

USE OF DCR-OPTIMIZED NATURAL SWITCHING SURFACES IN THE SECOND-ORDER SLIDING MODE CONTROL OF DC/DC BUCK AND BOOST CONVERTERS AND CURRENT LIMITING METHODS

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Second-order sliding mode control utilizing natural switching surfaces has demonstrated robust dynamic performance in the control of switching buck and boost converters for start-up, load changes, and large signal disturbances. However, practical implementation often encounters challenges such as the chattering phenomenon and large inductor currents during start-up or loading transients. The primary focus of this research is proposing potential solutions for these practical issues. Assuming the parasitic resistances, such as the on-state resistances of switches and the equivalent series resistances of output capacitors, are relatively low, the direct current resistance (DCR) of the inductor becomes the primary parasitic resistance that causes chattering due to its damping effect. For this reason, the DCR of the inductor is taken into account in the system dynamics from which natural switching surfaces are derived. By incorporating these surfaces into the control laws, the chattering effect caused by losses is mitigated while maintaining effective control. Furthermore, current limiting techniques are proposed for converter protection and reliability. These techniques are integrated into the control laws to manage large inductor currents effectively during transients, thereby preventing potential damage and enhancing the overall performance of the converter. Through theoretical analysis, geometrical representations, and simulation studies, a comprehensive framework for minimizing chattering and managing large inductor currents in the second-order sliding mode control of buck and converters integrating natural switching surfaces is established.

Keywords: DC/DC buck converter, DC/DC boost converter, second-order sliding mode control, boundary control, normalization, natural switching surface, chattering effect, current limiting, DC resistance.

ÖZET

DA/DA BUCK VE BOOST DÖNÜŞTÜRÜCÜLERİN İKİNCİ DERECE KAYAN KIPLI KONTROLÜNDE DAD-OPTİMİZE DOĞAL ANAHTARLAMA YÜZEYLERİNİN KULLANILMASI VE AKIM LİMITLEME METOTLARI

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Doğal anahtarlama yüzeylerini kullanan ikinci derece kayan kipli kontrol, başlama, yük değişimleri ve büyük sinyal bozulmaları için anahtarlama buck ve boost dönüştürücülerin kontrolünde etkili dinamik performans göstermiştir. Bununla birlikte, pratik uygulama chattering olgusu ve açılış, yük akımı artış durumları sırasında yüksek bobin akımları gibi zorluklarla sıklıkla karşılaşır. Bu araştırmanın temel odak noktası, bu pratik sorunlar için potansiyel çözümler önermektir. Anahtarların iletken durum dirençleri ve çıkış kapasitörlerin seri eşdeğer dirençleri gibi parazitik dirençlerin düşük olduğu varsayıldığında, indüktörün direkt akım direnci (DAD) damping etkisi nedeniyle chatteringe neden olan birincil parazitik direnç haline gelir. Bu sebeble, indüktörün DAD'sı doğal anahtarlama yüzeylerinin türetildiği sistem dinamiklerinde dikkate alınmıştır. Bu yüzeylerin kontrol yasalarına entegre edilmesiyle kayıpların neden olduğu chattering etkisi azaltılırken etkili kontrol korunmaktadır. Ayrıca, dönüştürücü koruması ve güvenilirliği için akım sınırlama teknikleri önerilmiştir. Bu teknikler, geçici büyük endüktör akımlarını etkili bir şekilde yönetmek için kontrol yasalarına entegre edilmiştir, böylece potansiyel hasar önlenir ve dönüştürücünün genel performansı artırılır. Teorik analiz, geometrik gösterimler ve simülasyon çalışmaları aracılığıyla, doğal anahtarlama yüzeylerini entegre eden buck ve boost dönüştürücülerin ikinci derece kayan kipli kontrolünde chatteringi en aza indirmek ve büyük endüktör akımlarını yönetmek için kapsamlı bir çerçeve oluşturulmuştur.

Anahtar sözcükler: DC/DC buck dönüştürücü, DC/DC boost dönüştürücü, ikinci derece kayan kipli kontrol, sınır kontrol, normalizasyon, doğal anahtarlama yüzeyi, chattering etkisi, akım limitleme, DC direnç.



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Chapter 1

Introduction

1.1 Motivation and Background

Modern electrical and electronic systems depend critically on power electronics, an area that deals with the control and conversion of electrical power. The use of converters to transform the form of electrical power is one of the fundamental concepts in power electronics. Two common types of converters are buck converters and boost converters. A buck converter steps down the input voltage to a lower output voltage, whereas a boost converter steps up the input voltage to a higher output voltage. Buck converters can regulate the output voltage using either linear or switching modes, whereas boost converters exclusively use switching regulation. Switching converters are widely favored because of their high conversion efficiency. The state of the switches must be controlled in order to achieve and maintain the target performance. The control methods are crucial for ensuring stability, efficiency, and dynamic performance. The literature review provided in the following chapter details control methods for switching buck and boost converters by focusing on their principles, benefits, and drawbacks. Classical linear control methods such as pulse width modulated voltage mode or current mode control are incapable of achieving satisfactory performance under large signal disturbances or load variations. Switching converters are non-linear

systems since their structure keeps varying during the control operation as the states of the switches change. Therefore, non-linear control methods improved the dynamic response of the converters [1]. The sliding mode control method, which incorporates first-order switching surfaces in control law, is widely studied and found to be well-suited for these systems [2]. This control method guarantees large signal stability and robustness against external perturbations and parameter variations [3, 4]. The studies with second-order switching surfaces demonstrated faster and better dynamic performance [5, 6]. A second-order switching surface obtained by the natural dynamics of these converters, called a natural switching surface, provides further improvements in dynamic response during start-up and load transients [7]. This work focuses on utilizing natural switching surfaces by overcoming practical issues such as the chattering caused by the damping effect resulting from parasitic resistances and dealing with large inductor currents, which can be observed during start-up or loading transients.

1.2 Thesis Overview

The organization of the rest of this thesis is as follows:

- In Chapter 2, the literature review on control methods for switch mode buck and boost converter is provided. By emphasizing their principles, advantages, disadvantages, and limitations, linear and non-linear control methods are investigated.
- In Chapter 3, the derivation of switching surfaces is done by taking the parasitic resistance of the inductor into account. Control laws using natural switching surfaces for optimizing dynamic performance are proposed. Control law for limiting inductor current at the desired value is also provided. Under these control laws, the natural trajectories are formulated and geometrically shown on the state plane. Start-up, loading, unloading, and steady-state operations are examined in detail. The performance of the proposed control method is demonstrated via ideal and realistic simulations,

and comparisons are made with theoretical calculations. The explanations for practical discrepancies are given heuristically.

- In Chapter 4, a study similar to the one conducted in the previous chapter is undertaken for the boost converter.
- In Chapter 5, final remarks are delivered, and future study directions are suggested.

1.3 Main Contributions

The main contributions of this thesis to buck and boost converter control literature can be listed below:

- The system dynamics of the buck and boost converters are modeled using the normalization technique by considering the parasitic resistance of the inductor in the switch ON and switch OFF cases for the buck converter and the switch OFF case for the boost converter. Eliminating the time allows natural switching surfaces to be developed for both cases with these dynamics. For steady-state operation, modifications for the on-state natural switching surface of the buck converter and the on and off-state natural switching surfaces of both converters are presented for controlling ripples and switching frequency.
- Control rules that use these natural switching surfaces to achieve high dynamic performance are provided. Additional control laws that allow the inductor current to be limited, as well as the switching frequency during the current limitation period, are proposed for both converters.
- Natural system trajectories under these laws are mathematically expressed and graphically shown on a state plane for start-up, loading, unloading, and steady-state operations. A theoretical basis is provided for the computation of the time duration, current, and voltage variations during these operations.

- The algorithms for determining design parameters for given requirements are provided in pseudo-code format.



Chapter 2

Literature Review and The Proposed Method

DC/DC converters are an important part of power electronics as they facilitate the transformation of electrical power from one voltage level to another. Among the many different converters, the buck and boost converters are two of the most commonly used types. A buck converter decreases the input voltage to produce a lower output voltage, whereas a boost converter raises the input voltage to generate a higher output voltage. There are two modes for regulating the output voltage of buck converters: linear and switching, whereas boost converters rely on switching regulation. Linear converters regulate output voltage by dissipating excess energy as heat [8]. They maintain a consistent output voltage by operating in the active region of their transistors, which are typically BJTs or MOSFETs [9]. On the other hand, transistors, typically MOSFETs, are employed by switching converters as switches [10] to regulate the energy flow and maintain a desired output voltage by periodically turning on and off. Switching converters are valued for their superior conversion efficiency and are frequently analyzed in literature and utilized in practice. The basic circuit diagrams of switching buck and boost converters are given in Figure 2.1 and Figure 2.2, respectively.

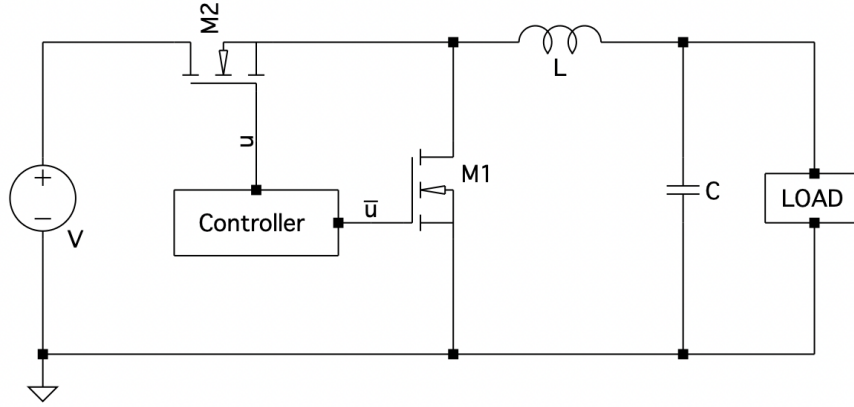


Figure 2.1: The schematic of the switching buck converter

The development of the switching buck converter allowed the effective reduction of voltage levels while preserving high efficiency. Its ability to quickly switch between on and off states in order to scale down DC voltages resulted in notable reductions in heat generation and power loss compared to linear buck converters [11], which made it ideal for a variety of applications, including automotive systems [12] and portable electronics [13]. On the other hand, the switching boost converter addresses the demand for an effective rise in DC voltage level. Switching boost converters can raise voltage levels while keeping high efficiency by storing energy in an inductor during switch-on periods and releasing it to the output during switch-off intervals [14]. Applications include renewable energy systems [15], battery-powered devices, and LED drivers [16] where the input voltage may be insufficient to power the load directly.

The basic control problem for switching buck and boost converters, which is considered in this thesis, is the following: maintaining the output voltage by adjusting the state of the switches despite variations in input voltage, load, and other disturbances while also addressing dynamic response, stability, and overall efficiency. The control strategies used by switching buck and boost converters have a significant impact on their reliability and performance. Stability, high efficiency, fast transient response, and robustness against parameter variations and external disturbances all depend on effective control strategies [3]-[7]. The control methods for these converters can be classified into two main categories:

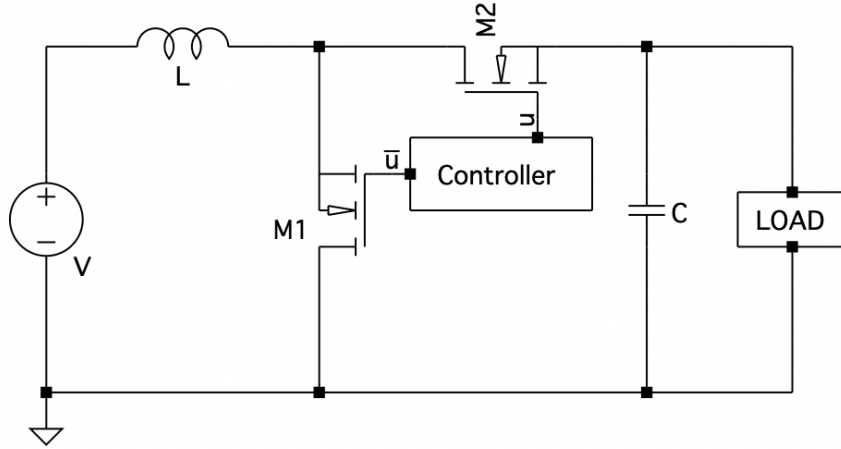


Figure 2.2: The schematic of the switching boost converter

linear and nonlinear. Each category has its own set of benefits and drawbacks.

2.1 Linear Control Methods

Linear control methods are widely used in the control of buck and boost converters because of their simplicity, ease of design, and solid theoretical foundation. These methods use linearized models of converters, which are inherently nonlinear systems. The state space averaging method introduced in [17] is the most common averaging technique for small signal modeling of these converters to implement linear control methods [18]. Common linear control methods are voltage mode control (VMC) and current mode control (CMC).

2.1.1 Voltage Mode Control

Voltage mode control is a method of managing the output voltage by modifying the duty cycle of the switching element in response to the difference between the output voltage and a reference voltage. The simplified block diagram for the voltage mode control of switching converters is illustrated in Figure 2.3, see [19]. It has only one control loop, which typically consists of a voltage error

generator that compares the output voltage with a reference voltage to generate an error signal, a compensator, which is often a type of proportional integral derivative (PID) controller that processes error signal to adjust the duty cycle of the switching signal, and a pulse width modulator (PWM) for controlling pulse width modulation of the switching elements according to the output of the compensator determining the duration of the on and off states [20]. Compared to the current-mode control method, VMC circuits are generally easier to design and implement. Despite its numerous advantages, VMC encounters several challenges. Due to their design based on linearized small-signal converter models, voltage mode controllers frequently are unable to perform satisfactorily in the presence of parameter or large load fluctuations [21]. Moreover, voltage mode control results in unstable zero dynamics, particularly for boost converters [22].

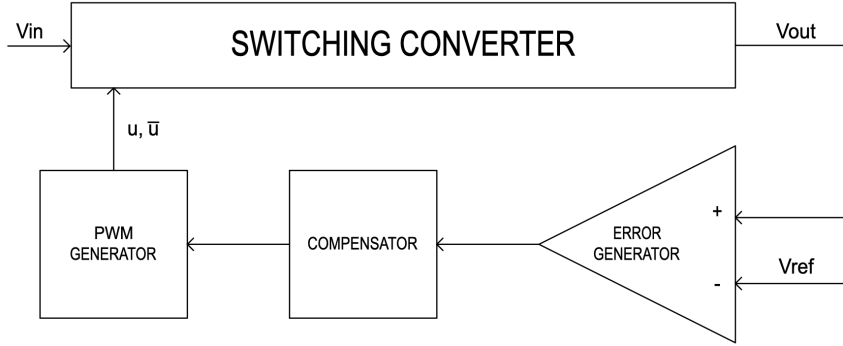


Figure 2.3: Basic voltage mode control block diagram

2.1.2 Current Mode Control

Current mode control is a technique that involves adding an extra control loop for the direct control of the inductor current. The simplified block diagram for current mode control of switching converters is constructed based on [19] and shown in Figure 2.4. The inner control loop measures the actual inductor current value, compares it with the reference current generated by the outer control loop, and adjusts the duty cycle of the switching signal using pulse width modulation. The outer control loop measures the output voltage, compares it to a reference value, and generates an error signal, then specifies reference inductor current

value by processing error through the voltage error amplifier and compensator [23, 24]. Since the inductor current is directly controlled by current mode control, it incorporates inherent overcurrent protection. Current mode control offers a more rapid reaction in comparison to VMC to sudden variations in load or input voltage due to its direct regulation of the inductor current by the inner loop [25]. The current mode control may cause oscillations at frequencies a fraction of the switching frequency in the output voltage or current if the duty cycle is greater than 0.5 [26]. The linearized model of the boost converter is a non-minimum phase system that has a right half plane zero so that the bandwidth of the closed loop must be kept low to maintain stability; this may slow down the rate at which the current loop can react to changes [27].

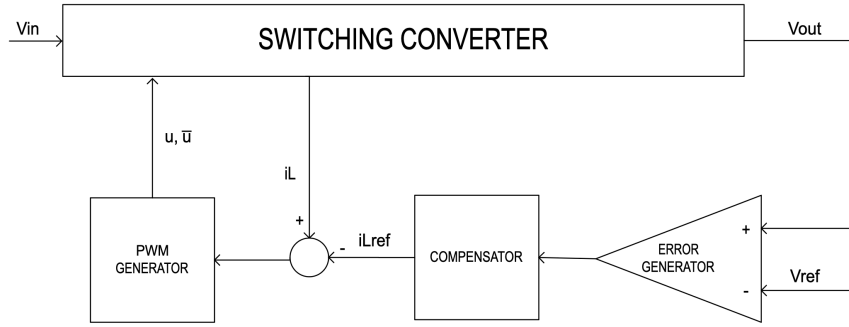


Figure 2.4: Basic current mode control block diagram

2.1.3 Compensation

The compensation networks in the closed loop of the converters can be designed using a variety of methods. The pole placement method is the most frequently employed technique, which involves using Bode plots to shape the loop transfer function to achieve the desired crossover frequency and phase margin [17]. The initial phase involves using a linearized small-signal model to plot the open-loop frequency response of the converter plant. Subsequently, an appropriate controller type of the compensation network is determined by analyzing both the ideal open loop response Bode plots and the converter response Bode plots. Then, the poles and zeroes of the compensation network are determined. A second method of designing the compensation network involves the computation of the controller

parameters to achieve an optimal closed-loop transfer function for the system [28]. This approach encompasses both the small signal model of the controller and the converter. The goal is to form a system that has an appropriately fast and stable response. Traditional controllers like proportional-integral (PI), proportional integral derivative (PID), lead, lag, etc., have been developed over time to achieve desired converter performance under specific conditions [29]. PI and PID controllers are among the most preferred controllers used in compensation networks. In [30], the PID controller outperformed the PI controller in terms of response time, steady state error, and settling time. However, the derivative term in the PID controller is sensitive to noise and measurement error of the system, which can lead to oscillation in the output voltage under steady-state conditions [31].

2.2 Nonlinear Control Methods

Linear control techniques encounter difficulties in dealing with the non-linear characteristics of buck and boost converters, especially under large-signal operating conditions or in the presence of significant disturbances [1]. Nonlinear control approaches have been developed as effective alternatives, providing upgraded performance in situations when linear models are insufficient.

2.2.1 Sliding Mode Control

Sliding Mode Control (SMC) includes moving the system states to a designed sliding surface in finite time and then maintaining a sliding motion using a discontinuous control rule so that the system performs as desired when restricted to the surface [32]. The sliding control method is found to be well-suited for variable structure systems [2]. Switching converters are this type of systems since their structures keep changing during the control operation. The application of sliding mode control for buck converter and boost converter are first shown in [33] and [34] respectively [35]. The application of sliding mode control to switching

converters ensures guaranteed stability and robustness against load and parameter uncertainties as well as external disturbances [3, 4]. Sliding mode controllers produce more consistent transient responses across a wider working range than standard linear controllers [35]. With a proper design of switching surfaces, SMC can provide a rapid response to changes in reference signals or external disturbances, making it suitable for applications that require fast transient responses.

2.2.1.1 Second-Order Sliding Mode Control

In the first-order sliding mode control (FOSC), the sliding surface is a linear function of the system states. The primary aim is to drive the system states to this surface and maintain them there. Its design is simple and easy to implement. FOSC offers decent large-signal performance and stability. However, its transient dynamics are not optimal [36]. In [36], a second-order switching surface that increases the tangential velocity of the trajectories so that the converter demonstrates better transient characteristics compared to the one with the first-order switching surface is proposed and experimentally tested for the buck converter. A detailed comparison of the dynamic performance of buck converters using first-order and second-order sliding mode control, as investigated in [5] and [6], shows that converters with second-order sliding surfaces achieve better dynamic responses compared to those with first order sliding surfaces. The sliding control method that uses a second-order (curved) switching surface is often referred to as boundary control [37]; in this thesis, the second-order sliding mode control (SOSC) term is preferred. [38] proposes a second-order switching surface on a state energy plane instead of a state plane for a boost converter. Using this surface provided a fast transient response under large signal disturbances. The dynamic response of the converter is enhanced by directing the state variables toward the target operating point faster through the use of second-order switching surfaces for both converters. However, the second-order surfaces used for controlling buck and boost converters showed overshoot issues during start-up and significant load transients, often requiring two switching actions to reach a steady state [7]. In [7], a second-order sliding surface based on the natural dynamics of

a boost converter with a constant current load was derived using a normalization technique to generalize the analysis. Using this surface, the converter achieved a steady state in only one switching operation with no overshoot by displaying an outstanding dynamic behavior and optimal response for start-up and load transients. This control technique demonstrated better performance than the first and second-order switching surfaces by geometrical comparison. A similar study is conducted for sliding mode control of buck converter in [39], and theoretical physical limits for the dynamic performance of buck converter are derived. Similarly, in [40], physical theoretical limits are derived for the dynamic performance of boost converters. [39] and [40] provide reference points for the optimal performance of both converters. In [41], the derivation of natural switching surfaces for the resistively loaded buck and boost converters are provided, aiming to reduce the damping effect caused by the resistor as a load. The performance of both converters with these surfaces is investigated via ideal and practice-oriented simulations.

2.2.1.2 Large Inductor Currents

In the sliding mode control of buck and boost converters, fast output voltage dynamics lead to an overshoot in the inductor current [42]. For two reasons, the converter devices could not withstand the high inductor current: either it would simply exceed the maximum permitted switch current, or it would cause the inductor core to saturate, resulting in an even higher peak current value. Therefore, it becomes reasonable to add a protection circuit to the controller to keep the inductor current from rising to hazardous levels. In [41], it is mentioned that the inductor and switches need to be larger in size to manage the peak current because the peak current during the start-up phase was significantly larger than its steady state value for both converters. [42] defines two distinct first-order sliding surfaces for cases where the inductor current is either above or below the limited value for the inductor current limitation of the buck converter. In [43], a nonlinear piecewise switching surface consisting of three different switching surfaces, one of them is for enforcing the system trajectories to follow a consistent

current path until the error in output voltage becomes small, is proposed for limiting inductor current of buck converter at the start-up. [37] is an example of the integration of current limiting to discrete-time sliding mode control of buck converters that use curved switching surfaces. In [44], a method is proposed for inductor current limiting during input voltage increase or loading transients in the sliding mode control of boost converter incorporating natural switching surfaces. This method adds an additional boundary to a part of the state plane when the switch is in the on-state.

2.2.1.3 Variable Switching Frequency and Chattering Effect

In order to accomplish the desired dynamic response and steady-state operation, the sliding mode controller should ideally operate at an infinite switching frequency, enforcing the controlled variables to follow a certain reference path[45]. However, practical sliding mode controllers operate at finite, inherently varying (usually high) switching frequencies. Nevertheless, this makes practical implementation difficult since converters that switch at extremely high speeds experience significant switching and inductor core losses as well as electromagnetic interference (EMI) noise issues [45, 46]. In [47], constant timer circuits were employed in the sliding mode control of the switching buck converter with a hysteresis band for constant switching frequency operation and experimentally tested. An adaptive hysteresis band that changes with parameter variation for the sliding control method is proposed for buck converters in [48]. [49] proposes pulse width modulation rather than hysteresis modulation for sliding mode voltage-controlled buck converter. [50] employs hysteresis modulation in order to achieve a finite switching frequency operation. [51] varies hysteresis band for accomplishing fixed frequency operation in the second order sliding mode control of buck converter. [52] introduces a hysteresis band for limiting switching frequency during start-up for the control of a boost converter that uses a second-order switching surface. In [7], a modification for the natural surface is presented for limiting switching frequency during the steady state for the boost converter.

The SMC method is afflicted by chattering in the state variables caused by

unmodeled dynamics and switching time delays. It also has a finite frequency and amplitude. Power converters experience losses and limited control accuracy as a consequence of chattering. Chattering is a significant impediment to the practical implementation of SMC in power converters; therefore, its effects must be mitigated to an acceptable level [53]. The methods indicated above for limiting switching frequency may help to reduce the chattering effect. Although the chattering effect can be eliminated entirely in theory, as demonstrated in studies utilizing constant current or resistive loaded natural surfaces for buck and boost converters [7, 39, 41], it can still be observed in practice-oriented simulations due to component parameter variations and the damping effect of the parasitic resistances [41]. If the system trajectories are derived by considering these resistances, the damping of parasitic resistances can be mitigated [41]. [54] proposes a continuous control strategy to replace inherited discontinuous control responsible for chattering in the sliding mode control. [55] introduces a continuous sigmoid function to replace the discontinuous sign function for eliminating the chattering effect in the sliding mode control. Disturbance observer-based integral sliding mode control methods are proposed for buck converter [56] and for boost converter [57] for chatter-free output voltages. A disturbance observer estimates the external disturbances that affect the system by counteracting the high-frequency components of the disturbance; the integral term reduces chattering, and a smoother control action can be achieved. [58] proposes the estimation of model uncertainties along with external disturbances in the sliding mode control of buck and boost converters to eliminate the chattering phenomenon completely.

2.2.2 Other Nonlinear Control Methods

[59] introduces fuzzy logic control algorithms for the control of buck and boost converters. [31] provides a comparative analysis of the PI/PID and fuzzy logic controller in terms of settling time and overshooting during large signal transients. Fuzzy logic control significantly improves large-signal transient performance in boost converters; however, in the case of buck converter, the two control mechanisms produce comparable outcomes. [60] presents a comparative analysis of

sliding mode, fuzzy logic, and proportional-integral controllers for a buck converter, highlighting the similarities in system behavior between the fuzzy logic and sliding mode controllers. Under different load variations, the sliding mode controlled boost converter achieves a shorter settling time compared to the fuzzy logic controlled one [61]. In [62], the performance of model predictive control is compared to PI control for the buck converter. The results indicate improved performance in terms of transient dynamics and disturbance rejection. In [63], the test results for variations in input voltage and output load of the buck converter demonstrate that both fuzzy logic and neural network control techniques outperform the standard PID control technique in terms of rise time and settling time. Notably, neural network control exhibited the fastest performance. In [64], the controller based on artificial neural networks demonstrates a rapid response in following the desired output voltage and is also effective in reducing overshoot, oscillations, and settling time compared to conventional PI control for the boost converter.

2.3 The Proposed Method

In the method proposed in this thesis, natural switching surfaces for the second-order sliding mode control of buck and boost converters are derived by considering the DCR of the inductor, which becomes the most significant parasitic contributing to chattering when other parasitic resistances, such as the on-state resistances of switches and the equivalent series resistances of output capacitors, are assumed to be relatively low. This study differs from [41] by deriving the natural trajectories of a buck converter for a constant current load instead of a resistive load, as done in [39]. Unlike both [39] and [41], this research takes into account the DCR of the inductor when obtaining the natural trajectories. This study also introduces special control laws for limiting large inductor currents for second-order sliding mode control of buck and boost converters with natural switching surfaces. Analogous to the theoretical analysis, geometrical representations, and simulation studies conducted in [41] for buck and boost converters with a resistive load, this study evaluates buck and boost converter models with constant

current loads that incorporate the inductor's DCR. Additionally, the study expands the analysis to include scenarios with current-limited loading and startup transients. Furthermore, the study offers alternative versions of current limiting control laws that are designed to limit the switching frequency during current limitation phases as well.



Chapter 3

Second-Order Sliding Mode Control of DC/DC Buck Converter

3.1 Modeling of Natural Trajectories

Figure 3.1 depicts the conventional schematic of a buck converter with direct current resistance (DCR) of the inductor. The system dynamics of the buck converter are modeled by the normalization technique to establish a simplified general representation that eliminates any combination of L and C component parameters. The normalization technique outlined below follows the method described in [7] and [41]. The DCR of the inductor, which is the most significant parasitic in the system responsible for the chattering effect (if the on-state resistances of the switch and the equivalent series resistances of output capacitors are considered relatively low), is also taken into account. Using these dynamics, the system's natural trajectories and the natural switching surfaces (which are natural trajectories of the converter crossing the target operating point) are derived in the normalized form. The derivation of these trajectories and surfaces employs mathematical methods similar to those used in [41]. Unlike [41], which uses a

resistive load, this study derives the natural trajectories of the buck converter for a constant current load, similar to the approach in [39]. In contrast to [39] and [41], the natural trajectories are obtained by considering the DCR of the inductor.

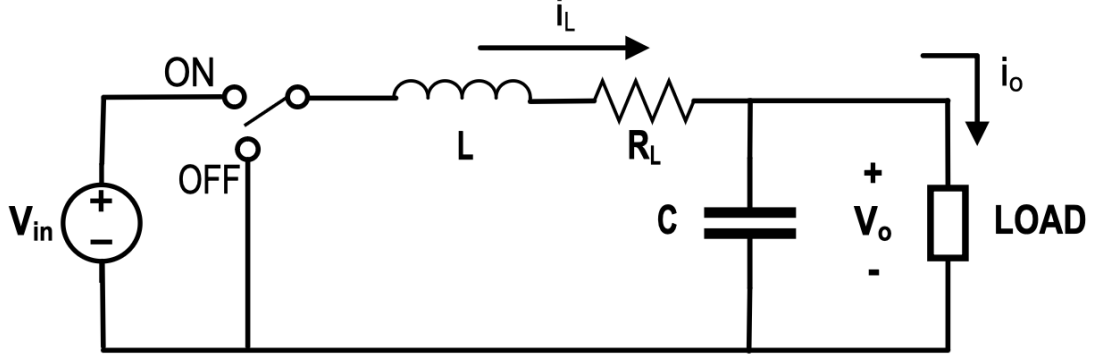


Figure 3.1: The schematic of the buck converter

The differential equations (3.1) represent the system dynamics of the buck converter with the position of the switch is denoted by u . On-state and Off-state dynamics can be obtained by replacing u with 1 and 0, respectively.

$$\begin{aligned} C \frac{dV_o}{dt} &= i_L - i_o, \\ L \frac{di_L}{dt} &= uV_{in} - R_L i_L - V_o. \end{aligned} \quad (3.1)$$

Note that, after obtaining the solutions $V_o(t)$ and $i_L(t)$ of (3.1) and eliminating time, the natural trajectories of the converter in $v_o - i_L$ plane can be obtained, which is essential in our approach to obtain a switching rule. To achieve this, a series of coordinate transformations are performed. First, a normalization is applied to simplify (3.1) using reference output voltage V_{ref} , the characteristic impedance Z_o , and the natural frequency f_o of L and C as described in (3.2).

$$Z_o = \sqrt{L/C}, \quad f_o = \frac{1}{2\pi\sqrt{LC}}. \quad (3.2)$$

The normalized form of the system variables can be defined as follows:

$$V_{xn} = \frac{V_x}{V_{ref}}, \quad i_{xn} = \frac{i_x Z_o}{V_{ref}}, \quad R_{Ln} = \frac{R_L}{Z_o}, \quad f_n = \frac{f}{f_o}, \quad t_n = f_o t. \quad (3.3)$$

Voltages, currents, frequency, and time are represented by V_x , I_x , f , and t , respectively; here, x is either in, L or o. The subscript n denotes the normalized representations of these variables. By using (3.2) and (3.3) in (3.1), we obtain the following normalized equations:

$$\begin{aligned}\frac{dV_{on}}{dt_n} &= 2\pi(i_{Ln} - i_{on}), \\ \frac{di_{Ln}}{dt_n} &= -2\pi(-uV_{inn} + R_{Ln}i_{Ln} + V_{on}).\end{aligned}\tag{3.4}$$

The new system variables $\hat{i}_{Ln}$ and $\hat{V}_{on}$ defined in (3.5) facilitate similarity transformation, which will be used in coordinate conversion.

$$\begin{aligned}\hat{i}_{Ln} &= i_{Ln} - i_{on}, \\ \hat{V}_{on} &= -uV_{inn} + R_{Ln}i_{on} + V_{on}.\end{aligned}\tag{3.5}$$

The normalized system equations (3.4) are rewritten in (3.6) with the new variables by using (3.5) in (3.4).

$$\begin{aligned}\frac{d\hat{V}_{on}}{dt_n} &= 2\pi\hat{i}_{Ln}, \\ \frac{d\hat{i}_{Ln}}{dt_n} &= -2\pi(R_{Ln}\hat{i}_{Ln} + \hat{V}_{on}).\end{aligned}\tag{3.6}$$

The analytical solution of (3.6) will be used for the derivation of the natural trajectories of the converter. The matrix form of (3.6):

$$\begin{bmatrix} \frac{d\hat{V}_{on}}{dt_n} \\ \frac{d\hat{i}_{Ln}}{dt_n} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 2\pi \\ -2\pi & -2\pi R_{Ln} \end{bmatrix}}_A \begin{bmatrix} \hat{V}_{on} \\ \hat{i}_{Ln} \end{bmatrix}.\tag{3.7}$$

The roots of the characteristic polynomial (3.8) of the system matrix A are the eigenvalues that will be used for solving equation (3.6).

$$\det(\lambda I - A) = \lambda^2 + 2\pi R_{Ln} + 4\pi^2.\tag{3.8}$$

The eigenvalues of the system matrix A can be found as:

$$\lambda_{1,2} = -\pi R_{Ln} \pm j\pi\sqrt{4 - R_{Ln}^2}.\tag{3.9}$$

Note that R_{Ln} must be less than 2 for complex eigenvalues. This is assumed for the rest of this thesis. The eigenvalues can be rewritten as follows:

$$\lambda_{1,2} = -\alpha \pm j\beta, \quad (3.10)$$

where

$$\alpha = \pi R_{Ln}, \quad \beta = \pi \sqrt{4 - R_{Ln}^2}. \quad (3.11)$$

A similarity transformation for utilizing the coordinate conversion from Cartesian to polar coordinates is performed as given in (3.12), where z_1 and z_2 denote the new system variables.

$$\begin{bmatrix} \hat{V}_{on} \\ \hat{i}_{Ln} \end{bmatrix} = \begin{bmatrix} -2\pi & 0 \\ \alpha & -\beta \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (3.12)$$

The system equations can be rewritten with the new variables by using (3.12) in (3.7):

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -\alpha & \beta \\ -\beta & -\alpha \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \quad (3.13)$$

The coordinate of the system can be converted from Cartesian (z_1, z_2) to polar coordinates (r, θ) by using the following transformation

$$\begin{aligned} z_1 &= r \cos \theta, \\ z_2 &= r \sin \theta. \end{aligned} \quad (3.14)$$

By using (3.14) in (3.13), we obtain:

$$\begin{aligned} \dot{r} &= \alpha r, \\ \dot{\theta} &= -\beta. \end{aligned} \quad (3.15)$$

The solutions of (3.15) can easily be obtained as follows:

$$r(t) = r_o e^{-\alpha t}, \quad (3.16)$$

$$\theta(t) = \theta(0) - \beta t. \quad (3.17)$$

Now, by using (3.16) and (3.17) in (3.14), we obtain the solutions in (z_1, z_2) coordinates. Similarly, using this solution in (3.12) and (3.5), we obtain V_{on} and i_{Ln} vs t . By using (3.17), time can be rewritten as:

$$t = \frac{\theta(0) - \theta}{\beta}. \quad (3.18)$$

By using (3.18) and the equations mentioned above, we could obtain V_{on} vs. i_L state trajectories, both analytically or numerically. To this end, by using (3.18) in (3.16) we obtain

$$r = r_0 e^{-\frac{\alpha}{\beta}(\theta(0) - \theta)}. \quad (3.19)$$

The system coordinates can be converted back to Cartesian (z_1, z_2) with the square of (3.19) represented in (3.20) using

$$\begin{aligned} r^2 &= z_1^2 + z_2^2, \\ \theta &= \text{atan2}(z_2, z_1). \end{aligned} \quad (3.20)$$

The equation (3.20) can be expressed with the original variables V_{on} and i_{Ln} using the similarity transformation in (3.12) and the variable definition in (3.5) as follows:

$$\begin{aligned} z_1 &= -\frac{1}{2\pi} \hat{V}_{on}, \\ z_2 &= -\frac{\alpha}{2\pi\beta} \hat{V}_{on} - \frac{1}{\beta} \hat{i}_{Ln}. \end{aligned} \quad (3.21)$$

Using (3.5), (3.21) can be rewritten with the original variables V_{on} and i_{Ln} as given below:

$$\begin{aligned} z_1 &= -\frac{1}{2\pi} (-uV_{inn} + R_{Ln}i_{on} + V_{on}), \\ z_2 &= -\frac{\alpha}{2\pi\beta} (-uV_{inn} + R_{Ln}i_{on} + V_{on}) - \frac{1}{\beta} (i_{Ln} - i_{on}). \end{aligned} \quad (3.22)$$

Then, the initial conditions can be expressed as:

$$\begin{aligned} z_1(0) &= -\frac{1}{2\pi} (-uV_{inn} + R_{Ln}i_{on} + V_{on}(0)), \\ z_2(0) &= -\frac{\alpha}{2\pi\beta} (-uV_{inn} + R_{Ln}i_{on} + V_{on}(0)) - \frac{1}{\beta} (i_{Ln}(0) - i_{on}). \end{aligned} \quad (3.23)$$

The solution (3.24) of (3.5) with the original variables V_{on} and i_{Ln} can be obtained by using (3.22) and (3.23) in (3.20).

$$r^2 - r_o^2 e^{-2\frac{\alpha}{\beta}(\theta(0)-\theta)} = 0. \quad (3.24)$$

The natural trajectories of the converter are the solution curves of (3.24) in the phase plane for given initial conditions. The natural trajectories can be named as λ and (3.24) now can also be given as $\lambda = 0$, where the latter is defined as follows:

$$\lambda = r^2 - r_o^2 e^{-2\frac{\alpha}{\beta}(\theta(0)-\theta)}. \quad (3.25)$$

The system variables expressed in (3.23), their polar coordinate conversion represented in (3.20), and the natural trajectories defined with the polar variables in (3.25) will be used for defining on and off-state natural trajectories throughout this chapter.

