

RAPID RECONSTRUCTION FOR PARALLEL MAGNETIC RESONANCE IMAGING WITH NON-CARTESIAN VARIABLE-DENSITY SAMPLING TRAJECTORIES

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By
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Rapid Reconstruction for Parallel Magnetic Resonance Imaging with
Non-Cartesian Variable-Density Sampling Trajectories

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January 2020

We certify that we have read this thesis and that in our opinion it is fully adequate,
in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

RAPID RECONSTRUCTION FOR PARALLEL MAGNETIC RESONANCE IMAGING WITH NON-CARTESIAN VARIABLE-DENSITY SAMPLING TRAJECTORIES

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Due to long acquisition times, the use of magnetic resonance imaging (MRI) remains challenging in some applications. Variable-density acquisitions enable scan acceleration while maintaining a desirable trade-off between signal-to-noise ratio (SNR) and spatial resolution. Several image-domain and k-space algorithms were previously proposed for parallel-imaging reconstructions of variable-density acquisitions. However, these methods involve iterative procedures for non-Cartesian data, resulting in substantial computational burden in particular for three-dimensional (3D) reconstructions. An efficient method based on partially parallel imaging with localized sensitivities (PILS) was recently proposed for fast reconstructions of 2D non-Cartesian data. This thesis introduces a generalized image-domain implementation for 3D non-Cartesian variable-density data, and compares it against conventional gridding, PILS, and ESPIRiT (iterative self-consistent parallel imaging reconstruction using eigenvector maps) reconstructions on brain and knee data accelerated at $R=2.5$ to 4.2. The results indicate that the proposed 3D variable-FOV method outperforms SOS (sum of squares) and PILS methods, and performs equally or better than ESPIRiT reconstruction at less than half of the processing time required by ESPIRiT. Thus, the proposed method provides fast, high-SNR, artifact-suppressed reconstructions.

Keywords: magnetic resonance imaging, parallel imaging, variable density.

ÖZET

DEĞİŞKEN YOĞUNLUKLU KARTEZYEN OLMAYAN PARALEL MANYETİK REZONANS GÖRÜNTÜLEMEDE HIZLI GERİÇATIM

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Uzun tarama süreleri manyetik rezonans görüntülemenin (MRG) bazı uygulamalarda kullanımını zorlaştırmaktadır. Değişken yoğunluklu taramalar, sinyal gürültü oranı (SNR) ve uzamsal çözünürlük arasında iyi bir denge sağlayacak şekilde taramayı hızlandırmaya olanak sağlamaktadır. Değişken yoğunluklu paralel görüntüleme geriçatımı için çeşitli görüntü uzayı ve k-uzayı algoritmaları önerilmiştir. Ancak bu yöntemler Kartezyen olmayan veriler için yinelemeli süreçler içerdiklerinden özellikle 3 boyutlu (3B) geriçatım için önemli bir hesaplama yüküne neden olmaktadır. Yakın zamanda 2 boyutlu Kartezyen olmayan verilerin hızlı geriçatımı için, bölgesel duyarlılıklı kısmî paralel görüntülemeye (PILS) dayalı etkin bir yöntem önerilmiştir. Bu tezde 3B Kartezyen olmayan değişken yoğunluklu veri için genelleştirilmiş bir görüntü uzayı uygulaması sunmakta ve bunu geleneksel ızgaralama, PILS ve ESPIRiT (Özvektör haritalarını kullanarak yinelemeli kendiyile tutarlı paralel görüntüleme geriçatımı) geriçatımları ile $R=2,5$ 'ten $4,2$ 'ye kadar hızlandırılmış beyin ve diz verisi üzerinde karşılaştırmaktadır. Sonuçlar önerilen 3B değişken görüntü alanı yönteminin SOS (karelerin toplamı) ve PILS yöntemlerinden üstün olduğunu, ESPIRiT'in yarısı kadar işleme süresinde ESPIRiT'e kıyasla eşit ya da daha iyi başarı sağladığını göstermektedir. Önerilen yöntem bu şekilde hızlı, yüksek SNR ile ve artefaktları bastırılmış geriçatımlar sağlamaktadır.

Anahtar sözcükler: manyetik rezonans görüntüleme, paralel görüntüleme, değişken yoğunluk.

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Chapter 1

Introduction

In magnetic resonance imaging (MRI), acquiring 3D data often requires long acquisition durations, which can limit its applicability. Attempts at decreasing the scan time or increasing the resolution come at the cost of additional aliasing artifacts. An effective trade-off between those employs Non-Cartesian variable-density k-space trajectories, which can cover the k-space more efficiently. Non-Cartesian parallel imaging help reduce the aliasing artifacts due to undersampling, just as parallel imaging can be used for undersampled Cartesian data [1].

Various techniques exist for reconstructing the missing data for parallel imaging, such as the simultaneous acquisition of spatial harmonics (SMASH) and generalized autocalibrating partially parallel acquisition (GRAPPA), which works in k-space; and sensitivity encoding (SENSE) and partially parallel imaging with localized sensitivities (PILS), which work in the image domain. While these techniques primarily work on Cartesian acquisitions, they have been adapted to work with non-Cartesian trajectories as well [2, 3, 4]. However, they normally require many iterations with high computational complexity, as successive forward and backward gridding operations are needed to ensure data consistency, making reconstruction times the limiting factor for their adoption.

In comparison, PILS achieves good quality reconstructions rapidly, not requiring such an iterative process, as long as the coil localization assumption is satisfied. In practice, coil sensitivities rarely have perfect localization; giving rise to aliasing artifacts. It is possible to obtain artifact-suppressed reconstructions by choosing an appropriate field-of-view (FOV) for each coil and ignoring all data outside it. Choosing FOV to suppress as much artifacts as possible, based on the lowest sampling density, does not utilize densely sampled data effectively; and more lenient FOVs do not adequately remove aliasing artifacts [5, 6].

To prevent this and make efficient use of all data in 3D variable-density trajectories, our proposed technique, 3D var-FOV, uses variable FOV for the 3D varying sampling density of the acquisition, instead of choosing a single FOV for the coil. This method, combined with a fast total-variation filtering to remove any remaining artifacts, provides a much faster alternative to other resource-intensive methods that render comparable artifact-suppressed reconstructions. We test it on brain and knee datasets, undersampled in non-Cartesian trajectories with different accelerations, and at various SNR levels; and compare its performance against three other reconstruction methods: SOS, PILS and ESPIRiT, in terms of reconstruction quality and processing times.

Chapter 2

Rapid Imaging Techniques in MRI

MRI has become an invaluable medical imaging tool by enabling non-invasive and safe measurements of tissue without requiring ionizing radiation. It is an extremely flexible imaging modality, where by changing properties of MR sequence, in particular the repetition time (TR) and the echo time (TE), or by using contrast agents, sensitivities to different tissues can be adjusted. One reason it has not gained even more widespread use than it already has is its lengthy acquisition time [7]. In addition to innovative methods that attempt to speed up the data acquisition, another approach is to acquire less samples and then reconstruct the full image from those, which is the subject of this thesis.

2.1 MRI with non-Cartesian trajectories

In MRI, data is obtained in k-space, the spatial frequency domain. When the obtained samples lie along a Cartesian grid, the image can be reconstructed efficiently using a fast Fourier transform (FFT). However, k-space can also be sampled along non-Cartesian trajectories. Various non-Cartesian trajectories has

been studied, including spiral [8, 9], radial [10], rosette [11] and stochastic [12] trajectories.

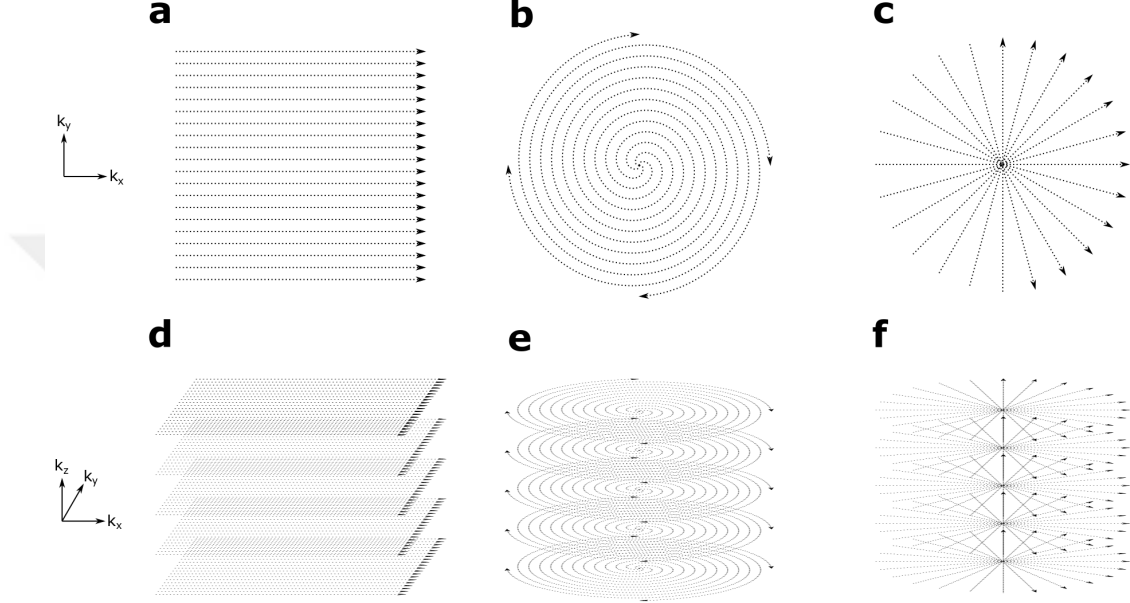


Figure 2.1: Some commonly used MRI trajectories: **(a)** Cartesian, **(b)** spiral, **(c)** radial. They can be used in 3D imaging by employing them in stacks **(d-f)**

Each of these non-Cartesian trajectories can have its own advantages. Fundamentally, they all try to provide better coverage of k-space by making more effective use of MR gradients. They can also reduce aliasing artifacts [13], they can be less susceptible to problems due to motion [14] and give higher temporal resolution, which is particularly useful for cardiac imaging and functional MRI (fMRI), they can have enhanced contrast [15] and a number of other benefits [16, 17].

