

**DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED
SCIENCES**

**COMPARISON OF CONTROL CHARTS FOR
AUTOCORRELATED DATA**

**by
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İZMİR**

COMPARISON OF CONTROL CHARTS FOR AUTOCORRELATED DATA

**A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
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**by
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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled “**COMPARISON OF CONTROL CHARTS FOR AUTOCORRELATED DATA**” completed by **ŞEBNEM DEMİRKOL** under supervision of **PROF.DR. G. MİRAC BAYHAN** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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Şebnem Demirkol

COMPARISON OF CONTROL CHARTS FOR AUTOCORRELATED DATA

ABSTRACT

As a result of improvements in measurement techniques, sampling intervals become shorter, and this causes serial correlation in data. Also, in some process industries like chemical manufacturing and refinery operations serial correlation is inherent in consecutive measurements. To deal with this challenge, the traditional control charts are improved or new control charts are developed in the last few decades. Residual control charts such as $\bar{X}$ residual and EWMA residual are widely used control charts for autocorrelated data. In recent years, EWMAST, ARMAST, and DFTC charts have been also introduced for this type of data. To compare the performances of control charts have attracted interest of researchers. In the relevant literature, although there have been a lot of comparison studies, in only few of them the first-order autoregressive moving average (ARMA(1,1)) process have been investigated.

The objective of this research is to compare performances of Shewhart $\bar{X}$, CUSUM, $\bar{X}$ residual, EWMA residual, EWMAST, ARMAST, and DFTC charts for ARMA(1,1) process when the mean shifts. Performance criterion used for this comparison is the average run length (ARL).

Keywords: Control charts, Serial correlation, Average run length, Autoregressive moving average process, Comparison

OTOKORELASYONLU VERİLER İÇİN KONTROL KARTLARININ KIYASLANMASI

ÖZ

Günümüzde ölçme tekniklerinin gelişmesiyle birlikte örnekleme aralıkları da kısalmıştır, bu durum veri seti içinde otokorelasyona yol açmaktadır. Ayrıca, kimyasal süreçlerde ve rafineri operasyonlarında olduğu gibi bazı endüstrilerde otokorelasyon sürecin doğasından kaynaklanmaktadır. Otokorelasyonla başa çıkabilmek için, son dönemlerde mevcut kalite kontrol kartları iyileştirilmekte veya yeni kontrol kartları geliştirilmektedir. X artık ve EWMA artık gibi kontrol kartları otokorelasyonlu gözlemler için sıkça kullanılan kartlardandır. Bunların dışında, son yıllarda bu tip gözlemler için EWMAST, ARMAST ve DFTC kartları geliştirilmiştir. Kontrol kartı kıyaslamaları araştırmacılar tarafından büyük ilgi görmüştür. Literatürde birçok kıyaslama çalışması bulunmasına rağmen, bu çalışmaların pek azında otokorelasyon yapısı birinci dereceden otoregresif hareketli ortalama modeline uymaktadır.

Bu araştırmanın amacı, Shewhart X , CUSUM, X artık, EWMA artık, EWMAST, ARMAST, ve DFTC kartlarını ARMA (1,1) süreci için ve süreç ortalamasından sapma olduğu durumda kıyaslamaktır. Kıyaslama için kullanılan performans ölçüsü ortalama koşum uzunluğudur.

Anahtar sözcükler: Kontrol kartları, Seri korelasyon, Ortalama koşum uzunluğu, Otoregresif hareketli ortalama, Kıyaslama

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CHAPTER ONE

INTRODUCTION

In this chapter, the background, motivation and objectives of this work are stated, and the organization of this dissertation is outlined.

1.1 Background and Motivation

The concept quality is as old as the industry itself. However, especially in last few decades, quality has taken much attention as a result of globalizing world. Due to the global economic conditions, companies compete in the overall world and customer satisfaction, which depends on high quality products and services, is vital for companies. Regardless of whether the consumer is an individual or an industrial organization quality is the key factor for customer satisfaction. Companies which want to survive in the competitive economy should regard this important customer decision factor, quality.

One of the essentials of producing high quality low cost products is to adopt and apply the Statistical Process Control (SPC) correctly. SPC is a tool for achieving and improving quality standards. One of the most useful properties of SPC is that it can be applied to any process. The most important and sophisticated tool of SPC is control charts.

In the use of traditional control charts the most important assumption is that the observations on process or product characteristics are independent. However, as a result of improvements in measurement techniques sampling intervals become shorter, and this causes serial correlation in data. Besides this, in some process industries like chemical manufacturing, refinery operations, wood product manufacturing, nuclear processes, and forging operations serial correlation is inherent in consecutive measurements. In this case, traditional control charts such as Shewhart $\bar{X}$ and CUSUM, estimate process parameters with bias, and this causes poor ARL performance such as high false alarm rates and slow detection of process

shifts. Under such conditions, the traditional control charts can still be used, but they will be ineffective.

Therefore, some modifications for traditional control charts are necessary. Residual control charts such as X residual and EWMA residual are widely used modified control charts for autocorrelated data (for further reading see Alwan & Roberts, 1988; Montgomery & Mastrangelo, 1991; Wardel et al., 1994; Runger & Willemain, 1995; Atienza et al., 1997; Reynolds & Lu, 1997; Zhang, 1997). In the last decade, EWMAST, ARMAST, and DFTsC charts have been also introduced for this type of data (see Zhang, 1998; Zhang, 2000; Jieng et al., 2000; Jiang & Tsui, 2001, Winkel & Zhang, 2004; Kim et al., 2006; Kim et al., 2007).

1.2 Research Objective

In the relevant literature, various control charts have been developed for monitoring autocorrelated processes, and their performances have been compared with each other. Although the first-order autoregressive moving average ARMA(1,1) process takes place commonly in real life industries, it has been investigated in only few of research studies. Furthermore, there seems to be no earlier study testing traditional, residual and distribution free control charts (DFTC) together for ARMA(1,1). In this study, we compare the performances of Shewhart X , CUSUM, X residual, EWMA residual, EWMAST, ARMAST, and DFTC control charts for an ARMA(1,1) process when the mean shifts. We use average run length (ARL) as performance criterion.

1.3 Organization of the Thesis

This dissertation is organized as follows. In chapter two, an overview of statistical process control and its tools are given. Chapter three includes detailed explanation of time series and control charts in the presence of data correlation. Also, a review of the recent works on control chart applications in autocorrelated processes are given, and both theoretical developments and practical experiences are discussed. Chapter 4

presents an extensive comparison of the different control chart implementations in the presence of autocorrelation. Finally, chapter five concludes the dissertation.

CHAPTER TWO

STATISTICAL PROCESS CONTROL

In this chapter, basic definitions and seven tools of statistical process control (SPC) which is called the “magnificent seven” are presented.

2.1 Statistical Process Control

It is better to define the basic concepts of statistical process control (SPC), before giving definition of it. In the following subsection we give the basic concepts of SPC, and then we explain what SPC means. After defining the SPC, we will give a brief overview on statistical process control charts in the next section.

2.1.1 The Basic Concepts

One of the fundamental concepts of SPC is *quality*. The word quality is often used to signify excellence of a product. The term product consists of manufactured goods such as refrigerators, computers, and automobiles and services such as hotel industry, transportation, and health care.

Basically, quality is the level of meeting the requirements of customer. On the other hand, by the customer’s point of view, quality is the customers’ perception of the value of the suppliers’ output. Here the term customer not only means the end user, it also involves the consecutive offices, stations, or departments of a company.

The term quality has been defined by many scientists. The most common definition of quality is given by Juran (1999) as “fitness for purpose or use”. Various scientist highlight different properties of quality. Walter Shewhart (1931), who introduced the idea of control charts, attracts attention in sides of quality. He states that quality has both an objective and a subjective part. While the objective side of quality deals with measurement specifications and minimizing variation, the subjective side deals with the commercial value and esthetics. In addition, W.

Edwards Deming (1988) emphasize on the beholder. According to him, “quality is in the eyes of the beholder”. He expresses that quality has different denotations for the end user and the industry.

Although there have been various definitions of quality in the literature, there are also some common items such as:

- *Customer satisfaction* is the fundamental value in quality whether the customer is an end user or not,
- *Measurement techniques* which are used for determining the quality level, and
- *Standards* facilitate to achieve and stable the desired quality level,

Another important concept is the process. A process is a set of causes, which work together to constitute a given result. In other words, a process is the transformation of a set of inputs, into desired outputs (Thomas et al. 1984, Oakland 2003).

The term output, which takes place in the above definitions, is processed inputs that are transferred to the customer. Through the definition of process, we can conclude that organizations have to monitor and analyze the process to fulfill customer satisfaction. Analyzing the process can only be formed by determining and controlling the inputs to the process perfectly. Also, specifying the purpose of the process is vital essential, because by this way the inputs can be set correctly, and the customer requirements can be achieved perfectly.

A simple model of a process is shown in Figure 2.1 (Oakland, 2003). It is shown that the inputs participate into the process and turn out as the outputs. Whether the process belongs to a manufacturing or service industry, the fundamental inputs are common: materials (paper, computers, raw materials, work in processes, etc...), methods / procedures (including instructions), information, people (skills, training, knowledge), and the environment. It is important to get feedback from the customer when setting the required inputs and executing the process.

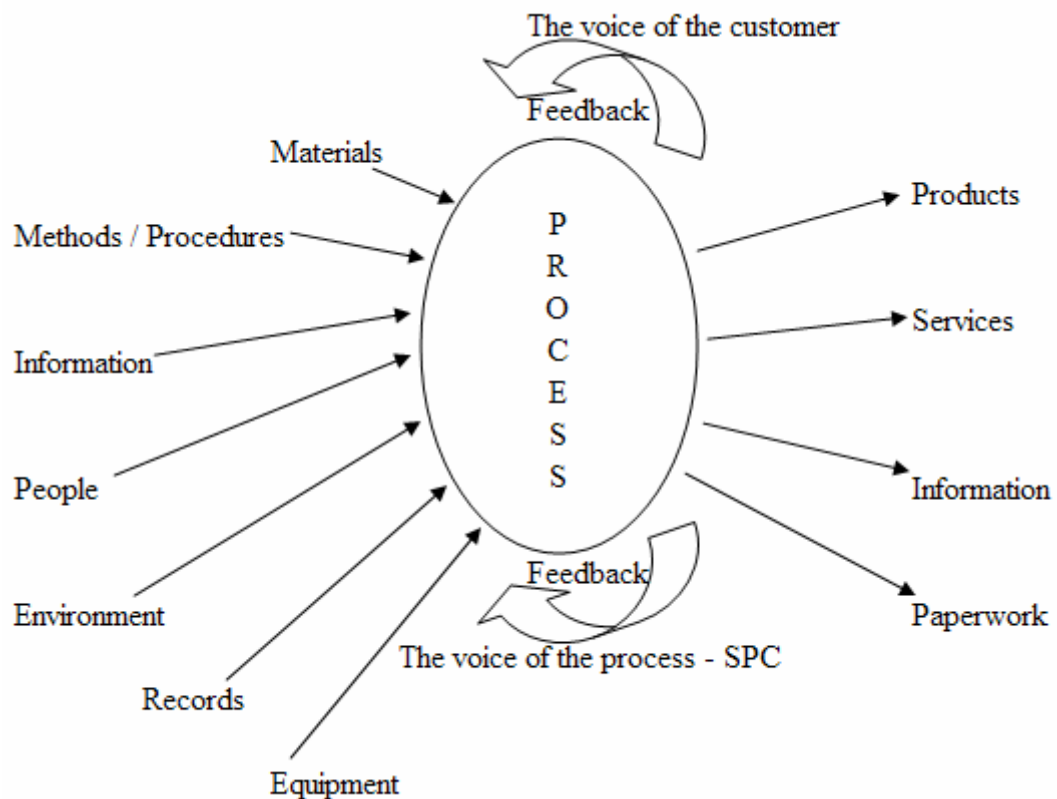


Figure 2.1 A process (Oakland, 2003).

Also, fundamental outputs of various processes are: products, services, information, or paperwork. The documentation of procedures about the process must be collected (the voice of the process) to monitor and control process correctly. The aim of monitoring and controlling a process is to reduce the *process variation*.

All processes exhibit variation. The probability of two things being exactly the same in a process is nearly zero. Even though the difference between two manufactured parts is very small, they are still different. Two units of a product which is produced by a manufacturing process can not be identical. There is always some process variation. For instance, the net content of a unit of toothpaste varies mildly from one to another (Montgomery, 1997; Griffith, 2000). There are two kinds of process variation in the literature: random variation and nonrandom variation.

Random variation occurs in every production process regardless of how well designed or adequately maintained it is. This natural variability or *background noise* is uncontrollable. These variations are referred as stable system of chance causes

(Banks, 1989). In other words, random variation is variability of a process caused by many fluctuations or chance factors that are *chance* (or *common*) *causes*. These common causes cannot be anticipated, detected, identified, or corrected easily. Random variation is inherent to a process.

On the other hand, *nonrandom variation* which is also said to be abnormal variation is fluctuations not inherent to a process. This kind of variability occurs in the output of the process. Such variability is generally large when compared to the random variation, and it usually represents an unacceptable level of process performance. *Assignable* (or *special*) *causes* engender this type of variation, and they can be detected, identified, and eliminated. These assignable causes in key quality characteristics are usually associated with the machines, the operators, or the materials of a process.

It can be concluded from the above definitions that quality is inversely proportional to variability. Hence, the way of improving quality is reducing the process variability.

Because of the reason that all of the manufacturing industry and the service industry processes have random or nonrandom variations, companies have to keep the variability of the processes at a reasonable level. The act of keeping the variation at a reasonable level is called *control*. Control is a management process which can be applied by using at least one of these following actions:

- i. actual performance is compared with planned performance,
- ii. difference between the two is measured,
- iii. causes contributing to the difference are identified, and
- iv. corrective action is taken to eliminate or minimize the difference.

There are two types of processes relevant to control: in control processes and out of control processes. A process which operates in the presence of only chance causes

of variation is said to be statistically in control, and a process which operates with assignable causes of variation is said to be statistically out of control.

There is also one more important term, *quality characteristics*. Quality characteristics are the parameters of quality which describe what the consumer thinks of as quality.

2.1.2 Definition of SPC

There are several definitions of SPC given by various authors in the literature. Montgomery (1997) defines SPC as a powerful collection of problem-solving tools which is useful in achieving process stability and improving capability through the reduction of variability. Ledolter, & Burrill (1999) also focuses on reducing the variability like Montgomery. They state that unusual process behavior can be determined and detected by implementing SPC. Smith (2004) describes SPC from another point of view. He denotes that SPC consists of collected, organized, analyzed, and interpreted data which can be fix a process at a desired level of quality.

It can be concluded from the above definitions that the major objective of SPC is the elimination or reduction of the variation in all processes such as in products, in times of deliveries, in ways of doing things, in materials, in people's attitudes, in equipment and its use, in maintenance practices, in everything. In addition to this main purpose, the followings are also fundamental objectives of SPC:

- Meeting the customer requirements and fulfilling customer satisfaction by achieving process stability.
- Minimizing production costs by embracing the “do it right the first time” philosophy. By avoiding defective products, costs associated with corrective actions are eliminated.

- Supporting the participation of all the employees from bottom to top into decisions and actions about the process.
- Include all members of the organization into continuous process improvement.

As a result, SPC is a tool for achieving and improving quality standards and it can be applied to any process. The basic SPC problem solving tools, the magnificent seven is described briefly in the following.

1. *Histogram*: Histogram is a bar graph which shows the frequency of the specific measurements of the quality characteristics.
2. *Check sheet*: A sheet which is used for collecting data in the early stages of an SPC implementation. It categorizes problems or defects by gathering information about everything relevant to the process; the type of the data, the operation number, the date, the analyst, etc... The check sheet output can be input for a Pareto chart, or for time series analysis.
3. *Pareto chart*: A chart which is simply a bar graph (histogram) of the number of occurrences of specific problems. Pareto chart determines not the most important, but the most frequently occurred defects. The largest bar indicates the most frequent problem. Pareto chart is commonly used in nonmanufacturing processes.
4. *Cause and effect diagram*: After a defect or a problem occurs, cause and effect diagram (or Ishikawa diagram, or fishbone diagram) analyzes the problem (effect) by considering potential causes. The diagram seeks for the root cause of the problem. Although it is suitable for any problem, it is commonly used for processes whose causes are not obvious. Cause and effect diagram is a powerful SPC tool which enables employees to participate into solution of the problem.

