A REPLENISHMENT POLICY FOR PERISHABLE ITEMS WITH COLD CHAIN TRANSPORTATION AND LEAD TIME REDUCTION OPTIONS

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Perishability relates to products that have limited shelf life and are prone to spoilage, decay, or becoming unsafe for use over time. Although perishability is a common aspect in several product categories such as fresh produce, pharmaceuticals, blood product and fashion industry, many inventory models assume that products have an infinite shelf life. In this work, we introduce a novel approach that integrates inventory replenishment and cold chain technology decisions to replenishment policy of inventories with the aim of better preserving the effective shelf life of products. We focus on a single product, single location, continuous review inventory model with positive lead time, where products have fixed lifetimes. Demand arrivals are assumed to follow a Poisson process. If there are two batches in stock, items in the old batch is disposed with a salvage value. We assume that a cold chain technology is available to maintain the items in a preservative environment, protecting them against temperature and moisture during lead time, thereby providing an extended shelf life when the items arrive in the inventory. In particular, we adopt a modified lot size/reorder point (Q,r) policy that allows for a cold chain technology. The model also allows for a scenario where lead time is reduced through expedited transportation, which directly increases shelf life of arriving batch while decreases the time spent during transportation. The objective is to minimize the expected cost per unit time over an infinite horizon the decision variables of order quantity, reorder level, and cold chain technology level. A numerical study is provided to demonstrate the model performance, its sensitivity to system parameters and to compare all the policies we present.

Keywords: inventory, perishables, cold chain technology, lead time reduction.

ÖZET

BOZULABİLİR ÜRÜNLER İÇİN SOĞUK ZİNCİR TAŞIMACILIĞI VE TESLİM SÜRESİ AZALTMA SEÇENEKLERİYLE TEDARİK POLİTİKASI

Atahan Bayır
Endüstri Mühendisliği, Yüksek Lisans
Tez Danışmanı: Ülkü Gürler
Temmuz 2023

Bozulabilirlik, sınırlı raf ömrüne sahip ve zamanla bozulma, çürüme veya kullanım için güvensiz hale gelme eğiliminde olan ürünlerle ilgilidir. Taze ürünler, ilaçlar, kan ürünleri ve moda endüstrisi gibi birçok ürün kategorisinde bozulma yaygın bir özelliktir, ancak birçok envanter modeli ürünlerin sonsuz raf ömrüne sahip olduğunu varsayar. Bu çalışmada, envanter doldurma politikasına soğuk zincir teknolojisi kararlarını entegre eden yeni bir yaklaşım sunuyoruz ve böylece ürünlerin etkin raf ömrünü daha iyi korumayı amaçlıyoruz. Tek bir ürün, tek bir konum, sürekli gözden geçirme envanter modelinde, pozitif ve sabit tedarik süresi olan ürünlerin sabit ömürleri olduğunu varsayıyoruz. Talep süreci Poisson dağılımına sahip varsayılmıştır. Önceki kafiledeki ürünlerin tamamı satılmadan önce yeni bir kafile gelirse eski kafiledeki ürünler hurda değeri ile atılır. Ürünleri tedarik süresince sıcaklık ve nemden koruyan bir ortam sağlamak için soğuk zincir teknolojisinin mevcut olduğunu varsayıyoruz ve bu şekilde, ürünler envantere geldiğinde daha uzun raf ömrü sağlanmış olur. Soğuk zincir teknolojisine olanak tanıyan modifiye edilmiş bir kafile büyüklüğü/yeniden sipariş noktası (Q,r) politikasını benimsemekteyiz. Model teslim süresinin hızlandırılmış taşımacılık yoluyla azaltılabileceği bir senaryoya izin verir, bu da doğrudan envantere varan ürünlerin raf ömrünü artırırken tedarik süresini azaltır. Hedef, sonsuz bir dönem boyunca birim zamana düşen beklenen maliyeti en aza indirmektir ve karar değişkenleri sipariş miktarı, yeniden sipariş seviyesi ve soğuk zincir teknolojisi seviyesidir. Modelin performansını, sistem parametrelerine olan duyarlılığını ve sunduğumuz tüm politikaları karşılaştırmak için sayısal bir çalışma sunulmuştur.

Anahtar sözcükler: envanter, bozulabilir ürünler, soğuk zincir yatırımı, tedarik süresi azaltma .

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NOTATION

$\boldsymbol{Q}$: Order quantity

$\boldsymbol{r}$: Reorder level

$\boldsymbol{\lambda}$: Demand rate

$\boldsymbol{\tau}$: Shelf lives of the items in a batch at the ordering instance

$\boldsymbol{\tau_l}$: Remaining shelf lives of the items in a batch at the instance of order arrival

$\boldsymbol{L}$: Lead time

$\boldsymbol{h}$: Holding cost per unit time

$\boldsymbol{K}$: Fixed ordering cost

$\boldsymbol{p}$: Perishing cost per unit

$\boldsymbol{s}$: Acquisiton cost per unit

$\boldsymbol{l}$: Cold chain technology level

$\boldsymbol{\theta}$: Cold chain technology cost coefficient

$\boldsymbol{\beta}$: Cold chain technology cost coefficient

$\boldsymbol{\alpha}$: Prespecified value for the average fraction of lost sales

$\boldsymbol{T_n}$: Time where inventory level hits Q for n^{th} time

$\boldsymbol{X_i}$: Random variable representing the arrival time of the $i'th$ consecutive demand

$\boldsymbol{N(t)}$: Number of Poisson arrivals in the interval $(0, t]$

$\boldsymbol{H_i(t)}$: Value of the cdf of the gamma r.v. with parameters i and λ at t

$\boldsymbol{E[CL]}$: Expected cycle length

$\boldsymbol{E[OH]}$: Expected on-hand inventory carried within a cycle

$\boldsymbol{E[P]}$: Expected number of units that perish per cycle

$\boldsymbol{E[SD]}$: Expected satisfied demand per cycle

$\boldsymbol{E[S]}$: Expected total salvage revenue from disposal for a cycle

Chapter 1

Introduction

Inventory management is one of the core areas of operations management that have attracted tremendous attention in the literature since it addresses fundemental questions. In very broad terms, inventory management aims to satisfy customer demands with least expected costs and achieves this by taking correct decisions about when to place replenishment orders and how much to order. In most of the existing inventory literature the items are assumed to have infinite shelf lives. Under this assumption and particular settings optimal replenishment policies are established, leading to the well known (s, S) or (Q, r) policies. There is a vast literature on the algorithms and approximations for computing optimal and near optimal solutions of these policies. The assumption of infinite shelf life of items on the other hand is violated in many practical situations. The concept of perishability pertains to products that have a limited shelf life and are susceptible to spoilage, decay, or becoming unsafe for use over time. However, most of the existing inventory models assume that products have an infinite shelf life. When products are perishable with finite usable lifetime, it is important to incorporate this aspect into models in operations management for more realistic, accurate and effective results. In the inventory management context, demand uncertainty, review policies as being continuous or periodic, structure of the lead time for replenishment, number of products and locations have been traditional factors that have impact on inventory control models. Perishability introduces significant additional complications due to the challenges introduced by finite shelf life of products in regards to keeping them on-hand in storage or in transit.

Examples of perishable products with a fixed finite lifetime include fresh foods, dairy products, pharmaceuticals, and newspapers. By considering perishability in inventory models, businesses can make informed decisions regarding replenishment, inventory levels, and other factors, leading to improved inventory management and reduced losses associated with perishable items. Consider for example raw fish: it is extremely perishable, mainly due to the vulnerability of its physical structure, which allows enzymes and spoilage bacteria to easily affect it. To preserve its quality, raw fish should be kept dry in refrigerators at temperatures below 0^oC. Even with optimal storage conditions, it typically has a storage life of around 10 days. Given its high perishability, it is essential to thermally package raw fish to prevent heat exchange and ensure careful distribution to retailers. With the transportation of items, the shelf life of the fish declines rapidly. Consequently, sellers face the challenge of optimizing management of the inventory for both fresh and older fish, as well as determining the appropriate quantity of new fish packages to order. There are numerous other perishable items that exhibit distinct rates of aging under various conditions. Kitinoja studies shelf life of the products such as milk, green vegetables and fruits that are experimented to stay in different tempereatures [2] (as indicated in Table 1.1). Effectively managing inventories of these perishable items is challenging due to their limited useful life, necessitating constant monitoring of their age throughout both the lead time and inventory stages. Failure to utilize these items before their expiry date leads to obsolescence, resulting in additional costs related to perished goods.

As environmental conditions, particularly temperature, worsen, the speed at which products deteriorate increases rapidly, significantly decreasing their chances of remaining usable. Regarding the impact of the temperature on shelf lives, Sumner and Williams [1] studied corn and soybeans in detail as a perishable product. Under different temperatures and moistures, they observed the shelf life of these products and concluded that high temperatures can hold more moisture which basically increases the deterioration rate. They showed that corn and soybeans exponentially decrease with respect to temperatures and moisture.

Food Product	**Storage potential**			
	at optimum cold temperature	**optimum temperature + 10**$^{\circ}C$	**optimum temperature + 20**$^{\circ}C$	**optimum temperature + 30**$^{\circ}C$
Fresh fish	10 days at 0°C	4 to 5 days at 10°C	1 to 2 days at 20°C	A few hours at 30°C
Milk	2 weeks at 0°C	7 days at 10°C	2 to 3 days at 20°C	A few hours at 30°C
Fresh green vegetables	1 month at 0°C	2 weeks at 10°C	1 week at 20°C	Less than 2 days at 30°C
Potatoes	5 to 10 months at 4 to 12 °C	Less than 2 months at 22 °C°	Less than 1 month at 32 °C	Less than 2 weeks at 42 °C
Mangoes	2 to 3 weeks at 13°C	1 week at 23°C	4 days at 33°C	2 days at 43°C
Apples	3 to 6 months at -1°C	2 months at 10°C	1 month at 20°C	A few weeks at 30°C

Table 1.1: Decrease in Shelf life w.r.t. Temperature [2]

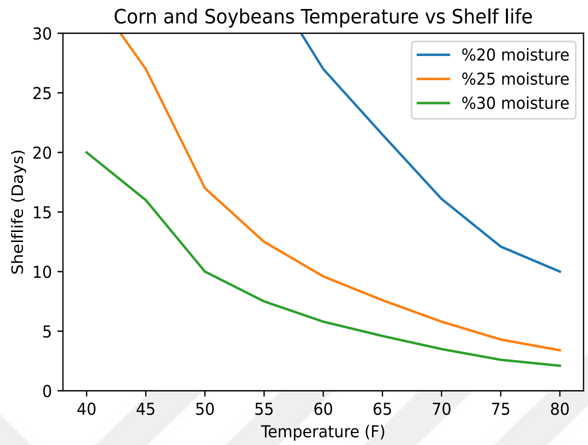

Figure 1.1: Shelf life of Corns and Soybeans w.r.t Temperature and Moisture [1]

In the literature on perishable inventories, especially in recent years, there has been a focus on controlling shelf life through cold chain preservation technology. This technology involves providing the necessary environment with appropriate temperature and moisture levels to enhance the shelf life of items in stock, albeit with an additional cost for the cold chain technology. However, these studies typically assume a zero lead time and do not take into account the aging process during the lead time. Building upon this concept, our goal is to adapt a cold chain technology with preservation techniques used during transportation to maintain suitable environmental conditions for perishable items. The aim is to slow down the aging process during the lead time and reduce the rate of deterioration, ensuring that products age less than the lead time during this period.

In this study, we introduce a novel approach that integrates inventory replenishment and cold chain technology decisions to increase the remaining lifetime of perishable products when they arrive to inventory. The objective is to minimize the expected cost per unit time over an infinite horizon. We also adopt a disposal of products in case of two batches on-hand. If the old batch is not depleted when new batch is arrived to inventory, products with low shelf life at the end of each cycle, are disposed with salvage value based on their remaining lifespan. This strategy allows us to utilize the items that are nearing expiration. In cases where perishing incurs particularly high costs, we further decrease the overall perishing instances. This joint model offers a framework for managing perishable inventory, making informed decisions regarding replenishment, cold chain technologys, and item valuation, ultimately leading to improved efficiency and cost reduction. We propose the implementation of "reefers" as a special environment during the lead time. Reefers are refrigerated containers utilized for transporting perishable products. They play a crucial role in preserving the products under optimal environmental conditions by regulating temperature and moisture levels. Reefers are commonly employed in both ships and trucks, effectively reducing the deterioration of perishable items until they are ready for sale. To achieve the objective of slowing down the aging process during the lead time, reefers can be used to maintain the appropriate environmental conditions. The temperature and moisture within the containers can be adjusted as needed, helping to extend the shelf life of the products. However, it's important to note that as we increase the age-slowing process through the use of reefers, it incurs additional costs. Therefore, the environmental conditions can be adjusted at a level that strikes a balance between preserving the products and managing the associated cost.

In inventory models the assumptions for unmet demands and the duration of lead times are crucial for the analysis of the resulting models. Positive lead times (vs. zero lead time) and the lost sales (vs. backordering) assumptions usually introduce more complications to the model development and the analysis but in most of the cases they reflect the reality better. The setting that we consider in our study is a single-location, single-item, continuous review inventory system with unit Poisson demand arrivals where the products have fixed lifetimes and unmet demands are lost and the replenishment lead time is positive. We propose a replenishment policy to determine the optimal ordering quantity and reorder level as well as the cold chain technology level for transportation during the lead time during lead time. We also consider a similar a model that has an option to reduce the replenishment lead time. In particular, we introduce two models, one of which focuses on better preservation of shelf life during lead time, and the other focuses on the reduction of chronological lead time during transportation. We derive the operating characteristics of the models and the long run expected profit rate. We present numerical results to gain insight about how the optimal cold chain technology levels and inventory levels are realised with different system parameters.

Our numerical findings indicate that both policies mentioned above outperform the model with no cold chain or lead time reduction policy particularly when shelf lives are short and service levels are high. We also observe that the policy with cold chain option performs better when demand rate is high, while expedited transportation policy performs better for low demand rates.

This thesis covers the following chapters. In Chapter 2, we present the related literature review for inventory systems with perishable items and cold chain technology. In Chapter 3, we explicitly introduce our policy and derive key operating characteristics. In Chapter 4, we present our numerical results on wide range of parameters settings in comparison with other models to find the key points of performance measure of the systems. In Chapter 5, we present the overall findings and conclusion for what we built in previous chapters as well as related future work.

Chapter 2

Literature Review

The studies on perishable inventories vary according to use of demand, shelf life, replenishment policies, diversity of items, etc. While considering how these factors affect the system, high demand correspondence with miminum cost rate is aimed. We will only talk about the research that are closely related to us.

Even though perishable products are widely used in real-world inventory management processes, majority of the literature considers durable products which do not age and have infinite shelf life to avoid mathematical difficulties brought by finite shelf life. Models that consider perishability may vary depending on the structure of shelf life definition and lead time structure. Some models assume that each item in a batch has the same shelf life, however; in some of them, items in a batch are assumed to have independent and identically distributions.

Liu [3] considers an (s, S) continuous review inventory system where items have random lifetimes. He optimize a closed form of long-run expected cost function while the lead time is zero and backlogs are allowed in his work. Then, Liu and Lian [4] extended the continuous review perishable inventory model with fixed lifetime by using a general renewal demand process and zero lead times. They derived the steady state probability distribution of the inventory level with a Markov renewal approach. Also, exponential lifetime is also used a lot in the literature. For instance, Lawrance and Saranya [5] analyze a stochastic inventory system with exponential lifetime and replacement for items in the inventory. They utilized matrix-geometric methods to find the optimal expected cost rate.

Weiss [6] considered a continuous review perishable inventory model with a Poisson process demand and zero lead time. He resulted that (s, S) policy is the optimal with if the demand process is compound Poisson. Chiu [7] proposed an approximate continuous review perishable inventory with (Q, r) policy. He assumes fixed shelf life, positive lead time and full backordering in the model. He also compares the same model with no perishability.

Inventory management choices on the best use or sale of perishable commodities that have passed their expiration date but still have some value are made when disposing of perishables with a salvage value. Numerous research has considered efficient disposal methods for items that have reached their expiration dates but still have some value.

