

DOES BITCOIN IMPROVE OPTIMAL PORTFOLIOS? A STOCHASTIC SPANNING APPROACH

A Master's Thesis

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September 2020

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Bilkent University 2020



To my family
and
to, the memory of my dear father,
Abdolhossein Rahiminejat,
who always encouraged me and my siblings to study hard

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The Graduate School of Economics and Social Sciences

of

İhsan Doğramacı Bilkent University

by

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In Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

THE DEPARTMENT OF

MANAGEMENT

İHSAN DOĞRAMACI BİLKENT UNIVERSITY

ANKARA

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ABSTRACT

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September 2020

The thesis evaluates the impact of Bitcoin as a means of portfolio diversification on different stochastically efficient portfolios. Here, the stochastic efficient portfolios are the results obtained by applying the stochastic spanning model on 11 different asset classes of various sectors of the financial market. Bitcoin exclusive and inclusive portfolios are compared with Sharpe ratio. Results reveal that in most of the cases, Bitcoin improves the optimal portfolio and should be considered as an asset to be included in investments.

Keywords: Bitcoin, Diversification, Optimal Portfolio, Stochastic Dominance, Stochastic Spanning

ÖZET

BITCOIN OPTİMAL PORTFÖYLERİ İYİLEŞTİRİR Mİ? BİR STOKASTİK YAYILMA YAKLAŞIMI

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Eylül 2020

Bu Tezde, Bitcoin'in çeşitli stokastik verimli portföylerde bir risk yayma aracı olarak etkisi değerlendirilmektedir. Burada kullanılan stokastik verimli portföyler, finansal piyasanın çeşitli sektörlerinin 11 farklı varlık sınıfı üzerinde stokastik yayılma modelinin uygulanması sonucu elde edilmiştir. Bitcoin'i içeren ve içermeyen bu portföylerin performansı Sharpe Oranı ile karşılaştırılmaktadır. Sonuçlar, çoğu durumda Bitcoin'in optimal portföyü geliştirdiğini ve yatırımlara dahil edilecek bir varlık olarak değerlendirilmesi gerektiğini ortaya koymaktadır.

Anahtar Kelimeler: Bitcoin, Optimal portföy, Riski yayma, Stokastik üstünlük, Stokastik yayılma

ACKNOWLEDGEMENT

First of all, I would like to thank my advisor, Asst. Prof. Dr. Ahmet Şensoy, for his excellence guidance throughout the M.Sc. study at Bilkent University. Without his continuous support, this accomplishment would not have been possible.

I would like to thank my thesis defense jury members, Assoc. Prof. Dr. Fehmi Tanrısever and Asst. Prof. Dr. Ali Coşkun, for reviewing my thesis and providing me with insightful advice and comments.

Also, I want to thank my family members for their relentless and spiritual love and support from the beginning years of my life up to now. They have always inspired me to stay motivated and overcome the difficulties. Furthermore, I want to give my deepest thanks and appreciation to my uncle, Kazem Rahiminejat, who has supported me in every way throughout these years.

I would like to thank my friends, specially Dr. Okan Demir and Dr. Raziye Javanmard, who supported and helped me a lot. I do also appreciate my dear uncle, Kazem Rahiminejat, for his immeasurable and tremendous help and support throughout my life.

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CHAPTER I: INTRODUCTION

Optimization has long been a part of our everyday life. Since the first days of the school, we were dealing with managing our pocket money to spend it in much more efficient ways and buy a variety of junky foods and not to be short in money soon. These days, we optimize our time to do all our duties and to follow our various aims at the same time.

In the finance area, also, optimization helps us in having the most efficient portfolio for a preferred level of risk. In that, optimization methods assist investors in picking the most optimal portfolios according to their level of risk tolerance. There are several methods of optimization where two famous ones are Markowitz Portfolio Theory (MPT) and stochastic dominance (Roman and Mitra, 2009).

According to MPT, mean and variance, or first and second moments of distributions, are enough to select the optimal portfolio. Therefore, the MPT method is considered as a simple scheme although it is a “normative theory”. In general, a “normative theory” characterizes a class of normal behavior that financiers should follow to optimize a portfolio (Fabozzi et al., 2002). This concept brings us to the shortfalls and problems of MPT which are the subjects of Chapter 2 of this study.

In Chapter 2, we introduce stochastic dominance which is a pairwise scheme for comparison of two alternatives. Stochastic dominance is defined by different degrees, and in those degrees the existence of the same order of the moment is assumed. The most applicable order of stochastic dominance in the financial world is the second-order stochastic dominance, (SSD) (Fábián et al., 2011). Although SSD fills some gaps of in the MPT method, it still has its own shortages. Deficiencies of SSD or in general stochastic dominance bring us to introduce the concept of stochastic spanning which is also introduced in detail with theorems behind it in Chapter 2.

In Chapter 3, we move to a totally different subject, cryptocurrencies and specifically Bitcoin. In this chapter, various aspects of this novel asset are defined and explained, including its efficiency, pricing, intrinsic value, and correlation with traditional assets. Finally, we review some studies that consider Bitcoin as a hedging tool, a safe haven, and a means of portfolio diversification. In that, properties of Bitcoin, such as negligible correlation with other asset classes, incur the idea of applying Bitcoin as a diversifier.

In this thesis, it is decided to include Bitcoin in an optimal portfolio and see if this virtual currency improves the performance of the optimal portfolio. We examine our hypothesis based on 11 different asset classes. Here, the formal way of constructing the optimal portfolio, pre- and post-Bitcoin inclusion, is the stochastic spanning method. In Chapter 4, we report the changes and improvements in the expected shortfall with the inclusion of Bitcoin in our asset universe. We also compare the optimal portfolios, before and after including Bitcoin, in terms of their performance.

The main contribution of the present study to literature is the inclusion of Bitcoin to an optimal portfolio, selected based on the stochastic spanning method, and checking the improvements and differences. Finally, in the conclusion chapter, we see that including Bitcoin prospers the performance of the portfolio in several asset classes under this study. We finish this document by coming up with some areas for further research.



CHAPTER II: DIVERSIFICATION AND POTFOLIO OPTIMIZATION METHODS

2.1 Diversification

Diversification is defined as money allocation among various investment opportunities. The term represents an attempt to include various assets in the portfolio to reach variety. In the science of finance and financial planning, diversification is a technique for risk management where reducing the risk of a portfolio is attainable through incorporating various types of assets with different ratios. Precisely, diversification is not only asset allocation but also portfolio selection. In this chapter, we explain the latter one and a couple of its methods in detail, and the former one is described in Chapter 5. These two steps assist in lowering the risk of the portfolio and in reducing bias toward the home country.

Portfolio's risk, volatility, is measured by returns' standard deviation, and it involves two adverse kinds of risks: a) idiosyncratic risk, also known as unsystematic and firm-specific risk, and b) systematic risk, which is also named as market and undiversifiable risk. The latter affects the whole market, and not a specific type of stock. The former, nonetheless, influences a certain kind of industry. The market risk accounts for instability in the

equity market, fluctuation of interest rates, and undesirable changes throughout financial systems. All these unwanted changes have, however, some features in common: they are unpredictable and completely avoidable. In contrast to the market risk, which is uncontrollable, idiosyncratic risk can be extremely reduced by the means of diversification. In that, the efficient market hypothesis suggests that tolerating unsystematic risk has no reward for the investors since this type of risk can be diversified away (Gold, 1995).

Unsystematic risk is a type of risk that is specific to a single asset, such as a particular asset class, stocks of a specified company, or a particular sector of the economy. The problem with unsystematic risk is that it works as a major source of uncertainty and fluctuation in the price of an asset. The good news is that the harmful effect of idiosyncratic risk can be reduced by diversification through an “equal-weighted portfolio variance measure” (Goyal and Santa-Clara, 2003). They also affirm that idiosyncratic risk constitutes a large portion of the stock’s total risk, namely volatility.

Being aware that not all industries and asset sectors change with the same magnitude and in the same direction will help us reduce the volatility of the portfolio via diversification. Precisely, including various non-correlated assets in a portfolio can nearly remove the firm-specific risk of the portfolio. Jacob (1974) insists that unsystematic risk can be reduced drastically with the inclusion of a few judiciously selected securities.

Another issue to be discussed about diversification is home country bias in which investors prefer to fund in the domestic market or even the market of their industry or state. French and Poterba (1991) show that investors in Japan, Britain, and the U.S. anticipate that the expected returns in their own

countries are by far higher than the returns in international markets. Yet, the great opportunities that international diversification provides for investors are undeniably tremendous because it lowers risk for periods of the domestic financial crisis. For instance, in their investigations, Driessen and Laeven (2006) have controlled two factors, the influence of currencies and constraints on short selling, in stock markets of developing countries. By controlling these two factors, they realize that the gains of investing internationally were higher for developing countries' investors.

Since the initial stages in which diversification theories were developed, there exist different methods and approaches for having the most optimal portfolios. The very first one is rules of thumb. According to this method, financiers should allocate 100 minus their age percent of their capital to stocks, and the rest is to be invested in secure A-grade bonds or very safe investment opportunities like government debt. However, since life expectancy has increased in recent years, the rules of thumb have been modified. In the modified version, financiers, who can bear a higher level of risk, invest an amount equal to 110 or 120 minus their age percent of their budget in the stocks.

The logic behind the rules of thumb method is that young financiers should allocate a greater ratio of their capital to risky assets like stocks because they have a long time ahead compared to older investors with shorter time horizons. In that, financiers with long time frames can tolerate short-run volatility of risky assets to appreciate the greater returns of them. The other way around stands for investors with short-term frames. To these investors, since they want to spend cash flows of the investment sooner, safer investments with lower volatility are much more appealing.

Picking a small number of randomly selected assets and assigning some weights to them is another scheme for portfolio diversification. Precisely, when randomly selected stocks, combined with equal weights, to form a portfolio, the higher the number of diverse assets, the lower the volatility of the portfolio is. It is important to notice that diversification power can restrict only the unsystematic risk but has no control over the systematic risk of the market which means that risk will never be omitted totally. Besides, by adding stocks, the risk of the portfolio declines at a decreasing pace. In that, It is indicated that a well-diversified portfolio of stocks consists of at least 30 stocks for a borrower investor and 40 stocks for a lender one (Statman, 1987).

Some other studies also emphasize the effects of diversification by mutual funds. For instance, O'neal (1997) indicates that the expected volatility of the last-period wealth is reduced significantly by including more than one mutual fund in the portfolio. In addition, adding funds decrease downside risk, which is the risk that realized returns being less than the expected returns. The problem with the method of picking randomly selected assets is adding the same types of stocks, for example, all technology stocks, or all financial ones. By picking randomly selected assets, the gains of diversification are significantly restricted. In some extreme cases, the risk of the portfolio, for instance, might be even raised. Nevertheless, risk reduction with forming a portfolio of a few numbers of stocks is almost implausible. As a result, individual investors are recommended to invest in index funds which are famous for limited costs of transaction and extensive diversification.

One of the traditional approaches to diversify a portfolio is no decision criteria. According to this scheme, multiple randomly selected assets are weighted to form a portfolio, and, of course, the portfolio might not be an

optimal one with the lowest possible level of risk. Here, we should point that benefits of the diversification method are subject to the correlation between included assets' expected returns. In other words, in a successful diversification, assets' returns are weakly correlated. To understand the point perfectly, consider an extreme case of a portfolio with only two perfectly positively correlated stocks. In this case, the risk of the portfolio is the summation of the risk of both stocks which means a higher level of risk through diversification. In fact, when returns of assets are uncorrelated, Kolm et al. (2014) assert that the higher the level of diversification, the more negligible the portfolio risk is. Nonetheless, when assets are correlated, even with an endless diversification, risk can remain considerable.

2.2 Mean-Variance Analysis, or MPT

Mean-Variance analysis, which is also called the Markowitz model or MPT, is the most famous technique of portfolio optimization. In this method, evaluation is based on two factors one of which is the expected return or mean and the other one is risk or variance portfolios. Mean or expected return is the sum of weighted returns of all assets included in the portfolio, and risk is the portfolio's standard deviation. In MPT analysis, risk-averse investors are helped to create a portfolio that have a maximized return for each level of market risk. Since risk is an intrinsic part of higher return, risk-averse financiers either prefer a less volatile portfolio to a more volatile one for each level of return or expect more return to bear more risk (Markowitz, 1978).

In fact, MPT maximizes return for a given level of risk and minimizes risk for a defined level of return to find the most efficient portfolios. To detect all these optimal portfolios, whole combinations of based assets are depicted

on a graph, where the X-axis is for risk of the portfolio and the Y-axis shows the portfolio's expected return. Among all these individual portfolios, ones with the highest return for the same level of risk, and the lowest risk for the same level of return are optimal. The set of all such portfolios is called Markowitz efficient frontier. Sub-optimal portfolios lie below or on the right side of this frontier since they do not come up with enough reward for a defined level of risk and have a higher risk for the same level of return, respectively.

Although the MPT scheme is easy to translate and does not demand complicated calculation, there are some main drawbacks with this method in practice. The more base assets in the portfolio, the more parameters to estimate, and the higher the estimation error will be. Besides, the portfolio weights are significantly affected by the estimation errors. For example, Black and Litterman (1990) claim that when a portfolio is optimized based on the Markowitz method, the weights of some assets are extreme or do not make sense.

The second problem with the MPT method is its unrealistic assumptions. In this method, investors are diversifiers, but not all financiers are into diversification in real life. Besides, according to MPT, the risk is identified by a single measure which is variance. This is unsound (Hanoch and Levy, 1969) since higher-order moments and their roles to define and determine the risk of a portfolio are ignored. In that, the mean and variance of distribution cannot explain all perspectives of returns' distribution. Another unrealistic assumption of Markowitz is the unlimited access of investors to borrow and lend capital at the risk-free rate.

All these barriers with MPT apart, the main problem with mean-variance is when we consider it in the context of expected utility hypothesis, EUH. Expected utility is “the utility that an entity or aggregate economy is expected to reach under any number of circumstances”. The expected utility is defined as the weighted average of whole feasible outcomes subject to certain conditions, and the weights, here, are probabilities of events. In that, EUH is a tool that helps individuals make decisions under uncertainty, and people pick the action with the greatest expected utility. Calculation of the expected utility requires, first, to compute the products of utility and probability for all outcomes. The summation of these products is expected utility which is also called von Neumann-Morgenstern function which has got different forms like quadratic function.

The expected utility of quadratic function can be formulated with terminal wealth's both mean (expected return) and the variance (or its square root, standard deviation) (Bailey, 2005). But, in general, all the moments of the probability distribution are required to determine a utility function (Hadar and Russell, 1969). To be exact, in the case of the polynomial utility function of degree n , the expected utility is determined by the first n moments (Richter, 1960). So, the utility function of degree two, which is quadratic, needs just mean (first moment) and variance (second moment) to be defined, and this function does not depend on other distribution's parameters.

The point is a great number of scientists disregard the quadratic function due to its special features. To be exact, with an increase in wealth, according to quadratic utility, risk aversion increases as well, which is contrary to what we see in the finance world. In fact, the wealthier investors are, the less they are willing to pay for insurance. Besides, for quadratic utility function, marginal utility is positive only for a limited range, and

marginal utility might not show rational behavior outside that range (Hanoch and Levy, 1970).

Despite all the deficiencies of quadratic utility, it is a sufficient and necessary condition based on which the MPT's outcome corresponds exactly with the result of utility maximization. Another condition under which the outcome of these two methods matches is when assets' returns have a multivariate normal distribution. To be exact, the distribution of assets' returns must be one that linear combinations of the returns have also the same two-parameter distribution (Feldstein, 1969). Levy and Markowitz (1979) also posit that the investors' expected utility can be maximized by a proper portfolio of the efficient set if and only if investors expect nearly normally distributed assets' returns, or if the utility function is approximately quadratic.

The normality of returns assumption is not what financiers realize in practice. In the finance real world, since negative deviations are considered with higher weights (De Giorgi and Post, 2008), returns are skewed or nonsymmetric, so normal distribution cannot capture all the features of the returns. To explain more, skewness is a characteristic that cannot be measured by mean and variance, but the third moment of a distribution. Thus, considering mean-variance as a special case of EUH is meaningless. For securities with nonsymmetric returns, as a result, some studies considered a third moment, namely skewness, along with two first moments. Simann (1993) develops a new method for portfolio diversification based on three first moments of a distribution. In this model, variance, and skewness, both determine the attitude of investors toward risk.

Kurtosis is another example of the distribution's characteristics that are not grabbed by the first and second moments of distributions. Kurtosis, or fourth moment, evaluates the inclination of returns to locate far from or close to their means. Some evidence asserts that rates of returns and asset prices have fat-tail distributions which means that outliers and extreme gains and losses are more than normal distribution's ones. As a result, the assumption of normality is not meaningful from an economic point of view (Bailey, 2005). Thereby, some studies discuss the optimal portfolio model when four first moments, or specifically kurtosis, are included in the model. The focus of variance and kurtosis is on dispersion. The only difference between these two is that kurtosis evaluates extreme values with higher weights. Based on this difference, Athayde and Flôres (2003) include kurtosis instead of variance to construct a portfolio frontier. This paper extends the model in Athayde (2001) which is just based on three first moments.

Lastly, it should be mentioned that MPT considers variance to measure risk although standard deviation is a much appropriate measure to quantify dispersion (Kolm et al., 2014). As of last words against the mean-variance theorem, it considers portfolios that no risk averter investors that into utility maximization would pick. Also, this MPT disregards some portfolios that plenty of risk averters may consider optimal, according to Aharony and Loeb (1977).

All in all, these drawbacks and problems with MPT persuaded scientists to search and follow more realistic and feasible models for portfolio optimization. One of these methods is the mean-semivariance (M-S) method which is similar to the M-V method, but, here, the risk factor is measured by semivariance. Semivariance is calculated in the same way as variance, yet semivariance is specifically for the outcomes below the expected return. Proter (1974) asserts that portfolios that are efficient

according to the M-S method are also efficient based on second-degree stochastic dominance.

Second-order stochastic dominance, SSD, which considers all moments of the return distribution, is also another substitute for the MPT method. SSD is a special case of stochastic dominance which is the subject of the next chapter.



CHAPTER III: STOCHASTIC DOMINANCE vs. STOCHASTIC SPANNING

3.1 Stochastic Dominance

Stochastic dominance, SD, is a concept or method about choice under risk, derived for the ordering of uncertain prospects by a particular set of investors. SD is specified by orders depending on the characteristics of decision-makers, which is defined by the utility functions of them. For example, first-order stochastic dominance, FSD, considers the set of decision-makers with weakly increasing utility functions, in that risk-loving investors who are rational. Rationality, here, means investors prefer higher return and wealth to the lower ones. In the same way, the second-degree stochastic dominance, SSD, characterizes risk-averse agents with a weakly increasing utility function. The utility set for SSD is a subset of FSD's one, and the difference is that this time, increasing in the wealth of investors results in decreasing marginal utility. In other words, utility function must be non-decreasing and concave in second-order stochastic dominance.

Finally, third-order stochastic dominance accounts for risk-averse investors with non-decreasing utility functions and decreasing absolute risk aversion (positively skewed utility function) (Dentcheva and Ruszczyński, 2010).

Decreasing absolute risk aversion (DARA) means that with an increase in wealth, investors are less willing to pay for insurance at a level of risk. So, the utility set, here, is a subset of the one for SSD. There are higher, n^{th} , - order stochastic dominance rules which do not have that much economic meaning, and their utility's shape is much more restricted (Levy, 1992). In the case of n^{th} order stochastic dominance, all odd derivatives are positive, and all even derivatives are negative.

In fact, when $F(x)$ dominates $G(x)$ by n^{th} - order stochastic dominance, all investors whose utility functions are in that class will achieve more or equal utility by investing in F option (Broske and Levy, 1989). These general definitions and features bring us to the point of the formal and mathematical definition of three first-order stochastic dominance. The note here is that the risk-less asset is not included in the set of base securities, and this case would be considered separately.

