

EXAMINING MATHEMATICS MENTOR TEACHER ONLINE PROFESSIONAL
DEVELOPMENT IN TERMS OF CONTENT, INTERACTION, FACILITATION

by

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B.S., Secondary School Mathematics Education, Boğaziçi University, 2021

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Master of Science

Graduate Program in Secondary School Science and
Mathematics Education
Boğaziçi University
2025

ACKNOWLEDGEMENTS

First of all, I would like to express my sincere and profound gratitude to my supervisor, Assoc. Prof. Fatma Aslan-Tutak, for her unwavering support, invaluable guidance, and encouragement throughout my research. This thesis would not have been possible without the guidance and expertise of my supervisor. Her insightful feedback and willingness to engage in detailed discussions greatly shaped this work.

I am also deeply grateful to my committee members, Assoc. Prof. Armağan Ateşkan and Assoc. Prof. Serkan Özel, for their thoughtful feedback and for challenging me to think more critically. I would like to thank them for their time and valuable suggestions that greatly improved this work.

Finally, I wish to thank my family, particularly my parents Meral and Ümit, and my sister Pelin, for their endless love, understanding, encouragement and patience during this challenging journey. Their support and belief in me have been a source of strength and motivation throughout my academic journey.

ABSTRACT

EXAMINING MATHEMATICS MENTOR TEACHER ONLINE PROFESSIONAL DEVELOPMENT IN TERMS OF CONTENT, INTERACTION, FACILITATION

The purpose of this study is to examine how an online professional development (OPD) program called “Mathematics Mentor Teacher Professional Development (MMPD)” operated in terms of content, interaction and collaboration, and facilitation to understand how it contributed to the learning of mentor mathematics teachers and how effective it was as an OPD program. The video recordings of the MMPD sessions were the main data source for analysis. The conversations and discussions that occurred during the sessions were analyzed by focusing on content, interaction and collaboration, and facilitation. The findings showed that the frequencies of opinions and views regarding the content categories are, from most to least, mentoring, preservice teacher learning, and mathematics. This sequence is parallel to the goals of the MMPD. The frequencies of mentoring and mathematics content across the sessions are also found parallel to the topics determined to be covered in each session. However, the frequencies of preservice teacher learning content do not show this distribution. In terms of interaction and collaboration, it is observed that the MMPD provided several opportunities for participant-facilitator, participant-content, and facilitator-content interactions, but it was limited in participant-participant interaction and collaboration. Regarding the facilitation dimension, it is observed that the facilitators used all effective facilitation moves when necessary to promote productive and meaningful discussions, to maintain the focus on the goals of the MMPD, and to support group collaboration with the contribution of all participants. The most frequently used facilitation moves throughout the MMPD are offering an explanation, highlighting, distributing participation, and clarifying, respectively. These facilitation moves by guiding mentor teachers toward the goals of the MMPD and the case discussions by stimulating the conversation of content focusing on important mentoring ideas and practices to contribute to preservice teachers’ learning to teach mathematics were influential in the development of mentor teachers.

ÖZET

ÇEVİRİMİÇİ MATEMATİK MENTÖR ÖĞRETMEN MESLEKİ GELİŞİM PROGRAMININ İÇERİK, ETKİLEŞİM, YÜRÜTÜLME BAKIMINDAN İNCELENMESİ

Bu çalışmanın amacı, “Matematik Mentör Öğretmen Mesleki Gelişim Programı (MMÖGP)” adlı çevrimiçi bir mesleki gelişim programının içerik, etkileşim ve iş birliği ve yürütülme açısından nasıl gerçekleştiğini inceleyerek, mentör matematik öğretmenlerinin öğrenmesine nasıl katkıda bulunduğunu ve bir çevrimiçi mesleki gelişim programı olarak ne kadar etkili olduğunu anlamaktır. MMÖGP oturumlarının video kayıtları analiz için temel veri kaynağını oluşturmaktadır. Oturumlar sırasında gerçekleşen konuşmalar ve tartışmalar, içerik, etkileşim ve iş birliği ve yürütülme bakımından analiz edilmiştir. Bulgular, içerik kategorilerine ilişkin görüş ve düşüncelerin sıklığının çoktan aza doğru mentörlük, hizmet öncesi öğretmen öğrenimi ve matematik olduğunu göstermiştir. Bu sıralama, MMÖGP’in hedefleriyle paralellik göstermektedir. Oturumlar genelinde mentörlük ve matematik içeriğinin sıklıkları da her oturumda ele alınması belirlenen konularla paralellik göstermektedir. Ancak, hizmet öncesi öğretmen öğrenimi içeriğinin sıklıkları bu dağılımı göstermemektedir. Etkileşim ve iş birliği açısından, MMÖGP’in katılımcı-yürütücü, katılımcı-içerik ve yürütücü-içerik etkileşimleri için çeşitli fırsatlar sunduğu, ancak katılımcı-katılımcı etkileşimi ve iş birliği açısından sınırlı kaldığı görülmektedir. Yürütülme boyutuyla ilgili olarak, yürütücülerin üretken ve anlamlı tartışmaları teşvik etmek, MMÖGP’in hedeflerine odaklanmayı sürdürmek ve tüm katılımcıların katkısıyla grup iş birliğini desteklemek için gerektiğinde tüm etkili yürütücü hamlelerini kullandıkları görülmektedir. MMÖGP boyunca en sık kullanılan yürütücü hamleleri sırasıyla açıklama sunma, vurgulama, katılımı dağıtma ve açıklığa kavuşturmaktır. Mentör öğretmenlerin MMÖGP hedeflerine doğru yönlendirilmelerini sağlayan bu yürütücü hamleleri ve hizmet öncesi öğretmenlerin matematik öğretmeyi öğrenmelerine katkı sağlayacak önemli mentörlük fikirleri ve uygulamalarına odaklanan içeriklerin konuşulmasına teşvik eden vaka tartışmaları, mentör öğretmenlerin gelişiminde etkili olmuştur.

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LIST OF ACRONYMS/ABBREVIATIONS

CCK	Common Content Knowledge
CK	Content Knowledge
COACTIV	Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy project
GPK	General Pedagogical Knowledge
HCK	Horizon Content Knowledge
IEA	International Association for the Evaluation of Educational Achievement
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
MCK	Mathematics Content Knowledge
MMPD	Mathematics Mentor Teacher Professional Development
MPCK	Mathematics Pedagogical Content Knowledge
OPD	Online Professional Development
PCK	Pedagogical Content Knowledge
PD	Professional Development
PISA	Programme for International Student Assessment
PST	Preservice Teacher
SCK	Specialized Content Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics

1. INTRODUCTION

Learning to teach is a complex process and teacher learning continues even after starting the profession. Learning to teach can be conceptualized around four general themes as “learning to *think* like a teacher, learning to *know* like a teacher, learning to *feel* like a teacher, and learning to *act* like a teacher” (Feiman-Nemser, 2008, p. 698). Such conceptualization of learning to teach having distinct dimensions from cognitive to emotional displays the complexity of it. However, many people and policy makers trivialize teaching profession by thinking that knowing subject matter and some general pedagogical knowledge is enough to teach well (Shulman, 1987; Darling-Hammond, 2000, 2006).

Knowing subject matter and general pedagogical skills is not sufficient to become an effective and competent teacher. It has been identified a list of knowledge types that a teacher needs to know in order to teach well (Shulman, 1986, 1987). Yet, sole knowledge is not enough. Teachers’ belief structures, their motivational orientations, and their self-regulation skills also play an important role in becoming an effective and competent teacher. Moreover, this complex cognitive and affective structure should be put into action in observable teaching behaviors. Thus, such a complicated nature of teaching makes learning to teach complicated as well and expecting a teacher become fully competent with only the education received in the teacher education program is not realistic. Teacher learning is a lifelong learning and professional development is crucial for this. Preservice teachers receive a substantial amount of theoretical knowledge at the university. However, this knowledge becomes useless if not act in practice. For preservice teachers to build the bridge between theory and practice, field experiences and mentor teachers’ role are quite important.

For this reason, this research aims to examine the online Mathematics Mentor Teacher Professional Development (MMPD) program to understand how it operated in terms of content, interaction and collaboration, and facilitation to contribute to mentor teachers’ learning.

2. REVIEW OF LITERATURE

2.1. Teacher Knowledge and Beliefs

The knowledge a teacher possesses is the primary qualification that makes them a teacher. A teacher needs to know not only subject matter but also how to represent, explain, and transfer it in a way that is understandable to students. For this, teachers need to consider the background of the learners and teaching context, resources and materials they have, and should be able to adapt their teaching styles and practices accordingly. A classification of teacher knowledge required for effective teaching has been made into seven categories as (a) content knowledge; (b) general pedagogical knowledge; (c) curriculum knowledge; (d) pedagogical content knowledge; (e) knowledge of learners and their characteristics; (f) knowledge of educational contexts; and (g) knowledge of educational ends, purposes, and values (Shulman, 1987). Among these, a particular emphasis is given to pedagogical content knowledge. It combines content and pedagogy in a special way that enables teachers to organize and use their content knowledge in pedagogically appropriate ways to teaching. Pedagogical content knowledge consists of a variety of representations such as examples, illustrations, or demonstrations useful to make the content comprehended by students as well as the conceptions and preconceptions of students that affect how students make sense of newly acquired information (Shulman, 1986).

Based on Shulman's classification of teacher knowledge, a survey measure of content knowledge for teaching in mathematics context was developed by examining teaching practices and a theoretical model for mathematical knowledge for teaching was proposed (Ball et al., 2008). Accordingly, it was claimed that content knowledge for teaching contains three dimensions as common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK) (Ball et al., 2008). Basically, CCK refers to the subject matter that the teacher is supposed to teach. A teacher needs to know the subject they teach accurately and this knowledge is not special to teaching. For instance, detecting mistaken information in textbooks can be considered as in CCK. On the other hand, SCK is special to teaching context and includes identifying patterns in students' errors and evaluating the generalizability of nonstandard methods that students use in solving problems.

That knowledge requires a special reasoning and unpacking of subject matter knowledge, which is unique to teaching context. Lastly, HCK can be defined as an understanding of how the topics included in the curriculum are related to each other.

Moreover, pedagogical content knowledge was divided into three subdomains as knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (Ball et al., 2008). The domains KCS and KCT correspond respectively to the knowledge of students' conceptions and preconceptions and of representing subject matter in ways that facilitate students' understanding as described by Shulman (1986). KCS consists of teachers' knowledge of anticipated misconceptions and common student errors and of what students find interesting, easy, or difficult. KCT is related to the design of instruction and includes the knowledge of how to sequence the content and select the tasks, examples, or demonstrations to be used during the instruction. Knowledge of content and curriculum corresponds to curricular knowledge in Shulman's classification (1986) and comprises the knowledge of available curriculum materials and how to use them, knowledge of how to link the topics of the subject with other subjects (lateral curriculum knowledge), and knowledge of how the topics in the subject area are organized over the years of teaching (vertical curriculum knowledge).

Teachers' mathematical content knowledge and pedagogical content knowledge are influential on their instructional quality and students' achievement in mathematics (Hill et al., 2005, 2008; Campbell et al., 2014; Copur-Genctürk, 2015). A study investigated the relationship between teachers' mathematical knowledge for teaching and students' mathematics achievement and found that teachers' mathematical knowledge for teaching is a significant factor of students' achievement in grades 1 and 3 (Hill et al., 2005). In another study, the relationship between teachers' mathematical content knowledge (CK), pedagogical content knowledge (PCK), teachers' perceptions including their beliefs about teaching and learning mathematics and their awareness of students' mathematical dispositions, and student achievement was examined (Campbell et al., 2014). The participants in that study were early-career (at most 6 years of teaching experience) 259 upper elementary (grades 4-5) and 184 middle school (grades 6-8) teachers from 23 school districts across three states in the USA. The data for teachers' CK, PCK, and perceptions were collected via surveys and their students' achievement data were provided by the

participating districts as their scores on the standardized state assessments. As a result, it was found that teachers' CK and PCK as a whole is significantly associated with students' achievement in mathematics for both upper elementary and middle school grades. However, when CK and PCK were analyzed separately, only CK was found an influential factor on student achievement in upper elementary grades while both CK and PCK were found influential in middle school grades. In addition, in terms of teachers' perceptions, no significant direct relationship with student achievement was found. In upper elementary grades, teachers' awareness of their students' mathematical dispositions was found interacted with teachers' mathematical knowledge. In middle school grades, teachers' beliefs about the importance of modeling for incremental mastery of procedural skills was found interacted with their knowledge (Campbell et al., 2014).

The impact of teachers' mathematical knowledge for teaching on student achievement can be explained by how their knowledge influences their teaching practices. A study exhibited the relationship between the mathematical knowledge for teaching and the mathematical quality of instruction of elementary school teachers by using a mixed method design (Hill et al., 2008). In this study, it was revealed that the mathematical quality of instruction is strongly correlated with teachers' mathematical knowledge for teaching and this relationship is mediated by teachers' beliefs about learning and teaching mathematics and about curriculum as well as the available curriculum materials and the way that teachers use those materials. The association between teachers' mathematical knowledge for teaching and their teaching practices was also displayed in a longitudinal study (Copur-Genctürk, 2015). In that study, how elementary and middle school teachers' knowledge is changed during a 3-year master program and how the changes in their knowledge are associated with their instructional practices were examined. The results showed that there were significant changes in teachers' mathematical knowledge for teaching and that change accounts for changes in several aspects of their instructional practices. Improvements in teachers' mathematical knowledge corresponded to improvements in their inquiry-based lessons, in their ability to encourage mathematical sense-making, and in promoting a positive classroom climate (Copur-Genctürk, 2015).

Teachers' professional knowledge, especially content knowledge and pedagogical content knowledge, is a fundamental factor on their actions and practices, which in turn has

an influence on student learning. Considering the significance of teacher knowledge on teaching practices, the International Association for the Evaluation of Educational Achievement (IEA) conducted an international study called Teacher Education and Development Study in Mathematics (TEDS-M) to measure and compare prospective elementary and lower secondary mathematics teachers' knowledge in 15 countries (Döhrmann et al., 2012). The aim of TEDS-M was to understand the effectiveness of mathematics teacher education programs and how national policies and practices affect mathematics teacher education by comparing data from different countries (Döhrmann et al., 2012). The focus of TEDS-M was mathematics content knowledge (MCK), mathematics pedagogical content knowledge (MPCK), and general pedagogical knowledge (GPK) as essential components of teachers' professional competencies. In addition to these cognitive components, affective-motivational traits such as teachers' professional beliefs, motivation and self-regulation were measured as complementary factors for their competencies (Döhrmann et al., 2012). Overall, the TEDS-M took a comprehensive view of cognitive and affective characteristics of teachers as influential on their teaching practices.

The importance of the knowledge base of teachers on their instructional decisions and teaching practices is evident. Teachers' beliefs are also important in shaping their teaching styles and the decisions they made. Beliefs are strong structures that affect how people perceive the phenomena they experience, understand themselves and others, process the new information they acquire, and eventually contribute to the formation of self (Pajares, 1992). Thus, teachers' beliefs about learning and teaching, about their students, about themselves, about the subject matter they teach are influential on their actions. However, it is difficult to reveal empirically how teachers' beliefs are related to their instructional practices. In a study, it was observed three middle school mathematics teachers' practices to investigate how their beliefs about mathematics and its teaching are influential on their instructional decisions and practices and it was denoted that the relationship between teachers' beliefs and practices has a complex structure including many interacting factors (Thompson, 1984). Yet, teachers' beliefs, whether conscious or unconscious, play a significant but subtle role in their specific teaching behaviors (Thompson, 1984).

2.2. Teacher Competence

The characteristics that affect teachers' decisions and practices have a complex structure and this structure has been conceptualized under the term teacher competence. The competency frameworks have been developed encompassing various teacher characteristics that are considered as essential for effective teaching. In these frameworks, teachers' professional knowledge stands out as a main component supplemented with affective-motivational aspects such as beliefs, values, and self-regulation. One of the frameworks for teachers' professional competencies has been proposed in the scope of TEDS-M as mentioned above to qualify the criteria for effective mathematics teacher education (Döhrmann et al., 2012). The competency framework of TEDS-M consists of teacher knowledge facets (MCK, MPCK, and GPK) as cognitive components and teachers' beliefs about mathematics and the teaching and learning of mathematics as well as professional motivation and self-regulation as affective-motivational components (Döhrmann et al., 2012).

A similar conceptualization of teacher competence has been presented in the Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy project (COACTIV) (Baumert et al., 2013). The COACTIV research project was conducted in Germany in conjunction with the 2003-2004 cycle of the OECD's Programme for International Student Assessment (PISA). The aim of the COACTIV project was to propose a framework for the characteristics that constitute teachers' professional competence and to measure the aspects of teacher competence and link them to teachers' instructional practices by developing and validating instruments (Baumert et al., 2013). The framework for teacher competence adopted in the COACTIV project involves professional knowledge of teachers; their values, beliefs, and goals; their motivational orientations; and their self-regulation skills (Baumert and Kunter, 2013a). In the COACTIV project, counseling and organizational knowledge was added as a part of teachers' professional knowledge in addition to content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK) (Baumert and Kunter, 2013a). In this comprehensive study of teacher competence, measures of various teacher characteristics and their students' PISA results were used to examine how these characteristics are influential on the quality of instruction and student learning outcomes.

The participants were 10th grade students who entered PISA and their mathematics teachers. The results of COACTIV study showed that teachers' knowledge especially PCK, their beliefs including epistemological beliefs (about the nature of knowledge) and beliefs about teaching and learning mathematics, and their enthusiasm for teaching are significant factors for their instructional quality and students' learning in mathematics (Baumert and Kunter, 2013b; Kunter, 2013; Voss et al., 2013).

The teacher competence frameworks used in the TEDS-M and COACTIV take an analytical stance towards competence as their aims were to measure and determine various teacher characteristics that are considered effective for high quality teaching. In other words, teacher competence was viewed as the combination of a set of cognitive and affective-motivational traits. The purpose in such analytical approach is to find ways to promote the development of competence by identifying the underlying characteristics that need to be improved (Blömeke et al., 2015). A contrasting perspective views competence as observable performance in real-life settings and argues that competence should be assessed by observing behaviors in real-life situations. Blömeke et al. (2015) emphasized that both perspectives should be taken into account for a thorough understanding of competence and developed a framework for teacher competence by combining the two perspectives. Accordingly, teacher competence is viewed as a continuum that begins with dispositional cognitive and affective-motivational characteristics that form the basis for observable behavior through the situation-specific skills of perception, interpretation, and decision making. Blömeke et al. (2015) viewed competence also as a vertical continuum progressing from being less competent to being more competent. The view of teacher competence as a process starting from teachers' knowledge and beliefs that lead to teachers' actions is crucial to understand how teachers learn and develop.

2.3. Teacher Learning

Teaching is a complex job that requires consideration of numerous factors in planning and implementing instruction. A sound knowledge of subject matter, albeit essential, is not enough to teach well. Teachers need to know not only what to teach, but also how to teach it regarding the curriculum and its goals, the theories of how students learn and develop, and the individual characteristics of their students. In addition, they need to know general

pedagogical skills to manage classroom effectively and to create a positive and interactive learning environment. Moreover, they must be able to put all this knowledge into action simultaneously and smoothly during the limited time of classroom instruction. Such a complicated nature of teaching makes learning to teach also complicated.

To date, different perspectives have emerged in research on teacher learning and education to understand how teachers learn to teach and to improve teacher education programs and professional development of teachers (Cochran-Smith and Fries, 2005; Russ et al., 2016). One of these perspectives is known as process-product research and focuses on observable teacher behaviors. In the process-product studies, the aim was to identify generic teacher actions that are thought to be effective on student achievement. In that perspective, teaching was considered as a “technical transmission activity” (Cochran-Smith and Fries, 2005) and performing a set of independent actions and routine procedures (Darling-Hammond, 2016; Russ et al., 2016). Moreover, teaching and learning were assumed directly and linearly connected as starting with teacher behaviors and ending with student learning (Cochran-Smith and Fries, 2005; Russ et al., 2016). Accordingly, teacher learning was viewed as the acquisition of general behaviors and actions that can be applicable in a range of contexts and stable over time, and teacher education was regarded as training of specific procedures and skills for the development of and change in teacher actions and behaviors that were found to be effective on student learning (Cochran-Smith and Fries, 2005; Russ et al., 2016). This view of teacher learning and education was prevalent until the early 1980s.

In the 1980s, the behaviorist perspective seen in the process-product studies has begun to diminish and a cognitive perspective towards teaching and learning has gained attention. The cognitive perspective focuses on teachers’ mental structures and orientations as underlying sources for their actions and emphasizes the domain-specific nature of teaching (Russ et al., 2016). In the process-product perspective, the focus was the general teaching behaviors regardless of the content of teaching. The cognitive perspective, on the other hand, takes into account the importance of teachers’ competence on the subject matter they teach for high-quality teaching. In addition to the subject matter knowledge, a variety of knowledge categories have been identified for teachers to perform teaching act effectively such as general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, and knowledge of learners and their characteristics (Shulman, 1987). Especially,

pedagogical content knowledge has gained importance with its peculiarity to teaching a particular content area. Thus, the view of teacher learning has changed from training teachers to perform specific behaviors to understanding change in their knowledge, attitudes, and beliefs (Cochran-Smith and Fries, 2005). In this respect, teacher learning takes on a more complex structure compared to the behaviorist perspective in terms of increasing demand for the knowledge and skills that teachers are expected to learn.