Tekin, Gürler, and Berk [8] conducted a study on a continuous review inventory policy with Poisson demand arrivals, constant shelf life, and lost sales. They also consider a service level constraint where lost sales rate is not allowed to exceed a certain threshold. Their research focused on investigating a time-based control policy for slow-moving perishable inventory systems with high service levels. They found that the time-based control policy outperforms the stock level policy, escpecially when short shelf life and high service levels are desired.

Gürler and Berk [9] studied the same setting and they modeled a system using an embedded Markov process aproach by introducing the concept of effective shelf life using a stationary distribution and compared their model with the time-based policy. Their analysis indicated that while their policy might not be practical in rare cases, it demonstrated strong overall performance.

Bayramoğlu [10], in her thesis, studied a joint pricing and replenishment policy with perishable items and positive lead times where demand is described as decreasing function of price. Haijema [11] also considers an inventory control system for perishable items with a cost reduction policy with disposal of the perishable products by using a stochastic dynamic programming. He demostrated that disposing items before the expiration date is optimal in some cases and optimal disposal policy can diminish the shortages and total cost significantly. Annadurai [12] analyzes replenishment policy with partial backloggings. He incorporate a salvage value into perishable items with the disposal process. The value of the product is assumed to be as the acquisition cost; thus, salvage value is decided based on acquisition cost and the remaining age during disposal.

In more recent studies, Sebatjane and Adetunji [13] consider lot-sizing policies for a three-echelon supply chain for growing and perishable items. It is assumed that there are three distinct stages involved in the system, namely, rearing (or growing), processing and selling, carried out by a farmer, a processor and a retailer, respectively. They also assume an inventory level and expiration date dependent demand. Relating inventory systems to environmental damage, Sepehri et al. [14] studied joint pricing and inventory model for deteriorating items. They assume a price-dependent demand with allowed delayed payment for the buyer. Moreover, carbon is assumed to emit due to ordering and storage operations and carbon cap and trade is regulated along with allowable carbon reduction investment. Boxma et al. [15] consider a stochastic EOQ model in which demand occurs in a two-state random environment alternating between periods of high demand and periods of low demand according to a continuous-time semi-Markov process. According to the demand level, they adopt a compound Poisson process for demand arrivals.

Although our model assumes a fixed shelf life during which the usability of the product stays constant, there is a sizeable literature for perishables where products deteriorate over time. We give some references below. The most commonly used deterioration function in inventory studies is constant decay. Bhunia and Maiti [16] assumed that on-hand inventory deteriorates at a constant rate per time unit. Chare and Schrader [17] were the first to propose a deterioration concept for perishable items as exponential decay. Afterward, more realistic deterioration concepts were proposed to increase applicability to real-life inventory models. Mukhopadhyay [18] considers a time-proportional deterioration rate with a power form of the price-dependent demand rate, where the deterioration of the inventory increases with time. Another type of inventory models is proposed with a time-dependent deterioration rate. In practice, the deterioration rate of items like foods, electronic components, fashionable goods, and drugs is suitable to express in terms of a Weibull distribution [19]. According to later studies, the deterioration rate increases with age. In other words, the longer on-hand items remain, the higher the rate at which they decay. Lo et al. [19] consider a two-parameter Weibull distribution, while Chakrabarty et al. [20] used a three-parameter Weibull distribution for the distribution of the time to deterioration. Moreover, different scenarios can be examined within one model using the Weibull distribution, such as proportional, inversely proportional, and constant deterioration with respect to time. Consequently, it is observed that perishing occurs in a non-linear and increasing structure according to time. Feng, L. [21] considers that perishable items result not only in direct quantitative losses but also in qualitative losses, which reduce the customer acceptability. A constant deterioration rate for quality is assumed to be between zero and one and an investment function on quality preservation is defined as a function of time. This directly affects the quality fuction which is a linear function of investment function.

We introduce below some of the studies focusing on preservation of the product lifetime with cold chain technologys with their assumptions. The deterioration of fresh food leads to substantial economic losses and food safety problems.

Hsu et al. [22] and Liu et al. [23] define a positive preservation technology cost. By integrating technology cost with total cost function, they aimed to decrease the deterioration with different levels of cost input.

Ma et al. [24] consider a two-echelon cold chain consisting of one supplier and one retailer. Regarding demand preferences and carbon policy, both supplier and retailer should improve the shelf life and diminish the environmental effects of perishable products. Ma et al. presents cold chain technology investment and carbon abatement to adress the needs of the suppliers and the retailer, as well as affecting the demand. On the other hand, further investment in cold chain technology increases carbon emission. His model tries to establish a balance between benefit and hazard on a long-term horizon and dynamic setup, as well as maximizing the profit. Overall, Mu et al. [24] showed a theoretical basis for long-term optimization and coordination of cold chains in low-carbon environments. Wang [25] utilizes Markov dynamic programming to solve a joint problem of replenishment, pricing, and cold chain technology for retailers on an indefinite period with objective of profit maximization. He considers perishable items in terms of both quality and quantity decay. He found that initial inventory can be essential to find the optimal strategies for replenishment, pricing and cold chain technology decisions. His study found that when quality loss increases, amount of a single replenishment and time for appearance preservation investment before cycle should be diminished, as well as prices should increase. Also, when quantity loss increases, quantity of single replenishment should be reduced.

Chapter 3

Cold Chain Technology and Inventory Control Policy

In this chapter, a replenishment policy and cold chain technology issues for a perishable inventory system under continuous review are considered.

3.1 Introduction and Description of the Model

The major concern in inventory management is to satisfy as many demand as possible while keeping the costs as small as possible. In this respect, replenishment policies with finite product lifetimes are considered to be difficult in general due to the risk of perishability on shelf or on transit that results in further costs both due to perishing items and also additional lost sales that may result from perishing. If the items in the inventory are subject to perishing after a constant time, neither the conventional (s, S) nor (Q, r) policies are optimal since the mentioned policies do not take into account the time that has passed since the order is placed or the items have joined the stock. In fact the structure of the optimal policies for perishable items is not known and it wold be expected that it would depend on the elapsed time after the shelf life has started. For technical convenience some studies with perishable products assume that the lead time is zero. For items with fixed shelf life, this assumption drastically simplifies the policy structure since one does not need to make advance ordering to hedge for the deman during the lead time. However, with positive lead times, the inventory management is more challenging, since hedging againts both potential lost sales during the lead time with conventional perspective and the additional risk of lost sales due to perishing are needed.

On the other hand, technologies for keeping perishable products or those subject to spoilage are continuously developing and are being used especially for pharmaceuticals, blood products, meats, fish and some green produce to keep the products in the best conditions possible during the transportation of them from one location to the other.

In this thesis we consider the incorporation of a technically available environment in the inventory replenishment policy of perishable products. A commonly used terminology is the *cold chain environment*. In our model we assumes that the aging of a fresh batch begins once it is ordered and as time passes by during the lead time when it is positive, the remaining shelf lives of the products reduce. Our aim is to consider a model where a temperature-controlled environment with cold chain technology is provided during the transportation of the orders.

We next will state the assumptions of our model.

We consider a single product, single location, continuous review inventory system with constant lead time $L > 0$, where the inventory levels are monitored continuously. It is assumed that demands arrive according to a Poisson Process with rate $\lambda > 0$. Replenishment orders are given in batches and all items in a batch have same fixed finite shelf life τ when order is placed. The items in the batch start aging once it is ordered and unsatisfied demands are lost.

Direct costs associated to the system are the linear holding cost, linear perishing cost, linear acquisition cost and the fixed ordering cost. The acquisition cost per unit is s; holding cost per unit held in stock per unit time is h ; perishing cost per unit is p ; fixed ordering cost per order is K . Since units are assumed to be equally useful throughout their lifetimes, a FIFO (first-in-first-out) issuing policy is exploited.

We assume that transportation of batches can be conducted with an age-slowing environment during lead time. We define cold chain technology level to be used in this environment as $l \in \mathbb{R}^+$ where $l \in [u_l, u_u]$ and $0 \leq u_l < u_u \leq 1$ where u_l is minimum and u_u is the maximum technology level available in practice. Then, if cold chain technology level of l is used, the batch will age $L(1-l)$ during lead time L, resulting a remaining shelf life of $\tau_l = \tau - L(1-l)$ when the batch is arrived to the inventory. The decision of cold chain technology level is made only once at time $t = 0$ but in each regenerative cycle, the same technology is utilized one order is placed. Thus, we allow ordered batches to undergo an aging process less than lead time L. In this respect, cold chain technology decision is equivalant to increasing remaining shelf life of arriving batch.

According to the literature, cold chain technology is associated with a cold chain cost function with a positive non-linear relationship with the age-preservation level of the cold chain environment [22]. Hence, we define an exponential cold chain cost function V w.r.t to cold chain technology level l as follows

$$V : \mathbb{R}^+ \rightarrow \mathbb{R}^+, \quad V(l) = \theta e^{\beta l}, \quad l \in [u_l, u_u], \theta, \beta \in R^+ \tag{3.1}$$

Our aim is to determine the appropriate cold chain technology level based on the ordering policy described below and observe its effect on the total cost function. Despite the fact that cold chain technology incurs an exponential cost with respect to the technology level, we will argue whether the resulting benefit of cold chain technology, namely the increased remaining shelf life of the arrival batch, will lead to a decrease in the total cost rate or not.

3.2 Replenishment Policy

The following lot size (Q) and reorder point (r) policy with cold chain technology level l is proposed where it is assumed that $r < Q$.

$\pi(Q, r, l)$ **Policy**: A replenishment order of Q units is placed when the inventory level hits r by demand or zero by perishing with cold chain technology level l and remaining shelf life τ_l. If the on-hand inventory is positive upon arrival of a new batch, the on-hand items are salvaged.

Lot size/reorder point policies are conventionally expressed in the literature in terms of *Inventory Position*, which is defined as the sum of the on-hand inventory and the outstanding orders minus the backorders. However, in our specific scenario, we do not allow backorders, resulting in lost sales. Additionally, we assume that $Q > r$, meaning that only one order is outstanding at any given time, and there may be a maximum of two batches on hand upon the arrival of an outstanding order. As we dispose the older batch when a new one arrives with a salvage, By focusing on the on-hand inventory level, on-hand inventory level provides a more straightforward and a sufficient description of the system.

In the literature, salvage costs are considered to be either linear or non-linear. Linear salvage cost implies that the cost of salvaging or disposing of excess inventory increases linearly with the quantity of items being salvaged. On the other hand, non-linear salvage cost indicates that the cost of salvaging or disposing of excess inventory is not constant and varies with the quantity of items being salvaged. In our study, we have opted for a linear salvage cost. This decision allows us to maintain simplicity and consistency in selling excess inventory, and to focus on the effect of cold chain technology regardless of the quantity salvaged. It ensures straightforward cost calculations and facilitates easy integration into our cold chain technology setting. Non-linear salvage cost may be more suitable in scenarios where economies of scale come into play, or when different methods are used for disposing of varying quantities of excess inventory. However, for our specific research objective of examining the effect of age-slowing environments with cold chain technology on the cost function, we aim to observe the impact of cold chain technology on cost function without complicating the salvage cost calculation. Since salvage revenue is an integral part of the cost function in our study, the choice of salvage cost behavior is crucial. If we were to use a non-linear salvage cost, the revenue from salvaged items would influence the optimal decision based on the order quantity. This could potentially make it difficult for us to isolate and analyze the direct effect of cold chain technology on the overall cost function. Therefore, we have deliberately chosen a linear salvage cost to maintain clarity and focus in our analysis, allowing us to examine the specific impact of cold chain technology on the cost function independently.

3.3 Basic Characteristics and Realizations

Consider the inventory system operating under the proposed $\pi(Q, r, l)$ replenishment and cold chain technology policy at time t = 0 with Q items on hand with remaining lifetime τ_l. According to the proposed policy, every time a new order joins the stock, if there is still items from the older batch, they are disposed with a salvage, so that the on-hand inventory becomes Q and the remaining shelf lives of the items in the most recent batch are τ_l. Hence at these instances, the system probabilistically renews itself. In Figure 3.1, we illustrate the possible realizations of the system dynamics. Hence the time intervals between two consecutive instances at which a new batch of size Q joins the onhand inventory constitute *Regenerative Cycles,* 'cycles' in short.

Let $\{T_n, n \geq 1\}$ be the sequence of times at which the inventory level hits Q for n'th time, with $T_1 = 0$. Also, for all $n \geq 1, I(T_n) = Q$, where $I(t)$ is the inventory level at time t. Let $\{X_j, j \geq 1\}$ be the sequences of Poisson demand arrival times with rate λ. Also, let $N(t)$ be the counting process associated with demand process with rate λ in $(0, t]$. We denote the time between T_n and T_{n+1} as n'th regenerative cycle for $n \geq 1$.

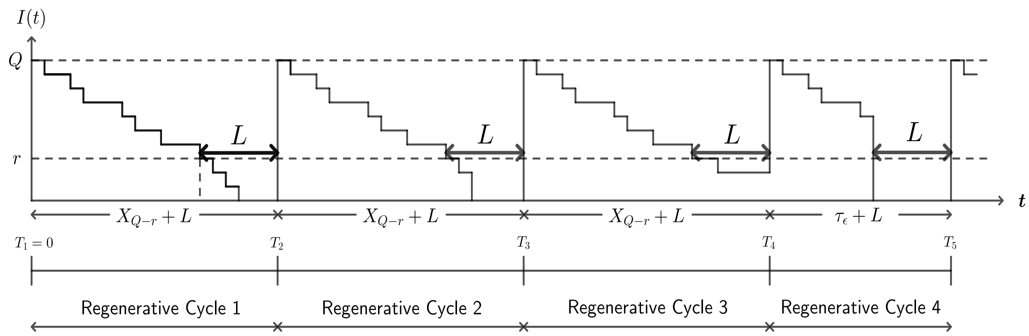

Figure 3.1: Possible Inventory Realizations

We start with Regenerative Cycle 1 at T_1 as the time origin, $t = 0$. In Regenerative Cycle 1, a replenishment order is given when the inventory level drops to r after $Q - r$ demands have arrived. During the lead-time period of length L, the remaining r units drop to zero by demand arrivals. Regenerative Cycle 1 is completed at the end of the lead time and the inventory level is increased to Q units by the arrival of the new batch and Regenerative Cycle 2 starts at time $X_{Q-r} + L$. Regenerative Cycle 2 is similar to Regenerative Cycle 1 with a minor difference. When inventory level drops to r, a replenishment order of Q units is placed; however, the items reach their shelf life during lead time and remaining units drop to zero by perishing. In Regenerative Cycle 3, the inventory drops to r by demand arrivals and a replenishment order is given X_{Q-r} time units after old batch's arrival. The order arrives when there are still some items on hand. These items are disposed with a salvage value of acquisition cost times ratio of remaining lifetime to τ. Regenerative Cycle 3 ends with the arrival of new batch making inventory hit level Q. In Regenerative Cycle 4, on the contrary than usual, the items on hand perish before inventory level drops to r. Thus, inventory level drops to zero instantly. In this case, we allow the model for ordering a replenishment at a time when inventory level hits zero. Regenerative Cycle 4 ends after L units of lead time. Therefore Regenerative Cycle 4 lasts $\tau_l + L$. The process continues in this fashion.

We present preliminary analysis of the model under consideration. In particular, typical behavior of the inventory process is displayed in detail. Now, we can list all possible realizations of the inventory level process with corresponding cycle length, CL and event, E.

