

DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**FUZZY SETS AND FUNCTIONS WITH
SELECTED PROBLEMS IN DECISION
ENGINEERING**



by
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İZMİR

FUZZY SETS AND FUNCTIONS WITH SELECTED PROBLEMS IN DECISION ENGINEERING

**A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Doctor of
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**by
İlker GÖLCÜK**

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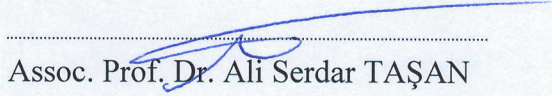
Ph.D. THESIS EXAMINATION RESULT FORM

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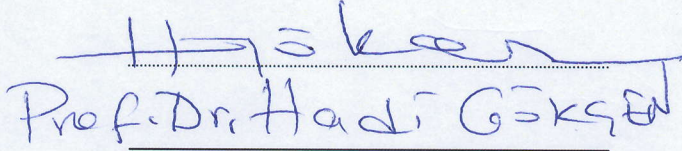
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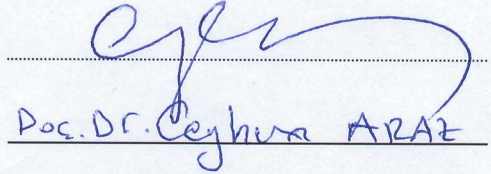
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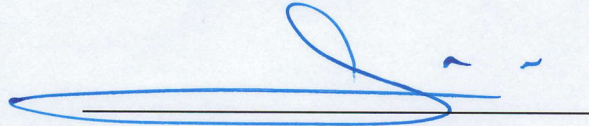
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İlker GÖLCÜK

FUZZY SETS AND FUNCTIONS WITH SELECTED PROBLEMS IN DECISION ENGINEERING

ABSTRACT

Decision engineering is an emerging multidisciplinary field in which systems analysis, uncertainty handling and decision theory have been utilized in combination. A decision engineer provides with a set of computer-assisted mathematical models to understand and model complex decision problems encountered in real-life. These problems are mostly characterized by imprecision, complexity and dynamism.

Understanding of complex interrelationships among decision criteria by managing complexity of the problem is of critical importance in real-life decision engineering problems. These interrelationships are hard to be articulated by means of exact terms. Interval type-2 fuzzy sets are more appropriate to model words due to their flexibility in assigning membership grades. First, interval type-2 fuzzy DEMATEL method has been used to demonstrate how the causal dependencies among criteria be managed within a hierarchically structured problem by means of interval type-2 fuzzy sets. Then, fuzzy cognitive maps, which are another advanced tools for modeling causal dependencies, are extended to the type-1 and interval type-2 fuzzy sets by proposing new alpha-cut based methods.

Another critical issue in decision engineering is about intelligent processing of past data for dynamic decision support. As ever-more data pour through the networks of organizations, considerable effort has been made in order to cultivate valuable information from these resources. In this thesis, two new models, based on fuzzy cognitive maps and interval type-2 fuzzy regression functions, have been developed to solve real-life dynamic multiple attribute decision making problems. Therefore, this thesis contributes to the decision engineering literature with four new decision models.

Keywords: Decision engineering, fuzzy cognitive maps, fuzzy regression functions

BULANIK KÜME VE FONKSİYONLARIN SEÇİLEN KARAR MÜHENDİSLİĞİ PROBLEMLERİNE UYGULANMASI

ÖZ

Karar mühendisliği, sistem analizi, belirsizlik yönetimi ve karar teorisinin bir arada kullanıldığı çok disiplinli bir alandır. Bir karar mühendisi, gerçek hayatta karşılaşılan karmaşık karar problemlerini anlamak ve modellemek için bir dizi bilgisayar destekli matematiksel modeli kullanır. Bu problemler çoğunlukla belirsizlik, karmaşıklık ve dinamizm özellikleriyle ile karşımıza çıkmaktadır.

Problemin karmaşıklığını yöneterek kriterler arasındaki karmaşık ilişkilerin anlaşılması, gerçek hayattaki karar mühendisliği problemlerinde kritik öneme sahiptir. Bu karşılıklı ilişkilerin kesin terimlerle ifade edilmesi zordur. Aralık tip-2 bulanık kümeleri, üyelik dereceleri vermedeki esneklikleri nedeniyle kelimeleri modellemek için daha uygundur. İlk olarak, aralık tip-2 bulanık kümeleriyle ölçütler arasındaki nedensel bağımlılıkların hiyerarşik olarak yapılandırılmış bir problemde nasıl yönetildiğini göstermek için aralık tip-2 bulanık DEMATEL yöntemi kullanılmıştır. Daha sonra, nedensel bağımlılıkları modellemek için bir başka gelişmiş araç olan bulanık bilişsel haritalar, yeni alfa kesim temelli yöntemler önerilerek tip-1 ve aralık tip-2 bulanık kümelere genişletilmiştir.

Karar mühendisliğinde bir diğer kritik husus, dinamik karar desteği için geçmiş verilerin akıllıca işlenmesiyle ilgilidir. Daha fazla veri kuruluşların ağları üzerinden akarken, bu kaynaklardan değerli bilgiler elde etmek için büyük çaba harcanmaktadır. Bu tezde, gerçek hayat dinamik çok kriterli karar verme problemlerini çözmek için bulanık bilişsel haritalar ve aralık tip-2 bulanık regresyon fonksiyonlarına dayalı iki yeni model geliştirilmiştir. Dolayısıyla bu tez, dört yeni karar modeliyle karar mühendisliği literatürüne katkıda bulunmaktadır.

Anahtar kelimeler: Karar mühendisliği, bulanık bilişsel haritalar, bulanık regresyon fonksiyonları

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CHAPTER ONE

INTRODUCTION

1.1 General Overview

Decision engineering is an emerging multidisciplinary field with opportunities and challenges. Many books have been written on decision engineering and the courses have been taught at industrial, system and management engineering. According to Barrager (2016), decision engineers have a degree in engineering or a strongly similar field, and have a profound understanding of systems analysis, uncertainty handling and decision theory. Therefore, decision engineers have ability to understand complex systems that are characterized by dynamism and uncertainty. The task of decision engineers is to design computer-assisted mathematical models for real-life complex decision problems.

A decision engineer brings processes, tools, facilitation and project management phases into the decision making projects (Barrager, 2012). The decision making situation is considered as a process, beginning from problem framing and alternative generation to the synthesis of the overall information. On the other hand, a decision engineer utilizes a set of computer-assisted mathematical models for the analysis. The frequently used decision engineering tools are illustrated in Figure 1 (adapted from De Smet (2019).

The decision engineering tools can be categorized as multiple attribute decision making (MADM), decision making under uncertainty and risk, game theory, and voting theory. The contributions of this thesis can be thought under the MADM and decision under uncertainty titles.

The field of MADM is one of the most significant components of decision engineering tools. A vast array of MADM approaches have been proposed in the literature in order to cope with real-life decision making problems. However, the proposed approaches have some drawbacks.

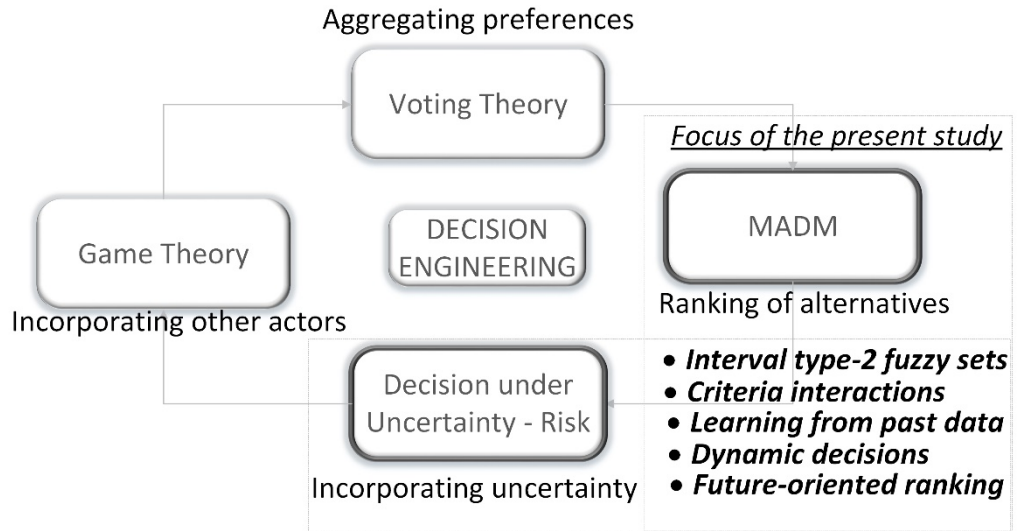


Figure 1.1 Decision engineering tools

One of the major tasks of a decision engineer is to manage the complexity of the problem. Complexity is mainly emerged from the interactions among the system components. These interactions cannot be quantified with exact terms, rather, they are characterized by linguistic terms which are inherently uncertain. Therefore, a decision engineer is expected to collect and process uncertain information while keeping the complexity of the problem manageable.

On the other hand, decision engineer is forward looking and a sound decision making requires incorporation of future-oriented thinking and being flexible to include change in preferences of decision makers (Barrager, 2016; Saaty, 2007). Therefore, learning from the past data is of crucial importance for tracking possible changes in the future preferences of decision makers. Data analytics have much to offer in this field by providing decision makers with a plethora of machine learning tools to extract valuable information from available data. Although the availability of information has been increased dramatically over the past decade, historical records regarding past decision making practices involve imprecision and uncertainty. Therefore, fuzzy system models and system identification tools come into play.

In this study, novel fuzzy MADM models have been proposed to meet the expectations of decision engineers coping with real-life problems. The research questions and motivation of present thesis are given in the next section.

1.2 Research Questions and Motivation

A plethora of approaches have been proposed to solve decision engineering problems in the literature. These approaches can be mainly categorized as expert-driven and data-driven approaches. In the expert-driven decision making, all of the required data is obtained from decision makers at the time of decision making. On the other hand, data-driven decision making models make use of historical records related to problem in hand and experts are elucidated about the temporal performances of the alternatives. The identified research gaps in terms of expert driven decision making models are twofold. A decision making model should be able to answer the following questions:

- How can we tackle causal dependencies among criteria in the form of IT2F sets?
- How can we consider both of the positive and negative causal relationships which are given in the form of IT2 sets?

In the literature, majority of the methods assume that the criteria are preferentially independent, which does not hold in the real life applications. Moreover, analysis of possible interactions and interdependencies helps decision makers in better understanding of the problem and facilitates articulation of preferences regarding set of criteria. Among available methods in resolving interactions among criteria, DEMATEL method has enjoyed a visible position in the literature. However, uncertainties regarding the strength of influences among criteria have generally modeled via type-1 fuzzy sets. In fact, interval type-2 fuzzy sets are more suitable to model words as it gives greater flexibility to define membership grades. Because the words mean different things to different people, IT2F sets are used to acquire the linguistic judgments of decision makers in this thesis. Based on the author's previous studies, a fully-fledged decision making model embodies hierarchical structure, criteria interactions and uncertain preferences of decision makers (Baykasoğlu & Gölcük, 2015a). This idea has been extended in such a way that causal relationships among criteria have been modeled via IT2F sets, which can be considered as a small

contribution to the author's previous work. Note that DEMATEL is not the only available tool for modeling interactions, FCM is the another tool which have superior capabilities in terms of reasoning with imprecise judgements.

FCMs are the extensions of the cognitive maps. FCMs have ability to consider both of the positive and negative causal relationships among concepts. Although FCMs have successfully applied for vast variety of complex problems, there are two questions that have not been answered yet in the literature:

- Are FCMs really fuzzy? Can FCMs represent fuzziness?
- Is it possible to simulate FCMs by using T1/IT2F sets without defuzzification?
- Can we apply FCMs for deriving criteria weights in MADM?

In the literature, concept values and degree of causal relationships are defuzzified, and crisp values are used to simulate the system. In the scope of this thesis, an extensive discussion of the current literature and the criticisms towards early-defuzzification in the FCMs have been given. Information theoretical measures are used to demonstrate the information-loss and incapability of defuzzified values in representing uncertainty. Therefore, there is a need for a new approach for manipulating uncertain information in T1/IT2F forms. Another research gap in the literature is about how to transform cause and effect relationships into meaningful priorities. With the recent advances in the convergence properties of the FCMs, a novel method is developed within the scope of this thesis in order to obtain criteria weights.

Apart from the expert-driven decision making, with the recent advances in the information technologies, significant amount of data accumulated in the datasets makes data-driven decision models an indispensable part of decision processes. MADM models, which take into account the past performance of alternatives, are called dynamic MADM. Generally, the dynamic MADM literature depends on aggregation operators which calculates averages of decision making matrices. There is no doubt that analytics-based solutions have much to offer in this field by incorporating learning and prediction aspects. By capturing the patterns and trends in

the historical records, one might be able to predict future performance of alternatives. Here, the major research questions can be given as follows:

- How can a model learn patterns and trends in the dataset by taking into account interactions among criteria?
- How can optimization of a learning model be done in a practical and straightforward manner with a parameter-free algorithm?
- How can we generate future decision making matrices by utilizing a learning algorithm?
- How can we use past, current and future decision matrices to rank the alternatives within a dynamic MADM?

Another critical issue in data-driven decision making is related to uncertainty. There is no doubt that uncertainty is prevalent in the historical records. High quality data cannot always be assured and data involves noise and imprecision. Therefore, predictions regarding the future performance of alternatives should be uncertain as well. Here, fuzzy logic comes into play with vast number of methods and tools for inferring valuable information from historical data. From the decision engineering point of view, there are important design considerations for building a fuzzy model for dynamic MADM as:

- Which fuzzy model is more suitable for inferring from past decision making matrices?
- Can we generate IT2F outputs by using a fuzzy system model?
- Can interval type-2 fuzzy regression model be used to make accurate predictions?
- Can fuzzy c-means clustering algorithm be used to increase prediction accuracy of fuzzy system models and how?
- Can we propose an integrated model by combining advantages of fuzzy functions concept of Turksen (2008) and fuzzy regression model of Lee and Tanaka (1999)?
- How can we increase the interpretability of the model outputs?

- How can we design an interactive procedure by providing decision makers with linguistic outputs for the sake of interpretability?
- How can we incorporate past performance for alternatives for the complete ranking?

All of the considerations given above have been issued and a new hybrid IT2F model is developed within an interactive dynamic MADM model.

1.3 Structure of the Thesis

The present thesis consists of six chapters. Chapter 1 presents the introductory information regarding the motivation and the scope of the thesis. Chapter 2 and chapter 3 are devoted to the expert-driven decision making models. Chapter 4 and chapter 5 introduce data-driven decision making models. The concluding remarks are given in chapter 6. The structure of the present thesis is illustrated in Figure 1.2.

As a research output of the present thesis, each chapter has written in an article format and chapters 2 to 5 have already been submitted to the respected journals in the fields of decision making and computational intelligence. Chapter 2 has been accepted by the journal “Expert Systems with Applications”. The other 3 chapters are currently under review. Details of the proposed models can be given as follows:

Chapter 2 proposes a new interval type-2 fuzzy multiple-attribute decision making model is developed by integrating Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Decision Making Trial and Evaluation Laboratory (DEMATEL). The proposed model utilizes hierarchical decomposition approach for reducing inherent complexity of the decision making problems. Additionally, interdependencies among problem attributes are taken into consideration by using interval type-2 fuzzy DEMATEL method. Finally, ranking orders of the alternatives are obtained by hierarchical interval type-2 fuzzy TOPSIS method.

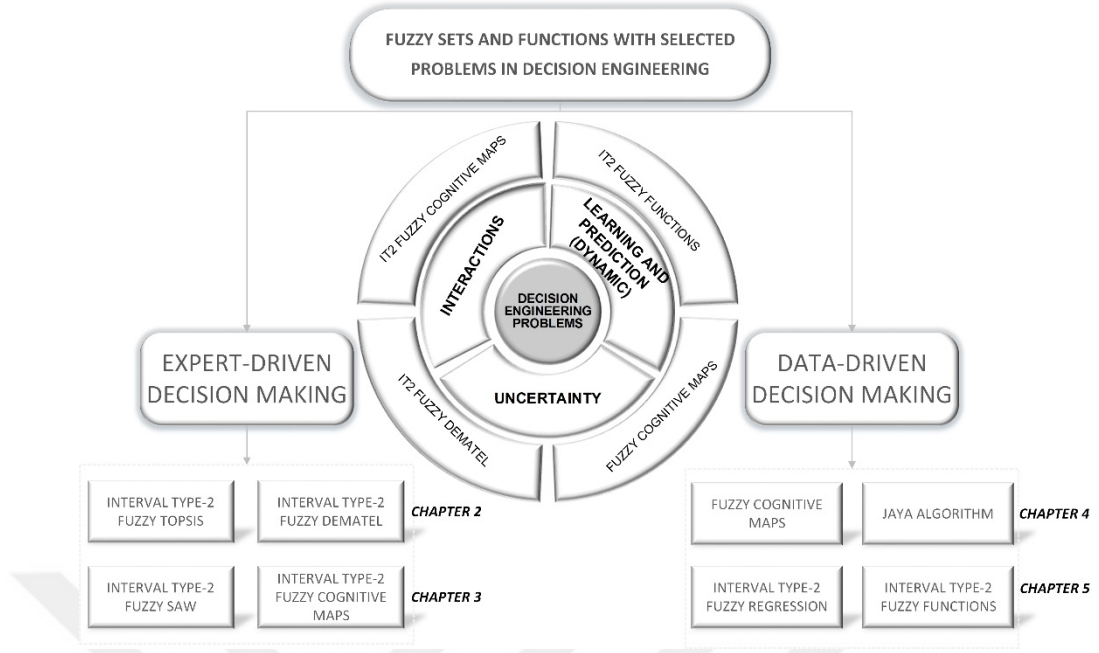


Figure 1.2 Structure of the thesis

As there are several forms of interactions among criteria in real life settings, decision makers should be provided with the expert and intelligent systems which can overcome the preferential independence assumption. The proposed method is able to model causal dependencies by using interval type-2 trapezoidal fuzzy sets. The proposed method is implemented in a Strengths, Weaknesses, Opportunities, and Threats (SWOT)-based strategy selection problem.

Chapter 3 proposes a new alpha-cut based computational procedures for simulation of IT2 FCMs with applications in MADM. Although FCMs have enjoyed a visible position in fuzzy modeling to date, concept values and relationships among concepts are usually defuzzified, and fuzzy singletons are used in the inference algorithm, which consequently evokes undesirable information loss. Despite its crucial importance, fuzzy number representation in concept values and connection matrix is very little issued in the literature. In the pursuit of enhancing modeling capabilities of FCMs, the inference algorithm of the FCMs is extended in such a way that both of the inputs and outputs of the system are represented as fuzzy numbers. To this end, an alpha-cut based computational procedure has been proposed for type-1 FCMs, and the proposed model is verified by using the extension principle. Then, natural generalization for the

case of interval type-2 fuzzy sets has been given. Furthermore, a new FCM-based objective weighting method is proposed for multiple attribute decision making problems. A real life case study, namely third party logistics service provider selection problem, is used to demonstrate the applicability and practicality of the proposed methods.

Chapter 4 concerns with a new multiple attribute decision making (MADM) model to cope with temporal performance of alternatives during different time periods. Dynamic MADM problems has grabbed remarkable attention of decision analysis community in recent years. In parallel with the recent advances in information technologies, firms steadily recognize the importance of data, and business analytics solutions is about to become standard business practice. Majority of the dynamic MADM literature deals with combining past and present data by means of aggregation operators. There is a research gap in developing data-driven methodologies to capture the patterns and trends in the historical data, and provide decision makers with meaningful insights in decision making practices. Analogous with the fact that style of decision making evolving from intuition-based to data-driven, this study proposes a new dynamic MADM model by learning of fuzzy cognitive maps (FCMs) to support decision makers in making informed decisions by considering future performance of alternatives. According to proposed model, Jaya algorithm, a simple and effective metaheuristic optimization method, is used to train FCMs to capture the patterns in historical data. Then, short-, medium-, and long-term future decision making matrices are generated. Finally, past, current and future decision making matrices are taken into consideration, and ranking of alternatives are obtained based on closeness coefficients. The proposed model is realized in a real-life supplier performance evaluation problem.

Chapter 5 proposes another data-driven MADM model by developing a new IT2F functions. A new multiple attribute decision making (MADM) model is proposed in this paper in order to cope with the temporal performance of alternatives during different time periods. Although dynamic MADM problems are enjoying a more visible position in the literature, majority of the applications deal with combining past and present data by means of aggregation operators. There is a research gap in

developing data-driven methodologies to capture the patterns and trends in the historical data. In parallel with the fact that style of decision making evolving from intuition-based to data-driven, the present study proposes a new interval type-2 fuzzy (IT2F) functions model in order to predict current performance of alternatives based on the historical decision matrices. As the availability of accurate historical data with desired quality cannot always be obtained and the data usually involves imprecision and uncertainty, predictions regarding the performance of alternatives are modeled as IT2F sets. These estimated outputs are transformed into interpretable forms by utilizing the vocabulary matching procedures. Then the interactive procedures are employed to allow decision makers to modify the predicted decision matrix based on their perceptions and subjective judgments. Finally, ranking of alternatives are performed based on past and current performance scores.

CHAPTER TWO

INTERVAL TYPE-2 FUZZY SETS BASED HIERARCHICAL MADM MODEL BY COMBINING DEMATEL AND TOPSIS

2.1 Chapter Introduction

Decision making is an omnipresent human activity that takes place in life of every individual very often. Making right decisions against complex problems yields significant influences on the future of the individuals or groups. Importance of making right decisions is not restricted to personal decisions but also for variety of organizations operating under intense pressure of competitive environment. Multiple-Attribute Decision Making (MADM), a field of operations research and management science, aims to help decision makers choose a best candidate from a set of alternatives by considering multiple, incommensurate, and conflicting criteria (Baykasoğlu & Gölcük, 2015a).

The long term impacts of the good quality decisions on the overall organizational performance have increased awareness to virtues of using MADM methods (Baykasoğlu & Gölcük, 2015a) . In the literature, a vast array of MADM methods, such as Multi-Attribute Utility Theory (MAUT) (Dyer, 2005; Fishburn, 1970; Keeney, 1988), Analytic Hierarchy Process (AHP) (Saaty, 1980), Analytic Network Process (ANP) (Saaty, 1996), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Elimination et Choice Translating Reality (ELECTRE) (Roy, 1990), VlseKriterijumska Optimizacija I Kompromisno Resenje technique (VIKOR) (Opricovic & Tzeng, 2004), Decision-Making Trial and Evaluation Laboratory (DEMATEL) (Gabus & Fontela, 1976), conjoint analysis (Krantz, 1964; Verma & Pullman, 1998), Choquet integral (Grabisch & Labreuche, 2010; Marichal, 2000, 2004), and etc., are proposed in order to support decision making processes. Although numerous methods have been proposed, some methods have grabbed remarkable attention due to their practicality in handling real world problems.

Developed by Hwang and Yoon (1981), TOPSIS method is one of the such practical methods based on the idea that the selected alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. TOPSIS method is successfully applied to wide variety of decision problems due to its simplicity and comprehensibility (Baykasoğlu, Kaplanoğlu, Durmuşoğlu, & Şahin, 2013). Since decision making requires linguistic assessments of the decision makers, fuzzy set theory (Zadeh, 1965) has been widely used within the TOPSIS algorithm. Many real world applications are reported where fuzzy TOPSIS and its extensions are deployed (Boran, Genç, Kurt, & Akay, 2009; Dağdeviren, Yavuz, & Kilinç, 2009; Wang & Elhag, 2006; Yang & Hung, 2007). Recently, hierarchical structure has been adopted to fuzzy TOPSIS algorithm (Kahraman, Ateş, Çevik, Gülbay, & Erdoan, 2007) which facilitates understanding of the problem and collecting preferences of decision makers. Hierarchical fuzzy TOPSIS method has been applied in different application domains, such as road safety assessment (Bao, Ruan, Shen, Hermans, & Janssens, 2012), green supply chain assessment (Wang & Chan, 2013), supplier selection (Roshandel, Miri-Nargesi, & Hatami-Shirkouhi, 2013), distribution channel management (Paksoy, Pehlivan, & Kahraman, 2012), new product pricing strategy selection (Baykasoğlu, Gölcük, & Akyol, 2017). One of the deficiencies of the hierarchical fuzzy TOPSIS method lies in its lack of ability to model criteria interactions. In the literature, there are variety of methods for modeling criteria interactions within MADM setting. For a detailed analysis of these methods, we refer to (Gölcük & Baykasoğlu, 2016).

DEMATEL method, developed in the Geneva Research Centre of the Battelle Memorial Institute, is one of the methods that can be used to model causal dependencies among criteria. DEMATEL method is able to visualize the complex cause and effect relationships in an understandable manner. DEMATEL can be used to construct a structural model for analyzing complex interrelationships among criteria (Büyükoçkan & Çifçi, 2012). Dalalah, Hayajneh, and Batieha (2011) used DEMATEL method for estimating criteria weights for supplier selection problem, Chou, Sun, and Yen (2012) combined fuzzy DEMATEL and fuzzy AHP for analyzing the human resource evaluation criteria, Lin and Wu (2008) applied fuzzy DEMATEL for the

R&D project selection problem. Tseng (2009) developed a grey-fuzzy DEMATEL method for real estate quality agent ranking problem. Lin (2013) evaluated green supply chain practices by using fuzzy DEMATEL method. Despite successful real world applications of the fuzzy DEMATEL method, inherent uncertainties in the decision makers' expressions can be modelled by the state of the art fuzzy technologies.

It is evident that the various facets of uncertainty are prevalent in the complicated process of decision making. Diverse manifestations of the uncertainty aspects are associated with the formalization of criteria, goal, system behavior, and the most importantly, with the preferences of decision makers. Because the decision makers' assessments are expressed in terms of subjective linguistic judgements, exact numerical values do not produce accurate results. For that reason, fuzzy set theory is incorporated into the decision making methods in order to cope with the linguistic uncertainties. However, majority of the methods make use of the type-1 fuzzy sets for modeling imprecise information of decision makers. However, type-1 fuzzy sets are not appropriate to model words (Mendel, 2003a, 2007). Because words mean different things to different people, they are uncertain. Specifically, there exist intra-uncertainty, which is the uncertainty a person has about the word, and inter-uncertainty, which refers to the uncertainty a group of people have about the word (Mendel, 2003a). Therefore, a word can be modeled by using an interval type-2 fuzzy sets (IT2FSs) which handle linguistic uncertainties appropriately. IT2FSs are the natural extensions of fuzzy sets, where the membership degrees are not exact (Mendel, 2003b). In this line of thought, there have been various attempts to integrate IT2FSs and MADM methods. A very detailed literature review regarding the IT2FSs-based MADM models is presented in (Celik, Gul, Aydin, Gumus, & Guneri, 2015).

In this paper, hierarchical IT2F-TOPSIS (Chen & Lee, 2010) method is combined with IT2F-DEMATEL (Abdullah & Zulkifli, 2015) method for the SWOT-based strategy selection problem. The preliminary version of the study (Baykasoğlu & Gölcük, 2015b) is extended, and the proposed model is discussed comprehensively. The rationale behind the proposed model is to provide decision makers with a practical

method, which is able to handle criteria interactions among hierarchically structured criteria under IT2F information.

This paper contributes to the literature in the following ways:

- Hierarchical fuzzy TOPSIS and DEMATEL methods are combined under IT2F environment. The preferential independence assumption is overcome by using IT2F-DEMATEL method.
- First time in the literature, the importance of SWOT factors are quantified by utilizing IT2FSs. Furthermore, IT2F-DEMATEL method is used to model causal dependencies among SWOT factors. A SWOT-based strategy selection problem is used to illustrate the proposed model.

In Section 2.2, the basics of IT2FSs and their arithmetic operations are given. The proposed methodology is introduced in Section 2.3, an illustrative SWOT-based strategy selection problem is given in Section 2.4, validation and discussions are covered in Section 2.5, and finally concluding remarks are given in Section 2.6.

2.2 Background Information

As a particular case of the ITFSs, trapezoidal membership functions are utilized throughout the presented paper. In this section, some important definitions and arithmetic operations related to trapezoidal IT2FSs are presented.

2.2.1 Interval Type-2 Fuzzy Sets

Definition 1 (Mendel, John, & Feilong, 2006; Mendel & Wu, 2010): A type-2 fuzzy set (T2FS) $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as in Equation 2.1.

$$\tilde{\tilde{A}} = \left\{ ((x, u), \mu_{\tilde{\tilde{A}}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1 \right\} \quad (2.1)$$

where J_x denotes an interval in $[0,1]$. Moreover, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as in Equation 2.2.

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x, u) / (x, u) \quad (2.2)$$

where $J_x \subseteq [0,1]$ and $\int \int$ denotes union over all admissible x and u .

Definition 2 (Mendel et al., 2006; Mendel & Wu, 2010): Let $\tilde{\tilde{A}}$ be a T2FS in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is called an IT2FS. An IT2FS $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad (2.3)$$

where $J_x \subseteq [0,1]$.

Definition 3 (Mendel et al., 2006; Mendel & Wu, 2010): The upper membership function and the lower membership function of an IT2FS are type-1 membership functions, respectively. Figure 2.1 shows a trapezoidal IT2FS $\tilde{\tilde{A}}$ as given in Equation 2.4.

$$\tilde{\tilde{A}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L) = \left((a_{i1}^U, a_{i1}^U, a_{i1}^U, a_{i1}^U; H_1(\tilde{\tilde{A}}_1^U), H_2(\tilde{\tilde{A}}_1^U)), \right. \quad (2.4)$$

$$\left. (a_{i1}^L, a_{i1}^L, a_{i1}^L, a_{i1}^L; H_1(\tilde{\tilde{A}}_1^L), H_2(\tilde{\tilde{A}}_1^L)) \right)$$

where $\tilde{\tilde{A}}_1^U$ and $\tilde{\tilde{A}}_1^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the reference points of the IT2FS $\tilde{\tilde{A}}$, $H_j(\tilde{\tilde{A}}_1^U)$ denotes the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function $\tilde{\tilde{A}}_1^U$, $1 \leq j \leq 2$, $H_j(\tilde{\tilde{A}}_1^L)$

denotes the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal membership function.

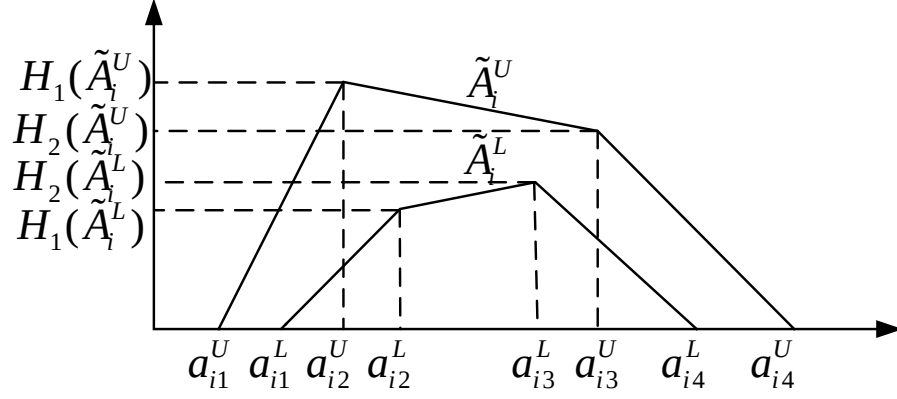


Figure 2.1 A trapezoidal type-2 fuzzy set (Lee & Chen, 2008b)

2.2.2 Arithmetic Operations

Definition 4 (Chen & Lee, 2010; Lee & Chen, 2008b): The addition operation between the trapezoidal IT2FSs is defined as in Equation 2.5:

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= \left(\begin{pmatrix} a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \\ \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \end{pmatrix}, \right. \\ &\quad \left. \begin{pmatrix} a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \\ \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)) \end{pmatrix} \right) \end{aligned} \quad (2.5)$$

Definition 5 (Chen & Lee, 2010; Lee & Chen, 2008b): The subtraction operation between the trapezoidal IT2FSs is defined as in Equation 2.6:

$$\begin{aligned}
\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \ominus (\tilde{A}_2^U, \tilde{A}_2^L) \\
&= \left(\begin{array}{l} \left(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \right. \\ \left. \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \right) \\ \left(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; \right. \\ \left. \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)) \right) \end{array} \right) \quad (2.6)
\end{aligned}$$

Definition 6 (Chen & Lee, 2010; Lee & Chen, 2008b): The multiplication operation between the trapezoidal IT2FSs is defined as in Equation 2.7:

$$\begin{aligned}
\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\
&= \left(\begin{array}{l} \left(a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \right. \\ \left. \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \right) \\ \left(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \right. \\ \left. \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)) \right) \end{array} \right) \quad (2.7)
\end{aligned}$$

Definition 7 (Chen & Lee, 2010; Lee & Chen, 2008b): The multiplication and division operations between IT2FS and a crisp value k is defined as in Equation 2.8-9, respectively:

$$\tilde{\tilde{A}} \otimes k = \left(\begin{array}{l} \left(a_{11}^U \times k, a_{12}^U \times k, a_{13}^U \times k, a_{14}^U \times k; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \right), \\ \left(a_{11}^L \times k, a_{12}^L \times k, a_{13}^L \times k, a_{14}^L \times k; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L) \right) \end{array} \right) \quad (2.8)$$

$$\frac{\tilde{\tilde{A}}}{k} = \left(\begin{array}{l} \left(a_{11}^U \times \frac{1}{k}, a_{12}^U \times \frac{1}{k}, a_{13}^U \times \frac{1}{k}, a_{14}^U \times \frac{1}{k}; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \right), \\ \left(a_{11}^L \times \frac{1}{k}, a_{12}^L \times \frac{1}{k}, a_{13}^L \times \frac{1}{k}, a_{14}^L \times \frac{1}{k}; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L) \right) \end{array} \right) \quad (2.9)$$

Definition 8 (Lee & Chen, 2008): The ranking value $Rank(\tilde{\tilde{A}}_i)$ of the trapezoidal IT2FS $\tilde{\tilde{A}}_i$ is defined as in the Equation 2.10:

$$\begin{aligned}
Rank(\tilde{\tilde{A}}_i) = & M_1(A_i^U) + M_1(A_i^L) + M_2(A_i^U) + M_2(A_i^L) + M_3(A_i^U) \\
& + M_3(A_i^L) - \frac{1}{4}(S_1(A_i^U) + S_1(A_i^L) + S_2(A_i^U) + S_2(A_i^L) \\
& + S_3(A_i^U) + S_3(A_i^L) + S_4(A_i^U) + S_4(A_i^L)) \\
& + H_1(A_i^U) + H_1(A_i^L) + H_2(A_i^U) + H_2(A_i^L)
\end{aligned} \quad (2.10)$$

where $M_p(\tilde{\tilde{A}}_i^j)$ denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$.

$$M_p(\tilde{\tilde{A}}_i^j) = (a_{ip}^j + a_{i(p+1)}^j) / 2, 1 \leq p \leq 3 \quad (2.11)$$

$S_q(\tilde{\tilde{A}}_i^j)$ denotes the standard deviation of the elements a_{iq}^j and $a_{i(q+1)}^j$.

$$S_q(\tilde{\tilde{A}}_i^j) = \sqrt{\frac{1}{2} \sum_{k=q}^{q+1} \left(a_{ik}^j - \frac{1}{2} \sum_{k=q}^{q+1} a_{ik}^j \right)^2}, 1 \leq p \leq 3 \quad (2.12)$$

$S_4(\tilde{\tilde{A}}_i^j)$ denotes the standard deviation of the elements $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j$.

$$S_4(\tilde{\tilde{A}}_i^j) = \sqrt{\frac{1}{4} \sum_{k=1}^4 \left(a_{ik}^j - \frac{1}{4} \sum_{k=1}^4 a_{ik}^j \right)^2} \quad (2.13)$$

$H_p(\tilde{\tilde{A}}_i^j)$ denotes the membership value of the element $a_{i(p+1)}^j$ in the trapezoidal membership function $\tilde{\tilde{A}}_i^j, 1 \leq p \leq 2, j \in \{U, L\}$, and $1 \leq i \leq n$.

Definition 9 (Hu, Zhang, Chen, & Liu, 2013): Expected value of a trapezoidal IT2FS $\tilde{\tilde{A}}$ is calculated by using Equation 2.14.

$$E(A) = \frac{1}{2} \left(\frac{1}{4} \sum_{i=1}^4 (a_i^L + a_i^U) \right) \times \frac{1}{4} \left(\sum_{i=1}^2 (H_i(A^L) + H_i(A^U)) \right) \quad (2.14)$$

where $\tilde{A} = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(\tilde{A}^L), H_2(\tilde{A}^L)))$.

2.3 The Proposed Methodology

In this section, the proposed methodology is introduced and the algorithmic steps are given. The proposed methodology consists of the following procedures:

- Problem structuring
- Modeling interactions among criteria by using IT2F-DEMATEL method.
- Deriving criteria weights based on IT2F-DEMATEL results.
- Ranking of alternatives by using IT2F-TOPSIS method.

The main steps and the flowchart of the proposed method are given in Figure 2.2.

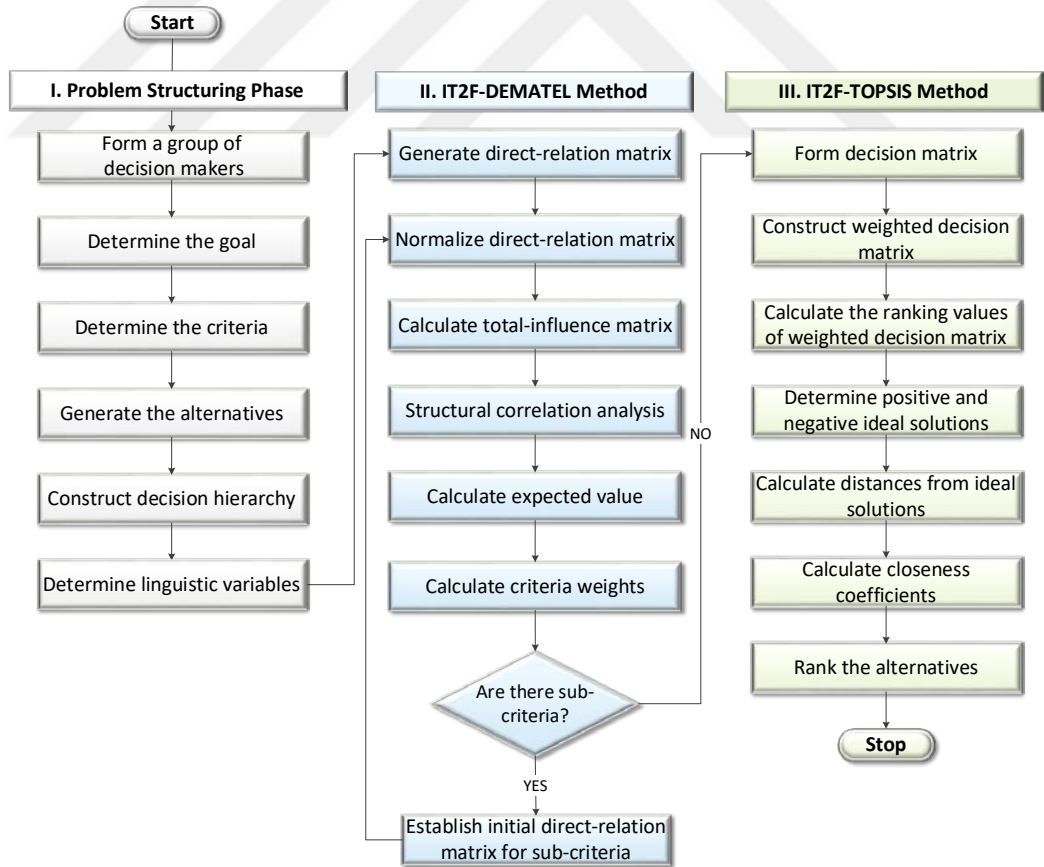


Figure 2.2 Proposed evaluation framework

The proposed method can be seen as the IT2F extension of the model presented in (Baykasoğlu et al., 2013). In the subsequent sections, the proposed model will be introduced in a step-by-step manner.

2.3.1 Problem Structuring Phase

In this step, the aim and the scope of the study, main-criteria, sub-criteria, and alternatives are determined. The stakeholders and the decision makers are identified. Hierarchical representation of the criteria along with the goal statement at the top of the hierarchy is determined at this phase. Also, the linguistic variables and their associated IT2FSs are formed.

2.3.2 Interval Type-2 Fuzzy DEMATEL Method

In this step, the IT2F-DEMATEL method is used to capture the relationships among criteria. Then, the relationships are transformed into local priorities. The main steps of the IT2F-DEMATEL method are briefly described as follows (Abdullah & Zulkifli, 2015):

Step 1: Acquiring the assessments of decision makers. The group of experts is asked to fill out the IT2F influence matrices. For that aim, total of k influence matrices, $\tilde{\mathbf{Z}}^{(1)}, \tilde{\mathbf{Z}}^{(2)}, \dots, \tilde{\mathbf{Z}}^{(k)}$ are collected.

Step 2: Calculating the average of the IT2F influence matrices. In this step, the average of the IT2F influence matrices is calculated as in the Equation 2.15.

$$\tilde{\mathbf{Z}} = \frac{\tilde{\mathbf{Z}}^{(1)} \oplus \tilde{\mathbf{Z}}^{(2)} \oplus \dots \oplus \tilde{\mathbf{Z}}^{(k)}}{k} \quad (2.15)$$

where $\tilde{\mathbf{Z}}$ denotes the initial direct-relation matrix. The initial direct-relation matrix takes the form as seen in Equation 2.16.

$$\tilde{\mathbf{Z}} = \begin{bmatrix} 0 & \tilde{z}_{12} & \cdots & \tilde{z}_{1m} \\ \tilde{z}_{21} & 0 & \cdots & \tilde{z}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{m1} & \tilde{z}_{m2} & \cdots & 0 \end{bmatrix} \quad (2.16)$$

where $\tilde{z}_{ij} = \left((a_{ij}, b_{ij}, c_{ij}, d_{ij}; H_1(\tilde{z}_{ij}^U), H_2(\tilde{z}_{ij}^U)), (e_{ij}, f_{ij}, g_{ij}, h_{ij}; H_1(\tilde{z}_{ij}^L), H_2(\tilde{z}_{ij}^L)) \right)$.

Step 3: Acquiring the normalized direct-relation matrix. In order to calculate the normalized direct-relation matrix, the trapezoidal IT2F initial direct-relation matrix is rearranged according to membership functions. For the sake of simplicity, the heights of the IT2FNs are omitted from the subsequent representations as they do not affect the calculations. Therefore, total of eight $m \times m$ matrices are constructed as in the Equation 2.17:

$$\mathbf{Z}_{a'} = \begin{bmatrix} 0 & a'_{12} & \cdots & a'_{1m} \\ a'_{21} & 0 & \cdots & a'_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a'_{m1} & a'_{m2} & \cdots & 0 \end{bmatrix}, \dots, \mathbf{Z}_{h'} = \begin{bmatrix} 0 & h'_{12} & \cdots & h'_{1m} \\ h'_{21} & 0 & \cdots & h'_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h'_{m1} & h'_{m2} & \cdots & 0 \end{bmatrix} \quad (2.17)$$

Therefore, $\mathbf{Z}_{a'}$ contains the greatest element which is used to calculate the normalization coefficient. The normalized direct-relation matrix is defined as in Equation 2.18.

$$\tilde{\mathbf{X}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mm} \end{bmatrix} \quad (2.18)$$

The elements of the normalized direct-relation matrix are calculated as in the Equation 2.19.

$$\tilde{\tilde{x}}_{ij} = \frac{\tilde{z}_{ij}}{s} = \left(\left(\frac{z_{a'_{ij}}}{s}, \frac{z_{b'_{ij}}}{s}, \frac{z_{c'_{ij}}}{s}, \frac{z_{d'_{ij}}}{s}; H_1(\tilde{z}_{ij}^U), H_2(\tilde{z}_{ij}^U) \right), \right. \\ \left. \left(\frac{z_{e'_{ij}}}{s}, \frac{z_{f'_{ij}}}{s}, \frac{z_{g'_{ij}}}{s}, \frac{z_{h'_{ij}}}{s}; H_1(\tilde{z}_{ij}^L), H_2(\tilde{z}_{ij}^L) \right) \right) \quad (2.19)$$

where the normalization coefficient s is calculated as in the Equation 2.20.

$$s = \max \left(\max_{1 \leq i \leq m} \sum_{j=1}^m z_{d'_{ij}}, \max_{1 \leq j \leq m} \sum_{i=1}^m z_{d'_{ij}} \right) \quad (2.20)$$

Step 4: Calculating the total-relation matrix. In a very similar fashion to the Step 3, the normalized direct-relation matrix can be represented by eight crisp matrices as in the Equation 2.21:

$$\begin{aligned} X_{a''} &= \begin{bmatrix} 0 & a''_{12} & \cdots & a''_{1m} \\ a''_{21} & 0 & \cdots & a''_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a''_{m1} & a''_{m2} & \cdots & 0 \end{bmatrix}, \\ X_{b''} &= \begin{bmatrix} 0 & b''_{12} & \cdots & b''_{1m} \\ b''_{21} & 0 & \cdots & b''_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b''_{m1} & b''_{m2} & \cdots & 0 \end{bmatrix}, \dots, \\ X_{h''} &= \begin{bmatrix} 0 & h''_{12} & \cdots & h''_{1m} \\ h''_{21} & 0 & \cdots & h''_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h''_{m1} & h''_{m2} & \cdots & 0 \end{bmatrix} \end{aligned} \quad (2.21)$$

The total relation matrix is represented by $\tilde{\tilde{T}}$ as:

$$\tilde{\mathbf{T}} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1m} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \cdots & \tilde{t}_{mm} \end{bmatrix} \quad (2.22)$$

where $\tilde{t}_{ij} = \left((a_{ij}''', b_{ij}''', c_{ij}''', d_{ij}'''; H_1(\tilde{t}_{ij}^U), H_2(\tilde{t}_{ij}^U)), (e_{ij}''', f_{ij}''', g_{ij}''', h_{ij}'''; H_1(\tilde{t}_{ij}^L), H_2(\tilde{t}_{ij}^L)) \right)$. The elements of the total relation-matrix are calculated as in the Equation 2.23.

Note that the heights of the IT2FNs are the same for each entry of the total-relation matrix; i.e. $H_1(\tilde{t}_{ij}^U) = H_1(\tilde{x}_{ij}^U)$.

$$\begin{aligned} [a_{ij}'''] &= \mathbf{X}_{a''} \times (\mathbf{I} - \mathbf{X}_{a''})^{-1} \\ [b_{ij}'''] &= \mathbf{X}_{b''} \times (\mathbf{I} - \mathbf{X}_{b''})^{-1}, \dots, \\ [h_{ij}'''] &= \mathbf{X}_{h''} \times (\mathbf{I} - \mathbf{X}_{h''})^{-1} \end{aligned} \quad (2.23)$$

Step 5: Conducting structural correlation analysis. The elements of the total-relation matrix $\tilde{t}_{ij}$ are used to conduct structural correlation analysis. Sum of the rows and sum of the columns of the $\tilde{\mathbf{T}}$ matrix, denoted as $\tilde{\tilde{D}}_i$ and $\tilde{\tilde{R}}_j$, respectively, can be obtained by using Equation 2.24-25.

$$\tilde{\tilde{D}}_i = \sum_{j=1}^m \tilde{t}_{ij} \quad (i = 1, 2, \dots, m) \quad (2.24)$$

$$\tilde{\tilde{R}}_j = \sum_{i=1}^m \tilde{t}_{ij} \quad (j = 1, 2, \dots, m) \quad (2.25)$$

In order to draw a causal diagram, expected values of the ordered pairs $(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i, \tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i)$ are calculated. The expected value of the $\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i$ is denoted by $E(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i)$ and called expected prominence. Similarly, $E(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i)$ is called expected relation.

Step 6: Calculating the weights of criteria. When the expected prominence and relation values are calculated, importance of each criterion is calculated by Equation 2.26:

$$w_i = \sqrt{\left(E\left(\tilde{D}_i \oplus \tilde{R}_i\right)\right)^2 + \left(E\left(\tilde{D}_i \ominus \tilde{R}_i\right)\right)^2} \quad (2.26)$$

Finally, the normalized importance degree of each criterion is calculated as in Equation 2.27.

The final weights are shown as in the Equation 2.28:

$$nw_i = \frac{w_i}{\sum_{i=1}^m w_i} \quad (2.27)$$

$$I_{MA} = \begin{bmatrix} MA_1 & nw_1 \\ MA_2 & nw_2 \\ \vdots & \vdots \\ MA_p & nw_p \\ \vdots & \vdots \\ MA_m & nw_m \end{bmatrix} \quad (2.28)$$

where I_{MA} denotes the main attribute weights. Similarly, sub-attribute weights are denoted as I_{SA} . The overall weight of the l th sub-attribute under p th main attribute is denoted by W_{pl} and calculated as in the Equation 2.29:

$$W_{pl} = w_p w_{pl} \quad (2.29)$$

where w_{pl} is the sub-attribute weight located in the I_{SA} , and the w_p is the weight of the p th main attribute.

2.3.3 Interval Type-2 Fuzzy TOPSIS Method

In this step, the alternatives are ranked by taking into account the weights of criteria which are obtained from IT2F-DEMATEL method. The main steps of the IT2F-TOPSIS algorithm are given as follows (Chen & Lee, 2010):

Assume that there is a set X of options or alternatives denoted by $X = \{x_1, x_2, \dots, x_n\}$. The attribute set, denoted by F , can be divided into two sets, F_1 and F_2 . Here F_1 represents the benefit attributes and F_2 denotes the cost attributes. Note that $F_1 \cap F_2 = \emptyset$ and $F_1 \cup F_2 = F$. The main steps of the IT2F-TOPSIS method are given as follows:

Step 1: Collecting the decision matrices. In this step, total of k decision matrices $\tilde{Y}^{(1)}, \tilde{Y}^{(2)}, \dots, \tilde{Y}^{(k)}$ are collected.

Step 2: Calculating the average of IT2F decision matrices. The average IT2F decision matrices is calculated as in the Equation 2.30

$$\tilde{Y} = \frac{\tilde{Y}^{(1)}, \tilde{Y}^{(2)}, \dots, \tilde{Y}^{(k)}}{k} \quad (2.30)$$

where $\tilde{Y}$ denotes the average IT2F decision matrices and takes the form as in the Equation 2.31:

$$\tilde{Y} = \begin{bmatrix} \tilde{f}_{ij} \end{bmatrix}_{m \times n} = \begin{matrix} & \begin{matrix} x_1 & x_2 & \cdots & x_n \end{matrix} \\ \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} & \begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} & \cdots & \tilde{f}_{1n} \\ \tilde{f}_{21} & \tilde{f}_{22} & \cdots & \tilde{f}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{f}_{m1} & \tilde{f}_{m2} & \cdots & \tilde{f}_{mn} \end{bmatrix} \end{matrix} \quad (2.31)$$

where $\tilde{f}_{ij}$ is an IT2F rating of the j th alternative with respect to i th attribute, and $1 \leq i \leq m, 1 \leq j \leq n$.

Step 3: Constructing weights of the attributes. In the previous sections, as a result of the DEMATEL-based calculations, the normalized weights have already been obtained. These weights are used in the IT2F-TOPSIS calculations as in the Equation 2.32.

$$W = [nw_i]_{1 \times m} = \begin{bmatrix} f_1 & f_2 & \cdots & f_m \\ nw_1 & nw_2 & \dots & nw_m \end{bmatrix} \quad (2.32)$$

Step 4: Constructing the weighted decision matrix. The weighted decision matrix is given in Equation 2.33.

$$\tilde{Y}_w = [\tilde{v}_{ij}]_{m \times n} = \begin{matrix} & \begin{matrix} x_1 & x_2 & \cdots & x_n \end{matrix} \\ \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} & \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mn} \end{bmatrix} \end{matrix} \quad (2.33)$$

where $\tilde{v}_{ij} = w_i \otimes \tilde{f}_{ij}$, $1 \leq i \leq m$, $1 \leq j \leq n$.

Step 5: Calculating the ranking value of the weighted decision matrix by using Equation 2.10-13. The ranking value of the entry in the i th row and j th column of the weighted decision matrix is denoted by $Rank(\tilde{v}_{ij})$.

Step 6: Determining the ideal solutions. In this step, positive and negative ideal solutions, denoted by $x^+ = (v_1^+, v_2^+, \dots, v_m^+)$ and $x^- = (v_1^-, v_2^-, \dots, v_m^-)$, are calculated, respectively. The ideal solutions are calculated as in the Equation 2.34-35:

$$v_i^+ = \begin{cases} \max_{1 \leq j \leq n} \{Rank(\tilde{v}_{ij})\}, & \text{if } f_i \in F_1 \\ \min_{1 \leq j \leq n} \{Rank(\tilde{v}_{ij})\}, & \text{if } f_i \in F_2 \end{cases} \quad (2.34)$$

$$v_i^- = \begin{cases} \min_{1 \leq j \leq n} \{Rank(\tilde{v}_{ij})\}, & \text{if } f_i \in F_1 \\ \max_{1 \leq j \leq n} \{Rank(\tilde{v}_{ij})\}, & \text{if } f_i \in F_2 \end{cases} \quad (2.35)$$

Step 7: Calculating the distance between each alternative and the ideal solutions. The distance between alternative x_j and the positive ideal solution x^+ , denoted by $d^+(x_j)$, is calculated as given in the Equation 2.36:

$$d^+(x_j) = \sqrt{\sum_{i=1}^m (Rank(\tilde{v}_{ij}) - v_i^+)^2} \quad (2.36)$$

where $1 \leq j \leq n$. The distance $d^-(x_j)$ between alternative x_j and the negative ideal solution x^- , is calculated by using Equation 2.37:

$$d^-(x_j) = \sqrt{\sum_{i=1}^m (Rank(\tilde{v}_{ij}) - v_i^-)^2} \quad (2.37)$$

Step 8: Calculating the relative degree of closeness (closeness coefficient) $C(x_j)$ of x_j with respect to the positive ideal solution x^+ and negative ideal solution x^- . The relative degree of closeness is calculated by using Equation 2.38:

$$C(x_j) = \frac{d^-(x_j)}{d^+(x_j) + d^-(x_j)} \quad (2.38)$$

Step 9: Sorting the values of $C(x_j)$ in a descending sequence, where $1 \leq j \leq n$. The larger the value of $C(x_j)$, the higher the preference of the alternative x_j .

2.4 An Illustrative Application

The solution procedures presented in the previous section are realized in an illustrative application. In the following case study, SWOT-based strategy selection problem by means of preparatory efforts to develop a renewed strategic plan for the industrial engineering department is revisited. The same data of (Baykasoğlu & Gölcük, 2015a) (except the linguistic scale, which is modified to the IT2FSs) is used in order to illustrate the proposed method.

In the case study, an external environment is analyzed by the group of experts. Hence, the SWOT sub-factors, which have considerable impact on the success of organization but cannot be controlled, are detected. Then, internal analysis is performed in order to capture the SWOT sub-factors that can be controlled and improved by the organization. Accordingly, sub-factors for strengths, weaknesses, opportunities, and threats dimensions are identified. In accordance with the identified sub-factors, alternative strategies are developed and deployed into the SWOT matrix as shown in Table 2.1. As can be seen from the SWOT matrix, four alternatives are determined as SO, WO, ST, and WT strategies.

In the SO strategy, the opportunities are fully exploited by the internal strengths of the organization. The WO strategy takes into account the weakness of the organization while aiming at the making good use of opportunities. The ST strategy utilizes organizational strengths in order to alleviate the effects of threats. Lastly, the WT strategy strives to diminish the effects of threats by considering organizational weaknesses.

2.4.1 Problem Structuring

The hierarchical decision problem is formed by using the identified SWOT-sub factors. The decision goal is determined as the “selecting the best strategy”. The main SWOT factors, which are strengths, weaknesses, opportunities, and threats are placed into the second level of the hierarchy. The total of 24 SWOT sub-factors are placed

into the third level of the decision hierarchy. In the fourth level, alternative strategies are presented. These strategies are determined as follows:

- SO strategy: Collaborating with industry for research/student projects
- WO strategy: Developing a research oriented academic program
- ST strategy: Establishing new laboratories
- WT strategy: Partnerships with foreign universities

The complete decision hierarchy is formed as seen in the Figure 2.3.

When the decision hierarchy is constructed, the linguistic variables and their associated IT2FSs are identified. The linguistic variables for the causal relationships among SWOT factors are given in Table 2.2. The linguistic variables for rating of the alternatives are determined as given in Table 2.3.

2.4.2 Evaluating Relationships Among SWOT Factors

In this section, the causal relationships among SWOT factors are modeled by using IT2F-DEMATEL method. The causal relationships among SWOT factors are expressed by means of IT2FSs. The intensities of relationships among SWOT factors are transformed into the local priorities.

Step 1: In order to calculate the IT2F-DEMATEL method, the degrees of causal dependencies among main factors are elicited from decision makers. Each decision maker is represented by DM_1 , DM_2 , and DM_3 , respectively. The IT2F influence matrices which indicate the degree of dependency among SWOT factors, are obtained as given in Tables 2.4-6.

Step 2: Employing Equation 2.15, the initial direct-relation matrix, $\tilde{Z}$, is constructed as given in Table 2.7.

