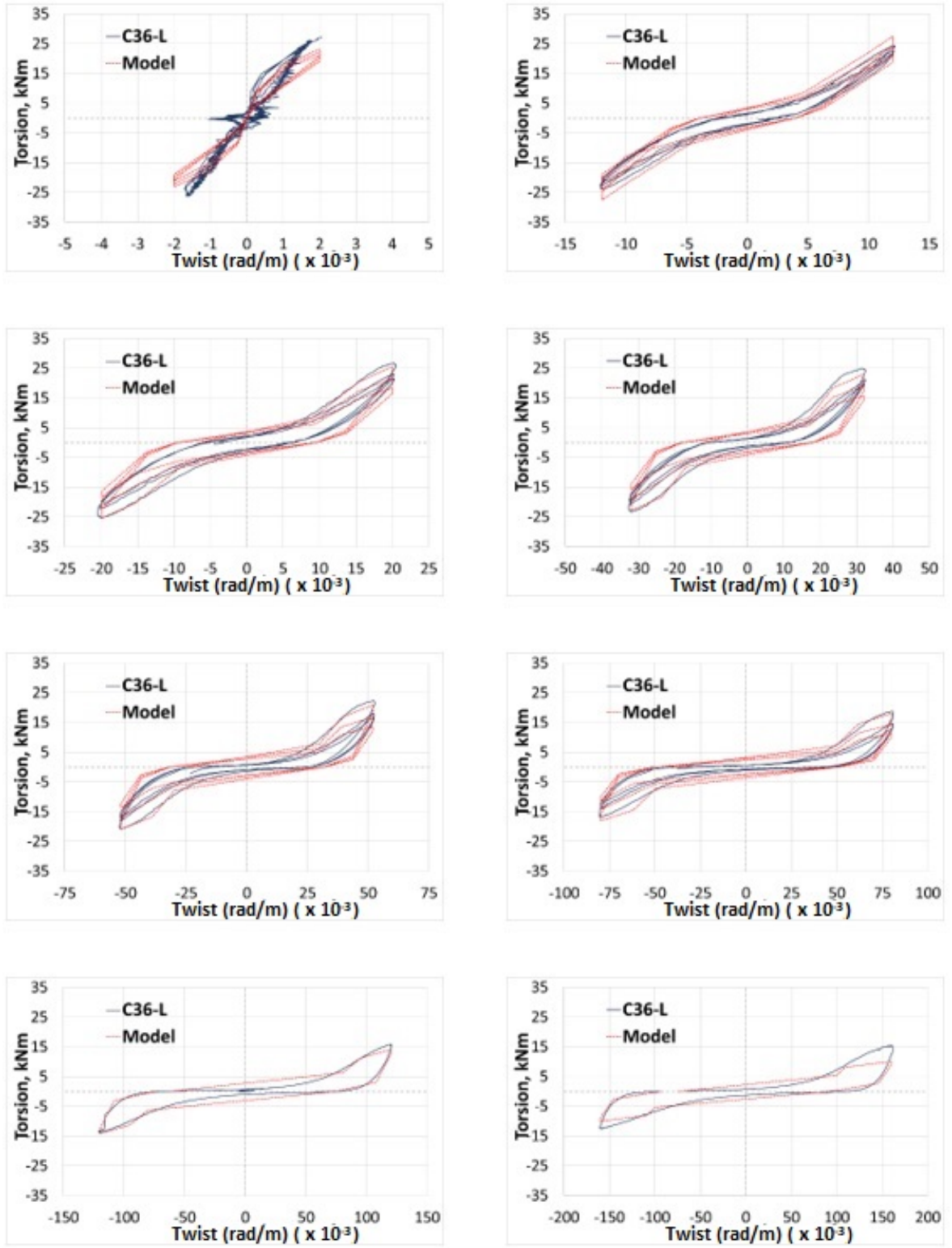


Figure C.4. Hysteresis loops for C36-L.

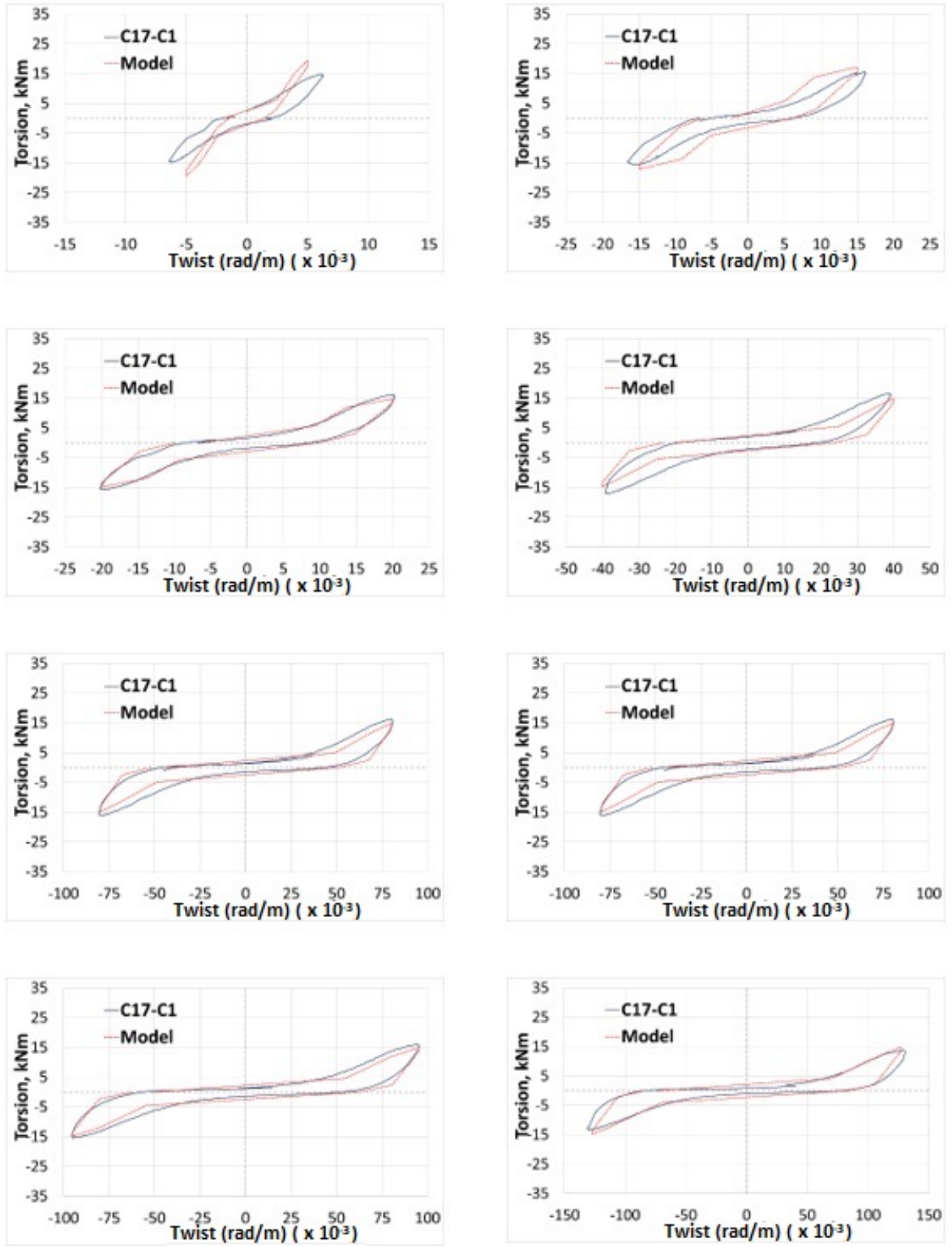


Figure C.5. Hysteresis loops for C17-C1.