Definition 3.1.1. Consider two lotteries A and B with cumulative distribution functions $F(x)$ and $G(x)$, respectively. The lottery A is first-order stochastically dominates the lottery B (Quirk and Saposnik, 1962) if and only if for every non-decreasing utility function $u(x)$, we have

- I. $F(x) \leq G(x)$ for all x , and $F(x) < G(x)$ for some x
- II. *Lottery A is preferred to lottery B under all non-decreasing utility functions, which means,*

$$\int u(x)dF \geq \int u(x)dG \text{ for all } x \Rightarrow E_F(u(x)) \geq E_G(u(x)) \text{ for all } x. \quad (1)$$

Definition 3.1.2. Cumulative distribution function F is second-degree stochastically dominate cumulative distribution function G (Fushburn, 1964) iff for all non-decreasing concave utility function $u(x)$

$$\int_{-\infty}^c G(x)dx \geq \int_{-\infty}^c F(x)dx \text{ for all } c, \text{ and } F(x) \neq G(x) \text{ for some } x$$

$$\Rightarrow E_F(u(x)) \geq E_G(u(x)) \text{ for all } x \quad (2)$$

Definition 3.1.3. Cumulative distribution function G is third-order stochastically dominated by another distribution F iff for each utility function with the negative second derivative and positive first and third derivatives we have

$$\int_{-\infty}^x \int_{-\infty}^v [G(t) - F(t)] dt dv \geq 0 \text{ for all } x. \quad (3)$$

What is obvious is FSD implies SSD, and SSD also implies TSD. As a result, FSD results in TSD (Levy, 1992). In fact, n^{th} order stochastic dominance implies $(n + 1)^{\text{th}}$ order stochastic dominance. Besides, for all SD orders, $E_F(x) \geq E_G(x)$ is a necessary condition. Till now, the disadvantage of SD is the big efficient set that this method results in, and not being able to compare two risky choices. For this reason, the case with a risk-free asset is considered to obtain a more reasonable conclusion. We call the stochastic dominance case with a risk-free asset SDR for short.

Definition 3.1.4. Consider X and Y as returns of two risky securities and r as the return of risk-less asset. $\{X_\alpha: X_\alpha = \alpha X + (1 - \alpha)r \text{ \& } \alpha > 0\}$ and $\{Y_\beta: Y_\beta = \beta Y + (1 - \beta)r \text{ \& } \beta > 0\}$ are linear combinations of risk-less asset and risky ones. The distribution function of $\{X_\alpha\}$, $F_\alpha(x)$, and $\{Y_\beta\}$, $F_\beta(y)$, are given by

$$F_\alpha(x) = F(X_\alpha \leq x), \text{ and } F_\beta(y) = F(Y_\beta \leq y) \quad (4)$$

X dominates Y in SDR frame iff for each element of $\{Y_\beta\}$, there exists at least one element in $\{X_\alpha\}$ that dominates it in the stochastic dominance framework (Levy and Kroll, 1976). In the same study, it is also asserted that SDR sets of efficient portfolios are not larger than the original SD efficient sets. In that, SDR efficient sets are markedly smaller in size than the SD's ones, and this result states for all three first stochastic dominance orders. Furthermore, the relationship between different orders of SD and SDR are as follow:

$$\begin{array}{ccccc} FSD & \Rightarrow & SSD & \Rightarrow & TSD \\ \Downarrow & & \Downarrow & & \Downarrow \\ FSDR & \Rightarrow & SSDR & \Rightarrow & TSDR \end{array}$$

Besides, the transitivity of these rules results that, for example, FSD concludes SSDR, TSD, and TSDR.

Another advantage of stochastic dominance is considering risk and reward at the same time and as a part of the distribution of returns. In that, unlike the mean-variance theorem that considers variance as a measure of risk, there is no need for a single Index of risk in SD scheme (Falk and Levy, 1989). Besides, SD considers no assumption for the distribution of returns and limited preference assumptions for the utility function. The limited assumptions of SD compared to other methods, especially the mean-variance method, make it easier to work with. For example, SD does not need to consider normally distributed returns to achieve the same result of utility maximization. In other words, this method is a distribution-free method, and the distribution of returns can be discrete, continuous, or a combination of both (Falk and Levy, 1989). Still, having considered a specific type of distributions, the obtained results of SD matches the other theories and methods. For instance, with the assumption of normally

distributed returns and risk aversion, the results of SD are in accordance with MPT.

Follows are some theorems and results obtained by stochastic dominance.

Theorem 3.1.1. *consider two nonnegative random variables with density functions of f and g , equal means, and finite variances. If f is greater than g in terms of SSD, then the variance of f is smaller than the one for f (Hadar and Russell, 1971).*

Theorem 3.1.2. *F and G are distribution functions of two random variables X and Y , respectively. Both X and Y are independent of random variable W . Consider random variables $aX + bW$, and $aY + bW$, for $a > 0$ and $b \geq 0$ and with distribution functions of F' and G' , respectively. So,*

If F is greater than G with respect to FSD (or SSD), then F' is greater than G' with respect to FSD (or SSD) (Hadar and Russell, 1971).

Theorem 3.1.3. *X and Y are returns on two investment opportunities with respectively F and G cumulative distribution functions, and they are normally distributed, where*

$$X \sim N(\mu_1, \sigma_1) \quad (5)$$

$$Y \sim N(\mu_2, \sigma_2) \quad (6)$$

F will first-order stochastically dominate G iff these conditions satisfy:

$$\mu_1 > \mu_2$$

$$\sigma_1 = \sigma_2.$$

What is obvious is that under these rules, we can get SSD and TSD, as well.

Theorem 3.1.4. *Let $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$ be random returns of two options for investment with cumulative distribution functions F and G , respectively. F second-degree stochastically dominates G iff F dominates G by the mean-variance rule.*

Application of the SD criterion needs identification of mean and variance of both all portfolios and the whole probability distribution (Aharony and Loeb, 1977). But another superiority of the SD method is that this method does not ask for a risk index. In that, since SD considers the entire distribution of returns in the process of assessment, the mean and variance are considered as a part of the distribution.

Now, we consider a case when risk-less asset is included.

Theorem 3.1.5. *Consider $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$ as returns on two investment opportunities, and for r , the risk free rate, we have $\mu_1 > r$ and $\mu_2 > r$. X FSDR dominates Y (Levy, 2015) iff*

$$\frac{\mu_1 - r}{\sigma_1} > \frac{\mu_2 - r}{\sigma_2} \quad (7)$$

There are several applications for SD in different fields of knowledge such as Economics, Agricultural Economics, Finance, and Medicine. Atkinson (1970) applies the SD method to measure inequality of wealth, consumption, or income. Jarrow (1986) states that if the market is complete (in complete markets, any contingent price is attainable), the first-order stochastic dominance is a sufficient and necessary condition for the existence of an arbitrage opportunity. In the case of Agricultural Economics, Harris and Mapp (1986), for example, apply stochastic dominance to assess

the water-conserving methods for irrigation, and find six methods of irrigation that first-order stochastically dominate the current methods.

Besides, Broske and Levy (1989) try first and second-order stochastic dominance to quantify the default risk of bonds. Also, Falk and Levy (1989) employ stochastic dominance rules of first, second, and third-degree to show that the market is efficient, the thing that was not shown in the CAPM-based framework of Watts's paper (Watts, 1978). In the R&D department of companies, SD rules are applied to select the optimal tactic among all efficient strategies determined by SSD (Arditti and Levy, 1980). Finally, to choose between two medications, Stinnett and Mullahy (1998) apply first- and second-degree stochastic dominance.

Despite all the advantages and applications of stochastic dominance, there are some drawbacks to this method. One of the main problems with the SD method is that this method is for pairwise comparison, but not to compare all feasible portfolios (Davidson and Duclos, 2000). Namely, SD can tell us if option 1 dominates options 2 and 3, but this method cannot help us to find a combination of all these options that dominates all other combinations of assets. Even in the case of comparing two options, on the other hand, this method gives us no clue about diversification. In other words, the percentage allocated to the dominant option is not determined by this method (Levy, 2015), and this is the second main problem of the SD approach. Furthermore, the SD method results in a large efficient set and lacks any algorithm to construct efficient portfolios.

As a result, plenty of studies try to find other schemes and alternatives that do not deal with these problems. (Rockafellar and Uryasev, 2000) introduce a new approach based on Value-at-Risk (VaR) computation and

Conditional Value-at-Risk (CVaR) minimization, instead of VaR optimization. This method is suitable for portfolio optimization when many base assets are available. The approach in CVaR method is to diminish the chance of extreme losses.

Anderson et al. (2019) introduce the concept of Utopian Index, which measures the distance to the lower bound of Integrated c.d.f., and Dystopian prospect, the least favorable available options. These measures are based on second-order stochastic dominance, and they rank options in a choice set. Besides, Anderson et al. generalized the concept of Almost Stochastic Dominance to make a comparison among any number of prospects, and to narrow the choice set of optimal options.

3.2 Stochastic Spanning

A recent study of Arvanitis et al. (2019) introduces a new concept and method for portfolio optimization for the very first time, “stochastic spanning”. As it is stated in their paper, stochastic spanning, “spanning occurs if introducing new securities or relaxing investment constraints does not improve the investment possibility set for a given class of investors” (Arvanitis et al., 2019: 573). This definition and the idea behind stochastic spanning is similar to ones for mean-variance spanning and intersection method.

As Huberman and Kandel (1987) show, if the efficient frontier of some base assets matches, in only one point, with the efficient set of the same base assets plus new ones, there exists one utility function, mean-variance one, that is called intersection. Intersection means that there is just one mean-

variance utility function that adds no benefit if we annex those new assets to our base assets. On the other hand, when the efficient frontier of these two sets, set of base assets and set of base assets plus new ones, coincides, spanning occurs. In this case, there is no advantage for a mean-variance financier to expand the set of base assets.

Nevertheless, the superiority of stochastic spanning is stochastic spanning can be applied to any number of base assets with any type of distribution, unlike the M-V method which must be applied to a market index with normally distributed returns. Besides, base assets can be either just risky securities or portfolios of different assets Arvanitis et al. (2019). In addition, this method is based on second-order stochastic dominance, and considers not only variance, but also moments with higher-order, and is a distribution-free model of mean-variance spanning.

Furthermore, unlike the SD method, which is to compare two given prospects such as two portfolios or two medical treatments, stochastic spanning considers the comparison of two sets of options and performs dominance analysis for each portfolio in these sets. Efficiency analysis is a special type of stochastic spanning where at least one of the sets consists of only one portfolio. Next, we consider the assumptions, theories, and measures behind the stochastic spanning method.

Consider random returns on M base securities as $X := (x_1, \dots, x_M)$ with a support set. The support is bounded by of $\mathcal{X}^M := [\underline{x}, \bar{x}]$, $-\infty < \underline{x} < \bar{x} < +\infty$. Let $\Lambda := \{\lambda \in \mathbb{R}_+^M : 1_M^T \lambda = 1\}$ be the set of all feasible portfolios. Through this definition of the opportunity set, limited risk-free borrowing (via longing risky securities and shorting a riskless asset) and bounded short selling (by longing risk-free security and shorting risky assets) are allowed.

Definition 3.2.1. Consider $F: \mathbb{R}^M \rightarrow [0,1]$ as joint cumulative distribution function of X which is also continuous and $F(y, \lambda) := \int 1(X^T \lambda \leq y) dF(X)$ as the marginal cumulative distribution function of $\lambda \in \Lambda$. The expected shortfall for $x \in \mathcal{X}$ level of return is defined as

$$F^{(2)}(x, \lambda) := \int_{-\infty}^x F(y, \lambda) dy = \int_{-\infty}^x (x - y) dF(y, \lambda). \quad (8)$$

If portfolio $\tau \in \Lambda$ is weakly second-degree stochastically dominated by portfolio $\lambda \in \Lambda$ or $\lambda \succeq_F \tau$ if

$$F^{(2)}(x, \lambda) \leq F^{(2)}(x, \tau) \quad \forall x \in \mathcal{X} \quad (9)$$

And if $\tau \in \Lambda$ is strictly second-degree stochastically dominated by portfolio $\lambda \in \Lambda$ or $\lambda \succ_F \tau$ if we have $\lambda \succeq_F \tau$ and

$$F^{(2)}(x, \lambda) < F^{(2)}(x, \tau) \text{ for some values of } x \in \mathcal{X}. \quad (10)$$

Definition 3.2.2. If portfolio $\tau \in \Lambda$ is not strictly second-degree stochastically dominated by any of the other portfolios in the investment set, then $\tau \in \Lambda$ is second-degree stochastically efficient. In other words, if and only if some risk-averse investors find portfolio $\tau \in \Lambda$ optimal, this portfolio is stochastically efficient. We show the set of all efficient portfolios of Λ by $E(\Lambda)$.

Definition 3.2.3. If all portfolios in Λ are weakly second-degree stochastically dominated by some portfolios in K , where $K \subset \Lambda$, set K second-order stochastically spans opportunity set Λ .

Notice that there exists always a set that spans Λ , which is Λ itself. Besides, a set K , which spans Λ , can itself be spanned by another set of K ". Furthermore, since two totally different portfolios can have equal returns, it is possible to find several immutable spans K ".

Proposition 3.2.1. Let $K \subset \Lambda$. If $\Lambda - K$ does not cause any change in the efficient set, K stochastic spans Λ .

To identify stochastic spanning for a given set, the below single-valued function is used:

$$\eta(F) := \sup_{\lambda \in \Lambda} \inf_{\kappa \in K} \sup_{x \in \mathcal{X}} F^{(2)}(x, \kappa) - F^{(2)}(x, \lambda) \quad (11)$$

In the case of stochastic spanning, K spans Λ , and $\eta(F) = 0$. But, if $\eta(F) > 0$, there exists no stochastic spanning. There is also a lower bound for this measure, and it can be reformulated as a function of the expected utility function in the below format

$$\eta(F) = \sup_{\lambda \in \Lambda; u \in \mathcal{U}_2} \inf_{\kappa \in K} \mathbb{E}_F[u(X^T \lambda) - u(X^T \kappa)]. \quad (12)$$

Proposition 3.2.2. A reformulation of stochastic spanning method is

$$\eta(F) = \sup_{\lambda \in \Lambda; w \in \mathcal{W}} \inf_{\kappa \in K} H(w, \kappa, \lambda; F); \quad (13)$$

Where

$$H(w, \kappa, \lambda; F) := \int_{\underline{x}}^{\bar{x}} w(x) (F^{(2)}(x, \kappa) - F^{(2)}(x, \lambda)) dx; \quad \text{and}$$

$$\mathcal{W} := \{w: \mathcal{X} \rightarrow [0,1]: \int_{\underline{x}}^{\bar{x}} w(x) dx = 1\}. \quad (14)$$

Assumption 3.2.1. α -mixing return sequence $(X_t)_{t \in \mathbb{N}_0}$ has mixing coefficients $(a_t)_{t \in \mathbb{N}_0}$ where $a_t = \mathcal{O}(t^{-\delta})$ for $\delta > 1$. Besides, the covariance matrix

$$\begin{aligned} & \mathbb{E}_F[(X_0 - \mathbb{E}_F[X_0])(X_0 - \mathbb{E}_F[X_0])^T] + \\ & \sum_{t=1}^{\infty} \mathbb{E}_F[(X_0 - \mathbb{E}_F[X_0])(X_t - \mathbb{E}_F[X_t])^T]. \end{aligned} \quad (15)$$

Is positive definite.

Since cumulative distribution function, F , is not given and is calculated through realized returns $(X_t)_{t=1}^T$, at first F must be determined. The empirical joint distribution function of $F_T(x)$ based on the sample $(X_t)_{t=1}^T$ is given by

$$F_T(x) := T^{-1} \sum_{t=1}^T 1(X_t \leq x). \quad (16)$$

Central Limit Theorem states that $\sqrt{T}(F_T - F)$ weakly converges to $\mathcal{B}_F$ which is a Gaussian process.

By having F_T , the test statistic of stochastic spanning is considered as the scaled version of the one in Proposition 3.2.1, which is

$$\begin{aligned} \eta_T &:= \sqrt{T} \eta(F_T) := \sqrt{T} \sup_{\lambda \in \Lambda} \inf_{\kappa \in K} \sup_{x \in \mathcal{X}} F^{(2)}(x, \kappa) - F^{(2)}(x, \lambda) \\ &= \sqrt{T} \sup_{\lambda \in \Lambda: w \in \mathcal{W}} \inf_{\kappa \in K} H(w, \kappa, \lambda; F_T). \end{aligned} \quad (17)$$

The stochastic spanning is tested by η_T , the null hypothesis is $\mathbf{H}_0: \boldsymbol{\eta}(F) = \mathbf{0}$, and the alternative hypothesis is $\mathbf{H}_1: \boldsymbol{\eta}(F) > \mathbf{0}$. The only thing left is to attain the limit distribution of the test statistic for the null hypothesis.

Proposition 3.2.3. If Assumption 3.2.1 holds, $\mathbf{H}_0$ stands, and $\mathcal{L} := \mathcal{W} \times \Lambda$. Then,

$$\eta_T \rightsquigarrow \eta_\infty := \sup_{(\omega, \lambda) \in \mathcal{L}} \inf_{\kappa \in K^\leq(\omega, \lambda)} H(\omega, \kappa, \lambda; \mathcal{B}_F)$$

where

$$\mathcal{L}^\leq := \{(\omega, \lambda) \in \mathcal{L} : \inf_{\kappa \in K} H(\omega, \kappa, \lambda; F) = 0\} \quad (18)$$

$$K^\leq(\omega, \lambda) := \{\kappa \in K : H(\omega, \kappa, \lambda; F) \leq 0(\omega, \lambda) \in \mathcal{L}\} \quad (19)$$

and $H(\cdot, \cdot, \cdot; \mathcal{B}_F)$ is a Gaussian process with mean of zero.

To develop a statistical test, first the latent c.d.f. F must be calculated since the critical value $\eta_T > q(\eta_\infty, 1 - \alpha)$ where $\alpha \in (0, 1)$ depends on it. To obtain F , we need to apply a subsampling method. Thus, we generate $(T - b_T + 1)$ subsamples that overlap maximally where $b_T \in \mathbb{N}_1$. These subsamples are taken from the return sequence $s_{b_T; T, t} := (X_s)_{s=t}^{t+b_T-1}$, $t = 1, \dots, T - b_T + 1$. Then, test scores are computed through $\eta_{b_T; T, t} = \sqrt{b_T} \eta(F_{b_T; T, t})$ for all subsamples where $F_{b_T; T, t}$ is empirical joint cumulative distribution function is created from $s_{b_T; T, t}$, $t = 1, \dots, T - b_T + 1$. Besides, to compute the distribution of tests scores, for subsamples, and quantile function, we have

$$S_{T, b_T}(y) := \frac{1}{T - b_T + 1} \sum_{t=1}^{T-b_T+1} 1(\eta_{b_T; T, t} \leq y) \quad (20)$$

$$q_{T, b_T}(1 - \alpha) := \inf_y \{y : S_{T, b_T}(y) \geq 1 - \alpha\} \quad (21)$$

If $\eta_T > q_{T, b_T}(1 - \alpha)$, we reject null $\mathbf{H}_0$ against alternative $\mathbf{H}_1$ at a significance level $\alpha \in (0, 1)$.

Despite the asymptotically accurate size of the test, quantile estimates may be sensitive to the size of subsamples, b_T , and as a result, biased. This problem is more likely to happen for subsamples with realistic dimensions of M and T . To fix these problems, we apply a regression method for bias correction. According to this method, we calculate $q_{T, b_T}(1 - \alpha)$ for a given

α and for some b_T s which are reasonable. Then, we calculate the slope and intercept of the below regression through the OLS method.

$$q_{T,b_T}(1 - \alpha) = \gamma_{0;T,1-\alpha} + \gamma_{1;T,1-\alpha}(b_T)^{-1} + v_{T;1-\alpha,b_T}. \quad (22)$$

In the last step, we calculate the biased-corrected version of $(1 - \alpha)$ -quantile for $b_T = T$ based on the following regression:

$$q_T^{BC}(1 - \alpha) := \hat{\gamma}_{0;T,1-\alpha} + \hat{\gamma}_{1;T,1-\alpha}(T)^{-1} \quad (23)$$

Therefore, we have necessary tools to apply this method to real financial data.

CHAPTER IV: BITCOIN AND ITS UNIQUE FEATURES

4.1 Cryptocurrencies: As a New Investment Opportunity

Cryptocurrencies are a form of currencies that are available digitally. Cryptos are structures permitting the safe transactions online by means of virtual vouchers. These vouchers or “tokens” are entitled by registry entries. In fact, cryptocurrencies are virtual assets that took their name from a variety of encryption techniques applied for network security. Through this medium of exchange, personal coin ownership records are saved on a computer-based database ledger to preserve the additional coins’ creation under control and to authorize the exchange of coin ownership.

This digital asset does not necessarily have the physical form of the paper money, and there is no central authority to issue it. In that, as a digital capital, cryptocurrency is designed to prevent fraudulent transactions. Besides, in contrast to centralized banking mechanism and central digital currency, cryptocurrencies make use of decentralized control. Generally, cryptos are assumed to be centralized once it is minted or produced, before being issued. Under decentralized control, cryptocurrencies are distributed through ledger technology, namely blockchain, that functions as the most required public database for financial transaction.

