

DEVELOPING A METHODOLOGY FOR THE DESIGN AND OPTIMIZATION OF THE PRESSURE-SWIRL ATOMIZERS

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DEVELOPING A METHODOLOGY FOR THE DESIGN AND OPTIMIZATION OF THE PRESSURE-SWIRL ATOMIZERS

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To my wonderful family for always supporting me.

To my beautiful girl, Lily. For being my loyal companion.

ABSTRACT

Atomization is the process of disintegration of bulk liquid into smaller droplets and has been an almost century-long research topic of fluid dynamics. Devices used for atomization are called atomizers and many different types of atomizers using different strategies to achieve atomization have been developed. Pressure-swirl atomizer is one of the most widely used types of atomizer due to its simplicity and ability to achieve a wide range of droplet sizes and coverage area.

Even though the geometry of the pressure-swirl atomizer is simple, its internal flow field includes complex phenomena such as turbulence, liquid/gas interface, recirculation zones, and instabilities. Modeling the performance of pressure-swirl atomizer have been challenged by many researchers, and different models have been developed. Out of these models, 2D and 3D numerical simulations were proposed to be the most accurate ones, even though the accuracy of 2D simulations was questioned by many researchers. Other models, 1D models, and semi-empirical correlations also exist in the literature yet the accuracy of the latter is reported to be low.

Models that could be used for the optimization of pressure-swirl atomizers, where it will be used to perform thousands of calculations while having low error values are lacking in the literature. Even though numerical modeling can result in accurate predictions, computation time prevents it to be used in optimization calculations. This study presents the developed models that could be used for such calculations and present their accuracy in comparison with conducted experiments and simulations.

In this study, first, inlet modeling of the 2D simulations is inspected. It was shown that the accuracy of the 2D simulations can be improved drastically by adjusting the inlet velocity components to include the effect of flow deformation that occurs due to

tangential ports. It is shown that a model developed in an earlier study can be used to describe this deformation and calculate the inlet velocity components.

Later, existing semi-empirical models in the literature along with the experimental data are presented and inspected. Overall 1,777 experimental data points, obtained from 34 different studies, are provided and cataloged. It is seen that, when the accuracy of semi-empirical correlations is evaluated in a global range, rather than in the range that they are developed at, error values increase significantly. A new set of semi-empirical correlations that are globally more accurate are obtained using experimental data. However, even with the developed correlations, error values in the calculation of pressure difference and droplet sizes are still higher than 50 %.

Due to the lack of the desired level of accuracy in the semi-empirical modeling, boundary layer modeling approach is adopted for the description of the internal flow field of the pressure-swirl atomizer. Two different models, one for the straight sections (swirl chamber and orifice) and another for the convergent or divergent sections (convergent section or trumpet), are obtained. Evaluation of the model for the straight section is done with the comparison of the experimental data of open-end pressure swirl atomizers. Comparison of convergent or divergent section models is done with the 3D simulation of the unique atomizer geometry that is developed in the framework of this dissertation. Finally, the accuracy of the combined model is evaluated with the experimental data of close-end pressure-swirl atomizers.

A large number of 3D full geometry simulations are conducted to reason the coupling of internal flow dynamics with the uniformity of the pressure-swirl atomizer spray. These simulations are done with atomizer geometry having a unique shape, and later three of the geometries are selected to be manufactured. Of the selected geometries, two have uniform sprays, while the third one has a non-uniform spray. Due to the unique shape of the inlets, these atomizers are manufactured with the method called Laser Lithography, and the manufacturing tolerance of the geometries

was less than $1\text{ }\mu\text{m}$. Experiments of the atomizers are done with Laser Induced Fluorescence method at the radial plane, and obtained results showed a non-uniform internal flow results in a spray where cluster of droplets with large diameters exist. A semi-empirical model for the description of the non-uniformity is obtained, and comparison with the existing model and experimental data in the literature is done.

Spray of the pressure-swirl atomizer takes different shapes as the pressure and flow rate through the atomizer is increased, and the final form of the spray is called fully-developed spray. Development of the spray of pressure-swirl atomizers is also modeled in this study. For this purpose, both experiments conducted in the framework of this study and experimental data obtained from the literature are used. A model that is based on the bulk Reynolds and Weber numbers is obtained using the gradient-descent method, and comparisons with the existing correlations in the literature are done.

Droplet diameter modeling is done by utilizing the model called Linear Instability Sheet Atomization (LISA), which is often used by commercial CFD codes. This model is analyzed by examining the equations, and eight different correlations, differing in the simplifications, are obtained. Evaluation of these correlations is done by making comparisons with the experimental data of this study, which is obtained with the Shadowgraphy method. All of the droplet diameter experiments are conducted in the primary break-up region of the atomization. Comparisons have shown that when the break-up length of the atomizer can be modeled accurately, the LISA model can be used to estimate the representative droplet diameter. For the description of the droplet diameter distribution, a new model is obtained for the calculation of the spreading parameter. This parameter is often taken as constant in the commercial CFD codes, yet it was shown that it significantly affects the obtained distributions. Comparison of obtained model with the model of commercial CFD codes is also presented.

Finally, combination of the obtained models and their order of use for performing an optimization study is presented. It was also presented that the obtained model can perform the calculation of single geometry in 20.0 seconds with a single core, whereas 2D and 3D simulations require 208 core-hour and 2,880 core-hour, respectively. It can be seen that the developed model is 37,440 times faster than the 2D simulation and 518,440 times faster than the 3D simulations.



ÖZETÇE

Atomizasyon, sıvının küçük damlacıklara parçalanması işlemidir ve neredeyse yüzyıldır akışkanlar dinamiğinin araştırma konusu olmuştur. Atomizasyon için kullanılan cihazlara atomizer denir ve atomizasyon elde etmek için farklı stratejiler kullanılan birçok farklı tipte atomizer geliştirilmiştir. Basınç girdaplı atomizer, basitliği ve çok çeşitli damlacık boyutu ve sprey kapsama alanı elde edebilme yeteneği nedeniyle en yaygın kullanılan atomizer türlerinden biridir.

Basınç girdaplı atomizerin geometrisi basit olsa da, dahili akış alanı türbülans, sıvı/gaz arayüzü, devir daim bölgeleri ve akış kararsızlığı gibi karmaşık olayları içerir. Basınç girdaplı atomizer performansının modellenmesi birçok araştırmacı tarafından çalışılmış ve farklı modeller geliştirilmiştir. Bu modeller arasından 2B ve 3B nümerik simülasyonların en doğru modeller olduğu öne sürülse de 2B simülasyonların doğruluğu birçok araştırmacı tarafından sorgulanmıştır. Literatürde başka modeller, 1B modeller ve yarı ampirik korelasyonlar da mevcuttur, ancak ikincisinin doğruluğunun düşük olduğu bildirilmektedir.

Basınç girdaplı atomizerlerin optimizasyonunda kullanılabilecek, hata değerleri düşük olan modeller literatürde bulunmamaktadır. Nümerik modelleme ile doğruluğu yüksek sonuçlar elde edilebilse de, hesaplama süresi bu modellerin optimizasyon hesaplamalarında kullanılmasını engeller. Bu çalışma, bu tür hesaplamalar için kullanılabilecek geliştirilmiş modelleri sunmakta ve yapılan deneyler ve simülasyonlarla karşılaştırmalı olarak doğruluklarını ortaya koymaktadır.

Bu çalışmada ilk olarak 2 boyutlu simülasyonların giriş kısmının modellenmesi incelenmiştir. Giriş hızı bileşenlerinin hesabında, teğet girişler nedeniyle oluşan akış deformasyonunun etkisini içerecek şekilde hesaplama yapıldığında, 2B simülasyonların

doğruluğunun büyük ölçüde iyileştirilebileceği gösterilmiştir. Deformasyon miktarı için daha önceki bir çalışmada geliştirilen bir modelin, giriş hızı bileşenlerini hesaplamak için kullanılabileceği gösterilmiştir.

Sonrasında, deneysel verilerle birlikte literatürdeki mevcut yarı ampirik modeller sunulmuş ve incelenmiştir. 34 farklı çalışmadan elde edilen toplam 1.777 deneysel veri noktası sağlanmış ve kataloglanmıştır. Yarı ampirik korelasyonların geliştirildikleri aralıkta değil, global bir aralıkta değerlendirildiğinde, hata değerlerinin önemli ölçüde arttığı görülmüştür. Deneysel veriler kullanılarak küresel olarak daha doğru olan yeni yarı ampirik korelasyon seti elde edilmiştir. Ancak, elde edilen korelasyonlarla bile, basınç farkı ve damlacık boyutlarının hesaplanmasındaki hata değerleri hala %50'den yüksektir.

Yarı ampirik modellemede istenen doğruluk seviyesinin elde edilememesi nedeniyle basınç girdaplı atomizerin iç akış alanının modellenmesi için sınır tabakası modelleme yaklaşımı benimsenmiştir. Atomizerin düz kısımları için (girdap odası ve orifis) ve yakınsak veya ıraksak kısımlar için (yakınsak kesit veya trompet) olmak üzere iki farklı model elde edilmiştir. Düz kısımlar için geliştirilen modelin değerlendirilmesi, açık uçlu basınçlı girdaplı atomizerlerin deneysel verileri ile yapılmıştır. Yakınsak veya ıraksak kısımlar için geliştirilen modelin değerlendirmesi ise, bu çalışma çerçevesinde geliştirilen atomizer geometrisinin 3 boyutlu simülasyonları ile yapılmıştır. Son olarak, birleştirilmiş modelin doğruluğu, kapalı uçlu basınç girdaplı atomizerlerin deneysel verileriyle değerlendirilmiştir.

Basınç girdaplı atomizer iç akış alanının, spreyn homojenliği üzerindeki etkisini anlayabilmek için çok sayıda 3B simülasyon gerçekleştirilmiştir. Bu simülasyonlar, farklı bir şekle sahip atomizer geometrisi ile yapılmıştır ve daha sonra bunların arasından üç geometri üretilmek üzere seçilmiştir. Seçilen geometrilerden ikisi homojen spreylere sahipken, üçüncüsü homojen olmayan spreye sahiptir. Girişlerin farklı şekli nedeniyle, bu atomizerler Lazer Litografi adı verilen yöntemle üretilmiştir

ve geometrilerin imalat toleransı $1\ \mu\text{m}$ 'den düşüktür. Atomizerlerin deneyleri, radyal düzlemde Laser Induced Fluorescence yöntemi ile yapılmıştır. Elde edilen sonuçlar, homojen olmayan iç akış yapısının spreyde büyük çaplı damlacık kümelerinin bulunmasına yol açtığını göstermiştir. İç akışın homojenliği hesaplayabilmek için yarı ampirik bir model geliştirilmiş ve literatürdeki mevcut model ve deneysel verilerle karşılaştırması yapılmıştır.

Basınç girdaplı atomizerin spreyi, atomizerden geçen akış debisi ve basınç arttıkça farklı şekiller alır ve spreyin son haline tam gelişmiş sprey denir. Basınç girdaplı atomizerlerin spreyinin gelişimi de bu çalışmada modellenmiştir. Bu amaçla hem bu çalışma çerçevesinde yapılan deneyler hem de literatürden elde edilen deneysel veriler kullanılmıştır. Gradient-descent yöntemi kullanılarak Reynolds ve Weber sayılarına dayalı bir model elde edilmiş ve literatürdeki mevcut korelasyonlarla karşılaştırması yapılmıştır.

Ticari CFD yazılımlarında sıklıkla kullanılan Linear Instability Sheet Atomization (LISA) adlı model kullanılarak damlacık çapı modellenmesi yapılmıştır. Denklemlerin incelenmesi ile bu modelin analizi yapılmıştır ve farklı varsayımlarla yapılan sadeleştirilmeler sonucu sekiz farklı korelasyon elde edilmiştir. Elde edilen korelasyonların değerlendirilmesi, bu çalışma kapsamında Shadowgraphy yöntemiyle elde edilen deneysel verileri ile yapılan karşılaştırmalar ile yapılmıştır. Damlacık çapı deneylerinin tamamı, atomizasyonun birincil parçalanma bölgesinde gerçekleştirilmiştir. Sonuçlar, spreyin parçalanma uzunluğunun doğru bir şekilde modellenebileceği durumlarda, temsili damlacık çapının elde edilmesi için LISA modelinin kullanılabileceğini göstermiştir. Damlacık çapı dağılımının elde edilebilmesi için, yayılma parametresinin hesaplanmasında kullanılabilecek yeni bir model elde edilmiştir. Bu parametre ticari CFD yazılımlarında sabit olarak alınsa dahi, elde edilen sonuçlar ile önemli bir etkiye sahip olduğu gösterilmiştir. Geliştirilen modelin ticari CFD yazılımlarında yer alan model ile karşılaştırılması da yapılmıştır.

Son olarak, elde edilen modellerin kombinasyonu ve bir optimizasyon çalışması yapmak için kullanım sırası sunulmuştur. Elde edilen modelin tek bir geometrinin hesabını tek işlemci çekirdeği kullanarak 20.0 saniyede yapabildiği belirtilmiştir. 2B ve 3B simülasyonların sırasıyla 208 çekirdek-saat ve 2.880 çekirdek-saat gerektirdiği bilgisi sunulmuştur. Geliştirilen modelin 2B simülasyona göre 37.440 kat, 3B simülasyona göre 518.440 kat daha hızlı olduğu görülmektedir.



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TABLE OF CONTENTS

DEDICATION	iii
ABSTRACT	iv
ÖZETÇE	viii
ACKNOWLEDGEMENTS	xii
LIST OF TABLES	xvii
LIST OF FIGURES	xxi
I INTRODUCTION	1
1.1 Motivation	6
1.2 Objectives	9
1.3 Research Scope	10
1.4 Thesis Organisation	10
II EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUES	12
2.1 Experimental Setup	15
2.2 Measurement Techniques	16
2.2.1 High Speed Imaging	16
2.2.2 Shadowgraphy	31
2.2.3 LIF/Mie	36
2.3 Atomizers	40
III NUMERICAL SETUP	42
3.1 2D Numerical Setup	42
3.2 3D Numerical Setup	52
IV EMPIRICAL MODEL	55
4.1 Introduction	55
4.2 Open Source Database	56
4.2.1 Close-End Pressure-Swirl Atomizer	58

4.2.2	Open-End Pressure-Swirl Atomizer	91
4.3	Available Models and Evaluation	95
4.3.1	Discharge Coefficient, Flow Number and Pressure Difference	96
4.3.2	Film Thickness	106
4.3.3	Spray Angle	113
4.3.4	Droplet Size	120
4.3.5	Break-up Length	126
4.4	Obtaining Empirical Model	130
4.4.1	Discharge Coefficient and Pressure Difference	133
4.4.2	Film Thickness	136
4.4.3	Spray Angle	137
4.4.4	Droplet Size	137
4.4.5	Break-up Length	138
V	INTERNAL FLOW MODEL	139
5.1	Introduction	139
5.2	Derivation of the Boundary Layer Model	141
5.2.1	Boundary Layer Model for Straight Section	142
5.2.2	Boundary Layer Model for Convergent and Divergent Section	176
5.3	Combined Boundary Layer Model	193
VI	UNIFORMITY ANALYSIS	201
6.1	Introduction	201
6.2	Design of the Atomizer	206
6.3	Results	207
6.4	Model	222
VII	SPRAY DEVELOPMENT ANALYSIS	225
7.1	Introduction	225
7.2	Previous Models	228
7.3	Experimental Data	230

7.4	Obtaining Spray Development Model	233
VII	DROPLET DIAMETER MODEL	245
8.1	Introduction	245
8.2	LISA Model and Analysis	245
8.2.1	Presentation of LISA Model	248
8.2.2	Analysis of LISA Model	262
8.3	Obtaining Droplet Diameter Model	269
8.3.1	Modeling of the Spreading Parameter	275
IX	PRESENTATION OF THE COMBINED MODEL	279
X	DISCUSSION AND CONCLUSIONS	282
APPENDIX A	— EXPERIMENTAL DATA OF LITERATURE .	288
REFERENCES		364
VITA		373

LIST OF TABLES

1	Experimental Apparatus	16
2	Validation Cases for 2D Axisymmetric Simulations	47
3	Results of Studied Cases for 2D Axisymmetric Simulations	51
4	Geometrical Parameters of Close-End Atomizers	59
5	Studied Atomizer	66
6	Break-up Length Values of Atomizer-19	90
7	Geometrical Parameters of Open-End Atomizers	93
8	Summary of the Experimental Data	95
9	Correlations for Calculating C_D	97
10	Correlations for Calculating FN	101
11	Error Values of Most Accurate C_D and FN Correlations - Close-End .	102
12	Error Values of Most Accurate Correlations for Pressure Difference - Close-End	103
13	Error Values of Most Accurate C_D and FN Correlations - Open-End .	105
14	Correlations for Calculating Film Thickness	107
15	Error Values of Most Accurate Film Thickness Correlations - Close-End	111
16	Error Values of Most Accurate Film Thickness Correlations - Open-End	112
17	Correlations for Calculating Spray Angle	114
18	Error Values of Most Accurate Spray Angle Correlations - Close-End	120
19	Error Values of Most Accurate Spray Angle Correlations - Open-End	121
20	Correlations for Calculating SMD	122
21	Error Values of Most Accurate SMD Correlations - Close-End	126
22	Correlations for Calculating Break-up Length	127
23	Comparison of Error Values of Most Accurate Break-up Correlations with the Derived Correlation	138
24	Error Values of Boundary Layer Model and Obtained Semi-Empirical Model - Open-End	175

25	Error Values of Boundary Layer Model and Obtained Semi-Empirical Model - Close-End	200
26	Geometrical Details of the Simulated Cases	208
27	Comparison of Numerical and Experimental Values at 3.34 g/s	214
28	Error Values of the Derived and Literature Spray Development Models	244
29	Largest Disturbance Values and Corresponding Wave Numbers for Atomizer-16	265
30	Effect of Spreading Factor on Representative Droplet Diameters . . .	269
31	List of the Atomizers used in Shadowgraphy Experiments and Flow Rates	270
32	Results of the Shadowgraphy Experiments	272
33	Error Values in Rosin-Rammler Diameter for the Obtained Model . .	273
34	Dimensions of the Atomizers [8]	288
35	Experimental Results - 1 [3]	293
36	Dimensions of the Atomizers - 1 [3]	293
37	Dimensions of the Atomizers - 2 [3]	294
38	Dimensions of the Atomizers - 3 [3]	295
39	Experimental Results - 2 [3]	296
40	Dimensions of the Atomizers - 4 [3]	296
41	Dimensions of the Atomizers - 5 [3]	297
42	Dimensions of the Atomizers - 6 [3]	298
43	Dimensions of the Atomizers - 7 [3]	300
44	Dimensions of the Atomizers - 8 [3]	300
45	Dimensions of the Atomizers [25]	301
46	Physical Properties of Working Liquid [26]	303
47	Dimensions of the Atomizers [26]	303
48	Experimental Results at 2 MPa [26]	303
49	Physical Properties of Working Liquid [28]	305
50	Dimensions of the Atomizers [28]	305
51	Dimensions of the Atomizers [29]	306

52	Dimensions of the Atomizers [30]	308
53	Experimental Results [30]	309
54	Dimensions of the Atomizers [18]	310
55	Constant Dimensions of the Atomizers [31]	311
56	Altered Geometrical Parameters and Results [31]	311
57	Physical Properties of Liquid [32]	320
58	Constant Dimension of the Atomizers [32]	320
59	Dimensions of the Atomizers [32]	321
60	Dimensions of the Atomizers [33]	323
61	Dimensions of the Atomizers [34]	325
62	Dimension of the Atomizer and Results [35]	326
63	Dimensions of the Atomizers [9]	327
64	Dimensions of the Atomizers [36]	329
65	Dimensions of the Atomizers [37]	332
66	Dimensions of the Atomizers [38]	333
67	Physical Properties of the Liquid [38]	334
68	Dimensions of the Atomizers [39]	337
69	Dimensions of the Atomizers [40]	339
70	Physical Properties of the Liquid [41]	341
71	Dimensions of the Atomizers [42]	342
72	Dimensions of the Atomizers [43]	343
73	Dimensions of the Atomizers [44]	344
74	Dimensions of the Atomizers [45]	345
75	Names and Physical Properties of Liquids [46]	347
76	Dimensions of the Atomizers [46]	347
77	Dimensions of the Atomizers [47]	349
78	Experimental Results [47]	349
79	Dimensions of the Atomizers [48]	350
80	Experimental Results [48]	351

81	Dimensions of the Atomizers [49]	352
82	Dimensions of the Atomizers [50]	354
83	Dimensions of the Atomizers [51]	356
84	Dimensions of the Atomizers [52]	357
85	Dimensions of the Atomizers [53]	359



LIST OF FIGURES

1	Processes that govern atomization	2
2	Droplet size distribution and representative diameters	3
3	Schematic of a)close-end PSA b)open-end PSA	4
4	Schematic for inlets having a)circular b)rectangular cross-section . . .	5
5	Schematic showing performance parameters of PSA	6
6	Design of the spray chamber	12
7	Frame of the spray chamber	14
8	Schematic of the experimental setup	16
9	Scaling plate used for the high-speed imaging	17
10	Limited region of the high-speed images	18
11	Mean image of the high-speed images	19
12	Obtained a)experimental image, b)binary image	20
13	Detection of the tip of the nozzle	21
14	Tracking the edges of the spray	22
15	Visual validation images for break-up length	23
16	Obtained values of break-up length	24
17	Statistical properties of obtained break-up length values	25
18	Effect of frame rate on the obtained break-up length values	26
19	Measured min. and max. angle values	27
20	Visual validation images for spray angle	28
21	Obtained values of spray angle	29
22	Statistical properties of obtained spray angle values	30
23	Effect of frame rate on the obtained spray angle length values	31
24	Image of the Shadowgraphy system	32
25	Droplet diameter and velocity calculation	33
26	Shadowgraphy experiment locations	34

27	Calculated volume distribution and Rosin-Rammler and Modified Rosin-Rammler diameter and spreading parameters	35
28	Effect of the image number on the obtained SMD values	36
29	Orientation of the CMOS cameras for radial LIF/Mie measurement .	37
30	3D calibration plate of LaVision company	38
31	Single and averaged experiment images	39
32	Raw and mapped experiment images	40
33	Flow characteristics of the atomizers	41
34	Boundary conditions for 2D axisymmetric simulations	43
35	Splitting the inlet velocity into radial and tangential components . . .	44
36	Experimental results and model fitted to the data [5]	45
37	Mesh sensitivity study for the 2D axisymmetric simulations	48
38	Spray angle calculation of 2D axisymmetric simulations	49
39	Calculated and experimental pressure difference of 2D axisymmetric simulations	50
40	Used meshes for 3D simulations	54
41	Range of the collected experimental data in terms of Bulk Reynolds and Weber numbers	57
42	Schematic of close-end pressure-swirl atomizer	58
43	Discharge coefficient of Atomizer-22, 23 and 24 of this study	66
44	Spray angle of Atomizer-22, 23 and 24 of this study	67
45	SMD of Atomizer-22, 23 and 24 of this study	67
46	Break-up length of Atomizer-22, 23 and 24 of this study	68
47	Diameter measurement of atomizers used in this study	76
48	Discharge coefficient of Atomizer-1, 2, and 3 of this study	77
49	Spray angle of Atomizer-1, 2, and 3 of this study	77
50	Breakup length of Atomizer-1, 2, and 3 of this study	78
51	Discharge coefficient of Atomizer-4, 5, and 6 of this study	79
52	Spray angle of Atomizer-4, 5, and 6 of this study	79
53	Breakup length of Atomizer-4, 5, and 6 of this study	80

54	Discharge coefficient of Atomizer-7, 8, and 9 of this study	81
55	Spray angle of Atomizer-7, 8, and 9 of this study	81
56	Breakup length of Atomizer-7, 8, and 9 of this study	82
57	Discharge coefficient of Atomizer-10, 11, and 12 of this study	83
58	Spray angle of Atomizer-10, 11, and 12 of this study	83
59	Breakup length of Atomizer-10, 11, and 12 of this study	84
60	Discharge coefficient of Atomizer-13, 14, and 15 of this study	85
61	Spray angle of Atomizer-13, 14, and 15 of this study	85
62	Breakup length of Atomizer-13, 14, and 15 of this study	86
63	Discharge coefficient of Atomizer-16, 17, and 18 of this study	87
64	Spray angle of Atomizer-16, 17, and 18 of this study	87
65	Breakup length of Atomizer-16, 17, and 18 of this study	88
66	Discharge coefficient of Atomizer-19, 20, and 21 of this study	89
67	Spray angle of Atomizer-19, 20, and 21 of this study	89
68	SMD of the selected atomizers	91
69	Schematic of open-end pressure-swirl atomizer	92
70	Close-end discharge coefficient and flow number error values	102
71	Close-end pressure difference error values	103
72	Effect of error in discharge coefficient on the error of pressure difference	104
73	Open-end discharge coefficient error values	105
74	Open-end pressure difference error values	106
75	Close-end film thickness error values	111
76	Open-end film thickness error values	112
77	Close-end spray angle error values	119
78	Open-end spray angle error values	120
79	Close-end SMD error values	126
80	Break-up error values	130
81	Proposed System of Calculations for Derived Semi-Empirical Correlations	133

82	Pareto front for close-end atomizer discharge coefficient and pressure difference	134
83	Pareto front for open-end atomizer discharge coefficient and pressure difference	136
84	Potential core and boundary layer schematic of PSA internal flow . .	140
85	Boundary layer thickness obtained in [13]	143
86	Comparison boundary layer velocity profiles	158
87	Comparison of boundary layer thickness values calculated with different velocity profiles	162
88	Comparison of calculated boundary layer thickness from obtained models	171
89	Comparison of calculated air core radius from 2D simulation and boundary layer model	172
90	Comparison of calculated velocity components from 2D simulation and boundary layer model	173
91	Comparison of calculated velocity components from 2D simulation and boundary layer model with initial air core value increased to 0.5R . .	174
92	Schematic of spherical coordinate system	177
93	Calculated boundary layer thickness from obtained convergent model	192
94	Comparison of calculated velocity components from 3D simulation and boundary layer model at the exit	193
95	Schematic of mass conversation between swirl chamber and convergent section	194
96	Calculated boundary layer thickness for close-end case	196
97	Comparison of velocity components at the start of the convergent section	197
98	Comparison of velocity components at the start of the orifice	198
99	Comparison of velocity components at the exit	199
100	Comparison of the distribution of arrays with same mean and standard deviation values	205
101	Surface of the fluid region of the designed atomizer	206
102	Image of the tested atomizer	207
103	Effect of convergence on angle on the mass flux distribution - constant swirl chamber diameter (Cases 1-9-10)	210

104	Effect of convergence on angle on the mass flux distribution - constant orifice diameter (Cases 11-12-13)	211
105	Effect of diameter of swirl chamber on the mass flux distribution - constant convergence angle (Cases 14-15-16)	212
106	Calculation of uniformity	212
107	Effect of geometrical parameters on the uniformity	213
108	Location of uniformity calculations made for the simulations and experiments	215
109	Instantaneous high-speed images of manufactured atomizers	215
110	Mean of the high-speed images of manufactured atomizers	216
111	Comparison of LIF/Mie experiment with the simulation for Case-1 . .	217
112	Comparison of LIF/Mie experiment with the simulation for Case-16 .	217
113	Comparison of LIF/Mie experiment with the simulation for Case-13 .	218
114	Obtained instantaneous high-speed images (row 1), average of high-speed images (row 2), obtained mass fluxes from the simulations at 5 mm downstream (row 3) and LIF/Mie experiment results (row 4) . .	219
115	Velocity distribution of Case-1 and Case-13 at the centerline of the inlets	220
116	Velocity vectors at tangential inlet ports for Case-1 and Case-13 . . .	221
117	Structure of air core and liquid sheet for Case-13	222
118	Comparison of semi-empirical correlations for calculating uniformity using simulations	223
119	Comparison of semi-empirical correlations for calculating uniformity using data of [30]	224
120	Schematic of spray development stages [2]	225
121	Schematic of spray development stages [3]	226
122	Obtained high-speed images of spray development stages	227
123	Stage-by-stage modeling of spray development [17]	229
124	Useful and non-useful spray points of this study	231
125	Useful and non-useful spray points of literature	232
126	Useful and non-useful spray points of combined data (present study and literature)	234
127	Separation line demonstration	235

128	Obtained separation line for combined data (present study and literature)	239
129	Comparison of literature with the combined spray development data [16]	240
130	Comparison of literature model with the combined spray development data [19]	241
131	Comparison of literature model with the combined spray development data [17]	242
132	Comparison of literature model with the combined spray development data [3]	243
133	Comparison of literature model with the combined spray development data [18]	244
134	Primary and secondary atomization regions of the spray	246
135	Sinusous mode	250
136	Varicose mode	250
137	$\tanh(kh)$	253
138	Difference between $\tanh(kh)$ and kh (left) and difference between the $\tanh(kh)$ and 1 (right)	254
139	$\coth(kh)$	254
140	Difference between $\coth(kh)$ and $1/kh$ (left) and difference between the $\coth(kh)$ and 1 (right)	255
141	Effect of density ratio on hyperbolic tangent function	256
142	Effect of density ratio on hyperbolic cotangent function	257
143	Calculated disturbance (w_r) for Atomizer-16	263
144	Calculated non-dimensional disturbance for Atomizer-16	264
145	Calculated break-up length values w.r.t to disturbance ratio for Atomizer-16	265
146	Calculated thickness of the sheet at break-up locations w.r.t to disturbance ratio for Atomizer-16	266
147	Calculated Rosin-Rammler droplet diameter w.r.t to disturbance ratio for Atomizer-16	267
148	Effect of Rosin-Rammler spreading parameter on the droplet size distribution	268
149	Comparison of Rosin-Rammler and modified Rosin-Rammler distributions with the experimental data	271

150	Calculated gas phase Weber number for Shadowgraphy experiments and comparison with limit value	274
151	Comparison of Rosin-Rammler (Eq. 2) and modified Rosin-Rammler (Eq. 3) spreading parameter values with value of CFD	275
152	Parametric evaluation of modified Rosin-Rammler spreading parameter with the flow parameters	276
153	Comparison of calculated and experimental values of modified Rosin-Rammler spreading parameter	277
154	Comparison of obtained model with the model of CFD softwares and experimental data	278
155	Order of the use of derived models	280
156	Experimental results - Geometry-1 [8]	289
157	Experimental results - Geometry-2,3,4 [8]	289
158	Experimental results - Geometry-5 [8]	290
159	Experimental results - Geometry-6 [8]	290
160	Experimental results - Geometry-5 and 7 [8]	291
161	Experimental results - Geometry-8 [8]	291
162	Experimental results - Geometry-9 [8]	292
163	Experimental results - Geometry-10 [8]	292
164	Experimental results - Geometry-5 [3]	293
165	Experimental results - Geometry-6-15 Discharge Coefficient [3]	294
166	Experimental results - Geometry-6-15 Spray Angle [3]	295
167	Experimental results - Geometry-16 and 17 [3]	296
168	Experimental results - Geometry-22 and 23 [3]	297
169	Experimental results - Geometry-24, 25, 26, 27 [3]	298
170	Experimental results - Geometry-28, 29, 30, 31, 32, 33 - Discharge Coefficient [3]	299
171	Experimental results - Geometry-28, 29, 30, 31, 32, 33 - Spray Angle [3]	299
172	Experimental results - Geometry-34, 35, 36 [3]	300
173	Experimental results - Geometry-37 and 38 [3]	301
174	Experimental results - 1 [25]	302

175	Experimental results - 2 [25]	302
176	Experimental results - 1 [27]	304
177	Experimental results - 2 [27]	304
178	Experimental results - 1 [28]	305
179	Experimental results - 2 [28]	306
180	Experimental results [29]	307
181	Experimental results [18]	310
182	Experimental results - 1 [32]	321
183	Experimental results - 2 [32]	322
184	Experimental results - 3 [32]	322
185	Experimental results - 4 [32]	323
186	Experimental results - 1 [33]	324
187	Experimental results - 2 [33]	325
188	Experimental results [34]	326
189	Experimental results - 1 [9]	328
190	Experimental results - 2 [9]	329
191	Experimental results - 1 [36]	330
192	Experimental results - 2 [36]	330
193	Experimental results - 3 [36]	331
194	Experimental results - 4 [36]	331
195	Experimental results - 1 [37]	332
196	Experimental results - 2 [37]	333
197	Experimental results - 1 [38]	335
198	Experimental results - 2 [38]	336
199	Experimental results - 3 [38]	337
200	Experimental results [39]	338
201	Experimental results - 1 [40]	339
202	Experimental results - 2 [40]	340
203	Experimental results - 3 [40]	340

204	Experimental results [41]	341
205	Experimental results [42]	342
206	Experimental results [43]	343
207	Experimental results [44]	344
208	Experimental results - 1 [45]	345
209	Experimental results - 2 [45]	346
210	Experimental results - 3 [45]	346
211	Experimental results - 1 [46]	347
212	Experimental results - 2 [46]	348
213	Experimental results - 3 [46]	348
214	Experimental results - 1 [49]	352
215	Experimental results - 2 [49]	353
216	Experimental results - 3 [49]	353
217	Experimental results - 4 [49]	354
218	Experimental results - 1 [50]	355
219	Experimental results - 2 [50]	355
220	Experimental results - 1 [51]	356
221	Experimental results - 2 [51]	357
222	Experimental results - 1 [52]	358
223	Experimental results - 2 [52]	358
224	Experimental results - 3 [52]	359
225	Experimental results - 1 [53]	360
226	Experimental results - 2 [53]	361
227	Experimental results - 3 [53]	362
228	Experimental results - 4 [53]	363

CHAPTER I

INTRODUCTION

The term atomization is used to describe the process of disintegration of the bulk liquid into large number of droplets. Devices used for this purpose are known as atomizers or injectors and the resultant flow is called spray.

Break-up of the liquid jet or sheet into many droplets leads to a higher surface area, enhancing the mass and heat transfer into or from the liquid. Due to this phenomenon sprays are used in many industries such as agriculture crop spraying, pharmaceutical industries, combustion devices such as internal combustion, liquid rocket, and gas turbine engines [1,2].

Sprays and atomizers have been an almost century-long research subject in the area of fluid dynamics. There are many pioneering works and many different types of atomizers have been proposed and utilized in different applications. Even though there are many different types of atomizers, atomization is achieved by one of the three different strategies. These can be listed as;

- kinetic energy of the liquid
- exposure to high-velocity air or gas
- applying mechanical (rotating or vibrating device) or electrical energy

Atomization of liquids is a complex physical process governed by many external and internal parameters as described in Figure 1. Depending on the type of atomizer, the internal flow field can be turbulent and unsteady, it could include complicated phenomena such as swirling motion, two-phase flow, cavitation, vibration, etc. At

the exit of the atomizer, the liquid interacts with the surrounding air, and aerodynamic forces develop on the surface of the liquid. The magnitude of these forces depends on the relative velocity, and the density of the ambient gas and these forces tend to deform and break-up the liquid, whereas, viscous and surface tension forces resist this. Development and growth of small disturbances lead to loss of stability and eventually disintegration of the liquid stream into ligaments and then drops. Moreover, atomization is not limited to the break-up of jets and sheets into drops but also comprises the disintegration of drops into smaller droplets and drop collision and coalescence.

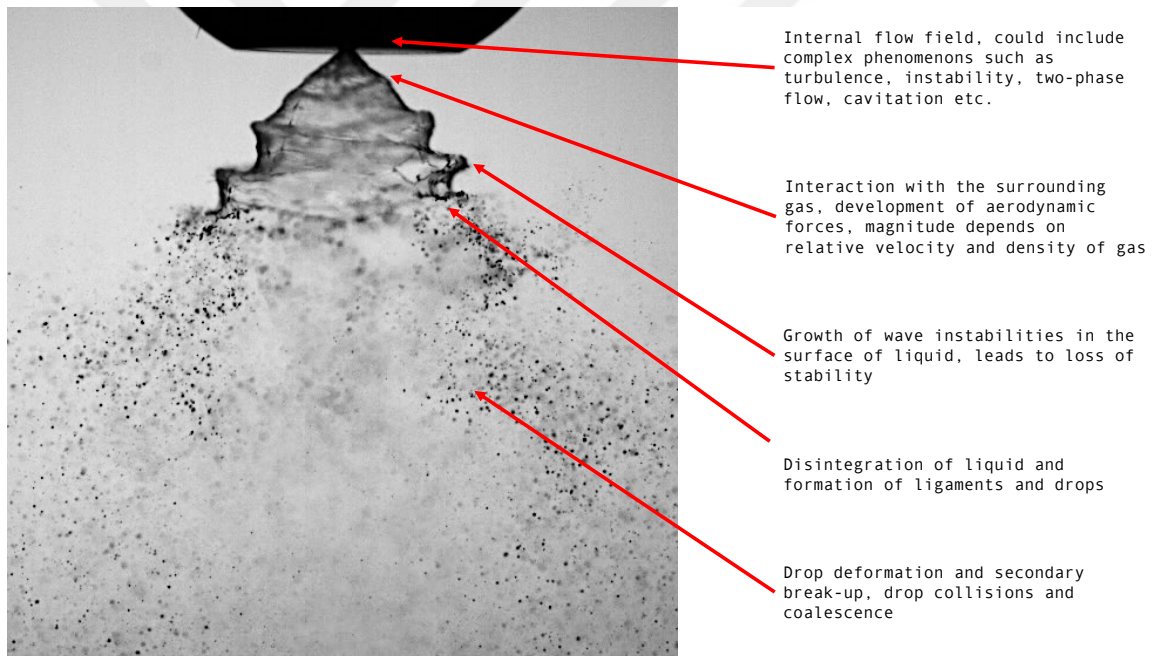


Figure 1: Processes that govern atomization

Due to the heterogeneous nature of the process, atomization could lead to droplets having sizes ranging from a few micrometers to a few millimeters. Therefore, mean droplet diameter values or graphical representation of the droplet sizes is used to describe the spray. A typical distribution is given in Figure 2. Distribution can be given in the form of frequency distribution or cumulative distribution. Four representative

diameter values are commonly used which are $D_{v,10}$, D_{32} , $D_{v,50}$ and $D_{v,90}$. $D_{v,10}$, $D_{v,50}$ and $D_{v,90}$ corresponds to the droplet diameter such that 10 %, 50 %, and 90 % of the total volume spray has a smaller diameter than these values. D_{32} , also named SMD, is calculated from Eq. 1 and, as can be seen, corresponds to the ratio of the total volume of the droplets to the total surface area of the droplets. Here N_i is the number of the droplet, and D_i is the diameter.

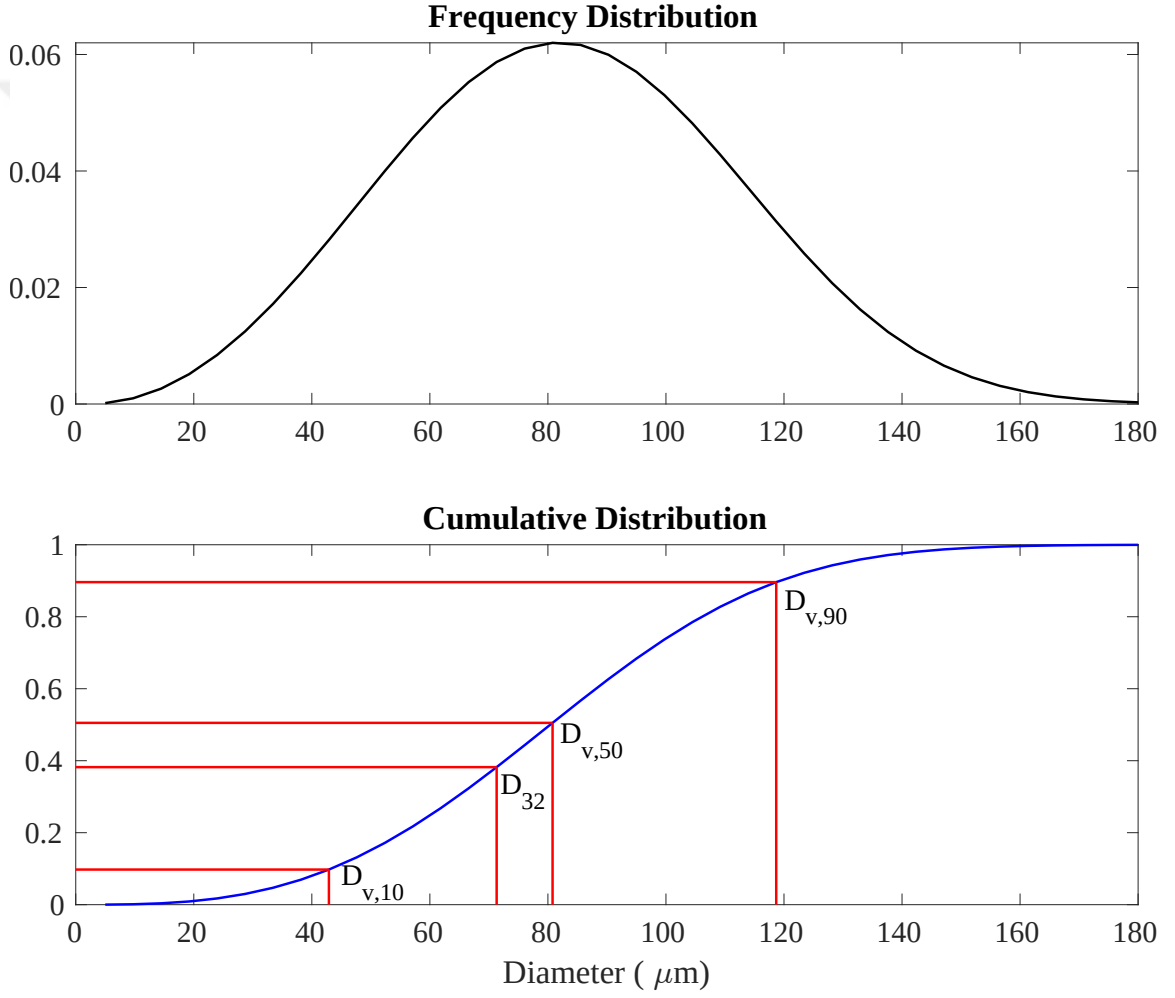


Figure 2: Droplet size distribution and representative diameters

$$D_{32} = SMD = \frac{\sum N_i D_i^3}{\sum N_i D_i^2} \quad (1)$$

Each individual type of atomizer or atomization method has specific advantages and disadvantages. However, the unified design aim of the atomizers is to achieve the spray having desired droplet sizes or mean droplet diameter at the desired pressure difference and flow rate and having a desired coverage area or spray angle. Therefore, it is critical to know the relation between these parameters with the properties of the working liquid, the geometry of the atomizer, and the conditions of the injection ambient. This requires a physical understanding of the internal flow field of the atomizer and the direct or indirect coupling between the internal flow with the formation of the spray and droplets.

A unique type of atomizing device is the pressure-swirl atomizer (PSA). This type of atomizer has the advantages of being simple, relatively easier to manufacture, and can achieve wide ranges of spray angles and droplet sizes. Schematic of the PSA is shown in Figure 3. There are two main different geometrical configurations of the PSA, namely open-end and close-end.

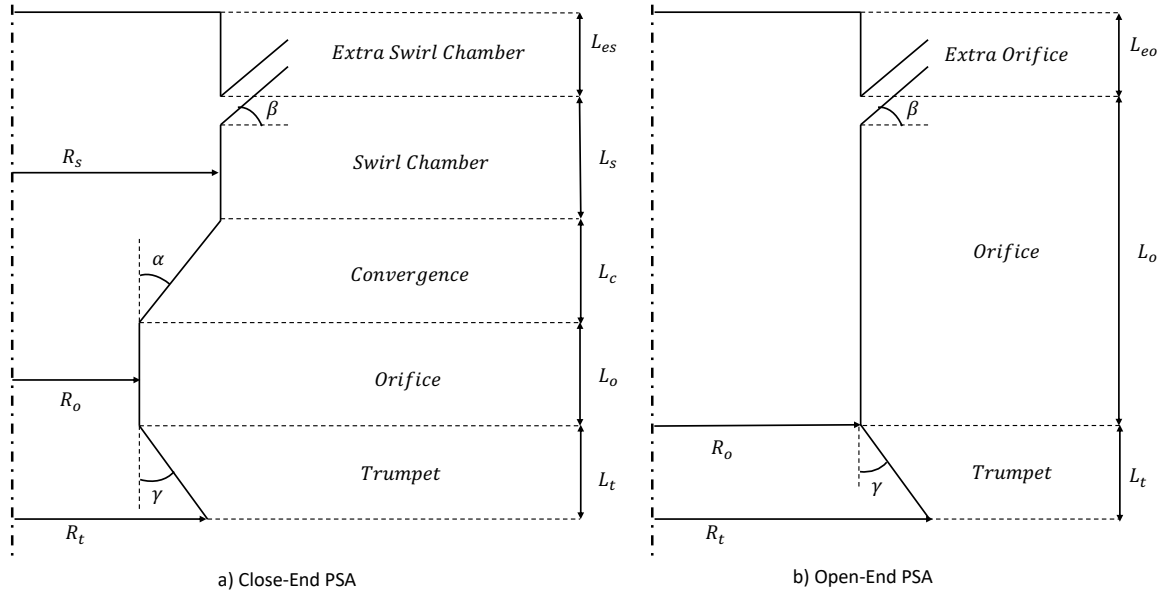


Figure 3: Schematic of a)close-end PSA b)open-end PSA

It should be noted that in addition to these two types, several variations exist such

as PSA with a divergent section (also known as trumpet) after the orifice to ensure desired spray angle is achieved, inlets having circular or square or rectangular cross-section or inlet ports that are not perpendicular to the nozzle axis are also employed. However, in all geometries, inlet ports impart a swirling motion as the liquid enters to the swirl chamber. Here, swirling motion leads to the decrease of the pressure along the center-line of the atomizer. Hence, the ambient gas is introduced into the center of the atomizer forming an air core. In the close-end atomizers, the liquid is accelerated by going through a convergent section and then later discharged from the orifice or trumpet.

Figure 4 shows the comparison of atomizer inlets of circular and rectangular cross-sections. It should be noted that when the width and height of the inlet are equal, it will be named as square inlet in this text.

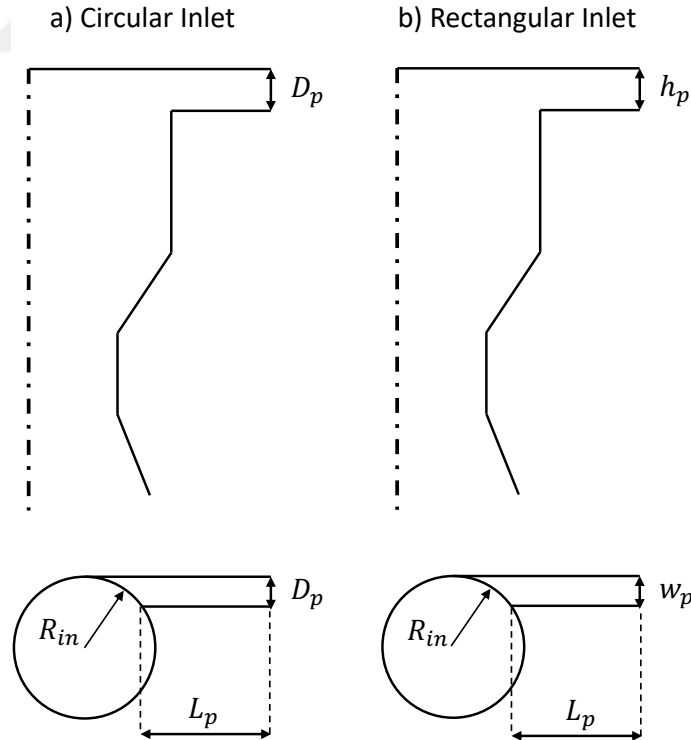


Figure 4: Schematic for inlets having a)circular b)rectangular cross-section

1.1 Motivation

Designing a PSA with the desired performance parameters is a difficult process despite its geometrical simplicity. The internal flow field is two-phase due to the formation of the air core, liquid/gas interface may lead to unsteadiness and instabilities, it is turbulent and unsteady, and depending on the design, re-circulation zones could exist.

Main performance parameters of the PSA can be listed as, pressure difference to achieve desired flow rate, film thickness, spray angle, breakup length, and droplet diameters or mean droplet diameter value. In addition to these parameters, it is known that circumferential distribution and development of the spray are also important [3]. Parameters related to spray shape are presented in Figure 5.

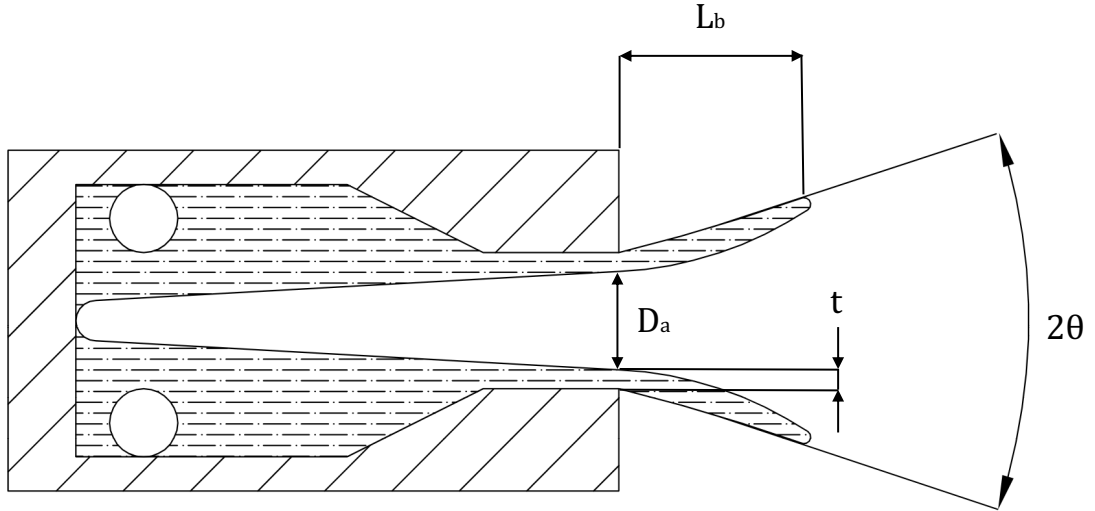


Figure 5: Schematic showing performance parameters of PSA

To predict the performance of PSA many different models are created. These models could be divided into four categories, namely,

- semi-empirical models,
- 1D models,

- 2D (two-dimensional) axisymmetric studies,
- 3D (three-dimensional) periodic studies and full 3D numerical studies.

Semi-empirical models are the most simple models that can be used or derived. Almost all of them are generated by conducting experiments with atomizers and then fitting correlations according to results and theoretical knowledge. In a previous study accuracy of these correlations have analyzed by comparing them with the experimental data [4]. Results showed high deviations between the correlations and data, leading to the conclusion that most of these correlations are viable in the range that they are derived at.

1D models are the models that are either based on the inviscid flow assumption or boundary layer modeling. Modeling of the internal flow with inviscid flow assumption is first proposed in an early study and later analyzed with greater detail [5, 6]. It should be noted that these models require adjustments, to account for the losses that occur in the PSA, in the form of introducing a constant or variable, which can only be obtained from the experimental data [7]. Moreover, obtained model is based on the geometrical parameters such as diameter of the tangential ports, swirl chamber, and orifice, and the length of them is not included in the calculations. However, it was experimentally shown that the length of the orifice and swirl chamber have an effect on the performance of the PSA [3, 8, 9]. In addition, models obtained with inviscid flow assumption are derived with the assumption called maximum flow principle which is originally developed for the hydraulics of rivers and states that air core will be formed such that it maximizes the flow rate of the atomizer [3].

Boundary layer based modeling was proposed much earlier than the inviscid flow models, and further improvements of the initial model also exist in the literature [10–12]. The accuracy of the boundary layer based model is evaluated, by the author of this dissertation, through the calculations performed with open-end PSA and it

was shown that this model can accurately predict the parameters such as pressure difference and film thickness [13]. However, it is also seen that calculated boundary layer thickness values were too small to be physically meaningful.

2D and 3D numerical simulations are becoming more and more widespread with the increase in computational power. However, there are debates regarding the accuracy of these studies due to the used viscous models [14]. Moreover, even with the increased computational power, conducting a full 3D numerical simulation of PSA is challenging. In addition, it should be noted that performing an optimization study requires, usually, thousands of cases to be calculated, and conducting this with 2D or 3D simulations is more time-consuming than using semi-empirical or 1D models. In a benchmark study solution time of 26 core hours for 2D simulation and solution time of 24,720 core hours for full 3D simulation is reported [15].

Circumferential distribution and development of the PSA spray are important yet often overlooked performance parameters of the PSA. Limited studies have been conducted for describing them and even less effort has been given to characterize these parameters. The only model to describe the circumferential distribution, that authors could find, exist in much earlier study [5]. Similar status is also valid for the development of the spray [3, 16–19].

Literature survey, have yielded that even though highly accurate 3D numerical methods were established for the simulation of PSA, performing an optimization study with them, in which thousands of geometries are evaluated, requires immense computational power. 2D numerical methods require much less computational resources yet the accuracy of this method was seem to be debatable. Semi-empirical models are easy to obtain and use in the calculations, yet correlations that exist in the literature have failed to make accurate predictions when compared with a wide range of experimental data. The boundary layer model seems to have promising results in terms of predicting the performance and requires much less computation time, yet in

the previous study of the author it was seen that obtained boundary layer thickness values were too small to be physically meaningful. Moreover, models for describing key performance parameters such as the development of the spray and the uniformity of the spray are limited to a few studies. All of the above-mentioned facts have encouraged the author to conduct the research presented in this dissertation.

1.2 Objectives

The objective of this dissertation is to develop a design methodology that will be used for the optimization of the PSA such that it takes the development of the spray and uniformity of the flow inside the atomizer into account and can accurately predict the performance parameters of the PSA. Hence, the following list can be made for the intentions of this study.

- To evaluate existing semi-empirical models and obtain new models, if necessary, for the prediction of the performance parameters that is based on the experimental data in the literature and experimental data of this dissertation.
- To develop an accurate internal flow model, based on the boundary layer, that is physically sensible and can accurately predict the internal flow.
- To obtain a model for describing the uniformity of the internal flow of the PSA.
- To establish a model and method for describing the transition point of the developed spray.
- To obtain a model that could accurately predict the droplet size distribution of the PSA.
- To present a systematic design methodology for the optimization of the PSA.

In order to achieve these intentions, an experimental setup and technique capable

of measuring the break-up length and spray angle of PSA were established, experiments were conducted to measure the droplet diameters in the primary break-up region, and a methodology is developed for the measurement of the uniformity of the spray. Moreover, 2D axisymmetric and full geometry 3D numerical methodologies were built for the prediction of the internal flow. Finally, all the available experimental data of the literature is obtained, presented, and used for the validation of the models.

1.3 Research Scope

The main interest of this research is to evaluate and model the performance parameters of PSA through numerical and experimental methods. Numerical studies will be conducted in the 2D (two-dimensional) and 3D (three-dimensional) domains. 2D studies will be used for the development of the internal flow model and 3D will be used for the circumferential distribution analysis, in addition to the validation of the internal flow model.

Experimental studies can also be divided into two parts. The first part focuses on the measurement of parameters that will be used for obtaining a method for the droplet sizes. These measurements consist of high-speed imaging and Shadowgraphy [20]. High-speed imaging is used for measuring the break-up length and spray angle whereas the latter mentioned is used for measuring the local droplet sizes. The second part of the experimental studies is conducted with the Laser Induced Fluorescence (LIF/Mie) method of LaVision company, also known as SprayMaster system [21]. These measurements will be done at the radial cut of the spray and will be used for developing a model for the circumferential distribution.

1.4 Thesis Organisation

This dissertation consists of 10 chapters. Chapter 1 begins with the introduction and motivation to the objectives of the study and the scope of the research. Chapter

2 describes the experimental methodologies used for measuring the parameters such as droplet diameter, break-up length, spray angle, and circumferential distribution. Chapter 3 provides information about the numerical methodologies used in this study. Chapter 4 consists of the presentation of the experimental data of the literature and the obtained semi-empirical model. Chapter 5 presents the developed model, based on the boundary layer, for describing the internal flow of the PSA. Chapter 6 deals with circumferential distribution modeling, consisting of 3D numerical studies, radial LIF/Mie experiments, and obtaining a semi-empirical model for the description of uniformity. Chapter 7 describes the effort made for modeling the transition point of developed spray and presents obtained model and experiments available in the literature. Chapter 8 focuses on the modeling studies conducted for describing the droplet sizes of the PSA. Chapter 9 presents the combination of the models obtained in Chapters 4-to-8. Chapter 10 gives the summary, conclusions, and suggestions for future research.

CHAPTER II

EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUES

Experiments in this dissertation are presented in three groups, namely high-speed imaging, Shadowgraphy, and LIF/Mie. High-speed imaging is used for obtaining the break-up length and spray angle of the spray. Shadowgraphy and LIF/Mie experiments are conducted to measure the local droplet sizes and the uniformity of the spray, respectively. For the experiments, a spray chamber is designed and built at Ozyegin University Fluid Dynamics and Spray Laboratory.

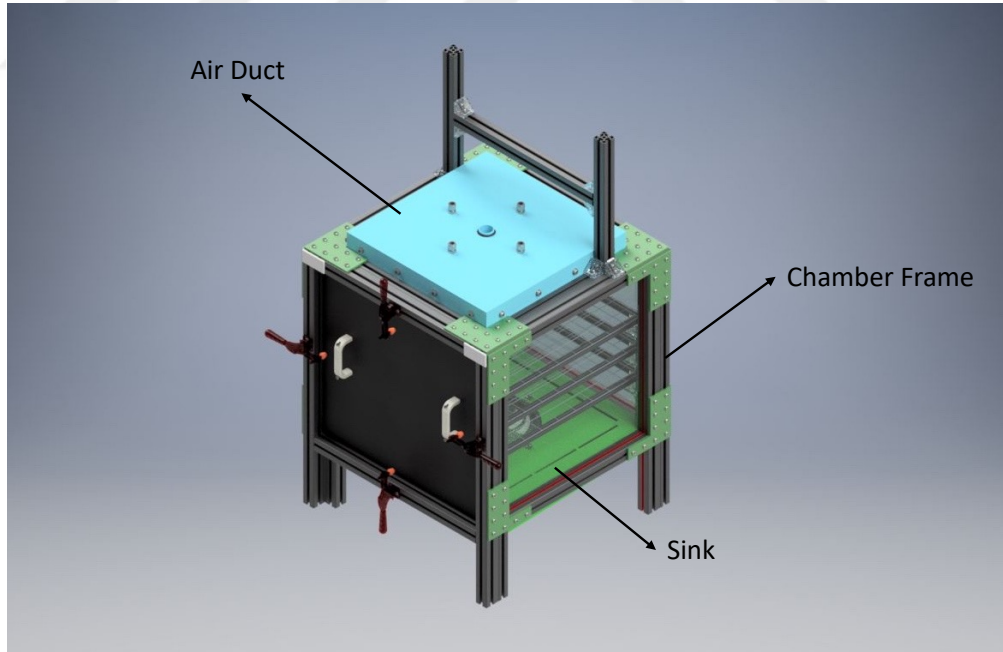


Figure 6: Design of the spray chamber

The designed chamber consists of three main parts which are, the frame, sink, and air duct, as shown in Figure 6.

Frame of the chamber consists of aluminum sigma profiles of sizes 45x90x90, 45x90, and 45x45. The 45x90x90 profiles are used for the legs, 45x90 profiles are used at the top side and 45x45 profiles are used at the bottom side.

Glass windows are used at three sides of the frame and plexiglass with handles is used at one side. Glass windows are assembled to the profiles with the mounting rims and plexiglass is pushed onto the profiles by means of four pushers.

This part is designed instead of a hinged door to eliminate leakage problems. Mounting rims, used for assembling the glasses, are specifically designed for the 10 mm slot profiles and provide the sealing for mounting elements up to 6 mm thickness.

Connection of the profiles is done with the corner connections and gussets. Since the used profiles have 2 slots, 90x90 corner connectors are used. These connections are made with M8x20 mm and M8x25 mm bolts, respectively, and 10 mm slot T-nuts. Moreover, to ensure tightness, gasket sealant was applied to the profile slots. The designed spray chamber has an overall height of 1300 mm and the observation area is 570 mm x 570 mm.

During the commissioning of the spray chamber it is seen that at high flow rates, due to the fine atomization capability of the atomizers and the high exit velocity of the liquid, small droplets hit the bottom part of the spray chamber and then float inside the chamber. Having these droplets scattered around the measurement area hampers the measurements. In order to eliminate them, three layers of stainless steel meshed surfaces are designed and mounted to the frame, as shown in Figure 7.

These meshes are installed such that the layer at the top (closer to the atomizer) has the highest open screening ratio and the one at the bottom has the smallest. These layers ensured that once droplets go through the openings, they will not be able to float after hitting the bottom of the spray chamber.

