PID CONTROLLER DESIGN FOR FIRST ORDER UNSTABLE TIME DELAY SYSTEMS

A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

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July 2009

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

PID CONTROLLER DESIGN FOR FIRST ORDER UNSTABLE TIME DELAY SYSTEMS

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In this thesis, problem of designing P, PI and PD-like controllers for switched first order unstable systems with time delay is studied. For each type of controller, the problem is solved in two steps. First, the set of stabilizing controllers for the class of plants considered is determined using different approaches. Then, an appropriate controller inside this set is chosen such that the feedback systems satisfies a desired property, which is for example gain and phase margin maximization or the dwell time minimization. In the first part, we focus on PI controllers and tune the PI controller parameters in order to maximize the gain and phase margins. The observations in this part show that a P controller is adequate to maximize gain and phase margins. Then, we move on to the problem of tuning P, PI and PD-like (first order stable) controller parameters such that the switched feedback system is stabilized and the dwell time (minimum required time between consequent switchings to ensure stability) is minimized. For this purpose, a dwell-time based stability condition of [39] is used for the class of switched time delay systems. We show that a proportional controller can be found with this method, but a PI controller is not feasible. Finally, we focus on the design of PD-like controllers for switched first order unstable systems with time delays. The proposed method finds the values of PD-like (first order stable) controller parameters which minimize an upper bound of the dwell time. The conservatism analysis of this method is done by time domain simulations. The results show that the calculated upper bound for the dwell time is close to the lower bound of the dwell time observed by simulations. In addition, we compare the obtained PD-like controller results with some alternative PD and first order controller design techniques proposed in the literature.

Keywords: Stability Analysis, Switched Systems, Time Delay, PID Control, Dwell Time, Gain Margin, Phase Margin

ÖZET

BİRİNCİ DERECEDEN ZAMAN GECİKMELİ VE KARARSIZ SİSTEMLER İÇİN PID DENETLEYİCİ TASARIMI

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Temmuz 2009

Bu tezin kapsamında birinci dereceden anahtarlamalı, kararsız ve zaman gecikmeli sistemler için orantısal denetleyici (P), orantısal-tümlevsel denetleyici (PI) ve orantısal-türevsel denetleyici benzeri (PD-like) denetleyici tasarımı problemlerinden bahsedilmiştir. Bu tip denetleyiciler için problemi iki adımda çözüyoruz. İlkinde, bahsedilen sistem grubu için farklı yaklaşımlar kullanarak geribesleme sisteminin kararlılığını sağlayacak denetleyici kümesi bulunmuştur. Daha sonra, bu kümenin içerisinden istenen özellikleri sağlayan uygun denetleyici seçilmiştir.

Tezin ilk kısmında kazanç ve faz paylarını maksimize edecek PI denetleyici tasarımı üzerine yoğunlaştık. Kazanç ve faz paylarını azami yapmak için P denetleyicinin yeterli olduğunu gözlemledik. Daha sonra anahtarlamalı ve zaman gecikmeli sistemler için denetleyici tasarımı konusuna geçtik. Bunun için, [39]'de verilen oturma zamanı hesabını temel alan kararlılık koşulları kullanıldı. Bu yöntem kullanılarak, uygun bir P ve PD benzeri denetleyici bulunabildi, ancak uygun bır PI denetleyici bulunamadı. Son olarak, PD benzeri denetleyiciler üzerinde yoğunlaştık. Önerilen yöntemle PD benzeri (birinci dereceden kararlı) denetleyici parametreleri bulundu, bu denetleyici tipi için zamanda benzetim yapılarak korunumluluk analizi yapıldı ve elde edilen PD benzeri denetleyiciler literatürde bulunan alternatif tasarım teknikleri ile karşılaştırıldı. Sonuçlar hesaplanan oturma zamanı üst sınırının benzetimler yardımıyla bulunan alt sınıra oldukça yakın olduğunu gösteriyor.

Anahtar Kelimeler: Kararlılık Analizi, Anahtarlamalı Sistemler, Zaman Gecikmesi, PID Denetleyici, Oturma Zamanı, Kazanç Payı, Faz Payı

ACKNOWLEDGMENTS

I would like to express my deep gratitude to my supervisor Prof. Dr. Hitay Özbay for his unlimited support and guidance.

I would like to thank Prof. Dr. Ömer Morgül and Assist. Prof. Uluç Saranlı for reading and commenting on this thesis and for being on my thesis committee.

I would also like to thank my dear friends Sevinç Figen Öktem, Esra Abacı and Gökhan Bora Esmer for their unlimited support.

I would like to thank Bilkent University EE Department and TÜBİTAK for their financal support.

Last but not the least, I would like to thank my husband and my parents for their support and endless love through my life.

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Dedicated to my husband and my parents

Chapter 1

Introduction

PID controllers offer the simplest and yet most efficient solution to many real world control problems. Therefore, they are the most widely used controller structures in the industry, [1]. The user has to tune three controller parameters in this setting. With the advances in the technology, automatic control area can now offer a wide range of controller structures. However, PID controllers are the most dominating. More than 90% of the controller loops used in the industry consists of PID controllers, [3]. PID controllers are used in various applications such as: process control, motor drives, magnetic and optic memories, automotive, flight control, instrumentation, etc. According to [38], PD control is most frequently used in robot position and force control because of its robustness to time delay and in addition, 98% of control loops in the pulp and paper industries are controlled by PI controllers, [23]. Another strength of the PID controller is that it deals with some practical issues such as actuator saturation and integrator windup. The PID controllers of interest are in the following form:

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\tau_d s + 1} \tag{1.1}$$

where K_p is the proportional constant, K_i is the integral constant, K_d is the derivative constant and $\tau_d > 0$ is a small time constant. The derivative part of the controller is implemented as in (1.1) to make it a proper transfer function.

Traditionally, the derivative part of a standard PID controller is filtered with a low-pass filter to prevent the high frequency gain of the controller growing too much (see [18]). The filter has not been regarded as a part of the design but added afterwards with the filter constant adjusted appropriately for the system to meet the specifications and small enough not to influence mid-frequency components. Also, note that when τ_d is an arbitrary positive number, (1.1) represents a stable controller structure. Such controllers are also have practical significant importance in the framework of low order strongly stabilizing controller design for unstable time delay systems, see e.g. [11] and [24].

In this thesis, we focus on PID (or stable) controller design for switched first order unstable systems with time delay. The transfer function of a first order time delayed unstable plant is as follows:

$$P(s) = \frac{e^{-hs}}{s-a} \tag{1.2}$$

where $h > 0$ is the time delay and $a > 0$ is the right half plane pole. A typical example of a first order unstable system with time delay is an aircraft model, [7]. The transfer function of the form (1.2) forms a distributed model of the aircraft for the purpose of controlling the longitudinal dynamics of an aircraft in the short time period. That means, only two parameters h and a model an infinite dimensional dynamics. In addition, the high frequency dynamics due to elasticity, actuators, sensors, computer and zero order hold contribute the effective time delay. Another example of such a system is the batch chemical reactor which has a strong nonlinearity due to heat generation term in the energy balance, see [21]. It is well known that we can effectively approximate a higher order transfer function with a first order time delayed transfer function. Thus, a wide range of plant structures can be handled by investigating the first order unstable plant with time delay.

It is difficult to control a plant if the product of the time delay and the right half plane (RHP) pole is large. A good example for demonstrating this fact is the

NASA X-29 forward-swept-wing aircraft, see [33]. The product of effective time delay and unstable pole was 0.37, see [21]. Although X-29 was built to illustrate the aerodynamic performance improvements, after lots of test, it was discovered that the vehicle was too unstable to control with the given hardware. Therefore, the product of effective time delay and unstable pole is a prominent parameter in terms of 'difficulty of control '. Moreover, [5] and [36] reported that a well tuned P or PI controller could stabilize a first order unstable plant with time delay if and only if the product of effective time delay and unstable pole $ah < 1$ is satisfied.

Over the last four decades, various methods were developed for setting the parameters of P, PI, PD and PID controllers. Some of these methods are modifications of the frequency response method introduced by Ziegler and Nichols, see [40], [12] and [4]. Some effort has been made to obtain an analytical formula which is only possible through rough approximations, see [13] and [17]. [37] developed tuning formulas based on minimization of integral performance criteria, [26] developed a method for design of controllers based on model matching in frequency domain, [27] proposed explicit tuning rules based on IMC design and [5] and [13] proposed tuning rules based on gain and phase margin specifications. See [23] to find an excellent collection of these tuning rules. However, very few of them investigated the set of all stabilizing controllers, see [31], [29] and [30]. As a part of this thesis, we investigated the parameter space which ensures closed-loop stability with PID controllers. The solution to the PID stabilization problem is based on determining appropriate intervals for proportional, integral, derivative and time constant given in (1.1) where the obtained PID controller stabilizes the feedback system. The method for finding this parameter space is as following:

- First an admissible range for proportional constant is found for which a stabilizing PID controller exists.

- For a fixed proportional constant value in this range, the set of stabilizing integral and derivative constant is found. It is either a trapezoid, a triangle or a quadrilateral.

- After determining the parameter space for proportional, integral and derivative constants, an admissible range for time constant is found to ensure stability.

As far as switched systems are concerned, we consider the switched feedback system shown in Fig. 1.1, where θ is an arbitrary piecewise switching signal taking values on the set

$$\mathcal{F} := \{1, \dots, l\}.$$

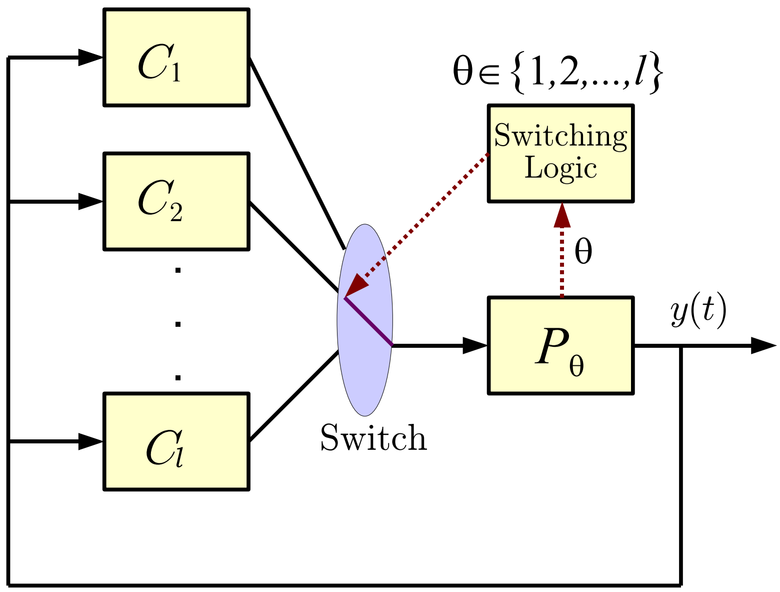

Figure 1.1: Typical Switched Feedback System

In this study, we assume that between switching time instants the plant is one of the elements of the following known set.

$$\mathcal{P} := \{P_1, \dots, P_l\}.$$

At each switching instant, the switching signal θ selects an index $\theta \in \mathcal{F}$, so a plant is selected from $\mathcal{P}$. Each $P_\theta \in \mathcal{P}$ is a first order unstable system with time delay and can be expressed in the following form:

$$P_\theta(s) = \frac{e^{-h_\theta s}}{s - a_\theta}. \tag{1.3}$$

where h_θ is the time delay and a_θ is the right half plane pole. As the plant switches according to the switching logic θ, controller has to switch in order to preserve stability. The controllers C_θ are proportional integral derivative (PID) controllers in the form:

$$C_\theta(s) = K_{p\theta} + \frac{K_{i\theta}}{s} + \frac{K_{d\theta}s}{\tau_{d\theta}s + 1} \tag{1.4}$$

where $K_{p\theta}$ is the proportional constant, $K_{i\theta}$ is the integral constant, $K_{d\theta}$ is the derivative constant and $\tau_{d\theta} > 0$ is a small time constant. A state-space realization of the closed loop dynamics can be written as follows:

$$\Sigma_\theta : \begin{cases} \dot{x}(t) = A_\theta x(t) + \bar{A}_\theta x(t - h_\theta) \\ y(t) = C_\theta x(t) \end{cases} \tag{1.5}$$

The triplet $\Sigma_\theta := (A_\theta, \bar{A}_\theta, h_\theta)$ is introduced to describe the θ^{th} candidate system of (1.5). Thus, $\forall t \geq 0$ we have

$$\Sigma_t \in \mathcal{A} := \{\Sigma_\theta : \theta \in \mathcal{F}\}$$

where $\mathcal{A}$ is the family of candidate systems of (1.5).

The switching signal θ causes an arbitrary selection between candidate systems and the selection of the switching signal for control purposes is out of the scope of this thesis. In other words, we deal with the system under arbitrary switchings in both the plant and the controller parameters, which are determined externally.

The feedback system shown in Fig. 1.1, runs with the initial conditions which means the reference input is zero. Since each candidate plant is stabilized with a corresponding controller, the switched system will preserve its stability if the candidate plant-controller pairs are running for a long enough time interval. In other words, if the switching intervals are sufficiently long, the overall switched system will be stable. On the other hand, frequent switching may cause instability, see e.g. [10], [22] and [34]. The minimum time needed between switching instants to maintain stability is called **dwell time**. An LMI-based stability condition is recently derived in [39], which also gives a dwell time expression. Using the derived LMI-based stability test, the set of stabilizing PID controllers for switched first order unstable plants is searched.

Our contributions can be summarized as follows:

- We investigate different PI controller design methods in literature and develop a method based on gain and phase margin maximization and a cost function minimization for first order unstable time delayed plants given in (1.2). For the beginning, we deal with some controller design criteria for non-switched plants, including the gain margin and the phase margin optimization and a cost function minimization which is defined as a linear combination of the weighted sensitivity function and the vector margin.

- Then, we find a stabilizing parameter space for P and PI controllers and develop a method using these parameter spaces for switched first order unstable plants with time delay using the LMI-based stability test derived in [39].

- Likewise, PD-like controller design approach is developed for the same class of plants using the LMI-based stability test given in [39]. The conservativeness of this method is tested using time domain simulations and the results of this methods are compared with some PID controller design methods mentioned above.

In Chapter 2, we provide the developed PI controller design method based on gain and phase margin maximization and a cost function minimization. Chapter 3 addresses P and PI controller design method we propose for switched first order unstable plants with time delay. In Chapter 4, the results of the PD-like controller design method for the same class of plants are given; the conservativeness analysis of the results are done and comparisons of the proposed approach with the existing methods are made. We give the concluding remarks in Chapter 5.

Chapter 2

PI Controller Design Based on Gain and Phase Margin Maximization and a Cost Function Minimization

For the plant of the form (1.2) and the PI controller of the form (1.1) where the derivative gain of the controller $K_d = 0$, the open-loop transfer function of the system is given by,

$$G(s) = C(s)P(s) = \frac{(K_p s + K_i)e^{-hs}}{s(s-a)} \tag{2.1}$$

which can be rewritten as;

$$G(s) = K\frac{(1+\tau\hat{s})e^{-\hat{h}\hat{s}}}{\hat{s}(\hat{s}-1)} \tag{2.2}$$

where $K = \frac{K_i}{a^2}$, $\tau = a\frac{K_p}{K_i}$, $\hat{h} = ha$ and $\hat{s} = \frac{s}{a}$. Therefore, in the rest of the chapter, we consider the generic form of the open-loop transfer function,

$$G(s) = K\frac{(1+\tau s)e^{-hs}}{s(s-1)} \tag{2.3}$$

We think of

$$C(s) = K\frac{1+\tau s}{s} \tag{2.4}$$

is the generic PI control and

$$P(s) = \frac{e^{-hs}}{s-1} \tag{2.5}$$

is the generic plant.

In this chapter, our aim is to design the controller parameters K and τ such that for a given plant of the form (2.5).

- The gain and phase margin are maximized separately.
- Gain and phase margin are optimized in a blended fashion.
- A cost function obtained from H_∞ robust performance problem is minimized.

The phase margin, denoted by ϕ_m, is defined as;

$$|G(j\omega_g)| = 1 \tag{2.6}$$

$$\phi_m = \angle G(j\omega_g) + \pi \tag{2.7}$$

where ω_g is called the gain crossover frequency. The magnitude of $G(j\omega)$ is a non-increasing function because $G(j\omega)$ has one pole at $\omega = 0$ and one real pole and zero, hence there exists a unique gain crossover frequency.

In order to ensure the feedback system stability, $1+G(j\omega)$ has to encircle -1 once in the counterclockwise direction from the Nyquist encirclement principle. That means $G(j\omega)$ have to intersect the negative real axis at least twice. This condition is satisfied if and only if

$$|G(j\omega_{p_1})| > 1 > |G(j\omega_{p_2})| \tag{2.8}$$

where ω_p is called the phase crossover frequency and $\omega_{p_1} < \omega_{p_2}$ are the smallest solutions of $G(j\omega) = -\pi$. The gain margin denoted by GM is defined as follows.

$$GM = min\{\sigma_1, \frac{1}{\sigma_2}\} \tag{2.9}$$

where $\sigma_1 = |G(j\omega_{p_1})|$ and $\sigma_2 = |G(j\omega_{p_2})|$. Note that, this definition is different from the classical gain margin definition (for example see [13])

$$GM = \frac{1}{\sigma_2}.$$

Our definition is the same as the gain margin definitions in [20] and [9]. This definition is more appropriate for unstable systems in terms of robustness analysis, see [20].

In addition, the cost function obtained from robust performance problem, denoted by J, is defined as;

$$J = (c\beta + e^{\alpha})^{-1} \tag{2.10}$$

where $0.01 \leq c \leq 40$ is a coefficient to be adjusted, S denotes the sensitivity function given as follows,

$$S(s) = \frac{1}{1 + C(s)P(s)} \tag{2.11}$$

α is a robust performance measure defined as follows which is the infinite norm of a weighted sensitivity function,

$$\frac{1}{\alpha} = ||W(s)S(s)||_{\infty} \tag{2.12}$$

and β is called vector margin as defined in (2.13), which is the distance of $G(j\omega)$ from -1.

$$\frac{1}{\beta} = \|S\|_{\infty} \tag{2.13}$$

The desired controller parameters should be chosen to satisfy robust stability and performance conditions. The necessary condition to satisfy the robust stability is to design a controller that stabilizes the nominal feedback system as well as all the possible plants with additive uncertainty bounded by $W(s)$. The vector margin β is a parameter related with the robust stability. For a robustly stable system, the controller should be designed to restrict the tracking error energy to satisfy robust performance criteria. Due to the robust performance constraint, the weight function is chosen as $W(s) = \frac{1}{s}$ in order to obtain a better tracking of step-like reference signals. Hence, α is a parameter related with the robust performance condition.

It is difficult to solve the above optimization problem analytically, hence we concentrate on numerical solutions.

2.1 Gain Margin Maximization

For a given plant (1.2), K and τ parameters in (2.3) are chosen such that the gain margin is maximized. The magnitude and phase expressions of the open-loop transfer function $G(s)$, as shown below, are required to find the gain margin.

$$|G(j\omega)| = \frac{K}{\omega}\sqrt{\frac{1+\tau^2\omega^2}{1+\omega^2}} \tag{2.14}$$

$$\angle G(j\omega) = -\frac{3\pi}{2} + \tan^{-1}(\tau\omega) + \tan^{-1}(\omega) - h\omega \tag{2.15}$$

Proposition 1. *For each fixed $h > 0$ and $\tau > 0$, the optimal K maximizing GM defined in (2.9) is:*

$$K = \left(\frac{1}{\omega_{p_1}\omega_{p_2}}\sqrt{\frac{(1+\tau^2\omega_{p_1}^2)(1+\tau^2\omega_{p_2}^2)}{(1+\omega_{p_1}^2)(1+\omega_{p_2}^2)}}\right)^{-\frac{1}{2}}. \tag{2.16}$$

Proof. Let ω_{p_1} and ω_{p_2} be the two smallest phase crossover frequencies satisfying $G(j\omega_p) = -\pi$. By substituting (2.14) into σ_1 and σ_2, we obtain the following equations.

$$\sigma_1 = |G(j\omega_{p_1})| = \frac{K}{\omega_{p_1}}\sqrt{\frac{1+\tau^2\omega_{p_1}^2}{1+\omega_{p_1}^2}}$$

$$\sigma_2 = |G(j\omega_{p_2})| = \frac{K}{\omega_{p_2}}\sqrt{\frac{1+\tau^2\omega_{p_2}^2}{1+\omega_{p_2}^2}}$$

Then define;

$$a = \frac{1}{\omega_{p_1}}\sqrt{\frac{1+\tau^2\omega_{p_1}^2}{1+\omega_{p_1}^2}} \tag{2.17}$$

$$b = \frac{1}{\omega_{p_2}}\sqrt{\frac{1+\tau^2\omega_{p_2}^2}{1+\omega_{p_2}^2}} \tag{2.18}$$

When a and b variables defined above are substituted into (2.8), we obtain $Ka > 1 > Kb$. This inequality can be rewritten as follows.

$$\frac{1}{b} > K > \frac{1}{a} \tag{2.19}$$

The optimal K value satisfying (2.19) maximizing the gain margin defined in (2.9) is $K = \frac{1}{\sqrt{ab}}$. If we substitute the a and b variables defined in (2.17) and (2.18), we obtain the optimal gain K as given in (2.16). □

For the generic plant, for each fixed τ under the above choice of K, the variation of GM is as shown in Figure 2.1. As illustrated in the Figures 2.1, 2.2 and 2.3, higher τ value yields better gain margin and smaller K value. The controller can be considered as follows,

$$C(s) = K\tau + \frac{K}{s} \tag{2.20}$$

where $K\tau$ is the proportional constant and K is the integral coefficient. From Figures 2.2 and 2.3, it could be seen that as $\tau \to \infty$, $K \to 0$ and $0 < K\tau < \infty$, which means the porportional constant is approaching to a constant finite value. Therefore, since the integral constant goes to 0, a proportional (P) controller is adequate to maximize the gain margin.

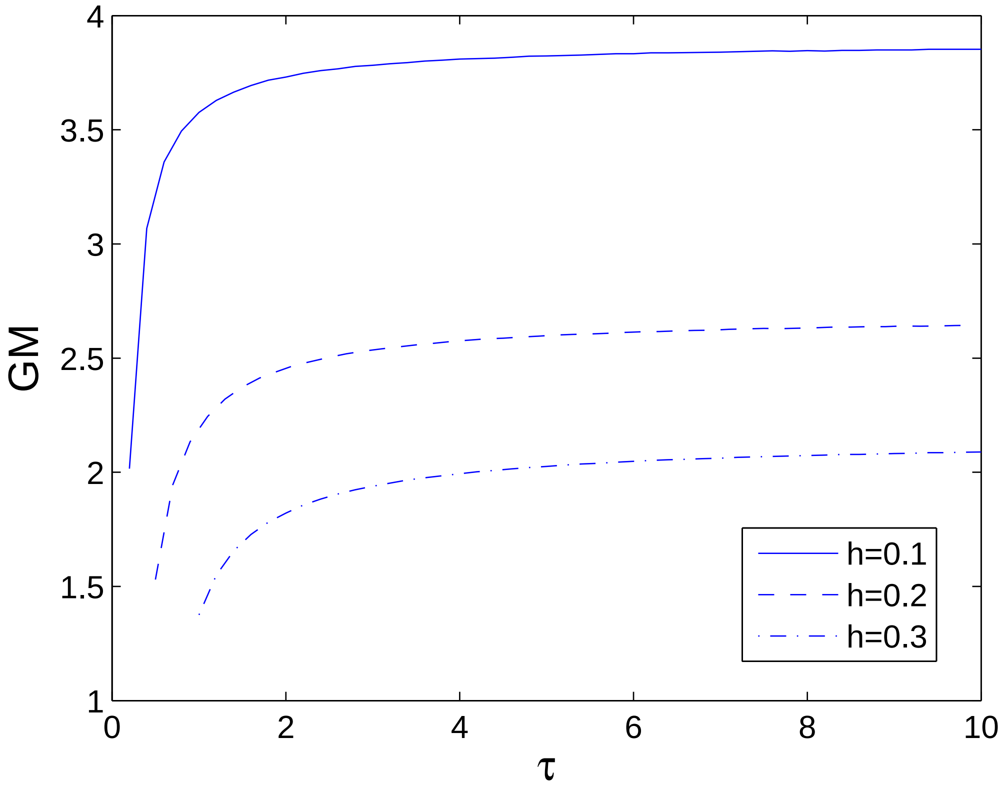

Figure 2.1: The Gain Margin versus τ

2.2 Phase Margin Maximization

Phase margin is a performance measure of the feedback system, which is related with the damping of the system, see [6]. In this section, we choose the controller parameters K and τ in (2.4) to maximize the phase margin, which is defined as follows.

