

COMPACT K_a -BAND FILTER APPLICATIONS BASED ON THE MULTIPLE MODE RECTANGULAR CAVITY

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By
Ceyhun Kelleci
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MULTIPLE MODE RECTANGULAR CAVITY

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We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Abdullah Atalar(Advisor)

H. Özlem Aydın Çivi

M. Gülbin Dural Ünver

Vakur Behçet Ertürk

Hayrettin Köymen

Approved for the Graduate School of Engineering and Science:

Ezhan Karahan
Director of the Graduate School

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ABSTRACT

COMPACT *Ka*-BAND FILTER APPLICATIONS BASED ON THE MULTIPLE MODE RECTANGULAR CAVITY

Ceyhun Kelleci

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Advisor: Abdullah Atalar

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Filters based on multiple mode cavity resonator technique have the advantage of realizing a given filter function in a reduced volume and weight with the drawback of increased complexity. In order to decrease the dependence on electromagnetic analysis software and to gain a better insight on the physics of the structure, the multiple mode single rectangular cavity filter structure is investigated with an analytical approach. Expressions are obtained for the modal frequency shifts and for the intermodal coupling due to various types of corner cuts. An algorithm is proposed predicting the physical dimensions of the final structure given the corresponding coupling matrix. Example designs are realized. The algorithm is able to determine the physical dimensions of the second and third-order filters within a few percent. The classical triple mode rectangular cavity filter structure is altered to form a triplet. The new triplet structure can be arranged to result in either a lower or higher sideband transmission zero. An example *Ka*-Band design is fabricated with both machining and a novel 3D printing technology. The results are in agreement with the expectations. The filter structure is further tailored to allow integration to *Ka*-Band waveguide output microwave modules without significant increase in the module's volume requirement.

Keywords: Cavity filter, multiple mode rectangular cavity, triplet, Ka-Band, 3D printing.

ÖZET

ÇOK MODLU DİKDÖRTGEN BOŞLUK REZONATÖRÜ TABANLI KOMPAKT Ka -BANT FİLTRE UYGULAMALARI

Ceyhun Kelleci

Elektrik ve Elektronik Mühendisliği, Doktora

Tez Danışmanı: Abdullah Atalar

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Çok modlu boşluk rezonatörü yöntemi kullanılarak tasarlanan filtreler hacim ve ağırlık açısından fayda sağlarken, yapıları görece karmaşıktır. Elektromanyetik analiz yazılımlarına olan bağımlılığı azaltmak ve yapının fiziği üzerine daha fazla bilgi sahibi olabilmek amacıyla, çok modlu tek dikdörtgen boşluk rezonatöründen oluşan filtre yapısı analitik olarak incelenmiş, farklı tiplerdeki köşe kesilerinin yol açtığı modal frekans kaymaları ve modlar arası kuplajı öngören matematiksel ifadeler elde edilmiştir. Belirli bir kuplaj matrisini gerçekleyen filtrenin fiziksel boyutlarını kestiren bir algoritma oluşturulmuş, örnek tasarımlar gerçekleştirilmiştir. Önerilen algoritma ikinci ve üçüncü seviye filtrelerin fiziksel boyutlarını yakınsayabilmektedir. Klasik üç modlu dikdörtgen boşluk rezonatör filtre yapısı üçlü oluşturacak biçimde değiştirilmiştir. Oluşan yeni yapı çalışma frekansının üst veya alt kısmına sıfır iletim noktası koyacak şekilde ayarlanabilmektedir. Örnek bir Ka -Bant tasarım hem talaşlı imalat hem de 3B yazıcı tabanlı yeni bir üretim teknolojisiyle üretilmiştir. Ölçüm sonuçları beklentileri karşılamaktadır. Filtre yapısı, dalga kılavuzu çıkışlı Ka -Bant mikrodalga modüllere, modülün hacim ihtiyacını kayda değer oranda artırmadan entegre edilebilecek şekilde uyarlanmıştır.

Anahtar sözcükler: Boşluk rezonatörü filtre, çok modlu dikdörtgen boşluk rezonatörü, üçlü, Ka -Bant, 3 boyutlu yazıcı.

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Dedicated to my beloved wife Yeliz ...

Contents

1	Introduction	1
2	Filter Design using the Coupling Matrix	3
3	The Multiple Mode Rectangular Cavity Filter	9
3.1	Multiple Mode Cavity Filter	10
3.2	Intermodal Coupling	12
3.2.1	Square Corner Cut	12
3.2.2	Triangular Corner Cut	15
3.2.3	Rounded Corner Cut	16
3.3	Waveguide to Cavity Coupling with a Rectangular Aperture . . .	16
3.4	Investigation of Dispersion	19
3.5	Filter Design Algorithm	21
3.6	Example Second-Order Filter Design	21
3.7	Example Third-Order Filter Design	22
3.8	The Triplet Structure	24
3.8.1	Transmission zero at the upper sideband	25
3.8.2	Transmission zero at the lower sideband	27
3.9	Experimental Verification	29
4	Filter Embedded Microstrip to Waveguide Transitions	33
4.1	Microstrip probe excitation of waveguides and cavities	34
4.2	Intermodal coupling mechanism	37
4.3	Second-order filter embedded microstrip to waveguide transition .	39
4.4	Third-order filter embedded microstrip to waveguide transition . .	41

5	Conclusion	44
A	Matlab Codes	49
A.1	Lowpass prototype element generation	49
A.1.1	Lowpass prototype for Butterworth filtering function . . .	49
A.1.2	Lowpass prototype for Chebyshev filtering function	50
A.2	Coupling matrix synthesis	51
A.2.1	Characteristic polynomial generation function	51
A.2.2	Coupling matrix generation function	52
A.2.3	Example coupling matrix generation main script	54
A.3	Coupling matrix reconfiguration	55
A.3.1	Reconfiguration to folded form	55
A.3.2	Reconfiguration to arrow form	58
A.4	Auxiliary functions	59
A.4.1	$U(w)$ - $V(w)$ polynomials generation	59
A.4.2	Polynomial coefficient generation	61
A.4.3	Transmission zero normalization	62
A.4.4	Coupling matrix cleaning	62
A.4.5	Coupling matrix reforming	63
A.4.6	Coupling matrix analysis	64

List of Figures

2.1	Example coupling matrix circuit model.	3
2.2	Filter design procedure using the coupling matrix approach. . . .	4
2.3	Common coupling configurations. The mainline couplings are presented with solid lines, whereas cross-couplings are shown with dashed lines. (S: Source, L: Load) (a) Folded form. (b) Arrow (wheel) form.	5
2.4	The response for the example fifth order filter.	6
3.1	The dual mode rectangular cavity filter structure. (a) Cavity. (b) Cavity and waveguide.	11
3.2	The triple mode rectangular cavity filter structure. (a) Cavity. (b) Cavity and waveguide.	12
3.3	Dual mode cavity with square corner cut.	12
3.4	Normalized resonance frequencies of the two coupled modes versus r with $a/c = 5/6$. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.	14
3.5	The physical coupling coefficient as a function of normalized corner cut dimension with $a/c = 5/6$. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.	15
3.6	Dual mode cavity with triangular corner cut.	15
3.7	Dual mode cavity with rounded corner cut.	16
3.8	The equivalent circuit for the waveguide-cavity aperture coupling structure.	16

3.9	Q_e of the example aperture coupled cavity for different aperture wall thicknesses, for $h = 1$ mm. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.	18
3.10	The normalized resonant frequency of the example aperture coupled cavity for different aperture wall thicknesses, for $h = 1$ mm. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.	19
3.11	The dispersion effects due to the waveguide and due to the aperture.	20
3.12	The initial response of the designed filter together with the theoretical response.	23
3.13	S-parameters of the predicted and optimized structures together with the theoretical response.	24
3.14	The triplet coupling diagram.	25
3.15	The triple mode triplet structure with an upper sideband finite frequency TZ.	26
3.16	The response for the triple mode triplet structure with an upper sideband finite frequency TZ.	27
3.17	The triple mode triplet structure with a lower sideband finite frequency TZ.	28
3.18	The response for the triple mode triplet structure with a lower sideband finite frequency TZ.	29
3.19	Chromate conversion coated parts of the machined filter. The scale is in centimeters.	30
3.20	Additive manufactured triple mode rectangular cavity filter.	30
3.21	The response for the fabricated triple mode triplet structure with a lower sideband finite frequency TZ.	32
4.1	The coaxial probe to waveguide transition structure.	35
4.2	The microstrip probe excited cavity structure.	35
4.3	Cut-out view of the GCPW structure.	36
4.4	External quality factor (Q_e) as a function of microstrip probe length.	37
4.5	Normalized resonant frequency of the coupled mode as a function of microstrip probe length.	37

4.6	Dual mode cavity with partial square corner cuts. The shape of the cuts can be triangular or rounded as well.	38
4.7	Normalized resonance frequencies of the two coupled modes versus r with $a/c = 5/6$	38
4.8	The physical coupling coefficient as a function of normalized corner cut dimension with $a/c = 5/6$	39
4.9	The second-order microstrip probe excited cavity filter structure. .	40
4.10	The initial response of the designed second-order filter together with the optimized response.	41
4.11	The microstrip probe excited third-order cavity filter structure. . .	42
4.12	The initial response of the designed third-order filter together with the optimized response.	43

List of Tables

3.1	The physical dimensions (in mm) of the designed second-order filter.	22
3.2	The physical dimensions (in mm) of the designed third-order filter.	24
3.3	The physical dimensions (in mm) of the designed triplet with upper sideband TZ.	27
3.4	The physical dimensions (in mm) of the designed triplet with lower sideband TZ.	29
4.1	The dimensions of the GCPW for a $50\ \Omega$ characteristic impedance.	36
4.2	The physical dimensions of the designed second-order asymmetric input/output filter.	40
4.3	The physical dimensions of the designed third-order asymmetric input/output filter.	42

Chapter 1

Introduction

Microwave filters have always played an important role in both civilian and military systems and this role becomes more critical with the ever increasing need for more bandwidth and channels in the spectrum, which is a very scarce resource. On the other hand, more compact, lightweight solutions are always welcome. One such filter solution is the multiple mode cavity filter.

The idea of using the degenerate modes of a multiple mode cavity (either rectangular or cylindrical) first appears in [1, p.675]. For the rectangular cavity case, the coupling between modes is realized either by using a tuning screw or by deforming the dual symmetry slightly so that the initially degenerate modes are no more degenerate and coincide with the filter resonance frequencies. Later on, the idea of deforming the square into a rectangle is also applied to the waveguide input/output case [2]. In fact, any type of asymmetry between the degenerate modes shifts their frequencies up and down, therefore it becomes possible to realize the filter. Over the years, the idea of using square corner cut is proposed [3]. The latter approach allows filter fabrication without tuning. The multiple mode technique is also applied to triple mode, quadruple mode [4] and hexa-mode [5] cavities as well.

The main advantage of the multiple mode filter approach is that, since multiple

resonators occupy the same space, the overall volume requirement is reduced (for a given filter order). As a result, the weight of the filter is reduced as well. A dual mode filter allows a reduction of almost twofold and with increased number of modes used, more reduction is obtained. The disadvantage is the need for analysis and fabrication of a relatively more complex structure. Note that, in applications where space and weight are strict requirements, e.g., portable devices, space applications, the complexity may become acceptable.

For relatively low frequency applications, filter design tables [6, ch. 4] can be used to calculate the inductor and capacitor values that realize a predefined filter function. When the application requires implementation in higher frequencies, especially where lumped elements can not be used anymore, the filter structure needs to be analyzed and modeled (if possible) carefully. With today's powerful computational tools, it is also possible to design microwave filters using specialized electromagnetic analysis software as well.

The aim of this work is to investigate the multiple mode rectangular cavity waveguide filter structure analytically. Such an effort would help to understand the physics of the structure better and would decrease design time and design dependence on computational tools as well. For that purpose, the direct relation between the coupling matrix entries and the physical parameters of the filter is sought for.

In Chapter 2, a brief introduction to filter design using the coupling matrix approach is provided. Chapter 3 contains analytical efforts to define the relation between a single rectangular cavity multiple mode filter and its coupling matrix entries. The classical triple mode cavity is altered to form a triplet, that in turn allows the addition of a transmission zero (TZ) to the response. Using the obtained analytical expressions, different designs are realized and an example design is fabricated with both machining and additive manufacturing [7]. In Chapter 4, the proposed filter structure is altered to be integrated in *Ka*-Band microwave modules. The last chapter is a discussion of the work done.

Chapter 2

Filter Design using the Coupling Matrix

It would take more than a lifetime to master all the aspects of filter design art. Although the focus of this work is mostly limited to multiple mode rectangular cavity filters, a very basic and generalized filter design guideline (via coupling matrix approach) is presented here for completeness. The reader is directed to appropriate references for detailed information.

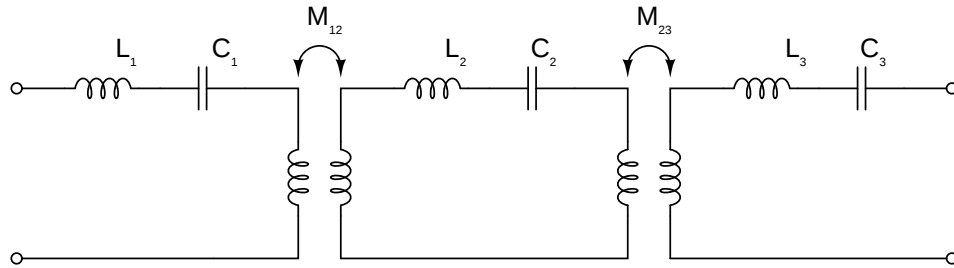


Figure 2.1: Example coupling matrix circuit model.

The coupling matrix approach is proposed by Atia and Williams in 1971 [3]. One of the major advantages of the method is its ability to relate matrix entries to some of the physical properties of the realized filter [8, p. 279]. An example coupling matrix circuit model containing three resonators and with nonzero couplings only among consecutive resonators is illustrated in Fig. 2.1. Note that, the

coupling matrix is only an intermediate step of the filter design procedure. The overall process is summarized in Fig. 2.2.

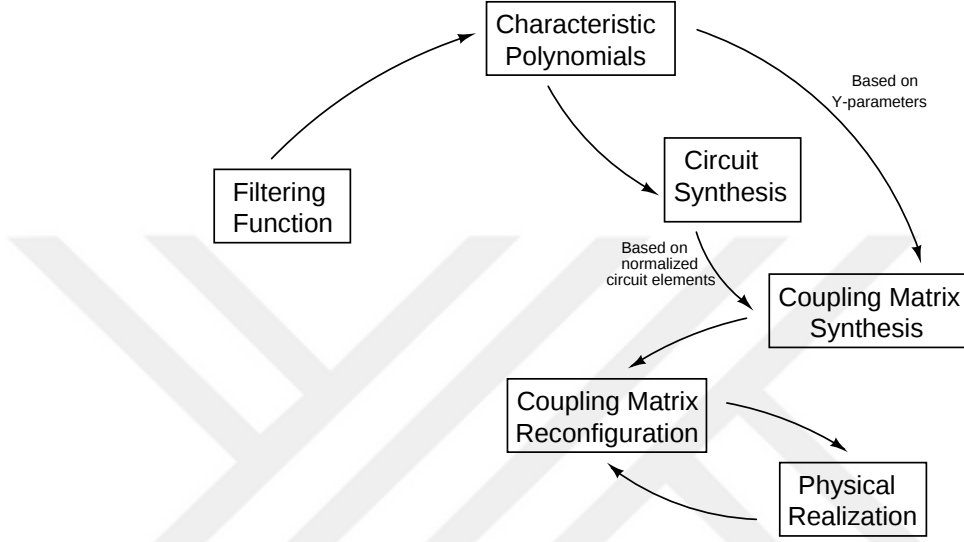


Figure 2.2: Filter design procedure using the coupling matrix approach.