3.1.1 On-state Model

The natural trajectories of the converter when the switch is in on-state can be obtained by replacing u with 1 in the formulation given above. The system variables can be revised as:

$$\begin{aligned} z_{1,on} &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on}), \\ z_{1,on}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on}(0)), \\ z_{2,on} &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on}) - \frac{1}{\beta}(i_{Ln} - i_{on}), \\ z_{2,on}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on}(0)) - \frac{1}{\beta}(i_{Ln}(0) - i_{on}). \end{aligned} \quad (3.26)$$

The polar coordinate variables used in the natural trajectories can be expressed as:

$$\begin{aligned}
r_{on}^2 &= z_{1,on}^2 + z_{2,on}^2, \\
r_{on,0}^2 &= z_{1,on}^2(0) + z_{2,on}^2(0), \\
\theta_{on} &= \text{atan2}(z_{2,on}^2, z_{1,on}^2), \\
\theta_{on}(0) &= \text{atan2}(z_{2,on}^2(0), z_{1,on}^2(0)).
\end{aligned} \tag{3.27}$$

Then, the on-state natural trajectories of the buck converter with variables in (3.27) can be defined as:

$$\lambda_{on} = r_{on}^2 - r_{on,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on}(0) - \theta_{on})}. \tag{3.28}$$

Figure 3.2 displays a selection of on-state natural trajectories plotted for 4 different initial conditions with distinct colors. These trajectories are the solution curves of (3.28) that start from initial conditions and end at the equilibrium point $(V_{inn} - R_{Ln}i_{on}, i_{on})$. To determine these trajectories, we can start with given $V_{on}(0)$ and $i_{Ln}(0)$ to calculate $z_{1,on}(0)$ and $z_{2,on}(0)$ using (3.27). Subsequently, we compute $r_{on,0}^2$ and $\theta_{on}(0)$ from (3.28). Next, solutions for r and θ can be found by equalizing (3.28) to zero which means $\lambda_{on} = 0$. Then, we can revert to the $(z1, z2)$ coordinates using (3.14), and subsequently determine i_{Ln} and V_{on} by applying (3.21) and (3.5). The blue curve passes through the target operating point $(V_{on,target}, i_{on})$ is a natural switching surface that will be used in the control law.

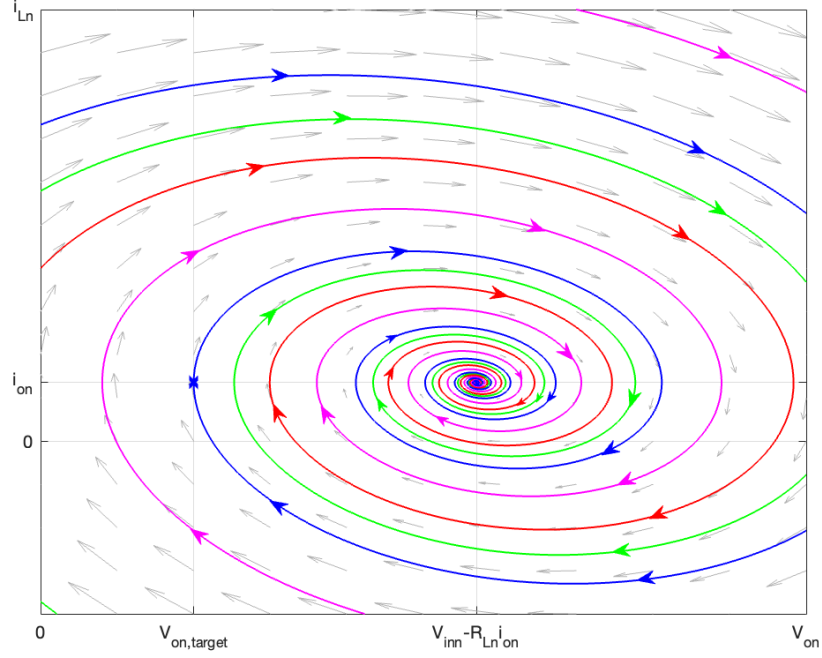


Figure 3.2: On-state natural trajectories of buck converter

3.1.2 Off-state Model

The natural trajectories of the converter when the switch is off position can be obtained by replacing u with 0. The system variables can be rewritten as:

$$\begin{aligned}
 z_{1,off} &= -\frac{1}{2\pi}(R_{Ln}i_{on} + V_{on}), \\
 z_{1,off}(0) &= -\frac{1}{2\pi}(R_{Ln}i_{on} + V_{on}(0)), \\
 z_{2,off} &= -\frac{\alpha}{2\pi\beta}(R_{Ln}i_{on} + V_{on}) - \frac{1}{\beta}(i_{Ln} - i_{on}), \\
 z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(R_{Ln}i_{on} + V_{on}(0)) - \frac{1}{\beta}(i_{Ln}(0) - i_{on}).
 \end{aligned} \tag{3.29}$$

The polar coordinate variables used in the natural trajectories can be expressed

as:

$$\begin{aligned}
r_{off}^2 &= z_{1,off}^2 + z_{2,off}^2, \\
r_{off,0}^2 &= z_{1,off}^2(0) + z_{2,off}^2(0), \\
\theta_{off} &= \text{atan2}(z_{2,off}^2, z_{1,off}^2), \\
\theta_{off}(0) &= \text{atan2}(z_{2,off}^2(0), z_{1,off}^2(0)).
\end{aligned} \tag{3.30}$$

Then, the off-state natural trajectories of the buck converter can be rewritten as:

$$\lambda_{off} = r_{off}^2 - r_{off,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}. \tag{3.31}$$

Similar to the on-state case, a selection of off-state natural trajectories are plotted for 4 different initial conditions with distinct colors in Figure 3.3. These spiral curves begin at the initial conditions and converge at the equilibrium point $(-R_{Ln}i_{on}, i_{on})$. They represent the solution curves of (3.31) and can be determined using (3.29)-(3.31), following similar steps as outlined in the previous section for the on-state case. The red curve passes through the target operating point $(V_{on,target}, i_{on})$ is a natural switching surface that will be used in the control law.

3.2 Control Law

The control law for buck converter determines the state of the switch during the operation to achieve and maintain the target performance. The control law is obtained with natural switching surfaces, which are the natural trajectories that cross the target operating point. At the target operating point, the output voltage matches the reference voltage, and the inductor current equals the load current, assuming the capacitor acts as an open circuit in steady-state conditions. The normalized form of these conditions can be expressed as:

$$\begin{aligned}
i_{Ln,target} &= i_{on}, \\
V_{on,target} &= 1.
\end{aligned} \tag{3.32}$$

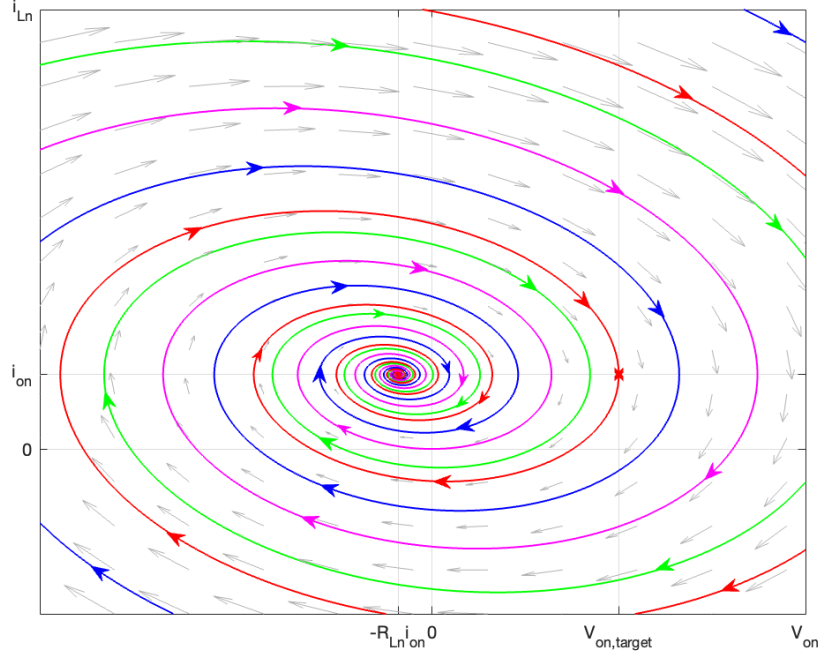


Figure 3.3: Off-state natural trajectories of buck converter

The natural switching surfaces can be determined using (3.32) by substituting $i_{Ln}(0)$ with i_{on} and $V_{on}(0)$ with 1 in the equations (3.26) and (3.29) of the natural trajectories. (3.33) and (3.34) highlight the modified equations of on-state and off-state natural trajectories with these initial conditions, respectively.

$$\begin{aligned} z_{1,on}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + 1), \\ z_{2,on}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + 1). \end{aligned} \quad (3.33)$$

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(R_{Ln}i_{on} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(R_{Ln}i_{on} + 1). \end{aligned} \quad (3.34)$$

By using (3.33) in (3.27), the on-state natural switching surface denoted by σ_{on} can be obtained and written as:

$$\sigma_{on} = r_{on}^2 - r_{on,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on}(0) - \theta_{on})}. \quad (3.35)$$

The off-state natural switching surface denoted by σ_{off} can be obtained by using (3.34) in (3.30) and written as:

$$\sigma_{off} = r_{off}^2 - r_{off,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}. \quad (3.36)$$

The control law (3.37) determines the position of the switch at any point on the state plane. If the states are above $i_{Ln} = i_{Ln,target}$ line, the control action is determined by the location of the states with respect to off-state natural switching surface σ_{off} . The switch is turned off if the result of solving equation (3.36) with current states is positive, the switch is turned on if it is negative. The control action is determined by the location of the states to the on-state switching surface σ_{on} if the states are below $i_{Ln} = i_{Ln,target}$ line. The switch is turned off if the result of solving equation (3.35) with current states is negative, the switch is turned on if it is positive.

Case 1 : $i_{Ln} < i_{Ln,target}$

$$u = \begin{cases} 1, & \sigma_{on} > 0 \\ 0, & \text{else} \end{cases} \quad (3.37)$$

Case 2 : $i_{Ln} > i_{Ln,target}$

$$u = \begin{cases} 0, & \sigma_{off} > 0 \\ 1, & \text{else.} \end{cases}$$

Figure 3.4 demonstrates the control action for any point on the state plane. The red curve is part of the off-state natural switching surface for $i_{Ln} > i_{Ln,target}$. Similarly, the part of the on-state natural switching surface for $i_{Ln} < i_{Ln,target}$ is the blue curve on the figure. The control law (3.37) ensures that if the states are above the $I_{Ln} = I_{Ln,target}$ line, the states will reach the off-state natural switching surface, causing the switch to turn off. Conversely, if the states are below this line, they will reach the on-state natural switching surface, causing the switch to turn on. The system reaches the target operating point with only one switching action.

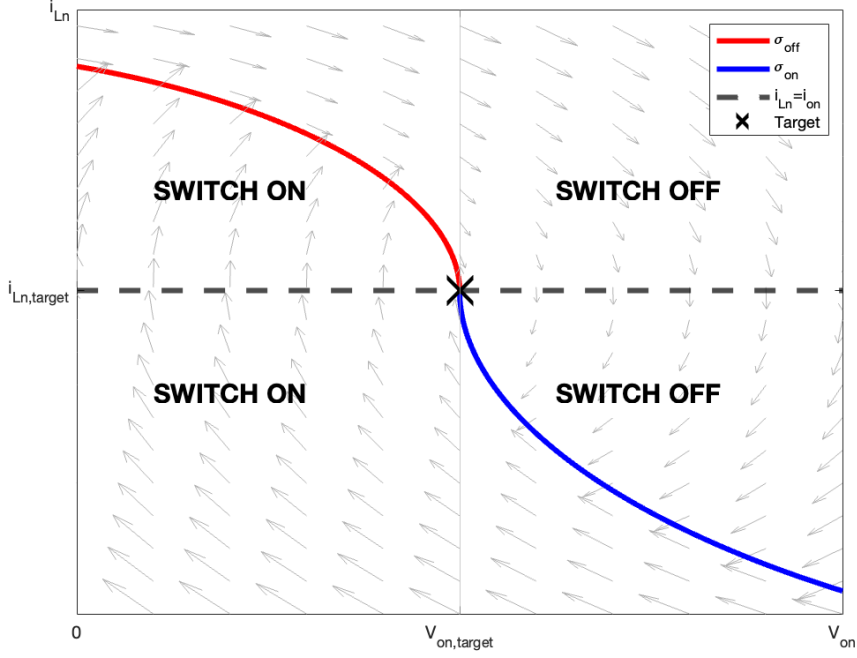


Figure 3.4: Buck converter control law operation in the state plane

3.2.1 Control Law for Limiting Inductor Current

The fast dynamics of this control method may cause high inductor currents. Limiting the inductor current may help to protect the converter components, including the inductor itself and the switch, from damage. Assume that an inductor current limit $i_{Ln,lim}$ is given. Naturally, we assume that $i_{Ln,lim} > i_{Ln,target}$. Note that the load current which is equal to $i_{Ln,target}$ is defined positive in the direction indicated in Figure 3.1, and the same applies to $i_{Ln,lim}$. Under this condition, the control law given by (3.37) can easily be modified to achieve current limitation. The control law (3.38) incorporates limiting the inductor current at $i_{Ln,lim}$. If the states are above $i_{Ln} = i_{Ln,lim}$ line, the switch is turned off to drive the states below the limited inductor current value. The control action is the same with (3.37) if the states are below $i_{Ln} = i_{Ln,lim}$ line.

$$\begin{aligned}
& \text{Case 1 : } i_{Ln,target} > i_{Ln} \\
& u = \begin{cases} 1, & \sigma_{on} > 0 \\ 0, & \text{else} \end{cases} \\
& \text{Case 2 : } i_{Ln,lim} > i_{Ln} > i_{Ln,target} \\
& u = \begin{cases} 0, & \sigma_{off} > 0 \\ 1, & \text{else} \end{cases} \\
& \text{Case 3 : } i_{Ln} > i_{Ln,lim} \\
& u = 0.
\end{aligned} \tag{3.38}$$

Figure 3.5 displays the control law (3.38) on the state plane. In comparison to law (3.37), the part of the off-state natural switching surface, the red curve now has an upper limit and is plotted for $i_{Ln,lim} > i_{Ln} > i_{Ln,target}$. The control action for the states above $I_{Ln,lim}$ line does not depend on their location with respect to the off-state natural switching surface anymore; the switch is turned off for this region. For the states below $i_{Ln,lim}$ line, the control law on the state plane is the same as that of (3.37). The control law (3.38) ensures that the system reaches the target operating point without exceeding the limited inductor current value.

It should be noted that the target operating point achieved by control laws (3.37) and (3.38) are not equilibrium solutions of the system. Therefore, it is impossible for the trajectories to remain at them unless on-off switching with an infinite frequency is employed; however, this is practically impossible. Additional control actions to prevent infinite frequency at steady state will be explained and employed later.

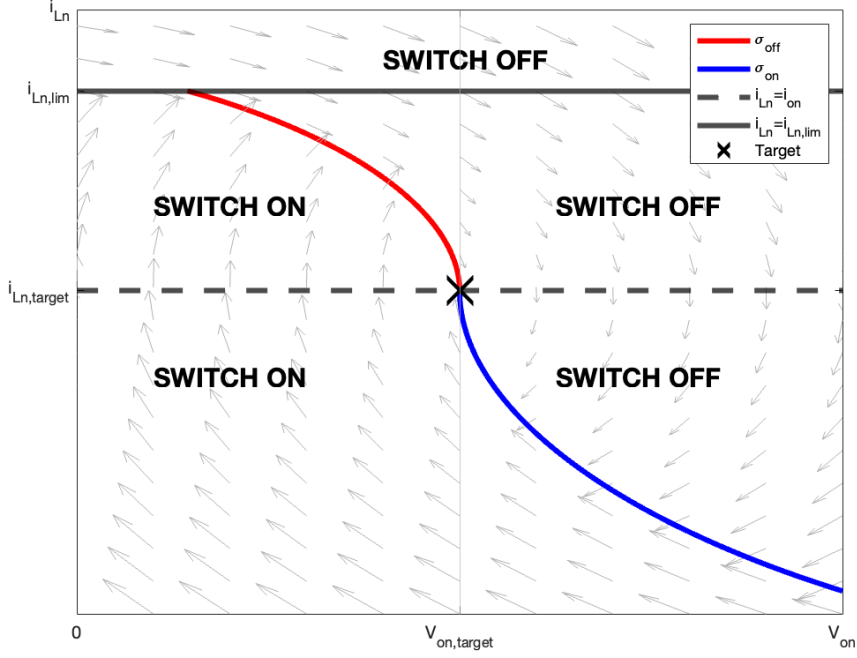


Figure 3.5: Buck converter control law incorporating inductor current limiting operation in the state plane

3.3 Transient Analysis

Analogous to the study described in [41] for a resistive loaded buck converter, similar evaluations are performed for the transient analysis of a constant current loaded buck converter model incorporating inductor DCR. Furthermore, this study extends the theoretical analysis to include current-limited loading and startup transients. In DC/DC converters, the overshoot of the inductor current and the settling time during start-up are critical issues that affect performance and reliability. Another important performance metric for switching converters is deviation of the output voltage from its reference value and the recovery time during load transients. The following section presents the theoretical framework for calculating the duration and deviations of output voltage and inductor current during transient events.

3.3.1 Start-up Transients

Figure 3.6 displays the start-up trajectories of the buck converter under the control law (3.37). The initial state of the system is $(0, 0)$ and the control law turns the switch on, the system follows the on-state natural trajectory $\lambda_{on,ss}$ until the off-state switching surface σ_{off} is reached. Then, the law turns the switch off, and the system follows the σ_{off} path and reaches the target operating point with only one switching action and zero overshoot in the output voltage. The on-state natural trajectory in start-up $\lambda_{on,su}$ can be derived as follows: First, substituting $i_{Ln}(0) = 0$ and $V_{on}(0) = 0$ in (3.26) results in

$$\begin{aligned} z_{1,on}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on}), \\ z_{2,on}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on}) + \frac{1}{\beta}i_{on}. \end{aligned} \quad (3.39)$$

Second, using (3.39) in (3.27) gives revised $r_{on,0}$ and $\theta_{on,0}$ which can be donated by $r_{on,su0}$ and $\theta_{on,su}(0)$ respectively. The on-state natural trajectory in start-up $\lambda_{on,ss}$ can be defined by using $r_{on,su0}^2$ and $\theta_{on,su}(0)$ in (3.28) as follows:

$$\lambda_{on,su} = r_{on}^2 - r_{on,su0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on,su}(0) - \theta_{on})}. \quad (3.40)$$

The on-state start-up natural trajectory hits the off-state natural switching surface at the maximum inductor current value, which is $i_{Ln,peak}$ on the start-up trajectories. $i_{Ln,peak}$ can be calculated using

$$\lambda_{on,su} = \sigma_{off}. \quad (3.41)$$

By reorganizing the second equation in (3.4), formally we obtain:

$$dt_n = -\frac{1}{2\pi(-uV_{inn} + R_{Ln}i_{Ln} + V_{on})}di_{Ln}. \quad (3.42)$$

By substituting u with 1 in (3.42) and integrating the right side between 0 and $i_{Ln,peak}$, the normalized on-state time during start-up can be expressed as:

$$t_{n,on(su)} = \int_0^{i_{Ln,peak}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})}di_{Ln}. \quad (3.43)$$

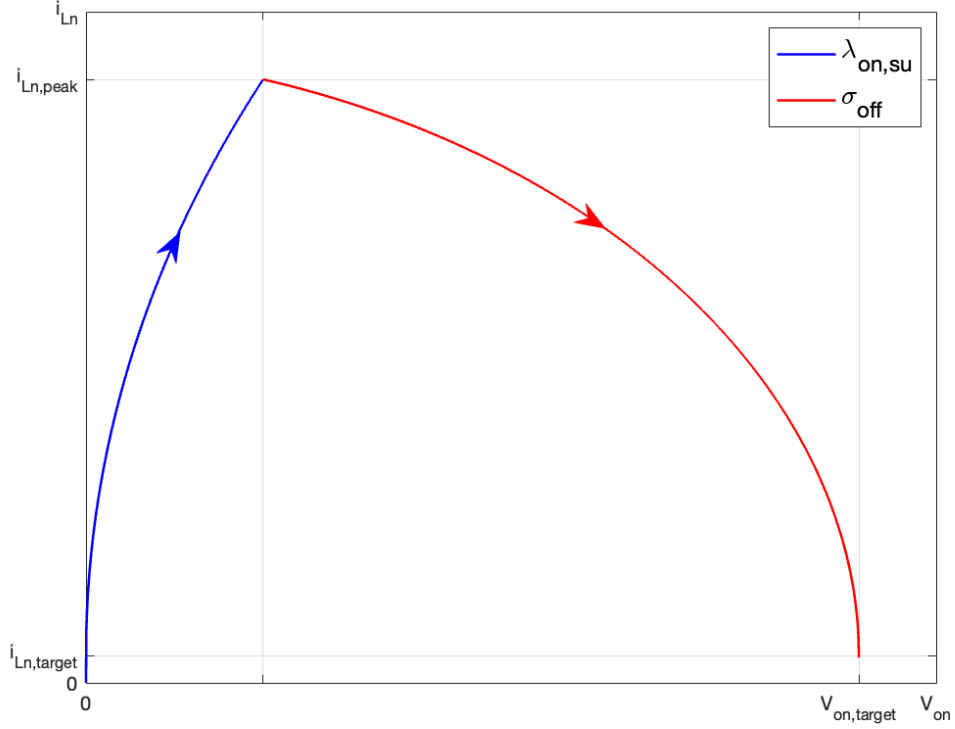


Figure 3.6: Buck converter start-up trajectories

Similarly, the normalized off-state time during start-up can be calculated by substituting u with 0 in (3.42) and integrating the right side between $i_{Ln,peak}$ and $i_{Ln,target}$ as follows:

$$t_{n,off(su)} = \int_{i_{Ln,peak}}^{i_{Ln,target}} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.44)$$

The trapezoidal rule is a numerical integration method that can be used for evaluating the integrals in (3.43), (3.44) and the integrals in subsequent sections due to the highly non-linear characteristics of the equations. The range of i_{Ln} values can be divided into smaller portions based on the limits of the integrals. For each portion of i_{Ln} , the corresponding V_{on} values that satisfy the on-state, off-state natural trajectory, or natural switching surface equations can be calculated, and these V_{on} values can then be used to evaluate the integrals [41].

Then, the total normalized time during start-up can be expressed as the sum of the on-state and off-state times below

$$t_{n,su} = t_{n,on(su)} + t_{n,off(su)}. \quad (3.45)$$

3.3.1.1 Start-up Transients with Current Limitation

The start-up trajectories of the buck converter with inductor current limitation under the control law (3.38) are shown in Figure 3.7. Once again, the system starts at $(0, 0)$, and the control law turns the switch on; the system follows the on-state natural trajectory $\lambda_{on,su}$ described in (3.40) until the maximum allowed inductor current value $i_{Ln,lim}$ is reached. Then, the system follows the $i_{Ln} = i_{Ln,lim}$ trajectory. Ideally, by turning the switch on and off continuously at an infinite frequency, the system stays at this line until it hits the off-state switching surface σ_{off} . The switch is turned off by the control law when σ_{off} is reached; then, the system states move along σ_{off} surface and converge to the target operating point.

The normalized output voltage value $V_{on,lim1}$ when the on-state natural start-up trajectory $\lambda_{on,su}$ intersects with the current limited line $i_{Ln} = i_{Ln,lim}$ can be calculated using

$$\lambda_{on,su}(i_{Ln} = i_{Ln,lim}) = 0. \quad (3.46)$$

The off-state natural switching surface σ_{off} crosses $i_{Ln} = i_{Ln,lim}$ line at normalized output voltage value of $V_{on,lim2}$ which can be found using

$$\sigma_{off}(i_{Ln} = i_{Ln,lim}) = 0. \quad (3.47)$$

Then, the voltage change during current limiting can be found as:

$$\Delta V_{on(su,lim)} = V_{on,lim2} - V_{on,lim1}. \quad (3.48)$$

Rearranging the first equation in (3.4) yields:

$$dt_n = \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (3.49)$$

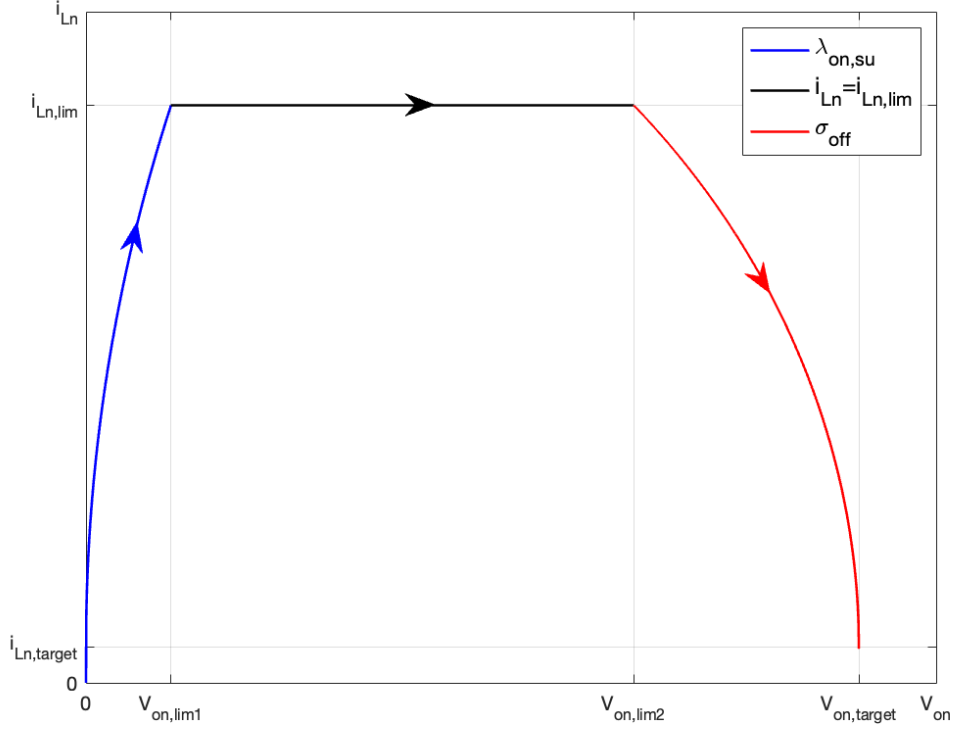


Figure 3.7: Buck converter start-up trajectories with current limitation

Integrating the right side of (3.49) between $V_{on,lim1}$ and $V_{on,lim2}$ gives the normalized time spent during current limiting, which can be found as:

$$t_{n,lim(su)} = \frac{\Delta V_{on(su,lim)}}{2\pi(i_{Ln,lim} - i_{on})}. \quad (3.50)$$

The normalized on-state time during start-up with current limitation can be found by substituting u with 1 in (3.42) and integrating the right side between 0 and $i_{Ln,lim}$ as below:

$$t_{n,on(su,lim)} = \int_0^{i_{Ln,lim}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.51)$$

By substituting u with 0 in (3.42) and integrating the right side between $i_{Ln,lim}$ and $i_{Ln,target}$ the normalized off-state time during start-up with current limiting

can be calculated as follows:

$$t_{n,off(su,lim)} = \int_{i_{Ln,lim}}^{i_{Ln,target}} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.52)$$

Then, the total time during start-up with current limiting is the sum of on-state, current limited, and off-state times as expressed below:

$$t_{n,su(lim)} = t_{n,on(su,lim)} + t_{n,lim(su)} + t_{n,off(su,lim)}. \quad (3.53)$$

The increase in the total start-up time with current-limiting with respect to the previous section can be found using

$$\Delta t_{n,su} = t_{n,su(lim)} - t_{n,su}. \quad (3.54)$$

(3.54) can be written more explicitly

$$\Delta t_{n,su} = \Delta t_{n,on(su)} + \Delta t_{n,off(su)} + t_{n,lim(su)}, \quad (3.55)$$

where

$$\begin{aligned} \Delta t_{n,on(su)} &= t_{n,on(su,lim)} - t_{n,on(su)}, \\ \Delta t_{n,off(su)} &= t_{n,off(su,lim)} - t_{n,off(su)}. \end{aligned} \quad (3.56)$$

(3.56) can be rewritten as:

$$\begin{aligned} \Delta t_{n,on(su)} &= \int_{i_{Ln,peak}}^{i_{Ln,Lim}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}, \\ \Delta t_{n,off(su)} &= \int_{i_{Ln,peak}}^{i_{Ln,Lim}} \frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \end{aligned} \quad (3.57)$$

Note that on-state and off-state times are reduced in comparison to the previous section; however, the total start-up is increased because of the time spent during limiting current. The difference in the total start-up time increases with decreasing $i_{Ln,lim}$ value.

3.3.2 Load Transients

DC/DC converters experience two forms of load transients: loading and unloading. Loading refers to an increase in the current demand of the load while unloading refers to the opposite. The natural trajectories, the deviations from reference output voltage, and the time duration it takes for the converter to reach the new target point are studied in subsequent sections for both types of load transients.