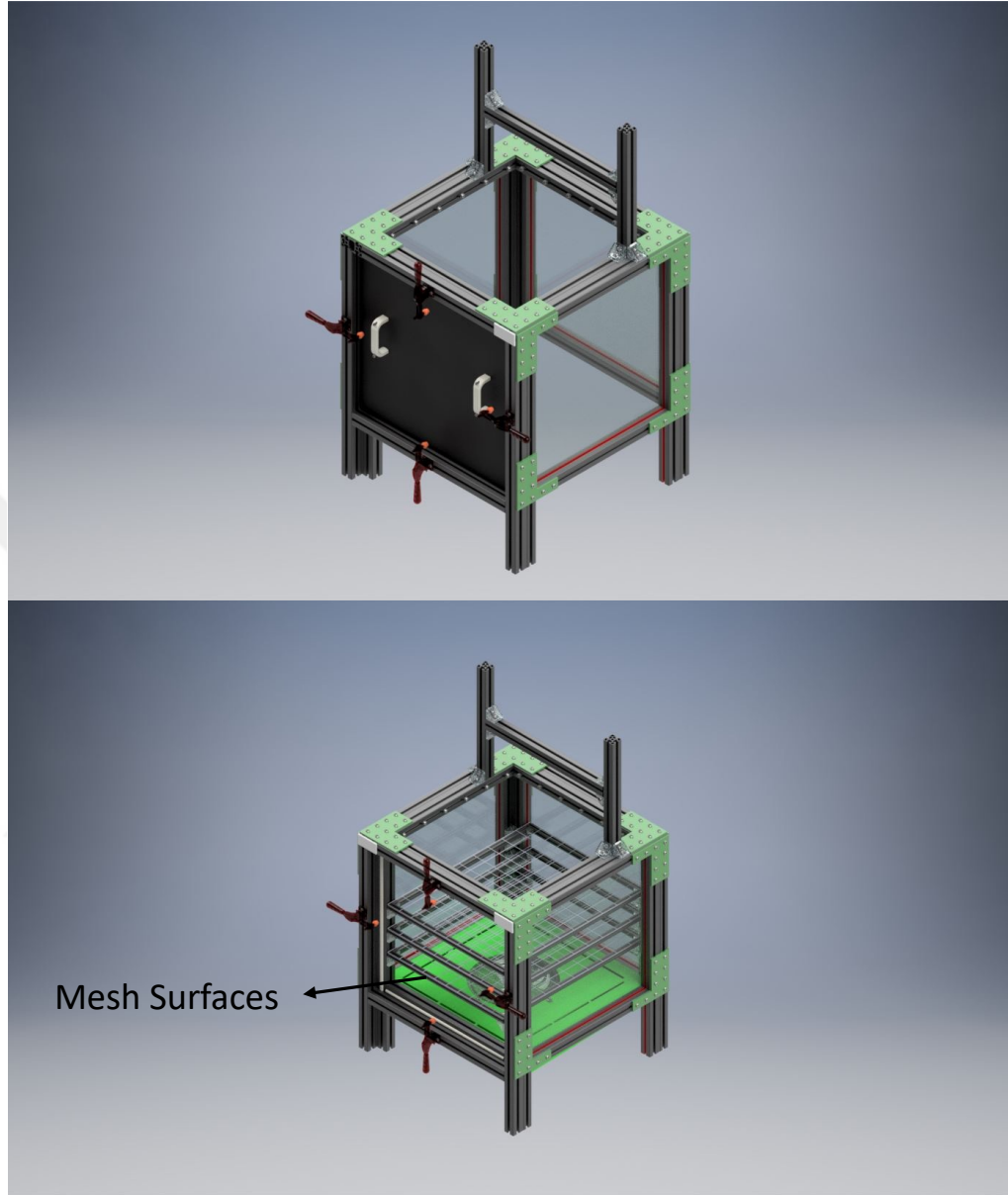


Figure 7: Frame of the spray chamber

The air duct is designed to supply airflow in front of the windows. It is foreseen that due to the high spray angle and exit velocities, droplets could hit the observation windows and this will hamper the measurements. This part is designed and manufactured from PMMA plastic. This part is designed to consist of two rectangular boxes, one larger than the other, placed into each other. Air enters from the four tapered

connections located at the top and leaves through a small slit region at the bottom. This region ensures that air leaving the air duct has a uniform velocity, by providing constant width all around the air duct. Moreover, a center pipe hole is used to insert the atomizer into the observation area. The total mass of the part is calculated to be 6.1 kg.

At the bottom of the spray chamber, a sink is designed to collect both the air and working liquid. It consists of three parts which are a plexiglass round sink, the sink frame, and the cover. Air suction holes are placed at both sides and liquid drainage holes are placed at the sink cover. The material of the sink frame and the cover is selected to be PMMA since it is cheap, light, and easy to machine. Mass of this part is calculated to be 5 kg. Moreover, the sink of the spray chamber has a slope in the favor of liquid drainage holes to ensure the liquid flow.

2.1 Experimental Setup

Schematic of the experimental setup is given in Figure 8. Water is chosen to be working liquid for all of the experiments. The temperature of the liquid is controlled by a thermostat which also acts as a water tank. The thermostat has a small pump that is used to feed the piston pump through a 50 μm filter. Danfoss PAH 2 piston pump is used to pressurize the liquid and this pump is capable of delivering a volume flow rate of up to 2.0 l/min. The maximum outlet pressure of the pump is 140 bar.

A pulsation damper is used to eliminate the oscillations of the system. The flow rate of the system is adjusted with a three-way needle valve. A small fan, in a closed circuit, is used to deliver air into the spray chamber which can achieve 200 m^3/h at 50 Pa and 10 m^3/h at 300 Pa.

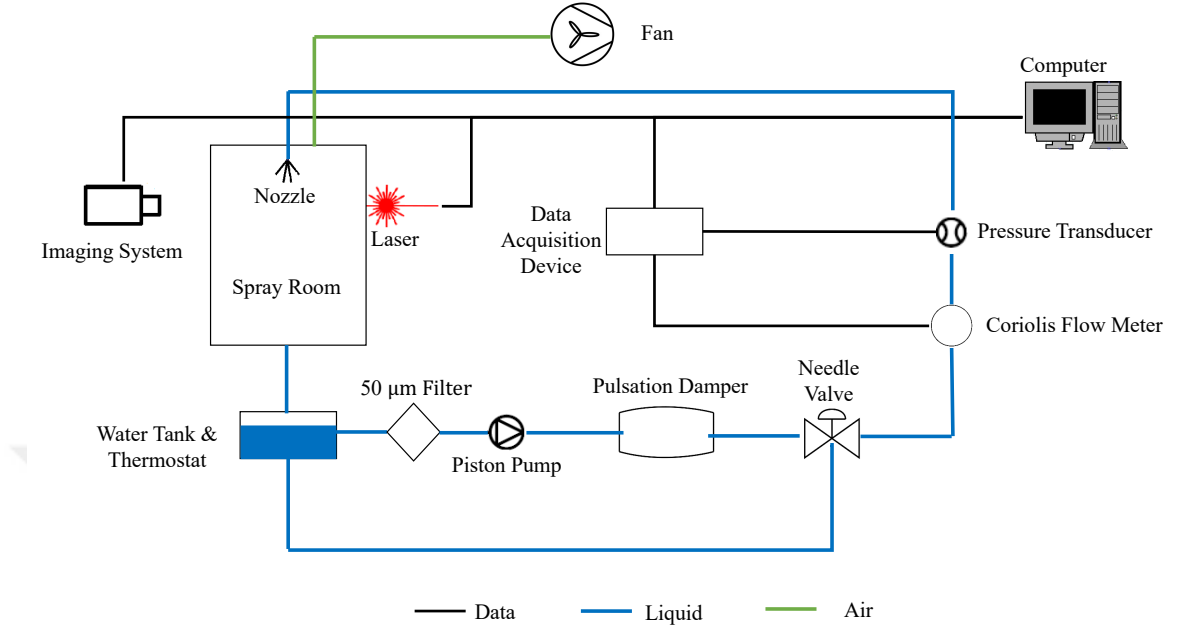


Figure 8: Schematic of the experimental setup

Flow rate, density, and temperature are measured with a Coriolis flow meter. Pressure values are obtained from the pressure transducer. Specifications of the transducers used are given in Table 1. Data from the measuring devices are obtained using a cRIO device of National Instruments and analyzed by in-house developed code using LabView software.

Table 1: Experimental Apparatus

Device	Measured Property	Range	Accuracy
Bronkhorst M14 Coriolis Flow Meter	Flowrate	0-8.3 g/s	$\pm 0.05\%$
	Density	-	$< \pm 5 \text{ kg/m}^3$
ABB 2600T Pressure Transducer	Pressure	0-200 bar	$\pm 0.05\%$

2.2 Measurement Techniques

2.2.1 High Speed Imaging

For high-speed imaging Photron Nova S6 camera and Canon EF 70-200 mm f/2.8 lens are used with continuous LED back-lighting. Images are captured with 6400 fps and

0.2 μs exposure time, to obtain the sharpest possible image. A total of 1000 images are taken at each experiment condition.

2.2.1.1 Breakup Length Measurement

It should be mentioned that there are two different approaches in defining the break-up length. One is the vertical distance from the nozzle tip to the point of break-up and the second is the diagonal distance from the nozzle tip to the point of break-up. In this study, diagonal distance from the nozzle tip is taken as the break-up length, as shown in Figure 5. Break-up length measurement from the obtained images is done with in-house written MATLAB code and consists of 8 steps.

1. Scaling the images according to the reference image

Scaling of the images is done using the scaling plate of LaVision company. This plate contains many circles with diameter values ranging from 10 μm to 2000 μm . Here, the circle with a 2 mm diameter is used, as shown in Figure 9. The red line is added in Matlab written code for the scaling and its length corresponds to 64 pixels. Knowing the diameter of the circle is 2 mm, the conversion rate is found to be 32 pixels to 1 mm.

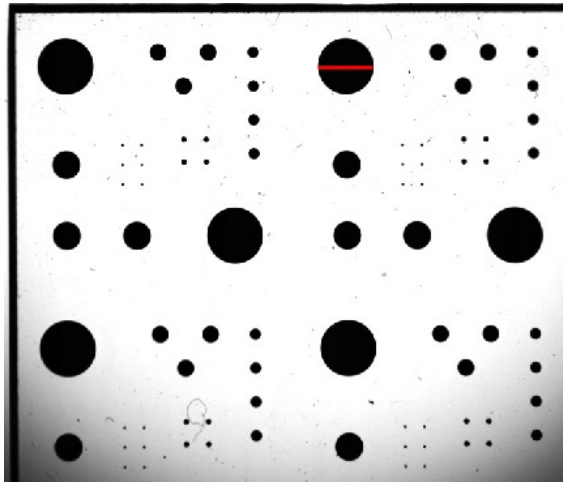


Figure 9: Scaling plate used for the high-speed imaging

2. Limiting the calculation area

In order to reduce the calculation time, the area of the investigation is reduced to only include the radially expanding liquid sheet, as shown in Figure 10.

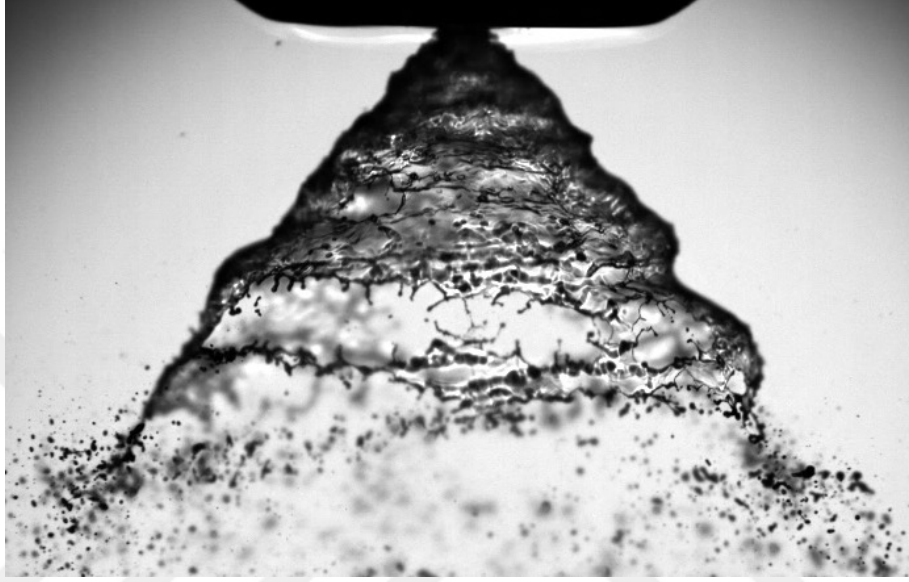


Figure 10: Limited region of the high-speed images

3. Calculating the mean image

The mean image is obtained by adding all of the obtained images and then dividing it by the number of images. The acquired image is shown in Figure 11 and this image is used for determining the location of the nozzle tip.

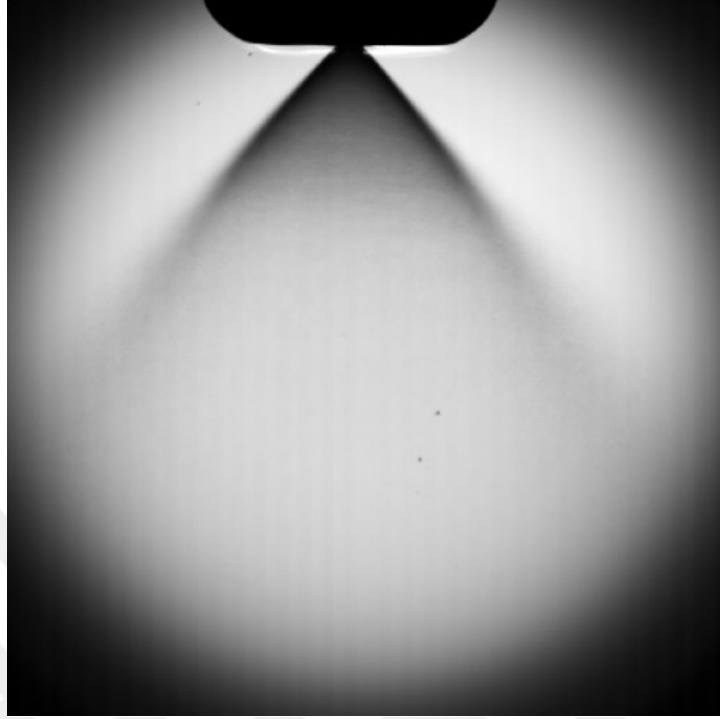


Figure 11: Mean image of the high-speed images

4. Making each image binary

A Matlab code utilizing the built-in function is used to make each obtained image binary. Comparison of the experimental and binary versions of experimental images are shown in Figure 12. From the figure, it can be seen that the binary version reflects the behavior of the liquid sheet seen in the experimental image.

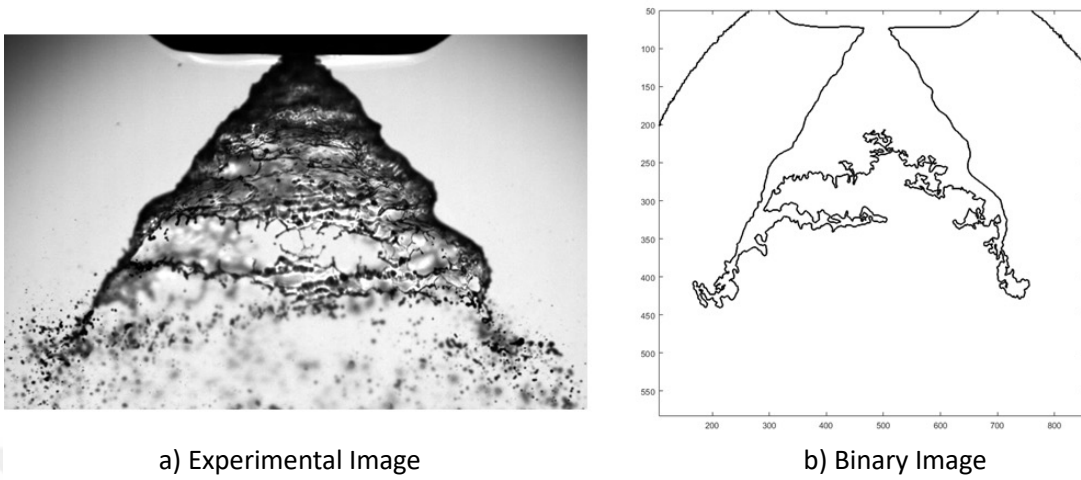


Figure 12: Obtained a)experimental image, b)binary image

5. Finding the location of the nozzle tip from the mean image

The location of the tip, or the exit coordinates of the liquid sheet, is found from the binary version of the mean image. Later, the left and right edges of the exit are selected manually and this location is used as the origin for all of the obtained images.

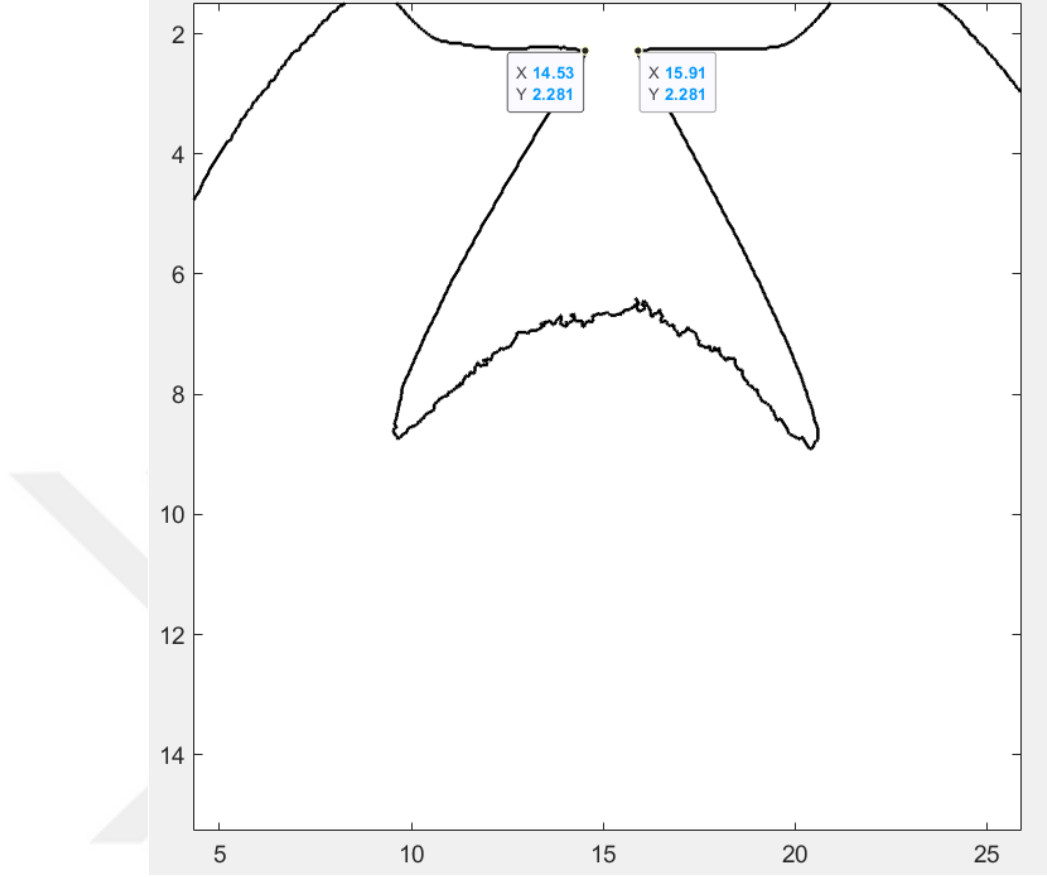


Figure 13: Detection of the tip of the nozzle

6. Tracking the edges of the spray up to the point of break-up

After determining the origin, the edges of the spray are tracked in each binary image, obtained in step 4. This is done using an in-house developed algorithm and the left and right sides of the sheet are tracked separately, as shown in Figure 14. Tracking started from the points found in step 5 and the distance from the start to the end point is taken as the break-up length for that side.



Figure 14: Tracking the edges of the spray

7. Selecting a number of images randomly and plotting the measured break-up length values for visual validation

In order to eliminate errors that could be made in the steps described above, a number of images are saved for visual validation. These images are created such that the obtained binary image is plotted on top of the experimental image along with the obtained break-up length values for each side of the spray.

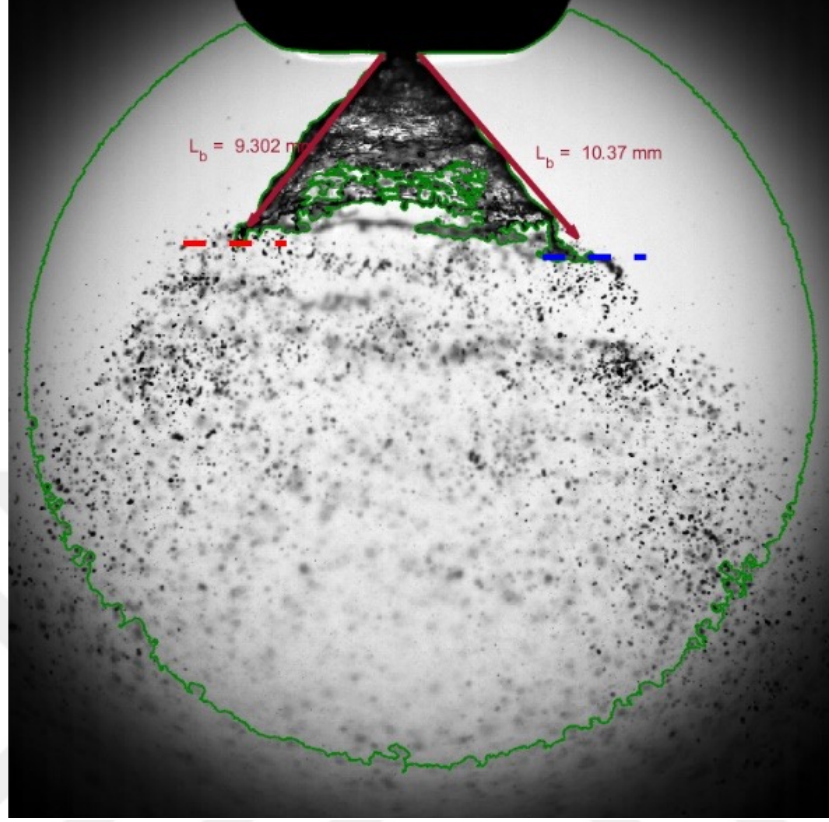


Figure 15: Visual validation images for break-up length

8. Obtaining results and statistical properties

As mentioned earlier, break-up length values are calculated for the left and right sides of the spray and for each image separately. Given that at each experimental condition, 1000 images are used and the two break-up length values are obtained from each image, a total of 2000 values is obtained.

Figure 16 shows how the obtained values for the left and right side changes with respect to the image number. For each side mean and standard deviation values are obtained and displayed on the plot.

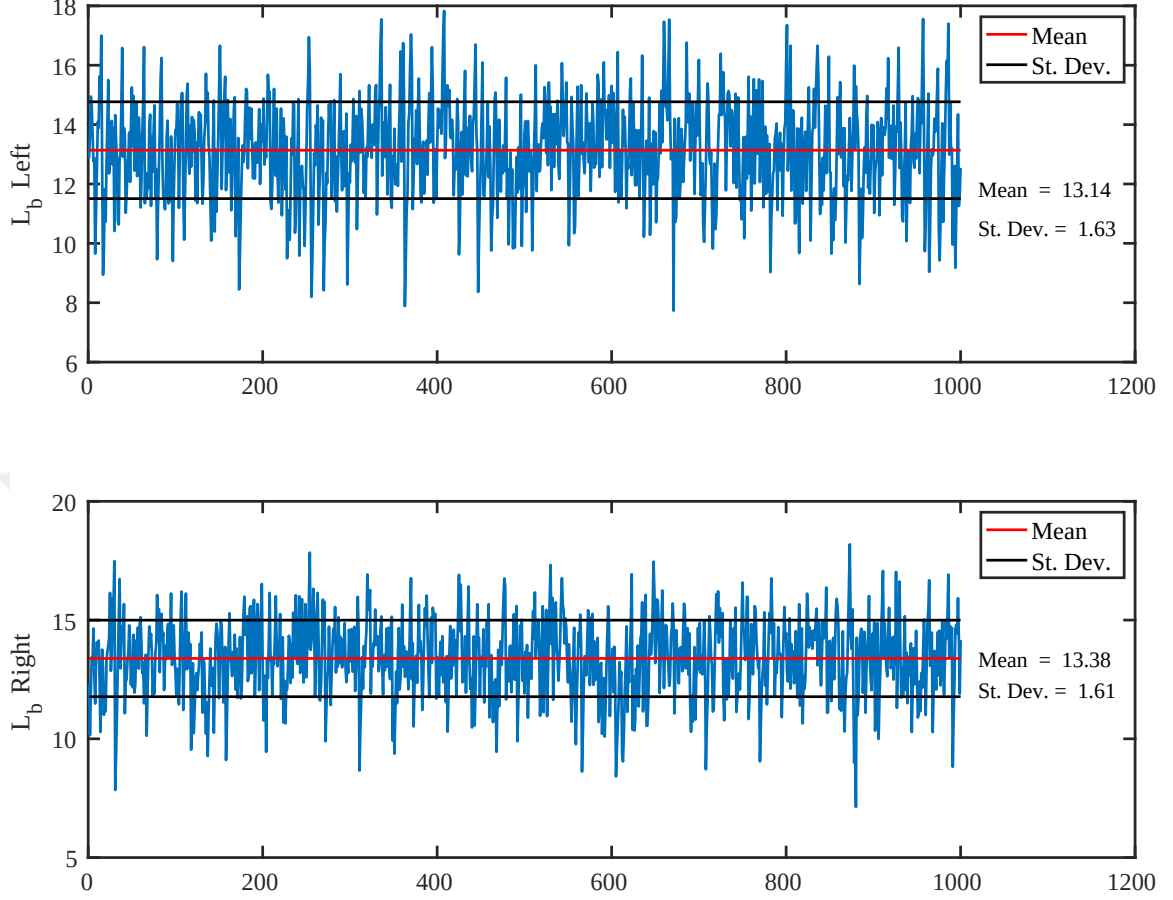


Figure 16: Obtained values of break-up length

In addition, the distribution of the obtained break-up length values is inspected as shown in Figure 17. Here, the percentage of the data that lies between single and double standard deviation values from the mean is shown. The mean value of obtained 2000 measurements is taken as the break-up length for that experiment condition.

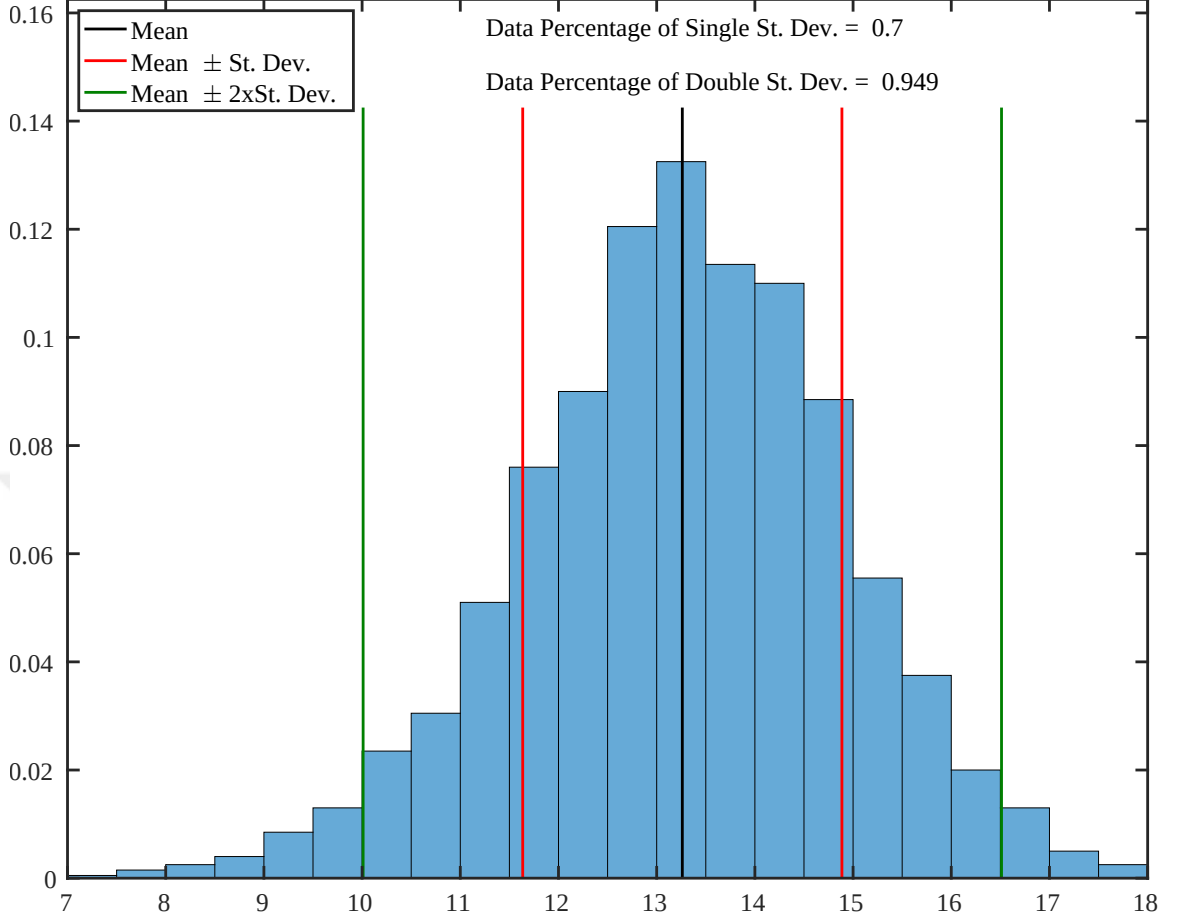


Figure 17: Statistical properties of obtained break-up length values

The effect of the frame rate on the obtained break-up length values is also inspected. Four experimental conditions are inspected by capturing the images of the spray at 400, 800, 1600, 3200, and 6400 frame rates. Figure 18 shows the comparison of break-up length values calculated with the steps described above. Results are presented as the ratio of the mean break-up length obtained with different frame rates to the obtained value with 6400 fps. It can be seen that the mean value has less than 4% difference even for the case of reducing the fps value to 1/16.

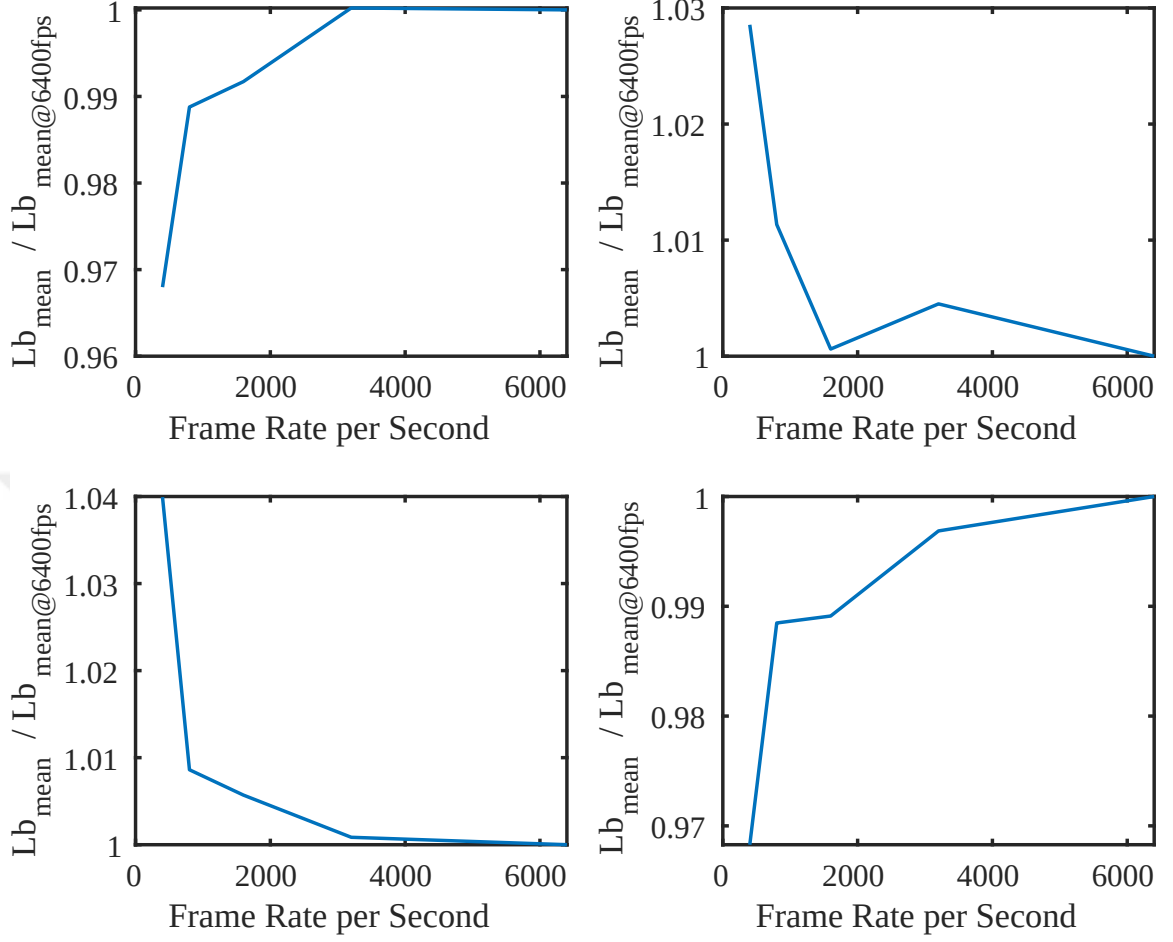


Figure 18: Effect of frame rate on the obtained break-up length values

2.2.1.2 Spray Angle Measurement

The second parameter measured from the high-speed images is the spray angle. This parameter is also attained from the binary version of the experimental image and tracking the edges of the liquid sheet, for the left and right sides separately, as described above.

The vertical distance from the nozzle tip at which the spray angle is calculated is selected based on the obtained break-up length values. To achieve a standard position for each experimental condition, minimum of the break-up length of all cases is used as the distance for the calculation. In this work, this value is 5 mm. In other words,

the spray angle is measured at 5 mm downstream of the atomizer for all cases.

An alternative method would be using the break-up location of each experimental condition and obtaining the angle at this position. However, this would mean that calculation of the spray angle will be done at different distances for each case and it is not desired.

As mentioned earlier, the spray angle is calculated from the tracked edges of the spray. Similar to the break-up length values, different angles are obtained from each image. For visual validation, minimum and maximum calculated spray angle values are plotted on the mean image, as shown in Figure 19.

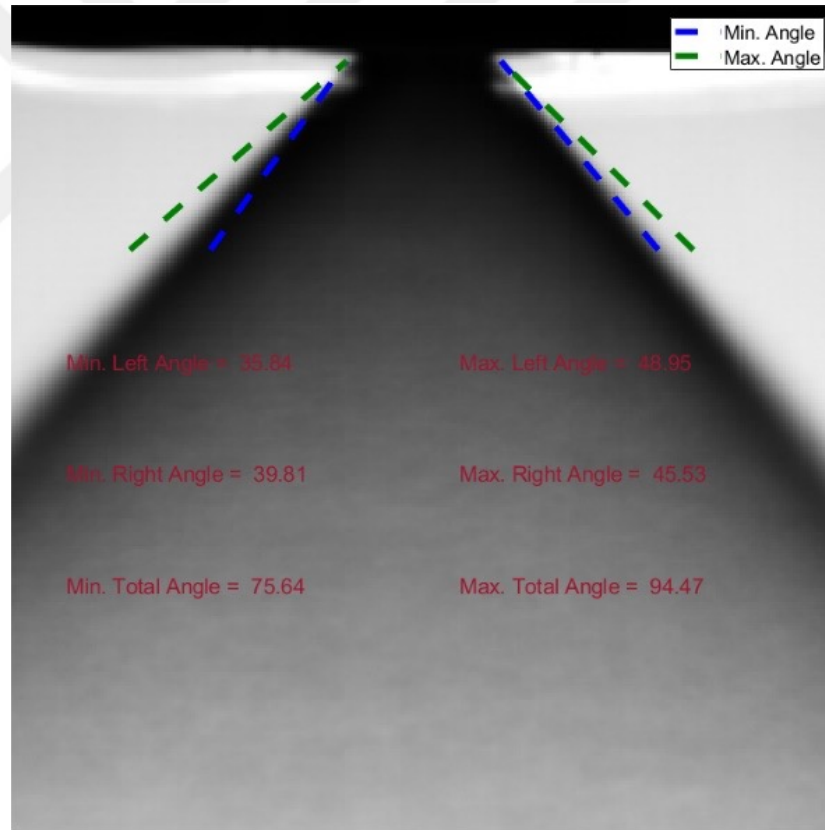


Figure 19: Measured min. and max. angle values

A number of randomly selected images, that show the calculated values of the spray angle on the experimental image, are saved for visual validation, as given in

Figure 20.

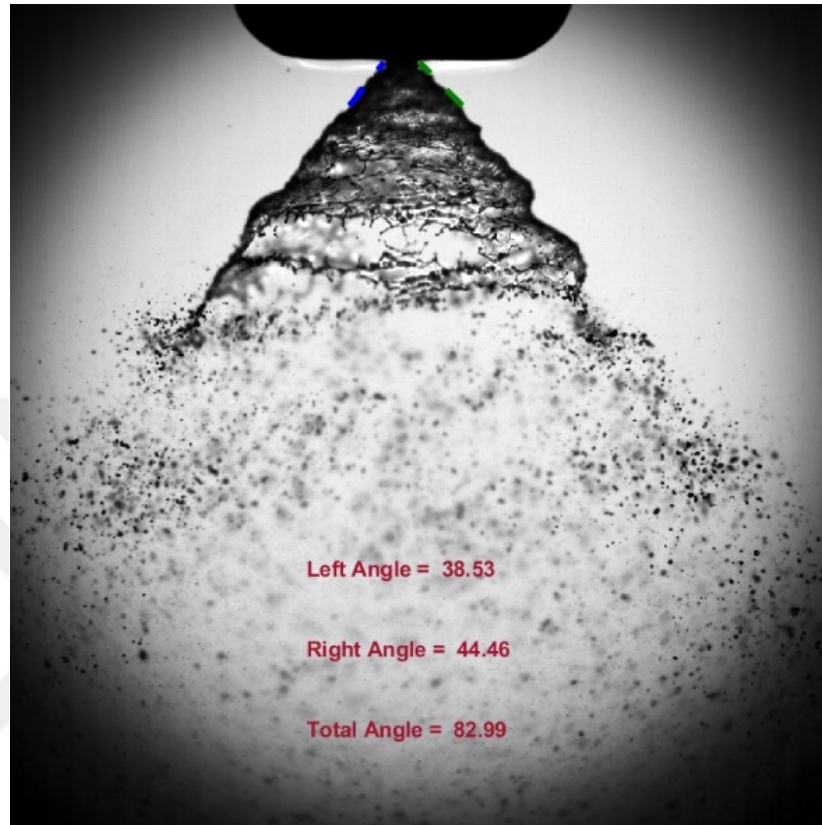


Figure 20: Visual validation images for spray angle

Moreover, obtained spray angle values are plotted against the image number for the left and right sides, and also total spray angle is plotted against the image number.

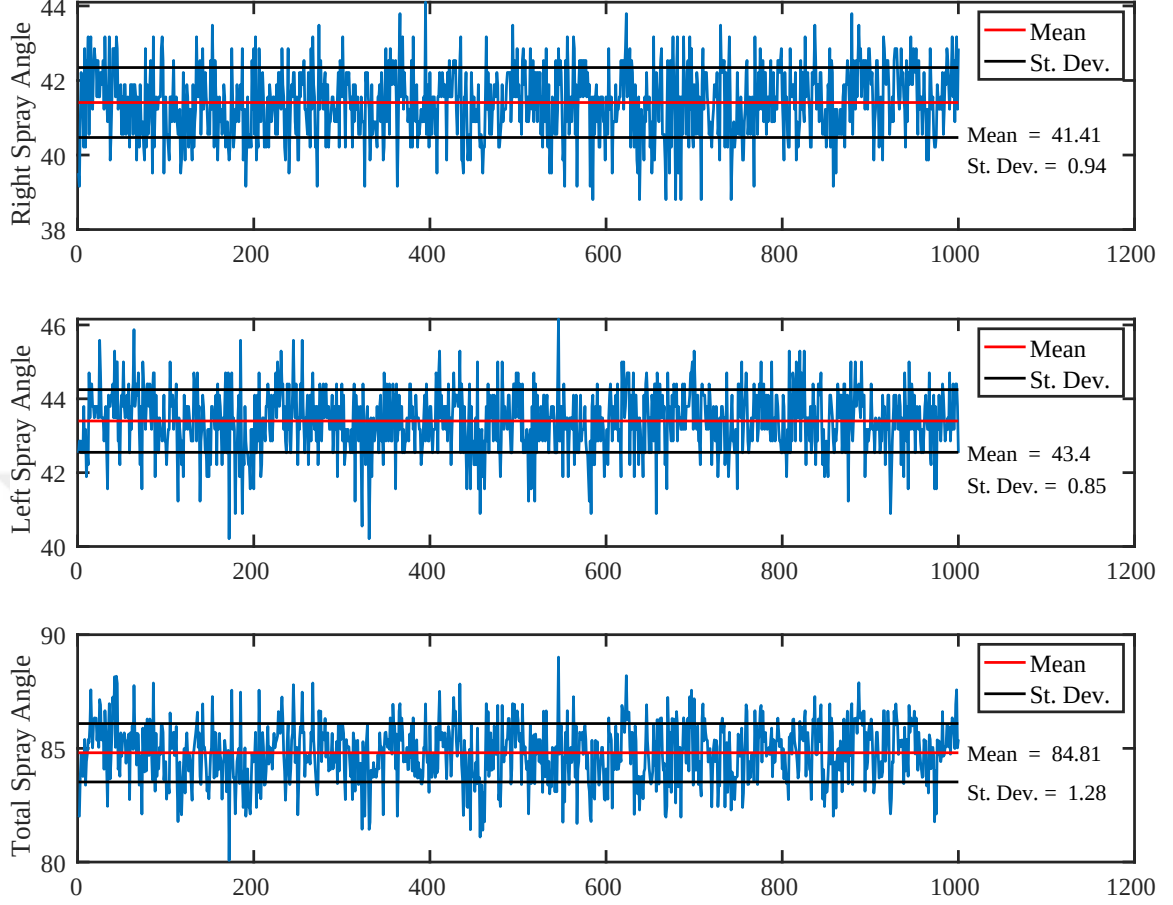


Figure 21: Obtained values of spray angle

Mean and standard deviation values for the total spray angle are also calculated and distribution is analyzed. Similar to the break-up length values, these are used for inspecting the distribution of the parameter and percentage of the data that lies between single and double standard deviation values from the mean value is found.

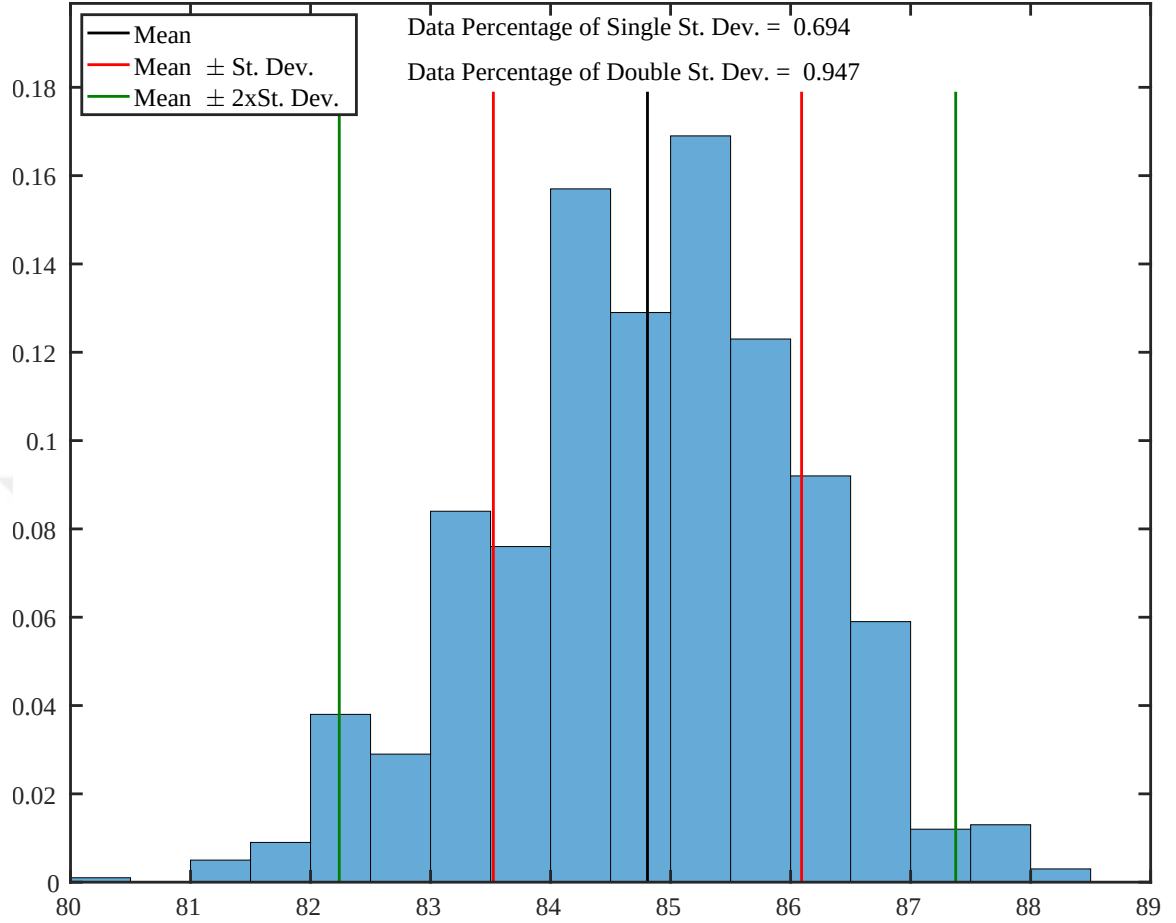


Figure 22: Statistical properties of obtained spray angle values

Finally, the effect of the frame rate on the calculated values of spray angle is also analyzed. Similar to the break-up length measurements mean of the obtained values with 5 different frame rates is compared with the one obtained at 6400 fps. Results show that difference between the mean spray angle values is even smaller than that of the break-up length.

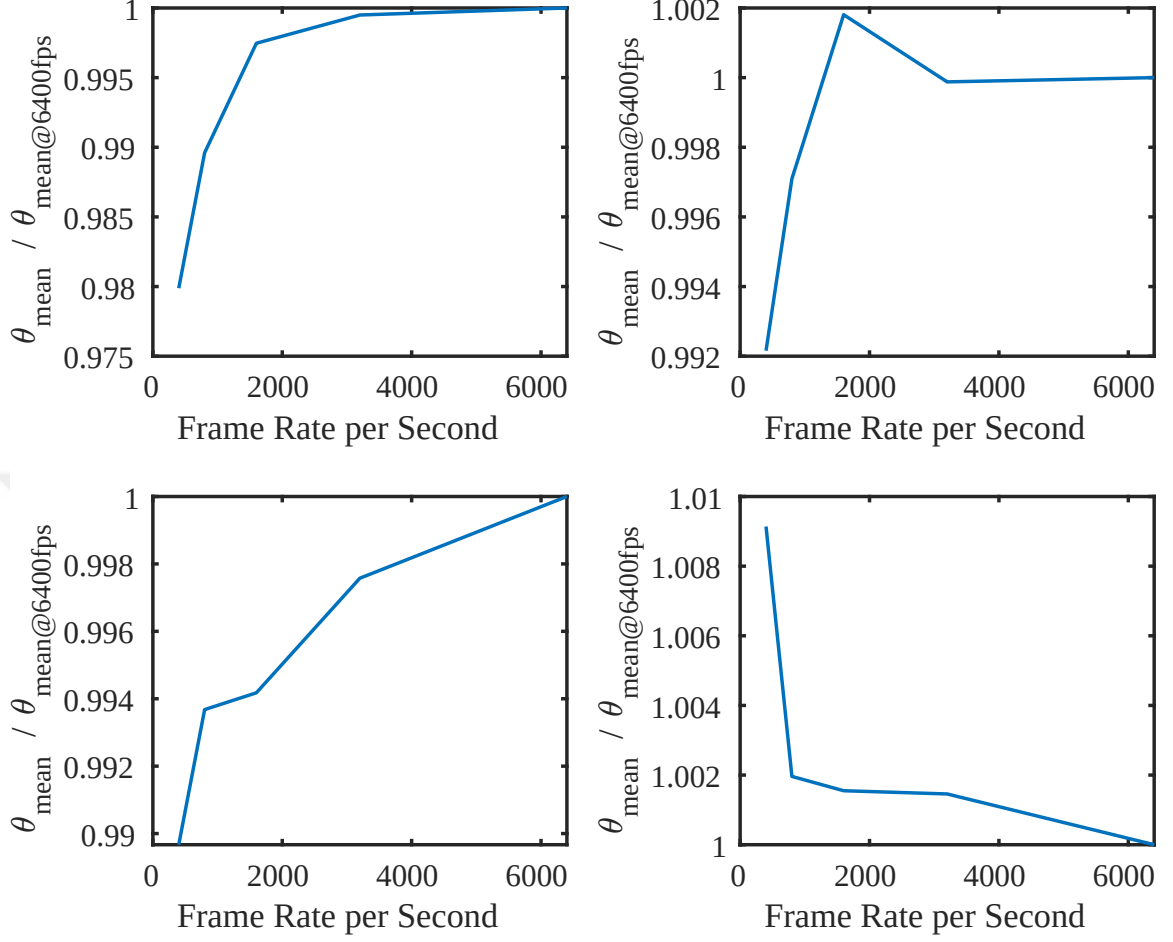


Figure 23: Effect of frame rate on the obtained spray angle length values

2.2.2 Shadowgraphy

For the measurement of the droplet sizes of the spray Shadowgraphy (back-lighting) technique, the ParticleMaster system of the LaVision company is used [20]. This system utilizes a laser, diffused through LaVision High-Efficiency Diffuser, as back-lighting and Imager CMOS camera with the QM1 long-distance microscope for capturing the images of the spray. It should be noted that to perform this measurement illumination and the detection system is aligned to be in one line, as shown in Figure 24.

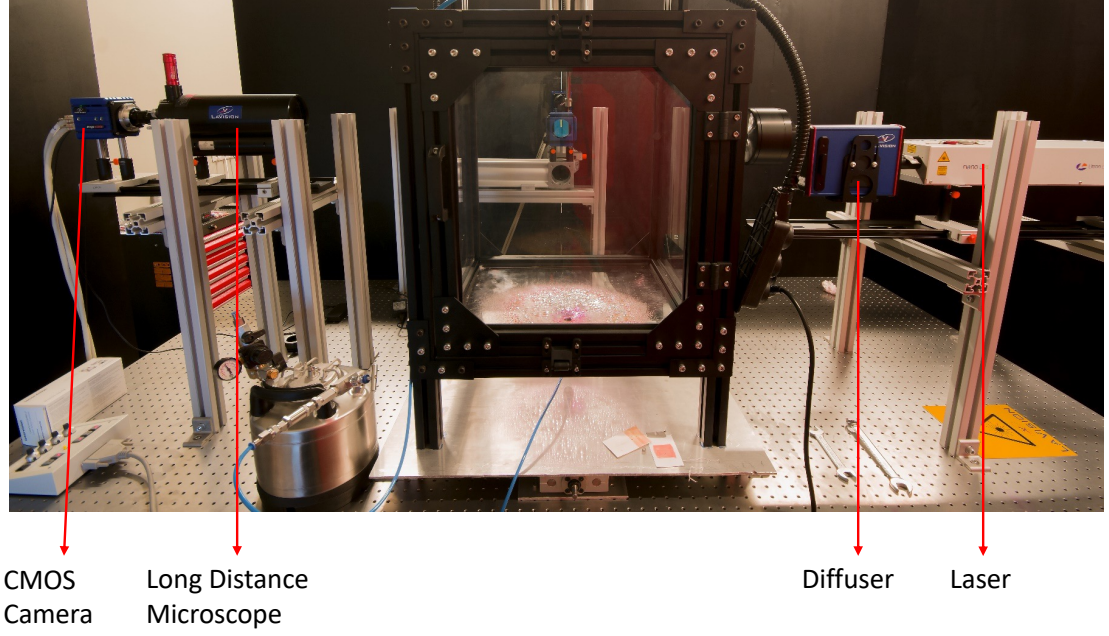


Figure 24: Image of the Shadowgraphy system

Double cavity Nd:YAG laser ($\lambda = 532 \text{ nm}$) working at 15 Hz is used in this study. Coupling the double-pulse laser with a double-frame camera allowed to investigate the droplet velocities as well as their diameters.

First, the position and diameter of the droplets are obtained. Velocity calculation is done after applying the sizing algorithm to each frame of the obtained images. Particles from the two frames are paired based on the size and the allowed shift of position. Since the pulse delay time between the two laser pulses is determined by the user, velocity values can be obtained. Figure 25 is an example of the above-explained method. Here, Laser A and Laser B are the two laser pulses. First frame is obtained at time zero and the second frame is obtained after a time of dt . The shift of the particles can be obtained from the position of the droplets and using this shift value with the determined dt time, velocity values are obtained. The result image is also provided in the same figure and as can be seen, both the diameter and the velocity magnitude and vector are displayed.

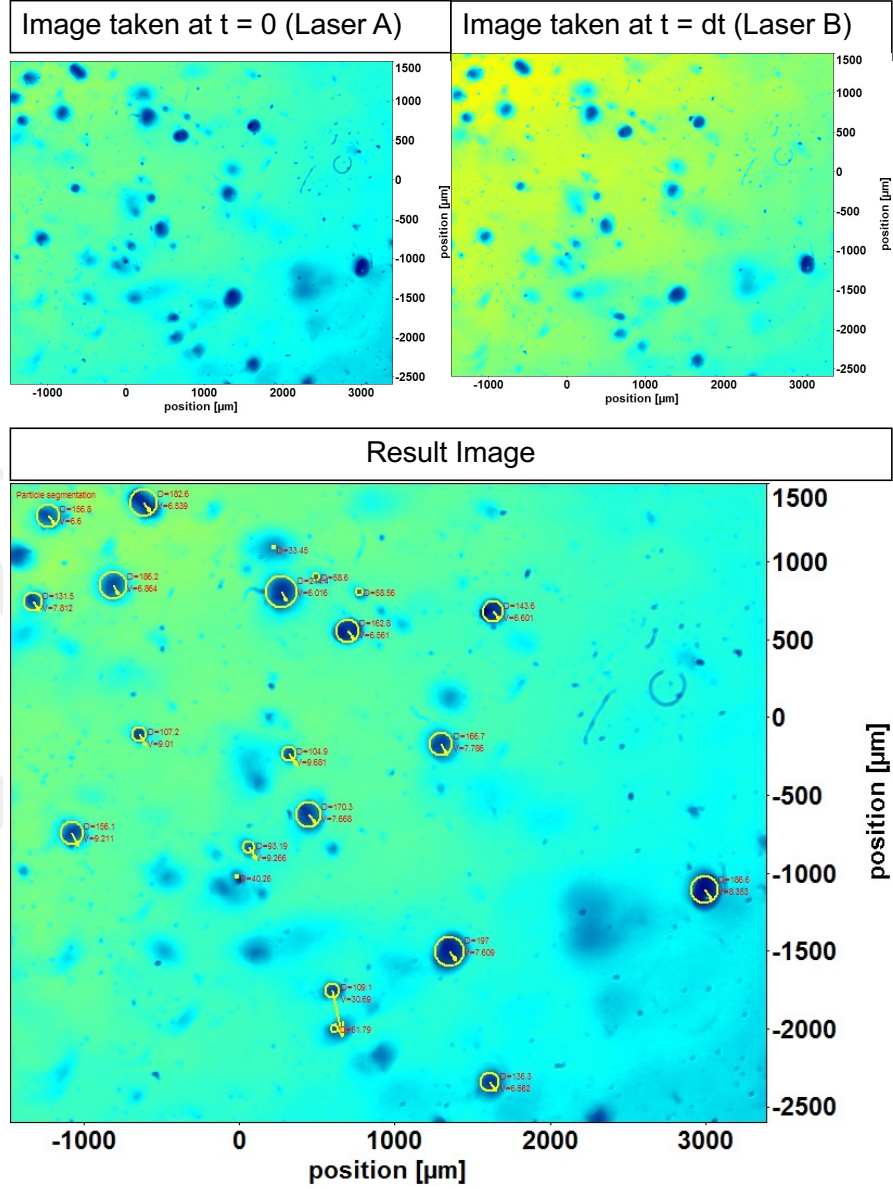


Figure 25: Droplet diameter and velocity calculation

Scaling of the obtained images is done with a scaling plate shown in Figure 9. With the long-distance microscope droplet diameters down to $3.5 \mu\text{m}$ were able to be measured in an interrogation area of 4 mm to 4 mm.

Measurement of droplet sizes for each experiment condition is performed in the region called as primary break-up region, which is described as the region where

ligaments and droplets are formed from the swirling sheet. For the atomizers used in this study, it is seen that the maximum vertical distance from the nozzle tip at which the droplets are formed was 9.8 mm, therefore closest location for the Shadowgraphy experiments was selected to be 10 mm downstream of the nozzle. Moreover, all of the measurements are conducted at the central plane of the nozzle and at 8 different locations, as shown in Figure 26.

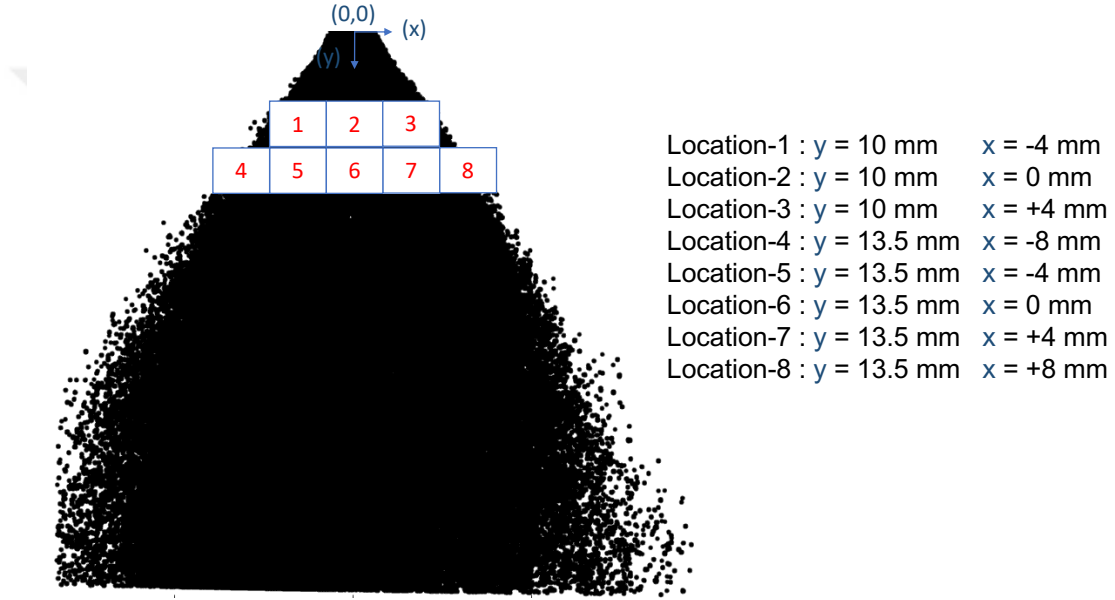


Figure 26: Shadowgraphy experiment locations

Obtaining the images and initial processing of them (calculating droplet diameters and velocities) is done using LaVision Davis software 8.3. Post-processing of the data is done with in-house developed Matlab code, which is used to obtain number distribution, volume distribution, and parameters of the Rosin-Rammler and Modified Rosin-Rammler distributions.

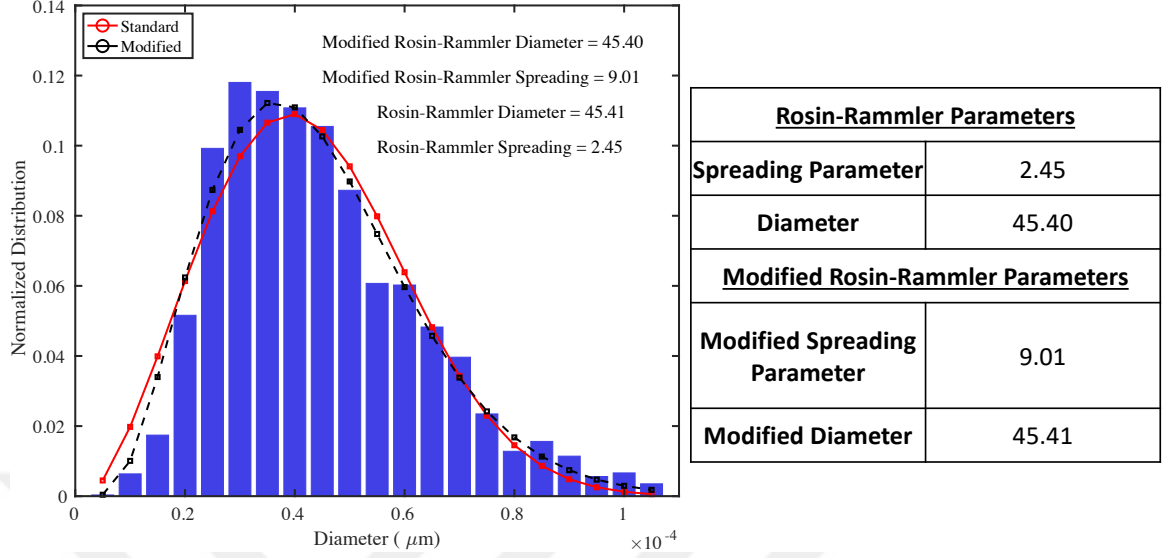


Figure 27: Calculated volume distribution and Rosin-Rammler and Modified Rosin-Rammler diameter and spreading parameters

Rosin-Rammler and Modified Rosin-Rammler distributions are described with Eq. 2 and Eq. 3, respectively.

$$Q = 1 - \exp\left(-\frac{D}{X}\right)^q \quad (2)$$

$$Q = 1 - \exp\left(-\frac{\ln D}{\ln X}\right)^q \quad (3)$$

where, Q is the cumulative fraction of the droplets having a smaller diameter than D , q is the spreading parameter and X is the representative diameter value.

