ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF ARTS AND SOCIAL SCIENCES

SEQUENTIAL AUCTIONS WITH BUDGET CONSTRAINED BIDDERS: TURKISH 4G SPECTRUM AUCTION

M.A. THESIS

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ SOSYAL BİLİMLER ENSTİTÜSÜ

BÜTÇE KISITI ALTINDA SIRALI İHALELER: TÜRKİYE 4N SPEKTRUM İHALESİ

YÜKSEK LİSANS TEZİ

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MAYIS 2015

Ömer Cem Ayaz, a **M.A.** student of ITU **Institute of Social Scneinces** student ID **412121019** successfully defended the **thesis/dissertation** entitled **"SEQUENTIAL AUCTIONS WITH BUDGET CONSTRAINED BIDDERS:TURKISH 4G SPECTRUM AUCTION"**, which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 4 May 2015
Date of Defense : 27 May 2015

FOREWORD

Although there are rumors about cancellation of the auction which is expected to be conducted on 26th of May, it is still one of the most popular topics in 2015 agenda of Turkey.

We have completed our preparations as if there was no problem in the auction timeline.

Without support of Assoc.Prof.Dr.Sencer Ecer, it would be quite hard for me to put a tangible output at the end of my studies. For this reason, I want to express my gratitude for his valuable insights and continuous support. I feel privileged to have such an adviser to work with.

I hope that this study will be helpful for many others to understand spectrum auctions in Turkey and to find basic information especially about design of 4G auction.

I want to thank all my instructors once again for their contributions and kind support during my studies.

May 2015 Ömer Cem AYAZ

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ABBREVIATIONS

2G	**:** Second-generation Wireless Telephone Technology
3G	**:** Third-generation Wireless Telephone Technology
4G	**:** Fourth-generation Wireless Telephone Technology
GSM	**:** Global System for Mobile Communications
UMTS	**:** Universal Mobile Telecommunications System
LTE	**:** Long-Term Evolution
SMRA	**:** Simultaneous Multiple Round Auction
CCA	**:** Combinatorial Clock Auction
FDD	**:** Frequency Division Duplexing
TDD	**:** Time Division Duplexing
ARPU	**:** Average Revenue per User

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SEQUENTIAL AUCTIONS WITH BUDGET CONSTRAINED BIDDERS: TURKISH 4G SPECTRUM AUCTION

SUMMARY

Auction is a method to find an answer for how to assign goods and at what prices. In auctions, sellers usually aim at maximizing auction revenue; whereas, the bidders concern with their own payoffs with respect to their valuations and object prices.

Auction theory is an interesting commercial application of game theory and spectrum auctions have a special position in it due to high amount of revenues and complex auction designs.

Basically, spectrum is a range of frequencies which is being used for different purposes such as TV broadcasting and mobile telecommunication services. Although it is not a depletable resource like oil reserves or mines, it is still scarce and valuable due to limited available range. For this reason, this strategic public resource should be assigned properly and contribute public welfare.

Telecommunications became the most noticeable area of spectrum usage due to its undeniable role in our daily routines or commercial activities. At the very beginning, 2G services introduced SMS and voice call and these services changed our daily life considerably with GSM 900. After the launch of 3G, mobile services gained a different dimension and data has become the main actor of the telecommunication market by using 2100MHz band for UMTS services. Increasing traffic and desire for higher speed resulted in implementation of a new technology called 4G and allocation of additional spectrum in 800MHz and 2600MHz bands for LTE services.

In order to assign spectrum to wireless communication providers, governments usually design and organize similar auctions. By auction design, governments aim at efficient usage of available spectrum, downstream competition in the market and reasonable amount of auction revenue which should be convincing for their tax-payers.

In 2015, new spectrum auction is planned to be conducted, which aims at providing LTE services in Turkey by next January. The design of Turkish 4G auction is quite similar with previous 3G auction which was conducted in 2008. However, this time everything is more complicated since the number of auctioned blocks increase from 3 to 20. In the previous auction, allocation of 2100 MHz band was the only concern , but now there are 5 different bands which are going to be allocated by sequential auctions. After September 2016, the license owners are allowed to use 900 MHz band for UMTS(3G) services. When we focus on each band separately 800MHz, 900MHz,and 2100MHz auctions might subject to low competition and final prices are expected to be more or less announced reserve prices. Usually, the highest competition occurs in 800MHz band but Turkish auction differs from other auctions in this part. Instead of 800MHz, 1800MHz and 2600MHz bands will be the challenging part of this huge auction which will assign 390MHz spectrum in total.

Second chapter of the thesis covers basic auction types and recently implemented some other models. It briefly explains simultaneous and sequential auction concepts. There is also some information about package bidding rather than selling objects individually. Combinatorial clock auction is introduced at the end of this section as the most well-known implementation of package based auction design.

In third chapter, the brief history of Turkish spectrum auctions and details of upcoming 4G auction could be found. Basically, upcoming 4G auction has two main stages. First, all participants will submit their sealed envelopes one week prior to auction and after that auction will start with open ascending phase on 26th of May. Activity rules, size of blocks, caps and all other information could be found within this part of the paper.

In last section, there is a model which resolves Turkish 4G auction based on some assumptions. Classical auction theory usually does not pay attention to budget constraints and overlooks this fact. On the other hand, there are still some models concerning budget constraints which illustrate that the sequence of auctioned objects affects auction revenue in multi-object auctions. It is better for auctioneer to sell more valuable object prior to others. The main goal of this study is to show that this argument still valid when each bidder has a different private value for auctioned objects. In spectrum auctions, private value assumption is necessary since these licenses would generate different revenues for different winners. Each company has different ARPU, market share and different expectations for future revenues. Prior to auction, each bidder sets its own budget constraint and price thresholds considering the net present values of LTE related future revenue along with its additional cost. In appendices, the proof of the claim is explained in detail.

BÜTÇE KISITI ALTINDA SIRALI İHALELER: TÜRKİYE 4N SPEKTRUM İHALESİ

ÖZET

Açık artırmalar ve ihaleler bir malın nasıl ve hangi fiyatlarda alıcısını bulacağını belirlemek için sıkça kullanılan yöntemlerdir. İhalelerde amaç satıcı için genellikle gelirini mümkün olabildiğince yukarı çekmekken, katılımcılarsa ürünün değerine ve fiyatına göre elde edecekleri karı optimize etmeye çalışırlar.

