DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

SPARE PART INVENTORY MANAGEMENT IN AN AUTOMOTIVE COMPANY

by

Nihan KALOĞLU

October, 2019
İZMİR

SPARE PART INVENTORY MANAGEMENT IN AN AUTOMOTIVE COMPANY

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science in Industrial Engineering, Industrial Engineering Program

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"SPARE PART INVENTORY MANAGEMENT IN AN AUTOMOTIVE COMPANY"** completed by **NİHAN KALOĞLU** under supervision of **ASSOC. PROF. DR. A. SERDAR TAŞAN** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ACKNOWLEDGEMENTS

Firstly, I would like to express my gratitude to my supervisor, Assoc. Prof. Dr. A. Serdar TAŞAN for his support, encouragement and advice throughout the preparation of this study. He helped me to overcome every challenge during the preparation of my M.Sc. thesis.

I would like to thank my brother, Mert Kaloğlu who helped me a lot to complete my study. I am very grateful for his continuing support, encouragement and understanding.

Besides, I would like also to thank to parents, Sami and Emine Kaloğlu for their love, trust, and endless support.

Finally, I could not have accomplished and completed without the support of Berkay Övez. Your encouragement is very much appreciated.

Nihan KALOĞLU

SPARE PART INVENTORY MANAGEMENT IN AN AUTOMOTIVE COMPANY

ABSTRACT

Total inventory cost minimization problem of a spare part company in the automotive sector is discussed in this study. When the firm's data is analyzed, this problem includes multiple products, multiple suppliers and multiple planning periods. Demand is deterministic. A product can be supplied from more than one supplier but ordering and purchasing costs vary accordingly. In addition, each supplier has a shipment capacity constraint and minimum ordering quantity policy.

A detailed literature has been reviewed about the thesis. Based on the definition of problem, a mathematical model is generated with an integer linear programming model. This model examines how many orders should be placed from which product, when, to which supplier. After the model was created, it was run with experimental data, analyzes and results were compared. Next, by integrating company data into this model, real life problem was solved.

After solving the real-life problem, data obtained from the company for the same period was compared. As a result of the comparison, the proposed mathematical model offers 6.02 percent less total inventory costs annually. Therefore, it has been observed that the proposed mathematical model is appropriate and can be integrated to the firm's inventory decision making process.

Keywords: Inventory management, multi-product, multi-supplier, multi-period, integer linear programming, deterministic inventory problem

BİR OTOMOTİV FİRMASINDA YEDEK PARÇA ENVANTER YÖNETİMİ

ÖZ

Bu çalışmada, otomotiv sektöründe faaliyet gösteren bir yedek parça firmasına ait toplam envanter maliyeti minimizasyonu problemi ele alınmıştır. Firmanın verileri incelendiğinde bu problem, çoklu ürün, çoklu tedarikçi ve çoklu planlama periyodunu içermektedir. Talep deterministiktir. Bir ürün birden fazla tedarikçiden karşılanabilmekte, ancak sipariş verme ve satınalma maliyetleri de bu doğrultuda değişmektedir. Ayrıca her bir tedarikçinin sevkiyat kapasite kısıtı ve minimum sipariş geçme politikası mevcuttur.

Konuyla ilgili detaylı literatür taraması yapılmıştır. Problemin tanımlanmasına istinaden bir tamsayılı doğrusal programlama modeli türetilmiştir. Bu model, hangi üründen, ne zaman, hangi tedarikçiye kaç adet sipariş verilmesi gerektiğini inceler. Model oluşturulduktan sonra deneysel verilerle çalıştırılıp, analizler yapılmış ve sonuçlar karşılaştırılmıştır. Daha sonra, firma verilerini bu modele entegre ederek gerçek hayat probleminin çözümü yapılmıştır.

Gerçek hayat probleminin çözümü yapıldıktan sonra, aynı dönem için firmadan alınan gerçek verilerle karşılaştırma yapılmıştır. Yapılan karşılaştırma neticesinde, önerilen matematiksel model, yıllık yüzde 6,02 daha az toplam envanter maliyeti sunmaktadır. Bu nedenle, önerilen matematiksel model, firmanın envanter yönetimi sistemine uygun ve entegre edilebilir olduğu gözlemlenmiştir.

Anahtar kelimeler: Envanter yönetimi, çoklu ürün, çoklu tedarikçi, çoklu periyod, tamsayılı doğrusal programlama, deterministik envanter problemi

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CHAPTER ONE
INTRODUCTION

1.1 Objectives and Motivation

Firms aim to maintain a high level of profitability in order to survive in today's industrial world. Product pricing is very important for high profitability. However, in the highly competitive environment brought about by globalization, the pricing of the product is usually determined by the market. Companies need to provide profitability according to the determined prices in order to take place in the market. In order to do this, costs must be kept to a minimum. For this reason, inventory management must be done effectively for firms of all sizes. With correct inventory management, both over and under stock can be prevented and customer satisfaction may be gained with efficient inventory management operations. Because the lack of stock at the right place at the right time leads to loss of customers and sales, while excess inventory leads to increased inventory costs. Thus, effective inventory management determines the future of firms and it directly affect in the long run.

Inventory has been regarded as a positive asset for many years and some of firms have not paid much attention to the inventory management issues. In our country, the high inflation and import difficulties in the past affected firms' outlook on inventory more and supported the evaluation of the inventory as an assurance. Inventory is an asset that is difficult to manage, especially in manufacturing companies and can be found in very complex chains. Demand, production, and supply planning in the supply triangle and sales points and relations with suppliers are added to this, the amount of stock in the system is in a decreasing trend. Maintaining inventory at the right place, and at the right time is a key issue in supply chain operations. There is now a very wide supply chain around the world. Therefore, inventory management has become increasingly critical. Harris (1915) studied the economic order quantity to minimize stock cost. In this model, lead time is zero and demand is known. This model was created for a single product without constraint. But when we implement some constraints, such as supplier capacity, and shipment constraints, problem turns more complicated and harder to find optimum solution.

The spare parts distributor operating in the automotive sector mentioned in the thesis attaches importance to inventory management in order to maintain its place in this wide supply chain network. The company produces products from its suppliers across the world and sells to domestic customers. Efficient inventory management is difficult due to the wide variety of products and supplier options of the relevant company. However, each supplier's order cost and purchase cost are different. The demands are determined at the beginning of the year. According to the demands of the products, it is decided how many orders need to be ordered from which supplier. Each supplier can produce some of product for the firm. Moreover, more than one product can be produced at one supplier. As a result of this decision, the total cost is changed due to different purchasing, ordering cost for each supplier. Therefore, the supply chain engineer has difficulty in analyzing how many orders to send to which supplier to fully meet customer demand. On the other hand, it should also consider the stock keeping, ordering and purchasing costs after orders sent. During the year-end assessments, it was observed that the cost of inventories was higher than the targeted costs. Therefore, a new method was searched for the firm.

According to the literature review, our problem is defined as multi-product, multi-period, multi-supplier inventory cost minimization problem with deterministic demand. Various studies have been carried out with more than one product, period or supplier. After the mathematical modeling of this problem, where all these elements occur at the same time, its application and evaluation of its results will be carried out in this thesis.

1.2 Research Methodology

We developed an integer linear programming (ILP) model for solving multi-product, multi-period, multi-supplier inventory cost minimization problem. Besides, we have supplier-based constraints which make problem more difficult. Supplier minimum order quantity and supplier shipment capacity are our supplier-based constraints. After creating proposed mathematical model, we firstly run the model with different data in order to check results and identify weather model is applicable for the firm or not. Besides, we determinate best planning horizon with comparing run results. Then, real data has been provided by firm and implement the data into our mathematical model. Results of the real data are summarized and compared with firm's annual results.

1.3 Outline of the Thesis

An inventory minimization problem is taken in hand based on a real-life problem of automotive spare part company. The thesis is organized in different chapters. Firstly, Chapter 1 presents an introduction of the problem and gives an idea of the thesis.

Chapter 2 contains description of inventory management, identification of stock concepts and stock costs. Some of key terms such as demand, lead time, planning horizon, product, capacity, and service level are defined in Chapter 2.

Detailed problem description is provided in Chapter 3. Review of past and recent studies in literature is summarized in Chapter 3. This review study focuses on the multi-product, multi-period, multi-supplier inventory cost minimization problem specifications of the published literature.

Proposed mathematical model is provided in Chapter 4. We develop an ILP model for the problem. We provide a brief information about the model's assumptions. Then details of the model is given such as parameters, sets, and decision variables. Besides, objective function and all constraints are explained in Chapter 4. We run some examples and analyze run results.

In Chapter 5, the modelling of multi-product, multi-period, multi-supplier inventory cost minimization problem is applied to real data of the company. Firstly, we analyze company and provide information about spare part sector. We explain how we analyze real data and select products for approach. Computational results are given in Chapter 5. It is crucial to state that comparison of proposed model's results and company's annual results is provided in Chapter 5.

Finally, the conclusion of the thesis is discussed in Chapter 6. The contribution of this thesis and future research opportunities are presented in Chapter 6.

CHAPTER TWO
ROLE OF INVENTORY MANAGEMENT WITHIN SUPPLY MANAGEMENT

2.1 Description of Inventory Management

Inventory management is defined as the making a plan and control of inventories for meeting organization priorities. It is important for a robust performance of any supply chain management. It is not to reduce the amount of stock to reduce the difficult costs, but it might the right amount to ensure that the enterprise reaches its competitive supply chain capability with most effective way (Doğan, 2006).

Inventory management is the most essential part of responsibility of a production manager. Inventory management is the function that regulates the number of inventories, which are the materials, tools and equipment that institutions hold to produce or sell inputs to production processes. For future, the best way to determine and supply the products and the required substances is the basis of stock management. An appropriate inventory management needs to be understood from the following main factors: Value of stock item, amount of stock item, and stock cost (Barutçu, 2007).

Different stock control policies are applied for these stocks that can be found in various cases in each firm. The challenge in determining these policies is the inventory control strategies that will ensure efficient production, reduce distribution and all system costs, and increase customer service levels.

In order to fully understand inventory management importance, it is needed to analyze the stock concept in detail. In following, we classify the stock, and the stock costs. Thus, relation between the stock concept and inventory management can be issued.

2.2 Stock Concepts

Doğan (2006) states that all physical elements that are both directly or indirectly involved in the production produced in a production system can be considered within the concept of stock itself. Stocks have physical elements that have the economic value of revolving capital and are accepted as a capital investment. In general terms, the stock is the material produced or purchased and kept for use. Inventories with economic value, defined as inert sources, represent large investments in many enterprises. Most of the time, stock and inventory words were used close to each other. However, these two concepts differ. Stock is an accumulated level with a certain limit (Doğan, 2006). The inventory is the measured stock size with a unit, which means the inventory determination by physical counting at the end of the year.

Stocks are investments in physical goods to meet demand in a given period. According to another definition, inventories are inert sources with potential economic value. However, other than the material, it would be more accurate to call resources such as equipment and labor (Top, 2006).

In order to understand stock concept well, it is easy to sum up with following statement. According to this definition, each stored value can be counted, and every counted value can also be measured by amount of money. (Kobu, 2006). We can define stock as measurable assets in each organization.

2.2.1 Classifying Stocks

Inventories vary according to their business types and sizes. While stocks of industrial enterprises consist of raw materials and materials and semi-finished products, the stocks of inventory are generally composed of trade goods (Kaya, 2004). In this section, stock types are classified as raw materials, semi-finished products, and finished products.

Examining whole properties that are in the meaning of stock together can be causation misconceptions. The inventory classification, which is considered appropriate in terms of production planning and control, procurement, sales and cost accounting departments (Kobu, 2006).

2.2.1.1 Raw Materials

All the assets that are entered into the production in the enterprise and are valued by the transaction. Raw material inventories consist of the basic materials purchased by other enterprises in order to realize the production. Examples of steel, timber, oil, cable and rubber raw materials. Regardless of type, all production enterprises must have raw materials. The purpose of holding the raw material stock is to prevent production disruption. If there is a delay in the purchase of raw materials or if there is difficulty in supplying raw materials, production can be continued by using raw material stocks. For this reason, raw material purchase and raw material stocks should be considered independent of each other (Kaya, 2004).

2.2.1.2 Semi-Finished Products

These are the assets that are expected to be delivered to the customer upon completion of the transactions on them. Semi-finished products consist of partially finished goods, which are not yet ready for use. Enterprises with more complex and long production processes invest more in semi-finished stocks. The purpose of holding a semi-finished stock is to prevent the disruption of an activity in the production process from affecting another activity. For example, suppose that each stage of a production process that consists of more than one stage depends on the previous stage. When a problem occurs in the first stage, semi-products cannot be produced, and other units will have to stop their activities. However, if the semi-finished products produced at each production stage are stocked, then the other units can continue their production by using semi-finished stocks when production stops at any stage (Berman & Evans, 2001).

2.2.1.3 Finished Products

All the operations to be completed are expected to be delivered to the customer. Product stocks consist of substances whose production has been completed but not yet sold. The purpose of keeping finished goods is to ensure coordination between production and sales activities. In many sectors sales are made from stocks. Sales are not expected to be made for production (Tarim & Kingsman, 2004).

Factors affecting finished products stocks (Tarim & Kingsman, 2004):

- Sales volume
- Structure of demand
- Competition conditions in the market
- Diversity of sales areas
- Structure and diversity of distribution channels
- Production for the order or market
- Physical properties of the product
- Diversity of production
- Protect against labor shortage or expectation of strike
- Cost of stock keeping
- Risks of holding and not holding stock

2.2.1.4 Indirect Materials

Indirect materials are the substances that do not play the basic role in the formation of the products produced as direct substances and they have indirect contributions in the production of the product or directly contribute to the production of the products, but they cannot be loaded directly into the manufactured goods (Brunner, 2002).

Indirect materials are divided into two groups which are auxiliary materials and operating materials.

- Auxiliary Materials: The excipients are inductive substances that play an adjunctive or complementary role in the production of the raw material in the production process. While the excipients are directly involved in the finished goods, their contribution rates are less than the direct substances (Brunner, 2002). For example, the button of the dress or the nail of the furniture.

- Operating Materials: Operating materials are various materials and materials required for the execution of production related activities, such as machinery spare parts, maintenance and repair materials, fuel materials which are not included in the products manufactured (Brunner, 2002). For example, lubricants, cleaning materials, chemicals, masks, worker aprons, fuel supplies.

2.2.2 Stock Costs

All businesses must keep some or all the stock types depending on their activities. Researches show that inventories create a cost of 25% of the annual income of the company. However, even though the inventories bear large costs to the enterprise, they bring some advantages, thus avoiding inventories (zero stock level) is contrary to the general policies of company. The main purpose of the business manager should be to see the inventories as the costs to be incurred and to minimize these costs (Eroğlu, 2002).

One of the most important problems in the accounting of inventories is the distribution of the cost of inventories between the inventories sold during the period and the remaining on the reporting date. When the cost of different inventory items changes during the period, the cost calculation can be used to affect the reported profit and asset values.

Inventory cost information is a valuable information for the purposes of product pricing, product profitability, production cost control and determination of strategic competition policies (Tekin, Gürler & Berk, 2001).

Businesses should adopt the most appropriate method of actual workflows from these methods. However, the same cost calculation method for most inventories with similar usage and understanding in the many firms. Thus, for different stock and business types, inventory calculation approached may also been utilized.

2.2.2.1 The Purchasing Cost

The purchasing cost is one of basic component from total inventory cost. The purchase cost of inventories contains the product price, import duties and other taxes and other costs that may be related with transportation, loading and unloading costs and the acquisition of goods, materials and services directly. Commercial discounts and other discounts are discounted in determining the purchase cost (Tan, 2001). Total purchasing cost can be calculated with multiplying of annual demand amount and purchase price per unit. In some business, purchasing cost can be hard to manage as supplier determinates this cost. Besides, currency fluctuation can directly affect the cost if the company is buying materials or products from different countries and selling them in domestic market.

2.2.2.2 The Ordering Cost

Ordering cost contains whole the costs that must be incurred in order to place the order, and the transportation costs that the products generate until they enter the warehouse. In some cases, quality control, delivery, deterioration and breakage in the entrance to the warehouse are also considered in order costs.

Costs occurred when a new order is placed from customer to supplier. Order costs generally depends on whether the ordered stock is produced in-house or purchased externally. If inventories are produced within the company, material transfer, labor, materials, stationery and overheads constitute the order cost. If it is purchased externally, the order confirmation, delivery, sending, receiving, acceptance inspection, expenses incurred as a result of work related to invoice and insurance and stationery expenses constitute the order cost (Bragg, 2005).

Order costs that may occur in a business include cost (Bragg, 2005):

- **Costs of Preparation**: It covers the costs incurred only for placing a new order, whether in or outside the establishment. For example, there is a cost for preparation of requests for products to be taken from outside such as e-mailing, and electronic data interface programs etc. On the other hand, approval of the necessary departments, conducting research between vendor companies, and acceptance tests. Similarly, upon the request from the sales, the order of work orders, loading and programming activities and manufacturing, mold, tooling, apparatus replacement and so on costs to be incurred costs. When each order is made, the preparation costs incurred in ordering the order must be compared with the benefit of frequent ordering.