Table 2.1 SWOT Matrix

External Factors	Internal Factors	
	Strengths (S)	Weaknesses (W)
	Appropriate physical environment	Campus location
	Student quality	Lack of communication among staff
	Dynamic research environment	Traditional undergraduate education
	Specialized academic staff	Library
	Brand status	Bureaucracy
	Working conditions	Arrogance
Opportunities (O)		
Talented new generation		
Government incentives		
Demand for re-skilling and training		
Assistant recruitments by government		
Entrepreneurial climate		
Need of interdisciplinary collaboration		
Threats (T)		
Increasing number of new universities		
Rapid technological change		
E-learning programs		
Employer legislations		
Corporate universities		
University budget cuts		

SO Strategy
Collaborating with industry for research/student projects

WO Strategy
Developing a research oriented academic program

ST Strategy
Establishing new laboratories

WT Strategy
Partnerships with foreign universities

Step 3: The normalized initial direct-relation matrix is calculated by using Equation 2.17-20. The normalization coefficient is found to be $s = 1.65$.

Step 4: The total-relation matrix is calculated by employing Equation 2.21-23. The total-relation matrix is given in Table 2.9.

The normalized initial direct-relation matrix is given in Table 2.8.

Step 5: Based on the total-relation matrix obtained in the previous step, the structural correlation analysis is performed. The row and column sums of the total relation matrix are calculated based on the Equation 2.24-25. The calculated $\tilde{D}$ and $\tilde{R}$ values for each SWOT factor are given in Table 2.10.

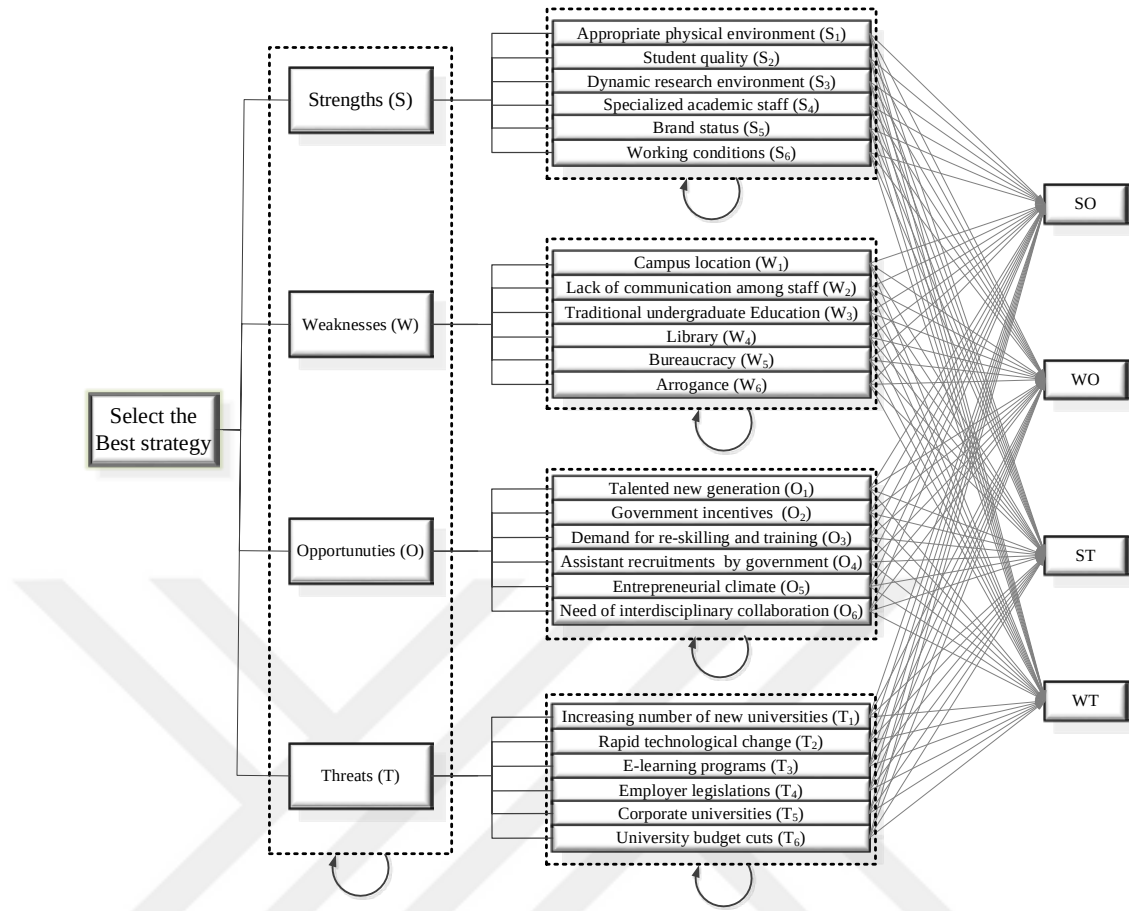


Figure 2.3 Hierarchical evaluation framework (Baykasoğlu & Gölçük, 2015a)

Table 2.2 Linguistic variables for causal relationships among attributes

Linguistic variable	ITFN
Very very low (VVL)	((0,0,1,0.1,0.2;1,1),(0.05,0.1,0.1,0.15;0.9,0.9))
Very low (VL)	((0.1,0.2,0.2,0.35;1,1),(0.15,0.2,0.2,0.3;0.9,0.9))
Low (L)	((0.2,0.35,0.35,0.5;1,1),(0.25,0.35,0.35,0.45;0.9,0.9))
Medium (M)	((0.35,0.5,0.5,0.65;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
High (H)	((0.5,0.65,0.65,0.8;1,1),(0.55,0.65,0.65,0.75;0.9,0.9))
Very high (VH)	((0.65,0.8,0.8,0.9;1,1),(0.7,0.8,0.8,0.85;0.9,0.9))
Very very high (VVH)	((0.8,0.9,0.9,1;1,1),(0.85,0.9,0.9,0.95;0.9,0.9))

Table 2.3 Linguistic variables for rating of alternatives

Linguistic variable	ITFN
Very poor (VP)	((0,0,0,0.1;1,1),(0.05,0,0,0.05;0.9,0.9))
Poor (P)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.25;0.9,0.9))
Medium poor (MP)	((0.1,0.3,0.3,0.5;1,1),(0.15,0.3,0.3,0.45;0.9,0.9))
Fair (F)	((0.3,0.5,0.5,0.7;1,1),(0.35,0.5,0.5,0.65;0.9,0.9))
Medium good (MG)	((0.5,0.7,0.7,0.9;1,1),(0.55,0.7,0.7,0.85;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1),(0.75,0.9,0.9,0.95;0.9,0.9))
Very good (VG)	((0.9,1,1,1;1,1),(0.95,1,1,0.95;0.9,0.9))

Table 2.4 Dependency degrees among main attributes obtained from DM₁

DM ₁	S	W	O	T
S	-	VVL	L	
W	L	-		H
O			-	VL
T		H	M	-

Table 2.5 Dependency degrees among main attributes obtained from DM₂

DM ₂	S	W	O	T
S	-	VL	H	
W	VL	-		M
O			-	VL
T		VH	VH	-

Table 2.6 Dependency degrees among main attributes obtained from DM₃

DM ₃	S	W	O	T
S	-	M	L	
W	VVL	-		H
O			-	M
T		VH	H	-

Table 2.7 Initial direct relation matrix

	S	W	O	T
S	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.15,0.27,0.27,0.4;1,1), (0.2,0.27,0.27,0.35;0.9,0.9))	((0.3,0.45,0.45,0.6;1,1), (0.35,0.45,0.45,0.55;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))
W	((0.1,0.22,0.22,0.35;1,1), (0.15,0.22,0.22,0.3;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.45,0.6,0.6,0.75;1,1), (0.5,0.6,0.6,0.7;0.9,0.9))
O	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.18,0.3,0.3,0.45;1,1), (0.23,0.3,0.3,0.4;0.9,0.9))
T	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.6,0.75,0.75,0.87;1,1), (0.65,0.75,0.75,0.82;0.9,0.9))	((0.5,0.65,0.65,0.78;1,1), (0.55,0.65,0.65,0.73;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))

Table 2.8 Normalized initial direct-relation matrix

	S	W	O	T
S	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.09,0.16,0.16,0.24;1,1), (0.12,0.16,0.16,0.21;0.9,0.9))	((0.18,0.27,0.27,0.36;1,1), (0.21,0.27,0.27,0.33;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))
W	((0.06,0.13,0.13,0.21;1,1), (0.09,0.13,0.13,0.18;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.27,0.36,0.36,0.45;1,1), (0.3,0.36,0.36,0.42;0.9,0.9))
O	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.11,0.18,0.18,0.27;1,1), (0.14,0.18,0.18,0.24;0.9,0.9))
T	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.36,0.45,0.45,0.53;1,1), (0.39,0.45,0.45,0.49;0.9,0.9))	((0.3,0.39,0.39,0.47;1,1), (0.33,0.39,0.39,0.44;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))

Table 2.9 Total relation matrix

	S	W	O	T
S	((0.01,0.03,0.03,0.1;1,1), (0.01,0.03,0.03,0.07;0.9,0.9))	((0.11,0.23,0.23,0.46;1,1), (0.15,0.23,0.23,0.36;0.9,0.9))	((0.2,0.34,0.34,0.57;1,1), (0.24,0.34,0.34,0.47;0.9,0.9))	((0.05,0.15,0.15,0.36;1,1), (0.08,0.15,0.15,0.27;0.9,0.9))
W	((0.07,0.16,0.16,0.32;1,1), (0.11,0.16,0.16,0.25;0.9,0.9))	((0.12,0.25,0.25,0.51;1,1), (0.16,0.25,0.25,0.39;0.9,0.9))	((0.11,0.24,0.24,0.51;1,1), (0.15,0.24,0.24,0.39;0.9,0.9))	((0.32,0.5,0.5,0.83;1,1), (0.37,0.5,0.5,0.69;0.9,0.9))
O	((0.01,0.01,0.01,0.05;1,1), (0.01,0.01,0.01,0.03;0.9,0.9))	((0.05,0.11,0.11,0.25;1,1), (0.07,0.11,0.11,0.19;0.9,0.9))	((0.04,0.1,0.1,0.23;1,1), (0.06,0.1,0.1,0.17;0.9,0.9))	((0.13,0.24,0.24,0.45;1,1), (0.17,0.24,0.24,0.36;0.9,0.9))
T	((0.03,0.08,0.08,0.19;1,1), (0.04,0.08,0.08,0.14;0.9,0.9))	((0.42,0.61,0.61,0.91;1,1), (0.48,0.61,0.61,0.77;0.9,0.9))	((0.35,0.54,0.54,0.85;1,1), (0.41,0.54,0.54,0.71;0.9,0.9))	((0.15,0.32,0.32,0.65;1,1), (0.2,0.32,0.32,0.5;0.9,0.9))

Then, IT2F prominence and relation values for each SWOT factor are calculated. The $\tilde{D} \oplus \tilde{R}$ and $\tilde{D} \ominus \tilde{R}$ values are calculated as given in the Table 2.11.

Step 6: Employing Equation 2.14, expected values of $\tilde{D} \oplus \tilde{R}$ and $\tilde{D} \ominus \tilde{R}$ are calculated.

Table 2.10 $\tilde{D}$ and $\tilde{R}$ values

	$\tilde{D}$	$\tilde{R}$
S	((0.37,0.75,0.75,1.49;1,1), (0.49,0.75,0.75,1.16;0.9,0.9))	((0.1,0.29,0.29,0.66;1,1), (0.17,0.29,0.29,0.49;0.9,0.9))
W	((0.62,1.16,1.16,2.17;1,1), (0.78,1.16,1.16,1.72;0.9,0.9))	((0.7,1.21,1.21,2.13;1,1), (0.86,1.21,1.21,1.71;0.9,0.9))
O	((0.22,0.47,0.47,0.98;1,1), (0.3,0.47,0.47,0.76;0.9,0.9))	((0.7,1.22,1.22,2.16;1,1), (0.86,1.22,1.22,1.75;0.9,0.9))
T	((0.96,1.56,1.56,2.6;1,1), (1.14,1.56,1.56,2.13;0.9,0.9))	((0.65,1.21,1.21,2.29;1,1), (0.83,1.21,1.21,1.82;0.9,0.9))

Based on the crisp $D + R$ and $D - R$ values, Equation 2.26-28 are used to calculate normalized weights for each SWOT factor. Table 2.12 summarizes the crisp prominence and relation values, and the weight of each SWOT factor.

Table 2.11 $\tilde{D} \oplus \tilde{R}$ and $\tilde{D} \ominus \tilde{R}$ values

	$\tilde{D} \oplus \tilde{R}$	$\tilde{D} \ominus \tilde{R}$
S	((0.47,1.04,1.04,2.15;1,1), (0.66,1.04,1.04,1.66;0.9,0.9))	((-0.3,0.46,0.46,1.39;1,1), (0.046,0.46,0.99;0.9,0.9))
W	((1.32,2.37,2.37,4.3;1,1), (1.64,2.37,2.37,3.43;0.9,0.9))	((-1.51,-0.05,-0.05,1.46;1,1), (-0.93,-0.05,-0.05,0.86;0.9,0.9))
O	((0.92,1.69,1.69,3.15;1,1), (1.16,1.69,1.69,2.51;0.9,0.9))	((-1.95,-0.76,-0.76,0.28;1,1), (-1.45,-0.76,-0.76,-0.1;0.9,0.9))
T	((1.61,2.77,2.77,4.89;1,1), (1.96,2.77,2.77,3.94;0.9,0.9))	((-1.33,0.35,0.35,1.95;1,1), (-0.68,0.35,0.35,1.3;0.9,0.9))

Table 2.12 Resulting weights of SWOT factors

	D+R	D-R	Weights	Normalized Weights
S	1.08	0.47	1.18	0.14
W	2.40	-0.04	2.40	0.29
O	1.72	-0.74	1.87	0.23
T	2.79	0.31	2.81	0.34

Figure 2.4 represents the causal diagram of the SWOT factors. The size of each bubble is relative to the normalized weight of each factor. For instance, threats factor has the relative importance of 30%, hence it is represented by the biggest bubble. Weaknesses factor has the relative importance of 29%, opportunities factor has the weight of 23%. The local priority of the strengths factor is found to be 14%. The causal

diagram of the SWOT factors does not only provide the relative importance of each factor, but it also helps identify the cause and effect groups. The cause group factors are known as the influencing factors, whereas the effect group denotes the meaning of the influenced factors (W.-W. Wu, 2012). In order to increase the system performance, one should focus on the cause group factors as they have considerable impact on the other factors. Analyzing the causal diagram of the SWOT factors, strengths and threats belong to cause group, and weaknesses and opportunities are the members of the effect group.

Threats factor has the highest prominence value (2.79) which means that it is the most important factor. Furthermore, it has a quite high relation value (0.31), which indicates that threats factor is one of the most influencing factors. The other cause group factor is strengths. Strengths factor has the greatest relation value (0.47) which implies that strengths factor is not easily influenced by the other factors. The effect group comprises of the weaknesses and opportunities factors. They have relatively low D-R values, which indicates that they are very easily influenced by the other factors.

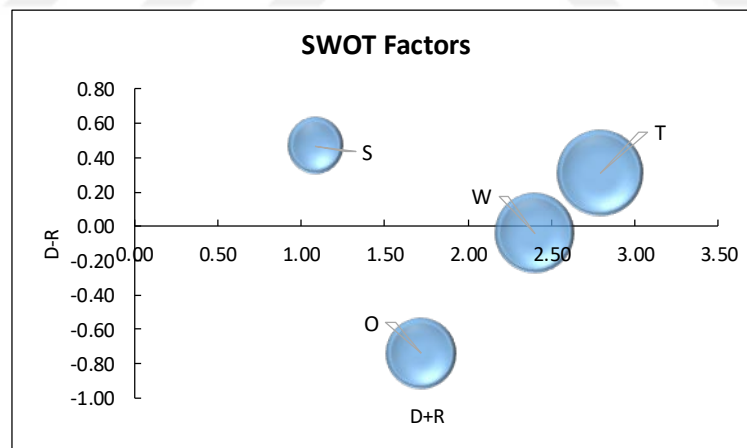


Figure 2.4 Causal diagram of SWOT factors

The performance of the weaknesses and opportunities factors can be easily improved as they are highly dependent on the other factors. On the other hand, the cause group factors are difficult to move (Hori & Shimizu, 1999). The opportunities factor has the lowest relation value, which indicates that its performance is highly dependent on the other factors.

The same calculations are performed for each SWOT sub-factor. Since strengths and threats factors belong to the cause group, their sub-factors are analyzed first. Figure 2.5a represents the causal diagram of the strengths sub-factors. In the strengths sub-factors, appropriate physical environment (S_1), dynamic research environment (S_3), and specialized academic staff (S_4) belong to cause group, while student quality (S_2), brand status (S_5), and working conditions (S_6) are members of the effect group. The most important factor is found to be dynamic research environment with the local priority of 22%. The least important factor is appropriate physical environment with the local priority of 13% due to the fact that it has the least prominence value (0.82). On the other hand, Figure 2.5b shows the causal diagram of threats sub-factors. Increasing number of new universities (T_1), rapid technological change (T_2), and corporate universities (T_5) are the members of the cause group, and e-learning programs (T_3), employer legislations (T_4), and university budget cuts (T_6) belong to effect group. Increasing number of universities has found to be the most important factor with the local priority of 25%. The corporate universities factor ranks second in terms of local priority of 20%. The least important factor is found to be employer legislations with the weight of 8%.

Figure 2.6 shows the causal diagram of the sub-factors weaknesses and opportunities. Because weaknesses and opportunities belong to the effect group, their sub-factors are analyzed separately. In the weaknesses sub-factors, campus location (W_1), traditional undergraduate education (W_3), and arrogance (W_6) belong to the cause group, whereas lack of communication among staff (W_2), library (W_4), and bureaucracy (W_5) belong to the effect group. The lack of communication among staff factor is found to be the most critical with local priority of 26%. Bureaucracy has the local priority of 25%, thereby it is the second most critical factor. The least important factor is found to be library which has the local priority of 4%. Campus location has the weight of 8% due to the relatively low prominence value. Figure 2.6b deals with the causal diagram of the opportunities sub-factors. Government incentives (O_2), demand for re-skilling and training (O_3), and need for interdisciplinary collaboration (O_6) are considered to be cause group, and talented new generation (O_1), assistant recruitments by government (O_4), and entrepreneurial climate (O_5) are the members of

the effect group. Government incentives factor is the most significant factor with the local priority of 27%. The government incentives factor is followed by the entrepreneurial climate and need of interdisciplinary collaboration factors, each one has the priority of 22%. The least significant factor has found to be increasing number of new universities with the local priority of 5%. As a member of the effect group, impact of this factor can be easily diminished by the other factors.

After calculating all factor and sub-factor weights, the overall priorities can be calculated by multiplying factor and sub-factor weights as in the Equation 2.29. For instance, the overall weight of the appropriate physical environment sub-factor is calculated by $w_{s_1} = 0.14 \times 0.13 = 0.018$. The overall weights of the each SWOT sub-factor is given in Table 2.13.

Once the overall sub-factor weights are calculated, the next step is to prioritize the strategy alternatives.

2.4.3 Ranking of alternatives

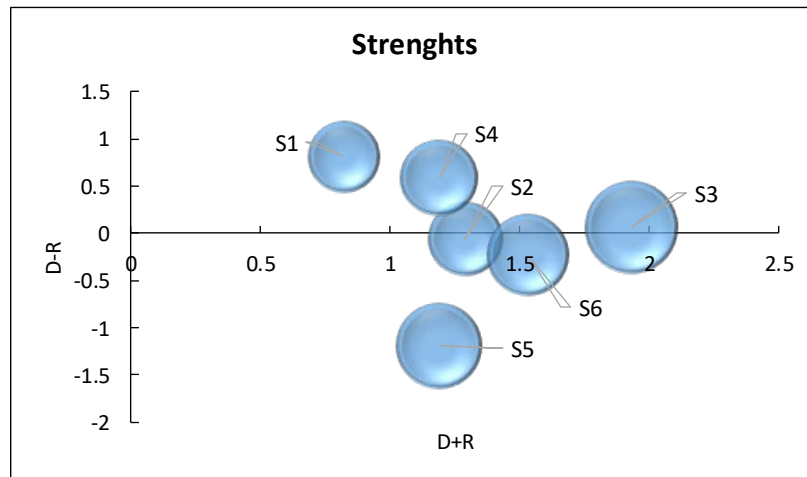
In this section, IT2F-TOPSIS calculations are given in a step by step manner.

Step 1: Decision matrices are collected from the three DMs. The collected data is given in Table 2.14.

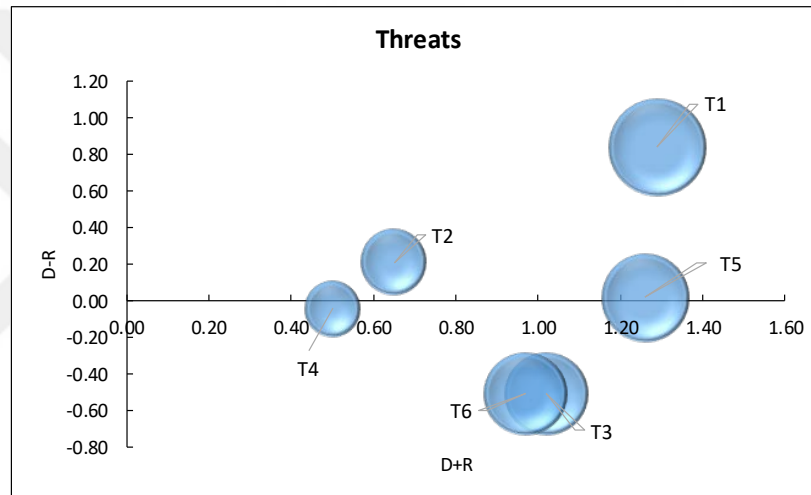
Step 2: The average IT2F decision matrix $\tilde{\bar{Y}}$ is calculated based on the Equation 2.30, which is given in Table 2.15.

Step 3: The overall priorities given in the Table 2.13 are considered as the sub-factor weights in the IT2F-TOPSIS algorithm.

Step 4: Weighted decision matrix is calculated based on the Equation 2.32-33. The weighted decision matrix is given in Table 2.16.



a. Strenghts



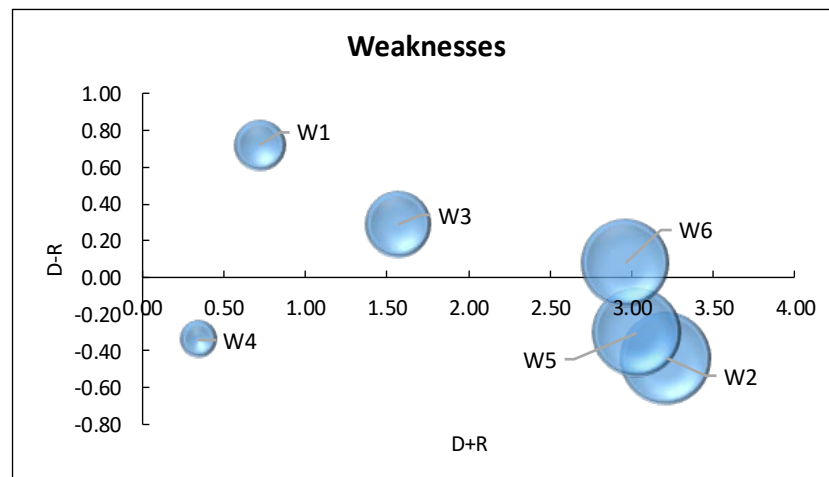
b. Threats

Figure 2.5 Causal diagram of strengths and threats factors

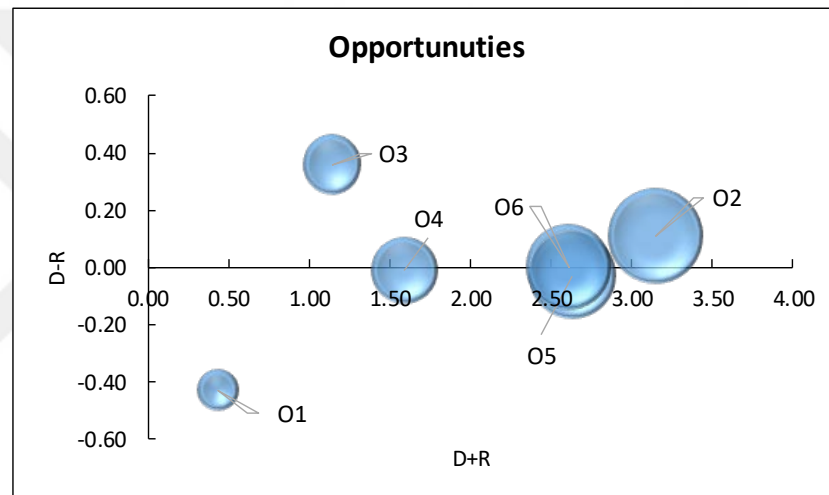
Step 5: The ranking values of the weighted decision matrix are calculated based on the Equation 2.10-13. The ranking values of the weighted decision matrix are given in Table 2.17.

Step 6: Positive and ideal solutions are determined based on the Equation 2.34-35.

Step 7: When the ideal solutions are determined, the distances between alternatives and the ideal solutions are calculated by using the Equation 2.36-37. The distances between alternatives and ideal solutions are given in Table 2.18.



a. Weaknesses



b. Opportunities

Figure 2.6 Causal diagram of weaknesses and opportunities factors

Table 2.13 Overall priorities of SWOT factors

SWOT factors	SWOT weights	SWOT sub-factors	Sub-factor weights	Overall weights
Strengths	0.14	Appropriate physical environment (S_1)	0.13	0.018
		Student quality (S_2)	0.14	0.020
		Dynamic research environment (S_3)	0.22	0.031
		Specialized academic staff (S_4)	0.15	0.021
		Brand status (S_5)	0.19	0.027
		Working conditions (S_6)	0.17	0.024
Weaknesses	0.29	Campus location (W_1)	0.08	0.023

Table 2.13 continues

SWOT factors	SWOT weights	SWOT sub-factors	Sub-factor weights	Overall weights
Opportunities	0.23	Lack of communication among staff (W ₂)	0.26	0.075
		Traditional undergraduate education (W ₃)	0.13	0.038
		Library (W ₄)	0.04	0.012
		Bureaucracy (W ₅)	0.25	0.073
		Arrogance (W ₆)	0.24	0.070
		Talented new generation (O ₁)	0.05	0.012
		Government incentives (O ₂)	0.27	0.062
		Demand for re-skilling and training (O ₃)	0.10	0.023
		Assistant recruitments by government (O ₄)	0.13	0.030
		Entrepreneurial climate (O ₅)	0.22	0.051
		Need of interdisciplinary collaboration (O ₆)	0.22	0.051
Threats	0.34	Increasing number of new universities (T ₁)	0.25	0.085
		Rapid technological change (T ₂)	0.11	0.037
		E-learning programs (T ₃)	0.18	0.061
		Employer legislations (T ₄)	0.08	0.027
		Corporate universities (T ₅)	0.20	0.068
		University budget cuts (T ₆)	0.18	0.061

Table 2.14 Fuzzy rating values of strategy alternatives

	SO			WO			ST			WT		
	DM ₁	DM ₂	DM ₃	DM ₁	DM ₂	DM ₃	DM ₁	DM ₂	DM ₃	DM ₁	DM ₂	DM ₃
S ₁	G	VG	MG	G	G	VG	VG	G	G	P	MP	F
S ₂	VG	G	MG	G	VG	G	G	MG	VG	F	MG	G
S ₃	MG	P	F	P	F	MP	MP	F	F	F	G	MG
S ₄	F	MG	G	P	MG	F	P	MP	F	P	P	F
S ₅	G	F	MP	P	F	MG	MP	F	P	G	VG	F
S ₆	P	VP	G	G	VG	G	G	VG	MG	G	F	F
W ₁	P	MP	P	VG	G	MG	G	MG	F	P	F	VG
W ₂	G	VG	MG	G	G	MG	VG	G	F	P	G	F
W ₃	VG	VG	G	VG	G	F	G	F	MG	P	MP	G
W ₄	F	MP	G	MG	F	G	P	MP	F	P	MP	F
W ₅	F	MP	P	G	VG	F	G	F	VG	P	F	MP
W ₆	G	MG	MG	F	MP	F	MP	P	P	MP	F	MG
O ₁	VG	G	VG	G	P	VP	F	MG	MG	G	MG	F
O ₂	G	MG	F	VG	G	MG	F	MP	F	G	F	MG
O ₃	G	MG	MG	F	F	G	G	G	F	VG	G	F
O ₄	P	MP	F	F	G	MG	MP	MP	P	F	MP	P

Table 2.14 continues

	SO			WO			ST			WT		
O ₅	G	VP	P	P	G	MG	G	G	MG	F	MG	MG
O ₆	VG	G	MG	VG	G	F	G	VG	MG	MG	G	G
T ₁	MG	G	MP	G	VG	G	MP	F	P	F	G	F
T ₂	G	MG	VG	G	VG	G	F	MP	MG	G	F	VP
T ₃	P	MP	F	P	MP	F	VG	G	VG	P	G	G
T ₄	F	MG	F	VG	G	MG	G	F	MG	P	F	VG
T ₅	F	MG	G	G	VG	G	MG	VG	G	VG	G	G
T ₆	P	MP	F	P	VP	MP	MP	VG	G	G	F	MG

Table 2.15 Average IT2F decision matrix

	SO		WO		ST		WT	
S ₁	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))		((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	
S ₂	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))		((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	
S ₃	((0.27,0.43,0.43,0.63;1,1), (0.32,0.43,0.43,0.58;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.23,0.43,0.43,0.63;1,1), (0.28,0.43,0.43,0.58;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	
S ₄	((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.27,0.43,0.43,0.63;1,1), (0.32,0.43,0.43,0.58;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.1,0.23,0.23,0.43;1,1), (0.15,0.23,0.23,0.38;0.9,0.9))	
S ₅	((0.37,0.57,0.57,0.73;1,1), (0.42,0.57,0.57,0.68;0.9,0.9))		((0.27,0.43,0.43,0.63;1,1), (0.32,0.43,0.43,0.58;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))	
S ₆	((0.23,0.33,0.33,0.47;1,1), (0.28,0.33,0.33,0.42;0.9,0.9))		((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))		((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.43,0.63,0.63,0.8;1,1), (0.48,0.63,0.63,0.75;0.9,0.9))	
W ₁	((0.03,0.17,0.17,0.37;1,1), (0.08,0.17,0.17,0.32;0.9,0.9))		((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.4,0.53,0.53,0.67;1,1), (0.45,0.53,0.53,0.62;0.9,0.9))	
W ₂	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.63,0.83,0.83,0.97;1,1), (0.68,0.83,0.83,0.92;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))		((0.33,0.5,0.5,0.67;1,1), (0.38,0.5,0.5,0.62;0.9,0.9))	
W ₃	((0.83,0.97,0.97,1;1,1), (0.88,0.97,0.97,0.95;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.27,0.43,0.43,0.6;1,1), (0.32,0.43,0.43,0.55;0.9,0.9))	
W ₄	((0.37,0.57,0.57,0.73;1,1), (0.42,0.57,0.57,0.68;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	
W ₅	((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	
W ₆	((0.57,0.77,0.77,0.93;1,1), (0.62,0.77,0.77,0.88;0.9,0.9))		((0.23,0.43,0.43,0.63;1,1), (0.28,0.43,0.43,0.58;0.9,0.9))		((0.03,0.17,0.17,0.37;1,1), (0.08,0.17,0.17,0.32;0.9,0.9))		((0.3,0.5,0.5,0.7;1,1), (0.35,0.5,0.5,0.65;0.9,0.9))	
O ₁	((0.83,0.97,0.97,1;1,1), (0.88,0.97,0.97,0.95;0.9,0.9))		((0.23,0.33,0.33,0.47;1,1), (0.28,0.33,0.33,0.42;0.9,0.9))		((0.43,0.63,0.63,0.83;1,1), (0.48,0.63,0.63,0.78;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	
O ₂	((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.23,0.43,0.43,0.63;1,1), (0.28,0.43,0.43,0.58;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	
O ₃	((0.57,0.77,0.77,0.93;1,1), (0.62,0.77,0.77,0.88;0.9,0.9))		((0.43,0.63,0.63,0.8;1,1), (0.48,0.63,0.63,0.75;0.9,0.9))		((0.57,0.77,0.77,0.9;1,1), (0.62,0.77,0.77,0.85;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))	
O ₄	((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))		((0.07,0.23,0.23,0.43;1,1), (0.12,0.23,0.23,0.38;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	
O ₅	((0.23,0.33,0.33,0.47;1,1), (0.28,0.33,0.33,0.42;0.9,0.9))		((0.4,0.57,0.57,0.73;1,1), (0.45,0.57,0.57,0.68;0.9,0.9))		((0.63,0.83,0.83,0.97;1,1), (0.68,0.83,0.83,0.92;0.9,0.9))		((0.43,0.63,0.63,0.83;1,1), (0.48,0.63,0.63,0.78;0.9,0.9))	
O ₆	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.63,0.8,0.8,0.9;1,1), (0.68,0.8,0.8,0.85;0.9,0.9))		((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))		((0.63,0.83,0.83,0.97;1,1), (0.68,0.83,0.83,0.92;0.9,0.9))	
T ₁	((0.43,0.63,0.63,0.8;1,1), (0.48,0.63,0.63,0.75;0.9,0.9))		((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))		((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))		((0.43,0.63,0.63,0.8;1,1), (0.48,0.63,0.63,0.75;0.9,0.9))	

Table 2.15 continues

	SO	WO	ST	WT
T ₂	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))	((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))	((0.3,0.5,0.5,0.7;1,1), (0.35,0.5,0.5,0.65;0.9,0.9))	((0.33,0.47,0.47,0.6;1,1), (0.38,0.47,0.47,0.55;0.9,0.9))
T ₃	((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	((0.83,0.97,0.97,1;1,1), (0.88,0.97,0.97,0.95;0.9,0.9))	((0.47,0.63,0.63,0.77;1,1), (0.52,0.63,0.63,0.72;0.9,0.9))
T ₄	((0.37,0.57,0.57,0.77;1,1), (0.42,0.57,0.57,0.72;0.9,0.9))	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))	((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	((0.4,0.53,0.53,0.67;1,1), (0.45,0.53,0.53,0.62;0.9,0.9))
T ₅	((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))	((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))	((0.7,0.87,0.87,0.97;1,1), (0.75,0.87,0.87,0.92;0.9,0.9))	((0.77,0.93,0.93,1;1,1), (0.82,0.93,0.93,0.95;0.9,0.9))
T ₆	((0.13,0.3,0.3,0.5;1,1), (0.18,0.3,0.3,0.45;0.9,0.9))	((0.03,0.13,0.13,0.3;1,1), (0.08,0.13,0.13,0.25;0.9,0.9))	((0.57,0.73,0.73,0.83;1,1), (0.62,0.73,0.73,0.78;0.9,0.9))	((0.5,0.7,0.7,0.87;1,1), (0.55,0.7,0.7,0.82;0.9,0.9))

Table 2.16 Weighted decision matrix

	SO	WO	ST	WT
S ₁	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.0,0.01,0.01,0.01;0.9,0.9))
S ₂	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.02,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))
S ₃	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.0,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.02,0.02,0.02,0.03;1,1), (0.02,0.02,0.02,0.03;0.9,0.9))
S ₄	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.01,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.0,0.01,0.01,0.01;0.9,0.9))	((0.0,0.0,0.01;1,1), (0.0,0.0,0.01;0.9,0.9))
S ₅	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.0,0.01,0.01,0.01;0.9,0.9))	((0.02,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))
S ₆	((0.01,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.02,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.02,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))
W ₁	((0.0,0.0,0.01;1,1), (0.0,0.0,0.01;0.9,0.9))	((0.02,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))
W ₂	((0.05,0.07,0.07,0.07;1,1), (0.06,0.07,0.07,0.07;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.05,0.06,0.06,0.07;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.05,0.06,0.06,0.06;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))
W ₃	((0.03,0.04,0.04,0.04;1,1), (0.03,0.04,0.04,0.04;0.9,0.9))	((0.02,0.03,0.03,0.03;1,1), (0.03,0.03,0.03,0.03;0.9,0.9))	((0.02,0.03,0.03,0.03;1,1), (0.02,0.03,0.03,0.03;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))
W ₄	((0.0,0.01,0.01,0.01;1,1), (0.0,0.01,0.01,0.01;0.9,0.9))	((0.01,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.0,0.0,0.01;1,1), (0.0,0.0,0.01;0.9,0.9))	((0.0,0.0,0.01;1,1), (0.0,0.0,0.01;0.9,0.9))
W ₅	((0.01,0.02,0.02,0.04;1,1), (0.01,0.02,0.02,0.03;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.05,0.06,0.06,0.06;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.05,0.06,0.06,0.06;0.9,0.9))	((0.01,0.02,0.02,0.04;1,1), (0.01,0.02,0.02,0.03;0.9,0.9))
W ₆	((0.04,0.05,0.05,0.06;1,1), (0.04,0.05,0.05,0.06;0.9,0.9))	((0.02,0.03,0.03,0.04;1,1), (0.02,0.03,0.03,0.04;0.9,0.9))	((0.0,0.01,0.01,0.03;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.02,0.03,0.03,0.05;1,1), (0.02,0.03,0.03,0.05;0.9,0.9))
O ₁	((0.01,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.0,0.0,0.01;1,1), (0.0,0.0,0.0;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.01,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))
O ₂	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))	((0.04,0.05,0.05,0.06;1,1), (0.05,0.05,0.05,0.06;0.9,0.9))	((0.01,0.03,0.03,0.04;1,1), (0.02,0.03,0.03,0.04;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))
O ₃	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))
O ₄	((0.0,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))	((0.01,0.02,0.02,0.03;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.0,0.01,0.01,0.01;0.9,0.9))	((0.0,0.01,0.01,0.01;1,1), (0.01,0.01,0.01,0.01;0.9,0.9))
O ₅	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.02,0.03,0.03,0.04;1,1), (0.02,0.03,0.03,0.03;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))	((0.02,0.03,0.03,0.04;1,1), (0.02,0.03,0.03,0.04;0.9,0.9))
O ₆	((0.04,0.04,0.04,0.05;1,1), (0.04,0.04,0.04,0.05;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.04;0.9,0.9))	((0.04,0.04,0.04,0.05;1,1), (0.04,0.04,0.04,0.05;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))
T ₁	((0.04,0.05,0.05,0.07;1,1), (0.04,0.05,0.05,0.06;0.9,0.9))	((0.07,0.08,0.08,0.09;1,1), (0.07,0.08,0.08,0.08;0.9,0.9))	((0.01,0.03,0.03,0.04;1,1), (0.02,0.03,0.03,0.04;0.9,0.9))	((0.04,0.05,0.05,0.07;1,1), (0.04,0.05,0.05,0.06;0.9,0.9))

Table 2.16 continues

	SO	WO	ST	WT
T ₂	((0.03,0.03,0.03,0.04;1,1), (0.03,0.03,0.03,0.03;0.9,0.9))	((0.03,0.03,0.03,0.04;1,1), (0.03,0.03,0.03,0.04;0.9,0.9))	((0.01,0.02,0.02,0.03;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))
T ₃	((0.01,0.02,0.02,0.03;1,1), (0.01,0.02,0.02,0.03;0.9,0.9))	((0.01,0.02,0.02,0.03;1,1), (0.01,0.02,0.02,0.03;0.9,0.9))	((0.05,0.06,0.06,0.06;1,1), (0.05,0.06,0.06,0.06;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.04;0.9,0.9))
T ₄	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.02,0.02,0.02,0.03;1,1), (0.02,0.02,0.02,0.02;0.9,0.9))	((0.01,0.02,0.02,0.02;1,1), (0.01,0.02,0.02,0.02;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))
T ₅	((0.03,0.05,0.05,0.06;1,1), (0.04,0.05,0.05,0.06;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.06,0.06,0.06,0.06;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.05,0.06,0.06,0.06;0.9,0.9))	((0.05,0.06,0.06,0.07;1,1), (0.06,0.06,0.06,0.06;0.9,0.9))
T ₆	((0.01,0.02,0.02,0.03;1,1), (0.01,0.02,0.02,0.03;0.9,0.9))	((0.01,0.01,0.01,0.02;1,1), (0.01,0.01,0.01,0.02;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.04,0.04,0.04,0.05;0.9,0.9))	((0.03,0.04,0.04,0.05;1,1), (0.03,0.04,0.04,0.05;0.9,0.9))

Table 2.17 Ranking values of the weighted decision matrix

	SO	WO	ST	WT
S ₁	3.89	3.9	3.9	3.83
S ₂	3.9	3.91	3.9	3.88
S ₃	3.88	3.85	3.88	3.92
S ₄	3.88	3.85	3.84	3.83
S ₅	3.89	3.87	3.85	3.92
S ₆	3.85	3.93	3.92	3.89
W ₁	3.82	3.92	3.89	3.87
W ₂	4.18	4.16	4.15	4.02
W ₃	4.01	3.97	3.95	3.89
W ₄	3.84	3.85	3.82	3.82
W ₅	3.92	4.14	4.14	3.92
W ₆	4.11	3.97	3.87	4
O ₁	3.86	3.82	3.84	3.85
O ₂	4.05	4.11	3.95	4.05
O ₃	3.9	3.88	3.9	3.91
O ₄	3.85	3.92	3.84	3.85
O ₅	3.9	3.97	4.04	3.98
O ₆	4.06	4.03	4.06	4.04
T ₁	4.11	4.26	3.94	4.11
T ₂	3.99	4	3.91	3.9
T ₃	3.9	3.9	4.15	4.02
T ₄	3.89	3.94	3.91	3.88
T ₅	4.07	4.17	4.14	4.17
T ₆	3.9	3.85	4.06	4.05

Step 8: The closeness coefficients of alternatives are calculated based on the Equation 2.38. The closeness coefficients are given in Table 2.19.

Step 9: The alternatives are ranked based on their closeness coefficients. Sorting the closeness coefficients in a descending order, final ranking of alternative strategies is obtained as:

Table 2.18 Distances of each alternative from ideal solutions

	d_i^+	d_i^-
SO	0.46	0.40
WO	0.38	0.52
ST	0.47	0.46
WT	0.42	0.38

Table 2.19 Closeness coefficients of alternatives

	CC_i
SO	0.46
WO	0.58
ST	0.50
WT	0.47

WO $\succ$ ST $\succ$ WT $\succ$ SO.

2.5 Validation and Discussion

In this section, the results of the proposed method are validated and tested. Although the case study has been adopted from (Baykasoğlu & Gölcük, 2015a), comparing the proposed method with the original work might yield erroneous results due to following reasons. First, the original work utilizes type-1 fuzzy (T1F) sets, while the proposed method utilizes different linguistic scale in the form of IT2FSs. Secondly, the original work collects relative importance of attributes and the degree of interdependencies spontaneously. However, the proposed method directly derives relative importance of attributes from the total relation matrix. Lastly, the original work utilizes fuzzy cognitive maps along with the hyperbolic tangent function in order to capture dynamic behavior. This means that the original work and proposed method have different purposes.

Because of the above mentioned reasons, three validation models are constructed and the results are compared with the proposed method. Changes in the overall weights of the attributes and final rankings are evaluated and discussed. The validation models and their descriptions are as follows:

- Model 1: As mentioned earlier, the case study has been adopted from (Baykasoğlu & Gölcük, 2015a). Because T1F sets are used in that study, a different linguistic scale (in the form of IT2FS) has been built and used in this paper. However, it is crucial to test the robustness of the proposed method against different linguistic scales. For that aim, based on the IT2FS representation method, the original T1F linguistic terms are adopted for IT2FSs as shown in Tables 2.20-21. Then, the proposed method is applied and the final rankings are obtained.
- Model 2: As there are many successful applications of T1F-DEMATEL and T1F-TOPSIS methods, the proposed method is compared with the T1F counterparts herein. Karnik-Mendel (KM) algorithm (Karnik & Mendel, 2001; Mendel, 2001) is carried out and the problem data is reduced to T1FSs. Then, T1F-DEMATEL (Lin & Wu, 2008) and T1F-TOPSIS (Chen, 2000) are implemented and the results are obtained.

Table 2.20 Linguistic variables for causal relationships in Model 1

Linguistic variable	ITFN
Very very low (VVL)	((0,0.1,0.1,0.2;1,1), (0,0.1,0.1,0.2;1,1))
Very low (VL)	((0.1,0.2,0.2,0.35;1,1), (0.1,0.2,0.2,0.35;1,1))
Low (L)	((0.2,0.35,0.35,0.5;1,1), (0.2,0.35,0.35,0.5;1,1))
Medium (M)	((0.35,0.5,0.5,0.65;1,1), (0.35,0.5,0.5,0.65;1,1))
High (H)	((0.5,0.65,0.65,0.8;1,1), (0.5,0.65,0.65,0.8;1,1))
Very high (VH)	((0.65,0.8,0.8,0.9;1,1), (0.65,0.8,0.8,0.9;1,1))
Very very high (VVH)	((0.8,0.9,0.9,1;1,1), (0.8,0.9,0.9,1;1,1))

Table 2.21 Linguistic variables for rating of alternatives in Model 1

Linguistic variable	ITFN
Very poor (VP)	((0,0,0,0.1;1,1), (0,0,0,0.1;1,1))
Poor (P)	((0,0.1,0.1,0.3;1,1), (0,0.1,0.1,0.3;1,1))
Medium poor (MP)	((0.1,0.3,0.3,0.5;1,1), (0.1,0.3,0.3,0.5;1,1))
Fair (F)	((0.3,0.5,0.5,0.7;1,1), (0.3,0.5,0.5,0.7;1,1))
Medium good (MG)	((0.5,0.7,0.7,0.9;1,1), (0.5,0.7,0.7,0.9;1,1))
Good (G)	((0.7,0.9,0.9,1;1,1), (0.7,0.9,0.9,1;1,1))
Very good (VG)	((0.9,1,1,1;1,1), (0.9,1,1,1;1,1))

- Model 3: In this case, the problem data is directly defuzzified. Then, DEMATEL and TOPSIS methods are implemented by using the crisp judgments. Distributive normalization and absolute ideal points are used to calculate final priorities in the crisp TOPSIS algorithm. For the purpose of defuzzification, centroid of a T2FSs (Karnik & Mendel, 2001) is utilized, and the results are verified by using

defuzzification based on ranking value (Lee & Chen, 2008) and Kahraman's approach (Kahraman, Öztayşi, Sarı, & Turanoğlu, 2014).

The most significant attributes and their overall weights are depicted in Figure 2.7.

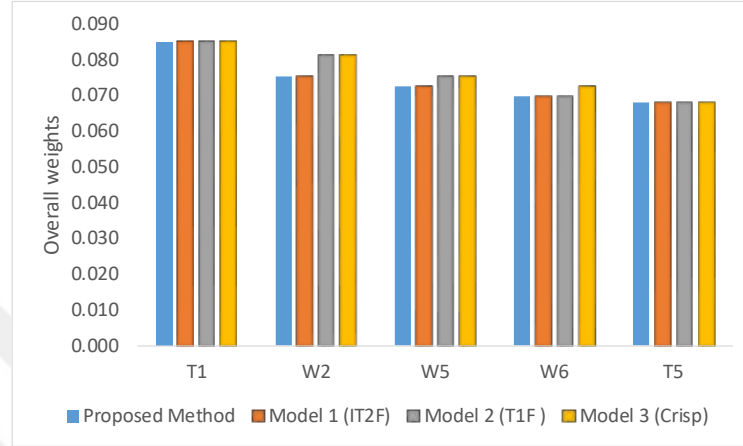


Figure 2.7 Overall weights in the verification models

Analysis of the changes in the overall attribute weights has revealed that increasing number of new universities (T₁), Lack of communication among staff (W₂), Bureaucracy (W₅), Arrogance (W₆), and Corporate universities (T₅) attributes have found to be the most important attributes in all of the verification models. In model 1, the overall attribute weights remain unchanged. In model 2, overall weights of the W₂ and W₅ are increased by 0.006 and 0.003, respectively. In model 3, overall weights of the W₂, W₅, W₆ are changed by 0.006, 0.003, 0.003, respectively. It has been observed that the ranking orders of the overall attribute weights remain the same for the top 12 attributes out of 24 attributes. The average amount of absolute change in the overall attribute weights is detected as 0.001.

The resulting ranking orders of the alternatives are given in the Table 2.22.

Analysis of the final rankings shows that the proposed method is robust enough to changes in the linguistic scale. In Model 1, the final ranking of the alternatives and the overall weights of the attributes remain the same as in the proposed method. The results of the Model 2 and Model 3 are quite consistent with the proposed method. The

only change in the ranking orders of the validation models is that of between SO and WT. In order to further analyze the produced results, normalized closeness coefficients are visualized as in the Figure 2.8.

Table 2.22 Comparison of the validation models

Model	Ranking orders	Goal	Validation Mechanism
Proposed method	$WO \succ ST \succ WT \succ SO$	-	The proposed method is implemented as described in this paper.
Model 1	$WO \succ ST \succ WT \succ SO$	Testing robustness of the proposed method against different linguistic scales	The proposed method is implemented by using IT2F representation of the linguistic scale used in the paper (Baykasoğlu & Gölcük, 2015a)
Model 2	$WO \succ ST \succ SO \succ WT$	Comparing with the well-established T1F-DEMATEL and T1F-TOPSIS methods	Decision makers' IT2F judgements are transformed into T1F values by using Karnik-Mendel (KM) type reduction algorithm. Then, T1F-DEMATEL and T1F-TOPSIS algorithms are applied.
Model 3	$WO \succ ST \succ SO \succ WT$	Comparing with crisp DEMATEL and crisp TOPSIS methods	Decision makers' IT2F judgements are directly defuzzified to crisp values. Three different defuzzification methods are used to validate the results.

It has been observed that normalized coefficients of the proposed method and the validation models are quite compatible. WO strategy has the highest closeness coefficient score in all of the models. Moreover, ST strategy is distinguished easily and ranks second. In model 2 and model 3, preference orders between SO and WT strategy differ from the proposed method. Actually, there is a very small difference in closeness coefficients of the SO and WT strategies.

The difference between the closeness coefficients of the SO and WT strategies can be distinguished in three decimal points. For instance, the difference between closeness coefficients of the SO and WT strategies is only 0.003. In other words, final

decision regarding SO and WT strategies requires expert intervention as the closeness coefficients are not distinctive enough.

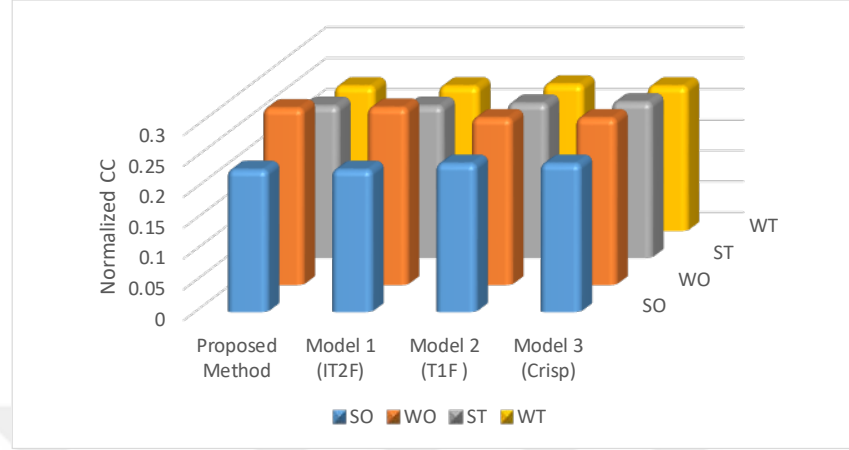


Figure 2.8 Normalized closeness coefficients of the validation models

2.6 Chapter Conclusions

In this study, a new hybrid MADM method is proposed by combining IT2F-DEMATEL and IT2F-TOPSIS methods. The proposed model is able to manage interactions among hierarchically structured criteria. Computational overload has been overcome by utilizing TOPSIS algorithm which increases practicality and applicability of the proposed method.

The main advantages of the proposed method can be counted as follows. In classical MADM methods, attributes are assumed to be preferentially independent of each other. However, there are various interdependencies and interactions among criteria. The proposed method is able to model causal dependencies among attributes by using IT2F-DEMATEL method. Moreover, the proposed method can deal with the hierarchically structured attributes. Hierarchy facilitates problem structuring and information acquisition from the decision makers. As the words mean different things to different people, it is more appropriate to use IT2FSs for modeling linguistic judgements of the decision makers. Therefore, proposed method can deal with hierarchy, imprecision, and causal dependencies.

Even though the computational cost of using IT2FSs can be seen as a disadvantage, we have observed that the proposed method can be implemented very practically. If the decision makers are elucidated about IT2FSs, the proposed method can be commonly used in real life problems. Main limitation of the proposed method is the lack of user friendly software. Also, it is assumed that the problem is hierarchically structured by the decision makers. Another limitation of the proposed method is that the decision makers are assumed to be informed about the IT2FSs and related topics.

Future studies can extend the proposed model in the following directions:

- Uncertainty modeling: Different variants of fuzzy sets (intuitionistic fuzzy sets, hesitant fuzzy sets, and etc.) can be used and the results can be compared.
- Algorithm improvement: Other ranking algorithms can be used instead of TOPSIS. For instance, IT2F-VIKOR, IT2F-ELECTRE, IT2F-PROMETHEE can be employed.
- Decision model improvement: Different attributes can be incorporated into the model. Furthermore, the proposed method can be used for wide variety of problems.
- Software with a graphical user interface (GUI) can be developed in order to easily implement the proposed method.

The proposed method addressed in the present paper can be applied in different problem domains, where the decision makers' perceptions have critical role on the final outcome.

CHAPTER THREE

DEVELOPMENT OF ALPHA-CUT BASED METHODS FOR FUZZY COGNITIVE MAPS WITH APPLICATIONS IN CRITERIA WEIGHTING

3.1 Introduction

Knowledge representation and modeling has been one of the most actively researched areas over the past decades. The cognitive maps, which were introduced by the political scientist Axelrod (1976) in the mid-seventies, can be regarded as a notable step in modeling social scientific knowledge. In the Axelrod's cognitive maps, nodes are variable concepts and directed edges are the causal influences between concepts. The causal connections can be defined as positive, negative or neutral. A serious shortcoming of the cognitive maps is their inability to come to grips with the issue of uncertainty and imprecision.

Kosko (1986) realized insufficiency of the conventional cognitive maps that they cannot provide adequate modes of reasoning. As most modes of human and commonsense reasoning is approximate, there is a need for a conceptual framework for dealing with the knowledge representation under uncertain environments. Ten years after the inception of cognitive maps, Kosko (1986) proposed fuzzy cognitive maps (FCMs). FCMs are graph-oriented fuzzy models describing causal relationships among system variables. The system variables are represented by concepts and the relationships between concepts are shown by directed arcs. Unlike the cognitive maps, concept values and causal relationships are described by linguistic labels in the form of fuzzy sets. The graph structure allows systematic causal propagation and inference (Kosko, 1986).

FCMs have enjoyed a visible position in the literature that wide variety of applications of FCMs have been reported in the fields of control, decision support, robotics, information technology modeling and etc. (Papageorgiou & Salmeron, 2013). FCMs are important decision support tools applied in vast array of problem domains such as reverse logistics performance evaluation (Trappey, Trappey, & Wu, 2010),

development of a knowledge-based system for emergency management (De Maio, Fenza, Gaeta, Loia, & Orciuoli, 2011), e-business maturity modeling (George Xirogiannis & Glykas, 2007), effect-based operations for military planning (Yaman & Polat, 2009), failure modes and effect analysis (Baykasoğlu & Gölcük, 2017a; Peláez & Bowles, 1996), criteria interaction modeling in multiple attribute decision making (MADM) problems (Baykasoğlu & Gölcük, 2015a; Baykasoğlu, Gölcük, & Akyol, 2017; Gölcük & Baykasoğlu, 2016), and etc. In our regard, FCMs have much to offer in MADM problems, and relevance and rationale of using FCMs within MADM approaches have been extensively discussed in (Baykasoğlu & Gölcük, 2015a).

Despite growing interest to FCMs, the salient deficiency of the current FCM applications is that collected fuzzy information is almost always defuzzified before inference computations. More importantly, the original FCM inference is compatible with the singleton fuzzy numbers in which iterative standard algebraic operations are performed. This has been a subject of severe criticism in the literature (Carvalho, 2013; Dodurka, Yesil, & Urbas, 2016; Dodurka, Yesil, & Urbas, 2017; Yesil, Dodurka, & Urbas, 2014). For instance, Carvalho (2013) states that “FCMs are not fuzzy” and the term “fuzzy” is not appropriate for describing FCMs. FCMs can be regarded as “weighted cognitive maps” as an extension of the Axelrod’s cognitive maps. Further discussions and details regarding uncertainty handling in FCMs are elaborated from the fuzziness and non-specificity perspectives in Section 3.3. Apart from the discussions whether FCMs are fuzzy or not, there has been a growing interest in granular computing (Pedrycz, 2013; Pedrycz & Homenda, 2014) which has potential to provide with the badly needed formalizations and computing frameworks in representation and processing of uncertain information. In parallel with the granular computing paradigm, several extensions of FCMs are reported in the literature by using interval numbers (Pedrycz & Homenda, 2014; Petalas, Papageorgiou, Parsopoulos, Groumpos, & Vrahatis, 2005), type-1 (T1) fuzzy sets (Pedrycz & Homenda, 2014; Yesil et al., 2014), interval-valued fuzzy sets (Hajek & Prochazka, 2016), intuitionistic fuzzy sets (Hajek & Prochazka, 2018; Iakovidis & Papageorgiou, 2011; Papageorgiou & Iakovidis, 2013), and etc. However, certain insufficiencies are

associated with the developed methods, viz. the curse of defuzzification, which will be given and discussed in the Section 3.3.