İhaleler uygulamalı oyun teorisinin en ilgi çekici örneklerinden biridir. İhale yöntemiyle satılan tüm mallar arasında spektrumun ayrı bir yeri vardır. Spektrum ihalelerinde ortaya çıkan yüksek gelir ve gelişmiş ihale tasarımları bu ihaleleri herkes için daha ilgi çekici hale getirmiştir. Spektrum ticari amaçlar için kullanılabilir bir frekans aralığıdır. Buna en iyi örnek olarak karasal televizyon yayınları ve mobil haberleşme hizmetleri verilebilir.Petrol ya da madenler gibi tükenen bir kaynak olmasada sınırlı miktarda olması nedeniyle ticari olarak oldukça değerlidir. Bu nedenle ihaleler düzgün bir şekilde yapılmalı ve toplumsal refaha katkıda bulunmalıdır.

Spektrumun tüm ticari kullanım alanları arasında günlük hayatımızdaki vazgeçilmez yeri nedeniyle mobil iletişim hizmetleri önplana çıkmaktadır. Başlangıçta 2N hizmetleriyle hayatımıza ses ve kısa mesajı dahil eden bu hizmetler 3N ile birlikte gelen mobil veri servisiyle telekomünikasyon sektörüne bambaşka bir boyut kazandırdı. Bu yeni dönemde katlanarak artan trafik ve yüksek hıza olan talep yeni bir teknolojinin geliştirilmesine neden oldu. 4N bu ithiyacı karşılmak için hayatımıza dahil olurken bununla birlikte tüm dünyada 800MHz ve 2600MHz bantları mobil servis sağlayıcılarına 4G hizmetleri için tahsis edilmeye başlandı.

Spektrumu doğru bir şekilde paylaştırabilmek için her ülke benzer ihaleler düzenlemektedir. İhaleyle birlikte yüksek gelirin yanı sıra bu kısıtlı kaynağın verimli kullanılcak şekilde dağıtması, ihale sırasında katılımcıların gerçek değerlemelerinin tekliflerine yansımasının sağlanması ve pazarda gerekli rekabet ortamının oluşturulması gibi bazı temel amaçlar göz önünde bulundurulmalıdır.

Yeni spektrum ihalesinin Mayıs 2015 döneminde yapılması planlanmaktadır. Tasarım olarak bu ihalenin yapısı 2008'deki 3N servisleri için yapılan 2100 MHz ihalesine oldukça benzemektedir. Diğer taraftan ihale edilen blokların sayısı üçten yirmiye çıktığı ve bu blokların sadece 2100MHz bandında değil 5 farklı bantta yer aldığı için durum 2008'e göre çok daha karmaşık gözükmektedir. Bu ihaleyle ilk kez tahsis edilecek bantlarla birlikte mevcut frekansların kullanımında da değişiklikler olacak ve 900MHz bandı 3N ,1800MHz bandı da 4N hizmetleri için kullanılabilecektir. Toplamda 390MHz'yi kapsayan bu büyük ihalenin 800MHz, 900MHz ve 2100MHz kısımlarında ciddi bir rekabet yaşanması beklenilmezken asıl rekabet 1800MHz ve 2600MHz bantlarında olacaktır.

Bu çalışmada öncelikle temel ihale tipleri ve son dönemlerde kullanılan gelişmiş bazı diğer ihale modellerinden kısaca bahsedilecektir. Temel ihale tiplerinin ötesinde tekliflerin tek tek ya da paketler için verilmesi esasına göre ihaleleri iki temel gruba ayırabiliriz. Bu tasarımlara örnek olarak ABD'de yapılan bazı ihale modelleri örnek olarak verilebilir ve bu kısımda bu modeller kısaca tanıtılmıştır. Bunlardan ilki çoklu nesnelerin eş zamanlı satımını esas alan modeldir. Burada teklifler paketler için değil tek tek ancak eş zamanlı yapılmaktadır. Bu modelin sıralı bir ihaleye göre bazı avantajları ve dezavantajları vardır. Bu modelin en güncel ve popüler örneklerinden biri Almanya'da yapılmış olan 4N spektrum ihalesidir. Diğer bir modelse paket esaslı saatli ihaledir. Burada fiyat zamanla artarken katılımcılar da bunula birlikte taleplerini düşürmek durumundadırlar. Saat aşaması talep arza eşit oluncaya kadar sürer ve sonraki aşamalara geçilir. Bu tasarımların hepsinin kendine göre avantajları ve dezavantajları vardır.

İkinci kısımda Türkiye'de düzenlenmiş olan önceki ihalelerin kısa bir özeti ve ağırlıklı olarak da 4N ihalesinin detaylarından bahsedilmektedir. Spektrum ihalesi iki aşamalı olarak yapılacaktır. İhaleden bir hafta önce tüm katılımcılar kapalı zarflardaki mühürlü tekliflerini regülasyona iletecek ve sonrasında ihale 26 Mayıs'da açık artırma usulüyle devame decektir. Bütün ihalelerde olduğu gibi burada da bazı kısıtlar,minimum artış bedelleri ve sürecin işleriyle ilgili bir takım kurallar bulunmaktadır. İhale sırayla 800MHz,900MHz,1800MHz,2100MHz ve 2600MHz bantları için yapılacak olup bunların hepsinin taban fiyat ve minimum artış miktarları resmi gazetede yayınlanmıştır. Bu ihaleyle birlikte Avea 900MHz bandında eşit bir servis sağlayıcı haline gelmektedir. Bu şekidle Vodafone ve Turkcell buradaki avantajlarını kaybetmektedirler.

