

Development of an Efficient Electromagnetic Simulation Algorithm for
Planar Geometries with Multiple Vertical Conductors

by

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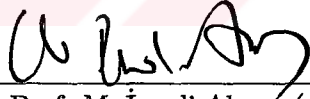
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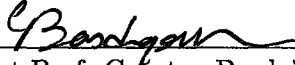
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To my mother and father



ABSTRACT

An efficient electromagnetic simulation algorithm, which is based on the solution of mixed-potential integral equation (MPIE) via the method of moments (MoM), is presented for the analysis of general planar geometries with multiple vertical metallizations. Although the algorithm itself is the main contribution of this thesis, there have been two additional contributions in the steps towards this goal. These contributions are in the form of thorough investigation of the two subtopics leading to the main algorithm, namely, closed-form Green's functions and MoM matrix entries.

Since the closed-form Green's functions are employed in and the vital part of the algorithm presented in this thesis, the steps of getting these Green's functions are critically reviewed, not only for the sake of completeness, but also for clarifying some misunderstandings in the derivation, and the limitations of the method. As it is well-known, the derivation of the closed-form Green's functions begins with the approximation of the spectral-domain Green's functions by complex exponentials, and continues with applying the Sommerfeld identity to cast these approximated spectral-domain Green's functions into the space domain in closed forms. During the last decade, several questions have been raised on the mathematical validity of some steps in the derivation, and remedies have been proposed. These questions can be categorized under the topics of branch point contributions, the effect of quasi-static terms, and the contributions of the surface wave poles to the closed-form Green's functions. The potential problems of the derivation of the closed-form Green's functions, with a priority given to the topics addressed in the literature, have been discussed in detail, and demonstrated that some are due to misunderstandings, some due to numerical errors interpreted as physical errors, and the others are the mere limitations of the method.

The numerical technique employed in this work for the analysis of planar geometries is based on the solution of MPIE using the spatial-domain MoM in conjunction with the closed-form Green's functions. The introduction of closed-form Green's functions into the MoM formulation has already resulted in a significant computational improvement for the

analysis of planar geometries. However, in cases of vertical metallization, the computational efficiency of the method cannot be improved by simply casting the Green's functions in closed forms, which is due to some difficulties encountered in incorporating the closed-form Green's functions into the MoM formulation for such geometries. In this work, a solution to circumvent these difficulties is proposed, and it is demonstrated that multiple vertical conductors can be efficiently modelled and incorporated into the simulation algorithm, provided all vertical metallization have the same dependence in the vertical direction. The results obtained from the algorithm proposed in this thesis are compared to those obtained from commercially available software and experiments.

Keywords: Green's functions, DCIM, Full-wave EM analysis, Method of Moments, 3-D Planarly Layered Media, Vertical Conductors

ÖZET

Çoklu Dikey İletkenli Düzlemsel Geometriler için Verimli Elektromagnetik Simulasyon Algoritmasının Geliştirilmesi

Çoklu dikey metalizasyonlu genel düzlemsel geometrilerin analizi için Moment Metodu uygulanmış MPIE (Karışık Potansiyel İntegral Denklemi) çözümü temelli bir verimli elektromagnetik simulasyon algoritması sunulmuştur. Bu tezin temel bilimsel katkısı (contribution) algoritmanın kendisi olmasına rağmen, bu amaç doğrultusunda ek olarak iki bilimsel katkı daha bulunmaktadır. Bu katkılar kapalı form Green fonksiyonları ve MoM matris girişleri olarak adlandırılan ana algoritmadaki iki adet alt başlığın araştırılması formundadır.

Bu tezde sunulan algoritmanın en önemli kısmında kapalı form Green fonksiyonları uygulandığı için bunlar kritik olarak gözden geçirilmiştir. Aynı zamanda sadece Green fonksiyonlarının elde ediliş aşamaları değil bunların türetilmesi ve metodun sınırlamalarındaki bazı yanlış anlaşılmalara da açıklık getirilmiştir. Bilindiği gibi kapalı form Green fonksiyonlarının türetilmesi kompleks üstelli frekans tanımlı (spektral domain) Green fonksiyonlarının yaklaşıklanması (veya yaklaşımı) ile başlar ve kapalı formdaki uzay tanımına (space domain) konan bu yaklaşık (veya yaklaşıklanmış) Green fonksiyonlarına Sommerfeld özdeşliğinin uygulanması ile devam eder. Son on yıl içerisinde, bu türetim işlemindeki bazı adımlarının matematiksel geçerliliği hakkında çeşitli sorular ortaya atılmış ve çözümler önerilmiştir. Bu sorular sapma noktalarının etkileri (branch point contribution), yarı durgun terimlerin etkileri, ve kapalı form Green fonksiyonlarına yüzey dalga kutuplarının katkısı başlıkları altında sınıflandırılabilir. Literatürde belirtilen başlıklarda verilen kapalı form Green fonksiyonlarının türetilmesindeki potansiyel problemler öncelikli olarak detaylı bir şekilde tartışıldı ve gösterildi ki, bu problemlerin bazıları yanlış anlamadan bazıları fiziksel hataların sayısal (nümerik) hata olarak gösterilmesinden ve bazıları da çok az da olsa metodun sınırlamalarından kaynaklanmaktadır.

Düzlemsel geometrilerin analizi için bu çalışmadaki uygulanan sayısal teknik kapalı form Green fonksiyonları ile ilişkili uzay tanımı MoM çözümünün kullanıldığı ve MPIE formülasyonu üzerine kurulmuştur. MoM formülasyonundaki kapalı form Green fonksiyonları-

nın tanımı düzlemsel geometrilerin analizi için yapılan etkili sayısal iyileştirmeler de hali hazırda sonuçlandırılmıştır. Ancak dikey metalizasyon durumlarında Green fonksiyonlarını, basitçe kapalı forma getirmek metodun hesap verimliliğini artırmaz. Çünkü bu geometriler için kapalı form Green fonksiyonlarının MoM ile formülasyonunda bazı zorluklarla karşılaşılır.

Bu çalışmada bu zorluklardan kaçınmak için bir çözüm önerilmiştir ve gösterilmiştir ki, çoklu dikey iletkenler verimli bir şekilde modellenebilir ve simulasyon algoritmasına konabilir. Verilen bütün dikey metalizasyonlar dikey doğrultuda aynı bağımlılıktadır. Bu tezde önerilen algorithmadan elde edilen sonuçlar ticari olarak kullanılan bilgisayar programları ve deneysel sonuçlarla karşılaştırılmıştır.

Anahtar kelimeler: Green fonksiyonları, Ayrık kompleks görüntü metodu, Tam dalga EM analiz, Moment Metodu, 3- boyutlu düzlemsel katmanlı ortam, Dikey iletkenler.

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NOMENCLATURE

CPU	Central Processing Unit
CFGF	Closed-form Green's Functions
DCIM	Discrete Complex Image Method
EFIE	Electric Field Integral Equation
EM	Electromagnetic
FEM	Finite Element Method
FDTD	Finite-Difference Time Domain
GPOF	Generalized Pencil of Functions
HED	Horizontal Electric Dipole
HMD	Horizontal Magnetic Dipole
LU	Lower Upper
MMIC	Monolithic Microwave Integrated Circuits
MoM	Method of Moments
MPIE	Mixed Potential Integral Equation
MFIE	Magnetic Field Integral Equation
ORGE	Order Recursive Gaussian Elimination
PCB	Printed Circuit Board
PEC	Perfect Electric Conductor
RF	Radio Frequency
SIP	Sommerfeld Integration Path
SWP	Surface Wave Pole
TE	Transverse Electric
TM	Transverse Magnetic
VED	Vertical Electric Dipole
VMD	Vertical Magnetic Dipole

Chapter 1

INTRODUCTION

During the last three decades, the use of multilayer printed geometries has been increasing steadily, especially in the applications of microstrip antennas and monolithic microwave integrated circuits (MMIC). This increase can be attributed partially to the physical features of printed structures, such as, being lightweight and conformable, and partially to the cost of the products (circuits), as printed circuits are cheaper to produce, easy to manufacture circuits with good reproducibility at low cost. To help understand this inevitable change towards printed circuit configuration, note that the products replaced mostly by the printed circuit elements were initially waveguides and coaxial lines, which are bulky and expensive to produce, and mostly used as interconnects.

With the increased use of printed structures in microwave and antenna applications, research for the development of simulation tools for such structures has significantly increased. As a result, plenty of methods have been proposed; some were tuned for some specific class of printed geometries, some were rigorous enough but not computationally efficient, and some others were not rigorous but very efficient. These methods can be mainly classified into two groups: i) approximate but numerically efficient methods like quasi-static methods [1, 2, 3]; ii) accurate but computationally expensive methods, such as the method of moments (MoM) [4, 5], the finite-difference time domain (FDTD) method [6] and the finite element method (FEM) [7]. Note that the methods in the second class are considered to be full-wave approaches in the EM literature. Since the main method of this thesis belongs to the second class, the commonly known and widely used methods in this class, FEM and FDTD method, are briefly reviewed.

The FEM is a general numerical technique to find the approximate solutions to the boundary value problems. In the application of FEM, the entire volume is divided into subvolumes in which the unknown function is represented by simple interpolating functions.

Then, a set of algebraic equations for the solution of the system is obtained by applying the Rayleigh-Ritz procedure. The main advantage of the FEM is that it is very versatile, and can be applied almost to any problem. However, the number of unknowns grows very rapidly with the fine features of the geometry, rendering the method computationally inefficient.

The FDTD method is based on the discretization of Maxwell's equations both in space and time domains, and the derivatives involved in Maxwell's equations are implemented by finite differencing [6], [8]. The method is very easy to implement, is made to converge almost by brute-force. With some available high-speed computer resources, there is actually no EM problem that this method cannot solve. One of the advantages of the FDTD method is that the frequency response of a circuit over a band of frequency can be extracted in one step from the response of the circuit to a narrow Gaussian pulse excitation in the time domain.

Both FEM and FDTD methods are quite general and versatile, and the resulting matrix equations are sparse. Due to the sparsity of the resulting matrices in these methods, they are considered to be efficient as far as the memory storage and the solution time of the resulting matrix equations are concerned. However, both methods have difficulties when open geometries are analyzed, such as radiating structures and not-shielded microwave and millimeter-wave circuit components. Since these techniques require discretization of the whole volume of interest, the difficulties arise from the discretization of an infinite region in cases of open structures. In other words, both FEM and FDTD method require finite domain to set up a mesh, i.e. the computational space must be of limited size, or must be approximated by a limited region by introducing some artificial boundaries, of course without altering the electromagnetic characteristics of the geometry. On the contrary, the MoM technique is applied to an integral equation that accounts for the geometry with its infinite extent. Therefore, the problem domain, where the unknown function of the integral equation is defined, is reduced to regions where the surface or volume current densities exist. As compared to the FEM and FDTD methods, MoM requires less number of unknowns and it is more suitable for planar geometries. However, in the conventional application of the MoM, numerical evaluations of the matrix entries are computationally very expensive, because of the double or triple integrals of complex functions involved.

It is widely accepted that the MoM based algorithms are the most suitable approaches for the rigorous analysis of layered printed structures, such as, moderate size microwave and

millimeter wave passive circuits fabricated in stratified media, when compared to the other rigorous techniques like FEM and FDTD method. Therefore, the main numerical technique that is used in this work is based on the MoM, and hence, a brief introduction of the method is provided to point out some numerical concerns and their remedies. This method, in essence, transforms an operator equation (differential, integral or integro-differential form) describing a physical problem into a matrix equation by following the steps given below: (i) approximate the unknown function in terms of known basis functions; and, (ii) apply the boundary conditions in integral sense with the use of some weighting functions, also called testing functions. This two-step procedure of the MoM can be implemented either in the spatial-domain or spectral-domain, using the spatial or spectral representations of the basis and testing functions, as well as the integral equation, respectively.

The MoM formulation in EM usually starts with writing an integral equation, which could be electric field integral equation (EFIE), magnetic field integral equation (MFIE) or mixed potential integral equation (MPIE). Traditionally, EFIE or MFIE is used for the application of the spectral-domain MoM, while MPIE is used for the spatial-domain MoM. Note that EFIE and MFIE use the Green's functions of the electric and magnetic fields as their kernels, respectively, whereas the MPIE uses the Green's functions of the scalar and vector potentials. Since the singularities of the Green's functions for the electric and magnetic fields are stronger than those of the vector and scalar potentials, the use of MPIE in conjunction with the spatial-domain MoM has been the preferred choice for the characterization of planar printed geometries in general [9, 10, 11, 12].

For the spectral-domain MoM used for the solution of EFIE, the MoM matrix entries involve two-dimensional integrals of complex, oscillatory and slow converging functions over an infinite domain [13]. Therefore, this formulation requires the use of computationally expensive numerical integration algorithms rendering the technique computationally inefficient. On the other hand, for the spatial-domain MoM in conjunction with MPIE, the MoM matrix entries consist of double integrals over finite domains, and the integrands include the spatial-domain Green's functions of the vector and scalar potentials. The spatial-domain Green's functions are obtained from the Hankel transforms of the corresponding spectral-domain Green's functions, and the integrands of this transformation, together with the Hankel function as its kernel, are usually quite oscillatory and slow-convergent in nature.

As a result, the spatial-domain MoM was not so popular until the recent introduction of the efficient approximation of the spatial-domain Green's functions in closed forms, which is called either the closed-form Green's functions (CFGF) or the method of discrete complex images (DCIM). The method of CFGFs was first demonstrated for a horizontal electric dipole on a single thick substrate [14], and then extended to multilayer geometries with arbitrary thicknesses and with arbitrary source types [15]. The efficiency and the robustness of the method have been significantly improved with the use of multi-level approximation scheme [16]. By using this method, the calculation of the spatial-domain Green's functions is improved significantly, and this is followed by a substantial improvement in the calculation of the MoM matrix entries in the spatial-domain [17]. Consequently, in this work, the MPIE is formulated as the governing operator equation of the printed structures in multilayer planar media, and the spatial-domain MoM is employed to solve for the unknown current densities in the structure.

Shorting pins or strips, via-holes, air-bridges and bond wires are indispensable parts of MMICs, and of some microstrip antennas. Perhaps, the indispensability of vertical metallization is best demonstrated by the use of via-holes in multilevel printed circuits. Nowadays, it is a common practice that the sizes of circuits are reduced by designing the circuits in a multilayer environment, where the signals and ground connections are carried from one layer to another with the use of via-holes. Another example is the design of dual-frequency operation of microstrip antennas, where some shorting pins or strips are used to adjust the ratio of the center frequencies of the two bands. Although the applications of vertical connections in the design of microwave circuits and antennas are quite extensive, the simulation tools of such circuits are not yet well-developed, or rather, not as developed as those for just planar circuits. Since the spatial-domain MoM is the preferred choice for the solution of MPIE, because of the availability of closed-form Green's functions, the same method will be extended to incorporate multiple vertical metallization efficiently in this thesis. However, it has been observed earlier that there were some difficulties in the implementation of the spatial-domain MoM in cases of vertical metallization. These difficulties mainly originate from the implementation of the algorithm for obtaining the spatial-domain Green's functions in closed forms. To overcome these difficulties, several methods involving the closed-form Green's functions have been proposed in [18], and demonstrated for a few

simple cases. In addition, the accuracy of the MPIE-MoM approach for the solution of vertical metallization has also been demonstrated in [19]. In this thesis, vertical conductors have been efficiently incorporated into the implementation of the spatial-domain MoM with the closed-form Green's functions, as proposed in [18]. Furthermore, the method is extended to geometries with multiple vertical conductors with no noticeable increase in the computation time. This point should be further emphasized by stating that additional vertical conductors do not increase the computational cost of the technique significantly, as would be the case with any other full-wave methods.

The organization of the thesis is provided here to guide the readers. In Chapter 2, the spectral and spatial-domain Green's functions for planarly layered media are introduced along with some comments on the problems of DCIM. Then, in Chapter 3, a general MPIE formulation for planarly layered media is demonstrated using the closed-form spatial-domain Green's functions. This is followed, in Chapter 4, by the arguments on and the demonstrations of the efficiency of the method when multiple vertical conductors are used in some realistic examples. In Chapter 5, the methods used to extract the electrical parameters of circuits are discussed, and several examples consisting of vertical conductors are analyzed. Moreover, the results are compared to those obtained from commercially available software and experiments in the same chapter. Finally, the conclusions are given in Chapter 6.

Chapter 2

GREEN'S FUNCTIONS IN PLANARLY LAYERED MEDIA

The application of the MoM for the solution of integral equations, either in spectral- or spatial-domain, requires the knowledge of the Green's functions in the corresponding domain. The spatial-domain Green's functions for multilayer media are obtained from the spectral-domain Green's functions by the Hankel transformation (Sommerfeld integral [20]) where the spectral-domain Green's functions are known in closed forms for layered media [15, 21]. Although these representations of the Green's functions are very well-known and well-studied, the resulting MoM algorithm in either domain becomes computationally expensive, in general. In the spatial-domain MoM, the computational inefficiency is caused by the oscillatory nature of the Sommerfeld integrals, while, in the spectral-domain MoM, the slow-decaying nature of the spectral-domain Green's functions results in slow-convergent MoM matrix entries [22, 23]. Although there are other methods to cast the Green's functions into closed forms, they provide either asymptotic approximations of the Green's functions or expressions only valid for a limited class of geometries, details of which are given in [24, 25].

Recently, a method has been proposed to efficiently approximate the spatial-domain Green's functions in terms of complex images, i.e., in terms of spherical waves originating from images in the complex plane (e^{-jkrc}/r_c , r_c is a complex distance), so it is called discrete complex image method (DCIM). The method was first proposed in [14] for a horizontal electric dipole over a thick substrate backed by a ground plane. Since then, it has been extended to two layer geometries with arbitrary thicknesses [26] first, and then, to multilayer geometries with horizontal electric dipole (HED), horizontal magnetic dipole (HMD), vertical electric dipole (VED) and vertical magnetic dipole (VMD) sources [15]. Throughout these improvements, it was observed that the robustness and the efficiency of the technique are directly related to the implementation of the exponential approximation of the spectral-domain Green's functions. In the originally proposed DCIM [14], the original Prony method was employed for the exponential approximations, and due to its

inadequacy, the overall technique was limited to thick and single layer structures. Although the use of the least-squares Prony method improved the technique in some extent [26], several trial-and-error iterations were still required due to the noise sensitivity of the Prony methods [27, 28], rendering the technique not robust. To alleviate this problem, another exponential approximation method, known as the generalized pencil of function (GPOF) method [29], was used in approximating the spectral-domain Green's functions [15]. The GPOF method has shown to be less noise sensitive and more robust as compared to the Prony methods. However, the DCIM is still computationally expensive because of the need to study the spectral-domain behavior of the Green's functions in advance in order to decide on the approximation parameters, such as, the number of sampling points and the maximum value of the sampling range. An additional difficulty arises from the fact that these exponential approximation methods, the Prony and GPOF, require uniform sampling of the function to be approximated along the approximation range. This restriction of uniform sampling may result in taking hundreds of samples in order to approximate a slow converging function with fast changes only over a small region. Consequently, these difficulties render the algorithm not robust and computationally expensive, and therefore, it cannot be used much for the development of a general purpose electromagnetic simulator. Recently, to eliminate these deficiencies in DCIM, a new approach has been proposed to accomplish a piecewise uniform sampling over two or multiple consecutive regions of the approximation domain [16, 30], which are called two-level and multi-level sampling algorithms, respectively. Hence, the DCIM, which employs the GPOF method for the exponential approximation and utilizes the two-level or multi-level algorithm for the sampling, becomes a robust and efficient algorithm to obtain the spatial-domain Green's functions in closed forms. Once the spatial-domain Green's functions are obtained efficiently, that is, eliminating the numerical evaluation of the time-consuming Sommerfeld integrals, the major numerical obstacle in the application of the spatial-domain MoM has been removed. Therefore, the spatial-domain MoM in conjunction with the closed-form Green's functions can be employed efficiently in the solution of MPIE for planarly layered geometries. It should be noted that the three-level sampling algorithm is employed in the derivation of the closed-form Green's functions in this thesis, and the closed-form Green's functions obtained are valid in all spatial regions of interest.

Although providing very accurate and efficient approximation for most practical problems within the range of validity of the approach, the closed-form Green's functions have some limitations as well as a range of validity as it is the case for most approximations. The major limitation comes from the accuracy of the closed-form representations of the Green's functions for large distances from the source, especially for distances beyond a few wavelengths. It is observed that the closed-form Green's functions are accurate for small and moderate distances, but for distances larger than a few wavelengths, they deteriorate violently. This was wrongly attributed to introducing non-physical branch points into the approximated spectral-domain Green's functions in the process of approximation [31, 32]. In addition, it was claimed that not extracting the quasi-dynamic terms in two-level approach contributes to the inaccuracies in the results [32]. However, the misinterpretation of the former claim was briefly discussed in [33] and the inaccuracy was attributed to the increased complexity due to the introduction of the extra branch cut. In [34] and [35], DCIM is critically studied to answer the problems of validity of the method beyond certain distance from the source, and to provide comments on the surface-wave and lateral-wave contributions to the Green's functions. In this chapter, the crucial steps of the DCIM are reviewed, and the validity of the method and its potential problems are discussed.

Since the DCIM begins with the spectral-domain Green's functions for planarly layered media, the chapter starts with the introduction of these Green's functions in Section 2.1. Then, in Section 2.2, the method for obtaining the closed-form spatial-domain Green's function is explained in detail. Following this section, some comments on the accuracy and the range of validity of the closed-form Green's functions are given in Section 2.3, and finally, Section 2.4 concludes this chapter.

2.1 Spectral-Domain Green's Functions

For the sake of illustration, a general planarly stratified medium is given in Fig. 2.1, where a general source (HED, VED, HMD or VMD) is embedded in region- i , and the observation point can be in any layer. The parameters of the layers that are relevant to the field calculations are denoted as similar to those of layer- i : the thickness by d_i , the relative permittivity by ϵ_{r_i} , the relative permeability by μ_{r_i} . Moreover, it is assumed that all layers extend to infinity in the horizontal plane, xy -plane in Fig. 2.1, and that the stratification of

the layers is in z -direction, with no loss of generality. Note that the origin of the coordinate axis is placed at the bottom of the source layer, and the time convention of $e^{j\omega t}$ has been adopted and suppressed throughout the thesis.

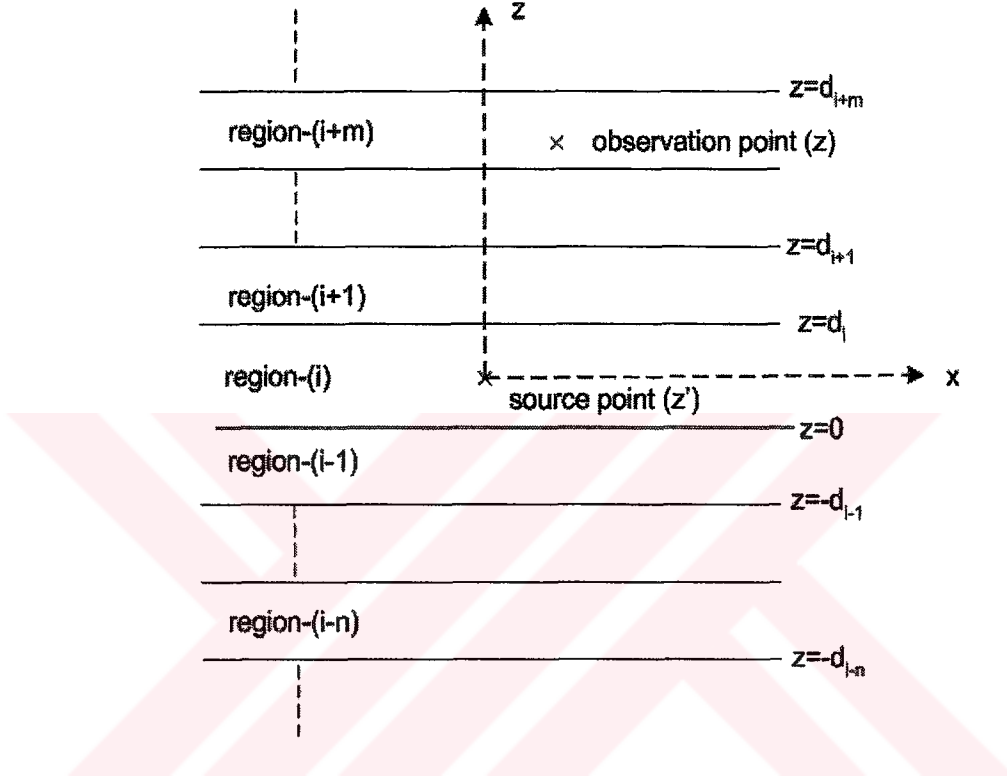


Figure 2.1: A typical multilayered medium with a source in region- i .

For the derivation of the spectral-domain Green's functions, the electric and magnetic field components due to a Hertzian dipole in a homogeneous and unbounded medium, and with a current moment $\hat{\alpha}Il$, need to be obtained first. Since the expressions of electric and magnetic fields in terms of vector potential are well-known and derived in Appendix A, the first step is to obtain the vector potential for the given Hertzian dipole. It is a well-known fact that the vector potential has only the vector component parallel to the dipole in an unbounded homogeneous medium. Therefore, the vector potential satisfies the scalar wave equation whose solution for a dipole source is a spherical wave, $(e^{-j\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}/|\mathbf{r}-\mathbf{r}'|)$. Hence, the electric and magnetic fields are written as

$$\mathbf{E}(\mathbf{r}, \mathbf{r}') = -j\omega\mu \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2} \right) \cdot \hat{\alpha} I l \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \quad (2.1)$$

$$\mathbf{H}(\mathbf{r}, \mathbf{r}') = \nabla \times \hat{\alpha} I l \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \quad (2.2)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field intensities, respectively. Remembering that the spherical wave solution for the vector potential used in (2.1) and (2.2) are only valid for an unbounded homogeneous medium, in cases of multilayer media, this solution needs to be modified in order to take the reflections from the layers above and below into account. Since we only know the reflection coefficients from a planar boundary for TE and TM waves, the spherical wave can be written in terms of plane waves by using the Weyl identity,

$$\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = -\frac{j}{2\pi} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \quad (2.3)$$

where $k_x^2 + k_y^2 + k_z^2 = k^2$. Then, it is a simple task to get the TE and TM wave components. Note that since the medium is translationally invariant in the xy plane, the phase matching condition requires that k_x and k_y are the same in all layers. After having substituted (2.3) into (2.1) and (2.2), only the exponential term with k_z needs to be modified by the reflections from the boundaries.

Once it is clarified how to get the spectral-domain Green's functions in a layer within a multilayer structure, let us briefly give the procedure of getting these Green's functions for a source in a given layer and observation in another. Spectral-domain Green's functions are first obtained in the source layer by considering the direct wave and up- and down-going waves due to the reflections from the boundaries. Then, the Green's functions in any other layer are obtained using an iterative algorithm applied to each TE and TM component of the Green's functions in the source layer. For the sake of completeness, the details of the derivation of spectral-domain Green's functions for HED and VED sources are provided in Appendix A.

2.1.1 Spectral-Domain Green's functions for electric dipoles

Following the procedure detailed in Appendix A, the spectral-domain Green's functions for electric dipoles, HED and VED, are obtained as follows [15]:

$$\tilde{G}_{xx}^A = \frac{\mu_i}{2jk_{z_i}} \left[e^{-jk_{z_i}|z-z'|} + A_h^e e^{jk_{z_i}(z-z')} + C_h^e e^{-jk_{z_i}(z-z')} \right] \quad (2.4)$$

$$\tilde{G}_{zx}^A = \frac{-\mu_i}{2jk_{z_i}} \left[\frac{k_x k_{z_i}}{k_\rho^2} (A_h^e + B_h^e) e^{jk_{z_i}(z-z')} + \frac{k_x k_{z_i}}{k_\rho^2} (D_h^e - C_h^e) e^{-jk_{z_i}(z-z')} \right] \quad (2.5)$$

$$\begin{aligned} \tilde{G}_x^{qe} &= \frac{1}{2j\epsilon_i k_{z_i}} \left[e^{-jk_{z_i}|z-z'|} + \frac{k_{z_i}^2 B_h^e + k_i^2 A_h^e}{k_\rho^2} e^{jk_{z_i}(z-z')} \right. \\ &\quad \left. + \frac{k_i^2 C_h^e - k_{z_i}^2 D_h^e}{k_\rho^2} e^{-jk_{z_i}(z-z')} \right] \end{aligned} \quad (2.6)$$

$$\tilde{G}_{zz}^A = \frac{\mu_i}{2jk_{z_i}} \left[e^{-jk_{z_i}|z-z'|} + A_v^e e^{-jk_{z_i}(z-z')} + B_v^e e^{jk_{z_i}(z-z')} \right] \quad (2.7)$$

$$\tilde{G}_z^{qe} = \frac{1}{2j\epsilon_i k_{z_i}} \left[e^{-jk_{z_i}|z-z'|} + C_v^e e^{-jk_{z_i}(z-z')} + D_v^e e^{jk_{z_i}(z-z')} \right] \quad (2.8)$$

where the superscript A and q_e denote the magnetic vector and electric scalar potentials, respectively. The coefficients $A_{h,v}^e$, $B_{h,v}^e$, $C_{h,v}^e$, and $D_{h,v}^e$ are the functions of generalized reflection coefficients [15], and are given as

$$A_h^e = \tilde{R}_{TE}^{i,i+1} M_i^{TE} \left[e^{-2jk_{z_i}(d_i-z')} + \tilde{R}_{TE}^{i,i-1} e^{-2jk_{z_i}d_i} \right] \quad (2.9)$$

$$B_h^e = \tilde{R}_{TM}^{i,i+1} M_i^{TM} \left[e^{-2jk_{z_i}(d_i-z')} - \tilde{R}_{TM}^{i,i-1} e^{-2jk_{z_i}d_i} \right] \quad (2.10)$$

$$C_h^e = \tilde{R}_{TE}^{i,i-1} M_i^{TE} \left[e^{-2jk_{z_i}z'} + \tilde{R}_{TE}^{i,i+1} e^{-2jk_{z_i}d_i} \right] \quad (2.11)$$

$$D_h^e = \tilde{R}_{TM}^{i,i-1} M_i^{TM} \left[-e^{-2jk_{z_i}z'} + \tilde{R}_{TM}^{i,i+1} e^{-2jk_{z_i}d_i} \right] \quad (2.12)$$

$$A_v^e = \tilde{R}_{TM}^{i,i-1} M_i^{TM} \left[e^{-2jk_{z_i}z'} + \tilde{R}_{TM}^{i,i+1} e^{-2jk_{z_i}d_i} \right] \quad (2.13)$$

$$B_v^e = \tilde{R}_{TM}^{i,i+1} M_i^{TM} \left[e^{-2jk_{z_i}(d_i-z')} + \tilde{R}_{TM}^{i,i-1} e^{-2jk_{z_i}d_i} \right] \quad (2.14)$$

$$C_v^e = \tilde{R}_{TM}^{i,i-1} M_i^{TM} \left[-e^{-2jk_{z_i}z'} + \tilde{R}_{TM}^{i,i+1} e^{-2jk_{z_i}d_i} \right] \quad (2.15)$$

$$D_v^e = \tilde{R}_{TM}^{i,i+1} M_i^{TM} \left[-e^{-2jk_{z_i}(d_i-z')} + \tilde{R}_{TM}^{i,i-1} e^{-2jk_{z_i}d_i} \right] \quad (2.16)$$

where

$$M_i^{TE,TM} = \left[1 - \tilde{R}_{TE,TM}^{i,i+1} \tilde{R}_{TE,TM}^{i,i-1} e^{-jk_{z_i}2d_i} \right]^{-1} \quad (2.17)$$

$$\tilde{R}_{TE,TM}^{j+1,j} = \frac{R_{TE,TM}^{j+1,j} + \tilde{R}_{TE,TM}^{j,j-1} e^{-jk_{z_j}2d_j}}{1 - R_{j,j+1} \tilde{R}_{TE,TM}^{j,j-1} e^{-jk_{z_j}2d_j}} \quad (2.18)$$

R and $\tilde{R}$ are the Fresnel and generalized reflection coefficients [21], respectively, and their subscripts TE and TM represent the polarization of the wave, and their superscripts show

the layer numbers. In addition, the subscripts h and v represent the orientation of the source, horizontal and vertical, respectively. It should be noted that the Green's functions for y -oriented dipoles can be obtained simply by setting $\tilde{G}_{yy}^A = \tilde{G}_{xx}^A$, $\tilde{G}_{zy}^A/k_y = \tilde{G}_{zx}^A/k_x$, and $\tilde{G}_y^{qe} = \tilde{G}_x^{qe}$.

When the observation layer is different from the source layer, the Green's functions are modified by using the appropriate boundary conditions and following iterative expressions [21]:

$$A_j^- = A_{j+1}^- \frac{T_{j+1,j} e^{-j(k_{z_{j+1}} - k_{z_j})(z' + z_{i-j+1})}}{1 - R_{j,j+1} \tilde{R}_{j,j-1} e^{-jk_{z_j} 2d_j}} \quad (2.19)$$

$$A_j^+ = A_{j-1}^+ \frac{T_{j-1,j} e^{-j(k_{z_{j-1}} - k_{z_j})(z_{i-j-1} + d_i - z')}}{1 - R_{j,j-1} \tilde{R}_{j,j+1} e^{-jk_{z_j} 2d_j}} \quad (2.20)$$

where A_j^- and A_j^+ are the amplitudes of the down- and up-going waves, respectively, and T is the transmission coefficient. So, the field expressions in any layer can be obtained iteratively starting from the source layer.

2.2 Closed-form Green's Functions in the Spatial-Domain

Spatial-domain Green's functions are represented by the inverse Hankel transforms of their spectral-domain counterparts as,

$$G = \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \tilde{G}(k_\rho) \quad (2.21)$$

where G and $\tilde{G}$ are the Green's functions in the spatial-domain and spectral-domain, respectively, $H_0^{(2)}$ is the Hankel function of the second kind, and SIP stands for Sommerfeld integration path. Note that this integral form is usually called the Sommerfeld integral in the literature [20]. Spectral-domain Green's functions, $\tilde{G}$, may have poles and branch-point singularities, from mathematical point of view, and these singularities are the mathematical representations of physically guided modes in the dielectric layers, and radiating modes in the outermost layers, respectively. Generally, exact analytical integration of Sommerfeld integrals is not possible, they are either evaluated by asymptotic methods, like the stationary phase method and the steepest descent path method [21], or by the numerical integration methods [36]. Since the asymptotic methods are case dependent and require reformulation for different geometries, they are not appropriate for the design of a general purpose EM

simulator. On the other hand, numerical integrations of Sommerfeld integrals are computationally quite expensive, mainly due to the oscillatory nature of the integrands and the infinite limits of the integration. In conclusion, these methods are not suitable for efficient evaluation of Sommerfeld integrals.

In [14], Chow et al. have demonstrated, for a horizontal electric dipole over a thick substrate, that the spectral-domain Green's functions are approximated by complex exponentials. Later, this method was further extended to geometries with a substrate and superstrate with arbitrary thicknesses in [26]. Since the complex exponentials are chosen in such a way that their Hankel transforms can be evaluated analytically, the spatial-domain Green's functions are written in closed forms. The original approach of getting the closed-form, spatial-domain Green's functions requires one to study, in advance, the spectral-domain behavior of the Green's functions, in order to be able to decide on the approximation parameters like the number of sampling points and the maximum range of approximation. Moreover, the quasi-dynamic images and the surface-wave poles need to be found and their contributions be extracted from the Green's functions prior to the approximation in order to ease the burden on the algorithm. But, with the use of the two-level approach proposed in [16], these difficulties are eliminated and the method becomes suitable for the development of a general purpose electromagnetic based simulation software. One of the main advantages of the two-level approach is that it requires much less samples of the function to be approximated than those required by the original method. To elucidate this advantage further, it should be noted that the spectral-domain Green's functions may have fast variations locally, and that the exponential approximation methods, Prony's and GPOF methods, require uniform sampling along the range of approximation. In the original one-level approach, to extract all the information accurately for such a function, the number of samples is determined by the fast-varying part of the function, resulting in an unnecessarily large number of samples for the slowly varying parts. However, for multi-level approaches, the number of samples over each level can be determined independently, which provides an additional degree of freedom for the choices of the number of samples over the subregions (levels) of approximations.

It should be noted that, during the approximation process, the sampling of the Green's function is performed along the SIP or along a path which is legitimately deformed from

SIP, details are given in [14, 16]. In this thesis, the two-level and three-level approaches are used interchangeably, depending on the behavior of the Green's functions [30]. In other words, it is not necessary to use three-level approach for smooth functions, the two-level approach would be sufficient to extract the behavior of such Green's functions. For the sake of completeness, the deformed SIP path and the equations representing the paths are given as follows:

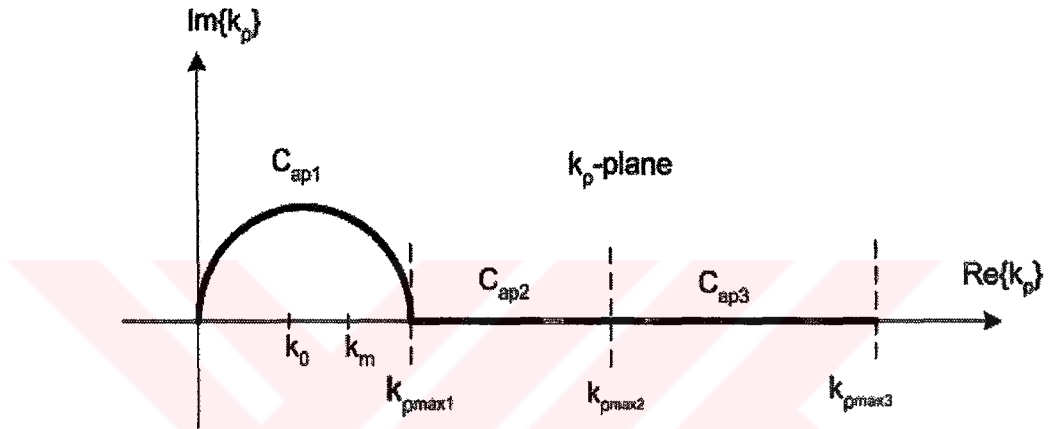


Figure 2.2: The deformed SIP path used in this thesis

$$\text{For } C_{ap3}: k_{z_i} = -jk_i [T_{o1} + T_{o2} + t] \quad 0 \leq t \leq T_{o3} \quad (2.22)$$

$$\text{For } C_{ap2}: k_{z_i} = -jk_i [T_{o1} + t] \quad 0 \leq t \leq T_{o2} \quad (2.23)$$

$$\text{For } C_{ap1}: k_{z_i} = k_i \left[-jt + \left(1 - \frac{t}{T_{o1}} \right) \right] \quad 0 \leq t \leq T_{o1} \quad (2.24)$$

where t is the running variable sampled uniformly on the corresponding ranges, T_{o1} , T_{o2} , and T_{o3} . The exponential approximation process begins with the sampling of the functions to be approximated, and then, an exponential approximation algorithm is employed for the sampled values of the function. In other words, the values of the function at the points of the samples should be known, which requires some of the parameters like z and z' in Eqs.(2.4)-(2.8) to be fixed. After the spectral-domain Green's function, apart from the term $1/j2k_{z_i}$, has been sampled, the GPOF method is used to obtain the exponential approximation. Then, the resulting approximation of the spectral-domain Green's function becomes

$$\tilde{G} \cong \frac{1}{j2k_{z_i}} \left\{ \sum_{n=1}^{N_1} a_{1n} e^{-b_{1n} k_{z_i}} + \sum_{n=1}^{N_2} a_{2n} e^{-b_{2n} k_{z_i}} + \sum_{n=1}^{N_3} a_{3n} e^{-b_{3n} k_{z_i}} \right\} \quad (2.25)$$

Once the spectral-domain Green's functions are represented as the sum of complex exponentials, each exponential term in (2.25) can be transformed to the spatial-domain via the Sommerfeld identity

$$\frac{e^{-jkr}}{r} = \frac{1}{2j} \int_{SIP} dk_{\rho} k_{\rho} H_0^{(2)}(k_{\rho} \rho) \frac{e^{-jk_z|z|}}{k_z} \quad (2.26)$$

yielding the following Green's function expression in the spatial-domain:

$$G \cong \sum_{n=1}^{N_1} a_{1n} \frac{e^{-jk_i r_{1n}}}{r_{1n}} + \sum_{n=1}^{N_2} a_{2n} \frac{e^{-jk_i r_{2n}}}{r_{2n}} + \sum_{n=1}^{N_3} a_{3n} \frac{e^{-jk_i r_{3n}}}{r_{3n}} \quad (2.27)$$

where $r_{1n} = \sqrt{\rho^2 - b_{1n}^2}$, $r_{2n} = \sqrt{\rho^2 - b_{2n}^2}$, $r_{3n} = \sqrt{\rho^2 - b_{3n}^2}$, and $\rho = \sqrt{x^2 + y^2}$, $k_i = \sqrt{k_{\rho}^2 + k_{z_i}^2}$, and, a_n 's and b_n 's are complex numbers, in general.

Although the extraction of the surface wave poles and the real images would have helped the exponential approximation technique, by making the Green's functions in the spectral-domain well-behaving (extraction of surface wave poles) and fast converging (extraction of real images), the algorithm would not be robust and general purpose. Because the contributions of some surface wave poles are small for geometries on a thin substrate, and because it is not possible to find the real images for multilayer geometries except for a few simple cases. Therefore, for a general-purpose algorithm, the spectral-domain Green's functions are approximated in closed forms for multilayer media, without extracting the surface wave poles and the real images. At this point, the procedure of getting the spatial-

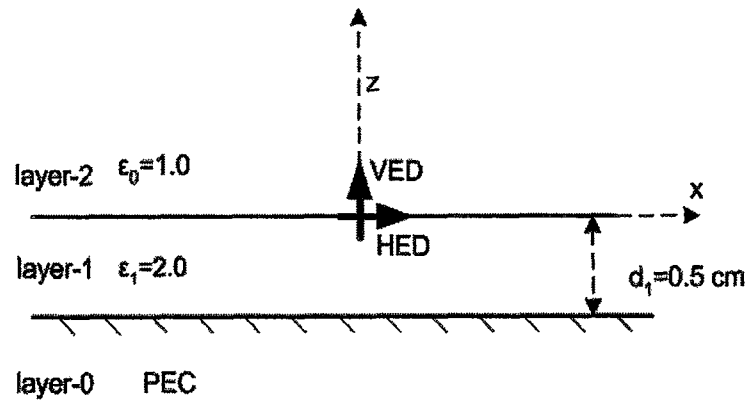


Figure 2.3: A typical three-layer geometry

domain Green's functions in closed forms has been completed. So, it would be instructive

to have examples for each Green's function to build an intuition on the behavior of the Green's functions in the spatial domain. For this purpose, a simple, three-layer geometry with the parameters as shown in Fig. 2.3 is studied at the frequency of 1.0 GHz, and for the sources and the observation points positioned in layer-2, at the interface between layer-1 and layer-2. As a result, the magnitude plots of the Green's functions, G_{xx}^A , G_{zx}^A , G_{zz}^A , G_x^q and G_z^q are given in Figs. 2.4 and 2.5. Note that k_0 and ρ used in the figures are the free space wavenumber and spatial distance from the source, respectively. There may be surface wave poles or branch points in each of the spectral-domain Green's functions and their effect start dominating in the region far from the source. Since their contributions are in the form of cylindrical waves and since the contribution of the real or complex images are in the form of spherical waves and dominates in the middle range, the breaks in some of the figures occur. For example, in G_{xx}^A plot, the break can be easily seen. Those breaks are the transitions of the waves from spherical to cylindrical forms.

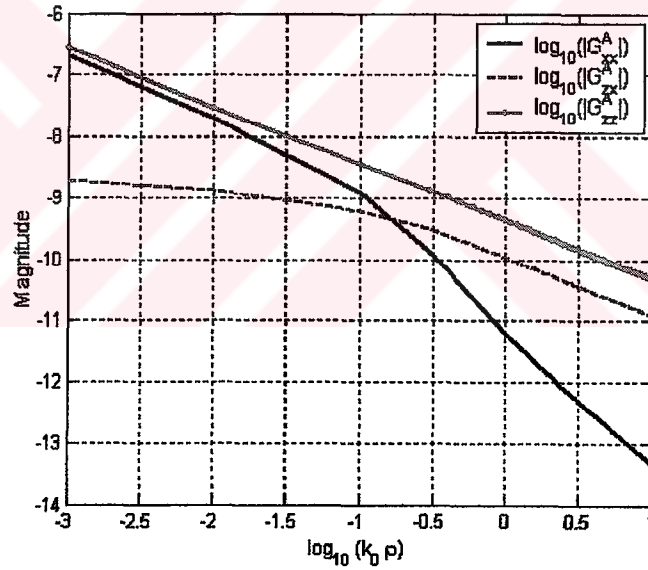


Figure 2.4: Magnitude of the Green's functions, G_{xx}^A , G_{zx}^A and G_{zz}^A , for the geometry given in Fig. 2.3. Frequency=1.0 GHz, Source layer= layer-2, $z = z' = 0$.

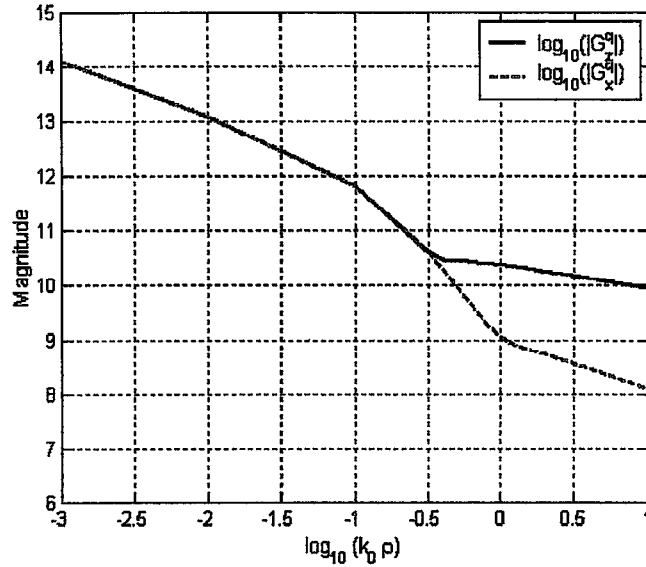


Figure 2.5: Magnitude of the Green's functions, G_x^q and G_z^q , for the geometry given in Fig. 2.3. Frequency=1.0 GHz, Source layer= layer-2, $z = z' = 0$.

2.3 Comments on the Closed-form Green's Functions

Following the first introduction of the closed-form Green's functions [14], there have been plenty of discussions, improvements and applications of the closed-form Green's functions. The discussions have been concentrated mainly on the approximation procedure, because, in general, it has been observed that the approximate closed-form Green's functions are accurate for distances as far as $k_0 \rho = 20 - 30$, and beyond they deteriorate significantly. In this section of the thesis, the attributed problems for the deterioration of the accuracy of the closed-form Green's functions in the literature are discussed, and the sources of these inaccuracies and their possible solutions are explained. Basically, the sources of the problems are attributed in the literature to the introduction of wrong branch points, and not extracting the quasi-static terms and the surface wave poles in the implementation of the two-level approach. Before going into the details of the discussions on these problems, it would be instructive to give some of the salient features of a typical N-layered medium, Fig. 2.6. For this geometry, since the uppermost and lowermost layers (layer-0 and $N-1$) are

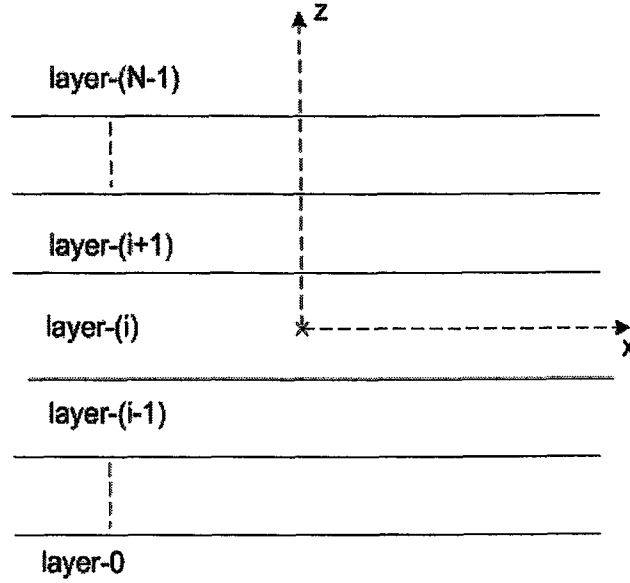


Figure 2.6: A typical N-layer planar medium, where the source is in i^{th} layer

semi-infinite dielectric materials, there are two branch point singularities in the spectral-domain expressions of the Green's functions at $k_\rho = k_0$ and $k_\rho = k_{N-1}$, where k_i is the wavenumber of the i th layer. The branch points exist in these semi-infinite layers, because the waves should satisfy the radiation condition at infinity. Therefore, by the appropriate choices of the branches of k_{z0} and $k_{z(N-1)}$, the waves in these layers are guaranteed to go to zero as the distance from the source goes to infinity. In addition, there exist a number of pole singularities in the spectral-domain expressions of the Green's functions, in the range of $k_{min} < k_\rho < k_{max}$ where k_{min} and k_{max} are the minimum and the maximum wavenumbers present in the geometry. These pole singularities are the mathematical manifestation of surface waves, so called surface-wave poles, and the number of these poles is dependent on the electrical thickness and the dielectric constants of the layers. Because branch points and surface-wave poles are usually located in a near vicinity of the real axis on k_ρ -plane, for low-loss dielectric materials, and because the exponential approximation methods require a mapping between a real variable t and k_{zi} , the SIP path is legitimately deformed to implement the approximation of the spectral-domain Green's functions in terms of complex exponentials. For the sake of completeness, the SIP, its deformed counterpart that are being used in the two-level approach, and branch cuts for a typical N-layer geometry, Fig. 2.6, are

shown in Fig 2.7.