3.3.2.1 Loading Transient

Figure 3.8 shows the loading trajectories of the buck converter under the control law (3.37). The states start at $(i_{Ln,initial}, V_{on,target})$, which is the initial target operating point that the system stays at before load current increases to $i_{on,l}$ which is the new target inductor current $i_{Ln,target(l)}$. When an increase in the load current takes place, the control law turns the switch on. The system follows the on-state natural trajectory $\lambda_{on,l}$ until the off-state switching surface $\sigma_{off,l}$ which crosses the new target point $(i_{Ln,target(l)}, V_{on,target})$ is hit. Then, the switch is turned off by the control law, and the system states move along σ_{off} surface and reach the target operating point with one switching action.

The on-state natural trajectory in loading transient $\lambda_{on,l}$ can be derived as follows: First, using $i_{Ln}(0) = i_{Ln,initial}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,l}$ in (3.26) gives

$$\begin{aligned} z_{1,on}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on,l} + 1), \\ z_{2,on}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on,l} + 1) - \frac{1}{\beta}(i_{Ln,initial} - i_{on,l}). \end{aligned} \quad (3.58)$$

Second, using (3.58) in (3.27) gives revised $r_{on,0}$ and $\theta_{on}(0)$ which can be represented by $r_{on,l0}$ and $\theta_{on,l}(0)$ respectively. The on-state natural trajectory in loading $\lambda_{on,l}$ can be defined by using $r_{on,l0}^2$ and $\theta_{on,l}(0)$ in (3.28) as follows:

$$\lambda_{on,l} = r_{on}^2 - r_{on,l0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on,l}(0) - \theta_{on})}. \quad (3.59)$$

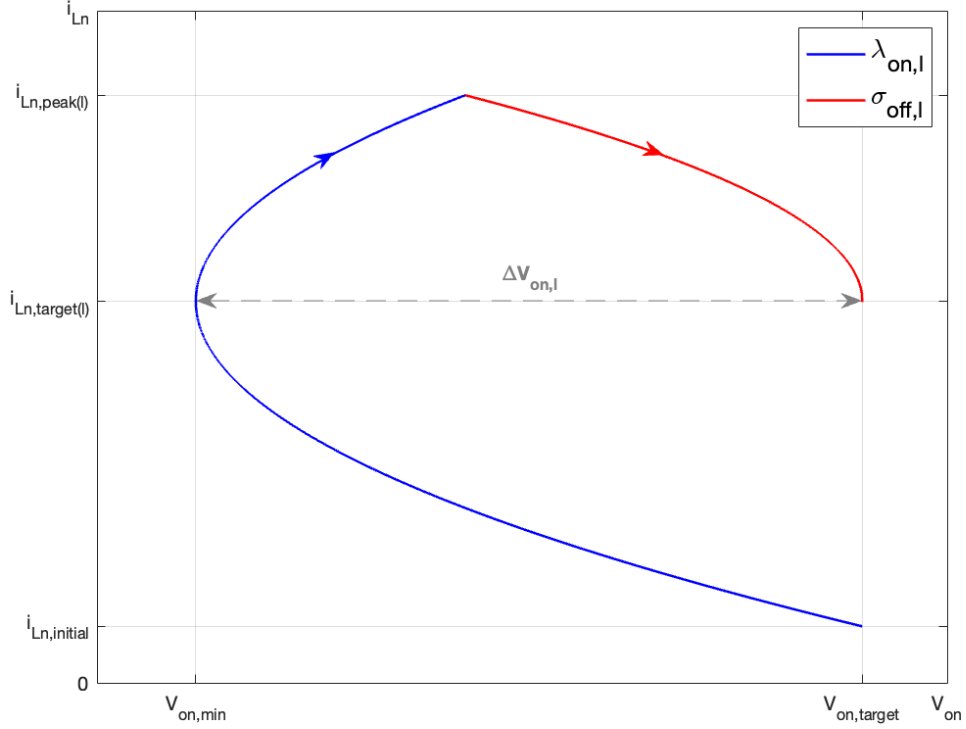


Figure 3.8: Buck converter loading trajectories

Using $i_{Ln}(0) = i_{Ln,target(l)}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,l}$ in (3.29) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(R_{Ln}i_{on,l} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(R_{Ln}i_{on,l} + 1) - \frac{1}{\beta}(i_{Ln,target(l)} - i_{on,l}). \end{aligned} \quad (3.60)$$

Revised $r_{off,0}$ and $\theta_{off}(0)$ represented by $r_{off,l0}$ and $\theta_{off,l}(0)$ can be obtained by using (3.60) in (3.30), then the new off-state natural switching surface denoted by $\sigma_{off,l}$ can be derived and written as:

$$\sigma_{off,l} = r_{off}^2 - r_{off,l0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off,l}(0) - \theta_{off})}. \quad (3.61)$$

$i_{Ln,peak(l)}$ is the maximum inductor current value on the loading trajectories

that occurs when the on-state loading trajectory hits the off-state natural switching surface, and it can be calculated using

$$\lambda_{on,l} = \sigma_{off,l}. \quad (3.62)$$

The increase in the inductor current during loading transient can be found as:

$$\Delta i_{Ln,l} = i_{Ln,peak(l)} - i_{Ln,initial}. \quad (3.63)$$

The minimum output voltage $V_{on,min}$ during loading transient can be found using

$$V_{on,min} = \min_{\lambda_{on,l}=0} V_{on} \quad for \quad i_{Ln,initial} < i_{Ln} < i_{Ln,peak}. \quad (3.64)$$

Then, the output voltage drop in the loading transient can be calculated as:

$$\Delta V_{on,l} = 1 - V_{on,min}. \quad (3.65)$$

By substituting u with 1 in (3.42) and integrating the right side between $i_{Ln,initial}$ and $i_{Ln,peak(l)}$, the normalized on-state time during loading can be expressed as:

$$t_{n,on(l)} = \int_{i_{Ln,initial}}^{i_{Ln,peak(l)}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.66)$$

Likewise, the normalized off-state time during loading can be calculated by substituting u with 0 in (3.42) and integrating the right side between $i_{Ln,peak}$ and $i_{Ln,target}$ as follows:

$$t_{n,off(l)} = \int_{i_{Ln,peak(l)}}^{i_{Ln,target(l)}} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.67)$$

Then, the total normalized time during loading can be expressed as the sum of the on-state and off-state times as given below:

$$t_{n,l} = t_{n,on(l)} + t_{n,off(l)}. \quad (3.68)$$

3.3.2.2 Loading Transient with Current Limitation

Figure 3.9 shows the loading trajectories of the buck converter under the control law (3.38). When an increase in the load current takes place, the control law turns the switch on, the states start at initial target operating point $(i_{Ln,initial}, V_{on,target})$ and follow the on-state natural trajectory $\lambda_{on,l}$ until the maximum allowed inductor current value $i_{Ln,lim}$ is reached. The system stays at this line until it hits the off-state switching surface $\sigma_{off,l}$ by turning the switch on and off continuously at an infinite frequency. After reaching σ_{off} , the switch is turned off by the control law, and then the system states move along $\sigma_{off,l}$ surface and converge the new target operating point.

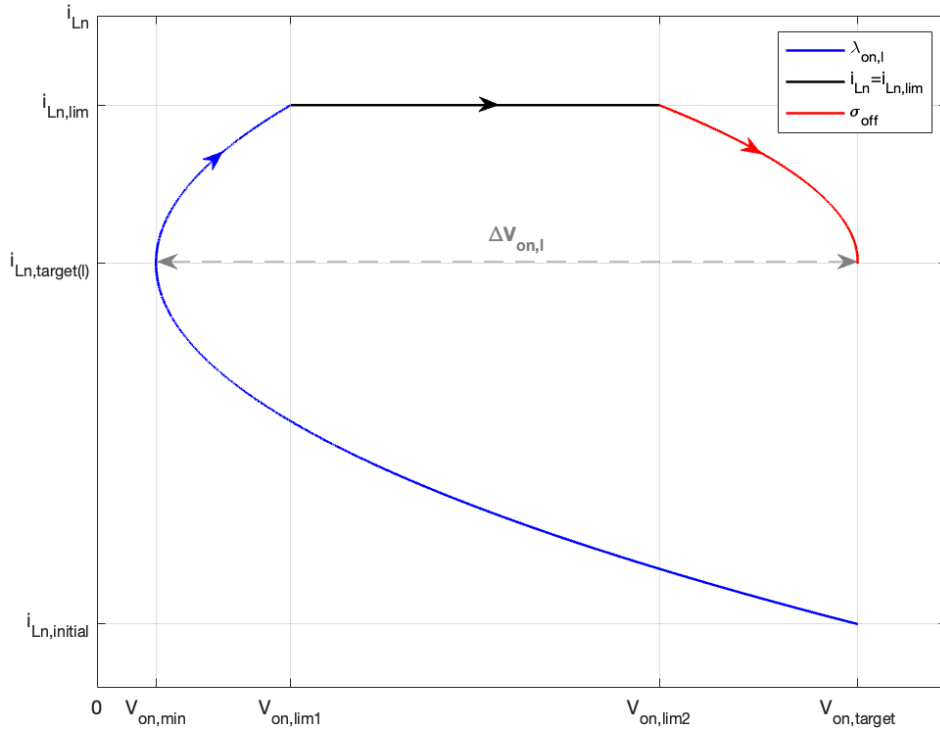


Figure 3.9: Buck converter loading trajectories with current limitation

The on-state natural start-up trajectory $\lambda_{on,l}$ intersects with the current limited line $i_{Ln} = i_{Ln,lim}$ at the normalized output voltage value of $V_{on,lim1}$ can be

calculated using

$$\lambda_{on,l}(i_{Ln} = i_{Ln,lim}) = 0. \quad (3.69)$$

The normalized output voltage value $V_{on,lim2}$ when the off-state natural switching surface $\sigma_{off,l}$ crosses $i_{Ln} = i_{Ln,lim}$ line can be found using

$$\sigma_{off,l}(i_{Ln} = i_{Ln,lim}) = 0. \quad (3.70)$$

Then, the normalized voltage change during current limiting in the loading trajectories can be found as:

$$\Delta V_{on(l,lim)} = V_{on,lim2} - V_{on,lim1}. \quad (3.71)$$

The minimum output voltage $V_{on,min}$ during loading transient can be found using

$$V_{on,min} = \min_{\lambda_{on,l}=0} V_{on} \quad for \quad i_{Ln,initial} < i_{Ln} < i_{Ln,lim}. \quad (3.72)$$

Then, the output voltage drop in the loading transient can be calculated as:

$$\Delta V_{on,l} = 1 - V_{on,min}. \quad (3.73)$$

By changing the upper limit of the integral in (3.66) with $i_{Ln,lim}$ the normalized on-state time during the current-limited loading transient can be found as

$$t_{n,on(l,lim)} = \int_{i_{Ln,initial}}^{i_{Ln,lim}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.74)$$

The normalized time spent during current limiting can be obtained by simply using $V_{on(l,lim)}$ in (3.50)

$$t_{n,lim(l)} = \frac{\Delta V_{on(l,lim)}}{2\pi(i_{Ln,lim} - i_{on})}. \quad (3.75)$$

The normalized off-state time during the current-limited loading transient can be found by changing the lower limit with $i_{Ln,lim}$ of the integral in (3.67).

$$t_{n,off(l,lim)} = \int_{i_{Ln,lim}}^{i_{Ln,target(l)}} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.76)$$

Then, the total time during the current-limited loading transient is the sum of on-state, current-limited, and off-state times as expressed below:

$$t_{n,l(lim)} = t_{n,on(l,lim)} + t_{n,lim(l)} + t_{n,off(l,lim)}. \quad (3.77)$$

The increase in the total time with current limiting with respect to the previous section can be expressed as:

$$\Delta t_{n,l} = \Delta t_{n,on(l)} + \Delta t_{n,off(l)} + t_{n,lim(l)}, \quad (3.78)$$

where

$$\begin{aligned} \Delta t_{n,on(l)} &= t_{n,on(l,lim)} - t_{n,on(l)}, \\ \Delta t_{n,off(l)} &= t_{n,off(l,lim)} - t_{n,off(l)}. \end{aligned} \quad (3.79)$$

(3.79) can be rewritten as similar to (3.57).

$$\begin{aligned} \Delta t_{n,on(l)} &= \int_{i_{Ln,peak(l)}}^{i_{Ln,Lim}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}, \\ \Delta t_{n,off(l)} &= \int_{i_{Ln,peak(l)}}^{i_{Ln,Lim}} \frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \end{aligned} \quad (3.80)$$

The total loading time is increased because of the time spent limiting current, while on-state and off-state times are reduced. Less $i_{Ln,lim}$ means more time spent during the loading transient.

3.3.2.3 Unloading Transient

Unloading trajectories of the buck converter under the control law (3.37) are shown in Figure 3.10. When a decrease in the load current takes place, the

control law turns the switch off; the states start at initial target operating point $(i_{Ln,initial}, V_{on,target})$, where the system remained before the load current decreased to $i_{on,ul}$ which is the new target inductor current $i_{Ln,target(ul)}$. The states follow the off-state natural trajectory $\lambda_{off,ul}$ until the off-state natural switching surface $\sigma_{on,ul}$ is reached. Then, the switch is turned on by the control law, the system states moves along $\sigma_{on,ul}$ surface and reaches the new target operating point $(i_{Ln,target(ul)}, V_{on,target})$.

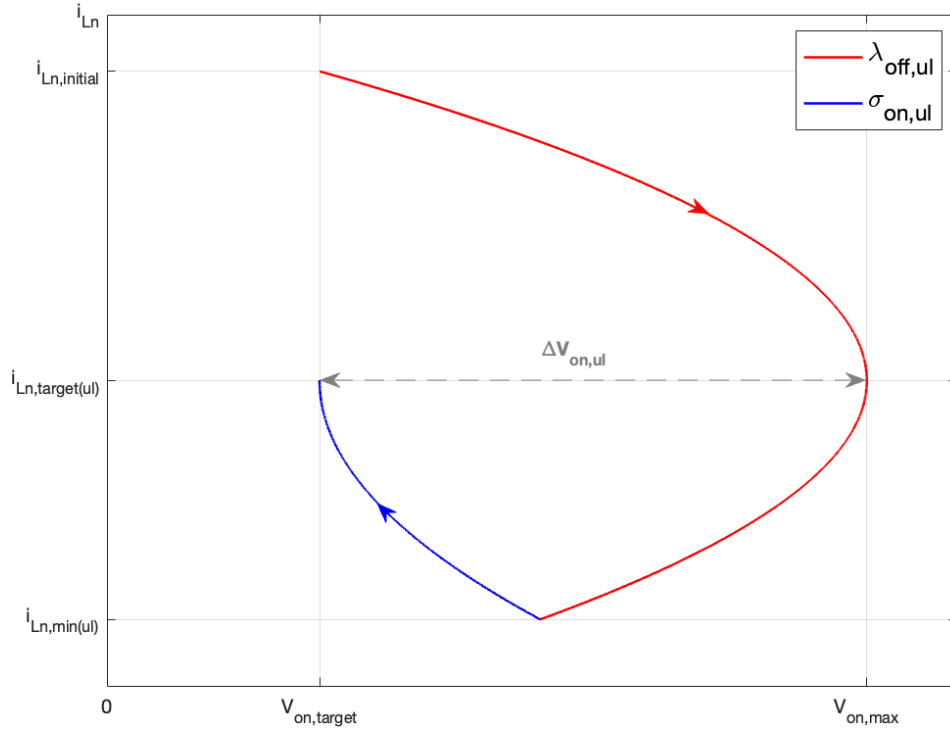


Figure 3.10: Buck converter unloading trajectories

The off-state natural trajectory in unloading transient $\lambda_{on,ul}$ can be obtained as follows: First, using $i_{Ln}(0) = i_{Ln,initial}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,ul}$ in (3.32) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(R_{Ln}i_{on,ul} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(R_{Ln}i_{on,ul} + 1) - \frac{1}{\beta}(i_{Ln,initial} - i_{on,ul}). \end{aligned} \quad (3.81)$$

Second, using (3.81) in (3.30) gives revised $r_{off,0}$ and $\theta_{off}(0)$ which can be donated by $r_{off,ul0}$ and $\theta_{off,ul}(0)$ respectively. The off-state natural trajectory in unloading transient $\lambda_{off,ul}$ can be defined by using $r_{off,ul0}^2$ and $\theta_{off,ul}(0)$ in (3.31) as follows:

$$\lambda_{off,ul} = r_{off}^2 - r_{off,ul0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off,ul}(0) - \theta_{off})}. \quad (3.82)$$

Using $i_{Ln}(0) = i_{Ln,target(ul)}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,ul}$ in (3.26) gives

$$\begin{aligned} z_{1,on}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on,ul} + 1), \\ z_{2,on}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on,ul} + 1) - \frac{1}{\beta}(i_{Ln,target,ul} - i_{on,ul}). \end{aligned} \quad (3.83)$$

Revised $r_{on,0}$ and $\theta_{on}(0)$ donated by $r_{on,ul0}$ and $\theta_{on,ul}(0)$ can be obtained by using (3.83) in (3.27), then the new on-state natural switching surface denoted by $\sigma_{on,ul}$ can be derived and written as:

$$\sigma_{on,ul} = r_{on}^2 - r_{on,ul0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on,ul}(0) - \theta_{on,ul})}. \quad (3.84)$$

$i_{Ln,min(ul)}$ is the minimum inductor current value on the unloading trajectories that occurs when the off-state loading trajectory hits the on-state natural switching surface, and it can be calculated using

$$\lambda_{on,l} = \sigma_{off,l}. \quad (3.85)$$

The decrease in the inductor current during loading transient can be found as:

$$\Delta i_{Ln,ul} = i_{Ln,initial} - i_{Ln,min(ul)}. \quad (3.86)$$

The maximum output voltage $V_{on,max}$ during unloading transient can be found using

$$V_{on,max} = \max_{\lambda_{off,ul}=0} V_{on} \quad for \quad i_{Ln,min} < i_{Ln} < i_{Ln,initial}. \quad (3.87)$$

Then, the output voltage rise in the unloading transient can be calculated as:

$$\Delta V_{on,ul} = V_{on,max} - 1. \quad (3.88)$$

By substituting u with 0 in (3.42) and integrating the right side between $i_{Ln,initial}$ and $i_{Ln,min}(ul)$, the normalized off-state time during loading can be expressed as:

$$t_{n,off}(ul) = \int_{i_{Ln,initial}}^{i_{Ln,min}(ul)} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.89)$$

Likewise, the normalized on-state time during loading can be calculated by substituting u with 1 in (3.42) and integrating the right side between $i_{Ln,min}(ul)$ and $i_{Ln,target}(ul)$ as follows:

$$t_{n,on}(ul) = \int_{i_{Ln,min}(ul)}^{i_{Ln,target}(ul)} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.90)$$

Then, the sum of the on-state and off-state times during unloading gives the total normalized unloading time.

$$t_{n,ul} = t_{n,on}(ul) + t_{n,off}(ul). \quad (3.91)$$

3.4 Steady-state Analysis

After the system reaches the target operating point, the switch turns on and off at an infinite frequency to maintain the trajectories at this point. So, the inductor current or output voltage ripple is zero in an ideal case. However, achieving this is not feasible in practical cases, as steady-state trajectories would contain uncertain high-frequency switching leading to uncontrollable ripple [7]. Using a similar approach to that suggested in [41], by incrementing in $r_{on,0}^2$ and $r_{off,0}^2$ of natural switching surfaces σ_{on} and σ_{off} as in (3.92) and (3.93), controllable ripple with finite switching frequency can be obtained as follows:

$$r_{on\Delta,0}^2 = r_{on,0}^2 + \Delta r^2, \quad (3.92)$$

$$r_{off\Delta,0}^2 = r_{off,0}^2 + \Delta r^2. \quad (3.93)$$

Using (3.92) in (3.28) and (3.93) in (3.31) gives modified natural switching surfaces $\sigma_{on\Delta}$ and $\sigma_{off\Delta}$

$$\begin{aligned}\sigma_{on\Delta} &= r_{on}^2 - r_{on\Delta,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{on}(0) - \theta_{on})}, \\ \sigma_{off\Delta} &= r_{off}^2 - r_{off\Delta,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}.\end{aligned}\tag{3.94}$$

Figure 3.11 shows the steady state trajectories of buck converter when (3.94) is used in the control laws (3.37) or (3.38). The states cycle between b and d during steady state operation. At point b, the switch turns off, and the states follow the $\sigma_{off\Delta}$ curve until reaching point d. At point d, the switch turns on, bringing the states back to point b, thus completing one switching cycle. For other parameters fixed, the points a, b, c, d, thus switching frequency and ripples are determined by Δr^2 . Lower ripple can be achieved by decreasing Δr^2 , resulting in higher switching frequency [41]. Δr^2 and the characteristic impedance Z_o determine ripples and switching frequency; a detailed explanation will be provided in the Section 3.5.

3.4.1 Ripple Calculations

Using a similar method described in [41], the normalized inductor current and output voltage ripple at the steady state, donated by $\Delta i_{Ln,ss}$ and $\Delta V_{on,ss}$ can be calculated by using the current values at points b and d and the voltage values at points a and c.

$$\sigma_{on\Delta} = \sigma_{off\Delta} \quad \text{for} \quad i_{Ln,target} < i_{Ln},\tag{3.95}$$

$$\sigma_{on\Delta} = \sigma_{off\Delta} \quad \text{for} \quad i_{Ln,target} > i_{Ln}.\tag{3.96}$$

The normalized inductor currents $i_{Ln,b}$ and $i_{Ln,d}$ at points b and d can be found using (3.95) and (3.96), respectively. These equations can be solved numerically by using the Newton-Raphson algorithm. Then, the inductor current ripple at steady state can be defined as:

$$\Delta i_{Ln,ss} = i_{Ln,b} - i_{Ln,d}.\tag{3.97}$$

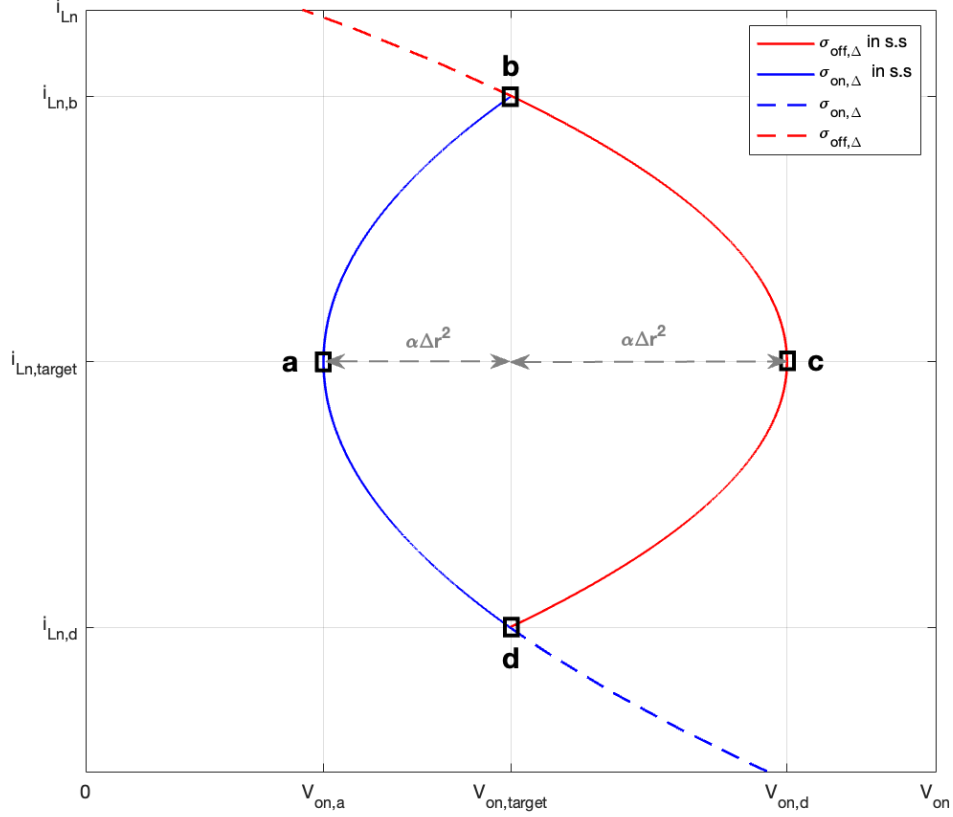


Figure 3.11: Buck converter steady-state trajectories

Once $\Delta i_{Ln,b}$ and $\Delta i_{Ln,d}$ are found, the normalized output voltages $\Delta V_{on,a}$ and $\Delta V_{on,c}$ at points a and c can be found using (3.98) and (3.99) respectively.

$$V_{on,a} = \min_{\sigma_{on\Delta}} V_{on} \quad for \quad i_{Ln,d} < i_{Ln} < i_{Ln,b}, \quad (3.98)$$

$$V_{on,c} = \max_{\sigma_{off\Delta}} V_{on} \quad for \quad i_{Ln,d} < i_{Ln} < i_{Ln,b}. \quad (3.99)$$

The solutions of (3.98) and (3.99) can be found by dividing i_{Ln} ranges into small parts and for each i_{Ln} part $\sigma_{on\Delta} = 0$ or $\sigma_{off\Delta} = 0$ must be solved numerically for V_{on} values. Then, the output voltage ripple at the steady state can be

found by

$$\Delta V_{on,ss} = V_{on,c} - V_{on,a}. \quad (3.100)$$

3.4.2 Frequency Calculation

The normalized on-state time during one switching cycle in steady state can be found by changing the lower limit with $i_{Ln,d}$ and the upper limit with $i_{Ln,b}$ in the integral (3.43).

$$t_{n,on(ss)} = \int_{i_{Ln,d}}^{i_{Ln,b}} -\frac{1}{2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.101)$$

By changing the lower limit with $i_{Ln,b}$ and the upper limit with $i_{Ln,d}$ in the integral in (3.44), the normalized off-state time during one switching cycle in the steady state can be found as:

$$t_{n,off(ss)} = \int_{i_{Ln,b}}^{i_{Ln,d}} -\frac{1}{2\pi(R_{Ln}i_{Ln} + V_{on})} di_{Ln}. \quad (3.102)$$

The normalized switching frequency in the steady state can be found using on-state and off-state times during one switching cycle

$$f_{n,ss} = \frac{1}{t_{n,on(ss)} + t_{n,off(ss)}}. \quad (3.103)$$

3.5 Controller Design

[41] served as an example for the studies in this and the subsequent section. An algorithm is implemented for determining design parameters, followed by ideal and practice-oriented simulations conducted with these parameters. The studies presented here are performed on a constant current-loaded buck converter model incorporating inductor DCR, and the simulations are extended to examine current limitations during start-up and loading transients.

An example design of a controller for the buck converter with the requirements given in Table 3.1 is presented in this section. The goal is to determine the parameters C and Δr^2 with a chosen inductor and its DCR so that design requirements are met. The ripple specifications in the design requirements are not exact values that must be met, but rather the maximum permissible values. The algorithm which is detailed in Appendix A outputs the minimum C and maximum Δr^2 values for L , R_L , and design requirements as inputs. The algorithm chooses minimum C because of potential space and cost constraints and maximum Δr^2 for its suitability with limited controller bandwidth in practical applications. The ripples decrease with a lower value of Δr^2 , the output voltage ripple increases and the inductor current ripple decreases with increasing Z_o for a given Δr^2 . The switching frequency increases with decreasing Δr^2 and decreases with increasing Z_o . The algorithm uses these facts for optimizing output parameters. The algorithm starts with the maximum Z_o in the given range; it searches for maximum Δr^2 that satisfies inductor ripple requirement by solving equations (3.95)-(3.97). If it is met, it solves (3.98)-(3.100) for determining output voltage ripple; if output voltage ripple is greater than the maximum allowed value, the algorithm lowers Z_o and repeats these steps again until both ripple requirements are met. When both ripple requirements are met, it solves (3.101)-(3.103) to determine the switching frequency; if it is higher than the desired, the algorithm decreases Z_o value and repeats this process. When the optimum Δr^2 and Z_o are determined, the algorithm calculates the capacitor value using (3.104) and outputs C and Δr^2 values. In comparison to the algorithm in [41], this algorithm does not specify L ; it determines only C and Δr^2 values for a chosen inductor and its DCR. This algorithm does not use the bisection search method. It searches for maximum Δr^2 and Z_o for satisfying requirements.

$$C = L/Z_o^2. \quad (3.104)$$

For the example design, the maximum steady state output voltage ripple is selected to be

$$\frac{\Delta V_{o,ss}}{V_{ref}} \times 100 = 0.0197\% \quad (3.105)$$

of the reference voltage and the maximum steady state inductor current ripple is selected as:

$$\frac{\Delta i_{L,ss}}{i_o} \times 100 = 90\% \quad (3.106)$$

of its target value. The inductor current ripple can be chosen high as long as it does not result in exceeding the limits of the chosen inductor so that the output capacitance can be minimized to satisfy the requirements.

Parameters	Value
V_{in}	10 V
V_{ref}	3.3 V
i_o	3 A
Maximum $\Delta V_{on,ss}$	0.065 V
Maximum $\Delta i_{L,ss}$	2.7 A
Maximum f_{ss}	18kHz

Table 3.1: Buck converter parameters

Buck converter design parameters are determined using Algorithm 1 given in Appendix A. The algorithm inputs the chosen value of the inductor, its DCR, and design requirements given in Table 3.1. The outputs of the algorithm for chosen $L = 47\mu H$, $R_L = 0.0469\Omega$ that satisfies the requirements are obtained as:

$$\begin{aligned} \Delta r^2 &= 6.73 \times 10^{-4}, \\ C &= 293.75\mu F. \end{aligned} \quad (3.107)$$

3.5.1 Simulations

LTspice software is used for the simulation of the buck converter. Figure 3.12 shows the circuit diagram, and the analytical expressions of the control law (3.37) and (3.38) are implemented in the hierarchical block, the netlist of the controller circuit can be found in Appendix B.1. The controller receives an input voltage and

a reference voltage; the value inductor current will be limited, and the feedback signals from the circuit, which are output voltage, inductor, and load current. It outputs the control signal u which will drive the switches on or off, one of them will be active while the other is not. In this section, the simulations are performed using ideal components and nearly ideal switches to validate the theory. The outcomes with non-ideal properties of the components will be covered in the next section.

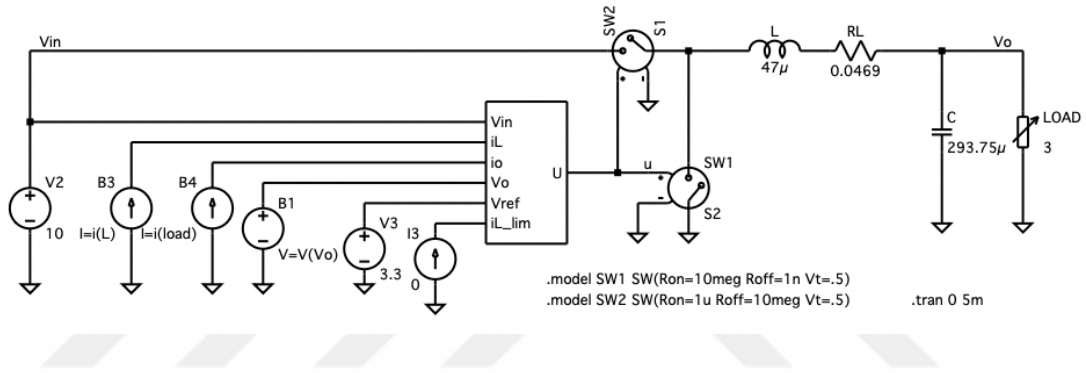


Figure 3.12: Buck converter schematic with ideal components

The simulations are performed using the input voltage, output voltage, load current given in Table 3.1, and the chosen inductor and its DCR. Furthermore, outputs of the algorithm Δr^2 and C given in (3.110) are used as design parameters. The steady-state simulation results for V_o and i_L are shown in Figure 3.13. $\Delta V_{o,ss}$, $\Delta i_{L,ss}$ and f_{ss} can be determined using the markings on the plot as follows:

$$\begin{aligned}\Delta V_{o,ss} &= 3.34165 - 3.2780569 = 0.0635931 \text{ V}, \\ \Delta i_{L,ss} &= 4.3462572 - 1.6614806 = 2.6847766 \text{ A}, \\ f_{ss} &= \frac{1}{(589.01088 - 533.36513)} \times 10^3 = 17.97082437 \text{ kHz}.\end{aligned}\tag{3.108}$$

The results are appropriate for the maximum limits of $\Delta V_{o,ss}$, $\Delta i_{L,ss}$ and f_{ss} given in Table 3.1 indicating that the Algorithm 1 operates successfully.

Figure 3.14 shows the start-up transient simulation of the buck converter. At $100\mu s$, the input voltage is applied, and the converter is started. It can be observed that the output voltage reaches the reference value with no overshoot.