Üçüncü kısımda Türkiye'deki 4N ihalesini göz önünde bulunduran bir model çözümlemesi yer almaktadır. Mevcut klasik modellerde bütçe kısıtlarına genellikle yer verilmemektedir. Tek bir ürün için pek çok model çözümü bulmak mümkünken ihaledeki ürün ve katılımcı sayısı arttıkça bu durum giderek daha zor bir hal almaktadır. Çoklu ürün ve katılımcı koşullarında bütçe kısıtını değerlendirmeye alan modellerde ortaya atılmıştır. Bu çalışmalarda da belirtildiği gibi ürünler değeri yüksek olandan düşük olana doğru bir sırayla satıldığında bunun gelir üzerinde olumlu bir etkisi olmaktadır. Bu hipotez ortak değer varsayımı yerine farklı değerlemelere göre yeniden çözülerek çalışmaya dahil edilmiştir. Bu lisanslara sahip olan operatörlerin ortalama kişi başına gelirleri, müşteri sayıları ve pazar payları gibi kriterleri farklı olduğu için bütçe kısıtları ve değerlemeleri de farklı olacaktır. İhale öncesinde tüm katılımcların kendi değerlemelerini ve bütçe sınırlarını belirlemiş olması beklenir. Bu değerler alacakları yeni frekanslarla elde etmeyi planladıkları gelecekteki gelirlerinin ve buna bağlı maliyetlerinin bugünkü değerlerine indirgenmesinin bir ifadesidir. Ürünlerin satış sırasının geliri nasıl etkilediğini gösteren bu modelle ilgili detaylı bilgi ve ispatlar ek kısmında bulunabilir. Modelin temelindeki varsayım 1800MHz ve 2600MHz bantları dışındaki tüm ihalelerin yaklaşık olarak taban fiyatlarla sonuçlanacağıdır. Modelde pazar lideri ihaledeki iki üründen ikisini ya da en az birisini alabiliyorken ikinci en fazla birini üçüncüyse hiçbirini alamamaktadır. Burada kastı geçen iki ürün 1800MHz ve 2600MHz bantlarındaki en geniş bloklardır. Tüm oyuncuların birbirlerinin bütçeleri ve değerlemeleri hakkında tam bilgi sahibi olduğu şartlarda en düşük bütçeli oyuncu yalnızca bir eşik değer belirleyebilmektedir. Yani modelde Turkcell ya da Vodafone en geniş bandı aldığında diğerinin en geniş ikinci bandı aldığı öngörülmüştür. Mevcut pazar payı ve karlılıklarıyla birlikte 900MHz bandı için ödeyeceği yüksek ücret göz önünde bulundurulduğunda Avea’nın 1800MHz ve 2600MHz bantları için kalan bütçesinin diğer iki operatörden daha düşük olması çok yanlış bir varsayım olmayacaktır.

1. INTRODUCTION

Auction theory could be considered as a popular area of applied game theory which is being used to assign valuable objects in a proper way. Although there are various auction designs to assign these goods, the number of main auction types is limited and rest of the models could be considered as augmented or hybrid versions of them. The prices might ascend or descend while the winner pays either the highest bid or the second highest price. The bidder could place bids either on individual objects or on package combinations. Sometimes prices increase by the bids in following rounds; sometimes clock automatically takes this responsibility to reduce total demand. The auctioneer might prefer to sell multiple objects in a sequence while it is also possible to assign them simultaneously. Auctioneer chooses a combination of these methods along with its own activity rules and designs a mechanism. In this thesis, the basic auction types and some multi-object designs will be introduced briefly. The main goal of the work conducted here is to analyze 4G spectrum auction in Turkey and show that 1800 MHz should be auctioned prior to 2600 MHz assuming that other blocks in remaining bands will be sold around their reserve prices and the budgets ,which are allocated for 1800 MHz and 2600 MHz,do not change by auction order.

1.1 Purpose of Thesis

In this thesis, it is intended to briefly explain basic auction types and recently developed auction models for multi-object auctions. There is also some information about history of Turkish telecommunication market and details of next spectrum auction. The main goal is to apply a model for Turkish 4G auction and show that seller benefits from selling the more valuable object early in the auction when each bidder has different private valuations and endogenous budget constraints in a perfect information game.

1.2 Literature Review

In the second section of the thesis, there is brief information about basic auction types and common multi-object auction models. It is easy to find examples and related studies for SMRA and CCA since there are many recent implementations in spectrum auctions. Cramton (2004) explains basic principles of SMRA and how the auction mechanism works. To gain more information about simultaneous and combinatorial auctions, his studies depict activity rules of these auctions along with his own recommendations and compare performances of different designs(Cramton,2013). He compares not only sequential and simultaneous auctions but also analyze advantages and disadvantages of CCA in comparison to SMRA. In addition to eligibility activity rule, which was adapted in English 4G auction, he proposes a new activity rule called revealed preferences and explains why it is superior to eligibility rule. Beyond activity rules, Cramton also explains how Vickrey core pricing mechanism works for a multi-object auction.

Klemperer (2002) tells us a story about European 3G auctions and explains why European auctions failed to generate sufficient revenues except England and Denmark. In his studies, he illustrates importance of proper reserve prices and of how collusion affected auction revenue while reserve prices were low in sequential auctions.

In the model section, previous studies related with budget-constrained bidders revisited. Classical auction theory usually does not pay attention to budget constraints; however, budget constraints play critical role in auction outcomes.

Pitchik and Schotter (1988) introduced a new concept while analyzing sequential auctions with budget-constrained bidders. In their model, there was no synergy between two objects and budget constraints were determined exogenously. Pitchik and Schotter show that there are some critical prices for each bidder beyond which bidder prefers to lose to auction rather than winning the related object.

Pitchik (2009) showed that order of objects in sequential auctions affects auction revenue when there are two bidders with private valuations and endogenous budgets in a perfect information game.

Benoit and Krishna (2001) resolved the model of Pitchik and extended it in many ways. They illustrated not only the correlation between revenue and sequence but also the comparison of simultaneous auctions to sequential auctions in terms of revenue. In their design, they also proposed a new hybrid model which might provide higher revenues than sequential and simultaneous auctions. Their model is built on 2 objects, n bidders perfect information game which has common value and endogenous budgets assumptions. In our model, it is intended to resolve this model with respect to private values while other assumptions are kept similar. Then, the model is implemented on Turkish 4G auction by transforming it into a 2 objects and 3 bidders perfect information.

1.3 Hypothesis

In a multi-object auction in which bidders are budget constrained and have different private valuations, it is better for auctioneer to sell more valuable object early in the auction.

2. AUCTION TYPES

2.1 Purpose

In this section, basic auction types are introduced which could be listed as English auction, Dutch auction, first sealed bid auction, and second sealed bid auction (Vickrey auction) along with some other assignment methods such as lottery and beauty contest. In the last part, simultaneous multiple round auction and combinatorial clock auction are explained briefly.

2.2 Lottery

All applicants are placed in a pool and the licenses are randomly assigned. This process has the advantage of being quick but there is only limited control of applicant capabilities. Thus, many bidders might have applied without the intention of using the licenses. Auctioneer goal is to assign them immediately.