Blockchain, as the name suggests, is “a chain of blocks” or several blocks. The blocks, here, contain information in a digital format and have three parts. One part is about transactional information such as date, dollar amount of the purchase, and time. Second part contains information of participants of transactions. But, instead of identification information of participants like their real names, purchase record uses a distinctive “digital signature”. Last part includes cryptographic codes which are called “hashes”. Hashes are to distinguish different blocks although they are quite similar.

Four phases or steps must be passed, so a new block will be joined to blockchain. At first, a transaction must be performed. Then, that transaction must be confirmed in detail such as parties, dollar amount, and time of transaction. After that, that transaction will be saved in a block. And finally, a distinctive hash must be allocated to that block. Having been hashed, the block can be a part of blockchain, and it would be accessible by the public (Meiklejohn et al. 2013).

Blockchain was invented as an attempt to create a structure which prevents timestamps of the documents to be falsified. In that, the creation of first version of blockchain was almost two decades before invention of Bitcoin, which is the first cryptocurrency based on blockchain. Bitcoin has remained the most valuable and famous cryptocurrency till now. Other popular cryptocurrencies are Ethereum, XRP, Tether, Chainlink, Bitcoin Cash, Bitcoin SV, Litecoin, Cardano, EOS, and Binance Coin, etc. In this study, our main concentration is on Bitcoin.

4.2 Bitcoin

Bitcoin, invented by Satoshi Nakamoto in August 2008 as a channel for direct electronic payments between two parties (Nakamoto, 2008), possesses the largest market capitalization of cryptocurrencies in the world. Bitcoins share is more than 68% of almost \$214 billion value of all cryptocurrencies market. To fasten the process of payments, Bitcoin operates by the means of peer-to-peer network. Besides, decentralized authority, or organizations and people who handle the transactions and trades in blockchain, are called “Miners” in this field. The “miners” are rewarded by either new bitcoin released, or transaction costs paid in the form of bitcoin. The process of Bitcoin mining releases more Bitcoins into circulation. Mining necessitates discovery of new blocks through solving complicated mathematical puzzles (Schilling and Uhlig, 2019). These new blocks, then, are attached to the blockchain, and then miners are rewarded Bitcoins for finding new blocks.

Main advantages of Bitcoin are transparent transactions without personal information disclosure, minimal transaction fees, no need for a third party or a central authority to monitor the transaction, and no possibility of counterfeit or fraud. Besides, since the number of Bitcoins cannot exceed 21 million bitcoins, there is no room for inflation, government pressure, or manipulation. On the other hand, the fact that virtual currencies are highly volatile, awfully risky for investment, threatened by hacking, and difficult to price are major disadvantages of them (Meiklejohn et al. 2013).

4.3 Bitcoin and Informational Efficiency

Informational efficiency of Bitcoin is another characteristic of this novel asset that requires a proper investigation and understanding. Urquhart (2016) applies five various methods to test the weak form efficiency of Bitcoin market and conclude that for the full sample, the Bitcoin market is inefficient. When two subsamples are tested, the result is efficiency for the second subsample, which means that Bitcoin new market is improving to reach efficiency level.

In another study based on the same data of Urquhart's (2016) paper, Nadarajah and Chu (2016) run eight different tests on transformed returns of Bitcoins and show the weak form informational efficiency of the Bitcoin market. Apart from two exceptional sub-periods, Tiwari et al. (2017) also assert that the Bitcoin market is efficient between July 2010 to June 2017. Furthermore, Vidal-Tomás and Ibañez (2018) perform an event study and state that the Bitcoin market is inefficient in semi-strong form and for the news related to monetary policies. Besides, they show that Bitcoin is more efficient than before for Bitcoin market related news.

Finally, Sensoy (2019) investigate efficiency of Bitcoin prices in US dollar, BTCUSD, and euro, BTCEUR, with using high-frequency data. His study shows that since 2016, both Bitcoin markets, BTCUSD and BTCEUR, have been more efficient, and BTCEUR is slightly less efficient than BTCUSD. This study also states that the efficiency of Bitcoin prices is significantly affected by the liquidity and volatility of Bitcoin in a positive way and negative one, respectively. Also, a reverse relationship between efficiency of Bitcoin prices and frequency of data is shown in this paper.

4.4 Bitcoin and Pricing: Are Bitcoins Speculative Bubbles?

Pricing of any type of asset is a hard task that financial analysts face, especially for Bitcoin, it can become a more challenging process since Bitcoin's attributes are so special and different from the other financial instruments before it. To be specific, not only does Bitcoin possess no underlying value, but also there is no monetary cashflow for cryptocurrencies, and the possible benefits an investor can get from Bitcoin are in the form of new Bitcoins. So, the traditional asset pricing models cannot explain the movements of Bitcoin's value and driving factors that cause its price fluctuations (Koutmos, 2019). So, to value Bitcoin, the first step is to identify potential determinants of Bitcoin price.

In one study, Kristoufek (2013) assumes that investors' sentiment is the only dominant element in determining the price of Bitcoin. To test this idea, he considers Wikipedia and Google Trends' search queries results as measures of investors' sentiment. To be exact, in this paper, any cryptocurrency related subject that is searched in these two search engines is considered as an investors' sentiment. Kristoufek (2013) realizes that the higher the recent price of Bitcoin, the more the investors' attention is captured. And then, the price of Bitcoin will be pushed up more. According to this paper, the reverse process is also valid. In another study, Kristoufek (2015) discusses some speculative, technical, and elementary factors that affect Bitcoin price, as well as the influence of index market of china on Bitcoin price which comes out to be the main influential factor.

Bouoiyour and Selmi (2015) evaluate three main sets of bitcoin price's determinants for both short-term and long-term periods. These

determinants are technical factors (namely, supply and demand of Bitcoin), macroeconomic determinants and attractiveness criterion. On the other hand, fast-increasing price and great volatility fluctuations of Bitcoin embolden the idea that bubbles can be an explanatory driving factor for Bitcoin price, and of course these bubbles are destined to burst eventually.

There are different definitions and measures for bubbles. In a traditional definition, bubbles are when the value of a financial asset deviates its fundamental value (Diba and Grossman, 1988). Nevertheless, in case of Bitcoin an asset with an ambiguous nature, no cash flow, and without intrinsic value, it is hard to determine the fundamental value. Therefore, to identify bubbles in cryptocurrency market, we must utilize other definitions and methods.

Cheung et al. (2015), based on an approach introduced in Phillips et al. (2013)'s paper, assert that there existed plenty of episodic bubbles in virtual-currency market between 2010 to 2014. Moreover, they detect three long-lasting bubbles, each lasted for almost 2-3 months, for period 2011-2013, and these bubbles' burst is at the same time of some major incidents in Bitcoin market. Finally, this paper indicates that bubble can be an explanation for the price of this novel speculative commodity (Cheung et al., 2015).

Furthermore, Cheah and Fry (2015) claim that the intrinsic value of Bitcoin is zero, and there are periods of bubble in Bitcoin's history. Having applied the methodology in Phillips et al. (2011) and considered the basic price determinants of virtual currencies, Corbet et al. (2018) also posited that Bitcoin and Ethereum show bubble behavior in some periods.

4.5 Bitcoin's and Conventional Assets: Correlation Analysis

Correlation between Bitcoin and other physical and virtual assets is another topic of interest in finance. Corbet et al. (2018) focus on the linkage between three famous cryptocurrencies, Bitcoin, Ripple, and Litecoin, and other classes of financial assets. It is shown that the prices of Ripple and Lite are affected by the price of Bitcoin, but the reverse relationship is comparatively limited. Moreover, these three virtual currencies are interconnected although they are almost isolated from main classes of financial assets.

Katsiampa (2019), on the other hand, examine the connection between Bitcoin and Ethereum and show that the conditional correlation between these two cryptos fluctuates over time and sometimes is negative. Finally, Aslanidis et al. (2019) evaluate the correlation of major financial assets such as stocks, bonds, gold, with major virtual currencies, and conclude that the correlations between all these assets and cryptocurrencies are negligible, but sometimes negative. Monero, as it is claimed in this paper, is an exception among cryptocurrencies since the correlation of this crypto with other assets is more stable across time. Besides, they show that correlations between virtual currencies are all positive which contradicts the result of Katsiampa (2019).

4.6 Bitcoin: A Diversifier, a Safe Haven, or a Hedging Tool?

The negligible correlation between Bitcoin and other assets makes it a suitable means of diversification, a candidate for hedge, and a possible safe haven. It is asserted that “this [cryptocurrencies] market could not be attractive for diversification purposes. The reason is that cryptocurrencies’ mean return and standard deviation are between 1 and 2 orders of magnitude larger than the other traditional assets. As a consequence, a small portion of cryptocurrency will dominate the stochastic dynamics of the whole market” (Aslanidis et al., 2019: 136). Albeit what Aslanidis et al. (2019) state, there are several studies that show the benefits of including Bitcoin in the portfolio.

First of all, there is a small difference among properties of a diversifier, hedge and safe haven asset. A diversifier, on average, is weakly positively correlated to another asset. A hedge, on the other hand, exhibits no correlation or negative correlation with other assets on average. Finally, an asset which is uncorrelated or negatively correlated with other assets during periods of market turmoil (Baur and Lucey, 2010).

Brière et al. (2015) add Bitcoin to portfolios consist of a) Bonds, equities and hard currencies, b) hedge funds, real estate, commodities, and c) a mixture of a and b assets to see if there exists any improvement in the performance of the portfolio. The included Bitcoin portfolios outperform the Bitcoin-exclusive portfolios in terms of expected return-volatility trade-offs. Besides, the efficient frontier of Bitcoin-inclusive portfolio is by far steeper than the Bitcoin-exclusive’s one.

Brière et al. (2015) also claim that the low correlation between Bitcoin and other assets might increase in times of crisis as it is the case for other assets. The fact that Bitcoin is in its early stage might provide the authors

with such result, they stated at the end. Therefore, although Bitcoin seems to be a safe haven now, it might not perform accordingly in near future.

Dyhrberg (2016) uses Bitcoin to hedge against equities in the Financial Times Stock Exchange Index and the American dollar. She show that Bitcoin can be used as a hedge tool, basically as a substitute for gold, to minimize the systematic risk of the market, and this virtual currency performs good in hedging American dollar in the short-term.

Bouri et al. (2017) examine daily and weekly data to see if Bitcoin can be considered as a portfolio diversifier, a hedging tool, or a safe haven for some major commodities and financial securities. They show that Bitcoin can be served as a portfolio diversifier in all cases of base assets, apart from Asia Pacific and Japanese stocks. Besides, depending on the time horizon and for some assets in this study, Bitcoin's hedge and safe haven properties might change. So, they classified Bitcoin as a distinctive asset class which is uncorrelated with the other assets.

Guesmi et al. (2019) introduce the best model that describes the joint dynamics of various financial assets and Bitcoin. They posit that investors could hedge the risk of the investment in all financial instruments by taking a short position in Bitcoin. In this study, it is discussed that the risk of a hedge scheme, including oil, gold, emerging stocks, and Bitcoin is much less than the risk of the same portfolio without Bitcoin, and these results offer some diversification and hedge benefits by including Bitcoin in the portfolio.

All these literature and results gave us an idea of finding the optimal portfolio of some base assets and Bitcoin based on new Stochastic Spanning method introduced in Chapter 2. In the next chapter, we are going to talk about the methodology, data, and results of implementing our new idea.



CHAPTER V: EMPIRICAL RESULTS

5.1 Asset Allocation

Portfolio selection and its methods are a part of the diversification concept, described and explained in Chapter 2. Now, we move one step back to define asset allocation and clarify the logic behind it. Asset allocation is the art of investing in different classes of assets, or in general, including a combination of bonds, equities, and cash in the portfolio. These three asset classes comprise some subclasses including, large-cap equities, mid-cap stocks, small-cap stocks, international assets, emerging market securities, fixed-income assets, money market securities, and real estate.

Asset allocation, as a part of diversification, is also to reduce the risk of the portfolio. All those different asset subclasses have different levels of risk, and since risk and returns go hand in hand, the riskier an asset subclass, the higher its expected return is. The assets classes, sorted in the order of highest to lowest volatility, are: small-cap securities, mid-cap securities, large-cap equities, corporate bonds, and cash or government treasuries. Nevertheless, all the asset classes have market swings that are specific to them. So, proper asset allocation protects the portfolio from fluctuations of a specific security or asset class. A combination of different assets with

different sensitivity toward the market provides investors with a more stable and optimal portfolio such that if one asset or components are more volatile with higher returns, the others provides a counter for adverse results.

To form an optimal portfolio, investors should first arrange assets according to asset class. The point here is that the risk and return of each asset class are various. Many factors determine which asset class is suitable to a financier, including investors' age, investment horizon, education, level of wealth and income, and the most important criterion, the investor's risk aversion level.

Riley Jr and Chow (1992) categorize assets into four main asset classes: real estate, personal property, risky assets; they show that the older the investors get, the less they invest in personal property. Besides, they state that until the age of 65, financiers increase their investment in risky assets like equities, but they reduce the equity investment after 65. Another study claims that Middle-age financiers invest more in equities in comparison with young and old investors (Bertaut and Starr-McCluer, 2000). Additionally, Dow (2009) asserts that there is almost no connection between age and time horizon of investment.

The investing horizon is another factor that affects the optimal portfolio's asset allocation. To begin, we should note that age is not always a proper proxy for the investment horizon (Dow, 2009). Besides, in a long-time horizon, since equities are less risky in comparison to bonds, long-term young financiers are advised to invest in equities (Cochrane, 1999). Furthermore, considering the impact of education on asset allocation, Riley Jr and Chow (1992) also indicate that there is no relationship between investment in different asset classes and the education level of investors.

This fact has an exception which is the equity investment; In general, the higher the level of education, the more financiers invest in equities.

Riley Jr and Chow (1992) also show that with higher income and wealth, financiers invest more in risky assets and less in bonds and personal properties. However, they could find no clear connection between wealth or income and real estate investment opportunities. As the last factor which affects asset allocation, risk aversion, we should first mention that risk tolerance has a strong relationship with age, education, income, and wealth of investors. In fact, in one study, Hariharan et al. (2000) detach the influence of risk aversion on portfolio selection and indicate that individuals are less inclined to invest in T-bills, or risk-free assets in general, as their level of risk aversion declines. Moreover, in the same study, the authors state that the portfolio composition of risky assets is not influenced by the lower level of risk aversion.

5.2 Bitcoin and Asset Allocation

To clarify the role of Bitcoin in asset allocation, we should notice that financial markets have become significantly cross-correlated in recent decades. The reasons behind the tighter connection between financial markets are financial crises, investor's herding behavior, and contagion effects. This higher cross-correlation between assets in different markets made portfolio construction task more arduous. Because of that, financiers are searching continuously for some safer investment opportunities with lower correlation with other assets. Thus, Bitcoin, with a low linkage with the traditional assets, can be a perfect substitute. However, the more investors are interested in Bitcoin, the stronger the connection between Bitcoin and

conventional assets, and the diversification benefits of Bitcoin inclusion, in turn, reduce (Demiralay and Bayraci, 2020).

So, Bitcoin, an asset that can be treated and traded just like other conventional assets, utilize us with a novel choice. Some studies, as explained in detail in the previous chapter, examined different features and usage of Bitcoin to see if Bitcoin can be a safe haven or a hedging tool. Along with these questions, the diversification benefits of Bitcoin were considered to see if Bitcoin improves a portfolio's performance. A couple of these studies, which consider the role of Bitcoin as a part of an optimal portfolio, were also introduced in the previous chapter.

To investigate the advantages of Bitcoin as an ingredient of a portfolio, Symitsi and Chalvatzis (2019) apply four varied strategies and form some portfolios based on five various asset classes. They find statistically significant gains from adding Bitcoin in these different portfolios. Moreover, they point that there is a decline in the benefits of including Bitcoin in non-bobble periods. Besides, they find out that the benefits of Bitcoin inclusion are much higher for the commodity asset class.

5.3 Data and Methodology

Having noticed portfolio optimization's new method, stochastic spanning, and the function of Bitcoin in improving the portfolio's performance, we decided to see the role of Bitcoin in the construction of optimal portfolios based on various asset universes. We considered 11 different categories of asset classes, including major asset classes; major sectors of USA stock market, which are DOW JONES Indices; developed markets; emerging

markets; frontier markets; a combination of developed, emerging, and frontier market; precious metals; energy commodities; agricultural commodities; all commodities; and a combination of all these assets. Since there exist different investor types with different needs in the market, we consider all these asset classes to cover almost all investor types. In that, almost all investors can be classified under these 11 classes.

Monthly asset prices are downloaded from Bloomberg for the period between 30 July 2010 to 29 May 2020, and monthly risk-free asset prices are obtained from the Kenneth R. French Data Library (French, K. R., March, 2020). Besides, the classification of countries into the developed, emerging, and frontier markets is based on MSCI website. We run the test of stochastic spanning for all 11 groups of the asset classes: first Bitcoin-excluded and then Bitcoin-included. We compare the optimal portfolios, pre- and post-inclusion of Bitcoin, to see if Bitcoin finds a place for itself in the optimal portfolio. We compare the old and new optimal portfolios in terms of performance to see if the portfolio with Bitcoin is significantly better. To test the hypothesis of stochastic efficiency, all returns must be excess returns, so we need to calculate risk-free rates for each month of the period studied. Besides, for each asset class, we need an appropriate index that plays the role of the market for that asset class. For the first 10 groups of assets, there exist proper indices that are considered as the market for that group. For 11th asset class, nevertheless, there is no such an index. So, in this category, we compute the Principal Component Analysis (PCA) as a benchmark for the market. Below, the results of each category are reported separately.

Each time we run the test, we have the market of the section on one side, and the base assets on the other side of the stochastic efficiency test. We run the test twice for each section: a) for Bitcoin-exclusive base assets and

T-bill (or risk-free asset) and b) for the same assets in item “a” plus Bitcoin. Then, we compare the results and changes in efficient portfolios and figures. Our aim is to obtain an optimal and efficient portfolio based on the stochastic spanning method and statistically test the hypothesis of stochastic efficiency.

In each section, three main figures exist. Two first figures consist of 4 panels A, B, C, and D. The first figure depicts the results of the test for the Bitcoin-exclusive efficient portfolio. The second figure is for the optimal portfolio which includes Bitcoin as well. In panel A of each part, the return PDF of the market portfolio, τ is compared with the stochastic efficient portfolio, λ , in terms of risk. Panel B displays the difference between the expected shortfall of the market portfolio and the stochastic efficient portfolio of the section for each return level. Panel C represents the decumulative subsampling distribution of the test statistic for two different subsample values, b_T 's. If there is a huge difference between these two curves, or more specifically if large values of test statistics mostly happen in smaller subsamples, there is room for quantile bias correction. Finally, panel D indicates the estimated OLS regression of equation (22) for the empirical quantiles $q_{T,b_T}(1 - \alpha)$ and the significance levels of $\alpha = 0.01$ and $\alpha = 0.1$. Note that the empirical quantiles are calculated based on different subsample sizes b_T . The point here being, since the number of available data in all sections is not fixed, we have to consider different b_T 's for each section. The third figure of each section is just to compare and see the differences between panels A and B of the two first figures perfectly. Now, the results for different asset classes are reported.

5.3.1 Major Asset Classes: SPX Index, Crude Oil, and Gold

In this category, the base assets are Crude Oil, Gold, and risk-free asset, and SPX Index is considered as the market. At first, we run the code for the data, excluding the Bitcoin. We aim to obtain an efficient portfolio based on the stochastic spanning method and statistically test the hypothesis of stochastic efficiency.

The results are depicted in Figure1. According to panel A of Figure 1, the optimal solution is riskier than the market. Also, panel B says that the expected shortfall of the optimal portfolio is more than the market's for some levels of expected returns. The optimal portfolio, here, is composed of 24% T-bills, 76% Gold, and no Crude Oil. In panel C, the decumulative subsampling distribution of the test statistic for two different subsamples, $b_T = 55$ and $b_T = 70$ is shown. Since two curves in panel C are quite close, there is no need for bias correction of quantile estimates. The test statistic is -5.6615, and we fail to reject the market portfolio efficiency at both 1% and 10% levels as can be seen in panel D. The poor test statistic here could be due to the restricted number of base assets. The mean return, standard deviation, and Sharpe ratio for the efficient portfolio are 0.2818%, 3.5265%, 0.0666, in the same order that we named them. We can compare these values with the market's, SPX Index, which are 0.8075%, 3.7279%, and 0.2040, respectively. What is obvious is that the optimal portfolio does not perform better than the market one, SPX.