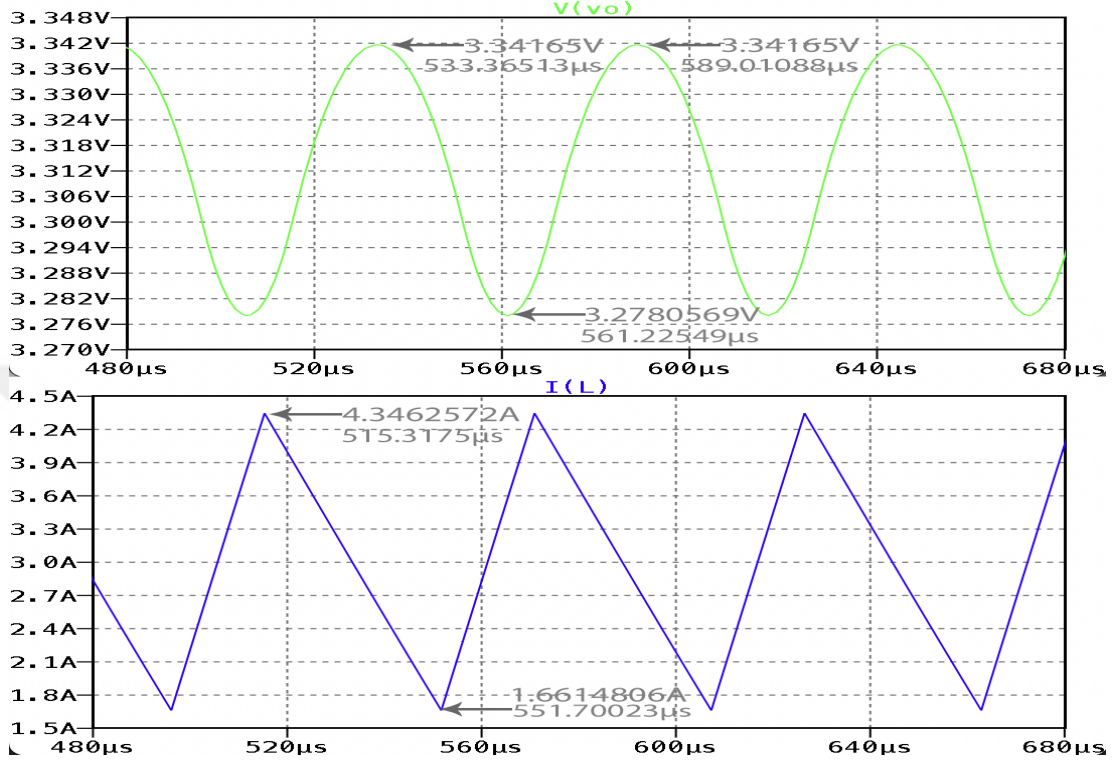


Figure 3.13: Ideal buck converter steady-state simulation results

The peak inductor current and the total time during start-up can be written as:

$$\begin{aligned} i_{L,peak} &= 12.023833 \text{ A}, \\ t_{su} &= 311.08052 - 100 = 211.08052 \text{ } \mu\text{s}. \end{aligned} \quad (3.109)$$

Similar to [41], it is noted that the peak inductor current is significantly higher during the start-up phase than it is in its steady state. This can be a problem in practical applications because the current handling capability of inductors and switches is limited. In order to solve this problem, the control law (3.39) can be used to limit the inductor current to the desired value.

Figure 3.15 shows the start-up transient simulation of the buck converter incorporating current limiting. The inductor current is limited to 6A during start-up using the control law (3.38). Total start-up time can be observed as:

$$t_{su(lim)} = 469.46441 - 100 = 369.46441 \text{ } \mu\text{s}. \quad (3.110)$$

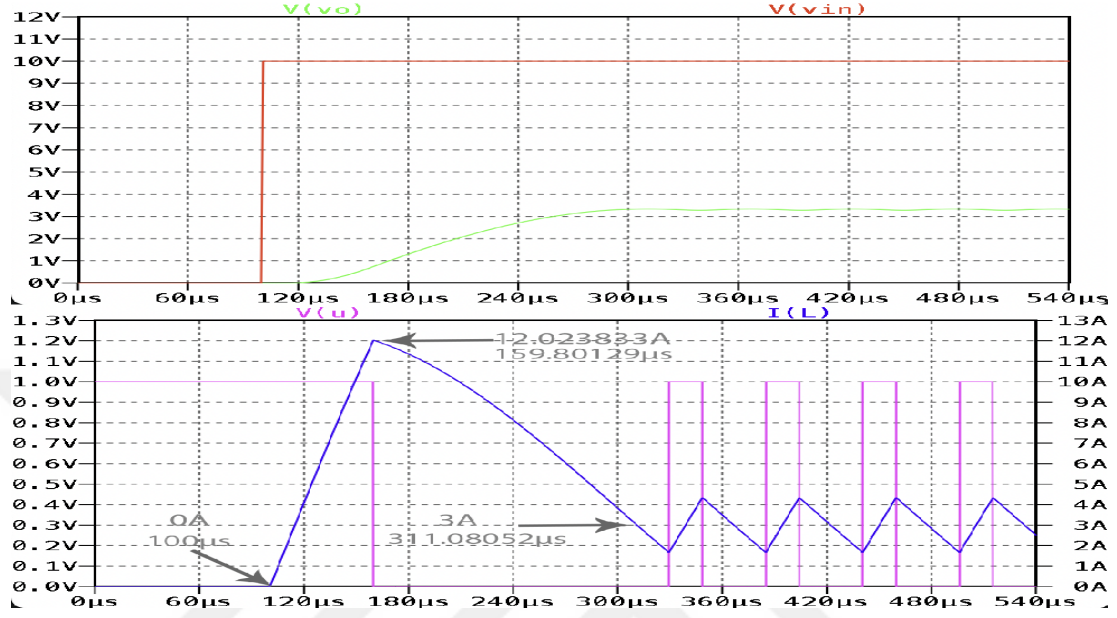


Figure 3.14: Ideal buck converter start-up simulation results

The start-up with current limiting is slower, but that is a price that must be paid if inductors and switches that can be used in the design do not have enough current handling capability. The dynamic response of the buck converter to unloading and loading transients is shown in Figure 3.16. The load current is decreased to 2.5A from 3A at time 369.405 μ s increased to 4.2A from 2.5A at time 576.149243 μ s. Recovery times, output voltage change, and minimum inductor current during loading and unloading transients can be written by using the data markings on the LTSpice simulation plots.

For unloading transient:

$$\begin{aligned}\Delta V_{o,ul} &= 3.3769858 - 3.3 = 0.0769858 \text{ V}, \\ i_{L,min(ul)} &= 0.83163409 \text{ A}, \\ t_{ul} &= 408.8728 - 359.1505 = 49.7223 \mu\text{s}.\end{aligned}\tag{3.111}$$

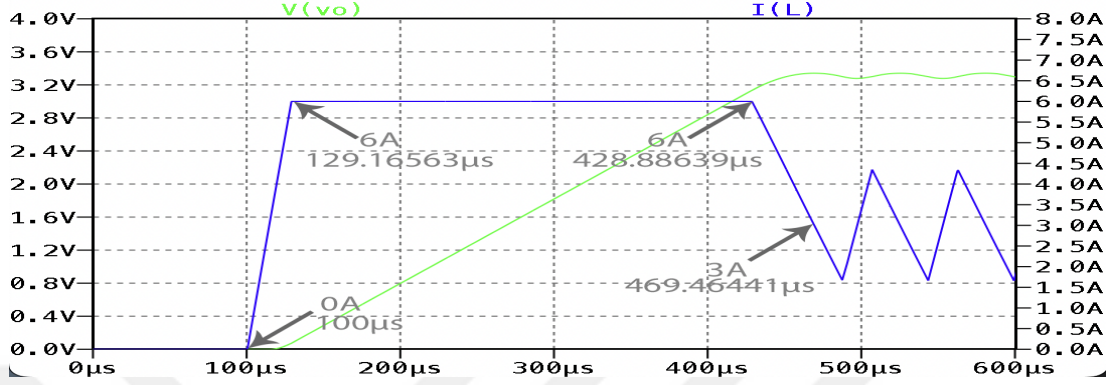


Figure 3.15: Ideal buck converter start-up incorporating current limitation simulation result

For loading transient:

$$\begin{aligned}\Delta V_{o,l} &= 3.3 - 3.18928 = 0.11072 \text{ V}, \\ i_{L,peak(l)} &= 6.2886459 \text{ A}, \\ t_l &= 622.36481 - 576.149243 = 46.215567 \mu s.\end{aligned}\tag{3.112}$$

It can be observed that the inductor current during the loading can exceed the current handling limit of inductors and switches in practical applications. Figure 3.17 shows the loading transient simulation of the buck converter incorporating current limiting. Once again, the inductor current is limited to 6A using control law (3.38). Total loading time can be observed as:

$$t_{l(lim)} = 612.51312 - 565.8215243 = 46.6915957 \mu s.\tag{3.113}$$

The loading time increases with the current limitation, as expected. The inductor current is limited at the expense of a slower loading transient. The control signal can be observed in Figures 3.14 and 3.16 to confirm that the target operating point is achieved with only one switching action during start-up and load transients. However, this is not the case with current limiting; ideally, the switch turns on and off infinitely many times to drive the states to the off-state natural switching surface by staying at the line at which the inductor current is limited.

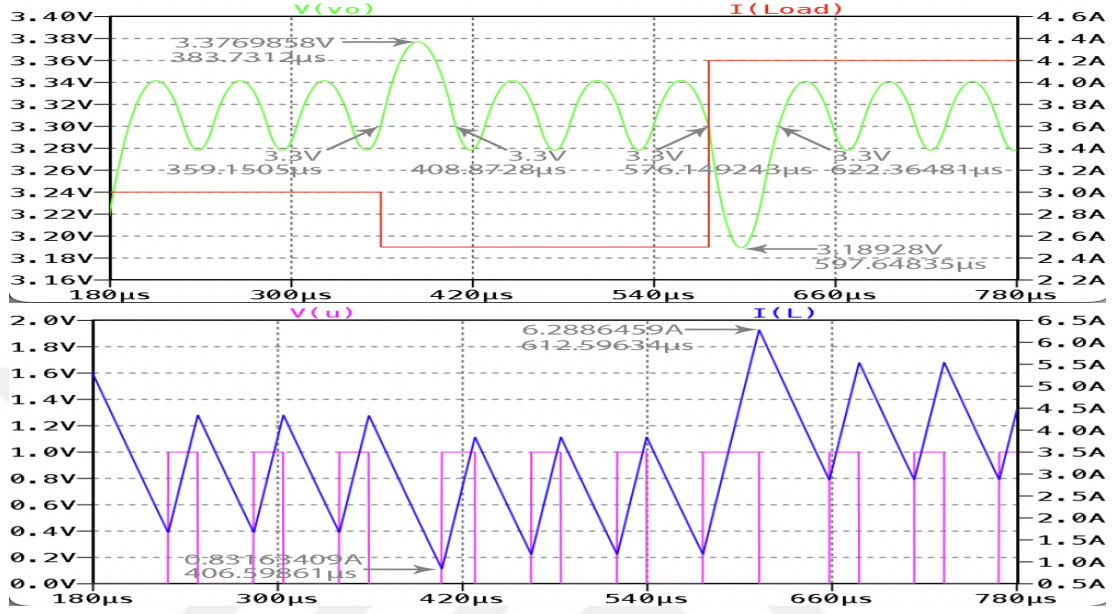


Figure 3.16: Ideal buck converter unloading and loading transients simulation results

Similar to [41], the results of ideal simulations presented in (3.108)-(3.113) are summarized in Table 3.2. The table includes the theoretical calculations done by solving related equations given in previous sections for the same conditions as well. The errors between the theoretical calculations and the ideal simulation results are presented in a separate column. It can be seen that errors are too small and on a negligible level for all performance measures of the buck converter. The differences between numerical solution methods may result in these minor variations between simulation and theoretical results.

3.6 Practical Considerations

There are primarily four factors that can lead to disparities between theory and practice for the buck converter. The first one is additional resistances in the circuit. The second one is variation in component values because of their tolerances and operating conditions. The third one is the delay of control action caused by the limited bandwidth of the controller [41]. The final one is high switching

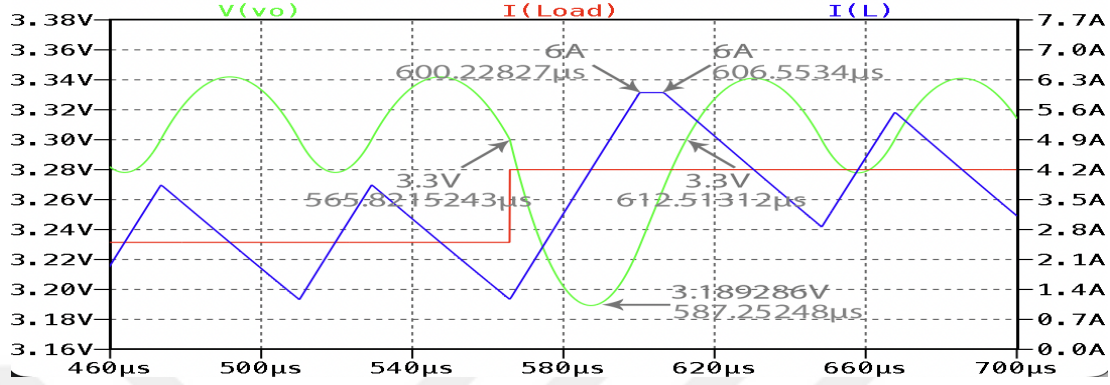


Figure 3.17: Ideal buck converter loading transient incorporating current limitation simulation results

frequency during inductor current limitation. These factors are not taken into account in the control laws, and their effects are investigated in this section.

Although the natural switching surfaces used in the control law incorporated the DC resistance of the inductor, capacitors have parasitic resistances as well. Additionally, there will be sense resistors for current measurements in the circuit, and the MOSFETs will have on-state resistances. Since these resistances are not modeled in the natural switching surfaces, they can cause additional damping effects, leading to faster-decaying trajectories [41]. Thus, the components with minimum parasitic resistances should be chosen, and sense resistors should be selected as low as possible, considering their tolerances for accurate measurement. The value of the components varies according to their tolerances and operating conditions which means more inconsistency between the actual circuit and the natural trajectories used in the control law. This problem can be reduced by selecting components with low tolerances and considering their derating for operating current, voltage, and temperature. Another practical concern is the finite bandwidth of the controller. Ideally, the controller applies the control law instantly; however, in reality, it takes time for the controller to solve the natural switching surface equations according to the measurements and apply the control law, so there will be a delay in switching actions. The final concern is about the high switching frequency during the inductor current limitation phase. The switching frequency can be quite high during current limiting, especially with a

Parameters	Theoretical Values	Ideal Simulation Results	Errors
$\Delta V_{o,ss}$	0.06359 V	0.0635931 V	0.005%
$\Delta i_{L,ss}$	2.68476	2.6847766 A	0.0006%
f_{ss}	17.99306 kHz	17.97082437 kHz	0.0012%
$i_{L,peak}$	12.023 A	12.023833 A	0.007%
$\Delta V_{o,ul}$	0.07695 V	0.0769858 V	0.0465%
$i_{L,min(ul)}$	0.831899 V	0.83163409 V	0.0318%
$\Delta V_{o,l}$	0.11071 V	0.11072 V	0.009%
$i_{L,peak(l)}$	6.28865 A	6.2886459 A	0.00007%
t_{su}	210.536 μs	211.08052 μs	0.25864%
$t_{su(lim)}$	368.769 μs	369.46441 μs	0.18858%
t_{ul}	49.71271 μs	49.7223 μs	0.01929%
t_l	46.214757 μs	46.215567 μs	0.00175%
$t_{l(lim)}$	46.66925 μs	46.6915957 μs	0.047888%

Table 3.2: Comparison of theoretical values and ideal simulation results of buck converter performance measures

high controller bandwidth, and this can cause transistors to heat up and even burn. This problem can be solved by using suitable transistors or using the new control law will be discussed in the next section.

Taking [41] as an example, the effects of these practical factors are examined by repeating the simulations in the previous section with realistic components and a controller. Figure 3.18 shows the buck converter schematic used for realistic simulations. The differences between this schematic with the ideal one can be expressed as:

1. 47 μH inductor is derated to 42 μH for its target RMS current value. In comparison to [41], inductor derating is taken into account as well.
2. 10 $m\Omega$ sense resistors R_{sense1} and R_{sense2} that can be used for measuring

inductor and load current are added.

3. 5 units of $47 \mu F$, 2 units of $22 \mu F$, 2 units of $4.7 \mu F$, 1 unit of $3.3 \mu F$ and 1 unit of $2.2 \mu F$, total $293.9 \mu F$ ceramic capacitors are used as output capacitors so that the total capacitance is very close to the calculated value. Their capacitances are derated for target output voltage, and their equivalent series resistances (ESR) are added to the schematic. Instead of one big capacitor, smaller ones are paralleled in order to reduce the effects of ESR, which became $0.454 m\Omega$ overall.
4. NMOS transistors M1 and M2 replaced the switches in the ideal schematic. Their on-state resistances are $0.7 m\Omega$.
5. In practice, the controller may not provide correct voltage and current levels for turning transistors on or off quickly; thus, a half-bridge driver integrated circuit is used to drive the transistors according to the control signal u in a realistic manner.
6. Controller bandwidth is limited to $1 MHz$. This means the control signal is updated every $1 \mu s$. It is assumed that it takes $1 \mu s$ for the controller to solve the natural switching surface equations and apply control law accordingly.

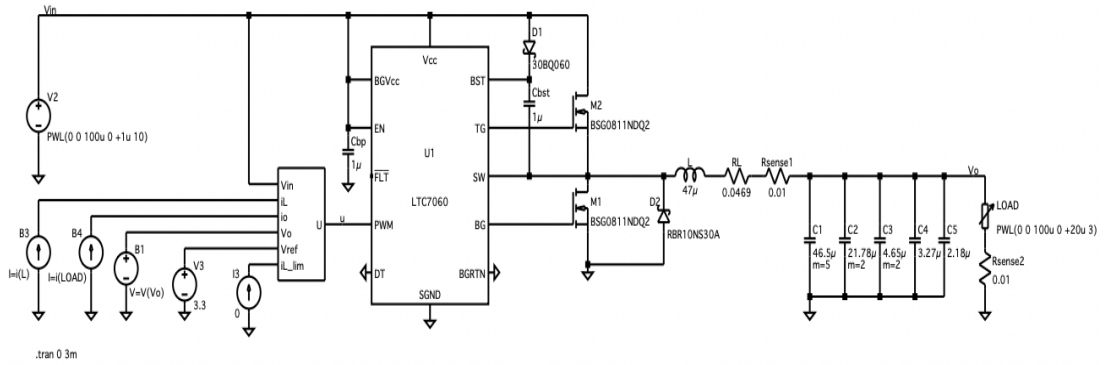


Figure 3.18: Buck converter circuit schematic with realistic components

Figures 3.19, 3.20, 3.21 3.22 and 3.23 show the simulation results for steady-state, start-up, start-up with current limitation, load transients and loading transient with current limitation of a realistic buck converter respectively. Data marking on

LTSpice plots is utilized to determine the performance metrics of the realistic buck converter, which are then summarized in Table 3.3, as done in [41]. Percentage errors between the theoretical calculations and the realistic simulation results are presented in a separate column in this table as well. The efficiency (η) of the converter is also added to this table.

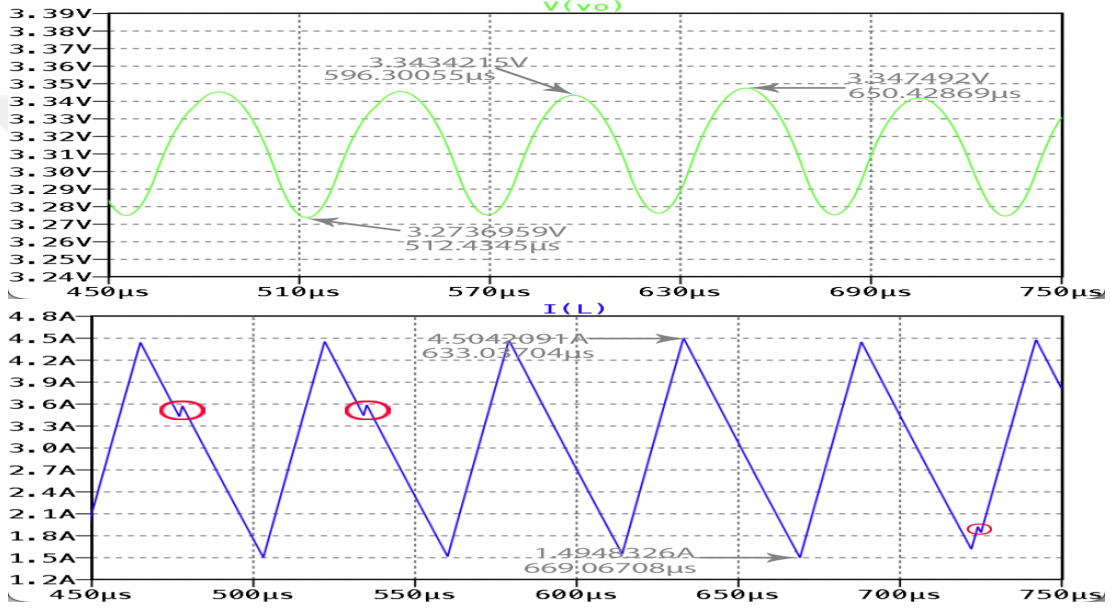


Figure 3.19: Realistic buck converter steady-state simulation results

By changing one non-optimal feature at a time, numerous simulations are done in the style of a controlled experiment. Since the ESR of output capacitors, on-state resistance of MOSFETs, and the resistance of current sense resistors are much less than the DC resistance of the inductor, which is taken into account in the natural trajectories, their effects are found to be negligible in the context of chattering and errors except efficiency. The main cause of errors was the finite bandwidth of the controller and variation in the component values, especially inductor derating, which caused load transients to be faster however, it is the primary reason for chattering, which is shown with red ellipses on the inductor current waveforms in Figures 3.19-3.23.

If the system states drop below $\sigma_{off}\Delta$ natural switching surface when it is in

off-state, the controller turns the switch on to bring the states to this surface and turns the switch off. Likewise, the controller turns the switch off if the system states raises above $\sigma_{on\Delta}$ natural switching surface when it is in on-state to bring the states to this surface and turns the switch on as it can be seen in Figure 3.22.

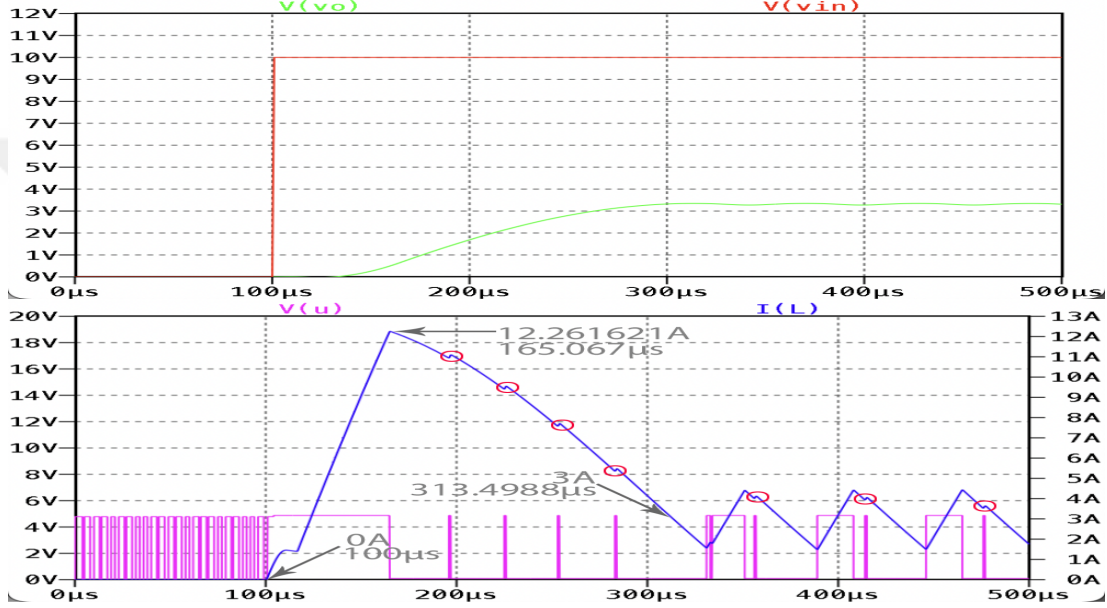


Figure 3.20: Realistic buck converter start-up simulation results

In the simulation trials, chattering is observed in the steady state when inductor derating is induced. This contrasts with [41], where chattering is observed even without inductor derating. However, if parasitic resistances not taken into account in the deriving of natural switching surfaces were high, chattering would be seen even without inductor derating since these parasitic resistances cause extra damping, leading the system states to drop below $\sigma_{off\Delta}$. The chattering frequency increases with higher losses and variations in the component values. Additionally, higher controller bandwidth results in checking the system states more frequently, causing more chattering. Therefore, attention should be paid to the controller bandwidth; it must be ensured that it is significantly greater than the natural frequency of L and C for reducing errors [41], and it does not cause high chattering frequency, which may lead components to heat up and even burn.

As observed in Figure 3.19, $\Delta V_{o,ss}$ and $\Delta i_{L,ss}$ are not constant for all cycles

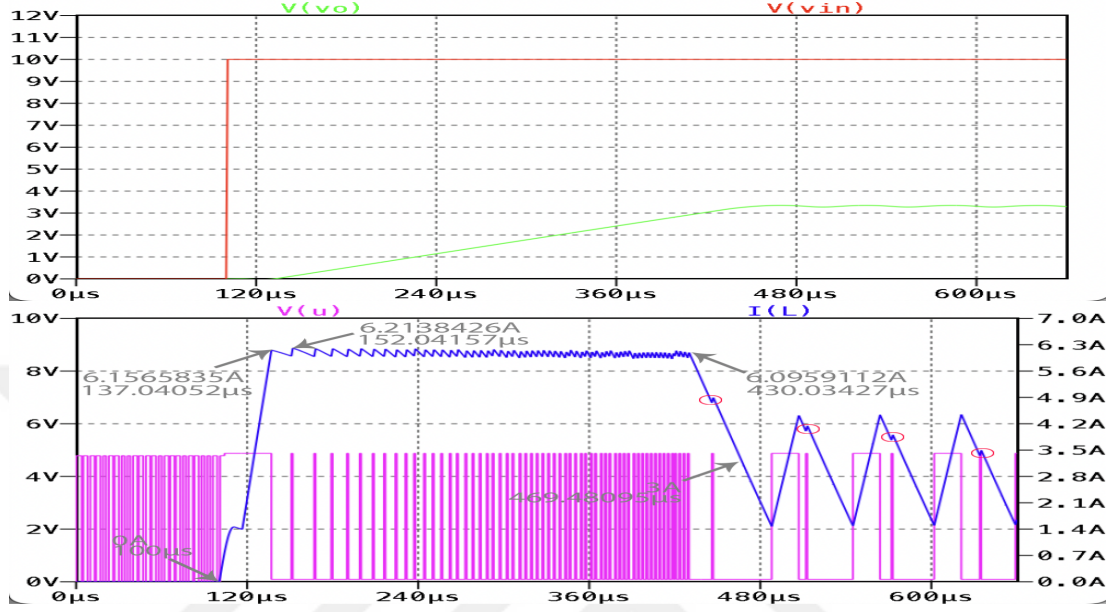


Figure 3.21: Realistic buck converter start-up with inductor current limitation simulation results

in the steady state due to limited controller bandwidth. Similar to [41], it is important to underline that the comparison in Table 3.3 is simply meant to provide a concept on the degree of inaccuracy between theory and practice. Since much of the simulation findings rely on from which switching cycle the data is gathered, the data in the table can not be regarded as exact values.

Figure 3.21 displays the simulation result of a buck converter with inductor current limitation in the start-up. It can be observed that the maximum inductor current exceeds the value used in the control law (3.38), which is the inductor current value desired to be limited. This is also the case for loading transient with current limitation, as can be seen in Figure 3.23. In general, the inductor current has cyclical variation from the desired limited value during the current limitation phase. The reason for this is the delay caused by limited controller bandwidth. Moreover, the switching frequency is high compared to its value in the steady state its maximum value is measured to be 333.7755 kHz during the current limiting phase in the start-up. The switching frequency during the current limitation phase is proportional to the bandwidth of the controller, and

this can be a problem with the high-bandwidth controller since the transistors used in the design may not be suitable for high-frequency switching. They may overheat and exceed their maximum operating temperature value. The solution to this problem is discussed in the next section.

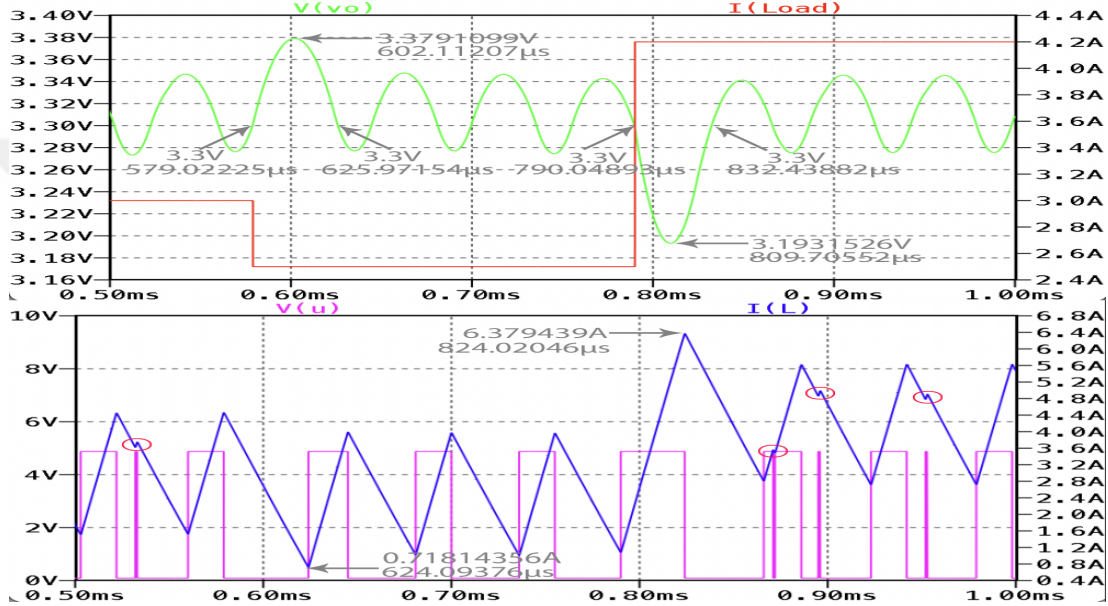


Figure 3.22: Realistic buck converter unloading and loading simulation results

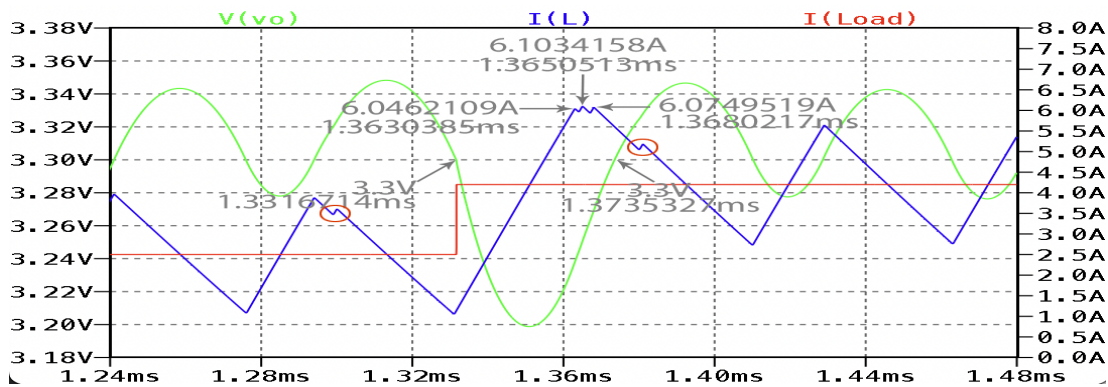


Figure 3.23: Realistic buck converter loading with inductor current limitation simulation results

Parameters	Theoretical Values	Realistic Simulation Results	Errors
$\Delta V_{o,ss}$	0.06359 V	0.0737961 V	16.04985%
$\Delta i_{L,ss}$	2.68476	3.00938 A	12.0912%
f_{ss}	17.99306 kHz	18.47468 kHz	2.6767%
$i_{L,peak}$	12.023 A	12.261621A	1.9847%
$\Delta V_{o,ul}$	0.07695 V	0.07911 V	2.80701%
$i_{L,min(ul)}$	0.831899 V	0.71814 V	13.67462%
$\Delta V_{o,l}$	0.11071 V	0.10685 V	3.48659%
$i_{L,peak(l)}$	6.28865 A	6.37944 A	1.44371%
t_{su}	210.536 μs	213.4988 μs	1.40727%
$t_{su(lim)}$	368.769 μs	369.48095 μs	0.19306%
t_{ul}	49.71271 μs	46.94904 μs	5.55928%
t_l	46.214757 μs	42.38989 μs	9.16955%
$t_{l(lim)}$	46.66925 μs	41.8613 μs	10.30218%
η	95.91% s	92.78%	3.263476%

Table 3.3: Comparison of theoretical values and realistic simulation results of buck converter performance measures

3.6.1 Switching Frequency Limitation During Current Limiting

The switching frequency during the current limitation phase can be restricted by defining a hysteresis band between two inductor current values. The system states will be in this band, swinging between the upper and lower limits of it until the system hits the off-state natural switching surface. This can be achieved by

the control law defined in (3.114).

$$\begin{aligned}
& \text{Case 1 : } i_{Ln,target} > i_{Ln} \\
& u = \begin{cases} 1, & \sigma_{on} > 0 \\ 0, & \text{else} \end{cases} \\
& \text{Case 2 : } i_{Ln} > i_{Ln,target} \\
& \text{Subcase 1 : } \sigma_{off} > 0 \\
& u = 0 \\
& \text{Subcase 2 : } \sigma_{off} < 0 \\
& u = \begin{cases} 1, & i_{Ln,liml} > i_{Ln} \\ 0, & i_{Ln} > i_{Ln,limh}. \end{cases}
\end{aligned} \tag{3.114}$$

Figure 3.24 displays the start-up trajectories of buck converter incorporating hysteresis band for current limiting under the control law (3.114). The system starts at $(0,0)$, and the control law turns the switch on. The system follows the on-state natural trajectory until the upper limit of the hysteresis band $i_{Ln,limu}$ is reached, then the switch turns off and stays off until the states reach the lower limit of the band $i_{Ln,liml}$. The switch is turned on by the control law when $i_{Ln,liml}$ is reached, and the switch stays on until the system reaches the upper limit of the band. This cycle continues until the system reaches the off-state natural switching surface. Then, the switch turns off if it is in on-state and stays off. If it is off-state, the system states move along the off-state natural switching surface and reaches the target operating point.