The design starts in mathematical domain. First, the characteristic polynomials resulting in the required transfer characteristics are formed. For widely used functions like Chebyshev and Butterworth, the process is relatively straight forward [8, pp. 111-114], whereas for special filter functions, e.g., containing TZ's, it may become necessary to use optimization via computational tools [8, p. 151]. In [8, ch. 6], a procedure to generate the transfer and reflection polynomials for a general class of Chebyshev filter function including prescribed TZ's is proposed and Fortran implementation of some steps is also provided. For this work, the overall procedure is implemented in Matlab. The corresponding functions as well as an example coupling matrix generating script are given in Appendix A.

The next step is the determination of the relation between actual circuit elements and mathematical functions. This step, the circuit synthesis, is a very old problem that is already studied extensively. The process is generally based on parameter extraction method presented in [8, ch. 7]. For the Chebyshev and Butterworth functions of various types, tables containing lowpass prototype element values are present [6, ch. 4]. These tables are used to design bandpass, lowpass, highpass or bandstop filters at any frequency and with any bandwidth

using normalization procedures provided in [9, pp. 398-405]. Matlab functions generating lowpass prototype tables are written and given in Appendix A based on the approximating functions proposed in [6, ch. 4].

Once the prototype element values are present, the corresponding coupling matrix can be computed as done in [8, pp. 286-288]. Note that, it is also possible to obtain the coupling matrix directly based on the characteristic polynomials using the Y-parameters [8, pp. 292-295]. The latter approach is computerized in Matlab and provided in Appendix A.

At this point, the synthesized coupling matrix may contain nonzero coupling values among any resonator and the source/load (transversal matrix). It may or may not be possible to realize such a structure physically. The coupling matrix reconfiguration step, a purely mathematical effort, is applied to modify the form of the coupling matrix via the use of similarity transforms that leaves the transfer characteristics of the filter unchanged [8, p. 295]. The aim is to obtain a coupling diagram that is more suitable for fabrication. Among preferred configurations are the folded and arrow (wheel) forms [8, p. 316] that are presented in Fig. 2.3 for a fifth order filter.

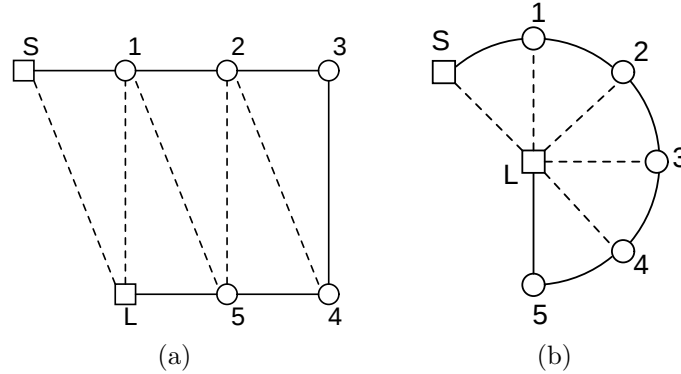


Figure 2.3: Common coupling configurations. The mainline couplings are presented with solid lines, whereas cross-couplings are shown with dashed lines. (S: Source, L: Load) (a) Folded form. (b) Arrow (wheel) form.

Let us illustrate this step with an example fifth order Chebyshev filter centered at 30 GHz, having a bandwidth of 1500 MHz, with a maximum in-band return loss of 25 dB. Let us add three prescribed TZ's at frequencies 27 GHz, 32 GHz

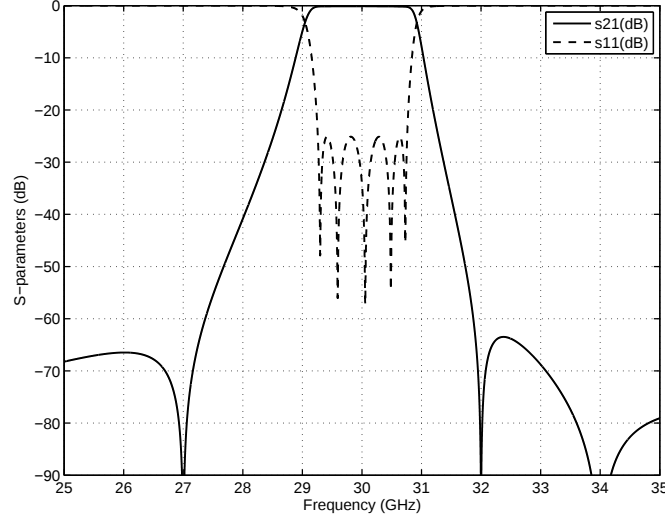


Figure 2.4: The response for the example fifth order filter.

and 34 GHz. The S-parameters of the described filter is depicted in Fig. 2.4. The generated transversal coupling matrix is:

$$M_{initial} = \begin{matrix} & \begin{matrix} S & R_1 & R_2 & R_3 & R_4 & R_5 & L \end{matrix} \\ \begin{matrix} S \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ L \end{matrix} & \begin{bmatrix} 0 & 0.4075 & 0.3715 & -0.5341 & -0.5829 & 0.5666 & 0 \\ 0.4075 & 1.3921 & 0 & 0 & 0 & 0 & 0.4075 \\ 0.3715 & 0 & -1.3232 & 0 & 0 & 0 & 0.3715 \\ -0.5341 & 0 & 0 & -1.0452 & 0 & 0 & 0.5341 \\ -0.5829 & 0 & 0 & 0 & 0.9271 & 0 & 0.5829 \\ 0.5666 & 0 & 0 & 0 & 0 & -0.1326 & 0.5666 \\ 0 & 0.4075 & 0.3715 & 0.5341 & 0.5829 & 0.5666 & 0 \end{bmatrix} \end{bmatrix} \quad (2.1)$$

Note that, there are no couplings among any of the resonators, whereas all of them are coupled to the load and to the source simultaneously. The procedure to obtain folded and arrow forms from the transversal coupling matrix are provided in [8, pp. 299-300] and in [8, p. 380]. (Matlab functions that implement these procedures are written and given in Appendix A.) The folded form corresponding

to the same response is:

$$M_{folded} = \begin{matrix} & S & R_1 & R_2 & R_3 & R_4 & R_5 & L \\ \begin{matrix} S \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ L \end{matrix} & \left[\begin{array}{ccccccc} 0 & 1.1181 & 0 & 0 & 0 & 0 & 0 \\ 1.1181 & 0.0183 & 0.9684 & 0 & 0 & -0.0086 & 0 \\ 0 & 0.9684 & 0.0248 & 0.6547 & 0.1749 & -0.0282 & 0 \\ 0 & 0 & 0.6547 & -0.2783 & 0.6741 & 0 & 0 \\ 0 & 0 & 0.1749 & 0.6741 & 0.0350 & 0.9680 & 0 \\ 0 & -0.0086 & -0.0282 & 0 & 0.9680 & 0.0183 & 1.1181 \\ 0 & 0 & 0 & 0 & 0 & 1.1181 & 0 \end{array} \right] \end{matrix} \quad (2.2)$$

In the folded form, there are couplings only among consecutive resonators and three cross-couplings corresponding to the TZ's are present. On the other hand, the arrow form is:

$$M_{arrow} = \begin{matrix} & S & R_1 & R_2 & R_3 & R_4 & R_5 & L \\ \begin{matrix} S \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ L \end{matrix} & \left[\begin{array}{ccccccc} 0 & 1.1181 & 0 & 0 & 0 & 0 & 0 \\ 1.1181 & 0.0183 & 0.9684 & 0 & 0 & 0 & 0 \\ 0 & 0.9684 & 0.0253 & 0.6761 & 0 & 0 & -0.0100 \\ 0 & 0 & 0.6761 & 0.0420 & 0.6737 & 0 & -0.0466 \\ 0 & 0 & 0 & 0.6737 & 0.3946 & 0.8258 & 0.4068 \\ 0 & 0 & 0 & 0 & 0.8258 & -0.6621 & 1.0404 \\ 0 & 0 & -0.0100 & -0.0466 & 0.4068 & 1.0404 & 0 \end{array} \right] \end{matrix} \quad (2.3)$$

This time, there are couplings among consecutive resonators, and three resonators (corresponding to the three TZ's) are coupled in extra to the load. Both folded and arrow forms are canonical forms, i.e., both forms realize the required response with minimum number of couplings [8, p. 316]. The investigation of possible coupling matrices, corresponding to various topologies and implementing the same response is still an interesting research subject ¹.

¹"Dedale-HF: coupling matrix synthesis, microwave filter synthesis." [Online]. <https://www-sop.inria.fr/apics/Dedale/WebPages/index.html>. Accessed: 07.04.2017

From a simplistic point of view, a filter is a network of resonators. If it is possible to couple energy to this network from the outside world, and if energy can be coupled among the resonators, then a filter can be realized. Appropriate resonator structures are needed to be used for different frequency, bandwidth, stopband characteristics, weight, size or cost requirements. Therefore, the last step in the design procedure, the physical implementation of the filter, is the effort of choosing an appropriate resonator structure and determining the relation between the coupling matrix equivalent circuit and the network of resonators that is to be formed. This relation can be investigated using CAD approaches as illustrated in [8, pp. 507-512] or analytically as attempted in the following chapter for the case of multiple mode rectangular cavity.

Chapter 3

The Multiple Mode Rectangular Cavity Filter

The idea of using the degenerate modes within a cavity in the synthesis of a filter was introduced in [1, pp. 673-677] in the late 1940s, whereas, a thorough analysis for the idea is carried out in [10]. The necessary coupling between the degenerate modes within the structure is realized by perturbing the geometry of the cavity. The corresponding perturbation elements can be tuning screws as well as square corner cuts [11]. In a more recent work [12], a specific sequence of intermodal couplings in a triple mode rectangular cavity, implemented with similar corner cuts, is proposed and an investigation is made on how a computer-aided design can be realized using the proposed structure.

The main contributions of this chapter to the cavity filter design techniques are as follows:

1. Simple closed-form expressions to determine the resonant frequency shifts of the modes of a multiple mode cavity, and the resulting coupling coefficient due to various types of corner cuts are obtained. The resulting expressions are combined with rectangular aperture-related expressions from the literature to form a complete set of equations that defines the connection between

the physical dimensions and the coupling matrix of a second or third-order multiple mode single cavity filter.

2. A filter design algorithm that uses the above-mentioned set of expressions is proposed. The algorithm provides a good starting point in obtaining the optimized dimensions of a single cavity multiple mode filter.
3. An analytical expression determining the dispersive behavior of the rectangular aperture is derived. This expression can be used directly with the coupling matrix to estimate the dispersion caused nonideal response of a filter.
4. The classical triple mode filter structure is altered to form a triplet. The new structure is able to generate either positive or negative cross couplings, hence allowing placement of a finite frequency transmission zero (TZ) either at the upper or lower sideband of the center frequency.
5. The additive manufacturing technology is combined with multiple mode cavity technique for the first time. The combination of the two techniques results in very lightweight filters. An example design illustrating the concept is realized in *Ka*-Band.

3.1 Multiple Mode Cavity Filter

The dual and triple mode cavity filter structures can be seen in Fig. 3.1 and Fig. 3.2. Both are composed of a single cavity with square corner cuts to generate the intermodal couplings and rectangular apertures to realize the source/load-to-resonator couplings.

4×4 coupling matrix corresponding to a Chebyshev second-order filter is in

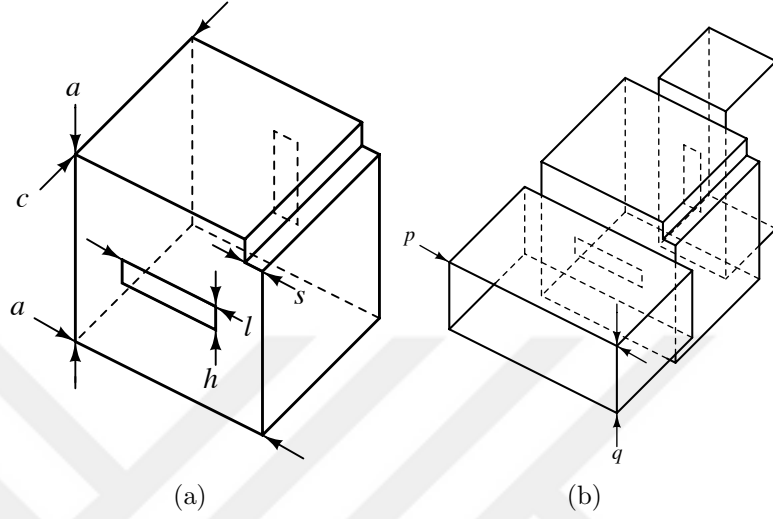


Figure 3.1: The dual mode rectangular cavity filter structure. (a) Cavity. (b) Cavity and waveguide.

the following form:

$$M = [M_{ij}] = \begin{matrix} & \begin{matrix} S & R_1 & R_2 & L \end{matrix} \\ \begin{matrix} S \\ R_1 \\ R_2 \\ L \end{matrix} & \begin{bmatrix} 0 & c_{S1} & 0 & 0 \\ c_{S1} & 0 & c_{12} & 0 \\ 0 & c_{12} & 0 & c_{2L} \\ 0 & 0 & c_{2L} & 0 \end{bmatrix} \end{matrix} \quad (3.1)$$

The only nonzero, thus to be defined, parameters of the coupling matrix are the source to the first resonator coupling c_{S1} , the inter-resonator coupling c_{12} , and the load to the last resonator coupling c_{2L} . Due to symmetry, $c_{S1} = c_{2L}$.

For a third-order Chebyshev filter, the only nonzero elements of 5×5 coupling matrix are $M_{12}=M_{21}=M_{45}=M_{54}=c_{S1}$ and $M_{23}=M_{32}=M_{34}=M_{43}=c_{12}$ with again only two independent parameters.

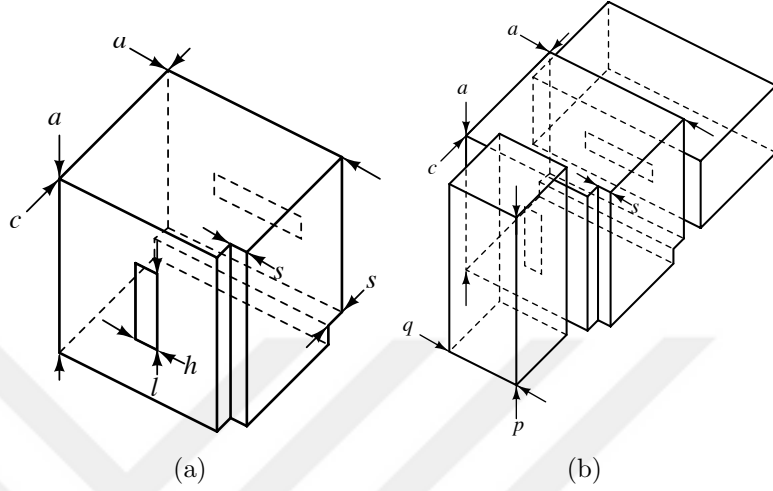


Figure 3.2: The triple mode rectangular cavity filter structure. (a) Cavity. (b) Cavity and waveguide.

3.2 Intermodal Coupling

3.2.1 Square Corner Cut

Atia *et al.* [11] proposed the idea of using square corner cuts instead of tuning screws to couple the degenerate modes of a square cross section waveguide/cavity. The same structure is also used for triple mode cavities [12]. The structure is illustrated in Fig. 3.3.