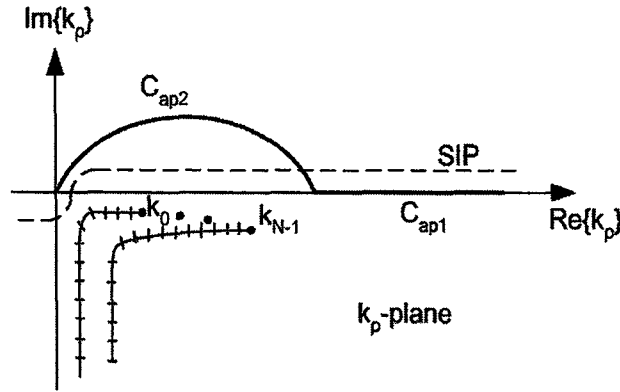


Figure 2.7: Integration paths on k_p -plane for Hankel transformation of spectral-domain Green's functions

2.3.1 Discussion on Branch Point Singularities and their Contributions

As it was already mentioned, for the geometry given in Fig. 2.6, there are only two branch point singularities at $k_p = k_0$ and $k_p = k_{N-1}$ corresponding to the wavenumbers of uppermost and lowermost layers. The mathematical and physical reasonings of this statement are not further discussed here, since it has already been clearly given in [21]. However, starting with [31], there have been a few discussion on the introduction of artificial branch points in the process of deriving the closed-form Green's functions, and in turn, on the mathematical validity of the method [32, 37].

The issue of incorrect complex-plane topology (due to the artificial introduction of branch points) was raised with an observation that the approximated scalar Green's function via the DCIM is violently off, as compared to the exact Green's function, even for small distances from the source [31], for the geometry given in Fig. 2.8 where HED is positioned in layer-2. Once the Fig. 10 of [31] is analyzed, it is observed that the deviation between the approximated and the quadrature values of the Green's function begins around $\rho \geq 0.02\lambda$. However, in this thesis, the two-level approach is applied without applying the modification procedure proposed in [31] for two cases, where HED is placed at the interface in layer-1 and in layer-2. As a consequence, it is observed that the deviations start beyond $\rho \geq 1.59\lambda$ and $\rho \geq 6\lambda$ for layer-1 and layer-2, respectively, as shown in Fig. 2.9.

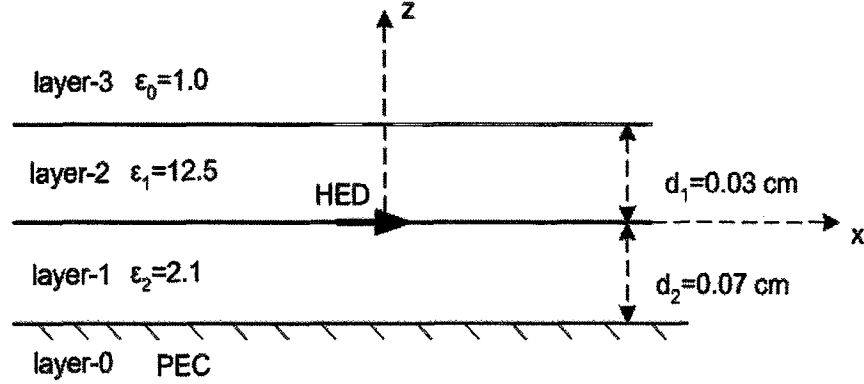


Figure 2.8: A four-layer structure with an embedded HED

Although the modification procedure proposed in [31] is not employed in this thesis, the results seem to be better than those of [31]. The difference in accuracy may be attributed to the use of the Prony's method in [31] and not using enough samples for small values when the original approach was used.

It has already been demonstrated that the Prony's method is very noise sensitive [27, 29] and therefore alternative methods such as GPOF have been proposed [15]. However, it would be instructive to explain the sampling issue further which seems to be the key factor for the early deterioration of the approximated Green's functions in [31].

The actual integration paths over k_ρ -plane for the choices of the following mappings are shown in Fig. 2.10 and their equations are given in Eqs.(2.28) and (2.29) .

$$k_{z3} = k_3 [-jt + (1 - t/T_0)] \quad \text{for } T_0 = 15 \quad (2.28)$$

$$k_{z2} = k_2 [-jt + (1 - t/T_0)] \quad \text{for } T_0 = 5 \quad (2.29)$$

where $k_2 = \sqrt{\epsilon_1} k_3 = \sqrt{12.5} k_3$. Note that the same number of samples which is 41 is used in both cases. Since k_2 is $\sqrt{12.5}$ times larger than k_3 , the sampling range for the mapping function of k_{z2} is reduced to 5 from 15 to bring the maximum of $Re(k_\rho)$ to the same neighborhood. Although this adjustment of the sampling range, T_0 , provides similar ranges on k_ρ -plane for both mappings, it cannot provide a similar set of sampling points, especially for small values, as evidenced from Fig. 2.10. For example, the first samples of both paths are zero but the second samples are at $k_\rho/k_3 = (0.6876 + j0.5318)$ and $k_\rho/k_3 = (1.4082 + j1.0819)$ for the mapping of k_{z3} and k_{z2} , respectively. Therefore, with

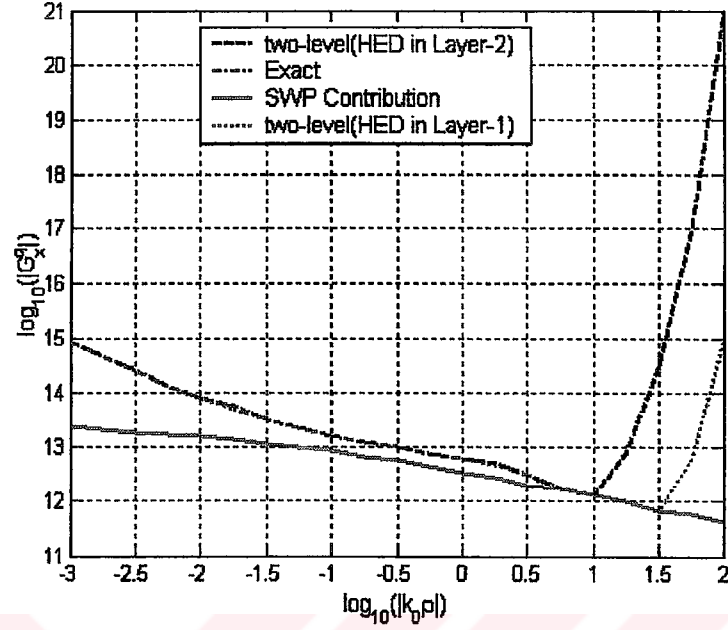


Figure 2.9: Magnitude of the approximated and exact scalar potential Green's function and the surface wave contribution for the geometry

the sampling using k_{z_2} , the fine features of the function to be approximated, for small k_ρ values, are not properly accounted for, which results in a poor approximation in the spatial-domain for moderate to large values of ρ . In addition, since sampling using k_{z_2} results in a path on k_ρ -plane with larger imaginary parts, that is, far from the surface wave poles, the effect of the surface wave poles along this path might be lost due to the finite precision in computer arithmetic, unless double precision arithmetic is used in the sampling and the subsequent calculations.

From the mathematical point of view, the spectral-domain Green's function for the geometry given in Fig. 2.8 has branch points at $\pm k_3 (=k_0$ free-space wave number in this case since $\epsilon_{r_3}=1.0$) and associated branch-cuts, not counting the branch point of Hankel function at $k_\rho = 0$. Once these branch-cuts are specified, the integration path of the Sommerfeld integral can be deformed to avoid these branch-cuts, and therefore, the integrand becomes a single valued function of $k_{z_3} (= \sqrt{k_0^2 - k_\rho^2})$ over the deformed integration path. The path

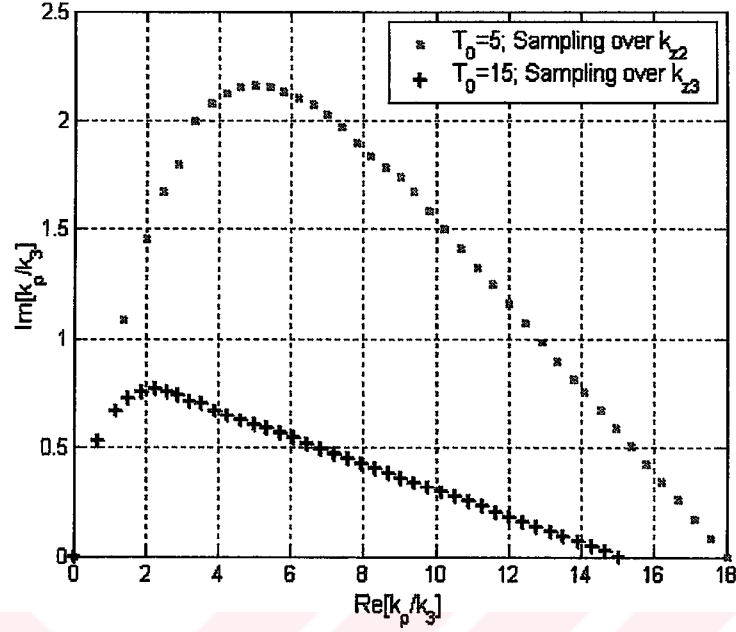


Figure 2.10: Deformed integration paths over k_p/k_3 - plane corresponding to the sampling over $k_{z2} = k_2[-jt + (1 - t/T_0)]$ for $T_0=5$ and $k_{z3} = k_3[-jt + (1 - t/T_0)]$ for $T_0=15$

used for the exponential approximation, defined by (2.28) or (2.29), is one of such paths, and hence the functions to be approximated by complex exponentials along any one of these paths, as well as the approximating functions, are single valued. Subsequently, the Sommerfeld identity can be used, with no problem, for these approximating functions.

Another point is that the branch-point contribution at the interfaces behaves like $1/\rho^2$, while the surface wave contribution behaves as $1/\sqrt{\rho}$. In other words, the branch point contribution decays much faster than the contributions of the surface wave poles. With the examination of Fig. 2.9, it is seen that the deviations for both cases can occur where the surface wave contribution starts dominating the Green's function, which should be way out of the influence range of the branch-point contribution.

2.3.2 Discussion on Quasi-Static Terms

This section is included to clarify the issues around the quasi-static terms in the application of the DCIM; there have been some concerns on the applicability of the DCIM for general geometries [38], and on the extraction of the quasi-static terms even when the two-level approach was employed [32]. Both of these concerns, actually, have been eliminated with the introduction of the two-level approach, as detailed in [16], and better emphasized here in this section.

The main reason for the extraction of the quasi-static terms from the spectral-domain Green's functions is to make them convergent or fast convergent to zero for large k_p values, to eliminate the need to sample the spectral-domain Green's functions over an extended range. The quasi-static terms of a spectral-domain Green's function are obtained in the form of exponentials by finding the limiting terms as $k_p \rightarrow \infty$, therefore in the two-level approach, the approximation of the spectral-domain Green's functions on the path via complex exponentials actually performs better than the quasi-static terms. In other words, once the approximation over the path C_{ap1} is subtracted from the spectral-domain Green's function, it can not only make the Green's function to smoothly converge to zero as $k_p \rightarrow \infty$ but it also makes it zero beyond a predefined value of k_p . Additionally, derivation or finding the quasi-static terms analytically for a Green's function of an arbitrary geometry is impossible in general; therefore, the approximation performed over the path C_{ap1} in the two-level approach is designed specifically for this purpose, as well as for enabling two different frequencies of sampling.

To demonstrate the effectiveness of the two-level approach, reconsider the geometry given in Fig. 2.8. In this case, the approximated scalar potential Green's function for a HED in layer-1 at the frequency of operation at 30 GHz is given in Fig. 2.11. It is obvious from the figure that extracting the exponentials that approximate the Green's function over large k_p make the Green's function converge to zero faster with the increased number of exponentials used. Therefore, approximation over the path C_{ap1} is much better than extracting the quasi-static terms, because it is robust, automatic and applicable to any number of layers.

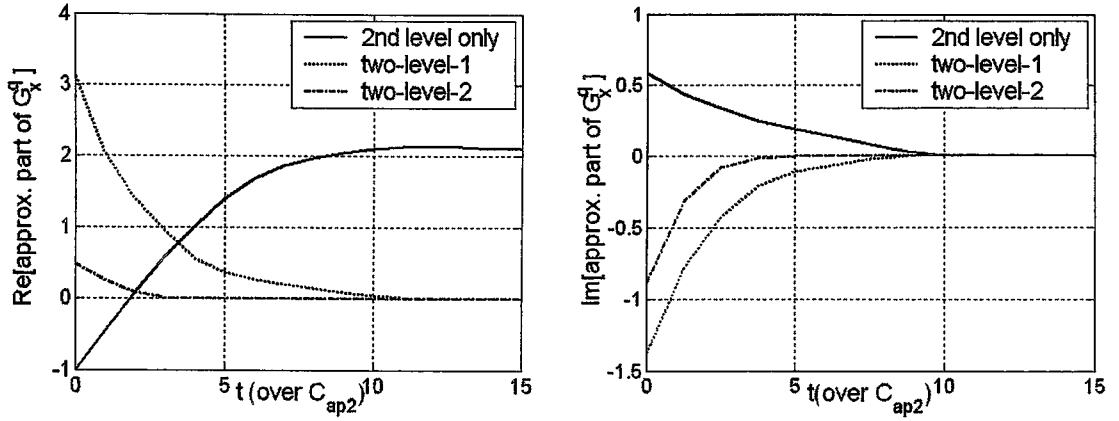


Figure 2.11: Effect of subtracting the exponentials of 2^{nd} level from the 1^{st} level for the spectral-domain Green's function $\tilde{G}_x^q$ for the geometry given in Fig. 2.8. In the “second level only” plot, the extraction is not performed whereas three exponentials in the “two level-1” case and ten exponentials in the “two level-2” case are extracted

2.3.3 Effects of Surface Wave Pole Contribution on the DCIM

In the previous subsection about the branch point singularities, it seems that the surface wave poles are responsible for the violent deterioration of the approximation of the Green's function for moderate distances from the source. This is attributed to the reason that the deviation of the approximation function from the actual one starts where SWP contributions become dominant as shown in Fig. 2.9. To verify this conclusion, another geometry where only one TM surface wave pole exists is chosen which is shown in Fig. 2.12. The surface wave

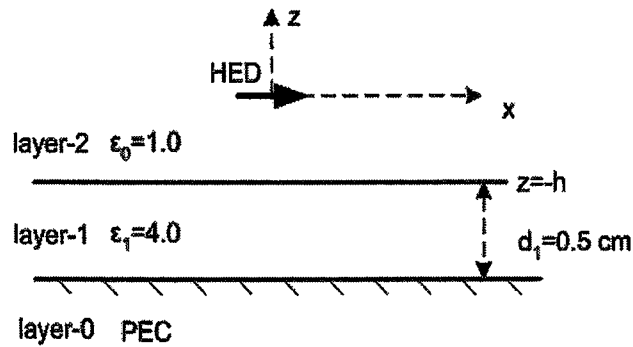


Figure 2.12: A three layered geometry where HED is placed in the uppermost layer

pole for the given parameters of the geometry at 1.0 GHz frequency is at $k_{\rho\rho}$. The scalar potential Green's function G_x^q is chosen for the approximation. The two-level approach is used in getting the closed-form Green's function for two cases where HED is placed at the interface for layer-1 and layer-2. As shown in Fig. 2.13, the approximation for both cases fail to approximate the Green's function for large distances from the source. It is obvious

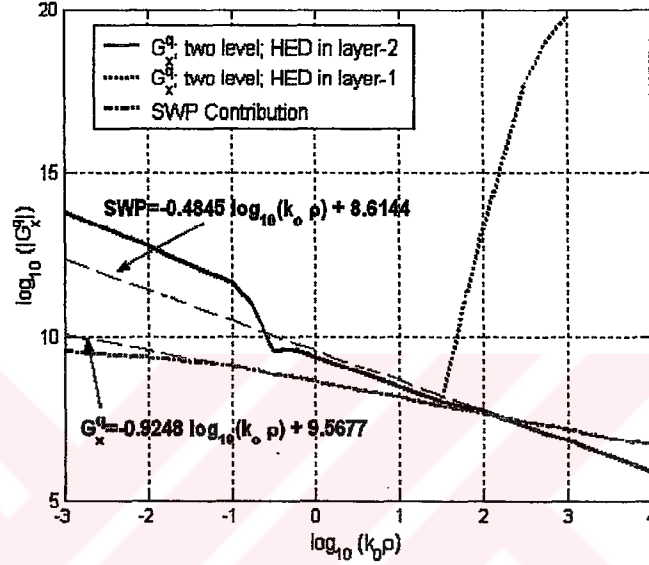


Figure 2.13: Magnitude of the scalar Green's function and the SWP contribution for the geometry given in Fig. 2.12 at 1.0 GHz, $h=0$ (HED is at interface).

from Fig. 2.13 that the deviation starts much before for the “HED in layer-1” case than the other. However, it is clear that both approximations cannot predict the dominant behavior of the surface wave poles. This can easily be seen when the behaviors of exact surface wave pole contribution and approximation for the “HED in layer-2” case are compared for large ρ values. For this purpose, SWP contribution and the mentioned approximation are line fitted and it is seen that as the SWP contribution approaches to its theoretical limit which is $1/\sqrt{\rho}$ or -0.5 in the log-scale, the approximation approaches to the limit of spherical wave which is $1/\rho$ or -1.0 in log-scale. These lines and their equations can be seen in Fig. 2.13. Actually, this should be the expected result when the approximating complex images are considered which are spherical waves coming from complex distances. It should

be noted that the approximating complex images, when HED is in layer-1, don't exhibit spherical wave behavior, even though they should be doing so. In the previous subsection, this was attributed to poor sampling of the spectral-domain Green's function in the case of HED being in layer-1, but now it will be demonstrated here that this is due to trying to approximate the SWP contributions only by spherical waves.

For the demonstration, it would be wise to give the steps in finding the SWP contribution for any Green's function. Consider that the chosen Green's function is the scalar potential Green's function due to a HED. First, the surface wave poles should be found and then the contribution is obtained by taking the following integral analytically:

$$G_x^{qsw} = \frac{1}{4\pi\epsilon} \sum_{i=1}^{N_{TE}+N_{TM}} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \left[\frac{2k_{pp(i)} Res^{(i)}}{k_\rho^2 - k_{pp(i)}^2} \right] \quad (2.30)$$

where $k_{pp(i)}$ and $Res^{(i)}$ are the i th SWP and corresponding residue. Since the term in the square bracket can be recognized as the spectral-domain representation of the SWPs, two-level approach can be used to approximate this term by complex exponentials. Once this is approximated by complex exponentials, using the Sommerfeld identity would provide a set of spherical waves approximating the SWP contributions, which actually exhibit cylindrical wave behavior. Note that, to perform the GPOF for the term in the square bracket-considering the terms in the numerator as constants-one needs $1/k_z$ term for the application of the Sommerfeld identity. Therefore, the term in the square bracket in Eq.(2.30) is multiplied and divided by k_z , resulting in:

$$G_x^{qsw} = \frac{1}{4\pi\epsilon} \sum_{i=1}^{N_{TE}+N_{TM}} 2k_{pp(i)} Res^{(i)} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \frac{1}{k_z} \left(\frac{k_z}{k_\rho^2 - k_{pp(i)}^2} \right) \quad (2.31)$$

Now the function $k_z/(k_\rho^2 - k_{pp(i)}^2)$ is approximated in terms of exponentials of k_z as follows:

$$\frac{k_z}{k_\rho^2 - k_{pp(i)}^2} = \sum_{n=1}^{N_{exp}} b_{in} e^{-\beta_{in} k_z} \quad (2.32)$$

where b_{in} and β_{in} are the coefficients and exponents for the i th SWP, respectively. Hence, the SWP contribution can be obtained in terms of spherical waves as

$$G_x^{qsw} = \frac{1}{4\pi\epsilon} \sum_{i=1}^{N_{TE}+N_{TM}} 2k_{pp(i)} Res^{(i)} (-2j) \sum_{n=1}^{N_{exp}} b_{in} \frac{e^{-jkr_{in}}}{r_{in}} \quad (2.33)$$

where $r_{in} = \sqrt{\rho^2 - \beta_{in}^2}$ and $k^2 = k_\rho^2 + k_z^2$. The spherical wave approximation of the SWP contribution and the SWP contribution itself are given in Figs. 2.14 and 2.15, for HED

in layer-2 and in layer-1, respectively. Note that k_{z_2} and k_{z_1} are the used wavenumbers in Eq.(2.31) for the cases of HED in layer-2 and HED in layer-1, respectively. The scalar Green's function, G_x^q , is also given for both cases to see and to compare the deviations in the approximation of the SWP contribution and the approximation of the Green's function. As

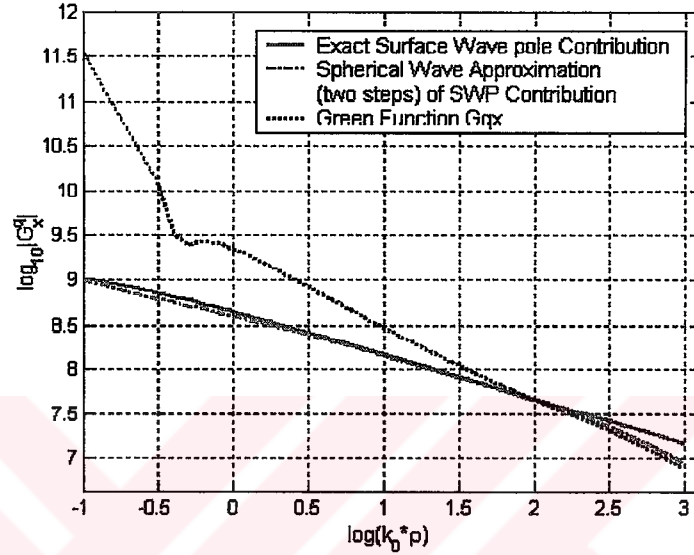


Figure 2.14: Magnitude of the SWP contribution and its spherical wave expansion, for the geometry shown in Fig. 2.12. Source and observation are in medium 2, $z=h=0$ and the frequency is 1.0 GHz.

shown in the Figs. 2.14 and 2.15, the deviations of the approximations of the Green's function start at the distances where the spherical wave approximations of the SWP contribution deviate from the exact SWP contribution.

Naturally, questions about the milder behavior of the Green's function for SWP contribution and "HED in layer-2" case as compared to the "HED in layer-1" case may arise or one may think that the Sommerfeld identity may not be applicable for some of the exponentials that GPOF provides and in turn the resulting exponentials cannot represent spherical waves that decay for large distances from the source. To clarify these issues, one should remember that sampled values of the spectral-domain Green's functions for both cases are very different, especially near the origin as shown in Fig. 2.16, which is due to sampling

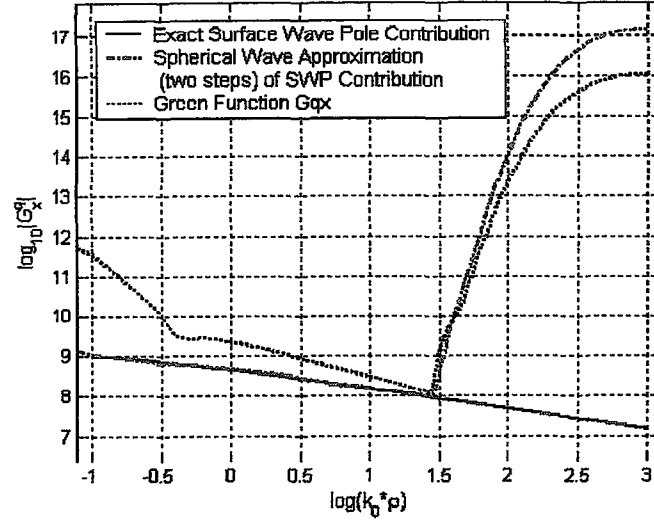


Figure 2.15: Magnitude of the SWP contribution and its spherical wave expansion, for the geometry shown in Fig. 2.12. Source and observation are in medium 1, $z=0$, $h=0.5$ and the frequency is 1.0 GHz.

over different paths as shown in Fig 2.10 and the slight differences in the functions to be approximated (k_z 's are different). It is obvious from Fig. 2.16 that the sampled data for HED in layer-1 are more difficult to approximate, especially to capture the fast varying part near the origin, which is also evidenced from the number of exponentials used for both cases, as provided in Table 2.3.3.

To answer the question whether the Sommerfeld identity is applicable for the exponentials obtained from the approximation when HED is in layer-1, these exponentials, as well as the exponentials for the case of HED being in layer-2, are given in Table 2.3.3. Note that these exponentials are only for the second level of the two-level algorithms, since the approximations in the first level, over C_{ap1} , always provide smoothly decaying exponentials of distance. It is obvious from Table 2.3.3 that the first 2-3 exponentials for the case of HED in layer-1 have very large coefficients and exponents, as compared to the rest of the exponentials for both cases. However, if the contribution of each term (image) is studied and plotted as a function of distance from the source, it is observed that the contributions in

HED in Layer-1			HED in Layer-2		
Coefficient (a_{2n})		Exponent (b_{2n})	Coefficients (a_{2n})		Exponent (b_{2n})
1	299034409+j 559243726	61.96+j 184.57	1	0.0+j 0.0	-496+j 1588
3	-479.4-j	28.3+j 124.6	2	0.009+j 0.014	59.85+j 60.3
3	-3.23-j 3.53	12.59+j 88.2	3	0.0009+j 0.009	39.1+j 49.2
4	0.0898+j 0.235	5.12+j 62.9	4	-0.0014+j 0.0055	25.5+j 39.7
5	0.026+j 0.05	1.56+j 45.0	5	-0.0019+j 0.0034	15.96+j 31.4
6	-0.025+j 0.007	0.06+j 32.3	6	-0.0018+j 0.002	9.28+j 24.04
7	0.023-j 0.0077	0.067+j 23.1	7	-0.0015+j 0.0012	4.67+j 17.73
8	-0.032+j 0.001	0.59+j 15.25	8	0.0+j 0.0	-4.73+j 7.08
9	0.0+j 0.0	-1.40-j 1.96	9	0.0+j 0.0	5.13+j 7.45
10	0.011-j 0.0007	-1.12+j 8.89	10	-0.0002+j 0.0	-2.31+j 9.76
11	0.0-j 0.0001	-2.99+j 4.48	11	-0.0014+j 0.0007	1.47+j 12.2
12	0.0026-j 0.005	-1.92+j 6.21	12	1.0+j 0.0	0.0+j 0.0
13	0.0039+j	1.75+j 6.53			
14	0.0+j 0.0001	3.47+j 4.64			
15	1.0+j 0.0	0.0+j 0.0			

Table 2.1: Coefficients and exponents of complex exponentials for the approximation of $\tilde{G}_x^q$ over C_{ap2} only, for the Geometry in Fig. 2.12.

the spatial domain converge to zero, the same way as a spherical wave does. The only difference for these images with large coefficients and exponents is that they have their maximum somewhere in the middle range (around $k_0\rho \geq 100$) rather than the origin ($k_0\rho = 0$). This is understandable because approximating functions (spherical waves) can not represent the true nature of the function to be approximated for large distances (cylindrical waves). If the algorithm is forced to do a better job, then it is inevitable for the algorithm to come up with larger and larger coefficients and exponents to fit the function in the region of interest, at the cost of sacrificing the accuracy everywhere else.

Up to this point, it has been discussed that approximations fail to approximate the true nature of the Green's functions for large distances from the source mainly because of

two reasons: i) poor sampling and the sampling frequency; and ii) trying to approximate SWP contributions by using spherical waves. The former can be improved by changing the parameters of the approximation algorithm, but the latter is inherent to the approach unless the SWP contributions are totally extracted from the functions to be approximated. To eliminate this rather inherent problem of poor approximation of SWP contributions via spherical waves, it may be suggested to subtract the surface wave poles from the spectral-domain Green's functions before the approximation, and to add their contribution analytically after the approximation. However, the precise extraction of surface wave poles is not possible; mainly because the exact pole locations are complex numbers and because all the algorithms to find them are numerical in nature, providing the locations with finite precision. Consequently, the extraction of the SWP contributions may improve the approximation up to a finite distance, beyond which the approximation is bound to fail due to the left-over of the SWPs. However, in most printed circuit applications, the acceptable distance of approximation is well within the region of interest.

A remedy for the logarithmic singularity introduced by the SWP contributions, due to Hankel function, around origin has been proposed in [39], by annihilating the behavior of the spectral-domain Green's function around surface wave pole by subtracting a special function consisting of two terms. One is used to represent the behavior around the surface wave pole while the other forces the resulting function to decay fast enough so that it will avoid the introduction of a new singularity at the origin in the spatial domain.

For the geometry presented in Fig. 2.12, and the scalar Green's function G_x^q , the surface wave contributions obtained as in [26] and annihilating terms proposed by [39] are presented for the frequencies 1.0 GHz and 3.0 GHz. The scalar Green's functions obtained by extracting surface wave contributions in the spectral domain and adding analytically obtained contributions is given in [31]. For both frequencies, Green's functions obtained using surface wave extraction in either method have been found in good agreement. The small deterioration occurring at 3.0 GHz for distances $k_o\rho > 100$ is due to inexact determination of the SWP location. For many applications, having approximate Green's functions smooth and well behaved in this region will be sufficient. However, for accurate representation in the far field, it is proposed to use the two level approach of Aksun [16] which does not possess the problem of logarithmic singularity around $\rho = 0$ and represent Green's functions

with surface wave contributions as in [40].

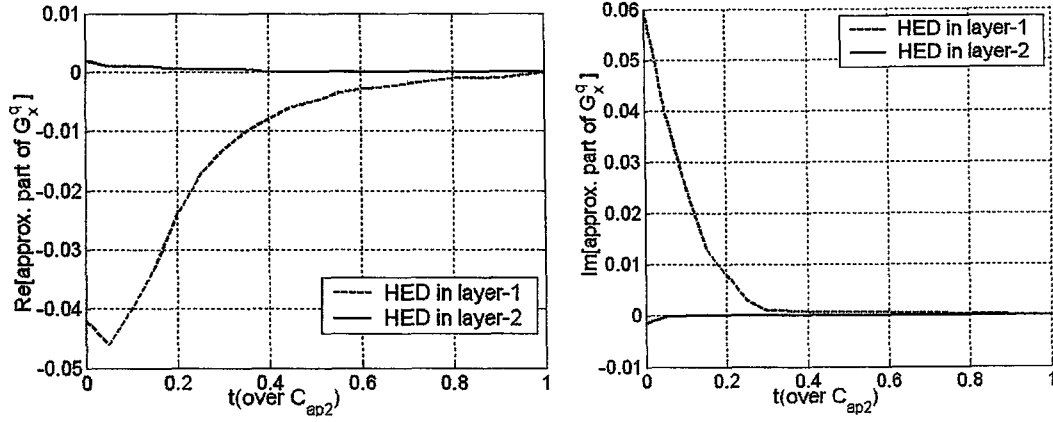


Figure 2.16: Real and Imaginary parts of the spectral-domain Green's function (parts that are approximated), G_x^q , for the geometry given in Fig. 2.12 at 1.0 GHz. For HED in layer-1, $z=0$, $h=0.5$ cm; for HED in layer-2, $z=h=0$.

2.4 Conclusions

With the use of the closed-form Green's functions, the computational efficiency of the spatial-domain MoM for planar geometries in multilayered media has improved. But the original approach requires the choosing of the proper approximation parameters for every different geometry. In addition to this, the extraction of surface wave poles and the quasi-dynamic images may not be possible or efficient for multilayer geometries when the original version of the approach is employed. But the extension of the original approach to two- or multi level approaches has made the use of the closed-form Green's functions attractive for the development of general purpose EM softwares.

However, as all approximations, this method also has a range of validity. It has been observed that closed-form approximations of the spatial-domain Green's functions deteriorate beyond certain distances and this was attributed to surface wave poles, to the use of incorrect complex-plane topology due to artificial introduction of branch points and not extracting the quasi-static images in the two level approach. In this chapter, these issues have been discussed and the reasons for the deterioration of the Green's functions are explained in detail along with some examples. It has been shown that the deterioration may

be attributed to the use of Prony's method or not using enough samples for small values. Also it has been demonstrated that the deterioration starts where the SWP contributions dominate. Therefore the approximation of the SWP contribution which exhibit cylindrical wave behavior via spherical waves may be the cause for the deterioration after a certain limit.



Chapter 3

FIELD ANALYSIS IN PLANARLY LAYERED MEDIA VIA METHOD OF MOMENTS

Because electric and magnetic fields are the fundamental vector parameters in high-frequency circuits and systems, and because the current density is induced on the conductors of the circuits to satisfy the boundary conditions of the fields, the analysis of high-frequency circuits requires the knowledge of the fields, or rather the knowledge of the induced current density. Once the current density is obtained in a circuit, the rest of the electrical characteristics of the circuit, such as input impedance, S-parameters, radiation patterns can easily be obtained. Therefore, any method proposed to analyze microwave circuits and antennas needs to write the field expressions, and then find the fields or the induced current density in the circuit. The method studied in this thesis is no exception: the MPIE is written as an expression for the electric field; and then, the current density is obtained by applying the boundary conditions on the circuit via the MoM procedure.

The first step of the spatial-domain MoM in the solution of MPIE for the analysis of planar stratified geometries is to calculate the Green's functions of the vector and scalar potentials. It should be re-emphasized here, as stated in Introduction, that this formulation has the advantage of using the Green's functions of vector and scalar potentials that are less singular than the Green's functions required by the EFIE. The derivations of the spatial-domain Green's functions for the vector and scalar potentials were given in detail in Chapter 2, and it was shown that it is quite robust and applicable for microwave components built on a planar multilayer medium, i.e., printed circuit board (PCB). Once the Green's functions are obtained, the solution of the MPIE via the MoM proceeds as follows: i) the unknown function of the MPIE, which is the current density in this work, is written in terms of known basis functions with unknown coefficients; ii) boundary conditions are implemented over the regions of PEC (small regions of PEC in a dominantly non-PEC layer) and/or over the regions of apertures (small regions of slots in a dominantly PEC layer) in integral sense,

which is generally known as testing procedure of the MoM; iii) the resulting set of linear equations is solved for the unknown coefficients of the basis functions, and hence the current density is obtained approximately by these coefficients and the used basis functions.

In this chapter, the formulation of the problem is given for a general planar multilayer circuit in Section 3.1. Then, comments on the used and other kinds of basis and attachment functions are discussed in Section 3.2. The interpretation of the MoM matrix entries are explained in details in Section 3.3 and finally section 3.4 concludes this chapter.

3.1 Formulation of MPIE in Planar Stratified Media

For the sake of illustration, a general 3-D microstrip structure is considered, as shown in Fig. 3.1, where it is assumed that all layers uniformly extend to infinity in transverse domain (xy -plane), conductors are lossless and infinitesimally thin, and the thickness and permittivity of layer- i are designated by d_i and ϵ_{r_i} , respectively. Note that when there are vertical metallizations in the circuit, the associated Green's functions, in addition to those for the horizontal conductors, need to be obtained prior to the application of the spatial-domain MoM for the solution of MPIE. In the rest of this chapter, it is assumed that all the necessary Green's functions have been obtained in closed forms by following the procedure detailed in Chapter 2. Therefore, this chapter mainly provides a detailed discussion on the formulation of MPIE and the implementation of the spatial-domain MoM. Note also that, throughout this work horizontal refers to the direction parallel to the plane of dielectric layers and vertical refers to the normal direction to the plane of layered media.

The formulation of MPIE begins with writing the electric field in terms of scalar and vector potentials as

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\phi \quad (3.1)$$

Since the Green's functions of the vector and scalar potentials have already been obtained and cast into closed forms, these potentials can be written as the convolution integrals involving surface current density $\mathbf{J}$ and charge density ρ on the metallizations as

$$\mathbf{A} = \mathbf{G}^A * \mathbf{J} \quad (3.2)$$

$$\phi = G^q * \rho = G^q * \left(\frac{-1}{j\omega} \nabla \cdot \mathbf{J} \right) \quad (3.3)$$

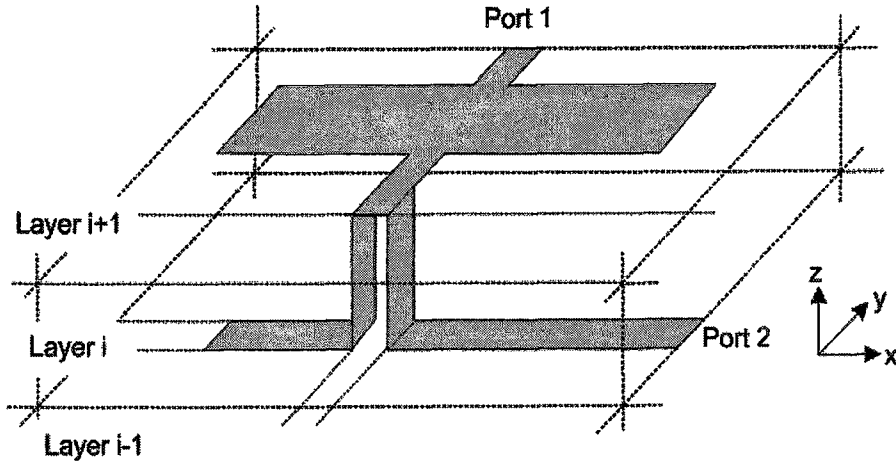


Figure 3.1: A general 3-D microstrip structure

where $\mathbf{G}^A$ is the dyadic Green's function of the vector potential, G^q is the Green's function of the scalar potential, $*$ denotes convolution, and the charge density ρ is written in terms of surface current density $\mathbf{J}$ using the continuity equation $\nabla \cdot \mathbf{J} + j\omega\rho = 0$. Note that the electric field expression given in (3.1) together with (3.2) and (3.3) is the MPIE, and the only unknown in this integral equation is the electric current density $\mathbf{J}$. For the sake of completeness, the block diagram of the MPIE representation is shown in Fig. 3.2. From equations (3.1), (3.2) and (3.3), the tangential components of the electric field on the plane of the horizontal and vertical metallizations can be written as

$$E_x = -j\omega G_{xx}^A * J_x + \frac{1}{j\omega} \frac{\partial}{\partial x} (G^q * \nabla \cdot \mathbf{J}) \quad (3.4)$$

$$E_y = -j\omega G_{yy}^A * J_y + \frac{1}{j\omega} \frac{\partial}{\partial y} (G^q * \nabla \cdot \mathbf{J}) \quad (3.5)$$

$$E_z = -j\omega G_{zx}^A * J_x - j\omega G_{zy}^A * J_y - j\omega G_{zz}^A * J_z + \frac{1}{j\omega} \frac{\partial}{\partial z} (G^q * \nabla \cdot \mathbf{J}) \quad (3.6)$$

where $G_{xx}^A = G_{yy}^A$. Note that the term G_{ij}^A represents the i -directed vector potential at $\mathbf{r}$ due to a j -directed electric dipole of unit strength located at $\mathbf{r}'$, and the term G^q represents the scalar potential of a unit point charge associated with an electric dipole. Since the traditional form of the Green's functions are employed in this formulation, the scalar potentials for the horizontal and vertical electric dipoles are not equal to each other, in other words, the scalar potential is not unique [41]. Therefore, the terms with the Green's function of the scalar

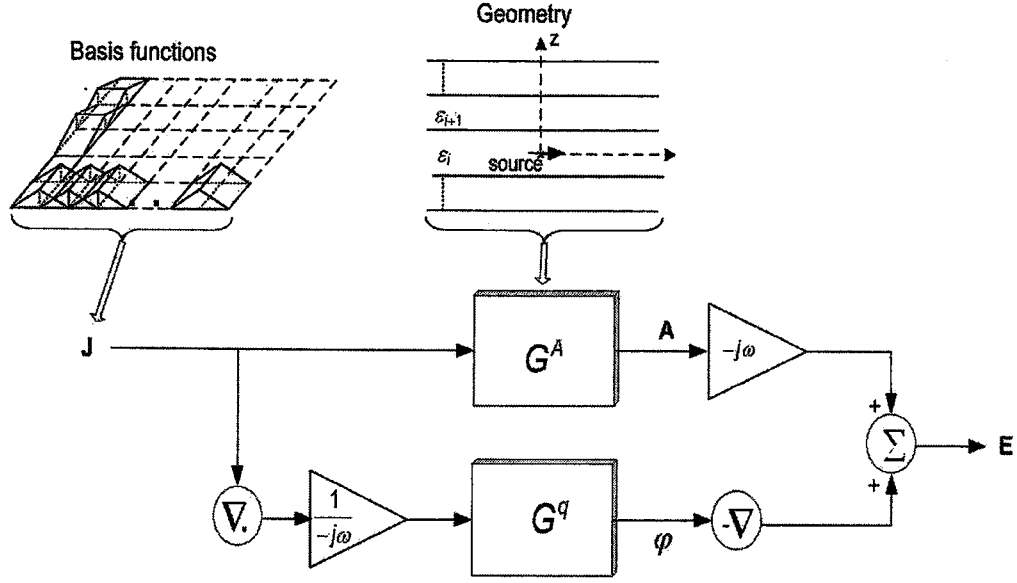


Figure 3.2: The block diagram for the solution of MPIE via MoM

potential in Eqs.(3.4-3.6) can be written explicitly as

$$G^q * \nabla \cdot \mathbf{J} = G_x^q * \frac{\partial J_x}{\partial x} + G_y^q * \frac{\partial J_y}{\partial y} + G_z^q * \frac{\partial J_z}{\partial z} \quad (3.7)$$

where G_x^q ($= G_y^q$) is the Green's function of the scalar potential for a horizontal electric dipole while G_z^q represents the scalar potential for a vertical electric dipole.

As the procedure of the MoM, to solve for the surface current density $\mathbf{J}$ on the metalizations, $\mathbf{J}$ is expanded in terms of known basis functions with unknown coefficients as

$$J_x(x, y, z) = \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)}(x, y, z) \quad (3.8)$$

$$J_y(x, y, z) = \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)}(x, y, z) \quad (3.9)$$

$$J_z(x, y, z) = \sum_l I_z^{(l)} B_z^{(l)}(x, y, z) \quad (3.10)$$

where $B_x^{(m,n)}$, $B_y^{(m,n)}$ and $B_z^{(l)}$ are the “known” basis functions with “unknown” amplitudes, or coefficients, $I_x^{(m,n)}$, $I_y^{(m,n)}$ and $I_z^{(l)}$, defined at the (m, n) -th position on the subdivided horizontal conductor and at (l) -th position on the subdivided vertical conductor. For the sake of illustration, Fig. 3.3 is provided to show the setup of discretization and the associated basis functions for a typical geometry involving vertical conductors in the form of strips and

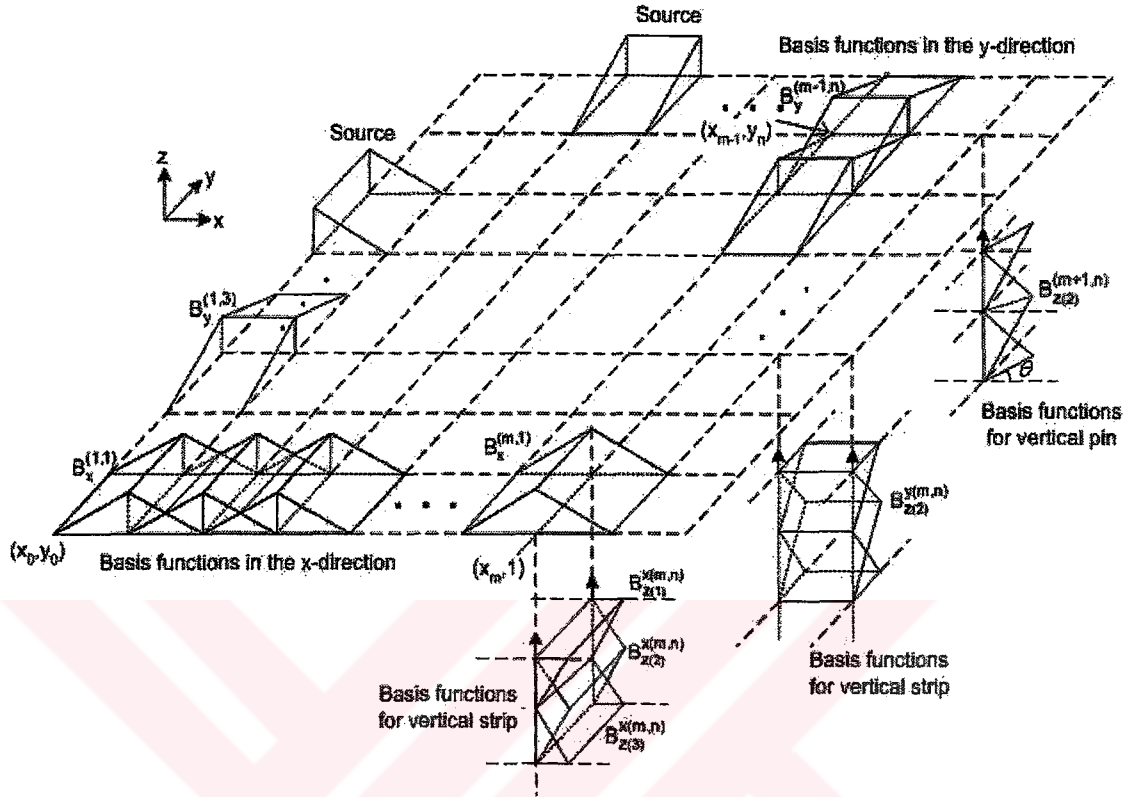


Figure 3.3: The discretization of the geometry and the basis functions

pins. Note that, the mathematical definitions of J_x and J_y , as given in Eqs.(3.8) and (3.9) are general, i.e. they can span any horizontal surface in the spatial domain. For the mathematical description of J_z , as in Eq.(3.10), it is assumed that there is only one vertical conductor and that its position in the horizontal plane is given by its x and y coordinates. To write a general expression for J_z , for geometries with vertical metallizations of different lengths and different locations on the horizontal plane, the following expression is suggested and used throughout this work:

$$\begin{aligned}
 J_z(x, y, z) &= \sum_m \sum_l I_{z(l)}^{x(m,n)} B_{z(l)}^{x(m,n)}(x, y, z) + \sum_n \sum_k I_{z(k)}^{y(m,n)} B_{z(k)}^{y(m,n)}(x, y, z) \\
 &+ \sum_l I_{z(l)}^{(m,n)} B_{z(l)}^{(m,n)}(x, y, z)
 \end{aligned} \quad (3.11)$$

where the mathematical expressions for the basis functions are given below:

$$B_{z(l)}^{x(m,n)}(x, y, z) = \begin{cases} \frac{1}{h_y} \left(\frac{1}{h_z} |z - z_l| + 1 \right) & \text{for } z_l - h_z \leq z \leq z_l + h_z, x = x' = x_m \\ & \text{and } y_n - h_y \leq y \leq y_n + h_y \\ 0 & \text{elsewhere} \end{cases} \quad (3.12)$$

$$B_{z(k)}^{y(m,n)}(x, y, z) = \begin{cases} \frac{1}{h_x} \left(\frac{1}{h_z} |z - z_l| + 1 \right) & \text{for } z_l - h_z \leq z \leq z_l + h_z, y = y' = y_n \\ & \text{and } x_m - h_x \leq x \leq x_m + h_x \\ 0 & \text{elsewhere} \end{cases} \quad (3.13)$$

$$B_{z(l)}^{(m,n)}(x, y, z) = \begin{cases} \left(\frac{1}{h_z} |z - z_l| + 1 \right) & \text{for } z_l - h_z \leq z \leq z_l + h_z, y = y' = y_n \\ & \text{and } x = x' = x_m \\ 0 & \text{elsewhere} \end{cases} \quad (3.14)$$

Note that h_x , h_y and h_z are the half spans of the basis functions in the x , y and z directions respectively as shown in Fig. 3.4.

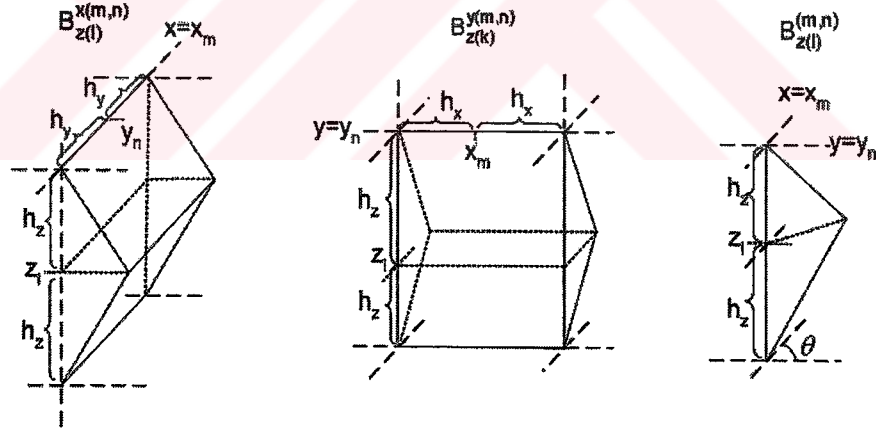


Figure 3.4: A detailed look at the basis functions in the vertical direction.