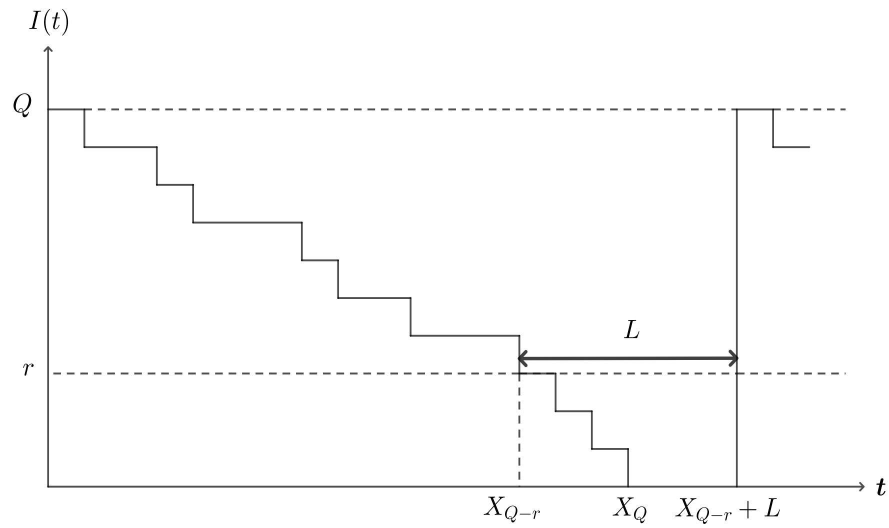

Figure 3.2: Inventory Levels in Realization 1

$$CL_1 = X_{Q-r} + L$$
$$E_1 = X_{Q-r} + X_r < \tau_l, X_r < L$$

It is important to note that all the items of a new batch start a regenerative cycle (after lead time) with an age of τ_l with cold chain level l because no time is lost between the arrival of the items and to start selling them.

Realization 1: In Figure 3.2, regenerative cycle begins with Q items on hand at $t = 0$. After X_{Q-r} units of time, the inventory level drops to r. A replenishment order of Q units is placed and the order arrives after L units of time. However, the inventory level drops to zero before the order arrives. This occurs at time X_Q and after a certain period of stock out state, the order of Q units arrives at time $X_{Q-r} + L$. Then, inventory level hits Q. At this point, Regenerative Cycle 2 begins. For this realization to occur, Q demand arrival should be depleted by demand before they perish ($X_{Q-r} + X_r < \tau_l$) and remaining on-hand inventory after the order is placed should be depleted by demand before the lead time ends ($X_r < L$).

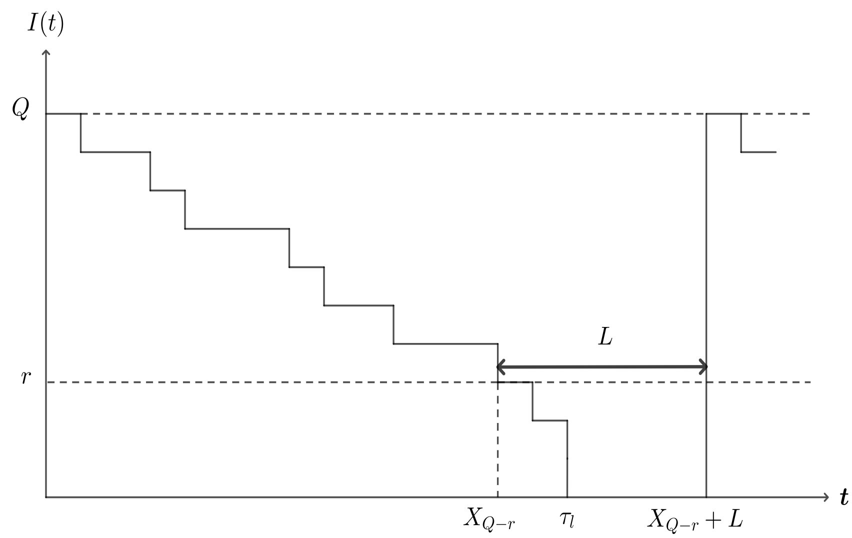

Figure 3.3: Inventory Levels in Realization 2

$$CL_2 = X_{Q-r} + L$$

$$E_2 = X_{Q-r} < \tau_l, X_{Q-r} + X_r > \tau_l, \tau_l - X_{Q-r} < L$$

Realization 2: In Figure 3.3, second realization is similar to Realization 1 with a small difference. In Realization 1, an order of Q units was placed once inventory level hits r, however in this realization the items reach shelf lies during the lead time and the units that have not been sold, perish at time τ_l. Again a stock out period is encountered and the order of Q units arrives at time $X_{Q-r} + L$. The new batch starts with shelf life of τ_l. For this realization to occur, $Q - r$ units of inventory should be depleted by demand before they perish ($X_{Q-r} < \tau_l$). Moreover, Q^{th} demand arrival should be occured after the items perish ($X_{Q-r} + X_r > \tau_l$). Also, we should eliminate the case where perishing occurs before inventory level hits r ($\tau_l - X_{Q-r} < L$).

Realization 3: In Realization 3 (see Figure 3.4), when the inventory level drops to r a replenishment order of quantity Q is placed. The differing characteristics

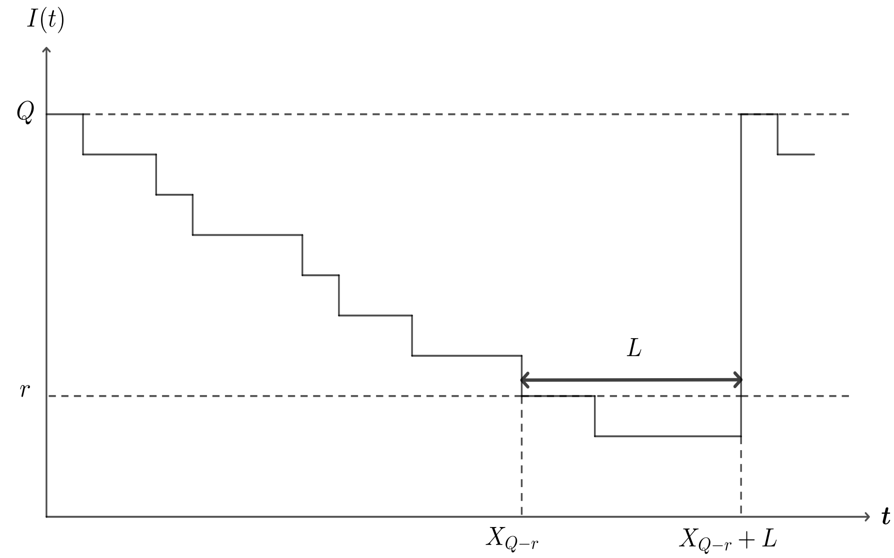

Figure 3.4: Inventory Levels in Realization 3

$$CL_3 = X_{Q-r} + L$$
$$E_3 = X_{Q-r} < \tau_l, X_r > L, \tau_l - X_{Q-r} > L$$

of this realization from the previous realizations is that inventory is not depleted during the lead time, neither by demand nor by perishing. Therefore, when the ordered quantity of Q arrives at point $X_{Q-r} + L$, the old batch is separated to be disposed according to the remaining lifetime and acquisition cost and inventory hits Q again with the arrival of new batch. For this realization to occur, perishing instance should be eliminated both before inventory level hits r ($X_{Q-r} < \tau_l$) and during lead time L ($\tau_l - X_{Q-r} > L$). Additionally, the remaining inventory level should not be depleted by demand during lead time L ($X_r > L$).

Realization 4: In Realization 4 (see Figure 3.5), remaining items in the inventory perish before inventory level drops to r by demand at time τ_l. A replenishment order of quantity Q is placed since inventory level drops to zero directly. Ordered quantity of Q arrives after L units of time. Then, inventory level hits Q. This ends Regenerative Cycle 4 at time $\tau_l + L$. For this realization to occur, all items should perish before inventory level hits r ($\tau_l < X_{Q-r}$).

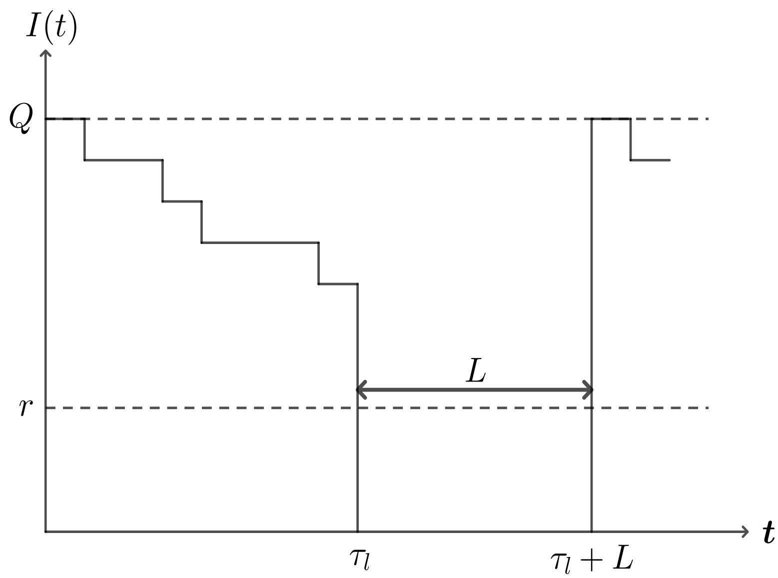

Figure 3.5: Inventory Levels in Realization 4

$$CL_4 = \tau_l + L$$
$$E_4 = \tau_l < X_{Q-r}$$

3.4 Operating Characteristics and Objective Function of Cold Chain Policy

In this chapter, we obtain the operating characteristics of the inventory system for the cold chain technology and replenishment policy and construct the objective function of the corresponding decision model.

3.4.1 Operating Characteristics

We will derive the expressions for the operating characteristics of cold chain policy, namely the expected cycle length, the expected holding cost, the expected perishing cost, and the expected salvage revenue. Then, these expressions are used to build the objective function which is explicitly discussed in the next section.

We formulate the expressions for the expected values of cycle length, on-hand inventory, number of satisfied demand, number of items that perish, cost reduction from salvage disposal as a function of the decision variables Q, r and l.

When a cycle begins there are four possible events. The first cycle type ($R1$) corresponds to fresh batch arrivals when inventory is depleted by arrivals. The second cycle type ($R2$) correspond to fresh batch arrivals when the inventory is perished during the lead time. The third one ($R3$) indicates that all Q units are not either depleted by demand or perishing after the lead time. Finally, the last one ($R4$) indicates that all items perish before the reorder point r is reached by demand.

We begin with the cycle length, CL, to derive the operating characteristics of the model of a cycle. X_{Q-r} and X_r indicate the for arrival time of $Q-r^{th}$ and r^{th} demand arrivals respectively, and they are independent.

$$CL = \begin{cases} X_{Q-r} + L \quad if & X_Q < \tau_l, X_r < L \qquad (R1) \\ X_{Q-r} + L \quad if & X_{Q-r} < \tau_l, \tau_l - X_{Q-r} < L, \\ & X_{Q-r} + X_r > \tau_l \qquad (R2) \\ X_{Q-r} + L \quad if & X_{Q-r} < \tau_l, X_r > L, \\ & \tau_l - X_{Q-r} > L \qquad (R3) \\ \tau_l + L \quad\quad\;\; if & \tau_l < X_{Q-r} \qquad (R4) \end{cases} \tag{3.2}$$

where the corresponding events are discussed above.

In order to find the holding cost we need the total stock time in a cycle. In other words we need the area under the inventory curve within a cycle which is denoted by OH. Recall that $N(t), t \geq 0$ is the number of demand arrivals with rate λ respectively in an interval of length t.

$$OH = \begin{cases} \sum_{i=1}^{Q} X_i & if \quad X_Q < \tau_l, X_r < L \quad (R1) \\ \sum_{i=1}^{N(\tau_l)} X_i + \tau_l(Q - N(\tau_l)) & if \quad X_{Q-r}<\tau_l, \tau_l - X_{Q-r}<L, \\ & \quad X_{Q-r} + X_r > \tau_l \quad (R2) \\ \sum_{i=1}^{Q-r+N(L)} X_i + (r - N(L))(X_{Q-r} + L) & if \quad X_{Q-r} < \tau_l, X_r > L, \\ & \quad \tau_l - X_{Q-r} > L \quad (R3) \\ \sum_{i=1}^{N(\tau_l)} X_i + \tau_l(Q - N(\tau_l)) & if \quad \tau_l < X_{Q-r} \quad (R4) \end{cases} \tag{3.3}$$

In the first event, inventory level drops to r by demand. Then, the remaining inventory is depleted by demand during lead time. In the second one, inventory level hits r by demand and remaining inventory is depleted by perishing during leadd time. The third one indicates that the inventory level is not depleted by demand or perishing throughout the cycle. Since there are two batches on-hand with the arrival of new batch, we disposed the old batch and continue with Q units of inventory. The fourth one indicates that items on hand perish before inventory level drops to r, resulting inventory level to be zero.

We have the number of perishing items in a cycle, P such that

$$P = \begin{cases} Q - N(\tau_l) & \text{if} \quad X_{Q-r} < \tau_l, \tau_l - X_{Q-r} < L, \\ & \quad X_{Q-r} + X_r > \tau_l \quad (R2) \\ Q - N(\tau_l) & \text{if} \quad \tau_l < X_{Q-r} \quad (R4) \\ 0 & \text{if} \quad \text{otherwise} \quad (R1, R3) \end{cases} \tag{3.4}$$

Items that have not depleted by demand in their lifetimes perish and this corresponds to the second and fourth events, relatively perishing after and before the reordering decision.

We continue with the number of satisfied demand in a cycle, SD such that

$$SD = \begin{cases} Q & \text{if} \quad X_Q < \tau_l, X_r < L \quad (R1) \\ N(\tau_l) & \text{if} \quad X_{Q-r} < \tau_l, \tau_l - X_{Q-r} < L, \\ & \quad X_{Q-r} + X_r > \tau_l \quad (R2) \\ Q - r + N(L) & \text{if} \quad X_{Q-r} < \tau_l, X_r > L, \\ & \quad \tau_l - X_{Q-r} > L \quad (R3) \\ N(\tau_l) & \text{if} \quad \tau_l < X_{Q-r} \quad (R4) \end{cases} \tag{3.5}$$

The first line corresponds to the event that inventory is depleted by demand during the lead time and after an order is placed and all Q items are depleted by demand. The second one corresponds to the events that all Q units perish during lead time and only items that have not reached their expiry age are depleted by demand. The third one indicates that there are on-hand items that aren't depleted by either perish or demand at the end of the lead time. The last one indicates that the batch perishes before the reorder point is reached and items that have not completed their lifetimes are depleted by demand.

Lastly, we have salvage revenue from disposal in a cycle, S such that

$$S = \begin{cases} \frac{s}{\tau}(\tau_l - L - X_{Q-r})(r - N(L)) & \text{if} \quad X_{Q-r} < \tau_l, X_r > L, \\ & \quad \tau_l - X_{Q-r} > L \quad (R3) \\ 0 & \text{if} \quad \text{otherwise} \quad (R1, R2, R4) \end{cases} \tag{3.6}$$

To calculate the expected values of operating characteristics, we present the following results of Tekin, Gürler and Berk [8].

Lemma

$$\sum_{i=1}^{Q} X_i = \sum_{i=1}^{Q-r} X_i + \sum_{Q-r+1}^{Q} X_i = \sum_{i=1}^{Q-r} X_i + rX_{Q-r} + \sum_{i=1}^{r} X_i \tag{3.7}$$

$$E\Big[\sum_{i=1}^{Q} X_i I(X_Q < \tau)\Big] = \frac{Q(Q+1)}{2\lambda} H_{Q+1}(\tau_l) \tag{3.8}$$

$$E\Big[\sum_{i=1}^{N(\tau)} X_i I(N(\tau) < Q)\Big] = \frac{\lambda}{2}\tau^2\big(1 - H_{Q-r}(\tau)\big) \tag{3.9}$$

$$\int_0^{\tau} u dH_Q(u) = \frac{Q}{\lambda} H_{Q+1}(\tau) \tag{3.10}$$

$$E\Big[\tau_l\big(Q - N(\tau)I(X_Q > \tau)\big)\Big] = \tau\big[Q\overline{H}_Q(\tau) - \lambda\tau\overline{H}_{Q-1}(\tau)\big] \tag{3.11}$$

Let us denote expected cycle length, expected on-hand inventory, expected number of units that perish, expected number of satisfied demand and expected salvage revenue per cycle by $E(CL), E(OH), E(P), E(SD)$ and $E(S)$, respectively. To exemplify our work, we present the derivation of $E(CL)$ and $E[P]$ in this section, others are given in the appendix.