- **Direct Labor Cost:** A person needs some time to learn and do his jobs properly, and this learning period very important. Complexity size of operation directly affect this cost type. It depends on person's ability and also good work environment. At the end of this period, the unit can be reduced by 20-80% when the unit is spent on the product. The decrease rate of labor-intensive jobs is higher. The smallest volume of the order, i.e. the frequent downtime, reduces the labor-time savings to be achieved by learning. It may be more economical to keep stock levels high and benefit from this advantage (Doğar, 2006).
- **Production Control Costs**: The annual cost and effort spent on production control depends on the orders' number placed. So, there is no relation to the quantity ordered. The lower the number of orders in a year, the lower the cost. These costs relate to costs such as opening and closing orders, scheduling planning, loading and finding.

2.2.2.3 The Holding Cost

These cost elements consist of the investment cost linked to the inventories, the cost of storage, the taxes paid for inventories, insurance, material transfer, loss of stocks, theft, loss, damage, etc. elements of inventory holding cost is given in Figure 2.1. Holding cost is one of main asset for inventory management operations. Many firms put their effort to minimize total holding cost. It also affects replenishment time of goods.

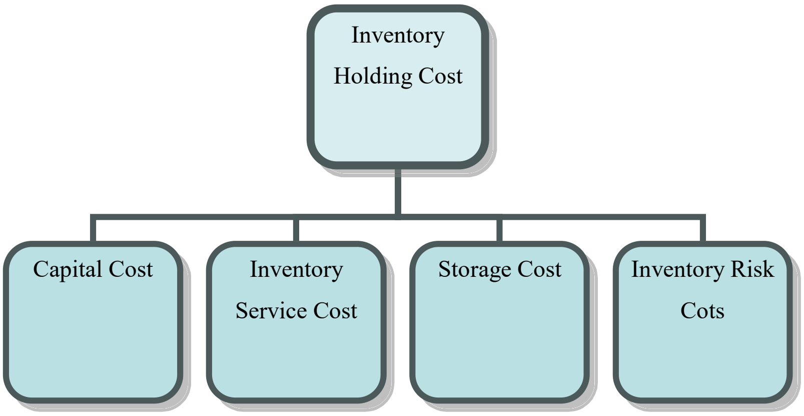

Figure 2.1 Stock holding cost components

The cost of capital, which constitutes the maximum portion of the cost of stock, is also called the opportunity cost. As the capital spent on stock investment is not available for other purposes, it is considered as a loss opportunity cost for the enterprise. If the investment for stock keeping is evaluated in other areas, the answer to the question is what would be the return on us. Therefore, the alternative cost of the money connected to the stock is calculated. For example, a portion of a working capital is not used for stock investment, but if government bonds are purchased, it can be measured as an opportunity cost.

The capital cost of holding a stock is calculated using the return on investment rate. Capital cost is related to cash value of total inventory. The cash value of the stock varies according to the type of business. If the business is a retailer or wholesaler, the cash value of the inventory is the current cost of replacing the stock. On the other hand, if a company is a manufacturer, the value is equal to the cost of producing the stock and making it ready for sale. For this reason, it should be noted that the cost method is used to determine the value of the inventory. In addition, when considering the opportunity cost of investment in stock, the opportunity cost must be taken into account or in the dead stock (Lambert et al., 1998).

Storage-area costs are known as the cost of use of warehouse equipment, but also include expenses for warehouses, workers and material handling equipment. Storage and space costs often increase as the stock level increases or the warehouse is full. As the stock level fluctuates, the stock value fluctuates at the portion of the area associated with its use. It is difficult to determine the cost of space as a percentage of the value of any product.

The factors of cost of using the storage:

- Tax: property tax of the warehouse
- Insurance: earthquake, property damage
- Public Services: heating, lighting, security
- Wear: repair and maintenance of warehouses and equipment

Measuring the cost of storage and space requires precision and mastery. This cost is generally composed of the machine fees used for the entry and exit of the storage, but the rent payments can be included in the costs incurred in criteria such as the wear of material handling equipment. The fastest way to measure such expenditures is to divide the general expenditures by rent, public services, repair and maintenance, the property tax of the building and the inventory according to the criteria for admission to the storage and the stowage (Tunç et al., 2013).

The stock service cost, which is another component of the cost of inventories, consists of the sum of the costs incurred by the inventories such as taxes, insurance and inventory counts. The tax rate varies depending on the state policy, product type, value and stock level. Depending on the type and value of the product, a high premium is usually required for insurance against fire, theft and damage. Insurance premiums should not be expressed as a direct percentage of the stock level as it covers the value of the product for a specific period of time. Since insurance and taxes vary from product to product, businesses should consider this when determining the cost of stock service for any product. However, since the insurance and tax premium amount varies according to the value of the stock, any reduction in average stock value directly changes savings in losses (Lord, 2006).

At this point, the value of the product which is important for the enterprises is to decide whether they will take the current production cost or the sale price. In calculating the cost of inventories, the second stage is to express each subcomponent that constitutes the cost of ownership as a percentage of the product value. The final stage is the multiplication of the total cost of ownership and the unit cost of the product. Thus, the cost of unit stocking will be calculated for any product in stock. When calculating stockholding costs for managers, using one-to-one industry averages or using the values in the literature can create some problems, rather than the percentage of stocking costs. First of all, the cost of ownership, which constitutes an important part of the cost of inventories, differs for the enterprises in dissimilar businesses, depend on the cash status of the enterprises and the opportunities to invest.

Koster et al. (2007) emphasized the importance of order collection, saying that the cost of collecting orders accounted for approximately 55% of the total warehouse operating cost. In these activities, the distance traveled, travel time and collection times should be minimized, and they worked on minimizing the cost of collecting orders.

2.2.2.4 The Stock Out Cost

The success of business strategies is closely related to the control of the stock. The stock, which is held more than necessary, increases the risk by decreasing capital, storage expenses, follow-up and monitoring costs. On the other hand, inadequate stock holding leads to inefficient supply, loss of sales and disruption of production. If the demand forecast during the supply period exceeds the forecast, a stock shortage is expected. When there is insufficient inventory in which we cannot meet customer demand, the negation situation occurs (Tan, 2001). The cost of inventories, which is also called as the cost of loss, consists of loss of customer loyalty, loss of good reputation of the enterprise, loss of sales due to not meeting the demand with the stock of hand.

Generally, the negation condition occurs at the time between ordering and arrival. This time consists of the preparation of the order and sending it to the supplier, the preparation of the order by the supplier and the sending of the product to the business and finally the total time required for the delivery of the product to the customer by the company. If the demand cannot be met by the business, the customer has two options. Either the order is completely lost, or the customer is prepared to wait until the product arrives. When the customer agrees to wait, he or she will either accept the normal time for the product to arrive in the enterprise or demand a better delivery time (Waters, 2003).

The essential factor for managers to determine the right stock level at the right time, to reduce their costs and to satisfy their customers. Destruction influences the future sales of the enterprise. Customers who are loyal to the enterprise are willing to wait for their requests to be met later when they encounter the negativity. However, the future behavior of the frustrated customer may change, and the customer may want to work with more reliable businesses. However, according to the sector in which the entity operates, the behaviors of customers in the case of neglect are different. In some sectors, for example in the automobile and furniture sector, customers accept the negligence and have to settle for the waiting period. Particularly in furniture and car galleries, samples are offered from different product options and there are not enough products to meet the demands of all customers. Considering all of these criteria, the reasons for letting enterprises to have the status of negligence are as follows (Tan, 2001):

- If the cost of holding stock is too high
- If there are too many different items to keep stock
- If the time to supply the supplier is short
- If the competition is limited
- If customers are willing to wait for delivery

A situation arises against the firm because of re-ordering costs, lost sales and possibly lost customers. In order to eliminate the damages caused by the non-stop situation when the demand is higher than the estimated time during the supply period, the enterprises can reduce the lack of products by keeping the extra stock. In addition, an urgent order can be sent by the company in order not to lose the customer. However, the improved delivery time that can be offered to the customer by the planner pushes the production process into confusion. Due to the urgent ordering, the production periods can be re-planned, and work can be done with other alternative and more expensive suppliers, extra effort and time is spent and the more expensive replacement of the product can be used (Waters, 1992).

2.3 Key Terms of Inventory Models

2.3.1 Demand

Businesses need to use the stocks they hold in the hands when it comes to demand. In the context of demand, as well as the demand from the customer for the final product, there may be demand for the intermediate products and the components of this product that are required to produce this product within the company. Demand for the final products offered by the firms to the market is expressed as independent demand. In order to make more accurate inventory plans, firms try to estimate the demands on their final products using mathematical or judicial methods. In this way, anticipating the demand in advance, however, provides a great advantage to firms. The demand for intermediate products used to produce the final product and the components for this product are expressed as dependent demand. Because the demand for these products is indirectly affected by the demand for the final product. In the end, production will be realized by taking into consideration the quantity of demand coming to the final product and intermediate products will be needed depending on this determined production. Therefore, firstly the independent demand for the final product is determined and then the dependent demand for the intermediates and the components is calculated.

Another important point in terms of inventory management is the fact that the demand is already known precisely. If the company knows the exact quantities and the times that it will encounter in the future, at the beginning of the planning horizon, there is deterministic demand. A model developed by Ladany & Sternlieb (1974) is one of the first deterministic models of stock models in terms of price impact on demand. The maximum profit in this model is targeted. The profitability of the entity is equal to the difference between the total revenue and the total cost and the total revenue is based on the sales price. The demand in the model is a function of the sales price.

On the contrary, stochastic demand is the case where demand is not known precisely beforehand, and only in the case of probabilities. In other words, in the case of stochastic demand, information on demand is when and how much demand, and what is likely to be encountered.

2.3.2 Lead Time

Lead time is that if the inventory is ordered from the outside, the period of time is the period between the order and the entry of the inventory into the warehouse. If the inventory is produced in the factory, then the lead time is a batch product production time. The time period in which the request is given is expressed in the same unit. If the demand is annual, the period of supply is also expressed in year (Karahan & Aslan, 2016).

The lead time is the time interval until an order is received. If demand is stable, what the actual demand is during the delivery period. However, if the demand is variable, we know the average demand in terms of supply time and variance. As demand variability increases during the delivery period, the need for additional inventory increases. These additional inventories help to reduce the risk of scarcity. Additional inventory is known as security stock. Therefore, as the variability of demand in delivery time increases, the required security stock increases. When variability is available on demand or lead time, the actual demand in the lead time will be different from the average demand in the lead time. The average delivery time may be greater or less than the demand (Karahan & Aslan, 2016). However, if there is no change in demand during the delivery period, the demand during the delivery period is equal to the average demand during the delivery period.

The period of supply is defined as the period from the time the product is placed in the stock to the order. Receiving the supply time positively, even assuming a random assumption is one of the factors that complicate the problem. The variability in the supply period brings with it the risk of planning and the difficulty in planning if the demand for the supply period is random (Silver, 2008).

It is known that inventory management in multistage supply chains is much more difficult than single stock point systems in business applications and modeling environment which reflects this. However, in this context, especially inventory problems involving non-stationary stochastic demand and fixed order costs include mathematical difficulties.

Supply means that raw materials, supplies, spare parts, products, etc. must be supplied by the enterprise. Therefore, supply is a feature that allows stocks to increase. The supply chain is a collection of activities that enable the materials to reach the end user from the supplier. For companies operating in almost all sectors, supply chain management is an important issue to be considered. The period from the start of purchase or production of the materials produced and ordered until the time it is ready to meet the demand is expressed as the time of supply. In inventory models, this period can be known as probable or probabilistic, but also in cases where supply time is completely uncertain (Silver, 2008).

It is often difficult to control the supply time and is one of the parameters of the inventory system. This time may be zero in the sudden supply or may be random according to a probability distribution. Procurement time has a significant impact on the solution of probabilistic inventory system.

A good inventory management is one of the most significant challenge in the management of good supply chain. On the other hand, when firms' cannot achieve good solutions in inventory management, they may face excess and/or stock-out issues in many times. Well the firm contains large product variety, they would face excess stock in each period. Yet, cutting edge technologies provides collaborative approaches for different types of inventory management problems, therefore, companies can make stock in large amounts of inventory. Because they have got a robust inventory management system. On the other side of excess stock issue, zero inventory is another business strategy. The aim of the zero inventory is to purchase only demand or required level to be needed and accordingly, it should be only ordered goods when they are needed. This also defined as exact time inventory approach. It provides zero inventory in more efficient, flexible and cheaper way rather than inventory holding. This approach is not suitable for all supply chain businesses, but it is might be best management system for some businesses in terms of geographical and technological point of view. Most Internet-based retailers work using a zero-inventory model, especially for a wide variety of perishable and fashionable consumer lines. To increase the rate of return and rate of companies by increasing the cash flow to ensure the highest level. As a result, the zero-inventory strategy is based on having an efficient supply chain that is completely reliable (Kaya, 2004). To facilitate the basics of zero inventory, the raw material and the inventory transport load are pushed down the supply chain. Suppliers must work with perfect efficiency to perfect their operations and create smaller, more frequent production processes. They must fulfill the orders individually and send them directly to the customer.

Most businesses are unlikely to reach zero inventory, because most will continue to have buffer stock for any immediate, late birth, natural disasters or any risk of customer service. They know that increasing customer satisfaction is the easiest way to find at the right time, at the right place, at the right time. However, the closer a business is to achieving zero inventory, the greater the cost savings and benefits of increased cash flow, reduced transport costs and stock wastage such as stock waste in maintaining large, redundant inventory levels.

When there is no price reduction, the economic order quantity (EOQ) can be ordered. Similarly, when there is a price reduction, EOQ is ordered or more is ordered from EOQ. As long as the cost is the main problem, never less orders than EOQ. However, if you are interested in flow time instead of production cost, you can order less than EOQ. When there is a periodic inventory system, it is always ordered at the beginning of the period. For example, the beginning of each week, the beginning of each month. However, in the continuous inventory system, the re-order point is defined in terms of quantity, not time. When the end of the period is not ordered, it is ordered that the stocks at hand reach a certain level. This defines the Reorder Point (ROP) (Silver, 2008).

In a periodic inventory system, ROP is the beginning of the period. However, in a continuous inventory system or in a continuous review inventory system, the re-order point is a reduction of the inventory to a predetermined amount. In the EOQ model, it is assumed that there is no change in demand, i.e. the demand is constant and remains constant. If there is no change in demand and the demand is only fixed, the ROP is equal to the average demand in the lead time of the inventory obtained. If the demand is fixed, demand will always be a certain number. Therefore, the average demand in delivery time is equal to the actual demand in the delivery time. On the contrary, if demand is variable, the average demand in delivery time is different from the actual demand in delivery time. However, if demand is stable without variability, demand in delivery time is the average demand in delivery time. The re-order point is a point where the inventory is equal to the lead in delivery time (Silver, 2008).

2.3.3 Planning Horizon

Inventory plans are created for the future. The planning horizon refers to the period that the inventory plan will cover, in other words, the impact. This time can be finite or infinite. However, inventory plans with finite planning horizon are generally made by enterprises. The reason for this is that as the planning horizon increases, the possibility of making mistakes in inventory plans increases. Finally, inventory plans include demand forecasting, so long-term errors are hard to predict (Lin & Hou, 2005).

Planning horizon is sometimes called a working time or a planning window. What the advisors make predictions means a window to the future where owners and managers take plans and decisions. The advisor's estimates are permanent costs, and the owners' decisions and preparations are related to capital projects, such as roofing or improving the boiler. This planning cycle often covers many calendar years or fiscal years and aims to provide enough lead time for the development of logistics to increase funds and manage assets responsibly (Eroğlu, 2002).

The planning horizon results in a spending plan that answers the question of how much money we need, and which constitutes the first part of the financial analysis. The spending plan helps to create financing models to answer the question of how much money we have.

For management purposes, the planning horizon can be divided into three adjacent phases. Combining the projects in all three stages ensures total capital burden. Projects in long-term phase have higher costs associated with the time value of money. Figure 2.2. is presented planning horizon. The stages are relative to each other and are not relative to the base year (Eroğlu, 2002):

- Short Term Planning (up to 3 months)
- Medium Term Planning (3 to 18 months)
- Long Term Planning (More than 1 year)

Figure 2.2 Planning tasks and horizon

The first classification of the plans is to classify the plans as short, medium and long term according to the time period covered. According to this classification, long-term plans are plans to cover 5 years, medium term plans between 1 and 5 years, and short-term plans covering up to 1 year. After the planning horizon is filled with events, usually as a result of the counseling report, experts can analyze patterns and trends in the horizontal distribution of events.

One of the most important uses of optimization is multi-term planning. Most of the problems that we think so far are basically one-term problems. The formulations act as if the decisions of this period were resolved from the decisions of the future. However, it is typically said that when we produce a particular product that is more than required by a restriction in this period, extra production will not be worthless, but it can probably be used in the future. These interactions between periods can be shown very easily in optimization models. In fact, the greatest linear programs encountered in practice are multi-term models. A common synonym for many periods is dynamic (Bordum, 2010).