This study attempts to propose alpha-cut based computation methods for simulating T1 and interval type-2 (IT2) FCMs without any loss of information. The proposed method overcomes the limitations of the previous studies by admitting T1/IT2 fuzzy number representation in FCMs by retaining the theoretical underpinnings of the classical FCMs without any modification. The present paper contributes to the literature in the following ways:

- Alpha-cut based inference method is proposed for T1 FCMs. Both of the concept values and connection matrix are allowed to be T1 fuzzy numbers. The proposed method constitutes of deconstruction, inference and reconstruction phases. Because defuzzification is not used, information loss is avoided. The proposed method is verified by using the extension-principle based method (Pedrycz & Homenda, 2014; Yesil et al., 2014).
- Alpha-cut based inference method is extended to IT2 FCMs. Here, both of the concept values and connection matrix are allowed to be defined by IT2 fuzzy sets. Therefore, IT2 concept values are produced as a result of FCM inference.
- Fully capitalizing the recent advances in the convergence properties of sigmoidal FCMs, an objective weighting method is proposed for MADM problems. The cause and effect relationships among decision criteria are transformed into fuzzy weights. To the best of our knowledge, the proposed approach is unique in dealing with both positive and negative relationships among criteria and producing objective weights as a result of dynamic analysis.
- The proposed methods are realized in the real-life third party logistic (3PL) service provider selection problem.

The paper is organized as follows: brief overview of the FCMs is given in Section 3.2. Recent developments, literature analysis, and discussions related to uncertainty propagation in FCMs are given in Section 3.3. Section 3.4 is dedicated to developed alpha-cut based T1 FCMs. Also details of the extension principle based computations

are provided. Developed alpha-cut based IT2 FCMs are given in Section 3.5. Extensive computational study is provided in Section 3.6. In Section 3.7, application of the developed method in objective weighting of criteria is given. Also, a real-life case study of 3PL service provider selection problem is solved. Finally, concluding remarks are given in Section 3.8.

3.2 Fuzzy Cognitive Maps

The mathematical modeling of human cognitive processes has been one of the primary concerns of the artificial and computational intelligence research. Cognitive maps, which were developed in the mid-1970s by Axelrod (1976) for knowledge representation of the political elites, can be considered as one of the initial steps toward this direction. Despite many successful applications of cognitive maps, they are inadequate to model and describe highly complex systems. Almost a decade later, FCMs were proposed by Kosko (1986) to cope with uncertain and imprecise knowledge.

FCMs ground on the cognitive mapping with the advantage that the relationships among system variables are not only described by sign, but strengths of influences are also taken into consideration by means of fuzzy sets. For instance, linguistic variables such as low, medium, high, etc. are exercised in order to quantify the degree of causal relationships. Another key advantage of FCMs is that FCMs exhibit desirable properties of artificial neural networks and fuzzy logic in a synergetic way. Also, neurons and connections do not have specific meanings in the neural networks; conversely, concepts and connections in FCMs have particular interpretations relying on the context of the study (Tsadiras, 2008).

The causal relationships among system variables are represented by directed arcs in the FCMs. Three possible relationships can be defined as: positive causality ($e_{ij} > 0$), negative causality ($e_{ij} < 0$), and no causality ($e_{ij} = 0$). The e_{ij} values denote the strength of influence exerted from C_i to C_j (Kosko, 1986). These causal dependencies are located in the connection (adjacency) matrix and they take the values between $[-1, +1]$.

The concept values are updated at each iteration by using the Equation 3.1 (Xirogiannis, Stefanou, & Glykas, 2004):

$$C_i^{t+1} = f \left(C_i^t + \sum_{\substack{j=1 \\ j \neq i}}^n e_{ji} C_j^t \right) \quad (3.1)$$

where C_i^t is the value of concept C_i at time t , C_i^{t+1} represent the value of concept C_i at time $t+1$, and e_{ji} is the weight of the relationship exerted from concept C_j to concept C_i . The symbol f is used to denote the transformation function. . The most commonly used transformation functions are sigmoid and hyperbolic tangent functions (Tsadiras, 2008). The inference mechanism is illustrated in Figure 3.1.

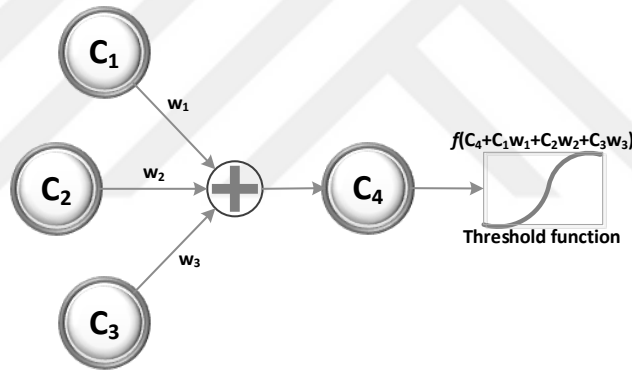


Figure 3.1 Inference mechanism in FCMs

An FCM is expected to converge one of the states: a fixed point, a limit cycle, or a chaotic behavior (Kosko, 1997). The concept vector remains the same for successive iterations in the fixed point state. In a limit cycle, concept vector keeps repeating indefinitely. In chaotic behavior, the concept vector keeps changing and exhibits a chaotic behavior.

Dynamic analysis is the most widely used technique in the FCM applications. FCM models provides with a precise way of formalization of concepts and reasoning about them with a required level of abstraction. When the relevant factors are expressed by means of concepts, what-if scenario analysis are used to simulate how the input factors

affect the target variables. In the vast majority of the literature, scenarios are generated with different concept values, and system response is observed as a result of FCM simulation. A good compilation of dynamic analysis applications can be found in (Papageorgiou & Salmeron, 2013). As the dynamic analysis is one of the most common application areas of FCMs, the proposed methods within the scope of this study are tested on several FCM configurations in the form of what-if scenario analysis.

3.3 Overview of the Recent Developments

Vast amount of effort is dedicated to development of granular cognitive maps recently (Pedrycz & Homenda, 2014). In granular cognitive maps, concept values and/or connection matrix take more general and abstract forms. For that purpose, interval and fuzzy number representations are used within FCMs. In this section, various extensions of FCMs are reviewed and shortcomings of each method are analyzed.

3.3.1 Literature Review

One of the most commonly used granulation mechanisms is to employ interval numbers in concept values and connection matrix. Osei-Bryson (2004) proposed an interval pairwise comparison technique for generating strength of causal relationships among concepts in FCMs. Outer interval, inner interval, and hybrid interval methods were compared in order to capture magnitudes of causal relationships. Petalas et al. (2005) proposed interval cognitive maps where concept and connection matrix values are interval numbers. The proposed method was applied on a medical decision problem for radiation therapy. Papageorgiou, Stylios, and Groumpos (2006) investigated different interval number representations in fuzzy cognitive maps. First, concept values were regarded as interval numbers and connection matrix was taken as crisp. In the second analysis, interval values for connection matrix and crisp values for concepts were considered. Then, both of the concept values and connection matrix were considered as interval numbers and the method is called “interval FCMs” instead of

“interval cognitive maps”. Kang, Mo, Sadiq, and Deng (2016) proposed a method called “generalized FCMs” by incorporating interval numbers in FCMs representation. In the study, different FCMs obtained from experts were aggregated by using genetic algorithm. Hajek and Prochazka (2016) introduced interval valued FCMs by using arithmetic operations defined for interval valued fuzzy sets (Deschrijver, 2007). The proposed method is illustrated in supplier selection and business performance modeling decisions.

Similar to the interval FCMs, Salmeron (2010) developed fuzzy grey cognitive maps in order to deal with grey uncertainty. Combining grey system theory and FCMs, concept values and connection matrix were defined as grey numbers in fuzzy grey cognitive maps. Information technology project risks were modelled in order to illustrate the proposed method. Salmeron and Papageorgiou (2012) used fuzzy grey cognitive maps in radiotherapy treatment planning selection. An improved construction process of fuzzy grey cognitive maps was also presented. Salmeron (2016) proposed an intelligent security system based on fuzzy grey cognitive maps. The structure of the fuzzy grey cognitive maps was improved based on nonlinear Hebbian learning. Papageorgiou and Salmeron (2012) implemented nonlinear Hebbian-based approach in order to determine the success of radiation therapy process that estimate the final dose delivered to the target volume. Froelich and Salmeron (2014) used fuzzy grey cognitive maps and evolutionary computation for predicting multivariate interval-valued time series forecasting.

Another extension of FCMs is to integrate intuitionistic fuzzy sets. Iakovidis and Papageorgiou (2011) proposed a novel approach called intuitionistic FCMs. Intuitionistic FCMs consider expert’s hesitancy in determination of the causal relationships between concepts. The proposed method was applied in a pneumonia severity assessment problem from the medical decision making domain. Papageorgiou and Iakovidis (2013) further extended the intuitionistic FCMs by enabling intuitionistic estimation of hesitancy at the output concepts. The proposed method exploits the mathematical operations of the intuitionistic fuzzy sets for reasoning, viz. intuitionistic fuzzy summation, intuitionistic fuzzy multiplication. Hajek and

Prochazka (2018) integrated interval-valued intuitionistic fuzzy sets with FCMs. In the inference equation, interval-valued intuitionistic fuzzy arithmetic was used. Supplier selection problem was used to illustrate the method.

Pedrycz and Homenda (2014) provided a comprehensive discussion on granular FCMs along with computation details. In the study, FCMs were generalized by admitting intervals and fuzzy sets to form causal dependencies linking the nodes of the map. In the interval granulation, interval arithmetic was employed to iterate FCM steps. In the fuzzy sets-based granulation, extension principle was carried out in order to calculate the output concepts in each iteration. In the study, calculation of an output concept after employing extension principle was shown. In these equations, concept values were assumed to be crisp. Algebraic equations for membership function calculations pertaining to triangular and parabolic fuzzy sets were provided. Yesil et al. (2014) extended FCMs by using triangular fuzzy numbers in forming connection matrix of the FCMs. In the study, extension principle was used as the main computational vehicle in calculating output concepts. When implementing extension principle, concept values become fuzzy sets after one iteration of the FCM inference algorithm, which brings about some computational difficulties. For that reason, concept values were defuzzified at the beginning of each iteration. In the inference algorithm, crisp concept values and fuzzy causal relationships were used. Four simulations were conducted and results were discussed. Dodurka, Sahin, Yesil, and Urbas (2015) proposed a learning method for FCMs by using Big Bang Big Crunch optimization algorithm. In the study triangular fuzzy numbers were used in the connection matrix of the FCM. Dodurka et al. (2016) discussed ambiguity and fuzziness representations in different granular FCMs. In the study, singleton, uniform and triangular fuzzy numbers were compared. Triangular fuzzy number representation was shown to be capable of modeling both ambiguity and fuzziness. Dodurka et al. (2017) provided static analysis of FCMs when the causal influences were represented by non-singleton fuzzy numbers. Experimental study on modeling ERP maintenance risks was presented.

Rickard, Aisbett, and Yager (2015a) developed an FCM structure based on computing with words called CWW FCMs. In this method, weighted power mean operator (Rickard, Aisbett, & Yager, 2015b) was extended to the domain where the FCM concepts and connection matrix were allowed to be IT2 fuzzy sets. The activation functions in traditional FCMs were replaced by linguistic weighted power mean operator (Rickard, Aisbett, Yager, & Gibbon, 2011). At each iteration, output concepts were mapped back into one of the predefined words based on similarity measure. The most similar word was selected as a surrogate for the concept value, thus only finite number of word combinations were produced. The proposed method was compared with the weighted power mean operator based FCMs. Najafi, Amirkhani, Papageorgiou, and Mosavi (2017) proposed a new application of CWW FCMs in medical decision making domain. In the study, one of the chronic diseases, celiac disease classification was studied based on the real dataset obtained from patients.

3.3.2 What Is Lacking in the Literature?

In the original cognitive maps, values of the concepts and connection matrices are regarded as negative, positive, or neutral. Kosko (1986) extended the previous work of Axelrod by introducing fuzzy causalities in the directed edges and concepts. Linguistic variables such as low, high, medium, and etc. are used to describe magnitude of influence among concepts. Although linguistic variables are modeled as fuzzy sets, they are defuzzified in order to calculate output concepts in FCM inference. In other words, FCMs do not admit the fuzzy character of their parameters along the computational steps, and thus precise numeric model is used in simulation and inference. It is needless to say that FCMs are not fuzzy as a result of defuzzification. This assertion has already been advocated by some researchers (Carvalho, 2010, 2013; Carvalho & Tome, 1999; Dodurka et al., 2015; Dodurka et al., 2016; Dodurka et al., 2017; Słóń & Yastrebov, 2017; Yesil et al., 2014).

Carvalho (2013) stated that “ the name FCM is definitely not the most appropriate since FCMs are not fuzzy and not adequate to model the dynamics of qualitative social cognitive maps as originally introduced by Axelrod...”. Also, “dynamic neural maps”,

“dynamic concept maps”, and “dynamic cognitive networks” are suggested instead of the name FCMs (Carvalho, 2010, 2013). Słóń and Yastrebov (2017) criticized partial introduction of fuzzy parameters in FCMs (viz. concept are crisp) and regarded this as a clear deviation from the fact that FCMs are fuzzy. As a matter of fact, because FCM inference makes use of standard algebraic operations with fuzzy singletons, fuzziness is sacrificed for the sake of computational purposes.

As mentioned in the previous section, wide variety of approaches has been developed in the last decade in order to overcome the limited representation capability of FCMs. However, there are drawbacks of these methods and some of them are still far from being fuzzy. These drawbacks are summarized in Table 3.1.

For instance, use of intervals and grey numbers help reflect experts’ uncertainty about magnitudes of causal dependencies and concept values. Unfortunately, intervals and grey numbers cannot model fuzziness. Let us give a small example to demonstrate our assertion by calculating commonly used uncertainty measures.

Example 1: Suppose that three linguistic variables are used to construct an FCM. These linguistic variables are determined as low, medium and high. Let us define these linguistic variables by using fuzzy singletons, interval/grey numbers, and T1 fuzzy numbers as given in Table 3.2.

The first examination is related to measure of fuzziness. Can fuzzy singletons or interval numbers represent and model fuzziness? In order to answer this question, fuzziness measure is required. Although there are number of ways to measure fuzziness (Klir & Folger, 1988; Klir & Yuan, 1995), measure of fuzziness of the fuzzy set $\tilde{A}$ can be calculated as (Klir & Folger, 1988):

$$F(\tilde{A}) = 1 - \int_0^1 |2\mu_{\tilde{A}}(x) - 1| dx \quad (3.2)$$

Table 3.1 Review of recent FCM extensions

Method	Reference	Concept	Connection Matrix	Inference mechanism	Output Concept	Drawbacks
Original FCMs	(Kosko, 1986, 1997)	Crisp	Crisp	Crisp Processing	Crisp	- Fuzziness is not considered. - Ambiguity is not considered. - All computations are performed based on crisp processing.
Interval FCMs	(Kang et al., 2016; Osei-Bryson, 2004; Papageorgiou et al., 2006; Petalas et al., 2005)	Interval	Interval	Interval arithmetic	Interval	- Fuzziness is not considered.
Grey FCMs	(Froelich & Salmeron, 2014; Papageorgiou & Salmeron, 2012; Salmeron, 2010, 2016; Salmeron & Papageorgiou, 2012)	Interval	Interval	Interval arithmetic	Interval	- Fuzziness is not considered.
Intuitionistic FCMs	(Hajek & Prochazka, 2018; Iakovidis & Papageorgiou, 2011; Papageorgiou & Iakovidis, 2013)	Intuitionistic fuzzy membership degree	Intuitionistic fuzzy membership degree	Intuitionistic fuzzy operations	Intuitionistic fuzzy membership degree	- Different levels of uncertainties are not considered.
Interval-valued FCMs	(Hajek & Prochazka, 2016)	Interval valued fuzzy membership degree	Interval valued fuzzy membership degree	Interval valued fuzzy operations (Deschrijver, 2007)	Interval valued fuzzy membership degree	- Different levels of uncertainties are not considered
Extension principle based T1-FCMs	(Dodurka et al., 2015; Pedrycz & Homenda, 2014; Yesil et al., 2014)	Crisp	T1 Fuzzy	Extension principle	T1 Fuzzy	- Concept values are defuzzified at each iteration. - Information loss due to defuzzification.
CWW-FCM	(Najafi et al., 2017; Rickard et al., 2015a)	IT2 Fuzzy	IT2 Fuzzy	Linguistic Weighted Power Mean Operator	IT2 Fuzzy	- The use of transformation functions is not allowed. Transformation function based inference equation is replaced with the linguistic weighted power mean operator. - At each iteration, concept values are mapped into finite number of words based on similarity measure which results in information loss due to similarity based mapping. - Only finite number of concept value combinations can be reached due to similarity mapping. - The use of defuzzified centroids of word memberships in calculating weighted power mean coefficients.
Proposed Method	Present study	T1/IT2 Fuzzy	T1/IT2 Fuzzy	Alpha-cut based decomposition and reconstruction	T1/IT2 Fuzzy	- Computational cost for very large FCMs

Table 3.2 Linguistic terms and their associated numbers

Linguistic Term	Fuzzy singleton	Interval / Grey number	T1 fuzzy number
Low (L)	0.15	[0.00 0.30]	(0.00, 0.15, 0.30)
Medium (M)	0.50	[0.40 0.60]	(0.40, 0.50, 0.60)
High (H)	0.90	[0.85 1.00]	(0.85, 0.90, 1.00)

where $F(\tilde{A})$ represents measure of fuzziness of the fuzzy set $\tilde{A}$. Note that fuzzy sets generalize crisp sets so the fuzzy singletons and intervals can be represented as fuzzy sets. Resulting fuzziness measures for fuzzy singletons, interval/grey numbers, and T1 fuzzy numbers are tabulated in Table 3.3.

Table 3.3 Fuzziness measures of the linguistic terms

Linguistic Term	Fuzziness Measure		
	Fuzzy singleton	Interval / Grey number	T1 fuzzy number
Low (L)	0.00	0.00	0.150
Medium (M)	0.00	0.00	0.100
High (H)	0.00	0.00	0.075

It is evident that fuzziness measures for fuzzy singletons and interval/grey numbers are equal to zero. In other words, classical FCMs and interval/grey FCMs cannot handle fuzziness. On the other hand, when the linguistic terms are modeled through T1 fuzzy numbers, non-zero fuzziness measures are obtained.

Another important uncertainty measure is related to non-specificity, which can be calculated by using generalized Hartley function of the classical set theory (Klir & Folger, 1988; Klir & Wierman, 1999). It has also been used as a measure of ambiguity (M. F. Dodurka et al., 2016) so that the term ambiguity is preferred in this study. Given the fuzzy set $\tilde{A}$, the measure of ambiguity $U(\tilde{A})$ can be calculated as:

$$U(\tilde{A}) = \frac{1}{h(\tilde{A})} \int_0^{h(\tilde{A})} \log(1 + \mu(\tilde{A}_\alpha)) d\alpha \quad (3.3)$$

Resulting ambiguity measures for fuzzy singletons, interval/grey numbers, and T1 fuzzy numbers are given in Table 3.4.

Table 3.4 Ambiguity measures of the linguistic terms

Linguistic Term	Ambiguity Measure		
	Fuzzy singleton	Interval / Grey number	T1 fuzzy number
Low (L)	0.00	0.262	0.137
Medium (M)	0.00	0.182	0.094
High (H)	0.00	0.139	0.072

After analyzing ambiguity measure results, it can be said that interval/grey and T1 fuzzy numbers can be used to reflect non-specificity or ambiguity in the modeled system. However, fuzzy singletons cannot accomplish this task. Despite the popular use of fuzzy singletons in FCM studies, fuzzy singletons are able to reflect neither fuzziness nor ambiguity. On the other hand, interval/grey FCMs can only be used to model ambiguity, not fuzziness.

In the intuitionistic FCMs (Iakovidis & Papageorgiou, 2011), the weights in the connection matrix are estimated by defuzzification of the aggregated weights, which results in information loss. In the improved version of the intuitionistic FCMs (Papageorgiou & Iakovidis, 2013), resulting output concepts are represented by intuitionistic membership grades, incorporating intuitionistic estimation of hesitancy degrees. However, the produced results do not cover the results for different uncertainty levels, viz. nested intervals to reconstruct the fuzzy set. Similarly problem is prevalent in interval-valued FCMs (Hajek & Prochazka, 2018) where different levels of uncertainties are not considered.

In the extension principle-based T1 FCMs (Dodurka et al., 2015; Yesil et al., 2014), defuzzification is applied to output concepts at each iteration of the FCM inference. The main reasons for using defuzzification are twofold. The first reason is that multiplication of two fuzzy numbers significantly changes the support of a fuzzy set, which makes iterative computations even more difficult. The second reason is that FCM inference checks a stopping condition, which is expressed based on the absolute difference between two subsequent concept values. Result of a fuzzy subtraction operation is another fuzzy set and thereby it is very challenging to decide whether the stopping condition is satisfied or not. A practical approach is to defuzzify concept values and retain a fuzzy connection. However, this approach leads to undesirable

information loss situation. In our regard, alpha-cut based computational approach seems to be the only choice to overcome aforementioned difficulties.

Analyzing the CWW-FCMs (Rickard et al., 2015a), transformation functions in the original FCMs are replaced by an aggregation operator, which can harm the neural network capabilities of the FCMs. Note that FCMs utilize fuzzy logic and artificial neural networks in a synergetic way (Baykasoğlu & Gölcük, 2017b). Choice of type and parameters of the transformation functions has significant impact on the learning capabilities of FCMs. Ignoring such transformation function can be seen as a clear departure from the fundamental conceptions of the original FCMs. In the computational steps of the CWW-FCMs, in a similar fashion with the extension principle-based T1 FCMs, output concept values are mapped into predefined linguistic variables based on similarity measure. The difference is that defuzzification is replaced by similarity-based mapping in the CWW-FCMs. However, information loss is still unavoidable mainly due to the fact that only finite number of concept value combinations can be observed by using similarity based mapping. In addition, coefficients of the weighted power mean operator are determined by using defuzzified centroids of the word memberships.

The principle motivation of the present study is to use fuzzy numbers in both of the concept and connection matrix values by keeping the fundamental computational procedures and theoretical underpinnings of the original FCMs. Pursuing such an aim brings about computational challenges. These difficulties have been reported by the earlier studies to some extent. In this study, an alpha-cut based computational procedure is proposed for T1 FCMs. Then, a natural extension to IT2 FCMs is developed based on the procedures developed for T1 FCMs.

3.4 Type-1 Fuzzy Cognitive Maps

In this section, basics of type-1 fuzzy sets are revised. Extension principle-based T1 FCMs are given. Then, the developed alpha-cut based T1 FCMs are introduced. The developed method is verified by using extension principle-based T1 FCMs.

3.4.1 Type-1 Fuzzy Sets

Some basic definitions of fuzzy sets and fuzzy numbers will be briefly reviewed based on the definitions given in (Klir & Yuan, 1995).

Definition 1: A type-1 fuzzy set $\tilde{A}$ is a set function on universe X into $[0,1]$, $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The value of $\mu_{\tilde{A}}(x)$ is termed the degree of membership x .

Definition 2: The support of a type-1 fuzzy set is the crisp set of all points such that $\mu_{\tilde{A}}(x) > 0$. A type-1 fuzzy set whose support is a single point with $\mu_{\tilde{A}}(x) = 1$ is called a fuzzy singleton.

Definition 3: Support of a fuzzy set $\tilde{A}$ is the ordinary subset of X that has nonzero membership.

Definition 4: A normal type-1 fuzzy set is the one for which $\sup_{x \in X} \mu_{\tilde{A}}(x) = 1$.

Definition 5: A type-1 fuzzy set is convex if and only if:

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)] \quad (3.4)$$

where $\lambda \in [0,1]$.

Definition 6: A type-1 fuzzy number is a fuzzy set $\tilde{A}$ which possess the following properties: $\tilde{A}$ is a normal fuzzy set, $\tilde{A}$ is a convex fuzzy set, support of $\tilde{A}$ is bounded, α -cut of $\tilde{A}$ is a close interval for every $\alpha \in (0,1]$.

Definition 7: A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a_1, a_2, a_3) . The membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} h(\tilde{A}), & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} h(\tilde{A}), & \text{if } a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (3.5)$$

where $h(\tilde{A})$ is the height of the $\tilde{A}$.

Considering two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, their operational laws are as follows:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3) + (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \end{aligned} \quad (3.6)$$

$$\begin{aligned} \tilde{A} \ominus \tilde{B} &= (a_1, a_2, a_3) \ominus (b_1, b_2, b_3) \\ &= (a_1 - b_3, a_2 - b_2, a_3 - b_1) \end{aligned} \quad (3.7)$$

$$\begin{aligned} \tilde{A} \otimes \tilde{B} &= (a_1, a_2, a_3) \times (b_1, b_2, b_3) \\ &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \end{aligned} \quad (3.8)$$

$$\begin{aligned} \tilde{A} \oslash \tilde{B} &= (a_1, a_2, a_3) \oslash (b_1, b_2, b_3) \\ &= (a_1 / b_3, a_2 / b_2, a_3 / b_1) \end{aligned} \quad (3.9)$$

3.4.2 Extension Principle-Based Type-1 Fuzzy Cognitive Maps

The extension principle-based T1 FCMs has recently been proposed by Yesil et al. (2014). In this section, the extension principle-based T1 FCMs are overviewed based on the papers (Pedrycz & Homenda, 2014; Yesil et al., 2014). Extension principle is one of the most significant concepts in fuzzy theory, which is used to generalize crisp mathematical concepts to fuzzy sets. Let X and Y be two universes of discourse, and

$f : X \rightarrow Y$ being a crisp function. The extension principle states how to induce a mapping $f : P(X) \rightarrow P(Y)$, where $P(X)$ and $P(Y)$ are the power sets of X and Y , respectively. The fuzzy extension principle is defined as follows: We have a mapping $f : X \rightarrow Y, y = f(x)$ which induces a function $f : \tilde{A} \rightarrow \tilde{B}$ such that

$$\tilde{B} = f(\tilde{A}) = \{(y, \mu_{\tilde{B}}(y)) | y = f(x), x \in X\} \quad (3.10)$$

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_{\tilde{A}}(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \quad (3.11)$$

where $\mu_{\tilde{B}}(y)$ is the membership function of the mapping.

Recalling the FCM inference equation given in Equation 3.1, fuzzy multiplication of the fuzzy weight matrix with the crisp concept values results in scaled versions of the fuzzy weight matrix itself. Suppose that adding up these partial results is represented by $\tilde{Z}_i$, which is a triangular fuzzy set. The membership function of $\tilde{C}_i^{t+1}$ can be determined by the use of extension principle. On the basis of transformation function f , the membership function of $\tilde{C}_i^{t+1}$ can be written as (Pedrycz & Homenda, 2014):

$$\tilde{C}_i^{t+1}(u) = \sup_{z: f(z)=u} \tilde{Z}_i^{t+1}(z) \quad (3.12)$$

where $\tilde{Z}_i$ has the following membership function:

$$\tilde{Z}_i(z) = \begin{cases} \frac{z-a}{m-a}, & \text{if } z \in [a, m] \\ \frac{b-z}{b-m}, & \text{if } z \in [m, b] \\ 0, & \text{otherwise} \end{cases} \quad (3.13)$$

Suppose that sigmoidal transformation function $f(z) = 1 / (1 + e^{-\lambda z})$ is selected. Based on the extension principle, membership function of the resulting concept value comes in the following form after processing with the transformation function (Pedrycz & Homenda, 2014):

$$f(\tilde{Z}_i)(u) = \begin{cases} \frac{\phi(u) - a}{m - a}, & \text{if } u \in [a_1, m_1] \\ \frac{b - \phi(u)}{b - m}, & \text{if } u \in [m_1, b_1] \\ 0, & \text{otherwise} \end{cases} \quad (3.14)$$

where $u \in [0, 1]$. The values of other parameters are calculated as: $a_1 = 1 / (1 + e^{-\lambda a})$, $b_1 = 1 / (1 + e^{-\lambda b})$, $m_1 = 1 / (1 + e^{-\lambda m})$, and $\phi(u) = -\log(1 / u - 1) / \lambda$. Note that different transformation functions are viable in this phase.

The extension principle-based T1-FCMs can be given in a more concise manner as follows (Yesil et al., 2014):

Step 1: Initial concept values are checked in this step. If the concept values are defined by using fuzzy numbers (viz. triangular), then a defuzzification method is employed in order to obtain a fuzzy singleton. For instance, center of gravity defuzzification method can be used as follows:

$$C_i(t) = \frac{\int \mu_{\tilde{C}_i(t)}(x) x dx}{\int \mu_{\tilde{C}_i(t)}(x) dx} \quad (3.15)$$

Step 2: One step iteration of the FCM inference is calculated as:

$$\tilde{C}_i(t+1) = C_i(t) + \sum_{\substack{j=1 \\ i \neq j}} \tilde{e}_{ji} C_j(t) \quad (3.16)$$

where $C_i(t)$ represents fuzzy singleton of the i th concept value at iteration t , and $\tilde{C}_i(t+1)$ is the resulting fuzzy output of the i th concept value at iteration $t+1$.

Step 3: The transformation function is employed to concept values. Here, extension principle calculations given earlier are performed in order to obtain membership function of the output concept value after nonlinear mapping.

$$\tilde{C}_i(t+1) = f(\tilde{C}_i(t+1)) \quad (3.17)$$

where f is the transfer function.

Step 4: The fuzzy concept values are obtained as a result of extension principle based mapping. In order to be able to continue FCM iterations via extension principle, one needs to defuzzify either connection matrix or concepts. Therefore, concept values are defuzzified and crisp numbers are obtained. The obtained crisp concept values are used in the FCM inference and the iterations are continued as mentioned in Step 2.

As mentioned earlier, extension principle-based T1 FCMs have a major disadvantage that information loss is inevitable due to defuzzification. Nevertheless, they are used to compare the results produced by the developed alpha-cut based computational procedure.

3.4.3 The Developed Alpha-Cut Based Type-1 Fuzzy Cognitive Maps

In this section, developed alpha-cut based computational approach is introduced. As alpha-cuts play a central role in the developed method, alpha-cut related definitions and theorems are given first.

Definition 8 (Klir & Yuan, 1995): The alpha-cut of type-1 fuzz set $\tilde{A}$ is an interval of real numbers, defined as:

$$A_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\} \quad (3.18)$$

where $\alpha \in [0,1]$ and A_α denotes alpha-cut of type-1 fuzzy set.

Given a type-1 fuzzy set, end-points of an alpha-cut can be calculated easily. Suppose that we have a type-1 fuzzy set $\tilde{A}$ as shown in Figure 3.2.

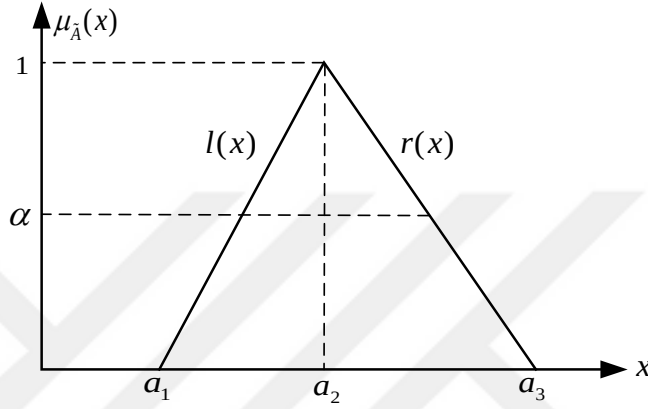


Figure 3.2 An alpha-cut of a type-1 fuzzy set

The alpha-cut of the fuzzy set $\tilde{A}$ can be calculated by solving $l(x) = \alpha$ and $r(x) = \alpha$, for left and right end-points, respectively.

Definition 9: The alpha-cut of a type-1 triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ can be calculated as:

$$A_\alpha = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha] \quad (3.19)$$

where A_α represents the alpha-cut of a type-1 fuzzy set $\tilde{A}$.

According to decomposition theorem, a fuzzy set can be represented by its alpha-cuts. Prior to present decomposition theorem, indicator and square-well functions need to be introduced. The indicator function is represented by (Mendel, 2017):

$$I_{A_\alpha}(x) = \begin{cases} 1, & x \in A_\alpha \\ 0, & x \notin A_\alpha \end{cases} \quad (3.20)$$

Square-well function can be represented based on the indicator function as:

$$\mu_A(x|\alpha) \equiv \alpha I_{A_\alpha}(x) = \alpha/A_\alpha \quad (3.21)$$

The square-well function raises the alpha-cut A_α of the x-axis to level alpha.

Decomposition Theorem (Klir & Yuan, 1995; Mendel, 2017; Zadeh, 1971, 1975):

A type-1 fuzzy set $\tilde{A}$ can be represented as:

$$\mu_{\tilde{A}}(x) = \bigcup_{\alpha \in [0,1]} \mu_{\tilde{A}}(x|\alpha) = \sup_{\alpha \in [0,1]} \{\alpha/A_\alpha\} \quad x \in X \quad (3.22)$$

where $\bigcup$ represents fuzzy union.

Based on the decomposition theorem, a type-1 fuzzy number $\tilde{A}$ can be reconstructed from its alpha-cuts $A_\alpha = [a_\alpha, b_\alpha]$ as expressed in Equation 3.23.

$$\mu_{\tilde{A}}(x) = \sup_{\alpha \in [0,1]} \{\alpha/[a_\alpha, b_\alpha]\}, \quad x \in X \quad (3.23)$$

where $\mu_{\tilde{A}}(x)$ is the reconstructed membership function of the fuzzy set $\tilde{A}$.

Another important concept in fuzzy arithmetic is that functions of type-1 fuzzy sets can be computed by using alpha-cut concept.

Alpha-cuts Decomposition Theorem (Klir & Yuan, 1995; Mendel & Wu, 2010; Nguyen, 1978; Zadeh, 1975): Let $B = G(A_1, A_2, \dots, A_r)$ be an arbitrary crisp function, where $\tilde{A}_i (i=1, 2, \dots, r)$ is a type-1 fuzzy set whose domain is D_{A_i} and alpha-cut is $(A_i)_\alpha$. Then, under the extension principle:

$$B_\alpha = G((A_1)_\alpha, \dots, (A_r)_\alpha) \quad (3.24)$$

and the height of $\tilde{B}$ equals the minimum height of all $\tilde{A}_i$.

The fuzzy set $\tilde{B}$ can be reconstructed from its alpha-cuts by using the alpha-cut decomposition theorem if the function G is cutworthy (Araabi, Kehtarnavaz, & Lucas, 2001; Rajati & Mendel, 2014; Wierman, 1997). Klir and Yuan (1995) defined cutworthy property as “Any property generalized from classical set theory into the domain of fuzzy set theory that is preserved in all alpha-cuts for $\alpha \in (0,1]$ ”. In mathematical terms, a function G is called cutworthy, if it preserves the subsethood order (Araabi et al., 2001; Wierman, 1997):

$$\begin{aligned} \forall i \in \{1, 2, \dots, n\}, D_i \subseteq E_i \\ \Rightarrow G(D_1, D_2, \dots, D_n) \subseteq G(E_1, E_2, \dots, E_n) \end{aligned} \quad (3.25)$$

In the present study, it has been found out that the FCM inference preserves the subsethood orders. It is known that arithmetic operations summation and multiplication preserves cutworthy property (Klir & Yuan, 1995). Additionally, sigmoidal and hyperbolic tangent transformation functions are monotone and non-decreasing functions that there exists one-to-one correspondence between input and output. Therefore, FCM inference can be extended to fuzzy numbers by fully capitalizing alpha-cut operations.

As mentioned earlier, interval cognitive maps can model ambiguity but this is not the case for fuzziness. The rationale behind the proposed method is that interval cognitive maps can be regarded as a special case of FCMs in which the alpha level is equal to zero. Because alpha-cuts of a fuzzy number are nested intervals, fuzzy parameters of FCMs are deconstructed into its alpha-cuts, and then interval cognitive maps are computed for each alpha-cut. Eventually, fuzzy sets are reconstructed based on their alpha-level results. This mechanism is illustrated in Figure 3.3.

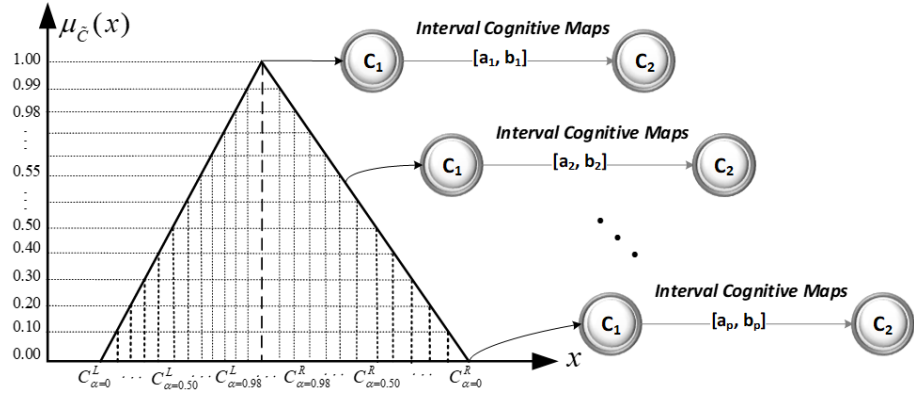


Figure 3.3 Representation of the proposed method

The developed alpha-cut based computations for T1 FCMs can be given in three phases.

Phase 1: Deconstruction Phase: In this phase, fuzzy parameters of the FCMs are deconstructed into their alpha-levels. As mentioned in the previous sections, FCMs have two components as concept values and the connection matrix. Here, decision makers are expected to determine number of alpha-cuts p , which affect output resolution. Based on the number of alpha-cuts, alpha-levels are computed as:

$$\alpha_s = h(s-1)/(p-1), \quad s = (1, 2, \dots, p) \quad (3.26)$$

where h represents the minimum height of the fuzzy sets. Unless otherwise stated, the heights of the type-1 fuzzy sets are taken as 1 in this study.

Then, concept values and connection matrix are formed for each α -level computed according to Equation 3.19. Alpha-cut of the concept values takes the form as given in Equation 3.27.

$$C_\alpha = ([C_{1_\alpha}^L, C_{1_\alpha}^R], [C_{2_\alpha}^L, C_{2_\alpha}^R], \dots, [C_{n_\alpha}^L, C_{n_\alpha}^R]) \quad (3.27)$$

where $C_{n_\alpha}^L$ and $C_{n_\alpha}^R$ represents left and right end-point of the n th alpha-level concept.

Similarly, alpha-cuts of the fuzzy numbers in the connection matrix are calculated as:

$$e_{\alpha} = \begin{pmatrix} \begin{bmatrix} e_{11_{\alpha}}^L, e_{11_{\alpha}}^R \end{bmatrix} & \begin{bmatrix} e_{12_{\alpha}}^L, e_{12_{\alpha}}^R \end{bmatrix} & \cdots & \begin{bmatrix} e_{1n_{\alpha}}^L, e_{1n_{\alpha}}^R \end{bmatrix} \\ \begin{bmatrix} e_{21_{\alpha}}^L, e_{21_{\alpha}}^R \end{bmatrix} & \begin{bmatrix} e_{22_{\alpha}}^L, e_{22_{\alpha}}^R \end{bmatrix} & \cdots & \begin{bmatrix} e_{2n_{\alpha}}^L, e_{2n_{\alpha}}^R \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} e_{n1_{\alpha}}^L, e_{n1_{\alpha}}^R \end{bmatrix} & \begin{bmatrix} e_{n2_{\alpha}}^L, e_{n2_{\alpha}}^R \end{bmatrix} & \cdots & \begin{bmatrix} e_{nn_{\alpha}}^L, e_{nn_{\alpha}}^R \end{bmatrix} \end{pmatrix} \quad (3.28)$$

where $e_{1n_{\alpha}}^L$ and $e_{1n_{\alpha}}^R$ represents alpha level result of the left and right end-point of the causal influence exerted from the first concept into n th concept.

Phase 2: Inference Phase: In this phase, interval concept values and connection matrices belonging to the same alpha-cut are used to form interval cognitive maps. Therefore, an interval cognitive map is simulated for each alpha-cut. The inference algorithm of the interval FCMs can be expressed as:

$$C_{i_{\alpha}}(t+1) = f \left(C_{i_{\alpha}}(t) \oplus \sum_{\substack{j=1 \\ j \neq i}}^n e_{ji_{\alpha}} \otimes C_{j_{\alpha}}(t) \right) \quad (3.29)$$

where $C_{i_{\alpha}}(t+1)$ represents i th concept value at iteration $t+1$ computed in terms of α -level.

In the inference computation, summation and multiplication operations are performed based on interval arithmetic. For instance, multiplication operation in Equation 3.29 is calculated as:

$$e_{ji_{\alpha}} \otimes C_{j_{\alpha}}(t) = \begin{bmatrix} \min(e_{ji_{\alpha}}^L C_{j_{\alpha}}^L(t), e_{ji_{\alpha}}^L C_{j_{\alpha}}^R(t), e_{ji_{\alpha}}^R C_{j_{\alpha}}^L(t), e_{ji_{\alpha}}^R C_{j_{\alpha}}^R(t)), \\ \max(e_{ji_{\alpha}}^L C_{j_{\alpha}}^L(t), e_{ji_{\alpha}}^L C_{j_{\alpha}}^R(t), e_{ji_{\alpha}}^R C_{j_{\alpha}}^L(t), e_{ji_{\alpha}}^R C_{j_{\alpha}}^R(t)) \end{bmatrix} \quad (3.30)$$

where interval multiplication is denoted by $\otimes$.

Phase 3: Reconstruction Phase: When the stopping criterion of the FCM simulation is satisfied, the fuzzy concept values are reconstructed in this phase. Stopping criterion is usually taken as reaching steady-state concepts or maximum number of iterations. The fuzzy sets of the resulting concept values are reconstructed based on the decomposition theorem as given in Equation 3.31.

$$\tilde{C}_i = \sup_{\alpha \in [0,1]} \{ \alpha / C_{i_\alpha} \} = \bigcup_{\alpha \in [0,1]} \alpha / [C_{i_\alpha}^L, C_{i_\alpha}^R] \quad (3.31)$$

where $\tilde{C}_i$ represents resulting fuzzy set of the i th concept value.

3.5. Interval Type-2 Fuzzy Cognitive Maps

In this section, IT2 fuzzy sets and FCMs are introduced. First, preliminaries related to IT2 fuzzy sets are given. Then, the developed method is presented.

3.5.1 Interval Type-2 Fuzzy Sets

Definition 10 (Mendel et al., 2006; Mendel & Wu, 2010): A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$ as given in Equation 3.32.

$$\tilde{\tilde{A}} = \left\{ ((x, u), \mu_{\tilde{\tilde{A}}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1 \right\} \quad (3.32)$$

where J_x denotes an interval in $[0,1]$. Moreover, type-2 fuzzy set $\tilde{\tilde{A}}$ can also be represented as:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x, u) / (x, u) \quad (3.33)$$

where $J_x \subseteq [0,1]$ and $\int \int$ denotes union over all admissible x and u .

Definition 11 (Mendel et al., 2006; Mendel & Wu, 2010): Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is called an IT2 fuzzy set. An IT2 fuzzy set $\tilde{\tilde{A}}$, which can be regarded as a special case of a type-2 fuzzy set, is represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad (3.34)$$

where $J_x \subseteq [0, 1]$.

The footprint of uncertainty (FOU) is represented by the lower and upper membership functions:

$$FOU(\tilde{\tilde{A}}) = \int_{x \in X} [\underline{\mu}_{\tilde{\tilde{A}}}(x), \bar{\mu}_{\tilde{\tilde{A}}}(x)] \quad (3.35)$$

where $\underline{\mu}_{\tilde{\tilde{A}}}(x)$ and $\bar{\mu}_{\tilde{\tilde{A}}}(x)$ represent lower and upper membership functions, respectively.

Definition 12 (Tahayori, Tettamanzi, & Antoni, 2006): The alpha-cut of the IT2 fuzzy set $\tilde{\tilde{A}}$ is represented as follows:

$$A_\alpha = \left\{ x \mid \underline{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha, \bar{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha \right\} \quad (3.36)$$

Definition 13 (Hamrawi & Coupland, 2009): An IT2 fuzzy set $\tilde{\tilde{A}}$ is said to be normal if $\sup \bar{\mu}_{\tilde{\tilde{A}}}(x) = 1$ and $\sup \underline{\mu}_{\tilde{\tilde{A}}}(x) = h < 1$, where h represents the height of the lower membership function.

Considering two triangular IT2 fuzzy numbers $\tilde{\tilde{A}} = (\bar{a}, \underline{a}) = ((\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{h}_A)(\underline{a}_1, \underline{a}_2, \underline{a}_3, \underline{h}_A))$ and $\tilde{\tilde{B}} = (\bar{b}, \underline{b}) = ((\bar{b}_1, \bar{b}_2, \bar{b}_3, \bar{h}_B)(\underline{b}_1, \underline{b}_2, \underline{b}_3, \underline{h}_B))$, their operational laws are as follows (Chen, Yang, Lee, & Yang, 2012):

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = \left(\begin{array}{l} (\bar{a}_1 + \bar{b}_1, \bar{a}_2 + \bar{b}_2, \bar{a}_3 + \bar{b}_3, \min(\bar{h}_{\tilde{A}}, \bar{h}_{\tilde{B}})), \\ (\underline{a}_1 + \underline{b}_1, \underline{a}_2 + \underline{b}_2, \underline{a}_3 + \underline{b}_3, \min(\underline{h}_{\tilde{A}}, \underline{h}_{\tilde{B}})) \end{array} \right) \quad (3.37)$$

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = \left(\begin{array}{l} (\bar{a}_1 - \bar{b}_3, \bar{a}_2 - \bar{b}_2, \bar{a}_3 - \bar{b}_1, \min(\bar{h}_{\tilde{A}}, \bar{h}_{\tilde{B}})), \\ (\underline{a}_1 - \underline{b}_3, \underline{a}_2 - \underline{b}_2, \underline{a}_3 - \underline{b}_1, \min(\underline{h}_{\tilde{A}}, \underline{h}_{\tilde{B}})) \end{array} \right) \quad (3.38)$$

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left(\begin{array}{l} (\bar{a}_1 \times \bar{b}_1, \bar{a}_2 \times \bar{b}_2, \bar{a}_3 \times \bar{b}_3, \min(\bar{h}_{\tilde{A}}, \bar{h}_{\tilde{B}})), \\ (\underline{a}_1 \times \underline{b}_1, \underline{a}_2 \times \underline{b}_2, \underline{a}_3 \times \underline{b}_3, \min(\underline{h}_{\tilde{A}}, \underline{h}_{\tilde{B}})) \end{array} \right) \quad (3.39)$$

$$\tilde{\tilde{A}} \oslash \tilde{\tilde{B}} = \left(\begin{array}{l} (\bar{a}_1 / \bar{b}_3, \bar{a}_2 / \bar{b}_2, \bar{a}_3 / \bar{b}_1, \min(\bar{h}_{\tilde{A}}, \bar{h}_{\tilde{B}})), \\ (\underline{a}_1 / \underline{b}_3, \underline{a}_2 / \underline{b}_2, \underline{a}_3 / \underline{b}_1, \min(\underline{h}_{\tilde{A}}, \underline{h}_{\tilde{B}})) \end{array} \right) \quad (3.40)$$

$$k \times \tilde{\tilde{A}} = \begin{cases} \left(\begin{array}{l} (k \times \bar{a}_1, k \times \bar{a}_2, k \times \bar{a}_3, \bar{h}_A), \\ (k \times \underline{a}_1, k \times \underline{a}_2, k \times \underline{a}_3, \underline{h}_A) \end{array} \right), & k \geq 0 \\ \left(\begin{array}{l} (k \times \bar{a}_3, k \times \bar{a}_2, k \times \bar{a}_1, \bar{h}_A), \\ (k \times \underline{a}_3, k \times \underline{a}_2, k \times \underline{a}_1, \underline{h}_A) \end{array} \right), & k \leq 0 \end{cases} \quad (3.41)$$

3.5.2 Developed Alpha-Cut Based Interval Type-2 Fuzzy Cognitive Maps

In a similar manner with the T1 FCMs, alpha-cut related definitions are introduced first. Then, T1 FCMs given in the previous section are extended to IT2 FCMs.

Definition 14 (Tahayori et al., 2006): The alpha-cut of the IT2 fuzzy set $\tilde{\tilde{A}}$ is represented as follows:

$$A_\alpha = \{x \mid \underline{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha, \bar{\mu}_{\tilde{\tilde{A}}}(x) \geq \alpha\} \quad (3.42)$$

The alpha-cut of the fuzzy set $\tilde{\tilde{A}}$ can be calculated by solving $\bar{\mu}_{\tilde{\tilde{A}}}(x) = \alpha$ and $\underline{\mu}_{\tilde{\tilde{A}}}(x) = \alpha$, and thus two consecutive intervals are obtained. In Figure 3.4., alpha-cut of a normal IT2 fuzzy set is illustrated. A fuzzy set $\tilde{\tilde{A}}$ can be represented by intervals with interval-valued bounds as $\left[[\bar{a}_\alpha^L, \underline{a}_\alpha^L], [\underline{a}_\alpha^R, \bar{a}_\alpha^R] \right]$.

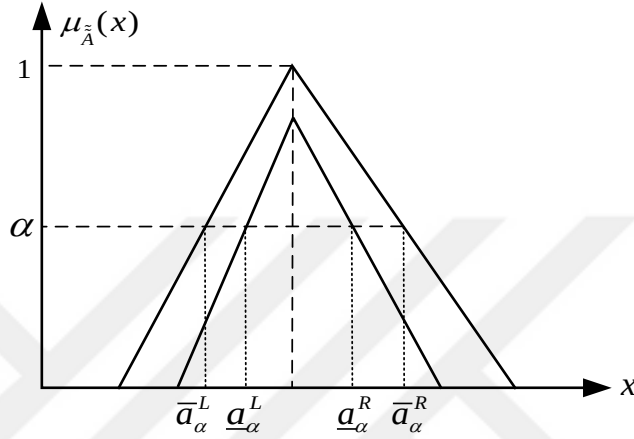


Figure 3.4 An alpha-cut of an IT2 fuzzy set

Given in the previous section, alpha-cuts decomposition theorem is applicable to IT2 fuzzy sets. Let G be a function extended to IT2 fuzzy sets $\tilde{\tilde{A}}_1, \tilde{\tilde{A}}_2, \dots, \tilde{\tilde{A}}_r$ and $\tilde{\tilde{B}} = G(\tilde{\tilde{A}}_1, \tilde{\tilde{A}}_2, \dots, \tilde{\tilde{A}}_r)$, then upper and lower membership functions of the fuzzy set $\tilde{\tilde{B}}$ can be calculated based on the alpha-cut concept as follows:

$$\begin{cases} \underline{B} = \bigcup_{\alpha \in [0, h]} \alpha / \underline{B}_\alpha \\ \bar{B} = \bigcup_{\alpha \in [0, 1]} \alpha / \bar{B}_\alpha \end{cases} \quad (3.43)$$

where $\underline{B}_\alpha$ and $\bar{B}_\alpha$ are calculated as

$$\begin{cases} \underline{B}_\alpha = G(\underline{A}_{1_\alpha}, \underline{A}_{2_\alpha}, \dots, \underline{A}_{r_\alpha}) \\ \bar{B}_\alpha = G(\bar{A}_{1_\alpha}, \bar{A}_{2_\alpha}, \dots, \bar{A}_{r_\alpha}) \end{cases} \quad (3.44)$$

Here, $\underline{B}_\alpha$ and $\bar{B}_\alpha$ denote lower and upper memberships at the alpha-level, respectively.

Along the same line as given for T1-FCMs, inference algorithm for IT2-FCMs can be calculated in three phases.

Phase 1: Deconstruction Phase: Concept values and connection matrix are deconstructed to their alpha-level intervals in this phase. Because much of the required material has already been given for T1-FCMs, only the necessary equations are introduced here.

Alpha-cuts of the concepts are represented as given in the Equation 3.45.

$$\begin{cases} \underline{C}_\alpha = \left([\underline{C}_{1_\alpha}^L, \underline{C}_{1_\alpha}^R], [\underline{C}_{2_\alpha}^L, \underline{C}_{2_\alpha}^R], \dots, [\underline{C}_{n_\alpha}^L, \underline{C}_{n_\alpha}^R] \right) \\ \bar{C}_\alpha = \left([\bar{C}_{1_\alpha}^L, \bar{C}_{1_\alpha}^R], [\bar{C}_{2_\alpha}^L, \bar{C}_{2_\alpha}^R], \dots, [\bar{C}_{n_\alpha}^L, \bar{C}_{n_\alpha}^R] \right) \end{cases} \quad (3.45)$$

where $\underline{C}_\alpha$ and $\bar{C}_\alpha$ are alpha-cuts of the lower and upper membership functions of the concepts, respectively. Similarly, alpha-cuts of the connection matrix can be arranged as given in the Equation 3.46.

$$\begin{cases} \underline{e}_\alpha = \begin{pmatrix} [\underline{e}_{1_\alpha}^L, \underline{e}_{1_\alpha}^R] & [\underline{e}_{2_\alpha}^L, \underline{e}_{2_\alpha}^R] & \cdots & [\underline{e}_{n_\alpha}^L, \underline{e}_{n_\alpha}^R] \\ [\underline{e}_{21_\alpha}^L, \underline{e}_{21_\alpha}^R] & [\underline{e}_{22_\alpha}^L, \underline{e}_{22_\alpha}^R] & \cdots & [\underline{e}_{2n_\alpha}^L, \underline{e}_{2n_\alpha}^R] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{e}_{n1_\alpha}^L, \underline{e}_{n1_\alpha}^R] & [\underline{e}_{n2_\alpha}^L, \underline{e}_{n2_\alpha}^R] & \cdots & [\underline{e}_{nn_\alpha}^L, \underline{e}_{nn_\alpha}^R] \end{pmatrix} \\ \bar{e}_\alpha = \begin{pmatrix} [\bar{e}_{1_\alpha}^L, \bar{e}_{1_\alpha}^R] & [\bar{e}_{12_\alpha}^L, \bar{e}_{12_\alpha}^R] & \cdots & [\bar{e}_{1n_\alpha}^L, \bar{e}_{1n_\alpha}^R] \\ [\bar{e}_{21_\alpha}^L, \bar{e}_{21_\alpha}^R] & [\bar{e}_{22_\alpha}^L, \bar{e}_{22_\alpha}^R] & \cdots & [\bar{e}_{2n_\alpha}^L, \bar{e}_{2n_\alpha}^R] \\ \vdots & \vdots & \ddots & \vdots \\ [\bar{e}_{n1_\alpha}^L, \bar{e}_{n1_\alpha}^R] & [\bar{e}_{n2_\alpha}^L, \bar{e}_{n2_\alpha}^R] & \cdots & [\bar{e}_{nn_\alpha}^L, \bar{e}_{nn_\alpha}^R] \end{pmatrix} \end{cases} \quad (3.46)$$

where $\underline{e}_\alpha$ and $\bar{e}_\alpha$ are alpha-cuts of the lower and upper membership functions of the connection matrices, respectively.

Phase 2: Inference Phase: In this phase, interval cognitive maps are simulated for alpha-cuts of the lower and upper membership functions, respectively. The inference equation of the interval FCMs which is constructed with respect to lower membership function is expressed as:

$$\underline{C}_{i_\alpha}(t+1) = f \left(\underline{C}_{i_\alpha}(t) \oplus \sum_{\substack{j=1 \\ j \neq i}}^n \underline{e}_{ji_\alpha} \otimes \underline{C}_{j_\alpha}(t) \right) \quad (3.47)$$

where summation and multiplication operations are performed based on interval arithmetic. Multiplication operation in the Equation 3.47 can be given as:

$$\underline{e}_{ji_\alpha} \otimes \underline{C}_{j_\alpha}(t) = \begin{bmatrix} \min(\underline{e}_{ji_\alpha}^L \underline{C}_{j_\alpha}^L(t), \underline{e}_{ji_\alpha}^L \underline{C}_{j_\alpha}^R(t), \underline{e}_{ji_\alpha}^R \underline{C}_{j_\alpha}^L(t), \underline{e}_{ji_\alpha}^R \underline{C}_{j_\alpha}^R(t)), \\ \max(\underline{e}_{ji_\alpha}^L \underline{C}_{j_\alpha}^L(t), \underline{e}_{ji_\alpha}^L \underline{C}_{j_\alpha}^R(t), \underline{e}_{ji_\alpha}^R \underline{C}_{j_\alpha}^L(t), \underline{e}_{ji_\alpha}^R \underline{C}_{j_\alpha}^R(t)) \end{bmatrix} \quad (3.48)$$

The inference equation of the interval FCMs belonging to upper membership function takes the form as:

$$\bar{C}_{i_\alpha}(t+1) = f \left(\bar{C}_{i_\alpha}(t) \oplus \sum_{\substack{j=1 \\ j \neq i}}^n \bar{e}_{ji_\alpha} \otimes \bar{C}_{j_\alpha}(t) \right) \quad (3.49)$$

Similarly, multiplication operation in the Equation 3.49 is calculated as:

$$\bar{e}_{ji_\alpha} \otimes \bar{C}_{j_\alpha}(t) = \begin{bmatrix} \min(\bar{e}_{ji_\alpha}^L \bar{C}_{j_\alpha}^L(t), \bar{e}_{ji_\alpha}^L \bar{C}_{j_\alpha}^R(t), \bar{e}_{ji_\alpha}^R \bar{C}_{j_\alpha}^L(t), \bar{e}_{ji_\alpha}^R \bar{C}_{j_\alpha}^R(t)), \\ \max(\bar{e}_{ji_\alpha}^L \bar{C}_{j_\alpha}^L(t), \bar{e}_{ji_\alpha}^L \bar{C}_{j_\alpha}^R(t), \bar{e}_{ji_\alpha}^R \bar{C}_{j_\alpha}^L(t), \bar{e}_{ji_\alpha}^R \bar{C}_{j_\alpha}^R(t)) \end{bmatrix} \quad (3.50)$$

Phase 3: Reconstruction phase: The IT2 fuzzy concept values are reconstructed from alpha-level results based on the decomposition theorem. The IT2 fuzzy concept values are given as:

$$\tilde{\bar{C}}_i = \begin{cases} \bar{C}_i = \bigcup_{\alpha \in [0,1]} \alpha / [\bar{C}_{i_\alpha}^L, \bar{C}_{i_\alpha}^R] \\ \underline{C}_i = \bigcup_{\alpha \in [0,h]} \alpha / [\underline{C}_{i_\alpha}^L, \underline{C}_{i_\alpha}^R] \end{cases} \quad (3.51)$$

where $\tilde{\bar{C}}_i$ represent the IT2 fuzzy value of the i th concept.