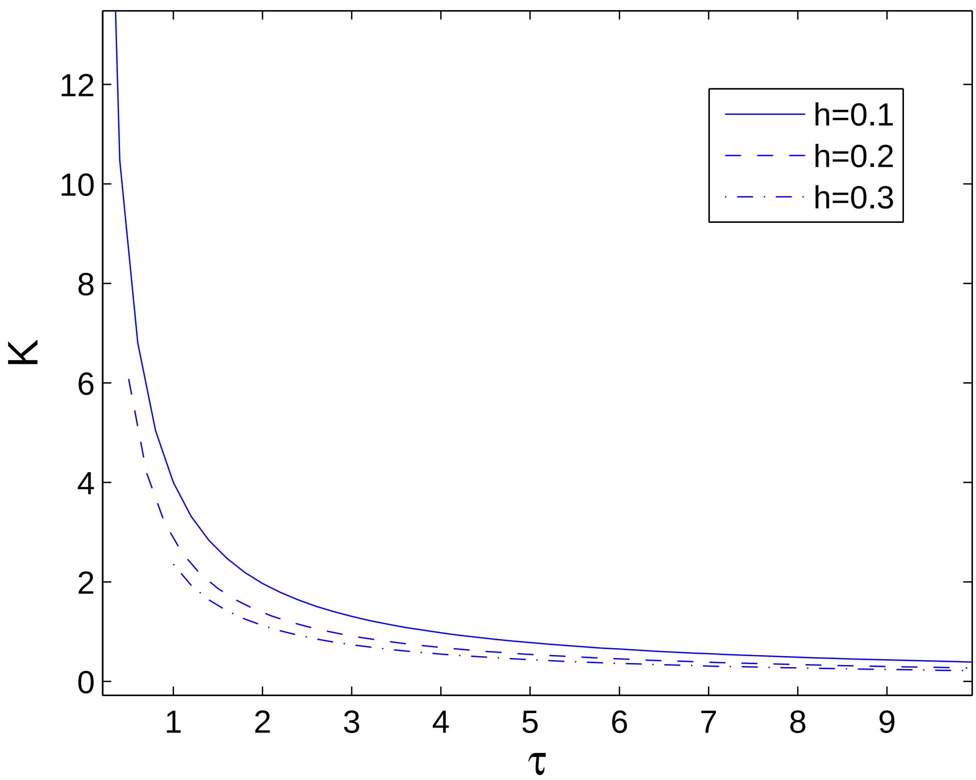

Figure 2.2: K versus τ

$$\phi_m = -\frac{\pi}{2} + tan^{-1}(\tau\omega_g) + tan^{-1}(\omega_g) - h\omega_g \tag{2.21}$$

For each fixed τ, the maximum ϕ value and the corresponding gain crossover frequency, ω_g, are found by evaluating (2.21) over a frequency range.

Proposition 2. *For each fixed $h > 0$ and $\tau > 0$, the optimal K maximizing the phase margin can be expressed as:*

$$K = \omega_g\sqrt{\frac{1+\omega_g^2}{1+\tau^2\omega_g^2}} \tag{2.22}$$

where the gain crossover frequency, ω_g, is obtained by setting the magnitude of $G(j\omega)$ in (2.14) to 1*.*

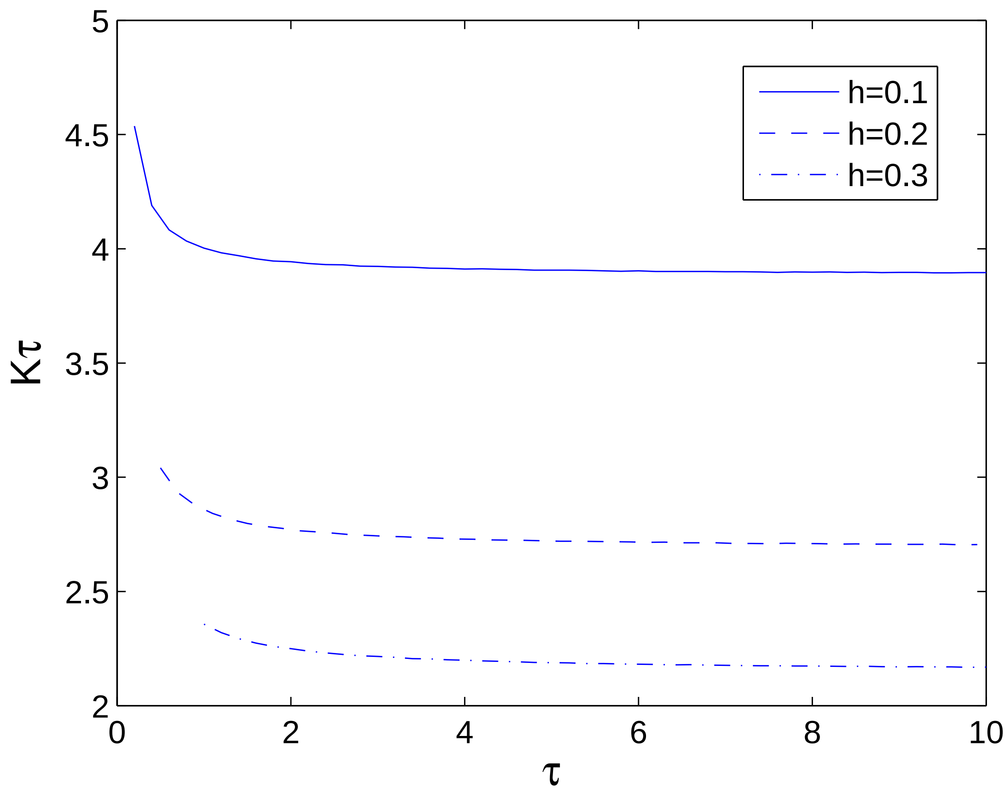

Figure 2.3: $K\tau$ versus τ

Proof. Let ω_g be the gain crossover frequency satisfying

$$|G(j\omega_g)| = 1. \qquad (2.23)$$

By substituting the magnitude of $G(j\omega)$ given in (2.14) into (2.23), we obtain the following equality.

$$\frac{K}{\omega_g}\sqrt{\frac{1+\tau^2\omega_g^2}{1+\omega_g^2}} = 1 \qquad (2.24)$$

Hence, the optimal K expression given in 2.22 for each fixed τ is derived from (2.24). □

For the generic plant, the graph of

$$\phi(\omega) = \tan^{-1}(\tau\omega) + \tan^{-1}(\omega) - h\omega - \frac{3\pi}{2} \qquad (2.25)$$

is shown in Fig. 2.4 for changing τ values. As we can see, $\phi(\omega)$ has a maximum point for each τ value. We should adjust ω_g such that

$$\omega_g = arg\max(\phi(\omega)) \tag{2.26}$$

The resulting phase margin ϕ_m versus τ and time delay h are illustrated in Fig. 2.5.

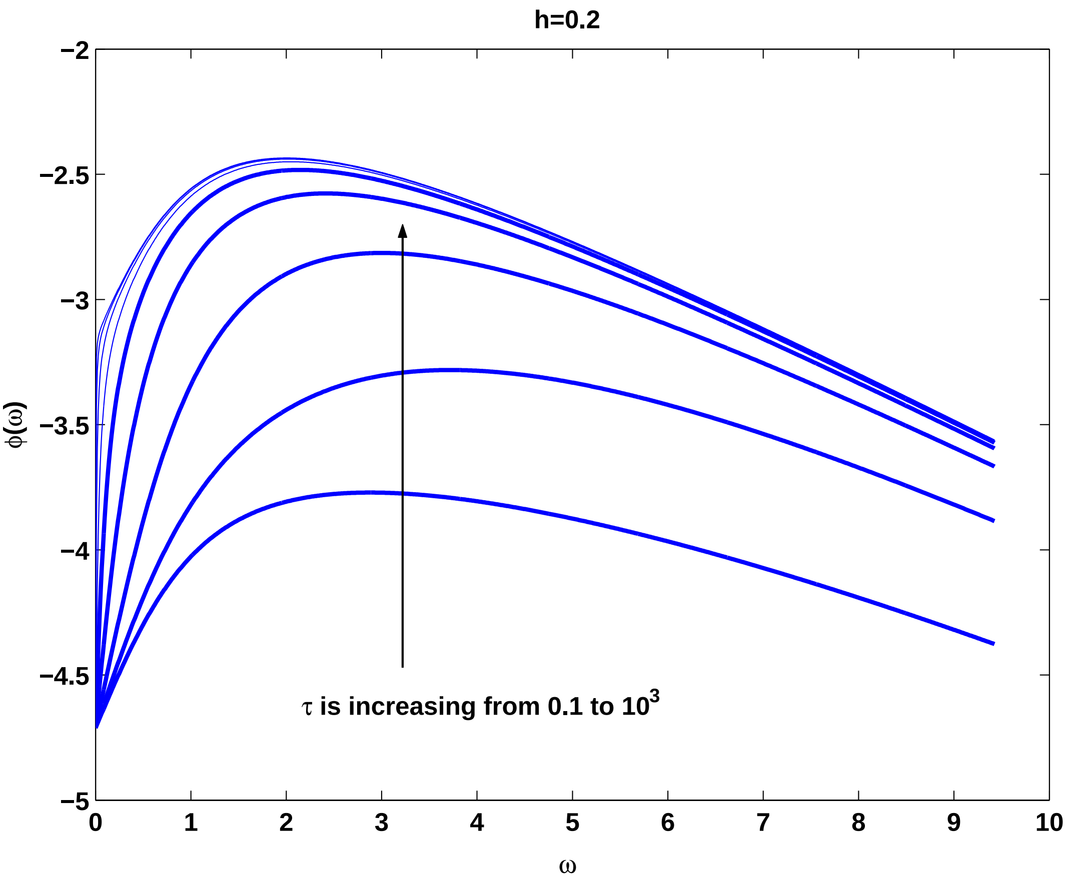

Figure 2.4: $\phi(\omega)$ versus frequency ω

The variation of ω_g versus τ with respect to different delay values is illustrated in Fig. 2.6 to show that as $\tau \to \infty$, the gain crossover frequency goes to a finite value.

In Figures 2.7 and 2.8, the graph of the proportional and integral constants versus τ are illustrated respectively with respect to time delay. As shown these

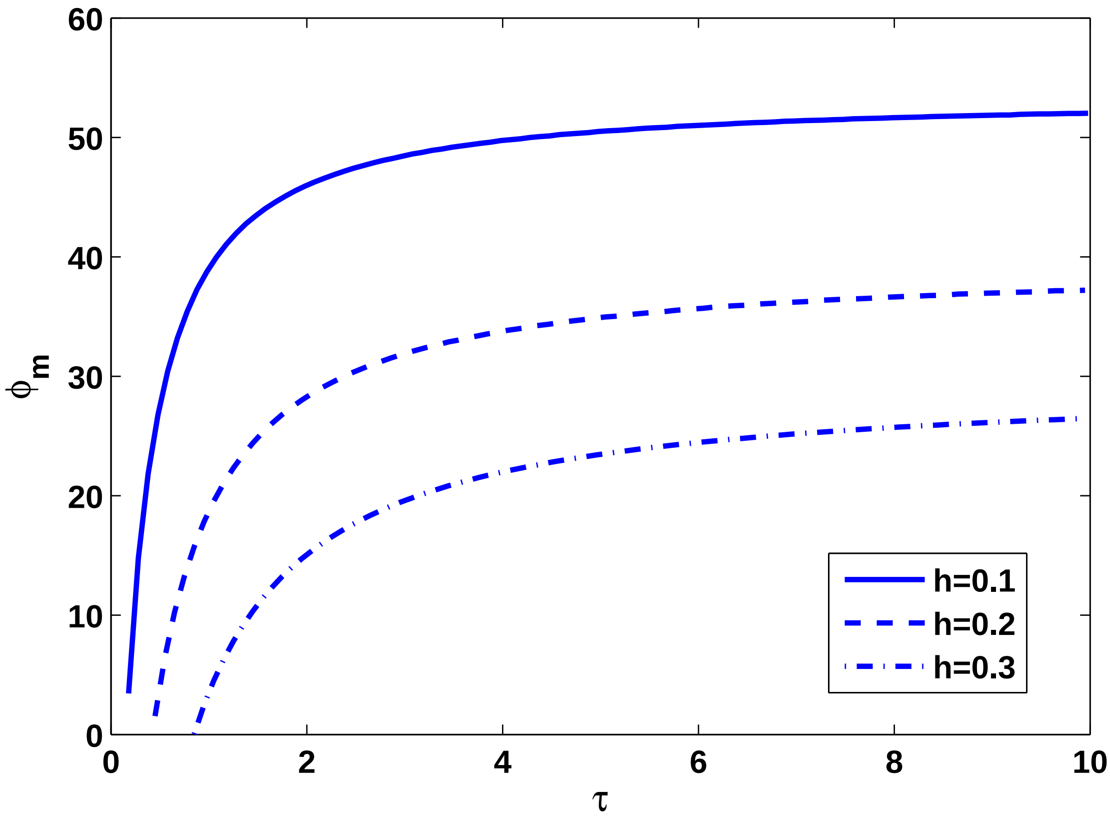

Figure 2.5: Phase margin ϕ_m versus τ

figures, higher τ value yields higher phase margin and smaller K. For the controller structure shown in (2.4), the same situation as the gain margin maximization problem occurs. As $\tau \to \infty$, a higher phase margin is obtained with $K \to 0$ and $K\tau$ approaches to a finite value. That means a P controller is adequate to obtain the maximum phase margin from a stable feedback system. Same observations are made in [14]. In addition, as we know, phase margin has to be positive to ensure the stability of the feedback system. Therefore, for each $h > 0$, we obtain a minimum $\tau > 0$ value which makes the phase margin greater or equal to zero. This is also the case in the gain margin maximization problem.

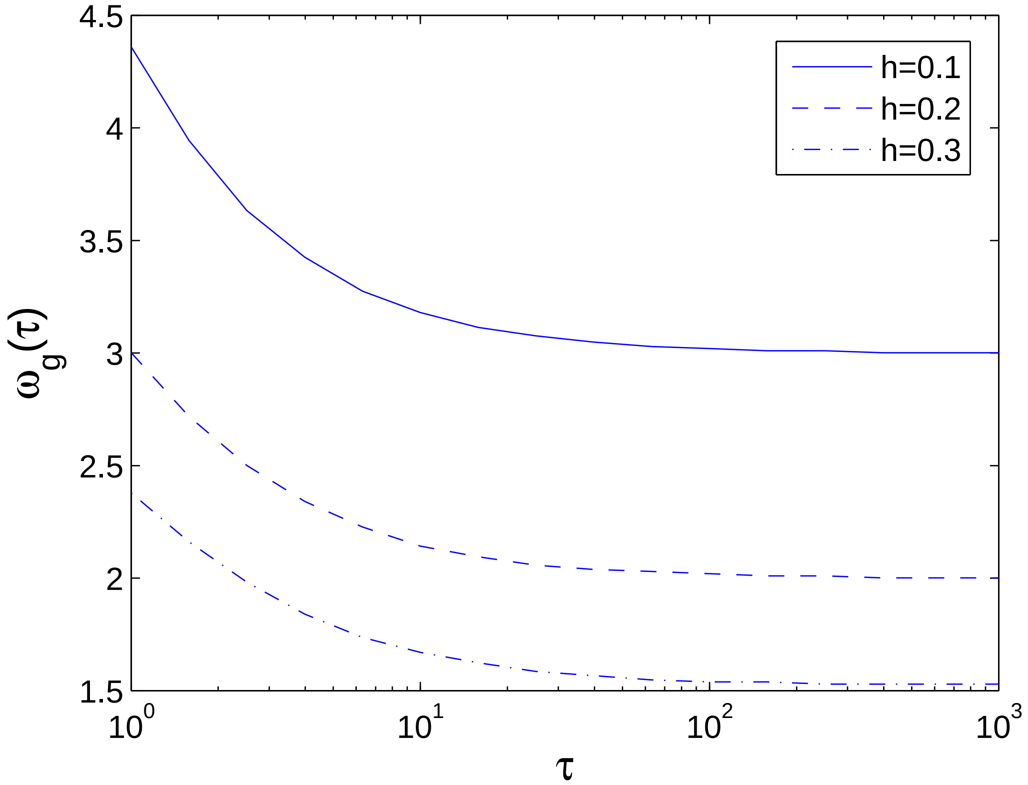

Figure 2.6: Gain crossover frequency ω_g versus τ

2.3 Gain and Phase Margin Optimization

As we stated in the previous sections, gain margin and phase margin parameters are good measures of robustness. Therefore the aim in this section is to design a controller that satisfies both gain margin and phase margin criteria. The phase margin definition in (2.21) and the gain margin definition in (2.9) are used to maximize the gain margin and the phase margin in a blended fashion.

In order to optimize gain and phase margin in a blended fashion, the optimal K value is chosen such that both gain and phase margin are maximized by equating the optimal K expression given in (2.16) and (2.22) for both gain and phase margin maximization problems. We can find the gain and the phase crossover frequencies in these expressions by evaluating the phase ϕ over a frequency range. Here, ω_1 and ω_2 are the phase crossover frequencies where $G(j\omega) = -\pi$ is satisfied

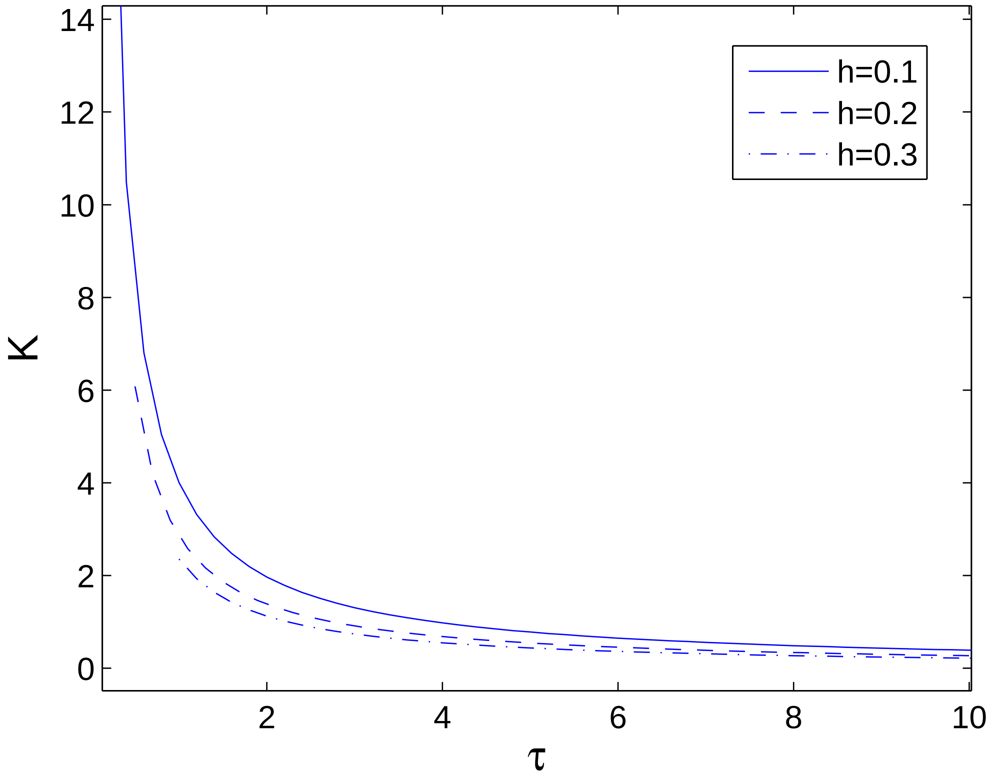

Figure 2.7: K versus τ

and ω_g is the gain crossover frequency where (2.23) is satisfied. A typical graph illustrating the phase crossover frequencies and the gain crossover frequency are as shown in Figure 2.9. By this way, the corresponding optimal K value is obtained.

Optimum gain and phase margin search explained above is accomplished over different τ values to choose corresponding τ that maximizes both parameters.

The maximum gain and phase margin values are indicated in Table 2.1 for each fixed τ and as $\tau \to \infty$, phase margin converges to 39.559 degrees and gain margin converges to 2.689.

Hence, the maximum gain and phase margins are obtained when τ goes to infinity and optimal K goes to zero which causes the integral action to disappear. In order not to get too small integral action gain, we may want to choose $\tau = 10$

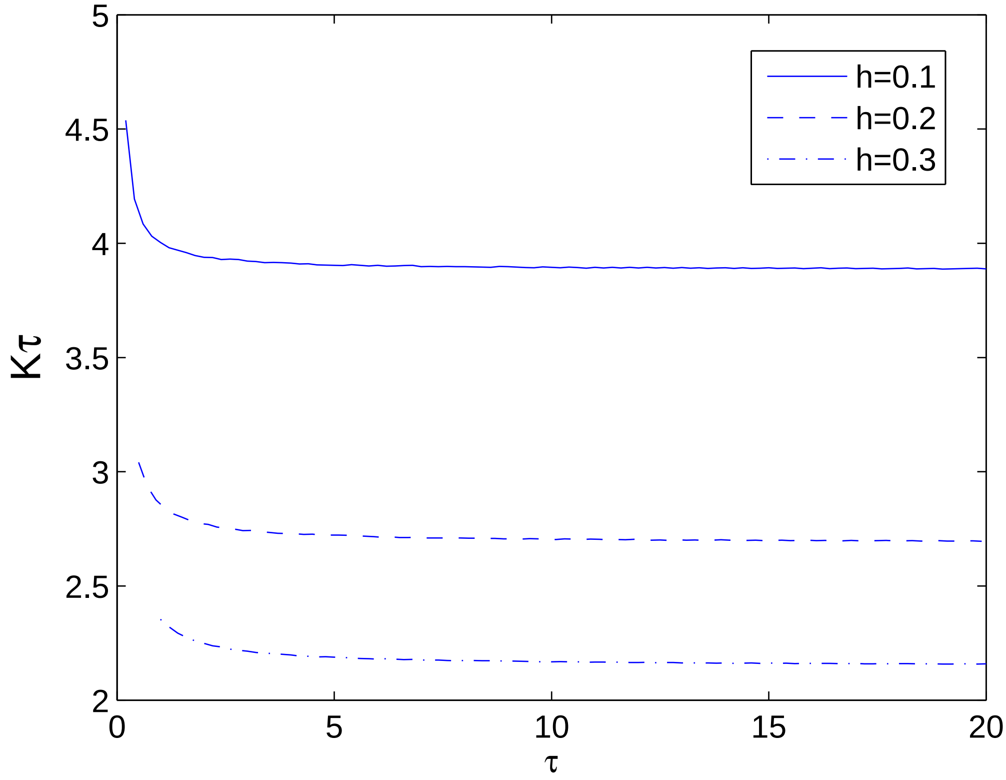

Figure 2.8: $K\tau$ versus τ

which yields $K = 0.543$, $GM = 37.214$ and $\phi_m = 2.645$. The step response of this system is as shown in Figure 2.10.

The Nyquist plots of the feedback system using the designed controller (under the choice of K defined above) for $\tau = 1$ and $\tau = 100$ are shown in Figures 2.11 and 2.12. Both of the Nyquist plots encircle -1 once in the counterclockwise direction, therefore feedback systems are stable. As illustrated in Figures 2.11 and 2.12, GM increases with increasing τ, because the magnitude of the negative

τ	ϕ_m (in degrees)	GM
1	18.680	2.196
5	34.913	2.600
10	37.214	2.645
50	39.103	2.683
100	39.333	2.683

Table 2.1: Maximum Gain Margin and Phase Margin Results For Corresponding τ values

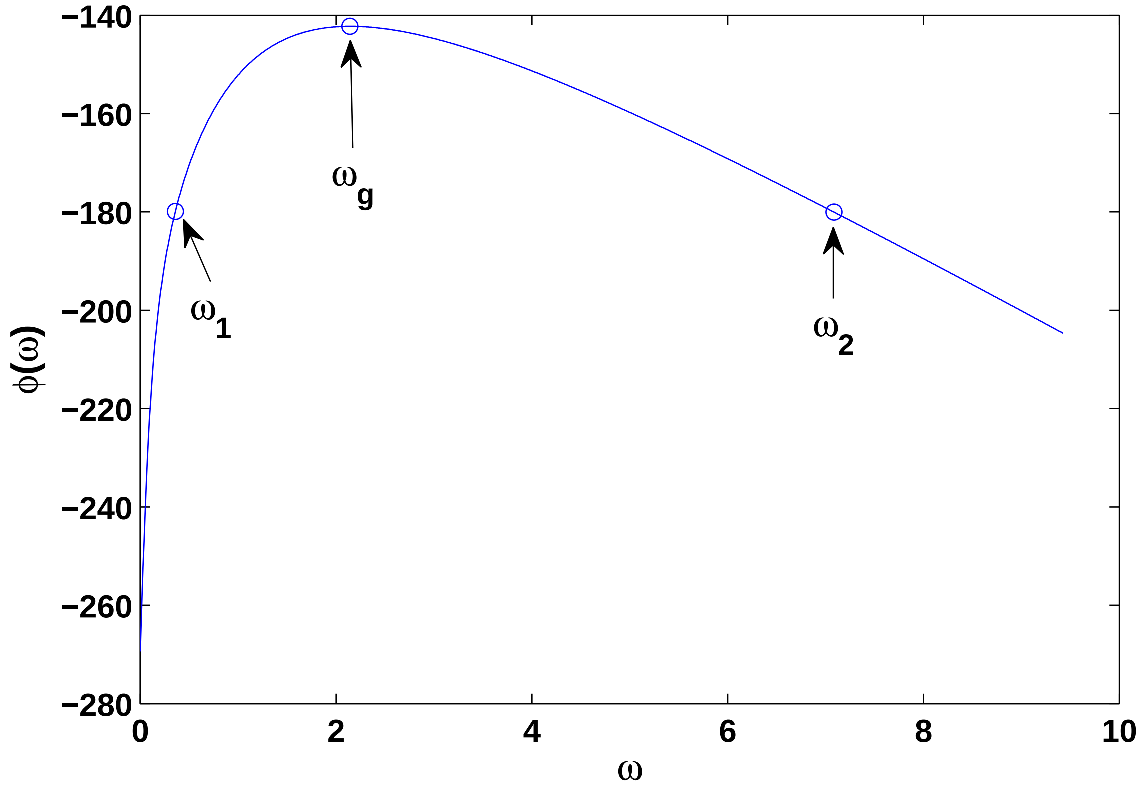

Figure 2.9: A typical phase $\phi(\omega)$ versus frequency ω graph

real axis crossing to the left of -1 increases and the angle of the point which Nyquist plot intersects the unit circle increases with increasing τ, therefore phase margin increases.

The problem of optimization of gain and phase margins in a blended fashion can be solved with a P-type controller, because the parameters of the desired controller in (2.4) are $\tau \to \infty$ and $K \to 0$ with $K\tau$ finite. Therefore, as in the gain margin maximization and phase margin maximization problems, the feedback system is stable due to the proportional control but the tracking performance to a step-like reference signal is not good due to the lack of integral action.

