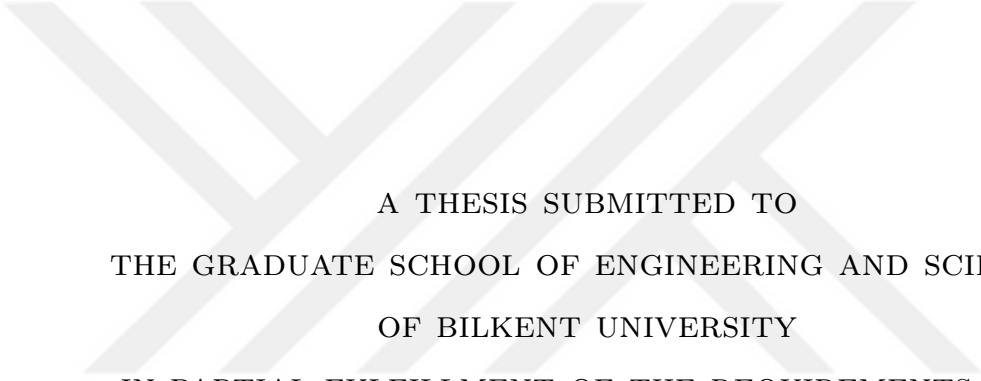


ASSORTMENT PLANNING CONSIDERING SPLIT ORDERS



A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF
MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

By
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August 2021

Assortment Planning Considering Split Orders

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August 2021

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

ASSORTMENT PLANNING CONSIDERING SPLIT ORDERS

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When multi-item orders cannot be satisfied through a single shipment stemming from not having all the items in an order in the same warehouse, the cost of packaging and transportation increases and the delivery of the orders can be delayed. In this regard, split order problem is one of the most significant challenges that the online retailers face. As the capacities of the warehouses are limited, it is not possible to stock every item in every warehouse. To minimize the number of orders that cannot be satisfied in a single shipment, it is important to determine how the limited capacities of the warehouses should be allocated to items or it is necessary to decrease the transportation costs through consolidating the split orders. Since this problem is NP-hard, the previous studies in the literature are based on heuristic algorithms. In this study, exact and heuristic methods have been examined to solve large scale problems. Some of the heuristic algorithms offered uses the LP relaxation of the model provided by Jehl et al. (2018). In this sense, the analytical characterization of the optimal solution of the LP relaxation has also been revealed. It is proved that the allocation variables can only take three different values at most one being fractional. It is shown that this solution can be found without actually solving the LP relaxation by benefiting from an algorithm offered in literature to solve 0-1 fractional programming problems. Moreover, it is proved that a similar characterization is preserved for multiple warehouses or when a central depot with unlimited capacity and a forward distribution center are considered together. Additionally, the working principle of the greedy ranking algorithm offered in the literature is theoretically justified and a dynamic version of this algorithm is developed. To evaluate the performance of the heuristic algorithms offered and the run time of the integer programming problem, an extensive numerical study has been conducted. The change in the difficulty level of the problem based on the plant capacity, the

number of orders, and the number of stock keeping units (SKU) is scrutinized. Furthermore, the assortment allocation problem is modeled together with the consolidation problem. The performance of the model is evaluated through comparing its solution to the solution obtained through solving two problems consecutively.



Keywords: Split order, Assortment Planning, Consolidation, Mathematical Programming, Heuristic Algorithms.

ÖZET

BÖLÜNMÜŞ SEVKİYAT MİKTARINI ENAZLAMAK İÇİN ÇEŞİT PLANLAMA

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Birden çok ürün içeren siparişlerin tüm ürünlerin aynı depoda bulunmamasına bağlı olarak tek seferde karşılanamaması paketleme ve taşımacılık masraflarını artırıp teslimatta gecikmelere sebep olmaktadır. Bu elektronik perakendecilerin karşılaştığı en önemli sorunlardan biridir. Depoların kapasitesi sınırlı olduğu için bütün ürünlerin bütün depolarda bulundurulması mümkün olmamaktadır. Tek bir sevkiyat ile karşılanamayan siparişlerin sayısının azaltılması için depoların kısıtlı kapasitelerinin hangi ürünler için kullanılmasına karar verilmesi veya bölünmüş siparişlerin konsolidasyon yöntemiyle birleştirilerek maliyetin düşürülmesi gerekmektedir. Bu problem NP-zor sınıfına ait olduğundan literatürdeki çözümler sezgisel algoritmalara dayanmaktadır. Bu çalışmada bu problemi büyük ölçeklerde çözebilmek için kesin ve sezgisel yöntemler incelenmiştir. Geliştirilen sezgisel yöntemlerden birkaçı Jehl ve ark. (2018) modelinin doğrusal gevşetme çözümünü kullanmaktadır. Bu bağlamda doğrusal gevşetme çözümünün analitik karakterizasyonu da yapılmış, ürünlerin en fazla üç farklı değer alabildiği ve bu değerlerden en fazla bir tanesinin kesirli olduğu kanıtlanmıştır. Literatürde ikili kesirli programlama problemlerinin çözümü için önerilen bir algoritmadan faydalanılarak bu çözümün doğrusal gevşetme modeli çözülmeksizin bulunabileceği gösterilmiştir. Ayrıca birden çok depo olduğunda veya bir merkez depo ile bir dağıtım deposu beraber düşünüldüğünde de benzer bir özelliğin varlığı ispatlanmıştır. Bunlara ek olarak literatürde önerilmiş olan ağgözlü sıralama algoritmasının çalışma mantığı teorik olarak temellendirilmiş, bu algoritmanın dinamik versiyonu geliştirilmiştir. Önerilen algoritmaların çözüm kalitesini ve tamsayı programlama modelinin çözüm zamanını değerlendirmek için geniş çaplı bir sayısal çalışma yapılmış ve problem zorluk seviyesinin kapasite, ürün ve sipariş sayısına göre değişimi irdelenmiştir. Ayrıca çeşit planlama

problemi konsolidasyon problemi ile birlikte modellenmiş, sayısal çalışmalarla modelin performansı bu iki problemin ardışık olarak çözülmesiyle elde edilen çözüm ile karşılaştırılmıştır.



Anahtar sözcükler: Bölünmüş Sevkiyat, Çeşit Planlama, Konsolidasyon, Matematiksel Programlama, Sezgisel Algoritmalar.

Acknowledgement

Firstly, I would like to express my gratitude to my advisors Oya Karařan and Alper řen for believing in and supporting me throughout the research that I have conducted. I had the chance to study under their guidance for three years starting from my senior year as an undergraduate student and I have learned a lot from them. They have shown me how a passionate researcher should be and have prepared me for the rest of my academic career. I feel so grateful for everything they have done for me.

I am indebted to řzlem řavuş İyigün and Seçil Savaşaneril Tüfekci for accepting to read and review this thesis. They have broadened my horizon with their comments and recommendations.

This thesis is a product of the knowledge that I have attained during my studies at Bilkent University. I am deeply thankful to all of my professors for their contributions. Especially, I would like to thank Selim Aktürk for guiding me in my decisions. It has been a motivation for me to feel his support and attention.

I want to acknowledge that TUBITAK supported my graduate studies financially as a part of 2210-A National Graduate Study Scholarship Program.

Additionally, I want to thank the people who have always been there for me. Without them, I would not manage to complete this thesis. My mother Nuray Söylemez has always been my greatest supporter. It is not possible to describe her love and how grateful I am for everything she has done for me. I want to thank my father Mehmet Ali Söylemez for his endless love. He always trust me more than myself. I am so thankful to my lovely sister and lifelong friend Damla Söylemez for being there for me whenever I need. It is a relief for me to feel her support. I could not be more grateful to my grandmother Aynur Can for accompanying me at every moment of my life. I want to thank my grandfather and first teacher Bekir Can for always believing in me. I am grateful to my

grandparents Emine and Salih Söylemez for their emotional support. Also, I am thankful to have my uncle Özgür Can and his wife Elif Can on my side while taking the greatest decisions of my life.

My special thanks go to Baha Erim Uzunoğlu for being the best friend that someone can ask for. I feel privileged to share every single moment of my life with him. As always, he was the source of my tranquility and he made me smile during my graduate studies. I am so grateful for his love and understanding. I look forward to experiencing the next stages of the life together.

I want to thank my friends Deniz Şimşek, İrem Nur Keskin, and Deniz Akkaya for their companionship in this journey, I am indebted to Parinaz Toufani for her guidance and friendship. I would like to thank Ege Bilaloğlu and his brother as they facilitated my work via their technical knowledge. I am also thankful to all other fellow graduate students for their contributions to my life.

I would like to thank Ezgi Şahin for her valuable friendship. Although Bilkent was not the same without her, she made me feel as if she was there thanks to “kırmızı yeşil şapkalı penguen”. As always, I felt the support of my dear friends Berivan Işık and Vuslat Sena Şener throughout the program. I am so lucky to have them in my life.

Lastly, I would like to thank myself for making a great effort and not giving up. It was not so easy, but I accomplished.

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Chapter 1

Introduction

In this chapter, the split order problem is introduced and our motivation to study this problem is provided. Contributions of this study are discussed. Lastly, the outline for the rest of the thesis is presented.

1.1 Motivation

Assortment is the set of products offered by a retailer to its customers, and when it is considered in the warehouse level it refers to the set of products assigned to one warehouse. Online retailers having a wide network of distribution centres may have a limited assortment in each distribution center. Initially, as the capacities of the warehouses are limited, it is not possible to carry all SKUs in every warehouse. Subsequently, retailers may want to benefit from economies of scale for their operations like inventory replenishment. However, having a limited assortment in warehouses results in the possibility of split orders. That is, some orders cannot be satisfied from the same warehouse in one shipment and these orders have to be split and fulfilled through several shipments from different warehouses. According to the analysis of Xu et al. [4] in a consolidated dataset, 37.6 % of a large online retailers orders include multiple items and approximately 12 % of

these orders could not be satisfied with a single delivery. Catalán and Fisher [2] also state that 13 % of multi-item orders received by the Chinese online grocery retailer Yihaodian needed to be split. Namely, the number of split orders is at a significant level for online retailers, and they increase the outbound shipping and packaging costs. In addition, they are also inconvenient for customers and cause environmental waste due to extra packaging material used.

1.2 Contributions

To begin with, in this study, we relate the split order problem for a single warehouse to some other problems by observing that it is a special version of the densest k -subhypergraph problem. Therefore, the theoretical findings and methods that we have offered for our problem in the context of single warehouse are also valid for the problems that can be modeled in this special form of the densest k -subhypergraph problem.

There are a some theoretical findings that we present in this work. The most significant one is that we characterize the solution of the LP relaxation of the benchmark model that we use for assortment planning in a single warehouse. We show that the allocation variables can only take three different values at most one being fractional. By benefiting from an algorithm offered in the literature, we also offer a combinatorial algorithm to find this solution. We extend this characterization to the multi-warehouse problem. Also, we prove a similar characterization for the optimal solution of the assortment planning problem where a central depot with unlimited capacity and a forward distribution center are considered together and the total transportation cost is minimized. We show that by using this solution, a solution can be developed for the problem where multiple forward distribution centers are considered.

As the densest k -subhypergraph problem is NP-hard [5], we also study on some heuristic algorithms for the single warehouse problem. Initially, we provide a theoretical justification for an algorithm offered in the literature and we develop

a dynamic version of this algorithm. Additionally, we offer three more algorithms based on the LP relaxation of our benchmark model. As one of these algorithms gives solutions whose objective function values are either close to or better than the solutions obtained through the benchmark model within a time limit and since the runtime of this algorithm is reasonable for large scale instances, we suggest this algorithm as an alternative to the benchmark model.

There are two approaches in the literature to solve the split order problem: assortment planning and order consolidation. However, there is not a study combining these two problems. As a crucial contribution of our study, we show that it is possible to reduce the total transportation cost by solving these problems together instead of solving them consecutively.

To sum up, the main contributions of this thesis are as follows.

- We prove that the optimal solution of the LP relaxation of the benchmark model, which we use for single warehouse split order problem, has a special structure.
- We extend this structure to some other problems.
- We offer a combinatorial algorithm to find the solution of the LP relaxation of the benchmark model.
- We provide a theoretical justification for an existing heuristic algorithm and we offer four other algorithms for single warehouse split order problem one of them being based on this existent algorithm.
- We show that solving assortment planning and order consolidation problems together can decrease the transportation costs significantly.
- As our problem is a special form of the densest k -subhypergraph problem, our theoretical results and the algorithms also apply to other problems that can be modeled in this form.

1.3 Outline

The organization of this thesis is as follows. In Chapter 2, a detailed literature review is conducted. The studies related to split order problem are categorized based on their approach. Also, the split order problem for a single warehouse is related to some other problems from the literature.

Chapter 3 focuses on the split order problem for a single warehouse. In this chapter, the problem is defined and the benchmark model from the literature is analysed. The special structure of the LP relaxation of this model is proven and it is extended for multiple warehouses and some other problem contexts. Also, a combinatorial algorithm is suggested to find this structured solution. A justification for a heuristic algorithm offered in the literature is provided. A dynamic version of this algorithm and some other heuristic algorithms based on the LP relaxation of the benchmark model are offered. Additionally, some valid inequalities for the problem are introduced. The extensive numerical experiments conducted and the data used in these experiments are described. The results obtained in an aggregate form and the conclusion reached from these results are given.

In Chapter 4, the problem is tackled for multiple warehouses. The aim of this chapter is to observe the effect of combining proactive and reactive approaches to either eliminate or solve the split order problem. For this, one benchmark model for each approach from the literature is introduced. A numerical study comparing the performances of solving assortment allocation and consolidation problems consecutively and combining these two problems in a single model is provided. The data used in this study is also described. Lastly, the conclusion reached through the numerical study is delivered.

A brief summary of this study is presented in Chapter 5. Additionally, the future research directions related to the split order problem are discussed.

Finally, in the appendix, the detailed numerical results for Chapter 3 and Chapter 4 are provided.

Chapter 2

Literature Review

This chapter provides a detailed literature review for the split order problem.

As Catalán and Fisher [2] state, despite the large impact of split orders on online retailers, this subject is barely addressed in the operations management literature. The studies directly related to the problem can be grouped into two categories based on their approach. While Xu et al. [4], Catalán and Fisher [2], Jehl et al. [1], and Zhu et al. [6] hold a proactive approach and try to eliminate the problem through assortment planning, Zhang et al. [7] and Zhang et al. [3] hold a reactive approach and deal with the problem after order splitting occurs through consolidation.

2.1 Studies on Assortment Planning

Xu et al. [4] assume that the allocation of the SKUs to warehouses and the inventory levels of the warehouses throughout the planning horizon are given and they determine the allocation of the orders to warehouses. They draw attention to the delay always existing between receiving an order and assembling it for shipping, and suggest that this delay can be used to re-evaluate real-time order-warehouse assignment decisions without violating estimated-to-ship dates

of the SKUs in the orders and question if this re-evaluation is beneficial in reducing the number of split shipments.

As they stated, they model the re-evaluation problem as a fixed-charge multi-commodity network flow problem with a time dimension. With the objective of minimizing the number of shipments, they decide the number of units of an SKU sent at a time epoch from a warehouse to satisfy a customer order and if there is a shipment at a time epoch from a warehouse to an order. By setting constraints, they ensure that the demand for each SKU in the orders is met before the estimated-to-ship date, shipments are limited by the available supply in the warehouses, and it is only possible to satisfy an order from a warehouse if there is a shipment from the warehouse to the order. As the general re-evaluation problem without time dimension is proved to be NP-hard by Xu [8] through showing some special simple cases of the problem is NP-hard, this problem is also NP-hard.

For this reason, Xu et al. [4] propose two heuristic algorithms based on neighbourhood search to solve the large instances of the problem: *order swap* and *SKU exchange*. Both algorithms take a feasible solution as input and improve this solution by reducing the number of shipments. *Order swap* algorithm is based on swapping the warehouses to which a single and a split order are assigned to minimize the number of shipments. In *SKU exchange* algorithm, a transportation problem for each SKU is solved and order allocations are updated. The numerical study conducted has shown that when these two algorithms are combined a near optimal solution can be obtained and 40% to 50% of the extra shipments can be eliminated.

Catalán and Fisher [2] define the problem different from Xu et al. [4] in terms of several aspects. First of all, they do not assume a given allocation of SKUs to warehouses and see allocation of SKUs as a decision by stating that it has a significant effect on the number of split orders. In addition to this, they consider the capacities of the warehouses, but include this constraint as a cardinality constraint. In other words, they assume that as long as an SKU is assigned to a warehouse, it can be used to satisfy an order ignoring the possibility of a stock-out. Lastly, the demand is assumed to be deterministic in this study, as

they believe that the demand will show resemblance to the demand observed in the recent past.

The problem is addressed by Catalán and Fisher [2] as determining the allocation of SKUs and orders to warehouses with the objective that the number of shipments is minimized. The model they build have the following constraints: Each SKU is assigned to a warehouse, the capacities of the warehouses are not exceeded, an SKU required by an order is shipped from exactly one warehouse, SKUs can be only shipped from the warehouses that they are assigned to, and an SKU in an order, which is satisfied from a warehouse means that there is a shipment from this warehouse to meet that order. A very special simple case of this problem is reduced to graph bisection problem, which is proved to be NP-hard by several studies.

Due to this high complexity of the split order problem, Catalán and Fisher [2] propose four heuristic algorithms based on the co-appearance matrix of SKUs to solve the problem with real life data: *greedy orders*, *greedy pairs*, *greedy seeds* and *bestsellers*. The main idea of the first two heuristics is to assign SKUs in the biggest orders or in the SKU pairs that appear in the most of the orders, respectively to the warehouses with the highest capacity. The *greedy seeds* algorithm attempts to assign the most popular SKUs having as small co-appearance as possible with each other to warehouses as seeds and allocate other SKUs according to the average co-appearance of the warehouses with these SKUs. *Bestsellers* algorithm allocates $\left\lfloor \frac{\sum_d k_d - S}{D-1} \right\rfloor$ best-selling SKUs to all warehouses, where k_d refers to the capacity of warehouse d , and S refers to the number of the SKUs and D refers to the number of warehouses. Other SKUs are allocated according to the average co-appearance of the warehouses with these SKUs. In all four algorithms, the remaining capacity is filled with respect to the co-appearance matrix. These heuristics are compared with each other and two benchmark algorithms applied by retailers by using actual transaction data of a Chinese retailer. The key performance indicators in the study are the average number of shipments per order, workload balance across the warehouses, and the number of warehouses that are capable of fulfilling a particular order. *Bestsellers*

algorithm has performed well in terms of all these criteria and has been found to be more advantages than other algorithms.

Jehl et al. [1] study the problem in a relatively small scale and define it as determining the assortment allocation for only one warehouse to maximize the number of orders that can be satisfied. They do not allow partial delivery and assume that if an order cannot be completely satisfied from the warehouse, then it will be satisfied from the central depot. Similar to Catalán and Fisher [2], they assume infinite inventory for the items in the assortment and a deterministic demand. Therefore, they only need to decide which SKUs should be assigned to the warehouse and which orders can be satisfied with this assortment. In their model, they make sure that the capacity of the warehouse is not exceeded and an order can only be satisfied if all the SKUs in it are assigned to the warehouse.

As the problem is NP-hard, Jehl et al. [1] suggest some different solution approaches instead of solving this mixed integer programming problem. They develop *greedy ranking* algorithm in which they try to prioritize the SKUs with high sales and less dependencies. *Parametric cut* algorithm is another solution method they offer. It is based on building a graph such that the optimum solution of the Lagrangian relaxation of the problem is a minimum cut. Lastly, they try to solve the problem by using *neural network framework* where forecasting and some other features can be built in. According to the numerical study they have conducted, *parametric cut* algorithm and *neural network framework* provide lower optimality gaps compared to *greedy ranking* algorithm.

Zhu et al. [6] reduce the complexity of the problem by assigning product categories to warehouses instead of individual SKUs. They claim that it is convenient for warehouse management because different product categories have different storage requirements. They study the problem in the context of a one large distribution center with several warehouses. For this reason, they also do not allow duplications of categories between warehouses. Therefore, they define the problem as distributing the product categories among warehouses and consequently allocating orders to them so as to minimize the number of additional shipments stemming from split orders. This problem definition only adds the

constraint that there is no duplication between the assortments of the warehouses on top of the problem definition made by Catalán and Fisher [2]. Therefore, the problem is NP-hard as proved by Catalán and Fisher [2].

The complexity of the problem motivates Zhu et al. [6] to find another method to solve the problem instead of solving the optimization problem. At first, they define weighted links between product categories and approximate the number of order splits by the sum of the out-links between multiple warehouses. They also state that weighted links provide a better upper bound than the co-appearance matrix used by Catalán and Fisher [2] for the number of order splits. In this way, they transform the order splitting minimization problem into out-link minimization problem and further view it as a K clusters problem. However, as K clusters problem is proved to be NP-hard by Feige and Lanberg [9], they develop two K-links clustering algorithms to improve a given assortment instead of solving the problem optimally from scratch. These algorithms are based on moving categories between two warehouses one at a time and exchanging two categories between two warehouses, respectively. The numerical study they have conducted shows that their algorithms reach near optimal or optimal solutions for out-link minimization problem and also outperform the *bestsellers* algorithm offered by Catalán and Fisher [2].

2.2 Studies on Order Consolidation

Zhang et al. [7] state that the assortment allocation studies do not eliminate the split orders completely and split orders are inevitable. For this reason, they suggest to consolidate the split orders instead of preventing orders from being split. They assume that there is only one consolidation warehouse and formulate the problem with a consolidation model and a packing model. The objective of the consolidation model is minimizing the total cost stemming from direct shipments from warehouses to customers and transshipments between warehouses for consolidation. In this model, they assume that each item is already assigned to a single warehouse and they try to determine which SKUs

in suborders should be transshipped for package consolidation. In the study, it is proven that the consolidation problem is NP-hard. In the packing model, they determine the assignment of SKUs to packages in a way that the number of packages is minimized. While doing this, they consider the capacity limitations of the packages in terms of volume and weight, and the conflict among the SKUs, which prevent them from being in the same package. This model can be seen as a bin-packing problem with conflicts that is an NP-hard problem.

Since both of the models they use while formulating the package consolidation problem is NP-hard, they suggest a solution procedure which is a combination of two heuristic algorithms. They solve the packing problem by using a modified first fit decreasing (FFD) heuristic algorithm and they solve the consolidation problem through a breadth-first search based algorithm which calls FFD algorithm. The numerical study conducted to test this method has shown that order fulfilment cost, which is the sum of shipment and transshipment costs is lower in the case that some of the suborders in split order are partially consolidated than in the cases of complete consolidation and not consolidating suborders. Also, this algorithm is used for split orders remaining in the system after the *bestsellers* heuristic suggested by Catalán and Fisher [2] is applied. The result has shown that total order fulfilment cost can be reduced by applying the algorithm even after an assortment allocation algorithm.

Zhang et al. [3] provides an extension of the study conducted by Zhang et al. [7]. Different from Zhang et al. [7], in this study the order consolidation can be done in all the warehouses instead of a prespecified one. They define the problem as determining which warehouses should be used as consolidation warehouses, which SKUs in suborders should be transshipped to these warehouses and which SKUs should be packed together so as to minimize the total order fulfilment cost. They formulate this problem as a combined multi-commodity network flow problem instead of providing separate consolidation and packing models. However, as even the package consolidation problem with one consolidation warehouse is proved to be NP-hard by Zhang et al. [7], this problem is also NP-hard. Yet, thanks to the special structure of the problem, they can develop an exact solution algorithm based on Bender's decomposition. A numerical study

shows that it is possible to obtain near optimal solutions for large scale problems in the existence of the time limit around 25% of the CPU time required by CPLEX to solve the mathematical model.

2.3 Relation to Densest k-Subhypergraph Problem

It is important to note that when the split order problem is defined as in Jehl et al. [1], it is actually a special form of the well-known densest k-subhypergraph problem, which is used as a modelling framework for capital budgeting problem [10], repair kit problem [11], and FMS part selection and tool loading problem [12]. The densest k-subhypergraph problem is proved to be strongly NP-hard by Gallo et al. [5].

Furthermore, our problem is equivalent to the non-negative monotone supermodular set function maximization under a uniform matroid constraint, and Feldman and Izsak [13] have shown that there cannot be a polynomial time algorithm for maximizing a non-negative monotone set function subject to a uniform matroid constraint which has a constant approximation ratio if Small-Set Expansion Hypothesis introduced by Raghavendra and Steurer [14] is not false.

Approach	Study	Problem Definition	Complexity	Solution Methods
Order and Assortment Allocation	Xu et al. [4]	Given the inventory levels, determine the order allocation for each time epoch to minimize the number of shipments.	NP-hard	Neighbourhood Search Heuristic Algorithm
	Catalán and Fisher [2]	Determine the allocation of SKUs and orders to warehouses so that the number of shipments is minimized.	NP-hard	Greedy Heuristic Algorithms Based on Co-Appearance Matrix
	Jehl et al. [1]	Determine the assortment allocation for only one warehouse to maximize the number of orders that can be satisfied.	Strongly NP-hard	*A Greedy Heuristic Algorithm *Parametric Cut Algorithm *Neural Network Framework
	Zhu et al. [6]	Distribute the product categories among warehouses and consequently allocate orders to them so as to minimize the number of additional shipments stemming from split orders.	NP-hard	K-Links Clustering Algorithms
Consolidation	Zhang et al. [7]	Determine which SKUs should be consolidated and packed together to minimize total shipment and transshipment costs.	NP-hard	*Modified First Fit Decreasing (FFD) Heuristic Algorithm for Packing Problem *Breadth-First Search Based Algorithm for Consolidation
	Zhang et al. [3]	Determine which warehouses should be used as consolidation warehouses, which SKUs in suborders should be transshipped to these warehouses and which SKUs should be packed together so as to minimize the total order fulfilment cost.	NP-hard	Exact Solution Algorithm Based on Bender's Decomposition

Table 2.1: Summary of the Literature Review

Chapter 3

Assortment Allocation Problem

In this chapter the assortment allocation problem for a single warehouse is tackled considering the split orders. Some theoretical findings and their proofs are provided. Also, heuristic algorithms and valid inequalities are suggested. Lastly, the results obtained from an extensive numerical study are discussed.

3.1 Problem Statement and Benchmark Model

The split order problem for a single warehouse can be defined as follows. Given a set of products I , a set of orders J , and the capacity of the warehouse c , find a subset $I' \subseteq I$ such that $|I'| \leq c$ and the number of orders that can be satisfied from the warehouse is maximized.

The following model provided by Jehl et al. [1] is used as a benchmark model in this study. Let for each $j \in J$, $f(j) \subseteq I$ represent its list of SKUs and $e_j \geq 0$ represent the expected number of such orders. The model decides if SKU i is selected for the assortment or not by the binary variable x_i for all $i \in I$, and whether order j is fulfilled from the warehouse or not by the binary variable y_j for all $j \in J$.

$$\begin{aligned}
& \max && \sum_{j \in J} e_j y_j \\
& \text{s.t.} && \sum_{i \in I} x_i \leq c,
\end{aligned} \tag{3.1}$$

$$y_j \leq x_i, \quad \forall j \in J, \forall i \in f(j), \tag{3.2}$$

$$x_i \in \{0, 1\}, \quad \forall i \in I, \tag{3.3}$$

$$y_j \in \{0, 1\}, \quad \forall j \in J. \tag{3.4}$$

In this model the objective is to maximize the number of orders that can be satisfied from the warehouse. Constraint (3.1) guarantees that the capacity of the warehouse is not exceeded. With constraint set (3.2), it is guaranteed if some of the SKUs included by an order are not selected for the assortment, then it is not possible to satisfy this order from the warehouse. Constraint sets (3.3) and (3.4) are domain restrictions for the binary variables.

3.2 Theoretical Findings

3.2.1 Optimal Solution Characterization for the LP Relaxation of the Benchmark Model

Now, consider the following LP relaxation of the benchmark model

$$\begin{aligned}
(P_1) \quad & z_1 = \max \sum_{j \in J} e_j y_j \\
& \text{s.t.} \quad \sum_{i \in I} x_i \leq c, \\
& y_j \leq x_i, \quad \forall j \in J, \forall i \in f(j), \\
& 0 \leq x_i \leq 1, \quad \forall i \in I, \\
& 0 \leq y_j \leq 1, \quad \forall j \in J,
\end{aligned}$$

whose feasible region is called S_1 .

Lemma 1. *Any extreme point optimal solution to P_1 , say $x^* \in \mathbb{R}_+^{|I|}$, $y^* \in \mathbb{R}_+^{|J|}$ is such that*

$$x_i^* \in \{0, \mu, 1\} \quad \text{for each } i \in I \text{ and } y_j^* \in \{0, \mu, 1\} \text{ for each } j \in J$$

for some fractional number $0 < \mu < 1$.

Proof. Note that it is enough to show that in an optimal solution $x_i^* \in \{0, \mu, 1\}$ for each $i \in I$ since $y_j^* = \min_{i \in f(j)} x_i^*$ for each $j \in J$ in any optimal solution (assuming without loss of generality that $e_j > 0$ for each $j \in J$). Assume to the contrary that there exists an extreme point optimal solution to the above LP that violates the stated condition. Say (x^*, y^*) is such an optimal solution.

Case (i): Assume only x^* violates the condition. In other words, there exists $0 < \mu < 1$ such that $y_j^* \in \{0, \mu, 1\}$ for each $j \in J$, but there is some $i' \in I$ for which $0 < x_{i'}^* < 1$ and $x_{i'}^* \neq \mu$. Assume without loss of generality that $x_{i'}^* < \mu$. Now, it is easy to check that the following two solutions, say (x^I, t^I) and (x^{II}, t^{II}) , are also feasible for the above LP where $I_\mu = \{i \in I : x_i^* = \mu\}$ for some small enough positive ϵ , say $\epsilon \in \left(0, \frac{|I_\mu|}{(|I_\mu|+1)}(\mu - x_{i'}^*)\right]$, and $(x^*, y^*) = \frac{1}{2}(x^I, t^I) + \frac{1}{2}(x^{II}, t^{II})$ contradicting the assumption that (x^*, y^*) is an extreme point.

$$x_i^I = \begin{cases} x_i^* & \text{if } x_i^* \in \{0, 1\} \\ x_i^* - \frac{\epsilon}{|I_\mu|} & \text{if } x_i^* = \mu \\ x_i^* + \epsilon & \text{if } i = i' \end{cases} \quad \text{and}$$

$$y_j^I = \begin{cases} y_j^* & \text{if } y_j^* \in \{0, 1\} \\ \mu - \frac{\epsilon}{|I_\mu|} & \text{if } y_j^* = \mu \\ x_{i'}^* + \epsilon & \text{if } y_j^* = x_{i'}^* \end{cases} \quad \text{for each } j \in J,$$

$$x_i^{II} = \begin{cases} x_i^* & \text{if } x_i^* \in \{0, 1\} \\ x_i^* + \frac{\epsilon}{|I_\mu|} & \text{if } x_i^* = \mu \\ x_i^* - \epsilon & \text{if } i = i' \end{cases} \quad \text{and}$$

$$y_j^{II} = \begin{cases} y_j^* & \text{if } y_j^* \in \{0, 1\} \\ \mu + \frac{\epsilon}{|I_\mu|} & \text{if } y_j^* = \mu \\ x_{i'}^* - \epsilon & \text{if } y_j^* = x_{i'}^* \end{cases} \quad \text{for each } j \in J,$$

Case (ii): Now assume there exist $j_p, j_r \in J$ such that $0 < y_{j_p}^* < 1$ and $0 < y_{j_r}^* < 1$ and $y_{j_p}^* \neq y_{j_r}^*$. Let

$$\lambda_1 = \max_{j \in J: 0 < y_j^* < 1} y_j^* \text{ and } \lambda_2 = \min_{j \in J: 0 < y_j^* < 1} y_j^*.$$

More specifically, for each $i \in I$, either $x_i^* \in \{0, 1, \lambda_1, \lambda_2\}$ or $\lambda_2 < x_i^* < \lambda_1$ and we have at least two fractional values for x variables. It is easy to check that the following two solutions, say (x^I, y^I) and (x^{II}, y^{II}) are also feasible for the above LP for some small enough positive ϵ , $l_1 = |\{i \in I : x_i^* = \lambda_1\}|$ and $l_2 = |\{i \in I : x_i^* = \lambda_2\}|$ and as in Case (i) we have a violation of (x^*, y^*) being extreme point since $(x^*, y^*) = \frac{1}{2}(x^I, y^I) + \frac{1}{2}(x^{II}, y^{II})$.

$$x_i^I = \begin{cases} x_i^* & \text{if } x_i^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 - \frac{\epsilon}{l_1} & \text{if } x_i^* = \lambda_1 \\ \lambda_2 + \frac{\epsilon}{l_2} & \text{if } x_i^* = \lambda_2 \end{cases} \quad \text{for each } i \in I \text{ and}$$

$$y_j^I = \begin{cases} y_j^* & \text{if } y_j^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 - \frac{\epsilon}{l_1} & \text{if } y_j^* = \lambda_1 \\ \lambda_2 + \frac{\epsilon}{l_2} & \text{if } y_j^* = \lambda_2 \end{cases} \quad \text{for each } j \in J,$$

$$x_i^{II} = \begin{cases} x_i^* & \text{if } x_i^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 + \frac{\epsilon}{l_1} & \text{if } x_i^* = \lambda_1 \\ \lambda_2 - \frac{\epsilon}{l_2} & \text{if } x_i^* = \lambda_2 \end{cases} \quad \text{for each } i \in I \text{ and}$$

$$y_i^{II} = \begin{cases} y_j^* & \text{if } y_j^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 + \frac{\epsilon}{l_1} & \text{if } y_j^* = \lambda_1 \\ \lambda_2 - \frac{\epsilon}{l_2} & \text{if } y_j^* = \lambda_2 \end{cases} \quad \text{for each } j \in J.$$

□

3.2.1.1 A Combinatorial Algorithm to Find the Optimal Solution

Lemma 1 suggests that the SKUs belong to a set of clusters. To find these clusters and ultimately find the optimal solution of the LP, we propose a combinatorial algorithm which is based on an algorithm offered to solve a 0-1 fractional programming problem by Picard and Queyranne [15] where they find the solution by iteratively solving maximum flow problems. Their algorithm starts with choosing an initial solution. Then, they form a mathematical programming model using this initial solution. Its objective is to maximize the the difference between the numerator of the fraction and the product of the value of the initial solution and the denominator of the fraction. Again, the variables are binary. (They have proven that the initial solution is an optimal solution for the original binary fractional programming problem if and only if the optimal objective value of this new problem is zero.) At this point, the value of the linear terms who has a positive coefficient in the objective function can be fixed to 1 as these terms always make a positive contribution to the value of the objective function. For solving the remaining problem as a maximum flow problem, they form a network. In this network, the capacity of the edges between the source node and the nodes corresponding to non-linear terms equals to the coefficient of these non-linear terms in the objective function, the capacity of the edges between the nodes of non-linear terms and the linear terms that they include equals to infinity, and the capacity of the edges between the nodes corresponding to linear terms and the sink node equals to the absolute value of the coefficients of these linear terms in the objective function. The capacity of all the other edges is equal to zero. Maximum

flow problem is solved in this network by using Ford-Fulkerson Algorithm [16]. The linear terms whose nodes are reachable from the source node in the residual network are also set to 1. If the optimal objective function value is zero, then the algorithm stops and returns the initial solution as an optimal solution of the original 0-1 fractional programming problem. Otherwise, the new solution is set as the initial solution and the algorithm runs for another iteration.

In our problem, when the warehouse has only 1 unit capacity, this capacity will be divided equally among the most valuable SKUs belonging the first cluster. So, these SKUs are the ones maximizing the ratio between the number of orders that can be satisfied and the number of SKUs in the assortment. In this sense, we can find the content of the clusters through solving the following binary fractional programming problem until all the SKUs are clustered. At most, we need to solve $|I|$ such problems

$$\begin{array}{ll}
\max & \frac{\sum_{j \in J} (e_j \prod_{i \in f(j)} x_i)}{\sum_{i \in I} x_i} \\
\text{s.t.} & x_i \in \{0, 1\}, \quad \forall i \in I, \\
& \sum_{i \in I} x_i \geq 1.
\end{array}$$

The following example illustrates how this algorithm works.

Example

Let $I = \{1, 2, 3, 4, 5, 6\}$ and

j	$i \in f(j)$	e_j
1	1,4	43
2	3,4	22
3	1,2,4	162
4	1,6	88
5	1,3	92
6	1,2	77
7	1	226
8	1,2,3	99
9	1,5	128

First Cluster

Consider the following problem (F):

$$\begin{aligned} \max \quad & z(x) = \frac{43x_1x_4 + 22x_3x_4 + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6} \\ & + 226x_1 + 99x_1x_2x_3 + 128x_1x_5 \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in I, \\ & \sum_{i \in I} x_i \geq 1. \end{aligned}$$

1st Iteration

Starting with $x^0 = (1, 1, 1, 1, 1, 1)$ and $z(x^0) = \frac{937}{6}$, we first solve the following 0-1 programming problem (F^0):

$$\begin{aligned} \max \quad & w^0(x) = \frac{43x_1x_4 + 22x_3x_4 + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 226x_1}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6} \\ & + 99x_1x_2x_3 + 128x_1x_5 - \frac{937}{6}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in I, \end{aligned}$$

that is

$$\begin{aligned} \max \quad & w^0(x) = 69.83x_1 - 156.1667(x_2 + x_3 + x_4 + x_5 + x_6) + 43x_1x_4 + 22x_3x_4 \\ & + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 99x_1x_2x_3 + 128x_1x_5 \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in I. \end{aligned}$$

The coefficient of x_1 is $69.83 > 0$. Fix $x_1 = 1$ and solve minimum cut problem in the following network.

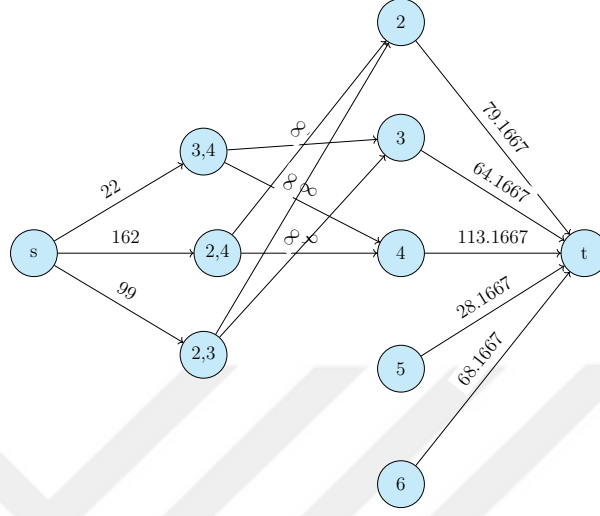


Figure 3.1: Example for the Combinatorial Algorithm 1st Cluster 1st Iteration Network

The residual network is as follows:

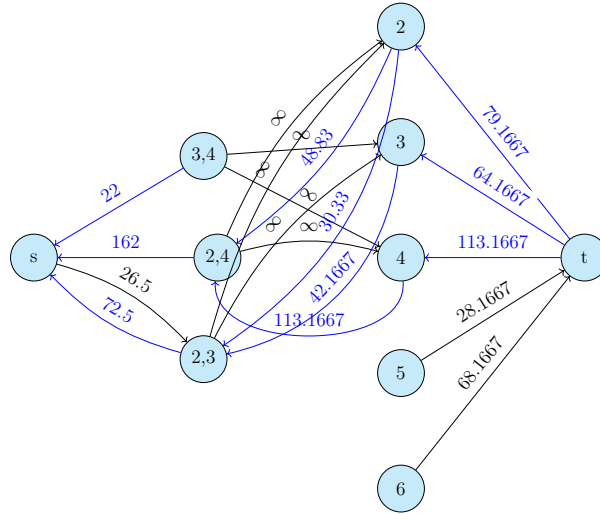


Figure 3.2: Example for the Combinatorial Algorithm 1st Cluster 1st Iteration Residual Network

In the residual network, nodes 2, 3, and 4 are reachable from the source node. Hence, an optimum solution to (F^0) is $x^1 = (1, 1, 1, 1, 0, 0)$ with $z(x^1) = \frac{721}{4} = 180.25 > z(x^0) = \frac{937}{6} = 156.1667$.

2nd Iteration

We have to solve (F^1):

$$\begin{aligned} \max \quad w^1(x) = & 43x_1x_4 + 22x_3x_4 + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 226x_1 \\ & + 99x_1x_2x_3 + 128x_1x_5 - \frac{721}{4}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \end{aligned}$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in I,$$

that is

$$\begin{aligned} \max \quad w^1(x) = & 45.75x_1 - 180.25(x_2 + x_3 + x_4 + x_5 + x_6) + 43x_1x_4 + 22x_3x_4 \\ & + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 99x_1x_2x_3 + 128x_1x_5 \end{aligned}$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in I.$$

The coefficient of x_1 is $45.75 > 0$. Fix $x_1 = 1$ and solve minimum cut problem in the following network.

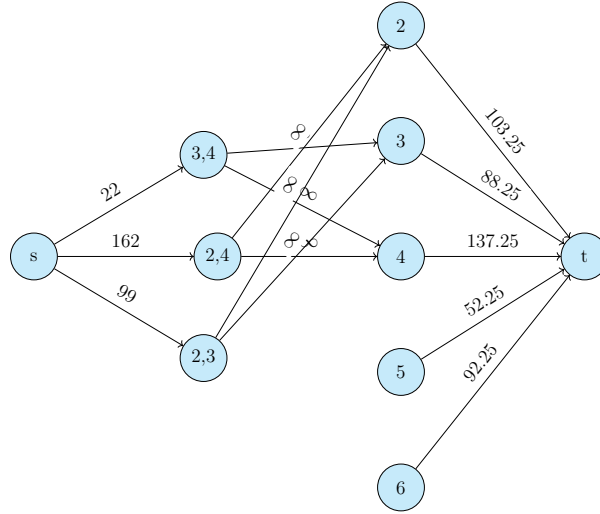


Figure 3.3: Example for the Combinatorial Algorithm 1st Cluster 2nd Iteration Network

The residual network is as follows:

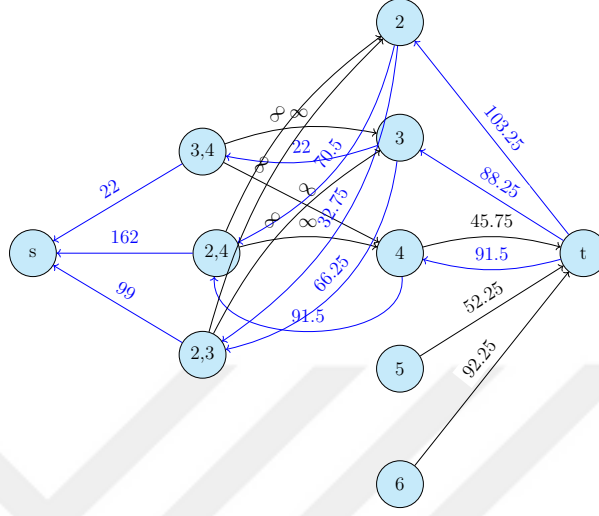


Figure 3.4: Example for the Combinatorial Algorithm 1st Cluster 2nd Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^1) is $x^2 = (1, 0, 0, 0, 0, 0)$ with $z(x^2) = 226 > z(x^1) = 180.25$.

3rd Iteration

We have to solve (F^2) :

$$\begin{aligned} \max \quad w^2(x) = & 43x_1x_4 + 22x_3x_4 + 162x_1x_2x_4 + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 226x_1 \\ & + 99x_1x_2x_3 + 128x_1x_5 - 226(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \end{aligned}$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in I,$$

that is

$$\begin{aligned} \max \quad w^2(x) = & 0x_1 - 226(x_2 + x_3 + x_4 + x_5 + x_6) + 43x_1x_4 + 22x_3x_4 + 162x_1x_2x_4 \\ & + 88x_1x_6 + 92x_1x_3 + 77x_1x_2 + 99x_1x_2x_3 + 128x_1x_5 \end{aligned}$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in I.$$

The coefficient of x_1 is $0 \geq 0$. Fix $x_1 = 1$ and solve minimum cut problem in the following network.

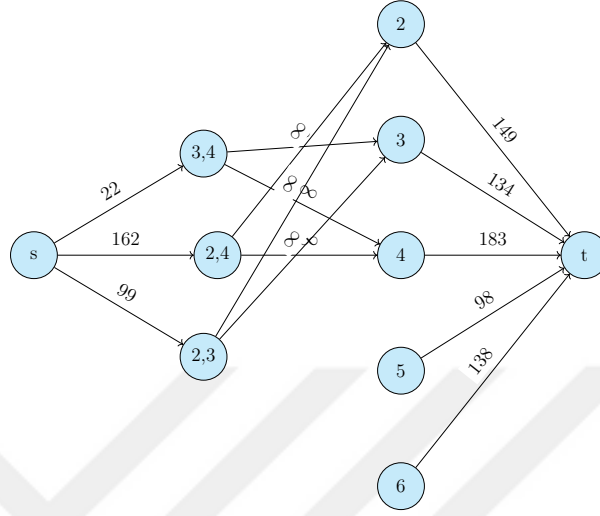


Figure 3.5: Example for the Combinatorial Algorithm 1st Cluster 3rd Iteration Network

The residual network is as follows:

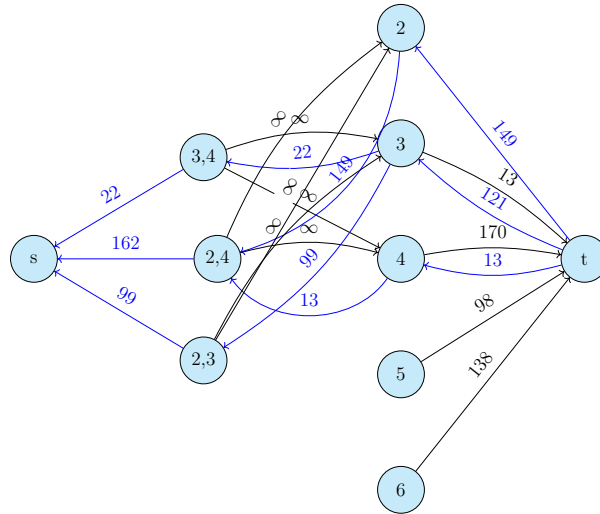


Figure 3.6: Example for the Combinatorial Algorithm 1st Cluster 3rd Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^2) is $x^3 = (1, 0, 0, 0, 0, 0)$ with $z(x^3) = 226 = z(x^2) = 226$ and the maximal value

of $w^2(x)$ is zero. An optimum solution to (F) is then $x^* = (1, 0, 0, 0, 0, 0)$ with $z(x^*) = 226$.

Second Cluster

Consider the following problem (F) :

$$\begin{aligned} \max \quad & z(x) = \frac{43x_4 + 22x_3x_4 + 162x_2x_4 + 88x_6 + 92x_3 + 77x_2 + 99x_2x_3 + 128x_5}{x_2 + x_3 + x_4 + x_5 + x_6} \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{2, 3, 4, 5, 6\}, \\ & \sum_{i \in \{2, 3, 4, 5, 6\}} x_i \geq 1. \end{aligned}$$

1st Iteration

Starting with $x^0 = (1, 1, 1, 1, 1)$ and $z(x^0) = \frac{711}{5}$, we first solve the following 0-1 programming problem (F^0) :

$$\begin{aligned} \max \quad & w^0(x) = \frac{43x_4 + 22x_3x_4 + 162x_2x_4 + 88x_6 + 92x_3 + 77x_2 + 99x_2x_3 + 128x_5}{x_2 + x_3 + x_4 + x_5 + x_6} \\ & - \frac{711}{5}(x_2 + x_3 + x_4 + x_5 + x_6) \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{2, 3, 4, 5, 6\}, \end{aligned}$$

that is

$$\begin{aligned} \max \quad & w^0(x) = -65.2x_2 - 50.2x_3 - 99.2x_4 - 14.2x_5 - 54.2x_6 + 22x_3x_4 + 162x_2x_4 \\ & + 99x_2x_3 \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{2, 3, 4, 5, 6\}. \end{aligned}$$

Solve minimum cut problem in the following network.

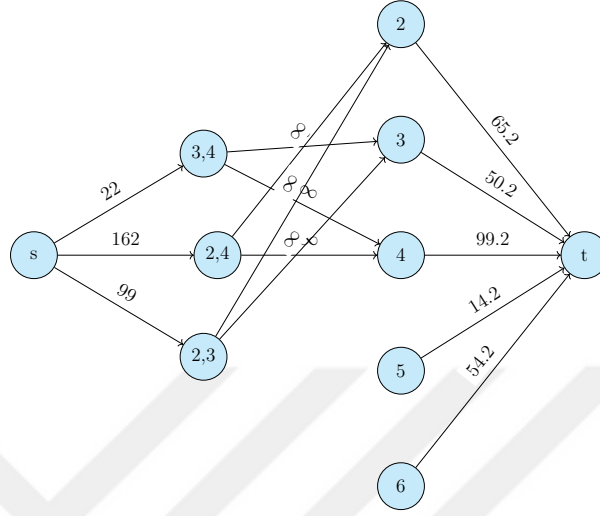


Figure 3.7: Example for the Combinatorial Algorithm 2nd Cluster 1st Iteration Network

The residual network is as follows:

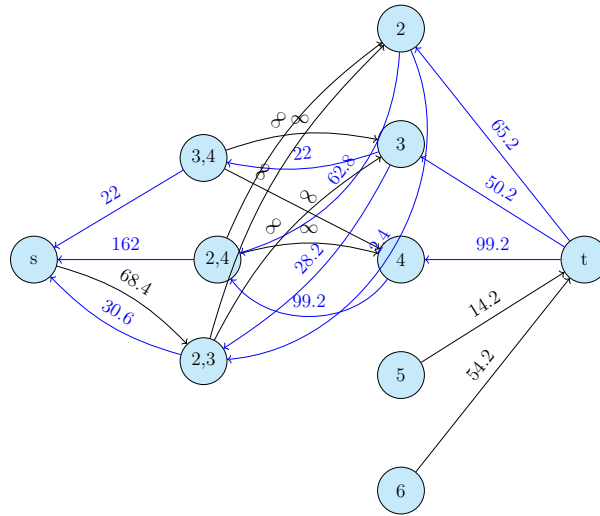


Figure 3.8: Example for the Combinatorial Algorithm 2nd Cluster 1st Iteration Residual Network

In the residual network, 2,3, and 4 are reachable from the source node. Hence, an optimum solution to (F^0) is $x^1 = (1, 1, 1, 0, 0)$ with $z(x^1) = \frac{495}{3} = 165 > z(x^0) = \frac{711}{5} = 142.2$.

2nd Iteration

We have to solve (F^1):

$$\begin{aligned} \max \quad w^1(x) = & 43x_4 + 22x_3x_4 + 162x_2x_4 + 88x_6 + 92x_3 + 77x_2 + 99x_2x_3 + 128x_5 \\ & - \frac{495}{3}(x_2 + x_3 + x_4 + x_5 + x_6) \end{aligned}$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in \{2, 3, 4, 5, 6\},$$

that is

$$\max \quad w^1(x) = -88x_2 - 73x_3 - 122x_4 - 37x_5 - 77x_6 + 22x_3x_4 + 162x_2x_4 + 99x_2x_3$$

$$\text{s.t.} \quad x_i \in \{0, 1\}, \quad \forall i \in \{2, 3, 4, 5, 6\}.$$

Solve minimum cut problem in the following network.

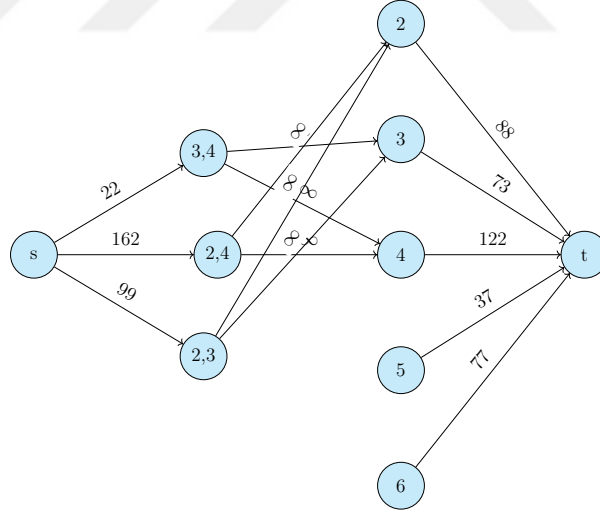


Figure 3.9: Example for the Combinatorial Algorithm 2nd Cluster 2nd Iteration Network

The residual network is as follows:

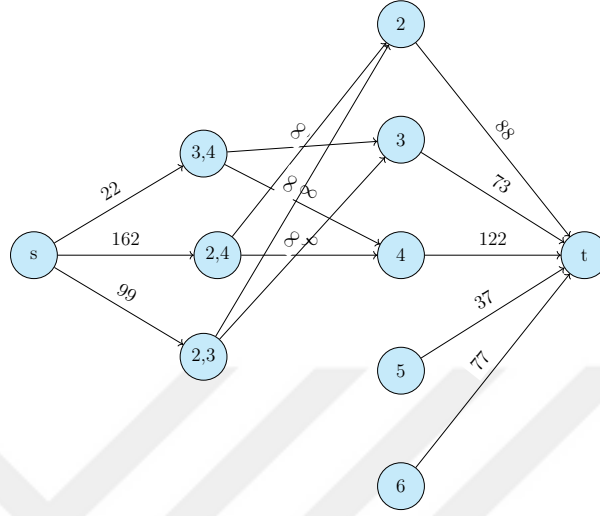


Figure 3.10: Example for the Combinatorial Algorithm 2nd Cluster 2nd Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^1) is $x^2 = (0, 0, 0, 0, 0)$ and the maximal value of $w^1(x)$ is zero. An optimum solution to (F) is then $x^* = x^1 = (1, 1, 1, 0, 0)$ with $z(x^*) = 165$.

Third Cluster

Consider the following problem (F) :

$$\begin{aligned}
 \max \quad & z(x) = \frac{88x_6 + 128x_5}{x_5 + x_6} \\
 \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{5, 6\}, \\
 & \sum_{i \in \{5, 6\}} x_i \geq 1.
 \end{aligned}$$

1st Iteration

Starting with $x^0 = (1, 1)$ and $z(x^0) = \frac{216}{2} = 108$, we first solve the following 0-1 programming problem (F^0) :

$$\begin{aligned}
 \max \quad & w^0(x) = 88x_6 + 128x_5 - 108(x_5 + x_6) \\
 \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{5, 6\},
 \end{aligned}$$

that is

$$\begin{array}{ll} \max & w^0(x) = 20x_5 - 20x_6 \\ \text{s.t.} & x_i \in \{0, 1\}, \quad \forall i \in \{5, 6\}. \end{array}$$

$a_5 = 20 > 0$. Fix $x_5 = 1$ and solve minimum cut problem in the following network.

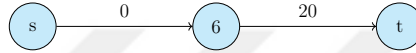


Figure 3.11: Example for the Combinatorial Algorithm 3rd Cluster 1st Iteration Network

The residual network is as follows:

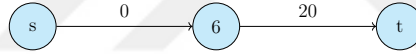


Figure 3.12: Example for the Combinatorial Algorithm 3rd Cluster 1st Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^0) is $x^1 = (1, 0)$ with $z(x^1) = 128 > z(x^0) = 108$.

2nd Iteration

We have to solve (F^1) :

$$\begin{array}{ll} \max & w^1(x) = 88x_6 + 128x_5 - 128(x_5 + x_6) \\ \text{s.t.} & x_i \in \{0, 1\}, \quad \forall i \in \{5, 6\}, \end{array}$$

that is

$$\begin{array}{ll} \max & w^1(x) = 0x_5 - 40x_6 \\ \text{s.t.} & x_i \in \{0, 1\}, \quad \forall i \in \{5, 6\}. \end{array}$$

The coefficient of x_5 is $0 \geq 0$. Fix $x_5 = 1$ and solve minimum cut problem in the following network.

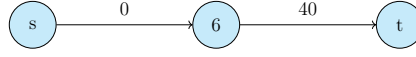


Figure 3.13: Example for the Combinatorial Algorithm 3rd Cluster 2nd Iteration Network

The residual network is as follows:

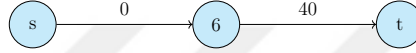


Figure 3.14: Example for the Combinatorial Algorithm 3rd Cluster 2nd Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^1) is $x^2 = (1, 0)$ and the maximal value of $w^1(x)$ is zero. An optimum solution to (F) is then $x^* = x^1 = (1, 0)$ with $z(x^*) = 128$.