Proposition 1 Let CL be the cycle length. Then, expected cycle length is

$$\begin{aligned}
E[CL] =&(\tau_l + L)\bar{H}_{Q-r}(\tau_l) + \int_0^{\tau_l} (X_{Q-r} + L) dH_{Q-r}(X_{Q-r}) \\
=&(\tau_l + L)\bar{H}_{Q-r}(\tau_l) + \int_0^{\tau_l} (u + L) dH_{Q-r}(u) \\
=&\tau_l - \tau_l H_{Q-r}(\tau_l) + L + \int_0^{\tau_l} u dH_{Q-r}(u) \\
=&\tau_l + L + \int_0^{\tau_l} H_{Q-r}(u) du \\
=&L + \int_0^{\tau_l} \bar{H}_{Q-r}(u) du
\end{aligned} \tag{3.12}$$

Proposition 2 Let P be the number of perished items per cycle. Then, expected number of perished items per cycle is

$$
\begin{aligned}
E[P] =&E[(Q-N(\tau_l))I(\tau_l < X_{Q-r}+L, \tau_l < X_{Q-r}+X_r)\\
=&E[(Q-N(\tau_l))I(\tau_l < X_{Q-r}+L, \tau_l < X_Q)]\\
=&E[(Q-N(\tau_l))I(N(\tau_l-L) < Q-r, N(\tau_l-L)+N(L) < Q)]\\
=&\sum_{i=0}^{Q-r-1}\sum_{j=0}^{Q-i-1}(Q-i-j)P(N(\tau_l-L)=i)P(N(L)=j)\\
=&\sum_{i=0}^{Q-r-1}\sum_{j=0}^{Q-i-1}(Q-i-j)\frac{e^{-\lambda(\tau_l-L)}(\lambda(\tau_l-L))^i}{i!}\frac{e^{-\lambda L}(\lambda L)^j}{j!}\\
=&\sum_{i=0}^{Q-r-1}\sum_{j=0}^{Q-i-1}(Q-i-j)\frac{e^{-\lambda\tau_l}\lambda^{i+j}(\tau_l-L)^i L^j}{i!\ j!}
\end{aligned}
\tag{3.13}
$$

Proposition 3 Let OH be the on-hand inventory per cycle. Then, expected number of on-hand inventory per cycle is

$$
\begin{aligned}
E[OH] =&\frac{(Q-r)(Q-r+1)}{2\lambda}\Bigg[H_r(L)H_{Q-r+1}(\tau_l-L)+H_{Q+1}(\tau_l)\\
&-\int_0^{\tau_l-L}H_r(\tau_l-u)dH_{Q-r+1}(u)\Bigg]\\
&+\frac{r(r+1)}{2\lambda}\Bigg[H_{r+1}(L)H_{Q-r}(\tau_l-L)+H_{Q+1}(\tau_l)\\
&-\int_0^{\tau_l-L}H_{r+1}(\tau_l-u)dH_{Q-r}(u)\Bigg]\\
&+\frac{(Q-r-1)(Q-r)}{2\lambda}\Bigg[H_{Q-r+1}(\tau_l)-H_{Q-r+1}(\tau_l-L)-H_{Q+1}(\tau_l)\\
&+\int_0^{\tau_l-L}H_r(\tau_l-u)dH_{Q-r+1}(u)\Bigg]\\
&+(r+1)\Bigg[\frac{Q-r}{\lambda}\big(H_{Q-r+1}(\tau_l)-H_{Q-r+1}(\tau_l-L)\big)-H_Q(\tau_l)\\
&+\int_0^{\tau_l-L}H_r(\tau_l)dH_{Q-r}(u)\Bigg]\\
&+\int_{\tau_l-L}^{\tau_l}\frac{\lambda(\tau_l-u)^2}{2}\bar{H}_{r-1}(\tau_l-u)dH_{Q-r}(u)
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{\tau_l-L}^{\tau_l}\Big[(\tau_l-u)r\bar{H}_r(\tau_l-u)-\lambda(\tau_l-u)^2\bar{H}_{r-1}(\tau_l-u)\Big]dH_{Q-r}(u)\\
&+\frac{(Q-r-1)(Q-r)}{2\lambda}\bar{H}_r(L)H_{Q-r+1}(\tau_l-L)\\
&+\frac{(r+1)(Q-r)}{\lambda}\bar{H}_r(L)H_{Q-r+1}(\tau_l-L)+\frac{\lambda}{2}L^2H_{Q-r}(\tau_l-L)\bar{H}_{r-1}(L)\\
&+LH_{Q-r}(\tau_l-L)\Big[rH_r(L)-\lambda L\bar{H}_{r-1}(L)\Big]+\frac{\lambda}{2}\tau_l^2\bar{H}_{Q-r-1}(\tau_l)\\
&+\tau_l\Big[Q\bar{H}_{Q-r}(\tau_l)-\lambda\tau_l\bar{H}_{Q-r-1}(\tau_l)\Big] \qquad (3.14)
\end{aligned}
$$

Prposition 4 Let SD be the number of satisfied demand per cycle. Then, expected number of satisfied demand per cycle is

$$
\begin{aligned}
E[SD]=&\int_0^L\int_0^{\tau_l-v}QdH_{Q-r}(u)dH_r(v)\\
&+\sum_{i=0}^{Q-r-1}\sum_{j=0}^{Q-i-1}(i+j)\frac{e^{-\lambda\tau_l}\lambda^{i+j}(\tau_l-L)^iL^j}{i!\ j!}\\
&+\sum_{i=0}^{r-1}\sum_{j=Q-r-1}^{\infty}(Q-r-i)\frac{e^{-\lambda\tau_l}\lambda^{i+j}(\tau_l-L)^jL^i}{i!\ j!} \qquad (3.15)
\end{aligned}
$$

Proposition 5 Let S be the salvage revenue from disposal per cycle. Then, expected salvage revenue from disposal per cycle is

$$
E[S]=\frac{s}{\tau}(r\bar{H}_r(L)-\lambda L\bar{H}_{r-1}(L))\int_{u=0}^{\tau_l-L}H_{Q-r}(u)du \qquad (3.16)
$$

3.4.2 Objective Function

We aim to minimize the long term expected cost per unit time. In our model, we have not imposed a lost sales cost but a service level constraint which does not allow the fraction of unmet demand per time unit exceed a threshold α. The cycles defined in previous sections are regenerative. By the renewal reward theorem [26], the expected cost in infinite horizon is equal to the ratio of expected cycle cost to expected cycle length. Therefore, our objective function becomes the result of equation 3.8.

$$\lim_{t\to\infty} \frac{E(\text{Cost}(0,t])}{t} = \frac{E(\text{Cycle Cost})}{E(\text{Cycle Length})} \tag{3.17}$$

We formally state our problem as follows.

$$\min_{Q,r,l} C(\pi(Q,r,l)) = \frac{sQ + K + V(l) + hE[OH] + pE[P] - E[S]}{E[CL]} \tag{3.18}$$

$$\text{subject to} \quad 1 - \frac{E[SD]}{\lambda E[CL]} \leq \alpha \tag{3.19}$$

Noting that $\lambda E[CL]$ is expected demand and $E[SD]$ is expected satisfied demand per cycle. The constraint corresponds to limitation of the ratio of expected unsatisfied demand per unit time.

Our goal in this study is to find whether cold chain technology offering an age-slowing environment during lead time cost effective or not. According to the technology level we utilize, the remaining shelf life of products when they arrived is increased. We want to observe the effect of saved part of shelf life on the inventory system. In this respect, cold chain technology policy is proposed to achieve a less cost rate compared to conventional inventory policies for perishables.

3.5 Lead Time Reduction Policy

In the previous model, we considered cold chain transportation options, which mainly reduces the effect of lead time on aging while the items are in transit, with a particular cost. Hence the trade-off was to balance the benefits from saving the remaining shelf lives of the items upon arrival to the stock and the cold chain costs. In a similar vein, we can consider expedited transportation, that reduces the actual chronological time of the lead time with different modes of transpostation. Since the resulting model would be technically similar to the previous one, we considered such a model, which allowed us to compare alternative policies that wold effectively increase the remaining shelflive of a batch upon arrival.

In the literature, lead time reduction is a well studied l area in inventory management, and numerous studies have investigated strategies and techniques to achieve this goal. Liao and Shyu [27] are responsible for one of the earliest articles addressing a variable lead time in an inventory model. The authors made the assumption that lead time can be divided into a number of components, each of which has a unique piecewise linear crashing cost function for lead time reduction, and that each component can be reduced to a specific minimum duration. With predetermined order quantity and normally distributed demand, they showed that lead time reduction can be effective for reducing total cost.

In their revision of Liao and Shyu's [27] work, Ben-Daya and Raouf [28] put forth a model that treats the lead time and order quantity as decision variables. They create two models: one that employs the negative exponential crashing cost function w.r.t lead time and the other that employs the lead time crashing cost function put out by Liao and Shyu. It is similar to our cold chain cost function we construct for the cold chain technology policy in previous sections. However, crashing cost function in this model is dependent on desired lead time whereas our model is not affected by the length of the lead time, but the cold chain technology level. Ben-Daya andd Raouf also offer models that employ various ways of representing the correlation between lead time and crashing cost. Furthermore, examining stochastic lead time and adding lead time as a choice variable in other inventory models are potential expansions of their work. After that it was anticipated that the inventory lead time could be controlled and Pan et al. [29] expressed the crashing cost as a function of the quantities in the orders and the reduced lead time.

Regarding the studies in the literature, we also present a replenishment policy and lead time reduction issues for perishable inventory system under continuous review.

We assume that a lead time reduction with expedited transportation can be achieved with additional crashing cost. We define lead time reduction level $l \in [u_l, u_u]$ and $0 \leq u_l < u_u \leq 1$ where u_l is minimum and u_u is the maximum lead time reduction level available in practice. Then, if lead time reduction level of l is made upon arrival, items will spend $L(1-l)$ during transportation, resulting a remaining shelf life of $\tau - L(1-i)$. Cost and parameter structure are identical to the previous policy. We use the same cost function from cold chain technology policy for crashing cost function as follows

$$V : \mathbb{R}^+ \to \mathbb{R}^+, \quad V(l) = \theta e^{\beta l}, \quad l \in [u_l, u_u], \theta, \beta \in R^+ \tag{3.20}$$

We utilize the same replenishment policy in the cold chain technology model.

$R(Q, r, l)$ **Policy**: A replenishment order of Q units is placed when the inventory level hits r units by demand or zero by perishing. If the old batch is still remaining when the new batch arrives, then the old batch is disposed with a salvage value.

Next we will present our numerical study in order to see the general behavior of the optimal policy parameters and expected cost rate with respect to different cost and system parameters. Then we will compare all models we developed until this point.

Chapter 4

Numerical Analysis

In this section we present the results of a numerical study for the performance of $\pi(Q,r,l)$, $R(Q,r,l)$ and (Q,r,l_0) policies developed in previous chapters, respectively, cold chain technology, expedited transportation and zero-level policies. In these policies, we assume that external demands are generated according to a stationary Poisson process with rate $\lambda > 0$.

$\pi(Q,r,l)$ policy aims to decrease the perishability during the lead time by incorporating a cold chain technology whereas $R(Q,r,l)$ policy focuses on increasing the effective shelf life and reducing real time with the lead time reduction. We propose 10 different technology levels l for the cold chain policy during lead time L: shelf life of the products can decrease L with zero technology level, $0.1L$, $0.2L$,... ,$0.8L$,$0.9L$ during lead time L. Similarly, for the expedited transportation policy has 10 options of lead time reduction in the same manner. Namely, lead time can be L with no reduction, $0.9L$, $0.8L$,... ,$0.2L$,$0.1L$ with different types of transportation.

Python is one of the most commonly used software to code mathematical models since it has built-in functions for probability distributions and allows probabilistic and mathematical operations. To build and anaylze both the theoretical and simulated approach, we utilize Python. Starting with non-perishables, we

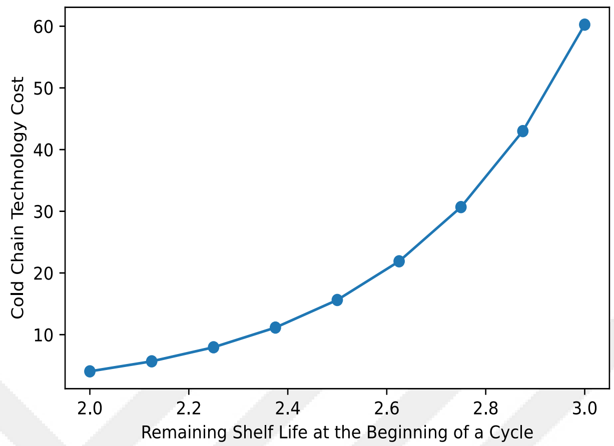

Figure 4.1: Remaining Shelf life vs Cold Chain Technology Cost ($\theta = 3, \beta = 3$)

firstly checked that whether our theoretical results are paired with the simulated policy. Then, we conducted the same analsis for the perishables. We see that after we run the simulation ten thousand units of time, the results converge to the theory. In this way, we validated our derivation of operating characteristics.

We conducted numerical studies to examine the sensitivity of the optimal policy parameters with respect to various parameters. To be consistent in our comparison of the two classes of the control policies, we maintain the restriction on $r(r < Q)$ and $\tau(\tau > L)$.

We first illustrate the impact of cold chain technology on total costs and feasible solutions in some settings. Figure 4.2 illustrates the average cost function with specific parameter values: $\lambda = 5, p = 10, K = 50, L = 1, h = 1, \alpha = 0.1, s = 10, \theta = 10, \beta = 2, l = 0$, while varying Q, r, and τ. Notably, total cost naturally increases with higher values of Q and r. However, reducing the shelf life without investing in the cold chain substantially reduces the number of feasible solutions due to service level constraints .Moving on to Figure 4.3, we explore total cost behavior with fixed $r = 15$ and parameters $\lambda = 5, K = 50, L = 1, p = 10, h = 1, s = 10, \theta = 10, \beta = 2, \alpha = 0.1$, but this time varying Q, l, and τ. Interestingly, there is no linear relationship between total cost and either Q or l. cold chain technology leads to an increase in the number of feasible solutions, as it enhances the service level and provides a wider range of viable Q and r values.Furthermore, cold chain technology significantly reduces the total cost, especially when dealing with items that have a short shelf life. This suggests that investing in a cold chain environment can lead to substantial cost savings and improved inventory management for perishable or time-sensitive goods.