5. *Defect concentration diagram*: A diagram which illustrates a picture of the unit by showing all relevant views. In this diagram different kinds of defects are drawn on the picture. It is used for determining the location and potential causes of the defects on the unit.
6. *Scatter diagram*: A useful diagram which plots pairs of measurements on a two dimensional coordinate system. It is used to determine the potential relationship between two variables.
7. *Control chart*: Control chart is the most important and sophisticated one of the magnificent seven. We state the definition of the control chart in the next section.

2.2 Statistical Process Control Charts

A control chart is a time-sequence plot of an important quality characteristic in a process which illustrates how the characteristic behaves over time. Samples are taken, checked, or measured at periodic intervals, and the results are plotted on the chart. The charts can show the change in quality characteristic, the variation in measurements, or the change in proportion of defective pieces over time (Ledolter & Burrill, 1999; Smith, 2004).

The major objective of a control chart is to find assignable cause, in other words nonrandom variation in the process. Control chart is a very powerful SPC tool that indicates the source of the variation, gives hints to the cause of the variation, and makes the employee take the action. By this way, control chart keeps the process statistically in control. Nevertheless, it should not be forgotten that the control chart only detects the assignable cause. It is up to management, engineer, and operator to eliminate the cause. Also, when eliminating the assignable cause, it is important to find the underlying root cause of the problem. Otherwise, cursory solutions will not improve the process in the long term.

Control charts are widely used as a SPC tool in manufacturing and service industries. They are also important tools in six sigma applications. They are applied in control part of the *DMAIC* methodology. DMAIC is the continuous improvement methodology of six sigma. It is an acronym of its steps: define, measure, analyze, improve, and control.

2.2.1 The Basic Principles

A typical control chart is shown in Figure 2.2, which is a graphical display of a quality characteristic that has been measured or computed from a sample versus to the sample number or time. The chart contains a center line (CL) that represents the average value of the quality characteristic corresponding to the in-control state. Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL), are also shown on the chart. Upper control limit (UCL) is the top limit for plotted quality characteristic before the process terminates in-control situation. Lower control limit (LCL) is the bottom limit for plotted quality characteristic before the process terminates in-control situation. Center line (CL) is the horizontal line which represents the mean of the quality characteristic.

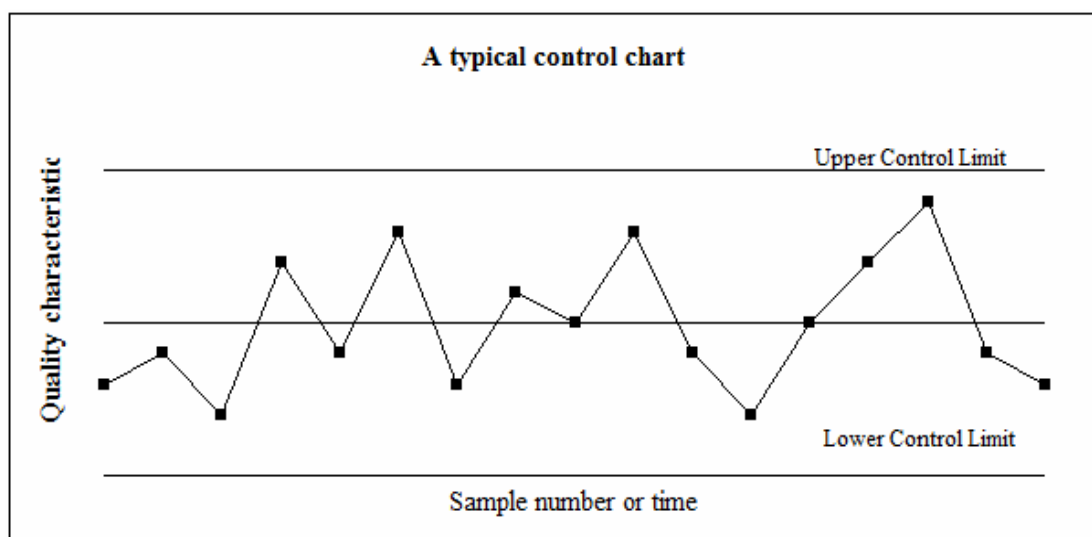


Figure 2.2 A typical control chart for in control processes (Montgomery, 1997).

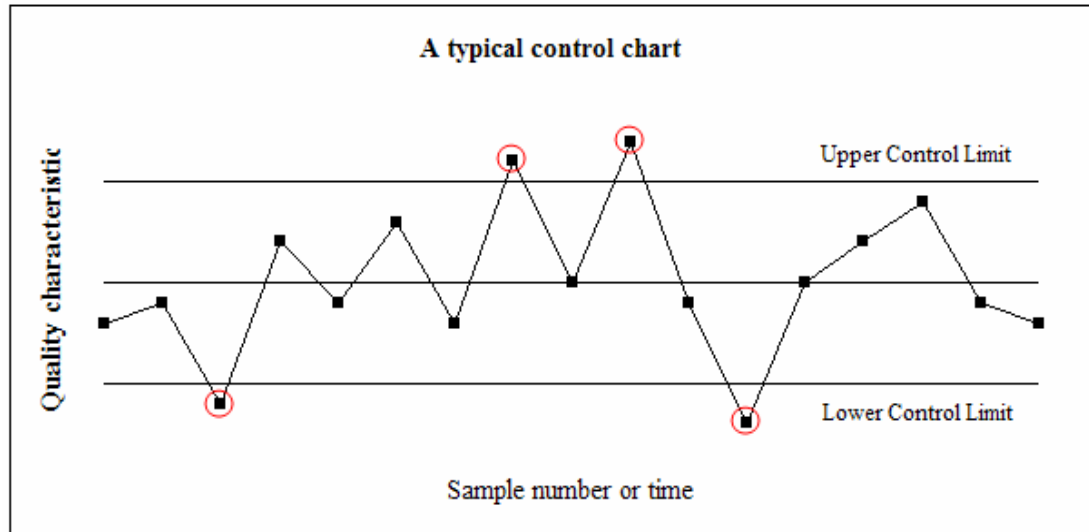


Figure 2.3 A typical control chart for out of control processes.

When a process is statistically in control, almost all of the plotted values of the quality characteristic should have a random pattern and have to fall between the control limits (Figure 2.2). If one or more of the points fall outside of the control limits, it denotes that the process is out of control (Figure 2.3). Then the special cause or causes of this out of control situation are must be investigated immediately, and the corrective action should be taken. This brings about the basic question: *‘When should we take corrective action, and when should we leave the process alone? In other words, how should we choose the control limits?’* Determining the control limits is vital because, if the control limits are estimated so far from the center line, there will be a great deal of risk of a point falling between the control limits when the process is out of control. On the other hand, there will be less risk of a point falling outside the control limits when the process is in control. Also, if the control limits are estimated so close to the center line, the opposite of the above effects will be occurred. Due to the fact that scientist want to minimize all of the above risks, “3-sigma” control limits are widely used in literature for calculating the control limits. If the process parameters are known, control limits can be calculated from the following equations:

$$UCL = \mu + 3\sigma \quad (2.1)$$

$$LCL = \mu - 3\sigma \quad (2.2)$$

where μ and σ are the process mean and the process standard deviation, respectively. Montgomery (1997) points that “sigma refers to the standard deviation of the statistic plotted on the chart ($\sigma_{\bar{x}}$), not the standard deviation of the quality characteristic”.

However, process parameters are unknown in practice and they have to be estimated. In such conditions that the process parameters are unknown estimators are used. Sample mean and sample standard deviation are suitable estimators for process mean (μ) and process standard deviation (σ), respectively. Let $z_1, z_2, \dots, z_n$ are observations of a sample of size n , the sample mean, variance and standard deviation are as follows:

Sample mean:

$$\bar{z} = \frac{z_1 + z_2 + \dots + z_n}{n} = \frac{\sum_{i=1}^n z_i}{n} \quad (2.3)$$

Sample variance:

$$s^2 = \frac{\sum_{i=1}^n (z_i - \bar{z})^2}{n-1} \quad (2.4)$$

Sample standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^n (z_i - \bar{z})^2}{n-1}} \quad (2.5)$$

The statistics $\bar{z}$ measures the central tendency of the sample while s (or s^2) measures variability.

Even though the control chart is a very powerful SPC tool, it is not recommended to use a control chart in every process. There is no need to use a control chart for a process which it is highly unlikely that the process could ever go out of control. A control chart should be used in a process where a problem is likely to occur. In other words, it should be worth to implement a control chart. Also, when a control chart is implemented it must reduce the costs relevant to the quality defects significantly. Thus, management can understand the importance of the control charts and support their continuously use.

We stated that the control charts are very powerful, important and sophisticated SPC tools in the beginning of this section. Now, we explain the reasons for that. Put another way, we state the benefits from using control charts in the following:

- The most important benefit is improving the process. Process improvement using the control chart is given substantially in figure 2.4 (Montgomery, 1997). Because most processes are statistically out of control, control charts improve their quality level by identifying assignable causes. Once assignable causes are identified, then the employees responsible for the process should take the action to eliminate them.

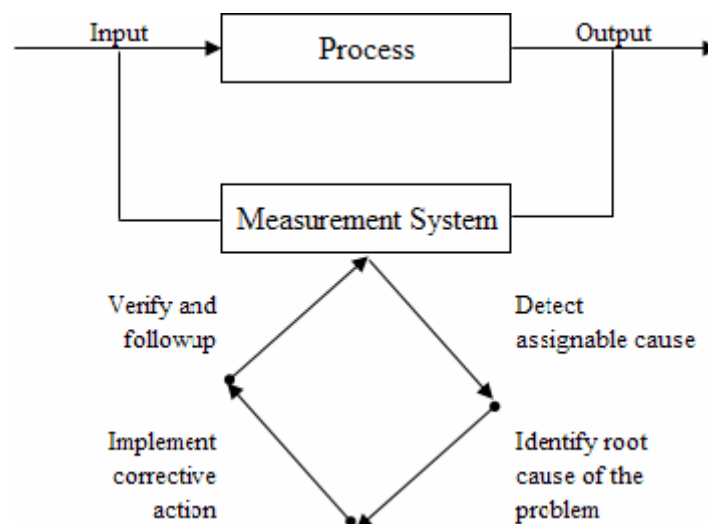


Figure 2.4 Process improvement using the control chart (Montgomery, 1997).

- Control charts reduce costs related with quality problems, because they are very effective in detect prevention. It is more expensive to identify defective goods from rest of them. So, it is more effective to prevent detects. Montgomery (1997) states that “if you do not have effective process control, you are paying someone to make a nonconforming product”.
- Control charts focus on the process rather than the product. Through this feature of control charts, they help us to conclude a defective product is due to defective process, or due to defective workmanship. In other words, control charts provide diagnostic information.
- Control charts prevent unnecessary process adjustments, because they can distinguish between the background noise and nonrandom variation. By this way control charts prevent employees from overreacting to the background noise.
- The use of control charts forces communication between important issues of a process such as standards, measurements, and so forth; and establishes responsibility on employees among these important issues.
- Process capability studies which have significant impact on many manufacturing decision problems can done by using control charts. That is through the control charts are also used as an estimating device. By this way, we can estimate certain process parameters such as the mean, standard deviation, fraction nonconforming, and so forth; and then we can determine the process capability by using these estimates. Consequently, control charts provide information about process capability.
- Control charts generate a set of techniques that are applied by the employees. It is better for employees to follow instructions of control charts during the process improvement.

The performances of control charts are measured via *average run length* calculations in the literature. *Run length* (RL) is the number of observations that have to be plotted until one of the plotted observations exceeds the control limits for the first time. Due to the random nature of observations, the run length follows a probability distribution which is called *run length distribution*.

Average run length (ARL) is the mean of the run length, which can be defined as the average number of observations before an out of control signal occurs. When a process is statistically in-control, namely has no shift from the mean, an out-of signal will be regarded to be false alarm. That means type I error, which is concluding the process is out of control when it is in control, occurs. Also, when a process is out-of-control, an out of control signal must warn as soon as possible to detect the shift quickly. By this way, occurrence of type II error, that is concluding the process is in control when it is out of control, can be prevented. So, the in-control ARL value must be large and the out-of-control ARL value must be small for avoiding both type I and type II errors. We will interpret the ARL calculations of various control charts in chapter four.

2.2.2 Development and Implementation of Control Charts

Control charts are classified into two general types in the literature: variables control charts and attributes control charts. In this section we clarify some control charts (Shewhart $\bar{X}$, EWMA, CUSUM) whose performances will be compared in chapter four.

A *variable* is a quality characteristic which can and expressed as a number on some continuous scale of measurement (Montgomery, 1997). A variable can be a dimension, weight, volume, and so forth. If the quality characteristic is a variable, it is suitable to control it with a measure of central tendency and a measure of variability. This kind of control charts are called variables control charts. Besterfield (2004) defines the control chart for variables as a means of visualizing the variations

that occur in the central tendency and dispersion of particular quality characteristic in a set of observations. The types of variables control charts are listed in the following:

- Shewhart $\bar{X}$ charts ($\bar{X}$)
- Average and range charts ($\bar{X}$ and R)
- Median and range charts ($\tilde{X}$ and R)
- Average and standard deviation charts ($\bar{X}$ and S)
- Individual and moving range charts (X and MR)
- Run charts
- S^2 charts

Shewhart $\bar{X}$ chart firstly introduced by Dr. Walter A. Shewhart in 1920s and is attracted many scientists' interest. Since the first statistical control charts, $\bar{x}$, $\bar{x}$ and R , $\bar{x}$ and S , were introduced by him, these charts have been also called the Shewhart control charts.

The Shewhart $\bar{X}$ chart which is the basis for many control charts is very simple and easy to use. . If $x_1, x_2, \dots, x_n$ is a sample of size n , the center line (CL), upper control limit (UCL), and lower control limit (LCL) of the Shewhart $\bar{X}$ chart is given, respectively in the following:

$$CL = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (2.6)$$

$$UCL = \bar{x} + 3\sigma \quad (2.7)$$

$$LCL = \bar{x} - 3\sigma \quad (2.8)$$

where $\bar{x}$ is the mean and σ is the standard deviation of the process. We assume that σ is known or an estimate is available.

We must treat the control limits as trial control limits. Trial control limits are useful for determining whether the process was in control or not when the initial observations were selected. According to this definition, it can be concluded that if all points plot inside the control limits, the process was in control in the past, and this means that the trial control limits can be used for controlling current and future production. Contrarily if some points plot outside the control limits, this means that the process was out of control in the past. In such a situation, every single assignable cause must be examined, and eliminated. After eliminating one of the assignable causes, the point belong to it must be ignored and the trial control limits should be recalculated. This process is continued until all points plot inside the limits. This analysis of past data is referred to *Phase I* analysis. After Phase I analysis, the obtained control limits are used in *Phase II*. Phase II is using the limits for current and future monitoring.

There is another concept in Shewhart $\bar{X}$ charts which named *rational subgroup* by Dr. Shewhart. Rational subgroup is the collection of sample data in subgroups or samples. The purpose of the rational subgroup concept is to maximize the differences between subgroups when assignable causes occur; and to minimize the differences within a subgroup. However, using rational subgroups are ineffective due to the lack of time and costs in industrial world. The case of individual observations occurs very often in practice. So, we prefer to use individual observations rather than rational subgroups in this dissertation.

Shewhart control charts have been used in practice for decades because; they do not need deep statistical knowledge, they are easy to use and interpret. Besides these advantages, Shewhart charts have also some disadvantages. Crucial issue of any Shewhart control chart is that it only takes into consideration the last plotted point, and can not contain information about the whole process. Because of this feature, Shewhart charts are usually effective for detecting large shifts but ineffective for detecting small shifts (about $1,5\sigma$ or less) in process parameters.

There is also one more type of control charts: attributes control charts, as it is expressed in the beginning of this section. Contrary to variables, many quality characteristics cannot be represented numerically. This type of quality characteristics is called attributes. “Attributes control charts use pass-fail information for charting. An item passes inspection when it conforms to the standards; a nonconforming item fails inspection” (Smith, 2004). The types of attributes control charts are listed in the following:

- Control charts for fraction nonconforming (p charts)
- Control charts for number nonconforming (np charts)
- Control charts for nonconformities (c charts)
- Control charts for nonconformities per unit (u charts)

As we mentioned before, there is an important shortcoming relevant to Shewhart charts which is they are ineffective for detecting small shifts. To overcome this disadvantage two different control charts, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA), are proposed. They are appropriate for detecting small shifts, because they give smaller weight to the past data. However, they do not react to large shifts as quickly as the Shewhart chart.

The cumulative sum (CUSUM) chart was firstly introduced by Page in 1954. The CUSUM chart was developed in Britain and is one of the most powerful management tools available for the detection of trends and slight changes in data.