Fourth Cluster

Consider the following problem (F) :

$$\begin{aligned} \max \quad & z(x) = \frac{88x_6}{x_6} \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{6\}, \\ & \sum_{i \in \{6\}} x_i \geq 1. \end{aligned}$$

1st Iteration

Starting with $x^0 = (1)$ and $z(x^0) = 88$, we first solve the following 0-1 programming problem (F^0) :

$$\begin{aligned} \max \quad & w^0(x) = 88x_6 - 88(x_6) \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{6\}, \end{aligned}$$

that is

$$\begin{aligned} \max \quad & w^0(x) = 0x_6 \\ \text{s.t.} \quad & x_i \in \{0, 1\}, \quad \forall i \in \{6\}. \end{aligned}$$

The coefficient of x_5 is $0 \geq 0$. Fix $x_6 = 1$ and solve minimum cut problem in the following network.

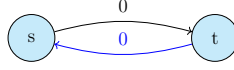


Figure 3.15: Example for the Combinatorial Algorithm 4th Cluster 1st Iteration Network

The residual network is as follows:



Figure 3.16: Example for the Combinatorial Algorithm 4th Cluster 1st Iteration Residual Network

A minimum cut is characterized by $N = \{s\}$; hence an optimum solution to (F^0) is $x^1 = (1)$ and the maximal value of $w^0(x)$ is zero. An optimum solution to (F) is then $x^* = x^0 = (1)$ with $z(x^*) = 88$.

Now, all the SKUs are clustered. The following table shows the values of assortment variables of the LP relaxation for different values of the capacity.

c	$x_1 \geq x_2 = x_4 = x_3 \geq x_5 \geq x_6$			
1	1	0	0	0
2	1	1/3	0	0
3	1	2/3	0	0
4	1	1	0	0
5	1	1	1	0
6	1	1	1	1

3.2.2 Extension of Assortment Optimization Problem for Multiple Warehouses

The definition of the split order problem can be extended for multiple warehouses as follows. Given a set of products I , a set of orders J , a set of warehouses K , and the capacity of each warehouse c_k $k \in K$, find a subset $I'_k \subseteq I$ for every $k \in K$ such that $|I'_k| \leq c_k$ and the number of orders that can be satisfied from a single warehouse is maximized.

The extension of the benchmark model for this problem is as follows. Define the binary variables w_{ik} , v_{jk} , and u_j which show if SKU $i \in I$ is selected for the assortment in the warehouse $k \in K$, if order $j \in J$ can be totally fulfilled from warehouse $k \in K$, and if order $j \in J$ can be satisfied without splitting the order respectively.

$$\begin{aligned} \max \quad & \sum_{j \in J} e_j u_j \\ \text{s.t.} \quad & \sum_{i \in I} w_{ik} \leq c_k, \quad \forall k \in K, \end{aligned} \quad (3.5)$$

$$v_{jk} \leq w_{ik}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \quad (3.6)$$

$$u_j \leq \sum_{k \in K} v_{jk}, \quad \forall j \in J, \quad (3.7)$$

$$v_{jk} \in \{0, 1\} \quad \forall j \in J, \forall k \in K, \quad (3.8)$$

$$u_j \in \{0, 1\}, \quad \forall j \in J, \quad (3.9)$$

$$w_{ik} \in \{0, 1\}, \quad \forall i \in I, \forall k \in K. \quad (3.10)$$

In this formulation, the objective is to maximize the number of orders that can be satisfied from a single warehouse without being split. Constraint set (3.5) makes sure that the capacities of the warehouses are not exceeded. (3.6) states that an order cannot be satisfied from a warehouse if all the SKUs in this order are not assigned to this warehouse. (3.7) means if an order can be fully satisfied from at least one warehouse, then this order can be satisfied with a single shipment. (3.8-3.10) are the integrality constraints for the binary decision variables.

Consider the following LP relaxation of this model.

$$\begin{aligned}
(P_2) \quad & z_2 = \max \sum_{j \in J} e_j u_j \\
& \text{s.t. } \sum_{i \in I} w_{ik} \leq c_k, & \forall k \in K, \\
& v_{jk} \leq w_{ik}, & \forall k \in K, \forall j \in J, \forall i \in f(j), \\
& u_j \leq \sum_{k \in K} v_{jk}, & \forall j \in J, \\
& v_{jk} \geq 0, & \forall j \in J, \forall k \in K, \\
& 0 \leq u_j \leq 1, & \forall j \in J, \\
& 0 \leq w_{ik} \leq 1, & \forall i \in I, \forall k \in K,
\end{aligned}$$

Let feasible region of this relaxation be called S_2 .

Let

$$c = \sum_{k \in K} c_k$$

Lemma 2. *Given an optimal solution to P_1 , a solution can be constructed for P_2 with the same objective value.*

Proof. Let (x^*, y^*) be an optimal solution of P_1 and let $w_{ik}^* = x_i^* \times \frac{c_k}{c} \forall i \in I, \forall k \in K, v_{jk}^* = y_j^* \times \frac{c_k}{c} \forall j \in J, \forall k \in K$, and $u_j^* = y_j^* \forall j \in J$. We shall show that $(w^*, v^*, u^*) \in S_2$:

i-) For all $k \in K$

$$\begin{aligned}
\sum_{i \in I} w_{ik}^* &= \sum_{i \in I} x_i^* \times \frac{c_k}{c} \\
&= \frac{c_k}{c} \times \sum_{i \in I} x_i^* \\
&\leq \frac{c_k}{c} \times c \\
&= c_k
\end{aligned}$$

where the inequality follows from the fact that (x^*, y^*) satisfies $\sum_{i \in I} x_i^* \leq c$. Hence,

$$\sum_{i \in I} w_{ik}^* \leq c_k \quad \forall k \in K \text{ is satisfied for } (w^*, v^*, u^*).$$

ii-) For all $k \in K, j \in J$ and $i \in f(j)$

$$\begin{aligned} v_{jk}^* &= y_j^* \times \frac{c_k}{c} \\ &\leq x_i^* \times \frac{c_k}{c} \\ &= w_{ik}^* \times \frac{c}{c_k} \times \frac{c_k}{c} \\ &= w_{ik}^* \end{aligned}$$

where the inequality follows from the fact that (x^*, y^*) satisfies $y_j^* \leq x_i^* \quad \forall j \in J, \forall i \in f(j)$. Hence, $v_{jk}^* \leq w_{ik}^* \quad \forall k \in K, \forall j \in J, \forall i \in f(j)$ is satisfied for (w^*, v^*, u^*) .

iii-) For all $j \in J$,

$$\begin{aligned} u_j^* &= y_j^* \\ &= y_j^* \times \sum_{k \in K} \frac{c_k}{c} \\ &= \sum_{k \in K} v_{jk}^* \end{aligned}$$

where the second equation follows from our assumption that $\sum_{k \in K} c_k = c$.

Therefore, $u_j^* \leq \sum_{k \in K} v_{jk}^* \quad \forall j \in J$ is satisfied for (w^*, v^*, u^*) .

iv-) Since (x^*, y^*) satisfies $y_j^* \geq 0 \quad \forall j \in J$, $v_{jk}^* = y_j^* \times \frac{c_k}{c}$ satisfies $v_{jk}^* \geq 0 \quad \forall j \in J, \forall k \in K$.

v-) Because (x^*, y^*) is the optimal solution of P_1 , it needs to satisfy $0 \leq x_i^* \leq 1 \quad \forall i \in I$ and $y_j^* \geq 0 \quad \forall j \in J$. Hence, $0 \leq u_j^* \leq 1 \quad \forall j \in J$ is satisfied.

vi-) As it is assumed that $\sum_{k \in K} c_k = c$ and (x^*, y^*) is the optimal solution of P_1 , y^* needs to satisfy $0 \leq x_i^* \leq 1 \quad \forall i \in I$. Therefore, $0 \leq w_{ik}^* \leq 1 \quad \forall i \in I, \forall k \in K$ is satisfied.

By i-vi, $(w^*, v^*, u^*) \in S_2$. Hence, it is possible to obtain a solution for the LP relaxation of the extended formulation, by using the optimal solution of the LP relaxation of the formulation provided by Jehl et al. [1].

It can be easily verified that the objective function values of (x^*, y^*) in P_1 and (w^*, v^*, u^*) in P_2 are the same. $\square$

Lemma 3. P_1 and P_2 share the same optimal value.

Proof. It is obvious that Lemma 3 holds true when it is possible to satisfy all the orders with the assumed capacity c since in this case z_1 is equal to the highest possible value z_2 can take.

Assume that the capacity is not enough to satisfy all the orders. Through Lemma 2, we know that $z_1 \leq z_2$. Therefore, it is only necessary to show that z_2 cannot be greater than z_1 to complete the proof.

Let (w^*, v^*, u^*) be an optimal solution of the extended formulation.

i-) (w^*, v^*, u^*) satisfies

$$\sum_{i \in I} w_{ik}^* \leq c_k \quad \forall k \in K$$

Adding these $|K|$ inequalities, we obtain

$$\sum_{k \in K} \sum_{i \in I} w_{ik}^* \leq \sum_{k \in K} c_k$$

Using the fact that $\sum_{k \in K} c_k = c$ and interchanging the summations, we attain

$$\sum_{i \in I} \sum_{k \in K} w_{ik}^* \leq c$$

ii-) (w^*, v^*, u^*) also satisfies

$$v_{jk}^* \leq w_{ik}^* \quad \forall k \in K, \forall j \in J, \forall i \in f(j)$$

Adding these $|K|$ inequalities, we obtain

$$\sum_{k \in K} v_{jk}^* \leq \sum_{k \in K} w_{ik}^* \quad \forall j \in J, \forall i \in f(j)$$

It is also known that

$$u_j^* \leq \sum_{k \in K} v_{jk}^* \quad \forall j \in J$$

Then,

$$u_j^* \leq \sum_{k \in K} w_{ik}^* \quad \forall j \in J, \forall i \in f(j)$$

iii-) As it is assumed that the capacity is not enough to satisfy all the orders, in an optimal solution, for all $i \in I$

$$\sum_{k \in K} w_{ik}^* \leq 1$$

To show this let J' be the set of orders that can be completely satisfied and I' be the set of SKUs such that for any $i \in I'$, $i \in j$ for some $j \in J'$.

From $0 \leq u_j^* \leq 1 \quad \forall j \in J$ and ii, it is known that

$$u_j^* \leq \sum_{k \in K} w_{ik}^* \quad \forall j \in J, \forall i \in f(j)$$

and

$$u_j^* \leq 1 \quad \forall j \in J$$

So,

$$u_j^* = \min \left\{ \min_{i \in f(j)} \left\{ \sum_{k \in K} w_{ik}^* \right\}, 1 \right\} \quad \forall j \in J$$

and only for some order $j \in J'$

$$u_j^* = 1$$

Then, it is only possible for some i that

$$\sum_{k \in K} w_{ik}^* > 1$$

if $i \in I'$ as in the other case this capacity can be used to satisfy some orders. However, it is possible to completely satisfy any order $j \in J'$ by setting $w_{ik} = \frac{c_k}{c}$ for $i \in I'$ and in such a solution

$$\sum_{k \in K} w_{ik} = 1$$

Hence, there cannot be some i such that

$$\sum_{k \in K} w_{ik}^* > 1$$

in an optimal solution because the capacity $\sum_{k \in K} w_{ik}^* - 1$ can be filled with some other SKU i' 's such that $i' \in I \setminus I'$ and the objective function value can be increased by satisfying more orders.

Now, (w^*, v^*, u^*) is in the feasible region of the following formulation.

$$\begin{aligned} \max \quad & \sum_{j \in J} e_j u_j \\ \text{s.t.} \quad & \sum_{i \in I} \sum_{k \in K} w_{ik} \leq c, \\ & u_j \leq \sum_{k \in K} w_{ik}, \quad \forall j \in J, \forall i \in f(j), \\ & 0 \leq \sum_{k \in K} w_{ik} \leq 1 \quad \forall i \in I, \\ & u_j \geq 0, \quad \forall j \in J. \end{aligned}$$

Let the feasible region of this formulation be S_3 . Then, $S_2 \subseteq S_3$. Also, by substituting $\sum_{k \in K} w_{ik} = x_i \forall i \in I$ and $u_j = y_j$, observe that this formulation is equivalent to the LP relaxation of the formulation provided by Jehl et al. [1]. Therefore, the optimal objective function value of the extended formulation cannot be greater than the first formulation. Hence, the optimal objective function values of the LP relaxation of the model built by Jehl et al. [1] and its multi-warehouse extension are equal to each other. $\square$

3.2.3 Assortment Optimization Problem in A Central Depot and Forward Distribution Centers (FDC)

3.2.3.1 Single FDC

Another interesting problem arises when a central depot with enough capacity to store every SKU and a forward distribution center (FDC) with a limited capacity are considered together if different variable and fixed shipment costs are charged depending on from which warehouse the orders are satisfied either as a single shipment or as split orders. As the FDC is closer to customers, it is meaningful to assume that fixed and variable shipment costs from the central depot is higher. Then, assortment optimization for the FDC is required to minimize the total shipment cost. We can define this problem mathematically as follows. Given a set of orders, a set of SKUs, and a set of warehouses, find an assortment for each warehouse so that all the orders are satisfied and the total transportation cost is minimized.

Define I as the set of SKUs, J as the set of orders, and $K = \{1, 2\}$ be the set of warehouses, where $k = 1$ refers to the central depot and $k = 2$ refers to the FDC. Let c be the capacity of the FDC. Define f_{kj} and c_{kj}^{ship} as fixed and variable shipping costs of SKUs from warehouse $k \in K$ to satisfy order $j \in J$. Let x_i represent the binary variable showing whether SKU i is in the assortment for the FDC. Additionally, let t_{ikj} and s_{kj} be binary variables indicating if SKU $i \in I$ ships from warehouse $k \in K$ to fulfil order $j \in J$ and if order $j \in J$ is fulfilled with a shipment from warehouse $k \in K$ respectively. Given these parameters and decision variables, the problem can be modeled as follows.

$$\begin{aligned}
\min \quad & \sum_{k \in K} \sum_{j \in J} f_{kj} e_j s_{kj} + \sum_{k \in K} \sum_{j \in J} \sum_{i \in f(j)} c_{kj}^{ship} e_j t_{ikj} \\
\text{s.t.} \quad & \sum_{i \in I} x_i \leq c, \tag{3.11} \\
& t_{i1j} + t_{i2j} = 1, \quad \forall j \in J, \forall i \in f(j), \tag{3.12} \\
& t_{i2j} \leq x_i, \quad \forall j \in J, \forall i \in f(j), \tag{3.13} \\
& t_{ikj} \leq s_{kj}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \tag{3.14} \\
& x_i \in \{0, 1\}, \quad \forall i \in I, \tag{3.15} \\
& s_{kj} \in \{0, 1\}, \quad \forall k \in K, \forall j \in J, \tag{3.16} \\
& t_{ikj} \in \{0, 1\}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j). \tag{3.17}
\end{aligned}$$

The objective of this problem is again to minimize the total transportation cost. (3.11) is the capacity constraint for the FDC. (3.12) makes sure that every order is satisfied. Thanks to (3.13), an SKU cannot be sent from the FDC if it is not assigned to the FDC. (3.14) states that if there is not a shipment from a warehouse to an order, then any of the SKUs in this order cannot be sent from this warehouse. (3.15-3.17) are the domain constraints for the decision variables.

Consider the LP relaxation of this model, denoted as P_3 .

$$\begin{aligned}
z_3 = \min \quad & \sum_{k \in K} \sum_{j \in J} f_{kj} e_j s_{kj} + \sum_{k \in K} \sum_{j \in J} \sum_{i \in f(j)} c_{kj}^{ship} e_j t_{ikj} \\
\text{s.t.} \quad & \sum_{i \in I} x_i \leq c, \\
& t_{i1j} + t_{i2j} = 1, \quad \forall j \in J, \forall i \in f(j), \\
& t_{i2j} \leq x_i, \quad \forall j \in J, \forall i \in f(j), \\
& t_{ikj} \leq s_{kj}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \\
& 0 \leq x_i \leq 1, \quad \forall i \in I, \\
& 0 \leq s_{kj} \leq 1, \quad \forall k \in K, \forall j \in J, \\
& 0 \leq t_{ikj} \leq 1, \quad \forall k \in K, \forall j \in J, \forall i \in f(j).
\end{aligned} \tag{P_3}$$

Lemma 4. Any extreme point optimal solution to P_3 , say (x^*, t^*, s^*) is such that

$$x_i^* \in \{0, \mu, 1\} \quad \text{for each } i \in I$$

$$t_{i1j}^* \in \{0, 1 - \mu, 1\} \quad \text{for each } j \in J, i \in f_j$$

$$t_{i2j}^* \in \{0, \mu, 1\} \quad \text{for each } j \in J, i \in f_j$$

$$s_{1j}^* \in \{0, 1 - \mu, 1\} \quad \text{for each } j \in J, \text{ and}$$

$$s_{2j}^* \in \{0, \mu, 1\} \quad \text{for each } j \in J$$

for some fractional number $0 < \mu < 1$.

Proof. As we have $t_{i1j} + t_{i2j} = 1 \quad \forall j \in J, i \in f_j$, if $t_{i2j}^* \in \{0, \mu, 1\} \quad \forall j \in J, i \in f_j$, then $t_{i1j}^* \in \{0, 1 - \mu, 1\} \quad \forall j \in J, i \in f_j$. Additionally, since $s_{1j} = \max_{i \in f(j)} t_{i1j}$ and $s_{2j} = \max_{i \in f(j)} t_{i2j} \quad \forall j \in J$, if $t_{i1j}^* \in \{0, 1 - \mu, 1\}$ and $t_{i2j}^* \in \{0, \mu, 1\} \quad \forall j \in J, i \in f_j$, then $s_{1j}^* \in \{0, 1 - \mu, 1\}$ and $s_{2j}^* \in \{0, \mu, 1\} \quad \forall j \in J$.

So, it is enough to show that in an optimal solution $x_i^* \in \{0, \mu, 1\} \quad \forall i \in I$, and $t_{i2j}^* \in \{0, \mu, 1\} \quad \forall j \in J, i \in f_j$. Assume to the contrary that there exists an extreme point optimal solution to P_3 that violates the stated condition. Say (x^*, t^*) is such an optimal solution.

Case (i): Assume only x^* violates the condition. In other words, there exists $0 < \mu < 1$ such that $t_{i2j}^* \in \{0, \mu, 1\} \quad \forall j \in J, i \in f_j$, but there is some $i' \in I$ for which $0 < x_{i'}^* < 1$ and $x_{i'}^* \neq \mu$. Assume without loss of generality that $x_{i'}^* < \mu$. Now, it is easy to check that the following two solutions, say (x^I, t^I) and (x^{II}, t^{II}) are also feasible for P_3 where $I_\mu = \{i \in I : x_i^* = \mu\}$ for some small enough positive ϵ , say $\epsilon \in \left(0, \frac{|I_\mu|}{(|I_\mu|+1)}(\mu - x_{i'}^*)\right]$, and $(x^*, t^*) = \frac{1}{2}(x^I, t^I) + \frac{1}{2}(x^{II}, t^{II})$ contradicting the assumption that (x^*, t^*) is an extreme point.

$$x_i^I = \begin{cases} x_i^* & \text{if } x_i^* \in \{0, 1\} \\ x_i^* - \frac{\epsilon}{|I_\mu|} & \text{if } x_i = \mu \\ x_i^* + \epsilon & \text{if } i = i' \end{cases} \quad \text{and}$$

$$t_{i2j}^I = \begin{cases} t_{i2j}^* & \text{if } t_{i2j}^* \in \{0, 1\} \\ \mu - \frac{\epsilon}{|I_\mu|} & \text{if } t_{i2j}^* = \mu \\ x_{i'}^* + \epsilon & \text{if } t_{i2j}^* = x_{i'}^* \end{cases} \quad \text{for each } j \in J, i \in f(j)$$

$$x_i^{II} = \begin{cases} x_i^* & \text{if } x_i^* \in \{0, 1\} \\ x_i^* + \frac{\epsilon}{|I_\mu|} & \text{if } x_i = \mu \\ x_i^* - \epsilon & \text{if } i = i' \end{cases} \quad \text{and}$$

$$t_{i2j}^{II} = \begin{cases} t_{i2j}^* & \text{if } t_{i2j}^* \in \{0, 1\} \\ \mu + \frac{\epsilon}{|I_\mu|} & \text{if } t_{i2j}^* = \mu \\ x_{i'}^* - \epsilon & \text{if } t_{i2j}^* = x_{i'}^* \end{cases} \quad \text{for each } j \in J, i \in f(j)$$

Case (ii): Now assume there exist $j', j'' \in J$ such that $0 < t_{i2j'}^* < 1$ and $0 < t_{i2j''}^* < 1$ and $t_{i2j'}^* \neq t_{i2j''}^*$. Let

$$\lambda_1 = \max_{j \in J: 0 < t_{i2j}^* < 1, i \in f_j} t_{i2j}^* \quad \text{and} \quad \lambda_2 = \min_{j \in J: 0 < t_{i2j}^* < 1, i \in f_j} t_{i2j}^*.$$

More specifically, for each $i \in I$, either $x_i^* \in \{0, 1, \lambda_1, \lambda_2\}$ or $\lambda_2 < x_i^* < \lambda_1$ and we have at least two fractional values for x variables. It is easy to check that the following two solutions, say (x^I, y^I) and (x^{II}, y^{II}) are also feasible for P_3 for some small enough positive ϵ , $l_1 = |\{i \in I : x_i^* = \lambda_1\}|$ and $l_2 = |\{i \in I : x_i^* = \lambda_2\}|$ and as in Case (i) we have a violation of (x^*, y^*) being extreme point since $(x^*, y^*) = \frac{1}{2}(x^I, y^I) + \frac{1}{2}(x^{II}, y^{II})$.

$$x_i^I = \begin{cases} x_i^* & \text{if } x_i^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 - \frac{\epsilon}{l_1} & \text{if } x_i^* = \lambda_1 \\ \lambda_2 + \frac{\epsilon}{l_2} & \text{if } x_i^* = \lambda_2 \end{cases} \quad \text{for each } i \in I \text{ and}$$

$$t_{i2j}^I = \begin{cases} t_{i2j}^* & \text{if } t_{i2j}^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 - \frac{\epsilon}{l_1} & \text{if } t_{i2j}^* = \lambda_1 \\ \lambda_2 + \frac{\epsilon}{l_2} & \text{if } t_{i2j}^* = \lambda_2 \end{cases} \quad \text{for each } j \in J, i \in f_j$$

$$x_i^{II} = \begin{cases} x_i^* & \text{if } x_i^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 + \frac{\epsilon}{l_1} & \text{if } x_i^* = \lambda_1 \\ \lambda_2 - \frac{\epsilon}{l_2} & \text{if } x_i^* = \lambda_2 \end{cases} \quad \text{for each } i \in I \text{ and}$$

$$t_{i2j}^{II} = \begin{cases} t_{i2j}^* & \text{if } t_{i2j}^* \notin \{\lambda_1, \lambda_2\} \\ \lambda_1 + \frac{\epsilon}{l_1} & \text{if } t_{i2j}^* = \lambda_1 \\ \lambda_2 - \frac{\epsilon}{l_2} & \text{if } t_{i2j}^* = \lambda_2 \end{cases} \quad \text{for each } j \in J, i \in f_j.$$

□

3.2.3.2 Multiple FDCs

The problem can be extended for multiple FDCs. The definition of the extended problem is the same as the definition of the problem for single FDC. Only the content of set K is different, but again, $k = 1$ is set as the central depot. Also, the definitions of the decision variables are the same. The mathematical formulation of the problem is as follows.

$$\begin{aligned} \min \quad & \sum_{k \in K} \sum_{j \in J} f_{kj} e_j s_{kj} + \sum_{k \in K} \sum_{j \in J} \sum_{i \in f(j)} c_{kj}^{ship} e_j t_{ikj} \\ \text{s.t.} \quad & \sum_{i \in I} w_{ik} \leq c_k \quad \forall k \in K \setminus \{1\} \end{aligned} \quad (3.18)$$

$$\sum_{k \in K} t_{ikj} = 1 \quad \forall j \in J, \forall i \in f(j) \quad (3.19)$$

$$t_{ikj} \leq w_{ik} \quad \forall k \in K \setminus \{1\}, \forall j \in J, \forall i \in f(j) \quad (3.20)$$

$$t_{ikj} \leq s_{k,j} \quad \forall k \in K, \forall j \in J, \forall i \in f(j) \quad (3.21)$$

$$w_{ik} \in \{0, 1\} \quad \forall i \in I, \forall k \in K \setminus \{1\} \quad (3.22)$$

$$s_{kj} \in \{0, 1\} \quad \forall k \in K, \forall j \in J \quad (3.23)$$

$$t_{ikj} \in \{0, 1\} \quad \forall k \in K, \forall j \in J, \forall i \in f(j) \quad (3.24)$$

The objective of this problem is again to minimize the total transportation cost. (3.18) is the capacity constraint for the FDCs. (3.19) makes sure that every order is satisfied. Thanks to (3.20), an SKU cannot be sent from a FDC if it is not assigned to this FDC. (3.21) states that if there is not a shipment from a warehouse to an order, then any of the SKUs in this order cannot be sent from this warehouse. (3.22-3.24) are the domain constraints for the decision variables.

Consider the following LP relaxation of the problem

$$\begin{aligned}
 (P_4) \quad z_4 = \min & \sum_{k \in K} \sum_{j \in J} f_{kj} e_j s_{kj} + \sum_{k \in K} \sum_{j \in J} \sum_{i \in f(j)} c_{kj}^{ship} e_j t_{ikj} \\
 \text{s.t.} \quad & \sum_{i \in I} w_{ik} \leq c_k, & \forall k \in K \setminus \{1\}, \\
 & \sum_{k \in K} t_{ikj} = 1, & \forall j \in J, \forall i \in f(j), \\
 & t_{ikj} \leq w_{ik}, & \forall k \in K \setminus \{1\}, \forall j \in J, \forall i \in f(j), \\
 & t_{ikj} \leq s_{kj}, & \forall k \in K, \forall j \in J, \forall i \in f(j), \\
 & 0 \leq w_{ik} \leq 1, & \forall i \in I, \forall k \in K \setminus \{1\}, \\
 & 0 \leq s_{kj} \leq 1, & \forall k \in K, \forall j \in J, \\
 & 0 \leq t_{ikj} \leq 1, & \forall k \in K, \forall j \in J, \forall i \in f(j),
 \end{aligned}$$

whose feasible region is called S_4 .

Let

$$c = \sum_{k \in K \setminus \{1\}} c_k$$

Lemma 5. *Given an optimal solution to P_3 , a solution can be constructed for P_4 .*

Proof. Let (x^*, s^*, t^*) be an optimal solution of P_3 and (w^*, s'^*, t'^*) be a solution of P_4 . Also, let $w_{ik}^* = x_i^* \times \frac{c_k}{c} \forall k \in K \setminus \{1\}, \forall i \in I$, $s'_{1j} = s_{1j}^* \forall j \in J$, $s'_{kj} = s_{2j}^* \times \frac{c_k}{c} \forall k \in K \setminus \{1\}, \forall j \in J$, $t'_{i1j} = t_{i1j}^* \forall i \in I \forall j \in J$, and $t'_{ikj} = t_{i2j}^* \times \frac{c_k}{c} \forall i \in I$,

$\forall j \in J, \forall k \in K \setminus \{1\}$. We shall show that $(w^*, s'^*, t'^*) \in S_4$:

i-) For all $k \in K \setminus \{1\}$

$$\begin{aligned} \sum_{i \in I} w_{ik}^* &= \sum_{i \in I} x_i^* \times \frac{c_k}{c} \\ &= \frac{c_k}{c} \times \sum_{i \in I} x_i^* \\ &\leq \frac{c_k}{c} \times c \\ &= c_k \end{aligned}$$

where the inequality follows from the fact that (x^*, s^*, t^*) satisfies $\sum_{i \in I} x_i^* \leq c$.

Hence, $\sum_{i \in I} w_{ik}^* \leq c_k \forall k \in K \setminus \{1\}$ is satisfied for (w^*, s'^*, t'^*) .

ii-) For all $j \in J$ and $i \in f(j)$

$$\begin{aligned} \sum_{k \in K} t'_{ikj} &= t'_{i1j} + \sum_{k \in K \setminus \{1\}} t'_{ikj} \\ &= t_{i1j}^* + \sum_{k \in K \setminus \{1\}} t_{i2j}^* \times \frac{c_k}{c} \\ &= t_{i1j}^* + t_{i2j}^* \times \sum_{k \in K \setminus \{1\}} \frac{c_k}{c} \\ &= t_{i1j}^* + t_{i1j}^* \\ &= 1 \end{aligned}$$

where the fourth equation follows from our assumption $\sum_{k \in K \setminus \{1\}} c_k = c$. Hence,

$\sum_{k \in K} t'_{ikj} = 1 \forall j \in J, \forall i \in f(j)$ is satisfied for (w^*, s'^*, t'^*) .

iii-) For all $k \in K \setminus \{1\}, j \in J$ and $i \in f(j)$,

$$\begin{aligned} t'_{ikj} &= t_{i2j}^* \times \frac{c_k}{c} \\ &\leq x_i^* \times \frac{c_k}{c} \\ &= w_{ik}^* \times \frac{c}{c_k} \times \frac{c_k}{c} \end{aligned}$$

where inequality follows from the fact that $t_{i2j}^* \leq x_i^* \forall j \in J$ and $\forall i \in f(j)$. Therefore, $t'_{ikj} \leq w_{ik}^* \forall k \in K \setminus \{1\}, \forall j \in J, i \in f(j)$ is satisfied for (w^*, s'^*, t'^*) .

iv-) As, $s'_{1j} = s_{1j}^*$ and $t'_{i1j} = t_{i1j}^*$, $t'_{i1j} \leq s'_{1j}$ following from the fact that $t_{i1j}^* \leq s_{1j}^*$. Additionally, for all $k \in K \setminus \{1\}$, $j \in J$ and $i \in f(j)$,

$$\begin{aligned} t'_{ikj} &= t_{i2j}^* \times \frac{c_k}{c} \\ &\leq s_{2j}^* \times \frac{c_k}{c} \\ &= s'_{kj} \end{aligned}$$

where inequality follows from the fact that $t_{i2j}^* \leq s_{2j}^* \forall j \in J$ and $\forall i \in f(j)$. Therefore, $t'_{ikj} \leq s'_{kj} \forall k \in K, \forall j \in J, i \in f(j)$ is satisfied for (w^*, s', t^*) .

v-) Since (x^*, s^*, t^*) satisfies $0 \leq x_i^* \leq 1 \forall i \in I$, $0 \leq x_i \times \frac{c_k^*}{c} = w_{ik} \leq \frac{c_k}{c} \leq 1 \forall i \in I, \forall k \in K \setminus \{1\}$.

vi-) Since (x^*, s^*, t^*) satisfies $0 \leq s_{kj}^* \leq 1 \forall k \in K, \forall j \in J$, $0 \leq s'_{kj} \leq 1 \forall k \in K, \forall j \in J$.

vii-) Since (x^*, s^*, t^*) satisfies $0 \leq t_{ikj}^* \leq 1 \forall k \in K, \forall j \in J, \forall i \in f(j)$, $0 \leq t'_{ikj} \leq 1 \forall k \in K, \forall j \in J, \forall i \in f(j)$.

By i-vii, $(w^*, s', t^*) \in S_4$. Hence, it is possible to obtain a solution for the LP relaxation of the extended formulation, by using the optimal solution of the LP relaxation of the formulation for single FDC.

□

3.2.4 Theoretical Justification for the Greedy Ranking Algorithm by Jehl et al. [1]

As a solution method for the single warehouse split order problem, Jehl et al. [1] suggest Greedy Ranking Algorithm. In this algorithm a G_i value, which is the summation of proportion between the expected number of order $j \in J$ is placed and the number of SKUs in this order for order j s including SKU i , is assigned to each SKU i . The G_i value is formally defined as follows.

$$G_i = \sum_{j \in J} \frac{e_j}{|f(j)|} 1_{i \in f(j)}.$$

The SKUs are sorted in the decreasing order based on their G_i values and they are added to the assortment until there is not space in the warehouse.

We show that this algorithm gives an optimal solution for the LP relaxation of an aggregated formulation obtained from the benchmark model. Therefore, we provide a mathematical justification for the algorithm. Also, we show that an optimal solution for P_2 can be obtained by using the solution provided by this algorithm.

3.2.4.1 Single Warehouse

It is possible to attain the following equivalent integer programming (IP) formulation by aggregating the second set of constraints in the model of Jehl et al. [1].

$$\begin{aligned}
& \max && \sum_{j \in J} e_j y_j \\
& \text{s.t.} && \sum_{i \in I} x_i \leq c, \\
& && |f(j)| y_j \leq \sum_{i \in f(j)} x_i, && j \in J, \\
& && x_i \in \{0, 1\}, && i \in I, \\
& && y_j \in \{0, 1\}, && j \in J.
\end{aligned}$$

It is easy to observe that in the linear programming (LP) relaxation of this aggregate model, each y_j variable will take its maximum possible value, i.e., $\frac{1}{|f(j)|} \sum_{i \in f(j)} x_i$. Thus, we get the following LP relaxation.

$$\begin{aligned}
& \max && \sum_{j \in J} \frac{e_j}{|f(j)|} \sum_{i \in f(j)} x_i \\
& \text{s.t.} && \sum_{i \in I} x_i \leq c \\
& && 0 \leq x_i \leq 1 && i \in I
\end{aligned}$$

Realizing $\sum_{j \in J: i \in f(j)} \frac{e_j}{|f(j)|} = G_i$, the above LP can be recast as:

$$\begin{aligned}
& \max && \sum_{i \in I} G_i x_i \\
& \text{s.t.} && \sum_{i \in I} x_i \leq c, \\
& && 0 \leq x_i \leq 1, \quad i \in I.
\end{aligned}$$

One can readily observe that this is simply a knapsack problem with cardinality constraint and the optimal solution will simply set the highest G valued c SKUs' x values to 1 and the remaining to zero. In other words, the LP relaxation of the aggregated formulation provides an integer solution which could be used as an initial heuristic solution for our problem. This is known as the Greedy Ranking Algorithm in the literature [1] as described above.

3.2.4.2 Multiple Warehouse

Similarly, by aggregating the second set of constraints in P_2 , the following formulation can be obtained.

$$\begin{aligned}
& \max && \sum_{j \in J} e_j u_j \\
& \text{s.t.} && \sum_{i \in I} w_{ik} \leq c_k, && \forall k \in K, \\
& && |f(j)| v_{jk} \leq \sum_{i \in f(j)} w_{ik}, && \forall k \in K, \forall j \in J, \\
& && u_j \leq \sum_{k \in K} v_{jk}, && \forall j \in J, \\
& && v_{jk} \geq 0, && \forall j \in J, \forall k \in K, \\
& && 0 \leq u_j \leq 1, && \forall j \in J, \\
& && 0 \leq w_{ik} \leq 1, && \forall i \in I, \forall k \in K.
\end{aligned}$$

In an optimal solution, every v_{jk} variable will take its maximum possible value which is $\frac{1}{|f(j)|} \sum_{i \in f(j)} w_{ik}$. Hence, every u_j will take the value $\frac{1}{|f(j)|} \sum_{k \in K} \sum_{i \in f(j)} w_{ik}$. So, when the first set of constraints of P_2 is aggregated, the following formulation is obtained.

$$\begin{aligned}
& \max && \sum_{j \in J} \frac{e_j}{|f(j)|} \sum_{k \in K} \sum_{i \in f(j)} w_{i,k} \\
& \text{s.t.} && \sum_{k \in K} \sum_{i \in I} w_{ik} \leq c, \\
& && 0 \leq w_{ik} \leq 1, \quad \forall i \in I, \forall k \in K.
\end{aligned}$$

Calling $x'_i = \sum_{k \in K} w_{ik}$, again the formulation P_3 can be obtained. So, in the optimal solution, the x' values of c SKUs with the highest G values are set to 1 and the remaining are set to zero. Then, we can get an optimal solution for the above LP by setting $w_{ik} = \frac{c_k}{c}$ for these SKU i s. Therefore, an optimal solution can be obtained by using the solution given by Greedy Ranking Algorithm.

3.3 Heuristic Algorithms

3.3.1 Dynamic Greedy Ranking Algorithm

This algorithm is basically the dynamic version of Greedy Ranking Algorithm developed by Jehl et al. [1]. Namely, G_i values are assigned to SKUs and the SKUs are sorted in the decreasing order based on these values. SKU i having the highest G_i value is added to the assortment and removed from the orders including it. This procedure is repeated until there is no space in the warehouse.

3.3.2 LP Relaxation Algorithm

This algorithm is related to Lemma 1. At first, we solve the LP relaxation of the benchmark model. Then add the SKUs for which the x variables take value 1 in the solution of LP relaxation to the assortment. Then we assign G_i values to the SKUs for which the x variables take a fractional value in the solution of LP relaxation. We sort these SKUs in the decreasing order based on their G_i values and add them to the assortment until there is no space in the warehouse.

3.3.3 Dynamic LP Relaxation Algorithm

This heuristic algorithm is the dynamic version of LP Relaxation Algorithm. We solve the LP relaxation of the benchmark model. We assign G_i values to the SKUs for which the x variables take a fractional value in the solution of LP relaxation. After that, we add the SKUs for which the x variables take value 1 to the assortment and set their x values to 1 while solving the LP relaxation again. We sort the SKUs for which the x variables take a fractional value in the solution of LP relaxation in the decreasing order based on their G_i values and add the one having the highest G_i value to the assortment and set its x value to 1 while solving the LP relaxation again. We repeat this procedure until there is no space in the warehouse.

3.3.4 Alternative Dynamic LP Relaxation Algorithm

This heuristic algorithm is an alternative Dynamic LP Relaxation Algorithm. We solve the LP relaxation of the benchmark model. We add the SKUs for which the x variables take value 1 to the assortment and set their x values to 1 while solving the LP relaxation again. For SKUs having fractional variable values we fix these variables to 1 one by one and solve the resulting LP. Subsequently, we choose the SKU with a fractional variable value which gives the highest objective function value. We add this SKU to the assortment and set its x value to 1 while

solving the LP relaxation again. We repeat this procedure until there is no space in the warehouse.

Example

On the following instance, we demonstrate how these algorithms work.

Let $I = \{1, 2, 3, 4, 5, 6\}$ and

j	$i \in f(j)$	e_j
1	1,4	43
2	3,4	22
3	1,2,4	162
4	1,6	88
5	1,3	92
6	1,2	77
7	1	226
8	1,2,3	99
9	1,5	128

Also, let $c = 3$.

Greedy Ranking Algorithm

Calculate the G_i value for every $i \in I$.

i	1	2	3	4	5	6
G_i	527	125.5	90	86.5	64	44

Sort SKUs in decreasing order based on their G_i values where $i \in I$.

i	1	2	3	4	5	6
G_i	527	125.5	90	86.5	64	44

Add the first $c = 3$ SKUs to the assortment.

$$assortment = \{1, 2, 3\}.$$

Orders 5, 6, 7 and 8 are satisfied and the total number of orders satisfied is 494.

Dynamic Greedy Ranking Algorithm

Calculate the G_i value for every $i \in I$.

i	1	2	3	4	5	6
G_i	527	125.5	90	86.5	64	44

Sort SKUs in decreasing order based on their G_i values where $i \in I$.

i	1	2	3	4	5	6
G_i	527	125.5	90	86.5	64	44

Add the first SKU to the assortment.

$$assortment = \{1\}.$$

Update the content of orders to calculate new G_i values for each remaining SKU i .

j	$i \in f(j)$	e_j
1	4	43
2	3,4	22
3	2,4	162
4	6	88
5	3	92
6	2	77
8	2,3	99
9	5	128

Calculate the G_i value for each remaining SKU i .

i	2	3	4	5	6
G_i	207.5	152.5	135	128	88

Sort the remaining SKUs indexed by i in decreasing order based on their G_i values.

i	2	3	4	5	6
G_i	207.5	152.5	135	128	88

Add the first SKU to the assortment.

$$assortment = \{1, 2\}.$$

Update the content of orders to calculate new G_i value for each remaining SKU i .

j	$i \in f(j)$	e_j
1	4	43
2	3,4	22
3	4	162
4	6	88
5	3	92
8	3	99
9	5	128

Calculate the G_i value for each remaining SKU i .

i	3	4	5	6
G_i	202	216	128	88

Sort the remaining SKUs indexed by i in decreasing order based on their G_i values.

i	4	3	5	6
G_i	216	202	128	88

Add the first SKU to the assortment.

$$assortment = \{1, 2, 4\}.$$

As the maximum possible number of SKUs is added to the assortment, stop the algorithm.

Orders 1, 3, and 6 are satisfied and the total number of orders satisfied is 508.

LP Relaxation Algorithm

Solve the LP relaxation of the benchmark model.

x_1	x_2	x_3	x_4	x_5	x_6
1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0

Add the SKUs for which the corresponding variables take value 1 to the assortment. Set x variables to 1 for these SKUs while solving the LP relaxation in the next iteration.

$$assortment = \{1\}.$$

Calculate the G_i value of each SKU i for which the corresponding variables take a fractional value.

i	2	3	4
G_i	125.5	90	86.5

Sort these SKUs indexed by i in decreasing order based on their G_i values.

i	2	3	4
G_i	125.5	90	86.5

Add the first remaining capacity $c_{remaining} = 2$ many SKUs to the assortment.

$$assortment = \{1, 2, 3\}.$$

Orders 5, 6, 7 and 8 are satisfied and the total number of orders satisfied is 494.

Dynamic LP Relaxation Algorithm

Solve the LP relaxation of the benchmark model.

x_1	x_2	x_3	x_4	x_5	x_6
1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0

Add the SKUs for which the corresponding variables take value 1 to the assortment. Set x variables to 1 for these SKUs while solving the LP relaxation in the next iteration.

$$assortment = \{1\}.$$

Update the content of orders.

j	$i \in f(j)$	e_j
1	4	43
2	3,4	22
3	2,4	162
4	6	88
5	3	92
6	2	77
8	2,3	99
9	5	128

Calculate the G_i value of each SKU i for which the corresponding variables take a fractional value.

i	2	3	4
G_i	207.5	152.5	135

Sort these SKUs indexed by i in decreasing order based on their G_i values.

i	2	3	4
G_i	207.5	152.5	135

Add the first SKU to the assortment. Set $x_2 = 1$ while solving the LP relaxation in the next iteration.

$$assortment = \{1, 2\}.$$

Update the content of orders.

j	$i \in f(j)$	e_j
1	4	43
2	3,4	22
3	4	162
4	6	88
5	3	92
8	3	99
9	5	128

Solve the LP relaxation of the benchmark model.

x_1	x_2	x_3	x_4	x_5	x_6
1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0

The SKUs for which the corresponding variables take value 1 are already added to the assortment.

Calculate the G_i value of each SKU i for which the corresponding variables take a fractional value.

i	3	4
G_i	202	216

Sort these SKUs indexed by i in decreasing order based on their G_i values.

i	4	3
G_i	216	202

Add the first SKU to the assortment.

$$assortment = \{1, 2, 4\}.$$

As the maximum possible number of SKUs is added to the assortment, stop the algorithm.

Orders 1, 3, and 6 are satisfied and the total number of orders satisfied is 508.

Alternative Dynamic LP Relaxation Algorithm

Solve the LP relaxation of the benchmark model.

x_1	x_2	x_3	x_4	x_5	x_6
1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0

Add the SKUs for which the corresponding variables take value 1 to the assortment. Set x variables to 1 for these SKUs while solving the LP relaxation in the next step.

$$assortment = \{1\}$$

For each SKU, whose variable takes a fractional value, calculate the value of the objective function when this SKU is added to the assortment.

SKU Added	Assortment	Objective Function Value
2	1,2	512
3	1,3	519.5
4	1,4	508

Sort these SKUs in decreasing order based on the objective function value.

SKU Added	Assortment	Objective Function Value
3	1,3	519.5
2	1,2	512
4	1,4	508

Add SKU 3 to the assortment and set $x_3 = 1$ while solving the LP relaxation in the next step.

$$assortment = \{1, 3\}.$$

Solve the LP relaxation of the benchmark model.

x_1	x_2	x_3	x_4	x_5	x_6
1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0

The SKUs for which the corresponding variables take value 1 are already added to the assortment.

For each SKU whose variable take a fractional value, calculate the value of the objective function when this SKU is added to the assortment.

SKU Added	Assortment	Objective Function Value
2	1,2,3	494
4	1,3,4	383

Sort these SKUs in decreasing order based on the objective function value.

SKU Added	Assortment	Objective Function Value
2	1,2,3	494
4	1,3,4	383

Add SKU 2 to the assortment.

$$assortment = \{1, 2, 3\}.$$

As the maximum possible number of SKUs is added to the assortment, stop the algorithm.

Orders 5, 6, 7, and 8 are satisfied and the total number of orders satisfied is 494.

3.4 Valid Inequalities

In this section, we suggest valid inequalities for the problem.

3.4.1 Dominance Cuts

As the partial delivery is not allowed by the benchmark model, if an order cannot be satisfied from the warehouse, the other orders which are the super sets of this order also cannot be satisfied. Let $f(j') \subseteq f(j'')$ for some $j', j'' \in J$. Then,

$$y_{j''} \leq y_{j'}.$$

It is possible to generate such valid inequalities for the benchmark model.

3.4.2 Capacity Related Cuts

Let J' be a minimal subset of orders such that the cardinality of the union of SKUs in its elements exceed the capacity of the warehouse, i.e.,

$$\left| \bigcup_{j \in J'} f(j) \right| > c.$$

Then, at least one of the orders in J' cannot be satisfied. In other words,

$$\sum_{j \in J'} \leq |J'| - 1$$

is a valid inequality for P_1 for this case.

Such inequalities can be added to P_1 and its optimal solution may be cut in some cases. To check whether there are any such cuts and to find the best possible one, a separation problem can be solved.

Let (x^*, y^*) be an optimum solution to P_1 . Construct the following IP to find the best cut.

$$\begin{array}{ll}
\max & \sum_{j \in J} y_j^* y_j - \sum_{j \in J} y_j + 1 \\
\text{s.t.} & y_j = 0 \text{ if } y_j^* = 0, \quad \forall i \in I, \\
& x_i \leq \sum_{j: i \in f(j)} y_j, \quad \forall i \in I, \\
& \sum_{i \in I} x_i \geq c, \\
& x_i \in \{0, 1\}, \quad \forall i \in I, \\
& y_j \in \{0, 1\}, \quad \forall j \in J.
\end{array}$$

If the optimal objective function value of this problem is strictly positive, then such a cut exists.

3.4.3 Stronger Capacity Related Cuts

If a subset of orders is such that at least two of them cannot be satisfied because of the capacity constraint, stronger capacity related cuts can be obtained.

Example

For the following instance, suggested valid inequalities are exemplified.

Let $I = \{1, 2, 3, 4, 5, 6\}$ and

j	$i \in f(j)$	e_j
1	1,4	43
2	3,4	22
3	1,2,4	162
4	1,6	88
5	1,3	92
6	1,2	77
7	1	226
8	1,2,3	99
9	1,5	128

Dominance Cuts:

$$y_1 \leq y_7, y_3 \leq y_1, y_3 \leq y_6, y_3 \leq y_7, y_4 \leq y_7, y_5 \leq y_7, y_6 \leq y_7, y_8 \leq y_5, y_8 \leq y_6, \\ y_8 \leq y_7, y_9 \leq y_7.$$

Capacity Related Cuts:

The optimal solution of P_1 for this instance is: As there are lots of such cuts, only the ones corresponding to two orders that cannot be met together are listed below.

$$y_1 + y_8 \leq 1, y_2 + y_3 \leq 1, y_2 + y_4 \leq 1, y_2 + y_6 \leq 1, y_2 + y_8 \leq 1, y_2 + y_9 \leq 1, \\ y_3 + y_4 \leq 1, y_3 + y_5 \leq 1, y_3 + y_8 \leq 1, y_3 + y_9 \leq 1, y_4 + y_8 \leq 1, y_8 + y_9 \leq 1.$$

Stronger Capacity Related Cuts:

Some of such cuts are listed below.

$$y_2 + y_3 + y_4 \leq 1, y_2 + y_3 + y_8 \leq 1, y_2 + y_3 + y_9 \leq 1, y_2 + y_4 + y_8 \leq 1.$$

3.5 Data

To be able to conduct numerical experiments, we have both generated some random instances. In addition, we also use the data published by JD.com [17].

3.5.1 Data Generation

While generating the random instances, we have calibrated some of the parameters using data reported in other studies with actual applications. According to the data used by Catalán and Fisher [2], the median for the number of SKUs per order is 4. This suggests that, in their data, at least half of the orders include 4 or less SKUs. The data published by JD.com [17] also verifies this distribution as 99.98% of the orders include 4 or less items. For this reason, we have assumed that orders include at most 4 items and generated orders based on this assumption. By this means, the number of split orders for a great proportion of the orders can be minimized.

In the data generation process, initially, we assign a popularity parameter between 0 and 1 to each SKU and check whether the proportion of this value to the number of SKUs is less than another random value between 0 and 1. If the random value assigned to the SKU satisfies this condition, we determine that the SKU will be in the order. If the number of SKUs in the order reaches to 4, we stop putting SKUs to the order. Following this procedure, some orders may not include any SKUs. We insert a randomly chosen SKU to these empty orders.

In this way, 60 instances with different number of orders and SKUs are generated. For these instances, the minimum, maximum, and average number of SKUs in orders (degree of orders) is as follows. It is important to note that in order not to increase the generation time the orders with the same content are not grouped and considered as different orders. Therefore, the number of unique orders can be less than the number shown in the table.

Instance No	Order Num.	SKU Num	Min Degree	Max Degree	Avg. Degree
1	7500	75	1	4	2.463333333
2			1	4	2.518133333
3			1	4	2.333333333
4			1	4	2.5268
5			1	4	2.385466667
6		93	1	4	2.4744
7			1	4	2.577333333
8			1	4	2.473066667
9			1	4	2.560266667
10			1	4	2.511466667
11		112	1	4	2.449066667
12			1	4	2.619066667
13			1	4	2.399333333
14			1	4	2.424533333
15			1	4	2.508266667
16		150	1	4	2.442
17			1	4	2.477866667
18			1	4	2.371066667
19			1	4	2.3252
20			1	4	2.372933333
21	10000	100	1	4	2.4528
22			1	4	2.5131
23			1	4	2.297
24			1	4	2.4143
25			1	4	2.3292
26		125	1	4	2.3889
27			1	4	2.5875
28			1	4	2.4741
29			1	4	2.3278
30			1	4	2.4034

Table 3.1: Degree of Orders in Generated Instances

31		150	1	4	2.4637
32			1	4	2.4696
33			1	4	2.4724
34			1	4	2.5186
35			1	4	2.4479
36		200	1	4	2.2997
37			1	4	2.3332
38			1	4	2.4073
39			1	4	2.3818
40			1	4	2.4356
41	12500	125	1	4	2.48384
42			1	4	2.39696
43			1	4	2.53744
44			1	4	2.45592
45			1	4	2.26384
46		156	1	4	2.41368
47			1	4	2.6064
48			1	4	2.38936
49			1	4	2.34512
50			1	4	2.54064
51		187	1	4	2.48464
52			1	4	2.55112
53			1	4	2.43528
54			1	4	2.41512
55			1	4	2.4656
56		250	1	4	2.4348
57			1	4	2.34224
58			1	4	2.51296
59			1	4	2.44584
60			1	4	2.49936

Table 3.1 Continued: Degree of Orders in Generated Instances

3.5.2 Description of Data from JD.com

JD.com [17] published a data set related to the demand that they have received for a product category during March, 2020. The data set provides the information related to the content of each order. In our study, we have only considered the retailer's own items which need to be stocked in the retailer's warehouses and have ignored the items sold by third-party merchants. In the data set, there are 328 SKUs managed by JD.com and there are 246912 orders including these items. When the same type of orders are grouped to assign frequency values of orders, 3559 unique orders are obtained. We have assumed that all these orders are received by a single warehouse. However, as instances of this size can be solved to optimality quickly, we have added another hypothetical product category with 300 SKUs and add at most 4 random products from this category to the actual orders for different possibilities of these two product categories appearing in a same order to obtain challenging problems.

3.6 Numerical Experiments

To be able to evaluate the performance of the heuristic algorithms and valid inequalities, an extensive numerical study has been conducted. The experiments have been performed on a Linux OS with Intel Core i7-3820 3.6 GHz processor. The models and the algorithms have been run using Python and the models have been solved via Gurobi 9.1.1. [18].

3.6.1 Experiments with Heuristic Algorithms

At first, for the instances that we have generated, we have solved the model provided by Jehl et al. [1] and its LP relaxation to be able to evaluate the performance of our heuristic algorithms. For all the instances we have considered four different capacity values as 10%, 25%, 50% and 75% of the total number of

SKUs. In order to ensure the consistency, each run having different parameters has been repeated 5 times with different order instances (detailed results can be found in the appendix for each instance.). As after some time the optimality gap of the problem remains the same for a long time, the runtime of the instances are limited to 3600 seconds. The numerical results can be found below. For each problem size, the columns of Table 3.2 show the number of orders ($|J|$), number of SKUs ($|I|$), plant capacity (c), the number of instances for which the problem can be solved optimally within the time limit, average runtime of the model for the instances that can be solved optimally, average remaining gap for the instances that cannot be solved optimally within the time limit, and the average percentage gap between the objective function value of the LP relaxation and the objective value given by the best known integer solution respectively.

$ J $	$ I $	c	# of instances solved optimally	avg. runtime of instances solved optimally	avg. remaining gap of instances unsolved optimally (%)	avg. LP gap in comparison to best known int. solution (%)
7500	75	7	5	204.27	-	90.3
		18	1	1531.75	5.4	58.1
		37	4	143.57	3.6	15.1
		56	5	4.34	-	0.9
	93	9	5	471.36	-	101.4
		23	0	-	9.7	66.0
		46	1	645.92	5.2	19.9
		69	5	5.09	-	0.9
	112	11	5	904.94	-	85.6
		28	0	-	10.7	55.4
		56	4	1681.78	4.4	13.8
		84	5	3.23	-	0.3
	150	15	4	1430.02	4.2	68.3
		37	0	-	15.2	45.7
		75	4	1539.33	5.1	9.4
		112	5	0.73	-	0.0

Table 3.2: Solution Parameters of the Benchmark Model for Generated Instances

$ J $	$ I $	c	# of instances solved optimally	avg. runtime of instances solved optimally	avg. remaining gap of instances unsolved optimally (%)	avg. LP gap in comparison to best known int. solution (%)
10000	100	10	5	707.15	-	77.3
		25	1	1214.84	7.0	45.4
		50	5	190.17	-	9.5
		75	5	1.42	-	0.0
	125	12	2	648.38	3.4	88.4
		31	0	-	12.5	54.0
		62	4	328.82	13.5	12.9
		93	5	5.04	-	0.6
	150	15	1	1429.91	4.5	86.0
		37	0	-	16.8	55.8
		75	0	-	4.8	12.9
		112	5	3.60	-	0.1
	200	20	0	-	6.9	62.6
		50	0	-	18.4	39.8
		100	2	2166.61	4.1	8.0
		150	5	5.32	-	0.0

Table 3.2 Continued: Solution Parameters of the Benchmark Model for Generated Instances

$ J $	$ I $	c	# of instances solved optimally	avg. runtime of instances solved optimally	avg. remaining gap of instances unsolved optimally (%)	avg. LP gap in comparison to best known int. solution (%)
12500	125	12	2	1962.95	4.0	87.2
		31	0	-	12.1	51.1
		62	3	375.61	7.1	11.0
		93	5	7.43	-	0.3
	156	15	2	633.83	4.5	82.1
		39	0	-	15.4	54.1
		78	1	1334.45	6.1	13.7
		117	5	11.03	-	0.4
	187	18	0	-	6.6	84.7
		46	0	-	20.6	55.7
		93	0	-	6.8	14.3
		140	5	8.66	-	0.1
	250	25	0	-	13.5	72.8
		62	0	-	28.3	49.7
		125	0	-	7.2	11.9
		187	5	47.77	-	0.1

Table 3.2 Continued: Solution Parameters of the Benchmark Model for Generated Instances

Among 240 instances, for 119 of them the benchmark model cannot be solved to optimality in 3600 seconds. For these instances the minimum optimality gap is 1.4%, maximum optimality gap is 36.3%, and average optimality gap is 10.1%.

Among 119 challenging instances, the proportion between the capacity of the plant and the number of SKUs is approximately 10% for 29 of them, 25% for 58 of them, and 50% for 32 of them. When this proportion is 75%, for all the instances the benchmark problem can be solved to optimality in 3600 seconds.

Again, among 119 challenging instances, the proportion between the number of SKUs and the number of orders is approximately 1% for 19 of them, 1.25% for 30 of them, 1.5% for 35 of them, and 2% for 35 of them.

The heuristic algorithms have also been run for the same instances. For each problem size, the columns of the table represent the number of orders ($|J|$), number of SKUs ($|I|$), plant capacity (c), the number of instances for which algorithms give the best solution, the average gap between the objective function value of the solutions given by algorithms and the best known solution of the benchmark model within the time limit, and the average runtime of the algorithms respectively. In the table, each algorithm is assigned to a number: 0 refers to Greedy Ranking Algorithm by Jehl et al. [1], 1 refers to Dynamic Greedy Ranking Algorithm, 2 refers to LP Relaxation Algorithm, 3 refers to Dynamic LP Relaxation Algorithm, and 4 refers to Alternative Dynamic LP Relaxation Algorithm.