Lastly, the other perspective considers how the physical, social, and cultural contexts in which people interact influence their learning. This approach is called the situative perspective and emphasizes that learning is situated in the physical and social contexts in which it occurs (Putnam and Borko, 2000). While, the cognitive perspective focuses on the changes in mental structures of individuals, the situative perspective underlines the social nature of learning and takes into consideration how the participants' roles and identities develop and change within communities as well as how communities develop and change through the contribution of participants (Putnam and Borko, 2000; Russ et al., 2016). From this perspective, learning can be understood as enculturation into communities of practice through accommodating to and adopting the community's rules, norms, tools, and the ways of thinking and behaving (Russ et al., 2016). According to this perspective, teacher learning can be considered to occur in two dimensions. One of which focuses on practice of teaching and the other on learning within communities.

The first dimension emphasizes that teacher learning is situated in teachers' practices. Ball and Cohen (1999) referred to this as "learning in and from practice". This requires teachers to use their teaching practices as sites for inquiry and systematically reflect on their teaching (Ball and Cohen, 1999; Cochran-Smith and Lytle, 1999). An important point to consider in situating teacher learning in their practices is that it does not mean to confine learning opportunities to just in classroom and school settings. Situating teacher learning in practice means that shaping learning opportunities around the fundamental tasks of teaching and the context of teachers' work (Feiman-Nemser, 2001a). For example, artifacts of practice such as video records of classroom teaching, samples of students' work, or curriculum materials can be used to ground teacher learning by providing teachers with a concrete base on which to discuss about their teaching, their students' learning, and the subject matter they teach (Ball and Cohen, 1999). In addition, out of classroom settings may provide valuable

experiences for teachers, allowing them to cross the boundaries of their classrooms and develop new ways of thinking and knowing (Putnam and Borko, 2000). However, for new knowledge and insights developed outside of the classroom to be effective they must be integrated into teachers' existing practices, and for this, use of multiple contexts for teacher learning can be beneficial such as a summer workshop combined with continuing support during the school year (Putnam and Borko, 2000).

An illustration of how to support teachers to learn from their practice is demonstrated in a study which used students' mathematical work to ground teachers' learning (Kazemi and Franke, 2004). The study was a year-long professional development for mathematics teachers in an elementary school that was carried out as monthly workgroup meetings and through informal observations and interactions with teachers in their classrooms. The researchers provided teachers with various word problems in different mathematical content areas. The teachers used these problems in their classrooms and brought samples of their students' work on those problems to the workgroups to discuss about students' mathematical thinking. Throughout the study, teachers developed more sophisticated knowledge about students' mathematical thinking and learned how to elicit and detail their students' mathematical thinking. This study can be a good example indicating how student work can be used to promote teachers' learning and how the combination of teachers' in and out of classroom experiences contributes to their learning.

Video recordings of instruction can also be powerful resources to promote teacher learning. For instance, a study with high school biology teachers showed that a video-based professional development in which teachers analyzed the cognitive demand of the tasks demonstrated in video cases helped teachers change their practices toward the use of practices that stimulate high-level student thinking and avoid the use of practices that might restrict student's sensemaking (Tekkumru-Kisa et al., 2018). In another study with preservice mathematics teachers, participation in a university course in which videotaped lessons are analyzed and discussed in terms of learning goals, students' thinking and learning, and alternative teaching strategies contributed to preservice teachers' ability to analysis of teaching as seen in their improved descriptions that turned out to be more detailed, critical and evidence-based, more attentive to teachers' and students' behaviors and thinking, and more focused on the mathematical content throughout the study (Santagata et al., 2007). This

study may be an example of how video cases can be used to ensure that preservice teachers develop the habit of continuous analysis and reflection on their teaching in the future.

These studies show how teachers' learning is embedded in their practice. Teachers have learned by analyzing, interpreting, and discussing about the artifacts of teaching practice such as student work and video recordings of instruction (Kazemi and Franke, 2004; Santagata et al., 2007; Tekkumru-Kisa et al., 2018). However, their learning did not occur in isolation, on the contrary, their learning was facilitated through their interactions with colleagues, researchers, and students. This brings out the other dimension of teacher learning from the situative perspective, which is learning within communities.

According to the situative perspective, learning is considered to occur as a social construction of knowledge within communities of practice. Cochran-Smith and Lytle (1999) have identified three conceptions of teacher learning based on the construction of knowledge in communities as "knowledge-for-practice", "knowledge-in-practice", and "knowledge-of-practice". Knowledge-for-practice can be understood as the formal knowledge and theories developed within the professional community of researchers for teachers to use and enhance their practice. Traditional teacher education is primarily based on this knowledge. The second conception, knowledge-in-practice, is the practical knowledge which is implicit in practice and emerges in the decisions and actions of competent teachers. Such knowledge is acquired through conscious reflection on and inquiry into practice. The last conception, knowledge-of-practice, does not make a distinction as formal and practical knowledge and emphasizes the essential and critical role of teachers in collectively producing knowledge within inquiry communities. The inquiry communities are seen as the fundamental contexts in which teachers learn. Accordingly, Cochran-Smith and Lytle (1999) suggested a view of teacher learning which adopts "inquiry as stance" in communities of practice where teachers share, discuss, and learn from their practice. This distinction on the relationship between teachers' knowledge and practice is significant for understanding how to support teachers' learning and development by considering the role of different communities and the ways that teachers learn.

The professional communities in which teachers take part not only affect their learning and development but also contribute to their students' learning. The review conducted by

Vescio et al. (2008) through the analysis of 11 studies that provided empirical data on the impact of professional learning communities on teachers' practices and students' learning has indicated the importance of these communities and revealed their features that lead to the improvement of teaching practices and student learning. Participating in a professional learning community in which student learning is the primary focus and teachers work collaboratively to support the needs of their students has been reported to help teachers shift their practices toward more student-centered approaches and increase students' achievement in the long term (Vescio et al., 2008). In addition, professional learning communities have found to be effective in changing school culture by promoting collaboration, teacher authority, continuous teacher learning and focus on student learning (Vescio et al., 2008).

2.4. Preservice Teachers' Learning

The education that preservice teachers receive at the university is extremely important because it constitutes the foundation of their professional identities, their views on teaching and learning, and shapes their teaching styles and practices. Thus, providing high-quality education to preservice teachers is the first step in preparing them to become competent and effective teachers. However, traditional teacher education programs have been criticized for being inefficient to prepare teachers for challenging tasks of teaching, distant from practice, incapable of promoting complex learning, inadequate in providing extensive field experiences, and unable to implement connected content and pedagogy courses (Ball and Cohen, 1999; Darling-Hammond, 2000; Feiman-Nemser, 2001a). Learning to teach is a complex process that involves developing competence in various cognitive, affective, and motivational aspects. Therefore, preparing teachers to be competent and effective is a challenging task, and it is not realistic to expect preservice teachers to be fully equipped to teach well only with the education they receive at the university.

Hammerness et al. (2005) pointed out that "to successfully prepare effective teachers, teacher education should lay a foundation for lifelong learning" (p. 359) and listed three issues that need to be considered in preservice teachers' learning to teach. The first issue emphasizes the importance of preservice teachers' preconceptions and experiences as students. Preservice teachers' observations and experiences as students lead them to develop various conceptions and beliefs about teaching and learning. These entering beliefs and

preconceptions act as a filter that may contribute to or restrain their learning and understanding of new knowledge and ideas (Feiman-Nemser, 2001a). Thus, for prospective teachers to develop new visions and learn to teach effectively, their initial understandings and beliefs should be addressed and their misconceptions about teaching and learning should be eliminated. The second issue underlines the importance of promoting prospective teachers' learning for understanding and enactment. To become successful teachers, they need to learn how to put into action what they know and learn. For this, case-based or problem-based approaches in preservice teacher education can be useful by providing the opportunity to examine and analyze rich and complex situations similar to the actual classroom situations they may encounter in the future (Putnam and Borko, 2000; Hammerness et al., 2005). The third issue highlights the need for developing metacognitive skills to handle the complexities in teaching. Preservice teachers should be prepared to deal with the multifaceted nature of teaching and to consider and attend to a variety of factors simultaneously during teaching. Therefore, it is important to help preservice teachers develop metacognitive skills that will enable them to think consciously and systematically about their teaching.

To prepare teachers for lifelong learning and help them become effective and competent teachers, teacher education programs should provide coherent and integrated learning experiences by addressing these issues (Darling-Hammond, 2006). When there are high coherence and integration among the courses that prospective teachers take at the university and between the practices of university and schools, prospective teachers can develop more consistent visions and understandings of teaching and learning. In addition, purposeful and extensive field experiences under intensive supervision are critical for prospective teachers to be able to learn how to enact theoretical knowledge in practice (Feiman-Nemser, 2001a; Darling-Hammond, 2006).

2.5. Professional Development

Teachers have significant impacts on students' learning and motivation. Therefore, having high quality and competent teachers is crucial for contributing to students' learning. For this, lifelong teacher learning should be promoted to ensure that teachers constantly improve themselves and adapt to changing student needs and profiles. Teacher education

programs prepare teachers to begin their careers with a certain level of knowledge and skills. Then, professional development (PD) plays a significant role in contributing to lifelong learning by enabling teachers to improve their skills and practices. PD is important for both beginner and experienced teachers. For beginner teachers, PD opportunities help them enhance their knowledge and learn new strategies to use in their teaching. For experienced teachers, PD can be used to keep them informed about the developments in the field of education and help them embrace the new knowledge and strategies in teaching and learning.

The meaning of PD is somewhat interesting in that it refers to both formal and informal learning activities in which teachers participate and the learning and development of teachers as a result of their participation in these activities (Feiman-Nemser, 2001a). PD is a broad umbrella encompassing a range of learning experiences of teachers, from formal and structured activities such as workshops and seminars to teachers' daily practices that contribute to their development such as discussions with their colleagues and reflections on their teaching (Borko, 2004; Desimone, 2009). This expanded view of PD is related to the shifts from behavioral to cognitive and situative perspectives in teacher learning. Traditional PD opportunities for teachers, often called in-service training or staff development, based on behavioral perspectives and take place in the form of one-shot workshops and seminars to give advice and deliver specific skills and strategies, have been criticized for being fragmented, disconnected to teachers' classroom practices, incompatible with current theories of learning and teaching, and lacking ongoing support and serious and complex learning for teachers (Ball and Cohen, 1999; Feiman-Nemser, 2001a; Borko et al., 2010).

Recent views on teacher PD adopting situative perspectives emphasize the importance of teachers' practices and professional learning communities for their learning and development. Accordingly, the scope of PD has expanded, including any activity embedded in teachers' practice that contributes to their learning and professional growth.

2.5.1. Characteristics of Effective PD

PD plays a significant role in enhancing teacher knowledge and practice, which consequently affects students' learning. However, not every PD activity is effective in improving teacher knowledge and changing instructional practices. Researchers have

identified five features of high-quality PD that is found effective in advancing teachers' knowledge and practices, which are *content focus*, *active learning*, *coherence*, *collective participation*, and *duration* (Garet et al., 2001; Penuel et al., 2007; Desimone, 2009). Content focus is related to the degree to which activities focus on subject matter content and teaching strategies based on how students learn. Active learning corresponds to the opportunities for discussion, presentation, obtaining and providing feedback, and reviewing and analyzing student work. Coherence means alignment with other PD activities, teachers' experiences and goals, and local and national standards. Collective participation refers to the participation of teachers from the same school, grade, or department to foster an interactive professional and learning community. Duration specifies that activities should be sustained over time to allow sufficient discussion of the content and teaching strategies and teachers to transform what they learn into practice and receive feedback.

2.5.2. Online Professional Development

The creation and proliferation of online PD have been induced in response to the need for PD that can accommodate teachers' hectic schedules, utilize strong resources that are frequently unavailable locally, and can pave the way for the provision of continuous, real-time, work-integrated support (Dede et al., 2009). Online PD provides teachers with convenience by allowing access from anywhere and at more flexible times. In terms of implementation, online PD activities can take place in three forms: *synchronous*, *asynchronous*, and *hybrid*. Synchronous online PD activities occur in real time with the participation of all learners simultaneously. Generally, technology is not used for transforming the learning experience in synchronous PD activities; rather, the learning experience resembles face-to-face PD in a virtual environment (Bates et al., 2016). Asynchronous online PD activities occur at different times for each participant. It allows participants to access the content and materials of PD at any time and progress at their own pace. Hybrid PD activities occur as the combination of face-to-face and online learning experiences with the use of synchronous or asynchronous activities depending on the purposes. Compared to asynchronous PD, synchronous PD provides more opportunity for peer interaction and promotes discussion and collaboration due to its nature of making participants gather in real time in a shared virtual environment often for prolonged durations (Koellner et al., 2023).

Ensuring interaction and collaboration in an online learning environment is quite important to contribute to the learning of participants. Bates et al. (2016) stated the significance of collaboration for any kind of learning and emphasize the necessity of making opportunities for collaboration in an online PD. Another important feature that should be ensured in an online learning environment is interaction. Moore (1989) has identified three types of interaction that need to be occur in distance education, which are learner-content interaction, learner-instructor interaction, and learner-learner interaction. Anderson and Garrison (1998) have extended the types of interactions by adding instructor-instructor, instructor-content, and content-content interactions.

2.5.3. Facilitation of Professional Development

Not only the design but also the facilitation of a PD program is substantially significant for PD to become successful. PD facilitators take a critical role in organizing and managing the circumstances to make sure all participants are engaged and have opportunities for meaningful learning and reflective inquiry (Koellner et al., 2023). A theoretical framework identifying effective facilitation moves that promotes teacher learning from PD has developed by van Es et al. (2014). This framework developed in the context of a video-based PD and classifies facilitation moves under four categories that contribute to producing effective discussions on the videos. Those categories are (a) orienting group to the video analysis task, (b) sustaining an inquiry stance, (c) maintaining a focus on the video and the mathematics, and (d) supporting group collaboration.

Another framework for planning and facilitating a video-based PD has developed by Tekkumru-Kisa and Stein (2017a). Their framework consists of five practices to be considered in planning and facilitating a video-based PD. The first two practices, *anticipating* responses and *sequencing* of videos and PD tasks, are related to planning phase of the PD. The last three practices, *monitoring* responses, *selecting* participant responses to share or highlight, and *connecting* participants' responses to the big idea are related to the facilitation phase of the PD. The facilitation moves under monitoring and selecting practices are adapted from van Es et al. (2014). Tekkumru-Kisa and Stein (2017a) has proposed a more extensive framework that includes both planning and facilitation phases of PD by combining the framework of van Es et al. (2014) and Five Practices Framework.

A revised version of the framework related to facilitation of PD developed by van Es et al. (2014) will be used in this study to examine the facilitation moves that occurred during the MMPD sessions. In addition to facilitation, content characteristics of the MMPD and how interaction and collaboration occurred during the MMPD sessions will be investigated. By approaching from three dimensions, *content*, *interaction and collaboration*, and *facilitation*, this research aims at presenting an extensive and in-depth description of the MMPD and how it contributed to the learning of mentor teachers. Investigating the MMPD from these three dimensions will help to grasp and acquire a complete understanding of how effective the online MMPD was.



3. STATEMENT OF THE PROBLEM

The purpose of this research is to investigate the online Mathematics Mentor Teacher Professional Development (MMPD) program which is designed to be case-based and by considering mentoring components and situated learning theory in terms of content, interaction and collaboration, and facilitation.

3.1. Research Questions

The research questions are as follows:

1. How did *content* emerge during the implementation of the online MMPD, designed based on case discussions, mentoring components, and situated learning theory?
2. How did *interaction and collaboration* occur during the implementation of the online MMPD, designed based on case discussions, mentoring components, and situated learning theory?
3. How did *facilitation* occur during the implementation of the online MMPD, designed based on case discussions, mentoring components, and situated learning theory?

4. METHODS

4.1. Research Design

The design of this research is a case study, which is a research method used to gain an in-depth understanding of a social phenomenon (Yin, 2018). Case study research aims to describe or explain a contemporary phenomenon on which researcher has little or no control (Yin, 2018). In general, case study research seeks answers to how and why questions. In a case study, a single case or multiple cases can be used depending on the purpose of the study. Examples of a case might be an individual, a community, an organization, a program, an event, or an activity. The important point to consider in defining a case, bounding the case with a timeframe and place in which it is studied (Creswell and Poth, 2018; Yin, 2018). In the case study research, multiple forms of data are collected and integrated by using interviews, observations, documents, or audiovisual materials to present an in-depth description of the case under study (Creswell and Poth, 2018). Yin (2006) states the strength of the case study method as “its ability to examine, in-depth, a case within its real-life context” (p. 111).

4.2. Research Setting

The case under study in this research is an online professional development program for mentor teachers called “Mathematics Mentor Teacher Professional Development (MMPD)” carried out with high school mathematics teachers from Istanbul, Ankara, and Trabzon provinces. The MMPD was selected as the case for this study to understand in detail and comprehensively how a specific OPD program operated in terms of content, interaction and collaboration, and facilitation so that how it contributed to the mentor teachers’ learning and how effective it was as an OPD program.

The MMPD was conducted during 2023 fall semester. The online MMPD sessions were held synchronously as approximately 1-hour weekly meetings via the online meeting platform Zoom and eight sessions in total were conducted. In addition to Zoom, an interactive presentation tool named Nearpod was used during sessions to increase

interaction. Nearpod is an instructional tool which allows to make interactive presentation of multimedia materials with a variety of activities like tasks that participants can write, add pictures, or solve multiple-choice questions. The MMPD materials and discussion cases were presented on the Nearpod and interactive activities were performed during the sessions via the Nearpod. During the MMPD two types of Nearpod activities were conducted as collaborative board and individual response activities. In the collaborative board activities, the participants wrote their responses to the Nearpod and could see each other's responses at the same time. In the individual response activities, first the participants wrote their individual responses to the questions and then they could see each other's answers when the facilitator shared their responses. The topics covered in the sessions of the MMPD and the activities performed during the sessions are given in Table 4.1.

The participants in the online MMPD were 10 high school mathematics teachers from Istanbul, Ankara, and Trabzon provinces with teaching experiences ranging from nine years to 24 years. There were three participants from Istanbul, two participants from Ankara, and five participants from Trabzon. In addition, while some participants had experiences in mentoring previously, it was the first mentoring experience for some.

Table 4.1. The topics and activities performed during the sessions.

	Topics of the sessions	The activities
Session 1	Content knowledge	A collaborative board activity on a given question A large group discussion on the responses to collaborative board activity A large group discussion on a case A large group discussion on given three questions A small group discussion
Session 2	Mentor-PST interaction (intervention, reflection, feedback)	A collaborative board activity on a given question A large group discussion on the responses to collaborative board activity

Table 4.1. The topics and activities performed during the sessions. (cont.)

	Topics of the sessions	The activities
Session 2	Mentor-PST interaction (intervention, reflection, feedback)	A large group discussion on a case An individual response activity on another case A large group discussion on the case An individual response activity on given questions
Session 3	Mentor-PST interaction (intervention, reflection, feedback)	A collaborative board activity on a given question A large group discussion on a given question An individual response activity on a case A large group discussion on the case Another collaborative board activity on a given question A large group discussion on the responses to collaborative board activity An individual response activity on a given question
Session 4	Mentor-PST interaction (modeling, sharing, co- designing)	An individual response activity on a given question A large group discussion on a case A collaborative board activity on the case A large group discussion on another case An individual response activity on a given question
Session 5	Practices on teaching (activity/task design, self- evaluation, and reflection)	A collaborative board activity on a given question A large group discussion on the responses to collaborative board activity A large group discussion on a case

Table 4.1. The topics and activities performed during the sessions. (cont.)

	Topics of the sessions	The activities
Session 5	Practices on teaching (activity/task design, self-evaluation, and reflection)	A large group discussion on another case An individual response activity on a given question
Session 6	Practices on teaching (teaching dynamics, self-evaluation, and reflection)	A large group discussion on a given question A small group discussion on a case and an individual response activity for the groups' responses A large group discussion on the groups' responses A small group discussion on the feedback samples of the participants A large group discussion on the responses of the groups
Session 7	Conceptual learning, mathematical skills, understanding student conceptions/misconceptions	A large group discussion on the responses of the groups from the previous session A large group discussion on a case A collaborative board activity on a case
Session 8	Understanding student conceptions/misconceptions, curriculum, use of tools and technology	A small group discussion on a given question and a collaborative board activity for the groups' responses A large group discussion on the groups' responses A large group discussion on a given question A collaborative board activity on a given question A large group discussion on a given question A collaborative board activity on a case

4.3. Data Analysis

The main data source of this study was the video recordings of the online MMPD sessions. In addition to the video recordings, the documents of Nearpod activities were used to support triangulation of data sources. First, the video recordings of the sessions of the online MMPD were transcribed. After the transcription process, the transcripts of the video recordings were separated into vignettes according to the flow of the conversations that occurred during the sessions. The vignettes were analyzed deductively by examining the conversations and discussions that occurred during the sessions from three dimensions, which are content, interaction and collaboration, and facilitation. In the analysis of small group discussions, only the video recordings of group discussions in which the primary facilitator was present were used. However, because of the recording problem that occurred in the first session, there is not a complete recording for any groups, only some parts of the discussions were recorded. For this reason, the small group discussion conducted in the first session was not included in the analysis of content and facilitation. For the analysis of the interaction and collaboration, the small group discussions are valuable data sources and the content of the discussion is not much important for this category. Because of this, the video recording that captures the most of a group's discussion was used in the analysis of interaction and collaboration. To support the analysis of video recordings of the sessions and for triangulation of data sources, the responses that participants wrote to the Nearpod activities were also analyzed by using the same dimensions determined to analyze the video recordings.