Models for planning over time represent the real world by dividing time into several periods. The part of the model that corresponds to a single period may be a combination of the product mixture, mixture and other models. It connects with single-period or static models by (Bordum, 2010):

- A link or inventory variable for each commodity and period. The binding variable represents the amount of commodity transferred from top to bottom.
- A material balance for every commodity and term or resources constraint. The simplest form of this restriction: "inventory + start production = ending inventory + sold goods".

Multi-period models are often used in the form of rolling or sliding. In this format, the model is solved at the beginning of each period. Recommendations for the first term of the solution are applied. As a period passes and better data and forecasts occur, the model shifts the period forward.

Bad or faulty production planning leads to excess stock, increased stocking costs or non-availability, and inability to meet consumer demands. Efficiency decreases.

2.3.4 Product

The concept of the product is the whole of which tangible and untouchable qualities. The concept of product can express different meanings to everyone. According to the producers, the product is the physical substances they produce by producing and marketing them. There are important things to be done for the product concept. According to sellers, the product is what they buy to sell to retailers.

According to the consumer, the product is the beneficiary objects by meeting personal needs. In general terms, the product is the objects offered by the enterprises to the consumers to meet the specific needs. It is everything that people want or don't want in the process of change. In the narrow sense, the product is a combination of various physical and chemical elements that can be easily seen and utilized. Only the concrete elements of the product cannot be mentioned. Contemporary consumers consider the manufacturer's prestige and brand image when purchasing products. Because consumers demand both physical and psychological benefits (Davis, 1993).

One of the keys to taking advantage of the power of product types is to learn when to use a simple and independent product. A simple product can be individually or grouped, configured or sold as part of the package product. A simple product with special options is sometimes referred to as a product that offers a variety of options to customers (Davis, 1993).

The complexity of multi-product models is known, and it is getting an imbalance in terms of different industries (Janssen, 2016). However, many companies have more than one product in the product range. In this case, determining the extent to which the business will produce or order from each type of product, and to what extent it should be controlled, and to classify them according to their criticality or values constitute an important problem area for the enterprise. On the other hand, it is not possible to control all inventory products at the same importance level. Creating a separate inventory control model for each product is quite difficult, time consuming and costly.

Failure to perform the inventory control properly results in an excessive accumulation of some inventory products and the imbalance in raw materials and semi-finished stocks required for production, thereby increasing the required financing requirement. For this reason, modeling approaches where multiple products are considered among the new approaches have an important place. Thus, the balance between the various inventory products can be prevented and the excessive swelling in some inventory items can be prevented (Choudhary & Shankar, 2011).

The stock problem in multi-product inventory control models under resource constraint is considered as a limited optimization model and solved by using the Lagrange multiplier method. In these problems, raw material constraint, storage space constraint, machinery and labor capacity and stock investments are considered as resource constraints. In many applications these models are used under one or two constraints.

Many businesses in real life are taking decisions about hundreds or even thousands of products. In this case, it is very difficult or impossible to follow a stock policy by developing a separate model for each product and bringing them together (Choudhary & Shankar, 2011). Therefore, different approaches have been developed for multiple products. The first one is resource-constrained models. Another approach is to determine the stock control policy by grouping the products or making a general aggregation.

ABC method is the most widely used method in grouping and controlling products. With this method, a small portion of stock items with a high share of value is determined as more important and more controlled. On the other hand, it is less important that most of the stock items, but the minority that contains very little of the stock value. In this way, the main purpose of grouping the products into groups is to determine the importance level for each product. In this way, businesses can save both time and cost and thus perform their stock control functions at a lower cost than controlling individual products (Haksever & Moussourakis, 2005).

All kinds of inventory in the inventory are not of the same importance. Therefore, inventories may be large in number and may constitute a large portion of your total inventory but may be more in number but less in inventory in terms of value and inventories that do not affect the inventory.

All inventories in the inventory do not have the same value. Stocks that are few but high in value should be controlled more than the others. The general rule is to keep high value stocks at minimum level. Moving costs will be high if the bulk of them are large, but less expensive than others. By making ABC analysis for inventory in stocks, stocks are classified according to their values. The ABC method is derived from the Pareto principle (20/80) and focuses on the few but most important ones. It is used for the purpose of diagnosing and analyzing process problems in the areas. Pareto analysis is an extremely simple but highly effective technique (Urban & Baker, 1997).

The application of Pareto's 20-80 principle to inventory management can be expressed as follows: most of the stocks (up to 80%) consist of items with regular tock inflow and outflow. A small portion (20%) is both highly mobile, i.e., stock turnover is high, and the economic return is high (Urban & Baker, 1997).

By applying ABC analysis to sales, it can be seen that generally 80% of sales are realized by 20% of the whole customer demand or sales amount, accordingly it may classify the suppliers or customers and build the purchasing amount, pricing operations, discount decisions, or campaign policies. Similarly, in the previous years, a product that sells in the last year cannot be sold or sold in the C group for the last one year, and perhaps the reason for this is that the price of this product is too high compared to the market or the sales orders could not be met due to the shortage of supply. The percentage and value percentages here are of course not a definite rule.

2.3.5 Capacity

Inventory management under capacity constraints represents an important part of inventory management. economic order quantity model is one of classical approach under the capacity criterion with the Lagrange method is among the models proposed in the literature. As the orders have different lengths, they are randomly distributed over time. In some studies, according to the random storage proposition, it was found that the capacity utilization methods used in the classical model caused the capacity to be used effectively (Tan, 2001).

According to capacity restricted party size-reorder point inventory model, the capacity constraint within the stochastic inventory management models based on continuous control where many products are controlled is the constraints that should not be ignored and should be included in the problem in determining the order quantities and reorder points. As in the classical EOQ model, the capacity constraint for the products to be checked in the batch size inventory order inventory model from the stochastic models based on continuous control should be included in the model. According to party size reorder quantity model in random storage, in the case of a capacity constraint for the classical EOQ model, it is known that a large part of the current capacity will remain idle for most of the time, as the capacity used is determined by the maximum amounts that will be achieved for each product. It was also determined that the same situation would occur when determining the capacity limit in order quantity reorder point model. By using the random storage-based inventory assignment model discussed in the previous model, the stock limitation of the products to a certain area is removed and the total stock capacity requirement is reduced, or the use of the existing capacity is increased (Kalınbacak, 2006).

There are two main reasons why products are ordered in coordinated order. The first reason is to ensure that products are coordinated in order to flow to a more uniform stock. The other main reason is to take advantage of the cost advantage resulting from the ordering of the products together in the opposite direction of the first reason. Because of the fact that coordinated order can be used in these systems in order to ensure smooth stock flow. With coordinated order, orders of products can be planned by manual or mathematical algorithms, and capacity utilization is distributed over time and more efficient use of capacity is ensured (Chen, 2013).

The most common use of product orders in the coordination of both manual and mathematical algorithms is the strength policies of the two. By multiplying the non-negative integer forces of the second by a determined basic period, the force coefficients of the two of the two basic periods are obtained, which is to limit the cycle times of the products with these coefficients. The cycle times of the products are limited so that they are simpler and offer a repeatable cycle time advantage over the collection. In addition, by limiting the cycle times of the products to the multiples of this basic period, it provides the appropriate planning and provides near-optimal results in cost. The planning of the product orders for the period length determined by the capacity utilization is done by mathematical algorithms or manually and a more uniform capacity utilization is achieved. In the case of a capacity constraint, a simple approach can be applied to reduce the order quantities of all products in proportion to the maximum capacity utilization (Chen, 2013).

2.3.6 Service Level

Service level indicates the possibility of not going out of stock. This percentage is required to calculate the security stock. Intuitively, the level of service is a tradeoff between the cost of inventories and the cost of inventories (which leads to missed sales, loss opportunities, and customer distress among others) (Ertuğrul, 2011).

In the supply chain, the cycle service level (or service level only) is the possibility of not coming out of a stock during the next refresh cycle and therefore not losing sales. Cycle time is indirect delivery time. The level of service can also be defined as the possibility that customers will be able to fulfill their claims that they are faced with any back order or loss of sales. 100% service level - that is to say, it always seems desirable - to serve each customer, is usually not a viable option. The service cycle should not be confused with the filling rate, which represents the rate of demand submitted without delay or loss of sales (Ertuğrul, 2011).

Customer satisfaction is the most valuable asset a business can register. The measurement of service level is important because of its link with inventory management, because it can affect its relationships with customers and can have a significant impact on profitability, depending on the nature of the business. While many companies see inventories as a cost element, there are companies that see them as active ingredients that contribute to the leadership of a particular market share with a high level of service and offer the availability of products in the current quantities (Ertuğrul, 2011).

In fact, the concept of service level only relates to situations where future demand is uncertain. Otherwise, reaching the 100% service level is only a matter of timing. When the demand for the future is uncertain, the only theoretical way to leave no room for stock is to choose an endless inventory. Therefore, in the application, the inventory manager needs to make a defective inventory swap. This swap is exactly measured by the service level concept.

Hunneman et al. (2015) found a strong impact between retail service level and customer satisfaction. By fulfilling customer demands, a higher service level corresponds to transport or slow-moving stock costs.

In inventory management, the level of service is the probability that it will not hit a share in the next refresh cycle or may not lose sales. The level of service is determined by the stock level in a company. Therefore, the security stock should be high enough to meet the vendor's delivery times, sufficient to meet customers' demand, but not high enough to lose money due to your company's high transport costs. The main reason for this is that there is not sufficient consistency to predict demand volatility and predict future volatility (Kasap et al., 2010).

Retailers and producers are trying to achieve a high level of satisfaction that maximizes sales on a customer basis. At the same time, it is expensive to maintain a high level of inventory and offers different risks such as: storage, expiration and lowering of prices. When stock level increases unpredictably, risks and costs. A high level of service may be in the risk and it is very essential in the retail sector. Companies that set targets at 95% do so because the level of service is a key factor in maintaining loyalty of customers (Kasap et al., 2010).

The level of service is a balance between opportunity costs and operating costs. Optimizing service levels to maximize company returns is often complex and domain specific. Analyzes are sensitive: lowering inventory levels may take years to observe a lower customer earned with more inventories, resulting in immediate availability of extra cash. Customer precision in product stocks varies from product to product, optimum service level is unique for each material.

The cost of stock-outs is completely different and more complex. The most obvious cost of inventories is the decline in sales, but this factor is important, even if it is important. Extensive research has shown that stock-outs are a major risk in terms of customer satisfaction and may cause serious erosion in the customer database in the long term (Kasap et al., 2010).

All efforts to improve the level of service within a company consist of a variety of sub-objectives that are monitored and must be part of the continuous improvement process. A company focuses on the following individual goals to meet the requirements of the level of service that it wants to provide: delivery preparation, delivery time, resilience, reliability and quality. Preparing for delivery is the ability to meet a requirement on time (Barutçu, 2007).

2.4 Chapter Conclusions

Inventory management problems have an important place in supply chain management. In this chapter, we explained the inventory management concept, and provided theoretical information about important terms in this area. In order to understand the subject of the thesis, we described key essentials in this chapter. We firstly descried inventory management definition. As we stated before, “stock” issue is the most important term in this area. That’s why the term of stock is scanned very well in order to understand the general idea of this thesis. Each defined term in this chapter is emphasized in the following sections.

The following chapter presents the problem description and literature review within this framework.

CHAPTER THREE
PROBLEM DESCRIPTION AND LITERATURE REVIEW

3.1 Problem Description

In this chapter, we provided a brief information about problem and literature review. Assumption of model are also presented in this section. Accordingly, we explained details of proposed model's indices, decision parameters, objective function and constraints. After a reviewing the proposed model, some examples generated in order to analyze solutions and calculated results are also given.

In this study; there is a supply chain with a variety of products and the same product can be purchased from more than one supplier. A single product is available from more than one supplier that can produce the same product. However, the basic cost elements such as ordering cost and purchasing cost of product are different from each other. Whereas some suppliers have limited capacity, some suppliers have minimum order quantity policy. Demand for the planning period is certain, a mathematical model has been proposed in order to fully and timely meet customer demand spread over multiple periods in this study. A shame of multi-period multi-supplier model is presented in Figure 3.1.

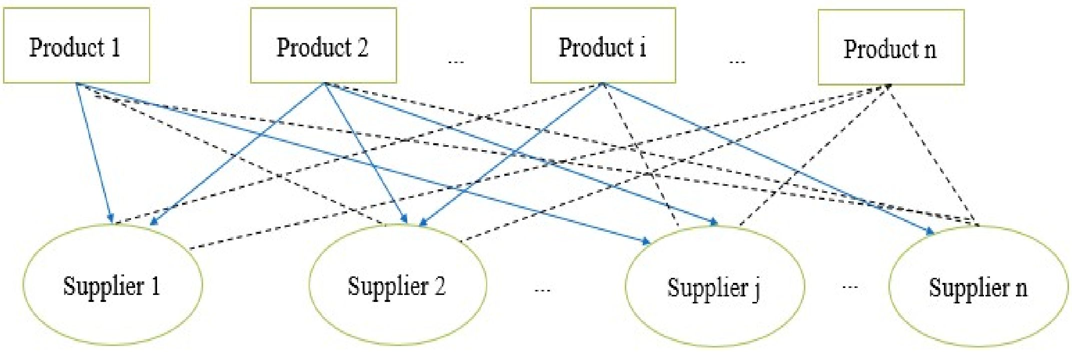

Figure 3.1 Multi-product multi-supplier model

3.2 Literature Review

Since our problem is multi-product, multi-supplier multi-period inventory cost minimization problem we need to check below three major selections which are associated with our study:

• *What product to order?* One product is utilized in many studies. In such cases, order prices may even be reduced by combining orders for many purchasing decisions into one single order. In these models, quality audits for various purchase can be done at the same time. Few models contemplate buying at the same time a collection of purchasing. What is more, several multiple products supply choice models handle the matter on an independent basis and ought to be applied iteratively to pick out suppliers for multiple product.

• *In how may quantities, from which suppliers?* Primarily, there are 2 forms of buying decisions which are single or multiple sourcing. On one hand, all suppliers will totally meet purchaser's needs in terms of quality, pricing, lot size etc. Therefore, purchasing decision is made for which supplier can provide a solution for the buyer's total demands (in terms of limitations on supplier's capability, quality, delivery, price, etc.). On the other hand, buyers want to avoid the dependency on one supply to shield from stockouts and maintaining strong competition with different vendors. Problem turns to choose of supplier and ordering decision in these situations.

• W*ithin which periods?* The other selection is planning horizon. First studies of inventory management problems are used only one period problem. Multi-period planning horizon provides that one or more supplier can be chosen in different horizon for ordering. Also, products can be held to following periods in order to reduce ordering or purchasing cost. Therefore, multiple planning horizon gives more complicated problems in literature, and more applicable for real life lot sizing problems.

Suppliers, producers and retailers for the most part make new buys in understanding with the deterministic continuous review stock approach as the stock level changes. The basic model for this strategy is the economic order quantity

(EOQ) model. It is required to begin from the EOQ model which is the first theorical step in inventory management and clarifies its basics. EOQ formula’s aim is to minimize total inventory cost for known demand. Deterministic models are the ones wherein every one of the parameters and factors are known and calculable. Lead time and demand are known in the begging of planning horizon. EOQ equation is first created by F. W. Harris in 1915. Afterward, Wilson (1934) has also worked this formula and has obtained a wide utilization in many industries. EOQ model is usable for single product single period inventory models.

Among the first studies made within the scope of deteriorated inventories are the model created by Ghare & Schrader (1963). In this model, the ongoing deterioration in inventories under constant demand is examined. The deterioration in stocks is a negative exponential function of time. In the period between orders, the inventories in the enterprise are continuously decreasing due to demand and deterioration. An economic order quantity formula has been obtained by considering these issues in the model.

Many stock control models deal with the problem of determining stock policy for only one product. The most common and simple models of order and production in terms of usage and application are based on this basic assumption. In a study by Bartmann & Beckmann (1992), a single-product order was compared in terms of unit costs of multi-product orders. Under the assumption that there is a fixed order cost for each product, it is concluded that multi-product orders do not have an advantage over a single product order. However, in cases where the fixed order cost is the same for single and multi-product orders, the unit cost of the multi-product order is shown numerically in the developed model, which is more advantageous than the unit cost of the single product order.

For the solution of multi-product inventory control problems under the single constraint, we can look at the work of Rosenblatt & Rothblum (1990), Maloney & Klein (1993). Haksever & Moussourakis (2005) and Bretthauer et al. (2006) developed an algorithm for the general solution of very limited and multi-product inventory control models. Sharma (2004) developed a mathematical model on a single product inventory control problem with a limited shelf life and aimed to determine the optimum cycle time and level of nonhomidity. Vairaktarakis (2000) extended the single product assumption in the classical single-period inventory problem and developed a budget-constrained multi-product inventory model. In her work, she developed a minimax regret approach to a multi-product newspaper dealer problem using two different ambiguous demand scenarios.