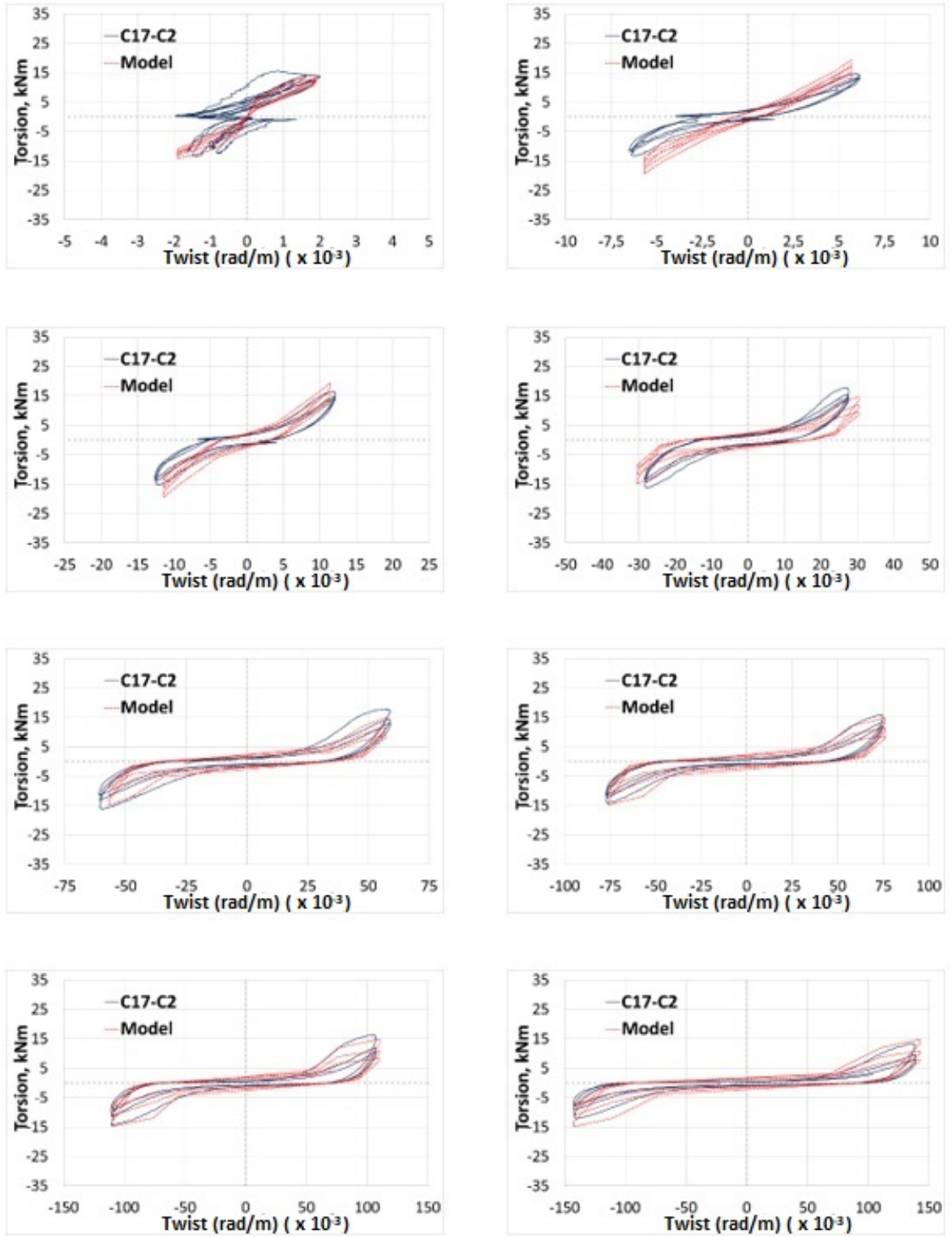


Figure C.6. Hysteresis loops for C17-C2.

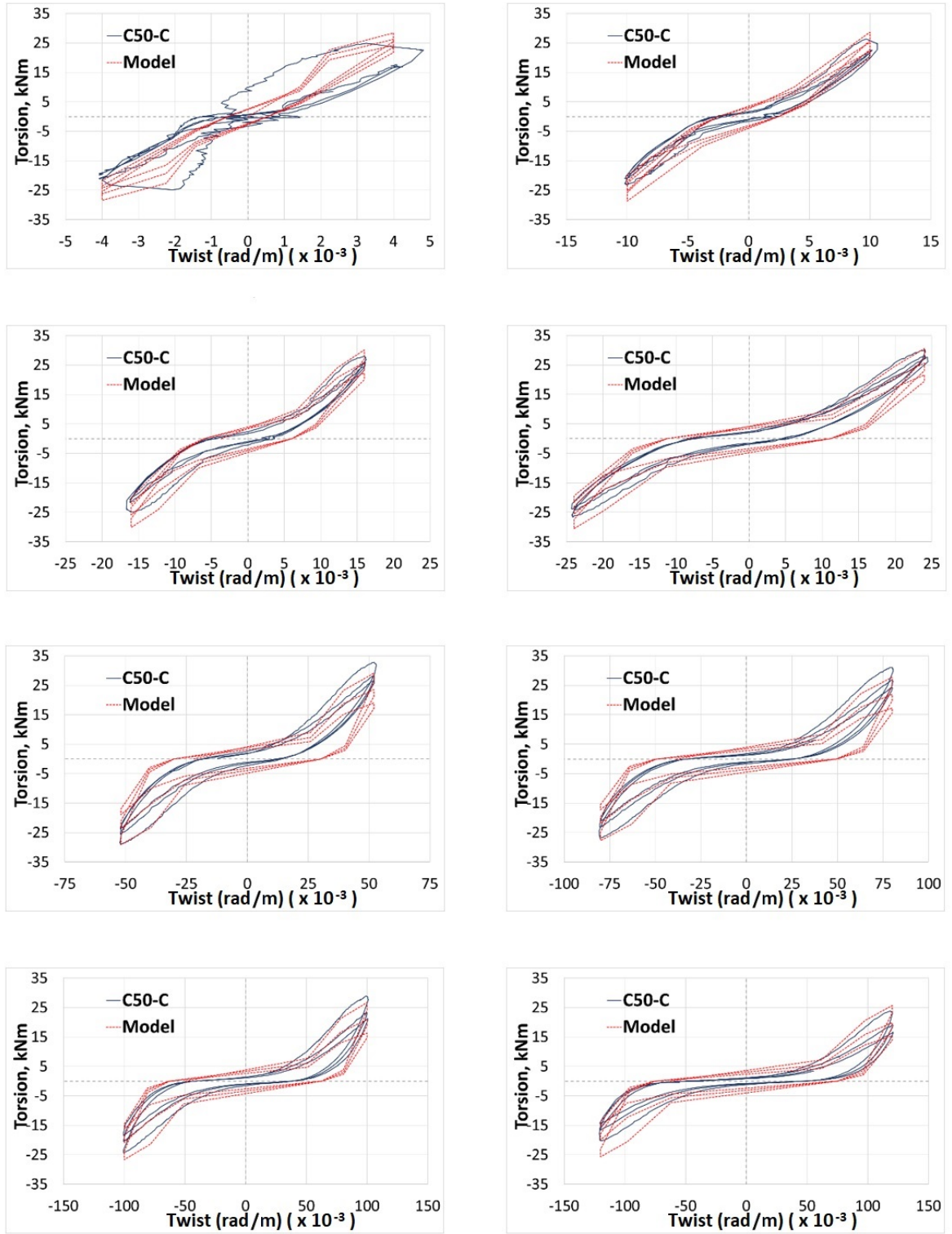


Figure C.7. Hysteresis loops for C50-C.