In this thesis, the basis functions used to approximate the induced current density on the conductors of the geometry are chosen to be rooftop functions. Since sources and sinks are modelled as current filaments connected to the conductor, half-rooftop basis functions are used to represent the induced currents at the points of connections of sources and sinks. In addition, special care should be exercised at the intersection regions of vertical

and horizontal conductors to satisfy the continuity equation. For such intersections, the attachment functions used in the horizontal directions are sawtooth functions and the basis functions used for the vertical direction are half-rooftop functions. The reasons for choosing such basis functions and comments on the other kinds of basis and testing functions will be discussed in the following section. But before proceeding further, it would be helpful to give the basis and testing function symbols and their definitions to be able to follow the derivations and comments easily throughout this work. Table 3.1 gives the notations and definition of the basis, attachment and source functions used for the formulation in this thesis. Note that symbols for the testing functions and the coefficients of the corresponding basis functions have the same notation, in other words it is sufficient to replace B by T or I to denote the corresponding testing function and coefficient for any basis function. Once

Basis Functions	Definition
$B_x^{(m,n)}(x, y)$	(m, n) -th x -directed xy spanning basis function
$B_y^{(m,n)}(x, y)$	(m, n) -th y -directed xy spanning basis function
$B_{z(l)}^{x(m,n)}(y, z)$	(l) -th z -directed yz spanning basis function at $x = x_m$
$B_{z(k)}^{y(m,n)}(x, z)$	(k) -th z -directed xz spanning basis function at $y = y_n$
$B_{z(l)}^{(m,n)}(z)$	(l) -th z -directed basis function at $x = x_m$ and $y = y_n$
$B_{xSP}^{(m,n)}(x, y)$	(m, n) -th x -directed xy spanning attachment function
$B_{ySP}^{(m,n)}(x, y)$	(m, n) -th y -directed xy spanning attachment function
$B_{xS}^{(m,n)}(x, y)$	(m, n) -th x -directed xy spanning source function(half rooftop)
$B_{yS}^{(m,n)}(x, y)$	(m, n) -th y -directed xy spanning source function(half rooftop)

Table 3.1: The symbols and definition of the basis and attachment functions used in this thesis

the unknown current density is expanded in terms of known basis functions with unknown coefficients, the next step of the spatial-domain MoM is to substitute the current densities in Eqs.(3.8), (3.9) and (3.11) into electric field expressions in Eqs.(3.4-3.6), and then it is followed by the application of the boundary conditions. Here the boundary conditions are applied in the integral sense through the well-known testing procedure of the MoM where the field expressions are multiplied by known testing functions $T_x^{(m',n')}$, $T_y^{(m',n')}$, T_z

and integrated on the conductors and set to zero. As a result of all these steps, a matrix equation for the unknown amplitudes of the basis functions is obtained and has the following form:

$$[\mathbf{Z}][\mathbf{I}] = [\mathbf{V}]$$

$$\begin{bmatrix} Z_{xx} & Z_{xy} & Z_{xz} \\ Z_{yx} & Z_{yy} & Z_{yz} \\ Z_{zx} & Z_{zy} & Z_{zz} \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \quad (3.15)$$

where $\mathbf{Z}$ is called the impedance matrix and its elements represent the mutual impedances between the testing and basis functions, $\mathbf{V}$ is the excitation matrix and its elements represent the excitation voltages due to the current sources in the system, and finally $\mathbf{I}$ is the unknown current amplitude matrix. Note that, the impedance and excitation matrix entries in Eq.(3.15) are matrices themselves and for the sake of completeness, their typical entries are given as follows:

$$Z_{xx}^{(m',n')(m,n)} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.16)$$

$$Z_{xy}^{(m',n')(m,n)} = \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \quad (3.17)$$

$$Z_{xz}^{(m',n')(l)} = \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (3.18)$$

$$Z_{yx}^{(m',n')(m,n)} = \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.19)$$

$$Z_{yy}^{(m',n')(m,n)} = \left\langle T_y^{(m',n')}, G_{yy}^A * B_y^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \quad (3.20)$$

$$Z_{yz}^{(m',n')(l)} = \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (3.21)$$

$$Z_{zx}^{(l')(m,n)} = \left\langle T_{z(l')}, G_{zx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.22)$$

$$Z_{zy}^{(l')(m,n)} = \left\langle T_{z(l')}, G_{zy}^A * B_y^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \quad (3.23)$$

$$Z_{zz}^{(l')(l)} = \left\langle T_{z(l')}, G_{zz}^A * B_{z(l)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (3.24)$$

$$V_x^{(m',n')(m,n)} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_{xs}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_{xs}^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.25)$$

$$V_y^{(m',n')(m,n)} = \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_{xs}^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.26)$$

$$V_z^{(l')(m,n)} = \left\langle T_{z(l')}^A, G_{zz}^A * B_{zs}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_{xs}^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.27)$$

where $B_{z(l)}$ and $T_{z(l')}$ are the general representations of the basis and testing functions in the vertical direction. According to the aforementioned definition of the current density J_z in Eq.(3.11), B_z and T_z in Eqs.(3.18),(3.21),(3.24) and (3.27) should be read $B_{z(l)}^{x(m,n)}$, $B_{z(k)}^{y(m,n)}$ or $B_{z(l)}^{(m,n)}$, and $T_{z(l')}^{x(m',n')}$, $T_{z(k')}^{y(m',n')}$ or $T_{z(l')}^{(m',n')}$, respectively. Note also that $\langle, \rangle$ and $*$ denote inner product and convolution operators, respectively, and they are defined as follows:

$$\langle f(x, y), g(x, y) \rangle = \iint dx dy f(x, y) \cdot g(x, y) \quad (3.28)$$

$$f(x, y) * g(x, y) = \iint dx' dy' f(x - x', y - y') \cdot g(x', y') \quad (3.29)$$

After having written the matrix entries explicitly in terms of known quantities, there are still two more major steps to find the unknown coefficients of the basis functions: i) evaluation of these matrix entries, which are quadruple integrals of complex and oscillatory functions; ii) solution of the matrix equation for the coefficients of the basis functions, as denoted by I_x , I_y and I_z in Eq.(3.15). Since the integrands of the matrix entries would be different for different types of basis and/or testing functions, the matrix entries should be classified first and the evaluation of the classes of matrix entries should be performed separately. Note that in the evaluation of these matrix entries, analytical methods are used which were introduced by Alatan et al. [17]. By the use of analytical evaluation instead of numerical techniques, a significant improvement in the computation time is achieved. Therefore, in Appendix D, the analytical evaluation of the matrix entries for different types of basis and/or testing functions are explained in detail for future reference. After the evaluation of the inner product terms and substituting them into Eq. 3.15, the current densities on the conductors are obtained by solving the matrix equations. Then, the circuit parameters such as the scattering parameters are extracted from the current distribution.

The use of closed-form Green's functions in conjunction with the MoM has been demonstrated to improve the computational efficiency of the MoM when applied to simple geometries like those consisting of only horizontal conductors. But in case of vertical metal-

lizations, there are some difficulties in the application of the spatial-domain MoM. These difficulties and their remedies are the subject of the next Chapter. Therefore in the following sections, comments on the choice of basis and testing functions and behavior of the MoM matrix entries will be presented along with their physical interpretations.

3.2 Comments on the Basis and Attachment Functions

As previously mentioned, the basis and testing functions used to approximate the induced current density along the horizontal and vertical conductors of the geometry are chosen to be rooftop functions throughout this thesis. In cases of some special geometries, like at the terminals of sources and sinks, and at the intersections of horizontal and vertical conductors, half-rooftop functions and sawtooth functions are employed, respectively. Since sources and sinks are modelled as filaments connected to the conductor(s) of the geometry, the induced current density on the conductor pieces right next to these filamentary connections is approximated by half-rooftop functions. The choices of half-rooftop and sawtooth functions as the basis functions defined at such connections can be justified by considering the actual physical current distribution on the conductor(s) next to these connections. The testing functions are always chosen to be rooftop functions whether they are implemented on the horizontal or vertical conductors.

The choice of basis and testing functions plays a crucial role in the solution of an operator equation via the MoM [22]. Therefore, it would be instructive to give a brief overview of the physical and mathematical importance of basis and testing functions, although they were chosen a priori, and have already been used throughout this work. While the basis functions are the “known” functions with “unknown” amplitudes to represent the current density on the regions where the current is induced, the testing functions are the weighting functions defined over the regions where the boundary conditions for the electric and/or magnetic fields are applied in integral sense. Considering only the electric current density in the geometry, i.e., the geometries with only conducting traces, the total electric field on a region specified by a testing function is the sum of the contributions of basis functions, each of which acts like a separate source. If the basis functions are chosen in such a way that the current density on the conductors of the geometry can be accurately represented by these basis functions, then the accuracy of the results and the computational cost of

the algorithm are improved significantly. In other words, if the functional form of the basis functions cannot match the physical form of the actual current density on the conductors of the geometry, then the MoM algorithm may not converge (or converge very slowly) with the increased number of basis functions, resulting in a very inefficient algorithm and inaccurate results. Therefore, for judicious choice of testing and basis functions for the solution of an integral equation via the MoM, one needs to have some intuition on the physical form of the current induced in the geometry, and to examine the resulting MoM matrix entries for the convergence of the integrals involved [22].

The choice of the basis functions for the conductors of the geometries, which may involve both horizontal and vertical conductors, is quite straightforward: i) one needs to decide if entire-domain basis functions or sub-domain basis functions are to be employed for the geometry to be analyzed; ii) if entire-domain basis functions are chosen for the problem at hand, then see if there is any edge-singularities of the current density that need to be included in the basis functions to accurately represent the current density; iii) if sub-domain basis functions are chosen, since edge-singularities can be predicted automatically by the appropriate amplitudes of the basis functions, the convergence of the resulting MoM matrix entries needs to be examined. However, when there are intersections of horizontal and vertical conductors in the geometry, the choice of the basis functions at these junctions is not as straightforward. There are mainly two major concerns: i) continuities of the current and charges at these junctions need to be satisfied; ii) singularities of the basis functions representing the horizontal and vertical current densities at these junctions need to be treated with utmost care. Note that these singularities are inevitable at these junctions because the flow of the currents on the vertical conductors is due to the discontinuities of the current on the horizontal conductors, and vice versa. Therefore, the basis functions chosen at these junctions should be able to model the physical current flow from the horizontal conductor to vertical conductor or vice versa in the mathematical world. Since the basis functions are written to approximate the unknown current density in the formulation of MPIE, their derivatives represent the charge density due to the continuity equation. Therefore, the chosen basis functions should ensure the continuity of the current as well as conservation of charges on the conductor. To see if the rooftop functions qualify as the mathematical model for the physical current density, the function itself and its derivative along a specified di-

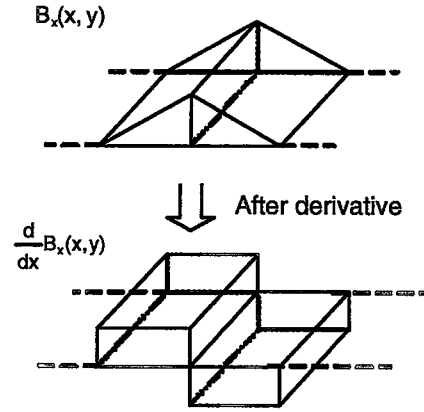


Figure 3.5: Rooftop basis function and its derivative

rection are shown in Fig. 3.5. Note that the total charges corresponding to a basis function representing a small part of the induced current along a conductor is zero, as it is observed from Fig. 3.5. This fact seems to be the result of the choice of the rooftop functions as the basis functions for the current density, but, as contrary, it is the requirement for the current and charge distributions over a conductor. Remembering that the rooftop basis functions are defined over a rectangular cell, the current entering into and exiting from this cell are equal, and hence there is no loss or gain of charges in this rectangular region, i.e., no sink or source in this particular cell. Therefore, the total charges in the circuit must be equal to zero, because the charges supplied by the source basis functions are taken out of the circuit by the sink basis functions. In other words, the rest of the basis functions defined over the conductor can neither introduce nor lose any charges, which becomes a requirement for the choice of the basis functions for the current density. Only source and sink basis functions, which are chosen to be half-rooftop functions in this work, result in finite total charges on the domain they are defined, provided that the impulse introduced by the differentiation of an half-rooftop function is ignored which is shown in Fig. 3.6.

After having clarified the issue of what introduces the charges in a circuit, a consistent mathematical model for the basis functions at the junctions of the vertical and horizontal conductors needs to be developed. As a first trial for the basis functions defined at the junctions, as the extension of the basis functions used everywhere else, one rooftop function is employed over the horizontal and one over the vertical conductor, as shown in Fig. 3.7.

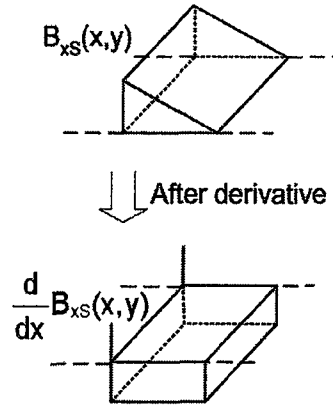


Figure 3.6: Half-rooftop basis function and its derivative

Obviously, the conservation of charge is satisfied, but in this case, the flow of charge, and in turn the flow of current, from the horizontal conductor to vertical conductor, or vice versa, is not achieved. Therefore, these basis functions can only model two non-touching conductors, i.e., vertical and horizontal conductors with an infinitesimal gap at the junction. As a conclusion of this trial, to achieve a flow of current between the horizontal and vertical

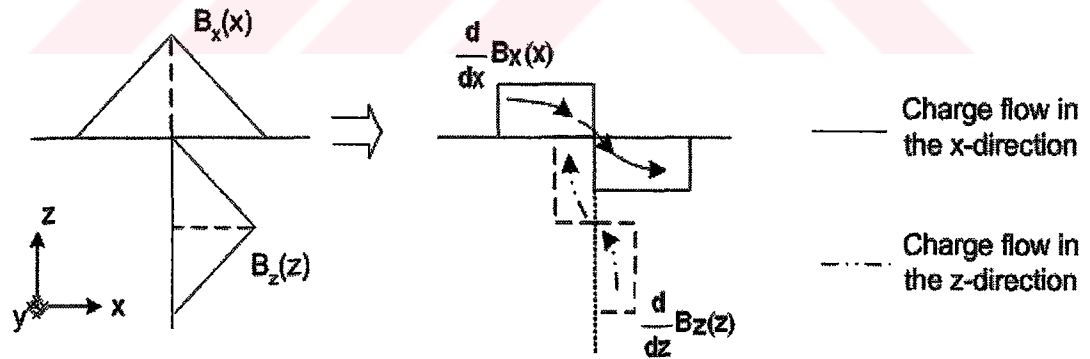


Figure 3.7: Basis functions for current and charge densities to model a non-touching vertical and horizontal conductors.

conductors, one needs to define a basis function over the domain, a part of which is on the horizontal conductor and the rest is on the vertical conductor. In addition, the requirement of zero total charge over the domain of the basis function must be satisfied in such a way

that there must be finite amount of charges on the horizontal and vertical conductors with opposite polarities. This additional requirement will ensure the flow of the current between these two conductors. When these requirements are examined carefully with a view that the proposed basis function is to be consistent with the rooftop functions used elsewhere on the geometry, a half-rooftop function is used as the basis function on the vertical part of the junction and a saw-tooth function as its attachment on the horizontal part of the junction. In other words, three different functions are used to represent the current density at every junction, which are, namely, i) a rooftop function on the horizontal conductor over the junction, ii) a half-rooftop function on the vertical conductor right next to the junction, and iii) a saw-tooth function on the horizontal conductor over the same cells as the rooftop function in item (i), which is shown in Fig. 3.8. Note that the attachment function, (iii), defined on the horizontal conductor, must be related to the basis function on the vertical conductor, (ii), with the use of the current continuity at the junction, i.e., the amplitude of the attachment function is not unknown but it is in terms of the unknown amplitude of the half-rooftop basis function defined on the vertical conductor at the junction.

Regarding the above choice of the basis functions for the current density at a junction, since they are discontinuous right at the junction, their derivatives become singular (impulse functions) at the positions of discontinuities as shown in Fig. 3.8. Physically, these singularities are interpreted as line charges located at the line of intersection of horizontal and vertical conductors. It should be noted that apart from the line charges resulting from the derivatives of the attachment and basis functions at the discontinuities, each function has a total net charge. Physically, the existence of the total net charge may be thought as the injection of charge on the conductor at the intersection regions which results in source behavior. Since actually, there is not a charge injection, the amplitude of the attachment function is chosen in such a way that the total net charge on the intersection due to both horizontal and vertical conductors is zero. As it can be seen from Fig. 3.8, the total net charges under the sawtooth attachment function and vertical half rooftop basis functions are proportional to $(I_{xSP}/h_x) 2h_x h_y = I_{xSP} 2h_y$ and $(I_z/h_z) h_z h_y = I_z h_y$, respectively. Since the total net charge on the intersection should be zero, the following equation is obtained:

$$I_z h_y + 2I_{xSP} h_y = 0 \quad (3.30)$$

Thus, choosing the amplitude of the sawtooth attachment function, I_{xSP} , -0.5 times the

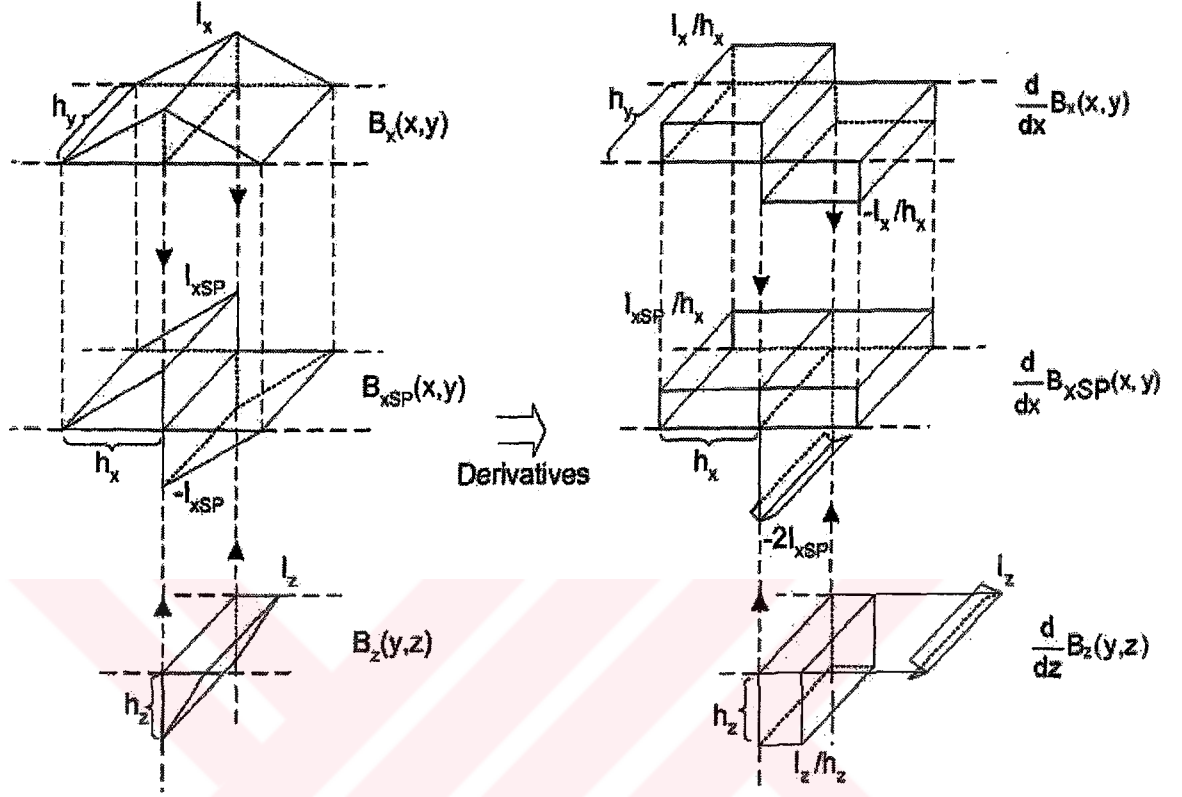


Figure 3.8: The rooftop basis function, sawtooth attachment function and the half rooftop basis function in the horizontal and vertical directions used to represent the current density at horizontal and vertical conductor junctions

amplitude of the vertical basis function, I_z , satisfies the conservation of charge at the junction. As a result, it can be seen that representing the current density at the junction by using this configuration is physically acceptable for the modelling of the charge flow.

Another point to mention here is that the line charges due to the discontinuities of the basis and attachment functions are not considered, because, at the region of intersection, their contributions are assumed to cancel each other owing to the fact that the Green's function of the scalar potentials due to horizontal and vertical dipoles, i.e. G_x^q and G_z^q are close to each other at $z = z'$ for $\rho \rightarrow 0$. To have a better understanding, the following matrix entry involving the vertical basis and horizontal attachment function is considered:

$$Z_{xz}^q = -\frac{1}{2j\omega} \left\langle T_x(x,y), \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_{xSP}(x,y)}{\partial x} \right] \right\rangle + \frac{1}{j\omega} \left\langle T_x(x,y), \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_z(y,z)}{\partial z} \right] \right\rangle \quad (3.31)$$

where $B_{xSP}(x, y)$ and $B_z(y, z)$ are the attachment and vertical basis function which are discontinuous at the junction. Therefore, by writing the basis and the attachment functions as the sum of their singular and continuous parts, the above expression can be written as follows:

$$\begin{aligned}
 Z_{xz}^q = & -\frac{1}{2j\omega} \left\langle \frac{\partial}{\partial x} T_x(x, y), G_x^q * \left\{ \frac{\partial B_{xSP}(x, y)}{\partial x} - 2\delta_y(x - x_i, z - z_i) \right\} \right. \\
 & + \left. G_x^q * 2\delta_y(x - x_i, z - z_i) \right\rangle + \frac{1}{j\omega} \left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \left\{ \frac{\partial B_z(y, z)}{\partial z} - \delta_y(x - x_i, z - z_i) \right\} \right. \\
 & + \left. G_z^q * \delta_y(x - x_i, z - z_i) \right\rangle
 \end{aligned} \quad (3.32)$$

where $\delta_y(x - x_i, z - z_i)$ represents a line source in the y -direction at the junction created by the derivatives of the discontinuous basis functions. Once the terms in the above equation are recombined, the following expression is obtained:

$$\begin{aligned}
 Z_{xz}^q = & -\frac{1}{2j\omega} \left\langle \frac{\partial}{\partial x} T_x(x, y), G_x^q * \left\{ \frac{\partial B_{xSP}(x, y)}{\partial x} - 2\delta_y(x - x_i, z - z_i) \right\} \right\rangle \\
 & + \frac{1}{j\omega} \left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \left\{ \frac{\partial B_z(y, z)}{\partial z} - \delta_y(x - x_i, z - z_i) \right\} \right\rangle \\
 & + \frac{1}{j\omega} \left\langle \frac{\partial}{\partial x} T_x(x, y), \{G_z^q - G_x^q\} * \delta_y(x - x_i, z - z_i) \right\rangle
 \end{aligned} \quad (3.33)$$

The last term in Eq. 3.33 can be neglected when the Green's functions are close to each other. Since at $z = z'$ as $\rho \rightarrow 0$ $G_x^q \approx G_z^q$, the contribution of the line sources resulted from the derivations of the basis functions is ignored. The behavior of the Green's function is demonstrated in Fig. 3.9 for a substrate backed by a ground layer whose thickness is 0.154 cm and dielectric constant is 4.76 at the frequency of 1.0 GHz where the source is at the substrate-air interface. As can be seen from the figure, the Green's functions G_x^q and G_z^q are close to each other for $\rho \rightarrow 0$.

The relations between the amplitudes of the vertical basis function and the attachment function at the junction given above is for the case where the vertical conductor is a vertical strip, i.e. the vertical conductor has a surface. But when the vertical conductor is a pin, i.e. there is a line current instead of surface current density, the basis functions representing the line current should be chosen accordingly. Since the pin structure can be considered as strips with zero widths, the basis functions can be chosen to be triangular functions that are obtained from the rooftop functions as their widths converge to zero as shown

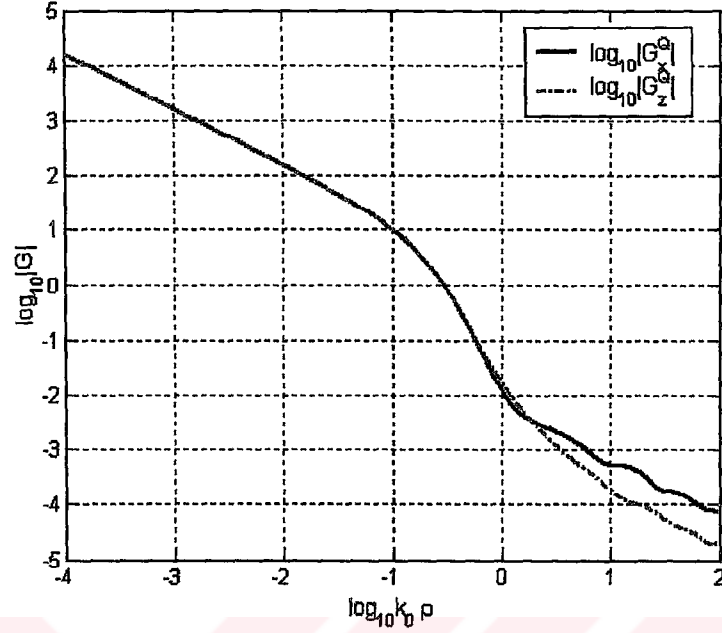


Figure 3.9: The magnitude of the Green's functions G_z^q and G_x^q for a substrate backed by a ground plane with a thickness of 0.152 cm and 4.76 dielectric constant. Note that the Green functions are close to each other at $z = z'$ for $\rho \rightarrow 0$

in Fig 3.10. Note that in this thesis, the vertical pins are always located at the corners of the horizontal cells used in the discretization of the horizontal conductor as shown in Fig. 3.3. As in the vertical strip case, the intersection of horizontal conductor and vertical pin needs special care. At the point of intersection, the conservation of charge and current flow should be satisfied. To satisfy these requirements, seven different functions are used to represent the current density at every intersection, four of which are the horizontal rooftop basis functions, two of them are horizontal sawtooth attachment functions and the last one is the half-triangular function on the vertical pin. As already mentioned, the vertical pins can be located at the common vertices of the four horizontal cells. A horizontal rooftop function is used for each of the adjacent two cell combinations. Therefore there are a total of four horizontal basis functions two of which are x -directed and located next to each other. Similarly the remaining two basis functions are y -directed and located next to each other. One of the sawtooth function is in the x -direction and occupies the same cells as the two x -directed horizontal basis functions occupy. Similarly, the other sawtooth

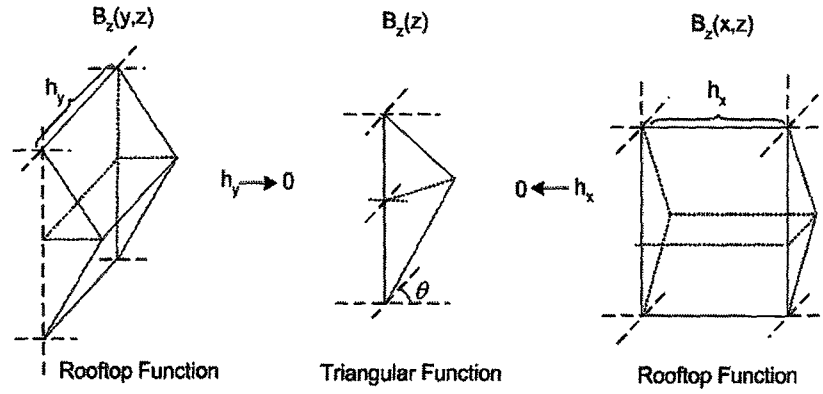


Figure 3.10: The triangular function used in the modelling of the current on vertical pins

function is in the y -direction spanning over the cells of the two y -directed horizontal basis functions. Finally the half triangular basis function on the vertical pin is located at the common vertices of these four cells. The overall configuration can be seen in Fig. 3.11. To

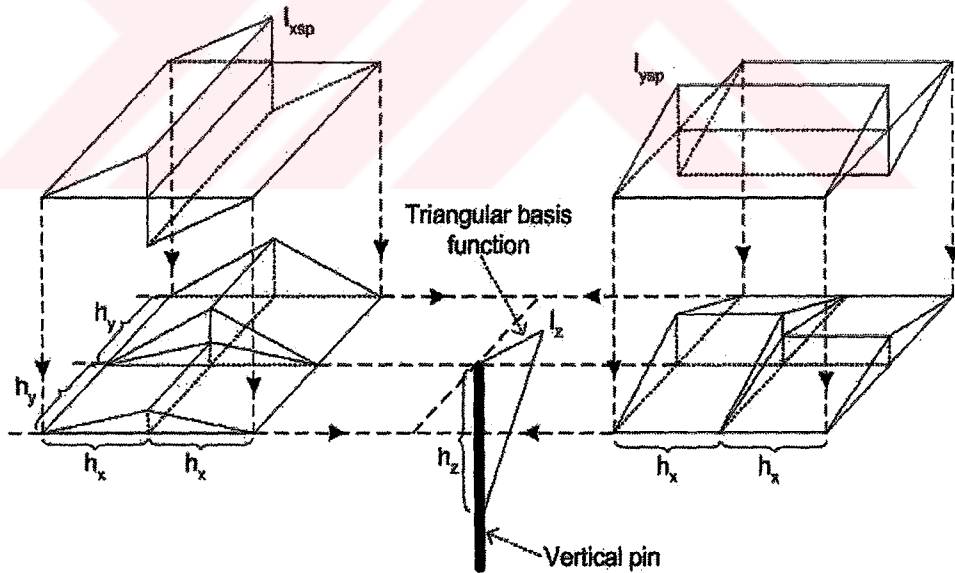


Figure 3.11: The vertical pin and its associated attachment functions

satisfy the conservation of charge at the intersection, the total charge under the attachment functions and the basis function on the vertical pin should be equal to each other. Taking the derivatives of the basis and attachment functions and equating the total charge under

the basis and attachment functions yield the following:

$$\begin{aligned}\frac{I_{xSP}}{h_x} 2h_x 2h_y + \frac{I_{ySP}}{h_y} 2h_y 2h_x + \frac{I_z}{h_z} h_z &= 0 \\ I_{xSP} 4h_y + I_{ySP} 4h_x + I_z &= 0\end{aligned}\quad (3.34)$$

where $2h_x$ and $2h_y$ are the x and y lengths of the attachment functions. Choosing the amplitudes of the attachment functions equal to each other, i.e. $I_{xSP} = I_{ySP}$, result in the following condition on the vertical basis and its associated attachment functions in case of vertical pins.

$$I_{xSP} 4(h_x + h_y) = -I_z \quad (3.35)$$

Therefore, when there exist a vertical pin and a horizontal conductor intersection, the contribution of the attachment functions should be accounted for the analysis by using the relation in Eq.(3.35).

Up to this point, the basis and attachment functions that will be used for the vertical strip and pin cases are given with their reasonings. But when the matrix entries given in Eqs.(3.16-3.27) are revisited, it may be thought that some choices of basis functions may decrease the computational complexity of the system. As an example consider that the vertical basis functions are chosen to be rectangular pulses instead of rooftop functions. In this case, the terms involving the derivative of the vertical basis function automatically become zero since the derivative of the rectangular pulse is zero. Therefore the computational complexity of the system may decrease significantly, since there is no need to evaluate the corresponding inner products. Although the computational efficiency seems to be increasing, the conservation of charge and current flow should be satisfied. If the attachment function is still kept as the sawtooth function, the flow of the charge may not be modelled accurately. This scenario is illustrated in Fig. 3.12 for the vertical strip case. As in the previous case, the amplitudes of the attachment and basis functions are related in such a way that the line charge (impulse function) from the sawtooth function eliminates the upper line charge from the rectangular pulse. But this time the flow of charge is not modelled accurately. Because, there is net total charge due to the derivative of the sawtooth function at the junction which acts as a source. Therefore, it is concluded that although this approach seems to decrease the computational complexity, the physical behavior is not modelled accurately and as a result of inaccurate modelling, there exists a charge injection

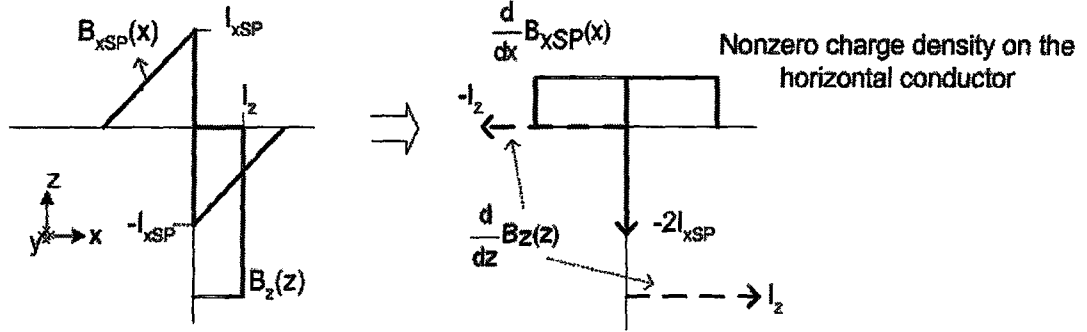


Figure 3.12: The sawtooth attachment function and the rectangular basis function in the horizontal and vertical directions, respectively

at the region of junction. Hence, choosing the attachment function as a sawtooth function is not a good choice for this kind of vertical basis function. The chosen attachment function should be able to model the conservation of charge and current flow. Since the problem with the sawtooth attachment function was due to the nonzero charge density obtained by the derivative of the sawtooth function, choosing the attachment function as a rectangular pulse as shown in Fig. 3.13 may get rid of the nonzero charge density under the sawtooth function. Because, there are only line charges in the derivative of the rectangular pulse attachment function. Again the amplitudes of the attachment and basis functions are cho-

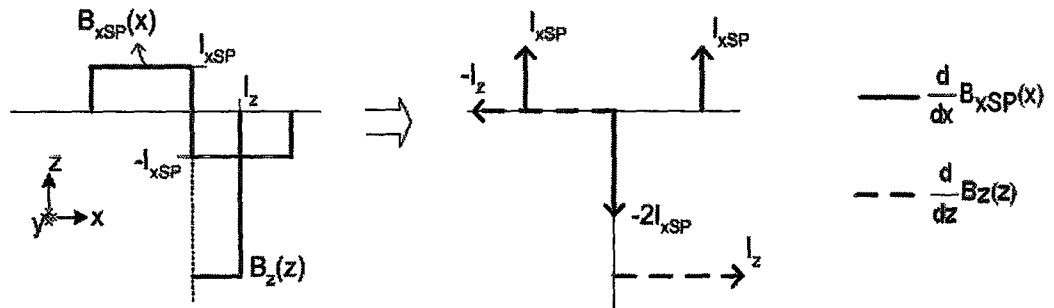


Figure 3.13: The rectangular pulse attachment function and the rectangular basis function in the horizontal and vertical directions, respectively

sen to satisfy the conservation of charge. The impulse functions in this case act as point sources at the points where they were defined. Therefore in this case, there is no physical

charge flow from the horizontal to vertical conductor or vice versa, i.e. the charge that should be flowing from the horizontal to vertical conductors is lost by choosing the basis and attachment functions as rectangular pulses. Hence, it is concluded that choosing basis and attachment functions as rectangular pulses is not appropriate for the modelling of the physical behavior of the current flow.

With the last two configurations of the basis and attachment functions, it has been observed that the choice of attachment and basis functions is quite important. The main criteria of choosing these functions is to look at their capability to model the physical behavior accurately. Once the accuracy of the model is guaranteed, the efficiency criteria should be considered. After a detailed analysis of various basis and attachment function configurations, half-rooftop and sawtooth functions are chosen and used throughout this work due to their accurate modelling and ability to be efficiently used in the analytical evaluation of the inner product terms. For the sake of completeness, the chosen basis and attachment functions in this thesis and the corresponding testing functions are given once more in Fig. 3.14.

3.3 Comments on MoM Matrix Entries

In the previous section, comments on the choices of basis and attachment functions in case of vertical metallization was explained. But except being rooftop functions for all the cases, not too many things were said about the testing functions. It is demonstrated in this section that choosing the basis and testing functions alike allows the usage of the similarity and interpretation of the terms in the MoM matrix entries. Interpreting the terms in the matrix entries can be very useful in guessing the system behavior, in obtaining an insight about the relations of these entries when the basis or testing functions change in a selected matrix entry term. It can also be used in the debugging and faster calculation of the MoM matrix entries. Therefore, in this section, the details of the interpretation of the MoM matrix entries are given.

As it was already mentioned, in the spatial-domain MoM formulation for the analysis of printed geometries, the procedure begins by writing the MPIE in terms of the Green's functions of vector and scalar potentials in a multilayered medium. After the discretization of the integral equation by expanding the unknown current densities in terms of known basis

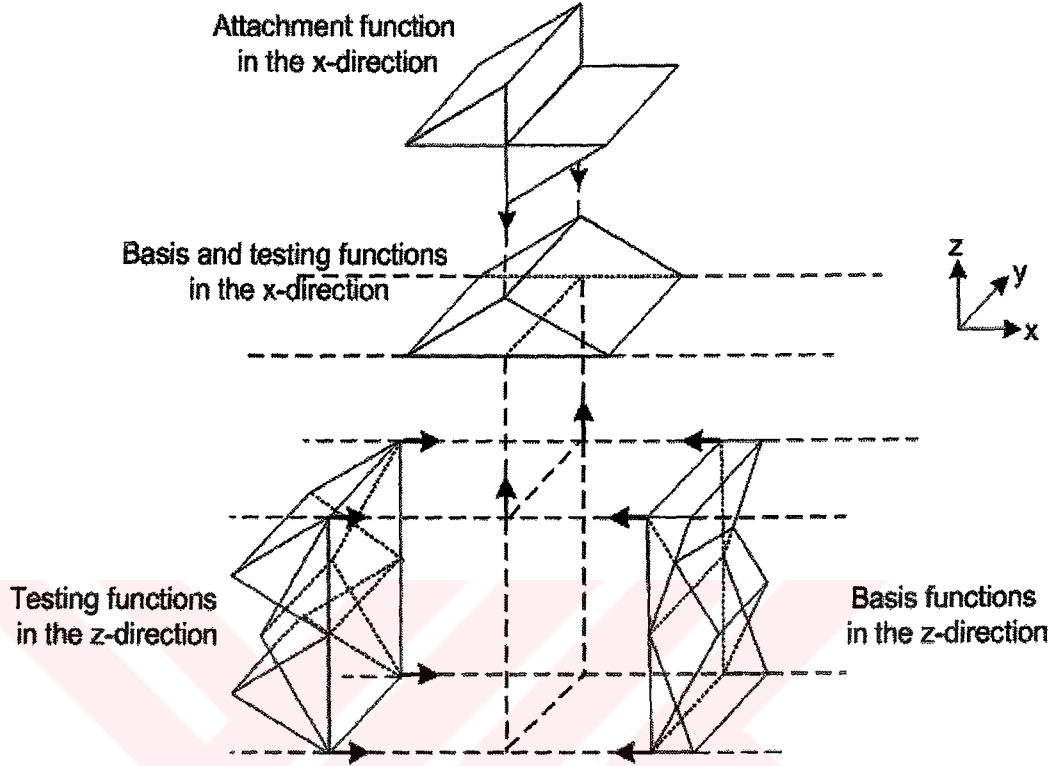


Figure 3.14: The basis, testing and attachment functions used in this thesis.

functions and application of boundary conditions in the integral sense, which is known as the testing procedure of the MoM, a linear system representing the current distribution on the conductor is found. This linear system is in the form of Eq.(3.15) whose details are given in Eqs.(3.16-3.27). But for the sake of completeness, the typical entries for the submatrices Z_{xx} , Z_{zx} and Z_{zz} are given below and their physical meanings are explained one more time to be able to interpret their behaviors.

$$(Z_{xx})^{(m',n'),(m,n)} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.36)$$

$$(Z_{zx})^{(l'),(m,n)} = \left\langle T_{z(l')}, G_{zx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (3.37)$$

$$(Z_{zz})^{(l'),(l)} = \left\langle T_{z(l')}, G_{zz}^A * B_{z(l)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (3.38)$$

When $(Z_{xx})^{(m',n'),(m,n)}$ term is considered, it is observed that there are two contribu-

tions to this term. The first contribution is due to the electric field generated by the current on the surface which is the $\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \rangle$ term and the second one is due to the electric field generated by the charge distribution on the surface which is shown as $\langle T_x^{(m',n')}, \frac{\partial}{\partial x} (G_z^q * \frac{\partial}{\partial z} B_z(l)) \rangle$ in Eq.(3.36). Similarly, all other terms in the impedance matrix entries containing the Green's function of the vector potential are due to the electric field generated by the current distribution on the surface whereas the ones containing the Green's functions of the scalar potential are due to the electric field generated by the charge distribution on the surface.

During the evaluation of the matrix entries, the identically chosen testing and basis functions (all are chosen to be rooftop or half-rooftop functions) allow the matrix filling algorithm to increase its efficiency by exploiting the similarity among these functions. To clarify the similarity, it is better to talk on an example: Consider the term $\langle T_x^{(m',n')}(x, y), G_{xx}^A * B_x^{(m,n)}(x, y) \rangle$. Physically this term is the application of the boundary condition in the integral sense on the region defined by the testing function $T_x^{(m',n')}(x, y)$ for the tangential electric field generated by the current distribution on the region defined by the basis function $B_x^{(m,n)}(x, y)$. When the above term is reexamined with the testing function being the basis function and vice versa (i.e. the basis and testing functions are switched), it is observed that due to reciprocity they yield the same results. This observation is valid for each pair of the testing and basis functions. Therefore, when a matrix entry due to a specific pair of basis and testing function is evaluated, the result can be used for the matrix entry where the basis and testing functions are switched.

It is better to give some examples on how to interpret the matrix entry terms for different cases. The relations between the matrix entry terms for a microstrip line over a ground plane is given below along with their physical interpretations. The microstrip line which is shown in Fig. 3.15 is fed from the edge port with a current source. Assume that in this microstrip line example, the width of the microstrip line is so small that the current can be assumed to be in the x -direction only, i.e. J_y does not exist. Consider the term $\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \rangle$ which is the application of the boundary condition in the integral sense over the region specified by the testing function $T_x^{(m',n')}$ for the electric field generated by the current density denoted by the basis function $B_x^{(m,n)}$. Physically speaking, this term should get smaller in magnitude as the distance between the basis and testing function increases. Likewise it

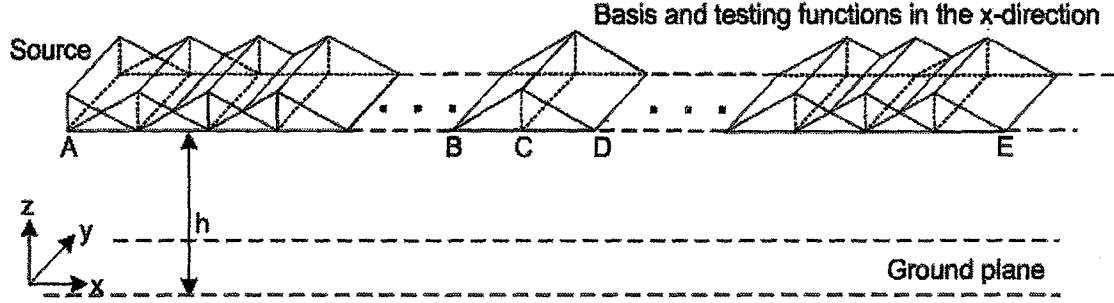


Figure 3.15: The microstrip line over an infinite ground plane

assumes the biggest value when the testing and basis functions overlap. This is consistent with the field theory which states that the field gets smaller as the observation point gets further away from the source point.

When the term $\left\langle T_x^{(m',n')}(x, y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)}(x, y) \right) \right\rangle$ which is the application of the boundary condition over the domain of the testing function $T_x^{(m',n')}(x, y)$ for the tangential electric field generated by the charge distribution specified by the derivative of the basis function $B_x^{(m,n)}(x, y)$ is considered, the magnitude decreases as the distance between the basis and testing function increases. This observation can be explained in a different way by remembering one of the fundamental rule of physics by asking how can one define the electric field. In the high school physics books, electric field is defined as the electric force per unit charge and its direction is taken to be the direction of the force it would exert on a positive test charge. So to use this definition for the interpretation of the relative behavior of the matrix entry terms, assume that in the microstrip line example, the basis function is chosen to be the one on the center of the microstrip line, i.e. point C is just on the center of the microstrip line whose length is defined from point A to point E in Fig. 3.15.

The charge density due to the basis function on the center of the conductor is simply found by the derivative of the basis function. Since the basis function is rooftop and therefore its derivative has positive and negative rectangular pulses, it means that on the positive pulse region positive charges are present whereas negative charges exist on the region of the negative rectangular pulse. The charge density exerts a force on a positive test charge. The location of the test charge is defined by the domain of the testing functions. When the

relation between the basis function on the center and the testing function on the two ends of the microstrip line is considered, the test charge is assumed to be close to points A or E. The scenario of the charge distribution due to the center basis function and the test charge is shown below in Fig. 3.16 in an exaggerated fashion. Note that the charge distribution due to the basis function is shown as two concentrated charge distribution one being positive and the other negative representing the negative and positive pulses of the derivative of the basis function.

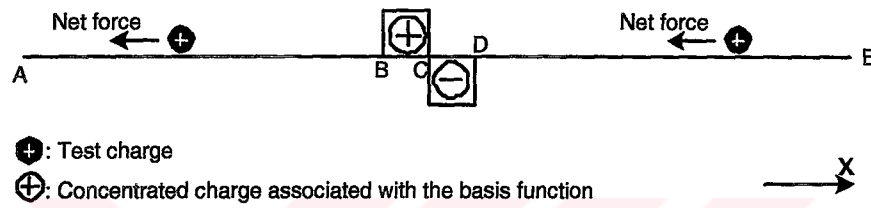


Figure 3.16: The positive test charge and the concentrated charge due to the center basis function in the microstrip line example. Note that the cross-section of the microstrip line is shown.

When the test charge is between points A and B, the force exerted on this unit charge due to the positive charge density is stronger than the force by the negative charge density because the positive charge density is closer to the unit charge. Since the positive charges try to push the test charge toward point A and the negative charges pull it toward point C, the net force is trying to push the test charge toward point A in the $-x$ -direction. Similarly when the test charge is between points D and E, the negative charges pull it toward point D whereas the positive charges push it toward point E and since the negative charges are close to the test charge resulting in a stronger pulling effect the net force is again in the $-x$ -direction.

As the test charge is moved closer to the basis function from point A to point B, the positive charge density pushes it more and more but at the same time the negative charge density pulls it more. Since their differences increase also, the net force exerted on the charge increases in magnitude. When the test charge is just on the point B, the positive charge density part totally pushes it in the $-x$ -direction whereas the negative charge density part pulls it in the $+x$ -direction.

Consider the case where the test charge is at the center of the center basis function, i.e

at the point C as shown in Fig. 3.17. In this case, the positive charge density pushes the test charge toward point E in the $+x$ -direction and the negative charges pulls it toward point D in the $+x$ -direction. Since the test charge is at such a position that the distance to positive and negative charge densities are the same, the pushing and pulling effects are the same and both of them are in the $+x$ -direction so the direction of the net total force is in the $+x$ -direction for this case. Actually this is the case where the basis and testing functions are overlapping, in other words when they are defined by the same mathematical formulas. It has been observed that the magnitude of the $\left\langle T_x^{(m',n')}(x,y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)}(x,y) \right) \right\rangle$ term assumes its biggest value when the testing and basis functions overlap. But since the total net force is in the other direction, the sign of this term for the overlapping basis and testing functions case is different from the non-overlapping basis and testing functions case.

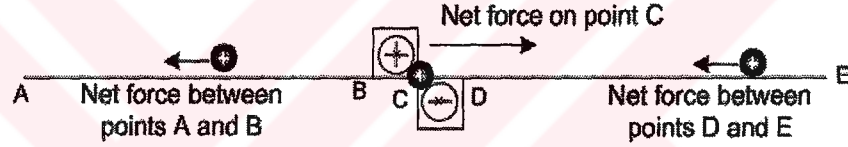


Figure 3.17: When the test charge is on the center, the magnitude of the total force assumes its biggest value but its direction is opposite of the force direction at any other point on the microstrip line.

Fig. 3.18 shows the amplitudes of the $\left\langle T_x^{(m',n')}(x,y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)}(x,y) \right) \right\rangle$ terms for a microstrip line example. In this example the microstrip line has been divided into 72 subcells in the x -direction resulting in a set of 71 basis functions to represent the current and charge distribution on the conductor. The boundary condition for the tangential electric field is applied at 71 different regions defined by the testing functions. In the evaluation of the above term, the basis function is chosen to be the one at the center of the microstrip line and all the 71 testing functions are used for the evaluation of the term. Since the magnitude of the term considered decays to zero very fast, only the 15 of these magnitudes has been shown in the figure to be able to observe the behavior in a better way.

The same interpretation can be used for the basis functions in the vertical direction. Assume that the microstrip line has been grounded with a vertical strip. The vertical strip is placed at the center of the microstrip line, i.e. at point C. The vertical strip and the associated basis and testing functions are shown in Fig. 3.19.

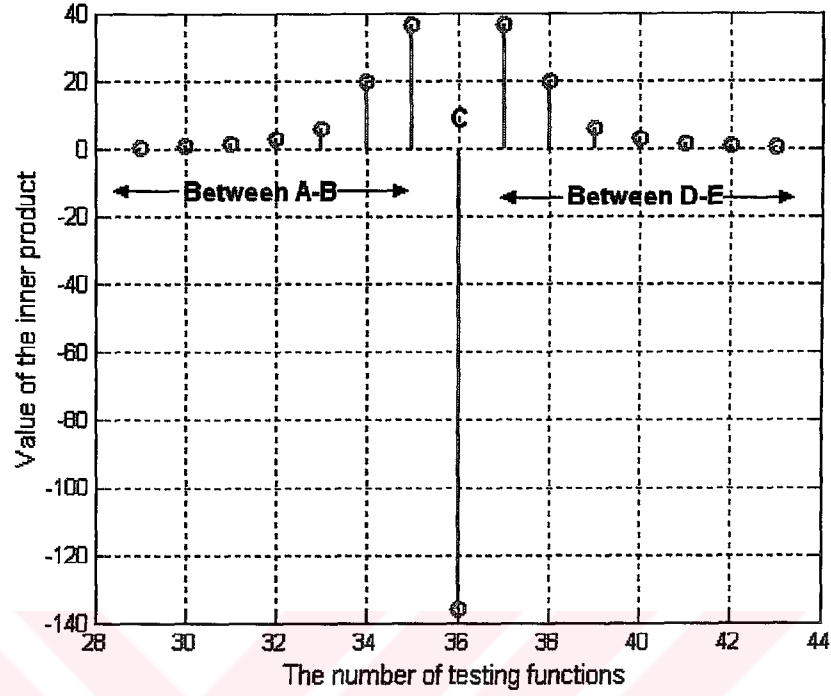


Figure 3.18: The real part of the term $\left\langle T_x^{(m',n')}(x,y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)}(x,y) \right) \right\rangle$ when the basis function is the center one. Note that at point C the magnitude is the biggest but since the force is in the opposite direction its sign is negative.