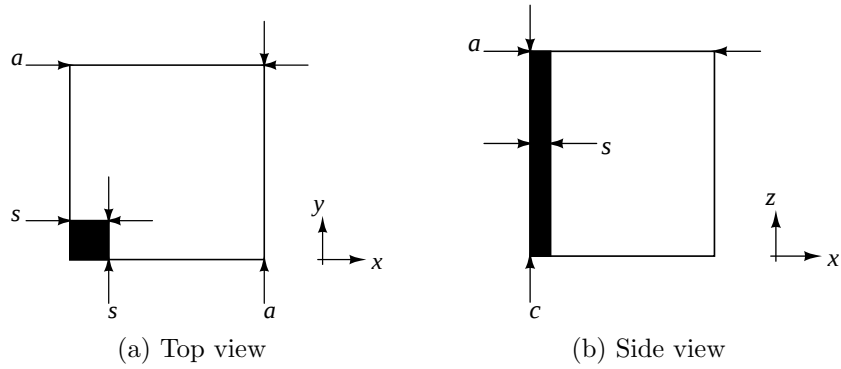


Figure 3.3: Dual mode cavity with square corner cut.

A square cross-sectioned cavity has dual symmetry, hence, it has two degenerate modes. In the presence of a square corner cut, these modes are no longer degenerate and can be interpreted as to be coupled to each other. The physical intermodal coupling coefficient, k_c , can be related to the resonant frequencies, f_1 and f_2 , of the coupled modes with [11]:

$$k_c = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2} \quad (3.2)$$

The presence of the square corner cut in a waveguide forces the fields to be diagonal [1, p. 675], [13]. The rectangular cavity field expressions of [14, pp. 155-157] are modified with the diagonal mode approach and are given as

$$\begin{cases} \mathbf{E}_{\{1,2\}t} = -\frac{\pi}{a} \sin(\frac{\pi z}{c}) [\sin(\frac{\pi y}{a}) \hat{\mathbf{x}} \pm \sin(\frac{\pi x}{a}) \hat{\mathbf{y}}] \\ \mathbf{H}_{\{1,2\}t} = j \frac{\pi^2}{k_0 a c} \sqrt{\frac{\epsilon_0}{\mu_0}} \cos(\frac{\pi z}{c}) [\sin(\frac{\pi y}{a}) \hat{\mathbf{x}} \pm \sin(\frac{\pi x}{a}) \hat{\mathbf{y}}] \\ \mathbf{H}_{\{1,2\}z} = -j \frac{\pi^2}{k_0 a^2} \sqrt{\frac{\epsilon_0}{\mu_0}} \sin(\frac{\pi z}{c}) [\cos(\frac{\pi x}{a}) \pm \cos(\frac{\pi y}{a})] \hat{\mathbf{z}} \end{cases} \quad (3.3)$$

where a is the cavity dimension in x - and y - directions, c is the cavity dimension in z -direction, $k_0 = \pi \sqrt{1/a^2 + 1/c^2}$ is the wavenumber of the unperturbed resonance, ϵ_0 and μ_0 are the dielectric permittivity and permeability in the vacuum.

Different approaches in predicting the resonant frequency of a perturbed cavity's TM_{110} mode are presented in [15]. In this work, Slater's perturbational formulation [9] is preferred:

$$\frac{f_{1,2} - f_0}{f_0} = \frac{\int_{\Delta V} (\mu_0 |H_{1,2}|^2 - \epsilon_0 |E_{1,2}|^2) dv}{\int_{V_0} (\mu_0 |H_{1,2}|^2 + \epsilon_0 |E_{1,2}|^2) dv} \quad (3.4)$$

where ΔV is the volume of the corner cut, f_0 and V_0 is the resonant frequency and volume of the unperturbed cavity, respectively. Replacing (3.3) in (3.4) and letting $r = s/a$ be the normalized corner cut dimension, the resonant frequencies are found as

$$\frac{f_{1,2}}{f_0} = 1 + \frac{r^2}{1 + a^2/c^2} [\text{sinc}(2\pi r) \pm \text{sinc}^2(\pi r)] \quad (3.5)$$

where $\text{sinc}(x) = \sin(x)/x$.

Replacing (3.5) in (3.2) and using the first two terms of the Taylor series around $r = 0$, we find the coupling coefficient, k_c , as

$$k_c \approx \frac{2}{1 + a^2/c^2} r^2 \quad (3.6)$$

The inverse function that can be used for design is

$$r \approx \left[\frac{k_c(1 + a^2/c^2)}{2} \right]^{\frac{1}{2}} \quad (3.7)$$

For the special case of a triple mode rectangular cavity with $a = c$, the expressions simplify to

$$k_c \approx r^2 \text{ or } r \approx \sqrt{k_c} \quad (3.8)$$

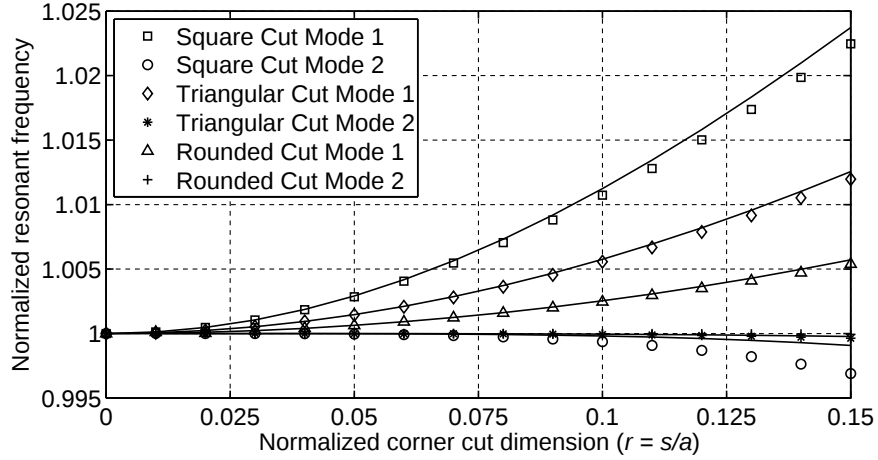


Figure 3.4: Normalized resonance frequencies of the two coupled modes versus r with $a/c = 5/6$. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.

To check their validity, the analytical expressions are plotted together with the values obtained from CST Microwave Studio's Eigenmode Solver for $a/c = 5/6$. The perturbed cavity's coupled modes' resonant frequencies are given in Fig. 3.4, whereas the resulting coupling coefficient is given in Fig. 3.5 as a function of r . The figures also display the results for the triangular and rounded corner cut cases as investigated below.

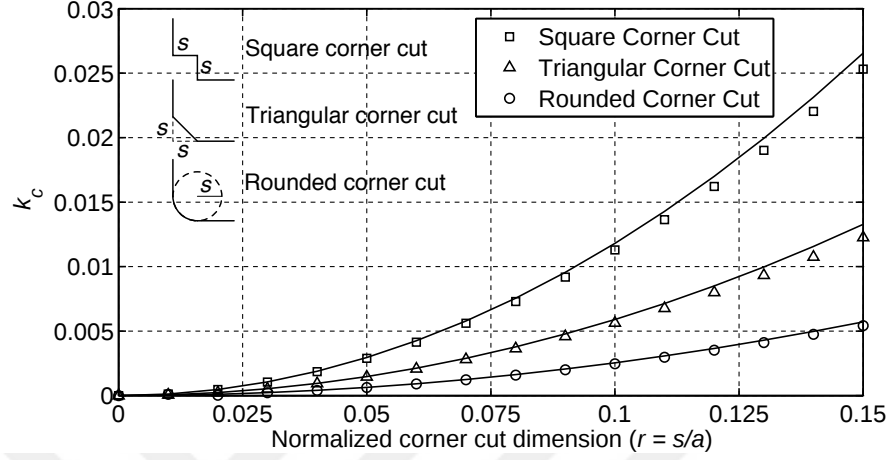


Figure 3.5: The physical coupling coefficient as a function of normalized corner cut dimension with $a/c = 5/6$. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.

3.2.2 Triangular Corner Cut

The perturbation resulting from a triangular corner cut, illustrated in Fig. 3.6, can be found using a similar procedure:

$$\frac{f_{1,2}}{f_0} = 1 + \frac{r^2}{2(1 + a^2/c^2)} [\text{sinc}^2(\pi r) \pm \text{sinc}(\pi r)] \quad (3.9)$$

$$k_c \approx \frac{1}{1 + a^2/c^2} r^2 \text{ or } r \approx \sqrt{k_c(1 + a^2/c^2)} \quad (3.10)$$

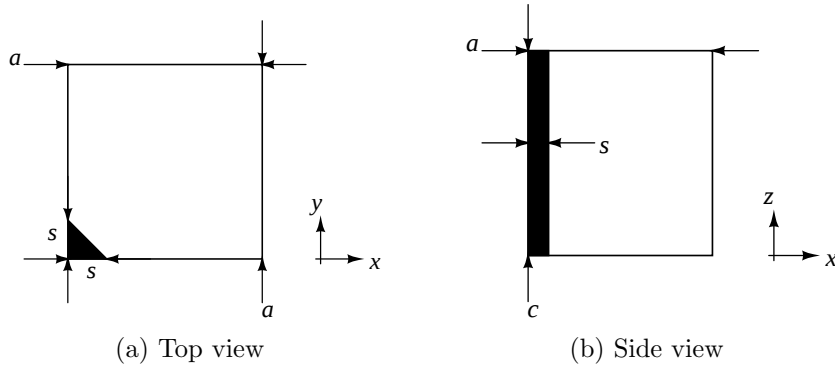


Figure 3.6: Dual mode cavity with triangular corner cut.

3.2.3 Rounded Corner Cut

For a rounded corner cut illustrated in Fig. 3.7, we apply the trapezoidal approximation with 64 partitions [16, p. 374] and taking up to the second-order terms, we find

$$\frac{f_1}{f_0} \approx 1 + 0.43 \frac{r^2}{(1 + a^2/c^2)} \quad \text{and} \quad \frac{f_2}{f_0} \approx 1 \quad (3.11)$$

$$k_c \approx 0.43 \frac{r^2}{(1 + a^2/c^2)} \quad \text{or} \quad r \approx \left[\frac{k_c(1 + a^2/c^2)}{0.43} \right]^{\frac{1}{2}} \quad (3.12)$$

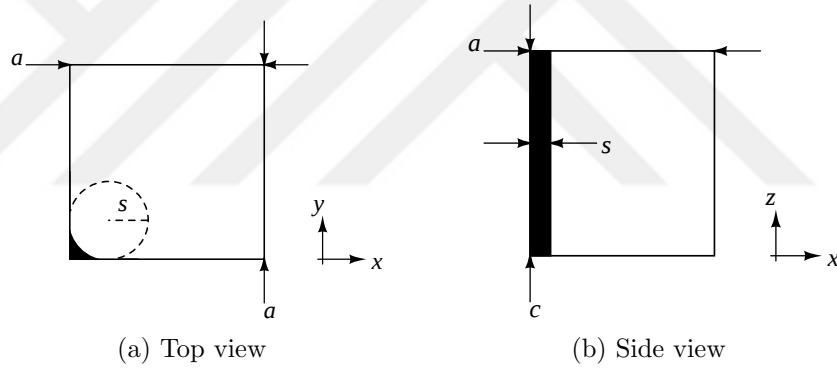


Figure 3.7: Dual mode cavity with rounded corner cut.

3.3 Waveguide to Cavity Coupling with a Rectangular Aperture

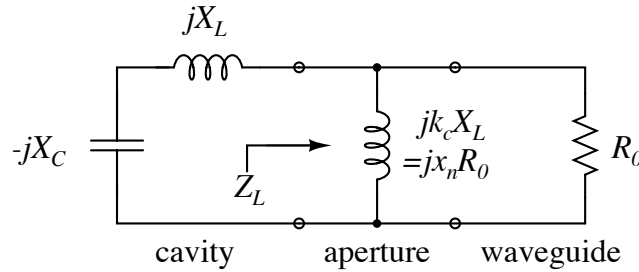


Figure 3.8: The equivalent circuit for the waveguide-cavity aperture coupling structure.

The waveguide to cavity coupling structure's equivalent circuit proposed by Wheeler [17] is given in Fig. 3.8, where X_L and X_C represents the unperturbed cavity, whereas R_0 shows the load resistance presented by the waveguide. The coupling between them is represented by an inductance with $x_n R_0 = k_c X_L$ [17]. Here x_n is the equivalent normalized shunt reactance due to an aperture placed in an infinitely long rectangular waveguide, whereas k_c is the coupling coefficient due to an aperture placed in the common wall of two identical cavities.

For x_n , combining the results from [18, 19, 20] we get

$$x_n = \frac{\alpha_m \beta_{10}}{pq} \left[1 + \text{sinc} \left(\frac{\pi l}{p} \right) \right] \left[\frac{2f_a}{\pi f} \tan \left(\frac{\pi f}{2f_a} \right) \right] \quad (3.13)$$

where p, q are waveguide cross sectional dimensions, β_{10} is the propagation constant of the waveguide's TE_{10} mode and α_m is the corresponding longitudinal magnetic polarizability given by [21, 22]

$$\alpha_m = \frac{0.132 l^3}{\ln(1 + 0.66 l/h)} e^{-\frac{\pi t}{l}} \quad (3.14)$$

t being the thickness of the aperture. The aperture resonance frequency f_a can be approximated by the cutoff frequency of the waveguide having the aperture's dimensions [19]: $f_a = 1/(2l\sqrt{\epsilon_0\mu_0})$.

On the other hand, for k_c , combining the results from [22, 19, 20] we get

$$k_c = \frac{2\alpha_m}{abc(1 + c^2/a^2)} \left[1 + \text{sinc} \left(\frac{\pi l}{a} \right) \right] \left[\frac{2f_a}{\pi f} \tan \left(\frac{\pi f}{2f_a} \right) \right] \quad (3.15)$$

Referring to Fig. 3.8, the cavity loading impedance, Z_L , is given by

$$Z_L = \frac{x_n^2 R_0}{1 + x_n^2} + j \frac{x_n R_0}{1 + x_n^2} \quad (3.16)$$

Using (3.16) and the equivalence of the two forms of the coupling inductance, we can find the quality factor, Q_e :

$$Q_e = \frac{X_L}{\text{Re}\{Z_L\}} = \frac{1 + x_n^2}{x_n k_c} \quad (3.17)$$

Its value can be found using (3.13) and (3.15). The new resonant frequency of the cavity is found by including the effect of coupling aperture. Setting the imaginary part of the overall structure's equivalent impedance to zero, the new resonant frequency can be approximated as:

$$\frac{f_r}{f_0} = \sqrt{\frac{Q_e x_n}{1 + Q_e x_n}} = \sqrt{\frac{1 + x_n^2}{1 + k_c + x_n^2}} \approx \sqrt{\frac{1}{1 + k_c}} \quad (3.18)$$

Using the first three terms of the Taylor series for the sine and cosine functions around $\pi f/(2f_a) = 0$:

$$\frac{f_r}{f_0} = \left[\frac{6A^2(1 + k_{cs}) + 3}{2(3 + k_{cs})} - \frac{\sqrt{36A^4(1 + k_{cs})^2 + 12A^2(k_{cs} - 3) + 9}}{2(3 + k_{cs})} \right]^{\frac{1}{2}} \quad (3.19)$$

where $A = 2f_a/(\pi f_0)$ and k_{cs} is the coupling expression provided in (3.15) without the Cohn's tangential frequency correction term [19].

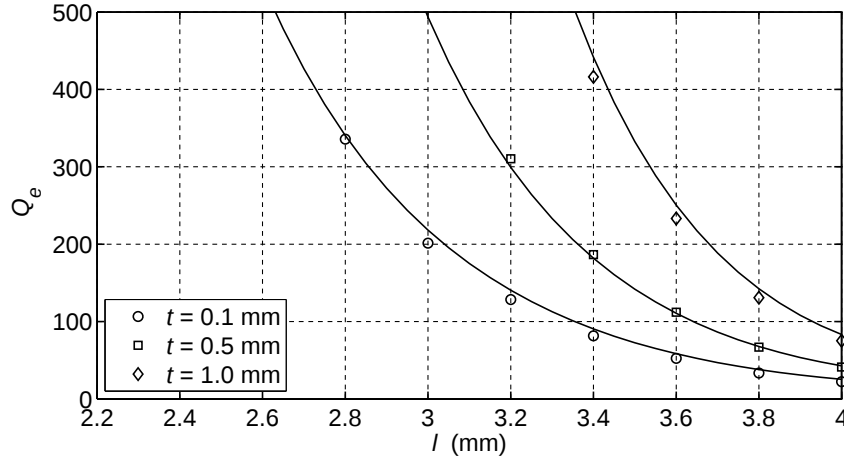


Figure 3.9: Q_e of the example aperture coupled cavity for different aperture wall thicknesses, for $h = 1$ mm. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.