In the experiments, integrated size distributions and the statistical diameter values are obtained from the ensemble of the particles detected in each image. Therefore, the number of images taken could affect the results. In other words, if the number of images is not sufficient, then the obtained statistical diameter values could lead to errors. On the other hand, if the number of images is too high, processing and post-processing time of the data will take too long which is not efficient.

Hence, analysis of having different numbers of images on the obtained mean diameter (D_{32}) value is conducted. Results of this analysis can be found in Figure 28,

for each 8 location, and it can be seen that having 100 images is appropriate since the mean diameter value is almost constant after 60-70 images.

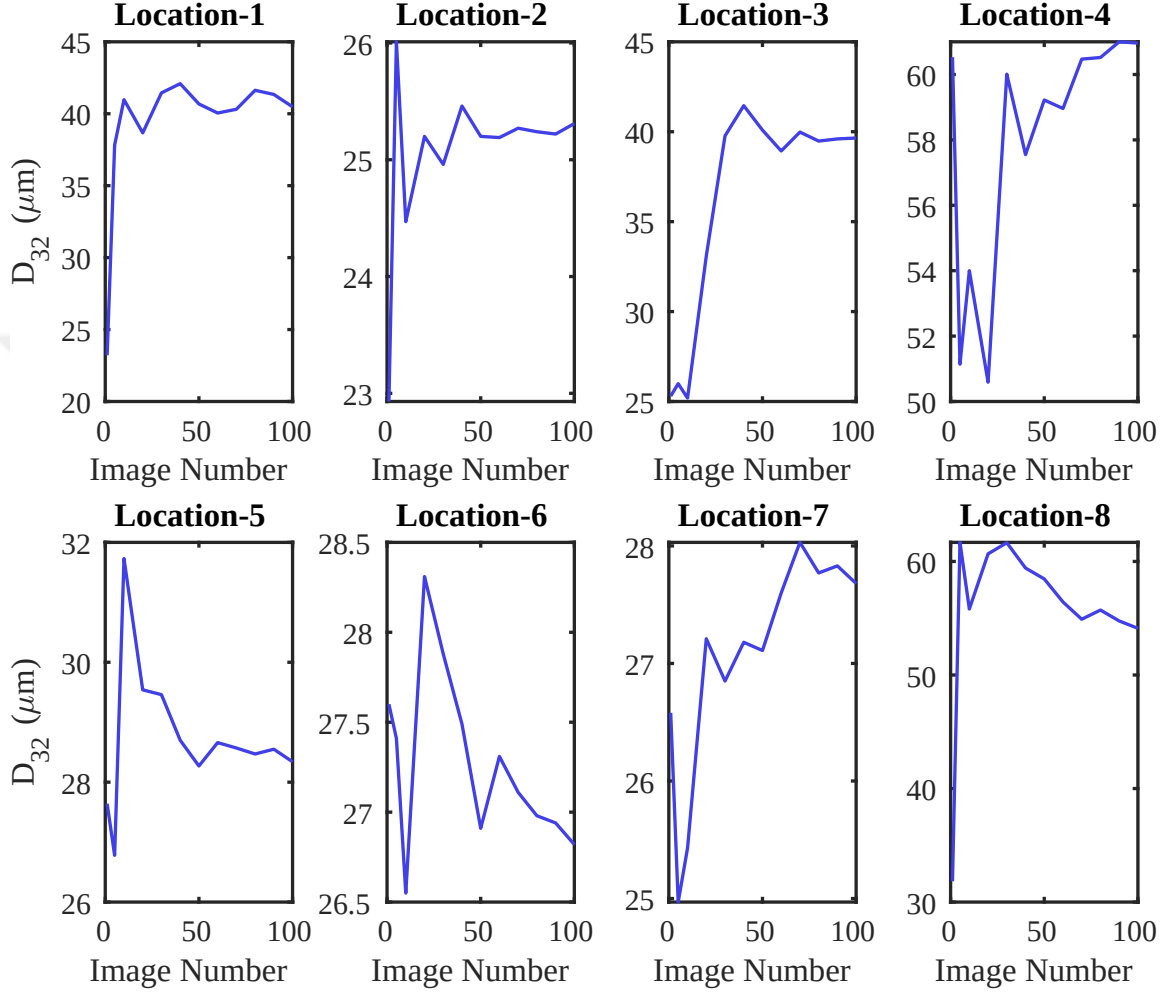


Figure 28: Effect of the image number on the obtained SMD values

2.2.3 LIF/Mie

For the uniformity assessment of the PSA, the SprayMaster system of LaVision is used [21]. Here, elastic scattering (Mie scattering) and LIF (Laser Induced Fluorescence) techniques are applied simultaneously. It should be noted that Mie scattering is caused by elastic scattering on the surface and hence, its intensity is proportional to the surface area. Whereas, LIF is based on the absorption and intensity of the signal

scales with the volume.

In this system, Nd:YAG laser is used with two CMOS cameras, and images of the spray at radial cut are taken. In order to obtain Mie and LIF images simultaneously, one of the cameras is attached with 540 nm filter to capture the LIF signal and the other is attached with 532 nm filter to capture the Mie signal. With the help of light steering optics, a light sheet with an approximate thickness of 1 mm is formed at the measurement location. The orientation of the cameras can be seen in Figure 29.

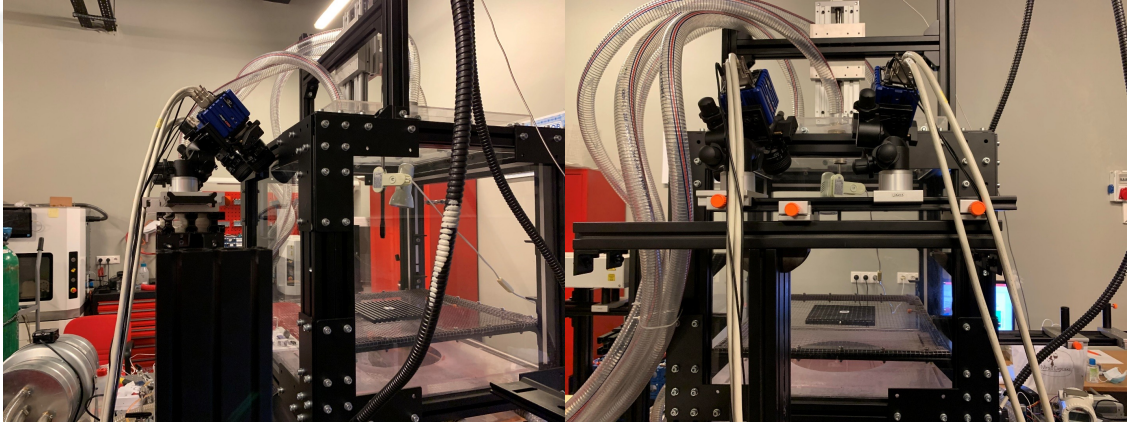


Figure 29: Orientation of the CMOS cameras for radial LIF/Mie measurement

To achieve radial planar cut, cameras overlook to spray with an angle below 45 degrees, as suggested by the system manual, using a special apparatus called Scheimpflug. Calibration of the cameras is done with a 3D calibration plate of the LaVision company, which is shown in Figure 30.

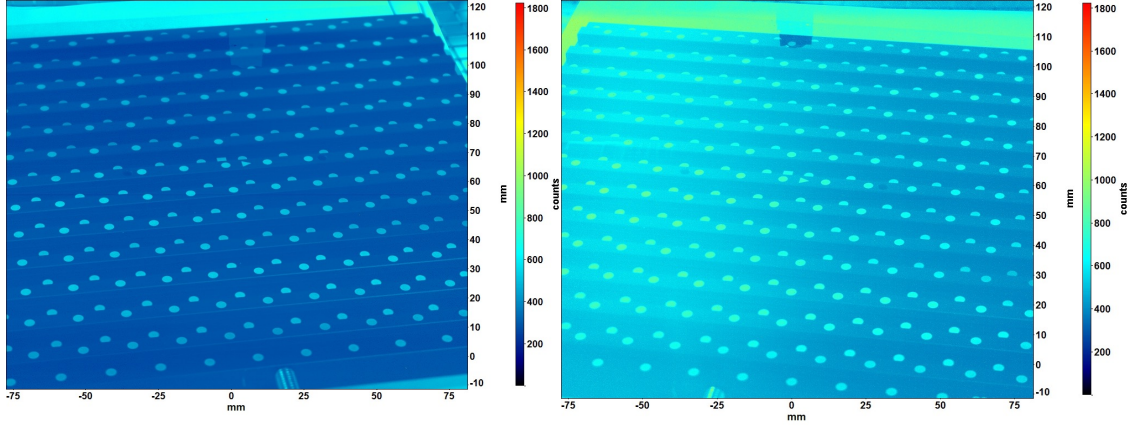


Figure 30: 3D calibration plate of LaVision company

Results are obtained after performing the steps listed below.

1. Obtaining and averaging background images

In order to eliminate the effect of the camera's dark image and the surrounding light, background images are taken. The average of these images is taken to be used in further steps.

2. Obtaining experimental images and averaging

At each experiment condition, 100 LIF and Mie images are obtained and later averaged. Figure 31 displays the example of an obtained image and the average experiment image.

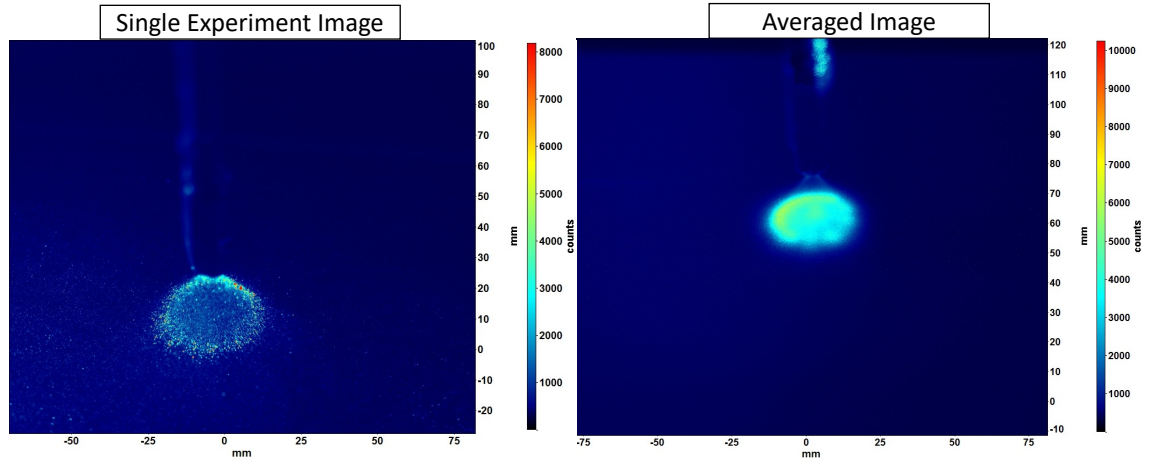


Figure 31: Single and averaged experiment images

3. Subtracting averaged background images from the averaged experiment images

As mentioned in step 1, the average background image is subtracted from the average experiment image.

4. Correcting (distorting) the images

In the last step, the images are mapped using the calibration function, as shown in Figure 32.

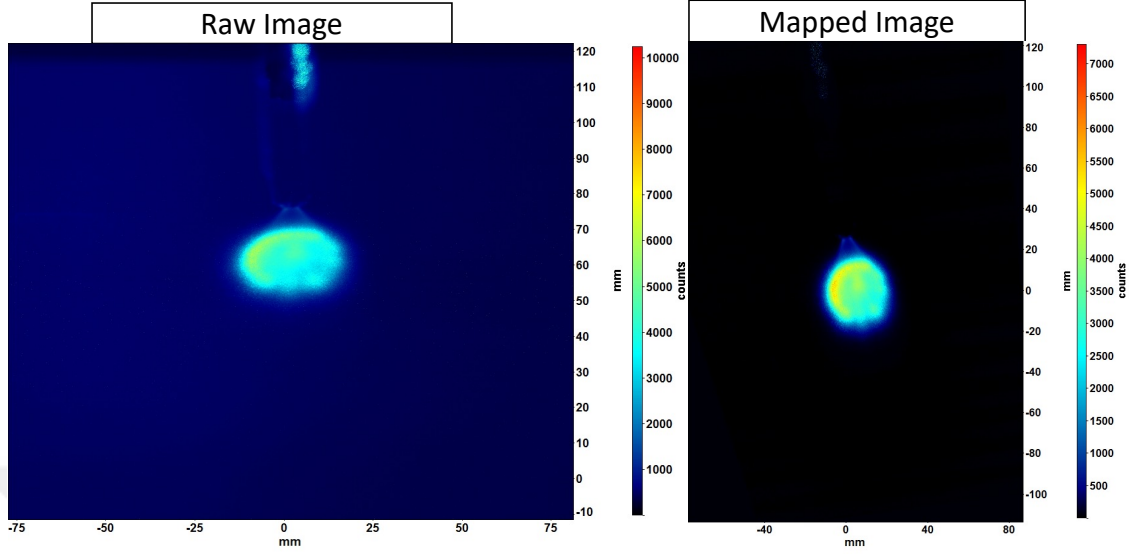


Figure 32: Raw and mapped experiment images

Analysis of the uniformity of PSA is done based on the obtained LIF images. As mentioned earlier, the intensity of the LIF signal is proportional to the volume of the droplets. According to the manual of the system, this image can be used for mapping the liquid phase alone. It should be noted that all of the experiments in this study are conducted at ambient conditions hence LIF image can be used for quantification of the uniformity. For the cases with evaporation or air flow use of the Mie images are suggested by the manual of the system [21].

2.3 Atomizers

In this work total of twenty-four atomizers are used for various experiments. Many of the atomizers are shelf products of companies such as Hago, Steinen, and Schlick. Flow rate vs pressure difference characteristics of the atomizers is given in Figure 33. As can be seen from the graph, some atomizers display very similar flow characteristics, such as Atomizer-4, Atomizer-5, and Atomizer-6. This is due to the fact that most of the atomizers are obtained as sets of 3. After obtaining the flow characteristic

of these atomizers only one of them, that have the most similar characteristic to the catalog values, is used in further experiments.

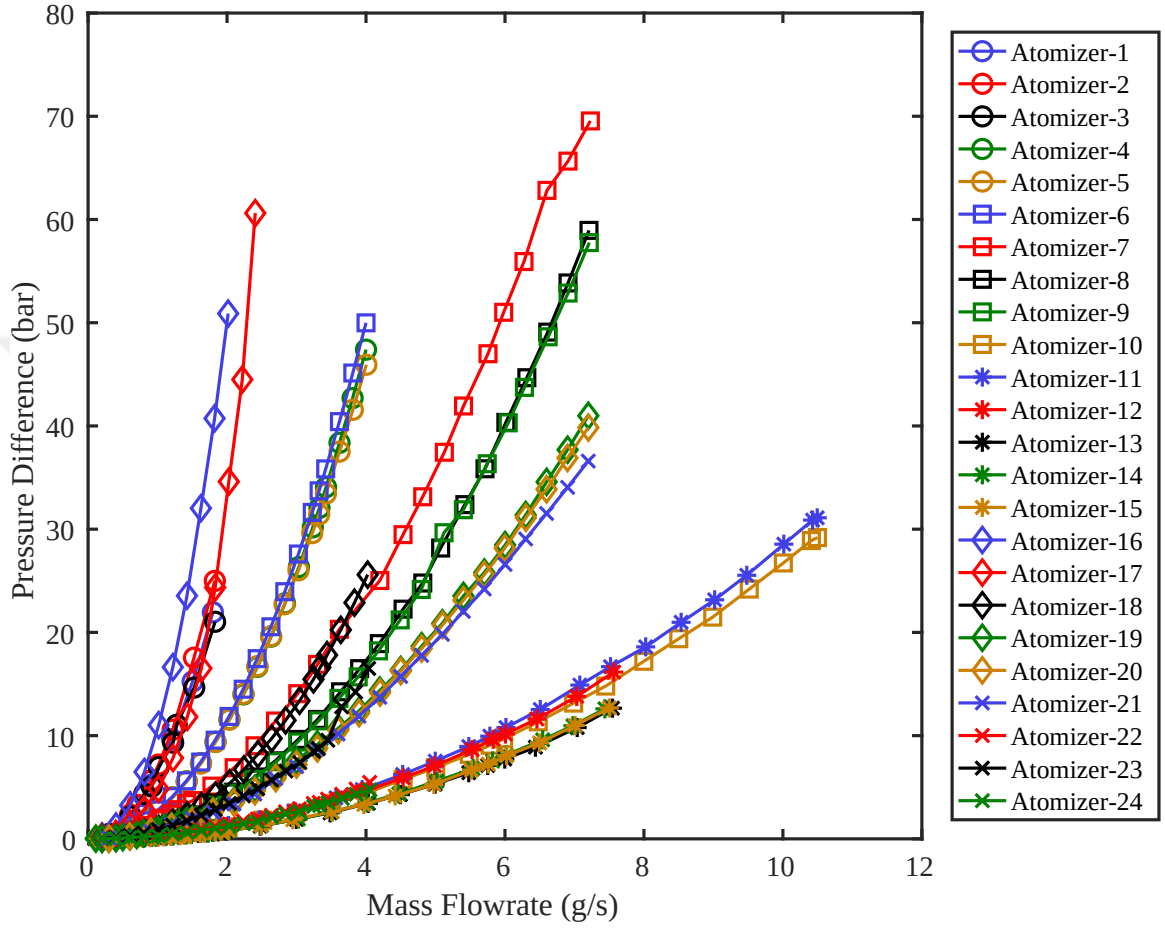


Figure 33: Flow characteristics of the atomizers

It should also be mentioned that three of the atomizers (Atomizer-22, 23, and 24) are designed in the framework of this study. A detailed description of these atomizers will be given in Chapter 6.

CHAPTER III

NUMERICAL SETUP

The internal flow of the PSA includes complex flow phenomenons such as liquid/gas interface, instabilities, and turbulence, as mentioned earlier. It is known that these phenomenons are closely coupled with the performance parameters of the atomizer, such as droplet sizes and spray angle. Due to the small dimensions of PSA, numerical studies are performed for a better understanding of the internal flow field.

For the modeling of the internal flow, many debating methodologies are proposed in the literature. Details of the conflicting studies can be found in literature [14]. To sum up, a large number of studies have proposed the laminar model to be more accurate than RANS models such as $k-\epsilon$, $k-\omega$ and RSM (Reynolds Stress Model) and it was claimed that the laminar model has similar order of accuracy as the Large Eddy Simulation. Therefore in this study, the laminar model is used for the simulations.

Also, numerical studies in the literature can be divided into three groups based on geometry, namely 2D (two-dimensional) axisymmetric studies, 3D (three-dimensional) periodic studies, and full 3D numerical studies. Details and comparisons of the developed methodologies and models can be found in the literature [14]. In this study, two different numerical methodologies, namely, 2D axisymmetric and full 3D are performed.

3.1 2D Numerical Setup

2D axisymmetric simulations have the benefit of requiring less computational power compared to the quarter model and 3D simulations yet, it also leads to less accurate results [14, 15].

One of the reasons is due to the finite number of tangential ports, PSA is prone to

exhibit non-uniformity in the internal flow field which defies the essential assumption of axisymmetry [22].

Another reason is due to the method of modeling the tangential inlets in 2D. For 2D axisymmetric simulations, tangential inlets are modeled as small slits on the swirl chamber wall, as shown in Figure 34.

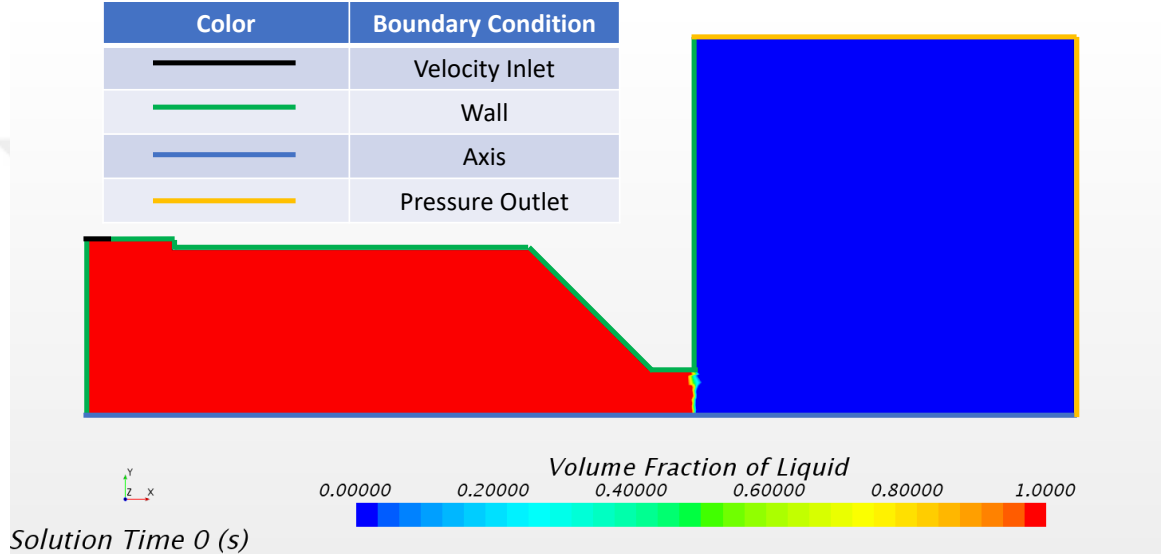


Figure 34: Boundary conditions for 2D axisymmetric simulations

This approach requires the splitting of the inlet port velocity into radial and tangential components to mimic the flow of the tangential port, as described in Figure 35.

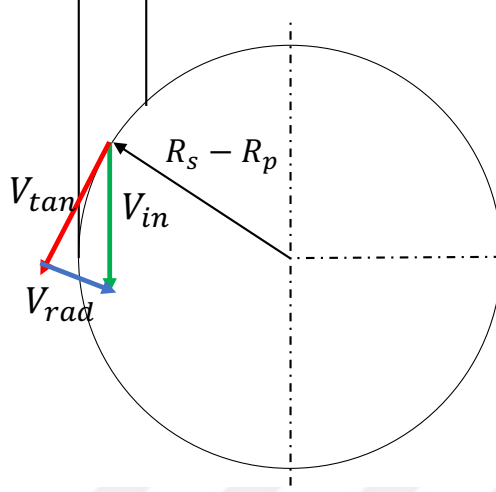


Figure 35: Splitting the inlet velocity into radial and tangential components

Splitting of the inlet velocity into radial and tangential components is done by conserving the angular momentum (Eq. 4) total kinetic energy (Eq. 6), and mass flow rate (Eq. 7) [23]. It should be noted that Eq. 7 shows the calculation of the width of the inlet opening to ensure mass flow rate is conserved.

$$V_{in} = \frac{\dot{Q}}{A_p} \quad (4)$$

$$V_{tan} = \frac{V_{in} \times (R_s - R_p)}{R_s} \quad (5)$$

$$V_{rad} = \sqrt{V_{in}^2 - V_{tan}^2} \quad (6)$$

$$L_{in} = \frac{\dot{Q}}{\pi \times D_s \times V_{rad}} \quad (7)$$

In two recently published studies, it was shown that obtained velocity components from the correlations above could lead to errors in the calculations [14, 15]. Both studies reported that calculated velocity values are smaller than that of 3D simulations and reasoned this with the distortion of the inlet velocity profile at the inlet ports.

During the literature survey, it is seen that similar phenomena is also described in much earlier study by the results of performed experiments [5]. In this study, authors have performed experiments with large-scale atomizers and placed pressure

transducers on various radial distances from the central axis. Based on the results it is reported that liquid flow from the inlet ports changes its shape before entering the swirl chamber. Moreover, it is stated that this is due to the fact that as the liquid flows from the inlet ports, due to the existing swirling motion, it will be pushed onto the walls of the swirl chamber and the distance from the inlet stream to the swirl chamber will increase. Based on the measurements author has provided a correction factor (ϵ) which is a function of the inlet and swirl chamber diameters (B) and can be used to calculate the real radius of the swirl. The experimental results of this study and the fitted correlation to the data are shown in Figure 36.

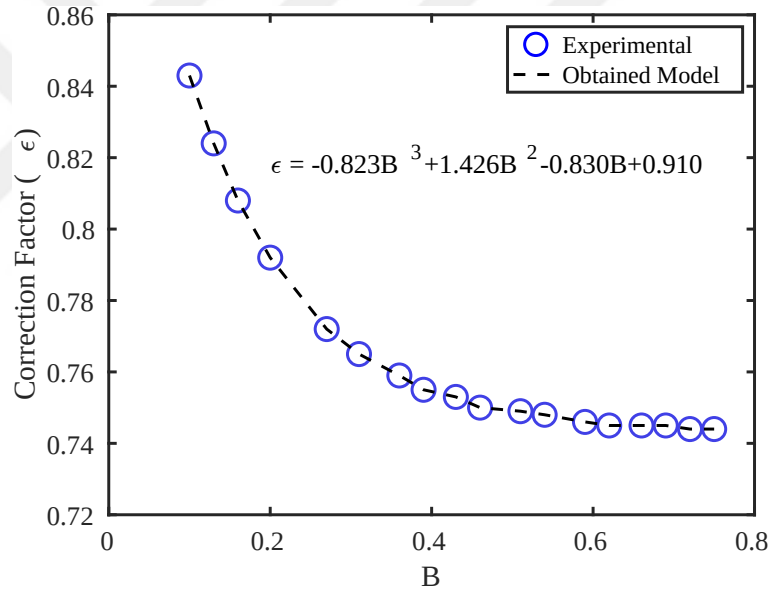


Figure 36: Experimental results and model fitted to the data [5]

$$B = \frac{R_p}{R_s - R_p} \quad (8)$$

It should be noted that in the above-mentioned study, it is not mentioned whether the increase only occurs in the tangential component or the total inlet velocity. One of the recently conducted studies found that the increase is in the total velocity [15]. Whereas other study suggested that only the tangential component increases

[14]. Therefore, a number of simulations are performed with calculating velocity components using:

1. Velocity components and inlet width calculated from Eq. 4 - 7
2. Increased tangential velocity component, same radial velocity component, and inlet width

$$V_{tan,new} = \frac{V_{tan}}{\epsilon} \quad (9)$$

3. Increased total inlet velocity

$$V_{tan,new} = \frac{V_{tan}}{\epsilon} \quad (10)$$

$$V_{rad,new} = \sqrt{\frac{V_{in}^2}{\epsilon} - V_{tan,new}^2} \quad (11)$$

$$L_{in,new} = \frac{\dot{Q}}{\pi \times D_s \times V_{rad,new}} \quad (12)$$

It should be emphasized that, since the radial velocity component changes in the third method, the width of the inlet is adjusted accordingly to ensure the same flow rate as the other models.

Comparisons of these methods are done with the experimental data obtained from the literature. Summary of the used data can be found in Table 2. Total of 17 cases was analyzed and considering that three different inlet velocity models are used, total of 51 simulations are conducted where inlet Reynolds numbers (Eq. 13) were ranging from 2,575 to 47,670.

$$Re_p = \frac{V_{in} \times D_{h,p} \times \rho_L}{\mu_L} \quad (13)$$

where, $D_{h,p}$ is the hydraulic inlet port diameter, ρ_L and μ_L are the density and absolute viscosity of the liquid, respectively.

Moreover, studied cases are selected to represent atomizers with different characteristics. The effect of atomizer scale (small or large), type of atomizer (open or close-end), shape and the number of the inlets, physical properties of the liquid, ambient pressures, and opening ratio (B) are analyzed.

Table 2: Validation Cases for 2D Axisymmetric Simulations

No.	Type	Flowrate (g/s)	Pressure Difference (bar)	Liquid	Ambient Pressure (bar)	Number of Inlets	Inlet Shape	B	Inlet Reynolds Number
1	Close-End	40.09	0.167	Water	1.0	2	Square	0.32	4,447
2	Close-End	60.19	0.390	Water	1.0	2	Square	0.32	6,677
3	Close-End	80.08	0.733	Water	1.0	2	Square	0.32	8,884
4	Close-End	97.98	1.156	Water	1.0	2	Square	0.32	10,869
5	Close-End	26.33	0.088	p-cymene	1.0	3	Rectangular	0.27	2,035
6	Close-End	52.33	0.363	p-cymene	1.0	3	Rectangular	0.27	4,044
7	Close-End	91.50	1.321	Water	1.0	3	Circular	0.18	12,924
8	Close-End	61.56	4.233	Water	1.0	3	Circular	0.18	8,694
9	Close-End	78.19	6.526	Water	1.0	3	Circular	0.18	11,044
10	Close-End	91.50	9.038	Water	1.0	3	Circular	0.18	12,924
11	Close-End	90.00	24.1	Water	3.4	3	Circular	0.48	24,446
12	Close-End	90.00	23.6	Water	24.0	3	Circular	0.48	24,446
13	Close-End	90.00	22.6	Water	48.1	3	Circular	0.48	24,446
14	Close-End	3.09	6.000	Water	1.0	4	Square	0.09	2,575
15	Close-End	3.66	8.350	Water	1.0	4	Square	0.09	3,045
16	Open-End	82.50	3.950	Water	1.0	1	Circular	0.38	47,670
17	Open-End	83.31	1.430	Water	1.0	1	Circular	0.51	39,224

Simulations are conducted with the Star-CCM+ 13.04.011 commercial software. A fixed time step of 1E-6 s is used for all of the cases. Simulations are started with atomizer filled with liquid up to the exit of the orifice to shorten the calculation time, as shown in Figure 34.

For all cases, simulation time is selected based on the pass time, which is defined as the ratio of atomizer volume to the ratio of volume flow rate (Eq. 14). It is reported that in 1.3 to 1.6 pass times, the internal flow of the atomizer develops [24]. Therefore, each simulation is run until three pass time is satisfied.

$$Pass\ Time = \frac{Atomizer\ Volume}{Volume\ Flow\ Rate} \quad (14)$$

Mesh sensitivity study is done with the case of the highest Reynolds number. Three different meshes, with 20,000, 30,000, and 40,000 cells are analyzed. As shown in Figure 37, increasing the mesh count from 20,000 to 30,000 drastically improves the calculated discharge coefficient value yet further increase only leads to a small change. Therefore, 30,000 cell count is used for all cases.

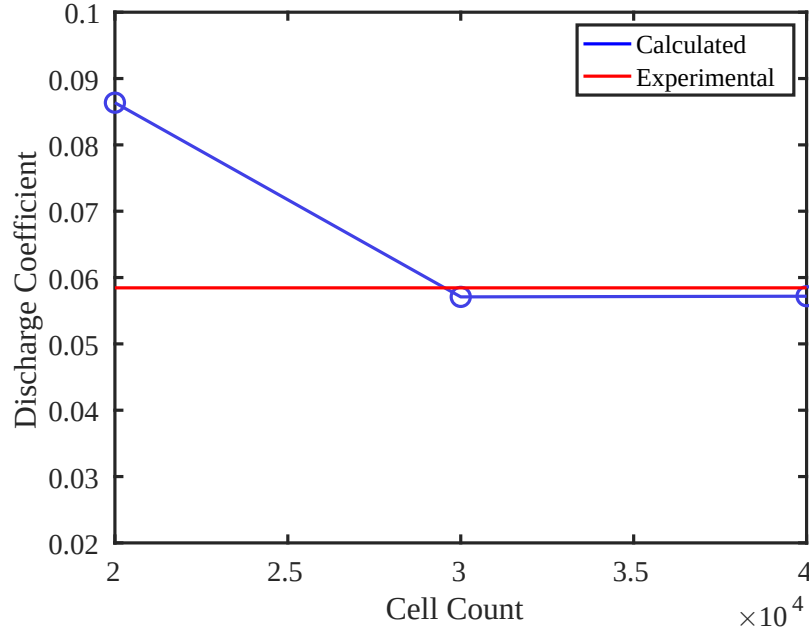


Figure 37: Mesh sensitivity study for the 2D axisymmetric simulations

Three methods are evaluated by comparing the calculated values of discharge coefficient, spray angle, and film thickness with the experimentally obtained values. These values are recorded for the last pass time duration and their average value is used for comparisons. The spray angle is calculated by exporting volume fraction information on two lines at distances of 3 mm and 6 mm downstream of the atomizer exit. Using this data the y-coordinate values corresponding to the maximum volume fraction are found. The tangent of the spray angle is obtained by the ratio of the difference between these two y-location values and the distance between the lines.

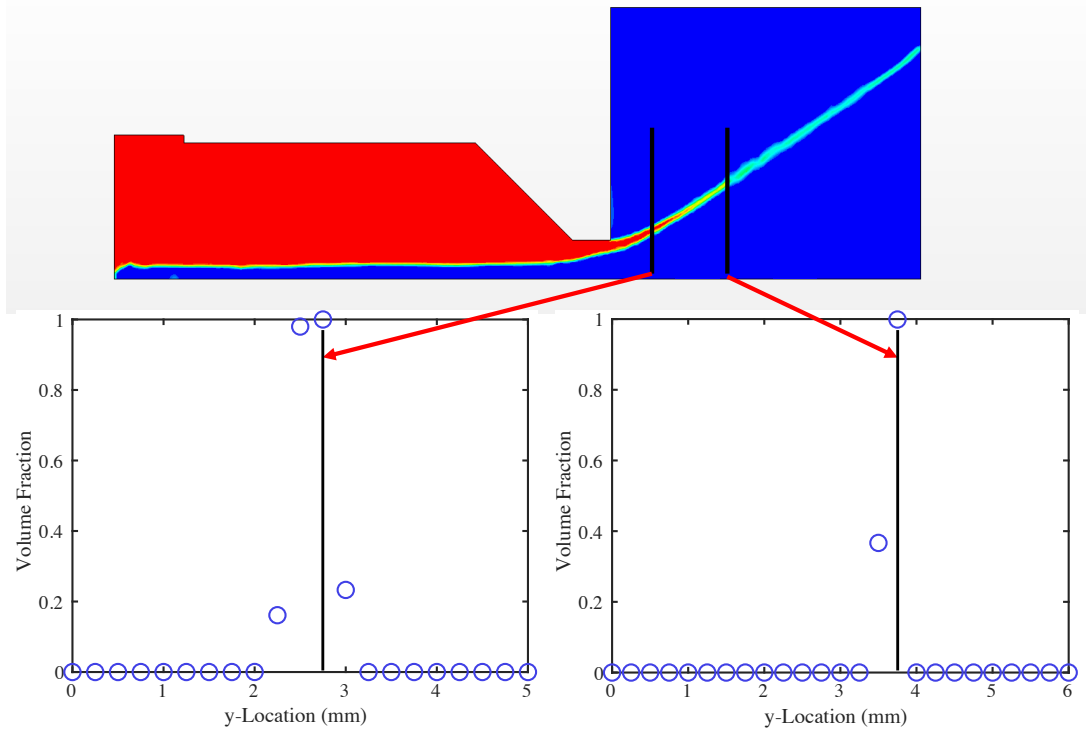


Figure 38: Spray angle calculation of 2D axisymmetric simulations

Obtained results for the pressure difference are shown in Figure 39. As can be seen, the proposed inlet velocity calculation models significantly improve the accuracy of the calculated values. Moreover, both models produced almost identical values with exception of cases 16 and 17.

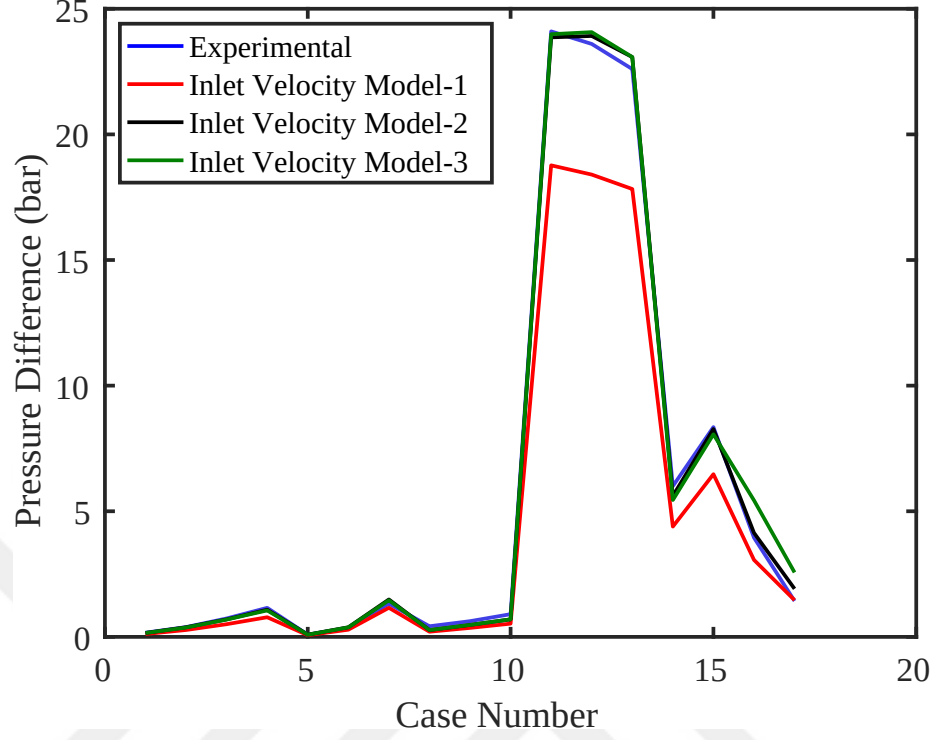


Figure 39: Calculated and experimental pressure difference of 2D axisymmetric simulations

Detailed evaluation of the models can be done by inspecting Table 3. Results showed that both of the velocity models drastically improve the accuracy of the 2D axisymmetric simulations. The biggest improvement is in the calculated discharge coefficient (Eq. 15) values. Moreover, the mean of the error values of calculated parameters are almost identical for both methods.

$$C_D = \frac{\dot{m}_l}{A_o \times \sqrt{2 \times \rho_L \times \Delta P_L}} \quad (15)$$

where A_o is the area of the orifice.

Table 3: Results of Studied Cases for 2D Axisymmetric Simulations

No.	Abs. Rel. Error of C_D			Abs. Rel. Error of t			Abs. Diff. of 2θ		
	Model-1	Model-2	Model-3	Model-1	Model-2	Model-3	Model-1	Model-2	Model-3
1	21.5	3.3	4.3	7.1	5.1	7.1	17.7	2.8	6.5
2	19.2	1.0	2.3	11.1	0.1	0.1	15.1	4.6	1.8
3	20.5	2.7	3.1	5.1	5.1	5.1	12.2	1.2	2.6
4	21.6	3.4	4.8	1.0	1.0	1.0	9.7	0.8	1.4
5	14.8	0.9	1.4	15.5	5.0	3.4	17.8	3.5	3.5
6	12.7	2.5	2.0	21.0	2.1	1.2	18.5	4.9	4.3
7	6.8	5.8	4.8	34.4	2.2	2.1	3.1	1.6	1.6
8	43.5	2.5	2.5	7.1	7.1	7.1	2.8	1.6	0.8
9	32.2	1.5	1.5	8.7	1.9	1.8	4.0	0.1	0.1
10	31.0	1.4	1.4	14.0	3.0	3.0	3.9	1.3	0.1
11	13.3	0.5	0.2	12.1	5.4	5.6	4.2	2.1	2.5
12	13.2	0.7	1.0	6.4	1.3	1.6	1.9	2.9	3.3
13	12.6	1.0	1.0	17.3	1.0	1.0	9.5	4.3	4.3
14	16.9	3.4	4.9	-	-	-	25.2	0.1	2.4
15	13.6	0.5	1.8	-	-	-	8.2	1.3	0.1
16	13.5	2.3	1.5	-	-	-	10.5	1.7	0.3
17	11.3	1.4	2.5	-	-	-	-	-	-
MEAN	18.7	2.1	2.2	12.4	3.1	3.1	10.3	2.2	2.2

3.2 3D Numerical Setup

3D full geometry numerical studies are conducted by solving continuity and Navier-Stokes equations using the finite volume method (FVM). Volume of Fluid (VOF) model is used to consider multiphase flow with High Resolution Interface Capturing (HRIC) method to precisely capture the liquid/gas interface.

The VOF model description assumes that all immiscible fluid phases present in a control volume share velocity, pressure, and temperature fields. Therefore, the same set of basic governing equations which describe momentum, mass, and energy transport in a single-phase flow, are solved.

$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_A \rho (v - v_g) \cdot d\mathbf{a} = \int_V S_u dV \quad (16)$$

$$\frac{\partial}{\partial t} \left(\int_V \rho v \right) dV + \oint_A \rho v \otimes (v - v_g) \cdot d\mathbf{a} = - \oint_A p I \cdot d\mathbf{a} + \oint_A \mathbf{T} \cdot d\mathbf{a} + \int_V f_g dV \quad (17)$$

In equations 16 and 17, fluid properties are assumed as that of an equivalent fluid in which its properties are stated as,

$$\rho = \sum_i \rho_i \alpha_i \quad (18)$$

$$\mu = \sum_i \mu_i \alpha_i \quad (19)$$

$$c_p = \sum_i \frac{(c_p)_i \rho_i}{\rho} \alpha_i \quad (20)$$

and,

$$\alpha_i = \frac{V_i}{V} \quad (21)$$

Where α_i is the volume fraction and ρ_i , μ_i and c_{p_i} are density, viscosity and specific heat of phases i , respectively.

The conservation equation that describes the transport of volume fractions α_i is stated as,

$$\frac{d}{dt} \int_V \alpha_i dV + \int_S \alpha_i (v - v_g) \cdot d\mathbf{a} = \int_V \left(s_{\alpha_i} - \frac{\alpha_i}{\rho_i} \frac{D\rho_i}{Dt} \right) dV \quad (22)$$

where s_{α_i} is the source or sink of the i^{th} phase, and $\frac{D\rho_i}{Dt}$ is the material or lagrangian derivative of the phase's density ρ_i .

If a non-zero sharpening factor is specified, an additional term is added to the VOF transport equation:

$$\nabla \cdot (v_{c_i} \alpha_i (1 - \alpha_i)) \quad (23)$$

where α_i is the volume fraction of phase i and v_{c_i} is defined as follows:

$$v_{c_i} = C_\alpha \times |v| \frac{|\nabla \alpha_i|}{|\nabla \alpha_i|} \quad (24)$$

C_α is the sharpening factor and v is the fluid velocity.

To assess the occurrence of the cavitation phenomenon, the Schnerr-Sauer cavitation model is used inside the atomizer.

In order to solve governing equations using FVM, two different unstructured grid is used to discretize the solution domain. First, calculations with a coarser (low-resolution) mesh are done until the air core inside the atomizer is formed. Later this mesh is refined to obtain the high-resolution solution. It should be noted that high-resolution mesh is only used inside the atomizer and region of spray, and for the regions close to the outlet coarser mesh is used. The transition time of the case is designated based on the pass time criteria described in the above section. Used meshes are shown in Figure 40.

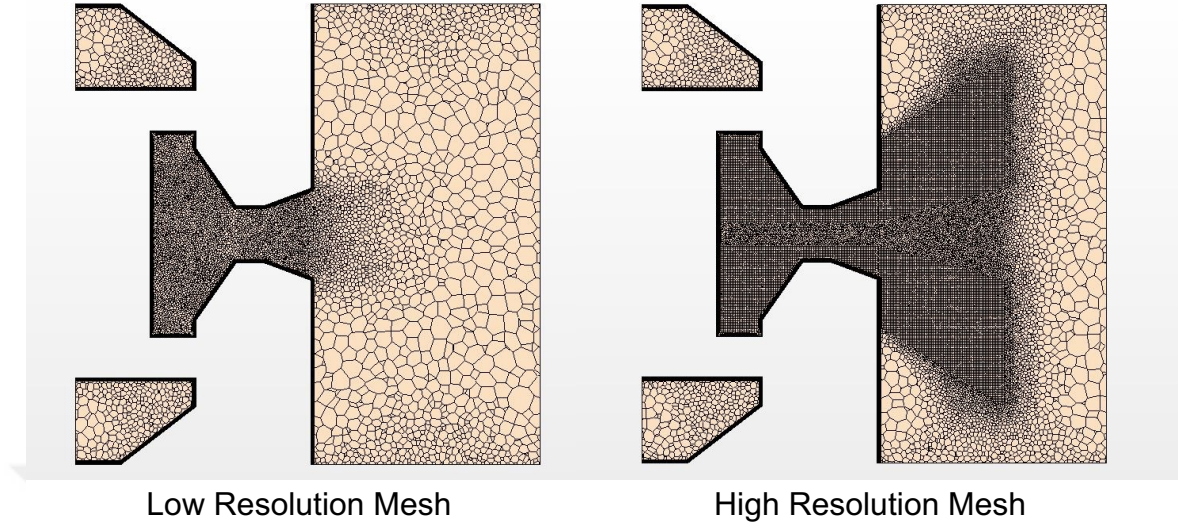


Figure 40: Used meshes for 3D simulations

Similar to the 2D axisymmetric studies, Star-CCM+ 13.04.011 is used for the simulations. Detailed explanation of the mesh sensitivity and validation of the used method is done in an earlier study [24]. Moreover, details of the studied cases and geometries are explained in greater detail in Chapter 6.

CHAPTER IV

EMPIRICAL MODEL

4.1 Introduction

In the past few decades, a large number of studies with many different methods are conducted to obtain simpler models (1D), compared to numerical methods, for predicting the performance parameters of the PSA.

In an earlier study of the author and coworkers experimental data extracted from the 17 different previous works along with the semi-empirical correlations are cataloged and the accuracy of these equations is quantified on a widest possible data range [4]. Results have shown that error values of semi-empirical correlations for predicting the discharge coefficient, pressure difference, film thickness, spray angle, break-up length, and droplet size are higher than 20 %.

From the literature survey, it is seen that although researchers have used many different methods for deriving semi-empirical correlations, in all of the studies experimental data is used for adjusting the equations [4]. Even though introducing constants or variables through the measurements allows to model the performance parameters, it results in models that are bounded with the range of experimental data. Furthermore, it is noticed that many of these studies include only a small number of experimental data, which is usually conducted by the same researchers. This is one of the reasons that the error values are increased, in comparison to the reported ones, when they are used in a wider range of experimental data.

Therefore, the intent of this chapter is to derive new models, that are more accurate than the previous ones, in terms of predicting the performance parameters of the PSA. For this purpose, a large number of experimental data that is reported in

the literature and conducted in this study is used for evaluating existing models and obtaining new models. Modeled performance parameters are discharge coefficient, pressure difference, film thickness, spray angle, break-up length, and droplet sizes.

4.2 Open Source Database

During the literature survey, it is noticed that in most of the studies, only a small number of geometrical parameters of the studied atomizers are provided. This could be due to the fact that atomizers are used in key technologies such as internal combustion and gas turbine engines. However, in order to perform initial dimensioning of the atomizer 1D models are crucial. Moreover, in a geometry optimization study, usually, thousands of geometries are evaluated and this can only be done with 1D models. It should be noted that numerical modeling can produce higher accuracy but the computational time of thousands of geometries will be massive.

Therefore creating an open-source database was one of the side objectives of this dissertation. Through the supplied information here, researchers can access a large number of experimental data and can make comparisons or determine the region of their study.

Figure 41 shows the range of the obtained experimental data in terms of Bulk Reynolds Number (Eq. 25) and Bulk Weber Number (Eq. 26). It should be noted that, in Figure 41, each box represents the limits of different studies and the thicker red box shows the range of the experiments conducted in this dissertation. As can be seen, experimental data covers a very wide range, and therefore, correlations will be tested comprehensively.

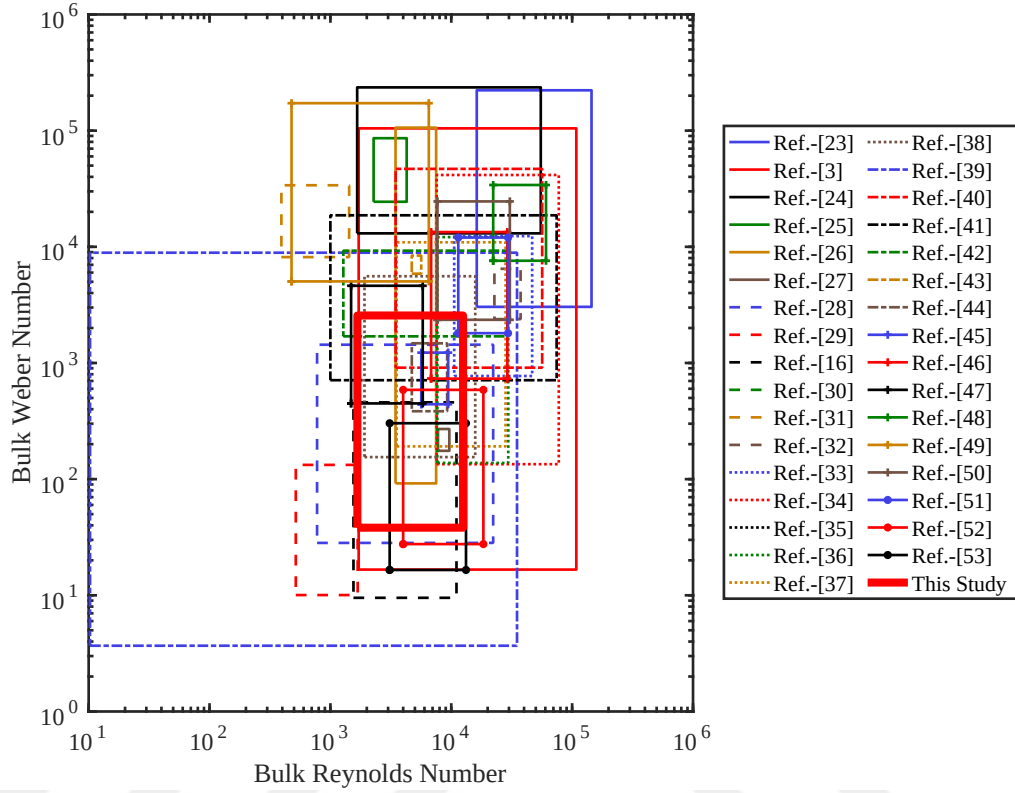


Figure 41: Range of the collected experimental data in terms of Bulk Reynolds and Weber numbers

$$Re_B = \frac{V_B \times \rho_L \times D_{exit}}{\mu_L} \quad (25)$$

$$We_B = \frac{V_B \times \rho_L \times D_{exit}}{\sigma_L} \quad (26)$$

where, σ_L is the surface tension of the liquid and D_{exit} is the exit diameter of the atomizer. For the cases without trumpet, this is equal to the diameter of the orifice and for the cases with trumpet, it is equal to the diameter of the trumpet. V_B is the bulk exit velocity as shown in Eq. 27.

$$V_B = \frac{\dot{m}_L}{A_{exit} \times \rho_L} \quad (27)$$

where, $\dot{m}_L$ is the mass flow rate and A_{exit} is the exit area.

4.2.1 Close-End Pressure-Swirl Atomizer

Before the experimental data, the geometrical parameters of the atomizers are listed. Schematic of the close-end atomizer geometry is given in Figure 42. For close-end, geometry can be divided into five regions, which are inlet, swirl chamber, convergence, orifice, and trumpet. From Table 4 it can be seen that 15 or 16 geometrical parameters (depending on the circular or rectangular inlet) are needed to describe the geometry of close-end PSA.

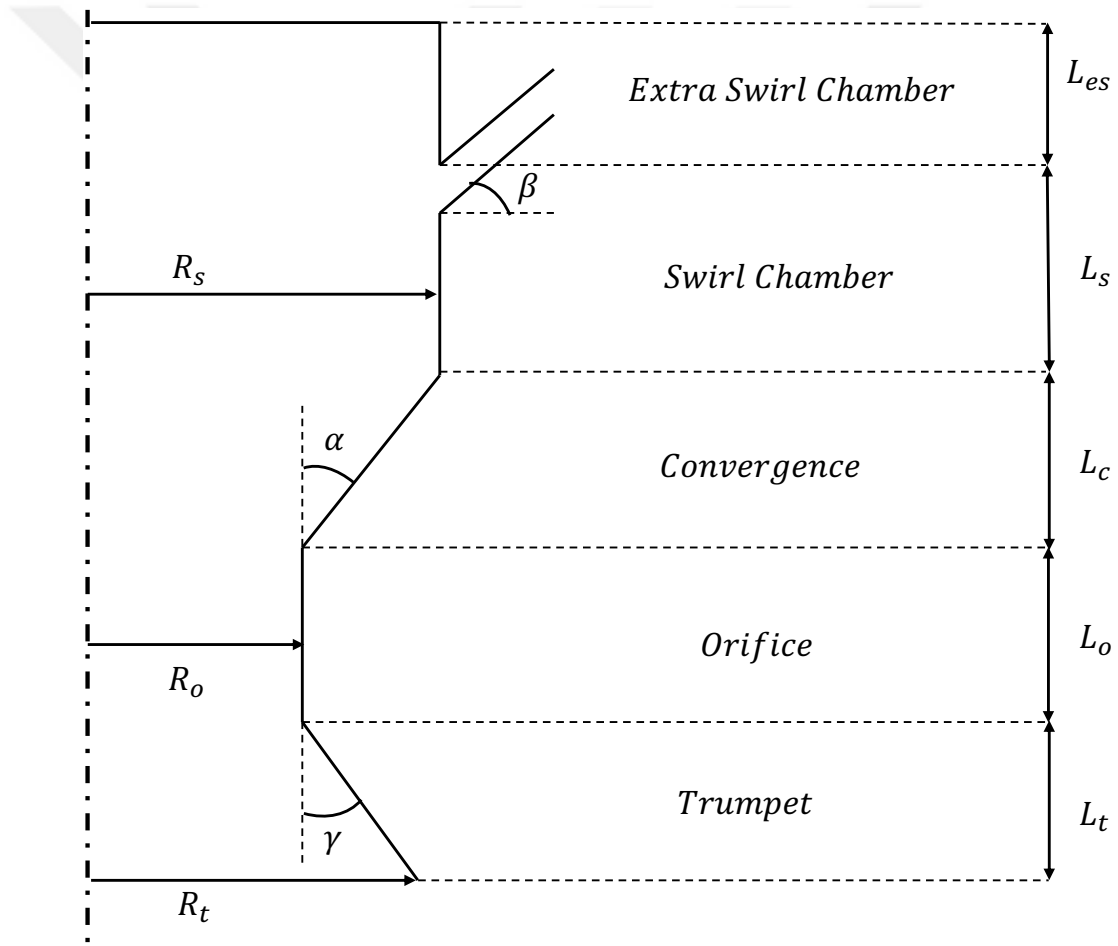


Figure 42: Schematic of close-end pressure-swirl atomizer

Table 4: Geometrical Parameters of Close-End Atomizers

Inlet	
N_p	Number of Inlets
D_p	Diameter of Inlet
w_p	Width of Inlet
h_p	Height of Inlets
L_p	Length of Inlets
$R_{in} = (R_s - 0.5 \times D_{h,p})$	Swirl Arm
β	Angle of Inlets
Swirl Chamber	
L_{es}	External Swirl Chamber Length
L_s	Swirl Chamber Length
D_s	Swirl Chamber Diameter
Convergence	
L_c	Convergence Length
α	Convergence Half Angle
Orifice	
L_o	Orifice Length
D_o	Orifice Diameter
Trumpet	
L_t	Trumpet Length
D_t	Trumpet Diameter
γ	Trumpet Half Angle

During the literature survey, it is seen that in most studies some of the geometrical parameters are not provided. Therefore, based on the level of presented geometrical parameters, studies are classified into four groups as listed below.

1. Complete Data: All of the geometrical parameters are provided. Data in this group can be used to perform 3D full geometry simulations, 2D axisymmetric simulations, and evaluation of the 1D models.
2. Quasi-Complete Data: Length of the inlets is not provided. Therefore 3D full geometry simulations cannot be done. 2D axisymmetric simulations and evaluation of the 1D models can be done.
3. Pseudo-Complete Data: In addition to length of the inlets, convergence angle is not provided for the data in this class. This prevents performing both 2D and 3D simulations. However, since the convergence angle is omitted in almost all of the semi-empirical correlations, this data can also be used to assess the accuracy of semi-empirical correlations.
4. Parametric Data: Data in this class is missing multiple geometrical parameters such as the length of the swirl chamber or orifice. It should be noted that these parameters are also lacking in some of the semi-empirical correlations, and hence data in this class can be used to evaluate such correlations. Moreover, parametric analysis can be performed with this data.

4.2.1.1 Complete Data

1. Swirl Atomizers Operating at High Water Pressure, 1996 [8]

In this study, authors have conducted experiments with 10 base geometries and inspected the effect of geometrical parameters, such as length of the inlets, length of the swirl chamber, length of extra swirl chamber, number and angle of the inlets, diameter of swirl chamber and orifice and length of the orifice. In the experiments, water is used and the reported temperature was 17⁰-to-21⁰. Experiments are conducted with pressure difference in the range of 7.0-to-151.7 bar. Reported accuracy for the flow rate measurements was $\pm 1\%$

and the accuracy of the pressure measurements was ± 1 bar. Spray angle measurements are done from the obtained images and calculated at 18 mm downstream. Droplet sizes are measured with Malvern 2600 system at 275 mm downstream. Moreover, the authors have stated that manufacturing tolerances of the atomizer geometries are $\pm 25 \mu\text{m}$. Geometries used in this study are given in Table 34.

Figure 156 shows the obtained discharge coefficient and SMD values for Geometry-1. As expected, SMD decreases with increasing pressure difference, and the discharge coefficient increases at first and then stays constant. Figure 157 displays the effect of inlet port length. Three geometries are shown here, namely, Geometry-2, Geometry-3, and Geometry-4. It can be seen that an initial increase in the length of the ports leads to a significant increase in spray angle and a decrease in the discharge coefficient. After $\frac{L_p}{D_p} = 2$ these parameters do not change significantly. Moreover, while having the same total inlet port area, small differences in the obtained values are observed for Geometry-2 and Geometry-3.

Figure 158 presents the effect of increasing the total swirl chamber length while the ratio of external to the swirl chamber length is constant. As expected, increasing this length leads to lower spray angle values and higher discharge coefficient values, which can be explained by the increase of the losses. In Figure 159, the effect of increasing the extra swirl chamber length is given. The effect of this parameter is found to be very small.

Effect of the inlet angle is shown in Figure 160. When the inlets are perpendicular to the nozzle axis it yielded a lower discharge coefficient and higher spray angle values. Figure 161 gives the results obtained with Geometry-8. Here, the diameter of the swirl chamber is increased while the diameter of the orifice is constant. It can be seen that at $\frac{D_s}{D_o} = 4.5$ a peak in the discharge coefficient

occurs. However, the change in the discharge coefficient and spray angle with respect to this parameter is considerably small. It should also be noted that in this geometry the ratio of swirl chamber length to diameter has remained constant. This means that, as the diameter of the swirl chamber is increased, its length increases as well. As shown in Figure 160 and Figure 158 increasing this parameter significantly increases the discharge coefficient. In this case, this may be prevented by the increase in the diameter of swirl chamber.

The effect of increasing the length of the orifice can be seen in Figure 162. Similar to the length of the inlet ports, the initial increase leads to a significant change in the discharge coefficient. Later this parameter becomes almost insensitive to the increase in the orifice length. Moreover, the spray angle decreases at first and then remains almost constant. Figure 163 displays the measurements conducted with Geometry-10. Here, both the ratio of the diameter of swirl chamber to the diameter of orifice and the length-to-diameter ratio of orifice is changed, while the diameter of the swirl chamber is constant. It should be noted that the change in the length-to-diameter ratio of orifice is considerably smaller. Moreover, as shown in Figure 162 effect of this parameter is considerably small. Therefore, it can be said that increasing the $\frac{D_s}{D_o}$ while D_s is constant, leads to considerably higher discharge coefficient values and considerably smaller spray angle values.

2. Theory and Practice of Swirl Atomizers, 2004 [3]

In this study, authors have conducted experiments with 38 base geometries and inspected the effect of many geometrical parameters. It should be noted that none of the geometries of this study had extra swirl chamber length. Experiments, again, are conducted to obtain discharge coefficient and spray angle values. Results of this study can be found in Table 35 - Table 44 and Figure

164 - Figure 173.

3. Effect of Chamber Back-pressure on Swirl Injector Fluid Mechanics, 2009 [25]

Here, the authors have performed experiments to measure the discharge coefficient, spray angle, and film thickness of PSA at elevated ambient pressures. Water was used as the working liquid and the temperature of the injection medium was reported as 300 K. For the film thickness measurements, a transparent adapter is designed and fitted to the atomizer, and hence the length of the orifice is elongated. Both, spray angle and film thickness are measured from the obtained images, which are taken at 4,000 fps and 10 μ s exposure time. Only single atomizer geometry is used, (Table 45) and it should be noted that all of the experiments are conducted at a constant mass flow rate of 0.09 kg/s. Results are shown in Figure 174 and Figure 175.

4. Optimization of Pressure-Swirl Atomizer for a Burner Refroit, 2013 [26]

Authors have conducted experimental studies using PSA with square inlets. Light heating oil is used as working liquid and the physical properties of the liquid can be found in Table 46. In the experiments discharge coefficient, spray angle, and droplet diameters are measured at a single pressure difference of 2 MPa. It should also be mentioned that authors measured the droplet diameters 100 mm downstream and used a 1-component PDA system of Dantec Dynamics. Geometries of the studied atomizers can be found in Table 47 and the results are shown in Table 48.

5. Energy Considerations in Spraying Process of a Spill-Return Pressure-Swirl Atomizer, 2014 [27]

This study is conducted by the same researchers as the above-mentioned study. Similarly, light heating oil is used for the experiments. Discharge coefficient,

spray angle, vertical break-up length, and droplet sizes are measured for single geometry at different pressure differences. The geometry of the atomizer is given in 47, as number 1. Results for the experiments can be found in Figure 176 and Figure 177. This study shows that with the increase in pressure difference, discharge coefficient decrease, spray angle increases at first then remains constant, break-up length significantly decreases, and SMD decreases.