Figure 3.25 shows the loading trajectories of buck converter under the control law (3.114). When the load current increases, the control law turns the switch on, and the states start at the initial target operating point $(i_{Ln,initial}, V_{on,target})$ and follow the on-state natural trajectory until the upper limit of the hysteresis band $i_{Ln,limu}$ is reached. Similarly, the system states stay in this band by swinging between the upper and lower limit until the off-state natural switching surface is reached. Then, the switch is turned off by the control law, and the system states move along this surface and converge to the target operating point.

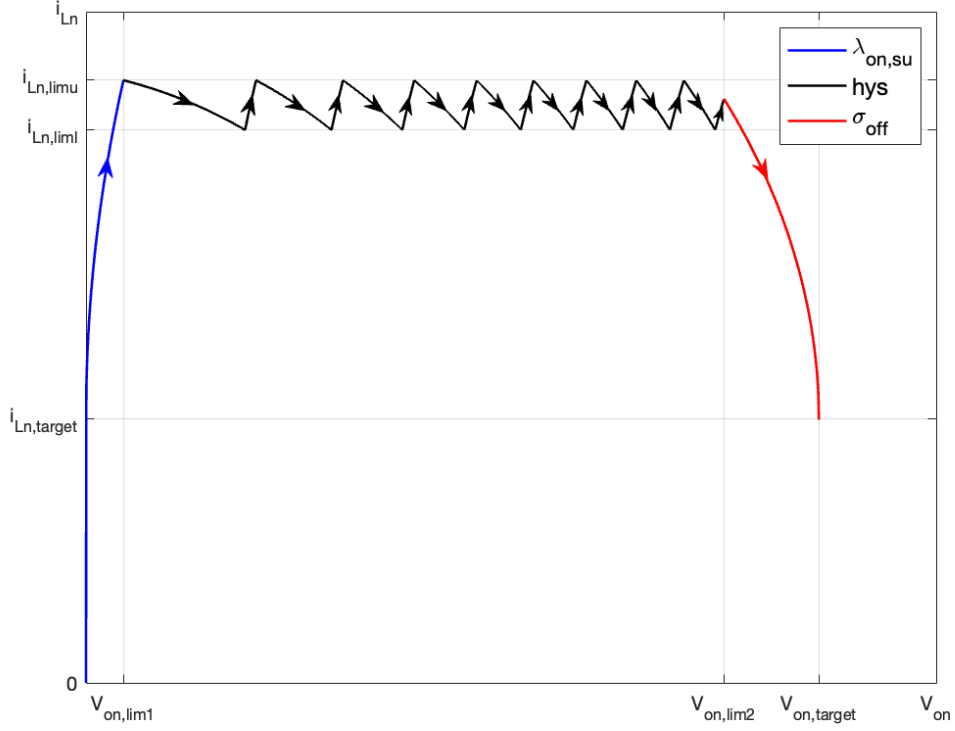


Figure 3.24: Buck converter start-up trajectories incorporating hysteresis band for current limitation

The maximum switching frequency during the current limitation stage can be determined by $i_{L,limu}$ and the width of the hysteresis band, which can be defined as

$$\Delta_{hys} = i_{L,limu} - i_{L,liml}. \quad (3.115)$$

Increasing Δ_{hys} decreases the maximum switching frequency during the current limitation whereas reducing $i_{L,limu}$ increases it. With appropriate selection of $i_{L,limu}$ and Δ_{hys} , the inductor current and the switching frequency limits of design may not be exceeded. In order to achieve this, an algorithm similar to the one used when determining the buck converter design parameters can be implemented. This algorithm would input all the design parameters and $i_{L,limu}$ which

is the desired maximum inductor current value, and outputs optimum Δ_{hys} for the maximum switching frequency allowed. The algorithm would start with the minimum value of Δ_{hys} in the given range, measure the maximum switching frequency in the current limitation phase, and increment Δ_{hys} until the optimum value is found. Then, it can calculate $i_{L,liml}$ value to be used in the control law by using (3.115).

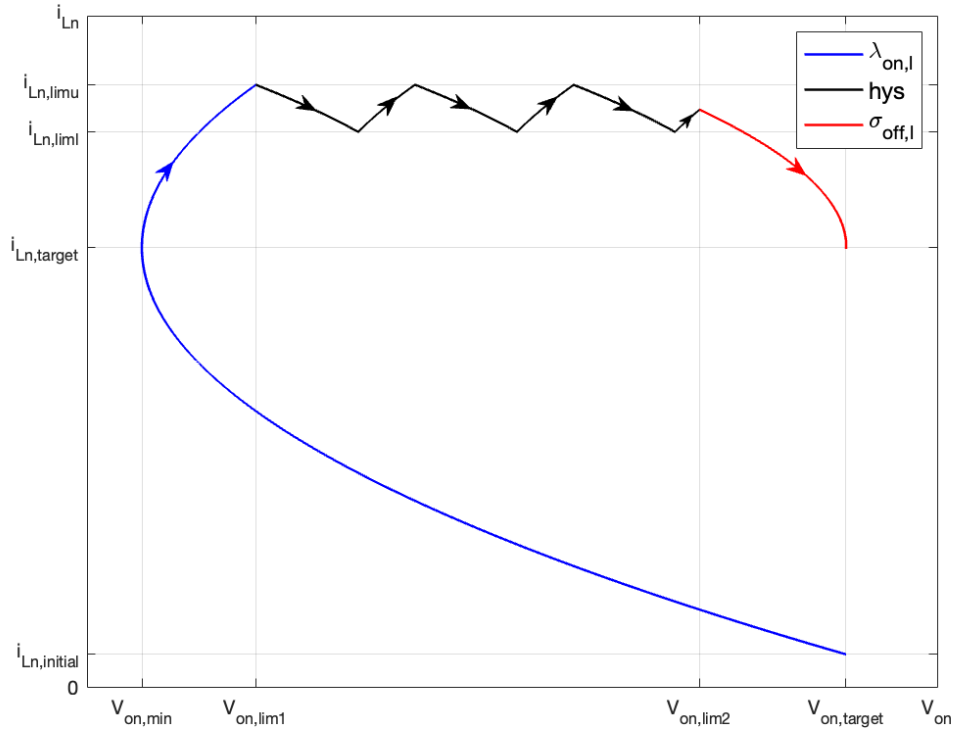


Figure 3.25: Buck converter loading trajectories incorporating hysteresis band for current limitation

In practice, there will be deviations from the limits of the hysteresis band because of the limited controller bandwidth, and the maximum switching frequency would be proportional to the bandwidth of the controller. An algorithm that takes into account the bandwidth of the controller should be implemented for exact results. The algorithm should determine $i_{L,limu}$ and $i_{L,liml}$ to be used in

the control law for allowed maximum current and switching frequency during the inductor current limiting phase.



Chapter 4

Second-Order Sliding Mode Control of Boost Converter

4.1 Modeling of Natural Trajectories

Figure 4.1 displays the schematic of the boost converter with the DC resistance of the inductor represented by R_L . As in the previous chapter, the normalized form of system variables is used for the modeling of the natural trajectories of the boost converter.

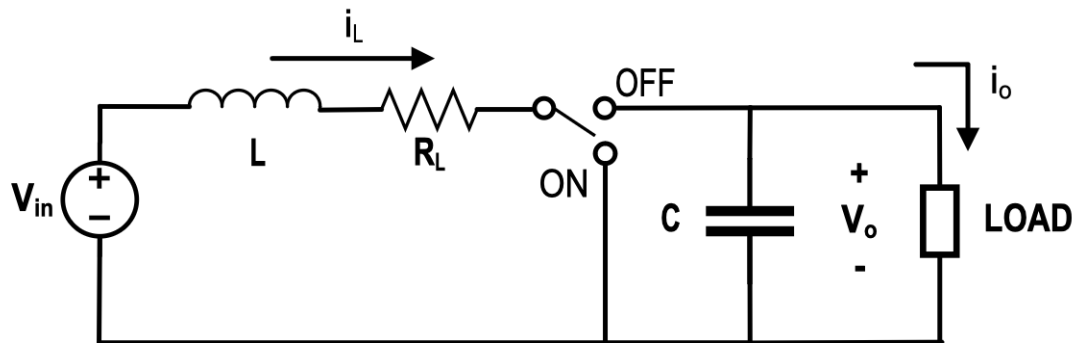


Figure 4.1: The schematic of the boost converter

4.1.1 Off-State Model

The system dynamics of the boost converter when the switch is off can be represented by the following differential equations:

$$\begin{aligned} C \frac{dV_o}{dt} &= i_L - i_o, \\ L \frac{di_L}{dt} &= V_{in} - R_L i_L - V_o. \end{aligned} \quad (4.1)$$

After solving (4.1) for $V_o(t)$ and $i_L(t)$ and eliminating the time variable, we can determine the off-state natural trajectories of the converter in the $v_o - i_L$ plane. This is a key step in our method for deriving a switching rule. To achieve this goal, the steps for deriving the natural trajectories of the buck converter, as outlined in the previous chapter, are repeated to obtain the off-state natural trajectories of the boost converter. First, equation (4.1) is simplified through normalization using the reference output voltage V_{ref} , the characteristic impedance Z_o , and the natural frequency f_o of L and C , as rewritten in Equation (4.2).

$$Z_o = \sqrt{L/C}, \quad f_o = \frac{1}{2\pi\sqrt{LC}}. \quad (4.2)$$

The normalized form of the system variables can be rewritten as follows:

$$V_{xn} = \frac{V_x}{V_{ref}}, \quad i_{xn} = \frac{i_x Z_o}{V_{ref}}, \quad R_{Ln} = \frac{R_L}{Z_o}, \quad f_n = \frac{f}{f_o}, \quad t_n = f_o t. \quad (4.3)$$

The variables for voltages, currents, frequency, and time are represented by V_x , I_x , f , and t , respectively, with x being either in, L or o. The subscript n indicates the normalized versions of these variables. Utilizing (4.2) and (4.3) in (4.1), we arrive at the following normalized equations:

$$\begin{aligned} \frac{dV_{on}}{dt_n} &= 2\pi(i_{Ln} - i_{on}), \\ \frac{di_{Ln}}{dt_n} &= -2\pi(-V_{inn} + R_{Ln}i_{Ln} + V_{on}). \end{aligned} \quad (4.4)$$

The newly defined system variables $\hat{i}_{Ln}$ and $\hat{V}_{on}$ in (4.5) enable a similarity

transformation that will be applied for coordinate conversion.

$$\begin{aligned}\hat{i}_{Ln} &= i_{Ln} - i_{on}, \\ \hat{V}_{on} &= -V_{inn} + R_{Ln}i_{on} + V_{on}.\end{aligned}\tag{4.5}$$

Using (4.5) in (4.4), the normalized system equations are rewritten in (4.6) with the new variables.

$$\begin{aligned}\frac{d\hat{V}_{on}}{dt_n} &= 2\pi\hat{i}_{Ln}, \\ \frac{d\hat{i}_{Ln}}{dt_n} &= -2\pi(R_{Ln}\hat{i}_{Ln} + \hat{V}_{on}).\end{aligned}\tag{4.6}$$

Note that equation (4.6) is the same as equation (3.6). By following similar steps and performing the coordinate conversions outlined in (3.7)-(3.19), the same solution can be obtained in the polar coordinates as given below:

$$r^2 - r_o^2 e^{-2\frac{\alpha}{\beta}(\theta(0)-\theta)} = 0.\tag{4.7}$$

The solution curves of equation (4.7) in the phase plane for specific initial conditions represent the natural trajectories of the converter. These natural trajectories can be denoted by λ , and equation (4.7) can now be expressed as $\lambda = 0$ with the latter being defined as follows:

$$\lambda = r^2 - r_o^2 e^{-2\frac{\alpha}{\beta}(\theta(0)-\theta)}.\tag{4.8}$$

The polar coordinates can be converted back to Cartesian (z_1, z_2) using

$$\begin{aligned}r^2 &= z_1^2 + z_2^2, \\ \theta &= \text{atan2}(z_2, z_1),\end{aligned}\tag{4.9}$$

where

$$\begin{aligned}z_1 &= -\frac{1}{2\pi}\hat{V}_{on}, \\ z_2 &= -\frac{\alpha}{2\pi\beta}\hat{V}_{on} - \frac{1}{\beta}\hat{i}_{Ln}.\end{aligned}\tag{4.10}$$

With the help of equation (4.5), equation (4.10) can be rewritten using the initial variables V_{on} and i_{Ln} , as given below:

$$\begin{aligned} z_1 &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on}), \\ z_2 &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on}) - \frac{1}{\beta}(i_{Ln} - i_{on}). \end{aligned} \quad (4.10)$$

Formulations in the Cartesian coordinates (z_1, z_2) can be rewritten for the off-state boost converter as below:

$$\begin{aligned} z_{1,off} &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on}), \\ z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on}(0)), \\ z_{2,off} &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on}) - \frac{1}{\beta}(i_{Ln} - i_{on}), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on}(0)) - \frac{1}{\beta}(i_{Ln}(0) - i_{on}). \end{aligned} \quad (4.11)$$

Then, the polar variables used in the natural trajectories can be expressed as:

$$\begin{aligned} r_{off}^2 &= z_{1,off}^2 + z_{2,off}^2, \\ r_{off,0}^2 &= z_{1,off}^2(0) + z_{2,off}^2(0), \\ \theta_{off} &= \text{atan2}(z_{2,off}^2, z_{1,off}^2), \\ \theta_{off}(0) &= \text{atan2}(z_{2,off}^2(0), z_{1,off}^2(0)). \end{aligned} \quad (4.12)$$

Thereafter, the off-state natural trajectories of the boost converter with variables in (4.12) can be expressed as:

$$\lambda_{off} = r_{off}^2 - r_{off,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}. \quad (4.13)$$

Figure 4.2 displays a selection of off-state natural trajectories plotted for 4 different initial conditions with distinct colors. These trajectories are the solution curves of equation (4.13); they are spiral curves that originate from initial conditions and converge to the equilibrium point. To determine these trajectories, we begin with the initial values $V_{on}(0)$ and $i_{Ln}(0)$ to compute $z_{1,off}(0)$ and

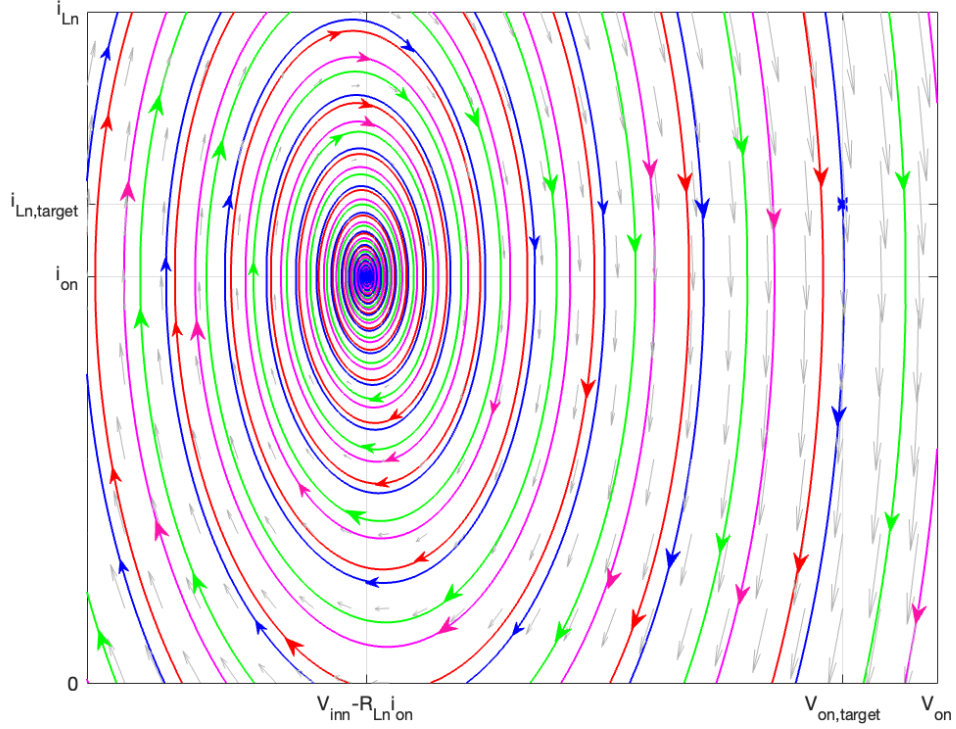


Figure 4.2: Off-state natural trajectories of the boost converter

$z_{2,off}(0)$ using (4.11). Next, we calculate $r_{off,0}^2$ and $\theta_{off}(0)$ from (4.12). Following this, we find the solutions for r and θ by setting (4.13) to zero, which implies $\lambda_{off} = 0$. Afterward, we transform back to the $(z1, z2)$ coordinates using (3.14), and finally, we determine i_{Ln} and V_{on} by applying (4.10). The blue curve is a natural switching surface that passes through the target operating point and will be used in the control law.

4.1.2 On-State Model

The system dynamics of the on-state boost converter are represented in Equation (4.14). The DCR of the inductor is neglected for the on-state trajectories of the

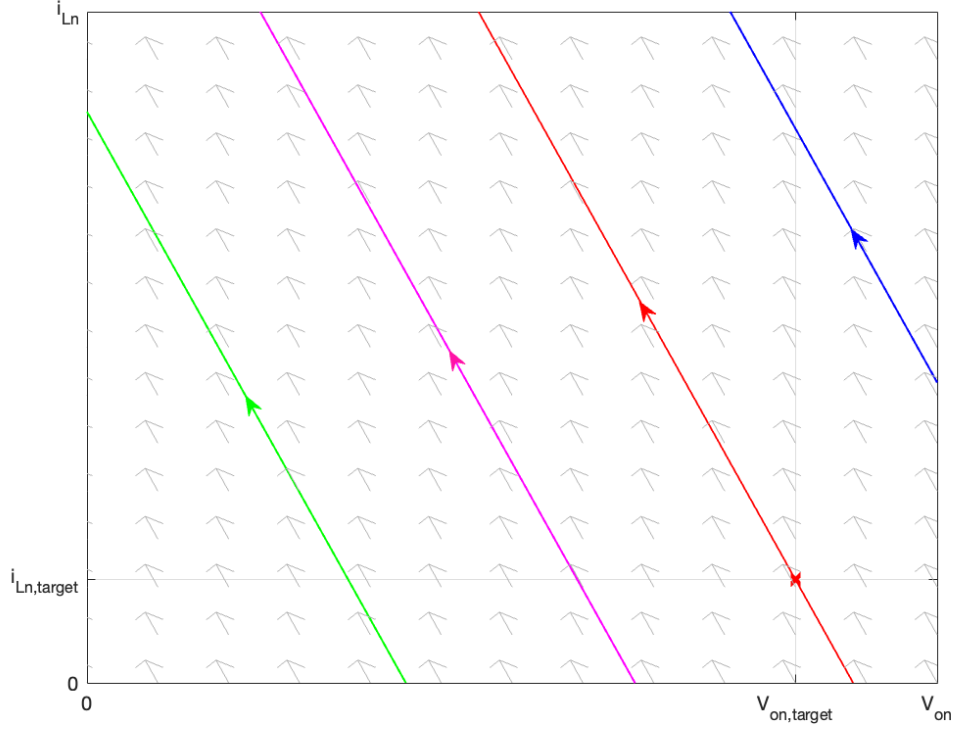


Figure 4.3: On-state natural trajectories of the boost converter

boost converter ($R_L = 0$), as done in [7]. In the on-state, the output voltage dynamics do not depend on the inductor current, and the inductor current dynamics do not depend on the output voltage. The DCR of the inductor is omitted in the on-state model of the boost converter for simplicity. However, in the off-state case, the damping effect caused by parasitics plays a crucial role, as it causes trajectories to decay more quickly.

$$\begin{aligned} C \frac{dV_o}{dt} &= -i_o, \\ L \frac{di_L}{dt} &= V_{in}. \end{aligned} \tag{4.14}$$

By using the normalization given in the preceding section, the normalized form

of (4.14) can be written as:

$$\begin{aligned}\frac{dV_{on}}{dt_n} &= -2\pi i_{on}, \\ \frac{di_{Ln}}{dt_n} &= 2\pi V_{inn}.\end{aligned}\tag{4.15}$$

Laplace transform of (4.15) can be written as:

$$\begin{aligned}sV_{on}(s) - V_{on}(0) &= -2\pi \frac{i_{on}}{s}, \\ si_{Ln}(s) - i_{Ln}(0) &= 2\pi \frac{V_{inn}}{s}.\end{aligned}\tag{4.16}$$

Taking the inverse Laplace transform of (4.16) gives

$$\begin{aligned}V_{on}(t) &= V_{on}(0) - 2\pi i_{on}t, \\ i_{Ln}(t) &= i_{Ln}(0) + 2\pi V_{inn}t.\end{aligned}\tag{4.17}$$

Time can be eliminated as follows:

$$t = \frac{i_{Ln} - i_{Ln}(0)}{2\pi V_{inn}}.\tag{4.18}$$

Substituting (4.18) in (4.17) gives

$$V_{on} = V_{on}(0) - \frac{i_{on}}{V_{inn}}(i_{Ln} - i_{Ln}(0)).\tag{4.19}$$

Then, the on-state natural trajectories of the boost converter can be defined as

$$\lambda_{on} = V_{on} + \frac{i_{on}}{V_{inn}}(i_{Ln} - i_{Ln}(0)) - V_{on}(0).\tag{4.20}$$

Similar to the off-state case, a selection of on-state natural trajectories is plotted for 4 different initial conditions with distinct colors in Figure 4.3. These trajectories can be easily found by setting (4.20) to zero, which means $\lambda_{on} = 0$ for given initial conditions $V_{on}(0)$ and $i_{Ln}(0)$. The red curve that passes through the target operating point is a natural switching surface that will be used in the control law.

4.2 Control Law

At the target operating point, normalized input power (P_{inn}) equals the sum of normalized output power (P_{on}) and power loss (P_{lossn}).

$$P_{on} + P_{lossn} - P_{inn} = 0, \quad (4.21)$$

where

$$\begin{aligned} P_{inn} &= V_{inn} i_{Ln,target}, \\ P_{on} &= V_{on,target} i_{on}, \\ P_{lossn} &= R_{Ln} i_{Ln,target}^2. \end{aligned} \quad (4.22)$$

Equation (4.21) can be rewritten as:

$$R_{Ln} i_{Ln,target}^2 - V_{inn} i_{Ln,target} + V_{on,target} i_{on} = 0. \quad (4.23)$$

At the target operating point $V_{on,target}$ is equal to 1, by substituting it in (4.23) normalized target inductor current can be found as:

$$i_{Ln,target} = \frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on}R_{Ln}}}{2R_{Ln}}. \quad (4.24)$$

Then, the on-state switching surface of the boost converter can be found using the target operating point as:

$$\sigma_{on} = V_{on} + \frac{i_{on}}{V_{inn}} \left(i_{Ln} - \left(\frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on}R_{Ln}}}{2R_{Ln}} \right) \right) - 1. \quad (4.25)$$

Using the target operating point as initial conditions in (4.11) the off-state switching surface can be found as follows:

$$\begin{aligned} V_{on}(0) &= 1, \\ i_{Ln}(0) &= \frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on}R_{Ln}}}{2R_{Ln}}. \end{aligned} \quad (4.26)$$

Using (4.26) in (4.11) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + 1) - \frac{1}{\beta}\left(\frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on}R_{Ln}}}{2R_{Ln}} - i_{on}\right). \end{aligned} \quad (4.27)$$

By using (4.27) in (4.13), the off-state switching surface of the boost converter can be derived as:

$$\sigma_{off} = r_{off}^2 - r_{off,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}. \quad (4.28)$$

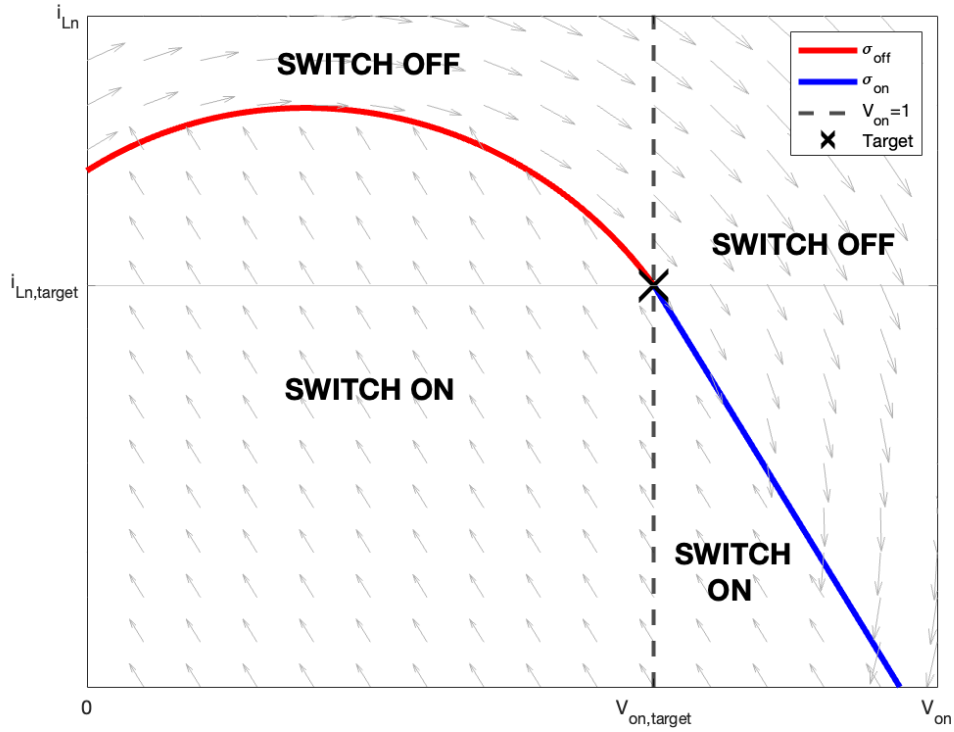


Figure 4.4: Control Law for boost converter in state plane

The position of the switch at any point on the state plane can be determined by the control law (4.29). $u = 1$ represents the switch being on, while $u = 0$

represents the switch being off. If the states are below $V_{on} = V_{on,target}$ line, the control action is determined by the location of the states with respect to off-state natural switching surface σ_{off} . The switch is turned off if the result of solving equation (4.28) with current states is positive, the switch is turned on if it is negative. If the states are above $V_{on} = V_{on,target}$ line, the control action is determined by the location of the states with respect to the on-state switching line σ_{on} . The switch is turned off if the result of solving equation (4.25) with current states is positive, the switch is turned on if it is negative.

$$\begin{aligned}
& \text{Case 1 : } V_{on} < 1 \\
& u = \begin{cases} 0, & \sigma_{off} > 0 \\ 1, & \text{else} \end{cases} \\
& \text{Case 2 : } V_{on} > 1 \\
& u = \begin{cases} 0, & \sigma_{on} > 0 \\ 1, & \text{else.} \end{cases}
\end{aligned} \tag{4.29}$$

The control action for any point on the state plane is demonstrated in Figure 4.4. The part of the off-state natural switching surface for $V_{on} < V_{on,target}$ is demonstrated using the color red. Likewise, the part of the on-state natural switching line for $V_{on} > V_{on,target}$ is the blue line on the figure. The control law (4.29) ensures that the states hit the switching surface, then the system reaches the target operating point with only one switching action.

4.2.1 Control Law for Limiting Inductor Current

On the off-state trajectories of the boost converter, the point where the maximum inductor current resides is $(V_{inn} - i_{Ln,peak}R_{ln}, i_{Ln,peak})$. The green curve demonstrates another off-state trajectory with a maximum inductor current point equal to the maximum allowed inductor current $i_{Ln,lim}$, which is utilized for limiting inductor current with the control law (4.29). Naturally, we assume that $i_{Ln,lim} > i_{Ln,target}$. It is important to note that $i_{Ln,target}$ is considered positive

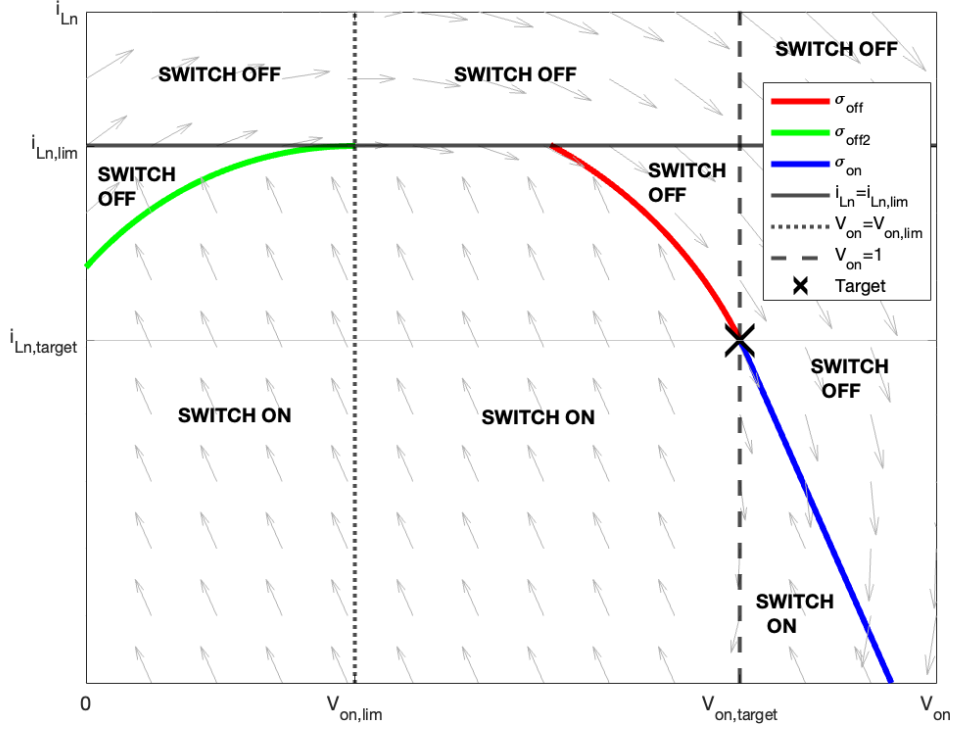


Figure 4.5: Control Law for boost converter incorporating current limiting

in the direction shown for the inductor current in Figure 4.1, and the same convention applies to $i_{Ln,lim}$. $(V_{inn} - i_{Ln,lim}R_{Ln}, i_{Ln,lim})$ is the target point for this new off-state trajectory, which will be used when V_{on} is less than or equal to $(V_{inn} - i_{Ln,lim}R_{Ln})$ for reaching $i_{Ln} = i_{Ln,lim}$ line. V_{on} point $(V_{inn} - i_{Ln,lim}R_{Ln})$ can be referred to as $V_{on,lim}$. The system stays at $i_{Ln} = i_{Ln,lim}$ line for reaching off-state natural trajectory σ_{off} represented by the red curve. Then the control action is the same as that of the previous section. The new off-state trajectory can be derived using the initial conditions as given below:

$$\begin{aligned} i_{Ln}(0) &= i_{Ln,lim}, \\ V_{on}(0) &= V_{on,lim}. \end{aligned} \tag{4.30}$$

Using (4.30) in (4.11) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on,lim}), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on,lim}) - \frac{1}{\beta}(i_{Ln,lim} - i_{on}). \end{aligned} \quad (4.31)$$

By using (4.31) in (4.12), $r_{off,20}^2$ and $\theta_{off2}(0)$ can be defined and used in the new off-state switching surface which can be expressed as

$$\sigma_{off2} = r_{off}^2 - r_{off2,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off2}(0) - \theta_{off})}. \quad (4.32)$$

Then, the inductor current limiting for the boost converter can be achieved by the control law below:

Case 1 : $V_{on} < 1$

Subcase 1 : $i_{Ln} < i_{Ln,lim}$ and $\sigma_{off} < 0$

$$u = \begin{cases} 0, & \sigma_{off2} > 0 \text{ and } V_{on} < V_{on,lim} \\ 1, & \text{else} \end{cases}$$

Subcase 2 : $i_{Ln} > i_{Ln,lim}$ and $\sigma_{off} < 0$

$$u = 0 \quad (4.33)$$

Subcase 3 : $\sigma_{off} > 0$

$$u = 0$$

Case 2 : $V_{on} > 1$

$$u = \begin{cases} 0, & \sigma_{on} > 0 \\ 1, & \text{else.} \end{cases}$$

In order to apply this method, indented limited inductor current value $i_{Ln,lim}$ must be greater than the maximum inductor current value of off-state natural trajectory and starts from the origin and can be derived as follows:

$$\begin{aligned} V_{on}(0) &= 0, \\ i_{Ln}(0) &= 0. \end{aligned} \quad (4.34)$$

Using (4.34) in (4.11) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on}), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on}) + \frac{1}{\beta}i_{on}. \end{aligned} \quad (4.35)$$

$r_{off,0}^2$ and $\theta_{offo}(0)$ can be defined by using (4.35) in (4.12). Then, the off-state natural trajectory that starts at the origin can be defined as:

$$\lambda_{offo} = r_{off}^2 - r_{off,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{offo}(0) - \theta_{off})}. \quad (4.36)$$

Then, the condition for applicability of (4.33) can be expressed as:

$$i_{Ln,lim} > \max_{\lambda_{offo}} i_{Ln}. \quad (4.37)$$

Equation (4.29) indicates that this current limitation method can be applied when the desired limited inductor current value, $I_{Ln,lim}$, must be greater than the maximum inductor current value along an off-state natural trajectory, which starts at the origin. Figure 4.5 displays the control law (4.33) on the state plane. For $V_{on} < V_{on,lim}$, the position of states with respect to σ_{off2} is used in the control law. $i_{Ln} = i_{Ln,lim}$ line and part of the σ_{off} between the target operating point and this line used in the control law for $V_{on,lim} < V_{on} < V_{on,target}$ region. The control action is the same previous section above $V_{on,target}$ line.