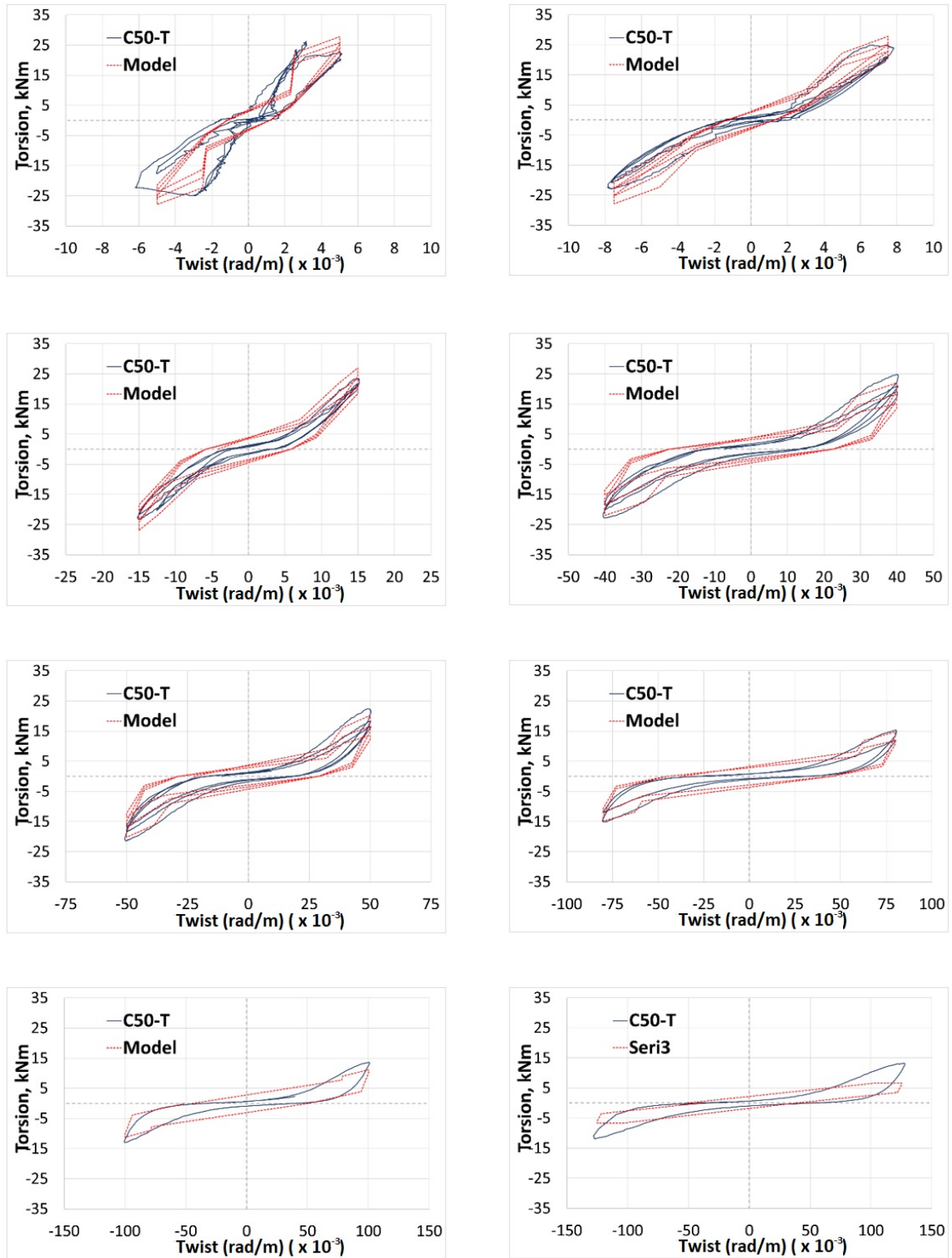


Figure C.8. Hysteresis loops for C50-T.

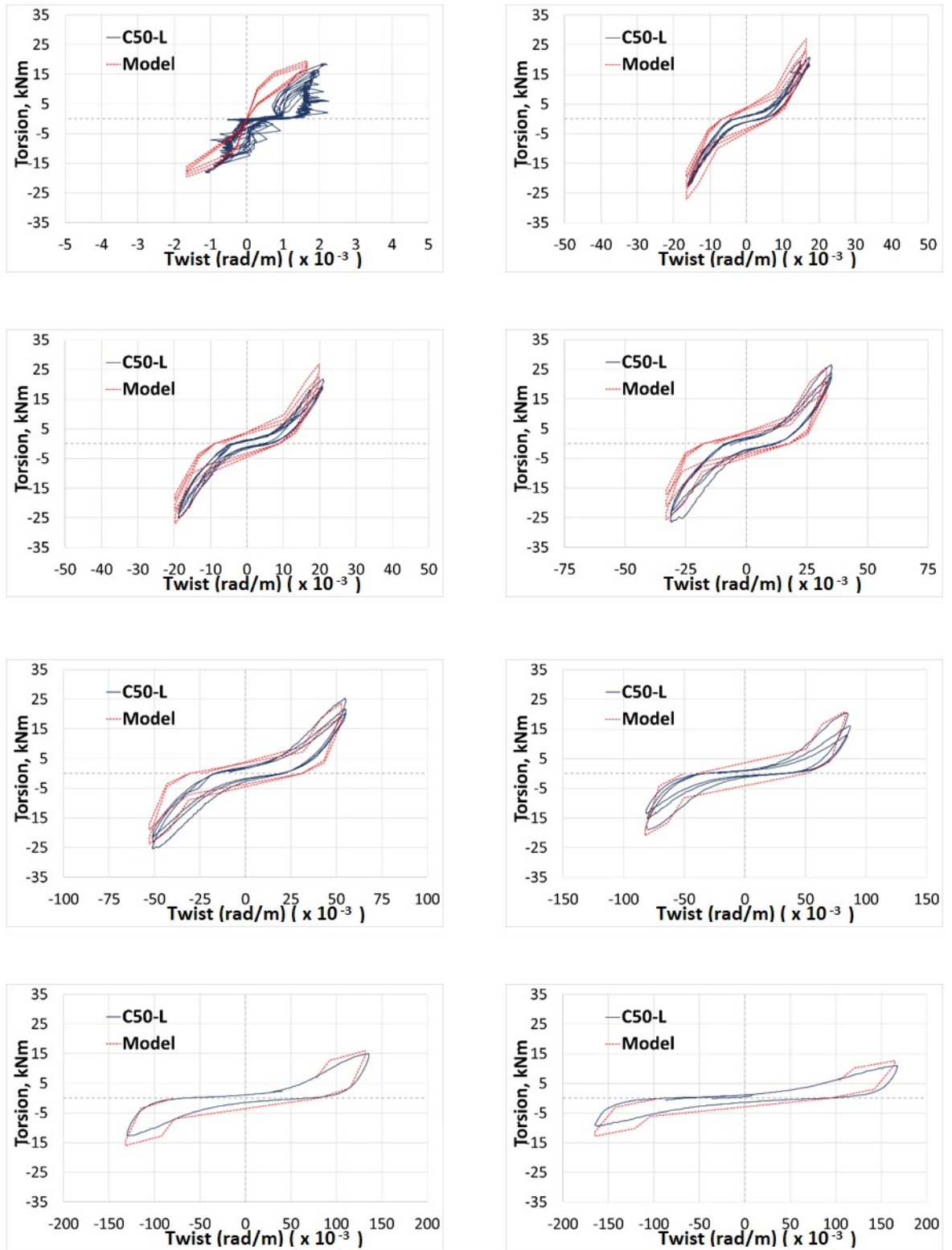


Figure C.9. Hysteresis loops for C50-L.

APPENDIX D: COMPARISON OF ENERGY CURVES

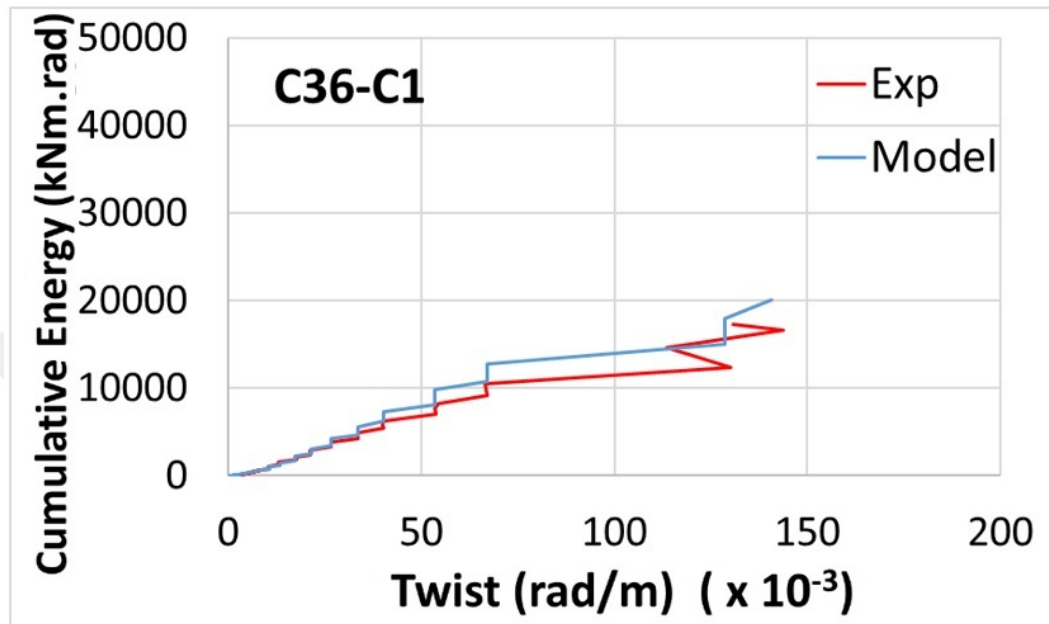


Figure D.1. Energy comparison for C36-C1.