2.3 Beauty Contest

The objects are being allocated by an administrative process. Proposals of candidates are expected to be evaluated according to their eligibility with respect to some predetermined criteria. However, this method would suffer lack of transparency and encounter with many objections at the end of license assignment.

2.4 English Auction

English auction or open ascending auction is a type auction in which bidding usually starts with a predetermined reserve price and increases till there is no higher bid. Most of the auctions could be classified under this category

.

2.5 Dutch Auction

Unlike English auction, Dutch auction starts with a high starting price and the price keeps descending till one of the participants become willing to pay the initial price. Although it is one of the main auction types it is not being used frequently in comparison to other alternative auctions.

2.6 First Sealed Bid Auction

The bids are being delivered in closed envelopes and the highest bid wins. The winner pays its final bid.

2.7 Second Sealed Bid Auction

The bids are being delivered in closed envelopes just like first sealed bid auction and similarly the highest bid wins; however, in this case the winner pays the second highest bid as the final price. This type of auction is also named as Vickrey auction and the final price is called as Vickrey price as well.

2.8 Simultaneous Multiple Round Auction

Simultaneous multiple round auction could be considered as multi-object version of English auction with special activity rules. After each round the winner and price information shared with all bidders. The bidding continues till no bidder wants to submit a higher bid on any object. To be able to bid on any object, the bidder must be active on that object in the previous round.

Unlike sequential auctions, SMRA enables bidders to switch between different combinations. When it is compared with sequential auctions, SMRA has less exposure risk in comparison to sequential auctions. To avoid exposure problems, limited withdrawal bids can be used in SMRA(Cramton 2004). Solution of exposure problem is not that easy in sequential auctions. Moreover, the sequential auctions might end up with low revenues in comparison to simultaneous auctions if reserve prices are not high enough. Most of the European 3G auctions suffered this problem except England and Denmark (Klemperer 2002). The reserve prices could ensure reasonable revenues and decrease the impact of successful collusion. SMRA has advantages in comparison to a sequential auction since it enables all bidders to present their preferences for substitute and complementary objects (McAfee and McMillan 1996). Cramton also states that guessing future prices and shaping initial strategies accordingly is another problem of sequential auctions. Sequential auction eliminates many strategies and causes high risk of regret (Cramton 2002). Bidders might regret having purchased early at high prices or not having purchased some other objects in earlier rounds.

SMRA could also be implemented as a clock auction which ends when the total demand is equal to the amount of available blocks. Clock version of SMRA has advantages to decrease signaling and prevent collusion among bidders.

In addition to revenue and efficiency concerns, an auction designer should consider creating market prices which means that similar objects should be sold for similar prices (Cramton 2002). However, in a sequential auction, similar objects might be sold at very different prices. Swiss 3G auction could be a very good example for this case. Larger blocks were sold at lower prices later in the auction. In a sequential auction, it is quite hard to determine market prices of the auctioned objects.

2.9 Combinatorial Clock Auction

CCA is a type of auction in which bidders place bids on packages rather than individual items. Combinatorial clock auction starts with a clock stage and the prices keep increasing by time till the demand is reduced to available spectrum amount. In the second round, bidders might submit new bids in supplementary stage. Finally, in the assignment stage winners and the final prices are being clarified. Auction revenue might change depending on the auction mechanism. For example, pay as bid method would provide higher revenues while discouraging truthful bidding. On the other hand, Vickrey core pricing method ensures more truthful bidding but generally ends up with lower revenues (Cramton 2013).

Combinatorial clock auctions might differ due to their availability rule. First, it could be based on a demand reduction mechanism in which bidders must have downward demand curve. English 4G auction could be one of the most notable implementations of CCA. Ofcom, which is the regulator of English 4G auction, implemented an activity rule which does not allow any demand increase in clock stage; however, it was possible to do so in supplementary stage if and only if the price at which a bidder reduced its demand creates an upper boundary for its new combinations. This activity rule is named as eligibility point rule and it sounds like a similar rule with demand reduction based activity rule in SMRA.

3. SPECTRUM AUCTIONS IN TURKEY

3.1 Purpose

The main goal of this section is to explain rules of Turkish 4G auction and express history Turkish telecommunication market briefly.

3.2 Previous Auctions in Turkey and Brief History of the Market

In 1994, Turkcell and Telsim acquired first licenses in Turkey for 900 MHz band which costed 500m USD for each operator. After that another auction has been made in 2000 and Aria (Telecom Italia) gained a license to provide wireless services in 1800 MHz band. The license costed 2.5billion USD and established a significant entry barrier for second auction which is planned to be conducted in the same year. As it is expected, no new entrant showed up in the second auction. In 2001, Aycell entered the market as a subsidy of national fixed service provider Turk Telekom. Turkish market had 4 four players only for a limited time; Aycell and Area merged as Avea in 2004. In fact, the case of Aria could be a good example of ”winner’s curse” due to its failure in entry strategy. In 2005, the market encountered with another change and Telsim was acquired by Vodafone at a price about 4.5 billion USD.

Three operators in the market experienced a breakthrough period in 2008 since both 3G auction and mobile number portability took place within this year. The design of 3G auction was quite simple and it was starting with sealed bid offers and then turning into an open ascending auction. There were three types of licenses called A, B and C blocks which have 45, 35, 30 MHz bandwidth respectively. In the sealed bid part of block A, Vodafone submitted the highest bid with 298m Euro whereas Turkcell and Avea prefers to bid 287m Euro and 285m Euro. After that in the English auction part Turkcell raised its highest bid to 358m Euro and outperform the highest bid of Avea which was 348m Euro. Vodafone did not submit any offer in this part of the auction. Thus, Turkcell acquired the largest spectrum, which was named as type A. In the following rounds the auction got even more interesting since neither Vodafone nor Avea wanted to bid for block B in order to be able to get last license at reserve price. Both offered 250m Euro, which was the reserve price, and did not make any higher bid afterwards in the ascending bidding part. In this case, auctioneer chose to make heads or tails to determine winner of second license that would be an unprecedented method in any similar auction. In the last round, Avea ensured to get last license by paying the reserve price, which was only 214m Euro (144m Euro less than the winning bid of Turkcell). Usually, there is a trade-off between complexity of auction design and strategies of participants. Apparently, it was not the case for this auction since both design and bidder strategies seem quite simple.