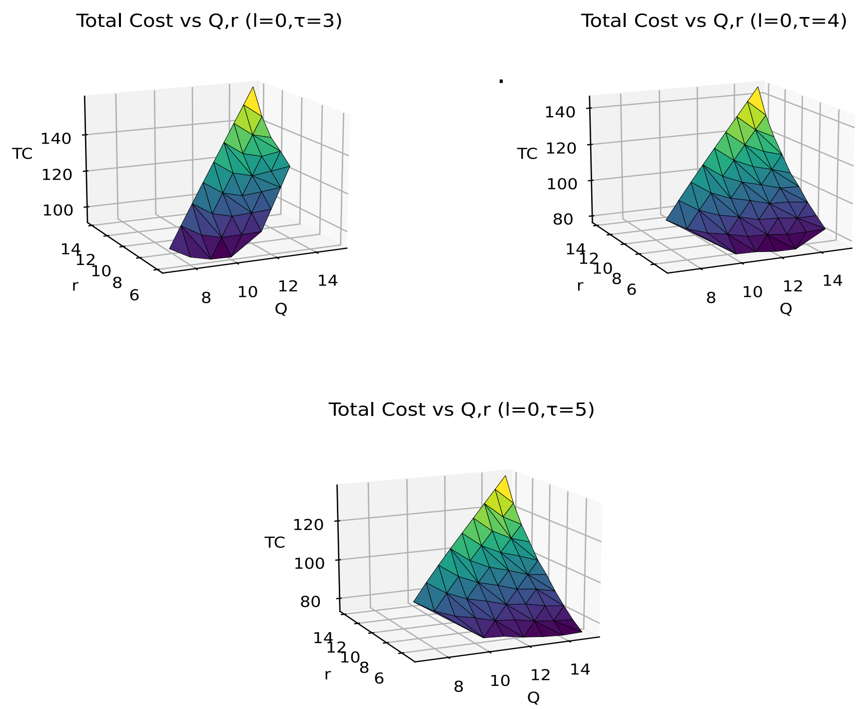

Figure 4.2: Average Cost vs Q,r,τ without cold chain technology level ($\lambda = 5, K = 50, L = 1, p = 10, h = 1, s = 10, \theta = 10, \beta = 2, \alpha = 0.1$)

4.1 Sensitivity Analysis for $\pi(Q, r, l)$ Policy

Next we discuss our findings about how the optimal policy parameters and average cost rate change with respect to ordering cost, perishing cost, permissible fraction of lost sales and shelf life. The reported experimental points represent a broad range of cases (Table 3.1). Disposal costs of some materials such as chemicals that harm the environment are high due to governmental regulations; therefore,

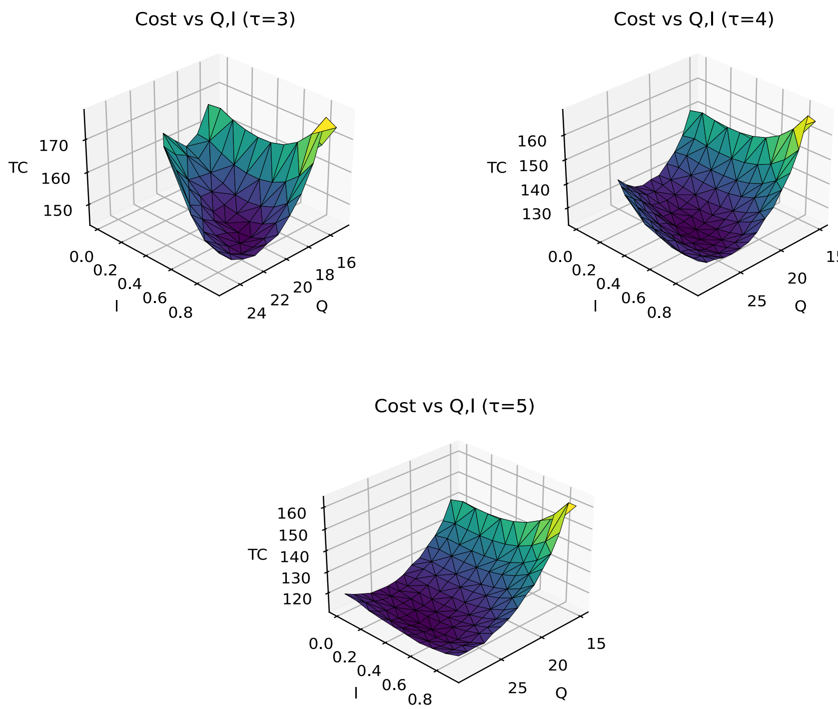

Figure 4.3: Average Cost vs Q,l with positive cold chain technology level ($r = 15, \lambda = 5, K = 50, L = 1, p = 10, h = 1, s = 10, \theta = 10, \beta = 2, \alpha = 0.1$)

high unit cost of perishing ($p = 50$) is an extent of our study to include such scenarios.

To compute the optimal average cost and the corresponding values for Q,r and l, we use an exhaustive search method where we enumerate all possible solutions and check each one to see if it is within the feasible region. Recall that both Q and r discrete variables. We also discretize l into ten equally spaced pieces and search for the best solution in three dimensional space.

In our numerical experiments, across a wide range of parameter settings, we consistently observed that the optimal order quantity Q never exceeds 30. Past this threshold, the cost function demonstrates an increasing behavior. Consequently, to streamline our search and optimize ordering decisions, we have confined our exploration to $Q < 30$ and $r < Q$ as the allowable decision variables. For both $\pi(Q, r, l)$ and $R(Q, r, l)$ policies, we also considered the case $l = 0$ indicating

Parameters	Symbol	Values tested
Allowable Loss Sales Rate	α	0.005, 0.01, 0.02, 0.05, 0.1
Ordering cost	K	50,100
Perishing cost	p	1,10,50
Acquisition cost	s	10
Shelf life	τ	3,4,5,13,16,21
Lead time	L	1,1.5,2
Demand rate	λ	0.5, 5
On-hand holding cost	h	1
Cold chain cost parameters	θ, β	(3,3),(10,2)

Table 4.1: Test parameters

neither a cold chain technology nor lead time reduction is used.

In Table 4.2, we present the results for the $\pi(Q, r, l)$ policy with fixed values of $\lambda = 5$, $L = 1$, $h = 1$,$s = 10$, $\theta = 3$, and $\beta = 3$. One notable finding is that the ordering quantity Q^* increases as the relax service level criterion α becomes more lenient. This implies that with a lower service level requirement, the system allows for higher inventory levels to reduce the risk of stockouts and meet customer demand more effectively. Conversely, the reorder level r^* is generally not significantly affected by changes in α, although in some cases, it may experience a slight decrease.

Another important aspect of the policy is the impact of increasing the reordering cost K. As we raise the reordering cost, the policy responds by increasing cold chain technology levels l. This decision allows for later reordering with larger ordering quantities, resulting in fresher products when dealing with high reordering costs. We observe that as K increased, average number of perished items is increased. $\pi(Q, r, l)$ policy tends to use less cold chain technology level choices which lead to products to have shorter shelf life with arrival to the inventory. It is expected that the average cost function would increase as a consequence of the greater cold chain technology level required for maintaining product freshness under such circumstances.

The length of shelf life τ has a considerable effect on the optimal ordering quantity Q^* and cold chain technology level l. As the shelf life increases, the policy increases the ordering quantity significantly. This adjustment ensures that more products are ordered to make the most of the extended shelf life, thus minimizing potential losses due to perishing. On the other hand, the reorder level r^* tends to experience a slight decrease when the shelf life increases. This change may be attributed to the strategy of optimizing the inventory level to accommodate the longer shelf life. Additionally, cold chain technology and lead time reduction levels tend to decrease as shelf life increases since the items on-hand begin to behave more like non-perishables, with most of them being depleted by customer demand.

The influence of perishing cost p is another factor affecting the policy's decisions. In some cases, an increase in perishing cost may lead to a decrease in the optimal ordering quantity Q^*, as the policy seeks to minimize the average number of perished items. However, the reorder level r^* remains relatively unaffected when dealing with products with short decay times and a small acceptable fraction of lost sales. Notably, higher perishing costs necessitate higher cold chain technology levels to maintain product freshness and meet the desired service level.

Overall, it is evident that reordering cost K result in higher average cost functions. Conversely, an increase in shelf life τ and cold chain technology level l leads to decreased average costs. Understanding these effects is vital in developing effective strategies to minimize costs and ensure the freshness of perishable products in inventory management. The insights gained from Table 4.2 shed light on the intricate relationships between various factors and the optimization of the $\pi(Q, r, l)$ policy, offering valuable guidance for decision-making in perishable inventory systems.

Table 4.2: $\pi(Q, r, l)$ Policy, Sensitivity Results w.r.t K, τ, p, α

$\lambda = 5$			$p = 1$					$p = 10$					$p = 50$				
		α	Q^*	r^*	τ_l^*	P	Cost	Q^*	r^*	τ_l^*	P	Cost	Q^*	r^*	τ_l^*	P	Cost
$K = 50$	$\tau = 3$	0.005	14	11	2.6	0.009	127.165	14	11	2.6	0.009	127.831	14	11	2.6	0.009	130.79
		0.01	14	10	2.6	0.016	118.309	14	10	2.6	0.016	119.416	13	9	2.8	0.008	122.906
		0.02	12	8	2.6	0.012	108.009	12	8	2.6	0.012	108.756	12	8	2.6	0.012	112.076
		0.05	12	7	2.5	0.041	96.862	12	7	2.5	0.041	99.076	10	7	2.4	0.011	105.058
		0.1	11	6	2.1	0.094	88.155	11	5	2.7	0.025	91.051	11	5	2.7	0.025	96.051
	$\tau = 4$	0.005	16	10	3.3	0.006	104.15	16	10	3.3	0.006	104.564	16	10	3.4	0.004	106.167
		0.01	16	9	3.5	0.014	98.772	16	9	3.5	0.014	99.599	15	9	3.3	0.006	101.654
		0.02	15	8	3.3	0.021	92.805	15	8	3.3	0.021	94.002	14	8	3.3	0.005	96.866
		0.05	13	6	3.4	0.012	84.944	13	6	3.4	0.012	85.52	12	6	3.3	0.003	87.03
		0.1	13	5	3.0	0.039	75.882	13	5	3.0	0.039	77.645	10	5	3.0	0.005	80.123
	$\tau = 5$	0.005	19	10	4.3	0.004	95.136	19	10	4.3	0.004	95.361	19	10	4.3	0.004	96.361
		0.01	20	9	4.5	0.008	90.901	20	9	4.5	0.008	91.342	18	9	4.3	0.003	92.617
		0.02	17	8	4.0	0.01	86.028	17	8	4.0	0.01	86.586	16	8	4.0	0.005	88.526
		0.05	17	6	4.0	0.027	76.956	17	6	4.0	0.027	78.225	14	6	4.0	0.004	80.039
		0.1	17	4	4.0	0.03	70.14	16	4	4.0	0.02	71.009	15	4	4.4	0.003	73.062
$K = 100$	$\tau = 3$	0.005	15	11	2.8	0.009	157.409	15	11	2.8	0.009	158.12	14	10	2.9	0.005	160.204
		0.01	14	10	2.6	0.016	146.204	14	10	2.6	0.016	147.311	13	9	2.8	0.008	150.801
		0.02	12	8	2.6	0.012	135.904	12	8	2.6	0.012	136.651	12	8	2.6	0.012	139.971
		0.05	12	7	2.5	0.041	121.761	12	7	2.5	0.041	123.975	12	7	2.6	0.031	132.807
		0.1	14	6	2.7	0.094	109.743	12	5	2.8	0.035	112.994	11	5	2.7	0.025	119.05
	$\tau = 4$	0.005	17	10	3.6	0.01	125.099	17	10	3.6	0.01	125.756	17	10	3.6	0.01	128.675
		0.01	16	9	3.5	0.014	119.618	16	9	3.5	0.014	120.446	16	9	3.5	0.014	124.125
		0.02	15	8	3.3	0.021	113.651	15	8	3.3	0.021	114.848	15	8	3.5	0.012	119.093
		0.05	17	7	3.4	0.059	103.131	16	7	3.3	0.048	105.95	14	6	3.6	0.013	108.769
		0.1	16	5	3.4	0.062	92.582	16	5	3.4	0.062	95.39	14	5	3.5	0.02	101.946
	$\tau = 5$	0.005	19	10	4.3	0.004	112.933	19	10	4.3	0.004	113.158	19	10	4.3	0.004	114.158
		0.01	21	9	4.5	0.016	106.086	20	9	4.5	0.008	106.839	20	9	4.5	0.008	108.799
		0.02	20	7	4.7	0.015	100.906	20	7	4.7	0.015	101.661	19	7	4.7	0.007	104.371
		0.05	19	6	4.1	0.038	92.428	17	6	4.0	0.027	93.722	17	6	4.4	0.007	96.92
		0.1	17	4	4.0	0.03	84.187	17	4	4.0	0.03	85.483	15	4	4.4	0.003	88.559

Table 4.3: $\pi(Q, r, l)$ Policy, Sensitivity Results w.r.t L, p, α

$\lambda = 5$		$p = 1$					$p = 10$					$p = 50$				
$\tau = 3$	α	Q^*	r^*	τ_l^*	P	Cost	Q^*	r^*	τ_l^*	P	Cost	Q^*	r^*	τ_l^*	P	Cost
$L = 0.5$	0.005	10.0	7.0	2.0	0.008	132.833	10.0	7.0	2.0	0.008	133.526	10.0	7.0	2.1	0.006	136.133
	0.01	11.0	6.0	2.3	0.021	119.062	11.0	6.0	2.3	0.021	120.466	11.0	6.0	2.3	0.021	126.705
	0.02	10.0	5.0	2.2	0.027	108.665	10.0	5.0	2.2	0.027	110.267	10.0	5.0	2.2	0.027	117.386
	0.05	10.0	4.0	2.1	0.062	96.97	9.0	4.0	2.0	0.043	99.473	8.0	4.0	2.0	0.021	108.345
	0.1	9.0	3.0	2.0	0.063	88.067	9.0	3.0	2.0	0.063	91.127	7.0	3.0	2.0	0.017	99.394
$L = 1$	0.005	14.0	11.0	2.6	0.009	127.165	14.0	11.0	2.6	0.009	127.831	14.0	11.0	2.6	0.009	130.79
	0.01	14.0	10.0	2.6	0.016	118.309	14.0	10.0	2.6	0.016	119.416	13.0	9.0	2.8	0.008	122.906
	0.02	12.0	8.0	2.6	0.012	108.009	12.0	8.0	2.6	0.012	108.756	12.0	8.0	2.6	0.012	112.076
	0.05	12.0	7.0	2.5	0.041	96.862	12.0	7.0	2.5	0.041	99.076	10.0	7.0	2.4	0.011	105.058
	0.1	11.0	6.0	2.1	0.094	88.155	11.0	5.0	2.7	0.025	91.051	11.0	5.0	2.7	0.025	96.051
$L = 1.5$	0.005	16.0	13.0	3.0	0.008	116.662	16.0	13.0	3.0	0.008	117.202	16.0	13.0	3.0	0.008	119.601
	0.01	15.0	12.0	3.0	0.007	112.133	15.0	12.0	3.0	0.007	112.583	15.0	12.0	3.0	0.007	114.583
	0.02	14.0	11.0	2.7	0.013	103.759	14.0	11.0	2.7	0.013	104.542	14.0	11.0	2.7	0.013	108.022
	0.05	14.0	10.0	2.6	0.053	94.63	14.0	10.0	2.7	0.034	97.483	12.0	9.0	3.0	0.004	99.228
	0.1	13.0	8.0	2.7	0.053	83.904	12.0	8.0	2.6	0.037	86.33	11.0	8.0	2.7	0.006	90.212

For the inventory system consideration, the ordering decisions depend not only on the demand rate but also on the lifetime of the items. In order to avoid loss sales during the lead time, perishing time is as crucial as the number of demands during lead time. More importantly, cold chain technology during lead time is the main part of this study; thus, we are particularly interested in interaction of the lead time and the product lifetime. For this purpose, we report the case with $\lambda = 5$,$K = 50$, $\tau = 3$,$p = 1$,$h = 1$,$\theta = 3$,$\beta = 3$ with different choices for L (Table 4.3).

For a fixed value of α, as the lead time increases, we expect to see an increase in the optimal order quantity Q^*, reorder point r^*, and cold chain technology level l. This phenomenon occurs because a larger lead time exposes the system to a higher risk of experiencing lost sales during that period. By allowing for higher cold chain technology levels, we can reduce the chances of stockouts and improve the overall performance of the inventory system. As a result, the average cost rate is decreased with an increased lead time, as it allows for more cold chain technology levels, thereby increasing the effective shelf life of the products.

On the other hand, if we fix the lead time and consider a tight allowable fraction of loss rate, we observe no change in Q^* and r^*. This outcome is expected because when the allowable fraction of loss is tight, the system is more conservative and will not increase the order quantity or reorder point to reduce the risk of lost sales. However, when the allowable fraction of loss rate is higher, it leads to a decrease in Q^* and r^*. This reduction in Q^* and r^* is beneficial as it lowers the average number of perishing items in the inventory, especially in cases with a high perishing cost ($p = 50$).

In summary, increasing the lead time allows for higher cold chain technology level and better preservation of products, resulting in a reduced average cost rate. On the other hand, tightening the allowable fraction of loss rate does not influence Q^* and r^*, but a higher allowable fraction of loss rate can lead to decreased Q^* and r^*, reducing the number of perishing items, especially when perishing cost is significant.