To check the validity of the expressions, an example cubic cavity of $a = b = c = 6.5$ mm attached to a WR-28 standard waveguide ($p=7.112$ mm, $q=3.556$ mm) via a rectangular aperture of $h=1$ mm is analyzed for different values of l and t . The structure is also simulated with CST Studio Eigenmode Solver for comparison. The results are plotted in Fig. 3.9 and Fig. 3.10. The predicted Q_e is in good agreement with the simulation results. On the other hand, the resonant frequency

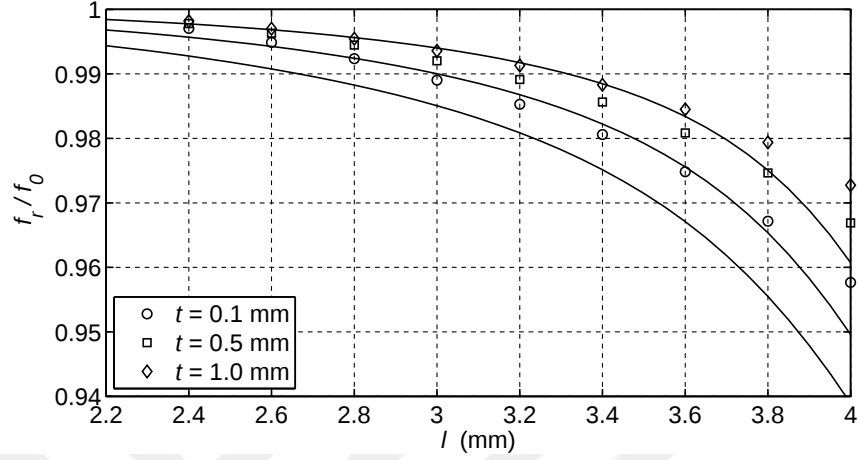


Figure 3.10: The normalized resonant frequency of the example aperture coupled cavity for different aperture wall thicknesses, for $h = 1$ mm. The solid lines correspond to the results obtained analytically, whereas the symbols correspond to simulation results.

prediction error increases with increasing aperture length but stays below 2% for all thickness values analyzed.

3.4 Investigation of Dispersion

In the ideal case, the elements of the coupling matrix are assumed frequency independent. In the previous section we showed that Q_e actually depends on frequency and this dependence is approximately given as

$$Q_e \propto \frac{1}{\beta_{10}} \left[\frac{2f_a}{\pi f} \tan\left(\frac{\pi f}{2f_a}\right) \right]^{-2} \quad (3.20)$$

Let us define the normalized dispersion factor η as the ratio of Q_e normalized to its value at the center frequency (f_0) of the filter. Then, η is given as

$$\eta \triangleq \frac{Q_e|_{f=f_0}}{Q_e} = \eta_{wg} \eta_{ap} \quad (3.21)$$

where η_{wg} is due to waveguide and η_{ap} is due to the rectangular aperture:

$$\eta_{wg} = \frac{\sqrt{f^2 - f_c^2}}{\sqrt{f_0^2 - f_c^2}} \quad \text{and} \quad \eta_{ap} = \left[\frac{f_0 \tan(\pi/2 \times f/f_a)}{f \tan(\pi/2 \times f_0/f_a)} \right]^2 \quad (3.22)$$

where f_c is the cutoff frequency of the waveguide. Assuming that the intermodal couplings are frequency independent (valid for narrow band filters), the effect of dispersion can be implemented in the coupling matrix by multiplying the corresponding coupling element (c_{S1}) with (3.21).

Both dispersion effects are given in Fig. 3.11 as a function of f/f_0 for different f_0/f_c and f_0/f_a ratios. It can be seen that the frequency dependence slope due

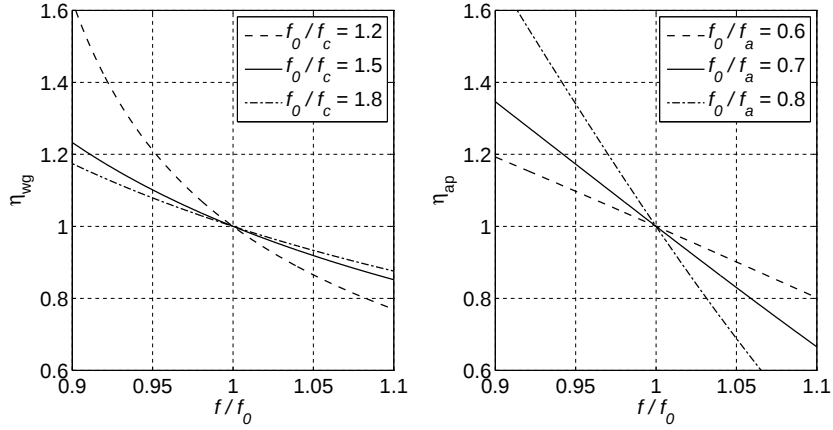


Figure 3.11: The dispersion effects due to the waveguide and due to the aperture.

to the waveguide gets higher when f_0 moves closer to f_c , hence, the distortion due to dispersion gets worse. For a specific filter, if the waveguide to be used is predefined, this effect can not be prevented. But note that, its value is relatively small as long as f_0 is sufficiently away from the waveguide's cutoff frequency (which generally is). On the other hand, the frequency dependence slope due to the aperture gets higher when the aperture resonance frequency gets closer to f_0 . The gravity of this dispersion effect can be lightened by using aperture structures resulting in the same amount of coupling but having a higher resonance frequency. The search for such aperture structure is not the subject of this work, hence rectangular shaped irises will be used.

3.5 Filter Design Algorithm

Using the results obtained so far, the physical dimensions of a dual (triple) mode second (third) order filter can be predicted with the following algorithm:

1. Calculate the cavity dimensions resonating at filter center frequency f_0 .
2. Find the required corner cut dimension using (3.7).
3. Compensate for the frequency shift of the modes, as proposed in [1, p. 676], with the use of (3.5). The blank cavity now resonates at $f'_0 < f_0$.
4. Go to step 2 until the corner cut dimension converges.
5. Calculate the aperture length corresponding to Q_e using (3.17).
6. Compensate for the frequency lowering effect of the input/output coupling irises by decreasing the length of the cavity [6, p. 463], so that the aperture coupled cavity still resonates at f'_0 using (3.18).
7. Go to step 5 until the aperture and cavity length dimensions converge.
8. Recompute the required corner cut dimension using (3.7), based on the final values of a and c .

3.6 Example Second-Order Filter Design

Let us design a second-order Chebyshev band-pass filter at 34 GHz, with a fractional bandwidth of 1% and an in-band return loss of 25 dB. The corresponding normalized coupling matrix of (3.1) has $c_{S1}=1.4312$ and $c_{12}=2.1670$.

The physical coupling coefficients k_{ij} and Q_e can be found from [8, pp. 516-520]:

$$k_{ij} = \frac{B}{f_0} M_{ij} \quad \text{and} \quad Q_e = \frac{f_0}{B} \frac{1}{R} \quad (3.23)$$

where B and R are the bandwidth and the coupling resistance respectively. Let the input/output be WR-28 standard waveguide, the height of the coupling aperture be 1 mm, whereas its thickness be 0.5 mm. Since a dual mode cavity resonating at f_0 is not unique, let us also specify the cavity ratio (a/c) as 1.25.

Since the resonant frequencies of the resonators of a second-order filter are located symmetrically around its center frequency, the third step of the algorithm is applied to find the length (c) of the cavity that results in $(f_1 + f_2)/2 = f_0$. Upon completion of all the steps, the dimensions given in Table 3.1 are obtained.

a	b	c	s	l
7.06	7.06	5.40	1.21	3.52

Table 3.1: The physical dimensions (in mm) of the designed second-order filter.

Fig. 3.12 depicts the result of CST Microwave Studio simulation run with the predicted dimensions. For this example, it can be seen that the initial response is very close to the dispersion included theoretical response without the need for optimization. In case it is not as close as desired, it is possible to do an optimization with only two parameters (c and l) assuming the intermodal coupling to be less susceptible to dimension changes.

3.7 Example Third-Order Filter Design

Let us consider a third-order filter at 34 GHz with a 1% fractional bandwidth and an in-band return loss of 25 dB. The normalized 5×5 coupling matrix has $c_{S1}=1.2214$ and $c_{12}=1.2197$. Again, let the input/output be WR-28 standard waveguide, the height of the coupling aperture be 1 mm, whereas its thickness be 0.5 mm. This time, there is no need to specify the cavity ratio since a triple mode cavity is cubic.

On the other hand, the application of the algorithm's third and fourth steps

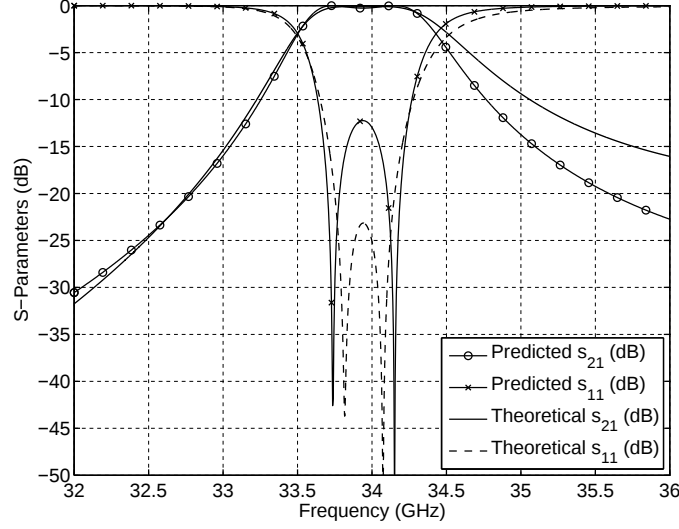


Figure 3.12: The initial response of the designed filter together with the theoretical response.

differs from that of the second-order case. For a third-order filter, two of the resonators' resonant frequencies (f_1 and f_2) are again located symmetrically around the center frequency, whereas the third resonator (f_3) resonates at the center frequency. Therefore, the algorithm needs to be applied to find the length (c) of the cavity that results in $(f_1 + f_2)/2 = f_0$, and the width (a) that results in $f_3 = f_0$ at the same time. (3.5) can be used to predict all three resonant frequencies assuming that the corner cut that couples two modes does not affect the third mode. But note that, taking into account the directions of the corner cuts, when $a \neq c$, the dual mode approximation does not hold anymore. Therefore, while keeping the cavity cubic, the frequency compensation can be based either on $(f_1 + f_2)/2 = f_0$ or $f_3 = f_0$. Here, the first one is chosen. With $a = c$, there is no need to perform the fourth step of the algorithm, and the value a that will be obtained will be a lower bound for the required value. Upon completion of all the steps, the dimensions given in Table 3.2 are obtained.

Using the initial results, an optimization is run in CST with the two cavity parameters (not three since the cavity has to be square cross sectioned) and the aperture length parameter. The corner cut dimension is left as is, assuming that the coupling coefficient will be less sensitive to dimension changes. The obtained

	a	b	c	s	l
Predicted	6.27	6.27	5.89	0.72	3.49
Optimized	6.34	6.34	5.86	0.72	3.49
Error (%)	1.1	1.1	0.5	0	0

Table 3.2: The physical dimensions (in mm) of the designed third-order filter.

result is given in Fig. 3.13 together with the theoretical response (including the dispersion effect).

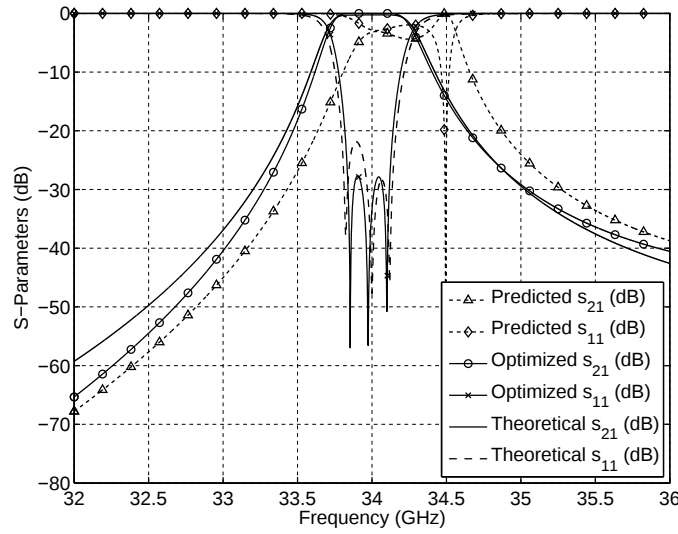


Figure 3.13: S-parameters of the predicted and optimized structures together with the theoretical response.

Note that the obtained response rejects better at lower frequencies but worse at higher frequencies as expected in light of the results obtained in the previous sections. An interesting point is that, this dispersion caused effect is similar to that of a capacitive cross-coupling between the first and third resonators.

3.8 The Triplet Structure

One way of generating a finite frequency TZ in a multiple mode cavity is via the use of nonresonating node concept [23]. On the other hand, in a filter of order

three, it is also possible to add the TZ by introducing a cross-coupling between the first and third resonators. The coupling diagram corresponding to this triplet structure can be seen in Fig. 3.14. Assuming k_{12} and k_{23} are positive, if the cross-coupling (k_{13}) is negative, the TZ is at the lower sideband and if it is positive, the TZ is at the upper sideband [24].

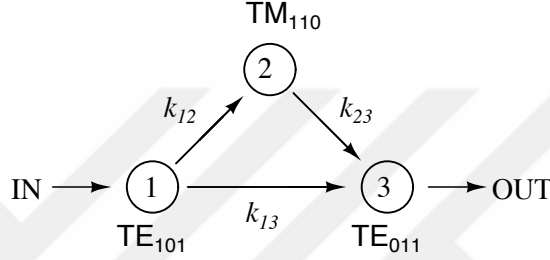


Figure 3.14: The triplet coupling diagram.

In a triple mode rectangular cavity, it is possible to implement the cross-coupling with a corner cut along the input-output direction. The sign of the cross-coupling can be alternated by rotating the corner that is cut by 90° , since the rotation interchanges the coupled modes' resonant frequencies. Hence, third-order filters containing either a lower or an upper sideband TZ can be designed. Both cases are considered below.

3.8.1 Transmission zero at the upper sideband

Consider a filter with the specifications of Section 3.7 and having a TZ at 34.75 GHz. The corresponding normalized coupling matrix is given by

$$M = \begin{matrix} & \begin{matrix} S & R_1 & R_2 & R_3 & L \end{matrix} \\ \begin{matrix} S \\ R_1 \\ R_2 \\ R_3 \\ L \end{matrix} & \begin{bmatrix} 0 & 1.2214 & 0 & 0 & 0 \\ 1.2214 & 0.0945 & 1.1841 & 0.3455 & 0 \\ 0 & 1.1841 & -0.3052 & 1.1841 & 0 \\ 0 & 0.3455 & 1.1841 & 0.0945 & 1.2214 \\ 0 & 0 & 0 & 1.2214 & 0 \end{bmatrix} \end{matrix} \quad (3.24)$$

Note that, there is an inductive (positive) coupling between the first and third

resonators. This coupling is realized with a corner cut along the input-output direction. The structure can be seen in Fig. 3.15. The cross-coupling corner cut is divided in two parts to prevent intersection with the main coupling corner cut. The two parts are located at opposite corners so that they both result in the same type of coupling.