Despite these advantages, non-Cartesian trajectories also present a computational challenge, as it is more difficult to reconstruct an image when the data does not fall on a Cartesian grid. A simple solution to this problem is simply interpolating between nearest data points for every grid point. While this approach works well, it does not make use of all the data as some data points may not be the nearest to any grid points. Therefore, usually a more sophisticated gridding algorithm is preferred for better accuracy. This approach calculates the contributions to all nearby grid points for every data point. This requires a

sampling density compensation as the first step [18, 19]. After that, the density compensated data is convolved with a finite kernel and sampled to a Cartesian grid. Then, performing an FFT and multiplying with an apodization correction function gives an estimate for the inverse Fourier transform [20].

2.2 Parallel MRI

Parallel MRI makes use of multiple spatially separated receiver coils, where each coil has different amplitude and phase sensitivity to different parts of the field of view. Thus, coils provide some localization. As undersampling in k-space during acquisitions will cause spatial aliasing, we can use this localization to remove aliasing during reconstruction, which is the principal benefit of using parallel MRI [21].

A number of reconstruction algorithms have been developed for parallel imaging. SENSE (Sensitivity encoding) is an image domain method that works on regularly sub-sampled Cartesian trajectories [22]. This method provides optimal results if coil sensitivities are known; however, it is not very robust if sensitivities are not estimated correctly. SMASH (Simultaneous acquisition of spatial harmonics) [23] and GRAPPA (Generalized auto calibrating partially parallel acquisition) [24] are frequency domain algorithms. In particular, GRAPPA provides robust reconstructions that auto-calibrate according to the data. SPIRiT (Iterative self-consistent parallel imaging reconstruction) further improves on this approach [25]. PILS (Partially Parallel Imaging with Localized Sensitivities) is another efficient image domain method discussed below [26].

2.2.1 PILS

If, during k-space acquisition, less samples than required by Nyquist criterion are acquired, the resulting image will have aliasing. For example, if a brain image as seen in Fig. 2.2a is undersampled in k_y , by skipping a line in k_y while acquiring

full data along k_x at Nyquist rate as in Fig. 2.2b, the reconstructed image will have aliasing in y-axis. If the coil used in the acquisition has full sensitivity in the FOV (Fig. 2.2c), the resulting image would have a second brain superimposed, shifted by $\text{FOV}_y/2$ (Fig. 2.2e). When a coil with a localized sensitivity along y-axis is used (Fig. 2.2d), an alias will occur at the same interval; however, as FOV_{coil} is more limited than the full FOV, the area of interest will be alias-free.

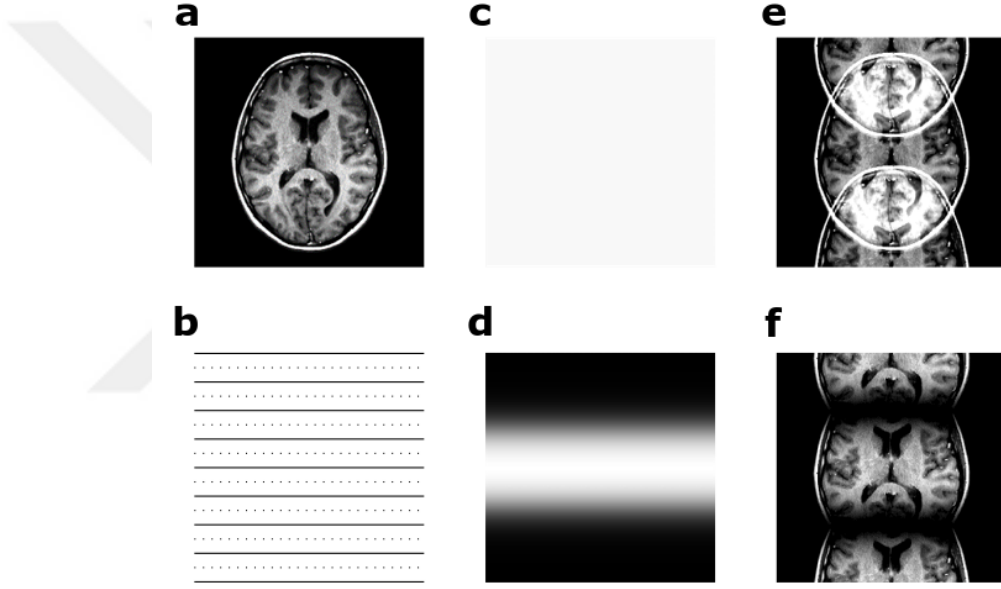


Figure 2.2: The effect of localized coil sensitivities on images received by coils with scanned acceleration: (a) original image, (b) acceleration pattern, with every other line in k_y skipped compared to a full Nyquist acquisition, (c,d) two different coil sensitivity profiles, with full and localized sensitivities, (e,f) respective image reconstructions using each of these coils.

Using this observation, artifact-free reconstructions can be obtained with an array of coils with localized sensitivities. A simple example with 3 coils, again with $R=2$ acceleration along k_y , is illustrated in Fig. 2.3. To remove aliases from Fig. 2.3b to Fig. 2.3c, the full FOV images obtained from each coil are limited by $\text{FOV}_{\text{recon}}$. Note that this is not necessarily equal to FOV_{coil} seen in Fig. 2.3a, as the rate of acceleration is also important, the effect of which is expressed by FOV_{acq} based on the sampling density in k-space. For example, if higher acceleration is applied along k_y , meaning lower sampling density in that direction; the full FOV image from each coil would have aliases at closer distance,

requiring a smaller FOV to eliminate them. Areas with missing data and black patches in the final reconstruction can be avoided as long as we have enough coils to have full coverage of the desired FOV.

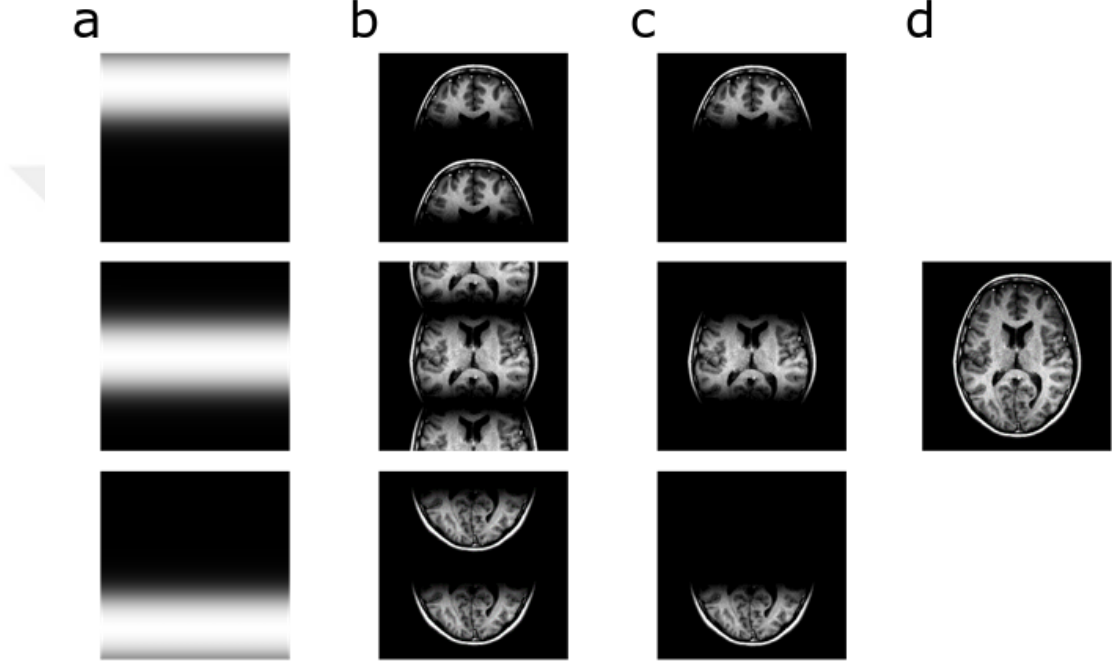


Figure 2.3: PILS reconstruction: **(a)** localized sensitivities of different coils, **(b)** image obtained from each coil, **(c)** reconstructed image from each coil, multiplied with the appropriate $\text{FOV}_{\text{recon}}$, **(d)** final combined reconstruction.

In general, it is desirable to use as large a $\text{FOV}_{\text{recon}}$ as possible while removing aliases, since this allows the most effective usage of acquired data, and improves the SNR, as utilizing data from different coils to estimate a single voxel helps reduce noise that is not correlated between the coils. Optimum selection of $\text{FOV}_{\text{recon}}$ for various cases is discussed in the next chapter of this thesis.

2.2.2 GRAPPA

Generalized auto calibrating partially parallel acquisition (GRAPPA) method makes use of the observation that k-space data is locally correlated. This can be noticed by observing that, smoothly changing coil sensitivities in image domain

correspond to a compact convolution kernel in k-space. This implies that the missing information can be estimated using the adjacent samples. If, for example, lines are skipped in k-space, the missing data can be calculated by a linear combination of its neighborhood points from all coils.