3.6 Experimental Study

In this section, extensive computational analysis has been performed and the proposed alpha-cut based FCM methods are tested on the case studies reported in the literature. First, a syntactic FCM model with six concepts and two input nodes is used to test inference capability of the proposed methods. Then, two real-life case studies are adopted from the literature and the proposed methods are validated. The first case study is the ERP maintenance risk model and the second case study concentrates on the radiotherapy treatment planning. Accordingly, FCM models are constructed by using crisp, T1 fuzzy, and IT2 fuzzy data for each case study, and 2 scenarios are simulated with 2 different activation functions (hyperbolic tangent and sigmoid) and 3 different activation function parameters ($\lambda = 1, \lambda = 3, \lambda = 5$) in each model. Totally, 108 different configurations are simulated and the results are reported.

3.6.1 Syntactic FCM Model

The syntactic FCM model involves 6 concepts and both positive and negative causal relationships. As the C_1 and C_3 affect the other concepts while they are not affected by others, C_1 and C_3 are considered as input nodes. The strengths of causal relationships among concepts are given in crisp, T1 fuzzy and IT2 fuzzy formats in Table 3.5.

The initial concept values for the scenario 1 and 2 are given in Table 3.6. In the first scenario, only the input nodes have negative values. In the second scenario, first three nodes are given negative values.

Table 3.5 Strengths of causal relationships in syntactic FCM model

Causal connection	Sign	Strength of causal relationship		
		Crisp	T1 fuzzy	IT2 fuzzy
$C_1 \rightarrow C_2$	+	0.67	(0.5,0.65,0.85)	((0.5,0.65,0.85;1),(0.55,0.65,0.80;0.8))
$C_1 \rightarrow C_4$	-	0.60	(0.5,0.6,0.7)	((0.5,0.6,0.7;1),(0.55,0.60,0.65;0.8))
$C_1 \rightarrow C_5$	+	0.58	(0.45,0.6,0.7)	((0.45,0.6,0.7;1),(0.50,0.6,0.65;0.8))
$C_1 \rightarrow C_6$	+	0.18	(0.1,0.2,0.25)	((0.1,0.2,0.25;1),(0.15,0.2,0.23;0.8))
$C_2 \rightarrow C_4$	+	0.65	(0.6,0.65,0.7)	((0.6,0.65,0.7;1),(0.62,0.65,0.68;0.8))
$C_2 \rightarrow C_6$	+	0.82	(0.65,0.85,0.95)	((0.65,0.85,0.95;1),(0.7,0.85,0.9;0.8))
$C_3 \rightarrow C_4$	+	0.85	(0.7,0.85,1)	((0.7,0.85,1;1),(0.75,0.85,0.95;0.8))
$C_3 \rightarrow C_6$	-	0.12	(0,0.1,0.25)	((0,0.1,0.25;1),(0.05,0.1,0.20;0.8))
$C_4 \rightarrow C_2$	+	0.85	(0.75,0.8,1)	((0.75,0.8,1;1),(0.72,0.8,0.95;0.8))
$C_4 \rightarrow C_5$	+	0.20	(0.1,0.2,0.3)	((0.1,0.2,0.3;1),(0.15,0.2,0.25;0.8))
$C_4 \rightarrow C_6$	+	0.30	(0.2,0.3,0.4)	((0.2,0.3,0.4;1),(0.25,0.3,0.35;0.8))
$C_5 \rightarrow C_4$	+	0.85	(0.6,0.95,1)	((0.6,0.95,1;1),(0.65,0.95,0.98;0.8))
$C_5 \rightarrow C_6$	-	0.20	(0.1,0.2,0.3)	((0.1,0.2,0.3;1),(0.15,0.2,0.25;0.8))
$C_6 \rightarrow C_2$	+	0.65	(0.45,0.65,0.85)	((0.45,0.65,0.85;1),(0.50,0.65,0.80;0.8))

Negative values in the concepts are intentionally used in the scope of this study. As the interrelationships among concepts have both positive and negative values, negative-valued concepts help investigate all possible situations as a consequence of fuzzy arithmetic operations.

Table 3.6 Initial concept values in syntactic FCM model

Concept	Scenario 1			Scenario 2		
	Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
C1	-0.2	(-0.3,-0.2,-0.1)	((-0.3,-0.2,-0.1;1), (-0.25,-0.2,-0.15;0.8))	-1	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;1))
C2	0	(0,0,0)	((0,0,0;1),(0,0,0;1))	-1	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;1))
C3	-0.6	(-0.7,-0.6,-0.5)	((-0.7,-0.6,-0.5;1), (-0.65,-0.6,-0.55;0.8))	-1	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;1))
C4	0	(0,0,0)	((0,0,0;1),(0,0,0;1))	0	(0,0,0)	((0,0,0;1), (0,0,0;1))
C5	0	(0,0,0)	((0,0,0;1),(0,0,0;1))	0	(0,0,0)	((0,0,0;1), (0,0,0;1))
C6	0	(0,0,0)	((0,0,0;1),(0,0,0;1))	0	(0,0,0)	((0,0,0;1), (0,0,0;1))

Results of syntactic FCM model simulated with hyperbolic tangent and sigmoid functions are given in Table 3.7-8, respectively. Note that larger values of activation function parameters increase the nonlinearity of the FCM outputs. For the sake of simplicity, left end-point, right end-point, and the point where the maximum membership degree is observed, are reported in tables throughout this section.

Table 3.7 Simulation results of syntactic FCM model with hyperbolic tangent function

λ	Concept	Scenario 1			Crisp	Scenario 2		
		Crisp	T1 fuzzy	IT2 fuzzy		T1 fuzzy	IT2 fuzzy	
1.0	C1	-0.031	(-0.031,-0.031,-0.03)	((-0.031,-0.031,-0.03;1), (-0.031,-0.031,-0.03;0.8))	-0.031	(-0.031,-0.031,-0.031)	((-0.031,-0.031,-0.031;1), (-0.031,-0.031,-0.031;0.8))	
	C2	-0.985	(-0.993,-0.984,-0.971)	((-0.993,-0.984,-0.971;1), (-0.992,-0.984,-0.973;0.8))	-0.986	(-0.993,-0.984,-0.971)	((-0.993,-0.984,-0.971;1), (-0.992,-0.984,-0.973;0.8))	
	C3	-0.031	(-0.031,-0.031,-0.031)	((-0.031,-0.031,-0.031;1), (-0.031,-0.031,-0.031;0.8))	-0.031	(-0.031,-0.031,-0.031)	((-0.031,-0.031,-0.031;1), (-0.031,-0.031,-0.031;0.8))	
	C4	-0.978	(-0.987,-0.981,-0.958)	((-0.987,-0.981,-0.958;1), (-0.985,-0.981,-0.966;0.8))	-0.978	(-0.987,-0.981,-0.958)	((-0.987,-0.981,-0.958;1), (-0.985,-0.981,-0.966;0.8))	
	C5	-0.743	(-0.81,-0.743,-0.627)	((-0.81,-0.743,-0.626;1), (-0.781,-0.743,-0.692;0.8))	-0.743	(-0.81,-0.744,-0.627)	((-0.81,-0.744,-0.627;1), (-0.781,-0.744,-0.693;0.8))	
	C6	-0.957	(-0.979,-0.96,-0.901)	((-0.979,-0.96,-0.901;1), (-0.971,-0.96,-0.93;0.8))	-0.957	(-0.979,-0.96,-0.901)	((-0.979,-0.96,-0.901;1), (-0.971,-0.96,-0.93;0.8))	
3.0	C1	-0.995	(-0.995,-0.995,-0.995)	((-0.995,-0.995,-0.995;1), (-0.995,-0.995,-0.995;0.8))	-0.995	(-0.995,-0.995,-0.995)	((-0.995,-0.995,-0.995;1), (-0.995,-0.995,-0.995;0.8))	
	C2	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C3	-0.995	(-0.995,-0.995,-0.995)	((-0.995,-0.995,-0.995;1), (-0.995,-0.995,-0.995;0.8))	-0.995	(-0.995,-0.995,-0.995)	((-0.995,-0.995,-0.995;1), (-0.995,-0.995,-0.995;0.8))	
	C4	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C5	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C6	-1.000	(-1,-1,0.91)	((-1,-1,0.91;1), (-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
5.0	C1	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C2	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C3	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C4	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C5	-1.000	(-1,-1,-1)	((-1,-1,-1;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	
	C6	-1.000	(-1,-1,0.995)	((-1,-1,0.995;1), (-1,-1,-1;0.8))	-1.000	(-1,-1,-1)	((-1,-1,-1;1), (-1,-1,-1;0.8))	

Table 3.8 Simulation results of syntactic FCM model with sigmoid function

λ	Concept	Scenario 1			Crisp	Scenario 2		
		Crisp	T1 fuzzy	IT2 fuzzy		T1 fuzzy	IT2 fuzzy	
1.0	C1	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	
	C2	0.938	(0.902,0.935,0.962)	((0.902,0.935,0.962;1), (0.91,0.935,0.956;0.8))	0.938	(0.902,0.935,0.962)	((0.902,0.935,0.962;1), (0.91,0.935,0.956;0.8))	
	C3	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	
	C4	0.914	(0.865,0.921,0.941)	((0.865,0.921,0.941;1), (0.882,0.921,0.934;0.8))	0.914	(0.865,0.921,0.941)	((0.865,0.921,0.941;1), (0.882,0.921,0.934;0.8))	
	C5	0.796	(0.758,0.799,0.828)	((0.758,0.799,0.828;1), (0.775,0.799,0.814;0.8))	0.796	(0.758,0.799,0.828)	((0.758,0.799,0.828;1), (0.775,0.799,0.814;0.8))	
	C6	0.856	(0.764,0.863,0.908)	((0.764,0.863,0.908;1), (0.807,0.863,0.889;0.8))	0.856	(0.764,0.863,0.908)	((0.764,0.863,0.908;1), (0.807,0.863,0.889;0.8))	
3.0	C1	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	
	C2	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	
	C3	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	
	C4	1.000	(0.999,1,1)	((0.999,1,1;1), (0.999,1,1;0.8))	1.000	(0.999,1,1)	((0.999,1,1;1), (0.999,1,1;0.8))	

Table 3.8 continues

Scenario 1					Scenario 2		
λ	Concept	Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
3.0	C5	0.995	(0.989,0.995,0.997)	((0.989,0.995,0.997;1), (0.992,0.995,0.996;0.8))	0.995	(0.989,0.995,0.997)	((0.989,0.995,0.997;1), (0.992,0.995,0.996;0.8))
	C6	0.997	(0.985,0.998,0.999)	((0.985,0.998,0.999;1), (0.993,0.998,0.999;0.8))	0.997	(0.985,0.998,0.999)	((0.985,0.998,0.999;1), (0.993,0.998,0.999;0.8))
	C1	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
5.0	C2	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
	C3	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
	C4	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))
	C5	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))
	C6	1.000	(0.999,1,1)	((0.999,1,1;1),(1,1,1;0.8))	1.000	(0.999,1,1)	((0.999,1,1;1),(1,1,1;0.8))

In the “crisp” column of the Tables 3.7-8, the resulting concept values of the classical FCM simulation are reported and their values are considered as a criterion for performance evaluation of the proposed methods. As a result of FCM simulations of the syntactic model, proposed alpha-cut based methods are in line with the conventional FCM results. The proposed methods successfully reflect the general behavior of the conventional FCM simulations. Among 36 simulation configurations for the syntactic FCM model, larger supports of fuzzy set have been observed in only two cases. These situations are observed in C_6 when the hyperbolic tangent function is used with the parameters 3 and 5. Although the crisp case returns the value of -1, the proposed method results in (-1,-1,0.91) and (-1,-1,0.995). When using sigmoid activation function, unexpected growth in the support of concept values is not observed.

3.6.2 ERP Maintenance Risk Model

The second case study concerns with ERP maintenance risks (Dodurka et al., 2017). There are 13 concepts in the model, and C_1 (continuous organizational changes) and C_{12} (the lack of active participation by external parties) are the input nodes of the FCMs as they are only influencers in the network. Strengths of causal relationships among concepts are given in Table 3.9. T1 fuzzy data are directly taken from the paper (Dodurka et al., 2017).

As Dodurka et al. (2017) employed static analysis, initial concept values for the scenarios are adopted from the first two scenarios of the work (López & Salmeron, 2014).

Table 3.9 Strengths of causal relationships in ERP risk model

Causal connection	Sign	Strength of causal relationship		
		Crisp	T1 fuzzy	IT2 fuzzy
C ₁ →C ₂	+	0.220	(0,0.06,0.6)	((0,0.06,0.6;1),(0.005,0.06,0.595;0.8))
C ₁ →C ₅	+	0.257	(0,0.07,0.7)	((0,0.07,0.7;1),(0.005,0.07,0.695;0.8))
C ₁ →C ₉	+	0.273	(0,0.12,0.7)	((0,0.12,0.7;1),(0.005,0.12,0.695;0.8))
C ₁ →C ₁₀	+	0.147	(0,0.04,0.4)	((0,0.04,0.4;1),(0.005,0.04,0.395;0.8))
C ₁ →C ₁₃	-	0.086	(0,0.05,0.208)	((0,0.05,0.208;1),(0.005,0.05,0.203;0.8))
C ₂ →C ₆	+	0.110	(0,0.03,0.3)	((0,0.03,0.3;1),(0.005,0.03,0.295;0.8))
C ₂ →C ₁₁	+	0.337	(0,0.01,1)	((0,0.01,1;1),(0.005,0.01,0.995;0.8))
C ₂ →C ₁₃	-	0.150	(0,0.108,0.342)	((0,0.108,0.342;1),(0.005,0.108,0.337;0.8))
C ₃ →C ₄	+	0.418	(0,0.255,1)	((0,0.255,1;1),(0.005,0.255,0.995;0.8))
C ₃ →C ₅	+	0.408	(0,0.325,0.9)	((0,0.325,0.9;1),(0.005,0.325,0.895;0.8))
C ₃ →C ₆	+	0.395	(0,0.185,1)	((0,0.185,1;1),(0.005,0.185,0.995;0.8))
C ₃ →C ₇	+	0.183	(0,0.05,0.5)	((0,0.05,0.5;1),(0.005,0.05,0.495;0.8))
C ₄ →C ₃	+	0.300	(0,0.15,0.75)	((0,0.15,0.75;1),(0.005,0.15,0.745;0.8))
C ₄ →C ₅	+	0.268	(0,0.105,0.7)	((0,0.105,0.7;1),(0.005,0.105,0.695;0.8))
C ₅ →C ₄	+	0.147	(0,0.04,0.4)	((0,0.04,0.4;1),(0.005,0.04,0.395;0.8))
C ₅ →C ₆	+	0.542	(0,0.625,1)	((0,0.625,1;1),(0.005,0.625,0.995;0.8))
C ₅ →C ₈	+	0.390	(0,0.17,1)	((0,0.17,1;1),(0.005,0.17,0.995;0.8))
C ₅ →C ₁₃	-	0.050	(0,0.018,0.133)	((0,0.018,0.133;1),(0.005,0.018,0.128;0.8))
C ₆ →C ₈	+	0.220	(0,0.06,0.6)	((0,0.06,0.6;1),(0.005,0.06,0.595;0.8))
C ₆ →C ₁₃	-	0.103	(0,0.109,0.2)	((0,0.109,0.2;1),(0.005,0.109,0.195;0.8))
C ₇ →C ₆	+	0.240	(0,0.12,0.6)	((0,0.12,0.6;1),(0.005,0.12,0.595;0.8))
C ₇ →C ₈	+	0.293	(0,0.08,0.8)	((0,0.08,0.8;1),(0.005,0.08,0.795;0.8))
C ₇ →C ₁₃	-	0.068	(0,0.038,0.167)	((0,0.038,0.167;1),(0.005,0.038,0.162;0.8))
C ₈ →C ₆	+	0.203	(0,0.11,0.5)	((0,0.11,0.5;1),(0.005,0.11,0.495;0.8))
C ₈ →C ₁₁	+	0.293	(0,0.08,0.8)	((0,0.08,0.8;1),(0.005,0.08,0.795;0.8))
C ₈ →C ₁₃	-	0.065	(0,0.029,0.167)	((0,0.029,0.167;1),(0.005,0.029,0.162;0.8))
C ₁₀ →C ₁₃	-	0.036	(0,0.025,0.083)	((0,0.025,0.083;1),(0.005,0.025,0.078;0.8))
C ₁₁ →C ₁₃	-	0.094	(0,0.032,0.25)	((0,0.032,0.25;1),(0.005,0.032,0.245;0.8))
C ₁₂ →C ₂	+	0.347	(0,0.14,0.9)	((0,0.14,0.9;1),(0.005,0.14,0.895;0.8))
C ₁₂ →C ₆	+	0.360	(0,0.28,0.8)	((0,0.28,0.8;1),(0.005,0.28,0.795;0.8))
C ₁₂ →C ₇	+	0.272	(0,0.315,0.5)	((0,0.315,0.5;1),(0.005,0.315,0.495;0.8))
C ₁₂ →C ₈	+	0.128	(0,0.035,0.35)	((0,0.035,0.35;1),(0.005,0.035,0.345;0.8))
C ₁₂ →C ₁₀	+	0.073	(0,0.02,0.2)	((0,0.02,0.2;1),(0.005,0.02,0.195;0.8))
C ₁₂ →C ₁₁	+	0.073	(0,0.02,0.2)	((0,0.02,0.2;1),(0.005,0.02,0.195;0.8))
C ₁₂ →C ₁₃	-	0.189	(0,0.134,0.433)	((0,0.134,0.433;1),(0.005,0.134,0.428;0.8))

In the first scenario, only the initial concept value of C₁₂ is 1 and the other concepts are set to 0. In the second scenario, only the initial concept values of C₃, C₄, and C₅

are set to 1 and the other concepts are set to zero. Results of the ERP risk model simulated with hyperbolic tangent and sigmoid functions are given in Table 3.10-11, respectively.

Table 3.10 Simulation results of ERP risk model with hyperbolic tangent function

λ	Concept	Scenario 1			Scenario 2		
		Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
1.0	C1	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C2	0.265	(0,0.198,0.346)	((0,0.198,0.346;1), (0.063,0.198,0.345;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C3	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.777	(0.031,0.625,0.931)	((0.031,0.625,0.931;1), (0.135,0.625,0.93;0.8))
	C4	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.868	(0.031,0.723,0.98)	((0.031,0.723,0.98;1), (0.164,0.723,0.98;0.8))
	C5	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.895	(0.031,0.789,0.987)	((0.031,0.789,0.987;1), (0.164,0.789,0.987;0.8))
	C6	0.770	(0,0.612,0.935)	((0,0.612,0.935;1), (0.164,0.612,0.933;0.8))	0.971	(0,0.932,0.999)	((0,0.932,0.999;1), (0.208,0.932,0.999;0.8))
	C7	0.245	(0,0.258,0.287)	((0,0.258,0.287;1), (0.063,0.258,0.286;0.8))	0.672	(0,0.436,0.871)	((0,0.436,0.871;1), (0.126,0.436,0.869;0.8))
	C8	0.765	(0,0.524,0.94)	((0,0.524,0.94;1), (0.153,0.524,0.939;0.8))	0.935	(0,0.751,0.997)	((0,0.751,0.997;1), (0.194,0.751,0.997;0.8))
	C9	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C10	0.160	(0,0.104,0.213)	((0,0.104,0.213;1), (0.063,0.104,0.211;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C11	0.809	(0,0.484,0.969)	((0,0.484,0.969;1), (0.151,0.484,0.968;0.8))	0.786	(0,0.53,0.94)	((0,0.53,0.94;1), (0.142,0.53,0.939;0.8))
	C12	0.019	(0.017,0.018,0.019)	((0.017,0.018,0.019;1), (0.017,0.018,0.019;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C13	-0.784	(-0.937,-0.661,0)	((-0.937,-0.661,0;1), (-0.934,-0.661,-0.214;0.8))	-0.815	(-0.95,-0.704,0)	((-0.95,-0.704,0;1), (-0.947,-0.704,-0.23;0.8))
3.0	C1	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C2	0.999	(0,0.998,1)	((0,0.998,1;1), (0.995,0.998,1;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C3	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.999	(0.995,0.998,1)	((0.995,0.998,1;1), (0.995,0.998,1;0.8))
	C4	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	1.000	(0.995,0.999,1)	((0.995,0.999,1;1), (0.995,0.999,1;0.8))
	C5	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	1.000	(0.995,1,1)	((0.995,1,1;1), (0.995,1,1;0.8))
	C6	1.000	(0,1,1)	((0,1,1;1),(0.995,1,1;0.8))	1.000	(0,1,1)	((0,1,1;1), (0.995,1,1;0.8))
	C7	0.999	(0,0.999,1)	((0,0.999,1;1), (0.995,0.999,1;0.8))	0.998	(0,0.996,1)	((0,0.996,1;1), (0.995,0.996,1;0.8))
	C8	1.000	(0,0.998,1)	((0,0.998,1;1), (0.995,0.998,1;0.8))	1.000	(0,0.999,1)	((0,0.999,1;1), (0.995,0.999,1;0.8))
	C9	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C10	0.997	(0,0.995,0.998)	((0,0.995,0.998;1), (0.995,0.995,0.998;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C11	1.000	(0,0.997,1)	((0,0.997,1;1), (0.995,0.997,1;0.8))	0.999	(0,0.997,1)	((0,0.997,1;1), (0.995,0.997,1;0.8))
	C12	0.995	(0.995,0.995,0.995)	((0.995,0.995,0.995;1), (0.995,0.995,0.995;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C13	-1.000	(-1,-1,0)	((-1,-1,0;1), (-1,-1,-0.996;0.8))	-0.999	(-1,-0.999,0)	((-1,-0.999,0;1), (-1,-0.999,-0.996;0.8))
5.0	C1	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C2	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C3	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))
	C4	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))
	C5	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))
	C6	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))
	C7	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))
	C8	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))
	C9	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C10	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C11	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))	1.000	(0,1,1)	((0,1,1;1),(1,1,1;0.8))
	C12	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))	0.000	(0,0,0)	((0,0,0;1),(0,0,0;0.8))
	C13	-1.000	(-1,-1,0)	((-1,-1,0;1),(-1,-1,-1;0.8))	-1.000	(-1,-1,0)	((-1,-1,0;1),(-1,-1,-1;0.8))

Table 3.11 Simulation results of ERP risk model with sigmoid function

λ	Concept	Scenario 1			Scenario 2		
		Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
1.0	C1	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))
	C2	0.756	(0.659,0.696,0.864)	((0.659,0.696,0.864;1), (0.661,0.696,0.864;0.8))	0.756	(0.659,0.696,0.864)	((0.659,0.696,0.864;1), (0.661,0.696,0.864;0.8))
	C3	0.721	(0.659,0.689,0.815)	((0.659,0.689,0.815;1), (0.66,0.689,0.814;0.8))	0.721	(0.659,0.689,0.815)	((0.659,0.689,0.815;1), (0.66,0.689,0.814;0.8))
	C4	0.766	(0.659,0.715,0.889)	((0.659,0.715,0.889;1), (0.661,0.715,0.888;0.8))	0.766	(0.659,0.715,0.889)	((0.659,0.715,0.889;1), (0.661,0.715,0.888;0.8))
	C5	0.815	(0.659,0.749,0.94)	((0.659,0.749,0.94;1), (0.662,0.749,0.94;0.8))	0.815	(0.659,0.749,0.94)	((0.659,0.749,0.94;1), (0.662,0.749,0.94;0.8))
	C6	0.909	(0.659,0.862,0.989)	((0.659,0.862,0.989;1), (0.665,0.862,0.989;0.8))	0.909	(0.659,0.862,0.989)	((0.659,0.862,0.989;1), (0.665,0.862,0.989;0.8))
	C7	0.741	(0.659,0.724,0.827)	((0.659,0.724,0.827;1), (0.661,0.724,0.826;0.8))	0.741	(0.659,0.724,0.827)	((0.659,0.724,0.827;1), (0.661,0.724,0.826;0.8))
	C8	0.840	(0.659,0.729,0.968)	((0.659,0.729,0.968;1), (0.663,0.729,0.967;0.8))	0.840	(0.659,0.729,0.968)	((0.659,0.729,0.968;1), (0.663,0.729,0.967;0.8))
	C9	0.709	(0.659,0.681,0.775)	((0.659,0.681,0.775;1), (0.66,0.681,0.774;0.8))	0.709	(0.659,0.681,0.775)	((0.659,0.681,0.775;1), (0.66,0.681,0.774;0.8))
	C10	0.699	(0.659,0.67,0.761)	((0.659,0.67,0.761;1), (0.661,0.67,0.759;0.8))	0.699	(0.659,0.67,0.761)	((0.659,0.67,0.761;1), (0.661,0.67,0.759;0.8))
	C11	0.793	(0.659,0.681,0.937)	((0.659,0.681,0.937;1), (0.662,0.681,0.937;0.8))	0.793	(0.659,0.681,0.937)	((0.659,0.681,0.937;1), (0.662,0.681,0.937;0.8))
	C12	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))
	C13	0.455	(0.19,0.536,0.659)	((0.19,0.536,0.659;1), (0.198,0.536,0.65;0.8))	0.455	(0.19,0.536,0.659)	((0.19,0.536,0.659;1), (0.198,0.536,0.65;0.8))
3.0	C1	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))
	C2	0.990	(0.944,0.97,0.999)	((0.944,0.97,0.999;1), (0.946,0.97,0.999;0.8))	0.990	(0.944,0.97,0.999)	((0.944,0.97,0.999;1), (0.946,0.97,0.999;0.8))
	C3	0.979	(0.944,0.966,0.995)	((0.944,0.966,0.995;1), (0.945,0.966,0.995;0.8))	0.979	(0.944,0.966,0.995)	((0.944,0.966,0.995;1), (0.945,0.966,0.995;0.8))
	C4	0.990	(0.944,0.978,0.999)	((0.944,0.978,0.999;1), (0.946,0.978,0.999;0.8))	0.990	(0.944,0.978,0.999)	((0.944,0.978,0.999;1), (0.946,0.978,0.999;0.8))
	C5	0.997	(0.944,0.988,1)	((0.944,0.988,1;1), (0.947,0.988,1;0.8))	0.997	(0.944,0.988,1)	((0.944,0.988,1;1), (0.947,0.988,1;0.8))
	C6	1.000	(0.944,0.999,1)	((0.944,0.999,1;1), (0.949,0.999,1;0.8))	1.000	(0.944,0.999,1)	((0.944,0.999,1;1), (0.949,0.999,1;0.8))
	C7	0.986	(0.944,0.982,0.997)	((0.944,0.982,0.997;1), (0.946,0.982,0.997;0.8))	0.986	(0.944,0.982,0.997)	((0.944,0.982,0.997;1), (0.946,0.982,0.997;0.8))
	C8	0.998	(0.944,0.981,1)	((0.944,0.981,1;1), (0.948,0.981,1;0.8))	0.998	(0.944,0.981,1)	((0.944,0.981,1;1), (0.948,0.981,1;0.8))
	C9	0.976	(0.944,0.962,0.993)	((0.944,0.962,0.993;1), (0.945,0.962,0.993;0.8))	0.976	(0.944,0.962,0.993)	((0.944,0.962,0.993;1), (0.945,0.962,0.993;0.8))
	C10	0.972	(0.944,0.954,0.991)	((0.944,0.954,0.991;1), (0.946,0.954,0.99;0.8))	0.972	(0.944,0.954,0.991)	((0.944,0.954,0.991;1), (0.946,0.954,0.99;0.8))
	C11	0.994	(0.944,0.961,1)	((0.944,0.961,1;1), (0.947,0.961,1;0.8))	0.994	(0.944,0.961,1)	((0.944,0.961,1;1), (0.947,0.961,1;0.8))
	C12	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))
	C13	0.104	(0.003,0.426,0.944)	((0.003,0.426,0.944;1), (0.003,0.426,0.936;0.8))	0.104	(0.003,0.426,0.944)	((0.003,0.426,0.944;1), (0.003,0.426,0.936;0.8))
5.0	C1	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
	C2	1.000	(0.993,0.997,1)	((0.993,0.997,1;1), (0.993,0.997,1;0.8))	1.000	(0.993,0.997,1)	((0.993,0.997,1;1), (0.993,0.997,1;0.8))
	C3	0.998	(0.993,0.997,1)	((0.993,0.997,1;1), (0.993,0.997,1;0.8))	0.998	(0.993,0.997,1)	((0.993,0.997,1;1), (0.993,0.997,1;0.8))
	C4	1.000	(0.993,0.998,1)	((0.993,0.998,1;1), (0.993,0.998,1;0.8))	1.000	(0.993,0.998,1)	((0.993,0.998,1;1), (0.993,0.998,1;0.8))
	C5	1.000	(0.993,0.999,1)	((0.993,0.999,1;1), (0.994,0.999,1;0.8))	1.000	(0.993,0.999,1)	((0.993,0.999,1;1), (0.994,0.999,1;0.8))
	C6	1.000	(0.993,1,1)	((0.993,1,1;1), (0.994,1,1;0.8))	1.000	(0.993,1,1)	((0.993,1,1;1), (0.994,1,1;0.8))
	C7	0.999	(0.993,0.999,1)	((0.993,0.999,1;1), (0.993,0.999,1;0.8))	0.999	(0.993,0.999,1)	((0.993,0.999,1;1), (0.993,0.999,1;0.8))
	C8	1.000	(0.993,0.999,1)	((0.993,0.999,1;1), (0.994,0.999,1;0.8))	1.000	(0.993,0.999,1)	((0.993,0.999,1;1), (0.994,0.999,1;0.8))
	C9	0.998	(0.993,0.996,1)	((0.993,0.996,1;1), (0.993,0.996,1;0.8))	0.998	(0.993,0.996,1)	((0.993,0.996,1;1), (0.993,0.996,1;0.8))
	C10	0.998	(0.993,0.995,1)	((0.993,0.995,1;1), (0.993,0.995,1;0.8))	0.998	(0.993,0.995,1)	((0.993,0.995,1;1), (0.993,0.995,1;0.8))
	C11	1.000	(0.993,0.996,1)	((0.993,0.996,1;1), (0.994,0.996,1;0.8))	1.000	(0.993,0.996,1)	((0.993,0.996,1;1), (0.994,0.996,1;0.8))
	C12	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
	C13	0.016	(0,0.099,0.993)	((0,0.099,0.993;1), (0,0.099,0.991;0.8))	0.016	(0,0.099,0.993)	((0,0.099,0.993;1), (0,0.099,0.991;0.8))

According to ERP risk model, results are in comply with the conventional FCM results. Among 36 cases of the ERP risk model, only the value of C_{13} is different from the conventional FCM result. This situation is observed when the activation function parameter is 3 and 5 for the sigmoid function. Note that although the output concepts have nonlinear forms, they are only represented by their end-points in the tables. Defuzzified values of these nonlinear fuzzy sets are equal to 0.4205 and 0.4122 for the activation function parameters 3 and 5, respectively. As a consequence, due to the incorporation of fuzzy concept values, simulation results might change in comparison with the crisp case for some activation function parameters.

3.6.3 Radiotherapy Treatment Planning

The third case study is about radiotherapy treatment planning where the nodes represent tumor localization (C_1), dose prescribed from treatment planning (C_2), machine factors (C_3), human factors (C_4), patient positioning and immobilization (C_5), quality assurance (C_6), and final dose to the target volume (C_7). This problem was modeled and solved by using grey FCMs (Salmeron & Papageorgiou, 2012). In this study, grey weights are assumed to be the end-points of triangular fuzzy numbers so that the crisp, T1 fuzzy, and IT2 fuzzy causal relationships are formed as given in Table 3.12.

Table 3.12 Strengths of causal relationships in radiotherapy planning model

Causal connection	Sign	Strength of causal relationship		
		Crisp	T1 fuzzy	IT2 fuzzy
$C_1 \rightarrow C_7$	+	0.5	(0.4,0.5,0.6)	((0.4,0.5,0.6;1),(0.45,0.5,0.55;0.8))
$C_2 \rightarrow C_1$	+	0.3	(0.05,0.3,0.55)	((0.05,0.3,0.55;1),(0.1,0.3,0.5;0.8))
$C_2 \rightarrow C_7$	+	0.6	(0.5,0.6,0.7)	((0.5,0.6,0.7;1),(0.55,0.6,0.65;0.8))
$C_3 \rightarrow C_2$	-	0.3	(0.2,0.3,0.4)	((0.2,0.3,0.4;1),(0.25,0.3,0.35;0.8))
$C_3 \rightarrow C_7$	-	0.25	(0.15,0.25,0.35)	((0.15,0.25,0.35;1),(0.2,0.25,0.30;0.8))
$C_4 \rightarrow C_5$	-	0.4	(0.3,0.4,0.5)	((0.3,0.4,0.5;1),(0.35,0.4,0.45;0.8))
$C_4 \rightarrow C_7$	-	0.3	(0.2,0.3,0.4)	((0.2,0.3,0.4;1),(0.25,0.3,0.35;0.8))
$C_5 \rightarrow C_4$	-	0.3	(0.2,0.3,0.4)	((0.2,0.3,0.4;1),(0.25,0.3,0.35;0.8))
$C_5 \rightarrow C_7$	+	0.6	(0.5,0.6,0.7)	((0.5,0.6,0.7;1),(0.55,0.6,0.65;0.8))
$C_6 \rightarrow C_2$	+	0.55	(0.3,0.55,0.8)	((0.3,0.55,0.8;1),(0.35,0.55,0.75;0.8))
$C_6 \rightarrow C_7$	+	0.5	(0.25,0.5,0.75)	((0.25,0.5,0.75;1),(0.3,0.5,0.7;0.8))
$C_7 \rightarrow C_1$	+	0.3	(0.05,0.3,0.55)	((0.05,0.3,0.55;1),(0.1,0.3,0.5;0.8))
$C_7 \rightarrow C_2$	+	0.7	(0.45,0.7,0.95)	((0.45,0.7,0.95;1),(0.5,0.7,0.9;0.8))
$C_7 \rightarrow C_5$	+	0.55	(0.45,0.55,0.65)	((0.45,0.55,0.65;1),(0.5,0.55,0.6;0.8))

Based on the original data reported in the paper (Salmeron & Papageorgiou, 2012), grey concept values are transformed into crisp, T1 fuzzy, and IT2 fuzzy formats as given in Table 3.13.

Table 3.13 Initial concept values in radiotherapy planning model

Concept	Scenario 1			Scenario 2		
	Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
C1	0.75	(0.75,0.75,0.75)	(0.75,0.75,0.75;1) (0.75,0.75,0.75;1)	0.80	(0.6,0.8,1)	((0.6,0.8,1;1), (0.65,0.8,0.95;0.8))
C2	0.8	(0.80,0.80,0.80)	(0.80,0.80,0.80;1) (0.80,0.80,0.80;1)	0.85	(0.7,0.85,1)	((0.7,0.85,1;1), (0.75,0.85,0.95;0.8))
C3	0.3	(0.3,0.3,0.3)	(0.3,0.3,0.3;1) (0.3,0.3,0.3;1)	0.25	(0,0.25,0.5)	((0,0.25,0.5;1), (0.05,0.25,0.45;0.8))
C4	0.6	(0.6,0.6,0.6)	(0.6,0.6,0.6;1) (0.6,0.6,0.6;1)	0.45	(0.3,0.45,0.6)	((0.3,0.45,0.6;1), (0.35,0.45,0.55;0.8))
C5	0.7	(0.7,0.7,0.7)	(0.7,0.7,0.7;1) (0.7,0.7,0.7;1)	0.60	(0.5,0.6,0.7)	((0.5,0.6,0.7;1), (0.55,0.6,0.65;0.8))
C6	0.5	(0.5,0.5,0.5)	(0.5,0.5,0.5;1) (0.5,0.5,0.5;1)	0.55	(0.4,0.55,0.7)	((0.4,0.55,0.7;1), (0.45,0.55,0.65;0.8))
C7	0.65	(0.65,0.65,0.65)	(0.65,0.65,0.65;1) (0.65,0.65,0.65;1)	0.25	(0,0.25,0.5)	((0,0.25,0.5;1), (0.25,0.25,0.45;0.8))

Results of the radiotherapy planning model simulated with hyperbolic tangent and sigmoid functions are given in Table 3.14-15, respectively.

Table 3.14 Simulation results of radiotherapy planning model with hyperbolic tangent function

λ	Scenario 1				Scenario 2		
	Concept	Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
1.0	C1	0.901	(0.598,0.901,0.967)	((0.598,0.901,0.967;1), (0.719,0.901,0.959;0.8))	0.901	(0.598,0.901,0.967)	((0.598,0.901,0.967;1), (0.719,0.901,0.959;0.8))
	C2	0.926	(0.861,0.926,0.958)	((0.861,0.926,0.958;1), (0.879,0.926,0.953;0.8))	0.927	(0.861,0.926,0.959)	((0.861,0.926,0.959;1), (0.879,0.926,0.953;0.8))
	C3	0.031	(0.031,0.031,0.031)	((0.031,0.031,0.031;1), (0.031,0.031,0.031;0.8))	0.000	(0,0.031,0.031)	((0,0.031,0.031;1), (0.026,0.031,0.031;0.8))
	C4	-0.792	(-0.842,-0.792,-0.716)	((-0.842,-0.792,-0.716;1), (-0.819,-0.792,-0.758;0.8))	-0.792	(-0.842,-0.792,-0.716)	((-0.842,-0.792,-0.716;1), (-0.819,-0.792,-0.758;0.8))
	C5	0.948	(0.917,0.948,0.966)	((0.917,0.948,0.966;1), (0.935,0.948,0.958;0.8))	0.948	(0.917,0.948,0.966)	((0.917,0.948,0.966;1), (0.935,0.948,0.958;0.8))
	C6	0.031	(0.031,0.031,0.031)	((0.031,0.031,0.031;1), (0.031,0.031,0.031;0.8))	0.031	(0.031,0.031,0.031)	((0.031,0.031,0.031;1), (0.031,0.031,0.031;0.8))
	C7	0.993	(0.978,0.993,0.997)	((0.978,0.993,0.997;1), (0.987,0.993,0.996;0.8))	0.993	(0.978,0.993,0.997)	((0.978,0.993,0.997;1), (0.987,0.993,0.996;0.8))
3.0	C1	1.000	(0.997,1,1)	((0.997,1,1;1), (0.998,1,1;0.8))	1.000	(0.997,1,1)	((0.997,1,1;1), (0.998,1,1;0.8))
	C2	1.000	(0.999,1,1)	((0.999,1,1;1), (1,1,1;0.8))	1.000	(0.999,1,1)	((0.999,1,1;1), (1,1,1;0.8))
	C3	0.995	(0.995,0.995,0.995)	((0.995,0.995,0.995;1), (0.995,0.995,0.995;0.8))	0.000	(0,0.995,0.995)	((0,0.995,0.995;1), (0.995,0.995,0.995;0.8))
	C4	0.963	(0.911,0.963,0.982)	((0.911,0.963,0.982;1), (0.945,0.963,0.975;0.8))	0.963	(-1,0.963,0.982)	((-1,0.963,0.982;1), (-0.999,0.963,0.975;0.8))
	C5	0.998	(0.993,0.998,0.999)	((0.993,0.998,0.999;1), (0.997,0.998,0.999;0.8))	0.998	(0.993,0.998,1)	((0.993,0.998,1;1), (0.997,0.998,1;0.8))
	C6	0.995	(0.995,0.995,0.995)	((0.995,0.995,0.995;1), (0.995,0.995,0.995;0.8))	0.995	(0.995,0.995,0.995)	((0.995,0.995,0.995;1), (0.995,0.995,0.995;0.8))
	C7	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
5.0	C1	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
	C2	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
	C3	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	0.000	(0,1,1)	((0,1,1;1), (1,1,1;0.8))
	C4	0.998	(0.995,0.998,0.999)	((0.995,0.998,0.999;1), (0.997,0.998,0.999;0.8))	0.998	(-1,0.998,0.999)	((-1,0.998,0.999;1), (0.997,0.998,0.999;0.8))
	C5	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
	C6	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))
	C7	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1), (1,1,1;0.8))

Table 3.15 Simulation results of radiotherapy planning model with sigmoid function

λ	Concept	Scenario 1			Scenario 2		
		Crisp	T1 fuzzy	IT2 fuzzy	Crisp	T1 fuzzy	IT2 fuzzy
1.0	C1	0.787	(0.681,0.787,0.868)	((0.681,0.787,0.868;1), (0.704,0.787,0.854;0.8))	0.787	(0.681,0.787,0.868)	((0.681,0.787,0.868;1), (0.704,0.787,0.854;0.8))
	C2	0.837	(0.739,0.837,0.9)	((0.739,0.837,0.9;1), (0.768,0.837,0.887;0.8))	0.837	(0.739,0.837,0.9)	((0.739,0.837,0.9;1), (0.768,0.837,0.887;0.8))
	C3	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))
	C4	0.592	(0.563,0.592,0.619)	((0.563,0.592,0.619;1), (0.578,0.592,0.606;0.8))	0.592	(0.563,0.592,0.619)	((0.563,0.592,0.619;1), (0.578,0.592,0.606;0.8))
	C5	0.729	(0.675,0.729,0.772)	((0.675,0.729,0.772;1), (0.702,0.729,0.752;0.8))	0.729	(0.675,0.729,0.772)	((0.675,0.729,0.772;1), (0.702,0.729,0.752;0.8))
	C6	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))	0.659	(0.659,0.659,0.659)	((0.659,0.659,0.659;1), (0.659,0.659,0.659;0.8))
	C7	0.902	(0.815,0.902,0.949)	((0.815,0.902,0.949;1), (0.854,0.902,0.934;0.8))	0.902	(0.815,0.902,0.949)	((0.815,0.902,0.949;1), (0.854,0.902,0.934;0.8))
3.0	C1	0.992	(0.96,0.992,0.998)	((0.96,0.992,0.998;1), (0.971,0.992,0.998;0.8))	0.992	(0.96,0.992,0.998)	((0.96,0.992,0.998;1), (0.971,0.992,0.998;0.8))
	C2	0.997	(0.982,0.997,0.999)	((0.982,0.997,0.999;1), (0.989,0.997,0.999;0.8))	0.997	(0.982,0.997,0.999)	((0.982,0.997,0.999;1), (0.989,0.997,0.999;0.8))
	C3	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))
	C4	0.837	(0.736,0.837,0.892)	((0.736,0.837,0.892;1), (0.795,0.837,0.868;0.8))	0.837	(0.736,0.837,0.892)	((0.736,0.837,0.892;1), (0.795,0.837,0.868;0.8))
	C5	0.972	(0.945,0.972,0.986)	((0.945,0.972,0.986;1), (0.961,0.972,0.98;0.8))	0.972	(0.945,0.972,0.986)	((0.945,0.972,0.986;1), (0.961,0.972,0.98;0.8))
	C6	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))	0.944	(0.944,0.944,0.944)	((0.944,0.944,0.944;1), (0.944,0.944,0.944;0.8))
	C7	1.000	(0.997,1,1)	((0.997,1,1;1),(0.999,1,1;0.8))	1.000	(0.997,1,1)	((0.997,1,1;1),(0.999,1,1;0.8))
5.0	C1	1.000	(0.996,1,1)	((0.996,1,1;1),(0.997,1,1;0.8))	1.000	(0.996,1,1)	((0.996,1,1;1),(0.997,1,1;0.8))
	C2	1.000	(0.999,1,1)	((0.999,1,1;1),(0.999,1,1;0.8))	1.000	(0.999,1,1)	((0.999,1,1;1),(0.999,1,1;0.8))
	C3	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
	C4	0.966	(0.936,0.966,0.98)	((0.936,0.966,0.98;1), (0.953,0.966,0.974;0.8))	0.966	(0.936,0.966,0.98)	((0.936,0.966,0.98;1), (0.953,0.966,0.974;0.8))
	C5	0.997	(0.991,0.997,0.999)	((0.991,0.997,0.999;1), (0.995,0.997,0.998;0.8))	0.997	(0.991,0.997,0.999)	((0.991,0.997,0.999;1), (0.995,0.997,0.998;0.8))
	C6	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))	0.993	(0.993,0.993,0.993)	((0.993,0.993,0.993;1), (0.993,0.993,0.993;0.8))
	C7	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))	1.000	(1,1,1)	((1,1,1;1),(1,1,1;0.8))

Analyzing the simulation results of the radiotherapy planning model, it is evident that when using sigmoid activation function, resulting concepts are heavily compatible with the conventional FCM results. Additionally, produced results are in line with the results of sigmoid FCMs with grey numbers reported by Salmeron and Papageorgiou (2012). Among 36 different simulation configurations, growth in the support of the fuzzy number is observed in only 2 cases of C_4 when hyperbolic tangent function is used with the parameters 3 and 5. Defuzzied values of these nonlinear fuzzy outputs are 0.523 and 0.900 for the activation function parameters 3 and 5, respectively.

It can be concluded that activation function parameter significantly influences the output concepts. It has been observed that when the activation function parameter is selected as a relatively large value, growth in the support of fuzzy concept values might be encountered. This is mainly due to fact that the interval arithmetic operations are performed in each alpha-level so that the increasing nonlinearity of the intermediate concept values during the iterations might enlarge the interval. Nevertheless, this

problem has been very rarely observed in the proposed methods. Overwhelming majority of the simulations, output fuzzy sets are highly smooth and in comply with the conventional FCM results.

3.7 Employing T1 and IT2 FCMs to Derive Objective Weights

In this section, the proposed T1 and IT2 FCMs are implemented in order to model criteria interactions in MADM to derive objective criteria weights. Computational steps are illustrated in a step-by-step manner.

3.7.1 The Proposed MADM Model

Majority of the MADM methods make use of criteria weights in some of the information processing or aggregation stages. Different MADM approaches might utilize weights in different ways. Despite the differences regarding how the weights are processed, criteria weights have a significant role in the final rankings of the alternatives. For that reason, variety of approaches have been developed for deriving criteria weights. According to Choo, Schoner, and Wedley (1999), decision makers should be informed about the meaning of the weights during the weight elicitation. Plethora of possible interpretations associated with the criteria weights are defined in the literature such as indifference trade-offs, gradient of the overall utility function, relative information content of criteria, relative functional importance, scaling factor for transforming into commensurate value and etc. (Choo et al., 1999). The most common interpretation is relative functional importance where considerable amount of research effort has been dedicated to elicit importance of criteria.

In the literature, weighting methods are divided into two categories: subjective weighting and objective weighting methods. In the subjective weighting methods, decision makers are asked to put forth their preferences about decision criteria. For instance, analytical hierarchy process is one of the subjective weighting methods that requires pairwise comparison matrices to be filled out. On the other hand, decision makers are not asked to assign importance degrees to criteria in the objective weighting

methods. Weights are obtained as a result of mathematical models without any consideration of the preferences of decision makers (Zardari, Ahmed, Shirazi, & Yusop, 2015). The most widely used objective weighting methods are entropy method (Deng, Yeh, & Willis, 2000), the Criteria Importance Through Inter-criteria Correlation, known as CRITIC (Diakoulaki, Mavrotas, & Papayannakis, 1995), standard deviation and variance methods (Zardari et al., 2015). These methods calculate some statistical measures such as correlation, standard deviation/variance by making use of the scores of alternatives with respect to criteria. In other words, objective weighting methods extract criteria weights from some statistical features of the decision matrix. This imposes a serious restriction on the applicability of these methods because scores of alternatives are expressed by means of information granules in many real life problems. Therefore, statistical measures might not be calculated for uncertain data. Additionally, statistical measures might not reflect the relative importance of criteria appropriately. Because weights are multiplied by the decision matrix in subsequent steps, extraction of weights from decision matrix causes a misleading manipulation of the scores of alternatives. Evidently, there is a need for different approaches to derive objective weights. Needless to say that FCMs have much to offer in this respect.

The questions posed to experts should be direct and simple enough to elicit accurate information. The notions of “causality” and “causal dependency” are those of which one is pretty much familiar with instinctively. Representing each criteria as a node and drawing arcs from one another to represent causal dependencies can flourish stakeholders’ understanding of the system and collaborative learning. In addition to advantages in participatory modeling and collaborative model framing, FCMs have some desirable features that no other method can cater that both of the positive and negative causal dependencies are taken into consideration in the FCMs. Majority of the methods in the literature, which can model interdependencies among criteria, admit use of only non-negative values such as analytical network process (ANP), decision making trial and evaluation laboratory (DEMATEL), interpretive structural modeling (ISM) and etc. (Gölcük & Baykasoğlu, 2016). The ability to process uncertain

information, dynamic simulation capabilities, and their neuro-fuzzy infrastructure make FCMs an indispensable part of the decision aid tools.

In this study, recent advances in the convergence properties of the sigmoidal FCMs have been capitalized in order to propose an objective weighting method. In recent years, existence and uniqueness of steady-state concept values with different activation functions have become a critical research topic. Existence of a stable fixed point is a desirable feature, as for long term decisions the initial transient dynamics are not of interest (Hobbs et al., 2002). In applications of the FCMs in decision making, it is important to know the existence of a stable fixed point since decision makers cannot take actions if the simulated FCMs exhibit chaotic behavior. A similar question arises regarding the effect of initial concept values on the existence of stable fixed points. In some FCM settings, it has been observed that FCM converges to the same equilibrium point regardless of the initial conditions (Karagiannis & Groumpos, 2013; Vergini & Groumpos, 2016). These input-insensitive FCMs handicap pattern recognition and related tasks that FCMs are unable to recognize patterns belonging to different decision classes (Nápoles, Concepción, Falcon, Bello, & Vanhoof, 2017; Nápoles, Papageorgiou, Bello, & Vanhoof, 2016). However, the input-insensitive FCMs have a great advantage for MADM problems because they guarantee convergence to a fixed point. The next important question is related to how one can check or control whether the FCM is input-insensitive or not. In order to answer this question, Boutalis, Kottas, and Christodoulou (2008); (Boutalis, Kottas, & Christodoulou, 2009) examined the equilibrium values of the concepts in sigmoidal FCMs by using the contraction-mapping theorem. Certain conditions related to connection matrix were identified that the FCMs converge to unique fixed points. Kottas, Boutalis, and Christodoulou (2010) investigated the conditions related to both connection matrix and parameter of the sigmoid transfer function. Although there are certain conditions related to connection matrix, strength of cause and effect relationships among criteria are elicited from decision makers in MADM problems. Therefore, it is not desirable to impose some constraints on the relationships in order to guarantee stable fixed points. Knight, Lloyd, and Penn (2014) proved that only the parameter of the sigmoid function λ can be used to guarantee unique fixed point attractor. In the study, upper bounds for λ value

of the sigmoid function are identified and mathematically proved that guarantee unique fixed points. In Figure 3.5., upper bounds of the λ parameter with respect to number of concepts are visualized. According to Figure 3.5., if there are three concepts, maximum lambda value is 1.2198. If the number of concepts are 4, 5, and 6, then the maximum λ values become 0.8624, 0.6624, and 0.5359, respectively. The λ parameter of the 0.0544 guarantees unique stable fixed points for up to 50 concepts. Penn et al. (2013) used λ value of 0.05 in order to guarantee convergence to a unique fixed point.

The sigmoidal FCMs with a proper selection of λ parameter have the following key advantages:

1. The resulting concept values lie between 0 and 1 so that they are suitable to be used as criteria weights without any transformation.
2. The FCM simulation produces steady-state concept values.
3. The produced steady-state concept values are unique.
4. The steady-state concept values are insensitive to initial concept values.

The above-mentioned factors have important ramifications for the objective weighting of criteria by using the proposed alpha-cut based computational procedure.

The resulting concept values are independent of initial concept values so only the strengths of influences among the criteria are elicited from the decision makers. Regardless of the initial concept values, the unique stable fixed points are obtained. These unique stable fixed points depend on the interdependencies among criteria so the positive and negative causal influences have direct impact on the results. By this way, influence of relationships among criteria can be reflected onto the decision model. Because interval cognitive maps are simulated for each alpha-level in the proposed computational procedures, there exists a solution for each alpha-level, which enables to reconstruct fuzzy concept values without any loss of information.

In the next section, a numerical study is given in order to show the proposed method. The main steps of the numerical study are illustrated in Figure 3.6.

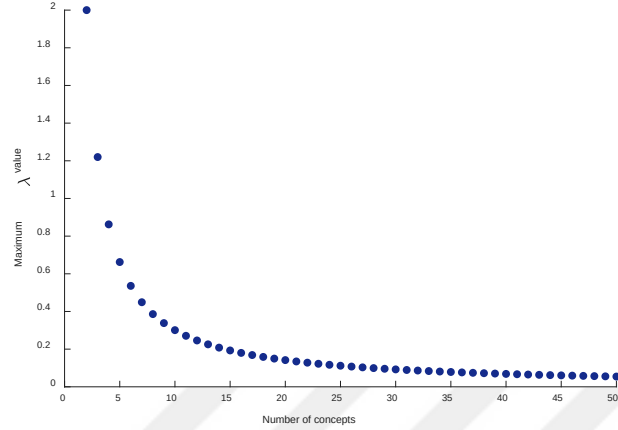


Figure 3.5 Maximum lambda value versus number of concepts

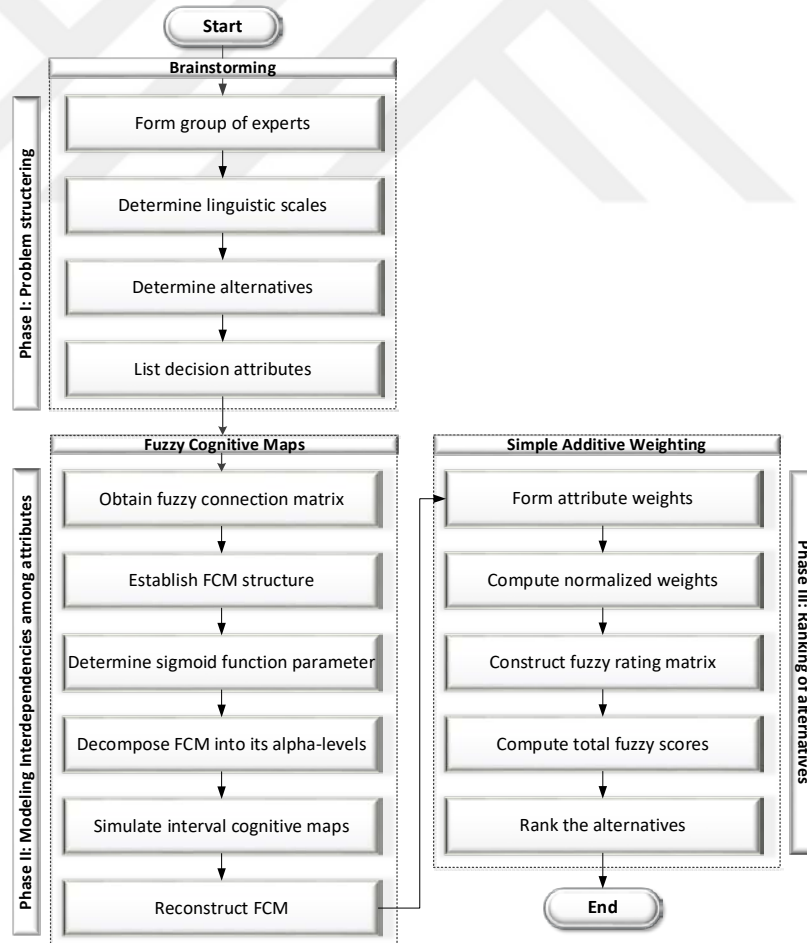


Figure 3.6 Flowchart of the numerical study

3.7.2 Case Study

In recent years, companies have increasingly embraced outsourcing activities in their global logistics services. Companies forming strategic alliances with 3PL providers have opportunities for establishing marketplace legitimacy, concentrating on the core competencies, reducing costs, improving operational efficiency and customer service. According to (Lieb & Miller, 2002), about 60% of the Fortune 500 companies have at least one logistics provider contract. With the growing trend of logistics outsourcing, providers are expanding their services for supporting their clients with expertise and experience.

Main sourcing services are transportation, warehousing, freight consolidation and distribution, packaging, carrier selection, rate negotiation, logistics information systems, cross docking, inventory management, and etc. (Rabinovich, Windle, Dresner, & Corsi, 1999). Among these wide array of logistics functions, warehousing, outbound transportation, custom brokerage and inbound transportation are the most common outsourced activities (Jharkharia & Shankar, 2007). These logistics services require critical examination of expectations of stakeholders as they involve business-to-business relationships. Hence, the user or customer should properly frame business expectations related to quality of the service of the provider. This entails identification of needs from the 3PL service provider. In other words, criteria for evaluating 3PL service providers should be determined in such a way that they suit the needs of outsourcing company.

In order to show the applicability of our proposed approach within a real life context, a company operating in manufacturing sector was contacted and managers' intention to participate in this study was seized. After receiving positive feedbacks and willingness to participate, the proposed method is applied in the company. Before starting the case study, a site visit was conducted to the company and relevant senior managers of the company were informed about this research. As the managers had no background knowledge about fuzzy sets, they were elucidated about the methodological aspects of the study. As the managers were unable to fill out survey

questionnaires, they decided to iteratively frame and refine the model with extensive engagements in the process in a participatory manner. For that reason, majority rule was used to obtain linguistic judgements of the group of managers in the brainstorming sessions.