$ J $	$ I $	c	best0	gap0	time0	best1	gap1	time1	best2	gap2	time2	best3	gap3	time3	best4	gap4	time4
7500	75	7	0	2.5	23.54	0	2.3	39.76	0	2.5	25.24	0	2.3	41.26	5	0.2	109.56
		18	2	2.3	23.59	2	2.1	61.86	2	2.3	25.39	2	2.0	62.54	3	0.2	173.06
		37	1	2.2	23.65	2	2.0	86.75	1	2.2	25.33	3	2.0	81.78	5	0.2	217.48
		56	1	2.4	23.73	2	2.2	97.85	2	2.4	25.50	4	2.2	52.57	5	0.2	128.97
	93	9	0	2.7	28.76	0	2.4	50.33	0	2.7	30.66	0	2.4	51.99	5	0.2	181.50
		23	0	2.1	28.77	0	1.8	78.97	0	2.1	30.69	0	1.8	80.09	5	0.2	302.80
		46	0	2.0	28.78	3	1.6	109.97	0	1.9	30.63	3	1.6	107.09	3	0.2	377.05
		69	0	2.3	28.92	4	2.0	124.45	1	2.3	30.77	4	2.0	66.80	5	0.2	223.02
	112	11	1	2.5	32.98	0	2.2	58.90	1	2.5	34.94	0	2.2	60.06	4	0.2	241.21
		28	0	2.1	33.33	0	1.8	92.05	0	2.1	34.89	0	1.8	91.94	5	0.2	409.43
		56	0	2.0	33.24	2	1.7	127.38	0	2.0	35.05	3	1.7	119.23	4	0.2	504.18
		84	1	2.4	33.22	4	2.0	141.96	3	2.3	34.97	5	2.0	59.52	5	0.2	178.28
	150	15	0	2.7	43.92	0	2.3	78.35	0	2.6	45.70	0	2.3	79.04	5	0.3	392.67
		37	0	2.3	43.97	0	2.0	119.81	0	2.3	45.64	0	1.9	117.67	5	0.2	687.36
		75	0	2.1	43.73	1	1.8	165.56	0	2.1	45.58	2	1.8	144.87	4	0.2	819.12
		112	0	2.3	43.93	4	2.0	184.22	3	2.3	45.73	4	2.0	56.65	5	0.2	50.37

Table 3.3: Solution Values and Times of the Heuristic Algorithms for Generated Instances

$ J $	$ I $	c	best0	gap0	time0	best1	gap1	time1	best2	gap2	time2	best3	gap3	time3	best4	gap4	time4
10000	100	10	0	2.7	40.74	0	2.5	71.64	0	2.7	43.10	0	2.5	72.99	5	0.3	227.37
		25	0	2.5	40.92	1	2.2	110.17	0	2.5	43.25	1	2.2	109.32	4	0.2	391.09
		50	0	2.4	40.82	0	2.1	151.07	0	2.4	43.25	0	2.1	137.88	5	0.2	475.54
		75	2	2.7	41.09	5	2.4	168.65	4	2.6	43.30	4	2.4	53.55	5	0.2	50.87
	125	12	0	2.9	49.99	0	2.6	87.39	0	2.9	52.64	0	2.6	89.10	5	0.2	383.50
		31	0	2.5	50.36	0	2.2	136.70	0	2.5	52.78	0	2.2	135.63	5	0.2	685.93
		62	1	2.3	50.31	1	2.1	187.59	1	2.3	52.60	0	2.1	170.07	4	0.2	810.09
		93	1	2.6	50.40	5	2.3	210.08	4	2.5	52.69	5	2.3	94.00	5	0.2	294.04
	150	15	0	2.8	59.99	0	2.7	107.40	0	2.8	62.37	0	2.7	109.07	5	0.2	565.75
		37	0	2.3	60.15	0	2.1	164.63	0	2.2	62.69	0	2.1	163.02	5	0.2	1013.06
		75	0	2.1	60.29	0	2.0	229.23	0	2.1	62.79	2	2.0	210.36	5	0.1	1229.82
		112	0	2.4	60.32	2	2.3	254.79	3	2.4	62.85	4	2.3	89.88	5	0.2	232.80
	200	20	0	2.8	78.74	0	2.7	139.81	0	2.7	81.11	0	2.7	140.18	5	0.2	939.97
		50	0	2.3	79.14	0	2.2	214.60	0	2.3	81.65	0	2.1	209.75	5	0.2	1671.09
		100	0	2.1	79.13	1	2.0	292.19	0	2.1	81.25	3	2.0	251.39	4	0.1	1959.21
		150	0	2.4	79.00	3	2.3	323.96	4	2.4	81.37	5	2.3	116.63	5	0.1	432.12

Table 3.3 Continued: Solution Values and Times of the Heuristic Algorithms for Generated Instances

$ J $	$ I $	c	best0	gap0	time0	best1	gap1	time1	best2	gap2	time2	best3	gap3	time3	best4	gap4	time4
12500	125	12	0	2.8	61.65	0	2.6	108.51	0	2.7	64.81	0	2.6	110.09	5	0.2	486.91
		31	1	2.4	62.03	0	2.3	170.45	1	2.4	65.14	0	2.2	167.46	4	0.2	858.76
		62	0	2.3	62.27	1	2.2	234.51	0	2.2	65.24	2	2.1	206.91	3	0.2	978.71
		93	1	2.4	62.17	4	2.3	261.26	3	2.3	65.44	4	2.2	107.09	5	0.1	357.49
	156	15	0	2.6	76.66	0	2.5	136.62	0	2.5	80.11	0	2.4	138.99	5	0.1	828.50
		39	0	2.3	76.71	0	2.0	214.62	0	2.3	79.72	0	2.0	213.56	5	0.1	1499.26
		78	0	2.3	76.85	1	2.0	297.84	0	2.3	80.03	2	2.0	276.94	5	0.1	1817.48
		117	2	2.5	76.94	3	2.2	333.35	4	2.5	79.98	5	2.2	140.67	5	0.1	664.10
	187	18	0	2.8	90.90	0	2.5	162.88	0	2.8	93.93	0	2.5	165.05	5	0.2	1194.06
		46	0	2.2	91.21	0	1.9	255.56	0	2.1	94.50	0	1.8	254.82	5	0.1	2206.54
		93	0	2.0	91.12	0	1.7	355.88	0	2.0	94.36	1	1.6	330.89	5	0.1	2733.12
		140	0	2.4	91.21	3	2.0	398.86	1	2.3	94.49	3	2.0	161.70	5	0.1	994.89
	250	25	0	2.9	120.79	0	2.5	220.19	0	2.9	124.34	0	2.4	221.50	5	0.2	2284.00
		62	0	1.3	120.82	0	0.9	339.54	0	1.2	123.95	0	0.9	334.76	5	0.2	4048.57
		125	0	0.4	120.51	0	0.1	470.32	0	0.3	123.87	1	0.0	423.38	4	0.0	4842.82
		187	0	0.3	121.22	2	0.0	526.19	2	0.1	124.27	5	0.0	212.74	5	0.0	1884.18

Table 3.3 Continued: Solution Values and Times of the Heuristic Algorithms for Generated Instances

As can be seen in the appendix, among 240 instances, Alternative Dynamic LP Relaxation Algorithm gives a solution with an equal or better objective function value than the value of the best integer solution obtained through the benchmark model in 3600 seconds for 128 instances. 39 of these instances are among the ones for which the benchmark model cannot be solved to optimality in 3600 seconds. For the rest of the instances, the algorithm also gives solutions with objective function values close to the value of the solutions found by the benchmark model. Also as shown in Table 3.3, this algorithm finds the best solution among all the algorithms for 226 of 240 instances. However, the time it requires is getting quite longer while the size of the instances is getting larger. Its average runtime for the generated instances is 917.45 seconds. Alternatively, Dynamic LP Relaxation Algorithm also gives solutions with objective function values close to or better than the value of the best integer solution found by the model in reasonable times. It comes up with a solution having an equal or better objective function value than the benchmark model for 65 instances. 9 of these instances are among the challenging instances for the benchmark model. As can be seen in Table 3.3, this algorithm gives the best solution among all the algorithms for 77 of 240 instances. Also, the average runtime of this algorithm is 143.59 seconds. So, this algorithm can be used for larger instances. Therefore, we have decided to set the time limit to the time that Dynamic LP Relaxation Algorithm is solved for the benchmark model and solve it for the same instances again.

The values shown in Table 3.4 are obtained through taking the average of corresponding values of 5 instances for each problem size. For each problem size, the columns of the table represent the number of orders $|J|$, the number of SKUs $|I|$, the plant capacity c , the objective function value for the solution obtained by Dynamic LP Relaxation Algorithm (DLPRval), the runtime of the algorithm (DLPRtime), objective function value of the best integer solution found in the time limit (objval), best known bound on the optimal objective value within the time limit (objbound), optimality gap when the time limit is reached (gap), problem generation time in terms of CPU time in seconds (gentime), runtime of the model in terms of CPU time in seconds (runtime), the number of nodes explored (nodenum), the number of simplex iterations performed (iternum), and

the gap between the objective function values of the best known integer solution of the benchmark model and the solution obtained via Dynamic LP Relaxation Algorithm (algogap). “algogap” takes a negative value when the algorithm gives a higher quality solution than the best known integer solution of the benchmark model, and vice versa.



$ J $	$ I $	c	DLPRval	DLPRtime	objval	objbound	gap (%)	runtime	nodenum	iternum	algogap (%)
7500	75	7	20242.4	41.257	21148.2	25331	20.8	41.263	1	42846.4	4.5
		18	64723.4	62.535	63894.4	81399.6	27.9	62.550	2	46422.2	-1.3
		37	183531.2	81.780	181200.2	191190.3798	6.1	70.843	437.8	280406.4	-1.5
		56	315057.6	52.574	315069.6	315069.6	0.0	4.342	0.8	6942.6	0.0
	93	9	18358.6	51.986	20032.2	23981	19.7	51.997	1	49433.6	8.4
		23	61330.6	80.089	61384	80714.8	31.6	80.111	1	59317.6	0.1
		46	172765.8	107.088	166569.6	188033	13.2	107.091	273.4	329509.4	-3.8
		69	307819.6	66.795	307840.8	307840.8	0.0	5.086	1	8928.2	0.0
	112	11	21218.2	60.056	22722	24879.2	10.0	60.064	1	55496.2	6.5
		28	68032.2	91.944	68497.4	82532.99449	21.0	91.948	1	64728.4	0.7
		56	186988.2	119.231	186001.4	197469	6.3	119.234	336	384137.6	-0.5
		84	317875	59.524	317875	317875	0.0	3.237	0.6	6301.8	0.0
	150	15	24035	79.036	25405.8	27443	8.0	79.038	3.8	75935.2	5.4
		37	70798.6	117.669	72376.2	87086.2	20.4	117.691	1	76075.8	2.2
		75	195633.2	144.875	195627	203454.4	4.1	144.880	727.6	522079	0.0
		112	318626.8	56.650	318630.8	318630.8	0.0	0.729	0.6	3788.2	0.0

Table 3.4: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Generated Instances

$ J $	$ I $	c	DLPRval	DLPRtime	objval	objbound	gap (%)	runtime	nodenum	iternum	algogap (%)
10000	100	10	30753.8	72.986	32062.6	38655.2	21.3	72.991	1	53681.4	4.2
		25	97116.8	109.318	96553.4	117916	22.8	109.353	1	65518.8	-0.7
		50	259458.8	137.876	259249.2	264949	2.2	135.223	440.8	375972.2	-0.1
		75	425032.6	53.548	425048.6	425048.6	0.0	1.421	0.4	5473.4	0.0
	125	12	27392.2	89.097	29181.4	34163.4	17.6	89.108	1	62271	6.3
		31	91524.2	135.627	91743.8	115088	26.5	135.633	1	76956.2	0.3
		62	251636	170.070	245235.6	263283.9524	9.9	155.095	369.8	318267.8	-3.8
		93	420859.4	93.996	420859.4	420859.4	0.0	5.066	0.2	7931.6	0.0
	150	15	28070.8	109.074	30491.2	35481	16.5	109.082	1	71024.6	8.0
		37	88500.2	163.018	89439.2	112869.9913	26.4	163.047	1	88357.6	1.1
		75	250523.6	210.355	247315.4	266199.948	7.8	210.360	224.4	387345	-1.4
		112	422067.8	89.882	422078	422078	0.0	3.597	0.6	7678.4	0.0
	200	20	33026.8	140.181	35964	39905.19928	11.1	140.184	1.8	81748.8	8.3
		50	102048.2	209.751	104390	126765.2	21.5	209.755	1.6	100235	2.3
		100	270382.8	251.392	270233	281154.1059	4.2	251.400	824.6	634213	-0.1
		150	434278.2	116.632	434278.2	434278.2	0.0	5.319	29.4	11822.4	0.0

Table 3.4 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Generated Instances

$ J $	$ I $	c	DLPRval	DLPRtime	objval	objbound	gap (%)	runtime	nodenum	iternum	algogap (%)
12500	125	12	34883.8	110.095	36946.2	47597.2	30.2	110.103	1	64422	5.7
		31	116779.8	167.456	116042	149678.7541	30.1	167.471	1	78684.8	-0.6
		62	319917.4	206.913	316697.8	333449.5445	5.9	180.230	195.8	275679.6	-1.2
		93	526873.6	107.093	526874.6	526874.6	0.0	7.428	0.4	9669	0.0
	156	15	34916.8	138.993	37538	44589.39423	19.7	139.008	1	74413	6.9
		39	112844.8	213.562	114462	145191.8	28.0	213.575	1	91687	1.4
		78	310603.2	276.942	304788.6	331753.4	9.5	276.957	192.2	333750	-2.1
		117	525533.8	140.669	525533.8	525533.8	0.0	11.082	0.6	11409.6	0.0
	187	18	34357.4	165.054	36799.4	44320.8	20.9	165.082	1	86635.6	6.6
		46	108382	254.823	110937.2	142765.2	29.1	254.866	1	110839	2.3
		93	306785.6	330.895	288714.6	332669.1692	15.4	330.901	123	380814.4	-6.3
		140	527411.8	161.697	527430.8	527430.8	0.0	8.647	1	10833.2	0.0
	250	25	38219.4	221.501	41133.4	48453.6	18.1	221.508	1	103083	7.1
		62	114744.4	334.764	117467.8	153702.4	31.3	334.777	1	128783	2.4
		125	315867.8	423.384	310414.2	340774.1895	10.0	423.390	566.6	652276.2	-1.9
		187	528107.4	212.740	528107.4	528107.4	0.0	47.754	314.8	114044.4	0.0

Table 3.4 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Generated Instances

Dynamic LP Relaxation Algorithm gives a solution with an equal or better objective value than the benchmark model for 110 of the 240 generated instances when the time limit of the model is set to the run time of the algorithm. In 60 of these instances, the benchmark model cannot be solved to optimality within the time limit and 45 of these instances are among the challenging 119 instances for the benchmark model. Also, the algorithm gives a similar or better quality solution than the benchmark model for 20 of the 48 different problem size and plant capacity combinations. In 11 of these combinations, the benchmark model cannot be solved to the optimality within the time limit and four of them are among the challenging 16 combinations for the benchmark model. So, for some challenging instances by running the Dynamic LP Relaxation Algorithm it is possible to obtain a solution with a higher objective function value than the value of the solution found by running the benchmark model for the same amount of time.

To further evaluate the performance of this algorithm, we have also tested it via the real data from which we have created challenging instances. For different possibilities (%25, %50, %75, and %100) of items from the actual product category and the hypothetical product category appearing in the same order, there are two tables below. The first one shows the solution parameters for the benchmark model when the runtime is limited to 3600 seconds. The columns of this table represent the instance number (no), number of unique orders ($|J|$), number of SKUs ($|I|$), minimum number of items in an order (mindeg), maximum number of items in an order (maxdeg), average number of items in an order (avgdeg), plant capacity (c), the objective function value for the best solution found (objval), best known bound for the objective function value (objbound), the optimality gap (gap), generation time of the model (gentime), runtime of the model in terms of CPU time in seconds (runtime), number of nodes explored (nodenum), and number of simplex iterations performed (iternum) respectively. The second table compares the quality of the solution obtained by the Dynamic LP Relaxation Algorithm and the benchmark model, when the runtime of the benchmark model is limited to the time in which the Dynamic LP Relaxation Algorithm terminates, for exactly the same instances. The columns of this table

show the instance number (no), objective function value attained by the Dynamic LP Relaxation Algorithm (DLPRval), time to run this algorithm (DLPRtime), the value of the solution parameters of the benchmark model when the runtime is limited to DLPRtime (objval, objbound, gap, runtime, nodenum, iternum).



no	$ J $	$ I $	mindeg	maxdeg	avgdeg	c	objval	objbound	gap	gentime	runtime	nodenum	iternum
real1	41225	628	1	9	3.090	62	145955	145955	0	7.47	3.68	0	42887
						157	179217	179217	0	7.47	14.00	1	47681
						314	216996	216996	0	7.47	14.98	1	81854
						471	241606	241606	0	7.47	5.75	0	56189
real2	40929	628	1	10	3.094	62	146169	146169	0	7.54	3.47	0	42621
						157	179540	179540	0	7.54	11.49	0	57160
						314	216855	216855	0	7.54	22.08	0	51273
						471	241658	241658	0	7.54	9.15	0	55349
real3	41191	628	1	8	3.100	62	146181	146181	0	7.61	3.62	0	42828
						157	179705	179705	0	7.61	6.95	0	59465
						314	216918	216918	0	7.61	25.51	1	61905
						471	241313	241313	0	7.61	9.89	0	56196
real4	41161	628	1	9	3.100	62	146010	146010	0	7.62	3.60	0	43131
						157	179359	179359	0	7.62	12.69	1	63282
						314	217557	217557	0	7.62	19.40	0	48486
						471	241250	241250	0	7.62	5.31	0	53337
real5	40616	628	1	9	3.104	62	146111	146111	0	7.56	3.48	0	42385
						157	180007	180007	0	7.56	10.90	1	61793
						314	218977	218977	0	7.56	19.96	0	48880
						471	242315	242315	0	7.56	6.01	0	51599

Table 3.5: Solution Parameters of the Benchmark Model for Modified Real Data with %25 Co-appearance Probability

no	c	DLPRval	DLPRtime	objval	objbound	gap	runtime	nodenum	iternum
real1	62	145955	46.11	145955	145955	0	3.57	0	42887
	157	179216	105.81	179217	179217	0	13.99	1	47681
	314	216994	203.36	216996	216996	0	15.26	1	81854
	471	241606	277.16	241606	241606	0	5.53	0	56189
real2	62	146169	46.14	146169	146169	0	3.50	0	42621
	157	179540	103.80	179540	179540	0	11.47	0	57160
	314	216855	190.48	216855	216855	0	21.99	0	51273
	471	241658	276.33	241658	241658	0	9.17	0	55349
real3	62	146181	46.29	146181	146181	0	3.54	0	42828
	157	179705	104.96	179705	179705	0	6.95	0	59465
	314	216918	193.82	216918	216918	0	25.59	1	61905
	471	241313	279.71	241313	241313	0	9.97	0	56196
real4	62	146010	46.73	146010	146010	0	3.68	0	43131
	157	179356	112.87	179359	179359	0	12.48	1	63282
	314	217557	190.18	217557	217557	0	19.39	0	48486
	471	241250	278.89	241250	241250	0	5.29	0	53337
real5	62	146111	45.90	146111	146111	0	3.47	0	42385
	157	180007	110.14	180007	180007	0	10.98	1	61793
	314	218977	188.55	218977	218977	0	20.01	0	48880
	471	242315	272.86	242315	242315	0	6.17	0	51599

Table 3.6: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Modified Real Data with %25 Co-appearance Probability

no	$ J $	$ I $	mindeg	maxdeg	avgdeg	c	objval	objbound	gap	gentime	runtime	nodenum	iternum
real6	67341	628	1	10	3.311	62	97344	97344	0	12.73	20.62	0	93093
						157	140921	140921	0	12.73	63.24	1	98119
						314	205129	205129	0	12.73	57.08	0	72882
						471	240460	240460	0	12.73	15.20	0	86737
real7	67320	628	1	12	3.313	62	97475	97475	0	12.83	22.49	0	98064
						157	141528	141528	0	12.83	96.07	1	166100
						314	205523	205523	0	12.83	70.89	1	91167
						471	240605	240605	0	12.83	16.88	0	92060
real8	67729	628	1	9	3.309	62	97353	97353	0	13.02	20.55	0	92546
						157	141185	141185	0	13.02	57.26	1	156267
						314	204488	204488	0	13.02	58.62	0	73723
						471	239848	239848	0	13.02	32.98	0	31882
real9	66850	628	1	12	3.302	62	97814	97814	0	13.00	23.80	1	94293
						157	141677	141677	0	13.00	65.25	1	108182
						314	205882	205882	0	13.00	53.10	1	66179
						471	240335	240335	0	13.00	14.42	0	85787
real10	67427	628	1	12	3.304	62	97864	97864	0	12.87	18.39	0	90753
						157	141092	141092	0	12.87	51.60	1	77716
						314	203364	203364	0	12.87	63.48	0	79375
						471	240365	240365	0	12.87	17.86	0	90971

Table 3.7: Solution Parameters of the Benchmark Model for Modified Real Data with %50 Co-appearance Probability

no	c	DLPRval	DLPRtime	objval	objbound	gap	runtime	nodenum	iternum
real6	62	97344	86.22	97344	97344	0	20.42	0	93093
	157	140919	226.39	140921	140921	0	63.43	1	98119
	314	205115	316.16	205129	205129	0	56.94	0	72882
	471	240460	448.84	240460	240460	0	14.33	0	86737
real7	62	97475	90.45	97475	97475	0	23.38	0	98064
	157	141524	325.91	141528	141528	0	95.03	1	166100
	314	205514	316.24	205523	205523	0	70.80	1	91167
	471	240605	459.01	240605	240605	0	16.85	0	92060
real8	62	97353	90.52	97353	97353	0	20.55	0	92546
	157	141185	206.55	141185	141185	0	57.17	1	156267
	314	204488	315.51	204488	204488	0	58.50	0	73723
	471	239848	455.20	239848	239848	0	32.91	0	31882
real9	62	97810	98.42	97814	97814	0	22.87	1	94293
	157	141676	176.69	141677	141677	0	65.37	1	108182
	314	205879	315.35	205882	205882	0	53.24	1	66179
	471	240335	448.24	240335	240335	0	14.45	0	85787
real10	62	97864	85.79	97864	97864	0	18.45	0	90753
	157	141084	192.38	141092	141092	0	52.38	1	77716
	314	203364	316.43	203364	203364	0	62.49	0	79375
	471	240365	456.64	240365	240365	0	17.83	0	90971

Table 3.8: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Modified Real Data with %50 Co-appearance Probability

no	$ J $	$ I $	mindeg	maxdeg	avgdeg	c	objval	objbound	gap	gentime	runtime	nodenum	iternum
real11	90742	328	1	10	3.445	62	55396	55396	0	17.32	904.59	1	491295
						157	111961	111961	0	17.32	972.54	109	529996
						314	198083	198083	2.35E-15	17.32	107.26	1	85351
						471	239773	239773	0	17.32	23.12	0	111287
real12	90753	328	1	12	3.438	62	55454	55454	0	17.45	936.97	1	618851
						157	111780	111780	0	17.45	3065.09	828	2170051
						314	196971	196971	0	17.45	129.27	0	123357
						471	239367	239367	0	17.45	24.73	0	116012
real13	90383	328	1	11	3.439	62	56291	56291	0	17.39	979.21	1	545577
						157	113279	113279	0	17.39	911.66	23	580235
						314	199980	199980	0	17.39	131.40	0	123224
						471	239699	239699	0	17.39	25.15	0	111447
real14	91068	328	1	12	3.454	62	55567	55567	0	17.87	1009.22	1	503829
						157	111146	111146	0	17.87	2049.30	111	895853
						314	197841	197841	0	17.87	129.33	0	126154
						471	239995	239995	0	17.87	24.99	0	114487
real15	90838	328	1	9	3.443	62	55497	55497	0	17.46	549.53	1	387891
						157	111104	111104	0	17.46	557.71	1	376370
						314	196531	196531	0	17.46	133.63	0	128588
						471	239670	239670	0	17.46	27.52	0	120756

Table 3.9: Solution Parameters of the Benchmark Model for Modified Real Data with %75 Co-appearance Probability

no	c	DLPRval	DLPRtime	objval	objbound	gap	runtime	nodenum	iternum
real11	62	55269	492.69	55396	55521	0.002256481	492.72	1	297350
	157	111937	914.77	111955	112013	0.000518065	914.79	1	466538
	314	198081	431.64	198083	198083	2.35E-15	108.49	1	85351
	471	239773	621.59	239773	239773	0	23.05	0	111287
real12	62	55315	521.76	55454	55541	0.001568868	521.78	1	309107
	157	111762	1100.93	111745	111927	0.001628708	1100.97	1	442154
	314	196971	435.61	196971	196971	0	128.08	0	123357
	471	239367	616.21	239367	239367	0	24.73	0	116012
real13	62	56225	348.79	0	56681	inf	348.94	0	116656
	157	113248	705.20	113278	113313	0.000308974	705.22	1	386187
	314	199980	429.61	199980	199980	0	131.78	0	123224
	471	239699	622.50	239699	239699	0	23.90	0	111447
real14	62	55528	429.13	0	56053	inf	429.29	0	117051
	157	111042	1181.82	111146	111210	0.000575819	1181.84	1	464304
	314	197841	436.37	197841	197841	0	130.99	0	126154
	471	239995	619.37	239995	239995	0	24.96	0	114487
real15	62	55398	468.74	55480	55562	0.00147801	468.82	1	296304
	157	111104	738.95	111104	111104	0	558.93	1	376370
	314	196531	433.63	196531	196531	0	133.53	0	128588
	471	239670	621.53	239670	239670	0	27.52	0	120756

Table 3.10: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Modified Real Data with %75 Co-appearance Probability

no	$ J $	$ I $	mindeg	maxdeg	avgdeg	c	objval	objbound	gap	gentime	runtime	nodenum	iternum
real16	111780	328	2	9	3.561	62	18007	36201	1.01038485	21.62	3600.04	1	1048431
						157	70630	95254	0.348633725	21.62	3600.03	1	715539
						314	193189	193189	0	21.62	247.26	0	163269
						471	239363	239363	0	21.62	87.83	0	59100
real17	111520	328	2	12	3.561	62	20240	35697	0.763685771	22.09	3600.12	1	938859
						157	70195	97056	0.382662583	22.09	3600.16	1	829161
						314	194790	194790	0	22.09	233.88	1	178152
						471	239380	239380	0	22.09	88.24	0	57313
real18	111815	328	2	12	3.558	62	18652	34842	0.868003431	22.19	3600.18	1	969168
						157	65227	94956	0.455777515	22.19	3600.12	1	783593
						314	192425	192425	0	22.19	232.96	0	159634
						471	239320	239320	0	22.19	89.39	0	58059
real19	111359	328	2	10	3.562	62	23057	35452	0.537580778	21.99	3600.15	1	1012672
						157	71234	95836.95113	0.345382137	21.99	3600.07	1	794283
						314	193371	193371	0	21.99	239.03	0	162075
						471	239763	239763	0	21.99	89.21	0	55405
real20	112399	328	2	12	3.552	62	17053	34863	1.044391016	22.35	3600.04	1	1160175
						157	57098	92838	0.625941364	22.35	3600.04	1	801698
						314	189172	189172	0	22.35	283.02	0	180228
						471	239562	239562	0	22.35	88.39	0	56807

Table 3.11: Solution Parameters of the Benchmark Model for Modified Real Data with %100 Co-appearance Probability

no	c	DLPRval	DLPRtime	objval	objbound	gap	runtime	nodenum	iternum
real16	62	23152	2669.83	17836	37191	1.085164835	2669.86	1	709459
	157	88232	4952.03	71935	95092	0.321915618	4952.07	1	828529
	314	193189	544.88	193189	193189	0	247.32	0	163269
	471	239363	770.50	239363	239363	0	87.73	0	59100
real17	62	23465	2672.44	20240	36677	0.812104743	2672.47	1	747250
	157	90168	5156.08	70495	96897	0.374523016	5156.11	1	1029222
	314	194790	547.36	194790	194790	0	233.94	1	178152
	471	239380	777.05	239380	239380	0	88.11	0	57313
real18	62	22294	2722.42	18652	36033	0.931857173	2722.45	1	748338
	157	87577	5061.17	65767	94588	0.438228899	5061.28	1	1202126
	314	192425	542.04	192425	192425	0	231.78	0	159634
	471	239320	778.27	239320	239320	0	89.31	0	58059
real19	62	24062	2678.76	23057	36320	0.575226612	2678.86	1	841141
	157	89431	4924.88	71234	95708	0.34357189	4924.90	1	1020847
	314	193371	543.59	193371	193371	0	238.44	0	162075
	471	239763	773.21	239763	239763	0	89.18	0	55405
real20	62	21439	2828.16	16216	36016	1.22101628	2828.19	1	878381
	157	83156	5843.81	61720	92281	0.495155541	5843.95	1	1212586
	314	189172	547.19	189172	189172	0	282.64	0	180228
	471	239562	786.02	239562	239562	0	88.59	0	56807

Table 3.12: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model for Modified Real Data with %100 Co-appearance Probability

As shown in tables, the benchmark model can be solved to optimality very quickly when the co-appearance probability of the actual and hypothetical product categories is less than 100%. But, the algorithm can still find a solution faster than the benchmark model for lower values of the proportion between the plant capacity and the number of SKUs when the co-appearance probability is 75%. Even among 20 such instances in 2 of them, the benchmark model cannot come up with an integer solution within the runtime of the algorithm. Also, for one of the 20 instances, the algorithm can find a better solution in the same amount of time.

When the co-appearance probability is 100% (in other words, when at least one SKU from the hypothetical product category is added to the actual orders), the problem becomes challenging for lower values of the proportion between the plant capacity and the number of SKUs. For these 10 instances, the problem cannot be solved to optimality in 3600 seconds. In all of the 20 instances having this co-appearance probability (including the challenging ones), the algorithm gives a same or better quality solution than the benchmark model when the time limit is set to the runtime of the algorithm for the benchmark model.

3.6.2 Experiments with Valid Inequalities

For the numerical experiments to evaluate the performance of valid inequalities, some other challenging instances (which are also generated through the procedure described in Data Generation section) are used. Again, for the sake of consistency, the average values for the same size instances are considered. The time limit for the models is set to 3600 seconds.

3.6.2.1 Dominance Cuts

All the dominance cuts are added to the benchmark model to see the effect of using these cuts on the runtime of the model. The objective functions value, optimality gap and the runtime of the model with and without the dominance cuts can be seen in Table 3.13.

$ J $	$ I $	c	lpval	objval w/out ineq.	gap w/out ineq.	runtime w/out ineq.	objval w/ ineq.	gap w/ ineq.	runtime w/ ineq.
12500	125	12	66494.097	34081.8	0.019990191	3195.92	34081.8	0.004390029	1350.43
		31	171776.418	106633.8	0.150650303	3600.01	106646.6	0.195319108	3600.07
		62	343552.836	297338.6	0.062909661	3134.05	297338.6	0.035527336	2859.85
		93	515080.109	513137.0	0	54.83	513137.0	0	8.93
	156	15	67235.898	36553.2	0.02616068	3465.59	36577.8	0.051886645	3600.05
		39	174813.335	113782.8	0.152099889	3600.01	113785.0	0.259634581	3600.06
		78	349626.670	309529.6	0.045896099	2948.13	309529.6	0.053535922	2996.68
		117	524349.888	522978.4	0	12.59	522978.4	0	11.18
	187	18	69145.980	41744.0	0.04591213	3600.01	41766.0	0.084124365	3600.04
		46	176706.392	126784.8	0.138835173	3600.01	126781.4	0.20908058	3600.05
		93	357254.228	330520.6	0.02513505	3101.47	330524.4	0.0296113	2914.75
		140	536288.222	535892.6	0	4.77	535892.6	0	5.04
	250	25	69870.161	39918.6	0.148597576	3600.02	39904.4	0.291025933	3600.03
		62	173277.999	115304.0	0.287267397	3600.02	115334.4	0.367147235	3600.03
		125	349350.805	310939.2	0.075236982	3600.02	310939.2	0.084015341	3600.03
		187	522536.532	522128.4	0	9.04	522128.4	0	7.33

Table 3.13: Effect of Dominance Cuts

When the dominance cuts are added to the formulation, the runtime of the benchmark model decreases for 15 of 80 instances. Also, the optimality gap of the benchmark model decreases with the use of cuts for 9 of the instances. When same size problems are averaged as in Table 3.13, the runtime can be decreased for 6 of 16 parameter combinations and the optimality gap can be decreased for 2 of them.

Surprisingly, the runtime of the benchmark model increases for 14 of 80 instances. This is also the case for 2 of 16 parameter combinations shown in Table 3.13. Additionally, for 46 of 80 instances the dominance cuts increases the optimality gap. This is also the case for 10 of 16 parameter combinations shown in the above table. This happens especially for the challenging instances that cannot be solved to optimality in 3600 seconds.

3.6.2.2 Capacity Related Cuts

The LP relaxation of the benchmark model is solved iteratively by adding the best capacity related cut, which is obtained through the separation problem, at each iteration to see whether it is possible to obtain an integer solution via this algorithm. Unlike the others Table 3.14 shows every instance and not the average values. The column “lpval” shows the value of the LP relaxation of the benchmark model. The column “stop code” shows how the algorithm is stopped for an instance. If it takes value 0, this means that an integer solution is obtained through the algorithm. If it takes value 1, then the last found fractional solution cannot be cut with the best capacity related cut. If it takes value 2, then the separation problem is infeasible for the last iteration. The column “objval” is the value of the solution obtained via the algorithm.

$ J $	$ I $	avg. degree	c	stop code	lpval	objval	algorithm time
12500	125	2.484	12	1	66514.041	66514.041	5.03
			31	1	171827.939	171827.939	4.99
			62	1	343655.878	343655.878	5.16
			93	1	515483.816	515483.816	7.12
		2.610	12	1	64326.291	64326.291	5.15
			31	1	166176.252	166176.252	5.14
			62	1	332352.505	332352.505	5.27
			93	1	498528.757	498528.757	4.65
		2.421	12	1	68132.455	68132.455	4.96
			31	1	176008.841	176008.841	4.99
			62	1	352017.682	352017.682	5.14
			93	0	527504.000	527504.000	2.99
		2.446	12	1	65811.093	65811.093	5.09
			31	1	170011.990	170011.990	5.01
			62	1	340023.979	340023.979	5.13
			93	1	510035.969	510035.938	60.86
		2.455	12	1	67686.607	67686.607	5.08
			31	1	174857.067	174857.067	5.09
			62	1	349714.135	349714.135	5.14
			93	0	523848.000	523848.000	2.86
	156	2.377	15	1	67646.870	67646.870	5.48
			39	1	175881.861	175881.861	5.55
			78	1	351763.722	351763.722	5.61
			117	0	527195.000	527195.000	3.07
		2.468	15	1	66791.441	66791.441	5.51
			39	1	173657.746	173657.746	5.64
			78	1	347315.492	347315.492	5.75
			117	1	520973.237	520963.542	32.01
		2.434	15	1	68479.375	68479.375	4.31
			39	1	178046.375	178046.375	5.57
			78	1	356092.750	356092.750	5.72
			117	1	534139.125	534133.125	11.38

Table 3.14: Do capacity related cuts lead to an integer solution?

$ J $	$ I $	avg. degree	c	stop code	lpval	objval	algorithm time
		2.443	15	1	66311.102	66311.102	5.51
			39	1	172408.866	172408.866	5.54
			78	1	344817.732	344817.732	5.71
			117	1	517226.598	517226.598	6.03
		2.548	15	1	66950.702	66950.702	5.53
			39	1	174071.826	174071.826	5.68
			78	1	348143.653	348143.653	5.85
			117	1	522215.479	522215.446	84.79
	187	2.444	18	1	67931.757	67931.757	6.03
			46	1	173603.378	173603.378	6.16
			93	1	350980.743	350980.743	6.44
			140	1	528358.108	528358.108	9.08
		2.327	18	1	70206.600	70206.600	6.01
			46	1	179416.867	179416.867	6.11
			93	1	362734.100	362734.100	6.24
			140	0	542262.000	542262.000	3.32
		2.299	18	1	69388.015	69388.015	5.95
			46	1	177324.926	177324.926	6.11
			93	1	358504.743	358504.743	6.33
			140	0	538688.000	538688.000	3.35
		2.341	18	1	69100.835	69100.835	5.97
			46	1	176591.022	176591.022	6.17
			93	1	357020.978	357020.978	6.17
			140	0	537183.000	537183.000	3.31
		2.348	18	1	69102.692	69102.692	6.06
			46	1	176595.769	176595.769	6.12
			93	1	357030.577	357030.577	6.81
			140	0	534950.000	534950.000	3.30
	250	2.467	25	1	69949.601	69949.601	5.68
			62	1	173475.011	173475.011	5.78
			125	1	349748.005	349748.005	5.98
			187	2	523223.016	523215.000	14415.29

Table 3.14 Continued: Do capacity related cuts lead to an integer solution?

$ J $	$ I $	avg. degree	c	stop code	lpval	objval	algorithm time
		2.465	25	1	69396.985	69396.985	5.68
			62	1	172104.523	172104.523	5.86
			125	1	346984.925	346984.925	6.06
			187	1	519089.447	519089.447	26.94
		2.442	25	1	70904.545	70904.545	5.79
			62	1	175843.273	175843.273	5.79
			125	1	354522.727	354522.727	5.96
			187	0	530366.000	530366.000	4.07
		2.456	25	1	69419.010	69419.010	5.70
			62	1	172159.146	172159.146	5.81
			125	1	347095.052	347095.052	6.01
			187	1	519254.198	519254.198	3605.69
		2.424	25	1	69680.663	69680.663	6.58
			62	1	172808.044	172808.044	5.70
			125	1	348403.315	348403.315	5.99
			187	0	520750.000	520750.000	4.03

Table 3.14 Continued: Do capacity related cuts lead to an integer solution?

This algorithm finds an integer solution for 9 of the 80 instances. It is stopped because the best cut found via separation problem cannot cut the current fractional solution for 70 of the 80 instances. Lastly, it is stopped for a single instance as the separation problem is infeasible.

The instances for which an integer solution can be found, the LP relaxation of the benchmark model has already given an integer solution with the same value.

3.7 Conclusion

Based on the results obtained through numerical experiments, it can be said that some instances of the problem are much more challenging than the others.

According to our analysis, the proportion between the plant capacity and the number of SKUs and the proportion between the number of SKUs and the number of orders are some factors affecting the difficulty of the problem. When the proportion of plant capacity to number of SKUs is lower, the problem is more challenging. Additionally, the problem gets harder for increasing values of the proportion of the number of SKUs to the number of orders for a while, but after some point the problem becomes simple as this proportion gets bigger. The underlying reason for this can be that the number of unique orders decreases when the number of SKUs is too low and the size of the problem is small enough to be solved quickly, or that the contents of orders do not intersect with each other when the number of SKUs is too high and problem is reduced to finding the SKUs in the most popular orders. In addition, as can be seen in the detailed results in the appendix, the problem is more challenging for higher values of the average degree of orders.

For challenging instances of the problem, it is possible to find solutions with a quality close to the solution found through solving the benchmark model within the time limit. Especially, the Alternative Dynamic LP Relaxation Algorithm gives quite high quality solutions for almost all of the instances. It can even come up with a solution better than the IP for most of the challenging cases. However, as the problem size gets bigger, the time it requires gets longer. As for some large scale instances it may require more time than the time limit set to benchmark model, it may not be meaningful to wait too long for a heuristic algorithm. Alternatively, the Dynamic LP Relaxation Algorithm also gives high quality solutions for most of the instances in reasonable times. Therefore it can be used for large scale challenging instances. It also can come up with better solutions than the benchmark model within the same time for these instances.

Although the dominance cuts can reduce the runtime and the optimality gap (when the time limit is reached) for some instances, they can also cause an increase in these two solution parameters by changing the root node of the search tree.

Finally, for most of the instances, it is not possible to reach an integer solution by adding the best capacity related cut obtained through the separation problem to the benchmark model.

Chapter 4

Combined Approach

This chapter discusses if it is advantageous to solve assortment allocation and consolidation problems together instead of solving them consecutively.

4.1 Problem Statement

Given a set of orders, a set of SKUs, and a set of warehouses, find an assortment for each warehouse which does not exceed the capacity and determine whether and how to consolidate each order so that all the orders are satisfied and the total transportation cost is minimized.

4.2 Benchmark Models

The formulation provided by Catalán and Fisher [2] and a modified version of the formulation provided by Zhang et al. [3] are used as benchmark models in this study.

4.2.1 Assortment Allocation Model Provided by Catalán and Fisher [2]

By using the parameters and decision variables defined in Chapter 3, the model provided by Catalán and Fisher [2] can be written as follows.

$$\min \quad \sum_{k \in K} \sum_{j \in J} s_{kj} \quad (4.1)$$

$$\text{s.t.} \quad \sum_{k \in K} w_{ik} \geq 1, \quad \forall i \in I, \quad (4.2)$$

$$\sum_{i \in I} w_{ik} \leq c_k, \quad \forall k \in K, \quad (4.3)$$

$$\sum_{k \in K} t_{ikj} = 1, \quad \forall j \in J, \forall i \in f(j), \quad (4.4)$$

$$t_{ikj} \leq w_{ik}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \quad (4.5)$$

$$t_{ikj} \leq s_{kj}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \quad (4.6)$$

$$t_{ikj} \in \{0, 1\}, \quad \forall k \in K, \forall j \in J, \forall i \in f(j), \quad (4.7)$$

$$w_{ik} \in \{0, 1\}, \quad \forall i \in I, \forall k \in K, \quad (4.8)$$

$$s_{kj} \in \{0, 1\}, \quad \forall k \in K, \forall j \in J. \quad (4.9)$$

The objective function is to minimize the number of shipments. Constraint set (4.2) guarantees that every SKU is assigned to at least one DC. Through (4.3), it is made sure that the capacities of the DCs are not exceeded. As stated in the literature review, Catalán and Fisher [2] model the capacity by using cardinality constraints instead of knapsack constraints. Constraint set (4.4) states that the items required by orders are shipped from exactly one DC. The formulation ensures that items can only be shipped from the DCs to which they are allocated via (4.5). (4.6) states if the demand for some items in an order are satisfied from a DC, then there must be a shipment from this DC to meet the order. Constraint sets (4.7-4.9) represent the integrality constraints for the binary decision variables.

We are using the output of this problem, which is the assortment decision for each warehouse, as an input for consolidation problems. As the objective of the consolidation problem is to minimize the total transportation cost, we have changed the objective of this assortment planning problem accordingly.

Also, to make the problem size smaller, we have assumed that the orders are assigned to some demand zones, aggregated the demand for each zone, and modified the constraints accordingly. We have defined M as the set of demand zones and dem_{im} as the number of orders including SKU i and coming from demand zone m . For this chapter, $f(m)$ refers to the set of SKUs in orders coming from demand zone m . Similarly, fixed and variable shipment cost parameters are redefined as costs of sending units from warehouses to demand zones. Binary decision variables decide if the aggregated demand of a demand zone (instead of a single order) will be satisfied from a warehouse. The final benchmark model for the assortment planning is as follows.

$$\min \sum_{m \in M} \sum_{k \in K} f_{km} s_{km} + \sum_{m \in M} \sum_{i \in f(m)} \sum_{k \in K} c_{km}^{ship} dem_{im} t_{ikm} \quad (4.10)$$

$$\text{s.t.} \quad \sum_{k \in K} w_{ik} \geq 1, \quad \forall i \in I, \quad (4.11)$$

$$\sum_{i \in I} w_{ik} \leq c_k, \quad \forall k \in K, \quad (4.12)$$

$$\sum_{k \in K} t_{ikm} = 1, \quad \forall m \in M, \forall i \in f(m), \quad (4.13)$$

$$t_{ikm} \leq w_{ik}, \quad \forall k \in K, \forall m \in M, \forall i \in f(m), \quad (4.14)$$

$$t_{ikm} \leq s_{km}, \quad \forall k \in K, \forall m \in M, \forall i \in f(m), \quad (4.15)$$

$$t_{ikm} \in \{0, 1\}, \quad \forall k \in K, \forall m \in M, \forall i \in f(m), \quad (4.16)$$

$$w_{ik} \in \{0, 1\}, \quad \forall i \in I, \forall k \in K, \quad (4.17)$$

$$s_{km} \in \{0, 1\}, \quad \forall k \in K, \forall m \in M. \quad (4.18)$$

4.2.2 Consolidation Model Provided by Zhang et al. [3]

Zhang et al. [3] formulates the consolidation problem for each order as a multi-commodity network flow problem. Although their formulation combines the consolidation problem and the packing problem, we are only interested in the consolidation part of the model which is given below.

Let $G = (V, A')$ be a directed network with the set of vertices V including

warehouses and the customer, and the set of arcs A' . A' is partitioned into the transshipment arc set A and shipping arc set B . a_i and b_i represent the source and destination vertices of SKU $i \in I$ respectively. The parameter d_{iv} denotes the flow differential for vertex $v \in V$ and SKU $i \in I$:

$$d_{iv} = \begin{cases} 1, & j = a_v, \\ -1, & j = b_v, \\ 0, & \text{otherwise.} \end{cases}$$

Both transshipment and shipping costs are taken into consideration in the formulation to find the consolidation plan with the lowest possible cost for an order. In addition to the fixed and variable shipment costs defined in Chapter 3, $c_{k_1 k_2}^{tran}$ is defined as the transshipment cost of one unit SKU from warehouse k_1 to warehouse k_2 .

The decision variable s_{kj} is defined as in Chapter 3 for all $k \in K$ and $j \in J$. Additionally, the decision variable $q_{iv_1 v_2}$, decides whether SKU $i \in I$ flows from vertex $v_1 \in V$ to vertex $v_2 \in V$.

The problem that is needed to be solved for every $j \in J$ is as follows.

$$\min \sum_{i \in f(j)} \sum_{(k_1, k_2) \in A} c_{k_1 k_2}^{tran} q_{ik_1 k_2} + \sum_{(k, j) \in B} f_{kj} s_{kj} + \sum_{i \in f(j)} \sum_{(k, j) \in B} c_{kj}^{ship} q_{ikj} \quad (4.19)$$

$$\text{s.t. } q_{ik_1 k_2} \leq d_{ik_1}, \quad \forall i \in f(j), (k_1, k_2) \in A, \quad (4.20)$$

$$q_{ikj} \leq s_{kj}, \quad \forall s \in f(j), (k, j) \in B, \quad (4.21)$$

$$\sum_{(v_1, v_2) \in A'} q_{iv_1 v_2} - \sum_{(v_2, v_1) \in A'} q_{iv_2 v_1} \leq d_{iv_1}, \quad \forall i \in f(j), v_1 \in V, \quad (4.22)$$

$$q_{iv_1 v_2} \in \{0, 1\}, \quad \forall i \in f(j), (v_1, v_2) \in A', \quad (4.23)$$

$$s_{kj} \in \{0, 1\}, \quad \forall (k, j) \in B. \quad (4.24)$$

The objective of this formulation is to minimize the sum of transshipment and shipment costs. Constraint (4.20) makes sure that SKUs can only be transshipped from the warehouses to which they are allocated. (4.21) states that there must

be a shipment from a warehouse to the customer to be able to satisfy the demand for SKUs in the order from this warehouse. Constraint set (4.22) represents the flow balance constraints for the vertices. Constraint sets (4.23) and (4.24) are the domain constraints for the decision variables.

We also solve the consolidation problems for each demand zone instead of each order. So, the final benchmark model for each demand zone $m \in M$ is as follows.

$$\min \sum_{i \in f(m)} \sum_{(k_1, k_2) \in A} c_{k_1 k_2}^{tran} dem_{im} q_{ik_1 k_2} + \sum_{(k, m) \in B} f_{km} s_{km} + \sum_{i \in f(m)} \sum_{(k, m) \in B} c_{km}^{ship} dem_{im} q_{ikm} \quad (4.25)$$

$$\text{s.t. } q_{ik_1 k_2} \leq d_{ik_1}, \quad \forall i \in f(m), (k_1, k_2) \in A, \quad (4.26)$$

$$q_{ikm} \leq s_{km}, \quad \forall i \in f(m), (k, m) \in B, \quad (4.27)$$

$$\sum_{(v_1, v_2) \in A'} q_{iv_1 v_2} - \sum_{(v_2, v_1) \in A'} q_{iv_2 v_1} \leq d_{iv_1}, \quad \forall i \in f(m), v_1 \in V, \quad (4.28)$$

$$q_{iv_1 v_2} \in \{0, 1\}, \quad \forall i \in f(m), (v_1, v_2) \in A', \quad (4.29)$$

$$s_{km} \in \{0, 1\}, \quad \forall (k, m) \in B. \quad (4.30)$$

4.3 Proposed Model

A formulation combining assortment allocation and consolidation can be developed. In addition to the already defined variables, let the binary variables $h_{imk_1 k_2}$ to decide if SKU i in orders coming from demand zone m is transshipped from warehouse k_1 to warehouse k_2 .

$$\min \sum_{m \in M} \sum_{i \in f(m)} c_{k_1 k_2}^{tran} h_{imk_1 k_2} + \sum_{m \in M} \sum_{k \in K} f_{km} s_{km} + \sum_{m \in M} \sum_{i \in f(m)} \sum_{k \in K} c_{km}^{ship} t_{ikm} \quad (4.31)$$

$$\text{s.t.} \quad \sum_{k \in K} w_{ik} \geq 1, \quad \forall i \in I, \quad (4.32)$$

$$\sum_{i \in I} w_{ik} \leq c_k, \quad \forall k \in K, \quad (4.33)$$

$$\sum_{k \in K} t_{ikm} = 1 \quad \forall m \in M, \forall i \in f(m), \quad (4.34)$$

$$t_{ikm} \leq s_{km}, \quad \forall m \in M, i \in f(m), k \in K, \quad (4.35)$$

$$h_{imk_1 k_2} \leq w_{ik_1}, \quad \forall j \in J, \forall i \in f(m), k_2 \in K \quad (4.36)$$

$$\sum_{k_1 \in K} h_{imk_1 k_2} - \sum_{k_1 \in K} h_{imk_2 k_1} - t_{ik_2 m} \geq -w_{ik_2}, \quad \forall m \in M, \forall i \in f(m), k_2 \in K, \quad (4.37)$$

$$t_{ikm} \in \{0, 1\}, \quad \forall m \in M, \forall i \in f(m), k \in K, \quad (4.38)$$

$$w_{ik} \in \{0, 1\}, \quad \forall i \in I, k \in K, \quad (4.39)$$

$$s_{km} \in \{0, 1\}, \quad \forall k \in K, m \in M, \quad (4.40)$$

$$h_{imk_1 k_2} \in \{0, 1\}, \quad \forall m \in M, \forall i \in f(m), k_1, k_2 \in K. \quad (4.41)$$

Similar to the formulation developed by Zhang et al. [3], the objective of this formulation is to minimize the total transshipment and shipment costs. (4.32) makes sure that every SKU is assigned to at least one warehouse. (4.33) is the capacity constraint for the warehouses. (4.34) states that the demand for any SKU coming from a demand zone has to be satisfied from exactly one warehouse. Constraint set (4.35) ensures that there is a shipment from a warehouse to satisfy orders coming from a demand zone if the demand for an SKU in these orders is satisfied from this warehouse. (4.36) states that for an SKU to be transshipped from a warehouse, it has to be allocated to this warehouse. Constraint set (4.37) represents the flow balance constraints for the warehouses. Constraint sets (4.38-4.41) are the integrality constraints for the binary variables.

4.4 Data Generation

To be able to evaluate the effect of solving the assortment planning and consolidation problems together, we have generated some random instances. After generating the demand data as described in Chapter 3, we have assigned each order to a demand zone. Then, we have randomly assigned coordinates between 0 and 100 to each demand zone and warehouse. We have set the transshipment costs between warehouses to the Euclidean distances between them. Similarly, the fixed shipment costs are assumed as the Euclidean distances between warehouses and demand zones, and the variable shipment costs are one-tenth of fixed shipment costs.

4.5 Numerical Experiments

An extensive numerical study has been conducted to understand whether there is room for improvement and it is possible to reduce the total transportation cost when the assortment planning and consolidation problems are solved together instead consecutively. The experiments have been performed on a Linux OS with Intel Core i7-3820 3.6 GHz processor. The models have been coded using Python and solved via Gurobi 9.1.1. [18]. For each instance the time limit for models is set to 3600 seconds. To make sure the consistency each run having different parameters has been repeated 5 times with different order instances (Detailed results can be found in the appendix for each instance.).

At first, it is assumed that the products can be assigned to more than one warehouse and the capacities of the warehouses are randomly set in a way that the multi-assignment is possible.

For each problem size, the columns of Table 4.1 represent the number of warehouses ($|K|$), the number of zones ($|M|$), the number of orders ($|J|$), the number of SKUs ($|I|$), average value of the total capacity, the number of instances for which the assortment planning problem is solved optimally within

the time limit, the number of instances for which the integrated problem is solved optimally within the time limit, the number instances for which the assortment problem gives a solution with the same objective function value as the solution given by the integrated problem, the number of instances for which the integrated problem gives a better solution than solving assortment planning and consolidation problems consecutively, and the improvement made through integrating assortment planning and consolidation problems in percentage respectively.



$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
3	6	7500	75	142	5	4	5	0	-
			93	179.6	5	5	5	0	-
			112	237.8	4	4	5	0	-
			150	322	4	3	5	0	-
		10000	100	179.8	3	3	5	0	-
			125	242.6	4	4	5	0	-
			150	324.8	5	5	5	0	-
			200	331.8	4	4	5	0	-
		12500	125	254.4	5	5	5	0	-
			156	294.4	3	3	5	0	-
			187	418.4	4	4	5	0	-
			250	512	5	5	4	1	0.018
	9	7500	75	132.2	5	5	5	0	-
			93	206.6	4	5	5	0	-
			112	239	5	5	3	1	0.009
			150	314.6	4	4	4	1	0.123
		10000	100	192	5	5	5	0	-
			125	250.4	4	5	5	0	-
			150	310	3	3	5	0	-
			200	358.2	4	4	4	1	0.026

Table 4.1: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		12500	125	265.6	5	5	5	0	-
			156	341.2	4	3	2	2	0.024
			187	449.4	5	5	4	1	0.017
			250	468.2	3	4	5	0	-
	12	7500	75	150.8	5	5	3	0	-
			93	228	4	5	3	2	0.047
			112	244.8	5	5	4	1	0.035
			150	313.8	5	5	3	2	0.011
		10000	100	203.2	5	5	4	1	0.001
			125	248.2	5	3	4	1	0.032
			150	318.6	5	5	5	0	-
			200	349.8	4	4	4	1	0.014
		12500	125	237	5	4	5	0	-
			156	343.4	5	5	4	0	-
			187	343.8	4	4	5	0	-
			250	438.2	3	4	5	0	-
	4	7500	75	205	5	5	4	0	-
			93	267.8	5	4	4	1	0.011
			112	266.8	4	4	4	0	-
			150	402.6	5	4	5	0	-

Table 4.1 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		10000	100	268	4	5	5	0	-
			125	289.8	5	5	4	1	0.007
			150	420.8	5	5	4	1	0.013
			200	584.2	4	3	3	1	0.008
		12500	125	304.4	4	4	5	0	-
			156	380.2	4	4	4	1	0.006
			187	446.4	4	4	5	0	-
			250	620.6	3	4	4	1	0.015
		7500	75	191.6	5	5	4	1	0.008
			93	257	5	5	5	0	-
			112	312	5	4	3	2	0.012
			150	378.2	5	5	3	1	0.057
		10000	100	278.4	5	5	4	1	0.020
			125	338.2	4	3	4	1	0.014
			150	406.8	5	4	5	0	-
			200	490.4	3	3	5	0	-
		12500	125	293.6	3	5	5	0	-
			156	406.2	4	4	5	0	-
			187	418.4	5	5	4	1	0.001
			250	642.2	3	3	4	0	-

Table 4.1 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
5	16	7500	75	206.4	5	5	3	1	0.031
			93	233.6	4	5	4	1	0.016
			112	309.6	5	4	3	1	0.043
			150	448.2	5	5	3	2	0.056
		10000	100	252.2	5	5	5	0	-
			125	297.6	5	5	4	1	0.041
			150	387.8	4	4	5	0	-
			200	493.4	3	4	5	0	-
		12500	125	321.2	5	3	4	1	0.032
			156	384.2	3	3	4	1	0.044
			187	446.4	3	3	3	2	0.003
			250	619.4	4	3	4	1	0.000
		7500	75	245.6	5	5	4	0	-
			93	284.8	4	4	4	1	0.008
			112	316	4	5	5	0	-
			150	490.4	5	4	4	0	-
		10000	100	302.2	5	5	3	1	0.041
			125	447.4	5	5	4	1	0.006
			150	488	5	5	3	1	0.002
			200	620.8	3	4	4	1	0.024

Table 4.1 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		12500	125	368.6	4	4	4	1	0.000
			156	520.2	4	4	5	0	-
			187	613.4	5	5	5	0	-
			250	692.2	4	4	3	0	-
	15	7500	75	244.6	5	5	3	2	0.133
			93	272	5	5	4	1	0.009
			112	352.2	5	5	3	2	0.034
			150	452	5	4	4	1	0.058
		10000	100	269.6	4	5	4	0	-
			125	316.4	4	4	5	0	-
			150	476.8	5	4	2	3	0.024
			200	621.4	4	4	3	2	0.012
		12500	125	384.6	4	5	4	1	0.083
			156	491	3	5	5	1	0.004
			187	536.4	4	5	4	1	0.016
			250	840.4	4	4	2	2	0.011
	20	7500	75	236.6	4	4	2	3	0.054
			93	297.4	5	5	5	0	-
			112	347.4	5	5	2	3	0.012
			150	459.4	5	3	3	1	0.047

Table 4.1 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. total capacity	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		10000	100	295.8	5	5	4	1	0.006
			125	375	3	4	4	1	0.000
			150	461.4	3	4	4	1	0.003
			200	591.4	3	5	5	0	-
		12500	125	343.4	5	5	5	0	-
			156	446.8	4	2	3	2	0.014
			187	606	4	4	2	3	0.016
			250	794.4	4	2	5	0	-

Table 4.1 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (multi-assignment)

Among 540 instances, in 76 of them, it is possible to reduce the transportation cost by combining assortment planning and consolidation problems in a single model instead of solving them consecutively.