On the other hand, multi-period studies are very common for inventory management problems. Firstly, Within (1954) suggested single-product multi-period inventory problem with (Q, r) policy, dynamic programming is used for solving. Veinott & Wager (1966) thought-about a dynamic multi-product inventory model during which the system was reviewed in equal lengths of time periods and shortage is allowed. They provide a literature overview in deterministic inventory lot size problem. In this environment the purchase price of an order is replaced by production costs. Liu (2003) analyzed an economic lot sizing (ELS) problem for single item and multiple periods with both upper and lower inventory bounds. Safety stock is defined as lowest bound in the problem in order not to face any shortage issue. Demand was deterministic and known for each period. Mathematical modeling has been suggested for minimizing total cost. They use holding and ordering cost in their study for their mathematical model.

Due to their ability to optimize the expressly declared objective, mathematical modelling is one of the most applicable technique that permits to formulate issues and find optimum solutions accordingly. It permits considering internal policy constraints associated outwardly obligatory system constraints placed on the shopping for method to work out an optimum ordering and inventory policy at the same time whereas choosing the simplest combination of suppliers. Gaballa (1974) was the primary author who applied this system to seller choice in an exceedingly real case. Mixed integer programming is used for the mathematical model to reduce total discounted value of allotted things to suppliers, under constraints of demand satisfaction and vendor's capability. Till the publication of the survey planned by Weber & Current (1993). A helpful approach to make sure the dependability of a manufacturer's offer stream is to follow a multiple sourcing policy. During this scenario, a purchaser purchases identical item from over a vendor and therefore the total demand is split among them.

Hopp et al. (1997) studied single and multi-product inventory problems with a continuous review inventory control policy (Q, r) under the number of orders and customer service level constraints. Due to the complexity and magnitude of the problem, 3 different heuristic algorithms have been developed.

Pirkul & Aras (1985) analyzed in addition the matter of deciding order quantities for multiple product with all-unit amount discount option. The target was to reduce the total holding, ordering and purchasing costs subject to a linear resource constraint. Their problem defined as a non-linear program and found a solution with Lagrangian relaxation. Nahmias & Schmidt (1984), under uncertain demand, have developed a single-period, multi-product inventory model based on storage capacity or budget constraints.

Dasgupta & Spulber (1989) studied single and multiple supplier selection problems. Their scenario is that purchaser knows all vendor's costs after purchaser order an item to vendor. With this scenario, their aim is to maximize purchaser's needs.

Researches Hong & Hayya (1992) have additionally argued that the utilization of multiple suppliers, also used just-in-time environment, reduces the inventory cost such as purchasing cost.

Kasilingam & Lee (1996) studied multiple product and single period inventory problem. They provided integer programming with selection best supplier in order to satisfy customer demand on time and minimize costs. Model also includes supplier lead time and supplier capacity with stochastic demand structure.

Wang et al. (2015) developed budget-constrained, single / multi-product, single-term inventory models, if the market demands were ambiguous random variables. Hnaien et al. (2016) have worked on a single-period inventory model for a single-level assembly system under stochastic delivery time and demand. As a solution approach, they proposed a new method based on branch boundary algorithm.

Chaudhry & Shankar (2011) used a mixed linear integer programming formulation to reduce the ordering cost. Constraint of supplier capacity is utilized with quantity discounts.

Regarding the second distinction associated with the third call of the provider choice problem (i.e. within which periods), the overwhelming majority of approaches that are investigated up to currently are single amount models determination solely a brief term coming up with drawback. In reality, once coming up with horizon covers many periods, the matter becomes more durable, however has the potential to yield far better procurement set up by considering inventory management. Multi-period inventory lot-sizing has been studied issues in production and inventory management literature.

Basnet & Leung (2005) worked multi-period and multi-item inventory cost minimization problem with considering purchasing, holding and ordering cost. They used mixed linear integer programming for inventory minimization problem. Their aim is to find demand satisfaction.

Abdelmaguid & Dessouky (2006) proposed a genetic algorithm approach for determination the integrated inventory distribution problem within the multiple periods, within which backorders are allowable. Backorder choices are typically established in two cases. Within the initial case is once there is short vehicle capability to deliver to a customer, whereas within the second case is once there's a transportation cost saving that's beyond the customer backorder cost.

Archetti et al. (2007) proposed a particular algorithm to the settled inventory management problem over a given time horizon. Every customer defines an inventory level. Supplier can control inventory level of every customer and determine replacement, accordingly there is no stock out happens at the customer due to control of supplier.

Nowadays, there are many metaheuristic approaches for multi-product inventory management problems. Huang & Lin (2010) have developed the ant colony optimization meta intuition for multiple product inventory management. Taleizadeh et al. (2008), multi-product, single-period inventory problem to increase the total profit and service speed. In this study, mixed unit integer nonlinear programming has been developed by including whole unit and incremental quantity discount policies and a mixed method consisting of genetic algorithm and target programming developed.

Das & Maiti (2007) have worked on a multi-purpose, single-period inventory problem with random demand involving a supplier and several retailers. They used a genetic algorithm to solve the model created under limited storage space constraint.

Kasap et al. (2010), in their work, a single-period - multi-product stochastic inventory model for maintenance repair warehouse of construction machinery. This model aims to determine the inventory parameters and re-order point of spare parts and minimize the total inventory cost. A nonlinear integer optimization model was created and solved by the developed heuristic method by including order frequency and service level constraints.

Karimi et al. (2016) studied the two-stage single-period inventory control problem under the budget constraint. However, they have included additional restrictions on the model, such as supplier and vehicle selection, green transport, quantity reduction and storage space, to make the model developed applicable to realistic problems.

Mousavi et al. (2013) studied a model which is included inflation and interest factors. They aimed to determine the optimum order quantities of the products and thus to minimize the net present value of the total system cost. Since the model NP was difficult, GA and simulated annealing algorithm were used to solve the proposed mathematical model and the results were compared.

Bahl et al. (1987) classified related literature about inventory management according to product. According to this information, we summarize literature review in Table 3.1 for single product models, and Table 3.2 for multi-product models

Table 3.1 Classification for single product models

Author (Year)	***Product***	***Period***	***Supplier***	***Demand***	***Policy***	***Constraints***
Sharma (2004)	Single	Single	Single	Deterministic	Mixed Integer Programming	Limited Shelf Life
Hnaien et al. (2016)	Single	Single	Single	Stochastic	Heuristics	Stochastic delivery time
Hadley & Within (1963)	Single	Multi	Single	Stochastic	Dynamic Programming	
Liu (2003)	Single	Multi	Single	Deterministic	Mixed Integer Programming	Lower and upper bound
Chaudhry & Shankar (2011)	Single	Multi	Single	Deterministic	Mixed Integer Programming	Rejections and late deliveries
Weber & Current (1993)	Single	Single	Multi	Deterministic	Multi Objective Programming	
Hong & Hayya (1992)	Single	Single	Multi	Deterministic	Non-linear Programming	
Chaudhry et al. (1993)	Single	Single	Multi	Deterministic	Mixed Integer Programming	Supplier capacity constraints
Mousavi et al. (2013)	Single	Single	Multi	Stochastic	Heuristics	Capacity constraint for allocation

Table 3.2 Classification for multi- product models

Author (Year)	*Product*	*Period*	*Supplier*	*Demand*	*Policy*	*Constraints*
Rosenblatt & Rothblum (1990)	Multi	Single	Single	Deterministic	EOQ	
Maloney & Klein (1993)	Multi	Single	Single	Deterministic	EOQ	
Vairaktarakis (2000)	Multi	Single	Single	Deterministic	Mixed Integer Programming	Budget constrained
Huang & Lin (2010)	Multi	Single	Single	Stochastic	Meta-heuristics	Vehicle capacity constrained
Taleizadeh et al. (2008)	Multi	Single	Single	Stochastic	Meta-heuristics	Incremental Discount and fuzzy cost
Kasap et al. (2010)	Multi	Single	Single	Stochastic	Heuristics	service level constraint
Wang et al. (2015)	Multi	Single	Single	Stochastic	Meta-heuristics	
Karimi et al. (2016)	Multi	Single	Single	Deterministic	Mixed Integer Programming	Vehicle selection, green transport constraint
Hopp et al. (1997)	Multi	Single	Single	Stochastic	Heuristics	Customer service level constraint
Das & Maiti (2007)	Multi	Single	Single	Stochastic	Heuristics	Limited storage space constraint
Nahmias & Schmidt (1984)	Multi	Single	Single	Stochastic	Mixed Integer Programming	Storage capacity or budget constraints
Veinott & Wagner (1966)	Multi	Multi	Single	Stochastic	Dynamic Programming	
Abdelmaguid & Dessouky (2006)	Multi	Multi	Single	Deterministic	Heuristics	Shortage, and vehicle capacity
Haksever & Moussourakis (2005)	Multi	Single	Multi	Deterministic	Mixed Integer Programming	Supplier constraints
Bretthauer et al. (2006)	Multi	Single	Multi	Deterministic	Mixed Integer Programming	Limits on raw materials, and machine capacity
Gaballa (1974)	Multi	Single	Multi	Deterministic	Mixed Integer Programming	Fixed all unit discount, tender capacity constraint
Pirkul & Aras (1985)	Multi	Single	Multi	Deterministic	Non-linear Programming	All-unit amount discounts
Kasilingam & Lee (1996)	Multi	Single	Multi	Stochastic	Mixed Integer Programming	Supplier capacity constraints, supplier lead time
Archetti et al. (2007)	Multi	Multi	Multi	Deterministic	Mixed Integer Programming	Shortage, and routing constraint
Basnet & Leung (2005)	Multi	Multi	Multi	Deterministic	Mixed Integer Programming	

3.3 Chapter Conclusions

Problem description is given in this chapter. Within the scope of this subject, literature is reviewed with a brief classification. This classification is given according to the product level. We started to check literature starting with EOQ models. It can be seen that; inventory cost minimization problems have an important place in literature. Each year, different methodologies are generated in order to find better solution in this area. Our aim is to check studies in single/multi product, supplier and period situations. When we check a proper literature review in inventory management area, we can see that various detailed approached have been studied for many different types of businesses. New approaches are utilized for real life problems and find better solutions for buyer’s better purchasing choices. To sum up,, every study may significant role to achieve firms’ goal in terms of supply chain operations.

It is beneficial to utilize a mathematical model in a real-life problem with considering those literature review. Next chapter proposed mathematical model is given and solutions are analyzed. Problem formulation is detailly provided in next chapter.

CHAPTER FOUR
PROBLEM FORMULATION

4.1 Proposed Mathematical Model

This model has mathematical formulas which minimize total inventory cost subject to demand constraint, minimum order quantity and supplier capacity. Fallowing notations are utilized to provide a formulation of the proposed multi-product, multi-supplier, multi-period inventory minimization model with deterministic demand environment. Firstly, we determinate assumptions, secondly sets, parameters and the decision variables are given. Accordingly, objective function and constraints are provided. Proposed mathematical model is checked with several random problems, and results are given in this chapter.

4.1.1 Assumptions

Fallowing assumptions are used to derive the Integer-Linear Programming (ILP) Model of the proposed mathematical model.

- Initial inventory of the first period is assumed as zero.
- Inventory level at the last period is assumed as zero.
- There are multiple supplier and product.
- Demand is deterministic which is known over a planning horizon.
- Backorders are not allowed.
- Demand must be satisfied in the period.
- Ordering cost is supplier dependent.
- Holding cost of product per period is product dependent.
- Purchasing cost of product is same in each period and it is supplier dependent.
- There are different minimum order quantities for each supplier. Minimum order quantities are supplier dependent.

- There is different total weight shipment capacity for each supplier, and it is supplier dependent.
- Planning horizon is finite, and planning is periodical.

4.1.2 Sets, Parameters and Decision Variables

The sets, parameters, and decision variables are given as follows:

<u>Sets</u>

t: set of time periods $\{1,\ldots,T\}$

p: set of products $\{1,\ldots,P\}$

s: set of suppliers $\{1,\ldots,S\}$

<u>Parameters</u>

$d_{p,t}$: demand for products p in period t

$PU_{p,s}$: Purchase price for product p from supplier s

OC_s: Ordering cost for supplier s

$HC_{p,t}$: Holding cost for product p in period t

$MOQ_{p,s}$: Minimum order quantity for product p in supplier s

W_p : Unit gross weight for product p

CAP_s: Shipment capacity for supplier s

M: Big number

Decision Variables

$I_{p,t}$: Ending inventory level for product p in period t

$Q_{p,t}$: Order quantity for product p in period t

$QS_{s,p,t}$: Order quantity from supplier s for product p in period t

$Y_{s,t}$: Binary variable: equal to one if an order is placed to supplier s in period t; otherwise, zero

$X_{s,p,t}$: Binary variable: equal to one if an order for product p is placed to supplier s in period t; otherwise, zero

4.1.3 Objective Function

Aim is to minimize total inventory cost which includes holding cost, purchasing cost and ordering cost. Equation 4.1 shows the objective function.

$$Min \sum_t \left[\sum_p (I_{p,t} \times HC_{p,t}) + \sum_s \sum_p (QS_{s,p,t} \times PU_{p,s}) + \sum_s (Y_{s,t} \times OC_{s,t}) \right] \quad (4.1)$$

Holding Cost:

$$\sum_t \sum_p (I_{p,t} \times HC_{p,t}) \quad (4.2)$$

Each product has a holding cost and this formula calculates total holding cost according to the stock quantity in each period. Equation 4.2 is holding cost formulation from the objective function.

Purchasing Cost:

$$\sum_t \sum_s \sum_p (QS_{s,p,t} \times PU_{p,s}) \quad (4.3)$$

In every period, if an order is placed in period, purchasing cost is calculated for each product. In this study, it is assumed that each product can be supplied from more than one supplier. As it can be seen in equation 4.3, this formula helps to calculate total purchasing cost based on orders in each supplier.

Ordering Cost:

$$\sum_{t}\sum_{s}(Y_{s,t} \times OC_{s,t}) \tag{4.4}$$

If $Y_{i,t} = 1$,then an order has been placed. If an order placed in period, an ordering cost occurs. Ordering cost is the products of the number of orders given during planning period and ordering cots. Equation 4.4 provides ordering cost formula from the objective function.

4.1.4 Constraints

$$I_{p,t} = I_{p,t-1} + Q_{p,t} - d_{p,t} \qquad \forall\, t = 2,..,T-1,\ \forall\, p = 1,..,P \tag{4.5}$$

$$I_{p,t} = Q_{p,t} - d_{p,t} \qquad t = 1,\ \forall\, p = 1,..,P \tag{4.6}$$

$$I_{p,t-1} + Q_{p,t} - d_{p,t} = 0 \qquad t = T,\ \forall\, p = 1,..,P \tag{4.7}$$

$$Q_{p,t} = \sum_{s} QS_{s,p,t} \qquad \forall\, t = 1..,T\ ,\ \forall\, p = 1,..,P \tag{4.8}$$

$$\sum_{p} QS_{s,p,t} \times W_p \le \left(CAP_s \times Y_{s,t}\right) \qquad \forall\, t = 1..,T\ ,\ \forall\, s = 1,..,S \tag{4.9}$$

$$QS_{s,p,t} \ge MOQ_{s,p} \times X_{s,p,t} \qquad \forall\, t = 1..,T,\ \forall p = 1..,P, \forall s = 1..,S \tag{4.10}$$

$$QS_{s,p,t} \le M \times X_{s,p,t} \qquad \forall\, t = 1..,T,\ \forall p = 1..,P, \forall s = 1..,S \tag{4.11}$$

$$Y_{s,t} \in \{0,1\} \tag{4.12}$$

$$X_{s,p,t} \in \{0,1\} \tag{4.13}$$

Constraint (4.5), provides that inventory level for product *p* in *t* period must be equal to inventory level of previous period, order quantity and demand in period t. As there is zero inventory in the beginning of first period, constraint (4.6) which is different from (4.5) has been added to model for only in first period. On the other hand, there is a constraint which provides that inventory level must be zero at the end of planning horizon. This constraint can be found in (4.7). Inventory level of the previous period which is *T-1* and order quantity in period T must be equal to demand for the last period *T*. Due to the fact that there are different conditions in the first and last period, constraint (4.5) is only been applied to the model from second period and the next to last period. According to the relation given in constraints (4.5) to (4.8), they are important the logic for inventory level accuracy in each period and product. Constraints (4.8) shows order can be placed from different supplier for each period. If an order has been raised for product *p* in the period, therefore there won't any need to place same order to another supplier. As there is a shipment capacity for suppliers, constraints (4.9) states that total gross weight of total order quantity must be less or equal to supplier's shipping capacity. It is assumed that one supplier has a capacity in each period, and every period it calculates once again. Constraint (4.9) and constraint (4.10) ensure that model implements minimum order quantity constraint if an order placed in the supplier. Order quantity must be greater or equal than minimum order quantity that is given to the model. In order to model those relations linearly, constraint (4.11) to (4.13) are integrated to the proposed model.

4.2 Computational Study for the Model

In order to find optimum solution of proposed mathematical model, some problem have been generated randomly and solutions have been analyzed. Each regarding problems different data. Demand quantities have been used between 0 and 500, and each demand quantity is integer for each period. Purchasing costs are between 3 and 35, and each purchasing costs is decimal for each supplier. Holding costs are generated from 0.1 to 2.0, they are decimal for each period. Ordering costs are generated between 20 and 120 and each ordering cost is integer for each supplier. Minimum order quantities are generated between 1 and 200 and they are integer for each supplier. Capacity of each supplier is set to 1000.