6. Experimental Assessment of a Pressure-Swirl Atomizer for Ethanol Operation on a Small Gas Generator, 2017 [28]

Atomizer working with ethanol is inspected in this study. The physical properties of the liquid and the geometry of the atomizer are given in Table 49 and Table 50, respectively. The authors have measured discharge coefficient, spray angle, and droplet sizes. It should be noted that droplet sizes are measured at four different downstream locations using Malvern Spray Analyzer. The spray angle is measured from the obtained images. Results can be found in Figure 178 and Figure 179.

7. Parametric Evaluation of Swirl Injector Dynamics in the High-Frequency Range, 2017 [29]

In this study, authors have conducted experiments with single atomizer geometry. Water was the working liquid and measurements of discharge coefficient and spray angle is done. Geometry of the atomizer can be found in Table 51 and results are shown in Figure 180

8. A Study of Spray Performance of MEMS based Microscale Pressure Swirl Atomizer, 2019 [30]

The authors have performed an experimental study with micro-scale atomizers. Overall 25 different geometries at a constant pressure difference of 8 bar

are inspected. Water was the working liquid. The discharge coefficient, spray angle, and uniformity of the atomizers are measured. Spray angle values are obtained from the images taken at 10,000 fps. Uniformity is obtained using SetScan optical patternator at 5 mm downstream. Researchers concluded that the discharge coefficient increases with increasing diameter of the orifice and total inlet port area. Moreover, it was found that as the number of inlets increases, spray becomes more uniform. It should be noted that these atomizers do not contain a convergent section and instead has an abrupt transition from the swirl chamber to the orifice. In other words, the convergence angle is zero. Uniformity is quantified by the term uniformity coefficient which is defined as [5]:

$$\vartheta = \frac{\max(Z) - \min(Z)}{\text{mean}(Z)} \quad (28)$$

where Z is the total liquid surface area per unit volume of the measurement sectors.

Geometry of the atomizers and the results are given Table 52 and 53

9. Dynamical Similarity and Universality of Drop Size and Velocity Spectra in Sprays, 2019 [18]

In this study, authors have inspected the effect of scaling the atomizers. For this purpose, an original atomizer and 3, 5, and 10 times scaled-up versions of this atomizer are manufactured. Water was selected to be the working liquid. Discharge coefficient and spray angle values are measured and reported. Dimensions of the atomizers and the experimental results are given in Table 54 and Figure 181. Similar to the above-mentioned study, these atomizers do not have a convergent section either.

10. Data in this study

As mentioned in Chapter 2, a total of 24 atomizers is used to conduct experiments in this study. Measurements of discharge coefficient, spray angle, diagonal break-up length, and droplet sizes are done. Geometries of the atomizers can be found in Table 5 and results can be found in Figure 43, Figure 44, Figure 45 and Figure 46.

Table 5: Studied Atomizer

No.	D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
22	0.30	1.00	4	1.20	0.30	5.00	1.03	0
23	0.25	1.00	4	1.20	0.25	13.00	0.74	0
24	0.30	1.00	4	1.00	0.30	2.86	0.90	0

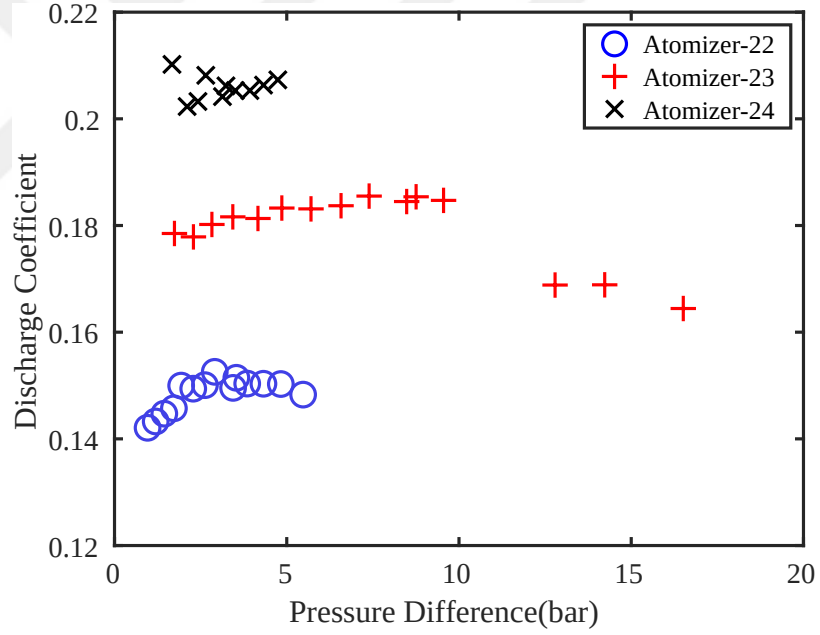


Figure 43: Discharge coefficient of Atomizer-22, 23 and 24 of this study

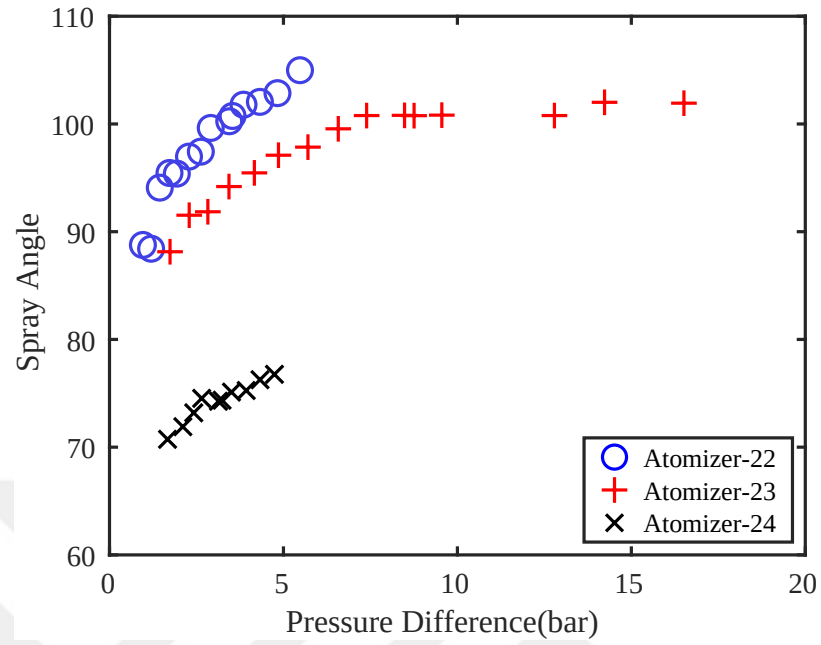


Figure 44: Spray angle of Atomizer-22, 23 and 24 of this study

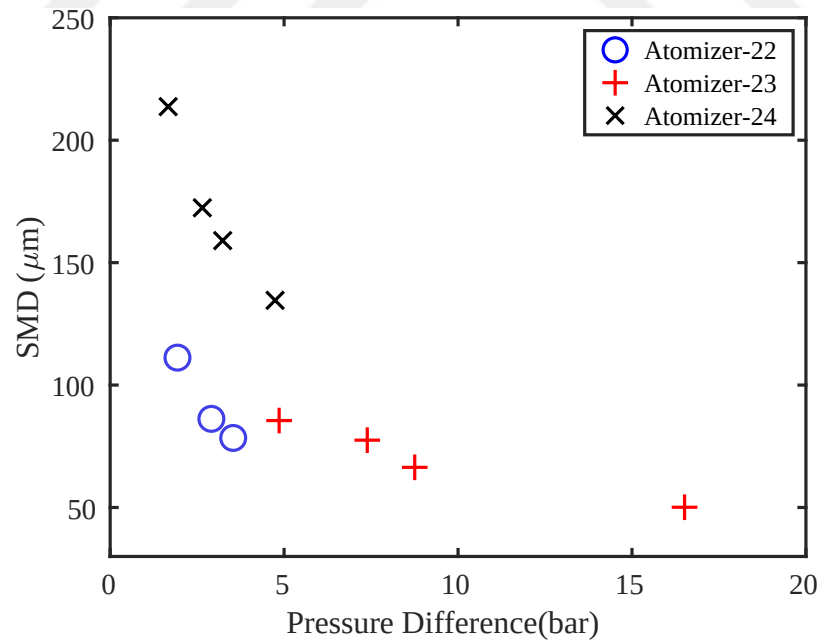


Figure 45: SMD of Atomizer-22, 23 and 24 of this study

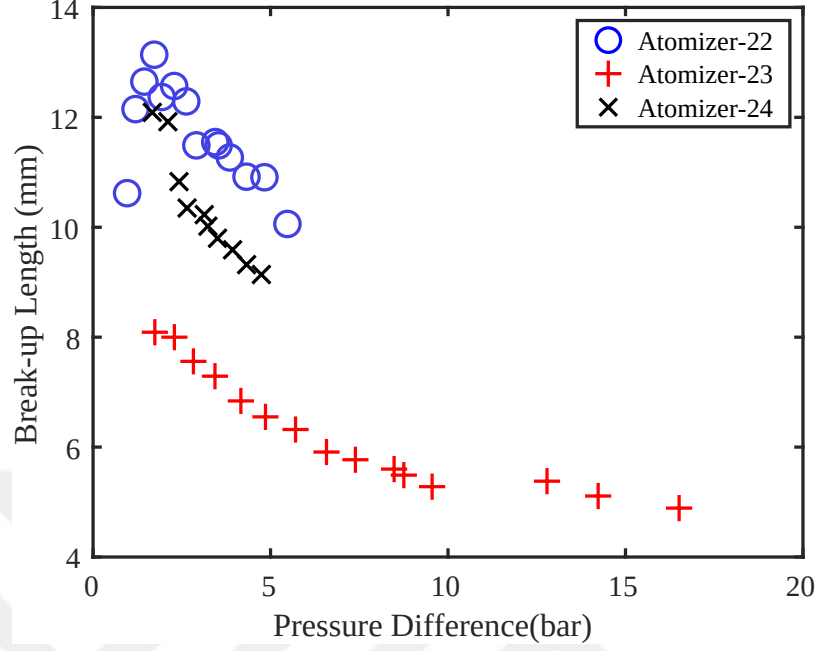


Figure 46: Break-up length of Atomizer-22, 23 and 24 of this study

4.2.1.2 Quasi-Complete Data

1. The Performance of a Type of Swirl Atomizer, 1953 [31]

In this study, author have performed experiments with 279 different atomizer geometries. All of the experiments are conducted at a constant pressure difference of 100 lb per sq. in (6.895 bar). The working liquid is not specified however, the density and the viscosity of the liquid are given as 800 kg/m³ and 2.15 cP, respectively. Only four geometrical parameters have changed and their effect on the discharge coefficient and spray angle is analyzed. Constant dimensions can be found in Table 55 and the obtained results are given in Table 56. It should be noted that Table 56 contains results of 222 geometries. This is due to the fact that in the experiments 77 of the geometries failed to form a spray therefore their results are not provided in the article. Moreover, in the table both the R_{in} and D_s parameters are provided since the author has altered the position of the inlets. In other words, for some cases, inlets are not located at

the edge of the swirl chamber.

2. Discharge Coefficient and Spray Angle Measurement for Small Pressure-Swirl Nozzles, 1994 [32]

This study is conducted to analyze the effect of orifice diameter on the discharge coefficient and the effect of liquid physical properties on the spray angle. For this purpose, experiments with 11 different atomizer geometries are done. The liquid is selected as heavy oil and experiments at four different temperatures are done. The spray angle is measured 3 mm downstream of the atomizer and authors reported the measurement variation to be less than 4^0 . Physical properties of the liquid with respect to temperature are given in Table 57 and the dimensions of the atomizers are given in Table 58 and Table 59. Results of the experiments can be found in Figure 182 - 185.

3. Effect of Inlet Slot Number on the Spray Cone Angle and Discharge Coefficient on Swirl Atomizer, 2012 [33]

In this study authors have conducted experiments with 4 different atomizers, each having different number of inlets. It should be noted that the diameter of inlets was kept constant, so the total inlet area changed with the number of inlets as well. Water was selected to be the working liquid. Measurements of the discharge coefficient and the spray angle are done. Geometry of the atomizers can be found in Table 60 and the results can be found in Figure 186 and Figure 187.

4. Experimental Investigation of Flow Through a Pressure Swirl Atomizer and the Resulting Hollow Cone Spray at Different Operating Conditions, 2013 [34]

This study contains single atomizer geometry which is made from plexiglass material and hence the measurement of film thickness could be performed. Water

was used as the working liquid. It was reported that the film thickness values are obtained from the images taken at 20,000 fps. Geometry and the results can be found in Table 61 and Figure 188.

5. Numerical and Experimental Investigation of a Pressure-Swirl Atomizer Flow Field, 2013 [35]

Similar to the above, this study also contains experiments with single atomizer geometry. Moreover, there was only a single data point consisting of discharge coefficient and spray angle. Results and geometry can be found in Table 62.

6. Spray Characteristics of Closed-Type Swirl Injectors with Varying Swirl Chamber Geometry, 2015 [9]

In this study, 7 different atomizers are used to inspect the effect of the diameter and length of the swirl chamber, separately. Measurements of discharge coefficient, spray angle, and film thickness are done. Water was the working liquid. Geometries can be found in Table 63 and results can be found in Figure 189 and Figure 190.

7. Effects of Orifice Divergence on Hollow Cone Spray at Low Injection Pressures, 2018 [36]

This study is conducted to inspect the effect of orifice divergence (trumpet) on the discharge coefficient, spray angle, break-up length, and droplet size. It should be noted that vertical break-up length values are reported here. Both the spray angle and break-up length values are obtained by taking the average of 100 images. Moreover, in all geometries, total length of the atomizer was kept constant. Geometries can be found in Table 64 and results can be found in Figure 191 - 194.

8. Experimental Study of Spray Break-up Phenomena in Small-Scale Simplex Atomizers with and without Air Swirl, 2018 [37]

In this study, authors have performed experiments with water and kerosene using three different atomizer geometries. Measurements of discharge coefficient, spray angle, and SMD are done. SMD is measured with Malvern Spray Analyzer 30 mm downstream. Table 65 presents the geometries of the atomizers, and Figure 195 and Figure 196 show the measured values.

9. Evaluation of the Atomization Characteristics of Aviation Fuels with Different Viscosities using a Pressure Swirl Atomizer, 2019 [38]

This study reports the results of experiments conducted with 4 different liquids. Authors have named the fluids as Jet-A1, Fuel-A, Fuel-B, and Fuel-C. Experiments for measuring discharge coefficient and spray angle are done at different liquid temperatures. Table 66 and 67 gives the geometry of the atomizer and the properties of the liquid, respectively. Results are given in Figure 197 - 199.

10. Experimental Investigations of Spray Characteristics of a Pressure Swirl Atomizer, 2019 [39]

This study contains the measurements of discharge coefficient, spray angle, and vertical break-up length. Experiments are conducted with RP3 aviation kerosene at 10 °C and single atomizer geometry is used. Spray angle and break-up length are obtained from the average of the 500 images which are taken at 10,000 fps. Geometry of the atomizer is given in Table 68 and results are shown in Figure 200.

11. Investigation of the Effect of Geometrical Parameters of Pressure Swirl Atomizer on the Hollow Cone, 2019 [40]

In this study experiments with 7 different geometries are done. Water is used

as working liquid and supplied to atomizers at three different pressures. Measurement of discharge coefficient, spray angle, and vertical break-up length is done. Both spray angle and break-up length are measured from the images taken at 20,000 fps. Average of 100 images were used for the calculations. Used geometries are provided in Table 69 and results can be found in Figure 201 - 203.

12. Viscosity Effect on the Pressure Swirl Atomization of an Alternative Aviation Fuel, 2019 [41]

In this study experiments with the above-mentioned geometry, given in Table 66, are conducted. The type or name of the working liquid is not mentioned yet, physical properties are supplied and can be found in Table 70. Measurements are done with single geometry and 8 different temperatures. Measurement of discharge coefficient is done and the results can be found in Figure 204.

13. Importance of Geometrical Factors on Spray Characteristics of Spill-Return Atomizers, 2021 [42]

This study contains the studies conducted with two atomizer geometries. Jet A-1 was used in the experiments and measurements of discharge coefficient, spray angle, and droplet sizes are done. Spray angle is obtained from images taken at 60,000 fps and 0.25 μ s shutter speed. Geometries of the atomizers are given in Table 71 and as can be seen, only difference of the geometries is the contraction angle (α). Results can be found in Figure 205.

4.2.1.3 *Pseudo-Complete Data*

1. Studies on Spray Behavior of a Pressure-Swirl Atomizer in Transition Regime, 2008 [43]

This study contains measurements of discharge coefficient, spray angle, and

droplet size for single atomizer geometry. Experiments are conducted with water. Spray angle is obtained by making average spray images binary and droplet sizes are measured with the Malvern system, 150 mm downstream. Geometry of the atomizers is given in Table 72 and results can be found in Figure 206.

2. Experimental Evaluation of Low Pressure Swirl Atomizer Applied Engineering Design Process, 2009 [44]

Similar to the above-mentioned study, this one also contains the results of experiments conducted with single atomizer geometry and using water. Also, measurements of discharge coefficient, spray angle, and droplet sizes are done and again, droplet sizes are measured with the Malvern system. It should be noted that droplet sizes are measured 40 mm downstream of the atomizer and spray angle values are obtained by making the long exposure image binary. Geometry of the atomizer is given in Table 73 and results are given in Figure 207.

3. Effect of Inlet Tangential Port Area on the Performance of Small-Scale Simplex Atomizer, 2012 [45]

In this study experiments with three different atomizers are done. Discharge coefficient, spray angle, and droplet sizes of the atomizers are measured. Effect of inlet area on the performance is studied by altering the width and number of the inlet ports. It should be noted that these parameters are altered simultaneously, as shown in Table 74. Water is used as the working liquid. Spray angle is measured from the images obtained and SMD values are obtained from the Malvern system. Results can be found in Figure 208 - 210.

4. Experimental Study of Spray Characteristics of Kerosene-Ethanol Blends from a Pressure-Swirl Nozzle, 2018 [46]

This article studied the effect of ethanol-kerosene mixtures on the performance of a PSA. Four different liquids are used in this study which are kerosene, and kerosene-ethanol mixtures at different volume ratios. Physical properties of the liquids are given in Table 75. Single atomizer geometry is used in this in this study, given in Table 76, and measurements of discharge coefficient, spray angle, and vertical break-up length are done. It is stated that spray angle is obtained from the Lavision PIV system and break-up length values are obtained from images taken. Results are presented in Figure 211 - 213.

4.2.1.4 *Parametric Data*

1. Design of Swirl Chamber Atomizers, 1987 [47]

This study contains the results of the experiments done with four different atomizer geometries. Experiments are done with water and measurements of the discharge coefficient, spray angle, and droplet sizes are done. Experiments are conducted at a constant pressure difference of 900 psi (approximately 62.1 bar). Geometry of the atomizers and the obtained results can be found in Table 77 and Table 78, respectively.

2. Drop Size Spectra in Sprays from Pressure-Swirl Atomizer, 2010 [48]

In this study experiments with 9 different atomizer geometries are done. All of the atomizers were products of Delavan company and all of them had single inlet ports. Measurements of discharge coefficient, spray angle, diagonal break-up length, and droplet sizes are done. Both spray angle and break-up length are obtained from the images taken. Droplet sizes are measured with the PDA system of Dantec company. It is reported that droplet sizes are measured 80 mm downstream of the atomizer. Moreover, mixtures of water-sucrose-ethanol at various rates are used as working liquid, and hence various density, viscosity, and surface tension values are obtained. Geometries of the atomizers are given

in Table 79 and results are presented in Table 80.

3. Improved Semi-empirical Correlation to Predict Sauter Mean Diameter for Pressure-Swirl Atomizers, 2014 [49]

This article studied the effect of geometrical parameters on the discharge coefficient and droplet sizes, using two different liquids. Experiments are conducted with water and Jet A-1. Malvern particle analyzer is used to measure the droplet sizes, 50 mm downstream of the atomizer. Geometries used in this study can be found in Table 81 and results can be found in Figure 214 - Figure 217.

4. Influence of Convergence Angle on Hollow Cone Spray Characteristics, 2019 [50]

This study is conducted to study the effect of convergence angle on the discharge coefficient and spray angle of the PSA. Water was selected to be the working liquid and experiments are done with 4 different atomizers. It should be noted that only the convergence angle, and hence the length of the convergence, is altered. Spray angle values are obtained from the images obtained at 7,000 fps. Moreover, it is stated that, reported spray angle values are the average of the angle values obtained from individual images. Geometry of the atomizers can be found in Table 82 and the results for discharge coefficient and spray angle are given in Figure 218 and Figure 219, respectively.

5. Data in this study

As mentioned earlier, of the 24 atomizers tested in this study 21 were shelf products of the companies such as Hago, Schlick, Steinen, etc. For these atomizers, only the exit diameter could be measured using microscope and compared to the reported exit diameter. An example of diameter measurement can be seen in Figure 47. Diameter value is measured from three different locations, and the average of these three values is used in further calculations. As can be seen, reported and measured dimensions can deviate from each other considerably.

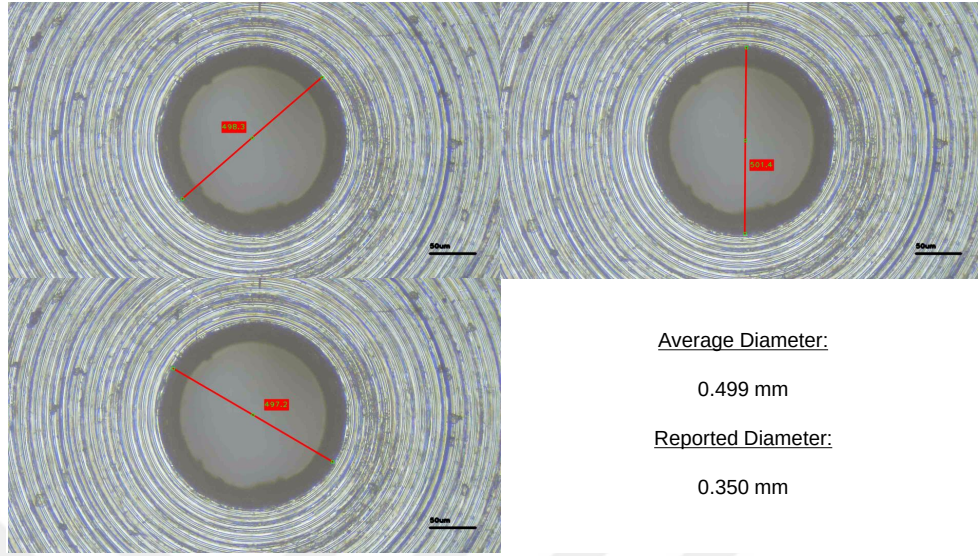


Figure 47: Diameter measurement of atomizers used in this study

Moreover, most of the atomizers are obtained as sets of 3 pieces. Measurements of discharge coefficient, spray angle, and diagonal break-up length are conducted for all of the atomizers whereas, measurements of droplet sizes are done only for a single atomizer of that set. This atomizer is selected based on the difference between the measured discharge coefficient and the reported discharge coefficient.

Results for the Atomizer-1, 2 and 3 are given in Figure 48 - 50. These atomizers are the same atomizers. Inspection of the results shows that Atomizer-1 and Atomizer-3 produce similar results whereas Atomizer-2 deviates from both in terms of discharge coefficient and spray angle.

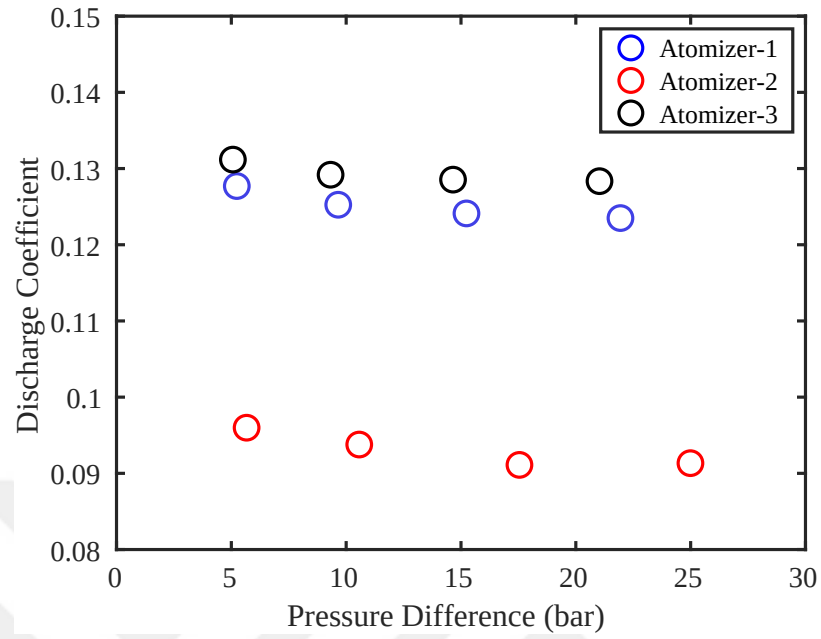


Figure 48: Discharge coefficient of Atomizer-1, 2, and 3 of this study

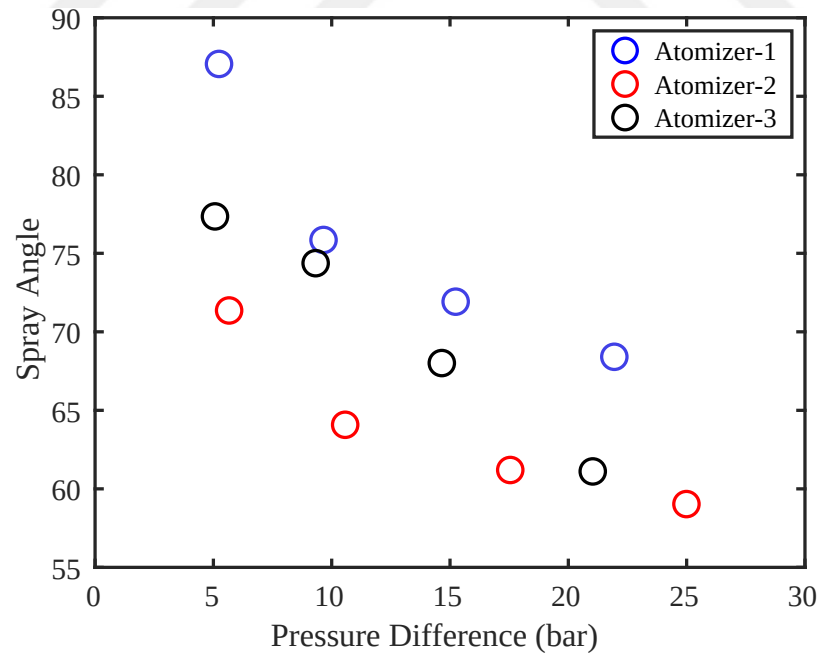


Figure 49: Spray angle of Atomizer-1, 2, and 3 of this study

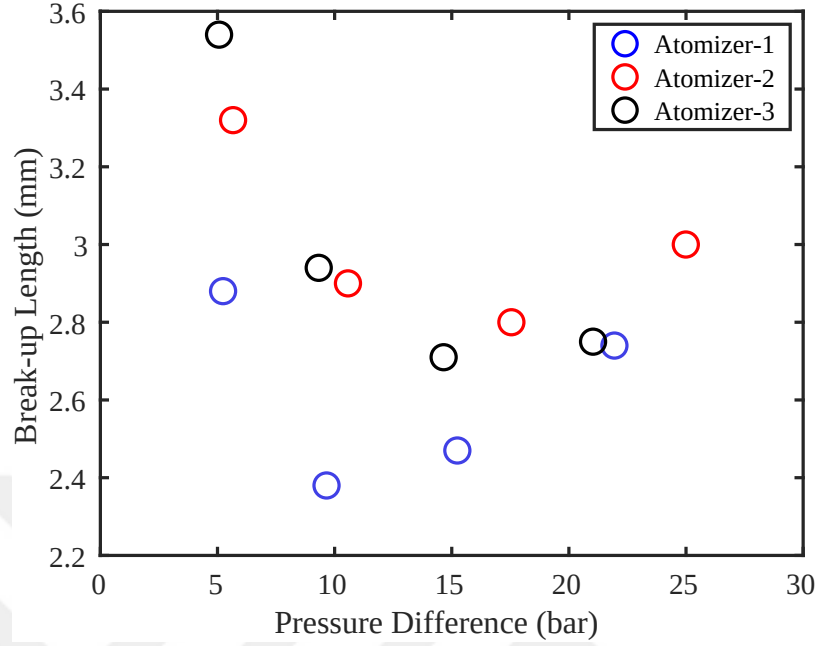


Figure 50: Breakup length of Atomizer-1, 2, and 3 of this study

Results for the Atomizer-4, 5 and 6 are given in Figure 51 - 53. These atomizers are also the same atomizers. Inspection of the results shows that Atomizer-4 and Atomizer-5 produce similar results whereas Atomizer-6 deviates from both in terms of discharge coefficient and spray angle. However, it is clear that all three follow similar trends in terms of how their performance changes with increasing pressure difference. For all three atomizers, discharge coefficient and spray angle decrease with pressure while break-up length decreases at first and then increases.

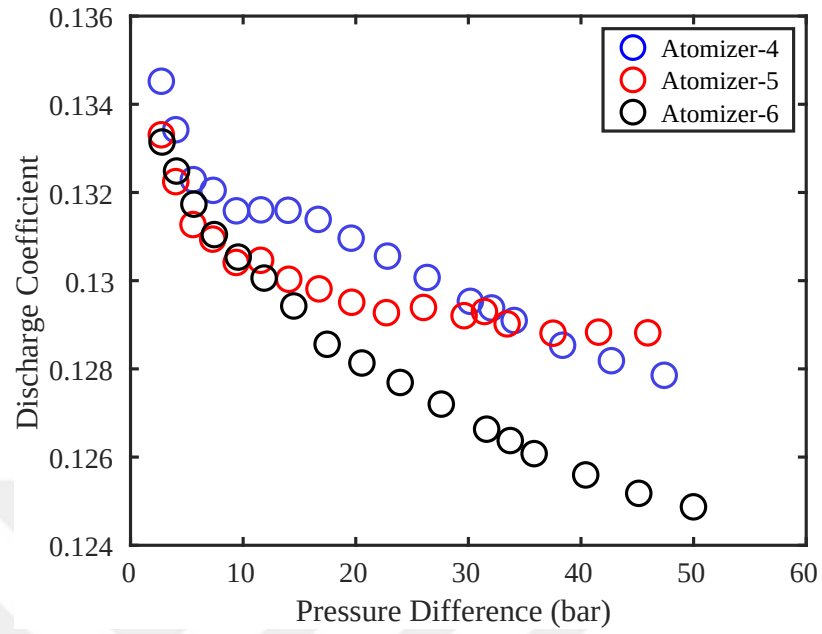


Figure 51: Discharge coefficient of Atomizer-4, 5, and 6 of this study

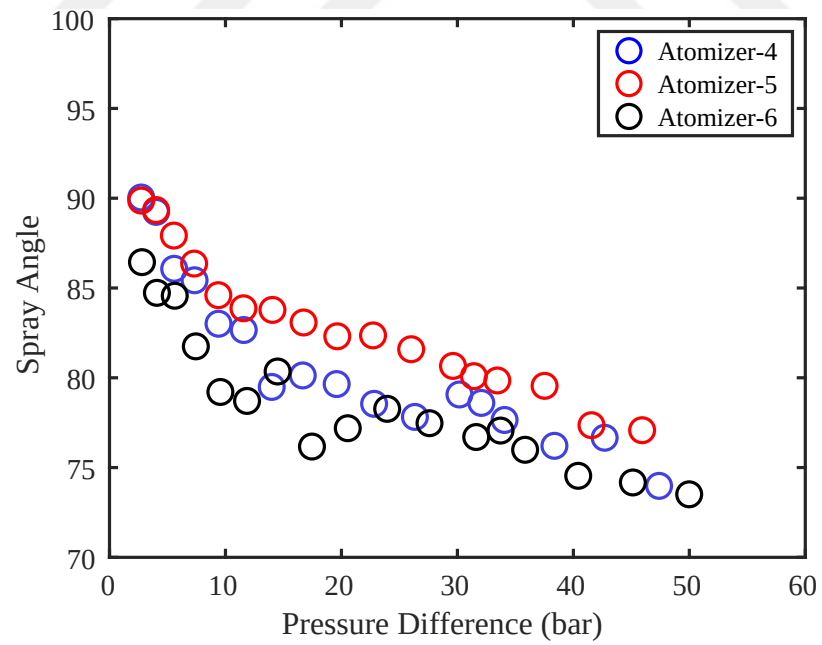


Figure 52: Spray angle of Atomizer-4, 5, and 6 of this study

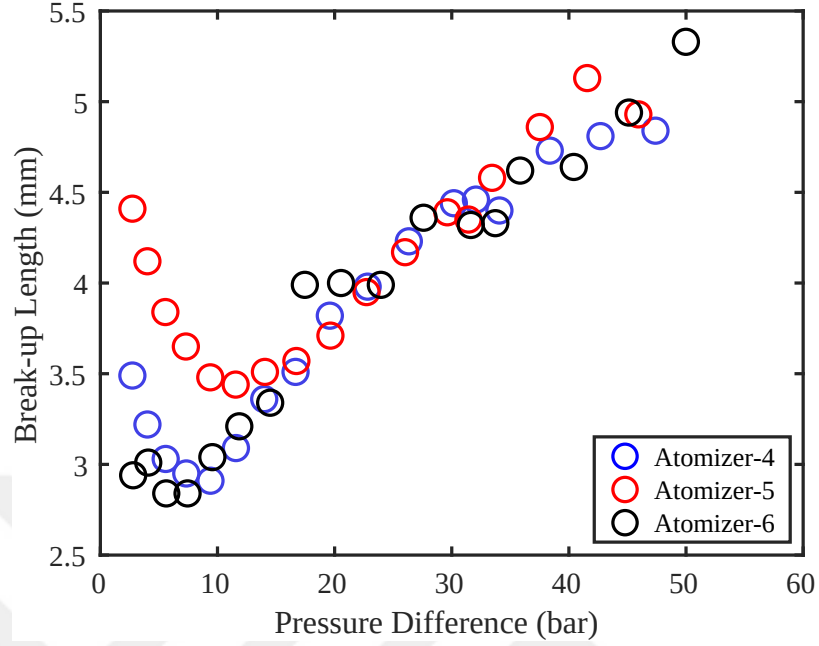


Figure 53: Breakup length of Atomizer-4, 5, and 6 of this study

Results for the Atomizer-7, 8 and 9 are given in Figure 54 - 56. Again, these atomizers are the same atomizers. Inspection of the results shows that Atomizer-8 and Atomizer-9 produce almost identical results, especially at low pressure values and Atomizer-7 deviates from both. Similar to the above-mentioned 3 atomizers, all three atomizers here also follow similar trends. Again, for all three atomizers, spray angle decreases with pressure while break-up length decreases at first and then increases.

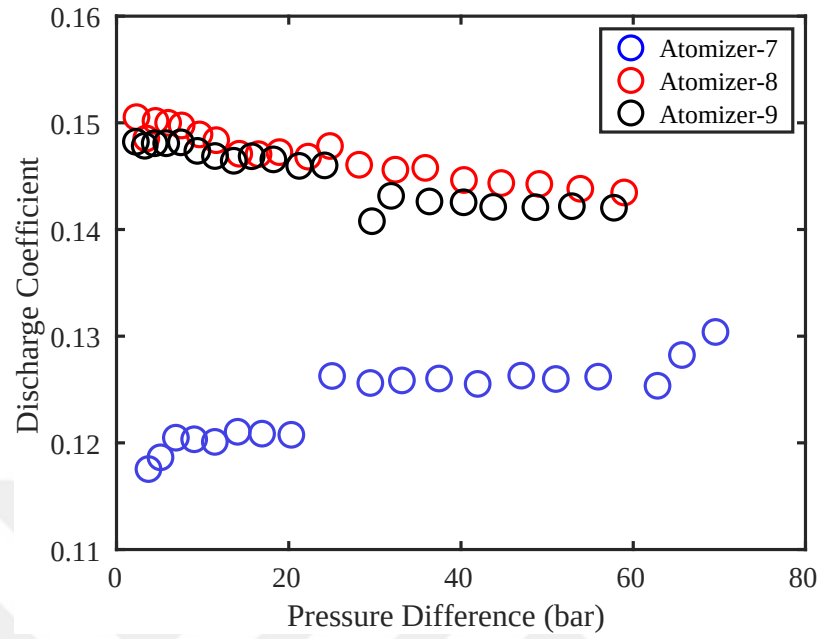


Figure 54: Discharge coefficient of Atomizer-7, 8, and 9 of this study

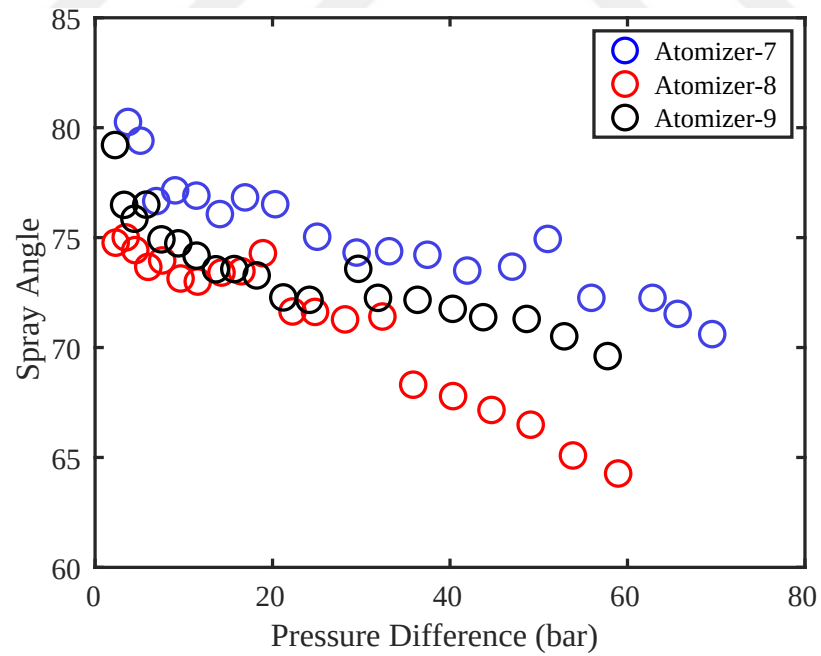


Figure 55: Spray angle of Atomizer-7, 8, and 9 of this study

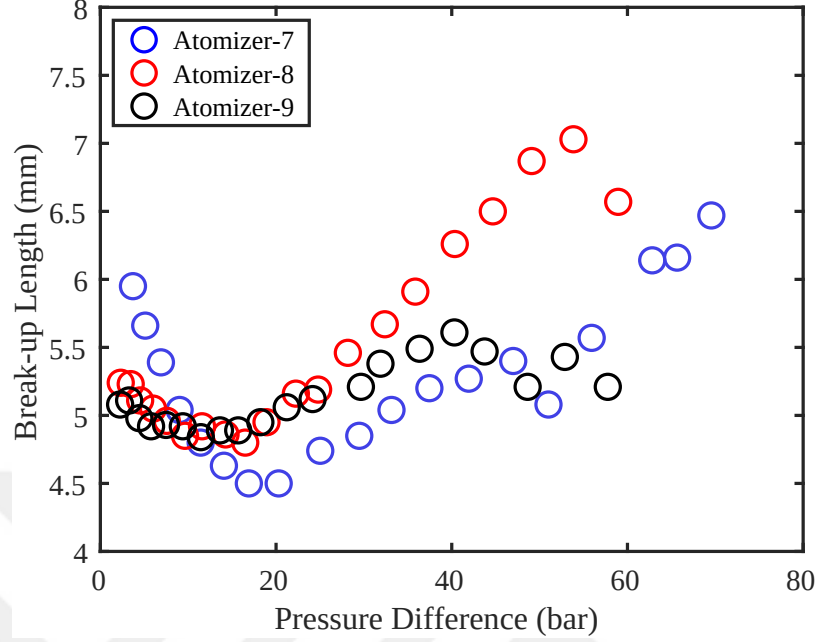


Figure 56: Breakup length of Atomizer-7, 8, and 9 of this study

Results for the Atomizer-10, 11 and 12 are given in Figure 57 - 59. These atomizers are also the same as each other. From the inspection of the results, it can be said that all three atomizers follow the same trends, except for the spray angle. As with the above cases, a decrease of the spray angle can be seen with the increase of the pressure, yet the pattern of this decrease is different for each atomizer. Discharge coefficient increases with the increasing pressure, although the change is very small. Again, for all three atomizers, break-up length decreases at first and then increases, as the pressure increases.

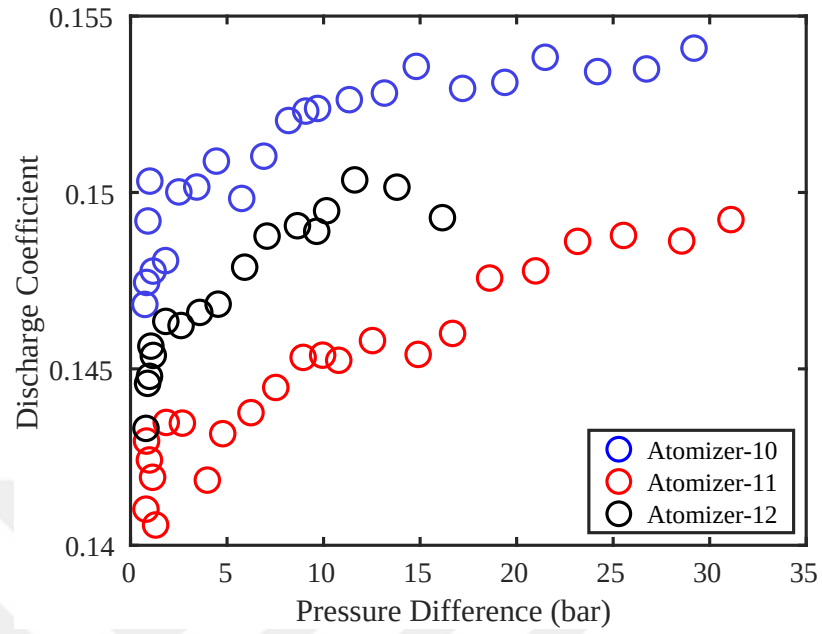


Figure 57: Discharge coefficient of Atomizer-10, 11, and 12 of this study

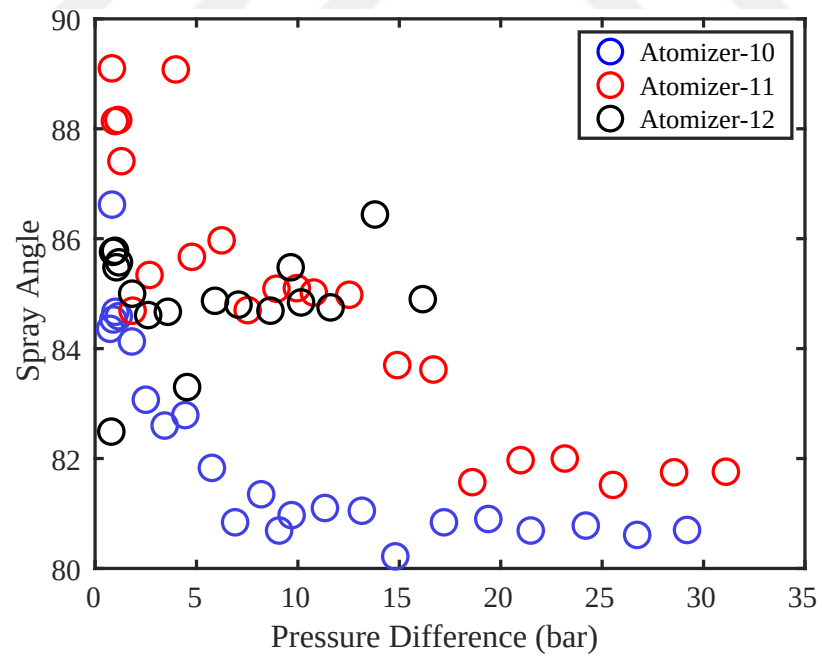


Figure 58: Spray angle of Atomizer-10, 11, and 12 of this study

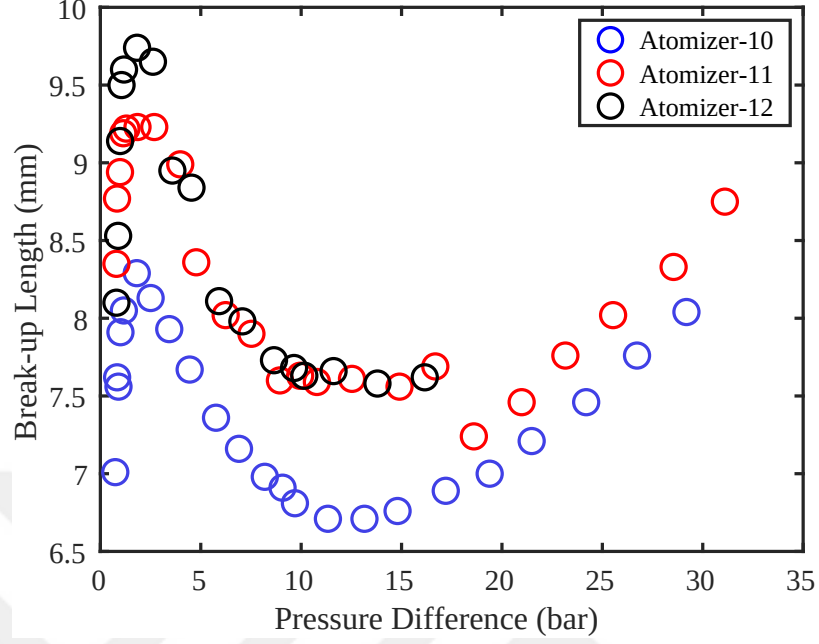


Figure 59: Breakup length of Atomizer-10, 11, and 12 of this study

Results for the Atomizer-13, 14 and 15 are given in Figure 60 - 62. The comments made for the above three atomizers can be repeated here. As the pressure difference increases, discharge coefficient increases for all three atomizers, spray angle decreases while Atomizer-14 and Atomizer-15 follow similar trends Atomizer-16 deviates from them. Moreover, as seen in almost all of the atomizers break-up length initially decreases and then increases. It should be noted that these atomizers are identical to each other.

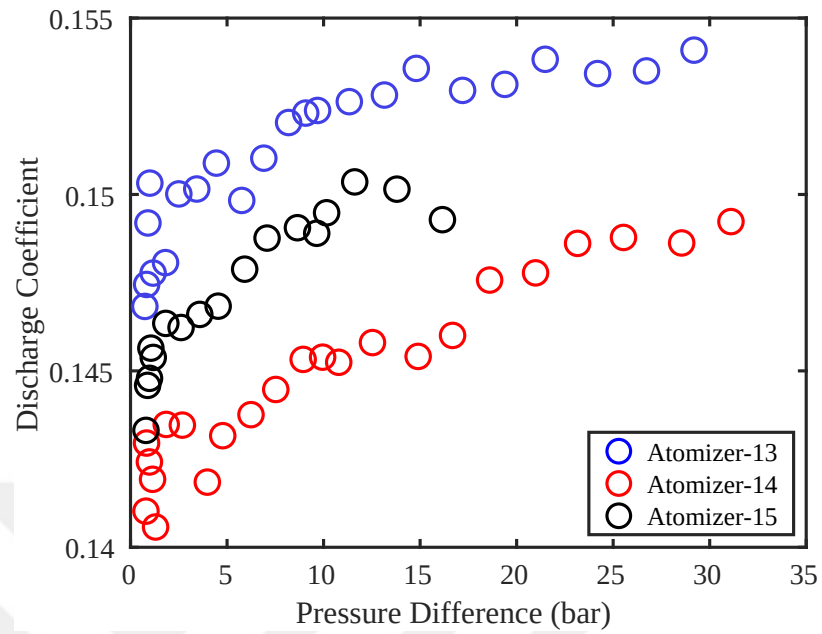


Figure 60: Discharge coefficient of Atomizer-13, 14, and 15 of this study

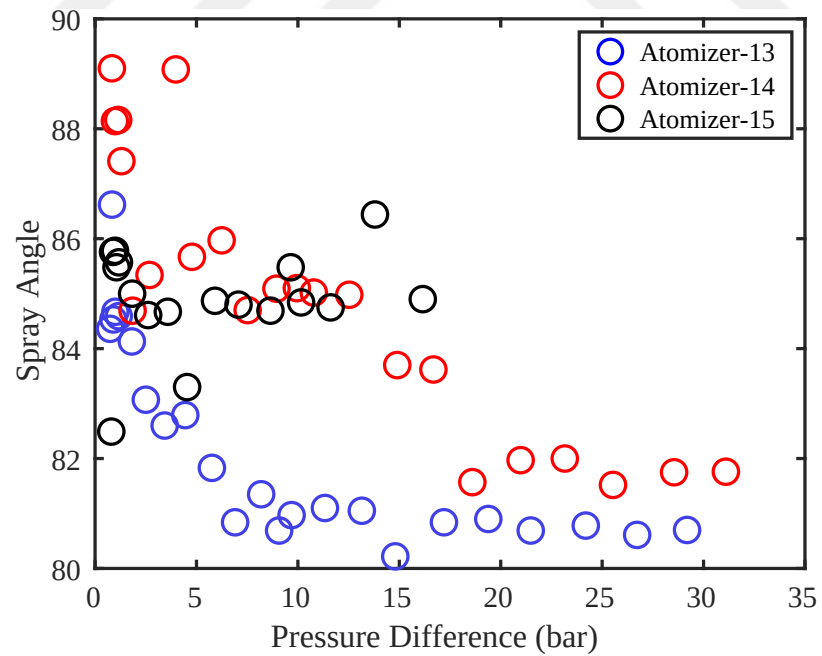


Figure 61: Spray angle of Atomizer-13, 14, and 15 of this study

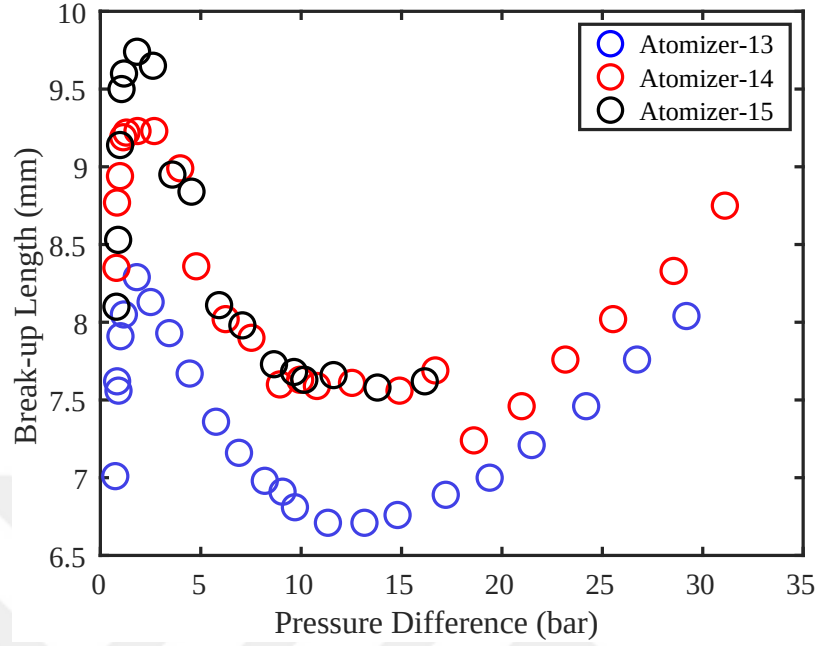


Figure 62: Breakup length of Atomizer-13, 14, and 15 of this study

Results for the Atomizer-16, 17 and 18 are given in Figure 63 - 65. An important note to be made is, these atomizers are different types of the same manufacturing company. Even though the atomizers are different, similar trends can be seen in the results. For all three atomizers, discharge coefficient decreases as the pressure difference increases. Spray angle increases at first, and then either decreases (Atomizer-16) or remains almost constant (Atomizer-17 and Atomizer-18). Break-up length decreases at first and then, similar to the spray angle, remains constant.

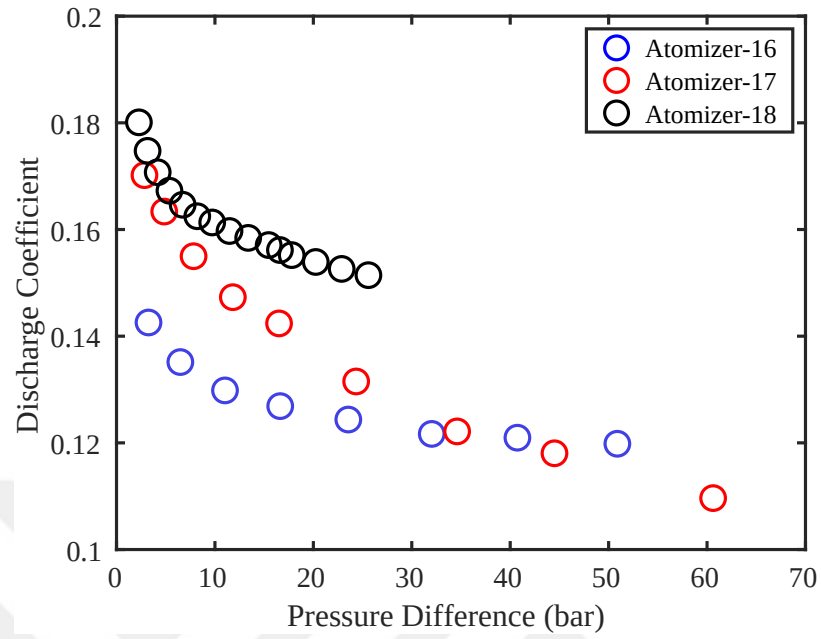


Figure 63: Discharge coefficient of Atomizer-16, 17, and 18 of this study

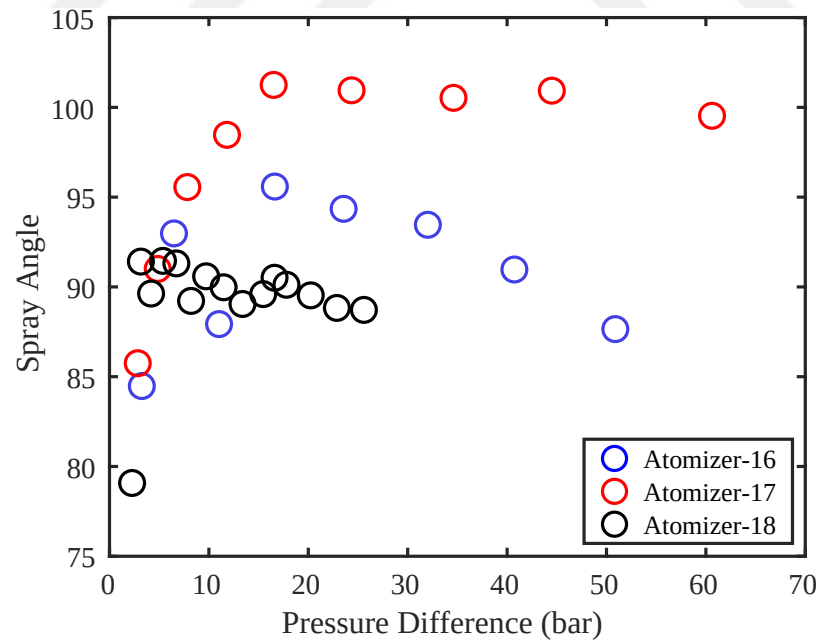


Figure 64: Spray angle of Atomizer-16, 17, and 18 of this study

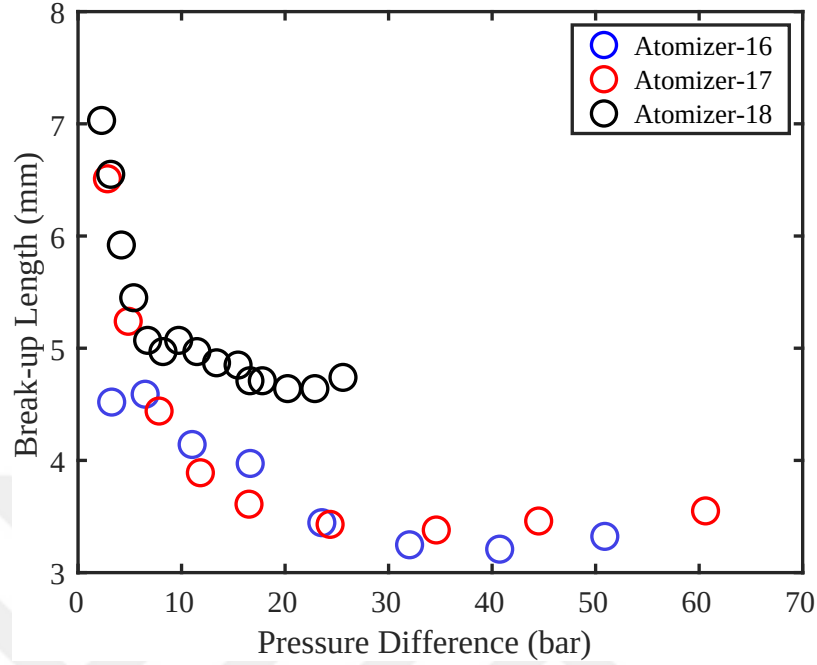


Figure 65: Breakup length of Atomizer-16, 17, and 18 of this study

Results for the Atomizer-19, 20 and 21 are given in Figure 66, Figure 67 and Table 6. Different from the last three atomizers, these are identical atomizers. From the results, it can be seen that Atomizer-19 and Atomizer-20 give similar discharge coefficient values, especially at high pressure difference values. Atomizer-21 has an increasing trend for discharge coefficient, with the increasing pressure, whereas it remains almost constant for other two atomizers. As seen in many of the above-mentioned atomizers, spray angle for all three atomizers decreases with increasing pressure.

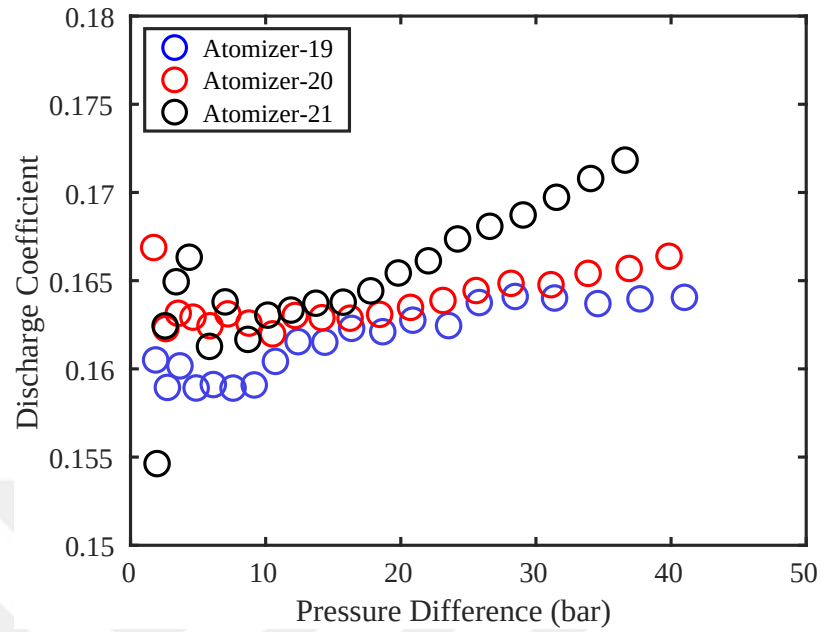


Figure 66: Discharge coefficient of Atomizer-19, 20, and 21 of this study

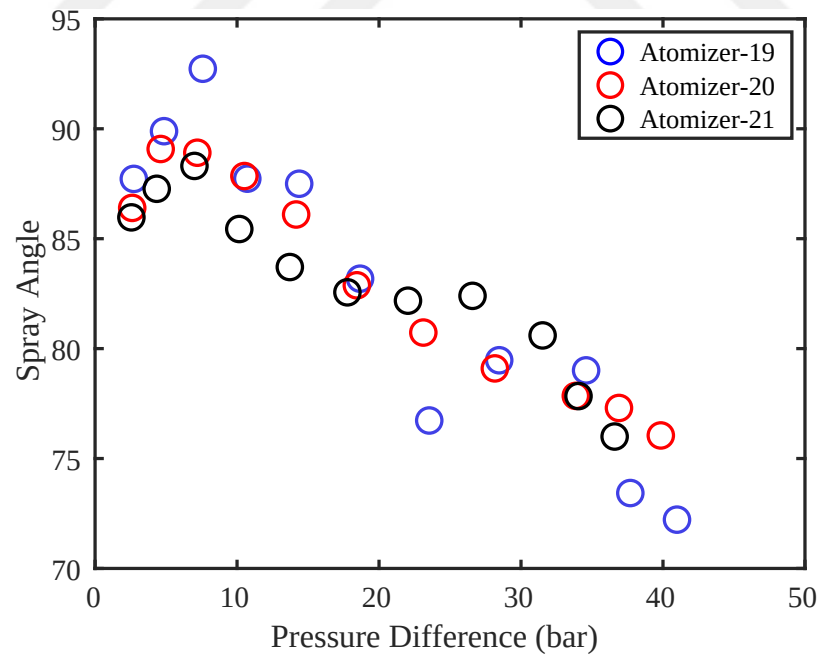


Figure 67: Spray angle of Atomizer-19, 20, and 21 of this study

Of the above-mentioned three atomizers, break-up length measurement is only

done for the Atomizer-19. Unlike other atomizers, break-up length is only measured at three flow rates which are also the same flow rates droplet size measurements are conducted. The relation between the break-up length and droplet size measurements will be explained in Chapter 8. Table 6 shows the measured values and as can be seen, even from the three measurements, it decreases at first and then increases.