Let $x_1, x_2, \dots, x_n$ is a sample of size n , $\bar{x}_j$ is the average of the j^{th} sample, and μ_0 is the target for the process mean. Then the CUSUM control chart is formed by plotting the following quantity:

$$C_i = \sum_{j=1}^i (\bar{x}_j - \mu_0) \quad (2.9)$$

C_i is called the cumulative sum of the i^{th} sample. As it is seen from the equation above CUSUM charts contain information from several samples. Consequently, CUSUM charts are more effective than Shewhart charts for detecting small process shifts.

Equation 2.9 above calculates the plotted points of CUSUM. Of course, it needs control limits to be a control chart. There are two representations of CUSUM charts related to control limits: the *tabular* (or *algorithmic*) CUSUM, and the *V-mask procedure*. Due to the various disadvantages of the V-mask procedure, it is recommended to use tabular CUSUM chart in the literature.

If X_i is the i^{th} observation of the process, it is assumed that X_i has a normal distribution with mean μ_0 and standard deviation σ when the process is in control. It is also assumed that σ is known or an estimate is available.

In general, μ_0 is interpreted as a target value for the quality characteristic x . The tabular CUSUM is formed by accumulating derivations from μ_0 that are above target with the statistic C^+ and accumulating derivations from μ_0 that are below target with the statistic C^- . The statistics C^+ and C^- are called *one sided upper and lower* CUSUMs, respectively. They are computed as follows:

$$C_i^+ = \max[0, x_i - (\mu_0 + K) + C_{i-1}^+] \quad (2.10)$$

$$C_i^- = \max[0, (\mu_0 - K) - x_i + C_{i-1}^-] \quad (2.11)$$

where K is the *reference value* (or the *allowance*, or the *slack value*). K is usually selected the half of the process shift in the literature, as the following equation:

$$K = \left| \frac{\mu_1 - \mu_0}{2} \right| = \frac{\delta}{2} \sigma \quad (2.12)$$

where μ_1 is the out of control value of the process mean ($\mu_1 = \mu_0 + \delta$), and δ is the amount of the process shift ($\delta = \left| \frac{\mu_1 - \mu_0}{\sigma} \right|$).

If neither C^+ nor C^- exceeds the decision interval H , the process will be statistically in control. The value of H is suggested as five times the process standard deviation (σ) in the literature.

When any out of control signal is occurred on a CUSUM control chart, on a CUSUM control one should search for the assignable cause, take any corrective action required, and then reinitialize the CUSUM at zero. In such situations which the process needs to be back to the target value μ_0 by a corrective action, it may be helpful to have an estimate of the new process mean including the shift. This can be computed as:

$$\hat{\mu} = \begin{cases} \mu_0 + K + \frac{C_i^+}{N^+}, & \text{if } C_i^+ > H \\ \mu_0 - K - \frac{C_i^-}{N^-}, & \text{if } C_i^- > H \end{cases} \quad (2.13)$$

It is obvious that when implementing the CUSUM chart the determination of K and H is vital. Let $H = h\sigma$ and $K = k\sigma$, Hawkins (1993) gives a table of k and the corresponding h values. We select values of k and the corresponding values of h as 0.5 and 4.77, respectively throughout this dissertation.

Various techniques have been introduced to calculate the ARL of a CUSUM, but the ARL approximation given by Siegmund (1985) is commonly used in the literature due to its simplicity. We also use the Siegmund's approximation in this study. The Siegmund's approximation is as follows:

$$ARL = \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2} \quad (2.14)$$

for $\Delta \neq 0$, where $\Delta = \delta^* - k$ for C_i^+ , and $\Delta = \delta^* - k$ for C_i^- , $b = h + 1.66$, and $\delta^* = (\mu_1 - \mu_0) / \sigma$. If $\Delta = 0$, $ARL = b^2$.

The ARL of the two sided CUSUM is:

$$\frac{1}{ARL} = \frac{1}{ARL^+} + \frac{1}{ARL^-} \quad (2.15)$$

where ARL^+ and ARL^- are one sided statistics.

CUSUM control charts are especially effective with processes whose sample sizes are one ($n = 1$). Due to this feature of CUSUM control charts, they are effectively used in individual observations one such as chemical and process industries, and discrete parts manufacturing with automatic measurement of each part.

The exponentially weighted moving average (EWMA) control chart was proposed by Roberts in 1959. Like CUSUM chart, EWMA is suitable for detecting small process shifts. Due to the structure of the EWMA chart, it gives less weight to farther past data. Even though the performance of the EWMA chart is similar to the corresponding CUSUM chart, EWMA chart is much easy to set up and operate. Such as the CUSUM chart, the EWMA chart is very effective particularly when used with individual observations.

The exponentially weighted moving average is defined as follows:

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1} \quad (2.16)$$

where λ is a constant between (0,1), and the starting value is the process target $z_0 = \mu_0$, or occasionally the average of preliminary data $z_0 = \bar{x}$.

If we substitute z_{i-j} for $j = 2, 3, \dots, t$ in equation 2.16, we get

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \quad (2.17)$$

Then the EWMA control chart is formed by plotting the z_i values versus the sample number i or time. It is understood from the above equation that weight assigned to the past data geometrically decreases. Because of this reason the EWMA control chart is also called the geometric moving average (GMA) control chart.

If the observations x_i are independent random variables with variance σ^2 , then the variance of z_i will be

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right) [1 - (1-\lambda)^{2i}] \quad (2.18)$$

Then the center line and the control limits for the EWMA chart are calculated as follows:

$$UCL = \mu_0 + L\sigma \sqrt{\left(\frac{\lambda}{2-\lambda} \right) [1 - (1-\lambda)^{2i}]} \quad (2.19)$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma \sqrt{\left(\frac{\lambda}{2-\lambda} \right) [1 - (1-\lambda)^{2i}]} \quad (2.20)$$

where L is the width of the control limits. Notice that the term $[1 - (1-\lambda)^{2i}]$ in equations 2.19 and 2.20 approaches unity as i gets larger. It means that if the amount of data is large enough, the variance and the control limits of EWMA will be,

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right) \quad (2.21)$$

$$UCL = \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad (2.22)$$

$$LCL = \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad (2.23)$$

When designing an EWMA control chart, determination of L and λ is vital. “the optimal design procedure would consist of specifying the desired in control and out of control average run lengths and the magnitude of the process shift that is anticipated, and then to select the combination of λ and L ” (Montgomery, 1997).

Generally, it is advised to choose λ between 0.05 and 0.25. Also $L = 3$ is suggested commonly.

Consequently, both of the CUSUM and EWMA control charts perform well in detecting small shifts, but they have poor performance with large shifts. nevertheless, EWMA is usually superior to CUSUM for large shifts particularly if $\lambda > 0.10$ (See Montgomery, 1997).

As mentioned above, the fundamental assumption of these charts is that the observations of the process are independent and identically distributed (iid) normal about a certain mean. However, the independency assumption is not realistic in practice due to various reasons. Control charts for such conditions (autocorrelated processes) will be examined in the next chapter.

CHAPTER THREE

TIME SERIES AND CONTROL CHARTS FOR AUTOCORRELATED PROCESSES

The purpose of this chapter is to present time series analysis, and give an overview of the control charts used for autocorrelated data. We explain time series before examining autocorrelation because time series analysis knowledge will enable someone better understanding of the autocorrelation structure of the process. Then, we review the literature of control charts for autocorrelated processes in a chronological order; discuss both theoretical developments and practical experiences; and also state historical progression of these charts.

3.1. Time Series

As it is stated in the previous chapter, many processes in the real manufacturing, or service environment are autocorrelated. The idea of describing an autocorrelated process by a time series model was firstly introduced by Alwan & Roberts in 1988. They use time series modeling to detect the nonrandom variation, namely assignable causes.

A time series is ordered sequence of observations. It is constructed by plotting the observed variable versus to time. If only one variable is observed, the time series is said to be univariate. Contrarily, if the time series involves simultaneous observations on several variables, it is called the multivariate time series. Multivariate time series analysis is beyond the purpose of this dissertation. A time series can be continuous, such as chemical processes or discrete, such as television manufacturing.

There are three general objectives for studying time series:

1. understanding and modeling of the underlying mechanism that generates the time series,
2. prediction of future values, and

3. control of some system for which the time series is a performance measure.

Examples of the third application occur frequently in industry. Almost all time series exhibit some structural dependency. That is, the successive observations are correlated over time, or autocorrelated. Special classes of statistical methods that take this autocorrelative structure into account are required (Mastrangelo et al., 2001).

For these autocorrelative cases, time series are analyzed and presented by Box, and Jenkins (1976), and are called *Box-Jenkins* (or *ARIMA* – AutoRegressive Integrated Moving Average) models. Box & Jenkins proposed a methodology to find an appropriate $ARIMA(p,d,q)$ model. This methodology consists of three steps: *i)* identification of the model, *ii)* estimation of the parameters, *iii)* diagnostic checking. It is recommended that one must have at least 50 observations available to identify the appropriate model structure.

In this study, we assume that the underlying process is best described by a stationary $ARMA(1,1)$ model. Thus, we give the definition of stationarity and discuss the $ARMA(1,1)$ model. We also study $AR(1)$, and $MA(1)$ processes for a better understanding of the $ARMA(1,1)$ structure in the following subsections.

3.1.1 Stationary and Nonstationary processes

The time series whose variable varies around a constant level, as in Figure 3.1, is stationary. On the contrary, if the variable drifts with no obvious fixed level, such as in Figure 3.2, this means that the process has nonstationary behaviour. Many time series behave as is they have no constant mean; that is, in any local segment of time the observations look like those in any other segment, apart from their average. Such a time series is called *nonstationary in mean* (Montgomery, & Johnson (1976)) Similarly, it is possible for a time series to exhibit nonstationary behavior in *both mean and slope*: that is, apart from the mean and the slope, observations in

different segments of time look very much alike. Examples of nonstationary time series are shown in Figure 3.2.

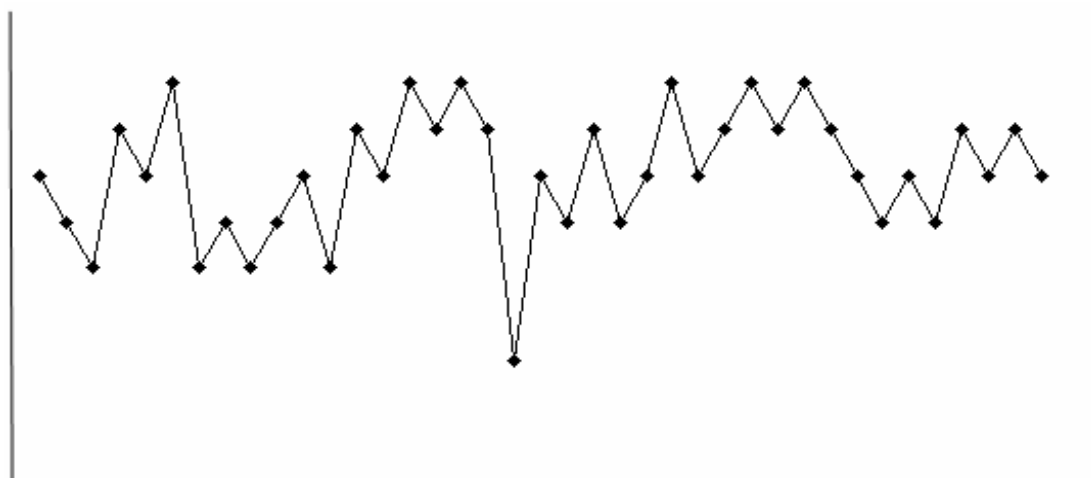
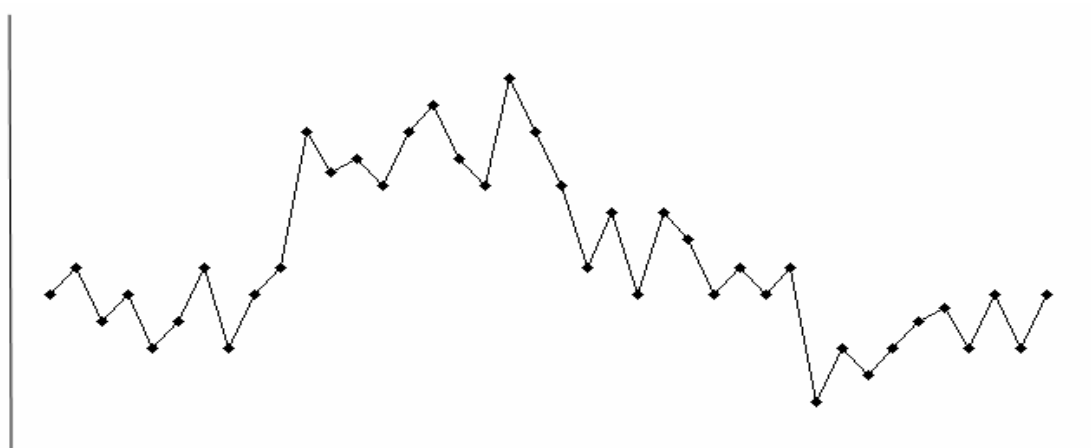
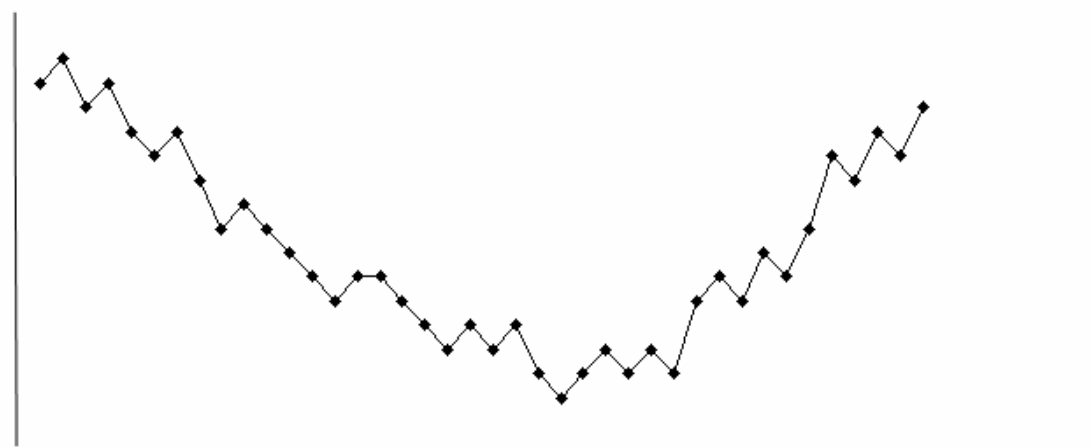


Figure 3.1 A stationary time series.



(a) Time series that is nonstationary in the mean.



(b) Time series that is nonstationary in mean and slope.

Figure 3.2 (a) and (b) Two nonstationary time series.

A general model of nonstationary time series is the autoregressive integrated moving average process of order (p,d,q) (ARIMA(p,d,q)). Nevertheless, a wide variety of time series in practice are from stationary processes. Therefore, we deal with the most common stationary autocorrelated process model, ARMA(1,1) throughout this work.

3.1.1.1 Stationarity and Invertibility

According to Cryer (1986) “the basic idea of stationarity is that the probability laws governing the process do not change with time that is the process in statistical equilibrium”.

“If $\{Z_t\}$ is a stationary series, the mean, variance, and autocorrelation can usually be well approximated by sufficiently long time averages based on a single realization.” (Enders, 1995).

$$E(Z_t) = E(Z_{t-s}) = \mu$$

$$E[(Z_t - \mu)^2] = E[(Z_{t-s} - \mu)^2] = \sigma_y^2 \text{ or } V(Z_t) = V(Z_{t-s}) = \sigma_y^2$$

$$E[(Z_t - \mu)(Z_{t-s} - \mu)] = E[(Z_{t-j} - \mu)(Z_{t-j-s} - \mu)] = \gamma_s$$

$$\text{or } Cov(Z_t, Z_{t-s}) = Cov(Z_{t-j}, Z_{t-j-s}) = \gamma_s \quad (3.1)$$

where μ , σ_y^2 and all γ_s are constants, representing mean, variance and lag s covariance of the process, respectively. “The series $\{Z_t\}$ is invertible if it can be represented by a finite-order or convergent autoregressive process” (Enders, 1995).

3.1.1.2 Autocorrelation and Partial Autocorrelation

Autocorrelation is the dependence in a time series, the term *serial dependence* is also used in the literature for autocorrelation. Autocorrelation is the correlation of one variable at one point in time with observations of the same variable at prior time points.