The first dimension of analysis is content and the framework in Table 4.2 was used to analyze the content of the discussions. This dimension of analysis was identified considering one of the important characteristics of an effective PD, which is content focus. The content of a PD is one of the most significant factors determining its effectiveness (Garet et al., 2001; Penuel et al., 2007; Desimone, 2009). Therefore, how the content of the discussions developed during the MMPD sessions was analyzed by focusing on mathematics content, mentoring content, preservice teacher learning content, divergence from the content, and content related to professional identity of teachers. The first three categories, mathematics, mentoring, and preservice teacher learning, were determined based on the goals and design of the MMPD. Since the goal of the MPPD is to develop mentoring conceptions and practices

of mathematics teachers to support the learning of the preservice teachers how to teach mathematics, these three categories were identified. Divergence was added to see if there is deviation from the discussion topics and the goals of the MMPD and whether they are consisted with the participants experiences, knowledge, and beliefs, which is another characteristic of an effective PD, coherence (Garet et al., 2001; Penuel et al., 2007; Desimone, 2009). The last category, professional identity, emerged during the analysis after noticing that the participants' teacher identities come to the fore time to time in their comments stating their ideas, views, or beliefs about teaching and their students' learning. These comments are important in that they reflect how the participants would guide preservice teachers regarding which teaching methods or strategies to use to support students' learning.

Table 4.2. Content categories and definitions.

Categories	Definitions
Mathematics	Addressing mathematical content; mathematical discussions; and expressing opinions and beliefs about mathematics
Mentoring	Expressing opinions and beliefs about mentoring; addressing mentoring functions and practices; experiences and expectations as mentors
Preservice teacher learning	Expressing opinions and beliefs about preservice teacher learning; addressing how preservice teachers learn to teach
Divergence	Deviating from the main idea, expressing an off-topic opinion, moments of distraction
Professional identity	Expressing opinions and beliefs about teaching and their students' learning

The second dimension that the MMPD was analyzed is how the interaction and collaboration occurred in an online learning environment. Ensuring interaction and collaboration in an online learning setting is significant to promote the formation of a group

culture and a supportive learning environment. The interaction types (Moore, 1989; Anderson and Garrison, 1998; Anderson, 2003) determined to be analyzed in the online MMPD are participant-participant interaction, facilitator-participant interaction, participant-content interaction, and facilitator-content interaction. In addition to these interaction types, how collaboration occurred during the MMPD was analyzed. Table 4.3 shows the definitions of how the interaction types and collaboration that occurred during the MMPD were identified. By analyzing the video recordings of the sessions and their transcripts, the occurrences of the interactions and collaboration were determined.

Table 4.3. Definitions of interaction types and collaboration.

Categories	Definitions
Participant-participant interaction	Communication between participants, addressing each other's comments, ideas, or answers, small group discussions
Facilitator-participant interaction	Communication between facilitators and participants
Participant-content interaction	Interaction with the technological tools and engaging with specific content
Facilitator-content interaction	Interaction with the technological tools and engaging with specific content
Collaboration	Work on a task together and create a product together

The last dimension that the MMPD was analyzed is facilitation. Facilitator roles are quite important in the effectiveness of a PD. Tekkumru-Kisa and Stein (2017b) stated that ignoring the roles of facilitators in a high-quality PD is like ignoring the roles of teachers in a high-quality instruction. The MMPD was analyzed by using a revised version of the effective facilitation moves framework developed by van Es et al. (2014) (see Table 4.4). The framework of van Es et al. was developed for discussion on video cases. In the online

MMPD, discussions occurred on written cases or discussion questions. Therefore, the parts related to videos and student ideas were changed to adapt the mentoring content and discussions around the written cases that occurred during the MMPD. The facilitation moves were categorized under three titles by slightly changing the categories in van Es et al.'s framework. The definitions of only three facilitation moves were revised, which are offering an explanation, highlighting, and redirecting. For instance, highlighting was changed from “direct attention to noteworthy student ideas in the videos” to “direct attention to noteworthy mentoring practices, ideas, or concepts”. The definitions of the other moves are taken directly from van Es et al. (2014).

Table 4.4. The framework for effective facilitation moves for the MMPD.

Categories	Facilitation Moves	Definitions
Sustaining an inquiry stance	Countering	Offer an alternative point of view
	Clarifying	Restate and revoice to ensure common understanding of an idea
	Pressing	Prompt participants to explain their reasoning and/or elaborate on their ideas
	Lifting up	Identify an important idea that a participant raised in the discussion for further discussion
	Offering an explanation	Provide an interpretation of an event, interaction, idea, concept, or question
Maintaining focus on the goals of the MMPD	Highlighting	Direct attention to noteworthy mentoring practices, ideas, and concepts related to preservice teacher learning
	Redirecting	Shift the discussion to maintain focus on the goals and main idea of the MMPD

Table 4.4. The framework for effective facilitation moves for the MMPD. (cont.)

Categories	Facilitation Moves	Definitions
Maintaining focus on the goals of the MMPD	Connecting ideas	Make connections between ideas raised in the discussion
Supporting group collaboration	Distributing participation	Invite participants to share different ideas based on who is (and is not) participating
	Validating participant ideas	Confirm and support participant contributions

5. FINDINGS

5.1. Content

The MMPD is an online professional development program designed to support and develop mentoring skills for mathematics teachers. Due to area specialized nature of the MMPD, one of the indicators of content was determined as *mathematics* and video recordings of the sessions were examined in terms of how discussions and conversations addressed mathematics. In addition, since the main goal of the MMPD is to improve mentoring for mathematics teacher learning, the other two indicators for content were identified as *mentoring* and *preservice teacher learning*. Other than these three categories regarding the aim and focus of the MMPD, another indicator for content was defined as *divergence* to determine how much the focus is maintained and if there are deviations and distractions from the focus and the goal of the MMPD. The last indicator for content has emerged during analysis, which is *professional identity*. Since the participants were teachers, they reflected on their professional identities time to time by sharing their opinions and beliefs about teaching and their students' learning. In total, the content of the MMPD was examined in terms of these five categories.

Table 5.1. Frequency of content categories in sessions.

Content Categories	Sessions								Total
	1	2	3	4	5	6	7	8	
Mathematics	34	15	17	14	50	30	39	28	227
Mentoring	80	151	160	67	109	120	100	89	876
Preservice teacher learning	61	81	57	48	62	33	56	64	462
Divergence	1	1	3	1	2	2	2	2	14
Professional identity	1	6	3	2	2	5	6	4	29
Total	177	254	240	132	225	190	203	187	1608

The analysis of the video recordings of the MMPD sessions indicated that conversations and discussions mostly took place around *mentoring* in compliance with the aims and design of the MMPD (see Table 5.1). The expression of opinions and discussions related to *preservice teacher learning* occurred approximately half as often as those related to *mentoring* across all sessions. While the total frequency of *mentoring* content in all eight sessions is 876, the total frequency of *preservice teacher learning* is 462. In addition, in terms of the *mathematics* content of the MMPD, mathematical discussions and views were found to be occurred about half the those of *preservice teacher learning*, with a total frequency of 227 across all sessions. The last two categories *divergence* and *professional identity* have frequencies 14 and 29 in total respectively showing that the focus was maintained on the goals of the MMPD and there were a few moments of distraction and deviation during the sessions.

5.1.1. Mathematics

When the sessions were examined separately in terms of mathematics content, it is seen that mathematical discussions and conversations occurred the most often in the fifth session. The frequencies of mathematical content are the lowest in the second, third, and fourth sessions. The difference in the frequency of mathematical discussions and conversations can be explained by the differences in the topics covered in each session. The topic of the first session was content knowledge and since the MMPD was designed to improve mentoring for mathematics teachers, the content knowledge discussed was mathematics. This may explain why the first session had one of the high frequencies of mathematics content. The following conversation, for example, shows how a mathematical discussion occurred during the first session. In this vignette, participants express mathematical views while discussing on a case which illustrates a teaching practice of a preservice teacher on the topic of solving system of linear equations. Since the discussion topic of this week was content knowledge, the participants expressed what preservice teachers should do mathematically in teaching of this topic.

Ahmet: She could be asked to make a generalization from this, for example, instead of directly saying that zero equals zero all real numbers, she could have expressed it as a generalization about when it occurs for all real numbers.

Facilitator: Exactly. Does anyone else want to comment? What could she have done differently?

Salih: I would like her to graph it, have it plotted, and question what it means for it to be real.

Facilitator: Ok.

Salih: Maybe, she could have demonstrated visually that they are coincident lines and actually have infinite common points.

The following excerpts are also from the same case discussion as above vignette. In these excerpts, two participants' mathematical expressions are seen related to the case. In the first comment, the participant expresses mathematically how she would guide the preservice teacher to understand her mistake in solving the system of linear equations. In the second comment, the participant approaches the situation from a different perspective and emphasizes the importance of specifying the number set in which the problem is to be solved.

Zehra: ... I would prefer to examine these two lines with Merve. Perhaps she will notice the properties of these two lines when she looks at them one by one, yes, these are coincident lines. Therefore, I think she will notice that the solution is the set of points on those lines, on themselves, those ordered pairs.

İsmail: ... I would ask in which number set she solved this equation. Because unfortunately, including in our books, we are making a solution like this, but we do not tell which number set to make the solution in.

The second, third, and fourth sessions were related to the interactions between mentor teachers and preservice teachers. The topics covered in the second and third sessions were intervention, reflection, and feedback. While the second session focused more on the intervention and reflection part, the third session focused more on the feedback part. The topics discussed in the fourth session were modeling and sharing. These are more general topics related to mentoring, so this may indicate why mathematics content was less frequent during these three sessions. In these three sessions, instead of mathematical discussions, mathematics was more frequently used as a context for discussions regarding mentoring and preservice teacher learning. An example from the second session below demonstrates how a

participant interacts mathematically with preservice teachers. In this example, rather than a mathematical view, the participant's expression of how she interacts with preservice teachers to improve their mathematics is seen.

Melek: For example, today there were questions about powers of zero. I mean, we talked about it during the break. There was a definition of rational numbers in the book. There was an interesting section. I asked "What do you think about that"? I said that for next week, look at this, research it, come back next week and let's talk. I wondered their views on the concepts of indeterminate and undefined. We'll talk about them next week, for example.

The following conversation from the third session illustrates the statement of a participant about how he gave preservice teachers feedback on how to start teaching the topic of sum and difference formulas in trigonometry. In this example, the participant does not state mathematical ideas about these formulas, but rather focuses on how to teach them. Mathematics content was addressed in this vignette as an example of the interaction between mentor teachers and preservice teachers. Like in the above example from the second session, mathematics content was served as a context, the participant does not discuss mathematics but exemplifies how he discusses mathematics with preservice teachers.

İsmail: In the twelfth grade, we start the second part of trigonometry. There are sum and difference formulas. I asked how they would start this topic. You know, when we were discussing among ourselves...

Facilitator: So, you started with a reflection like that, right? They gave an answer. Then how did you proceed? How did you give feedback?

İsmail: I also said that the starting can differ from class to class when I taught a lesson. You know, it can also change depending on the situation in the classroom. In some cases, ... in high school, we don't do proofs like in university, but we still show how the rule of the function came out, more or less. In some cases, you start like that. In some cases, you first approach it geometrically. For example, this becomes a preliminary talk.

Since the focus of the second, third, and fourth sessions is the interaction between mentor teachers and preservice teachers, mathematics content appeared in the discussions mostly as either examples from the participants' experiences or the context of the discussion cases. A similar occurrence of mathematics content in the fourth session is given below. Here, a participant remembers a question related to cosine theorem that came up in the national university entrance exam a few years ago while discussing about a case which shows a dialog between a mentor teacher and a preservice teacher, and mentor teacher telling the preservice teacher that the questions she selects regarding cosine theorem do not include daily life questions.

Sibel: ... Again, related to the cosine theorem, there is a compass question that came up. It was like drawing a circle with a compass. Actually, it was both a question from daily life, and a question on cosine...

In this example, mathematics content is not the focus of the discussion like the previous examples from the second and third sessions. The case reminds the participant of a question, and the participant then comments about the importance of talking to preservice teachers about solving such questions in the classroom. In general, mathematics content mostly remained in the background in the second, third, and fourth sessions and occurred less frequently compared to other sessions since the purpose and focus of these sessions was on the general mentoring practices and the interaction between mentor teachers and preservice teachers.

The fifth session has the highest frequency of mathematics content. The topics discussed in this session were about designing teaching tasks and activities and preparing and selecting questions to use during the lesson. These issues are closely related to the content of teaching. In other words, in this session, it was discussed how mentor teachers can help preservice teachers learn how to select and prepare mathematical tasks, how to start teaching a mathematics topic, and what type of mathematics questions to solve according to students' level. Therefore, in this session, mathematics was addressed not only as a context but also as the focus of the discussions. That is why mathematical discussions and conversations may have the highest frequency in this session. For instance, the following conversation occurred while discussing on a case regarding an activity a preservice teacher

prepared about the topic of interpreting parabolas. It demonstrates how a participant would guide the preservice teacher in teaching this topic by explaining mathematically what they need to consider. What is important here in terms of mathematics content is that although a mathematical content was given in the case discussions in the second, third, and fourth sessions, mathematics remained only as a context in those sessions. However, as seen in the example below, in this session, in addition to mentoring the preservice teachers, a mathematical discussion was also carried out based on the case. This shows that according to the purpose and focus of the sessions, the cases provided for discussions facilitated the occurrence of mathematical discussions.

Melek: I would say like this. I mean, I would expect them to start by showing the relationship between the change in the degree of the function and the graph. Like what I wrote a moment ago.

Facilitator: Like transition from a line graph to a parabola.

Melek: Yes, for example, there could be a ready-made function in a graphing program. I mean, showing students there how the graph changes as the degree of the function changes, what shape it takes when it becomes second degree, that is, focusing on the general representation and then the details and drawing the graph.

Facilitator: Are those details something like the relationship between the equation and the graph?

Melek: Well, I mean the details in the drawing like finding the vertex, the points where it intersects with the x-axis and y-axis, we show how to find these before that.

The following excerpt from the same case discussion as above example also exhibits a participant's mathematical views regarding the teaching the topic of interpreting parabolas. As these instances show, in parallel with the fact that the topics discussed in this session are more subject specific issues, the participants also expressed mathematical ideas through the given cases while discussing how they would mentor preservice teachers.

Ahmet: I think he needs to understand the quadratic equation and how to convert it to a perfect square, why does it have its smallest or largest value at the vertex? Why would its graph be upwards or downwards?

The sixth session was related to how to give preservice teachers feedback about their lesson plans and teaching practices and help them make reflection and self-evaluation after they prepare and implement their lesson plans. Two small group discussions were carried out in this session. The first was a discussion on a case describing the observation notes taken by a mentor teacher while a preservice teacher was teaching a lesson on the topic of inverse functions. In the second one, the examples that the participants brought about their own preservice teachers' lesson plans and teaching practices were discussed. This session is one of the sessions with high frequency of mathematics content, as mentor teachers also give mathematical feedback to the preservice teachers' lesson plans and teaching practices. For instance, the following vignette from the first group discussion indicates how two participants discuss mathematically about the observation notes given in the case. Just as mathematical thoughts were expressed during the discussion of preparing an activity on the topic of parabola in the fifth session, mathematical thoughts emerged during this small group discussion while discussing the observation notes about a preservice teacher's teaching practice on the topic of inverse function. These examples indicate that the given cases facilitated the discussion of mathematical content in both sessions.

İsmail: What dimensions did they focus on? For example, while explaining the inverse function, she overlooked the basic properties of the inverse function, being one-to-one and onto.

Zeynep: Okay, so she focused on the shape, on the graph, and lacked its meaning in the introduction? Did she fail to make sense of it? So, perhaps she couldn't make sense of the inverse function in the initial interpretation then. She showed the graph. By directing that this is the shape of this and its inverse is that?

İsmail: I mean, the thing Bilge teacher probably means is this: Here you wrote it as an algebraic mathematical expression, but you did not focus on drawing the graph of the function and taking its inverse, or on which interval the inverse can be taken, in a way that will make it understandable to the student in terms of the shape by using a diagram or graphic method.

The topics covered in the seventh session were conceptual learning, mathematical skills, and students' conceptions. These topics have high relevance to mathematics, which may explain why this session had the second highest frequency of mathematics content. For

instance, the following conversation taken from this session on a case discussion related to a lesson plan prepared by a preservice teacher on the topic of logarithm illustrates how a mentor teacher would provide feedback to the preservice teacher to help him facilitate conceptual learning in this topic. This vignette is also an example of how cases promoted the discussion of mathematics content. The mathematical topics presented in the cases played a stimulating role for mathematical discussions, depending on the focuses and objectives of the sessions.

Sevim: Why didn't he start by solving an equation, for example? Do we expect it to be conceptual through directly on function?

Facilitator: Exactly. That's probably what I would say to Can.

Sevim: So first, for example, an exponential function, an equation, for example, say $3^x = 9$. How can he make solved this equation to find what x should be and then express it as a logarithm? Logarithm nine in base three. That equals two. So, after showing with a few examples that the base of the exponential function is the base of the logarithm, look, x and y switch places. Wouldn't it be more efficient to have students put the points on the graph from there, put the points of the exponential function, put the points of the logarithm function, draw them, and then do a transition like this, say, "Look, this is the inverse of this, y to the x " and then connect it to automatic mode and go there? I mean, at least I would have preferred to start this way. To make the transition a bit easier.

The eighth session started with a small group discussion regarding how to develop preservice teachers in noticing students' conceptions, which was planned to be held in the seventh session but was postponed to this week due to time constraints. This lasted for about half of the session. In the second half, the topics of curriculum and the use of tools and technology were discussed. During the first half of this session, mathematical content was not addressed much. The conversations and discussions occurred more on the general methods for helping preservice teachers notice students' conceptions. During the second half, mathematical content was addressed more while discussing the curriculum and the use of tools and technology. The following excerpt, for example, displays a participant's views about the importance of preservice teachers having a solid knowledge of the curriculum and how she provided feedback on this issue.

Melek: For example, last week the intern teacher was going to explain this, explain the intervals in which the function has positive and negative values. They took a quadratic function and said that students determine where it is positive and negative by drawing it, for example. First of all, they must have known that students had not learnt the topic of parabola yet, that they cannot draw one, or that they cannot solve a quadratic inequality. Because those are later topics. For example, I warned him about that.

In general, the sessions of the MMPD can be divided into two groups in terms of mathematics content. It was observed that mathematical discussions and expressions of ideas regarding mathematics were significantly less in the second, third and fourth sessions than in the other sessions. In these sessions, not area specific, more general issues related to mentoring were discussed, such as whether mentor teachers should intervene or not during preservice teachers' teaching practices, how to help them make reflection, how to give effective feedback, and how to be a model for them and share their experiences. Therefore, mathematics served more as a context for the discussions in these sessions. In the other sessions, in addition to being a context, mathematics was appeared as the subject of the discussions while discussing about content knowledge, conceptual learning, mathematical skills, knowledge of curriculum, use of mathematical tools and technology, how to guide preservice teachers in preparing mathematical tasks and activities, and how to give them feedback about their lesson plans and teaching practices. This distinction between the sessions also demonstrates that even if a specific mathematical topic was not given in the cases, the participants would need to discuss mathematics in the sessions with high frequency of mathematics content because even if mathematics was present in the cases discussed in the second, third, and fourth sessions, mathematics remained only as a context there due to the focus and aims of these sessions. However, the topics discussed in the sessions with high frequency of mathematics content were more area specific issues, so even if a general discussion question, not including a specific mathematics content, was given, participants would need to bring mathematical content to the discussions. On the other hand, the cases provided for discussions played a facilitative role for mathematical discussions and if they had not been provided, the frequency of mathematical content might not have been as high as it is now.

5.1.2. Mentoring

When the sessions were examined in terms of mentoring content, it is seen that conversations and discussions about mentoring occurred most in the second and third sessions, with a frequency of 151 and 160, respectively. They are followed by the sixth session with a frequency of 120, the fifth session with a frequency of 109, and the seventh session with a frequency of 100. The sessions where mentoring content was addressed least were the fourth, first and eighth sessions with frequencies of 67, 80 and 88, respectively. The category with the highest frequency in all the sessions was mentoring content since the aim of the online MMPD was to improve mentoring conceptions and practices of mentor mathematics teachers. The topics that were chosen to be discussed in each session was determined according to different components and practices of mentoring. Therefore, in each session the mostly emphasized issues were about mentoring. In the first session, the topic covered was content knowledge. It was discussed how mentor teachers can detect preservice teachers' deficiencies in content knowledge and what they can do to help them improve their content knowledge. This session is one of the sessions with a low frequency of mentoring content. The small group discussion carried out at the end of this session could not be included in the analysis because of the recording problem. This may have had an influence on the frequency of mentoring content determined in this session. For instance, the following excerpt from the first session displays the views of a participant regarding what mentor teachers can do to detect preservice teachers' deficiencies in content knowledge and how they can guide them to develop their content knowledge.