3.3 4G and New Spectrum Auction

4G Auction is going to take place in 2015 and cover 390 MHz of the available spectrum in 800, 900, 1800, 2100, 2600 frequency bands as 20 different blocks.

900 MHz and 1800 MHz bands will be available for GSM and UMTS services after 01.09.2015 which means new network dimensioning opportunities for operating companies 1. New frequencies in 2100 MHz band will be also available for operation after 01.09.2015; however, LTE services will be provided by 01.01.2016. Current spectrum holdings of 3 bidders can be found in Table 3.1.

Table 3.1. : Current spectrum holdings.

Band	Turkcell	Vodafone	Avea
900MHz	11MHz	11MHz	2.4MHz
1800MHz	-	-	15MHz
2100MHz	45MHz	35MHz	30MHz

Auction is designed as a sequential English auction which starts with a sealed bid round. Each participant has to submit their sealed bids one week before the auction and these bids are not allowed to be less than the reserve prices. For this reason, sealed bid phase introduces an activity rule for open ascending auction part. Without submitting a proper sealed envelope for each block bidders could not be active in the second phase.

Ascending phase considers the highest sealed bid as the new base price and the owner of the lowest sealed bid makes the first move. If the bids are equal the bidding will follow the same order with the revealing of the sealed envelopes. In case of having 2 or 3 equal bids as the highest sealed bid in the first part , If no bidder wants to participate in the open ascending part, then the winner will be determined by lottery and auction continues with next block. Summary of auction content and caps are given in below Table 3.2.

Table 3.2 : Available blocks in auction.

Band	**Packages**	**Caps**
800 MHz(A1,A2,A3)	2x10 3 blocks	Each bidder gets one block
900 MHz(B1,B2,B3)	2x7.6 / 2x1.4 2 blocks	12.5MHz cap,each bidder gets one block
1800 MHz(C1,C2,C3)	2x30 / 2x20 / 2x10	35MHz cap,each bidder gets one block
2100 MHz(D1,D2,D3)	2x5 MHz / 10 MHz TDD	No cap
2600 MHz(E1,E2,E3,E4)	2x25 / 2x15 / 2x10 2 blocks	Each bidder gets one block
2600 MHz TDD(F1,F2,F3,F4)	15/ 10 2 blocks/5	Each bidder gets one block

3.3.1 800 MHz Auction

There are 3 equal blocks and 3 bidders in 800 MHz phase. Each bidder can win at most one block. These blocks are abstract at the beginning of the auction and, first the winner of A1 will be choosing its location within the available spectrum. After removing the envelopes of the winner of A1 auction continues for A2 and A3 respectively. When the last standing bidder for A2 is determined it will be picking its place on remaining available spectrum. Finally, the last bidder will be acquiring A3 by paying the first bid it submitted in sealed bid phase, which is most probably the reserve price, and rest of the available frequencies within 800 MHz will be dedicated to 3rd bidder. In this phase, bidders must submit three different envelops for each abstract block to be active during all rounds of the auction. Although the bandwidth is same for all blocks, A1 is the most valuable block since it could be placed within the clean zone in which the risk of interference will be lower in comparison to lower frequencies within 800 MHz band.

3.3.2 900MHz Auction

Currently, Vodafone and Turkcell possess 11 MHz each whereas Avea has only 2,4 MHz in 900 MHz band. In this auction, there will be an additional 10 MHz to be auctioned which would make Avea at par with its competitors and make it capable of using double carrier technology. However, Vodafone and Turkcell are not happy with the way how 10 MHz is divided into 3 different blocks since they are going to lose their advantage in this area.

There are 3 asymmetric blocks and 3 bidders in 900 MHz phase. Each bidder could have at most 2x12,5 MHz within 900 MHz range and win at most one of the three available blocks in 900 MHz auction. All blocks are predetermined before the auction and, Avea will be the only available bidder for B1(7.6MHz) ,which ensures to get this block at reserve price, while either Turkcell or Vodafone could get only additional 1.4 MHz spectrum. In this part of the auction, there will be no competition among the bidders and the only ambiguity is whether Turkcell or Vodafone will show any interest in B2 and B3.

3.3.3 1800MHz Auction

Currently, Only Avea has 15 MHz bandwidth in 1800 MHz which is being used for GSM1800 services. In this auction, there are three asymmetric abstract blocks and 3 bidders which are constrained with 35 MHz cap. Each bidder could win at most one package within 1800 MHz range and Avea is not able to bid for C1 due to its previous holdings.

All blocks are abstract before the auction. First, Avea will be choosing its place on spectrum since it is eventually win C2 or C3. When Avea determines its place on the spectrum, winner of C1 will pick its place on remaining available spectrum.

Since Turkish 4G spectrum auction enables license owners to use 900 MHz band for UMTS services, 1800 MHz band is quite critical due to its usage in GSM 1800 and LTE 1800 services. 1800 MHz band offers not only a fair tradeoff between coverage and capacity but also gives chance to be more flexible in network dimensioning.

3.3.4 2100 MHz Auction

In this part of the auction, there are 3 predetermined blocks (D1-5MHz FDD,D2-5MHz FDD,D3-10MHz TDD) and no cap is implemented. If the winners wish so new packages and current holdings could be put in order again in order to place all frequency holdings consecutively.

Like 900 MHz auction, there will be no significant competition among bidders, and available blocks will be acquired around their reserve prices.

3.3.5 2600 MHz Auction

In 2600 MHz frequency band there will be a allocated block for a new entrant in addition to existing operators. There are three blocks for existing license owners and one block for a new comer. All blocks are predetermined and there is only one block that is greater 20 MHz FDD, which is defined as the minimum required bandwidth in 2600 MHz range for advanced LTE services in future.

Unlike other auctions, 2600 MHz auction is divided into two parts for FDD and TDD blocks. There are four more blocks in second auction which are available for all bidders without any parking place. Even if there is no new comer in the first part, no bidder will be able to gain more than one block in second part.

4. THE MODEL

In the model, study of Benoit and Krishna is resolved for private value assumption and implemented for Turkish 4G auction. Proof of the model and related assumptions are given in appendices.