The 3PL provider selection problem has been formulated in three stages as problem structuring, resolving interrelationships among the criteria and weighting through FCMs, and ranking of alternatives.

3.7.2.1 Problem Structuring

In the first stage, a team of managers, fully supported by the company top management, is created with the aim of identification of suitable alternatives and criteria for selection. The team is headed by five members constituting production, marketing, human resources, finance and information technology managers. In this stage, three possible alternatives are determined based on the discussions in terms of economic considerations to the existing in-house logistics operations. Then, selection criteria are adopted with the help of the factors reported in the literature. The list of criteria compiled from the literature has been examined and the decision hierarchy is drawn as seen in Figure 3.7. The following attributes are determined in the evaluation (Boyson, Corsi, Dresner, & Rabinovich, 1999; Jharkharia & Shankar, 2007; Rabinovich et al., 1999; Razzaque & Sheng, 1998; Simchi-Levi, Kaminsky, & Simchi-Levi, 2000):

1. Cost of service refers to the total cost of the logistics services, which needs to be minimized.
2. Company reputation of a 3PL provider refers to general perception of the people regarding the provider's success in satisfying customer needs. Company reputation is also related to the notion of trust that this criterion has significant role in the initial screening of the alternatives.

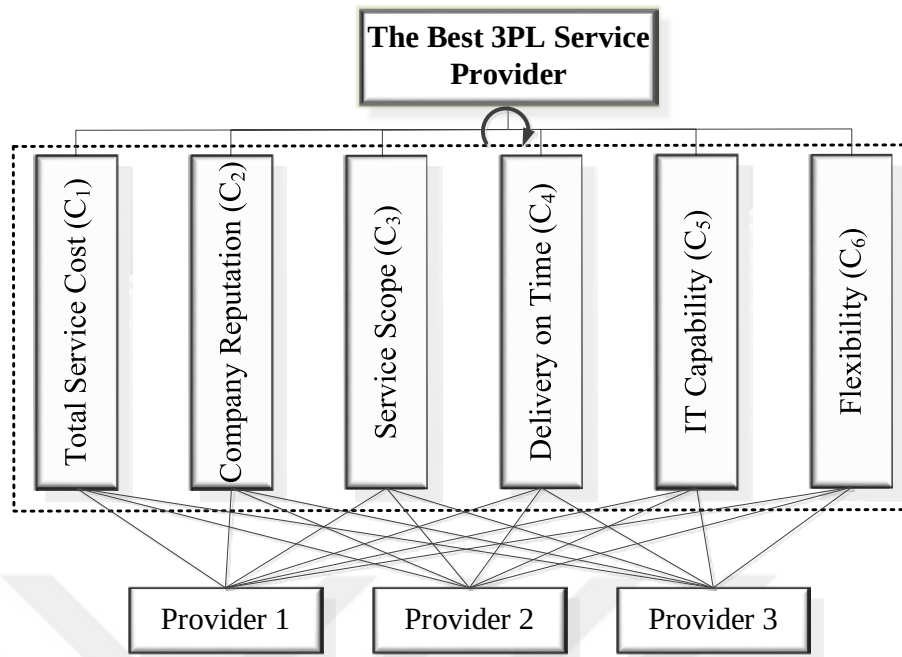


Figure 3.7 3PL service provider selection hierarchy

3. Service scope refers to the spread and diversity of services that a 3PL provider offers. Wider service scope brings about enhanced access to markets and new avenues for the company. Also, distribution and marketing costs can be reduced.
4. Delivery performance is related to speed and reliability of the provided services. Delivery performance plays a significant role on the performance of a 3PL company.
5. IT capability means capabilities in information technologies for tracing flow of information and materials. IT technology helps reduce the uncertainties regarding companies' inventory levels. In the case of higher IT capability of a 3PL service provider, user company does not need to invest in advanced IT technologies for tracing goods and supplying services, which inevitably results in reduced costs and higher service quality.
6. Flexibility refers to adaptability to the changing situations. When the non-routine requests are delivered, flexible providers can quickly respond to the customer needs and enable company to provide customized services.

3.7.2.2 Resolving Interdependencies via FCMs

In this stage, the proposed computational procedures for T1 and IT2 FCMs have been implemented in order to simulate FCMs without information loss. Initially, collected data regarding the problem in hand is described and then results of the proposed methods are given in detail.

The managers are asked to articulate possible relationships among 3PL service provider selection criteria. Here, only positive and negative relationships are identified without strength of influences. As a result, the cognitive map, as can be seen in Figure 3.8., is drawn.

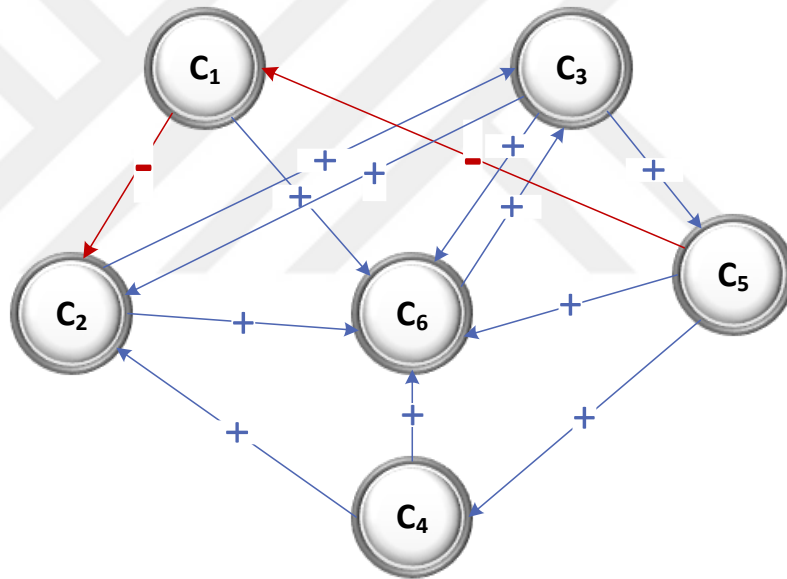


Figure 3.8 Cognitive map of the 3PL provider selection criteria

According to managers, the company reputation is negatively influenced by the cost of the service, which implies that the increased cost of the service damages company reputation. On the other hand, IT capability of the 3PL induces lower operating costs. For these reasons, directed arcs $C_1 \rightarrow C_2$ and $C_5 \rightarrow C_1$ are interpreted as negative causality and indicated by red lines. Blue lines show the positive causality and 11 relationships are defined as positive.

Although the positive and negative influences are defined for the relationships between criteria, they do not give any information about the degree of influence. In the next step, the strengths of influences are defined as given in Table 3.16.

As quantitative and empirically tested information is not available for the degree of influences, this information is collected by means of fuzzy sets. First, linguistic labels are defined as extremely low, very low, low, medium, high, very high and extremely high. Then, the managers are asked to assign linguistic labels to the strengths of influences. Here, majority rule is employed. If the linguistic label “very low” is assigned to the relationships between criteria by the majority of the decision makers, then the fuzzy number corresponding to the linguistic value “very low” is used as the strength of influence. If majority rule cannot be satisfied, then group discussions are promoted and consensus is sought.

Table 3.16 Strengths of causal relationships

Causal connection	Sign	Strength of causal relationship		
		Crisp	T1 fuzzy	IT2 fuzzy
$C_1 \rightarrow C_2$	-	0.5	(0.35,0.5,0.65)	((0.35,0.5,0.65;1),(0.4,0.5,0.6;0.8))
$C_1 \rightarrow C_6$	+	0.1	(0,0.1,0.2)	((0,0.1,0.2;1),(0.05,0.1,0.15;0.8))
$C_2 \rightarrow C_3$	+	0.35	(0.2,0.35,0.5)	((0.2,0.35,0.5;1),(0.25,0.35,0.45;0.8))
$C_2 \rightarrow C_6$	+	0.65	(0.5,0.65,0.8)	((0.5,0.65,0.8;1),(0.55,0.65,0.75;0.8))
$C_3 \rightarrow C_2$	+	0.35	(0.2,0.35,0.5)	((0.2,0.35,0.5;1),(0.25,0.35,0.45;0.8))
$C_3 \rightarrow C_5$	+	0.23	(0.1,0.2,0.35)	((0.1,0.2,0.35;1),(0.15,0.2,0.3;0.8))
$C_3 \rightarrow C_6$	+	0.65	(0.5,0.65,0.8)	((0.5,0.65,0.8;1),(0.55,0.65,0.75;0.8))
$C_4 \rightarrow C_2$	+	0.65	(0.5,0.65,0.8)	((0.5,0.65,0.8;1),(0.55,0.65,0.75;0.8))
$C_4 \rightarrow C_6$	+	0.65	(0.5,0.65,0.8)	((0.5,0.65,0.8;1),(0.55,0.65,0.75;0.8))
$C_5 \rightarrow C_1$	-	0.9	(0.8,0.9,1)	((0.8,0.9,1;1),(0.85,0.9,0.95;0.8))
$C_5 \rightarrow C_4$	+	0.50	(0.35,0.5,0.65)	((0.35,0.5,0.65;1),(0.4,0.5,0.6;0.8))
$C_5 \rightarrow C_6$	+	0.50	(0.35,0.5,0.65)	((0.35,0.5,0.65;1),(0.4,0.5,0.6;0.8))
$C_6 \rightarrow C_3$	+	0.50	(0.35,0.5,0.65)	((0.35,0.5,0.65;1),(0.4,0.5,0.6;0.8))

Having obtained the degree of relationships among criteria, sigmoid function parameter is determined and the FCM is simulated. As mentioned in the previous section, maximum λ value of the sigmoid function for 6 concepts is 0.5359. For the sake of simplicity, λ parameter is defined as 0.50 that guarantees unique stable fixed-points. Referring to the previous section, this λ value also ensures that unique stable fixed points are independent of the initial concept values.

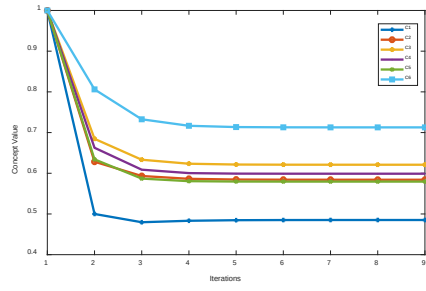
Given the strengths of influences in Table 3.5., both of the T1 and IT2 FCMs are simulated. First, results of the proposed alpha-cuts based computational procedures for T1 FCMs are given. The produced results are compared with the extension principle based method in order to verify the results. Then, IT2 FCMs are simulated and the objective weights are derived.

As mentioned earlier, resulting unique concept values are independent of the initial concept values. In order to prove that three different initialization strategies have been carried out. In the first strategy, unity activations are used which means that all of the concept values are drivers of the system (Penn et al., 2013). In the second strategy, zero activations are used referring the fact that all of the initial concept values are set to zero. The last strategy is using random activations. Here, initial concept values are randomly assigned fuzzy numbers. The simulation results belonging to left and right end-points for the alpha level zero are given in in Figure 3.9.

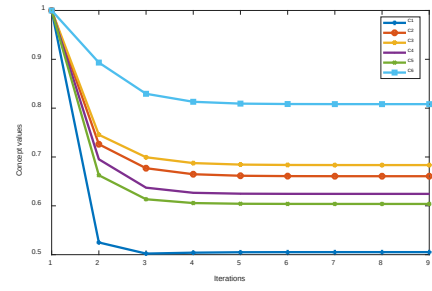
Analyzing the results in Figure 3.9. show that the resulting concept values are the same regardless of the initial concept values. For instance, resulting left end-point of the C_6 is 0.7128 for unity activation, zero activation, and random activation. Similarly, it can be easily seen that the resulting right end-point of the C_6 is 0.808 for all of the three activation strategies. Therefore, independence from the initial concept values is verified for all of the concepts.

Due to the ease of understanding, all of the criteria are considered as drivers of the system and unity activation is employed in the rest of the calculations. Table 3.17 shows the concept values at each iteration when the alpha-level equals to zero. It takes nine iterations to reach steady-state concept values.

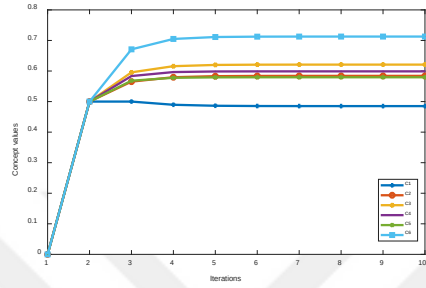
According to Table 3.17, all initial values are represented by ones. In other words, all initial values are considered a triangular fuzzy number as (1,1,1). Although initial concept values have no impact on the resulting concept values due to the proper selection of sigmoid function parameter, unity activation has the advantage of interpretability.



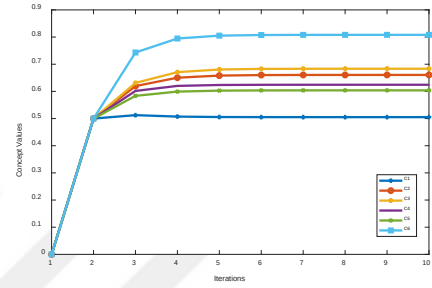
a) Unity activations (left end-points)



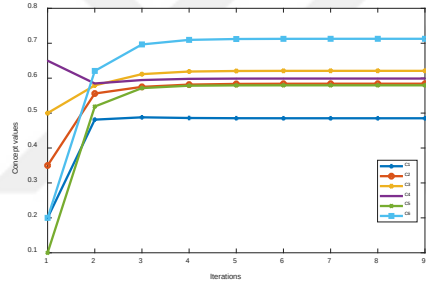
b) Unity activations (right end-points)



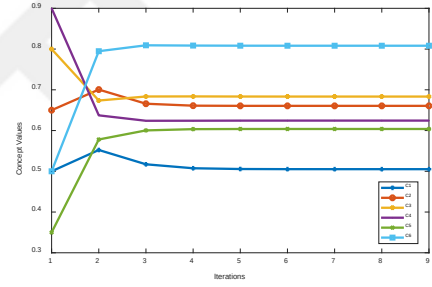
c) Zero activations (left end-points)



d) Zero activations (right end-points)



e) Random activations (left end-points)



f) Random activations (right end-points)

Figure 3.9 Convergence graphs for different activation levels at alpha-level zero

Table 3.17 Concept values of criteria at alpha-level zero

Iteration	Concepts					
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
1	[1, 1]	[1, 1]	[1, 1]	[1, 1]	[1, 1]	[1, 1]
2	[0.5, 0.525]	[0.628, 0.726]	[0.685, 0.746]	[0.663, 0.695]	[0.634, 0.663]	[0.806, 0.893]
3	[0.48, 0.502]	[0.593, 0.677]	[0.633, 0.699]	[0.609, 0.637]	[0.587, 0.613]	[0.733, 0.829]
4	[0.483, 0.504]	[0.586, 0.665]	[0.623, 0.688]	[0.6, 0.627]	[0.581, 0.606]	[0.717, 0.813]
5	[0.485, 0.505]	[0.585, 0.662]	[0.621, 0.684]	[0.599, 0.625]	[0.58, 0.604]	[0.714, 0.809]
6	[0.485, 0.505]	[0.584, 0.661]	[0.621, 0.684]	[0.599, 0.625]	[0.58, 0.604]	[0.713, 0.808]
7	[0.485, 0.505]	[0.584, 0.661]	[0.621, 0.683]	[0.599, 0.624]	[0.58, 0.604]	[0.713, 0.808]
8	[0.485, 0.505]	[0.584, 0.661]	[0.621, 0.683]	[0.599, 0.624]	[0.58, 0.604]	[0.713, 0.808]
9	[0.485, 0.505]	[0.584, 0.661]	[0.621, 0.683]	[0.599, 0.624]	[0.58, 0.604]	[0.713, 0.808]

Hence, all criteria are considered as drivers of the system, and thus it is advisable to use unity activations. Similar to the data obtained in Table 3.6, resulting concept

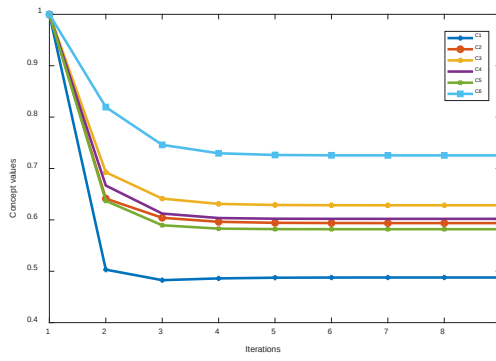
values are computed for the total of 101 alpha-levels where $\alpha \in \{0, 0.01, 0.02, \dots, 0.99, 1\}$. Convergence graphs for alpha levels 0.25, 0.50, 0.75, and 1 are visualized in Figure 3.10. Note that convergence graph for the alpha level 0 has already been given in Figure 3.9.

According to Figure 3.10., smaller alpha levels demonstrates greater uncertainty as seen from the left and right end-points of the resulting concepts. When the alpha level approaches to 1, uncertainty disappears as left and right end-points equal to each other. For instance, as it can be seen from the topmost lines of the Figure 3.10a and Figure 3.10b, left and right end points of the C_6 at alpha level 0.25 equal to 0.7255 and 0.7972. At alpha level 0.50, they become 0.7380 and 0.7859, respectively. The uncertainty boundaries come to 0.7503 and 0.7743 at alpha level 0.75. Finally, left and right end-points become 0.7624 for both of them at alpha level 1.

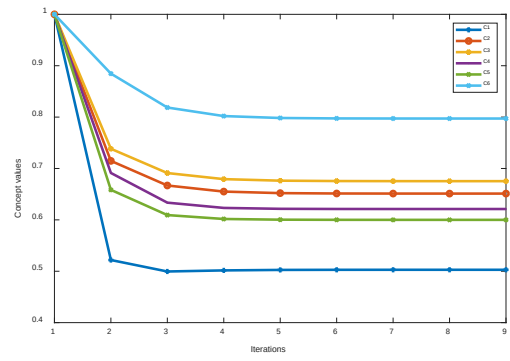
As in the given alpha levels in Figure 3.10, the interval cognitive map simulations are conducted for total of 101 alpha levels. Based on the resulting concept values, fuzzy sets are reconstructed. Alpha-cut based inference computations for the T1 FCMs result in the triangular fuzzy numbers as given in Figure 3.11.

As it can be seen from the Figure 3.11., linearity is endured in the resulting fuzzy sets due to the relatively small λ parameter of the sigmoid function. This is an advantage for decision making applications as there is no compelling evidence to use highly nonlinear mappings in FCM simulation. Also, resulting T1 fuzzy numbers are computationally undemanding which can be managed to steer standard fuzzy arithmetic operations in a simple manner. The produced results are compared with the extension principle based inference method as elaborated in Section 3.4. The resulting concept values for the extension principle based inference method is visualized in Figure 3.12.

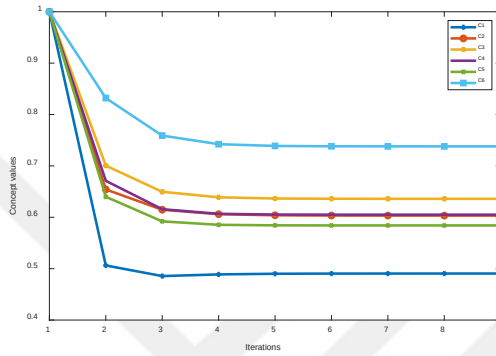
Analysis of the resulting concept values reveals that the produced results are quite similar even though the extension principle based method employs defuzzification at the end of each iteration.



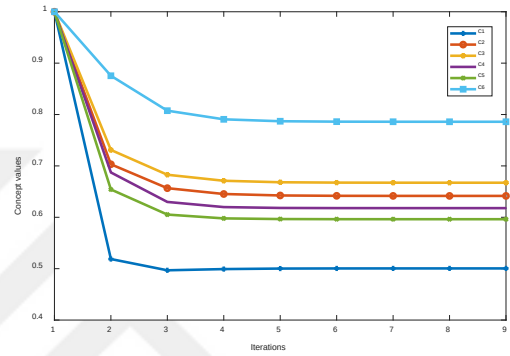
a) Alpha level 0.25 (left end-point)



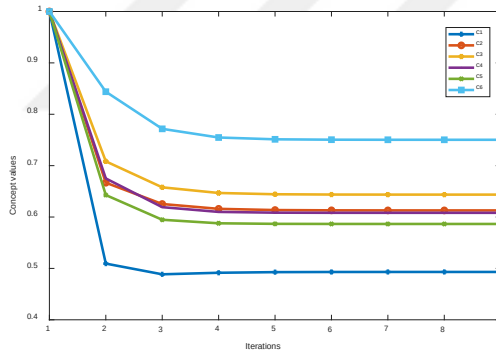
b) Alpha level 0.25 (right end-point)



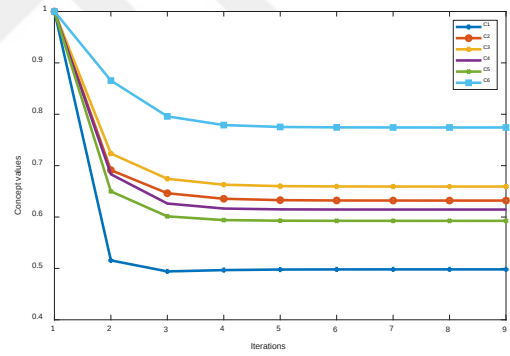
c) Alpha level 0.50 (left end-point)



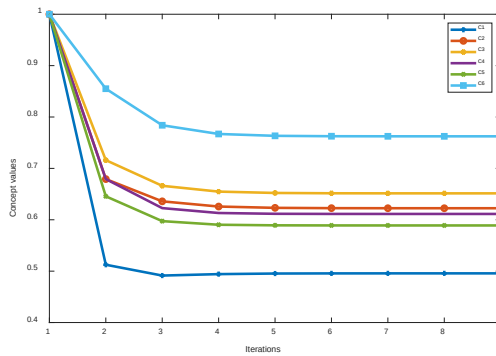
d) Alpha level 0.50 (right end-point)



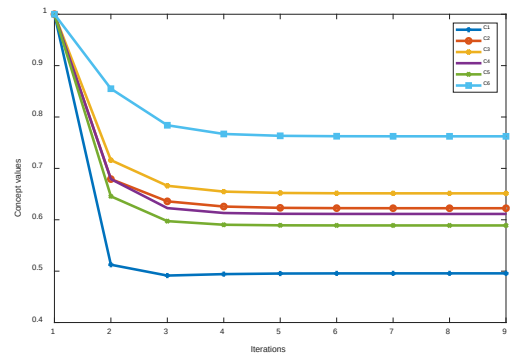
e) Alpha level 0.75 (left end-point)



f) Alpha level 0.75 (right end-point)



g) Alpha level 1 (left end-point)



h) Alpha level 1 (right end-point)

Figure 3.10 Convergence graphs at different alpha-levels

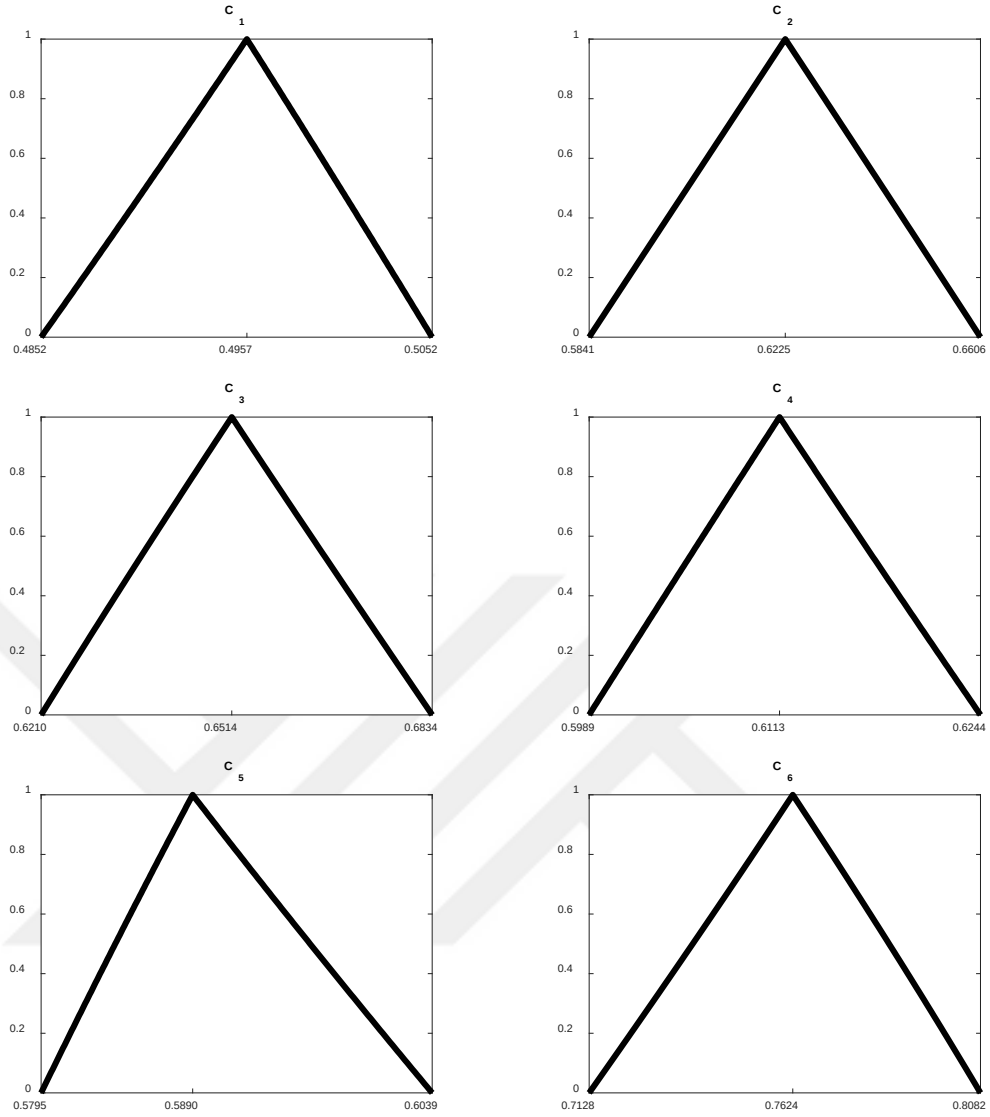


Figure 3.11 Resulting concepts for alpha-cut based inference method

The central difference between two results is that produced fuzzy sets derived from extension principle based inference method have narrower support. This can be attributed to the potential information loss due to the defuzzification operations.

The produced results are compared by using the fuzziness and ambiguity measures. Comparison of fuzziness measures are given in Table 3.18.

According to Table 3.18, alpha-cut based methods produce higher fuzziness measures in comparison to extension principle based method. The biggest difference of 0.01 is observed in the C_6 .

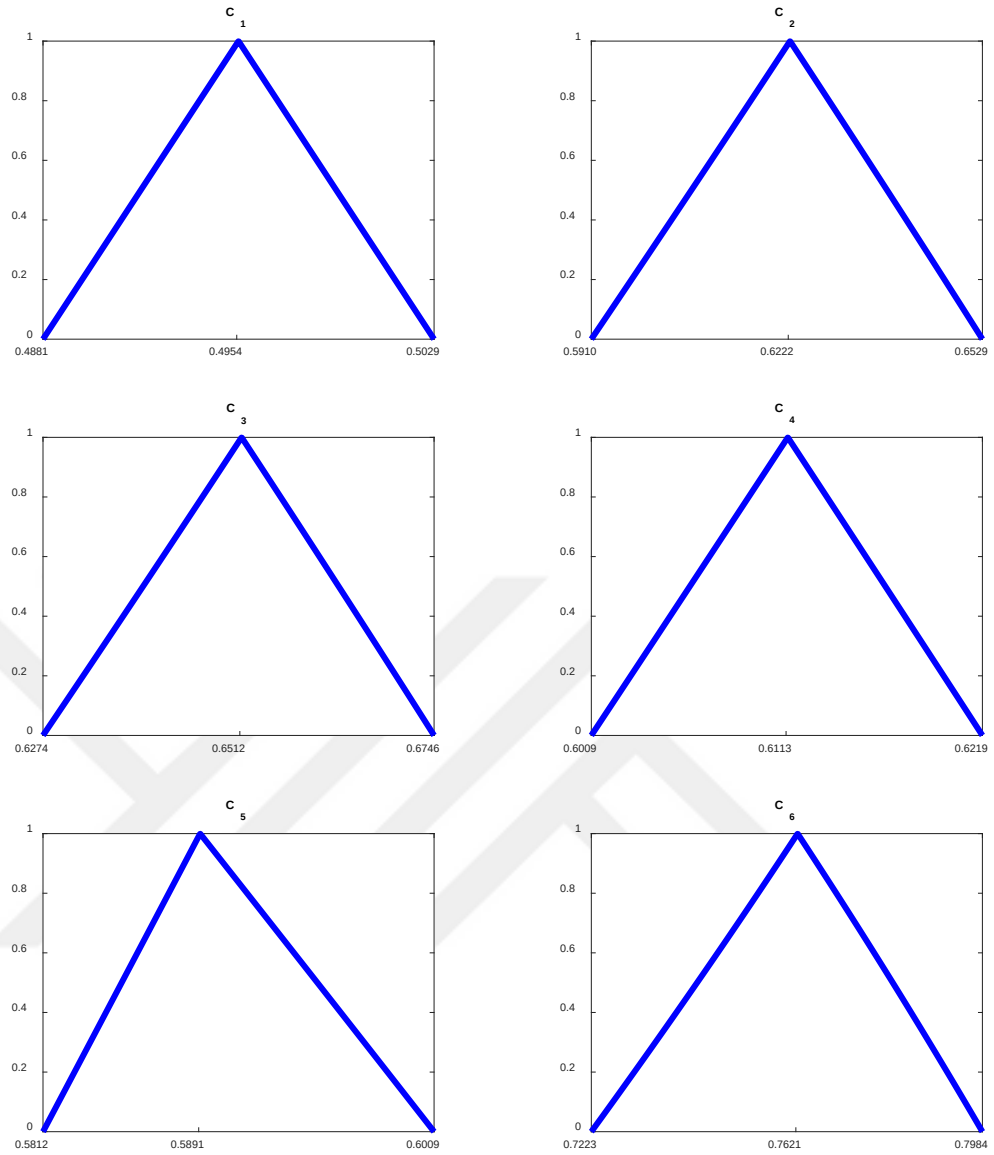


Figure 3.12 Resulting concepts for extension principle based inference method

Table 3.18 Comparison of resulting concepts in terms of fuzziness measure

Concepts	Inference method	
	Alpha-cut based method	Extension principle based method
C_1	0.010	0.007
C_2	0.038	0.031
C_3	0.031	0.024
C_4	0.013	0.011
C_5	0.012	0.010
C_6	0.048	0.038

The difference between alpha cut based and extension principle based method for C_2 and C_3 are 0.007. The differences for C_1 , C_4 , C_5 are 0.003, 0.002, 0.002,

respectively. Similarly, comparison of the two methods in terms of ambiguity measure is given in Table 3.19.

Table 3.19 Comparison of resulting concepts in terms of ambiguity measure

Concepts	Inference method	
	Alpha-cut based method	Extension principle based method
C ₁	0.010	0.007
C ₂	0.037	0.030
C ₃	0.031	0.023
C ₄	0.013	0.010
C ₅	0.012	0.010
C ₆	0.046	0.037

Analyzing the ambiguity measure results, the difference between produced ambiguity measures for C₆ is 0.009. This quantity is followed by 0.008 for the C₃. The difference between ambiguity measures of the two methods for C₂ turns out to be 0.007. The difference of 0.003 is calculated for C₁ and C₄. Finally, 0.002 is found for the C₅.

Note that alpha-cut based method produces higher fuzziness and ambiguity measures in all of the concepts, which is due to the fact that defuzzification is not employed and information loss is not matter of discussion. It has been observed that produced output concepts as a result of extension-principle based FCMs are slightly narrower than those of alpha-cut based results. As the alpha-cut based computations do not employ defuzzification, all of the possible situations are captured by interval arithmetic operations. Due to this property, alpha-cut based methods are able to model ambiguity more precisely. On the other hand, results produced by alpha-cut based method are verified via extension principle based method as the resulting fuzzy sets are quite similar. As the alpha-cut based method is validated by the extension principle based method, natural generalization of the alpha-cut based method to the IT2 FCMs is computationally examined.

Applying the computational steps given in Section 3.5, the resulting concepts for IT2 FCMs are given in Figure 3.13. When the IT2 fuzzy sets are obtained as a result of FCM simulation, they are considered as the criteria weights. The next step is to normalize these weights. In this study, alpha-cut based normalization method which

was proposed by Mendel and Rajati (2014) has been utilized. The computational details of the normalization operation are given as follows:

Suppose that there are n criteria in the decision making model and weights are represented by IT2 fuzzy numbers as $\tilde{W}_i$ where $i \in \{1, 2, \dots, n\}$. As with the equations given in the alpha cut based inference computations, weights are deconstructed into their alpha levels by choosing an appropriate number of alpha-cuts. Let the weight of the i th criterion is represented by lower and upper membership functions $[\underline{W}_i, \bar{W}_i]$. Alpha cut based normalization formula for the lower membership function can be given as (Mendel & Rajati, 2014):

$$\begin{cases} \underline{N}_{i_\alpha}^L = \frac{\underline{W}_{i_\alpha}^L}{\underline{W}_{i_\alpha}^L + \sum_{j=1, j \neq i}^n \underline{W}_{j_\alpha}^R} \\ \underline{N}_{i_\alpha}^R = \frac{\underline{W}_{i_\alpha}^R}{\underline{W}_{i_\alpha}^R + \sum_{j=1, j \neq i}^n \underline{W}_{j_\alpha}^L} \end{cases} \quad (3.52)$$

where $\underline{N}_{i_\alpha}^L$ and $\underline{N}_{i_\alpha}^R$ denotes left and right end points of the lower membership function belonging to normalized weight of the i th criterion at alpha level.

Similarly, alpha cut based normalization formula for the upper membership function can be given as:

$$\begin{cases} \bar{N}_{i_\alpha}^L = \frac{\bar{W}_{i_\alpha}^L}{\bar{W}_{i_\alpha}^L + \sum_{j=1, j \neq i}^n \bar{W}_{j_\alpha}^R} \\ \bar{N}_{i_\alpha}^R = \frac{\bar{W}_{i_\alpha}^R}{\bar{W}_{i_\alpha}^R + \sum_{j=1, j \neq i}^n \bar{W}_{j_\alpha}^L} \end{cases} \quad (3.53)$$

where $\bar{N}_{i_\alpha}^L$ and $\bar{N}_{i_\alpha}^R$ denotes left and right end points of the upper membership function.

Then, all of the left and right coordinates are combined in order to form lower and upper membership functions as shown in the decomposition theorem.

Because T1 fuzzy set is the special case of the IT2 fuzzy sets, the same normalization operation has been used for T1 fuzzy sets as well.

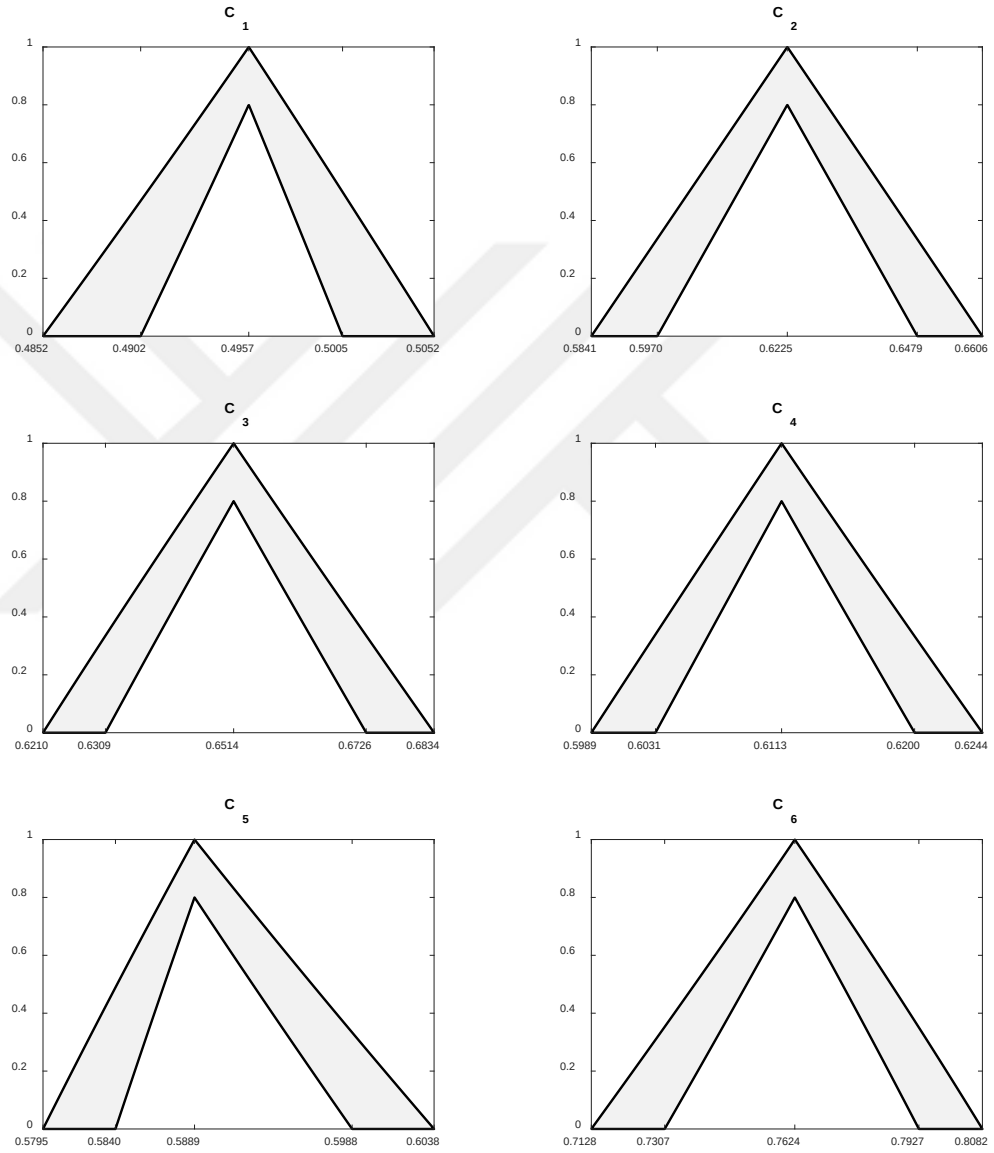


Figure 3.13 Resulting concepts for IT2 FCMs

The normalized weights are given in Table 3.20.

Table 3.20 Normalized weights of criteria

Concept	Crisp	T1 Fuzzy	IT2 Fuzzy
C1	0.133	(0.12551,0.13282,0.14028)	((0.12551,0.13282,0.14028;1), (0.12825,0.13282,0.13727;0.8))
C2	0.167	(0.15334,0.16678,0.1806)	((0.15334,0.16678,0.1806;1), (0.15788,0.16678,0.17573;0.8))
C3	0.174	(0.16242,0.17453,0.18755)	((0.16242,0.17454,0.18755;1), (0.16644,0.17454,0.18289;0.8))
C4	0.164	(0.15515,0.1638,0.17312)	((0.15515,0.1638,0.17312;1), (0.15806,0.1638,0.16974;0.8))
C5	0.158	(0.15008,0.1578,0.16747)	((0.15008,0.1578,0.16747;1), (0.15298,0.1578,0.16401;0.8))
C6	0.204	(0.18805,0.20428,0.2198)	((0.18805,0.20428,0.2198;1), (0.19379,0.20428,0.21436;0.8))

The normalized weights are used in the final ranking of the alternatives.

3.7.2.3 Ranking of Alternatives

In the ranking of alternatives, fuzzy SAW method has been utilized (S.-Y. Chou, Chang, & Shen, 2008). The primary reason to choose the SAW method is its inherent practicality. The computational details of the fuzzy SAW method are given as follows:

Suppose that the fuzzy rating matrix based on the IT2 fuzzy sets are given as

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad (3.54)$$

where $\tilde{x}_{ij}$ represents fuzzy rating value of the i th alternative.

Based on the fuzzy simple additive weighting, total fuzzy scores for individual alternatives are calculated as (Chou et al., 2008):

$$\begin{aligned}
\tilde{\tilde{F}} &= \tilde{\tilde{D}} \otimes \tilde{\tilde{N}}^T = \begin{bmatrix} \tilde{\tilde{x}}_{11} & \tilde{\tilde{x}}_{12} & \cdots & \tilde{\tilde{x}}_{1n} \\ \tilde{\tilde{x}}_{21} & \tilde{\tilde{x}}_{22} & \cdots & \tilde{\tilde{x}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{m1} & \tilde{\tilde{x}}_{m2} & \cdots & \tilde{\tilde{x}}_{mn} \end{bmatrix} \otimes \begin{bmatrix} \tilde{\tilde{N}}_1 \\ \tilde{\tilde{N}}_2 \\ \vdots \\ \tilde{\tilde{N}}_n \end{bmatrix} \\
&= \begin{bmatrix} \tilde{\tilde{x}}_{11} \otimes \tilde{\tilde{N}}_1 \oplus \tilde{\tilde{x}}_{12} \otimes \tilde{\tilde{N}}_2 \oplus \cdots \oplus \tilde{\tilde{x}}_{1n} \otimes \tilde{\tilde{N}}_n \\ \tilde{\tilde{x}}_{21} \otimes \tilde{\tilde{N}}_1 \oplus \tilde{\tilde{x}}_{22} \otimes \tilde{\tilde{N}}_2 \oplus \cdots \oplus \tilde{\tilde{x}}_{2n} \otimes \tilde{\tilde{N}}_n \\ \vdots \\ \tilde{\tilde{x}}_{m1} \otimes \tilde{\tilde{N}}_1 \oplus \tilde{\tilde{x}}_{m2} \otimes \tilde{\tilde{N}}_2 \oplus \cdots \oplus \tilde{\tilde{x}}_{mn} \otimes \tilde{\tilde{N}}_n \end{bmatrix} \quad (3.55)
\end{aligned}$$

where $\tilde{\tilde{F}} = [\tilde{\tilde{f}}_1, \tilde{\tilde{f}}_2, \dots, \tilde{\tilde{f}}_m]^T$ represents total fuzzy scores of the alternatives.

First, fuzzy ratings matrices are collected from decision makers. Majority rule is applied for each rating of the alternatives. The SAW results are computed for the T1 and IT2 fuzzy numbers corresponding to linguistic labels of the ratings. To avoid repetition, only the fuzzy numbers corresponding to the linguistic labels are given. Fuzzy rating matrix comprising of T1 fuzzy numbers is given in Table 3.21.

Table 3.21 T1 fuzzy rating matrix

	Provider 1	Provider 2	Provider 3
C ₁	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.7,0.9,1)
C ₂	(0.5,0.7,0.9)	(0.7,0.9,1)	(0.9,1,1)
C ₃	(0,0.1,0.3)	(0.9,1,1)	(0.9,1,1)
C ₄	(0,0.1,0.3)	(0.3,0.5,0.7)	(0.7,0.9,1)
C ₅	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0.1,0.3,0.5)
C ₆	(0.7,0.9,1)	(0.7,0.9,1)	(0.7,0.9,1)

The fuzzy rating matrix is weighted by the normalized weights as given in Table 3.20. The weighted rating matrix is calculated as given in Table 3.22.

Table 3.22 T1 fuzzy weighted rating matrix

	Provider 1	Provider 2	Provider 3
C ₁	(0.087854,0.11954,0.14028)	(0.012551,0.039845,0.070138)	(0.087854,0.11954,0.14028)
C ₂	(0.076672,0.11674,0.16254)	(0.10734,0.1501,0.1806)	(0.13801,0.16678,0.1806)
C ₃	(0,0.017453,0.056266)	(0.14617,0.17453,0.18755)	(0.14617,0.17453,0.18755)
C ₄	(0,0.01638,0.051937)	(0.046544,0.081898,0.12119)	(0.1086,0.14742,0.17312)
C ₅	(0.045024,0.0789,0.11723)	(0.045024,0.0789,0.11723)	(0.015008,0.04734,0.083734)
C ₆	(0.13163,0.18385,0.2198)	(0.13163,0.18385,0.2198)	(0.13163,0.18385,0.2198)

In a similar fashion with the T1 fuzzy sets, fuzzy ratings matrix in the form of IT2 fuzzy sets is formed as given in Table 3.23.

Table 3.23 IT2 fuzzy rating matrix

	Provider 1	Provider 2	Provider 3
C ₁	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))	((0.1,0.3,0.5;1), (0.15,0.3,0.45;0.8))	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))
C ₂	((0.5,0.7,0.9;1), (0.55,0.7,0.85;0.8))	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))	((0.9,1,1;1), (0.95,0.95,1;0.8))
C ₃	((0.0,0.1,0.3;1), (0.05,0.1,0.25;0.8))	((0.9,1,1;1), (0.95,0.95,1;0.8))	((0.9,1,1;1), (0.95,0.95,1;0.8))
C ₄	((0.0,0.1,0.3;1), (0.05,0.1,0.25;0.8))	((0.3,0.5,0.7;1), (0.35,0.5,0.65;0.8))	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))
C ₅	((0.3,0.5,0.7;1), (0.35,0.5,0.65;0.8))	((0.3,0.5,0.7;1), (0.35,0.5,0.65;0.8))	((0.1,0.3,0.5;1), (0.15,0.3,0.45;0.8))
C ₆	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))	((0.7,0.9,1;1), (0.75,0.9,0.95;0.8))

The IT2 fuzzy rating matrix is weighted by the normalized IT2 fuzzy weights given in Table 3.20. The weighted rating matrix is calculated as given in Table 3.24.

Table 3.24 IT2 fuzzy weighted rating matrix

	Provider 1	Provider 2	Provider 3
C ₁	((0.087857,0.11954,0.14028;1), (0.096188,0.11954,0.13041;0.8))	((0.012551,0.039846,0.07014;1), (0.019238,0.039846,0.061772;0.8))	((0.087857,0.11954,0.14028;1), (0.096188,0.11954,0.13041;0.8))
C ₂	((0.07667,0.11675,0.16254;1), (0.086834,0.11675,0.14937;0.8))	((0.10734,0.1501,0.1806;1), (0.11841,0.1501,0.16694;0.8))	((0.13801,0.16678,0.1806;1), (0.14999,0.15844,0.17573;0.8))
C ₃	((0.0,0.017454,0.056265;1), (0.008322,0.017454,0.045722;0.8))	((0.14618,0.17454,0.18755;1), (0.15812,0.16581,0.18289;0.8))	((0.14618,0.17454,0.18755;1), (0.15812,0.16581,0.18289;0.8))
C ₄	((0.0,0.01638,0.051936;1), (0.007903,0.01638,0.042435;0.8))	((0.046545,0.0819,0.12118;1), (0.055321,0.0819,0.11033;0.8))	((0.10861,0.14742,0.17312;1), (0.11855,0.14742,0.16125;0.8))
C ₅	((0.045024,0.0789,0.11723;1), (0.053543,0.0789,0.10661;0.8))	((0.045024,0.0789,0.11723;1), (0.053543,0.0789,0.10661;0.8))	((0.015008,0.04734,0.083735;1), (0.022947,0.04734,0.073804;0.8))
C ₆	((0.13163,0.18385,0.2198;1), (0.14534,0.18385,0.20364;0.8))	((0.13163,0.18385,0.2198;1), (0.14534,0.18385,0.20364;0.8))	((0.13163,0.18385,0.2198;1), (0.14534,0.18385,0.20364;0.8))

After obtaining the weighted rating matrix, summation of the performance scores of the alternatives are calculated. The results are tabulated in Table 3.25. Total fuzzy scores of the alternatives with respect to IT2, T1, and defuzzified FCMs are visualized in Figure 3.14.

Evidently, IT2 FCMs provide more information to the decision makers. Making use of crisp numbers in modeling MADM problems (defuzzified FCM), final rankings seem to be clearly separated which implies exact dominance of one alternative over another due to information loss (which is not the case for this problem).

Table 3.25 Total scores of the alternatives

	Crisp	T1 fuzzy	IT2 fuzzy
Provider 1	0.533	(0.34118,0.53286,0.74805)	((0.34119,0.53287,0.74805;1), (0.39813,0.53287,0.67818;0.8))
Provider 2	0.691	(0.48927,0.70913,0.89651)	((0.48927,0.70914,0.8965;1), (0.54997,0.70041,0.83218;0.8))
Provider 3	0.812	(0.62728,0.83945,0.98509)	((0.62729,0.83947,0.98508;1), (0.69113,0.8224,0.92773;0.8))

However, results produced in the form of T1 fuzzy sets reveal that there are intersections and overlapping between total scores of alternatives. While the intersecting point is a single numeric value in T1 fuzzy sets, IT2 FCMs provide uncertainty bounds where uncertainty is characterized by FOU. These uncertainty bounds promote freedom and flexibility in decision aiding as decision makers are given leeway to choose and make the final decision, which gives rise to versatility of the produced results.

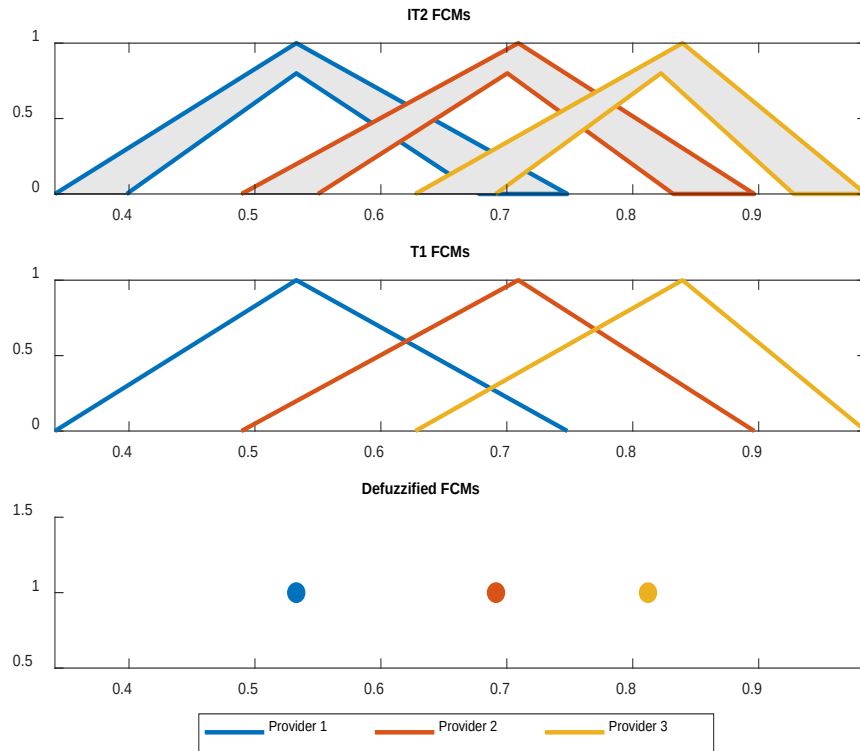


Figure 3.14 Total scores of alternatives and provided information

3.8 Chapter Conclusions

In this study, a new alpha-cut based method is proposed for simulating FCMs. The proposed method is able to admit T1/IT2 fuzzy sets in concept values and the connection matrix. Because defuzzification is not used in the proposed method, there is no information loss in the resulting fuzzy concept values. Another advantage is that the methods rely on the alpha-cut computations, hence it is quite straightforward and easy to be implemented. Additionally, a novel application of the proposed method is developed for objective weighting of criteria in MADM problems. The objective weighting method transforms negative and positive causal relationships among criteria into relative priorities. The method makes use of the convergence properties of sigmoidal FCMs in which proper selection of sigmoidal function parameter guarantees fixed point convergence. The presented paper can be extended in the following directions:

- Although sigmoidal FCMs are the main focus of the present paper, alpha-cut based approaches can be used with other continuous and nonlinear transformation functions. Computational study given in Section 3.6 shows that the proposed method works well with hyperbolic tangent functions. However, convergence properties of the FCMs with hyperbolic tangent functions need considerable further research.
- The proposed alpha-cut based method can be implemented for other fuzzy sets, viz. intuitionistic fuzzy sets.
- The obtained objective weights can be integrated with the subjective weights. There are many approaches in the literature for dealing with synthesis of objective and subjective weights. For instance, an additive synthesis with fuzzy weighted aggregation operator might produce integrated weights in a practical manner.
- Hierarchical MADM problems with sub-attributes can be modeled via the proposed methods. Also, wide variety of MADM problems can be solved.

- As fuzzy SAW method is employed in ranking of the alternatives in the present study, more sophisticated MADM methods can also be easily integrated into the hybrid MADM model.
- Learning of FCMs can be reconsidered in the light of the findings of the present study. Learning algorithms can be implemented in such a way that optimization is performed in each alpha level.

We believe that the proposed methods addressed in the present paper can be extended in many different ways and bring new insights on FCMs and their applications in MADM problems.



CHAPTER FOUR

A DYNAMIC MULTIPLE ATTRIBUTE DECISION MAKING MODEL WITH LEARNING OF FUZZY COGNITIVE MAPS

4.1 Chapter Introduction

The increasing competition of the rapidly changing business environment entails effective use of resources such as people, material, capital, information in order to attain organizational goals. To carry out business functions effectively, managers are continuously engaged in a process of making decisions. These decisions involve consideration of diverse set of factors such as environmental, social, economic and etc., which are characterized by their multiple and conflicting nature. Decision making is becoming more complicated and difficult due to the dramatic increase in the availability of information obtained from diverse resources. Therefore, Multiple Attribute Decision Making (MADM) offers set of sophisticated techniques to help decision makers to select the best alternative by considering multiple, conflicting and incommensurate criteria.

With the continuing proliferation of decision making methods, the field of MADM is rapidly growing. Many state of the art methods have been proposed such as Multi-Attribute Utility Theory (MAUT) (Dyer, 2005; Fishburn, 1970; Keeney, 1988), Analytic Hierarchy Process (AHP) (Saaty, 1980), Analytic Network Process (ANP) (Saaty, 1996), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Elimination et Choice Translating Reality (ELECTRE) (Roy, 1990), VlseKriterijumska Optimizacija I Kompromisno Resenje technique (VIKOR) (Opricovic & Tzeng, 2004), Decision-Making Trial and Evaluation Laboratory (DEMATEL) (Fontela & Gabus, 1976). Although many successful applications of MADM methods are reported in the literature, the salient deficiency of these methods is their incapability to handle temporal profiles of alternatives. It is assumed that alternatives and criteria are fixed, and decision making situation occurs only at once, that is to say, past performance of an alternative is never taken into account. Recently, with the rapid advances in the information technologies

and digital transformations, data is becoming an indispensable part of decision making practices, and the field of MADM undergo a paradigm shift towards data-driven and dynamic decision making.

Dynamic MADM problems have grabbed considerable attention in the literature recently. In the dynamic MADM problems, at least two-period decision making information is considered. In other words, in addition to attributes and alternatives, time dimension is considered in dynamic MADM problems. Campanella and Ribeiro (2011) proposed a framework for dynamic MADM and majority of the studies in the literature employs this framework. The resolution process in this framework can be summarized as follows (Campanella & Ribeiro, 2011; Zulueta, Martínez-Moreno, Pérez, & Martínez, 2014): 1) non-dynamic ratings are computed by means of weighted sum as in the classical MADM. 2) dynamic ratings are computed by utilizing an aggregation operator which makes use of current period ratings and the previous performance ratings. 3) alternatives are ranked and the historical set is updated which will be used in calculating dynamic ratings of the subsequent periods. The proposed framework highly relies on the selection of an appropriate associative aggregation operator. Many time-dependent aggregation operators are proposed in the literature; the most widely known ones are dynamic weighted averaging operator and dynamic intuitionistic fuzzy weighted averaging operator (Xu, 2008; Xu & Yager, 2008). Despite the importance of effectively aggregating past data by using aggregation operators in dynamic MADM, there are great research avenues to make informed decisions by extracting valuable information from seamless flow of data, and implement business analytics solutions.

As ever-more data pour through the networks of organizations, considerable effort has been made in order to cultivate valuable information from these resources. As the notions of big data, digital revolution or internet of things pervade most organizations, firms steadily recognize the importance of data, and mining of firms' data to identify patterns and trends is about to become standard business practice. In parallel with these efforts, the culture of decision making is changing as well. Style of decision making evolving from intuition-based to data-driven. The proposed dynamic decision making

model in this study can be seen as a step towards this direction in which past data is fully capitalized by using intelligent algorithms.

It is evident that sound decision making not only considers past data but also looks ahead as a process of thinking and planning (Saaty, 2007). In this study, a new dynamic MADM model is proposed by Jaya algorithm based-learning of Fuzzy Cognitive Maps (FCMs) and trained FCMs are used to predict future performance scores of alternatives. Furthermore, past, current, and future performance scores of alternatives are used to obtain ranking orders of alternatives. Short-, medium-, and long-term decision horizons are considered and the changes in priorities of alternatives through different time periods are provided to decision makers. The proposed dynamic MADM model contributes to the literature in the following ways:

- FCM learning is employed to capture patterns and trends in historical decision matrices. Jaya algorithm, a simple and effective metaheuristic optimization algorithm, is used to determine degree of interconnections and activation function parameters in the FCM. FCMs are able to model interactions among attributes while learning of the underlying structure, and thus performance patterns of alternatives are captured by means of multiple-attributes simultaneously. The present study is the first paper which uses FCM in the domain of dynamic MADM. Moreover, first time in the literature, a learning algorithm is used for dynamic MADM.
- Considering different decision horizons, future decision matrices are generated via trained FCMs. First time in the literature, future decision making matrices generated by means of an intelligent algorithm are analyzed in decision making.
- Short-, medium-, and long-term rankings of alternatives are provided to decision makers by utilizing past, current, and future information about alternatives.
- A real-life supplier performance evaluation problem is used to demonstrate practicality of the proposed model.