Also, among 540 instances, the benchmark model used for assortment planning can find a solution with the same value of the solution given by the combined model for 444 of them.

It is important to note that no correlation between any parameter and the event that combined model gives a better solution than consecutively solving two problems is observed.

By turning constraints (4.11), (4.28), (4.37) into equations, we have modified our problems so that the SKUs can be assigned to a single warehouse. We have also changed the capacities of the warehouses in way that the total capacity equals to the number of SKUs. The numerical results for single-assignment are as follows.

For each problem size, the columns of Table 4.2 represent the number of warehouses ($|K|$), the number of zones ($|M|$), the number of orders ($|J|$), the number of SKUs ($|I|$), the number of instances for which the assortment planning problem is solved optimally within the time limit, the number of instances for which the integrated problem is solved optimally within the time limit, the number instances for which the assortment problem gives a solution with the same objective function value as the solution given by the integrated problem, the number of instances for which the integrated problem gives a better solution than solving assortment planning and consolidation problems consecutively, and the improvement made through integrating assortment planning and consolidation problems in percentage respectively.

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
3	6	7500	75	5	5	5	0	-
			93	5	5	5	0	-
			112	5	4	5	0	-
			150	5	5	4	1	0.002
		10000	100	5	5	5	0	-
			125	4	2	5	0	-
			150	5	4	5	0	-
			200	5	4	5	0	-
		12500	125	5	4	5	0	-
			156	5	5	5	0	-
			187	5	4	4	0	-
			250	5	3	5	0	-
	9	7500	75	5	5	4	0	-
			93	5	3	5	0	-
			112	5	5	4	1	0.176
			150	5	5	5	0	-
		10000	100	5	4	5	0	-
			125	5	4	5	0	-
			150	5	4	4	0	-
			200	5	4	4	1	0.032

Table 4.2: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		12500	125	5	3	5	0	-
			156	3	4	5	0	-
			187	5	4	5	0	-
			250	4	4	4	1	0.011
	12	7500	75	5	4	4	0	-
			93	5	5	5	0	-
			112	5	3	5	0	-
			150	5	5	5	0	-
		10000	100	5	5	5	0	-
			125	4	3	5	0	-
			150	5	5	3	2	0.018
			200	5	4	3	2	0.019
		12500	125	5	4	4	0	-
			156	5	3	5	0	-
			187	4	4	5	0	-
			250	4	2	4	0	-
	4	7500	75	5	5	4	0	-
			93	5	5	3	2	0.094
			112	5	5	4	0	-
			150	5	5	5	0	-

Table 4.2 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		10000	100	5	5	5	0	-
			125	4	5	2	3	0.034
			150	4	4	3	1	0.118
			200	5	5	4	1	0.006
		12500	125	4	3	2	1	0.045
			156	5	4	5	0	-
			187	5	4	5	0	-
			250	5	5	3	2	0.020
		7500	75	5	5	2	3	0.024
			93	5	5	4	1	0.143
			112	5	5	5	0	-
			150	5	5	3	2	0.085
		10000	100	5	5	1	3	0.077
			125	3	4	3	1	0.083
			150	5	5	4	1	0.018
			200	5	4	3	2	0.041
		12500	125	5	4	5	0	-
			156	5	5	4	1	0.013
			187	5	3	3	2	0.009
			250	3	4	3	1	0.002

Table 4.2 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)		
	16	7500	75	5	5	3	2	0.083		
			93	5	4	0	3	0.059		
			112	5	5	4	1	0.397		
			150	4	5	3	2	0.004		
		10000	100	5	5	5	0	-		
			125	5	5	5	0	-		
			150	5	5	3	1	0.036		
			200	5	5	5	0	-		
		12500	125	5	4	2	3	0.075		
			156	5	5	3	1	0.129		
			187	5	5	3	1	0.136		
			250	4	5	4	1	0.030		
		5	10	7500	75	5	5	1	3	0.055
					93	5	4	2	3	0.068
					112	5	5	1	2	0.167
					150	5	5	2	3	0.066
10000	100			5	5	1	4	0.035		
	125			5	5	2	3	0.020		
	150			5	5	2	1	0.074		
	200			5	4	5	0	-		

Table 4.2 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
	15	12500	125	5	5	2	1	0.026
			156	5	4	3	2	0.087
			187	4	4	3	1	0.016
			250	5	5	2	2	0.028
		7500	75	5	5	4	1	0.075
			93	5	4	2	3	0.022
			112	5	5	0	4	0.061
			150	5	3	1	4	0.058
		10000	100	5	5	2	2	0.010
			125	5	5	2	3	0.092
			150	4	5	2	2	0.015
			200	4	4	1	2	0.108
		12500	125	5	5	2	2	0.021
			156	4	3	3	0	-
			187	5	5	5	0	-
			250	5	2	2	3	0.073
	20	7500	75	5	4	2	1	0.003
			93	5	3	2	2	0.057
			112	5	0	1	4	0.067
			150	5	1	0	4	0.092

Table 4.2 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

$ K $	$ M $	$ J $	$ I $	# of instances assortment problem solved optimally	# of instances integrated problem solved optimally	# of instances assortment problem = integrated problem	# of instances integrated problem > consecutive problems	improvement through integration (%)
		10000	100	5	5	1	1	0.006
			125	5	3	2	2	0.143
			150	5	3	2	2	0.125
			200	4	2	4	1	0.076
		12500	125	5	0	1	4	0.152
			156	4	2	0	3	0.067
			187	5	2	1	3	0.054
			250	2	4	4	0	-

Table 4.2 Continued: Solving Assortment Planning and Consolidation Problems Consecutively vs. Combined (single-assignment)

Among 540 instances, combining two problems gives a better solution than solving them consecutively for 128 of them. As the problem size gets larger, the combined model becomes more advantageous.

Additionally, among 540 instances, the benchmark model used for assortment planning gives a solution with the same value as the solution given by the combined model for 363 instances. As the problem size gets larger, the number of instances for which the assortment planning model gives a same value solution as the combined model decreases.

Again, no correlation is observed between any parameter and the event that combined model is more advantageous.

4.6 Conclusion

Although the assortment planning cannot totally eliminate the split order problem as stated by Zhang et al. [7], considering the transportation costs while making assortment planning decisions can decrease the cost significantly. In some cases, it may even eliminate the need for consolidation.

Also, even when the transportation costs are considered in assortment planning, consecutively solving assortment planning and consolidation problems may not give the best solution. The solution can be improved through combining these two problems in a single model for a non-ignorable portion of instances. This improvement can be greater for bigger instances.

Chapter 5

Conclusions and Future Work

In this chapter, a brief summary of this study is provided and possible future research directions are discussed.

5.1 Summary

In the first part of this thesis, the split order problem is addressed for a single warehouse. A model from the literature is chosen as a benchmark model and the special structure of the optimal solution of its LP relaxation is proven. This structure is extended for some other related problems.

Also, a mathematical justification for a heuristic algorithm from the literature is provided and a dynamic version of this algorithm is developed. In addition, three other heuristic algorithms based on the LP relaxation of the benchmark model is suggested.

Through numerical experiments, it is observed that the difficulty of the problem depends on some parameters and some instances of the problem are more challenging than the others. The proportion between the number of SKUs and the number of orders, the proportion between the capacity of the warehouse

and the number of SKUs, and the average number of SKUs in orders are some of the parameters affecting the difficulty of the problem.

According to the results, the suggested heuristic algorithms come up with good quality solutions. For some challenging instances, they may even find a better solution than the solution given by the benchmark model in 3600 seconds. Especially, the Dynamic LP Relaxation Algorithm both gives similar or better results than the benchmark model in reasonable times for challenging instances.

Although some valid inequalities are suggested for the problem, it is observed that they do not facilitate solving the problem.

In the second part of the problem, the effect of combining assortment allocation and consolidation problems instead of solving them separately is analysed via numerical experiments. According to the results, it is possible to reduce the total transportation cost in this manner especially for some bigger and challenging instances. This is the case even if the shipment costs are taken into consideration in the assortment planning while solving two problems consecutively.

5.2 Future Work

As discussed in Chapter 3, the heuristic algorithms that we have developed for a single warehouse give good enough solutions in reasonable times compared to the benchmark model. As a further study, these algorithms can be extended for the multi-warehouse problem.

Additionally, our numerical experiments have shown that combining assortment planning and consolidation problems in a single model can decrease the total transportation cost. However, the runtime of this model gets quite longer as the problem size gets bigger. For this reason, some heuristic algorithms can be developed to solve these two problems concurrently.

Lastly, the capacity of the warehouse(s) can be modeled by a knapsack

constraint instead of a cardinality constraint. In other words, the model can determine the optimum number of each SKU to be assigned to the warehouse or warehouses to minimize the number of split orders.



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Appendix A

Detailed Results of Numerical Experiments in Chapter 3

no	$ J $	$ I $	c	lpval	objval	objbound	gap	gentime	runtime	nodenum	iternum
1	7500	75	7	38590.42623	20087	20087	0	1.45	174.99	47	305088
			18	99232.52459	61302	65354	0.066098985	1.49	3600.00	4902	11804640
			37	203977.9672	173111	173111	0	1.47	354.56	3426	1674656
			56	308723.4098	307420	307420	0	1.50	5.24	1	8465
2			7	38423.21212	17546	17546	0	1.52	394.32	273	1101525
			18	98802.54545	56748	61728	0.087756397	1.52	3600.01	3225	9287093
			37	203094.1212	162206	168020	0.03584331	1.48	3600.01	7216	12791417
			56	307385.697	296033	296033	0	1.52	10.53	1	12426
3			7	42351.6383	25307	25307	0	1.49	106.63	17	113016
			18	108904.2128	75327	75327	0	1.45	1531.75	9270	11680046
			37	223858.6596	202602	202602	0	1.48	88.07	1546	513392
			56	333508	333508	333508	0	1.49	0.44	0	1608
4			7	39265.08772	20581	20581	0	1.49	179.74	33	226557
			18	100967.3684	64070	66349	0.03557047	1.49	3600.01	19427	23044217
			37	207544.0351	189202	189202	0	1.52	16.85	1	16894
			56	314120.7018	313467	313467	0	1.48	4.70	1	8525
5			7	40680.62963	22323	22323	0	1.49	165.68	39	263645
			18	104607.3333	68566	70486	0.028002217	1.49	3600.01	6563	14906894
			37	215026.1852	191432	191432	0	1.50	114.82	960	568520
			56	324952	324920	324920	0	1.50	0.80	1	3689

Table A.1: Solution Parameters of the Benchmark Model for Each Generated Instance

6	93	9	41554.54286	20260	20260	0	1.51	802.38	1410	3772332
		23	106194.9429	62374	72520	0.16266393	1.51	3600.00	1094	6853337
		46	212389.8857	182274	189039	0.037114454	1.47	3600.01	7367	12228186
		69	318584.8286	318342	318342	0	1.51	2.92	1	7461
7		9	39793.31507	19494	19494	0	1.53	389.77	331	1034653
		23	101694.0274	60098	66266	0.102632367	1.53	3600.00	1956	11219654
		46	203388.0548	167913	180047	0.072263613	1.49	3600.00	6755	12578588
		69	305082.0822	304675	304675	0	1.54	3.03	1	6227
8		9	39948.63158	21346	21346	0	1.51	211.74	236	602497
		23	102090.9474	66352	68338	0.029931276	1.48	3600.01	4899	15932260
		46	204181.8947	174594	174594	0	1.49	645.92	4758	2904855
		69	306272.8421	302957	302957	0	1.51	7.30	1	10812
9		9	40245.80769	19163	19163	0	1.48	340.63	141	752717
		23	102850.3974	59262	64674	0.091323276	1.52	3600.00	1560	10482180
		46	205700.7949	160185	171475	0.070481006	1.53	3600.03	1926	7411914
		69	308551.1923	299090	299090	0	1.54	10.36	1	14098
10		9	40986.9	20450	20450	0	1.53	612.26	436	1891362
		23	104744.3	64164	70340	0.096253351	1.49	3600.01	2660	14966666
		46	209488.6	179716	184540	0.026842351	1.49	3600.01	4615	10978037
		69	314232.9	314140	314140	0	1.53	1.85	1	6043

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

11		112	11	41783.4321	23524	23524	0	1.44	182.89	121	435876
			28	106357.8272	70957	77880	0.097566132	1.46	3600.01	1937	9639053
			56	212715.6543	191525	191525	0	1.43	559.89	3156	2293746
			84	318766	318766	318766	0	1.46	0.72	0	4285
12		11	41496.15556	19778	19778	0	1.49	1236.51	2131	5421114	
		28	105626.5778	60506	71244	0.177470003	1.49	3600.01	1360	9073700	
		56	211253.1556	176211	183974.8359	0.044059882	1.47	3600.02	2453	8261338	
		84	316879.7333	313409	313409	0	1.50	10.88	1	13679	
13		11	44463.83333	26763	26763	0	1.47	154.93	123	243106	
		28	113180.6667	80514	83457	0.036552649	1.43	3600.00	4233	12611511	
		56	226361.3333	204801	204801	0	1.46	442.40	2039	1826637	
		84	337448	337448	337448	0	1.44	0.36	0	1156	
14		11	40975.90588	23508	23508	0	1.46	186.75	53	284607	
		28	104302.3059	69042	75879	0.099026679	1.47	3600.02	2355	9745475	
		56	208604.6118	184440	184440	0	1.43	2795.34	12621	11157465	
		84	312906.9176	312809	312809	0	1.44	2.04	1	5445	
15		11	40245.66292	20115	20115	0	1.48	2763.63	9953	17636376	
		28	102443.5056	63671	71615	0.124766377	1.49	3600.02	1879	8447636	
		56	204887.0112	178742	178742	0	1.49	2929.50	7133	9270334	
		84	307330.5169	306943	306943	0	1.48	2.13	1	6944	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

16	150	15	42242.24299	24345	24345	0	1.43	3087.71	9745	18461475
		37	104197.5327	70220	81764	0.164397608	1.48	3600.00	2596	9165978
		75	211211.215	192735	192735	0	1.48	3493.97	14013	11149829
		112	314653	314635	314635	0	1.45	0.67	1	3617
17		15	42976.75676	24202	24202	0	1.45	982.18	1971	4282561
		37	106009.3333	67217	80625	0.199473348	1.49	3600.02	2132	9515980
		75	214883.7838	187491	197096	0.051229126	1.45	3600.00	7388	10569789
		112	320827	320820	320820	0	1.45	0.79	1	4728
18		15	41720.30973	26298	26298	0	1.46	469.36	2161	2676348
		37	102910.0973	73100	82028	0.122134063	1.48	3600.01	3863	10638996
		75	208601.5487	190750	190750	0	1.43	1562.01	5752	5027724
		112	311511.646	311510	311510	0	1.43	1.19	1	4661
19		15	44405.49505	26191	27300	0.04234279	1.45	3600.01	7649	20124914
		37	109533.5545	77271	89327	0.156022311	1.46	3600.01	3149	9401430
		75	222027.4752	210321	210321	0	1.43	692.10	3770	2205771
		112	328416	328416	328416	0	1.42	0.44	0	2432
20		15	42579.54545	26285	26285	0	1.46	1180.84	5561	7434156
		37	105029.5455	75021	83978	0.119393237	1.48	3600.02	4502	15351759
		75	212897.7273	197108	197108	0	1.43	409.24	3433	1724283
		112	317773	317773	317773	0	1.48	0.55	0	3503

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

21	10000	100	10	56224.49275	30326	30326	0	1.92	996.96	1122	3065501
			25	140561.2319	95262	101485	0.065325103	1.94	3600.02	2086	6901325
			50	281122.4638	256174	256174	0	1.93	248.34	728	750885
			75	420120	420120	420120	0	1.93	1.29	0	6561
22			10	55956.13333	29628	29628	0	1.98	443.13	45	551198
			25	139890.3333	90116	97519	0.082149674	2.00	3600.01	1237	7916054
			50	279780.6667	253521	253521	0	2.01	152.25	920	495822
			75	419671	419671	419671	0	1.96	1.50	0	6834
23			10	57115.69444	35789	35789	0	1.95	221.91	176	419496
			25	142789.2361	108278	108278	0	1.96	1214.84	1160	2597328
			50	285578.4722	262258	262258	0	1.91	125.43	697	406451
			75	426572.5	426563	426563	0	1.91	0.72	1	1816
24			10	57727.76119	30329	30329	0	1.98	1611.46	1746	5555333
			25	144319.403	94121	103577	0.100466421	1.97	3600.01	1454	7962856
			50	288638.806	264182	264182	0	1.93	246.80	914	680996
			75	428965	428965	428965	0	1.94	1.07	0	4980
25			10	57345.31646	35315	35315	0	1.92	262.30	23	230794
			25	143363.2911	103185	106390	0.031060716	1.90	3600.02	3181	8600530
			50	286726.5823	262184	262184	0	1.91	178.03	551	478545
			75	430089.8734	429924	429924	0	1.93	2.53	1	7176

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

26		125	12	54808.91566	30885	31779	0.02894609	1.92	3600.01	4785	12136541
			31	141589.6988	101174	109690	0.084171823	1.93	3600.01	1584	7329942
			62	283179.3976	266041	266041	0	1.92	86.88	644	275819
			93	422124	422124	422124	0	1.92	1.01	0	5692
27		12	51963.96226	23476	24460	0.041915147	1.97	3600.01	1835	10146430	
		31	134240.2358	71667	89038	0.242384919	1.99	3600.01	527	5171910	
		62	268480.4717	206105	233920	0.134955484	1.99	3600.00	1085	3217950	
		93	402720.7075	391518	391518	0	1.98	21.06	1	17315	
28		12	54083.22581	29372	29372	0	1.96	702.78	467	1558020	
		31	139715	91587	100945	0.102176073	1.96	3600.02	1024	6032343	
		62	279430	250509	250509	0	1.99	706.71	1959	1729849	
		93	419145	419145	419145	0	2.02	1.14	0	5814	
29		12	57338.10989	31413	32356	0.030019419	1.97	3600.01	5169	13719745	
		31	148123.4505	103079	112538	0.091764569	1.97	3600.02	1213	5747296	
		62	296246.9011	275643	275643	0	1.96	142.59	402	334905	
		93	443869	443869	443869	0	1.97	0.97	0	5634	
30		12	55197.46667	31233	31233	0	1.99	593.99	346	1307677	
		31	142593.4556	97287	107302.9974	0.102953091	1.99	3600.03	1124	5155610	
		62	285186.9111	262635	262635	0	1.99	379.09	1300	985477	
		93	427641	427641	427641	0	1.93	1.00	0	5203	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

31	150	15	55995.66372	31405	32076	0.021366025	1.94	3600.00	5638	13991064
		37	138122.6372	92046	105163	0.142504835	2.01	3600.01	1108	5713485
		75	279978.3186	254331	257970.9074	0.014311694	1.96	3600.01	4385	7669931
		112	418100.9558	418087	418087	0	1.96	2.06	1	6627
32		15	56145.87379	32247	32247	0	2.01	1429.91	1079	3837783
		37	138493.1553	91637	105771	0.154239008	2.02	3600.02	1041	6048196
		75	280729.3689	249656	262133.8643	0.04998023	1.95	3600.00	3126	7022362
		112	418598	418598	418598	0	1.96	0.92	0	4669
33		15	57213.61607	30387	31842	0.047882318	1.95	3600.00	2624	11726721
		37	141126.9196	92017	105983	0.151776302	1.95	3600.02	1037	6200465
		75	286068.0804	255210	269572	0.056275224	2.01	3600.02	3013	7037030
		112	427195	427195	427195	0	1.97	1.32	0	7271
34		15	56469.02542	27674	29627	0.070571656	1.97	3600.01	1127	6618395
		37	139290.2627	81466	100341	0.231691749	1.98	3600.01	809	5736629
		75	282345.1271	237886	257895	0.084111717	1.98	3600.01	2253	6631440
		112	421635.3898	419905	419905	0	1.98	12.28	1	13325
35		15	57153.13636	30854	32113	0.040805082	1.96	3600.01	3529	12302501
		37	140977.7364	91965	106551	0.158603817	1.95	3600.01	1058	5862681
		75	285765.6818	257052	266217	0.035654265	1.96	3600.00	3186	7280363
		112	426633	426605	426605	0	2.01	1.42	1	6500

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

36		200	20	61203.52	39989	41343	0.033859311	1.92	3600.01	3907	12228997
			50	153008.8	114033	132525	0.162163584	1.93	3600.00	2498	6778454
			100	306017.6	295399	295399	0	1.91	1373.26	7391	4135632
			150	448723	448723	448723	0	1.95	0.40	0	1059
37		20	58530.07194	36814	39360	0.069158472	1.96	3600.01	3189	11046744	
		50	146325.1799	108399	125727.9455	0.159862596	1.96	3600.00	1981	6990155	
		100	292650.3597	271216	279417	0.030237892	1.99	3600.01	13048	10028216	
		150	437233	437233	437233	0	1.98	0.66	0	3601	
38		20	58442.42857	33979	37555	0.105241473	2.00	3600.00	1769	8059916	
		50	146106.0714	101490	121884	0.200945906	1.98	3600.03	1459	6350690	
		100	292212.1429	268212	276790	0.031982163	1.98	3600.02	7768	7699878	
		150	435719	435719	435719	0	2.00	0.66	0	3209	
39		20	57174.02597	35082	37283	0.062738726	1.98	3600.00	2856	10374054	
		50	142935.0649	99722	119440	0.197729689	1.99	3600.01	1770	6910390	
		100	285870.1299	253105	268502	0.060832461	1.99	3600.01	4283	7112411	
		150	428805.1948	428044	428044	0	1.93	24.15	147	47131	
40		20	56343.47222	33956	36462	0.07380139	1.96	3600.00	2909	10509031	
		50	140858.6806	98671	118435	0.200302014	1.94	3600.00	1469	6219080	
		100	281717.3611	264169	264169	0	1.95	2959.96	6251	6280517	
		150	421672	421672	421672	0	1.94	0.73	0	4112	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

41	12500	125	12	67087.53191	34259	35076	0.023847748	2.38	3600.01	1629	8329299
			31	173309.4574	107784	123653	0.147229644	2.39	3600.01	461	4500743
			62	346618.9149	302765	322511	0.065218899	2.39	3600.03	2216	5735265
			93	519928.3723	519789	519789	0	2.46	5.07	1	9940
42		12	70503.81176	39649	39649	0	2.43	2899.57	3545	8825166	
		31	182134.8471	130207	139897	0.074419962	2.44	3600.02	1038	4367188	
		62	364269.6941	342300	342300	0	2.44	199.03	393	348125	
		93	541319	541319	541319	0	2.43	1.44	0	6562	
43		12	65642.60377	31047	32341	0.041678745	2.42	3600.00	1072	7219619	
		31	169576.7264	100806	115846.9824	0.149207214	2.43	3600.01	305	3783535	
		62	339153.4528	278626	300088	0.077027987	2.41	3600.01	1079	2438225	
		93	508730.1792	501068	501068	0	2.48	27.75	1	19610	
44		12	69960.84706	36046	37969	0.053348499	2.46	3600.01	1158	7665145	
		31	180732.1882	114547	135629	0.184046723	2.47	3600.01	435	4657142	
		62	361464.3765	333929	333929	0	2.45	903.90	2771	2024730	
		93	538880	538880	538880	0	2.42	1.47	0	5902	
45		12	69008.6747	44014	44014	0	2.40	1026.33	1652	3487372	
		31	178272.4096	138406	145662	0.052425473	2.39	3600.01	1391	5198930	
		62	356544.8193	342703	342703	0	2.41	23.89	1	18535	
		93	533317	533317	533317	0	2.41	1.41	0	6331	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

46		156	15	69062.5	39194	40483	0.032887687	2.45	3600.00	2001	9480917
			39	179562.5	121207	137884	0.137591063	2.40	3600.00	767	4912827
			78	359125	324166	336142.725	0.036946271	2.45	3600.01	2324	5198694
			117	538517	538517	538517	0	2.46	1.48	0	7280
47			15	64597.10526	30712	32764	0.066814275	2.50	3600.01	1190	7130576
			39	167952.4737	96562	117773	0.219661979	2.45	3600.00	176	2991276
			78	335904.9474	269565	300058.7428	0.11312204	2.46	3600.02	476	3785981
			117	503857.4211	494316	494316	0	2.52	44.98	1	27373
48			15	70946.78571	41983	41983	0	2.45	853.24	375	1514116
			39	184461.6429	125671	141716	0.127674643	2.45	3600.03	726	6840403
			78	368923.2857	335858	346682.7187	0.032230046	2.39	3600.01	3300	6083191
			117	550578	550578	550578	0	2.38	1.11	0	4792
49			15	66540.38136	41941	41941	0	2.42	414.43	121	470090
			39	173004.9915	126033	135959	0.078757151	2.44	3600.02	1147	5686261
			78	346009.9831	317426	317426	0	2.39	1334.45	3901	3003035
			117	519014.9746	518977	518977	0	2.37	2.41	1	7565
50			15	67355.59322	34295	35530	0.03601108	2.48	3600.00	1337	7450694
			39	175124.5424	106489	128396	0.205720779	2.50	3600.03	526	5554888
			78	350249.0847	307130	326137	0.061885846	2.43	3600.01	2245	5180872
			117	525373.6271	525281	525281	0	2.45	5.15	1	10038

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

51		187	18	68333.41259	39061	39894	0.021325619	2.48	3600.00	2088	8469546
			46	174629.8322	115174	135117	0.1731554	2.49	3600.00	648	4127686
			93	353055.965	310810	330022	0.061812683	2.46	3600.01	1155	3115033
			140	531482.0979	531332	531332	0	2.42	4.15	1	8768
52		18	68294.17021	33421	36700	0.098111966	2.48	3600.01	751	6138735	
		46	174529.5461	102093	130886	0.282027171	2.49	3600.01	424	4170756	
		93	352853.2128	298463	326783	0.094886133	2.51	3600.01	1080	2377241	
		140	531176.8794	530914	530914	0	2.44	14.19	1	12238	
53		18	69132.08759	35208	39500	0.121904113	2.45	3600.01	1089	6374305	
		46	176670.8905	110984	137665	0.240404022	2.47	3600.01	579	4281881	
		93	357182.4526	313808	336275	0.071594733	2.46	3600.01	1696	5324041	
		140	537237	537215	537215	0	2.47	1.54	1	7205	
54		18	66830.375	39754	41004	0.031443377	2.45	3600.00	2951	9757582	
		46	170788.7361	116718	135179	0.158167549	2.41	3600.01	768	4785966	
		93	345290.2708	308759	323877	0.048963755	2.41	3600.03	2060	6497168	
		140	519791.8056	519403	519403	0	2.42	6.42	1	9232	
55		18	66945.42857	37242	39379	0.057381451	2.49	3600.04	2081	7793420	
		46	171082.7619	113620	133827	0.17784721	2.49	3600.05	680	4527732	
		93	345884.7143	302775	322399	0.064813806	2.49	3600.02	1150	2880361	
		140	520686.6667	518290	518290	0	2.43	17.00	1	16723	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

56		250	25	71565.37356	42993	47783	0.111413486	2.46	3600.01	1179	6857364
			62	177482.1264	122637	154275	0.25798087	2.46	3600.03	1110	4687474
			125	357826.8678	328077	344158.7913	0.049018344	2.47	3600.00	4784	5871211
			187	534044	534044	534044	0	2.41	0.94	0	5096
57		25	70826.08696	45124	50215	0.112822445	2.36	3600.02	1426	6590744	
		62	175648.6957	129256	154860	0.198087516	2.44	3600.02	1479	5496616	
		125	354130.4348	329692	340254.9946	0.032038978	2.45	3600.02	2985	5049404	
		187	529763	529763	529763	0	2.42	1.00	0	5720	
58		25	70197.71574	37923	43845	0.156158532	2.48	3600.00	940	6647285	
		62	174090.335	109365	145316	0.328724912	2.45	3600.01	900	4409329	
		125	350988.5787	305057	333500	0.093238313	2.42	3600.01	1802	3998334	
		187	525078.9137	523693	523693	0	2.49	189.56	1214	457519	
59		25	70160.64103	42259	46870	0.109112852	2.47	3600.01	1031	5984887	
		62	173998.3897	118642	150141	0.265496199	2.42	3600.04	1143	4394618	
		125	350803.2051	313188	337458	0.077493391	2.41	3600.03	2201	5471026	
		187	524801.5949	524345	524345	0	2.48	5.52	1	9719	
60		25	70754.38144	37368	44367	0.187299294	2.49	3600.01	955	6878659	
		62	175470.866	108428	147772	0.362858302	2.49	3600.01	804	4226455	
		125	353771.9072	304859	337107.9182	0.105783061	2.49	3600.02	2108	5825969	
		187	529242.7732	528692	528692	0	2.44	41.84	359	92168	

Table A.1 Continued: Solution Parameters of the Benchmark Model for Each Generated Instance

no	c	GRval	GRtime	DGRval	DGRtime	LPRval	LPRtime	DLPRval	DLPRtime	ADLPRval	ADLPRtime
1	7	19573	23.15	18431	39.35	19573	24.96	18431	41.20	20087	119.87
	18	60709	23.61	60709	62.10	60709	25.37	60709	63.12	60568	184.17
	37	172101	23.49	172653	87.88	172101	25.20	172653	84.67	173111	227.78
	56	306961	23.77	307360	99.66	306961	25.52	307360	55.49	307420	150.20
2	7	16333	23.73	16333	40.23	16333	25.38	16333	42.41	17546	130.79
	18	54668	23.51	56078	62.71	54668	25.24	56078	65.29	56748	215.82
	37	162186	23.81	161804	90.02	162186	25.41	161804	92.80	162206	296.18
	56	295632	23.74	295632	103.44	295632	25.64	296033	93.90	296033	308.51
3	7	23783	23.52	24768	39.40	23783	25.11	24768	39.96	25307	82.82
	18	75002	23.53	74796	60.15	75002	25.27	74796	59.69	75181	134.11
	37	202335	23.53	202567	83.20	202335	25.26	202567	79.22	202567	170.22
	56	332518	23.55	333508	92.16	333508	25.39	333508	30.30	333508	23.23
4	7	19682	23.80	19805	40.53	19682	25.50	19805	41.99	20581	115.60
	18	64070	23.69	64070	63.21	64070	25.63	64070	62.90	63565	170.04
	37	189178	23.75	189200	87.04	189178	25.30	189200	71.47	189200	188.17
	56	313303	23.79	313467	98.19	313303	25.44	313467	52.38	313467	139.40
5	7	21875	23.49	21875	39.26	21875	25.26	21875	40.72	22323	98.72
	18	68024	23.60	67964	61.12	68024	25.45	67964	61.67	68490	161.14
	37	191432	23.69	191171	85.59	191432	25.46	191432	80.74	191432	205.06
	56	324920	23.77	324862	95.79	324920	25.53	324920	30.79	324920	23.53

Table A.2: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

6	9	19102	28.84	19102	50.13	19102	30.80	19102	51.73	20260	175.78
	23	60499	28.82	60326	78.35	60499	30.74	60326	79.07	62099	290.30
	46	182100	28.86	182274	108.17	182100	30.45	182274	101.98	182274	329.44
	69	317585	28.84	318342	121.18	317585	30.71	318342	51.33	318342	145.62
7	9	17609	28.64	17478	50.50	17609	30.64	17478	52.19	19494	183.74
	23	59706	28.77	59548	79.52	59706	30.70	59548	79.89	59846	302.66
	46	167673	28.71	167798	111.31	167673	30.77	167798	109.14	167714	383.46
	69	303649	29.01	304675	127.12	303649	31.04	304675	49.60	304675	132.66
8	9	19236	28.82	19143	50.28	19236	30.71	19143	51.85	21346	190.55
	23	64588	28.71	64995	78.50	64588	30.54	64995	79.57	66352	303.17
	46	173076	28.68	174330	108.26	173076	30.50	174330	104.65	174594	366.85
	69	301582	29.02	302757	122.71	301582	30.49	302851	83.31	302873	324.56
9	9	16991	28.84	17172	50.54	16991	30.61	17172	52.47	19079	191.47
	23	58054	28.64	58009	79.77	58054	30.77	58009	82.51	59262	342.78
	46	159289	28.84	159711	113.23	159289	30.85	159711	115.26	160012	455.06
	69	298725	28.96	299090	129.09	298725	30.76	299090	105.37	299090	432.06
10	9	18898	28.65	18898	50.23	18898	30.54	18898	51.68	20247	165.97
	23	63775	28.93	63775	78.74	63775	30.70	63775	79.40	63836	275.12
	46	179059	28.80	179716	108.90	179059	30.60	179716	104.41	179633	350.46
	69	313262	28.78	314140	122.14	314140	30.87	314140	44.37	314140	80.20

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

11	11	20143	32.54	20174	58.42	20143	34.87	20174	59.17	23524	222.11
	28	70002	33.26	70002	91.19	70002	34.77	70002	89.56	70957	371.05
	56	190795	33.26	191313	125.83	190795	35.10	191313	115.93	191525	464.72
	84	318175	33.25	318583	139.55	318766	34.94	318766	42.32	318766	32.64
12	11	18544	33.30	18544	60.48	18544	35.27	18544	62.01	19721	299.01
	28	59057	33.13	59013	95.03	59057	34.87	59013	96.84	60214	496.23
	56	174832	33.50	175822	133.06	174832	35.28	176211	127.75	176156	591.44
	84	312273	33.46	313409	148.74	312273	35.05	313409	103.38	313409	530.72
13	11	26635	33.05	25381	58.44	26635	34.81	25381	59.17	26583	207.36
	28	80149	33.30	79708	89.95	80149	34.93	79708	89.75	80514	354.46
	56	204300	33.01	204801	123.30	204300	34.86	204801	114.67	204801	449.82
	84	336772	33.24	337448	136.24	337448	35.04	337448	42.62	337448	32.58
14	11	22484	33.00	22484	58.24	22484	34.79	22484	59.58	23508	229.70
	28	67499	33.87	68250	91.39	67499	34.83	68250	90.69	68979	402.60
	56	183878	33.29	183878	126.09	183878	35.07	183878	117.09	184440	494.43
	84	312779	33.04	312809	141.18	312779	34.95	312809	54.41	312809	144.14
15	11	18429	33.02	19508	58.93	18429	34.94	19508	60.34	19995	247.86
	28	62692	33.11	63188	92.68	62692	35.03	63188	92.87	63757	422.81
	56	178588	33.15	178738	128.63	178588	34.93	178738	120.71	178738	520.51
	84	306943	33.09	306943	144.09	306943	34.88	306943	54.88	306943	151.31

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

16	15	22619	44.03	23261	78.92	22619	45.75	23261	79.68	23669	404.32
	37	68251	43.51	69373	120.49	68251	45.40	69373	118.65	70165	700.18
	75	192187	44.08	192682	168.98	192187	45.78	192682	147.04	192735	826.18
	112	314432	43.95	314542	187.08	314621	45.67	314621	56.56	314635	43.43
17	15	22007	43.96	22007	79.51	22007	45.71	22007	80.45	24202	417.25
	37	64724	43.98	64889	122.40	64724	45.71	64889	121.36	67217	753.58
	75	187243	43.42	187027	168.83	187243	45.39	187388	156.31	187546	953.24
	112	320594	44.03	320814	188.88	320814	45.69	320814	56.43	320814	43.33
18	15	25924	43.82	25440	77.71	25924	45.60	25440	78.50	26269	397.03
	37	71216	44.59	71838	118.59	71216	45.82	71838	116.79	73066	707.85
	75	190026	43.61	190680	164.32	190026	45.51	190680	145.67	190677	858.72
	112	310467	43.77	311510	183.39	311346	45.69	311510	58.27	311510	78.24
19	15	23716	44.06	24370	77.68	23716	45.85	24370	77.95	26053	354.54
	37	75232	44.08	74729	118.40	75232	45.64	74729	114.28	76376	594.42
	75	209345	43.67	210308	161.91	209345	45.69	210308	135.35	210321	671.03
	112	327964	43.99	328416	178.25	328416	45.75	328416	55.87	328416	43.24
20	15	24958	43.73	25097	77.92	24958	45.57	25097	78.59	26200	390.21
	37	72477	43.70	73164	119.19	72477	45.63	73164	117.26	74211	680.75
	75	196367	43.86	196773	163.73	196367	45.52	197108	140.01	197108	786.45
	112	317312	43.90	317773	183.50	317773	45.86	317773	56.12	317773	43.58

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

21	10	29010	40.09	29454	71.71	29010	42.98	29454	72.99	30257	217.75
	25	93385	40.91	94724	110.39	93385	42.95	94724	109.24	95262	384.54
	50	255675	40.57	256041	151.30	255675	42.91	256041	140.36	256174	484.34
	75	419990	40.79	420120	168.48	420120	42.87	420120	52.25	420120	40.02
22	10	28020	40.78	27777	72.31	28020	42.86	27777	73.96	29445	260.83
	25	88313	41.15	88277	113.18	88313	43.37	88277	111.66	89924	407.96
	50	252621	41.05	253396	156.70	252621	43.52	253396	137.13	253431	480.91
	75	419671	41.40	419671	175.24	419671	43.58	419671	52.86	419671	40.62
23	10	34212	40.84	34619	70.76	34212	43.08	34619	72.33	35789	213.81
	25	105873	40.81	106307	107.92	105873	43.34	106307	107.91	108278	385.47
	50	260170	41.08	261943	147.05	260170	43.30	261943	138.30	262258	472.99
	75	426494	41.13	426563	164.01	426494	43.35	426494	53.32	426563	40.53
24	10	27643	41.19	27643	72.28	27643	43.49	27643	72.96	30329	220.29
	25	93093	41.32	93761	110.82	93093	43.47	93761	109.11	91893	375.53
	50	263490	40.84	264123	151.48	263490	43.40	264123	138.23	264182	443.90
	75	428625	41.14	428965	167.93	428965	43.37	428965	52.74	428965	40.47
25	10	34276	40.82	34276	71.13	34276	43.12	34276	72.69	35024	224.17
	25	102391	40.42	102515	108.54	102391	43.11	102515	108.66	102921	401.95
	50	260023	40.56	261408	148.84	260023	43.12	261791	135.36	262184	495.58
	75	429913	40.99	429913	167.58	429913	43.32	429913	56.56	429913	92.72

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

26	12	29038	49.06	29038	85.64	29038	51.86	29038	86.38	30821	315.76
	31	96421	49.84	98564	133.83	96421	52.38	98564	129.84	101171	565.74
	62	264946	49.94	265636	181.88	264946	52.15	265636	150.16	266020	641.88
	93	421537	50.12	422124	202.29	422124	52.08	422124	63.80	422124	49.37
27	12	21008	50.19	21008	89.76	21008	53.13	21008	92.58	23476	501.05
	31	70110	50.60	69814	142.88	70110	53.27	69814	145.66	71707	908.62
	62	204722	50.15	204032	201.56	204722	52.78	204032	204.35	206105	1173.94
	93	390835	50.53	391518	230.62	390835	53.03	391518	212.23	391518	1270.95
28	12	27013	50.38	27522	87.74	27013	52.48	27522	89.71	29242	396.42
	31	91145	50.35	91145	136.92	91145	52.59	91145	136.60	91587	713.97
	62	250217	50.38	250340	188.16	250217	52.85	250340	174.56	250509	819.38
	93	419145	50.49	419145	211.07	419145	52.69	419145	64.74	419145	50.15
29	12	29929	50.06	29929	86.54	29929	52.99	29929	88.05	31186	336.76
	31	101484	50.48	102337	133.75	101484	52.77	102337	132.29	103079	610.76
	62	275332	50.63	275642	181.57	275332	52.72	275642	154.46	275643	688.05
	93	443742	50.31	443869	200.99	443869	53.02	443869	64.84	443869	50.14
30	12	29464	50.29	29464	87.27	29464	52.74	29464	88.76	30827	367.50
	31	95761	50.52	95761	136.14	95761	52.87	95761	133.76	97053	630.56
	62	262635	50.42	262635	184.77	262635	52.50	262530	166.83	262530	727.20
	93	427080	50.53	427641	205.43	427641	52.61	427641	64.36	427641	49.61

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

31	15	28775	59.81	29610	106.98	28775	62.19	29610	108.69	30937	557.31
	37	90657	59.75	90470	163.87	90657	62.30	90470	162.27	91332	1011.58
	75	253941	60.13	254220	227.99	253941	62.74	254298	200.45	254298	1195.71
	112	417071	60.44	417949	254.57	417376	63.08	418087	79.72	418087	109.02
32	15	29691	59.88	29691	107.37	29691	62.47	29691	107.88	32036	519.42
	37	90543	59.74	90224	163.85	90543	62.68	90224	160.58	91438	938.80
	75	249116	60.39	248645	228.62	249116	62.87	249573	211.54	249656	1177.11
	112	417844	60.43	418598	253.09	418598	62.75	418598	76.40	418598	59.70
33	15	27097	59.98	27086	107.33	27097	62.69	27086	109.33	30094	570.90
	37	91161	60.28	91819	165.09	91161	63.22	91819	161.76	92012	983.47
	75	252784	60.35	254677	227.93	252784	62.96	254677	212.71	255210	1163.54
	112	425814	60.54	427174	252.22	427195	63.04	427195	77.11	427195	60.04
34	15	25141	60.10	25451	107.91	25141	62.05	25451	110.67	27502	639.24
	37	79035	60.53	80228	166.04	79035	62.63	80228	167.35	80653	1148.89
	75	236091	60.18	237018	233.68	236091	62.77	237018	225.56	237775	1423.81
	112	419011	60.28	419905	261.00	419011	62.68	419905	138.64	419905	875.39
35	15	28690	60.16	28516	107.39	28690	62.46	28516	108.80	30717	541.87
	37	89364	60.45	89760	164.29	89364	62.63	89760	163.14	91594	982.54
	75	256941	60.39	256941	227.94	256941	62.61	257052	201.51	257052	1188.91
	112	426413	59.91	426561	253.08	426605	62.72	426554	77.54	426605	59.87

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

36	20	37796	78.83	37973	138.51	37796	80.92	37973	137.81	39743	764.11
	50	112081	79.42	111609	210.82	112081	81.89	111609	202.34	114024	1341.15
	100	294620	79.65	295118	282.83	294620	81.77	295399	220.69	295399	1487.53
	150	447842	78.56	448694	306.12	448723	80.95	448723	100.39	448723	77.69
37	20	35189	78.90	35019	138.58	35189	81.07	35019	138.41	36703	913.82
	50	106148	79.36	106468	211.90	106148	81.54	106468	207.89	108368	1601.25
	100	269240	79.50	270936	289.10	269240	81.18	271216	257.41	270746	1933.47
	150	435902	78.89	437233	320.44	437233	81.28	437233	100.78	437233	78.84
38	20	31706	79.36	31163	141.52	31706	81.60	31163	142.09	33916	938.03
	50	99121	79.48	99739	217.22	99121	81.91	99739	213.18	100534	1693.98
	100	266365	79.55	268090	294.56	266365	81.32	268090	260.86	268212	2040.26
	150	434994	79.35	435626	327.84	435719	81.92	435719	100.70	435719	78.52
39	20	32360	78.14	32391	139.59	32360	80.88	32391	141.69	35082	1115.58
	50	96365	78.17	98132	214.55	96365	81.12	98132	213.81	99552	2003.36
	100	251158	78.60	252691	296.74	251158	80.59	253040	276.66	253105	2441.53
	150	427493	78.98	428044	334.47	427493	81.56	428044	180.73	428044	1847.28
40	20	29019	78.45	28588	140.83	29019	81.10	28588	140.90	33662	968.31
	50	94181	79.24	94293	218.49	94181	81.79	94293	211.54	98427	1715.72
	100	263844	78.35	264169	297.71	263844	81.37	264169	241.34	264169	1893.27
	150	421453	79.24	421672	330.94	421672	81.16	421672	100.56	421672	78.25

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

41	12	32264	60.67	32264	108.35	32264	64.52	32264	110.53	34259	527.74
	31	104036	61.92	104503	172.99	104036	65.17	104503	171.96	107784	910.42
	62	302231	62.16	302212	240.10	302231	65.17	302462	228.40	302765	1108.01
	93	519259	62.00	519660	269.49	519784	65.19	519784	95.93	519789	272.88
42	12	36192	62.37	36775	108.45	36192	64.89	36775	109.16	39649	445.76
	31	129026	61.90	129026	167.77	129026	64.94	129026	164.30	130207	764.73
	62	340554	62.44	341481	228.78	340554	65.14	342300	191.60	338606	865.05
	93	540251	62.66	541319	253.03	541319	65.73	541319	79.73	541319	61.56
43	12	28738	61.92	28922	110.66	28738	64.77	28922	114.55	30923	656.28
	31	98680	61.99	99492	175.84	98680	65.58	99492	179.70	100417	1226.86
	62	277347	62.26	278512	247.33	277347	65.14	278512	245.07	277514	1392.32
	93	500425	61.71	501068	278.91	500425	65.38	501068	200.94	501068	1330.11
44	12	32952	61.71	33351	109.49	32952	65.35	33351	110.25	36046	430.17
	31	113163	62.73	112519	172.30	113163	65.15	112519	167.15	113003	746.75
	62	332951	62.22	333727	234.54	332951	65.50	333727	212.14	333929	877.87
	93	538880	62.27	538880	258.70	538880	65.35	538880	79.25	538880	61.30
45	12	42441	61.58	43107	105.57	42441	64.53	43107	105.99	43944	374.60
	31	137264	61.60	138359	163.33	137264	64.86	138359	154.17	138406	645.05
	62	342334	62.27	342243	221.78	342334	65.23	342586	157.36	342644	650.30
	93	532068	62.19	533317	246.17	533317	65.55	533317	79.62	533317	61.61

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

46	15	37272	76.04	37105	134.72	37272	79.46	37105	136.44	39077	790.74
	39	118479	77.01	118289	212.17	118479	79.42	118289	209.48	120681	1404.81
	78	323078	76.26	323822	291.72	323078	79.46	324119	265.56	324166	1670.93
	117	538096	76.75	538517	327.02	538517	80.05	538517	97.96	538517	76.12
47	15	29248	77.30	29177	140.49	29248	80.90	29177	144.50	30617	1053.69
	39	93839	77.20	93809	222.90	93839	80.36	93809	227.13	96090	1912.75
	78	269227	77.51	268570	314.44	269227	80.71	268570	315.89	269523	2430.05
	117	494316	77.11	494316	357.31	494316	80.09	494316	283.00	494316	2451.42
48	15	38819	76.69	38871	135.22	38819	80.19	38871	137.87	41773	713.88
	39	123111	76.52	123549	211.37	123111	80.15	123549	208.50	125502	1304.73
	78	334383	76.51	335858	288.35	334383	79.87	335858	262.95	335858	1563.35
	117	550046	76.83	550498	317.52	550578	79.62	550578	98.04	550578	75.68
49	15	39447	76.31	37998	133.98	39447	79.68	37998	135.35	41941	747.28
	39	123865	75.99	124322	207.98	123865	79.17	124322	205.86	126033	1371.91
	78	316052	76.92	316699	288.77	316052	79.88	317339	260.03	317426	1642.62
	117	518977	76.69	518977	323.57	518977	79.70	518977	105.17	518977	273.70
50	15	32737	76.96	31433	138.70	32737	80.34	31433	140.81	34210	836.89
	39	104347	76.81	104255	218.69	104347	79.49	104255	216.83	106462	1502.10
	78	305961	77.05	306587	305.92	305961	80.24	307130	280.28	307130	1780.44
	117	524842	77.31	525037	341.35	525219	80.43	525281	119.18	525281	443.58

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

51	18	35534	90.82	36164	162.68	35534	93.95	36164	165.47	38994	1171.27
	46	112892	90.90	112657	255.71	112892	94.25	112657	254.34	115157	2181.05
	93	310059	91.38	310425	357.94	310059	94.33	310658	334.13	310810	2728.08
	140	530625	90.86	531332	397.63	530919	93.93	531332	135.95	531332	570.38
52	18	31916	90.78	31398	164.52	31916	93.87	31398	166.80	33421	1300.01
	46	98555	91.51	97884	259.81	98555	94.57	97884	260.33	101420	2329.94
	93	297335	90.66	297904	362.72	297335	94.06	298105	344.86	298463	2867.03
	140	530531	90.83	530914	406.18	530531	94.09	530914	180.33	530914	1417.73
53	18	32767	90.66	32911	162.02	32767	93.79	32911	163.18	35024	1098.70
	46	106965	90.92	107754	254.47	106965	94.74	107754	252.71	110570	2072.31
	93	312893	91.29	313109	353.01	312893	94.62	313767	320.24	313808	2581.06
	140	536407	91.42	537196	394.65	537196	94.53	537196	116.89	537215	90.65
54	18	37143	91.04	37582	161.79	37143	94.03	37582	163.63	39660	1127.65
	46	112356	92.24	113301	252.60	112356	94.41	113301	250.49	116501	2129.32
	93	307125	90.88	307822	348.38	307125	93.91	308623	322.90	308698	2619.78
	140	517949	91.40	519403	394.51	519403	94.86	519403	156.21	519403	916.74
55	18	33776	91.21	33732	163.37	33776	94.01	33732	166.18	36926	1272.68
	46	111076	90.48	110314	255.19	111076	94.54	110314	256.23	113620	2320.07
	93	301455	91.42	301995	357.36	301455	94.89	302775	332.34	302775	2869.62
	140	518047	91.55	518168	401.35	518047	95.04	518214	219.10	518270	1978.95

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

56	25	40394	120.71	40903	219.86	40394	123.74	40903	219.04	42883	2031.23
	62	119109	120.38	119157	337.81	119109	123.16	119157	328.02	122354	3564.54
	125	327081	119.91	326897	464.74	327081	123.36	327908	407.88	328000	4149.44
	187	532233	120.83	534044	516.95	534044	124.34	534044	152.98	534044	119.41
57	25	41001	120.65	42031	216.94	41001	124.52	42031	217.00	44867	2007.11
	62	126193	120.25	126953	329.30	126193	123.00	127212	318.65	129093	3493.54
	125	326494	120.81	328609	452.45	326494	123.45	328609	388.62	329692	4174.60
	187	528110	121.16	529543	504.41	529763	124.13	529763	152.98	529763	119.03
58	25	34231	121.67	34445	222.28	34231	124.45	34445	225.60	37917	2595.24
	62	106018	121.11	105624	345.42	106018	124.63	105624	345.41	108716	4619.36
	125	303411	119.60	304813	477.73	303411	123.19	305050	434.67	305054	5528.37
	187	521862	121.11	523693	536.35	522749	123.63	523693	285.01	523693	4048.07
59	25	38605	119.65	39161	218.70	38605	123.80	39161	220.42	42259	2376.47
	62	115019	120.55	116699	338.72	115019	123.86	116699	334.35	118266	4158.84
	125	311871	121.13	312273	472.93	311871	124.72	313060	432.13	313188	4991.63
	187	523200	120.84	524290	532.45	523247	124.65	524345	190.55	524345	1298.07
60	25	35132	121.28	34557	223.14	35132	125.22	34557	225.44	37335	2409.93
	62	104507	121.77	105030	346.46	104507	125.09	105030	347.38	107197	4406.57
	125	303153	121.09	304433	483.76	303153	124.63	304712	453.61	304572	5370.07
	187	528476	122.14	528661	540.81	528476	124.61	528692	282.18	528692	3836.33

Table A.2 Continued: Solution Values and Times of the Heuristic Algorithms for Each Generated Instance

no	c	DLPRval	DLPRtime	objval	objbound	gap	runtime	nodenum	iternum
1	7	18431	41.20	20038	24636	0.229464018	41.20	1	45602
	18	60709	63.12	60096	77965	0.297340921	63.13	1	43086
	37	172653	84.67	172788	183743	0.063401394	84.67	367	312016
	56	307360	55.49	307420	307420	0	5.25	1	8465
2	7	16333	42.41	17492	23079	0.319403156	42.42	1	43128
	18	56078	65.29	56154	76348	0.359618193	65.35	1	52188
	37	161804	92.80	152300	179246	0.176927118	92.80	113	218031
	56	296033	93.90	296033	296033	0	10.50	1	12426
3	7	24768	39.96	25307	27115	0.071442684	39.97	1	42860
	18	74796	59.69	74881	89011	0.188699403	59.69	5	57802
	37	202567	79.22	202602	205075.899	0.012210634	79.23	1234	471388
	56	333508	30.30	333508	333508	0	0.44	0	1608
4	7	19805	41.99	20581	25757	0.251494096	42.00	1	41790
	18	64070	62.90	62253	79320	0.274155462	62.90	1	41683
	37	189200	71.47	189202	189202	0	16.77	1	16894
	56	313467	52.38	313467	313467	0	4.71	1	8525
5	7	21875	40.72	22323	26068	0.167764189	40.72	1	40852
	18	67964	61.67	66088	84354	0.276389057	61.68	2	37352
	37	191432	80.74	189109	198685	0.050637463	80.74	474	383703
	56	324920	30.79	324920	324920	0	0.81	1	3689

Table A.3: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

6	9	19102	51.73	20092	24846	0.236611587	51.73	1	50223
	23	60326	79.07	61324	84581	0.379247929	79.08	1	59632
	46	182274	101.98	179542	194772	0.084826949	101.98	401	388516
	69	318342	51.33	318342	318342	0	2.92	1	7461
7	9	17478	52.19	19494	23471	0.204011491	52.20	1	48516
	23	59548	79.89	59248	79999	0.350239671	79.90	1	60691
	46	167798	109.14	165366	184249	0.114189132	109.14	187	312624
	69	304675	49.60	304675	304675	0	3.03	1	6227
8	9	19143	51.85	21126	24545	0.161838493	51.86	1	51569
	23	64995	79.57	64697	79521	0.229129635	79.58	1	57974
	46	174330	104.65	171606	186106	0.08449588	104.65	422	365144
	69	302851	83.31	302957	302957	0	7.29	1	10812
9	9	17172	52.47	18999	22369	0.177377757	52.51	1	48992
	23	58009	82.51	58574	77177	0.317598252	82.52	1	59594
	46	159711	115.26	148446	185031	0.246453256	115.27	195	313652
	69	299090	105.37	299090	299090	0	10.33	1	14098
10	9	18898	51.68	20450	24674	0.206552567	51.69	1	47868
	23	63775	79.40	63077	82296	0.304691092	79.48	1	58697
	46	179716	104.41	167888	190007	0.131748547	104.42	162	267611
	69	314140	44.37	314140	314140	0	1.86	1	6043