4.3 Transient Analysis

Similar to the study reported in [41] for a resistive loaded boost converter, analogous evaluations are undertaken for the transient analysis of a constant current loaded boost converter model considering the inductor DCR in the off-state. Furthermore, this study broadens the theoretical analysis by including current-limited loading and start-up transients. In DC/DC converters, the inductor current overshoot and settling time during start-up are crucial issues that can impact

both performance and reliability. Another key performance measure for switching converters is how much the output voltage departs from its reference value and the time it takes for the converter to recover during load transients. The following section outlines the theoretical framework for calculating the duration, output voltage, and inductor current deviations during transient events.

4.3.1 Start-up Transients

The start-up trajectories of the boost converter under the control law (4.29) are shown in Figure 4.6. The control law turns the switch on, and the system starts at $(0, 0)$ follows $V_{on} = 0$ line until it reaches the off-state switching surface σ_{off} . Then, the switch is turned off, and the system reaches the target operating point by following the σ_{off} path.

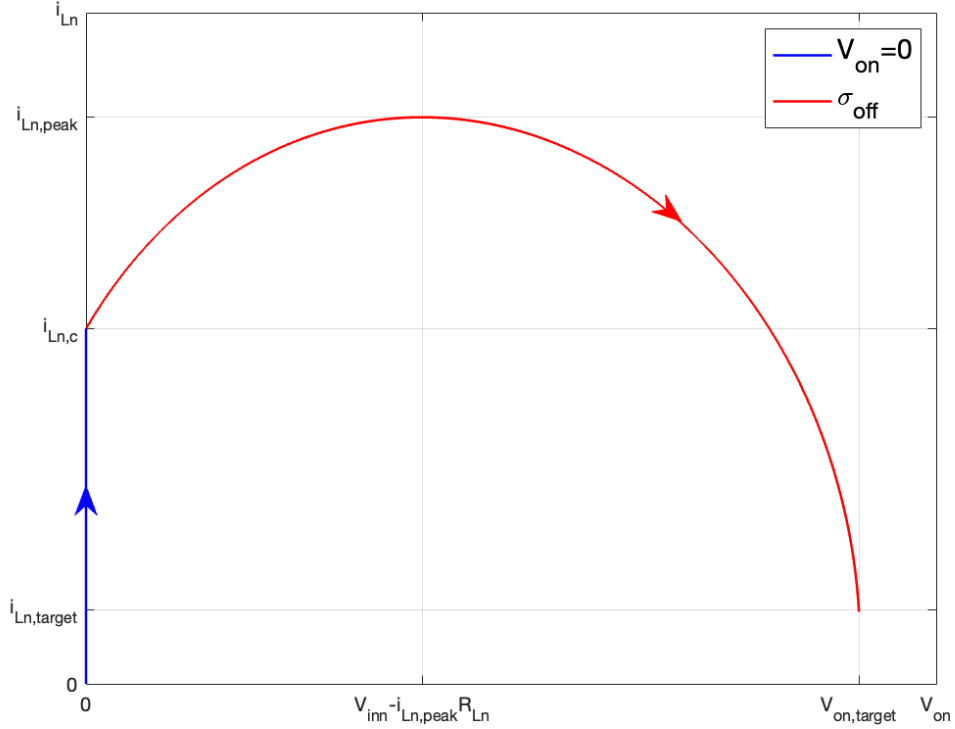


Figure 4.6: Boost converter start-up trajectories

The on-state startup trajectories intersect off-state natural switching surface σ_{off} at inductor current value of $i_{Ln,c}$, which can be found by solving the equation given below:

$$\sigma_{off}(V_{on} = 0) = 0. \quad (4.38)$$

Isolating dt_n term in the inductor current equation of (4.7) gives

$$dt_n = \frac{1}{2\pi V_{inn}} di_{Ln}. \quad (4.39)$$

By integrating the right side between 0 and $i_{Ln,c}$, the normalized on-state time during start-up can be found as:

$$t_{n,on(su)} = \int_0^{i_{Ln,c}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,c}}{2\pi V_{inn}}. \quad (4.40)$$

Similarly, dt_n term can be isolated in the output voltage equation in (4.4)

$$dt_n = \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.41)$$

Then, the normalized off-state time during start-up can be found by integrating the right side between 0 and $V_{on,target}$

$$t_{n,off(su)} = \int_0^{V_{on,target}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.42)$$

Due to the highly non-linear nature of the equations, the trapezoidal rule can be employed to evaluate the integral in (4.42) and the integrals for calculating off-state times in subsequent sections. The range of V_{on} values is determined by the limits of the integrals, which can be divided into smaller segments. Subsequently, i_{Ln} values that satisfy the corresponding off-state natural trajectory or natural switching surface equations can be calculated for each V_{on} portion, and the integrals can be evaluated using these i_{Ln} values [41].

The total normalized time during start-up can be expressed as the sum of the on-state and off-state times can be calculated as given below:

$$t_{n,su} = t_{n,on(su)} + t_{n,off(su)}. \quad (4.43)$$

4.3.2 Start-up Transients with Current Limitation

Figure 4.7 shows the start-up trajectories of a boost converter with inductor current limitation under the control law (4.33). Once again, the control law turns the switch on, and the system starts at $(0,0)$, following the $V_{on} = 0$ line until it reaches the σ_{off2} surface. Then, it moves along the surfaces until the inductor current reaches the value of $i_{Ln,lim}$. Thereafter, the system follows the $i_{Ln} = i_{Ln,lim}$ line by continuously turning the switch on and off at an infinite frequency until it reaches the off-state switching surface σ_{off} , at which point the switch turns off. Next, the system states move along σ_{off} surface and converge to the target operating point.

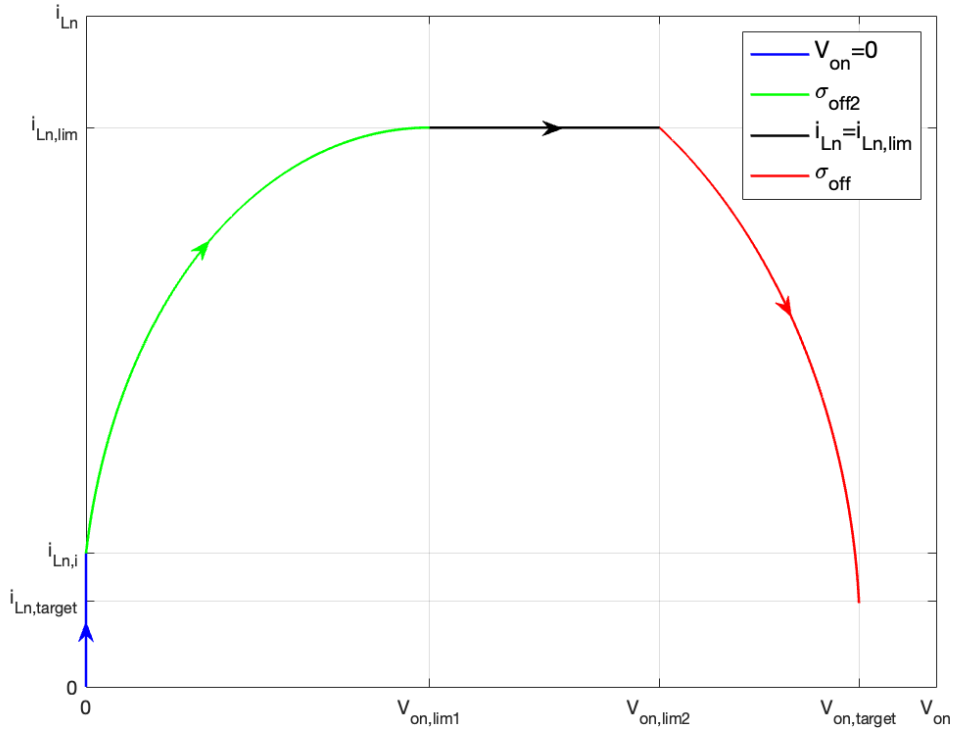


Figure 4.7: Boost converter start-up trajectories incorporating current limiting

On-state startup trajectory intersects σ_{off2} surface at inductor current value

of $i_{Ln,i}$ which can be found by solving the following equation

$$\sigma_{off2}(V_{on} = 0) = 0. \quad (4.44)$$

Then, the normalized on-state time during start-up can be found by integrating the right side between 0 and $i_{Ln,i}$ in (4.39).

$$t_{n,on(su,lim)} = \int_0^{i_{Ln,i}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,i}}{2\pi V_{inn}}. \quad (4.45)$$

$V_{on,lim1}$ point when σ_{off2} curve and the maximum allowed current line $i_{Ln,lim}$ intersects can be found by using

$$\sigma_{off2}(i_{Ln} = i_{Ln,lim}) = 0. \quad (4.46)$$

Then, the normalized off-state time during the system follows σ_{off2} curve can be calculated as:

$$t_{n,off(su,lim1)} = \int_0^{V_{on,lim1}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.47)$$

Off switching surface σ_{off2} and the $i_{Ln} = i_{Ln,lim}$ line intersects at the normalized output voltage value of $V_{on,lim2}$ which can be found by using

$$\sigma_{off2}(i_{Ln} = i_{Ln,lim}) = 0. \quad (4.48)$$

Then, the voltage change during current limiting can be found as:

$$\Delta V_{on(su,lim)} = V_{on,lim2} - V_{on,lim1}. \quad (4.49)$$

The average rate of change of the inductor current when the switch is off and on position during the current limiting region can be expressed as:

$$\Delta i_{n,offavg} = \frac{1}{\Delta V_{on(su,lim)}} \int_{V_{on,lim1}}^{V_{on,lim2}} 2\pi(V_{inn} + R_{Ln}i_{Ln,lim} + V_{on}) dV_{on}, \quad (4.50)$$

$$\Delta i_{n,onavg} = 2\pi V_{inn}.$$

The change of inductor current during the current limiting region is zero.

$$\Delta i_{n,offavg} \Delta t_{offavg} = \Delta i_{n,onavg} \Delta t_{onavg}. \quad (4.51)$$

The proportion α can be introduced to present the equation more clearly.

$$\alpha = \frac{\Delta t_{offavg}}{\Delta t_{onavg}} = \frac{\Delta i_{n,onavg}}{\Delta i_{n,offavg}}. \quad (4.52)$$

The normalized time spent during current limiting can be found as:

$$t_{n,lim(su)} = \frac{\Delta V_{on(su,lim)}}{(\frac{\alpha}{1+\alpha})2\pi(i_{Ln} - i_{on}) - (\frac{\alpha}{1+\alpha})(2\pi i_{on})}. \quad (4.53)$$

The normalized off-state time during the system follows σ_{off} curve to reach the target operating point can be expressed as:

$$t_{n,off(su,lim2)} = \int_{V_{on,lim2}}^{V_{on,target}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.54)$$

Then, the total time during start-up with a current limitation can be found as follows:

$$t_{n,su(lim)} = t_{n,on(su,lim)} + t_{n,off(su,lim1)} + t_{n,lim(su)} + t_{n,off(su,lim2)}. \quad (4.55)$$

The increase in the total start-up time with a current limitation with respect to the previous section can be found using

$$\Delta t_{n,su} = t_{n,su(lim)} - t_{n,su}. \quad (4.56)$$

(4.56) can be written more explicitly

$$\Delta t_{n,su} = \Delta t_{n,on(su)} + \Delta t_{n,off(su)} + t_{n,lim(su)}, \quad (4.57)$$

where

$$\begin{aligned} \Delta t_{n,on(su)} &= t_{n,on(su,lim)} - t_{n,on(su)}, \\ \Delta t_{n,off(su)} &= t_{n,off(su,lim1)} + t_{n,off(su,lim2)} - t_{n,off(su)}. \end{aligned} \quad (4.58)$$

(4.58) can be rewritten as:

$$\begin{aligned}\Delta t_{n,on(su)} &= \int_{i_{Ln,c}}^{i_{Ln,i}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,i} - i_{Ln,c}}{2\pi V_{inn}}, \\ \Delta t_{n,off(su)} &= \int_{V_{on,target}}^{V_{on,lim1}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on} + \int_{V_{on,lim2}}^{V_{on,target}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}.\end{aligned}\tag{4.59}$$

The off-state time integrals in (4.59) can be numerically solved using the trapezoidal rule outlined earlier. The range of V_{on} values is defined by the integral limits, which can be broken down into smaller intervals. For each segment of V_{on} , i_{Ln} values that satisfy the off-state natural switching surface equations can be computed, and the integrals can then be evaluated using these i_{Ln} values. To repeat, This method can be utilized for all off-state time integrals discussed throughout the chapter.

Although on-state and off-state times are reduced, the total start-up time is increased because of the time spent during limiting current. The difference in the total start-up time increases with decreasing $i_{Ln,lim}$ value.

4.3.3 Load Transients

4.3.3.1 Loading Transient

Figure 4.8 shows the loading trajectories of the boost converter under the control law (4.29). When an increase in the load current occurs, the control law activates the switch, starting the states at $(i_{Ln,initial}, V_{on,target})$, which is the initial target operating point that the system maintains before the load current increases to $i_{on,l}$. The system follows the on-state natural trajectory $\lambda_{on,l}$ until the off-state switching surface $\sigma_{off,l}$ which crosses the new target point $(i_{Ln,target(l)}, V_{on,target})$ is hit. Then, the switch is turned off, and the system states moves along $\sigma_{off,l}$ surface and reaches the target operating point with one switching action. The on-state natural trajectory in loading transient $\lambda_{on,l}$ can be derived using $i_{Ln}(0) =$

$i_{Ln,initial}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,l}$ in (4.20).

$$\lambda_{on,l} = V_{on} + \frac{i_{on,l}}{V_{inn}}(i_{Ln} - i_{Ln,initial}) - 1. \quad (4.60)$$

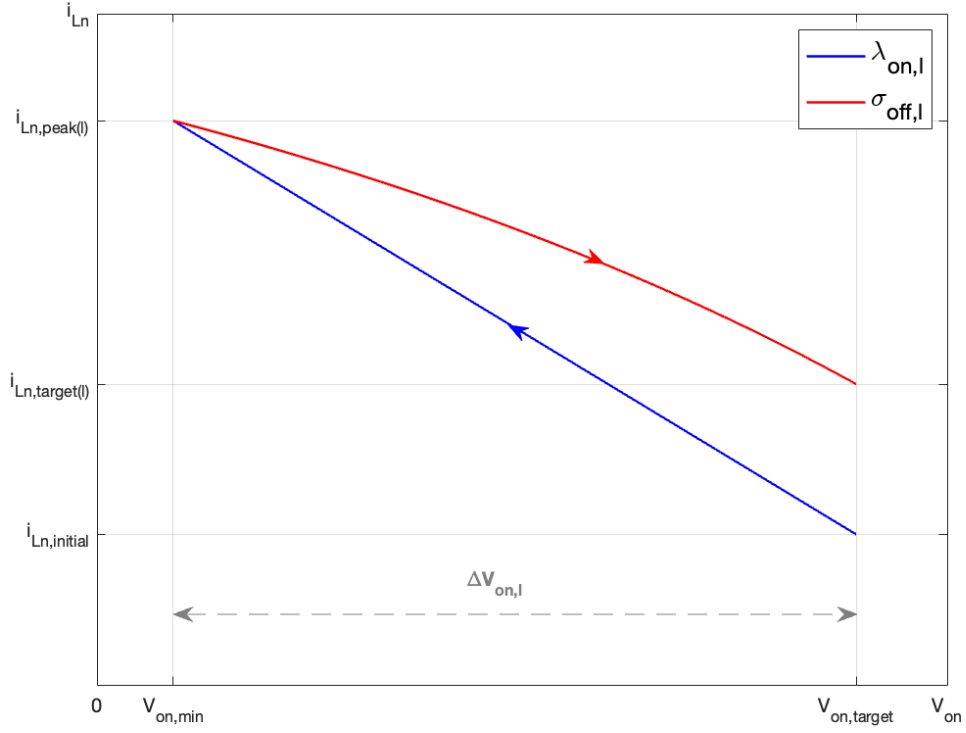


Figure 4.8: Boost converter loading trajectories

The new inductor current value at the target operating point can be calculated with target load current $i_{on,l}$.

$$i_{Ln,target(l)} = \frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on,l}R_{Ln}}}{2R_{Ln}}. \quad (4.61)$$

Using $i_{Ln}(0) = i_{Ln,target(l)}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,l}$ in (4.11) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on,l} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on,l} + 1) - \frac{1}{\beta}(i_{Ln,target(l)} - i_{on,l}). \end{aligned} \quad (4.62)$$

The revised $r_{off,0}$ and $\theta_{off}(0)$, represented by $r_{off,l0}$ and $\theta_{off,l}(0)$ can be obtained by using (4.62) in (4.12), then the new off-state natural switching surface denoted by $\sigma_{off,l}$ can be derived and written as

$$\sigma_{off,l} = r_{off}^2 - r_{off,l0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off,l}(0) - \theta_{off})}. \quad (4.63)$$

The maximum inductor current $i_{Ln,peak(l)}$ and the minimum output voltage $V_{on,min}$ on the loading trajectories occurs when the on-state loading trajectory hits the off-state natural switching surface. They can be calculated by using

$$\lambda_{on,l} = \sigma_{off,l}. \quad (4.64)$$

The increase in the inductor current during loading transient can be found as:

$$\Delta i_{Ln,l} = i_{Ln,peak(l)} - i_{Ln,initial}. \quad (4.65)$$

The output voltage drop in the loading transient can be calculated as:

$$\Delta V_{on,l} = 1 - V_{on,min}. \quad (4.66)$$

Then, the normalized on-state time during loading transient can be found by integrating the right side between $i_{Ln,initial}$ and $i_{Ln,peak(l)}$ in (4.39)

$$t_{n,on(l)} = \int_{i_{Ln,initial}}^{i_{Ln,peak(l)}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,peak(l)} - i_{Ln,initial}}{2\pi V_{inn}}. \quad (4.67)$$

Then, the normalized off-state time during the system follows σ_{off} curve can be calculated by integrating the right side between $V_{on,min}$ and $V_{on,target}$ in (4.41).

$$t_{n,off(l)} = \int_{V_{on,min}}^{V_{on,target}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.68)$$

The total normalized time during loading can be expressed as the sum of the on-state and off-state times as given below:

$$t_{n,l} = t_{n,on(l)} + t_{n,off(l)}. \quad (4.69)$$

4.3.3.2 Loading Transient with Current Limitation

Figure 4.9 shows the loading trajectories of the boost converter under the control law (4.33). The control law turns the switch on when the load current increases. Then, the states start at initial target operating point $(i_{Ln,initial}, V_{on,target})$ and follow the on-state natural trajectory $\lambda_{on,l}$ until the maximum allowed inductor current value $i_{Ln,lim}$ is reached. Thereafter, the system stays at the $i_{Ln} = i_{Ln,lim}$ line until it reaches the off-state switching surface $\sigma_{off,l}$ by turning the switch on and off continuously at an infinite frequency. Next, the switch is turned off, and the system moves along σ_{off} surface and converges to the new target operating point.

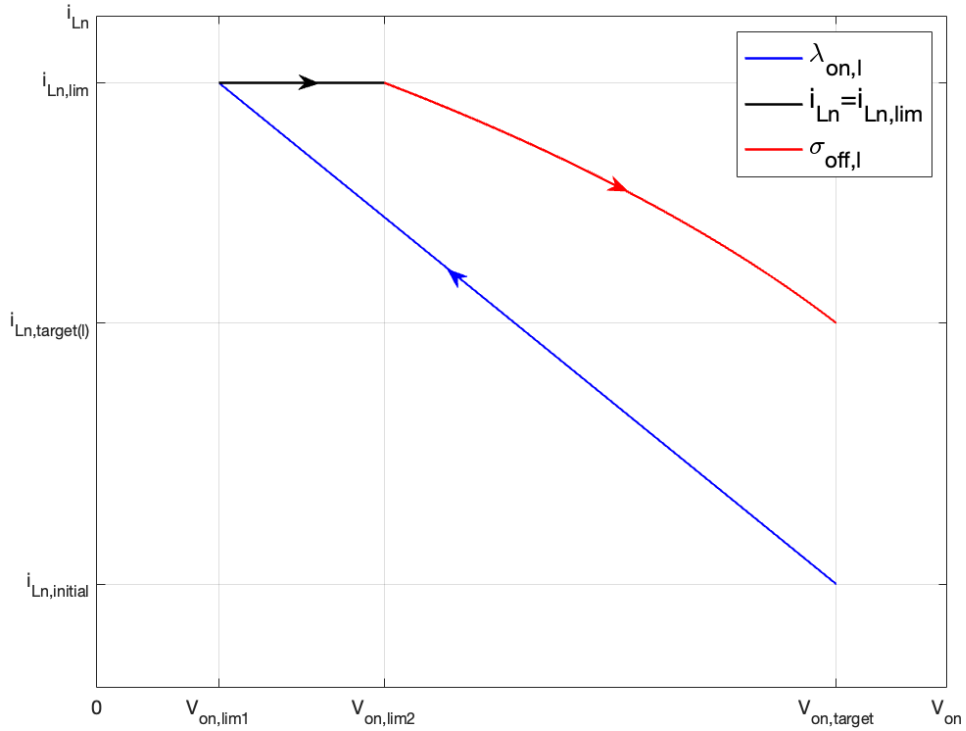


Figure 4.9: Boost converter loading trajectories incorporating current limiting

The normalized output voltage value $V_{on,lim1}$ where on-state natural trajectory and maximum allowed current $i_{Ln,lim}$ line intersects is the minimum output

voltage during loading transient, which can be found by using

$$V_{on,lim1} = 1 - \frac{i_{on,l}}{V_{inn}}(i_{Ln,lim} - i_{Ln,initial}). \quad (4.70)$$

Then, the voltage drop in the loading transient can be expressed as

$$\Delta V_{on,l} = 1 - V_{on,lim1}. \quad (4.71)$$

The off-state switching surface crosses $i_{Ln,lim}$ line at the normalized output voltage value of $V_{on,lim2}$ which can be found by using

$$\sigma_{off,l}(i_{Ln} = i_{Ln,lim}) = 0. \quad (4.72)$$

Then, the voltage change during current limiting can be found as

$$\Delta V_{on(l,lim)} = V_{on,lim2} - V_{on,lim1}. \quad (4.73)$$

The normalized on-state time during loading transient can be found by integrating the right side between $i_{Ln,initial}$ and $i_{Ln,lim}$ in (4.39)

$$t_{n,on(l,lim)} = \int_{i_{Ln,initial}}^{i_{Ln,lim}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,lim} - i_{Ln,initial}}{2\pi V_{inn}}. \quad (4.74)$$

The average rate of change of the inductor current when the switch is off and on position during the current limiting region can be found as follows:

$$\begin{aligned} \Delta i_{n,offavg} &= \frac{1}{\Delta V_{on(l,lim)}} \int_{V_{on,lim1}}^{V_{on,lim2}} 2\pi(V_{inn} + R_{Ln}i_{Ln,lim} + V_{on}) dV_{on}, \\ \Delta i_{n,onavg} &= 2\pi V_{inn}. \end{aligned} \quad (4.75)$$

The average change of inductor current during the current limiting region is zero.

$$\Delta i_{n,offavg} \Delta t_{offavg} = \Delta i_{n,onavg} \Delta t_{onavg}. \quad (4.76)$$

The proportion α can be included to improve the clarity of the equation.

$$\alpha = \frac{t_{offavg}}{t_{onavg}} = \frac{\Delta i_{n,onavg}}{\Delta i_{n,offavg}}. \quad (4.77)$$

The normalized time spent in the current limited region can be found as

$$t_{n,lim(l)} = \frac{\Delta V_{on(l,lim)}}{(\frac{\alpha}{1+\alpha})2\pi(i_{Ln} - i_{on}) - (\frac{\alpha}{1+\alpha})(2\pi i_{on})}. \quad (4.78)$$

The normalized off-state time during the system follows σ_{off} curve can be calculated by integrating the right side between $V_{on,lim2}$ and $V_{on,target}$ in (4.41).

$$t_{n,off(l,lim)} = \int_{V_{on,lim2}}^{V_{on,target}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.79)$$

Then, the total time during loading with current limiting is can found as

$$t_{n,l(lim)} = t_{n,on(l,lim)} + t_{n,lim(l)} + t_{n,off(l,lim)}. \quad (4.80)$$

The increase in the total time with current limiting with respect to the previous section can be expressed as

$$\Delta t_{n,l} = \Delta t_{n,on(l)} + \Delta t_{n,off(l)} + t_{n,lim(l)}, \quad (4.81)$$

where

$$\Delta t_{n,on(l)} = t_{n,on(l,lim)} - t_{n,on(l)}, \quad (4.82)$$

$$\Delta t_{n,off(l)} = t_{n,off(l,lim)} - t_{n,off(l)}.$$

(4.82) can be rewritten as

$$\begin{aligned} \Delta t_{n,on(l)} &= \int_{i_{Ln,peak(l)}}^{i_{Ln,lim}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,lim} - i_{Ln,peak(l)}}{2\pi V_{inn}}, \\ \Delta t_{n,off(l)} &= \int_{V_{on,lim2}}^{V_{on,min}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \end{aligned} \quad (4.83)$$

Although on-state and off-state times are reduced, the total loading time is increased because of the time spent during limiting current. The difference in the loading time increases with decreasing $i_{Ln,lim}$ value.

4.3.3.3 Unloading Transient

Unloading trajectories of the boost converter under the control law (4.29) are shown in Figure 4.10. When a decrease in the load current takes place, the control law turns the switch off; the states start at initial target operating point $(i_{Ln,initial}, V_{on,target})$, where the system remained before the load current decreased to $i_{on,ul}$. The states follow the off-state natural trajectory $\lambda_{off,ul}$ until the off-state natural switching surface $\sigma_{on,ul}$ is reached. Then, the switch is turned on by the control law, the system states moves along $\sigma_{on,ul}$ surface and reaches the new target operating point $(i_{Ln,target(ul)}, V_{on,target})$.

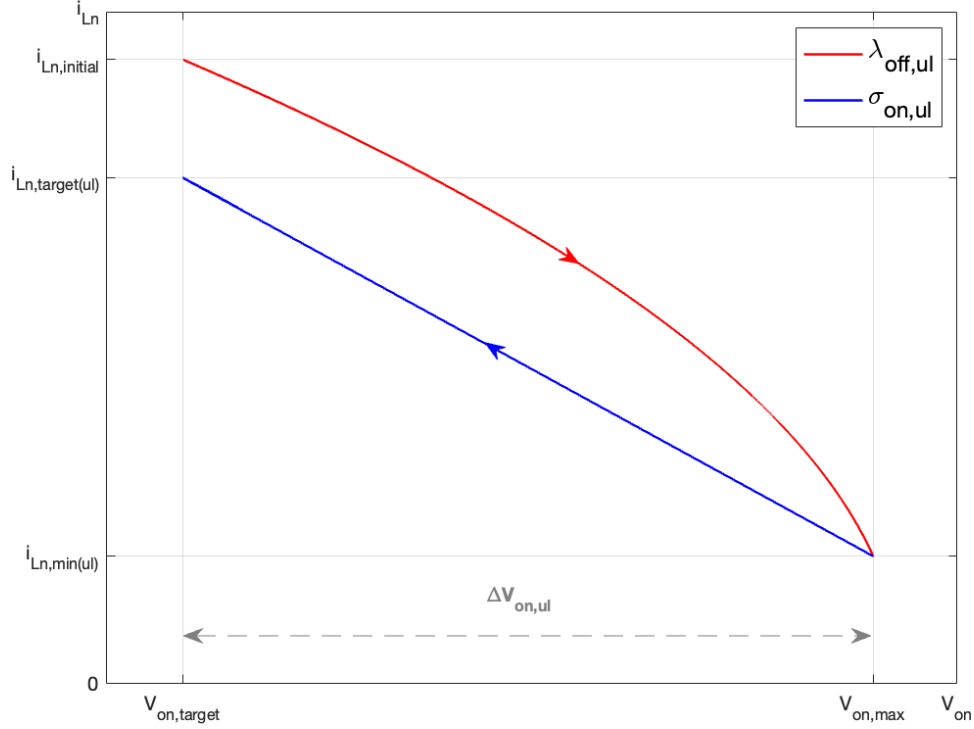


Figure 4.10: Boost converter unloading trajectories

Using $i_{Ln}(0) = i_{Ln,initial}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,ul}$ in (4.11)

gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on,ul} + 1), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on,ul} + 1) - \frac{1}{\beta}(i_{Ln,initial} - i_{on,ul}). \end{aligned} \quad (4.84)$$

The revised $r_{off,0}$ and $\theta_{off}(0)$, represented by $r_{off,ul0}$ and $\theta_{off,ul}(0)$ can be obtained by using (4.84) in (4.4), then the off-state natural trajectory denoted by $\lambda_{off,ul}$ can be derived and written as:

$$\lambda_{off,ul} = r_{off}^2 - r_{off,ul0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off,ul}(0) - \theta_{off})}. \quad (4.85)$$

The new target inductor current value at the target operating point can be calculated with target load current $i_{on,ul}$

$$i_{Ln,target(ul)} = \frac{V_{inn} - \sqrt{V_{inn}^2 - 4i_{on,ul}R_{Ln}}}{2R_{Ln}}. \quad (4.86)$$

The on-state natural switching line in loading transient $\sigma_{on,l}$ can be derived using $i_{Ln}(0) = i_{Ln,target(ul)}$, $V_{on}(0) = 1$ and substituting i_{on} with $i_{on,ul}$ in (4.20)

$$\sigma_{on,l} = V_{on} + \frac{i_{on,ul}}{V_{inn}}(i_{Ln} - i_{Ln,target(ul)}) - 1. \quad (4.87)$$

The off-state unloading trajectory hits the on-state natural switching surface on the point where the maximum output voltage $V_{on,max}$ and the minimum inductor current $i_{Ln,min(ul)}$ occurs, which can be calculated by using

$$\lambda_{off,ul} = \sigma_{on,ul}. \quad (4.88)$$

Then, the output voltage rise in the unloading transient can be calculated as:

$$\Delta V_{on,ul} = V_{on,max} - 1. \quad (4.89)$$

The decrease in the inductor current during the unloading transient can be found as follows:

$$\Delta i_{Ln,ul} = i_{Ln,initial} - i_{Ln,min(ul)}. \quad (4.90)$$

The normalized off-state time during the system follows σ_{off} curve can be calculated by integrating the right side between $V_{on,target}$ and $V_{on,max}$ in (4.41)

$$t_{n,off(ul)} = \int_{V_{on,target}}^{V_{on,max}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.91)$$

Then, the normalized on-state time during loading transient can be found by integrating the right side between $i_{Ln,min(ul)}$ and $i_{Ln,target(ul)}$ in (4.39).

$$t_{n,on(ul)} = \int_{i_{Ln,min(ul)}}^{i_{Ln,target(ul)}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,target(ul)} - i_{Ln,min(ul)}}{2\pi V_{inn}}. \quad (4.92)$$

The total normalized time during loading can be expressed as the sum of the on-state and off-state times as given below:

$$t_{n,ul} = t_{n,on(ul)} + t_{n,off(ul)}. \quad (4.93)$$

4.4 Steady State Analysis

Ideally, the switch turns on and off at an infinite frequency to maintain the trajectories at the equilibrium point, leading to zero ripple. Practically, this is not feasible; steady-state trajectories would contain uncertain high-frequency switching, leading to uncontrollable ripple [7]. Incrementing the radius $r_{off,0}^2$ of the off-state natural switching surface σ_{off} according to (4.94) enables the achievement of a controllable ripple with a finite switching frequency, as outlined in [7] and [41].

$$r_{off\Delta,0}^2 = r_{off,0}^2 + \Delta r^2. \quad (4.94)$$

By using (4.94) in (4.12) modified off-state natural switching surface $\sigma_{off\Delta}$ can be obtained as:

$$\sigma_{off\Delta} = r_{off}^2 - r_{off\Delta,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off}(0) - \theta_{off})}. \quad (4.95)$$

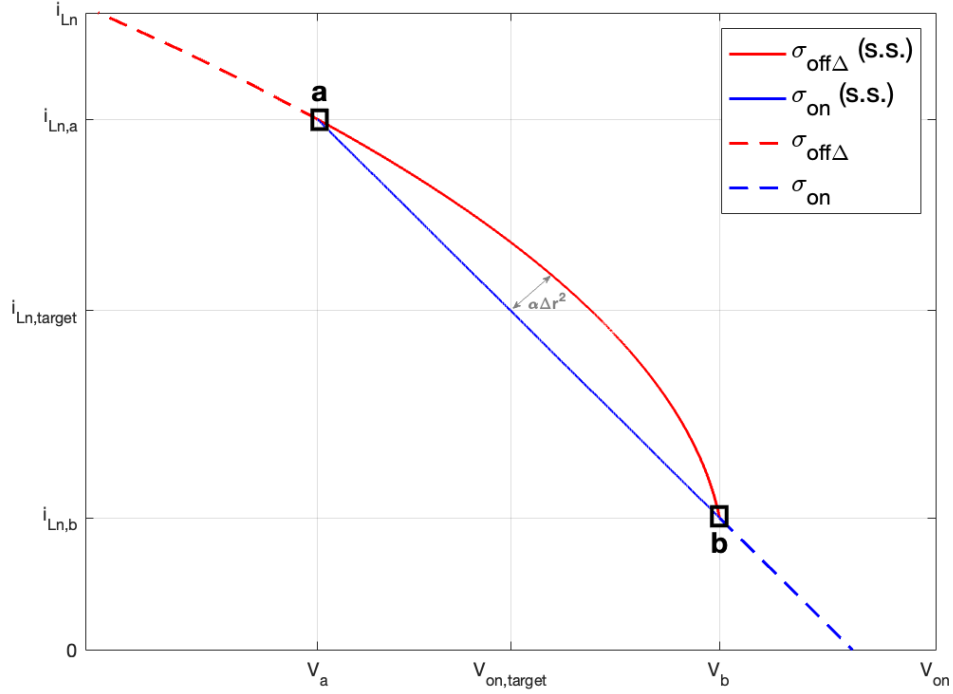


Figure 4.11: Boost converter steady-state trajectories

Figure 4.11 shows the steady-state trajectories of the boost converter when either control law (4.29) or (4.33) is used with (4.95). The states cycle between a and b during steady state operation. At point a the switch turns off, the states follow $\sigma_{off\Delta}$ curve and reach b. At point b, the switch turns on, and the system follows σ_{on} line and reaches point a again, completing one switching cycle. For other parameters fixed, the points a and b, thus switching frequency and ripples, are determined by Δr^2 . Lower ripples can be achieved by decreasing Δr^2 , resulting in higher switching frequency.