LINGO 9.0 software has been used for solving the proposed ILP Model. We have tried to solve experimental problems on AMD FX™ 6300 CPU @3.50 GHz and 8.00 GB RAM personal computer. The experimental studies and results are given in Table 4.1. Each problem's product, period, and supplier quantity are defined in the Table 1. After running the model, number of variables, number of constraints, total run time, and run results are also included to the table. We studied 24 different problems, as the number of products, number of period and number of period increase, run time affects directly. Due to the fact that, problem has some integer values, it is more difficult to solve this optimization problem. That's why computational time grows exponentially. Pro15, Pro16, Pro19, Pro20, Pro23, and Pro24 has been run 10 hours and software couldn't find optimum solution. Yet, there are small differences between best objective and objective bound. For instance, Pro15's best objective value is 110095, and objective bound is 109581. According to these values, the difference can be found as 515. This difference is divided by best objective; therefore, we find 0.47% optimality gap ratio which is very small gap. On the other hand, Pro20's best objective value is 259729, and objective bound is 254095. According to these values, the difference can be found as 5634. This difference is divided by best objective; therefore, we find 2.17% optimality gap ratio. This ratio is the biggest gap according to other run results. Product, period, and supplier set numbers are directly affecting results. This Table 4.2 presents optimality gap between best objective and objective bounds for solutions which are feasible.

Table 4.1 Computational results of generated problems

Problem Number	p	t	s	Number of Variable	Number of Constraint	Run Time (sec)	Optimality Result	Run Result
Pro1	2	4	2	59	68	2	Global Optimal	5363.44
Pro2	2	8	2	115	132	2	Global Optimal	7688.23
Pro3	2	12	2	171	196	3	Global Optimal	10742.3
Pro4	2	24	2	339	388	4	Global Optimal	19204.97
Pro5	4	4	2	107	124	2	Global Optimal	14795.6
Pro6	4	8	2	211	244	2	Global Optimal	14795.60
Pro7	4	12	3	423	472	3	Global Optimal	25627.98
Pro8	4	24	4	1059	1156	56	Global Optimal	51154.09
Pro9	6	4	2	155	180	3360	Global Optimal	21036.48
Pro10	6	8	3	411	460	2	Global Optimal	26967.60
Pro11	6	12	4	771	844	10	Global Optimal	37031.27
Pro12	6	24	6	2163	2236	3660	Global Optimal	78220.04
Pro13	8	8	3	539	604	2	Global Optimal	33597.73
Pro14	8	12	4	1011	1108	1800	Global Optimal	48356.82
Pro15	8	24	4	2019	2212	36000	Feasible	110067.8
Pro16	8	48	6	5667	6052	36000	Feasible	222189.5
Pro17	10	8	4	835	916	4	Global Optimal	37815.99
Pro18	10	12	4	1251	1372	4	Global Optimal	56452.99
Pro19	10	24	6	3507	3748	36000	Feasible	128638.5
Pro20	10	48	8	9027	9508	36000	Feasible	259729.1
Pro21	15	8	4	1235	1356	4	Global Optimal	48629.02
Pro22	15	12	6	2595	2776	1500	Global Optimal	71241.11
Pro23	15	24	8	6675	7036	36000	Feasible	165482.1
Pro24	15	48	8	13347	14068	36000	Feasible	333311.9

Table 4.2 Optimality gap for feasible solutions

Problem Number	p	t	s	Best Objective	Objective Bound	Optimality Gap	Optimality Gap Ratio
Pro15	8	24	4	110096	109581	515	0.47%
Pro16	8	48	6	217065	212713	4352	2.00%
Pro19	10	24	6	128639	127381	1258	0.98%
Pro20	10	48	8	259729	254095	5634	2.17%
Pro23	15	24	8	165482	163236	2246	1.36%
Pro24	15	48	8	333312	325393	7919	2.38%

We checked results, especially number of periods affect computational time due to growing integer variables. Figure 4.1 shows total running time by second according to changes of number of periods. This figure presents how number of periods affect total run time basically.

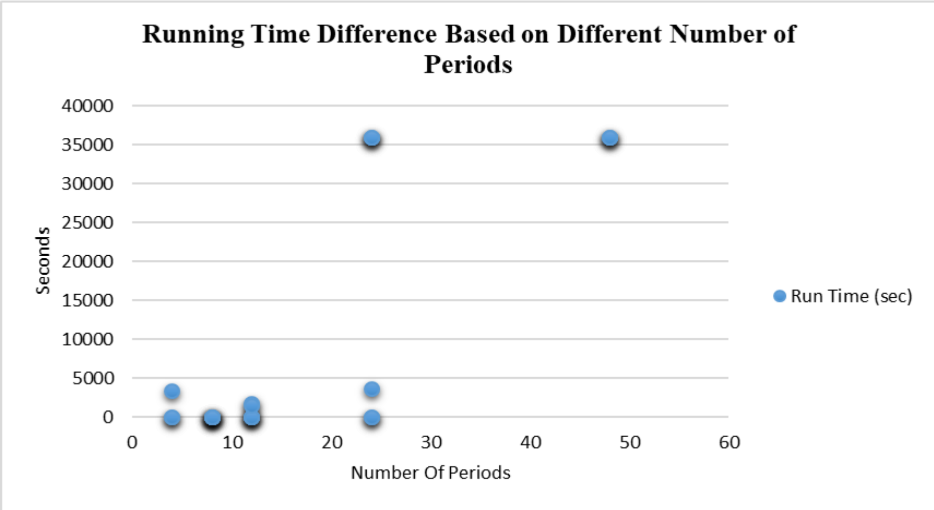

Figure 4.1 Model's running time for different number of periods

In order to analyze model results, one of experimental problems is given. For example, Pro14 provides a data set for 8-product, 12-period and 4-supplier model. It is assumed that demand is known and deterministic in each period and beginning and final inventory level is zero. Tables 4.3, 4.4, 4.5, 4.6, 4.7 show all data of this problem.

Table 4.3 provides demand quantity for the example. Table 4.4 presents purchase prices of 8-product and 4-supplier for the problem of pro14. Table 4.5 shows ordering costs and capacity of the problem. Holding costs are shoed in the Table 4.6. Minimum order quantities of 8-product and 4-supplier for the problem of pro14 are presented in the Table 4.7.

Table 4.3 Demand quantities of 8-product and 12-period for the problem of pro14

Demand	*p=1*	*p=2*	*p=3*	*p=4*	*p=5*	*p=6*	*p=7*	*p=8*
t=1	171	26	74	93	42	63	34	66
t=2	11	6	0	5	0	47	16	126
t=3	55	15	85	16	48	28	13	193
t=4	213	27	317	88	30	2	2	60
t=5	0	1	26	5	0	10	7	4
t=6	60	32	47	28	14	0	0	22
t=7	120	24	16	29	8	10	4	105
t=8	4	0	3	1	4	1	0	25
t=9	56	52	2	53	9	22	19	215
t=10	82	37	97	16	40	17	11	162
t=11	33	6	20	15	10	0	2	40
t=12	38	0	37	20	80	22	17	73

Table 4.4 Purchase prices of 8-product and 4-supplier for the problem of pro14

Purchase Price	*s=1*	*s=2*	*s=3*	*s=4*
p=1	999999	9.46	11.82	12
p=2	999999	9.21	11.5	12
p=3	10	11.98	14.97	12
p=4	10	17.94	22.42	999999
p=5	999999	13.05	16.3	999999
p=6	999999	999999	32.57	999999
p=7	999999	999999	32.57	30
p=8	6.35	999999	7.17	7

Table 4.5 Ordering costs and capacity of 8-product and 4-supplier for the problem of pro14

	s=1	*s=2*	*s=3*	*s=4*
Ordering Cost	100	80	20	60
Capacity	999999	999999	10000	999999

Table 4.6 Holding costs of 8-product and 4-supplier for the problem of pro14

Holding Cost	*p=1*	*p=2*	*p=3*	*p=4*	*p=5*	*p=6*	*p=7*	*p=8*
t=1	0.36	0.36	0.47	0.7	0.51	1.19	0.86	0.21
t=2	0.36	0.36	0.47	0.7	0.51	1.19	0.86	0.21
t=3	0.36	0.36	0.47	0.7	0.51	1.19	0.86	0.21
t=4	0.36	0.36	0.47	0.7	0.51	1.19	0.86	0.2
t=5	0.38	0.38	0.49	0.71	0.54	1.19	0.86	0.2
t=6	0.38	0.38	0.49	0.71	0.54	1.19	0.86	0.2
t=7	0.38	0.38	0.49	0.71	0.54	1.19	0.85	0.2
t=8	0.38	0.38	0.49	0.71	0.54	1.19	0.85	0.2
t=9	0.38	0.38	0.49	0.71	0.53	1.19	0.85	0.21
t=10	0.38	0.38	0.49	0.71	0.53	1.19	0.85	0.21
t=11	0.38	0.38	0.49	0.71	0.53	1.19	0.85	0.21
t=12	0.38	0.38	0.49	0.71	0.53	1.19	0.85	0.21

Table 4.7 Minimum order quantities of 8-product and 4-supplier for the problem of pro14

Minimum Order Quantity	*s=1*	*s=2*	*s=3*	*s=4*
p=1	-	20	1	20
p=2	200	100	1	20
p=3	200	100	1	20
p=4	-	80	1	-
p=5	-	108	1	-
p=6	-	-	1	-
p=7	-	-	1	20
p=8	200	-	1	20

Proposed ILP Model's results are given in Figure 4.2. Problem-14's data set is given to understand the main idea of the study. According to the objective function, model found the minimum total inventory cost. By summing holding, purchasing and ordering cost, model found the objective value as 48,356.82 with 1800 sec. computational time.

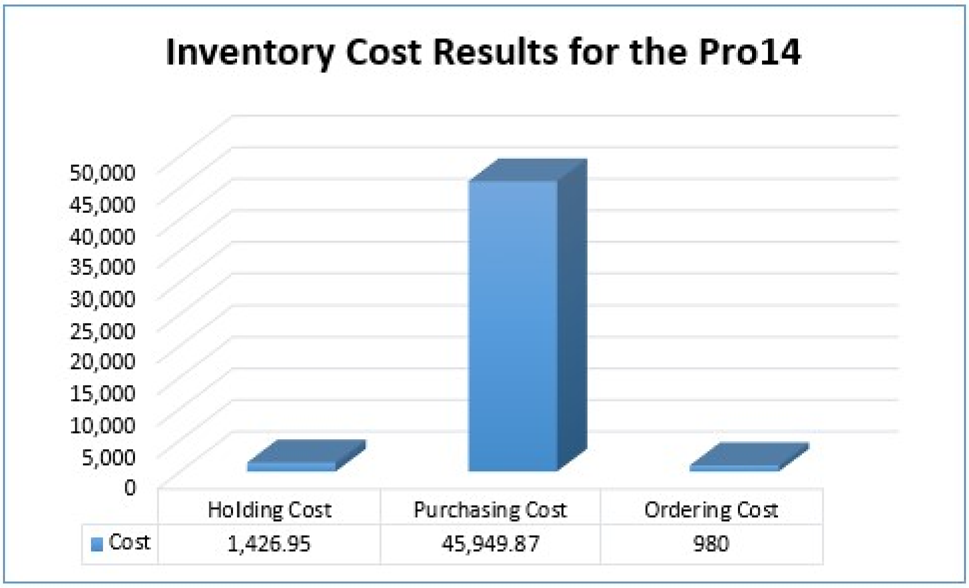

Figure 4.2 Inventory cost results of the pro14

Table 4.8 shows inventory quantity for product 1 to 8 for each period end. Table 4.9 presents total order quantities for all products in each period. Therefore, we can see which order is placed in which supplier in the Table 4.9. Table 4.10 shows suppliers' situation in each period. If order placed for a supplier, then results is 1 in the period. According to run results, this model can be applicable for our problem.

Table 4.8 Inventory quantity results for products in each period end

Inventory Quantity	*t=1*	*t=2*	*t=3*	*t=4*	*t=5*	*t=6*	*t=7*	*t=8*	*t=9*	*t=10*	*t=11*	*t=12*
p=1	66	55	0	60	60	0	4	0	0	71	38	0
p=2	81	75	60	33	32	0	95	95	43	6	0	0
p=3	126	126	41	94	68	21	5	2	0	57	37	0
p=4	21	16	0	63	58	30	1	0	51	35	20	0
p=5	104	104	56	26	26	12	4	0	0	90	80	0
p=6	0	0	2	0	0	0	1	0	0	0	0	0
p=7	31	15	2	0	0	0	0	0	13	2	0	0
p=8	319	193	0	156	152	130	25	0	275	113	73	0

Table 4.9 Order quantity results for products in each period

Order Quantity	***t=1***	***t=2***	***t=3***	***t=4***	***t=5***	***t=6***	***t=7***	***t=8***	***t=9***	***t=10***	***t=11***	***t=12***
p=1	237	0	0	273	0	0	124	0	56	153	0	0
p=2	107	0	0	0	0	0	119	0	0	0	0	0
p=3	200	0	0	370	0	0	0	0	0	154	0	0
p=4	114	0	0	151	0	0	0	0	104	0	0	0
p=5	146	0	0	0	0	0	0	0	9	130	0	0
p=6	63	47	30	0	10	0	11	0	22	17	0	22
p=7	65	0	0	0	7	0	4	0	32	0	0	17
p=8	385	0	0	216	0	0	0	0	490	0	0	0

Table 4.10 Supplier based order placed or not table for each period

Y	*t=1*	*t=2*	*t=3*	*t=4*	*t=5*	*t=6*	*t=7*	*t=8*	*t=9*	*t=10*	*t=11*	*t=12*
s=1	1	0	0	1	0	0	0	0	1	0	0	0
s=2	1	0	0	1	0	0	1	0	1	1	0	0
s=3	1	1	1	0	1	0	1	0	1	1	0	1
s=4	1	0	0	0	0	0	0	0	1	0	0	0

4.3 Chapter Conclusions

In this chapter, we aim to develop an ILP model for the multi-product, multi-supplier, and multi-period total cost minimization problem with a deterministic demand environment. Firstly, proposed model's definition is given the begging of chapter. Sets, parameters and decision variables are also defined. Besides, in order to fully understand the problem, assumptions are detailly defined. Accordingly, objective function and its main parts are explained. Each constraint is formulized and defined in this chapter. Secondly, we run this model with different problems and solutions are checked. Generally, we run this problem with 24 different type of problem and detailed results are provided. We faced both optimal and feasible solution according to type of problem. An example problem data set, and its solution are provided in order to see mathematical model performance. As the results are satisfactory, case study can be implemented to the model. Next chapter shows information about the case study and the solution approach of this mathematical model to the problem. The details of the case study are explained in the next chapter.

CHAPTER FIVE
CASE STUDY IN AUTOMOTIVE SPARE PART COMPANY

5.1 Introduction

We represented a multi-product, multi-supplier, multi-period inventory problem with providing mathematical model. With this model, we proposed a solution with using mathematical model for deterministic demand. In this section, we implemented this mathematical model into real life problem. Our real-life problem is coming from an automotive aftermarket company.

The automotive spare part is that the secondary market of the automotive business, involved with the producing, remanufacturing, distribution, retailing, and installation of all vehicle components, equipment, and accessories. After the sale of the car by the original equipment manufacturer (OEM) to the buyer, some parts of the car should be changed in the process of time. This changed part is known as spare part or aftermarket product in automotive literature.

In this chapter, key topics of automotive aftermarket have been discussed and provided the importance of good inventory management in aftermarket organizations. Firstly, we explained a brief information about the sector and inventory management key topics for spare part industries. Secondly, we analyzed current inventory management process and determined the best data for our optimization model. Lastly, proposed model is represented for the case study, and results are showed. Results is also compared with company annual inventory costs. In this chapter, company data has been masked due to company data protection policy.

5.2 Inventory Management in Automotive Spare Part

Spare part inventory management have a vital role in several industries, like automotive, airline, and white goods. Spare part management started to evolve into another line of business. The importance of the spare part management can be expressed as the increasing number of the companies and the distributors focusing on the spare part management. Service parts for products like home appliances, vehicles and automotive sector have grown up into a business value more than a $200 billion worldwide (Gallagher et al., 2005). Spare parts inventories must be compelled to be offered at acceptable points among the availability chain, to produce after-sales services and to ensure the required service level (Botter & Fortuin, 2000).

New methods are developed and implemented in industrial manufacturing systems in order to ensure the sustainability and increase the rate of profit for spare part management. The increasing demand for the efficiency in the spare part industries induces new research and development projects. Keeping the spare part stock for the sold parts has become a very important aspect for the companies to be able to compete in the market and take role as a leading company in long term.