4.2 Comparison of $\pi(Q, r, l)$, $R(Q, r, l)$ and (Q, r, l_0) Policies

As we propose new policies for controlling perishable inventories, it is interest of to compare the performance of $\pi(Q, r, l)$ and $R(Q, r, l)$ policies with benchmark zero-level (Q, r, l_0) policy. For this purpose, we tested three policies over a wide range of parameter setting. We report our findings on the representative cases of low and relatively high demand rates (λ=0.5,5), short and long shelf life τ=(3,4,5,13,16,21), low and high perishing costs (p=1,10,50), and tight and relaxed service levels (α=0.005,0.01,0.02,0.05,0.1). The fixed parameters $K = 50$, $L = 1$, $h = 1$,$s = 10$,θ=3 and $\beta = 3$. The parameter range is carefully chosen to examine the model from a different perspective. The policies we present can outperform under various conditions, such as high and low demand rates. Our aim is to find the paramater settings where cold chain and lead time reduction models outperforms the zero-level model in terms of corresponding expected cost rates. To compare $\pi(Q, r, l)$ and $L(Q, r, l)$ policies with (Q, r, l_0) policy, we present the percentage improvement in the average cost rate $\Delta_1\%$ for $\pi(Q,r,l)$ policy and Δ_2 for $R(Q,r,l)$ policy as follows:

$$\Delta_1\% = \frac{C_1(Q_1^*, r_1^*) - C_2(Q_2^*, r_2^*, l^*)}{C_1(Q_1^*, r_1^*)} \text{ x } 100 \tag{4.1}$$

$$\Delta_2\% = \frac{C_1(Q_1^*, r_1^*) - C_3(Q_2^*, r_2^*, l^*)}{C_1(Q_1^*, r_1^*)} \text{ x } 100 \tag{4.2}$$

where C_1, C_2 and C_3 are the cost functions of (Q,r,l_0), $\pi(Q,r,l)$ and $R(Q,r,l)$ policies.

In table 4.4, we present the optimal parameters for products high demand ($\lambda = 5, K = 50, L = 1, h = 1, s = 10, \theta = 3, \beta = 3$). Note that the improvements obtained by both $\pi(Q,r,l)$ and $R(Q,r,l)$ policies are higher when α is low ($\alpha < 0.02$) which corresponds to higher service levels. When service level constraint is tight, all policies increase the reorder level r^* in order to prevent the likelihood of loss sales during the lead time. It is essential to emphasize that both $\pi(Q,r,l)$ and (Q,r,l) policies prioritize high service levels achieved through two key strategies: adopting smaller values for the reorder point (r) and optimizing the ordering time to maximize effective shelf life, facilitated by cold chain technology and lead time reduction. As the service level constraint gets looser, superiority of cold chain technology and lead time reduction policies decreases. We observe that non of the policies are highly sensitive to changes in p. Both cold chain technology and lead time reduction policies exploit increased remaining shelf life with l rather than changing Q and r. Consequently, both policies, do not require adjusting order amount and reorder level variables. Instead, cost minimization is attained by deciding l.

In all policies, the shelf life of the products plays a significant role in determining the optimal ordering quantity. As the shelf life increases, Q^* increases, while r^* decreases noticeably. Furthermore, when handling more durable products ($\tau = 5$), the order amount increases while the reorder point decreases, resulting in longer cycle times. It is worth noting that cold chain technology and lead time reduction policies exhibit the same average number of perished items, indicating comparable performance in minimizing losses.

In terms of cost minimization, both $\pi(Q, r, l)$ and $R(Q, r, l)$ policies surpass the zero-level policy. However, the cold chain technology model outperforms the others in most scenarios, achieving a cost-efficiency of 5-7% in most cases compared to zero-level model. We observe that highest percentage difference in the average costs of cold chain technology and lead time reduction policies with zero-level policy when lifetime of the items is short and service level is high. This advantage becomes more pronounced, reaching 16% in extreme cases involving short-lived products ($\tau = 3$). For instance, as we decrease α, $\pi(Q, r, l)$ policy demonstrates a diminishing benefit while $R(Q, r, l)$ policy shows an irregular way of benefit.

The expedited transportation stands out with the lowest order amount Q^* and reorder point r^*, making it a cost-effective option in terms of holding costs. On the other hand, the cold chain model requires the highest order amount Q^* and reorder point r^*, indicating a more cautious approach with higher stock levels and early replenishment as well as reordering decisions get to be made sooner, that is at larger r^*. As τ decreases, the cold chain model tends to require greater technology levels.

In table 4.5, we present the optimal parameters for products with low demand rate ($\lambda = 0.5, K = 50, L = 1, h = 1, s = 10, \theta = 3, \beta = 3$), the corresponding expected cost rates of the three policies, and the percentage improvement in the average cost rate under the $\pi(Q, r, l)$ and $R(Q, r, l)$ over the optimal cost under the (Q, r, l_0) policy. The first thing we notice is that both order quantity Q^* and reorder point r^* significantly decrease compared to products with high demand rate, as one expects. Also, we observe the same structures and effects as in the products with high demand rate throughout three models. However, as K and τ increases, $\pi(Q, r, l)$ policy operates with no cold chain technology option and benefit of technology starts to diminish. In other words, it converges to zero-level policy. As K and τ increases, $R(Q, r, l)$ policy still tends to use lead time reduction choices to increase remaining shelf life when the items are arrived to the inventory even though it decreases. It is important to note that in products with high demand rates, when service level constraint is tight and lifetime of the products are low, $\pi(Q, r, l)$ and $R(Q, r, l)$ policies perform better than zero-level policy. One crucial difference in products with low demand rate, $R(Q, r, l)$ policy outperforms the other policies and achieves more than 17% improvement in case of high service levels and low shelf life.

Table 4.6 presents an analysis of the effect of cold chain and lead time reduction levels on the aging and sustainability of products, focusing on the number of perished items and the associated perishing cost under different parameter settings.

The goal of implementing cold chain and lead time reduction levels is to mitigate the aging of products and ensure their sustainability to decrease the overall cost function. The results show that in some cases, the average number of perished items decrease with the adoption of cold chain technology and lead time reduction strategies. However, there are instances where the opposite occurs. We resulted that neither $\pi(Q, r, l)$ nor $L(Q, r, l)$ policy decrease the number of perished items in any structure. However, as service level constraint gets tighter and shelf life decreases, both policies can produce great number of feasible solutions which cannot be attained in zero-level policy.

When Q and r values are fixed, cold chain and lead time reduction models generally lead to fewer instances of perished items. This suggests that both model help reduce the number of products that go bad due to aging, leading to better inventory management.

However, to minimize overall costs, cold chain and lead time reduction models tend to increase the Q and r values to better match demand patterns. This, in turn, can result in an increase in the number of perished items since larger quantities are ordered, and some products may expire before they are sold. This trade-off between minimizing costs and reducing perished items is an important consideration in inventory optimization.

As perishing cost (p) increases, the average number of perished items decreases for all policies. This is expected, as higher perishing costs incentivize more careful inventory management and reduce the wastage of perishable products.

Interestingly, the behavior of the $\pi(Q, r, l)$ and $R(Q, r, l)$ policies appears to contradict each other concerning perishability. In many cases, perishing cost increases in the cold chain policy while decreasing in the lead time reduction policy, and vice versa. This observation suggests that cold chain and lead time reduction strategies have different effects on perishing items, and their effectiveness can depend on specific parameter settings and demand characteristics.

In Figure 4.4, we present the cost behavior of all policies w.r.t shelf life, allowable loss sales rate and different demand rates while other parameters are fixed ($K = 50$,$p = 1$,$h = 1$,$\theta = 3$,$\beta = 3$). It demonstrates the impact of shelf life on various inventory management policies. Specifically, the figure illustrates how the difference between different policies changes with increasing shelf life for products with low and high demand rates.

For products with high demand rate, the results indicate that when shelf life is relatively long or infinite, cold chain technology or lead time reduction level do not lead to significant cost savings. This suggests that in systems where products have long lifetimes, the benefits of implementing cold chain technology and lead time reduction policies are minimal. However, as the perishability of products increases (i.e., shorter shelf life), the $\pi(Q, r, l)$ policy outperforms the other policies. This indicates that cold chain technology in cold chain preservation, combined with the specific ordering and replenishment policy, leads to cost savings and more efficient inventory management in cases where products have a higher likelihood of perishing.

Conversely, for items with low demand rate, the difference between policies increases with increasing shelf life. In this scenario, having a short shelf life becomes less beneficial, as the products are likely to perish before a significant portion of the demand arrives. The results indicate that the $R(Q, r, l)$ policy becomes more advantageous for such slow-moving items.

In general, for a fixed demand rate λ and a tighter service level criterion of $\alpha = 0.005$, both $\pi(Q, r, l)$ and $R(Q, r, l)$ policies (cold chain and lead time reduction) demonstrate superior performance in terms of achieving a lower average cost rate compared to the zero-level policy. This highlights the importance of considering cold chain technology and lead time reduction options in inventory management to optimize costs and improve overall efficiency, especially in scenarios with stringent service level requirements.

The figure's insights provide valuable guidance for decision-makers in choosing appropriate inventory management policies based on the perishability of products, their demand patterns, and the desired service levels.

Table 4.4: Total Cost Comparison (Q, r, l_0) vs $\pi(Q, r, l)$ vs $R(Q, r, l)$, $\lambda = 5, K = 50, L = 1, h = 1, s = 10, \theta = 3, \beta = 3$

			(Q, r, l_0)			$\pi(Q, r, l)$					$R(Q, r, l)$				
$\lambda = 5$		α	Q^*	r^*	Cost	Q^*	r^*	τ_l^*	Cost	$\Delta_1\%$	Q^*	r^*	L^*	Cost	$\Delta_2\%$
$p = 1$	$\tau = 3$	0.005	14	13	151.579	14	11	2.6	127.165	16.11	11	10	0.9	141.969	6.34
		0.01	13	11	128.628	14	10	2.6	118.309	8.02	13	11	1.0	128.628	0.0
		0.02	11	9	115.205	12	8	2.6	108.009	6.25	11	9	1.0	115.205	0.0
		0.05	9	7	101.59	12	7	2.5	96.862	4.65	9	7	1.0	101.59	0.0
		0.1	10	6	88.93	11	6	2.1	88.155	0.87	10	6	1.0	88.93	0.0
	$\tau = 4$	0.005	14	10	107.289	16	10	3.3	104.15	2.93	14	10	1.0	107.289	0.0
		0.01	13	9	102.336	16	9	3.5	98.772	3.48	13	9	1.0	102.336	0.0
		0.02	14	8	93.138	15	8	3.3	92.805	0.36	14	8	1.0	93.138	0.0
		0.05	14	7	86.078	13	6	3.4	84.944	1.32	14	6	0.9	85.379	0.81
		0.1	13	5	75.882	13	5	3.0	75.882	0.0	13	5	1.0	75.882	0.0
	$\tau = 5$	0.005	17	10	96.267	19	10	4.3	95.136	1.17	15	7	0.7	94.63	1.7
		0.01	16	9	92.404	20	9	4.5	90.901	1.63	16	7	0.8	89.726	2.9
		0.02	17	8	86.028	17	8	4.0	86.028	0.0	15	6	0.8	85.884	0.17
		0.05	17	6	76.956	17	6	4.0	76.956	0.0	17	6	1.0	76.956	0.0
		0.1	17	4	70.14	17	4	4.0	70.14	0.0	17	4	1.0	70.14	0.0
$p = 10$	$\tau = 3$	0.005	14	13	152.343	14	11	2.6	127.831	16.09	11	10	0.9	142.265	6.62
		0.01	11	10	130.757	14	10	2.6	119.416	8.67	10	7	0.6	130.25	0.39
		0.02	11	9	116.753	12	8	2.6	108.756	6.85	11	9	1.0	116.753	0.0
		0.05	9	7	102.526	12	7	2.5	99.076	3.37	9	7	1.0	102.526	0.0
		0.1	10	6	92.565	11	5	2.7	91.051	1.64	10	6	1.0	92.565	0.0
	$\tau = 4$	0.005	14	10	107.604	16	10	3.3	104.564	2.83	14	10	1.0	107.604	0.0
		0.01	13	9	102.606	16	9	3.5	99.599	2.93	13	9	1.0	102.606	0.0
		0.02	14	8	94.236	15	8	3.3	94.002	0.25	14	8	1.0	94.236	0.0
		0.05	14	7	87.896	13	6	3.4	85.52	2.7	12	5	0.8	87.512	0.44
		0.1	13	5	77.645	13	5	3.0	77.645	0.0	13	5	1.0	77.645	0.0
	$\tau = 5$	0.005	17	10	96.339	19	10	4.3	95.361	1.02	15	7	0.7	94.711	1.69
		0.01	16	9	92.467	20	9	4.5	91.342	1.22	16	7	0.8	90.203	2.45
		0.02	17	8	86.586	17	8	4.0	86.586	0.0	15	6	0.8	86.262	0.37
		0.05	17	6	78.225	17	6	4.0	78.225	0.0	17	6	1.0	78.225	0.0
		0.1	16	4	71.009	16	4	4.0	71.009	0.0	16	4	1.0	71.009	0.0
$p = 50$	$\tau = 3$	0.005	14	13	155.743	14	11	2.6	130.79	16.02	11	10	0.9	143.585	7.81
		0.01	11	10	133.196	13	9	2.8	122.906	7.73	11	10	1.0	133.196	0.0
		0.02	11	9	123.632	12	8	2.6	112.076	9.35	11	9	1.0	123.632	0.0
		0.05	9	7	106.685	10	7	2.4	105.058	1.53	9	7	1.0	106.685	0.0
		0.1	8	6	98.38	11	5	2.7	96.051	2.37	8	6	1.0	98.38	0.0
	$\tau = 4$	0.005	14	10	109.004	16	10	3.4	106.167	2.6	14	10	1.0	109.004	0.0
		0.01	13	9	103.806	15	9	3.3	101.654	2.07	13	9	1.0	103.806	0.0
		0.02	12	8	98.553	14	8	3.3	96.866	1.71	12	8	1.0	98.553	0.0
		0.05	12	7	91.49	12	6	3.3	87.03	4.87	11	6	0.9	90.804	0.75
		0.1	10	5	80.123	10	5	3.0	80.123	0.0	10	5	1.0	80.123	0.0
	$\tau = 5$	0.005	17	10	96.659	19	10	4.3	96.361	0.31	15	7	0.7	95.071	1.64
		0.01	16	9	92.746	18	9	4.3	92.617	0.14	16	7	0.8	92.322	0.46
		0.02	16	8	88.526	16	8	4.0	88.526	0.0	14	6	0.8	87.404	1.27
		0.05	14	6	80.039	14	6	4.0	80.039	0.0	14	6	1.0	80.039	0.0
		0.1	16	4	74.849	15	4	4.4	73.062	2.39	14	3	0.8	73.656	1.59

Table 4.5: Total Cost Comparison (Q, r, l_0) vs $\pi(Q, r, l)$ vs $R(Q, r, l)$, $\lambda = 0.5, K = 50, L = 1, h = 1, s = 10, \theta = 3, \beta = 3$