Ahmet: So, maybe we can ask them to explain the lesson to us before teaching it to the students in the class. We can identify there, but a deficiency might still emerge due to different situations that the intern teachers will encounter in the class, such as student questions. It is not possible to completely eliminate this, but we can basically predict the difficulties they may encounter. As a teacher who has taught that topic hundreds of times before, we can tell them to pay attention to this, we can point out the areas they should focus on. However, we can't completely prevent any shortcomings because in a new situation that arises at that moment, for example, a student's question might reveal a shortcoming.

The second and third sessions have the highest frequencies of the mentoring content. The theme of these two sessions was the interaction between mentor teachers and preservice teachers. In the second session, it was discussed whether mentor teachers should intervene or not during preservice teachers' teaching practices and it was emphasized that mentor teachers should not intervene but instead give opportunities for preservice teachers to reflect. In the third session, what is effective and ineffective feedback, how and on what topics mentor teachers should give feedback to preservice teachers were discussed. It is obvious that these issues are predominantly related to mentoring, which may be the reason why the discussions and ideas put forward about mentoring had the highest frequencies in these sessions. The theme of the fourth session was also the interaction between mentor teachers and preservice teachers and, in this session, the importance of mentor teachers being models for preservice teachers and sharing their experiences were emphasized as well as what kind of studies mentor teachers can do with preservice teachers were discussed. In fact, these issues are also closely related to mentoring. However, the fourth session has the lowest frequency of mentoring content. The reason for this may be that the duration of this session was shorter than the other sessions. While the duration of the other sessions was approximately 1 hour and 10 minutes, the duration of this session was 53 minutes and last 10 minutes were allocated for what participants want to share or discuss, so the issues determined to discuss in this session were covered in 43 minutes. This may have had an impact on the frequency of mentoring content discussed in this session.

For example, the following conversation from the second session shows a part of the discussion on a case illustrating how a mentor teacher intervened when a preservice teacher's voice began to tremble during his teaching practice. Here, participants share their opinions on whether they would intervene or not if they were in this situation. This discussion reflects participants' mentoring approaches and beliefs and expectations as mentors related to preservice teachers' learning. For instance, all the three participants think that mentor teachers should not intervene during preservice teachers' teaching practices to support their formation of professional identities and to enable them to see themselves as the teacher of that class. Their comments indicate that one of the mentor teacher's duties is to prepare preservice teachers for the problems they may encounter in the classroom and to teach them that they should find a solution to these problems themselves.

Salih: So, I wouldn't intervene. I would expect them to confront the situation and the excitement they experienced and draw a conclusion from it. After all, they are the teacher of the class, and they'll need to confront this process and deal with it. That's why I wouldn't intervene.

...

Sevim: I agree with the previous teacher. I would try not to intervene as much as possible, but I would try to maintain eye contact with the intern teacher to offer support. I would stand up and smile in a way that the students couldn't see, nodding my head to show my approval, trying to reassure the teacher with eye contact... I would try to encourage them to continue teaching as if I weren't even in the classroom.

Facilitator: I understand. Melek teacher?

Melek: I was going to say something similar. I wouldn't intervene either. I would wait for the teacher candidate to solve the problem themselves somehow. Because, yes, it may be the beginning, but I think they need to figure out how to solve the problem themselves. They need to find the solution themselves.

The following excerpt, also taken from the second session, includes the views of a participant on mentoring and reflection, stating her opinions about the importance of reflection for preservice teachers and mentor teachers' guidance for this.

Sevim: ... I think reflection is very effective in terms of both guiding preservice teachers about how they see themselves as a teacher and if there are any shortcomings, by using Socratic questioning, that is, gradually guiding them to question and discover themselves or to find out how they could have done better.

The example from the third session below is related to providing feedback to preservice teachers which is one of the mentoring practices. It indicates how a participant provided feedback to preservice teachers related to grading exam papers and use of answer key. This sharing of the participant includes a significant mentoring practice. Assessment and evaluation are a crucial part of teaching, and as experienced teachers, supporting and giving opportunity for preservice teachers to practice this is one of the important mentoring practices.

Ahmet: Today, for example, we graded the exam papers. ... Each intern teacher graded them. We evaluated them, for example. We looked at the average, and there were two students who graded too high or too low. One had graded much higher than the average, and one had graded much lower. We discussed them, for example. Why did you grade so low? Why is there such a deviation from the answer key? We gave them feedback. We also provided feedback on how to use the answer key and what to pay attention to when grading.

The following example, also from the third session, demonstrates a participant's comment on a case that illustrates the feedback a mentor teacher gave to a preservice teacher. In this example, the mentoring content occurs in the participant's comments on how the feedback could be given to the preservice teacher. The comment reflects the participant's approach to mentoring, how to interact with preservice teachers to support their learning and to motivate them.

Zehra: ... The direction could have been like this. I saw that you had problems with classroom management. How could we have handled this with a different approach? It doesn't necessarily have to be related to voice. Maybe the content was too simple or too difficult for that class. It didn't capture their interest. Maybe at that moment, something else could have captured their interest, even something outside of class, I don't know. I think we could have tried different methods.

The following excerpt shows a sharing of a participant related to the same case as above example. It reflects the participant's views on mentoring based on her own experiences. This comment elucidates one of the important mentor roles. As experienced teachers, sharing their experiences and guiding preservice teachers based on their experiences is one of the primary roles of mentoring.

Sibel: Absolutely, I've experienced this myself. I used to go and get injections every two or three weeks because my vocal cords were severely damaged when I worked at a boarding school for boys in Şanlıurfa. Believe me, I think this is the first thing we should say and teach. Without compromising ourselves too much, meaning our voice, when we go home and can no longer speak, or can no longer stand up, I think this is

one of the things we should teach them in teaching. I wish someone had told me that personally.

The fifth and sixth sessions were about applications on teaching. In the fifth session, the issues of selecting and preparing questions and designing teaching activities and tasks were discussed. The sixth session was about how to give preservice teachers feedback on their lesson plans and after implementation of their lesson plans as well as help them reflect on and evaluate themselves about their lesson plans and teaching practices. After the second and third sessions, these two sessions have the highest frequency of mentoring content. For instance, the following conversation from the fifth session displays a practice that a participant used to guide preservice teachers in preparing questions and designing teaching tasks. This vignette also indicates a significant mentoring practice. Including preservice teachers among the discussions is quite important in terms of preparing them for the profession and getting them accustomed to the school culture and community. Providing such an opportunity to preservice teachers help them get used to work and practices of a teacher and see how teachers prepare for lessons and exams.

İsmail: We couldn't do it with this year's intern teachers, but last year, for example, in our school's math department... Before explaining the topic to the eleventh graders, we first discuss it among ourselves. We prepare our preliminary questions. Once the topic progresses, we discuss among ourselves which question papers to distribute and which questions to choose. We've included them in those processes.

Facilitator: Well, then they are getting into a mathematical discussion.

İsmail: So, they didn't speak up. They didn't speak up, but when they saw the conversation among us, they were able to figure out how to prepare questions and what to do. ... Later, when I asked them what they thought, they listened and agreed, or expressed their opinions if they wanted to.

The excerpts below, also from the fifth session, demonstrate two participants' comments on a case depicting a preservice teacher who prepared questions that were only difficult and related to finding the vertex of a parabola. Their comments include their views about how they would guide preservice teachers in such a situation. Two different mentoring approaches are evident in their comments. The first participant prefers a more direct

approach, talking to the preservice teacher about the selection of questions. The second participant prefers a more indirect approach, aiming to have the preservice teacher discover themselves what type of questions to select.

Melek: They should always be aware that there are students at different levels in the classroom. ... Not everyone's mathematical perception is the same. Therefore, knowing that there are students at different levels, we need to talk about not choosing questions that are all at the same level.

Salih: So, I would probably act a little bit here. ... For example, if they asked me a question about parabolas, about finding the vertex, I would first make them feel that I didn't know about parabolas. ... I would act like a student. The intern teacher would realize that these questions were not appropriate for my level at that moment and would make an adjustment accordingly.

Another excerpt from the fifth session below shows a participant's view on mentoring who experienced a similar situation as above case and states the importance of keeping interaction with preservice teachers and studying with them, and the supportive attitude of mentor teachers for this to happen.

Salih: Yes, we actually need to convey the message as the mentor of the teacher candidate, you know, let's come and change this, improve it, come again, change it again, improve it again. That's what I mean by communication. ... Actually, we experienced the previous example. First, I discussed whether students could answer the questions they sent. Then we decided together to assign them as homework later. We set those questions aside and wrote new ones, for example. Perhaps through such continuous communication and mutual development, we can reach an ideal state.

The sixth session was related to giving preservice teachers feedback on their lesson plans and after their teaching practices. In addition, how to guide them to make reflection and self-evaluation was discussed. For example, the following excerpt is from a large group discussion that occurred after a small group discussion related to what was talked in the groups. The small group discussion was about a case involving a mentor teacher's

observation notes on a preservice teacher's teaching practice. The excerpt includes one of the groups' evaluations of these notes and how they would give feedback if they were in the place of the mentor teacher in the case.

Melek: We would express that we expected her to discuss the circumstances under which the functions could be reversed. It seems she had some difficulty with that. She didn't start that way. We would probably start like that. Which functions have inverse? We would start from that part. There are no evaluation of classroom management and the connection she established with the students. Salih teacher said she only took notes on the presentation of the lesson.

The issues discussed in the seventh session were conceptual learning, mathematical skills, and students' conceptions. The frequency of mentoring content in this session is somewhere in the middle among the other sessions. For instance, the following excerpt indicates how a participant would guide preservice teachers in beginning to teach logarithmic functions so that mathematical skills are promoted. The participant's comment here addresses how to start teaching logarithmic functions to improve students' problem-solving skills.

Melek: For example, I'll ask why we needed something like logarithms. I'll start with that. Why do we need logarithmic numbers? Let's say $3^x = 2$. We're actually trying to find the x there. Why is it needed? Maybe we'll get to the function part later, but as an introduction, I want to address the need for it, what gap does this topic fill? I'd like them to address that.

The eighth session started with a small group discussion on guiding preservice teachers in noticing students' conceptions which was postponed from previous week to this week due to time constraints. After this, the topics of the curriculum and the use of mathematical tools and technology were discussed. The frequency of mentoring content in this session is relatively low compared to other sessions. The following conversation taken from the small group discussion carried out at the beginning of this session illustrates the participants' views on how mentor teachers can help preservice teachers improve their noticing of students' conceptions.

Sevim: Student conceptions. So how do we start? For example, they can take notes on questions asked by the students during the lesson.

Melek: Yes. I'll write it down then. During the lesson...

...

Sevim: They can take notes or write down the answers students give to the teacher's questions. This can reveal misconceptions and help determine the student's understanding. Students...

Melek: Wait a minute, Sevim teacher, what did you say?

Sevim: They can also note how the students responded to the questions asked by the teacher.

The example below taken from the eight session shows what participants wrote on a Nearpod activity regarding how they would guide preservice teachers to improve their knowledge of curriculum. Figure 5.1 shows a screenshot during this Nearpod activity.

Zeynep: They should look at the curriculum and annual plans.

Melek: They cannot examine all the grade levels, but they must examine according to the grade level they will teach.

Zehra: During a conversation, I would mention that the Ministry of Education published the curriculum. I would recommend them to check it out. We might ask them to have a knowledge at the grade level that they are teaching.

Melek: The boundaries of the subject, what the students know and do not know up to that point, horizontally and vertically.

Ahmet: Even if they do not look at all the outcomes in the new mathematics curriculum, they need to know the main unit objectives at each grade level and which units are taught at which grade level.

Sevim: They can look at the content of the subjects at the grade level in the horizontal curriculum and prepare a sample lesson plan or worksheet for each grade level.

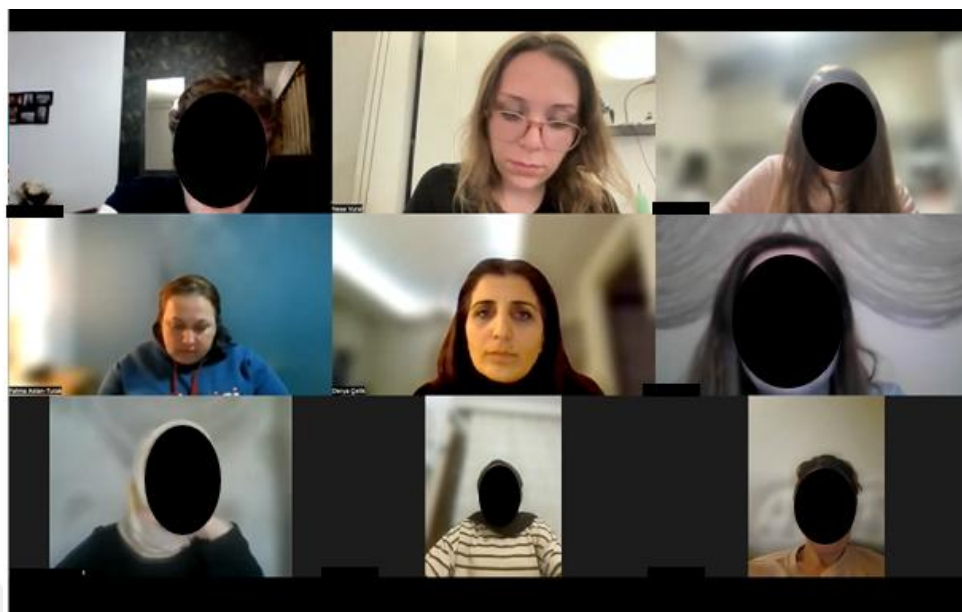


Figure 5.1. A screenshot during collaborative Nearpod activity.

Mentoring was the mostly discussed and emphasized content during the sessions as expected in parallel with the primary aim and design of the MMPD. In each session, depending on the session's topic, participants shared their experiences and expectations as mentor teachers and expressed their opinions and views regarding discussion questions and cases from their mentoring perspectives. In addition, facilitators played an important role in connecting participants' different ideas related to mentoring and guiding the discussions toward the objectives of the MMPD by emphasizing noteworthy ideas and mentoring practices.

5.1.3. Preservice Teacher Learning

Preservice teacher learning was not an explicit topic determined to be discussed during the sessions. However, during the MMPD, aiming to improve the practices of mentor teachers and advance their understanding of mentoring, it was also aimed to develop how they can contribute more effectively to the learning of preservice teachers. Therefore, opinions and beliefs about preservice teacher learning were discussed during the sessions independent of the topics of the sessions.

In terms of preservice teacher learning content, the conversations and views that addressed preservice teacher learning occurred the most often in the second session with the frequency of 81. The session that the discussions and opinions regarding preservice teacher learning emerged the least often is the sixth session with the frequency of 33. Other than these two sessions, the frequencies of preservice teacher learning content in the other six sessions are close to each other, ranging from 48 to 64. While the frequencies of mathematics and mentoring contents are more or less in parallel with the issues discussed in the sessions, making such a relation for preservice teacher learning content does not seem possible. The frequencies of mathematics and mentoring contents that occurred during the MMPD could be somehow grouped as high and low based on the topics discussed in the sessions. However, the frequencies of preservice teacher learning content do not show such a distribution. It was obvious that some topics covered in the sessions are more related to mathematics like content knowledge, designing and preparing teaching tasks and activities, conceptual learning, and mathematical skills. The frequencies of mathematics content were higher in the sessions in which these topics were discussed. The issues of intervention, reflection, and feedback have a more direct relevance to mentoring, but they do not have much relation to mathematics. In the sessions where these issues were discussed, the mentoring content was the highest while the mathematics content was the lowest. However, there is not such a relation for preservice teacher learning content.

Since the aim of the MMPD was to improve participants' mentoring practices and extend their visions of mentoring, the conversations and discussions occurred more on mentoring, but the behind of this aim is to develop mentor teachers' practices so that they can be more effective in preservice teachers' learning. Therefore, while the focus of the discussions and activities were mostly on mentoring, the discussions and opinions addressing preservice teacher learning were also considerably high and occurred approximate frequencies in the sessions regardless of the topic of the session. Only for the sessions with the highest and the lowest frequencies, it can be said that there is a connection with the topics discussed in the sessions. The session with the highest frequency of preservice teacher learning content is the second session. The focus of this session was guiding preservice teachers in making reflection to help them learn to teach. Therefore, there was more emphasis on preservice teacher learning compared to other sessions, this may explain why the frequency is the highest in this session. The session with the lowest

frequency of preservice teacher learning content is the sixth session. The focus of this session was how to give feedback to preservice teachers related to their lesson plans and teaching practices. In addition, there were two small group discussions in this session. Because of the nature of the topic and the conversations that occurred between participants in the small groups, the discussions and opinions occurred more from mentoring point of view. This may explain the reason why this session has the lowest frequency of preservice teacher learning content.

In order to see how the discussions and views related to preservice teacher learning occurred during the sessions, an example is given below from the first session. The first session was about content knowledge. In this session, how participants evaluate preservice teachers' content knowledge and what they can do to determine the deficiencies in preservice teachers' content knowledge, to help them be aware of that, and what practices they can use to help preservice teachers develop their content knowledge were discussed. The example below indicates a participant's views about the role of internship in preservice teachers' learning and how they can realize their deficiencies in content knowledge during lesson observations. The participant sees the internship period as an opportunity for preservice teachers to realize what they are lacking in and what areas they need to improve.

Salih: Yes, for example, we use calculators in our lessons. For example, if they are going to teach a lesson, they need to use the calculator. Therefore, by recognizing their weaknesses, actually I think the main reason they are at school, do an internship is to prepare in advance. Because during lesson observations, they also sense that, for example, I don't have a full knowledge on this; I need to work on it.

The second session has the highest frequency of preservice teacher learning content. The following example reflects the views of a participant on preservice teacher learning concerning the importance of giving them feedback about their mistakes and making reflection and self-evaluation for their development.

Ahmet: ... An outside perspective is always better. I mean, it's good for the intern teacher also. I think it's very much normal to tell them about their mistakes. After all, they're just learning. There is no harm in them knowing their mistakes. They need to

think that they can make mistakes. Telling them about their mistakes isn't wrong either. Of course, not during the lesson as I said, but in very urgent situations, perhaps intervention is possible. I think that an outside perspective, telling them, making them evaluate themselves, and knowing how to criticize themselves is necessary.

The following excerpt from the third session demonstrates a participant's comment about a case which illustrates how a mentor teacher gave feedback to a preservice teacher. The participant's views related to preservice teacher learning are seen in her comment regarding how feedback should be given to preservice teachers so that their learning can be promoted. In her comment, the participant emphasizes the importance of mentor teachers' approach to preservice teachers to support their learning.

Sevim: I didn't like the mentor teacher's tone at all. She is very direct, her criticisms are harsh, and she speaks in a commanding tone. In other words, the other party might not be very inclined to receive feedback from someone who uses this tone anyway. ... And the approach. I mean, they should find it together. The intern teacher should also think for herself. You know, it was like "I'm like this, I'm very good" and so on. She's just out to protect herself. Like, "I think everything was fine. I don't think there was any problem". Because she's defending herself there. She's not actually listening at the time. She's not getting feedback.

An example from the fourth session below shows a participant's comment on a case which illustrates the conversation between a mentor teacher and a preservice teacher related to selecting questions about cosine theorem. The participant states how the behavior of the mentor teacher in the case contributes to the learning and development of the preservice teacher. Her comment indicates based on her experiences that preservice teachers need to learn how to think like a teacher and it is important to guide them in the process of transitioning from thinking like a student to thinking like a teacher.

Zehra: I think that solving problems is not exactly equivalent to teaching. It is only a part of teaching. It's as if Armağan teacher told her to solve these questions, she would only solve the questions, that is, as a contribution to the lesson. But if she found them

herself, if she learned from other sources, it would be like she was learning how to fish, not buying the fish directly, I thought when I read.

Facilitator: I think the fish she catches will be pedagogical content knowledge.

Zehra: Exactly. I mean, by researching this herself, she will improve herself as a teacher like what is in the subject, she will primarily develop herself mathematically. Also, when I was started my teaching career, I experienced something like this. I'd initially solve the problem, then I'd think, "Wait a minute, I'm not the student, I need to think about how to explain this now". You go through that process while you're learning. If I had given them the questions, I think the intern teacher would have seen herself as someone who was still solving problems like she did when she was a student.

In the fifth session, how to guide preservice teachers in selecting and preparing teaching tasks and activities was discussed. For instance, a participant's comment was given below after a case which demonstrates a dialog between a mentor teacher and a preservice teacher where the mentor teacher asks the preservice teacher how students may answer the questions she prepared. The participant states regarding this situation that preservice teachers have difficulty in how to engage students when designing teaching tasks and activities based on her experiences.

Zeynep: I experienced this in the class when my intern teacher was teaching the lesson. They're thinking as you said, "I'll ask a question like this, and they'll answer". Now, the intern teacher asks something. Let's think about it, they say, what could the answer be? There's no response from the class. I was waiting for them to do something, to guide them. What kind of answer do I want? The intern teacher has no idea how to channel on this issue.