The k^{th} autocorrelation (denoted by ρ_k) of a covariance stationary process is defined as its autocovariance divided by the variance. The autocorrelation at lag k (ρ_k) refers to the correlation between any two observations in a time series that are k periods apart (Hamilton, 1994; Montgomery, & Johnson, 1976). The autocorrelation at lag k is given in the following equation:

$$\rho_k = \frac{\text{Cov}(z_t, z_{t+k})}{\sqrt{V(z_t)V(z_{t+k})}} = \frac{\gamma_k}{\gamma_0} \quad (3.2)$$

Autocorrelation function $\{\rho_k\}$ of a process is the graphical display of ρ_k versus the lag k . Autocorrelation function is symmetric which results in $\rho_k = \rho_{-k}$. The autocorrelation function value is between -1 and 1 ($-1 \leq \rho_k \leq 1$) and it is dimensionless. Mostly, if observations k lags apart are close together in value, the autocorrelation function value ρ_k will be close to 1. Also, if a large observation at time t is followed by a small observation at time $t+k$, the ρ_k value will be close to -1. Furthermore, if there is little relationship between observations k lags apart, the ρ_k value will be close to 0 (Box & Jenkins, 1976; Montgomery & Johnson, 1976).

Partial Autocorrelation is the correlation of one variable at one point in time with another observation of the same variable at prior time point. In other words, “the partial autocorrelation at lag k is the correlation between z_t and z_{t+k} with the effects of the intervening observations $(z_{t+1}, z_{t+2}, \dots, z_{t+k-1})$ removed” (Montgomery, & Johnson, 1976). k^{th} partial autocorrelation is denoted as the coefficient ϕ_{kk} . *Partial*

autocorrelation function $\{\phi_{kk}\}$ of a process is the graphical display ϕ_{kk} versus the lag k .

3.1.2 The First-Order Autoregressive Process

The first-order autoregressive (AR(1)) process is:

$$Z_t = \xi + \phi Z_{t-1} + \varepsilon_t \quad (3.3)$$

where the distribution of $\{\varepsilon_t\}$ is normal with mean 0 and variance σ_ε^2 , and the sequence of random variables $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots$ is called a white noise process.

The mean, variance, and autocovariance of the AR(1) process are given, respectively in the following:

$$\mu = \frac{\xi}{1 - \phi} \quad (3.4)$$

$$\gamma_0 = E(Z_t - \mu)^2 = \frac{\sigma_\varepsilon^2}{1 - \phi^2} \quad (3.5)$$

$$\gamma_k = E(Z_t - \mu)(Z_{t-k} - \mu) = \left[\frac{\phi^k}{1 - \phi^2} \right] \sigma_\varepsilon^2, \quad k \geq 0 \quad (3.6)$$

Then the autocorrelation function can be found easily from the equations (3.5) and (3.6):

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k, \quad k \geq 0 \quad (3.7)$$

3.1.3 The First-Order Moving Average Process

The first-order moving average (MA(1)) process is:

$$Z_t = \mu + \varepsilon_t - \theta\varepsilon_{t-1} \quad (3.8)$$

where the distribution of $\{\varepsilon_t\}$ is normal with mean 0 and variance σ_ε^2 .

The mean, and variance of the MA(1) process are given, respectively in the following:

$$E(Z_t) = \mu \quad (3.9)$$

$$\gamma_0 = \sigma_\varepsilon^2(1 + \theta^2) \quad (3.10)$$

and using (3.10), the autocorrelation function is:

$$\rho_k = \begin{cases} \frac{-\theta}{1+\theta^2}, & k = 1 \\ 0, & k > 1 \end{cases} \quad (3.11)$$

3.1.4 The First-Order Autoregressive Moving Average Process

While building empirical models of actual time series, scientists (Box & Jenkins, 1976; Montgomery & Johnson, 1976) found that a major part of processes do not fit in a pure autoregressive or a pure moving average forms. Many industrial processes' behavior includes both autoregressive and moving average terms. Consequently, the mixed autoregressive moving average model is suggested.

The first-order autoregressive moving average (ARMA(1,1)) process is:

$$Z_t = \xi + \phi Z_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1} \quad (3.12)$$

where the distribution of $\{\varepsilon_t\}$ is normal with mean 0 and variance σ_ε^2 . The process is stationary and invertible if $|\phi| < 1$ and $|\theta| < 1$, respectively.

The mean, variance, and autocovariance of ARMA(1,1) are:

$$\mu \equiv E(Z_t) = \frac{\xi}{1-\phi} \quad (3.12)$$

$$\gamma_0 = \frac{(1-2\theta\phi+\theta^2)}{1-\phi^2} \sigma_\varepsilon^2 \quad (3.13)$$

$$\gamma_k = \left[\frac{(1-\theta\phi)(\phi-\theta)}{1-\phi^2} \right] \phi^{k-1} \sigma_\varepsilon^2, \quad k \geq 1 \quad (3.14)$$

The autocorrelation function is calculated from (3.13) and (3.14):

$$\rho_1 = \frac{(1-\phi\theta)(\phi-\theta)}{1+\theta^2-2\phi\theta} \quad (3.15)$$

$$\rho_k = \phi \rho_{k-1}, \quad k \geq 2 \quad (3.16)$$

From the equation (3.15) and (3.16) we see that the moving average component affects only the autocorrelation of the first lag, after the autocorrelation function for the first lag autocorrelation decays exponentially from ρ_1 . Thus, the autocorrelation function tails off after lag 1 in ARMA(1,1) processes. Since ρ_1 is composed of two parameters (ϕ, θ) , the values of (ϕ, θ) determine the autocorrelation function value, namely serial correlation of a process. If the value of ϕ is close to 1 when θ is smaller than 0, the process will have high positive autocorrelation even if the magnitude of θ is high or small. If ϕ value is close to -1 when θ value is greater than zero, the process will have high negative autocorrelation. Besides, autocorrelation gets close to zero when the autoregressive and the moving average

process parameters close up each other. When $\phi = \theta$, there will be no autocorrelation. If $\phi = 0$ the process is purely moving average (MA(1)), and if $\theta = 0$ then the process is purely autoregressive (AR(1)). Furthermore, when a process is said to be highly positive (negative) autocorrelated, it means that the first lag autocorrelation coefficient is close to 1 (-1).

As a result, the first q lags of the autocorrelation function of a $ARMA(p,q)$ process is affected by the moving average parameters, and lags greater than q is affected only by the autoregressive parameters (Montgomery, & Johnson, 1976).

3.2 Control Charts for Autocorrelated Processes

In this section, we can examine the need for control charts when processes are autocorrelated and define particular charts used for this purpose.

As it is mentioned in chapter two, the fundamental assumption of traditional control charts is that the observations of the process are independent and identically distributed (iid) normal about a certain mean. However, the independency assumption is not realistic in practice. High speed automatic data collection techniques make the sampling intervals shorter. Shorter sampling intervals cause significant serial correlation (autocorrelation) in observations. Besides this, in continuous flow processes such as chemical manufacturing, refinery operations, wood product manufacturing and nuclear processes serial correlation is inherent in consecutive measurements. When there is significant autocorrelation in a process, traditional control charts with iid assumption will estimate biased process parameters which results in poor ARL performance like high false alarm rates and slow detection of process shifts. Under such conditions, the traditional control charts can still be used, but they will be ineffective. Because of this reason some modifications for traditional control charts are necessary if autocorrelation cannot be ignored. Therefore, various control charts have been developed for monitoring autocorrelated processes. These control charts used for autocorrelated processes have attracted many scientists' interest especially in the last two decades. In the following

subsections we define residual, EWMAST, ARMAST, and DFTC control charts whose performances will be compared later.

3.2.1 Residual Control Charts

The first residual control chart, namely special cause chart (SCC), was introduced by Alwan & Roberts in 1988. The SCC chart is also known as the X residual chart in the literature. In residual charts, forecast errors, namely residuals, are assumed to be statistically uncorrelated. An appropriate time series model is fitted to the autocorrelated observations and the residuals are plotted in a control chart. For this reason all of the well-known control schemes can be transformed to the residual control scheme.

Alwan & Roberts (1988) suggested that in a wide range of applications in which processes are not in control in the sense of iid random variables, one can use relatively elementary regression techniques to identify and fit appropriate time series models. If one succeed in finding such a model, he have reached a negative verdict about statistical control in the sense of iid and can obtain fitted values and residuals along with probabilistic assessments of uncertainty as follows:

$$\text{Actual} = \text{Fitted} + \text{Residual} \quad 3.17$$

When constructing the X residual chart, the centerline is at μ , and the 3-sigma control limits are as follows:

$$UCL = \bar{x} + 3\sigma_{\varepsilon} \quad 3.18$$

$$LCL = \bar{x} - 3\sigma_{\varepsilon} \quad 3.19$$

After Alwan & Roberts (1988), many scientists have interested in residual control charts. Literature survey will be given at the end of this chapter. EWMA residual charts have attracted many scientists' interest among residual control charts.

Reynolds & Lu (1997) defined the EWMA residual which uses a control statistic of the form:

$$Z_t = (1 - \lambda)Z_{t-1} + \lambda e_t \quad (3.20)$$

where e_t is the residual for observation t . The EWMA residual chart is constructed by charting Z_t the centerline is at μ , and the 3-sigma control limits are:

$$UCL = +L\sigma_e \sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad (3.21)$$

$$LCL = -L\sigma_e \sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad (3.22)$$

where L is a constant and σ_e is the standard deviation of e_t .

3.2.2 The Exponentially Weighted Moving Average Control Chart for Stationary Processes (EWMAST)

Exponentially Weighted Moving Average for Stationary Process (EWMAST) control chart has been introduced by Nien Fan Zhang in 1998 to deal with the disadvantages of the residual charts. EWMAST chart is an extension of the traditional EWMA chart and basically constructed by charting the EWMA statistics for stationary process.

Zhang (1998) remarked that the limits of the EWMAST chart are different from that of the traditional EWMA chart when the data are autocorrelated. When the process is positively autocorrelated, the limits of the EWMAST chart are wider than that of the ordinary EWMA chart.

Suppose $\{X_t\}$ is a discrete stationary process with constant mean and autocovariance function, γ_k . That is,

$$E(X_t) = \mu \quad t = 0, 1, 2, \dots \quad (3.23)$$

$$\gamma_k = \text{cov}[X_t, X_{t+\tau}] = E[(X_t - \mu)(X_{t+\tau} - \mu)] \quad (3.24)$$

The autocovariance function of Z_t is given by Zhang (1998) as follows:

$$\begin{aligned} \text{cov}(Z_t, Z_{t+\tau}) = & \frac{\lambda}{2-\lambda} \sigma_x^2 \left\{ \sum_{k=0}^{t-1} \rho_{k+\tau} (1-\lambda)^k [1 - (1-\lambda)^{2(t-k)}] \right. \\ & + \sum_{k=1}^{\tau} \rho_{t-k} (1-\lambda)^k [1 - (1-\lambda)^{2t}] \\ & \left. + \sum_{k=\tau+1}^{t-1+\tau} \rho_{t-k} (1-\lambda)^k [1 - (1-\lambda)^{2(t+\tau-k)}] \right\} \end{aligned} \quad (3.25)$$

where ρ_k is the autocorrelation of X_t at lag k and $\rho_k = \gamma_k / \gamma_0$ from (3.7). Then the variance of Z_t is given below, when $\tau = 0$:

$$\sigma_z^2 = \text{var}(Z_t) = \frac{\lambda}{2-\lambda} \sigma_x^2 \left\{ 1 - (1-\lambda)^{2t} + 2 \sum_{k=1}^{t-1} \rho_k (1-\lambda)^k [1 - (1-\lambda)^{2(t-k)}] \right\} \quad (3.26)$$

The approximate variance of Z_t when t is large enough ($t \rightarrow \infty$) is written as follows:

$$\sigma_z^2 = \text{var}(Z_t) \approx \frac{\lambda}{2-\lambda} \sigma_x^2 \left\{ 1 + 2 \sum_{k=1}^M \rho_k (1-\lambda)^k [1 - (1-\lambda)^{2(M-k)}] \right\} \quad (3.27)$$

where M is a large integer. Assuming that X_t is normally distributed, Z_t is also normally distributed with mean = μ and variance given in (3.26). The EWMAST chart is constructed by charting Z_t . the centerline is at μ , and the 3-sigma control limits are:

$$UCL = \mu + 3\sigma_z$$

$$LCL = \mu - 3\sigma_z$$

where σ_z is the standard deviation of Z_t which can be calculated by taking the square root of (3.26). The approximation given in (3.27) can be used for large t .

When observations are from an iid process, namely $\rho_k = 0$ for $k \geq 1$, the term in braces in (3.26) will be $1 - (1 - \lambda)^{2t}$, and the term in braces in (3.27) will be 1. In this case, equations (2.18) and (2.21) which calculate the standard deviation of EWMA statistic, Z_t , are equal to equations (3.26) and (3.27), respectively. Thus, the ordinary EWMA chart is a special case of the EWMAST chart when the sequence of observations, X_t , are independent.

As it seen from (3.27), the choice of M is important. Box & Jenkins (1976) proposed that useful estimates of ρ_k can only be made if the data size N is roughly 50 or more and $k < N / 4$. Consequently, Zhang (1998) suggested that “ M should be large enough to make the approximation in (3.27) usable and at the same time less than $N / 4$ to avoid the large estimation errors of the autocorrelations”.

3.2.3 The Autoregressive Moving Average (ARMA) Control Chart

The autoregressive moving average (ARMA) chart has been proposed by Jiang et al. (2000). The ARMA chart monitors the successive values of an ARMA statistic which is obtained by the application of generalized first order autoregressive moving average (ARMA(1,1)) process applied to the iid process. Jiang et al. (2000) use the same notation of the EWMAST chart proposed by Zhang (1998), and denote the ARMA chart as the ARMAST chart for stationary processes.

Suppose $\{X_t\}$ is a series of autocorrelated observations with normality, an in control mean of 0, and variance σ_x^2 . Jiang et al. (2000) applied the ARMA statistic to the underlying process, X_t , as follows:

$$\begin{aligned} Z_t &= \theta_0 X_t - \theta X_{t-1} + \phi X_{t-1} \\ &= \theta_0 (X_t - \beta X_{t-1}) + \phi X_{t-1} \end{aligned} \quad (3.28)$$

where $\beta = \theta / \theta_0$ and $\theta_0 = 1 + \theta - \phi$.

Stationarity and invertibility constraints of the process are $|\phi| < 1$, and $|\beta| < 1$, respectively.

Jiang et al. (2000) assume that the underlying process, X_t , is characterized by the autocorrelation structure ρ_τ with $\rho_\tau = \gamma_\tau / \gamma_0$ and $\gamma_\tau = \text{cov}(X_t, X_{t+\tau})$. And they represented the ARMA statistic as follows:

$$Z_t = \theta_0 X_t + \alpha \sum_{k=1}^{t-1} \phi^{k-1} X_{t-k} \quad (3.29)$$

where $\alpha = \phi \theta_0 - \theta$ and $\theta_0 = 1 + \theta - \phi$.