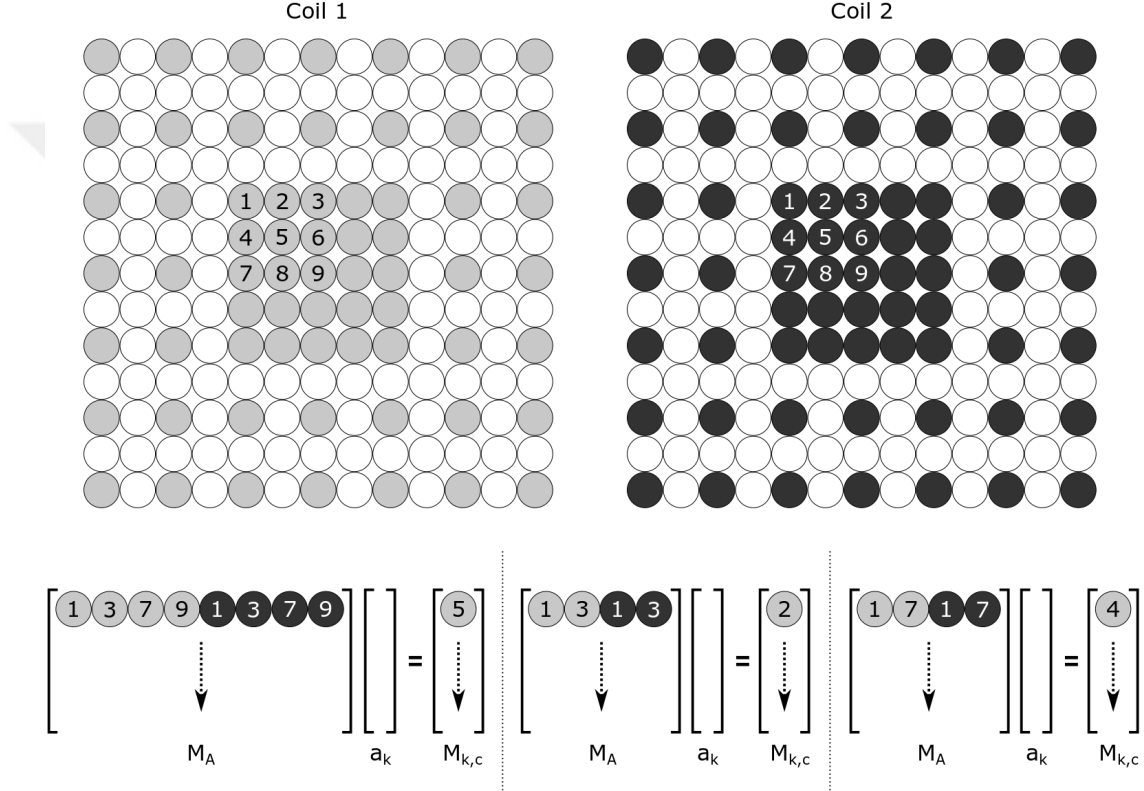


Figure 2.4: GRAPPA method. Shaded circles correspond to acquired samples; empty circles are points that were not acquired for acceleration. For every possible geometry due to sampling patterns, coefficients to estimate unsampled points from known samples in their neighborhood will be determined. In this example, lines were skipped both along k_x and k_y , therefore, some samples only have known adjacent samples in one direction, some in the other direction, and others have neither, with only diagonally adjacent samples. Coefficients are estimated for each case. Then, these coefficients will be used to calculate unknown points from their acquired neighborhood data.

In order to estimate the missing data, the relationship between adjacent samples needs to be known. For this purpose, a fully sampled calibration region is used, by acquiring the central part of k-space fully. This is used to solve for the relevant GRAPPA weights, as shown in Fig. 2.4.

Thus, this relationship can be written as the matrix equation:

$$\vec{M}_{k,c} = \mathbf{M}_A \cdot \vec{a}_k \quad (2.1)$$

where $M_{k,c}$ is the calibration data for k^{th} coil, M_A expresses neighborhood samples from all coils, and a_k are the GRAPPA coefficients from all coils to the k^{th} coil. Then, the GRAPPA weights can be estimated as:

$$\vec{a}_k = (\mathbf{M}_A^* \mathbf{M}_A + \lambda \mathbf{I})^{-1} \mathbf{M}_A^* \vec{M}_{k,c} \quad (2.2)$$

where λ is the Tikhonov regularization parameter that needs to be tuned.

GRAPPA method is sample geometry dependent, as GRAPPA weights depend on which samples are acquired and which are estimated. Depending on the acceleration, different sets of weights will be necessary for samples. This becomes particularly unmanageable for irregular sampling.

2.2.3 SPIRiT

SPIRiT is an alternative reconstruction that is based on similar ideas to GRAPPA [25]. However, instead of estimating only the missing data, SPIRiT applies the kernel to estimate all data based on all neighborhood points, which makes it geometry independent, allowing it to be applied to any trajectory. Afterwards, the known data is restored, and this process is repeated until data stops changing. Thus, SPIRiT enforces data consistency and calibration consistency iteratively.

A major benefit of SPIRiT is its flexibility; however, this comes at the cost of long reconstruction times due to iterations that can be computationally expensive. ESPIRiT is a computationally efficient successor to SPIRiT that makes use of similar ideas while achieving clinically feasible runtimes [27].

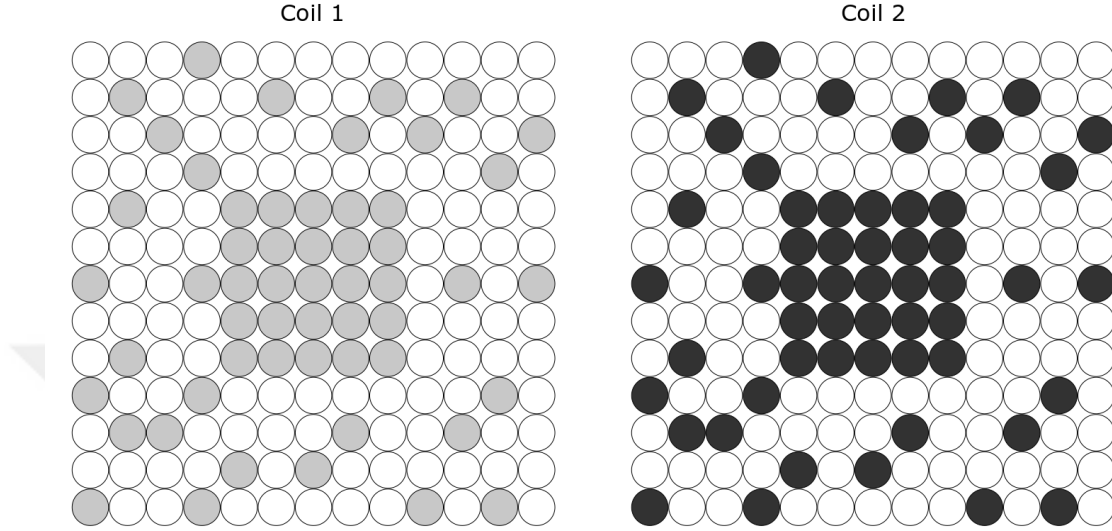


Figure 2.5: SPIRiT has the ability to do reconstructions for arbitrary sampling trajectories.

2.3 Non-Cartesian Parallel Imaging

Parallel imaging methods have initially been used for Cartesian undersampled trajectories. Applying these algorithms to non-Cartesian data combines the benefits of both methods, allowing high accelerations. Non-Cartesian methods generally use the same approach as Cartesian parallel imaging; however, applying Cartesian algorithms to non-Cartesian data is not always trivial. Aliasing patterns are more complex in non-Cartesian trajectories, which can make SENSE-based algorithms difficult to apply; and nonuniform undersampling of non-Cartesian data can make GRAPPA-based algorithms challenging; therefore, non-Cartesian trajectories require more complex reconstruction techniques [1].

Non-Cartesian parallel imaging also provides lower noise amplification, called g-factor, compared to Cartesian methods, and is able to better preserve SNR at comparable accelerations [28]. In addition, non-Cartesian trajectories typically contain more samples at the center of k-space, and most of the acceleration is done on higher frequency areas, which further helps reducing noise amplification.

Some non-Cartesian parallel imaging algorithms that have been proposed are

CG SENSE (Conjugate Gradient SENSE) [2], non-Cartesian GRAPPA [29, 30, 31]. SPIRiT and ESPIRiT are readily applicable to non-Cartesian trajectories. However, large reconstruction times for non-Cartesian data has been an important barrier for the adaptation of these methods in clinical settings. For on-line use, any delay longer than a few minutes makes its use unpractical; moreover, in clinical scans, the results are often needed promptly to determine a successive scan that may be required [32]. Therefore, quicker runtimes are necessary for widespread adoption of non-Cartesian parallel imaging methods. In this thesis, 3D var-FOV, an alternative such method, is explained and compared to these existing methods.

Chapter 3

3D Variable-FOV Reconstruction for Parallel Imaging with Non-Cartesian Variable-Density Sampling Trajectories

3.1 Introduction

Variable density trajectories have different sampling densities for different frequencies. Low frequencies are sampled densely while the density decreases towards high frequencies. 3D variable density stack of spirals trajectory (Fig. 3.1) and variable density cones [33] are some examples of 3D variable density trajectories. They can be used to densely sample low frequencies and reduce sampling density as the distance from k space origin increases.