In the sixth session, how to give preservice teachers feedback about their lesson plans and after their teaching practices was discussed. In the following example, a participant expresses a view similar to the comment in the example above. She states that preservice teachers focus more on themselves and have difficulty in engaging students in the lesson, capturing students' thoughts and how to respond to them.

Zehra: ... If a good answer comes up, how should they respond to it? For example, should they use positive reinforcement, or not? Or when a question comes up, should they answer it themselves, or can they redirect it back to the class? Can they turn it into an opportunity? We didn't talk about these things. Of course, teacher candidates tend to focus more on themselves, like, "I'm going to do this". "Now I'll ask this question, and then I'll move on to this activity". Because they always had this plan in mind, they didn't really consider the questions or the good answers. But actually, some great doors opened during the lesson. I concluded that I would focus on them in our next plan.

The following excerpt from the seventh session demonstrates a participant's evaluation of what preservice teachers need to learn and develop based on his observations. The participant expresses his views regarding preservice teacher learning from a broad perspective by evaluating them in terms of their pedagogical knowledge, content knowledge, and pedagogical content knowledge.

Ahmet: ... In general, I see this in the intern teachers: There is an academic part and a classroom management part. Their classroom management is generally good, I mean, they try to communicate. ... But of course, they always play in front of the goal, they are always in front of the board. I want them to move to the back for a while, walk around in the back while someone else is solving a problem, take a look, but they generally don't leave the board right now. Maybe it's because of the initial fear. ... About the teaching of the lesson topic: They prepare it, but they cannot prepare examples from easy to difficult, or after explaining a topic, I always say, it's not like in my first years of teaching, that I gave it, and the student immediately understood the formula, but it doesn't work. ... Partly because they don't have a full grasp of the subject matter, because they don't fully understand how the students learn. We know that what we teach, if we explain ten things, the student doesn't get through right away, maybe even fewer.

The following conversation is taken from the small group discussion conducted at the beginning of the eighth session. It shows two participants' views concerning what preservice

teachers can do to notice students' conceptions while observing their mentor teachers' teaching.

Sevim: While working independently or when given a problem, how did the student begin the problem? What do we expect? To raise awareness about how the student begins with their knowledge up to that point, intern teachers can also note the beginnings and stages of their problem-solving techniques if there is an opportunity, I would suggest that.

Melek: For example, the person sitting there can actually observe the mistakes made by the students very well. Now, as I listen to them, I also have the opportunity to see the many mistakes made by the students. They probably see in the same way. For example, the student writes down mistakenly, transfers them incorrectly to their paper, the intern teacher would see these probably. They can also take notes on them.

In the MMPD, although preservice teacher learning was not explicitly addressed as a topic of discussion, expressions of views, opinions, and beliefs about preservice teacher learning occurred in considerably high frequencies in each session. The cases provided for discussions contributed to the emergence of ideas and views regarding preservice teacher learning by demonstrating the mistakes that preservice teachers can make and the areas they may need to develop. In addition, the cases helped participants recall their similar experiences with preservice teachers and stimulated the discussions that include views and beliefs about preservice teacher learning. Facilitators' guidance was also influential to make participants think about preservice teacher learning and consider the impact of their practices on preservice teachers' learning. Moreover, the opinions and views stated during the discussions regarding preservice teachers' learning shifted from expressing ideas on more general issues to focusing more on improving content knowledge and pedagogical content knowledge of preservice teachers across the sessions. In this shift, the facilitators' guidance was quite effective in directing participants to what to consider while studying with preservice teachers.

5.1.4. Divergence

The first three categories of content; mathematics, mentoring, and preservice teacher learning, are about the aims and the design of the MMPD. The fourth category, divergence, is related to deviance from the main idea and moments of distractions. This category was added considering the importance of maintaining coherence with the participants' prior experiences, knowledge, and beliefs, which is one of the characteristics of an effective PD (Desimone, 2009). Accordingly, by analyzing the moments where participants deviated from the discussion topics, stated an off-topic comment, and lost their attention, it is aimed at understanding if the objectives of the MMPD was consistent with the participants' objectives. Divergence occurred when the participants' deviated from the discussion topics or lose their attention. During the MMPD, the total frequency of divergence is 14 and the highest frequency that occurred in a session is three, which is the third session. In each session, there was at least one divergence moment and the sessions which have only one divergence moment are the first, second, and fourth sessions. The remaining four sessions have two divergence moments. It is good that the frequency of divergence is low, which shows that there was not much deviation from the goals and focus of the MMPD and that the coherence with participants' knowledge and beliefs could be maintained. For instance, the following excerpt from the second session shows how a participant's comment deviated from the discussion topic while discussing on how to guide preservice teachers to make reflection. This comment is not much related to reflection. It focuses more on the importance of selecting and preparing questions, which is not a topic of this session.

İsmail: So, I asked our interns to prepare questions and send them to me via WhatsApp. They prepared and sent them, but, well, they had prepared good questions in their own way, but I wrote underneath them, "Where did you get this from"? So, where did you get the questions from? You also need to write that down. They didn't realize the importance of that. ... We usually distort the questions or spread them around too much. What I'm saying is, this also affects our reliability. They might go and buy that book. They might not even follow your class.

The example from the third session below demonstrates a question that a participant asked based on her experience in the class. The topic discussed in this session was related to

giving feedback to preservice teachers and the participant was supposed to write an answer to a Nearpod activity when she asked this question. In this question, the participant asks whether there is any prejudice against preservice teachers in the classroom, based on an incident she experienced that day. This question deviates from the aim and the focus of the session.

Zeynep: Today, we had the following experience. My students asked where the intern teachers were. They asked why they weren't teaching. I said that they would start teaching after the holiday. One said, "I don't think they have much knowledge in the field". I said, "Why did you say that? How can you know whether someone knows something or not"? They said, "Teacher, we had a conversation. I asked something". They said, "I understood that from their answer". So, is there generally a prejudice against intern teachers in the classrooms? Did it happen here? That's why I actually want to get information from you.

The following example related to divergence is from the small group discussion carried out in the eighth session. Here, the participants deviated from the topic they were supposed to discuss and talked about a different practice that was applied at the school that one of the participants work.

Melek: Then these students are not your intern teachers, but the interns of all mathematics teachers?

Sevim: There is one mentor.

Melek: They attend every lesson.

Sevim: There is one mentor. They attend every lesson, yes but they can also take notes from other teachers. That teacher gives them feedback. ... Then they discuss all the ideas they have gathered with their mentor...

Facilitator: May I make a suggestion? Now, actually, the majority of the group is subject to the internship implemented by the Ministry of National Education. Perhaps we should develop an application based on this existing internship. So...

In general, the participants did not deviate from the discussion topics and stated ideas, views, or comments consistent with the goals and focus of the MMPD. This may indicate

that the MMPD was coherent with the participants' experiences, beliefs, and background knowledge to a large extent. In the moments of divergence, the facilitators redirected the participants toward the goals and main ideas of the MMPD by highlighting effective mentoring strategies and practices to promote the development of preservice teachers.

5.1.5. Professional Identity

The last category of content is professional identity. This category has come from the data during the analysis regarding the views of participants on teaching and the learning of their students. Although the focus of the discussions and activities carried out during the MMPD was about participants' mentoring identities and for improving their mentoring practices, the participants' teacher identities also came to the fore at some times. The underlying aim in developing mentor teachers' mentoring practices is actually to enable them, as experienced teachers, to become models for preservice teachers to help them become competent teachers who can teach effectively in the classroom. Therefore, these views regarding teaching and students' learning are significant in that they reflect how participants can be models for preservice teachers to contribute to student learning.

In terms of the frequency of the views and opinions stated regarding professional identity, the total frequency of all sessions is 29. The highest frequency of views and opinions reflecting professional identity is six and occurred in the second and the seventh sessions. The sixth session follows them with the frequency of five and after it, the eighth and third sessions comes with the frequencies of four and three. The lowest frequency of professional identity is one and occurred in the first session. The fourth and fifth sessions have the frequency of two. For instance, the following comment from the second session indicates how a participant uses constructivist learning theory to promote the learning of his students. In a similar way, he states helping preservice teachers make reflection can contribute to their learning.

Salih: Sometimes, when we realize something ourselves, its impact lasts longer and becomes more meaningful. It's also like, for example, when we're teaching a topic, instead of directly answering a student's question, we try to help them find the answer

themselves. Because they need more permanent learning. I think we can communicate with the intern teachers and make them reflect in a similar way.

The following example from the third session demonstrates how a participant starts the topic of trigonometry in grade 12 considering what his students know and do not know. He states the importance of identifying students readiness levels and giving preservice teachers feedback about that.

Ahmet: ... For example, let's take the example of trigonometry again. In the twelfth grade, trigonometry starts with the sum and difference formulas, but I don't start with that. Because I know that the students don't have readiness for it. They learned that trigonometry topic a year ago and either forgot most of it or didn't learn it then. The intern teacher may not know this. They might say, "I've prepared for this topic, so I'll start from here". They might write down the formula, but they need to know if the student has the prerequisite knowledge. Do they know the unit circle? Do they know what trigonometric functions represent? Every year, I start from scratch with the twelfth-grade students. Even if I go through it quickly, I explain to the students what trigonometry is, what trigonometric functions are, how the signs in the regions are determined, and how to simplify them.

Another example below from the sixth session shows a participant's view on the importance of visualization in teaching geometry.

Zeynep: For example, right now they taught geometry to eleventh graders, and when we teach geometry, we try to make the drawings, points, and lines to visualize them in the students' minds, even if they're not perfectly accurate. ... At least we try to explain the questions by visualizing in the students' minds.

The following example from the seventh session displays a participant's opinions about the importance of using real life problems in teaching mathematics and how this can be motivating for students to learn.

Sevim: Well, I'll speak in general. Not just about logarithms. If students can make sense of it, connect it to real life, and understand where it will be useful in real life or why such a thing has come about, then they will be more eager to learn mathematics...

The ideas of participants regarding how they teach and contribute to the learning of their students are important in the sense that they indirectly include information about how they would guide preservice teachers. These views indicate their teaching styles and approaches to teaching and learning, which also indicate what practices and teaching strategies they would model for preservice teachers. Therefore, these views are indirectly related to mentoring and preservice teacher learning. Mentor teachers should make a bridge between their teacher roles and mentor roles considering both the learning of students and the learning of preservice teachers. These views provide clues about how they would affect preservice teachers in becoming a mathematics teacher.

5.2. Interaction and Collaboration

Another dimension from which the online MMPD was examined is interaction and collaboration. In online learning settings, ensuring interaction and collaboration is a significant factor for creating an effective learning environment. The types of interaction that were analyzed in the online MMPD were *participant-participant*, *facilitator-participant*, *participant-content*, and *facilitator-content* interaction. In the analysis of the video recordings of the MMPD sessions, participant-participant interaction was identified as communication between participants, and attending to or addressing each other's ideas, comments, or answers. Facilitator-content interaction was determined as communication between facilitators and participants. Participant-content and facilitator-content interactions were described as interaction with the technological tools used during the sessions and engaging with the specific content presented in the sessions. In addition to these types of interaction, how *collaboration* of participants occurred during the sessions were analyzed. Collaboration of participants was identified as working on a task together and creating a product.

Table 5.2. Frequency of interaction and collaboration in sessions.

Interaction and Collaboration	Sessions								Total
	1	2	3	4	5	6	7	8	
Participant-participant interaction	12	10	6	6	6	22	7	16	85
Facilitator-participant interaction	35	40	25	18	31	55	24	43	271
Participant-content interaction	35	33	40	26	25	30	17	45	251
Facilitator-content interaction	28	22	22	22	17	28	13	32	184
Collaboration	3	0	0	0	0	13	0	9	25
Total	113	105	93	72	79	148	61	145	816

In the online MMPD, the type of interaction that occurred most is the facilitator-participant interaction with a total frequency of 271 (see Table 5.2). Participant-content and facilitator-content interactions followed it with total frequencies of 251 and 184 respectively. Participant-participant interaction remained limited during the MMPD compared to the other types of interaction with a total frequency of 85. Collaboration in the online MMPD occurred only in the small group discussions carried out in the first, sixth, and eighth sessions. Therefore, collaboration has the lowest frequency in the MMPD, which is 25.

5.2.1. Participant-Participant Interaction

In general, the online MMPD was limited in terms of participant-participant interaction compared to the other types of interaction. In the sixth and eighth sessions, the frequency of participant-participant interaction is relatively higher than the other sessions. In these sessions, small group discussions were conducted which allowed participants interact more with each other. In the first session, a small group discussion was also held, but because of the recording problem occurred during this session, there is not a complete recording of the group discussions. Therefore, the recording which captured the most of one of the groups' discussions was used in the analysis of interaction and collaboration. That is why participant-

participant discussion occurred during this session found in the analysis may be less than what it actually was. Especially, participant-participant interaction occurred at most in the sixth session in which two small group discussions were held. In the other sessions, participant-participant interaction occurred approximately at the same frequencies. Participant-participant interactions that took place out of the small group discussions was mostly in the form of attending to and agreeing with each other's views, opinions, or comments, or sharing an opinion or a question that came to their minds based on a comment made by another participant rather than a dialogue between the participants. In addition, interactive Nearpod activities where participants could simultaneously post and like each other's answers also contributed to participant-participant interaction during the sessions. For example, the following excerpt demonstrates a part of the dialogue between the two participants occurred during the small group discussion carried out in the first session.

Zehra: Salih teacher, we have talked about the upcoming questions, you can write them down if you want.

Salih: I wrote like this. We defined content knowledge as focusing on the subject being taught and deepening in it, rather than thinking more generally.

Zehra: Now, how would we plan for this? So, what would be our action? How should we think about this?

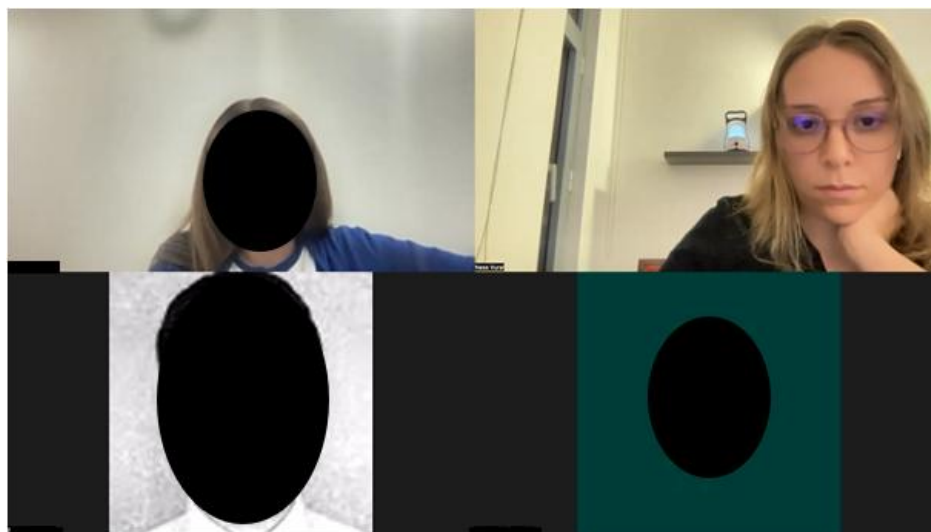


Figure 5.2. A screenshot during the small group discussion in the first session.

The following excerpt from the second session is an example of participant-participant interaction occurred while discussing about whether mentor teachers should intervene or not during a problem happened in the teaching of preservice teachers.

Sevim: I agree with the previous teacher. I would try to avoid interfering as much as possible...

...

Melek: I was going to say similar things. I wouldn't intervene either...

The following example from the second session also indicates a question that a participant asked based on another participant's comment about a mathematical mistake that a preservice teacher made during a teaching practice. In this example, participant-participant interaction did not take place in direct way like a dialogue. The interaction between the participants in this example occurred in a more indirect way, with a participant's comment influencing another participant and thinking about it.

Salih: ... Now that Ahmet teacher has spoken, a question comes to my mind. Of course, no matter how experienced we are, sometimes we make mistakes, even when adding two and three...

The excerpts from the fifth and the seventh sessions below also show participant-participant interactions through the way that a participant made a comment that agree with and support the comment of another participant.

Ahmet: Students really do not answer, as Ms. Zeynep said. My intern teachers had the same reaction. "They don't ask any questions" they said.

İsmail: ... As Sevim teacher said, after an equation, making any equivalence with an algebraic expression, check whether its inverse can be found, and then make the graph of the logarithm function plotted...

In general, participant-participant interaction occurred limitedly during the online MMPD. The facilitators tried to compensate this after they noticed this limitation by adding

more small group discussions toward the last sessions. However, only the small group discussions were not sufficient for participant-participant interaction. Ensuring participant-participant interaction during the large group discussions is also quite important. For this, the facilitators might have encouraged the participants to ask questions each other or make comments to each other's ideas or views during the large group discussions.

5.2.2. Facilitator-Participant Interaction

The type of interaction that occurred the most in the online MMPD is facilitator-participant interaction. The session in which facilitator-participant interaction took place at most is the sixth session and the session in which facilitator-participant interaction took place at least is the fourth session. During the MMPD, almost all discussions consisted of a dialogue between the facilitator and the participant, with one participant responding to a question posed and the facilitator asking for clarification or further details or commenting on the participant's answer, and then moving on to another participant in a similar way. For instance, the following dialogue shows how a facilitator-participant interaction occurred during the MMPD.

Ahmet: So, the questions need to be diversified. It wouldn't be correct to solve questions based on just one thing. Just the vertex...

Facilitator: But would we say that it won't be correct? What should we say to him? How can we explain?

Ahmet: I mean, it could be said that it should cover all the objectives. We could say that it shouldn't cover just the objective on the vertex.

Facilitator: And if he asked such difficult questions, it will probably be possible to answer only one or two questions.

Ahmet: Or maybe, if he is going to prepare a difficult question, he can leave it for last, and if it is not solved, he can leave that question as homework.

Facilitator: Yes, right.

In general, all the other facilitator-participant interactions occurred in a similar way to this dialogue. The dialogues did not last as long as in this vignette all the times. Sometimes,

after the facilitator provided a comment or feedback on a participant's answer, the discussion continued with another participant's answer.

5.2.3. Participant-Content Interaction

Participant-content interaction is the second highest type of interaction that occurred in the MMPD after facilitator-participant interaction. The session with the highest frequency of participant-content interaction is the eighth session. The session with the lowest frequency of participant-content interaction is the seventh session. In the other sessions, the frequencies of participant-content interaction are close to each other in general. It can be said that participant-content interaction took place in two ways in the online MMPD. One of these is the participants' interaction with the technological tools that were used during the sessions. The other is the participants' interaction with the teaching materials that were presented and discussed during the sessions. Two technological tools were used during the sessions, one of which is the online meeting platform Zoom where the sessions were held and the other is the interactive presentation tool Nearpod where the teaching materials and discussion cases were presented and interactive activities were performed. Participants interaction with the technological tools occurred in general when they wrote answers to the Nearpod activities, read each other's answers, and followed the presentation via the Nearpod. For instance, Figure 5.3 displays the screenshot of the participants' responses to a Nearpod activity during the first session. The participants posted their responses so that everyone can read them.

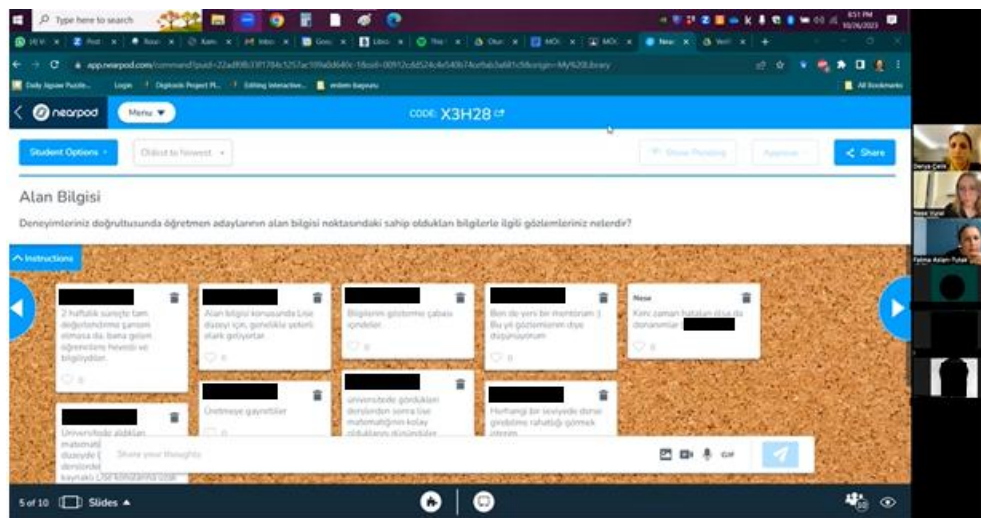


Figure 5.3. A screenshot of participants' responses to a Nearpod activity.

The following excerpt from the eighth session demonstrates a problem when participants encountered when writing their responses to a Nearpod activity.

Ahmet: There is probably a word limit there. You can't write more than a certain limit on the corkboard; you have to write in pieces.

Facilitator: It may be in piece by piece.

Ahmet: Because when I pasted them altogether, it said minus sixty. There is two hundred and fifty letters limit.

Facilitator: Salih teacher, should we send this to Nearpod?