The current distribution on the surface of the vertical conductor is modelled with half and full rooftop functions. Also to satisfy the conservation of charge and continuity of current at the point of intersection, an attachment function in the form of a sawtooth function is used in the horizontal conductor. It is known that the charge density associated with the attachment function is always positive since the derivative of the sawtooth function is a positive pulse function. The charge distribution associated with the attachment function and the force exerted on the test charge by this charge distribution is given in Fig. 3.20.

When the positive test charge is considered in the existence of the attachment function, the following observations are obtained. The positive charge distribution pushes the test charge in the $-x$ -direction if the test charge is between the points A and B. When it is in between the points D and E, the positive charge is pushed toward point E in the $+x$ -direction. When the positive test charge is at point C, there is no net force acting on the charge since the charge distribution between points B and C pushes in the $+x$ -direction with

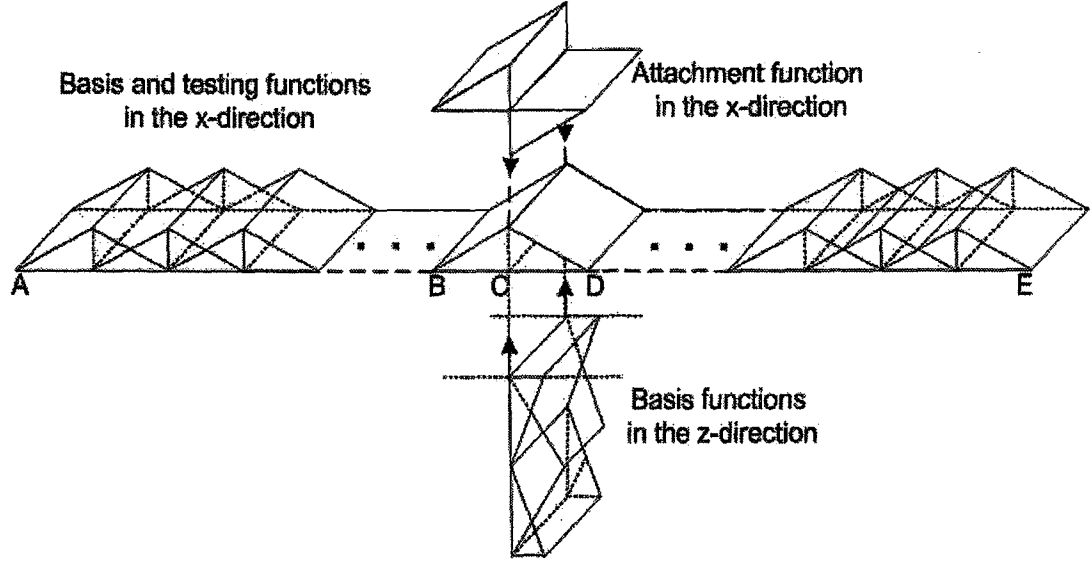


Figure 3.19: The microstrip line over infinite ground plane grounded via vertical strip at the center of the line

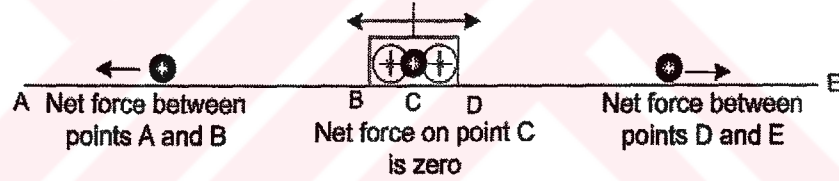


Figure 3.20: The charge distribution associated with the sawtooth attachment function and the force exerted on a positive test charge at different points.

the same force as the charge distribution between points C and D pushes in the $-x$ -direction. Therefore the total net force is zero when the positive test charge is at point C. Since in the regions specified by the points A-B and D-E the forces exerted on the positive test charge are in different directions, the signs of the terms should be opposite. This is shown in Fig. 3.21 where the $\left\langle T_x^{(m',n')}(x,y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_{xSP}^{(m,n)}(x,y) \right) \right\rangle$ term is evaluated for different testing functions.

Once the similar interpretation is given for the term including the basis function in the vertical direction and testing functions on the horizontal conductor, the procedure and the intuition for the interpretation of all terms will be complete. So consider the vertically directed basis function on the point of intersection. This basis function is chosen to be

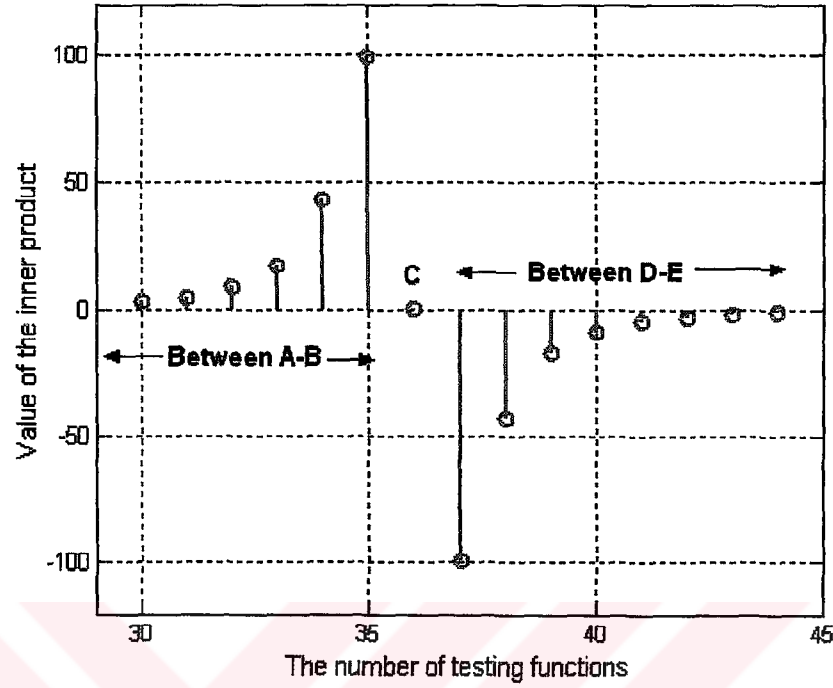


Figure 3.21: The real part of the term $\left\langle T_x^{(m',n')}(x,y), \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} B_{xSP}^{(m,n)}(x,y) \right) \right\rangle$ for different testing functions. Note that at point C the magnitude is zero since there is no net total force.

half-rooftop. So the charge associated with it should be only the positive charge since its derivative will yield in a positive rectangular pulse in the z -direction. The charge distribution due to this half rooftop function and the force exerted to the positive test charge at various points on the microstrip line is shown in Fig. 3.22.

The vertical strip grounds the microstrip line at the center. It has a width in the y -direction and infinitesimally small thickness in the x -direction. Therefore it can be assumed that the charge is distributed on the surface of the basis function. When the test charge is at the region defined by the points A and B, the net force is in the $-x$ -direction whereas it is in the $+x$ -direction when the test charge is between D and E. When the test charge is at point C, i.e. it is just above the basis function, then the net force is zero in the $-/+x$ -direction. therefore one can conclude that the general behavior is similar to the attachment function case.

As seen in several examples above, the magnitudes and directions of the forces due to

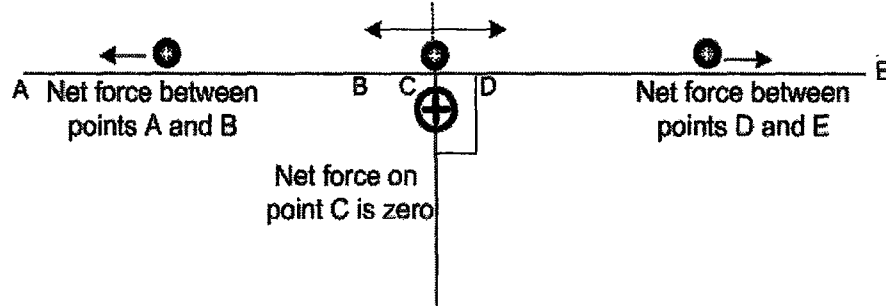


Figure 3.22: The charge distribution associated with the half-rooftop basis function in the z -direction and the force exerted on a positive test charge at different points.

the charge distribution defined by the derivatives of the basis functions on the test charges which is defined by the testing functions in this case exhibit a systematic behavior. Having a strong insight to these behaviors can be used in the debugging and faster calculation of the MoM matrix entries.

3.4 Conclusions

In this chapter, the formulation of the spatial domain MoM formulation using the MPIE representation is given for general planar stratified geometries involving vertical strips or pins. The details of evaluating the MoM matrix entries and the inner product terms encountered in these terms are left to Appendices. The physics under the choice of several basis and attachment functions are explained in details and it was shown that using half rooftop basis function in the vertical direction along with a sawtooth attachment function in the horizontal direction models the current flow and conservation of charge at the intersection points. Additionally, the interpretation of the MoM matrix entries are explained in details along with several examples which can be used in debugging and faster calculation of the MoM matrix entries.

Chapter 4

EFFICIENT USE OF MULTIPLE VERTICAL CONDUCTORS

The use of the closed-form spatial-domain Green's functions in conjunction with the MoM for the solution of MPIE has significantly improved the computational efficiency of the solution, even before the introduction of the robust multi-level approach, and before getting the matrix entries analytically [11, 17, 42]. Therefore, the introduction of the analytical integration of the MoM entries, in addition to the use of closed-form Green's functions, has further improved the computational efficiency of the spatial-domain MoM, to the extent that the method may now be considered to have reached to its ultimate efficiency. Moreover, the use of the two-level approach (or multi-level extension) in the derivation of the closed-form Green's functions makes the whole algorithm very robust, rendering the method to become a powerful candidate for a general computer aided design software for planar EM problems. However, it was shown that, by simply casting the spatial-domain Green's functions into closed forms, the computational efficiency of this method can not be improved significantly for planarly layered geometries with vertical conductors. The difficulties involved in using the spatial-domain Green's functions for such geometries and their possible solutions were addressed in [18].

Since the efficiency of the method, that is developed and used throughout this work, is the main topic of this chapter, the components of the method that contribute to the efficiency of the whole algorithm need to be explicitly defined. In a numerical algorithm, it is inevitable to define the efficiency in terms of its computational time and the accuracy of the results. Assuming and guaranteeing that the accuracy of the algorithm used in this thesis is equivalent to the algorithms that the efficiency is compared, the computational time becomes the only factor in assessing the efficiency of the overall method. Of course, the accuracy of the method is not guaranteed by just words, but it is also demonstrated by comparing the results to those obtained from well-known full-wave EM simulators like *Ensemble* from ANSOFT and *em* from SONNET. Once the criterion for the efficiency of

the method is decided as the computational time, basically, there are two main components that dominate the overall solution time for a geometry: Matrix fill time, which can be the dominating portion of the overall CPU time for small to moderate size geometries like those requiring several hundreds of unknowns; and, the matrix solution time that dominates for large geometries with thousands of unknowns. Therefore, any attempt to improve the efficiency of the technique should try to find better treatments for these components.

Regarding the matrix fill time, the use of the closed-form Green's functions in the characterization of printed geometries with multiple vertical metallizations is formulated such that the matrix entries corresponding to the vertical strips are obtained in closed-forms. Not only these entries are cast in closed-forms, but also they include the parameter defining the location of the strip explicitly. Therefore, if the matrix entries for only one vertical metallization are obtained, adding new vertical metallizations with the same length does not require any additional computational cost rather than evaluation of the already obtained entries at the location of the new vertical metallization(s). For the matrix solution time of the resulting matrix equation, the well-known, computationally expensive ($O(n^3)$ operation), LU decomposition is used. Although nothing is added to LU decomposition algorithm to make it more efficient, the solution time is improved for additional vertical metallizations once the resulting matrix equation is solved for only one vertical metallization. This is because, since adding new vertical metallizations does not alter the horizontal geometry, the part of the matrix equation does not change, and therefore, the part of the original LU can be used repeatedly. This is especially important in the design process of microwave components, such as filters or antennas with prescribed characteristics, via an optimization algorithm. In this chapter, these algorithms are reviewed, and their contributions to the efficiency of the overall algorithm are assessed in terms of matrix fill and matrix solution times.

Since one of the main contributions of this thesis is the improvement of the matrix fill time when vertical metallizations are involved, the discussion on this issue is given first in section 4.1. Then, it is followed by the theory and demonstration of the method in Section 4.2. In section 4.3, a brief discussion on the matrix solution time is provided with a view of having multiple vertical metallizations in conjunction with an optimization algorithm. The efficiency of the method is supported via some representative examples in Section 4.4 and

finally in Section 4.5 conclusions are given.

4.1 Discussions on Matrix Fill Time

Since matrix fill time is generally the dominant part of the overall solution time for small to moderate size geometries, development of efficient matrix filling algorithms has been one of the major research topics in computational electromagnetics. In this context, the use of closed-form Green's functions in conjunction with the MoM has improved the computational cost of matrix fill time significantly for planar geometries. However, in cases of vertical metallizations, the use of closed-form Green's functions was not as straightforward and easy to implement, and its advantages were not as obvious. There was no problem with the derivation of the Green's functions for VED, but there were problems in incorporating them into the MoM formulation, originated from the implementation of the method that provides the closed-form Green's functions in the spatial domain. With the method proposed in [18], these difficulties have been alleviated, and as a result, the method has become an efficient method for multilayer planar geometries even with vertical metallizations. In this thesis, this improved version of the MoM with the closed-form Green's functions is used, and it is demonstrated that the matrix fill time has been improved not only because the closed-form Green's functions are used but because the additional vertical metallizations can introduce no significant additional cost provided that z or z' dependence of the vertical conductors are the same.

Prior to any discussion and comments on the effects of the technique on the matrix fill time, for the sake of coherence and completeness of the chapter, the main steps of the formulation are briefly given here, as its details were given in Chapter 3, as follows: i) the tangential components of the electric field, E_x , E_y and E_z , on the horizontal and vertical conductors are written in terms of current densities, J_x , J_y and J_z , with the use of the associated Green's functions, as given in Eqs.(3.4-3.6); ii) the unknown current densities are expanded in terms of known basis functions with unknown coefficients; then, iii) the matrix equation for the unknown coefficients of the basis functions is obtained by applying the boundary conditions on the conductors via the testing procedure of the MoM, as $[\mathbf{Z}][\mathbf{I}] = [\mathbf{V}]$. Note that the elements of matrix $\mathbf{Z}$ that correspond to horizontal conductors are denoted by Z_{xx} , Z_{xy} , Z_{yx} and Z_{yy} , whereas the elements that correspond to

vertical conductors are designated by Z_{xz} , Z_{zx} , Z_{yz} , Z_{zy} and Z_{zz} , all of which are explicitly written in [10], [17], [18]. It has been demonstrated in [18] that some difficulties exist for the evaluation of the terms corresponding to vertical conductors, and these difficulties are originated from the way the closed-form Green's functions are derived; namely, from the exponential approximation of the spectral-domain Green's functions, which requires fixing of z and z' for the sampling of the function. To put it another way, the exponential approximation, and in turn the resulting closed-form Green's function, is only valid for those fixed values of z and z' . Since the horizontal conductors are already placed at constant z -planes and the MoM requires the evaluation of the Green's functions at these planes only, there is no problem for the matrix terms corresponding to horizontal conductors. But, the MoM matrix entries corresponding to vertical metallizations require the convolution and the inner-product integrals to be performed over z or z' , which is the source of the problem. This problem has been circumvented by the method given in [18], and hence, the efficient calculation of the matrix entries has been achieved for geometries involving vertical metallizations too. The approach proposed and implemented in [18] has not only made it possible to use the closed-form Green's functions with vertical metallizations, but, in addition, it has facilitated the addition of multiple vertical metallizations with almost no computational cost, provided that all vertical strips employ the same number of basis functions with same z and z' dependence. The working mechanism of this approach and its use with multiple vertical metallizations are the subject matters of this chapter. Therefore, the technique is explained in detail and demonstrated on an example where the inner-product integral is to be integrated over z -variable in the following section.

4.2 Theory and Demonstration

For a general 3-D geometry, as depicted in Fig. 3.1, the matrix entries that require integrations over z and/or z' correspond to the vertical metallizations, and the calculation of these entries requires special attention. There are mainly five different inner-product terms involving z -directed basis and/or testing functions among the MoM matrix entries, which are $\langle \frac{\partial}{\partial x} T_x, G_x^q * \frac{\partial}{\partial z} B_z \rangle$, $\langle T_z, G_{zx}^A * B_x \rangle$, $\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial}{\partial x} B_x \rangle$, $\langle T_z, G_{zz}^A * B_z \rangle$ and $\langle \frac{\partial}{\partial z} T_z, G_z^q * \frac{\partial}{\partial z} B_z \rangle$. For the purpose of demonstration, the following inner-product term

is chosen, and the rest of the terms are cast into their final forms in Appendix D:

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint dz dy T_z(y) \frac{\partial}{\partial z} T_z(z) \cdot \iint dx' dy' G_x^q(x - x', y - y', z, z' = \text{cons.}) \frac{\partial}{\partial x'} B_x(x', y') \quad (4.1)$$

Note that for the above expression, the separability of the basis functions is used, i.e. $\frac{\partial}{\partial z} T_z(y, z) = T_z(y) \frac{\partial}{\partial z} T_z(z)$. After some simple manipulations, this expression can be written as

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \int dy T_z(y) \iint dx' dy' \frac{\partial}{\partial x'} B_x(x', y') \cdot \int dz \frac{\partial}{\partial z} T_z(z) G_x^q(x - x', y - y', z, z' = \text{cons.}) \quad (4.2)$$

When the spatial-domain Green's function G_x^q is written in terms of its spectral-domain representation, the innermost integral in the above expression becomes

$$F_x^q \stackrel{\text{def}}{=} \int dz \frac{\partial}{\partial z} T_z(z) \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho |\rho - \rho'|) \tilde{G}_x^q(k_\rho, z, z' = \text{cons}) \\ \cong \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho |\rho - \rho'|) \cdot \text{GPOF} \left\{ \int dz \frac{\partial}{\partial z} T_z(z) \tilde{G}_x^q(k_\rho, z, z' = \text{cons}) \right\} \quad (4.3)$$

where $\text{GPOF}\{\}$ represents the approximation process with complex exponentials by using the GPOF method. When F_x^q is substituted into Eq.(4.2), the following expression is obtained:

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \int dy T_z(y) \cdot \iint dx' dy' \frac{\partial}{\partial x'} B_x(x', y') F_x^q(x - x', y - y', z' = \text{cons}) \quad (4.4)$$

After the substitutions of $x - x' = u$ and $y - y' = v$, the chosen inner product term is finally cast into the form of

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint du dv F_x^q(u, v, z' = \text{cons}) \cdot \int dy T_z(y) \frac{\partial}{\partial u} B_x(x - u, y - v) \quad (4.5)$$

where $x = x_{sp}$, i.e. the x -coordinate of the vertical metallization which is constant for this case. Note that, in the evaluation of F_x^q in Eq.(4.3), the GPOF is applied to the integral of the spectral-domain Green's function multiplied with the z -derivative of the testing function over z -variable. This integral - the term that the GPOF is applied - can always be evaluated

analytically because the z and z' dependences of a spectral-domain Green's function are always in exponential forms, and therefore, their products with most of the basis and testing functions used in EM simulations, and with their derivatives, are analytically integrable over z and z' . Consequently, the integrations over z and z' can be evaluated analytically once the spatial-domain Green's functions in the inner-product expressions are written in terms of their inverse transforms of their spectral-domain representations. Then, the exponential approximation procedure using the GPOF method is invoked for the resulting spectral-domain function.

Note that the basis and testing functions representing the current densities on the vertical strips/pins may have different spans laterally, i.e., some may span in x -direction, some in y -direction while the rest may be just pins with no span in either direction as shown in Fig. 4.1. Although this approach is quite general for any set of lengths of vertical metallizations, the

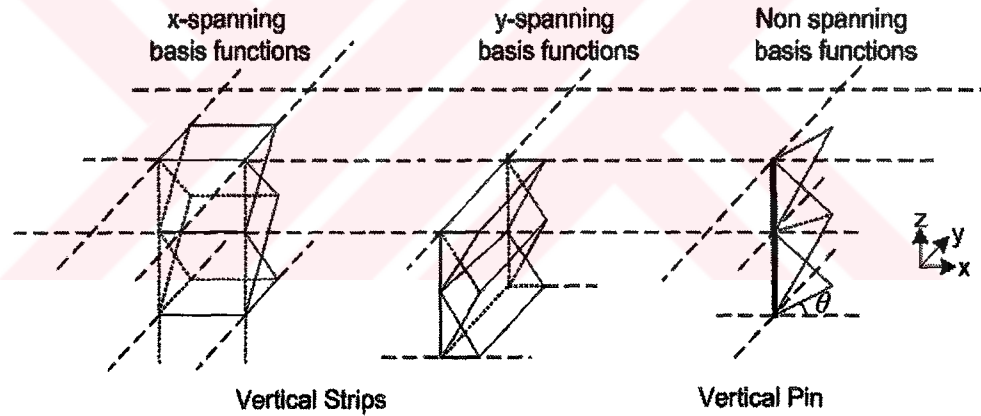


Figure 4.1: The basis functions on the vertical strips may span in the x or y direction whereas the basis functions on the vertical pins do not have spans in the lateral directions

efficiency of the method for multiple vertical metallizations can only be achieved when z and z' dependences of the basis and testing functions for all vertical metallizations are similar. To clarify this issue, i.e. why the efficiency is achieved for identical z and z' dependent basis and testing functions, one should refer to Eq.(4.3). In this equation, GPOF is applied to the integral of the spectral domain Green's function multiplied with the z -derivative of the testing function over z -variable. Therefore for every testing function with a different z domain representation (i.e. $T_z(z)$), the GPOF should be called separately which increases

the matrix filling time in proportion to the number of distinct testing functions with different z domain representations. The same argument also applies to the other matrix entry terms which involve z directed basis and/or testing functions. Hence, the efficiency of the method improves when different z or z' domain representations of the distinct basis and testing functions are kept at minimum. Because, the more the different z domain functions, the bigger the matrix filling time.

At this point, a question can come to mind on how to keep the number of different z domain representations at minimum for basis and testing functions when vertical strips in both lateral directions and vertical pins are used at the same time. To answer this question one should remember the definitions of the basis and testing functions used in this thesis. Recall that the chosen and used basis and testing functions are separable functions. For example, the horizontal basis function $B_x(x, y)$ can be written as the multiplication of two independent functions which are $B_x(x)$ and $B_x(y)$. The same approach is applicable to the vertically directed basis or testing functions. Therefore, they can be written in terms of two independent functions; one of which is the z domain representation whereas the other one is the lateral direction representation for vertical strips. The basis or testing functions on a vertical pin is written only in terms of their z domain representations since they do not span in the lateral directions. Hence, here it can be observed that, the z domain representations of the basis or testing functions on the vertical strips or pins can be chosen to be same since they are not related to the lateral directions. Therefore using vertical pins or vertical strips with different spans at the same time does not pose a problem provided that their z and z' dependences are same. Once the z and z' dependences are chosen to be identical, the GPOF method used in the evaluation of the auxiliary functions can be called only once for every different z domain representation of the basis or testing functions. For example, consider the testing function representation in Eq.(4.3). This z domain representation may be the vertical domain part of a vertical strip spanning in the x direction or y direction or it can be the testing function on a vertical pin. But since the z domain representations are chosen to be same, the evaluation of this auxiliary function, hence the calling of GPOF method, need to be carried out only once. As the auxiliary function is written in closed-form after the exponential approximation using the GPOF method, it can be used repeatedly in the evaluation of the matrix entries corresponding to different testing functions on different

vertical conductors as given in Eq.(4.5). The innermost integral in Eq.(4.5) can be evaluated analytically and it can be seen from this equation that the MoM entries corresponding to the basis or testing functions on a vertical strip are obtained as a function of ρ and the domains of integrations along the vertical directions are same if the used basis and testing functions have the same z and z' dependence. Also, by employing the analytical evaluation of MoM entries presented in [17], the interactions between different basis and testing functions in the vertical direction can be easily found by simply calculating the inner product integral for the corresponding values of ρ .

To demonstrate the efficiency of the method in the use of multiple vertical conductors on a typical geometry, the microstrip line whose geometry given in Fig. 4.2 is chosen. Assume

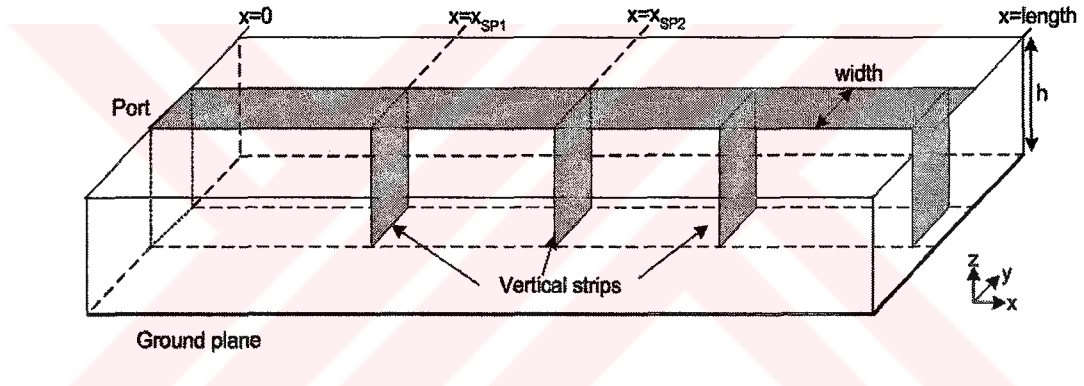


Figure 4.2: A microstrip line with multiple vertical strips over a substrate backed by a ground plane. The z or z' dependence of the vertical basis and testing functions are chosen to be same.

that the microstrip line surface and the vertical strips are discretized and assigned basis functions to accurately model the current distribution on the conductors. The width of the stripline is short enough to assume that the current density in the y -direction is constant so that no y -directed horizontal basis functions are used. The thickness of the geometry is chosen in such a way that three basis functions are sufficient to represent the current density on each vertical strip. Moreover, the basis functions and the corresponding testing functions on each vertical strip are chosen to have identical z or z' dependence. At first, assume that there is no vertical strip in the geometry. In this case, the impedance matrix of the system is only due to horizontal basis and testing functions and it can be represented as $[Z_{xx}]$. Note that, the number of columns and rows of this matrix is equal to the number of

horizontal basis functions and it is enough to evaluate the vector potential Green's function G_{xx}^A and scalar potential Green's function G_x^q for the evaluation of the matrix entries. For the evaluation of each of these Green's functions, the GPOF method is called once. Therefore, for a geometry with only horizontal conductors, the GPOF method is called twice in total; one for G_{xx}^A and the other for G_x^q . But when a vertical conductor is added to the geometry, due to the z directed current, additional matrix entry terms are included in the impedance matrix which becomes as follows:

$$\begin{bmatrix} Z_{xx} & Z_{xz} \\ Z_{zx} & Z_{zz} \end{bmatrix} \quad (4.6)$$

The number of included rows and columns to the impedance matrix are equal to the number of basis functions on the vertical strip. Since it was already mentioned that there are three basis functions on each vertical strip in this case, for the inclusion of each vertical strip three rows and columns are added to the impedance matrix. For the new matrix entry terms, five inner product terms, which are $\langle \frac{\partial}{\partial x} T_x, G_z^q * \frac{\partial}{\partial z} B_z \rangle$, $\langle T_z, G_{zx}^A * B_x \rangle$, $\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial}{\partial x} B_x \rangle$, $\langle T_z, G_{zz}^A * B_z \rangle$ and $\langle \frac{\partial}{\partial z} T_z, G_z^q * \frac{\partial}{\partial z} B_z \rangle$, should be evaluated for each distinct basis and testing functions. Since these inner product terms involve integration and/or derivation along z and/or z' , instead of closed-form Green's function, the closed-form auxiliary functions whose details are given before in this chapter and in Appendix D are evaluated. For each distinct testing and basis functions, these auxiliary functions are evaluated. Since there are 3 distinct basis and testing functions for a vertical strip, 27 different auxiliary functions are evaluated with the inclusion of a vertical strip. These are: 3 for $\langle \frac{\partial}{\partial x} T_x, G_x^q * \frac{\partial}{\partial z} B_z \rangle$, $\langle T_z, G_{zx}^A * B_x \rangle$ and $\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial}{\partial x} B_x \rangle$ which involve one horizontal and one vertically directed basis and testing function couples and 9 for $\langle T_z, G_{zz}^A * B_z \rangle$ and $\langle \frac{\partial}{\partial z} T_z, G_z^q * \frac{\partial}{\partial z} B_z \rangle$ with vertical directed basis and testing function couples. Evaluation of each auxiliary function requires the calling of GPOF method separately, therefore with the inclusion of a vertical strip (with three basis functions) to the horizontal geometry, the GPOF method is called 27 times more. Now consider that another vertical strip is added to the geometry and for the sake of illustration, assume that the basis and testing functions on this vertical strip have different z and z' dependences from the basis and testing functions on the previous vertical strip. In this case, three more rows and columns are added to the impedance matrix of the system. For the evaluations of these matrix entry terms, new

auxiliary functions should be calculated because the z and/or z' dependences of these new basis and testing functions are different from the previous ones. Therefore 45 new auxiliary functions are to be evaluated, 27 of which are due to the interactions between the basis and testing functions of the new vertical strip and 18 of them are due to the interaction between the basis and testing functions of the two distinct vertical strips. As a result, the GPOF method is called 45 times more than the previous case. Similarly, if the number of basis and testing functions with different z and/or z' dependences are added to the geometry, the number of GPOF method calling increases which is not desired in terms of efficiency. On the other hand, if any other number of included vertical strips or pins have identical z and z' dependent basis and testing functions, the auxiliary functions evaluated for the first vertical strip can be used repeatedly. In this case, the only difference will be the different ρ values used in the analytical evaluation of the resulting integrals. Therefore for vertical strips or pins represented by 3 basis functions, evaluation of 27 auxiliary functions will be sufficient for any other number of included vertical strips or pins. As a conclusion, in terms of exponential approximation via GPOF method, inclusion of another or any other number of vertical conductors does not increase the computational complexity of the technique provided that the z and z' dependence of the basis and testing functions are chosen to be identical.

4.3 Discussion on Matrix Solution Time

In the previous section, it has been demonstrated that addition of multiple vertical strips or pins with identical z or z' dependences does not require the exponential approximation during the evaluation of the closed-form auxiliary functions more than once for each distinct basis and/or testing functions. As a result, improvement in the matrix fill time of the spatial domain MoM with multiple vertical metallizations is observed. However as it is well known, the matrix fill time, which is the dominating factor for small to moderate size geometries, is not the only factor that affects the computational efficiency of an analysis method. The matrix solution time which starts dominating for large geometries with thousands of unknowns should also be considered.

Once the MoM matrices are filled, the solution of the matrix equations starts by using the computationally expensive LU decomposition. For small to moderate size geometries,

the solution time does not dominate but as the geometry gets larger, the solution time starts to dominate. If the optimization of the geometry is also considered in addition to the analysis, then the use of LU decomposition for each iteration in the optimization steps will dominate the overall solution time. But as it is well known, for typical optimization problems the geometry, therefore the matrices representing the system are not changed at each iteration. Instead at each step, new parts are added or deleted which correspond to addition or deletion of new rows and columns. Using this fact, the LU decomposition of the original geometry can be used repeatedly for each iteration of the optimization process. As an example, assume that the addition or deletion of new parts in an optimization procedure are carried out by using vertical conductors. In other words, the locations or the numbers of the vertical strips or pins are changed at every iteration in the optimization. There is no alterations made on the horizontal geometry, therefore the matrix entries of the system matrices corresponding to the horizontal basis and testing functions will always be the same. For this case, if the number of horizontal basis functions is much more than the number of basis functions on the added vertical conductors, then LU decomposition of the whole system matrices at each iteration will dominate the overall solution time. But once the LU decomposition for the system with no vertical conductors is obtained, this can be used repeatedly in the LU decomposition of the system matrices obtained by including the vertical strips to the horizontal geometry. This can be achieved by using the Order Recursive Gaussian Elimination (ORGE) method [43] which is developed to improve the computational efficiency of solving the matrix equations for modified geometries. Since optimization is not studied in this thesis, the use of ORGE method is not employed. But since the optimization is the next step after developing an efficient method for the solution of 3-D general planar geometries, ORGE method should be used in conjunction with an optimization algorithm in the future works.

4.4 Numerical Examples

In the previous sections, it has been shown that the use of multiple vertical strips or pins with the same z and z' dependences does not increase the matrix fill time. In addition to this, the matrix solution time can be reduced by using the ORGE method in optimization problems by using the LU decomposition of the original geometry to be optimized. The

mathematical validations of these observations have been given in the previous sections. In this section, the method used in this thesis is compared in terms of efficiency with the results obtained from well-known commercially available full-wave EM simulators like *Ensemble* from ANSOFT and *em* from SONNET.

As the first example, consider the microstrip line whose geometry is given in Fig. 4.2. The length and width of the microstrip line are 18 cm and 0.1 cm, respectively. The frequency of operation is 1.0 GHz and the thickness of the dielectric medium is 0.4 cm. Assume that at first the dielectric constant of the substrate is 1.0, i.e. the medium is air and there is no vertical conductors. For this particular case, the surface current density on the microstrip line is represented by 359 basis functions, therefore the impedance matrix should have 359 rows and columns. Since at first, there is no vertical conductors, the Green's functions G_x^q and G_{xx}^A which do not involve z or z' integrations are sufficient for the filling of the system matrices. For the evaluations of these Green's functions the GPOF method is called two times in total. But when a vertical conductor which is a vertical strip spanning in the y direction with a width of 0.1 cm is added to the geometry, the impedance matrix is augmented with three more rows and columns, since each vertical strip is represented by three distinct basis functions for this geometry. The new matrix entries are due to the z directed testing and basis functions. To evaluate these matrix entries, the auxiliary functions for each of the distinct basis and/or testing functions should be evaluated, therefore the exponential approximation via GPOF method needs to be called as the number of different auxiliary functions which is 27 for three vertical basis and testing functions case. Table 4.1 shows the time it takes for the evaluation of the Green's or

Green's or Auxiliary functions involving	Time (sec) (for $\epsilon_r = 1.0$)	Time (sec) (for $\epsilon_r = 2.2$)	Time (sec) (for $\epsilon_r = 4.0$)
No z or z' integrations	1.041	1.862	1.972
Only z or only z' integrations	2.774	4.667	4.988
Both z and z' integrations	25.071	28.182	29.594

Table 4.1: The computer times for the evaluation of Green's functions for the geometry of the microstrip line with thickness 0.4 cm at frequency of 1.0 GHz by using 1.4 GHz Pentium CPU.

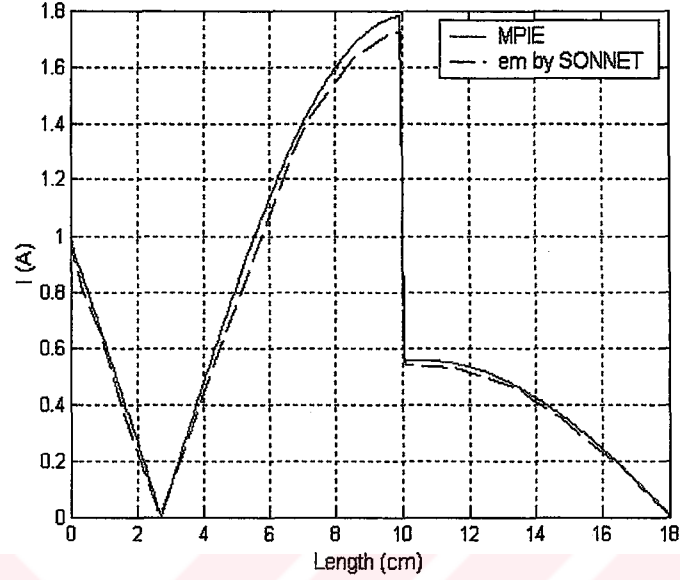


Figure 4.3: The current distributions obtained from the MPIE method described in this thesis and *em* by SONNET on the microstrip line whose geometry is given in Fig. 4.2 when vertical strip of width 0.1 cm at $x = 10$ cm is inserted

auxiliary functions for the given geometry by using 1.4 GHz Pentium CPU for different dielectric constants. As it can be seen from the table, the evaluation of the auxiliary functions involving z and/or z' integrations takes more time than the evaluation of the Green's functions without z and z' integrations. Therefore, the inclusion of new vertical conductors should be carried out with special care. Because, when the number of basis and testing functions with different z domain representations increases, more and more auxiliary functions need to be evaluated for each new basis and testing function which further increases the matrix filling time. Therefore, to improve this criteria, one should try to minimize the number of the basis and testing functions with different z domain representations in the case of multiple vertical conductors. Hence, the evaluated auxiliary functions can be used repeatedly for different basis and/or testing functions at different lateral locations with identical z and z' dependences.

For the microstrip line geometry given, when the first vertical strip of width 0.1 cm is inserted at $x = 10$ cm, the current distribution on the strip becomes as given in Fig. 4.3.

Since with the inclusion of this vertical strip, the auxiliary functions involving z and

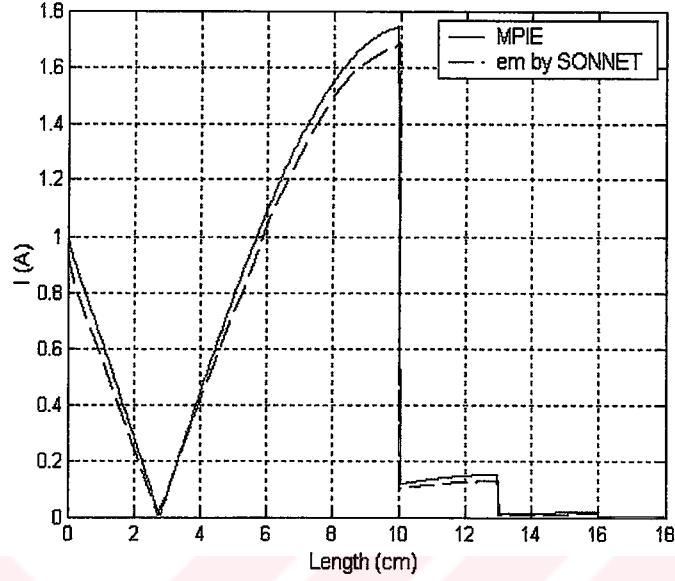


Figure 4.4: The current distributions obtained from the MPIE method described in this thesis and *em* by SONNET on the microstrip line whose geometry is given in Fig. 4.2 when four vertical strips of width 0.1 cm at $x = 10$ cm, $x = 13$ cm, $x = 16$ cm and $x = 18$ cm are inserted

z' integrations are evaluated for the first time, the time spent on the evaluations of these auxiliary functions is inevitable. But when additional vertical strips with identical z and z' dependences are added, these already evaluated auxiliary functions can be used repeatedly and therefore there is no need for the evaluation of new auxiliary functions. As a conclusion, if the auxiliary functions for one vertical metallization is obtained, adding new vertical metallizations with the same z and z' dependence does not increase the matrix filling time. This is illustrated for the inclusion of additional vertical strips at $x = 13.0$ cm, $x = 16.0$ cm and at $x = 18.0$ cm by comparing the current distribution on the microstrip line and the percentage increase in the matrix filling times for different cases with the results obtained from *em* by SONNET as shown in Fig. 4.4 and in Table 4.2.

In this table, the matrix filling times are compared in terms of percentage increases with respect to the previous value of the matrix filling times for a vertical strip, e.g. the matrix filling time for only the second vertical strip is compared with the matrix filling time of the first vertical strip, the third one is compared with the second one and so on.

Number of vertical strips	Percentage increase in Matrix fill time	
	<i>em</i> by SONNET	MPIE method
1	100	100
2	15.30	0.805
3	19.92	0.852
4	24.41	0.718

Table 4.2: The percentage increases in the matrix fill times for addition of multiple vertical strips for *em* by SONNET and MPIE method for the geometry in Fig. 4.2 with dielectric constant of 1.0 at frequency of 1.0 GHz

The reason for comparing in terms of percentage increase is that the same geometry may be discretized in different number of basis functions for different frequencies. For any geometry, there is a bottom limit on the number of basis functions chosen to represent the current distribution on the conductors. When the number of basis functions are less than this limit, then the circuit parameters cannot be accurately extracted. Therefore the analysis using insufficient number of basis functions would yield to misleading results. To circumvent this problem, the geometry should be discretized in such a way that the number of basis functions are above this limit. It should be noted that once a particular number of basis functions which accurately represent the current distribution on the conductors is chosen for a geometry, any number of basis functions more than this value would also represent the current distribution on the conductors accurately. But in this case, it would take more time to fill the system matrices due to the increased number of basis functions. As a conclusion, the overall matrix filling time changes for every different number of basis functions for a particular geometry. In cases of vertical metallizations, the matrix entries of the Z_{zx} and Z_{xz} submatrices should be calculated as many times as the number of horizontal basis functions. Therefore, the time it takes for the filling of the submatrices Z_{zx} and Z_{xz} is different for different number of horizontal basis functions. But since the number of entries evaluated in these submatrices are directly proportional to the number of horizontal basis functions, the overall percentages of the filling times for these submatrices remain the same. For instance, consider the microstrip line example given before in this chapter. For this example, the

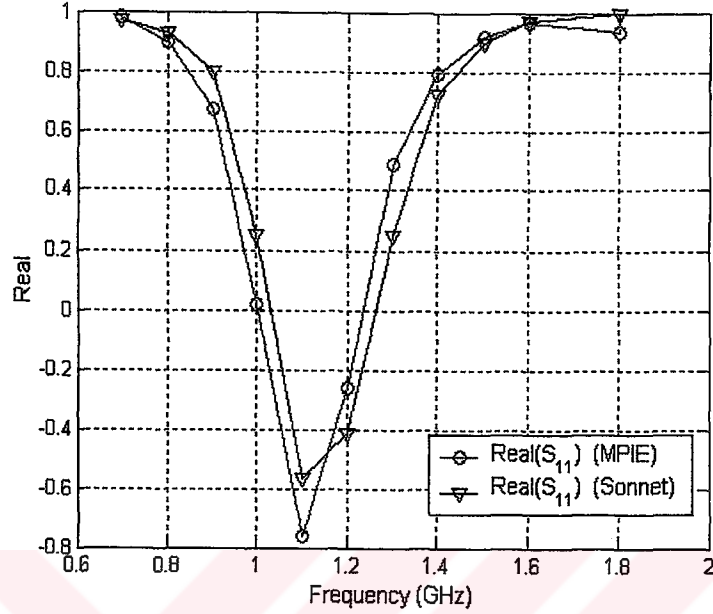


Figure 4.5: The real part of S_{11} for the microstrip line given in Fig. 4.2 when the substrate has a dielectric constant of 2.0.

number of horizontal basis functions used in the simulations is 359. This number of basis functions is more than enough for this frequency of operation for accurate representation of the circuit parameters, but, one can use more basis functions to represent the current distribution on the conductors at the expense of increased overall matrix filling times. If, for example, 600 basis functions were used instead of 359, then the filling time of the impedance matrix will be inevitably more. Because the evaluations of the matrix entry terms would be carried out for each 600 horizontal basis functions instead of 359. Similarly, *em* by SONNET and *Ensemble* from ANSOFT discretize the geometry and the discretization information is not explicitly provided in these softwares. Therefore, the chosen number of basis functions in these softwares and in the method explained in this thesis may not be similar for many cases. As a result, instead of overall times, the percentage increases are considered in the comparisons. As shown in Table 4.2, the percentage increase in matrix fill time for the method used in this thesis is less than the results obtained from *em* by SONNET. But there is one thing that should be noted here. The use of identical z and z' dependent basis and testing functions decrease the matrix filling time provided that the analytical integrations

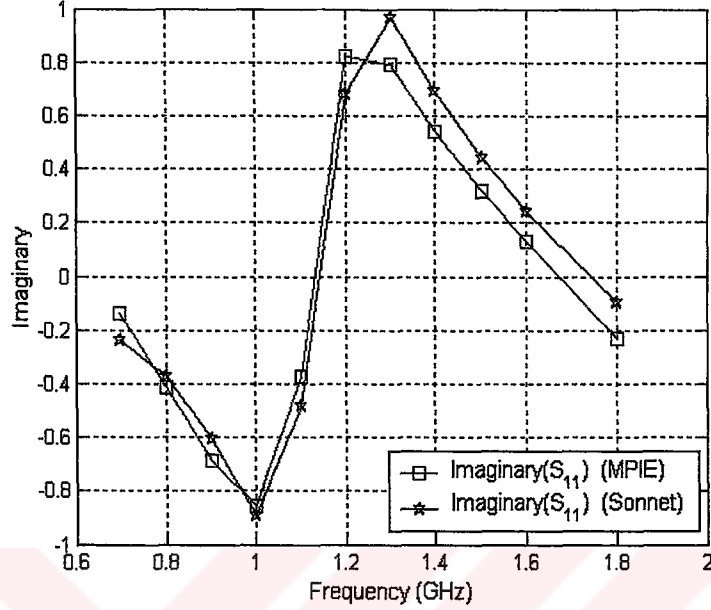


Figure 4.6: The imaginary part of S_{11} for the microstrip line given in Fig. 4.2 when the substrate has a dielectric constant of 2.0.

involving auxiliary function are taken and they are kept in a look up table to be used in and the ρ values corresponding to each different basis and testing function is substituted to these analytically evaluated integrals. Otherwise, for the evaluation of the analytical integrations for each basis function, the matrix filling time would increase. Therefore the values given in Table 4.2 are evaluated according to this criteria.

The results obtained here are for the no dielectric case, i.e. the dielectric constant of the substrate is 1.0. When the substrate of Fig. 4.2 has a dielectric constant of 2.0, the S_{11} -parameter of the circuit and the percentage increases in the matrix filling time information are given in Figs. 4.5-4.6 and Table 4.3, respectively.

Another example demonstrating the efficiency of using multiple vertical conductors is the patch antenna structure whose geometry and dimensions are given in Fig. 4.7. The surface of the patch antenna is discretized in such a way that the total number of horizontal basis functions used on the patch surface is 442. In this example, the S-parameter and the far field pattern of the antenna is analyzed for different number of vertical strips. The far field patterns are compared with those obtained from *Ensemble* by ANSOFT. One of the

Number of vertical strips	Percentage increase in Matrix fill time	
	<i>em</i> by SONNET	MPIE method
1	100	100
2	7.00	0.391
3	9.67	0.367
4	11.66	0.378

Table 4.3: The percentage increases in the matrix fill times for addition of multiple vertical strips for *em* by SONNET and MPIE method for the geometry in Fig. 4.2 with dielectric constant of 2.0 at frequency of 1.40 GHz

Number of vertical strips	Percentage increase in Matrix fill time	
	<i>em</i> by SONNET	MPIE method
1	100	100
2	31.37	0.512
3	34.48	0.593
4	37.98	0.616

Table 4.4: The percentage increases in the matrix fill times for addition of multiple vertical strips for *em* by SONNET and MPIE method for the geometry in Fig. 4.7 at frequency of 0.90 GHz

reasons for using *Ensemble* for the far field comparisons is the fact that *em* by SONNET analyzes the planar structures inside a shielding box, therefore the radiating structures are not accurately analyzed by using *em*.

For this example, five different vertical strips are added one by one to the horizontal geometry. All the vertical strips are chosen to be spanning in the y -direction with a width of 0.2572 cm and all have the same x -coordinate which is $x = 8.7234$ cm. All the vertical strips have the same length and therefore the basis and testing functions have the same z and z' dependences which will enable the efficient use of multiple vertical strips. At first, it is assumed that there is only one vertical strip which is spanning between $y = 0 - 0.25728$. The real and imaginary parts of the S-parameters for this case are shown in Figs. 4.8-

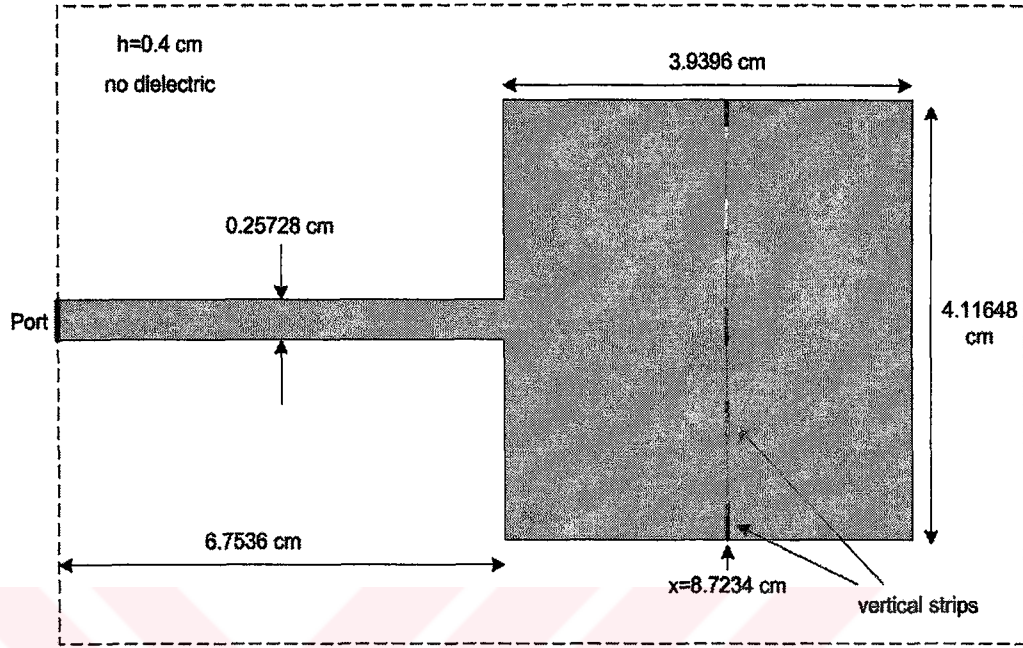


Figure 4.7: Geometry of a patch antenna using multiple vertical conductors.