Automotive supply industry takes a key-role in the economy of all developed countries all over the world. Product diversity of the automotive supply industry and the target market being wide places the automotive supply industry into an important place in the economy. The companies which produce and provide equipment to the automotive manufactures are classified as “OEM” companies which stands for “Original Equipment Manufacturer” in the literature. Aftermarket services have had increasing interest among producing companies because it brings advantages in profitableness, customer loyalty, and is seen as a good supply of revenue. Automotive spare parts business is also defined as aftermarket. Larger product market is one of key aspect in this area, and the turnover of the new product is significantly more with in aftermarket services (Bundschuh & Dezvane, 2003). Firms inside the aftermarket can so not contemplate the aftermarket as an afterthought (Cohen et al., 2006). They must be compelled to specialize in enhancing the worth of the core product wherever value adding services need to be provided to customer through the life-cycle of the product in order to achieve long run business relationship in the competitive aftermarket area. (Cavalieri et al., 2007). Companies need operational excellence in supply chain management of spare parts at completely different locations round the world.

Products are finished goods with aftermarket packaging. However, supply chain management is more difficult than that of finished product in aftermarket area. The quality of the elements business is generated by its own distinctive attributes. The life cycle of spare elements is longer than that of vehicles and therefore the total variety of SKUs is incredibly large. All the on top of create huge challenges to elements coming up with purchasing, ordering, and supply among alternative operations. In order to achieve good service level, stock availability is very critical in the organization. For this reason, there are many suppliers which are producing same aftermarket products with different prices across the world. Purchasers should pay more attention multiple supplier selections in terms of quality, prices and lead time perspective. Without some good relations with different suppliers globally, aftermarket companies can lose sales opportunities easily. As multiple supplier is one of most important dynamic for most of spare part company, our proposed total inventory minimization model can be applicable for companies which provide spare parts services.

5.3 Application of the Case Study

The address company in this thesis is one of the leading automotive aftermarket company in the world. When we analyze their structure in supply chain management, we faced that customer demand is deterministic and it come from the beginning of the planning horizon. According to lead time of suppliers, 12 weeks planning horizon is used in current system. When demand is provided from sales team, a material planner check which product can be purchased in which supplier. Besides, suppliers have their own capacity constraints for shipping. Gross weight of the product is considered for capacity in this case study. Because when we checked size and dimension of products, only weight is a main issue for container capacity constraints. Products have generally small sizes but heavy.

We analyzed the company’s inventory management system, and processed data from the company database. After checking all data, we showed that there were more than 7000 SKU exist in the system. In order to do better analyses, it was needed to classify stocks and then check demand structure. After investigating current process and determine products. ABC analyses has been studied.

ABC analyses is very important for inventory management. Because this analysis provides a classification of products. With a good study. we can able to determine product groups which need to be focused more on their inventory level or less. Therefore. deriving an impressive inventory management can only be possible with classifying inventories and deciding which inventory will be analyzed and studied more frequently or needs to check. Tersine (1994) summarized inventory management procedure with classifying product as ABC and it is given in table 5.1.

Table 5.1 Analogy of A, B, and C classes

Classification	Degree of Control	Importance	Frequency of Review
A	Tight	High	Continuous
B	Moderate	Medium	Occasional
C	Loose	Low	Infrequent

ABC analysis is utilized for a determination and better understanding a level of service for different types of product groups. It is possible to obtain optimum service level for each product in the theory (Kasap et al. 2010). However, an efficient approach should be considered in order to find the most appropriate service level according to the business perspective in the long term (Barutçu, 2007). The company has 7380 different products in their inventory management system. Frequently ABC analyses is mandatory for high rate of availability in stocks. Products are classified as sales value in last one year. The highest sales value is considered as "A" class. medium sales value is considered as "B" class. and lastly sales value is considered as "C" class. A final list of ABC classification is given in table 5.2.

Table 5.2 ABC classification results in the company

Classification	Sales (USD)	% Sales	Count of Item	% of Item
A	11,810,498.00	85%	300	4%
B	1,390,124.00	10%	904	12%
C	657,460.00	5%	6176	84%
Total	13,858,082.00	100%	7380	100%

Results shows that there are 300 SKUs for class "A" products. Whereas class "A" products contain only 4% of total SKU, it underlines 85% of total sales. Class "B" products consist of 904 SKU and this also means 12% of total SKU. Class “B” contain 10% of total cost. Only 5% of total sales are from Class “C” products, yet there are 6176 SKU which is 84% total SKU. Figure 5.1 provides a comparison A, B, and C classes differences in terms of SKU and sales levels. The data has been provided by company’s database and studied in Microsoft Excel Spreadsheets.

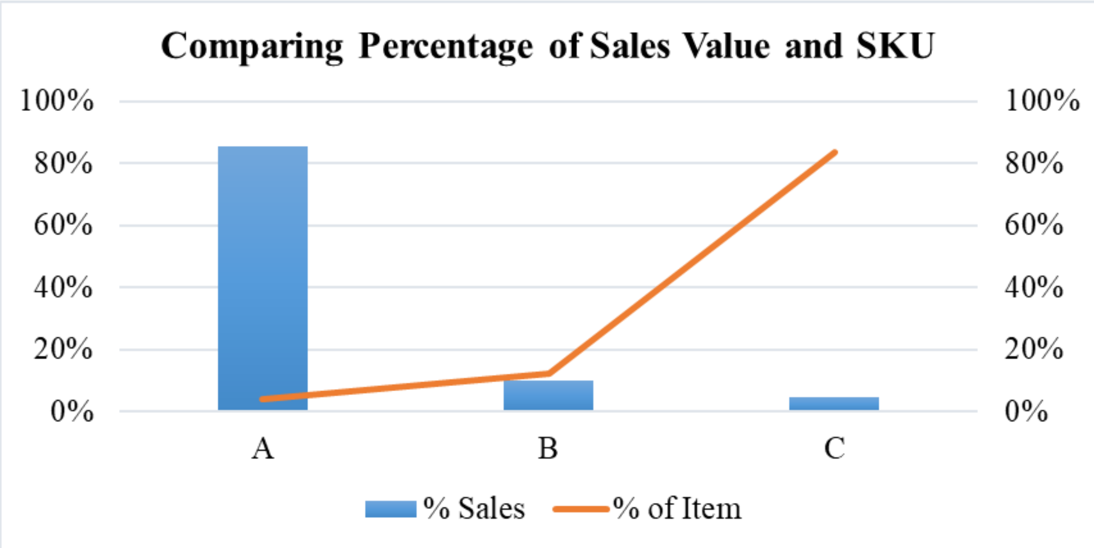

Figure 5.1 Comparing A, B, and C classes in terms of sales percentages

After comparing ABC classification results, it is more efficient to focus on class “A” products. Class “A” products’ demand structure is deterministic and planning horizon is 12 weeks. On the other hand, each class “A” products can be purchased from more than one supplier. As stock availability and high service level are very important aspects in automotive aftermarket area, proposed mathematical model can be helpful in order to optimize current inventory management system.

5.4 Computational Study

We have selected class “A” products, which are totally 300 SKUs, from automotive spare part company. We generated inventory minimization model for one-year period. Thus, we did medium term planning for the company. We have modeled the problem as multi-product, multi-period, and multi-supplier environment. There are 300 different products, and their demand is deterministic. On the other hand, we have 18 different suppliers for regarding 300 products. An order can be satisfied from more than one supplier. According to company structure, each product has dual source. This means, products can be supplied from two different suppliers.

Firstly, we run the mathematical model for 12 weeks planning horizon, where $t=\{1,\ldots,12\}$. In order to reach one-year planning horizon, we generated the model four times. 12-week model is called Model-1 in this chapter.

Weekly demand that is taken from the company has been separated into four different planning horizons. Concerning four different horizons is shown in Table 5.3. All planning horizon provide yearly demand, therefore total ordering, holding, and purchasing cost show yearly total inventory cost. Therefore, sets can be applied to the model as below:

Model-1

- $t = \{1,\ldots,12\}$
- $p = \{1,\ldots,300\}$
- $s = \{1,\ldots,18\}$

Table 5.3 Separation of the taken weekly demands into the model-1

Planning Horizon for Model-1	*Weeks of Demand*
Planning Horizon-1	From Week-1 to Week-12
Planning Horizon-2	From Week-13 to Week-24
Planning Horizon-3	From Week-25 to Week-36
Planning Horizon-4	From Week-37 to Week-48

We generated 4 models and run in LINGO 9.0 software. Each model has run 15 hours, and software found feasible results. Figure 5.2 shows differences between best objective and objective bound for each run. Optimization gap ratio is found between 0.19% and 0.29%. Every result is very close the optimum solution.

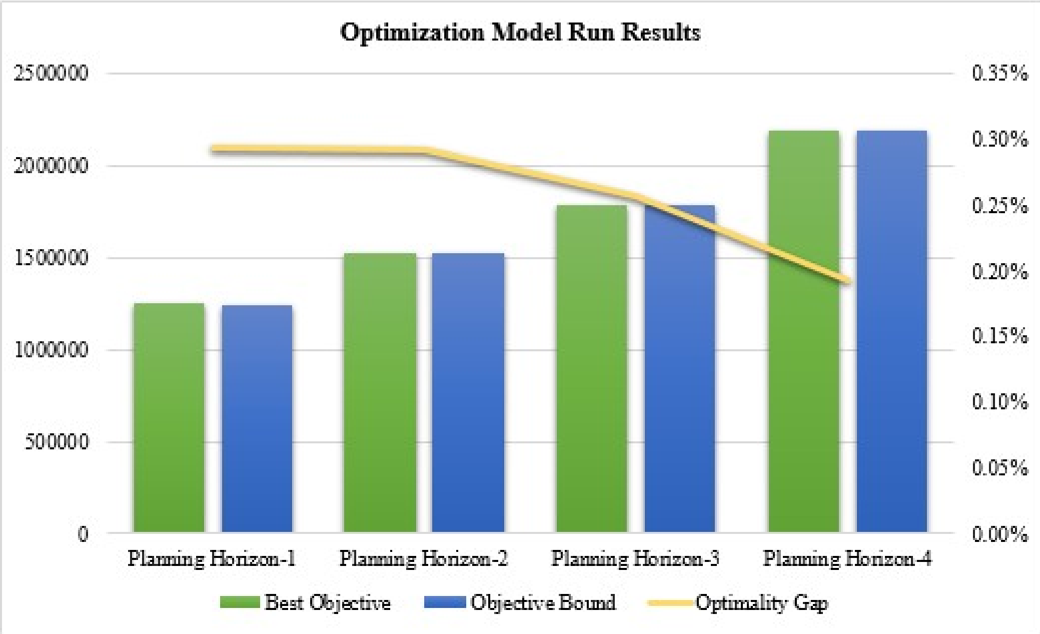

Figure 5.2 Optimality gap between best objective and objective bound for the model-1

Model-1's solutions for holding cost, ordering cost and purchasing cost are given in Table 5.4. Total cost is found $6,758,988.

Table 5.4 Model-1 all inventory cost results for 48-week demand

	Holding Cost	*Purchasing Cost*	*Ordering Cost*	*Total Cost*
Planning Horizon-1	$ 17,533	$ 1,220,224	$ 10,769	$1,248,526
Planning Horizon-2	$ 24,401	$ 1,488,401	$ 13,314	$1,526,116
Planning Horizon-3	$ 22,033	$1,756,589	$ 12,662	$1,791,284
Planning Horizon-4	$ 22,792	$ 2,155,213	$ 15,057	$2,193,062
Total 48-week Cost	$ 86,759	$ 6,620,427	$ 51,802	$6,758,988

Our aim is to minimize total inventory cost. Total cost includes holding cost, purchasing cost, and ordering cost. Figure 5.3 show total inventory cost values after running Model-1. According to rising demand level on yearlong, total cost is increasing. It is normal to see this type of results.

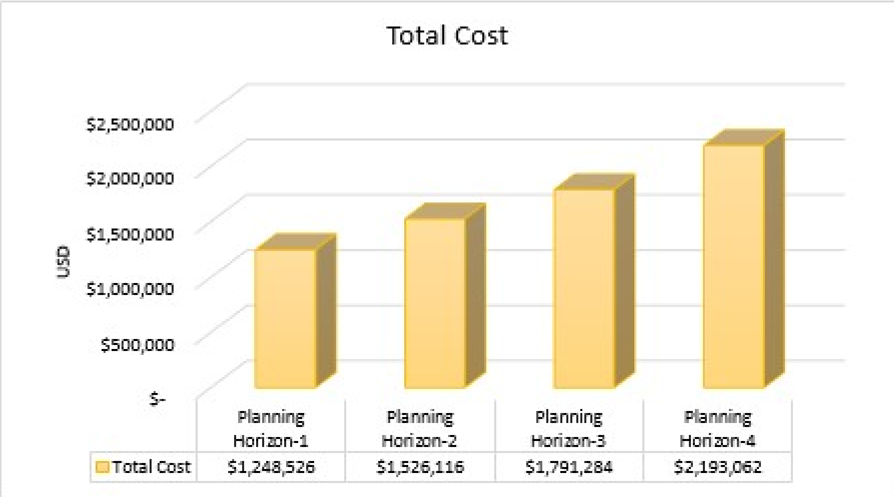

Figure 5.3 Total inventory cost figure for the model-1

Secondly, we run same data for 6-week periods where t= {1,…,6}. In order to reach one-year planning horizon, we generated the model eight times. 6-week model is called Model-2 in this chapter.

Model-2

t = {1,…,6}

p = {1,…,300}

s = {1,…,18}

Weekly demand that is taken from the company has been separated into eight different planning horizons. Concerning four different horizons is shown in Table 5.5. In this model, we have eight small model, and each model is run in the computer and results are analyzed in the point of inventory cost view.

Table 5.5 Separation of the taken weekly demands into the model-2

Planning Horizon for Model-2	*Weeks of Demand*
Planning Horizon-1	From Week-1 to Week-6
Planning Horizon-2	From Week-7 to Week-12
Planning Horizon-3	From Week-13 to Week-18
Planning Horizon-4	From Week-19 to Week-24
Planning Horizon-5	From Week-25 to Week-30
Planning Horizon-6	From Week-31 to Week-36
Planning Horizon-7	From Week-37 to Week-42
Planning Horizon-8	From Week-43 to Week-48

We generated 8 models and run in LINGO 9.0 software. We found optimum solution for each run except Planning Horizon-7 run. Figure 5.4 shows differences between best objective and objective bound for each run. Optimization gap ratio is found 0.02% for Planning Horizon-7 run. Results show that Model-2 is very applicable for our model when we compare with previous model.

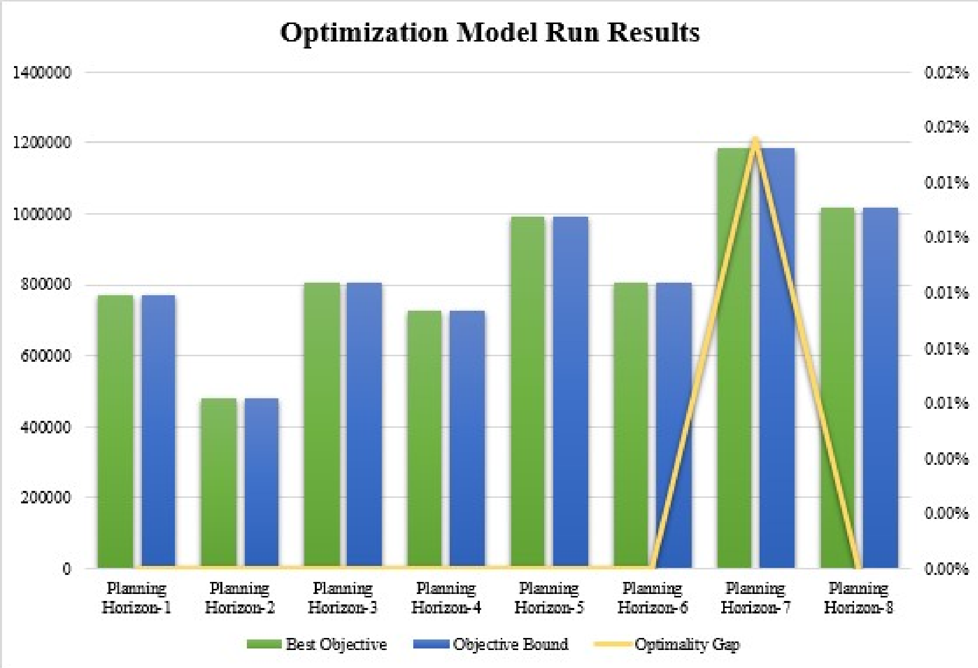

Figure 5.4 Optimality gap between best objective and objective bound for the model-2

Model-2's solutions for holding cost, ordering cost and purchasing cost are given in Table 5.6. Total cost is found $ 6,778,582.

Table 5.6 Model-2 all inventory cost results for 48-week demand

	Holding Cost	***Purchasing Cost***	***Ordering Cost***	***Total Cost***
Planning Horizon-1	$ 6,382	$ 760,059	$ 5,679	$ 772,120
Planning Horizon-2	$ 6,278	$ 467,073	$ 4,538	$ 477,889
Planning Horizon-3	$ 8,210	$ 790,515	$ 6,657	$ 805,383
Planning Horizon-4	$ 10,137	$ 709,681	$ 6,820	$ 726,638
Planning Horizon-5	$ 5,612	$ 978,983	$ 6,983	$ 991,578
Planning Horizon-6	$ 11,178	$ 786,744	$ 5,842	$ 803,764
Planning Horizon-7	$ 9,104	$ 1,166,148	$ 7,422	$ 1,182,674
Planning Horizon-8	$ 8,874	$ 1,001,859	$ 7,798	$ 1,018,531
Total 48-week Cost	$ 65,778	$ 6,661,064	$ 51,739	$ 6,778,582

Figure 5.5 Total inventory cost figure for the model-2

Figure 5.5 presents total cost in each planning horizon. We can also see same inventory increasing in this model. However, there is more balanced pattern in successive planning horizons.