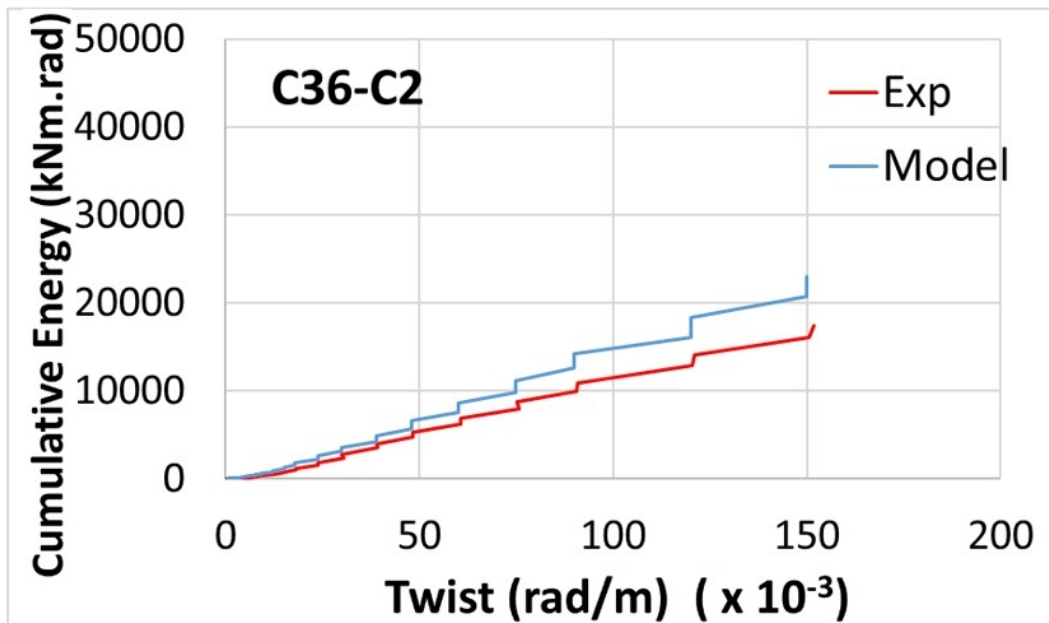


Figure D.2. Energy comparison for C36-C2.

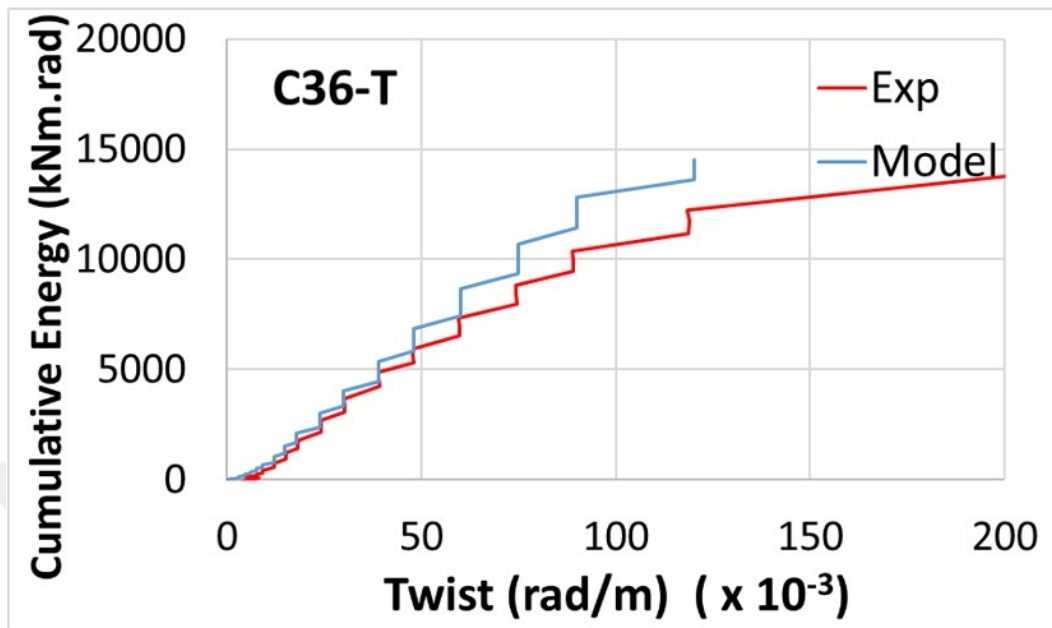


Figure D.3. Energy comparison for C36-T.

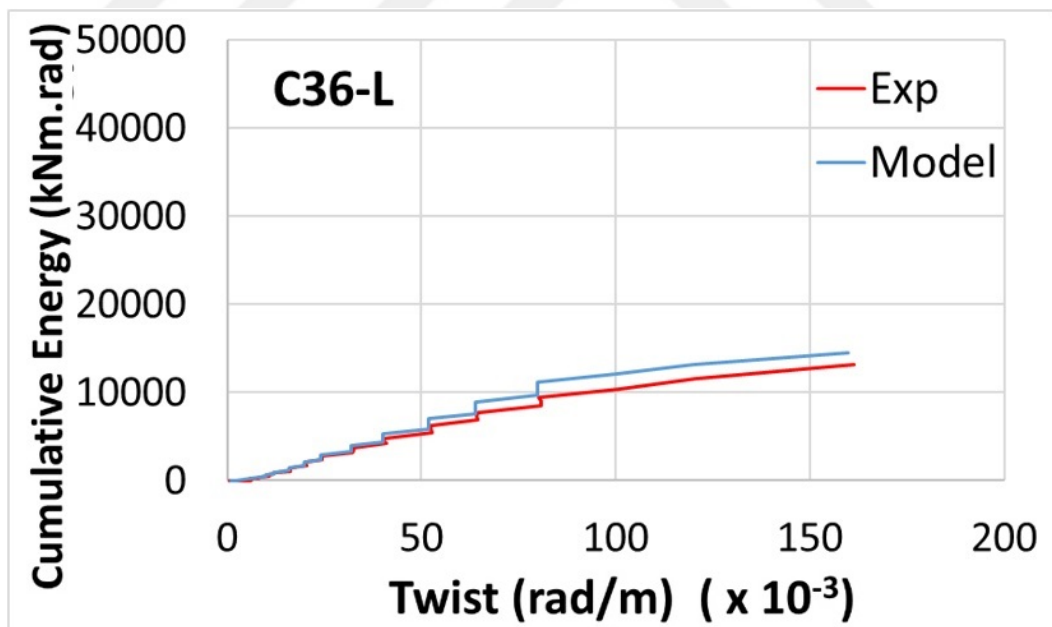


Figure D.4. Energy comparison for C36-L.

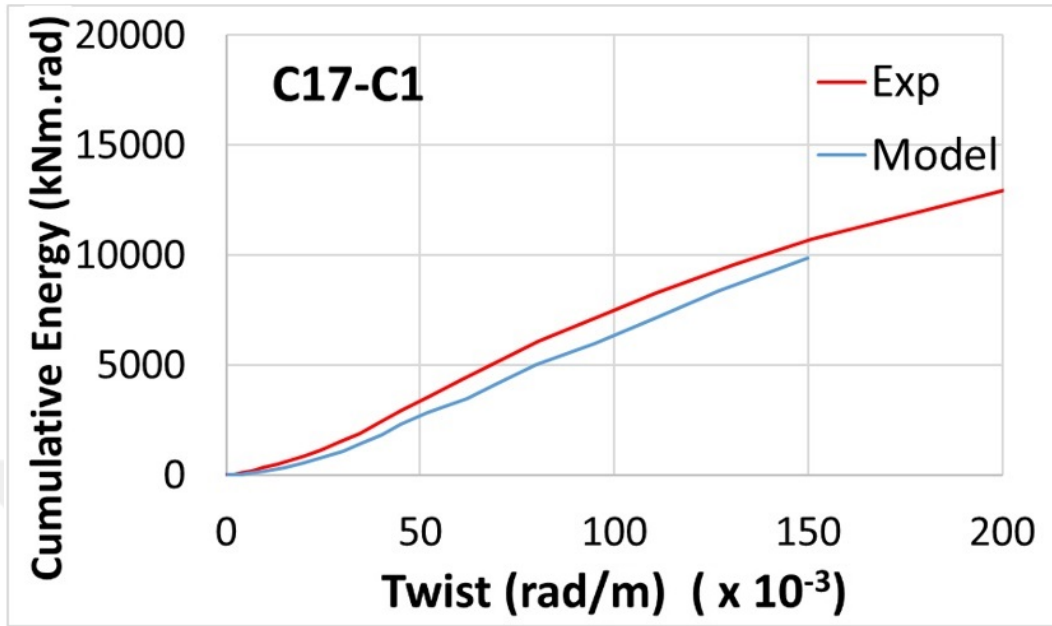


Figure D.5. Energy comparison for C17-C1.