The covariance function given by Jiang et al. (2000) is:

$$\begin{aligned} \text{cov}(Z_t, Z_{t+\tau}) &= \text{cov} \left[\theta_0 X_t + \alpha \sum_{k=1}^{t-1} \phi^{k-1} X_{t-k}, \theta_0 X_{t+\tau} + \alpha \sum_{k=1}^{t+\tau-1} \phi^{k-1} X_{t+\tau-k} \right] \\ &= \theta_0^2 \gamma_\tau + \theta_0 \alpha \left[\sum_{k=1}^{t-1} \phi^{k-1} \gamma_{\tau+k} + \sum_{k=1}^{t+\tau-1} \phi^{k-1} \gamma_{\tau-k} \right] + \alpha^2 \sum_{i=1}^{t-1} \sum_{j=1}^{t+\tau-1} \phi^{i+j-2} \gamma_{\tau-j+i} \\ &= \sigma_x^2 \left\{ \theta_0^2 \rho_\tau + \theta_0 \alpha \left[\sum_{k=1}^{t-1} \phi^{k-1} \rho_{\tau+k} + \sum_{k=1}^{t+\tau-1} \phi^{k-1} \rho_{\tau-k} \right] \right. \\ &\quad \left. + \alpha^2 \sum_{i=1}^{t-1} \sum_{j=1}^{t+\tau-1} \phi^{i+j-2} \rho_{\tau-j+i} \right\} \end{aligned} \quad (3.30)$$

When $\tau = 0$, the variance of Z_t is:

$$\sigma_Z^2 = \left\{ \theta_0^2 + 2\theta_0\alpha \sum_{k=1}^{t-1} \phi^{k-1} \rho_k + \alpha^2 \sum_{i=1}^{t-1} \sum_{j=1}^{t-1} \phi^{i+j-2} \rho_{j-i} \right\} \sigma_X^2 \quad (3.31)$$

Similarly, the variance of Z_t for large t ($t \rightarrow \infty$), namely steady state variance is:

$$\begin{aligned} \sigma_Z^2 &= \left\{ \theta_0^2 + 2\theta_0\alpha \sum_{k=1}^{\infty} \phi^{k-1} \rho_k + \alpha^2 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \phi^{i+j-2} \rho_{j-i} \right\} \sigma_X^2 \\ &= \left\{ \theta_0^2 + 2\theta_0\alpha \sum_{k=1}^{\infty} \phi^{k-1} \rho_k + \frac{\alpha^2}{1-\phi^2} \rho_0 + 2\alpha^2 \phi^{-2} \sum_{k=1}^{\infty} \phi^k \rho_k \sum_{l=1}^{\infty} \phi^{2l} \right\} \sigma_X^2 \\ &= \left\{ \theta_0^2 + \frac{\alpha^2}{1-\phi^2} + 2 \left(\theta_0\alpha + \frac{\phi\alpha^2}{1-\phi^2} \right) \sum_{k=1}^{\infty} \phi^{k-1} \rho_k \right\} \sigma_X^2 \end{aligned} \quad (3.32)$$

where $\sum_{k=1}^{\infty} \phi^k \rho_k$ converges because $|\rho_k| < 1$ ($k > 0$).

The ARMA chart is constructed based on (3.30), and (3.32) for large t . then, the 3-sigma control limits are:

$$\text{UCL} = \mu + 3\sigma_Z$$

$$\text{LCL} = \mu - 3\sigma_Z$$

where σ_Z is the standard deviation of Z_t which can be calculated by taking the square root of (3.30). The approximation given in (3.32) can be used for large t .

ARMA chart reduces to the EWMA chart when $\theta = 0$ with $\phi = 1 - \lambda$. Therefore, the EWMA chart can be considered as a special case of the ARMA chart.

As it is stated above, ARMA statistic is $Z_t = \theta_0 X_t - \theta X_{t-1} + \phi X_{t-1}$. Since $\theta_0 = 1 + \theta - \phi$, the equation can be written as follows:

$$Z_t = X_t + \theta X_t - \phi X_t - \theta X_{t-1} + \phi Z_{t-1}$$

If $\phi = 1 - \lambda$, and $\theta = 0$, then

$$Z_t = X_t - (1 - \lambda)X_t + (1 - \lambda)Z_{t-1} = X_t - X_t + \lambda X_t + (1 - \lambda)Z_{t-1}$$

Consequently, ARMA statistic becomes

$$Z_t = (1 - \lambda)Z_{t-1} + \lambda X_t$$

which is equal to the EWMA statistic given previously.

3.2.4 A Distribution Free Tabular CUSUM Chart for Autocorrelated Processes

Distribution free charts are suggested for distribution free processes. Kim et al. (2007) proposed a distribution free tabular CUSUM (DFTC) chart to detect mean shifts of autocorrelated observations. Kim et al. (2007) defined the proposed chart as “a generalization of the conventional tabular CUSUM chart that is designed for iid normal random variables”.

Suppose $\{X_t\}$ is a discrete-time stochastic process which has steady state distribution with mean μ , marginal variance σ_x^2 . Let the in control process mean be μ_0 . The standardized CUSUM, $C_i(t)$, is defined as:

$$C_i(t) = \frac{\sum_{t=1}^{[ni]} X_t - nt\mu}{\Omega_x \sqrt{n}} \quad 3.33$$

where $[.]$ is the “floor” (greatest integer) function so that $[z]$ denotes the largest integer not exceeding z , and Ω_X^2 is the variance parameter for the process $\{X_t\}$, defined as:

$$\Omega_X^2 = \lim_{n \rightarrow \infty} n \text{var}[\bar{X}(n)] = \sum_{l=-\infty}^{\infty} \text{cov}(X_t, X_{t+l}) \quad 3.34$$

For the one sided tabular CUSUM statistics, C_i^+ and C_i^- , defined in equations (2.10) and (2.11), Kim et al. (2007) used the notations S_n^+ and S_n^- corresponding times at which an alarm is raised,

$$T_X^\pm = \min\{n : S_n^\pm \geq H \quad \text{and} \quad n = 1, 2, \dots\} \quad 3.35$$

$$S_n = \begin{cases} 0, & \text{if } n = 0 \\ S_{n-1} + (X_n - \mu_0) - K, & \text{if } n = 1, 2, \dots \end{cases} \quad 3.36$$

where $K = k\sigma_X$ and $H = h\sigma_X$

Kim et al. (2007) formulate the procedure of DFTC by computing the one sided ARL $E[T_X^+]$ for the in-control condition $E[X_t] = \mu_0$.

H. Kim et al. (2007) formulate a distribution-free generalization of Siegmund's approximation to handle the case of observations that may be correlated or nonnormal and develop a procedure for DFTC as follows:

Step 1. Choose K and the target two-sided ARL_0 . Then, calculate H and the solution,

$$\frac{\Omega_X^2}{2K^2} \left\{ \exp \left[\frac{2K(H + 1,166\Omega_X)}{\Omega_X^2} \right] \right\} - 1 - \frac{2K(H + 1,166\Omega_X)}{\Omega_X^2} = 2ARL_0 \quad 3.37$$

Step 2. Set an out of control alarm after the n^{th} observation if $S_n^+ \geq H$ or $S_n^- \geq H$.

3.2.5 A Review on Statistical Process Control Charts for Autocorrelated Processes

The fundamentals of SPC and control charts were firstly introduced by Dr. Walter A. Shewhart in 1920s while he was working for Bell Telephone Laboratories. Since then the first statistical control charts such as $\bar{X}$, $\bar{X}$ and R, $\bar{X}$ and S, are called Shewhart charts due to the scientist who developed them. Shewhart control charts are widely used in both theoretical researches and practical applications, because they are the basis of many control charts and easy to use and interpret. Nevertheless, there is also a main disadvantage relevant to any type of a Shewhart chart that is it only uses the information about the process contained in the last plotted point. So, it has lack of information about the entire process. As a result, Shewhart charts are effective when the shift in process parameters are large, but are ineffective when the shift is small (about $1,5\sigma$ or less).

Two different control charts are proposed to overcome this disadvantage: cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts that are introduced by Page (1954) and Roberts (1959), respectively. The CUSUM and EWMA charts give smaller weight to the past data, and much weight to the recent data. Consequently, they are appropriate for detecting small shifts, but inappropriate for large ones.

All of the above traditional control charts perform under iid assumption, but we mentioned previously that the iid assumption is not realistic in practice. In the last two decades, the scientists' interest in autocorrelated processes have improved, and two general approaches are proposed to deal with autocorrelated processes: i) residual control charts, and ii) modified control charts.

Residual control charts have been studied quite extensively. In these charts, forecast errors, namely residuals, are assumed to be statistically uncorrelated. An appropriate time series model is fitted to the autocorrelated observations and the

residuals are plotted in a control chart. For this reason all of the well-known control schemes can be transformed to the residual control scheme. The main advantage of a residual chart is that it can be applied to any autocorrelated data whether the process is stationary or not. However, there are also some disadvantages of the residual control charts such as:

- time series modeling knowledge is needed for constructing the ARIMA model,
- some residual charts which based on two valid time series models signal differently, and
- they do not detect a possible mean shift quickly (i.e. for some cases, the detection capability of an X residual chart is smaller than the traditional X chart and other residual charts as EWMA and CUSUM) (for further reading see Harris & Ross 1991, Longnecker & Ryan 1992, Wardell et al. 1994, and Zhang 1998).

Alwan & Roberts (1988) introduced the common cause chart (CCC) which is applied by forming an ARIMA model for the autocorrelated process. CCC is not a control chart actually, because it does not have any control limits, it consists of only plotted data which have been modeled with an ARIMA model. Even though CCC has no control limits, it gives information about variation of the modeled data. Furthermore, Alwan & Roberts (1988) developed a residual Shewhart chart and called it the special cause chart (SCC). SCC is an individual control chart (individual control chart is constructed by plotting single observations and setting control limits on these observations) applied to the residuals.

Yourstone & Montgomery (1989), studied geometric moving average (GMA), geometric moving range, and Shewhart X control charts; and two years later (1991), they proposed two innovative control charts, the sample autocorrelation chart (SACC), and the group autocorrelation chart (GACC) that are based on the

autocorrelation function of autocorrelated data. The authors claimed that the charts are capable of detecting changes in mean, variance and stochastic behavior of a series. Later, in the study of Atienza et al. (1997), the performance of SACC was compared with SCC and residual CUSUM chart, and the authors concluded that SACC does not have a better performance than both of the residual CUSUM and SCC charts, and SACC is appropriate only for medium shifts in mean. Furthermore, SACC outperforms SCC and CUSUM charts when detecting changes in model parameters. In the following year, Atienza et al. (1998) proposed a chart which can detect level shifts in time series data effectively.

Lucas & Saccucci (1990) showed that CUSUM and EWMA charts perform almost the same. Harris & Ross (1991) studied the impact of autocorrelation on CUSUM and EWMA charts for AR(1) processes. Montgomery & Mastrangelo (1991) proposed an EWMA chart which is constructed by plotting one step ahead prediction errors on the chart. Wardell et al. (1992) compared the traditional Shewhart and EWMA charts with CCC and SCC charts for ARMA(1,1) process. They examined the robustness of the Shewhart and EWMA charts, in order to figure out their cost effectiveness and thus their ability in competing with the residual control charts. They observed that the EWMA chart is more robust to serial correlation than the Shewhart chart. Yashchin (1993) discussed the run length characteristics of a CUSUM chart when the underlying process is autocorrelated. Reynolds & Lu (1997) compared various types of EWMA control charts based on the original observations or on the residuals from an AR(1) model. Wardell et al. (1994) and Zhang (1997) remarked that the detection capability of an $\bar{X}$ residual chart was poor for small mean shifts compared to the traditional $\bar{X}$ chart, EWMA, and CUSUM charts. Lu and Reynolds (1999) discussed the residual EWMA chart.

Zhang, N.F. (1997) studied the detection capability of the $\bar{X}$ residual chart, and the following year (1998) he introduced the Exponentially Weighted Moving Average for Stationary Process (EWMAST) control chart which is constructed by charting the EWMA statistics for stationary process. Also, Zhang (2000) pointed out that the performances of CUSUM residual and EWMA residual control charts are almost the

same. Winkel & Zhang (2004) defined EWMAST chart as the extension of the traditional EWMA chart designed to monitor a stationary process. Even though the EWMAST chart is very similar to the traditional EWMA chart, its control limits depend on the autocorrelation of the process and it is not necessary to fit a time series model to data. Some advantages of the EWMAST chart are: it is easy to implement, and no time series modeling effort is needed.

Jieng et al. (2000) proposed the ARMA chart. They used the ARMA (1,1) statistics obtained by the fitting generalized ARMA(1,1) model to data. They called the proposed chart ARMAST chart for stationary processes. The EWMAST chart is a special case of the ARMA chart. If $\phi = 1 - \lambda$ and $\theta=0$, ARMA and EWMA statistics will be the same. Jiang & Tsui (2001) discussed computing ARL for ARMA charts. Furthermore, Pacella & Semeraro (2007) introduced the Elman's neural network (ENN) chart, and compared it to Shewart $\bar{X}$, SCC, EWMAST, ARMAST charts for ARMA(1,1) autocorrelation structure.

The second approach, the modified control chart, is based on applying the original control chart methodology with a little modification. Autocorrelated data is used in original control chart by adjusting its control limits. Since rearrangement of the control limits for autocorrelated data is not so easy, application of modified charts is more complicated than the first approach. Besides, the model-based charts have the disadvantage that the underlying model may be inappropriate. In this case, distribution free SPC charts are proposed to overcome the distribution based procedures. Distribution free charts have been used in SPC problems when there is limited or lack of knowledge about the underlying process distribution. Johnson & Bagshaw (1974), and Bagshaw & Johnson (1975) examined the performance of CUSUM control chart under autocorrelation and they found CUSUM is not robust under autocorrelation. They presented CUSUM based methods that use raw (unbatched) observations instead of batch means in 1974. Vasilopoulos & Stamboulis (1978) introduced the $\bar{X}$ chart with modified limits when the underlying process is described by an AR(2) model, but they ignore the autocorrelation between subsample means. Alwan & Radson (1992) investigate the time series behavior of

subsample means for AR(1) process. Runger & Willemain (1995) studied the sequence of observations from the monitored process into adjacent nonoverlapping batches of equal size; and initially developed the Unweighted Batch Means (UBM) chart which aims to diminish the autocorrelation between subsample means or batch means; and also compared with the X residual and CUSUM residual charts. Furthermore, the following year (1996) they proved that larger batch sizes give better performance for AR(1) process. Kim et al. (2006) presented a model free CUSUM (MFC) chart by unbatched CUSUM based methods, and in 2007 introduced a distribution free tabular CUSUM chart (DFTC).

In addition to above charts, Schmid (1997), and VanBrackle & Reynolds (1997) also discussed the application of the EWMA chart to autocorrelated processes.

As can be seen from the literature review given above, performances of several control charts for ARMA(1,1) process was firstly compared by Wardel et al. in 1992. Later, this process model was employed by several authors in comparison studies. Even though there exist much amount of comparison studies for ARMA(1,1) model, there seems to be no earlier study testing traditional, residual and distribution free control charts (DFTC) together for ARMA(1,1). DFTC have attracted the interests of scientists, because of process data are generally not independent and identically distributed normal about a certain mean. DFTC have been included by many comparison studies, but in only few of them ARMA(1,1) process structure is considered. In the current study, we examine Shewhart X , CUSUM, X residual, EWMA residual, EWMAST, ARMAST, and DFTC for ARMA (1,1) process.

3.2.6 Summary

Over the last two decades, control charts for autocorrelated observations have been applied to an increasing number of real-world problems. We tried to provide an extensive literature review on the applications of control charts to processes with various autocorrelation structures, in previous subsection. In Table 3.1, we summarize the recent research studies in a chronological order, in order to see the

gradual development in these works. Our survey is limited with the publications appearing in refereed journals and conference proceedings between 1974 and 2007.

Table 3.1 Evolution of control charts for autocorrelated processes

Year	Author(s)	Control Charts	Auto-correlation Structure
1974	Johnson & Bagshaw	J&B	AR(1)
1975	Bagshaw & Johnson	J&B	AR(1)
1978	Vasilopoulos & Stamboulis	modified $\bar{X}$ chart	AR(2)
1988	Alwan & Roberts	Shewhart $\bar{X}$, CCC, SCC	ARIMA(0,1,1), ARIMA(1,0,1)
1989	Yourstone & Montgomery	GMA, GMR, Shewhart $\bar{X}$	AR(2), ARMA(2,1)
1990	Lucas & Saccucci	CUSUM, EWMA	
1991	Harris & Ross	CUSUM, EWMA	AR(1)
1991	Montgomery & Mastrangelo	EWMA residual	
1991	Yourstone & Montgomery	SACC, GACC	AR(4)
1992	Alwan & Radson	modified $\bar{X}$ chart	AR(1)
1992	Wardel et al.	EWMA, Shewhart, SCC, CCC	ARMA(1,1)
1993	Yashchin	CUSUM	
1994	Wardel et al.	Shewhart $\bar{X}$, $\bar{X}$ residual	ARMA(1,1)
1995	Runger & Willemain	$\bar{X}$ residual, CUSUM residual, UBM chart	AR(1)
1996	Runger & Willemain	UBM chart	AR(1)
1997	Atienza et al.	SACC, SCC, CUSUM residual	AR(1), MA(1)

1997	Reynolds & Lu	EWMA residual	AR(1)
1997	VanBrackle & Reynolds	CUSUM, EWMA	AR(1), ARMA(1,1)
1997	Zhang	Shewhart $\bar{X}$, $\bar{X}$ residual, adjusted $\bar{X}$ chart	AR(1), AR(2)
1998	Zhang	EWMAST, Shewhart $\bar{X}$, $\bar{X}$ residual	AR(1), AR(2), ARMA(1,1)
1999	Lu and Reynolds	EWMA, EWMA residual	AR(1)
2000	Zhang	Shewhart $\bar{X}$, $\bar{X}$ residual, CUSUM, EWMA, EWMA residual, CUSUM residual, EWMAST	AR(1)
2000	Jieng et al.	SCC, CUSUM, EWMA, EWMAST, ARMA, ARMAST	AR(1), ARMA(1,1)
2001	Jiang & Tsui	ARMA	AR(1), AR(2)
2004	Winkel & Zhang	EWMA, EWMAST	AR(1)
2006	Kim et al.	CUSUM, J&B, MFC	AR(1)
2007	Kim et al.	J&B, MFC, DFTC	AR(1), AR(2)
2007	Pacella & Semeraro	ENN, SCC, Shewart $\bar{X}$, EWMAST, ARMAST	ARMA(1,1)

We briefly summarize our conclusions from this detailed review in the following:

- Fundamentals of control charts are constituted in 1920s.
- Control charts for autocorrelated processes attracted scientists' attention in 1970s. Scientists studied the effect of autocorrelation on the existing charts, initially. Later, they proposed charts for autocorrelated data.