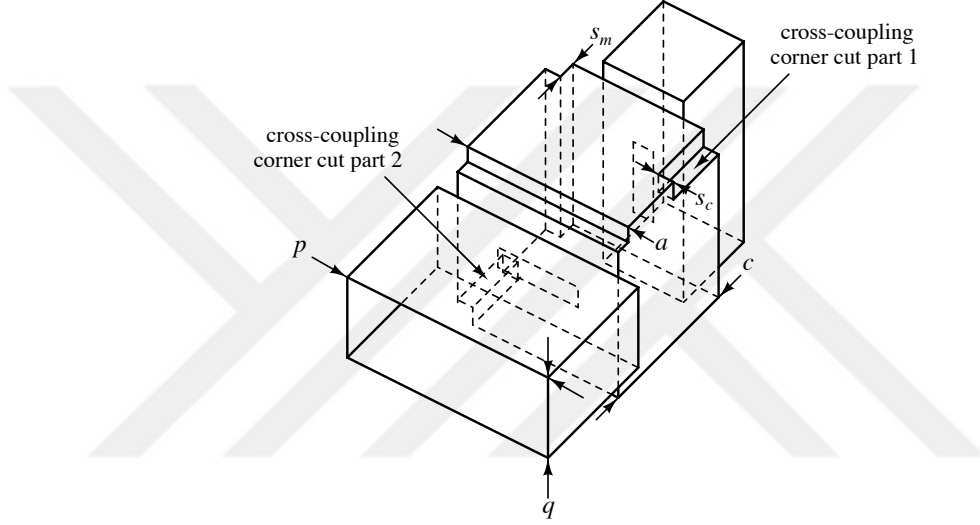


Figure 3.15: The triple mode triplet structure with an upper sideband finite frequency TZ.

To determine the dimensions, other than the cross-coupling corner cut dimension (s_c), the proposed filter algorithm in the form described in Section 3.7 is used. The deviations of the modes' resonant frequencies from the case with no TZ are assumed small, therefore neglected. On the other hand, the intermodal coupling expression in (3.7) is used to determine s_c . Note that, the obtained value is a lower bound for the actual s_c needed, because the generated inductive coupling needs to counteract the capacitive effect of dispersion as well. A full-wave optimization is performed based on the initial values and the final dimensions are attained. (The main coupling corner cut parameter s_m is left out of optimization assuming that intermodal coupling is less susceptible to dimension changes.) Both predicted and optimized values are given in Table 3.3, whereas Fig. 3.16 depicts the corresponding responses together with the theoretical response.

	a	b	c	s_m	s_c	l
Predicted	6.27	6.27	5.88	0.70	> 0.38	3.49
Optimized	6.33	6.33	5.89	0.70	0.40	3.47
Error (%)	1.0	1.0	0.2	0	5.3	0.6

Table 3.3: The physical dimensions (in mm) of the designed triplet with upper sideband TZ.

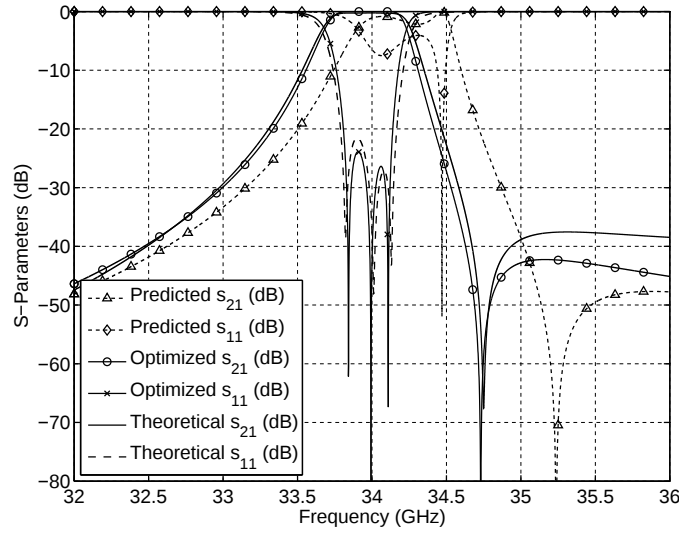


Figure 3.16: The response for the triple mode triplet structure with an upper sideband finite frequency TZ.

3.8.2 Transmission zero at the lower sideband

Let us move the TZ of the filter in Section 3.8.1 to lower sideband, at 33.25 GHz. The corresponding normalized coupling matrix is

$$M = \begin{matrix} & \begin{matrix} S & R_1 & R_2 & R_3 & L \end{matrix} \\ \begin{matrix} S \\ R_1 \\ R_2 \\ R_3 \\ L \end{matrix} & \begin{bmatrix} 0 & 1.2214 & 0 & 0 & 0 \\ 1.2214 & -0.0925 & 1.1857 & -0.3377 & 0 \\ 0 & 1.1857 & 0.2985 & 1.1857 & 0 \\ 0 & -0.3377 & 1.1857 & -0.0925 & 1.2214 \\ 0 & 0 & 0 & 1.2214 & 0 \end{bmatrix} \end{matrix} \quad (3.25)$$

This time, there is a capacitive (negative) coupling between the first and third

resonators. This coupling is realized with a corner cut along the input-output direction rotated by 90° compared to the earlier design. The new structure can be seen in Fig. 3.17.

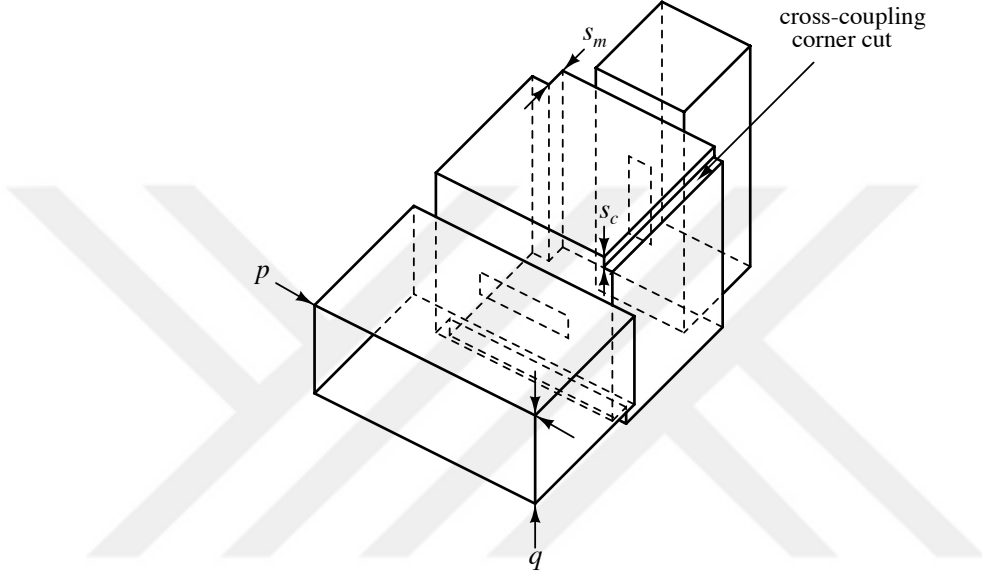


Figure 3.17: The triple mode triplet structure with a lower sideband finite frequency TZ.

The dimensions are determined with the same procedure described in the previous section. Note that, because the frequency dependence of the external couplings acts like a capacitive cross-coupling, the overall effective coupling becomes higher than intended. Therefore, the determined cross-coupling corner cut dimension (s_c) is an upper bound for the actual value required. A full-wave optimization is performed based on the predicted dimensions and final values are obtained. (The main coupling corner cut parameter s_m is left out of optimization.) Both the predicted and optimized dimensions can be seen in Table 3.4, whereas the corresponding responses and the theoretical response are given in Fig. 3.18 for comparison.

	a	b	c	s_m	s_c	l
Predicted	6.27	6.27	5.88	0.71	< 0.38	3.49
Optimized	6.35	6.35	5.87	0.71	0.34	3.47
Error (%)	1.3	1.3	0.2	0	10.5	0.6

Table 3.4: The physical dimensions (in mm) of the designed triplet with lower sideband TZ.

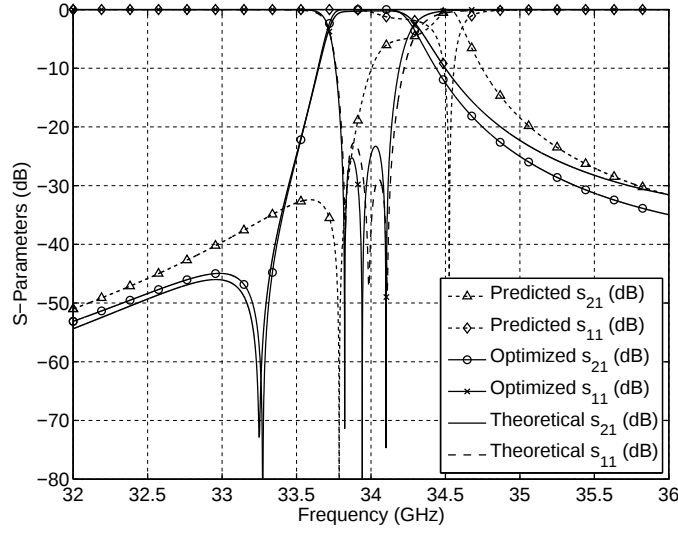


Figure 3.18: The response for the triple mode triplet structure with a lower sideband finite frequency TZ.

3.9 Experimental Verification

A *Ka*-Band filter with 2% fractional bandwidth, having a lower sideband TZ is designed and fabricated with CNC milling. This type of filter can be used to increase selectivity in the lower sideband, or to obtain high rejection at a specific frequency, e.g., the TZ can be adjusted to coincide with the local oscillator (LO) frequency to prevent leakage at the output of a transmitter. The chromate conversion coated parts of the fabricated filter can be seen in Fig. 3.19. The assembled structure weighs 10.2 g.

Fig. 3.21 depicts the measurement results compared to the simulation results of the machined filter as well as the additive manufactured filter that is discussed

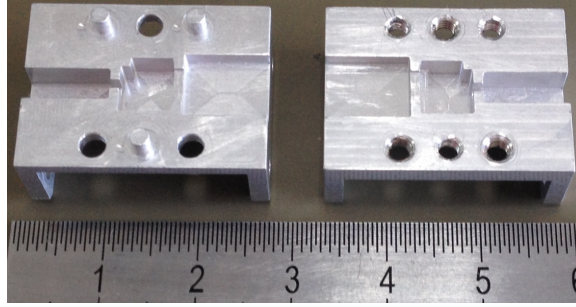


Figure 3.19: Chromate conversion coated parts of the machined filter. The scale is in centimeters.

below. The measurement results of the machined filter are in perfect agreement with the simulation results and the measured in-band insertion loss is below 0.7 dB.

One interesting alternative manufacturing possibility is the use of novel 3D printing technology. It is possible to print the structure's main body with the use of a polymer (wall thickness down to 0.5 mm attainable) and then metal coat the printed bare body¹.

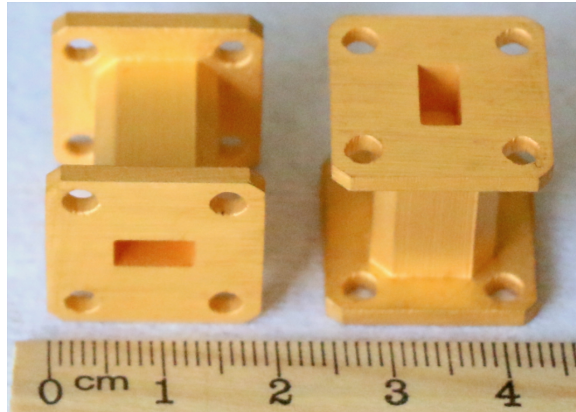


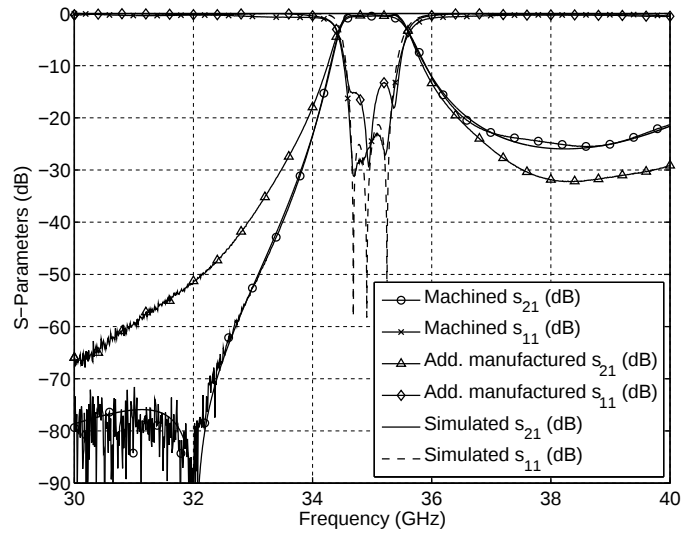
Figure 3.20: Additive manufactured triple mode rectangular cavity filter.

The same filter is also fabricated with additive manufacturing for comparison. The fabricated filter, having a minimum wall thickness of 1 mm and Cu plating of 5 μm with 100 nm Au flash on top of it¹, can be seen in Fig. 3.20. The input

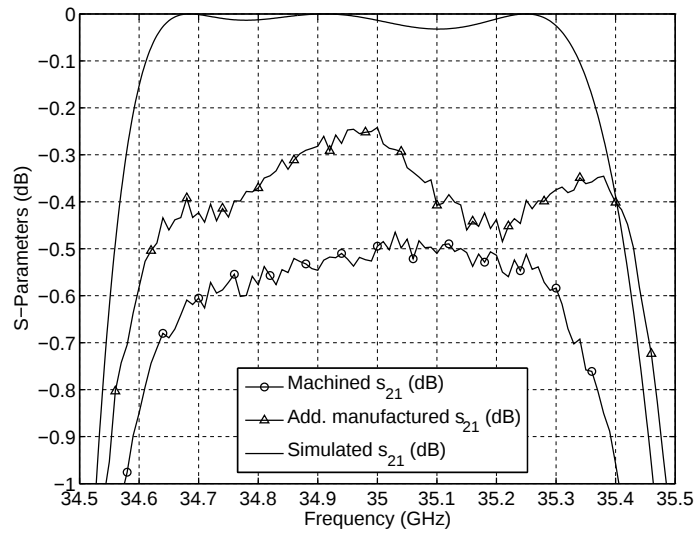
¹"Home swissto12." [Online]. Available: <http://www.swissto12.com/>. Accessed: 2016-05-20.

and output of the filter are designed to be used with standard WR - 28 waveguide flanges. The structure weighs only 4.4 g.

The measurement results are given in Fig. 3.21. The in-band insertion loss is below 0.5 dB. Note that the insertion loss of the additive manufactured filter is lower compared to the machined filter. This is an expected result since copper is a better conductor than aluminum. The usage of gold for the 3D printed filter and chromium for the machined filter as the coating material further favors the printed filter in terms of loss. On the other hand, although the in-band responses are in agreement, the TZ in the response of the printed filter is missing. This is probably because the dimensions of the cross-coupling corner cut are too small to be implemented with this technology in its present state. Nonetheless, the results are very promising: additive manufacturing together with the multiple mode cavity technique are advantageous in filtering applications especially if weight and space have limited budgets.



(a) S-parameters



(b) Close up view of the insertion loss in the pass band

Figure 3.21: The response for the fabricated triple mode triplet structure with a lower sideband finite frequency TZ.

Chapter 4

Filter Embedded Microstrip to Waveguide Transitions

In the previous chapter, the analyzed filter structure had waveguide input/output coupled to the cavity via apertures. It is also possible to excite a waveguide/cavity using coaxial [25, pp. 276-281] or microstrip [26] probes. Hence, it should be possible to form a cavity filter having a microstrip line input and waveguide output (or vice versa).