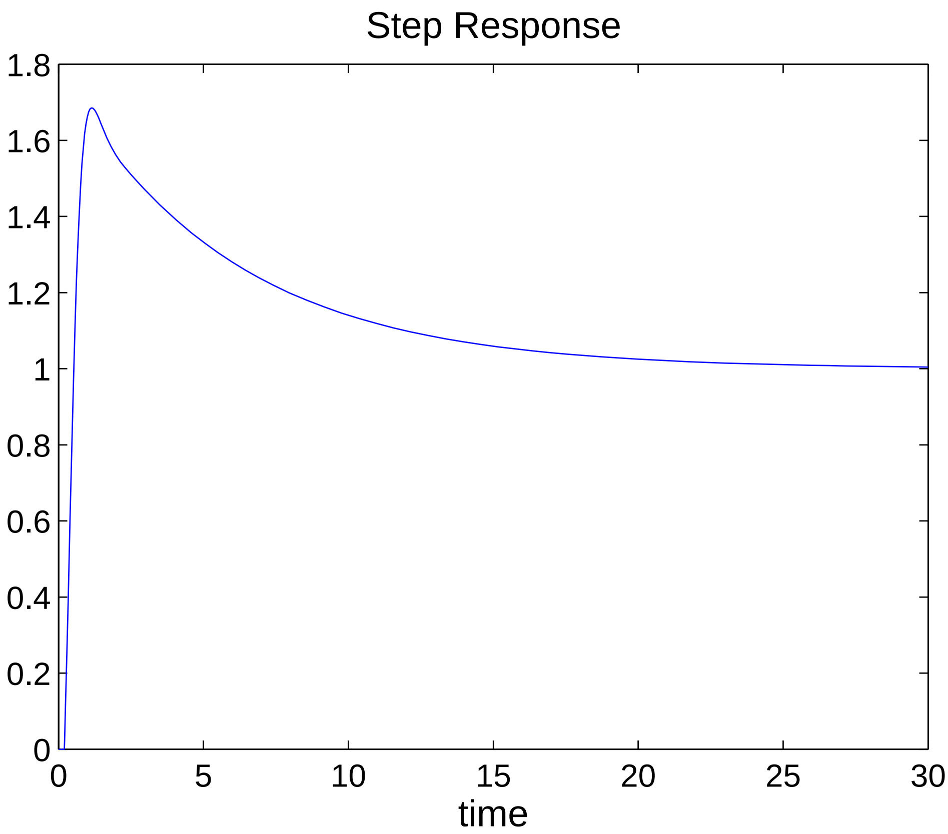

Figure 2.10: Step response of the obtained system

2.4 Cost Function Minimization

Solutions to gain and phase margin maximization problems yields a P-type controller, which is insufficient to obtain a good transient performance. Hence, by defining a cost function to minimize, we tried to put a bound on τ in (2.4) such that the designed controller is a PI-type controller and due to integral control, the feedback system provides a good tracking performance to step-like reference signals.

The cost function, denoted by J, is defined as follows,

$$J = (c\beta + e^{\alpha})^{-1} \tag{2.27}$$

where α and β are defined as,

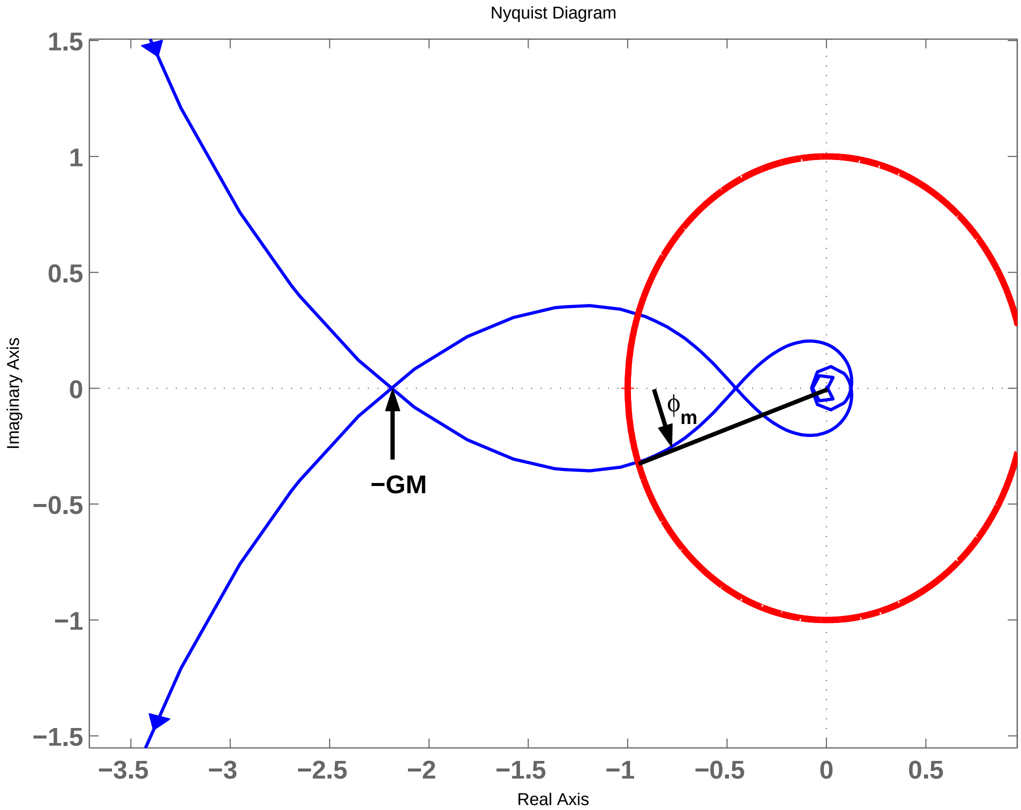

Figure 2.11: The Nyquist Plot of the Feedback System for $\tau = 1$

$$\frac{1}{\beta} = \|S\|_{\infty} \tag{2.28}$$

$$\frac{1}{\alpha} = ||W(s)S(s)||_{\infty} \tag{2.29}$$

Here, S is the sensitivity function defined in (2.11), β is vector margin which is defined as the distance of $G(j\omega)$ from -1. For each fixed τ, the gain crossover frequency satisfying stability conditions ($\phi_m > 0$) and optimal K in (2.22) are found by evaluating $G(j\omega)$ over a frequency range. From these values, vector margin is evaluated by taking supremum over ω of the sensitivity function and α is found by taking supremum over ω of the weighting function $W(s) = \frac{1}{s}$ times the sensitivity function S.

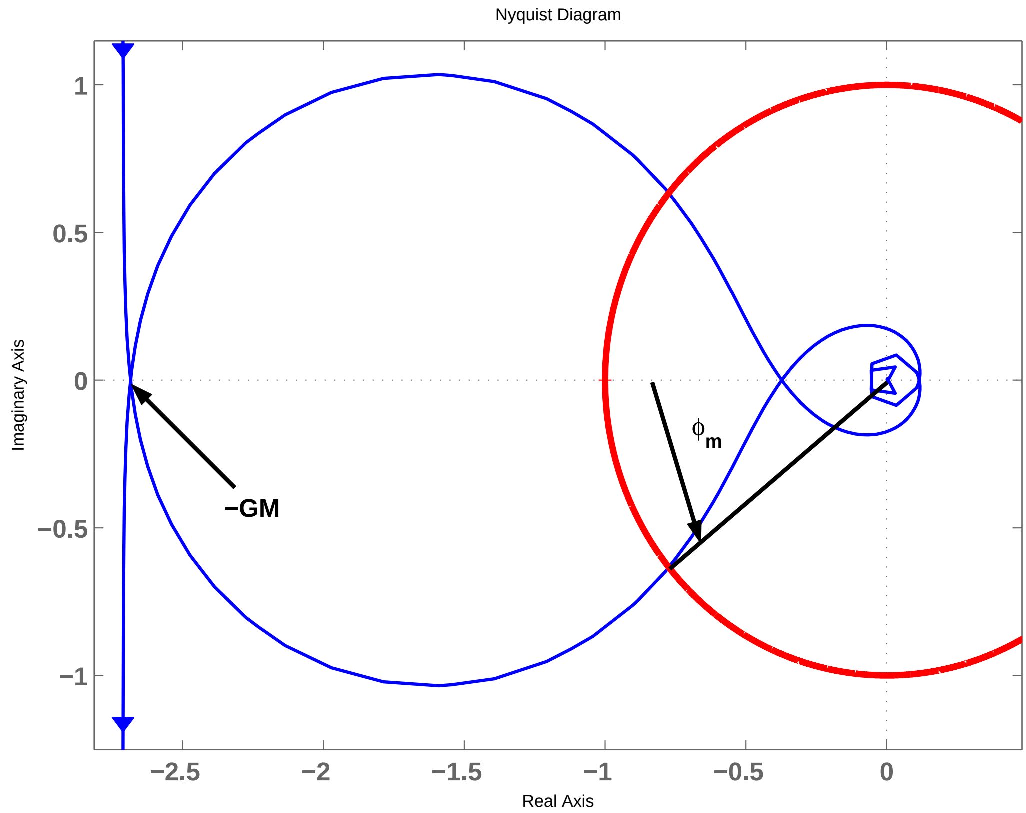

Figure 2.12: The Nyquist Plot of the Feedback System for $\tau = 100$

For the generic plant, the variation of β and α over different τ values is as shown in Figures 2.13 and 2.14. As τ increases, vector margin increases and α increases up to a point then decreases. In these figures, τ is greater than a certain value for each time delay $h > 0$, because below these values the system becomes unstable.

Then, by combining α and β with $c = 1$ such that

$$J = (e^{\alpha} + \beta)^{-1}.$$

For the generic plant with $h = 0.2$, the variation of J versus τ is as shown in Fig. 2.15.

The optimum values of parameters satisfying the specified cost minimization problem is $\tau = 2.24$, maximum gain margin $GM = 2.48$, maximum phase margin

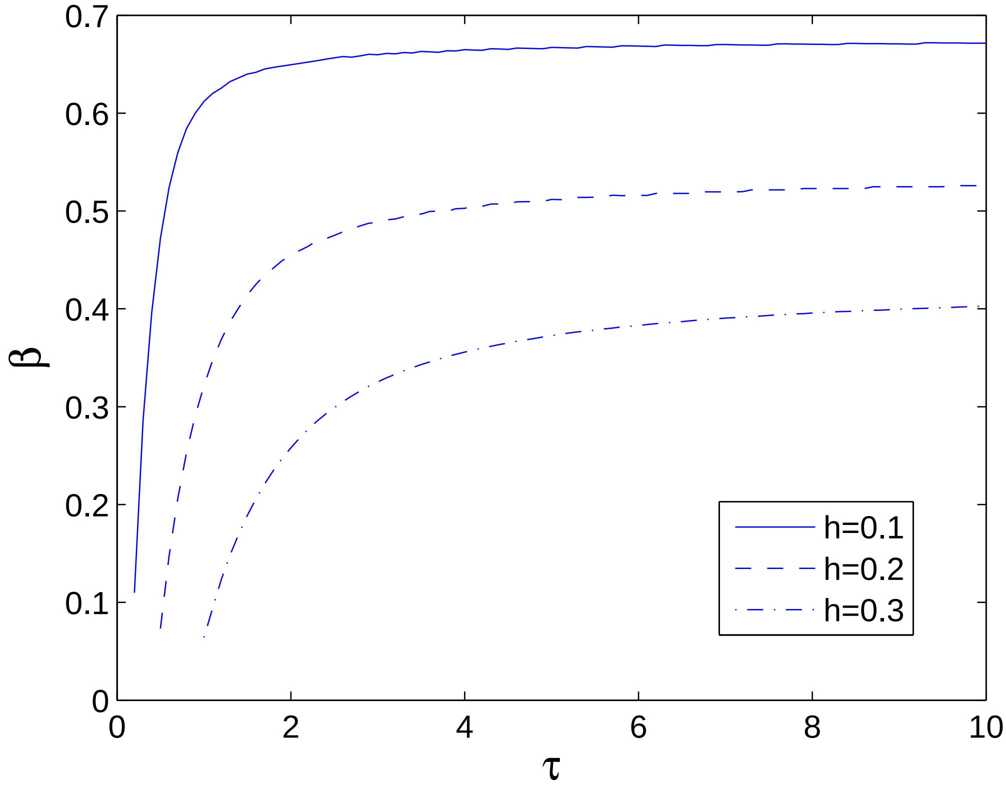

Figure 2.13: Vector Margin β versus τ Graph

obtained $\phi_m = 29.41$ degrees and optimal $K = 1.23$. For the same system, the maximum gain and phase margins with changing c values are indicated in Table 2.2.

c	ϕ_m (deg.)	GM	Optimal τ	Optimal K
0.05	38.25	2.66	17.78	0.15
0.06	34.92	2.60	5.01	0.54
0.07	30.46	2.52	2.51	1.09
0.1	30.46	2.52	2.51	1.09
0.2	29.41	2.48	2.24	1.23
0.3	29.41	2.48	2.24	1.23
1	29.41	2.48	2.24	1.23

Table 2.2: Maximum Gain Margin GM and Phase Margin ϕ_m Results For Corresponding c

The cost function does not have a minimum point for $c > 40$. The step response of this system with $c = 1$ is as shown in Figure 2.16. We can obtain the same results if we design a controller using the gain and phase margin results

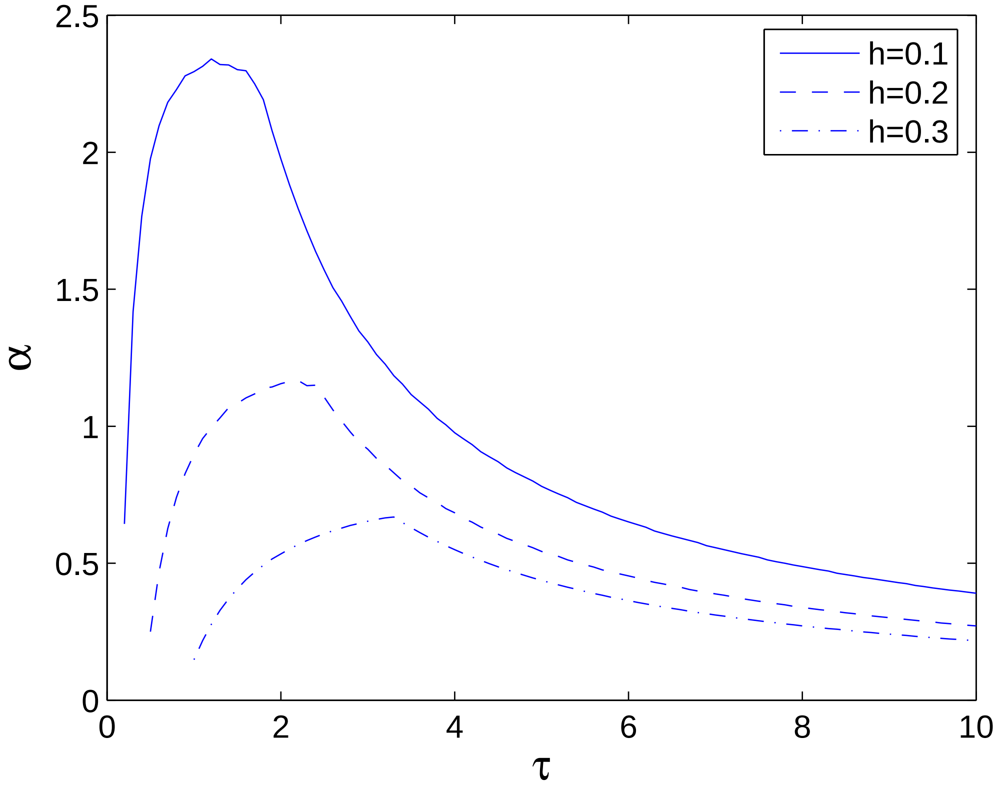

Figure 2.14: α versus τ Graph

obtained in Table 2.1 with the design methods given in [14]. If we compare our design with $c = 1$ and the design in [14] under 3 dB gain margin and 30 degrees phase margin specifications, we observe that the step responses of these designs which are illustrated in Figure 2.16 and the Figure 3 in [14] are similar. That means we obtained a step response with about 83% overshoot and 4 seconds of settling time in our design. Likewise, they obtained approximately 90% overshoot and 5 seconds of settling time. In addition, [14] showed that this design gives better results than the designs given in [5], [36] and [25].

If we compare the results obtained in Sections 2.3 and 2.4, the system designed in Section 2.3 has higher gain and phase margins, but the settling time of the system designed in Section 2.4 is lower which means this system has a better transient performance. In addition, the overshoot of the step response in Figure 2.16 is higher than the step response in Figure 2.10.

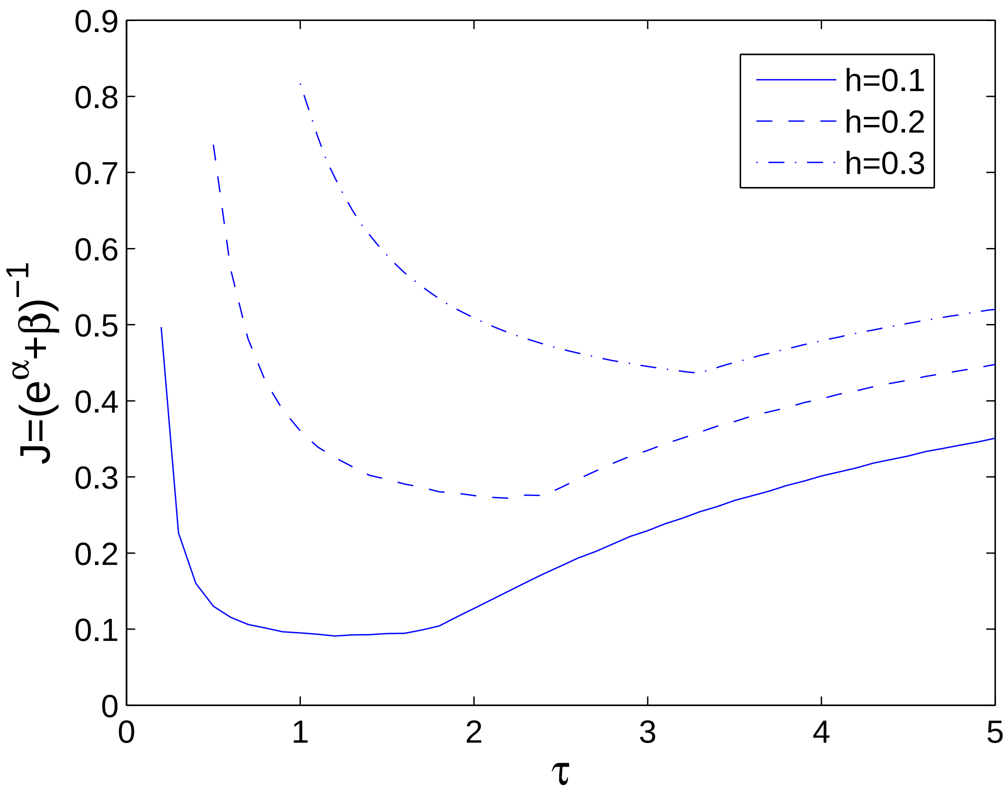

Figure 2.15: Cost function J versus τ Graph

2.5 Transient Response Optimization

Another method to improve the transient response is to adjust the dominant poles of the closed-loop system in order to obtain fast tracking of the reference signal. If the poles of the closed-loop system are denoted as p_i's, then we define a parameter related to the settling time as follows:

$$\sigma_s^{-1} = (\max_{p_i}\{Re\{p_i\}\})^{-1} \tag{2.30}$$

We can minimize the σ_s^{-1} parameter by placing the dominant poles far away from the imaginary axis. The aim in this section is to choose the controller parameters in order to place the dominant poles of the closed-loop system away from the imaginary axis.

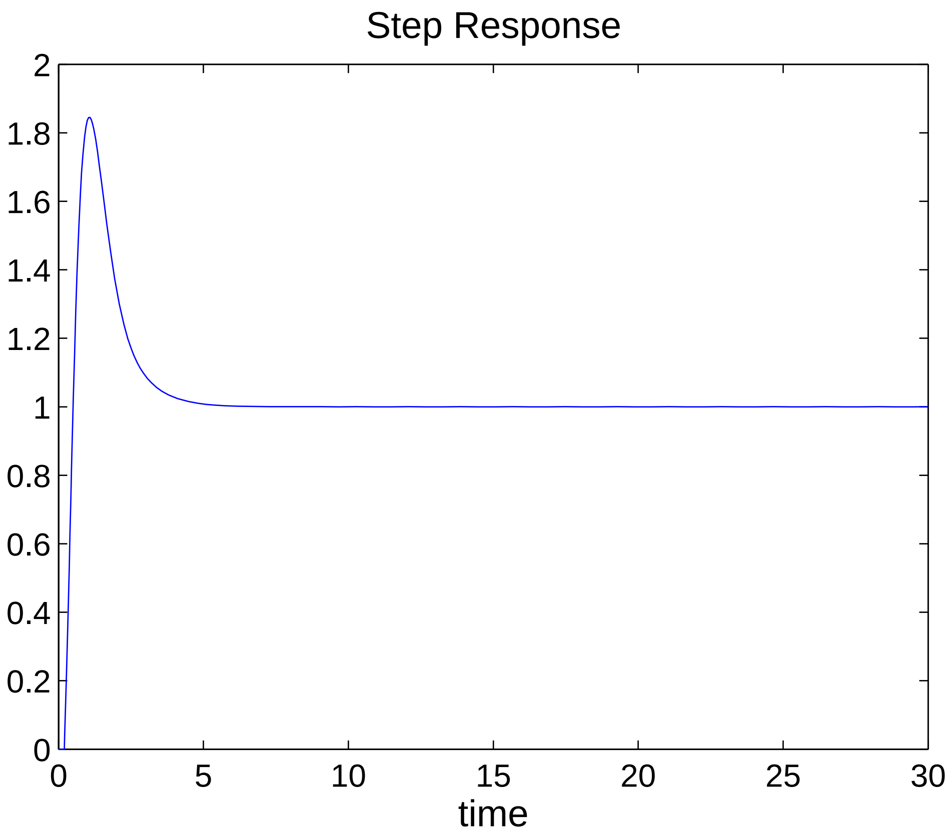

Figure 2.16: Step response of the system when $c = 1$

For the generic plant in (1.2) with $h = 0.2$, when we choose optimal K to maximize both gain and phase margins, the root locus of the closed-loop system is as shown in Fig. 2.17.

As we can see from 2.17, there are infinitely many complex conjugate poles due to the time delay and a real pole. For small τ values, the real pole is very close to origin and for increasing τ values, the real pole moves away from the origin. In contrast, the complex conjugate poles move towards the imaginary axis as τ is increasing. Initially, the real pole is the dominant pole and it moves away from the imaginary axis with the increasing τ. After a certain τ value, one of the complex conjugate poles become dominant which move toward the imaginary axis with the increasing τ. Therefore, the settling time decreases up to the certain τ value for each time delay and then increases, which is illustrated in Figure 2.18.

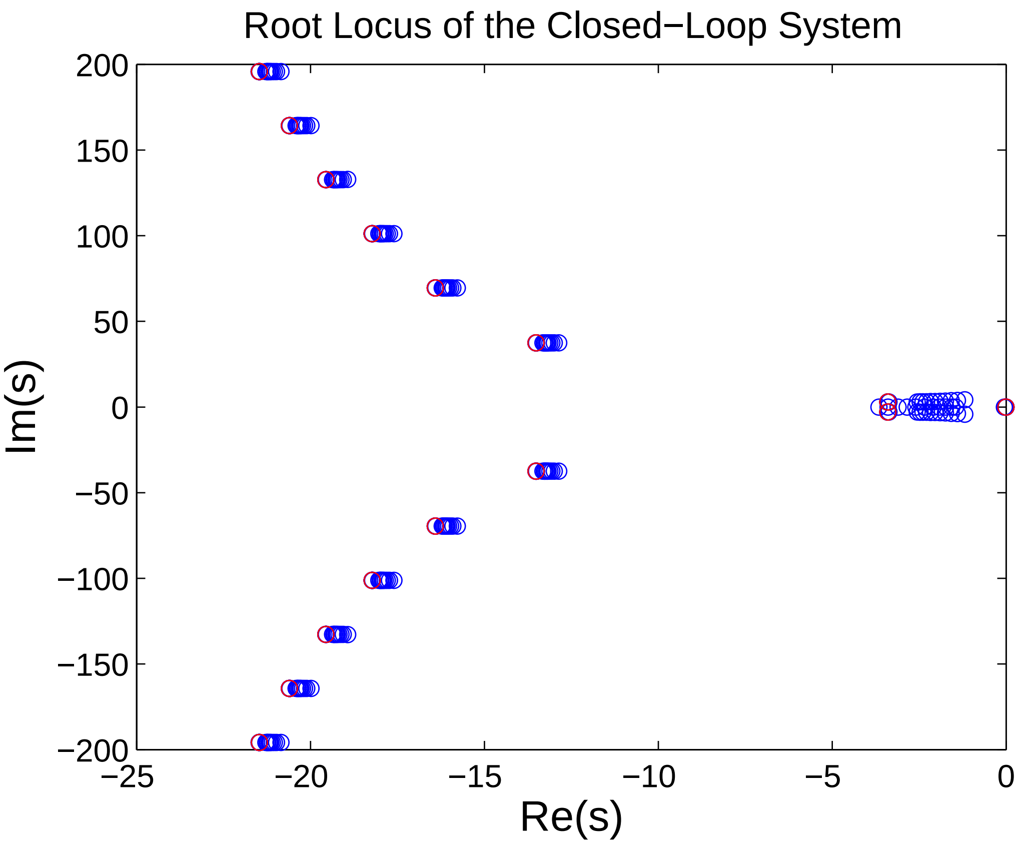

Figure 2.17: The Root Locus of the Closed-Loop System

For $h = 0.2$, the minimum σ_s^{-1} obtained is $\sigma_s^{-1} = 0.477$ with $GM = 2.242$, $\phi_m = 20.29$ degrees, $\tau = 1.1$ and $K = 2.589$. The step response of this system is as shown in Figure 2.19.