- Many scientists have studied the residual control charts more than modified charts due to their simplicity.
- The first residual control chart was applied to Shewhart $\bar{X}$. After the first residual control chart introduced at the end of the 1980s, scientists compared the residual chart with traditional charts at early 1990s. Because of the reason that the residual chart perform well, then the scientists become interested in residual charts of well-known control schemes. Many research studies that compare residual charts have been issued during the last decade. From the middle of 1990s to present, many scientists have studied various types of residual control charts.
- Scientists proposed original and innovative control charts in earlier years. But in recent years most of the proposed control charts are enhanced versions of existing charts. However, some sophisticated control charts such as ENN and DFTC were also introduced by the year 2000.
- As it can be seen from Table 3.1, most of the scientists have chosen an appropriate $AR(p)$ model, especially $AR(1)$, for their data set. Few of the researchers have studied $ARMA(1,1)$ model.

In this section, control charts for autocorrelated processes were reviewed, and the historical progression in this field was emphasized. We compare the performances of control charts in the presence of autocorrelation in the following chapter.

CHAPTER FOUR

ARL PERFORMANCES OF CONTROL CHARTS FOR AUTOCORRELATED DATA

4.1 Introduction

In this chapter, ARL performances of Shewhart $\bar{X}$, CUSUM, $\bar{X}$ residual, EWMA residual, EWMAST, ARMAST, and DFTC charts are compared in the presence of data correlation. Firstly, the desired ARL value (ARL_0) is given, and parameters of the above control charts are selected relevant to it. Subsequently, the impact of the different levels of autocorrelation and different magnitudes of process shift is interpreted; and ARL results of the charts are concluded.

4.2 Problem Statement

As it is stated previously, the independency assumption is not realistic in practice due to internal or external reasons. Having numerous potential applications in real life, in recent years, various research works have been carried out to deal with the autocorrelation problem. Although various control charts have been developed for monitoring autocorrelated processes and their performances have been compared with each other, the first-order autoregressive moving average ARMA(1,1) process has been considered in only a few comparison study. Because of this reason we compare the ARL performances of several charts for the first order autoregressive moving average (ARMA(1,1)) process, in this dissertation. Furthermore, many industrial processes can be presented by ARMA(1,1) model. Especially in process industries first order autoregressive moving average process behavior is fairly common (see Wardel et al., 1992, Montgomery, 1997). It is a stationary process, and contains both autoregressive and moving average components.

The model structure of ARMA(1,1) process is explained in section 3.1.4 and defined as follows:

$$Z_t = \xi + \phi Z_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1}$$

where

Z_t = the observation at time t ,

ε_t = random noise term with mean 0 and variance σ_ε^2 at time t ,

ϕ = the autoregressive parameter,

θ = the moving average parameter,

ξ = the mean of the process.

It is also assumed that distribution of the $\{\varepsilon_t\}$ is normal with mean 0, and a common variance σ_ε^2 (Gaussian white noise).

4.3 Design of the Control Charts

In this study, we compare the performances of Shewhart $\bar{X}$, CUSUM, $\bar{X}$ residual, EWMA residual, EWMAST, ARMAST, and DFTC control charts for an ARMA(1,1) process, as mentioned before. We use average run length (ARL) as performance criterion.

Firstly, we generate data that fits the ARMA(1,1) model. The autoregressive and moving average process parameters (ϕ, θ) are chosen as following: (0.95, -0.90), (0.00, -0.90), (-0.95, 0.90), (-0.95, 0.00), and (-0.95, 0.45). These combinations of (ϕ, θ) have been also used by many authors (Wardell et al., 1992; Zhang, 1998; Jiang et al., 2000; Pacella & Semeraro, 2007). Various magnitudes of process shifts (δ) are also considered as 0σ , 0.5σ , 1σ , 2σ , 3σ .

Let X_t be the random time series of the quality characteristic measurements:

$$X_t = Z_t + \delta \quad \text{for } t = 1, 2, \dots \quad (5.1)$$

where δ is a shift due to special causes and Z_t is the first-order autoregressive moving average process which defined in equation (3.12). Shift from the process mean can be defined as the difference between the shifted process mean and target process mean divided by the process standard deviation:

$$\delta = \frac{\mu_1 - \mu_0}{\sigma} \quad (5.2)$$

where

δ = magnitude of the shift due to special causes,

μ_0 = target mean,

μ_1 = out-of-control value of the process mean,

σ = standard deviation of in-control process.

In simulations 5000 runs are generated, 500 run lengths are collected to calculate an ARL, and 10 ARLs are used to calculate a grand ARL. When comparing ARL values of various control charts, initially a target ARL for iid situation (ARL_0) is set for all compared charts. In this study, we take ARL_0 is equal to 370.

Different types of control charts can be compared each other for various levels of autocorrelation and mean shift by adjusting the constants such as L , k , h , λ , M , R_S in the control limits of each chart to give the same in control ARL in iid case. Then the ARLs of the charts can be interpreted. In ARL calculations, 3σ control limits are used for the $\bar{X}$ and $\bar{X}$ Residual charts. For CUSUM and DFTE charts, the ARL_0 values of k and the corresponding values of h are selected as 0.5 and 4.77, respectively (see Hawkins, 1993). As mentioned before, we use Siegmund's (1985) approximation for calculating the ARL values of CUSUM and DFTE charts. In addition, for the EWMA residual chart, $\lambda=0.2$ and 3σ control limits were used, and for the EWMAST chart $M = 25$ (see Zhang, 1998). For the ARMAST chart, selected value of $R_S = 3$ (Jiang et al., 2000).

4.4 Experimental Results

In the light of foregoing, the calculated ARL values are given in Table 2.

Table 4.1 Comparisons of ARL's: Shewhart $\bar{X}$, CUSUM, $\bar{X}$ residual, EWMA residual, EWMAST, ARMAST, and DFTC

Shift	ϕ	θ	ρ_1	Shewhart $\bar{X}$	CUSUM	$\bar{X}$ Residual	EWMA Residual	EWMAST	ARMAST	DFTC
0	0	0	0	370	371	370	370	370	370	370
0.5				155	38	158	41	46	36	39
1				44	10	43	9	11	6	8
2				6	4	5	3	3	2	3
3				2	2	2	2	2	1	1
0	0.95	-0.9	0.97	839	27	370	370	370	370	370
0.5				418	21	42	39	240	43	165
1				158	14	1	1	110	1	24
2				32	5	1	1	26	1	1
3				6	3	1	1	9	1	1
0	0	0.9	-0.50	401	51	370	370	370	370	370
0.5				181	26	278	235	216	221	230
1				56	12	145	116	89	90	92
2				9	4	1	19	24	22	27
3				3	3	1	12	8	9	14
0	-0.95	0.9	-0.97	839	28	370	370	370	370	370
0.5				418	23	24	18	21	10	17
1				159	16	21	6	10	3	6
2				31	5	2	3	6	2	4
3				6	3	1	2	3	1	2
0	-0.95	0	-0.95	837	29	370	370	370	370	370
0.5				416	22	11	6	14	3	8
1				155	15	5	2	2	2	3
2				29	5	3	1	1	1	2
3				7	3	2	1	1	1	2
0	-0.95	0.475	-0.97	839	28	370	370	370	370	370
0.5				417	24	32	11	15	8	11
1				160	15	18	5	7	2	5
2				31	5	2	2	4	2	3
3				6	3	1	1	2	2	2

The first column of this table denotes magnitude of the mean shift. Mean shifts branch into three classes relevant to their magnitudes such as, small, medium and large shifts. In the literature, small shifts are said to be less than or equal to 1.5 ($\delta \leq 1.5$); medium shifts are between 1.5 and 2.5 ($1.5 \leq \delta \leq 2.5$); and large shifts are greater than or equal to 2.5 ($\delta \geq 2.5$). Moreover, the second and third columns represent parameters of the autocorrelation structure, and autocorrelation function value is given in the fourth column. Recall from chapter three that, if the autocorrelation function value is smaller than 0, then the process will be negatively correlated; and if it is smaller than 0, the process will be positively correlated. Like

mean shift, autocorrelation is also classified according to absolute value of the correlation as follows:

$$0 \leq \rho_1 \leq 0.3 \rightarrow \text{low autocorrelation}$$

$$0.3 \leq \rho_1 \leq 0.7 \rightarrow \text{moderate autocorrelation}$$

$$0.7 \leq \rho_1 \leq 1 \rightarrow \text{high autocorrelation}$$

Finally, the rest of the columns of Table 4.1 show the calculated ARL values of relevant charts. Furthermore, we clarified the term ARL in chapter two and stated that the in-control ARL value must be large and the out-of-control ARL value must be small. Thus, when there is no shift ($\delta = 0$) in the mean the charts with large ARL values (in-control ARL) performed better, and when there is shift ($\delta \neq 0$) the charts with small ARL values (out-of-control ARL) are better.

It can be seen from the table, when the process is under iid condition CUSUM, EWMA residual, EWMAST, ARMAST, and DFTEC control charts perform almost the same, especially for medium to large shifts. Particularly, DFTEC performed similar to the CUSUM in iid situation. Figure 4.1 illustrates this situation. The colored bars represent the corresponding chart's ARL value. Shewhart $\bar{X}$ chart and $\bar{X}$ residual chart perform exactly the same for iid situation and they are not effective for detecting small shifts as expected, as it seen from the figure.

ARL performances of control charts are figured in Figure 4.2 when the process is highly positive correlated. It is seen from Table 4.1 and Figure 4.2 that the CUSUM chart has the worst in-control ARL performance. Furthermore, the Shewhart $\bar{X}$ chart performs worst and the EWMA residual chart performs best especially for small shifts. $\bar{X}$ residual, EWMA residual and ARMAST charts present similar performances as for small shifts, and the DFTEC chart also performs like them for large shifts. Moreover, residual charts show better performance than the EWMAST and DFTEC for small to medium shifts.

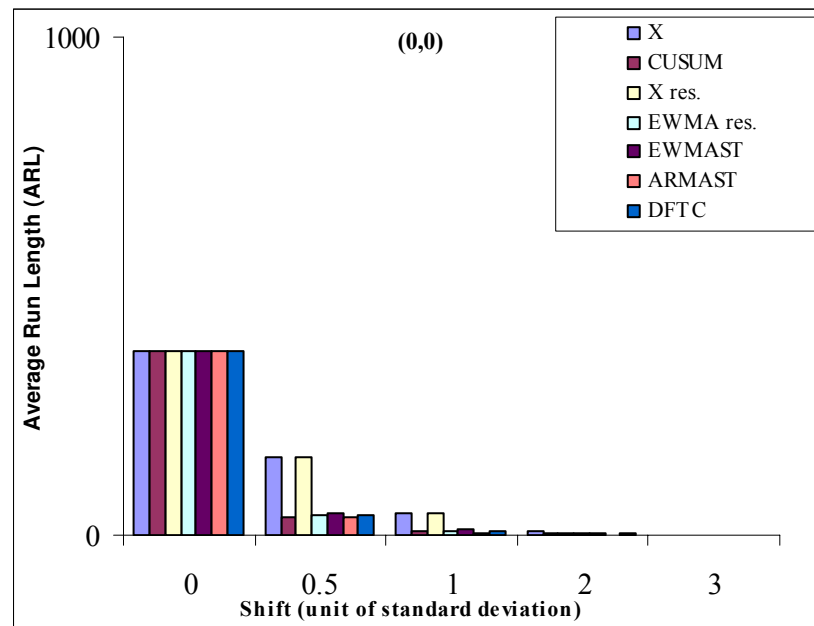


Figure 4.1. ARL bars of uncorrelated process. $(\phi, \theta) = (0, 0)$

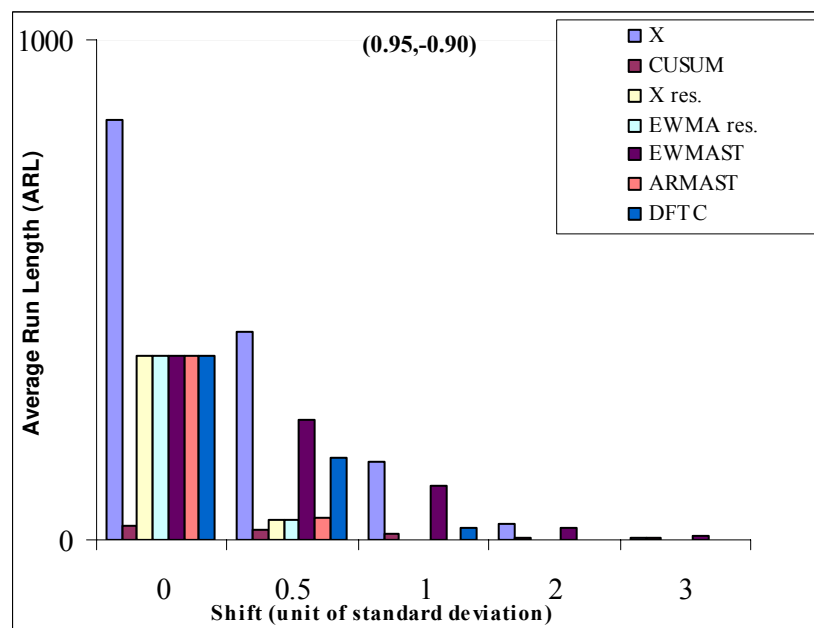


Figure 4.2 ARL bars of highly positive correlated process. $(\phi, \theta) = (0.95, -0.90)$

ARL bars of a moderate negatively correlated process versus shift of the mean are represented in Figure 4.3. Except the CUSUM chart, all of the charts have analogous and satisfactory in-control ARL performances. Even though the in-control ARL performance of the CUSUM chart is significantly worst of all charts, and its out-of-

control performance is the best for small to medium shifts. The $\bar{X}$ residual chart performs best for large shifts. The EWMAST chart perform better than $\bar{X}$ residual, EWMA residual, ARMAST and DFTEC charts for small to medium shifts. ARMAST and DFTEC charts perform like each other for medium shifts.