Basically, there are three bidders with budgets $y_1 > y_2 > y_3$ and two objects A and B to be auctioned in sequence of AB and BA respectively. In this model, A represents the largest block in 1800 MHz Band while B is representing the largest block in 2600 MHz band similarly. Since it is very complicated to solve an open ascending sequential auction with six objects and three bidders, the model should be simplified as much as possible.

To reduce number of objects, lets assume that y_3 is relatively small and bidder 3 always wins remaining third block at reserve prices in both 1800 MHz and 2600 MHz auctions. While either bidder 1 or bidder 2 wins A, other bidder gets the second largest block in 1800MHz band automatically. Likewise, winner of B gets the largest bandwidth in 2600MHz range and the other bidder secures the second largest block within this range. In table 4.1, possible winner scheme for each block is depicted in order to summarize above scenario.

Table 4.1 : Possible winners of each block.

1800MHz		2600MHz	
Package	Winner	Package	Winner
A	y_1 or y_2	B	y_1 or y_2
Second block	y_1 or y_2	Second Block	y_1 or y_2
Third block	y_3	Third Block	y_3

In the model, each bidder has a private valuation and endogenous budget. Value of object A is greater than value of object B for all bidders. This is true for our model since 1800 MHz band is definitely more valuable than 2600 MHz band. If we consider the current market shares and forecast expected future revenues accordingly, it should be reasonable to put bidder valuations in a sequence as it is given in equations **(4.1)** and **(4.2)**.

$$V_{A,1} > V_{A,2} > V_{A,3} \tag{4.1}$$

$$V_{B,1} > V_{B,2} > V_{B,3} \tag{4.2}$$

In the auction, the main goal of all bidders is to optimize their payoffs. However, the main concern of the model is to analyze auction revenue rather than payoffs and bidder strategies.

While building this model, it is assumed that other bands except 1800 MHz and 2600 MHz will not subject to any competition and players will be acquiring all blocks within 800, 900, 2100MHz bands approximately at reserve prices. Even if we change order of only these 2 bands rather than entire sequence, it still should give the same result since reserve prices and outcomes in other bands are almost certain.

1800MHz and 2600MHz are complements and benefit of gaining of both A and B is represented with α. Although there are relatively small synergies among other combinations, they will be neglected in the model.

The auction is solved as a perfect information game which means that endogenous budgets and private valuations are commonly known by all bidders. No bidder could place a bid which is higher than its own budget. Similarly, bidders could bid up to their own valuations since any bid beyond object value means a negative payoff.

Since the budget $y_1 > y_2$ only bidder 1 could acquire both A and B while its closest competitor gets A or B in best case. Rest of the model is solved concerning only A and B without mentioning real blocks in the auction. In table 4.2, possible winning scenario of each bidder is illustrated.

Table 4.2 : Possible subsets of victory for each bidder.

Bidder	2 objects
1	(A,B),(A), (B)
2	Ø,(A), (B)
3	Ø

The solution of this model could be divided into 3 parts in which bidder 1 wins both A and B or only one of them. After solving all three cases, it is conjectured that selling objects in order of AB generates revenue which is at least as high as that of order BA. For this reason, auctioneer should prefer to sell more valuable object early in the auction.

5. CONCLUSIONS AND RECOMMENDATIONS

The budget constraints provide a different approach for auction theory so that the sequence of the auctioned objects causes different revenues in multi-object auctions. In a sequential auction, auctioneer should prefer to sell more valuable object early in the auction and this result still holds when the bidders have private valuations. Turkish spectrum auction fits in this rule and sells 1800MHz blocks before 2600 MHz. In the model, budget and valuations of Vodafone play crucial role in total revenue. Thus, BTK should reconsider its reserve prices paying much more attention to Vodafone' s critical values. Moreover, Avea sets only a threshold to limit payoffs of other players. It does not have any further impact on auction revenue. Turkish 4G auction achieves to sell 1800MHz blocks before 2600 MHz. However, it still suffers some disadvantages of sequential auctions such as lack of flexibility to switch between different objects and possibility of regret due to outcomes in previous rounds. Unlike 3G auctions in Europe, this auction ensures generating high revenue since reserve prices are set high enough when they are compared with outcomes of similar auctions after adjusting per capita MHz cost of each band with respect to ARPU of each market. Another disadvantage of this auction design is lack of price discovery. Most probably, similar items will not be sold for similar prices and there will be no market price for a certain spectrum block.

Another interesting part of Turkish 4G spectrum auction is the sealed bid stage. Since all bidders would prefer to submit bids at reserve prices, there is no benefit to add such a part into the auction. It just creates an activity rule to be able to active in next stage and obligate bidders to acquire all available blocks at their sealed prices even if they do not want to place any higher bid in the open auction.

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APPENDICES

APPENDIX A: Proof of the model

APPENDIX A

A.1.Proof of the Model

It is assumed that bidder 1 has the highest budget and higher private valuations in comparison to other players in the market. Each player values A higher than B. Only bidder 1 might have sufficient budget to acquire both objects. For this reason, budget of bidder 2 should be smaller than some of its own valuations for A and B; otherwise, it is causing a trivial unconstrained case. In the model, it is also assumed that valuation of bidder 3 for less valuable object is higher than its own budget so that threshold price for getting second object after depletion is being set as y_3. All assumptions are given below in equation form in addition to their explanations in the model.

$y_1 > y_2 > y_3$

$V_{A,1} > V_{A,2} > V_{A,3}$

$V_{B,1} > V_{B,2} > V_{B,3}$

$V_{A,1} > V_{B,1}$

$V_{A,2} > V_{B,2}$

$V_{A,3} > V_{B,3}$

$V_{B,3} > y_3$

$y_2 < V_{A,2} + V_{B,2}$

$V_{B,2} > y_1 - y_3$

A.1.1.Case 1

If bidder 1 wins both A and B in the order BA then the revenue gained by order of AB is at least as large as revenue of order BA.

Bidder 1 wins A and B.