On the other hand, the transition from a microstrip line to a rectangular waveguide is an important subject especially in millimeter-wave frequencies where antennae are mostly fed by waveguides, whereas active elements are placed on a printed board. It is common to have microwave modules that contain various types of microstrip line to waveguide transitions. One of these structures is the right angle transition formed by a microstrip probe fed directly into the waveguide [27].

The two structures discussed above, i.e., the asymmetrically fed cavity filter and the microstrip to waveguide transition, are similar in the sense that they have a microstrip line input/output and a waveguide output/input. Therefore, it should be possible to replace the transition with the asymmetrically excited filter

structure, especially in millimeter-wave applications where cavity dimensions are relatively small. The advantage would be the additional capability of filtering without significant increase in volume requirement, while the transition goal is also achieved.

This chapter contains the implementation of the above idea with dual/triple mode cavities that result in second/third-order filters that can be integrated in microwave modules. The discussion starts with microstrip excitation of cavities, continues with intermodal coupling mechanisms and concludes with the example designs of second and third-order filters.

4.1 Microstrip probe excitation of waveguides and cavities

The coaxial probe excited rectangular waveguide structure, analyzed in [25, pp. 276-281], is illustrated in Fig. 4.1. Assuming a filamentary, sinusoidal probe current, the radiation resistance of the probe is approximated as:

$$R_0 = \frac{Z_w}{2abk_0^2} |1 - e^{-j2\beta d}|^2 \tan^2\left(\frac{k_0 l}{2}\right) \quad (4.1)$$

where a, b are waveguide dimensions, Z_w is the wave impedance, d is the back-short depth, l is the probe length, β is the propagation constant and k_0 is the wavenumber. The analysis is also valid for the microstrip probe excitation under the filamentary current assumption, i.e., infinitesimal probe width. From the designer's point of view, the implication of (4.1) is that the radiation resistance can be set to the required value by playing with the probe length l and the back-short depth d . On the other hand, the reactance caused by the excitation of evanescent modes needs to be tuned by applying an appropriate reactance [25, p. 281]. A similar approach is taken in [27] with a microstrip probe.

As for the probe excitation of a cavity, although there are excellent discussions as in [9, 18, 14], there is no exact analytical solution for the external Q of the structure. Nonetheless, it is a known practice to use probes to excite cavities [9,

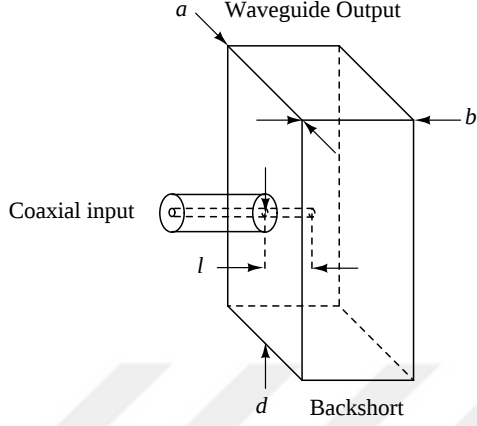


Figure 4.1: The coaxial probe to waveguide transition structure.

p. 291]. As in the waveguide excitation case, using a microstrip probe instead of a coaxial one, it should be possible to adjust the probe's length and position to reach a required level of coupling. The proposed structure is given in Fig. 4.2. The lower half of the cavity can be carved in the body of a microwave module whereas the upper half can be formed in the lid. The shift in resonant frequency of the coupled mode can be compensated by adjusting the corresponding dimension of the cavity as done in the previous chapter.

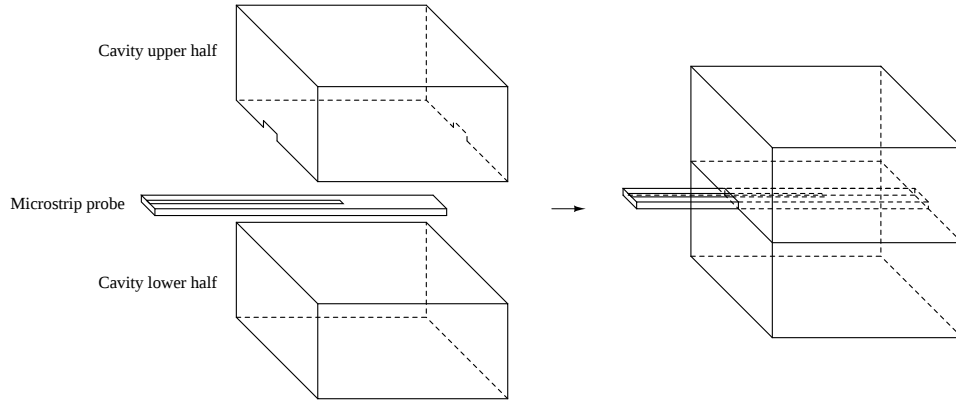


Figure 4.2: The microstrip probe excited cavity structure.

Since no analytic solution is present, computer-aided design (CAD) tools can be used to check the feasibility of the approach. Let us assume that the width of the probe does not affect Q_e significantly as in the waveguide case [27]. Let us also fix the probe position to the center of the cavity's wall from which the

excitation occurs.

Let the substrate be 5 mils thick RT6002 with a dielectric constant $\epsilon_r = 2.94$. Let the input line be a grounded coplanar waveguide (GCPW). For the sake of simplicity, the probe width is chosen to result in a 50Ω microstrip line. The required width is calculated with AWR Microwave Office's TXLine tool. The structure is illustrated in Fig. 4.3, whereas the corresponding dimensions are summarized in Table 4.1.

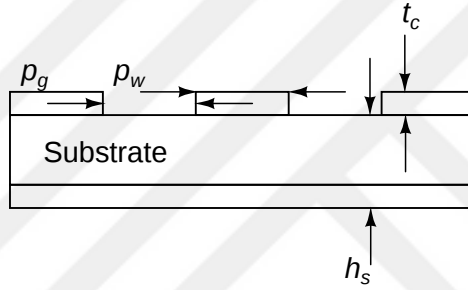


Figure 4.3: Cut-out view of the GCPW structure.

ϵ_r	p_w (mil)	p_g (mil)	h_s (mil)	t_c (μm)
2.94	12	14	5	35

Table 4.1: The dimensions of the GCPW for a 50Ω characteristic impedance.

Under the above mentioned assumptions, an example microstrip probe fed cavity is analyzed using CST Studio's Eigenmode Solver. The dimensions of the example cavity are chosen as $a = b = c = 6.5$ mm. Let the probe opening window width and height be 1 mm both and the cavity wall thickness be 2 mm. The resulting external Q plot can be seen in Fig. 4.4, whereas Fig. 4.5 depicts the resonant frequency shift of the coupled mode. The results show that it is possible to excite a rectangular cavity for a required Q_e value with a microstrip probe. On the other hand, cavity probe coupling results in the lowering of the coupled mode's resonant frequency as in the aperture coupling case.

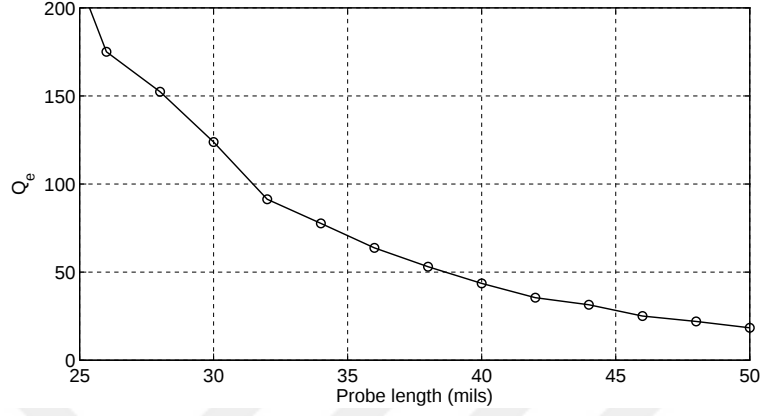


Figure 4.4: External quality factor (Q_e) as a function of microstrip probe length.

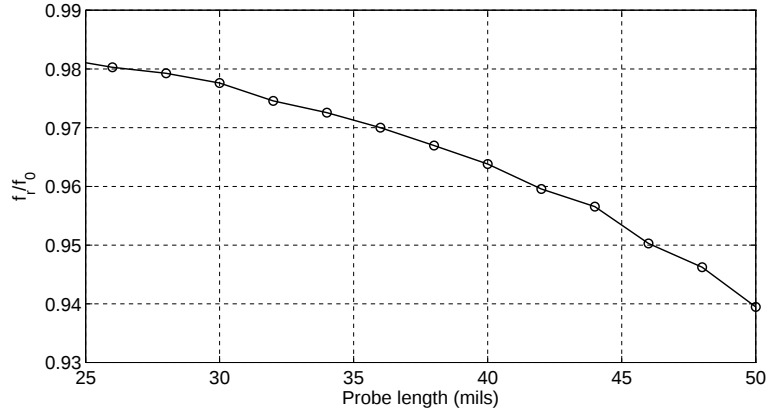


Figure 4.5: Normalized resonant frequency of the coupled mode as a function of microstrip probe length.

4.2 Intermodal coupling mechanism

The coupling corner cut along an axis needs not to go all the way from one wall to the other. An interesting design possibility arises if partial corner cuts as in [13] are used. The idea is illustrated in Fig. 4.6. Within a microwave module, this structure can be realized easily with a metal-backed printed circuit board (or even with the classical two sided printed circuit board) and the module's body.

Unfortunately, the resulting structure can not be investigated analytically using the method presented in Section 4.2, because the E-fields are not uniform along the direction of the corner cut, hence field expressions of (3.3) are no more

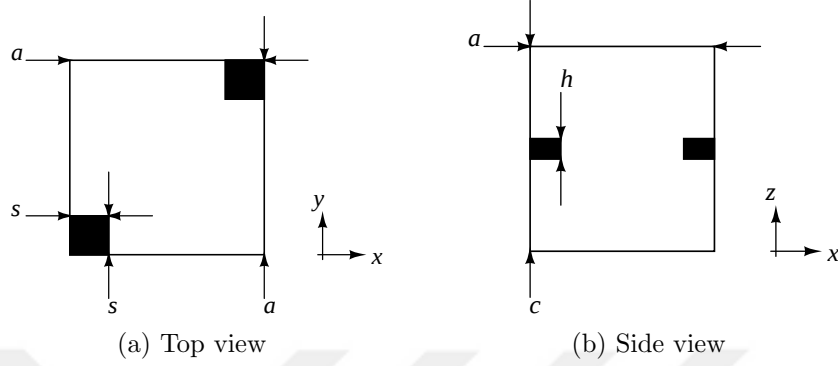


Figure 4.6: Dual mode cavity with partial square corner cuts. The shape of the cuts can be triangular or rounded as well.

valid. Therefore, in order to find out the amount of coupling corresponding to various lengths and heights of corner cut, CAD tools can be used. An example cavity of $a/c = 5/6$ with partial square corner cuts is analyzed with CST Eigenmode Solver and the corresponding coupling values are calculated. The resulting normalized resonant frequencies are given in Fig. 4.7, whereas the calculated coupling coefficient (k) is presented in Fig. 4.8. The results show that increasing the normalized thickness (h/c) of the coupling corner cuts increases k . On the other hand, for a given h/c ratio, the required coupling value can be obtained with the correct choice of normalized corner cut parameter r .

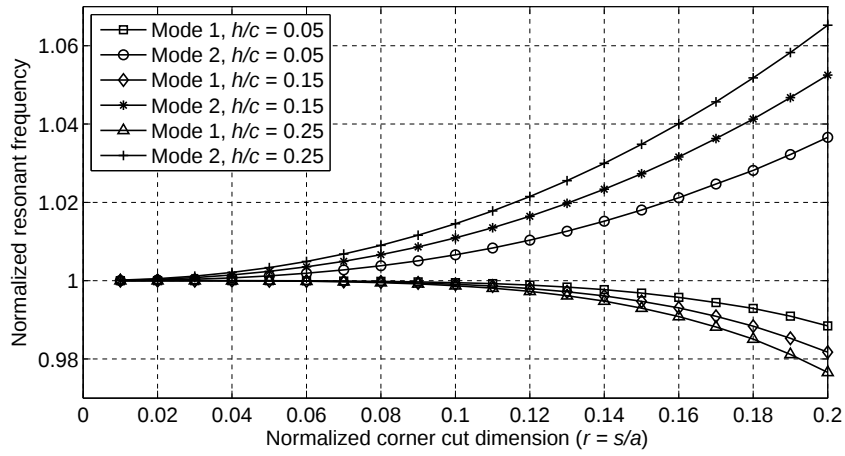


Figure 4.7: Normalized resonance frequencies of the two coupled modes versus r with $a/c = 5/6$.

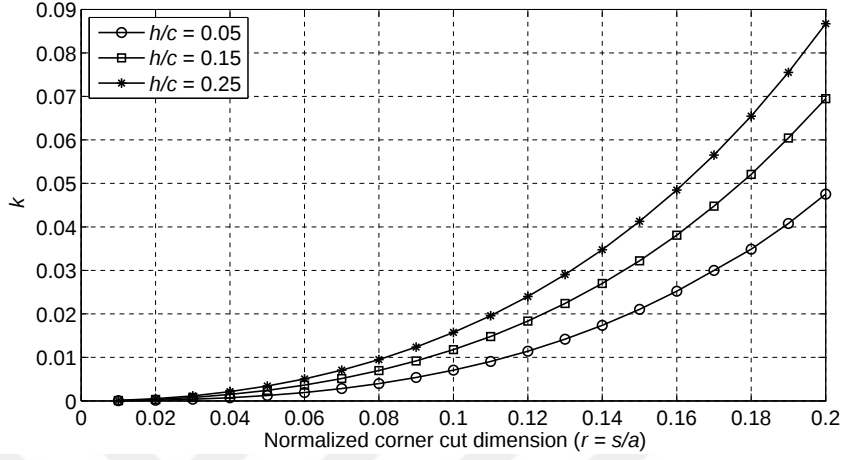


Figure 4.8: The physical coupling coefficient as a function of normalized corner cut dimension with $a/c = 5/6$.

4.3 Second-order filter embedded microstrip to waveguide transition

The cavity microstrip excitation structure discussed in Section 4.1, the inter-modal coupling structure investigated in Section 4.2 and the aperture coupling structure presented in Section 3.3 can be combined to result in a dual mode second-order filter that can replace the classical microstrip line to rectangular waveguide transition. The resulting filter structure can be seen in Fig. 4.9.

The design procedure follows the algorithm proposed in the previous chapter. The analytical expressions obtained are used wherever possible. Let us realize the same filter designed in Section 3.6. The blank cavity with aspect ratio $a/c = 1.25$, resonating at 34 GHz has dimensions of $a = b = 7.06$ mm and $c = 5.65$ mm.

For such a cavity, using the CST Eigenmode Solver, the required triangular corner cut (realized with the metal back of the PCB that has 0.5 mm thickness) is found to be 1.56 mm. The cavity's c dimension is increased to 5.72 mm so that $(f_1 + f_2)/2 = f_0$.