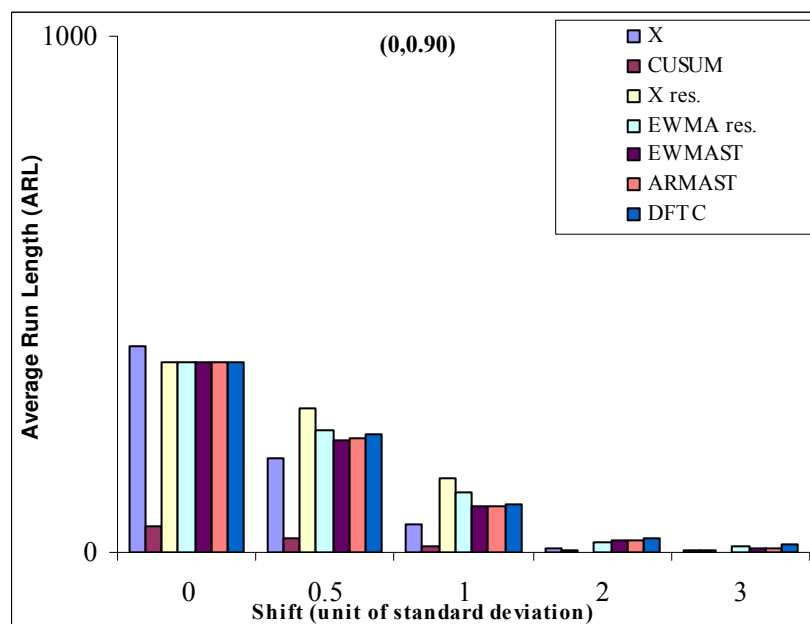
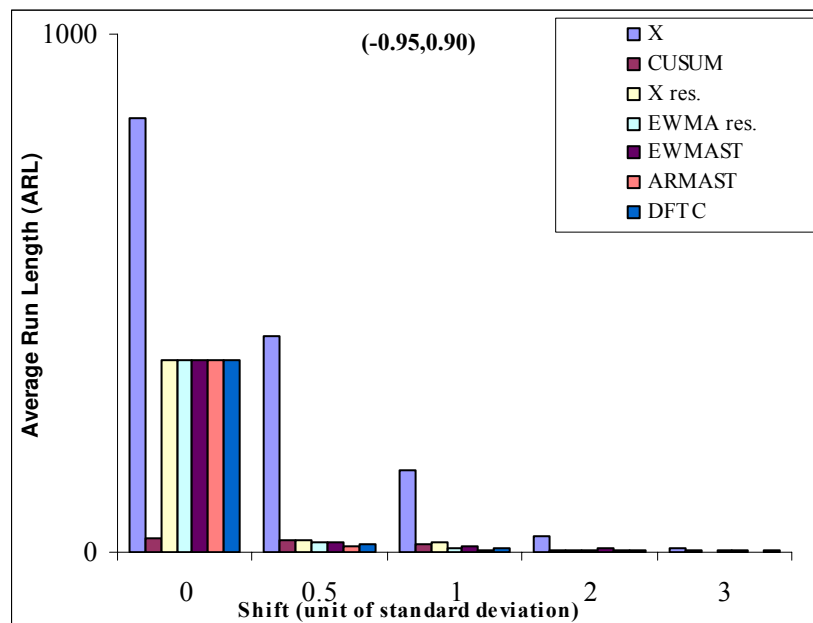
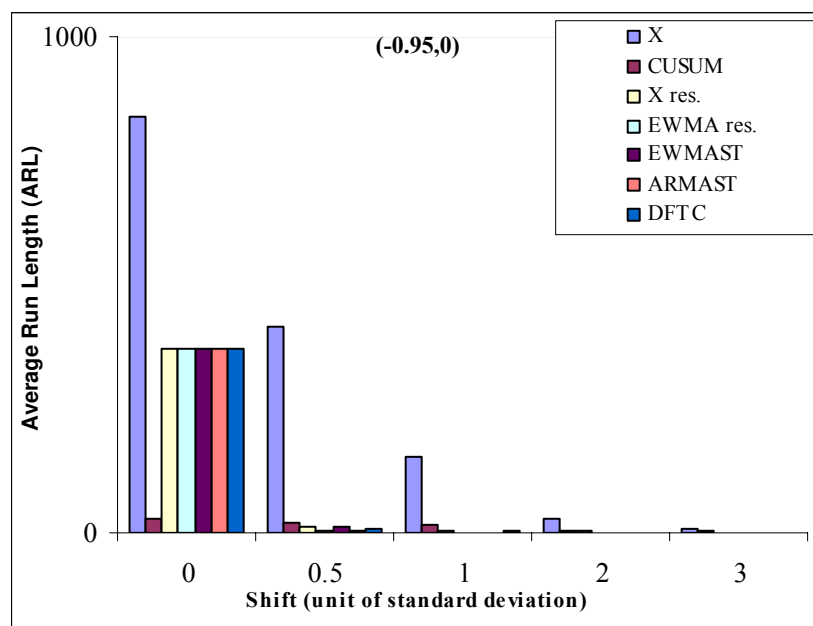
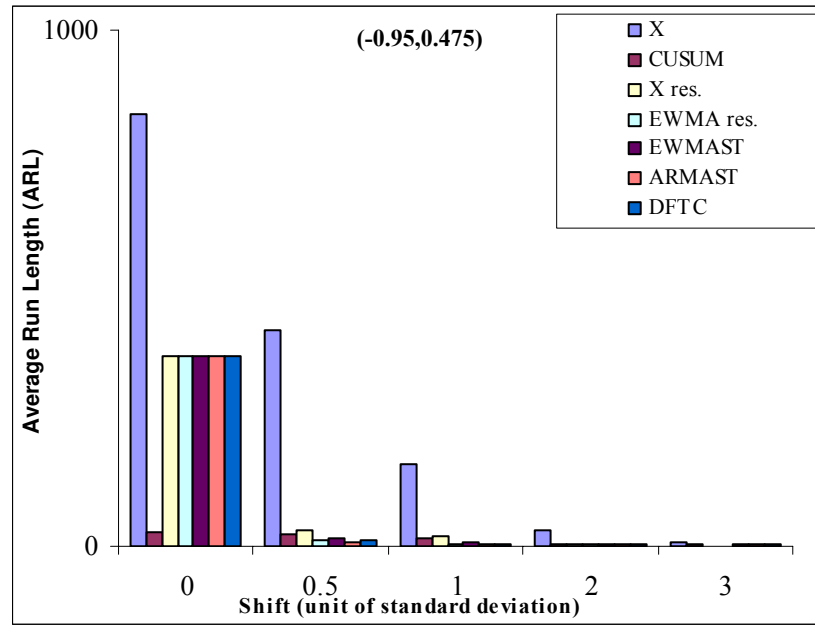


Figure 4.3 ARL bars of moderate negatively correlated process. $(\phi, \theta) = (0, 0.90)$

ARL bars of highly negative correlated processes are illustrated in Figure 4.4. Three different ARMA(1,1) processes such as $(-0.95, 0.90)$, $(-0.95, 0)$ and $(-0.95, 0.475)$ form high negative correlation in this study. It is concluded that when there is high negative correlation in the process, the ARMAST chart has an overall performance. Especially for small shifts the only one who performs best is ARMAST. $\bar{X}$ residual and EWMA residual charts perform well for medium to large shifts, and the DFTEC charts performance is well for large shifts. The Shewhart $\bar{X}$ achieved the worst out-of-control ARL performance for high autocorrelation whether it is positive or negative, and small shifts.

(a) $(\phi, \theta) = (-0.95, 0.90)$ (b) $(\phi, \theta) = (-0.95, 0)$



(c) $(\phi, \theta) = (-0.95, 0.475)$

Figure 4.4 (a), (b) and (c) ARL bars of highly negative correlated processes.

We have compared various control charts via ARL calculations, and interpreted them in respect of autocorrelation structure and magnitude of shift. Now, we explain control charts behavior one by one, in the following. Firstly, we discuss the Shewhart X chart in Figure 4.5. The autocorrelation has big impact on the traditional control charts like Shewhart X chart. When there is high correlation, whether it is positive or negative, out-of-control ARL increases significantly. This effect can be seen in Figure 4.5 that the ARL bars of highly correlated processes such as $(0.95, -0.90)$, $(-0.95, 0.90)$, $(-0.95, 0)$, and $(-0.95, 0.475)$ are higher than the blue painted iid bar. It means that the Shewhart X chart perform well for in-control ARL, but awful for out-of-control ARL due to the reason that bars belong to highly autocorrelated processes decrease very slowly when there is mean shift.

It is obvious from Table 4.1 that, when the process is autocorrelated, whether it is positive or negative, the in-control ARL of CUSUM chart drops dramatically. For instance, when process parameters changed from $(\phi, \theta) = (0, 0)$ to $(\phi, \theta) = (0.95, -0.90)$, the in-control ARL of the CUSUM reduced from 371 to 27. Thus, when a process is autocorrelated, even it is weak, CUSUM control charts chart give false alarms frequently when the process is in control. These explanations are illustrated in

Figure 4.6. The impact of autocorrelation on in-control ARL performance of CUSUM takes place in figure by a sudden decrease in 0-shift area which means poor in-control ARL performance. On the other hand, as mentioned in the previous studies (Johnson & Bagshaw, 1974; Bagshaw & Johnson, 1975; Harris & Ross, 1991) autocorrelation has relatively small effect on the out-of-control ARL of CUSUM. CUSUM chart responds quickly to the shifts from various levels of autocorrelation.

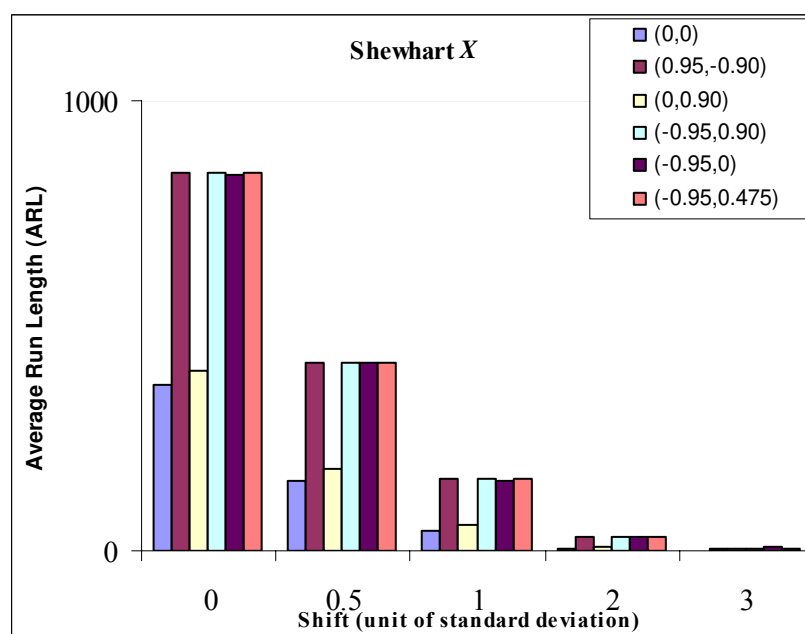


Figure 4.5 ARL bars of Shewhart $\bar{X}$ chart for different levels of autocorrelation versus mean shift.

ARL bars of $\bar{X}$ residual and EWMA residual charts are given in Figures 4.7 and 4.8, respectively. The $\bar{X}$ residual chart does not perform well when the process is moderately negative correlated which is represented by the yellow bar in Figure 4.7. The yellow bar does not decay significantly especially for small to medium shifts. However, all of the bars are close to zero for large shifts. Thus, the $\bar{X}$ residual chart is appropriate for large shifts.

Moreover, the EWMA residual chart has quick response to autocorrelation. All of the bars except the yellow one (moderate negative autocorrelation) declines quickly when the process mean shifts. It means that the EWMA residual chart gives better

performance than the X residual in detecting small shifts, just like traditional control charts.

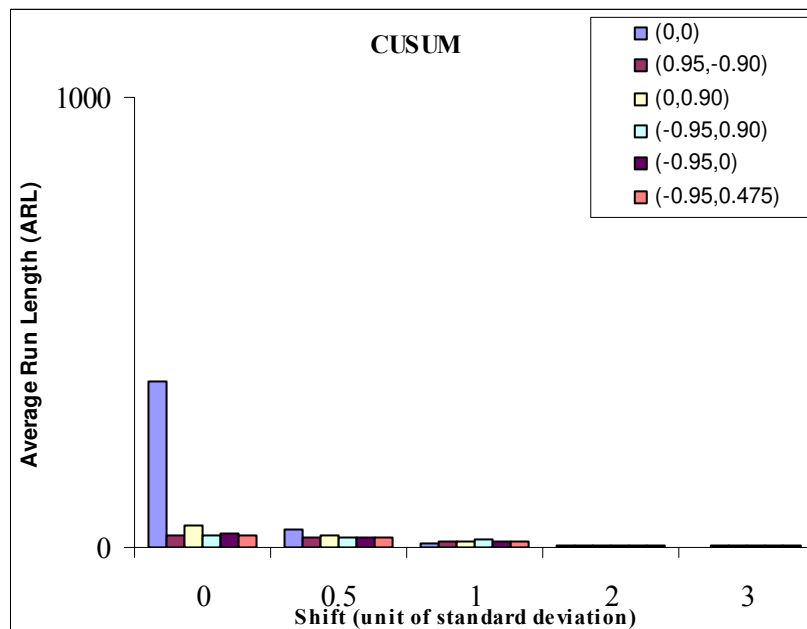


Figure 4.6 ARL bars of CUSUM chart for different levels of autocorrelation versus mean shift.

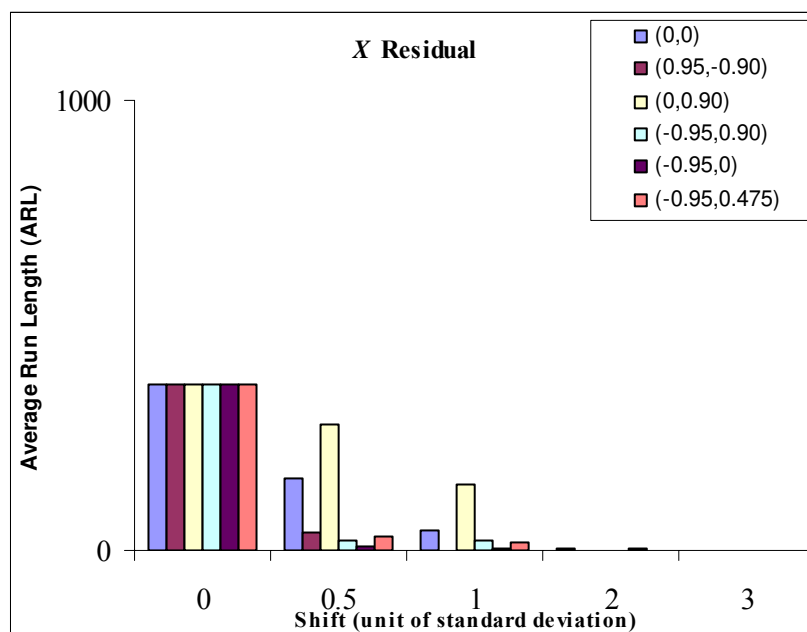


Figure 4.7 ARL bars of X Residual chart for different levels of autocorrelation versus mean shift.

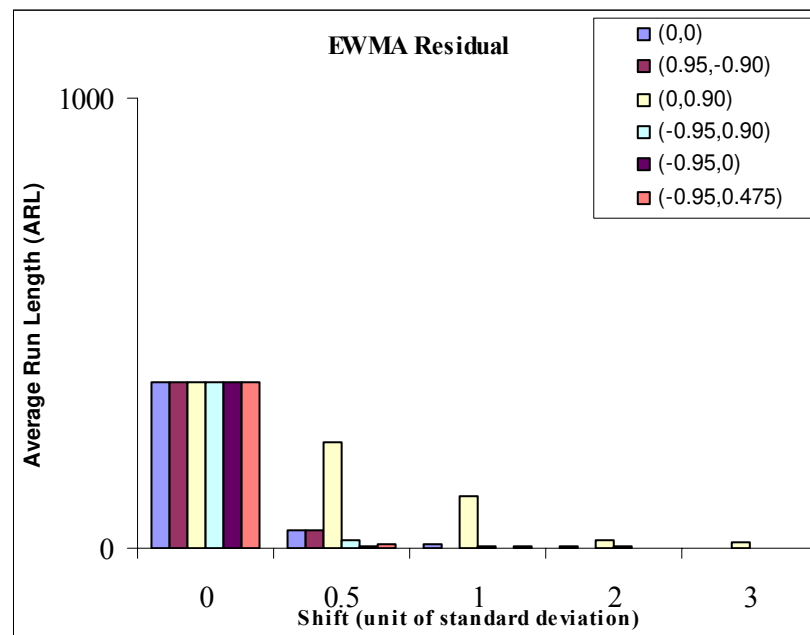


Figure 4.8 ARL bars of EWMA Residual chart for different levels of autocorrelation versus mean shift.

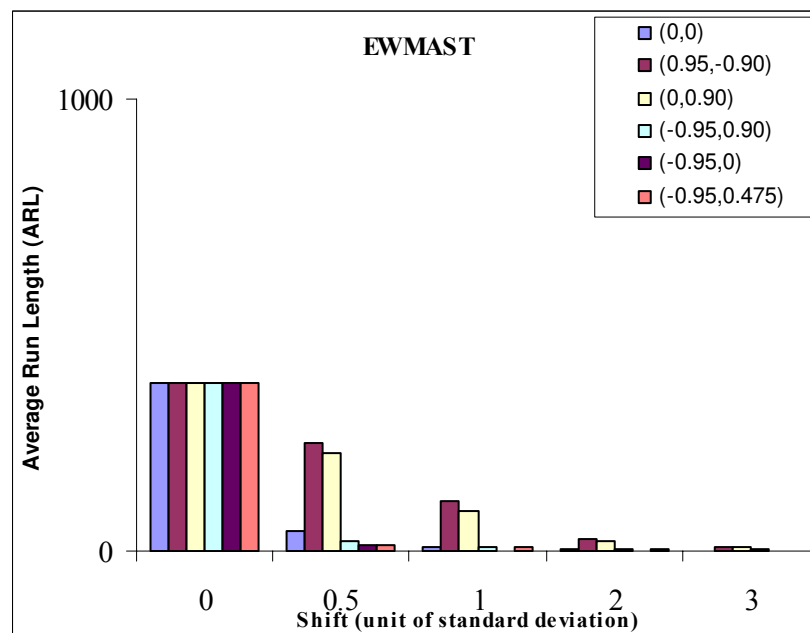


Figure 4.9 ARL bars of EWMAST chart for different levels of autocorrelation versus mean shift.