When the highest sampling density in the trajectory is used to select the reconstruction FOV ($\text{FOV}_{\text{recon}}$), aliasing artifacts are introduced. PILS solves this problem by selecting a $\text{FOV}_{\text{recon}}$ that is supported by the lowest sampling density. This ensures that no aliasing artifacts will be introduced. However, this approach

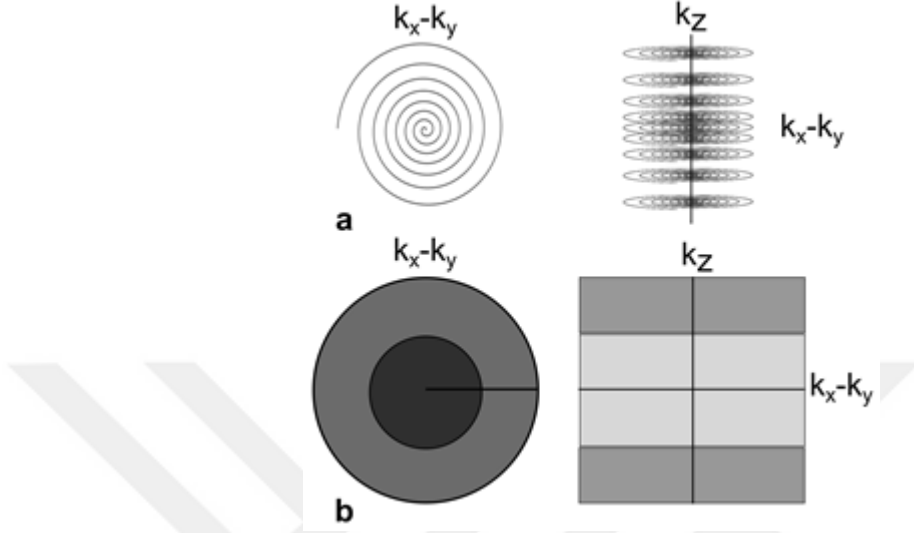


Figure 3.1: **a)** A stack of spirals k-space trajectory with varying sampling density in both the transverse k_x-k_y plane and the k_z dimension. The sampling density gradually decreases with increasing distance from the k-space origin. **b)** A representative separation of the non-Cartesian trajectory into annuli reflecting different sampling densities is shown. A total of 4 annuli are created by splitting k-space into two segments in the k_x-k_y plane and two segments along the k_z dimension.

causes the problem of not using the acquired data effectively. Moreover, for trajectories with very low sampling density towards the k-space periphery, i.e. high acceleration, selecting the $\text{FOV}_{\text{recon}}$ as the FOV supported by the lowest sampling density may cause the method to fail to cover the whole ROI. By increasing $\text{FOV}_{\text{recon}}$, a suboptimal balance can be maintained between aliasing and signal utilization. A different approach would be to fully exploit the variance in the supported FOV. Since the sampling density is varies across spatial frequencies, $\text{FOV}_{\text{recon}}$ also varies. As a result, densely sampled low frequency samples can be used to reconstruct a larger FOV. Therefore, the low-frequency data from each coil will be averaged in a larger spatial extent, thereby improving SNR.

For the trajectory shown in Fig. 3.1, the variable density FOV recon approach divides the stack of spirals in several non intersecting cylindrical shells. For example, k_x-k_y plane and k_z direction can each be divided to 2 annuli. This produces 4 annuli in total. The innermost annulus is a solid cylinder with its center of gravity at the origin. Within the same k_x-k_y plane projection, the rest

of the k_z direction can be combined into one subset. This subset covers two solid cylinders with one standing on top of the innermost and one standing below the innermost. These two subsets have the same average sampling density, hence the same supported FOV, in xy plane and in z direction. The outer k_x - k_y plane can be covered in the same sense, this time in cylindrical shell shape. By dividing k-space in this manner and reconstructing each annuli with the FOV supported by the average sampling density, or minimum sampling density to completely avoid aliasing artifacts, the sampled data is utilized more effectively than PILS.

The ideal FOV_{recon} should satisfy the following condition in order to avoid artifacts completely [3].

$$FOV_{recon} = \begin{cases} FOV_{acq} & FOV_{coil} \leq FOV_{acq} \\ 2FOV_{acq} - FOV_{coil} & FOV_{coil}/2 \leq FOV_{acq} \leq FOV_{coil} \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

These conditions ensure the complete removal of aliasing artifacts in the worst case; however, they are often too strict, and prevent all of the obtained data from being effectively utilized. In particular, given the SNR advantage of 3D acquisition over 2D, this condition can be relaxed [34]. Therefore, FOV_{recon} is bounded with $FOV_{coil}/2$.

3.2 Methods

3.2.1 3D Variable-FOV Reconstruction

Sampling density in k-space determines the corresponding FOV_{acq} . If a certain frequency component is used to reconstruct a FOV that is larger than the supported FOV, aliasing will occur. Therefore, by restricting the reconstruction FOV to the corresponding FOV_{acq} of the frequency component, aliasing artifacts can

be avoided. Moreover, this will lead to a substantial improvement in SNR.

The aforementioned idea can be applied to whole k-space to effectively utilize the data. By reconstructing every frequency component in FOV that is supported by the local sampling density artifacts can be avoided while minimizing signal loss. This requires a continuous change in $\text{FOV}_{\text{recon}}$ as the k-space is traversed. However, a good enough result can be obtained through a discretized implementation. Dividing k-space into several annuli and reconstructing these annuli in the FOV they support leads to an improvement over selecting a single $\text{FOV}_{\text{recon}}$ for the whole k-space.

3.2.2 Gridding

Although the operation described above can be performed by dividing the k-space into several annuli and then performing gridding and apodization for each annuli, the more computationally efficient way is first gridding the whole k-space and then dividing the resulting image into frequency annuli. Therefore, the first step in the method is a conventional gridding operation:

$$m_o(r) = \frac{1}{c(r) + g} \left\{ [c(r)m_s(r)] * \text{III} \left(\frac{r}{\alpha \text{FOV}_{\text{max}}} \right) \right\} \quad (3.2)$$

where $m_o(r)$ is the reconstructed image with the spatial coordinate r , $m_s(r)$ is the Fourier transform of the density-compensated k-space data, g is a constant added to prevent division by zero, $c(r)$ is the Fourier transform of the gridding kernel $C(k)$, and $\text{III}(r)$ is the Shah sampling function, where α is the oversampling factor and FOV_{max} is FOV_{acq} supported by the highest sampling density.

Afterwards, the data are partitioned into cylindrical shells for stack-of-spirals. A low resolution image is obtained either from the innermost annulus or a separate Fourier domain truncation. This low resolution image is used to estimate coil sensitivities. After that, the $\text{FOV}_{\text{recon}}$ for each annulus should be determined. For stack of spirals, the supported FOV in transversal plane can be determined by

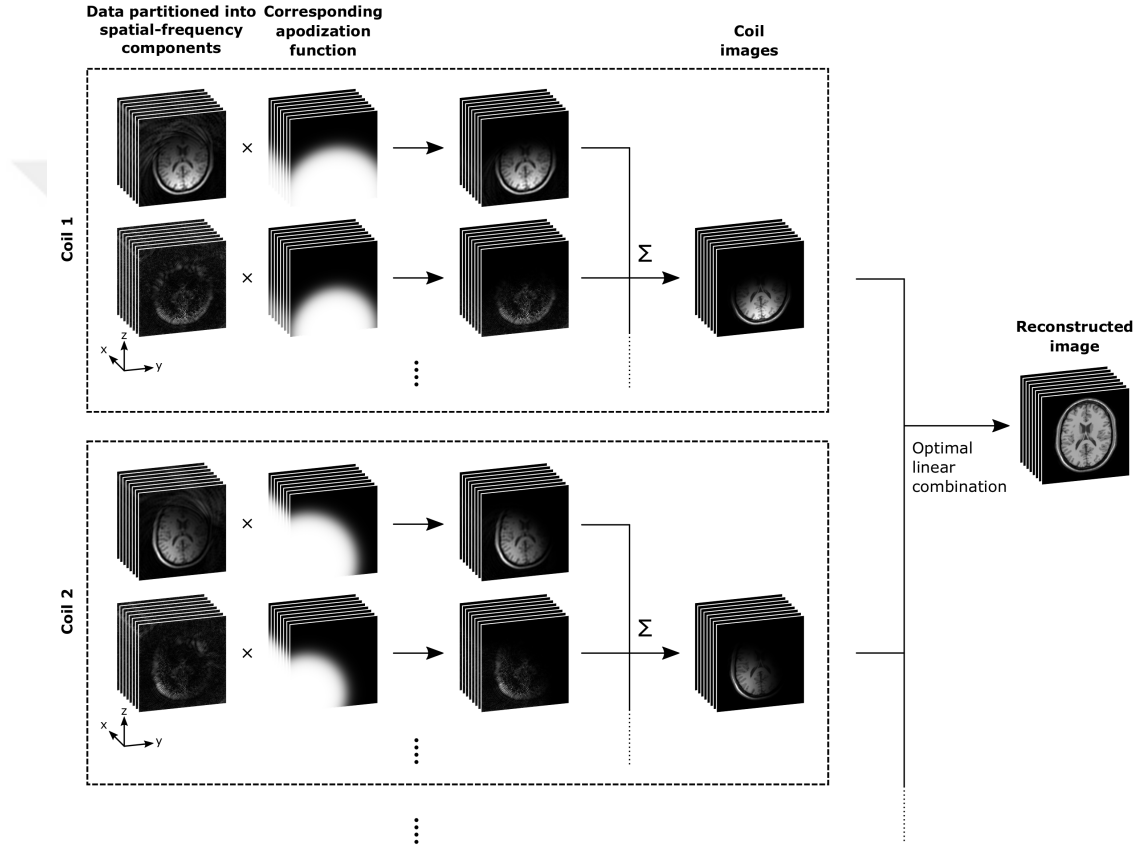


Figure 3.2: The combination of the reconstructed frequency bands from different coils and the summation of the combined bands to form the resulting image. For each coil, the reconstructed frequency bands and the estimated coil sensitivities are restricted with corresponding $\text{FOV}_{\text{recon}}$. For each frequency band, SOS combination of the restricted sensitivities are taken over all coils. The restricted sensitivities are normalized by their corresponding SOS combination. The resulting normalized sensitivities are used to weight the image from the corresponding band and coil. The combined frequency bands are summed to form the reconstructed image.

taking a slice from the k-space stack-of-spirals and finding the distance between neighbor points of the two dimensional variable density spiral annuli. While finding the supported FOV at each annuli, average sampling density or the minimum sampling density in that annuli can be used. The latter ensures artifact removal but in terms of signal utilization the former is superior. The z direction FOV_{acq} can be determined from the distance between the slices of k-space stack of spirals in the corresponding annulus. From the determined, average or maximum, FOV_{acq} , the corresponding $\text{FOV}_{\text{recon}}$ can be calculated according to the condition given in Eq. 3.1. The effective output can be expressed as:

$$m_v(r) = \sum_{b=1}^F a_b(r) [t_b(r) * m_o(r)] \quad (3.3)$$

where $t_b(r)$ is an ideal band-pass filter that passes only the spatial frequencies in the corresponding k-space partition Z_b , and $a_b(r)$ is an ideal apodization function that only supports $\text{FOV}_{\text{recon}}^b$ associated with Z_b . The resulting image $m_v(r)$ has a full-resolution central region, and the resolution gradually decreases toward the edges of the image. The k-space data is divided into 2 annuli in k_x - k_y plane, similar to shown in Fig. 3.1. While low frequency data is preserved throughout ROI, high frequency is restricted to the region near the center of the coil, which in this case coincides with the center of the image.