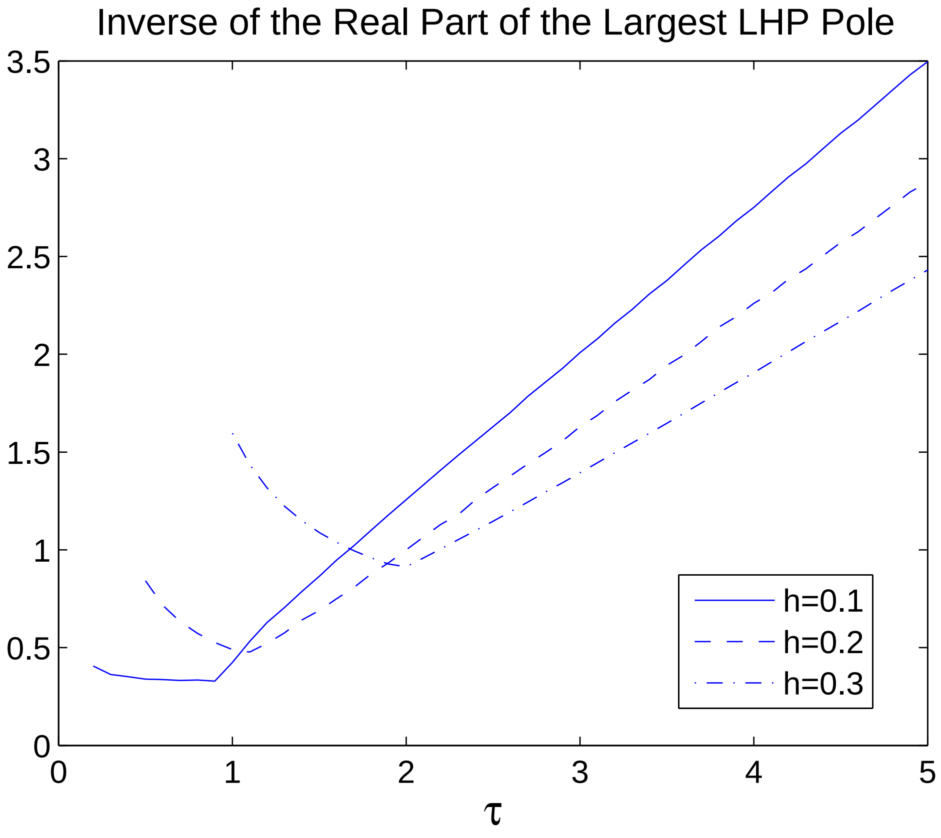

Figure 2.18: Settling time versus τ

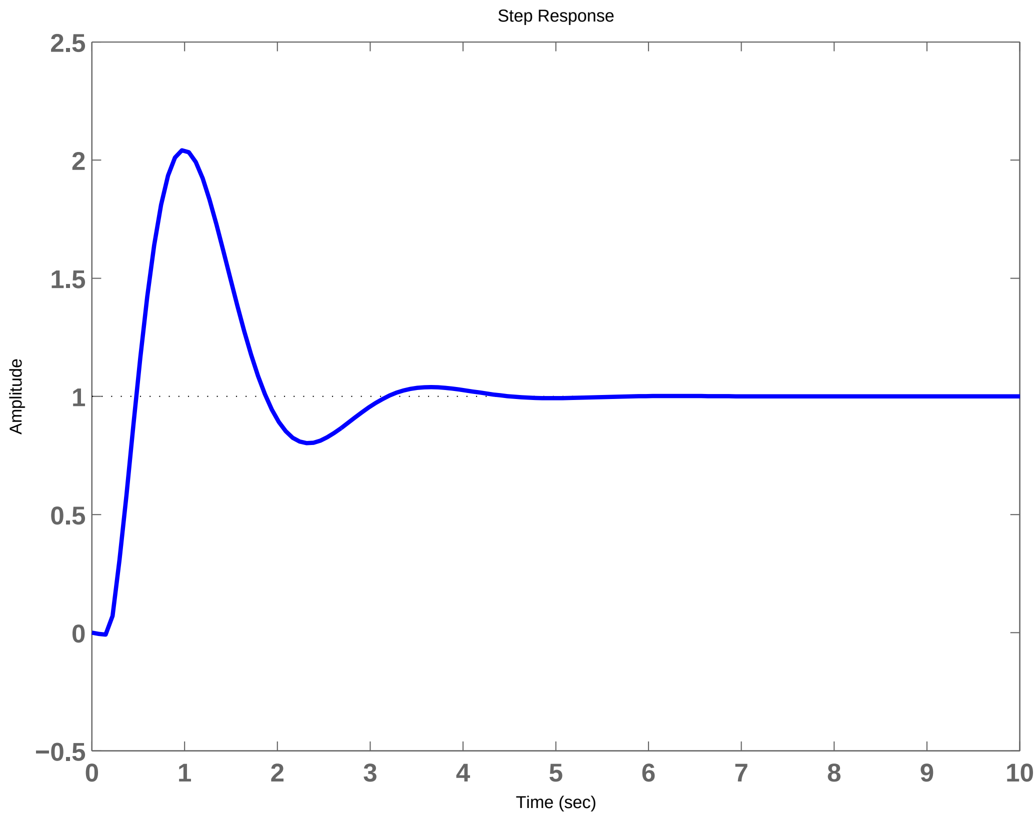

Figure 2.19: The σ_s^{-1} of the system

Chapter 3

P and PI Controller Design for A Switched System Using the LMI-based Stability Test Given in [39]

In this chapter, we first review some preliminaries from Linear Algebra.

Definition 1 (Principal Leading Minor)**.** *The k^{th} order principal leading minor of an $n \times n$ matrix $\mathcal{X}$, denoted by $|M_k|$, is the determinant of the first k rows and columns of the matrix $\mathcal{X}$.*

Fact 1. *A $n \times n$ matrix is negative definite if and only if $\forall k \in \{1, \ldots, n\}$ $(-1)^k |M_k| > 0$, where M_k's are the principal leading minors of the matrix.*

Fact 2. *Consider a second order polynomial with coefficients a, b and c. ($P(x) = ax^2 + bx + c$)*

- *$\frac{c}{a}$ is the multiplication of the roots $P(x) = 0$.*

- $-\frac{b}{a}$ *is the sum of the roots* $P(x) = 0$.

- *If the discriminant of the polynomial* ($\Delta = b^2 - 4ac$) *is negative and* $a > 0$*, then the polynomial is always positive for all* x.

- *If the discriminant of the polynomial* ($\Delta = b^2 - 4ac$) *is positive and* $a > 0$*, then the polynomial is intersects the x-axis and becomes negative for some* x.

The linear matrix inequality (LMI) based stability test derived in [39] for switched time delay systems is stated as follows.

Lemma 1. *For each fixed time delay system of the form (1.5), the triplet defined as,*

$$\Sigma : (A_\theta, \bar{A}_\theta, h_\theta) \in \mathbf{R}^{n \times n} \times \mathbf{R}^{n \times n} \times \mathbf{R}^{+} \tag{3.1}$$

is asymptotically stable dependent of delay if the following inequality holds.

$$\mathcal{X} := \begin{bmatrix} \Lambda_\theta & P_\theta \bar{A}_\theta M_\theta \\ M_\theta^T \bar{A}_\theta^T P_\theta & -R_\theta \end{bmatrix} < 0 \tag{3.2}$$

where

$$\begin{aligned} &\Lambda_\theta = (A_\theta + \bar{A}_\theta)^T P_\theta + P_\theta (A_\theta + \bar{A}_\theta) + \tau_\theta p_\theta (\alpha_\theta + \beta_\theta) P_\theta, \\ &M_\theta = [A_\theta \quad \bar{A}_\theta], \\ &R_\theta = diag(\alpha_\theta P_\theta, \beta_\theta P_\theta), \\ &\alpha_\theta > 0, \quad \beta_\theta > 0, \quad p_\theta > 1 \quad are \quad scalars \quad and \\ &P_\theta \in \mathbf{R}^{n \times n} \quad is \quad a \quad symmetric \quad positive \quad definite \quad matrix. \end{aligned} \tag{3.3}$$

The variables α_θ, β_θ, p_θ and P_θ are the decision variables of the inequality given in (3.2). If any three of these variables are fixed, this inequality becomes an LMI with the fourth decision variable. Initially, the aim is to find a feasible $(\alpha_\theta, \beta_\theta, p_\theta, P_\theta)$ set which ensures stability of a candidate system by satisfying Lemma 1. In order to satisfy this lemma, we do analytical derivations using the inequality (3.2) and find feasible intervals for each variable and controller parameter which ensures feedback system stability. All the feasible intervals for decision variables (which are $\alpha_\theta, \beta_\theta, p_\theta, P_\theta$) and the controller parameters defined in (1.1) (which are $K_{p\theta}, K_{i\theta}, K_{d\theta}, \tau_{d\theta}$) form the parameter space.

For stability of the switched system, an upper bound for the dwell time τ derived in [39] is given as follows,

$$\tau := T_d + 2h_{max}, \qquad h_{max} = \max_{i \in \mathcal{F}} \{h_i\} \tag{3.4}$$

where

$$T_d \leq \mu_d = \max_{i \in \mathcal{F}} \frac{1}{\sigma_{min}(S_i)} \tag{3.5}$$

$$\begin{aligned} S_i = & - \{(A_i + \bar{A}_i) + (A_i + \bar{A}_i)^T + h_i \alpha_i^{-1} \bar{A}_i A_i A_i^T \bar{A}_i^T \\ & + h_i \beta_i^{-1} (\bar{A}_i)^2 (\bar{A}_i^T)^2 + h_i p_i (\alpha_i + \beta_i)\}. \end{aligned} \tag{3.6}$$

Using the dwell time expression given in (3.4), the minimum of the dwell time inside this parameter space is searched by the developed MATLAB scripts which are given in Appendix.

3.1 Proportional Control

In this section, the analysis of the system with proportional controller is given in details. A candidate system defined in (1.5) with proportional controller can explicitly be expressed as follows.

$$u(t) = K_\theta e(t) = -K_\theta y(t)$$

$$y(t) = x(t)$$

$$\dot{x}(t) = a_\theta x(t) + u(t - h_\theta) = a_\theta x(t) - K_\theta x(t - h_\theta) \tag{3.7}$$

Since the plant and the controller are first order, the coefficients of state space equations are scalars which are the following.

$$A_\theta = a_\theta, \quad \bar{A}_\theta = -K_\theta \quad and \quad C_\theta = 1 \tag{3.8}$$

Lemma 1 and Eqn. 3.6 are subsequently constructed to investigate the asymptotic stability of the switched system as follows.

$$\mathcal{X} := \begin{bmatrix} 2h^{-1}(a_i - K_i) + p_i(\alpha_i + \beta_i) & -a_i K_i & K_i^2 \\ -a_i K_i & -\alpha_\theta & 0 \\ K_\theta^2 & 0 & -\beta_\theta \end{bmatrix} < 0 \tag{3.9}$$

$$S_\theta := -\{2(a_\theta - K_\theta) + h_\theta[(\alpha_\theta^{-1} + \beta_\theta^{-1})a_\theta^2 K_\theta^2 + p_\theta(\alpha_\theta + \beta_\theta)]\} \tag{3.10}$$

Since all the entries of the matrix $\mathcal{X}$ given in (3.2) are scalars and P_θ multiplies each non-zero entry, the P_θ multipliers of all the entries are eliminated. The remaining problem is such that if we could find some $\alpha_\theta > 0, \beta_\theta > 0$ and $p_\theta > 1$ values satisfying (3.9), then asymptotic stability of the candidate system is guaranteed. In addition, along with the candidate system stability, if all of the switching intervals between consequent switchings are longer than the dwell time obtained, then stability of the overall switched system is guaranteed. Therefore, our aim is to find an appropriate proportional constant K_θ value for each candidate system such that (3.9) is satisfied and at the same time the dwell time defined in (3.4) is minimized.

Using Fact 1, negative definiteness of $\mathcal{X}$ is investigated as follows:

- The determinant of the first leading principal minor has to be negative. (i.e $|M_1| < 0$)

$$\begin{aligned} & 2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta(\alpha_\theta + \beta_\theta) < 0 \\ \Rightarrow \quad & 0 < p_\theta(\alpha_\theta + \beta_\theta) < -2h_\theta^{-1}(a_\theta - K_\theta) \\ \Rightarrow \quad & K_\theta > a_\theta \end{aligned} \tag{3.11}$$

Since α_θ, β_θ and p_θ are positive, $p_\theta(\alpha_\theta + \beta_\theta)$ is positive and consequently $K_\theta > a_\theta$.

- The determinant of the second principal leading minor has to be positive. (i.e $|M_2| > 0$)

$$\begin{aligned} & -\alpha_\theta(2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta(\alpha_\theta + \beta_\theta)) - a_\theta^2 K_\theta^2 > 0 \\ \Rightarrow \quad & p_\theta \alpha_\theta^2 + (2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta \beta_\theta)\alpha_\theta + a_\theta^2 K_\theta^2 < 0 \end{aligned} \tag{3.12}$$

If we denote left hand side of the Eqn. (3.12) with the polynomial $P(\alpha) = a\alpha^2 + b\alpha + c$ that is equivalent to a parabola in 2D, we know that $a > 0$, which means the parabola is turned upwards as shown in Fig. 3.1 and $c > 0$, which means multiplication of the roots of $P(\alpha)$ are positive.

Since, α_θ could only take positive values, the sum of the roots of the polynomial has to be positive, therefore $b < 0$.

$$2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta \beta_\theta < 0$$

Moreover, if $a > 0$ and the discriminant of the polynomial is negative, then $P(\alpha)$ always takes positive values. Therefore, it could take negative values if and only if the discriminant of the polynomial is positive and equivalently, if the polynomial has two real roots as shown in Figure 3.1.

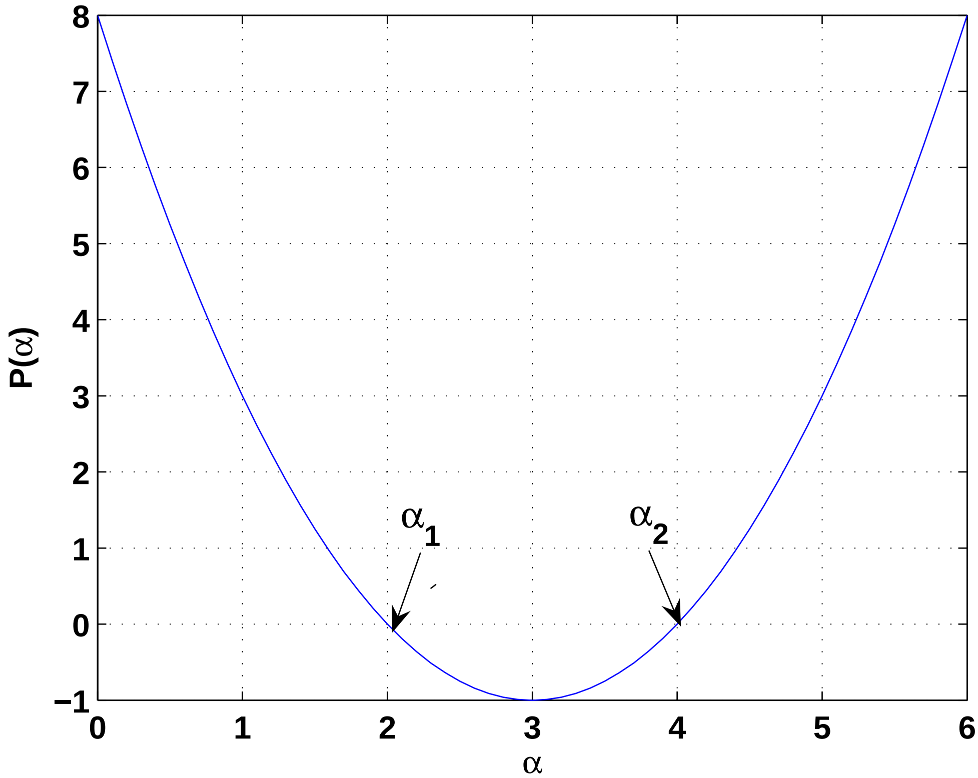

Figure 3.1: A Typical Parabola of $P(\alpha) = a\alpha^2 + b\alpha + c$ with $a > 0$

The discriminant of the polynomial in (3.12) can be expressed with the left hand side of following inequality.

$$
\begin{aligned}
(2h_\theta^{-1}(a_\theta - K_\theta) &+ p_\theta\beta_\theta)^2 - 4p_\theta a_\theta^2 K_\theta^2 > 0 \\
&\Rightarrow (2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta\beta_\theta - 2\sqrt{p_\theta} a_\theta K_\theta) \times \\
&\quad\; (2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta\beta_\theta + 2\sqrt{p_\theta} a_\theta K_\theta)) > 0
\end{aligned}
$$

The second inequality is the expanded version of the first inequality using $A^2 - B^2 = (A-B)(A+B)$. We know that $2h_\theta^{-1}(a_\theta - K_\theta) + p_\theta\beta_\theta < 0$ and if a positive term is subtracted from both sides, left hand side remains negative. Therefore, the first multiplier of the expanded inequality is negative. To ensure the positiveness of the product, the second multiplier has to be negative too and consequently we can deduce the inequality below for β_θ.

$$\beta_\theta < \frac{2h_\theta^{-1}(K_\theta - a_\theta) - 2\sqrt{p_\theta}a_\theta K_\theta}{p_\theta} \tag{3.13}$$

Since $\beta_\theta > 0$, we can find an interval for p_θ, which satisfies (3.9) as follows;

$$p_\theta < \left(\frac{K_\theta - a_\theta}{h_\theta a_\theta K_\theta}\right)^2 \tag{3.14}$$

and using the inequality $p_\theta > 1$, the following feasible interval for the time delay is obtained.

$$h_\theta < \frac{K_\theta - a_\theta}{a_\theta K_\theta} \tag{3.15}$$

- The determinant of the third principal leading minor has to be negative. (i.e $|M_3| < 0$)

$$\begin{aligned} &\left(p_\theta(\alpha_\theta + \beta_\theta) + \frac{2}{h_\theta}(a_\theta - K_\theta)\right)\alpha_\theta\beta_\theta + a_\theta^2K_\theta^2\beta_\theta + \alpha_\theta K_\theta^4 < 0 \\ \Rightarrow\quad & p_\theta(\alpha_\theta + \beta_\theta) + \frac{a_\theta^2K_\theta^2}{\alpha_\theta} + \frac{K_\theta^4}{\beta_\theta} + \frac{2}{h_\theta}(a_\theta - K_\theta) < 0 \end{aligned} \tag{3.16}$$

Obviously, the negative quantity obtained in Eqn. (3.16) is equal to

$$\frac{-S_\theta}{h_\theta} = \frac{-1}{h_\theta T_{d_\theta}} \tag{3.17}$$

which is defined in (3.5). Therefore, the dwell time for first order systems is formulated as follows:

$$T_{d_\theta} = \frac{-1}{2(a_\theta - K_\theta) + h_\theta\alpha_\theta^{-1}a_\theta^2K_\theta^2 + h_\theta\beta_\theta^{-1}K_\theta^4 + h_\theta p_\theta(\alpha_\theta + \beta_\theta)} \tag{3.18}$$

The following algorithm is developed for finding the minimum dwell time. Given a_θ and h_θ satisfying

$$0 < a_\theta h_\theta < 1$$

1. Fix p in the interval $\left(1, \frac{1}{a_\theta^2 h_\theta^2}\right)$

2. Fix $K_\theta \in \left(\frac{a_\theta}{1-a_\theta h_\theta \sqrt{p_\theta}}, \infty\right)$

3. Search upon $\alpha_1 < \alpha_\theta < \alpha_2$ and $0 < \beta_\theta < \beta_{max}$ variables to find positive T_{d_θ}'s satisfying Eqn. (3.18), where

$$\alpha_{1,2} = \left(\frac{K_\theta - a_\theta}{h_\theta p_\theta} - \frac{\beta_\theta}{2}\right) \pm \sqrt{\left(\frac{K_\theta - a_\theta}{h_\theta p_\theta} - \frac{\beta_\theta}{2}\right)^2 - \frac{a_\theta^2 K_\theta^2}{p_\theta}} \quad (3.19)$$

$$\beta_{max} = 2\left(\frac{K_\theta(1 - a_\theta h_\theta \sqrt{p_\theta}) - a_\theta}{h_\theta p_\theta}\right) \quad (3.20)$$

4. After each of the above search is completed for fixed p_θ and K_θ, the minimum value for T_{d_θ} is held and search is continued with the whole available space of parameters p_θ and K_θ. Finally, find the global minimum among held T_{d_θ} values.

Let us illustrate the algorithm on an example. For the generic plant of the form (1.2) where the location of the unstable pole is at +1, the point in the parameter space which the minimum dwell time is obtained is as shown in Table 3.1 in details. Since we focus on each candidate non-switched plant, the subscript θ's are dropped.

h	τ	p	K	β	α
0.01	0.0860	1.01	9.6043	93.1153	9.4450
0.05	0.2471	1.01	8.1322	64.5463	8.0566
0.1	0.6988	1.01	4.1823	17.6770	3.8344
0.12	1.1055	1.01	3.1875	10.2178	3.1542
0.15	3.2442	1.01	3.2279	10.3012	3.0392
0.16	7.4380	1.01	2.2218	4.8631	2.0975
0.165	14.6192	1.01	2.2290	5.0000	2.3306
0.17	91.0673	1.01	2.4721	6.0825	2.3979
0.1705	431.3492	1.01	2.4728	6.0610	2.3952

Table 3.1: The minimum dwell time τ with changing time delay h

As we can see from Table 3.1, the minimum dwell time increases with the increasing time delay h. If h is increased further, we could not find a finite dwell time and corresponding $\alpha > 0$, $\beta > 0$ or $p > 1$ satisfying Eqn. (3.9). The variation of the dwell time τ versus time delay h is as shown in Figure 3.2.

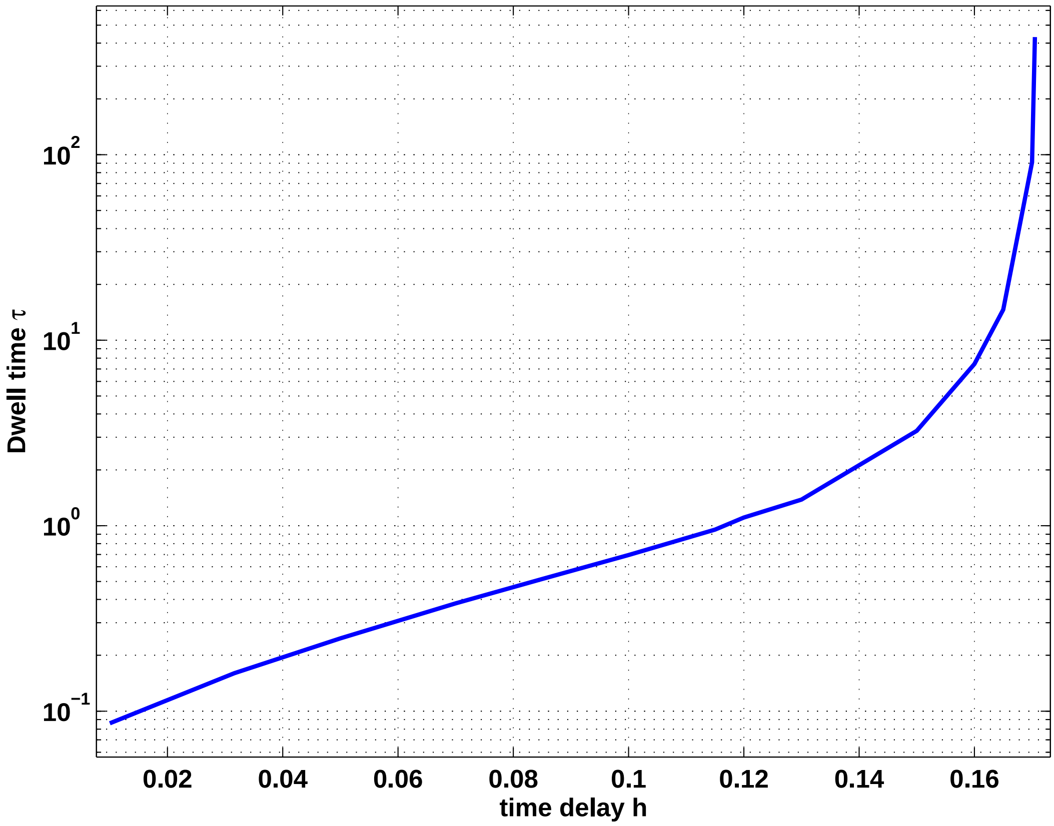

Figure 3.2: The minimum dwell time τ with changing time delay h

3.2 Proportional-Integral Control

The controller is of the following form for proportional-integral type controllers:

$$C_\theta(s) = K_{p\theta} + \frac{K_{i\theta}}{s} \tag{3.21}$$

where $K_{p\theta}$ is the proportional constant and $K_{i\theta}$ is the integral constant. A closed-loop state-space representation of PI controller of the form (3.21) and first order delayed unstable plant of the form (1.2) can be written similarly as follows.

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_c(t) \\ \dot{x}_p(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}}_{A_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ K_{i\theta} & -K_{p\theta}\tau \end{bmatrix}}_{\bar{A}_\theta} \begin{bmatrix} x_c(t-h_\theta) \\ x_p(t-h_\theta) \end{bmatrix} \\
y(t) &= \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix}
\end{aligned} \tag{3.22}
$$

where $x_c(t)$ and $x_p(t)$ are the states of the controller and plant respectively. The sufficient condition for closed-loop system stability is to satisfy the linear matrix inequality given in Lemma 1. The controller parameters are chosen such that this LMI-based stability condition is satisfied and dwell time expression is minimized among these parameter pairs. The first 2×2 portion of the LMI can be written as follows provided that $p_\theta > 1$, $\alpha_\theta > 0$ and $\beta_\theta > 0$:

$$
\begin{bmatrix} p_\theta\left(\alpha_\theta + \beta_\theta\right) & h_i^{-1}\left(K_{i\theta} - 1\right) \\ h_i^{-1}\left(K_{i\theta} - 1\right) & 2h_i^{-1}\left(1 - K_{p\theta}\right) + p_\theta\left(\alpha_\theta + \beta_\theta\right) \end{bmatrix} < 0 \tag{3.23}
$$

In order to satisfy the negative definiteness of this matrix above, the first leading principal minor of it has to be negative which is supported by the fact 1. However, the first leading principal minor of this matrix is:

$$
M_1 = p_\theta\left(\alpha_\theta + \beta_\theta\right) > 0
$$

which is positive due to definitions of p_θ, α_θ and β_θ. Therefore, we conclude that a stabilizing PI controller could not be found using the LMI-based stability test derived in [39].