4.9. After the inclusion one vertical strip, four more vertical strips spanning between $y = 1.02912 - 1.2864$, $y = 1.80096 - 2.05824$, $y = 2.83008 - 3.0873$ and $y = 3.8592 - 4.11648$ are added to the geometry. The resulting S-parameters and the far fields obtained from the patch antenna are given in Figs. 4.10-4.12. The percentage increases in the matrix filling times are also given in Table 4.4.

As it can be seen from Fig. 4.12, the results for the far field patterns from *Ensemble* and the method used in this thesis are similar to each other for multiple vertical conductors. The S-parameters obtained from the MPIE method and *em* for different frequencies are also in good agreement as shown in the corresponding figures. It is shown in this chapter that choosing the z and z' dependences of the vertical basis and testing functions identical results in an improvement in the matrix fill time of the MPIE equation method for multiple vertical metallizations. The timing information are compared with those obtained from *em*. Because *Ensemble* does not give the timing information explicitly.

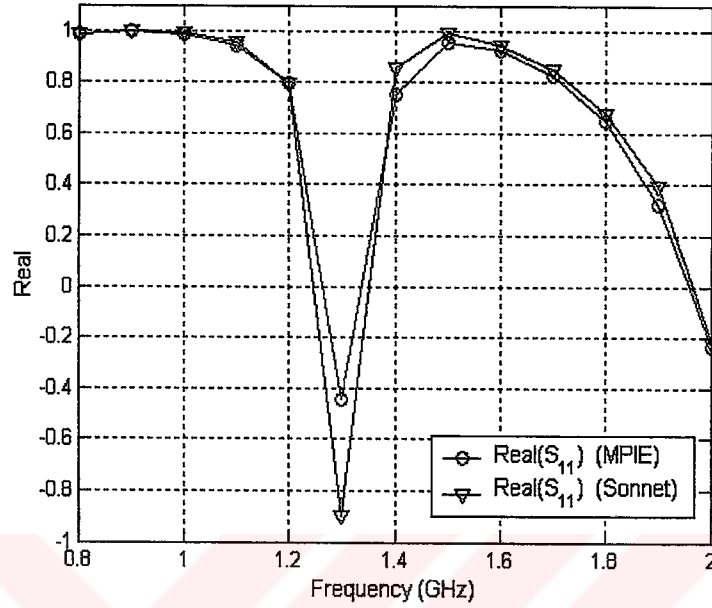


Figure 4.8: The real part of S_{11} for the patch antenna in Fig. 4.7 when there is only one vertical conductor spanning between $y = 0 - 0.25728$

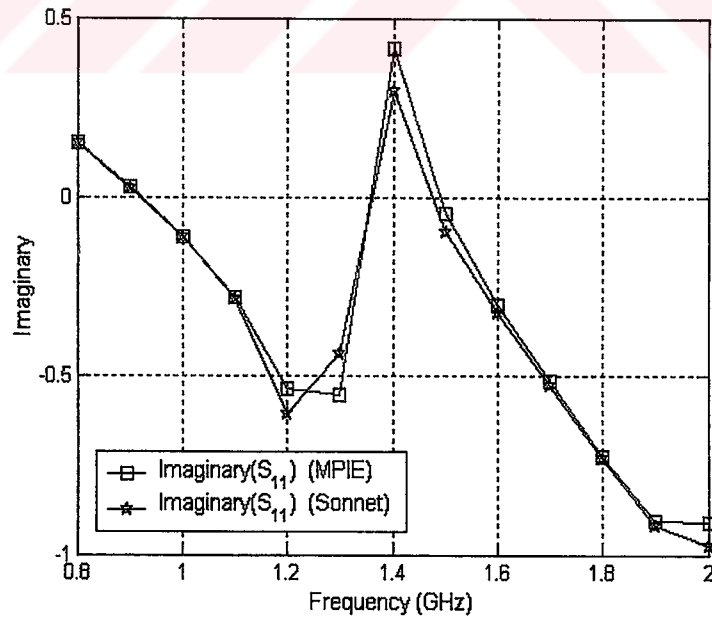


Figure 4.9: The imaginary part of S_{11} for the patch antenna in Fig. 4.7 when there is only one vertical conductor spanning between $y = 0 - 0.25728$

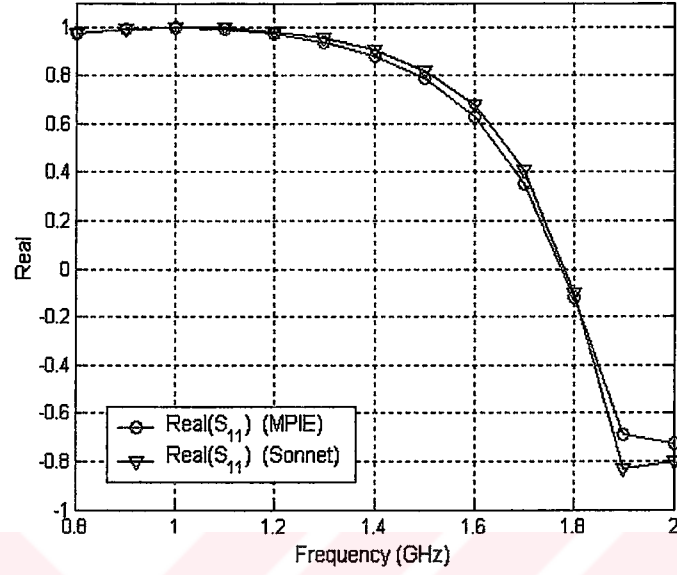


Figure 4.10: The real part of S_{11} for the patch antenna in Fig. 4.7 when there are five vertical strips at $x = 8.7234$ spanning between $y = 0 - 0.25728$, $y = 1.02912 - 1.2864$, $y = 1.80096 - 2.05824$, $y = 2.83008 - 3.0873$ and $y = 3.8592 - 4.11648$

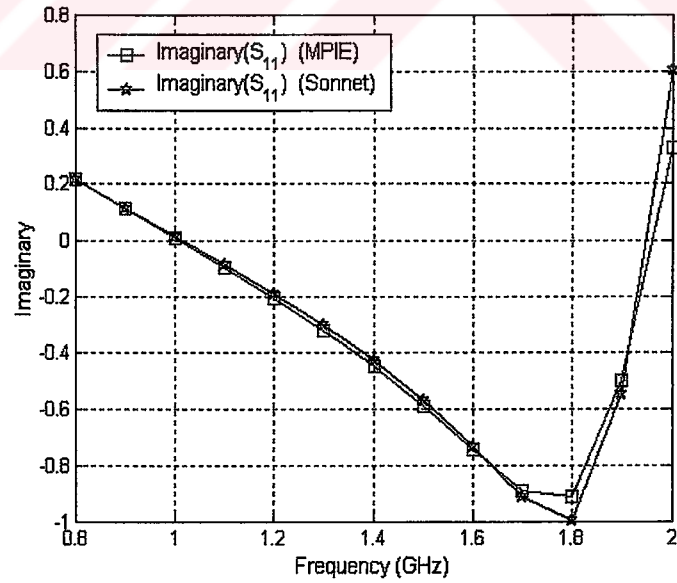


Figure 4.11: The imaginary part of S_{11} for the patch antenna in Fig. 4.7 when there are five vertical strips at $x = 8.7234$ spanning between $y = 0 - 0.25728$, $y = 1.02912 - 1.2864$, $y = 1.80096 - 2.05824$, $y = 2.83008 - 3.0873$ and $y = 3.8592 - 4.11648$

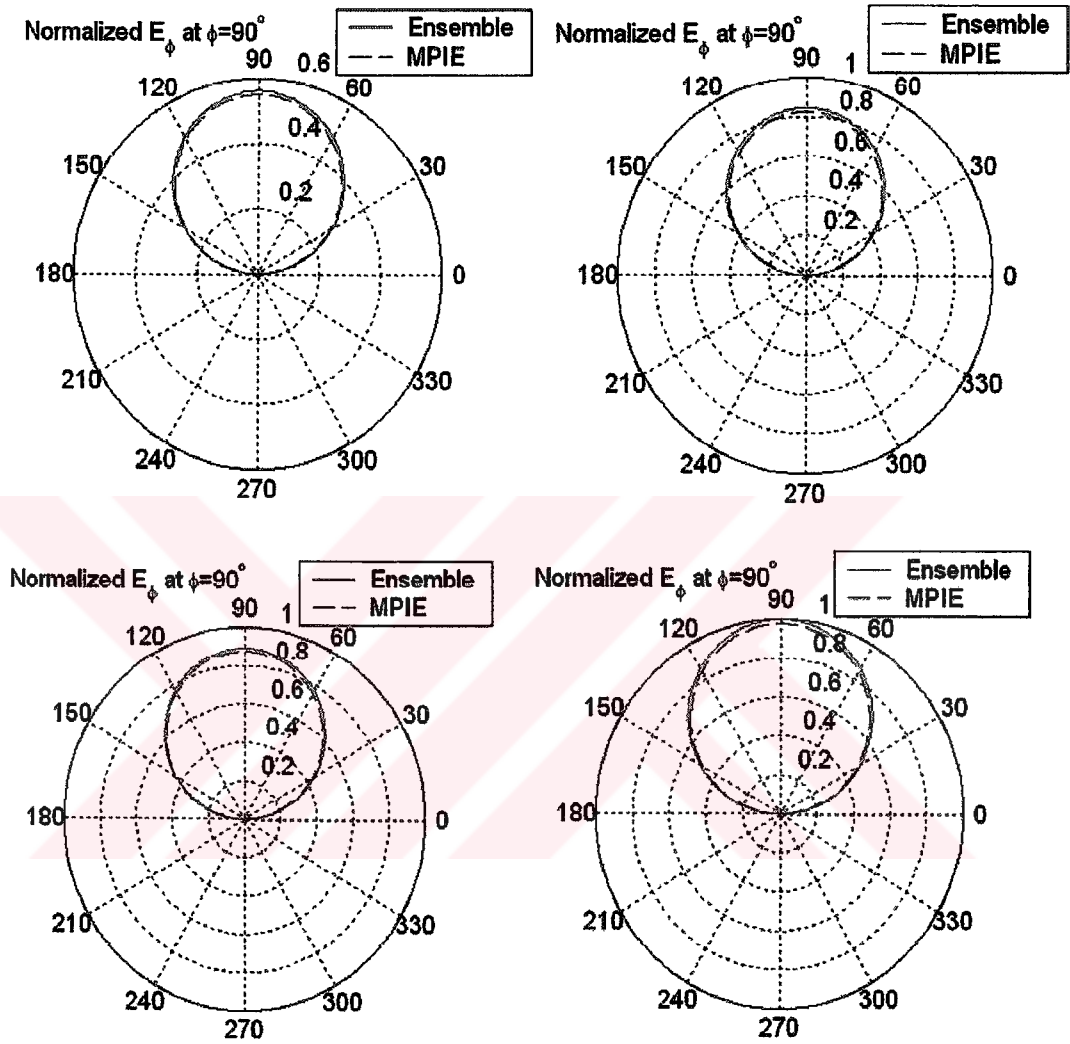


Figure 4.12: The comparisons of the far field patterns for the patch antenna with multiple vertical strips obtained from *Ensemble* and MPIE method. All the strips are at $x = 8.7234$. a) One vertical strip spanning between $y = 0 - 0.25728$ cm b) Two vertical strips spanning between $y = 0 - 0.25728$ and $y = 1.80096 - 2.05824$ c) Three vertical strips spanning between $y = 0 - 0.25728$, $y = 1.80096 - 2.05824$ and $y = 2.83008 - 3.0873$ d) Five vertical strips spanning between $y = 0 - 0.25728$, $y = 1.80096 - 2.05824$, $y = 2.83008 - 3.0873$, $y = 1.02912 - 1.2864$ and $y = 3.8592 - 4.11648$

4.5 Conclusions

The use of closed-form Green's functions in conjunction with the MoM formulation has improved the matrix fill time significantly for planar geometries. But in the case of vertical conductors, there were problems in incorporating the closed-form Green's functions into the MoM formulation. The problems originated from the implementation of the method that provides the closed-form Green's functions in the spatial domain. With the method explained in this chapter, these problems are solved and the method has become an efficient method for multilayer planar geometries even with vertical metallizations. The efficiency of the method has been supported by some examples. It was shown that once a vertical conductor is present in the geometry and the corresponding auxiliary functions are evaluated, incorporating additional vertical metallizations with same z and z' dependent basis and testing functions requires almost no computational cost. Therefore, this approach is suitable for the purpose of optimization of a geometry involving vertical metallizations.

Chapter 5

RESULTS AND DISCUSSIONS

In the previous chapters, the formulation for the spatial domain MoM in the analysis of planar multilayered geometries was studied in detail and in this chapter, the techniques presented are demonstrated on several examples and the results are compared with those obtained from commercial EM analysis softwares, such as *em* from SONNET and *Ensemble* from ANSOFT. The technique presented in this thesis and *Ensemble* use a structure based on the MPIE and solve the open geometry, but *em* puts the circuit in a conducting box whose default wall conductivities are infinite [44]. For additional information on the operation of *em* and *Ensemble*, one can refer to the manuals provided with these softwares [44], [45].

For the sake of completeness, it would be wise to give the techniques in finding the circuit parameters such as scattering parameters or input impedance briefly.

5.1 Scattering Parameter Analysis

The formulation for finding the scattering parameters will be presented for a general two port network as shown in Fig. 5.1. It will be demonstrated that by using this method the input

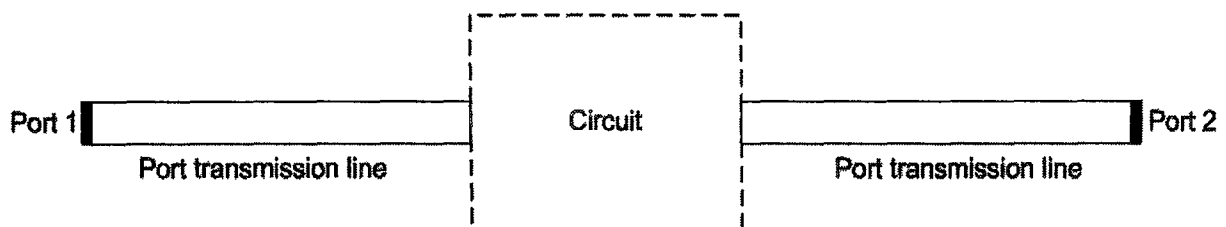


Figure 5.1: A general two port network

impedances and the mutual couplings can also be calculated. Since the current densities on the conductors, including the port transmission lines have already been calculated, the current on each transmission line is written as a linear combination of complex exponentials

by using the GPOF method as

$$I(l) \approx \sum_{i=1}^N I_i e^{(\alpha_i + j\beta_i)l} \quad (5.1)$$

where l is the distance along the port transmission line, α_i is the attenuation and β_i is the propagation constants of the i^{th} mode of the current wave.

This approximation is equivalent to numerically separating the incident wave, the reflected wave, the evanescent waves and the higher order propagating modes at the reference plane $l = 0$. However, since there exist fringing, reactive and evanescent fields in the vicinity of the source and load terminals, S-parameters can be obtained by removing these higher order effects from the calculations, which is called the de-embedding [46]. Therefore, the current densities are expressed by only two exponentials with complex coefficients corresponding to the incident and reflected current waves at the corresponding ports. Then, the S-parameters can be calculated and transferred to the desired reference planes. However, the propagation constant in the above equation is also not known and it should be determined. Although it is possible to find both the coefficients of the exponentials and the propagation constant simultaneously using the same transmission line, this process is not always numerically stable. This is because it is difficult to determine the propagation constant and the complex coefficients simultaneously when the standing-wave pattern on the line is not revealing; a situation that occurs for electrically short circuits. Therefore, the propagation constant is first found from the solution of an open-ended microstrip transmission line which is called the test transmission line which has the same characteristics as the original line. It is important to select the length of the test transmission line sufficient enough so that the standing wave pattern becomes revealing. This process does not affect the propagation constant since it only changes the length of the line. After determining the propagation constant, the current on the original port transmission lines can now be accurately approximated by complex exponentials. After approximating the current on the original port transmission lines by complex exponentials, S-parameters of the network are obtained from the coefficients of the exponentials by following the procedure described as follows. First, port 1 is excited and the current densities on the transmission lines are written as below:

$$I_1(l) = I_{11}^+ e^{-j\beta_1 l} + I_{11}^- e^{+j\beta_1 l}$$

$$I_2(l) = I_{21}^+ e^{-j\beta_2 l} + I_{21}^- e^{+j\beta_2 l} \quad (5.2)$$

where I_{ij} is the current wave coefficient on the i^{th} port transmission line when port j is excited. Note that α_i 's are not written in the above equations because at the end of approximations it is found that α_i 's are small compared to β_i 's. These current wave coefficients can be related to each other by using the S-parameter matrix

$$\begin{bmatrix} -I_{11}^- \\ I_{21}^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} I_{11}^+ \\ -I_{21}^+ \end{bmatrix} \quad (5.3)$$

Then, port 2 is excited and the current distribution on the entire structure is again solved and the port transmission line current densities are expressed as follows

$$\begin{aligned} I_1(l) &= I_{12}^+ e^{-j\beta_1 l} + I_{12}^- e^{+j\beta_1 l} \\ I_2(l) &= I_{22}^+ e^{-j\beta_2 l} + I_{22}^- e^{+j\beta_2 l} \end{aligned} \quad (5.4)$$

Similarly, these current wave coefficients can be written in terms of S-parameters as

$$\begin{bmatrix} -I_{12}^- \\ I_{22}^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} I_{12}^+ \\ -I_{22}^+ \end{bmatrix} \quad (5.5)$$

After combining Eqs. 5.3 and 5.5, the matrix relation which is used in finding the S-parameters for a general two port network can be written in the following form:

$$\begin{bmatrix} I_{11}^+ & I_{12}^+ & 0 & 0 \\ 0 & 0 & I_{11}^+ & I_{12}^+ \\ I_{21}^+ & I_{22}^+ & 0 & 0 \\ 0 & 0 & I_{21}^+ & I_{22}^+ \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{bmatrix} = \begin{bmatrix} -I_{11}^- \\ -I_{12}^- \\ -I_{21}^- \\ -I_{22}^- \end{bmatrix} \quad (5.6)$$

Note that the above expression can be easily generalized for a N-port network.

This expression provides the generalized S-parameters of the network, i.e. the network parameters are automatically normalized to the characteristic impedances of the transmission lines, which are not known. However, it is more practical to express the S-parameters normalized to known reference impedance (usually 50 Ω). To convert the generalized S-parameters to normalized ones, the characteristic impedances of the port transmission lines should be known. Since the rigorous calculation of the characteristic impedance of a microstrip line in a multilayer medium is quite time consuming, a method which provides a

closed-form expression for the characteristic impedance of a microstrip line is used [47]. The characteristic impedance is calculated by using the following equation

$$Z_c = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{eff}}} \log \left\{ F_1 \frac{h}{w} + \sqrt{1 + \left(2 \frac{h}{w} \right)^2} \right\}$$

$$F_1 = 6 + (2\pi - 6) \exp\left\{ \left(-30.666 \frac{h}{w} \right)^{0.7528} \right\} \quad (5.7)$$

where h is the thickness of the geometry, w is the width of the port transmission line and ϵ_{eff} is the effective dielectric constant. Note that the characteristic impedance found from this approach does not change with frequency, whereas in reality it changes. Therefore the change in the frequency is taken into account by using a frequency dependent effective permittivity ϵ_{eff} in the closed-form expression for the characteristic impedance. The concept of effective dielectric constant is shown in Fig. 5.2. After finding the characteristic impedances of port transmission lines, renormalized S-parameters are found by first calculating a unique impedance matrix associated with the geometry. The unique impedance matrix, Z is

$$Z = \sqrt{Z_0} (I + S) (I - S)^{-1} \sqrt{Z_0} \quad (5.8)$$

where S is the generalized S-matrix, I is the identity matrix and Z_0 is a matrix with the characteristic impedance of each port as a diagonal value. The renormalized S-matrix is then calculated from Z using the following relationship:

$$S_\Omega = \sqrt{Y_\Omega} (Z - Z_\Omega) (Z + Z_\Omega)^{-1} \sqrt{Z_\Omega} \quad (5.9)$$

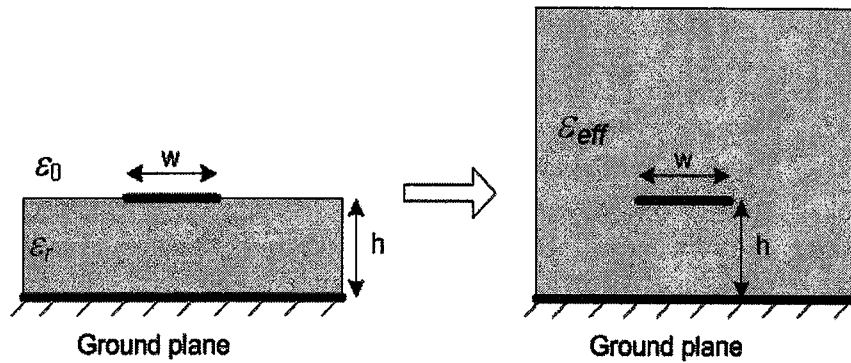


Figure 5.2: Concept of effective dielectric constant

where Z is the unique impedance matrix of the geometry, Z_Ω and Y_Ω are matrices with the desired impedance and admittance as diagonal values. Note that the generalized S-matrix can be visualized as an S-matrix that has been renormalized to the characteristic impedances of the geometry.

5.2 Impedance and Admittance Matrices

The impedance matrix, Z , is calculated from S-matrix as follows:

$$Z = \sqrt{Z_0} (I + S) (I - S)^{-1} \sqrt{Z_0} \quad (5.10)$$

where S is the generalized S-matrix, I is the identity matrix and Z_0 is a diagonal matrix having the characteristic impedance of each port as a diagonal value.

The admittance matrix, Y , is the inverse of the impedance matrix, Z .

5.3 Numerical Examples

As mentioned in the previous chapters, to be able to apply the spatial domain MoM using the MPIE representation for the analysis of planar multilayered geometries, the current distribution on the conductors is expanded in terms of basis functions, in other words, the known basis functions are assigned on the conductors to represent the current distribution. A typical uniform gridding of a planar circuit is shown in Fig. 5.3. Here Δx and Δy are constant through the whole geometry. By using this gridding structure, the basis functions are assigned to the current distribution. Note that a rooftop basis function is assigned over every two cells. Similarly, for half-rooftop only one cell is used. Although non-uniform gridding is possible, it should be remembered that the matrix fill time may be increased due to the increased complexity of the evaluation of the reaction integrals. Additionally, the advantages associated with the symmetry may not be utilized with non-uniform gridding. Therefore, it is recommended that the uniform gridding be used unless the other one is necessary. As for the gridding of the vertical conductors, two identical cells are used in this thesis. Therefore, three basis functions are used to represent the current distribution on the vertical conductors. The arrows on the port transmission lines in Fig. 5.3 show the reference planes used in the calculation and de-embedding of the circuit parameters. The

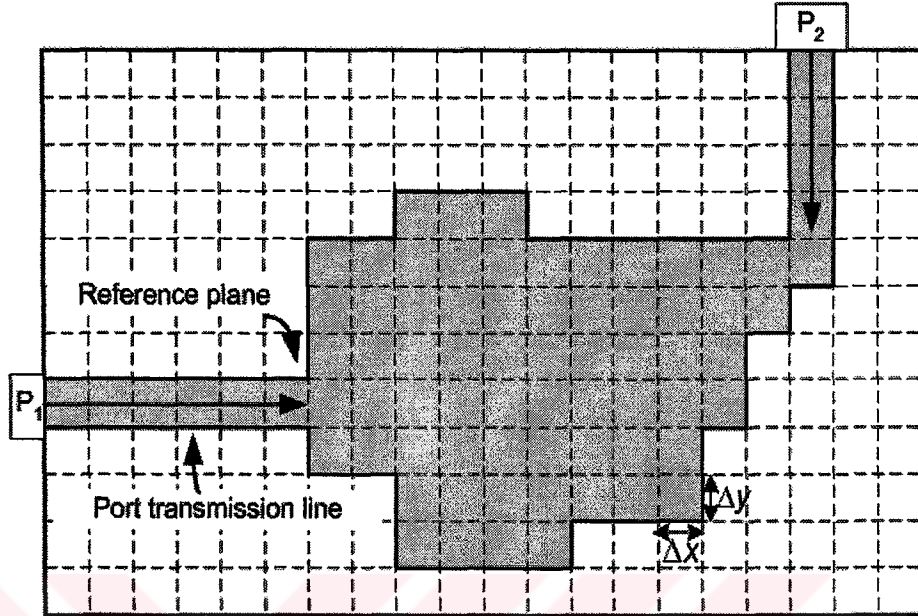


Figure 5.3: Typical gridding used in the solution process

S-parameters provided in these examples are normalized with respect to 50Ω reference impedance. The conductors used in the simulations are assumed to be lossless.

5.3.1 Microstrip Branchline Coupler

Branchline coupler is a passive microwave component having 90 degrees phase difference between its two output ports which are named through and coupled arms. It can be used in the splitting of power in antenna arrays. Branchline couplers are often made in microstrip or stripline form. Since there are x and y directed basis functions, multiports and no vertical metallizations in its geometry, branchline coupler is a good example to start with to show the validity of the technique presented in this thesis for geometries including only horizontal conductors. The geometry of the branchline coupler analyzed is given in Fig. 5.4. As it can be seen from the figure, branchline coupler has four symmetrical ports. The substrate used in this case is alumina with a dielectric constant of 9.8. The conductors are placed 25 mils above the ground plane. Note that in this example, mils unit is used where 1 mils is equal to $2.54 \mu m$.

The S-parameters obtained from the simulation software and *Ensemble* are compared

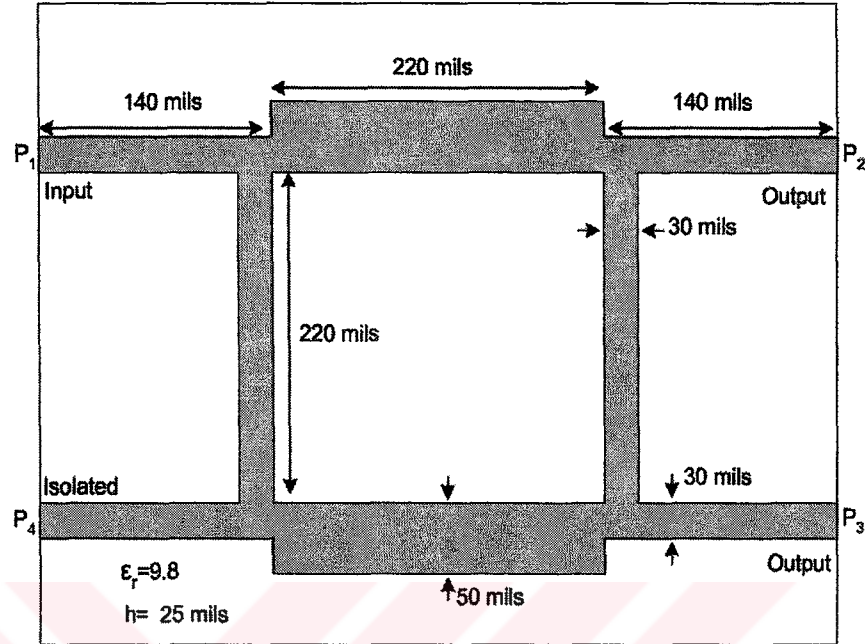
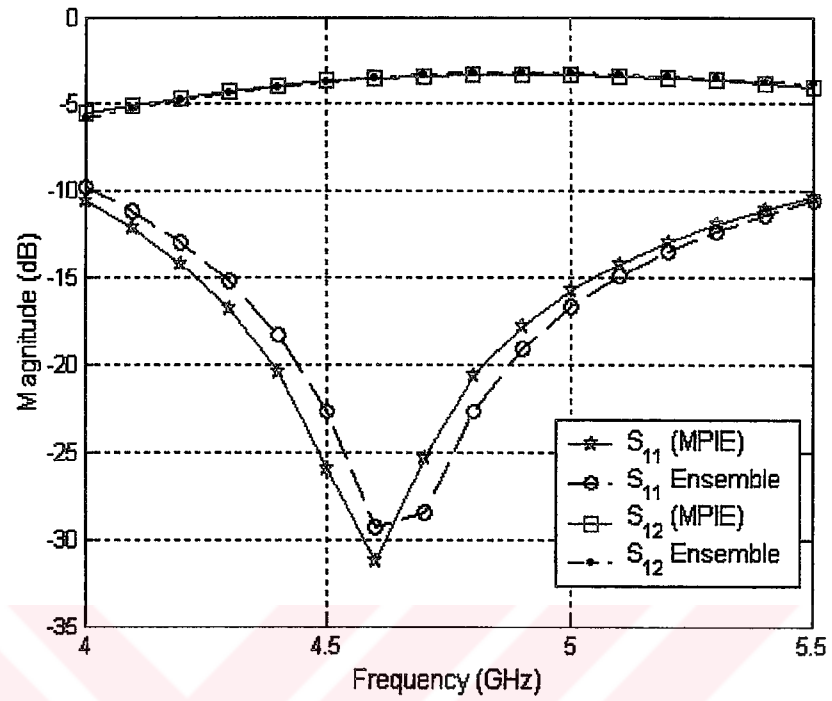
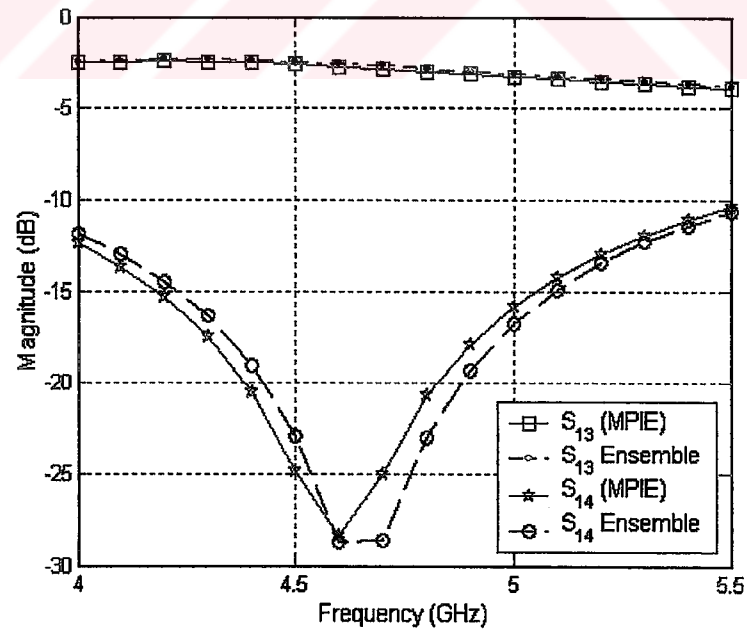


Figure 5.4: The geometry of the branchline coupler analyzed. Note that 1 mils= 25.4 μm

and given in Figs. 5.5 and 5.6. Input return loss (S_{11}), balanced quadrature ports (S_{12} and S_{13}) and isolation between ports 1 and 4 (S_{14}) are shown in the figures. Note that the frequency of optimum performance is around 4.65 GHz. Due to the symmetry of the coupler, any of the ports can be used as the input port but the corresponding output ports should be changed accordingly.

Figure 5.5: Magnitude of S_{11} and S_{12} of the branchline coupler.Figure 5.6: Magnitude of S_{13} and S_{14} of the branchline coupler.

5.3.2 Via to Ground

In the branchline coupler example, the formulation presented in this thesis has been applied to a geometry with no vertical conductors and it has been shown that the spatial domain MoM solution of the MPIE employing the closed form Green's functions gives accurate results for such a geometry. Now it is time to apply the algorithm to planar structures involving vertical metallizations. The use of vertical metallizations can be best demonstrated by the use of via-holes in multilevel printed circuits. Nowadays, the signals and ground connections are carried from one layer to another with the use of via-holes to reduce the sizes of the circuits. Therefore, in this example, a via connected to ground will be analyzed. Fig. 5.7 shows a via connected to ground in the form of a rectangular box. The via has a pad

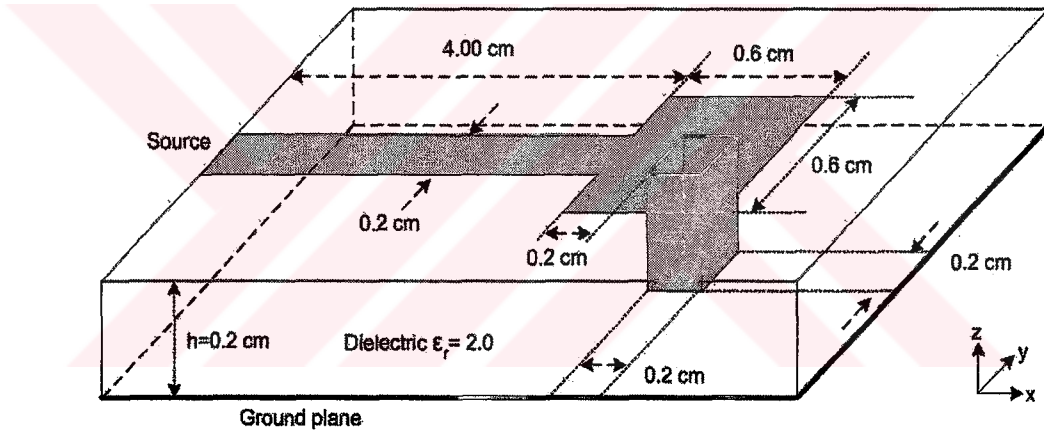


Figure 5.7: Geometry of a microstrip line connected to a via to ground. Note that the via has a pad larger than the via itself.

larger than itself and a microstrip line is connected to the pad. In the simulation software, the via is implemented by using four vertical strips. The basis and testing functions on each of the vertical strip are chosen to have the same z and z' dependence so that the generated spatial domain auxiliary functions can be used repeatedly. To be able to show the efficiency of using basis and testing functions with same z and z' dependency, the circuit is first analyzed when there is only the vertical strip at $x = 4.2$ forming the leftmost wall of the rectangular box. Then other strips forming the via is included and the results are compared with those obtained from *em* by SONNET.

Number of vertical strips	Percentage increase in Matrix fill time	
	<i>em</i> by SONNET	MPIE method
1	100	100
2	12.50	2.98
3	18.75	2.82
4	20.31	2.77

Table 5.1: The percentage increases in the matrix fill times for addition of multiple vertical strips for *em* by SONNET and MPIE method which is used in this thesis for via to ground example.

Fig. 5.8 shows the S-parameter when there is only one vertical strip and Fig. 5.9 shows when four vertical strip are used. Table 5.1 shows the percentage increases in the matrix fill times.

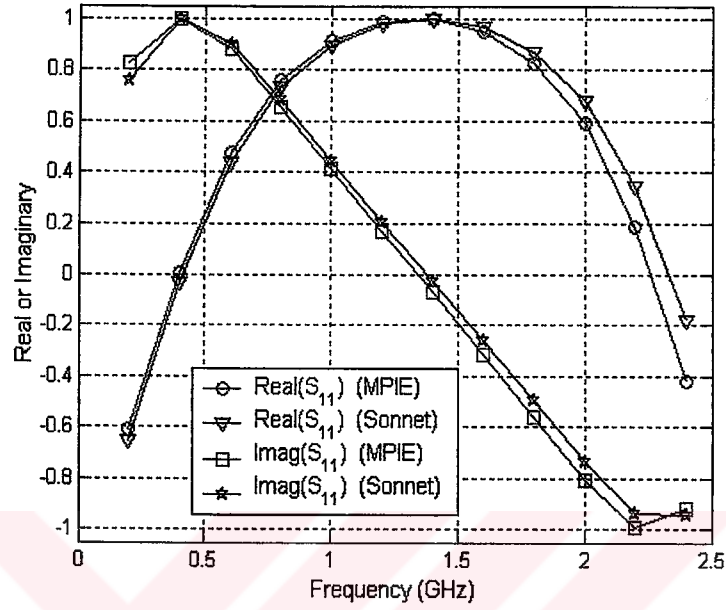


Figure 5.8: Real and imaginary parts of S_{11} when there is only one vertical strip is used.

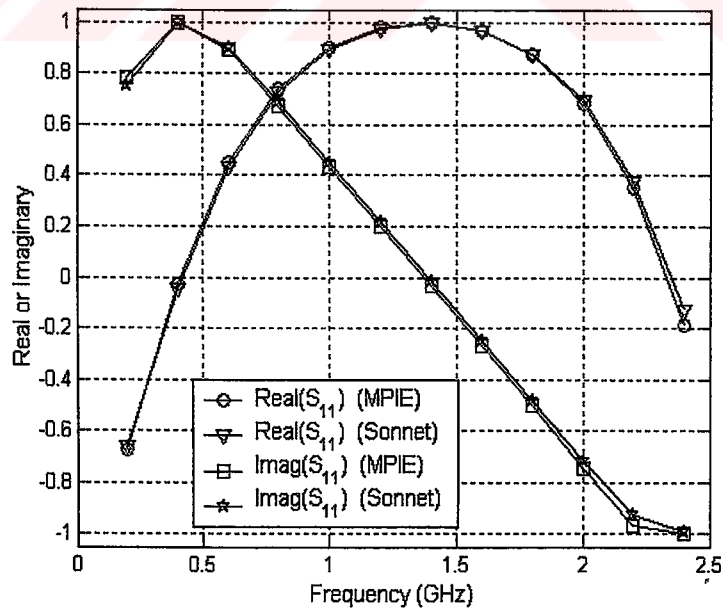


Figure 5.9: Real and imaginary parts of S_{11} when the via with four vertical strips is used

5.3.3 Air Bridge

An air bridge is simply a metallic bridge that permits two transmission lines to cross over each other without forming an electrical short circuit. In MMICs air bridges are commonly used to carry signal paths over other signal paths. For example, in the case of series connected spiral inductor air bridge is used. Since vertical metallization is required in the implementation of the air bridge, it is a good example to show the validity of the technique presented in this thesis. For this purpose, the circuit given in Fig. 5.10 is analyzed. To implement the vertical metallization shown in the figure, two vertical strips are used. As in the via to ground example, the z and z' dependence of the basis and testing functions on the vertical strips are chosen to be same to be able to exploit the efficient use of auxiliary functions. In this example, first the vertical strip at $x = 8.0$ cm is included and the

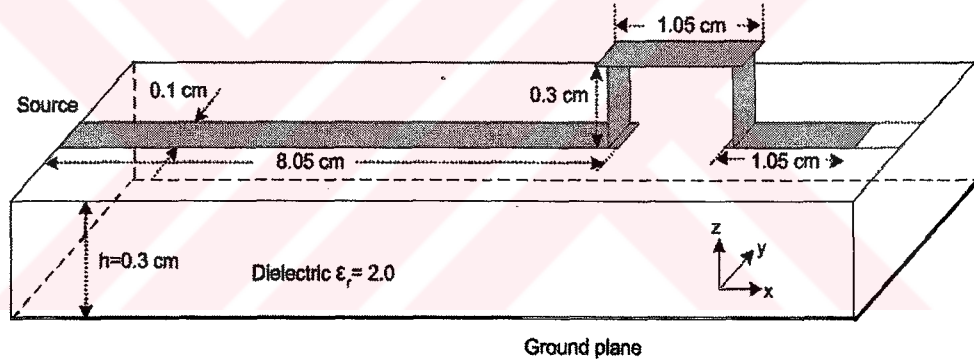


Figure 5.10: Geometry of the air bridge analyzed

corresponding S-parameters are shown in Fig. 5.11. Then the second vertical strip at $x = 9.0$ cm is added and the S-parameters are found and shown in Fig. 5.12. Additionally, the current distribution on the vertical conductors are found and compared for the two vertical strip case at the frequency of 1.0 GHz. The current distribution is shown in Fig. 5.13. As it can be seen from the current distribution figure, at the points of vertical metallizations there are glitches. The reason for these glitches are the two basis functions on the ends of the horizontal conductors. Because the vertical metallization is just over those basis functions, their coefficients become smaller as compared to the others. Therefore the glitches occur.

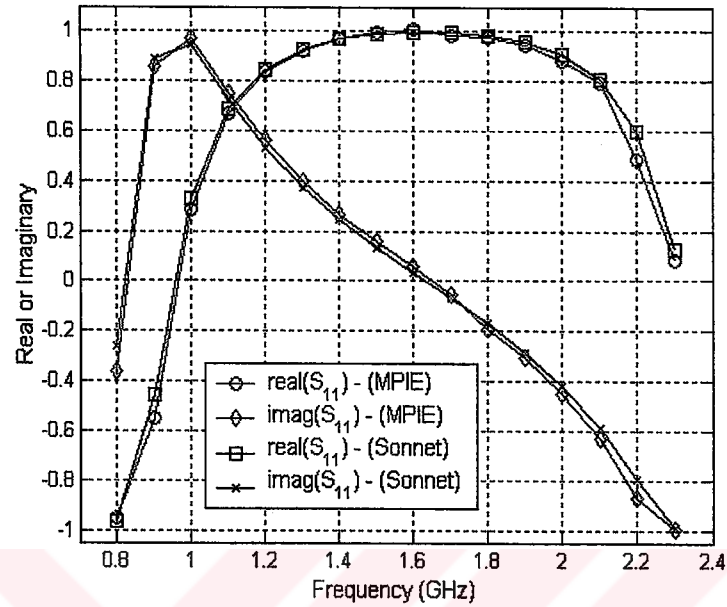


Figure 5.11: S-parameter for the air bridge when there is only one vertical strip at $x = 8.0$ cm.

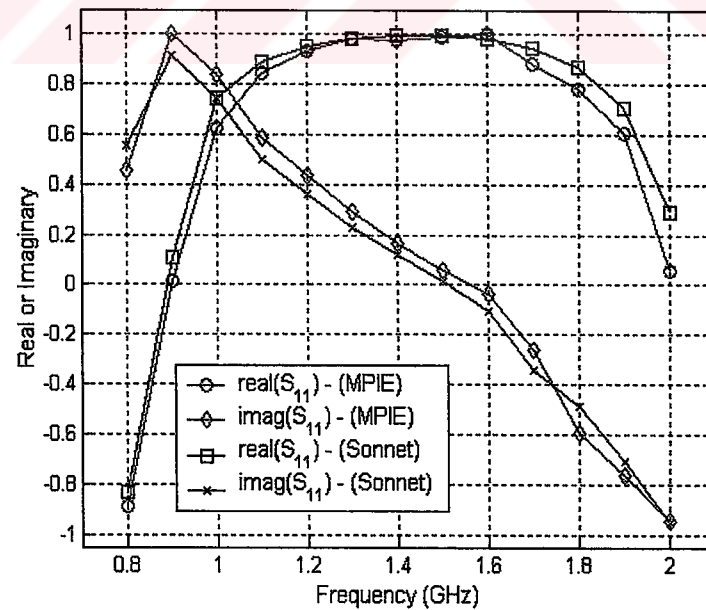


Figure 5.12: S-parameter for the air bridge with two vertical strips

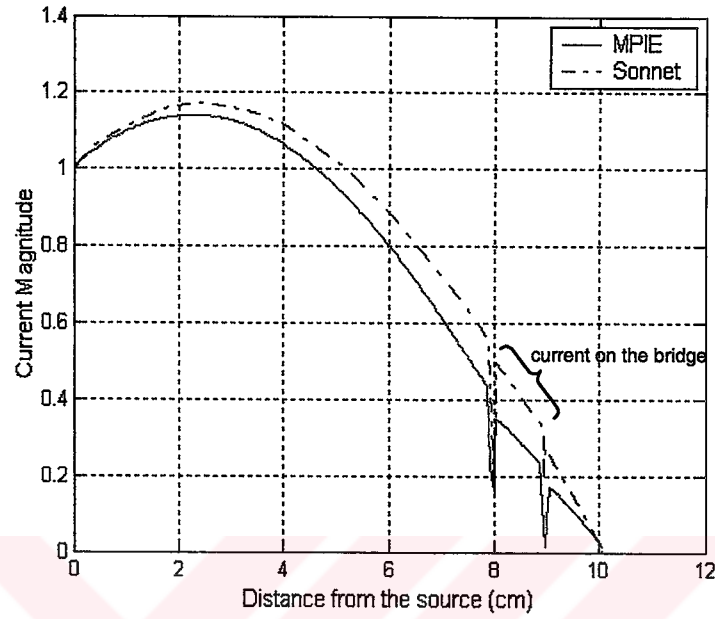


Figure 5.13: The current distribution on the vertical conductors at the frequency 1.0 GHz

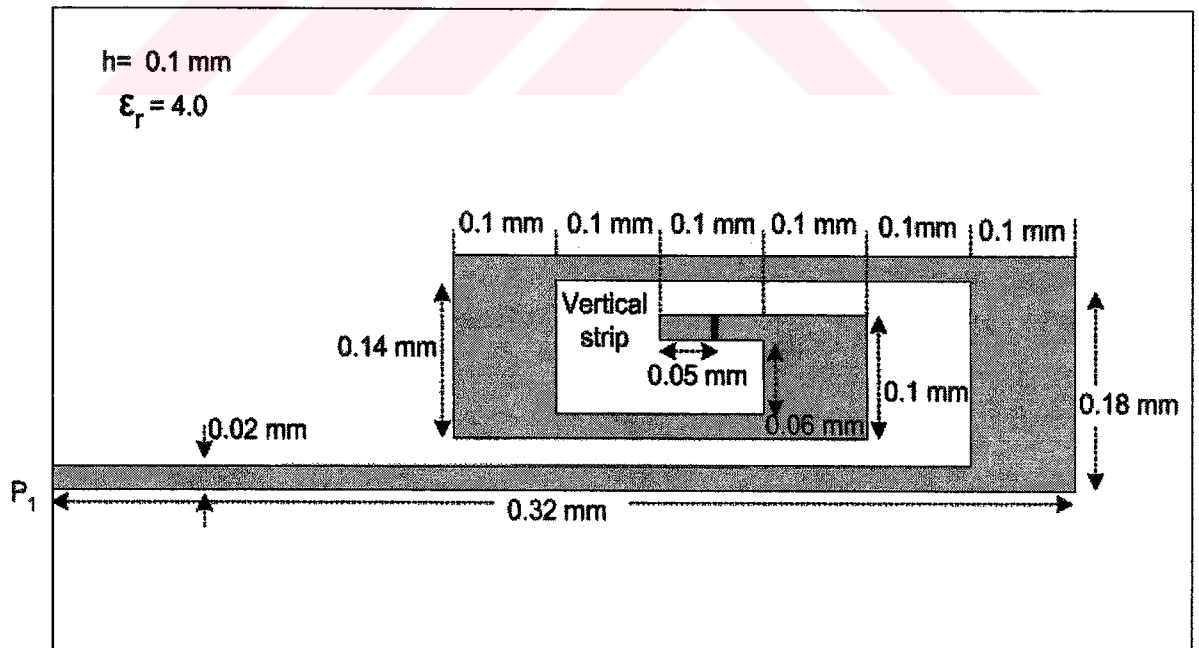


Figure 5.14: The geometry of the spiral inductor analyzed.

5.3.4 Spiral Inductor

In MMICs inductors can be implemented by forming spirals of conductors in planar media. Some may have vertical metallizations for grounding. Therefore in this example, a spiral inductor terminated in a short circuit, shown in Fig. 5.14 printed on a substrate with a dielectric constant of 4.0 with a thickness of 0.1 mm is analyzed. The S_{11} parameter and the input impedance are calculated and plotted in Figs. 5.15 and 5.16 as functions of frequency. The S parameter plot is given only for the purpose to show that the calculated values are equal to those obtained from *em*. Since this is an inductor, it is best to analyze its input impedance behavior with respect to frequency which is done in Fig. 5.16.