4.4.1 Ripple Calculations

Employing a method analogous to the one outlined in [41], the normalized inductor current and output voltage steady-state ripples, represented by $\Delta i_{Ln,ss}$ and

$\Delta V_{on,ss}$ can be calculated by using the inductor current and output voltage values at points a and b. At point b, $V_{on,b}$ and $i_{Ln,b}$ can be found using

$$\sigma_{on} = \sigma_{off\Delta} \quad \text{for} \quad i_{Ln} < i_{Ln,target} \text{ and } V_{on} > 1. \quad (4.96)$$

Likewise, $V_{on,a}$ and $i_{Ln,a}$ can be found by using

$$\sigma_{on} = \sigma_{off\Delta} \quad \text{for} \quad i_{Ln} > i_{Ln,target} \text{ and } V_{on} < 1. \quad (4.97)$$

The solutions of equations (4.96) and (4.97) can be found by solving them numerically via the Newton-Raphson method. The normalized inductor current ripple at steady-state can be expressed as:

$$\Delta i_{Ln,ss} = i_{Ln,a} - i_{Ln,b}. \quad (4.98)$$

The normalized output voltage ripple at steady-state can be found as:

$$\Delta V_{on,ss} = V_{on,b} - V_{on,a}. \quad (4.99)$$

The relationship between the normalized output voltage ripple and inductor current ripple at the steady state can be given using (4.19) as:

$$\Delta V_{on,ss} = \frac{i_{on}}{V_{inn}} \Delta i_{Ln,ss}. \quad (4.100)$$

4.4.2 Frequency Calculation

The normalized on-state time during steady state can be found by integrating the right side between $i_{Ln,b}$ and $i_{Ln,a}$ in (4.39)

$$t_{n,on(ss)} = \int_{i_{Ln,b}}^{i_{Ln,a}} \frac{1}{2\pi V_{inn}} di_{Ln} = \frac{i_{Ln,a} - i_{Ln,b}}{2\pi V_{inn}}. \quad (4.101)$$

The normalized off-state time during the system follows $\sigma_{off\Delta}$ curve in steady state can be calculated by integrating the right side between $V_{on,a}$ and $V_{on,bx}$ in

(4.41)

$$t_{n,off(ss)} = \int_{V_{on,a}}^{V_{on,b}} \frac{1}{2\pi(i_{Ln} - i_{on})} dV_{on}. \quad (4.102)$$

The normalized switching frequency in the steady state can be found by using on-state and off-state times during one switching cycle

$$f_{n,ss} = \frac{1}{t_{n,on(ss)} + t_{n,off(ss)}}. \quad (4.103)$$

4.5 Controller Design

The work in [41] serves as a model for the investigations in this section and the next. An algorithm is used to establish the design parameters, and simulations—both ideal and practical—are performed with these parameters. The study examines a boost converter model with a constant current load, incorporating inductor DCR in the off-state. The simulations extend to evaluate current limitations during startup and under transient loading conditions. An example design of a controller for the boost converter with the requirements given in Table 4.1 is presented in this section. The goal is the same as that of the buck converter, which is to determine the parameters C and Δr^2 for the chosen inductor and its DCR so that design requirements are met. For the example design, the maximum steady-state output voltage ripple is selected as

$$\frac{\Delta V_{o,ss}}{V_{ref}} \times 100 = 3\% \quad (4.104)$$

of the reference voltage and the maximum ripple for inductor current at the steady state is selected as:

$$\frac{\Delta i_{L,ss}}{i_{L,target}} \times 100 = 19.3546\% \quad (4.105)$$

of the target DC inductor current.

Boost converter design parameters are determined by using Algorithm 2 given in Appendix A. The algorithm is nearly identical to the one employed in the

Parameters	Value
V_{in}	3.3 V
V_{ref}	10 V
i_o	3 A
Maximum $\Delta V_{on,ss}$	0.3 V
Maximum $\Delta i_{Ln,ss}$	2 A
Maximum f_{ss}	14kHz

Table 4.1: Boost converter parameters

example design of the buck converter. The only distinction is that the equations provided for the boost converter in this chapter are used to calculate the parameters. The outputs of the algorithm for chosen $L = 82\mu H$, $R_L = 0.0384\Omega$ and the design requirements are obtained as:

$$\begin{aligned}\Delta r^2 &= 1.47 \times 10^{-5}, \\ C &= 512.5\mu F.\end{aligned}\tag{4.106}$$

4.5.1 Simulations

LTspice software is used to simulate the boost converter with the design parameters that have been determined. Similar to the case of the buck converter, simulations are conducted in this section to verify the theory by employing ideal components and nearly ideal switches. Outcomes with components exhibiting non-ideal properties will be addressed in the subsequent section.

Figure 4.12 displays the schematic used for the simulation of the boost converter. The hierarchical block implements the analytical expressions of the control law (4.29) and (4.33). The netlist of the controller circuit is available in Appendix B.2. The controller receives the input voltage, reference voltage, the value at which the inductor current will be restricted, and the feedback signals from the circuit, which include the output voltage, inductor current, and load current. It generates a control signal u that controls the activation of the switches. One switch will be active, while the other will be inactive.

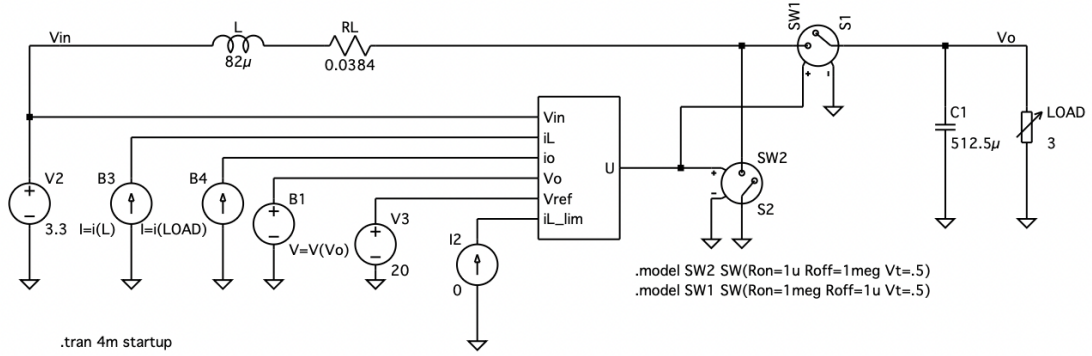


Figure 4.12: Boost converter schematic with ideal components

The steady-state simulation results for V_o and i_L are shown in Figure 4.13. Peak-to-peak ripples and f_{ss} can be determined using the markings on the plot as follows:

$$\begin{aligned}\Delta V_{o,ss} &= 10.2601 - 9.9732 = 0.2869 \text{ V}, \\ \Delta i_{L,ss} &= 4.3462572 - 1.6614806 = 1.9722059 \text{ A}, \\ f_{ss} &= \frac{1}{(1.4112393 - 1.3397421)} = 13.98656171 \text{ kHz}.\end{aligned}\tag{4.107}$$

The results show that the algorithm operates appropriately; the results are suitable for the maximum limits of $\Delta V_{o,ss}$, $\Delta i_{L,ss}$ and f_{ss} in Table 4.1.

The start-up transient simulation of the boost converter is displayed in Figure 4.14. The converter is initiated when the input voltage is applied at $100\mu s$. It is evident that the output voltage reaches the reference value with one switching action without any overshoot. The peak inductor current and the total time spent during start-up can be expressed as

$$\begin{aligned}i_{L,peak} &= 23.01943 \text{ A}, \\ t_{su} &= 961.883 - 100 = 861.883 \mu s.\end{aligned}\tag{4.108}$$

As in the case of the buck converter, it is observed that the peak inductor current is substantially higher during the start-up phase than it is in its steady state. This can be a challenge in practical applications due to the limited current handling capacity of inductors and switches. The control law (4.25) can be

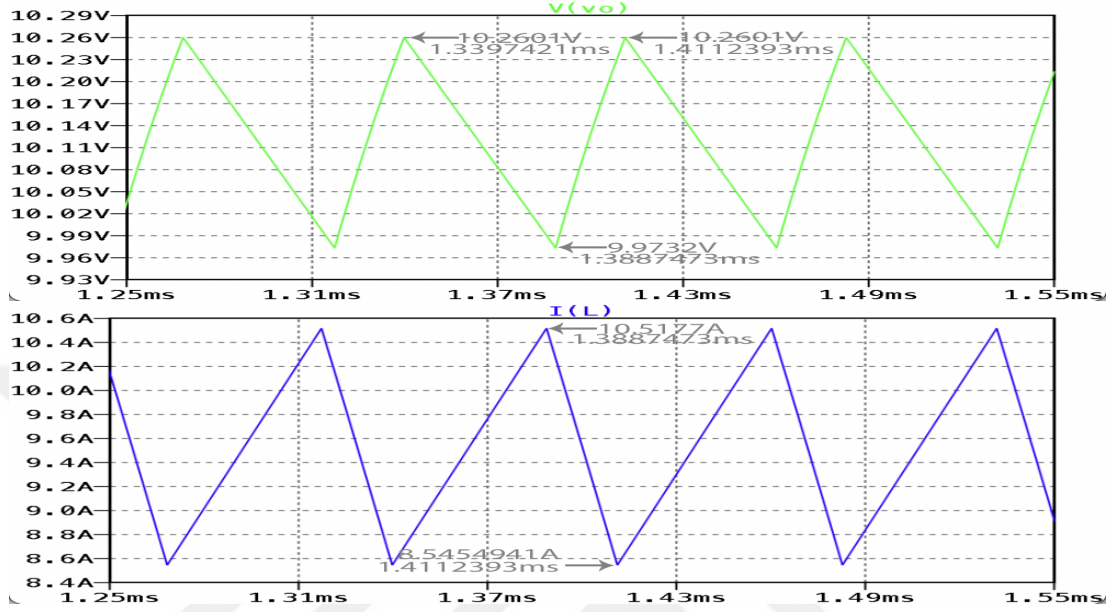


Figure 4.13: Ideal boost converter steady-state simulation results

employed to restrict the inductor current to the desired value in order to resolve this issue.

The start-up transient simulation of a boost converter that incorporates current limiting is depicted in Figure 4.15. The control law (4.33) is employed to restrict the inductor current to 15A during start-up. The total start-up time can be determined as

$$t_{su(lim)} = 1.3277809 - 0.1 = 1.2277809 \text{ ms}. \quad (4.109)$$

The start-up process is slowed by limiting inductor current. However, this is a necessary trade-off if inductors and switches are not capable of managing sufficient current. Figure 4.16 illustrates the dynamic response of the boost converter to unloading and loading transients. At 1.390598ms, the load current is reduced to 2.5A from 3A, and at 1.785608ms, it is increased to 3.5A from 2.5A. Using the data markings on the plot, it is possible to calculate the recovery times, output voltage change, and maximum and minimum inductor currents during loading and unloading transients.

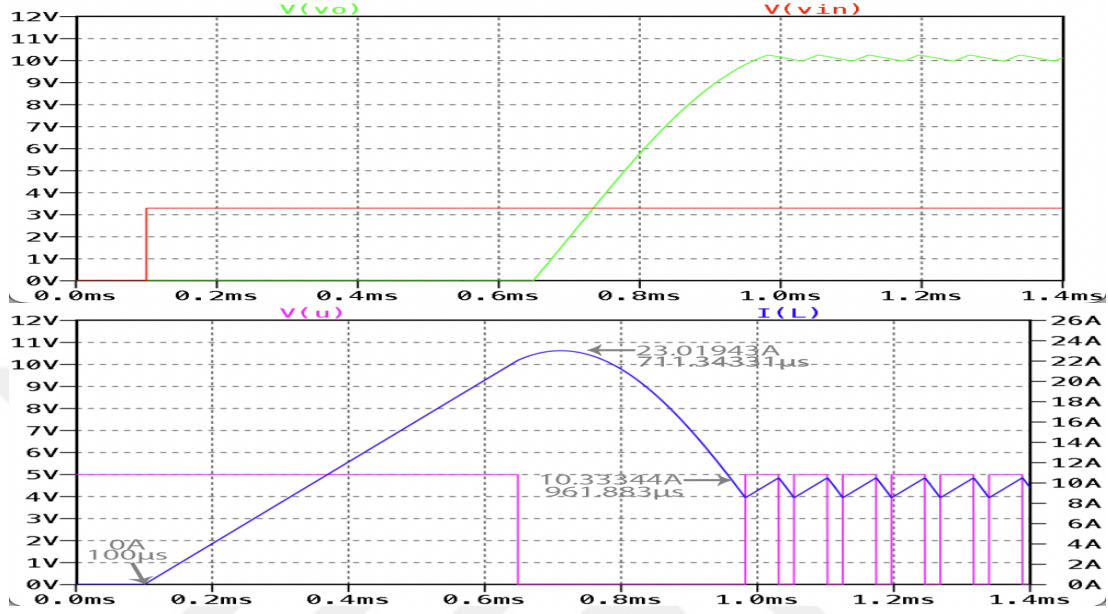


Figure 4.14: Ideal boost converter start-up simulation results

For unloading transient:

$$\begin{aligned}
 \Delta V_{o,ul} &= 10.6722 - 10 = 0.6722 \text{ V}, \\
 i_{L,min(ul)} &= 2.85077 \text{ A}, \\
 t_{ul} &= 1611.6539 - 1390.598 = 221.0559 \mu s.
 \end{aligned} \tag{4.110}$$

For loading transient:

$$\begin{aligned}
 \Delta V_{o,l} &= 10 - 8.35752 = 1.64248 \text{ V}, \\
 i_{L,peak(l)} &= 18.075 \text{ A}, \\
 t_l &= 2097.17 - 1785.608 = 311.562 \mu s.
 \end{aligned} \tag{4.111}$$

It is evident that the current can surpass the maximum capacity of inductors and switches in practical scenarios during loading. Figure 4.17 depicts the simulation of the loading transient of the boost converter that includes current limiting. Once again, the inductor current is restricted to a maximum of 15A by employing control law (4.33). Total loading time can be calculated as

$$t_{l(lim)} = 2796.32 - 2437.5418 = 358.7782 \mu s. \tag{4.112}$$

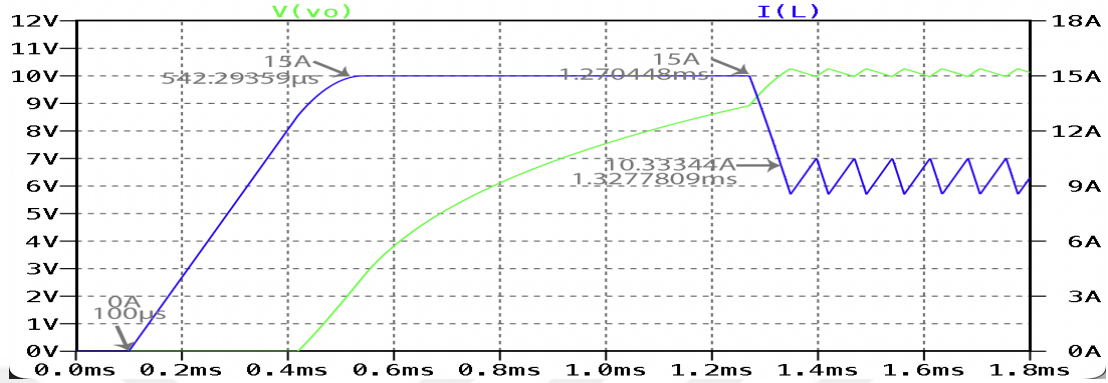


Figure 4.15: Ideal boost converter start-up with inductor current limitation simulation results

As expected, the loading time increases due to current limitation. The inductor current is limited at the cost of a little bit slower loading transient.

Table 4.2 summarizes the ideal simulation results from (4.107)-(4.112). Similar to [41], the table also provides theoretical calculations performed by solving related equations given in earlier sections under the same conditions. The errors between theoretical calculations and ideal simulation results are shown in a separate column. For all boost converter performance measures, errors are too minor, as expected, since simulations are conducted using ideal component models.

4.6 Practical Considerations

Similar to the buck converter, there are four main issues that can cause differences between theory and practice for the boost converter. One of the factors to consider is the presence of additional resistances in the circuit. The second factor is the variability in component values due to their tolerances and operating conditions. The third issue is the delay in control action due to the restricted bandwidth of the controller. The last factor is a high switching frequency when limiting the inductor current. Due to the fact that these concerns are identical to those that were raised in the buck converter case, and because they have already been

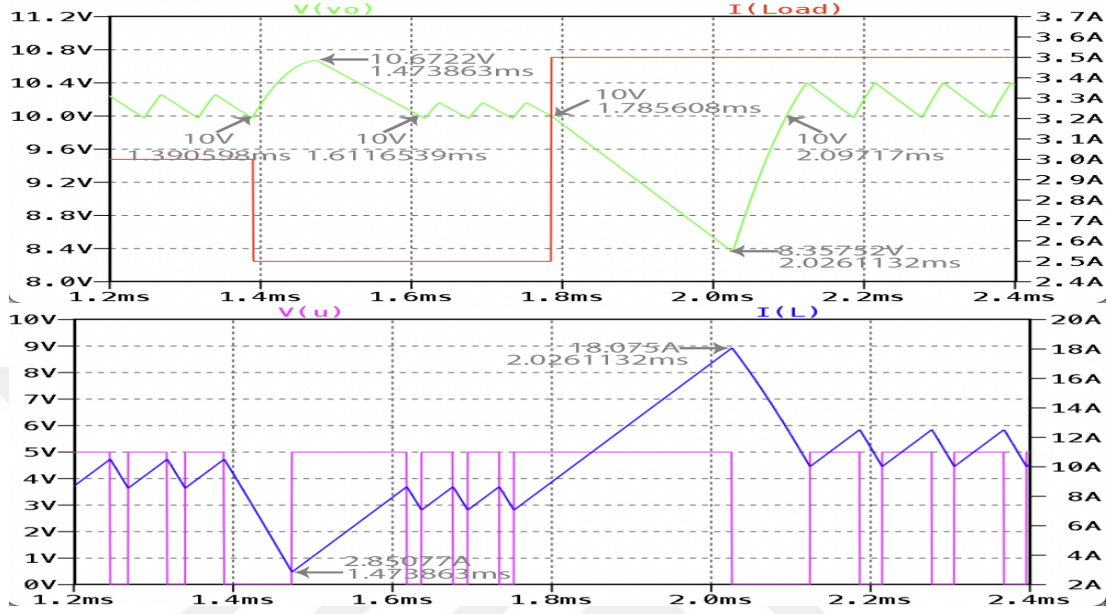


Figure 4.16: Ideal boost converter unloading and loading simulation results

described in Section 3.6, detailed explanations are not provided here.

Figure 4.18 depicts the schematic of the boost converter that is utilized for the purpose of conducting realistic simulations. With this setup, to correctly realize the switch-on and switch-off scenarios, the output of the controller block, represented by u in the schematic, is set to 0 when the control law indicates a switch-on condition and to 1 in all other cases. The differences that separate realistic simulations from the ideal can be listed below:

1. $82 \mu H$ inductor is derated to $66 \mu H$ for its target RMS current value. Unlike [41], this study also includes the consideration of inductor derating.
2. $10 m\Omega$ sense resistors R_{sense1} and R_{sense2} that can be used for measuring inductor and load current are added.
3. 10 units of $47 \mu F$, 1 unit of $33 \mu F$, 1 unit of $3.3 \mu F$, 2 units of $2.2 \mu F$ and 1 unit of $1.8 \mu F$ total $512.5 \mu F$ ceramic capacitors are used as output capacitors in order to ensure that the total capacitance is nearly identical to the calculated value. The capacitances are derated for target output

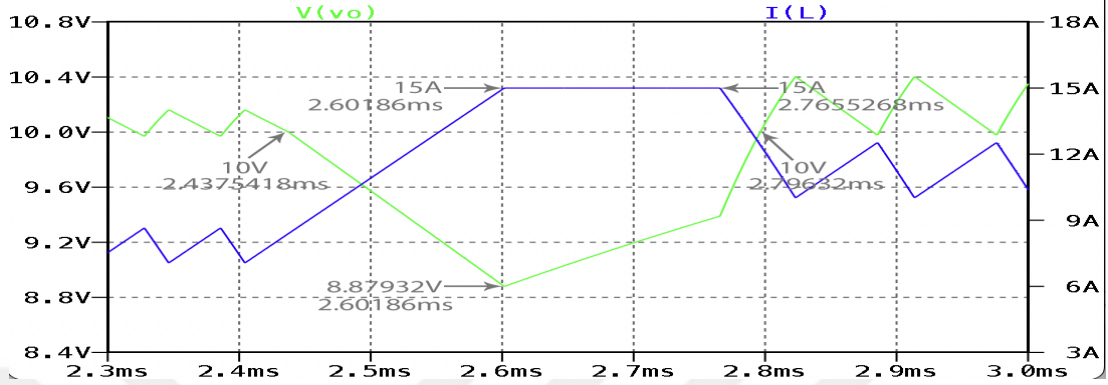


Figure 4.17: Ideal boost converter loading transient with current limiting simulation results

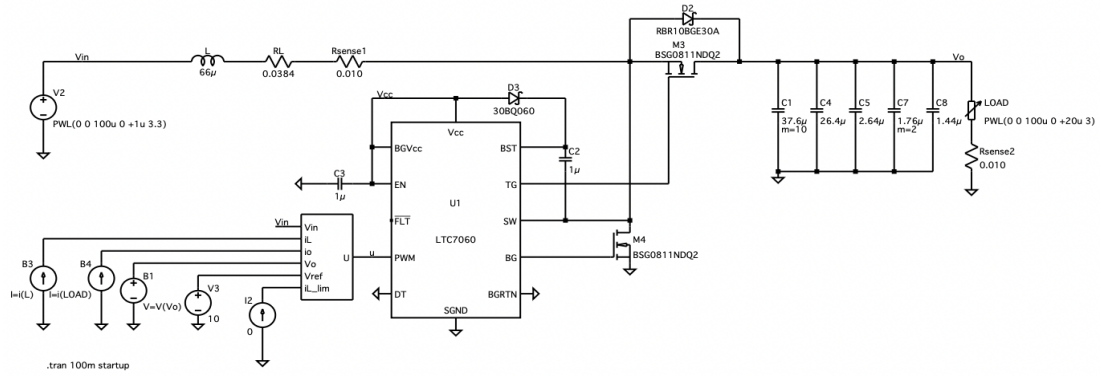


Figure 4.18: Boost converter circuit schematic with realistic components

voltage, and their ESRs are added to the schematic. In order to mitigate the effects of ESR, which is 0.333Ω in total, smaller capacitors were paralleled in place of a single large capacitor.

4. NMOS transistors M1 and M2 replaced the switches in the ideal schematic. Their on-state resistances are $0.7\text{ m}\Omega$.
5. Since the controller may not be able to provide the appropriate voltage and current levels to quickly turn transistors on or off in practice, a half-bridge driver integrated circuit is employed to drive the transistors in accordance with the control signal u in a proper manner.
6. The bandwidth of the controller is restricted to 1 MHz . The controller is

Parameters	Theoretical Values	Ideal Simulation Results	Errors
$\Delta V_{o,ss}$	0.2868603V	0.2869 V	0.01383%
$\Delta i_{L,ss}$	1.972119	1.9722059 A	0.00441%
f_{ss}	13.98602 kHz	13.98656 kHz	0.00386%
$i_{L,peak}$	23.0194 A	23.01943 A	0.00013%
$\Delta V_{o,ul}$	0.672151 V	0.6722 V	0.00729%
$i_{L,min(ul)}$	2.8507705 V	2.85077 V	0.00175%
$\Delta V_{o,l}$	1.6424799 V	1.64248 V	0.00001%
$i_{L,peak(l)}$	18.074951 A	18.075 A	0.00027%
t_{su}	861.88247 μs	861.883 μs	0.00006%
$t_{su(lim)}$	1.22778 ms	1.2277809 ms	0.00007%
t_{ul}	221.05571 μs	221.0559 μs	0.00009%
t_l	311.557 μs	311.562 μs	0.00151%
$t_{l(lim)}$	358.7738 μs	358.7782 μs	0.00123%

Table 4.2: Comparison of theoretical values and ideal simulation results of boost converter performance measures

presumed to solve the natural switching surface equations and apply the control law in a time of 1 μs .

The simulation results for steady-state, start-up, start-up with current limitation, load transients, and loading transient with current limitation of a realistic boost converter are shown in Figures 4.19, 4.20, 4.21 4.22 and 4.23 respectively. Similar to what is done in [41], performance measures of realistic boost converter are computed and compiled in Table 4.3 with data marks on these graphs. Additionally shown in another column in this table are percentage deviations between the theoretical calculations and the realistic simulation results. The table also includes the efficiency (η) of the converter.

Similar to [41], limited controller bandwidth causes $\Delta V_{o,ss}$ and $\Delta i_{L,ss}$ to vary for every cycle in the steady state as seen in Figure 4.19. It is necessary to

highlight that the comparison in Table 4.3 is just aimed to offer a concept on the degree of inaccuracy between theory and practice. The data in the table cannot be considered as exact values since most of the simulation results depend on from which switching cycle the data is obtained.

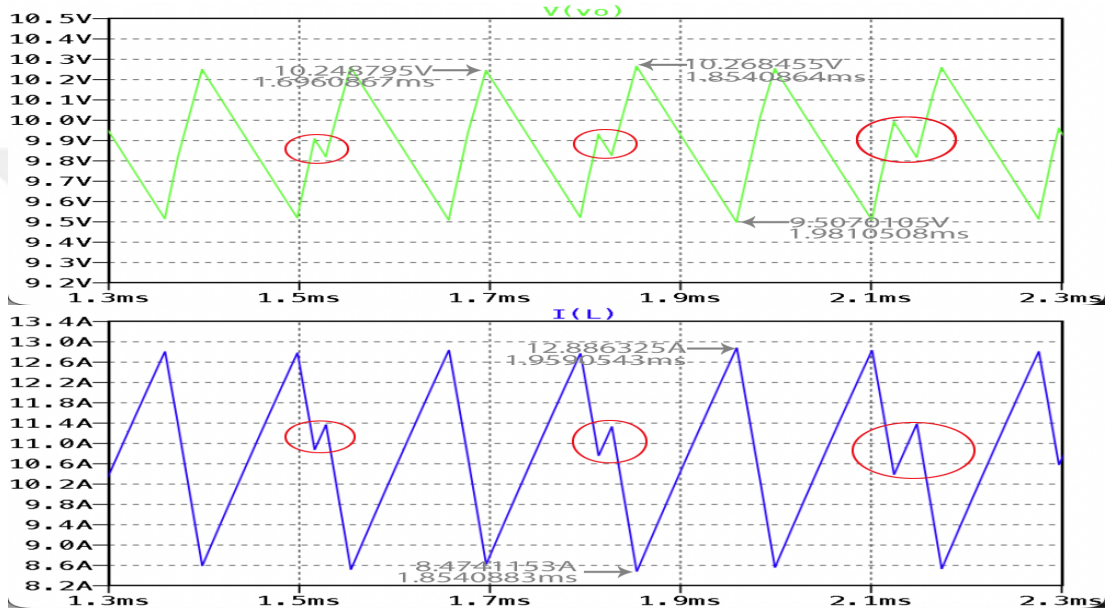


Figure 4.19: Realistic boost converter steady-state simulation results

Many simulations were conducted in the style of a controlled experiment by modifying one non-optimal feature at a time. As in the case of the buck converter, chattering was observed in the steady state when inductor derating was induced. This is illustrated with red ellipses on the inductor current and output voltage waveforms in the figures. This differs from [41], where chattering occurs even without inductor derating. It can be concluded that the parasitic resistances of the system if they are chosen low enough, have negligible effect in the context of chattering when the DCR of the inductor is taken into account in deriving of natural switching surfaces used in the control law. Again, the primary factors contributing to the errors in Table 4.3 were the limited bandwidth of the controller and the variation of component values under operating conditions. A higher amount of reduction of inductance and capacitance due to higher target inductor current and output voltage resulted in greater deviation from theoretically calculated values in comparison to Section 3.6. The remarks on chattering,

the effects of non-ideal components, and the limited bandwidth of the controller made in the case of the buck converter also apply to the boost converter.

Parameters	Theoretical Values	Realistic Simulation Results	Errors
$\Delta V_{o,ss}$	0.2868603V	0.76144 V	165.4391%
$\Delta i_{L,ss}$	1.972119	4.4122072 A	223.72926%
f_{ss}	13.98602 kHz	6.32913 kHz	54.74674%
$i_{L,peak}$	23.0194 A	23.042449 A	0.10013%
$\Delta V_{o,ul}$	0.672151 V	0.6663 V	0.87049%
$i_{L,min(ul)}$	2.8507705 V	2.86152 V	0.37707%
$\Delta V_{o,l}$	1.6424799 V	2.45874 V	49.69681%
$i_{L,peak(l)}$	18.074951 A	19.85254 A	9.83454%
t_{su}	861.88247 μs	839.70084 μs	2.57363%
$t_{su(lim)}$	1.22778 ms	1.22996 ms	0.17756%
t_{ul}	221.05571 μs	175.3394 μs	20.6809%
t_l	311.557 μs	403.6147 μs	27.76527%
$t_{l(lim)}$	358.7738 μs	692.1975 μs	99.93424%
η	87.98% s	84.65%	3.78495%

Table 4.3: Comparison of theoretical values and realistic simulation results of boost converter performance measures

Similar to the buck converter, the inductor current exhibits cyclical fluctuation from the intended limited value during the current limitation phase, as can be seen in Figure 4.21 and 4.23. The maximum inductor current surpasses the value applied in the control law (4.33), the inductor current value intended to be constrained at. This results from the delay limited controller bandwidth causes. Moreover, the switching frequency is high compared to its value in the steady state; its maximum value is measured to be 332.32 kHz during the current limiting phase in the start-up. The switching frequency during the current limiting phase is proportional to the bandwidth of the controller; however, this can be problematic with the high bandwidth controller since the transistors employed

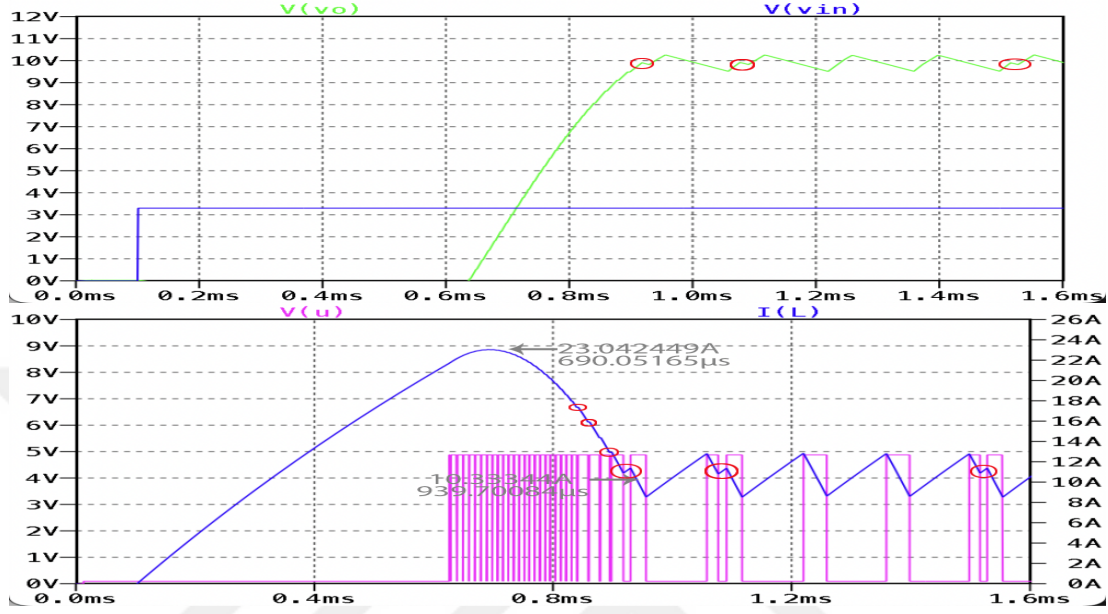


Figure 4.20: Realistic boost converter realistic start-up simulation results

in the design might not be fit for high-frequency switching. They might over-heat and surpass their value of maximum working temperature. The next section addresses the remedy of this issue.