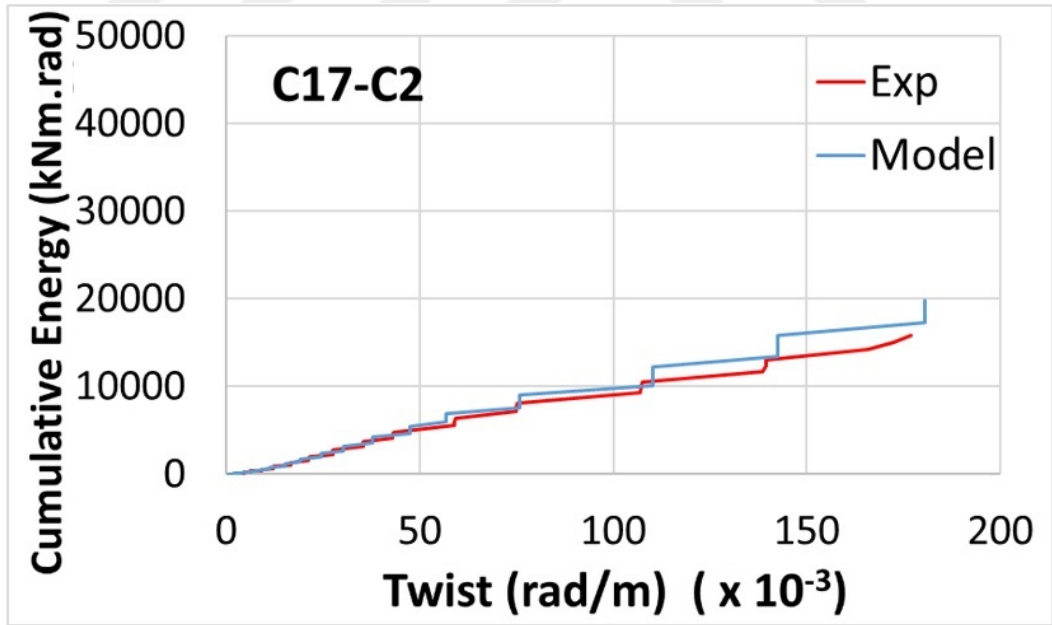


Figure D.6. Energy comparison for C17-C2.

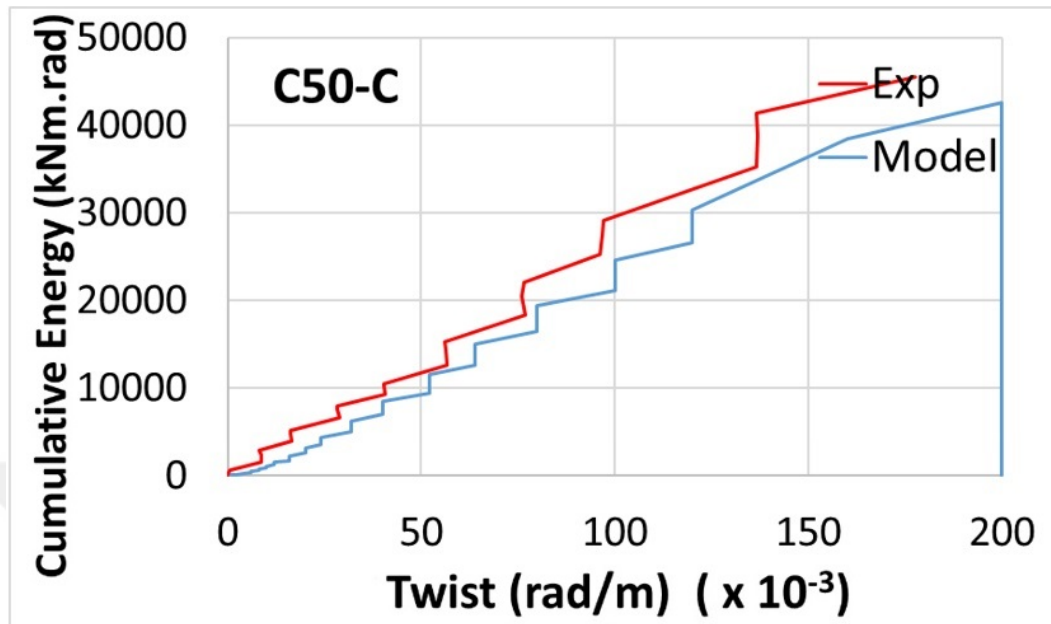


Figure D.7. Energy comparison for C50-C.

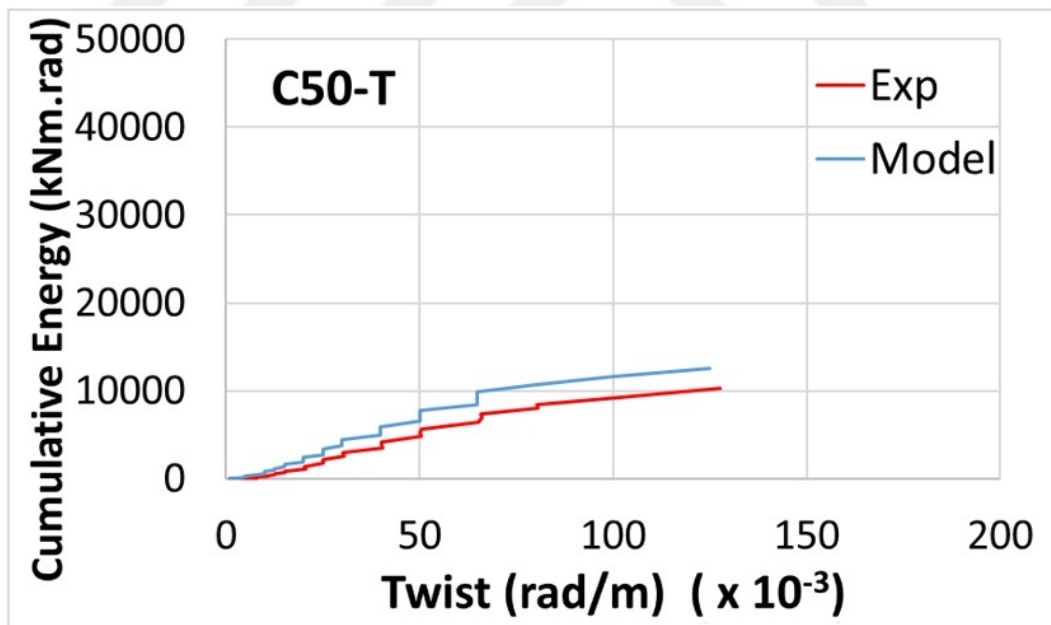


Figure D.8. Energy comparison for C50-T.

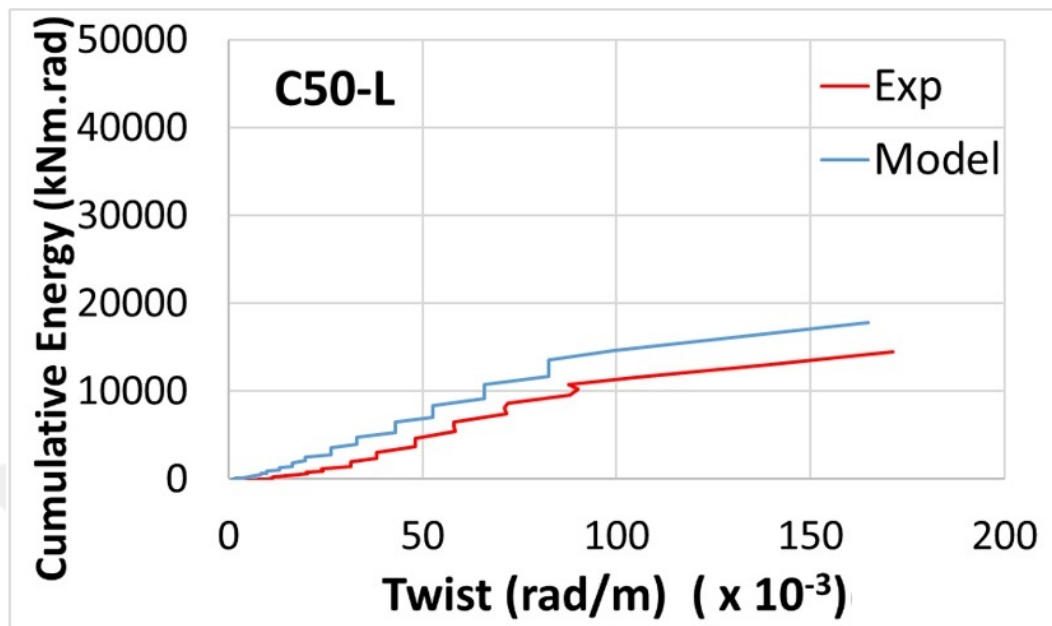


Figure D.9. Energy comparison for C50-L.