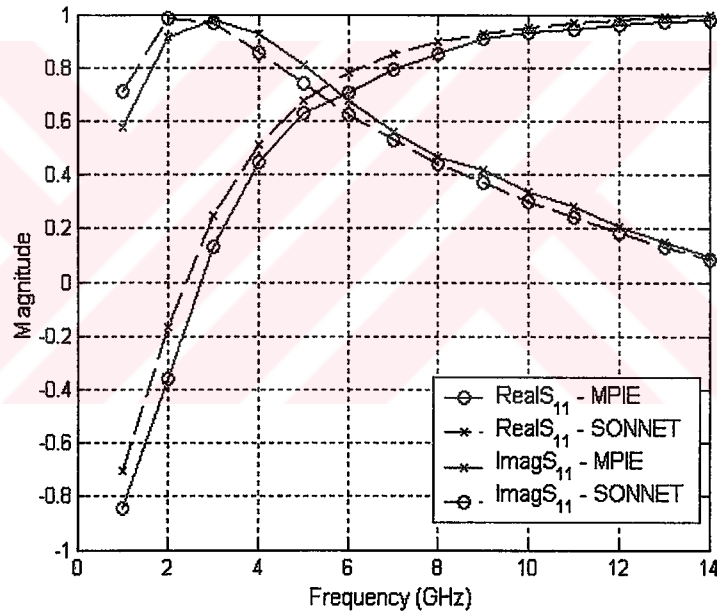


Figure 5.15: S_{11} parameter of the spiral inductor

5.3.5 Interdigital Bandpass Filter

Passive bandpass filter design in MMIC technology offers many advantages in layout and fabrication such as reduction of size or ease of connection. Therefore in this example, an interdigital bandpass filter with vertical strips is analyzed. The geometry of the circuit is shown in Fig. 5.17. As shown in the figure, five vertical strips may be included in the

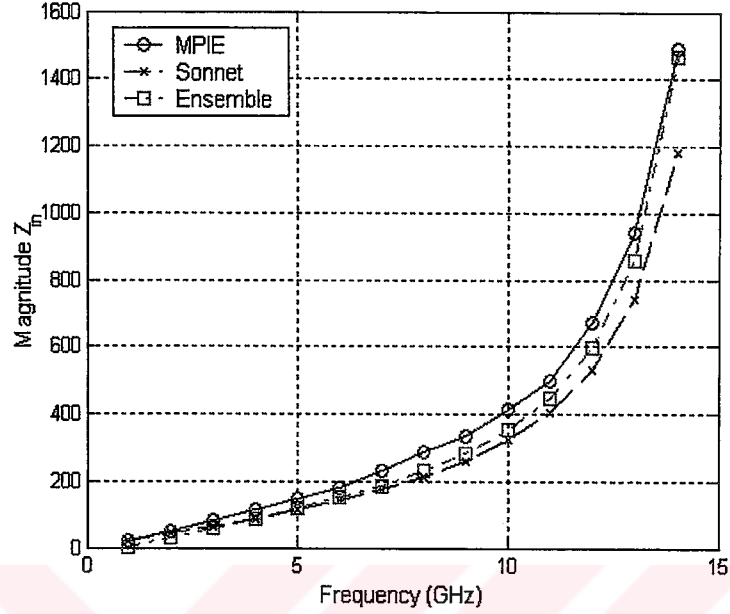


Figure 5.16: Input impedance of the spiral inductor

geometry. The S-parameters for the case when there is only one vertical strip between $x = 1.0$ and $x = 1.05$ cm are found and compared with those obtained from *em* from Sonnet. The real and imaginary parts of S_{11} and the magnitude of S_{12} are shown in Fig. 5.18, Fig. 5.19 and Fig. 5.20, respectively. Similarly, the S-parameters for the case with five vertical strips are shown in Figs. 5.21 and 5.22. Note that to be able to utilize the auxiliary functions generated for the vertically directed basis functions, the z and z' dependences of the basis and testing functions on the vertical strips are chosen to be same.

When Fig. 5.20 is analyzed, the patterns of the S_{12} parameter obtained from the MPIE and *em* seem to be showing the same behavior. But there exists an approximately 10 dB difference between the plots. This difference comes from the fact that *em* analyzes the circuits in a shielding box. When there is a shielding box over the conductors and if it is chosen to be close to the actual circuit, then the coupling of the electric field from one conductor to others are not realized accurately. Consider the case when the shielding metal is taken very close to the actual circuit. In this case, the shielding metal may behave as a shorting component. On the other hand, since all the effects of the geometry is taken into

account with the use of Green's functions, the technique presented in this thesis analyzes the circuit in an open volume. Therefore, the coupling effects are taken into account accurately by using this technique. Since the coupling is less in *em* case, the difference between the plots occurs.

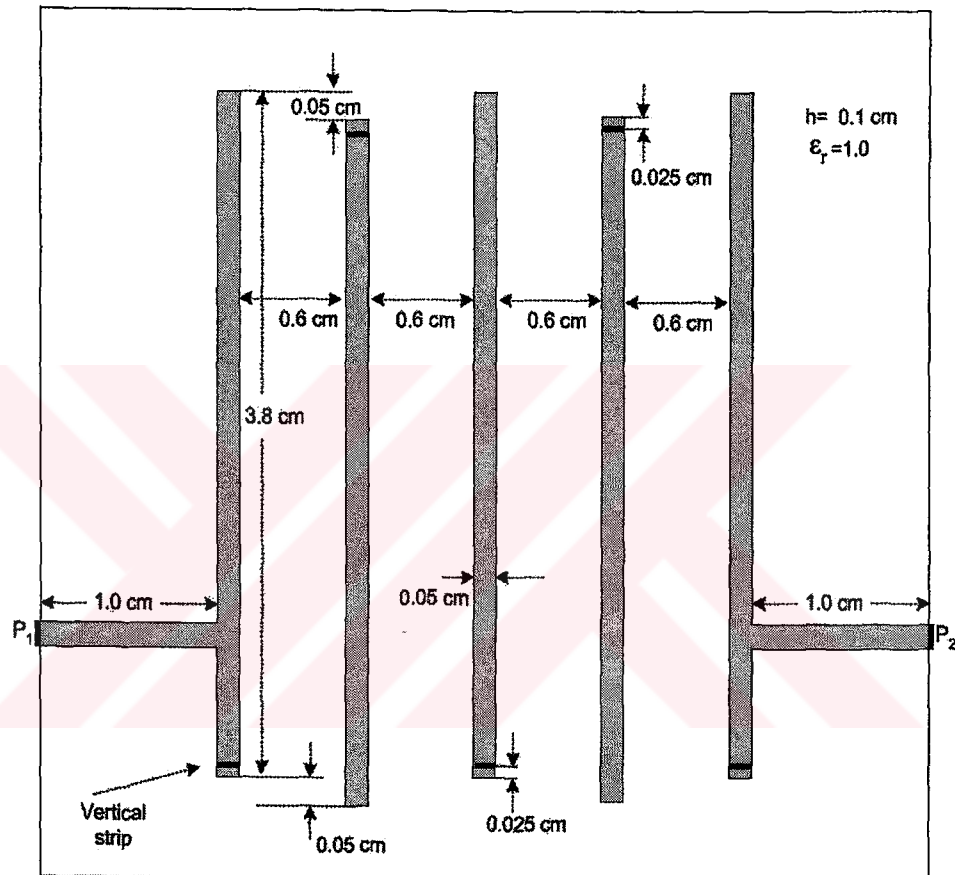
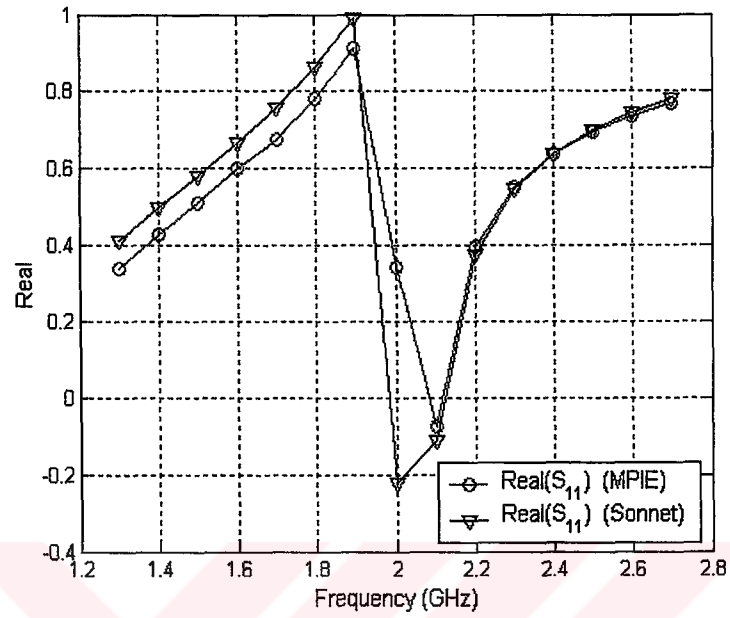
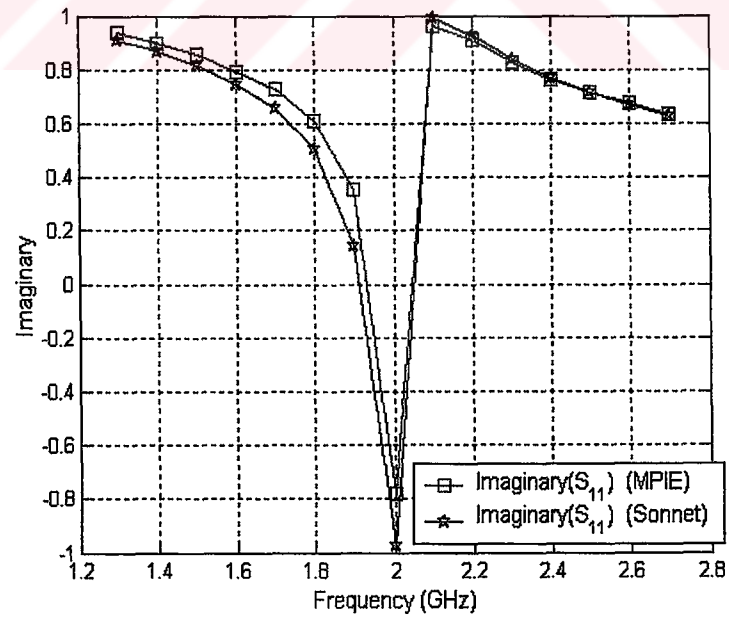
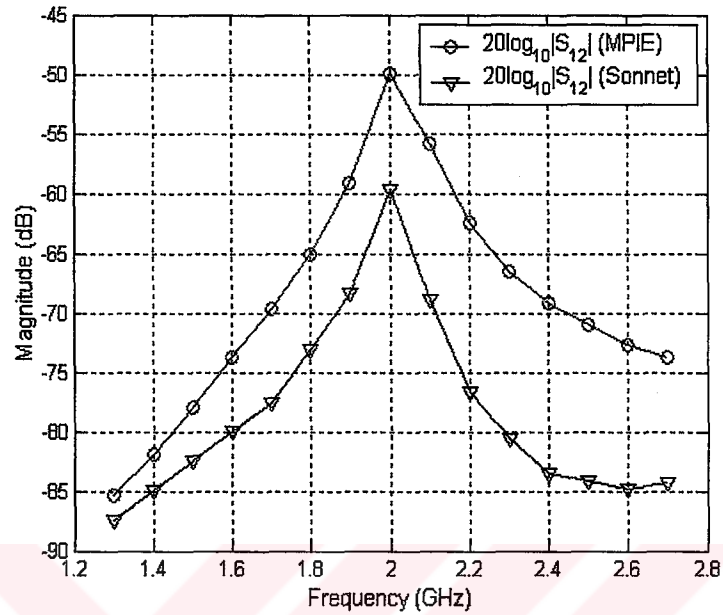
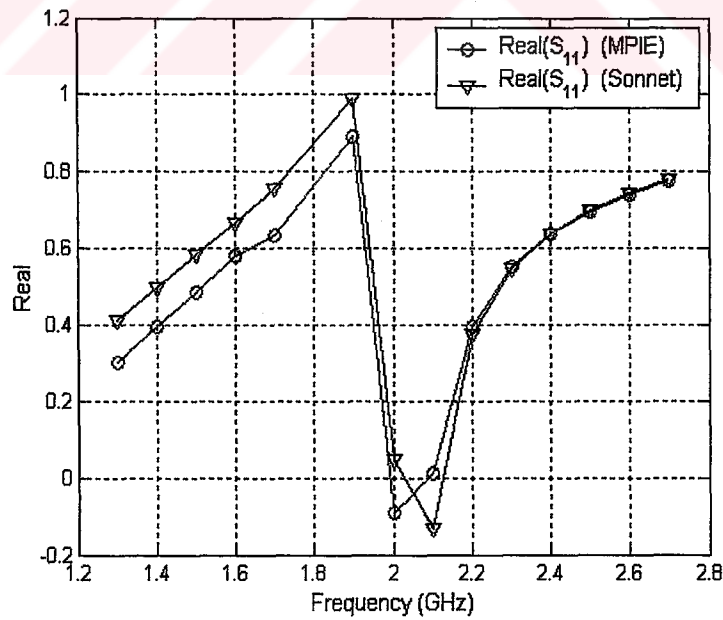


Figure 5.17: The geometry of the interdigital bandpass filter. Note that there are five vertical strips on the edges of the microstrips

Figure 5.18: The real part of S_{11} for one vertical strip caseFigure 5.19: The imaginary part of S_{11} for one vertical strip case

Figure 5.20: Magnitude of S_{12} for one vertical strip caseFigure 5.21: The real part of S_{11} for five vertical strips case

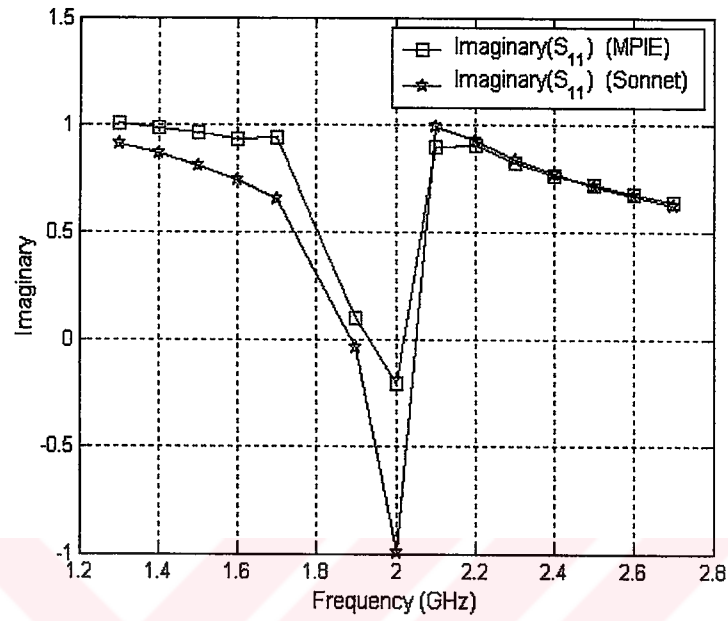


Figure 5.22: The imaginary part of S_{11} for five vertical strips case

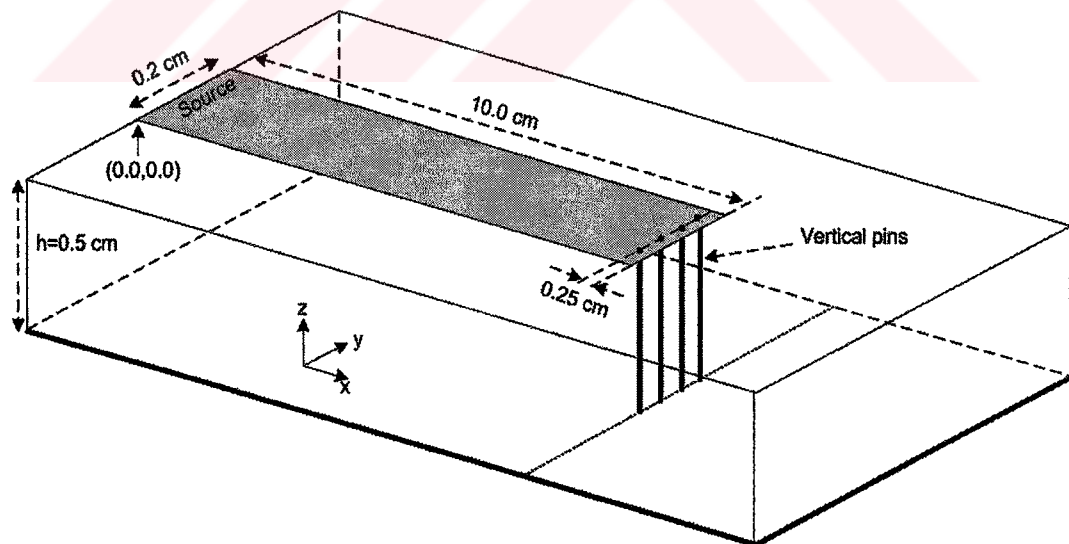


Figure 5.23: The microstrip circuit analyzed for the vertical pin implementation. Note that in this case there are four vertical pins used in total. The distance between each vertical pin is 0.04 cm.

5.3.6 Vertical Pin Implementation Using a Microstrip Line

As mentioned in the previous sections of this thesis, the vertical conductors used in this work are in the form of strips and pins where strips span in one of the lateral direction and pins have no spanning. Up to this example, all the vertical conductors used were in the form of vertical strips, therefore to show that the formulation presented in this thesis works for vertical pins, the circuit given in Fig. 5.23 is analyzed. First only one vertical pin at $y = 0.04$

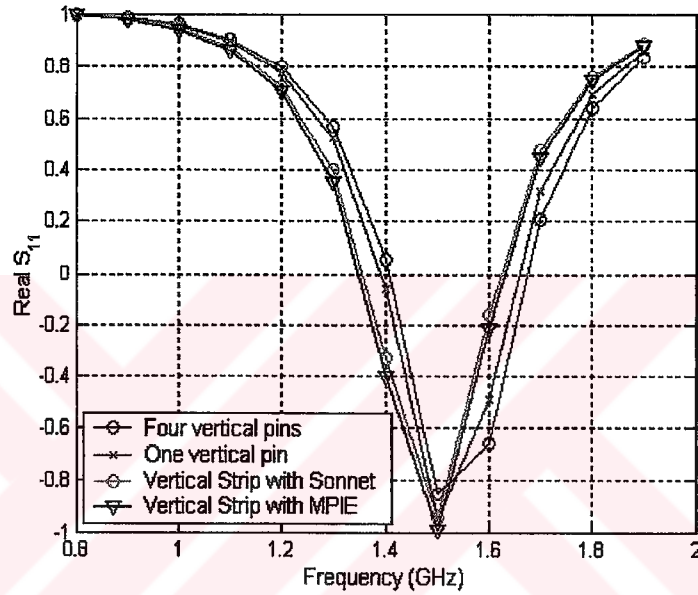


Figure 5.24: The real part of S_{11} for case with four vertical pins

cm is included and the geometry is analyzed. Then four vertical pins are located with a 0.04 cm distance between each other. The reason behind this configuration is to compare the S-parameters of the microstrip line with those obtained from a full vertical strip case. After four pins implementation, a vertical strip spanning 0.2 cm in the y -direction at $x = 9.75$ cm is included and the geometry is analyzed. Finally *em* by SONNET is invoked with a vertical strip and the corresponding results are obtained. The simulation results are given in Figs. 5.24 and 5.25. Note that the vertical strip simulations obtained from the MPIE method and *em* agree well. But when the pins are used there are slight differences. But in general they behave as grounding components. For the sake of completeness the current distribution in the case of four vertical pins is given in Fig. 5.26.

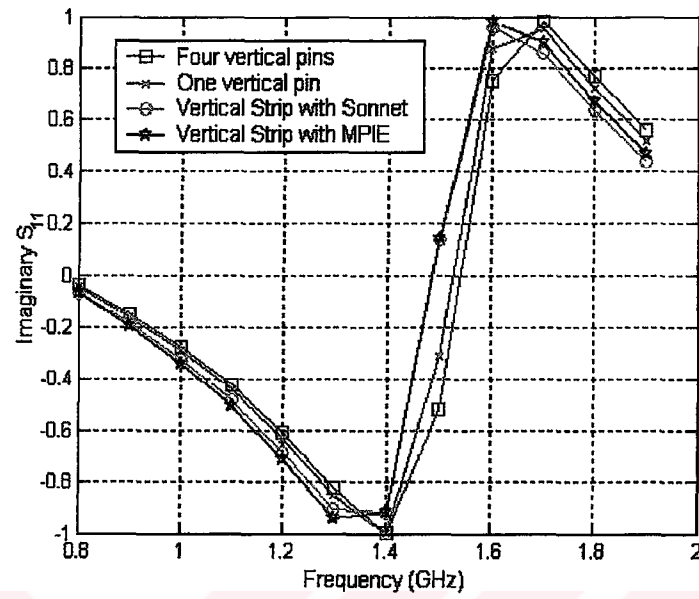


Figure 5.25: The imaginary part of S_{11} for the case with four vertical pins

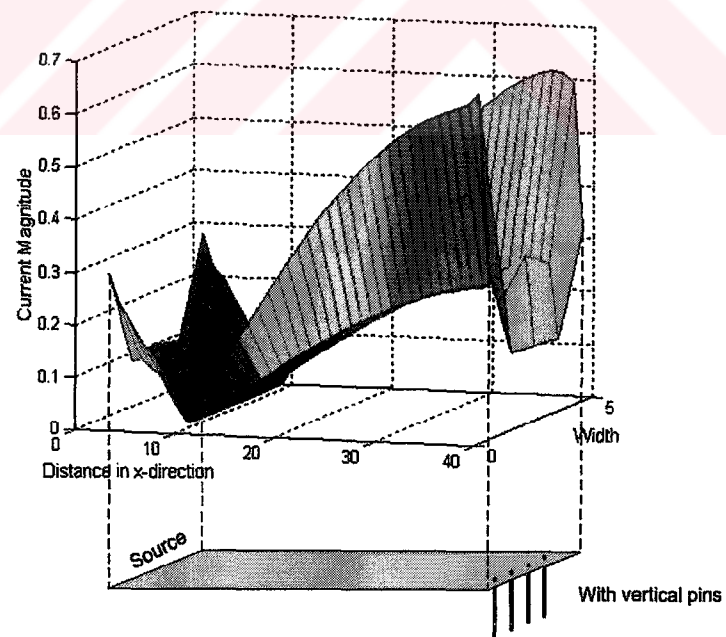


Figure 5.26: The current distribution on the microstrip line for the four vertical pin case

5.3.7 Experiment with a Corner Circuit

In order to verify the technique presented in this thesis further, an experiment is also carried out using a corner structure whose geometry is shown in Fig. 5.27. The corner is on

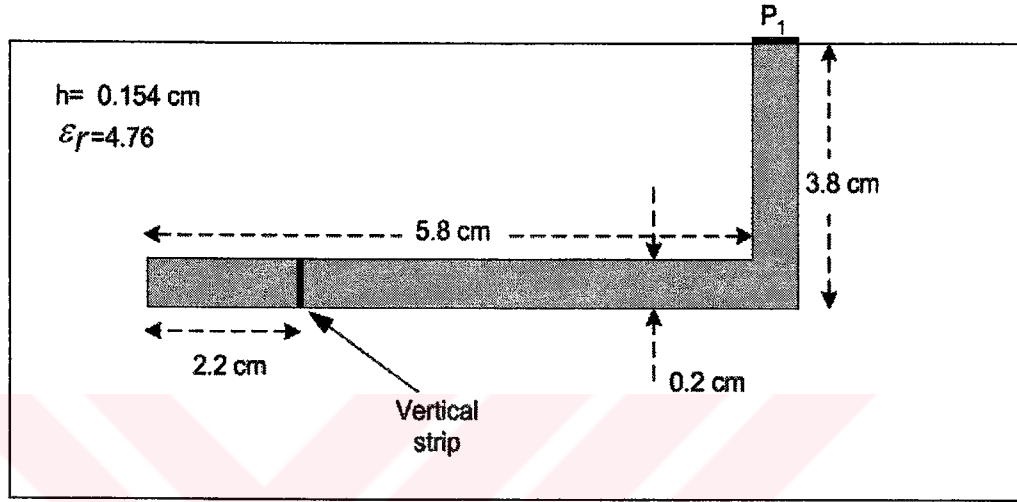


Figure 5.27: The geometry of the Corner circuit used in the experimental verification

a substrate with a dielectric constant of 4.76 and thickness of 0.154 cm. The results obtained from the experiment and those from the MPIE representation technique and SONNET are compared and given in Figs. 5.28 and 5.29. During the simulations using the MPIE representation and SONNET all the conditions have been assumed to be ideal, e.g. the thickness of the substrate is constant for all points or the conductivity of the conductors are infinity etc. But for the experiment, the thickness of the substrate is not constant, i.e. at some point it is 0.154 cm and it is 0.152 cm at another point. Also the conductivity of the conductor is finite. Additionally, during the inclusion of the vertical strip, the soldering causes the thickness of the metal to increase at the corresponding regions and also during the chemical etch of the circuit the borders of the corner circuit is not as accurate as they are entered in the simulation softwares.

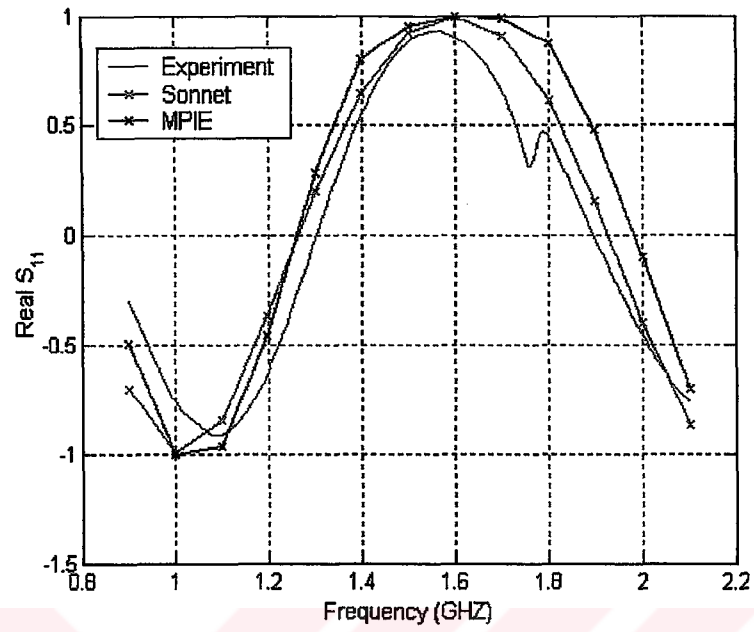


Figure 5.28: The real part of the S_{11} for the corner geometry

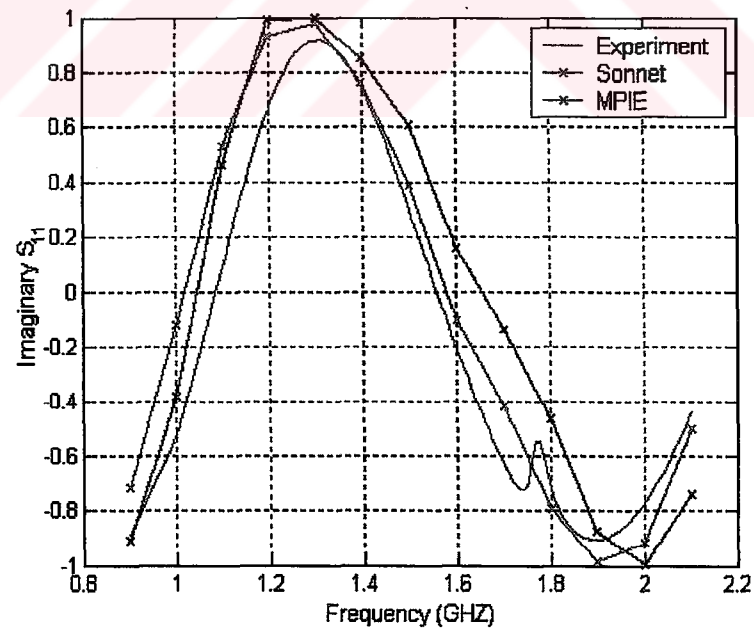


Figure 5.29: The imaginary part of the S_{11} for the corner geometry

5.3.8 Short-circuited Microstrip Line Experiment

In this example, another experiment is carried out for a simple short circuited microstrip line. The geometry of the microstrip line is shown in Fig. 5.30. The shorting of the microstrip line is achieved by using a vertical strip whose width is equal to the width of the microstrip line. The dielectric constant of the substrate is given to be 4.76 and the thickness is 0.154 cm. The microstrip line is excited with a current source from the left end and it is short circuited in the right end. Since the network analyzer used in the experiment gives the

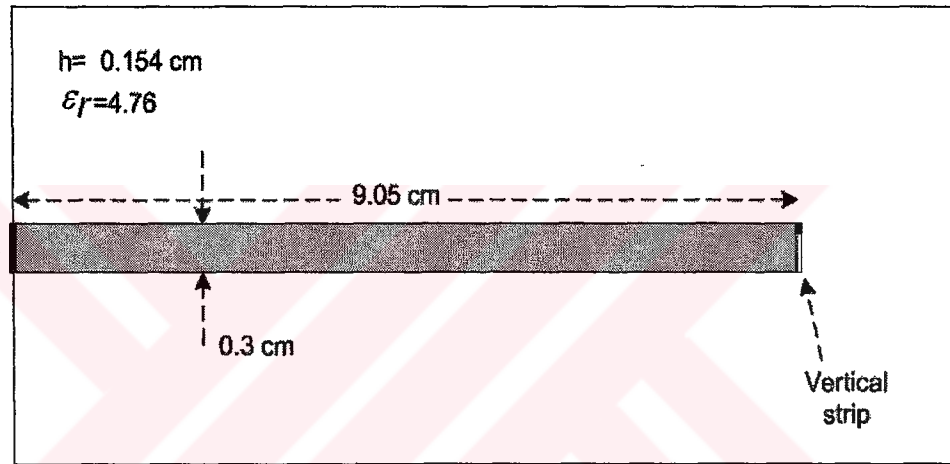


Figure 5.30: The geometry of the short circuited microstrip line used in the experiment

S-parameters of the circuit, the real and imaginary parts of the S-parameters are compared and they are given as in Figs. 5.31-5.32. In addition to these values, the input impedance values obtained from the MPIE method and em are compared and the results are given in Fig. 5.33. As can be seen from the figures, the results agree well for this thin substrate. But it should be noted that the results of em starts to deviate as the thickness of the substrate is increased because em assumes constant current distribution on the vertical connections [44].

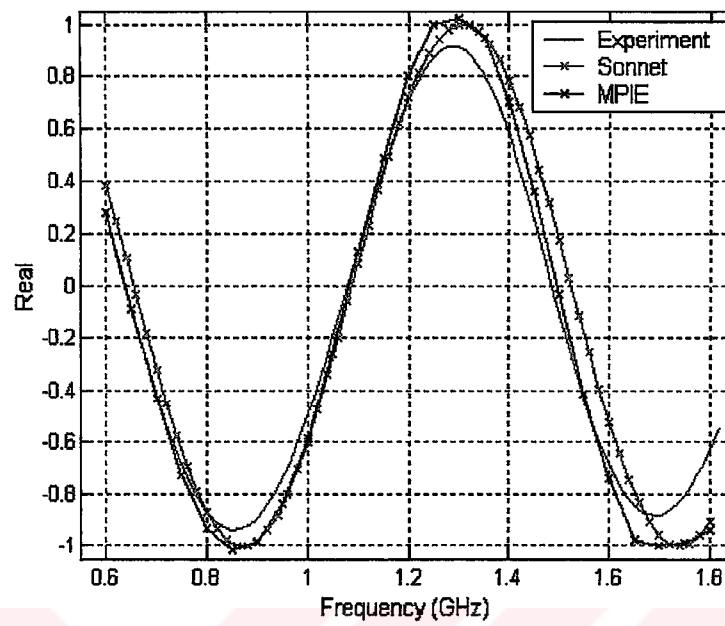


Figure 5.31: The real part of the S_{11} for the short circuited microstrip line experiment

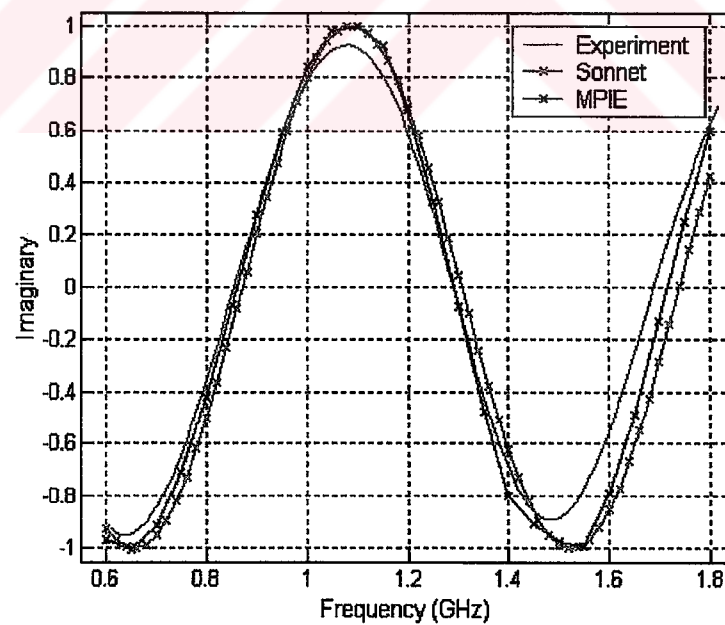


Figure 5.32: The imaginary part of the S_{11} for the short circuited microstrip line experiment

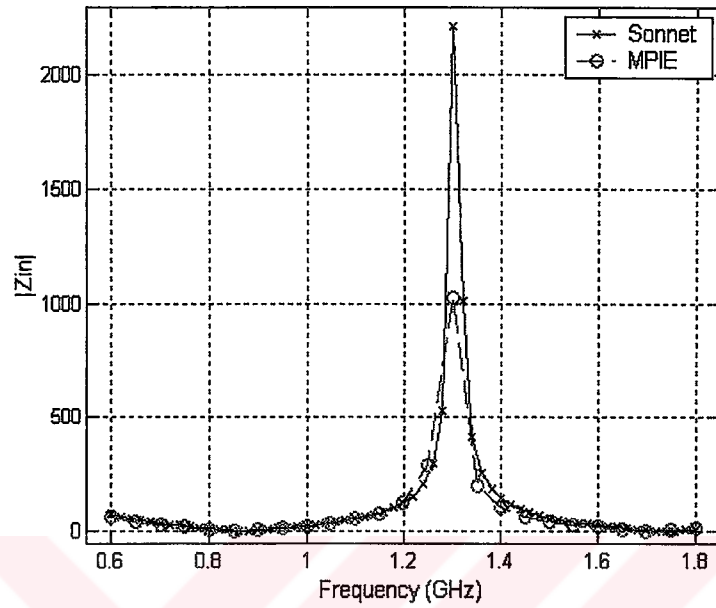


Figure 5.33: The input impedance of the short circuited microstrip line

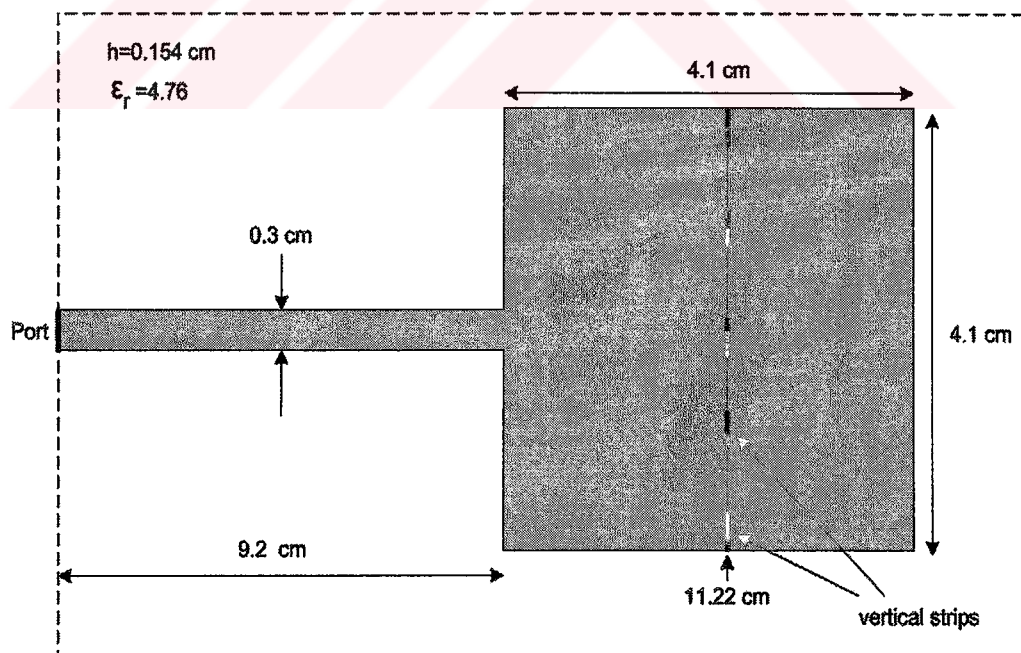


Figure 5.34: The geometry of the patch antenna used in the experiment

5.3.9 Short-circuited Microstrip Patch Antenna Experiment

Microstrip patch antennas are among the simplest radiating structures that can be built on a multilayer medium. Single patch antennas and patch antenna arrays are widely used in mobile microwave communication systems [48, 49, 50, 51]. Additionally, the possibility of incorporating the antenna with active circuits on the same substrates increases the importance of microstrip antennas. Several methods have been proposed to analyze the patch antennas. With the formulation presented in this thesis, the radiating structures can be analyzed rigorously since the radiation condition is automatically satisfied. Moreover, the surface waves are considered inherently in this formulation, therefore a rigorous analysis of coupling between the antenna array elements is achieved. Therefore, by using the method presented in this thesis, the microstrip patch antennas can be rigorously analyzed by considering the radiation, electromagnetic couplings and fringing fields.

With the inclusion of shorting posts in a microstrip patch antenna, the resonant frequency of the antenna may be shifted without affecting the radiation pattern significantly [52, 53, 54]. Therefore, shorting components can be employed in the design of dual-frequency antennas. In this example, a microstrip patch antenna circuit whose geometry is given in Fig. 5.34 is analyzed in the case of vertical strips. As it is the case for the previous experiments, the dielectric constant of the substrate is 4.76 and the thickness is 0.154 cm. The vertical metallizations are in the form of vertical strips and all have width of 0.3 cm. At first only one vertical strip spanning between $y = 1.9 - 2.2$ at $x = 11.22$ cm is included. The resulting S-parameters are given in Figs. 5.35 and 5.36.

Then four more vertical strips at the same x -coordinate is included to the geometry. The new vertical strips span between $y = 0.0 - 0.3$, $y = 0.95 - 1.25$, $y = 2.55 - 2.85$ and $y = 3.8 - 4.1$. For this case, the S-parameters of the circuit are shown in Figs. 5.37 and 5.38.

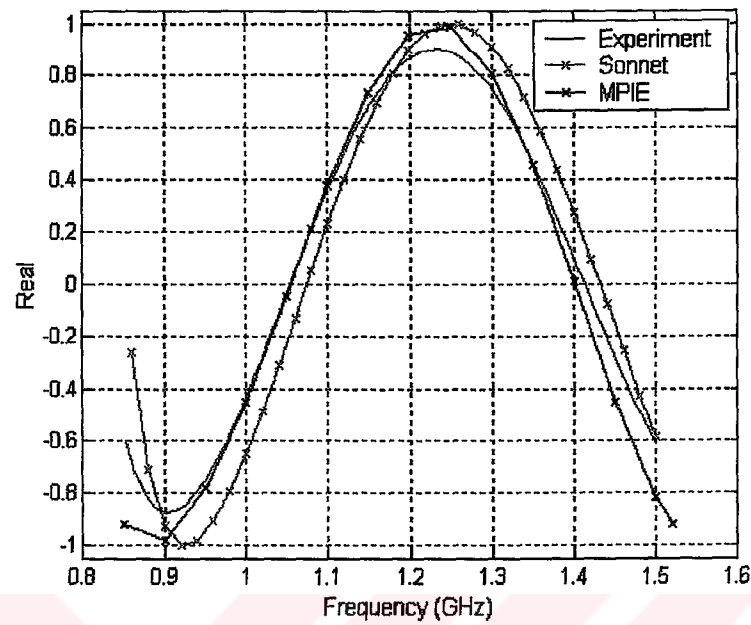


Figure 5.35: The real part of the S_{11} for the microstrip patch antenna when there is only one vertical strip between $y = 1.9 - 2.2$ at $x = 11.22$ cm

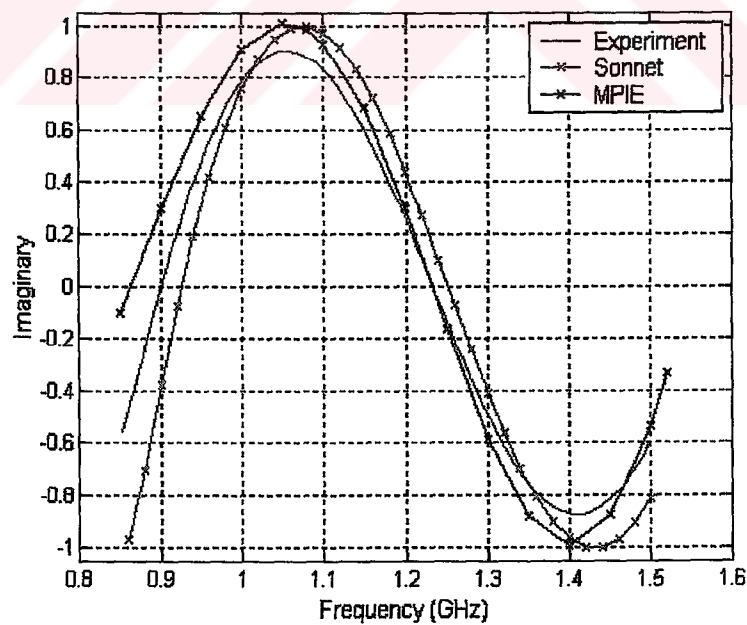


Figure 5.36: The imaginary part of the S_{11} for the microstrip patch antenna when there is only one vertical strip between $y = 1.9 - 2.2$ at $x = 11.22$ cm

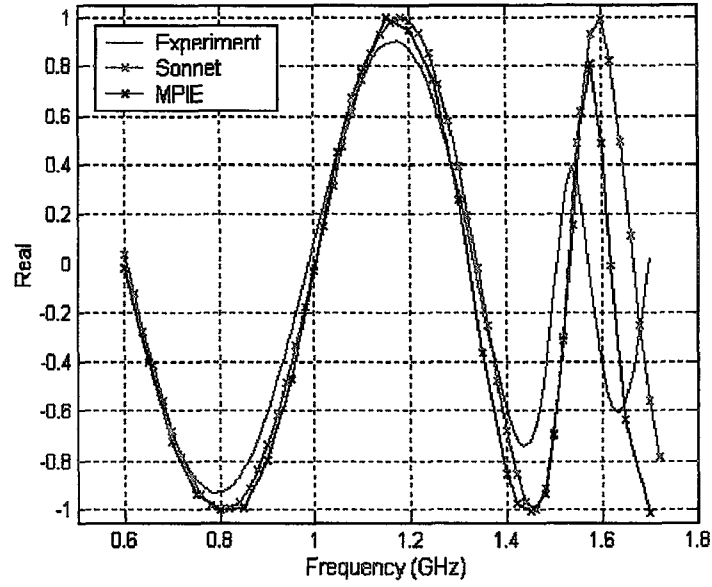


Figure 5.37: The real part of the S_{11} for the microstrip patch antenna when there are five vertical strips at $x = 11.22$ plane between $y = 0.0 - 0.3$, $y = 0.95 - 1.25$, $y = 1.9 - 2.2$, $y = 2.55 - 2.85$ and $y = 3.8 - 4.1$

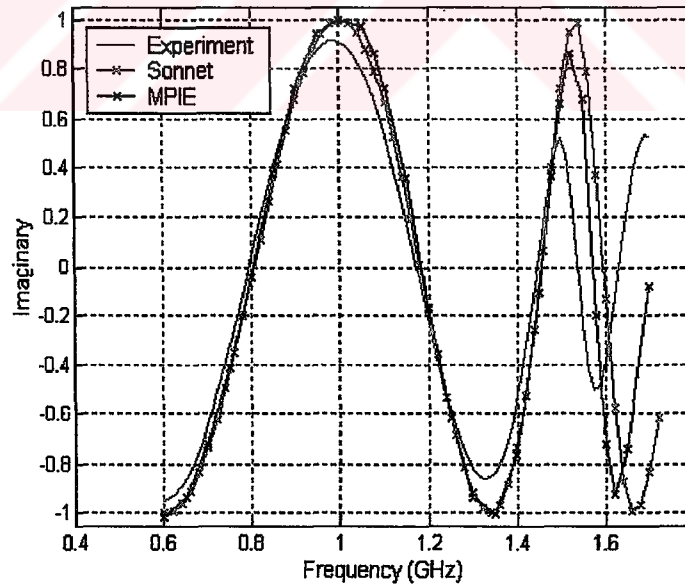


Figure 5.38: The imaginary part of the S_{11} for the microstrip patch antenna when there are five vertical strips at $x = 11.22$ plane between $y = 0.0 - 0.3$, $y = 0.95 - 1.25$, $y = 1.9 - 2.2$, $y = 2.55 - 2.85$ and $y = 3.8 - 4.1$

5.4 Conclusions

In this chapter, the de-embedding method employed in the algorithm is explained along with the methods used to extract the circuit parameters such as S-parameters or Z-parameters. It has been shown that scattering parameters are found accurately given that the correct impedance values for the port transmission lines are used and that the current on a port transmission line is sampled starting from a sufficiently distant point from the discontinuity. After the presentation of the algorithms used to extract the circuit parameters, several circuits are analyzed to assess the validity of the method presented in this thesis. The validation is carried out by comparing the results obtained from the method explained in this thesis with those obtained from commercially available softwares such as *em* by SONNET or *Ensemble* by ANSOFT and from experiments. The common property of most of the circuits used in these examples are the vertical metallizations they contain. It has been shown that the results obtained agree very well for thin substrates, in other words for short metallizations. But for longer metallizations *em* cannot give accurate results due to the fact that it uses only one basis function for the vertical metallizations. On the other hand, the technique presented here is not limited with the length of the vertical conductors. Any number of layers and any length for vertical metallizations can be accurately analyzed with the technique presented here by using enough numbers of basis functions. Since the method is capable of incorporating multiple vertical metallizations with almost no computational cost, this can be suitable for the purpose of optimization of a geometry involving multiple vertical metallizations with same z and z' dependences.

Chapter 6

CONCLUSIONS

In this thesis, a full-wave CAD algorithm for the analysis of planar geometries with vertical metallizations was presented and the closed-form Green's functions derived for scalar and vector potentials in planar multilayer media have been revisited to clarify some questions and misunderstandings on the derivation of these Green's functions.

Efficient derivation of closed-form Greens functions in the spatial-domain using the discrete complex image method (DCIM) has increased the computational efficiency of the MoM for planar geometries in multilayer media. However there have been some issues and misunderstandings related to the deterioration of the approximation as compared to exact Greens functions for moderate to large distances. The basically three attributable problems presented in the literature, which are (i) not extracting the quasi-static terms, (ii), introducing a wrong branch point in the process of approximation, and (iii) not extracting the surface wave poles, in the implementation of two-level approach, have been addressed with a detailed discussion to clarify some of the misunderstandings.

The method presented here for the analysis of planar geometries with vertical metallizations is based on the use of the closed-form Green's functions in conjunction with the MoM technique. It has already been demonstrated that the use of closed-form spatial-domain Green's functions into the MoM formulation has significantly improved the computational efficiency of the solution for the analysis of planar geometries, even before the use of the robust multi-level approach [16] and analytical integrations of the MoM entries. By integrating the MoM entries analytically, the computational efficiency of the spatial-domain MoM has further improved and with the use of multi-level approach for the derivation of the closed-form Green's functions, the method has proved to be very robust and become one of the most powerful candidate for general CAD software for planar geometries. But in the case of vertical metallizations, such as shorting pins or via holes, etc., it was realized that the computational efficiency of the method can not be improved significantly by simply

casting the spatial-domain Green's functions in the closed-forms. The difficulties involved in using the Green's functions for such geometries have been addressed and a possible solution is given in this thesis. It has been shown that the solution may be extended to multiple vertical metallizations with almost no computational cost. The vertical metallizations may be strips spanning in different directions or pins with no widths, but as far as they have the same dependence in the vertical direction, which is the z -direction in this work, the inclusion of additional vertical metallizations is efficient. The application of the formulation has been demonstrated on some microwave circuits containing horizontal and vertical conductors. The results were also verified by with the results obtained from commercially available softwares, such as *em* by SONNET or *Ensemble* by ANSOFT and experiments.

The method developed and analyzed in this thesis can be applied to optimization problems involving multiple vertical metallizations with great efficiency. As it is well-known, in optimization problems, the original geometry of the circuit is not changed at all. New parts are added or deleted and the geometry is solved at each iteration. Once the iteration number gets larger, the matrix solution time begins to dominate the overall solution, because for the solution of the system matrices, computationally expensive LU decomposition method is used. Therefore to be able to make the method efficient for the optimization problems, the LU decomposition of the original geometry can be used by employing the ORGE method. Once this method is used, the optimization of structures with multiple vertical metallizations becomes very efficient. The optimization problem is the next step after developing this efficient algorithm. Therefore, in the future works the optimization problems should be carried out. Also, the vertical metallizations used in this thesis are in one dielectric layer only. The formulation should be extended for vertical metallizations passing through two or more layers with different dielectric constants. Once this is completed the metallizations should be extended to any direction or in other words, the metallizations in the oblique directions should also be considered. With the development of this formulation, the usage of spatial-domain MoM for MPIE would extend its generality.

Appendix A

DERIVATION OF SPECTRAL-DOMAIN GREEN'S FUNCTIONS FOR VECTOR AND SCALAR POTENTIALS IN PLANARLY STRATIFIED MEDIA

A typical planarly stratified medium is demonstrated in Fig. A.1, where it is assumed that all layers are isotropic and homogeneous, and extend to infinity in the transverse directions (xy -plane in this case). In addition, horizontal and vertical electric dipoles are considered as the sources located in an arbitrary layer i , whose thickness, relative permittivity and permeability are denoted by d_i , ϵ_i and μ_i , respectively. Also note that the origin of the coordinate system is always set to the bottom of the source layer, and the distance of the source from the origin is denoted by z' .