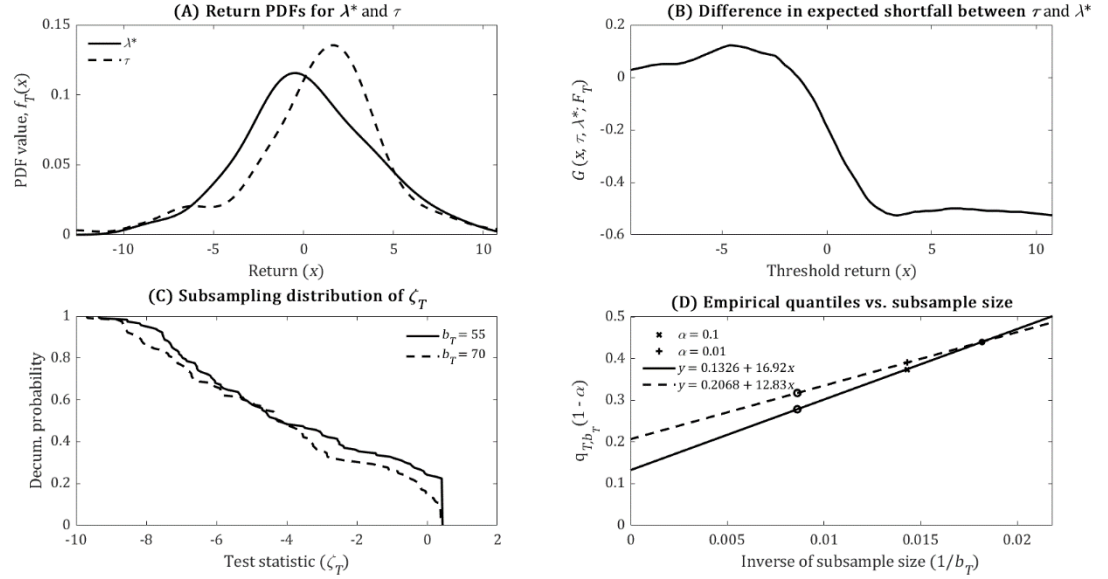


Figure 1. Results of the stochastic efficiency test for the major asset classes; Bitcoin-exclusive portfolio.

When Bitcoin is added to the base assets, the portfolio changes dramatically. The optimal portfolio has 85% T-bill, almost 1% Crude Oil, 7.1% Gold, and 6.9% Bitcoin. The t-statistic is 0.3031, and again, we fail to reject the market efficiency at both critical levels. As can be seen in panel B of Figure 2, the expected shortfall of the portfolio with Bitcoin is always less than the market's expected shortfall. The mean return, standard deviation, and Sharpe ratio for the efficient portfolio including Bitcoin are 1.3814%, 4.2811%, 0.3117, respectively. Here, the optimal portfolio including Bitcoin outperforms both the market portfolio, SPX Index, and the optimal portfolio excluding Bitcoin.

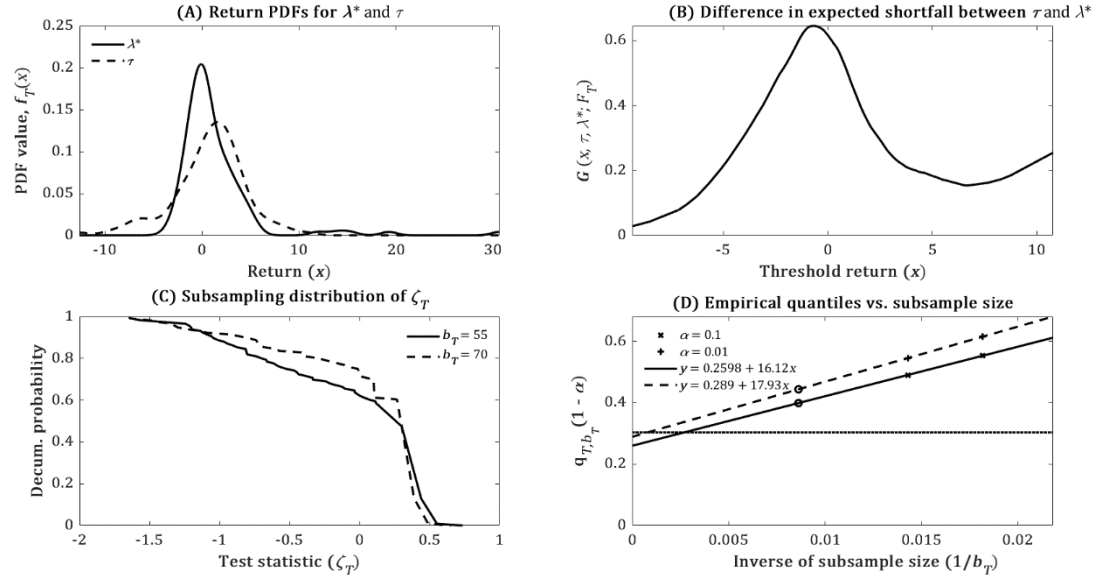


Figure 2. Results of the stochastic efficiency test for the major asset classes; Bitcoin-inclusive portfolio.

In Figure 3, the difference between portfolios in terms of return and shortfall, pre- and post- inclusion Bitcoin, is much more evident. So, adding Bitcoin improves the performance of the efficient portfolio obtained by the stochastic spanning method.

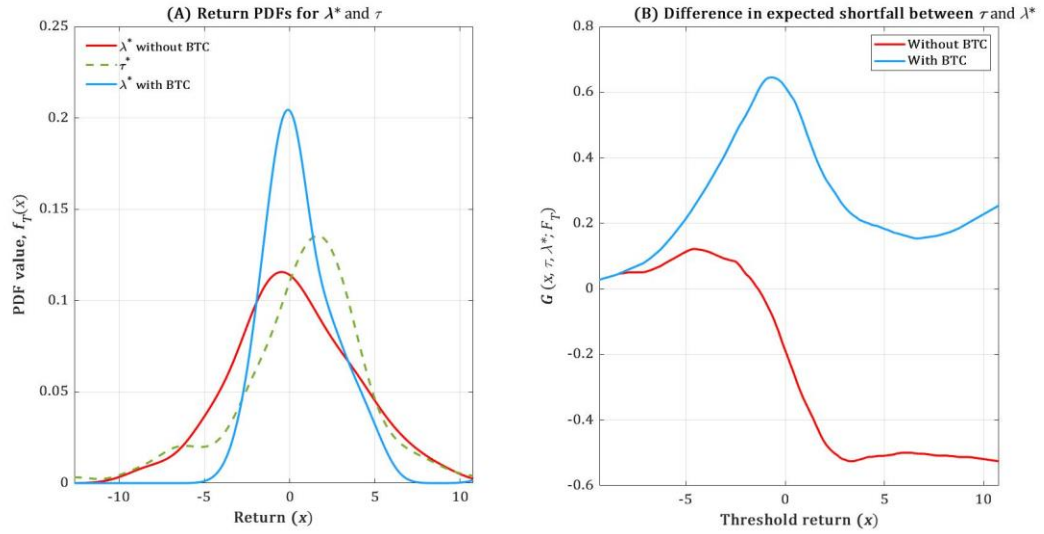


Figure 3. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on major asset classes.

5.3.2 Major Sectors of USA Stock Market, DOW JONES Indices

In this part, the base assets are T-bill and 10 indices of DOW Jones, including basic materials, consumer services, consumer goods, energy, financials, health care, industrials, technology, telecommunications, and utilities plus T-bill. On one side, these base assets exist, and on the other side, an index that represents the whole market of this section.

When the code is run for the exclusive-Bitcoin set of the assets, the optimal portfolio includes 31% T-bill, 0.3% basic materials, 1.1% consumer services, 0.3% consumer goods, 0.2% energy, 51% health care, 16.1% technologies, and almost no utilities. The test statistic is 0.419, and we fail to reject the market efficiency at both 90% and 99% confidence levels. The mean return for the optimal portfolio is 0.6592%, the standard deviation of

the optimal portfolio is 2.3857%, and the Sharpe ratio of the optimal portfolio is 0.2566. These values are much higher than the market's which are 0.3560%, 4.6386%, and 0.0666, respectively. These values show that the performance of the Bitcoin-exclusive efficient portfolio is superior to the performance of the market portfolio.

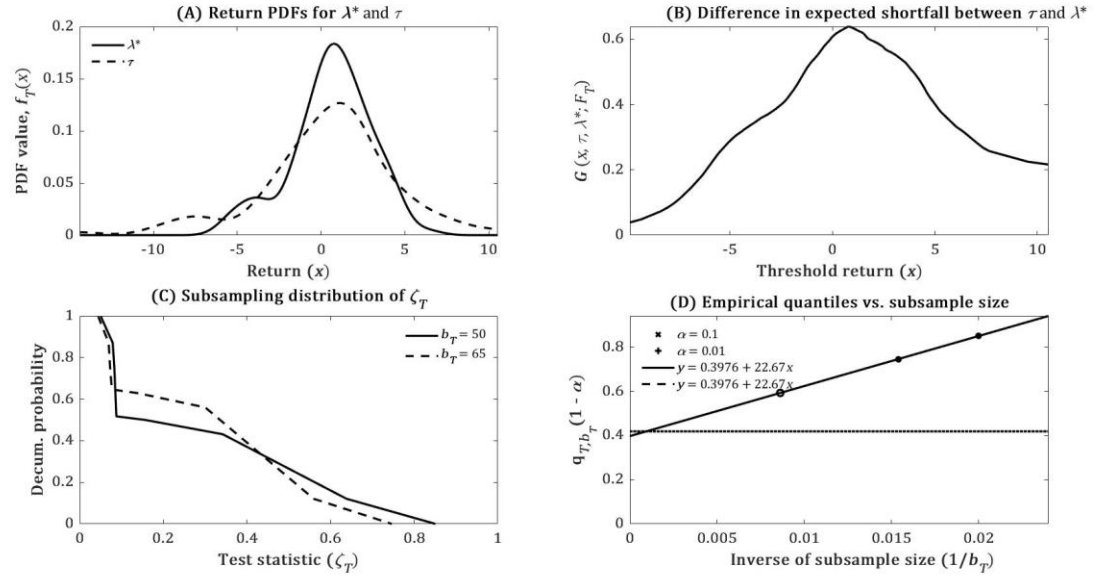


Figure 4. Results of the stochastic efficiency test for the major sectors of USA stock market; Bitcoin-exclusive portfolio.

By adding Bitcoin to the base assets, the optimal portfolio changes to 76.2% T-bill, 0.3% consumer services, 0.2% financials, 6.2% health care, 12.2% technology, 0.1% utilities, and 4.8% Bitcoin. Test statistic is the same as the case without Bitcoin, but critical values are different. We fail to reject the market efficiency for the both significance levels of 1% and 10%. The return, risk, and Sharpe ratio for this portfolio are 1.1422%, 3.1482%, and 0.3479, respectively. Clearly, these values are much higher than the equivalent for both the market and the efficient portfolio without Bitcoin. Besides, As can be seen in Figure 4 panel B, the expected shortfall of the efficient portfolio has improved by adding Bitcoin.

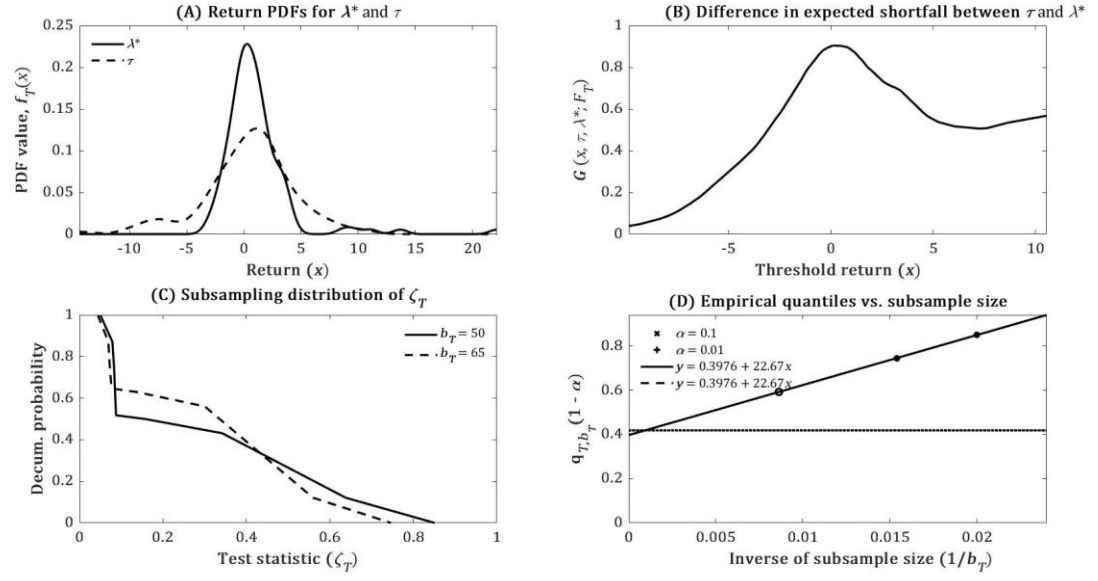


Figure 5. Results of the stochastic efficiency test for the major sectors of USA stock market; Bitcoin-inclusive portfolio.

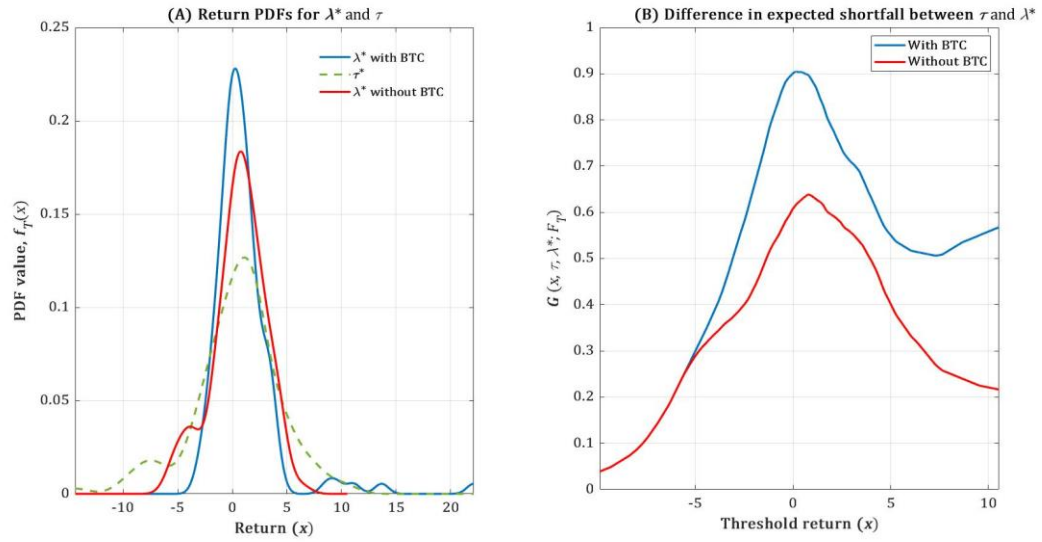


Figure 6. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on major sectors of USA stock market.

5.3.3 Developed Markets

On one side of the stochastic efficiency test, T-bill and all developed markets, classified by MSCI, are located. On the other side, we have the MSCI World Index which captures large and mid-cap representation across all 23 Developed Markets (DM) countries. The Developed Markets are the markets of U.S.A., Canada, Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, U.K., Australia, Hong Kong, Japan, New Zealand, and Singapore.

In the case where Bitcoin is excluded from the set of assets, the optimal portfolio consists of 61.5% USA, 10.4% Denmark, 0.1% Germany, and 27.8% T-bill. The share for all other countries is less than 0.2% each. The test statistic is 0.4259, and we cannot reject the market efficiency at the significance levels of 0.1 and 0.01. The mean return, standard deviation, and Sharpe ratio of the Bitcoin excluded efficient portfolio are 0.3541%, 4.7898%, and 0.0641, respectively. These values for the market, MSCI World Index in the category, are 0.5071%, 3.8879%, 0.1184, respectively, and show that this Bitcoin-exclusive optimal portfolio is not better than the market one.

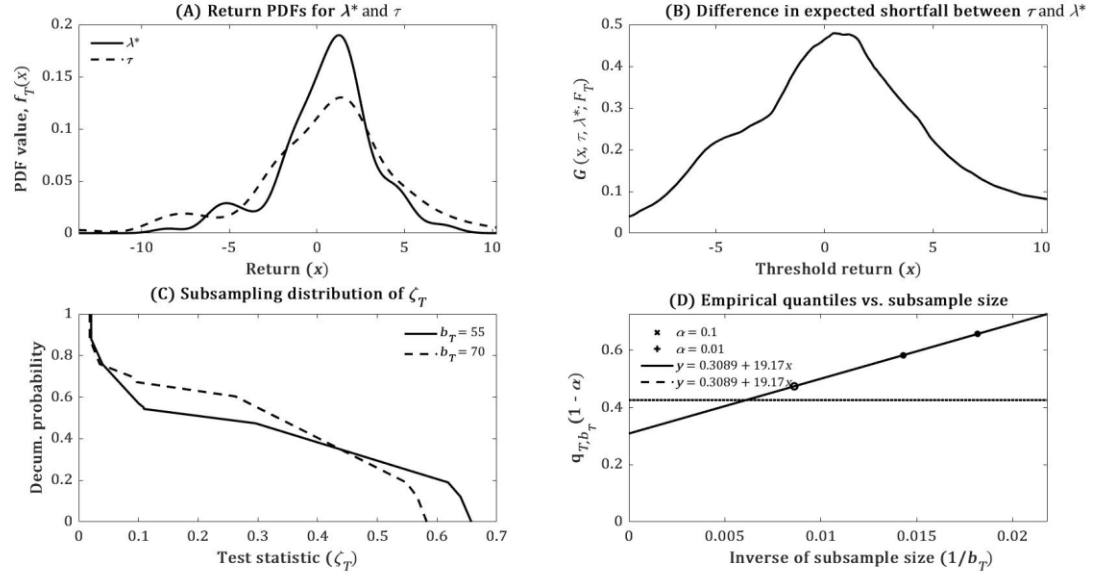


Figure 7. Results of the stochastic efficiency test for the developed markets; Bitcoin exclusive portfolio.

By adding Bitcoin to the base assets and running the tests, the optimal portfolio accounts for 17.1% USA, 0.1% Austria, 1.4% Denmark, 0.2% France, 0.1% Netherlands, 0.1% Norway, 0.2% Sweden, 0.1% Japan, 0.2% New Zealand, 0.1% Singapore, 75.2% T-bills, 5.1% Bitcoin, and almost no other developed markets. Test statistic here is the same as previous case, and we, again, fail to reject the market efficiency for both significance levels. The mean return of the portfolio including Bitcoin is 14.4087%, 46.7498% is its standard deviation, and Sharpe ratio of this portfolio is 0.3072. According to the Sharpe measure, not only does the portfolio including Bitcoin exceeds the one excluding Bitcoin, but also it defeats the market portfolio. Besides, the portfolio with Bitcoin has got a better expected shortfall compared to the market.

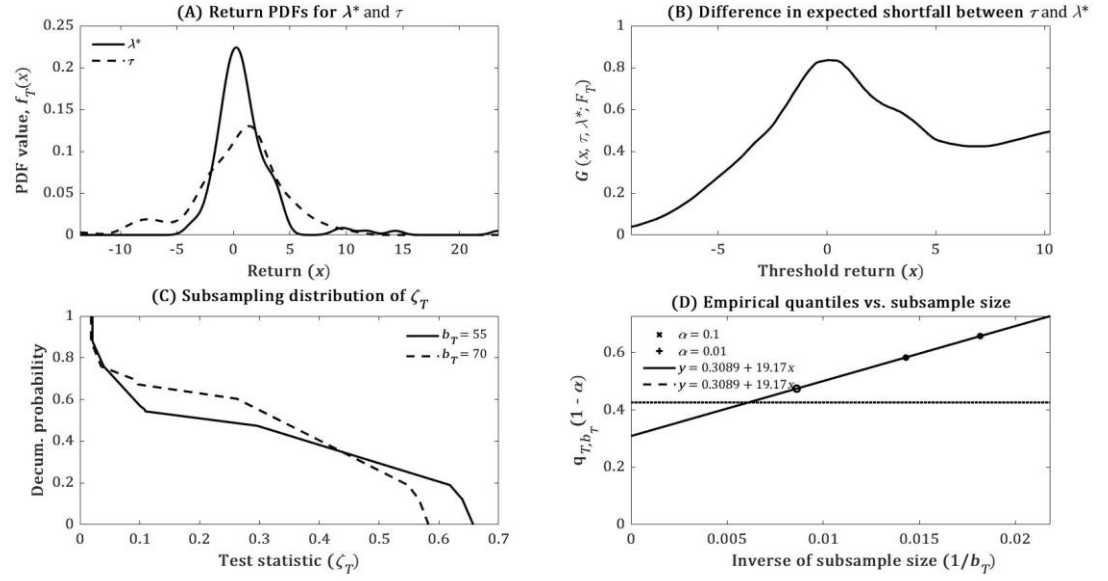


Figure 8. Results of the stochastic efficiency test for the developed markets; Bitcoin-inclusive portfolio.