The paper is organized as follows; brief review of the literature on dynamic MADM methods is given in Section 4.2. Theoretical background on the methodologies used within the scope of this paper are given in Section 4.3. The proposed model is provided in Section 4.4. The real-life application of the proposed dynamic MADM model is illustrated in Section 4.5. The concluding remarks are given in Section 4.6.

4.2 Related Work

Xu and Yager (2008) investigated dynamic MADM problems under intuitionistic fuzzy environment. Two new aggregation operators were developed; namely dynamic intuitionistic fuzzy weighted averaging and uncertain dynamic intuitionistic fuzzy weighted averaging. Also, weight vector generation functions for the proposed aggregation operators were given. In the first step of the proposed models, dynamic aggregation operators were used to aggregate all the intuitionistic fuzzy decision matrices pertaining to past periods, and then the classical methods are used to rank the alternatives. Bali, Dagdeviren, and Gumus (2015) integrated AHP and dynamic intuitionistic fuzzy weighted averaging operator in dynamic personnel promotion problem in the same manner with Xu and Yager's approach.

Park, Cho, and Kwun (2013) proposed dynamic intuitionistic fuzzy weighted geometric and uncertain dynamic intuitionistic fuzzy weighted geometric operators for dynamic MADM problems. After aggregation past decision matrices by using the proposed aggregation operators, VIKOR method was used to obtain rankings of alternatives.

Chen and Li (2011) proposed a dynamic MADM model based on triangular intuitionistic fuzzy numbers and a new distance measure is suggested. In this paper, decision matrices at the different periods were aggregated by using weighted arithmetic averaging operator for triangular intuitionistic fuzzy numbers, and then the ranking orders of alternatives were obtained by using closeness coefficients. An investment decision making example was used to illustrate the proposed method where three periods were considered. Similarly, Liang, Zhang, Qi, and Cai (2010)

implemented evidential reasoning approach to aggregate decision matrices with incomplete information.

Xu (2009) proposed dynamic weighted geometric aggregation operator for dynamic MADM problems. Three-period investment decision making was used to show methodological steps. Crisp, interval, and fuzzy number representations were used to compute ranking orders. The aggregation was performed based on the closeness coefficients of the different periods.

Lin, Lee, and Ting (2008) employed grey numbers and Minkowski distance to deal with multi-period and uncertain decision matrices. The TOPSIS method was used as a main MADM method. Rather than aggregating all of the decision matrices at the beginning, the proposed model calculated period-weighted distances to ideal and anti-ideal solutions. The proposed model was implemented in subcontractor selection problem with three periods.

Zulueta et al. (2014) criticized associative aggregation operators proposed by Campanella and Ribeiro (2011) as equal dynamic ratings could be obtained even if the non-dynamic ratings were different. They proposed a reinforcement supported aggregation operator called discrete time variable index, which admits bipolar values. Supplier selection problem with five periods was used to illustrate the proposed operator.

There are similar aggregation operator-based studies in the literature which utilize intuitionistic fuzzy numbers (Wei, 2009; Zhang, 2016), 2-tuple linguistic representation (Ai & Yang, 2014; Liu, Jiang, & Martínez, 2018; Liu, 2014; Xu, 2009; Zulueta-Veliz & Sanchez, 2018), grey numbers (Cui, Liu, Dang, Xie, & Zeng, 2011; Shen, Dang, Zhou, & Li, 2015), and etc. For more information about dynamic aggregation operators, we refer to a review paper (Mardani et al., 2018).

Unlike aggregation operators, future concentration in dynamic MADM has not received enough attention from the research community. Saaty (2007) studied time-

dependent eigenvectors and approximating functional forms of relative priorities in dynamic MADM. Hashemkhani Zolfani, Maknoon, and Zavadskas (2016) emphasized the relevance and necessity of future studies in MADM problems. In the paper, scenario-based MADM papers were reviewed and analyzed. Possible changes in the experts' evaluations were expressed by using probabilities. Orji and Wei (2015) integrated fuzzy logic and system dynamics simulation to sustainable supplier selection problem. In the system dynamics simulation, future time horizons were considered. Jassbi, Ribeiro, and Varela (2014) improved the model of Campanella and Ribeiro (2011) by taking into account expert predictions about future periods. In the paper, yearly data of supplier selection problem between 2009 and 2012 was used to show computational steps. In computing dynamic rating related to year 2010, past data of 2009, current data of 2010, and data of 2011 as a future data, were aggregated by using an associative aggregation operator. Although the model considered past, current, and future decision matrices, the proposed model in this study differs from the literature with respect to following aspects;

- The proposed model provides an explicit methodology to generate future decision matrices. In the literature, how to generate future decision matrices has not been fully explored yet.
- The proposed model not only considers 1-period past and future decision matrices, but it also generates future projections with multiple-period performance values. Then, past, current, and future decision matrices are resolved simultaneously.
- Time series weights are associated with the past, current, and future decision matrices.
- The proposed model is more appropriate to business analytics applications as historical records are used to train learning models.

In the next section, the theoretical background is given.

4.3 Theoretical Background

In this section, methodologies used within the present study are overviewed. First, the traditional dynamic MADM problem and its necessary notations are given in Section 4.3.1. As the proposed method employs FCMs and their training via a metaheuristic algorithm, FCMs and Jaya algorithm are introduced in Sections 4.3.2 and Section 4.3.3, respectively.

4.3.1 Traditional Dynamic Multiple Attribute Decision Making

A dynamic MADM problem under study can be described as follows. Let $A = \{A_1, A_2, \dots, A_M\}$ be a discrete set of M feasible alternatives and $C = \{C_1, C_2, \dots, C_N\}$ be a finite set of all attributes. The set of all periods is denoted by $t = \{t_1, t_2, \dots, t_p\}$. Each alternative is evaluated in terms of N attributes and p periods. Each period is associated with weights that these time series weights are denoted by $\xi(t) = [\xi(t_1), \xi(t_2), \dots, \xi(t_p)]^T$ where $\xi(t_k) \geq 0$ and $\sum_{k=1}^p \xi(t_k) = 1$. The weight vector of attributes are given by $[w_1(t_k), w_2(t_k), \dots, w_N(t_k)]^T$ where $w_i(t_k) \geq 0$ and $\sum_{i=1}^N w_i(t_k) = 1$. The decision matrix at the period t_k is denoted by $A(t_k) = (a_{ij}(t_k))_{N \times M}$ where $a_{ij}(t_k)$ is the value of alternative A_j with respect to attribute C_i at period t_k . Let Ω_b and Ω_c be the set of benefit and cost attributes, respectively. $A(t_k)$ is normalized to corresponding dimensionless decision matrix $R(t_k) = (r_{ij}(t_k))_{N \times M}$ by using the following formulas:

$$r_{ij}(t_k) = \frac{a_{ij}(t_k)}{\max_j \{a_{ij}(t_k)\}}, j = 1, 2, \dots, M; k = 1, 2, \dots, p; i \in \Omega_b \quad (4.1)$$

$$r_{ij}(t_k) = \frac{\min_j \{a_{ij}(t_k)\}}{a_{ij}(t_k)}, j = 1, 2, \dots, M; k = 1, 2, \dots, p; i \in \Omega_c \quad (4.2)$$

Hence, the normalized decision matrix is obtained as:

$$\mathbf{R}(t_k) = \begin{bmatrix} r_{11}(t_k) & r_{12}(t_k) & \cdots & r_{1M}(t_k) \\ r_{21}(t_k) & r_{22}(t_k) & \cdots & r_{2M}(t_k) \\ \vdots & \vdots & \ddots & \vdots \\ r_{N1}(t_k) & r_{N2}(t_k) & \cdots & r_{NM}(t_k) \end{bmatrix} \quad (4.3)$$

The overall assessment value of the j th alternative is calculated by:

$$y_j = \sum_{k=1}^p \sum_{i=1}^N \xi(t_k) w_i(t_k) r_{ij}(t_k), \quad j = 1, 2, \dots, M \quad (4.4)$$

Therefore, alternatives are ranked based on y_j in which the best alternative is with the highest overall assessment value.

4.3.2 Fuzzy Cognitive Maps

From the advent of cognitive mapping by Axelrod (1976) to date, many successful applications for the analysis of the social systems have been reported in the literature. However, dealing with much more complicated systems requires manipulation of uncertain information. In order to overcome the limited inference and knowledge representation capabilities of cognitive maps, Kosko (1986) introduced FCMs.

The FCMs can be regarded as combination of artificial neural networks (ANNs) and fuzzy logic (Baykasoğlu & Gölcük, 2017c). Comparing with ANNs, FCMs are open to interpretation and can be easily implemented. In the conventional ANNs, neurons and connections do not have particular meanings; on the contrary, concepts and connections have certain meanings in the FCMs (Tsadiras, 2008). In addition, the causal dependencies among system variables are not only depicted by sign, but their strengths are also asserted by means of linguistic variables. These linguistic variables such as very low, fair, extremely high, etc., are used to quantify the degree of causal relationships among the system variables.

In FCMs, concepts are used to describe the important elements of the system, and the causal relationships are represented by directed arcs (Papageorgiou & Salmeron, 2014). Three different relationships as positive causality ($e_{ij} \geq 0$), negative causality ($e_{ij} \leq 0$), and no causality ($e_{ij} = 0$) are used to describe the influences among system variables. The e_{ij} values represent how strongly concept C_i exerts influence on the concept C_j (Kosko, 1986). The causal dependencies are stored in the connection matrix and they are usually normalized into $[-1, +1]$.

The values of the concepts are being updated at each time step by the Equation 4.5 (Xirogiannis et al., 2004):

$$C_i^{t+1} = f \left(C_i^t + \sum_{\substack{j=1 \\ j \neq i}}^N e_{ji} C_j^t \right) \quad (4.5)$$

where C_i^t is the value of concept C_i at time t , C_i^{t+1} represent the value of concept C_i at time $t + 1$, and e_{ji} is the weight of the causal influence exerted from concept C_j to concept C_i . The symbol f is used to denote the transformation function. The most widely used transformation functions are the bivalent, trivalent, sigmoid (logistic), and hyperbolic tangent functions (Bueno & Salmeron, 2009; Tsadiras, 2008). In the literature, sigmoid and hyperbolic tangent functions are commonly employed.

One deficiency of the sigmoidal FCMs is that spurious influence of inactive concepts on the other concepts can be observed. As the sigmoid function maps 0 value to 0.5, the concept value of 0.5 cannot be distinguished from inactive concepts or missing values so that the conflicts emerge (Papageorgiou, Roo, Huszka, & Colaert, 2012). On the other hand, activation function parameter λ has considerable impact on the simulation results and on the rate of convergence. Assuming that all of the concepts have the same λ value is not rapport with reality. In real life applications, each concept

can have different activation degree that characterizes the complex phenomenon observed in nature.

Therefore, in order to overcome the abovementioned drawbacks, modified inference equation is employed in this study as:

$$C_i^{t+1} = f_i \left((2C_i^t - 1) + \sum_{\substack{j=1 \\ j \neq i}}^N e_{ji} (2C_j^t - 1) \right) \quad (4.6)$$

where f_i represents the activation function of the i th concept. In this study, sigmoid activation function is used for all of the concepts with varying degrees of λ parameter. Network topology and inference scheme of an FCM are given in Figure 4.1a and Figure 4.1b, respectively.

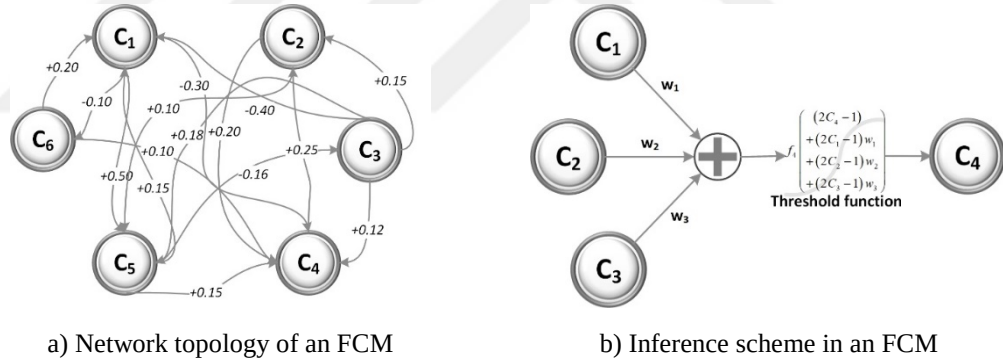


Figure 4.1 FCM representation

An FCM exhibits one of the behaviors: a fixed point, a limit cycle, or a chaotic behavior (Kosko, 1997). In the fixed point state, the concept vector remains the same for successive iterations. In a limit cycle, concept vector keeps repeating indefinitely. In chaotic behavior, the concept vector keeps changing and exhibits a chaotic behavior.

4.3.3 Jaya Algorithm

Jaya algorithm, recently developed by Rao (2016), is a member of population-based metaheuristic algorithms. Jaya is based on the idea that individuals try to approach the

best solutions and escape from the non-promising solutions. Jaya algorithm has a great advantage that parameter setting is not a cumbersome process as it only requires population size and number of generations to be set. Jaya algorithm is used in the present study as it does not require any algorithm-specific parameters. It is also member of parameter-free algorithms Rao (2016). Moreover, it has not been used for FCM training before. Therefore, this can also be considered as a small contribution. Additionally, search procedure is very straightforward and computationally efficient. The Jaya algorithm can be briefly summarized in four steps as follows:

Step 1: Initialization of the population. At the beginning of the algorithm, initial positions of individuals are determined randomly within the range of boundaries as given in Equation 4.7.

$$x_{ij} = x_{\min,j} + rand(0,1)(x_{\max,j} - x_{\min,j}) \quad (4.7)$$

Population of solution vectors $\{x_1, x_2, \dots, x_{nPop}\}$ are generated where $x_i = [x_{i1}, \dots, x_{ij}, \dots, x_{iD}]$, $i = 1, 2, \dots, nPop$, $j = 1, 2, \dots, D$, $rand(0,1)$ represents a uniform distributed random number between 0 and 1, x_{ij} is the value of the i th individual at the j th dimension, $x_{\min,j}$ and $x_{\max,j}$ are lower and upper bounds for the j th dimension, respectively. Note that number of search agents are denoted as $nPop$ and number of optimization parameters are represented by D .

After initialization, fitness of each solution vector is evaluated. Then, the best and worst solutions are determined. Then, population of search agents are subjected to repeat cycles of the search processes until the termination criterion (maximum number of iterations) is met.

Step 2: Generating new solutions. New solution vectors are generated as:

$$x'_i = x_i + r_1(x_{best} - |x_i|) - r_2(x_{worst} - |x_i|) \quad (4.8)$$

where $\mathbf{x}'_i \in \{\mathbf{x}'_1, \mathbf{x}'_2, \dots, \mathbf{x}'_{nPop}\}$, $\mathbf{r}_1$ and $\mathbf{r}_2$ are vectors of uniform random numbers, $\mathbf{x}_{best}$ and $\mathbf{x}_{worst}$ are the best and worst solutions at given iteration, respectively. The term $\mathbf{r}_1(\mathbf{x}_{best} - |\mathbf{x}_i|)$ guides the solution towards the best solution, whereas the term $-\mathbf{r}_2(\mathbf{x}_{worst} - |\mathbf{x}_i|)$ implies the tendency that search procedure try to get away from the worst solution.

Step 3: Checking for boundary conditions. Generated solutions are checked whether the allowable regions are exceeded.

$$\mathbf{x}'_{i,j} = \begin{cases} \mathbf{x}'_{\min,j} & \text{if } \mathbf{x}'_{i,j} < \mathbf{x}'_{\min,j} \\ \mathbf{x}'_{\max,j} & \text{if } \mathbf{x}'_{i,j} > \mathbf{x}'_{\max,j}, \forall i \in \{1, \dots, nPop\}, j \in \{1, \dots, D\} \\ \mathbf{x}'_{i,j} & \text{otherwise} \end{cases} \quad (4.9)$$

Step 4: Solution acceptance: Objective function is calculated for the new solutions, and new and current solutions are compared against each other. If the new solution is better than the current solution, than the new solution is accepted and the individual in the population is replaced by the new solution. Otherwise, current solution survives in the population and new solutions is not accepted.

$$\mathbf{x}_i = \begin{cases} \mathbf{x}'_i & \text{if } fobj(\mathbf{x}'_i) \leq fobj(\mathbf{x}_i) \\ \mathbf{x}_i & \text{otherwise} \end{cases} \quad (4.10)$$

where $fobj$ denotes the objective function to be minimized.

4.4 Proposed Dynamic MADM Model

This section presents the main framework of the proposed dynamic MADM model. The proposed model is introduced and the algorithmic steps are provided. The proposed model consists of following phases:

- Problem structuring
- Learning of FCMs
- Ranking of alternatives by considering past, current and future performance values

The proposed hybrid model is depicted in Figure 4.2.

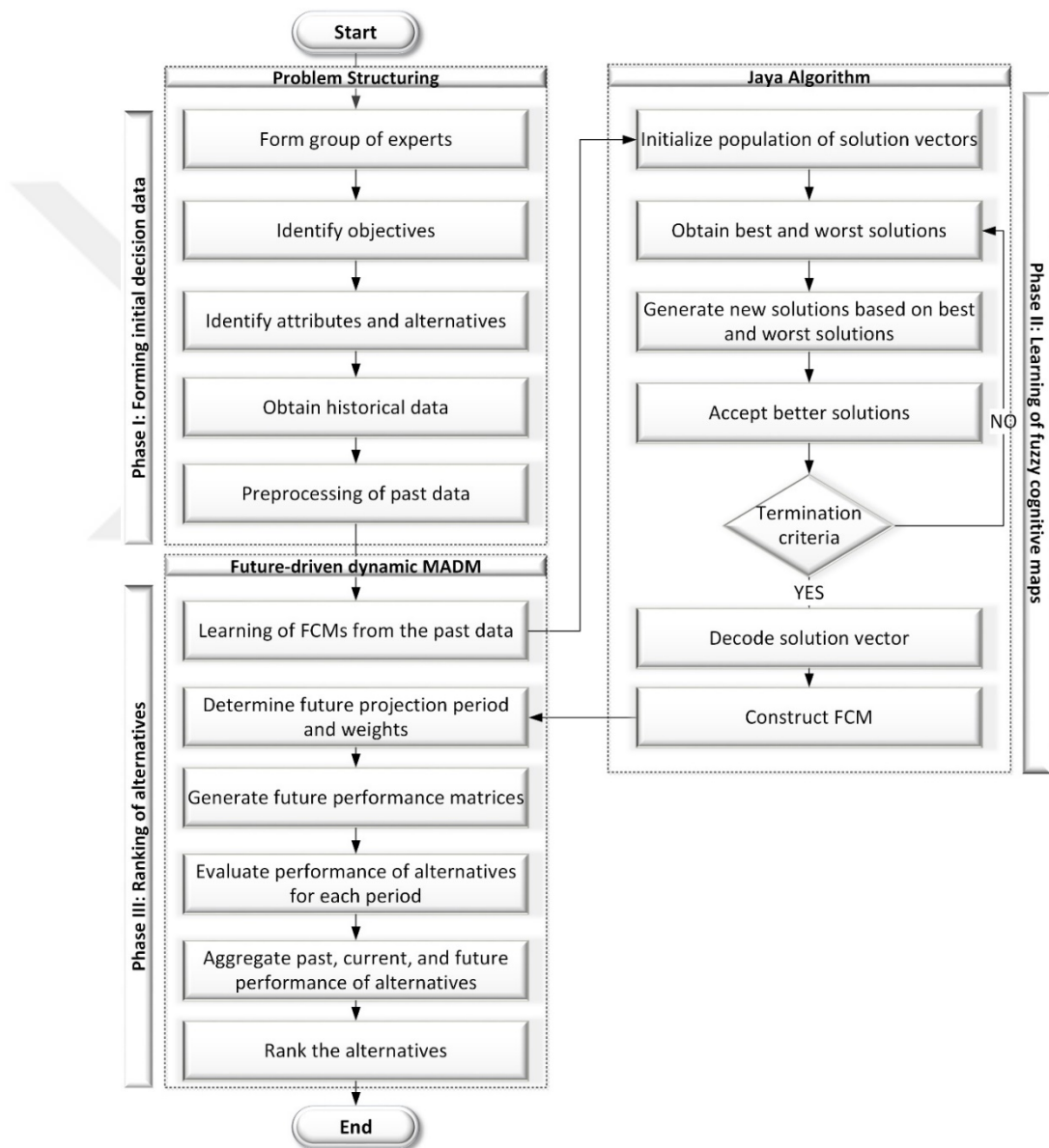


Figure 4.2 Flowchart of the proposed model

4.4.1 Phase I: Problem Structuring

Step 1: Problem-framing: In this step, group of experts decide on the objective of the study, and the attributes and alternatives are identified. Here, expert opinions and the literature surveys help arrive at problem-framing.

Step 2: Obtaining historical data: Historical records are identified and the past data are fetched from the databases. Past data contain performance values of alternatives with respect to attributes at different periods as given in Equation 4.11.

$$\begin{aligned} \mathbf{A}(t_1) &= \begin{bmatrix} a_{11}(t_1) & a_{12}(t_1) & \cdots & a_{1M}(t_1) \\ a_{21}(t_1) & a_{22}(t_1) & \cdots & a_{2M}(t_1) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(t_1) & a_{N2}(t_1) & \cdots & a_{NM}(t_1) \end{bmatrix}, \\ &\vdots, \\ \mathbf{A}(t_p) &= \begin{bmatrix} a_{11}(t_p) & a_{12}(t_p) & \cdots & a_{1M}(t_p) \\ a_{21}(t_p) & a_{22}(t_p) & \cdots & a_{2M}(t_p) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(t_p) & a_{N2}(t_p) & \cdots & a_{NM}(t_p) \end{bmatrix} \end{aligned} \quad (4.11)$$

where $\mathbf{A}(t_1)$ and $\mathbf{A}(t_p)$ are the decision matrices at the first and last period of the historical data, respectively.

This data can be unstructured so that the preprocessing is required.

Step 3: Preprocessing of historical data: In order to ensure accurate and meaningful analysis, data cleaning and preprocessing techniques are implemented in this step. Bad or missing data are eliminated by removing or replacing. Also, abrupt changes and local optima values identified. Smoothing or de-trending methods can be applied to remove noise. Then, normalization is used. Conventional min-max normalization can be used to map the data into the range $[0,1]$ by using the formula

$(x - \min)/(\max - \min)$ where x is the input numeric value, $\min$ and $\max$ are the minimum and the maximum of the dataset, respectively.

4.4.2 Phase II: Learning of FCMs

Step 4: Determining optimization variables: The objective of the FCM learning is to identify the connection matrix. Assume that number of concepts are denoted by N , and sequence of activation degrees are given by $\mathbf{C}(t)$ where $t \in \{0, \dots, t_k\}$. Given an initial condition $\hat{\mathbf{C}}(0) = \mathbf{C}(0)$, the sequence $\hat{\mathbf{C}}(t)$ is obtained after running the FCM model. The objective of learning is to minimize the difference between $\mathbf{C}(t)$ and $\hat{\mathbf{C}}(t)$.

The main merit of FCM learning is to reduce dependence on expert intervention and fully exploit the benefits of data-driven modeling. Capitalizing the information from historical data, candidate FCMs are generated with different connection matrices in which the aim is to produce similar results with the historical data by mimicking the same behavior. The number of consecutive points in historical data is called data length.

Under the assumption that a concept cannot cause itself, which means that the directed edges are only drawn between different concepts, an FCM is characterized by $N(N-1)$ variables. These values are represented in Equation 4.12.

$$\mathbf{E} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \cdots & C_{N-1} & C_N \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_{N-1} \\ C_N \end{matrix} & \begin{pmatrix} 0 & e_{1,2} & \cdots & e_{1,N-1} & e_{1,N} \\ e_{2,1} & 0 & \cdots & e_{2,N-1} & e_{2,N} \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ e_{N-1,1} & e_{N-1,2} & \cdots & 0 & e_{N-1,N} \\ e_{N,1} & e_{N,2} & \cdots & e_{N,N-1} & 0 \end{pmatrix} \end{matrix} \quad (4.12)$$

In order to capture the direction and magnitude of interrelationships among concepts, historical data is used to generate sequence of state vectors. A metaheuristic algorithm, namely Jaya algorithm, is used to train FCM so that the expert involvement is eliminated.

Each solution is encoded as a floating-point vector. Dimension of each solution vector corresponds to number of variables in a given problem. In this problem, entries of the connection matrix and sigmoid function parameters are the decision variables. Each variable in the connection matrix ranges from -1 to 1. On the other hand, sigmoid function parameter λ ranges from 1 to N . The solution representation is given in Equation 4.13.

$$\left[e_{1,2}, e_{1,3}, \dots, e_{1,N}, e_{2,1}, e_{2,3}, \dots, e_{2,N}, \dots, e_{N,N-1}, \lambda_1, \lambda_2, \dots, \lambda_N \right] \quad (4.13)$$

where e_{ij} is the strength of influence i th concept exerts on the j th concept in the connection matrix of the candidate FCMs, and λ_i is the sigmoid function parameter associated with the i th concept. Having a different λ parameter for each concept better represents the intelligent behavior in the FCMs.

Each solution vector is decoded back into a square matrix for fitness evaluation. Number of solutions is determined by the *population size* parameter.

Step 5: Determining objective function: Fitness functions evaluation is one of the most crucial parts in any metaheuristic optimization algorithm. In the case of FCM learning, entries of connection matrix are subject to optimization that the quality of solutions are evaluated based on a specific property of FCMs. When simulating FCM model, state vector $C(t+1)$ depends on the concept values at preceding iteration only. Assuming that the historical dataset consists of L data points, then $L-1$ state vectors can be formed as:

$$C(t) \rightarrow C(t+1) \quad \forall t = 0, \dots, L-1 \quad (4.14)$$

Defining initial state vector $\mathbf{C}(t)$ and the system response $\hat{\mathbf{C}}(t)$, fitness function can be calculated based on the difference between system response generated by the candidate FCM and the initial state vector defined by the historical data. The solution vector is decoded back to the connection matrix, and then the candidate FCM is simulated for one iteration in order to obtain system response. The difference between system response and the initial state vector is computed across all $L-1$ data points. The measure of error based on L_2 norms is given as:

$$Error_{-L_2} = \frac{1}{(L-1) \times N} \sum_{t=1}^{L-1} \sum_{i=1}^N |C_i(t) - \hat{C}_i(t)|^2 \quad (4.15)$$

where $\mathbf{C}(t) = [C_1(t), C_2(t), \dots, C_N(t)]$ and $\hat{\mathbf{C}}(t) = [\hat{C}_1(t), \hat{C}_2(t), \dots, \hat{C}_N(t)]$ represent state vectors for the historical data and the system response, respectively. Note that the system response $\hat{\mathbf{C}}(t)$ is generated by the candidate FCM with the initial state vector $\mathbf{C}(t-1)$.

The stopping condition is defined by the maximum number of generations parameter.

Step 6: Establishing optimized FCMs: As a result of learning of FCMs, optimized connection matrix and sigmoid function parameters are determined. The optimized FCMs can be represented by 3-tuple $(\mathbf{C}, \mathbf{E}^{opt}, \mathbf{f}^{opt})$ where $\mathbf{C}$ is the concept vector, $\mathbf{E}^{opt}$ is the optimized connection matrix, and $\mathbf{f}^{opt}$ is the optimized activation function pertaining to each concept. The optimized activation function is characterized by the λ parameter. As a result, optimized FCMs can be described by the expressions given in Equation 4.16.

$$\begin{aligned}
\mathbf{E}^{opt} &= \begin{matrix} & C_1 & C_2 & \cdots & C_{N-1} & C_N \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_{N-1} \\ C_N \end{matrix} & \begin{pmatrix} 0 & e_{1,2}^{opt} & \cdots & e_{1,N-1}^{opt} & e_{1,N}^{opt} \\ e_{2,1}^{opt} & 0 & \cdots & e_{2,N-1}^{opt} & e_{2,N}^{opt} \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ e_{N-1,1}^{opt} & e_{N-1,2}^{opt} & \cdots & 0 & e_{N-1,N}^{opt} \\ e_{N,1}^{opt} & e_{N,2}^{opt} & \cdots & e_{N,N-1}^{opt} & 0 \end{pmatrix} \end{matrix}, \\
\boldsymbol{\lambda}^{opt} &= \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_{N-1} \\ C_N \end{matrix} \begin{bmatrix} \lambda_1^{opt} \\ \lambda_2^{opt} \\ \cdots \\ \lambda_{N-1}^{opt} \\ \lambda_N^{opt} \end{bmatrix}
\end{aligned} \tag{4.16}$$

4.4.3 Phase III: Ranking of Alternatives

Step 7: Obtaining subjective evaluations: In this step, subjective evaluation of the decision makers are collected at the current period t_C .

$$\mathbf{A}(t_C) = \begin{bmatrix} a_{11}(t_C) & a_{12}(t_C) & \cdots & a_{1M}(t_C) \\ a_{21}(t_C) & a_{22}(t_C) & \cdots & a_{2M}(t_C) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(t_C) & a_{N2}(t_C) & \cdots & a_{NM}(t_C) \end{bmatrix} \tag{4.17}$$

where $a_{NM}(t_C)$ denotes the performance value of the alternative M with respect to attribute N at period t_C . The role of current data in the proposed model is illustrated in Figure 4.3. Note that if the decision makers are not able to provide subjective evaluations, decision matrix of the current period can be estimated based on historical data as well.

The proposed model simultaneously considers historical data, expert judgements and predicted performance of alternatives.



Figure 4.3 The role of current data in the proposed model

Step 8: Identifying future projection period and weights: In this step, time horizon for the decision making problem is decided. Time horizon determines the periods which will be used to rank alternatives in the dynamic MADM context. For instance, if the decision makers choose the time horizon as eleven periods, it means that only five past and future periods are considered with the current decision matrix. The past data period in the time horizon is denoted by t_p , and current period is represented by t_c . Illustration of time horizon is given in Figure 4.4.

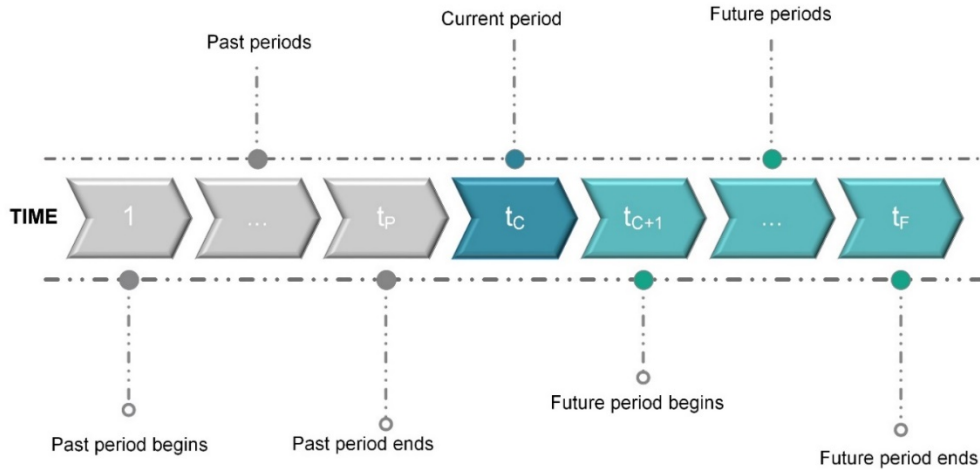


Figure 4.4 Time horizon for a decision problem

In order to consider future performance of alternatives, projection period t_F needs to be determined. Number of future periods might vary according to problem in hand. After the projection period is determined, time series weights are associated with past, current, and future data points. The total number of periods in the time horizon is

denoted by H . Different weight generation methods are reported in the literature (Xu, 2008). In this study, normal distribution-based method is adopted to generate time series weights (Xu, 2008). Normal distribution can be used to derive time series weights as:

$$\xi(t_k) = \frac{1}{\sqrt{2\pi}\sigma_H} e^{-\frac{(k-\mu_H)^2}{2\sigma_H^2}}, \quad k = 1, 2, \dots, H \quad (4.18)$$

where μ_H is the mean of the collection of $1, 2, \dots, H$ and σ_H ($\sigma_H > 0$) is the standard deviation of the collection of $1, 2, \dots, H$. Mean and standard deviations are computed as follows:

$$\mu_H = \frac{1}{H} \frac{H(1+H)}{2} = \frac{1+H}{2} \quad (4.19)$$

$$\sigma_H = \sqrt{\frac{1}{H} \sum_{k=1}^H (k - \mu_H)^2} \quad (4.20)$$

Because the weights sum up to 1, normal distribution based time series weights can be written as:

$$\xi(t_k) = \frac{e^{-\frac{(k-\mu_H)^2}{2\sigma_H^2}}}{\sum_{j=1}^H e^{-\frac{(j-\mu_H)^2}{2\sigma_H^2}}}, \quad k = 1, 2, \dots, H \quad (4.21)$$

If the number of periods is an odd number, maximum weight is obtained when $k = \text{round}\left(\frac{1+H}{2}\right)$. On the other hand, if the number of periods is an even number, maximum weight is reached when $k = \text{round}\left(\frac{1+H}{2}\right)$ or $1 + \text{round}\left(\frac{1+H}{2}\right)$. The prominent feature of the normal distribution based method is that the midmost period

takes the largest weight, and weights are gradually decreased while departing the mid-period. Because credibility of the estimation decreases in parallel with the number of projection periods, using normal distribution based method can be considered as a reasonable choice for generating time series weights.

Step 9: Obtaining future performance of alternatives: As the optimized FCM is obtained in the Step 6, future performance of each alternative is generated by using the FCM, that is to say, FCM simulation is run for t_F periods and results are collected. Here, the current data is given as the initial conditions of the FCM, and iterations are carried on until the decision matrices are generated for all of the future periods. All necessary equations are the same with those given in Section 4.3.2, in particular Equation 4.6.

Step 10: Evaluating performance of alternatives at each period: Performance of each alternative is calculated for different periods separately. In this step, variety of MADM methods can be employed to obtain a performance indicator. For the sake of simplicity, well-known closeness coefficient measures are used to evaluate performance of alternatives at each period. Assuming that the decision matrices are normalized as given earlier in Equation 4.1-2., weighted normalized decision matrices are calculated as:

$$v_{ij}(t_k) = r_{ij}(t_k) \times w_i, \quad i = 1, 2, \dots, N, \quad j = 1, 2, \dots, M, \quad k = 1, 2, \dots, H \quad (4.22)$$

Then, positive ideal solution (PIS) and negative ideal solutions (NIS) are calculated:

$$\begin{aligned} PIS &= (v_1^+, v_2^+, \dots, v_N^+) \\ &= \left\{ \left(\max_j \{v_{ij}(t_k)\} \mid i \in \Omega_b \right), \left(\min_j \{v_{ij}(t_k)\} \mid i \in \Omega_c \right) \right\}, \quad k = 1, 2, \dots, H \end{aligned} \quad (4.23)$$

$$NIS = (v_1^-, v_2^-, \dots, v_N^-)$$

$$= \left\{ \left(\min_j \{v_{ij}(t_k)\} \right) \middle| i \in \Omega_b, \right. \\ \left. \left(\max_j \{v_{ij}(t_k)\} \right) \middle| i \in \Omega_c \right\}, k = 1, 2, \dots, H \quad (4.24)$$

Then, separation values are measure by using the Euclidean distance which is given as:

$$D_j^+(t_k) = \sqrt{\sum_{i=1}^N w_i(t_k) (v_{ij}(t_k) - v_i^+(t_k))^2} \quad (4.25)$$

$$D_j^-(t_k) = \sqrt{\sum_{i=1}^N w_i(t_k) (v_{ij}(t_k) - v_i^-(t_k))^2} \quad (4.26)$$

Finally, closeness coefficients are calculated as:

$$CC_j(t_k) = \frac{D_j^-(t_k)}{(D_j^+(t_k) + D_j^-(t_k))} \quad (4.27)$$

where $CC_j(t_k)$ represents the closeness coefficients.

Step 11: Aggregating future performance of alternatives: Finally, Dynamic Weighted Averaging (DWA) operator is utilized to obtain final ranking values of alternatives.

$$DWA_{\xi(t)}(CC_j(t_1), CC_j(t_2), \dots, CC_j(t_H)) = \sum_{k=1}^H \xi(t_k) CC_j(t_k) \quad (4.28)$$

Step 12: Ranking of alternatives: When the past, current and future performance of alternatives are aggregated, alternatives are ranked based on their ranking values. Higher ranking value implies superiority of an alternative.

4.5 Case Study

The role of supply chain management on the business performance of the firms is rapidly expanding. In parallel with the increase in complexity of products, services, and customer expectations, the compelling market pressures direct firms to collaborate and establish long term relationships with business partners. Supplier evaluation and improvement is one of the critical processes, which is very commonly encountered in real life practice.

Despite the growing interest to the supplier selection problem in the literature, there are numerous insufficiencies of the adopted methodologies in practice. Unfortunately, many enterprises are not aware of methodologies and tools to utilize historical records regarding their operations with suppliers or business partners. As the supplier evaluation programs are time consuming and cognitively demanding undertakings, firms should be supported with appropriate data-driven decision support tools. Evidently, awareness regarding the importance of data-driven management tools for tracing and evaluating performance of suppliers has become crucial. The company which is contacted within the scope of this study is well aware of the significance of such tools. Due to the firm policy, it is named as company A.

The case company A represents a typical case in testing the proposed approach. Company A is a medium-sized company involving in automobile subsidiary industry and one of its manufacturing facilities is located in Turkey. The authors of this study have contacted with company A in order to implement the proposed dynamic MADM approach.

4.5.1 Problem Structuring

Step 1: In this first step of the proposed MADM approach, experts are identified based on their professional backgrounds. Assuring the heterogeneity of experts, number of participants are reduced and five experts are determined. The experts consist of a purchasing manager, two purchasing assistants, production planning and

quality control directors. After identification of the experts, participants are elucidated about the scope and details of the study. Then, the supplier evaluation program of the company is investigated and the attributes which are used to evaluate performance of suppliers are detected from the historical records. Also, three most widely collaborated suppliers are selected in order to implement the proposed approach. Decision hierarchy is formed as illustrated in Figure 4.5.

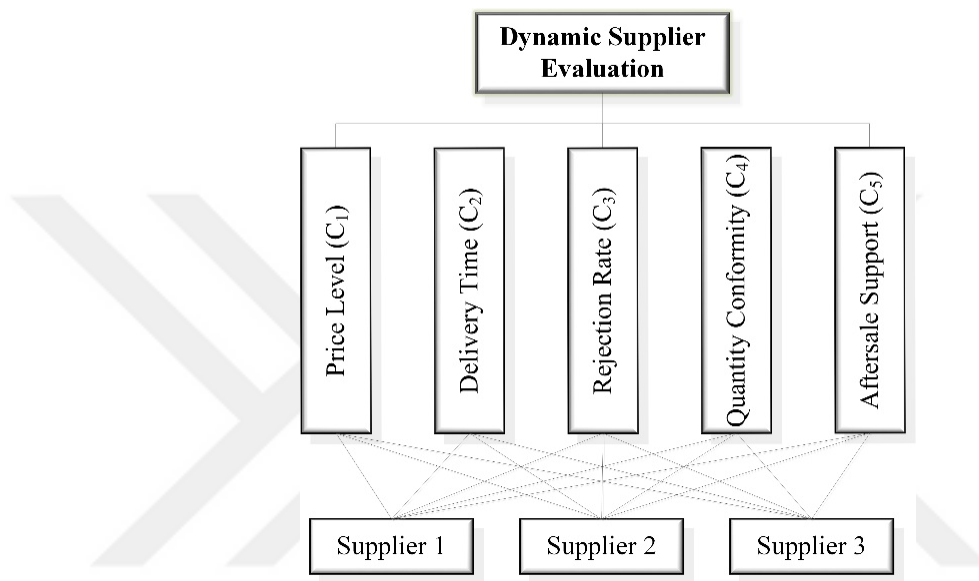


Figure 4.5 Decision hierarchy of supplier evaluation

Step 2: Historical data regarding performance of suppliers with respect to identified attributes are obtained. The company has a performance evaluation system that assigns performance scores between 0-100. As an example, delivery time is assessed with respect to delayed weeks. If the delay of delivery is 1 week, delivery score is 75 out of 100. If the delay of delivery is 2 weeks, then the score becomes 50. Similarly, just-in-time delivery implies full score without any penalty. In the scope of this study, performance values of the past 3 years (36 months) between 2015-2017 are directly used. The historical data of the performance scores of the supplier 1 with respect to decision attributes is given in Table 4.1.

Table 4.1 Historical data of supplier 1

Period	C ₁	C ₂	C ₃	C ₄	C ₅
1	54.69	66.18	67.92	69.17	61.77
2	48.3	65.64	68.43	66.17	64.48
3	58.83	63.85	71.62	68.32	65.21
4	51.51	64.55	66.31	74.74	64.63
5	51.09	63.39	64.08	74.19	64.76
6	49.18	62.91	61.85	75.44	61.49
7	49.68	64.44	60.54	74.76	58.61
8	42.52	65.53	60.14	70.37	56.55
9	46.6	64.95	57.49	66.48	54.36
10	46.4	65.24	57.12	60.69	58.76
⋮	⋮	⋮	⋮	⋮	⋮
35	79.17	55.75	84.41	84.88	55.18
36	84.89	55	86.52	82.09	56.08

Step 3: When the historical data are organized in tables, data preprocessing techniques are applied. The total of 12 missing values is detected within 4 periods, and missing values are filled by the technique of linear interpolation of neighboring non-missing values. Afterwards, the historical dataset is normalized by using min-max standardized algorithm to obtain the concept values lying between $[0,1]$. In the Table 4.2, normalized historical data of the supplier 1 is given.

Table 4.2 Normalized historical data of supplier 1

Period	C ₁	C ₂	C ₃	C ₄	C ₅
1	0.26	1.00	0.36	0.24	0.60
2	0.12	0.95	0.38	0.17	0.82
3	0.34	0.79	0.49	0.22	0.88
4	0.19	0.85	0.31	0.39	0.83
5	0.18	0.75	0.23	0.37	0.84
6	0.14	0.71	0.16	0.40	0.58
7	0.15	0.84	0.12	0.39	0.35
8	0.00	0.94	0.10	0.27	0.18
9	0.09	0.89	0.01	0.17	0.00
10	0.08	0.92	0.00	0.03	0.36
⋮	⋮	⋮	⋮	⋮	⋮
35	0.89	0.00	0.99	0.57	0.14
36	0.89	0.00	0.99	0.57	0.14

4.5.2 Learning of FCMs

Step 4: As there are five attributes in the decision problem, the connection matrix of FCMs have $5 \times (5 - 1) = 20$ decision variables. Additionally, five λ parameters pertaining to sigmoid function need to be optimized. Therefore, total dimension of the decision variables is 25. The encoding of each solution in the metaheuristic optimization is given in Equation 4.29.

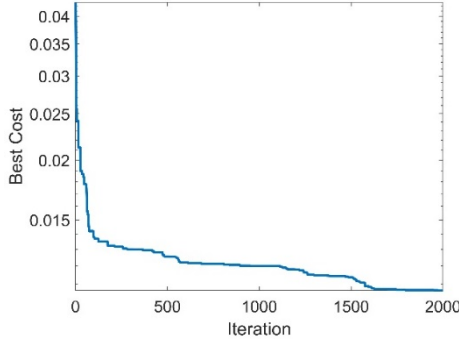
$$\left[e_{1,2}, e_{1,3}, \dots, e_{1,5}, e_{2,1}, e_{2,3}, \dots, e_{2,5}, \dots, e_{5,4}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \right] \quad (4.29)$$

Step 5: When the decision variables are determined, objective function is defined with respect to decision variables. The objective function is defined based on L_2 – norm as described earlier in Section 4.4.

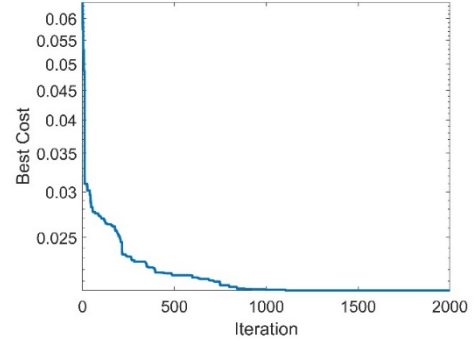
Step 6: In this step, FCM training is performed and the structure of the FCM is established based on available data. First, data set is divided into training and test sets. Training set consists of the first 70% part of the historical data which accounts for 26 data points. The rest of the data points are allocated to the test set. Jaya algorithm is used to train FCMs. The main advantage of Jaya algorithm is that decision makers are not asked to tedious work of determining optimization parameters which are quite difficult to set in real-life practice. The only parameters of Jaya algorithm are *population size* and *number of generations*. It has been reported in the previous works that the population size parameter taken in the range between 10-50 results in high quality solutions (Du, Vinh, Trung, Hong Quyen, & Trung, 2018; Rao, 2016). Based on the preliminary experiments, *population size* is set to 10 and *number of generations* is set to 2000. Therefore, 20,000 function evaluations are performed, and 30 Monte-Carlo simulations are conducted to collect statistical information. The convergence graphs of the best solutions are depicted in Figure 4.6.

As a result of the Jaya training, best solutions of 30 replications are found, and they are used in the latter steps of the proposed approach. The best solutions are decoded

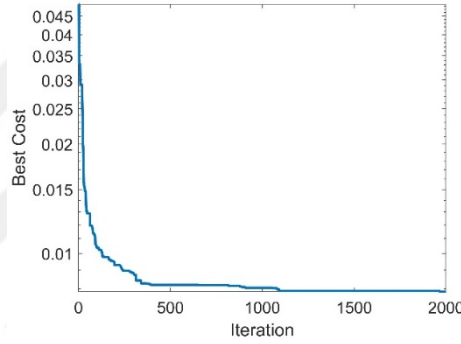
back to connection matrix and parameter forms. The optimized FCMs for supplier 1, supplier 2, and supplier 3 are given in Equation 4.30-32 respectively.



a) Convergence plot of supplier 1



b) Convergence plot of supplier 2



c) Convergence plot of supplier 3

Figure 4.6 Convergence plots of Jaya training

$$\begin{aligned}
 \mathbf{E}^{Supplier_1} &= \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.0000 & -0.0100 & 0.2635 & 0.1537 & 0.4663 \\ 0.0008 & 0.0000 & -0.0513 & 0.1023 & 0.0232 \\ -0.0994 & -0.1727 & 0.0000 & 0.2591 & 0.2731 \\ 0.2636 & 0.1146 & -0.0076 & 0.0000 & -1.0000 \\ 0.0607 & 0.1672 & 0.0100 & 0.2803 & 0.0000 \end{pmatrix} \end{matrix}, \\
 \boldsymbol{\lambda}^{Supplier_1} &= \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} 2.0240 \\ 1.9593 \\ 1.9996 \\ 1.7233 \\ 1.7779 \end{bmatrix}
 \end{aligned} \tag{4.30}$$

$$\begin{aligned}
& \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.0000 & -0.0050 & 0.4756 & 0.0210 & 0.3700 \\ -0.1996 & 0.0000 & 0.2462 & 0.2217 & -0.5451 \\ 0.0271 & 0.0805 & 0.0000 & 0.1122 & -0.1283 \\ -0.1583 & 0.0124 & -0.0382 & 0.0000 & -0.1974 \\ -0.0244 & -0.0563 & -0.2220 & 0.2549 & 0.0000 \end{pmatrix} \end{matrix} \\
\mathbf{E}^{Supplier_2} &= \\
& \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} 1.7911 \\ 2.0935 \\ 1.2022 \\ 2.8247 \\ 1.0012 \end{bmatrix} \\
\boldsymbol{\lambda}^{Supplier_2} &=
\end{aligned} \tag{4.31}$$

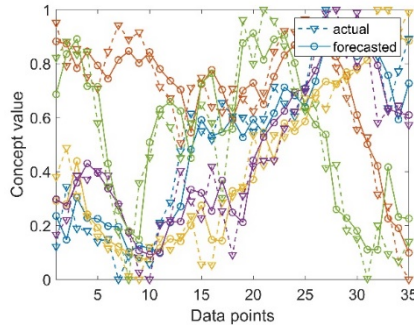
$$\begin{aligned}
& \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.0000 & 0.1163 & -0.2282 & 0.1808 & -0.0373 \\ -0.0541 & 0.0000 & 0.6068 & 0.0755 & -0.0743 \\ 0.0412 & -0.1362 & 0.0000 & -0.3351 & -0.0005 \\ 0.0989 & -0.0255 & 0.1154 & 0.0000 & 0.0015 \\ 0.2890 & -0.0035 & 0.2955 & -0.5975 & 0.0000 \end{pmatrix} \end{matrix} \\
\mathbf{E}^{Supplier_3} &= \\
& \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} 1.8653 \\ 2.4212 \\ 1.3578 \\ 1.5901 \\ 2.5474 \end{bmatrix} \\
\boldsymbol{\lambda}^{Supplier_3} &=
\end{aligned} \tag{4.32}$$

Automated construction of FCMs helps decision makers fully exploit the available data and eliminate expert intervention during the development stage. Although the constructed FCMs are used to make future projections in this study, FCMs established by training historical data can also be used as weight determination with interacting criteria in MADM problems (Baykasoğlu & Gölcük, 2015a; Gölcük & Baykasoğlu, 2016). Statistical summary of the FCM training is given in Table 4.3. As the Jaya algorithm is computationally efficient, solutions can be obtained within seconds.

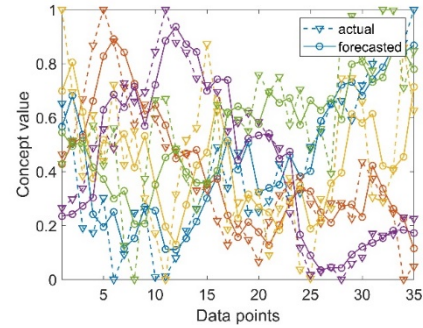
Table 4.3 Performance of FCM training

<i>Alternative</i>	<i>Perf.</i>	<i>Jaya Obj. Func.</i>	<i>CPU (sec.)</i>
<i>Supplier₁</i>	<i>best</i>	1.06E-02	1.61E+00
	<i>mean</i>	1.12E-02	1.77E+00
	<i>std</i>	2.12E-03	1.24E-01
<i>Supplier₂</i>	<i>best</i>	2.00E-02	1.63E+00
	<i>mean</i>	2.12E-02	1.65E+00
	<i>std</i>	3.48E-03	1.71E-02
<i>Supplier₃</i>	<i>best</i>	8.14E-03	1.65E+00
	<i>mean</i>	9.19E-03	1.67E+00
	<i>Std</i>	2.74E-03	3.31E-02

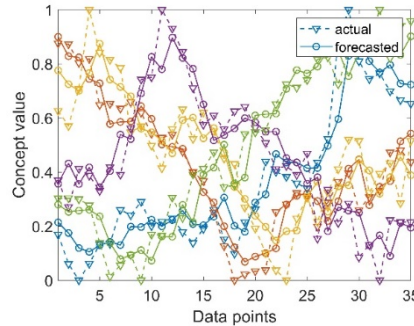
Time series plots of the actual and forecasted performance values are depicted in Figure 4.7. As shown in the Figure 4.7, FCMs successfully capture the system behavior in a relatively low computational cost. The merit of the FCM is that FCMs take into account interactions among concepts/attributes and grasp underlying pattern in a straightforward manner.



a) Time series of Supplier 1



b) Time series of Supplier 2



c) Time series of Supplier 3

Figure 4.7 Time series plots of alternatives

In the following steps, current and future performance scores are obtained and dynamic MADM is employed to rank the alternatives.

4.5.3 Ranking of Alternatives

Step 7: Subjective evaluation is an indispensable part of the any MADM method. Regardless of what the available data points out, there can be some aspects in the decision problem that cannot be inferred from data and expert judgements can be badly needed. For that reason, the proposed dynamic MADM model takes into account subjective judgements of the decision makers. Note that historical data helps decision makers articulate or calibrate the preferences regarding current period as well. In this study, experts are elucidated about the historical data on hand and they are asked to provide their subjective judgement regarding performance of suppliers with respect to predefined attributes. The simple mean of the decision matrices provided by group of decision makers is considered as the performance scores of the current period. In a similar manner, weights of attributes are defined as given in Equation 4.33.

$$A(t_c) = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 86 & 84 & 65 \\ 57 & 63 & 60 \\ 84 & 75 & 62 \\ 70 & 67 & 63 \\ 56 & 66 & 71 \end{pmatrix} \end{matrix}, \begin{matrix} \mathbf{w} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{pmatrix} 0.20 \\ 0.25 \\ 0.25 \\ 0.20 \\ 0.10 \end{pmatrix} \quad (4.33)$$

where t_c represent current period and $\mathbf{w}$ is the weights of attributes. As the weight information is not available for different time periods, it is assumed that attribute weights are the same throughout different periods.

Step 8: In this step, time horizon for decision making is determined and time series weights are generated. Communicating with the decision makers, three different time horizons are determined as short-, medium-, and long-term. Short-term consists of 3 months past and future periods, medium-term consist of 6 months, and long-term deals with 12 months past and future periods. As the subjective scores of decision makers

are taken into consideration in the current period, there are 7, 13, and 25 time periods in short-, medium-, and long-term decision horizons, respectively. Normal distribution based time series weights are generated for short-, medium-, and long-term decision horizons as illustrated in Figure 4.8.

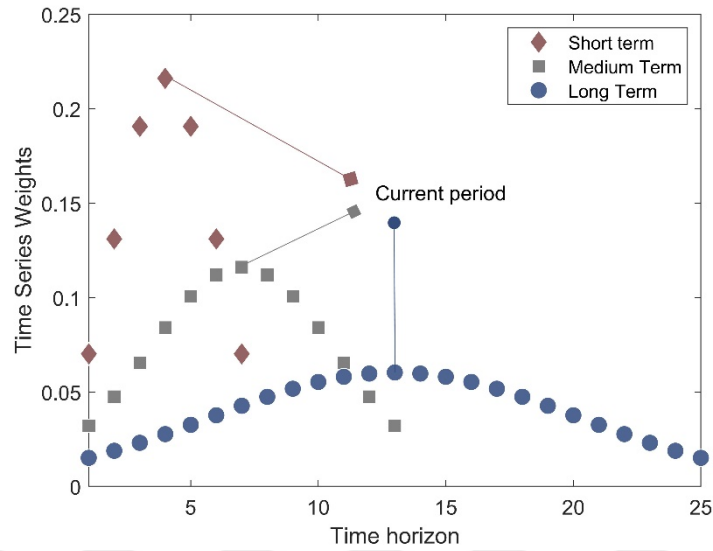


Figure 4.8 Generated time series weights

As is seen from the Figure 4.8., the highest weight is assigned to the current period. While departing from the current period, weights are gradually decreased so that the past, present, and future data points are associated with relative priorities. Note that sum of the time series weights equals to 1.

Step 9: Future performance of alternatives are predicted by using FCMs. The decision matrix of the current period is considered as the initial concept values and FCM simulation is run for 3, 6, and 12 iterations in order to obtain short-, medium-, and long-term predictions, respectively. As an example, future performance of suppliers with respect to the medium-term decision period are given in Table 4.4.

When the future performances are obtained, they are transformed into the decision matrices. For example, based on the data in Table 4.4, decision matrix at the first future period is constructed as given in Equation 4.34.

Table 4.4 Medium-term future performance of suppliers

		Attributes				
		C ₁	C ₂	C ₃	C ₄	C ₅
Supplier 1	t _{F=1}	78.8171	56.4612	83.6271	72.6594	62.2794
	t _{F=2}	74.9500	56.7205	83.0719	78.4500	65.0616
	t _{F=3}	73.8109	57.2415	82.5126	84.9028	65.2303
	t _{F=4}	74.7557	57.7839	82.0731	89.4950	64.2930
	t _{F=5}	76.6821	58.2274	81.8505	91.7198	62.7604
	t _{F=6}	78.4897	58.4686	81.8420	92.3051	61.0776
Supplier 2	t _{F=1}	65.5267	65.3243	62.3787	80.8626	59.2960
	t _{F=2}	59.4786	65.9749	60.2044	82.7120	56.8730
	t _{F=3}	55.6288	66.3760	59.0628	82.5874	55.5709
	t _{F=4}	52.7575	66.6416	58.3286	82.1564	54.7812
	t _{F=5}	50.4341	66.8186	57.8012	81.7141	54.2571
	t _{F=6}	48.4648	66.9324	57.3925	81.3063	53.8826
Supplier 3	t _{F=1}	67.5492	61.3104	63.5256	63.6140	70.3616
	t _{F=2}	69.6992	62.0150	64.3392	63.9991	70.0560
	t _{F=3}	71.5167	62.5442	64.7941	64.2905	69.8711
	t _{F=4}	72.9460	63.0316	65.0725	64.5551	69.7380
	t _{F=5}	73.9634	63.5338	65.2913	64.8079	69.6295
	t _{F=6}	74.6172	64.0797	65.5295	65.0389	69.5335

$$A(t_{F=1}) = \begin{matrix} & A_1 & A_2 & A_3 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 78.8171 & 65.5267 & 67.5492 \\ 56.4612 & 65.3243 & 61.3104 \\ 83.6271 & 62.3787 & 63.5256 \\ 72.6594 & 80.8626 & 63.6140 \\ 62.2794 & 59.2960 & 70.3616 \end{pmatrix} \end{matrix} \quad (4.34)$$

where $t_{F=1}$ represents the first future period.