For each individual coil image, resolution will decrease as a given point's distance to the center of coil increases. However, this resolution loss can be compensated by the high frequency data from another coil. Therefore, as long as the high frequency $\text{FOV}_{\text{recon}}$ s of the coils cover the entire ROI, there will be no resolution loss in the final image. For heavily undersampled variable density trajectories, $\text{FOV}_{\text{recon}}$ should be selected to cover the whole ROI, possibly at the expense of introducing high frequency artifacts in the final image. Therefore, the minimum $\text{FOV}_{\text{recon}}$ is limited to $\text{FOV}_{\text{coil}}/2$ in Eq. 3.1.

3.2.3 Combination of Variable-FOV Images Across Coils

After all the frequency bands of each coil are reconstructed, they are assembled with optimal linear combination in order to maximize SNR. First, the reconstructions of the same frequency bands from all coils are combined. In this step, the reconstructed images are weighted with complex conjugate of its sensitivity normalized by the sum-of-squares (SOS) combination of all sensitivities [cite]. Sensitivities can be estimated by a low-resolution image of the coil. The resulting images are then summed to produce the reconstructed image.

Since the $\text{FOV}_{\text{recon}}$ is different for different frequency bands, the weights should be computed separately. $\text{FOV}_{\text{recon}}$ s of same frequency bands from different coils overlap and because of different $\text{FOV}_{\text{recon}}$ s for frequency bands, at a given pixel more coils contribute to low-frequency data than that for high frequencies, resulting in an overweighting of low frequencies accompanied by image blurring. In order to solve this problem, the frequency bands should be weighted and combined separately.

Given the number of frequency bands F and number of coils C , the resulting image P can be expressed in terms of the unprocessed image of frequency band b and from coil i S_{bi} , corresponding weight W_{bi} , low resolution image D_i and windowing ($\text{FOV}_{\text{recon}}$ restricting) function M_{bi} .

$$P(r) = \sum_{b=1}^F \sum_{i=1}^C K(r)_{bi} W(r)_{bi} \quad (3.4)$$

$$K(r)_{bi} = S(r)_{bi} M(r)_{bi} \quad (3.5)$$

$$W(r)_{bi} = \frac{M(r)_{bi} D(r)_i}{\sqrt{\sum_{i=1}^C M(r)_{bi} |D(r)_i|^2}} \quad (3.6)$$

3.2.4 Total Variation Filtering

Finally, total variation filtering is applied on the resulting image to reduce noise and aliasing artifacts. Minimizing the objective $J(x) = \|P - x\|_2^2 + \lambda \|\nabla x\|_1$ using iterative-clipping algorithm:

$$x^{(i+1)} = P - \nabla^t z^{(i)} \quad (3.7)$$

$$z^{(i+1)} = \text{clip} \left(z^{(i)} + \frac{\nabla x^{(i+1)}}{\alpha}, \frac{\lambda}{2} \right) \quad (3.8)$$

where ∇^t is the adjoint finite-difference operator, $z^{(1)} = 0$ and $\alpha = 4$. 5 iterations was used in our implementation. The important parameter to choose here is λ , which is selected based on the estimated noise of the image. The noise is estimated by calculating the variance at a part of the image known to correspond to background. For example, at 20 SNR noise levels, 0.02 times the maximum absolute value of the image was chosen as λ .

3.2.5 Experiments

The 3D var-FOV method was demonstrated on 3D GRE acquisition of the knee (0.5 mm isotropic) [35] and a simulation of T1-weighted acquisition of the brain (1 mm isotropic) based on anatomical model of normal brain [36]. Knee data is from an 8-channel phased-array, and with a FOV = 160 mm x 160 mm x 110 mm. The datasets were resampled on 3D stack-of-spirals trajectories of varying acceleration factors (R). FOV_{acq} was designed to linearly decrease from 10 cm to 1 cm for knee and 18 cm to (9, 4, 2) cm for the brain. The trajectories had R = 4.1 with 6 interleaves for the knee, and R = (2.5, 3.7, 4.2) with (12, 8, 7) interleaves for the brain. The reconstructed images have matrix size of (320x320x220) and (240x240x240), for knee and brain respectively. In order to demonstrate the effect of noise on the performances of the reconstruction methods, different amounts of

bivariate complex noise was added to brain data in image domain, simulating 20 and 10 SNR levels. For the knee dataset, the data used was already at 20 SNR.

As a first step for 3D var-FOV reconstruction, gridding reconstructions within full FOV were performed on each coil. An oversampling factor of $a=2$ and a 4 pixel wide 3D Kaiser-Bessel kernel were used. After the gridding step, the coil images were separated into non-intersecting annuli in Fourier domain. The radii of the annuli vary linearly. For each annulus, the $\text{FOV}_{\text{recon}}$ was found by the average sampling density and the condition provided in Eq. 3.1. For enhanced performance 12 annuli were used for all datasets.

The k-space data in each partition were then transformed into the image domain. In order to achieve a smooth transition in FOV restriction, the apodization functions ($a(r)$) were chosen as Fermi windows with a full width at half maximum (FWHM) equal to the corresponding $\text{FOV}_{\text{recon}}$ and a transition width of 6 pixels. Meanwhile, the coil sensitivities were estimated from the central k-space samples within a diameter of 0.04, assuming a normalized maximum of 1. Finally, separate sets of spatial-frequency component images were combined using the weights calculated from the estimated sensitivities.

SOS reconstructions were obtained via sum-of-squares combination of gridding reconstructions within the full FOV for non-Cartesian datasets and for Cartesian dataset it was obtained via sum-of-squares combination of channel images, again within full FOV, reconstructed by inverse Fourier transform of the k-space data. For PILS reconstructions, the $\text{FOV}_{\text{recon}}$ is set to the largest supported FOV, which is 18 cm for the brain dataset, and 10 cm for the knee dataset. All ESPIRiT reconstructions were obtained using a kernel size of $[6, 6, 6]$, a Tykhonov parameter of 0.01 for sensitivity estimation, an eigenvalue threshold of 0.001. 20 iterations were used for both datasets, as increasing the number of iterations beyond that did not provide meaningful improvement in performance despite increased computation time.

All data were processed using MATLAB code (R2015b, MathWorks) on a workstation with a 24-core Intel Xeon E5 CPU and 192 GB RAM. ESPIRiT

reconstructions for comparison were done with Berkeley Advanced Reconstruction Toolbox (BART) [37].

3.3 Results

Fig. 3.3 shows the SOS, PILS, ESPIRiT, and 3D variable-FOV reconstructions of the aforementioned brain data collected with $R = 2.5$ at $\text{SNR} = 20$ and $\text{SNR} = 10$, as well as the error maps for those reconstructions. The figures show that SOS images have severe aliasing artifacts, and while PILS has less, artifacts due to undersampling in spiral trajectories are still visible; and error is still especially high in the central region, where sensitivities of the coils are low. Both ESPIRiT and the variable-FOV method effectively suppress these artifacts. However, 3D var-FOV is better at removing the artifacts, as well the noise, which is especially visible at the higher noise levels. var-FOV's better performance compared to ESPIRiT can also be observed from their SSIM and PSNR values (Tables 3.2 and 3.3): At $R = 2.5$, the PSNR of the 3D var-FOV reconstruction is approximately 2.5 dB higher than that of ESPIRiT for $\text{SNR} = 20$, and 4.5 dB higher for $\text{SNR} = 10$. For SSIM, the differences are again considerable, 0.944 vs. 0.970 for $\text{SNR} = 20$, and 0.849 vs. 0.912 for $\text{SNR} = 10$, again in favor of 3D var-FOV.

Fig. 3.4 and 3.5 show the same comparison at higher accelerations, at $R = 3.7$ and 4.2. As the acceleration increases, ESPIRiT's and 3D var-FOV's reconstructions start to become more comparable, though the var-FOV reconstructions are generally still better, especially at higher noise levels. The regime where ESPIRiT outperforms 3D var-FOV is at high acceleration with little noise, as seen in Table 3.1, but their performance remains comparable, and both are significantly better than SOS and PILS reconstructions. At higher noise, 3D var-FOV outperforms ESPIRiT, with a PSNR difference of 4 dB, and SSIM of 0.921 vs. 0.854, for $R = 4.2$ and $\text{SNR} = 10$.

Fig. 3.6 shows the reconstructions for the knee data at $r = 4.1$ and at 20 SNR. Again, 3D var-FOV and ESPIRiT perform similarly, also as evidenced by their

similar PSNR and SSIM values (Table 3.4 – both has PSNR around 30 dB and SSIM at 0.94s, 3D var-FOV being slightly better in average), but var-FOV is better at preserving details, while removing noise more effectively.