			(Q,r,l_0)			$\pi(Q,r,l)$					$R(Q,r,l)$				
$\lambda = 0.5$		α	Q^*	r^*	Cost	Q^*	r^*	τ_l^*	Cost	$\Delta_1\%$	Q^*	r^*	L^*	Cost	$\Delta_2\%$
$p = 1$	$\tau = 13$	0.005	5	3	21.241	5	3	12.0	21.241	0.0	6	0	0.1	19.243	9.41
		0.01	5	3	21.241	5	2	12.3	18.107	14.75	6	1	0.4	17.531	17.47
		0.02	5	2	17.117	6	2	12.1	16.868	1.45	6	1	0.5	17.017	0.58
		0.05	5	1	14.558	5	1	12.0	14.558	0.0	5	1	1.0	14.558	0.0
		0.1	5	0	13.005	5	0	12.0	13.005	0.0	5	0	1.0	13.005	0.0
	$\tau = 16$	0.005	7	3	17.573	8	2	15.6	17.012	3.19	7	2	0.9	16.113	8.31
		0.01	6	2	15.881	6	2	15.0	15.881	0.0	6	2	1.0	15.881	0.0
		0.02	7	2	15.604	7	1	15.5	14.998	3.88	6	1	0.7	14.949	4.2
		0.05	6	1	13.898	6	1	15.0	13.898	0.0	6	1	1.0	13.898	0.0
		0.1	7	0	13.113	5	0	15.1	12.499	4.68	5	0	0.9	12.638	3.62
	$\tau = 21$	0.005	6	3	17.966	7	2	20.8	17.839	0.71	8	1	0.5	15.126	15.81
		0.01	9	2	15.27	9	2	20.0	15.27	0.0	8	1	0.8	14.392	5.75
		0.02	8	1	13.77	8	1	20.0	13.77	0.0	8	1	1.0	13.77	0.0
		0.05	7	1	13.227	7	1	20.0	13.227	0.0	7	0	0.8	12.837	2.95
		0.1	6	0	11.609	6	0	20.0	11.609	0.0	6	0	1.0	11.609	0.0
$p = 10$	$\tau = 13$	0.005	5	3	21.322	5	3	12.0	21.322	0.0	6	0	0.1	20.196	5.28
		0.01	5	3	21.322	5	2	12.3	18.422	13.6	6	1	0.4	18.331	14.03
		0.02	5	2	17.503	5	2	12.0	17.503	0.0	5	2	1.0	17.503	0.0
		0.05	5	1	15.025	5	1	12.0	15.025	0.0	5	1	1.0	15.025	0.0
		0.1	5	0	13.598	5	0	12.0	13.598	0.0	5	0	1.0	13.598	0.0
	$\tau = 16$	0.005	7	3	18.004	8	2	15.6	17.722	1.57	7	2	0.9	16.661	7.46
		0.01	6	2	16.214	6	2	15.0	16.214	0.0	5	1	0.6	16.212	0.01
		0.02	7	2	16.197	7	1	15.5	15.537	4.07	6	1	0.7	15.353	5.21
		0.05	6	1	14.33	6	1	15.0	14.33	0.0	6	1	1.0	14.33	0.0
		0.1	7	0	13.751	5	0	15.1	12.751	7.27	5	0	0.9	12.881	6.33
	$\tau = 21$	0.005	6	3	18.002	7	2	20.8	17.955	0.26	8	1	0.5	15.431	14.28
		0.01	9	2	15.693	7	2	20.1	15.52	1.1	8	1	0.8	14.752	6.0
		0.02	8	1	14.085	8	1	20.0	14.085	0.0	8	1	1.0	14.085	0.0
		0.05	7	1	13.451	7	1	20.0	13.451	0.0	7	0	0.8	13.071	2.83
		0.1	6	0	11.735	6	0	20.0	11.735	0.0	6	0	1.0	11.735	0.0
$p = 50$	$\tau = 13$	0.005	5	3	21.682	5	3	12.0	21.682	0.0	5	3	1.0	21.682	0.0
		0.01	5	3	21.682	5	2	12.3	19.82	8.59	5	2	0.9	19.937	8.05
		0.02	5	2	19.221	5	2	12.0	19.221	0.0	4	1	0.6	18.946	1.43
		0.05	4	1	16.476	4	1	12.0	16.476	0.0	4	1	1.0	16.476	0.0
		0.1	5	0	16.235	5	0	12.0	16.235	0.0	4	0	0.8	15.325	5.61
	$\tau = 16$	0.005	6	3	19.35	5	2	15.3	18.335	5.25	6	2	0.9	18.089	6.52
		0.01	5	2	17.387	5	2	15.0	17.387	0.0	4	1	0.8	16.705	3.92
		0.02	5	2	17.387	5	2	15.0	17.387	0.0	4	1	0.9	16.285	6.34
		0.05	4	1	15.385	4	1	15.0	15.385	0.0	4	1	1.0	15.385	0.0
		0.1	4	1	15.385	5	0	15.1	13.869	9.85	4	0	0.9	13.65	11.28
	$\tau = 21$	0.005	6	3	18.161	6	3	20.0	18.161	0.0	6	2	0.9	16.254	10.5
		0.01	5	2	16.56	7	2	20.2	16.359	1.21	6	1	0.4	15.998	3.39
		0.02	8	1	15.483	8	1	20.0	15.483	0.0	5	1	0.8	15.033	2.91
		0.05	5	1	14.098	5	1	20.0	14.098	0.0	5	0	0.4	14.055	0.31
		0.1	6	0	12.294	6	0	20.0	12.294	0.0	6	0	1.0	12.294	0.0

Table 4.6: Perishing Item Comparison, (Q, r, l_0) vs $\pi(Q, r, l)$ vs $R(Q, r, l)$, $\lambda = 5, K = 50, L = 1, h = 1, s = 10, \theta = 3, \beta = 3$

			(Q, r, l_0)			$\pi(Q, r, l)$				$R(Q, r, l)$			
$\lambda = 5$		α	Q^*	r^*	P	Q^*	r^*	τ_l^*	P	Q^*	r^*	L^*	P
$p = 1$	$\tau = 3$	0.005	14	13	0.7	14	11	2.6	0.9	11	10	0.9	0.3
		0.01	13	11	2.6	14	10	2.6	1.6	13	11	1.0	2.6
		0.02	11	9	2.2	12	8	2.6	1.2	11	9	1.0	2.2
		0.05	9	7	1.6	12	7	2.5	4.1	9	7	1.0	1.6
		0.1	10	6	7.2	11	6	2.1	9.4	10	6	1.0	7.2
	$\tau = 3$	0.005	14	10	0.4	16	10	3.3	0.6	14	10	1.0	0.4
		0.01	13	9	0.4	16	9	3.5	1.4	13	9	1.0	0.4
		0.02	14	8	1.99	15	8	3.3	2.1	14	8	1.0	1.9
		0.05	14	7	3.5	13	6	3.4	1.2	14	6	0.9	4.3
		0.1	13	5	3.9	13	5	3.0	3.9	13	5	1.0	3.9
	$\tau = 3$	0.005	17	10	0.1	19	10	4.3	0.4	15	7	0.7	0.1
		0.01	16	9	0.1	20	9	4.5	0.8	16	7	0.8	0.9
		0.02	17	8	1.0	17	8	4.0	1.0	15	6	0.8	0.7
		0.05	17	6	2.7	17	6	4.0	2.7	17	6	1.0	2.7
		0.1	17	4	3.0	17	4	4.0	3.0	17	4	1.0	3.0
$p = 10$	$\tau = 3$	0.005	14	13	0.7	14	11	2.6	0.9	11	10	0.9	0.3
		0.01	11	10	0.7	14	10	2.6	1.6	10	7	0.6	1.5
		0.02	11	9	2.2	12	8	2.6	1.2	11	9	1.0	2.2
		0.05	9	7	1.6	12	7	2.5	4.1	9	7	1.0	1.6
		0.1	10	6	7.2	11	5	2.7	2.5	10	6	1.0	7.2
	$\tau = 3$	0.005	14	10	0.4	16	10	3.3	0.6	14	10	1.0	0.4
		0.01	13	9	0.4	16	9	3.5	1.4	13	9	1.0	0.4
		0.02	14	8	1.9	15	8	3.3	2.1	14	8	1.0	1.9
		0.05	14	7	3.5	13	6	3.4	1.2	12	5	0.8	1.9
		0.1	13	5	3.9	13	5	3.0	3.9	13	5	1.0	3.9
	$\tau = 3$	0.005	17	10	0.1	19	10	4.3	0.4	15	7	0.7	0.1
		0.01	16	9	0.1	20	9	4.5	0.8	16	7	0.8	0.9
		0.02	17	8	1.0	17	8	4.0	1.0	15	6	0.8	0.7
		0.05	17	6	2.7	17	6	4.0	2.7	17	6	1.0	2.7
		0.1	16	4	2.0	16	4	4.0	2.0	16	4	1.0	2.0
$p = 50$	$\tau = 3$	0.005	14	13	0.7	14	11	2.6	0.9	11	10	0.9	0.3
		0.01	11	10	0.7	13	9	2.8	0.8	11	10	1.0	0.7
		0.02	11	9	2.2	12	8	2.6	1.2	11	9	1.0	2.2
		0.05	9	7	1.6	10	7	2.4	1.1	9	7	1.0	1.6
		0.1	8	6	1.4	11	5	2.7	2.5	8	6	1.0	1.4
	$\tau = 3$	0.005	14	10	0.4	16	10	3.4	0.4	14	10	1.0	0.4
		0.01	13	9	0.4	15	9	3.3	0.6	13	9	1.0	0.4
		0.02	12	8	0.4	14	8	3.3	0.5	12	8	1.0	0.4
		0.05	12	7	0.8	12	6	3.3	0.3	11	6	0.9	0.3
		0.1	10	5	0.5	10	5	3.0	0.5	10	5	1.0	0.5
	$\tau = 3$	0.005	17	10	0.1	19	10	4.3	0.4	15	7	0.7	0.1
		0.01	16	9	0.1	18	9	4.3	0.3	16	7	0.8	0.9
		0.02	16	8	0.5	16	8	4.0	0.5	14	6	0.8	0.2
		0.05	14	6	0.4	14	6	4.0	0.4	14	6	1.0	0.4
		0.1	16	4	2.0	15	4	4.4	0.3	14	3	0.8	0.6

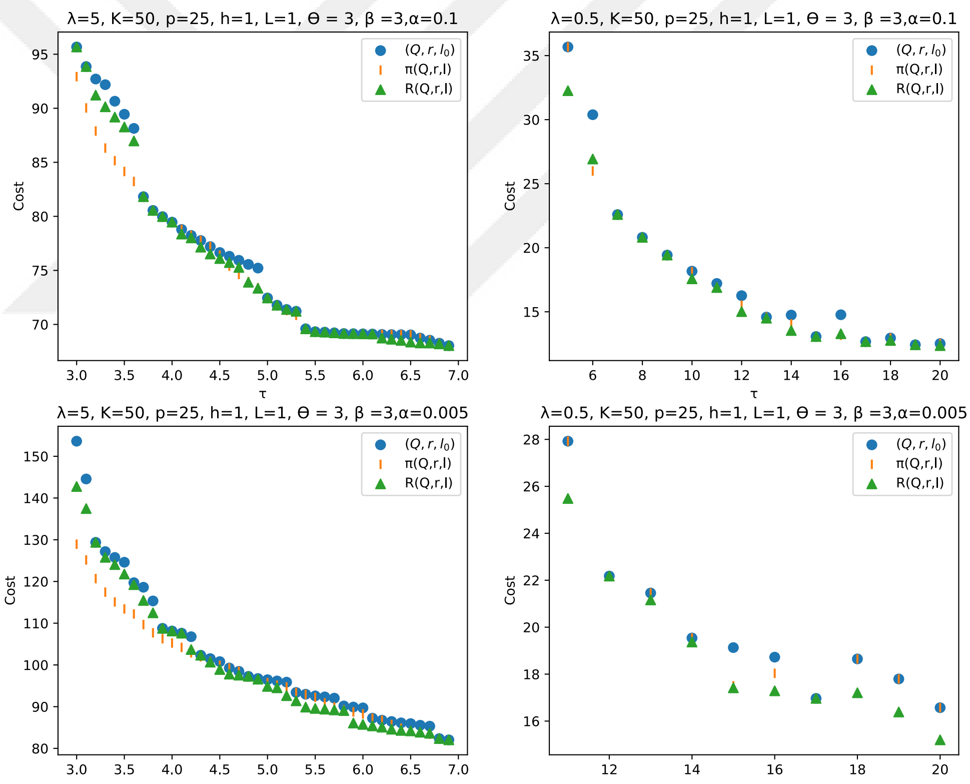

Figure 4.4: Average cost vs Shelf life

Chapter 5

Conclusion

In this thesis, we proposed inventory replenishment policies for perishable items where cold chain facilities during transportation or expedited transportation options are available at additional costs, with the aim of analysing the tradeoff between increased remaining shelf life and technology costs. We considered a setting under continuous review, with a single-item, single-location with perishable items having fixed shelf life and where unmet demands are lost. The lead time is assumed to be positive and the demands are generated with respect to a Poisson process.

We introduced two policies that aim to reduce aging during transpostation: the cold chain technology policy and the lead time reduction policy. The objective of these joint policies, denoted as $\pi(Q, r, l)$ and $R(Q, r, l)$ respectively, is to replenish the inventories with the order quantity-reorder point policies, while adopting alternative technologies that enable the products to join the stock with higher remaining shelf lives at a certain cost. The technology level l is considered as a decision variable to control the aging process during the lead time. We derived the operating characteristics of the models resulting from the two policies. For both $\pi(Q, r, l)$ and $R(Q, r, l)$ policies, the objective is to minimize the expected cost rate with service level constraints .

To illustrate the effectiveness of the policies, we conducted a numerical analysis to explore the sensitivity of the models to various inventory system factors across different parameter values as well as the comparison of the policies with alternative options of increasing the remaining shelf life when the order joins the stocks or reducing the lead time with expedited transportation.

We compared the performances of $\pi(Q, r, l)$ and $R(Q, r, l)$ policies against a zero-level model (Q, r, l_0) to identify the parameter settings where cold chain technology or lead time reduction offer superior results. Our findings indicate that the suggested policies outperform the model with no cold chain or expedited transportation options (benchmark case) when shelf life of items are short ($\tau = 3$) and high service levels ($\alpha \leq 0.05$). However, as we increase the shelf life of the items, the performance of suggested models tends to converge to that of the benchmark model. We also observe that the $\pi(Q, r, l)$ policy performs better for items with high demand rates (λ=5), while the $R(Q, r, l)$ policy outperforms for items with low demand rates (λ=0.5).

Cold chain technology policies are relatively novel concepts in inventory systems and hold significant importance in real-world applications. As a potential extension of this thesis, research can be expanded to encompass multiple products in the analysis. Although this may introduce additional complexities, it would be valuable for practical inventory management scenarios and contribute substantially to the literature. Moreover, incorporating a two echelon environment into this setting can enable us to observe the benefits of cold chain technology in more complex systems.

Overall, this thesis provides an modeling approach for effective management of perishable inventory systems by allowing the adoption of technologic advancements to the commonly used (Q, r) policies, and its findings can further be extended to address more complex and practical scenarios in the future.

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Appendix A

Operating Characteristics

We derive the operating characteristics according to the each realization type separately. Recall that X_{Q-r} and X_r are independent.

Following expressions are for the expected On Hand Inventory per cycle $E(OH)$:

$$OH = \begin{cases} \sum_{i=1}^{Q} X_i & if \quad X_{Q-r} + X_r < \tau_l, \\ & \quad X_r < L \quad (R1) \\ \sum_{i=1}^{N(\tau_l)} X_i + \tau_l(Q - N(\tau_l) & if \quad X_{Q-r}<\tau_l, \tau_l - X_{Q-r}<L, \\ & \quad X_{Q-r} + X_r > \tau_l \quad (R2) \\ \sum_{i=1}^{Q-r+N(L)} X_i + (r - N(L))(X_{Q-r} + L) & if \quad X_{Q-r} < \tau_l, X_r > L, \\ & \quad \tau_l - X_{Q-r} > L \quad (R3) \\ \sum_{i=1}^{N(\tau_l)} X_i + \tau_l(Q - N(\tau_l)) & if \quad \tau_l < X_{Q-r} \quad (R4) \end{cases} \tag{A.1}$$