Table 6: Break-up Length Values of Atomizer-19

Pressure Difference (bar)	Break-up Length (mm)
1.85	6.55
7.59	5.41
34.58	6.37

Figure 68 shows the measured values SMD for the selected atomizers. As expected, for all cases, SMD values decrease with increasing pressure. Atomizer-13 produced the highest SMD value at low pressure difference but decreases sharply as the pressure is increased. The smallest change in the SMD occurs for the Atomizer-17. As the pressure difference increases, only a small decrease occurs. Moreover, Atomizer-8 produces the smallest SMD value of all, however, it is tested at a considerably higher pressure difference than the others.

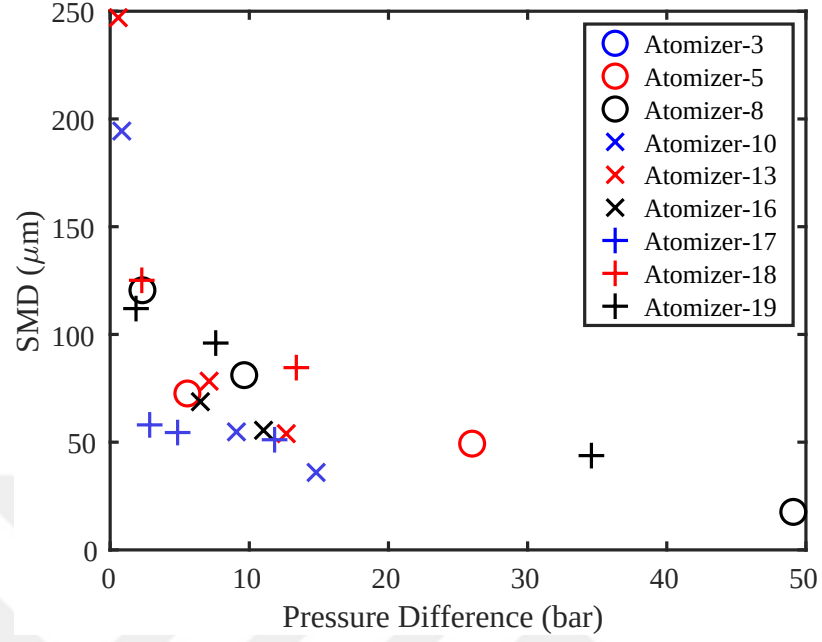


Figure 68: SMD of the selected atomizers

4.2.2 Open-End Pressure-Swirl Atomizer

List of the geometrical parameters is also prepared for the open-end PSA. Schematic of the open-end PSA geometry is given in Figure 69.

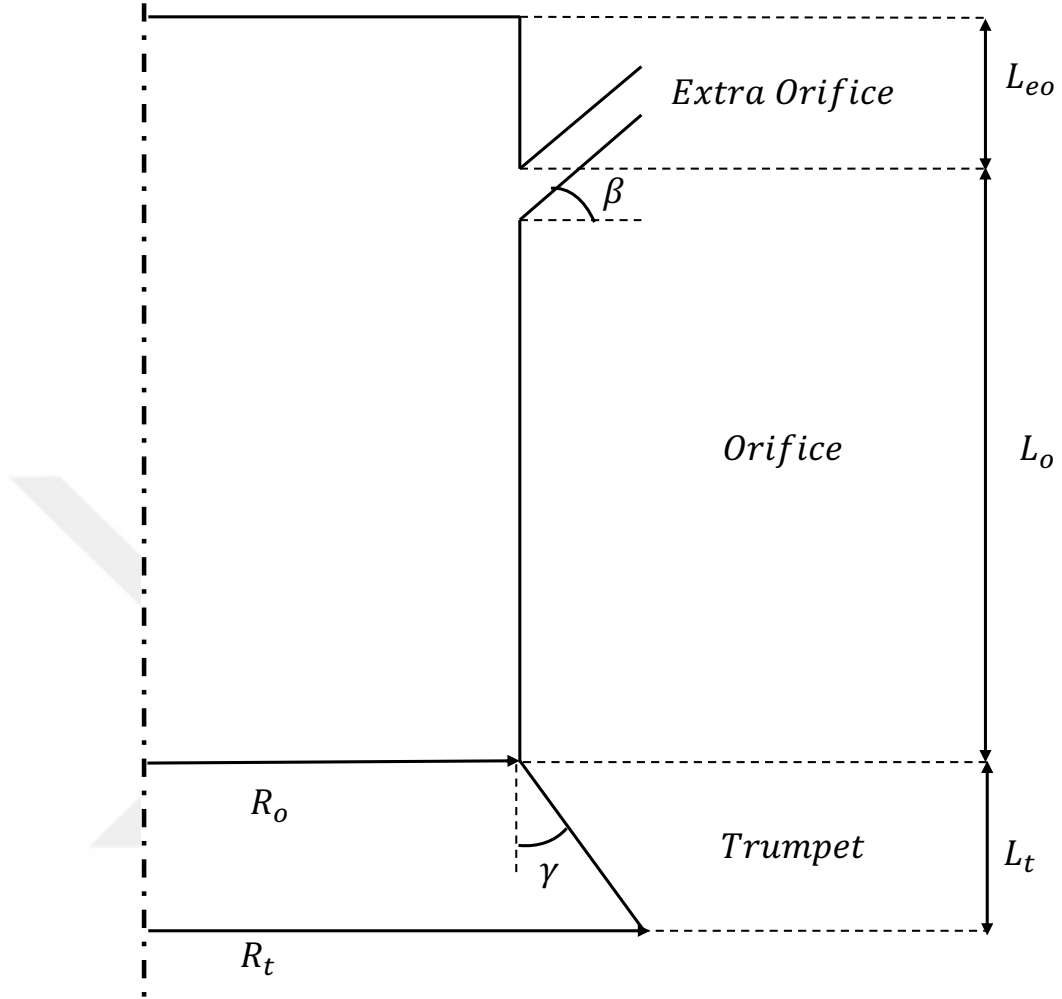


Figure 69: Schematic of open-end pressure-swirl atomizer

Here, also depending on the cross-section of the inlet 11 or 12 geometrical parameters are required, which is considerably less than that of the close-end atomizer. List of the geometrical parameters is presented in Table 7.

Table 7: Geometrical Parameters of Open-End Atomizers

Inlet	
N_p	Number of Inlets
D_p	Diameter of Inlet
w_p	Width of Inlet
h_p	Height of Inlets
L_p	Length of Inlets
$R_{in} = (R_s - 0.5 \times D_{h,p})$	Swirl Arm
β	Angle of Inlets
Orifice	
L_{eo}	Extra Orifice Length
L_o	Orifice Length
D_o	Orifice Diameter
Trumpet	
L_t	Trumpet Length
D_t	Trumpet Diameter
γ	Trumpet Half Angle

Literature survey showed that for the open-end atomizers, the length of the inlets is not provided while the other geometrical parameters are given. Therefore all of the data is classified as Quasi-Complete Data.

4.2.2.1 Quasi-Complete Data

1. Geometrical Effects on the Fluid Dynamics of an Open-End Swirl Injector, 2011 [51]

This study contains experiments conducted with 4 different open-end atomizer geometries, given in Table 83, using water. As can be seen, only the diameter of

inlet ports is altered in this study. Measurement of discharge coefficient, spray angle, and film thickness is done. Spray angle is obtained from the average images which are taken at 60 fps. Authors have reported the accuracy as $\pm 2^\circ$. Conductance method is used for the measurement of liquid film thickness. For this measurement, authors have reported the uncertainty as 0.038 mm. Results of this study can be found in Figure 220 and Figure 221.

2. Spray Characteristics of an Open-End Swirl Injector, 2012 [52]

This study is conducted by the same researchers as the above-mentioned study. This time, 5 different open-end atomizers are tested using water. Geometries of the atomizers can be seen in Table 84, and as can be seen, Atomizer-1 and Atomizer-4 of this article are also used in the aforementioned study. In this study, measurement of discharge coefficient, spray angle, and vertical break-up length is done. Authors have utilized a high-speed camera and obtained images at 6,000 fps and 10 μ s exposure time. Both the spray angle and break-up length are measured from the average of the images obtained. Results of this study can be found in Figure 222 - 224.

3. The Spray Characteristics of an Open-End Swirl Injector at Ambient Pressure, 2017 [53]

In this study, effect of diameter of inlet ports and length of the atomizer is studied at atmospheric and increased ambient pressures. Water is used as the working liquid and 8 different atomizer geometries are tested. Geometrical details of the studied atomizers can be found in Table 85. Measurement of discharge coefficient and spray angle is done. Spray angle is measured with images obtained at 10,000 fps and 2.7 μ s exposure time. Authors reported that the presented spray angle values are the average of the angle values obtained from 100 images. Results of this study can be found in Figure 225 - 228.

4.3 Available Models and Evaluation

This section presents and evaluates the obtained semi-empirical correlations from the literature. These correlations will be evaluated based on the comparisons made with the above-presented experimental data. To sum up, in the above section, a total of 1,777 data points are given in Table 8.

Table 8: Summary of the Experimental Data

Parameter Data Type	Discharge Coefficient	Spray Angle	Film Thickness	SMD	Break-up Length
Close-End Atomizer					
Complete	493	369	15	58	57
Quasi-Complete	605	494	53	47	81
Pseudo-Complete	67	66	0	42	24
Parametric	407	349	0	25	269
Open-End Atomizer					
Quasi-Complete	205	131	5	0	19

It should be noted that of the 493 data points in the complete-data class, 39 are from this study, and of the 407 data points in the parametric data class 327 are from this study.

To quantify the error values, absolute mean relative error (AMRE) and standard error (SE) are used. In the preliminary study of the author, root mean squared error (RMSE) was used to quantify the error values [4]. However, in the RMSE, error values are squared before they are averaged; hence, RMSE gives a relatively high weight to large error values, whereas AMRE is more robust to the outlier values. Definitions of them are given below.

$$AMRE = \frac{1}{n} \sum_{i=1}^n \frac{|Calculated_i - Experimental_i|}{Experimental_i} \quad (29)$$

where, n is the number of data, $Calculated_i$ is the calculated values for that parameter, and $Experimental_i$ is the experimental values for that parameter.

$$SE = \frac{\sigma}{\sqrt{n}} \quad (30)$$

where, σ is the standard deviation of the absolute relative error values.

It should be mentioned that experimental values of performance parameters are used for error analysis of the correlations. For example, there are correlations for calculating spray angle that requires the film thickness. Instead of calculating film thickness from a correlation, experimental values of this parameter are used. Obtained correlations and their results are given below.

4.3.1 Discharge Coefficient, Flow Number and Pressure Difference

Equations for calculating discharge coefficient and flow number are given below. It should be noted that for the PSA, usually discharge coefficient (Eq. 15) or flow number (Eq. 31) is calculated and then pressure difference is calculated from these parameters.

$$FN = \frac{\dot{m}_l}{\sqrt{\rho_L \times \Delta P_L}} \quad (31)$$

4.3.1.1 Models

Equations found from the literature for the calculation of discharge coefficient are listed below, along with their labels and references.

Table 9: Correlations for Calculating C_D

Cd-1	[54]	$C_D = \frac{0.432}{A^{0.64}}$	(32)
Cd-2	[55]	$C_D = 1 - \frac{2}{\pi} \times \tan^{-1} \left(2.13 \times \left\{ \frac{(4/\pi) \times K + 1.2}{[(4/\pi) \times K + 1]^2 - 1} \right\} \times e^{-0.12 \times (D_s/\sqrt{A_p})} \right)$	(33)
Cd-3	[56]	$C_D = 0.45 \times \left(\frac{D_o \times \rho_L \times V_B}{\mu_L} \right)^{-0.02} \times \left(\frac{L_o}{D_o} \right)^{-0.03} \times \left(\frac{L_s}{D_s} \right)^{0.05} \times (K)^{0.52} \times \left(\frac{D_s}{D_o} \right)^{0.23}$	(34)
Cd-4	[2]	$C_D = 0.35 \times (K)^{0.5} \times \left(\frac{D_s}{D_o} \right)^{0.25}$	(35)
Cd-5	[32]	$C_D = 1.323 \times 10^{-3} \times (K)^{0.29} \times D_0^{-0.82} \times \Delta P_L^{0.03}$	(36)
Cd-6	[32]	$C_D = 1.335 \times 10^{-3} \times (K)^{0.29} \times D_0^{-0.82} \times \Delta P_L^{0.07}$	(37)

Cd-7	[57]	$C_D = 0.466 \times \left(\frac{D_o \times \rho_L \times V_B}{\mu_L} \right)^{-0.027} \times \left(\frac{L_o}{D_o} \right)^{0.229} \times \left(\frac{L_s}{D_s} \right)^{0.091} \times (K)^{0.517} \times \left(\frac{D_s}{D_o} \right)^{0.187} \quad (38)$
Cd-8	[3]	$C_D = \frac{0.489}{A^{0.619}} \quad (39)$
Cd-9	[3]	$C_D = \frac{0.47}{(A')^{0.76}} \quad (40)$
Cd-10	[58]	$C_D = 0.44 \times \left(\frac{A_p}{D_0^2} \right)^{0.84 \times C^{-0.52}} \times C^{-0.59} \quad (41)$
Cd-11	[52]	$C_D = \frac{0.4354}{A^{0.877}} \quad (42)$
Cd-12	[52]	$C_D = 0.19 \times \left(\frac{A_p}{D_0^2} \right)^{0.65} \times C^{-2.13} \quad (43)$

Cd-13	[59]	$C_D = 0.721 \times \left(\frac{1}{A}\right)^{0.416} \times \left(\frac{L_o}{D_o}\right)^{-0.0558} \times \left(\frac{D_s}{D_o}\right)^{0.147}$	(44)
Cd-14	[3]	$C_D = \frac{0.3844}{A^{0.866}}$	(45)
Cd-15	[10]	$C_D = \sqrt{0.225 \times (K')}$	(46)
Cd-16	[31]	$C_D = \sqrt{0.0616 \times (K') \times \frac{D_s}{D_o}}$	(47)
Cd-17	[53]	$C_D = 0.331 \times P_a^{-0.018} \times \Delta P_L^{0.053} \times \left(\frac{L}{D}\right)^{-0.013} \times A^{0.602}$	(48)

where,

$$A_p = 0.25 \times \pi \times D_p^2 \times N_p \quad (49)$$

for atomizers with circular inlet and,

$$A_p = w_p \times h_p \times N_p \quad (50)$$

for atomizers with rectangular and square inlet.

$$A = \frac{\pi \times D_o \times R_{in}}{2 \times A_p} \quad (51)$$

$$K = \frac{A_p}{D_o \times D_s} \quad (52)$$

for close-end atomizer and,

$$K = \frac{A_p}{D_o^2} \quad (53)$$

for open-end atomizer. Also,

$$A' = \left(\frac{D_s \times D_o}{D_p^2} \right) \times \left(\frac{D_o}{2 \times D_{in}} \right)^{0.5} \quad (54)$$

$$C = \frac{R_{in}}{R_o} \quad (55)$$

Inspection of these correlations showed that even though there are correlations that consider many geometrical parameters (Eq. 34 and Eq. 38), there are also more simple correlations (Eq. 35, Eq. 46 and Eq. 47). Moreover, there are also correlations which are dimensionally inconsistent (Eq. 37 and Eq. 38). It should be noted that, as stated by the researchers who derived the correlation, unit of the geometrical parameters are in meter, and unit of the pressure difference is Pascal for these equations.

In addition, it can be seen that not all of the correlations are applicable to all of the obtained experimental data. For example, Eq. 34, Eq. 38 and Eq. 44 includes the term length of the swirl chamber, which is not available for the parametric data class. Also, these correlations are not suitable for the open-end atomizers since they include the term ratio of diameter of swirl chamber to the diameter of orifice. On the contrary, Eq. 48 can only be used for the open-end atomizers.

As mentioned earlier, flow number is also used to calculate the pressure difference. Correlations obtained for the calculation of the flow number are given below.

Table 10: Correlations for Calculating FN

FN-1	[2]	$FN = 0.0308 \times \frac{A_p^{0.5} \times d_o}{D_s^{0.45}} \quad (56)$
FN-2	[2]	$FN = 0.395 \times \frac{A_p^{0.5} \times d_o^{1.25}}{D_s^{0.25}} \quad (57)$
FN-3	[57]	$FN = 0.371 \times \frac{A_p^{0.5} \times d_o^{1.25}}{D_s^{0.25}} \quad (58)$

Similar to the Eq. 37 and Eq. 38, it can be seen that Eq. 56 is not dimensionally consistent. As suggested in the study from which the correlation is obtained from, the units of all geometrical parameters are taken as meter.

4.3.1.2 Evaluation

First, evaluation of the correlations is done for the close-end atomizers. As mentioned earlier, Eq. 48 cannot be used for the close-end atomizers. Hence total of 16 discharge coefficients and 3 flow number correlations are evaluated. Figure 70 shows the obtained error values of the correlations. Inspection of the results shows that equations with the label Cd-16, Cd-3, Cd-4, FN-2, and FN-3 have the smallest error values in the given order, as listed in Table 11. Moreover, it was seen that equation with label Cd-2 produces almost 200% error.

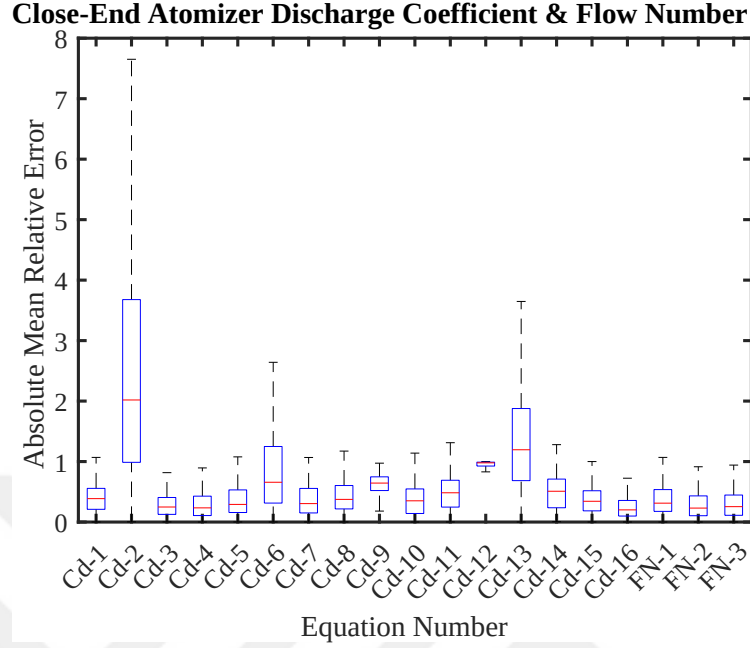


Figure 70: Close-end discharge coefficient and flow number error values

Table 11: Error Values of Most Accurate C_D and FN Correlations - Close-End

Equation	AMRE	SE
Cd-16	0.284	0.018
Cd-3	0.312	0.014
Cd-4	0.330	0.022
FN-2	0.337	0.022
FN-3	0.345	0.021

Later, calculation of the pressure difference, from the calculated discharge coefficient and flow number values, is done, and obtained values are compared with the experimental pressure difference values is done. Figure 71 presents these results.

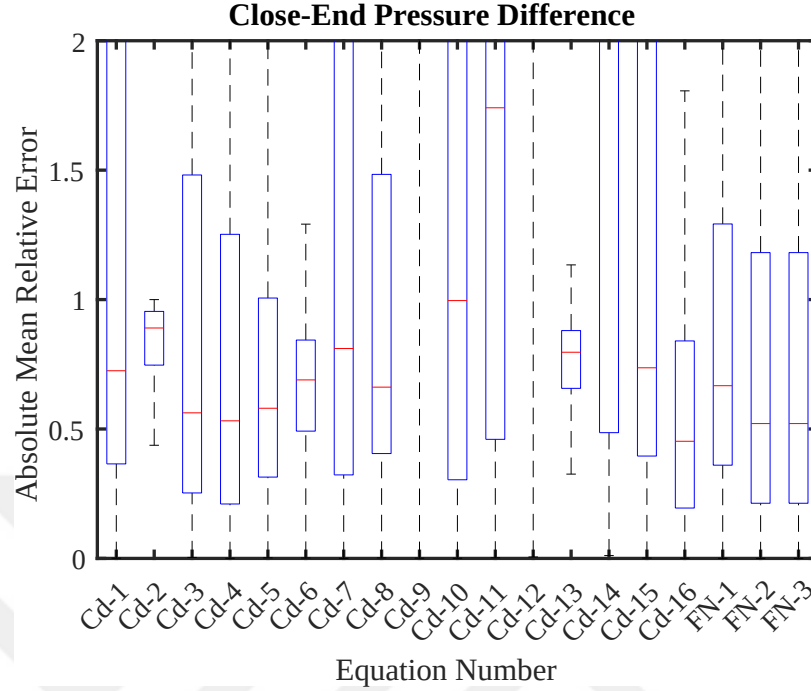


Figure 71: Close-end pressure difference error values

Inspection of the results shows that when the calculated values of discharge coefficient and flow number are used to calculate pressure difference, error values increase drastically. Moreover, instead of the five correlations listed earlier, Cd-13, Cd-2, Cd-6, and Cd-16 have the smallest error values. In other words, the error values increase in the pressure difference calculation, and the equations that produce the smallest errors change. Error values of the aforementioned 4 correlations are listed in Table 12.

Table 12: Error Values of Most Accurate Correlations for Pressure Difference - Close-End

Equation	AMRE	SE
Cd-13	0.730	0.007
Cd-2	0.801	0.006
Cd-6	0.942	0.074
Cd-16	1.081	0.063

A small study is conducted to understand the increase in the error of pressure

difference and the change of the correlations that produce the smallest error values. Here, certain error values are introduced to the discharge coefficient, and the effect of these error values on the pressure difference error is analyzed. Total of 10 cases is inspected. First, C_d values are increased by 5, 10, 20, 40, and 80 % and then decreased by 5, 10, 20, 40, and 80 %. Results are shown in Figure 72.

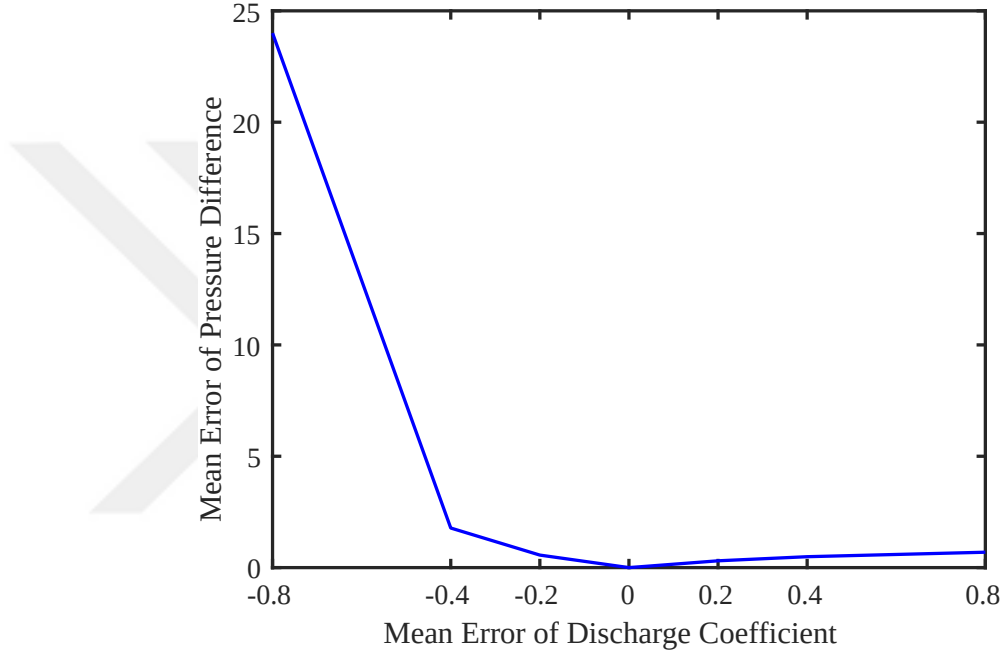


Figure 72: Effect of error in discharge coefficient on the error of pressure difference

This small study showed that under-predicting the discharge coefficient value leads to very high error values when it is used for the pressure difference calculation. In other words, when the discharge coefficient value is over-predicted, the error value of the pressure difference is much smaller. This peculiarity comes from the relation of discharge coefficient and pressure difference (Eq. 15) and can explain the situation described above.

Calculations to assess the accuracy of discharge coefficient equations are also done for the open-end atomizers. Obtained error values for the discharge coefficient and pressure difference are given in Figure 73 and 74, respectively. For the discharge

coefficient, it can be seen that Cd-11, Cd-12, and Cd-14 have the smallest error values. Moreover, unlike the close-end atomizer calculations, these equations produce the smallest error values in the pressure difference calculation. Obtained error values for these correlations are given in Table 13.

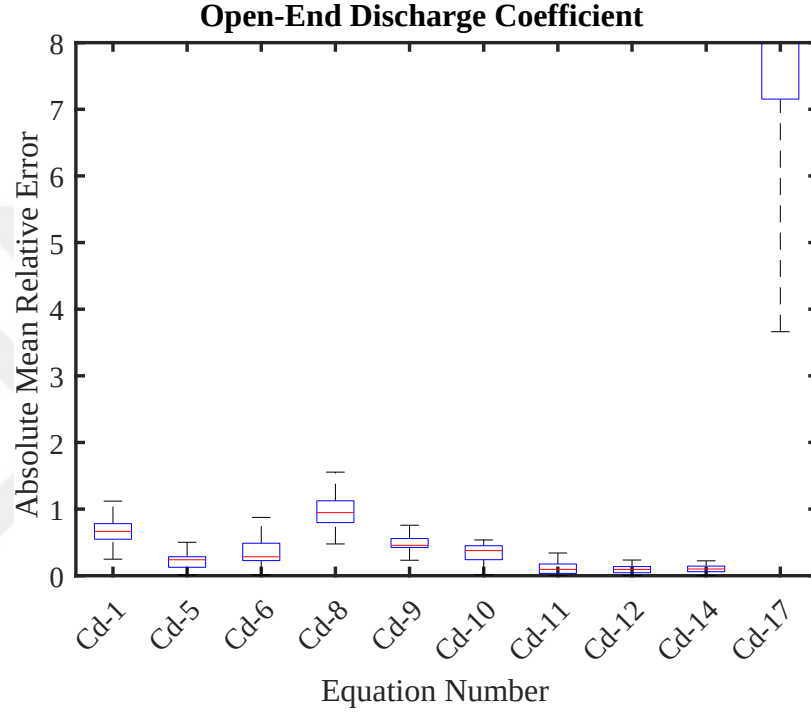


Figure 73: Open-end discharge coefficient error values

Table 13: Error Values of Most Accurate C_D and FN Correlations - Open-End

Equation	AMRE	SE
Discharge Coefficient		
Cd-11	0.105	0.006
Cd-12	0.124	0.008
Cd-14	0.113	0.005
Pressure Difference		
Cd-11	0.205	0.010
Cd-12	0.287	0.023
Cd-14	0.253	0.012

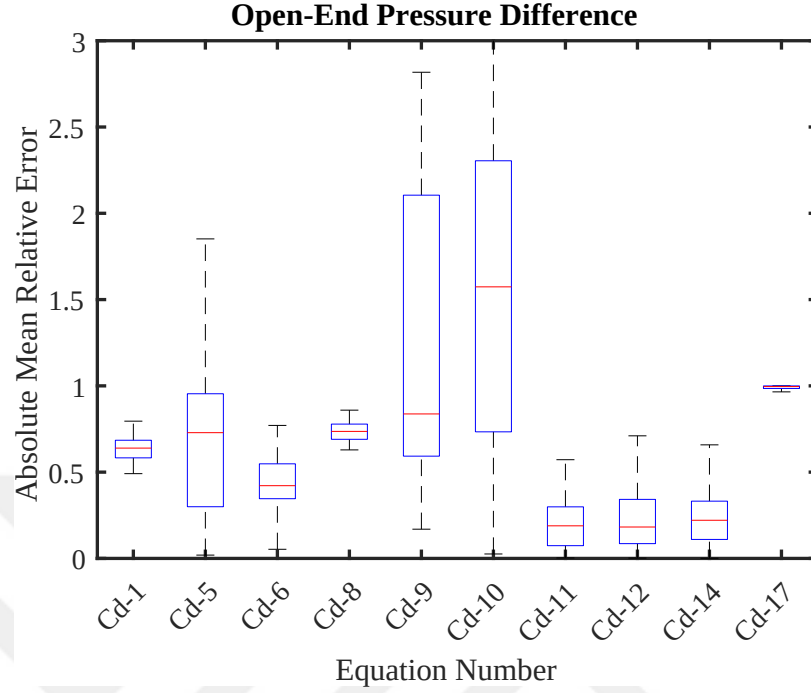


Figure 74: Open-end pressure difference error values

4.3.2 Film Thickness

4.3.2.1 Models

Equations found in the literature for calculating the film thickness are provided below, along with their references and labels. As can be seen, there are also correlations for calculating the diameter of the air core. The relation of air core and film thickness is shown in Eq. 59.

$$t = \frac{D_o - D_a}{2} \quad (59)$$

Moreover, there are also complex correlations, such as Eq. 61, Eq. 63, Eq. 64, Eq. 73 and Eq. 74. In the calculations, these correlations are converted to the form of polynomial equations and solved by calculating their roots.

Table 14: Correlations for Calculating Film Thickness

t-1	[60]	$t = \frac{71.2 \times FN_{dim}}{D_0 \times \cos \theta}$	(60)
t-2	[2]	$t^2 = \frac{1560 \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L \times D_0} \times \frac{1+X}{(1-X)^2}$	(61)
t-3	[2]	$t = 3.66 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25}$	(62)
t-4	[61]	$K^2 = \frac{\pi^2}{32} \times \frac{(1-X)^3}{X^2}$	(63)
t-5	[61]	$0.09 \times K \times \left(\frac{D_s}{D_o} \right)^{0.5} = \frac{(1-X)^3}{1+X}$	(64)
t-6	[60]	$t = 0.00805 \times \frac{\sqrt{\rho_L} \times FN}{D_o \times \cos \theta}$	(65)

t-7	[2]	$t = 2.7 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25}$	(66)
t-8	[2]	$X = 1 - 0.03265 \times \frac{FN \times \sqrt{\rho_L}}{K_v \times D_0^2 \times \cos \theta}$	(67)
t-9	[57]	$t = 3.78 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25}$	(68)
t-10	[3]	$t = 0.28 \times C_D \times D_o$	(69)
t-11	[49]	$\frac{t}{d_0} = 1.44 \times \left[\frac{\dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L \times D_o^3} \right]^{0.3} \times \left(\frac{L_o}{D_o} \right)^{0.6}$	(70)
t-12	[62]	$\frac{D_a}{D_o} = 0.338 \times (1 - \exp(-1.45 \times 10^{-4} \times Re_p)) \times \alpha^{0.073} \times \left(\frac{D_o}{D_s} \right)^{0.424} \times \left(\frac{D_p}{D_s} \right)^{-0.732} \times \left(\frac{L_o}{D_s} \right)^{-0.252}$	(71)

t-13	[51]	$t = 3.1 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25}$	(72)
t-14	[61]	$C_D = \sqrt{\frac{(1-X)^3}{1+X}}$	(73)
t-15	[61]	$C_D = 1.17 \times \sqrt{\frac{(1-X)^3}{1+X}}$	(74)
t-16	[56]	$t = A_p^{0.08} \times \left(\frac{\mu_L \times d_o}{\rho_L} \right)^{0.25} \times \left(\frac{L_o}{D_o} \right)^{0.76} \times (\tan \theta)^{-0.12}$	(75)

where, FN_{dim} in Eq. 60 is the dimensional form of the flow number and is defined as;

$$FN_{dim} = \frac{\dot{m}_L}{\sqrt{\Delta P_L}} \quad (76)$$

It should be noted that the unit of mass flow rate is kg/hr, and the unit of pressure difference is kPa in Eq. 60. Moreover, for this correlation unit of the geometrical parameter is mm, and obtained film thickness has the unit of μm .

In addition,

$$X = \frac{D_a^2}{D_o^2} \quad (77)$$

and for Eq. 67

$$K_v = 0.0037 \times \left(\frac{A_P}{D_s \times d_0} \right)^{0.29} \times \left(\frac{\Delta P_L \times \rho_L}{\mu_L^2} \right)^{0.11} \quad (78)$$

4.3.2.2 Evaluation

Evaluation of the accuracy of the film thickness equations is done for the close-end atomizer first. Figure 75 shows the obtained error values of the correlations. Inspection of the results shows that equations with the labels t-9, t-3, and t-13 have the smallest error values, in this order, as listed in Table 15.

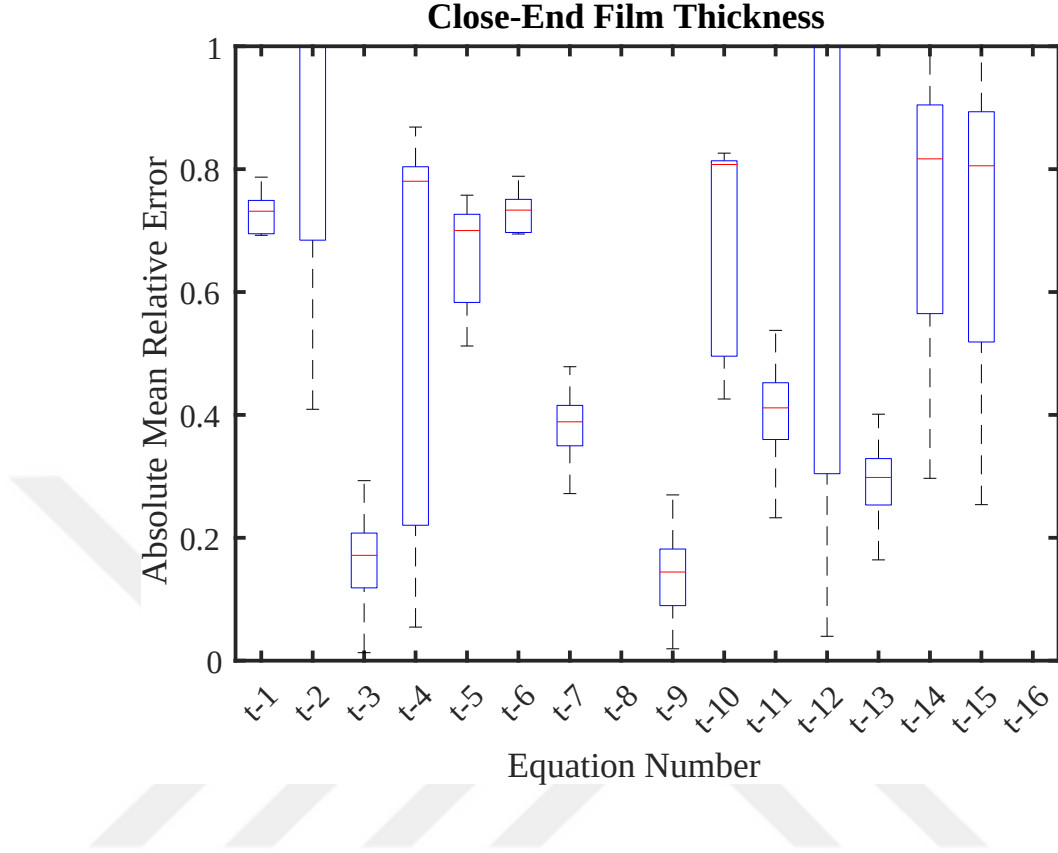


Figure 75: Close-end film thickness error values

Table 15: Error Values of Most Accurate Film Thickness Correlations - Close-End

Equation	AMRE	SE
t-9	0.137	0.008
t-3	0.163	0.007
t-13	0.291	0.006

It should be noted that all three of the listed correlations in Table 15 have the same form. Only the constant is changing. Another note that can be made is, there are only 68 film thickness measurements present in the experimental data set which is small to make comprehensive comparisons.

Same calculations are done for the open-end atomizer as well. It should be pointed out that, Eq. 64, Eq. 71 and Eq. 75 cannot be used for the open-end atomizers. Therefore 13 equations are evaluated. Results of the calculation are shown in Figure

76. It can be seen that equations with the labels t-7 and t-13 are the most accurate correlations. Error values of these correlations are listed in Table 16.

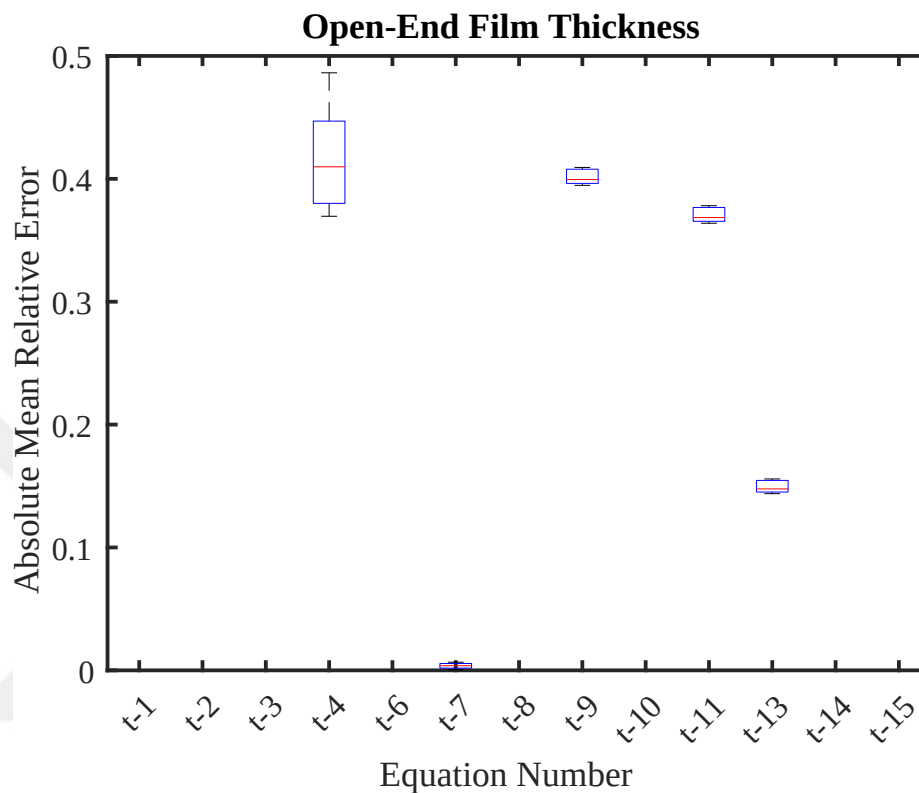


Figure 76: Open-end film thickness error values

Table 16: Error Values of Most Accurate Film Thickness Correlations - Open-End

Equation	AMRE	SE
t-7	0.040	0.001
t-13	0.149	0.002

A very important note that has to be made is there are only 5 measurements of film thickness for the open-end atomizers. Therefore, the results can be misleading. However, Eq. 72 is in the list of most accurate correlations for both close-end and open-end atomizers.

4.3.3 Spray Angle

4.3.3.1 Models

Equations for calculating spray angle values are given in Table 17. References and labels of the correlations are also listed here. Similar to the film thickness correlations, there are complex and simple equations for calculating the spray angle. Moreover, some of the correlations are used for calculating the spray angle whereas some are used for calculating the sin, cos, or tan of the half spray angle. In addition, dimensionally inconsistent equations are also present here.

Table 17: Correlations for Calculating Spray Angle

$\theta-1$	[55]	$\theta = 90 - \tan^{-1} \left[\frac{4}{\pi} \times K \times \left(1.37 + 26.9 \times e^{-11.1 \times (\sqrt{A_p/D_s})} \right) \right]$	(79)
$\theta-2$	[2]	$(\cos \theta)^2 = \frac{2 \times \dot{m}_L \times \mu_L \times (A/B)}{\pi \times \rho_L \times \Delta P_L \times t \times (1 - X)}$	(80)
$\theta-3$	[47]	$\theta = \tan^{-1} \left(\frac{\pi \times (1 - X) \times K_\theta}{4 \times B} \right)$	(81)
$\theta-4$	[32]	$\theta = 8.078 \times K^{-0.39} \times D_0^{1.13} \times \mu_L^{-0.9} \times \Delta P_L^{0.39}$	(82)
$\theta-5$	[32]	$\theta = 0.10985 \times K^{-0.39} \times D_0^{0.63} \times \mu_L^{-0.91} \times \Delta P_L^{0.42}$	(83)
$\theta-6$	[57]	$2\theta = 9.75 \times (K)^{-0.237} \times \left[\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2} \right]^{0.067}$	(84)

θ -7	[63]	$2\theta = 70.2 - 30.2 \times e^{-18.05 \times \Delta P_L}$	(85)
θ -8	[53]	$2\theta = 150.92 \times P_a^{-0.06} \times \Delta P_L^{0.12} \times \left(\frac{L}{D}\right)^{0.31} \times A^{-0.34}$	(86)
θ -9	[61]	$\tan \theta = \frac{2\sqrt{2} \times (X)}{\sqrt{1 - \bar{X}} \times (1 + \sqrt{\bar{X}})}$	(87)
θ -10	[61]	$\tan \theta = \frac{\pi \times (1 - \sqrt{\bar{X}})}{2K}$	(88)
θ -11	[5]	$\tan \theta = \frac{2 \times C_D \times A}{\sqrt{(1 + S)^2 - 4 \times C_D \times A^2}}$	(89)
θ -12	[2]	$\cos \theta = \frac{C_D}{K_v \times (1 - X)}$	(90)

θ -13	[2]	$(\cos \theta)^2 = \frac{1 - X}{(1 + X)}$	(91)
θ -14	[59]	$\cos \theta = 0.302 \times (1 + \tan \beta)^{0.414} \times \left(\frac{1}{A}\right)^{0.35} \times \left(\frac{L_o}{D_o}\right)^{0.043} \times \left(\frac{D_s}{D_o}\right)^{0.026} + 0.612$	(92)
θ -15	[61]	$\sin \theta = \frac{\pi \times C_D \times \sin \beta}{2K \times (1 + \sqrt{X})} \times \frac{R_{in}}{R_s}$	(93)
θ -16	[3]	$\tan \theta = \frac{2 \times (1 - X)}{1 + \sqrt{X}} \times \frac{R_s \times R_o}{n \times R_p^2}$	(94)
θ -17	[3]	$\tan \theta = 0.033 \times A^{0.338} \times Re_p^{0.249}$	(95)
θ -18	[2]	$2\theta = 6 \times (K)^{-0.15} \times \left(\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2}\right)^{0.11}$	(96)

θ -19	[57]	$2\theta = 9.75 \times (K)^{-0.237} \times \left(\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2} \right)^{0.067}$	(97)
θ -20	[64]	$2\theta = \tan^{-1} \sqrt{\frac{1 + \left(1 - 61.68 \times \frac{\rho_L^{0.1} \times \mu_L^{0.2} \times L_s^{0.8}}{\Delta P_L^{0.1}} \times \left(1 + \frac{D_s - D_o}{L_s \times \tan \alpha} \right)^2 \times \left(\frac{W}{U} \right)_{inv}^2 \right)}{\exp \left(0.1023 \times \left(\mu_L \times \sqrt{\frac{\rho_L}{\Delta P_L L}} \right)^{0.25} \times R_o^{0.75} \times L_o \right)}} - 1}$	(98)

For Eq. 80;

$$A/B = 400 \quad (99)$$

For Eq. 81; For $\Delta P_L \leq 2.76$ MPa (400 psi)

$$K_\theta = 0.00812 \times \frac{A_p^{0.034}}{A_s^{0.0245} \times A_0^{0.176}} \quad (100)$$

$$n = 28 \times \frac{A_0^{0.142} \times A_0^{0.270}}{A_s^{0.176}} \quad (101)$$

and For $\Delta P_L > 2.76$ MPa (400 psi)

$$K_\theta = 0.0831 \times \frac{A_p^{0.349}}{A_s^{0.327} \times A_0^{0.263}} \quad (102)$$

$$n = 28 \times \frac{A_0^{0.140} \times A_0^{0.234}}{A_s^{0.178}} \quad (103)$$

where,

$$B = \frac{A_p}{D_s \times d_0} \times \left(\frac{D_s}{d_0} \right)^{1-n} \quad (104)$$

For Eq. 85, pressure difference has the unit of MPa and for Eq. 89;

$$S = \frac{R_a}{R_o} \quad (105)$$

For Eq. 90;

$$K_v = 0.0037 \times (K)^{0.29} \times \left(\frac{\Delta P_L \times \rho_L}{\mu_L^2} \right)^{0.11} \quad (106)$$

Lastly, for Eq. 98;

$$\left(\frac{W}{U} \right)_{inv} = \frac{0.5 \times \pi \times (1 - X)^{1.5}}{\sqrt{K^2 \times (1 + \sqrt{X})^2 \times (1 + X) - \left(\frac{\pi}{2} \right)^2 \times (1 - X)^3}} \quad (107)$$

where K is the geometric constant (Eq. 52 or Eq. 53) and X (ratio of air core area to the area of discharge orifice) is calculated from;

$$K = \frac{(1 - X)^{1.5}}{\sqrt{2} \times X} \quad (108)$$

4.3.3.2 Evaluation

Results for the close-end atomizers are given in Figure 77. It can be seen that, equations with the label θ -7, θ -18, θ -9, θ -3 and θ -17 has the smallest error values, in this order, as listed in Table 18. One important thing that has to be pointed out is θ -9, θ -3, and θ -17 include a term for the film thickness. As mentioned earlier, in the calculations for the performance parameters, experimental values of them are used. Since there are only 68 experimental data for film thickness of close-end atomizers, these equations are only evaluated with 68 data. Whereas other two correlations are compared against 979 data.

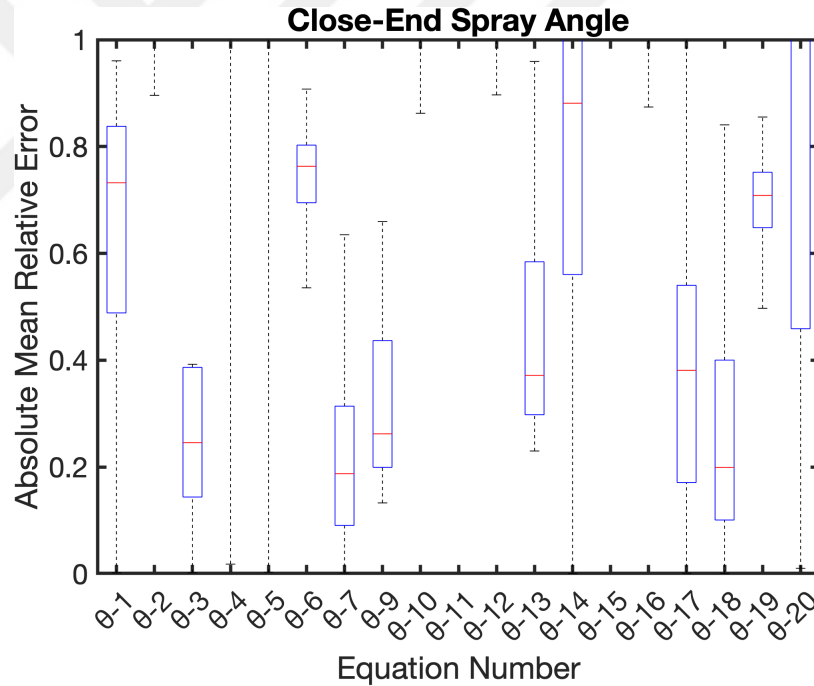


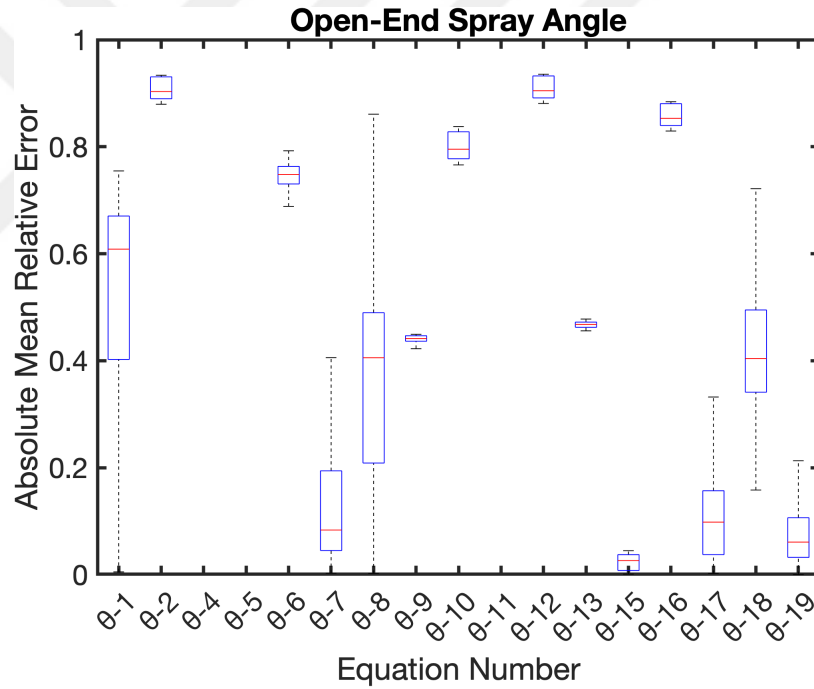
Figure 77: Close-end spray angle error values

Same calculations are also done for the open-end atomizers and results are shown in Figure 78. It can be seen that equations with the label θ -15, θ -19, θ -17, and θ -7 are the most accurate correlations, in this order, for spray angle calculation as listed in Table 19. However, as stated above, equation θ -15 has a term for the film

Table 18: Error Values of Most Accurate Spray Angle Correlations - Close-End

Equation	AMRE	SE
θ -7	0.266	0.010
θ -18	0.288	0.007
θ -9	0.319	0.021
θ -3	0.356	0.036
θ -17	0.364	0.007

thickness and since there are only 5 measurements of film thickness for the open-end atomizer, this correlation is compared to a total of 5 angle data. Whereas the other 3 correlations are compared against 131 data.

**Figure 78:** Open-end spray angle error values

4.3.4 Droplet Size

4.3.4.1 Models

Semi-empirical correlations that are derived for calculating the SMD of the PSA are presented below. It can be seen that there are correlations of the same form such as

Table 19: Error Values of Most Accurate Spray Angle Correlations - Open-End

Equation	AMRE	SE
θ -15	0.024	0.008
θ -19	0.089	0.010
θ -17	0.109	0.007
θ -7	0.131	0.012

Eq. 111, Eq. 112, Eq. 113, Eq. 119 and Eq. 115, Eq. 118 and Eq. 122. Moreover, there are also correlations that are dimensionally inconsistent.



Table 20: Correlations for Calculating SMD

SMD-1	[31]	$SMD = 360 \times \frac{m_L^{0.3}}{\Delta P_L^{0.5}}$	(109)
SMD-2	[31]	$SMD = 325 \times \frac{m^{0.318}}{\Delta P_L^{0.530}}$	(110)
SMD-3	[65]	$SMD = 7.3 \times \sigma_L^{0.6} \times \mu_L^{0.2} \times \rho_L^{-0.2} \times m_L^{0.25} \times \Delta P_L^{-0.4}$	(111)
SMD-4	[5]	$SMD = 52 \times \mu_L^{0.204} \times \rho_L^{-0.204} \times m_L^{0.282} \times \Delta P_L^{-0.397}$	(112)
SMD-5	[66]	$SMD = 4.4 \times \sigma_L^{0.6} \times \mu_L^{0.16} \times \rho_L^{-0.16} \times m_L^{0.22} \times \Delta P_L^{-0.43}$	(113)
SMD-6	[32]	$SMD = 0.436 \times D_0^{-0.05} \times \mu_L^{0.55} \times \Delta P_L^{-0.74} \times A_p^{-0.24}$	(114)

SMD-7	[57]	$SMD = 1.072 \times \left(\frac{\sigma^{0.5} \times \mu_L}{\rho_a^{0.5} \times \Delta P_L} \right)^{0.108} \times (t \times \cos \theta)^{0.946} + 0.2146 \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{0.0523} \times (t \times \cos \theta)^{0.9477} \quad (115)$
SMD-8	[48]	$\frac{SMD}{D_s} = 3.074 \times Re_{port}^{-0.8505} \times Oh^{-0.7538} \times \left(\frac{L_S}{D_s} \right)^{-0.0574} \times \left(\frac{D_o}{D_s} \right)^{-0.3496} \times \left(\frac{w_p}{D_s} \right)^{-0.0426} \quad (116)$
SMD-9	[49]	$SMD = C_L \times \left[\frac{K \times D_0 \times (1 + \sqrt{X})}{1 - X} \right] \times \left[\frac{t^{1.17} \times (d_0 - t)^{0.67}}{\dot{m}_L^{0.67}} \right] \times \left(\frac{\sigma}{\rho_L \times \rho_a} \right)^{1/6} \quad (117)$
SMD-10	[9]	$SMD = A \times \left(\frac{\sigma \times \mu_L^2}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.25} + B \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.75} \quad (118)$
SMD-11	[2]	$SMD = 2.25 \times \sigma_L^{0.25} \times \mu_L^{0.25} \times \rho_a^{-0.25} \times \dot{m}_l^{0.25} \times \Delta P_L^{-0.5} \quad (119)$
SMD-12	[2]	$SMD = 3.29 \times 10^4 \times \left(\frac{\sigma \times \mu_L \times \dot{m}_L}{\rho_a} \right)^{0.25} \times \Delta P_L^{-0.5} \quad (120)$

SMD-13	[67]	$SMD = 0.5536 \times \left[\frac{\sigma_L \times \mu_L \times \rho_l^{0.5} \times D_o^2}{\rho_a \times \Delta P_L^{1.5}} \right]^{0.25} \times \left[\left(\frac{L_o}{D_o} \right)^{0.122} \times \left(\frac{D_s}{D_o} \right)^{0.33} \times (1 + \tan \theta)^{1.38} \right] \quad (121)$
SMD-14	[2]	$SMD = 4.52 \left(\frac{\sigma \times \mu_L^2}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.25} + 0.39 \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.75} \quad (122)$
SMD-15	[57]	$SMD = 2.11 \left(\frac{\sigma \times \mu_L^2}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.25} + 0.62 \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos \theta)^{0.75} \quad (123)$
SMD-16	[64]	$SMD = 10^{-3} \times \sigma_L \times \left(6.11 + 0.32 \times 10^5 \times FN \times \sqrt{\rho_L} - 6.973 \times 10^{-3} \times \sqrt{\Delta P_L} + 1.89 \times 10^{-6} \times \Delta P_L \right) \quad (124)$

It should be noted that Eq. 109 and Eq. 110 mass flow rate has the unit of Lb. per hr. and pressure difference have the unit lb per sq in.

For Eq. 116;

$$Re_{port} = \frac{\sqrt{\rho_L \times \Delta P_L} \times D_s}{\mu_L} \quad (125)$$

For Eq. 117;

$$C_L = \frac{D_d}{D_L} = 22 \quad (126)$$

For Eq. 118

$$A = 2.11 \times [\cos 2 \times (\theta - 30)]^{2.25} \times \left(\frac{3.4 \times 10^{-3}}{d_0} \right)^{0.4} \quad (127)$$

and

$$B = 0.635 \times [\cos 2 \times (\theta - 30)]^{2.25} \times \left(\frac{3.4 \times 10^{-3}}{d_0} \right)^{0.2} \quad (128)$$

4.3.4.2 Evaluation

Evaluation of the correlations is done for the close-end atomizers, due to the lack of the measurement of this parameter for the open-end atomizers. Results are presented in Figure 79. It can be seen that equations with labels SMD-11 and SMD-8 are the most accurate ones. Their AMRE and SE values are also provided in Table 21. It should be pointed out that equations label SMD-7, SMD-9, SMD-10, SMD-14, and SMD-15 include a term for the film thickness and hence requires the experimental film thickness value as well. In the database of experimental data, there is only a single data point where both experimental SMD and experimental film thickness value is present. Therefore, these correlations can only be compared with single data.

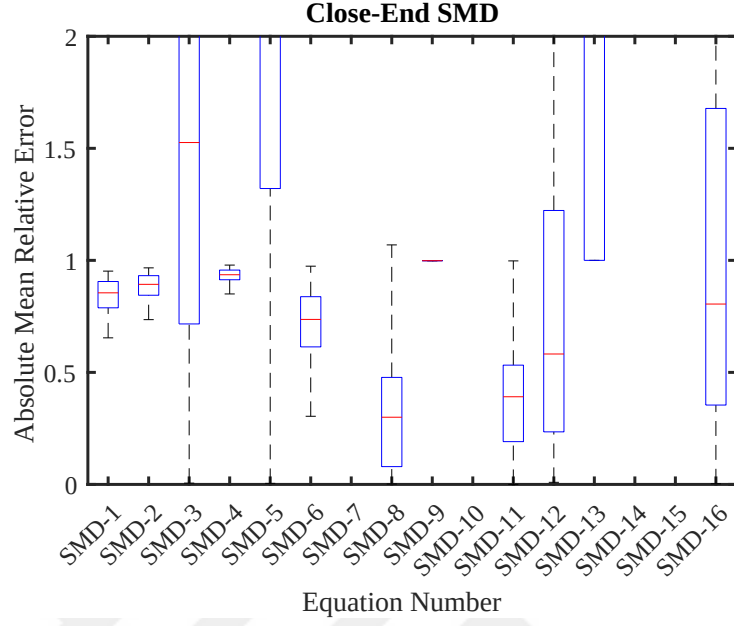


Figure 79: Close-end SMD error values

Table 21: Error Values of Most Accurate SMD Correlations - Close-End

Equation	AMRE	SE
SMD-11	0.483	0.037
SMD-8	0.511	0.086

4.3.5 Break-up Length

4.3.5.1 Models

Obtained semi-empirical correlations for the calculation of break-up length are given below. It should be pointed out that all of these correlations are derived for the vertical break-up length. For the data including diagonal break-up length, such as data from this study, an approximation given in Eq. 129 is done. Here, it is foreseen that if the diagonal break-up length is multiplied with the sine of the half spray angle, vertical break-up length can be obtained.

$$(L_b)_{Vertical} = (L_b)_{Diagonal} \times \sin \theta \quad (129)$$

Table 22: Correlations for Calculating Break-up Length

L _b -1	[68]	$L_b = 15.8 \times 10^{-3} \times D_o \times \left(\frac{\rho_L}{\rho_a} \right)^{0.5}$	(130)
L _b -2	[69]	$L_b = 7 \times 10^{-3} \times D_o \times \left(\frac{\rho_L}{\rho_a} \right)^{0.5}$	(131)
L _b -3	[70]	$L_b = 2.59 \times 10^5 \times D_o \times We_g^{-0.46} \times Re_L^{-0.39} \times \left(\frac{\rho_L}{\rho_a} \right)^{0.2}$	(132)
L _b -4	[71]	$L_b = 2.65 \times 10^{-3} \times D_o \times We_g^{-0.1} \times Re_L^{-0.3} \times \left(\frac{\rho_L}{\rho_a} \right)^{0.08}$	(133)
L _b -5	[72]	$L_b = 100.8 \times D_o \times We^{-0.215} \times \left(\frac{L_o}{D_o} \right)^{0.0424} \times \left(\frac{A_p}{A_o} \right)^{0.395}$	(134)
L _b -6	[73]	$L_b = \frac{1}{2 \times \sin \theta} \times \left[3 \times V \times \sin \theta \times \ln (\eta_b / \eta_0) \times \sqrt[2/3]{\frac{t \times (D_o - t)}{Q \times V^2 \times k - \sigma \times k^2 / \rho_L}} \right] + (D_o - t)^{3/2}$	(135)

L _b -7	[74]	$L_b = \frac{1}{2 \sin \theta} \times \left[3\sqrt{2} \times V \times \sin \theta \times \ln(\eta_b/\eta_0) \times \sqrt{\frac{\sigma \times t \times (D_o - t) \times \cos \theta}{Q^2 \times V^4 \times \rho_L}} \right]^{2/3} + (D_o - t)^{3/2} - (D_o - t) \quad (136)$
L _b -8	[75]	$L_b = 0.82 \times \left[\frac{\rho_l \times \sigma \times \ln(\eta_b/\eta_0) \times t \times \cos \theta}{\rho_a^2 \times V^2} \right]^{0.5} \quad (137)$
L _b -9	[76]	$L_b = 0.2175 \times \left[\frac{\rho_l \times \sigma \times \ln(\eta_b/\eta_0) \times t \times \cos \theta}{\rho_a^2 \times V^2} \right]^{0.3} \quad (138)$

where,

$$We_g = \frac{\rho_a \times V^2 \times t}{\sigma} \quad (139)$$

$$Re_L = \frac{\rho_L \times V \times t}{\mu_L} \quad (140)$$

For Eq. 135;

$$k = \frac{Q \times V^2 \times \rho_L}{2\sigma} \quad (141)$$

For Eq. 136 and Eq. 138;

$$\ln(\eta_b/\eta_0) = 12 \quad (142)$$

For Eq. 137;

$$\ln(\eta_b/\eta_0) = 2.5 \quad (143)$$

Inspection of the equations for calculating break-up length and their terms showed that, except for Eq. 130 and Eq. 131, the exit velocity of the liquid from the atomizer needed to be known. In the collected experimental such data is not present. Therefore, this parameter is needed to be calculated. For this purpose, the LISA model which is widely used by commercial CFD software such as Star-CCM+ and ANSYS Fluent is used [77, 78]. Equations for the calculation of exit velocity are given below.

First, the calculation of the velocity coefficient is done.

$$K_v = \max \left[0.7, \frac{4 \times \dot{m}_L}{\pi \times \rho_L \times D_0^2 \times \cos \theta} \times \sqrt{\frac{\rho_L}{2 \times \Delta P_L}} \right] \quad (144)$$

Velocity of the liquid exiting the atomizer is found as:

$$V = K_v \times \sqrt{\frac{2 \times \Delta P_L}{\rho_L}} \quad (145)$$

Similarly, for the film thickness, Eq. 68, which was the most accurate correlation to calculate film thickness, is used.