Chapter 4

PD-like Controller Design for A Switched System Using the LMI-based Stability Test Given in [39]

In this chapter, we derive sufficient conditions upon the system parameters to guarantee the switched system stability with PD-like controllers of the following form.

$$C_\theta(s) = K_{p\theta} + \frac{K_{d\theta} s}{\tau_{d\theta} s + 1} \tag{4.1}$$

Then, among this derived parameter space, the point which minimizes the dwell time is searched. Hence, the controller for each candidate system which stabilizes the feedback system and minimizes the dwell time is designed. Then, the conservativeness of this design is investigated. Lastly, we compare this design with other design methods in terms of dwell time.

A state-space representation of the closed-loop dynamics can be written as follows:

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_c(t) \\ \dot{x}_p(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} a_{c\theta} & -a_{c\theta}K_{d\theta} \\ 0 & a_\theta \end{bmatrix}}_{A_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix} \\
&+ \underbrace{\begin{bmatrix} 0 & 0 \\ -a_{c\theta} & a_{c\theta}K_{d\theta} - K_{p\theta} \end{bmatrix}}_{\bar{A}_\theta} \begin{bmatrix} x_c(t-h_\theta) \\ x_p(t-h_\theta) \end{bmatrix} \\
y(t) &= \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_\theta} \begin{bmatrix} x_c(t) \\ x_p(t) \end{bmatrix}
\end{aligned} \tag{4.2}
$$

where $x_c(t)$ and $x_p(t)$ are the states of the controller and the plant respectively. Consequently, the triplet $(A_\theta, \bar{A}_\theta, h_\theta)$ defines a candidate system of the form (4.2) from the set $\mathcal{A} := \{(A_\theta, \bar{A}_\theta, h_\theta) : i \in \mathcal{F}\}$.

In (3.6), the free parameters $p_\theta > 1, \alpha_\theta > 0$ and $\beta_\theta > 0$ are found by satisfying the LMI's of Lemma 1. A sufficient condition on asymptotic stability of the switched system is that for any switching rule, the switching intervals $[t_{j-1} \quad t_j), \quad j \in \mathcal{F}$ should be longer than the dwell time τ.

Our aim is to investigate the conditions on $K_{p\theta}, K_{d\theta}$ and $a_{c\theta} = -\tau_{d\theta}^{-1}$ for each candidate system to ensure the stability of the switched system and obtain the corresponding values of these parameters to minimize the upper bound of the dwell time, given by (3.4), (3.5) and (3.6).

First, the matrix inequality given in Lemma 1 has to be satisfied and can be expressed in terms of plant and controller parameters as follows:

$$X = \begin{bmatrix} X_{11} & X_{21} & 0 & 0 & 0 & 0 \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} \\ 0 & X_{23} & -\alpha_\theta & 0 & 0 & 0 \\ 0 & X_{24} & 0 & -\alpha_\theta & 0 & 0 \\ 0 & X_{25} & 0 & 0 & -\beta_\theta & 0 \\ 0 & X_{26} & 0 & 0 & 0 & -\beta_\theta \end{bmatrix} < 0 \tag{4.3}$$

$$\begin{aligned} X_{11} &= 2h_\theta^{-1} a_{c\theta} + p_\theta(\alpha_\theta + \beta_\theta) \\ X_{21} &= -a_{c\theta}\,(1 + K_{d\theta})h_\theta^{-1} \\ X_{22} &= 2h_\theta^{-1}\,(a_\theta - K_{p\theta} + a_{c\theta}K_{d\theta}) + p_\theta(\alpha_\theta + \beta_\theta) \\ X_{23} &= -a_{c\theta}^2 \\ X_{24} &= a_{c\theta}^2 K_{d\theta} - a_\theta\,(K_{p\theta} - a_{c\theta}K_{d\theta}) \\ X_{25} &= a_{c\theta}\,(K_{p\theta} - a_{c\theta}K_{d\theta}) \\ X_{26} &= (K_{p\theta} - a_{c\theta}K_{d\theta})^2 \end{aligned}$$

In order to satisfy the negative definiteness of the matrix X, Fact 1 is used.

$*$ The determinant of the first leading minor has to be negative. (i.e $|M_1| < 0$)

$$\begin{aligned} -\frac{2h_\theta^{-1}}{\tau_{d\theta}} &+ p_\theta(\alpha_\theta + \beta_\theta) < 0 \Rightarrow 0 < p_\theta(\alpha_\theta + \beta_\theta) < \frac{2h_\theta^{-1}}{\tau_{d\theta}} \\ &\Rightarrow 0 < \tau_{d\theta} < \frac{2h_\theta^{-1}}{p_\theta(\alpha_\theta + \beta_\theta)} \end{aligned} \tag{4.4}$$

$*$ The determinant of the second leading minor has to be positive. (i.e $|M_2| > 0$)

$$\begin{aligned} &\Rightarrow p_\theta^2\alpha_\theta^2 + 2p_\theta\left[p_\theta\beta_\theta + h_\theta^{-1}\left(a_\theta - K_{p\theta} - \frac{K_{d\theta}+1}{\tau_{d\theta}}\right)\right]\alpha_\theta \\ &+ p_\theta^2\beta_\theta^2 + 2h_\theta^{-1}p_\theta\left(a_\theta - K_{p\theta} - \frac{K_{d\theta}+1}{\tau_{d\theta}}\right) \\ &- h_\theta^{-2}\left[\frac{4}{\tau_{d\theta}}\left(a_\theta - K_{p\theta} - \frac{K_{d\theta}}{\tau_{d\theta}}\right) + \frac{(1+K_{d\theta})^2}{\tau_{d\theta}^2}\right] > 0 \end{aligned} \tag{4.5}$$

Using Fact 2, since the discriminant and the coefficient of the second order term of the polynomial in (4.5) are positive, it has two real roots. By definition α is positive and consequently multiplication of the roots of the polynomial in (4.5) is positive which means the constant term of the polynomial is positive.

$$\begin{aligned} p_\theta^2 \quad & \beta_\theta^2 + 2h_\theta^{-1} p_\theta \left(a_\theta - K_{p\theta} - \frac{K_{d\theta}+1}{\tau_{d\theta}} \right) \\ - \quad & h_\theta^{-2} \left[\frac{4}{\tau_{d\theta}} \left(a_\theta - K_{p\theta} - \frac{K_{d\theta}}{\tau_{d\theta}} \right) + \frac{(1+K_{d\theta})^2}{\tau_{d\theta}^2} \right] > 0 \end{aligned} \tag{4.6}$$

Similarly, by definition β is positive; the discriminant and the coefficient of the second order term of the polynomial in (4.6) are positive, then it has two positive real roots. Therefore, multiplication of the roots of the polynomial in (4.6) is positive which means the constant term of the polynomial is positive. Since $h_\theta > 0$ and $\tau_{d\theta} > 0$, this term can be expressed as follows:

$$4(a_\theta - K_{p\theta})\tau_{d\theta} + (1 - K_{d\theta})^2 < 0 \tag{4.7}$$

In order to satisfy the inequality (4.7), $K_{p\theta} > a_\theta$ must hold. Similarly, a bound for $K_{d\theta}$ could be found from inequality (4.7) which is as follows:

$$1 - 2\sqrt{(K_{p\theta} - a_\theta)\tau_{d\theta}} < K_{d\theta} < 1 + 2\sqrt{(K_{p\theta} - a_\theta)\tau_{d\theta}} \tag{4.8}$$

It can be shown that a P controller stabilizes a first order unstable process with time delay if and only if $a_\theta h_\theta < 1$, ([16]). Thus, the sufficient conditions upon the plant and the controller parameters are defined and the remaining problem is to find the values of these parameters in the defined intervals which minimizes the pre-defined dwell time expression. Since the expressions given are too complex to solve analytically, we tried to find the set of values of the corresponding parameters which minimizes the dwell time by a numerical search in the parameter space restricted by the inequalities derived above.

Our first assumption was that the candidate systems inside the set $\mathcal{A}$ are known, which means the plant parameters a_θ and h_θ are known. By dividing the intervals for controller parameters in (4.4), (4.7) and (4.8) into certain number of points, a set of parameters is obtained consisting of values of $(K_{p\theta}, K_{d\theta}, \tau_{d\theta})$. We tried to reach positive T_d values defined in (3.5) and store them by searching upon the variables $\alpha_\theta, \beta_\theta$ and p_θ. After the search is completed among the whole parameter space, global minimum point for T_d and the corresponding parameters are obtained.

Let us illustrate the results on an example with the plant

$$P(s) = \frac{e^{-h_\theta s}}{s-1}$$

which means the right half plane pole of the plant is set to 1 and only the delay parameter of the plant switches. Note that the plant (1.3) with an arbitrary a_θ, for any $\theta = i \in \mathcal{F}$ can be written as:

$$P_\theta(\hat{s}) = \frac{e^{-h_\theta a_\theta \hat{s}}}{\hat{s}-1} \tag{4.9}$$

where $\hat{s} = \frac{s}{a_\theta}$ is the normalized Laplace transform variable. Therefore, without the loss of generality, we can consider $a_\theta = 1$ and discuss controllers for switched parameter

$$\hat{h}_\theta = h_\theta a_\theta \tag{4.10}$$

Our numerical calculations for minimizing the upper bound of the dwell time show that the controller can be written in the following form which is valid for $h_\theta \in (0.0032, 0.155)$:

$$C_\theta(s) = \frac{R_\theta s + K_{p\theta}}{\tau_{d\theta} s + 1} \tag{4.11}$$

where $R_\theta = (\tau_{d\theta} + 1.65 + 3h_\theta)$. Note that the controller is determined by two parameters $K_{p\theta}$ and $\tau_{d\theta}$ whose values are shown in Table 4.1.

h_θ	τ	$K_{p\theta}$	$\tau_{d\theta}$
0.0032	0.0188	172	0.0155
0.01	0.0591	54.6	0.05
0.0316	0.2040	16.4	0.175
0.07	0.575	7.22	0.461
0.1	1.068	4.77	0.766
0.13	2.469	3.55	1.2
0.15	8.696	3.04	1.522
0.155	22.003	2.89	1.7

Table 4.1: The minimum dwell time τ versus delay

For small delay values, the time constant of the system is small and hence the system response is fast. Therefore, dwell time obtained is obviously small. As delay is increasing, the time constant of the system is higher which results in a slower system and hence dwell time gets larger. The parameters of the controller which are $K_{p\theta}$ and $\tau_{d\theta}$ are shown in Fig. 4.1 and 4.2. It can be seen from the figures that $K_{p\theta}$ is rapidly decreasing while $\tau_{d\theta}$ is increasing with the increasing delay.

The minimum dwell time calculated versus time delay graph is as shown in Fig. 4.3. From this figure, we can conclude that as the delay is increasing, the dwell time is increasing exponentially and for $h_\theta > 0.155$, a finite dwell time can not be found with this approach.

4.1 Conservatism Analysis and Simulations

In this section, the conservativeness of the LMI-based stability test suggested in [39] for the switched time delay system is analyzed. We search for a particular switching with the highest dwell time (the minimal time interval between consequent switchings) leading to an unstable system, where each candidate systems

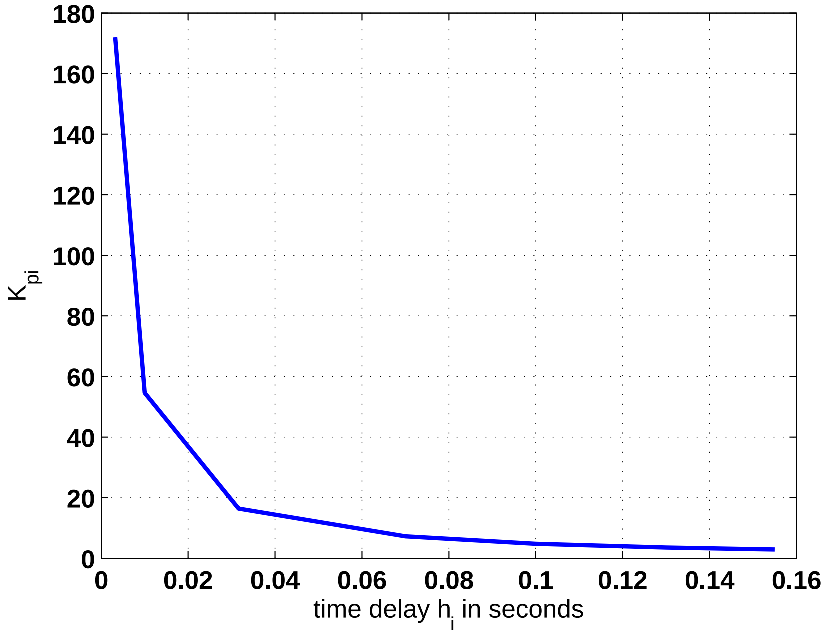

Figure 4.1: The parameters of the controller versus delay

are stable and by this way the conservativeness of the calculated value is realized. Time domain simulations and analysis are carried out in order to accomplish this goal.

The closed loop system in (4.2) is simulated in time domain with nonzero initial conditions and this simulation could not be done precisely with internal time delay. Therefore, for simplicity as the first step, the time delay of the plant is approximated by 2^{nd} order Pade approximation, as follows:

$$\begin{aligned} e^{-h\theta s}X(s) &\approx \left(\frac{1 - \frac{h_\theta}{2}s + \frac{h_\theta^2}{12}s^2}{1 + \frac{h_\theta}{2}s + \frac{h_\theta^2}{12}s^2} I \right) X(s) \\ &= (C_{d\theta} + (sI - A_{d\theta})^{-1}B_{d\theta} + D_{d\theta})X(s) \end{aligned} \tag{4.12}$$

where

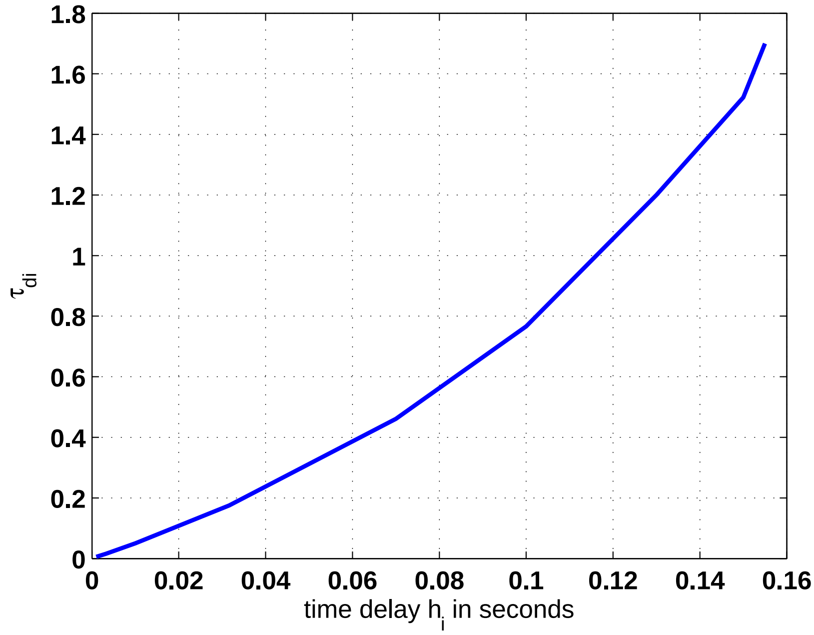

Figure 4.2: The parameters of the controller versus delay

$$
\begin{aligned}
A_{d\theta} &= \begin{bmatrix} 0 & I \\ -\frac{12}{h_\theta^2} I & -\frac{6}{h_\theta} I \end{bmatrix} \quad B_{d\theta} = \begin{bmatrix} 0 \\ I \end{bmatrix} \\
C_{d\theta} &= \begin{bmatrix} 0 & -\frac{12}{h_\theta^2} I \end{bmatrix} \quad D_{d\theta} = I
\end{aligned}
$$

Then, the time delay part is converted to state space with internal state $z(t)$ by the following equations;

$$
\begin{aligned}
\dot{z}(t) &= A_{d\theta} z(t) + B_{d\theta} x(t) \\
x(t - h_\theta) &= C_{d\theta} z(t) + D_{d\theta} x(t)
\end{aligned} \tag{4.13}
$$

and the overall switched system can be expressed as follows:

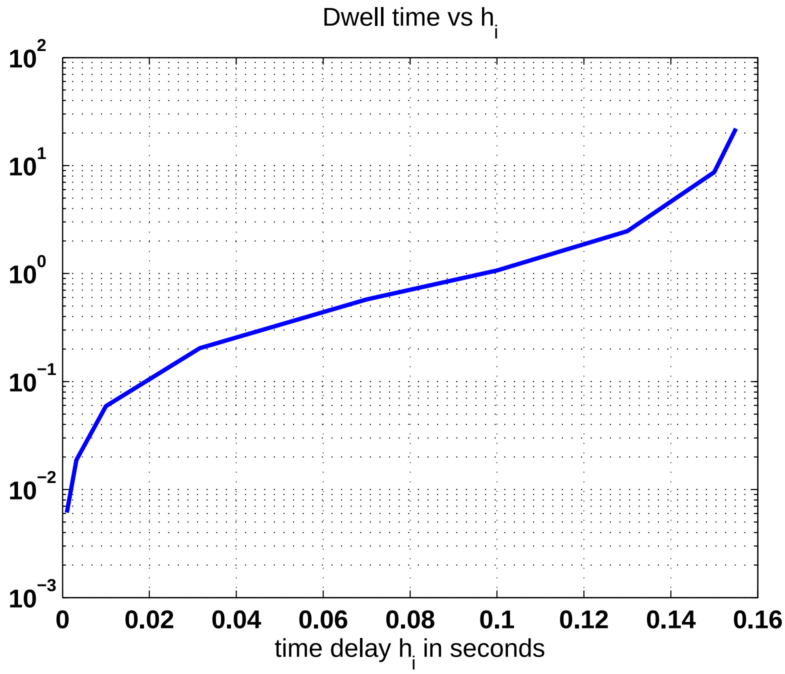

Figure 4.3: The minimum dwell time versus delay

$$\begin{bmatrix} \dot{z}(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} A_{d\theta} & B_{d\theta} \\ \bar{A}_i C_{d\theta} & (a_\theta + \bar{A}_i D_{d\theta}) \end{bmatrix} \begin{bmatrix} z(t) \\ x(t) \end{bmatrix} \tag{4.14}$$

The instability of the system can be realized from the norm of the state vector. If the norm of the states goes to infinity as time goes to infinity, then system is unstable and if the norm of the states goes to zero as time goes to infinity, then system is stable.

Two systems are selected from the set $\mathcal{A}$ and simulations are started with arbitrary initial conditions for $x(t)$ and zero initial condition for $z(t)$. At the beginning, the first system runs t_1 seconds with the specified initial conditions. When $t = t_1$, the plant and the controller are switched to the second system in the set, which then runs t_2 seconds with the states at $t = t_1$ as initial condition. This is an infinite loop, meaning that switching from one system to the other continues as time goes to infinity. Actually, the switching intervals should be arbitrary. But in this case, we applied this constant interval switching rule to find a lower bound of the dwell time.

The minimum of t_1 and t_2 values for which the system goes from instability to stability yields the dwell time. This can be illustrated on an example of the previous section. Assume the plant is $P(s) = \frac{e^{-h_\theta s}}{s-1}$, the delay parameters that construct the set of candidate plants are $h_1 = 0.01$ and $h_2 = 0.07$.

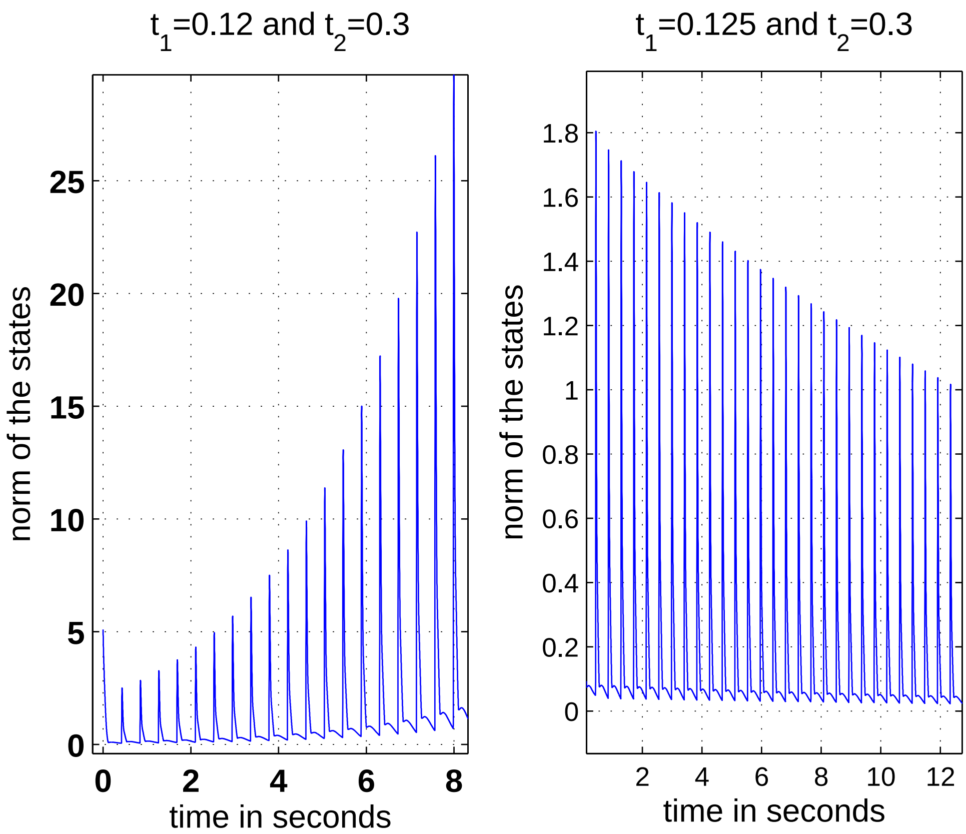

Figure 4.4: Dwell time from simulations

From Fig. 4.4, it is obvious that the graph on the left belongs to an unstable system and the graph on the right belongs to a stable system and a lower bound of the dwell time is between 0.12 and 0.125 seconds for this example. Whereas the computed dwell time from [39] is 0.575.

The difference between the dwell time from calculation and simulation could be due to the Pade approximation or the conservativeness of the LMI-based analysis. Therefore, we have investigated the role of the Pade approximation by increasing the Pade order and applying the same process.

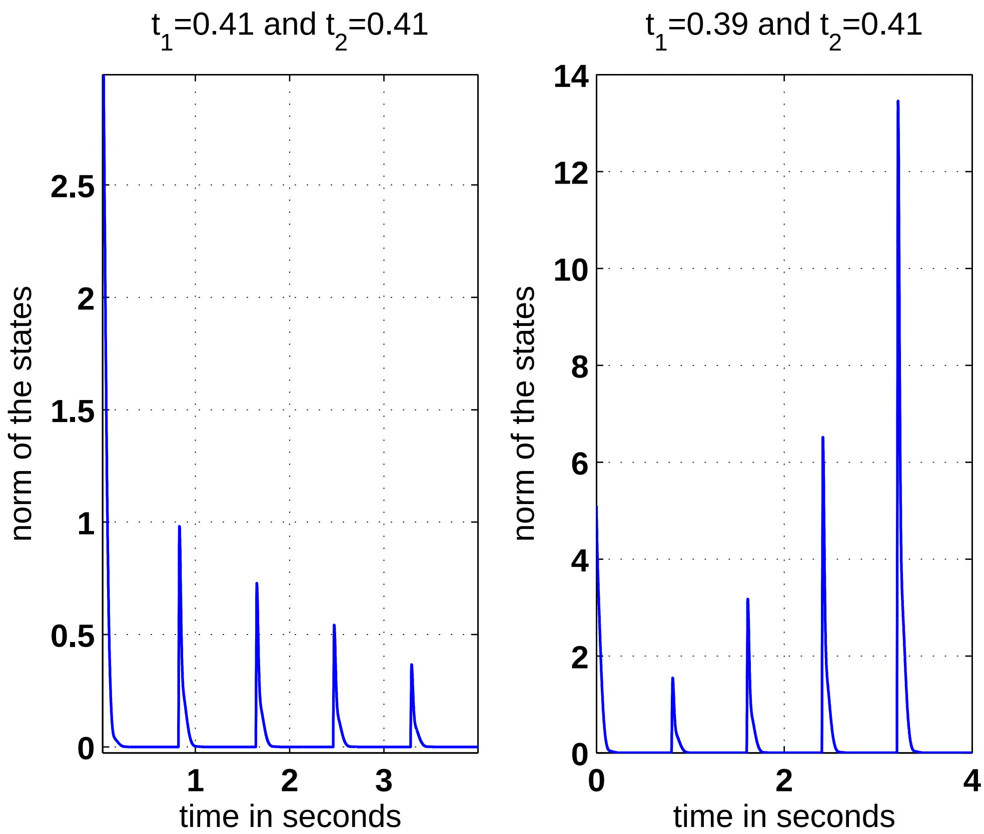

Figure 4.5: Dwell time from simulations when Pade order=8

From Fig. 4.5, a lower bound of the dwell time is between 0.39 sec. and 0.41 sec. and as we can see from Figure 4.6, as the Pade order increases, the dwell time value from simulations get closer to the calculated dwell time. In conclusion, for this example, the exact minimum dwell time is between 0.39 (lower bound found from simulations) and 0.57 (upper bound found from the formula given in [39]). This illustrates the level of conservativeness in the dwell time computation for this type of plants and controllers.