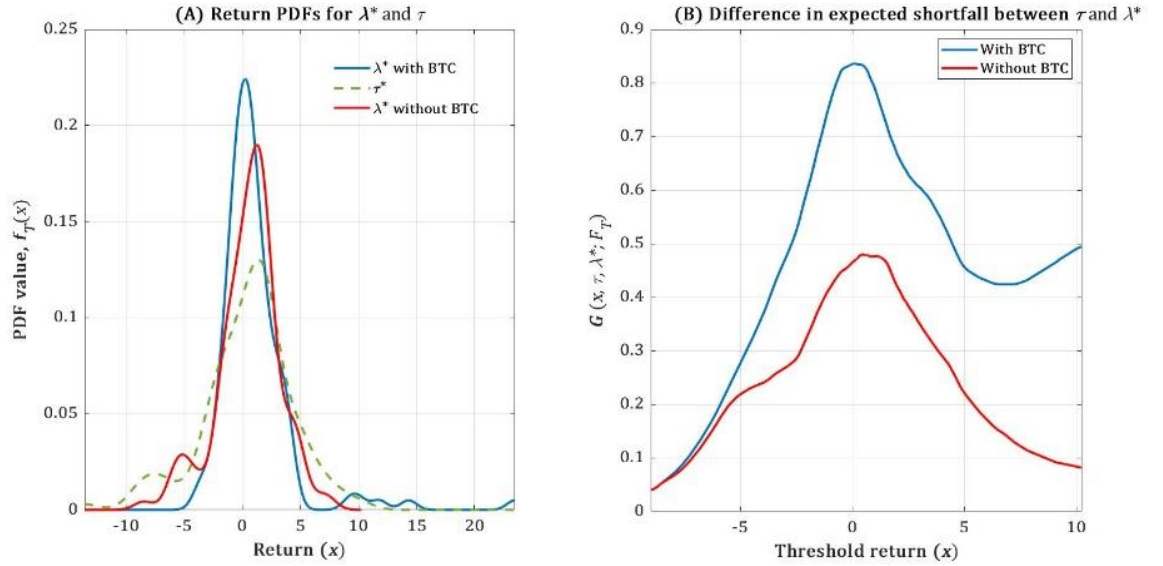


Figure 9. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on the developed markets.

5.3.4 Emerging Markets

In the same way, we did the tests for the emerging markets which consist of 26 different countries' markets. These countries are Argentina, Brazil, Chile, Colombia, Mexico, Peru, Czech Republic, Egypt, Greece, Hungary, Poland, Qatar, Russia, Saudi Arabia, South Africa, Turkey, United Arab Emirates, China, India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, and Thailand.

On one side of stochastic efficiency test, there are emerging markets and T-bill, and on the other side, the market, i.e. an index which represents the all emerging markets. There is just one point to note; since the prices of Saudi Arabia's market has no data listing prior to August 2014, we consider two cases: In the first case, the Saudi Arabia is excluded, the starting date coincides with the emergence of Bitcoin; In the second case, we include the Saudi Arabia, and consider the starting date August 2014.

Case I:

The efficient portfolio is based on 81.4% T-bill, 2.5% Qatar, almost 2% United Arab Emirates, 1.1% Philippines, almost 12% Taiwan, and less than 0.1% for the other countries. The test statistic of the sample is 0.0878, and we fail to reject the null of hypothesis of market efficiency at both 10% and 1% significance levels. The mean return for the optimal portfolio is 0.1053%, the standard deviation of the optimal portfolio is 0.7773%, and Sharpe ratio of the optimal portfolio is 0.0751. These values are much higher than the same ones for the market which are -0.0043%, 5.0906%, and -0.0101, respectively. This comparison shows the better performance of the stochastic efficient portfolio.

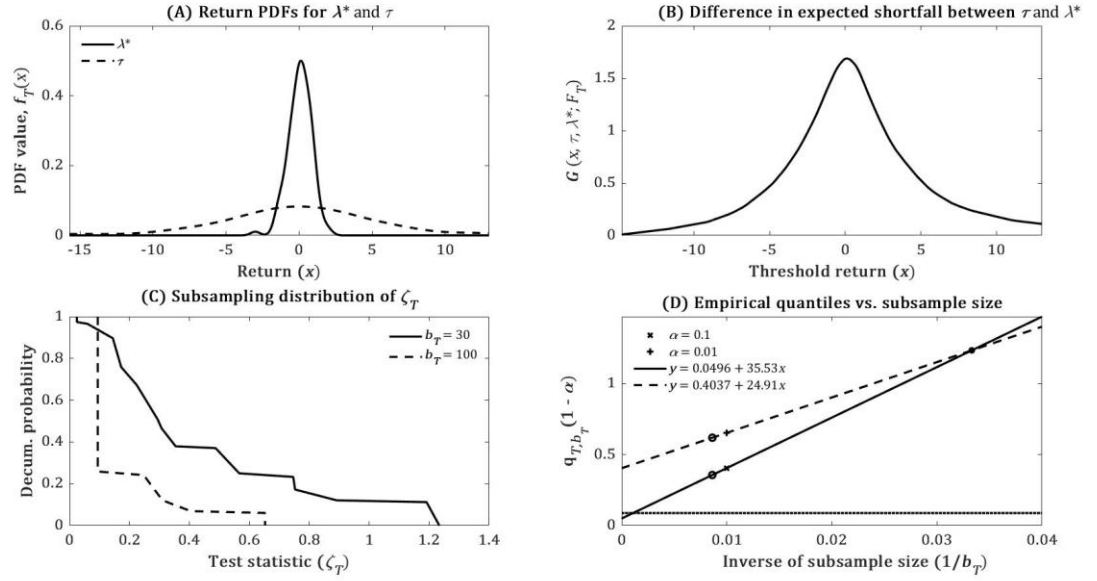


Figure 10. Results of the stochastic efficiency test for the emerging markets; Bitcoin and Saudi Arabia exclusive.

As can be seen in the panel A of Figure 4, the efficient portfolio has lower risk compared to the market one. Furthermore, its expected shortfall is less than the market, but still, for some levels of returns, the differences are not significant.

When Bitcoin is added to the set of base assets, the efficient portfolio changes for 91.6% of T-bill, almost 6.1% of Bitcoin, and less than 0.5% of some other markets. Test statistic and the critical values remain unchanged as in the case with non-Bitcoin efficient portfolio. Again, we fail to reject the efficiency of the market portfolio at both significance levels. The mean return for the efficient Bitcoin-inclusive portfolio is 1.2092%, the standard deviation of this portfolio is 3.7692%, and Sharpe ratio of this efficient portfolio is 0.3084. It is evident that the Bitcoin-inclusive efficient portfolio is dominant over both the market and the Bitcoin-exclusive efficient portfolio. Besides, our new portfolio has lower risk compared to the market in this section. As can be seen in panel B of Figure 11, by having Bitcoin as a part of the

portfolio, the expected shortfall of the efficient portfolio will be much lower than the market's.

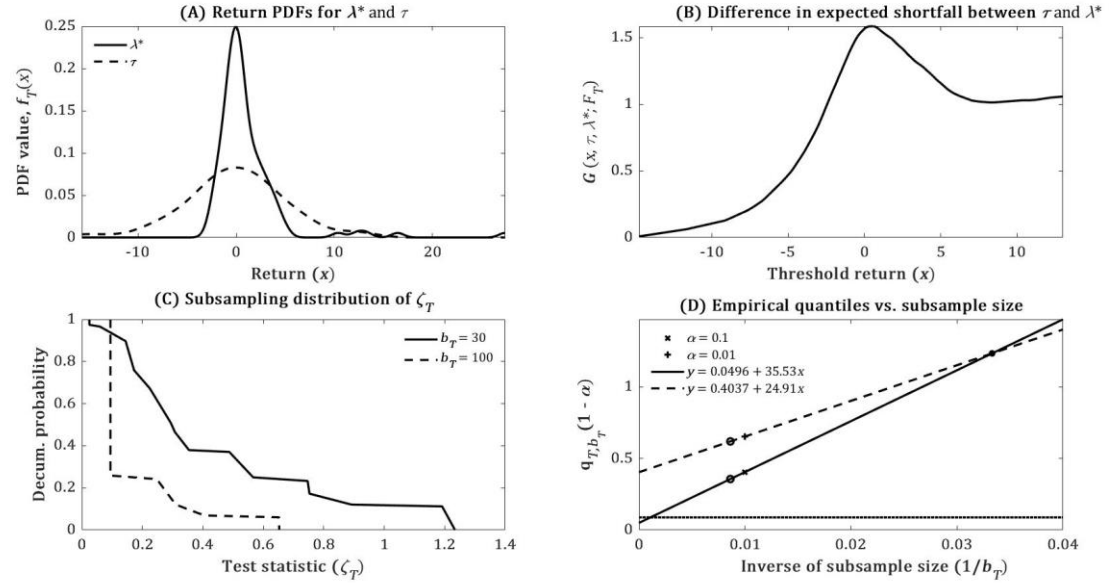


Figure 11. Results of the stochastic efficiency test for the emerging markets; Bitcoin inclusive and Saudi Arabia Exclusive.

In the Figure 12, we can easily see the difference between these two efficient portfolios of this section.

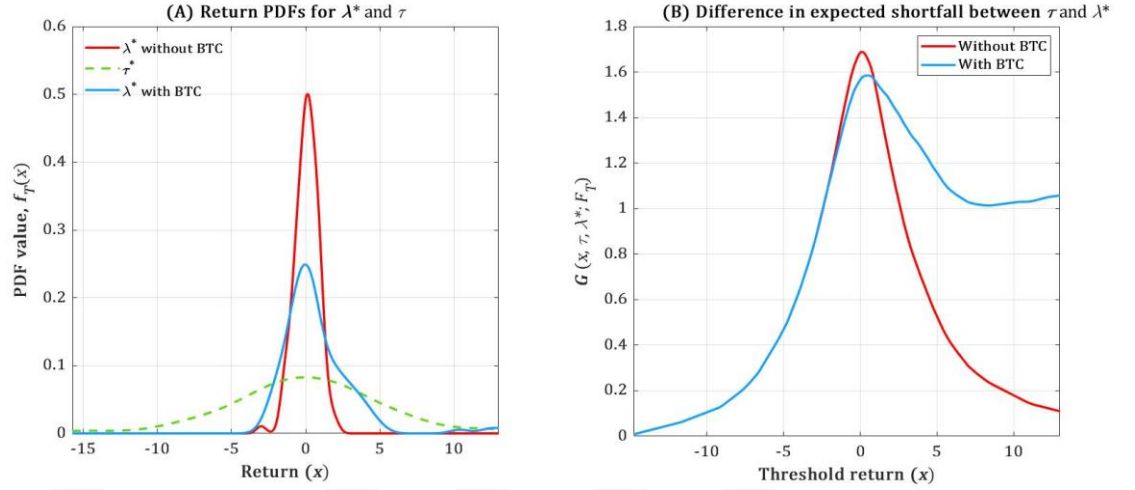


Figure 12. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on the emerging markets; Saudi Arabia exclusive.

Case II:

If Saudi Arabia is considered as a part of our base assets, and data is as of 2014 August, the resulted Bitcoin-excluded efficient portfolio accounts for 87.6% T-bill, 9.5% Hungary, 1.3% China, 0.4% Russia, 0.4% Philippines, and 0.4% Taiwan. Test statistic is 0.7976 and we cannot reject the market efficiency at both significance levels. The mean return for the optimal portfolio is -0.1724%, the standard deviation of the optimal portfolio is 1.2368%, and Sharpe ratio of the optimal portfolio is -0.2032. This optimal portfolio is much worse than the market one. The poor result might be a result of limited data used in this case.

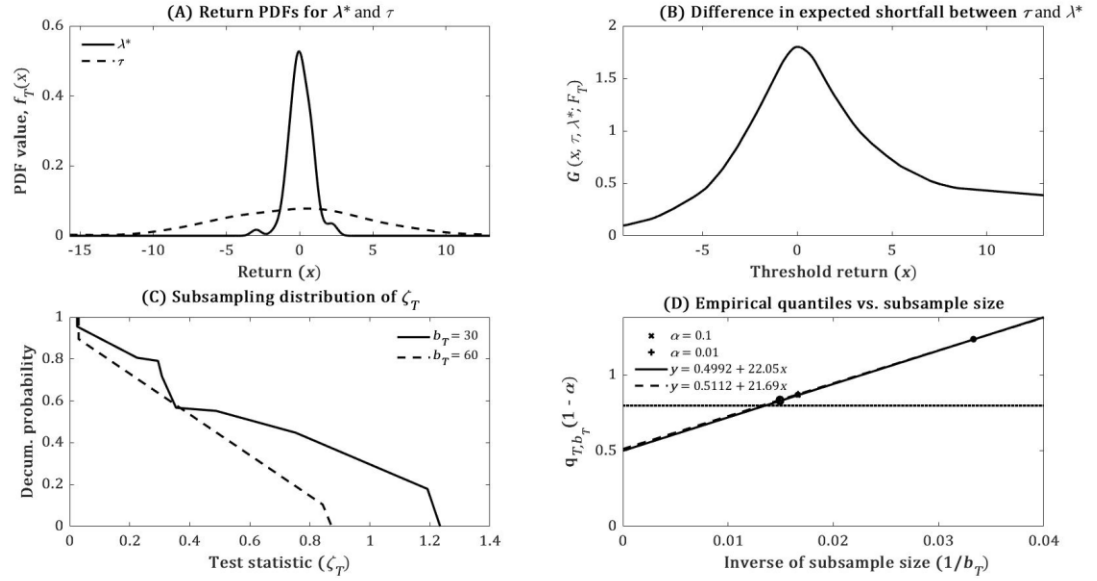


Figure 13. Results of the stochastic efficiency test for the emerging markets; Bitcoin exclusive and Saudi Arabia inclusive.

When Bitcoin is considered in the base assets, the efficient portfolio mainly involves of 88.1% T-bill, 2.5% China, 0.4% Taiwan, 0.4% South Africa, and 8% Bitcoin. Test statistic is the same as that without Bitcoin, and we fail to reject the market efficiency. The mean return for the efficient portfolio is 0.5569%, the standard deviation of the efficient portfolio is 2.023%, and Sharpe ratio of the efficient portfolio is 0.2363. This time, the efficient portfolio exceeds the market and the efficient portfolio without Bitcoin according to the Sharpe ratio criterion. Besides, the expected shortfall of the portfolio with Bitcoin improves, yet the portfolio seems riskier than the one without Bitcoin although both efficient portfolios dominate the market (Figure 14, panel A).

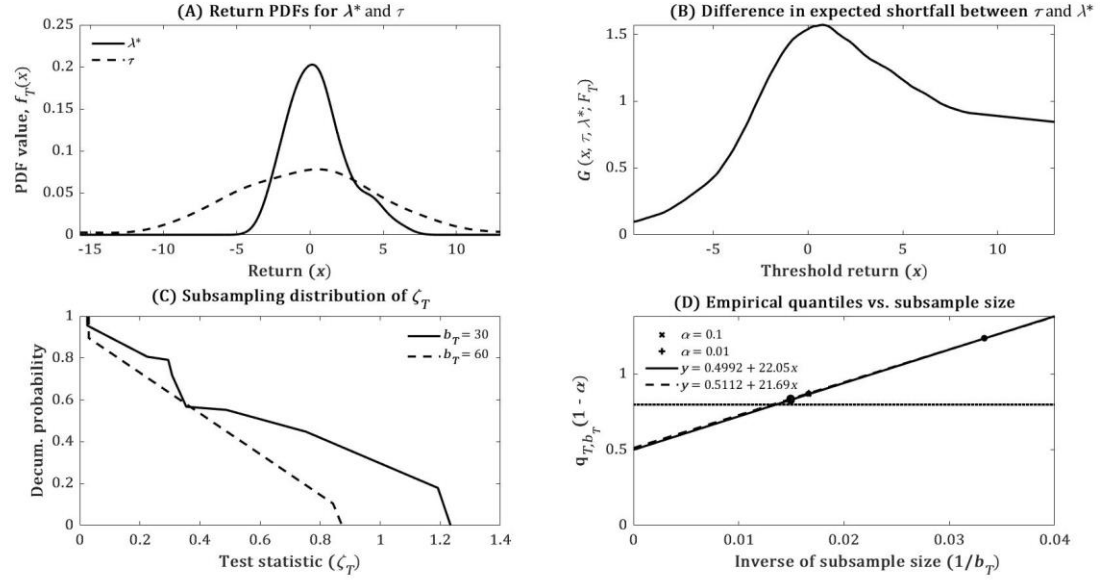


Figure 14. Results of the stochastic efficiency test for the emerging markets; Bitcoin and Saudi Arabia inclusive.

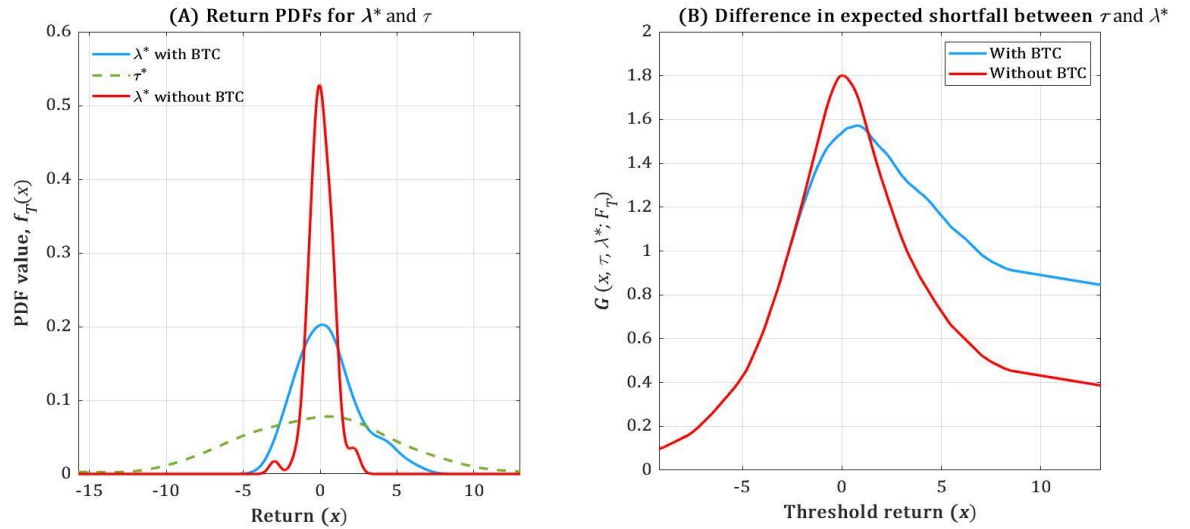


Figure 15. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on the emerging markets; Saudi Arabia inclusive.

5.3.5 Frontier Markets

In this part, we consider countries' markets listed as the frontier. The countries here are Croatia, Estonia, Lithuania, Kazakhstan, Romania, Serbia, Slovenia, Kenya, Mauritius, Morocco, Nigeria, Tunisia, WAEMU, Bahrain, Jordan, Kuwait, Lebanon, Oman, Bangladesh, Sri Lanka, and Vietnam. In this section and as before, I needed the data as of the first date of Bitcoin. we have problem wherein the West African Economic and Monetary Union (WAEMU) consists of Benin, Burkina Faso, Ivory Coast, Guinea-Bissau, Mali, Niger, Senegal and Togo, however, currently the MSCI WAEMU Indexes only include securities from Senegal, Ivory Coast and Burkina Faso; the data listed are subsequent January 2017. So, we decide to consider two cases: one excluding the WAEMU, and the other to including it. The problem is that the data is not sufficient to run the code if we consider only the data subsequent January 2017. As a result, we did the test excluding the WAEMU. We have the returns of these countries' markets and T-bill, on one side of the stochastic efficiency test, and on the other side, the market of this section.