Derivation of Expected On-hand Inventory (Proposition 3)

$$
\begin{aligned}
E[OH]_1 =& E\Bigg[\Bigg(\sum_{i=1}^{Q-r-1} X_i + (r+1)X_{Q-r} + \sum_{i=1}^{r} Y_i\Bigg) I(X_{Q-r} + X_r < \tau_l, X_r < L)\Bigg] \\
=& E\Bigg[\Bigg(\sum_{i=1}^{Q-r-1} X_i + (r+1)X_{Q-r}\Bigg) I(X_{Q-r} + X_r < \tau_l, X_r < L)\Bigg] \quad (a) \\
+& E\Bigg[\sum_{i=1}^{r} Y_i I(X_{Q-r} + X_r < \tau_l, X_r < L)\Bigg] \quad (b)
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
E[OH]_{1a} =& E\Bigg[\Bigg(\sum_{i=1}^{Q-r-1} X_i + (r+1)X_{Q-r}\Bigg) I(X_{Q-r} + X_r < \tau_l, X_r < L)\Bigg] \\
=& \int_{u=0}^{\tau_l} \Bigg[(Q-r-1)\frac{u}{2} + (r+1)u\Bigg] H_r(min(L, \tau_l - u)) dH_{Q-r}(u) \\
=& \int_{u=0}^{\tau_l} \frac{(Q-r+1)(Q-r)}{2\lambda} H_r(min(L, \tau_l - u)) dH_{Q-r+1}(u) \\
=& \frac{(Q-r)(Q-r+1)}{2\lambda} \Bigg[H_r(L) \int_{0}^{\tau_l - L} dH_{Q-r+1}(u) \\
+& \int_{\tau_l - L}^{\tau_l} H_r(\tau_l - u) dH_{Q-r+1}(u)\Bigg] \\
=& \frac{(Q-r)(Q-r+1)}{2\lambda} \Bigg[H_r(L) H_{Q-r+1}(\tau_l - L) + H_{Q+1}(\tau_l) \\
-& \int_{0}^{\tau_l - L} H_r(\tau_l - u) dH_{Q-r+1}(u)\Bigg]
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
E[OH]_{1b} =& E\Bigg[\sum_{i=1}^{r} Y_i I(X_{Q-r} + X_r < \tau_l, X_r < L)\Bigg] \\
=& \int_{0}^{\tau_l} E\Bigg[\sum_{i=1}^{r} Y_i I(Y_r < min(L, \tau_l - u))\Bigg] dH_{Q-r}(u) \\
=& \int_{0}^{\tau_l} \frac{r(r+1)}{2\lambda} H_{r+1}(min(L, \tau_l - u)) dH_{Q-r}(u) \\
=& \frac{r(r+1)}{2\lambda} \Bigg[H_{r+1}(L) \int_{0}^{\tau_l - L} dH_{Q-r}(u) + \int_{\tau_l - L}^{\tau_l} H_{r+1}(\tau_l - u) dH_{Q-r}(u)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{r(r+1)}{2\lambda}\bigg[H_{r+1}(L)H_{Q-r}(\tau_l-L)+H_{Q+1}(\tau_l)\\
&-\int_0^{\tau_l-L}H_{r+1}(\tau_l-u)dH_{Q-r}(u)\bigg] \qquad \text{(A.4)}
\end{aligned}
$$

$$
\begin{aligned}
E[OH]_2 =&E\bigg[\bigg(\sum_{i=1}^{Q-r-1}X_i+(r+1)X_{Q-r}+\sum_{i=1}^{N(\tau_l-X_{Q-r})}Y_i+(\tau_l-X_{Q-r})\\
&(r-N(\tau_l-X_{Q-r}))I(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r})\bigg]\\
=&E\bigg[\bigg(\sum_{i=1}^{Q-r-1}X_iI(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r})\bigg] \quad (a)\\
+&E\bigg[(r+1)X_{Q-r}I(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r}\bigg] \quad (b)\\
+&E\bigg[\sum_{i=1}^{N(\tau_l-X_{Q-r})}Y_iI(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r})\bigg] \quad (c)\\
+&E\bigg[(\tau_l-X_{Q-r})(r-N(\tau_l-X_{Q-r})\\
&I(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r})\bigg] \quad (d) \qquad \text{(A.5)}
\end{aligned}
$$

$$
\begin{aligned}
E[OH]_{2a} =&E\bigg[\sum_{i=1}^{Q-r-1}X_iI(\tau_l-L<X_{Q-r}<\tau_l,X_r>\tau_l-X_{Q-r})\bigg]\\
=&\int_{u=\tau_l-L}^{\tau_l}(Q-r-1)\frac{u}{2}\bar{H}_r(\tau_l-u)dH_{Q-r}(u)\\
=&\frac{(Q-r-1)(Q-r)}{2\lambda}\int_{\tau_l-L}^{\tau_l}\bar{H}_r(\tau_l-u)dH_{Q-r+1}(u)\\
=&\frac{(Q-r-1)(Q-r)}{2\lambda}\bigg[H_{Q-r+1}(\tau_l)-H_{Q-r+1}(\tau_l-L)-H_{Q+1}(\tau_l)\\
&+\int_0^{\tau_l-L}H_r(\tau_l-u)dH_{Q-r+1}(u)\bigg] \qquad \text{(A.6)}
\end{aligned}
$$

$$
\begin{aligned}
E[OH]_{2b} =& E\Big[(r+1)X_{Q-r}I(\tau_l - L < X_{Q-r} < \tau_l, X_r > \tau_l - X_{Q-r})\Big] \\
& (r+1)\int_{\tau_l-L}^{\tau_l} u\bar{H}_r(\tau_l - u)dH_{Q-r}(u) \\
=& (r+1)\Big[\int_{\tau_l-L}^{\tau_l} udH_{Q-r}(u) - \int_{\tau_l-L}^{\tau_l} H_r(\tau_l - u)udH_{Q-r}(u))\Big] \\
=& (r+1)\Big[\frac{Q-r}{\lambda}\big(H_{Q-r+1}(\tau_l) - H_{Q-r+1}(\tau_l - L)\big) - H_Q(\tau_l) \\
& + \int_0^{\tau_l-L} H_r(\tau_l)dH_{Q-r}(u)\Big]
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
E[OH]_{2c} =& E\Big[\sum_{i=1}^{N(\tau_l - X_{Q-r})} Y_i I(\tau_l - L < X_{Q-r} < \tau_l, X_r > \tau_l - X_{Q-r})\Big] \\
=& \int_{\tau_l-L}^{\tau_l} E\big[\sum_{i=1}^{N(\tau_l-u)} Y_i I(N(\tau_l - u) < r)dH_{Q-r}(u)\big] \\
=& \int_{\tau_l-L}^{\tau_l} \frac{\lambda(\tau_l - u)^2}{2}\bar{H}_{r-1}(\tau_l - u)dH_{Q-r}(u)
\end{aligned} \tag{A.8}
$$

$$
\begin{aligned}
E[OH]_{2d} =& E\Big[(\tau_l - X_{Q-r})(r - N(\tau_l - X_{Q-r})) \\
& I(\tau_l - L < X_{Q-r} < \tau_l, X_r > \tau_l - X_{Q-r})\Big] \\
=& \int_{u=\tau_l-L}^{\tau_l} (\tau_l - u)E\Big[(r - N(\tau_l - u))I(N(\tau_l - u) < r)\Big]dH_{Q-r}(u) \\
=& \int_{\tau_l-L}^{\tau_l} \Big[(\tau_l - u)r\bar{H}_r(\tau_l - u) - \lambda(\tau_l - u)^2\bar{H}_{r-1}(\tau_l - u)\Big]dH_{Q-r}(u)
\end{aligned} \tag{A.9}
$$

$$
\begin{aligned}
E[OH]_3 =&E\Bigg[\Big[\sum_{i=1}^{Q-r-1} X_i + (r+1)X_{Q-r} + \sum_{i=1}^{N(L)} Y_i + (r-N(L))L\Big] \\
&I(X_{Q-r} < \tau_l - L, X_r > L)\Bigg] \\
=&E\Bigg[\sum_{i=1}^{Q-r-1} X_i I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \quad (a) \\
+&E\Bigg[(r+1)X_{Q-r} I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \quad (b) \\
+&E\Bigg[\sum_{i=1}^{N(L)} Y_i I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \quad (c) \\
+&E\Bigg[L(r-N(L)) I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \quad (d)
\end{aligned}
\tag{A.10}
$$

$$
\begin{aligned}
E[OH]_{3a} =&E\Bigg[\sum_{i=1}^{Q-r-1} X_i I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \\
=&\int_0^{\tau_l - L} (Q-r+1)\frac{u}{2}\bar{H}_r(L) dH_{Q-r}(u) \\
=&\frac{(Q-r-1)(Q-r)}{2\lambda}\bar{H}_r(L) H_{Q-r+1}(\tau_l - L)
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
E[OH]_{3b} =&E\Bigg[(r+1)X_{Q-r} I(X_{Q-r} < \tau_l - L, N(L) < L)\Bigg] \\
=&(r+1)\bar{H}_r(L)\int_0^{\tau_l - L} u dH_{Q-r}(u) \\
=&\frac{(r+1)(Q-r)}{\lambda}\bar{H}_r(L) H_{Q-r+1}(\tau_l - L)
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
E[OH]_{3c} =& E\left[\sum_{i=1}^{N(L)} Y_i I(X_{Q-r} < \tau_l - L, N(L) < L)\right] \\
=& \frac{\lambda}{2} L^2 H_{Q-r}(\tau_l - L)\bar{H}_{r-1}(L)
\end{aligned} \tag{A.13}
$$

$$
\begin{aligned}
E[OH]_{3d} =& E\left[L(r - N(L))I(X_{Q-r} < \tau_l - L, N(L) < L)\right] \\
=& LrH_{Q-r}(\tau_l - L)\bar{H}_r(L) - LH_{Q-r}(\tau_l - L)\sum_{k=0}^{r-1}\frac{e^{-\lambda L}(\lambda L)^k}{k!}k \\
=& LrH_{Q-r}(\tau_l - L)\bar{H}_r(L) - LH_{Q-r}(\tau_l - L)(\lambda L\bar{H}_{r-1}(L)) \\
=& LH_{Q-r}(\tau_l - L)\left[rH_r(L) - \lambda L\bar{H}_{r-1}(L)\right]
\end{aligned} \tag{A.14}
$$

$$
\begin{aligned}
E[OH]_4 =& E\left[\big[\sum_{i=1}^{N(\tau_l)} X_i + \tau_l\big[Q - N(\tau_l)\big]\big]I(X_{Q-r} > \tau_l)\right] \\
=& E\left[\sum_{i=1}^{N(\tau_l)} X_i I(N(\tau_l) < Q - r)\right] \quad (a) \\
&+ E\left[\tau_l\big[Q - N(\tau_l)\big]I(N(\tau_l) < Q - r)\right] \quad (b)
\end{aligned} \tag{A.15}
$$

$$
\begin{aligned}
E[OH]_{4a} =& E\left[\sum_{i=1}^{N(\tau_l)} X_i I(N(\tau_l) < Q - r)\right] \\
=& \frac{\lambda}{2}\tau_l^2 \bar{H}_{Q-r-1}(\tau_l)
\end{aligned} \tag{A.16}
$$

$$
\begin{aligned}
E[OH]_{4b} =& E\Bigg[\tau_l\big[Q - N(\tau_l)\big]I(N(\tau_l) < Q - r)\Bigg] \\
=& \tau_l\Bigg[Q\bar{H}_{Q-r}(\tau_l) - \lambda\tau_l\bar{H}_{Q-r-1}(\tau_l)\Bigg]
\end{aligned} \tag{A.17}
$$

The following expressions are for the expected number of satisfied demand, $E(SD)$:

$$
SD = \begin{cases}
Q & \text{if} \quad X_Q < \tau_l, X_r < L \quad (R1) \\
N(\tau_l) & \text{if} \quad X_{Q-r} < \tau_l, \tau_l - X_{Q-r} < L, \\
& \qquad X_{Q-r} + X_r > \tau_l \quad (R2) \\
Q - r + N(L) & \text{if} \quad X_{Q-r} < \tau_l, X_r > L, \\
& \qquad \tau_l - X_{Q-r} > L \quad (R3) \\
N(\tau_l) & \text{if} \quad \tau_l < X_{Q-r} \quad (R4)
\end{cases} \tag{A.18}
$$

Derivation of Expected Number of Satisfied Demand (Proposition 4)

$$
\begin{aligned}
E[SD] =& E[QI(X_{Q-r} + X_r < \tau_l, X_r < L)] \quad (a) \\
&+ E[(N(\tau_l - L) + N(L)) \\
&\quad I(N(\tau_l - L < Q - r, N(\tau_l - L) + N(L) < Q))] \quad (b) \\
&+ E[(Q - r + N(L))I(X_{Q-r} + L < \tau_l, X_r > L)] \quad (c)
\end{aligned} \tag{A.19}
$$

$$
\begin{aligned}
E[SD]_a =& E[QI(X_{Q-r} + X_r < \tau_l, X_r < L)] \\
=& \int_0^L \int_0^{\tau_l - v} Q dH_{Q-r}(u) dH_r(v)
\end{aligned} \tag{A.20}
$$

$$
\begin{aligned}
E[SD]_b =& E[(N(\tau_l - L) + N(L)) \\
& I(N(\tau_l - L < Q - r, N(\tau_l - L) + N(L) < Q))] \\
=& \sum_{i=0}^{Q-r-1} \sum_{j=0}^{Q-i-1} (i+j) P(N(\tau_l - L = i)) P(N(L) = j) \\
=& \sum_{i=0}^{Q-r-1} \sum_{j=0}^{Q-i-1} (i+j) \frac{e^{-\lambda(\tau_l - L)}(\lambda(\tau_l - L))^i}{i!} \frac{e^{-\lambda L}(\lambda L)^j}{j!} \\
=& \sum_{i=0}^{Q-r-1} \sum_{j=0}^{Q-i-1} (i+j) \frac{e^{-\lambda \tau_l} \lambda^{i+j} (\tau_l - L)^i L^j}{i!\, j!}
\end{aligned} \tag{A.21}
$$

$$
\begin{aligned}
E[SD]_c =& E[(Q - r + N(L)) I(X_{Q-r} + L < \tau_l, X_r > L)] \\
=& \sum_{i=0}^{r-1} \sum_{j=Q-r}^{\infty} (Q - r + i) P(N(L) = i) P(N(\tau_l - L) = j) \\
=& \sum_{i=0}^{r-1} \sum_{j=Q-r}^{\infty} (Q - r + i) \frac{e^{-\lambda(\tau_l - L)}(\lambda(\tau_l - L))^j}{j!} \frac{e^{-\lambda L}(\lambda L)^i}{i!} \\
=& \sum_{i=0}^{r-1} \sum_{j=Q-r}^{\infty} (Q - r + i) \frac{e^{-\lambda \tau_l} \lambda^{i+j} (\tau_l - L)^j L^i}{i!\, j!}
\end{aligned} \tag{A.22}
$$

$$
\begin{aligned}
E[SD] =& \int_0^L \int_0^{\tau_l - v} Q dH_{Q-r}(u) dH_r(v) \\
& + \sum_{i=0}^{Q-r-1} \sum_{j=0}^{Q-i-1} (i+j) \frac{e^{-\lambda \tau_l} \lambda^{i+j} (\tau_l - L)^i L^j}{i!\, j!} \\
& + \sum_{i=0}^{r-1} \sum_{j=Q-r}^{\infty} (Q - r - i) \frac{e^{-\lambda \tau_l} \lambda^{i+j} (\tau_l - L)^j L^i}{i!\, j!}
\end{aligned} \tag{A.23}
$$

The following expressions are for the expected cost reduction from salvage, $E(S)$:

$$S = \begin{cases} \frac{s}{\tau}(\tau_l - L - X_{Q-r})(r - N(L)) & \text{if} \quad X_{Q-r} < \tau_l, X_r > L, \\ & \quad \tau_l - X_{Q-r} > L \quad (R3) \\ 0 & \text{if} \quad \text{otherwise} \quad (R1, R2, R4) \end{cases} \tag{A.24}$$

Derivation of Expected Salvage Revenue from Disposal (Proposition 5))

$$\begin{aligned} E[S] =& \frac{s}{\tau}(\tau_l - L - X_{Q-r})(r - N(L))I(X_{Q-r} < \tau_l, X_r > L, \tau_l - X_{Q-r} > L) \\ =& \frac{s}{\tau}((\tau_l - L)r - (\tau_l - L)N(L) - X_{Q-r}r + N(L)X_{Q-r}) \\ & I(X_r > L, X_{Q-r} < \tau_l - L, N(L) \leq r - 1) \\ =& \frac{s}{\tau}(\tau_l - L)H_{Q-r}(\tau_l - L)(r\bar{H}_r(L) - \sum_{k=0}^{r-1} \frac{e^{-\lambda L}(\lambda L)^k}{k!}k) \\ +& \int_{u=0}^{\tau_l - L} udH_{Q-r}(u)(\sum_{k=0}^{r-1} \frac{e^{-\lambda L}(\lambda L)^k}{k!}k - r\bar{H}_r(L)) \\ =& \frac{s}{\tau}(\tau_l - L)H_{Q-r}(\tau_l - L)(r\bar{H}_r(L) - \lambda L\bar{H}_{r-1}(L)) \\ +& \frac{Q-r}{\lambda}H_{Q-r+1}(\tau_l - L)(\lambda L\bar{H}_{r-1}(L) - r\bar{H}_r(L)) \\ =& (r\bar{H}_r(L) - \lambda L\bar{H}_{r-1}(L)) \\ & \frac{s}{\tau}((\tau_l - L)H_{Q-r}(\tau_l - L) - \frac{Q-r}{\lambda}H_{Q-r+1}(\tau_l - L)) \\ =& \frac{s}{\tau}\int_{u=0}^{\tau_l - L} H_{Q-r}(u)du(r\bar{H}_r(L) - \lambda L\bar{H}_{r-1}(L)) \end{aligned} \tag{A.25}$$