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

11	11	20174	59.17	23524	25271	0.074264581	59.18	1	55024
	28	70002	89.56	70270	84066	0.196328447	89.57	1	66905
	56	191313	115.93	188199	198328	0.05382069	115.93	411	438217
	84	318766	42.32	318766	318766	0	0.71	0	4285
12	11	18544	62.01	19725	22980	0.165019011	62.02	1	52772
	28	59013	96.84	60058	78176.97244	0.301691239	96.85	1	62178
	56	176211	127.75	174838	193442	0.106407074	127.75	278	335333
	84	313409	103.38	313409	313409	0	10.94	1	13679
13	11	25381	59.17	26763	27984	0.045622688	59.18	1	54729
	28	79708	89.75	80111	90196	0.125887831	89.75	1	65783
	56	204801	114.67	204382	212286	0.038672682	114.68	303	381813
	84	337448	42.62	337448	337448	0	0.36	0	1156
14	11	22484	59.58	23508	25263	0.074655436	59.58	1	59348
	28	68250	90.69	68430	81716	0.194154611	90.70	1	64845
	56	183878	117.09	183850	194206	0.056328529	117.09	392	433632
	84	312809	54.41	312809	312809	0	2.04	1	5445
15	11	19508	60.34	20090	22898	0.13977103	60.36	1	55608
	28	63188	92.87	63618	78510	0.234084693	92.87	1	63931
	56	178738	120.71	178738	189083	0.057878011	120.72	296	331693
	84	306943	54.88	306943	306943	0	2.12	1	6944

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

16	15	23261	79.68	24250	26541	0.094474227	79.68	1	70291
	37	69373	118.65	70206	84743	0.207062074	118.65	1	77478
	75	192682	147.04	192735	200450	0.040029055	147.04	655	524628
	112	314621	56.56	314635	314635	0	0.67	1	3617
17	15	22007	80.45	24202	25890	0.069746302	80.46	1	68489
	37	64889	121.36	67203	83675	0.245108105	121.46	1	80510
	75	187388	156.31	187234	202793	0.083099223	156.32	548	506338
	112	320814	56.43	320820	320820	0	0.79	1	4728
18	15	25440	78.50	26298	27510	0.046087155	78.51	9	98408
	37	71838	116.79	72647	85947	0.183077071	116.80	1	68696
	75	190680	145.67	190750	198441	0.04031979	145.67	731	547860
	112	311510	58.27	311510	311510	0	1.19	1	4661
19	15	24370	77.95	26044	29052	0.115496851	77.95	7	88434
	37	74729	114.28	76961	92899	0.207091904	114.28	1	79611
	75	210308	135.35	210308	213059	0.013080815	135.36	1030	568681
	112	328416	55.87	328416	328416	0	0.44	0	2432
20	15	25097	78.59	26235	28222	0.075738517	78.59	1	54054
	37	73164	117.26	74864	88167	0.177695555	117.26	1	74084
	75	197108	140.01	197108	202529	0.027502689	140.02	674	462888
	112	317773	56.12	317773	317773	0	0.55	0	3503

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

21	10	29454	72.99	30238	39887	0.319101792	72.99	1	54096
	25	94724	109.24	92757	116776	0.258945417	109.28	1	63489
	50	256041	140.36	255914	265008	0.035535375	140.37	245	356780
	75	420120	52.25	420120	420120	0	1.29	0	6561
22	10	27777	73.96	29254	37846	0.293703425	73.97	1	52704
	25	88277	111.66	86038	115765	0.345510123	111.67	1	66309
	50	253396	137.13	253420	259671	0.024666561	137.13	673	448616
	75	419671	52.86	419671	419671	0	1.49	0	6834
23	10	34619	72.33	35476	39291	0.10753749	72.34	1	53638
	25	106307	107.91	107657	118930	0.104712188	107.96	1	67489
	50	261943	138.30	262258	262258	0	125.02	697	406451
	75	426494	53.32	426563	426563	0	0.73	1	1816
24	10	27643	72.96	30186	37750	0.250579739	72.97	1	52809
	25	93761	109.11	93279	119730	0.283568649	109.19	1	66357
	50	264123	138.23	263534	269701	0.023401155	138.23	331	356879
	75	428965	52.74	428965	428965	0	1.07	0	4980
25	10	34276	72.69	35159	38502	0.09508234	72.69	1	55160
	25	102515	108.66	103036	118379	0.148909119	108.67	1	63950
	50	261791	135.36	261120	268107	0.026757813	135.37	258	311135
	75	429913	56.56	429924	429924	0	2.52	1	7176

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

26	12	29038	86.38	30847	35340	0.145654359	86.39	1	61098
	31	98564	129.84	101016	117486	0.163043478	129.85	1	76355
	62	265636	150.16	266041	266041	0	86.98	644	275819
	93	422124	63.80	422124	422124	0	1.00	0	5692
27	12	21008	92.58	23242	29590	0.273126237	92.61	1	61497
	31	69814	145.66	71610	101985	0.424172602	145.66	1	68813
	62	204032	204.35	171350	244784	0.428561424	204.35	27	216371
	93	391518	212.23	391518	391518	0	21.21	1	17315
28	12	27522	89.71	29372	34247	0.165974397	89.72	1	60283
	31	91145	136.60	88615	115128	0.299193139	136.60	1	84450
	62	250340	174.56	250509	260633	0.040413718	174.56	286	327787
	93	419145	64.74	419145	419145	0	1.13	0	5814
29	12	29929	88.05	31247	36584	0.170800397	88.06	1	64476
	31	102337	132.29	101491	123247	0.214363835	132.29	1	77039
	62	275642	154.46	275643	275643	0	142.75	402	334905
	93	443869	64.84	443869	443869	0	0.98	0	5634
30	12	29464	88.76	31199	35056	0.123625757	88.76	1	64001
	31	95761	133.76	95987	117594	0.225103399	133.76	1	78124
	62	262530	166.83	262635	269318.7619	0.025448862	166.83	490	436457
	93	427641	64.36	427641	427641	0	1.00	0	5203

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

31	15	29610	108.69	31405	36020	0.146951122	108.71	1	74982
	37	90470	162.27	91473	111059	0.214117827	162.27	1	85253
	75	254298	200.45	254104	263267	0.036060038	200.46	272	387847
	112	418087	79.72	418087	418087	0	2.06	1	6627
32	15	29691	107.88	32138	36150	0.124836642	107.88	1	71387
	37	90224	160.58	91206	114436	0.254698156	160.58	1	87616
	75	249573	211.54	248609	265733	0.068879244	211.54	276	464033
	112	418598	76.40	418598	418598	0	0.91	0	4669
33	15	27086	109.33	30387	36421	0.198571758	109.34	1	67194
	37	91819	161.76	91903	116240	0.264811812	161.76	1	88522
	75	254677	212.71	254516	269572	0.059155417	212.72	262	416914
	112	427195	77.11	427195	427195	0	1.32	0	7271
34	15	25451	110.67	27674	33653	0.216051167	110.68	1	70363
	37	80228	167.35	81157	109239.9565	0.346032461	167.37	1	93586
	75	237018	225.56	227482	262949	0.155911237	225.57	70	292296
	112	419905	138.64	419905	419905	0	12.29	1	13325
35	15	28516	108.80	30852	35161	0.139666796	108.80	1	71197
	37	89760	163.14	91457	113375	0.239653608	163.25	1	86811
	75	257052	201.51	251866	269478.74	0.06992901	201.52	242	375635
	112	426554	77.54	426605	426605	0	1.41	1	6500

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

36	20	37973	137.81	39989	43351	0.08407312	137.81	1	66650
	50	111609	202.34	113780	135966	0.194990332	202.34	3	108341
	100	295399	220.69	295399	298343.7895	0.009968854	220.71	1023	598491
	150	448723	100.39	448723	448723	0	0.39	0	1059
37	20	35019	138.41	36814	40906	0.111153366	138.42	1	86709
	50	106468	207.89	108399	128544	0.185841198	207.89	1	87053
	100	271216	257.41	271216	282163	0.040362663	257.41	865	608826
	150	437233	100.78	437233	437233	0	0.65	0	3601
38	20	31163	142.09	33979	38986.99641	0.147385044	142.10	1	85638
	50	99739	213.18	101378	125334	0.236303735	213.19	1	111263
	100	268090	260.86	268212	280451.9706	0.045635432	260.86	926	659999
	150	435719	100.70	435719	435719	0	0.66	0	3209
39	20	32391	141.69	35082	38480	0.096858788	141.70	1	83826
	50	98132	213.81	99722	122206	0.225466798	213.82	2	89387
	100	253040	276.66	252169	273868	0.086049435	276.66	610	687802
	150	428044	180.73	428044	428044	0	24.16	147	47131
40	20	28588	140.90	33956	37802	0.113264224	140.90	5	85921
	50	94293	211.54	98671	121776	0.234162013	211.54	1	105131
	100	264169	241.34	264169	270943.7692	0.025645588	241.35	699	615947
	150	421672	100.56	421672	421672	0	0.73	0	4112

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

41	12	32264	110.53	34150	46921	0.373967789	110.54	1	63276
	31	104503	171.96	106663	143696	0.34719631	171.98	1	74281
	62	302462	228.40	298592	323980	0.085025721	228.40	246	403060
	93	519784	95.93	519789	519789	0	5.05	1	9940
42	12	36775	109.16	39649	49964	0.260157885	109.17	1	67717
	31	129026	164.30	128050	157336.8629	0.228714275	164.31	1	78735
	62	342300	191.60	342300	344721	0.007072743	191.61	340	336266
	93	541319	79.73	541319	541319	0	1.45	0	6562
43	12	28922	114.55	30921	44139	0.427476472	114.56	1	61228
	31	99492	179.70	98640	136775.9075	0.386617067	179.74	1	78470
	62	278512	245.07	265965	311985.9412	0.173033825	245.07	33	206403
	93	501068	200.94	501068	501068	0	27.76	1	19610
44	12	33351	110.25	36046	48709	0.351301115	110.26	1	62667
	31	112519	167.15	110665	155842	0.408232052	167.16	1	85020
	62	333727	212.14	333929	343857.7813	0.029733211	212.14	359	414134
	93	538880	79.25	538880	538880	0	1.47	0	5902
45	12	43107	105.99	43965	48253	0.097532128	105.99	1	67222
	31	138359	154.17	136192	154743	0.136212112	154.17	1	76918
	62	342586	157.36	342703	342703	0	23.92	1	18535
	93	533317	79.62	533317	533317	0	1.42	0	6331

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

46	15	37105	136.44	39194	45484	0.160483748	136.44	1	73392
	39	118289	209.48	121207	148887	0.228369649	209.51	1	92906
	78	324119	265.56	323639	340260	0.051356604	265.57	222	378987
	117	538517	97.96	538517	538517	0	1.48	0	7280
47	15	29177	144.50	30691	41513	0.352611515	144.51	1	71661
	39	93809	227.13	94079	136595	0.451918069	227.14	1	90183
	78	268570	315.89	252572	309916	0.22704021	315.96	1	103593
	117	494316	283.00	494316	494316	0	45.27	1	27373
48	15	38871	137.87	41713	49034	0.175508834	137.87	1	77241
	39	123549	208.50	125207	152942	0.221513174	208.51	1	91089
	78	335858	262.95	334079	350570	0.049362576	262.95	293	431613
	117	550578	98.04	550578	550578	0	1.10	0	4792
49	15	37998	135.35	41797	45246.97114	0.082541119	135.36	1	77983
	39	124322	205.86	125603	144030	0.146708279	205.87	1	91403
	78	317339	260.03	317116	328015	0.034369127	260.03	314	399008
	117	518977	105.17	518977	518977	0	2.41	1	7565
50	15	31433	140.81	34295	41669	0.215016766	140.86	1	71788
	39	104255	216.83	106214	143505	0.351093076	216.84	1	92854
	78	307130	280.28	296537	330006	0.112866185	280.28	131	355549
	117	525281	119.18	525281	525281	0	5.14	1	10038

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

51	18	36164	165.47	39061	45924	0.175699547	165.48	1	86772
	46	112657	254.34	113837	142115	0.248407811	254.43	1	117681
	93	310658	334.13	291365	334670.8462	0.148630914	334.14	103	377231
	140	531332	135.95	531332	531332	0	4.13	1	8768
52	18	31398	166.80	33195	43566	0.31242657	166.81	1	84550
	46	97884	260.33	100535	144198	0.43430646	260.34	1	112634
	93	298105	344.86	277874	333232	0.19921979	344.87	55	299017
	140	530914	180.33	530914	530914	0	14.16	1	12238
53	18	32911	163.18	35122	43710	0.244519105	163.26	1	86037
	46	107754	252.71	110622	146324	0.322738696	252.79	1	109056
	93	313767	320.24	296451	339291	0.144509548	320.24	108	403071
	140	537196	116.89	537215	537215	0	1.53	1	7205
54	18	37582	163.63	39754	43859	0.103260049	163.68	1	84507
	46	113301	250.49	116408	139924	0.202013607	250.50	1	105570
	93	308623	322.90	302891	328357	0.08407645	322.90	265	482329
	140	519403	156.21	519403	519403	0	6.43	1	9232
55	18	33732	166.18	36865	44545	0.208327682	166.19	1	91312
	46	110314	256.23	113284	141265	0.246998694	256.28	1	109254
	93	302775	332.34	274992	327795	0.192016495	332.35	84	342424
	140	518214	219.10	518290	518290	0	16.99	1	16723

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

56	25	40903	219.04	42993	49423	0.149559231	219.05	1	106140
	62	119157	328.02	122480	157604	0.286773351	328.03	1	135021
	125	327908	407.88	328077	346773	0.056986622	407.89	757	642431
	187	534044	152.98	534044	534044	0	0.94	0	5096
57	25	42031	217.00	45124	51429	0.139726088	217.00	1	104747
	62	127212	318.65	128998	157854	0.223693391	318.65	1	117653
	125	328609	388.62	327969	342787	0.045181099	388.63	597	671627
	187	529763	152.98	529763	529763	0	0.99	0	5720
58	25	34445	225.60	37923	46255	0.219708356	225.60	1	100113
	62	105624	345.41	108867	148893	0.367659621	345.43	1	127232
	125	305050	434.67	293487	336328	0.145972394	434.67	411	577909
	187	523693	285.01	523693	523693	0	189.48	1214	457519
59	25	39161	220.42	42259	48788	0.154499633	220.43	1	107093
	62	116699	334.35	118635	153184	0.291220972	334.39	1	132257
	125	313060	432.13	312347	338314	0.083135103	432.14	586	691779
	187	524345	190.55	524345	524345	0	5.53	1	9719
60	25	34557	225.44	37368	46373	0.240981589	225.45	1	97322
	62	105030	347.38	108359	150977	0.39330374	347.39	1	131752
	125	304712	453.61	290191	339668.9474	0.170501316	453.61	482	677635
	187	528692	282.18	528692	528692	0	41.82	359	92168

Table A.3 Continued: Comparison of Dynamic LP Relaxation Algorithm and the Benchmark Model Each Generated Instance

$ J $	$ I $	c	avg. Degree	lpval	objval w/out ineq.	gap w/out ineq.	runtime w/out ineq.	objval w/ ineq.	gap w/ ineq.	runtime w/ ineq.
12500	125	12	2.484	66514.041	34441	0	3478.34	34441	0	542.82
		31	2.484	171827.939	109529	0.124624529	3600.01	109529	0.154780926	3600.06
		62	2.484	343655.878	303733	0.038346179	3600.03	303733	0	2586.21
		93	2.484	515483.816	514610	0	9.47	514610	0	7.78
		12	2.610	64326.291	29567	0.030033483	3600.00	29567	0.021950147	3600.06
		31	2.610	166176.252	93015	0.1864753	3600.02	93015	0.298618502	3600.06
		62	2.610	332352.505	262991	0.139472453	3600.01	262991	0.132430387	3600.05
		93	2.610	498528.757	491481	0	248.11	491481	0	19.53
		12	2.421	68132.455	37572	0.02472586	3600.01	37572	0	332.87
		31	2.421	176008.841	116762	0.126059848	3600.00	116762	0.138692383	3600.07
		62	2.421	352017.682	320374	0	1270.20	320374	0	912.84
		93	2.421	527504.000	527504	0	1.46	527504	0	4.15
		12	2.446	65811.093	33192	0.045191612	3600.00	33192.00007	0	1916.85
		31	2.446	170011.990	106341	0.148879548	3600.00	106341	0.175275764	3600.06
		62	2.446	340023.979	297309	0.056798748	3600.01	297309	0.015939306	3600.06
		93	2.446	510035.969	508242	0	13.62	508242	0	9.18
		12	2.455	67686.607	35637	0	1701.23	35637	0	359.55
		31	2.455	174857.067	107522	0.167212291	3600.01	107586	0.209227966	3600.07
		62	2.455	349714.135	302286	0.079930926	3600.02	302286	0.029266986	3600.07
		93	2.455	523848.000	523848	0	1.51	523848	0	4.00

Table A.4: Effect of Dominance Cuts for Each Instance

156	15	2.377	67646.870	39325	0.031837254	3600.01	39325	0.033846154	3600.05
	39	2.377	175881.861	122097	0.130748503	3600.01	122097	0.2068765	3600.12
	78	2.377	351763.722	329206	0	340.57	329206	0	583.21
	117	2.377	527195.000	527195	0	1.06	527195	0	3.39
	15	2.468	66791.441	36250	0.011255172	3600.00	36250	0.041517241	3600.05
	39	2.468	173657.746	112661	0.144255776	3600.01	112666	0.258622832	3600.05
	78	2.468	347315.492	301865	0.0754998	3600.02	301865	0.071641297	3600.05
	117	2.468	520973.237	520798	0	8.94	520798	0	5.09
	15	2.434	68479.375	37600	0.019654255	3600.00	37600	0.038643617	3600.05
	39	2.434	178046.375	115375	0.155943662	3600.02	115375	0.26575948	3600.05
	78	2.434	356092.750	314914	0.0514996	3600.00	314914	0.056744806	3600.06
	117	2.434	534139.125	533856	0	4.82	533856	0	6.53
	15	2.443	66311.102	36163	0	2927.95	36163	0.049304538	3600.05
	39	2.443	172408.866	112258	0.13428851	3600.02	112258	0.245264589	3600.05
	78	2.443	344817.732	297898	0.051214544	3600.04	297898	0.072877035	3600.05
	117	2.443	517226.598	511775	0	33.99	511775	0	20.26
	15	2.548	66950.702	33428	0.068056719	3600.00	33551	0.096121676	3600.05
	39	2.548	174071.826	106523	0.195262995	3600.01	106529	0.321649504	3600.05
	78	2.548	348143.653	303765	0.051266551	3600.01	303765	0.066416473	3600.05
	117	2.548	522215.479	521268	0	14.14	521268	0	20.65

Table A.4 Continued: Effect of Dominance Cuts for Each Instance

187	18	2.444	67931.757	38438	0.055283834	3600.02	38541	0.109857035	3600.04
	46	2.444	173603.378	114848	0.192619811	3600.01	114848	0.297235546	3600.05
	93	2.444	350980.743	311594	0.05661213	3600.01	311613	0.065218358	3600.04
	140	2.444	528358.108	526380	0	19.74	526380	0	12.38
	18	2.327	70206.600	43788	0.04256874	3600.01	43788	0.065132	3600.05
	46	2.327	179416.867	132281	0.136928206	3600.00	132281	0.18394176	3600.06
	93	2.327	362734.100	349895	0	1107.29	349895	0	173.55
	140	2.327	542262.000	542262	0	0.97	542262	0	3.19
	18	2.299	69388.015	43198	0.046066948	3600.00	43198	0.072225566	3600.05
	46	2.299	177324.926	132781	0.105534677	3600.04	132781	0.155383677	3600.05
	93	2.299	358504.743	333885	0.013840029	3600.01	333885	0.024834087	3600.05
	140	2.299	538688.000	538688	0	0.99	538688	0	3.27
	18	2.341	69100.835	42296	0.032958199	3600.01	42296	0.074546056	3600.04
	46	2.341	176591.022	127375	0.125950932	3600.01	127375	0.198696762	3600.04
	93	2.341	357020.978	322927	0.044451977	3600.03	322927	0.043505188	3600.06
	140	2.341	537183.000	537183	0	1.20	537183	0	3.23
	18	2.348	69102.692	41000	0.052682927	3600.03	41007	0.09886117	3600.04
	46	2.348	176595.769	126639	0.133142239	3600.01	126622	0.210145156	3600.07
	93	2.348	357030.577	334302	0.010771113	3600.00	334302	0.014498866	3600.04
	140	2.348	534950.000	534950	0	0.93	534950	0	3.11

Table A.4 Continued: Effect of Dominance Cuts for Each Instance

	250	25	2.467	69949.601	38596	0.166571665	3600.02	38596	0.328608766	3600.03
		62	2.467	173475.011	112849	0.308518463	3600.03	112849	0.397672997	3600.04
		125	2.467	349748.005	310044	0.083333333	3600.01	310044	0.08830037	3600.03
		187	2.467	523223.016	523215	0	2.64	523215	0	4.11
		25	2.465	69396.985	38672	0.178061647	3600.03	38573	0.321131361	3600.04
		62	2.465	172104.523	112298	0.30573118	3600.03	112450	0.382257506	3600.03
		125	2.465	346984.925	307772	0.073232783	3600.01	307772	0.085371292	3600.03
		187	2.465	519089.447	517693	0	14.84	517693	0	14.90
		25	2.442	70904.545	41237	0.126706598	3600.03	41237	0.260615467	3600.03
		62	2.442	175843.273	116807	0.295735701	3600.01	116807	0.379574854	3600.03
		125	2.442	354522.727	310304	0.099934258	3600.00	310304	0.106937068	3600.03
		187	2.442	530366.000	530366	0	1.07	530366	0	2.54
		25	2.456	69419.010	38981	0.159718837	3600.01	39009	0.306262657	3600.03
		62	2.456	172159.146	114857	0.27824164	3600.00	114857	0.355093725	3600.03
		125	2.456	347095.052	308044	0.073642068	3600.03	308044	0.091629767	3600.03
		187	2.456	519254.198	518618	0	25.72	518618	0	12.70
		25	2.424	69680.663	42107	0.111929133	3600.01	42107	0.238511411	3600.03
		62	2.424	172808.044	119709	0.24811	3600.03	119709	0.321137091	3600.03
		125	2.424	348403.315	318532	0.04604247	3600.03	318532	0.047838208	3600.03
		187	2.424	520750.000	520750	0	0.91	520750	0	2.41

Table A.4 Continued: Effect of Dominance Cuts for Each Instance



Appendix B

Detailed Results of Numerical Experiments in Chapter 4

$ K $	$ M $	$ J $	$ I $	total capacity	avg. degree of orders	assort. objval	assort. objbound	assort. gap	assort. gentime	assort. runtime	assort. nodenum	assort. iternum	consecutive run cost	consolid. time	total time
3	6	7500	75	133	2.472	86377.368	86377.368	0	0.16	0.08	1	1335	86377.368	0.03	0.11
				120	2.368	82722.803	82722.803	0	0.22	0.28	1	2642	82722.803	0.03	0.31
				171	2.605	60423.469	60423.469	0	0.16	0.07	1	1298	60423.469	0.03	0.10
				123	2.177	95246.018	95246.018	0	0.16	0.22	1	2605	95246.018	0.03	0.25
				163	2.525	51602.068	51602.068	0	0.16	0.04	0	1274	51602.068	0.03	0.07
			93	172	2.536	107246.460	107246.460	0	0.20	2.21	1132	15449	107246.460	0.04	2.25
				218	2.356	56103.591	56103.591	0	0.20	0.15	1	1622	56103.591	0.04	0.19
				123	2.353	73863.347	73863.347	0	0.22	0.05	0	1839	73863.347	0.04	0.09
				205	2.453	70246.725	70246.725	0	0.20	0.12	1	1597	70246.725	0.04	0.15
				180	2.266	81438.230	81438.230	0	0.20	0.14	1	1341	81438.230	0.04	0.18
			112	236	2.366	84289.453	84289.453	0	0.26	0.04	0	892	84289.453	0.04	0.08
				260	2.374	58089.381	58089.381	0	0.24	0.18	1	2150	58089.381	0.05	0.22
				301	2.516	73709.383	73709.383	0	0.26	0.28	1	1937	73709.383	0.04	0.33
				226	2.523	85319.604	85319.604	1.36E-15	0.24	0.36	1	2576	85319.604	0.04	0.41
				166	2.368	114394.550	114394.550	0	0.26	0.06	0	2275	114394.550	0.04	0.10
			150	364	2.508	86312.922	86312.922	0	0.32	0.45	1	3187	86312.922	0.06	0.51
				287	2.420	59946.543	59946.543	0	0.32	0.32	1	3917	59946.543	0.06	0.38
				365	2.426	94556.417	94556.417	0	0.32	6.30	2179	63470	94556.417	0.05	6.35
				316	2.391	63326.762	63326.762	2.64E-15	0.32	0.06	0	2431	63326.762	0.06	0.12
				278	2.479	65239.833	65239.833	0	0.32	0.08	0	2790	65239.833	0.06	0.14
		10000	100	186	2.290	104764.307	104764.307	0	0.22	0.22	1	2343	104764.307	0.05	0.27
				164	2.393	84481.095	84481.095	0	0.24	0.05	0	1890	84481.095	0.04	0.09
				181	2.490	99435.722	99435.722	1.61E-15	0.22	0.05	0	2118	99435.722	0.04	0.09
				215	2.332	80100.231	80100.231	1.45E-15	0.22	0.05	0	1923	80100.231	0.04	0.09
				153	2.436	97868.134	97868.134	0	0.24	0.11	1	1664	97868.134	0.04	0.15
			125	200	2.400	87785.132	87785.132	0	0.27	0.06	0	2332	87785.132	0.05	0.11
				222	2.408	54100.753	54100.753	0	0.27	0.36	1	3381	54100.753	0.05	0.41
				211	2.486	80110.782	80110.782	4.00E-15	0.29	0.32	1	2402	80110.782	0.05	0.37

Table B.1: Solution Parameters for Consecutive Runs (multi-assignment)

				313	2.359	72891.256	72891.256	0	0.27	0.33	1	2123	72891.256	0.05	0.38
				267	2.386	134511.383	134511.383	0	0.27	0.23	1	2417	134511.383	0.05	0.28
			150	213	2.417	96936.318	96936.318	0	0.32	0.08	0	3222	96936.318	0.06	0.13
				341	2.551	64036.554	64036.554	0	0.32	0.07	0	2369	64036.554	0.06	0.12
				341	2.471	62150.433	62150.433	0	0.32	0.07	0	2751	62150.433	0.06	0.13
				386	2.515	62543.399	62543.399	0	0.32	0.06	0	2145	62543.399	0.06	0.12
				343	2.386	118840.869	118840.869	0	0.32	0.21	1	3015	118840.869	0.06	0.27
			200	297	2.448	87989.649	87989.649	0	0.42	0.11	0	4106	87989.649	0.07	0.18
				398	2.322	36547.267	36547.267	0	0.43	0.09	0	3611	36547.267	0.07	0.16
				290	2.382	122621.713	122621.713	0	0.42	0.11	0	4125	122621.713	0.07	0.18
				342	2.464	88298.413	88298.413	0	0.45	0.13	0	4165	88298.413	0.07	0.20
				332	2.439	88849.709	88849.709	1.80E-15	0.45	0.96	1	5224	88849.709	0.07	1.03
		12500	125	249	2.419	77490.714	77490.714	0	0.27	0.15	1	2130	77490.714	0.05	0.20
				175	2.470	131173.728	131173.728	0	0.27	0.06	0	2560	131173.728	0.05	0.11
				345	2.405	74952.981	74952.981	0	0.29	0.03	0	964	74952.981	0.04	0.08
				278	2.378	129199.684	129199.684	0	0.29	0.06	0	2175	129199.684	0.05	0.11
				225	2.321	82740.208	82740.208	0	0.27	0.13	1	2697	82740.208	0.05	0.18
			156	256	2.362	127480.737	127480.737	1.14E-15	0.34	0.33	1	3290	127480.737	0.06	0.39
				347	2.504	75775.602	75775.602	0	0.34	0.07	0	2680	75775.602	0.06	0.13
				314	2.419	134834.164	134834.164	0	0.33	0.08	0	3062	134834.164	0.06	0.13
				290	2.490	105701.416	105701.416	0	0.36	0.18	1	2403	105701.416	0.06	0.24
				265	2.506	149835.939	149835.939	7.77E-16	0.36	0.07	0	3184	149835.939	0.06	0.13
			187	510	2.404	72551.373	72551.373	0	0.40	0.08	0	2685	72551.373	0.07	0.15
				455	2.375	84313.967	84313.967	0	0.43	0.09	0	2594	84313.967	0.07	0.15
				333	2.414	106388.980	106388.980	2.05E-15	0.40	0.09	0	3853	106388.980	0.07	0.15
				374	2.512	98688.177	98688.177	0	0.40	0.84	1	5652	98688.177	0.07	0.90
				420	2.239	100166.339	100166.339	0	0.43	0.22	1	2831	100166.339	0.06	0.28
			250	589	2.327	59112.350	59112.350	0	0.53	0.43	1	3019	59112.350	0.08	0.51
				448	2.427	94266.324	94266.324	0	0.56	0.13	0	4557	94266.324	0.09	0.22
				514	2.322	83829.378	83829.378	0	0.53	0.32	1	3475	83829.378	0.08	0.40
				366	2.362	119116.382	119116.382	0	0.54	0.33	1	5586	119116.382	0.09	0.42

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

9	7500	75	643	2.330	145564.469	145564.469	0	0.56	0.70	1	3403	145501.719	0.09	0.79
			133	2.472	87501.326	87501.326	0	0.24	0.19	1	2010	87501.326	0.04	0.23
			143	2.283	96331.858	96331.858	0	0.24	0.09	0	2483	96331.858	0.04	0.13
			158	2.538	48832.862	48832.862	0	0.24	0.58	1	3343	48832.862	0.05	0.63
			121	2.299	64733.312	64733.312	0	0.24	0.08	0	2373	64733.312	0.05	0.13
			106	2.371	59107.423	59107.423	0	0.24	2.67	2493	29813	59107.423	0.05	2.72
		93	250	2.501	58443.360	58443.360	3.49E-11	0.30	5.82	1202	34119	58443.360	0.05	5.87
			169	2.502	112076.766	112076.766	0	0.30	13.56	1964	106572	112076.766	0.06	13.62
			211	2.398	48894.828	48894.828	0	0.30	0.12	1	1993	48894.828	0.06	0.17
			207	2.321	57676.581	57676.581	0	0.30	0.16	1	2539	57676.581	0.06	0.21
			196	2.565	54894.708	54894.708	0	0.30	0.19	1	2536	54894.708	0.06	0.24
		112	170	2.540	79487.104	79487.104	0	0.37	1.80	1	6943	79487.104	0.07	1.87
			187	2.366	73259.606	73259.606	0	0.35	0.08	0	2933	73259.606	0.06	0.15
			266	2.420	75573.325	75573.325	0	0.35	1.36	1	4245	75573.325	0.06	1.42
			318	2.085	45954.622	45954.622	0	0.35	0.40	1	2364	45939.813	0.06	0.46
			254	2.466	91213.401	91213.401	0	0.38	0.28	1	2403	91113.872	0.07	0.34
		150	349	2.425	66983.454	66983.454	1.52E-15	0.47	0.14	0	3512	66983.454	0.08	0.23
			324	2.423	72327.615	72327.615	0	0.47	0.33	1	4072	72327.615	0.09	0.41
			281	2.425	64672.114	64672.114	0	0.47	0.86	1	5319	64672.114	0.08	0.95
			344	2.493	72251.464	72251.464	0	0.47	1.61	131	11245	72251.464	0.08	1.69
			275	2.292	63195.541	63195.541	0	0.47	0.99	1	4649	63195.541	0.09	1.07
	10000	100	183	2.220	61513.644	61513.644	0	0.32	0.08	0	2460	61513.644	0.06	0.14
			188	2.379	92161.667	92161.667	0	0.32	0.26	1	3106	92161.667	0.06	0.32
			179	2.391	98144.909	98144.909	0	0.32	0.08	0	3148	98144.909	0.06	0.14
			191	2.582	111863.973	111863.973	0	0.32	1.46	18	6239	111863.973	0.06	1.52
			219	2.392	88040.469	88040.469	0	0.32	0.19	1	2709	88040.469	0.06	0.26
		125	327	2.379	58578.067	58578.067	0	0.40	3.10	1764	61023	58578.067	0.07	3.17
			226	2.322	80202.717	80202.717	0	0.42	0.61	1	4539	80202.717	0.07	0.69
			185	2.346	80829.367	80829.367	0	0.40	2.18	51	8389	80829.367	0.07	2.25
			190	2.335	98770.769	98770.769	1.03E-15	0.42	1.82	57	8542	98770.769	0.07	1.89
			324	2.400	61676.618	61676.618	0	0.40	0.38	1	2793	61676.618	0.07	0.46

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

			150	290	2.398	109379.153	109379.153	9.31E-16	0.47	0.17	0	4655	109379.153	0.09	0.25
				369	2.539	140189.176	140189.176	0	0.47	0.09	0	2502	140189.176	0.08	0.17
				329	2.436	97461.655	97461.655	0	0.47	0.26	1	4019	97461.655	0.08	0.35
				261	2.390	126752.180	126752.180	0	0.48	0.19	0	4443	126752.180	0.09	0.28
				301	2.391	78890.988	78890.988	1.29E-15	0.48	0.08	0	2703	78890.988	0.08	0.17
			200	357	2.391	98187.295	98187.295	4.74E-15	0.63	0.16	0	5902	98187.295	0.11	0.27
				400	2.448	76380.941	76380.941	0	0.62	10.74	1737	43510	76380.941	0.11	10.85
				255	2.420	88765.713	88765.713	0	0.62	7.89	1234	34436	88765.713	0.11	7.99
				382	2.475	72894.738	72894.738	0	0.64	0.39	1	5294	72894.738	0.11	0.50
				397	2.409	92878.181	92878.181	0	0.65	0.45	1	5801	92878.181	0.11	0.56
		12500	125	237	2.370	82128.218	82128.218	0	0.40	0.09	0	2459	82128.218	0.07	0.16
				252	2.458	98236.322	98236.322	0	0.40	0.12	0	3393	98236.322	0.07	0.19
				259	2.313	111694.377	111694.377	0	0.43	0.44	1	4296	111694.377	0.07	0.51
				272	2.393	179588.892	179588.892	0	0.40	0.11	0	3462	179588.892	0.07	0.19
				308	2.258	71821.161	71821.161	0	0.42	0.10	0	2982	71821.161	0.07	0.17
			156	411	2.572	108310.817	108310.817	0	0.50	0.63	1	3631	108310.817	0.09	0.71
				350	2.339	130475.215	130475.215	0	0.52	0.32	1	4049	130475.215	0.09	0.41
				202	2.549	166026.873	166026.873	1.75E-15	0.52	4.58	1	10834	166000.785	0.09	4.67
				345	2.433	94037.149	94037.149	0	0.53	0.16	0	4003	94037.149	0.09	0.25
				398	2.450	132211.144	132211.144	0	0.52	4.35	1061	73228	132211.144	0.09	4.44
			187	461	2.295	63593.724	63593.724	0	0.59	0.41	1	4051	63593.724	0.10	0.51
				507	2.364	76894.165	76894.165	0	0.62	2.53	176	11859	76894.165	0.10	2.63
				482	2.417	76272.958	76272.958	0	0.60	0.81	1	4183	76272.958	0.10	0.91
				431	2.390	106092.133	106092.133	0	0.59	0.15	0	4516	106092.133	0.10	0.25
				366	2.428	101536.906	101536.906	0	0.62	0.37	1	4425	101536.906	0.10	0.48
			250	468	2.388	71514.421	71514.421	0	0.80	0.72	1	5508	71514.421	0.13	0.85
				450	2.440	95684.060	95684.060	0	0.78	0.18	0	6290	95684.060	0.13	0.32
				565	2.473	151641.399	151641.399	2.11E-15	0.79	0.79	1	6235	151641.399	0.13	0.92
				519	2.373	146571.843	146571.843	0	0.79	2.10	1	10553	146571.843	0.13	2.23
				339	2.396	162337.605	162337.605	5.38E-16	0.82	0.22	0	6354	162337.605	0.13	0.35
	12	7500	75	133	2.472	78043.047	78043.047	0	0.31	0.17	1	2565	78043.047	0.06	0.23

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

					163	2.347	83520.777	83520.777	0	0.30	11.35	4157	206883	83435.711	0.06	11.41
					150	2.600	68432.154	68432.154	0	0.32	0.10	0	2827	68432.154	0.06	0.17
					137	2.414	63303.223	63303.223	0	0.31	0.73	1	3315	63303.223	0.06	0.79
					171	2.351	48665.429	48665.429	0	0.31	7.52	1776	64386	48619.666	0.06	7.58
				93	228	2.350	63786.355	63786.355	0	0.40	0.21	1	2879	63786.355	0.07	0.28
					226	2.441	73858.746	73858.746	1.58E-15	0.39	3.05	2128	25514	73858.746	0.07	3.12
					212	2.441	55875.639	55875.639	0	0.41	0.39	1	4370	55875.639	0.08	0.47
					240	2.541	54345.259	54345.259	0	0.38	1.36	216	11185	54345.259	0.07	1.44
					234	2.353	45040.402	45040.402	0	0.40	7.53	2204	68000	45040.402	0.07	7.60
				112	185	2.476	70709.285	70709.285	0	0.46	0.16	0	4290	70709.285	0.09	0.24
					300	2.520	56315.576	56315.576	0	0.46	0.46	1	3134	56315.576	0.09	0.55
					236	2.414	69726.732	69726.732	0	0.45	2.50	1	4923	69726.732	0.09	2.59
					228	2.451	70948.528	70948.528	0	0.46	18.07	2879	120829	70948.528	0.09	18.16
					275	2.324	68916.303	68916.303	0	0.49	7.76	5915	93743	68916.303	0.09	7.86
				150	384	2.311	59257.105	59257.105	0	0.63	0.50	1	4007	59257.105	0.11	0.61
					323	2.590	78392.774	78392.774	0	0.65	0.23	0	5497	78392.774	0.11	0.35
			345		2.294	61553.308	61553.308	0	0.65	3.27	402	23644	61553.308	0.11	3.38	
			209		2.417	77038.357	77038.357	0	0.65	0.78	1	6336	77038.357	0.11	0.89	
			308		2.441	64823.775	64823.775	0	0.63	0.38	1	5409	64823.775	0.11	0.50	
			10000	100	228	2.219	78533.308	78533.308	0	0.42	0.19	1	3063	78533.308	0.08	0.27
					144	2.412	85714.535	85714.535	0	0.44	0.11	0	3402	85714.535	0.08	0.19
					257	2.531	79631.044	79631.044	0	0.42	0.52	1	4622	79631.044	0.08	0.60
					209	2.514	75234.878	75234.878	0	0.42	0.13	0	3430	75234.878	0.08	0.21
					178	2.328	90663.523	90663.523	0	0.44	5.78	1217	25377	90663.523	0.08	5.86
				125	182	2.487	119348.413	119348.413	0	0.51	0.47	1	4892	119348.413	0.10	0.57
					284	2.540	59871.834	59871.834	0	0.55	0.86	1	3892	59871.834	0.10	0.96
					246	2.429	94534.168	94534.168	0	0.54	0.12	0	3649	94534.168	0.10	0.22
					235	2.328	77051.771	77051.771	0	0.52	9.72	1241	45022	77008.143	0.10	9.82
					294	2.407	90319.234	90319.234	0	0.54	0.49	1	3983	90319.234	0.10	0.59
				150	325	2.401	91061.683	91061.683	0	0.65	1.62	1	6365	91061.683	0.11	1.74
					327	2.418	81925.570	81925.570	0	0.65	15.94	1175	40733	81925.570	0.12	16.05

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

			200	336	2.509	75128.065	75128.065	0	0.65	0.32	1	5383	75128.065	0.12	0.43
				356	2.399	73646.811	73646.811	0	0.62	0.20	0	4265	73646.811	0.11	0.31
				249	2.515	129740.849	129740.849	0	0.62	0.20	0	5878	129740.849	0.11	0.31
				328	2.504	82430.149	82430.149	0	0.82	0.25	0	6658	82430.149	0.15	0.39
				387	2.501	73024.026	73024.026	0	0.82	38.04	1181	90220	73024.026	0.15	38.18
				389	2.504	84510.044	84510.044	0	0.86	1.00	1	5906	84510.044	0.14	1.14
				265	2.495	130688.437	130688.437	3.01E-15	0.86	1.69	1	13014	130688.437	0.14	1.83
				380	2.308	75903.025	75903.025	0	0.81	25.42	1242	89233	75903.025	0.14	25.56
		12500	125	293	2.511	99316.408	99316.408	0	0.52	0.25	1	3828	99316.408	0.10	0.35
				235	2.411	129271.068	129271.068	0	0.52	34.76	10620	139345	129271.068	0.10	34.86
				158	2.411	132745.906	132745.906	0	0.55	0.31	1	5207	132745.906	0.10	0.40
				210	2.316	91408.052	91408.052	0	0.53	2.59	1	8928	91408.052	0.10	2.69
				289	2.390	88077.808	88077.808	0	0.56	1.97	1	6610	88077.808	0.10	2.07
			156	259	2.408	105689.686	105689.686	0	0.68	0.39	1	6330	105689.686	0.12	0.51
				359	2.458	124509.404	124509.404	0	0.67	0.48	1	5633	124509.404	0.12	0.60
				360	2.425	82554.459	82554.459	0	0.67	0.20	0	4806	82554.459	0.12	0.32
				366	2.431	110671.730	110671.730	0	0.68	11.70	1559	138604	110556.380	0.12	11.81
				373	2.325	94924.823	94924.823	0	0.69	0.19	0	4065	94924.823	0.12	0.31
			187	290	2.582	111335.693	111335.693	0	0.80	0.66	1	6999	111335.693	0.13	0.79
				242	2.482	136689.097	136689.097	2.56E-15	0.80	0.55	1	7595	136689.097	0.14	0.68
				425	2.313	92156.965	92156.965	0	0.82	0.64	1	6258	92156.965	0.14	0.78
				490	2.410	115662.933	115662.933	0	0.79	2.42	236	14720	115662.933	0.13	2.55
				272	2.400	152827.532	152827.532	0	0.78	6.73	117	23124	152827.532	0.13	6.86
			250	490	2.381	74941.167	74941.167	0	1.04	0.30	0	7484	74941.167	0.18	0.48
				332	2.459	128598.701	128598.701	2.72E-15	1.08	3.13	1	15081	128598.701	0.17	3.31
				477	2.463	87356.346	87356.346	1.33E-15	1.03	1.15	1	10429	87356.346	0.18	1.33
				374	2.399	135311.201	135311.201	0	1.07	0.78	1	9030	135311.201	0.17	0.96
				518	2.406	108788.592	108788.592	0	1.05	1.68	1	10382	108788.592	0.17	1.85
4	8	7500	75	188	2.472	61281.311	61281.311	0	0.27	0.10	1	1559	61281.087	0.06	0.16
				192	2.430	53400.557	53400.557	0	0.27	0.45	1	3445	53400.557	0.06	0.51
				211	2.562	43755.821	43755.821	0	0.28	0.28	1	2156	43755.821	0.06	0.34

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

			93	258	2.257	38661.347	38661.347	0	0.28	0.10	1	966	38661.347	0.06	0.15
				176	2.507	58835.492	58835.492	0	0.28	0.18	1	2526	58835.492	0.06	0.24
				263	2.438	55837.003	55837.003	0	0.36	0.39	1	2863	55837.003	0.07	0.46
				247	2.315	78725.470	78725.470	0	0.34	0.52	1	4169	78725.470	0.07	0.59
				318	2.359	44992.127	44992.127	0	0.36	0.21	1	2086	44992.127	0.07	0.28
				254	2.340	48128.050	48128.050	0	0.34	1.01	1	6683	48128.050	0.07	1.08
				257	2.285	45895.376	45895.376	0	0.37	5.88	1181	38051	45895.376	0.06	5.95
			112	301	2.465	62001.898	62001.898	0	0.42	0.22	1	2813	62001.898	0.07	0.29
				223	2.463	66117.316	66117.316	0	0.41	0.16	1	2971	66117.316	0.08	0.24
				279	2.427	67543.657	67543.657	2.59E-15	0.41	0.41	1	3120	67543.657	0.08	0.49
				219	2.299	58161.044	58161.044	0	0.41	1.61	20	8575	58161.044	0.08	1.68
				312	2.518	68572.798	68572.798	0	0.41	0.55	1	3268	68534.137	0.07	0.63
			150	412	2.601	61966.907	61966.907	0	0.56	0.50	1	4147	61966.907	0.10	0.60
				438	2.480	51326.905	51326.905	0	0.57	0.54	1	4320	51326.905	0.09	0.63
				408	2.385	50262.290	50262.290	0	0.57	0.14	0	2937	50262.290	0.09	0.23
				439	2.521	38288.229	38288.229	0	0.57	0.47	1	4223	38288.229	0.10	0.57
				316	2.410	89237.120	89237.120	0	0.54	0.32	1	4084	89237.120	0.10	0.41
		10000	100	265	2.468	72910.901	72910.901	0	0.37	0.09	0	2689	72910.901	0.07	0.16
				213	2.442	64422.849	64422.849	1.81E-15	0.39	0.70	196	6122	64422.849	0.07	0.77
				256	2.423	69332.249	69332.249	0	0.39	3.86	191	11145	69332.249	0.07	3.93
				248	2.457	64830.612	64830.612	0	0.37	3.19	196	10497	64830.612	0.07	3.26
				358	2.361	75701.187	75701.187	0	0.37	0.06	0	1631	75701.187	0.07	0.13
			125	226	2.431	96734.588	96734.588	0	0.46	1.74	1	8574	96734.588	0.08	1.83
				360	2.454	70473.695	70473.695	0	0.48	0.46	1	4139	70473.695	0.09	0.55
				265	2.278	87093.010	87093.010	0	0.46	4.47	407	19504	87093.010	0.08	4.55
				306	2.505	75599.612	75599.612	0	0.49	4.16	34	13017	75599.612	0.08	4.25
				292	2.488	96140.228	96140.228	0	0.46	0.27	1	3116	96140.228	0.08	0.36
			150	430	2.536	70181.399	70181.399	0	0.58	0.32	1	3915	70181.399	0.09	0.41
				354	2.252	59317.192	59317.192	0	0.58	15.04	1178	51380	59317.192	0.10	15.14
				427	2.492	79780.243	79780.243	0	0.55	0.17	1	2067	79780.243	0.09	0.25
				403	2.413	97559.271	97559.271	0	0.54	2.21	93	11434	97559.271	0.10	2.31

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

		12500	200	490	2.444	62825.269	62825.269	0	0.55	3.24	345	15940	62825.269	0.10	3.34
				511	2.381	64480.371	64480.371	3.05E-15	0.72	1.31	1	5677	64480.371	0.12	1.43
				581	2.535	77575.893	77575.893	0	0.76	0.68	1	5329	77575.893	0.13	0.81
				669	2.395	68071.229	68071.229	0	0.77	1.57	93	9819	68033.660	0.12	1.69
				615	2.488	68854.233	68854.233	0	0.75	0.36	1	4421	68854.233	0.13	0.49
				545	2.412	70818.754	70818.754	0	0.73	19.66	1450	143594	70818.754	0.14	19.80
			125	270	2.321	94255.996	94255.996	2.01E-15	0.47	0.15	0	3642	94255.996	0.09	0.23
				305	2.364	77572.419	77572.419	0	0.47	0.35	1	3407	77572.419	0.09	0.44
				304	2.463	77676.024	77676.024	0	0.49	0.11	0	2670	77676.024	0.08	0.19
				287	2.646	92306.418	92306.418	0	0.46	0.21	1	3143	92306.418	0.08	0.29
				356	2.416	143109.415	143109.415	0	0.49	0.50	1	2946	143109.415	0.08	0.58
			156	401	2.369	82927.798	82927.798	2.63E-15	0.57	0.18	0	3726	82927.798	0.09	0.28
				414	2.406	106630.997	106630.997	0	0.60	2.41	815	29062	106630.997	0.11	2.52
				392	2.433	92801.906	92801.906	0	0.57	0.18	0	4329	92801.906	0.10	0.28
				391	2.285	110399.292	110399.292	0	0.57	0.57	1	4690	110399.292	0.10	0.66
				303	2.405	103757.701	103757.701	0	0.57	0.98	1	5391	103757.701	0.10	1.08
			187	444	2.345	131512.241	131512.241	0	0.68	0.43	1	5645	131512.241	0.13	0.56
				345	2.395	116110.772	116110.772	5.38E-12	0.72	1.33	1	9756	116110.772	0.13	1.46
				500	2.366	97998.011	97998.011	0	0.71	0.17	0	4323	97998.011	0.12	0.29
				544	2.459	78242.818	78242.818	0	0.68	0.78	1	4710	78242.818	0.12	0.90
				399	2.314	103895.879	103895.879	0	0.69	0.36	1	4212	103895.879	0.13	0.49
			250	492	2.297	94476.377	94476.377	0	0.94	4.70	24	17098	94431.111	0.16	4.86
				689	2.488	93303.200	93303.200	1.56E-15	0.94	0.54	1	5448	93303.200	0.16	0.71
				640	2.373	76795.229	76795.229	0	0.91	0.53	1	5776	76795.229	0.16	0.69
				620	2.497	96876.404	96876.404	0	0.95	2.96	153	16939	96876.404	0.16	3.12
				662	2.526	100294.205	100294.205	2.03E-15	0.91	0.80	1	7957	100294.205	0.16	0.96
		7500	75	188	2.472	60356.386	60356.386	0	0.41	0.47	1	3252	60329.423	0.08	0.55
				178	2.356	39194.131	39194.131	0	0.41	0.56	1	4603	39194.131	0.09	0.64
				206	2.579	47202.056	47202.056	0	0.41	0.09	0	2451	47202.056	0.09	0.18
				194	2.584	64699.135	64699.135	0	0.41	0.10	0	2835	64699.135	0.09	0.20
				192	2.412	60057.012	60057.012	0	0.41	1.88	52	10439	60057.012	0.09	1.96

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

			93	292	2.435	54849.815	54849.815	0	0.53	19.96	1357	76616	54849.815	0.10	20.05
				195	2.479	73715.479	73715.479	0	0.50	0.16	0	4224	73715.479	0.10	0.26
				224	2.590	65614.799	65614.799	0	0.53	2.84	1	9774	65614.799	0.11	2.94
				284	2.649	68140.009	68140.009	0	0.53	0.23	1	3210	68140.009	0.10	0.33
				290	2.429	55267.092	55267.092	0	0.50	0.35	1	3529	55267.092	0.09	0.45
			112	370	2.257	40585.784	40585.784	0	0.59	20.27	1202	69088	40585.784	0.11	20.38
				303	2.320	55612.299	55612.299	0	0.63	1.34	1	9423	55612.299	0.12	1.46
				364	2.433	84531.656	84531.656	0	0.63	82.52	3555	639867	84471.692	0.11	82.62
				236	2.508	70117.281	70117.281	0	0.64	11.26	1316	86450	70117.281	0.11	11.37
				287	2.335	58280.375	58280.375	0	0.64	4.57	78	15218	58280.375	0.12	4.69
			150	269	2.475	76011.449	76011.449	0	0.79	27.91	2096	76848	76011.449	0.15	28.05
				511	2.476	39221.008	39221.008	0	0.82	2.03	1	5315	39221.008	0.15	2.18
				517	2.309	60136.651	60136.651	0	0.81	24.14	3494	404405	60072.305	0.14	24.28
				326	2.417	87288.077	87288.077	0	0.83	0.69	0	7731	87288.077	0.15	0.84
				268	2.389	69761.425	69761.425	0	0.81	13.18	1238	58048	69761.425	0.14	13.32
		10000	100	327	2.311	56104.821	56104.821	0	0.55	12.71	2376	129784	56104.821	0.11	12.82
				255	2.646	94999.150	94999.150	0	0.55	0.13	0	3495	94999.150	0.10	0.23
				319	2.437	81354.803	81354.803	0	0.58	0.12	1	1605	81354.803	0.10	0.22
				203	2.388	66898.278	66898.278	0	0.58	24.93	1217	47330	66898.278	0.11	25.04
				288	2.378	59082.359	59082.359	0	0.58	3.06	336	19928	59082.359	0.10	3.16
			125	278	2.255	85903.171	85903.171	0	0.71	18.47	3971	161552	85903.171	0.13	18.59
				366	2.325	86946.055	86946.055	0	0.70	0.96	1	3684	86927.270	0.12	1.08
				358	2.411	88094.591	88094.591	0	0.71	2.46	1	12305	88094.591	0.13	2.59
				331	2.376	77630.260	77630.260	1.31E-15	0.68	0.95	1	7355	77630.260	0.13	1.07
				358	2.359	74540.346	74540.346	0	0.70	0.81	1	5247	74540.346	0.13	0.94
			150	470	2.397	62896.337	62896.337	0	0.82	0.27	0	4760	62896.337	0.13	0.40
				465	2.367	91916.492	91916.492	0	0.85	2.98	162	16618	91916.492	0.14	3.12
				432	2.392	63025.243	63025.243	0	0.84	0.41	1	4604	63025.243	0.15	0.55
				318	2.586	62365.164	62365.164	0	0.82	6.50	299	24317	62365.164	0.15	6.64
				349	2.467	87793.879	87793.879	0	0.85	0.27	0	5611	87793.879	0.15	0.42
			200	460	2.374	82586.380	82586.380	0	1.07	0.77	1	7852	82586.380	0.19	0.96

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

				347	2.415	93543.965	93543.965	1.09E-15	1.10	0.94	1	9731	93543.965	0.18	1.12
				435	2.421	59955.634	59955.634	0	1.08	0.60	0	8955	59955.634	0.19	0.79
				703	2.554	76188.221	76188.221	0	1.08	1.05	1	6804	76188.221	0.19	1.24
				507	2.439	85195.821	85195.821	4.61E-15	1.07	1.08	1	6664	85195.821	0.19	1.27
		12500	125	360	2.436	94719.599	94719.599	0	0.69	4.97	478	30360	94719.599	0.12	5.09
				242	2.514	137760.778	137760.778	6.34E-16	0.72	1.31	1	6984	137760.778	0.12	1.44
				370	2.324	77693.648	77693.648	0	0.69	3.31	48	11518	77693.648	0.12	3.43
				315	2.559	114614.752	114614.752	2.67E-15	0.72	0.71	1	5715	114614.752	0.13	0.84
				181	2.355	115584.521	115584.521	0	0.68	45.20	1203	81908	115584.521	0.12	45.32
			156	339	2.488	111086.456	111086.456	2.23E-15	0.89	0.68	1	6534	111086.456	0.15	0.84
				455	2.374	69101.422	69101.422	0	0.87	0.51	1	5122	69101.422	0.15	0.67
				478	2.448	72282.560	72282.560	0	0.85	4.92	608	25154	72282.560	0.15	5.07
				425	2.539	148677.458	148677.458	0	0.88	3.96	128	16996	148677.458	0.14	4.10
				334	2.361	78811.429	78811.429	0	0.88	0.74	1	7671	78811.429	0.15	0.89
			187	447	2.458	90569.991	90569.991	0	1.02	20.19	2326	122387	90569.991	0.19	20.38
				341	2.417	106863.265	106863.265	0	1.05	83.02	1580	159057	106863.265	0.18	83.21
				415	2.225	104773.842	104773.842	0	1.02	4.32	1	18193	104773.842	0.18	4.51
				467	2.425	95209.597	95209.597	0	1.02	0.40	0	6673	95209.597	0.18	0.58
				422	2.314	90787.935	90787.935	0	1.02	0.47	0	8992	90787.935	0.19	0.66
			250	555	2.315	107741.021	107741.021	0	1.37	0.49	0	10959	107741.021	0.23	0.72
				681	2.373	69882.075	69882.075	0	1.35	1.76	1	17434	69882.075	0.24	2.00
				668	2.369	88270.509	88270.509	3.46E-15	1.37	0.49	0	8748	88270.509	0.23	0.72
				756	2.454	111152.498	111152.498	2.62E-15	1.39	3.77	1	21454	111066.150	0.23	4.00
				551	2.328	98981.212	98981.212	0	1.38	1.53	1	14060	98981.212	0.23	1.76
	16	7500	75	188	2.472	64244.957	64244.957	0	0.53	1.64	58	10694	64244.957	0.11	1.76
				216	2.421	54345.398	54345.398	0	0.54	3.31	64	12398	54345.398	0.11	3.42
				217	2.429	50873.338	50873.338	0	0.54	1.46	1	6438	50763.363	0.11	1.57
				181	2.475	55381.952	55381.952	0	0.56	0.42	1	3998	55381.952	0.12	0.54
				230	2.374	59524.845	59524.845	0	0.57	36.27	7160	650028	59524.845	0.12	36.39
			93	185	2.305	58547.601	58547.601	0	0.69	1.71	1	11446	58547.601	0.13	1.85
				230	2.434	56136.525	56136.525	0	0.67	0.48	1	6075	56136.525	0.14	0.62