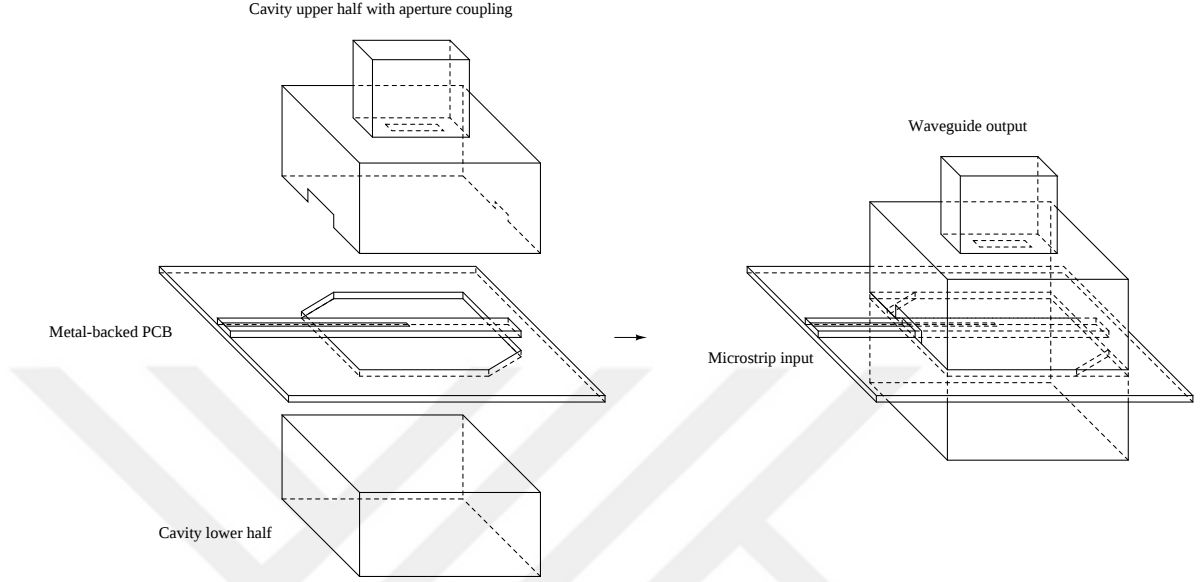


Figure 4.9: The second-order microstrip probe excited cavity filter structure.

In the proposed algorithm, the input and output couplings have the same structure, hence the resulting design has $a = b$. This is no longer the case since the filter is not physically symmetric. Since analytical expressions are already obtained for the aperture coupling case, let us assume for a moment that the structure is symmetric and apply the algorithm's steps 5 through 7. Assuming an aperture thickness of 0.5 mm and height of 1 mm, the resulting aperture length (l_a) is 3.46 mm whereas $c = 5.35$ mm.

On the other hand, the required probe length (l_p) value is found to be 36 mils again using the Eigenmode Solver. The determined initial dimensions as well as the optimized dimensions are summarized in Table 4.2.

	a (mm)	b (mm)	c (mm)	s (mm)	l_a (mm)	l_p (mils)
Initial	7.06	7.06	5.35	1.56	3.46	36
Optimized	7.17	7.04	5.35	1.56	3.53	36
Error (%)	1.6	0.3	0	0	2.0	0

Table 4.2: The physical dimensions of the designed second-order asymmetric input/output filter.

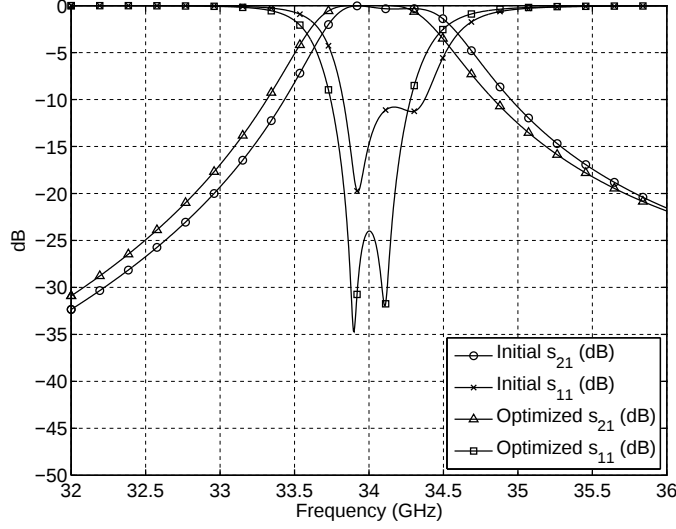


Figure 4.10: The initial response of the designed second-order filter together with the optimized response.

The S-parameters corresponding to initial and optimized dimensions are given in Fig. 4.10. The results show that the same response obtained in Section 3.6 is achieved with the new structure as well.

4.4 Third-order filter embedded microstrip to waveguide transition

The advantage of the second-order filter designed in the previous section is that the cavity carved in the module is square cross-sectioned, lacking the corner cuts used in Chapter 3, hence the fabrication would be relatively easy. But the partial corner cuts realized by the metal-backed board can couple only two modes, therefore partial corner cuts are not sufficient for a higher order filtering function. If a third-order filter is to be designed, the structure investigated in Section 3.7 has to be used with an alteration: the aperture coupled waveguide input replaced with the microstrip probe. The resulting structure is presented in Fig. 4.11.

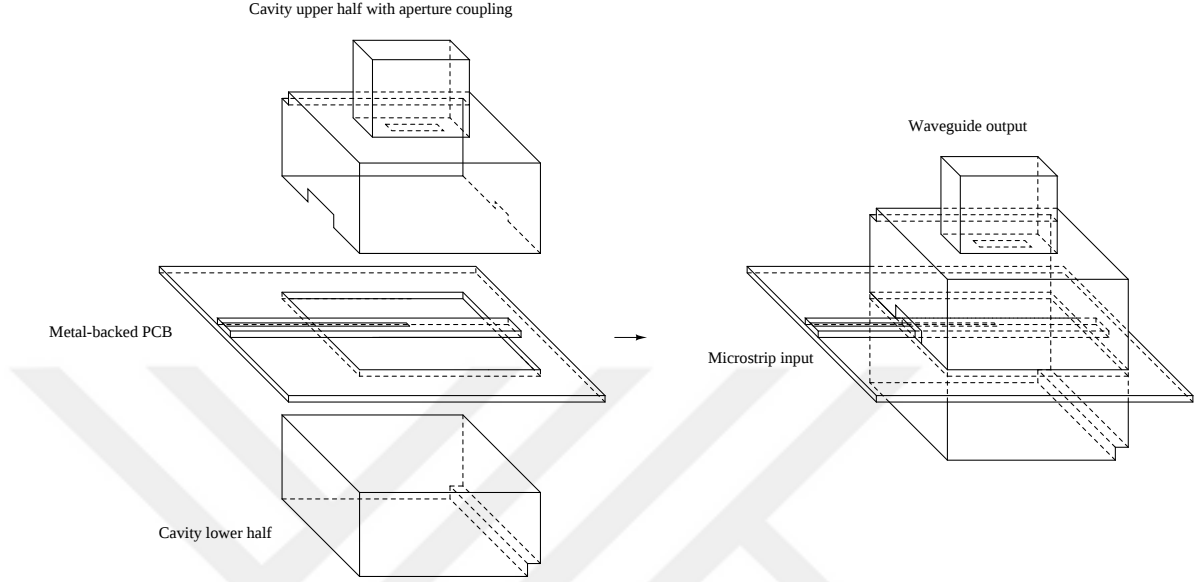


Figure 4.11: The microstrip probe excited third-order cavity filter structure.

In order to design a filter with the same characteristics as in Section 3.7, the previously predicted dimensions can be used assuming that the two structures are similar. Under this assumption, the required probe length is 30 mils resulting in a Q_e of 85.22. The optimization process is more complex compared to the previous case, since now $a \neq b$ and an extra optimization variable comes into play.

	a (mm)	b (mm)	c (mm)	s (mm)	l_a (mm)	l_p (mils)
Initial	6.27	6.27	5.89	0.72	3.49	30
Optimized	6.26	6.33	5.89	0.72	3.46	31
Error (%)	0.16	0.96	0	0	0.86	3.33

Table 4.3: The physical dimensions of the designed third-order asymmetric input/output filter.

The obtained optimized dimensions are given in Table 4.3, whereas the initial and optimized responses are depicted in Fig. 4.12. It can be seen that a similar response can be achieved with the new structure as well. Note that, combining the third-order asymmetrically fed filter structure with the additional coupling capability of the metal-backed board (as a cross-coupling), it is also possible to form triplets as well.

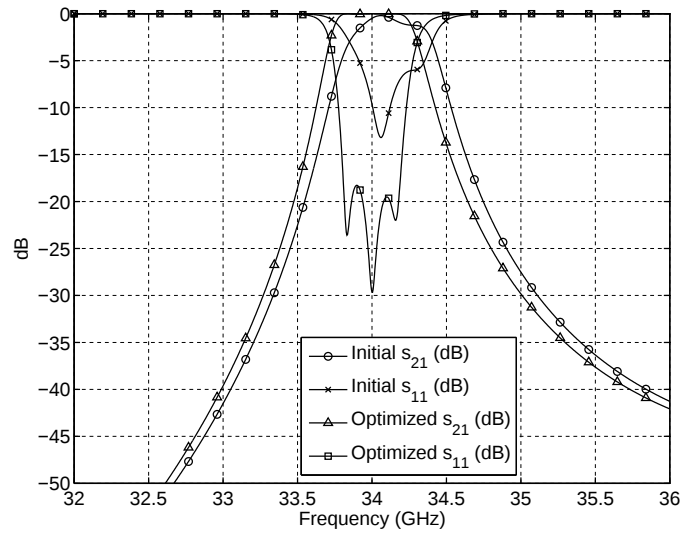


Figure 4.12: The initial response of the designed third-order filter together with the optimized response.

Chapter 5

Conclusion

In this work, a brief introduction to filter design via the coupling matrix approach is given. MatLab codes corresponding to some design steps are provided. The main focus of the work, the multiple mode single rectangular cavity structure is investigated analytically. As a result, expressions corresponding to different entries of the coupling matrix are obtained. An algorithm is proposed making use of the obtained expressions. The proposed algorithm is able to determine the physical dimensions of second and third-order filters within a few percent.

The dispersion mechanism leading to asymmetrical filter response is also studied. The dispersive behavior due to rectangular aperture coupling can be injected to the theoretical response easily by multiplying the corresponding coupling matrix entries with the expression proposed.

The classical triple mode rectangular cavity structure is altered with an additional corner cut that realizes a cross-coupling. The cross-coupling's sign can be alternated easily by rotating the corner that is cut by 90° . As a result, the newly formed triplet's response displays a TZ either at the upper or lower sideband of the operating frequency, based on the sign of the cross-coupling.

An example design with a lower sideband TZ is fabricated with both machining and additive manufacturing. The machined filter's measured response is in perfect

agreement with the expectation. On the other hand, the response of the printed filter is in good agreement with the simulation result except for the presence of the TZ. Nonetheless, the results show that the usage of the two techniques together, i.e., multiple mode cavity and 3D printing, very lightweight filters can be designed and fabricated. The printed example third-order filter weighs only 4.4 g.

Since the size of the resonating cavity decreases with increasing frequency, at *Ka*-Band frequencies, it becomes possible to fit it within a microwave module. It is shown that second and third-order filters (as well as triplets) can be integrated to *Ka*-Band microwave modules incorporating waveguide outputs, without significant increase in volume requirement.

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Appendix A

Matlab Codes

A.1 Lowpass prototype element generation

A.1.1 Lowpass prototype for Butterworth filtering function

```
1 function g = maxflat(n)
2 %   maxflat: Maximally flat low pass prototype filter
3 %coefficient calculation
4 %   The function takes the filter order (n) as input
5 %and generates the n+1 filter elements. The formulae
6 %are based on Matthaei's book.
7 %   n: Filter order
8 %   g: Prototype element values
9 g = ones(1, n+2);
10 for i = 2:n+1
11     g(i) = 2*sin((2*(i-1)-1)*pi/2/n);
12 end
```

A.1.2 Lowpass prototype for Chebyshev filtering function

```
1 function g = chebyshev(n, Lar)
2 %   chebyshev: Chebyshev low pass prototype filter coefficient
3 %calculation
4 %   The function takes the filter order (n) and the passband
5 %ripple (Lar in dB) as input and generates the n+1 filter
6 %elements. The formulae are based on Matthaei's book.
7 %   n: Filter order
8 %   Lar: Passband ripple
9 %   g: Prototype element values
10 g = ones(1, n+2);
11 beta = log(coth(Lar/17.37));
12 gamma = sinh(beta/(2*n));
13 a = zeros(1, n);
14 b = zeros(1, n);
15 for i = 1:n
16     a(1,i) = sin((2*i-1)*pi/2/n);
17     b(1,i) = gamma^2 + (sin(i*pi/n))^2;
18 end
19 g(1,2) = 2*a(1,1)/gamma;
20 for i = 2:n
21     g(1,i+1) = 4*a(1, i-1)*a(1,i)/b(1, i-1)/g(1,i);
22 end
23 if rem(n,2) == 1
24     g(1, n+2) = 1;
25 else
26     g(1, n+2) = (coth(beta/4))^2;
27 end
28 end
```

A.2 Coupling matrix synthesis

A.2.1 Characteristic polynomial generation function

```
1 function [E F P eps] = calcEFP(N, fz, RL)
2 % calcEFP: Characteristic polynomials generation
3 % The function calculates E, F and P polynomials'
4 %coefficients. The inputs are the filter order (N),
5 %the normalized TZ vector (fz) and maximum in-band
6 %input return loss (RL) in dB. The procedure is based on
7 %Cameron's book pp. 225-228.
8 %N: Filter order
9 %fz: Normalized TZ vector
10 %RL: Maximum in-band return loss
11 %E,F,P: Characteristic polynomials' coefficient vector
12 %eps: Normalization constant
13 %-----
14 %Number of prescribed zeros
15 nfz = length(fz);
16 %Calculate the roots for U and V functions
17 [U V] = calcUV(N,fz);
18 Froots = 1j*roots(U);
19 Vroots = 1j*roots(V);
20 %eps and er calculation
21 if (nfz < N)
22     er = 1;
23 %Needs to be calculated if nfz = N, this part is not implemented
24 else
25     er = 1;
26 end
27 %Value of P(w) at 1j
28 p3dB = 1;
29 for i = 1:length(fz)
30     p3dB = p3dB*(1j-1j*fz(i));
31 end
32 %Value of F(w) at 1j
33 f3dB = 1;
```

```

34 for i = 1:length(Froots)
35     f3dB = f3dB*(1j-Froots(i));
36 end
37 %The normalization constant e
38 eps = 1/(sqrt(10^(RL/10)-1))*abs((p3dB)/(f3dB)/er);
39 %Form P(s)
40 P = roots2poly(1j*fz, N);
41 if rem(N-nfz,2) == 0
42     P=P*1j;
43 end
44 %Form F(s)
45 F = roots2poly(Froots, N);
46 %Find the roots of E polynomial
47 E = P/eps + F/er;
48 Eroots = roots(E);
49 %Play with the roots so that E(s) is Hurwitz
50 for i=1:length(Eroots)
51     if (real(Eroots(i)) > 0)
52         Eroots(i) = -conj(Eroots(i));
53     end
54 end
55 %Form E(s)
56 E = roots2poly(Eroots, N);
57 end

```

A.2.2 Coupling matrix generation function

```

1 function [MM] = calcNP2(E, F, P, N, eps)
2 % calcNP2: Coupling matrix generation
3 % The function takes characteristic polynomials'
4 %coefficients and filter order as input and calculates
5 %the N+2 coupling matrix
6 %E, F, P: Characteristic polynomials
7 %eps: Normalization constant for the polynomials
8 %N: Filter order
9 %MM: Generated coupling matrix
10 %-----