One advantage of FCM is that it allows investigation of system behavior via simulation. If the concept vector remains the same for successive iterations, it means that system has reached steady-state. The steady-state circumstance represents the system state that does not change anymore. In the context of present study, steady-state behavior is interpreted as performance scores that can be observed in the very long term. Therefore, in addition to short, medium and long term estimations, steady state matrix of the FCM simulation is obtained as given in Table 4.5. It has been taken 45 iterations to reach steady-state for the supplier 1. On the other hand, 70 and 43 iterations are needed to obtain steady-state concept values in supplier 2 and supplier 3 simulations, respectively.

Table 4.5 Steady-state performance of suppliers

Alternatives	Attributes				
	C ₁	C ₂	C ₃	C ₄	C ₅
Supplier ₁	76.5860	57.1272	81.9131	87.3584	58.7737
Supplier ₂	41.6373	65.6946	55.4520	76.8119	53.6791
Supplier ₃	73.5189	70.5373	73.6507	64.1521	67.5256

The steady-state performance values are transformed into a decision matrix as in the case of other future decision matrices.

Step 10: In this step, performance of each alternative is evaluated at each period. Here, past, current, and future decision matrices are evaluated one-by-one separately. As mentioned earlier, closeness coefficient is very practical to use so that for each period distance to positive and negative ideal solutions are calculated. These separation measures for the medium-term decision horizon are given in Table 4.6-7.

Table 4.6 Distances between performance of suppliers and positive ideal solution

Periods	Distance to Positive Ideal Solution		
	Supplier 1	Supplier 2	Supplier 3
t _{P=1}	0.0363	0.1459	0.0861
t _{P=2}	0.0352	0.1426	0.0871
t _{P=3}	0.0375	0.1306	0.0794
t _{P=4}	0.0357	0.1256	0.0699
t _{P=5}	0.0448	0.1171	0.0744
t _{P=6}	0.0453	0.1196	0.0713
t _C	0.0318	0.0294	0.0849
t _{F=1}	0.0412	0.0736	0.0805
t _{F=2}	0.0372	0.0824	0.0751
t _{F=3}	0.0350	0.0890	0.0741
t _{F=4}	0.0341	0.0971	0.0774
t _{F=5}	0.0336	0.1051	0.0788
t _{F=6}	0.0339	0.1118	0.0786

Based on the distances from positive and negative ideal solutions, closeness coefficients are calculated as given in Table 4.8.

Step 11: In this step, closeness coefficients pertaining to past, current, and future performance scores of alternatives are aggregated by taking into account time series

weights. As there are numerous aggregation operators, the most practical one, namely DWA operator, is used in this study.

Table 4.7 Distances between performance of suppliers and negative ideal solution

Periods	Distance to Negative Ideal Solution		
	Supplier 1	Supplier 2	Supplier 3
$t_{p=1}$	0.1450	0.0311	0.0728
$t_{p=2}$	0.1415	0.0280	0.0643
$t_{p=3}$	0.1298	0.0320	0.0601
$t_{p=4}$	0.1250	0.0301	0.0703
$t_{p=5}$	0.1131	0.0168	0.0625
$t_{p=6}$	0.1172	0.0241	0.0651
t_C	0.0841	0.0659	0.0243
$t_{F=1}$	0.0754	0.0545	0.0251
$t_{F=2}$	0.0883	0.0572	0.0407
$t_{F=3}$	0.1001	0.0551	0.0545
$t_{F=4}$	0.1095	0.0515	0.0647
$t_{F=5}$	0.1169	0.0489	0.0719
$t_{F=6}$	0.1226	0.0473	0.0775

Table 4.8 Closeness coefficients of suppliers

Periods	Closeness Coefficients		
	Supplier 1	Supplier 2	Supplier 3
$t_{p=1}$	0.7999	0.1756	0.4581
$t_{p=2}$	0.8006	0.1642	0.4245
$t_{p=3}$	0.7757	0.1968	0.4308
$t_{p=4}$	0.7778	0.1935	0.5014
$t_{p=5}$	0.7161	0.1254	0.4563
$t_{p=6}$	0.7211	0.1675	0.4772
t_C	0.7254	0.6918	0.2221
$t_{F=1}$	0.6470	0.4254	0.2376
$t_{F=2}$	0.7034	0.4099	0.3512
$t_{F=3}$	0.7408	0.3825	0.4240
$t_{F=4}$	0.7623	0.3466	0.4552
$t_{F=5}$	0.7767	0.3176	0.4773
$t_{F=6}$	0.7835	0.2975	0.4963

As a result of DWA operator, short-term, medium-term, and long-term rankings are obtained.

Moreover, for the purpose of comparison, decision matrix pertaining to current period only is resolved and closeness coefficients are obtained. Similarly, steady-state

FCM results are used to obtain rankings of suppliers and all of the results are given in Table 4.9.

Table 4.9 Ranking results of alternatives w.r.t. different time horizons

Time Horizon	Overall Assessment					
	Supplier 1		Supplier 2		Supplier 3	
	CC	Rank	CC	Rank	CC	Rank
Only Current	0.7254	1	0.6918	2	0.2221	3
Short-term ($t_F=3$)	0.7103	1	0.3732	2	0.3551	3
Medium-term ($t_F=6$)	0.7347	1	0.3228	3	0.3966	2
Long-term ($t_F=12$)	0.7756	1	0.2644	3	0.4552	2
Steady-state FCM ($t_F=\infty$)	0.7299	1	0.2483	3	0.6550	2

Step 12: The alternatives are ranked and the results are interpreted in this step. As can be seen from the Table 4.9, supplier 1 dominates the other alternatives regardless of the decision horizon. However, the performance of supplier 2 needs more attention here as the ranking order differs by the considered periods. The performance of supplier 2 has been visualized in Figure 4.9.

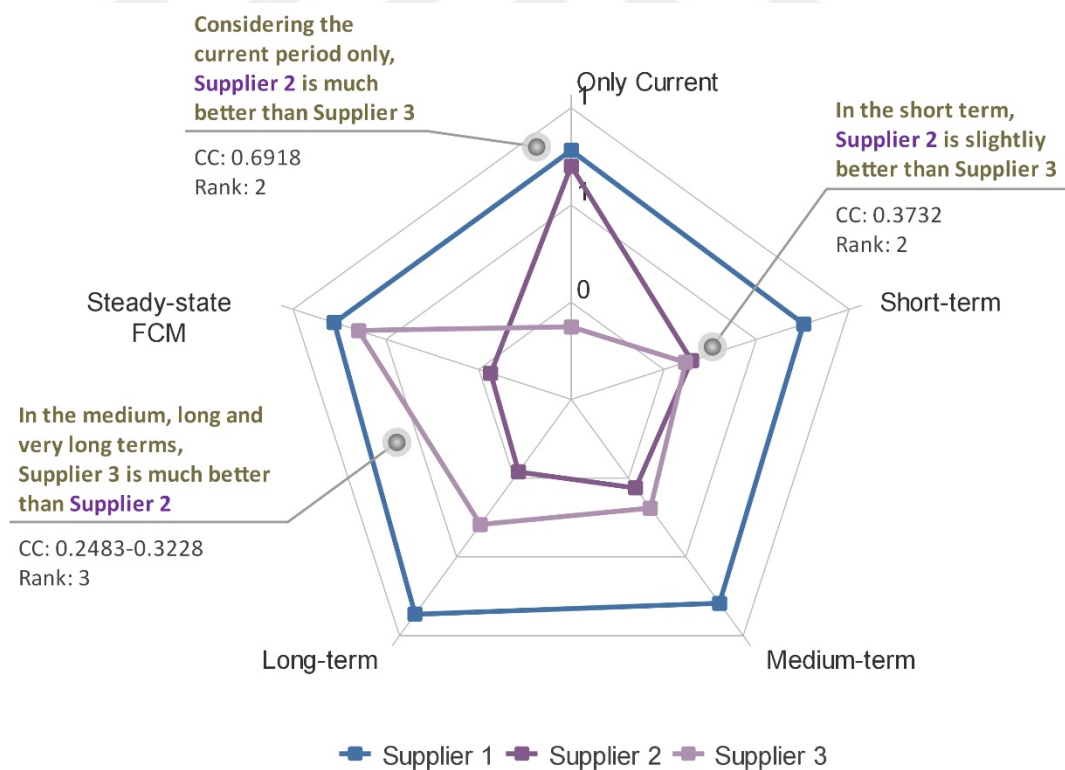


Figure 4.9 Changes in the ranking of supplier 2

It has been observed that the performance of supplier 2 is better than supplier 3. Also, in the short term, dynamic MADM ranks supplier 2 as second, whereas supplier 3 is ranked third. However, considering medium- and long-term periods, it is estimated that the performance of supplier 3 will be improved and supplier 3 will outperform supplier 2. Furthermore, steady-state FCM simulation reveals that the gap between supplier 2 and 3 is even larger in the very long period. Therefore, the present study provides policy makers with a broader view regarding performance of suppliers by considering different decision horizons.

4.6 Chapter Conclusions

In this study, a new hybrid dynamic MADM model, which combines Jaya algorithm, FCMs, time series weights and closeness coefficients, is proposed. The proposed model exhibits desirable properties that help overcome the drawbacks of the traditional dynamic MADM approaches. The proposed model is realized in a real-life supplier performance evaluation problem. The results have showed that the supplier 1 dominates other alternatives in all of the periods. On the other hand, although the supplier 2 seems to perform better in the current period, its performance decreases in accordance with future periods. As a result, supplier 3 becomes more viable alternative in the medium-, and long-term perspectives.

The proposed model provides decision makers with more information about temporal performance of alternatives by considering past, current, and future performance values. The main limitation of the presented study is that the results produced by the proposed model is highly dependent on the data obtained from the case company. Accordingly, results are difficult to be generalized. Secondly, prediction performance highly depends on available data. Nevertheless, the presented study can be improved in terms of many aspects. The higher order uncertainties, such as type-2 fuzzy sets, hesitant fuzzy sets, etc. can be used in the current decision matrix. Also, different strategies can be imposed to generate time series weights, and results can be compared. Last but not least, different MADM approaches can be used to rank the alternatives. In future studies, these considerations will be at the top of our agenda.

CHAPTER FIVE

AN INTERACTIVE DYNAMIC MULTIPLE ATTRIBUTE DECISION MAKING MODEL VIA INTERVAL TYPE-2 FUZZY FUNCTIONS

5.1 Chapter Introduction

Managers are continuously engaged in a process of making decisions in a rapidly changing business environment. Making right decisions is crucial in order to attain organizational goals and effective use of resources. Quality of decisions relies heavily on the information processing capabilities by considering multiple and conflicting criteria. With the dramatic increase in the availability of information obtained from diverse set of resources, decision making becomes much more complicated and difficult. Multiple Attribute Decision Making (MADM) offers a set of sophisticated techniques to help decision makers in selecting the best alternative by considering multiple, conflicting, and incommensurate criteria.

The field of MADM is rapidly expanding with the continuing proliferation of new techniques and applications. Many state of the art methods have been proposed such as Multi-Attribute Utility Theory (MAUT) (Dyer, 2005; Fishburn, 1970; Keeney, 1988), Analytic Hierarchy Process (AHP) (Saaty, 1980), Analytic Network Process (ANP) (Saaty, 1996), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), Elimination et Choice Translating Reality (ELECTRE) (Roy, 1990), VlseKriterijumska Optimizacija I Kompromisno Resenje technique (VIKOR) (Opricovic & Tzeng, 2004), Decision-Making Trial and Evaluation Laboratory (DEMATEL) (Fontela & Gabus, 1976). Despite many successful applications of the MADM methods currently available in the literature, the salient deficiency of these methods is their incapability to handle temporal profiles of alternatives. Unfortunately, static MADM methods cannot deal with the temporal profiles of alternatives, that is, past performance scores are not taken into consideration. In order to overcome this deficiency, data-driven and dynamic MADM methods have been developing along with diverse applications.

In the dynamic MADM, at least two-period decision making information is considered. In addition to the alternative and criteria dimensions, time is considered as a third dimension in the dynamic MADM. With the recent advances in the information technologies, data is becoming an indispensable part of the decision making practices, which forces the pace of a paradigm shift towards data-driven decision making. As ever-more data pour through the networks of organizations, collecting and storing performance scores of alternatives with time stamps are not cumbersome procedures anymore. As the style of decision making evolving from intuition based to data-driven, the decision makers should be supported with relevant methodologies and tools to fully capitalize the available data. However, it is evident that the field of dynamic MADM is in its infancy and the current literature is far from meeting the requirements of a fully-fledged data-driven methodology.

Despite many new developments, the literature of the dynamic MADM field is dominated by the aggregation-operator based models. Campanella and Ribeiro (2011) proposed a framework for dynamic MADM where the aggregation operators were the main computation tools, and majority of the studies in the literature employ this framework. Xu and Yager (2008) proposed dynamic intuitionistic fuzzy weighted averaging and uncertain dynamic intuitionistic fuzzy weighted averaging operators. According to their model, decision matrices of the past periods are aggregated into a decision matrix, and classical MADM techniques were implemented afterwards. Park et al. (2013) proposed dynamic intuitionistic fuzzy weighted geometric and uncertain dynamic intuitionistic fuzzy weighted geometric operators for dynamic MADM problems. The past decision matrices were aggregated and then the VIKOR method was used to rank the alternatives. Chen and Li (2011) proposed a new distance measure for triangular intuitionistic fuzzy sets with an application in dynamic MADM. The weighted arithmetic averaging operator for triangular intuitionistic fuzzy numbers was used to aggregate the decision matrices of the past periods. The ranking orders were obtained by using closeness coefficients. An investment decision making was used to illustrate the proposed method. Liang et al. (2010) employed evidential reasoning approach to aggregate decision matrices with incomplete information. The enterprise evaluation in a technological zone was used to illustrate the model. Bali et al. (2015)

proposed an integrated model based on AHP and dynamic intuitionistic fuzzy weighted averaging operator under intuitionistic fuzzy environment. The proposed model was implemented in personnel promotion problem.

In some of the studies, the aggregation operators were not used to aggregate the decision matrices of the past periods at the very beginning. Xu (2009) proposed dynamic weighted geometric aggregation operator along with an illustrative three-period investment decision making model. Rather than aggregating the decision matrices of the past periods, aggregation was conducted based on the closeness coefficients of the different periods. Zulueta et al. (2014) proposed discrete time variable index by admitting bipolar values in the aggregation. A five-period supplier selection problem was used to illustrate the proposed aggregation operator. Lin et al. (2008) used grey numbers and Minkowski distance in dealing with dynamic subcontractor selection problem. The proposed method calculated the period weighted distances to the ideal and anti-ideal solutions. Similar aggregation operator-based studies in the literature which utilize intuitionistic fuzzy numbers (Wei, 2009; Zhang, 2016), 2-tuple linguistic representation (Ai & Yang, 2014; Liu et al., 2018; Liu, 2014; Xu, 2009; Zulueta-Veliz & Sanchez, 2018), grey numbers (Cui et al., 2011; Shen et al., 2015), and etc. can be found. For more information about dynamic aggregation operators, we refer to a review paper (Mardani et al., 2018). On the other hand, non-aggregation operator based studies can be summarized as follows: Saaty (2007) studied time-dependent eigenvectors and approximating functional forms of relative priorities in dynamic MADM. Hashemkhani Zolfani et al. (2016) emphasized the relevance and necessity of future studies in MADM problems. In the paper, scenario-based MADM papers were reviewed and analyzed. Possible changes in the experts' evaluations were expressed by using probabilities. Orji and Wei (2015) integrated fuzzy logic and system dynamics simulation to sustainable supplier selection problem.

Although wide range of applications have been provided in the context of dynamic MADM, the literature still lacks the following considerations:

- Decision makers are expected to fill out tedious questionnaires to articulate their preferences over alternatives at each period. This is especially very time consuming and demanding when the number of criteria and alternatives are high, and the decision points are frequent, i.e. performance evaluation, risk assessment, and etc.
- The models do not provide any mechanism to help decision makers making use of past decision making matrices when articulating their preferences at the current period. An interactive mechanism is needed to facilitate preference elicitation in the light of historical performance of alternatives.

Because of the availability of accurate historical data with high quality and quantity cannot always be assured and the data is usually affected by imprecision and noise, the predictions regarding the performance of alternatives should handle uncertainty properly. Interval type-2 fuzzy (IT2F) sets are very suitable tools for manipulating and reasoning with uncertain information. For that reason, the present study makes use of IT2F regression (Lee & Tanaka, 1999) to predict the current decision matrix. In order to enhance the prediction capability of the IT2F regression, a new hybrid IT2F model is proposed on the basis of highly practical method of so called “fuzzy functions” (Turksen, 2008). The proposed model is able to capture nonlinearities more successfully than the traditional fuzzy regression models, thanks to its unique and intelligent way of integrating membership grades of data points into the prediction problem.

The proposed dynamic MADM model contributes to the literature with its following features:

- A dynamic MADM model is proposed based on a new IT2F functions approach.
- An interactive procedure is provided that the current decision making matrix is predicted in forms of IT2F sets. Moreover, vocabulary matching procedure is developed so that the predicted performance scores of alternatives are

recommended to the decision makers through linguistic terms such as low, medium, high, and etc.

- The proposed model interacts with decision makers whose subjective judgements are combined with the notion of “let the data speak for itself”. By providing decision makers with a data-driven suggestions regarding the performance of alternatives, preference elicitation effort at each period is considerable reduced.
- The proposed model does not require any technical knowledge such as fuzzy sets, t-norms, t-conorms, implication functions, and etc. The proposed model can be easily integrated into the legacy systems of the firms, since the crisp values are processed when providing IT2F outputs.
- A real-life personnel promotion problem is used to demonstrate the applicability of the proposed model. Rankings of employees are calculated based on past and current performance matrices with appropriate time series weights.

This paper is organized as follows: theoretical background on the methodologies used within the scope of this paper are given in Section 5.2. The proposed model is provided in Section 5.3. The real life application of the proposed dynamic MADM model is illustrated in Section 5.4. The concluding remarks are given in Section 5.5.

5.2 Theoretical Background

5.2.1 Traditional Dynamic Multiple Attribute Decision Making

A dynamic MADM problem under study can be described as follows. Let $A = \{A_1, A_2, \dots, A_M\}$ be a discrete set of M feasible alternatives and $C = \{C_1, C_2, \dots, C_N\}$ be a finite set of all attributes. The set of all periods is denoted by $t = \{t_1, t_2, \dots, t_H\}$. Each alternative is evaluated in terms of N attributes and H periods. Each period is associated with weights that these time series weights are denoted by $\xi(t) = [\xi(t_1), \xi(t_2), \dots, \xi(t_H)]^T$ where $\xi(t_H) \geq 0$ and $\sum_{k=1}^H \xi(t_k) = 1$. The weight

vector of attributes are given by $[w_1(t_k), w_2(t_k), \dots, w_N(t_k)]^T$ where $w_i(t_k) \geq 0$ and $\sum_{i=1}^N w_i(t_k) = 1$. The decision matrix at the period t_k is denoted by $A(t_k) = (a_{ij}(t_k))_{N \times M}$ where $a_{ij}(t_k)$ is the value of alternative A_j with respect to attribute C_i at period t_k . Let Ω_b and Ω_c be the set of benefit and cost attributes, respectively. Due to the immensurability of the different attributes, decision matrix $A(t_k)$ is normalized to corresponding dimensionless decision matrix $R(t_k) = (r_{ij}(t_k))_{N \times M}$ by using the following formulas:

$$r_{ij}(t_k) = \frac{a_{ij}(t_k)}{\max_j \{a_{ij}(t_k)\}}, j = 1, 2, \dots, M; k = 1, 2, \dots, H; i \in \Omega_b \quad (5.1)$$

$$r_{ij}(t_k) = \frac{\min_j \{a_{ij}(t_k)\}}{a_{ij}(t_k)}, j = 1, 2, \dots, M; k = 1, 2, \dots, H; i \in \Omega_c \quad (5.2)$$

Hence, the normalized decision matrix is obtained as:

$$R(t_k) = \begin{bmatrix} r_{11}(t_k) & r_{12}(t_k) & \cdots & r_{1M}(t_k) \\ r_{21}(t_k) & r_{22}(t_k) & \cdots & r_{2M}(t_k) \\ \vdots & \vdots & \ddots & \vdots \\ r_{N1}(t_k) & r_{N2}(t_k) & \cdots & r_{NM}(t_k) \end{bmatrix} \quad (5.3)$$

The overall assessment value of the j th alternative is calculated by:

$$y_j = \sum_{k=1}^H \sum_{i=1}^N \xi(t_k) w_i(t_k) r_{ij}(t_k), j = 1, 2, \dots, M \quad (5.4)$$

Therefore, alternatives are ranked based on y_j in which the best alternative is with the highest overall assessment value.

5.2.2 Possibilistic Fuzzy Regression

In this section, possibilistic fuzzy regression analysis with asymmetric fuzzy numbers is overviewed based on (Lee & Tanaka, 1999). Although there are variety of fuzzy regression approaches developed during the last two decades, regression models relying on possibility and necessity concepts have pivotal role in the current literature. Possibilistic models strive to minimize sum of spreads in such a way that the estimated outputs must include the given targets. It is indeed advantageous to have asymmetric fuzzy numbers in possibilistic models as the lower and upper bounds of the estimated model are not necessarily equidistant from the center which implies superior capability to capture central tendency. A fuzzy regression model can be formalized as:

$$Y(\mathbf{x}) = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \cdots + \tilde{\beta}_{nv} x_{nv} = \tilde{\boldsymbol{\beta}} \mathbf{x} \quad (5.5)$$

where the input vector is represented by $\mathbf{x} = (1, x_1, \dots, x_{nv})^t$ and $\tilde{\boldsymbol{\beta}} = (\tilde{\beta}_0, \tilde{\beta}_1, \dots, \tilde{\beta}_{nv})$ is a vector of fuzzy coefficients, and $Y(\mathbf{x})$ is the estimated fuzzy output. The coefficients $\tilde{\beta}_i$ are denoted as $\tilde{\beta}_i = (a_i, c_i, d_i)$ can be defined by:

$$\mu_{\tilde{\beta}_i}(x) = \begin{cases} 1 - (a_i - x) / c_i, & \text{if } a_i - c_i \leq x \leq a_i \\ 1 - (x - a_i) / d_i, & \text{if } a_i \leq x \leq a_i + d_i \\ 0, & \text{otherwise} \end{cases} \quad (5.6)$$

where a_i represents center, and c_i and d_i left- and right-spreads, respectively.

Given the input-output data as $(\mathbf{x}_j, y_j) = (1, x_{j,1}, \dots, x_{j,nv}; y_j)$, $j = 1, 2, \dots, nd$, where $x_{j,nv}$ being value of the variable nv of the j th data-point among the total of nd data-points, the estimated output $Y(\mathbf{x}_j)$ can be calculated by using fuzzy arithmetic. Representing regression coefficients as $\tilde{\beta}_i = (a_i, c_i, d_i)$, $(i = 0, 1, \dots, nv)$, fuzzy regression model can be expressed as:

$$Y(\mathbf{x}_j) = (a_0, c_0, d_0) + (a_1, c_1, d_1)x_{j,1} + \dots + (a_n, c_n, d_n)x_{j,nv} \quad (5.7)$$

Equation 5.7 can be written in a more compact form as given in Equation 5.8.

$$Y(\mathbf{x}_j) = (\theta_C(\mathbf{x}_j), \theta_L(\mathbf{x}_j), \theta_R(\mathbf{x}_j)) \quad (5.8)$$

where the terms $\theta_C(\mathbf{x}_j), \theta_L(\mathbf{x}_j), \theta_R(\mathbf{x}_j)$ are calculated as given in Equation 5.9.

$$\begin{aligned} \theta_C(\mathbf{x}_j) &= \sum_{i=0}^n a_i x_{ji} \\ \theta_L(\mathbf{x}_j) &= \sum_{x_{ji} \geq 0}^n c_i x_{ji} - \sum_{x_{ji} < 0}^n d_i x_{ji} \\ \theta_R(\mathbf{x}_j) &= \sum_{x_{ji} \geq 0}^n d_i x_{ji} - \sum_{x_{ji} < 0}^n c_i x_{ji} \end{aligned} \quad (5.9)$$

Finally, the possibilistic fuzzy regression model can be written as:

$$\begin{aligned} J &= \sum_{j=1}^{nd} (y_j - \mathbf{a}^t \mathbf{x}_j)^2 + (1-h) \sum_{j=1}^{nd} (\mathbf{c}^t |\mathbf{x}_j| + \mathbf{d}^t |\mathbf{x}_j|) + \xi (\mathbf{c}^t \mathbf{c} + \mathbf{d}^t \mathbf{d}) \\ \text{subject to } & \theta_C(\mathbf{x}_j) + (1-h)\theta_R(\mathbf{x}_j) \geq y_j \\ & \theta_C(\mathbf{x}_j) - (1-h)\theta_L(\mathbf{x}_j) \leq y_j \\ & c_i \geq 0, d_i \geq 0, i = 0, 1, \dots, nv \end{aligned} \quad (5.10)$$

where ξ is a small positive number.. The term $\xi(\mathbf{c}^t \mathbf{c} + \mathbf{d}^t \mathbf{d})$ is added to objective function so that the objective function becomes a quadratic function with respect to decision variables $\mathbf{a}$, $\mathbf{c}$, and $\mathbf{d}$. The resulting optimization problem is a quadratic optimization problem, which involves minimizing a quadratic objective function subject to linear constraints. The possibilistic fuzzy regression analysis will be detailed in the subsequent sections.

5.2.3 Turksen's Fuzzy Functions Approach

Turksen (2008) proposed a new fuzzy modeling technique as an alternative to classical fuzzy rule bases (FRB). Fuzzy rule bases (FRB) have been effectively used as a facilitator to decision makers' problem solving activities. FRBs are the one of the best currently available means to codify human knowledge. This knowledge is represented by "IF...THEN" rule structures. The "IF" part represents the antecedents and "THEN" part represents consequents. In the literature, there are different FRB system modeling strategies with unique antecedent and consequent parameter formation approaches. In these systems, membership values have also different interpretations such as "degree of fire", "degree of compatibility", "degree of belongingness" or "weight or strength of local functions". Fuzzy functions approach adds a new means of membership degrees to the list by exploiting the predictor power of membership grades (Çelikyilmaz & Turksen, 2007).

The representation of each unique rule of an FRB system by means of fuzzy functions is the governing idea of fuzzy functions approach. In the fuzzy functions approach, the membership degree of each sample vector directly affects the local fuzzy functions. One of the advantages of fuzzy functions approach is that even non-experts can build fuzzy models as there are lower steps and parameters. It is quite practical to identify and reason with fuzzy functions approach that some technical information regarding constructing fuzzy system models is not required such as fuzzification, t-norms and t-conorms, modus ponens, and etc.

Vast array of fuzzy modeling approaches has been developed in the literature where the expert knowledge is encoded to define linguistic variables characterized by fuzzy sets. However, majority of the approaches suffers from a major drawback of being subjective and not generalizable. In order to reduce expert intervention into fuzzy system modeling, more objective methods have been developed (Babuška & Verbruggen, 1997; Emami, Türksen, & Goldenberg, 1998; Kasabov & Song, 2002; Kilic, Sproule, Turksen, & Naranjo, 2004; Zarandi, Turksen, & Rezaee, 2004). In these systems, membership grades are not defined by decision makers, on the contrary, they

are extracted from the dataset. There are also some approaches where some sophisticated techniques are integrated into fuzzy models so that the hybrid fuzzy system models are built. The prominent examples of these methods are neuro-fuzzy systems (Jang, 1993) and genetic-fuzzy systems (Cordón, Gomide, Herrera, Hoffmann, & Magdalena, 2004).

There are still enduring challenges in the mentioned fuzzy system modeling approaches. Main disadvantages of the classical fuzzy system modeling approaches can be listed as follows:

- Membership functions pertaining to antecedent and consequent parts of the fuzzy rules should be identified.
- Aggregation of antecedents requires selection of suitable conjunction and disjunction operators (t-norms, t-conorms).
- Proper implication operators should be identified for representation of the rules, which can be a challenging issue.
- A suitable defuzzification method should be identified.

The fuzzy functions approach mainly reduces number of fuzzy operators by taking advantage of data-driven modeling. For instance, fuzzy operators in determination of the membership functions in antecedents and consequents, fuzzification, aggregation of antecedents, implication, and in aggregation of consequents. It can be said that the fuzzy functions are more practical than their counterpart FRB models. Fuzzy function can be simply described as follows:

- The training dataset is partitioned into C overlapping clusters where each cluster center is represented by $\mathbf{v}_i$, $i = 1, 2, \dots, C$.
- For each one of the clusters, a local fuzzy model $f_i : \mathbf{v}_i \rightarrow \mathfrak{R}$ is built and one output is produced for each cluster. Here, memberships and their several transformations are added into the input space and the augmented input matrix is generated. Membership grades and their transformations are considered as new variables in the regression matrix. Practically, least square estimation is

used to derive regression coefficients. Then, degree of belongingness of each given input vector is used to aggregate the local model outputs and the estimated values are produced.

General steps of the fuzzy functions approach can be given as:

Step 1: Matrix $\mathbf{Z}$ comprises of inputs and output of the system. Inputs and output of the system are clustered by using fuzzy c-means clustering algorithm. Fuzzy c-means clustering method can be applied by using the formulas given as:

$$\mathbf{v}_i = \frac{\sum_{j=1}^{nd} \mu_{ij}^m \mathbf{z}_j}{\sum_{j=1}^{nd} \mu_{ij}^m}, i = 1, 2, \dots, c \quad (5.11)$$

$$\mu_{ij} = \frac{1}{\sum_{h=1}^c \left(\frac{\|\mathbf{v}_i - \mathbf{z}_j\|}{\|\mathbf{v}_h - \mathbf{z}_j\|} \right)^{\frac{2}{m-1}}}, i = 1, 2, \dots, c; j = 1, 2, \dots, nd \quad (5.12)$$

where $\|\cdot\|$ represents the Euclidean distance.

Step 2: In the second step, membership values of the input space are calculated. Here, the cluster centers identified in the previous step are used to calculate membership grades of the input data. Membership construction from the identified cluster centers is performed based on Equation 5.13.

$$\mu_{ij} = \frac{1}{\sum_{h=1}^c \left(\frac{\|\mathbf{v}_i - \mathbf{x}_j\|}{\|\mathbf{v}_h - \mathbf{x}_j\|} \right)^{\frac{2}{m-1}}}, i = 1, 2, \dots, c; j = 1, 2, \dots, nd \quad (5.13)$$

Step 3: For each cluster i , membership values of each input data sample, μ_{ij} and original inputs are gathered together, and i th local fuzzy function is obtained by

predicting $\mathbf{Y}^{(i)} = \mathbf{X}^{(i)}\boldsymbol{\beta}^{(i)} + \boldsymbol{\varepsilon}^{(i)}$ based on least squares estimation. When the number of inputs is nv , $\mathbf{X}^{(i)}$ and $\mathbf{Y}^{(i)}$ matrices are as follows:

$$\mathbf{X}^{(i)} = \begin{bmatrix} \mu_{i,1} & x_{1,1} & \cdots & x_{nv,1} \\ \mu_{i,2} & x_{1,2} & \cdots & x_{nv,2} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{i,nd} & x_{1,nd} & \cdots & x_{nv,nd} \end{bmatrix}, \mathbf{Y}^{(i)} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{nd} \end{bmatrix} \quad (5.14)$$

Step 4: Output values are calculated by aggregating the results of the local fuzzy functions as follows:

$$\hat{y}_j = \frac{\sum_{i=1}^c \hat{y}_{ij} \mu_{ij}}{\sum_{i=1}^c \mu_{ij}}, \quad j = 1, 2, \dots, nd \quad (5.15)$$

5.3 Developed IT2F Model

In this section, developed IT2F model for dynamic MADM problems is given. First, the basics of the IT2F sets and necessary equations are overviewed. Afterwards, the IT2F regression model is revisited based on (Lee & Tanaka, 1999). Then, the procedural steps of the proposed dynamic MADM model is given.

5.3.1 Interval Type-2 Fuzzy Sets

Definition 1 (Mendel et al., 2006; Mendel & Wu, 2010): A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$ as:

$$\tilde{\tilde{A}} = \left\{ ((x, u), \mu_{\tilde{\tilde{A}}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1 \right\} \quad (5.16)$$

where J_x denotes an interval in $[0,1]$. Moreover, type-2 fuzzy set $\tilde{\tilde{A}}$ can also be represented as:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x, u) / (x, u) \quad (5.17)$$

where $J_x \subseteq [0,1]$ and $\int \int$ denotes union over all admissible x and u .

Definition 2 (Mendel et al., 2006; Mendel & Wu, 2010): Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is called an IT2F set. An IT2F set $\tilde{\tilde{A}}$, which can be regarded as a special case of a type-2 fuzzy set, is represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad (5.18)$$

where $J_x \subseteq [0,1]$.

The footprint of uncertainty (FOU) is represented by the lower and upper membership functions:

$$FOU(\tilde{\tilde{A}}) = \int_{x \in X} [\underline{\mu}_{\tilde{\tilde{A}}}(x), \bar{\mu}_{\tilde{\tilde{A}}}(x)] \quad (5.19)$$

where $\underline{\mu}_{\tilde{\tilde{A}}}(x)$ and $\bar{\mu}_{\tilde{\tilde{A}}}(x)$ represent lower and upper membership functions, respectively.

Definition 3 (Hamrawi & Coupland, 2009): An IT2F set $\tilde{\tilde{A}}$ is said to be normal if $\sup \bar{\mu}_{\tilde{\tilde{A}}}(x) = 1$ and $\sup \underline{\mu}_{\tilde{\tilde{A}}}(x) = h < 1$, where h represents the height of the lower

membership function. An IT2F set $\tilde{\tilde{A}}$ is said to be perfectly normal if $\sup \bar{\mu}_{\tilde{\tilde{A}}}(x) = \sup \underline{\mu}_{\tilde{\tilde{A}}}(x) = 1$.

In this study, perfectly normal IT2F sets are employed so that the basic definitions and operational laws regarding perfectly normal triangular IT2F sets are overviewed.

Considering perfectly normal triangular IT2F numbers $\tilde{\tilde{A}} = (\bar{A}, \underline{A}) = ((\bar{a}_1, \bar{a}_2, \bar{a}_3; 1)(\underline{a}_1, \underline{a}_2, \underline{a}_3; 1))$ and $\tilde{\tilde{B}} = (\bar{b}, \underline{b}) = ((\bar{b}_1, \bar{b}_2, \bar{b}_3; 1)(\underline{b}_1, \underline{b}_2, \underline{b}_3; 1))$, their operational laws are as follows (Chen et al., 2012):

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = \left(\begin{array}{c} (\bar{a}_1 + \bar{b}_1, \bar{a}_2 + \bar{b}_2, \bar{a}_3 + \bar{b}_3; 1), \\ (\underline{a}_1 + \underline{b}_1, \underline{a}_2 + \underline{b}_2, \underline{a}_3 + \underline{b}_3; 1) \end{array} \right) \quad (5.20)$$

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = \left(\begin{array}{c} (\bar{a}_1 - \bar{b}_1, \bar{a}_2 - \bar{b}_2, \bar{a}_3 - \bar{b}_3; 1), \\ (\underline{a}_1 - \underline{b}_1, \underline{a}_2 - \underline{b}_2, \underline{a}_3 - \underline{b}_3; 1) \end{array} \right) \quad (5.21)$$

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left(\begin{array}{c} (\bar{a}_1 \times \bar{b}_1, \bar{a}_2 \times \bar{b}_2, \bar{a}_3 \times \bar{b}_3; 1), \\ (\underline{a}_1 \times \underline{b}_1, \underline{a}_2 \times \underline{b}_2, \underline{a}_3 \times \underline{b}_3; 1) \end{array} \right) \quad (5.22)$$

$$\tilde{\tilde{A}} \oslash \tilde{\tilde{B}} = \left(\begin{array}{c} (\bar{a}_1 / \bar{b}_1, \bar{a}_2 / \bar{b}_2, \bar{a}_3 / \bar{b}_3; 1), \\ (\underline{a}_1 / \underline{b}_1, \underline{a}_2 / \underline{b}_2, \underline{a}_3 / \underline{b}_3; 1) \end{array} \right) \quad (5.23)$$

$$k \times \tilde{\tilde{A}} = \begin{cases} \left(\begin{array}{c} (k \times \bar{a}_1, k \times \bar{a}_2, k \times \bar{a}_3; 1), \\ (k \times \underline{a}_1, k \times \underline{a}_2, k \times \underline{a}_3; 1) \end{array} \right), & k \geq 0 \\ \left(\begin{array}{c} (k \times \bar{a}_3, k \times \bar{a}_2, k \times \bar{a}_1; 1), \\ (k \times \underline{a}_3, k \times \underline{a}_2, k \times \underline{a}_1; 1) \end{array} \right), & k \leq 0 \end{cases} \quad (5.24)$$

Definition 4: The ranking value $Rank(\tilde{\tilde{A}})$ of a IT2F set $\tilde{\tilde{A}} = (\bar{A}, \underline{A})$ can be defined via the concept of centroid as (Karnik & Mendel, 2001):

$$C_{\tilde{\tilde{A}}}^L = \min_{\xi \in [a,b]} \frac{\int_a^{\xi} x \bar{\mu}_A(x) dx + \int_{\xi}^b x \underline{\mu}_A(x) dx}{\int_a^{\xi} \bar{\mu}_A(x) dx + \int_{\xi}^b \underline{\mu}_A(x) dx} \quad (5.25)$$

$$C_{\tilde{\tilde{A}}}^R = \max_{\xi \in [a,b]} \frac{\int_a^{\xi} x \underline{\mu}_A(x) dx + \int_{\xi}^b x \bar{\mu}_A(x) dx}{\int_a^{\xi} \underline{\mu}_A(x) dx + \int_{\xi}^b \bar{\mu}_A(x) dx} \quad (5.26)$$

where $C_{\tilde{\tilde{A}}}^L$ and $C_{\tilde{\tilde{A}}}^R$ are the endpoints of the centroid. The ranking value of the IT2F set $\tilde{\tilde{A}}$ is calculated as:

$$Rank(\tilde{\tilde{A}}) = \frac{C_{\tilde{\tilde{A}}}^L + C_{\tilde{\tilde{A}}}^R}{2} \quad (5.27)$$

where $Rank(\tilde{\tilde{A}})$ is the centroid-based ranking value of $\tilde{\tilde{A}}$.

Definition 5 (Wu & Mendel, 2009): The Jaccard similarity measure for fuzzy sets $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is defined by

$$SM(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = \frac{\int_X \min(\bar{\mu}_A(x), \bar{\mu}_B(x)) dx + \int_X \min(\underline{\mu}_A(x), \underline{\mu}_B(x)) dx}{\int_X \max(\bar{\mu}_A(x), \bar{\mu}_B(x)) dx + \int_X \max(\underline{\mu}_A(x), \underline{\mu}_B(x)) dx} \quad (5.28)$$

where SM represent similarity degree of fuzzy sets with respect to Jaccard measure.

5.3.2 IT2F Regression Model

In this section, IT2F regression model is revisited based on (Lee & Tanaka, 1999). IT2F regression model will be utilized within the fuzzy functions approach (Turksen, 2008) in order to increase its performance in the next section. In this section, the necessary equations are derived for IT2F regression in a step-by-step approach. IT2F

regression models can be constructed based on the concepts of possibility and necessity. Here, possibility and necessity concepts are used to build upper approximation model (UAM) and lower approximation model (LAM), respectively (Bajestani, Kamyad, Esfahani, & Zare, 2018). Building an integrated model, UAM and LAM are used to form upper and lower membership functions of the IT2F coefficients, respectively.

In mathematical terms, LAM and UAM models can be written as:

$$\begin{aligned} LAM : \tilde{Y}_*(\mathbf{x}_j) &= \tilde{\beta}_{*0} + \tilde{\beta}_{*1}x_{j1} + \cdots + \tilde{\beta}_{*n}x_{j,nv} = \tilde{\beta}_*\mathbf{x}_j, j = 1, \dots, nd \\ UAM : \tilde{Y}^*(\mathbf{x}_j) &= \tilde{\beta}_0^* + \tilde{\beta}_1^*x_{j1} + \cdots + \tilde{\beta}_n^*x_{j,nv} = \tilde{\beta}^*\mathbf{x}_j, j = 1, \dots, nd \end{aligned} \quad (5.29)$$

where coefficients $\tilde{\beta}_{*j}$ and $\tilde{\beta}_j^*$ are non-symmetric triangular fuzzy numbers. The regression coefficients $\tilde{\beta}_{*j}$ and $\tilde{\beta}_j^*$ are shown in Figure 5.1.

As shown in Figure 5.1., $\tilde{\beta}_{*i}$ and $\tilde{\beta}_i^*$ can be defined as

$$\begin{aligned} \tilde{\beta}_{*i} &= (b_i - f_i, b_i, b_i + g_i; 1) \\ \tilde{\beta}_i^* &= (b_i - f_i - p_i, b_i, b_i + g_i + q_i; 1) \end{aligned} \quad (5.30)$$

where the condition $\tilde{\beta}_i^* \supseteq \tilde{\beta}_{*i}$, is satisfied for $i = 0, \dots, nv$.

In order to increase readability of the formulations, conventional representation of the literature is adopted.

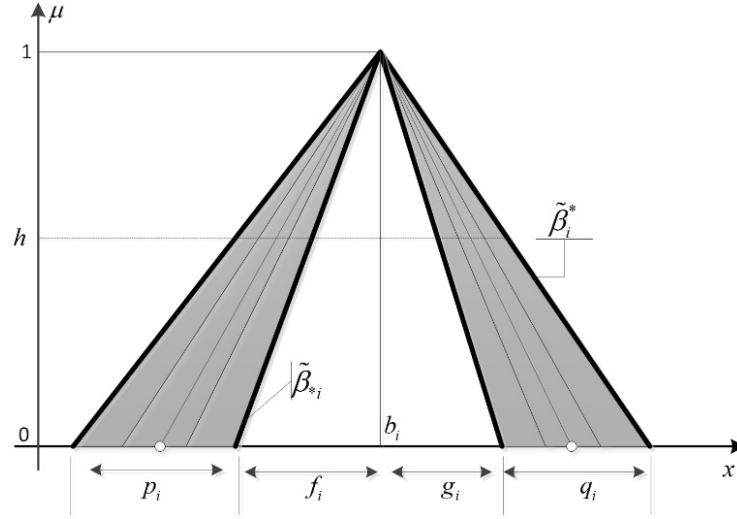


Figure 5.1 Representation of regression coefficients

Here, $(center, left_spread, right_spread)$ is used to show the LAM and UAM as given by $\tilde{\beta}_{*i} = (b_i, f_i, g_i)$ and $\tilde{\beta}_i^* = (b_i, f_i + p_i, g_i + q_i)$, respectively.

The inclusion relation between $\tilde{\beta}_{*i}$ and $\tilde{\beta}_i^*$ can be extended to $\tilde{Y}_*(\mathbf{x}_j)$ and $\tilde{Y}^*(\mathbf{x}_j)$, that is

$$\tilde{Y}^*(\mathbf{x}) \supseteq \tilde{Y}_*(\mathbf{x}) \text{ for any } \mathbf{x} = (1, x_1, \dots, x_{nv})^t \text{ if } \tilde{\beta}_i^* \supseteq \tilde{\beta}_{*i} \quad (5.31)$$

Using the coefficients of the LAM model $\tilde{A}_{*i} = (b_i, f_i, g_i)$, $\tilde{Y}_*(\mathbf{x}_j)$ can be expressed as:

$$\begin{aligned} \tilde{Y}_*(x_j) &= (b_0, f_0, g_0) + (b_1, f_1, g_1)x_{j1} + \dots + (b_{nv}, f_{nv}, g_{nv})x_{j,nv} \\ &= \left(\sum_{i=0}^n b_i x_{ji}, \sum_{x_{ji} \geq 0} f_i x_{ji} - \sum_{x_{ji} \leq 0} g_i x_{ji}, \sum_{x_{ji} \geq 0} g_i x_{ji} - \sum_{x_{ji} \leq 0} f_i x_{ji} \right) \\ &= (\mathbf{b}^t \mathbf{x}_j, \theta_{*L}(\mathbf{x}_j), \theta_{*R}(\mathbf{x}_j)) \end{aligned} \quad (5.32)$$

where $\mathbf{b} = (b_0, b_1, \dots, b_{nv})^t$.

Similarly, $\tilde{Y}^*(\mathbf{x}_j)$ can be expressed as:

$$\begin{aligned}
\tilde{Y}^*(x_j) &= (b_0, f_0, g_0) + (b_1, f_1 + p_1, g_1 + q_1)x_{j1} + \dots + (b_{nv}, f_{nv} + p_{nv}, g_{nv} + q_{nv})x_{jnv} \\
&= \left(\sum_{i=0}^{nv} b_i x_{ji}, \sum_{x_{ji} \geq 0} f_i x_{ji} + \sum_{x_{ji} \leq 0} p_i x_{ji} - \sum_{x_{ji} \leq 0} g_i x_{ji} - \sum_{x_{ji} \leq 0} q_i x_{ji}, \right. \\
&\quad \left. \sum_{x_{ji} \geq 0} g_i x_{ji} + \sum_{x_{ji} \geq 0} q_i x_{ji} - \sum_{x_{ji} \leq 0} f_i x_{ji} - \sum_{x_{ji} \leq 0} p_i x_{ji}, \right) \\
&= (\mathbf{b}^t \mathbf{x}_j, \theta_L^*(\mathbf{x}_j), \theta_R^*(\mathbf{x}_j))
\end{aligned} \tag{5.33}$$

As the possibility and necessity concepts are employed, observed outputs are transformed into granular constructs by admitting a tolerance level for the left- and right-spreads. A user-defined *tolerance_level* is set, thereby possibilistic relationships between observed and estimated outputs can be defined. Generally, *tolerance_level* is a percentage type, i.e., assigning 20% of each output y_j as the corresponding spread. In this study, left- and right-spreads of the observed outputs are called tolerance levels. The tolerance level for the j th data-point is denoted by e_j .

The possibilistic model states that the resulting output from the UAM should cover all of the observed data-points within the given tolerance- and h -level. In other words, $[\tilde{Y}^*(\mathbf{x}_j)]_h$ should approach to $[Y_j]_h$ from the upper side; i.e., $[\tilde{Y}^*(\mathbf{x}_j)]_h$ should be the least interval among all feasible solutions. This brings up the following constraints

$$\begin{aligned}
[\tilde{Y}^*(\mathbf{x}_j)]_h \supseteq [Y_j]_h &\Leftrightarrow \begin{cases} \mathbf{b}^t \mathbf{x}_j + (1-h)\theta_R^*(\mathbf{x}_j) \geq y_j + (1-h)e_j \\ \mathbf{b}^t \mathbf{x}_j - (1-h)\theta_L^*(\mathbf{x}_j) \leq y_j - (1-h)e_j \end{cases} \\
&\mathbf{f} \geq 0, \mathbf{g} \geq 0, \mathbf{p} \geq 0, \mathbf{q} \geq 0, j = 1, \dots, nd
\end{aligned} \tag{5.34}$$

where $\mathbf{x}_j = (1, x_{j,1}, x_{j,2}, \dots, x_{j,nv})$, $j = 1, \dots, nd$ and the term $x_{j,nv}$ denotes the value of the variable nv of the j th data point.

On the other hand, according to necessity model, the h -level set of the $\tilde{Y}_*(\mathbf{x}_j)$ should be included in the h -level set of the given output Y_j . In other words, $[\tilde{Y}_*(\mathbf{x}_j)]$

should approach to $[Y_j]_h$ from the lower side, i.e., $[\tilde{Y}_*(\mathbf{x}_j)]$ should be the greatest interval among all feasible solutions. This can be written in a constraint form as:

$$[\tilde{Y}_*(\mathbf{x}_j)]_h \subseteq [Y_j]_h \Leftrightarrow \begin{cases} \mathbf{b}^t \mathbf{x}_j + (1-h)\theta_{*R}(\mathbf{x}_j) \leq y_j + (1-h)e_j \\ \mathbf{b}^t \mathbf{x}_j - (1-h)\theta_{*L}(\mathbf{x}_j) \geq y_j - (1-h)e_j \end{cases}, \quad (5.35)$$

$$\mathbf{f} \geq 0, \mathbf{g} \geq 0, i = 0, \dots, n, j = 1, \dots, nd$$

Integrating the possibility and necessity models by taking into account the inclusion relation $[\tilde{Y}_*(\mathbf{x}_j)]_h \subseteq [\tilde{Y}^*(\mathbf{x}_j)]_h$ a quadratic programming formulation of the IT2F regression model is formulated as:

$$\begin{aligned} \min_{\mathbf{b}, \mathbf{f}, \mathbf{g}, \mathbf{p}, \mathbf{q}} \quad & J = \sum_{j=1}^{nd} (y_j - \mathbf{b}^t \mathbf{x}_j)^2 + (1-h) \sum_{j=1}^{nd} (\mathbf{p}^t |\mathbf{x}_j| + \mathbf{q}^t |\mathbf{x}_j|) \\ & + \xi (\mathbf{f}^t \mathbf{f} + \mathbf{g}^t \mathbf{g} + \mathbf{p}^t \mathbf{p} + \mathbf{q}^t \mathbf{q}) \end{aligned} \quad (5.36)$$

$$\text{Subject to } \begin{cases} \mathbf{b}^t \mathbf{x}_j + (1-h)\theta_R^*(\mathbf{x}_j) \geq y_j + (1-h)e_j \\ \mathbf{b}^t \mathbf{x}_j - (1-h)\theta_L^*(\mathbf{x}_j) \leq y_j - (1-h)e_j \\ \mathbf{b}^t \mathbf{x}_j + (1-h)\theta_{*R}(\mathbf{x}_j) \leq y_j + (1-h)e_j \\ \mathbf{b}^t \mathbf{x}_j - (1-h)\theta_{*L}(\mathbf{x}_j) \geq y_j - (1-h)e_j \end{cases},$$

$$\mathbf{f} \geq 0, \mathbf{g} \geq 0, \mathbf{p} \geq 0, \mathbf{q} \geq 0, j = 1, \dots, nd$$

where ξ is a small positive number. The term $\xi (\mathbf{f}^t \mathbf{f} + \mathbf{g}^t \mathbf{g} + \mathbf{p}^t \mathbf{p} + \mathbf{q}^t \mathbf{q})$ is inserted to objective function so that the objective function becomes a quadratic function with respect to decision variables $\mathbf{b}, \mathbf{f}, \mathbf{g}, \mathbf{p}, \mathbf{q}$. The obtained UAM and LAM by solving the above integrated quadratic programming model always satisfy inclusion relation $Y_*(\mathbf{x}) \subseteq Y^*(\mathbf{x})$ at h -level.

5.3.3 Dynamic MADM Model via Proposed IT2F Functions

In this section, the proposed IT2F functions approach has been given in a step-by-step manner. The flowchart of the proposed model is illustrated in Figure 5.2.

5.3.3.1 Phase-I: Problem Structuring

Step 1: Problem-framing: In this step, group of experts decide on the objective of the study, and the attributes and alternatives are identified. Here, expert opinions and the literature surveys help to arrive at problem-framing.

Step 2: Obtaining historical data: Historical records are identified and the past data are fetched from the databases. Past data contain performance values of alternatives with respect to attributes at different periods as given in Equation 5.37.

$$\mathbf{A}(t_1) = \begin{bmatrix} a_{11}(t_1) & a_{12}(t_1) & \cdots & a_{1M}(t_1) \\ a_{21}(t_1) & a_{22}(t_1) & \cdots & a_{2M}(t_1) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(t_1) & a_{N2}(t_1) & \cdots & a_{NM}(t_1) \end{bmatrix},$$

$$\mathbf{A}(t_H) = \begin{bmatrix} a_{11}(t_H) & a_{12}(t_H) & \cdots & a_{1M}(t_H) \\ a_{21}(t_H) & a_{22}(t_H) & \cdots & a_{2M}(t_H) \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}(t_H) & a_{N2}(t_H) & \cdots & a_{NM}(t_H) \end{bmatrix} \quad (5.37)$$

where $\mathbf{A}(t_1)$ and $\mathbf{A}(t_H)$ are the decision matrices at the first and last period of the historical data, respectively.

This data can be unstructured so that the preprocessing is required.

Step 3: Preprocessing of historical data: In order to ensure accurate and meaningful analysis, data cleaning and preprocessing techniques are implemented in this step. Bad or missing data are eliminated by removing or replacing.

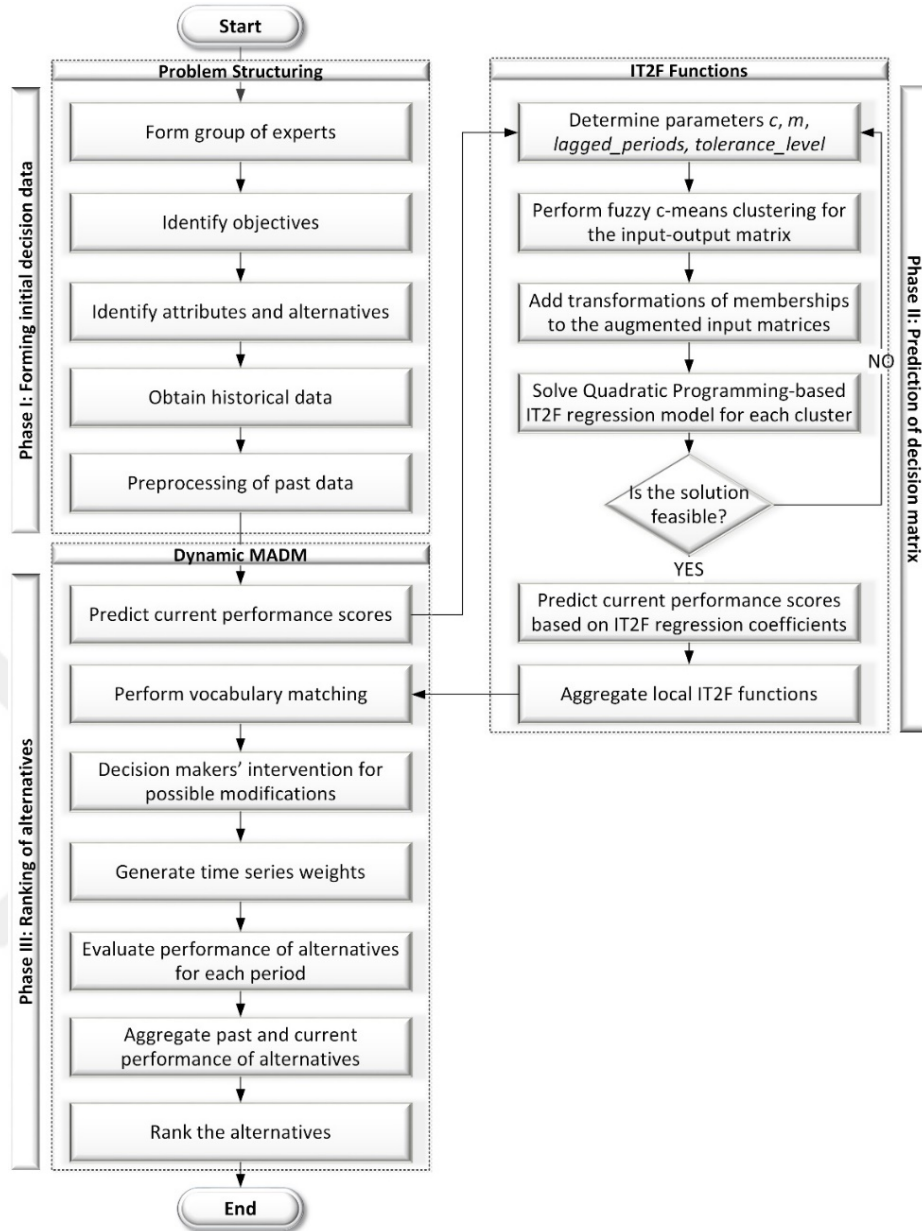


Figure 5.2 Flowchart of the proposed model

Also, abrupt changes and local optima values are identified. Smoothing or de-trending methods can be applied to remove noise. Moreover, the historical decision making matrices are arranged as time series data. Here, for each alternative and attribute pair, time series data are formed. Mathematically speaking, the performance scores for a particular alternative and attribute $(a_{ij}(t_1), a_{ij}(t_2), \dots, a_{ij}(t_H))$ are collected from each period and the time series $\mathbf{y} = (y_1, y_2, \dots, y_H)^t$ is formed, where the number of points is equal to number of periods H .

Then, the lagged matrices are constructed where number of lagged periods is denoted by p . Note that j th data point in the input matrix $\mathbf{x}_j = (x_{t-1,j}, x_{t-2,j}, \dots, x_{t-p,j})^t$ will be used to estimate y_j , $j = 1, 2, \dots, nd$, where nd is equal to $H - p$. The inputs and the outputs of the system are given as:

$$\mathbf{X} = \begin{bmatrix} x_{t-1,1} & x_{t-2,1} & \cdots & x_{t-p,1} \\ x_{t-1,2} & x_{t-2,2} & \cdots & x_{t-p,2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{t-1,nd} & x_{t-2,nd} & \cdots & x_{t-p,nd} \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{nd} \end{bmatrix} \quad (5.38)$$

where $x_{t-p,nd}$ represents the value of variable $t - p$ of the data point nd .

Let the number of past decision matrices are four, and the lagged periods are determined as two. Suppose that decision makers are concerned with the past performance of the second alternative with respect to the first attribute. The corresponding time series data, input and output matrices are illustrated in Table 5.1.

Note that because the lagged periods are two, number of data points are $H - p = 4 - 2 = 2$.

5.3.3.2 Phase-II: Training of Fuzzy Functions Approach

Step 4: Determining parameters of the IT2F functions model: In this step, parameters of the fuzzy c-means clustering algorithm are determined. The number of clusters c , fuzzification coefficient m , *lagged_periods*, and *tolerance_level* for calculating possibility- and necessity-based constraints in IT2F regression are defined.