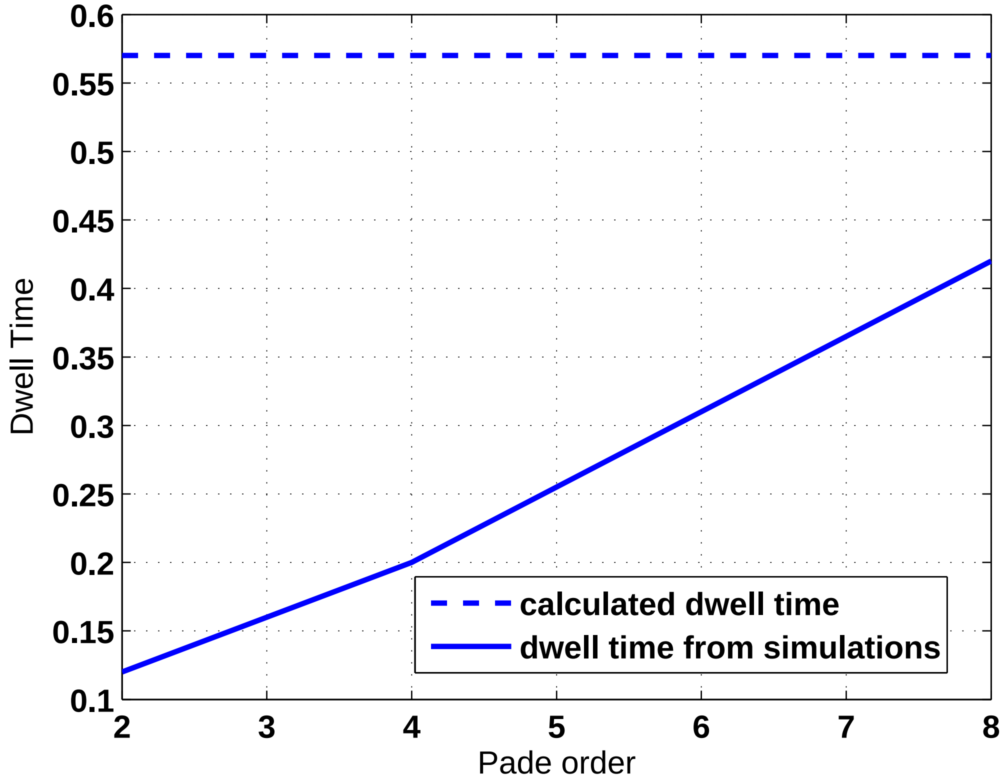

Figure 4.6: Dwell time versus Pade order

4.2 Comparison of the Results with Alternative Design Methods

In this section, the results of the LMI-based stability test suggested in [39] for the switched time delay systems is analyzed with respect to alternative first order and PD controller design methods. That means the LMI-based stability test is applied on the system with the same plant and controller structure, but the controller parameters would be determined by the alternative controller design methods. An upper bound of the dwell time is calculated for each of these controllers. But, since the other design methods do not consider possible switchings in the plant parameters, comparison is done using the non-switched plants which are first order unstable plants with time delay of the form (4.9). Consequently, by comparing the upper bounds of the dwell time, the behavior of the designs under arbitrary switching is pointed out.

Standard PD controller design methods of the form $C(s) = K_p + K_d s$ which stabilizes the non-switched plant of the form (4.9) are suggested in [15], [31] and [16]. We filtered the derivative part of the standard PD controller structures as proposed in [18] and three parameters K_p, K_d and τ_d are left for tuning when the integral part of (1.1) is discarded. Also [30] suggested a method to stabilize the first order unstable plant with a first order stable controller of the form $C(s) = \frac{\alpha_2 s+\alpha_3}{s+\alpha_1}$ and [24] proposed a different design method with the same controller structure given in (4.1).

The alternative controller design methods mentioned above generally tried to find a set of stabilizing controllers for plant structures of the form (4.9)and the procedures of finding the stabilizing sets can briefly be explained as follows:

- [15] investigated the stability of the feedback control system for first order unstable systems with time delay of the form (4.9) with standard P, PI, PD and PID controllers using the D-partition technique and we focus on stabilization using standard PD controllers. That means using the D-partition boundary equations, stability domains in $(K_p - K_d)$ plane for various values of time delay h are constructed as shown in Figure 4.7.

  Since K_p and K_d are assumed to be nonnegative, the D-partition boundaries are $0 \leq K_d < 1$, $K_p > 1$ and the upper bound of K_p to ensure the system stability for various delay values are as shown in Figure 4.7.

- [31] considers the problem of finding the complete set of stabilizing PID parameters for a first order open loop unstable plant with time delay. First, the range of admissible proportional gains is determined in closed form and for the plant of the form (4.9), the admissible range for K_p is as follows:

$$1 < K_p < \frac{\alpha_1}{h} \sin(\alpha_1) + \cos(\alpha_1) \tag{4.15}$$

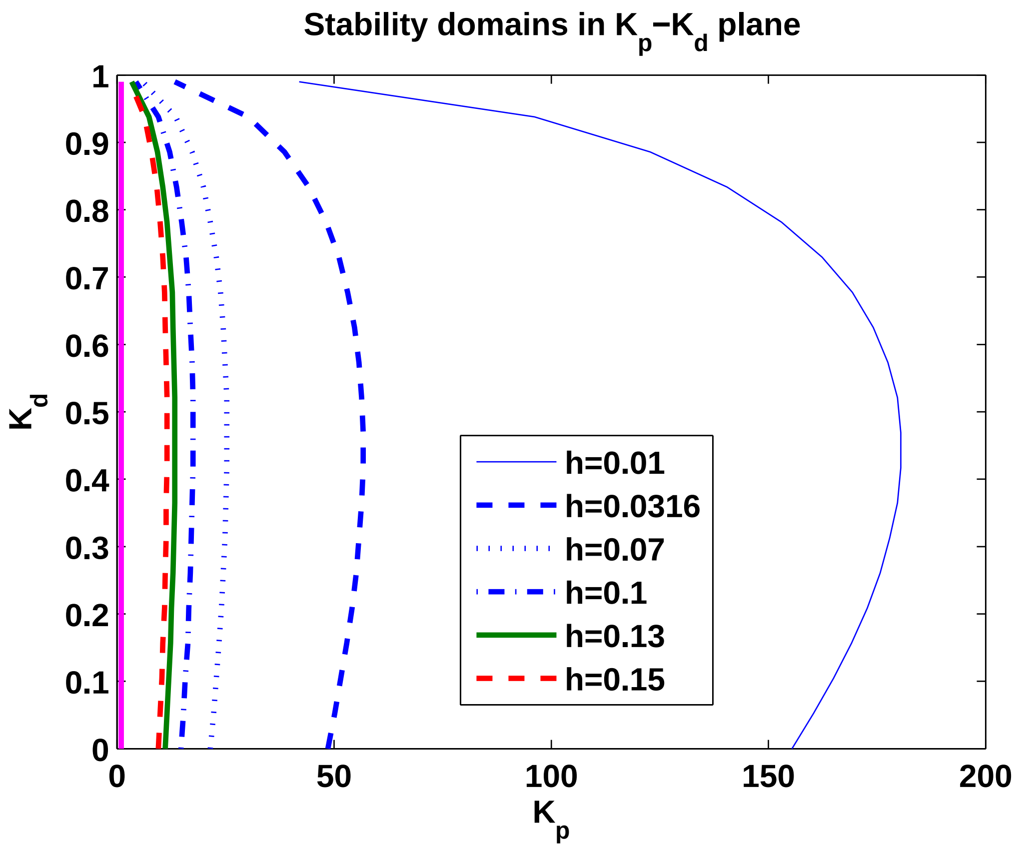

Figure 4.7: Stability domains in $(K_p - K_d)$ plane with various delays

where α_1 is the solution of the equation

$$\tan(\alpha) = \frac{\alpha}{h-1}$$

in the interval $(0, \pi)$. Then, for each proportional gain in this range, an interval of derivative gains for stabilizing the closed-loop system is found assuming the integral gain is zero.

For a specific proportional gain in this range, assuming the roots of

$$K_p + \cos(z) + \frac{1}{h} z \sin(z) = 0$$

are z_1 and z_2 $(z_1 < z_2)$, the bounds of derivative gain are as follows:

$$b(z) = \frac{-h}{z} \left[\sin(z) - \frac{1}{h} z \cos(z) \right]$$
$$b_j = b(z_j) \qquad j = 1, 2 \tag{4.16}$$

Thereby, for each K_p in the range stated in (4.15), a derivative gain interval $b_1 < K_d < b_2$ which stabilizes the feedback system is found by [31].

A typical stabilizing set in the space of proportional and derivative gains K_p and K_d is shown in Figure 4.8.

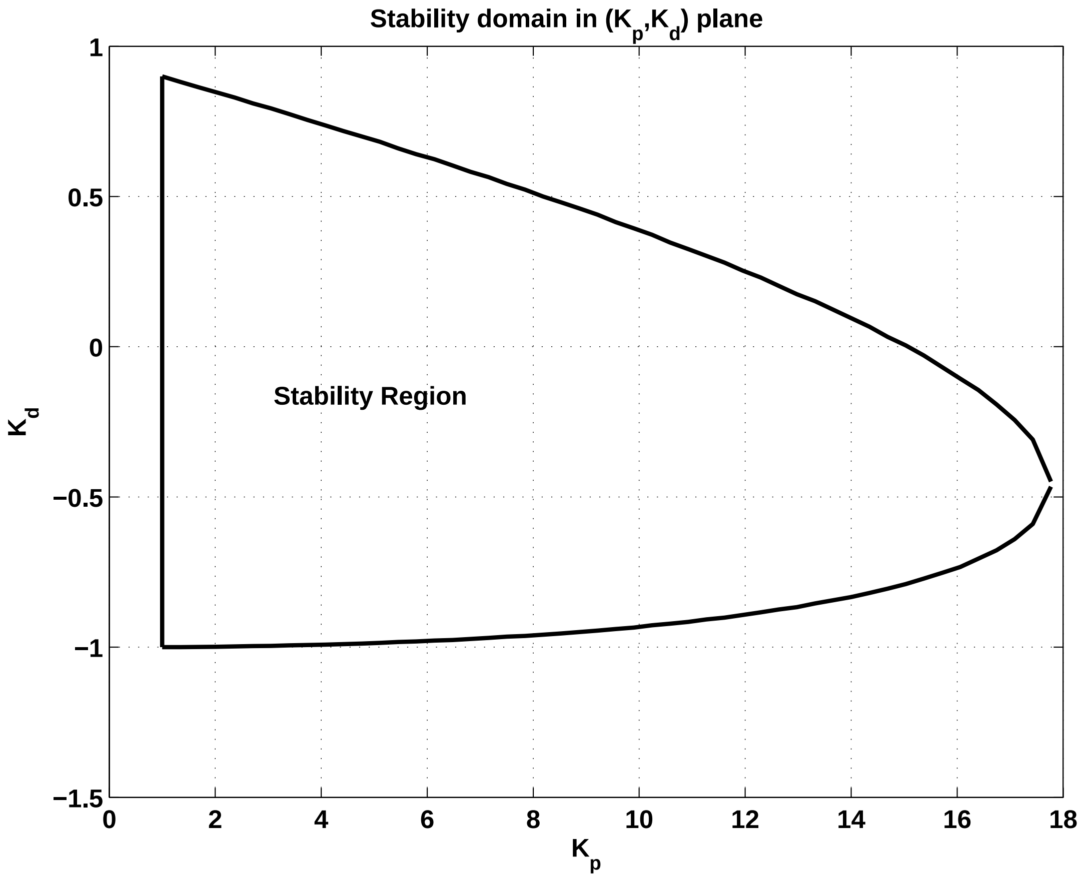

Figure 4.8: A typical stabilizing region in $(K_p - K_d)$ plane

- [16] had chosen a PD controller of the form $C(s) = K(bs + 1)$ to stabilize the feedback system with an unstable plant. The loop gain K which stabilizes the system is bounded from below and above according to the phase criterion of root loci. For the plant structure in (4.9), assuming that the solutions of

$$h\omega = \arctan(\omega)$$

are $\omega_1 = 0$ and ω_2 in the interval $\omega \in [0, \pi/h)$, the loop gain bounds are as follows:

$$K_j = \sqrt{1 + \omega_j^2} \quad j = 1, 2 \tag{4.17}$$

It can be inferred from the equation above that the lower bound of the loop gain $K_1 = 1$ and the upper bound K_2 rapidly decreases with the growing delay. [16] had also given the relation between the optimal b value and the delay. That is, for small delays ($h < 0.1$), the optimal b value is very small. However, the loop gain can be very large for small delays which results in a reasonable derivative gain.

- Having found intervals for K_p and K_d for a standard PD controller, the derivative part is filtered using a low-pass filter. The filter constant is $T_f = cK_d$, where c is often chosen as 0.1, see [18]. It is obvious that the filter constant T_f is exactly the same parameter as the time constant of the controller τ_d mentioned in the previous section.

- [24] suggested a PD controller of the form

$$C(s) = K_p \left(1 + \frac{\hat{K}_d s}{\tau_d s + 1}\right) \tag{4.18}$$

for a class of plants with time delay. Define $\Phi(s)$ for the plant of the form (4.9) as follows.

$$\Phi(s) := s^{-1} \left[e^{-hs} \left(1 + \frac{\hat{K}_d s}{\tau_d s + 1}\right) - 1\right] \tag{4.19}$$

If $||\Phi||_\infty < 1$, then for any $\hat{K}_d \in \mathbf{R}$, $\tau_d > 0$ and K_p satisfying

$$1 < K_p < ||\Phi||_\infty^{-1} \tag{4.20}$$

and the controller of the form (4.18) stabilizes the feedback system. Then, [24] fixed $\hat{K}_d$ and τ_d in order to design a resilient controller by maximizing the size of the K_p interval, which is equivalent to finding a $\hat{K}_d$ and τ_d such that:

$$[\hat{K}_d \quad \tau_d] = arg\min ||\Phi||_\infty \tag{4.21}$$

- A parametric method using extensions of the Hermite-Biehler theorem to find the stabilizing regions of a first order controller for an all-pole system with time delay is proposed in [30]. The first order controller structure is as following:

$$C(s) = \frac{\alpha_2 s + \alpha_3}{s + \alpha_1} \tag{4.22}$$

and can be expressed as a PD-like controller of the form (4.1) with the following parameters:

$$K_p = \frac{\alpha_3}{\alpha_1}, \quad K_d = \frac{\alpha_2}{\alpha_1} - \frac{\alpha_3}{\alpha_1^2}, \quad \tau_d = \frac{1}{\alpha_1} \tag{4.23}$$

The ranges for α_1, α_2 and α_3 are found using the fact that a non-zero polynomial $\Psi \in \mathbf{R}[s]$, such that $\Psi(0) \neq 0$, has r real roots without counting the multiplicity if and only if the signature of the polynomial $\Psi(s^2)+\Psi'(s^2)$ is $2r$ and the odd part of the polynomial $[q(s) + \alpha p(s)]p(-s)$ has at least $\bar{r} = \lfloor \frac{deg(q)-\sigma(p)-1}{2} \rfloor$ real negative roots with odd multiplicity, where the signature $\sigma(\Psi)$ of a polynomial $\Psi \in \mathbf{R}[s]$ is defined as the difference between the number of its $\mathbf{C}_-$ roots and $\mathbf{C}_+$ roots.

From the characteristic polynomial of the closed-loop system with the plant and controller are of the form (4.9) and (4.22), the polynomial is separated to its even and odd parts and we tried to eliminate the unknown parameters α_2 and α_3 in the odd part in order to use the facts stated above. When just α_1 is left in the odd part, the odd part of the polynomial is such that

$$\phi_n(s, \alpha_1) = q_n(s) + \alpha_1 p_n(s) \tag{4.24}$$

And the admissible range of α_1 is determined by searching the α_1 values which lead (4.24) to have $\bar{r} = \lfloor \frac{deg(q_n)-\sigma(p_n)-1}{2} \rfloor$ real, negative and distinct roots.

Assuming the denominator polynomial of an all pole plant is $q(s)$, the real and imaginary parts of $q(s)$ are defined as $R(s)$ and $I(s)$ respectively. Replacing s with $j\omega$, for $\omega > 0$, the plane of (α_2, α_3) can be partitioned into root-invariant regions by sweeping over values of $\omega > 0$ according to the following pair of equations with a fixed α_1 in the range found above:

$$\begin{aligned}\alpha_2 &= \left[I(jh\omega) - \alpha_1 \frac{R(jh\omega)}{\omega}\right]\sin(h\omega) \\ &\quad - \left[R(jh\omega) + \alpha_1 \frac{I(jh\omega)}{\omega}\right]\cos(h\omega) \\ \alpha_3 &= \left[\omega I(jh\omega) - \alpha_1 R(jh\omega)\right]\cos(h\omega) \\ &\quad + \left[\omega R(jh\omega) + \alpha_1 I(jh\omega)\right]\sin(h\omega)\end{aligned} \tag{4.25}$$

By this way, stabilizing region in the (α_2, α_3) plane is found with a fixed α_1 by checking the stability of a point inside the region and the space of stabilizing controllers of the form (4.22) is constituted. A typical stabilizing set of controller parameters is as shown in Figure 4.9 for delay value $h = 0.1$.

After finding the space of stabilizing controllers with respect to each design method mentioned above, an upper bound for the dwell time is searched by applying the LMI-based stability test to the system at each point in the parameter space of the stabilizing controllers.

In the first three methods, for the defined (K_p, K_d) plane the filter constant can be chosen to be a constant $\tau_d = 0.1K_d$ as suggested in [18] or can be chosen to be a variable defined in an interval where the minimum upper bound of the dwell time is searched. When τ_d is fixed, the LMI's of the stability test derived in [39] can be satisfied for small delay values ($h \leq 0.01$). However for $h > 0.01$ the filter constant τ_d has to be adjusted accordingly to satisfy the sufficient conditions of the stability test.

Due to the necessity of choosing τ_d as a variable, the search of an upper bound for the dwell time is carried out upon K_p, K_d and τ_d parameters in the intervals

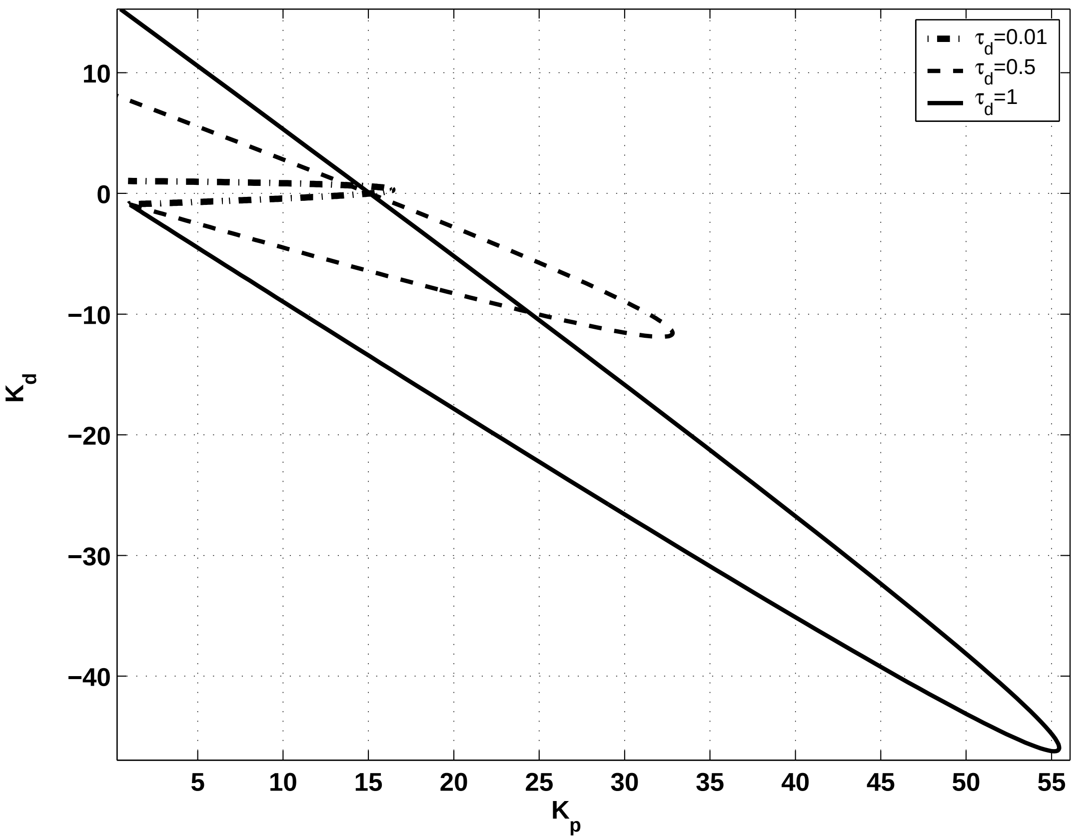

Figure 4.9: The stabilizing region of (α_2, α_3) when $\alpha_1 = 1$

determined by the design methods stated above. The determined intervals of the controller parameters for each design method is as shown in Table 4.2 for delay value $h = 0.1$.

By using the program we had developed for calculating the dwell time bound and checking the sufficient conditions of the stability test, the minimum upper bound of the dwell time is searched over these parameter spaces and the results according to different delay values are as shown in Table 4.3, Figures 4.10, 4.11 and 4.12.

As we can see from Table 4.3, the minimum dwell time is obtained when controller parameter K_d is around -1, which is the case in our design, the design proposed by [30] and [31]. These approaches give similar results, because the stabilizing parameter spaces proposed by these methods are approximately the

Design Method	K_p	K_d	τ_d
Hwang and Hwang (2004)	$1 < K_p < 17.52$	$0 < K_d < 1$	$0 < \tau_d < 2$
Silva et al. (2002)	$1 < K_p < 17.769$	$-1 \leq K_d \leq 1$	$0 < \tau_d < 2$
Huang and Chen(1997)	$1 < K_p < 15.04$	$0 < K_d < K_p$	$0 < \tau_d < 2$
Gundes and Ozbay (2007)	$1 < K_p < 11.889$	$-K_p < K_d < K_p$	$0 < \tau_d < 2$
Saadaoui et al. (2008)	$0 < K_p < \infty$	$0 < K_d < K_p$	$0 < \tau_d < 2$

Table 4.2: The minimum and maximum bounds of the controller parameters K_p and K_d for $h = 0.1$

Design Method	Dwell time	K_p	K_d	τ_d
Our design	0.951	5.081	-1	0.745
Saadaoui et al.(2008)	1.085	4.618	-1	0.823
Silva et al. (2002)	1.078	4.531	-1	0.8
Huang and Chen(1997)	4.924	2.727	0.001	1.247
Gundes and Ozbay (2007)	5.597	2.668	0.04	1.27
Hwang and Hwang (2004)	5.04	2.714	0.001	1.242

Table 4.3: The minimum dwell time τ of different design methods with time delay $h = 0.1$

same. Likewise, K_p, K_d and τ_d intervals are similar for the design methods [15] and [16]. Since the parameter spaces are very close, the obtained minimum dwell time and the optimal values of the controller parameters, which can be seen from Table 4.3, are similar. For the design method suggested in [24], since K_d is fixed to obtain a resilient controller, the search of the upper bound of the dwell time is carried out upon the (K_p, τ_d) plane. Hence, although the stabilizing controller parameter intervals defined in [24] are close to [30] and [31], larger upper bounds of dwell time are observed. The small differences between the observed results regarding the same parameter spaces are probably due to the insensitivity of the search.