A.1.1.1. $y_2 \geq V_{A,2} + V_{B,2} + \alpha$

Order of AB

Bidder 1 wins both objects

$V_{A,2} + \alpha < y_2$

$p_A = \min(V_{A,2} + \alpha, y_2) = V_{A,2} + \alpha$

$y_2 > V_{A,2} + V_{B,2}$

$V_{B,2} < y_2$

$p_B = \min(y_2, V_{B,2}) = V_{B,2}$

$R_{AB} = V_{A,2} + V_{B,2} + \alpha$

$$R_{AB} = V_{A,2} + V_{B,2} + \alpha \qquad \textbf{(A.1)}$$

Order of BA

Similarly,

$p_B = \min(V_{B,2} + \alpha, y_2) = V_{B,2} + \alpha$

$p_A = \min(V_{A,2}, y_2) = V_{A,2}$

$y_1 - p_B > \min(V_{A,2}, y_2)$

$R_{BA} = V_{A,2} + V_{B,2} + \alpha$

$$R_{BA} = V_{A,2} + V_{B,2} + \alpha \qquad \textbf{(A.2)}$$

By solving equations **(A.1)** and **(A.2)** together, equation **(A.3)** shows that revenue of order AB is at least as high as that of BA.

$$R_{BA} = R_{AB} \qquad \textbf{(A.3)}$$

A.1.1.2. $y_2 < V_{A,2} + V_{B,2}$

In this case, it is not known that whether y_2 is greater than $V_{A,2}$ or $V_{B,2}$.

Order of AB

Bidder 1 wins both objects.

$p_A = \min(V_{A,2}, y_2)$

$p_B = \min(V_{B,2}, y_2)$

$R_{AB} = \min(V_{B,2}, y_2) + \min(V_{A,2}, y_2)$

$$R_{AB} = \min(V_{B,2}, y_2) + \min(V_{A,2}, y_2) \tag{A.4}$$

Order of BA

Similarly,

$P_B = \min(V_{B,2}, y_2)$

$P_A = \min(V_{A,2}, y_2)$

$R_{BA} = \min(V_{B,2}, y_2) + \min(V_{A,2}, y_2)$

$$R_{BA} = \min(V_{B,2}, y_2) + \min(V_{A,2}, y_2) \tag{A.5}$$

If equations **(A.4)** and **(A.5)** are solved together equation **(A.6)** verifies that revenue of order AB is at least as high as that of order BA.

$$R_{AB} = R_{AB} \tag{A.6}$$

$R_{AB} = V_{A,2} + V_{B,2} + \alpha \qquad y_2 > V_{A,2} + V_{B,2}$

or

$R_{AB} = \min(V_{B,2}, y_2) + \min(V_{A,2}, y_2) \qquad y_2 < V_{A,2} + V_{B,2}$

A.1.2. Case 2

Bidder 1 wins only A.

A.1.2.1. $y_2 > V_{A,2} + V_{B,2}$

Order of AB

Since bidder 1 wins only A

$y_2 > V_{A,2} + V_{B,2}$

$V_{A,2} + \alpha < y_2$

$p_A = \min(V_{A,2} + \alpha, y_2) = V_{A,2} + \alpha$

Bidder 2 wins object B which means

$y'_1 = y_1 - (V_{A,2} + \alpha)$

$y_1 > y_2 > V_{A,2} + V_{B,2} + \alpha$

$y'1 > V_{B,2}$

Since $y'_1 > V_{B,2}$ and $V_{B,1} > V_{B,2}$ bidder 2 could not win B and drops at $V_{B,2}$. Bidder 1 wins both A and B.

Order of BA

Similarly,

$P_B = \min(V_{B,2} + \alpha, y_2) = V_{B,2} + \alpha$

$y'_2 = y_2 - (V_{B,2} + \alpha)$

$y'_2 > V_{A,2}$

Bidder 1 wins A.

$y'_2 < P_B$

$y'_2 > V_{A,2}$

$p_A = \min(V_{A,2}, y'_2) = V_{A,2}$

In this case, it is necessary to check whether $\pi_{B,1} \geq \pi_{AB,1}$ since bidder 1 has sufficient budget to acquire both A and B.

$\pi_{B,1} = V_{B,1} - (V_{B,2} + \alpha)$

$\pi_{AB,1} = V_{A,1} + V_{B,1} + \alpha - (V_{A,2} + V_{B,2} + \alpha)$

$\pi_{AB,1} \geq \pi_{B,1}$

Thus, bidder 1 would not drop at $V_{B,2} + \alpha$ and gets both A and B.

$R_{BA} = V_{A,2} + V_{B,2} + \alpha$

A.1.2.2. $y_2 < V_{A,2} + V_{B,2}$

In this case, it is not known that whether y_2 is greater than $V_{A,2}$ or $V_{B,2}$.

Order of AB

Bidder 1 wins only A.

$p_A = \min(V_{A,2}, y_2)$

Bidder 2 wins B.

$y'_1 < p_B$

Bidder 2 should not drop

$y'_1 < V_{B,2}$

$y'_1 < y_2$

$p_B = \max(y_3, (y_1 - \min(V_{A,2}, y_2))$

$R_{AB} = p_A + \max(y_3, (y_1 - p_A))$

$$R_{AB} = \min(V_{A,2}, y_2) + \max(y_3, (y_1 - \min(V_{A,2}, y_2)) \qquad \textbf{(A.7)}$$

Order of BA

Similarly,

$p_B = \min(V_{B,2}, y_2)$

$y'_2 < p_A$

$p_A = \max(y_3, y_2 - \min(V_{B,2}, y_2))$

$$R_{BA} = p_B + \max(y_3, y_2 - p_B) \qquad \textbf{(A.8)}$$

Equations **(A.7)** and **(A.8)** should be solved for different values of y_2, $V_{A,2}$ and $V_{B,2}$ since we do not know their sequence in case 2. To prove $R_{AB} \geq R_{BA}$, there are there possible sequences for y_2, $V_{A,2}$ and $V_{B,2}$.

If $y_2 > V_{A,2} > V_{B,2}$ and equations **(A.7)** and **(A.8)** are solved accordingly we see that $R_{AB} \geq R_{BA}$ holds for case 2.

$V_{A,2} + \max(y_3, y_1 - V_{A,2}) \geq V_{B,2} + \max(y_3, y_2 - V_{B,2})$

$R_{AB} = V_{A,2} + \max(y_3, y_1 - V_{A,2}) = V_{A,2} + y_3 = p_A + y_3$

$R_{BA} = V_{B,2} + \max(y_3, y_2 - V_{B,2}) = V_{B,2} + y_{3=}\, p_B + y_3$

$$R_{AB} \geq R_{BA} \qquad \textbf{(A.9)}$$

If $V_{A,2} > y_2 > V_{B,2}$ and **(A.7)** and **(A.8)** are solved accordingly we see that $R_{AB} \geq R_{BA}$

holds for case 2.