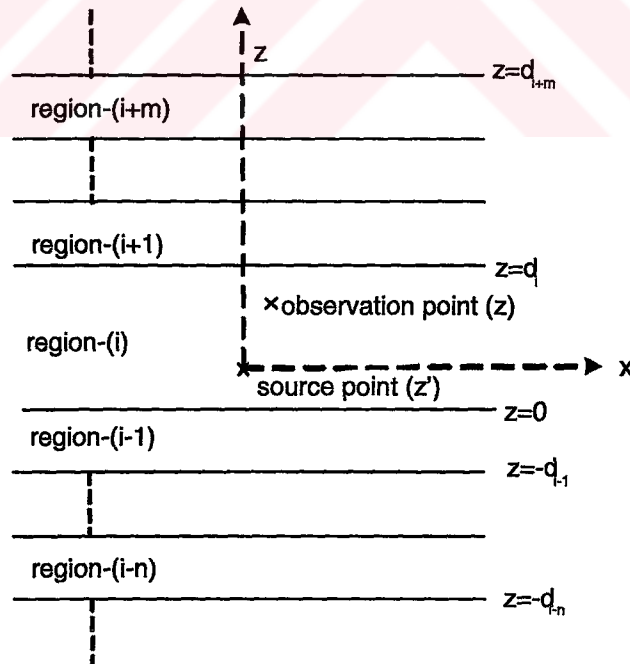


Figure A.1: A general source embedded in a multilayer medium

A.1 Derivation of TE and TM wave components for the fields in planarly stratified media

First of all, to find the Green's function of the vector wave equation for the electric field in this geometry, it would be instructive to start with the source-free case and then apply the appropriate boundary conditions. Using the Maxwell's equations in homogeneous, isotropic and source-free region, the following vector wave equations are obtained:

$$\mu \nabla \times \frac{1}{\mu} \nabla \times \mathbf{E}(\mathbf{r}) - \omega^2 \mu \epsilon \mathbf{E}(\mathbf{r}) = 0 \quad (\text{A.1})$$

$$\epsilon \nabla \times \frac{1}{\epsilon} \nabla \times \mathbf{H}(\mathbf{r}) - \omega^2 \mu \epsilon \mathbf{H}(\mathbf{r}) = 0 \quad (\text{A.2})$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields intensities, respectively. Since for this structure and other planarly layered medium, the electrical properties change only in one direction, which is the z -direction in this case, the vector wave equations need not be solved in their full forms. In fact, in the source-free case, the vector wave equations can be reduced to two scalar equations representing transverse electric (TE) and transverse magnetic (TM) waves to z -direction which are decoupled from each other. As it is well known for a TE wave, electric field can only be in the transverse direction whereas for a TM wave, magnetic field is only in the transverse direction. In other words, z component of the electric field for a TE wave to z -direction is zero, similarly $H_z = 0$ for a TM wave to z -direction. The reflection of TE and TM waves in a planarly layered media is already well-known and therefore they can be used in the solution of these two decoupled scalar wave equations. But how can one be sure that these waves exist and they are decoupled? To prove this, one can first assume that TE- z and TM- z waves exist and then derive the scalar wave equations for both cases. If the solutions of these two equations are shown to be independent of each other, in other words, the two wave forms are decoupled, then the existence of such waves are guaranteed. So, one can start the procedure by choosing the electric and magnetic field components appropriate for the TE and TM waves. Since for TE- z wave, the electric field is in the transverse direction (xy plane), it can be assumed to be in the y -direction only, because the coordinate system can always be rotated about the z -axis to align the polarization of the electric field in the y -direction. Similarly, for TM- z formulation, only the y component of the magnetic field is assumed to exist. Therefore the electric and magnetic fields can be

written as follows:

$$\mathbf{E}(\mathbf{r}) = \hat{y}E_y(\mathbf{r}) \quad (\text{A.3})$$

$$\mathbf{H}(\mathbf{r}) = \hat{y}H_y(\mathbf{r}) \quad (\text{A.4})$$

where $\hat{\alpha}$ represents the unit vector in the α direction. After substituting Eqs.(A.3-A.4) into Eqs.(A.1-A.2) and simplifications, the following scalar wave equations for the electric and magnetic fields are obtained:

$$\left[\frac{\partial^2}{\partial x^2} + \mu(z) \frac{\partial}{\partial z} \mu^{-1}(z) \frac{\partial}{\partial z} + \omega^2 \mu \epsilon \right] E_y = 0 \quad (\text{A.5})$$

$$\left[\frac{\partial^2}{\partial x^2} + \epsilon(z) \frac{\partial}{\partial z} \epsilon^{-1}(z) \frac{\partial}{\partial z} + \omega^2 \mu \epsilon \right] H_y = 0 \quad (\text{A.6})$$

Now it has been shown that, for a source-free and planar medium, the vector wave equations for electric and magnetic fields can be reduced down to two scalar equations, as given in Eqs.(A.5-A.6). However, nothing has been done to demonstrate that these two scalar wave equations are independent, in other words, decoupled. This can be demonstrated by finding the field components available for TE-z and TM-z formulations and by showing that they form mutually exclusive vector components. For TE-z, the available vector components are as follows:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \hat{y} E_y(x, z) \\ \nabla \times \hat{y} E_y &= -j\omega\mu \mathbf{H}(x, z) \Rightarrow \mathbf{H}(x, z) = \hat{x} H_x(x, z) + \hat{z} H_z(x, z) \\ \nabla \times (\hat{x} H_x + \hat{z} H_z) &= j\omega\epsilon \mathbf{E}(x, z) \Rightarrow \mathbf{E}(x, z) = \hat{y} E_y(x, z) \end{aligned} \quad (\text{A.7})$$

Similarly for TM-z the available field components are:

$$\begin{aligned} \mathbf{H}(\mathbf{r}) &= \hat{y} H_y(x, z) \\ \nabla \times \hat{y} H_y &= j\omega\epsilon \mathbf{E}(x, z) \Rightarrow \bar{\mathbf{E}}(x, z) = \hat{x} E_x(x, z) + \hat{z} E_z(x, z) \\ \nabla \times (\hat{x} E_x + \hat{z} E_z) &= -j\omega\mu \mathbf{H}(x, z) \Rightarrow \mathbf{H}(x, z) = \hat{y} H_y(x, z) \end{aligned} \quad (\text{A.8})$$

As a summary, the field components for TE-z are E_y , H_x and H_z whereas for TM-z they are H_y , E_x and E_z . As it can be easily seen, the field components of two formulations are mutually exclusive, hence decoupled from each other.

So it has been shown here that the vector wave equation is reduced to two independent scalar wave equations. Further simplification to these scalar wave equations is also possible

by considering the salient feature of the geometry. Since the geometry, as depicted in Fig. A.1, is translationally invariant on xy -plane, in other words, there is no boundary or discontinuity on xy plane that would force the field to vary, the magnitude of the fields should be constant while there should be a phase progress with x like $e^{\pm jk_x x}$. Consequently, due to the boundary condition on the tangential fields at the interfaces of the layers (E_y and H_y are tangential at the interfaces) the solution for all z values must have the same phase variation, $e^{\pm jk_x x}$, which is called as the phase matching condition. Therefore the two dimensional scalar wave equations are reduced down to one-dimensional scalar wave equations.

$$\left[\mu(z) \frac{\partial}{\partial z} \mu^{-1}(z) \frac{\partial}{\partial z} + \omega^2 \mu \epsilon - k_x^2 \right] E_y = 0 \quad (\text{A.9})$$

$$\left[\epsilon(z) \frac{\partial}{\partial z} \epsilon^{-1}(z) \frac{\partial}{\partial z} + \omega^2 \mu \epsilon - k_x^2 \right] H_y = 0 \quad (\text{A.10})$$

From these two equations, it can be observed that by writing the fields in terms of TE and TM wave components, all field components can be found by following the Maxwell's equations. Therefore in finding the spectral domain Green's functions due to a dipole, the electric field and magnetic field need to be written in such a way that TE and TM field components can easily be extracted.

A.2 Relationship between the Green's Functions of Scalar and Vector Potentials

As it is already known, in the MPIE formulation, the Green's functions of both vector and scalar potentials are used. Although these Green's functions are not as singular as the electric field Green's functions, they are not uniquely defined in stratified media [41, 55]. Hence, different sets of Green's functions for the vector and scalar potentials can be used to satisfy the same boundary conditions. The following components of the Green's function for the vector potential are commonly used and referred to as the traditional form:

$$\mathbf{G}^A = (\hat{x}\hat{x} + \hat{y}\hat{y}) G_{xx}^A + \hat{z}\hat{x} G_{zx}^A + \hat{z}\hat{y} G_{zy}^A + \hat{z}\hat{z} G_{zz}^A \quad (\text{A.11})$$

where $\mathbf{G}^A$ denotes the dyadic form of the Green's function for the vector potential. The scalar potential due to a single point charge associated with a time-harmonic Hertzian dipole can be derived from the vector potential as in [41]. But even in [41], the scalar potentials

due to a point charge and dipole have been confused and used in a wrong fashion. Therefore to clarify this issue, the derivation of this fact is given here provided that the vector and scalar potentials can be written in terms of their Green's functions as follows:

$$\mathbf{A} = \int_V d\mathbf{r}' \mathbf{G}^A(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') \quad (\text{A.12})$$

$$\phi = \int_V d\mathbf{r}' G^q(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') \quad (\text{A.13})$$

where G^q is the Green's function due to a single point charge. Once these equations are substituted into Lorentz Gauge expression ($\nabla \cdot \mathbf{A} = -j\omega\mu\epsilon\phi$), the following equations are obtained:

$$\nabla \cdot \int_V d\mathbf{r}' \mathbf{G}^A(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') = -j\omega\mu\epsilon \int_V d\mathbf{r}' G^q(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}') \quad (\text{A.14})$$

By using the continuity equation and taking the divergence into the integral, the following expression is achieved:

$$\int_V d\mathbf{r}' [\nabla \cdot \mathbf{G}^A(\mathbf{r}, \mathbf{r}')] \cdot \mathbf{J}(\mathbf{r}') = -j\omega\mu\epsilon \int_V d\mathbf{r}' G^q(\mathbf{r}, \mathbf{r}') \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}')}{-j\omega} \quad (\text{A.15})$$

Since $\nabla' \cdot [G^q(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}')] = G^q(\mathbf{r}, \mathbf{r}') [\nabla' \cdot \mathbf{J}(\mathbf{r}')] + \nabla' G^q(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')$, the above expression can be written as follows:

$$\int_V d\mathbf{r}' [\nabla \cdot \mathbf{G}^A(\mathbf{r}, \mathbf{r}')] \cdot \mathbf{J}(\mathbf{r}') = \mu\epsilon \int_V d\mathbf{r}' \nabla' \cdot [G^q(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}')] - \mu\epsilon \int_V d\mathbf{r}' \nabla' G^q(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') \quad (\text{A.16})$$

By using the divergence theorem, the first integral term on the left hand side can be written as:

$$\oint_S d\mathbf{s}' \cdot [G^q(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}')] = 0 \quad (\text{A.17})$$

where S is the surface of the volume V . Since there cannot be any current normal to the surface of the volume, this integral is zero. Hence, finally, the following expression is obtained:

$$\int_V d\mathbf{r}' [\nabla \cdot \mathbf{G}^A(\mathbf{r}, \mathbf{r}')] \cdot \mathbf{J}(\mathbf{r}') = -\mu\epsilon \int_V d\mathbf{r}' \nabla' G^q(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') \quad (\text{A.18})$$

Since the integrands are equal to each other, the following expression can be written for the Green's function of the scalar potential due to a single point charge:

$$\frac{j\omega}{k^2} \nabla \cdot \mathbf{G}^A = \frac{1}{j\omega} \nabla' G^q \quad (\text{A.19})$$

However, since the scalar potential depends on the chosen form of the magnetic vector potential in a layered medium, it is not unique for the traditional choice of the vector potential, that is, the scalar potential associated with a vertical dipole is different from that of a horizontal dipole when there are at least two media with different dielectric constants. Therefore, in general, a single scalar potential G^q satisfying (A.19) does not exist when the traditional form of the vector Green's function $\mathbf{G}^A$ is used. The proof for the non-uniqueness of the scalar potential Green's functions is given in Appendix B for further reference.

A.3 Derivation of the Green's Functions

Now after having found the relations between the Green's functions of the vector and scalar potentials, it is time to find them. To be able to find the spectral-domain Green's functions, first the field components of a dipole in the $\hat{\alpha}$ direction in a homogeneous and unbounded region should be written and then by using the TE and TM waves components of the fields the effects of the reflections from the layers should be accounted for. So it is better to start with the MPIE representation as given below:

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\phi \quad (\text{A.20})$$

Using the Lorentz Gauge, the above expression is written as:

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla \left(\frac{\nabla \cdot \mathbf{A}}{-j\omega\mu\epsilon} \right) \quad (\text{A.21})$$

which is equal to

$$\mathbf{E} = -j\omega \left[\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2} \right] \cdot \mathbf{A} \quad (\text{A.22})$$

where $\bar{\mathbf{I}} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$ is an idem factor. Since the vector potential for a point current source directed in $\hat{\alpha}$ direction, $\mathbf{J} = \hat{\alpha}Il\delta(\mathbf{r} - \mathbf{r}')$, in an homogeneous and unbounded region is given as follows

$$\mathbf{A}(\mathbf{r}, \mathbf{r}') = \hat{\alpha}\mu Il \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} \quad (\text{A.23})$$

where Il is the current moment, the electric and the magnetic fields can be written as follows:

$$\mathbf{E}(\mathbf{r}, \mathbf{r}') = -j\omega\mu \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2} \right) \cdot \hat{\alpha}Il \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|} \quad (\text{A.24})$$

$$\mathbf{H}(\mathbf{r}, \mathbf{r}') = \nabla \times \hat{\alpha}Il \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|} \quad (\text{A.25})$$

Note that $\mathbf{H}$ field is found from $\mu\mathbf{H} = \nabla \times \mathbf{A}$. From these equations, the TM and TE field components of the dipole can be easily derived. The spherical wave behavior in (A.24) and (A.25) must be modified since the dipole is in a layered medium. This can be achieved by expanding the spherical wave terms in terms of plane waves using the Weyl identity (A.26).

$$\frac{e^{-jk|r-r'|}}{|r-r'|} = -\frac{j}{2\pi} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \quad (\text{A.26})$$

where $k_x^2 + k_y^2 + k_z^2 = k^2$. Then, the reflections due to the planar boundaries of the stratified medium, can be easily accounted for the field expressions. It should also be noted that since the medium is translationally invariant in the xy plane, the phase matching condition requires that k_x and k_y are the same in all layers.

The derivation of the spectral-domain Green's functions starts by first deriving the Green's functions in the source layers by considering the direct wave, and the reflected waves from the boundaries, as shown in the next sections. Then, Green's functions in any other layer are obtained using an iterative algorithm applied to each TE and TM component of the Green's functions in the source layer.

A.3.1 Horizontal Electric Dipole (HED)

Derivation of the spectral-domain Green's functions for an HED pointing in the x direction in layer- i for a homogeneous medium starts with writing E_z (TM-z) and H_z (TE-z) components of the fields in the source layer. Since $\hat{\alpha} = \hat{x}$, Eqs.(A.24-A.25) can be written as:

$$\mathbf{E}(\mathbf{r}, \mathbf{r}') = -j\omega\mu \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2} \right) \cdot \hat{x} Il \frac{e^{-jk|r-r'|}}{4\pi|r-r'|} \quad (\text{A.27})$$

$$\mathbf{H}(\mathbf{r}, \mathbf{r}') = \nabla \times \hat{x} Il \frac{e^{-jk|r-r'|}}{4\pi|r-r'|} \quad (\text{A.28})$$

After simplifying the above equations, E_z and H_z can be written as

$$E_z = -\frac{jIl}{4\pi\omega\epsilon} \frac{\partial^2}{\partial z \partial x} \frac{e^{-jk|r-r'|}}{|r-r'|} \quad (\text{A.29})$$

$$H_z = -\frac{Il}{4\pi} \frac{\partial}{\partial y} \frac{e^{-jk|r-r'|}}{|r-r'|} \quad (\text{A.30})$$

Substituting the Weyl identity (Eq.(A.26)) into E_z and H_z expressions results the following:

$$E_z = -\frac{jIl}{4\pi\omega\epsilon} \frac{\partial^2}{\partial z \partial x} \left(\frac{-j}{2\pi} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \right) \quad (\text{A.31})$$

$$H_z = -\frac{Il}{4\pi} \frac{\partial}{\partial y} \left(\frac{-j}{2\pi} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \right) \quad (\text{A.32})$$

After a little bit of manipulation, the spherical wave terms in Eqs.(A.29) and (A.30) are expressed as an integral summation of the plane waves propagating in all directions as follows:

$$E_z = \frac{Il}{8\pi^2 \omega \epsilon} \iint_{-\infty}^{\infty} dk_x dk_y k_x e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|} \quad (\text{A.33})$$

$$H_z = \frac{Il}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y k_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \quad (\text{A.34})$$

These expressions are valid in a homogeneous and unbounded medium. Therefore to obtain the fields in the layered medium along the z -direction, the z -dependence of the fields in the source region (layer- i) is written as the sum of the reflected waves from the boundaries as

$$E_z = \frac{Il}{8\pi^2 \omega \epsilon_i} \iint_{-\infty}^{\infty} dk_x dk_y k_x e^{-jk_x(x-x')-jk_y(y-y')} F_{TM} \quad (\text{A.35})$$

$$H_z = \frac{Il}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y k_y \frac{e^{-jk_x(x-x')-jk_y(y-y')}}{k_{z_i}} F_{TE} \quad (\text{A.36})$$

where

$$F_{TE} = e^{-jk_{z_i}|z-z'|} + A_h^e e^{jk_{z_i}(z-z')} + C_h^e e^{-jk_{z_i}(z-z')} \quad (\text{A.37})$$

$$F_{TM} = e^{-jk_{z_i}|z-z'|} + B_h^e e^{jk_{z_i}(z-z')} + D_h^e e^{-jk_{z_i}(z-z')} \quad (\text{A.38})$$

and A_h^e , B_h^e , C_h^e and D_h^e are the coefficients of the down-going and up-going waves. Since for TM- z wave, the z component of the electric field exists and the z component of the magnetic field is zero, F_{TM} is used as the notation in the spectral-domain representation of E_z field. Similarly for H_z , F_{TE} is used. Note that source is located at point z' as shown in Fig. A.1. Recall that these coefficients were already given in Eqs.(2.9-2.12), but for the sake of completeness, the procedure followed to find these coefficients is demonstrated here once more for B_h^e and D_h^e .

Since the origin of the coordinate system is defined at the bottom of the source layer for this derivation (see Fig. A.1), the down-going wave in the source layer is the consequence of the reflection of the up-going wave at $z = d_i$, similarly, the up-going wave is the consequence

of the reflection of the down-going wave at $z = 0$. Hence,

$$B_h^e e^{jk_{z_i}(d_i - z')} = \tilde{R}_{TM}^{i,i+1} \left(e^{-jk_{z_i}(d_i - z')} + D_h^e e^{-jk_{z_i}(d_i - z')} \right) \quad (\text{A.39})$$

$$D_h^e e^{jk_{z_i}z'} = \tilde{R}_{TM}^{i,i-1} \left(-e^{-jk_{z_i}z'} + B_h^e e^{-jk_{z_i}z'} \right) \quad (\text{A.40})$$

where $\tilde{R}_{TM}$ is the generalized reflection coefficient at the boundaries [21]. By solving (A.39) and (A.40) simultaneously, the unknown coefficients B_h^e and D_h^e are found as

$$B_h^e = \frac{\tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i}(d_i - z')} - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i}d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i}d_i}} \quad (\text{A.41})$$

$$D_h^e = \frac{-\tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i}z'} + \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i}d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i}d_i}} \quad (\text{A.42})$$

Similarly by using the same approach, the other coefficients A_h^e and C_h^e are found as follows:

$$A_h^e = \frac{\tilde{R}_{TE}^{i,i+1} e^{-j2k_{z_i}(d_i - z')} + \tilde{R}_{TE}^{i,i+1} \tilde{R}_{TE}^{i,i-1} e^{-j2k_{z_i}d_i}}{1 - \tilde{R}_{TE}^{i,i+1} \tilde{R}_{TE}^{i,i-1} e^{-j2k_{z_i}d_i}} \quad (\text{A.43})$$

$$C_h^e = \frac{\tilde{R}_{TE}^{i,i-1} e^{-j2k_{z_i}z'} + \tilde{R}_{TE}^{i,i-1} \tilde{R}_{TE}^{i,i+1} e^{-j2k_{z_i}d_i}}{1 - \tilde{R}_{TE}^{i,i+1} \tilde{R}_{TE}^{i,i-1} e^{-j2k_{z_i}d_i}} \quad (\text{A.44})$$

Now at this point, E_z and H_z are known and it is time to find the Green's functions by using these field expressions. The first step for the derivation of the vector Green's functions is to write the horizontal and vertical components of the vector potential in terms of E_z and H_z . Since for an isotropic and homogeneous medium $\mathbf{B} = \mu\mathbf{H} = \nabla \times \mathbf{A}$, the following equation can be written for A_x :

$$\hat{z} \mu_i H_z = \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \times \hat{x} A_x \quad (\text{A.45})$$

which results in

$$\mu_i H_z = -\frac{\partial}{\partial y} A_x \quad (\text{A.46})$$

Then the vector Green's function G_{xx}^A is derived by proceeding as follows:

$$\begin{aligned} A_x &= -\mu_i \int H_z dy \\ &= \frac{Il}{j8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x') - jk_y(y-y')}}{k_{z_i}} F_{TE} \end{aligned} \quad (\text{A.47})$$

$$\Rightarrow \tilde{G}_{xx}^A = \frac{\mu_i}{j2k_{z_i}} \left\{ e^{-jk_{z_i}|z-z'|} + A_h^e e^{jk_{z_i}(z-z')} + C_h^e e^{-jk_{z_i}(z-z')} \right\} \quad (\text{A.48})$$

Similarly, to find the vector Green's function G_{zx}^A , the vector potential A_z should be written in terms of E_z and H_z . Using $\mu\mathbf{H} = \nabla \times \mathbf{A}$ once more, the following relation is obtained:

$$\mu_i H_x = \frac{\partial}{\partial y} A_z \quad (\text{A.49})$$

Since H_x can be written in terms of E_z and H_z , the vector Green's function G_{zx}^A is derived by using A_z as follows:

$$A_z = \mu_i \int H_x dy = \mu_i \int \frac{1}{k_\rho^2} \left[\frac{\partial^2}{\partial z \partial x} H_z + j\omega\epsilon \frac{\partial}{\partial y} E_z \right] dy \quad (\text{A.50})$$

$$\begin{aligned} &= \frac{\mu_i Il}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \left\{ \frac{k_x}{k_{zi}k_\rho^2} \frac{\partial}{\partial z} F_{TE} + \frac{jk_x}{k_\rho^2} F_{TM} \right\} \\ \Rightarrow \tilde{G}_{zx}^A &= -\frac{\mu_i k_x}{2jk_\rho^2} \left\{ (A_h^e + B_h^e) e^{jk_{zi}(z-z')} + (D_h^e - C_h^e) e^{-jk_{zi}(z-z')} \right\} \end{aligned} \quad (\text{A.51})$$

The derivation of the Green's function of the scalar potential ϕ_q due to a single point charge use the result given in Eq. (A.19) which can be written as follows:

$$\nabla' G^q = \frac{-1}{\mu\epsilon} \nabla \cdot \mathbf{G}^A \quad (\text{A.52})$$

Since for this case, the x and z components of the vector potential Green's function exist and the dipole is in the x -direction, Eq. (A.52) can be written as:

$$\frac{\partial}{\partial x'} G_x^q = \frac{-1}{\mu_i \epsilon_i} \left[\frac{\partial}{\partial x} G_{xx}^A + \frac{\partial}{\partial z} G_{zx}^A \right] \quad (\text{A.53})$$

Note that the above Green's functions are in the spatial-domain. Therefore to find their spectral-domain counterparts, the plane wave expansions of the Green's functions are substituted in the above equation and the following expression is achieved:

$$\begin{aligned} &\frac{\partial}{\partial x'} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_x^q = \\ &-\frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial x} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_{xx}^A - \frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial z} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_{zx}^A \end{aligned} \quad (\text{A.54})$$

Because the spatial-domain Green's functions are written in terms of their spectral-domain representations in the above expressions, the coefficient $-j/8\pi^2$ is required for all the double integral terms. But since all the terms have the same coefficient, they cancel each other and therefore they are not shown in the above expression. The following facts can be used for rearranging the terms in the above expression:

$$\begin{aligned} \frac{\partial}{\partial x} &= -\frac{\partial}{\partial x'} \\ e^{-jk_x(x-x')} &= \frac{1}{jk_x} \frac{\partial}{\partial x'} e^{-jk_x(x-x')} \end{aligned}$$

Hence, the right hand side of Eq. (A.54) can be rearranged as follows:

$$= -\frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial x'} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} (-\tilde{G}_{xx}^A) - \frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial x'} \iint_{-\infty}^{\infty} dk_x dk_y \frac{1}{jk_x} e^{-jk_x(x-x')-jk_y(y-y')} \frac{\partial}{\partial z} \tilde{G}_{zx}^A \quad (\text{A.55})$$

By comparing the above equation with the left hand side of Eq. (A.54) the following expression is obtained for the spectral-domain Green's function of the scalar potential due to a single charge:

$$\tilde{G}_x^q = -\frac{1}{\mu_i \epsilon_i} \left[-\tilde{G}_{xx}^A + \frac{1}{jk_x} \frac{\partial}{\partial z} \tilde{G}_{zx}^A \right] \quad (\text{A.56})$$

Substituting $\tilde{G}_{xx}^A$ and $\tilde{G}_{zx}^A$ from Eqs. (A.48) and (A.51) into Eq. (A.56), the following expression is obtained:

$$\begin{aligned} \tilde{G}_x^q &= -\frac{1}{\mu_i \epsilon_i} \left[-\frac{\mu_i}{j2k_{z_i}} \left\{ e^{-jk_{z_i}|z-z'|} + A_h^e e^{jk_{z_i}(z-z')} + C_h^e e^{-jk_{z_i}(z-z')} \right\} \right] \\ &\quad - \frac{1}{\mu_i \epsilon_i} \left[\frac{1}{jk_x} \left(\frac{-\mu_i k_x}{2jk_\rho^2} \right) \left\{ (A_h^e + B_h^e) e^{jk_{z_i}(z-z')} + (D_h^e - C_h^e) e^{-jk_{z_i}(z-z')} \right\} \right] \quad (\text{A.57}) \end{aligned}$$

Once the terms in the above equation are rearranged, the Green's function for scalar potential due to a single point charge G_x^q is easily found as:

$$\begin{aligned} \Rightarrow \tilde{G}_x^q &= \frac{-j}{2\epsilon_i k_\rho^2} \left\{ \frac{k_\rho^2}{k_{z_i}} e^{-jk_{z_i}|z-z'|} + \frac{k_i^2 A_h^e + k_{z_i}^2 B_h^e}{k_{z_i}} e^{jk_{z_i}(z-z')} \right. \\ &\quad \left. + \frac{k_i^2 C_h^e - k_{z_i}^2 D_h^e}{k_{z_i}} e^{-jk_{z_i}(z-z')} \right\} \quad (\text{A.58}) \end{aligned}$$

A.3.2 Vertical Electric Dipole (VED)

Similarly the procedure for deriving the spectral-domain Green's function for a VED starts by writing the E_z and H_z components of the fields. Since $\hat{\alpha} = \hat{z}$ for a VED, Eqs. (A.24-A.25) are written as

$$\mathbf{E}(\mathbf{r}, \mathbf{r}') = -j\omega\mu \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2} \right) \cdot \hat{z} Il \frac{e^{-jk|r-r'|}}{4\pi|r-r'|} \quad (\text{A.59})$$

$$\mathbf{H}(\mathbf{r}, \mathbf{r}') = \nabla \times \hat{z} Il \frac{e^{-jk|r-r'|}}{4\pi|r-r'|} \quad (\text{A.60})$$

Therefore, E_z and H_z are found as follows:

$$E_z = -\frac{jIl\omega\mu}{4\pi k^2} \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \frac{e^{-jk|r-r'|}}{|r-r'|} \quad (\text{A.61})$$

$$H_z = 0 \quad (\text{A.62})$$

Then the spherical wave term in Eq.(A.61) is written as the summation of plane waves propagating in all directions using the Weyl identity and E_z becomes

$$\begin{aligned} E_z &= -\frac{jIl\omega\mu}{4\pi k^2} \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \left\{ \frac{-j}{2\pi} \iint_{-\infty}^{\infty} dk_x dk_y \frac{e^{-jk_x(x-x')-jk_y(y-y')-jk_z|z-z'|}}{k_z} \right\} \\ &= -\frac{Il\omega\mu}{8\pi^2 k^2} \iint_{-\infty}^{\infty} dk_x dk_y \left(\frac{k^2}{k_{z_i}} - k_{z_i} \right) e^{-jk_x(x-x')-jk_y(y-y')-jk_{z_i}|z-z'|} \end{aligned} \quad (\text{A.63})$$

This expression is valid in an homogeneous and unbounded medium. Therefore, the field expression in a layered medium is obtained by considering the reflected waves as

$$E_z = \frac{Il}{8\pi^2 \omega \epsilon_i} \iint_{-\infty}^{\infty} dk_x dk_y k_x e^{-jk_x(x-x')-jk_y(y-y')} F_{TM} \quad (\text{A.64})$$

where

$$F_{TM} = e^{-jk_{z_i}|z-z'|} + A_v^e e^{-jk_{z_i}(z-z')} + B_v^e e^{jk_{z_i}(z-z')} \quad (\text{A.65})$$

and A_v^e and B_v^e are the coefficients of down-going and up-going waves. By applying the boundary conditions at the interface of the layers, the unknown coefficients A_v^e and B_v^e are found as

$$A_v^e = \frac{\tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} z'} + \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i} d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}} \quad (\text{A.66})$$

$$B_v^e = \frac{\tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i}(d_i-z')} + \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}} \quad (\text{A.67})$$

Now, to find the Green's functions due to VED, the vector and scalar potentials should be written in terms of E_z . As it was already showed for HED case, the vector potential A_z is used to find G_{zz}^A as follows:

$$\begin{aligned} A_z &= \mu_i \int H_x dy = \mu_i \int \frac{1}{k_\rho^2} \left[j\omega\epsilon \frac{\partial}{\partial y} E_z \right] dy \\ &= j \frac{\mu_i Il}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x x - jk_y y} \left(\frac{k^2}{k_{z_i} k_\rho^2} - \frac{k_{z_i}}{k_\rho^2} \right) F_{TM} \end{aligned} \quad (\text{A.68})$$

$$\Rightarrow \tilde{G}_{zz}^A = \frac{\mu_i}{2jk_{z_i}} \left\{ e^{-jk_{z_i}|z-z'|} + A_v^e e^{-jk_{z_i}(z-z')} + B_v^e e^{jk_{z_i}(z-z')} \right\} \quad (\text{A.69})$$

Similarly, to find the Green's function of the scalar potential ϕ_q due to a single point charge associated with a VED, Eq. (A.52) is written as follows:

$$\frac{\partial}{\partial z'} G_z^q = \frac{-1}{\mu_i \epsilon_i} \left[\frac{\partial}{\partial z} G_{zz}^A \right] \quad (\text{A.70})$$

The spatial-domain Green's functions in the above equation are written in terms of their spectral-domain representations and the following expression is obtained:

$$\frac{\partial}{\partial z'} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_z^q = -\frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial z} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_{zz}^A \quad (\text{A.71})$$

Once $\tilde{G}_{zz}^A$ given in Eq.(A.69) is substituted, the left hand side of the above expression can be written as follows

$$= -\frac{1}{\mu_i \epsilon_i} \frac{\partial}{\partial z} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \frac{\mu_i}{2jk_{z_i}} F_{TM} \quad (\text{A.72})$$

Note that for this expression, the z derivative can be taken over F_{TM} , since it is the only term that has z variable. Therefore the above equation now becomes:

$$= -\frac{1}{\mu_i \epsilon_i} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \frac{\mu_i}{2jk_{z_i}} \frac{\partial}{\partial z} F_{TM} \quad (\text{A.73})$$

The explicit forms of the coefficients A_v^e and B_v^e which are given in Eqs.(A.66) and (A.67) are substituted into F_{TM} and after arranging the terms it can be written as:

$$\begin{aligned} F_{TM} = & e^{-jk_{z_i}|z-z'|} + \tilde{R}_{TM}^{i,i-1} M_i^{TM} e^{-jk_{z_i}(z+z')} + \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} M_i^{TM} e^{-jk_{z_i}(z-z'+2d_i)} \\ & + \tilde{R}_{TM}^{i,i+1} M_i^{TM} e^{-jk_{z_i}(z+z'-2d_i)} + \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} M_i^{TM} e^{-jk_{z_i}(z-z'-2d_i)} \end{aligned} \quad (\text{A.74})$$

where $\tilde{R}_{TM}$ is the generalized reflection coefficients and

$$M_i^{TM} = \left[1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-jk_{z_i} 2d_i} \right]^{-1} \quad (\text{A.75})$$

The z derivative of F_{TM} is given as follows

$$\begin{aligned} \frac{\partial F_{TM}}{\partial z} = & \frac{\partial}{\partial z} e^{-jk_{z_i}|z-z'|} + \tilde{R}_{TM}^{i,i-1} M_i^{TM} \frac{\partial}{\partial z} e^{-jk_{z_i}(z+z')} + \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} M_i^{TM} \frac{\partial}{\partial z} e^{-jk_{z_i}(z-z'+2d_i)} \\ & + \tilde{R}_{TM}^{i,i+1} M_i^{TM} \frac{\partial}{\partial z} e^{-jk_{z_i}(z+z'-2d_i)} + \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} M_i^{TM} \frac{\partial}{\partial z} e^{-jk_{z_i}(z-z'-2d_i)} \end{aligned} \quad (\text{A.76})$$

which can also be written in terms of z' derivative as

$$\begin{aligned} \frac{\partial F_{TM}}{\partial z} = & -\frac{\partial}{\partial z'} e^{-jk_{z_i}|z-z'|} + \tilde{R}_{TM}^{i,i-1} M_i^{TM} \frac{\partial}{\partial z'} e^{-jk_{z_i}(z+z')} \\ & - \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} M_i^{TM} \frac{\partial}{\partial z'} e^{-jk_{z_i}(z-z'+2d_i)} + \tilde{R}_{TM}^{i,i+1} M_i^{TM} \frac{\partial}{\partial z'} e^{-jk_{z_i}(z+z'-2d_i)} \\ & - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} M_i^{TM} \frac{\partial}{\partial z'} e^{-jk_{z_i}(z-z'-2d_i)} \end{aligned} \quad (\text{A.77})$$

Hence substituting Eq.(A.77) into Eq.(A.73) and equating to the right hand side of Eq.(A.71), the following expression is obtained:

$$\begin{aligned} \frac{\partial}{\partial z'} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \tilde{G}_z^q = \\ -\frac{1}{\mu_i \epsilon_i} \iint_{-\infty}^{\infty} dk_x dk_y e^{-jk_x(x-x')-jk_y(y-y')} \frac{\mu_i}{2jk_{z_i}} \frac{\partial}{\partial z'} F_{TM_{new}} \end{aligned} \quad (\text{A.78})$$

where

$$\begin{aligned} F_{TM_{new}} = & -e^{-jk_{z_i}|z-z'|} + \tilde{R}_{TM}^{i,i-1} M_i^{TM} e^{-jk_{z_i}(z+z')} - \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} M_i^{TM} e^{-jk_{z_i}(z-z'+2d_i)} \\ & + \tilde{R}_{TM}^{i,i+1} M_i^{TM} e^{-jk_{z_i}(z+z'-2d_i)} - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} M_i^{TM} e^{-jk_{z_i}(z-z'-2d_i)} \end{aligned} \quad (\text{A.79})$$

After simplifications, $F_{TM_{new}}$ can be written as

$$F_{TM_{new}} = -\left(e^{-jk_{z_i}|z-z'|} + C_v^e e^{-jk_{z_i}(z-z')} + D_v^e e^{jk_{z_i}(z-z')}\right) \quad (\text{A.80})$$

where C_v^e and D_v^e are

$$C_v^e = \frac{-\tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} z'} + \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i} d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}} \quad (\text{A.81})$$

$$D_v^e = \frac{-\tilde{R}_{TM}^{i,i+1} e^{-j2k_{z_i}(d_i-z')} + \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}}{1 - \tilde{R}_{TM}^{i,i+1} \tilde{R}_{TM}^{i,i-1} e^{-j2k_{z_i} d_i}} \quad (\text{A.82})$$

Once Eq.(A.78) is analyzed, it can be seen that the scalar potential Green's function can be written as follows:

$$\tilde{G}_z^q = \frac{-1}{2j\epsilon_i k_{z_i}} F_{TM_{new}} \quad (\text{A.83})$$

and therefore, the Green's function for the scalar potential due to a single charge can be found as follows:

$$\Rightarrow \tilde{G}_z^q = \frac{1}{2j\epsilon_i k_{z_i}} \left\{ e^{-jk_{z_i}|z-z'|} + C_v^e e^{-jk_{z_i}(z-z')} + D_v^e e^{jk_{z_i}(z-z')} \right\} \quad (\text{A.84})$$

The above derivations of the Green's functions for HED and VED are for the dipole located in the x -direction. The horizontal Green's functions for the y -oriented dipoles can be obtained simply by setting $\tilde{G}_{yy}^A = \tilde{G}_{xx}^A$, $\tilde{G}_{zy}^A/k_y = \tilde{G}_{zx}^A/k_x$, and $\tilde{G}_y^{qe} = \tilde{G}_x^{qe}$.

Note also that these Green's functions are for the case where the observation and source layers are the same. But when the observation layer is different from the source layer, the Green's functions should be modified according to the appropriate boundary conditions.

The amplitudes of the up- and down-going waves in a layer different from the source layer are related to those in the adjacent layer by

$$A_j^- = A_{j+1}^- \frac{T_{j+1,j} e^{-j(k_{z_{j+1}} - k_{z_j})(z' + z_{i-j+1})}}{1 - R_{j,j+1} \tilde{R}_{j,j-1} e^{-jk_{z_j} 2d_j}} \quad (\text{A.85})$$

where A_j^- and A_{j+1}^- are the amplitudes of the down-going waves in layer j and $j + 1$, respectively, T is the transmission coefficient and where z_{i-j} is the distance between the lower boundary of the source layer- i and the lower boundary of layer- j . Similarly the amplitudes of the up-going waves in layer- j can be written as

$$A_j^+ = A_{j-1}^+ \frac{T_{j-1,j} e^{-j(k_{z_{j-1}} - k_{z_j})(z_{i-j-1} + d_i - z')}}{1 - R_{j,j-1} \tilde{R}_{j,j+1} e^{-jk_{z_j} 2d_j}} \quad (\text{A.86})$$

Therefore, starting from the source layer, the field expressions for any layer can be obtained iteratively [21].

Appendix B

NON-UNIQUENESS OF THE GREEN'S FUNCTIONS

The following components of the Green's function for the vector potential are commonly used and referred to as the traditional form:

$$\mathbf{G}^A = (\hat{x}\hat{x} + \hat{y}\hat{y}) G_{xx}^A + \hat{z}\hat{x}G_{zx}^A + \hat{z}\hat{y}G_{zy}^A + \hat{z}\hat{z}G_{zz}^A \quad (\text{B.1})$$

When this traditional form of the Green's functions are used, the scalar potential due to horizontal and vertical electric dipoles are not equal to each other, i.e. one needs to use both G_x^q and G_z^q at the same time which are not equal to each other. The proof of this fact is derived on a specific geometry which is shown in Fig. B.1. It is a three layer geometry

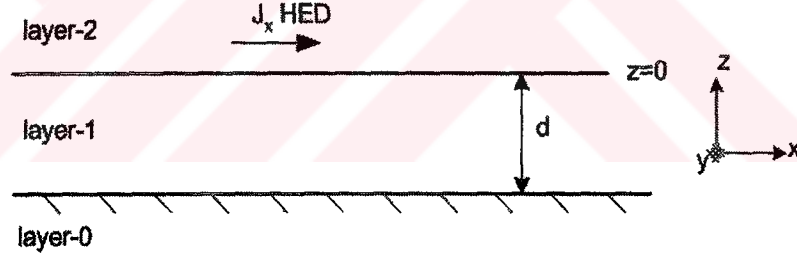


Figure B.1: Geometry of single layer geometry with a horizontal electric dipole

with the following properties: Bottom layer is PEC, middle layer is a dielectric material with dielectric constant of ϵ_r and thickness d , the top layer is free-space and finally the free-space dielectric layer interface is at $z = 0$.

The electric field at any point of the geometry can be represented in terms of vector and scalar potentials as follows:

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\phi \quad (\text{B.2})$$

The vector and scalar potential are related to each other via the following equation which is known as Lorentz Gauge:

$$\nabla \cdot \mathbf{A} = -j\omega\mu\epsilon\phi \quad (\text{B.3})$$

The vector and scalar potentials can be represented in terms of convolution integrals involving surface current density $\mathbf{J}$ and charge density ρ on the metallizations as

$$\mathbf{A} = \bar{\mathbf{G}}^A * \mathbf{J} \quad (\text{B.4})$$

$$\phi = G^q * \rho \quad (\text{B.5})$$

The charge density ρ in the scalar potential equation can be written in terms of surface current density $\mathbf{J}$ using the continuity equation $\nabla \cdot \mathbf{J} + j\omega\rho = 0$ as

$$\phi = G^q * \left(\frac{-1}{j\omega} \nabla \cdot \mathbf{J} \right) \quad (\text{B.6})$$

For the above geometry, if one can show that both A_x and A_z are needed to satisfy the boundary conditions for a HED, then the reason for using different G_x^q and G_z^q will be clear. First assume that there exists a horizontal surface current density in the x -direction, i.e. J_x , and it creates only horizontal component of the vector potential, A_x . The boundary conditions should be satisfied at the interface of the dielectric layer and the free-space at $z = 0$. The boundary condition states that the tangential electric fields in the dielectric layer, E_{x_1} and E_{y_1} , should be equal to tangential electric fields in the free-space, E_{x_2} and E_{y_2} , at the dielectric layer free-space interface. The field expressions at the interface are as follows:

$$E_{x_2} = -j\omega A_{x_2} - \frac{\partial}{\partial x} \phi_2 \quad (\text{B.7})$$

$$E_{y_2} = -\frac{\partial}{\partial y} \phi_2 \quad (\text{B.8})$$

$$E_{z_2} = -\frac{\partial}{\partial z} \phi_2 \quad (\text{B.9})$$

$$E_{x_1} = -j\omega A_{x_1} - \frac{\partial}{\partial x} \phi_1 \quad (\text{B.10})$$

$$E_{y_1} = -\frac{\partial}{\partial y} \phi_1 \quad (\text{B.11})$$

$$E_{z_1} = -\frac{\partial}{\partial z} \phi_1 \quad (\text{B.12})$$

When E_{y_1} and E_{y_2} are considered, it is concluded that the scalar potentials at $z = 0$, i.e. $\phi_1(z = 0)$ and $\phi_2(z = 0)$ should be equal to each other so that E_{y_1} and E_{y_2} can be equal to each other at the interface. From another point of view, if the scalar potentials at the interface were not equal to each other, the derivative terms in Eq.(B.9) and Eq.(B.12) for the z components of the electric field would result in infinite electric field E_z at the interface

which is unreasonable. Therefore, the conclusion that $\phi_2(z=0)$ and $\phi_1(z=0)$ are equal to each other is verified for this case.

For the tangential components of the electric field in the x -direction, i.e. E_{x1} and E_{x2} , to be equal to each other at the interface, the vector potentials in the x -direction at the interface should be equal to each other which can easily be seen from Eq.(B.10) and Eq.(B.11). Therefore, one can conclude that $A_{x1}(z=0) = A_{x2}(z=0)$.

In this case, by using the Lorentz Gauge Eq.(B.3), the scalar potentials in different layers can be written as:

$$\phi_2 = -\frac{1}{j\omega\mu\epsilon_0} \frac{\partial A_{x2}}{\partial x} \quad (\text{B.13})$$

$$\phi_1 = -\frac{1}{j\omega\mu\epsilon_1} \frac{\partial A_{x1}}{\partial x} \quad (\text{B.14})$$

Since it has already been stated that $A_{x1}(z=0)$ is equal to $A_{x2}(z=0)$, their derivatives should also be equal to each other. When this fact is considered by analyzing Eq.(B.13) and Eq.(B.14), the scalar potentials $\phi_2(z=0)$ and $\phi_1(z=0)$ seem to be unequal at the interface because of the different permittivities of the dielectric layer and free-space which are ϵ_1 and ϵ_0 , respectively. Therefore at this point, there is a contradiction between the conclusions reached for the scalar potentials. Consequently, the assumption made in the beginning of this discussion, i.e. J_x creates only A_x , does not satisfy the boundary conditions. In other words, it is not sufficient to have only A_x to satisfy the boundary conditions for a HED.

Since only A_x is not sufficient, now assume that horizontal surface current density in the x -direction, J_x , creates A_x and A_z at the same time. Note that this assumption conforms to rules of the traditional form for the Green's functions. If the electric field expressions at the free-space and dielectric layer interface are written for this case, it will be observed that the Eqs.(B.7-B.12) are still valid except for the expressions for the electric field in the z -direction which are written as

$$E_{z1} = -j\omega A_{z1} - \frac{\partial}{\partial z} \phi_1 \quad (\text{B.15})$$

$$E_{z2} = -j\omega A_{z2} - \frac{\partial}{\partial z} \phi_2 \quad (\text{B.16})$$

Similarly, from the expressions of the tangential electric field in the y -direction, the scalar potentials in different layers at the interface are found to be equal to each other, i.e. $\phi_1(z=0) = \phi_2(z=0)$. Using this fact in the other electric field expressions, it is concluded that

$A_{x_2} = A_{x_1}$ and $A_{z_2} = A_{z_1}$. Another point to note is that the z -derivatives of the scalar potentials at the interface must be discontinuous to ensure the continuity of the normal components of the $\mathbf{D}$ -field, or in other words, the discontinuity of the normal electric field components at the interface should be ensured, which is given as:

$$E_{z_1} = \frac{\epsilon_0}{\epsilon_1} E_{z_2} \quad (\text{B.17})$$

Therefore J_x is assumed to be at the interface for this case, then $\mathbf{H}$ -field which is $\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A}$ becomes

$$\begin{aligned} \mathbf{H} &= \frac{1}{\mu} \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \times (A_x \hat{x} + A_z \hat{z}) \\ \mathbf{H} &= \frac{1}{\mu} \left(-\frac{\partial}{\partial x} A_z + \frac{\partial}{\partial z} A_x \right) \hat{y} + \frac{1}{\mu} \frac{\partial}{\partial y} A_z \hat{x} - \frac{1}{\mu} \frac{\partial}{\partial y} A_x \hat{z} \end{aligned} \quad (\text{B.18})$$

From the boundary condition relation of the tangential $\mathbf{H}$ -fields, it is known that

$$H_{y_1} - H_{y_2} = J_x \quad (\text{B.19})$$

which can be evaluated as follows:

$$\frac{1}{\mu} \left(-\frac{\partial}{\partial x} A_{z_1} + \frac{\partial}{\partial z} A_{x_1} \right) \Big|_{z=0} - \frac{1}{\mu} \left(-\frac{\partial}{\partial x} A_{z_2} + \frac{\partial}{\partial z} A_{x_2} \right) \Big|_{z=0} = J_x \quad (\text{B.20})$$

and after arranging the terms, the above relation becomes

$$-\frac{\partial}{\partial x} A_{z_1} \Big|_{z=0} + \frac{\partial}{\partial x} A_{z_2} \Big|_{z=0} + \frac{\partial}{\partial z} A_{x_1} \Big|_{z=0} - \frac{\partial}{\partial z} A_{x_2} \Big|_{z=0} = \mu J_x \quad (\text{B.21})$$

In this expression the terms with z -derivatives are equal to each other because of the phase matching condition along the x -direction and the previously made conclusion $A_{z_2} = A_{z_1}$ and therefore they cancel each other. So the following expression is achieved:

$$\frac{\partial}{\partial z} A_{x_1} \Big|_{z=0} - \frac{\partial}{\partial z} A_{x_2} \Big|_{z=0} = \mu J_x \quad (\text{B.22})$$

To validate this conclusion one can check whether the scalar potentials are equal to each other at the interface. Note that this is one of the first conclusions made but it has to be verified and for the previous assumption there was a contradiction at this point. To check for the validity of this conclusion, the scalar potentials in the free-space and dielectric potentials are written below by using the Lorentz gauge.

$$\phi_2 = \frac{1}{-j\omega\mu\epsilon_0} \left(\frac{\partial A_{x_2}}{\partial x} + \frac{\partial A_{z_2}}{\partial z} \right) \Big|_{z=0} \quad (\text{B.23})$$

$$\phi_1 = \frac{1}{-j\omega\mu\epsilon_1} \left(\frac{\partial A_{x_1}}{\partial x} + \frac{\partial A_{z_1}}{\partial z} \right) \Big|_{z=0} \quad (\text{B.24})$$

Equating these two expressions gives

$$\frac{1}{\epsilon_0} \frac{\partial A_{z_2}}{\partial z} \Big|_{z=0} - \frac{1}{\epsilon_1} \frac{\partial A_{z_1}}{\partial z} \Big|_{z=0} = \frac{1}{\epsilon_1} \frac{\partial A_{x_1}}{\partial x} \Big|_{z=0} - \frac{1}{\epsilon_0} \frac{\partial A_{x_2}}{\partial x} \Big|_{z=0} \quad (\text{B.25})$$

Since $\frac{\partial A_{x_2}}{\partial x} \Big|_{z=0} = \frac{\partial A_{x_1}}{\partial x} \Big|_{z=0}$, the above expression can be written as follows:

$$\frac{1}{\epsilon_0} \frac{\partial A_{z_2}}{\partial z} \Big|_{z=0} - \frac{1}{\epsilon_1} \frac{\partial A_{z_1}}{\partial z} \Big|_{z=0} = \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_0} \right) \frac{\partial A_{x_2}}{\partial x} \Big|_{z=0} \quad (\text{B.26})$$

If the right-hand side of the above expression is not zero, then $\frac{\partial A_{z_2}}{\partial z} \Big|_{z=0}$ and $\frac{\partial A_{z_1}}{\partial z} \Big|_{z=0}$ terms cannot be equal to each other, otherwise the scalar potentials would not be equal to each other which suggest yet another contradiction to the previous assumption. So it is concluded that for the assumptions made in the beginning be valid, the z -derivatives of the A_{z_2} and A_{z_1} at the interface should be different from each other which means that the A_z term is different than zero. So when the traditional form for the Green's functions is used one needs to use different Green's functions for the scalar potentials for horizontal and vertical electric dipoles.