Comparison of reconstruction times (Table 3.5) indicates that, while 3D var-FOV requires more processing time than SOS and PILS, its reconstruction times are less than half of ESPIRiT’s for all accelerations. In our experiment, 3D var-FOV on average took 139 seconds, compared with ESPIRiT’s 353 seconds, for the acceleration $R = 2.5$, with similar ratios at other acceleration levels. In terms of computational complexity, 3D var-FOV has an additional factor of **number of annuli** $\times$, in addition to PILS’s **number of channels** $\times$ **matrix size**; however, the initial gridding constitutes an important portion of both reconstruction times, thus their time differences is not as stark and ESPIRiT is at greater disadvantage due to being iterative.

To demonstrate the benefit of varying the FOV over the PILS method, results comparison with no filtering is given in Fig. 3.7 The advantage of varying the FOV can be seen especially at the center, as var-FOV makes better use of the data in that part; and overall it is a significant improvement over PILS.

3.4 Discussion

Variable density acquisitions generally have high sampling density at lower frequencies, while density decreases towards the k-space periphery. Different frequency bands support different FOVs; thus, by reconstructing each frequency component in the FOV supported by the component, artifacts can be avoided. Variable FOV method was proposed to discretize this process and reconstruct the lower frequencies at a larger extent while confining under-sampled high frequency to a smaller extent with high coil sensitivity. In this paper, a generalized and more efficient implementation of this method for 3D variable density data was presented.

In 3D, compared to PILS, var-FOV provides a more efficient way of utilizing data since frequency components are reconstructed in the FOV they support. In PILS, the $\text{FOV}_{\text{recon}}$ the images will be cropped to can be chosen $\text{FOV}_{\text{acq-high}}$ or $\text{FOV}_{\text{acq-low}}$ or any value in between. The first option will perform better at removing artifacts since no frequency component will be reconstructed in a FOV larger than supported, but this will prevent the data from being utilized efficiently. The second option will use the most of the data but it will result in artifacts. Choosing a value in between can be a way to balance the trade-off, but the selection of this value is an issue. Var-FOV utilizes the data, and the $\text{FOV}_{\text{recon}}$ selection of the annuli is easier. While PILS is faster than var-FOV, the speed difference is compensated with higher SNR and better artifact suppression.

Comparing var-FOV and ESPIRiT, they have varying degrees of success under different regimes of noise and acceleration, and in different parts of the reconstructed image. While one sometimes outperforms the other as discussed in the results, their overall performance is comparable. Therefore, for general applications, their difference lies in their substantial difference in computational burden, as var-FOV requires much less processing time.

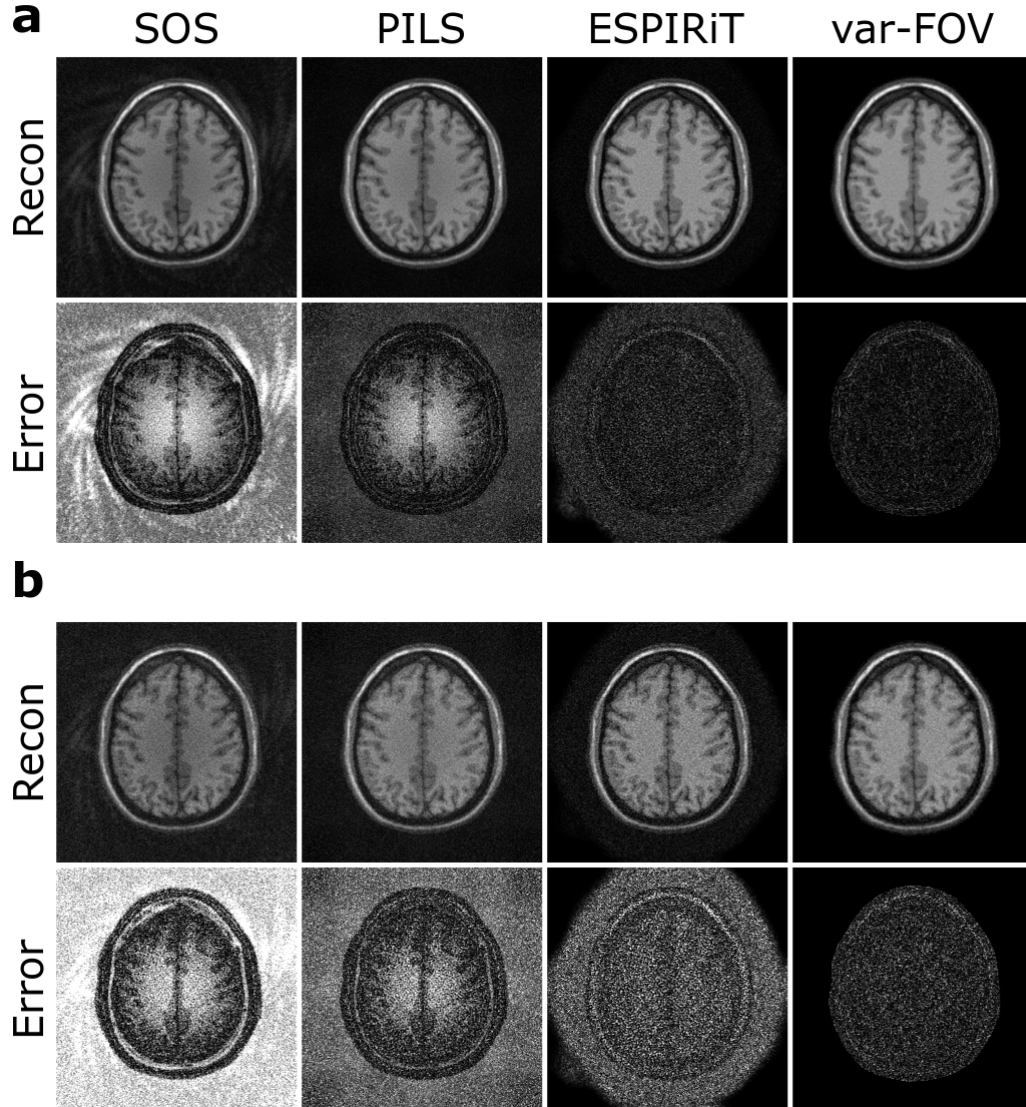


Figure 3.3: SOS, PILS, ESPIRiT and 3D var-FOV reconstructions of brain data at $R=2.5$ for (a) $\text{SNR}=20$, (b) $\text{SNR}=10$. Error is shown in the range $[0 \ 0.2]$. SOS images have severe aliasing, and while PILS has less, artifacts due to under-sampling in spiral trajectories is still visible; and error is still especially high in the central region, where sensitivities of the coils are low. Both ESPIRiT and the 3D variable-FOV method effectively suppress these artifacts. However, var-FOV is better at removing the artifacts, as well the noise, which is especially visible at the higher noise levels.

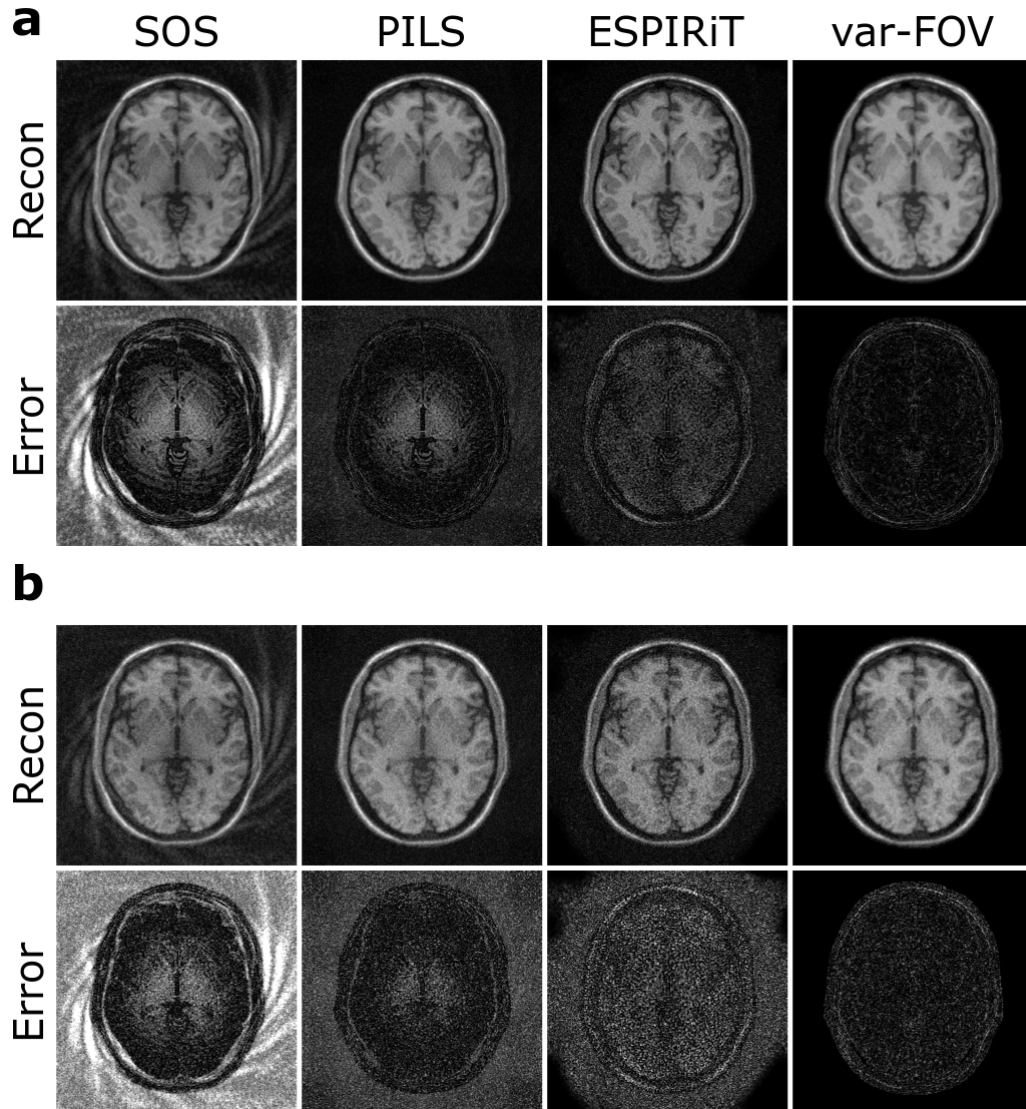


Figure 3.4: SOS, PILS, ESPIRiT and var-FOV reconstructions of brain data at $R=3.7$ for (a) $\text{SNR}=20$, (b) $\text{SNR}=10$. Error is shown in the range $[0 \ 0.3]$.