The efficient portfolio of this section is based on 67% T-bill, 9% Romania, 12% Kenya, and almost 12% Tunisia. Test statistic is 1.2425, and we, again, cannot reject the market efficiency at both significance levels. The mean return, standard deviation, and Sharpe ratio of the efficient portfolio are, 0.2401%, 1.0915%, and 0.1770, respectively. In the same order, these values for the market portfolio are -0.0881%, 3.9767%, and -0.0339. So, the efficient portfolio possesses a higher mean return and Sharpe ratio compared to the market. Besides, the optimal portfolio is less risky in comparison with the market of this section. Finally, according to Figure 16 panel B, the expected shortfall of the efficient portfolio is not that different from the market's for some scenarios with negative returns.

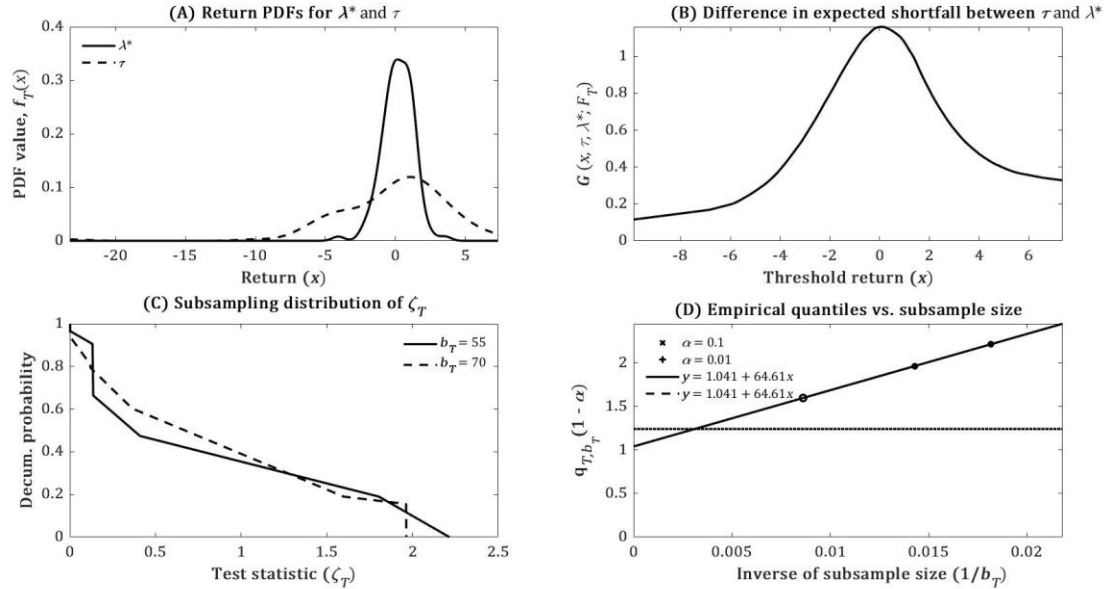


Figure 16. Results of the stochastic efficiency test for the frontier markets; Bitcoin and WAEMU exclusive.

When Bitcoin is added to the efficient portfolio, it finds its place in the portfolio. The optimal portfolio with Bitcoin contains 45% T-bills, 8.5% Romania, 3.3% Kenya, 36.6% Tunisia, 0.1% Bangladesh, 0.1% Lithuania, and 6.4% Bitcoin. The test statistic and the critical values are the same, and we fail to reject the market efficiency.

As can be seen in Figure 17 panel A, the risk of the efficient portfolio with Bitcoin is less than the market's. Besides, the expected shortfall of the efficient portfolio improves a lot with Bitcoin. The mean return, standard deviation, and Sharpe ratio of the efficient portfolio are, 1.4873%, 4.0192%, and 0.3584, respectively. As it is obvious, Sharpe ratio of Bitcoin-included portfolio is almost twice of the Bitcoin exclusive one's. therefore, adding Bitcoin to the optimal portfolio of frontier markets does increase the risk and mean return of the portfolio, but at the same time improves the expected shortfall and Sharpe ratio of the efficient portfolio.

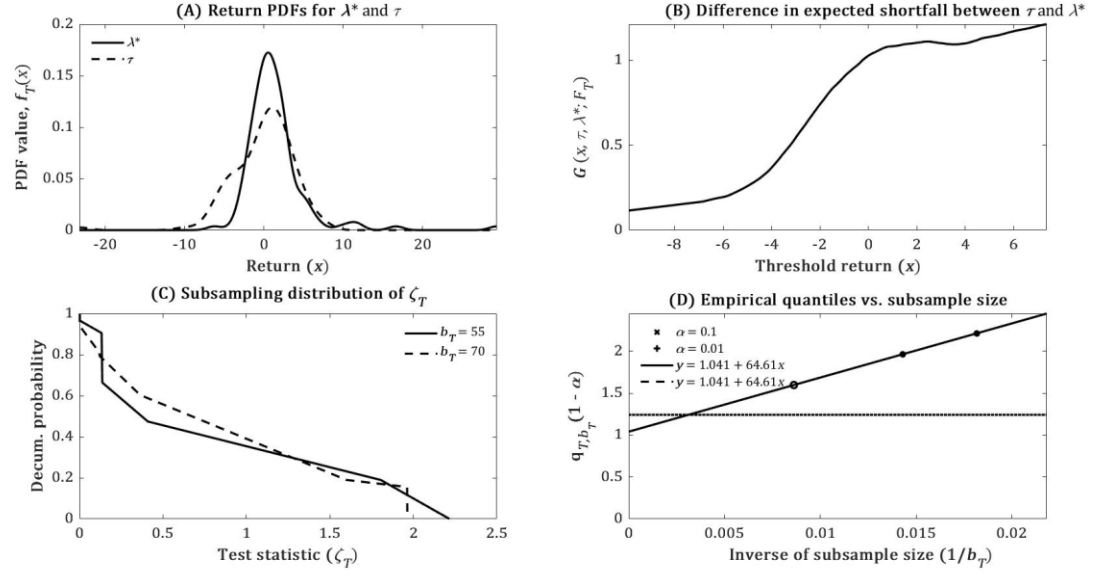


Figure 17. Results of the stochastic efficiency test for the frontier markets; Bitcoin-inclusive and WAEMU-exclusive portfolio.

In Figure 18, the comparison between the efficient portfolios, including and excluding the Bitcoin, is much more apparent.

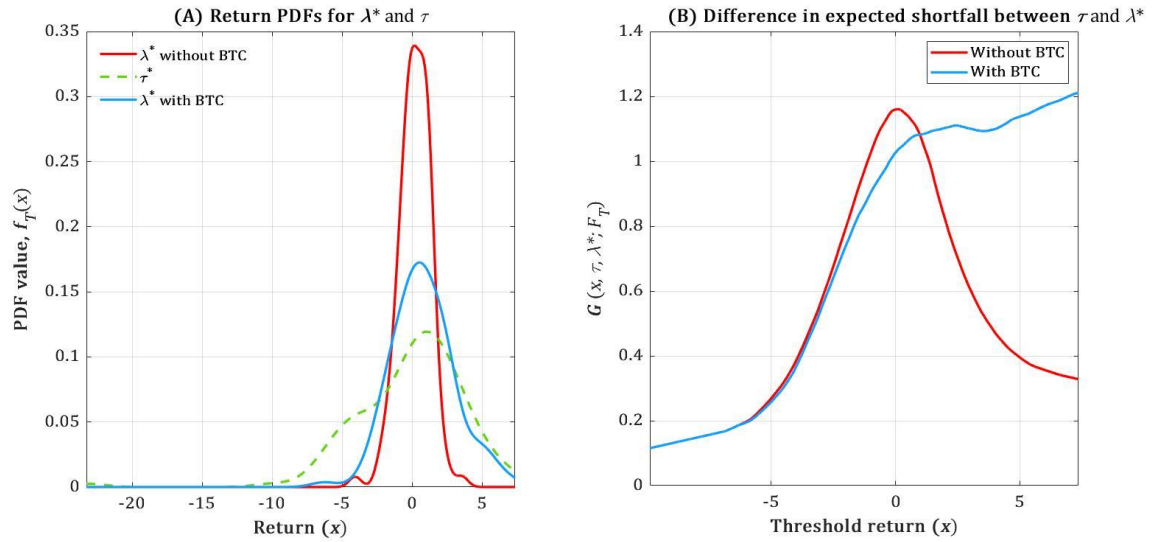


Figure 18. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on the frontier markets; WAEMU inclusive.

5.3.6 A combination of all stock markets; developed, emerging, and frontier markets

In this part, we combine all markets in the developed, emerging, and frontier markets to do the test again. The 70 countries' indices plus T-bill on one side, and on the other side, the market for all countries exist. The closing price of WAEM and Saudi Arabia is available after January 2017 and August 2014, respectively. Due to great number of base assets in this section, the test has no result in the case of including these two specific indices. As a result, these markets are simply removed from the base assets of this section. Consequently, 68 indices and the T-bill form the base assets set.

In excluding Bitcoin case, the efficient portfolio consists of 19.6% T-bill, 48.3% USA, 3.3% Denmark, 0.1% Mexico, 8.3% Kenya, and 20% Tunisia. The test statistic here is 0.39, and the market efficiency is rejected at 99% level of confidence, but we fail to reject the market efficiency at 90% level.

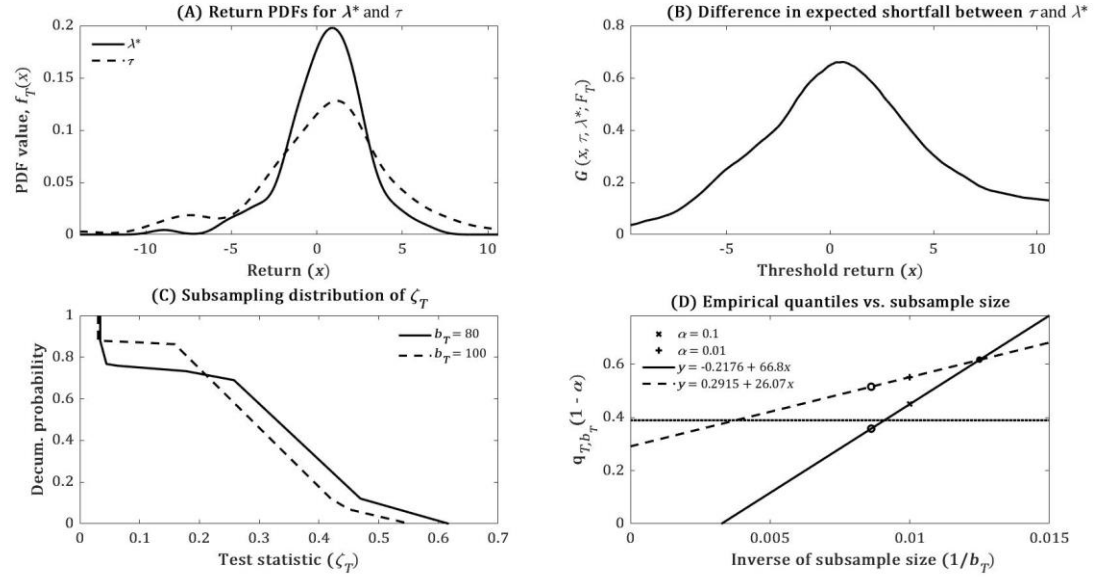


Figure 19. Results of the stochastic efficiency test for a combination of all stock markets; Bitcoin, Saudi Arabia, and WAEMU exclusive.

As can be seen in the Figure 1 panel A, the optimal portfolio is less risky than the market portfolio. In panel B, we can see that the expected shortfall of the efficient portfolio is not so different from the market's for some extremely negative returns. Besides, the rejection of market efficiency at 99% level of confidence is obvious in panel D. The mean, standard deviation, and Sharpe ratio of the optimal portfolio are 0.5743%, 2.2867%, and 0.2306, respectively. For the market, these values are 0.4435% mean return, 3.9431% standard deviation, and 0.1006 Sharpe ratio. So, the optimal portfolio's Sharpe ratio is more than twice that of the market.

By including Bitcoin to our base assets and running the tests, the optimal portfolio consists of 41.4% T-bill, 15% USA, 1.2% Denmark, 0.2% Finland, almost 0.2% Ireland, almost 0.2 Israel, almost 0.1% Portugal, almost 0.2% New Zealand, almost 0.2% Singapore, almost 0.2% Poland, almost 0.3% China, 0.1% Thailand, almost 0.2% Estonia, 0.4% Romania, 0.3% Kenya, 32.1% Tunisia, 0.2% Bahrain, 0.2% Bangladesh, 0.1% Sri Lanka, and 7.2%

Bitcoin. Test statistic, here, is the same as without Bitcoin case, 0.0362, and we rejected the efficiency of the market for 99% level of confidence and failed to reject for 90% level of confidence.

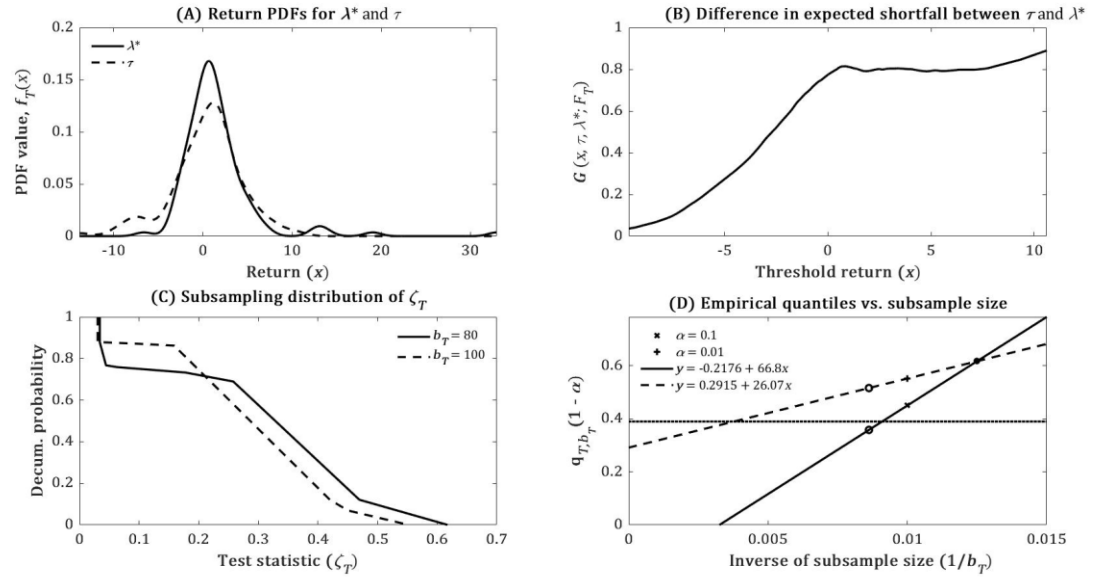


Figure 20. Results of the stochastic efficiency test for a combination of all stock markets; Bitcoin-inclusive portfolio, Saudi Arabia and WAEMU exclusive.

As can be seen in Figure 2 panels A and B, the optimal portfolio with Bitcoin has both lower risk and lower expected shortfall compared to the market. The mean, standard deviation, and Sharpe ratio of the optimal portfolio are 1.6644%, 4.5067%, and 0.3589, respectively. The Sharpe ratio of the portfolio with Bitcoin is not only higher than the portfolio without Bitcoin, but also 3.5 times of the market's Sharpe ratio. So, the performance of the efficient portfolio, obtained by stochastic spanning method based on all countries' market Bitcoin. In Figure 21 panels A and B, this improvement can be seen easily.

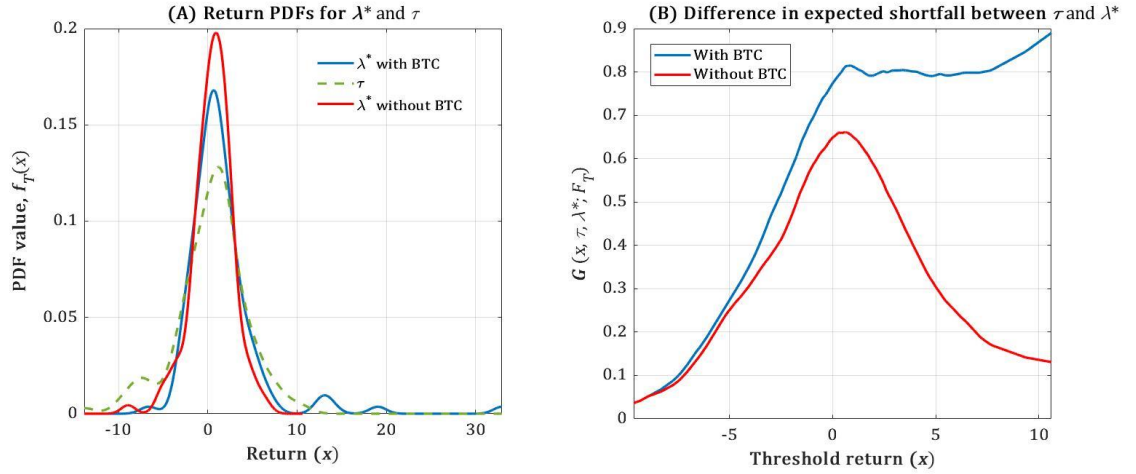


Figure 21. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on a combination of all stock markets; Saudi Arabia and WAEMU exclusive.

5.3.7 Precious Metals

In this section, we considered four famous precious metals, gold, silver, palladium, and platinum, plus T-bill as the base assets. The market is with SPGCPM Index ticker. When we run the code for Bitcoin-exclusive case, the optimal portfolio consists of 44.4% T-bill, 27.6% gold, and 28% palladium. The test statistic is 0.1742, and we fail to reject the market efficiency at both significance levels. The mean return, standard deviation, and Sharpe ratio of the optimal portfolio are 0.5811%, 2.9128%, and 0.1834, respectively. For the market, these values are 0.3366%, 5.0201%, and 0.0577, respectively. Thus, Sharpe ratio of the optimal portfolio is almost three times of the market's.

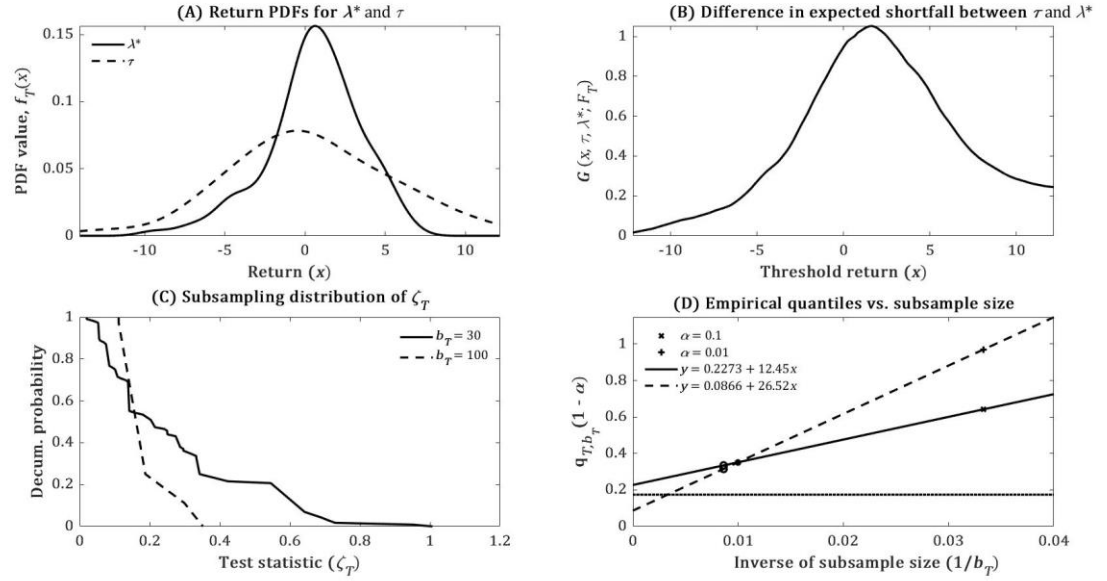


Figure 22. Results of the stochastic efficiency test for precious metals; Bitcoin-exclusive portfolio.

When Bitcoin is added to the base assets, the new efficient portfolio accounts for 85.4% T-bill, almost 0.6% gold, no silver and platinum, 9.1% palladium, and 4.9% Bitcoin. Test statistic here is the same as the previous Bitcoin-exclusive case, and it is 0.0162. As for the previous case, we fail to reject the market efficiency at both significance levels.