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

				251	2.397	44494.820	44494.820	0	0.71	28.79	3200	207569	44444.730	0.14	28.92
				285	2.315	48668.507	48668.507	4.04E-15	0.67	0.56	1	5298	48668.507	0.13	0.69
				217	2.468	72999.643	72999.643	0	0.67	11.79	1478	77883	72999.643	0.14	11.93
			112	330	2.415	61170.473	61170.473	0	0.83	0.94	1	6229	61170.473	0.16	1.09
				346	2.351	61972.532	61972.532	0	0.83	0.62	1	5778	61947.652	0.15	0.77
				337	2.319	44350.761	44350.761	0	0.79	20.05	8717	330844	44347.259	0.14	20.19
				234	2.417	55840.477	55840.477	0	0.82	399.98	33547	2166912	55840.477	0.15	400.13
				301	2.457	59868.980	59868.980	0	0.81	0.21	0	5501	59868.980	0.15	0.36
			150	418	2.537	54475.203	54475.203	0	1.06	20.06	2427	114536	54475.203	0.20	20.26
				439	2.534	44260.639	44260.639	0	1.10	24.00	3373	264445	44260.639	0.18	24.19
				540	2.572	72775.444	72775.444	0	1.08	43.68	3430	287674	72764.200	0.19	43.87
				486	2.498	57958.412	57958.412	0	1.11	8.10	3672	39546	57958.412	0.19	8.29
				358	2.312	61527.168	61527.168	0	1.09	4.02	1	14646	61527.168	0.18	4.20
		10000	100	210	2.575	108031.493	108031.493	0	0.73	207.04	32419	2044688	108031.493	0.15	207.19
				306	2.268	113376.330	113376.330	0	0.74	0.71	1	4726	113376.330	0.13	0.85
				309	2.410	72731.619	72731.619	0	0.73	21.30	1826	144756	72731.619	0.15	21.45
				241	2.384	66529.240	66529.240	0	0.75	46.26	2774	119400	66529.240	0.15	46.40
				195	2.466	86932.376	86932.376	0	0.72	97.74	1558	228321	86932.376	0.15	97.89
			125	388	2.312	60453.804	60453.804	0	0.93	3.42	231	15822	60453.804	0.16	3.58
				286	2.337	103272.192	103272.192	0	0.88	2.09	1	15481	103272.192	0.17	2.26
				291	2.393	79037.273	79037.273	0	0.90	0.28	0	6351	79037.273	0.17	0.45
				246	2.385	114394.579	114394.579	0	0.89	11.44	1199	40447	114394.579	0.17	11.61
				277	2.435	91425.999	91425.999	0	0.94	14.56	1256	52272	91425.999	0.17	14.73
			150	300	2.354	86010.508	86010.508	0	1.10	1.89	1	14653	86010.508	0.20	2.09
				429	2.345	107516.284	107516.284	0	1.06	2.60	1	17351	107516.284	0.18	2.78
				416	2.334	65743.942	65743.942	0	1.06	5.54	1	17001	65743.942	0.19	5.73
				398	2.315	61399.533	61399.533	0	1.10	3.34	1	16992	61399.533	0.18	3.53
				396	2.505	69279.155	69279.155	2.31E-15	1.13	0.28	0	5638	69279.155	0.20	0.48
			200	490	2.438	65569.213	65569.213	4.88E-15	1.45	0.47	0	9512	65569.213	0.24	0.72
				589	2.383	87646.547	87646.547	1.16E-15	1.46	3.31	1	16618	87646.547	0.26	3.57
				278	2.334	91015.182	91015.182	0	1.39	64.13	1188	136277	91015.182	0.24	64.37

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

		12500	125	472	2.326	76516.560	76516.560	0	1.40	82.62	3467	223368	76516.560	0.26	82.88
				638	2.517	77551.511	77551.511	0	1.47	15.02	1408	58539	77551.511	0.27	15.29
				274	2.512	109171.990	109171.990	0	0.97	87.03	11672	386279	109155.777	0.17	87.20
				222	2.411	123941.067	123941.067	0	0.96	0.67	1	8668	123941.067	0.17	0.84
				367	2.362	79483.663	79483.663	0	0.97	1.66	1	10042	79483.663	0.16	1.82
				398	2.424	85383.123	85383.123	0	0.91	1.84	1	8479	85383.123	0.16	2.00
				345	2.494	91660.727	91660.727	0	0.92	0.21	0	5452	91660.727	0.16	0.37
			156	365	2.301	76785.317	76785.317	0	1.13	7.06	471	39603	76785.317	0.19	7.26
				439	2.408	148046.965	148046.965	5.90E-16	1.17	0.44	1	5003	148046.965	0.20	0.65
				434	2.433	85972.175	85972.175	6.77E-15	1.16	1.58	1	9496	85972.175	0.19	1.77
				283	2.436	106965.528	106965.528	0	1.16	10.08	119	25877	106965.528	0.20	10.28
				400	2.615	100466.736	100466.736	0	1.18	7.13	285	31507	100466.736	0.21	7.34
			187	572	2.348	59493.237	59493.237	0	1.38	60.14	1778	203952	59493.237	0.24	60.38
				428	2.400	106990.585	106990.585	0	1.34	0.48	0	10049	106990.585	0.25	0.73
				315	2.345	141738.469	141738.469	8.21E-16	1.38	1.69	1	13246	141738.469	0.23	1.93
				475	2.476	102938.909	102938.909	1.98E-15	1.39	2.00	1	15656	102938.909	0.25	2.25
				442	2.442	94034.215	94034.215	0	1.39	1.87	1	12039	94034.215	0.24	2.11
			250	717	2.302	85112.165	85112.165	0	1.79	5.08	1	20405	85112.165	0.32	5.40
				671	2.277	77891.842	77891.842	0	1.83	7.81	1	22634	77891.842	0.31	8.12
				623	2.398	108890.982	108890.982	1.07E-15	1.76	4.88	1	21637	108890.982	0.32	5.19
				495	2.409	115381.928	115381.928	0	1.81	17.04	1	42612	115381.928	0.32	17.36
				591	2.463	103086.136	103086.136	0	1.81	2.01	1	14081	103086.136	0.31	2.32
5	10	7500	75	239	2.472	54144.650	54144.650	0	0.41	1.59	275	14008	54135.678	0.09	1.68
				269	2.503	35326.863	35326.863	0	0.44	1.40	1	5909	35326.863	0.09	1.50
				222	2.398	57875.507	57875.507	0	0.41	0.40	1	3339	57875.507	0.09	0.49
				261	2.235	34907.426	34907.426	0	0.42	0.38	1	2824	34907.426	0.09	0.47
				237	2.333	47491.472	47491.472	0	0.44	1.61	44	8681	47491.472	0.09	1.70
			93	281	2.515	40674.080	40674.080	0	0.51	0.60	1	3795	40665.948	0.11	0.71
				168	2.554	70224.541	70224.541	0	0.54	3.16	597	21942	70224.541	0.12	3.28
				357	2.171	27421.403	27421.403	0	0.54	0.23	1	2458	27421.403	0.12	0.35
				337	2.388	34165.193	34165.193	1.67E-11	0.55	10.44	3611	176751	34165.193	0.11	10.55

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

				281	2.551	40781.767	40781.767	0	0.52	3.86	382	17595	40781.767	0.11	3.97
			112	374	2.333	42965.145	42965.145	0	0.62	0.58	1	5343	42965.145	0.13	0.71
				324	2.530	75855.490	75855.490	1.53E-15	0.62	0.74	1	5767	75855.490	0.13	0.87
				272	2.379	72623.687	72623.687	0	0.62	17.68	2234	83135	72623.687	0.14	17.82
				314	2.560	54212.439	54212.439	0	0.66	1.12	1	8390	54212.439	0.13	1.26
				296	2.562	45344.074	45344.074	0	0.62	1.69	1	7259	45344.074	0.12	1.81
			150	439	2.239	42717.874	42717.874	0	0.84	42.48	2780	404123	42717.874	0.18	42.66
				461	2.567	63998.950	63998.950	0	0.87	6.31	677	34069	63998.950	0.16	6.47
				551	2.627	38783.497	38783.497	0	0.83	0.49	1	3790	38777.838	0.16	0.65
				515	2.283	38006.685	38006.685	0	0.82	1.38	1	6201	38006.685	0.16	1.53
				486	2.229	68540.378	68540.378	0	0.82	29.34	2727	153630	68540.378	0.17	29.51
	10000	100	400	2.167	58926.885	58926.885	0	0.56	3.98	472	26289	58926.885	0.12	4.10	
			268	2.317	97987.368	97987.368	0	0.56	2.12	1	6933	97987.368	0.12	2.24	
			277	2.409	68604.088	68604.088	0	0.59	0.71	1	4124	68593.023	0.12	0.83	
			267	2.473	73109.763	73109.763	0	0.55	1.62	84	11652	73109.763	0.13	1.74	
			299	2.297	62178.356	62178.356	0	0.56	0.56	1	4068	62178.356	0.13	0.69	
			125	338	2.343	60347.116	60347.116	0	0.72	0.77	1	8042	60347.116	0.15	0.91
				454	2.185	43728.476	43728.476	0	0.69	1.34	1	6960	43728.476	0.14	1.48
				523	2.494	60773.644	60773.644	0	0.70	2.79	228	14726	60773.644	0.13	2.92
				459	2.440	51995.107	51995.107	0	0.69	0.34	1	3403	51995.107	0.13	0.47
				463	2.392	63390.296	63390.296	0	0.72	3.55	1	9931	63390.296	0.14	3.69
			150	517	2.454	75183.101	75183.101	0	0.83	0.33	1	4776	75183.101	0.16	0.49
				414	2.242	60155.366	60155.366	0	0.87	0.50	1	4547	60155.366	0.16	0.67
				539	2.449	68793.518	68793.518	0	0.83	0.96	1	6373	68793.518	0.16	1.12
				443	2.430	65545.364	65545.364	0	0.82	0.75	1	5807	65527.119	0.16	0.91
				527	2.432	75956.292	75956.292	0	0.83	3.52	1	10594	75956.292	0.16	3.68
			200	505	2.410	75833.842	75833.842	3.65E-15	1.09	0.64	0	9184	75833.842	0.21	0.85
				767	2.343	45001.649	45001.649	0	1.10	10.15	781	109198	45001.649	0.24	10.39
				750	2.367	65916.980	65916.980	0	1.09	0.81	1	5604	65916.980	0.20	1.01
				554	2.504	62503.123	62503.123	3.84E-15	1.09	5.48	88	23988	62503.123	0.21	5.69
				528	2.354	56117.691	56117.691	0	1.10	0.48	0	7248	56117.691	0.21	0.69

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

		12500	125	355	2.402	80749.733	80749.733	0	0.70	1.04	1	7367	80749.733	0.15	1.20
				416	2.440	74211.668	74211.668	2.55E-15	0.73	0.44	1	4957	74211.668	0.15	0.59
				374	2.375	90177.056	90177.056	0	0.70	0.20	0	4358	90177.056	0.14	0.33
				382	2.375	111914.644	111914.644	0	0.73	4.97	405	28591	111914.644	0.13	5.10
				316	2.546	96264.727	96264.727	0	0.70	2.08	1	11923	96264.727	0.15	2.23
			156	586	2.414	69661.016	69661.016	0	0.87	5.77	242	25366	69661.016	0.17	5.94
				562	2.311	87679.180	87679.180	1.49E-15	0.86	2.99	10	10980	87679.180	0.16	3.15
				614	2.450	79108.895	79108.895	0	0.87	0.94	1	7406	79108.895	0.17	1.11
				413	2.404	94234.326	94234.326	0	0.87	6.11	15	15492	94234.326	0.18	6.29
				426	2.406	106143.522	106143.522	0	0.91	1.76	1	11014	106143.522	0.17	1.94
			187	551	2.331	126377.320	126377.320	0	1.08	7.57	41	16766	126377.320	0.19	7.76
				473	2.494	102062.323	102062.323	0	1.09	5.71	38	17474	102062.323	0.20	5.91
				618	2.592	109457.678	109457.678	0	1.09	1.35	1	9838	109457.678	0.20	1.55
				786	2.404	71912.370	71912.370	0	1.09	0.78	1	4760	71912.370	0.20	0.98
				639	2.366	101751.294	101751.294	0	1.08	0.33	0	5427	101751.294	0.19	0.52
			250	618	2.383	90228.826	90228.826	0	1.40	0.69	0	10526	90228.826	0.26	0.95
				793	2.391	67247.553	67247.553	0	1.44	2.37	1	17114	67230.834	0.27	2.65
				562	2.403	100229.919	100229.919	0	1.38	1.27	1	11940	100229.919	0.27	1.53
				858	2.392	104630.492	104630.492	0	1.43	6.45	35	24133	104628.639	0.27	6.72
				630	2.348	112074.644	112074.644	7.53E-15	1.43	2.54	1	16232	112074.644	0.26	2.81
	15	7500	75	239	2.472	51317.102	51317.102	0	0.61	2.96	439	20461	51303.705	0.13	3.09
				207	2.375	60426.461	60426.461	0	0.64	0.62	1	5551	60426.461	0.13	0.75
				258	2.462	58613.798	58613.798	0	0.61	3.15	673	22050	58613.798	0.14	3.28
				253	2.761	73785.749	73785.749	0	0.62	1.09	1	6073	73785.749	0.14	1.23
				266	2.485	46687.916	46687.916	0	0.65	0.38	1	3018	46687.916	0.14	0.52
			93	257	2.375	46318.447	46318.447	0	0.74	57.03	2554	388007	46318.447	0.16	57.20
				302	2.566	70666.882	70666.882	0	0.76	21.73	7857	330355	70666.882	0.17	21.90
				203	2.350	69143.164	69143.164	0	0.76	0.52	1	6139	69143.164	0.18	0.70
				320	2.445	54081.877	54081.877	0	0.76	2.31	1	8646	54081.877	0.17	2.48
				278	2.373	44898.711	44898.711	0	0.79	0.56	1	5517	44898.711	0.19	0.74
			112	291	2.344	55238.809	55238.809	0	0.89	13.85	949	62417	55238.809	0.21	14.06

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

					301	2.279	47528.036	47528.036	0	0.94	0.82	1	6860	47528.036	0.21	1.03	
					370	2.431	48510.633	48510.633	0	0.95	3.69	81	18103	48510.633	0.18	3.87	
					388	2.421	45372.061	45372.061	0	0.93	3.30	160	19374	45372.061	0.19	3.49	
					411	2.520	57630.625	57630.625	0	0.96	0.90	1	7020	57630.625	0.18	1.09	
				150	444	2.287	48741.890	48741.890	0	1.20	2.01	1	8336	48741.890	0.23	2.24	
					437	2.372	51106.646	51106.646	0	1.24	1.30	1	8923	51106.646	0.24	1.53	
					545	2.451	49077.768	49077.768	0	1.20	2.28	1	12154	49077.768	0.24	2.53	
					409	2.479	57794.175	57794.175	0	1.22	1.41	1	7998	57794.175	0.23	1.63	
				425	2.392	45715.698	45715.698	0	1.24	16.35	19	33496	45715.698	0.26	16.61		
				10000	100	278	2.499	74893.668	74893.668	0	0.83	28.05	2457	169614	74888.477	0.19	28.24
						253	2.522	75982.889	75982.889	0	0.82	0.56	1	6313	75982.889	0.20	0.76
						287	2.379	70786.218	70786.218	1.44E-15	0.82	1.01	1	7047	70786.218	0.18	1.19
			188			2.264	97963.729	97963.729	0	0.87	62.09	1347	136519	97963.729	0.19	62.29	
			342			2.446	90402.688	90402.688	0	0.87	0.19	0	4132	90402.688	0.18	0.37	
			125			365	2.426	86607.897	86607.897	0	1.06	53.77	4074	232994	86607.897	0.21	53.98
				326	2.277	62313.250	62313.250	0	1.07	0.57	1	7278	62313.250	0.23	0.80		
				267	2.406	67218.825	67218.825	0	1.07	1.04	1	10345	67218.825	0.22	1.26		
				342	2.299	61808.159	61808.159	0	1.07	6.34	431	31819	61808.159	0.20	6.54		
				282	2.297	105700.714	105700.714	9.64E-16	1.07	3.54	1	18134	105700.714	0.21	3.75		
				150	332	2.453	72681.760	72681.760	0	1.24	2.00	1	11668	72681.760	0.25	2.25	
			367		2.404	86124.168	86124.168	0	1.26	103.23	1679	221599	86083.302	0.23	103.47		
			588		2.297	73296.366	73296.366	0	1.24	15.58	2101	114154	73296.366	0.25	15.83		
			572		2.518	54931.712	54931.712	0	1.24	1.00	1	6572	54931.712	0.24	1.24		
			525		2.415	87803.376	87803.376	0	1.29	7.51	1	15166	87803.376	0.24	7.75		
			200		484	2.247	66405.037	66405.037	2.41E-15	1.67	2.42	1	14968	66405.037	0.33	2.75	
				703	2.458	87486.090	87486.090	0	1.69	5.47	107	25483	87486.090	0.32	5.79		
				664	2.418	81078.578	81078.578	0	1.65	32.21	1030	134087	81078.578	0.32	32.53		
				521	2.467	95732.907	95732.907	0	1.72	0.79	0	12822	95732.907	0.33	1.12		
				735	2.427	48680.475	48680.475	0	1.64	4.90	1	11822	48680.475	0.34	5.24		
				12500	125	441	2.477	79082.932	79082.932	0	1.04	16.88	1114	83889	79082.932	0.21	17.09
			349			2.471	101432.174	101432.174	1.72E-15	1.03	0.66	1	8350	101432.174	0.22	0.88	

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

				296	2.502	104274.252	104274.252	0	1.03	0.35	0	7807	104274.252	0.22	0.57
				452	2.249	63727.257	63727.257	0	1.03	47.40	2775	311139	63727.257	0.23	47.63
				385	2.488	80127.615	80127.615	0	1.04	22.62	2585	166340	80127.615	0.21	22.83
			156	479	2.428	77439.124	77439.124	0	1.34	9.23	965	56505	77439.124	0.25	9.48
				463	2.493	96528.245	96528.245	0	1.29	4.16	1	15985	96528.245	0.25	4.40
				585	2.453	77962.285	77962.285	0	1.33	0.91	1	6431	77962.285	0.25	1.16
				476	2.432	93080.529	93080.529	1.75E-12	1.30	33.51	1166	88923	93084.601	0.27	33.77
				452	2.399	75772.819	75772.819	1.54E-15	1.34	1.15	1	6496	75772.819	0.24	1.39
				486	2.366	88153.995	88153.995	0	1.50	10.66	124	29016	88153.995	0.30	10.96
				652	2.285	88562.046	88562.046	5.59E-15	1.52	1.46	1	8559	88562.046	0.30	1.76
			187	464	2.428	85240.914	85240.914	0	1.57	2.23	1	16695	85240.914	0.29	2.51
				481	2.525	94945.177	94945.177	0	1.59	25.97	1406	124753	94945.177	0.31	26.28
				599	2.380	74690.609	74690.609	0	1.60	8.85	251	35870	74690.609	0.31	9.16
				889	2.429	100144.846	100144.846	0	2.08	16.99	1690	110260	100144.846	0.37	17.36
			250	730	2.513	114687.092	114687.092	0	2.05	444.26	41055	1608378	114687.092	0.39	444.65
				664	2.439	77564.545	77564.545	2.25E-15	2.04	3.04	1	16286	77564.545	0.36	3.41
				877	2.305	69490.995	69490.995	0	1.98	3.31	1	16429	69490.995	0.37	3.68
				1042	2.339	73225.330	73225.330	0	2.07	49.60	249	39194	73224.222	0.38	49.98
	20	7500	75	239	2.472	46347.686	46347.686	0	0.82	11.51	2852	184175	46334.289	0.17	11.69
				229	2.234	57331.216	57331.216	7.36E-11	0.81	453.90	10732	4028793	57236.648	0.19	454.09
				235	2.307	48860.329	48860.329	0	0.82	31.60	3723	253096	48860.329	0.19	31.78
				226	2.359	44849.463	44849.463	0	0.83	36.05	5200	275937	44849.463	0.19	36.24
				254	2.468	51242.790	51242.790	0	0.82	64.35	1970	294923	51242.790	0.21	64.56
			93	325	2.507	50829.276	50829.276	0	0.99	44.81	1941	187145	50829.276	0.22	45.03
				364	2.365	32757.821	32757.821	0	1.01	6.14	116	20470	32757.821	0.23	6.37
				258	2.451	56480.632	56480.632	0	0.99	1.90	1	9402	56480.632	0.21	2.11
				317	2.486	48887.526	48887.526	0	1.02	9.77	571	48655	48887.526	0.22	9.99
				223	2.278	70880.863	70880.863	0	0.98	13.77	223	38905	70880.863	0.23	14.00
			112	392	2.425	43399.451	43399.451	0	1.18	82.04	6709	859470	43396.931	0.27	82.31
				292	2.230	51990.892	51990.892	0	1.19	785.39	7979	4274918	51990.892	0.26	785.64
				437	2.211	35908.445	35908.445	0	1.22	11.67	532	38998	35908.445	0.28	11.95

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

			150	281	2.422	60113.357	60113.357	0	1.24	99.71	1475	370940	60113.357	0.26	99.98
				335	2.586	57350.992	57350.992	0	1.27	2.94	1	10546	57350.992	0.29	3.23
				440	2.409	59382.133	59382.133	0	1.62	33.76	1440	238421	59382.133	0.32	34.08
				483	2.523	53078.578	53078.578	0	1.59	52.34	1167	120041	53042.122	0.31	52.65
				559	2.316	55872.579	55872.579	0	1.61	23.53	826	78217	55872.579	0.33	23.86
				413	2.268	59993.545	59993.545	0	1.57	382.76	3809	1778503	59993.545	0.36	383.12
				402	2.423	42906.543	42906.543	0	1.62	21.95	4434	80571	42906.543	0.36	22.31
		10000	100	347	2.520	73714.021	73714.021	0	1.15	79.14	6897	519733	73714.021	0.24	79.38
				293	2.410	61105.889	61105.889	0	1.11	114.40	5059	531803	61105.889	0.24	114.64
				221	2.453	77478.810	77478.810	0	1.10	0.76	0	9747	77478.810	0.27	1.03
				223	2.543	87932.766	87932.766	0	1.15	3.49	1	15484	87932.766	0.27	3.76
				395	2.553	63491.247	63491.247	0	1.14	13.68	815	92415	63491.247	0.24	13.92
			125	400	2.343	88300.469	88300.469	0	1.45	12.11	4430	81503	88300.469	0.26	12.38
				326	2.413	82167.822	82167.822	0	1.39	358.69	4886	1297598	82167.822	0.29	358.98
				278	2.460	68987.859	68923.672	0.000930408	1.41	3600.01	47285	10180497	68987.859	0.30	3600.31
				272	2.519	92476.875	92476.875	1.34E-11	1.43	17.05	1	33656	92476.877	0.29	17.34
				599	2.350	67596.394	67596.394	0	1.43	89.15	10133	915924	67566.264	0.29	89.44
			150	521	2.294	64174.811	64174.811	0	1.66	38.18	1251	176407	64158.059	0.34	38.52
				467	2.371	63785.555	63785.555	0	1.67	46.41	2717	342677	63785.555	0.32	46.73
				474	2.335	63137.287	63137.287	2.33E-11	1.62	89.62	5715	346500	63137.287	0.32	89.94
				325	2.419	92407.164	92407.164	2.68E-15	1.62	3.11	1	15154	92407.164	0.35	3.45
				520	2.448	70013.955	70013.955	0	1.66	6.82	1	28588	70013.955	0.32	7.14
			200	719	2.537	67781.512	67781.512	0	2.22	305.56	6094	1204664	67781.512	0.42	305.98
				689	2.379	119853.575	119853.575	8.50E-16	2.09	25.87	1479	117335	119853.575	0.41	26.27
				470	2.500	92962.901	92962.901	1.41E-15	2.10	27.28	919	137438	92962.901	0.42	27.70
				481	2.452	98144.849	98144.849	0	2.07	60.51	410	128258	98144.849	0.42	60.93
				598	2.304	66866.825	66866.825	0	2.11	394.08	12213	1623246	66866.825	0.43	394.51
		12500	125	324	2.229	91704.871	91704.871	0	1.38	53.63	1651	162395	91704.871	0.30	53.93
				389	2.292	67237.229	67237.229	0	1.35	9.49	66	25389	67237.229	0.27	9.76
				316	2.521	125065.531	125065.531	0	1.38	87.52	3840	429998	125065.531	0.32	87.84
				388	2.466	72608.242	72608.242	0	1.44	2.85	1	12616	72608.242	0.31	3.17

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

				300	2.485	96903.203	96903.203	0	1.37	18.73	320	41576	96903.203	0.30	19.02
			156	479	2.455	91068.435	91068.435	0	1.76	1.20	1	10387	91068.435	0.33	1.53
				435	2.435	88064.565	88064.565	0	1.70	26.68	1439	87642	88064.565	0.35	27.03
				312	2.431	121092.643	121092.643	3.12E-15	1.71	4.27	1	27019	121092.643	0.34	4.62
				453	2.510	87322.198	87322.198	0	1.76	438.53	5790	1781362	87322.198	0.35	438.88
				555	2.448	93530.135	93530.135	0	1.76	415.17	4244	2472528	93473.342	0.33	415.50
			187	679	2.472	90763.206	90763.206	0	2.10	202.52	5652	1094221	90763.206	0.38	202.91
				489	2.516	94823.656	94823.656	4.76E-15	2.09	10.76	1	36765	94823.656	0.42	11.18
				566	2.320	70644.137	70644.137	0	2.06	2.97	1	16762	70644.137	0.41	3.38
				649	2.662	101152.158	101152.158	0	2.07	8.91	52	30302	101152.158	0.39	9.30
				647	2.375	100908.262	100908.262	0	2.01	16.78	1023	83277	100908.262	0.38	17.16
			250	671	2.397	84587.404	84587.404	0	2.58	96.66	5156	396894	84587.404	0.48	97.14
				734	2.447	74722.086	74722.086	2.73E-15	2.61	23.02	1	42751	74722.086	0.51	23.53
				808	2.297	98885.102	98885.102	0	2.68	3.34	1	10295	98885.102	0.51	3.85
				803	2.431	93000.291	93000.291	0	2.75	129.62	2344	320888	93000.291	0.52	130.15
				956	2.426	69198.945	69198.945	0	2.72	14.82	1	28464	69198.945	0.55	15.38

Table B.1 Continued: Solution Parameters for Consecutive Runs (multi-assignment)

$ K $	$ M $	$ J $	$ I $	total capacity	avg. degree of orders	combined objval	combined objbound	combined gap	combined gentime	combined runtime	combined nodenum	combined iternum
3	6	7500	75	133	2.472	86377.368	86377.368	1.35E-15	0.53	0.12	1	1389
				120	2.368	82722.803	82722.803	0	0.53	0.33	1	2377
				171	2.605	60423.469	60423.469	0	0.54	0.15	1	1348
				123	2.177	95246.018	95246.018	0	0.54	0.48	1	2955
				163	2.525	51602.068	51602.068	0	0.54	0.06	0	1259
			93	172	2.536	107246.460	107246.460	0	0.66	1.45	85	6960
				218	2.356	56103.591	56103.591	0	0.65	0.27	1	1643
				123	2.353	73863.347	73863.347	0	0.67	0.08	0	1879
				205	2.453	70246.725	70246.725	0	0.67	0.21	1	1703
				180	2.266	81438.230	81438.230	0	0.65	0.26	1	1337
			112	236	2.366	84289.453	84289.453	0	0.79	0.06	0	889
				260	2.374	58089.381	58089.381	0	0.78	0.42	1	2208
				301	2.516	73709.383	73709.383	0	0.80	0.36	1	2185
				226	2.523	85319.604	85319.604	1.71E-15	0.78	0.41	1	2513
				166	2.368	114394.550	114394.550	0	0.80	0.11	0	2419
			150	364	2.508	86312.922	86312.922	0	1.06	0.82	1	3522
				287	2.420	59946.543	59946.543	1.82E-15	1.06	0.47	1	3896
				365	2.426	94556.417	94556.417	0	1.06	2.77	938	37689
				316	2.391	63326.762	63326.762	2.53E-15	1.05	0.14	0	2516
				278	2.479	65239.833	65239.833	0	1.06	0.16	0	2842
		10000	100	186	2.290	104764.307	104764.307	0	0.78	0.39	1	2386
				164	2.393	84481.095	84481.095	0	0.72	0.09	0	1885
				181	2.490	99435.722	99435.722	1.61E-15	0.72	0.08	0	2086
				215	2.332	80100.231	80100.231	1.45E-15	0.70	0.08	0	1967
				153	2.436	97868.134	97868.134	0	0.72	0.20	1	1639
			125	200	2.400	87785.132	87785.132	0	0.87	0.12	0	2449
				222	2.408	54100.753	54100.753	0	0.87	0.71	1	3428
				211	2.486	80110.782	80110.782	4.54E-15	0.89	0.43	1	2348
				313	2.359	72891.256	72891.256	0	0.90	0.50	1	2231
				267	2.386	134511.383	134511.383	0	0.87	0.57	1	4675

Table B.2: Solution Parameters for Combined Problem (multi-assignment)

			150	213	2.417	96936.318	96936.318	0	1.06	0.13	0	3068
				341	2.551	64036.554	64036.554	0	1.06	0.12	0	2369
				341	2.471	62150.433	62150.433	0	1.06	0.16	0	2923
				386	2.515	62543.399	62543.399	0	1.07	0.11	0	2124
				343	2.386	118840.869	118840.869	0	1.07	0.44	1	3035
			200	297	2.448	87989.649	87989.649	0	1.41	0.23	0	4087
				398	2.322	36547.267	36547.267	0	1.42	0.18	0	3670
				290	2.382	122621.713	122621.713	0	1.41	0.24	0	4238
				342	2.464	88298.413	88298.413	0	1.40	0.27	0	4174
				332	2.439	88849.709	88849.709	1.64E-15	1.41	2.63	1	10093
		12500	125	249	2.419	77490.714	77490.714	0	0.90	0.38	1	2240
				175	2.470	131173.728	131173.728	0	0.87	0.11	0	2547
				345	2.405	74952.981	74952.981	0	0.89	0.06	0	961
				278	2.378	129199.684	129199.684	0	0.89	0.11	0	2170
				225	2.321	82740.208	82740.208	0	0.90	0.25	1	2649
			156	256	2.362	127480.737	127480.737	1.03E-15	1.11	0.49	1	3980
				347	2.504	75775.602	75775.602	0	1.11	0.15	0	2824
				314	2.419	134834.164	134834.164	0	1.10	0.16	0	3105
				290	2.490	105701.416	105701.416	0	1.11	0.35	1	2513
				265	2.506	149835.939	149835.939	7.77E-16	1.11	0.16	0	3425
			187	510	2.404	72551.373	72551.373	0	1.32	0.16	0	2698
				455	2.375	84313.967	84313.967	0	1.33	0.18	0	2624
				333	2.414	106388.980	106388.980	1.91E-15	1.32	0.16	0	3648
				374	2.512	98688.177	98688.177	0	1.32	1.89	44	10099
				420	2.239	100166.339	100166.339	0	1.33	0.41	1	2842
			250	589	2.327	59112.350	59112.350	0	1.75	0.99	1	5847
				448	2.427	94266.324	94266.324	0	1.76	0.25	0	4505
				514	2.322	83829.378	83829.378	0	1.77	1.18	1	7065
				366	2.362	119116.382	119116.382	0	1.76	0.87	1	6051
				643	2.330	145474.939	145474.939	0	1.80	1.81	1	6870
	9	7500	75	133	2.472	87501.326	87501.326	0	0.77	0.35	1	2019

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

				143	2.283	96331.858	96331.858	0	0.79	0.12	0	2405
				158	2.538	48832.862	48832.862	0	0.79	1.03	1	5491
				121	2.299	64733.312	64733.312	0	0.80	0.11	0	2294
				106	2.371	59107.423	59107.423	0	0.80	3.18	11	8143
			93	250	2.501	58443.360	58443.360	0	0.98	8.64	4774	149313
				169	2.502	112076.766	112076.766	0	1.00	19.39	3667	236914
				211	2.398	48894.828	48894.828	0	0.99	0.34	1	2543
				207	2.321	57676.581	57676.581	0	0.97	0.28	1	2551
				196	2.565	54894.708	54894.708	0	0.97	0.28	1	2510
			112	170	2.540	79487.104	79487.104	0	1.17	2.35	29	10049
				187	2.366	73259.606	73259.606	0	1.18	0.15	0	2999
				266	2.420	75573.325	75573.325	0	1.17	2.05	11	6592
				318	2.085	45935.901	45935.901	0	1.17	1.07	1	5124
				254	2.466	91113.872	91113.872	0	1.17	0.45	1	2340
			150	349	2.425	66983.454	66983.454	1.52E-15	1.54	0.26	0	3433
				324	2.423	72327.615	72327.615	0	1.59	0.54	1	4099
				281	2.425	64672.114	64672.114	0	1.59	1.36	1	5528
				344	2.493	72162.785	72162.785	0	1.57	1.58	1	3820
				275	2.292	63195.541	63195.541	0	1.58	1.60	1	8620
		10000	100	183	2.220	61513.644	61513.644	0	1.07	0.14	0	2423
				188	2.379	92161.667	92161.667	0	1.07	0.44	1	3381
				179	2.391	98144.909	98144.909	0	1.07	0.17	0	3262
				191	2.582	111863.973	111863.973	0	1.07	1.04	1	6066
				219	2.392	88040.469	88040.469	0	1.07	0.33	1	2706
			125	327	2.379	58578.067	58578.067	0	1.32	3.74	73	14679
				226	2.322	80202.717	80202.717	0	1.32	1.13	1	9678
				185	2.346	80829.367	80829.367	0	1.34	2.14	29	8380
				190	2.335	98770.769	98770.769	0	1.32	2.77	1	8769
				324	2.400	61676.618	61676.618	0	1.33	0.47	1	3032
			150	290	2.398	109379.153	109379.153	9.31E-16	1.58	0.39	0	5026
				369	2.539	140189.176	140189.176	0	1.56	0.16	0	2499

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

				329	2.436	97461.655	97461.655	0	1.60	0.48	1	4118	
				261	2.390	126752.180	126752.180	0	1.61	0.31	0	4461	
				301	2.391	78890.988	78890.988	1.29E-15	1.61	0.15	0	2822	
				200	357	2.391	98187.295	98187.295	4.74E-15	2.11	0.33	0	6089
				400	2.448	76380.941	76380.941	0	2.06	6.90	1264	42155	
				255	2.420	88742.433	88742.433	0	2.08	8.82	242	30881	
				382	2.475	72894.738	72894.738	0	2.12	0.83	1	5234	
				397	2.409	92878.181	92878.181	0	2.08	0.91	1	5965	
		12500	125	237	2.370	82128.218	82128.218	0	1.37	0.14	0	2490	
				252	2.458	98236.322	98236.322	0	1.33	0.23	0	3417	
				259	2.313	111694.377	111694.377	0	1.34	1.12	1	7735	
				272	2.393	179588.892	179588.892	0	1.34	0.18	0	3391	
				308	2.258	71821.161	71821.161	0	1.33	0.17	0	3023	
			156	411	2.572	108304.725	108304.725	0	1.66	2.32	1	10161	
				350	2.339	130475.215	130475.215	0	1.64	0.48	1	3968	
				202	2.549	166000.785	166000.785	7.01E-16	1.66	4.06	82	17048	
				345	2.433	94037.149	94037.149	0	1.66	0.24	0	3780	
				398	2.450	132156.132	132156.132	8.81E-16	1.64	5.06	360	28600	
			187	461	2.295	63593.724	63593.724	0	1.97	1.06	1	7679	
				507	2.364	76881.341	76881.341	0	2.00	4.19	74	18082	
				482	2.417	76272.958	76272.958	0	1.98	2.76	1	11263	
				431	2.390	106092.133	106092.133	0	1.97	0.33	0	4433	
				366	2.428	101536.906	101536.906	0	2.01	0.74	1	4313	
			250	468	2.388	71514.421	71514.421	0	2.57	2.39	1	11736	
				450	2.440	95684.060	95684.060	0	2.61	0.41	0	6459	
				565	2.473	151641.399	151641.399	4.03E-15	2.64	2.51	1	10865	
				519	2.373	146571.843	146571.843	0	2.65	4.13	1	20107	
				339	2.396	162337.605	162337.605	0	2.63	0.53	0	6690	
	12	7500	75	133	2.472	78043.047	78043.047	0	1.02	0.42	1	2471	
				163	2.347	83435.711	83435.711	0	1.01	2.33	89	12321	
				150	2.600	68432.154	68432.154	0	1.04	0.17	0	2835	

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			93	137	2.414	63303.223	63303.223	0	1.01	1.25	1	4229
				171	2.351	48619.666	48619.666	0	1.02	3.34	483	19372
				228	2.350	63786.355	63786.355	0	1.28	0.36	1	2997
				226	2.441	73858.746	73858.746	0	1.28	2.92	422	16232
				212	2.441	55875.639	55875.639	0	1.28	0.63	1	5010
				240	2.541	54322.621	54322.621	0	1.27	3.62	646	27117
			112	234	2.353	45017.130	45017.130	0	1.26	2.86	249	14992
				185	2.476	70709.285	70709.285	0	1.56	0.25	0	4105
				300	2.520	56315.576	56315.576	0	1.54	0.74	1	3605
				236	2.414	69726.732	69726.732	0	1.51	5.16	802	30040
				228	2.451	70923.680	70923.680	0	1.54	20.87	1986	124139
				275	2.324	68916.303	68916.303	0	1.55	4.23	1340	34601
			150	384	2.311	59257.105	59257.105	0	2.04	1.37	1	6649
				323	2.590	78383.962	78383.962	0	2.07	0.39	0	5463
				345	2.294	61553.308	61553.308	0	2.07	2.91	1	15560
				209	2.417	77038.357	77038.357	0	2.09	2.37	1	11603
				308	2.441	64817.052	64817.052	0	2.05	0.38	0	5271
			100	228	2.219	78532.651	78532.651	0	1.39	0.34	1	3045
				144	2.412	85714.535	85714.535	0	1.39	0.19	0	3300
				257	2.531	79631.044	79631.044	0	1.40	1.31	1	7952
				209	2.514	75234.878	75234.878	0	1.40	0.21	0	3285
				178	2.328	90663.523	90663.523	0	1.39	7.77	1	20697
			125	182	2.487	119348.413	119348.413	2.07E-15	1.70	1.25	1	9812
				284	2.540	59871.834	59871.834	0	1.73	1.38	1	7096
				246	2.429	94534.168	94534.168	0	1.74	0.22	0	3662
				235	2.328	76983.289	76983.289	0	1.74	4.51	156	16470
				294	2.407	90319.234	90319.234	5.94E-12	1.71	0.90	1	5173
			150	325	2.401	91061.683	91061.683	0	2.06	2.06	1	7992
				327	2.418	81925.570	81925.570	0	2.08	5.97	586	49053
				336	2.509	75128.065	75128.065	0	2.11	0.57	1	5645
				356	2.399	73646.811	73646.811	0	2.08	0.33	0	4338

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

4	8	7500	200	249	2.515	129740.849	129740.849	0	2.05	0.35	0	5769
				328	2.504	82430.149	82430.149	0	2.74	0.44	0	6592
				387	2.501	73024.026	73024.026	0	2.75	14.17	46	30510
				389	2.504	84510.044	84510.044	0	2.76	3.06	1	12349
				265	2.495	130688.437	130688.437	1.00E-15	2.74	5.38	1	29574
			125	380	2.308	75892.216	75892.216	0	2.73	12.46	177	34260
				293	2.511	99316.408	99316.408	0	1.76	0.47	1	3836
				235	2.411	129271.068	129271.068	0	1.74	10.23	796	61034
				158	2.411	132745.906	132745.906	0	1.73	0.76	1	5065
				210	2.316	91408.052	91408.052	1.75E-15	1.75	3.62	1	14898
			156	289	2.390	88077.808	88077.808	0	1.75	4.31	62	13017
				259	2.408	105689.686	105689.686	0	2.15	1.12	1	12639
				359	2.458	124509.404	124509.404	0	2.15	0.74	1	5592
				360	2.425	82554.459	82554.459	0	2.14	0.34	0	4706
				366	2.431	110556.380	110556.380	0	2.16	1.56	1	8036
			187	373	2.325	94924.823	94924.823	0	2.18	0.38	0	4131
				290	2.582	111335.693	111335.693	0	2.59	1.54	1	6973
				242	2.482	136689.097	136689.097	2.77E-15	2.59	1.07	1	7467
				425	2.313	92156.965	92156.965	0	2.61	1.05	1	6299
				490	2.410	115662.933	115662.933	0	2.53	8.14	528	48147
			250	272	2.400	152827.532	152827.532	0	2.60	242.16	34624	2002601
				490	2.381	74941.167	74941.167	0	3.39	0.63	0	4333
				332	2.459	128598.701	128598.701	0	3.50	3402.50	643209	18754938
				477	2.463	87356.346	87356.346	7.33E-15	3.47	1.64	1	5249
				374	2.399	135311.201	135311.201	0	3.41	1.28	1	6143
			75	518	2.406	108788.592	108788.592	0	3.36	3.23	1	14932
				188	2.472	61281.087	61281.087	0	1.10	0.24	1	1604
				192	2.430	53400.557	53400.557	0	1.13	0.49	1	2550
				211	2.562	43755.821	43755.821	0	1.13	0.50	1	2133
				258	2.257	38661.347	38661.347	0	1.13	0.23	1	1006
				176	2.507	58835.492	58835.492	0	1.13	0.39	1	2508

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

	8	10000	93	263	2.438	55837.003	55837.003	1.82E-15	1.40	0.99	1	4775
				247	2.315	78725.470	78725.470	0	1.39	1.31	1	5574
				318	2.359	44992.127	44992.127	0	1.40	0.81	1	4192
				254	2.340	48128.050	48128.050	0	1.40	6.94	1	20653
				257	2.285	45890.460	45890.460	0	1.40	4.38	427	23565
			112	301	2.465	62001.898	62001.898	2.23E-15	1.69	0.63	1	5289
				223	2.463	66117.316	66117.316	0	1.66	0.79	1	6418
				279	2.427	67543.657	67543.657	0	1.68	1.25	1	6139
				219	2.299	58161.044	58161.044	0	1.66	2.23	11	9780
				312	2.518	68534.137	68534.137	0	1.67	1.70	1	8485
			150	412	2.601	61966.907	61966.907	0	2.21	1.48	1	7954
				438	2.480	51326.905	51326.905	0	2.21	1.91	1	8560
				408	2.385	50262.290	50262.290	0	2.25	0.31	0	2889
				439	2.521	38288.229	38288.229	0	2.23	1.00	1	3883
				316	2.410	89237.120	89237.120	5.71E-15	2.24	1.88	1	9661
			100	265	2.468	72910.901	72910.901	0	1.52	0.23	0	2844
				213	2.442	64422.849	64422.849	0	1.50	1.35	22	5732
				256	2.423	69332.249	69332.249	0	1.50	2.66	228	11227
				248	2.457	64830.612	64830.612	0	1.51	6.01	147	13849
				358	2.361	75701.187	75701.187	0	1.54	0.15	0	1628
			125	226	2.431	96734.588	96734.588	0	1.88	1.94	1	7531
				360	2.454	70469.107	70469.107	0	1.86	1.28	1	7758
				265	2.278	87093.010	87093.010	0	1.89	10.62	213	26866
				306	2.505	75599.612	75599.612	0	1.86	5.95	466	34516
				292	2.488	96140.228	96140.228	0	1.88	0.86	1	4985
			150	430	2.536	70181.399	70181.399	0	2.27	1.06	1	6817
				354	2.252	59317.192	59317.192	0	2.27	15.05	3700	211760
				427	2.492	79780.243	79780.243	0	2.24	0.51	1	2022
				403	2.413	97559.271	97559.271	0	2.24	3.10	1	10400
				490	2.444	62817.191	62817.191	0	2.24	5.14	630	26418
			200	511	2.381	64480.371	64480.371	4.18E-15	2.98	1.42	1	3492

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

		12500		581	2.535	77575.893	77575.893	0	3.01	1.49	1	3959
				669	2.395	68033.660	68033.660	0	3.02	4.00	1	10292
				615	2.488	68854.233	68854.233	1.48E-15	2.93	1.11	1	3180
				545	2.412	70813.320	70813.320	0	2.99	21.51	681	135099
			125	270	2.321	94255.996	94255.996	2.01E-15	1.91	0.33	0	3568
				305	2.364	77572.419	77572.419	0	1.89	0.79	1	3526
				304	2.463	77676.024	77676.024	0	1.91	0.24	0	2695
				287	2.646	92306.418	92306.418	0	1.89	0.53	1	3299
			156	356	2.416	143109.415	143109.415	0	1.91	1.49	1	5560
				401	2.369	82927.798	82927.798	2.63E-15	2.36	0.37	0	3642
				414	2.406	106630.997	106630.997	0	2.37	5.22	2371	58994
				392	2.433	92801.906	92801.906	0	2.38	0.43	0	4382
			187	391	2.285	110392.233	110392.233	0	2.38	2.06	1	8368
				303	2.405	103757.701	103757.701	0	2.34	3.36	1	9826
				444	2.345	131512.241	131512.241	0	2.83	1.70	1	9693
				345	2.395	116110.772	116110.772	1.00E-15	2.81	2.62	1	7814
			250	500	2.366	97998.011	97998.011	0	2.76	0.39	0	2462
				544	2.459	78242.818	78242.818	0	2.81	2.10	1	7768
				399	2.314	103895.879	103895.879	0	2.82	0.88	1	2742
				492	2.297	94416.860	94416.860	0	3.71	3.51	1	5860
		7500	75	689	2.488	93303.200	93303.200	1.56E-15	3.67	1.44	1	3479
				640	2.373	76795.229	76795.229	0	3.74	1.26	1	3545
				620	2.497	96876.404	96876.404	0	3.74	8.53	244	26938
				662	2.526	100294.205	100294.205	0	3.75	2.00	1	8415
			93	292	2.435	54849.815	54849.815	0	2.10	21.38	1802	131003
				195	2.479	73715.479	73715.479	0	2.08	0.34	0	3955
			75	188	2.472	60324.893	60324.893	0	1.65	1.31	1	6023
				178	2.356	39194.131	39194.131	0	1.70	1.26	1	4254
				206	2.579	47202.056	47202.056	0	1.68	0.20	0	2387
				194	2.584	64699.135	64699.135	0	1.69	0.23	0	2774
			93	192	2.412	60057.012	60057.012	0	1.68	3.85	81	15453
				292	2.435	54849.815	54849.815	0	2.10	21.38	1802	131003
				195	2.479	73715.479	73715.479	0	2.08	0.34	0	3955
			93	292	2.435	54849.815	54849.815	0	2.10	21.38	1802	131003
				195	2.479	73715.479	73715.479	0	2.08	0.34	0	3955
				292	2.435	54849.815	54849.815	0	2.10	21.38	1802	131003
				195	2.479	73715.479	73715.479	0	2.08	0.34	0	3955

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

				224	2.590	65614.799	65614.799	0	2.11	3.23	1	11230		
				284	2.649	68140.009	68140.009	0	2.11	0.50	1	3249		
				290	2.429	55267.092	55267.092	0	2.07	0.82	1	3612		
				112	370	2.257	40578.638	40578.638	0	2.48	33.38	2455	147463	
					303	2.320	55612.299	55612.299	0	2.51	2.80	1	14106	
					364	2.433	84465.796	84465.796	1.21E-15	2.47	2.30	1	10767	
					236	2.508	70117.281	70117.281	0	2.50	10.10	322	42283	
					287	2.335	58280.375	58280.375	0	2.52	6.40	152	21241	
				150	269	2.475	76011.449	76011.449	0	3.29	9.60	154	24724	
					511	2.476	39198.743	39198.743	0	3.31	4.16	1	8363	
					517	2.309	60072.305	60072.305	0	3.34	15.90	1250	109919	
					326	2.417	87288.077	87288.077	0	3.26	1.01	0	5340	
					268	2.389	69761.425	69761.425	0	3.33	104.47	1902	319663	
				10000	100	327	2.311	56093.639	56093.639	0	2.26	8.93	2782	101611
						255	2.646	94999.150	94999.150	0	2.28	0.29	0	3392
						319	2.437	81354.803	81354.803	0	2.25	0.31	1	1593
						203	2.388	66898.278	66898.278	0	2.26	32.85	6100	234757
						288	2.378	59082.359	59082.359	0	2.25	7.50	1447	78961
					125	278	2.255	85903.171	85903.171	5.20E-12	2.81	33.12	3683	235863
						366	2.325	86915.128	86915.128	0	2.79	8.13	1453	52826
						358	2.411	88094.591	88094.591	0	2.80	2.80	1	12673
						331	2.376	77630.260	77630.260	1.31E-15	2.81	2.08	1	9977
						358	2.359	74540.346	74540.346	0	2.77	1.56	1	5799
				150	470	2.397	62896.337	62896.337	0	3.38	0.59	0	3025	
					465	2.367	91916.492	91916.492	0	3.39	9.74	495	42375	
					432	2.392	63025.243	63025.243	4.50E-15	3.34	1.78	1	8160	
					318	2.586	62365.164	62365.164	0	3.39	18.67	219	41994	
					349	2.467	87793.879	87793.879	0	3.35	0.62	0	3779	
				200	460	2.374	82586.380	82586.380	0	4.42	3.71	1	10759	
					347	2.415	93543.965	93543.965	0	4.38	2.12	1	6243	
					435	2.421	59955.634	59955.634	0	4.45	1.85	0	6387	

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

	12	12500	125	703	2.554	76188.221	76188.221	1.34E-15	4.44	4.70	1	11982
				507	2.439	85195.821	85195.821	8.54E-15	4.41	2.25	1	4048
				360	2.436	94719.599	94719.599	0	2.80	10.08	2769	86841
				242	2.514	137760.778	137760.778	0	2.85	2.53	1	6692
				370	2.324	77693.648	77693.648	0	2.83	5.06	1	13543
				315	2.559	114614.752	114614.752	0	2.80	2.09	1	11740
				181	2.355	115584.521	115584.521	0	2.82	32.19	2129	125190
			156	339	2.488	111086.456	111086.456	2.10E-15	3.52	1.81	1	9308
				455	2.374	69101.422	69101.422	0	3.42	2.02	1	8268
				478	2.448	72282.560	72282.560	0	3.50	6.61	783	30921
				425	2.539	148677.458	148677.458	0	3.48	5.20	200	16579
				334	2.361	78811.429	78811.429	0	3.48	2.33	1	13598
			187	447	2.458	90569.991	90569.991	0	4.20	20.62	1348	83820
				341	2.417	106862.571	106862.571	0	4.17	72.18	2037	212787
				415	2.225	104773.842	104773.842	0	4.21	30.00	340	53138
				467	2.425	95209.597	95209.597	0	4.18	0.87	0	4482
				422	2.314	90787.935	90787.935	0	4.14	1.00	0	6492
			250	555	2.315	107741.021	107741.021	0	5.51	1.73	0	8434
				681	2.373	69882.075	69882.075	0	5.57	4.16	1	16864
				668	2.369	88270.509	88270.509	3.46E-15	5.49	1.26	0	6483
				756	2.454	111066.150	111066.150	3.80E-15	5.54	0.63	0	3216
				551	2.328	98981.212	98981.212	0	5.51	3.56	1	15010
	16	7500	75	188	2.472	64225.120	64225.120	0	2.19	5.07	52	11757
				216	2.421	54345.398	54345.398	0	2.22	6.78	540	40662
				217	2.429	50763.363	50763.363	0	2.21	1.02	1	5569
				181	2.475	55381.952	55381.952	0	2.22	0.72	1	3977
				230	2.374	59524.845	59524.845	0	2.26	17.93	1494	158734
			93	185	2.305	58547.601	58547.601	0	2.77	3.92	1	11260
				230	2.434	56136.525	56136.525	0	2.76	1.08	1	7684
				251	2.397	44437.560	44437.560	0	2.77	16.79	1249	121484
				285	2.315	48668.507	48668.507	0	2.76	1.29	1	5327

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

				217	2.468	72999.643	72999.643	0	2.74	30.79	1804	132136
			112	330	2.415	61170.473	61170.473	0	3.30	2.86	1	10451
				346	2.351	61947.652	61947.652	0	3.30	2.51	1	9654
				337	2.319	44328.330	44328.330	2.80E-12	3.23	34.14	5643	265867
				234	2.417	55840.477	55840.477	0	3.29	244.22	12230	1153849
				301	2.457	59868.980	59868.980	0	3.30	0.47	0	5347
			150	418	2.537	54475.203	54475.203	0	4.38	14.56	285	36364
				439	2.534	44260.639	44260.639	0	4.36	11.33	369	36375
				540	2.572	72699.831	72699.831	0	4.29	19.44	133	30713
				486	2.498	57944.482	57944.482	0	4.39	10.35	635	25683
				358	2.312	61527.168	61527.168	0	4.32	14.63	1	27251
		10000	100	210	2.575	108031.493	108031.493	0	2.97	184.05	19459	1180721
				306	2.268	113376.330	113376.330	0	3.01	3.32	1	11127
				309	2.410	72731.619	72731.619	0	2.99	9.43	844	48348
				241	2.384	66529.240	66529.240	0	2.98	2179.54	234931	12188516
				195	2.466	86932.376	86932.376	0	2.97	104.74	3541	578895
			125	388	2.312	60453.804	60453.804	0	3.68	4.29	121	12738
				286	2.337	103272.192	103272.192	0	3.63	5.20	1	23783
				291	2.393	79037.273	79037.273	0	3.70	0.73	0	4341
				246	2.385	114347.433	114347.433	0	3.66	17.45	116	32305
				277	2.435	91425.999	91425.999	0	3.69	19.09	3688	141342
			150	300	2.354	86010.508	86010.508	0	4.34	4.10	1	11723
				429	2.345	107516.284	107516.284	0	4.38	4.57	1	18597
				416	2.334	65743.942	65743.942	0	4.33	5.11	1	10392
				398	2.315	61399.533	61399.533	0	4.35	4.42	1	15349
				396	2.505	69279.155	69279.155	2.31E-15	4.48	0.68	0	3020
			200	490	2.438	65569.213	65569.213	4.88E-15	5.82	1.11	0	6331
				589	2.383	87646.547	87646.547	0	5.82	6.06	1	14383
				278	2.334	91015.182	91015.182	0	5.73	78.57	506	117889
				472	2.326	76516.560	76516.560	0	5.75	36.70	143	59427
				638	2.517	77551.511	77551.511	0	5.83	15.21	115	31279

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

	16	12500	125	274	2.512	109120.499	109120.499	0	3.80	25.22	904	63806
				222	2.411	123941.067	123941.067	0	3.80	2.26	1	9795
				367	2.362	79483.663	79483.663	1.46E-15	3.80	4.00	21	9489
				398	2.424	85383.123	85383.123	1.53E-15	3.77	4.62	124	17357
				345	2.494	91660.727	91660.727	0	3.78	0.61	0	3595
			156	365	2.301	76751.166	76751.166	0	4.61	13.27	375	37033
				439	2.408	148046.965	148046.965	5.90E-16	4.66	2.02	1	7344
				434	2.433	85972.175	85972.175	5.75E-15	4.60	3.50	1	10446
				283	2.436	106965.528	106965.528	0	4.61	18.31	256	51465
				400	2.615	100466.736	100466.736	0	4.68	8.89	426	28834
			187	572	2.348	59492.052	59492.052	0	5.46	110.21	2388	320700
				428	2.400	106990.585	106990.585	0	5.55	1.23	0	7107
				315	2.345	141738.469	141738.469	3.70E-15	5.44	5.32	1	22546
				475	2.476	102938.909	102938.909	2.97E-15	5.50	4.91	1	21986
				442	2.442	94030.771	94030.771	0	5.53	6.62	1	19344
			250	717	2.302	85112.165	85112.165	4.10E-15	7.17	8.98	1	15661
				671	2.277	77891.842	77891.842	1.20E-11	7.33	10.63	1	17668
				623	2.398	108890.982	108890.982	0	7.23	6.97	1	16480
				495	2.409	115381.388	115381.388	0	7.22	94.81	2275	221534
				591	2.463	103086.136	103086.136	0	7.17	5.20	1	13543
5	10	7500	75	239	2.472	54135.678	54135.678	0	2.00	4.24	451	19650
				269	2.503	35326.863	35326.863	0	2.04	1.80	200	9458
				222	2.398	57875.507	57875.507	0	2.00	1.13	1	3194
				261	2.235	34907.426	34907.426	0	2.04	1.06	1	2778
				237	2.333	47491.472	47491.472	0	2.06	3.90	59	9289
			93	281	2.515	40662.542	40662.542	0	2.50	1.22	1	4488
				168	2.554	70224.541	70224.541	0	2.52	13.80	4209	148366
				357	2.171	27421.403	27421.403	0	2.49	0.67	1	2363
				337	2.388	34165.193	34165.193	2.77E-11	2.52	21.68	1534	78934
				281	2.551	40781.767	40781.767	0	2.55	6.96	89	18226
			112	374	2.333	42965.145	42965.145	0	3.07	2.11	1	8311