APPENDIX E: COMPARISON OF STIFFNESS DEGRADATION

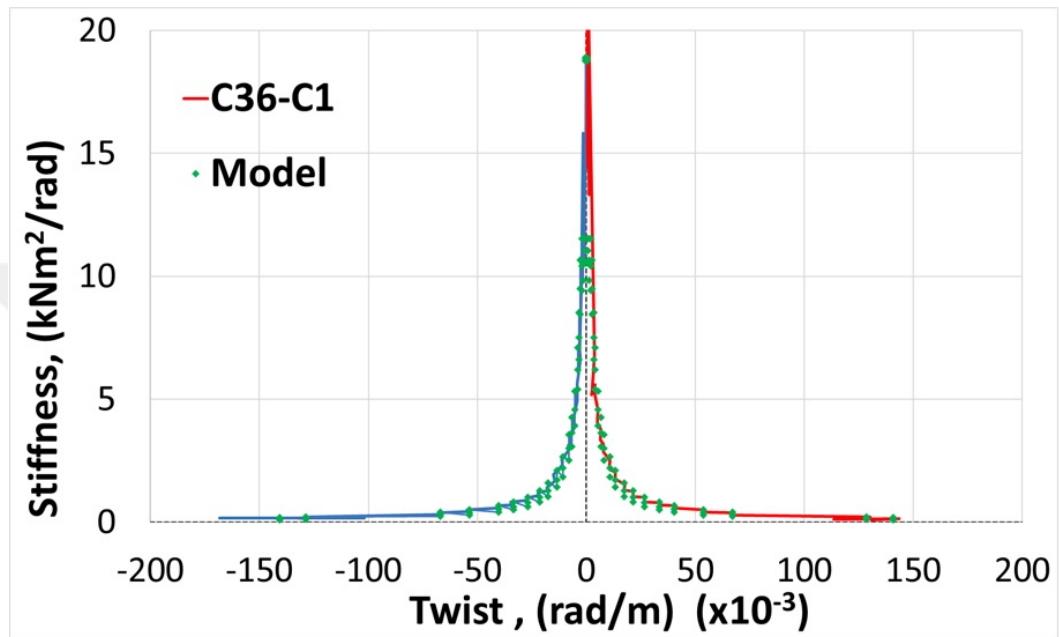


Figure E.1. Stiffness comparison for C36-C1.

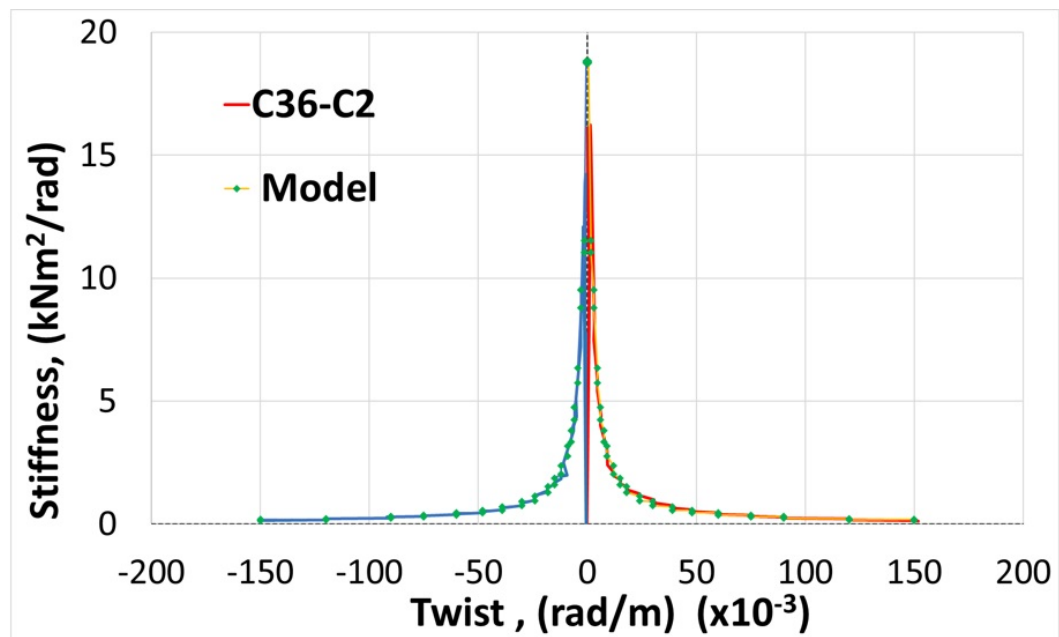


Figure E.2. Stiffness comparison for C36-C2.

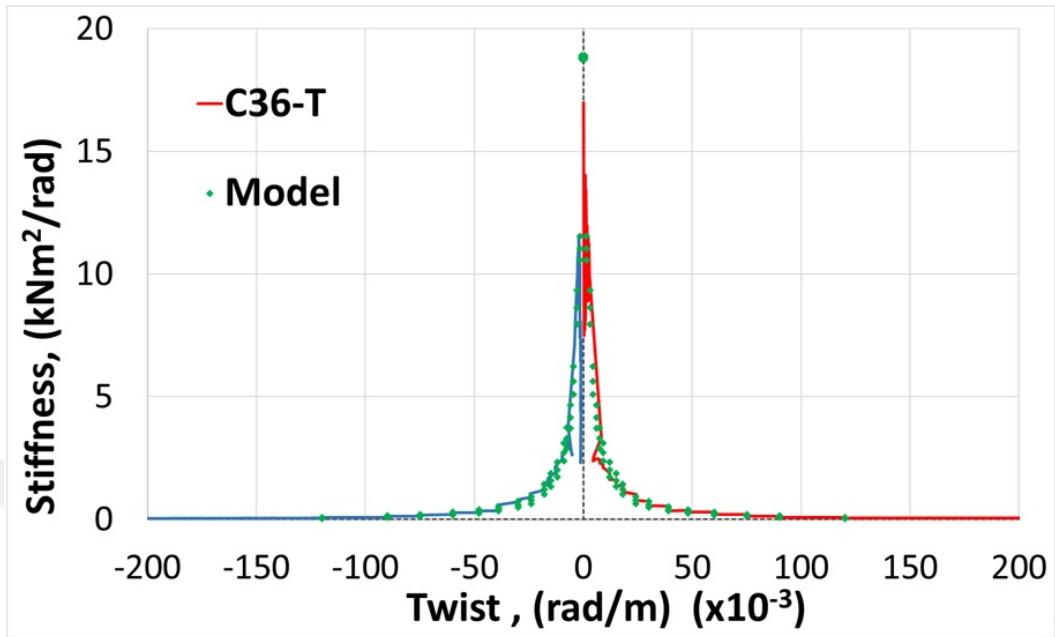


Figure E.3. Stiffness comparison for C36-T.

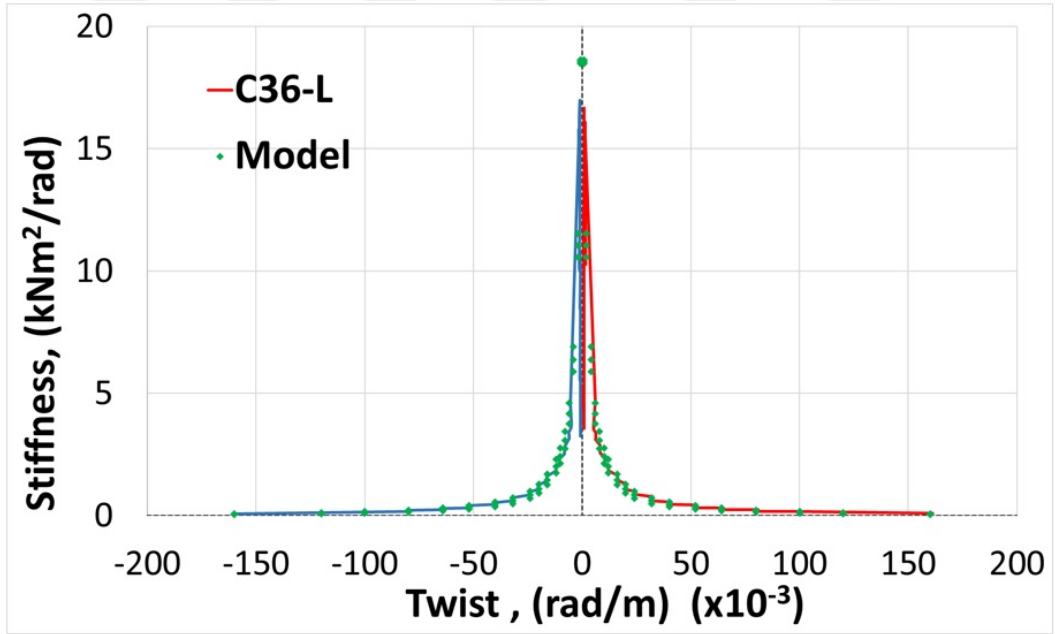


Figure E.4. Stiffness comparison for C36-L.