Finally, we run same data for 8-week periods where $t = \{1,\ldots,8\}$. In order to reach one-year planning horizon, we generated the model eight times. 8-week model is called Model-3 in this chapter.

Model-3

$t = \{1,\ldots,8\}$

$p = \{1,\ldots,300\}$

$s = \{1,\ldots,18\}$

Weekly demand that is taken from the company has been separated into six different planning horizon. Concerning four different horizons is shown in Table 5.7.

Table 5.7 Separation of the taken weekly demands into the model-3

Planning Horizon for Model-3	*Weeks of Demand*
Planning Horizon-1	From Week-1 to Week-8
Planning Horizon-2	From Week-9 to Week-16
Planning Horizon-3	From Week-17 to Week-24
Planning Horizon-4	From Week-25 to Week-32
Planning Horizon-5	From Week-33 to Week-40
Planning Horizon-6	From Week-41 to Week-48

Model-3 is run in the software, and results are analyzed. We have feasible solution after run them in software. Best objective and objective bound differences is shown in Figure 5.6. Optimization gap ratio is found between 0.02% and 0.13%. Feasible solutions are very close with optimality.

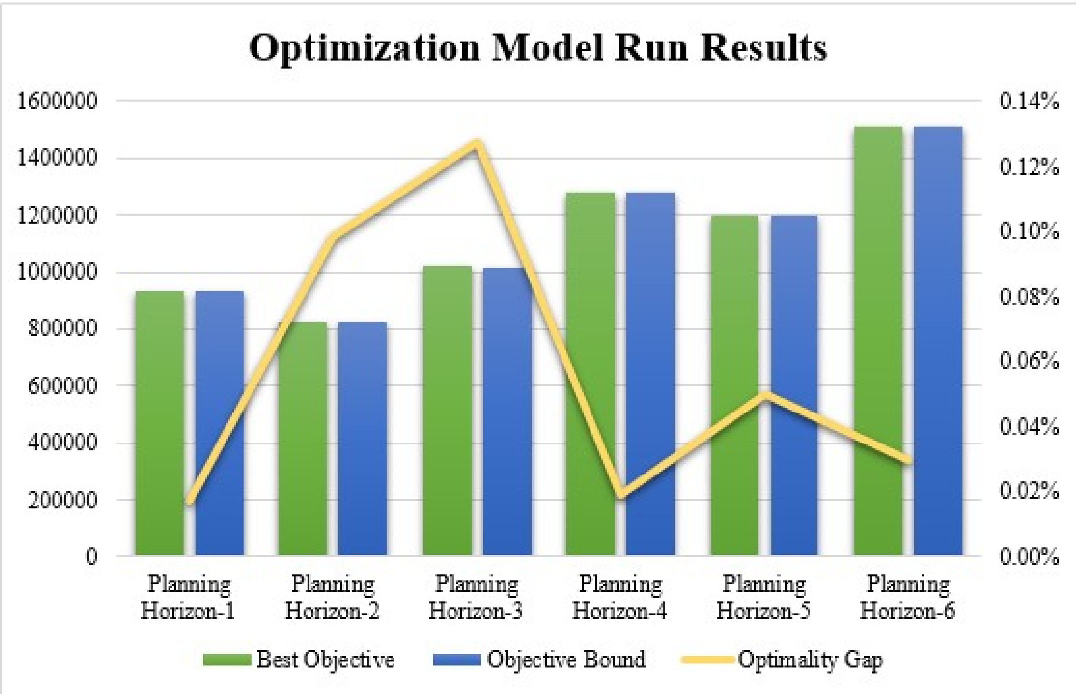

Figure 5.6 Optimality gap between best objective and objective bound for the model-3

Model-3's solutions for holding cost, ordering cost and purchasing cost are given in Table 5.8. Total cost is found $ 6,768,147.

Table 5.8 Model-3 all inventory cost results for 48-week demand

	Holding Cost	***Purchasing Cost***	***Ordering Cost***	***Total Cost***
Planning Horizon-1	$ 9,212	$ 919,159	$ 7,083	$ 935,454
Planning Horizon-2	$ 13,010	$ 804,946	$ 8,061	$ 826,017
Planning Horizon-3	$ 13,958	$ 995,936	$ 8,713	$ 1,018,606
Planning Horizon-4	$ 13,465	$ 1,256,223	$ 9,854	$ 1,279,542
Planning Horizon-5	$ 14,923	$ 1,174,137	$ 8,876	$ 1,197,936
Planning Horizon-6	$ 11,780	$ 1,488,632	$ 10,180	$ 1,510,592
Total 48-week Cost	$ 76,347	$ 6,639,033	$ 52,767	$ 6,768,147

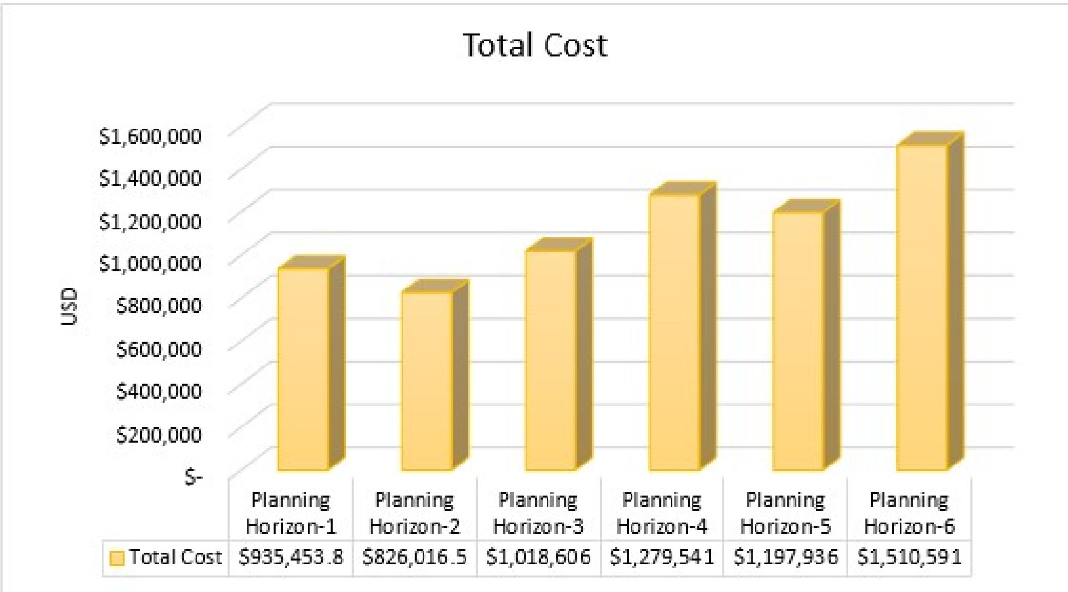

Figure 5.7 Total inventory cost figure for the model-3

Figure 5.7 presents total cost in each planning horizon. There is balanced pattern in successive planning horizons.

5.4.1 Comparison of Results

Three model has been run in order to find best solution for the inventory minimization problem of the automotive spare part company. Main difference in those three approaches is planning horizon. In previous section, we have run every scenario, and checked 48-week results individually. In the solution of this inventory problem, it is necessarily required to find best approach for the company and decide an ordering policy which is suitable for the company supply chain operations. Thus, it is needed to compare models in terms of holding cost, purchasing cost and holding cost.

Total holding cost in the result of applied models are given in Figure 5.8. In the results of 48-week total holding cost, Model-1 is found the highest holding cost among other models. Whereas Model-1 includes 12-week demand, others includes 6-week and 8-week demand. Thus Model-1 holds more inventory during each twelve periods. On the other hand, Model-2, which includes 6-week demand, holds minimum inventory which is sensible.

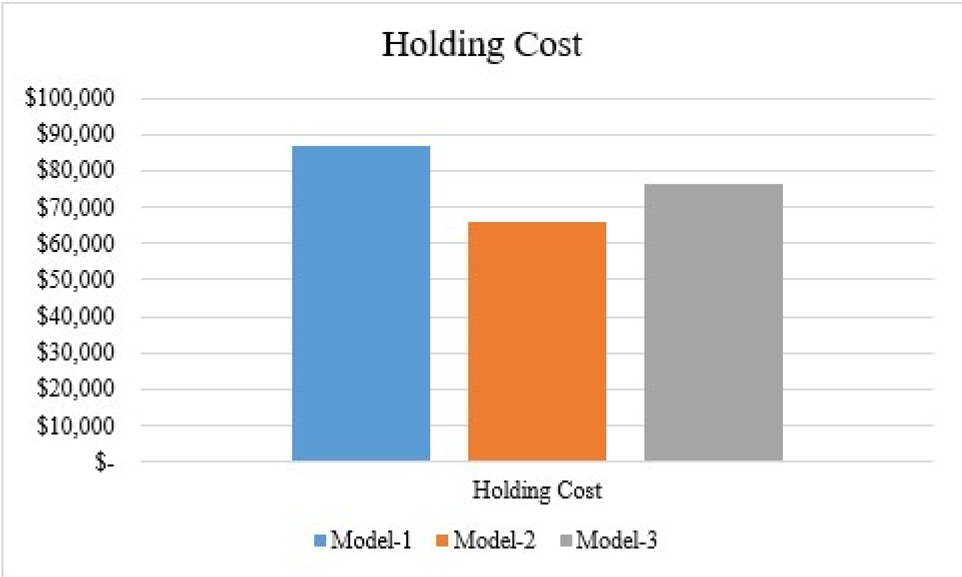

Figure 5.8 Comparison of holding cost

Total purchasing cost in the result of applied models are given in Figure 5.9. When purchasing costs are analyzed, it is found that goods have high purchasing cost by considering other costs. Hence, purchasing cost results are higher than holding and ordering cost. As Model-2 has only 6-week planning horizon, model decided to purchase product rather than keeping stock on hand. On the other side, Model-1 has lowest purchasing cost result.

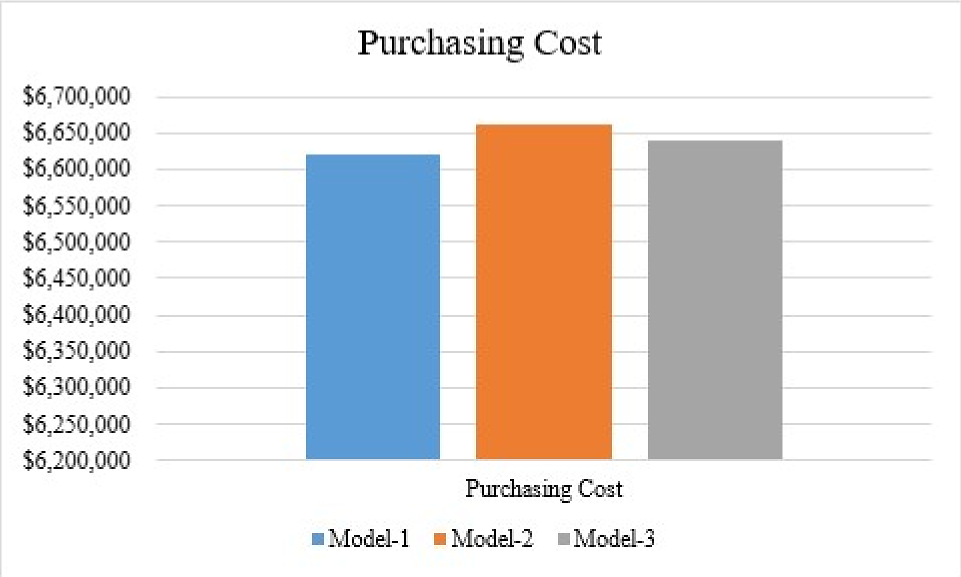

Figure 5.9 Comparison of purchasing cost

Figure 5.10 shows total ordering cost in the result of applied models in this study. Model-3 has biggest ordering cost level. Thus, Model-3 provides more ordering policy in order to find optimum inventory management. If we decide to use 8-week planning horizon approach, we need to order more frequently to suppliers.

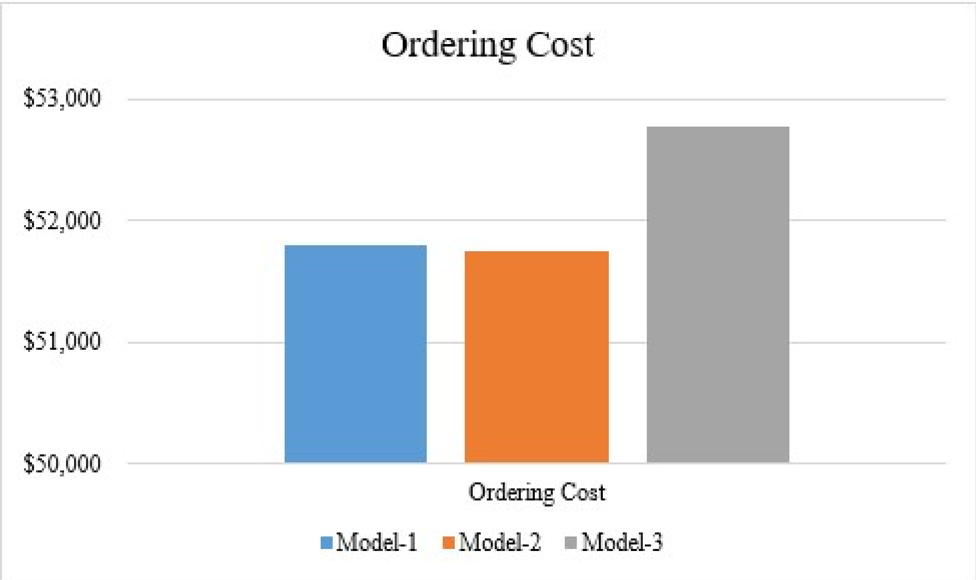

Figure 5.10 Comparison of ordering cost

It is crucial to state that total costs for each approach is very close to each other. There isn't any big gap between models. Nevertheless, Model-1 gives best solution among other approaches. Total cost has been found as $6,758,988. According to results, it is better to utilize 12-week planning horizon for real life application. Model-1 is used for comparison with company results in next section. Table 5.9 shows overall comparison of applied models.

Table 5.9 Comparison of applied models

Applied Model	*Total Cost*
Model-1 (12-week planning horizon)	$ 6,758,988
Model-2 (6-week planning horizon)	$ 6,778,582
Model-3 (8-week planning horizon)	$ 6,768,147

5.5 Comparison with Company Annual Costs

In the previous section, the proposed mathematical model has been utilized in order to find optimum total cost. Different planning horizons have been compared, and it is found that 12-week planning horizon finds best solution regarding to company data. This section, we analyzed real annual data which is provided by company. Thereafter, annual data and computational results are compared. Accordingly, summarized results are given to decide inventory management policy for company.

Holding, purchasing and ordering cost of regarding 48 weeks have been summarized for 300 products. As we used real data from previous years, therefore it is easier to check annual cost data which is provided by finance department of the automotive spare part company. Table 5.10 presents annual real life results. Total 48-week planning horizon cost is $ 7,191,664. Purchasing, ordering and holding costs are included into this result.

Table 5.10 Company results in same period

Planning Horizon	**Value (USD)**
Planning Horizon-1	1,394,314
Planning Horizon-2	1,614,381
Planning Horizon-3	1,885,837
Planning Horizon-4	2,297,132
Total Cost	7,191,664

When we compared with computational study, Model-1 results have lower cost than company results in same horizon, same demand, and same costs. Figure 5.11 shows full comparison between computational results and company results. Whereas orange bars present proposed mathematical model results, grey bars present company financial results with same data. In the planning horizon-1, there is 10.46% better results if we use computational study. There is only one exception which should be clarified. We assumed that inventory model starts with zero stocks. However, company has some stocks in the begging of the period. Especially planning horizon-1 of the company result are affected from stock cost in the begging of period. In the period horizon-2, ratio of difference is 5.47%. After, we found 5.01% better results in planning horizon-3 and 4.53% in planning horizon-4. If we compare total cost over all 48-week data, proposed mathematical model outperformed with 6.02%. Because whereas, company result is $ 7,191,664, Model-1 result is $ 6,758,988. This means, if company has used proposed model for deciding how many products needed to order to which supplier and when, company would have reported less inventory cost annually. When we check the comparison of real life versus computational study results, mathematical model may support during decision making of inventory lot size problem. Firm's supply chain engineer should use mathematical model in order to achieve less inventory cost in the medium level planning horizon.