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

				324	2.530	75855.490	75855.490	0	3.06	2.23	1	5230
				272	2.379	72623.687	72623.687	0	3.06	11.24	1590	61894
				314	2.560	54212.439	54212.439	0	3.07	4.02	1	14441
				296	2.562	45344.074	45344.074	0	3.04	4.13	1	8942
			150	439	2.239	42717.874	42717.874	0	3.97	46.95	623	177555
				461	2.567	63998.950	63998.950	0	4.13	9.97	204	28688
				551	2.627	38777.838	38777.838	0	4.08	1.32	1	2688
				515	2.283	38006.685	38006.685	0	4.04	4.75	1	5314
				486	2.229	68540.378	68540.378	1.40E-11	3.99	45.70	1661	148341
			100	400	2.167	58926.885	58926.885	0	2.76	7.46	378	39711
				268	2.317	97987.368	97987.368	0	2.76	3.57	1	8008
				277	2.409	68593.023	68593.023	0	2.73	2.32	1	1633
				267	2.473	73079.871	73079.871	0	2.70	2.52	41	5440
				299	2.297	62178.356	62178.356	0	2.77	2.98	1	9889
			125	338	2.343	60347.116	60347.116	0	3.40	1.81	1	7544
				454	2.185	43728.476	43728.476	0	3.38	3.97	1	13354
				523	2.494	60770.206	60770.206	0	3.42	6.07	686	28632
				459	2.440	51995.107	51995.107	0	3.37	1.37	1	2420
				463	2.392	63390.296	63390.296	0	3.41	7.11	45	17374
			150	517	2.454	75181.327	75181.327	0	4.06	0.90	1	3138
				414	2.242	60155.366	60155.366	0	4.04	1.22	1	3075
				539	2.449	68793.518	68793.518	0	4.06	3.52	1	8395
				443	2.430	65527.119	65527.119	0	4.06	2.19	1	8052
				527	2.432	75956.292	75956.292	0	4.12	5.70	1	13428
			200	505	2.410	75833.842	75833.842	3.65E-15	5.33	1.43	0	6624
				767	2.343	44990.988	44990.988	0	5.43	30.40	1988	308876
				750	2.367	65916.980	65916.980	0	5.32	3.44	1	8245
				554	2.504	62503.123	62503.123	0	5.35	20.08	1299	53302
				528	2.354	56117.691	56117.691	0	5.34	1.11	0	5084
		12500	125	355	2.402	80749.733	80749.733	0	3.44	2.86	1	9633
				416	2.440	74211.668	74211.668	0	3.43	1.06	1	3699

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			156	374	2.375	90177.056	90177.056	1.13E-15	3.42	0.49	0	2958
				382	2.375	111914.644	111914.644	0	3.41	11.73	355	38639
				316	2.546	96264.454	96264.454	0	3.42	7.47	47	24415
				586	2.414	69661.016	69661.016	0	4.27	9.65	132	28240
				562	2.311	87679.180	87679.180	1.99E-15	4.21	6.90	9	13006
				614	2.450	79108.895	79108.895	0	4.26	1.77	1	3880
				413	2.404	94234.326	94234.326	0	4.25	13.09	55	27149
				426	2.406	106143.522	106143.522	0	4.26	5.24	1	16299
			187	551	2.331	126377.320	126377.320	0	5.07	13.19	38	21010
				473	2.494	102062.323	102062.323	0	5.09	10.92	17	15065
				618	2.592	109457.678	109457.678	0	5.09	4.11	1	13474
				786	2.404	71912.370	71912.370	0	5.09	1.95	1	2777
				639	2.366	101751.294	101751.294	0	5.08	0.79	0	3522
			250	618	2.383	90228.826	90228.826	0	6.62	1.36	0	7350
				793	2.391	67230.834	67230.834	0	6.79	5.11	1	11976
				562	2.403	100229.919	100229.919	0	6.71	3.58	1	9566
				858	2.392	104628.639	104628.639	0	6.76	13.51	9	23653
				630	2.348	112074.644	112074.644	4.54E-15	6.74	7.27	1	14452
	15	7500	75	239	2.472	51292.566	51292.566	0	2.96	8.82	1239	56412
				207	2.375	60426.461	60426.461	0	3.01	1.64	1	7292
				258	2.462	58470.376	58470.376	0	3.02	1.27	1	6825
				253	2.761	73785.749	73785.749	0	3.07	2.57	1	5931
				266	2.485	46687.916	46687.916	0	3.05	0.98	1	1831
			93	257	2.375	46314.464	46314.464	0	3.70	43.89	2142	264520
				302	2.566	70666.882	70666.882	0	3.77	34.88	5521	211196
				203	2.350	69143.164	69143.164	0	3.75	1.74	1	6967
				320	2.445	54081.877	54081.877	0	3.73	4.05	1	7309
				278	2.373	44898.711	44898.711	0	3.71	2.26	1	9896
			112	291	2.344	55228.513	55228.513	0	4.35	14.69	178	28150
				301	2.279	47528.036	47528.036	0	4.42	2.87	1	11124
				370	2.431	48486.560	48486.560	0	4.48	4.34	1	10021

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			150	388	2.421	45372.061	45372.061	0	4.36	22.65	4967	161980	
				411	2.520	57630.625	57630.625	0	4.51	3.59	1	9072	
				444	2.287	48741.890	48741.890	0	5.71	7.24	1	11949	
				437	2.372	51106.646	51106.646	1.99E-15	5.88	6.37	1	16523	
				545	2.451	49077.768	49077.768	0	5.90	7.38	1	14731	
				409	2.479	57794.175	57794.175	0	6.00	6.39	1	18728	
				425	2.392	45689.112	45689.112	0	5.88	20.58	56	29064	
			10000	100	278	2.499	74888.477	74888.477	0	4.08	76.97	10466	526376
					253	2.522	75982.889	75982.889	0	4.03	1.55	1	4396
					287	2.379	70786.218	70786.218	0	4.02	3.57	1	11935
					188	2.264	97963.729	97963.729	0	4.08	632.78	67546	3437627
					342	2.446	90402.688	90402.688	0	4.08	0.68	0	2993
				125	365	2.426	86607.897	86607.897	0	5.00	213.48	15987	1064365
					326	2.277	62313.250	62313.250	0	5.02	1.77	1	5364
					267	2.406	67218.825	67218.825	0	5.04	4.21	1	18220
					342	2.299	61808.159	61808.159	0	5.04	22.98	2654	94394
					282	2.297	105700.714	105700.714	1.24E-15	5.07	7.77	70	15155
				150	332	2.453	72651.519	72651.519	4.21E-15	6.06	5.19	1	9127
					367	2.404	86077.898	86077.898	0	6.16	72.48	3206	207268
					588	2.297	73277.967	73277.967	0	6.11	76.39	1898	247260
					572	2.518	54931.712	54931.712	0	6.15	3.20	1	4544
					525	2.415	87803.376	87803.376	0	6.14	25.77	1256	67442
				200	484	2.247	66405.037	66405.037	4.82E-15	8.01	4.92	1	10635
					703	2.458	87486.090	87486.090	0	8.13	11.47	443	32716
					664	2.418	81073.626	81073.626	0	8.02	516.78	6224	1114958
					521	2.467	95732.907	95732.907	0	8.22	2.01	0	8618
					735	2.427	48671.366	48671.366	0	8.03	15.65	1	18191
				12500	125	441	2.477	79017.679	79017.679	0	4.99	18.90	918
			349			2.471	101432.174	101432.174	0	5.03	3.84	1	12397
			296			2.502	104274.252	104274.252	0	5.09	1.07	0	5494
			452			2.249	63727.257	63727.257	0	5.09	31.41	1116	117495

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			156	385	2.488	80127.615	80127.615	0	5.10	28.77	3971	168151
				479	2.428	77439.124	77439.124	0	6.42	20.12	1931	90849
				463	2.493	96528.245	96528.245	0	6.34	11.16	1	20392
				585	2.453	77962.285	77962.285	0	6.38	2.69	1	4050
				476	2.432	93080.529	93080.529	0	6.37	109.63	3181	269862
			187	452	2.399	75772.819	75772.819	0	6.34	2.56	1	5259
				486	2.366	88153.995	88153.995	0	7.41	18.93	100	40010
				652	2.285	88562.046	88562.046	0	7.46	6.15	1	10647
				464	2.428	85240.914	85240.914	0	7.50	6.64	1	17822
				481	2.525	94929.793	94929.793	0	7.47	97.75	1380	199208
			250	599	2.380	74690.609	74690.609	0	7.52	12.51	1	20843
				889	2.429	100137.824	100137.824	0	9.86	54.26	2597	172644
				730	2.513	114669.120	114669.120	0	10.01	788.96	24914	1715121
				664	2.439	77564.545	77564.545	4.69E-15	10.01	7.52	1	6901
				877	2.305	69490.995	69490.995	0	9.73	6.74	1	12471
			75	1042	2.339	73224.222	73224.222	0	9.74	39.62	1138	110962
				239	2.472	46313.999	46313.999	0	3.95	13.20	1158	81431
				229	2.234	57179.476	57171.266	0.000143592	4.01	3600.04	114639	22674431
				235	2.307	48850.836	48850.836	0	4.03	49.21	7901	337308
				226	2.359	44849.463	44849.463	0	4.07	32.93	1122	98405
				254	2.468	51242.790	51242.790	0	4.03	132.85	6031	673398
			93	325	2.507	50829.276	50829.276	0	4.90	20.41	207	40838
				364	2.365	32757.821	32757.821	0	4.99	13.66	227	36598
				258	2.451	56480.632	56480.632	0	4.88	5.96	1	6852
				317	2.486	48887.526	48887.526	0	4.99	18.46	170	33738
				223	2.278	70880.863	70880.863	0	4.76	40.94	1367	105921
			112	392	2.425	43384.162	43384.162	0	5.85	136.35	4289	447370
				292	2.230	51989.358	51989.358	0	5.84	2278.71	20769	5525348
				437	2.211	35908.445	35908.445	0	5.77	26.35	1223	64225
				281	2.422	60111.694	60111.694	0	5.78	1963.31	24598	2959850
				335	2.586	57350.992	57350.992	0	5.96	3.52	1	9533

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			150	440	2.409	59354.224	59354.224	4.17E-15	7.71	44.48	1436	233014
				483	2.523	53042.122	53042.122	0	7.73	25.02	46	36632
				559	2.316	55872.579	55872.579	0	7.60	45.43	844	130651
				413	2.268	59993.545	59679.681	0.005231626	7.73	3600.02	21294	3770950
				402	2.423	42906.543	42906.543	0	7.70	39.03	1512	64131
		10000	100	347	2.520	73714.021	73714.021	0	5.40	1150.49	34815	2588987
				293	2.410	61105.889	61105.889	0	5.44	345.90	24883	2214361
				221	2.453	77478.810	77478.810	0	5.40	1.78	0	7154
				223	2.543	87932.766	87932.766	0	5.40	10.09	1	21935
				395	2.553	63487.644	63487.644	0	5.42	71.19	4048	329422
			125	400	2.343	88300.469	88300.469	0	6.60	19.04	301	33410
				326	2.413	82167.822	82167.822	0	6.59	315.84	1391	449790
				278	2.460	68987.859	68846.783	0.00204494	6.74	3600.01	19855	6433449
				272	2.519	92476.875	92476.875	0	6.70	42.24	25	57328
				599	2.350	67566.264	67566.264	0	6.75	58.27	4216	387753
			150	521	2.294	64156.017	64156.017	0	7.87	19.71	515	32992
				467	2.371	63785.555	63785.555	0	7.93	56.59	1536	190366
				474	2.335	63137.287	63137.287	0	7.92	42.83	875	114995
				325	2.419	92407.164	92407.164	0	7.95	7.87	1	20118
				520	2.448	70013.955	70013.955	1.87E-15	7.90	15.17	1	24015
			200	719	2.537	67781.512	67781.512	0	10.67	488.59	5306	866292
				689	2.379	119853.575	119853.575	0	10.26	51.78	1840	131820
				470	2.500	92962.901	92962.901	0	10.41	114.52	2090	445888
				481	2.452	98144.849	98144.849	0	10.33	94.16	364	140147
				598	2.304	66866.825	66866.825	0	10.38	652.39	13539	1567089
		12500	125	324	2.229	91704.871	91704.871	0	6.79	153.98	4637	470356
				389	2.292	67237.229	67237.229	0	6.68	15.28	58	19997
				316	2.521	125065.531	125065.531	0	6.81	145.50	3055	315733
				388	2.466	72608.242	72608.242	0	6.83	10.08	1	15534
				300	2.485	96903.203	96903.203	0	6.68	354.26	8830	1270865
			156	479	2.455	91068.435	91068.435	0	8.50	3.33	1	7649

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

			187	435	2.435	88052.806	88052.806	4.30E-15	8.29	12.68	1	40865
				312	2.431	121092.643	121092.643	3.61E-15	8.40	9.46	1	26470
				453	2.510	87322.198	87255.033	0.000769162	8.35	3600.75	20869	7211225
				555	2.448	93460.526	93460.526	0	8.36	17.80	1	32723
				679	2.472	90724.175	90724.175	0	9.85	902.13	23920	2955711
				489	2.516	94823.058	94823.058	7.06E-15	9.84	27.74	1	52111
				566	2.320	70644.137	70644.137	0	9.83	10.09	1	23556
				649	2.662	101148.243	101148.243	0	10.08	17.37	98	26099
			250	647	2.375	100908.262	100908.262	0	9.86	88.48	2068	199529
				671	2.397	84587.404	84514.257	0.000864753	12.86	3600.07	21460	3694798
				734	2.447	74722.086	74722.086	9.52E-12	13.00	38.16	43	46989
				808	2.297	98885.102	98885.102	1.18E-15	12.81	12.03	1	21017
				803	2.431	93000.291	93000.291	0	13.12	92.74	2627	228211
				956	2.426	69198.945	69198.945	0	13.00	23.21	1	28178

Table B.2 Continued: Solution Parameters for Combined Problem (multi-assignment)

$ K $	$ M $	$ J $	$ I $	avg. degree of orders	assortment objval	assortment objbound	assortment gap	assortment gentime	assortment runtime	assortment nodenum	assortment iternum	consecutive run cost	consolidation time	total time
3	6	7500	75	2.472	90372.308	90372.308	0	0.16	0.04	0	155	90372.308	0.02	0.06
				2.429	101523.304	101523.304	0	0.16	0.05	1	148	101523.304	0.02	0.07
				2.398	63320.704	63320.704	0	0.16	0.05	1	153	63320.704	0.02	0.07
				2.252	72321.353	72321.353	0	0.16	0.04	0	134	72321.353	0.02	0.06
				2.287	86470.596	86470.596	0	0.16	0.04	0	163	86470.596	0.02	0.06
			93	2.424	82254.675	82254.675	0	0.19	0.56	1167	8203	82254.675	0.02	0.58
				2.500	118592.467	118592.467	0	0.20	0.06	1	177	118592.467	0.02	0.09
				2.590	94931.317	94931.317	0	0.21	0.10	1	305	94931.317	0.02	0.13
				2.627	96141.696	96141.696	0	0.19	0.08	1	232	96141.696	0.02	0.11
				2.456	75312.416	75312.416	0	0.20	0.05	0	185	75312.416	0.02	0.08
			112	2.468	71688.192	71688.192	0	0.25	0.43	1	1112	71688.192	0.03	0.46
				2.396	85501.859	85501.859	0	0.23	0.15	1	532	85501.859	0.03	0.18
				2.591	104510.595	104510.595	0	0.25	0.07	0	215	104510.595	0.03	0.10
				2.405	78809.238	78809.238	0	0.23	0.12	1	243	78809.238	0.03	0.14
				2.410	81046.420	81046.420	0	0.25	0.12	1	347	81046.420	0.03	0.15
			150	2.503	115587.503	115587.503	0	0.31	0.09	0	255	115587.503	0.04	0.12
				2.411	85110.366	85110.366	0	0.31	0.22	1	389	85110.366	0.04	0.26
				2.539	113704.446	113704.446	0	0.31	0.15	1	378	113676.627	0.04	0.19
				2.275	101760.519	101760.519	0	0.31	0.29	126	1567	101760.519	0.04	0.33
				2.418	111355.595	111355.595	0	0.31	0.39	1	896	111355.595	0.04	0.42
		10000	100	2.553	108537.213	108537.213	0	0.21	0.06	0	152	108537.213	0.03	0.08
				2.191	118997.435	118997.435	0	0.23	0.05	0	163	118997.435	0.03	0.08
				2.467	130299.122	130299.122	0	0.21	0.08	1	203	130299.122	0.02	0.11
				2.609	121736.496	121736.496	0	0.21	0.09	1	262	121736.496	0.03	0.12
				2.559	94306.484	94306.484	0	0.23	0.06	0	200	94306.484	0.03	0.08
			125	2.331	117590.373	117590.373	0	0.26	0.07	0	212	117590.373	0.03	0.10
				2.602	122302.335	122302.335	0	0.26	0.12	1	304	122302.335	0.03	0.15
				2.396	77865.137	77865.137	0	0.28	0.08	0	231	77865.137	0.03	0.11
				2.573	153451.890	153451.890	7.59E-16	0.26	0.10	1	205	153451.890	0.03	0.13
				2.310	108998.027	108998.027	0	0.26	0.07	0	224	108998.027	0.03	0.10

Table B.3: Solution Parameters for Consecutive Runs (single-assignment)

			150	2.450	124720.724	124720.724	0	0.31	0.09	0	230	124720.724	0.04	0.13
				2.507	146070.385	146070.385	0	0.31	0.16	1	410	146070.385	0.04	0.20
				2.459	116724.628	116724.628	0	0.31	0.14	1	351	116724.628	0.04	0.18
				2.474	171991.121	171991.121	0	0.31	0.20	1	517	171991.121	0.04	0.24
				2.521	182028.813	182028.813	0	0.31	0.09	0	271	182028.813	0.04	0.13
			200	2.325	96943.480	96943.480	0	0.41	0.13	0	328	96943.480	0.05	0.18
				2.445	100620.795	100620.795	0	0.41	0.15	1	310	100620.795	0.05	0.20
				2.502	160560.262	160560.262	0	0.41	0.13	0	405	160560.262	0.05	0.18
				2.505	85432.376	85432.376	0	0.44	0.13	0	291	85432.376	0.05	0.18
				2.341	128136.446	128136.446	0	0.44	0.13	0	328	128136.446	0.05	0.18
		12500	125	2.326	156948.732	156948.732	0	0.26	0.07	0	202	156948.732	0.03	0.10
				2.435	216585.344	216585.344	0	0.26	0.08	1	265	216585.344	0.03	0.11
				2.419	142980.900	142980.900	0	0.28	0.14	1	395	142980.900	0.03	0.17
				2.463	251643.673	251643.673	0	0.28	0.14	1	396	251643.673	0.03	0.17
				2.409	126321.081	126321.081	0	0.26	0.13	1	411	126321.081	0.03	0.17
			156	2.547	120327.133	120327.133	0	0.32	0.09	0	221	120327.133	0.04	0.13
				2.407	164527.934	164527.934	0	0.32	0.13	1	297	164527.934	0.04	0.17
				2.536	173869.027	173869.027	0	0.32	0.13	1	358	173869.027	0.04	0.17
				2.491	174129.831	174129.831	0	0.35	0.12	1	258	174129.831	0.04	0.16
				2.348	129519.871	129519.871	0	0.35	0.14	1	344	129519.871	0.04	0.18
			187	2.417	134666.257	134666.257	0	0.39	0.12	0	354	134666.257	0.05	0.16
				2.322	99244.555	99244.555	0	0.42	0.20	1	337	99198.747	0.05	0.25
				2.364	184526.638	184526.638	0	0.39	0.27	1	662	184526.638	0.05	0.32
				2.446	192121.048	192121.048	0	0.41	0.63	11	1519	192121.048	0.05	0.68
				2.488	144846.174	144846.174	0	0.42	0.50	13	1029	144846.174	0.05	0.55
			250	2.396	152210.899	152210.899	0	0.52	0.21	1	354	152210.899	0.06	0.27
				2.382	114709.556	114709.556	0	0.54	0.18	0	415	114709.556	0.06	0.24
				2.491	140329.234	140329.234	0	0.52	0.22	1	421	140329.234	0.06	0.28
				2.361	163782.780	163782.780	0	0.51	0.18	0	387	163782.780	0.06	0.24
				2.504	183209.748	183209.748	0	0.55	0.87	1	1387	183209.748	0.06	0.93
	9	7500	75	2.472	99548.393	99548.393	0	0.23	0.06	0	191	99548.393	0.03	0.09

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			93	2.539	101987.195	101987.195	0	0.24	1.11	1070	7720	101649.210	0.03	1.14
				2.309	112489.644	112489.644	0	0.24	0.08	1	162	112489.644	0.03	0.11
				2.297	78836.833	78836.833	0	0.24	0.09	1	195	78836.833	0.03	0.12
				2.501	103481.553	103481.553	0	0.24	0.10	1	256	103481.553	0.03	0.13
				2.424	78645.910	78645.910	0	0.29	0.09	0	277	78645.910	0.04	0.12
				2.652	102275.114	102275.114	0	0.29	0.08	0	206	102275.114	0.04	0.12
				2.558	159192.048	159192.048	0	0.29	0.12	1	228	159192.048	0.03	0.16
				2.676	81468.999	81468.999	0	0.29	0.12	1	265	81468.999	0.04	0.16
			112	2.556	114039.109	114039.109	0	0.29	0.08	0	180	114039.109	0.04	0.12
				2.568	88794.783	88794.783	0	0.37	0.22	1	446	88794.783	0.04	0.27
				2.324	87419.169	87419.169	0	0.34	0.67	40	1556	87419.169	0.04	0.71
				2.370	105484.510	105484.510	0	0.35	2.11	1558	15636	105484.510	0.04	2.15
				2.368	95556.440	95556.440	0	0.35	0.34	1	566	95556.440	0.04	0.38
				2.327	93458.891	93458.891	0	0.37	3.13	1398	16709	93288.758	0.05	3.17
			150	2.434	88638.565	88638.565	0	0.46	0.16	0	237	88638.565	0.06	0.21
				2.144	71372.055	71372.055	0	0.47	5.64	5045	74041	71372.055	0.06	5.70
				2.530	89316.271	89316.271	0	0.46	0.51	178	2403	89316.271	0.06	0.57
				2.261	82362.085	82362.085	0	0.45	0.86	684	11597	82362.085	0.06	0.91
				2.401	61964.769	61964.769	0	0.46	0.43	1	804	61964.769	0.06	0.49
		10000	100	2.503	122177.677	122177.677	0	0.32	0.09	0	185	122177.677	0.04	0.13
				2.364	114903.634	114903.634	0	0.32	0.13	1	244	114903.634	0.04	0.16
				2.263	138542.514	138542.514	0	0.32	0.09	0	191	138542.514	0.04	0.13
				2.374	93132.379	93132.379	0	0.32	0.09	0	202	93132.379	0.04	0.13
				2.377	116410.671	116410.671	0	0.32	0.52	12	1080	116410.671	0.04	0.56
			125	2.454	94144.795	94144.795	0	0.40	0.13	0	243	94144.795	0.05	0.18
				2.401	122945.512	122945.512	0	0.42	2.25	1404	14335	122945.512	0.05	2.30
				2.426	109341.535	109341.535	0	0.40	0.13	0	202	109341.535	0.05	0.18
				2.519	202521.985	202521.985	0	0.42	0.17	1	303	202521.985	0.05	0.21
				2.443	93212.180	93212.180	0	0.39	0.18	1	324	93212.180	0.05	0.23
			150	2.388	122574.151	122574.151	0	0.49	0.29	1	465	122574.151	0.06	0.34
				2.501	112451.809	112451.809	0	0.46	0.23	1	393	112451.809	0.06	0.29

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			200	2.439	115368.167	115368.167	0	0.47	0.17	0	280	115368.167	0.06	0.22
				2.516	103723.072	103723.072	0	0.47	0.83	30	1419	103723.072	0.06	0.88
				2.418	87312.556	87312.556	0	0.47	1.99	1237	9419	87300.480	0.06	2.05
				2.533	132889.452	132889.452	0	0.62	0.23	0	309	132889.452	0.07	0.30
				2.404	83360.386	83360.386	0	0.62	0.23	0	382	83360.386	0.07	0.31
				2.393	135235.450	135235.450	0	0.62	0.23	0	344	135235.450	0.08	0.31
				2.434	100188.741	100188.741	0	0.65	0.37	1	589	100188.741	0.07	0.44
				2.431	90747.233	90747.233	0	0.65	7.34	1404	35648	90747.233	0.07	7.41
		12500	125	2.455	118201.393	118201.393	0	0.40	0.13	0	245	118201.393	0.05	0.18
				2.443	146332.542	146332.542	0	0.40	0.13	0	220	146332.542	0.05	0.18
				2.423	137699.882	137699.882	0	0.42	0.18	1	157	137699.882	0.05	0.22
				2.376	144850.755	144850.755	0	0.40	0.13	0	238	144850.755	0.05	0.18
				2.477	152499.112	152499.112	0	0.42	0.66	860	9101	152499.112	0.05	0.71
			156	2.455	200487.338	200487.338	5.81E-16	0.49	0.28	1	577	200487.338	0.06	0.34
				2.515	181522.723	181522.723	4.81E-16	0.49	2.67	1242	19721	181522.723	0.06	2.73
				2.481	169036.256	169036.256	0	0.52	1.95	1311	13236	169036.256	0.06	2.01
				2.315	162837.903	162837.903	0	0.52	0.18	0	332	162837.903	0.06	0.24
				2.553	187633.122	187633.122	0	0.52	0.17	0	222	187633.122	0.06	0.23
			187	2.315	157625.120	157625.120	0	0.61	0.21	0	269	157625.120	0.07	0.28
				2.371	216674.719	216674.719	0	0.61	0.31	1	340	216674.719	0.07	0.38
				2.441	182306.730	182306.730	0	0.58	0.27	1	402	182306.730	0.07	0.34
				2.436	118893.809	118893.809	0	0.61	1.25	80	3396	118893.809	0.07	1.32
				2.412	156522.859	156522.859	0	0.62	0.70	380	4418	156522.859	0.07	0.77
			250	2.447	126933.207	126933.207	0	0.78	0.63	1	462	126933.207	0.09	0.72
				2.476	122558.594	122558.594	0	0.81	0.33	0	419	122558.594	0.09	0.42
				2.292	172140.940	172140.940	0	0.78	0.33	0	570	172140.940	0.09	0.43
				2.474	175778.681	175778.681	0	0.77	3.62	1125	13382	175778.681	0.09	3.71
				2.472	201243.122	201243.122	4.34E-16	0.81	0.33	0	441	201243.122	0.09	0.42
	12	7500	75	2.472	95973.182	95973.182	0	0.31	0.09	0	202	95973.182	0.03	0.13
				2.413	74397.871	74397.871	0	0.31	0.47	1	875	74397.871	0.03	0.50
				2.464	98169.680	98169.680	0	0.31	0.14	1	226	98169.680	0.04	0.18

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				93	2.454	106210.655	106210.655	0	0.32	0.16	1	233	106210.655	0.04	0.19
					2.301	60436.010	60436.010	0	0.31	0.18	1	370	60428.426	0.03	0.22
					2.505	91030.161	91030.161	0	0.38	0.27	1	368	91030.161	0.05	0.32
					2.384	124826.254	124826.254	0	0.39	1.40	1089	13952	124826.254	0.05	1.44
					2.408	79476.038	79476.038	0	0.41	0.14	1	217	79476.038	0.05	0.18
					2.471	101177.754	101177.754	0	0.38	0.12	0	221	101177.754	0.05	0.17
					2.441	112145.604	112145.604	0	0.41	1.43	4990	43591	112145.604	0.05	1.47
				112	2.454	85865.879	85865.879	0	0.46	0.23	1	278	85865.879	0.06	0.28
					2.463	85630.256	85630.256	0	0.46	0.20	1	307	85630.256	0.06	0.26
					2.270	103052.798	103052.798	0	0.46	0.15	0	262	103052.798	0.06	0.21
					2.429	98174.810	98174.810	0	0.46	0.28	1	428	98174.810	0.06	0.33
					2.311	106753.554	106753.554	0	0.46	0.27	1	502	106753.554	0.06	0.33
				150	2.539	89031.031	89031.031	0	0.62	2.34	3763	37283	89031.031	0.08	2.41
					2.485	77700.946	77700.946	0	0.61	0.66	1	901	77700.946	0.08	0.74
					2.401	71860.621	71860.621	0	0.64	0.23	0	351	71860.621	0.08	0.31
					2.336	96013.221	96013.221	0	0.64	1.75	2509	25419	96013.221	0.08	1.83
					2.418	81859.599	81859.599	0	0.64	0.36	1	435	81859.599	0.08	0.44
			10000	100	2.436	118070.971	118070.971	0	0.42	0.14	0	266	118070.971	0.05	0.19
					2.461	123840.789	123840.789	0	0.42	0.17	1	273	123840.789	0.05	0.22
					2.537	174605.521	174605.521	0	0.45	0.24	1	389	174605.521	0.05	0.29
					2.454	97598.265	97598.265	0	0.42	0.16	1	242	97598.265	0.05	0.21
					2.390	134056.683	134056.683	0	0.44	0.16	1	280	134056.683	0.05	0.21
				125	2.517	114841.470	114841.470	0	0.55	0.77	133	4044	114841.470	0.06	0.84
					2.446	146636.372	146636.372	9.92E-16	0.55	0.23	1	257	146636.372	0.06	0.30
					2.365	103809.514	103809.514	0	0.51	2.87	5856	49311	103809.514	0.06	2.94
					2.443	147166.898	147166.898	0	0.52	0.27	1	373	147166.898	0.06	0.34
					2.305	87108.741	87108.741	0	0.55	0.23	1	282	87108.741	0.06	0.30
				150	2.390	112289.852	112289.852	0	0.62	1.45	1741	18912	112279.919	0.08	1.53
					2.488	121520.071	121520.071	0	0.62	4.74	2468	27130	121520.071	0.08	4.82
					2.459	127754.015	127754.015	0	0.63	0.28	1	310	127754.015	0.08	0.36
					2.343	150262.897	150262.897	0	0.65	0.33	1	453	150262.897	0.08	0.41

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			200	2.468	144134.695	144134.695	0	0.65	0.46	1	560	144134.695	0.08	0.54
				2.401	114765.748	114765.748	0	0.85	3.42	1455	14546	114765.748	0.10	3.52
				2.318	118810.468	118810.468	0	0.85	12.57	3984	80724	118567.655	0.11	12.67
				2.327	90719.369	90719.369	0	0.85	0.51	1	527	90719.369	0.10	0.60
				2.450	130060.530	130060.530	0	0.82	0.42	1	418	130060.530	0.10	0.51
				2.474	104165.703	104165.703	0	0.86	0.45	1	479	104165.703	0.10	0.54
		12500	125	2.506	186499.944	186499.944	0	0.56	0.55	161	2531	186442.834	0.06	0.62
				2.378	99899.583	99899.583	0	0.56	0.69	15	1073	99899.583	0.06	0.76
				2.404	168030.908	168030.908	0	0.52	0.19	0	206	168030.908	0.07	0.25
				2.296	199348.411	199348.411	0	0.55	0.28	1	371	199348.411	0.06	0.34
				2.529	134609.502	134609.502	0	0.55	2.13	1630	12699	134609.502	0.06	2.19
			156	2.455	183396.139	183396.139	0	0.65	0.35	1	315	183396.139	0.08	0.43
				2.524	243225.780	243225.780	0	0.65	0.37	1	441	243225.780	0.08	0.45
				2.619	181483.419	181483.419	0	0.65	0.25	0	368	181483.419	0.08	0.33
				2.563	150692.922	150692.922	0	0.65	0.25	0	305	150692.922	0.08	0.33
				2.437	141834.030	141834.030	0	0.65	0.30	1	259	141834.030	0.08	0.38
			187	2.483	136827.117	136827.117	8.51E-16	0.77	0.33	0	386	136827.117	0.09	0.42
				2.408	159642.372	159642.372	0	0.81	1.55	26	2833	159642.372	0.09	1.64
				2.346	131161.981	131161.981	0	0.78	0.48	1	546	131161.981	0.09	0.58
				2.374	111054.657	111054.657	0	0.78	0.41	1	463	111054.657	0.09	0.50
				2.475	185615.181	185615.181	0	0.81	1.98	322	5728	185615.181	0.09	2.07
			250	2.370	164502.303	164502.303	0	1.05	1.97	909	17106	164502.303	0.12	2.09
				2.352	135281.863	135281.863	0	1.06	0.48	0	477	135281.863	0.12	0.60
				2.466	112033.831	112033.831	9.09E-16	1.05	1.24	1	1112	112033.831	0.12	1.36
				2.421	146798.529	146798.529	0	1.02	1.91	120	3961	146798.529	0.12	2.03
				2.348	122977.544	122977.544	0	1.02	2.25	20	3684	122878.603	0.13	2.38
4	8	7500	75	2.472	104705.503	104705.503	0	0.27	0.22	1	459	104705.503	0.04	0.26
				2.345	93078.501	93078.501	0	0.27	0.14	1	360	93078.501	0.04	0.18
				2.610	88992.016	88992.016	0	0.27	0.20	1	471	88992.016	0.04	0.24
				2.549	107067.175	107067.175	0	0.27	2.12	2976	33838	107067.175	0.04	2.16
				2.195	80016.557	80016.557	0	0.28	0.08	0	195	79941.574	0.04	0.12

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			93	2.413	103322.791	103322.791	0	0.36	0.65	1	1297	103322.791	0.04	0.69
				2.304	83388.214	83388.214	0	0.34	0.16	1	374	83388.214	0.04	0.20
				2.436	118601.939	118601.939	0	0.36	0.31	1	623	118601.939	0.04	0.36
				2.406	75553.437	75553.437	0	0.34	0.26	1	538	75553.437	0.04	0.30
				2.266	74213.692	74213.692	0	0.36	1.53	129	4517	74213.692	0.04	1.57
			112	2.176	62848.618	62848.618	0	0.40	0.87	19	1450	62848.618	0.05	0.92
				2.576	86419.214	86419.214	0	0.41	4.96	2047	31774	86419.214	0.05	5.01
				2.425	111975.918	111975.918	0	0.41	0.19	1	305	111975.918	0.05	0.24
				2.245	96353.827	96353.827	0	0.40	0.24	1	486	96353.827	0.05	0.29
				2.372	85788.028	85788.028	0	0.43	1.26	1051	9738	85700.294	0.05	1.31
			150	2.488	79361.963	79361.963	0	0.57	2.34	1240	12707	79361.963	0.07	2.41
				2.480	98819.872	98819.872	0	0.57	3.55	1177	10639	98819.872	0.07	3.61
				2.417	114406.529	114406.529	0	0.56	3.28	1221	19593	114406.529	0.07	3.35
				2.567	126472.011	126472.011	0	0.57	0.43	1	812	126472.011	0.07	0.50
				2.354	72144.405	72144.405	0	0.54	0.89	55	1881	72144.405	0.07	0.95
		10000	100	2.548	135918.205	135918.205	0	0.36	2.61	2145	29260	135918.205	0.05	2.66
				2.372	135473.803	135473.803	0	0.36	0.63	41	1715	135473.803	0.05	0.68
				2.572	92228.657	92228.657	0	0.39	0.14	1	336	92228.657	0.05	0.19
				2.468	84224.788	84224.788	0	0.37	0.67	1	1607	84224.788	0.05	0.72
				2.440	102098.386	102098.386	0	0.36	0.23	1	497	102098.386	0.05	0.27
			125	2.429	143313.347	143313.347	0	0.45	0.25	1	391	143313.347	0.06	0.31
				2.449	120911.939	120911.939	0	0.48	3.11	1591	16008	120838.387	0.06	3.16
				2.555	171724.785	171724.785	5.08E-16	0.45	0.27	1	538	171724.785	0.06	0.33
				2.448	116856.323	116856.323	0	0.50	0.16	0	324	116856.323	0.06	0.22
				2.388	106127.310	106127.310	0	0.45	2.37	1133	9632	106127.310	0.06	2.43
			150	2.469	105843.854	105843.854	1.10E-15	0.57	0.45	1	759	105762.772	0.07	0.52
				2.590	117763.252	117763.252	0	0.57	0.31	1	466	117650.245	0.07	0.38
				2.475	107694.657	107694.657	0	0.58	4.14	2263	37620	107694.657	0.07	4.21
				2.347	164166.830	164166.830	0	0.55	2.41	1460	16676	164166.830	0.07	2.47
				2.514	123354.585	123354.585	0	0.54	0.32	1	577	123354.585	0.07	0.38
			200	2.412	86923.996	86923.996	0	0.72	5.85	1828	17938	86923.996	0.09	5.94

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				2.487	124068.122	124068.122	0	0.72	0.36	1	482	124068.122	0.09	0.45
				2.459	131486.267	131486.267	0	0.72	1.46	114	4070	131486.267	0.09	1.55
				2.346	106706.323	106706.323	0	0.74	4.67	1140	11558	106706.323	0.09	4.75
				2.463	107212.420	107212.420	0	0.72	0.70	1	893	107212.420	0.09	0.78
		12500	125	2.408	164396.786	164396.786	5.31E-16	0.46	0.29	1	472	164358.362	0.06	0.35
				2.347	154835.525	154835.525	0	0.46	0.16	0	280	154835.525	0.06	0.22
				2.486	116438.749	116438.749	0	0.49	1.14	29	2936	116438.749	0.06	1.20
				2.544	154785.767	154785.767	0	0.46	1.51	1127	7416	154718.342	0.06	1.57
				2.386	149535.383	149535.383	0	0.49	0.33	1	653	149535.383	0.06	0.39
			156	2.306	136391.557	136391.557	0	0.57	0.35	1	661	136391.557	0.07	0.42
				2.443	151873.355	151873.355	0	0.57	0.25	1	464	151873.355	0.07	0.32
				2.407	129817.316	129817.316	0	0.57	1.63	2050	26434	129817.316	0.07	1.70
				2.411	185083.109	185083.109	0	0.56	0.35	1	585	185083.109	0.07	0.42
				2.349	71646.964	71646.964	0	0.57	0.64	1	1202	71646.964	0.07	0.70
			187	2.523	212033.484	212033.484	0	0.68	0.52	1	669	212033.484	0.08	0.60
				2.468	175586.433	175586.433	0	0.71	3.94	2043	25975	175586.433	0.08	4.02
				2.469	142055.766	142055.766	0	0.67	0.25	0	404	142055.766	0.08	0.33
				2.409	157975.554	157975.554	0	0.68	0.43	1	531	157975.554	0.08	0.52
				2.409	138270.666	138270.666	0	0.71	3.94	2218	31158	138270.666	0.08	4.02
			250	2.481	174249.722	174249.722	0	0.94	0.63	1	762	174249.722	0.11	0.73
				2.516	220679.545	220679.545	0	0.94	3.75	1457	19843	220679.545	0.11	3.86
				2.392	132209.655	132209.655	0	0.94	0.90	1	1207	132209.655	0.11	1.00
				2.375	158258.088	158258.088	0	0.90	8.46	1371	20696	158258.088	0.11	8.57
				2.374	164867.620	164867.620	0	0.93	0.62	1	631	164867.620	0.11	0.72
	12	7500	75	2.472	103081.625	103081.625	0	0.41	2.55	1223	16381	103081.625	0.06	2.61
				2.347	75370.961	75370.961	0	0.39	0.76	1	934	75370.961	0.06	0.82
				2.583	73907.966	73907.966	0	0.41	3.38	2631	51422	73907.966	0.07	3.45
				2.253	78581.180	78581.180	0	0.41	2.97	1354	14230	78581.180	0.07	3.04
				2.540	99006.669	99006.669	0	0.41	3.27	2237	25836	99006.669	0.07	3.33
			93	2.154	91135.979	91135.979	0	0.50	3.93	1121	13672	91135.979	0.07	3.99
				2.554	112490.056	112490.056	0	0.50	7.46	6864	150689	112490.056	0.07	7.52

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				2.569	117191.752	117191.752	0	0.53	1.50	1619	23626	117191.752	0.07	1.57
				2.283	80313.763	80313.763	0	0.53	3.29	4582	60897	80313.763	0.07	3.36
				2.541	112360.858	112360.858	0	0.51	0.44	1	788	112360.858	0.07	0.51
			112	2.470	94485.452	94485.452	0	0.60	3.21	1381	22700	94485.452	0.08	3.29
				2.193	75313.512	75313.512	0	0.62	10.23	14506	174357	75313.512	0.08	10.30
				2.462	116735.913	116735.913	0	0.63	0.27	1	412	116735.913	0.08	0.35
				2.466	117671.867	117671.867	0	0.63	0.63	1	1237	117671.867	0.08	0.71
				2.348	100864.172	100864.172	0	0.62	3.31	1181	12500	100864.172	0.08	3.38
				2.542	96881.585	96881.585	0	0.79	0.80	1	761	96881.585	0.10	0.90
			150	2.556	87656.905	87656.905	0	0.80	1.61	23	2772	87567.659	0.10	1.71
				2.414	84474.897	84474.897	0	0.79	11.48	1711	29719	84474.897	0.10	11.58
				2.359	90770.797	90770.797	0	0.83	1.23	308	6059	90770.797	0.10	1.33
				2.393	65354.397	65354.397	0	0.81	4.52	1099	20514	65264.645	0.11	4.62
				2.474	98093.021	98093.021	0	0.58	1.07	189	2920	97938.574	0.08	1.14
		10000	100	2.490	126587.002	126587.002	0	0.54	5.64	3707	48558	126587.002	0.07	5.72
				2.468	118123.999	118123.999	0	0.55	2.22	1119	10179	118122.857	0.07	2.30
				2.423	123310.205	123310.205	0	0.54	3.00	1108	10755	123310.205	0.07	3.07
				2.335	142049.500	142049.500	0	0.55	3.27	1074	9547	142005.605	0.07	3.34
				2.423	129283.057	129283.057	0	0.67	0.76	82	2350	129283.057	0.09	0.84
			125	2.427	147070.619	147070.619	5.94E-16	0.69	0.41	1	549	146998.238	0.09	0.50
				2.433	110835.225	110835.225	0	0.67	4.71	2623	42015	110835.225	0.09	4.79
				2.492	104391.915	104391.915	0	0.70	0.36	1	487	104391.915	0.09	0.45
				2.447	136457.043	136457.043	8.53E-16	0.68	0.66	1	921	136457.043	0.09	0.75
				2.385	119194.694	119194.694	0	0.83	12.29	1535	40123	119194.694	0.10	12.39
			150	2.389	94888.109	94888.109	0	0.80	8.16	1339	14254	94772.605	0.11	8.26
				2.411	137008.492	137008.492	0	0.84	0.67	1	780	137008.492	0.10	0.77
				2.305	94666.972	94666.972	0	0.80	5.92	2303	29043	94666.972	0.11	6.03
				2.459	119203.595	119203.595	0	0.84	0.52	1	483	119203.595	0.10	0.63
			200	2.412	145588.035	145588.035	0	1.08	18.39	8034	173938	145513.144	0.13	18.52
				2.466	99644.996	99644.996	0	1.06	15.41	5233	106687	99644.996	0.13	15.54
				2.510	151451.373	151451.373	0	1.08	7.42	2931	48595	151451.373	0.13	7.55

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

		12500	125	2.431	115841.095	115841.095	0	1.07	2.12	24	2777	115749.660	0.14	2.26
				2.347	133753.805	133753.805	0	1.06	0.53	0	498	133753.805	0.13	0.66
				2.397	183170.354	183170.354	0	0.68	0.47	1	585	183170.354	0.09	0.56
				2.466	133785.094	133785.094	0	0.71	0.68	1	927	133785.094	0.09	0.77
				2.445	171073.988	171073.988	0	0.67	1.66	958	13768	171073.988	0.09	1.75
				2.375	168662.036	168662.036	0	0.68	1.28	12	2233	168662.036	0.09	1.36
				2.542	163060.183	163060.183	0	0.71	0.65	1	764	163060.183	0.09	0.73
			156	2.459	116522.098	116522.098	0	0.83	2.11	147	4217	116355.562	0.11	2.22
				2.413	163965.116	163965.116	0	0.85	0.53	1	562	163965.116	0.11	0.63
				2.489	159535.071	159535.071	0	0.85	8.63	1085	12901	159535.071	0.10	8.74
				2.357	117398.694	117398.694	0	0.88	9.05	4275	69754	117398.694	0.11	9.15
				2.444	164514.726	164514.726	0	0.86	1.06	1	1469	164514.726	0.11	1.16
			187	2.298	138867.622	138867.622	0	1.03	0.70	1	789	138867.622	0.12	0.82
				2.481	145704.523	145704.523	0	1.02	2.04	313	6508	145563.529	0.13	2.17
				2.466	170741.664	170741.664	0	1.01	5.23	2837	36347	170741.664	0.12	5.35
				2.515	149325.606	149325.606	0	1.02	0.68	1	651	149325.606	0.13	0.80
				2.506	177467.750	177467.750	0	1.00	9.34	1714	27178	177262.928	0.13	9.47
			250	2.392	196818.558	196818.558	0	1.31	1.36	1	931	196818.558	0.16	1.52
				2.346	163296.689	163296.689	0	1.36	10.27	1362	19434	163296.689	0.16	10.43
				2.274	104329.944	104329.944	0	1.33	16.76	3799	97436	104074.849	0.18	16.94
				2.261	168471.702	168471.702	6.91E-16	1.36	8.75	3042	64581	168471.702	0.16	8.91
				2.446	153872.629	153872.629	5.67E-16	1.40	7.38	1739	22547	153872.629	0.16	7.54
	16	7500	75	2.472	106655.731	106655.731	0	0.54	5.01	2298	32496	106655.731	0.08	5.09
				2.411	81497.668	81497.668	0	0.53	1.55	1029	9039	81131.163	0.09	1.64
				2.246	65498.085	65498.085	0	0.52	0.47	1	838	65448.683	0.08	0.56
				2.530	104479.866	104479.866	0	0.56	0.56	1	567	104479.866	0.08	0.64
				2.314	76209.226	76209.226	0	0.57	4.94	3864	48663	76209.226	0.09	5.03
			93	2.195	95558.684	95558.684	0	0.68	1.12	1324	18612	95443.828	0.09	1.21
				2.226	90474.250	90474.250	0	0.66	5.08	2334	36519	90474.250	0.09	5.17
				2.063	63987.803	63987.803	0	0.71	10.19	3222	68931	63987.803	0.09	10.28
				2.451	98307.085	98307.085	0	0.68	0.31	1	321	98307.085	0.09	0.40

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				2.464	97538.149	97538.149	0	0.69	3.15	2078	25225	97456.591	0.10	3.24
		112		2.459	95572.673	95572.673	0	0.82	8.70	5058	99532	95572.673	0.11	8.81
			2.317	96498.615	96498.615	0	0.83	10.71	4951	120977	96498.615	0.10	10.81	
			2.408	83593.452	83593.452	0	0.82	18.10	7851	189681	83593.452	0.11	18.21	
			2.327	102840.644	102840.644	0	0.81	32.06	30352	497787	102840.644	0.11	32.17	
			2.515	79696.033	79696.033	0	0.85	7.04	1649	23986	79696.033	0.10	7.15	
		150		2.521	81581.783	81581.783	0	1.06	30.72	24100	399309	81581.783	0.13	30.85
			2.407	94458.555	94458.555	1.23E-15	1.11	1.39	1	1038	94458.555	0.13	1.52	
			2.431	81529.734	81529.734	0	1.05	4.10	1098	14419	81348.707	0.14	4.24	
			2.534	92988.772	92988.772	0	1.04	24.60	8780	239469	92988.772	0.13	24.73	
			2.450	73267.467	73267.467	0	1.36	6.85	11101	86238	73267.467	0.14	6.99	
	10000	100		2.550	119809.826	119809.826	0	0.72	7.93	1298	24674	119809.826	0.09	8.02
			2.477	139724.756	139724.756	0	0.77	1.80	1072	14247	139724.756	0.10	1.90	
			2.353	113336.987	113336.987	0	0.72	4.77	1193	18976	113336.987	0.10	4.87	
			2.341	122993.759	122993.759	0	0.77	5.22	3211	52484	122993.759	0.10	5.32	
			2.551	124131.905	124131.905	0	0.73	1.86	1490	21967	124131.905	0.10	1.96	
		125		2.419	132556.513	132556.513	0	0.92	0.66	1	737	132556.513	0.11	0.77
			2.393	142645.067	142645.067	0	0.90	7.23	2098	45190	142645.067	0.11	7.34	
			2.430	129581.393	129581.393	0	0.92	11.96	1622	31576	129581.393	0.12	12.08	
			2.360	103312.107	103312.107	0	0.90	13.63	11078	182437	103312.107	0.12	13.75	
			2.486	105094.767	105094.767	0	0.95	15.54	2428	58282	105094.767	0.12	15.66	
		150		2.384	129602.142	129602.142	0	1.11	6.77	1306	18406	129602.142	0.14	6.91
			2.411	94380.377	94380.377	0	1.09	6.28	2106	28598	94380.377	0.14	6.42	
			2.325	165039.160	165039.160	0	1.06	7.86	4446	71872	164945.680	0.14	7.99	
			2.290	91891.931	91891.931	0	1.10	7.68	1206	15090	91891.931	0.13	7.82	
			2.405	116449.530	116449.530	0	1.12	6.65	2425	54326	116263.589	0.14	6.80	
		200		2.449	100329.975	100329.975	0	1.45	14.05	2079	47620	100329.975	0.19	14.24
			2.354	145611.213	145611.213	0	1.45	12.06	4170	92062	145611.213	0.17	12.23	
			2.536	128864.617	128864.617	0	1.46	6.69	2231	35941	128864.617	0.17	6.86	
			2.386	117701.594	117701.594	0	1.47	11.24	1395	26950	117701.594	0.17	11.42	
			2.375	141414.811	141414.811	0	1.43	6.43	1234	15700	141414.811	0.17	6.60	

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

		12500	125	2.354	162534.609	162534.609	0	0.97	3.81	1056	15681	162534.609	0.12	3.92
				2.291	129221.268	129221.268	0	0.94	10.17	13163	180202	129077.141	0.12	10.29
				2.450	159303.366	159303.366	0	0.93	13.22	3965	83081	159137.393	0.13	13.35
				2.295	129274.009	129274.009	0	0.92	24.14	16552	346928	129230.206	0.12	24.26
				2.369	134317.205	134317.205	0	0.92	0.87	1	1069	134317.205	0.12	0.98
			156	2.514	147929.891	147929.891	0	1.12	19.56	5615	105493	147625.503	0.16	19.73
				2.454	149624.797	149624.797	0	1.17	0.69	1	513	149624.797	0.15	0.84
				2.396	146974.320	146974.320	0	1.15	10.13	1270	23527	146759.409	0.15	10.28
				2.444	143093.172	143093.172	0	1.12	6.37	1288	16688	143093.172	0.14	6.51
				2.405	145881.250	145881.250	0	1.13	1.12	1	856	145881.250	0.14	1.26
			187	2.397	129378.305	129378.305	0	1.35	5.80	1523	23045	129334.512	0.16	5.97
				2.458	198285.752	198285.752	0	1.38	12.94	3084	63980	198285.752	0.16	13.10
				2.443	168219.242	168219.242	0	1.39	19.42	2077	44759	168219.242	0.17	19.59
				2.424	179840.827	179840.827	0	1.39	34.08	16969	425682	179840.827	0.18	34.27
				2.425	151554.165	151554.165	0	1.39	11.36	1466	20392	151542.807	0.17	11.53
			250	2.288	139165.998	139165.998	0	1.72	15.26	4433	83868	139165.998	0.22	15.48
				2.475	184135.284	184135.284	0	1.84	19.21	3309	70897	184135.284	0.21	19.42
				2.386	195597.652	195597.652	5.95E-16	1.73	5.17	531	16495	195588.086	0.22	5.39
				2.367	167402.805	167402.805	0	1.80	1.40	1	705	167402.805	0.22	1.61
				2.467	139301.977	139301.977	0	1.83	11.48	1219	14066	139301.977	0.22	11.69
5	10	7500	75	2.472	106348.713	106348.713	0	0.41	2.80	1072	21137	106348.713	0.06	2.86
				2.472	75021.856	75021.856	0	0.44	1.11	293	3950	75018.455	0.07	1.18
				2.326	67887.914	67887.914	0	0.41	0.42	1	892	67713.570	0.07	0.50
				2.156	85540.689	85540.689	0	0.42	3.80	4236	45433	85183.678	0.08	3.87
				2.413	70089.816	70089.816	0	0.44	0.91	1022	13738	70089.816	0.07	0.98
			93	2.534	109357.275	109357.275	0	0.52	0.52	1	802	109357.275	0.09	0.61
				2.397	102357.688	102357.688	0	0.55	3.57	2008	32112	102357.688	0.09	3.66
				2.368	78144.892	78144.892	0	0.55	2.16	1095	15083	78059.214	0.09	2.25
				2.437	79927.544	79927.544	0	0.55	4.65	1383	14235	79839.874	0.09	4.74
				2.430	95227.367	95227.367	0	0.56	3.31	1312	20558	95227.367	0.09	3.40
			112	2.182	72453.572	72453.572	0	0.64	8.67	5026	90541	72453.572	0.09	8.76

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				2.408	103735.121	103735.121	0	0.66	11.99	14106	233718	103638.568	0.10	12.09	
				2.435	93461.587	93461.587	0	0.64	0.84	1	881	93350.034	0.09	0.93	
				2.373	71789.577	71789.577	0	0.62	1.04	109	2370	71716.782	0.09	1.14	
				2.495	95090.185	95090.185	0	0.66	0.66	1	986	94841.651	0.10	0.75	
				150	2.391	103283.260	103283.260	0	0.83	1.43	33	2439	103163.083	0.12	1.54
				2.467	74027.151	74027.151	0	0.83	5.89	1153	12100	73926.888	0.12	6.01	
				2.387	96343.715	96343.715	0	0.81	8.34	3146	80046	96343.715	0.12	8.45	
				2.483	65472.661	65472.661	0	0.88	11.26	1252	16823	65420.794	0.11	11.37	
			2.402	76903.396	76903.396	0	0.87	7.92	2319	43608	76903.396	0.11	8.03		
			10000	100	2.509	123688.979	123688.979	0	0.59	3.52	1097	12848	123688.979	0.10	3.62
					2.346	125814.806	125814.806	0	0.56	4.95	2714	40009	125814.806	0.10	5.05
					2.467	96279.660	96279.660	0	0.57	0.28	1	461	96041.833	0.11	0.39
					2.266	136387.780	136387.780	0	0.56	6.69	9349	117838	136387.780	0.10	6.79
					2.487	140545.213	140545.213	0	0.57	10.85	14909	202773	140151.007	0.11	10.96
				125	2.502	127838.071	127838.071	0	0.71	1.14	1	1745	127428.906	0.11	1.25
					2.539	118082.679	118082.679	0	0.70	10.90	6249	85099	117766.155	0.11	11.01
		2.408			113823.478	113823.478	0	0.74	2.68	1100	12100	113823.478	0.10	2.77	
		1500		2.454	177877.766	177877.766	0	0.71	0.40	1	638	177877.766	0.10	0.50	
				2.361	121749.286	121749.286	0	0.73	5.19	1069	16060	121552.869	0.10	5.30	
				150	2.424	92019.299	92019.299	0	0.87	4.42	1047	9503	92019.299	0.11	4.53
				2.297	114019.145	114019.145	0	0.87	8.73	3051	49246	113939.711	0.12	8.85	
				2.447	125639.309	125639.309	0	0.84	2.54	1032	9731	125504.318	0.12	2.65	
				2.342	107998.673	107998.673	0	0.84	0.67	1	836	107998.673	0.11	0.78	
				2.324	110130.341	110130.341	0	0.82	10.22	2958	57706	110069.618	0.11	10.33	
				200	2.504	113782.819	113782.819	0	1.09	19.88	9809	141543	113782.819	0.14	20.02
				2.365	130222.067	130222.067	0	1.14	5.69	1796	28097	130222.067	0.14	5.83	
				2.437	92774.358	92774.358	0	1.17	22.70	6715	167934	92774.358	0.14	22.84	
				2.439	139547.869	139547.869	0	1.11	6.78	1074	26824	139547.869	0.14	6.92	
				2.296	117276.348	117276.348	0	1.11	8.21	1946	37950	117276.348	0.14	8.35	
				125	2.361	134390.710	134390.710	0	0.73	0.71	1	933	134390.710	0.10	0.81
					2.468	150813.347	150813.347	0	0.73	1.40	141	2972	150429.907	0.11	1.52