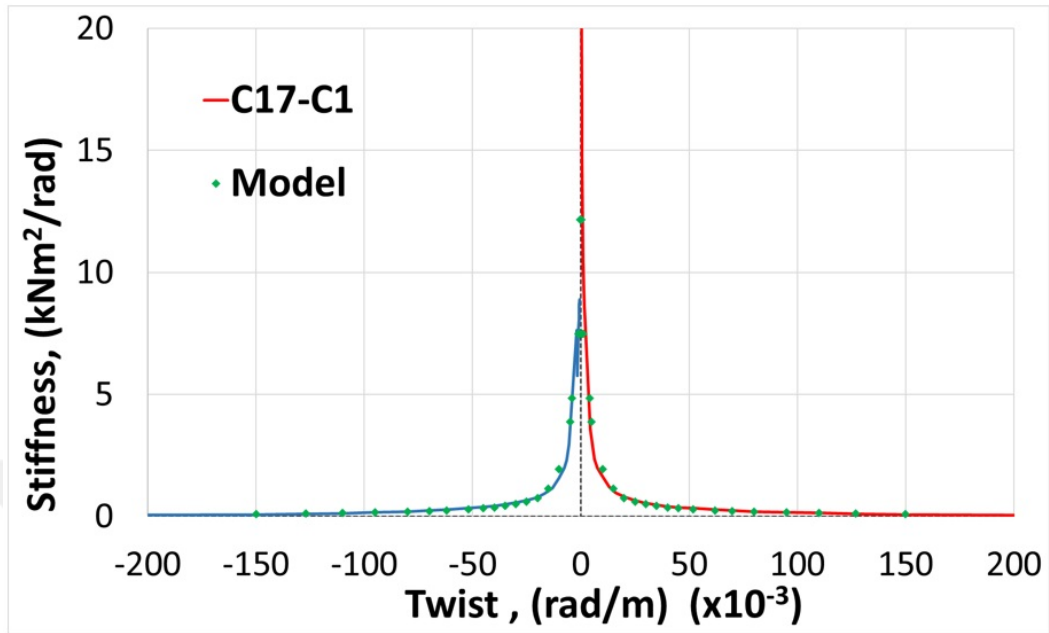


Figure E.5. Stiffness comparison for C17-C1.

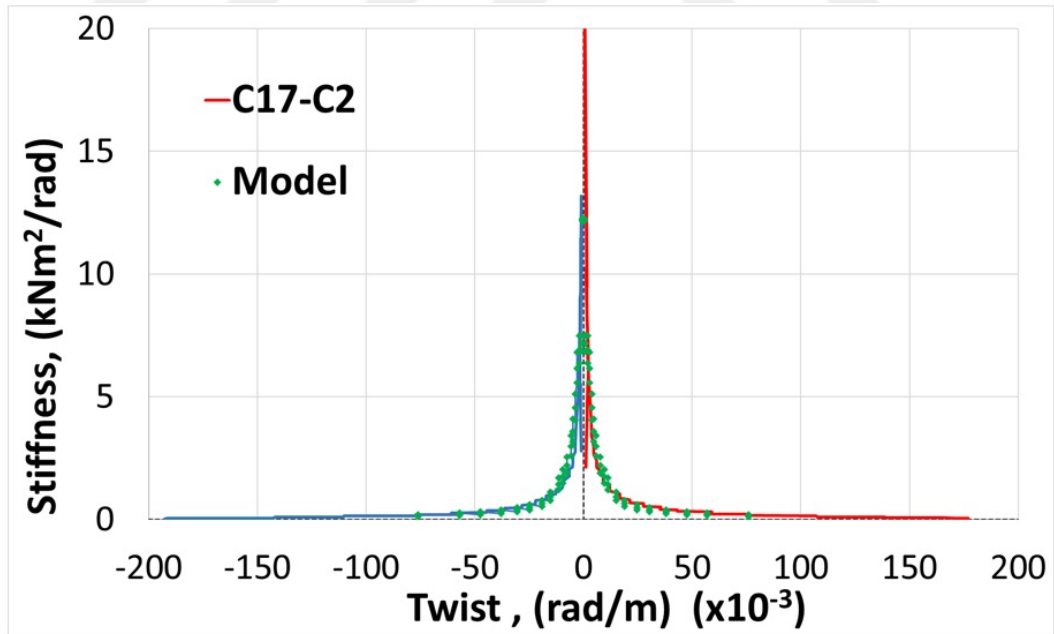


Figure E.6. Stiffness comparison for C17-C2.

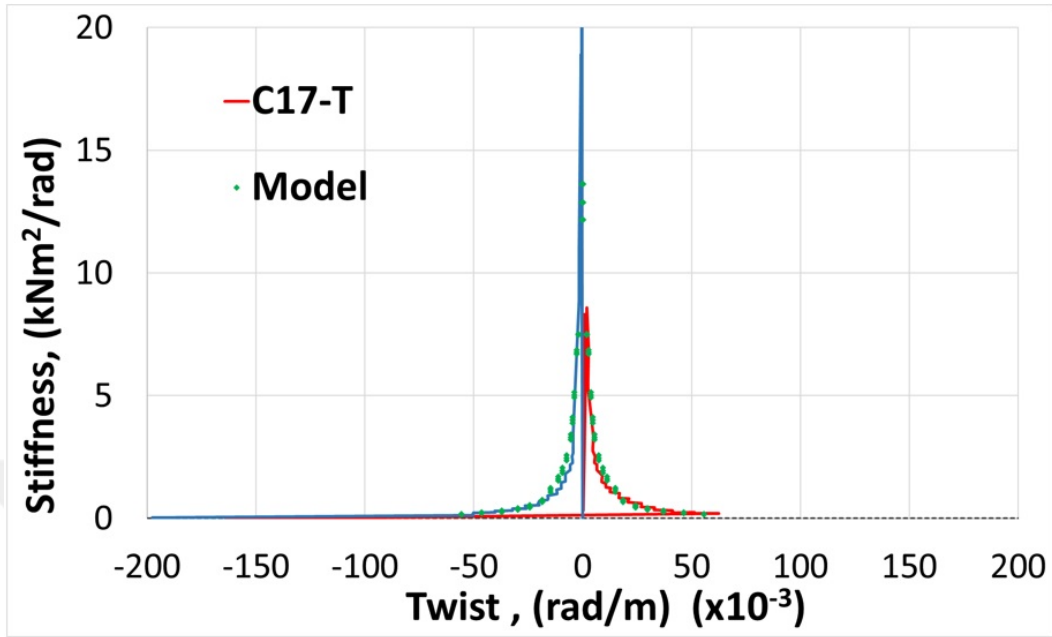


Figure E.7. Stiffness comparison for C17-T.

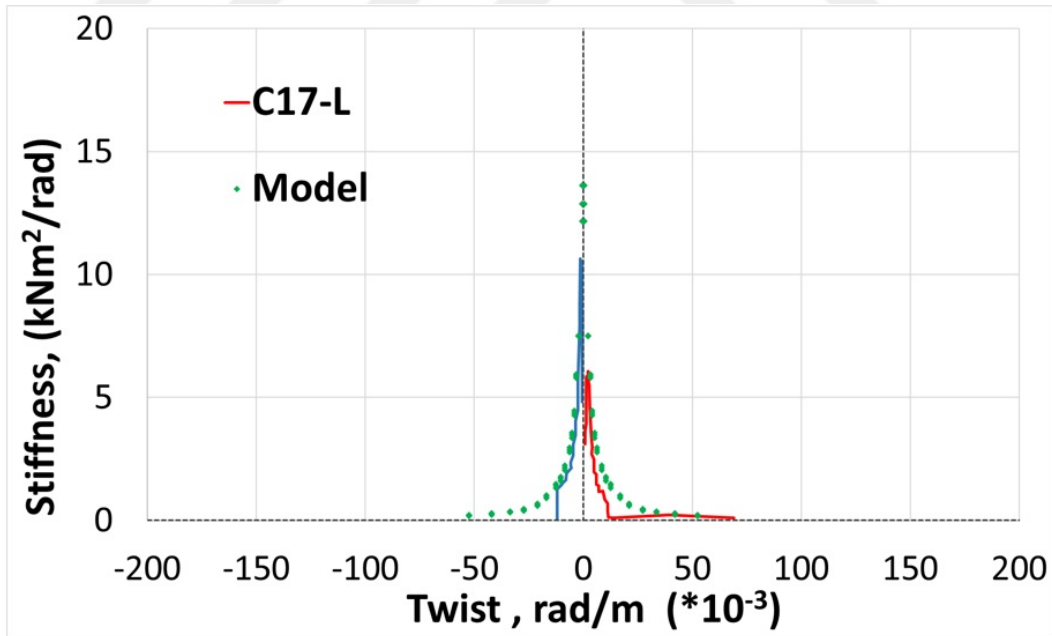


Figure E.8. Stiffness comparison for C17-L.