Step 5: Performing fuzzy c-means clustering to input-output model: In this step, inputs and outputs of the system are used to carry out fuzzy c-means clustering. Having the inputs of the system in the form of lagged variables, the next step is to form input-output matrix $\mathbf{Z}$.

Table 5.1 Illustration of the four-period example

Period	Past decision matrices	Corresponding Time series	Input Matrix	Output Matrix
t_1	$A_1 \quad A_2 \quad A_3$ $C_1[\dots \quad a_{12}(t_1)=5 \quad \dots]$			
t_2	$A_1 \quad A_2 \quad A_3$ $C_1[\dots \quad a_{12}(t_2)=3 \quad \dots]$	$\begin{bmatrix} y_1=5 \\ y_2=3 \\ y_3=6 \\ y_4=8 \end{bmatrix}$	$\mathbf{X} = \begin{bmatrix} y_1 & y_2 \\ y_2 & y_3 \end{bmatrix}$	$\mathbf{Y} = \begin{bmatrix} y_3 \\ y_4 \end{bmatrix}$
t_3	$A_1 \quad A_2 \quad A_3$ $C_1[\dots \quad a_{12}(t_3)=6 \quad \dots]$		$= \begin{bmatrix} 5 & 3 \\ 3 & 6 \end{bmatrix}$	$= \begin{bmatrix} 6 \\ 8 \end{bmatrix}$
t_4	$A_1 \quad A_2 \quad A_3$ $C_1[\dots \quad a_{12}(t_4)=8 \quad \dots]$			

The matrix $\mathbf{Z} = (\mathbf{X}, \mathbf{Y})$ is composed of the input matrix $\mathbf{X}$ and output matrix $\mathbf{Y}$. Then, elements of the $\mathbf{Z}$ matrix $\mathbf{z}_j$ are clustered by using FCM algorithm. FCM is applied by using the following formulas:

$$\mathbf{v}_i = \frac{\sum_{j=1}^{nd} \mu_{ij}^m \mathbf{z}_j}{\sum_{j=1}^{nd} \mu_{ij}^m}, \quad i = 1, 2, \dots, c \quad (5.39)$$

$$\mu_{ij} = \frac{1}{\sum_{h=1}^c \left(\frac{\|\mathbf{v}_i - \mathbf{z}_j\|}{\|\mathbf{v}_h - \mathbf{z}_j\|} \right)^{\frac{2}{m-1}}}, \quad i = 1, 2, \dots, c; \quad j = 1, 2, \dots, nd \quad (5.40)$$

where $\|\cdot\|$ represents the Euclidian distance.

Step 6: Generating augmented input matrices: The augmented input matrix is obtained by adding memberships and their transformations into the original input matrix. Based on the cluster centers found in the Step 5, membership values of the input space are calculated as:

$$\mu_{ij} = \frac{1}{\sum_{h=1}^c \left(\frac{\|\mathbf{v}_i - \mathbf{x}_j\|}{\|\mathbf{v}_h - \mathbf{x}_j\|} \right)^{\frac{2}{m-1}}}, i = 1, 2, \dots, c; j = 1, 2, \dots, nd \quad (5.41)$$

where $\mathbf{x}$ denotes input matrix.

Membership values of each input data sample, μ_{ij} and their transformations are augmented to the original input matrix for the i th cluster as:

$$\phi_i = \begin{bmatrix} 1 & \mu_{i,1} & \exp(\mu_{i,1}) & (\mu_{i,1})^p & x_{t-1,1} & \cdots & x_{t-p,1} \\ 1 & \mu_{i,2} & \exp(\mu_{i,2}) & (\mu_{i,2})^p & x_{t-1,2} & \cdots & x_{t-p,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \mu_{i,j} & \exp(\mu_{i,j}) & (\mu_{i,j})^p & x_{t-1,j} & \cdots & x_{t-p,j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \mu_{i,nd} & \exp(\mu_{i,nd}) & (\mu_{i,nd})^p & x_{t-1,nd} & \cdots & x_{t-p,nd} \end{bmatrix} \quad (5.42)$$

where j th data point is represented by $\phi_{i,j} = (1, \mu_{i,j}, \exp(\mu_{i,j}), \mu_{i,j}^2, x_{t-1,j}, \dots, x_{t-p,j})^t$.

The schematic representation of the proposed IT2F functions is given in Figure 5.3.

Step 7: Solving quadratic programming model for each cluster: Fuzzy regression coefficients are calculated for each cluster by solving a quadratic programming model as given in Equation 5.43.

$$\begin{aligned} \min_{b, f, g, p, q} J &= \sum_{j=1}^{nd} (y_j - \mathbf{b}^t \phi_{i,j})^2 + (1-h) \sum_{j=1}^{nd} (p^t |\phi_{i,j}| + q^t |\phi_{i,j}|) \\ &+ \xi (\mathbf{f}^t \mathbf{f} + \mathbf{g}^t \mathbf{g} + \mathbf{p}^t \mathbf{p} + \mathbf{q}^t \mathbf{q}) \end{aligned} \quad (5.43)$$

$$\text{Subject to } \begin{cases} \mathbf{b}^t \phi_{i,j} + (1-h)\theta_R^*(\phi_{i,j}) \geq y_j + (1-h)e_j \\ \mathbf{b}^t \phi_{i,j} - (1-h)\theta_L^*(\phi_{i,j}) \leq y_j - (1-h)e_j \\ \mathbf{b}^t \phi_{i,j} + (1-h)\theta_{*R}(\phi_{i,j}) \leq y_j + (1-h)e_j \\ \mathbf{b}^t \phi_{i,j} - (1-h)\theta_{*L}(\phi_{i,j}) \geq y_j - (1-h)e_j \end{cases},$$

$$\mathbf{f} \geq 0, \mathbf{g} \geq 0, \mathbf{p} \geq 0, \mathbf{q} \geq 0, j = 1, \dots, nd$$

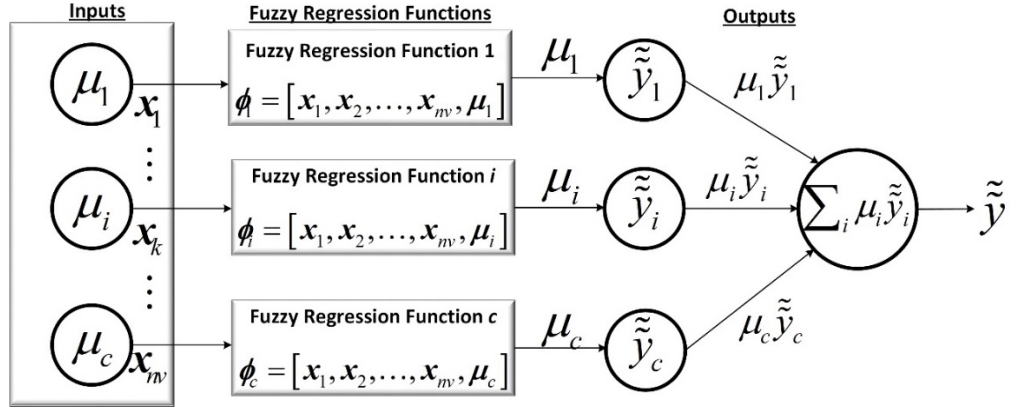


Figure 5.3 Proposed IT2F functions approach

where regression coefficients are IT2F numbers represented by $\tilde{\beta} = ((b, f + p, g + q), (b, f, g))$,

Step 8: Collecting predictions of local fuzzy functions: Predicted output values are calculated as:

$$\tilde{y}_i = \phi \otimes \tilde{\beta}_i \quad (5.44)$$

where $\tilde{y}_i = (\tilde{y}_{i,1}, \tilde{y}_{i,2}, \dots, \tilde{y}_{i,j}, \dots, \tilde{y}_{i,nd})^t$, $\tilde{\beta}_i$ is the regression coefficients of the i th local fuzzy function and $\otimes$ denotes fuzzy matrix multiplication.

Step 9: Aggregating local IT2F functions: Finally, outputs of the local fuzzy functions $\tilde{y}_i$ are weighted by the corresponding membership values and predicted IT2F output is calculated:

$$\tilde{Y}_j = \frac{\sum_i^c \tilde{y}_{i,j} \mu_{i,j}}{\sum_{i=1}^c \mu_{i,j}}, j = 1, 2, \dots, nd \quad (5.45)$$

where $\tilde{Y}_j$ is the predicted value of the j th data point.

5.3.3.3 Phase-III: Ranking of Alternatives

Step 10: Performing vocabulary matching: The resulting values of the IT2F functions are inherently IT2F sets. Because experts often linguistically evaluate the objects in the decision making applications and it is difficult to analytically interpret the obtained numerical values, there is a need for transforming IT2F functions results into the linguistic terms. For that aim, similarity-based vocabulary matching is implemented in this step.

Let $\mathbf{V} \in \{V_1, V_2, \dots, V_U\}$ represents the vocabulary of the linguistic terms, i.e., V_U denotes the linguistic term *very good*, V_{U-1} denotes the term *good*, and etc. The linguistic outputs of the IT2F functions approach can be given as:

$$\tilde{\mathbf{A}}'(t_C) = \begin{bmatrix} \tilde{a}'_{11}(t_C) & \tilde{a}'_{12}(t_C) & \cdots & \tilde{a}'_{1M}(t_C) \\ \tilde{a}'_{21}(t_C) & \tilde{a}'_{22}(t_C) & \cdots & \tilde{a}'_{2M}(t_C) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}'_{N1}(t_C) & \tilde{a}'_{N2}(t_C) & \cdots & \tilde{a}'_{NM}(t_C) \end{bmatrix} \quad (5.46)$$

The $\tilde{a}'_{ij}$ values are calculated as:

$$\tilde{a}'_{ij} = \arg \max_{l \in \{1, 2, \dots, U\}} SM(\tilde{a}'_{ij}, V_l) \quad (5.47)$$

where SM represents the Jaccard similarity measure given earlier, and V_l is the l th linguistic term in the vocabulary.

Step 11: Modifying solutions if necessary: In this step, the results of IT2F functions in the form of linguistic variables are presented to the decision makers. In other words, the current decision matrix is automatically generated based on the past data. The decision makers evaluate the results and make necessary modifications if needed. The illustration of this process is given in Figure 5.4. Here, the decision makers' perceptions have pivotal role. For example, decision makers might decide on

the fact that the performance of the first alternative with respect to the first and second attributes needs to be modified. Then the decision matrix takes the form as given in Equation 5.48.

$$\tilde{\tilde{A}}''(t_C) = \begin{bmatrix} \tilde{\tilde{a}}''_{11}(t_C) & \tilde{\tilde{a}}'_{12}(t_C) & \cdots & \tilde{\tilde{a}}'_{1M}(t_C) \\ \tilde{\tilde{a}}''_{21}(t_C) & \tilde{\tilde{a}}'_{22}(t_C) & \cdots & \tilde{\tilde{a}}'_{2M}(t_C) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{a}}'_{N1}(t_C) & \tilde{\tilde{a}}'_{N2}(t_C) & \cdots & \tilde{\tilde{a}}'_{NM}(t_C) \end{bmatrix} \quad (5.48)$$

where $\tilde{\tilde{a}}''_{ij}(t_C)$ represents the subjective judgements of the decision makers and $\tilde{\tilde{a}}'_{ij}(t_C)$ denotes IT2F function result.

Step 12: Generating time series weights: In this step, time series weights are generated. A basic unit-interval monotonic (BUM) function is used to generate weights. Yager (1988) defined the BUM function as $Q:[0,1] \rightarrow [0,1]$, where the weights are considered as quantifiers underlying the information fusion process.

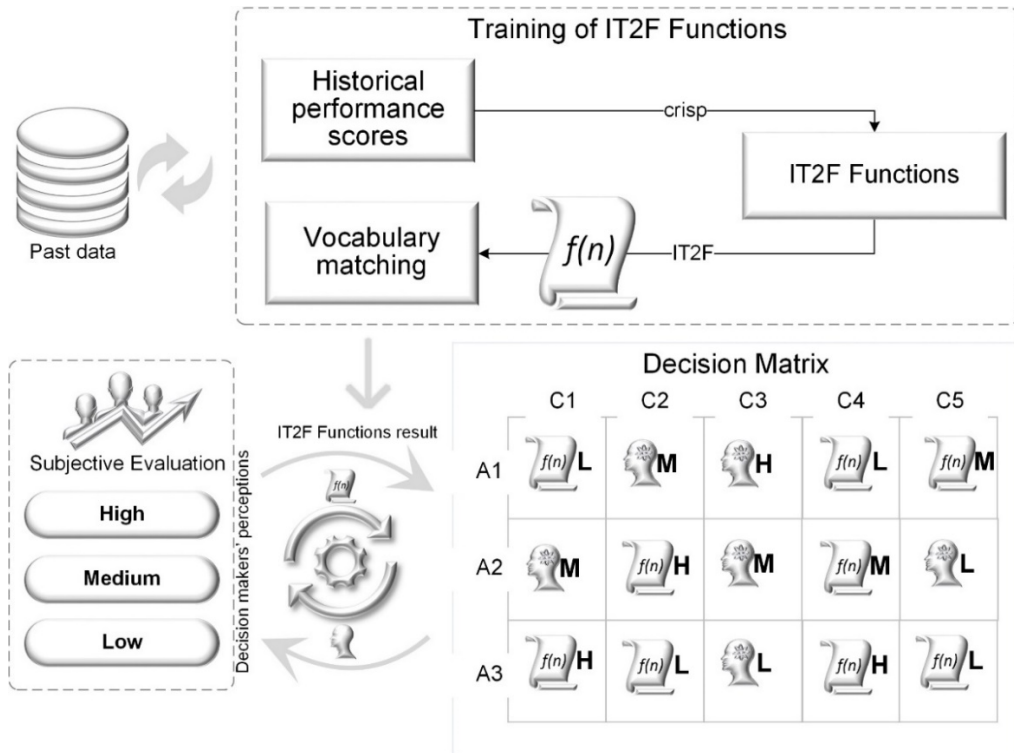


Figure 5.4 Interactive process of the proposed model

- $Q(0) = 0$
- $Q(1) = 1$
- $Q(x) \geq Q(y)$, if $x > y$

where $Q(x)$ is a monotonically non-decreasing function defined in the unit interval $[0,1]$.

Based on the BUM function, the time series weights are generated as:

$$\xi(t_k) = Q\left(\frac{k}{p}\right) - Q\left(\frac{k-1}{p}\right), k = 1, 2, \dots, p \quad (5.49)$$

where $Q(x) = \frac{e^{\alpha x} - 1}{e^{\alpha} - 1}$, $\alpha > 0$.

According to BUM function, the more the period is closer to the current period, the higher the weight of that period, which is a desired behavior for real-world applications.

Step 13: Evaluating performance of alternatives at each period: Performance of each alternative is calculated for different periods separately. In this step, variety of MADM methods can be employed to obtain a performance indicator. For the sake of simplicity, well-known closeness coefficient measures are used to evaluate performance of alternatives at each period. As the current decision matrix comprises of IT2F evaluations, computational steps for the IT2FSs are given in this section in order to avoid repetition. Note that the required computations for the past data are the same, except the fact that numerical values are crisp. First, the decision matrices related to past periods are normalized as given earlier in Equation 5.1-2. As the current decision matrix consists of IT2F evaluations, normalization is conducted based on the Equation 5.50-51.

$$\tilde{r}_{ij}^{\approx}(t_k) = \begin{cases} \left(\frac{\bar{a}_{ij1}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}, \frac{\bar{a}_{ij2}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}, \frac{\bar{a}_{ij3}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}; 1 \right), \\ \left(\frac{\underline{a}_{ij1}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}, \frac{\underline{a}_{ij2}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}, \frac{\underline{a}_{ij3}(t_k)}{\max_j \{\bar{a}_{ij3}(t_k)\}}; 1 \right) \end{cases}, \text{if } i \in \Omega_b \quad (5.50)$$

$$\tilde{r}_{ij}^{\approx}(t_k) = \begin{cases} \left(\frac{\min_j \{\bar{a}_{ij1}(t_k)\}}{\bar{a}_{ij3}(t_k)}, \frac{\min_j \{\bar{a}_{ij2}(t_k)\}}{\bar{a}_{ij2}(t_k)}, \frac{\min_j \{\bar{a}_{ij1}(t_k)\}}{\bar{a}_{ij1}(t_k)}; 1 \right), \\ \left(\frac{\min_j \{\bar{a}_{ij1}(t_k)\}}{\underline{a}_{ij3}(t_k)}, \frac{\min_j \{\bar{a}_{ij2}(t_k)\}}{\underline{a}_{ij2}(t_k)}, \frac{\min_j \{\bar{a}_{ij1}(t_k)\}}{\underline{a}_{ij1}(t_k)}; 1 \right) \end{cases}, \text{if } i \in \Omega_c \quad (5.51)$$

Then, the weighted normalized decision matrices are calculated as:

$$\tilde{v}_{ij}(t_k) = \tilde{r}_{ij}^{\approx}(t_k) \times w_i, \quad i = 1, 2, \dots, N, \quad j = 1, 2, \dots, M, \quad k = 1, 2, \dots, H \quad (5.52)$$

When the weighted normalized decision matrices are constructed, the next step is to calculate the positive ideal solutions (PIS) and negative ideal solutions (NIS) as:

$$\begin{aligned} PIS &= (\nu_1^+, \nu_2^+, \dots, \nu_N^+) \\ &= \left\{ \left(\max_j \{Rank(\tilde{v}_{ij}(t_k))\} \right) \middle| i \in \Omega_b, \right. \\ &\quad \left. \left(\min_j \{Rank(\tilde{v}_{ij}(t_k))\} \right) \middle| i \in \Omega_c \right\}, \quad k = 1, 2, \dots, H \end{aligned} \quad (5.53)$$

$$\begin{aligned} NIS &= (\nu_1^-, \nu_2^-, \dots, \nu_N^-) \\ &= \left\{ \left(\min_j \{Rank(\tilde{v}_{ij}(t_k))\} \right) \middle| i \in \Omega_b, \right. \\ &\quad \left. \left(\max_j \{Rank(\tilde{v}_{ij}(t_k))\} \right) \middle| i \in \Omega_c \right\}, \quad k = 1, 2, \dots, H \end{aligned} \quad (5.54)$$

Then, separation measures are calculated by using the Euclidean distance as:

$$D_j^+(t_k) = \sqrt{\sum_{i=1}^N \left(\text{Rank}(\tilde{v}_{ij}(t_k)) - v_i^+(t_k) \right)^2} \quad (5.55)$$

$$D_j^-(t_k) = \sqrt{\sum_{i=1}^N \left(\text{Rank}(\tilde{v}_{ij}(t_k)) - v_i^-(t_k) \right)^2} \quad (5.56)$$

Finally, closeness coefficients are calculated as:

$$CC_j(t_k) = \frac{D_j^-(t_k)}{(D_j^+(t_k) + D_j^-(t_k))} \quad (5.57)$$

where $CC_j(t_k)$ represents the closeness coefficients.

Step 14: Aggregating past and current performance of alternatives: Finally, Dynamic Weighted Averaging (DWA) operator is utilized to obtain final ranking values of alternatives.

$$DWA_{\xi(t)}(CC_j(t_1), CC_j(t_2), \dots, CC_j(t_H)) = \sum_{k=1}^H \xi(t_k) CC_j(t_k) \quad (5.58)$$

Step 15: Rank the alternatives: When the past and current performance scores of alternatives are aggregated, alternatives are ranked based on their ranking values. Higher ranking value implies superiority of an alternative.

5.4 Case Study

One of the most important assets of a company is undoubtedly the human resources (HRs). Regardless of how the other resources are managed in an organization, inadequacies in the management of HRs result in poor performance of many operations. Therefore, firms have steadily recognized the importance of HRs and have been taking necessary actions to increase overall performance.

Personnel promotion is a significant task in HR management that aims to select the right person for the right job. Despite its similarity with the personnel selection problem, personnel promotion problem deals with selecting appropriate personnel for higher positions within the firms' current personnel rather than evaluating the applicants from outside the firm. Personnel promotion problem can be defined as selecting the most qualified employee among the available candidates for a vacant position by considering their performance during their employment at the firm. Hence, personnel promotion problem is inherently dynamic MADM problem as the temporal performance of employees are taken into consideration with respect to predetermined attributes. Unfortunately, many enterprises are not aware of the methodologies and tools to utilize historical records in their HR practices. As evaluating personnel with respect to their performance on diverse set of criteria during their employment is cognitively demanding, firms should be supported with relevant data-driven tools.

In this study, a real-life personnel promotion problem is considered and the proposed model is implemented. The company which is contacted within the scope of this study is a medium sized firm involved in automobile subsidiary industry. Due to the firm policy, it is named as company A. Company A has hired 3 students as a part-time employee for their continuous improvement project. Upon graduation, at most two students with qualified skills will be offered to full-time job so that the supervisors have been evaluating students' performance in a monthly basis. Therefore, performance data has been used to demonstrate the procedural steps of the proposed model.

5.4.1 Structuring Personnel Promotion Problem

Step 1: In this step, the experts are identified based on their professional backgrounds. The heterogeneity of the experts is assured and three experts are identified. The experts are the directors of HR and production and quality control departments. After identification of the experts, participants are elucidated about the scope and details of the study. The evaluation criteria determined by the experts are employed within the scope of this study. The evaluation criteria consist of content-

specific knowledge (C_1), communication skills (C_2), job involvement (C_3), organizational commitment (C_4), problem solving skills (C_5). The decision hierarchy is given in Figure 5.5.

Step 2: Historical performance data of the employees with respect to predefined attributes are obtained. The company has a performance evaluation system that assigns performance scores between 0-10, 0 and 10 represent the worst and the best values, respectively. In the scope of this study, performance values of the past 1.5 years (18 months) are directly used. The historical data of the performance scores of the employee 1 with respect to decision attributes is given in Table 5.1. Similarly, the historical records for the employee 2 and employee 3 are obtained.

Step 3: Historical data is organized in tables, and missing values are sought for. No missing values are identified within the 18-month data period. On the other hand, it has been decided not to implement normalization techniques as the proposed model can handle numeric values in the range 0-10. Also, the historical decision matrices are transformed into time series vectors so that the IT2F functions method can be implemented.

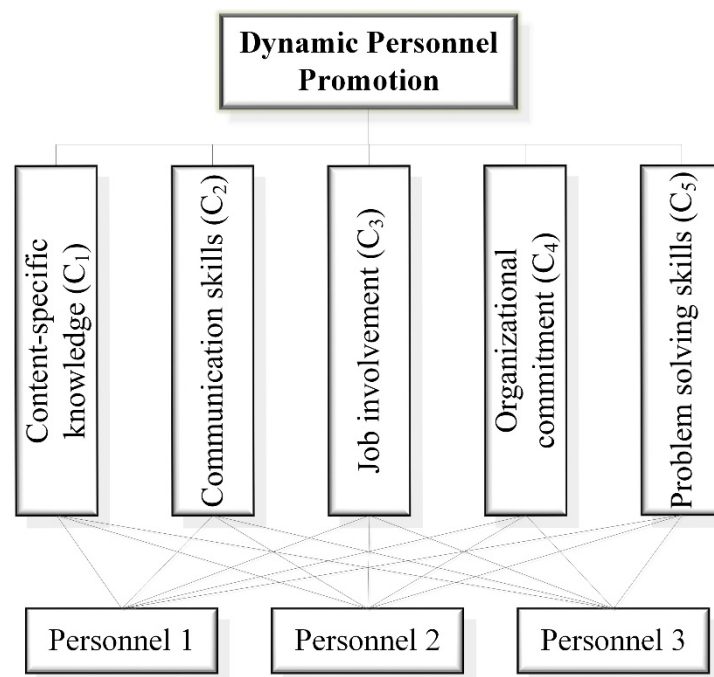


Figure 5.5 Decision hierarchy

Afterwards, the data set is divided into training and test sets. Training set consists of the first 80% part of the historical data and the rest of the data points are allocated to the test set.

5.4.2 Estimating the Current Decision Matrix

Having obtained the historical data, the next step is to employ proposed IT2F functions approach to estimate the current decision matrix based on past data.

Table 5.2 Historical data of employee 1

Period	C ₁	C ₂	C ₃	C ₄	C ₅
1	6	7	3	3	7
2	6	7	2	2	6
3	7	8	3	3	5
4	6	9	4	5	5
5	6	8	3	6	5
6	6	9	4	8	4
7	7	8	4	8	4
8	6	7	5	7	4
9	6	6	6	8	5
10	6	5	6	8	5
11	7	4	7	9	6
12	6	3	6	10	6
13	5	4	6	10	6
14	5	4	8	8	6
15	4	4	6	7	8
16	4	3	6	7	7
17	4	3	6	7	8
18	4	2	6	9	8

Step 4: In this step, parameters of the model are identified. The model parameters are number of *lagged_periods*, *tolerance_levels*, number of cluster and degree of fuzzification. Instead of using cluster validity indices in order to select the optimum *c* and *m* parameters, the model parameters are selected by using a grid search for each parameter based on RMSE performance metric. The *tolerance_levels* are set to 30% for being able to find feasible solutions for every combination of parameters when solving quadratic programming models. The identified parameters are given in Table 5.3.

Step 5: In this step, the input and output matrices are combined and the FCM clustering algorithm is performed based on the parameters identified in the previous step. By utilizing FCM clustering algorithm, cluster centers are identified. These cluster centers have a key role in Turksen's fuzzy system model in which the clusters are used as a fuzzification engine of the classical FRB systems.

Table 5.3 Parameters of the model

Alternative	Criteria	Parameters		
		c	m	$lagged_periods$
1	1	2	1.6	5
	2	5	1.6	5
	3	4	2.1	4
	4	5	2.1	5
	5	4	1.6	4
2	1	4	1.6	5
	2	5	2.1	5
	3	2	1.1	5
	4	5	2.1	5
	5	5	1.6	5
3	1	2	2.1	5
	2	2	1.6	5
	3	3	1.6	5
	4	5	2.1	5
	5	5	1.6	5

Step 6: Having identified cluster centers, the next step is to find membership grades of the input data and to form the augmented input matrices by integrating membership transformations as explanatory variables. In this study, integration of membership degrees, exponentials of the memberships and the square of the memberships have been found to exhibit good performance so that these transformations are used to augment the input space.

Step 7: When the input and output matrices are formed by means of membership grades, quadratic programming model has been solved in order to obtain fuzzy regression coefficients. The models have been written in MATLAB 9.5.0 and quadratic programming models are solved via quadratic programming solver function *cplexqp* of the Cplex Optimization Studio 12.8. As the regression coefficients have

been defined as the distance to the center (b) earlier, their corresponding IT2F number representations are given in Table 5.4-5 for the case of first alternative and first criteria.

Table 5.4 Cluster 1 results for the 1st alternative and 1st criteria

Variable	IT2F Regression Coefficients					IT2F coefficients
	b	f	g	p	q	
1	10.572	8.65E-09	0.031	9.30E-11	9.13E-11	((10.572,10.572,10.603;1), (10.572,10.572,10.603;1))
μ	-8.524	3.39E-09	0.070	6.45E-11	6.44E-11	((-8.524,-8.524,-8.454;1), (-8.524,-8.524,-8.454;1))
$\exp(\mu)$	-4.550	2.04E-09	0.016	2.79E-11	2.78E-11	((-4.55,-4.55,-4.534;1), (-4.55,-4.55,-4.534;1))
μ^2	13.466	7.27E-09	0.085	7.34E-11	7.34E-11	((13.466,13.466,13.551;1), (13.466,13.466,13.551;1))
x_{t-1}	0.017	0.146043	0.010	2.67E-11	2.59E-11	((-0.129,0.017,0.027;1), (-0.129,0.017,0.027;1))
x_{t-2}	-0.613	1.24E-09	0.005	0.1	0.185714	((-0.713,-0.613,-0.423;1), (-0.613,-0.613,-0.609;1))
x_{t-3}	-0.211	1.44E-09	0.004	1.67E-11	1.63E-11	((-0.211,-0.211,-0.207;1), (-0.211,-0.211,-0.207;1))
x_{t-4}	0.822	0.078111	0.126	1.48E-11	1.42E-11	((0.744,0.822,0.948;1), (0.744,0.822,0.948;1))
x_{t-5}	0.038	1.44E-09	0.006	1.36E-11	1.34E-11	((0.038,0.038,0.044;1), (0.038,0.038,0.044;1))

Table 5.5 Cluster-2 results for the 1st alternative and 1st criteria

Variable	IT2F Regression Coefficients					IT2F coefficients
	b	f	g	p	q	
1	-10.907	1.28E+00	1.018	4.25E-09	4.59E-09	((-12.184,-10.907,-9.889;1), (-12.184,-10.907,-9.889;1))
μ	-18.964	3.03E-09	0.000	1.44E-08	1.44E-08	((-18.964,-18.964,-18.964;1), (-18.964,-18.964,-18.964;1))
$\exp(\mu)$	15.077	1.79E-09	0.000	3.39E-09	3.61E-09	((15.077,15.077,15.077;1), (15.077,15.077,15.077;1))
μ^2	-3.447	3.12E-09	0.000	7.20E-01	1.16E+00	((-4.167,-3.447,-2.289;1), (-3.447,-3.447,-3.447;1))
x_{t-1}	-0.107	1.50E-09	0.000	8.08E-10	8.56E-10	((-0.107,-0.107,-0.107;1), (-0.107,-0.107,-0.107;1))
x_{t-2}	-0.725	9.06E-10	0.000	3.64E-07	0.024974	((-0.725,-0.725,-0.7;1), (-0.725,-0.725,-0.725;1))
x_{t-3}	-0.262	3.18E-09	0.000	6.04E-10	6.37E-10	((-0.262,-0.262,-0.262;1), (-0.262,-0.262,-0.262;1))
x_{t-4}	0.735	3.18E-09	0.000	5.26E-10	5.27E-10	((0.735,0.735,0.735;1), (0.735,0.735,0.735;1))
x_{t-5}	0.150	1.16E-05	0.001	5.74E-10	6.10E-10	((0.15,0.15,0.151;1), (0.15,0.15,0.151;1))

Similarly, the same computations are performed for all of the alternative and criteria pairs. Note that number of clusters are based on the Table 5.2.

Step 8: When all of the IT2F regression coefficients are calculated, the output is predicted by using the obtained regression coefficients for each cluster. For the time series data of the alternative 1 and criteria 1, 2 local fuzzy functions are calculated. Since there are 5 clusters in the time series data of the alternative 1 and criteria 2, the total of 5 local fuzzy function results are obtained.

Step 9: In this step, the local fuzzy functions are aggregated and the estimated outputs are calculated. Performance of the proposed IT2F functions approach is compared with the IT2F regression model by means of RMSE and MAPE metrics. Table 5.6 shows the performance comparison of the proposed IT2F functions approach.

Table 5.6 Performance comparison of proposed method and IT2F Regression

Alternative	Criteria	Proposed IT2F Functions		IT2F Regression	
		RMSE	MAPE	RMSE	MAPE
1	1	0.2307	2.3883	0.6974	11.4445
	2	0.1948	3.3149	0.7844	15.1122
	3	0.2519	4.0679	0.9828	14.0217
	4	0.375	3.4459	0.6952	7.2826
	5	0.3606	5.6463	0.8976	16.7376
2	1	0.2894	6.0687	0.4449	13.6456
	2	0.2392	2.4306	0.6793	7.5079
	3	0.3733	4.2811	0.9343	11.083
	4	0.4283	5.1677	0.5934	7.8132
	5	0.3035	4.0027	0.5657	7.9788
3	1	0.1457	1.722	0.7285	8.0463
	2	0.1686	3.9912	0.5833	17.4631
	3	0.2417	2.6302	0.6196	6.6876
	4	0.2142	2.135	0.3221	3.4379
	5	0.2205	2.7494	0.5567	7.9517

Despite its practicality, IT2F regression, which does not make use of FCM clustering and augmented input matrix as in the case of proposed IT2F functions, cannot capture the trends and patterns in the historical dataset to the desired extent. Hence, both of the performance metrics are quite high. On the other hand, the proposed IT2F functions approach has successfully captured the patterns in the data thanks to its membership processing mechanisms. Figure 5.6 shows the estimated performance

scores of the alternative 1 with respect to 5 criteria by means of IT2F functions approach.

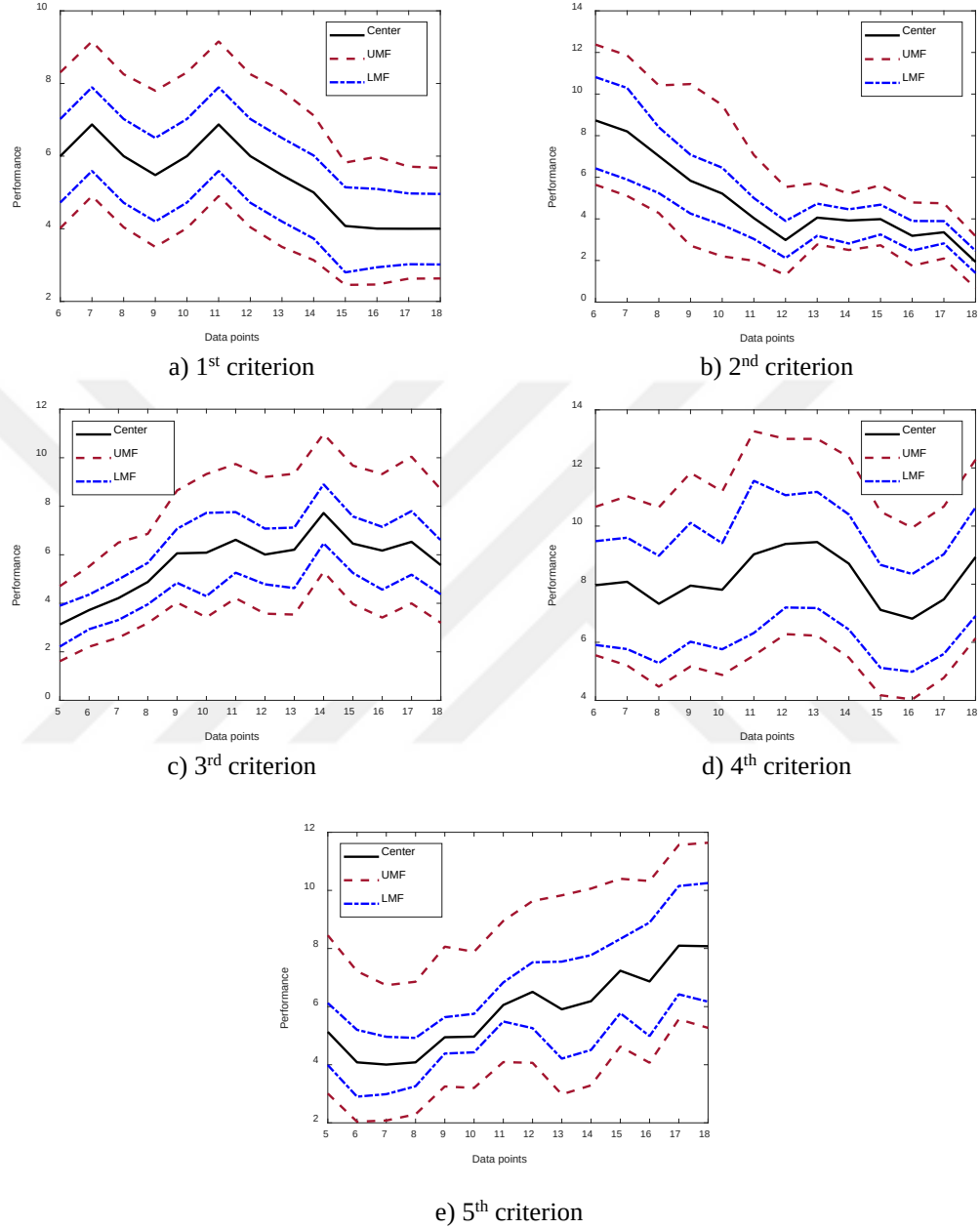
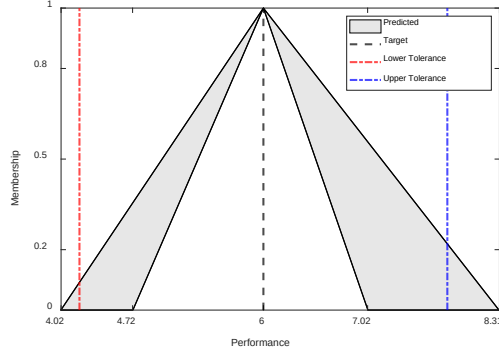


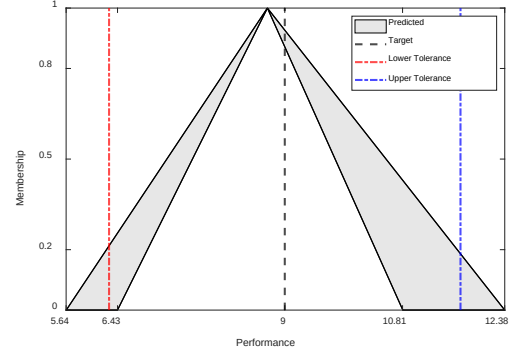
Figure 5.6 Estimations for the 1st alternative

Note that the produced results are IT2F numbers. Another critical point is about whether the produced results satisfy the imposed constraints by possibility and necessity relationships. Figure 5.7 digs into the first data points of the predictions which are illustrated in Figure 5.6. Note that lower and upper tolerances cover the

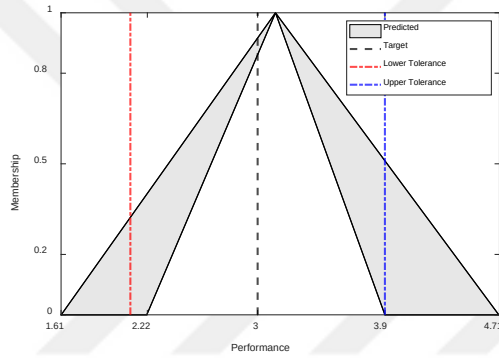
lower membership function which is the property of the necessity constraints in the mathematical model.



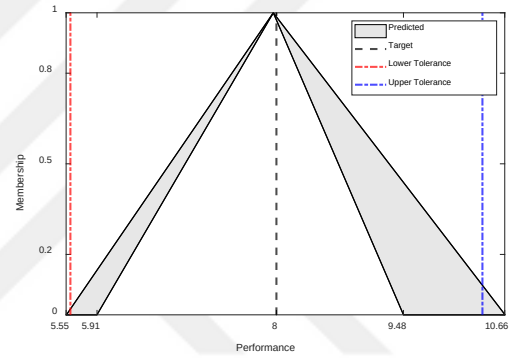
a) 6th data point w.r.t. 1st criterion



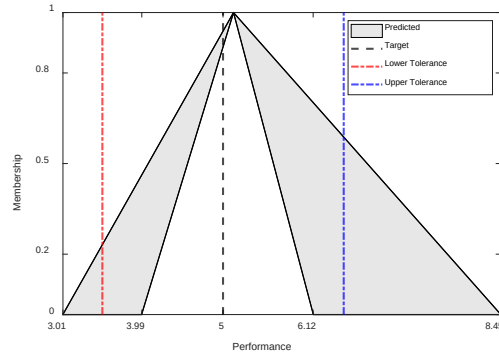
b) 6th data point w.r.t. 2nd criterion



c) 5th data point w.r.t. 3rd criterion



d) 6th data point w.r.t. 4th criterion



e) 5th data point w.r.t. 5th criterion

Figure 5.7 Details of the predictions for the 1st alternative

Furthermore, lower and upper tolerances are covered by the upper membership function, which is due to the possibility constraints. It can be seen that both of the lower and upper tolerances lie within the FOU of the IT2F outputs. Also, estimated

outputs are inside the FOU and the centers of the IT2F outputs are quite close to the target values, which indicates the predictive power of the IT2F functions.

Based on the trained IT2F functions model, the current decision matrix is predicted. The resulting decision matrix is shown in Table 5.7.

Table 5.7 Resulting decision matrix of the IT2F functions

Criteria	Alternatives		
	A ₁	A ₂	A ₃
C ₁	((1.694,2.99,4.515;1), (2.064,2.99,3.822;1))	((2.163,3.73,5.516;1), (2.864,3.73,4.204;1))	((5.223,7.902,11.27;1), (5.601,7.902,10.396;1))
C ₂	((1.354,2.255,3.33;1), (1.891,2.255,2.623;1))	((4.992,8.419,12.522;1), (5.964,8.419,10.802;1))	((2.662,3.919,5.449;1), (3.204,3.919,4.596;1))
C ₃	((2.986,5.358,8.46;1), (4.13,5.358,6.362;1))	((2.194,4.143,5.991;1), (3.144,4.143,5.238;1))	((6.016,9.229,12.788;1), (6.68,9.229,11.589;1))
C ₄	((7.187,10.353,14.08;1), (8.084,10.353,12.123;1))	((0.469,1.617,3.242;1), (0.933,1.617,2.178;1))	((5.678,8.415,11.417;1), (6.071,8.415,10.508;1))
C ₅	((2.409,5.252,9.04;1), (3.372,5.252,7.208;1))	((4.427,6.446,9.052;1), (5.168,6.446,7.487;1))	((3.715,6.246,8.94;1), (4.566,6.246,7.472;1))

5.4.3 Ranking of Employees

Step 10: In this step, similarity-based vocabulary matching is performed. First, the linguistic terms and their corresponding IT2F number are determined. Table 5.8 shows the linguistic terms of the vocabulary. Based on the linguistic terms given in Table 5.8, similarity-based vocabulary matching is carried out. As a result, the linguistic decision matrix is given in Table 5.9.

Table 5.8 Vocabulary of linguistic terms

Linguistic Terms		IT2F number
Symbol	Explanation	
VL	Very Low	((0,0,1;1),(0,0,0.5;1))
L	Low	((0,1,3;1),(0.5,1,2;1))
ML	Medium Low	((1,3,5;1),(2,3,4;1))
M	Medium	((3,5,7;1),(4,5,6;1))
MH	Medium High	((5,7,9;1),(6,7,8;1))
H	High	((7,9,10;1),(8,9,9.5;1))
VH	Very High	((9,10,10;1),(9.5,10,10;1))

Step 11: When the linguistic decision matrix is formed, the results are presented to the decision makers. Then, the decision makers are asked to change the performance values of the alternatives based on their perceptions.

Table 5.9 Linguistic decision matrix

Criteria	Alternatives		
	A ₁	A ₂	A ₃
C ₁	ML	ML	MH
C ₂	ML	H	ML
C ₃	M	M	H
C ₄	H	L	H
C ₅	M	MH	MH

The decision makers may accept the decision matrix as it is or alter the whole performance values depending on their judgements. In this study, decision makers decided to change the performance values of the second and third alternatives with respect to first, fourth and fifth criteria. The modified decision matrix is represented in Table 5.10. The modified linguistic terms are highlighted by gray shading.

Table 5.10 Modified decision matrix

Criteria	Alternatives		
	A ₁	A ₂	A ₃
C ₁	ML	MH	MH
C ₂	ML	H	ML
C ₃	M	M	H
C ₄	H	L	MH
C ₅	M	H	MH

The decision makers have accepted the linguistic terms produced by IT2F functions model for the rest of the evaluations. By this way, required expert judgements have been decreased by 80%, which dramatically increases speed of decision making.

Step 12: In this step, time series weights are generated based on BUM function. The only parameter required by BUM function is α . Figure 5.8 illustrates the time series weights for different α values.

In this study, α is selected as 0.5. Furthermore, results for the different α values are examined as well.

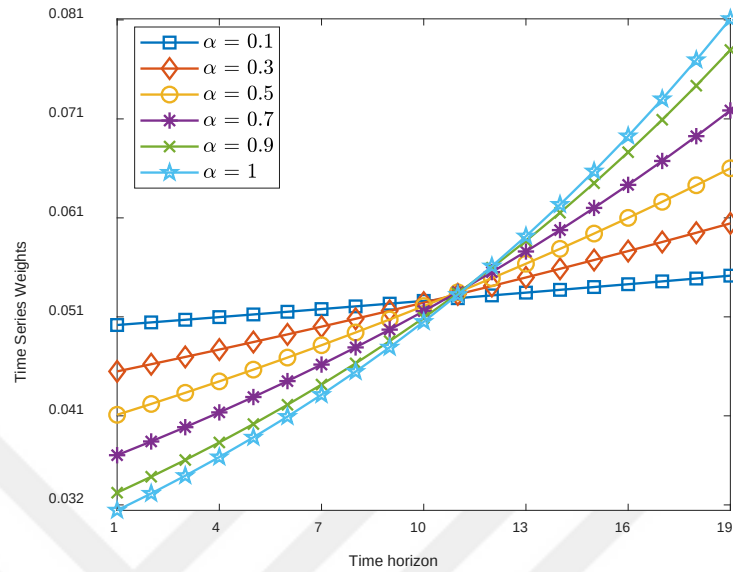


Figure 5.8 Time series weights

Step 13: In this step, performance of each employee is evaluated at each period. Here, past and current decision matrices are evaluated one-by-one separately. As mentioned earlier, closeness coefficient is very practical to use so that for each period distance to positive and negative ideal solutions are calculated. The weights of criteria are assumed to be fixed throughout the periods and given as 0.10, 0.10, 0.15, 0.25, 0.35, respectively. The separation measures are given in Table 5.11-12.

Based on the distances from positive and negative ideal solutions, closeness coefficients are calculated as given in Table 5.13.

Step 14: In this step, closeness coefficients pertaining to past and current performance scores of employees are aggregated by taking into account time series weights. As there are numerous aggregation operators, the most practical one, namely DWA operator, is used in this study. As a result of DWA operator, rankings of employees are obtained. Moreover, for the purpose of comparison, different model components are activated and the results are examined. Table 5.14 summarizes the computational setting with different model components.

Table 5.11 Distances between performance of employees and positive ideal solution

Periods	Distance to Positive Ideal Solution		
	Employee 1	Employee 2	Employee 3
t ₁	0.1458	0.1101	0.1111
t ₂	0.2259	0.1569	0.1084
t ₃	0.2155	0.1546	0.1161
t ₄	0.1604	0.1203	0.0850
t ₅	0.1730	0.1313	0.0938
t ₆	0.1700	0.0919	0.0833
t ₇	0.1700	0.0977	0.0750
t ₈	0.1714	0.0709	0.0643
t ₉	0.1256	0.0872	0.0750
t ₁₀	0.1303	0.0874	0.0750
t ₁₁	0.0960	0.0797	0.1086
t ₁₂	0.1146	0.0901	0.1397
t ₁₃	0.1204	0.0961	0.1393
t ₁₄	0.1004	0.0914	0.1050
t ₁₅	0.1226	0.1401	0.0797
t ₁₆	0.1264	0.1330	0.0972
t ₁₇	0.1422	0.1873	0.1001
t ₁₈	0.1404	0.2335	0.0981
t _C	0.1725	0.1955	0.1143

Table 5.12 Distances between performance of employees and negative ideal solution

Periods	Distance to Negative Ideal Solution		
	Employee 1	Employee 2	Employee 3
t ₁	0.1409	0.1556	0.1541
t ₂	0.1169	0.1524	0.2462
t ₃	0.1161	0.1334	0.2155
t ₄	0.0898	0.0698	0.1613
t ₅	0.0995	0.0805	0.1762
t ₆	0.1011	0.1636	0.1836
t ₇	0.0983	0.1278	0.1857
t ₈	0.0744	0.1697	0.1807
t ₉	0.0684	0.1259	0.1385
t ₁₀	0.0563	0.0950	0.1336
t ₁₁	0.0811	0.1102	0.0717
t ₁₂	0.0997	0.1397	0.0750
t ₁₃	0.0817	0.1311	0.0750
t ₁₄	0.0680	0.1050	0.0914
t ₁₅	0.0927	0.0850	0.1247
t ₁₆	0.0810	0.1000	0.1244

Table 5.12 continues

Periods	Distance to Negative Ideal Solution		
	Employee 1	Employee 2	Employee 3
t ₁₇	0.1218	0.1050	0.1719
t ₁₈	0.2143	0.1173	0.1966
t _C	0.1873	0.1631	0.1751

Table 5.13 Closeness coefficients of employees

Periods	Closeness coefficients		
	Employee 1	Employee 2	Employee 3
t ₁	0.4915	0.5856	0.5811
t ₂	0.3411	0.4928	0.6942
t ₃	0.3501	0.4633	0.6499
t ₄	0.3588	0.3671	0.6549
t ₅	0.3652	0.3801	0.6527
t ₆	0.3729	0.6405	0.6878
t ₇	0.3664	0.5666	0.7123
t ₈	0.3028	0.7053	0.7376
t ₉	0.3525	0.5907	0.6487
t ₁₀	0.3016	0.5208	0.6404
t ₁₁	0.4580	0.5802	0.3978
t ₁₂	0.4652	0.6078	0.3493
t ₁₃	0.4043	0.5769	0.3500
t ₁₄	0.4039	0.5347	0.4653
t ₁₅	0.4304	0.3776	0.6099
t ₁₆	0.3906	0.4291	0.5614
t ₁₇	0.4613	0.3593	0.6321
t ₁₈	0.6041	0.3345	0.6671
t _C	0.5206	0.4547	0.6051

Table 5.14 Different model configurations

Model components	Models			
	Proposed Model	Model 1	Model 2	Model 3
IT2F Functions	Yes	Yes	Yes	Yes
Vocabulary Matching	Yes	Yes	No	Yes
Modified preferences (interactive)	Yes	No	No	Yes
DWA Operator	Yes	Yes	Yes	No

The proposed model integrates all of the model components. Different from the proposed model, the predicted decision matrix is not modified in the model 1. In model 2, neither linguistic decision matrix is generated nor the decision makers are allowed to change the decision matrix. Finally, Model 3 makes use of IT2F functions,

vocabulary matching and modified preferences, but only the decision matrix of the current period is considered for ranking of alternatives, which corresponds to the conventional MADM approach.

Table 5.15 Overall results

Model	Overall Assessment					
	Employee 1		Employee 2		Employee 3	
	CC	Rank	CC	Rank	CC	Rank
Proposed Model	0.4138	3	0.4984	2	0.5895	1
Model 1	0.4176	3	0.4926	2	0.5967	1
Model 2	0.4185	3	0.4905	2	0.5951	1
Model 3	0.5206	2	0.4547	3	0.6051	1

Step 15: The employees are ranked and the results are interpreted in this step. As can be seen from the Table 5.15, employee 3 dominates the other candidates regardless of the model configurations. However, the performance of employee 1 needs more attention here as the ranking order differs by the static or dynamic MADM aspects. The performance of employee 1 has been visualized in Figure 5.9.

It has been observed that the performance of employee 2 is better than employee 1 for all of the dynamic MADM models. However, considering only the current decision matrix in the Model 3, static MADM ranks employee 1 as second and employee 2 as third. Although the performance of the employee 1 is higher than the employee 2 in the current period, considering historical performance values changes the overall rankings. Therefore, the present study provides policy makers with a broader view regarding performance of employees by considering different modeling aspects.

5.5 Chapter Conclusions

In this study, a new interactive dynamic MADM model, which combines IT2F regression and fuzzy functions approaches, is proposed. The proposed model exhibits desirable properties that help overcome the drawbacks of the traditional dynamic MADM approaches. The proposed model is realized in a real-life personnel promotion problem. The proposed IT2F functions approach has improved the performance of

IT2F regression by successfully capturing the trends and patterns in the historical records.

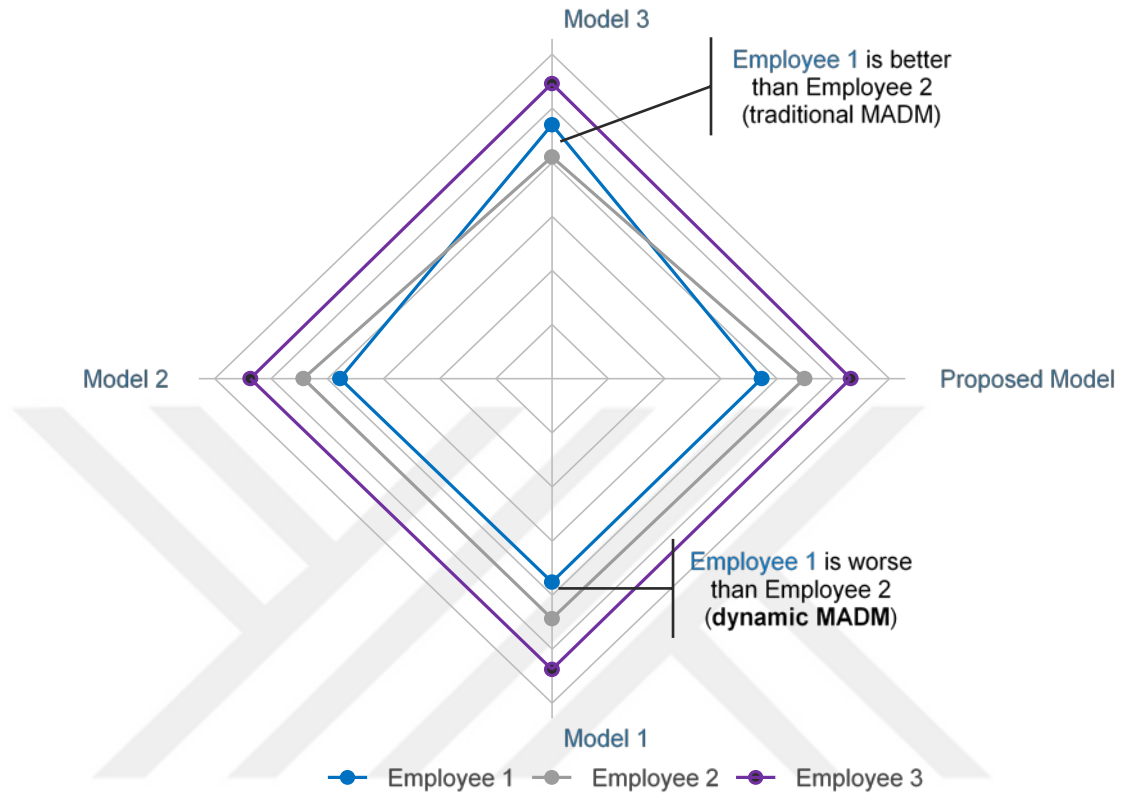


Figure 5.9 Changes in the ranking of employee 1

Moreover, current decision matrix is automatically generated based on past data and the decision makers have modified 3 performance scores out of 15, which dramatically facilitates preference elicitation procedures.

The results have showed that the employee 3 has ranked first according to all of the dynamic MADM models. Although the employee 1 has better performance than employee 2 in terms of current period performance, consideration of the past data makes the employee 1 the worst candidate for the full-time position among other employees. As a result, taking into account the past and current decision matrices together provides more information to the decision makers and leads to more accurate rankings.

The main limitation of the presented study is that the results produced by the proposed model is highly dependent on the data obtained from the case company. Accordingly, results are difficult to be generalized. Secondly, prediction performance highly depends on available data. Nevertheless, the presented study can be improved in terms of many aspects. Hyper parameter optimization can be used to tune the parameters of the model automatically. Also, different strategies can be imposed to generate time series weights, and results can be compared. Last but not least, different MADM approaches can be used to rank the alternatives. In future studies, these considerations will be at the top of our agenda.



CHAPTER SIX

CONCLUSIONS

Decision engineering is a vivid research area that the real-life applications are pervasive. In the literature, a plethora of decision engineering tools have been developed in order to help decision makers in problem framing, alternative generation, and synthesis. Despite continuing proliferation of the new methods and tools, there are some inadequacies in the current state-of-the-art. First, uncertainty handling in modeling interdependencies among criteria is a critical research area. However, reasoning with uncertain interactions is not a trivial problem. Especially when the interval type-2 fuzzy sets are concerned, new computational methods are needed. On the other hand, past data is almost always not taken into consideration in decision making practices. However, data is an invaluable asset of any firm so that the firms steadily recognize the importance of data-driven decision making. Integrating past data into the decision making practices is one of the challenging problems in decision engineering.

In this thesis, IT2F DEMATEL and IT2 FCMs have been developed for expert-driven decision making applications. Here, new computational methods are proposed for the case of IT2 FCMs. Both of the T1F and IT2F sets can be used to model interactions among criteria by using the proposed alpha-cut based computational methods. The proposed models are able to consider interactions among hierarchy of criteria under IT2F environment. These models are realized in strategy selection and third party logistics service provider selection problems, respectively.

Furthermore, data-driven decision making models are developed based on learning of FCMs and IT2F regression functions methods. The distinctive feature of FCM learning is that the algorithm considers interactions among criteria while learning from past data. Here, one of the parameter-free metaheuristic algorithms, namely Jaya algorithm, has been used to capture the trends and patterns in the past data. Then, future projections are performed based on the trained model and future decision matrices are generated. Finally, dynamic MADM module has been used to rank the alternatives

based on past, current and the future decision making matrices. The proposed model is implemented in dynamic supplier selection problem.

Another contribution of the present thesis is to develop a new fuzzy functions architecture based on Türkşen's fuzzy functions approach and provide its real-life application in dynamic personnel promotion problem. In fuzzy functions approach, LSE has been implemented and the results are crisp values. There are two drawbacks of this approach. First, LSE requires statistical assumptions to be satisfied. Moreover, decision makers' preferences are represented linguistically in decision making applications so that the crisp estimations do not yield desired results. In this thesis, a quadratic programming-based local IT2F regression functions are embedded into the Türkşen's fuzzy functions in order to produce IT2F outputs. The proposed model is realized in dynamic interactive decision making environment in which the IT2F output are transformed into linguistic terms via vocabulary matching procedures. The decision makers are allowed to alter the automatically generated linguistic terms based on estimated performance of alternatives.

To sum up, present thesis contributes to the literature with four decision models. The future studies include incorporation of general type-2 fuzzy sets and fuzzy random variables into the decision engineering problems. Moreover, big data analytics solutions can be integrated into the workflow of the decision engineering applications. Last but not least, vast array of machine learning algorithms can be used within the proposed models.

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