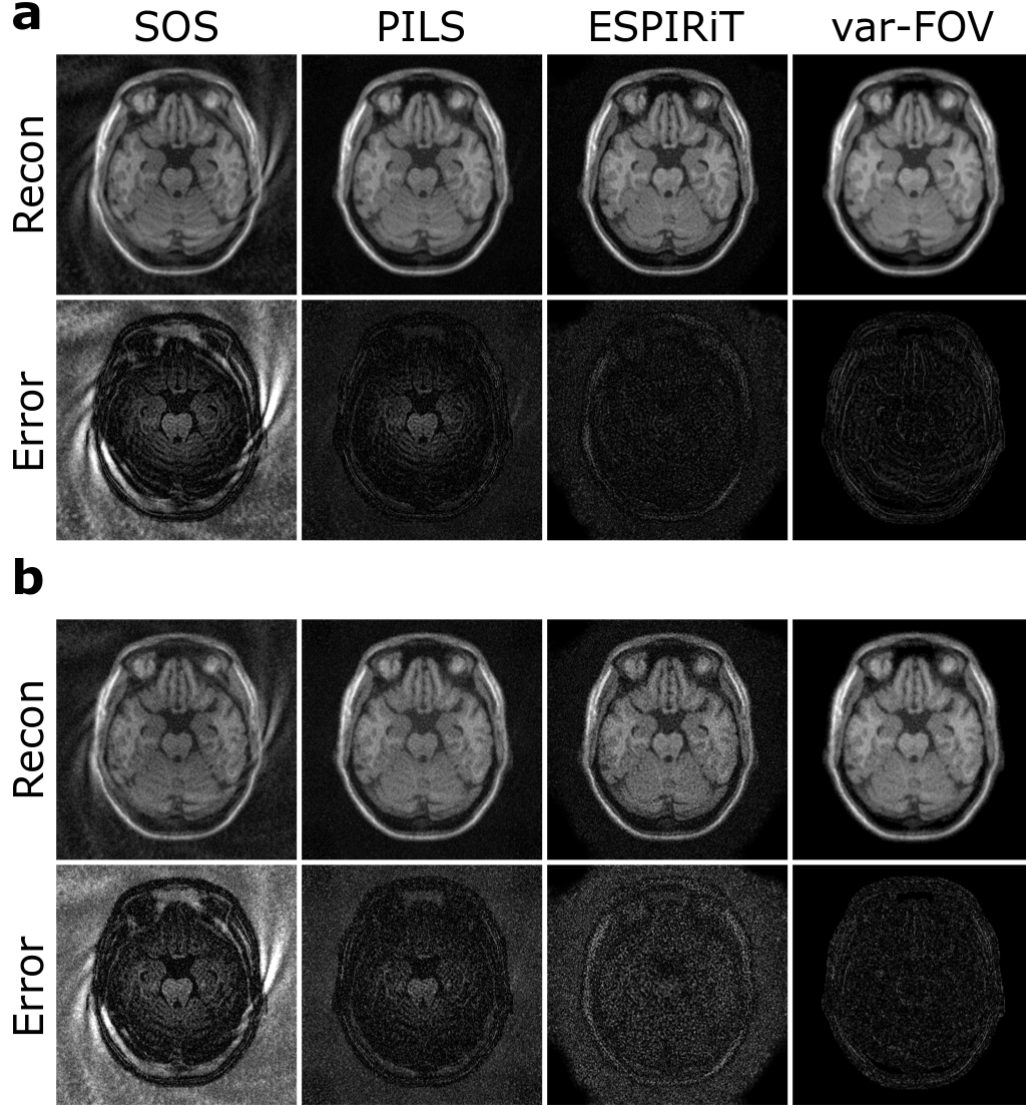


Figure 3.5: SOS, PILS, ESPIRiT and var-FOV reconstructions of brain data at $R=4.2$ for (a) $\text{SNR}=20$, (b) $\text{SNR}=10$. Error is shown in the range $[0 \ 0.4]$. At high acceleration, ESPIRiT's and 3D var-FOV's reconstructions start to become more comparable, though the var-FOV reconstructions are generally still better, especially at higher noise levels.

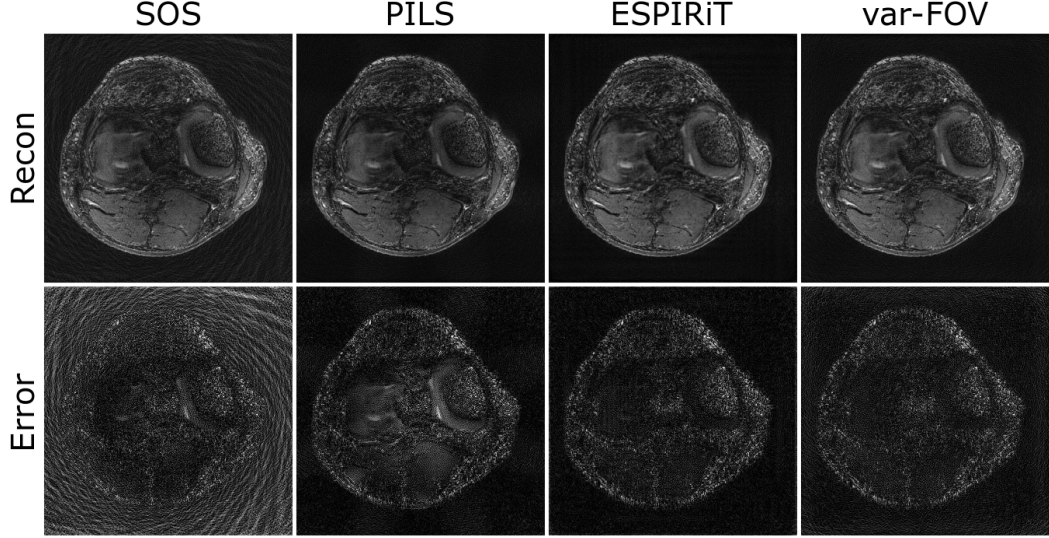


Figure 3.6: SOS, PILS, ESPIRiT and var-FOV reconstructions of knee data at $R=4.1$ for $\text{SNR}=20$. Error is shown in the range $[0 \ 0.2]$. 3D var-FOV and ESPIRiT perform similarly, but var-FOV is better at preserving details, while removing noise more effectively.

Table 3.1: PSNR and SSIM measurements on brain data with added noise – $\text{SNR}=30$

		SOS	PILS	ESPIRiT	Var-FOV
$R = 2.5$	PSNR	24.88 ± 1.52	27.84 ± 1.29	32.92 ± 1.24	32.20 ± 2.28
	SSIM	0.937 ± 0.014	0.967 ± 0.005	0.972 ± 0.008	0.983 ± 0.005
$R = 3.7$	PSNR	23.51 ± 1.90	27.77 ± 1.19	31.79 ± 1.54	30.15 ± 1.70
	SSIM	0.917 ± 0.032	0.961 ± 0.008	0.970 ± 0.008	0.974 ± 0.009
$R = 4.2$	PSNR	23.67 ± 1.79	28.11 ± 0.71	32.08 ± 1.24	28.20 ± 3.03
	SSIM	0.914 ± 0.032	0.958 ± 0.009	0.970 ± 0.010	0.970 ± 0.010

PSNR and SSIM measurements on SOS, PILS, ESPIRiT and var-FOV reconstructions of brain data. PSNR in dB is listed as mean $\pm$ s.e. across 7 axial cross-sections spanning the volume.

Table 3.2: PSNR and SSIM measurements on brain data with added noise – SNR=20

		SOS	PILS	ESPIRiT	Var-FOV
$R = 2.5$	PSNR	23.85 ± 1.35	26.24 ± 1.40	29.26 ± 1.00	31.72 ± 1.46
	SSIM	0.913 ± 0.019	0.943 ± 0.010	0.944 ± 0.016	0.970 ± 0.008
$R = 3.7$	PSNR	22.94 ± 1.60	26.80 ± 0.76	28.52 ± 1.63	30.13 ± 1.07
	SSIM	0.901 ± 0.033	0.944 ± 0.011	0.943 ± 0.016	0.964 ± 0.011
$R = 4.2$	PSNR	23.22 ± 1.51	27.43 ± 0.42	28.97 ± 1.31	28.47 ± 2.46
	SSIM	0.900 ± 0.034	0.943 ± 0.012	0.945 ± 0.017	0.962 ± 0.012

PSNR and SSIM measurements on SOS, PILS, ESPIRiT and var-FOV reconstructions of brain data. PSNR in dB is listed as mean $\pm$ s.e. across 7 axial cross-sections spanning the volume.

Table 3.3: PSNR and SSIM measurements on brain data with added noise – SNR=10

		SOS	PILS	ESPIRiT	Var-FOV
$R = 2.5$	PSNR	21.00 ± 0.57	23.10 ± 0.89	22.28 ± 0.85	26.69 ± 0.83
	SSIM	0.838 ± 0.036	0.868 ± 0.027	0.849 ± 0.041	0.912 ± 0.024
$R = 3.7$	PSNR	21.31 ± 0.77	24.46 ± 0.62	22.48 ± 0.72	27.69 ± 0.85
	SSIM	0.844 ± 0.042	0.882 ± 0.026	0.849 ± 0.041	0.920 ± 0.023
$R = 4.2$	PSNR	21.56 ± 0.83	24.99 ± 0.51	22.97 ± 0.43	27.01 ± 1.18
	SSIM	0.847 ± 0.043	0.886 ± 0.025	0.854 ± 0.040	0.921 ± 0.024

PSNR and SSIM measurements on SOS, PILS, ESPIRiT and var-FOV reconstructions of brain data. PSNR in dB is listed as mean $\pm$ s.e. across 7 axial cross-sections spanning the volume.