Appendix C

DERIVATION OF MOM MATRIX ENTRIES

In the application of the spatial-domain MoM to the solution of MPIE for the analysis of planar stratified geometries the surface current densities on the conductors are written in terms of linear combination of known basis functions and unknown amplitudes corresponding to each basis function. After the testing procedure of the MoM the amplitudes of the current densities are found using the matrix equation given in Eq.(3.15). Note that for the following derivations, source terms are also added to the current density expressions in Eqs.(3.8, 3.9, 3.11) and written as follows:

$$J_x(x, y) = \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)}(x, y) + \sum_n I_{xS}^{(n)} B_{xS}^{(n)}(x, y) + \sum_m \sum_n I_{xSP}^{(m,n)} B_{xSP}^{(m,n)}(x, y) \quad (C.1)$$

$$J_y(x, y) = \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)}(x, y) + \sum_m I_{yS}^{(m)} B_{yS}^{(m)}(x, y) + \sum_m \sum_n I_{ySP}^{(m,n)} B_{ySP}^{(m,n)}(x, y) \quad (C.2)$$

$$J_z(x, y, z) = \sum_m \sum_l I_{z(l)}^{x(m,n)} B_{z(l)}^{x(m,n)}(y, z) + \sum_n \sum_k I_{z(k)}^{y(m,n)} B_{z(k)}^{y(m,n)}(x, z) + \sum_l I_{z(l)}^{(m,n)} B_{z(l)}^{(m,n)}(z) \quad (C.3)$$

where $B_{xS}^{(n)}$ and $B_{yS}^{(m)}$ represent the source terms in the x - and y -directions, respectively whereas $B_{xSP}^{(m,n)}$ and $B_{ySP}^{(m,n)}$ denote the attachment functions introduced for the intersection of horizontal and vertical conductors to satisfy the charge conservation in the system. Note that in Eq.(C.3), the first two terms represent the current distribution on vertical strips and the last term represents the current distribution on vertical pins if they exist. Note also that the mathematical formulations for the basis functions in the z -direction are already given in Eqs.(3.12-3.14).

Using the current densities given above and the electrical field expressions, the derivation of the matrix entries corresponding to horizontal and vertical metallizations are evaluated as shown in the following sections.

C.1 Matrix Entries Obtained from E_x

Remember that the tangential electric field on the x -direction is given as below:

$$E_x = -j\omega G_{xx}^A * J_x + \frac{1}{j\omega} \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} J_x + G_y^q * \frac{\partial}{\partial y} J_y + G_z^q * \frac{\partial}{\partial z} J_z \right) \quad (C.4)$$

Substituting Eqs.(C.1-C.3) into Eq.(C.4) results in the following expression for E_x :

$$\begin{aligned} E_x = & -j\omega G_{xx}^A * \left\{ \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)} + \sum_n I_{xS}^{(n)} B_{xS}^{(n)} + \sum_m \sum_n I_{xSP}^{(m,n)} B_{xSP}^{(m,n)} \right\} \\ & + \frac{1}{j\omega} \frac{\partial}{\partial x} \left(G_x^q * \frac{\partial}{\partial x} \left\{ \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)} + \sum_n I_{xS}^{(n)} B_{xS}^{(n)} + \sum_m \sum_n I_{xSP}^{(m,n)} B_{xSP}^{(m,n)} \right\} \right) \\ & + \frac{1}{j\omega} \frac{\partial}{\partial x} \left(G_y^q * \frac{\partial}{\partial y} \left\{ \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)} + \sum_m I_{yS}^{(m)} B_{yS}^{(m)} + \sum_m \sum_n I_{ySP}^{(m,n)} B_{ySP}^{(m,n)} \right\} \right) \\ & + \frac{1}{j\omega} \frac{\partial}{\partial x} \left(G_z^q * \frac{\partial}{\partial z} \left\{ \sum_m \sum_l I_z^{(m,n)} B_{z(l)}^{(m,n)} + \sum_n \sum_k I_z^{(m,n)} B_{z(k)}^{(m,n)} + \sum_l I_z^{(n)} B_{z(l)}^{(n)} \right\} \right) \end{aligned} \quad (C.5)$$

After arranging the terms, the above equation can be written as:

$$\begin{aligned} E_x = & -j\omega \sum_m \sum_n I_x^{(m,n)} \left\{ G_{xx}^A * B_x^{(m,n)} + \frac{1}{\omega^2} \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)} \right] \right\} \\ & - j\omega \sum_m \sum_n I_{xSP}^{(m,n)} \left\{ G_{xx}^A * B_{xSP}^{(m,n)} + \frac{1}{\omega^2} \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_{xSP}^{(m,n)} \right] \right\} \\ & - j\omega \sum_n I_{xS}^{(n)} \left\{ G_{xx}^A * B_{xS}^{(n)} + \frac{1}{\omega^2} \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_{xS}^{(n)} \right] \right\} \\ & + \frac{1}{j\omega} \sum_m \sum_n I_y^{(m,n)} \left\{ \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_y^{(m,n)} \right] \right\} + \frac{1}{j\omega} \sum_m \sum_n I_{ySP}^{(m,n)} \left\{ \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_{ySP}^{(m,n)} \right] \right\} \\ & + \frac{1}{j\omega} \sum_m I_{yS}^{(m)} \left\{ \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_{yS}^{(m)} \right] \right\} + \frac{1}{j\omega} \sum_m \sum_l I_z^{(m,n)} \left\{ \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{(m,n)} \right] \right\} \\ & + \frac{1}{j\omega} \sum_m \sum_l I_z^{(m,n)} \left\{ \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(k)}^{(m,n)} \right] \right\} + \frac{1}{j\omega} \sum_m \sum_l I_z^{(m,n)} \left\{ \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{(m,n)} \right] \right\} \end{aligned} \quad (C.6)$$

The next step is to test E_x with all the testing functions $T_x^{(m',n')}$ and set the resulting equation to zero. Testing of E_x , i.e. applying boundary conditions in the integral sense on the horizontal conductor is denoted as follows:

$$\langle T_x^{(m',n')}, E_x \rangle = 0 \quad (C.7)$$

where $\langle \cdot, \cdot \rangle$ is the inner product and it is defined as Eq.(C.8)

$$\langle f(x, y), g(x, y) \rangle = \iint dx dy f^*(x, y) g(x, y) \quad (C.8)$$

After multiplying E_x with $T_x^{(m',n')}$, the following equation is obtained

$$\begin{aligned}
0 = & \sum_m \sum_n I_x^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_m \sum_n I_y^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_y^{(m,n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_m \sum_l I_{z(l)}^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{(m,n)} \right] \right\rangle \right\} \\
& + \sum_m \sum_n I_{xSP}^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, G_{xx}^A * B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_{xSP}^{(m,n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_n \sum_k I_{z(k)}^{y(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(k)}^{y(m,n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_m \sum_n I_{ySP}^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_{ySP}^{(m,n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_l I_{z(l)}^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{(m,n)} \right] \right\rangle \right\} \\
& + \sum_n I_{xS}^{(n)} \left\{ \left\langle T_x^{(m',n')}, G_{xx}^A * B_{xS}^{(n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial}{\partial x} B_{xS}^{(n)} \right] \right\rangle \right\} \\
& + \frac{1}{\omega^2} \sum_m I_{yS}^{(m)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial}{\partial y} B_{yS}^{(m)} \right] \right\rangle \right\} \tag{C.9}
\end{aligned}$$

In this expression, the terms with unknown coefficients $I_x^{(m,n)}$, (i.e. the first line) are the elements of the Z_{xx} submatrix. Similarly the terms with coefficients $I_y^{(m,n)}$ are the entries of the Z_{xy} matrix. The terms with coefficients $I_{z(l)}^{x(m,n)}$, $I_{z(k)}^{y(m,n)}$ and $I_{z(l)}^{(m,n)}$ are the Z_{xz} matrix entries. The terms with coefficients $I_{xSP}^{(m,n)}$ are the field expressions due to the x -directed attachment function on the region specified with the testing function $T_x^{(m',n')}$. Note that these coefficients are related to the unknown coefficients of the vertical basis functions $I_{z(l)}^{x(m,n)}$ or $I_{z(l)}^{(m,n)}$ which are defined at the horizontal and vertical conductor intersections in order to satisfy the conservation of charge at the junction region. Therefore their contributions are included in the Z_{xz} submatrix entries of the impedance matrix. The same rule applies to the terms with amplitudes $I_{ySP}^{(m,n)}$ and $I_{z(k)}^{y(m,n)}$ or $I_{z(l)}^{(m,n)}$. Recall that the relations between the attachment functions and the vertical basis functions at the junctions are already given in Chapter 3. Therefore interested readers should refer to that chapter. Finally the terms with coefficients $I_{xS}^{(n)}$ and $I_{yS}^{(m)}$ constitute the elements of the excitation matrix V_x . To summarize, the matrix entries obtained due to E_x are given as follows:

$$Z_{xx}^{(m',n')(m,n)} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \tag{C.10}$$

$$Z_{xy}^{(m',n')(m,n)} = \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \quad (\text{C.11})$$

$$Z_{xz}^{(m',n')(l)} = \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (\text{C.12})$$

$$\begin{aligned} V_x^{(m',n')(m,n)} &= -\left\langle T_x^{(m',n')}, G_{xx}^A * B_{xs}^{(n)} \right\rangle - \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_{xs}^{(n)}}{\partial x} \right] \right\rangle \\ &\quad - \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial B_{ys}^{(m)}}{\partial y} \right] \right\rangle \end{aligned} \quad (\text{C.13})$$

where basis functions in the z -direction are shown as $B_{z(l)}$ without using any superscripts to avoid confusion and address that the direction of the basis function (which is the z -direction) is important not the type. As it was mentioned before, in the case of vertical to horizontal conductor junctions, attachment functions on the horizontal conductors are used and their contributions are included in the contribution of the z -directed basis functions right next to the junction. Therefore, in case of attachment functions, the elements of the Z_{xz} matrix can be written as follows:

$$\begin{aligned} Z_{xz}^{(m',n')(l)} &= \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_{z(l)}^{x(m,n)}}{\partial z} \right] \right\rangle \\ &\quad - \frac{1}{2} \left\{ \left\langle T_x^{(m',n')}, B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial x} \right] \right\rangle \right\} \end{aligned} \quad (\text{C.14})$$

$$\begin{aligned} Z_{xz}^{(m',n')(k)} &= \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_{z(k)}^{y(m,n)}}{\partial z} \right] \right\rangle \\ &\quad - \frac{1}{2} \left\{ \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial y} \right] \right\rangle \right\} \end{aligned} \quad (\text{C.15})$$

$$\begin{aligned} Z_{xz}^{(m',n')(l)} &= \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_z^q * \frac{\partial B_{z(l)}^{(m,n)}}{\partial z} \right] \right\rangle \\ &\quad - \frac{1}{4h_x} \left\{ \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial y} \right] \right\rangle \right\} \\ &\quad - \frac{1}{4h_y} \left\{ \left\langle T_x^{(m',n')}, B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial x} \right] \right\rangle \right\} \end{aligned} \quad (\text{C.16})$$

where h_x and h_y are the lengths of a cell used in the discretization in the x - and y -directions, respectively. Note that Eq.(C.14) and Eq.(C.15) are for the basis functions on the vertical strips spanning in the yz - and xz -directions, respectively and Eq.(C.16) is for the basis functions on vertical pins.

C.2 Matrix Entries Obtained from E_y

The procedure followed for the derivation of the matrix entries obtained from E_y is similar for the E_x case. Therefore only the resulting matrix entries are given here.

$$Z_{yy}^{(m',n')(m,n)} = \left\langle T_y^{(m',n')}, G_{yy}^A * B_y^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \quad (C.17)$$

$$Z_{yx}^{(m',n')(m,n)} = \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (C.18)$$

$$Z_{yz}^{(m',n')(l)} = \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \quad (C.19)$$

$$\begin{aligned} V_y^{(m',n')(l)} &= - \left\langle T_y^{(m',n')}, G_{yy}^A * B_y^{(n)} \right\rangle - \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_y^q * \frac{\partial B_{ys}^{(n)}}{\partial y} \right] \right\rangle \\ &\quad - \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_{xs}^{(m)}}{\partial x} \right] \right\rangle \end{aligned} \quad (C.20)$$

Similar to the Z_{xz} case, the matrix elements of the Z_{yz} matrix can also be written in terms of three components, two of which are due to yz and xz spanning basis functions on the vertical strips and the last one is due to basis functions on the vertical pins as follows:

$$\begin{aligned} Z_{yz}^{(m',n')(k)} &= \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_z^q * \frac{\partial B_{z(k)}}{\partial z} \right] \right\rangle \\ &\quad - \frac{1}{2} \left\{ \left\langle T_y^{(m',n')}, B_{ySP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial x} \right] \right\rangle \right\} \end{aligned} \quad (C.21)$$

$$\begin{aligned} Z_{yz}^{(m',n')(l)} &= \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle \\ &\quad - \frac{1}{2} \left\{ \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial y} \right] \right\rangle \right\} \end{aligned} \quad (C.22)$$

$$\begin{aligned} Z_{yz}^{(m',n')(l)} &= \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_z^q * \frac{\partial B_{z(l)}}{\partial z} \right] \right\rangle - \frac{1}{4h_y} \left\{ \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_x^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial y} \right] \right\rangle \right\} \\ &\quad - \frac{1}{4h_x} \left\{ \left\langle T_y^{(m',n')}, B_{ySP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_y^{(m',n')}, \frac{\partial}{\partial y} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial x} \right] \right\rangle \right\} \end{aligned} \quad (C.23)$$

Note that entries involving attachment functions exist only if there is a vertical and horizontal conductor intersections.

C.3 Matrix Entries Obtained from E_z

The tangential electric field on the vertical metallizations are given as below:

$$E_z = -j\omega G_{zx}^A * J_x - j\omega G_{zy}^A * J_y - j\omega G_{zz}^A * J_z + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_x^q * \frac{\partial J_x}{\partial x} \right) + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_y^q * \frac{\partial J_y}{\partial y} \right) + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_z^q * \frac{\partial J_z}{\partial z} \right) \quad (C.24)$$

Substituting Eqs.(C.1-C.3) into Eq.(C.24) results in the following expression:

$$\begin{aligned} E_z = & -j\omega G_{zx}^A * \left\{ \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)} + \sum_n I_{xS}^{(n)} B_{xS}^{(n)} + \sum_m \sum_n I_{xSP}^{(m,n)} B_{xSP}^{(m,n)} \right\} \\ & - j\omega G_{zy}^A * \left\{ \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)} + \sum_n I_{yS}^{(n)} B_{yS}^{(n)} + \sum_m \sum_n I_{ySP}^{(m,n)} B_{ySP}^{(m,n)} \right\} \\ & - j\omega G_{zz}^A * \left\{ \sum_m \sum_l I_{z(l)}^{x(m,n)} B_{z(l)}^{x(m,n)} + \sum_n \sum_k I_{z(k)}^{y(m,n)} B_{z(k)}^{y(m,n)} + \sum_l I_{z(l)}^{(m,n)} B_{z(l)}^{(m,n)} \right\} \\ & + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_x^q * \frac{\partial}{\partial x} \left\{ \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)} + \sum_n I_{xS}^{(n)} B_{xS}^{(n)} + \sum_m \sum_n I_{xSP}^{(m,n)} B_{xSP}^{(m,n)} \right\} \right) \\ & + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_y^q * \frac{\partial}{\partial y} \left\{ \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)} + \sum_m I_{yS}^{(m)} B_{yS}^{(m)} + \sum_m \sum_n I_{ySP}^{(m,n)} B_{ySP}^{(m,n)} \right\} \right) \\ & + \frac{1}{j\omega} \frac{\partial}{\partial z} \left(G_z^q * \frac{\partial}{\partial z} \left\{ \sum_m \sum_l I_{z(l)}^{x(m,n)} B_{z(l)}^{x(m,n)} + \sum_n \sum_k I_{z(k)}^{y(m,n)} B_{z(k)}^{y(m,n)} + \sum_l I_{z(l)}^{(m,n)} B_{z(l)}^{(m,n)} \right\} \right) \end{aligned} \quad (C.25)$$

Now, test E_z with testing functions $T_{z(l')}^{x(m',n')}$, $T_{z(k')}^{y(m',n')}$ or $T_{z(l')}^{(m',n')}$ and set the resulting equation to zero to satisfy the boundary conditions. In Eq.(C.26), the result after using the testing function $T_{z(l')}^{x(m',n')}$ is given.

$$\begin{aligned} 0 = & \sum_m \sum_n I_x^{(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)} \right] \right\rangle \right\} \\ & + \sum_m \sum_n I_y^{(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zy}^A * B_y^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial}{\partial y} B_y^{(m,n)} \right] \right\rangle \right\} \\ & + \sum_m \sum_l I_{z(l)}^{x(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(l)}^{x(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{x(m,n)} \right] \right\rangle \right\} \\ & + \sum_m \sum_n I_{xSP}^{(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial}{\partial x} B_{xSP}^{(m,n)} \right] \right\rangle \right\} \\ & + \sum_m \sum_k I_{z(k)}^{y(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(k)}^{y(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial}{\partial z} B_{z(k)}^{y(m,n)} \right] \right\rangle \right\} \\ & + \sum_m \sum_n I_{ySP}^{(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{ySP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial}{\partial y} B_{ySP}^{(m,n)} \right] \right\rangle \right\} \end{aligned}$$

$$\begin{aligned}
& + \sum_l I_{z(l)}^{(m,n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(l)}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial}{\partial z} B_{z(l)}^{(m,n)} \right] \right\rangle \right\} \\
& + \sum_n I_{xS}^{(n)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{xS}^{(n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial}{\partial x} B_{xS}^{(n)} \right] \right\rangle \right\} \\
& + \sum_m I_{yS}^{(m)} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zy}^A * B_{yS}^{(m)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial}{\partial y} B_{yS}^{(m)} \right] \right\rangle \right\}
\end{aligned} \tag{C.26}$$

In this equation the terms with coefficients $I_x^{(m,n)}$ gives the elements of Z_{zx} matrix, likewise the terms with coefficients $I_y^{(m,n)}$ gives the Z_{zy} matrix entries. The terms with the source or in other words with the amplitudes $I_{xS}^{(n)}$ or $I_{yS}^{(m)}$ constitute the entries of the excitation matrix V_z . Other terms give the Z_{zz} matrix entries of impedance matrix. Note that the contribution of the attachment functions (if available) are included in the Z_{zz} matrix entries of the vertical basis functions right next to horizontal to vertical conductor junctions.

Therefore as a summary, the matrix entries due to E_z are given as follows.

$$Z_{zx}^{(l')(m,n)} = \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \tag{C.27}$$

$$Z_{zy}^{(l')(m,n)} = \left\langle T_{z(l')}^{x(m',n')}, G_{zy}^A * B_y^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_y^{(m,n)}}{\partial y} \right] \right\rangle \tag{C.28}$$

$$Z_{zz}^{(l')(l)} = \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(l)}^{x(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(l)}^{x(m,n)}}{\partial z} \right] \right\rangle \tag{C.29}$$

$$\begin{aligned}
V_z^{(l')(m,n)} &= - \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{xS}^{(n)} \right\rangle - \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_{xS}^{(n)}}{\partial x} \right] \right\rangle \\
&- \left\langle T_{z(l)}^{x(m',n')}, G_{zy}^A * B_{yS}^{(m)} \right\rangle - \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_{yS}^{(m)}}{\partial y} \right] \right\rangle
\end{aligned} \tag{C.30}$$

Similarly the contribution of attachment functions can be included in the Z_{zz} term of the impedance matrix as shown below:

$$\begin{aligned}
Z_{zz}^{(l')(l)} &= \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(l)}^{x(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(l)}^{x(m,n)}}{\partial z} \right] \right\rangle \\
&- \frac{1}{2} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_x^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial x} \right] \right\rangle \right\}
\end{aligned} \tag{C.31}$$

$$\begin{aligned}
Z_{zz}^{(l')(k)} &= \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(k)}^{y(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l)}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(k)}^{y(m,n)}}{\partial z} \right] \right\rangle \\
&- \frac{1}{2} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zy}^A * B_{ySP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial y} \right] \right\rangle \right\}
\end{aligned} \tag{C.32}$$

$$\begin{aligned}
Z_{zz}^{(l')(l)} &= \left\langle T_{z(l')}^{x(m',n')}, G_{zz}^A * B_{z(l)}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_z^q * \frac{\partial B_{z(l)}^{(m,n)}}{\partial z} \right] \right\rangle \\
&- \frac{1}{4h_x} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{ySP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_{ySP}^{(m,n)}}{\partial y} \right] \right\rangle \right\} \\
&- \frac{1}{4h_y} \left\{ \left\langle T_{z(l')}^{x(m',n')}, G_{zx}^A * B_{xSP}^{(m,n)} \right\rangle + \frac{1}{\omega^2} \left\langle T_{z(l')}^{x(m',n')}, \frac{\partial}{\partial z} \left[G_y^q * \frac{\partial B_{xSP}^{(m,n)}}{\partial y} \right] \right\rangle \right\}
\end{aligned} \tag{C.33}$$

Note that Eq. C.31 and Eq. C.32 are for basis functions on the vertical strips in yz and xz direction respectively whereas Eq. C.33 is for the basis functions on the vertical pins.

The above matrix entries are for the testing function $T_{z(l')}^{x(m',n')}$. When the matrix entries for the testing functions $T_{z(k')}^{y(m',n')}$ and $T_{z(l')}^{(m',n')}$ are required, it is enough to replace $T_{z(l')}^{x(m',n')}$ with the testing functions $T_{z(k')}^{y(m',n')}$ or $T_{z(l')}^{(m',n')}$. Since similar equations are obtained for the elements of the Z_{zz} matrix when the other testing functions are used, they are not given here.

Appendix D

ANALYTICAL EVALUATION OF THE INNER PRODUCT TERMS

The general formulation for the spatial domain MoM by using the aforementioned basis and testing functions is demonstrated in Chapter 3. The MoM entries of the formulation are given in Appendix C. It is observed that in case of vertical metallizations, various basis and testing function configurations exist in the formulation. Now in this appendix, the evaluation of these MoM matrix entries will be given.

D.1 Evaluation of $\left\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial}{\partial x} B_x \right\rangle$

In this term, integration and derivation with respect to z should be taken. The evaluation for $T_z(y, z)$ starts by writing

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint dz dy \frac{\partial}{\partial z} T_z(y, z) \cdot \iint dx' dy' G_x^q(x - x', y - y', z, z') \frac{\partial}{\partial x'} B_x(x', y') \quad (\text{D.1})$$

After a little bit of manipulation this equation becomes

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint dx' dy' \frac{\partial}{\partial x'} B_x(x', y') \cdot \int dy T_z(y) \int dz \frac{\partial}{\partial z} T_z(z) G_x^q(x - x', y - y', z, z') \quad (\text{D.2})$$

The innermost integral in Eq.(D.2) is taken by using the method explained in Chapter 4 by writing the spatial domain Green's function in terms of its spectral domain counterpart.

The innermost integral results in the auxiliary function

$$\begin{aligned} F_x^q &\stackrel{\text{def}}{=} \int dz \frac{\partial}{\partial z} T_z(z) \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \tilde{G}_x^q(k_\rho) \\ &= \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \int dz \frac{\partial}{\partial z} T_z(z) \tilde{G}_x^q(k_\rho) \end{aligned} \quad (\text{D.3})$$

where $\tilde{G}_x^q$ is the spectral domain Green's function. After the substitution of the auxiliary function in Eq.(D.2), the equation becomes

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \int dy T_z(y) \iint dx' dy' F_x^q(x - x', y - y', z') \frac{\partial}{\partial x'} B_x(x', y') \quad (D.4)$$

Then by making the substitutions $x - x' = u$ and $y - y' = v$, the following expression is obtained:

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint du dv F_x^q(u, v, z') \int dy T_z(y) \frac{\partial}{\partial x'} B_x(x - u, y - v) \quad (D.5)$$

where x is the x -coordinate of the vertical strip and $\partial/\partial x' = -\partial/\partial u$. Similarly, for $T_z(x, z)$ (testing function on the xz spanning vertical strip) this inner product becomes

$$\left\langle \frac{\partial}{\partial z} T_z(x, z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint du dv F_x^q(u, v, z') \int dx T_z(x) \frac{\partial}{\partial x'} B_x(x - u, y - v) \quad (D.6)$$

where y is the y -coordinate of the vertical strip and for $T_z(z)$ (testing function on the vertical pin) it is as follows:

$$\left\langle \frac{\partial}{\partial z} T_z(z), G_x^q * \frac{\partial}{\partial x} B_x(x, y) \right\rangle = \iint du dv F_x^q(u, v, z') \frac{\partial}{\partial x'} B_x(x - u, y - v) \quad (D.7)$$

where x and y are the x and y coordinates of the vertical pin respectively.

D.2 Evaluation of $\langle T_z, G_{zx}^A * B_x \rangle$

This term involves integration in the z direction. For the testing functions on the yz spanning vertical strip (i.e. $T_z(y, z)$) the inner product becomes

$$\langle T_z(y, z), G_{zx}^A * B_x(x, y) \rangle = \iint dz dy T_z(y, z) \iint dx' dy' G_{zx}^A(x - x', y - y', z, z') B_x(x', y') \quad (D.8)$$

Then this equation becomes

$$\langle T_z(y, z), G_{zx}^A * B_x(x, y) \rangle = \int dy T_z(y) \iint dx' dy' B_x(x', y') \int dz T_z(z) G_{zx}^A(x - x', y - y', z, z') \quad (D.9)$$

To evaluate the innermost integral the following definition is made:

$$\begin{aligned} F_{zx}^A &\stackrel{\text{def}}{=} \int dz T_z(z) \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \frac{\tilde{G}_{zx}^A(k_\rho)}{-jk_x} \\ &= \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \int dz T_z(z) \frac{\tilde{G}_{zx}^A(k_\rho)}{-jk_x} \end{aligned} \quad (D.10)$$

The reason for dividing the spectral domain Green's function G_{zx}^A by $-jk_x$ is to eliminate the multiplicative $-jk_x$ term existing in the Green's function G_{zx}^A to be able to apply the GPOF method. Therefore its effect is added as the derivative with respect to x' in the spatial domain. Then the inner product becomes

$$\langle T_z(y, z), G_{zx}^A * B_x(x, y) \rangle = - \int dy T_z(y) \iint dx' dy' B_x(x', y') \frac{\partial}{\partial x'} F_{zx}^A(x - x', y - y', z') \quad (\text{D.11})$$

The following chain rule is applied to the inner product.

$$\frac{\partial}{\partial x'} [B_x(x', y') F_{zx}^A] = \frac{\partial}{\partial x'} B_x(x', y') F_{zx}^A + B_x(x', y') \frac{\partial}{\partial x'} F_{zx}^A \quad (\text{D.12})$$

and the inner product reduces to

$$\begin{aligned} \langle T_z(y, z), G_{zx}^A * B_x(x, y) \rangle &= - \int dy T_z(y) \int dy' [B_x(x', y') F_{zx}^A(x - x', y - y', z')] \Big|_{x' \in \Omega_B} \\ &\quad + \int dy T_z(y) \iint dx' dy' F_{zx}^A(x - x', y - y', z') \frac{\partial}{\partial x'} B_x(x', y') \end{aligned} \quad (\text{D.13})$$

where Ω_B is the boundary of the domain of the basis function. After the substitutions $x - x' = u$ and $y - y' = v$, the inner product term becomes

$$\begin{aligned} \langle T_z(y, z), G_{zx}^A * B_x(x, y) \rangle &= \int dv F_{zx}^A(u, v, z') \int dy [T_z(y) B_x(x - u, y - v)] \Big|_{x' \in \Omega_B} \\ &\quad + \iint du dv F_{zx}^A(u, v, z') \int dy T_z(y) \frac{\partial}{\partial x'} B_x(x - u, y - v) \end{aligned} \quad (\text{D.14})$$

where x is the x coordinate of the yz spanning vertical strip. Similarly, for testing function $T_z(x, z)$ the inner product becomes

$$\begin{aligned} \langle T_z(x, z), G_{zx}^A * B_x(x, y) \rangle &= \int dv F_{zx}^A(u, v, z') \int dx [T_z(x) B_x(x - u, y - v)] \Big|_{y' \in \Omega_B} \\ &\quad + \iint du dv F_{zx}^A(u, v, z') \int dx T_z(x) \frac{\partial}{\partial x'} B_x(x - u, y - v) \end{aligned} \quad (\text{D.15})$$

where y is the y -coordinate of the vertical strip. Finally for the testing function on the vertical pin, i.e. for $T_z(z)$, the inner product becomes

$$\begin{aligned} \langle T_z(z), G_{zx}^A * B_x(x, y) \rangle &= \int dv F_{zx}^A(x - x', v, z') [B_x(x', y - v)] \\ &\quad + \iint du dv F_{zx}^A(u, v, z') \frac{\partial}{\partial x'} B_x(x - u, y - v) \end{aligned} \quad (\text{D.16})$$

where x and y are the x and y coordinates of the vertical pin.

D.3 Evaluation of $\left\langle \frac{\partial}{\partial x} T_x, G_z^q * \frac{\partial}{\partial z} B_z \right\rangle$

In this term, integration and derivation with respect to z' should be taken. The evaluation for the basis function on the vertical strip $B_z(y, z)$ starts by writing

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \iint dx dy \frac{\partial}{\partial x} T_x(x, y) \cdot \iint dz' dy' G_z^q(x - x', y - y', z, z') \frac{\partial}{\partial z'} B_z(y', z') \quad (D.17)$$

After a little bit of algebraic manipulation this equation becomes

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \iint dx dy \frac{\partial}{\partial x} T_x(x, y) \cdot \int dy' B_z(y') \int dz' \frac{\partial}{\partial z'} B_z(z') G_z^q(x - x', y - y', z, z') \quad (D.18)$$

The innermost integral is written in terms of the following auxiliary function

$$\begin{aligned} F_z^q &\stackrel{\text{def}}{=} \int dz' \frac{\partial}{\partial z'} B_z(z') \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \tilde{G}_z^q(k_\rho) \\ &= \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \int dz' \frac{\partial}{\partial z'} B_z(z') \tilde{G}_z^q(k_\rho) \end{aligned} \quad (D.19)$$

After the substitution of the auxiliary function in Eq.(D.18), the equation becomes

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \int dy' B_z(y') \iint dx dy F_z^q(x - x', y - y', z) \frac{\partial}{\partial x} T_x(x, y) \quad (D.20)$$

Then by making the substitutions $x - x' = u$ and $y - y' = v$, the following expression is obtained:

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \iint du dv F_z^q(u, v, z) \int dy B_z(y - v) \frac{\partial}{\partial x} T_x(x' + u, y) \quad (D.21)$$

where $x' = x_i$, i.e. x - coordinate of the basis function. Similarly, for the basis function $B_z(x, y)$ the inner product becomes as follows:

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(x, z) \right\rangle = \iint du dv F_z^q(u, v, z) \int dx B_z(x - u) \frac{\partial}{\partial x} T_x(x, y' + v) \quad (D.22)$$

where $y' = y_i$, i.e. y -coordinate of the basis function. Finally for the basis function $B_z(z)$, the inner product term reduces to

$$\left\langle \frac{\partial}{\partial x} T_x(x, y), G_z^q * \frac{\partial}{\partial z} B_z(z) \right\rangle = \iint du dv F_z^q(u, v, z) \frac{\partial}{\partial x} T_x(x' + u, y' + v) \quad (D.23)$$

where $x' = x_i$ and $y' = y_i$, i.e. the x and y coordinates of the vertical pin.

D.4 Evaluation of $\langle T_z, G_{zz}^A * B_z \rangle$

This term involves integration in the z and z' directions. For the testing and basis functions on the yz spanning vertical strip (i.e. for $T_z(y, z)$ and $B_z(y, z)$) the inner product becomes

$$\langle T_z(y, z), G_{zz}^A * B_z(y, z) \rangle = \iint dz dy T_z(y, z) \iint dz' dy' G_{zz}^A(x - x', y - y', z, z') B_z(x', y') \quad (D.24)$$

Then after manipulating the equation, the following is obtained

$$\langle T_z(y, z), G_{zz}^A * B_z(y, z) \rangle = \int dy T_z(y) \int dy' B_z(y') \iint dz dz' T_z(z) B_z(z') G_{zz}^A(x - x', y - y', z, z') \quad (D.25)$$

To evaluate the innermost integral the following definition is made:

$$\begin{aligned} F_{zz}^A &\stackrel{\text{def}}{=} \iint dz dz' T_z(z) B_z(z') \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \tilde{G}_{zz}^A(k_\rho) \\ &= \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \iint dz dz' T_z(z) B_z(z') \tilde{G}_{zz}^A(k_\rho) \end{aligned} \quad (D.26)$$

After substituting the auxiliary function into Eq.(D.25), the inner product term reduces to

$$\langle T_z(y, z), G_{zz}^A * B_z(y, z) \rangle = \int dy T_z(y) \int dy' B_z(y') F_{zz}^A(x - x', y - y') \quad (D.27)$$

where x and x' are the x coordinates of the testing and basis functions respectively. By making the substitution $y - y' = v$, the following equation is obtained:

$$\langle T_z(y, z), G_{zz}^A * B_z(y, z) \rangle = \int dv F_{zz}^A(u, v) \int dy T_z(y) B_z(y - v) \quad (D.28)$$

Similarly for testing function $T_z(x, z)$ and basis function $B_z(x, z)$ the inner product becomes

$$\langle T_z(x, z), G_{zz}^A * B_z(x, z) \rangle = \int du F_{zz}^A(u, v) \int dx T_z(x) B_z(x - u) \quad (D.29)$$

where y and y' are the y coordinates of the basis and testing functions respectively.

Likewise, for testing function $T_z(x, z)$ and basis function $B_z(y, z)$, the inner product is

$$\langle T_z(x, z), G_{zz}^A * B_z(y, z) \rangle = \int du \int dv F_{zz}^A(u, v) T_z(x' + u) B_z(y - v) \quad (D.30)$$

where x' is the x coordinate of the basis function whereas y is the y coordinate of the testing function. Similarly for testing function $T_z(y, z)$ and basis function $B_z(x, z)$ the inner product is as follows:

$$\langle T_z(y, z), G_{zz}^A * B_z(x, z) \rangle = - \int du \int dv F_{zz}^A(u, v) T_z(y' + v) B_z(x - u) \quad (D.31)$$

where x is the x coordinate of the testing function and y' is the y coordinate of the basis function.

D.5 Evaluation of $\left\langle \frac{\partial}{\partial z} T_z, G_z^q * \frac{\partial}{\partial z} B_z \right\rangle$

In this term, integration and derivation with respect to z and z' should be taken. The evaluation for testing function $T_z(y, z)$ and basis function $B_z(y, z)$ starts by writing

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \iint dz dy \frac{\partial}{\partial z} T_z(y, z) \cdot \iint dy' dz' G_z^q(x - x', y - y', z, z') \frac{\partial}{\partial z'} B_z(y', z') \quad (\text{D.32})$$

This equation can be written as follows:

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \int dy T_z(y) \int dy' B_z(y') \cdot \iint dz dz' \frac{\partial}{\partial z} T_z(z) \frac{\partial}{\partial z'} B_z(z') G_z^q(x - x', y - y', z, z') \quad (\text{D.33})$$

The innermost integral is written as the following auxiliary function

$$\begin{aligned} F_z^q &\stackrel{\text{def}}{=} \iint dz dz' \frac{\partial}{\partial z} T_z(z) \frac{\partial}{\partial z'} B_z(z') \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \tilde{G}_z^q(k_\rho) \\ &= \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \iint dz dz' \frac{\partial}{\partial z} T_z(z) \frac{\partial}{\partial z'} B_z(z') \tilde{G}_z^q(k_\rho) \end{aligned} \quad (\text{D.34})$$

Then by substituting the auxiliary function into Eq.(D.33), the following equation is obtained

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \int dy T_z(y) \int dy' B_z(y') F_z^q(x - x', y - y') \quad (\text{D.35})$$

where x and x' are the x -coordinates of the testing and basis functions respectively. By making the substitution $y - y' = v$, the above equation is reduced to

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \int dv F_z^q(x - x', v) \int dy T_z(y) B_z(y - v) \quad (\text{D.36})$$

Similarly for testing function $T_z(x, z)$ and basis function $B_z(x, z)$ the inner product becomes

$$\left\langle \frac{\partial}{\partial z} T_z(x, z), G_z^q * \frac{\partial}{\partial z} B_z(x, z) \right\rangle = \int du F_z^q(u, y - y') \int dx T_z(x) B_z(x - u) \quad (\text{D.37})$$

where y and y' are the y coordinates of the basis and testing functions respectively and for the testing function $T_z(x, z)$ and basis function $B_z(y, z)$, the inner product is

$$\left\langle \frac{\partial}{\partial z} T_z(x, z), G_z^q * \frac{\partial}{\partial z} B_z(y, z) \right\rangle = \int du \int dv F_z^q(u, v) T_z(x' + u) B_z(y - v) \quad (\text{D.38})$$

where x' is the x coordinate of the basis function whereas y is the y coordinate of the testing function. Similarly for testing function $T_z(y, z)$ and basis function $B_z(x, z)$ the inner product is as follows:

$$\left\langle \frac{\partial}{\partial z} T_z(y, z), G_z^q * \frac{\partial}{\partial z} B_z(x, z) \right\rangle = \int du \int dv F_z^q(u, v) T_z(y' + v) B_z(x - u) \quad (\text{D.39})$$

where x is the x coordinate of the testing function and y' is the y coordinate of the basis function.

D.6 Other Terms

Evaluation of the terms involving basis functions in the y direction or attachment functions and source functions such as $\left\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial}{\partial x} B_{xSP} \right\rangle$, $\left\langle T_z, G_{zx}^A * B_{xSP} \right\rangle$, $\left\langle T_z, G_{zx}^A * B_{xS} \right\rangle$, and $\left\langle \frac{\partial}{\partial z} T_z, G_x^q * \frac{\partial B_{xS}}{\partial x} \right\rangle$ are similar to the ones discussed in this appendix. The only difference is in the definition of the functions. Therefore they are shown here.

BIBLIOGRAPHY

- [1] P. Silvester and P. Benedek, "Equivalent capacitance of microstrip open circuits," *IEEE Trans. on Microwave Theory Tech.*, vol. 20, pp. 511–516, August 1972.
- [2] —, "Equivalent capacitance of microstrip gaps and steps," *IEEE Trans. on Microwave Theory Tech.*, vol. 20, pp. 729–733, November 1972.
- [3] —, "Equivalent discontinuities capacitances for right-angle bends, t-junctions and crossings," *IEEE Trans. on Microwave Theory Tech.*, vol. 21, pp. 341–346, May 1973.
- [4] R. F. Harrington, "Matrix methods for field problems," *IEEE Proceedings*, vol. 55, pp. 136–149, Feb. 1967.
- [5] —, *Field Computation by Moment Methods*. The Macmillan Company, 1968.
- [6] K. Kunz and R. Luebber, *The Finite Difference Time Domain method for Electromagnetics*. Boca Raton, FL: CRC press, 1993.
- [7] J. Jin, *The Finite Element Method in Electromagnetics*. Newyork: John Wiley & Sons, Inc., 1993.
- [8] A. Taflove and M. E. Brodwin, "Numerical solution of steady state electromagnetic scattering problems using the time dependent maxwell's equations," *IEEE Trans. on Microwave Theory Tech.*, vol. 23, pp. 623–630, August 1975.
- [9] Juan R. Mosig, "Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation," *IEEE Trans. on Microwave Theory Tech.*, vol. MTT-36, no. 2, pp. 314–323, February 1988.
- [10] Ikmo Park, R. Mittra, and M. I. Aksun, "Numerically efficient analysis of planar microstrip configurations using closed-form green's functions," *IEEE Trans. on Microwave Theory Tech.*, vol. 43, no. 2, pp. 394–400, February 1995.

- [11] Krishna Naishadham and Todd W. Nuteson, "Efficient analysis of passive microstrip elements in mmics," *Int. J. MIMICAE*, vol. 4, no. 3, pp. 219–229, July 1994.
- [12] N. Kinayman and M. I. Aksun, "Efficient and accurate em simulation technique for analysis and design of mmics," *Int. J. MIMICAE*, vol. 7, pp. 344–358, September 1997.
- [13] J. R. Mosig and F. E. Gardiol, "General integral equation formulation for microstrip antennas and scatterers," *IEE Proceedings*, vol. 132, no. 7, pp. 424–432, December 1985.
- [14] Y. L. Chow, J. J. Yang, D. G. Fang, and G. E. Howard, "A closed-form spatial green's function for the thick microstrip substrate," *IEEE Trans. on Microwave Theory Tech.*, vol. 39, pp. 588–592, March 1991.
- [15] G. Dural and M. I. Aksun, "Closed-form green's functions for general sources and stratified media," *IEEE Trans. on Microwave Theory Tech.*, vol. 43, pp. 1545–1552, July 1995.
- [16] M. I. Aksun, "A robust approach for the derivation of closed-form green's functions," *IEEE Trans. on Microwave Theory Tech.*, vol. 44, no. 5, pp. 651–658, May 1996.
- [17] K. M. L. Alatan, M. I. Aksun and T. Birand, "Analytical evaluation of the mom matrix elements," *IEEE Trans. on Microwave Theory Tech.*, vol. 44, pp. 519–525, Apr 1996.
- [18] N. Kinayman and M. I. Aksun, "Efficient use of closed-form green's functions for the analysis of planar geometries with vertical connections," *IEEE Trans. on Microwave Theory Tech.*, vol. 45, pp. 593–603, May 1997.
- [19] T. M. Grzegorzcyk and J. R. Mosig, "Line charge distributions arising in the integral equation treatment of bent scatterers in stratified media," *IEE Proceedings*, vol. 148, pp. 365–368, December 2001.
- [20] A. Sommerfeld, *Partial Differential Equations in Physics*. New York : Academic, 1949.

- [21] W. C. Chew, *Waves and Fields in Inhomogeneous Media*. Van Nostrand Reinhold, 1990.
- [22] M. I. Aksun and R. Mittra, "Choices of expansion and testing functions for the method of moments applied to a class of electromagnetics problem," *IEEE Trans. on Microwave Theory Tech.*, vol. 41, pp. 503–509, March 1993.
- [23] N. Kinayman and M. I. Aksun, "Comparative study of acceleration techniques for integrals and series in electromagnetic problems," *Radio Science*, vol. 30, pp. 1713–1722, Nov-Dec 1995.
- [24] M. Marin, S. Barkeshli, and P. H. Pathak, "Efficient analysis of planar microstrip geometries using a closed-form asymptotic representation of the grounded dielectric slab green's function," *IEEE Trans. on Microwave Theory Tech.*, vol. 37, pp. 669–679, Apr. 1989.
- [25] M. Marin and P. H. Pathak, "An asymptotic closed-form representation for the grounded double-layer surface green's function," *IEEE Trans. on Antennas and Propagation*, vol. 37, pp. 1357–1366, Nov. 1992.
- [26] M. I. Aksun and R. Mittra, "Derivation of closed-form green's functions for a general microstrip geometry," *IEEE Trans. on Microwave Theory Tech.*, vol. 40, pp. 2055–2062, Nov. 1992.
- [27] A. J. Mackay and A. McCowan, "An improved pencil-of-function method for extracting poles of an em system from its transient response," *IEEE Trans. on Antennas and Propagation*, vol. 35, pp. 435–441, April 1987.
- [28] D. G. Dudley, "Parametric modeling of transient electromagnetic systems," *Radio Science*, vol. 14, pp. 387–396, 1979.
- [29] H. Yingbo and T. K. Sarkar, "Generalized pencil-of-function method for extracting poles of an em system from its transient response," *IEEE Trans. on Antennas and Propagation*, vol. AP-37, pp. 229–234, Feb. 1989.

- [30] N. Kınayman, *A Novel CAD Algorithm for the Analysis of Printed Geometries*. PhD thesis, Graduate College of the Bilkent University, Ankara, Turkey, 1997.
- [31] R. A. Kipp and C. H. Chan, "Complex image method for sources in bounded regions of multilayer structures," *IEEE Trans. on Microwave Theory Tech.*, vol. 42, pp. 860–865, May 1994.
- [32] J. Bernal, F. Medina, and R. R. Boix, "Full-wave analysis of nonplanar transmission lines on layered medium by means of mpie and complex image theory," *IEEE Trans. on Microwave Theory Tech.*, vol. 49, pp. 177–185, January 2001.
- [33] N. Hojjat, S. Safavi-Naeini, and Y. L. Chow, "Numerical computation of complex image green's functions for multilayer dielectric media: near-field zone and the interface region," *IEE Proceedings*, vol. 145, pp. 449–454, December 1998.
- [34] M. E. Yavuz, M. I. Aksun, and G. Dural, "Critical study of the problems in discrete complex image method," *IEEE Proc. Int. Symp. on Electromagnetic Compat.*, Istanbul, Turkey, May 2003.
- [35] M. I. Aksun, M. E. Yavuz, and G. Dural, "Comments on the problems in dcim," *IEEE Proc. Int. Symp. on Electromagnetic Compat.*, Columbus, OH, USA, June 22-27 2003.
- [36] J. R. Mosig and F. E. Gardiol, "Analytical and numerical techniques in the green's function treatment of microstrip antennas and scatterers," *IEE Proceedings*, vol. 130, no. 2, pp. 175–182, March 1983.
- [37] J. Bernal, F. Medina, R. R. Boix, and M. Horno, "Fast full-wave analysis of multistrip transmission lines on mpie and complex image theory," *IEEE Trans. on Microwave Theory Tech.*, vol. 48, pp. 445–452, March 2000.
- [38] K. A. Michalski and J. R. Mosig, "Multilayered media green's functions in integral equation formulations," *IEEE Trans. on Antennas and Propagation*, vol. 45, pp. 508–519, March 1997.

- [39] F. Demuyne, G. A. E. Vandenbosch, and A. den Capelle, "The expansion of wave concept-part i: Efficient calculation of spatial green's functions in stratified dielectric medium," *IEEE Trans. on Antennas and Propagation*, vol. 46, no. 3, pp. 397–406, March 1998.
- [40] N. Hojjat, S. Safavi-Naeini, R. Faraji-Dana, and Y. L. Chow, "Fast computation of the nonsymmetrical components of the green's functions for multilayer media using complex images," *IEE Proceedings*, vol. 145, pp. 285–288, August 1998.
- [41] K. A. Michalski, "On the scalar potential of a point charge associated with a time-harmonic dipole in a layered medium," *IEEE Trans. on Microwave Theory Tech.*, vol. 35, no. 5, pp. 1299–1301, Nov 1987.
- [42] M. I. Aksun and R. Mittra, "Estimation of spurious radiation from microstrip etches using closed-form green's functions," *IEEE Trans. on Microwave Theory Tech.*, vol. 40, pp. 2063–2069, November 1992.
- [43] K. Naishadham and P. Misra, "Order recursive gaussian elimination and efficient cad of microwave circuits," *IEEE Proceedings MTT-S International Microwave Symposium*, vol. 3, pp. 1435–1438, Orlando, May 16-20 1995.
- [44] S. S. Inc., *Sonnet User Guide*, 2003.
- [45] A. Corporation, *Ansoft Ensemble Online Help System*, 2000.
- [46] James C. Rautio, "A de-embedding algorithm for electromagnetics," *Int. J. MIMICAE*, vol. 1, pp. 282–287, 1991.
- [47] A. Sommerfeld, *Microstrip Circuits*. New York : John Wiley and Sons, Inc, 1994.
- [48] J. R. James, "Printed antennas," *SBMO Proceedings International Microwave Symposium*, pp. 597–606, Rio de Janeiro, Brazil 1987.

- [49] J. R. James and G. J. Wilson, "Microstrip antennas and arrays, pt. i- fundamental action and limitations," *IEE J. Microwaves, Optics, and Acoustics*, vol. 1, pp. 165–174, 1977.
- [50] K. R. Carver and J. W. Mink, "Microstrip antenna technology," *IEEE Trans. on Antennas and Propagation*, vol. 29, pp. 2–24, January 1981.
- [51] R. J. Mailloux, J. F. McIlvanna, and N. P. Kernweis, "Microstrip array technology," *IEEE Trans. on Antennas and Propagation*, vol. 29, pp. 25–37, January 1981.
- [52] D. H. Schaubert, F. G. Farrar, A. Sindoris, and S. T. Hayes, "Microstrip antennas with frequency agility and polarization diversity," *IEEE Trans. on Antennas and Propagation*, vol. 29, pp. 118–123, January 1981.
- [53] W. F. Richards, Y. T. Lo, and D. Harrison, "An improved theory for microstrip antennas and applications," *IEEE Trans. on Antennas and Propagation*, vol. 29, pp. 38–46, January 1981.
- [54] W. F. Richards and Y. T. Lo, "Theoretical and experimental investigation of a microstrip radiator with multiple lumped linear loads," *Electromagnetics*, vol. 3, pp. 371–387, 1983.
- [55] K. A. Michalski and D. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media, part i: Theory," *IEEE Trans. on Antennas and Propagation*, vol. 38, pp. 335–344, March 1990.

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