4.6.1 Switching Frequency Limitation During Current Limiting

By establishing a hysteresis band between two inductor current values, the switching frequency can be limited during the current limitation phase. The system states will be within this band, fluctuating between the upper and lower limits until the system reaches the off-state natural switching surface. This can be

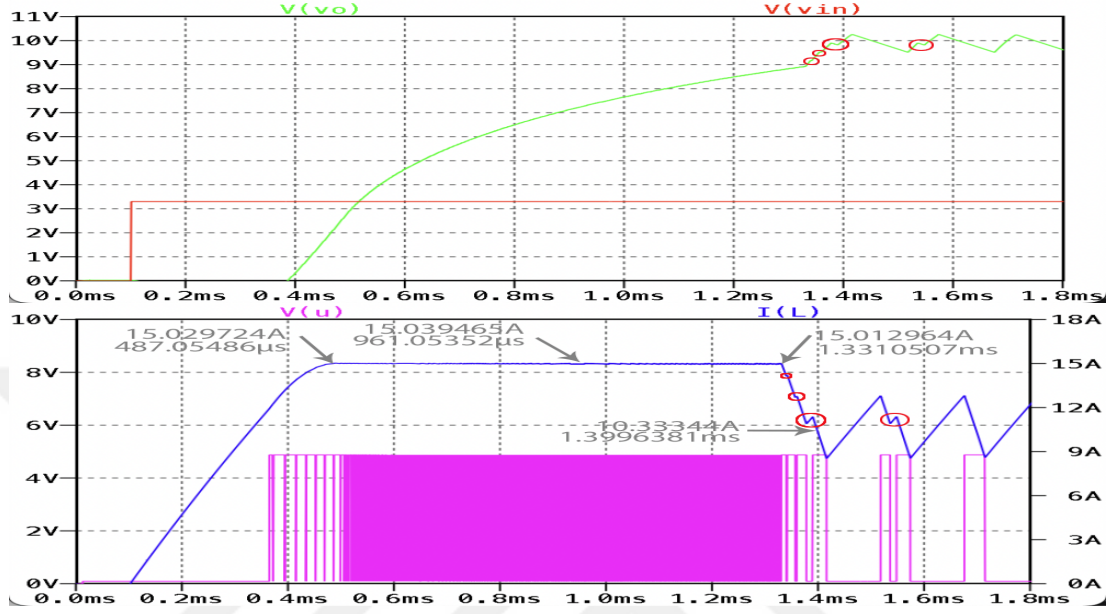


Figure 4.21: Realistic boost converter start-up with inductor current limitation simulation results

accomplished by the control law described in (4.113).

Case 1 : $V_{on} < 1$

Subcase 1 : $i_{Ln} < i_{Ln,liml}$ and $\sigma_{off} < 0$

$$u = \begin{cases} 0, & \sigma_{off2} > 0 \text{ and } V_{on} < V_{on,lim} \\ 1, & \text{else} \end{cases}$$

Subcase 2 : $i_{Ln} > i_{Ln,limu}$ and $\sigma_{off} < 0$

$$u = 0$$

(4.113)

Subcase 3 : $\sigma_{off} > 0$

$$u = 0$$

Case 2 : $V_{on} > 1$

$$u = \begin{cases} 0, & \sigma_{on} > 0 \\ 1, & \text{else.} \end{cases}$$

σ_{off2} must be derived with $i_{Ln,liml}$ as below

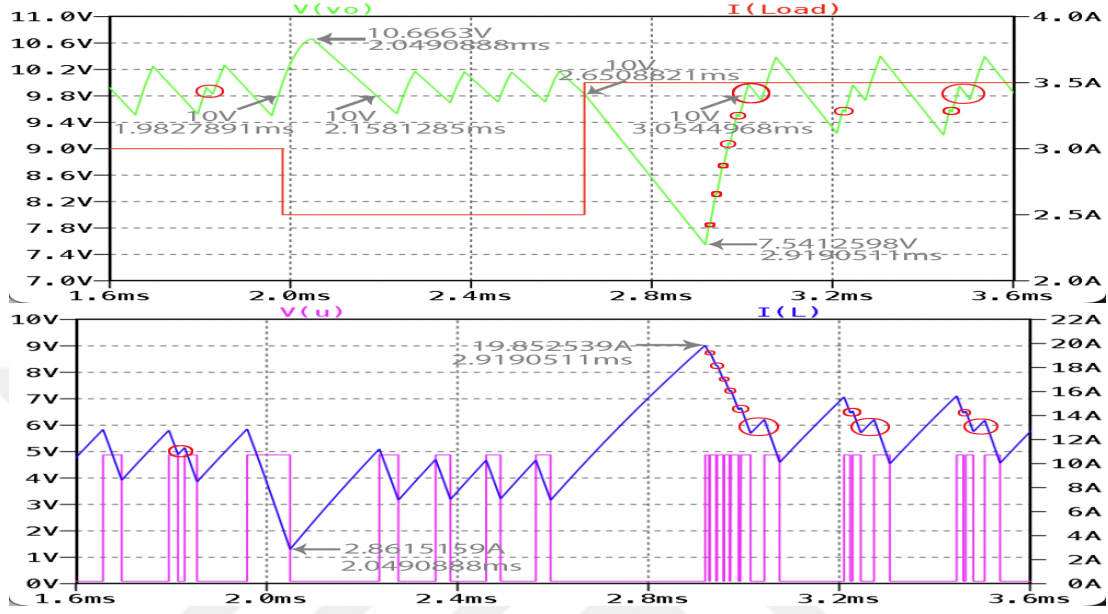


Figure 4.22: Realistic boost converter unloading and loading simulation results

$$\begin{aligned} i_{Ln}(0) &= i_{Ln,liml}, \\ V_{on}(0) &= V_{on,lim} = V_{inn} - i_{Ln,liml}R_{Ln}. \end{aligned} \quad (4.114)$$

Using (4.114) in (4.11) gives

$$\begin{aligned} z_{1,off}(0) &= -\frac{1}{2\pi}(-V_{inn} + R_{Ln}i_{on} + V_{on,lim}), \\ z_{2,off}(0) &= -\frac{\alpha}{2\pi\beta}(-V_{inn} + R_{Ln}i_{on} + V_{on,lim}) - \frac{1}{\beta}(i_{Ln,liml} - i_{on}). \end{aligned} \quad (4.115)$$

By using (4.115) in (4.12), $r_{off,20}^2$ and $\theta_{off2}(0)$ can be defined and utilized in the new off-state switching surface, which can be expressed as

$$\sigma_{off2} = r_{off}^2 - r_{off2,0}^2 e^{-2\frac{\alpha}{\beta}(\theta_{off2}(0) - \theta_{off})}. \quad (4.116)$$

The start-up trajectories of the boost converter with hysteresis band for current limiting are shown in Figure 4.24, obeying the control law given by equation (4.113). The system begins at $(0,0)$, and the control law activates the switch.

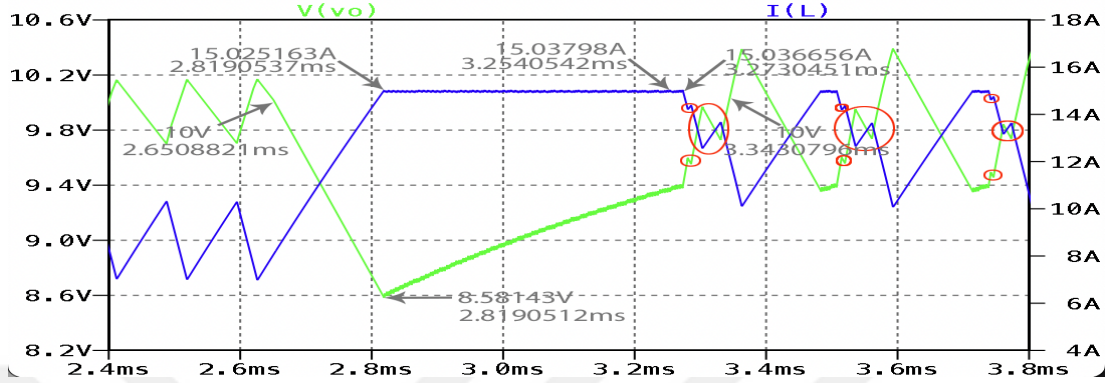


Figure 4.23: Realistic boost converter loading with inductor current limitation simulation results

The system then follows the natural trajectory of the on-state until it reaches the upper limit of the hysteresis band, $i_{Ln,limu}$. At this point, the switch turns off and remains off until the states reach the lower limit of the band, $i_{Ln,liml}$. When the control law detects that $i_{Ln,liml}$ has been reached, it turns on the switch. The switch remains on until the system reaches the upper limit of the band. This cycle continues until the system reaches the off-state natural switching surface. Then, the switch turns off if it is in the on-state and stays off if it is in the off-state. The system states move along the off-state natural switching surface and eventually reach the target operating point.

Figure 4.25 displays the loading trajectories of a buck converter using the control law (4.113). As the load current increases, the control law activates the switch, and the system starts at the initial target operating point $(i_{Ln,initial}, V_{on,target})$. It then follows the natural trajectory until it reaches the upper limit of the hysteresis band $i_{Ln,limu}$. In the same way, the system maintains its state within this band by bouncing between the upper and lower limits until it reaches the off-state natural switching surface. Then, the switch turns off the system states move along this surface and converge towards the desired operating point.

The highest switching frequency during current limitation can be restricted by

$i_{L,limu}$ and the width of the hysteresis band, which is

$$\Delta_{hys} = i_{L,limu} - i_{L,liml}. \quad (4.117)$$

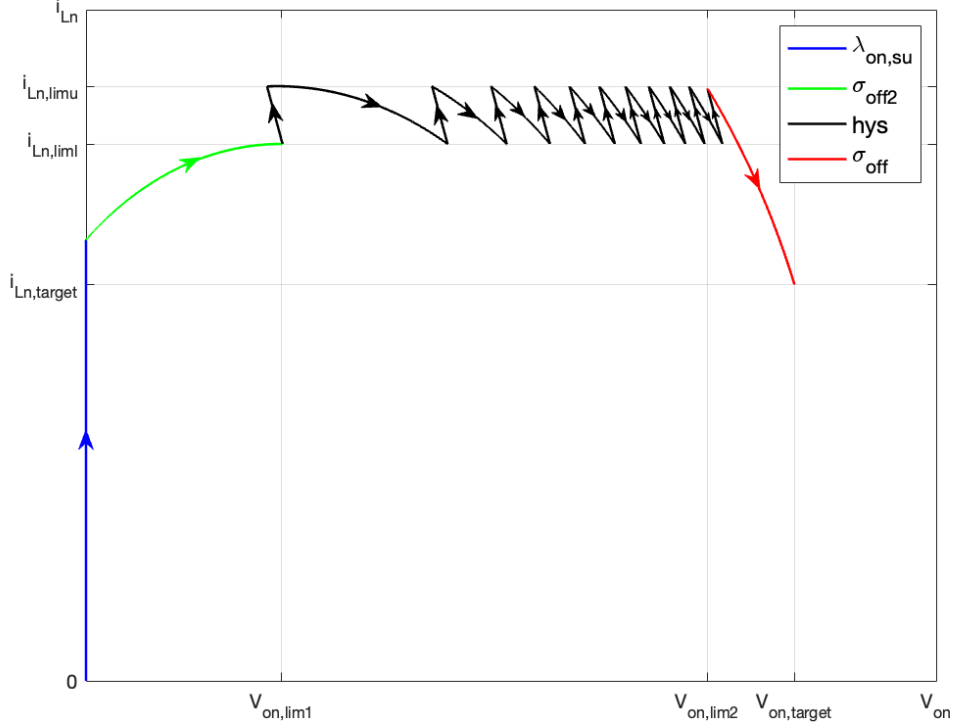


Figure 4.24: Boost converter control law start-up trajectories incorporating hysteresis band for current limitation

While lowering $i_{L,limu}$ raises the maximum switching frequency during the current limiting region, increasing Δ_{hys} lowers it. The current and switching frequency limits of design may not be exceeded with a suitable choice of $i_{L,limu}$ and Δ_{hys} . This can be accomplished with an algorithm similar to the one applied to decide the buck converter design parameters. The algorithm can input all the design parameters and $i_{L,limu}$, the intended maximum inductor current value, then produces optimum Δ_{hys} for the maximum switching frequency permitted. Starting with the smallest value of Δ_{hys} in the specified range, the method would

measure the highest switching frequency in the current limitation phase and increase Δ_{hys} until the optimal value is specified. Then, using (4.117), it may determine $i_{L,liml}$ value to be applied in the control law. The limits of the hysteresis band will deviate from calculated values in practice due to the limited controller bandwidth, and the maximum switching frequency will be proportional to the controller's bandwidth. The implementation of an algorithm that considers the bandwidth of the controller is necessary to achieve precise results. $i_{L,limu}$ and $i_{L,liml}$ should be determined by the algorithm for use in the control law for the maximum allowed current and switching frequency during the inductor current limiting phase.

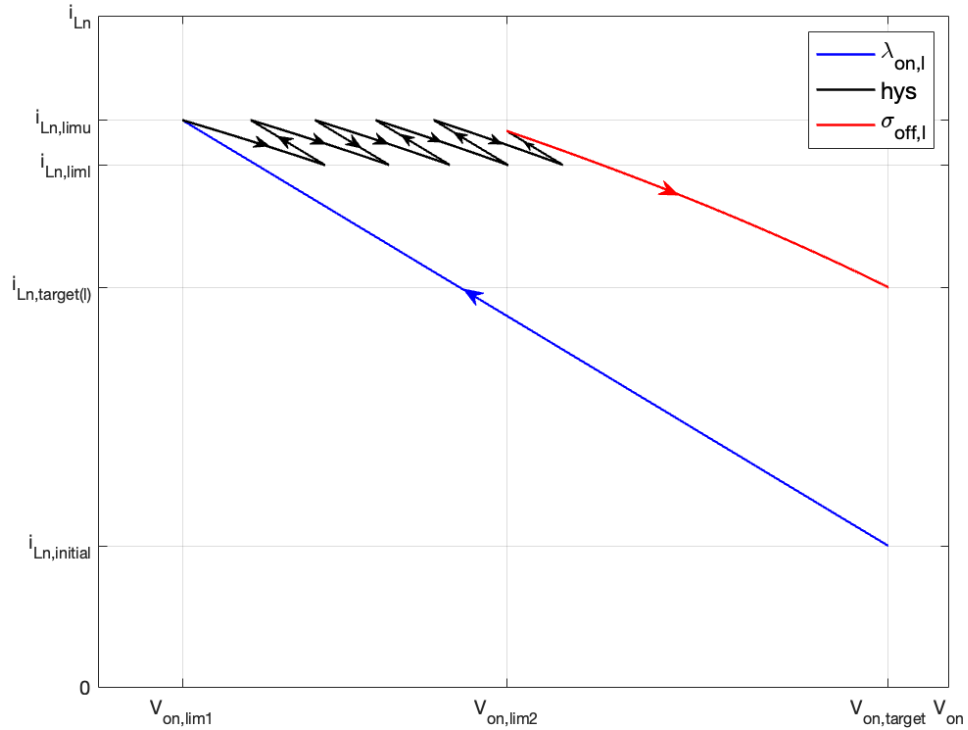


Figure 4.25: Boost converter loading trajectories incorporating hysteresis band for current limitation

Chapter 5

Conclusions and Future Work

It has already been proven that satisfactory dynamic performance can be achieved by using natural switching surfaces in the sliding mode control of buck and boost converters. Natural switching surfaces used in the control laws makes it possible to reach a steady state without overshoot or error by performing a single switching action during start-up, load changes, and large signal disturbances. The proposed methods are developed by focusing on the practical aspects, opening ways to use surfaces by overcoming practical problems. Natural switching surfaces that take into account the DCR of the inductor, which has a significant impact on the chattering effect, are derived. Modifications for controlling switching frequency and ripples in steady-state operation are presented. The control laws that enable the limit of the inductor current at an intended value and limit switching frequency during the current limiting phase are proposed for both converters. The system trajectories are geometrically represented on the state plane, offering a virtual understanding of the behavior of the converters under these control laws. The performance of the proposed control methods is demonstrated via simulations. From a theoretical standpoint, the chattering effect is fully eliminated. This effect can be minimized in practice by careful selection of components considering the variation of their values due to operating conditions and tolerances. Since the DCR of the inductor, which is the most significant one among parasitic resistances, is already considered in deriving NSS, chattering caused by them can be

easily minimized by keeping other parasitic resistances low. To ensure that the practical results are close to the theoretical ones, the controller bandwidth must be as high as possible, and the suggestions for decreasing the chattering effect also apply to minimizing errors.

In summary, the work presented in this thesis has made the following contributions to the buck and boost converter control literature:

- The normalization technique is employed to model the system dynamics of the buck and boost converters by taking the parasitic resistance of the inductor into account in the switch ON and switch OFF cases for the buck converter and the switch OFF case for the boost converter. Natural switching surfaces are derived for both cases with these dynamics by eliminating the time. Modifications for the on-state NSS of the buck converter and the on and off-state NSS of both converters are presented for controlling switching frequency and ripples during steady-state operation.
- Control laws utilizing these natural switching surfaces for satisfying dynamic performance are presented. Control laws that enable the limit of the inductor current and both inductor current and switching frequency during the current limitation period are proposed for both converters as well.
- Trajectories under these control laws are derived and graphically shown for start-up, loading, unloading and steady-state operations. A theoretical foundation is established for the computation of the time duration, current and voltage variations during these operations.
- The algorithms for determining design parameters for given requirements are presented in pseudo-code format.

In the future, an experimental hardware setup for testing of the proposed controllers can be built. Studies that analytically explain the practical deviations caused by limited bandwidth of controller can be carried out. The design algorithms for specifying design parameters can be updated with taking limited controller bandwidth into account.

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Appendix A

Converter Design Algorithms

Algorithm 1 Buck Converter Design Algorithm

```
1: procedure BUCK( $\Delta V_{o,maximum}, \Delta i_{L,maximum}, f_{maximum}, V_{in}, V_o, i_o, L, R_L$ )
2:   set A search array  $\Delta r^2(i_r)$  starts at  $\Delta r_{upper}^2$  ends at  $\Delta r_{lower}^2$  with a
   decrement size of  $\Delta r_{dec}^2$ 
3:   set A search array  $Z_o(i_z)$  starts at  $Z_{o,upper}$  ends at  $Z_{o,lower}$  with a decre-
   ment size of  $Z_{o,dec}$ 
4:   initialize  $i_r = 1$  and  $i_z = 1$ 
5:   repeat
6:      $\Delta r^2 \leftarrow \Delta r^2(i_r)$ 
7:      $Z_o \leftarrow Z_o(i_z)$ 
8:     Calculate inductor current ripple,  $\Delta i_{L,ss}$  (see equation (3.97))
9:     if  $\Delta i_{L,ss} < \Delta i_{L,maximum}$  then
10:      Calculate output voltage ripple,  $\Delta V_{o,ss}$  (see equation (3.100))
11:      if  $\Delta V_{o,ss} < \Delta V_{o,maximum}$  then
12:        Calculate switching frequency,  $f_{ss}$  (see equation (3.103))
13:        if  $f_{ss} < f_{maximum}$  then
14:          break
15:        else
16:           $i_z \leftarrow i_z + 1$ 
17:        end if
18:      else
19:         $i_z \leftarrow i_z + 1$ 
20:      end if
21:    else
22:       $i_r \leftarrow i_r + 1$ 
23:    end if
24:    print  $\Delta r^2$ 
25:    Calculate capacitance C using (3.104)
26:    print C
27: end procedure
```

Algorithm 2 Boost Converter Design Algorithm

```
1: procedure BOOST( $\Delta V_{o,maximum}, \Delta i_{L,maximum}, f_{maximum}, V_{in}, V_o, i_o, L, R_L$ )
2:   set A search array  $\Delta r^2(i_r)$  starts at  $\Delta r^2_{upper}$  ends at  $\Delta r^2_{lower}$  with a
   decrement size of  $\Delta r^2_{dec}$ 
3:   set A search array  $Z_o(i_z)$  starts at  $Z_{o,upper}$  ends at  $Z_{o,lower}$  with a decre-
   ment size of  $Z_{o,dec}$ 
4:   initialize  $i_r = 1$  and  $i_z = 1$ 
5:   repeat
6:      $\Delta r^2 \leftarrow \Delta r^2(i_r)$ 
7:      $Z_o \leftarrow Z_o(i_z)$ 
8:     Calculate inductor current ripple,  $\Delta i_{L,ss}$  (see equation (4.98))
9:     if  $\Delta i_{L,ss} < \Delta i_{L,maximum}$  then
10:      Calculate output voltage ripple,  $\Delta V_{o,ss}$  (see equation (4.99))
11:      if  $\Delta V_{o,ss} < \Delta V_{o,maximum}$  then
12:        Calculate switching frequency,  $f_{ss}$  (see equation (4.103))
13:        if  $f_{ss} < f_{maximum}$  then
14:          break
15:        else
16:           $i_z \leftarrow i_z + 1$ 
17:        end if
18:      else
19:         $i_z \leftarrow i_z + 1$ 
20:      end if
21:    else
22:       $i_r \leftarrow i_r + 1$ 
23:    end if
24:    print  $\Delta r^2$ 
25:    Calculate capacitance C using (3.104)
26:    print C
27: end procedure
```

Appendix B

Controller Circuit Netlists

B.1 Netlist of the Buck Converter Controller

```

B1 ron_02_a 0 V=pwr((-1/(2*pi))*((-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref))+1),2)
B2 ron_02_b 0 V=pwr((-a/(2*pi*b))*((-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref))+1),2)
B3 ron_02 0 V=V(ron_02_a)+V(ron_02_b)+V(dr2)
B4 roff_02_a 0 V=pwr((-1/(2*pi))*((RLn*i(R5)*Zo/V(Vref))
+1),2)
B5 roff_02_b 0 V=pwr((-a/(2*pi*b))*((RLn*i(R5)*Zo/V(Vref))
+1),2)
B6 roff_02 0 V=V(roff_02_a)+V(roff_02_b)+V(dr2)
B7 teta_on_0_a 0 V=(-1/(2*pi))*((-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref))+1)
B8 teta_on_0_b 0 V=(-a/(2*pi*b))*((-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref))+1)
B9 teta_on_0 0 V=atan2(V(teta_on_0_b),V(teta_on_0_a))
B10 teta_off_0_a 0 V=(-1/(2*pi))*((RLn*i(R5)*Zo/V(Vref))
+1)
B11 teta_off_0_b 0 V=(-a/(2*pi*b))*((RLn*i(R5)*Zo/V(Vref))
+1)
B12 teta_off_0 0 V=atan2(V(teta_off_0_b),V(teta_off_0_a))
B13 ron_2_a 0 V=pwr((-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref)+V(Vo)/V(Vref)),2)
B14 ron_2_b 0 V=pwr((-a/(2*pi*b))*(-V(Vin)/V(Vref)+RLn*i(R5)
)*Zo/V(Vref)+V(Vo)/V(Vref))-(1/b)*(i(R7)-i(R5))*Zo/V(Vref),2)
B15 ron_2 0 V=V(ron_2_a)+V(ron_2_b)
B16 teta_on_a 0 V=(-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)*
Zo/V(Vref)+V(Vo)/V(Vref))

```

B17 $\text{teta_on_b} = 0$ $V = (-a/(2*\pi*b)) * (-V(\text{Vin})/V(\text{Vref}) + R_{Ln} * i(R5) * Z_o/V(\text{Vref}) + V(\text{Vo})/V(\text{Vref})) - (1/b) * (i(R7) - i(R5)) * Z_o/V(\text{Vref})$
 B18 $\text{teta_on} = 0$ $V = \text{atan2}(V(\text{teta_on_b}), V(\text{teta_on_a}))$
 B19 $\text{roff_2_a} = 0$ $V = \text{pwr}((-1/(2*\pi)) * (R_{Ln} * i(R5) * Z_o/V(\text{Vref}) + V(\text{Vo})/V(\text{Vref})), 2)$
 B20 $\text{roff_2_b} = 0$ $V = \text{pwr}((-a/(2*\pi*b)) * (R_{Ln} * i(R5) * Z_o/V(\text{Vref}) + V(\text{Vo})/V(\text{Vref})) - (1/b) * (i(R7) - i(R5)) * Z_o/V(\text{Vref})), 2)$
 B21 $\text{roff_2} = 0$ $V = V(\text{roff_2_a}) + V(\text{roff_2_b})$
 B22 $\text{teta_off_a} = 0$ $V = (-1/(2*\pi)) * (R_{Ln} * i(R5) * Z_o/V(\text{Vref}) + V(\text{Vo})/V(\text{Vref}))$
 B23 $\text{teta_off_b} = 0$ $V = (-a/(2*\pi*b)) * (R_{Ln} * i(R5) * Z_o/V(\text{Vref}) + V(\text{Vo})/V(\text{Vref})) - (1/b) * (i(R7) - i(R5)) * Z_o/V(\text{Vref})$
 B24 $\text{teta_off} = 0$ $V = \text{atan2}(V(\text{teta_off_b}), V(\text{teta_off_a}))$
 B25 $\text{s_on} = 0$ $V = V(\text{ron_2}) - V(\text{ron_02}) * \exp(-2*(a/b) * (V(\text{teta_on_0}) - V(\text{teta_on})))$
 B26 $\text{s_off} = 0$ $V = V(\text{roff_2}) - V(\text{roff_02}) * \exp(-2*(a/b) * (V(\text{teta_off_0}) - V(\text{teta_off})))$
 B27 $u1 = 0$ $V = \text{if}(V(\text{s_on}) > 0, 1, 0)$
 B28 $u2 = 0$ $V = \text{if}(V(\text{s_off}) > 0, 0, 1)$
 B29 $uwol = 0$ $V = \text{if}(i(R7) < i(R5), V(u1), V(u2))$
 B30 $u = 0$ $V = \text{if}(0 < i(R1), V(uw1), V(uwol))$
 B31 $uw1 = 0$ $V = \text{if}(i(R7) \leq i(R5), V(u14), V(u12))$
 B32 $ul2 = 0$ $V = \text{if}(i(R7) \leq i(R1), V(u13), 0)$
 B33 $ul3 = 0$ $V = \text{if}(V(\text{s_off}) > 0, 0, 1)$
 B34 $ul4 = 0$ $V = \text{if}(V(\text{s_on}) > 0, 1, 0)$
 V1 $\text{dr2} = 0.000673$
 R1 $iL_lim = 0$ 10k
 R3 $V_o = 0$ 10k
 R4 $V_{ref} = 0$ 10k
 R5 $i_o = 0$ 10k
 R6 $V_{in} = 0$ 10k
 R7 $i_L = 0$ 10k

```

.param RL=0.0469
.param b=pi*sqrt(4-RLn*RLn)
.param a=pi*RLn
.param L=47u
.param C=293.75u
.param Zo=sqrt(L/C)
.param RLn=RL/Zo
.backanno
.end

```

B.2 Netlist of the Boost Converter Controller

```

B1 roff_2_a 0 V=pwr((-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)
*Zo/V(Vref)+V(Vo)/V(Vref)),2)
B2 roff_2_b 0 V=pwr((-a/(2*pi*b))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+V(Vo)/V(Vref))-(1/b)*(i(R7)-i(R5))*Zo/V(Vref),2)
B3 roff_2 0 V=V(roff_2_a)+V(roff_2_b)
B4 teta_off_a 0 V=(-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+V(Vo)/V(Vref))
B5 teta_off_b 0 V=(-a/(2*pi*b))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+V(Vo)/V(Vref))-(1/b)*(i(R7)-i(R5))*Zo/V(Vref)
B6 teta_off 0 V=atan2(V(teta_off_b),V(teta_off_a))
B7 roff_02_a 0 V=pwr((-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+1),2)
B8 roff_02_b 0 V=pwr((-a/(2*pi*b))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+1)-(1/b)*(i(R1)-i(R5))*Zo/V(Vref),2)
B9 roff_02 0 V=V(roff_02_a)+V(roff_02_b)+V(dr2)
B19 teta_off_0_a 0 V=(-1/(2*pi))*(-V(Vin)/V(Vref)+RLn*i(R5)*Zo/V(Vref)+1)

```

B11 $\text{teta_off_0_0 } V = \text{atan2}(V(\text{teta_off_0_b}), V(\text{teta_off_0_a}))$
 B12 $\text{0_N002 } I = (((V(Vin)/V(Vref)) - \text{sqrt}(V(Vin)*V(Vin)/(V(Vref)*V(Vref))) - 4*i(R5)*RLn*Zo/V(Vref)))/(2*RLn))*V(Vref)/Zo$
 B13 $\text{teta_off_0_b_0 } V = (-a/(2*pi*b))*(-V(Vin)/V(Vref) + RLn*i(R5)*Zo/V(Vref) + 1) - (1/b)*(i(R1) - i(R5))*Zo/V(Vref)$
 B14 $\text{s_off_0 } V = V(\text{roff_2}) - V(\text{roff_02})*\exp(-2*(a/b)*(V(\text{teta_off_0}) - V(\text{teta_off})))$
 B15 $\text{s_on_0 } V = V(Vo)/V(Vref) + (i(R5)*Zo*Zo/(V(Vin)*V(Vref)))*(i(R7) - i(R1)) - 1$
 B16 $\text{u1_0 } V = \text{if}(V(\text{s_off}) > 0, 0, 1)$
 B17 $\text{u2_0 } V = \text{if}(V(\text{s_on}) > 0, 0, 1)$
 B18 $\text{u_wol_0 } V = \text{if}((V(Vo)/V(Vref)) < 1, V(u1), V(u2))$
 B19 $\text{roff2_02_a_0 } V = \text{pwr}((-1/(2*pi))*(-V(Vin)/V(Vref) + RLn*i(R5)*Zo/V(Vref) + V(Von_lim)), 2)$
 B20 $\text{roff2_02_b_0 } V = \text{pwr}((-a/(2*pi*b))*(-V(Vin)/V(Vref) + RLn*i(R5)*Zo/V(Vref) + V(Von_lim)) - (1/b)*(i(R2) - i(R5))*Zo/V(Vref), 2)$
 B21 $\text{roff2_02_0 } V = V(\text{roff2_02_a}) + V(\text{roff2_02_b})$
 B22 $\text{teta_off2_0_a_0 } V = (-1/(2*pi))*(-V(Vin)/V(Vref) + RLn*i(R5)*Zo/V(Vref) + V(Von_lim))$
 B23 $\text{teta_off2_0_0 } V = \text{atan2}(V(\text{teta_off2_0_b}), V(\text{teta_off2_0_a}))$
 B24 $\text{teta_off2_0_b_0 } V = (-a/(2*pi*b))*(-V(Vin)/V(Vref) + RLn*i(R5)*Zo/V(Vref) + V(Von_lim)) - (1/b)*(i(R2) - i(R5))*Zo/V(Vref)$
 B25 $\text{0_N001 } I = i(R2)*Zo/V(Vref)$
 B26 $\text{Von_lim_0 } V = V(Vin)/V(Vref) - i(R8)*RLn$
 B27 $\text{s_off2_0 } V = V(\text{roff_2}) - V(\text{roff2_02})*\exp(-2*(a/b)*(V(\text{teta_off2_0}) - V(\text{teta_off})))$
 B28 $\text{Vo_lim_0 } V = (V(Vin)/V(Vref) - i(R8)*RLn)*V(Vref)$
 B29 $\text{ul1_0 } V = \text{if}(V(\text{s_on}) > 0, 0, 1)$
 B30 $\text{ul2_0 } V = \text{if}(V(\text{s_off}) > 0, 0, V(\text{ul3}))$
 B31 $\text{u_0 } V = \text{if}(V(Vo)/V(Vref) > 1, V(\text{ul1}), V(\text{ul2}))$

```

B32 ul3 0 V=if(i(R7)>i(R2), 0, V(ul5))
B33 ul5 0 V=if((V(s_off2)>0 & V(Vo)<V(Vo_lim)), 0, 1)
B34 dr2 0 V=0.0000147
R1 N002 0 10k
R2 iL_Lim 0 10k
R3 Vo 0 10k
R4 Vref 0 10k
R5 io 0 10k
R6 Vin 0 10k
R7 iL 0 10k
R8 N001 0 10k
.param RL=0.0384
.param b=pi*sqrt(4-RLn*RLn)
.param a=pi*RLn
.tran 20m
.param L=82u
.param C=512.5u
.param Zo=sqrt(L/C)
.param RLn=RL/Zo
.backanno
.end

```