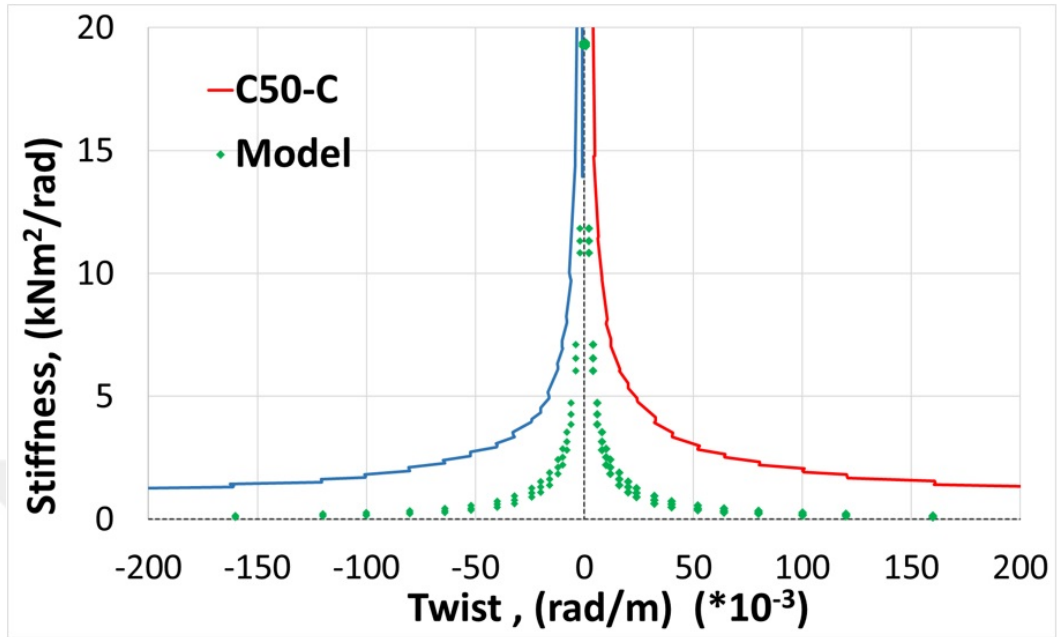


Figure E.9. Stiffness comparison for C50-C.

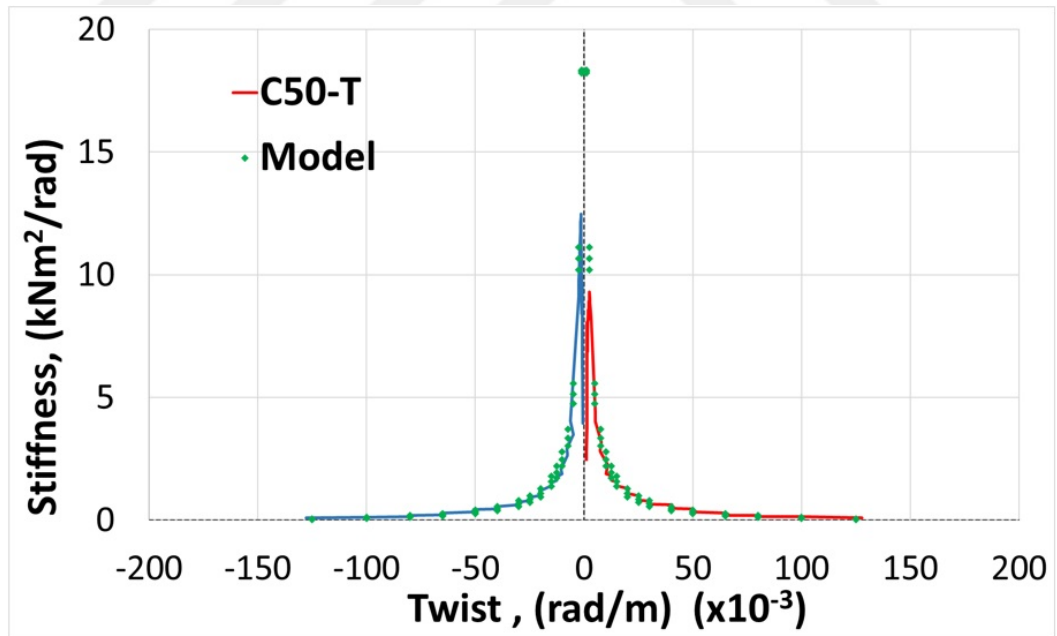


Figure E.10. Stiffness comparison for C50-T.

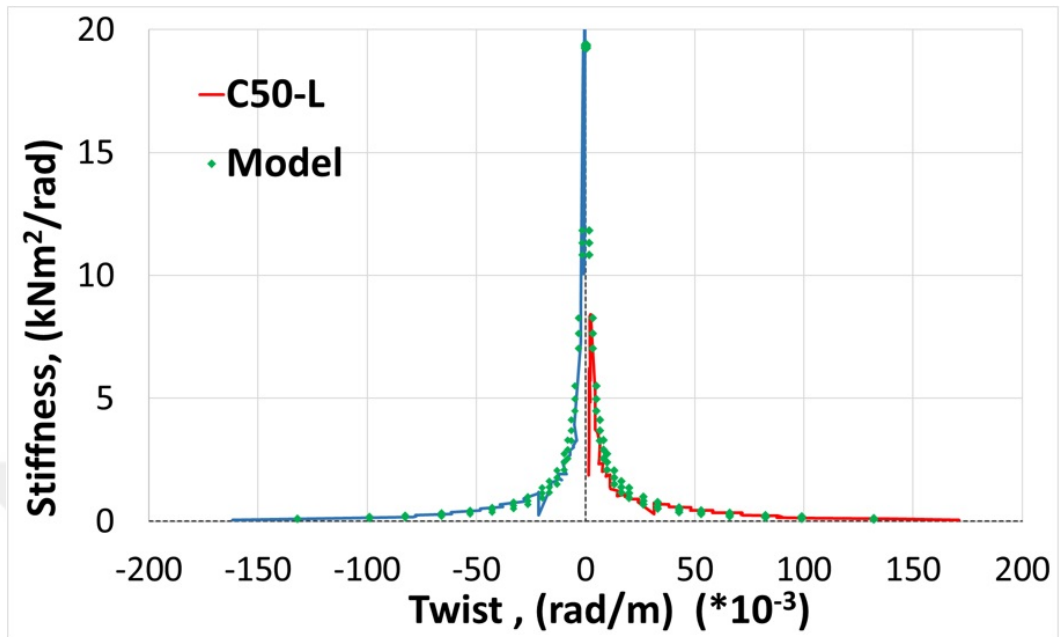


Figure E.11. Stiffness comparison for C50-L.

APPENDIX F: COPYRIGHTS OF FIGURES

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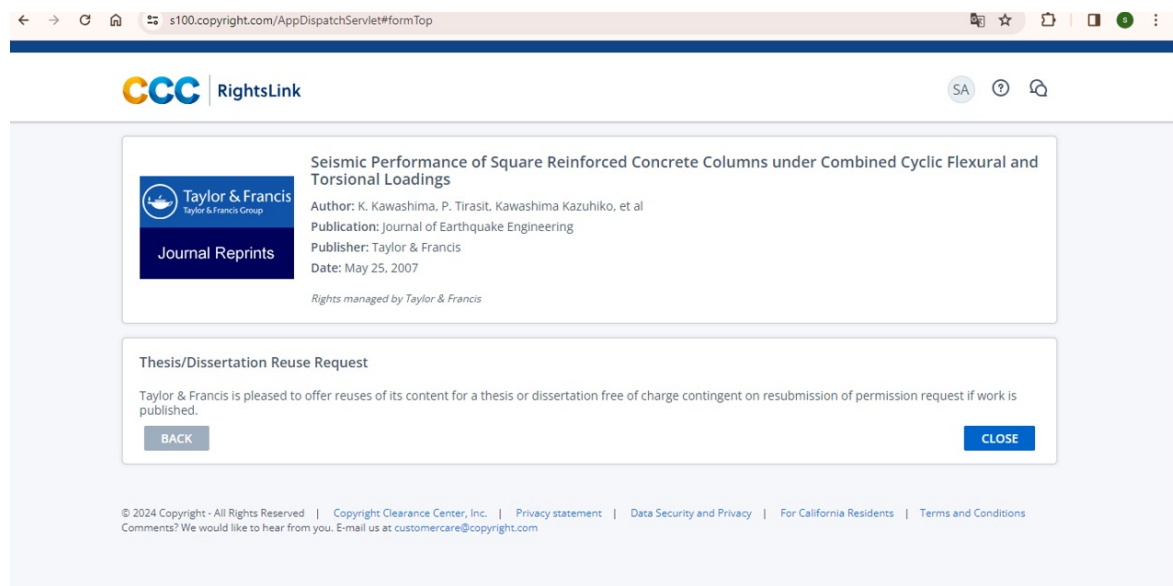


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