The mean return, standard deviation, and Sharpe ratio of the optimal portfolio are 1.1351%, 3.1751%, and 0.3427, respectively. The Sharpe ratio here is more than twice of the Bitcoin-exclusive efficient portfolio. Also, the expected shortfall of the optimal portfolio including Bitcoin has improved (Figure 23, panel B). Therefore, the Bitcoin-inclusive stochastically efficient portfolio has got a really high Sharpe ratio and lower risk in comparison with the market's.

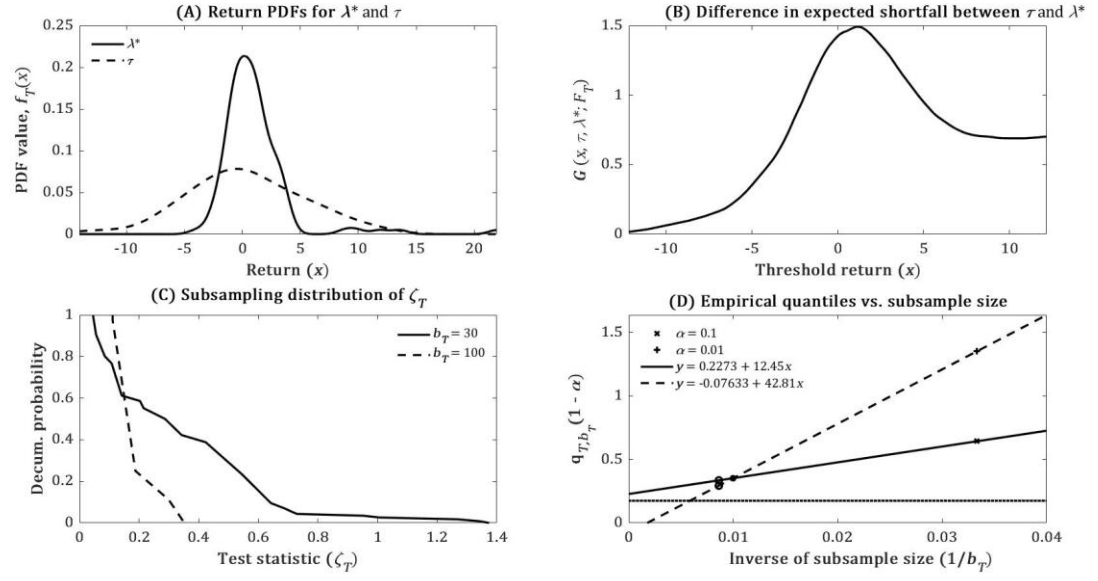


Figure 23. Results of the stochastic efficiency test for precious metals; Bitcoin-inclusive portfolio.

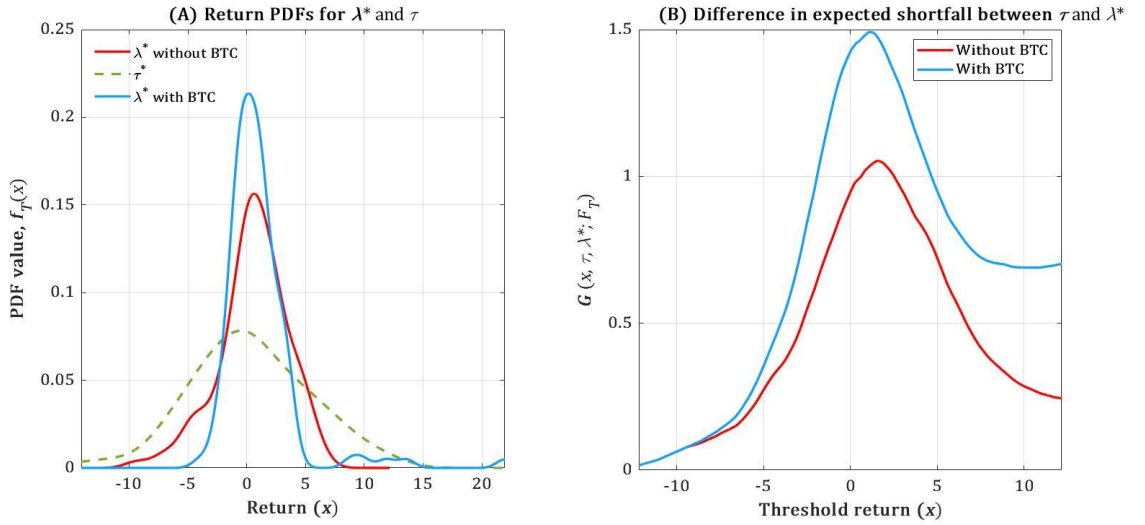


Figure 24. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on precious metals.

5.3.8 Energy Commodities

Four energy commodities, crude oil, gasoline, heating oil, and natural gas, are considered in this section, and our aim is to see if the efficient portfolio obtained based on these for commodities plus T-bill dominates the market portfolio. Then, we add the Bitcoin to this portfolio, running the test again, to see the effect of the Bitcoin on the optimal portfolio.

When the base assets exclude Bitcoin, the efficient portfolio consists of 93% T-bill, 0.2% gasoline, 5.1% heating oil, and 1.7% natural gas. Test statistic is 2.5910, and we fail to reject the market efficiency at the edge of both critical values (90% critical value is 2.6147, and 99% one is 2.751). The mean return, standard deviation, and the Sharpe ratio of the optimal portfolio are, 0.0211%, 0.4878%, and -0.0528, respectively. These values for the market portfolio are -0.5728%, 8.6944%, and -0.0713.

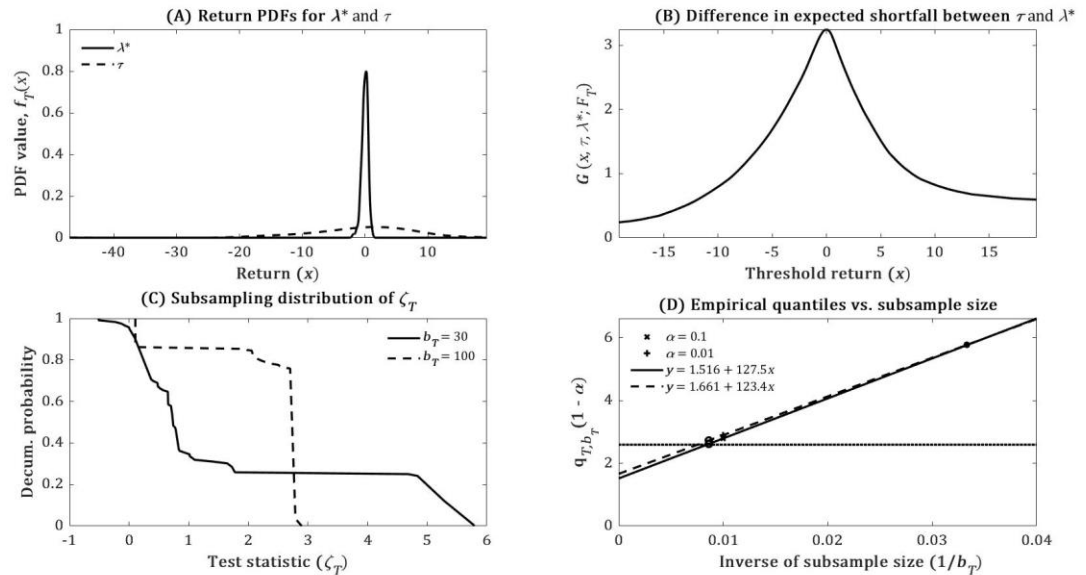


Figure 25. Results of the stochastic efficiency test for energy commodities; Bitcoin-exclusive portfolio.

When Bitcoin is added to the base assets, the efficient portfolio accounts for almost 90% T-bill, 0.5% heating oil, and 9.5% Bitcoin. Test statistic is the same as the case without Bitcoin, 0.2426. As can be seen in panel B of Figure 26, the expected shortfall of the portfolio including Bitcoin is much higher than the Bitcoin-exclusive one. The mean return, standard deviation, and the Sharpe ratio of the optimal portfolio are, 1.8521%, 5.8575%, and 0.3082, respectively. So, the Sharpe ratio improves a lot by adding Bitcoin as one of our base assets.

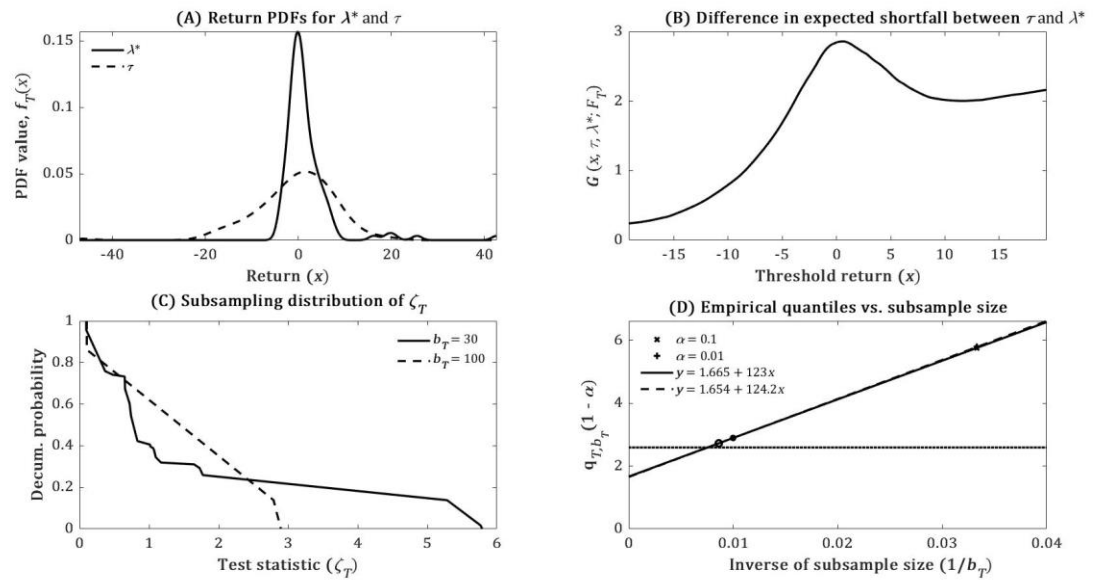


Figure 26. Results of the stochastic efficiency test for energy commodities; Bitcoin-inclusive portfolio.

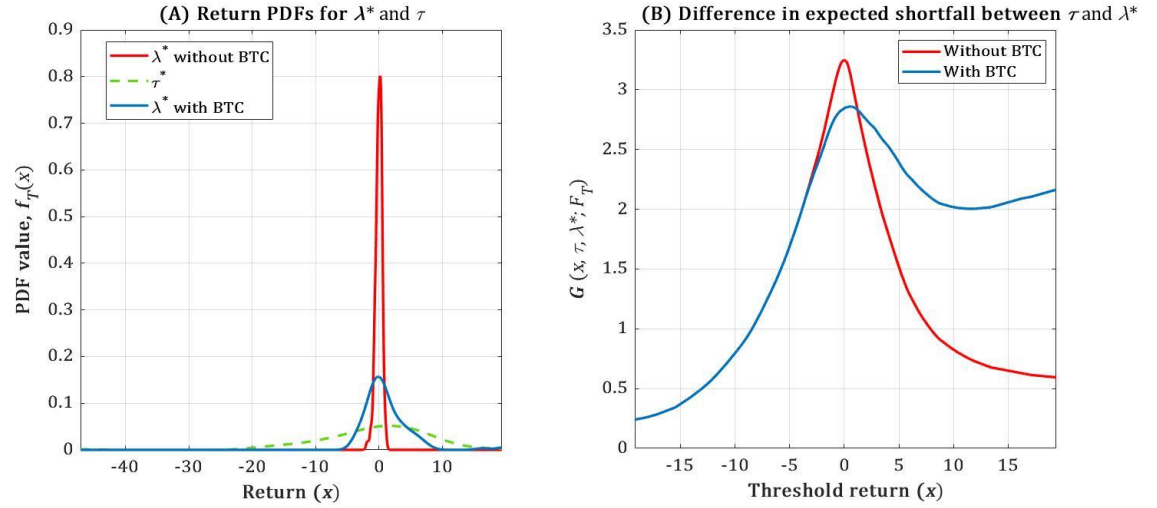


Figure 27. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on energy commodities.

5.3.9 Agricultural Commodities

In this part, we replace the base assets by seven agricultural commodities: corn, soybean, wheat, sugar, coffee, cocoa, and cotton, and our aim is to obtain the efficient portfolio based on stochastic spanning method.

The optimal portfolio here possesses 89.3% T-bill, 9.5% wheat, 0.7% coffee, and almost 0.5% sugar. Besides, we fail to reject the market efficiency at both significance levels, and the test statistic is 0.6062. The mean return 0.0620%, the standard deviation 0.8306%, and the Sharpe ratio 0.0181 are values for the optimal portfolio of this part. We can compare these values with the market's in this section which are -0.4274%, 5.8278%, and -0.0814, respectively. It is obvious that the optimal portfolio here outperforms the market one.

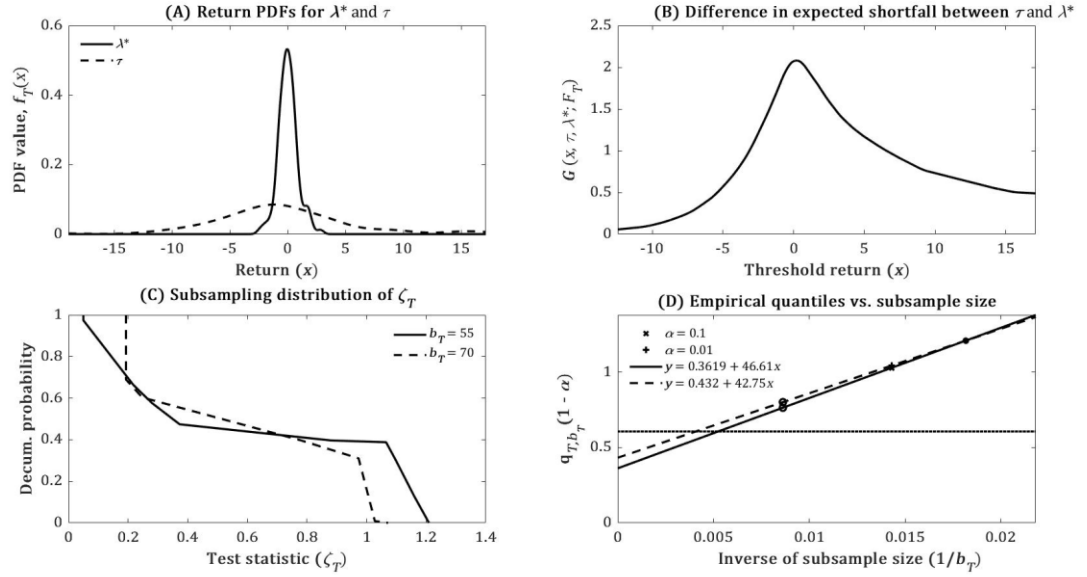


Figure 28. Results of the stochastic efficiency test for agricultural commodities; Bitcoin-exclusive portfolio.

Having added Bitcoin to the base assets, the optimal portfolio changes for 0.91.2% T-bill, 0.1% soybean, 0.2% sugar, 0.1% coffee, 0.1% cocoa, almost zero percent of cotton, wheat, and corn, and 8.1% Bitcoin. Now, the mean return is 1.6012%, standard deviation is 5.0347%, and the Sharpe ratio is 0.3087. These values show an improvement of the portfolio with Bitcoin compared to the market portfolio and the equivalent Bitcoin-exclusive efficient portfolio. Furthermore, the expected shortfall of the portfolio compared to that of the market improves as a result of adding Bitcoin to the portfolio. The test statistic here is the same as the Bitcoin-exclusive case, and we, again, fail to reject the market efficiency at both significance levels (Figure 29 panel D).

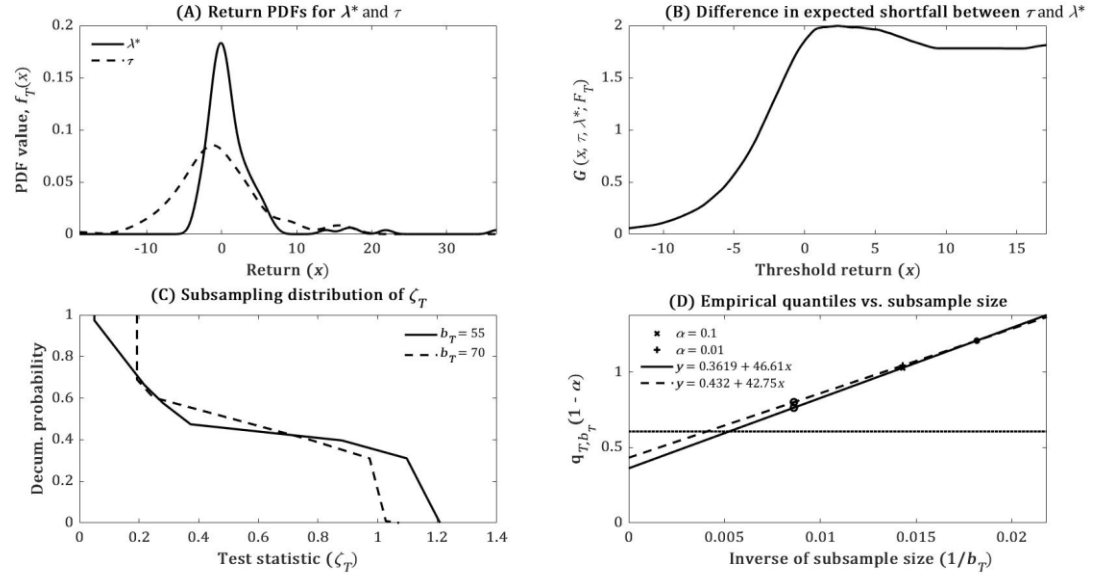


Figure 29. Results of the stochastic efficiency test for agricultural commodities; Bitcoin-inclusive portfolio.

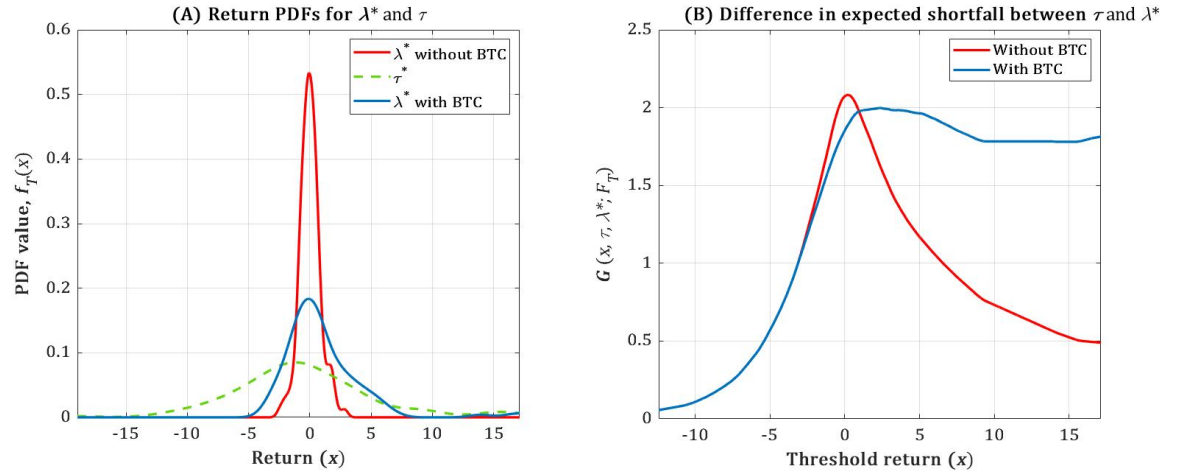


Figure 30. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on agricultural commodities

5.3.10 All Commodities

In this part, all commodities in sections 7, 8, and 9 plus T-bill are considered as the base assets. The market is the Goldman Sachs Commodity Index. The optimal portfolio consists of 0.5% gold, 20.4% palladium, 0.1% crude oil, 0.1% natural gas, 0.1% corn, 0.2% wheat, 0.1% coffee, almost 80% T-bill, and almost zero percent of all other assets. The test statistic here is 1.3639, and we fail to reject the market efficiency at 1% and 10% significance levels. The mean return, the standard deviation, and the Sharpe ratio for the efficient portfolio vs. for the market are 0.3736% vs. -0.4277%, 1.6647% vs. 6.0237%, and 0.1962 vs. -0.0788, respectively. What is obvious is the better performance of the efficient portfolio based on stochastic spanning method.