Salih: Yes, there is, as Ahmet teacher said, there is a character limit.

In addition to the interaction with the technological tools, participant-content interaction occurred when participants engaged with the teaching materials by commenting about an event occurred in the cases or expressing what they attracted their attention in the cases. For example, the following excerpts from different sessions demonstrate the participants' interaction with the discussion cases by offering their comments and pointing out what they attracted their attention.

Ahmet: Actually, she should have said the solution set has infinite elements. Merve confused those concepts there.

Zehra: Another thing that caught my attention is that it started with something like, "I think you have a problem with these students".

İsmail: She provides guidance on how to engage students in the lesson. This is very good.

Sevim: For example, she said that she was weak in relating the graph to the algebraic formula.

Melek: We'd express our expectation that he'd discuss the situations in which functions have inverse. He had difficulties about this.

5.2.4. Facilitator-Content Interaction

In terms of facilitator-content interaction, the sessions with the highest frequencies are the first, sixth, and eighth sessions and the session with the lowest frequency is the seventh session. In general, the frequencies of facilitator-content interaction in the sessions are close to each other. Facilitator-content interaction during the MMPD also occurred in two ways like participant-content interaction. One of which is the facilitators' interaction with the technological tools and the other is the facilitators' interactions with specific content in teaching materials and discussion cases. The facilitators interaction with the technological tools occurred when the facilitators shared screen on the Zoom or used other features of the Zoom like chat or discussion rooms, presented the slides on the Nearpod, and checked and read the participants' responses to the Nearpod activities. Figure 5.4, for example, shows a moment of the facilitator shared screen to show the entrance code for the Nearpod presentation in the third session.

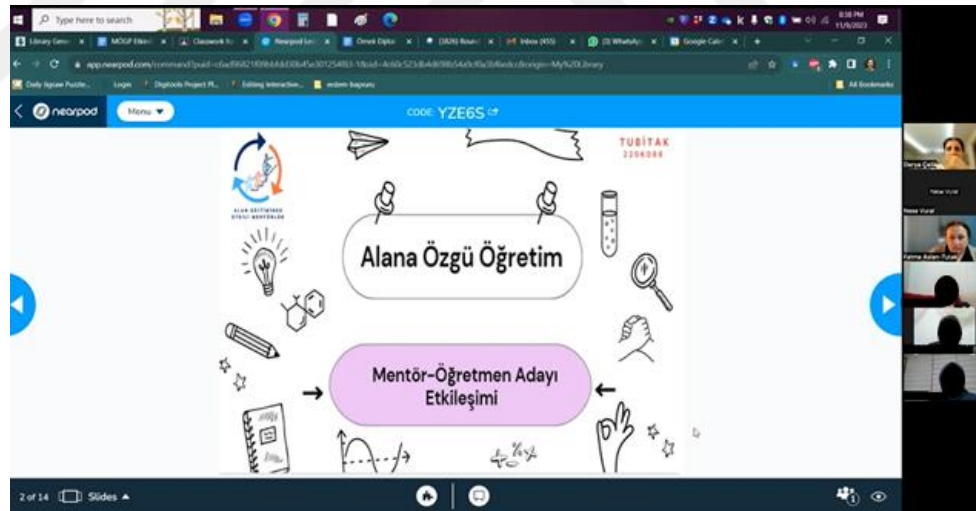


Figure 5.4. A screenshot from the third session.

The other type of facilitator-content interaction occurred when the facilitators commented about a specific event in the cases, or directed attention to what they found important in the cases. For instance, the following excerpts indicate facilitator-content interaction that occurred regarding the discussion cases.

Didn't she ask the question to Burak like that? She did not say, Burak teacher got excited and he can't speak, right?

She actually said something, right? For example, towards the end, she says, "You've thought it well, you've prepared well".

Yes, the prospective teacher may not even be able to use the term "visualization". The teacher tells her "Look, you can use this to visualize".

He's already missed here that if the number a is negative, the arms of the graph will open as it gets smaller, right? So, mathematically, he should say that the arms open as its absolute value increases.

So, for example, I felt like he had mentioned content knowledge, right? He said it should be a graphic representation or something.

5.2.5. Collaboration

During the MMPD, collaboration occurred only in the first, sixth, and eighth sessions in which small group discussions were carried out. During the small group discussion in the first session, the participants discussed what kinds of knowledge a mathematics teacher need to have and they designed an activity or task to help preservice teachers acquire this knowledge. In the sixth session, two small group discussions were held. In the first one, the participants worked on a case illustrating the observation notes of a mentor teacher related to a preservice teacher's teaching practice. The participants identified on which dimensions of mathematics teacher learning the observation notes focused and discussed how they would give feedback to the preservice teacher if they were in the place of the mentor teacher. In the second one, the participants worked on a feedback sample they gave to preservice teachers on their lesson plans and teaching. They discussed how they guided preservice teachers to improve their lesson plans, how they gave feedback to preservice teachers to promote their pedagogical content knowledge, and what they do the same and different to support preservice teachers in learning how to teach mathematics. In the eighth session, the participants worked on three suggested methods to help preservice teachers notice students'

conceptions. By selecting one method or proposing a new method, they discussed how they can use it. Figure 5.5 displays a screenshot including the groups' responses to this discussion. In an online learning setting, collaboration is quite important in creating a professional learning community and supportive group dynamics. For instance, as seen in Figure 5.6 which demonstrates the moment that participants returned to the main room after the small group discussion in the sixth session, the smiles on the faces of the participants illustrates how working in small groups and collaborating with each other contributed to the formation of a supportive learning community.

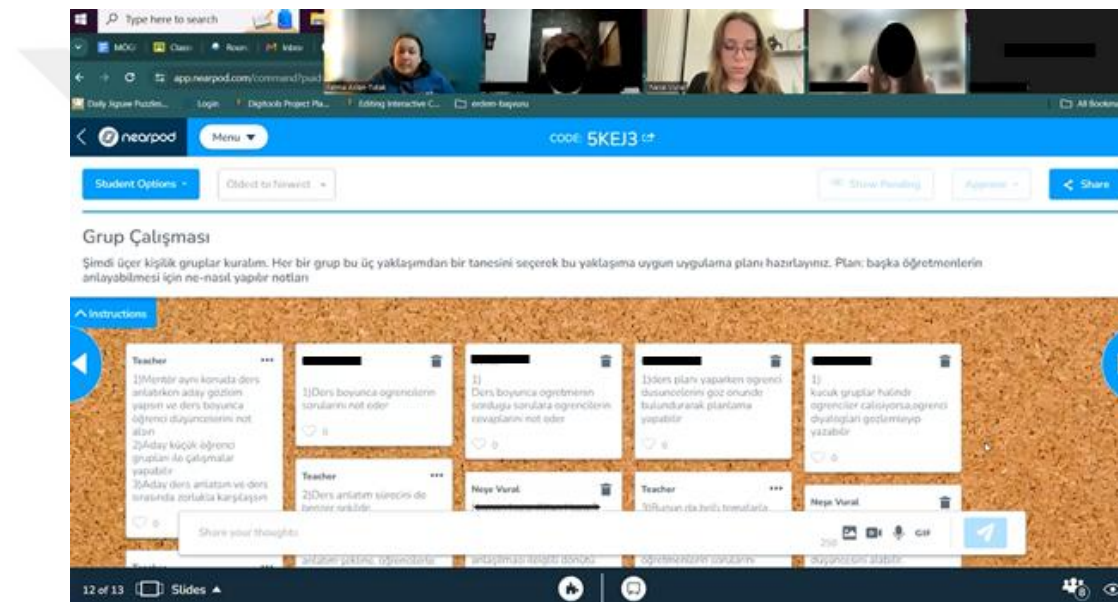


Figure 5.5. Participants' responses from the small group discussion.



Figure 5.6. A screenshot from the sixth session after the small group discussion.

5.3. Facilitation

The third dimension in which the online MMPD was examined is facilitation. The role of facilitators is crucial in directing participants' attention and guiding discussions toward the goals and focus of the PD. The online MMPD was analyzed by using a revised version of the facilitation framework developed by van Es et al. (2014). The descriptions of some of the facilitation moves are adapted to the mentoring content and design of the MMPD. The frequency of facilitation moves occurred across the sessions of the MMPD is given in Table 5.3. The facilitation moves are grouped under three categories by adapting the categories used by van Es et al. (2014). The first five moves, countering, clarifying, pressing, lifting up, and offering an explanation, are under the category sustaining an inquiry stance as in the framework of van Es et al. (2014). The definitions of these moves are taken directly from van Es et al. except offering an explanation. Its definition was slightly revised to suit the content and design of the MMPD. Countering occurs when facilitators offer an alternative point of view to the participants' ideas. Clarifying means restating and revoicing to ensure common understanding of an idea. Pressing means asking participants to explain their reasoning and/or elaborate on their ideas. Lifting up occurs when facilitators identify an important idea that a participant raised in the discussion and elevate for further discussion. Offering an explanation means providing interpretation about an event, interaction, idea, concept, or question.

The next three moves, highlighting, redirecting, and connecting ideas, are grouped under the category maintaining focus on the goals of the MMPD. This category was slightly revised to make it more suitable to the content and design of the MMPD. Highlighting was in the first category in the van Es et al.'s framework (2014). However, after its definition was revised based on the content and design of the MMPD, it seemed more suitable to the category of maintaining focus on the goals of the MMPD. Highlighting was defined as drawing attention to noteworthy mentoring practices, conceptions, and ideas related to preservice teacher learning. Redirecting means shifting the discussion to maintain focus on the goals and main ideas of the MMPD. Connecting ideas occurs when facilitators make connections between ideas raised in the discussion.

The last two moves, distributing participation and validating participants ideas, are under the category supporting group collaboration as in van Es et al.'s framework (2014). Distributing participation means inviting participants to share different ideas based on who is (and is not) participating. Validating participant ideas occurs when facilitators confirm and support participant contributions.

Table 5.3. Frequency of facilitation moves in sessions.

Facilitation Moves	Sessions								Total
	1	2	3	4	5	6	7	8	
Sustaining an inquiry stance									
Countering	4	4	3	1	1	3	1	2	19
Clarifying	5	13	11	6	16	13	6	12	82
Pressing	2	5	6	1	9	7	3	5	38
Lifting up	4	4	3	4	5	0	6	0	26
Offering an explanation	13	21	25	23	31	18	28	20	179
Total									344
Maintaining focus on the goals of the MMPD									
Highlighting	10	17	25	12	22	4	16	9	115
Redirecting	1	1	2	1	2	3	2	1	13
Connecting ideas	5	8	9	5	5	5	3	4	44
Total									172
Supporting group collaboration									
Distributing participation	18	17	13	10	16	10	9	6	99
Validating participant ideas	4	4	9	9	10	7	12	3	58
Total									157
Total of all moves	66	94	106	72	117	70	86	62	673

5.3.1. Sustaining an Inquiry Stance

The facilitation moves under this group are effective in promoting meaningful and productive discussions (van Es et al., 2014). Among the moves in this group, in fact among all the facilitation moves, the most frequently used across the sessions of the MMPD is offering an explanation. During the MMPD, offering an explanation occurred in various ways. Facilitators' comments and opinions regarding discussion cases or topics, about participants' ideas or questions, or related to an event or interaction occurred during the sessions were identified as offering an explanation moves. For instance, the following excerpt from the first session shows the facilitator's explanation after a participant wrote an answer to a discussion question which asks participants' observations regarding preservice teachers' content knowledge. The facilitator reinterpreted the question by expanding its scope to promote a productive discussion with the contributions of all participants.

Facilitator1: You said you were novice in mentoring Zehra teacher, actually, maybe, let's think about what your expectations would be while we're discussing.

A similar occurrence of offering an explanation move from the third session is given below. The following excerpt demonstrates the facilitator's comment regarding a discussion question asking participants to what they would do the same or differently based on sample feedback they gave preservice teachers. Like in the above example; to encourage the involvement of all participants, the facilitator provided an interpretation to the question by explaining in detail what the question asks and how they can answer this question if they have not given feedback to preservice teachers.

Facilitator1: Now, think about it. What would you do differently from now on? Like "I used to do it this way. Now I realize it would have been better if I had done it this way". Think of it like, "I used to do this. I realized that I should continue doing this". ... Zeynep teacher is also having a student for the first time this year. She said they have not taught a lesson yet, but you've certainly had conversations like this, Zeynep teacher. You could write along those lines, too.

Similarly, the example from the sixth session below indicates how the facilitator offered an explanation related to the components of effective mathematics teaching to help the participant understand the question. By providing an interpretation to the question, the facilitator made it more understandable for the participant.

Facilitator1: What could be the components of effective mathematics teaching, how does it happen?

Melek: Components of effective mathematics teaching.

Facilitator1: For example, you entered a classroom and said that the teacher taught math very well. What happened? What did you see that made it a very good lesson?

Another way that offering an explanation moves identified during the MMPD is about the cases provided for discussions or related to the discussion topics of the sessions. An example of this from the first session below demonstrates the facilitator's comment regarding a case which illustrates teaching practice of a preservice teacher about solving the system of linear equations. After participants expressed their opinions and views related to the case, the facilitator offered her explanation about the case by explaining mathematically in detail what mistake that preservice teacher did. In addition to offering an explanation, there is another facilitation move in this comment, which is validating participant ideas. By saying that "as others have just stated, graphically, they are coincident lines" the facilitator confirmed participants' contributions.

Facilitator2: Students generally don't experience problems when there's only one solution. However, they do experience problems when there are infinitely many solutions, or no solutions at all. Why? ... That's where we talk about content knowledge, that's where the problem lies. What's actually being done here? Well, in the process of producing an equivalent system, the first equation was added to the second equation. ... We actually obtained an equation of the form $0x + 0y = 0$. So, this leads us to the following: every x - y pair that satisfies the first equation, that is, every x - y pair that algebraically satisfies the first equation, is a solution set for this system. Because all x 's and y 's satisfy the second equation. ... Or, as others have just stated, graphically, they are coincident lines. Therefore, the solution is neither the set of real numbers nor the entirety of R^2 , but rather the set of points on this line.

Another example of offering an explanation move related to discussion cases or topics from the second session is given below. It indicates the facilitator's interpretation about reflection and a discussion case in which a mentor teacher asks a preservice teacher if he had taught the topic of logarithmic functions to the same class again, what she would have done differently or the same. First, the facilitator proposes an interpretation about reflection by giving an example from private lessons to make more understandable the idea behind the giving preservice teachers opportunity to make reflection. Second, she comments about the case by giving an example how the mentor teacher might have continue her question according to the answer of the preservice teacher. In this excerpt, there is also a validating participant ideas move. In the comment, the facilitator confirms a participant idea by saying that "its purpose is this as Zehra teacher said".

Facilitator1: Just as when we explain it to the class, we explain it to thirty people, but in private lessons, we ask a question. Depending on the student's answer, we ask them a new one, right? It's like that. She's asking something, and what she's asking is a reflection question. Its purpose is this as Zehra teacher said. Let him think about himself, let him evaluate himself. If he can grasp it when he evaluated, she'll proceed accordingly; if not, she'll proceed accordingly. Perhaps she'll ask, "If he doesn't, do you remember how we defined the sets, domain, and range of logarithms? Do you remember what you said or how you explained it in class"?

Similarly, the following excerpt from the fifth session shows how the facilitator provided an interpretation about a case which includes a mathematical mistake of a preservice teacher. The facilitator commented on the mathematical mistake and also about how preservice teachers have difficulty in determining the main idea of a lesson. The facilitator offered this explanation while introducing the case for a discussion about how to guide preservice teachers in such a situation to help them catch the main idea in interpreting parabolas so that participants can understand better what they are expected to discuss and a meaningful and productive discussion can be ensured.

Facilitator1: So, he should have said that the vertex is symmetrical with respect to the line parallel to y, passing through the vertex. He just thought about $y = ax^2$, thought about the one symmetrical around y. And that's not the only thing, it's not that it's

symmetrical, I mean, the main idea can't be that simple. In fact, they are having difficulty from the very beginning. I mean, since you have been teaching for years, it is easy to get the main idea straight away, but they are having difficulty.

In the MMPD, offering an explanation moves were also identified in some of the comments of the facilitators about the participants' ideas. For example, the following excerpt from the seventh session, demonstrates an explanation that the facilitator provided regarding a participant's comment about how many questions to be solved in a lesson. The facilitator provided an interpretation of this issue by giving examples of mathematics topics.

Sibel: ... We discussed among ourselves whether we would complete the lesson with just one question, or whether we should allocate a certain amount of time for ten questions and arrange the minutes accordingly.

...

Facilitator1: This also varies depending on the subject, so you might need to go slower on some subjects. For example, on a subject, I'm thinking, maybe you need to go faster on the topic of ninth-grade problems, I don't know yet.

While the facilitators were offering an explanation regarding participants' ideas or comments, it happened together with clarifying moves sometimes. In other words, this situation occurred when the facilitators provided a remark on a participant idea to clarify its meaning but at the same time by adding their interpretation of it. For instance, in the following vignette from the fourth session, the facilitator expresses a comment about a participant's idea regarding the discussion case. While the facilitator clarified what the participant said, she also presented her interpretation about catching fish metaphor that the participant expressed.

Zehra: ... It's as if Armağan teacher told her to solve these questions, she would only solve the questions, that is, as a contribution to the lesson. But if she found them herself, if she learned from other sources, it would be like she was learning how to fish, not buying the fish directly, I thought when I read.

Facilitator1: I think the fish she catches will be pedagogical content knowledge.

Zehra: Exactly. I mean, by researching this herself, she will improve herself as a teacher like what is in the subject, she will primarily develop herself mathematically.

Similarly, the following excerpt from the eighth session is an example of both offering an explanation and clarifying moves. While trying to clarify the participant's idea about giving some themes to preservice teachers to observe in the classroom to help them notice students' conceptions, the facilitator also added her interpretation by giving some examples about how the method that participant suggest can be used. The clarifying move includes only restating or revoicing a participant idea to make its meaning clear without adding an interpretation. Therefore, in the MMPD, the facilitators' remarks that aim to clarify what participants says but also include a substantive comment of the facilitator were coded as both moves.

Facilitator1: Sometimes universities may assign such tasks, but they may not, or the university's observation task may be different, but considering what Salih teacher said, since you know what's happening in the classroom, specific to that subject, for example, there is a topic we all struggle with, like probability, for example... or you might say there is something in functions that students often make mistakes about. Maybe then you could say, "Look, pay attention to that this week, pay attention to student thinking". That's how I understood it, Salih teacher. Did I understand correctly?

Lastly, offering an explanation moves were identified during the MMPD when the facilitators proposed an interpretation of an event, an interaction, or a situation occurred during the sessions. For instance, the following excerpt from the second session involves the facilitator's comment about a discussion regarding whether participants would intervene or not during preservice teachers' teaching practice. The offering an explanation move in this excerpt occurs by interpreting the situation after some participants expressed their opinions. For instance, some participants said they would not intervene, some said there is no problem in intervening, some said they would intervene in a way that students do not see the intervention. By stating the situation about this issue as a dilemma, the facilitator stimulated the discussion and encouraged participants to think more about this issue.

Facilitator1: We actually ... discussed this in Istanbul. Even though that wasn't our topic. It was as if we were divided in two there. So, there are always those among us who say, "I would intervene" and those who say, "No, I wouldn't intervene". I feel like we're caught in this dilemma.

In short, offering an explanation is the most frequently used move during the MMPD to support productive and meaningful discussions. In the MMPD, the interpretations of the facilitators about the discussion cases, questions, or topics; related to participants' ideas, comments, or views; or regarding a situation, an event, or interaction occurred during the sessions were identified as offering an explanation moves as they contribute to effective discussions by helping participants understand what to focus and encouraging them to think more and from various perspectives. In the sustaining an inquiry stance category, after offering an explanation, the second most frequently used move is clarifying. Clarifying occurred when the facilitators rephrased a participant idea or asked questions to make its meaning clear and ensure a shared understanding among the group. As mentioned above, it sometimes occurred with offering an explanation move. The remarks of the facilitators aiming to clarify a participant idea but also include a substantive interpretation about it were identified as both moves. An example of clarifying move from the first session is given below. The facilitator clarified a long comment of a participant regarding preservice teachers' content knowledge by specifying the main idea of what the participant said.

Facilitator1: That's how I understood. I mean, advanced mathematics is fine, but I understood from you that they might make mistakes when defining and explaining concepts in high school mathematics.

Another example of clarifying move is seen in the following vignette from the third session. Here, the facilitator actually started with a pressing move by asking the participant what he meant by knowing the applications on the board. Since the facilitator asked this question to prompt the participant elaborate on his answer, this is a pressing move. After the participant's explanation, the facilitator clarified the participant idea by emphasizing its essence.

Facilitator1: Ahmet teacher, can I ask you a question quickly? You wrote this answer here, saying that they should know the applications on the board well. What did you mean by that? Let's hear it together.

Ahmet: For example, we use Starboard on the smart board. It's a blank board that allows you to draw shapes, measure angles, have a ruler, and a compass. I don't normally use it for writing. That's something different.

Facilitator1: So, it's something mathematical again. I mean, in order to properly address the content of mathematics.

Similarly, the following excerpt from the fifth session shows the facilitator's clarification of a participant's comment to make it clear with no misunderstanding and by underlining the main idea of the comment.