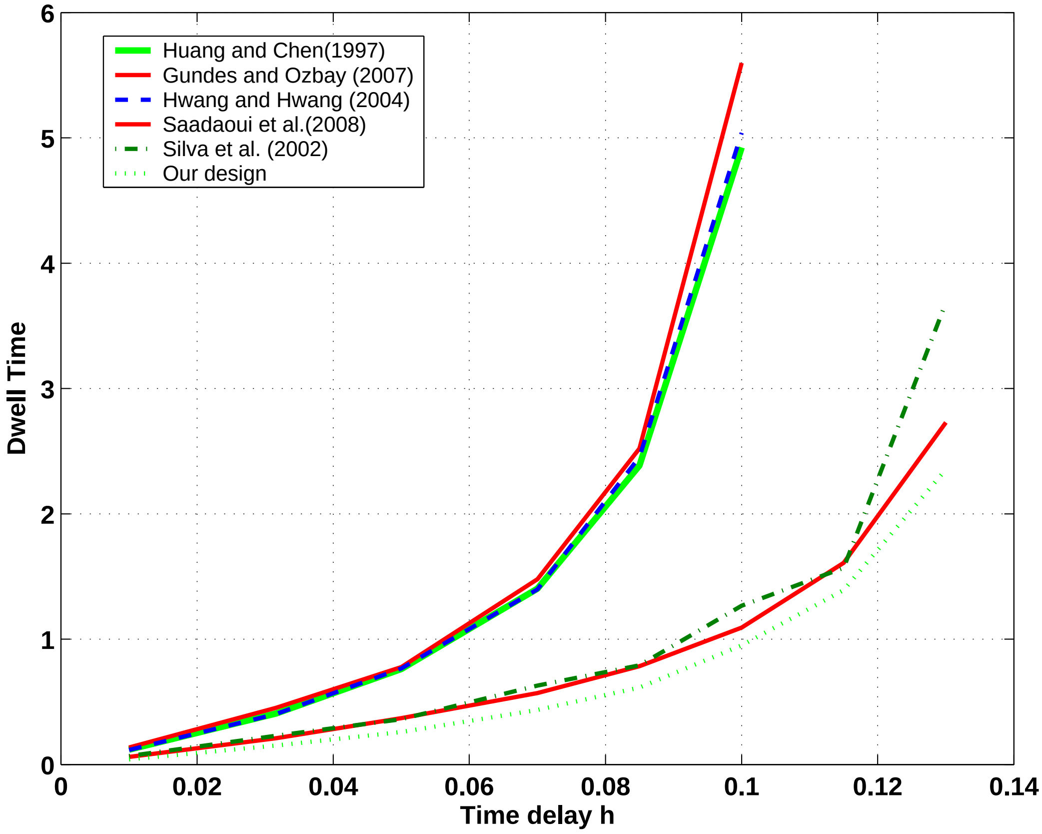

Figure 4.10: The dwell time τ versus time delay h

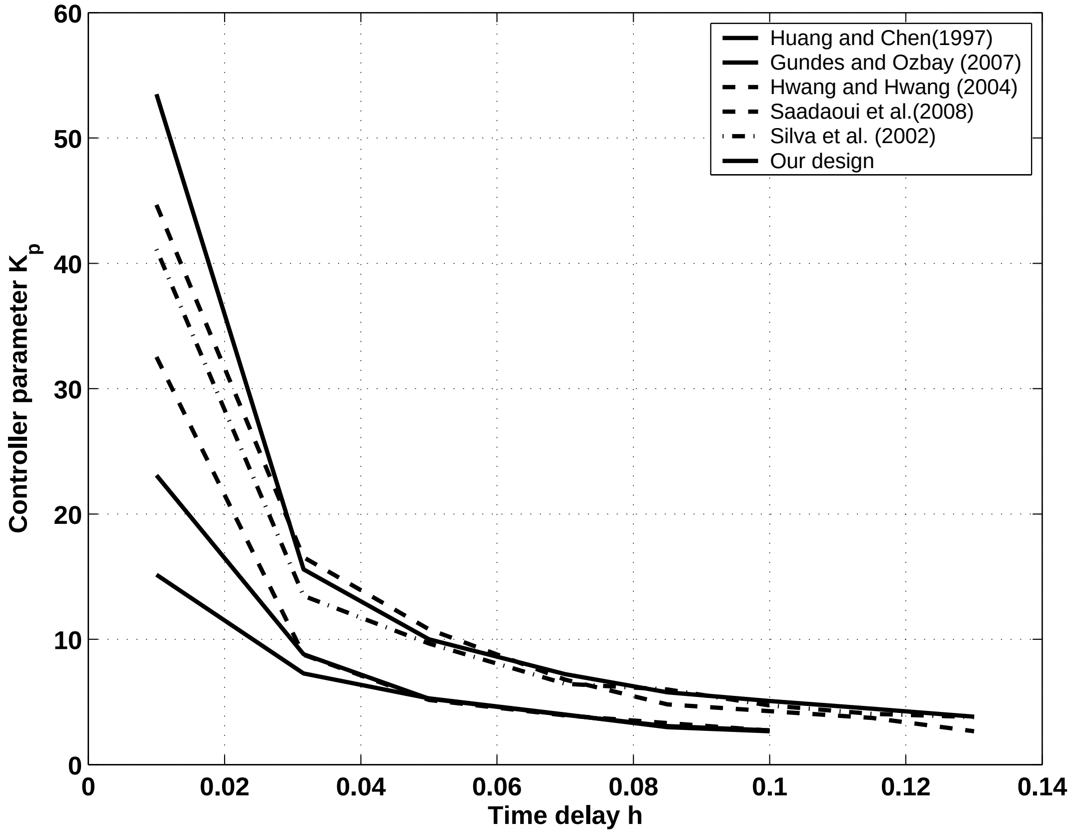

Figure 4.11: The controller parameter K_p versus time delay h

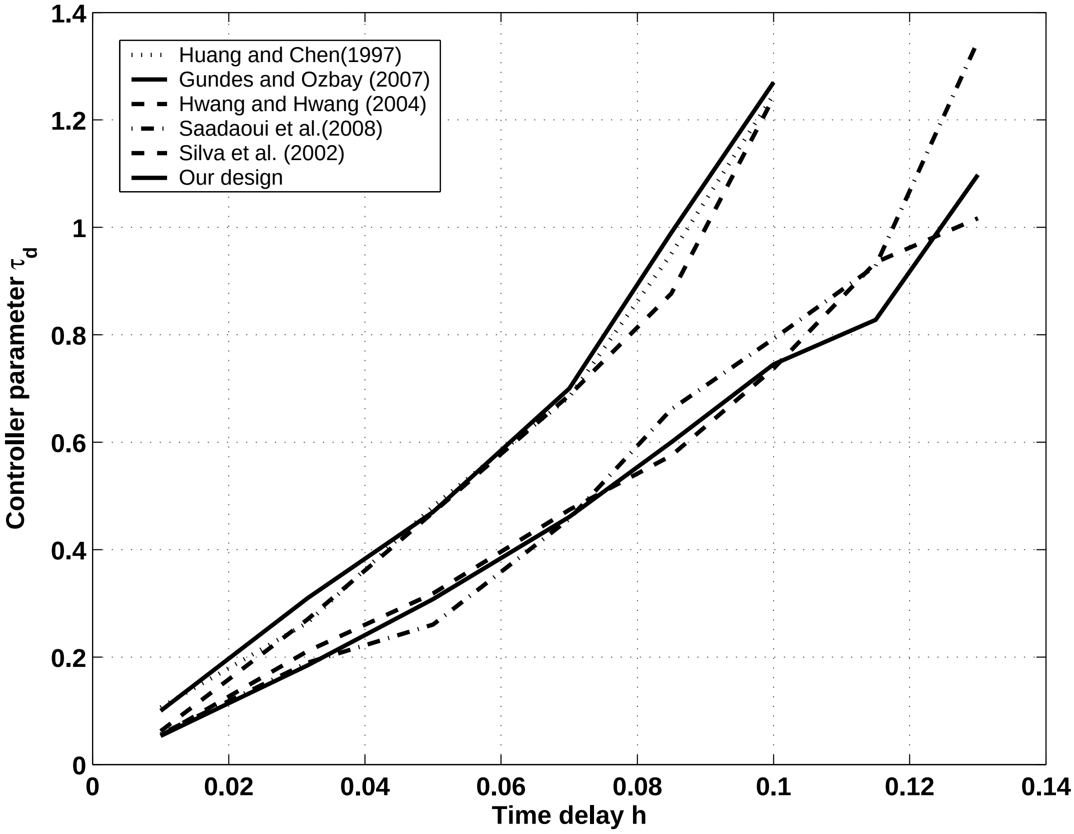

Figure 4.12: The controller parameter τ_d versus time delay h

Chapter 5

Conclusions

In this thesis, we investigated PID controller design techniques for first order unstable systems with time delay. The problem of designing PID controllers is separated into two parts. As a first step, the set of all stabilizing controllers with fixed structures is determined. Then, a controller in this stabilizing set is searched such that the feedback system satisfies a desired property, i.e. a specific gain or phase margin or a fast tracking of the output for a reference input. Different methods have been proposed in order to tune the PID controller coefficients for linear time-invariant plants. However, these results are generally not extended to time delay systems. Therefore, in this work, we focus on linear time delay systems which are first order and unstable.

In the first part of the thesis, we dealt with the problem of designing a PI controller for a first order unstable system with time delay such that the system will have the maximum gain and phase margins and the output of the system tracks the reference input fast and accurately. The Nyquist encirclement principal is used to determine the stabilizing set of PI controllers. That means finite intervals for controller coefficients are found which stabilizes the feedback system. These intervals form the parameter space of coefficients and an appropriate controller is numerically searched in this parameter space. We observed that a proportional controller is adequate to maximize both gain and phase margins of the class of plants considered here. However, we know that the output can not track the reference input accurately with a proportional controller. Therefore, the tracking response is tried to be improved with the pre-defined cost function minimization and the transient response optimization techniques. A more accurate tracking response is obtained by using these techniques which loses from gain and phase margins at the same time. As a result, these observations brought us to a trade-off between the gain and phase margins and tracking response of the output to the reference input.

In Chapter 3, we analyzed the stability requirements of a switched time delay system with P and PI controllers. The analysis is done in terms of minimization of the dwell time. By using the matrix inequality based sufficient conditions of [39], we determine the parameter space that stabilizes the switched feedback system. Then, the controller coefficients which minimizes the dwell expression is searched in this parameter space. We observed that the switched time delay system could be stabilized with a proportional controller and as time delay increases, minimum dwell time increases while the proportional constant decreases. In addition, a stabilizing PI controller could not be found using the LMI-based stability analysis derived in [39].

In the Chapter 4, we designed PD-like controllers for switched time delay systems. Similar to Chapter 3, the parameter space of the controller coefficients for which the switched system is stable, is determined by using the matrix inequalities derived in [39]. The appropriate controller parameters are chosen from this parameter space to minimize the dwell time expression. The dwell time increases very fast with the increasing time delay which is also the case in Chapter 3. Then, we checked our results with time domain simulations and realized that the calculated dwell time is an upper bound of the minimum time required between consequent switchings. Therefore, we found the lower bound of the dwell time with the simulations in the conservativeness analysis section of Chapter 4. The results show that the upper bound calculated is close to the lower bound found from the simulations. Since the calculated dwell time expression is an upper bound, a lower bound of the dwell time can be formulated in further studies. Finally, we compared this PD-like controller design with the alternative PD and first order controller design methods for the same class of plants and observed that the set of stabilizing controllers obtained in this work is similar with the one obtained by alternative methods.

For the class of first order unstable time delayed systems, we tried to solve the dwell time minimization problem with P, PI and PD-like controllers. This solution could further be solved with PID controller or other types of controller structures. In addition, the problem of PID controller design for first order systems could be extended to higher order systems, which is also left as an open problem.

APPENDIX A

The Matlab Codes

A.1 PI Control for Gain and Phase Margin Maximization

phaseopt.m

```
\% Phase margin optimization of a time-delay system with an unstable pole
\% using PI controller
\% Define the plant P(s)=e^(-hs)/(s-a)
h = [0.1 0.2 0.3];
\% tau= a*Kp/Ki
tau =[0.5:0.1:1.5 10.^[1.5:0.5:4]];
for (ii=1:length(h))
    for(jj=1:length(tau))
        [wc,fmx]=findcutoff(h(ii),tau(jj));
        [a b K(jj)]=gainopt(tau(jj),h(ii));
        x=K(jj)^2;
        wc=sqrt((x*tau(jj)^2-1)/2+sqrt(((x*tau(jj)^2-1)/2)^2+x));
        phi(jj)=atan(wc)+atan(tau(jj)*wc)-h(ii)*wc-pi/2;
```

```
        gm(jj)=K(jj)*a;
    end
[phimax index]=max(phi);
phimax= phimax*180/pi
gmmax=gm(index)
tau_opt=tau(index)
Kopt=K(index)
end
```

findcutoff.m

```
\% To find cut-off frequency for given h and tau by maximizing f(wc)
function [wc,fmx]=findcutoff(h,tau)
N=1000;
w = linspace(0.001,3*pi,N);
for(ind=1:length(w))
    f(ind)=atan(w(ind))+atan(tau*w(ind))-h*w(ind)-pi/2;
end
plot(w,f)
[fmx index]=max(f);
wc=w(index);
```

gainopt.m

```
\% Gain margin optimization of a time-delay system with an unstable pole
\% using PI controller
\% Define the plant P(s)=e^(-hs)/(s-a)
\% Define the PI controller C(s)=Kp+Ki/s
function [a b K]=gainopt(tau,h)
h = [0.1 0.2 0.3];
\% tau= a*Kp/Ki
tau =10.^[0:0.2:4];
```

```
for(ii=1:length(h))
    for(jj=1:length(tau))
        \% Find w1 and w2
        [w1,w2]=findw(h(ii),tau(jj));
        \%Define q(h,t)
        q(ii,jj) = (w1/w2)^2*((1+w1^2)/(1+w2^2))*((1+tau(jj)^2*w2^2)/
        (1+tau(jj)^2*w1^2));
        a(ii,jj) = sqrt((1+tau(jj)^2*w1^2)/(1+w1^2))/w1;
        b(ii,jj) = sqrt((1+tau(jj)^2*w2^2)/(1+w2^2))/w2;
        K(ii,jj) = 1/sqrt(a(ii,jj)*b(ii,jj));
    end
figure
plot(tau,q(ii,:))
xlabel('\tau')
ylabel('q(h,\tau)')
[qmin(ii) index] = min(q(ii,:));
taumin(ii)=tau(index);
end
```

findw.m

```
% To find w1 and w2 that satisfies
function [w1,w2]=findw(h,tau)
iter=1;
soln=[];
N = 5000;
w = linspace(0.00001,15,N);
for(ind=1:length(w))
    f(ind)=log(atan(tau*w(ind))+atan(w(ind))-h*w(ind)-pi/2);
    if(ind>=2)
        grad(ind-1)=f(ind)-f(ind-1);
    end
```

```
    if(ind>=3)
        if(grad(ind-1)>0 && grad(ind-2)<0)
            soln=[soln w(ind-1)];
        end
    end
end
w1=soln(1);
w2=soln(2);
```

cost.m

```
% Phase margin optimization of a time-delay system with an unstable pole
% using PI controller
% Define the plant P(s)=e^(-hs)/(s-a)
% Define the PI controller C(s)=Kp+Ki/s
h = [0.1 0.2 0.3];
% tau= a*Kp/Ki
tau =[0.5:0.05:10];
c=10.^[-1:0.1:3];
for(ind=1:length(c))
for (ii=1:length(h))
    for(jj=1:length(tau))
        findcutoff(h(ii),tau(jj));
        [a b K(jj)]=gainopt(tau(jj),h(ii));
        x=K(jj)^2;
        wc=sqrt((x*tau(jj)^2-1)/2+sqrt(((x*tau(jj)^2-1)/2)^2+x));
        phi(jj)=atan(wc)+atan(tau(jj)*wc)-h(ii)*wc-pi/2;
        gm(jj)=K(jj)*a;
        [beta(jj),alpha_max(jj)]=vect_margin(tau(jj),h(ii),K(jj));

    end
    cost=1./(exp(alpha_max)+c(ind).*beta);
```

```
    [Jmax index]=min(cost);
    phimax(ind)= phi(index)*180/pi
    gmmax(ind)=gm(index)
    tau_opt(ind)=tau(index)
    Kopt(ind)=K(index)
end
end
% For drawing Nyquist graphs
Kopt=wc(ind)*sqrt((1+wc(ind)^2)/(1+tau(ind)^2*wc(ind)^2));
end
[a b K]=gainopt(tau,h);
gm=K*a
wc=sqrt((tau^2*K^2-1)/2+sqrt(((tau^2*K^2-1)/2)^2+K^2))
pm=abs((-h*wc+atan(wc)+atan(tau*wc)-pi/2)*180/pi)
P=tf(K*[tau 1],[1 -1 0],'iodelay',0.2)
close all
nyquist(P)
hold on;
x=-1:0.001:1;
y=sqrt(1-x.^2);
plot(x,y,'r',x,-y,'r')
hold off
```

vect_margin.m

```
function [beta,alpha_max]=vect_margin(tau,h,K)
N=1000;
w = linspace(0.01,100,N);
for(ind=1:length(w))
    C(ind)=K*(1+j*tau*w(ind))/(j*w(ind));
    P(ind)=exp(-j*h*w(ind))/(j*w(ind)-1);
    S(ind)=abs(1/(1+P(ind)*C(ind)));
```

```
% G(ind)=abs(C(ind))*S(ind);
    WS(ind)=abs(j*w(ind)/S(ind));
end
alpha_max=min(WS);
beta=1/max(S);
```

A.2 P Control for Dwell Time Minization

alternative_min_dwelltime_1storder.m

```
% First order plus time delay system which is unstable
% P(s)=e^{-hs}/(s-a)
% C(s)=Kp   proportional controller
% First, fix a and h such that 0<ah<1
a=1;
for(dd=1:length(a))
h=[0.01 0.05 0.1 0.15 0.17 0.18];
for(cc=1:length(h))
index=1;
clear beta alpha K p feas_soln
% Then, fix p in (1,1/(ah)^2)
pmax=1/(a(dd)*h(cc))^2;
% p=linspace(1.01,pmax-1,50);
p=1.01;
for( aa=1:length(p))
    % Fix K in the range (a/(1-ah*sqrt(p)),infty)
    Kmin=a(dd)/(1-a(dd)*h(cc)*sqrt(p(aa)));
    K=linspace(Kmin+0.01,Kmin+6,40);
    for(bb=1:length(K))
        % Find the maximum value of 1/(h*Td) by iterating alpha and beta over the
        % allowable ranges
```

```
        betamax=2*((K(bb)*(1-a(dd)*h(cc)*sqrt(p(aa)))-a(dd))/(h(cc)*p(aa)));
        %0<beta<betamax
        N=100;
        beta=linspace(0.0001,betamax*0.9999,50);
        for (ii=1:length(beta))
            % alpha1 < alpha < alpha2
            alpha1=(K(bb)-a(dd))/(h(cc)*p(aa))-beta(ii)/2-
            sqrt(((K(bb)-a(dd))/(h(cc)*p(aa))-beta(ii)/2)^2-a(dd)^2*K(bb)^2/p(aa));
            alpha2=(K(bb)-a(dd))/(h(cc)*p(aa))-beta(ii)/2+
            sqrt(((K(bb)-a(dd))/(h(cc)*p(aa))-beta(ii)/2)^2-a(dd)^2*K(bb)^2/p(aa));
            %     X=sqrt(p)/a/K*(betamax-beta(ii))/2;
            %     alpha1=a*K/sqrt(p)*X*(1-sqrt(1-(1/(1+X))^2));
            %     alpha2=a*K/sqrt(p)*X*(1+sqrt(1-(1/(1+X))^2));
            if (alpha2>alpha1+20)
                alpha=linspace(alpha1+0.001,alpha1+20,50);
            else
                alpha=linspace(alpha1+0.001,alpha2-0.001,50);
            end
            for(jj=1:length(alpha))
                % Calculate 1/(h*Td)
                var=2/h(cc)*(K(bb)-a(dd))-a(dd)^2*K(bb)^2/alpha(jj)-K(bb)^4/beta(ii)
                -p(aa)*(alpha(jj)+beta(ii));
                if (var>0)
                    feas_soln(index,:)=[1/(h(cc)*var) p(aa) K(bb) beta(ii) alpha(jj)
                    index=index+1;
                end
            end
        end
    end
end
[minn ind]=min(feas_soln,[],1);
```

```
min_Td(cc,:)=feas_soln(ind(1),:)
end
end
```

A.3 PD-like Control for Dwell Time Minization

min_dwelltime.m

```
% First order plus time delay system which is unstable
% P(s)=e^{-hs}/(s-a)
% C(s)=Kp+Kd*s   PD controller
eps=0.01;
a=1; % Assumed to be 1
h=[0.001 0.0032 0.01 0.0316 0.1 0.13 0.155 0.16 0.165];
N=10;
%************************
% Assume P=lambda*[1 x2;x2 x1]
%************************
for(aa=1)
    index=1;
    clear Td ind beta alpha Kp Kd tau\_d p
    Kp=linspace(a,a+300,N);
    for(cc=1:length(Kp))
        % Fix x1
        x1=1+eps;
        % x1=linspace(x1min,x1max-eps,N/2);
        for(ee=1:length(x1))
            % Then, fix p
            % p=linspace(1.01,20,2*N);
            p=(x1(ee)-eps)/(1-eps);
            for(bb=1:length(p))
```

```
% Fix Kdmax>Kd>Kdmin
Kdmin=-1.1;
% Kdmax=-0.17*x1(ee);
Kdmax=-0.9;
Kd=linspace(Kdmin,Kdmax,N/2);
for(dd=1:length(Kd))
    tau\_d=linspace(0.01,2,2*N);
    for(gg=1:length(tau\_d))
        betamin=0;
        betamax=2/(h(aa)*p(bb)*tau\_d(gg));
        if (betamax<10)
            beta=linspace(betamin+eps,betamax-eps,N);
        elseif(betamax<100 \&\& betamax>10)
            beta=linspace(betamin+eps,betamax-eps,2*N);
        else
            beta=linspace(betamin+eps,betamax-eps,3*N);
        end
        for (hh=1:length(beta))
            alphamin=0;
            alphamax=2/(p(bb)*h(aa)*tau\_d(gg))-beta(hh);
            if (alphamax<0)
                break;
            else
                if (alphamax <10)
                    alpha=linspace(alphamin+eps,alphamax-eps,N);
                elseif(alphamax <100 \&\& alphamax>10)
                    alpha=linspace(alphamin+eps,alphamax-eps,2*N);
                else
                    alpha=linspace(alphamin+eps,alphamax-eps,3*N);
                end
            end
```

```
                        for (ii=1:length(alpha))
                            P=[1 0;0 x1(ee)];
                            lambda=max(svd(P))/min(svd(P));
                            A=[-1/tau\_d(gg) Kd(dd)/tau\_d(gg);0 a];
                            Abar=[0 0;1/tau\_d(gg) -(Kp(cc)+Kd(dd)/tau\_d(gg))];
                            S=-(P*(A+Abar)+(A+Abar)'*P+h(aa)*(P*Abar*As*...
                            inv(P)*A'*Abar'*P/alpha(ii)+P*(Abar)^2*inv(P)*...
                            (Abar')^2*P/beta(hh)+p(bb)*(alpha(ii)+beta(hh))*P));
                            w=min(svd(S));
                            X=[((A+Abar)'*P+P*(A+Abar))/h(aa)+p(bb)*(alpha(ii)..
                            +beta(hh))*P P*Abar*[A Abar];[A Abar]'*Abar'*P...
                            *[-alpha(ii)*P zeros(2);zeros(2) -beta(hh)*P]];
                            eigenX=eig(X);
                            if ( w>0)
                                if( eigenX(1)<0\&\&eigenX(2)<0\&\&eigenX(3)<0
                                \&\&eigenX(4)<0\&\&eigenX(5)<0\&\&eigenX(6)<0)
                                    mu=max(svd(P))/w;
                                    Td(index)=lambda*mu*floor((lambda-1)/...
                                    (p(bb)-1)+1);
                                    ind(index,:)=[h(aa) p(bb) x1(ee) Kp(cc)
                                    Kd(dd) tau\_d(gg) beta(hh) alpha(ii)];
                                    index=index+1;
                                end
                            end
                        end
                    end
                end
            end
        end
    end
end
```

```
    [minn min\_index]=min(Td);
    result(aa,:)=[minn ind(min\_index,:)]
end
```

A.3.1 Conservatism Analysis

switched.m

```
N=5;   % number of switching
t1=1;    % switching time for sys1
t2=1;    % switching time for sys2
pade_ord=15;
init=[zeros(2*pade_ord,1);5;1];
time=[];
x=[];
for(ind=1:N)
%     t1=rand(1)/5;
    [t,y]=ode45('eqn4',[0.01:0.01:t1],init);
    init=y(size(y,1),:)';
    x=[x;y];
    time=[time t1];
%     t2=rand(1)/5;
    [t,y]=ode45('eqn5',[0.01:0.01:t2],init);
    init=y(size(y,1),:)';
    x=[x;y];
    time=[time t2];
end
for row=1:length(x)
    nrm(row)=norm(x(row,:));
end
tf=sum(time);
```

```
plot(linspace(0,tf,length(nrm)),nrm)
grid
title(['t_1=' num2str(t1) ' and t_2=' num2str(t2)])
xlabel('time in seconds')
ylabel('norm of the states')
time;
```

eqn4.m

```
function xdot=eqn4(t,x,pade_ord);
% First system
a=1;
h=0.01;
Kp=53.5;
Kd=-0.987;
tau_d=0.0533;
if(pade_ord==1)
    % 1st order pade approxiamtion
    Ad=-2/h*eye(2);
    Bd=eye(2);
    Cd=4/h*eye(2);
    Dd=-eye(2);
elseif(pade_ord==2)
    % 2nd order pade approxaimation
    Ad=[zeros(2) eye(2);-12/h^2*eye(2) -6/h*eye(2)];
    Bd=[zeros(2);eye(2)];
    Cd=[zeros(2) -12/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==3)
    % 3rd order pade approxiamtion
    Ad=[zeros(4,2) eye(4);-120/h^3*eye(2) -60/h^2*eye(2) -12/h*eye(2)] ;
    Bd=[zeros(4,2); eye(2)];
```

```
    Cd=[240/h^3*eye(2) zeros(2) 24/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==4)
    % 4rd order pade approxiamtion
    Ad=[zeros(6,2) eye(6);-1680/h^4*eye(2) -840/h^3*eye(2) -180/h^2*eye(2)
    -20/h*eye(2)] ;
    Bd=[zeros(6,2); eye(2)];
    Cd=[zeros(2) -1680/h^3*eye(2) zeros(2) -40/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==5)
    % 5rd order pade approxiamtion
    Ad=[zeros(8,2) eye(8);-30240/h^5*eye(2) -15120/h^4*eye(2) -3360/h^3*eye(2)
    -420/h^2*eye(2) -30/h*eye(2)] ;
    Bd=[zeros(8,2); eye(2)];
    Cd=[60480/h^5*eye(2) zeros(2) 6720/h^3*eye(2) zeros(2) 60/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==6)
    % 5rd order pade approxiamtion
    Ad=[zeros(10,2) eye(10);-665280/h^6*eye(2) -332640/h^5*eye(2) -75600/h^4*eye(2)
     -10080/h^3*eye(2) -840/h^2*eye(2) -42/h*eye(2)] ;
    Bd=[zeros(10,2); eye(2)];
    Cd=[zeros(2) -665280/h^5*eye(2) zeros(2) -20160/h^3*eye(2) zeros(2)
     -84/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==7)
    % 7rd order pade approxiamtion
    Ad=[zeros(12,2) eye(12);-1.7297e7/h^7*eye(2) -8.6486e6/h^6*eye(2)
    -1.9958e6/h^5*eye(2) -277200/h^4*eye(2) -25200/h^3*eye(2) -1512/h^2*eye(2)
     -56/h*eye(2)] ;
    Bd=[zeros(12,2); eye(2)];
    Cd=[34594560/h^7*eye(2) zeros(2) 3991700/h^5*eye(2) zeros(2) 50400/h^3*eye(2)
```

```
     zeros(2) 108/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==8)
    % 8rd order pade approxiamtion
    Ad=[zeros(14,2) eye(14);-5.1892e8/h^8*eye(2) -2.5946e8/h^7*eye(2)
    -6.054e7/h^6*eye(2) -8.6486e6/h^5*eye(2) -831600/h^4*eye(2) -55440/h^3*eye(2)
    -2520/h^2*eye(2) -72/h*eye(2)] ;
    Bd=[zeros(14,2); eye(2)];
    Cd=[zeros(2) -5.1892e8/h^7*eye(2) zeros(2) -17297200/h^5*eye(2) zeros(2)
     -110880/h^3*eye(2) zeros(2) -144/h*eye(2)];
    Dd=eye(2);
end
A=[-1/tau_d Kd/tau_d; 0 a];
Abar=[0 0;1/tau_d -(Kp+Kd/tau_d)];
M=[Ad Bd;Abar*Cd A+Abar*Dd];
xdot=M*x;
```