4.3.5.2 Evaluation

Results for the equations are shown in Figure 80. It should be noted that the correlations are tested on the open-end and close-end atomizer experimental data simultaneously. Inspection of the results showed that most correlations, except equation

with label Lb-8, have AMRE values higher than 0.9. Equation Lb-8 has produced the smallest AMRE value of 0.595.

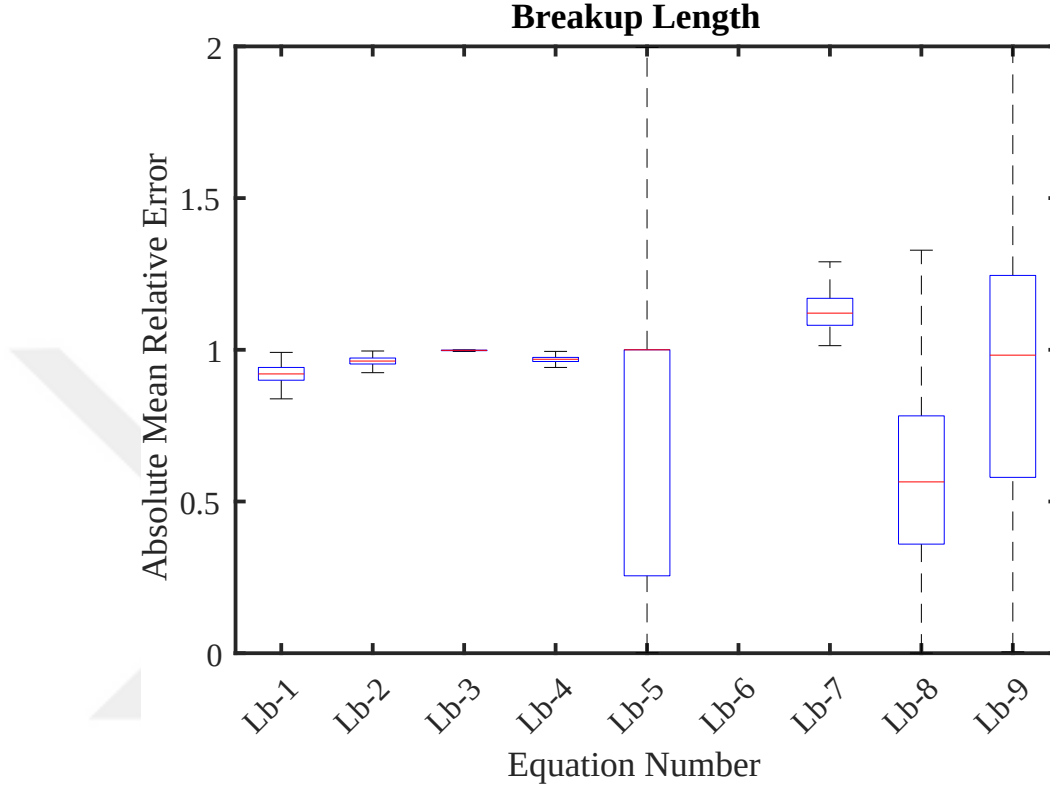


Figure 80: Break-up error values

4.4 Obtaining Empirical Model

Assessing the accuracy of the semi-empirical correlations has shown that large error values are present, especially for the calculation of the pressure difference, SMD, and break-up length. Moreover, due to lack of data, most of the correlations could not be used with the complete data set or could not be used at all.

In addition, as mentioned earlier, evaluation of the previously derived correlations is done by using the experimental value of the performance parameters. For example, if an equation included a term for the pressure difference, spray angle, or film thickness, the experimental values of these parameters are used. However, these correlations are used for, whether initial or final, the design of the PSA. During

this process, the experimental values of these parameters are unknown. Therefore, additional correlation(s) is needed.

It is foreseen that, using the experimental data presented here, a system of semi-empirical correlations that are more accurate than previous ones can be derived. Here, system of semi-empirical correlations refers to a system that each performance parameter will be calculated with a semi-empirical correlation and the calculated value will be used in further calculations. Therefore, the only required input will be the physical properties of the liquid, the dimensions of the PSA, and the mass flow rate. Moreover, with this system and the correlations derived here, optimization studies for the PSA could be performed.

First, with the help of the above calculations, form of the correlations are determined and is shown below. It should be noted that Eq. 146 and Eq. 150 are not the form of the most accurate correlations for these parameters. However, they are more complete in terms of the parameters that they include. With the Eq. 146, almost all of the geometrical parameters can be included in the calculation and with Eq. 150, film thickness and spray angle can be included in SMD calculation. As it is experimentally shown that these parameters have an effect on SMD.

As can be seen, all of the correlations are dimensionally consistent. It should be noted that the constants and powers (a_1 to a_{21}) are variables that will be determined in sections below.

For close-end atomizer pressure difference:

$$C_D = a_1 \times \left(\frac{D_o \times \rho_L \times V_B}{\mu_L} \right)^{a_2} \times \left(\frac{L_o}{D_o} \right)^{a_3} \times \left(\frac{L_s}{D_s} \right)^{a_4} \times (K)^{a_5} \times \left(\frac{D_s}{D_o} \right)^{a_6} \quad (146)$$

For open-end atomizer pressure difference:

$$C_D = a_7 \times \left(\frac{A_p}{D_o^2} \right)^{a_8} \times C^{a_9} \quad (147)$$

For film thickness:

$$t = a_{10} \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25} \quad (148)$$

For spray angle:

$$2\theta = a_{11} \times (K)^{a_{12}} \times \left(\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2} \right)^{a_{13}} \quad (149)$$

For SMD:

$$\begin{aligned} SMD = a_{14} \times \left(\frac{\sigma^{0.5} \times \mu_L}{\rho_a^{0.5} \times \Delta P_L} \right)^{a_{15}} \times (t \times \cos \theta)^{1-1.5 \times a_{15}} \\ + a_{16} \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{a_{17}} \times (t \times \cos \theta)^{1-a_{17}} \end{aligned} \quad (150)$$

For break-up length:

$$L_b = a_{18} \times We_g^{a_{19}} \times Re_L^{a_{20}} \times \left(\frac{C_D}{\cos \theta} \right)^{a_{21}} \quad (151)$$

where We_g and Re_L will be calculated from Eq. 139 and Eq. 140, respectively and the velocity will be calculated from Eq. 144 and Eq. 145.

Proposed system of equations is shown in Figure 81. As can be seen first, calculation of the discharge coefficient, from the liquid properties, mass flow rate, and atomizer dimensions, will be done. This parameter will be used to calculate pressure difference using Eq. 15. Later, calculated pressure difference will be inserted into film thickness and spray angle correlations. At last, these two calculated values will be used to calculate SMD and break-up length.

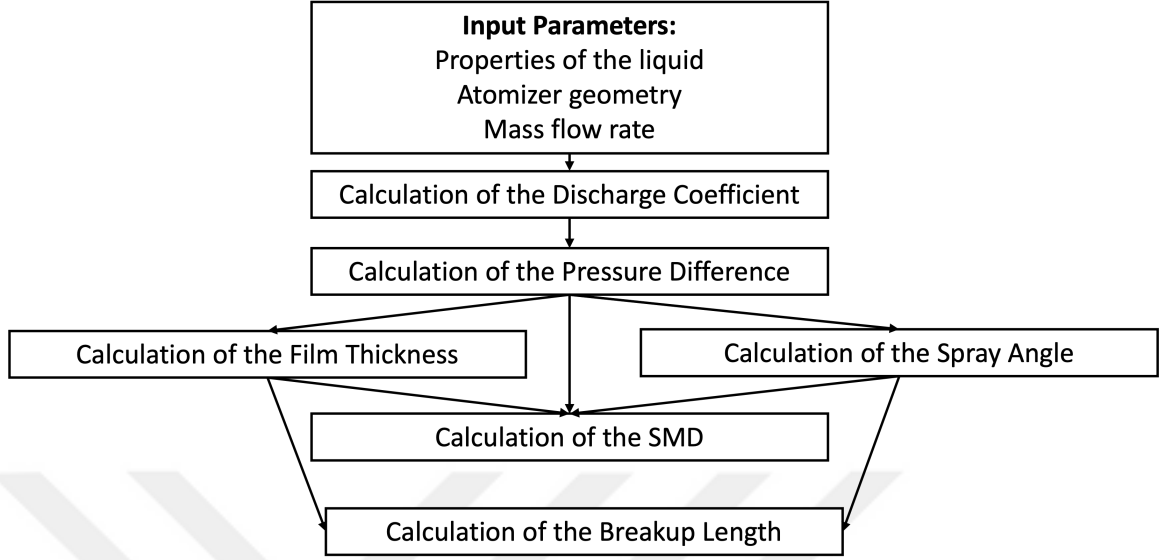


Figure 81: Proposed System of Calculations for Derived Semi-Empirical Correlations

It should be mentioned that all of the calculations are obtained using the built-in optimization functions of MATLAB.

4.4.1 Discharge Coefficient and Pressure Difference

It was mentioned and shown earlier that, predicting discharge coefficient accurately does not necessarily mean that pressure difference can be predicted accurately as well. Since discharge coefficient is the only way to relate the pressure difference with the geometry of the atomizer, it is desired to have a discharge coefficient correlation that can produce low error values for the pressure difference calculation as well. Therefore, a multi-objective optimization study is set up for deriving a correlation for the discharge coefficient. For the calculations, built-in MATLAB function named 'multi-objective optimization with generic algorithm' is used. Derived correlations are assessed from the plotted Pareto front.

The optimization problem can be written as:

$$\min_{\forall a_i} \begin{matrix} Error\ C_D \\ Error\ \Delta P_L \end{matrix} \quad (152)$$

As mentioned earlier, AMRE (Eq. 29) and SE (Eq. 30) values are used to evaluate correlations. Here mean squared error (MSE) is used as the function to minimize. Definition of MSE is given in Eq. 153. MSE is used instead of AMRE since it is easier to implement. It should be noted that AMRE needs more complicated tools such as linear programming to compute the gradients.

$$MSE = \frac{1}{n} \sum_{i=1}^n (Calculated_i^2 - Experimental_i^2) \quad (153)$$

Pareto front obtained from the calculations for the close-end atomizers is given in Figure 82. Overall 2,000 iterations are done and 35 different points on the Pareto front are obtained. It should be noted that each of these 35 points corresponds to a different semi-empirical correlation for calculating discharge coefficient and pressure difference.

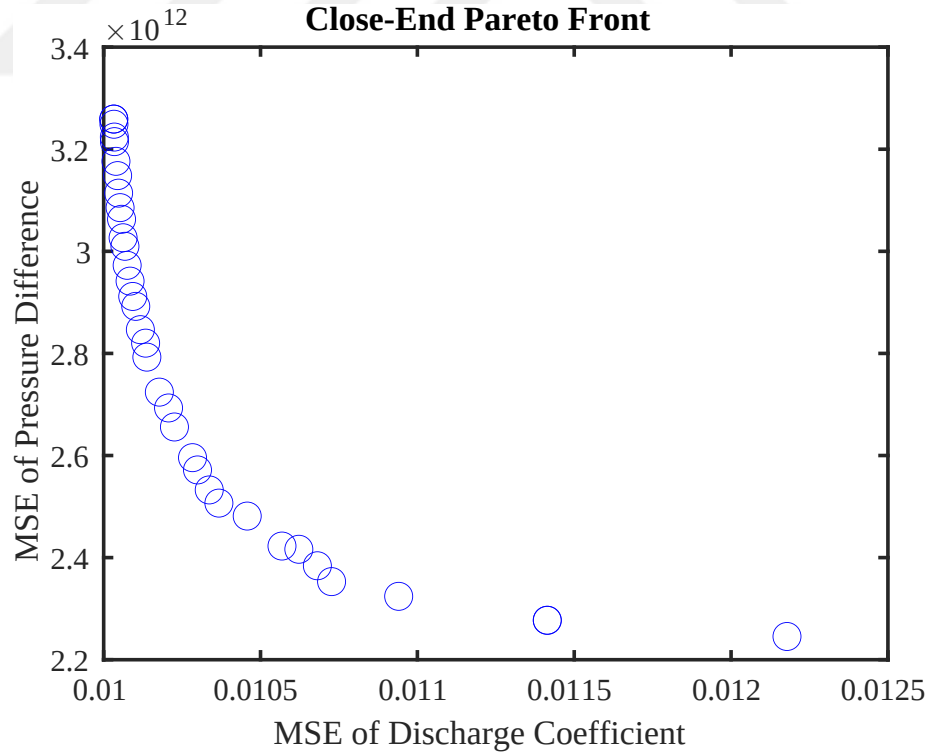


Figure 82: Pareto front for close-end atomizer discharge coefficient and pressure difference

Final correlation is selected based on the distance to the origin on the Pareto front, where the origin represents the error being zero for both discharge coefficient and pressure difference. Obtained correlation is given in Eq. 154.

$$C_D = 0.253 \times \left(\frac{D_o \times \rho_L \times V_B}{\mu_L} \right)^{0.014} \times \left(\frac{L_o}{D_o} \right)^{-0.021} \times \left(\frac{L_s}{D_s} \right)^{-0.016} \times (K)^{0.390} \times \left(\frac{D_s}{D_o} \right)^{0.446} \quad (154)$$

Obtained correlation has an AMRE value of 0.375 and SE value of 0.001 for discharge coefficient and an AMRE value of 0.547 and SE value of 0.001 for the calculation of pressure difference. Comparison of these values with Table 11 and Table 12 shows a slight increase in the error of discharge coefficient and a substantial decrease in pressure difference error.

Same calculations are done for the open-end atomizers as well. Obtained Pareto front is given in Figure 83.

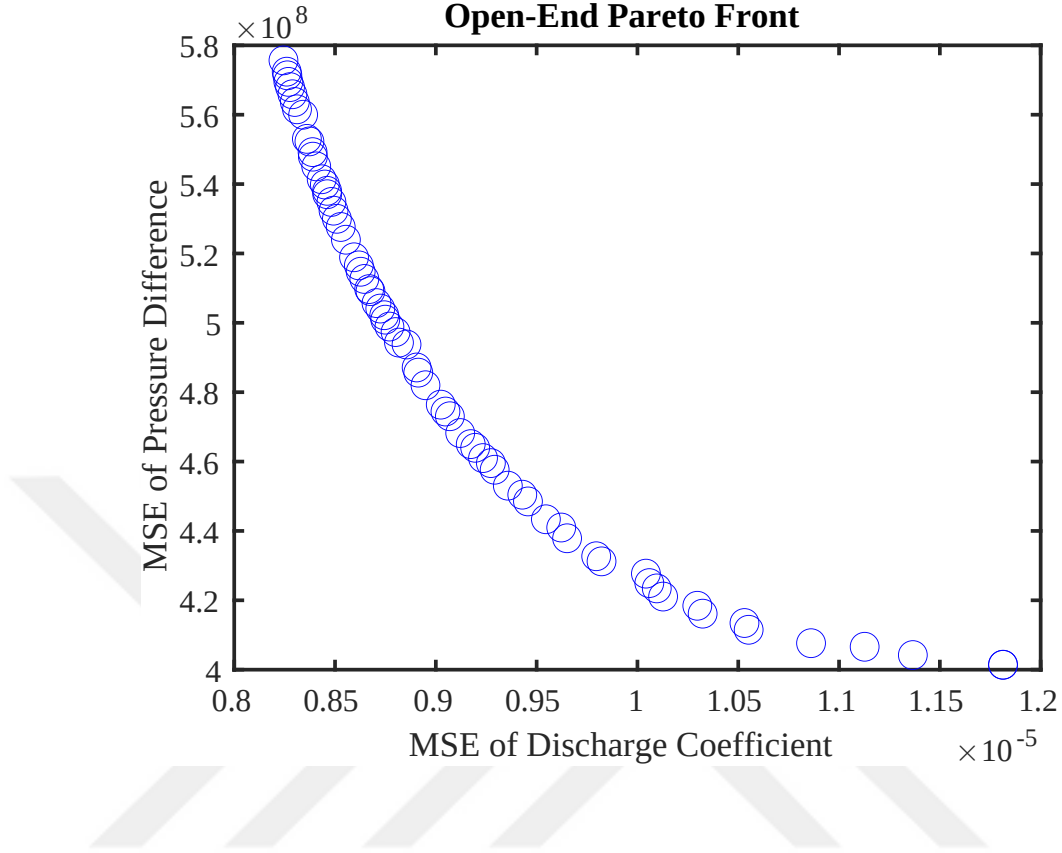


Figure 83: Pareto front for open-end atomizer discharge coefficient and pressure difference

Obtained correlation is given in Eq. 155. This correlation has the AMRE value of 0.072 and SE value of 4.9E-4 for the discharge coefficient. For the pressure difference obtained correlation has an AMRE value of 0.140 and an SE value of 0.001. Comparing these values with the ones in Table 13 shows improvement in error values for both pressure difference and discharge coefficient.

$$C_D = 0.609 \times (K)^{0.781} \times (C)^{0.531} \quad (155)$$

4.4.2 Film Thickness

For the rest of the parameters, built-in 'patternsearch' function of MATLAB is used. Obtained correlations for calculating the film thickness are given in Eq. 156 and Eq.

157 for close-end and open-end, respectively.

$$t = 3.596 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25} \quad (156)$$

$$t = 2.606 \times \left[\frac{D_o \times \dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L} \right]^{0.25} \quad (157)$$

Correlation for calculating the close-end atomizer film thickness has the AMRE value of 0.117 and SE value of 0.001. Correlation derived for the open-end atomizer has the AMRE value of 0.040 and SE value of 3.7E-4. Comparing these values with the previous ones shows improvement in the accuracy of film thickness calculation for the close-end atomizer. Error values of the open-end atomizer have remained constant and the inspection of the best literature equation and the derived correlation shows that the two correlations are very similar. Moreover, it should be mentioned again that, for the open-end atomizer there are only 5 data points.

4.4.3 Spray Angle

Obtained semi-empirical correlations for calculating the spray angle of close-end and open-end atomizers are given in Eq. 158 and Eq. 159, respectively.

$$2\theta = 10.656 \times (K)^{-0.139} \times \left(\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2} \right)^{0.079} \quad (158)$$

$$2\theta = 10.629 \times (K)^{-0.176} \times \left(\frac{\Delta P_L \times D_o^2 \times \rho_L}{\mu_L^2} \right)^{0.068} \quad (159)$$

Correlation for the close-end atomizer has the AMRE value of 0.224 and SE value of 2.7E-4. It can be seen that obtained correlation is more accurate than the literature correlation. AMRE and SE values of open-end atomizer correlation are found to be 0.074 and 7.4E-4, respectively. Again, the obtained correlation is more accurate than the best correlation in the literature.

4.4.4 Droplet Size

As mentioned earlier, droplet size measurement data only exist for the close-end atomizer and hence only one semi-empirical equation is obtained. Correlation is

given in Eq. 160. This correlation has AMRE and SE values of 0.420 and 0.003. As can be seen, improvement in both of these values is obtained in comparison to the best correlation of the literature.

$$SMD = 0.913 \times \left(\frac{\sigma^{0.5} \times \mu_L}{\rho_a^{0.5} \times \Delta P_L} \right)^{0.200} \times (t \times \cos \theta)^{0.700} + 0.133 \times \left(\frac{\sigma \times \rho_L}{\rho_a \times \Delta P_L} \right)^{0.117} \times (t \times \cos \theta)^{0.883} \quad (160)$$

4.4.5 Break-up Length

Semi-empirical equation for calculating the break-up length is given in Eq. 161. It should be noted that the form of this correlation is different from the ones in the literature. This correlation has an AMRE value of 0.270 and an SE value of 0.003. Comparing these values with the ones in the literature shows significant improvement in the accuracy of calculating this parameter.

$$L_b = 0.075 \times We_g^{-0.507} \times Re_L^{0.697} \times \left(\frac{C_D}{\cos \theta} \right)^{0.364} \quad (161)$$

Table 23: Comparison of Error Values of Most Accurate Break-up Correlations with the Derived Correlation

Equation	AMRE	SE
Eq. 161	0.270	0.003
Lb-8	0.595	0.018
Lb-1	0.922	0.016
Lb-2	0.964	0.001
Lb-4	0.968	0.001

CHAPTER V

INTERNAL FLOW MODEL

5.1 Introduction

Studies conducted for determining the accuracy of the semi-empirical correlations have shown that large error values are present for predicting the performance parameters. Even though derived semi-empirical correlations had lower error values, using them to perform design optimization will lead to significant deviations. It should be noted that, even with the globally most accurate correlation sets, the error of pressure difference calculation is around 50 %. This indicates that a new model, that is more based on the physics of flow, is required for achieving higher accuracy.

Literature review has shown that there are two main methods for obtaining such model, namely inviscid flow based modeling and boundary layer based modeling. The former method consists of two steps. First, internal flow is modeled with an inviscid flow assumption, and later, based on the experimental data, adjustment of the model is done by introducing a constant or variable. This constant or variable is usually a function of the Reynolds number and is introduced to compensate for the error made with neglecting the losses in the internal flow. This shows that the accuracy of this model will be bounded with the introduced constant or variable, which is dependent on the experimental data. In other words, with this method, a completely physics-based model cannot be obtained [3, 6, 7].

The latter, however, allows obtaining such model. In this model, the internal flow field is divided into two regions, namely the potential core and boundary layer. Potential core is assumed to have inviscid flow, and Bernoulli's equation could be used to determine the pressure distribution in the axial direction. Schematic of these

regions is shown in Figure 84.

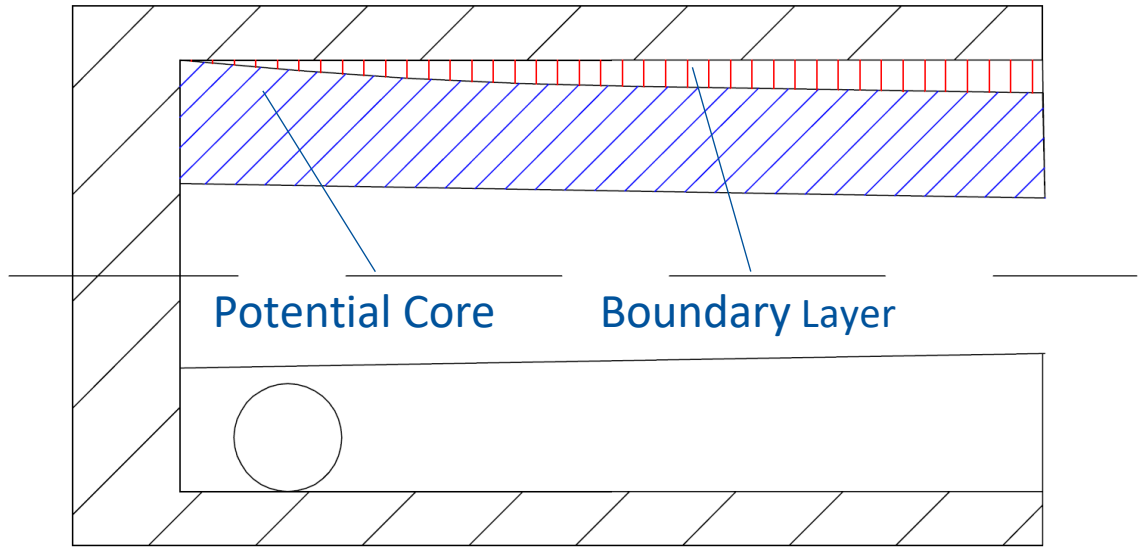


Figure 84: Potential core and boundary layer schematic of PSA internal flow

It should be mentioned that only a small number of studies have used this method. The earliest study in this field is done with the assumption of free vortex flow in the potential core and neglected axial velocity outside the boundary layer [10]. In other words, the author has assumed a boundary layer such that its thickness reaches the film thickness value at the exit, and all of the mass goes through the boundary layer. Later, a similar approach is used, but this time, uniform axial velocity outside the boundary layer is considered [11]. Authors have stated that having no axial velocity outside the boundary layer will result in very high velocities inside the boundary layer since its thickness is small. It should be noted that these two studies are conducted for geometries that only have a convergent section. Similar modeling is also done for the swirling pipe flow where no air core exists [79]. Later, above-mentioned models have been combined to obtain a model based on the boundary layer for predicting the internal flow of close-end PSA [12]. In preliminary work, the author of the present

work has developed a boundary layer model for the open-end atomizers [13]. Even though there is small number of studies conducted for the boundary layer modeling of the internal flow of PSA, it was shown that it can accurately predict the internal flow parameters such as discharge coefficient, pressure difference, and film thickness.

In this chapter, first, the derivation of a boundary layer model for PSA is explained. It should be noted that two different models, one for the straight section of PSA and one for the convergent and divergent sections, are obtained. For the straight section model, use of the Navier-Stokes equations in cylindrical coordinates is done. As shown in Figure 3 straight parts correspond to the swirl chamber and orifice of close-end PSA. Moreover, this model can be used to predict the internal flow of the open-end atomizers. Therefore, validation of the straight section model is done using the experimental data of open-end atomizers, presented in Chapter 4. For the convergent and divergent section model, Navier-Stokes equations in spherical coordinates are used. Two models are combined with mapping of the velocity components, and evaluation of the accuracy of the combined model is done using close-end atomizer experimental data.

5.2 *Derivation of the Boundary Layer Model*

First, the difference between the free vortex and the forced vortex is explained. Equations for free vortex and forced vortex are given Eq. 162 and Eq. 163, respectively.

$$U_{\theta} = \frac{\Omega_{free}}{r}. \quad (162)$$

$$U_{\theta} = \Omega_{forced} \times r. \quad (163)$$

where, U_{θ} is the tangential velocity component, Ω_{free} is the circulation constant for the free vortex with the unit of m^2/s , Ω_{forced} is the circulation constant for the forced vortex with the unit of $1/s$ and r is the radius. It can be seen that tangential velocity decreases with the radius for free vortex and reaches infinity at the center (where

$r=0$). For the forced vortex reverse trend can be seen, where tangential velocity increases with the radius and have the value of 0 at the center.

In this study free vortex is assumed for the potential core, which is also done in the earlier studies [10–13, 79]. In addition, the following assumptions are made for the flow in the potential core.

1. Axisymmetric,
2. Steady,
3. Inviscid flow,
4. Uniform axial velocity

5.2.1 Boundary Layer Model for Straight Section

As mentioned earlier, derivation of the correlations for describing the boundary layer thickness of straight section of the PSA was done before [12]. In the earlier study of the author, same model has been applied to open-end atomizers [13]. The only difference between the model of the applied model was the method of calculation of the size of the air core and the pressure difference. Instead of using the maximum flow principle, use of the mass flow rate was done for calculating the size of the air core, and the pressure difference losses occurring in the tangential inlet ports are added through hydraulic loss equations. Comparison with the experimental data has shown that pressure difference is calculated with AMRE (Eq. 29) value of 0.102 and film thickness is calculated with AMRE value of 0.037 [13].

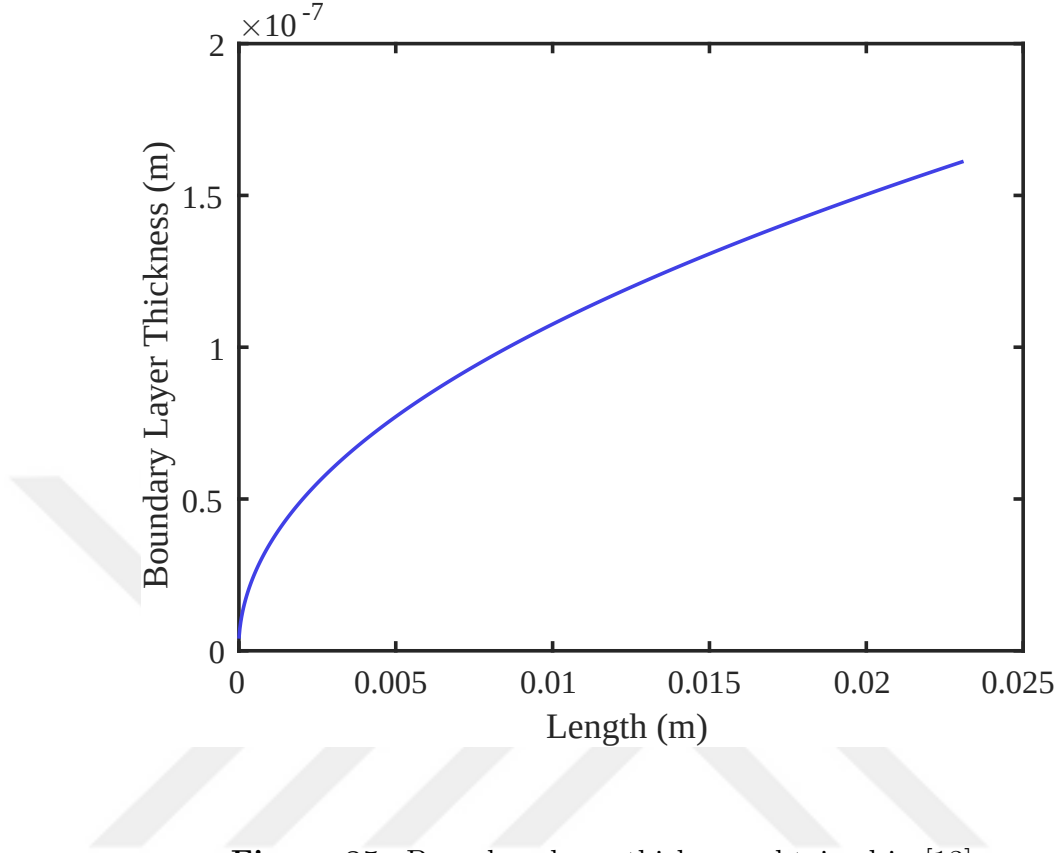


Figure 85: Boundary layer thickness obtained in [13]

However, when the obtained boundary layer thickness values are inspected (Figure 85), it is seen that it is in the order of nanometers. This has shown that obtained model produces physically-wrong boundary layer thickness values and therefore new models are derived.

Navier-Stokes equation in cylindrical coordinates is used for the model of straight section. For incompressible flow these equations are given in Eq. 164 - 167.

- Continuity:

$$\frac{1}{r} \frac{\partial (rU_r)}{\partial r} + \frac{1}{r} \frac{\partial U_\theta}{\partial \theta} + \frac{\partial U_z}{\partial z} = 0 \quad (164)$$

- r-Momentum:

$$\rho \left[\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\theta}{r} \frac{\partial U_r}{\partial \theta} - \frac{U_\theta^2}{r} + U_z \frac{\partial U_r}{\partial z} \right] = -\frac{\partial P}{\partial r} + \rho g_r + \mu \times \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U_r}{\partial r} \right) - \frac{U_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 U_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial U_\theta}{\partial \theta} + \frac{\partial^2 U_r}{\partial z^2} \right] \quad (165)$$

- θ -Momentum:

$$\rho \left[\frac{\partial U_\theta}{\partial t} + U_r \frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r} \frac{\partial U_\theta}{\partial \theta} + \frac{U_r U_\theta}{r} + U_z \frac{\partial U_\theta}{\partial z} \right] = -\frac{\partial P}{\partial \theta} + \rho g_\theta +$$

$$\mu \times \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U_\theta}{\partial r} \right) - \frac{U_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 U_\theta}{\partial \theta^2} + \frac{2}{r^2} + \frac{\partial U_r}{\partial \theta} + \frac{\partial^2 U_\theta}{\partial z^2} \right] \quad (166)$$

- z-Momentum:

$$\rho \left[\frac{\partial U_z}{\partial t} + U_r \frac{\partial U_z}{\partial r} + \frac{U_\theta}{r} \frac{\partial U_z}{\partial \theta} + U_z \frac{\partial U_z}{\partial z} \right] = -\frac{\partial P}{\partial z} + \rho g_z +$$

$$\mu \times \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U_z}{\partial \theta^2} + \frac{\partial^2 U_z}{\partial z^2} \right] \quad (167)$$

where, U_r , U_θ and U_z are the velocity components in r , θ , and z -directions, respectively, and P is the pressure, ρ is the density, g is the gravitational acceleration and μ is the absolute viscosity. These equations are simplified using the assumptions listed below.

1. Axisymmetric,
2. Steady,
3. Velocity component in r -direction is neglected,
4. Momentum equation in r -direction is neglected,
5. Thickness of the boundary layer is considered to be small

$$\frac{\partial U_r}{\partial r} \gg \frac{U_r}{r} \quad (168)$$

$$\frac{\partial U_\theta}{\partial r} \gg \frac{U_\theta}{r} \quad (169)$$

$$\frac{\partial U_z}{\partial r} \gg \frac{U_z}{r} \quad (170)$$

which simply means, velocity gradient in the boundary layer is larger than the ratio of velocity to the radius.

6.

$$\frac{\partial^2}{\partial r^2} \gg \frac{\partial^2}{\partial z^2} \quad (171)$$

With these assumptions, Navier-Stokes equations become:

• Continuity:

$$\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z} = 0 \quad (172)$$

• θ -Momentum:

$$U_r \frac{\partial U_\theta}{\partial r} + U_z \frac{\partial U_\theta}{\partial z} = \frac{\mu}{\rho} \frac{\partial^2 U_\theta}{\partial r^2} \quad (173)$$

• z-Momentum:

$$U_r \frac{\partial U_z}{\partial r} + U_z \frac{\partial U_z}{\partial z} = -\frac{\partial P}{\partial z} + \rho g_z + \frac{\mu}{\rho} \frac{\partial^2 U_z}{\partial r^2} \quad (174)$$

Moreover, for the velocity distributions inside the boundary layer, the following correlations are used.

$$u_z = U_z \Psi_1(\eta) \quad (175)$$

$$u_\theta = U_\theta \Psi_2(\eta) \quad (176)$$

where, U is the velocity at the edge of the boundary layer and u is the velocity inside the boundary layer, Ψ_1 and Ψ_2 are the dimensionless velocity profiles inside the boundary layer for the axial component and tangential component, respectively and,

$$\eta = \frac{R - r}{\delta(z)} \quad (177)$$

and R is the radius of the straight part (for swirl chamber it is R_s and for orifice it is R_o), r is the radius and δ is the boundary layer thickness.

Additionally, since uniform axial velocity and free vortex is assumed for the potential core, Eq. 175 and Eq. 176 can be written as:

$$u_z(z) = U_z(z) \Psi_1(\eta) \quad (178)$$

$$u_{\theta}(z) = \frac{\Omega}{R-\delta} \Psi_2(\eta) \quad (179)$$

It should be noted that Eq. 178 shows that axial velocity is only a function of z (axial coordinate) and not a function of the radial coordinate. Also, in the previously obtained model instead of Eq. 179, Eq. 180 is used for u_{θ} . As can be seen, this equation neglects the effect of boundary layer thickness on tangential velocity. It is suspected that this could lead to thickness values being very small therefore, its effect is included. Similarly, effect of gravity in the axial direction is included which was neglected in previous models.

$$u_{\theta} = \frac{\Omega}{R} \Psi_2(\eta) \quad (180)$$

Final equations are obtained by integrating momentum equations over the boundary layer using the continuity equation and integration of parts.

First, θ -momentum equation is integrated. Integral form of Eq. 173 is given in Eq. 181.

$$\int_{R-\delta}^R u_r \frac{\partial u_{\theta}}{\partial r} dr + \int_{R-\delta}^R u_z \frac{\partial u_{\theta}}{\partial z} dr = \int_{R-\delta}^R \frac{\mu}{\rho} \frac{\partial^2 u_{\theta}}{\partial r^2} dr \quad (181)$$

First term on the left hand side of Eq. 181 can be written as:

$$u_r \frac{\partial u_{\theta}}{\partial r} = \frac{\partial (u_r u_{\theta})}{\partial r} - u_{\theta} \frac{\partial u_r}{\partial r} \quad (182)$$

From the continuity equation (Eq. 172):

$$\frac{\partial u_r}{\partial r} = -\frac{\partial u_z}{\partial z} \quad (183)$$

Re-writing Eq. 182 with the help of Eq. 183 result in:

$$u_r \frac{\partial u_{\theta}}{\partial r} = \frac{\partial (u_r u_{\theta})}{\partial r} + u_{\theta} \frac{\partial u_z}{\partial z} \quad (184)$$

Integrating Eq. 184 over the boundary layer:

$$\int_{R-\delta}^R u_r \frac{\partial u_{\theta}}{\partial r} dr = \int_{R-\delta}^R \frac{\partial u_r u_{\theta}}{\partial r} dr + u_{\theta} \frac{\partial u_z}{\partial z} dr = u_r u_{\theta} \Big|_{R-\delta}^R + \int_{R-\delta}^R u_{\theta} \frac{\partial u_z}{\partial z} dr \quad (185)$$

Inserting Eq. 185 into Eq. 181:

$$u_r u_\theta|_{R-\delta}^R + \int_{R-\delta}^R u_\theta \frac{\partial u_z}{\partial z} dr + \int_{R-\delta}^R u_z \frac{\partial u_\theta}{\partial z} dr = \int_{R-\delta}^R \frac{\mu}{\rho} \frac{\partial^2 u_\theta}{\partial r^2} dr = \frac{\mu}{\rho} \frac{\partial u_\theta}{\partial r} \Big|_{R-\delta}^R \quad (186)$$

From the integration of Eq. 183, following can be obtained.

$$u_r = - \int_{R-\delta}^R \frac{\partial u_z}{\partial z} dr \quad (187)$$

With the help of Eq. 187, Eq. 179 and Eq. 178, θ -momentum equation can be written as:

$$\begin{aligned} - \frac{\Omega}{R-\delta} \int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta))}{dz} dr \\ + \int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta) (\Omega/(R-\delta)) \Psi_2(\eta))}{dz} dr \\ = \frac{\mu}{\rho} \frac{d}{dr} \left(\frac{\Omega}{R-\delta} \Psi_2(\eta) \right) \Big|_{R-\delta}^R \end{aligned} \quad (188)$$

First, calculation of first term on the left hand side is done. Naming this term as θ_1 :

$$\theta_1 = - \frac{\Omega}{R-\delta} \int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta))}{dz} dr \quad (189)$$

From Eq. 177, $dr = -\eta d\eta$ can be obtained. Replacing r with η in Eq. 189, θ_1 becomes:

$$\theta_1 = - \frac{\Omega}{R-\delta} \left[\int_0^1 \frac{dU_z(z)}{dz} \Psi_1(\eta) \delta d\eta + \int_0^1 U_z(z) \frac{d\Psi_1(\eta)}{dz} \delta d\eta \right] \quad (190)$$

Arranging Eq. 190:

$$\theta_1 = - \frac{\Omega}{R-\delta} \left[\frac{dU_z(z)}{dz} \delta \int_0^1 \Psi_1(\eta) d\eta + U_z(z) \delta \int_0^1 \frac{d\Psi_1(\eta)}{dz} d\eta \right] \quad (191)$$

Using chain rule as:

$$\frac{d\Psi_1(\eta)}{dz} = \frac{d\Psi_1(\eta)}{d\eta} \frac{d\eta}{d\delta} \frac{d\delta}{dz} \quad (192)$$

where,

$$\frac{d\eta}{d\delta} = - \frac{R-r}{\delta^2} = - \frac{\eta}{\delta} \quad (193)$$

Inserting Eq. 192 and Eq. 193 into Eq. 191:

$$\theta_1 = -\frac{\Omega}{R-\delta} \left[\frac{dU_z(z)}{dz} \delta \int_0^1 \Psi_1(\eta) d\eta + U_z(z) \delta \int_0^1 \frac{d\Psi_1(\eta)}{d\eta} \left(-\frac{\eta}{\delta}\right) \frac{d\delta}{dz} d\eta \right] \quad (194)$$

Arranging Eq. 194:

$$\theta_1 = -\frac{\Omega}{R-\delta} \left[\frac{dU_z(z)}{dz} \delta \int_0^1 \Psi_1(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \int_0^1 \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta \right] \quad (195)$$

where,

$$\int_0^1 \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta = \int_0^1 \frac{d(\eta\Psi_1(\eta))}{d\eta} d\eta - \int_0^1 \Psi_1(\eta) \frac{d\eta}{d\eta} d\eta \quad (196)$$

furthermore,

$$\int_0^1 \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta = \eta\Psi_1(\eta)|_0^1 - \int_0^1 \Psi_1(\eta) d\eta \quad (197)$$

First term on the right hand side can be evaluated as:

$$\eta\Psi_1(\eta)|_0^1 = (1)\Psi_1(1) - (0)\Psi_1(0) = \Psi_1(1) \quad (198)$$

Using Eq. 197 and Eq. 198, Eq. 195 can be written as:

$$\theta_1 = -\frac{\Omega}{R-\delta} \left[\frac{dU_z(z)}{dz} \delta \int_0^1 \Psi_1(\eta) d\eta + U_z(z) \frac{d\delta}{dz} \int_0^1 \frac{d\Psi_1(\eta)}{d\eta} d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \right] \quad (199)$$

Final form of the θ_1 obtained as:

$$\theta_1 = -\frac{\Omega}{R-\delta} \left[\frac{d(U_z(z)\delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \right] \quad (200)$$

Calculating second term on the left hand side of Eq. 188:

$$\theta_2 = \int_{R-\delta}^R \frac{d(U_z(z)\Psi_1(\eta)(\Omega/(R-\delta))\Psi_2(\eta))}{dz} dr \quad (201)$$

Replacing r with η in Eq. 201, θ_2 becomes:

$$\theta_2 = \int_0^1 \frac{d}{dz} (U_z(z)\Psi_1(\eta)(\Omega/(R-\delta))\Psi_2(\eta)) \delta d\eta \quad (202)$$

Expanding Eq. 202:

$$\begin{aligned}\theta_2 = & \int_0^1 \delta \frac{\Omega}{(R-\delta)^2} \frac{d\delta}{dz} U_z(z) \Psi_1(\eta) \Psi_2(\eta) d\eta \\ & + \int_0^1 \delta \frac{\Omega}{R-\delta} \frac{dU_z(z)}{dz} \Psi_1(\eta) \Psi_2(\eta) d\eta \\ & + \int_0^1 \delta \frac{\Omega}{R-\delta} U_z(z) \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{dz} d\eta \quad (203)\end{aligned}$$

Using chain rule as:

$$\frac{d\Psi_1(\eta) \Psi_2(\eta)}{dz} = \frac{d\Psi_1(\eta) \Psi_2(\eta)}{d\eta} \frac{d\eta}{d\delta} \frac{d\delta}{dz} \quad (204)$$

Using Eq. 193 and Eq. 204, Eq. 203 can be written as:

$$\begin{aligned}\theta_2 = & \delta \frac{\Omega}{(R-\delta)^2} \left(\frac{U_z(z) d\delta}{R-\delta dz} + \frac{dU_z(z)}{dz} \right) \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \\ & - \frac{\Omega}{R-\delta} U_z(z) \frac{d\delta}{dz} \int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{d\eta} \eta d\eta \quad (205)\end{aligned}$$

where,

$$\int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{d\eta} \eta d\eta = \int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta) \eta)}{d\eta} d\eta - \int_0^1 \Psi_1(\eta) \Psi_2(\eta) \frac{d\eta}{d\eta} d\eta \quad (206)$$

and,

$$\int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{d\eta} \eta d\eta = (\Psi_1(\eta) \Psi_2(\eta) \eta)|_0^1 - \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \quad (207)$$

First term on the right hand side can be evaluated as:

$$(\Psi_1(\eta) \Psi_2(\eta) \eta)|_0^1 = (1) \Psi_1(1) \Psi_2(1) - (0) \Psi_1(0) \Psi_2(0) = \Psi_1(1) \Psi_2(1) \quad (208)$$

Inserting Eq. 207 and Eq. 208 into Eq. 205:

$$\begin{aligned}\theta_2 = & \delta \frac{\Omega}{(R-\delta)^2} \left(\frac{U_z(z) d\delta}{R-\delta dz} + \frac{dU_z(z)}{dz} \right) \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \\ & + \frac{\Omega}{R-\delta} U_z(z) \frac{d\delta}{dz} \int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta) \eta)}{d\eta} d\eta \\ & - \frac{\Omega}{R-\delta} U_z(z) \frac{d\delta}{dz} \Psi_1(1) \Psi_2(1) \quad (209)\end{aligned}$$

Arranging Eq. 209:

$$\theta_2 = \frac{\Omega}{R-\delta} \left[\left(\frac{\delta}{R-\delta} U_z(z) \frac{d\delta}{dz} + U_z(z) \frac{d\delta}{dz} + \frac{dU_z(z)}{dz} \delta \right) \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \Psi_2(1) \right] \quad (210)$$

Considering:

$$U_z(z) \frac{d\delta}{dz} \gg \frac{\delta}{R-\delta} U_z(z) \frac{d\delta}{dz} \Rightarrow \frac{\delta}{R-\delta} U_z(z) \frac{d\delta}{dz} + U_z(z) \frac{d\delta}{dz} \cong U_z(z) \frac{d\delta}{dz} \quad (211)$$

Eq. 210 becomes:

$$\theta_2 = \frac{\Omega}{R-\delta} \left[\left(U_z(z) \frac{d\delta}{dz} + \frac{dU_z(z)}{dz} \delta \right) \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \Psi_2(1) \right] \quad (212)$$

Lastly, right hand side term of Eq. 188 is evaluated.

$$\theta_3 = \frac{\mu}{\rho} \frac{d}{dr} \left(\frac{\Omega}{R-\delta} \Psi_2(\eta) \right) \Big|_{R-\delta}^R = \frac{\mu}{\rho} \frac{\Omega}{R-\delta} \frac{d\Psi_2(\eta)}{dr} \Big|_{R-\delta}^R \quad (213)$$

Replacing r with η in Eq. 213 and using chain rule as:

$$\frac{d\Psi_2(\eta)}{dr} = \frac{d\Psi_2(\eta)}{d\eta} \frac{d\eta}{dr} \quad (214)$$

where,

$$\frac{d\eta}{dr} = \frac{-1}{\delta} \quad (215)$$

Eq. 211 becomes:

$$\theta_3 = -\frac{\mu}{\rho} \frac{\Omega}{R-\delta} \frac{1}{\delta} \frac{d\Psi_2(\eta)}{d\eta} \Big|_1^0 \quad (216)$$

$$\theta_3 = \frac{\mu}{\rho} \frac{\Omega}{R-\delta} \frac{1}{\delta} \frac{d\Psi_2(\eta)}{d\eta} \Big|_0^1 \quad (217)$$

Finally, θ -momentum equation becomes:

$$\begin{aligned}
& -\frac{\Omega}{R-\delta} \left[\frac{d(U_z(z)\delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \right] \\
& + \frac{\Omega}{R-\delta} \left[\left(U_z(z) \frac{d\delta}{dz} + \frac{dU_z(z)}{dz} \delta \right) \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \right. \\
& \quad \left. - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \Psi_2(1) \right] \\
& = \frac{\mu}{\rho} \frac{\Omega}{R-\delta} \frac{1}{\delta} \frac{d\Psi_2(\eta)}{d\eta} \Big|_0^1 \quad (218)
\end{aligned}$$

Arranging Eq. 218:

$$\begin{aligned}
& \frac{d\delta}{dz} \left[U_z(z) \left(\Psi_1(1) - \int_0^1 \Psi_1(\eta) d\eta + \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - \Psi_1(1) \Psi_2(1) \right) \right] \\
& + \frac{d(U_z(z))}{dz} \left[\delta \left(\int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - \int_0^1 \Psi_1(\eta) d\eta \right) \right] \\
& = \frac{\mu}{\rho} \frac{1}{\delta} \frac{d\Psi_2(\eta)}{d\eta} \Big|_0^1 \quad (219)
\end{aligned}$$

Second, z-momentum equation is integrated. Integral form of Eq. 174 is given in Eq. 220.

$$\int_{R-\delta}^R u_r \frac{\partial u_z}{\partial r} dr + \int_{R-\delta}^R u_z \frac{\partial u_z}{\partial z} dr = \int_{R-\delta}^R -\frac{1}{\rho} \frac{\partial P}{\partial z} dr + \int_{R-\delta}^R g_z dr + \int_{R-\delta}^R \frac{\mu}{\rho} \frac{\partial^2 u_z}{\partial r^2} dr \quad (220)$$

First term on the left hand side of Eq. 220 can be written as:

$$u_r \frac{\partial u_z}{\partial r} = \frac{\partial (u_r u_z)}{\partial r} - u_z \frac{\partial u_r}{\partial r} \quad (221)$$

Using Eq. 183:

$$u_r \frac{\partial u_z}{\partial r} = \frac{\partial (u_r u_z)}{\partial r} + u_z \frac{\partial u_z}{\partial z} \quad (222)$$

Integrating Eq. 222 over boundary layer:

$$\int_{R-\delta}^R u_r \frac{\partial u_z}{\partial r} dr = \int_{R-\delta}^R \frac{\partial u_r u_z}{\partial r} dr + \int_{R-\delta}^R u_z \frac{\partial u_z}{\partial z} dr = u_r u_z \Big|_{R-\delta}^R + \int_{R-\delta}^R u_z \frac{\partial u_z}{\partial z} dr \quad (223)$$

Using Eq. 223 in Eq. 220:

$$u_r u_z \Big|_{R-\delta}^R + 2 \int_{R-\delta}^R u_z \frac{\partial u_z}{\partial z} dr = -\frac{1}{\rho} \int_{R-\delta}^R \frac{\partial P}{\partial z} dr + \int_{R-\delta}^R g_z dr + \frac{\mu}{\rho} \int_{R-\delta}^R \frac{\partial^2 u_z}{\partial r^2} dr \quad (224)$$

Last term on the right hand side of Eq. 224 can be evaluated as:

$$\frac{\mu}{\rho} \int_{R-\delta}^R \frac{\partial^2 u_z}{\partial r^2} dr = \frac{\mu}{\rho} \frac{\partial u_z}{\partial r} \Big|_{R-\delta}^R \quad (225)$$

Also, first term on the left hand side of Eq. 224 can be written as (with the help of Eq. 175 and Eq. 187):

$$u_r u_z \Big|_{R-\delta}^R = -U_z(z) \Psi_1(1) \int_{R-\delta}^R \frac{\partial u_z}{\partial z} dr \quad (226)$$

Eq. 224 becomes (using Eq. 175):

$$\begin{aligned} & -U_z(z) \Psi_1(1) \int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta))}{dz} dz \\ & + 2 \int_{R-\delta}^R U_z(z) \Psi_1(\eta) \frac{d(U_z(z) \Psi_1(\eta))}{dz} dz \\ & = -\frac{1}{\rho} \int_{R-\delta}^R \frac{dP}{dz} dz + \int_{R-\delta}^R g_z dz + \frac{\mu}{\rho} \frac{d(U_z(z) \Psi_1(\eta))}{dz} \Big|_{R-\delta}^R \end{aligned} \quad (227)$$

First term on the left hand side of the Eq. 227 can be calculated as:

$$Z_1 = -U_z(z) \Psi_1(1) \int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta))}{dz} dz \quad (228)$$

From the solution of θ_1 (Eq. 200) it is known that:

$$\int_{R-\delta}^R \frac{d(U_z(z) \Psi_1(\eta))}{dz} dz = \frac{d(U_z(z) \Psi_1(\eta))}{dz} \int_0^1 d\Psi_1(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \Psi_1(1) \quad (229)$$

Hence, term z_1 becomes:

$$Z_1 = -U_z(z) \Psi_1(1) \frac{d(U_z(z) \delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta + U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(\eta) \quad (230)$$

Second term on the left hand side of the Eq. 227 can be calculated as:

$$Z_2 = 2 \int_{R-\delta}^R U_z(z) \Psi_1(\eta) \frac{d(U_z(z) \Psi_1(\eta))}{dz} dz \quad (231)$$

Expanding right hand side of Eq. 231:

$$Z_2 = 2U_z(z) \left[\frac{dU_z(z)}{dz} \int_{R-\delta}^R \Psi_1^2(\eta) dz + U_z(z) \int_{R-\delta}^R \Psi_1(\eta) \frac{d\Psi_1(\eta)}{dz} dz \right] \quad (232)$$

Using Eq. 192 and Eq. 193:

$$Z_2 = 2U_z(z) \left[\delta \frac{dU_z(z)}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \int_0^1 \Psi_1(\eta) \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta \right] \quad (233)$$

Moreover, noticing that:

$$\frac{d(\Psi_1(\eta) \eta)}{d\eta} = \Psi_1^2(\eta) + 2\Psi_1(\eta) \eta \frac{d\Psi_1(\eta)}{d\eta} \quad (234)$$

$$\Psi_1(\eta) \eta \frac{d\Psi_1(\eta)}{d\eta} = \frac{1}{2} \left[\frac{d\Psi_1^2(\eta) \eta}{d\eta} - \Psi_1^2(\eta) \right] \quad (235)$$

Integrating Eq. 235:

$$\begin{aligned} \int_0^1 \Psi_1(\eta) \eta \frac{d\Psi_1(\eta)}{d\eta} d\eta &= \frac{1}{2} \left[\int_0^1 \frac{d(\Psi_1^2(\eta) \eta)}{d\eta} d\eta - \int_0^1 \Psi_1^2(\eta) d\eta \right] \\ &= \frac{1}{2} \left[\Psi_1^2(1) - \int_0^1 \Psi_1^2(\eta) d\eta \right] \end{aligned} \quad (236)$$

Using Eq. 236 in Eq. 233:

$$Z_2 = 2U_z(z) \left[\delta \frac{dU_z(z)}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z(z) \frac{d\delta}{dz} \frac{1}{2} \left(\Psi_1^2(1) - \int_0^1 \Psi_1^2(\eta) d\eta \right) \right] \quad (237)$$

Arranging Eq. 237:

$$Z_2 = \frac{d(U_z^2(z) \delta)}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \quad (238)$$

Pressure term in the Eq. 227 can be calculated as:

$$Z_3 = -\frac{1}{\rho} \int_{R-\delta}^R \frac{dP}{dz} dr \quad (239)$$

Neglecting the variation of the pressure inside the boundary layer and with the inviscid flow assumption in the potential core, following can be written:

$$\frac{P}{\rho} + \frac{U_\theta^2}{2} + \frac{U_z^2}{2} = Constant \quad (240)$$

Taking the derivative with respect to axial coordinate and evaluating it at the edge of the boundary will result in:

$$\frac{1}{\rho} \frac{dP}{dz} = -U_z(z) \frac{dU_z(z)}{dz} - \frac{\Omega^2}{(R-\delta)^3} \frac{d\delta}{dz} \quad (241)$$

Therefore, Z_3 becomes:

$$Z_3 = \int_{R-\delta}^R U_z(z) \frac{dU_z(z)}{dz} dr + \int_{R-\delta}^R \frac{\Omega^2}{(R-\delta)^3} \frac{d\delta}{dz} dr \quad (242)$$

Integrating Eq. 242 results in:

$$Z_3 = U_z(z) \frac{dU_z(z)}{dz} \delta + \frac{\Omega^2}{(R-\delta)^3} \frac{d\delta}{dz} \delta \quad (243)$$

Gravity term in the Eq. 227 can be calculated as:

$$Z_4 = \int_{R-\delta}^R g_z dr = \delta g_z \quad (244)$$

Finally, the last term on right hand side of the Eq. 227 can be calculated as:

$$Z_5 = \frac{\mu}{\rho} \frac{d(U_z(z) \Psi_1(\eta))}{dr} \Big|_{R-\delta}^R = \frac{\mu}{\rho} U_z(z) \frac{d\Psi_1(\eta)}{dr} \Big|_{R-\delta}^R \quad (245)$$

Replacing r with η using Eq. 214 and Eq. 215:

$$Z_5 = \frac{\mu}{\rho} \frac{U_z(z)}{\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \quad (246)$$

z-Momentum equation can be written as:

$$\begin{aligned} & -U_z(z) \Psi_1(1) \frac{d(U_z(z) \delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta + U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & + \frac{d(U_z^2(z) \delta)}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & = U_z(z) \frac{dU_z(z)}{dz} \delta + \frac{\Omega^2}{(R-\delta)^3} \frac{d\delta}{dz} \delta + \delta g_z + \frac{\mu}{\rho} \frac{U_z(z)}{\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \end{aligned} \quad (247)$$

Arranging Eq. 247:

$$\begin{aligned} & \frac{d\delta}{dz} \left[U_z(z) \delta \int_0^1 \Psi_1^2(\eta) d\eta - \Psi_1(1) \int_0^1 \Psi_1(\eta) d\eta - \frac{\Omega^2}{(R-\delta)^3} \frac{\delta^2}{U_z(z)} \right] \\ & + \frac{dU_z(z)}{dz} \left[\delta^2 \left(2 \int_0^1 \Psi_1^2(\eta) d\eta - \Psi_1(1) \int_0^1 \Psi_1 d\eta - 1 \right) \right] \\ & = \frac{\delta^2 g_z}{U_z^2(z)} + \frac{\mu}{\rho} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \end{aligned} \quad (248)$$

As mentioned earlier, instead of maximum flow rate principle use of the flow rate equation is done for calculating the size of the using air core. Flow rate can be calculated from Eq. 249.

$$\dot{Q} = \int_{R_a}^{R-\delta} U_z(z) (2\pi r) dr + \int_{R-\delta}^R U_z(z) \Psi_1(\eta) (2\pi r) dr \quad (249)$$

It can be seen that flow rate is calculated by considering both flow rate through the potential core (first term on the right hand side of Eq. 249) and flow rate through the boundary layer (second term on the right hand side of Eq. 249). Since the flow rate is constant with respect to axial coordinate, taking the derivative of Eq. 249 will result in:

$$\begin{aligned} \frac{d\dot{Q}}{dz} = 0 = & \pi \left(-2 \frac{d\delta}{dz} (R - \delta) - 2R_a(z) \frac{dR_a(z)}{dz} \right) U_z(z) \\ & + \pi \left((R - \delta)^2 - R_a(z)^2 \right) \frac{dU_z(z)}{dz} \\ & + \pi \left(2 \frac{d\delta}{dz} (R - \delta) U_z(z) \int_0^1 \Psi_1(\eta) d\eta \right) \\ & + \pi \left((R^2 - (R - \delta)^2) \frac{dU_z(z)}{dz} \int_0^1 \Psi_1(\eta) d\eta \right) \quad (250) \end{aligned}$$

Arranging Eq. 250 result in :

$$\begin{aligned} \frac{d\delta}{dz} \left[2U_z(z) (R - \delta) \left(\int_0^1 \Psi_1(\eta) d\eta - 1 \right) \right] \\ + \frac{dU_z(z)}{dz} \left[(R^2 - (R - \delta)^2) \int_0^1 \Psi_1(\eta) d\eta + ((R - \delta)^2 - R_a^2(z)) \right] \\ + \frac{dR_a(z)}{dz} [-2R_a(z) U_z(z)] = 0 \quad (251) \end{aligned}$$

To use a Pohlhausen type of approach on the obtained θ -momentum (Eq. 219), z -momentum (Eq. 248) and flow rate (Eq. 251) equations, velocity profiles across the boundary layer must be chosen. It is been stated that, if these profiles can satisfy more boundary conditions, accuracy will be better [80]. These boundary conditions are:

- At the wall ($\eta = 0$):

$$u_z|_{\eta=0} = u_\theta|_{\eta=0} = 0$$

$$\Psi_1(0) = \Psi_2(0) = 0$$

- At the boundary layer edge ($\eta = 1$):

$$u_z|_{\eta=1} = U_z(z) \Rightarrow \Psi_1(1) = 1$$

$$\text{Similarly, } u_\theta|_{\eta=1} = \frac{\Omega}{R-\delta} \Rightarrow \Psi_2(1) = 1$$

Also,

$$\frac{\partial \Psi_1(\eta)}{\partial \eta} = \frac{\partial^2 \Psi_1(\eta)}{\partial \eta^2} = \dots = \frac{\partial^m \Psi_1(\eta)}{\partial \eta^m}$$

and,

$$\frac{\partial \Psi_2(\eta)}{\partial \eta} = \frac{\partial^2 \Psi_2(\eta)}{\partial \eta^2} = \dots = \frac{\partial^m \Psi_2(\eta)}{\partial \eta^m}$$

It should be noted that $\Psi_1(1) = 1$ comes from the assumption of axial flow exists outside the boundary layer as well. As stated earlier, it is been shown that this assumption leads to better results in comparison to assuming axial flow only occurs inside the boundary layer [11, 12].

For the velocity profile inside the boundary layer polynomial form is assumed and using the boundary conditions above, system of equations is obtained as shown with an example below.

For $m=3$, velocity profile will have the form:

$$\Psi(\eta) = x_1\eta + x_2\eta^2 + x_3\eta^3 + x_4\eta^4 \quad (252)$$

where, x_1, x_2, x_3 and x_4 are constants to be determined. It can be seen that with this form, $\Psi(0) = 0$ is satisfied for every x_1, x_2, x_3 , and x_4 value. Using boundary conditions at the edge of the boundary layer following equations are obtained.

$$\Psi(1) = x_1(1) + x_2(1)^2 + x_3(1)^3 + x_4(1)^4 = 1 \quad (253)$$

$$\left. \frac{\partial \Psi(\eta)}{\partial \eta} \Psi(1) \right|_{\eta=1} = (x_1 + 2x_2(\eta) + 3x_3(\eta)^2 + 4x_4(\eta)^3)|_{\eta=1} = 0 \quad (254)$$

$$\left. \frac{\partial^2 \Psi(\eta)}{\partial \eta^2} \Psi(1) \right|_{\eta=1} = (2x_2 + 6x_3(\eta) + 12x_4(\eta)^2)|_{\eta=1} = 0 \quad (255)$$

$$\left. \frac{\partial^3 \Psi(\eta)}{\partial \eta^3} \Psi(1) \right|_{\eta=1} = (6x_3 + 24x_4(\eta))|_{\eta=1} = 0 \quad (256)$$

Eq. 253 - 256 can also be written as:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 2 & 6 & 12 \\ 0 & 0 & 6 & 24 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (257)$$

Solving Eq. 257 will result in:

$$\Psi(\eta) = 4\eta - 6\eta^2 + 4\eta^3 - \eta^4 \quad (258)$$

Using the above-demonstrated method, following velocity profile equations are obtained.