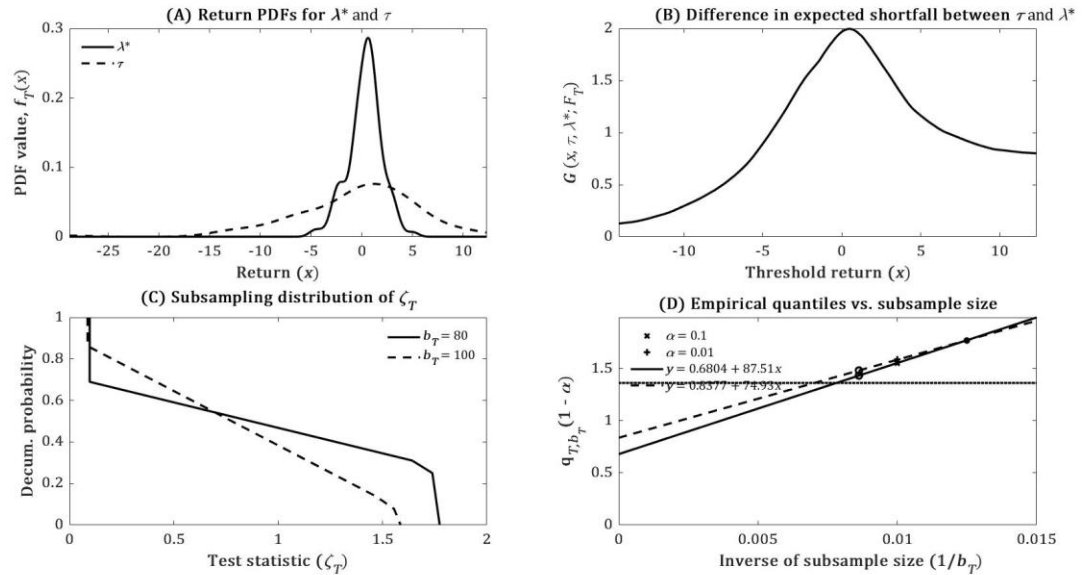


Figure 31. Results of the stochastic efficiency test for all commodities; Bitcoin-exclusive portfolio.

When we add Bitcoin to the base assets and perform the stochastic efficiency test again, the optimal portfolio opens a space for Bitcoin. The efficient portfolio, now, is formed of 0.3% gold, 10.5% palladium, 0.1%

natural gas, 0.1% soybean, 0.1% wheat, 82.8% T-bill, 5.6% Bitcoin, and almost zero percent of other assets in this category. Test statistic is the same as the bitcoin-exclusive portfolio of this section, and we fail to reject the market efficiency at both significance levels. The mean return, standard deviation, and the Sharpe ratio of the Bitcoin-inclusive efficient portfolio are 1.2926%, 3.6447%, and 0.3418, respectively. The optimal portfolio with Bitcoin is riskier than the one excludes Bitcoin, Bitcoin-inclusive portfolio has a higher Sharpe ratio, and both efficient portfolios outperform the market one. Besides, an improvement in the expected shortfall of the Bitcoin-inclusive efficient portfolio is evident, compared to the efficient portfolio without Bitcoin (Figure 33, panels A and B).

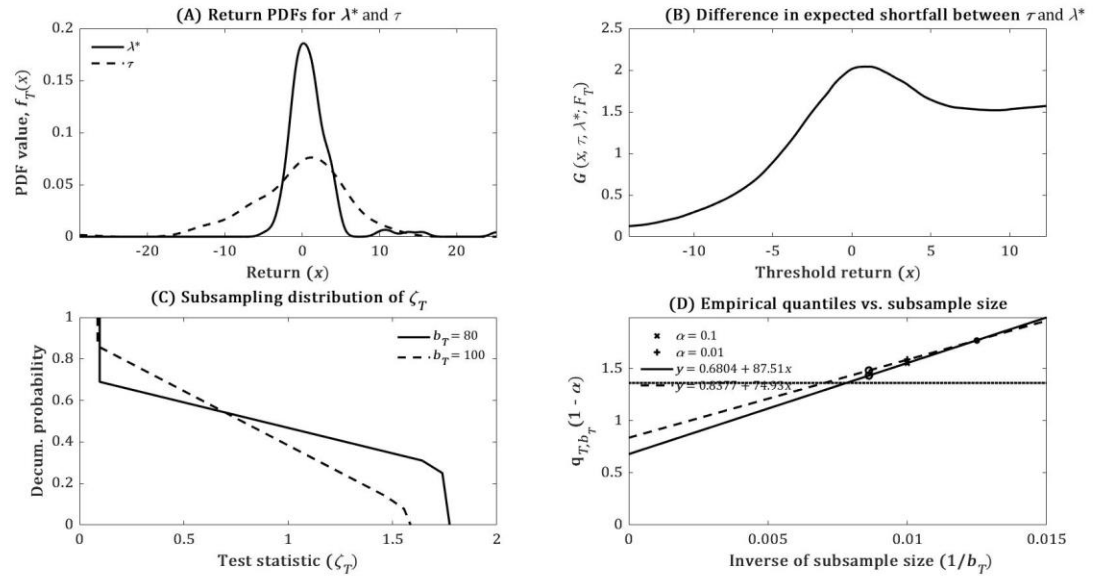


Figure 32. Results of the stochastic efficiency test for all commodities; Bitcoin-inclusive portfolio.

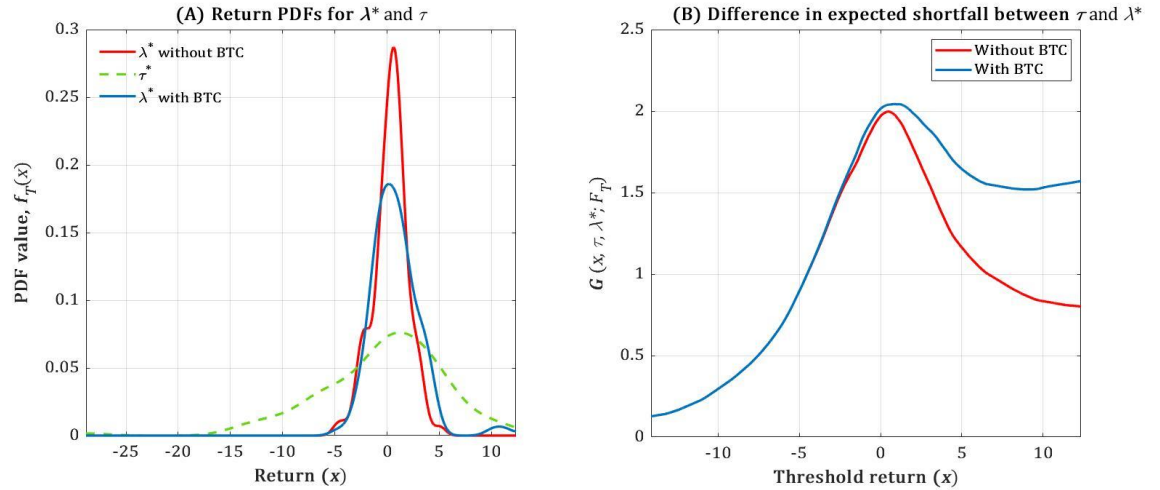


Figure 33. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on all commodities

5.3.11 All Assets

In the last part, we consider all assets in the other sections. Thus, on one side, we have T-bills plus all the assets in other sections, apart from section one which has already been included in the other sections, and on the other side, the market. The problem here is Saudi Arabia and WAEMU. The prices of these two markets are not available from the very first day of Bitcoin. Besides, due to a number of base assets in this category, we do not include these two assets in our test since the data is inadequate. Also, since there is no index to be considered as the market of this section, we apply Principle Component Analysis, PCA, to find a proper benchmark for the market. Furthermore, in the calculation of PCA, these two specific markets, Saudi Arabia and WAEMU, are excluded.

In the case Bitcoin is excluded, the optimal portfolio consists of 0.2% consumer services, 40.5% health care, 0.3% industrials, 9.5% technology, 0.6% telecommunications, 1.3% USA, 0.4% Germany, 0.1% Ireland, 0.1%

Israel, 0.4% Spain, 0.3% Switzerland, 0.2% Hong Kong, 0.2% Japan, 0.2% New Zealand, 0.1% Colombia, 0.1% Egypt, 0.2% Greece, 0.5% South Africa, 0.2% China, 0.3% Korea, 0.6% Lithuania, 0.1% Kazakhstan, 2.6% Kenya, 22.4% Tunisia, 0.2% Oman, 0.1% Bangladesh, 0.1% Sri Lanka, 0.3% Vietnam, 0.6% gold, 0.2% silver, 0.1% platinum, 12.9% palladium, 0.1% gasoline, 0.3% corn, 0.1% soybean, 0.4% wheat, 0.2% sugar, 0.4% coffee, 1.6% T-bills, and almost nothing for the other assets.

Test statistic for this class is 1.0547, and we reject the market efficiency at both confidence levels. The mean return, the standard deviation, and the Sharpe ratio of the optimal portfolio are 0.8072%, 2.7236%, and 0.2795, respectively. We can compare these values with the market's which are 1.83e-16%, 6.4086%, and -0.0072, in the same order. So, the efficient portfolio, here, outperforms the market portfolio.

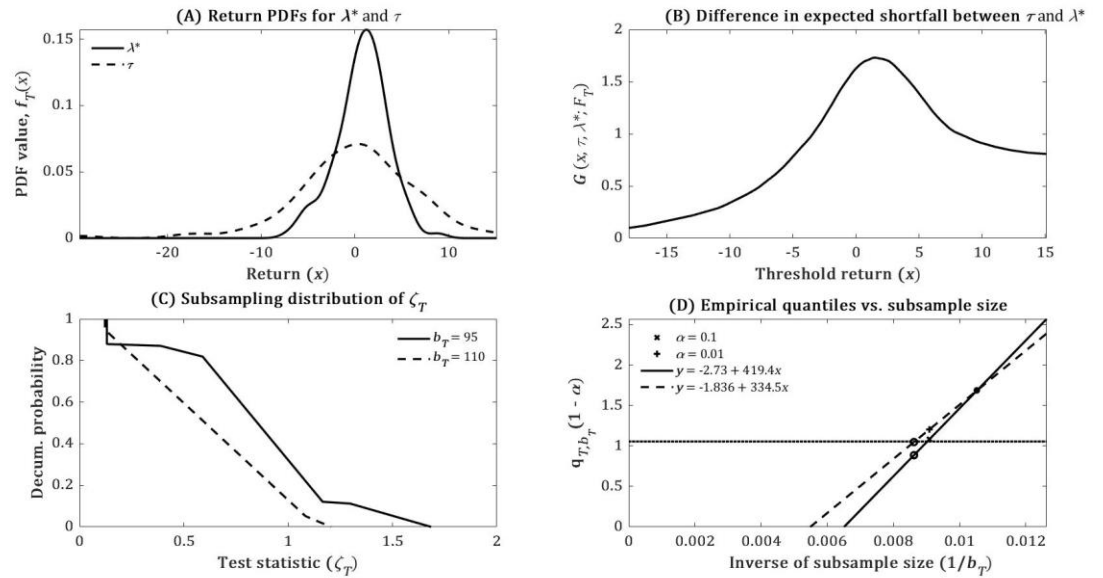


Figure 34. Results of the stochastic efficiency test for all assets; Bitcoin-exclusive portfolio.

When Bitcoin is added to the base assets, the combination of assets changes; however, the Bitcoin has no space in the optimal portfolio. The

optimal portfolio accounts for 0% Bitcoin, 1% consumer services, 1% consumer goods, 1.3% financials, 33.2% health care, 0.5% industrials, 6.5% technology, 0.3% telecommunications, 1.1% USA, 0.2% Belgium, 0.7% Denmark, 0.2% Germany, 0.1% Ireland, 0.2% Israel, 0.8% Portugal, 0.3% Spain, 0.7% Hong Kong, 0.9% Japan, 0.3% Mexico, 0.4% Peru, 0.3% Czech Republic, 0.1% Turkey, 0.5% United Arab Emirates, 1.6% China, 0.3% India, 0.5% Korea, 0.5% Malaysia, 0.4% Pakistan, 0.7% Thailand, 0.1% Estonia, 0.4% Lithuania, 0.2% Kazakhstan, 0.3% Romania, 0.1% Serbia, 0.5% Slovenia, 2.2% Kenya, 0.3% Mauritius, 0.6% Morocco, 20% Tunisia, 0.5% Bahrain, 0.4% Jordan, 0.2% Kuwait, 0.1% Lebanon, 0.2% Bangladesh, 0.1% Sri Lanka, 1% gold, 0.1% silver, 13.2% palladium, 0.5% crude oil, 0.2% natural gas, 0.5% corn, 0.2% soybean, 0.4% wheat, 0.2% sugar, 0.3% coffee, 0.4% cocoa, and 19.7% T-bills.

Test statistic is 1.0547, and we reject the market efficiency at both significance levels. The mean return of the efficient portfolio including Bitcoin is 0.7166%, the standard deviation of the efficient portfolio is 2.7573%, and its Sharpe ratio is 0.2432. These numbers show that the efficient Bitcoin-exclusive portfolio outperforms both the one including Bitcoin and the market. Therefore, including Bitcoin does not improve the performance of the efficient portfolio. The comparison between these two portfolios, with and without Bitcoin, is much more evident in the Figure 36. Besides, including Bitcoin does not even improve the expected shortfall of the efficient portfolio either.

A part of poor results of this section can be due to the effect of Covid19 pandemic on the economy. In Figure 37, which shows returns for the market of this section till March 2020, the return for the March 2020 is extremely negative (almost -30%), almost an outlier compared to the others. As we mentioned in the previous chapter, in times of crisis, the correlation between

Bitcoin and other conventional assets increases, so adding Bitcoin to the optimal portfolio does not necessarily improve the performance of the portfolio.

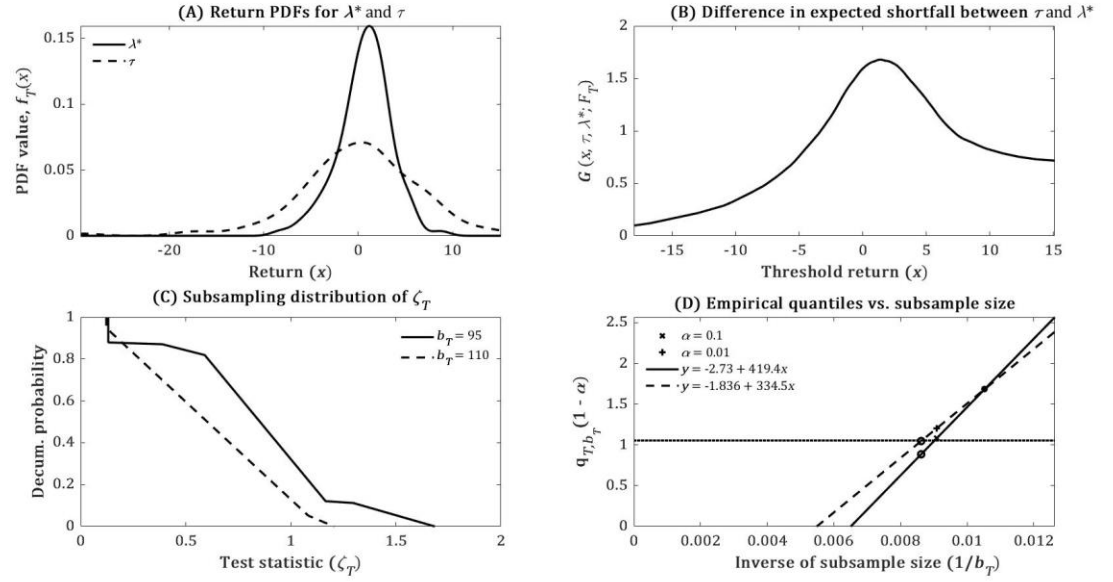


Figure 35. Results of the stochastic efficiency test for all assets; Bitcoin-inclusive portfolio.

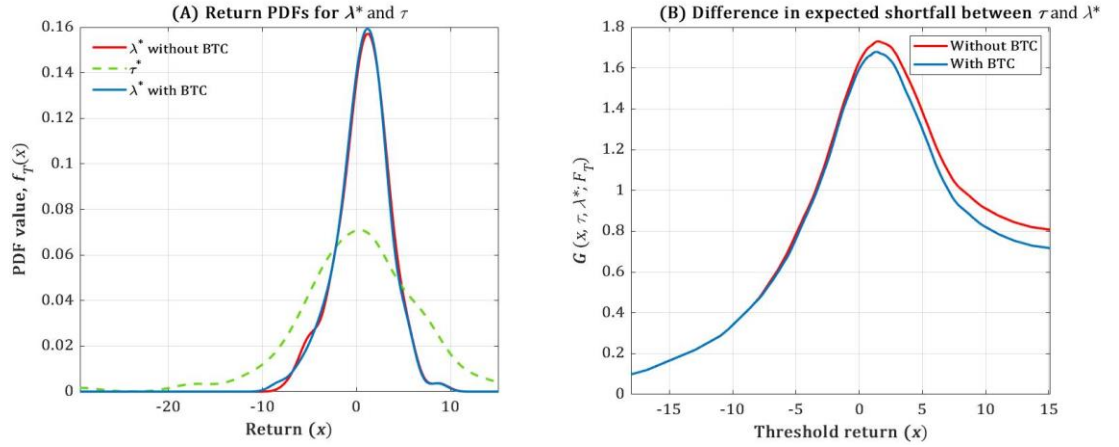


Figure 36. The comparison between the return PDF (panel A) and expected shortfall (panel B) for portfolios with (blue curve) and without the Bitcoin (red curve), based on all assets

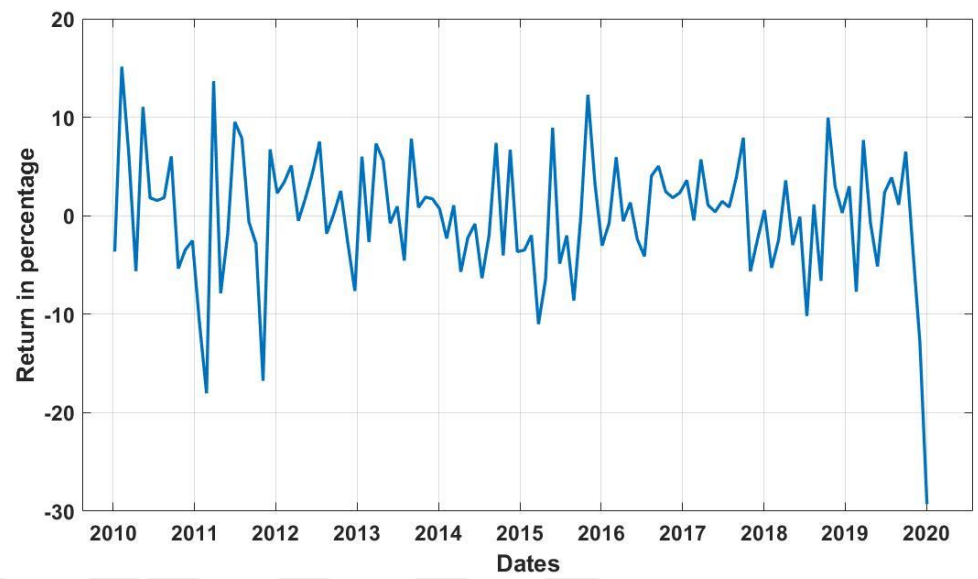


Figure 37. PCA return percentage.

CHAPTER VI: CONCLUSION

In this thesis, we have considered the famous methods of diversification and portfolio optimization, including MPT and stochastic dominance, and we went over the superiorities and shortages of these methods. We have also reviewed the novel method of stochastic spanning. Besides, the Bitcoin, as a new asset and an investment opportunity, has been evaluated. We have explained some of the main features of this virtual currency, as well.

We have applied the stochastic spanning scheme to 11 different asset classes and in two steps: pre- and post-Bitcoin inclusion to see if Bitcoin finds a place in the efficient portfolio of each section. The Sharpe ratio has been utilized to compare Bitcoin's inclusive and exclusive portfolios.

We have realized that the Bitcoin inclusive portfolio outperforms the Bitcoin-exclusive ones for all first 10 asset classes. In the last category, the poor result could be because of the market crash at the end of March 2020 or applying PCA analysis instead of considering a proper index for the market.

In future research, we hope to do the following:

1. Doing the same analysis with means of other cryptocurrencies
2. Evaluating the efficient portfolios, pre- and post-Bitcoin inclusion, by other methods, such as Treynor ratio and Jensen measure
3. Applying different kinds of subsampling methods like bootstrap and comparing the results with this study's ones



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