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

				2.249	131859.041	131859.041	0	0.71	3.22	2066	33951	131859.041	0.10	3.31
				2.457	180407.284	180407.284	0	0.74	0.63	1	839	180393.378	0.10	0.73
				2.388	144868.538	144868.538	0	0.70	6.61	1876	21927	144537.215	0.11	6.73
			156	2.208	162022.863	162022.863	0	0.91	0.79	1	1133	162022.863	0.11	0.91
				2.300	166052.722	166052.722	0	0.87	8.18	1920	29166	166052.722	0.11	8.30
				2.443	134423.223	134423.223	0	0.92	0.56	1	763	134423.223	0.12	0.68
				2.544	163660.032	163660.032	0	0.89	8.71	1304	20554	163651.616	0.12	8.83
				2.462	195009.486	195009.486	0	0.87	0.71	1	1090	195009.486	0.11	0.83
			187	2.330	100785.022	100785.022	0	1.03	9.69	1897	30734	100759.126	0.14	9.82
				2.400	180053.802	180053.802	6.47E-16	1.08	1.19	1	1213	180053.802	0.14	1.32
				2.565	168830.086	168830.086	0	1.09	10.57	5126	111471	168830.086	0.14	10.70
			250	2.402	117542.817	117542.817	0	1.08	1.11	1	1550	117407.356	0.14	1.25
				2.392	115260.269	115260.269	0	1.05	0.80	1	931	115260.269	0.14	0.94
				2.414	182334.462	182334.462	0	1.41	10.44	2371	37181	182334.462	0.17	10.61
				2.379	148935.093	148935.093	0	1.45	13.56	2030	40560	148935.093	0.18	13.74
				2.440	174142.874	174142.874	0	1.42	59.83	29181	960268	174142.874	0.17	60.01
				2.322	124927.311	124927.311	0	1.38	10.82	1157	16208	124927.311	0.18	11.00
				2.403	144302.944	144302.944	0	1.38	19.36	8308	172939	144167.290	0.19	19.55
			75	2.472	109843.162	109843.162	0	0.61	3.95	1117	23515	109843.162	0.10	4.04
				2.342	84121.974	84121.974	0	0.64	11.62	14201	205642	84121.974	0.10	11.72
				2.371	86914.620	86914.620	0	0.63	5.60	1207	16523	86914.620	0.10	5.70
				2.548	111995.547	111995.547	0	0.66	22.36	51680	539864	111995.547	0.10	22.46
				2.541	105356.263	105356.263	0	0.64	8.80	10918	182730	105356.263	0.10	8.90
			93	2.297	82290.339	82290.339	0	0.82	8.54	3601	68989	82290.339	0.14	8.68
				2.397	86611.399	86611.399	0	0.82	23.44	21868	448319	86611.399	0.13	23.57
				2.560	95288.281	95288.281	0	0.79	8.42	2265	45547	95219.448	0.13	8.55
				2.433	90205.724	90205.724	0	0.78	23.79	23707	397148	90205.724	0.13	23.92
				2.498	87156.576	87156.576	0	0.77	12.54	7989	196224	87016.270	0.14	12.68
			112	2.312	95130.832	95130.832	0	0.98	8.51	3165	60440	95130.832	0.15	8.67
				2.496	86996.896	86996.896	0	1.17	32.05	30404	482414	86866.090	0.17	32.23
				2.392	99808.347	99808.347	0	0.93	13.54	6096	163281	99808.347	0.15	13.69

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			150	2.256	68941.797	68941.797	0	0.93	31.13	24277	555773	68907.768	0.16	31.30
				2.420	87602.164	87602.164	0	0.95	21.83	15396	248083	87512.987	0.15	21.99
				2.489	90479.391	90479.391	0	1.26	27.33	5847	212664	90372.029	0.19	27.52
				2.522	123819.621	123819.621	0	1.29	10.90	3308	77820	123819.621	0.17	11.07
				2.310	74286.207	74286.207	0	1.27	34.03	10948	260315	74286.207	0.19	34.22
				2.367	100615.191	100615.191	0	1.25	54.04	30565	569027	100550.926	0.20	54.24
				2.318	82674.962	82674.962	0	1.29	27.02	5022	139906	82569.924	0.18	27.20
		10000	100	2.409	127236.395	127236.395	0	0.91	18.05	12155	271219	127230.513	0.16	18.21
				2.602	132523.490	132523.490	0	0.94	4.53	1052	20214	132523.490	0.15	4.68
				2.565	141208.476	141208.476	0	0.88	3.57	1032	19427	141208.476	0.15	3.72
				2.430	113362.340	113362.340	0	0.89	25.13	24311	334099	113235.403	0.15	25.28
				2.499	102191.263	102191.263	0	0.87	6.54	2035	30537	102169.401	0.16	6.70
			125	2.504	134973.945	134973.945	0	1.04	24.08	23602	418437	134717.972	0.17	24.25
				2.456	137238.674	137238.674	0	1.05	25.34	7088	130413	137144.352	0.15	25.49
				2.342	112354.242	112354.242	0	1.06	9.49	4700	80691	112354.242	0.15	9.64
				2.490	137150.704	137150.704	0	1.07	14.28	5390	132819	137150.704	0.15	14.43
				2.346	103417.891	103417.891	0	1.05	3.85	1072	10026	103414.905	0.15	4.00
			150	2.389	101552.211	101552.211	0	1.26	15.95	3956	84187	101371.988	0.19	16.15
				2.599	121652.118	121652.118	0	1.29	4.95	1095	20193	121427.771	0.18	5.14
				2.473	178578.623	178578.623	6.52E-16	1.24	4.39	1035	19955	178578.623	0.19	4.58
				2.547	153704.667	153704.667	0	1.31	1.64	1	1707	153383.351	0.18	1.83
				2.332	135822.000	135822.000	0	1.31	12.83	4550	131812	135822.000	0.17	13.00
			200	2.424	124302.842	124302.842	0	1.71	15.11	1891	38536	124013.555	0.23	15.34
				2.440	147318.264	147318.264	5.93E-16	1.71	22.80	3532	81104	147123.121	0.22	23.02
				2.443	158871.186	158871.186	0	1.65	63.13	16489	518120	158377.686	0.23	63.36
				2.473	106719.189	106719.189	0	1.64	31.04	2065	53184	106702.208	0.23	31.27
				2.436	127148.294	127148.294	0	1.74	38.51	12799	241652	127148.294	0.24	38.76
		12500	125	2.411	139982.874	139982.874	0	1.06	21.15	8789	248622	139982.874	0.15	21.30
				2.482	165206.114	165206.114	0	1.09	16.82	9440	177566	165206.114	0.15	16.97
				2.537	147365.200	147365.200	0	1.07	2.43	1041	12634	147329.657	0.16	2.60
				2.307	176968.245	176968.245	0	1.05	4.03	1193	11176	176618.834	0.16	4.18

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			156	2.392	164828.444	164828.444	0	1.08	12.53	3342	53837	164828.444	0.15	12.68
				2.244	154521.463	154521.463	0	1.34	15.13	1152	23803	154024.438	0.19	15.33
				2.512	149848.850	149848.850	0	1.31	15.46	1229	25646	149817.085	0.17	15.63
				2.559	167455.699	167455.699	0	1.39	13.09	1814	35085	167455.699	0.18	13.27
				2.400	122385.943	122385.943	0	1.39	9.18	1315	22016	122385.943	0.18	9.36
				2.444	163856.497	163856.497	5.33E-16	1.32	3.18	1069	15861	163856.497	0.17	3.35
			187	2.465	132405.478	132405.478	0	1.60	223.72	21524	823036	132405.478	0.20	223.92
				2.366	151255.478	151255.478	0	1.62	26.34	8737	177357	151255.478	0.21	26.55
				2.451	148110.183	148110.183	0	1.57	17.14	2471	60712	148110.183	0.20	17.35
				2.361	126327.212	126327.212	0	1.59	28.80	7218	156741	126327.212	0.20	29.00
				2.466	135648.740	135648.740	0	1.56	13.49	1764	31284	135648.740	0.21	13.69
			250	2.344	158319.469	158319.469	0	2.11	41.42	5609	148793	158319.469	0.27	41.68
				2.339	145540.249	145540.249	0	2.10	6.96	479	11644	145377.081	0.27	7.23
				2.404	167557.038	167557.038	0	2.05	20.63	2822	77803	167248.949	0.28	20.91
				2.430	156872.425	156872.425	0	2.06	32.10	4038	107825	156872.425	0.30	32.40
				2.310	144926.384	144926.384	0	2.07	64.82	10086	416295	144741.094	0.28	65.09
	20	7500	75	2.472	109341.674	109341.674	0	0.82	6.04	2150	48822	109341.674	0.13	6.16
				2.453	87793.802	87793.802	0	0.81	24.56	12915	344801	87084.063	0.14	24.71
				2.469	97596.898	97596.898	0	0.83	7.18	2777	58604	97494.045	0.15	7.33
				2.336	99826.025	99826.025	0	0.84	5.05	1100	32546	99826.025	0.14	5.19
				2.480	95666.290	95666.290	0	0.84	3.91	1030	22799	95493.880	0.14	4.05
			93	2.220	70213.852	70213.852	0	0.99	68.86	30062	789915	69941.287	0.19	69.04
				2.207	86920.034	86920.034	0	1.06	34.22	25753	565998	86920.034	0.18	34.40
				2.652	111629.161	111629.161	0	1.05	12.67	1360	25447	111556.135	0.18	12.86
				2.565	121614.532	121614.532	0	1.08	19.85	9313	187976	121614.532	0.19	20.04
				2.464	104319.552	104319.552	0	1.07	7.67	3339	69622	104008.140	0.20	7.88
			112	2.406	90470.248	90470.248	0	1.28	84.35	30257	546796	90387.101	0.23	84.58
				2.414	95178.376	95178.376	0	1.23	200.12	21818	1136215	95069.017	0.23	200.35
				2.220	90563.113	90563.113	0	1.34	65.03	20801	518790	90563.113	0.22	65.25
				2.403	77543.947	77543.947	0	1.23	62.71	39311	866910	77461.216	0.23	62.94
				2.591	107537.742	107537.742	0	1.24	141.23	21333	1043995	107060.807	0.22	141.45

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			150	2.497	91545.102	91545.102	0	1.70	68.22	34356	888282	91021.776	0.25	68.47
				2.423	90833.135	90833.135	0	1.62	344.45	40112	1924271	90537.135	0.25	344.70
				2.351	81125.782	81125.782	0	1.61	161.92	21410	793306	80989.854	0.29	162.21
				2.296	82338.658	82338.658	0	1.71	7.79	1878	69953	82248.430	0.24	8.03
				2.470	72785.094	72785.094	0	1.65	352.38	28277	1383117	72732.156	0.25	352.63
		10000	100	2.634	160049.635	160049.635	0	1.12	18.70	16286	275825	159954.166	0.20	18.90
				2.304	123185.438	123185.438	0	1.21	4.03	1042	27977	123185.438	0.21	4.23
				2.409	113447.684	113447.684	0	1.11	13.22	2112	43709	113447.684	0.21	13.42
				2.333	112554.003	112554.003	0	1.11	9.56	3166	80024	112312.266	0.21	9.77
				2.489	125323.256	125323.256	0	1.12	1.10	1	1149	125298.081	0.21	1.31
			125	2.434	122945.082	122945.082	0	1.43	33.98	9638	225105	122945.082	0.22	34.20
				2.451	129692.662	129692.662	0	1.40	14.74	4611	112227	129675.241	0.20	14.93
				2.419	111246.263	111246.263	0	1.43	21.33	4250	94657	111246.263	0.23	21.56
				2.419	121917.210	121917.210	0	1.43	47.60	20535	486208	121871.242	0.21	47.81
				2.381	129199.154	129199.154	0	1.36	6.00	2143	59092	129070.212	0.20	6.20
			150	2.321	145434.521	145434.521	0	1.66	165.40	20987	554230	145307.780	0.25	165.65
				2.310	129006.022	129006.022	0	1.62	212.72	21363	787595	129006.022	0.24	212.96
				2.349	113572.024	113572.024	0	1.78	83.39	41662	812703	113572.024	0.24	83.64
				2.462	115929.536	115929.536	0	1.69	53.06	21014	357841	115929.536	0.23	53.29
				2.511	122174.570	122174.570	0	1.69	49.96	8656	269573	122058.738	0.28	50.24
			200	2.341	124570.995	124570.995	0	2.16	296.18	24320	1093085	124417.716	0.31	296.49
				2.409	116151.166	116151.166	0	2.31	63.60	11343	435421	116151.166	0.32	63.92
				2.552	162969.402	162969.402	0	2.17	61.95	21995	567795	162969.402	0.31	62.25
				2.512	138467.165	138467.165	8.41E-16	2.25	335.44	23403	844473	138467.165	0.32	335.76
				2.460	97569.187	97569.187	0	2.20	51.42	8038	214940	97569.187	0.29	51.71
		12500	125	2.354	158889.296	158889.296	0	1.40	39.81	15746	455827	158889.296	0.23	40.04
				2.547	190515.644	190515.644	0	1.46	26.80	6760	177383	190515.644	0.20	27.01
				2.473	172658.626	172658.626	0	1.47	102.88	21507	690453	172597.568	0.22	103.10
				2.375	167595.444	167595.444	0	1.55	18.50	6189	149491	167595.444	0.21	18.72
				2.342	154216.739	154216.739	0	1.40	41.20	15038	382057	154110.655	0.22	41.42
			156	2.471	167851.202	167851.202	0	1.79	48.88	23206	353915	167851.202	0.25	49.13

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

			187	2.377	144265.197	144265.197	0	1.77	27.57	4094	146986	144006.588	0.27	27.85
				2.284	144048.979	144048.979	0	1.70	218.70	21139	657284	144048.979	0.23	218.93
				2.495	156161.532	156161.532	0	1.74	115.96	31644	991196	155848.565	0.26	116.22
				2.447	183018.965	183018.965	6.36E-16	1.77	176.96	21695	489875	182884.276	0.23	177.19
				2.488	129402.468	129402.468	0	2.15	28.23	5211	114241	129245.671	0.36	28.58
				2.452	169274.249	169274.249	0	2.12	31.25	10511	212460	169274.249	0.31	31.56
				2.461	156622.422	156622.422	0	2.12	229.50	21477	614824	156135.642	0.32	229.82
				2.398	149947.277	149947.277	0	2.22	83.69	21973	614412	149922.332	0.29	83.98
				2.470	143613.363	143613.363	0	2.06	16.93	2312	44893	143466.453	0.29	17.22
			250	2.416	147291.299	147291.299	7.90E-16	2.76	568.24	30076	2202920	147291.299	0.35	568.59
				2.460	128287.678	128287.678	9.07E-16	2.72	104.33	12769	484614	128132.427	0.39	104.72
				2.319	159539.403	159539.403	9.12E-16	2.75	44.21	2922	76505	159539.403	0.39	44.60
				2.327	133171.380	133171.380	0	2.72	125.06	24609	1067139	133171.380	0.39	125.45
				2.448	185245.322	185245.322	0	2.76	92.39	29126	659489	185245.322	0.38	92.78

Table B.3 Continued: Solution Parameters for Consecutive Runs (single-assignment)

$ K $	$ M $	$ J $	$ I $	avg. degree of orders	combined objval	combined objbound	combined gap	combined gentime	combined runtime	combined nodenum	combined iternum
3	6	7500	75	2.472	90372.308	90372.308	0	0.51	0.10	0	1765
				2.429	101523.304	101523.304	0	0.52	0.13	1	1539
				2.398	63320.704	63320.704	0	0.52	0.19	1	1496
				2.252	72321.353	72321.353	0	0.52	0.32	1	1860
				2.287	86470.596	86470.596	0	0.53	0.07	0	1625
			93	2.424	82254.675	82254.675	0	0.64	0.59	1	3716
				2.500	118592.467	118592.467	0	0.63	0.21	1	1910
				2.590	94931.317	94931.317	0	0.64	0.40	1	2224
				2.627	96141.696	96141.696	0	0.64	0.26	1	2110
				2.456	75312.416	75312.416	0	0.63	0.12	0	2124
			112	2.468	71688.192	71688.192	0	0.77	0.64	1	2645
				2.396	85501.859	85501.859	2.21E-15	0.76	0.46	1	2908
				2.591	104510.595	104510.595	0	0.78	0.12	0	2301
				2.405	78809.238	78809.238	0	0.76	0.50	1	2885
				2.410	81046.420	81046.420	0	0.77	0.44	1	2413
			150	2.503	115587.503	115587.503	0	1.03	0.17	0	3194
				2.411	85110.366	85110.366	0	1.03	0.79	1	3675
				2.539	113673.954	113673.954	0	1.03	0.72	1	3425
				2.275	101760.519	101760.519	0	1.03	0.61	1	3757
				2.418	111355.595	111355.595	0	1.02	0.75	1	3912
		10000	100	2.553	108537.213	108537.213	0	0.69	0.12	0	2062
				2.191	118997.435	118997.435	0	0.70	0.09	0	2015
				2.467	130299.122	130299.122	0	0.70	0.30	1	2051
				2.609	121736.496	121736.496	0	0.68	0.34	1	2229
				2.559	94306.484	94306.484	0	0.70	0.13	0	2424
			125	2.331	117590.373	117590.373	1.98E-15	0.85	0.13	0	2626
				2.602	122302.335	122302.335	0	0.84	0.50	1	3058
				2.396	77865.137	77865.137	1.50E-15	0.86	0.16	0	2903
				2.573	153451.890	153451.890	0	0.87	0.24	1	2368
				2.310	108998.027	108998.027	1.20E-15	0.84	0.16	0	2926

Table B.4: Solution Parameters for Combined Problem (single-assignment)

			150	2.450	124720.724	124720.724	0	1.03	0.17	0	3222
				2.507	146070.385	146070.385	0	1.03	0.41	1	3503
				2.459	116724.628	116724.628	0	1.03	0.66	1	3240
				2.474	171991.121	171991.121	3.38E-15	1.03	0.72	1	3218
				2.521	182028.813	182028.813	0	1.03	0.17	0	3035
			200	2.325	96943.480	96943.480	2.10E-15	1.37	0.23	0	4307
				2.445	100620.795	100620.795	0	1.37	0.51	1	3942
				2.502	160560.262	160560.262	0	1.36	0.22	0	4780
				2.505	85432.376	85432.376	0	1.36	0.22	0	3832
				2.341	128136.446	128136.446	0	1.37	0.20	0	3893
		12500	125	2.326	156948.732	156948.732	0	0.88	0.16	0	2828
				2.435	216585.344	216585.344	0	0.85	0.30	1	2583
				2.419	142980.900	142980.900	0	0.87	0.53	1	3099
				2.463	251643.673	251643.673	6.94E-16	0.87	0.52	1	2862
				2.409	126321.081	126321.081	0	0.87	0.46	1	2810
			156	2.547	120327.133	120327.133	0	1.08	0.16	0	3294
				2.407	164527.934	164527.934	0	1.07	0.44	1	3394
				2.536	173869.027	173869.027	0	1.07	0.33	1	3176
				2.491	174129.831	174129.831	0	1.08	0.35	1	3464
				2.348	129519.871	129519.871	0	1.07	0.64	1	3591
			187	2.417	134666.257	134666.257	0	1.28	0.23	0	4316
				2.322	99198.747	99198.747	0	1.28	1.32	56	5009
				2.364	184526.638	184526.638	7.89E-16	1.29	0.96	1	4601
				2.446	192121.048	192121.048	0	1.28	1.21	1	5056
				2.488	144846.174	144846.174	0	1.29	1.91	189	7800
			250	2.396	152210.899	152210.899	0	1.71	0.58	1	4777
				2.382	114709.556	114709.556	1.27E-15	1.71	0.28	0	4552
				2.491	140329.234	140329.234	0	1.72	0.64	1	5110
				2.361	163782.780	163782.780	1.42E-15	1.70	0.29	0	5709
				2.504	183209.748	183209.748	0	1.71	2.05	36	7426
	9	7500	75	2.472	99548.393	99548.393	0	0.75	0.14	0	2573

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

				2.539	101649.210	101649.210	0	0.78	1.41	1	6252
				2.309	112489.644	112489.644	0	0.79	0.43	1	2298
				2.297	78836.833	78836.833	0	0.78	0.27	1	2239
				2.501	103481.553	103481.553	0	0.79	0.45	1	2915
			93	2.424	78645.910	78645.910	1.67E-15	0.97	0.18	0	3323
				2.652	102275.114	102275.114	9.96E-16	0.97	0.17	0	3029
				2.558	159192.048	159192.048	0	0.97	0.53	1	2802
				2.676	81468.999	81468.999	0	0.98	0.45	1	3103
			112	2.556	114039.109	114039.109	0	0.96	0.16	0	2887
				2.568	88794.783	88794.783	0	1.17	1.07	1	5403
				2.324	87419.169	87419.169	0	1.15	2.09	533	13660
				2.370	105484.510	105484.510	0	1.17	2.49	383	12996
				2.368	95556.440	95556.440	0	1.16	1.00	1	4634
			150	2.327	93124.774	93124.774	0	1.15	5.97	1628	50744
				2.434	88638.565	88638.565	0	1.54	0.30	0	4595
				2.144	71372.055	71372.055	0	1.57	12.59	2230	73321
				2.530	89316.271	89316.271	0	1.57	1.13	1	5573
				2.261	82362.085	82362.085	0	1.53	3.33	93	9481
				2.401	61964.769	61964.769	0	1.57	2.72	65	7240
		10000	100	2.503	122177.677	122177.677	8.34E-16	1.07	0.18	0	3169
				2.364	114903.634	114903.634	0	1.06	0.32	1	3336
				2.263	138542.514	138542.514	0	1.05	0.84	1	3391
				2.374	93132.379	93132.379	0	1.06	0.19	0	3285
				2.377	116410.671	116410.671	0	1.05	1.23	169	7881
			125	2.454	94144.795	94144.795	2.16E-15	1.32	0.22	0	3875
				2.401	122945.512	122945.512	0	1.30	1.92	209	8557
				2.426	109341.535	109341.535	0	1.32	1.27	1	6398
				2.519	202521.985	202521.985	0	1.31	0.56	1	3981
				2.443	93212.180	93212.180	0	1.30	0.43	1	4236
			150	2.388	122574.151	122574.151	0	1.54	4.60	1	10880
				2.501	112451.809	112451.809	1.16E-15	1.54	0.89	1	4231

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			200	2.439	115368.167	115368.167	0	1.59	0.31	0	4764
				2.516	103723.072	103723.072	0	1.58	3.38	1509	27562
				2.418	87300.480	87300.480	0	1.60	20.42	9677	150839
				2.533	132889.452	132889.452	0	2.09	0.43	0	6622
				2.404	83360.386	83360.386	1.40E-15	2.07	0.52	0	5904
				2.393	135235.450	135235.450	0	2.08	0.37	0	6165
				2.434	100188.741	100188.741	0	2.05	1.05	1	6342
			12500	2.431	90718.156	90718.156	0	2.10	32.87	3216	120670
				2.455	118201.393	118201.393	1.97E-15	1.35	0.23	0	4208
				2.443	146332.542	146332.542	7.96E-16	1.32	0.23	0	3982
				2.423	137699.882	137699.882	0	1.32	0.87	1	3469
				2.376	144850.755	144850.755	0	1.33	0.22	0	4038
				2.477	152499.112	152499.112	0	1.32	0.97	38	5557
				2.455	200487.338	200487.338	8.71E-16	1.63	0.94	1	5548
				2.515	181522.723	181522.723	0	1.63	2.97	128	9679
				2.481	169036.256	169036.256	0	1.63	3.13	63	9899
				2.315	162837.903	162837.903	0	1.65	0.30	0	5467
				2.553	187633.122	187633.122	0	1.63	0.23	0	4184
				2.315	157625.120	157625.120	1.29E-15	1.93	0.31	0	5225
				2.371	216674.719	216674.719	0	1.96	1.19	1	6388
				2.441	182306.730	182306.730	0	1.95	0.94	1	5823
				2.436	118893.809	118893.809	0	1.95	3.18	300	15027
				2.412	156522.859	156522.859	0	1.94	1.93	24	7512
				2.447	126918.997	126918.997	0	2.62	2.78	1	9867
				2.476	122558.594	122558.594	0	2.59	0.49	0	8087
				2.292	172140.940	172140.940	6.76E-16	2.63	0.56	0	8202
				2.474	175778.681	175778.681	0	2.59	4.35	190	13433
				2.472	201243.122	201243.122	0	2.58	0.52	0	8172
			75	2.472	95973.182	95973.182	0	1.03	0.22	0	3449
				2.413	74397.871	74397.871	0	1.04	1.88	236	9738
				2.464	98169.680	98169.680	0	1.04	0.73	1	3517
	12	7500	75	2.472	95973.182	95973.182	0	1.03	0.22	0	3449
				2.413	74397.871	74397.871	0	1.04	1.88	236	9738
				2.464	98169.680	98169.680	0	1.04	0.73	1	3517

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			93	2.454	106210.655	106210.655	1.78E-15	1.04	0.51	1	3489
				2.301	60428.426	60428.426	0	1.02	1.92	737	12293
				2.505	91030.161	91030.161	0	1.28	0.73	1	4180
				2.384	124826.254	124826.254	0	1.29	1.90	51	9210
				2.408	79476.038	79476.038	0	1.27	0.46	1	3936
				2.471	101177.754	101177.754	0	1.27	0.25	0	4060
			112	2.441	112145.604	112145.604	0	1.28	1.72	1	6319
				2.454	85865.879	85865.879	0	1.56	0.60	1	4950
				2.463	85630.256	85630.256	0	1.55	0.60	1	4423
				2.270	103052.798	103052.798	1.13E-15	1.54	0.31	0	5038
				2.429	98174.810	98174.810	0	1.54	0.88	1	5330
			150	2.311	106753.554	106753.554	1.23E-15	1.54	1.41	1	6229
				2.539	89031.031	89031.031	0	2.05	9.22	1458	36304
				2.485	77700.946	77700.946	0	2.04	2.59	27	9707
				2.401	71860.621	71860.621	0	2.03	0.47	0	6321
				2.336	96013.221	96013.221	0	2.06	6.89	85	17373
		10000	100	2.418	81859.599	81859.599	0	2.07	1.27	1	7364
				2.436	118070.971	118070.971	0	1.40	0.24	0	4115
				2.461	123840.789	123840.789	0	1.40	0.51	1	4380
				2.537	174605.521	174605.521	0	1.41	0.76	1	5040
				2.454	97598.265	97598.265	0	1.39	0.47	1	4067
			125	2.390	134056.683	134056.683	0	1.38	0.60	1	4132
				2.517	114841.470	114841.470	0	1.73	5.85	93	13287
				2.446	146636.372	146636.372	9.92E-16	1.77	0.59	1	5299
				2.365	103809.514	103809.514	0	1.73	14.67	7174	114083
				2.443	147166.898	147166.898	1.38E-15	1.76	0.96	1	5315
			150	2.305	87108.741	87108.741	0	1.76	0.97	1	5258
				2.390	112270.319	112270.319	0	2.09	8.41	1238	44243
				2.488	121487.851	121487.851	0	2.06	31.48	5271	165476
				2.459	127754.015	127754.015	0	2.10	1.12	1	6037
				2.343	150262.897	150262.897	0	2.06	1.60	1	10228

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			200	2.468	144134.695	144134.695	0	2.07	1.27	1	6698
				2.401	114750.751	114750.751	0	2.74	16.86	1845	42152
				2.318	118538.434	118538.434	0	2.73	61.76	3944	157571
				2.327	90719.369	90719.369	0	2.69	1.27	1	7661
				2.450	130060.530	130060.530	8.95E-16	2.75	1.28	1	8802
				2.474	104165.703	104165.703	0	2.76	0.98	1	8197
		12500	125	2.506	186442.834	186442.834	0	1.77	10.59	2252	35380
				2.378	99899.583	99899.583	0	1.79	1.56	1	6300
				2.404	168030.908	168030.908	6.93E-16	1.74	0.29	0	5025
				2.296	199348.411	199348.411	0	1.74	1.07	1	4747
				2.529	134609.502	134609.502	0	1.78	3.30	312	13536
			156	2.455	183396.139	183396.139	0	2.19	1.05	1	6926
				2.524	243225.780	243225.780	0	2.20	1.18	1	7965
				2.619	181483.419	181483.419	1.28E-15	2.17	0.41	0	6462
				2.563	150692.922	150692.922	2.32E-15	2.17	0.49	0	6329
				2.437	141834.030	141834.030	0	2.15	0.60	1	5683
			187	2.483	136827.117	136827.117	2.34E-15	2.60	0.57	0	7696
				2.408	159642.372	159642.372	0	2.57	8.00	333	23085
				2.346	131161.981	131161.981	0	2.61	1.65	1	7350
				2.374	111054.657	111054.657	0	2.61	1.34	1	8048
				2.475	185615.181	185615.181	0	2.60	9.65	19	14751
			250	2.370	164502.303	164502.303	5.31E-16	3.40	29.72	3414	114033
				2.352	135281.863	135281.863	1.72E-15	3.43	0.79	0	6824
				2.466	112033.831	112033.831	0	3.37	4.51	120	9618
				2.421	146798.529	146798.529	0	3.40	6.00	238	10918
				2.348	122878.603	122878.603	1.66E-15	3.42	8.31	1189	80305
4	8	7500	75	2.472	104705.503	104705.503	0	1.10	0.88	1	3576
				2.345	93078.501	93078.501	0	1.10	0.65	1	3335
				2.610	88992.016	88992.016	0	1.12	1.05	1	4095
				2.549	107067.175	107067.175	0	1.12	3.19	290	15881
				2.195	79941.574	79941.574	0	1.12	0.81	1	3875

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

		93	2.413	103322.791	103322.791	0	1.38	1.84	28	5224	
			2.304	83388.214	83388.214	0	1.37	0.54	1	3888	
			2.436	118601.939	118601.939	0	1.39	1.51	1	4441	
			2.406	75456.926	75456.926	0	1.37	5.85	639	21466	
			2.266	74168.572	74168.572	0	1.38	9.49	3888	53807	
		112	2.176	62848.618	62848.618	0	1.66	1.99	188	8210	
			2.576	86419.214	86419.214	0	1.67	81.26	40134	554978	
			2.425	111975.918	111975.918	0	1.69	0.73	1	4348	
			2.245	96353.827	96353.827	0	1.65	2.17	1	7538	
			2.372	85700.294	85700.294	0	1.67	33.87	7082	165372	
		150	2.488	79361.963	79361.963	0	2.21	4.35	523	16473	
			2.480	98819.872	98819.872	0	2.20	4.07	94	10416	
			2.417	114406.529	114406.529	0	2.20	12.19	1	22152	
			2.567	126472.011	126472.011	0	2.22	1.83	1	6728	
			2.354	72144.405	72144.405	0	2.21	2.26	1	7068	
		10000	100	2.548	135918.205	135918.205	0	1.50	1.99	1	5573
				2.372	135473.803	135473.803	0	1.49	1.26	1	5795
				2.572	92228.657	92228.657	0	1.50	0.57	1	4100
				2.468	84224.788	84224.788	0	1.49	3.17	792	17953
				2.440	102098.386	102098.386	0	1.51	1.50	1	7388
			125	2.429	143280.608	143280.608	0	1.84	2.90	82	6863
	2.449			120775.579	120775.579	0	1.87	5.26	1101	25586	
	2.555			171724.785	171724.785	0	1.86	1.15	1	4865	
	2.448			116856.323	116856.323	0	1.89	0.47	0	5357	
	2.388			106098.209	106098.209	0	1.85	4.91	119	12627	
	150		2.469	105637.564	105637.564	0	2.20	11.12	1985	65033	
			2.590	117650.245	117650.245	0	2.25	2.80	1	8339	
			2.475	107694.657	107694.657	0	2.26	16.57	3562	56763	
			2.347	164166.830	164166.830	7.20E-13	2.25	22.12	3977	295687	
			2.514	123354.585	123354.585	0	2.21	1.16	1	6125	
	200		2.412	86923.996	86923.996	0	2.98	46.43	6147	189310	

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

				2.487	124068.122	124068.122	0	3.00	1.27	1	5669
				2.459	131486.267	131486.267	0	2.96	4.95	1	9852
				2.346	106706.323	106706.323	0	2.90	5.15	25	5996
				2.463	107206.316	107206.316	0	2.98	6.66	318	14447
		12500	125	2.408	164358.362	164358.362	7.08E-16	1.91	8.25	265	17970
				2.347	154835.525	154835.525	1.50E-15	1.87	0.40	0	5002
				2.486	116438.749	116438.749	0	1.89	1.46	1	6197
				2.544	154718.342	154718.342	0	1.87	4.24	3072	44835
			156	2.386	149468.674	149468.674	0	1.89	3.57	340	12641
				2.306	136391.557	136391.557	0	2.35	1.29	1	6529
				2.443	151873.355	151873.355	0	2.37	0.91	1	6029
				2.407	129817.316	129817.316	0	2.34	2.05	1	7298
			187	2.411	185083.109	185083.109	1.10E-15	2.34	1.57	1	5440
				2.349	71646.964	71646.964	0	2.31	1.76	1	6100
				2.523	212033.484	212033.484	0	2.77	3.05	1	2644
				2.468	175586.433	175586.433	0	2.78	17.37	2227	79175
			250	2.469	142055.766	142055.766	1.64E-15	2.78	0.54	0	1187
				2.409	157975.554	157975.554	0	2.81	3.20	1	2803
				2.409	138270.666	138270.666	0	2.78	11.14	1891	44817
				2.481	174249.722	174249.722	0	3.70	11.39	423	23826
			75	2.516	220679.545	220679.545	0	3.70	36.30	4584	140885
				2.392	132209.655	132209.655	0	3.64	3.33	1	8706
				2.375	158235.682	158235.682	0	3.68	37.39	6851	134312
				2.374	164826.244	164826.244	0	3.63	1.77	1	6117
	12	7500	75	2.472	103081.625	103081.625	0	1.64	2.08	1	6822
				2.347	75335.168	75335.168	0	1.63	1.97	1	5950
				2.583	73907.966	73907.966	0	1.67	43.39	14326	326138
				2.253	78566.334	78566.334	0	1.67	11.22	2928	45040
			93	2.540	99002.193	99002.193	0	1.65	8.79	4444	81312
				2.154	91135.979	91135.979	0	2.06	8.37	498	24325
				2.554	112490.056	112490.056	0	2.04	2011.67	329942	13781280

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

					2.569	117024.213	117024.213	0	2.11	57.81	15306	305128
					2.283	80313.763	80313.763	0	2.08	981.35	165781	8335812
					2.541	112360.858	112360.858	0	2.07	1.88	1	7561
					2.470	94485.452	94485.452	0	2.46	12.93	2746	53277
					2.193	75313.512	75313.512	0	2.43	67.99	12746	336842
					2.462	116735.913	116735.913	0	2.45	1.05	1	7598
					2.466	117671.867	117671.867	0	2.45	3.77	1	13006
					2.348	100864.172	100864.172	0	2.44	44.10	6928	240402
				150	2.542	96881.585	96881.585	0	3.28	3.68	1	3935
					2.556	87541.335	87541.335	0	3.26	90.98	7017	334626
					2.414	84474.897	84474.897	0	3.25	43.58	4194	116360
					2.359	90770.797	90770.797	0	3.22	6.93	1	10315
					2.393	65172.824	65172.824	0	3.32	516.67	21858	640011
		10000	100	2.474	97938.574	97938.574	0	2.26	22.30	3971	86957	
				2.490	126571.973	126571.973	0	2.18	18.65	5034	141788	
				2.468	118093.061	118093.061	0	2.23	6.21	774	21231	
				2.423	123310.205	123310.205	0	2.21	13.81	2006	43685	
				2.335	141730.703	141730.703	0	2.24	27.69	5257	112449	
			125	2.423	129283.057	129283.057	0	2.74	6.48	823	24097	
				2.427	146998.238	146998.238	0	2.74	16.04	295	27623	
				2.433	110835.225	110835.225	0	2.75	38.10	4147	121889	
				2.492	104305.343	104305.343	0	2.75	15.85	404	33911	
				2.447	136457.043	136457.043	6.40E-16	2.78	2.95	1	6486	
			150	2.385	119194.694	119194.694	0	3.30	102.86	21699	418881	
				2.389	94755.687	94755.687	0	3.27	19.23	2864	62798	
				2.411	137008.492	137008.492	0	3.34	2.69	1	1407	
				2.305	94666.972	94666.972	0	3.31	35.33	4059	103711	
				2.459	119203.595	119203.595	0	3.33	2.65	1	2222	
			200	2.412	145413.904	145413.904	0	4.30	153.80	27425	381932	
				2.466	99644.996	99644.996	0	4.31	70.98	8718	217978	
				2.510	151451.373	151451.373	0	4.31	352.13	84799	930200	

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

		12500	125	2.431	115733.865	115733.865	0	4.39	53.04	531	37184
				2.347	133753.805	133753.805	1.09E-15	4.35	1.26	0	2225
				2.397	183170.354	183170.354	0	2.78	1.64	1	1575
				2.466	133785.094	133785.094	0	2.77	3.08	1	2788
				2.445	171073.988	171073.988	5.10E-16	2.79	5.46	1	8881
				2.375	168662.036	168662.036	0	2.80	5.53	56	7341
				2.542	163060.183	163060.183	0	2.77	1.96	1	1507
			156	2.459	116340.673	116340.673	0	3.42	144.33	17913	944587
				2.413	163965.116	163965.116	0	3.46	1.98	1	7147
				2.489	159535.071	159535.071	0	3.46	261.24	35330	1316959
				2.357	117398.694	117398.694	0	3.48	24.17	1565	56318
				2.444	164514.726	164514.726	0	3.41	3.43	1	4088
			187	2.298	138867.622	138867.622	0	4.13	3.78	1	2763
				2.481	145540.280	145540.280	6.00E-16	4.15	207.54	17162	359885
				2.466	170741.664	170741.664	3.63E-13	4.12	43.69	1721	49374
				2.515	149325.606	149325.606	0	4.06	3.44	1	8401
				2.506	177260.006	177260.006	0	4.11	364.31	21489	527077
			250	2.392	196818.558	196818.558	0	5.41	4.76	1	12970
				2.346	163293.976	163293.976	0	5.44	76.18	5885	317006
				2.274	104074.849	104074.849	0	5.44	192.41	11712	629102
				2.261	168471.702	168471.702	0	5.42	56.94	4292	141000
				2.446	153872.629	153872.629	7.57E-16	5.45	11.67	1	11841
	16	7500	75	2.472	106655.731	106655.731	0	2.18	16.45	4330	89880
				2.411	80999.443	80999.443	0	2.20	37.45	6320	231237
				2.246	65446.367	65446.367	0	2.16	62.26	10961	250638
				2.530	104479.866	104479.866	0	2.25	2.08	1	7022
				2.314	76209.226	76209.226	0	2.24	147.43	34077	656919
			93	2.195	95443.828	95443.828	0	2.75	80.97	8308	239679
				2.226	90464.434	90464.434	0	2.74	104.98	23837	740158
				2.063	63957.617	63793.019	0.002573547	2.75	3600.03	376433	29086168
				2.451	98190.675	98190.675	0	3.06	13.07	1	20178

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

					2.464	97456.591	97456.591	0	2.78	100.18	10459	271832	
				112	2.459	95572.673	95572.673	0	3.27	115.89	24637	453298	
					2.317	96115.865	96115.865	0	3.28	43.98	4851	114746	
					2.408	83593.452	83593.452	0	3.27	634.62	122000	3037952	
					2.327	102840.644	102840.644	0	3.31	84.66	10743	275827	
					2.515	79696.033	79696.033	0	3.30	60.64	10354	301703	
				150	2.521	81578.707	81578.707	0	4.33	140.05	12432	459239	
					2.407	94458.555	94458.555	0	4.37	8.62	1	5283	
					2.431	81344.466	81344.466	0	4.34	107.90	8291	221350	
					2.534	92988.772	92988.772	0	4.32	264.30	27735	1002326	
					2.450	73267.467	73267.467	0	4.37	101.17	10556	326561	
				10000	100	2.550	119809.826	119809.826	0	2.96	249.70	44159	1037010
						2.477	139724.756	139724.756	0	3.02	14.54	1077	30111
						2.353	113336.987	113336.987	0	2.99	8.56	1	12432
						2.341	122993.759	122993.759	0	2.98	17.95	791	32209
						2.551	124131.905	124131.905	0	2.99	7.17	1	7314
					125	2.419	132556.513	132556.513	0	3.66	80.96	4324	85212
						2.393	142645.067	142645.067	0	3.73	33.02	2608	61520
						2.430	129581.393	129581.393	0	3.68	73.45	6979	272215
						2.360	103312.107	103312.107	0	3.68	219.14	31456	716232
						2.486	105094.767	105094.767	0	3.71	93.72	19539	380497
					150	2.384	129602.142	129602.142	0	4.37	7.89	18	12869
						2.411	94380.377	94380.377	0	4.46	12.68	77	10433
						2.325	164945.680	164945.680	0	4.32	135.81	9064	245925
						2.290	91891.931	91891.931	0	4.39	40.02	3205	112663
						2.405	116221.875	116221.875	0	4.38	54.16	2868	210348
					200	2.449	100329.975	100329.975	0	5.83	210.92	18799	468158
						2.354	145611.213	145611.213	0	5.81	111.38	9144	272727
						2.536	128864.617	128864.617	0	5.71	21.74	520	29967
						2.386	117701.594	117701.594	0	5.81	91.05	3139	142611
						2.375	141414.811	141414.811	0	5.72	247.05	33275	583735

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

		12500	125	2.354	162534.609	162534.609	5.37E-16	3.71	23.78	1218	56567
				2.291	128980.353	128980.353	0	3.71	274.33	24207	499423
				2.450	158970.468	158970.468	0	3.73	601.42	30544	1162439
				2.295	129170.381	129170.381	0	3.73	1434.70	86702	5236545
				2.369	134317.205	134317.205	0	3.74	3.33	1	3186
			156	2.514	147435.461	147435.461	0	4.63	255.16	18240	631797
				2.454	149624.797	149624.797	0	4.66	5.89	1	7649
				2.396	146759.409	146759.409	0	4.61	249.93	23121	511638
				2.444	143093.172	143093.172	0	4.68	73.41	12268	213144
				2.405	145881.250	145881.250	0	4.60	5.88	1	5034
			187	2.397	129334.512	129334.512	0	5.35	45.93	4187	123853
				2.458	198285.752	198285.752	0	5.53	147.99	5457	213870
				2.443	168219.242	168219.242	0	5.53	33.75	1168	76674
				2.424	179840.827	179840.827	0	5.53	2172.99	45507	3853111
				2.425	151336.797	151336.797	0	5.50	64.78	1838	122023
			250	2.288	139165.998	139165.998	0	7.09	32.00	1	31857
				2.475	184135.284	184135.284	0	7.22	293.63	27139	488224
				2.386	195529.263	195529.263	0	7.16	265.39	10988	396465
				2.367	167402.805	167402.805	0	7.21	4.83	1	2324
				2.467	139301.977	139301.977	0	7.30	106.19	4591	131821
	10	7500	75	2.472	106348.713	106348.713	0	2.00	19.79	2664	58433
				2.472	74964.724	74964.724	0	2.04	4.75	1462	23375
				2.326	67713.570	67713.570	0	2.02	20.06	2614	92287
				2.156	85174.154	85174.154	0	2.04	412.06	31176	1324228
				2.413	70031.920	70031.920	0	2.05	125.09	21011	518229
			93	2.534	109357.275	109357.275	0	2.57	4.49	1	9755
				2.397	102247.985	102247.985	7.36E-11	2.54	71.38	5778	167967
				2.368	78007.142	78007.142	0	2.55	362.54	29475	1098295
				2.437	79815.927	79815.927	0	2.52	58.10	16645	261601
				2.430	95227.367	95227.367	0	2.55	6.74	1	16036
			112	2.182	72453.572	72453.572	0	3.06	302.66	70505	1357400

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

					2.408	103397.190	103397.190	0	3.10	37.30	2034	119070			
					2.435	93255.227	93255.227	0	3.08	276.40	21335	639831			
					2.373	71716.782	71716.782	0	3.03	199.36	21560	679539			
					2.495	94841.651	94841.651	0	3.06	123.77	6567	304046			
				150		2.391	103146.037	103146.037	0	4.05	161.81	16798	500952		
						2.467	73821.566	73821.566	0	4.07	272.57	37678	907040		
						2.387	96343.715	96343.715	0	3.98	63.22	6373	180639		
						2.483	65394.989	65394.989	0	4.02	127.18	18682	497269		
						2.402	76903.396	76903.396	0	4.08	45.50	3282	120102		
					10000	100		2.509	123688.979	123688.979	0	2.76	28.84	1793	49985
								2.346	125781.826	125781.826	0	2.75	37.05	10564	180230
								2.467	95996.237	95996.237	0	2.78	334.18	21782	832166
							2.266	136375.115	136375.115	0	2.75	25.34	1411	34700	
							2.487	140069.889	140069.889	0	2.76	69.42	14370	318325	
				125			2.502	127394.517	127394.517	0	3.42	7.76	84	11396	
							2.539	117747.932	117747.932	0	3.38	245.73	38684	1479295	
							2.408	113823.478	113823.478	0	3.43	9.21	21	14222	
							2.454	177877.766	177877.766	0	3.47	4.17	1	7175	
							2.361	121530.638	121530.638	0	3.42	187.75	33300	1005664	
				150		2.424	92019.299	92019.299	0	4.10	234.63	43021	800874		
						2.297	113855.618	113855.618	0	4.03	281.15	60027	1298749		
						2.447	125504.318	125504.318	0	4.09	7.17	1	9460		
						2.342	107998.673	107998.673	0	4.09	3.42	1	2121		
						2.324	110069.618	110069.618	0	4.02	228.43	38117	921760		
				200		2.504	113782.819	113782.819	0	5.35	99.27	6707	206017		
						2.365	130222.067	130222.067	0	5.37	31.70	2408	63741		
						2.437	92774.358	92774.358	0	5.44	98.65	5012	285372		
						2.439	139547.869	139547.869	6.26E-16	5.40	263.73	20980	390138		
						2.296	117276.348	117276.348	0	5.36	18.04	828	29166		
				12500	125		2.361	134390.710	134390.710	0	3.43	2.71	1	2153	
							2.468	150429.907	150429.907	0	3.42	50.97	2269	127852	

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

				2.249	131859.041	131859.041	0	3.46	98.44	31737	520886
				2.457	180393.378	180393.378	0	3.48	13.81	189	18568
				2.388	144499.390	144499.390	0	3.43	733.35	25015	1426343
			156	2.208	162022.863	162022.863	0	4.25	3.26	1	2710
				2.300	166044.640	166044.640	0	4.24	67.79	11911	266946
				2.443	134423.223	134423.223	0	4.28	2.00	1	1735
				2.544	163374.750	163374.750	0	4.30	116.91	18288	528888
				2.462	195009.486	195009.486	1.04E-15	4.28	5.30	1	7493
			187	2.330	100743.448	100743.448	0	5.02	49.72	4724	123100
				2.400	180053.802	180053.802	6.47E-16	5.06	5.23	1	11914
				2.565	168830.086	168830.086	0	5.11	45.56	3115	85300
				2.402	117407.356	117407.356	0	5.07	58.79	4785	166533
				2.392	115260.269	115260.269	0	5.11	33.72	1589	59258
			250	2.414	182288.004	182288.004	0	6.70	77.45	4137	159465
				2.379	148935.093	148935.093	0	6.78	92.99	6648	247293
				2.440	174090.787	174090.787	0	6.68	333.53	17010	977914
				2.322	124927.311	124927.311	0	6.68	73.50	6048	146298
				2.403	144167.290	144167.290	0	6.72	288.42	27387	543599
	15	7500	75	2.472	109843.162	109843.162	0	3.00	71.64	13970	389820
				2.342	84121.974	84121.974	0	3.03	626.76	33663	1368809
				2.371	86914.620	86914.620	0	3.10	82.30	24140	459737
				2.548	111911.694	111911.694	0	3.08	460.44	20813	798135
				2.541	105356.263	105356.263	0	3.08	70.10	15659	305798
			93	2.297	82261.867	82261.867	0	3.78	232.12	24315	832680
				2.397	86611.399	86611.399	0	3.79	194.87	29137	724477
				2.560	95219.016	95219.016	0	3.74	62.46	4734	175623
				2.433	90205.724	90205.724	0	3.79	779.43	103292	2711034
				2.498	86990.394	86761.767	0.002628193	3.76	3600.05	510270	13433622
			112	2.312	95118.960	95118.960	0	5.01	249.08	22966	642464
				2.496	86745.962	86745.962	0	4.60	1725.21	59170	4087651
				2.392	99760.841	99760.841	0	4.48	238.01	29697	1087161

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			2.256	68907.768	68907.768	0	4.48	450.68	52464	1322907		
			2.420	87474.047	87474.047	0	4.51	1654.16	173012	5238044		
			150	2.489	90350.016	90350.016	0	5.94	3437.03	57205	7385613	
				2.522	123692.275	123617.927	0.000601079	6.04	3600.09	85277	9349434	
				2.310	74286.207	74286.207	0	6.05	1465.63	144774	3351716	
				2.367	100452.248	100051.597	0.003988471	6.04	3600.05	116315	10409651	
				2.318	82563.087	82563.087	0	5.93	193.20	10010	479135	
		10000	100	2.409	127224.591	127224.591	0	4.14	1206.03	27550	1464186	
				2.602	132523.490	132523.490	0	4.14	200.36	30450	784753	
				2.565	141208.476	141208.476	0	4.11	480.98	21839	682726	
				2.430	113235.403	113235.403	0	4.61	802.13	30661	1585530	
				2.499	102154.282	102154.282	0	4.14	29.47	5960	109899	
			125	2.504	134525.198	134525.198	0	5.06	1162.09	43618	1737511	
				2.456	137028.684	137028.684	0	5.07	1743.49	71261	4615753	
				2.342	112354.242	112354.242	0	5.10	88.55	5597	192744	
			2.490	137150.704	137150.704	0	5.09	1026.56	40993	2503278		
			2.346	103363.749	103363.749	0	5.06	938.88	45256	2351059		
			150	2.389	101354.041	101354.041	0	6.09	1149.75	32372	1545026	
				2.599	121427.771	121427.771	0	6.18	1141.81	22740	1090412	
				2.473	178578.623	178578.623	0	6.01	402.72	23714	488080	
		2.547		153365.797	153365.797	0	6.12	736.76	27008	1190463		
		200	2.332	135822.000	135822.000	0	6.14	233.30	19195	656970		
			2.424	123764.502	123764.502	0	7.96	493.68	11554	831795		
			2.440	147101.128	146886.366	0.001459956	8.00	3600.08	99387	11015295		
			2.443	158377.686	158377.686	0	8.08	794.39	23914	1280961		
	2.473		106702.208	106702.208	0	7.92	952.06	21791	428130			
			2.436	127148.294	127148.294	0	7.86	354.77	26589	465098		
			12500	125	2.411	139982.874	139982.874	0	5.13	553.28	56674	1576162
					2.482	165206.114	165206.114	0	5.17	1121.65	249868	4056054
					2.537	147329.657	147329.657	0	5.21	101.28	10285	282447
		2.307	176610.270		176610.270	0	5.13	1935.04	250287	6613327		

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			156	2.392	164768.484	164768.484	0	5.19	104.85	6422	202417
				2.244	154024.438	153363.886	0.004288619	6.39	3600.05	31098	5648441
				2.512	149817.085	149817.085	0	6.37	277.35	10930	608057
				2.559	167455.699	167455.699	0	6.36	72.60	3416	145613
				2.400	122385.943	122385.943	0	6.42	1451.25	148164	4066829
			187	2.444	163856.497	163856.497	5.33E-16	6.36	39.91	672	43155
				2.465	132405.478	132405.478	0	7.58	2800.29	126518	7851470
				2.366	151255.478	151255.478	0	7.54	349.73	28158	566743
				2.451	148110.183	148110.183	0	7.55	52.55	1205	49601
				2.361	126327.212	126327.212	0	7.52	116.24	3752	145518
			250	2.466	135648.740	135648.740	0	7.53	124.87	8027	195716
				2.344	158319.469	158319.469	0	10.02	659.74	25087	940347
				2.339	145252.186	145017.842	0.001613359	9.94	3600.09	20963	2013609
				2.404	167097.583	167097.583	0	10.04	592.61	9753	656031
				2.430	156872.425	156794.468	0.000496945	10.08	3600.08	252275	3521522
			75	2.310	144680.786	144434.497	0.001702297	10.20	3600.12	39431	341
				2.472	109341.674	109341.674	0	3.99	2457.47	398382	10001795
				2.453	87081.379	87081.379	0	4.03	1108.74	27218	2775014
				2.469	97494.045	97494.045	0	4.06	514.38	21010	699698
				2.336	99826.025	99658.900	0.001674169	4.12	3600.05	89590	12393264
				2.480	95493.880	95493.880	0	4.10	440.81	21240	515015
			93	2.220	69893.046	69893.046	0	4.88	1656.21	23327	1727676
				2.207	86920.034	86723.544	0.002260584	5.03	3600.05	142984	10520077
				2.652	111556.135	111556.135	0	5.12	2269.64	42375	1526654
				2.565	121614.532	121131.944	0.003968174	5.02	3600.07	98156	10486786
				2.464	103961.760	103961.760	0	5.10	2120.34	118619	8690669
			112	2.406	90364.775	90302.058	0.000694044	5.99	3600.11	266846	8590559
				2.414	95057.477	94440.442	0.006491187	5.97	3600.11	31323	5378700
				2.220	90563.113	90192.208	0.004095533	5.94	3600.07	36744	4378773
				2.403	77417.587	77104.482	0.00404436	6.10	3600.07	83775	7971736
				2.591	106872.181	106728.795	0.001341664	6.04	3600.06	183213	10822692

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

			150	2.497	91000.712	90828.212	0.001895591	7.93	3600.06	52250	6161477
				2.423	90423.218	89194.678	0.013586553	7.94	3600.12	41756	2845984
				2.351	80946.111	80748.115	0.002446022	7.81	3600.08	113825	5626409
				2.296	82248.430	82248.430	0	7.86	3103.95	146354	4194593
				2.470	72611.802	71750.440	0.011862562	7.80	3600.07	41719	2377056
		10000	100	2.634	159954.166	159954.166	0	5.49	1270.41	27776	1069768
				2.304	123185.438	123185.438	0	5.55	1184.79	23336	1168153
				2.409	113440.603	113440.603	0	5.41	600.24	71797	1654911
				2.333	112312.266	112312.266	0	5.43	148.87	13750	402076
				2.489	125298.081	125298.081	0	5.43	1064.38	27931	1365351
			125	2.434	122945.082	122756.646	0.001532686	6.78	3600.10	267171	5928848
				2.451	129634.245	129634.245	0	6.90	2048.84	189185	5139307
				2.419	111246.263	111246.263	0	6.67	743.83	47409	1427149
				2.419	121560.437	121450.812	0.000901815	6.71	3600.05	145636	10577857
				2.381	129070.212	129070.212	0	6.72	62.90	2527	77797
			150	2.321	145307.780	145307.780	0	7.92	2919.78	200607	5372372
				2.310	128820.338	128576.331	0.001894164	7.91	3600.07	168313	5672523
				2.349	113572.024	113095.288	0.004197657	8.10	3600.07	155744	8220449
				2.462	115929.536	115929.536	0	8.11	542.68	25551	854193
				2.511	121928.465	121928.465	0	8.07	523.73	24918	878412
			200	2.341	124322.695	123469.059	0.006866295	10.38	3600.09	44117	3034735
				2.409	116151.166	115591.261	0.00482048	10.49	3600.09	74820	5442693
				2.552	162969.402	162969.402	0	10.71	1168.39	41112	2077252
				2.512	138467.165	137729.620	0.005326491	10.55	3600.37	31119	2685483
				2.460	97569.187	97569.187	0	10.44	253.10	3333	230910
		12500	125	2.354	158626.813	158504.357	0.000771981	6.77	3600.06	197154	12415870
				2.547	190316.008	190118.532	0.001037623	6.96	3600.07	65758	8678997
				2.473	172163.336	171517.925	0.003748831	6.95	3600.14	49331	4085716
				2.375	167595.444	167595.444	8.68E-16	7.04	676.67	21291	682079
				2.342	153977.251	153376.712	0.003900179	6.79	3600.11	32049	2399765
			156	2.471	167847.232	167847.232	0	8.38	1900.80	158736	3916578

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)

				2.377	144006.588	144006.588	0	8.34	2964.20	32091	3596291
				2.284	143954.772	143551.067	0.002804385	8.37	3600.08	75369	6220885
				2.495	155639.427	154572.761	0.006853445	8.50	3600.06	194857	8458394
				2.447	182884.276	182321.518	0.003077123	8.62	3600.11	30993	3737156
			187	2.488	129136.183	128947.720	0.001459418	9.94	3600.09	56821	8198178
				2.452	169274.249	169274.249	0	9.99	703.79	32768	1044079
				2.461	156061.995	156061.995	0	10.06	3083.76	121206	6026033
				2.398	149922.332	149038.171	0.005897461	10.13	3600.19	41500	2228523
			250	2.470	143423.299	142902.580	0.003630644	10.09	3600.09	37298	4399731
				2.416	147291.299	145858.153	0.009730015	13.18	3600.17	42131	2598260
				2.460	128132.427	128132.427	0	13.25	824.97	16985	1067526
				2.319	159539.403	159539.403	0	13.03	2201.36	102911	2331356
				2.327	133171.380	133171.380	0	12.92	2920.20	87016	3701620
				2.448	185245.322	185245.322	0	13.06	1042.76	40375	1153120

Table B.4 Continued: Solution Parameters for Combined Problem (single-assignment)