- m = 1

$$\Psi(\eta) = 2\eta - \eta^2 \quad (259)$$

- m = 2

$$\Psi(\eta) = 3\eta - 3\eta^2 + \eta^3 \quad (260)$$

- m = 3

$$\Psi(\eta) = 4\eta - 6\eta^2 + 4\eta^3 - \eta^4 \quad (261)$$

- m = 5

$$\Psi(\eta) = 6\eta - 15\eta^2 + 20\eta^3 - 15\eta^4 + 6\eta^5 - \eta^6 \quad (262)$$

- m = 6

$$\Psi(\eta) = 7\eta - 21\eta^2 + 35\eta^3 - 35\eta^4 + 21\eta^5 - 7\eta^6 + \eta^7 \quad (263)$$

Comparison of the obtained velocity profiles is given in Figure 86. Inspection of the results has shown that for $m=5$ and $m=6$ profile is very similar.

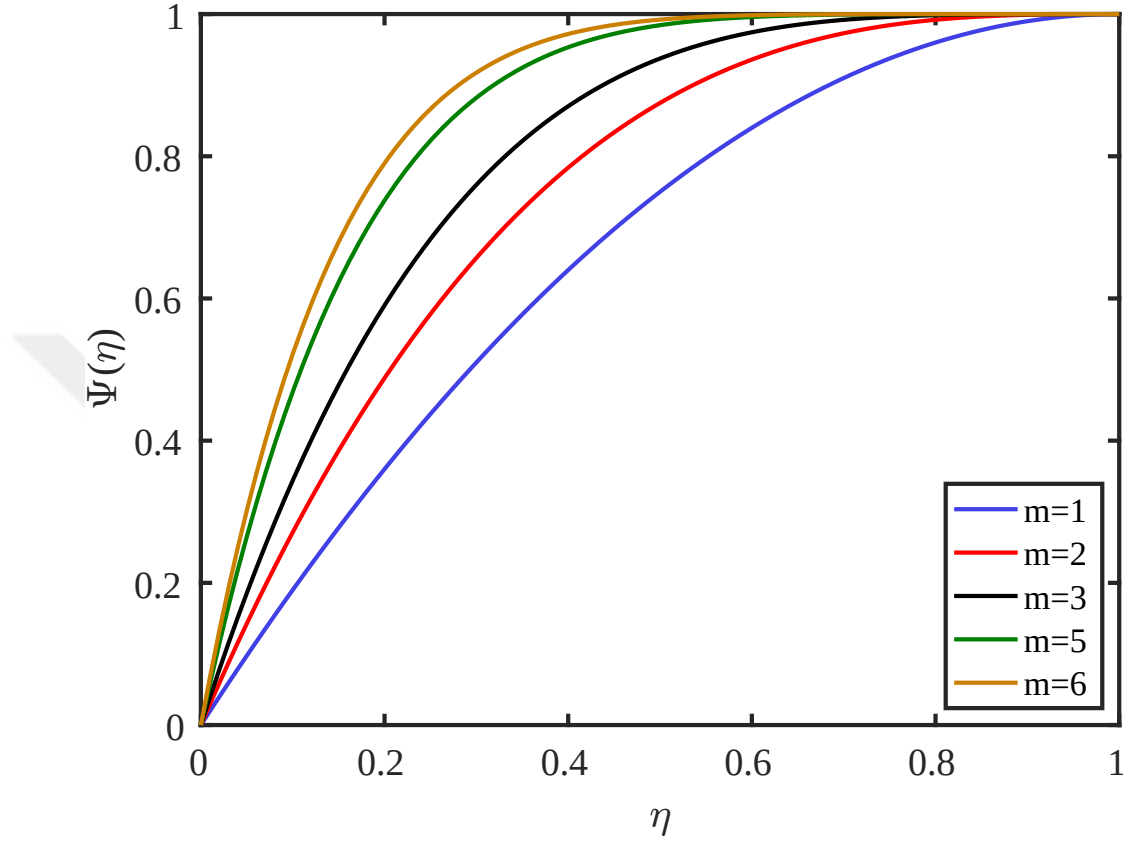


Figure 86: Comparison boundary layer velocity profiles

Moreover, Eq. 219, Eq. 248 and Eq. 251 can be re-written using:

$$\Psi_1(1) = 1 \quad (264)$$

$$\Psi_2(1) = 1 \quad (265)$$

$$A_1 = \int_0^1 \Psi_1(\eta) d\eta \quad (266)$$

$$A_2 = \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \quad (267)$$

$$A_3 = \int_0^1 \Psi_1^2(\eta) d\eta \quad (268)$$

$$B_1 = \left. \frac{d\Psi_1(\eta)}{d\eta} \right|_0^1 \quad (269)$$

$$B_2 = \left. \frac{d\Psi_2(\eta)}{d\eta} \right|_0^1 \quad (270)$$

Following correlations are obtained:

$$\frac{d\delta}{dz} [U_z(z) \delta (A_2 - A_1)] + \frac{dU_z(z)}{dz} [\delta^2 (A_2 - A_1)] = \frac{\mu}{\rho} B_2 \quad (271)$$

$$\begin{aligned} \frac{d\delta}{dz} \left[U_z(z) \delta (A_3 - A_1) - \frac{\Omega^2}{(R - \delta)^3} \frac{\delta^2}{U_z^2(z)} \right] + \frac{dU_z(z)}{dz} [\delta^2 (2A_3 - A_1 - 1)] \\ = \frac{\delta^2 g_z}{U_z(z)} + \frac{\mu}{\rho} B_1 \end{aligned} \quad (272)$$

$$\begin{aligned} \frac{d\delta}{dz} [2U_z(z) (R - \delta) (A_1 - 1)] \\ + \frac{dU_z(z)}{dz} [(R^2 - (R - \delta)^2) A_1 + ((R - \delta)^2 - R_a^2(z))] \\ + \frac{dR_a(z)}{dz} [-2R_a(z) U_z(z)] = 0 \end{aligned} \quad (273)$$

Obtained equations can be made non-dimensional using the:

$$U_z^*(z^*) = \frac{U_z(z) R}{\Omega} Re^{0.2} \quad (274)$$

$$\delta^* = \frac{\delta}{R} Re^{0.4} \quad (275)$$

$$z^* = \frac{z}{R} \quad (276)$$

$$Re = \frac{\Omega \rho}{\mu} \quad (277)$$

$$g_z^* = g_z \frac{R^3}{\Omega^2} Re^{0.4} \quad (278)$$

$$R_a^* = \frac{R_a}{R} Re^{0.2} \quad (279)$$

It should be mentioned that $Re^{0.2}$ and $Re^{0.4}$ are used to eliminate the Reynolds number from the right hand side of equations.

Obtained non-dimensional equations are:

$$\frac{d\delta^*}{dz^*} [U_z^*(z^*)\delta^*(A_2 - A_1)] + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2}(A_2 - A_1)] = B_2 \quad (280)$$

$$\begin{aligned} \frac{d\delta^*}{dz^*} \left[U_z^*(z^*)\delta^*(A_3 - A_1) - \frac{\delta^{*2}}{(1 - \delta^*)^3} \right] \\ + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2}(2A_3 - A_1 - 1)] \\ = \frac{\delta^{*2}g_z^*}{U_z^*(z^*)} + B_1 \end{aligned} \quad (281)$$

$$\begin{aligned} \frac{d\delta^*}{dz} \left[\frac{2U_z^*(z)(1 - \delta^*)(A_1 - 1)}{Re^{0.4}} \right] \\ + \frac{dU_z^*(z^*)}{dz} [(1 - (1 - \delta^{*2}))A_1 + (1 - \delta^*)^2 - (R_a^*)^2 Re^{0.4}] \\ + \frac{dR_a^*}{dz^*} [-2R_a^*U_z^*(z)^* Re^{0.4}] = 0 \end{aligned} \quad (282)$$

For comparison, non-dimensional θ -momentum and z -momentum equations in the previous study are given below [13].

$$\frac{d\delta^*}{dz^*} [U_z^*(z^*)\delta^*(A_2 - A_1)] + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2}(A_2 - A_1)] = B_2 \quad (283)$$

$$\frac{d\delta^*}{dz^*} [U_z^*(z^*)\delta^*(A_3 - A_1) - \delta^{*2}] + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2}(2A_3 - A_1 - 1)] = B_1 \quad (284)$$

Instead of 4th Order Runge-Kutta method, which was used in previous studies, a more stable method is used [12,13]. Utilized method is a predictor-correcter method known as Adams Bashfort-Moulton. This method is combination of two methods namely, Adams-Bashfort and Adams-Moulton. Algorithm for this method is given below.

Predicted $w_{i+1} =$

$$w_i + \frac{h}{24} [55f(t_i, w_i) - 59(t_{i-1}, w_{i-1}) + 37(t_{i-2}, w_{i-2}) - 9(t_{i-3}, w_{i-3})] \quad (285)$$

Corrected $w_{i+1} =$

$$w_i + \frac{h}{24} [9f(t_{i+1}, \text{Predicted } w_{i+1}) + 19(t_i, w_i) - 5(t_{i-1}, w_{i-1}) + (t_{i-2}, w_{i-2})] \quad (286)$$

It should be noted that this method requires 4 initial values to perform the calculation. Therefore, 4th Order Runge-Kutta method is used to obtain the values at the first 4 steps, and later, Adams Bashfort-Moulton method is employed.

Initial values are selected such that:

$$\Omega|_{z=0} = V_{in}R_{in} \quad (287)$$

$$\delta|_{z=0} = 0.01R \quad (288)$$

$$R_a|_{z=0} = 0.2R \quad (289)$$

$$U_z|_{z=0} = \frac{\dot{m}}{\rho\pi (R^2 - R_a|_{z=0}^2)} \quad (290)$$

$$g_z = 9.81 \quad (291)$$

$$Re|_{z=0} = \frac{\rho\Omega|_{z=0}}{\mu} \quad (292)$$

Values of non-dimensional parameters (Eq. 274 - Eq. 279) are calculated from these initial values. These equations are first applied to an open-end geometry which was also calculated with 2D numerical method. Selected case was Case-16 in Table 2. Obtained values of boundary layer thickness are given in Figure 87.

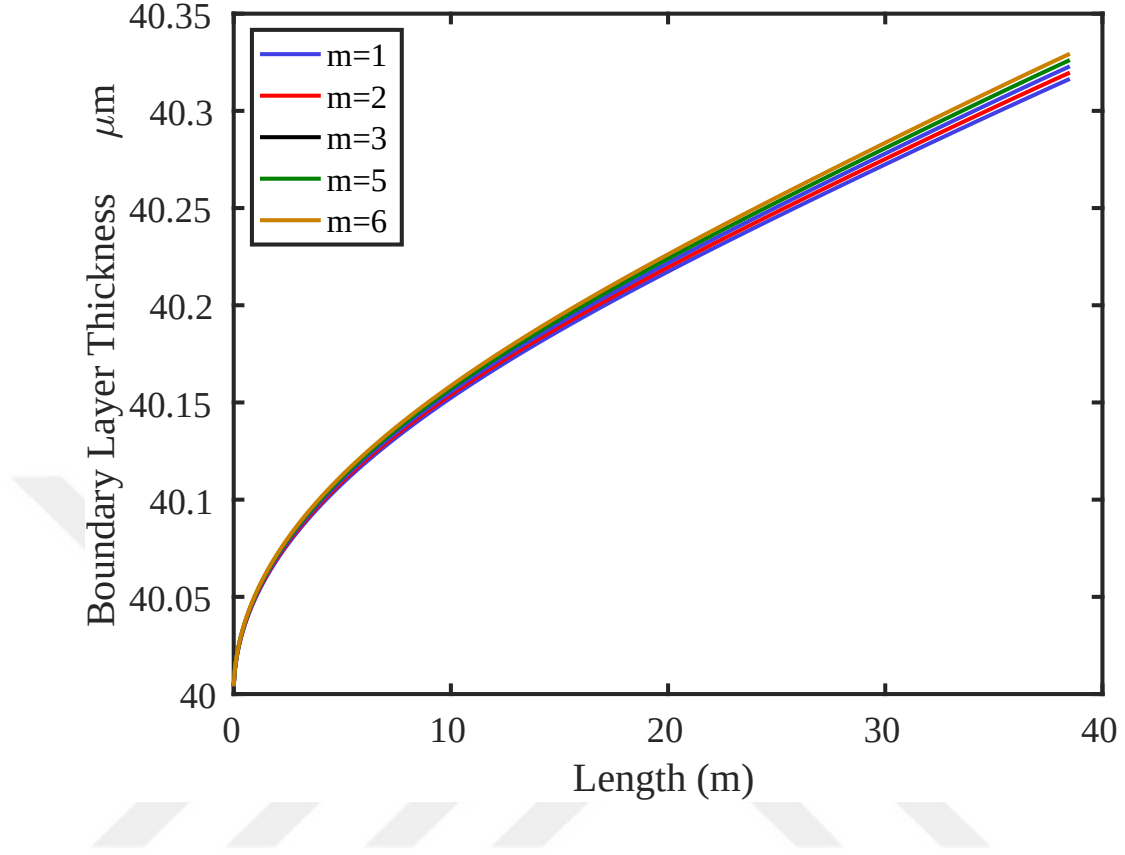


Figure 87: Comparison of boundary layer thickness values calculated with different velocity profiles

Inspection of the results has shown that calculated values of boundary layer thickness are still too small to be physically sensible. Initial value of the boundary layer thickness was $40 \mu\text{m}$ and it increases less than $0.35 \mu\text{m}$. Moreover, it can be seen that using different velocity profiles has only a small effect on the calculated thickness values.

Analysis of the obtained θ -momentum and z -momentum equations and the assumptions made during deriving them have indicated two assumptions that might have errors. One of them is the modeling of the pressure term in the z -momentum equation (Eq. 239 - Eq. 243). Literature review has shown that modeling of this term is done in the same way [12,79]. However, it should be noted that one of these studies is conducted for swirling pipe flow where the air core does not exist, while the

rest of the studies are performed for PSA where an air core exists. Second assumption that might lead to errors is neglecting the variation of the pressure inside the boundary layer. It should be mentioned that it was suggested that this could lead to under-predicting the boundary layer thickness [79]. Therefore, two new models are obtained such that, first one acknowledges the presence of air core. Second model is obtained such that both the presence of air core and the variation of pressure inside the boundary layer is acknowledged.

For the model acknowledges the presence of air core, a new expression is obtained for the term Z_3 (Eq. 239) with the assumption of inviscid flow inside the potential core. Neglecting radial velocity component and using steady and axisymmetric assumptions, r-momentum equation for the potential core can be written as:

$$-\frac{U_\theta^2}{r} = \frac{1}{\rho} \frac{\partial P}{\partial r} \quad (293)$$

Since free vortex flow is assumed in the potential core, Eq. 293 becomes:

$$U_\theta = \frac{\Omega}{r} \Rightarrow \frac{\partial P}{\partial r} = \rho \frac{\Omega^2}{r^3} \quad (294)$$

Integrating Eq. 294 through the potential core:

$$\int_{R_a}^{R-\delta} \frac{\partial P}{\partial r} dr = \int_{R_a}^{R-\delta} \rho \frac{\Omega^2}{r^3} dr \quad (295)$$

It should be noted that integration is done with the limits of R_a and $R-\delta$ which is the limit of radius of the potential core. Evaluating Eq. 295 result in:

$$P|_{R-\delta}^{R_a} = \frac{1}{2} \rho \Omega^2 \frac{1}{r^2} \Big|_{R-\delta}^{R_a} \quad (296)$$

$$P_{R-\delta} - P_{R_a} = \frac{1}{2} \rho \Omega^2 \left[\frac{1}{R_a^2} - \frac{1}{(R-\delta)^2} \right] \quad (297)$$

Noting that at $r=R_a$, pressure will be equal to the ambient pressure, hence using $P_{R_a} = 0$:

$$P_{R-\delta} = \frac{1}{2} \rho \Omega^2 \left[\frac{1}{R_a^2} - \frac{1}{(R-\delta)^2} \right] \quad (298)$$

where, term $P_{R-\delta}$ is the pressure at the edge of the boundary layer. Since the variation of the pressure inside the boundary layer is neglected, it is also the pressure inside the boundary layer.

Taking derivative of Eq. 298 with respect to axial coordinate:

$$\frac{dP_{R-\delta}}{dz} = \frac{1}{2}\rho\Omega^2 \left[-2\frac{dR_a}{dz} \frac{1}{R_a^3} - 2\frac{d\delta}{dz} \frac{1}{(R-\delta)^3} \right] \quad (299)$$

Using Eq. 299 in Eq. 239:

$$Z_3 = -\frac{1}{\rho} \int_{R-\delta}^R \frac{dP}{dz} dr = -\frac{1}{\rho} \int_{R-\delta}^R \frac{1}{2}\rho\Omega^2 \left[-2\frac{dR_a}{dz} \frac{1}{R_a^3} - 2\frac{d\delta}{dz} \frac{1}{(R-\delta)^3} \right] \quad (300)$$

Integrating Eq. 300:

$$Z_3 = \Omega^2\delta \left[\frac{dR_a}{dz} \frac{1}{R_a^3} + \frac{d\delta}{dz} \frac{1}{(R-\delta)^3} \right] \quad (301)$$

Using Eq. 301 in z-momentum equation instead of Eq. 243 result in:

$$\begin{aligned} & -U_z(z)\Psi_1(1) \frac{d(U_z(z)\delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta + U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & + \frac{dU_z^2(z)\delta}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & = \Omega^2\delta \left[\frac{dR_a}{dz} \frac{1}{R_a^3} + \frac{d\delta}{dz} \frac{1}{(R-\delta)^3} \right] + \delta g_z + \frac{\mu}{\rho} \frac{U_z(z)}{\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \end{aligned} \quad (302)$$

Arranging Eq. 302:

$$\begin{aligned} & \frac{d\delta}{dz} \left[U_z(z)\delta \left(A_3 - A_1 - \frac{\Omega^2}{U_z^2(z)} \frac{\delta}{(R-\delta)^3} \right) \right] \\ & + \frac{dU_z(z)}{dz} [\delta^2(2A_3 - A_1)] + \frac{dR_a(z)}{dz} \left[-\frac{\Omega^2}{U_z(z)} \frac{\delta^2}{R_a^3} \right] \\ & = \frac{\delta^2 g_z}{U_z(z)} + \frac{\mu}{\rho} B_1 \end{aligned} \quad (303)$$

Second model is obtained by considering the variation of pressure inside the boundary layer as well as the presence of the air core as noted earlier. If the radial component of velocity is neglected, with the assumptions led to Eq. 172 - 174, r-momentum equation can be written as:

$$\frac{\partial P}{\partial r} = \rho \frac{u_\theta^2}{r} \quad (304)$$

If we approximate left hand side Eq. 304 as:

$$\frac{dP}{dr} = \frac{P_{R-\delta} - P}{R - \delta - r} \quad (305)$$

where $P_{R-\delta}$ is the pressure at the edge of the boundary layer, which corresponds to $r = R - \delta$. Also, P is the pressure inside the boundary layer at a location r such that $R - \delta < r < R$.

Right hand side of Eq. 305 can be evaluated at these points as (with Eq. 179):

$$\frac{P_{R-\delta} - P}{R - \delta - r} = \rho \frac{\Omega^2}{(R - \delta)^3} \Psi_2^2(1) - \rho \frac{\Omega^2}{(R - \delta)^3} \Psi_2^2(\eta) \quad (306)$$

Since $\Psi_2(1) = 1$:

$$P_{R-\delta} - P = \rho \frac{\Omega^2}{(R - \delta)^3} (1 - \Psi_2^2(\eta)) (R - \delta - r) \quad (307)$$

Arranging Eq. 307:

$$P = P_{R-\delta} - \rho \frac{\Omega^2}{(R - \delta)^3} (1 - \Psi_2^2(\eta)) (R - \delta - r) \quad (308)$$

If Eq. 308 is evaluated at two points, namely at the boundary layer edge and at the wall:

- At the boundary layer edge ($r = R - \delta$, $\eta = 1$, $\Psi_2^2(\eta) = 1$):

$$P = P_{R-\delta} - \rho \frac{\Omega^2}{(R - \delta)^3} (1 - 1) (R - \delta - (R - \delta)) \Rightarrow P = P_{R-\delta} \quad (309)$$

- At wall ($r = R$, $\eta = 0$, $\Psi_2^2(\eta) = 0$):

$$P = P_{R-\delta} - \rho \frac{\Omega^2}{(R - \delta)^3} (1 - 0) (R - \delta - R) \Rightarrow P = P_{R-\delta} + \rho \frac{\Omega^2}{(R - \delta)^3} \delta \quad (310)$$

It can be seen that Eq. 308 can be used to represent pressure distribution inside the boundary layer if $P_{R-\delta}$ is known. Here, Eq. 298 is used to calculate the pressure value at the edge of the boundary layer. Eq. 308 becomes:

$$P = \frac{1}{2} \rho \Omega^2 \left[\frac{1}{R_a^2} - \frac{1}{(R - \delta)^2} \right] - \rho \frac{\Omega^2}{(R - \delta)^3} (1 - \Psi_2^2(\eta)) (R - \delta - r) \quad (311)$$

Term Z_3 (Eq. 239) the rewritten as:

$$Z_3 = -\frac{1}{\rho} \int_{R-\delta}^R \frac{dP}{dz} dr = -\frac{1}{\rho} \int_{R-\delta}^R \frac{d}{dz} \left[\frac{1}{2} \rho \Omega^2 \left[\frac{1}{R_a^2} - \frac{1}{(R-\delta)^2} \right] - \rho \frac{\Omega^2}{(R-\delta)^3} (1 - \Psi_2^2(\eta)) (R - \delta - r) \right] dr \quad (312)$$

$$Z_3 = -\frac{1}{\rho} \int_{R-\delta}^R \frac{1}{2} \rho \Omega^2 \left[-\frac{2}{R_a^3} \frac{dR_a}{dz} - \frac{2}{(R-\delta)^3} \frac{d\delta}{dz} - 3 \frac{\Omega^2}{(R-\delta)^4} \frac{d\delta}{dz} (1 - \Psi_2^2(\eta)) (R - \delta - r) - \frac{\Omega^2}{(R-\delta)^3} \left(-2\Psi_2(\eta) \frac{d\Psi_2(\eta)}{dz} (R - \delta - r) - (1 - \Psi_2^2(\eta)) \frac{d\delta}{dz} \right) \right] dr \quad (313)$$

Arranging Eq. 313:

$$Z_3 = \int_{R-\delta}^R \Omega^2 \left[\frac{1}{R_a^3} \frac{dR_a}{dz} + \frac{1}{(R-\delta)^3} \frac{d\delta}{dz} + \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} (1 - \Psi_2^2(\eta)) (R - \delta - r) + \frac{1}{(R-\delta)^3} \left(-2\Psi_2(\eta) \frac{d\Psi_2(\eta)}{dz} (R - \delta - r) - (1 - \Psi_2^2(\eta)) \frac{d\delta}{dz} \right) \right] dr \quad (314)$$

$$Z_3 = \Omega^2 \left[\int_{R-\delta}^R \frac{1}{R_a^3} \frac{dR_a}{dz} dr + \int_{R-\delta}^R \frac{1}{(R-\delta)^3} \frac{d\delta}{dz} dr + \int_{R-\delta}^R \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} (1 - \Psi_2^2(\eta)) (R - \delta - r) dr - \frac{1}{(R-\delta)^3} \int_{R-\delta}^R \left(2\Psi_2(\eta) \frac{d\Psi_2(\eta)}{dz} (R - \delta - r) \right) dr - \frac{1}{(R-\delta)^3} \int_{R-\delta}^R \left(1 - \Psi_2^2(\eta) \frac{d\delta}{dz} \right) dr \right] dr \quad (315)$$

Representing Eq. 315 as:

$$Z_3 = \Omega^2 \left[\int_{R-\delta}^R E_1 dr + \int_{R-\delta}^R E_2 dr + \int_{R-\delta}^R E_3 dr + \int_{R-\delta}^R E_4 dr + \int_{R-\delta}^R E_5 dr \right] \quad (316)$$

Evaluating the first term:

$$\int_{R-\delta}^R E_1 dr = \int_{R-\delta}^R \frac{1}{R_a^3} \frac{dR_a}{dz} dr = \frac{dR_a}{dz} \frac{\delta}{R_a^3} \quad (317)$$

Evaluating the second term:

$$\int_{R-\delta}^R E_2 dr = \int_{R-\delta}^R \frac{1}{(R-\delta)^3} \frac{d\delta}{dz} dr = \frac{d\delta}{dz} \frac{\delta}{(R-\delta)^3} \quad (318)$$

Evaluating the third term:

$$\int_{R-\delta}^R E_3 dr = \int_{R-\delta}^R \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} (1 - \Psi_2^2(\eta)) (R - \delta - r) dr \quad (319)$$

$$\int_{R-\delta}^R E_3 dr = \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} \left[\int_{R-\delta}^R (R - \delta - r) dr - \int_{R-\delta}^R \Psi_2^2(\eta) (R - \delta - r) dr \right] \quad (320)$$

First integral on the right hand side can be evaluated as:

$$\int_{R-\delta}^R (R - \delta - r) dr = Rr - \delta r - \frac{r^2}{2} \Big|_{R-\delta}^R = \frac{1}{2} [(R - \delta)^2 - R^2] + \delta (R - \delta) \quad (321)$$

Second integral on the right hand side can be evaluated as:

$$- \int_{R-\delta}^R \Psi_2^2(\eta) (R - \delta - r) dr = - \int_{R-\delta}^R \Psi_2^2(\eta) (R - \delta) dr + \int_{R-\delta}^R \Psi_2^2(\eta) r dr \quad (322)$$

Replacing r with η in Eq. 322:

$$\begin{aligned} - \int_{R-\delta}^R \Psi_2^2(\eta) (R - \delta - r) dr = \\ - \int_0^1 \Psi_2^2(\eta) (R - \delta) \delta d\eta + \int_0^1 \Psi_2^2(\eta) (R - \eta\delta) \delta d\eta \end{aligned} \quad (323)$$

Arranging Eq. 323:

$$\begin{aligned} - \int_{R-\delta}^R \Psi_2^2(\eta) (R - \delta - r) dr = \\ - (R - \delta) \delta \int_0^1 \Psi_2^2(\eta) d\eta + R\delta \int_0^1 \Psi_2^2(\eta) d\eta - \delta^2 \int_0^1 \Psi_2^2(\eta) \eta d\eta \end{aligned} \quad (324)$$

Final form of Eq. 319:

$$\begin{aligned} \int_{R-\delta}^R E_3 dr = \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} \left[\left(\frac{1}{2} ((R - \delta)^2 - R^2) + \delta (R - \delta) \right) \right. \\ \left. + \delta^2 \int_0^1 \Psi_2^2(\eta) d\eta - \delta^2 \int_0^1 \Psi_2^2(\eta) \eta d\eta \right] \end{aligned} \quad (325)$$

Fourth term of Eq. 316 is evaluated as:

$$\int_{R-\delta}^R E_4 dr = -\frac{1}{(R-\delta)^3} \int_{R-\delta}^R 2\Psi_2(\eta) \frac{d\Psi_2(\eta)}{dz} (R-\delta-r) dr \quad (326)$$

Replacing r with η in Eq. 326:

$$\int_{R-\delta}^R E_4 dr = \frac{1}{(R-\delta)^3} \int_0^1 2\Psi_2(\eta) \frac{d\Psi_2(\eta)}{d\eta} \frac{d\delta}{dz} (\eta^2 - \eta) \delta d\eta \quad (327)$$

$$\int_{R-\delta}^R E_4 dr = \frac{2\delta}{(R-\delta)^3} \frac{d\delta}{dz} \left[\int_0^1 \Psi_2(\eta) \frac{d\Psi_2(\eta)}{d\eta} \eta^2 d\eta - \int_0^1 \Psi_2(\eta) \frac{d\Psi_2(\eta)}{d\eta} \eta d\eta \right] \quad (328)$$

Noting that,

$$\int_0^1 \Psi_2(\eta) \frac{d\Psi_2(\eta)}{d\eta} \eta^2 d\eta = \int_0^1 \frac{1}{2} \frac{d(\eta^2 \Psi_2^2(\eta))}{d\eta} d\eta - \int_0^1 \frac{d\Psi_2^2(\eta)}{d\eta} \eta d\eta \quad (329)$$

$$\int_0^1 \Psi_2(\eta) \frac{d\Psi_2(\eta)}{d\eta} \eta d\eta = \frac{1}{2} \left[\int_0^1 \frac{d(\eta \Psi_2^2(\eta))}{d\eta} d\eta - \int_0^1 \frac{d\Psi_2^2(\eta)}{d\eta} d\eta \right] \quad (330)$$

Eq. 326 becomes:

$$\int_{R-\delta}^R E_4 dr = \frac{2\delta}{(R-\delta)^3} \frac{d\delta}{dz} \left[\frac{1}{2} \int_0^1 \Psi_2^2(\eta) d\eta - \int_0^1 \eta \Psi_2^2(\eta) d\eta \right] \quad (331)$$

Finally:

$$\int_{R-\delta}^R E_5 dr = -\frac{1}{(R-\delta)^3} \frac{d\delta}{dz} \int_{R-\delta}^R (1 - \Psi_2^2(\eta)) dr \quad (332)$$

Replacing r with η :

$$\int_{R-\delta}^R E_5 dr = -\frac{1}{(R-\delta)^3} \frac{d\delta}{dz} \delta \left[1 - \int_0^1 \Psi_2^2(\eta) d\eta \right] \quad (333)$$

Term Z_3 becomes:

$$\begin{aligned} Z_3 = \Omega^2 & \left[\frac{dR_a}{dz} \frac{\delta}{R_a^3} + \frac{d\delta}{dz} \frac{\delta}{(R-\delta)^3} \right. \\ & + \frac{3}{(R-\delta)^4} \frac{d\delta}{dz} \left[\left(\frac{1}{2} ((R-\delta)^2 - R^2) + \delta(R-\delta) \right) + \delta^2 \int_{R-\delta}^R \Psi_2^2(\eta) d\eta \right. \\ & - \delta^2 \int_{R-\delta}^R \Psi_2^2(\eta) \eta d\eta \left. \right] + \frac{2\delta}{(R-\delta)^3} \frac{d\delta}{dz} \left[\frac{1}{2} \int_0^1 \Psi_2^2(\eta) d\eta - \int_0^1 \eta \Psi_2^2(\eta) d\eta \right] \\ & \left. - \frac{1}{(R-\delta)^3} \frac{d\delta}{dz} \delta \left[1 - \int_0^1 \Psi_2^2(\eta) d\eta \right] \right] \quad (334) \end{aligned}$$

using,

$$A_4 = \int_0^1 \Psi_2^2(\eta) d\eta \quad (335)$$

$$A_5 = \int_0^1 \eta \Psi_2^2(\eta) \Psi_2(\eta) d\eta \quad (336)$$

$$Z_3 = \Omega^2 \left[\frac{dR_a}{dz} \frac{\delta}{R_a^3} + \frac{d\delta}{dz} \left(\frac{\delta}{(R-\delta)^3} (2A_4 - 2A_5 + 3) + \frac{3}{(R-\delta)^4} \left(\frac{1}{2} ((R-\delta) - R^2) + \delta^2 (A_4 - A_5) \right) \right) \right] \quad (337)$$

Using Eq. 337 in z-momentum equation result in:

$$\begin{aligned} & -U_z(z) \Psi_1(1) \frac{d(U_z(z) \delta)}{dz} \int_0^1 \Psi_1(\eta) d\eta + U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & + \frac{dU_z^2(z) \delta}{dz} \int_0^1 \Psi_1^2(\eta) d\eta - U_z^2(z) \frac{d\delta}{dz} \Psi_1^2(1) \\ & = \Omega^2 \left[\frac{dR_a}{dz} \frac{\delta}{R_a^3} + \frac{d\delta}{dz} \left(\frac{\delta}{(R-\delta)^3} (2A_4 - 2A_5 + 3) + \frac{3}{(R-\delta)^4} \left(\frac{1}{2} ((R-\delta) - R^2) + \delta^2 (A_4 - A_5) \right) \right) \right] \\ & + \delta g_z + \frac{\mu}{\rho} \frac{U_z(z)}{\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \end{aligned} \quad (338)$$

Arranging Eq. 338:

$$\begin{aligned} & \frac{d\delta}{dz} \left[U_z(z) \delta (A_3 - A_1) - \frac{\Omega^2 \delta}{U_z(z)} \left(\frac{\delta}{(R-\delta)^3} (2A_4 - 2A_5 + 3) + \frac{3}{(R-\delta)^4} \left(\frac{1}{2} ((R-\delta) - R^2) + \delta^2 (A_4 - A_5) \right) \right) \right] + \frac{dU_z(z)}{dz} [\delta^2 (2A_3 - A_1)] \\ & + \frac{dR_a(z)}{dz} \left[-\frac{\Omega^2}{U_z(z)} \frac{\delta^2}{R_a^3} \right] = \frac{\delta^2 g_z}{U_z(z)} + \frac{\mu}{\rho} B_1 \end{aligned} \quad (339)$$

Non-dimensional forms of these models are given below. It should be noted that since only z-momentum equation changes, only non-dimensional z-momentum equations are given. Eq. 280 and Eq. 281 is used for θ -momentum and flow rate, respectively.

The model obtained with neglecting variation of pressure inside the boundary layer:

$$\begin{aligned} \frac{d\delta^*}{dz^*} \left[U_z^*(z^*) \delta^* (A_3 - A_1) - \frac{\delta^{*2}}{1 - \delta^{*3}} \right] \\ + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2} (2A_3 - A_1)] \\ + \frac{dR_a^*}{dz^*} \left[-\frac{1}{U_z^*} \frac{\delta^{*2}}{R_a^{*3}} \right] = \frac{\delta^{*2} g_z^*}{U_z^*(z^*)} + B_1 \quad (340) \end{aligned}$$

The model obtained with considering the variation of pressure in the boundary layer:

$$\begin{aligned} \frac{d\delta^*}{dz^*} \left[U_z^*(z^*) \delta^* (A_3 - A_1) \right. \\ \left. - \frac{\delta^*}{U_z^*} \left(\frac{\delta^{*2}}{1 - \delta^{*3}} (2A_4 - 2A_5 + 3) + \frac{3}{(1 - \delta^*)^4 \left(\frac{1}{2} ((1 - \delta^*)^2 - 1) + \delta^* (A_4 - A_5) \right)} \right) \right] \\ + \frac{dU_z^*(z^*)}{dz^*} [\delta^{*2} (2A_3 - A_1)] \\ + \frac{dR_a^*}{dz^*} \left[-\frac{1}{U_z^*} \frac{\delta^{*2}}{R_a^{*3}} \right] = \frac{\delta^{*2} g_z^*}{U_z^*(z^*)} + B_1 \quad (341) \end{aligned}$$

Comparison of these models with the model obtained earlier is done using Case-16 in Table 2. It should be mentioned that, since in the earlier calculations, it was seen that the velocity profile has a small effect on the calculations, velocity profile corresponding to $m = 3$ is used.

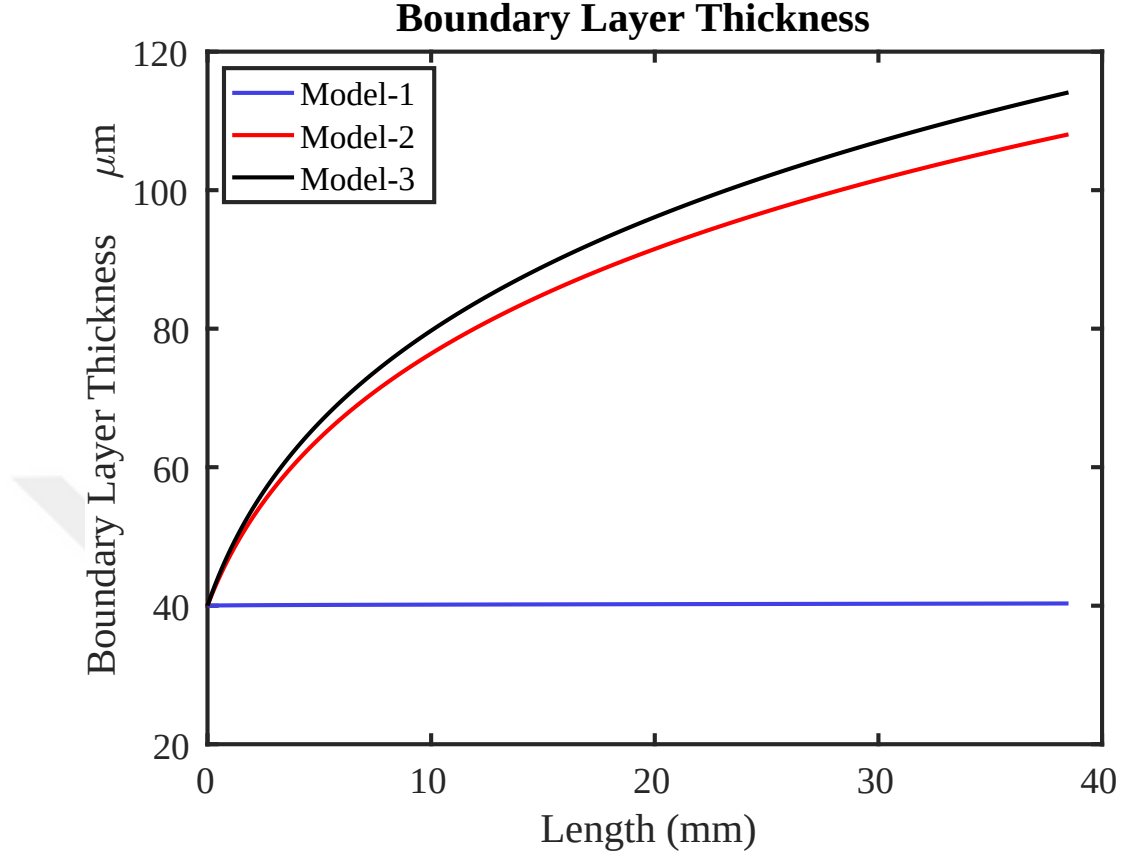


Figure 88: Comparison of calculated boundary layer thickness from obtained models

It can be seen that both the second model (including the effect of air core) and the third model (including the variation of pressure through the boundary layer) produced higher boundary layer values. It should be noted that initial value of the boundary layer thickness was $40 \mu\text{m}$. For model-1 this value remains almost constant. For the other two models, increase in the boundary layer is apparent. Moreover, including the variation of pressure inside the boundary layer has led to higher values as suggested in the literature [79]. However, since it has a simpler form, second model is selected to be used in further calculations. Moreover, it is seen that the thickness of the boundary layer is almost an order of magnitude smaller than the thickness of the hence, mass through the boundary layer is small to be neglected. With this assumption, a new

flow rate equation is derived, Eq. 342, which is used in further calculations.

$$\frac{dU_z^*(z^*)}{dz} [1 - R_a^{*2} Re^{0.4}] + \frac{dR_a^*}{dz^*} [-2R_a^* U_z^*(z)^* Re^{0.4}] = 0 \quad (342)$$

In order to make further evaluation, comparison of values of axial and tangential velocity and film thickness obtained from 2D simulations and the second model is done. Figure 89 shows the comparison for the obtained radius of air core values. It can be seen that the boundary layer model produces similar values to the 2D simulation.

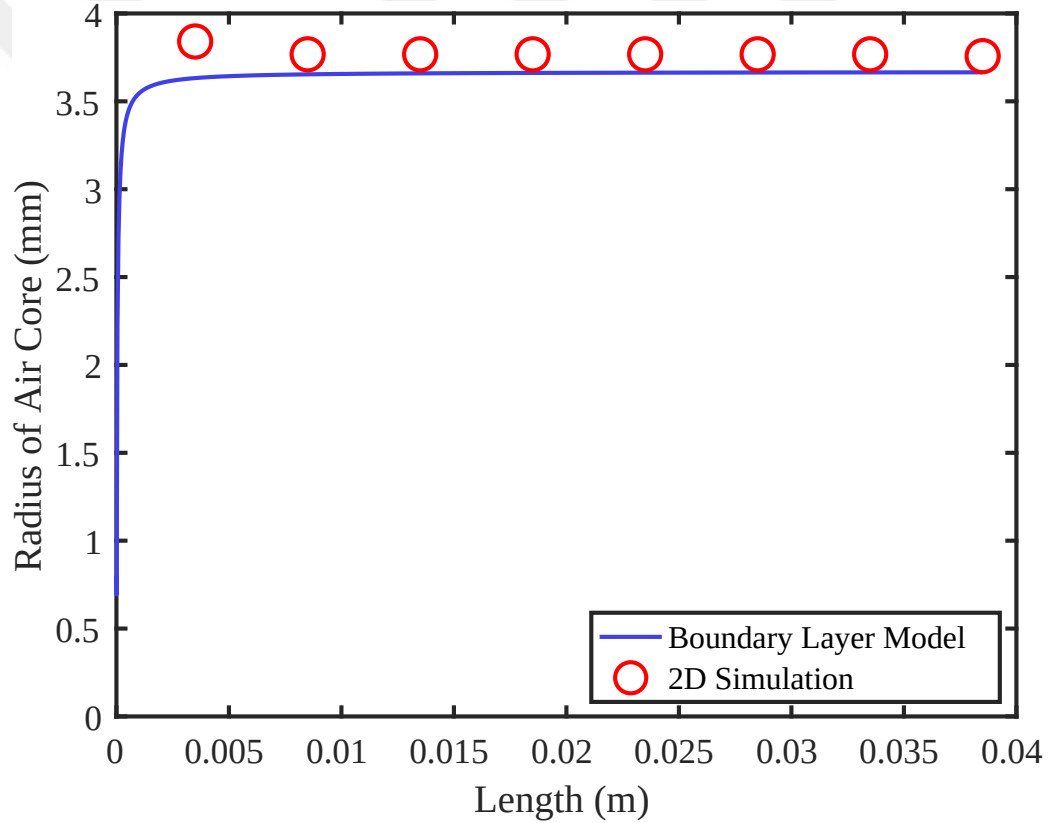


Figure 89: Comparison of calculated air core radius from 2D simulation and boundary layer model

Comparison of calculated velocity components is given in Figure 90. Similar to the air core calculation, obtained velocity components are close to that of 2D simulation. In the development of the boundary layer model, it is often mentioned that the radial

component of the velocity is neglected. Inspection of the results of 2D simulation has shown that even though the radial velocity component is not equal to zero, it is an order of magnitude smaller than other two components. Moreover, it can be seen that obtained tangential velocity values are close to each other while axial component is under-predicted. It should be noted that this is caused by the under-predicting the radius of the air core and the difference between the obtained values is 0.1 mm.

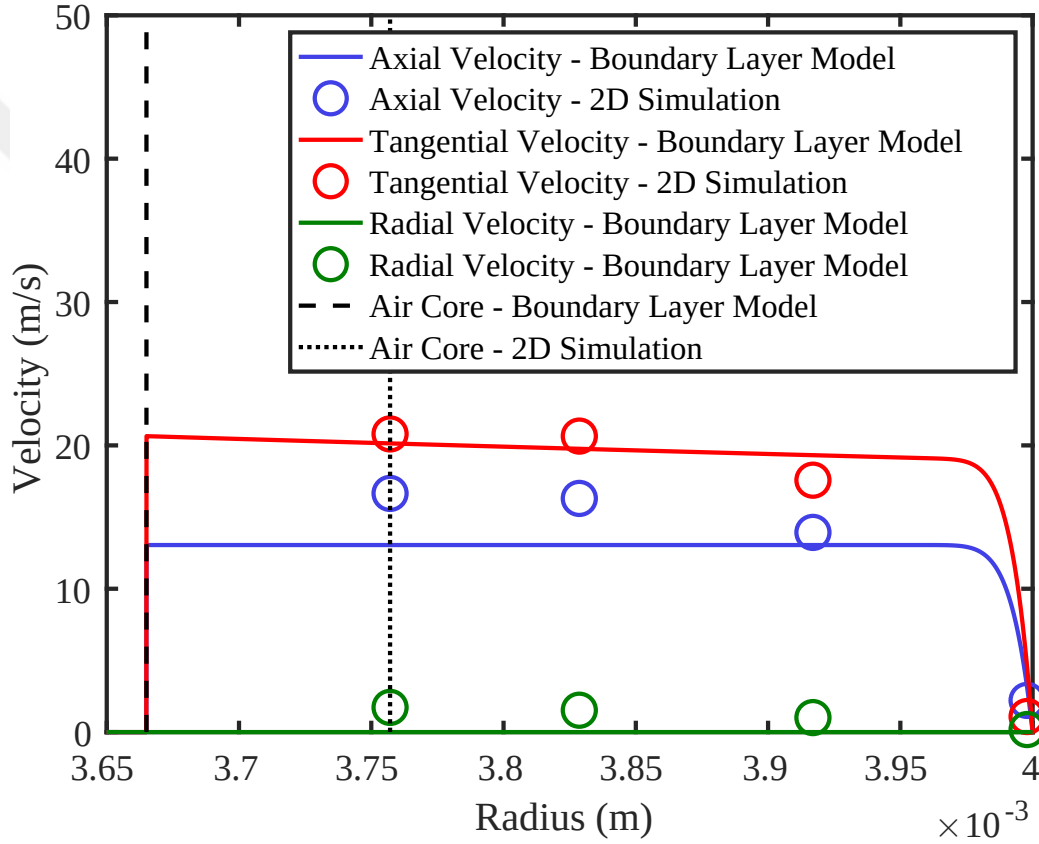


Figure 90: Comparison of calculated velocity components from 2D simulation and boundary layer model

In order to improve the accuracy of obtained air core values, the initial value selected for the radius of air core is altered. Since 2D simulation of this geometry is performed, the air core value at the top wall is known and can be used for the calculations. Hence, initial condition for the air core becomes $R_a|_{z=0} = 0.5R$. Obtained

air core value and the velocity components at the exit plane are given in Figure 91.

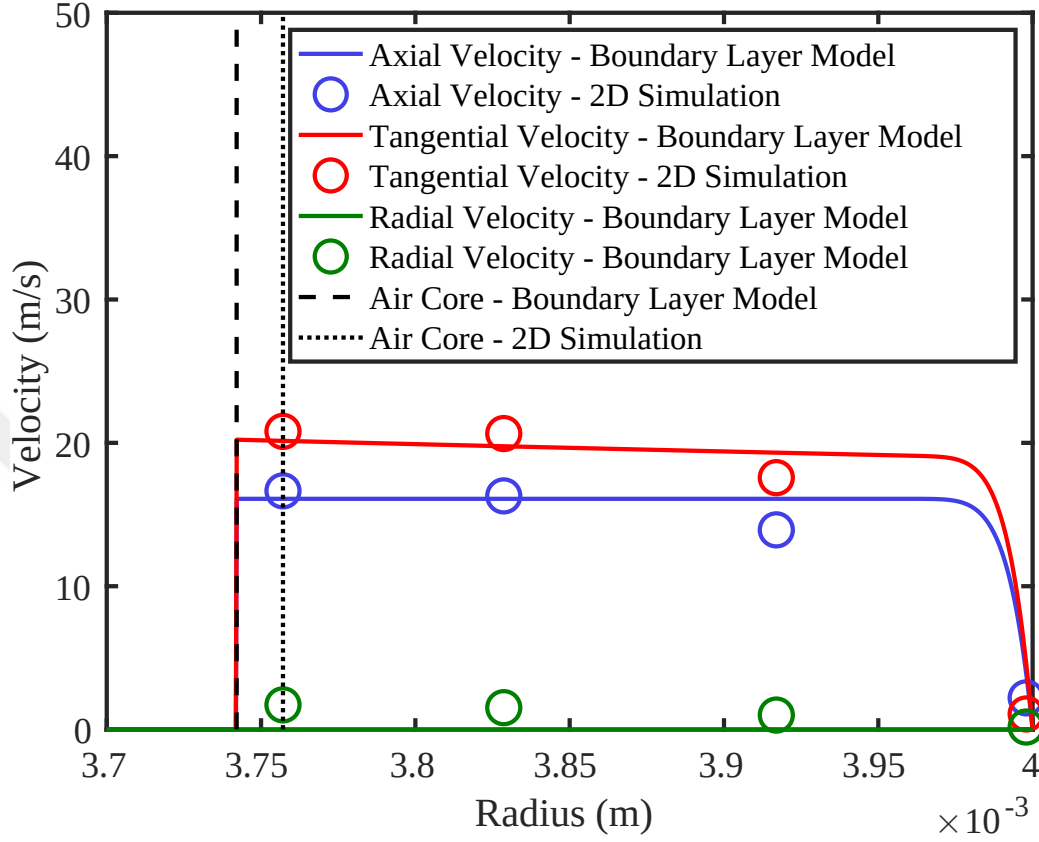


Figure 91: Comparison of calculated velocity components from 2D simulation and boundary layer model with initial air core value increased to $0.5R$

It can be seen that for the simulated case, both radius of air core and velocity components are more accurately predicted with the increased initial air core value. However, when both initial values are used for the calculation of the air core values of open-end atomizers presented in Chapter 4, it is seen that when $R_a|_{z=0} = 0.2R$ is used as the initial condition calculated air core values has AMRE values of 0.030 and underpredicts the radius of air core for all cases. When $R_a|_{z=0} = 0.5R$ is used AMRE value increases to 0.035 and over-predicts the radius of air core. Since it was more accurate, even though difference is small, initial value of $R_a|_{z=0} = 0.2R$ is used in the further calculations.

Finally, obtained model is compared with the open-end PSA experimental data presented in Chapter 4. Comparisons are done with calculated pressure difference, film thickness, and spray angle values. Pressure difference is calculated from Eq. 343 and spray angle is calculated from Eq. 344, where $\bar{U}_\theta$ and $\bar{U}_z$ is the mean tangential and mean axial velocity at the exit, respectively. An important point that has to be made is since the axial velocity component is obtained from the derived boundary layer viscous effects are considered in the calculation of this parameter. Moreover, in the calculation of the mean velocities at the exit, effect of the boundary layer is included as well. Hence, viscous effects are considered in the calculation of the pressure difference and spray angle.

$$\Delta P_L = \frac{1}{2}\rho \left(\bar{U}_z^2 + \bar{U}_\theta^2 \right) \quad (343)$$

$$\theta = \tan^{-1} \left(\frac{\bar{U}_\theta}{\bar{U}_z} \right) \quad (344)$$

Obtained AMRE (Eq. 29) and SE (Eq. 30) values of the boundary layer model are given in Table 24. For comparison, AMRE and SE values of the derived semi-empirical correlation are also presented in the same table. It can be seen that, with the boundary layer model, more accurate calculations for pressure difference, hence discharge coefficient, and film thickness can be performed in comparison to using semi-empirical correlation. However, it should be mentioned again that, there are only 5 experimental data for the film thickness. Moreover, a slight increase in AMRE is seen in the calculation of spray angle however, this increase is only 2.3 %.

Table 24: Error Values of Boundary Layer Model and Obtained Semi-Empirical Model - Open-End

Parameter	Boundary Layer Model		Obtained Semi-Empirical Correlation	
	AMRE	SE	AMRE	SE
Pressure Difference	0.102	0.001	0.140	0.001
Discharge Coefficient	0.056	3.1E-4	0.072	4.9E-4
Film Thickness	0.030	2.2E-4	0.040	3.7E-4
Spray Angle	0.097	0.001	0.074	7.4E-4

5.2.2 Boundary Layer Model for Convergent and Divergent Section

Here, obtained boundary layer model for the convergent and divergent sections of the PSA will be presented. To obtain the model Navier-Stokes equation in spherical coordinates is used. These equations, for incompressible flow, are given in Eqs. 345 - 348.

- Continuity:

$$\frac{\partial U_R}{\partial R} + 2\frac{U_R}{R} + \frac{1}{R \sin \theta} \frac{\partial U_\phi}{\partial \phi} + \frac{\partial U_\theta}{R} \cot \theta + \frac{1}{R} \frac{\partial U_\theta}{\partial \theta} = 0 \quad (345)$$

- R-Momentum:

$$\begin{aligned} \rho \left[\frac{\partial U_R}{\partial t} + U_R \frac{\partial U_R}{\partial R} + \frac{U_\phi}{R \sin \theta} \frac{\partial U_R}{\partial \phi} + \frac{U_\theta}{R} \frac{\partial U_R}{\partial \theta} - \frac{U_\phi^2 + U_\theta^2}{R} \right] = \\ - \frac{\partial P}{\partial R} + \rho g_R + \\ \mu \times \left[\frac{2}{R} \frac{\partial U_R}{\partial R} + \frac{\partial^2 U_R}{\partial R^2} + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 U_R}{\partial \phi^2} + \frac{\cot \theta}{R^2} \frac{\partial U_R}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_R}{\partial \theta^2} - \right. \\ \left. 2 \frac{U_R + \frac{\partial U_\theta}{\partial \theta} + U_\theta \cot \theta}{R^2} - \frac{2}{R^2 \sin \theta} \frac{\partial U_\phi}{\partial \phi} \right] \quad (346) \end{aligned}$$

- ϕ -Momentum:

$$\begin{aligned} \rho \left[\frac{\partial U_\phi}{\partial t} + U_R \frac{\partial U_\phi}{\partial R} + \frac{U_\phi}{R \sin \theta} \frac{\partial U_\phi}{\partial \phi} + \frac{U_\theta}{R} \frac{\partial U_\phi}{\partial \theta} + \frac{U_R U_\phi + U_\phi U_\theta \cot \theta}{R} \right] = \\ - \frac{1}{R \sin \theta} \frac{\partial P}{\partial \phi} + \rho g_\phi + \\ \mu \times \left[\frac{2}{R} \frac{\partial U_\phi}{\partial R} + \frac{\partial^2 U_\phi}{\partial R^2} + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 U_\phi}{\partial \phi^2} + \frac{\cot \theta}{R} \frac{\partial U_\phi}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_\phi}{\partial \theta^2} + \right. \\ \left. \frac{2 \sin \theta \frac{\partial U_R}{\partial \phi} + 2 \cos \theta \frac{\partial U_\theta}{\partial \phi} - U_\phi}{R^2 \sin^2 \theta} \right] \quad (347) \end{aligned}$$

- θ -Momentum:

$$\begin{aligned} \rho \left[\frac{\partial U_\theta}{\partial t} + U_R \frac{\partial U_\theta}{\partial R} + \frac{U_\phi}{R \sin \theta} \frac{\partial U_\theta}{\partial \phi} + \frac{U_\theta}{R} \frac{\partial U_\theta}{\partial \theta} + \frac{U_R U_\theta - U_\phi^2 \cot \theta}{R} \right] = \\ - \frac{1}{R} \frac{\partial P}{\partial \theta} + \rho g_\theta + \\ \mu \times \left[\frac{2}{R} \frac{\partial U_\theta}{\partial R} + \frac{\partial^2 U_\theta}{\partial R^2} + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 U_\theta}{\partial \phi^2} + \frac{\cot \theta}{R^2} \frac{\partial U_\theta}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_\theta}{\partial \theta^2} + \right. \\ \left. \frac{2}{R^2} \frac{\partial U_R}{\partial \theta} - \frac{U_\theta + 2 \cos \theta \frac{\partial U_\phi}{\partial \phi}}{R^2 \sin^2 \theta} \right] \quad (348) \end{aligned}$$

where, U_R , U_ϕ and U_θ are the velocity components in radial, azimuthal angle, and polar angle directions, respectively, and P is the pressure ρ is the density, g is the gravitational acceleration and μ is the absolute viscosity. Moreover, R is the distance of a point from the origin which is taken at the vertex of the cone and θ is the angle a radius vector makes with the axis. Schematic of the coordinate system is given in Figure. 92.

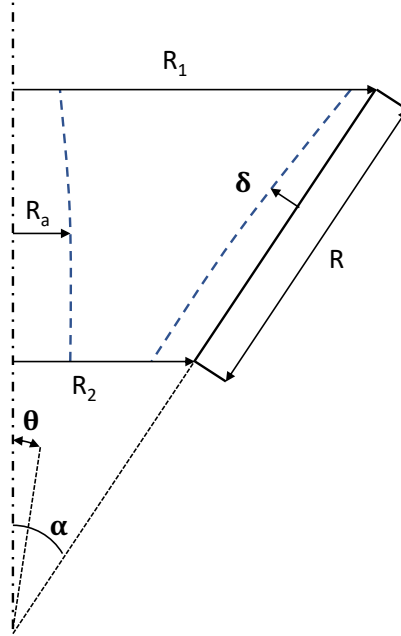


Figure 92: Schematic of spherical coordinate system

For steady and symmetrical flow with respect to $\theta=0$, above equations become:

- Continuity:

$$\frac{\partial U_R}{\partial R} + 2\frac{U_R}{R} + \frac{\partial U_\theta}{R} \cot \theta + \frac{1}{R} \frac{\partial U_\theta}{\partial \theta} = 0 \quad (349)$$

- R-Momentum:

$$U_R \frac{\partial U_R}{\partial R} + \frac{U_\theta}{R} \frac{\partial U_R}{\partial \theta} - \frac{U_\phi^2 + U_\theta^2}{R} = -\frac{1}{\rho} \frac{\partial P}{\partial R} + g_R + \frac{\mu}{\rho} \times \left[\frac{2}{R} \frac{\partial U_R}{\partial R} + \frac{\partial^2 U_R}{\partial R^2} + \frac{\cot \theta}{R^2} \frac{\partial U_R}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_R}{\partial \theta^2} - 2 \frac{U_R + \frac{\partial U_\theta}{\partial \theta} + U_\theta \cot \theta}{R^2} \right] \quad (350)$$

- ϕ -Momentum:

$$U_R \frac{\partial U_\phi}{\partial R} + \frac{U_\theta}{R} \frac{\partial U_\phi}{\partial \theta} + \frac{U_R U_\phi + U_\phi U_\theta \cot \theta}{R} = \frac{\mu}{\rho} \times \left[\frac{2}{R} \frac{\partial U_\phi}{\partial R} + \frac{\partial^2 U_\phi}{\partial R^2} + \frac{\cot \theta}{R} \frac{\partial U_\phi}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_\phi}{\partial \theta^2} - \frac{U_\phi}{R^2 \sin^2 \theta} \right] \quad (351)$$

- θ -Momentum:

$$U_R \frac{\partial U_\theta}{\partial R} + \frac{U_\theta}{R} \frac{\partial U_\theta}{\partial \theta} + \frac{U_R U_\theta - U_\phi^2 \cot \theta}{R} = -\frac{1}{\rho} \frac{1}{R} \frac{\partial P}{\partial \theta} + \rho g_\theta + \frac{\mu}{\rho} \times \left[\frac{2}{R} \frac{\partial U_\theta}{\partial R} + \frac{\partial^2 U_\theta}{\partial R^2} + \frac{\cot \theta}{R^2} \frac{\partial U_\theta}{\partial \theta} + \frac{1}{R^2} \frac{\partial^2 U_\theta}{\partial \theta^2} - \frac{2}{R^2} \frac{\partial U_R}{\partial \theta} - \frac{U_\theta}{R^2 \sin^2 \theta} \right] \quad (352)$$

For the flow inside the boundary layer, Eqs. 349 - 352 using the assumptions listed below can be approximated as:

1. Velocity component in θ -direction is neglected,
2. Momentum equation in θ -direction is neglected,
3. Right hand side of R-momentum and ϕ -momentum equations can be represented with their largest terms which are, $\frac{1}{R^2} \frac{\partial^2 U_R}{\partial \theta^2}$ and $\frac{1}{R^2} \frac{\partial^2 U_\phi}{\partial \theta^2}$, respectively.

Final form of the equations becomes:

- Continuity:

$$\frac{\partial U_R}{\partial R} + 2\frac{U_R}{R} + \frac{1}{R} \frac{\partial U_\theta}{\partial \theta} = 0 \quad (353)$$

- R-Momentum:

$$U_R \frac{\partial U_R}{\partial R} + \frac{U_\theta}{R} \frac{\partial U_R}{\partial \theta} - \frac{U_\phi^2}{R} = -\frac{1}{\rho} \frac{\partial P}{\partial R} + g_R + \frac{\mu}{\rho} \frac{1}{R^2} \frac{\partial^2 U_R}{\partial \theta^2} \quad (354)$$

- ϕ -Momentum:

$$U_R \frac{\partial U_\phi}{\partial R} + \frac{U_\theta}{R} \frac{\partial U_\phi}{\partial \theta} + \frac{U_R U_\phi}{R} = \frac{\mu}{\rho} \frac{1}{R^2} \frac{\partial^2 U_\phi}{\partial \theta^2} \quad (355)$$

Similar to the model obtained for the straight section, momentum equations are integrated through the boundary layer. Here, boundary layer thickness is δ and is a function of R . Also, integration of the momentum equations will be done with the limits of $\theta = \alpha$ (at the wall) and $\theta = \alpha - \frac{\delta}{R}$ (at the edge of the boundary layer) where α is the convergence angle as shown in Figure 92. Moreover, for the velocity distributions inside the boundary layer, following correlations are used.

$$u_R = U_R \Psi_1(\eta) \quad (356)$$

$$u_\phi = U_\phi \Psi_2(\eta) \quad (357)$$

where, similar to the straight model, U is the velocity at the edge of the boundary layer and u is the velocity inside the boundary layer, Ψ_1 and Ψ_2 is the velocity profile inside the boundary layer and,

$$\eta = \frac{R}{\delta} (\alpha - \theta) \quad (358)$$

It can be seen that value of η is 0 at the wall and its value is 1 at the edge of the boundary layer. Finally, similar to the straight section model, free vortex and uniform velocity in R -direction are assumed for the potential core hence, Eq. 356 and Eq. 357 can be written as:

$$u_R = U_R(R) \Psi_1(\eta) \quad (359)$$

$$u_\phi = \frac{\Omega}{R \sin \alpha} \Psi_2(\eta) \quad (360)$$

First, R-momentum equation is integrated. Integral form of Eq. 354 is given in Eq. 361.