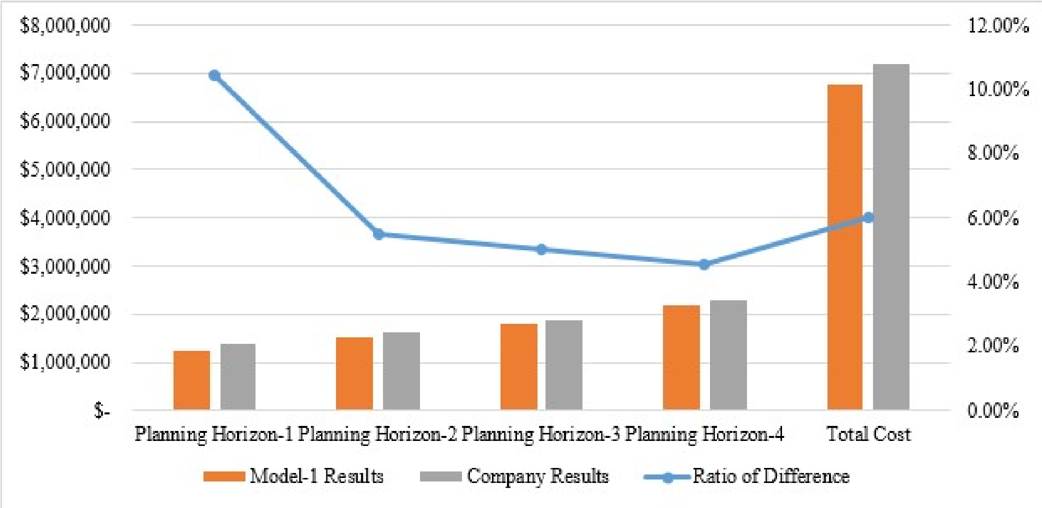

Figure 5.11 Comparison between computational results and company annual cost results

5.6 Chapter Conclusions

In this chapter, we provided a brief information about automotive spare part industry and the key points of this sector. In order to fully understand the case study and provide a good solution, data, which is provided from the company, has been analyzed in this chapter. Because the background of the inventory management is defined in the first chapter, real life problem analyzed in a better way in this chapter. As a nature of automotive spare part industry, the company has more than 7000 different products. This is very high product variety according to normal production industries, it is very hard to obtain the best purchase decision in this case. An ABC analyzed has been utilized, and item "A" demand has been integrated to the proposed mathematical model. We use different planning horizon in order to determinate best solution for this case study. In this chapter we use three different planning horizon and we run model several times. Model results are compared and we determinate which planning horizon could be better according to company structure. Firm's planning horizon is fine in their current structure, yet proposed model's result should be taken in to consideration by comparing current operation. Company annual results and ILP model results are also compared and mathematical model outperformed with 6.02%.

CHAPTER SIX
CONCLUSION

6.1 Summary of the Thesis

The overall objective of inventory management models is to determinate ordering time and quantities of all inventories in order to meet the inventory level by taking into account many similar parameters such as purchasing, ordering and holding cost. Models used in this point are shaped according to the structure of the current system. Within the scope of this subject, literature is checked with starting basic models. It can be seen that; inventory cost minimization problems have an important place in literature. Each year, different methodologies are generated in order to find better solution in this area. Our aim is to check studies in single/multi product, supplier and period situations. When we check a proper literature review in inventory management area, we can see that various detailed approached have been studied for many different types of businesses.

In this study, we investigate a spare part company inventory management system. Our aim is to provide better solution for the company's current ordering decision. Our proposed model is one of inventory total cost minimization problem for deterministic demand structure. We have multiple product, planning horizon, and supplier. In order to minimize total cost of inventory we proposed a mathematical model. This model has been generated according to firm's needs.

Generated ILP model is analyzed with 24 different type of examples. Each example has different number of products, supplier and period. We faced both optimal and feasible solution according to type of problem. An example problem data set, and its solution are provided in order to see mathematical model performance. Results run times have been compared. Accordingly, we continue with the case study.

Thereafter, real problem is employed on a real-life problem data. Company's inventory management system is analyzed. After checking all data, there were more than 7000 SKU exist in the system. In order to do better analyses, it was needed to classify stocks An ABC analyzed has been utilized, and item “A” demand has been integrated to the proposed mathematical model.

We have 3 different approaches in terms of planning horizon. We run same data with different planning periods, then we check results. When we compare 3 scenarios with their total holding, ordering and purchasing cost, the model of 12-week period shows best results.

Proposed model’s results and company’s annual inventory costs have been compared. Holding, purchasing and ordering cost of regarding 48 weeks have been summarized for 300 products. As we used real data from previous years, therefore it is easier to check annual cost data which is provided by finance department of the automotive spare part company. ILP model finds 6.02% better results comparing with real life results. Therefore, should implement the proposed mathematical model in order to decide their ordering strategy. Moreover, 12-week periods should be the best planning horizon according to current costs of holding, purchasing and ordering. Company’s current planning horizon strategy is good for purchasing decision. Proposed model provides minimum inventory level according to customer demand in the case study.

6.2 Contributions

Main contributions of this thesis are summarized as below:

- A new mathematical model is developed for multi-product, multi-period and multi-supplier inventory cost minimization problem.
- Model found 6.02% less annual inventory cost according to company annual results.
- By current ordering, purchasing and holding costs, company can use 12-week planning horizon in order to minimize total cost.

6.3 Future Research Directions

The main aim of this thesis is to develop a suitable model for company structure. Therefore, our proposed model has deterministic demand with multiple supplier selection for multiple products. Since only class “A” items are included to the case study, more product can be applied to the model in future research. Model results for more item can be analyzed.

As company have a large variety of product range, it is not possible to find optimum solution with the ILP model. On the other hand, when planning horizon is getting longer, it is getting hard to find optimum solution. That’s why, heuristic and/or meta-heuristic models can be applied to this case study in order to obtain better solution, and results can be compared with the proposed mathematical model and also company’s annual results.

A decision support system can be designed for the firm. It would be an easier way for firm’s utilization during ordering process.

REFERENCES

Abdelmaguid, T.F., & Dessouky M.M. (2006). A genetic algorithm approach to the integrated inventory distribution problem. *International Journal of Production Research*, *44* (21), 4445-4464.

Archetti, C., Bertazzi, L., Hertz, A., & Speranza, M. (2012). A hybrid heuristic for an inventory routing problem. *INFORMS Journal on Computing*, *24* (1), 101-116.

Archetti, C., Bertazzi, L., Laporte, G., & Speranza, M.G. (2007). A branch-and-cut algorithm for a vendor managed inventory-routing problem. *Transportation Science*, *41*, 382-391.

Bahl H.C., Ritzman L.P., & Gupta J.N.D. (1987). Determining lot sizes and resource requirements: a review. *Operations Research*, *35*, 329–345.

Bartmann, D., & Beckmann M. (1992). *Inventory control: models and methods* (1st ed.), Heidelberg: Springer Verlag.

Barutçu, S. (2007). İnternet tabanlı tedarik zinciri yönetimi (denizli tekstil işletmelerinin internet tabanlı tedarik zinciri yönetiminden yararlanma durumuna yönelik bir araştırma. *Selçuk Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *18*, 133-150.

Basnet C., & Leung J.M.Y. (2005). Inventory lot-sizing with supplier selection. *Computers and Operations Research, 32,* 1-14

Berman, B., & Evans, J.R. (2001). *Retail management a strategic approach* (8th ed.). New Jersey: Prentice Hall.

Bordum, A. (2010). The strategic balance in a change management perspective. *Society and Business Review*, *5* (3), 245.

Botter R., & Fortuin L. (2000). Stocking strategy for service parts: a case study. *International Journal of Operation and Production Management*, *20*, 656-674.

Bragg, S.M. (2005). *Inventory Accounting: a comprehensive guide* (1st ed.)., New York: John Wiley&Sons.

Bretthauer K.M., Shettyb B., Syamc S., & Vokurka R.J. (2006). Production and inventory management under multiple resource constraints. *Mathematical and Computer Modelling*, *44*, 85–95.

Brunner, P. (2002). Beyond materials flow analysis. *Journal of Industrial Ecology, 6* (1), 8-10.

Bundschuh, R.G., & Dezvane, T.M. (2003). How to make after services pay off. *The McKinsey Quarterly*, *4*(4), 116-128.

Cavalieri, S., Gaiardelli, P., & Ierace, S. (2007). Aligning strategic profiles with operational metrics in after-sales service. *International Journal of Productivity and Performance Management*, *56*, 436-455.

Chaudhry D., & Shankar R. (2011). Modeling and analyzing of single item multi-period procurement lot-sizing problem considering rejections and late deliveries. *Computers & Industrial Engineering, 6,* 1318-1323.

Chaudhry S.S, Forst F.G., & Zydiak J.L. (1993). Vendor selection with price breaks. *European Journal of Operational Research*, *70*, 52–66.

Chen L. (2013). A multi-product multi-echelon inventory control model with joint replenishment strategy. *Applied Mathematical Modelling*, *37* (4), 2039-2050.

Choudhary D., & Shankar R. (2011). Modeling and analysis of single item multi-period procurement lot-sizing problem considering rejections and late deliveries. *Computers & Industrial Engineering*, *61*, 1318–1323.

Cohen M.A., & Agrawal N. (2006). Winning in the aftermarket. *Harvard Business Review*, *84* (5), 129-138.

Das B., & Maiti M. (2007). An application of bi-level newsboy problem in two substitutable items under capital cost. *Applied Mathematics and Computation, 190* (1), 410–422.

Dasgupta S., & Spulber D. (1989). Managing procurement actions. *Information Economics and Policy, 4*, 5-29.

Davis, T. (1993). Effective supply chain management. *Sloan Management Review*, Summer, 35-46.

Doğan, G. (2006). *Envanter ve stok kontrol modellerinin incelenmesi ve en iyi sipariş miktarının belirlenmesi üzerine bir uygulama*. MSc Thesis, Marmara University, Istanbul.

Doğar, A. (2006). *Tedarik zinciri'nde stok yönetimi*. Phd Thesis, Istanbul Thecnical University, Istanbul

Eroğlu, A. (2002). *Deterministik envanter modelleri* (1st ed.). Isparta: Fakülte Kitabevi.

Ertuğrul, K. (2011). A Vendor-buyer lot sizing problem with stochastic demand: An exact procedure under service level approach. *European Journal of Industrial Engineering*, *5* (1), 101-110.

Gaballa A.A. (1974). Minimum cost allocation of tenders. *Operational Research Quarterly*, *25* (3), 89–98.

Gallagher T., Mitchke M.D., & Rogers M.D. (2005). Profiting from spare parts. *The McKinsey Quarterly, 4,* 1-4.

Ghare P. M. & Schrader G. P. (1963). A model for exponentially decaying inventory, *Journal of Industrial Engineering*, *14*, 238-243.

Haksever C., & Moussourakis J. (2005). Model for optimizing multi-product inventory systems with multiple constraints. *International Journal of Production Economics, 97*, 18–30.

Harris, F. W. (1915). What quality to make it once. *Operation and Cost, 5,* 47-52.

Hnaien F., Dolgui A., & Wu D.D. (2016). single-period inventory model for one-level assembly system with stochastic lead times and demand. *International Journal of Production Research, 54*, 186–203.

Hong J., & Hayya JC. (1992). Just-in-time purchasing: single or multiple sourcing. *International Journal of Production Economics*, 27 (1), 75–81.

Hopp W.J., Spearman M.L., & Zhang R.Q. (1997). Easily implementable inventory control policies. *Operations Research, 45*, 327–340.

Huang, S. H., & Lin, P. C. (2010). A modified ant colony optimization algorithm for multi item inventory routing problems with demand uncertainty. *Transportation Research Part E: Logistics and Transportation Review, 46*, 598-611.

Hunneman, A., Verhoef, P.C., & Sloot, L.M. (2015), The impact of consumer confidence on store satisfaction and share of wallet formation. *Journal of Retailing*, *91* (3), 516-532.

Kalınbacak, S. (2006). *Stok kontrol.* Retrieved March 30, 2019, from http://www.ie.sakarya.edu.tr/pano.php?no=203.

Karahan, M., & Aslan, Ş. (2016). Bir çimento fabrikasında hammadde stok kontrolü uygulaması. *Süleyman Demirel Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, 21* (3), 773-783.

Karimi M., Niknamfar A.H., & Pasandideh S.H.R. (2016). Two-stage single period inventory management for a manufacturing vendor under green-supplier supply chain. *International Journal of Systems Assurance Engineering and Management*, *28* (4), 1-15.

Kasap N., Biçer İ., & Yüksel Özkaya B. (2010). Stokastik envanter model kullanılarak iş makinelerinin onarımında kullanılanılan kritik yedek parçalar için envanter yönetim sistemi oluşturulması. *İstanbul Üniversitesi İşletme Fakültesi Dergisi*, *39*(2), 310-334.

Kasilingham R.G., & Lee C.P. (1996). Selection of vendors- a mixed-integer programming approach. *Computers and Industrial Engineering, 31,* 347-350.

Kaya, N. (2004*). Etkin stok yönetimi ve türkiye'de bir uygulama*. Phd Thesis, Atatürk University, Erzurum.

Kobu, B. (2006). *Üretim yönetimi* (13th ed.). Istanbul: Beta Basım Dağıtım.

Koster, R., & Roodbergen, K. J. (2007). Design and control of warehouse order picking: a literature review, *European Journal of Operational Research, 182* (2), 481-501.

Ladany, S., & Sternlieb, A. (1974). The interaction of economic erdering quantities and marketing policies. *AIIE Transactions*, *6* (1), 35-40.

Lambert, D. M., Stock, J. R., & Ellram, L. M. (1998). *Fundamentals of logistics management* (1st ed.). London: Mc Graw Hill.

Lin L.C., & Hou K.L. (2005). An inventory system with investment to reduce yield variability and set-up cost. *Journal of the Operation Research Society, (56),* 67-74.

Liu, S. C., & Lee, S. B. (2003). A two-phase heuristic method for the multi-depot location routing problem taking inventory control decisions into consideration. *The International Journal of Advanced Manufacturing Technology*, *22*, 941-950.

Lord, D. (2006). *The real cost of carrying inventory*. Retrieved March 29, 2019, from http://www.inventoryguru.com/article.php.

Maloney, B.M., & Klein, C.M. (1993). Constrained multi-item inventory systems: An implicit approach. *Operations Research*, *20* (6), 639-649.

Nahmias S., & Schmidt C. P. (1984). An efficient heuristic for the multi-item newsboy problem with a single constraint. *Naval Research Logistics Quarterly, 31* (3), 463-474.

Pirkul H., & Aras O. A. (1985). Capacitated multiple item ordering problem with quantity discounts. *IIE Transactions ,17* (2), 06–11.

Rosenblatt, M. J., & Rothblum, U. G. (1990). On the single resource capacity problem for multi-item inventory systems. *Operations Research, 38,* 686-693.

Sharma S. (2004). optimal production policy with shelf life including shortages. The *Journal of the Operational Research Society, 55*(8), 902–909.

Silver, E.A. (2008). Inventory management: an overview, Canadian publications, practical applications and suggestions for future research. *INFOR, 46* (1), 15-28.

Taleizadeh, A. A., Aryanezhad, M-B., & Niaki, S. T. A. (2008). Optimizing multiproduct multi-constraint inventory control systems with stochastic replenishment. *Journal of Applied Sciences*, *8* (7), 1228-1234.

Tan, K.C. (2001). A framework of supply chain management literature. *European Journal of Purchasing & Supply Chain Management*, *7* (1), 39-48.

Tarim, S.A. & Kingsman B.G. (2004). The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics*, *88*, 105-119.

Tekin E., Gürler Ü., & Berk E. (2001). Age-based vs. stock level control policies for a perishable inventory system. *European Journal of Operational Research*, *134* (2), 309-329.

Tersine R.J. (1994). Principles of inventory and materials management (4th ed.). New Jersey: PTR Prentice-Hall.

Top, A. (2006*). Üretim yönetimi* (4th ed.). Ankara: Nobel Yayın Dağıtım.

Tunç, H., Kilic O.A., Tarim S.A., & Eksioglu B. (2013). A simple approach for assessing the cost of system nervousness. *International Journal of Production Economics*, *141*(2), 619-625.

Urban, T. L., & Baker, R. C. (1997). Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns. *European Journal of Operational Research*, *103*, 573-583.

Vairaktarakis, G.L. (2000). Robust multi-item newsboy models with a budget constraint. *International Journal of Production Economics, 66*, 213-226.

Veinott A.F., & Wagner, H.M. (1966). Computing optimal (s, S) inventory policies. *Management Science, 11*, 525–552.

Wang D., Qin Z., & Kar S. (2015). A novel single-period inventory problem with uncertain random demand and its application. *Applied Mathematics and Computation, 269*, 133-145.

Waters, C.D.J. (1992). *Inventory control and management* (2nd ed.). Chichester: Wiley.

Waters, W. (2003). *Inventory control and management*. New Jersey: J. Wiley.

Weber C.A., & Current J.R. (1993). A multiobjective approach to vendor selection. *European Journal of Operational Research*, *68*, 173–184.

Wilson R.H. (1934). A scientific routine for stock control. *Harvard Business Review, 13,* 116-128.

Within, T.M. (1954). Inventory control research: a survey. *Management Science, 1,* 32-40.