İsmail: We prepare our preliminary questions. Once the topic progresses, we discuss among ourselves which question papers to distribute and which questions to choose. We've included them in those processes.

Facilitator1: Well, then they are getting into a mathematical discussion.

İsmail: So, they didn't speak up. They didn't speak up, but when they saw the conversation among us, they were able to figure out how to prepare questions and what to do.

Clarifying moves also occurred during the MMPD when the facilitators read and commented on what the participants wrote on the Nearpod activities. In such cases, it sometimes occurred together with connecting ideas and offering an explanation moves. For instance, in the following excerpt from the second session, the facilitator's comment on the answers of the participants related to the interactions between mentor teachers and preservice teachers includes the three moves. Here, the facilitator connected the ideas of the participants by clarifying the common meaning in their comments and including her interpretation about it. Therefore, connecting ideas, clarifying, and offering an explanation moves occurred at the same time.

Facilitator1: So, are our interactions always on practices? Am I understanding correctly? There is an action. It's not like we're sitting down and having a chat. We

could do that too... After that, it's like we're talking about something related to teaching.

Similarly, the following example from the fourth session demonstrates how the facilitator connected the participants' answers to a question which asks how they would continue to the given case by suggesting methods to the preservice teacher to improve herself in teaching the cosine theorem. The facilitator connected the ideas by restating what is in common the answers and asking for clarification. Therefore, this excerpt includes connecting ideas and clarifying moves at the same time, but there is no offering an explanation here because the facilitator did not include her interpretation about the answers, only restated them by focusing on the common feature in them.

Facilitator1: Exactly, you said they could look at proof techniques. They do first themselves. Sibel teacher suggested that they work on the proof together first, and then they can identify areas where students might struggle and work through them. So, the teacher and the preservice teacher work together, right? That's what I've got from you now. From Sevim teacher, too.

Clarifying is the second most frequently used move during the MMPD from the sustaining an inquiry stance category. By restating and revoicing participants' comments, the facilitators revealed and emphasized the main idea in their comments and ensured the common understanding for all without any misunderstanding. It sometimes occurred with offering an explanation and connecting moves indicating that facilitators used clarifying move also to ensure whether their interpretations about participant ideas reflected what they meant to say or whether their connection between the participant ideas included the essence of all ideas.

Other than offering an explanation and clarifying, the remaining three facilitation moves in this category are countering, pressing, and lifting up. These moves did not occur as frequent as offering an explanation and clarifying moves during the sessions of the MMPD. Countering moves were identified as the facilitators' comments which suggests an alternative point of view to a participant idea. For instance, the following excerpt from the first session shows an example of countering. After a participant stated that to improve

preservice teachers' content knowledge, mentor teachers could say what to pay attention to in a mathematics topic as they taught it about hundred times, the facilitator countered the participant's idea by pointing out that even experienced teachers may have shortcomings in some areas.

Facilitator1: I think we may also have shortcomings sometimes. Perhaps we have taught that topic hundreds of times, as Ahmet teacher mentioned. Perhaps it will be a topic we are teaching for the first time, or perhaps we have not addressed it before, or perhaps the curriculum has changed. We may also have shortcomings sometimes, or we may have forgotten something.

Another example of countering from the second session below demonstrates the facilitator's alternative idea to a participant's idea regarding how to prevent the mistakes that preservice teachers could make in their teaching practices. The participant states that giving opportunity to solve a few questions before teaching a whole lesson may prevent the mistakes that they could do. The facilitator drew attention to that preservice teachers could make the same mistakes even when solving just two questions. In this statement of the facilitator, there is also a clarifying move. Before countered the participant's idea, the facilitator clarified by restating the participant's idea.

Facilitator1: So, I understand that you are managing this like that. So, your goal is to avoid intervention, so you let them solve two questions before giving a full lesson, but you can experience the same thing in those two questions as you did in the lesson plan. I mean, they might make mistakes while solving those two questions, right?

Pressing move occurred when the facilitators asked participants to explain their ideas, to further their reasoning and for more details about their comments. The following excerpts from the third sessions are examples of pressing questions that the facilitator asked participants to elaborate their comments and ideas.

Remember we were talking about mathematics knowledge in the first weeks. At that time, we said that it's not really about science and literature mathematics knowledge,

but rather that the nature of knowing mathematics for teaching is different. Did you need to give feedback to preservice teachers on mathematics as well?

Well, you have just mentioned content knowledge. If you were to give such feedback to a prospective teacher, could you give us an example or experience? Is there such a thing? What would it be like? What kind of feedback would you give on this issue?

Now, İsmail teacher, you mentioned preparation before class. How do you approach in that?

The following examples from the fifth session are pressing questions to make the participants think further about how to guide preservice teachers related to the issues they expressed. In the first example, the participant expressed her idea about how preservice teachers should start teaching the topic of interpreting parabolas. In the second example, the participants stated that preservice teachers should solve different types of questions in teaching a topic. By asking the pressing questions, the facilitator prompted the participants to think further what they can do as mentor teachers to guide preservice teachers about these issues.

Melek: I want this to be the basic starting point. I want them to start here.

Facilitator1: What shall we do about this? So, should we say to the prospective teacher, for example, “Deniz, come here, Deniz. I’m explaining this topic this way. You should explain it this way and that way too”? What should we say?

Ahmet: So, the questions need to be diversified. It wouldn’t be correct to solve questions based on just one thing. Just the vertex...

Facilitator1: But would we say that it won’t be correct? What should we say to him? How can we explain?

The last facilitation move in the sustaining an inquiry stance category is lifting up. It occurred when the facilitators emphasized and elevated an important idea or issue that a participant raised in the discussion for further discussion. For instance, the following excerpt from the second session shows an example of lifting up. Here, the facilitator lifted up a

participant's idea regarding whether mentor teachers should intervene or not when there is a problem during preservice teachers teaching practices.

İsmail: ... Unless there's a health issue, the mentor teacher absolutely shouldn't interfere with the intern teacher's instruction until the bell rings. The reason is, they need to find a way out of the situation themselves. They'd lose classroom management, that is, if we were to say a word there. That's my opinion.

Facilitator1: I think Sevim teacher said. In a way that the students wouldn't see. Is the whole point here perhaps to avoid interfering in front of the students?

In the following example from the fourth session, the facilitator lifted up the three participants' ideas they wrote at the beginning of the session in answering a question which asks for their views regarding what preservice teachers should do during the process of learning to teach mathematics. Later, during a case discussion, while presenting her comment about the case, the facilitator elevated two important ideas raised by the participants. In this excerpt, there are also other facilitation moves. For example, while lifting up the idea related to mathematics knowledge, the facilitator connected the two participants ideas by focusing the common point in them. Also, there is offering an explanation here as the facilitator proposed her interpretations about the case. Lastly, there is distributing participation move by asking participants what else attracted their attention in the case.

Facilitator1: We often encounter this: They only studied the problems they were going to solve. I mean, not thinking how to involve students. For example, I think Zehra teacher wrote about it in the previous question. She wrote something about students. For example, Armağan teacher caught it here. She said, "Look, you haven't considered the student aspect at all", and actually guided her. Maybe there are things we missed. Did you notice anything else? I thought the preservice teacher didn't know the proof of the cosine theorem very well. First, she studied on it herself. You mentioned this in the previous question, too. I think both Ahmet teacher and İsmail teacher wrote about it. I mean, related to language in mathematics, about mathematical knowledge...

Another lifting up example from the fifth session is given below. Here, the facilitator lifted up a question a participant asked while discussing about how to guide preservice

teachers in preparing an activity or questions about the topic of parabola. The participant asked when introducing this topic, whether they should ask preservice teachers to focus on the graph first or on the concept of second-degree functions first.

Facilitator2: What fundamental mathematical ideas are there? We can do when identifying these that “together look what we’ll focus on...” About the order of these, I think Zeynep teacher asked a moment ago. Which one should I give first? Although sometimes, giving which one first depends on the situation, there’s no single correct answer; but for example, after defining, they sometimes say in the literature that since graphs are a more concrete way to interpret the characteristics of functions, it can be start there after defining the function.

The following excerpt from the seventh session demonstrates the lifting up of a participant comment stating a situation that the participant realized after he had given feedback to preservice teachers. The participant stated that preservice teachers got demoralized and the relationship between them became like a student-teacher relationship instead of a mentor-mentee relationship after his feedbacks. The facilitator elevated the comment of the participant by asking other participants if they had experienced a similar situation.

Facilitator1: Has anyone experienced a similar situation? I have an idea, but I won’t say it right away. This is actually a very important question, İsmail teacher. So, has anyone experienced something similar? And let me tell you, the prospective teachers were not like this ten years ago. So, I inevitably notice a generation gap there, too.

Overall, during the MMPD, by using the five facilitation moves under the sustaining an inquiry stance category in various ways, when necessary, the facilitators ensured to maintain productive and meaningful discussions toward the goals and main ideas of the MMPD. By using these facilitation moves, they also modeled the participants how to approach and engage with the discussion questions and cases from an inquiry point of view. Among the five facilitation moves under this category, offering an explanation was used the most. The second most frequently used one is clarifying and the remaining three moves,

countering, pressing, and lifting up, occurred substantively less frequent during the MMPD than offering an explanation and clarifying moves.

5.3.2. Maintaining Focus on the Goals of the MMPD

The facilitation moves in this category are highlighting, redirecting, and connecting ideas. These moves were used to guide the participants toward the goals and main ideas of the MMPD and ensure that the focus of discussions occurred on those goals and main ideas. The most frequently used facilitation move from this category is highlighting, which is also the second most frequently used facilitation move among all the moves. The least frequently used facilitation move from this category, and also among all the moves, is redirecting. Highlighting occurred during the MMPD when facilitators directed attention to noteworthy mentoring practices and significant concepts and ideas associated with preservice teacher learning. For instance, the following excerpt from the first session shows an example of how a highlighting move occurred in the MMPD. Here, the facilitator highlighted which issues mentor teachers should pay attention to when guiding preservice teachers to improve their content knowledge. There are also lifting up and connecting ideas moves in this excerpt. The facilitator lifted up two participant ideas that were raised in the previous discussions when emphasizing the importance of conceptual understanding and use of multiple representations. In addition, the facilitator also connected these ideas expressed by the participants in a meaningful way to attract attention to what to consider in content knowledge.

Facilitator1: We can say that a teacher fundamentally has two types of knowledge: One is content knowledge and the other is pedagogical content knowledge. Therefore, when designing field-specific instruction, a teacher must first understand the mathematics in it. I think Melek teacher also said, they shouldn't make such conceptual mistakes, they need to understand it conceptually, connect it to other subjects, and I think someone else also mentioned a moment ago, saying, "I would graph these lines". They also need to be able to use multiple representations. In other words, prospective teachers may know mathematics, but when they need to know it for teaching, we need to pay attention to these three aspects: understanding it conceptually, being able to connect it to other subjects, and being able to use different representations.

Similarly, the following excerpt from the second session demonstrates how the facilitator highlighted what mentor teachers should do to improve preservice teachers' understanding of mathematics concepts before starting teaching. In general, highlighting occurred, like in the example above and this example, when the facilitators directly stated what mentor teachers should pay attention to, take into account, or do to contribute to the learning of preservice teachers.

Facilitator2: ... As mentors, the thing we should do, they need to focus on the concept and analyze it thoroughly. One of the most important things for prospective teachers here is that they are often limited by textbooks. However, there are many resources available to help them understand these concepts, analyze them in detail, and make sense of them. As mentors, we can support them in that regard. In other words, they will analyze the concept thoroughly and then develop a plan. If we provide this kind of guidance before class, they can do things more confidently and feel more secure.

The following excerpt from the third session also shows an example of highlighting. This excerpt is from the closing part of the session. At the end of the session, the facilitator highlighted significant and effective mentoring practices to guide preservice teachers in learning how to teach mathematics. The facilitator also connected the ideas and issues talked during the session by highlighting what mentor teachers need to consider while studying with preservice teachers. In general, highlighting occurred during the MMPD, like in this example, when facilitators summarized and connected the ideas and issues at the end of the sessions or at the end of a discussion before starting a new discussion.

Facilitator1: ... We don't intervene, even if they make a mistake, whenever possible. Unless there is a threat to the safety of the students, or unless they say, "I couldn't turn on the smartboard, help me", we actually watch them from behind and as a teacher, we expect them to take responsibility and test their teaching skills. So, if we don't intervene, what are we going to do? We are going to make them reflect. ... Isn't it actually looking back at their own practices, evaluating them, and thinking about them? ... They may not be accustomed to or know which practices to think about. It's perfectly normal for them not to know. ... Then we will guide them with feedback. ... There was a gap in what we were providing feedback on. And that was related to

pedagogical content knowledge. How can I teach mathematics? Prospective teachers should constantly ask themselves this. ... So, when giving feedback to a prospective teacher, we need to consider both of these dimensions. It's not just about classroom management or tone of voice, but also how they reflect on mathematics on the smart board, whether they use mathematical definitions correctly, and whether they integrate students' thinking into the lesson so they can learn to think that way. Let this become a habit for them.

Highlighting is the most frequently occurred facilitation move to maintain the focus on the goals of the MMPD. The facilitators highlighted noteworthy effective mentoring practices and concepts and ideas regarding preservice teacher learning. In general, after the participants discussed their ideas and views regarding a question or a case before starting a new discussion or at the end of the sessions while summarizing and connecting the ideas, the facilitators highlighted what mentor teachers need to pay attention and should do while studying with preservice teachers. The issues that the facilitators highlighted the most during the MMPD are that mentor teachers should not intervene in the teaching of preservice teachers, instead they should ask them questions that will prompt reflection and provide them feedback to support their development, they should focus on the points that will improve preservice teachers' content knowledge and pedagogical content knowledge when giving feedback, they should also help them determine the goal and main idea of the lesson when preparing a lesson plan, and they should support them in noticing, considering, and understanding student conceptions.

The next facilitation move under this category is redirecting. Redirecting is the least frequently used move during the MMPD. It occurred when the participants stated ideas or comments outside of the discussion topics or diverged from the main ideas and goals of the MMPD. For example, the following excerpt from the first session is an example of redirecting move which occurred when a participant diverged from the discussion topic of how to detect preservice teachers' deficiencies in content knowledge and what mentor teachers can do to improve their content knowledge. The participant diverged from the topic by stating that he did not find it right to measure preservice teachers' content knowledge in high school level, and the mistakes they can do in the classroom should not be seen as a lack of content knowledge by stating that he encountered a student, even themselves know the

answer, asked a preservice teacher whether there are other types of geometries by leading to leave the preservice teacher in a difficult situation. After the comment of the participant, the facilitator redirected the focus on the importance of having content knowledge for teaching by offering an explanation how it needed in teaching.

Facilitator1: So, actually, what we are looking at here is knowing mathematics like a teacher rather than advanced mathematical knowledge. So, when I know something like a teacher, I go down to its capillaries, like to make sense of it, to relate the concepts.

Another example of redirecting from the sixth session is given below. During a small group discussion related to the feedbacks that the participants gave preservice teachers to improve their pedagogical content knowledge and promote their learning of how to teach mathematics, the facilitator redirected the participants when their examples deviated from pedagogical content knowledge to general pedagogical knowledge.

Facilitator1: So, since we're focusing on pedagogical content knowledge, we can't write everything as you said. Okay, we've mainly written about these, right?

Similarly, during a small group discussion in the eighth session, the facilitator redirected the participants as in the following example. This small group discussion was about the methods to guide preservice teachers to notice students' conceptions. When the participants started to talk about a different internship program which is applied in the school one of the participants works, the facilitator redirected them toward the discussion topic by telling them to develop a method by considering the regular internship program of the National Ministry of Education.

Facilitator1: ... Now, actually, the majority of the group is subject to the internship implemented by the Ministry of National Education. Perhaps we should develop an application based on this existing internship. So...

The last facilitation move in this category is connecting ideas. During the MMPD, connecting ideas move occurred in three ways in general. The first one occurred when the

facilitators linked a participant's idea with another participant's idea. The second one occurred when the facilitators connected the ideas in the participants' responses to Nearpod activities. The last one occurred when the facilitators summarized the ideas talked in a discussion or during the session in general. The following excerpts from different sessions are examples of the first way of the use of connecting ideas move. In these examples, the facilitator makes a connection between the ideas of two participants.

So, isn't that like what Melek teacher said? It seems like studying those conceptual things was more beneficial for her.

I think that's why Sevim teacher made such a suggestion. I saw a connection between what you said and what Sevim teacher wrote.

Actually, it's like Salih teacher's suggestion.

This actually sounds similar to what İsmail teacher wrote, doesn't it? He said he would give feedback during pre-lesson preparation. Just like you said.

Do you know what this could be? Zehra teacher wrote something about flexibility on the board. This flexibility skill is required in math.

I think Melek teacher also wrote what you said, right, Melek teacher?

The following excerpts demonstrate the use of connecting ideas move in the second way. The facilitator summarized and connected the ideas in the participants' responses to the Nearpod activities by focusing on and interpreting their commonalities and emphasizing their main ideas related to the goals of the MMPD.

Yes, we talked about the sandwich method. Sibel teacher made a correct point. We sometimes provide feedback by asking questions, right? We let them find it. Perhaps by reflecting. Exactly. Zeynep teacher said the same thing. We make sure they realize it. Ahmet teacher said something similar.

When I look at, everyone comes up with something related to mathematics. Some of us focus on mathematical knowledge. Some of us focus on mathematics being a universal language, for example, but we always talk about mathematics. Lesson planning, resource review, and so on are actually all related to mathematical content knowledge and pedagogical content knowledge.

It's not about knowing the exact learning outcomes, but through an application you're trying to help them develop a skill. ... That's what I understand from what you've written. It's actually a very valuable skill they can transfer to later. It's not knowledge what you're focusing on. Exactly. Sevim teacher also mentioned the horizontal curriculum. Ahmet teacher also said that. We all seem to agree.

The following excerpts are examples of the use of connecting ideas move in the third way. The facilitator connected the ideas talked in a discussion or during the session by clarifying the main idea of the discussion and emphasizing the ideas related to the goals of the MMPD.

So, what I understand from all of you is that we're having Merve talk a little math. That's how I understand it. I mean, the definitions in mathematics, and then that coincident lines, I mean, it seems like we haven't said much of a pedagogical thing here, right? It's not like it can be explained this way or that way, but we talked about the mathematics a little bit.

Now, if we put all these together, this person in front of us, a prospective teacher, is going through a learning process. When we give feedback, it's as if we can no longer separate feedback and reflection. In other words, they go hand in hand. Part of our feedback is having them reflect. We do this through conversation, but what about the things we reflect on? I feel like we've talked about that a bit, too.

Let me summarize everything you said, everything all the groups said. Actually, it's not like directly just go and teach a lesson from the beginning to the end in front of the class, but rather, like come and do something with this group over there while I am solving problems. That's how I understand from you now. Let's prepare a quiz before

going to the front of the class, Melek teacher, for example, said it today. Come, prepare a quiz and see the question types. After that, let's go from easy to difficult, or as İsmail teacher said, for example, when I choose those questions, on what basis do I choose them?

The facilitation moves in this category; highlighting, redirecting, and connecting ideas, were used during the MMPD when necessary to sustain the focus of the discussions on the goals of the MMPD. Among these moves, the most frequently used during the MMPD is highlighting, which is also the second most frequently used move among all the facilitation moves. By highlighting the practices, ideas, and concepts that participants need to consider in guiding preservice teachers, the facilitators helped participants understand better how they can contribute to the learning of preservice teachers. By connecting the ideas, the facilitators identified the common points in the participants' ideas and pointed out the main idea of them related to the goals of the MMPD. Among the moves in this group and all the facilitation moves, the least frequently occurred is redirecting, indicating that the participants' ideas and comments did not diverge from the topic of the discussions in general.

5.3.3. Supporting Group Collaboration

The last two facilitation moves are distributing participation and validating participant ideas. These two moves are important to support the participation of all participants and encourage the participants to express their ideas freely. During the MMPD, both distributing participation and validating participant ideas moves were used frequently to create an interactive discussion environment. Among all the facilitation moves, distributing participation is the third most frequently occurred move during the MMPD. Since the MMPD was carried out online, this facilitation move also helped the discussions proceed smoothly. The following excerpts from different sessions are examples of how the facilitators distributed the participation to sustain efficient and fluent discussions with the contributions of different participants.

Does anyone else have any opinions on this? Or experience?

For example, Salih teacher shared something. If you'd like to share it again in this context, I think it was about the topics he didn't know.

Zehra teacher, let me give the floor to you. Then to Salih teacher, and then Sevim teacher.

Anything else?

Melek teacher, were you going to say something?

Is there anything else you'd like to add? Shall I move on to the next group?

Zehra teacher, Sibel teacher, what have you discussed in your group?

During the MMPD, the facilitators validated participants' contributions by giving positive feedback and confirming their comments. The following examples from different sessions demonstrate how the facilitators supported their contributions.

Sevim teacher summarized it very well, didn't she? We give feedback on that pedagogical approach in the classroom.

Sibel teacher's closing remark is very nice, as you may have noticed.

Ahmet teacher shared a very good example.

I agree with you. We shouldn't say, "Look, this is how I do it, do exactly what I do".

Sibel teacher, your answer is very interesting.

As Zehra teacher said, perhaps it would be enough for the prospective teacher to simply be aware of this.