eqn5.m

```
function xdot=eqn5(t,x,pade_ord);
% Second system
a=1;
h=0.07;
Kp=7.223;
Kd=-1.01;
tau_d=0.461;
if(pade_ord==1)
    % 1st order pade approxiamtion
    Ad=-2/h*eye(2);
    Bd=eye(2);
    Cd=4/h*eye(2);
    Dd=-eye(2);
```

```
elseif(pade_ord==2)
    % 2nd order pade approxaimation
    Ad=[zeros(2) eye(2);-12/h^2*eye(2) -6/h*eye(2)];
    Bd=[zeros(2);eye(2)];
    Cd=[zeros(2) -12/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==3)
    % 3rd order pade approxiamtion
    Ad=[zeros(4,2) eye(4);-120/h^3*eye(2) -60/h^2*eye(2) -12/h*eye(2)] ;
    Bd=[zeros(4,2); eye(2)];
    Cd=[240/h^3*eye(2) zeros(2) 24/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==4)
    % 4th order pade approxiamtion
    Ad=[zeros(6,2) eye(6);-1680/h^4*eye(2) -840/h^3*eye(2) -180/h^2*eye(2)
    -20/h*eye(2)] ;
    Bd=[zeros(6,2); eye(2)];
    Cd=[zeros(2) -1680/h^3*eye(2) zeros(2) -40/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==5)
    % 5th order pade approxiamtion
    Ad=[zeros(8,2) eye(8);-30240/h^5*eye(2) -15120/h^4*eye(2) -3360/h^3*eye(2)
    -420/h^2*eye(2) -30/h*eye(2)] ;
    Bd=[zeros(8,2); eye(2)];
    Cd=[60480/h^5*eye(2) zeros(2) 6720/h^3*eye(2) zeros(2) 60/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==6)
    % 6th order pade approxiamtion
    Ad=[zeros(10,2) eye(10);-665280/h^6*eye(2) -332640/h^5*eye(2) -75600/h^4*eye(2)
    -10080/h^3*eye(2) -840/h^2*eye(2) -42/h*eye(2)] ;
    Bd=[zeros(10,2); eye(2)];
```

```
    Cd=[zeros(2) -665280/h^5*eye(2) zeros(2) -20160/h^3*eye(2) zeros(2)
    -84/h*eye(2)];
    Dd=eye(2);
elseif(pade_ord==7)
    % 7th order pade approxiamtion
    Ad=[zeros(12,2) eye(12);-1.7297e7/h^7*eye(2) -8.6486e6/h^6*eye(2)
    -1.9958e6/h^5*eye(2) -277200/h^4*eye(2) -25200/h^3*eye(2) -1512/h^2*eye(2)
     -56/h*eye(2)] ;
    Bd=[zeros(12,2); eye(2)];
    Cd=[34594560/h^7*eye(2) zeros(2) 3991700/h^5*eye(2) zeros(2) 50400/h^3*eye(2)
    zeros(2) 108/h*eye(2)];
    Dd=-eye(2);
elseif(pade_ord==8)
    % 8th order pade approxiamtion
    Ad=[zeros(14,2) eye(14);-5.1892e8/h^8*eye(2) -2.5946e8/h^7*eye(2)
    -6.054e7/h^6*eye(2)-8.6486e6/h^5*eye(2) -831600/h^4*eye(2) -55440/h^3*eye(2)
     -2520/h^2*eye(2) -72/h*eye(2)];
    Bd=[zeros(14,2); eye(2)];
    Cd=[zeros(2) -5.1892e8/h^7*eye(2) zeros(2) -17297200/h^5*eye(2) zeros(2)
     -110880/h^3*eye(2) zeros(2) -144/h*eye(2)];
    Dd=eye(2);
end
A=[-1/tau_d Kd/tau_d; 0 a];
Abar=[0 0;1/tau_d -(Kp+Kd/tau_d)];
M=[Ad Bd;Abar*Cd A+Abar*Dd];
xdot=M*x;
```

A.3.2 Comparisons with Different Methods

ozbay_gundes.m

```
disp('PD controller design derived in Ozbay-Gundes 2007')
% Plant parameters Gp(s)=e^{-hs}/(s-p)
% Controller parameters C(s)=Kp+Kd*s/(tau_d*s+1)
p=1;
% h=[ 0.01 0.0316 0.07 0.1 0.13 0.15];
tau_d=0.01:0.01:1.5;
for (aa=1:length(h))
    for(kk=1:length(tau_d))
        % Find the minimum of mu over q
        q=0.001:0.005:0.5;
        w=0.01:0.1:10;
        for(ii=1:length(q))
            for(jj=1:length(w))
                phi(ii,jj)=abs((exp(-j*w(jj)*h(aa))-1)/(j*w(jj))+q(ii)*exp(-j*
                w(jj)*h(aa))/(tau_d(kk)*j*w(jj)+1));
            end
            mu(ii)=max(phi(ii,:));
        end
        [min_mu index]=min(mu);
        qopt=q(index);
        % Controller parameters are:
        Kp_max(kk)=1/min_mu;
        Kp_min=p;
        Kp=linspace(Kp_min+0.01,Kp_max(kk)-0.01,50);
        Kd=qopt.*Kp;
        % filtered derivative part
        % b=10;
        % Tf=qopt/b;
        for (ind=1:length(Kp))
            % Tf=Kd(ind)/b;
            dwell(kk,ind)=calc_dwelltime(h(aa),Kp(ind),Kd(ind),tau_d(kk));
```

```
        end
    end
    globalmin=500;
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
            if (dwell(ii,jj)==-1)
                dwell(ii,jj)=100;
            end
        end
    end
    for(ii=1:size(dwell,1))
        [minn index]=min(dwell(ii,:));
        if(minn<globalmin)
            globalmin=minn;
            ind=[index ii];
        end
    end
    result(aa,:)=[globalmin Kp_max(ind(1)) Kp(ind(1)) qopt tau_d(ind(2))];
end
```

silva.m

```
disp('PD controller design derived in Silva-Datta 2002')
% Plant parameters Gp(s)=Ke^{-Ls)/(Ts+1)
K=1;
T=-1;
L=[0.01 0.0316 0.07 0.1 0.13 0.15 0.155];
b=10;
N=10;
Tf=linspace(0.001,1,20);
for(aa=1:length(L))
    % Controller is in the following form: Cpd(s)=Kp+Kds
```

```
% alpha1 is the solution of tan(alpha1)=-T*alpha1/(T+L)
alpha=1:0.01:3;
for(ind=1:length(alpha))
    f(ind)= tan(alpha(ind))+T*alpha(ind)/(T+L(aa));
    if(f(ind)<0.05 && f(ind)>-0.05)
        alpha1=alpha(ind);
    end
end
% Kpmin<Kp<Kpmax
Kpmin(aa)=1/K*(T/L(aa)*alpha1*sin(alpha1)-cos(alpha1));
Kpmax(aa)=-1/K;
Kp=linspace(Kpmin(aa)+0.001,Kpmax(aa)-0.001,2*N);
for (ii=1:length(Kp))
    % Step 2: Find the roots of z1 and z2 of y(z)=0
    if (Kp(ii)<-14)
        z=0:0.005:pi;
    elseif(Kp(ii)>-4)
        z=0:0.005:pi;
    else
        z=0:0.005:pi;
    end
    clear y
    soln=[];
    for(ind=2:length(z))
        y(ind)=cos(z(ind))-T/L(aa)*z(ind)*sin(z(ind))+K*Kp(ii);
        if(y(ind)>0 && y(ind-1)<0)
            soln(1)=z(ind);
        elseif(y(ind)<0 && y(ind-1)>0)
            soln(2)=z(ind);
        end
    end
```

```
        z1=soln(1);
        z2=soln(2);
        for(ind=1:length(soln))
            if(soln(ind)-soln(1)>0.1)
                z2=soln(ind);
            end
        end
        % Step 3:Compute m1,m2,b1,b2,w1,w2
        m1=L(aa)^2/z1^2;
        m2=L(aa)^2/z2^2;
        b1=-L(aa)*(sin(z1)+T/L(aa)*z1*cos(z1))/(K*z1);
        b2=-L(aa)*(sin(z2)+T/L(aa)*z2*cos(z2))/(K*z2);
        w1=z1*(sin(z1)+T/L(aa)*z1*(cos(z1)+1))/(K*L(aa));
        w2=z2*(sin(z2)+T/L(aa)*z2*(cos(z2)+1))/(K*L(aa));
        Kdmin(aa,ii)=b1;
        Kdmax(aa,ii)=b2;
        Kd=linspace(Kdmin(aa,ii),Kdmax(aa,ii),2*N);
% Kd=mean(Kd)
        for(jj=1:length(Kd))
            % a standart PD controller is designed. Then add a filter to
            % derivative term Tf=Kd/Kp/b
            % Tf=Kd(kk)/b/-Kp(jj);
            Tf=linspace(0.7,0.85,N);
            for(kk=1:length(Tf))
                dwell(ii,jj,kk)=calc_dwelltime(L(aa),-Kp(ii),Kd(jj),Tf(kk));
            end
        end
    end
    globalmin=500;
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
```

```
            for(kk=1:size(dwell,3))
                if (dwell(ii,jj,kk)==-1)
                    dwell(ii,jj,kk)=100;
                end
            end
        end
    end
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
            [minn index]=min(dwell(ii,jj,:));
            if(minn<globalmin)
                globalmin=minn;
                ind=[ii jj index];
            end
        end
    end
    Kd=linspace(Kdmin(aa,ind(1)),Kdmax(aa,ind(1)),2*N);
    result(aa,:)=[L(aa) globalmin Kp(ind(1)) Kd(ind(2)) Tf(ind(3))];
end
result
```

huang.m

```
disp('PD controller design derived in Huang-Chen (1997)')
% Plant parameters Gp(s)=Ke^{-thetas)/(Ts-1)
K=1;
T=1;
eps=0.01;
N=100;
theta=[0.01 0.0316 0.07 0.1 0.13 0.15 ];
for(aa=1:length(theta))
    ind=1;
```

```
    clear y soln
    w=0:0.1:pi/theta(aa);
    for(ii=2:length(w))
        y(ii)=theta(aa)*w(ii)-atan(T*w(ii));
        if(y(ii)>0 && y(ii-1)<0)
            w2(aa)=w(ii);
        end
    end
    w1(aa)=0;
    % Lower and upper bounds of Ki
    Km(aa)=sqrt(1+T^2*w1(aa)^2);
    KM(aa)=sqrt(1+T^2*w2(aa)^2);
    % Choose b from the graph: For small delay b is quarter of the delay
% b(aa)=theta(aa)/4;
    % Controller parameters Cpd=Kc(bs+1)
    Kc(aa)=(Km(aa)+KM(aa))/2;
% Cpd(aa,:)=[Kc(aa)*b(aa) Kc(aa)];
    Kp=linspace(Km(aa)+eps,KM(aa)-eps,N/2);
    b=linspace(0.001,1,N/5);
    Tf=linspace(0.01,2,20);
    for(ii=1:length(Kp))
        for(jj=1:length(b))
            Kd=Kp(ii)*b(jj);
            for(kk=1:length(Tf))
                dwell(ii,jj,kk)=calc_dwelltime(theta(aa),Kp(ii),Kd,Tf(kk));
            end
        end
    end
    globalmin=500;
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
```

```
            for(kk=1:size(dwell,3))
                if (dwell(ii,jj,kk)==-1)
                    dwell(ii,jj,kk)=100;
                end
            end
        end
    end
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
            [minn index]=min(dwell(ii,jj,:));
            if(minn<globalmin)
                globalmin=minn;
                ind=[ii jj index];
            end
        end
    end
    result(aa,:)=[globalmin Kp(ind(1)) b(ind(2)) Tf(ind(3))];
end
result
```

hwang.m

```
disp('PD controller design derived in Hwang-Hwang 2004')
% Plant parameters Gp(s)=e^{-hs)/(s-1)
% Controller parameters C(s)=Kp+Kd*s
h=[0.01 0.0316 0.07 0.1 0.13 0.15 ];
eps=0.01;
% Filtered part of the derivative term
Tf=linspace(0.01,2,40);
N=50;
for(aa=1:length(h))
    Kd=linspace(0.01,1-eps,20);  % 0<Kd<1
```

```
Kp=linspace(1.2,50,50); % Kp>1
for(jj=1:length(Kd))
    for(ii=1:length(Kp))
        w=sqrt(-(1-Kp(ii)^2)/(1-Kd(jj)^2));
        f(jj,ii)=Kd(jj)-Kp(ii)+(1-Kp(ii)*Kd(jj))*cos(h(aa)*w)+
        (1-Kd(jj)^2)*w*sin(h(aa)*w);
        if (f(jj,ii)>-0.01)
            Kpmax(jj)=Kp(ii);
            break;
        end
    end
end
clear Kp
for(jj=1:length(Kd))
    Kp(jj,:)=linspace(1+eps,Kpmax(jj),N);
    for(ii=1:length(Kp))
        for(kk=1:length(Tf))
            dwell(ii,jj,kk)=calc_dwelltime(h(aa),Kp(jj,ii),Kd(jj),Tf(kk));
        end
    end
end
globalmin=500;
for(ii=1:size(dwell,1))
    for(jj=1:size(dwell,2))
        for(kk=1:size(dwell,3))
            if (dwell(ii,jj,kk)==-1)
                dwell(ii,jj,kk)=100;
            end
        end
    end
end
```

```
    for(ii=1:size(dwell,1))
        for(jj=1:size(dwell,2))
            [minn index]=min(dwell(ii,jj,:));
            if(minn<globalmin)
                globalmin=minn;
                ind=[ii jj index];
            end
        end
    end
    result(aa,:)=[globalmin Kp(ind(2),ind(1)) Kd(ind(2)) Tf(ind(3))];
end
```

saadaoui.m

```
disp('First order controller design derived in Saadaoui et al. 2008')
% Plant parameters Gp(s)=e^{-Ls)/(s-1)
L=[ 0.01 0.0316 0.05 0.07 0.085 0.1 0.115 0.13 0.15];
q=[1 -1];
% Do the pade approximation of order l
l=3;
for(aa=1:length(L))
    [pp,p]=pade(L(aa),l);
    % p=p./p(length(p));
    q0=conv(q,p);   % q0(s)=q(s)*p(s)
    p0=negate(p);   % p0=p(-s)
    % (h,g) & (f,e) are the even odd components of q0(s) and p0(s) respectively
    [h g]=even_odd(q0);
    [f e]=even_odd(p0);
    % (H,G) is the even-odd component of q0(s)p0(-s)
    % F(s^2):=p0(s)p0(-s)
    diff=length(conv(h,f))-length([conv(g,e) 0]);
    if (diff>0)
```

```
    H=conv(h,f)-[zeros(1,diff) conv(g,e) 0];
elseif(diff<0)
    H=[zeros(1,abs(diff)) conv(h,f)]- [conv(g,e) 0];
else
    H=conv(h,f)-[conv(g,e) 0];
end
diff=length(conv(g,f))-length(conv(h,e));
if (diff>0)
    G=conv(g,f)-[zeros(1,diff) conv(h,e)];
elseif(diff<0)
    G=[zeros(1,abs(diff)) conv(g,f)]- conv(h,e);
else
    G=conv(g,f)-conv(h,e);
end
diff=length(conv(f,f))-length([conv(e,e) 0]);
if (diff>0)
    F=conv(f,f)-[zeros(1,diff) conv(e,e) 0];
elseif(diff<0)
    F=[zeros(1,abs(diff)) conv(f,f)]- [conv(e,e) 0];
else
    F=conv(f,f)-[conv(e,e) 0];
end
% Find H1,H2,G1,G2,F1
H1=conv(H,F)-[zeros(1,length(conv(H,F))-length(conv(derivative(H),
derivative(F)))-1) conv(derivative(H),derivative(F)) 0];
H2=conv(G,F)-[zeros(1,length(conv(G,F))-length(conv(derivative(G),
derivative(F)))-1) conv(derivative(G),derivative(F)) 0];
G1=conv(derivative(H),F)-conv(H,derivative(F));
G2=conv(derivative(G),F)-conv(G,derivative(F));
F1=conv(F,F)-[zeros(1,length(conv(F,F))-length(conv(derivative(F),
derivative(F)))-1) conv(derivative(F),derivative(F)) 0];
```

```
% p1 and p2 polynomials, rbar
deg_q1=max(length(expand(H))-1,length(expand(G))-1);
p1=expand(F)+[0 derivative(expand(F))];
q2=expand(G1)+[0 derivative(expand(G1))];
p2=expand(G2)+[0 derivative(expand(G2))];
sigma=signature(p1);
rbar=floor((deg_q1-sigma-1)/2);
% Lastly, find the interval for alpha1 such that phi2=q2+alpha1*p2 has a
% signature (RHP roots-LHP roots) equal to 2*rbar
alpha=-100:10:200;
index=1;
for(ind=1:length(alpha))
    diff=length(q2)-length(p2);
    if (diff>0)
        phi2=q2+alpha(ind)*[zeros(1,diff) p2];
    else
        phi2=[zeros(1,diff) q2]+alpha(ind)*p2;
    end
    % find the signature
    sign_phi2(ind)=signature(phi2);
    if(sign_phi2(ind)==2*rbar)
        alpha1_range(index)=alpha(ind);
        index=index+1;
    end
end
% Algorithm 3.2 from his thesis
m=length(q2)-1;
n=length(p2)-1;
r=2*rbar;
[q2p2_even q2p2_odd]=even_odd(conv(q2,negate(p2)));
root=roots(q2p2_odd);
```

```
k=0;
for(ind=1:length(root))
    if(isreal(root(ind)) && root(ind)<0)
        v(ind)=root(ind);
        k=k+1;
    end
end
v=[sort(v) 0];
% Step 1
p2p2=conv(p2,negate(p2));
for (ind=1:length(v))
    if (polyval(p2p2,v(ind))~=0)
        alpha(ind)=-polyval(q2p2_even,v(ind))/polyval(p2p2,v(ind));
    end
end
% step 2: Fix alpha1 and find alpha2 range given q1 and p1
alpha1=0.35;
q1=expand(H)+[0 derivative(expand(H))]+alpha1*([zeros(1,length(expand(H))-
length(expand(G))) expand(G)]+[zeros(1,length(expand(H))-
length(derivative(expand(G)))) derivative(expand(G))]);
% Lastly, find the interval for alpha1 such that phi1=q1+alpha2*p1 has a
% signature (RHP roots-LHP roots) equal to 2*rbar
alpha=-23:30;
index=1;
for(ind=1:length(alpha))
    diff=length(q1)-length(p1);
    if (diff>0)
        phi1=q1+alpha(ind)*[zeros(1,diff) p1];
    else
        phi1=[zeros(1,diff) q1]+alpha(ind)*p1;
    end
```

```
    % find the signature
    sign_phi1(ind)=signature(phi1);
    if(sign_phi1(ind)==2*rbar)
        alpha2(index)=alpha(ind);
        index=index+1;
    end
end
% calculate the dwelltime
alpha1=1./linspace(0.65,0.9,30);
for(ii=1:length(alpha1))
    w=0.01:0.1:30;
    ind=1;
    fin=0;
    while(fin~=-1)
        qjw(ind)=polyval(q,j*w(ind));
        R(ind)=real(qjw(ind));
        I(ind)=imag(qjw(ind));
        alpha2_range(ind)=(I(ind)-alpha1(ii)*R(ind)/w(ind))*sin(L(aa)*w(ind))-
        (R(ind)+alpha1(ii)*I(ind)/w(ind))*cos(L(aa)*w(ind));
        alpha3_range(ind)=(w(ind)*I(ind)-alpha1(ii)*R(ind))*cos(L(aa)*w(ind))+
        (w(ind)*R(ind)+alpha1(ii)*I(ind))*sin(L(aa)*w(ind));
        if(ind>1)
            if(alpha3_range(ind)<alpha1(ii) && alpha3_range(ind-1)>alpha1(ii))
                fin=-1;
            end
        end
        ind=ind+1;
    end
    alpha2=linspace(alpha2_range(1),alpha2_range(ind-1),30);
    alpha3=linspace(alpha3_range(1),max(alpha3_range),50);
    for(jj=1:length(alpha2))
```

```
        for(kk=1:length(alpha3))
            Kp=alpha3(kk)/alpha1(ii);
            Kd=alpha2(jj)/alpha1(ii)-alpha3(kk)/alpha1(ii)^2;
            tau_d=1/alpha1(ii);
            dwell(ii,jj,kk)=calc_dwelltime(L(aa),Kp,Kd,tau_d);
        end
    end

end
globalmin=500;
for(ii=1:size(dwell,1))
    for(jj=1:size(dwell,2))
        for(kk=1:size(dwell,3))
            if (dwell(ii,jj,kk)==-1)
                dwell(ii,jj,kk)=100;
            end
        end
    end
end
for(ii=1:size(dwell,1))
    for(jj=1:size(dwell,2))
        [minn index]=min(dwell(ii,jj,:));
        if(minn<globalmin)
            globalmin=minn;
            ind=[ii jj index];
        end
    end
end
result(aa,:)=[L(aa) globalmin alpha3(ind(3))/alpha1(ind(1))
alpha2(ind(2))/alpha1(ind(1))-alpha3(ind(3))/alpha1(ind(1))^2
1/alpha1(ind(1))];
```

```
end
result
```

derivative.m

```
function derP=derivative(P)
% derivative of a polynomial whose coefficients are as a vector P
N=length(P);
for (ind=1:N-1)
    derP(ind)=P(ind)*(N-ind);
end
```

expand.m

```
function new_poly=expand(poly)
N=length(poly);
new_poly=[];
for(ind=1:N-1)
    new_poly=[new_poly poly(ind) 0];
end
new_poly=[new_poly poly(N)];
```

negate.m

```
function polyout=negate(polyin)
N=length(polyin);
for(ind=1:N)
    if (rem(N,2)==0)
        polyout(ind)=polyin(ind)*(-1)^ind;
    else
        polyout(ind)=polyin(ind)*(-1)^(ind+1);
    end
end
```

even_odd.m

```
function [even odd]=even_odd(poly);
N=length(poly);
ii=1;
jj=1;
for (ind=1:N)
    if(rem(N,2)==0)
        % if the length is even, then odd elements of the poly constructs
        % the odd poynomial
        if(rem(ind,2)==1)
            odd(ii)=poly(ind);
            ii=ii+1;
        else
            even(jj)=poly(ind);
            jj=jj+1;
        end
    else
        % if the length is odd, then odd elements of the poly constructs
        % the even poynomial
        if(rem(ind,2)==1)
            even(ii)=poly(ind);
            ii=ii+1;
        else
            odd(jj)=poly(ind);
            jj=jj+1;
        end
    end
end
```

signature.m

```
function sigma=signature(p1)
% Find the difference between the LHP and the RHP roots of a polynomial
root=roots(p1);
sigma=0;
for(ind=1:length(root))
    if(real(root(ind))<0)
        sigma=sigma+1;
    elseif(real(root(ind))>0)
        sigma=sigma-1;
    end
end
```

calc_dwelltime.m

```
function dwelltime=calc_dwelltime(h,Kp,Kd,tau_d)
eps=0.01;
a=1; % Assumed to be 1
N=10;
Td=[];
% fix p
p=1.01;
index=1;
betamin=0;
betamax=2/(h*p*tau_d);
if (betamax<10)
    beta=linspace(betamin+eps,betamax-eps,2*N);
elseif(betamax<100)
    beta=linspace(betamin+eps,betamax-eps,2*N);
else
    beta=linspace(betamin+eps,betamax-eps,3*N);
end
for (aa=1:length(beta))
```

```
alphamin=0;
alphamax=2/(p*h*tau_d)-beta(aa);
if (alphamax<0)
    break;
else
    if (alphamax <10)
        alpha=linspace(alphamin+eps,alphamax-eps,2*N);
    elseif(alphamax <100)
        alpha=linspace(alphamin+eps,alphamax-eps,2*N);
    else
        alpha=linspace(alphamin+eps,alphamax-eps,3*N);
    end
end
for (bb=1:length(alpha))
    P=[1 0;0 1];
    lambda=max(svd(P))/min(svd(P));
    A=[-1/tau_d Kd/tau_d;0 a];
    Abar=[0 0; 1/tau_d -(Kp+Kd/tau_d)];
    S=-(P*(A+Abar)+(A+Abar)'*P+h*(P*Abar*A*inv(P)*A'*Abar'*P/alpha(bb)+P*
    (Abar)^2*inv(P)*(Abar')^2*P/beta(aa)+p*(alpha(bb)+beta(aa))*P));
    w=min(svd(S));
    X=[((A+Abar)'*P+P*(A+Abar))/h+p*(alpha(bb)+beta(aa))*P P*Abar*[A Abar];
    [A Abar]'*Abar'*P [-alpha(bb)*P zeros(2);zeros(2) -beta(aa)*P]];
    eigenX=eig(X);
    if ( w>0)
        if( eigenX(1)<0 && eigenX(2)<0 && eigenX(3)<0 && eigenX(4)<0 &&
        eigenX(5)<0 && eigenX(6)<0)
            mu=max(svd(P))/w;
            Td(index)=lambda*mu*floor((lambda-1)/(p-1)+1);
            ind(index,:)=[beta(aa) alpha(bb)];
            index=index+1;
```

```
            end
        end
    end
end
if (~isempty(Td))
    [minn min_index]=min(Td);
    result=[minn ind(min_index,:)];
    dwelltime=minn+2*h;
else
    dwelltime=-1;
end
```

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