$y_2+\max(y_3, y_1-y_2)\geq V_{B,2}+\max(y_3, y_2-V_{B,2})$

$R_{AB}=y_2+\max(y_3, y_1-y_2)=y_2+y_3=p_A+y_3$

$R_{BA}=V_{B,2}+\max(y_3,y_2-V_{B,2})=V_{B,2}+y_3=p_B+y_3$

$$R_{AB}\geq R_{BA} \qquad \textbf{(A.20)}$$

If $V_{A,2}>V_{B,2}>y_2$ and **(A.7)** and **(A.8)** are solved accordingly we see that $R_{AB}\geq R_{BA}$ in case 2.

$y_2+\max(y_3, y_1-y_2)\geq y_2+\max(y_3, y_2-y_2)$

$R_{AB}=y_2+\max(y_3, y_1-y_2)=y_2+y_3=p_A+y_3$

$R_{BA}=y_2+\max(y_3,y_2-y_2)=y_2+y_3=p_B+y_3$

$$R_{AB}\geq R_{BA} \qquad \textbf{(A.31)}$$

Thus, **(A.9), (A.10)** and **(A.11)** depict that $R_{AB}\geq R_{BA}$ for case 2 in which bidder 1 wins only object A.

A.1.3. Case 3

Bidder 1 wins only B.

A.1.3.1 $y_2>V_{A,2}+V_{B,2}$

Order of AB

Bidder 2 wins A.

$y_2>V_{A,2}+V_{B,2}+\alpha$

$V_{A,2}+\alpha<y_2$

$p_A=\min(V_{A,2}+\alpha,y_2)=V_{A,2}+\alpha$

Bidder 1 wins only B.

$y'_2>V_{B,2}$

$p_B=\min(V_{B,2}, y_2)=V_{B,2}$

$$R_{AB}= V_{A,2}+ V_{B,2}+\alpha \quad \textbf{(A.42)}$$

Order of BA

Similarly,

$p_B= \min(V_{B,2}+ \alpha,y_2)= V_{B,2}+ \alpha$

$y'_1= y_1- (V_{B,2}+ \alpha)$

$y'_1> V_{A,2}$

Bidder 2 wins only A.

$y'_2< p_A$

$p_A =\min(V_{A,2}, y'_2)= V_{A,2}$

$R_{BA}= V_{A,2}+ V_{B,2}+ \alpha$

$$R_{AB}= V_{A,2}+ V_{B,2}+\alpha \quad \textbf{(A.53)}$$

If we put equations **(A.12)** and **(A.13)** together $R_{AB}\geq R_{BA}$ holds for case 3.

A.1.3.2. $y_2< V_{A,2}+ V_{B,2}$

In this case, it is not known that whether y_2 is greater than $V_{A,2}$ or $V_{B,2}$.

Order of AB

Bidder 2 wins A.

$p_A= \min(V_{A,2},y_2)$

Bidder 1 wins B.

$y'_2<p_B$

$y'_2< V_{B,2}$

$y'_1< y_2$

$p_B= \max(y_3,(y_1- \min(V_{A,2},y_2))$

$R_{AB=}\ p_A+ \max(y_3,(y_2- p_A))$

$$R_{AB}= \min(V_{A,2},y_2)+\max(y_3,(y_2- \min(V_{A,2},y_2)) \quad \textbf{(A.64)}$$

Order of BA

Similarly,

$p_B= \min(V_{B,2},y_2)$

$y'_1 < p_A$

$p_A = \max(y_3, y_1- \min(V_{B,2},y_2))$

$R_{BA}= p_B+ \max(y_3, y_1- p_B))$

$$R_{BA}= \min(V_{B,2},y_2)+ \max(y_3, y_1- \min(V_{B,2},y_2)) \quad \textbf{(A.75)}$$

To prove $R_{AB}\geq R_{BA}$, 3 different scenarios should be checked due to possible sequences of y_2, $V_{A,2}$ and $V_{B,2}$.

If $y_2> V_{A,2}> V_{B,2}$ and **(A.14)** and **(A.15)** are solved accordingly we see that $R_{AB}\geq R_{BA}$ holds for case 3.

$p_A+\max(y_3, y_2-p_A) \geq p_B+\max(y_3, y_1- p_B)$

$V_{A,2}+\max(y_3, y_2-V_{A,2}) \geq V_{B,2}+\max(y_3, y_1-V_{B,2})$

$V_{A,2}+y_3\geq V_{B,2}+y_3$

$$R_{AB}\geq R_{BA} \quad \textbf{(A.16)}$$

If $V_{A,2} >y_2>V_{B,2}$ and **(A.14)** and **(A.15)** are solved accordingly we see that $R_{AB}\geq R_{BA}$ holds for case 3.

$p_A+\max(y_3, y_2-p_A) \geq p_B+\max(y_3, y_1- p_B)$

$y_2+\max(y_3, y_2-y_2) \geq V_{B,2}+\max(y_3, y_1-V_{B,2})$

$y_2+y_3\geq V_{B,2}+y_3$

$$R_{AB}\geq R_{BA} \quad \textbf{(A.17)}$$

If $V_{A2}>V_{B,2}>y_2$ and **(A.14)** and **(A.15)** are solved accordingly we see that $R_{AB}\geq R_{BA}$ holds for case 3.

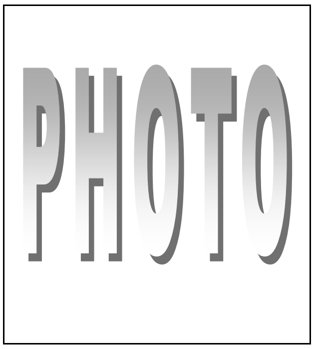
$p_A+\max(y_3, y_2-p_A) \geq p_B+\max(y_3, y_1- p_B)$

$y_2+\max(y_3, y_2-y_2) \geq y_2+\max(y_3, y_1-y_2)$

$y_2+y_3 \geq y_2+y_3$

$$R_{AB} \geq R_{BA} \qquad \textbf{(A.18)}$$

(A.16), (A.17) and **(A.18)** verify that $R_{AB} \geq R_{BA}$ for case 3 in which bidder 1 wins only object B. Finally, it could be conjectured that order of AB generates revenue which is at least as high as that of order BA for all possible outcomes of the auction.



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