Table 3.4: PSNR and SSIM measurements on knee data with no added noise

		SOS	PILS	ESPIRiT	Var-FOV
$R = 4.1$	PSNR	28.36 ± 1.04	28.91 ± 1.03	29.56 ± 1.04	29.91 ± 1.05
	SSIM	0.927 ± 0.003	0.940 ± 0.002	0.940 ± 0.003	0.944 ± 0.002

PSNR and SSIM measurements on SOS, PILS, ESPIRiT and var-FOV reconstructions of knee data. PSNR in dB is listed as mean $\pm$ s.e. across 7 axial cross-sections spanning the volume.

Table 3.5: Reconstruction times for the brain dataset

	SOS	PILS	ESPIRiT	Var-FOV
$R = 2.5$	86 s	93 s	353 s	139 s
$R = 3.7$	77 s	86 s	356 s	131 s
$R = 4.2$	75 s	77 s	343 s	127 s

Reconstruction time measurements in seconds for (240x240x240) pixels matrix size brain data at different accelerations.

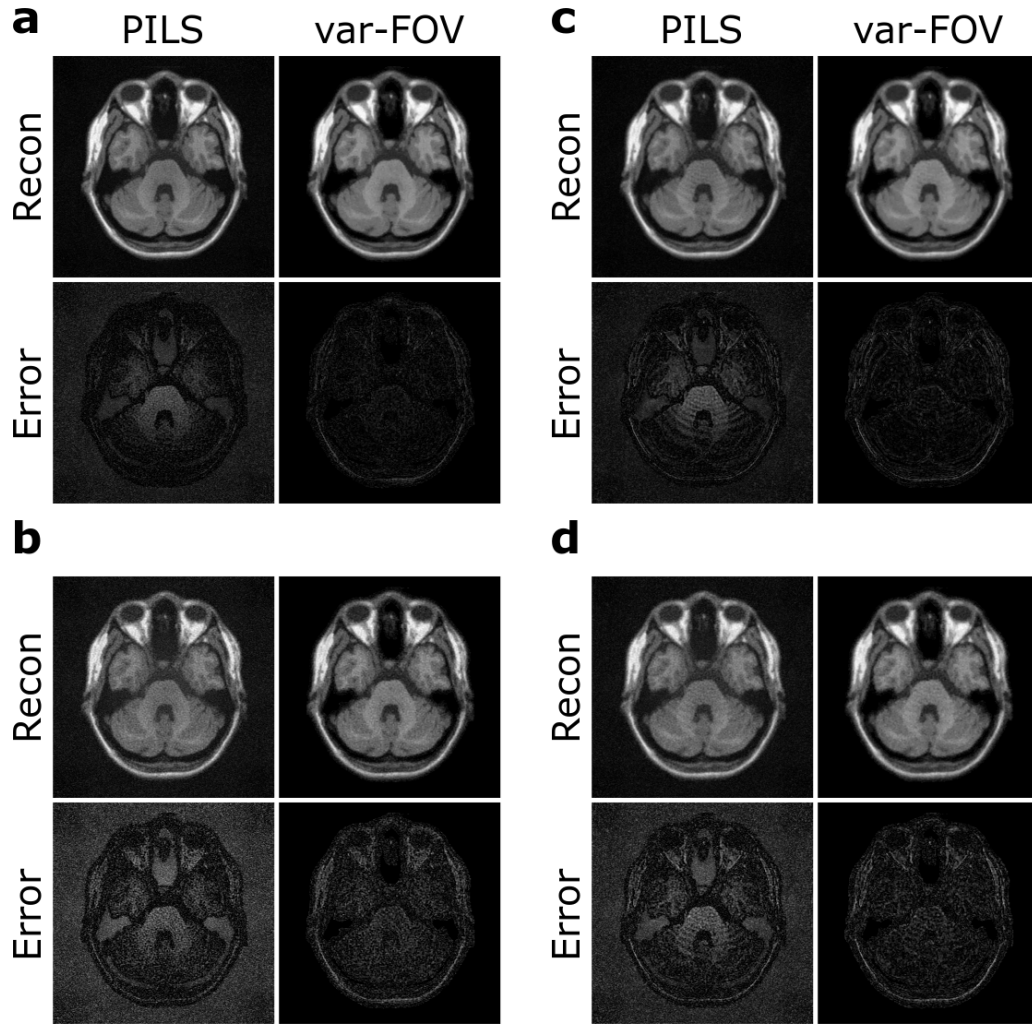


Figure 3.7: Comparison of PILS and var-FOV with no filters for brain data at $R=2.5$ for (a) $\text{SNR}=20$, (b) $\text{SNR}=10$, and at $R=4.2$ for (c) $\text{SNR}=20$, (d) $\text{SNR}=10$. Error is shown in the range $[0 \ 0.4]$.

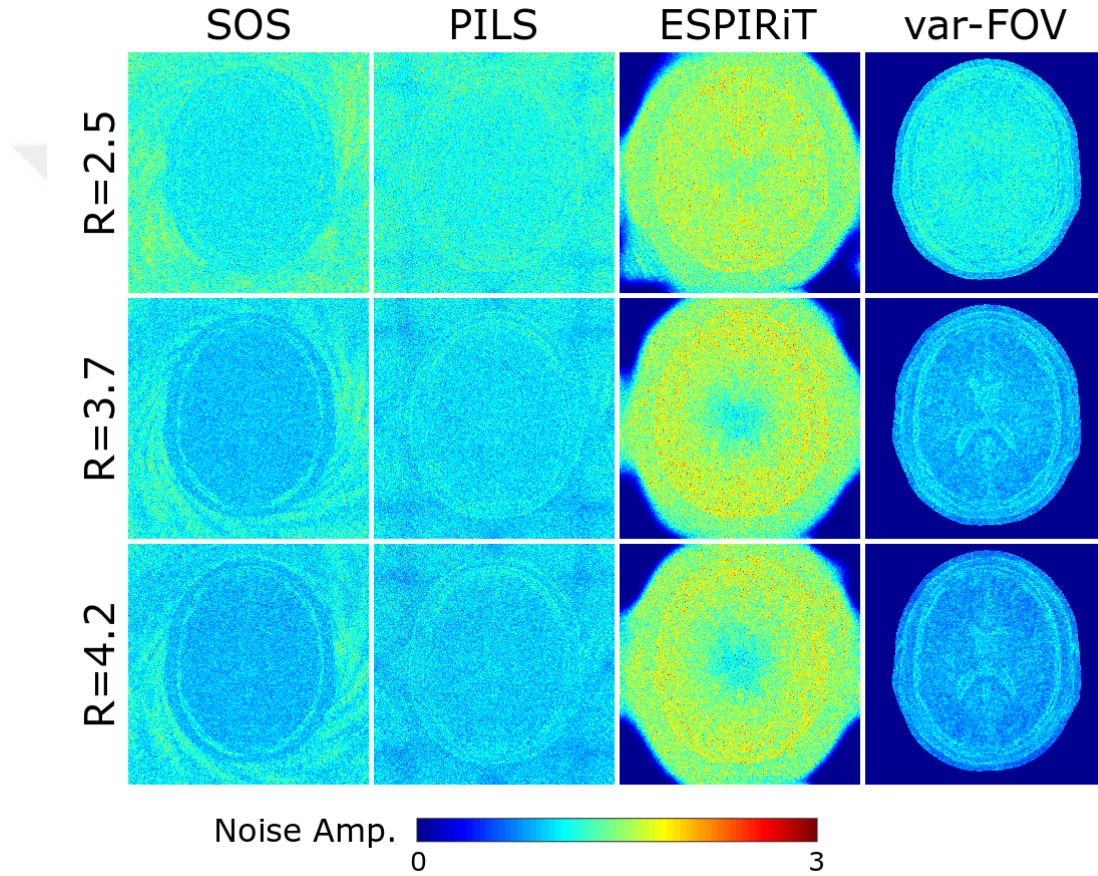


Figure 3.8: Noise amplification maps, scale $[0-3]$. For each reconstruction method, identical datasets were given. Datasets included the same brain data, corrupted by 40 different 4% bivariate complex noise for each coil in image domain. For each dataset that was corrupted by one of 40 noise instances, a sum-of-squares combination of coil images was obtained as a reference. Each reconstruction, including the ones forming the reference, is normalized to have maximum of 1. Then, for each reconstruction method, standard deviation maps were obtained. These maps were divided by the standard deviation map of the reference.

Chapter 4

Conclusion

The image domain variable-FOV method proposed and examined in this thesis provides high SNR and artifact-free reconstructions of 3D variable density data, and it is a fast and computationally efficient alternative to existing methods. It does not have the problem of finding optimal combination of parameters and $\text{FOV}_{\text{recon}}$ calculations can be modified to control the tradeoff between artifacts and SNR for specific data. Moreover, its speed makes real-time reconstructions of 3D non-Cartesian datasets feasible.

4.1 Future Work

Here, the proposed method has been applied to non-Cartesian data, as that is where its advantages are most immediately self-evident. Its performance compared to other methods for Cartesian data still needs to be investigated. Further improvement to its speed might be achieved by implementing it with more lower level optimizations comparable to what was used for ESPIRiT. Better performance might be achieved by making use of deep neural networks for denoising [38, 39]. Another question that needs to be explored is how it compares against

the compressed sensing reconstruction method for 3D non-Cartesian data, recently proposed by Baron et al. [40].



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