```

```

11 %Calculate m and n polynomials
12 m1 = zeros(1, N+1);
13 n1 = zeros(1, N+1);
14 for i=0:N
15     if rem(i,2) == 0 %even
16         m1(N+1-i) = real(E(N+1-i) + F(N+1-i));
17         n1(N+1-i) = 1j*imag(E(N+1-i) + F(N+1-i));
18     else %odd
19         m1(N+1-i) = 1j*imag(E(N+1-i) + F(N+1-i));
20         n1(N+1-i) = real(E(N+1-i) + F(N+1-i));
21     end
22 end
23 if rem(N,2) == 1 %odd
24     y21den = n1;
25     y21num = P/eps/y21den(1);
26     y22num = m1/y21den(1);
27     y22den = n1/y21den(1);
28     y21den = y21den/y21den(1);
29 else %even
30     y21den = m1;
31     y21num = P/eps/y21den(1);
32     y22num = n1/y21den(1);
33     y22den = m1/y21den(1);
34     y21den = y21den/y21den(1);
35 end
36 %Find the residues, eigenvalues, eigen vectors
37 MSL = y21num(1)/y21den(1)/(1j);
38 [r22, p22, ~] = residue(y22num, y22den);
39 [r21, ~, ~] = residue(y21num, y21den);
40 lambdak = imag(p22);
41 TNk = sqrt(r22);
42 T1k = r21./sqrt(r22);
43 %Form the coupling matrix
44 MM = zeros(N+2);
45 for i=2:N+1
46     MM(i,i) = -lambdak(i-1);
47 end
48 MM(2:N+1,1) = real(T1k);
49 MM(1,2:N+1) = real(T1k);
50 MM(2:N+1,N+2) = real(TNk);

```

```

51 MM(N+2,2:N+1) = real(TNk);
52 MM(1,N+2) = MSL;
53 MM(N+2,1) = MSL;
54 MM = cleanM(MM);
55 end

```

A.2.3 Example coupling matrix generation main script

```

1 clear
2 clc
3 close all
4 %-----INPUTS-----
5 %Filter order
6 N = 5;
7 %Return loss in dB
8 RL = 25;
9 %Prescribed zeros
10 Qu = 4000;
11 %Filter center frequency
12 fo = 30e9;
13 %Filter bandwidth
14 bw = 1500e6;
15 %Analysis lower and upper bounds
16 lb = 25e9;ub = 35e9;
17 %Prescribed transmission zero normalization
18 tz1 = normTZ(fo, bw, 32e9);
19 tz2 = normTZ(fo, bw, 34e9);
20 tz3 = normTZ(fo, bw, 27e9);
21 %Normalized prescribed zero vector
22 fz = [tz1 tz2 tz3];
23 %-----CALCULATE THE CHARACTERISTIC POLYNOMIALS-----
24 [E F P eps] = calcEFP(N, fz, RL);
25 %-----CALCULATE THE COUPLING MATRIX-----
26 NP2 = calcNP2(E,F,P,N,eps)
27 %-----FOLDED FORM-----
28 %Reduce the N+2 coupling matrix to folded configuration
29 NP2folded = reduce2FoldedNP2(NP2);

```

```

30 NP2folded = reformCM(NP2folded);
31 NP2folded = cleanM(NP2folded)
32 %Plot the s-parameters corresponding to the folded
33 %coupling matrix
34 figure('position', [100, 100, 800, 600])
35 [s21 s11 f] = sparamNP2(NP2folded, fo, bw, lb, ub, Qu);
36 plot(f/1e9, 20*log10(abs(s21)), 'k-', ...
37      f/1e9, 20*log10(abs(s11)), 'k--', ...
38      'linewidth', 1.5)
39 grid on
40 axis([(min(f)/1e9) (max(f)/1e9) -90 0])
41 xlabel('Frequency (GHz)', 'fontsize', 12);
42 ylabel('S-parameters (dB)', 'fontsize', 12);
43 legend('s21(dB)', 's11(dB)')
44 set(gca, 'fontsize', 12)
45 set(gca, 'linewidth', 1)
46 %-----ARROW FORM-----
47 NP2Arrow= reduce2ArrowNP2(NP2);
48 NP2Arrow = reformCM(NP2Arrow);
49 NP2Arrow = cleanM(NP2Arrow)

```

A.3 Coupling matrix reconfiguration

A.3.1 Reconfiguration to folded form

```

1 function [NP2new] = reduce2FoldedNP2( NP2 )
2 %   reduce2FoldedNP2: N+2 coupling matrix reconfiguration
3 %to folded form
4 %   The function takes N+2 coupling matrix of any form as
5 %input and generates reconfigurates it to folded form.
6 %The procedure is based on Cameron's book.
7 %   NP2: N+2 coupling matrix of any form
8 %   NP2new: N+2 coupling matrix of folded form
9 NP2new = NP2;
10 %N is the filter order

```

```

11 N = length(NP2)-2;
12 %Start with the first row
13 i = 1;
14 for b =1:N-1
15     j = N+2-b;
16     p1 = j-1;
17     p2 = j;
18     teta = atan(-NP2new(i, j)/NP2new(i, j-1));
19     R = eye(N+2);
20     R(p1,p1) = cos(teta);
21     R(p2,p2) = cos(teta);
22     R(p2,p1) = sin(teta);
23     R(p1,p2) = -sin(teta);
24     Rt = transpose(R);
25     NP2new = R*NP2new*Rt;
26 end
27 %Continue with last column
28 j = N+2;
29 for b =3:N
30     i = b;
31     p1 = i;
32     p2 = i+1;
33     teta = atan(NP2new(i, j)/NP2new(i+1, j));
34     R = eye(N+2);
35     R(p1,p1) = cos(teta);
36     R(p2,p2) = cos(teta);
37     R(p2,p1) = sin(teta);
38     R(p1,p2) = -sin(teta);
39     Rt = transpose(R);
40     NP2new = R*NP2new*Rt;
41 end
42 %Continue with other rows and columns
43 if rem(N,2)==1 %If odd
44     ub = floor((N+2)/2-1);
45 else
46     ub = floor((N+2)/2-2);
47 end
48 for a = 2:ub
49     i=a;
50     %Rows

```

```

51     for b = i+2:(N+2-i)
52         j = N+4-b;
53         p1 = j-1;
54         p2 = j;
55         teta = atan(-NP2new(i, j)/NP2new(i, j-1));
56         R = eye(N+2);
57         R(p1,p1) = cos(teta);
58         R(p2,p2) = cos(teta);
59         R(p2,p1) = sin(teta);
60         R(p1,p2) = -sin(teta);
61         Rt = transpose(R);
62         NP2new = R*NP2new*Rt;
63     end
64     %Columns
65     j = N+3-a;
66     for b = (N+5-j):(j-2)
67         i = b;
68         p1 = i;
69         p2 = i+1;
70         teta = atan(NP2new(i, j)/NP2new(i+1, j));
71         R = eye(N+2);
72         R(p1,p1) = cos(teta);
73         R(p2,p2) = cos(teta);
74         R(p2,p1) = sin(teta);
75         R(p1,p2) = -sin(teta);
76         Rt = transpose(R);
77         NP2new = R*NP2new*Rt;
78     end
79 end
80 if rem(N,2)==0 %If even
81     %End with the last row
82     i=floor((N+2)/2-1);
83     for b = i+2:N+2-i
84         j = N+4-b;
85         p1 = j-1;
86         p2 = j;
87         teta = atan(-NP2new(i, j)/NP2new(i, j-1));
88         R = eye(N+2);
89         R(p1,p1) = cos(teta);
90         R(p2,p2) = cos(teta);

```

```

91         R(p2,p1) = sin(teta);
92         R(p1,p2) = -sin(teta);
93         Rt = transpose(R);
94         NP2new = R*NP2new*Rt;
95     end
96 end
97 end

```

A.3.2 Reconfiguration to arrow form

```

1 function [ NP2new ] = reduce2ArrowNP2 (NP2)
2 %   reduce2ArrowNP2: N+2 coupling matrix reconfiguration
3 %to folded form
4 %   The function takes N+' coupling matrix of any form as
5 %input and generates reconfigurates it to folded form.
6 %The procedure is based on Cameron's book.
7 %   NP2: N+2 coupling matrix of any form
8 %   NP2new: N+2 coupling matrix of folded form
9 %Filter order
10 n = length(NP2) - 2;
11 NP2new = NP2;
12 for i=1:n-1
13     for j=i+1:n
14         %The pivot coordinates
15         a = i+1; b = j+1; k = i;
16         %Rotation angle
17         teta = -atan(NP2new(k,b)/NP2new(k,a));
18         %Perform rotation
19         R = eye(n+2);
20         R(a,a) = cos(teta); R(b,b) = cos(teta);
21         R(b,a) = sin(teta); R(a,b) = -sin(teta);
22         NP2new = R*NP2new*transpose(R);
23     end
24 end
25 end

```

A.4 Auxiliary functions

A.4.1 $U(w)$ - $V(w)$ polynomials generation

```
1 function [U V] = calcUV(N, fz)
2 % calcUV: U(w)-V(w) polynomials generation
3 % The function calculates U(w) and V(w) polynomials'
4 %coefficients that are used as an intermediate step in
5 %calculating the characteristic polynomials. The inputs
6 %are the filter order (N) and the normalized TZ vector
7 %(fz). The recursive procedure is based on Cameron's book
8 %pp. 225-228.
9 %N: Filter order
10 %fz: Normalized TZ vector
11 %U: U(w) polynomial's coefficients
12 %V: V(w) polynomial's coefficients
13 %-----
14 %Number of prescribed zeros
15 nfz = length(fz);
16 %Zeros vector
17 z = zeros(1, N);
18 z = z + inf;
19 for i = 1:nfz
20     z(i) = fz(i);
21 end
22 %Calculate the initial U and V coefficients
23 a = -1/z(1);
24 b = sqrt(1-1/(z(1)^2));
25 U = zeros(1,N+1);
26 V = zeros(1,N+1);
27 wp = zeros(1,N+1);
28 wp(1) = -1;
29 wp(3) = 1;
30 U(1)=a;
31 U(2)=1;
32 V(1)=b;
33 %Perform the remaining iterations
```

```

34 for i=2:N
35     P = zeros(4,2*N+1);
36     temp = zeros(1,2*N+1);
37     %Calculate the a and b values for the the new TZ
38     a = -1/z(i);
39     b = sqrt(1-1/(z(i)^2));
40     %Calculate the new U and V coefficients
41     Unew = zeros(1,N+1);
42     Vnew = zeros(1,N+1);
43     Unew(1)=a;
44     Unew(2)=1;
45     Vnew(1)=b;
46     for x = 0:N
47         for y = 0:N
48             %Compute U(i)*Unew
49             P(1,x+y+1) = U(x+1) * Unew(y+1) + P(1,x+y+1);
50             %Compute U(i)*Vnew
51             P(2,x+y+1) = U(x+1) * Vnew(y+1) + P(2,x+y+1);
52             %Compute V(i)*Unew
53             P(3,x+y+1) = V(x+1) * Unew(y+1) + P(3,x+y+1);
54             %Compute V(i)*Vnew
55             P(4,x+y+1) = V(x+1) * Vnew(y+1) + P(4,x+y+1);
56         end
57     end
58     %Update V
59     V = P(2,1:length(V))+P(3,1:length(V));
60     %Replace w'2 by w^2-1
61     for x = 0:N
62         for y = 0:N
63             %Compute V(i)*Vnew
64             temp(x+y+1) = P(4,x+1) * wp(y+1) + temp(x+y+1);
65         end
66     end
67     %Update U
68     U = P(1,1:length(U))+temp(1:length(U));
69 end
70 %Reverse the coeff order of U and V polynomials
71 temp1 = zeros(1,N+1);
72 temp2 = zeros(1,N);
73 for i = 1:N;

```

```

74     temp1(i) = U(N+2-i);
75     temp2(i) = V(N+1-i);
76 end
77 temp1(N+1) = U(1);
78 U = temp1;
79 V = temp2;
80 return;

```

A.4.2 Polynomial coefficient generation

```

1 function [poly] = roots2poly(rts, N)
2 % roots2poly: Polynomial coefficients generation based on its
3 %roots
4 % The function takes the roots of a polynomial as input and
5 %calculates its coefficients. It acts as the inverse function
6 %of roots().
7 % N: Polynomial order
8 % rts: Roots vector
9 % poly: Polynomial coefficients vector
10 poly = zeros(1,2*N+1);
11 poly(1) = -rts(1);
12 poly(2) = 1;
13 for i=2:length(rts)
14     temp = zeros(1,N+1);
15     temp(1) = -rts(i);
16     temp(2) = 1;
17     temp2 = zeros(1,2*N+1);
18     for x=0:N
19         for y = 0:N
20             temp2(x+y+1) = poly(x+1) * temp(y+1) + temp2(x+y+1);
21         end
22     end
23     poly = temp2;
24 end
25 %Reverse the coeff order of F polynomial
26 temp1 = zeros(1,N+1);
27 for i = 1:N;

```

```

28     temp1(i) = poly(N+2-i);
29 end
30 temp1(N+1) = poly(1);
31 poly = temp1;
32 end

```

A.4.3 Transmission zero normalization

```

1 function [ntz] = normTZ(fo, bw, tz)
2 % normTZ: Transmission zero (TZ)normalization.
3 % The function takes the TZ frequency, filter center
4 %frequency and filter bandwidth as input and calculates
5 %the normalized value of the TZ.
6 %fo: Filter center frequency in Hz
7 %bw: Filter bandwidth in Hz
8 %tz: TZ frequency in Hz
9 %ntz: Normalized TZ value
10 %-----
11 ntz = fo/bw*(tz/fo - fo/tz); %See Cameron p.281
12 end

```

A.4.4 Coupling matrix cleaning

```

1 function [ cNP ] = cleanM( NP )
2 % cleanM: Coupling matrix cleaning.
3 % The function takes a coupling matrix as input and equates
4 %entries that are less than 0.0005 to zero.
5 %NP: input coupling matrix
6 %cNP: cleaned coupling matrix
7 %-----
8 cNP = NP;
9 th = 0.0005;
10 s = size(NP);

```

```

11 for i=1:s(1)
12     for j=1:s(2)
13         if abs(cNP(i,j)) < th
14             cNP(i,j) = 0;
15         end
16     end
17 end
18 end

```

A.4.5 Coupling matrix reforming

```

1 function [ rCM ] = reformCM( CM )
2 % reformCM: Coupling matrix reforming
3 % The function takes a coupling matrix as input and tries
4 %to decrease the number of negative couplings via similarity
5 %transforms.
6 %CM: input coupling matrix
7 %rCM: reformed coupling matrix
8 rCM = CM;
9 for i = 1:length(CM)-1
10     j = i+1;
11     if (rCM(i,j) < 0)
12         rCM(j,:) = -rCM(j,:);
13         rCM(:,j) = -rCM(:,j);
14     end
15 end

```

A.4.6 Coupling matrix analysis

```
1 function [s21 s11 f] = sparamNP2( NP2, fo, bw, lb, ub, Qu)
2 % sparamNP2: Coupling matrix analysis
3 %   The function takes a N+2 coupling matrix, analysis lower
4 %and upper bounds, filter center frequency, bandwidth and
5 %unloaded Q as input and calculates the corresponding
6 %s-parameters. The analysis is based on Cameron's book.
7 %NP2: N+2 coupling matrix
8 %fo: Filter center frequency in Hz
9 %bw: Filter bandwidth in Hz
10 %lb: Analysis lower bound in Hz
11 %ub: Analysis upper bound in Hz
12 %Qu: Unloaded Q
13 %s21: s21 vector
14 %s11: s11 vector
15 %f: Frequency vector
16 %-----
17 %Filter Order
18 N = length(NP2)-2;
19 %Source and load impedances
20 Rs = 1;
21 Rl = 1;
22 %Identity matrix
23 I = eye(N+2);
24 I(1,1) = 0;
25 I(N+2, N+2) = 0;
26 %Input and output impedances
27 R = zeros(N+2);
28 R(1,1) = Rs;
29 R(N+2, N+2) = Rl;
30 %-----
31 %Frequency mapping
32 f = lb:1e5:ub;
33 %Complex frequency variable with d loss
34 d = fo/bw/Qu;
35 s = d+1j*fo/bw*(f/fo - fo./f); %See Cameron p.281
36 %-----S-parameter calculation-----
37 s11 = zeros(1, length(f));
```

```
38 s21 = zeros(1, length(f));
39 for i=1:length(s)
40     P = (1j*NP2 + s(i)*I + R)^-1;
41     s11(i) = 1 - 2*Rs*P(1,1);
42     s21(i) = 2*sqrt(Rs*Rl)*P(end,1);
43 end
44 end
```