The effect of different autocorrelation structures on EWMAST and ARMAST charts are attached below in Figures 4.9 and 4.10. It is obvious that the ARMAST eliminates the disadvantage of EWMAST control chart when the process is highly

correlated. For instance, when $(\phi, \theta) = (-0.95, 0.90)$ the light green bar of ARMAST in Figure 4.9 decays faster than the EWMAST's bar in Figure 4.9 which means that ARMAST performed better than EWMAST. It is seen that the ARMAST has been more effective especially for small shifts. These observations are consistent with the results obtained by Jiang et al. (2000). Moreover, it is concluded from Figures 4.9 & 4.10 that both of the EWMAST and ARMAST perform better when the process is highly negative correlated, because the last three bars of corresponding charts which represent highly negative autocorrelation decline suddenly when the process mean shifts.

Finally, ARL bars for DTFC versus mean shift are illustrated in Figure 4.11. DTFC displays better performance than CUSUM in detecting small shifts for highly negative autocorrelation. However, it performs worse than CUSUM when the process is positively correlated with small shifts. In addition, we conclude that the DTFC chart performs best when the process is negatively correlated.

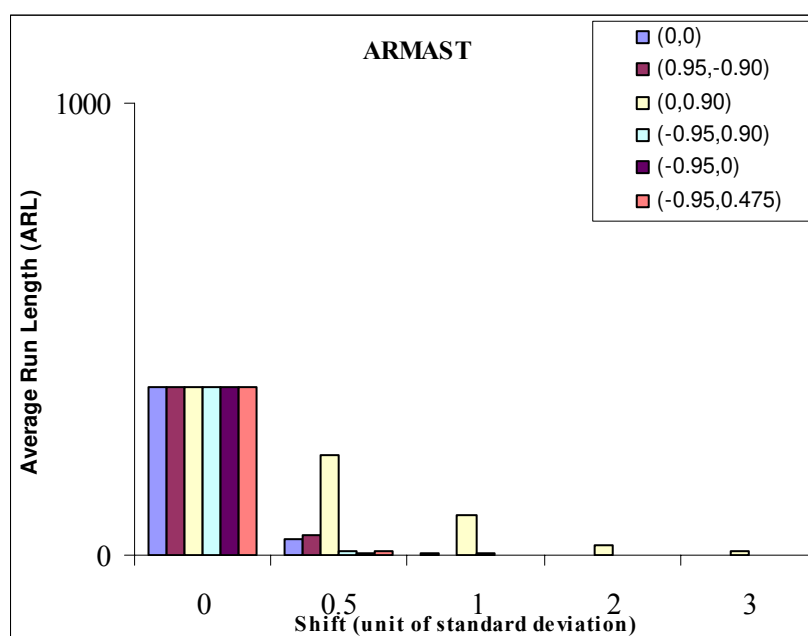


Figure 4.10 ARL bars of ARMAST chart for different levels of autocorrelation versus mean shift.

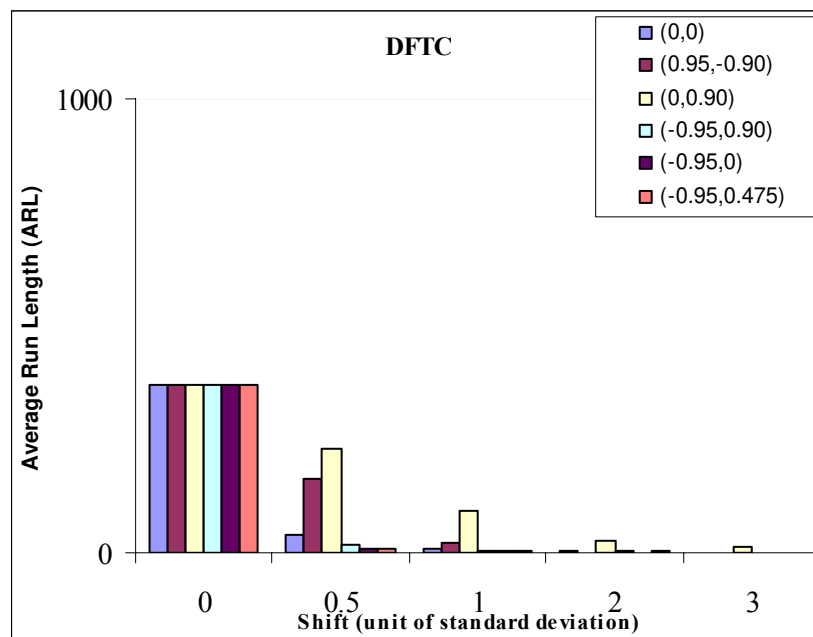


Figure 4.11 ARL bars of DTFC chart for different levels of autocorrelation versus mean shift.

CHAPTER FIVE

CONCLUSION

In this concluding chapter, we summarize what has been accomplished in this thesis, and describe some potential future work to extend the present results for the discussed autocorrelation problem.

Control charts are one of the most powerful tools of Statistical Process Control (SPC) and a fundamental assumption of traditional control charts is that the observations of the process are independent and identically distributed (iid) normal about a certain mean. Because the independency assumption is not realistic in practice, during the last decades many researchers have proposed some modifications for traditional control charts when processes are autocorrelated. Therefore, the aim of this thesis is to compare various charts which have been developed for monitoring autocorrelated processes.

In this thesis, firstly, an overview of SPC and its tools are given. Then, behavior of traditional control charts in the presence of serial correlation and the charts that deal with autocorrelation problem are studied. A detailed literature review of control chart applications for autocorrelated processes is provided.

In chapter four, performances of Shewhart $\bar{X}$, CUSUM, $\bar{X}$ residual, EWMA residual, EWMAST, ARMAST, and DFCT charts are compared by using average run length (ARL) performance criterion. In this comparison study, the first-order autoregressive moving average ARMA(1,1) model is considered as autocorrelation structure. Even though there exist much amount of comparison studies for ARMA(1,1) model, there seems to be no earlier study examining traditional, residual and distribution free control charts together for ARMA(1,1).

It is concluded in chapter four that autocorrelation has big impact on traditional control charts. When there is high correlation, whether it is positive or negative, the out-of-control ARL of Shewhart $\bar{X}$ chart increases significantly, and the in-control ARL of CUSUM chart drops dramatically. So, autocorrelation causes poor ARL performances for out-of-control ARL of Shewhart $\bar{X}$ and in-control ARL of CUSUM. Control charts which are proposed for autocorrelation problem, perform differently for various levels of autocorrelation and various magnitudes of mean shifts. The ARL results of these charts and interpretation of them are given in chapter four. In Table 5.1, a brief overview about the ARL performance of control charts compared in this study is given. For each autocorrelation-shift combination the best performed control charts are recorded.

Table 5.1 Best Performed Control Charts

autocorrelation shift		high positive	moderate negative	high negative
small		CUSUM EWMA Residual	CUSUM EWMAST	ARMAST EWMA Residual
medium		ARMAST EWMA Residual	CUSUM EWMAST	ARMAST EWMA Residual
large		ARMAST DFTC $\bar{X}$ Residual	$\bar{X}$ Residual	ARMAST $\bar{X}$ Residual EWMAST DFTC

It can be concluded that none of the compared charts have an overall performance. Every single control chart has good performances for some situations, and bad performances for some. However, in general, the ARMAST chart has best

performance in most of the autocorrelation-shift combinations. The Shewhart $\bar{X}$ chart achieved the worst ARL performance, so that it does not even take place in the table. CUSUM performs well when there is high positive or moderate negative autocorrelation in the process and mean shifts are small to medium. Residual control charts are appropriate for highly correlated processes, and the $\bar{X}$ residual control chart is suitable for large mean shifts while the EWMA residual chart is suitable for small ones. Also, the $\bar{X}$ residual chart and the ARMAST chart are suitable for moderately negative correlated processes with large mean shifts, and highly negative correlated processes with small mean shifts, respectively. EWMAST performs well when the process is highly negative correlated and the shifts from mean are large. Furthermore, The EWMAST is much more effective than Shewhart $\bar{X}$, residual charts, ARMAST and DFCTC when the absolute value of the lag-one autocorrelation coefficient is not high, and the shift from the mean is small. When the process is highly correlated and mean shifts are large, the DFCTC chart performed as well as the ARMAST.

Processes studied throughout this thesis were stationary. It may also be interesting to extend ARL calculations for processes which are nonstationary in mean.

REFERENCES

- Alwan, L. C., & Radson, D. (1992). Time-series investigation of subsample mean charts. *IIE Transactions*, 24 (5), 66-80.
- Alwan, L. C., & Roberts, H. V. (1988). Time-series modeling for statistical process control. *Journal of Business and Economic Statistics*, 6, 86-95.
- Atienza, O. O., Tang, L. C., & Ang, B. W. (1997). ARL properties of a sample autocorrelation chart. *Computers & Industrial Engineering*, 33, 733-736.
- Atienza, O. O., Tang, L. C., & Ang, B. W. (1998). SPC procedure for detecting level shifts of autocorrelated processes. *Journal of Quality Technology*, 30 (4), 340-351.
- Bagshaw, M. & Johnson, R. A. (1975). The effect of serial correlation on the performance of CUSUM Tests II. *Technometrics*, 17, 73-80.
- Banks, J. (1989). *Principles of quality control*. Canada: John Wiley & Sons
- Besterfield, D.H. (2004). *Quality Control* (7th ed.). New Jersey: Pearson Prentice Hall.
- Box, G. E. P. & Jenkins, G. M. (1976). *Time series analysis forecasting and control*. San Francisco, California: Holden-Day.
- Cryer, J. D. (1986), *Time series analysis*. Boston, PWS-KENT.
- Deming, W. E. (1988). *Out of the crisis*. Cambridge, MA: MIT Center for Advanced Engineering Study.
- Enders, W. (1995). *Applied econometric time series*. NY: John Wiley & Sons

- Griffith, G. K. (2000). *The quality technician's handbook* (4th ed.). New Jersey: Prentice Hall.
- Hamilton, J. D. (1994). *Time series analysis*. New Jersey: Princeton University Press.
- Harris, T. J., & Ross, W. H. (1991). Statistical process control procedures for correlated observations. *Canadian Journal of Chemical Engineering*, 69, 48-57.
- Hawkins, D. M. (1993). Cumulative sum control charting: an underutilized SPC tool. *Quality Engineering*, 5.
- Jiang, W., Tsui, K. L., & Woodall, W. H. (2000). A new SPC monitoring method: the ARMA chart. *Technometrics*, 42, 4, 399-410.
- Jiang, W., & Tsui, K. L. (2001). Some properties of the ARMA control chart. *Nonlinear Analysis*, 47, 2073-2088.
- Johnson, R. A., & Bagshaw, M. (1974). The effect of serial correlation on the performance of CUSUM tests. *Technometrics*, 16, 103-112.
- Juran, J. M. (1999). *Quality handbook* (5th ed.). NY: McGraw-Hill.
- Kim, S.H., Alexopoulos, C., Goldsman, D., & Tsui, K. L. (2006). A new model-free CUSUM procedure for autocorrelated processes. *Technical report*, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA.
- Kim, S. H., Alexopoulos, C., Tsui, K. L., & Wilson, J. R. (2007). A distribution – free tabular CUSUM chart for autocorrelated data. *IEE Transactions*, 39, 317-330.
- Ledolter, J., & Burrill, C. W. (1999). *Statistical quality control strategies and tools for continual improvement*. NY: John Wiley & Sons.

- Longnecker, M. T., & Ryan, T. P. (1992). Charting correlated process data. *Technical Report 166*, Texas A&M University, Dept. of Statistics.
- Lu, C. W., & Reynolds, M. R. Jr. (1999). EWMA control charts for monitoring the mean of autocorrelated processes. *Journal of Quality Technology*, 31 (2).
- Lucas, J. M., & Saccucci, M. S. (1990). Exponentially weighted moving average control schemes. Properties and enhancements. *Technometrics*, 32 (1), 1-12.
- Mastrangelo, C. M., Simpson, J. R. & Montgomery, D. C. (2001). Time series analysis. *Encyclopedia of Operations Research and Management Science*, 6, 828-833.
- Montgomery, D. C. (1997). *Introduction to statistical quality control* (3rd ed.). New York: John Wiley & Sons.
- Montgomery, D. C., & Johnson, L.A. (1976). *Forecasting and time series analysis*. New York, McGraw-Hill.
- Montgomery, D. C., & Mastrangelo, C. M. (1991). Some statistical process control methods for autocorrelated data. *Journal of Quality Technology*, 23, 179-193.
- Oakland, J. S. (2003). *Statistical Process Control* (5th ed.). Oxford: Butterworth-Heinemann.
- Pacella, M., & Semeraro, Q. (2007). Using recurrent neural networks to detect changes in autocorrelated processes for quality monitoring. *Computers & Industrial Engineering*, 52, 502-520.

- Reynolds, M. R. Jr., & Lu, C.W. (1997). Control charts for monitoring processes with autocorrelated data. *Nonlinear Analysis, Theory, Methods & Applications*, 30 (7).
- Runger, G. C., & Willemain, T. R. (1995). Model-based and model-free control of autocorrelated processes. *Journal of Quality Technology*, 27 (4), 283-292.
- Runger, G. C., & Willemain, T. R. (1996). Batch-means control charts for autocorrelated data. *IIE Transactions*, 28, 483-487.
- Schmid, W. (1997). On EWMA charts for time series. In *Frontiers of Statistical Quality Control*, 5, H.J. Lenz, P.-Th. Wilrich, Heidelberg: Pyhsica-Verlag, 115-137.
- Shewhart, W.A. (1931). *Economic control of quality of a manufactured product*. NY: Van Nostrand.
- Siegmund, D. (1985). *Sequential analysis: tests and confidence intervals*. NY: Springer-Verlag.
- Smith, G. M. (2004). *Statistical process control and quality improvement* (5th ed.). New Jersey: Pearson Prentice Hall
- Thomas, D. W., Barrett, C. S., Blankenstein, E. E., Chapman, A. T., Cole, C. C., Drummond, F. H., et al. (1984). *Statistical quality control handbook*. Indiana: AT&T Technologies.
- VanBrackle, L. N., III, and Reynolds, M. R., Jr. (1997). EWMA and CUSUM control charts in the presence of correlation. *Communications in Statistics – Simulation and Computation*, 26, 979-1008.

- Vasilopoulos, A. V., & Stamboulis, A. P. (1978). Modification of control chart limits in the presence of data correlation. *Journal of Quality Technology*, 10 (1), 20-30.
- Wardell, D. G., Moskowitz, H., & Plante, R. D. (1992). Control Charts in the Presence of Data Correlation. *Management Science* 38, 1084-1105.
- Wardell, D. G., Moskowitz, H., & Plante, R. D. (1994). Run-length distributions of special-cause control charts for correlated observations. *Technometrics*, 36, 3- 17.
- Winkel, P., & Zhang, N. F. (2004). Serial correlation of quality control data – on the use of proper control charts. *Scandinavian Journal of Clinical and Laboratory Investigation*, 64, 195-204.
- Yashchin, E. (1993). Performance of CUSUM control schemes for serially correlated observations. *Technometrics*, 35 (1), 37-52.
- Yourstone, S. A., and Montgomery, D. C. (1989). A Time Series Approach to Discrete Real-Time Process Quality Control. *Quality and Reliability Engineering International*, 5, 309-317.
- Yourstone, S. A., and Montgomery, D. C. (1991). Detection of process upsets-sample autocorrelation control chart and group autocorrelation control chart applications. *Quality and Reliability Engineering International*, 7, 133-140.
- Zhang, N. F. (1997). Detection capability of residual control chart for stationary process data. *Journal of Applied Statistics*, 24 (4), 475-492.
- Zhang, N. F. (1998). A statistical control chart for stationary process data. *Technometrics*, 40 (1).
- Zhang, N. F. (2000). Statistical control charts for monitoring the mean of a stationary Process. *Journal of Statistical Computation and Simulation*, 66, 249-258.

Zhang, N. F. (2004). Serial correlation of quality control data-on the use of proper control charts. *Scandinavian Journal of Clinical and Laboratory Investigation*, 64, 195-204.