Facilitation is a crucial element for the effectiveness of a PD. Therefore, examining the effective facilitation moves that contribute to sustaining productive and meaningful discussions on the goals of the PD and supporting group collaboration is quite important. The MMPD was analyzed by using a revised version of the effective facilitation moves framework developed by van Es et al. (2014). The analysis indicated that the facilitators used all the facilitation moves when necessary to ensure that the discussions were carried out effectively and efficiently by directing the participants toward the goals and objectives of the MMPD.



6. DISCUSSION

This study examined the online Mathematics Mentor Teachers Professional Development (MMPD) program in terms of content, interaction and collaboration, and facilitation. In this section, the findings regarding how content, interaction and collaboration, and facilitation occurred during the implementation of the MMPD are discussed in two parts considering the dimensions of mentor teacher learning and online professional development. In the first part, the findings are discussed to present an understanding of how the MMPD contributed to the mentor teachers' learning. In the second part, the findings are discussed in terms of how effective the MMPD as an online professional development (OPD) program.

6.1. Mentor Teacher Learning

The MMPD is designed to improve mentoring practices and conceptions of high school mathematics teachers to enable them to contribute effectively to preservice teachers' learning to teach mathematics. There are various aspects that preservice teachers need to develop from cognitive to affective and motivational to become competent teachers who can teach effectively for student learning. Several types of knowledge have been identified (e.g., Shulman, 1986, 1987; Ball et al., 2008) that teachers need to be mastered to accomplish the task of teaching in the best possible way to promote student learning. Within the teacher knowledge base, the pedagogical content knowledge (PCK) stands out as a special knowledge for teaching that preservice teachers need to be supported to develop regarding how to design teaching to represent and transfer the content knowledge in a manner that students can comprehend. Ball et al. (2008) proposed a model elaborating on the PCK for mathematics teaching by identifying what knowledge and skills that mathematics teachers should have.

The examination of the MMPD indicated that the mentor teachers as experienced teachers have strong PCK on mathematics as seen in their expressions including their views about teaching mathematics and what preservice teachers should do when teaching a mathematics topic. However, what is missing in their explanations for the most part and showing what they need to improve as mentor teachers is that what their role is as mentors

to develop preservice teachers' PCK and how they support them in learning to teach mathematics. In general, their comments about how they gave feedback to preservice teachers and how they guided them in preparing lesson plans were focused on what they told them to revise not to have a problem during their teaching practices in the process of internship and did not include a reasoning of why they guided preservice teachers in the way they did. In other words, with their competence in PCK, they could identify what preservice teachers need to do, but they have difficulty in unpacking their PCK while guiding them.

Mentor teachers should do more than provide emotional support or give advice to promote the learning of preservice teachers; they should be able to express their reasoning in their instructional decisions to help them understand not only what of teaching but also how and why of teaching (Stanulis et al., 2019). Mentor teachers need to see themselves as teacher educator to be able to guide them effectively in learning to teach. Feiman-Nemser (1998, 2001b) proposed the term “educative mentoring” to differentiate this kind of mentoring, which focuses on the professional growth of preservice teachers from more conventional approaches that underline supervisory roles of mentors. During the MMPD, the facilitators' moves highlighting the roles of mentor teachers as teacher educators who are responsible for helping preservice teachers learn to teach and prompting mentor teachers to think about how they can guide preservice teachers so that they can contribute to the development of their PCK were observed to be effective in developing mentor teachers' conceptions of mentoring throughout the sessions.

In mentoring to preservice teachers, mentor teachers need a different kind of knowledge or skill to be able to reflect their PCK to preservice teachers to promote their development in learning to teach. Being a good teacher does not make someone automatically a good mentor (Schwille, 2008). Mentor teachers need to build a bridge between their teacher identities and roles as mentors to transform their knowledge of mathematics teaching in a way that they can transfer that knowledge to preservice teachers. Similarly, the need for learning such a different skill or knowledge in mentoring was stated by Orland-Barak (2001) in the context of English mentor teachers by resembling the learning of mentor teachers as “learning a second language of teaching” to emphasize the transition from a teacher role to a mentor teacher role requires redefining the understanding of teaching in a new context.

Across the sessions of the MMPD, it is observed that the mentor teachers developed an awareness about the importance of guiding preservice teachers to help them develop their PCK. The facilitators' directions during the sessions emphasizing that their role as mentors is to help preservice teachers to learn how to teach mathematics and that knowing mathematics for teaching is different than the knowledge of mathematics were substantially effective in contributing to the mentor teachers' understanding of this. Across the sessions, the mentor teachers started to focus more on the issues and practices related to the PCK like the importance of noticing students' conceptions and misconceptions during teaching, determining the main idea of a mathematical topic when to teach it, and using multiple representations to support students' learning.

An improvement also occurred in the mentor teachers' understanding of the effective mentoring practices that contribute to the development of preservice teachers. Across the sessions, the mentor teachers started to include the ideas and practices that were emphasized during the sessions in their comments and responses. For instance, the importance of giving opportunities for reflection and providing constructive feedback to preservice teachers, working together with preservice teachers in lesson planning, sharing their experiences regarding what difficulties they encountered and about the students' conceptions and misconceptions, and approaching preservice teachers like a colleague from their department were some of the issues highlighted during the MMPD. The facilitators' moves to highlight, explain, clarify, and connect these important ideas throughout the sessions helped the mentor teachers adopt these ideas as seen in their responses and comments developed across the sessions.

These mentoring practices frequently mentioned and highlighted by both the mentor teachers and the facilitators during the MMPD show similarities with the characteristics identified by Glenn (2006) through the observation of two mentor teachers who are found effective by their student teachers and through the interviews with these mentor teachers and student teachers pairs. Accordingly, the characteristics of effective mentor teachers were identified as collaborating rather than dictating, relinquishing an appropriate level of control, allowing for personal relationships, sharing constructive feedback, and accepting differences (Glenn, 2006). For instance, the mentor teachers in the MMPD stated how they approach preservice teachers like a colleague from their school by including them in the discussions

among themselves in their departments and asking their ideas or suggestions regarding planning for teaching or preparing exam questions. This practice overlaps with the first characteristic identified by Glenn (2006). Providing constructive feedback was also one of the mostly emphasized issues during the MMPD by both the mentor teachers and the facilitators. Other than that, regarding the relinquishing the control of the classroom, giving authority to preservice teachers and not intervening during their teaching practices unless there is an emergent situation were also substantially discussed during the MMPD.

Co-planning and co-teaching are also effective mentoring practices that contribute to preservice teachers' learning (Feiman-Nemser, 1998; Schwille, 2008; Stanulis et al., 2019). During the MMPD, the importance of co-planning was discussed, however co-teaching was not mentioned much. The mentor teachers and the facilitators expressed views regarding working together with preservice teachers in lesson planning, however views or ideas regarding co-teaching were not observed much during the MMPD. One of the mostly emphasized mentoring practices during the MMPD was reflection. The views articulating the importance of guiding and modeling preservice teachers to make continuous reflection on their teaching practices as well as those of their mentors were expressed by the mentor teachers and highlighted by the facilitators frequently. Talking with preservice teachers before and after their teaching practices and after they observe their mentor teachers' lessons, and asking questions that lead them to reflect on their lesson plans, their teaching practices, or their mentor teachers' lesson were some of the views stated by the mentor teachers during the MMPD. Schwille (2008) also stated the importance of implementation of similar practices by highlighting purposeful demonstration of teaching by making preservice teachers consciously observe by taking notes and making reflection as well as making debriefing sessions to guide preservice teachers to reflect on the areas they need to improve.

Another issue that frequently came up during the MMPD was how to guide preservice teachers to notice students' conceptions and misconceptions. The mentor teachers in the MMPD stated based on their observations of preservice teachers' teaching practices that preservice teachers have difficulty in noticing student thinking as they focus more on themselves and what to do according to their lesson plans. A variety of methods that mentor teachers can use to help preservice teachers notice student thinking and their conceptions and misconceptions discussed during the MMPD. For instance, talking to preservice teachers

the misconceptions that mentor teachers encountered, asking preservice teachers to guess how students answer to a question that may reveal some misconceptions, taking notes of student thinking while observing mentor teachers' lessons, making preservice teachers study with a small group of students, or talking with them about how they would address the students' answers or questions after preservice teacher' practice teaching were some of the methods discussed during the MMPD. Feiman-Nemser (1998) also stated based on a cross-cultural study that mentor teachers who see themselves as teacher educators give importance in student thinking and in helping preservice teachers to elicit and analyze student thinking to guide their planning and instruction accordingly.

The analysis of the MMPD from content, interaction and collaboration, and facilitation showed that all the three dimensions were important factors that contribute to the improvement in the mentor teachers' understanding of effective mentoring practices that are conducive to the development of preservice teachers' PCK and learning to teach during the MMPD. The cases presented for discussion were an effective tool that stimulated the discussions on how practices of mentor teachers can affect preservice teachers' learning and what practices mentor teachers should do to contribute preservice teachers' learning to teach mathematics. The cases also became helpful to remind the mentor teachers their similar experiences with preservice teachers and lead them to think on them so that they contribute to the mentor teachers' reflection on their practices. The facilitators' moves highlighting, explaining, clarifying, and connecting important ideas and views were also highly effective by guiding the mentor teachers what to focus on to promote preservice teachers' learning to teach. The opportunity to interact and collaborate with each other and with the facilitators throughout the MMPD contributed to the formation of a professional learning community in which they discuss and learn from their own and each other's experiences as mentors. These opportunities to interact and collaborate with each other and with the facilitators to discuss, analyze, and reflect on their practices were important experiences that facilitate the learning of mentor teachers (Feiman-Nemser, 1998; Stanulis and Ames, 2009; Stanulis et al., 2019).

In addition, making connection between mentor teachers' knowledge of student teaching and learning and the effective mentoring practices and ideas for preservice teacher learning is another important factor that contribute to the learning of mentor teachers (Stanulis and Ames, 2009; Kasten et al., 2024). During the MMPD, it is observed that the

facilitators helped the mentor teachers make such a connection time to time. For instance, the facilitators highlighted the importance of asking questions to guide preservice teachers to make reflection effectively by connecting it the practice of asking questions to students to guide their learning. The mentor teachers also make connection with their teaching practices and mentoring practices sometimes. For example, a mentor teacher stated that just as guiding students to discover something on their own contributes to their learning, making reflection can contribute to preservice teachers' learning in a similar way by enabling them to discover the areas they need to improve themselves. Furthermore, integrating research-based knowledge and practice of mentoring is also a significant factor for the development of mentor teachers (Tang and Choi, 2005). Since the MMPD was conducted during the first semester of the school year of 2023 while the mentor teachers were working with intern teachers, they had the opportunity to apply the practices and ideas discussed during the sessions with their intern teachers, and during this process, they brought up and discussed their experiences as mentors with each other and with the facilitators during the sessions. This cooccurrence of their mentoring and the MMPD might have helped them make connection between the ideas presented in the MMPD and what they learned from their experiences as mentors.

The analysis of the discussions and conversations occurred in the MMPD indicated that the participated mentor teachers have strong CK and PCK as experienced mathematics teachers. However, it is observed that they had difficulty in articulating the reasoning of their actions as mentors to promote the development of preservice teachers' CK and PCK. To effectively mentoring preservice teachers, they need different kind of knowledge and skills that make a bridge between their role as mentors and their knowledge and skills as effective mathematics teachers. At the beginning of the MMPD, the mentor teachers were not much aware of their role as teacher educators who are responsible for continuous professional growth of preservice teachers. Throughout the MMPD, they developed an awareness about the importance of promoting preservice teachers' learning to teach and showed improvement in understanding of effective mentoring practices to support the development of their CK and PCK. The contextually rich case discussions and the facilitators' moves that guide and promote productive discussions were observed as the most important factors for the learning of the mentor teachers throughout the MMPD.

6.2. The Online Professional Development Dimension

Online professional development programs provide convenience for teachers by allowing them to access professional development opportunities at more flexible times and from anywhere during their busy work schedule. Comparison of face-to-face and online professional development programs showed that both types of professional development have equivalently positive outcomes on teachers (Russell et al., 2009; Fishman et al., 2013). Considering the convenience that they provide to teachers and their potential to be as effective as face-to-face professional development programs, examining the factors that make online professional development programs effective is significant for designing and scaling up such OPD opportunities for teachers.

The MMPD as an online professional development program was examined in terms of interaction and collaboration which are considered as two of the important qualities of an effective OPD (Elliott, 2017). During the MMPD, how collaboration and four types of interaction as participant-participant, participant-facilitator, participant-content, and facilitator-content occurred were examined to see whether these features could be assured to contribute to the active engagement of the participants and the creation of a positive and supportive learning community among the participants so that effective and productive discussions could be made.

The findings indicated that participant-content and facilitator-content interactions could be maintained effectively during the sessions with the use of interactive activities and presentations via the Nearpod and through the cases presented for discussions. Sometimes, there were participants who did not write an answer to the interactive Nearpod activities. However, in general most of the participants wrote their answers, and the discussions were occurred through the contributions of different participants. During the discussions, facilitator-participant interactions were much more than the participant-participant interactions. Participant-participant interactions occurred more efficiently during the small group discussions. In this regard, the MMPD was limited to offer opportunities for participant-participant interaction. Similarly, collaboration was limitedly occurred during the MMPD when the participants worked on a case or discussion questions in small groups.

The cases provided for discussions were effective for both participant-content and facilitator-content interactions. The cases stimulated active engagement of the participants by providing a concrete basis for the discussions through the illustration of situations that the mentor teachers may have experienced before or may encounter in their practices with preservice teachers. In this regard, it can be said that the quality of the cases provided for discussions was an influential factor for the participant-content interaction. Another study of a synchronous OPD for mathematics teachers also showed that the quality and nature of the mathematical tasks was quite effective for the quality of the interactions and professional learning in a synchronous online learning setting (Francis and Jacobsen, 2013). In the MMPD, the use of contextually rich cases fostered meaningful discussions on the effective mentoring practices and contributed to the professional development of the mentor teachers.

A study conducted by Meyer et al. (2023) indicated that effective OPD programs ensure cognitive activation of teachers, encourage collaboration, and have clear goals and well-structured activities. Especially cognitive activation and collaboration were found effective in changing teachers' professional practices. During the MMPD, cognitive activation of participants was especially promoted through case discussions that lead them to think about how effective mentor teachers act and to reflect on their practices as mentors. However, collaboration between participants was limited during the MMPD. As the facilitators realized that collaboration among participants was lacking, more small group activities were conducted toward the end of the MMPD. After the small group activity at the end of the first session, there were no further small group activities until the sixth session. There were two small group activities in the sixth session and one in the eighth session. If these small group activities had been conducted more periodically across all the sessions, it could have been more beneficial. Given the diverse learning needs and preferences of participants, facilitating collaboration in online settings is a challenging task (Teräs, 2016), and OPD programs in general are inadequate in providing opportunities for collaboration among participants (Powell and Bodur, 2019; Meyer et al., 2023).

Teachers' internal factors, like their motivations, interests or needs, affect their learning experiences and play a crucial role in their interactions with the content, facilitators, and each other (Teräs, 2016; Dille and Røkenes, 2021). In the MMPD, the effects of participants' personal differences were also observed. For instance, while some participants were more

willing to participate in the discussions, some were more reluctant. Furthermore, during the writing tasks, it was observed that some participants preferred to express their thoughts verbally instead of writing. The facilitators' moves were quite significant in adjusting these differences. The facilitators in general managed to contribution of all participants in discussions by distributing participation taking into account who participated and who did not. To ensure engagement of all participants, the facilitators asked participants to elaborate or add on what they wrote on writing tasks so that both those who did not like to write got opportunity to express their ideas in more detail and those who did not like to talk could articulate their thought based on what they wrote.

In general, the MMPD provided participants a rich experience in terms of interaction although collaboration and interaction between them were somehow limited. Examining the MMPD in terms of interaction and collaboration provided a view of how it operated from a technological and contextual perspective. However, this is not sufficient to understand the effectiveness of a PD. In fostering those interactions and collaboration and in generating and sustaining discussions on meaningful content, the roles of the PD facilitators are substantially influential. Facilitators play a critical role for the quality of a PD by affecting the learning of teachers (Tekkumru-Kisa and Stein, 2017b; Dille and Røkenes, 2021). Therefore, to get a more comprehensive view on the MMPD, facilitator moves were analyzed in addition to interaction and collaboration. The examination of the effective facilitation moves by using a revised version of the framework developed by van Es et al. (2014) indicated that the facilitators used all the facilitation moves when necessary to sustain effective and meaningful discussions on the goals of the MMPD through the contribution of different participants.

The four most commonly used facilitation moves throughout the MMPD were offering an explanation, highlighting, distributing participation, and clarifying, respectively. Offering an explanation and clarifying moves are in the sustaining an inquiry stance category, highlighting move is in the maintaining focus on the goals of the MMPD category, and distributing participation move is in the supporting group collaboration category. This shows that the facilitators' moves during the MMPD encompassed all the three categories to promote productive discussions, keep the focus on the goals, and ensure the engagement of all participants. In another study examining the facilitation moves by using van Es et al.'s

(2014) framework, the three moves used the facilitators most were found as clarifying, pressing, and offering an explanation (González et al., 2016). Tekkumru-Kisa and Stein (2017a) also found the mostly used facilitation moves as clarifying, pressing, and distributing participation. In general, the moves used by the facilitators the most show similarity. Tekkumru-Kisa and Stein (2017a) did not include offering an explanation move in their analysis, so it is not possible to make a comparison for that move including their study. Both González et al. (2016) and Tekkumru-Kisa and Stein (2017a) found the most commonly used moves as clarifying and pressing. During the MMPD, clarifying was also used by the facilitators frequently, however, pressing was not observed much compared to other moves. During the MMPD, the most frequently used move is offering an explanation. González et al. (2016) also found offering an explanation as one of the mostly used moves. Highlighting is the second most frequently used move in the MMPD differently from González et al. (2016) and Tekkumru-Kisa and Stein (2017a).

The reason why offering an explanation and highlighting moves are the most observed moves in the MMPD may be that the definitions of these moves were changed and their scope expanded according to the context of the MMPD. Originally, these moves refer to providing an interpretation of event or interaction occurred in the videos and directing attention to noteworthy ideas in the videos. In the context of the MMPD, their definitions were changed and expanded including the interpretation of any event, interaction, or idea related to cases or emerged during the discussions and emphasizing effective mentoring practices and ideas that contribute to the learning of preservice teachers.

In the MMPD, the sequence of the total frequency of the moves in the three categories from highest to lowest is sustaining an inquiry stance, maintaining focus on the goals of the MMPD, and supporting group collaboration. González et al. (2016) also found the same sequence regarding the total frequency of the moves in the categories. The use of moves in sustaining an inquiry category was essential to promote effective discussions regarding how mentor teachers should act and what practices to perform to promote preservice teachers' learning to teach mathematics. Especially offering an explanation and clarifying moves were used by the facilitators the most to ensure a common understanding of effective mentoring practices and ideas among all the mentor teachers. van Es et al. (2014) suggest that the coordination of the moves in each category is quite important to effectively facilitate the

productive discussions. Throughout the MMPD, it is observed that the facilitators used and combined several moves from each category to facilitate efficient discussions with the engagement of different participants. In this study, it is important that a framework originally developed for the facilitation of video-based discussions (van Es et al., 2014) could be adapted with slight revisions in a different context and it provided crucial insights regarding how the facilitators' actions are influential in promoting discussions and learning of the mentor teachers.

Overall, as an online professional development, the MMPD provided opportunities for the participants to actively engage with the content through the interactions with the technological tools, with the facilitators, and, albeit limitedly, with each other. The facilitators in general effectively guided and sustained the discussions to contribute the learning of the mentor teachers by offering an explanation, highlighting noteworthy mentoring practices and ideas, clarifying and connecting the ideas, pressing for more details and further reasoning, lifting up important participant ideas, countering ideas by offering an alternative perspective, redirecting the focus on the goal of the MMPD when there is a divergence from the objectives, and having them engaged by distributing participation and validating their contributions.

7. LIMITATIONS AND SUGGESTIONS

This study is based on the analysis of the video recordings of the online sessions of the MMPD. The technical problems occurred in the recording of some sessions is a limitation that affected the analysis slightly. There were missing parts in the video recordings of small group discussions. Therefore, only the video recordings of the small groups in which the primary facilitator was present were used in the analysis because only these recordings were not missing and capture the most part of the discussions. This limitation especially affected the analysis of the participant-participant interactions and collaboration between them.

In addition, since this study is only based on the mentor teachers' expressions of ideas and views during the MMPD sessions and does not include an observation of how the mentor teachers interact with preservice teachers, these inferences were based on their comments and it may not imply that they will change their practices according to what they have learned during the MMPD. Further research is needed including observations of the interactions between the mentor teachers and the preservice teachers to see whether they apply the ideas and practices highlighted during the MMPD. Further research is also needed to elaborate on the structure of the knowledge and skills that mentor teachers need to be able to guide preservice teachers effectively in learning to teach mathematics.

This study provided significant insights regarding the factors that are influential in mentor teachers' learning and for the effectiveness of the OPD programs. Taking into account of these factors in the design and implementation of other OPD programs for mentor teachers can foster the proliferation of high-quality OPD opportunities for mentor teachers.

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