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_R}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_R}{\partial \theta} d\theta - \int_{\alpha-\delta/R}^{\alpha} \frac{u_\phi^2}{R} d\theta = \\ - \frac{1}{\rho} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial P}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} g_R d\theta + \frac{\mu}{\rho} \frac{1}{R^2} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial^2 u_R}{\partial \theta^2} d\theta \end{aligned} \quad (361)$$

Second term on the left hand side can be written as:

$$\int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_R}{\partial \theta} d\theta = \frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_R u_\theta)}{\partial \theta} d\theta - \frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_\theta}{\partial \theta} d\theta \quad (362)$$

From the continuity equation:

$$-\frac{1}{R} \frac{\partial u_\theta}{\partial \theta} = \frac{\partial u_R}{\partial R} + 2 \frac{u_R}{R} \quad (363)$$

Hence, Eq. 362 becomes:

$$\int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_R}{\partial \theta} d\theta = \frac{1}{R} (u_R u_\theta) \Big|_{\alpha-\delta/R}^{\alpha} + \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R^2}{R} d\theta \quad (364)$$

Integrating Eq. 363 through boundary layer will result in:

$$-\frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_\theta}{\partial \theta} d\theta = \int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \quad (365)$$

$$-\frac{1}{R} u_\theta \Big|_{\alpha-\delta/R}^{\alpha} = -\frac{1}{R} (u_\theta \Big|_{\alpha} - u_\theta \Big|_{\alpha-\delta/R}) = \int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \quad (366)$$

Since all velocities at the wall ($\theta = \alpha$) is zero:

$$\frac{1}{R} u_\theta \Big|_{\alpha-\delta/R} = \int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \quad (367)$$

Using Eq.367, first term on the right hand side of Eq.364 can be written as:

$$\frac{1}{R} u_R u_\theta \Big|_{\alpha-\delta/R}^{\alpha} = \frac{1}{R} [u_R u_\theta \Big|_{\alpha} - u_R u_\theta \Big|_{\alpha-\delta/R}] = -\frac{1}{R} u_R \Big|_{\alpha-\delta/R} u_\theta \Big|_{\alpha-\delta/R} \quad (368)$$

$$\frac{1}{R} u_R u_\theta \Big|_{\alpha-\delta/R}^{\alpha} = -u_R \Big|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] \quad (369)$$

Eq. 364 becomes:

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\theta}}{R} \frac{\partial u_R}{\partial \theta} d\theta = \\ - u_R|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] + \\ \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R^2}{R} d\theta \quad (370) \end{aligned}$$

Eq. 361 can be written as:

$$\begin{aligned} 2 \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R^2}{R} d\theta - \\ u_R|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] - \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\phi}^2}{R} d\theta = \\ - \frac{1}{\rho} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial P}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} g_R d\theta + \frac{\mu}{\rho} \frac{1}{R^2} \frac{\partial u_R}{\partial \theta} \Big|_{\alpha-\delta/R}^{\alpha} \quad (371) \end{aligned}$$

With the help of Eq. 359 and Eq. 360, R-momentum equation becomes:

$$\begin{aligned} 2 \int_{\alpha-\delta/R}^{\alpha} U_R(R) \Psi_1(\eta) \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta - \\ U_R(R) \Psi_1(1) \left[\int_{\alpha-\delta/R}^{\alpha} \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta + \int_{\alpha-\delta/R}^{\alpha} \frac{2U_R(R) \Psi_1(\eta)}{R} d\theta \right] + \\ 2 \int_{\alpha-\delta/R}^{\alpha} \frac{U_R(R)^2 \Psi_1(\eta)^2}{R} d\theta - \int_{\alpha-\delta/R}^{\alpha} \frac{\Omega^2}{R^3 \sin \alpha} \Psi_2(\eta)^2 d\theta = \\ - \frac{1}{\rho} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial P}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} g_R d\theta + \frac{\mu}{\rho} \frac{1}{R^2} \frac{d(U_R(R) \Psi_1(\eta))}{d\theta} \Big|_{\alpha-\delta/R}^{\alpha} \quad (372) \end{aligned}$$

Arranging Eq. 372:

$$\begin{aligned} U_R(R) \left[2 \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta - \right. \\ \left. \Psi_1(1) \int_{\alpha-\delta/R}^{\alpha} \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta - \Psi_1(1) \frac{2U_R(R)}{R} \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) d\theta + \right. \\ \left. 2 \frac{U_R(R)}{R} \int_{\alpha-\delta/R}^{\alpha} \Psi_1^2(\eta) d\theta \right] - \int_{\alpha-\delta/R}^{\alpha} \frac{\Omega^2}{R^3 \sin \alpha} \Psi_2^2(\eta) d\theta = \\ - \frac{1}{\rho} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial P}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} g_R d\theta + \frac{\mu}{\rho} \frac{U_R(R)}{R^2} \frac{d\Psi_1(\eta)}{d\theta} \Big|_{\alpha-\delta/R}^{\alpha} \quad (373) \end{aligned}$$

First term on the left hand side of Eq. 373 can be evaluated as:

$$R_1 = 2 \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta \quad (374)$$

$$R_1 = 2 \int_{\alpha-\delta/R}^{\alpha} \frac{dU_R(R)}{dR} \Psi_1^2(\eta) d\theta + 2U_R(R) \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) \frac{d\Psi_1(\eta)}{dR} d\theta \quad (375)$$

Using Eq. 358, following expression can be obtained:

$$\frac{d\eta}{d\theta} = -\frac{R}{\delta} \Rightarrow d\theta = -\frac{R}{\delta} d\eta \quad (376)$$

Replacing θ with η in Eq. 375:

$$R_1 = 2 \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1^2(\eta) d\eta + 2U_R(R) \frac{\delta}{R} \int_0^1 \Psi_1(\eta) \frac{d\Psi_1(\eta)}{dR} d\eta \quad (377)$$

Using chain rule as:

$$\frac{d\Psi_1(\eta)}{dR} = \frac{d\Psi_1(\eta)}{d\eta} \frac{d\eta}{d\delta} \frac{d\delta}{dR} \quad (378)$$

where,

$$\frac{d\eta}{d\delta} = -\frac{R}{\delta^2} (\alpha - \theta) = -\frac{\eta}{\delta} \quad (379)$$

Eq. 377 becomes:

$$R_1 = 2 \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1^2(\eta) d\eta - 2 \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1(\eta) \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta \quad (380)$$

Using Eqs. 234 - 236, Eq. 380 becomes:

$$R_1 = 2 \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1^2(\eta) d\eta - \frac{U_R(R)}{R} \frac{d\delta}{dR} \Psi_1^2(1) + \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1^2(\eta) d\eta \quad (381)$$

Second term on the left hand side of Eq. 373 can be evaluated as:

$$R_2 = -\Psi_1(1) \int_{\alpha-\delta/R}^{\alpha} \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta \quad (382)$$

$$R_2 = -\Psi_1(1) \frac{dU_R(R)}{dR} \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) d\theta - \Psi_1(1) U_R(R) \int_{\alpha-\delta/R}^{\alpha} \frac{d\Psi_1(\eta)}{dR} d\theta \quad (383)$$

Replacing θ with η in Eq. 383:

$$R_2 = -\Psi_1(1) \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta - \Psi_1(1) U_R(R) \frac{\delta}{R} \int_0^1 \frac{d\Psi_1(\eta)}{dR} d\eta \quad (384)$$

Using Eq. 378 and Eq. 379:

$$R_2 = -\Psi_1(1) \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta + \Psi_1(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \frac{d\Psi_1(\eta)}{d\eta} \eta d\eta \quad (385)$$

With the help of Eqs. 196 - 198, Eq. 385 becomes:

$$R_2 = -\Psi_1(1) \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta + \Psi_1^2(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} - \Psi_1(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1(\eta) d\eta \quad (386)$$

Evaluating third term on the left hand side of Eq.373 as:

$$R_3 = -\Psi_1(1) \frac{2U_R(R)}{R} \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) d\theta \quad (387)$$

Replacing θ with η in Eq. 387:

$$R_3 = -\Psi_1(1) \frac{2U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta \quad (388)$$

Fourth term on the left hand side of Eq. 373 evaluated as:

$$R_4 = \frac{2U_R(R)}{R} \int_{\alpha-\delta/R}^{\alpha} \Psi_1^2(\eta) d\theta \quad (389)$$

Replacing θ with η in Eq. 389:

$$R_4 = \frac{2U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1^2(\eta) d\eta \quad (390)$$

Final term on the left hand side of Eq. 373 evaluated as:

$$R_5 = - \int_{\alpha-\delta/R}^{\alpha} \frac{\Omega^2}{R^3 \sin^2(\alpha)} \Psi_2^2(\eta) d\theta \quad (391)$$

Replacing θ with η in Eq. 391:

$$R_5 = - \frac{\Omega^2}{R^3 \sin^2(\alpha)} \frac{\delta}{R} \int_0^1 \Psi_2^2(\eta) d\eta \quad (392)$$

Now, integration of the pressure term in Eq. 373 is done. As shown in the section above, this term needs to be evaluated by acknowledging the existence of air

core. Moreover, it was shown that considering the variation of pressure inside the boundary layer has a small effect on the boundary layer thickness and leads to much more complex equations. Here, same approach is followed, and pressure is modeled with neglecting its variation inside the boundary layer and considering the effect of air core.

For potential core θ -momentum equation can be written as:

$$-\frac{U_\phi^2 \cot \theta}{R} = -\frac{1}{\rho} \frac{1}{R} \frac{\partial P}{\partial \theta} \quad (393)$$

where,

$$U_\phi = -\frac{\Omega \cot \theta}{R} \quad (394)$$

Eq. 393 becomes:

$$\frac{\Omega^2}{R^2} \frac{\cot \theta}{\sin^2(\theta)} = \frac{1}{\rho} \frac{\partial P}{\partial \theta} \quad (395)$$

Integrating Eq. 395 through potential core:

$$\int_{\theta_\alpha}^{\alpha-\delta/R} \frac{\Omega^2}{R^2} \frac{\cot \theta}{\sin^2(\theta)} d\theta = \frac{1}{\rho} \int_{\theta_\alpha}^{\alpha-\delta/R} \frac{\partial P}{\partial \theta} d\theta \quad (396)$$

$$-\frac{1}{2} \frac{\Omega^2}{R^2} \frac{1}{\sin^2(\theta)} \Big|_{\theta_\alpha}^{\alpha-\delta/R} = \frac{1}{\rho} [P_{\alpha-\delta/R} - P_{\theta_\alpha}] \quad (397)$$

Here, $P_{\alpha-\delta/R}$ is the pressure at the edge and inside the boundary layer (since variation through the boundary layer is neglected). Moreover, P_{θ_α} is the pressure at the surface of the air core. Noting that pressure at the surface of the air core is equal to the ambient pressure and considering that the thickness of the boundary layer is small, Eq. 397 can be approximated as:

$$\frac{1}{2} \rho \frac{\Omega^2}{R^2} \left[\frac{1}{\sin^2(\theta_\alpha)} - \frac{1}{\sin^2(\alpha)} \right] = P_{\alpha-\delta/R} \quad (398)$$

Taking derivative with respect to R will yield:

$$\frac{dP_{\alpha-\delta/R}}{dR} = -\rho \frac{\Omega^2}{R^2} \left[\frac{\cos \theta_\alpha}{\sin^3(\theta_\alpha)} \frac{d\theta_\alpha}{dR} \right] \quad (399)$$

Using Eq. 399 for the pressure term of the R-momentum equation, Eq. 373, will give:

$$R_6 = \frac{1}{\rho} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial P}{\partial R} d\theta = \frac{\Omega^2}{R^2} \int_{\alpha-\delta/R}^{\alpha} \frac{\cos\theta_{\alpha}}{\sin^3(\theta_{\alpha})} \frac{d\theta_{\alpha}}{dR} d\theta \quad (400)$$

$$R_6 = \frac{\Omega^2}{R^2} \frac{\cos\theta_{\alpha}}{\sin^3(\theta_{\alpha})} \frac{d\theta_{\alpha}}{dR} \frac{\delta}{R} \quad (401)$$

Gravitational term in the Eq. 373 is evaluated as:

$$R_7 = \int_{\alpha-\delta/R}^{\alpha} g_R d\theta = g_R \frac{\delta}{R} \quad (402)$$

where,

$$g_R = g_z \cos \alpha \quad (403)$$

Hence, Eq. 402 can be written as:

$$R_7 = g_z \frac{\delta}{R} \cos \alpha \quad (404)$$

Finally, viscous term of the Eq. 373 can be calculated as:

$$R_8 = \frac{\mu}{\rho} \frac{U_R(R)}{R^2} \frac{d\Psi_1(\eta)}{d\theta} \Big|_{\alpha-\delta/R}^{\alpha} \quad (405)$$

Replacing θ with η will result in:

$$R_8 = \frac{\mu}{\rho} \frac{U_R(R)}{R\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \quad (406)$$

With the help of Eqs. 381, 386, 388, 390, 392, 401, 404 and 406, R-momentum equation can be written as:

$$\begin{aligned} & U_R(R) \left[2 \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1^2(\eta) d\eta - \frac{U_R(R)}{R} \frac{d\delta}{dR} \Psi_1^2(1) + \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1^2(\eta) d\eta - \right. \\ & \Psi_1(1) \frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta + \Psi_1^2(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} - \Psi_1(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1(\eta) d\eta - \\ & \left. - \Psi_1(1) \frac{2U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta + \frac{2U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta \right] \\ & - \frac{\Omega^2}{R^3 \sin^2(\alpha)} \frac{\delta}{R} \int_0^1 \Psi_2^2(\eta) d\eta \\ & = \frac{\Omega^2}{R^2} \frac{\cos\theta_{\alpha}}{\sin^3(\theta_{\alpha})} \frac{d\theta_{\alpha}}{dR} \frac{\delta}{R} + g_z \frac{\delta}{R} \cos \alpha + \frac{\mu}{\rho} \frac{U_R(R)}{R\delta} \frac{d\Psi_1(\eta)}{d\eta} \Big|_0^1 \quad (407) \end{aligned}$$

With the help of Eq. 264, Eq. 266, Eq. 268, Eq. 269 and Eq. 270, Eq. 407 can be written as:

$$\begin{aligned} \frac{d\delta}{dR} \left[\frac{U_R^2(R)}{R} (A_3 - \Psi_1(1) A_1) \right] + \frac{dU_R(R)}{dR} \left[U_R(R) \frac{\delta}{R} (2A_3 - \Psi_1(1) A_1) \right] \\ + \frac{d\theta_\alpha}{dR} \left[-\frac{\Omega^2}{R^2} \frac{\delta \cos \theta_\alpha}{R \sin^3 \theta_\alpha} \right] = \\ \frac{\delta}{R} \left[2 \frac{U_R^2(R)}{R} (\Psi_1(1) A_1 - A_3) + \frac{\Omega^2}{R^3 \sin^2 \alpha} A_4 + g_z \cos \alpha \right] + \frac{\mu}{\rho} \frac{U_R(R)}{\delta R} B1 \quad (408) \end{aligned}$$

Final form of the R-momentum equation is:

$$\begin{aligned} \frac{d\delta}{dR} [U_R(R) \delta (A_3 - \Psi_1(1) A_1)] + \frac{dU_R(R)}{dR} [\delta^2 (2A_3 - \Psi_1(1) A_1)] \\ + \frac{d\theta_\alpha}{dR} \left[-\frac{\Omega^2}{U_R(R) R^2} \frac{\delta^2 \cos \theta_\alpha}{\sin^3 \theta_\alpha} \right] = \\ \delta^2 \left[2 \frac{U_R(R)}{R} (\Psi_1(1) A_1 - A_3) + \frac{\Omega^2}{U_R(R) R^3 \sin^2 \alpha} A_4 + g_z \frac{\cos \alpha}{U_R(R)} \right] + \frac{\mu}{\rho} B1 \quad (409) \end{aligned}$$

Second, integration of the θ -momentum equation is done. Integral form of Eq. 355 is given in Eq. 410.

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_\phi}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_\phi}{\partial \theta} d\theta + \int_{\alpha-\delta/R}^{\alpha} \frac{u_R u_\phi}{R} d\theta = \\ \frac{\mu}{\rho} \frac{1}{R^2} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial^2 u_\phi}{\partial \theta^2} d\theta \quad (410) \end{aligned}$$

Second term on the left hand side of the Eq. 410 can be written as:

$$\int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_\phi}{\partial \theta} d\theta = \frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_\theta u_\phi)}{\partial \theta} d\theta - \frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} u_\phi \frac{\partial u_\theta}{\partial \theta} d\theta \quad (411)$$

Using Eq. 363:

$$\int_{\alpha-\delta/R}^{\alpha} \frac{u_\theta}{R} \frac{\partial u_\phi}{\partial \theta} d\theta = \frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_\theta u_\phi)}{\partial \theta} d\theta + \int_{\alpha-\delta/R}^{\alpha} u_\phi \frac{\partial u_R}{\partial R} d\theta + \int_{\alpha-\delta/R}^{\alpha} u_\phi \frac{2u_R}{R} d\theta \quad (412)$$

First term on the right hand side of Eq. 412 can be evaluated as (with the help of Eq. 365 - Eq. 368):

$$\frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_\theta u_\phi)}{\partial \theta} d\theta = \frac{1}{R} u_\theta u_\phi |_{\alpha-\delta/R}^{\alpha} = -\frac{1}{R} u_\theta u_\phi |_{\alpha-\delta/R} \quad (413)$$

Using Eq. 367:

$$\frac{1}{R} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_{\theta} u_{\phi})}{\partial \theta} d\theta = u_{\phi}|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] \quad (414)$$

Eq. 412 becomes:

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\theta}}{R} \frac{\partial u_{\phi}}{\partial \theta} d\theta = & -u_{\phi}|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] + \\ & \int_{\alpha-\delta/R}^{\alpha} u_{\phi} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\phi} u_R}{R} d\theta \end{aligned} \quad (415)$$

Introducing Eq. 415 into Eq. 410:

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} u_R \frac{\partial u_{\phi}}{\partial R} d\theta - u_{\phi}|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] + \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \\ + \int_{\alpha-\delta/R}^{\alpha} u_{\phi} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\phi} u_R}{R} d\theta = \frac{\mu}{\rho} \frac{1}{R^2} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial^2 u_{\phi}}{\partial \theta^2} d\theta \end{aligned} \quad (416)$$

Eq. 416 can be arranged as:

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial (u_R u_{\phi})}{\partial R} d\theta + 3 \int_{\alpha-\delta/R}^{\alpha} \frac{u_{\phi} u_R}{R} d\theta \\ - u_{\phi}|_{\alpha-\delta/R} \left[\int_{\alpha-\delta/R}^{\alpha} \frac{\partial u_R}{\partial R} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{u_R}{R} d\theta \right] = \frac{\mu}{\rho} \frac{1}{R^2} \int_{\alpha-\delta/R}^{\alpha} \frac{\partial^2 u_{\phi}}{\partial \theta^2} d\theta \end{aligned} \quad (417)$$

Using Eq. 359 and Eq. 360, Eq. 417 becomes:

$$\begin{aligned} \int_{\alpha-\delta/R}^{\alpha} \frac{d \left(\frac{\Omega}{R \sin \alpha} U_R(R) \Psi_1(\eta) \Psi_2(\eta) \right)}{dR} d\theta \\ + 3 \int_{\alpha-\delta/R}^{\alpha} \frac{1}{R} \frac{\Omega}{R \sin \alpha} U_R(R) \Psi_1(\eta) \Psi_2(\eta) d\theta \\ - \frac{\Omega}{R \sin \alpha} \Psi_2(1) \left[\int_{\alpha-\delta/R}^{\alpha} \frac{d (U_R(R) \Psi_1(\eta))}{dR} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{U_R(R) \Psi_1(\eta)}{R} d\theta \right] \\ = \frac{\mu}{\rho} \frac{1}{R^2} \frac{d \left(\frac{\Omega}{R \sin \alpha} \Psi_2(\eta) \right)}{d\theta} \Big|_{\alpha-\delta/R}^{\alpha} \end{aligned} \quad (418)$$

Evaluating first term on the left hand side of the Eq. 418:

$$\phi_1 = \int_{\alpha-\delta/R}^{\alpha} \frac{d \left(\frac{\Omega}{R \sin \alpha} U_R(R) \Psi_1(\eta) \Psi_2(\eta) \right)}{dR} d\theta \quad (419)$$

$$\phi_1 = \frac{\Omega}{R \sin \alpha} \left[\frac{dU_R(R)}{dR} \int_{\alpha-\delta/R}^{\alpha} \Psi_1(\eta) \Psi_2(\eta) d\theta + U_R(R) \int_{\alpha-\delta/R}^{\alpha} \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{dR} d\theta \right] \quad (420)$$

Replacing θ with η in Eq. 420:

$$\phi_1 = \frac{\Omega}{R \sin \alpha} \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta + U_R(R) \frac{\delta}{R} \int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{dR} d\eta \right] \quad (421)$$

Using Eq. 378 and Eq. 379, Eq. 421 becomes:

$$\phi_1 = \frac{\Omega}{R \sin \alpha} \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta + U_R(R) \frac{\delta}{R} \int_0^1 \frac{d(\Psi_1(\eta) \Psi_2(\eta))}{dR} d\eta \right] \quad (422)$$

With the help of Eq. 206, Eq. 207 and Eq. 208, Eq. 422 becomes:

$$\phi_1 = \frac{\Omega}{R \sin \alpha} \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta + \frac{U_R(R)}{R} \frac{d\delta}{dR} \left(\int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - \Psi_1(1) \Psi_2(1) \right) \right] \quad (423)$$

Second term on the left hand side of the Eq. 418 can be evaluated as:

$$\phi_2 = 3 \int_{\alpha-\delta/R}^{\alpha} \frac{1}{R} \frac{\Omega}{R \sin \alpha} U_R(R) \Psi_1(\eta) \Psi_2(\eta) d\theta \quad (424)$$

Replacing θ with η in Eq. 424:

$$\phi_2 = 3 U_R(R) \frac{\delta}{R} \frac{\Omega}{R^2 \sin \alpha} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta \quad (425)$$

Final term on the left hand side of the Eq. 418 can be evaluated as:

$$\phi_3 = -\frac{\Omega}{R \sin \alpha} \Psi_2(1) \left[\int_{\alpha-\delta/R}^{\alpha} \frac{d(U_R(R) \Psi_1(\eta))}{dR} d\theta + 2 \int_{\alpha-\delta/R}^{\alpha} \frac{U_R(R) \Psi_1(\eta)}{R} d\theta \right] \quad (426)$$

Using Eq. 386 and Eq. 388, Eq. 426 becomes:

$$\phi_3 = -\frac{\Omega}{R \sin \alpha} \Psi_2(1) \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta - \Psi_1(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} + \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1(\eta) d\eta + 2 \frac{U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta \right] \quad (427)$$

Finally, viscous term of the Eq. 418 evaluated as:

$$\phi_4 = \frac{\mu}{\rho} \frac{1}{R^2} \frac{\Omega}{R \sin \alpha} \frac{d\Psi_2(\eta)}{d\theta} \Big|_{\alpha-\delta/R}^{\alpha} \quad (428)$$

Replacing θ with η in Eq. 428:

$$\phi_4 = \frac{\mu}{\rho} \frac{1}{\delta} \frac{\Omega}{R^2 \sin \alpha} \frac{d\Psi_2(\eta)}{d\eta} \Big|_0^1 \quad (429)$$

With the help of Eqs. 423, 425, 427 and 429 ϕ -momentum equation can be written as:

$$\begin{aligned} \frac{\Omega}{R \sin \alpha} \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta + \frac{U_R(R)}{R} \frac{d\delta}{dR} \left(\int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - \Psi_1(1) \Psi_2(1) \right) \right] + \\ 3U_R(R) \frac{\Omega}{R \sin \alpha} \frac{\delta}{R^2} \int_0^1 \Psi_1(\eta) \Psi_2(\eta) d\eta - \\ \frac{\Omega}{R \sin \alpha} \Psi_2(1) \left[\frac{dU_R(R)}{dR} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta - \Psi_1(1) \frac{U_R(R)}{R} \frac{d\delta}{dR} + \frac{U_R(R)}{R} \frac{d\delta}{dR} \int_0^1 \Psi_1(\eta) d\eta + \frac{2U_R(R)}{R} \frac{\delta}{R} \int_0^1 \Psi_1(\eta) d\eta \right] = \\ \frac{\mu}{\rho} \frac{1}{\delta} \frac{\Omega}{R^2 \sin \alpha} \frac{d\Psi_2(\eta)}{d\eta} \Big|_0^1 \quad (430) \end{aligned}$$

With the help of Eq. 264, Eq. 266, Eq. 268, Eq. 269 and Eq. 270 and noting that $\Psi_2(1) = 1$, Eq. 430 can be written as:

$$\begin{aligned} \frac{d\delta}{dR} [\delta U_R(R) (A_2 - A_1)] + \frac{dU_R(R)}{dR} [\delta^2 (A_2 - A_1)] = \\ U_R(R) \frac{\delta^2}{R} (2A_1 - 3A_2) + \frac{\mu}{\rho} B_2 \quad (431) \end{aligned}$$

Finally, mass flow equation is derived. From the results obtained in the section above it is seen that thickness of the boundary layer is almost an order of magnitude smaller than the thickness of the liquid film. Hence, it is seen that mass flow through the boundary layer is small when compared with the mass flow through the potential core. Here, mass flow through the boundary layer is neglected to obtain the mass flow equation. For the spherical coordinates, area can be calculated from Eq. 432.

$$A = 2\pi R^2 \int \sin \theta d\theta \quad (432)$$

With the assumption of mass flow through boundary layer is small:

$$\dot{m}_L = \rho U_R(R) A_{potential} \quad (433)$$

where, $A_{potential}$ is the area of the potential core. Eq. 433 can be approximated as:

$$\dot{m}_L = \rho U_R(R) 2\pi R^2 \int_{\theta_\alpha}^{\alpha} \sin \theta d\theta \quad (434)$$

Eq. 434 can be arranged as:

$$\frac{\dot{m}_L}{2\pi\rho R^2} = U_R(R) \int_{\theta_\alpha}^{\alpha} \sin \theta d\theta = U_R(R) (\cos \theta_\alpha - \cos \alpha) \quad (435)$$

Taking derivative with respect to R-direction will result in:

$$\frac{d}{dR} \left(\frac{\dot{m}_L}{2\pi\rho R^2} \right) = 0 = \frac{dU_R(R)}{dR} [\cos \theta_\alpha - \cos \alpha] + \frac{d\theta_\alpha}{dR} [-U_R(R) \sin \theta_\alpha] \quad (436)$$

Similar to the straight section model, obtained R-momentum (Eq. 418), ϕ - momentum (Eq. 431) and flow rate (Eq. 436) equations are made non-dimensional using the following terms.

$$\delta^* = \frac{\delta}{R_1} \left(\frac{Re}{\sin \alpha} \right)^{0.5} \quad (437)$$

$$R^* = \frac{R}{R_1} \quad (438)$$

$$Re = \frac{\Omega}{\mu} \quad (439)$$

$$U_R^*(R^*) = \frac{U_R(R) R_1 \sin \alpha}{\Omega} \quad (440)$$

$$g_z^* = g_z \frac{R^3}{\Omega^2} \quad (441)$$

Obtained non-dimensional equations are:

$$\begin{aligned} \frac{d\delta^*}{dR^*} [U_R^*(R^*) \delta^*(A_2 - A_1)] + \frac{dU_R^*(R^*)}{dR^*} [\delta^{*2}(A_2 - A_1)] = \\ U_R^*(R^*) \frac{\delta^{*2}}{R^*} (2A_1 - 3A_2) + B_2 \end{aligned} \quad (442)$$

$$\begin{aligned} \frac{d\delta^*}{dR^*} [U_R^*(R^*) \delta^*(A_3 - A_1)] + \frac{dU_R^*(R^*)}{dR^*} [\delta^{*2}(2A_3 - A_1)] + \\ \frac{d\theta_\alpha}{dR^*} \left[-\frac{\delta^{*2}}{R^{*2}} \frac{1}{U_R^*} \sin^2 \alpha \frac{\cos \theta_\alpha}{\sin^3 \theta_\alpha} \right] = \\ \delta^{*2} \left[2 \frac{U_R^*}{R^*} (A_1 - A_3) + \frac{A_4}{U_R^* R^{*3}} + \frac{g_z^*}{U_R^*} \sin^2 \alpha \cos \alpha \right] + B_1 \end{aligned} \quad (443)$$

$$\frac{dU_R^*(R^*)}{dR} [\cos \theta_\alpha - \cos \alpha] + \frac{d\theta_\alpha}{dR^*} [-U_R^* \sin \theta_\alpha] = 0 \quad (444)$$

Eqs. 442 - 444 are solved with Adams Bashfort-Moulton method. For the validation of these equations, they are first applied to the Atomizer-23 described in Chapter 4. It should be noted that there is no atomizer geometry consisting of only convergent or divergent sections. However, Atomizer-23 is designed in the scope of this dissertation and has a different geometry than the classical PSA geometry shown in Figure 3. This atomizer has a very small swirl chamber length, which is equal to the diameter of the inlet ports. Then the convergent section begins, and also, this atomizer does not have an orifice section. Hence, it is the closest geometry that could be used to validate the model obtained for convergent or divergent sections. Details of the geometry of this atomizer and its schematic are discussed and presented with detail in Chapter 6. Moreover, as stated in Chapter 3, 3D numerical simulation of this geometry is done and will be used for the comparison.

It should be noted that Eq. 343 and Eq. 344 are used for calculating the pressure difference and spray angle, respectively. Since only the comparison of the model for

convergent or divergent model is done, initial conditions are given in Eqs. 287 - 292 is used and velocity profile corresponding to $m=3$ is used.

Calculated boundary layer thickness for the Atomizer-23 is given in Figure 93. From the results, it was found that the thickness of the boundary layer at the exit is calculated to be around $20 \mu\text{m}$. Inspection of the results of 3D simulations has yielded that film thickness at the exit is $85 \mu\text{m}$.

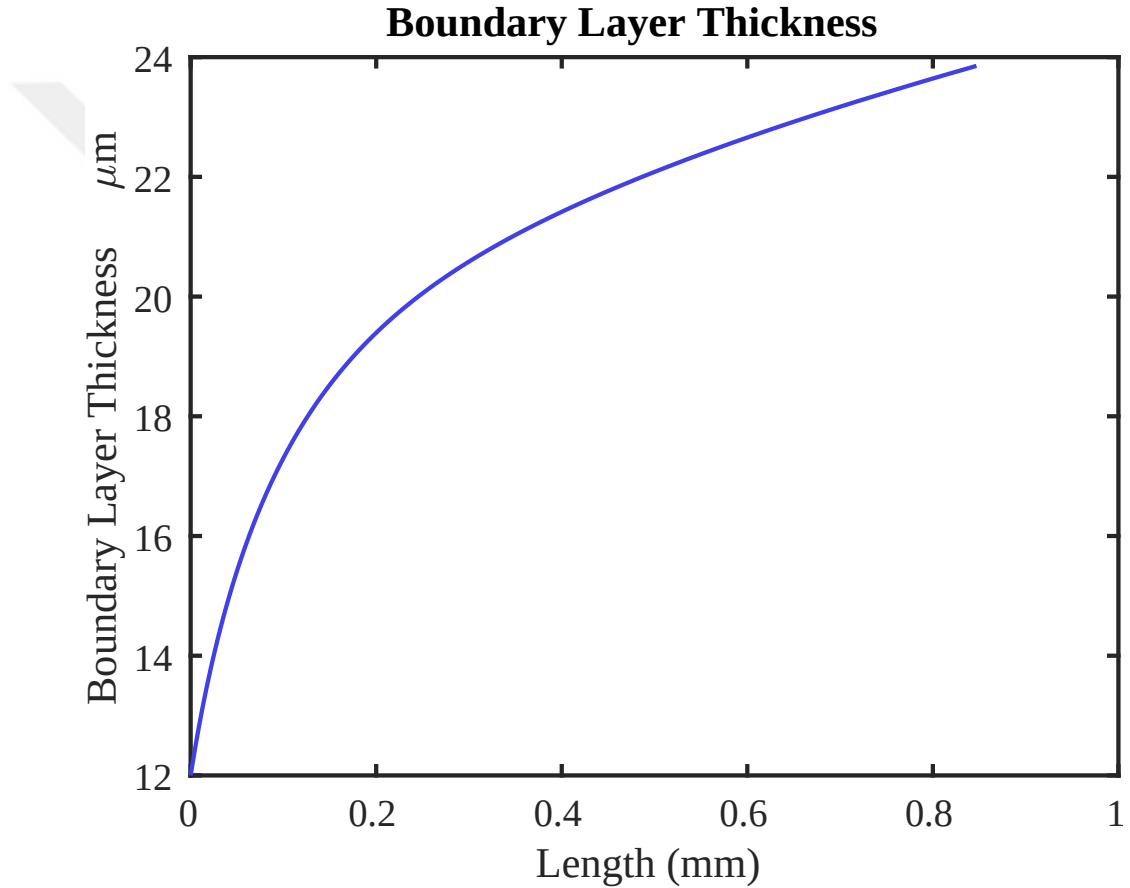


Figure 93: Calculated boundary layer thickness from obtained convergent model

Comparison of the velocity components obtained from the 3D simulations and the boundary layer is given in Figure 94. It is seen that obtained model could accurately predict the velocity components. The biggest difference in the results is in the calculated values of the radius of the air core. It can be seen that boundary layer model

under-predicts the radius of air core. Moreover, similar to the results of the straight section, polar component of the velocity is almost an order of magnitude smaller than the axial and azimuthal components. Moreover, it should be noted that axial velocity refers to the R-component.

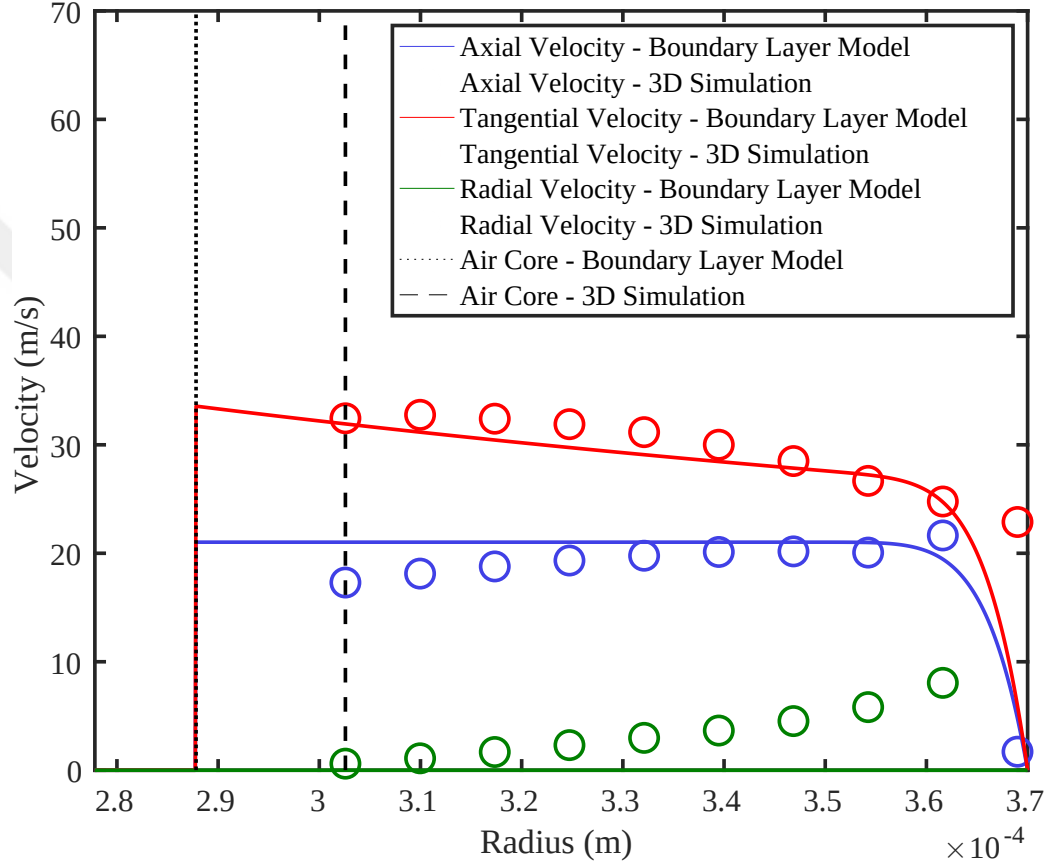


Figure 94: Comparison of calculated velocity components from 3D simulation and boundary layer model at the exit

Finally, deviation of pressure difference is found to be 15.2%, and the difference in spray angle is 3.4 degrees.

5.3 Combined Boundary Layer Model

After obtaining valid boundary layer models for the straight section and convergent (and divergent) section of the PSA, assessment of the combined model is done. It

should be noted that these models are obtained with two different coordinate systems, and therefore a correlation to transform velocity components between them is needed. In this study, the method proposed in the literature is used [12]. Here, author has derived a transformation method based on the conversation of mass, as presented below.

At the interconnecting surface of the swirl chamber with the convergent section, conservation of the mass can be expressed using Eq. 445 and is shown schematically in Figure 95.

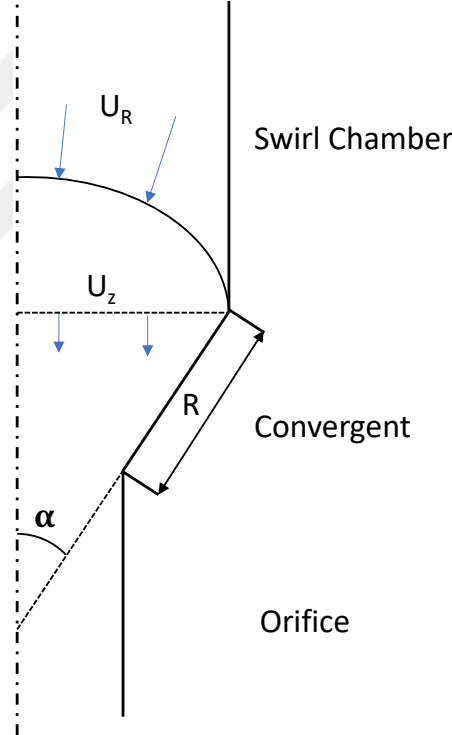


Figure 95: Schematic of mass conversation between swirl chamber and convergent section

$$A_c U_R = A_s U_Z \quad (445)$$

where A_c is the area of the spherical cap at the start of the convergent section and

A_s is the area of circular cross-section of swirl chamber. With the help of Eq. 432 surface area of the spherical cap can be calculated as:

$$A_c = 2\pi R (R - R \cos \alpha) \quad (446)$$

Surface area of the cross-section of swirl chamber is:

$$A_s = \pi (R \sin \alpha)^2 \quad (447)$$

Using Eq. 446 and Eq. 447 in Eq. 445 will yield the following correlation for the velocity in R-direction.

$$U_R = \frac{1 + \cos \alpha}{2} U_Z \quad (448)$$

As can be seen, this method allows to transform z-component of velocity in the cylindrical coordinate to R-component of velocity in the spherical coordinate without needing the calculation of radial component in cylindrical and polar angle component in spherical coordinate. With the help of this relation comparison of the combined model with the 2D simulation and experimental data is done.

First, comparison of the combined model is done with the 2D simulations. Selected case for the calculations is Case-6 in Table 2. Validation of the model is done with comparing the calculated velocity components at the beginning of the convergence, beginning of the orifice, and exit of the orifice.

Figure 96 presents the calculated boundary layer thickness values. Inspection of the results has shown that thickness of the boundary layer increases through the atomizer. Increase is more apparent for the swirl chamber and convergent section. Increase of the boundary layer in the orifice is very small; however, it should be noted that for this atomizer, length of the orifice is smaller than 2 mm, which is considerably less than the length of the swirl chamber and convergent section.

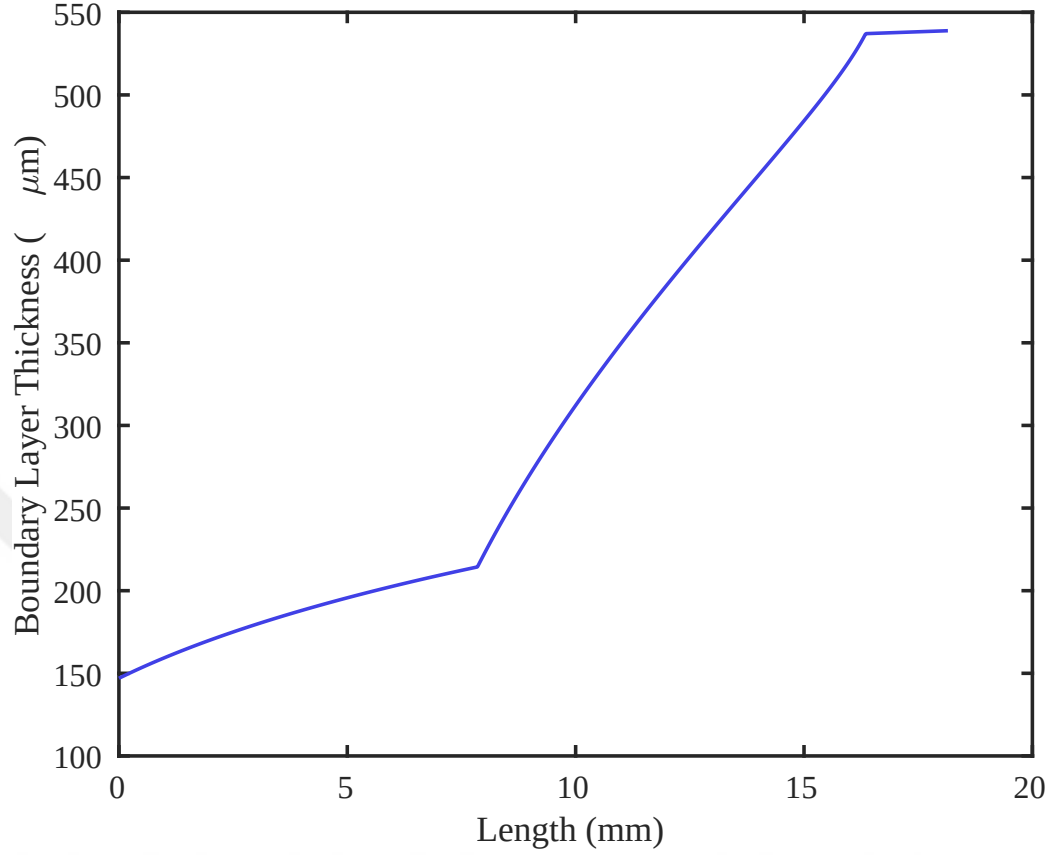


Figure 96: Calculated boundary layer thickness for close-end case

In Figure 97 calculated tangential and axial components of the velocity at the start of the convergent section is given. It can be seen that obtained values from the 2D simulation and the boundary layer model are very close to each other. It should be mentioned that, for the calculated case (Case-6 of 2D simulation), diameter of the swirl chamber is almost an order of magnitude higher than the diameter of the orifice, and considering the flow rate, very small axial velocity components are calculated at the beginning of the convergent section.

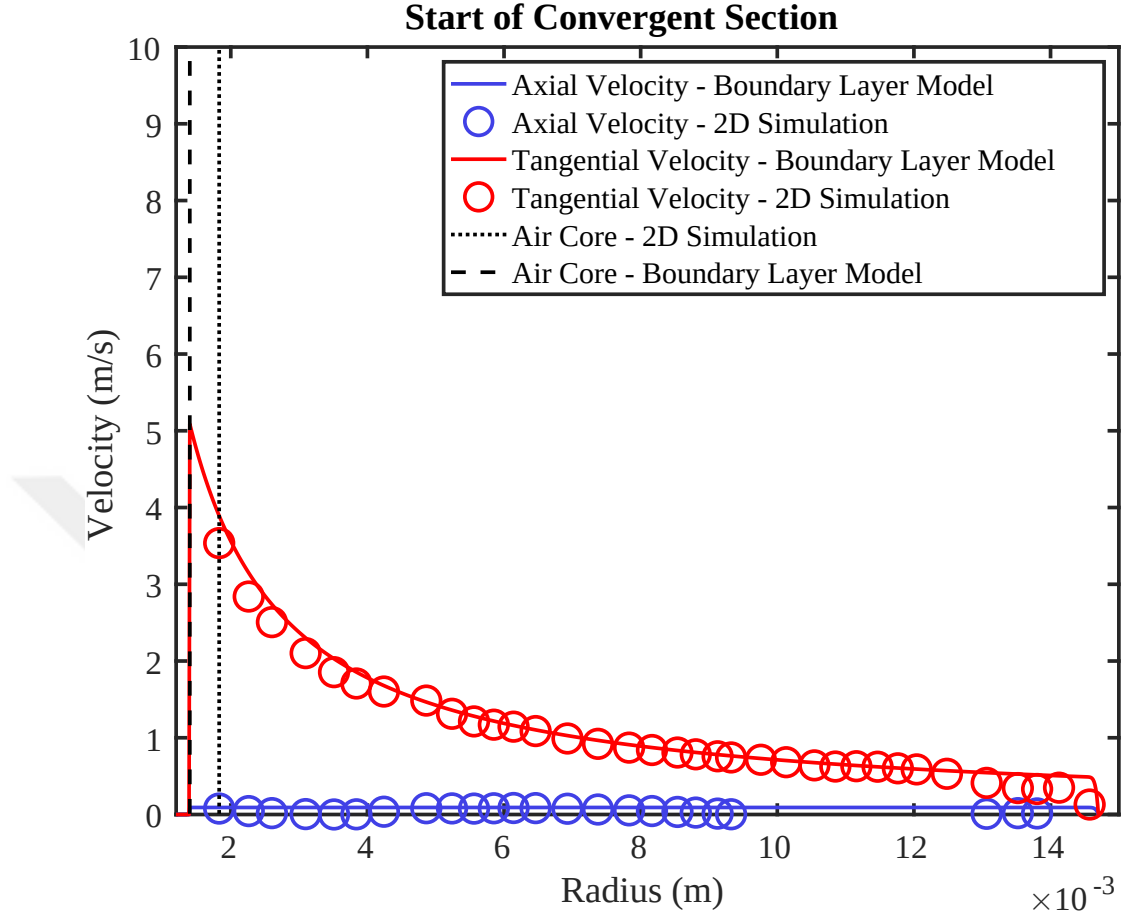


Figure 97: Comparison of velocity components at the start of the convergent section

Figure 98 shows the comparison of velocity components at the start of the orifice section. Significant increase in the axial velocity component can be seen which due to the reason explained above. Moreover, it can be seen that boundary layer model under-predicts the radius of air core. In 2D simulations radius of air core was calculated to be 1.45 mm, whereas with the boundary layer model this parameter is found as 1.40 mm.

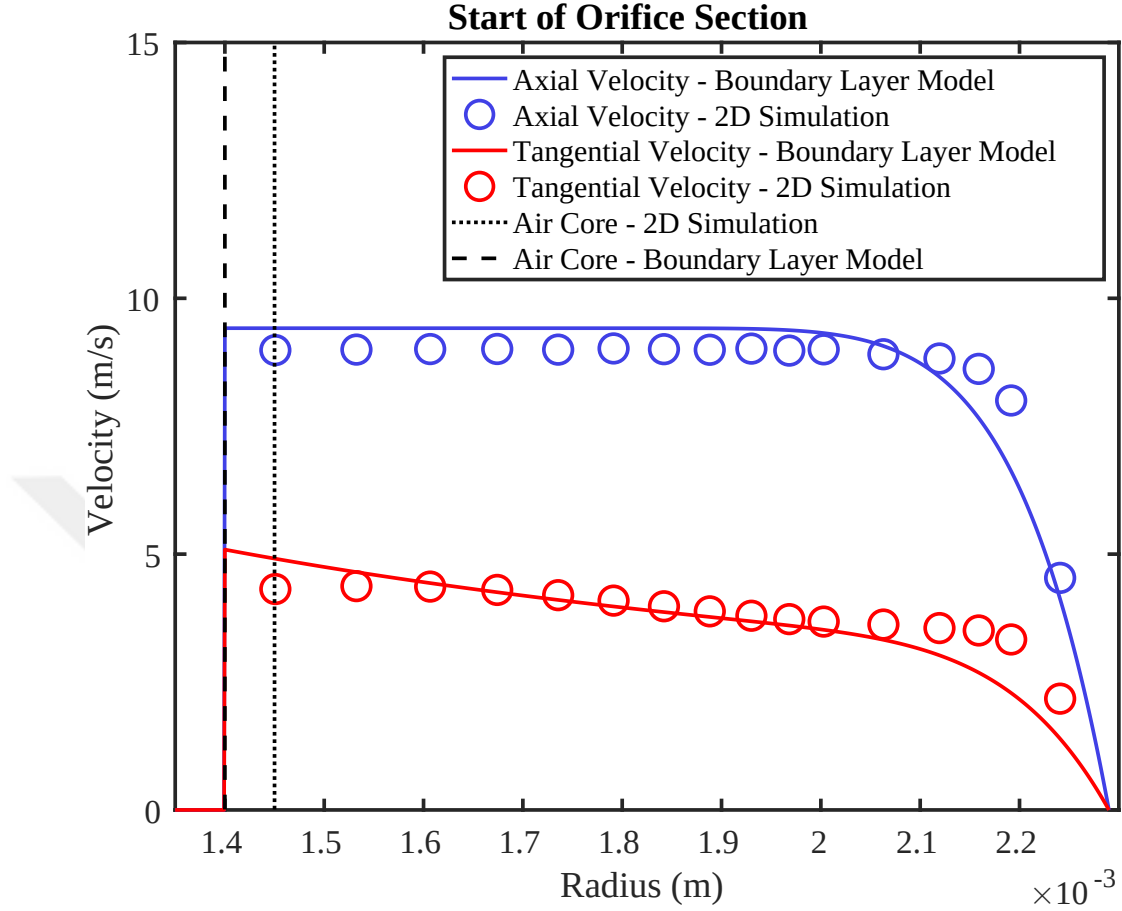


Figure 98: Comparison of velocity components at the start of the orifice

Finally, Figure 99 compares velocity components at the exit of the orifice. It can be seen that even though there is a small deviation in the calculated axial velocity components, the tangential velocity is accurately calculated. The reason for the deviation in the axial component is due to the difference in the calculated film thickness values, where the film thickness value from the experiments is 0.670 mm, film thickness value from the 2D simulation is 0.690 mm, and the calculated film thickness from the boundary layer model is 0.720 mm which leads to lower axial velocity component in the potential core.

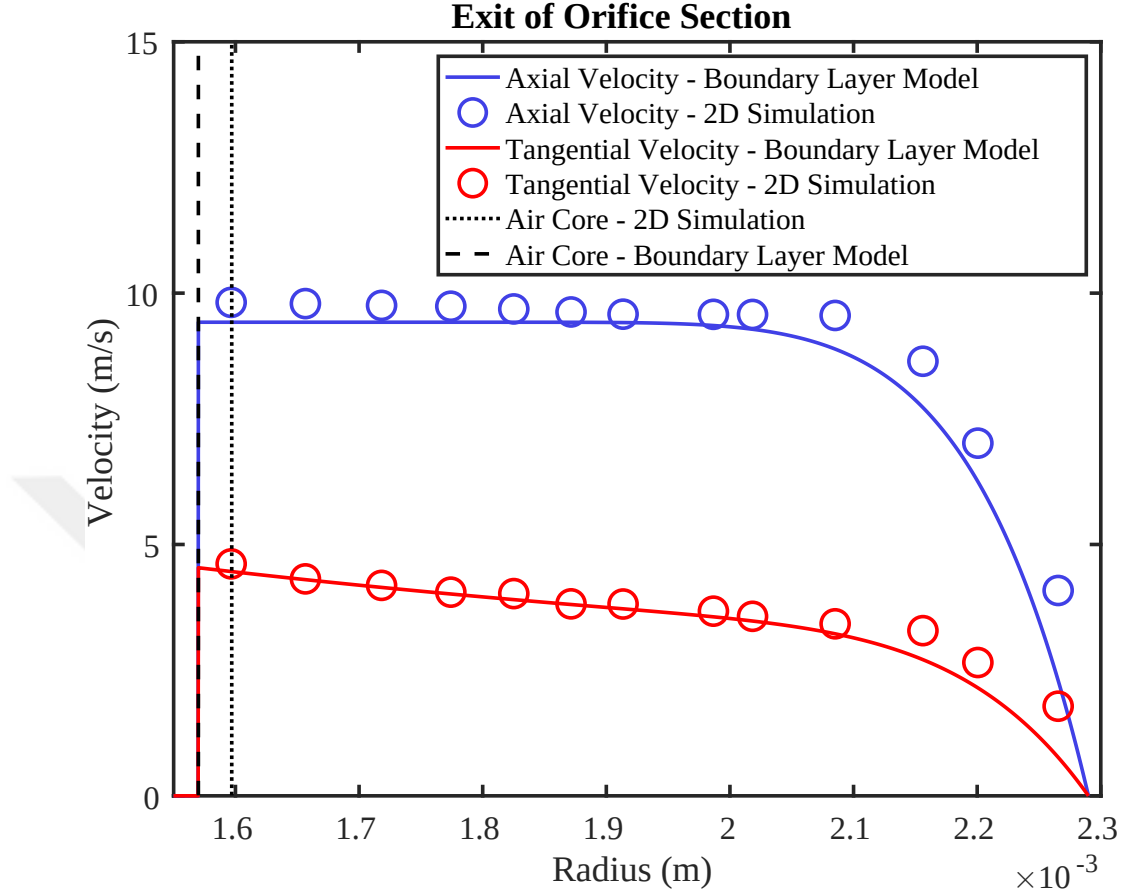


Figure 99: Comparison of velocity components at the exit

Lastly, obtained boundary layer is compared with the experimental data of close-end atomizers. As mentioned in Chapter 4, experimental data is divided into 4 groups. In order to perform calculations with the boundary layer model, all of the geometrical parameters except the length of the inlets are required. Hence, comparison is done with the experimental data in the groups complete data and quasi-complete data. Again, calculated values of pressure difference, film thickness and spray angle are compared with the experimental data, and data set used for the validation consists of 1,098 pressure difference, 68 film thickness, and 863 spray angle measurements. Obtained AMRE and SE values of the boundary layer are presented in Table 25 along with the results of obtained semi-empirical model in Chapter 4.

Table 25: Error Values of Boundary Layer Model and Obtained Semi-Empirical Model - Close-End

Parameter	Boundary Layer Model		Obtained Semi-Empirical Correlation	
	AMRE	SE	AMRE	SE
Pressure Difference	0.214	0.001	0.547	0.001
Discharge Coefficient	0.136	4.3E-4	0.375	0.001
Film Thickness	0.108	3.1E-4	0.117	0.001
Spray Angle	0.193	0.001	0.224	2.7E-4

Obtained AMRE and SE values are given in Table 25. It can be seen that significant improvement in the accuracy of the pressure difference calculation is obtained. Similarly, discharge coefficient values can be calculated with higher accuracy when boundary layer model is used. Moreover, improvement in the accuracy of film thickness and spray angle calculations is insignificant. It should be noted that number of data for the film thickness is very small compared to other parameters and similar behavior for the spray angle was also present in the calculations done for the open-end atomizer.

Additionally, it is clear that obtained boundary layer model has higher error values when compared with that of the 2D and 3D numerical calculations. However, it should also be mentioned that calculation time of the boundary layer model implemented on MATLAB is found to be 18 seconds on a single core. Whereas 2D simulation used for the comparison of combined model is calculated with Star-CCM+ on 8 cores at 26 hours and finally, computation time of the 3D simulation is around 6 days with 20 cores on the same workstation. Considering the improvement in the calculation time, it can be stated that obtained model is more suitable to be used for the optimization calculations.

CHAPTER VI

UNIFORMITY ANALYSIS

6.1 Introduction

Uniformity of the internal and external flow of PSA is one of the major topics to be investigated. It is known that, because of the defects of the internal flow, the resultant spray may not produce desired spray angle, droplet size, and uniformity. There are several reasons to explain why these defects occur, such as errors during manufacturing, improper design, or operating conditions. In an earlier study, it was reported that non-uniform distribution of the mass flux could lead to the appearance of larger droplet sizes than predicted and could also lead to regions of droplet clusters with large sizes [81].

Previous studies have linked the non-uniformity of the spray with the streaks and voids present in the images of the spray and/or visual inspection of the spray [8,60,82,83]. Moreover, it was experimentally shown that the existence of the streaks in the spray is directly related to the areas with high concentrations of droplets [84]. As a result of the presence of such areas, instability of combustion, decrease in combustion efficiency, and increased emission could occur [2].

Several attempts have been made to interpret the reasons for the uniformity and characterize it. One of the earliest studies in this area has studied the relation between uniformity and the number of sectors present in the patternator. It was reported that having less than six sectors in the patternator could lead to inaccurate results [85]. It should be noted that in this study, uniformity is described by the term Patternation Index (PI), which is defined as:

$$PI = \frac{Q_{max}}{Q_{min}} \quad (449)$$

where, Q_{max} and Q_{min} are used to represent maximum and minimum volume of the liquid in the sectors of the patternator, respectively.

In a later study, the relation between the geometrical parameters of the PSA and uniformity is studied. Measurement of uniformity is done with a 12-sector patternator [5]. It should be noted that instead of Eq. 449 this study has used a different method for quantifying the non-uniformity, given in Eq. 28, named uniformity coefficient.

Through the obtained experimental data, a semi-empirical correlation is formed to describe the uniformity from the geometrical parameters such as number of inlets, opening ratio (C), length and diameter of swirl chamber, and the size of the air core. This correlation is given in 450.

$$\vartheta = \frac{23.7}{C^{2.7} \times N_p^{1.4} \times \phi^{1.1} \times (L_s/D_s)^{0.15}} \quad (450)$$

It should be noted that size of the air core developed inside the atomizer is a function of geometrical and operating parameters. Authors have also stated that, based on the performed experiments, due to the finite number of tangential ports of PSA, ϑ value of 10% is inevitable. In other words, non-uniformity corresponding to ϑ value of 10% will always be present in the spray due to the finite number of inlets. In addition, for the number of inlets, it is stated that having higher than three tangential ports will not significantly improve the uniformity of the spray as long as the total area of the ports kept constant.

Uniformity is also described by standard deviation of the volumes of the liquid collected in each sector [22]. In this study, experiments are conducted with a 16-sector patternator, and the effect of geometrical parameters (diameter-to-length ratio of orifice and the number of inlets) on the uniformity is analyzed. Authors have stated that, since the liquid leaves the atomizer from the orifice, this section of the atomizer will have the most effect on the uniformity. Results of the conducted experiments have shown that increasing the length-to-diameter ratio from 0.5 to 2.0 improves the uniformity while further increase have the reverse effect. It should be mentioned that

this ratio is altered by changing the length of the orifice while the diameter is constant. It was also reported that increase of tangential ports from one to two improves the uniformity, while increase from two to three does not have a significant effect.

Effect of number of tangential ports is also studied in more recent studies [86,87]. In the former study, this effect is studied through a bi-propellant atomizer having two, four, and six tangential inlet ports. A circular patternator consisting of 91 tubes is used in this study. Results have shown that increasing the number of ports from two to four significantly improves the uniformity, and the most uniform distribution is obtained with atomizer having six ports. The latter study utilized atomizer with two, three, and four inlets. Moreover, effect of the shape of the convergent section is studied. For this purpose, four atomizers with different shapes (hemispheric, conical, curved convex, and flat conical) of the convergent section are used. Experimental results have shown that uniformity of the atomizer increases as the number of tangential ports increases, and the most uniform spray is obtained with atomizer having four tangential inlet ports and convergent section having a conical shape. It can be seen that while more recent studies conclude that uniformity is improved with the continuous increase of number of tangential ports, earlier studies have found that uniformity does not significantly change after a certain number of tangential ports.

In addition to the patternators, laser-based techniques are also utilized for quantifying the uniformity of the spray. It was shown that results of the laser-based technique, based on the extraction of Mie scattering, coincide with the results of planar laser techniques [88]. Fuel Planar Laser-Induced-Fluorescence technique is also applied to atomizer operating with kerosene RP-3 [89]. In this study, non-uniformity is defined as:

$$Z = \frac{S_{max} - S_{min}}{S_{max}} \quad (451)$$

where, S_{max} and S_{min} refers to the maximum and minimum intensity of light.

Different optical methods, based on determining the local liquid surface area per

unit volume, have also used to study the uniformity and effect of geometrical parameters such as diameter of orifice, number, and width of the tangential inlet ports is studied [30,90]. Uniformity is quantified using Eq. 450. Obtained results have shown that while the number and width of the tangential ports and pressure difference are constant, increasing diameter of the orifice improves the uniformity of the spray in some cases, whereas in others it hampers the uniformity. In other words, increasing the diameter of orifice leads to conflicting results.

Inspection of the previously conducted studies has shown that, while the effect of some geometrical parameters, namely diameter and length of orifice, opening ratio, and number of tangential ports, have studied yet cause of the non-uniformity is not reasoned. This chapter focuses on understanding the effect of internal flow dynamics on uniformity and obtaining a model for describing it. 3D full geometry simulations (explained in Chapter 3.2) of different geometries are done, and then experimental inspection by means of high-speed imaging (explained in Chapter 2.2.1) and LIF/Mie (explained in Chapter 2.2.3) of the selected atomizers is done.

First, determination of how to quantify the non-uniformity is done. Of the many proposed methods to quantify uniformity, such as Eqs. 449, 28, 451, two are used more commonly. These are the use of standard deviation [22] and use of Eq. 28, [5]. Due to the definition of standard deviation, it is foreseen that it cannot truly reflect the non-uniformity since it does not describe the maximum and minimum values. In order to test this hypothesis, a small calculation is conducted. Total of nine one-dimensional arrays created such that each array contains 16 elements with the mean of 1 and standard deviation of 0.4. These arrays replicate the volume values obtained from 16-sector patternator, where each element corresponds to a sector. It should be emphasized that elements of each array are different yet have the same mean and standard deviation values.

Results are shown in Figure 100. Inspection of the graph shows that even though

each array has the same mean and standard deviation, they are significantly different from each other. Obtained ϑ values of each array are also shown, and it can be seen that this parameter is sensitive to the changes of array elements.

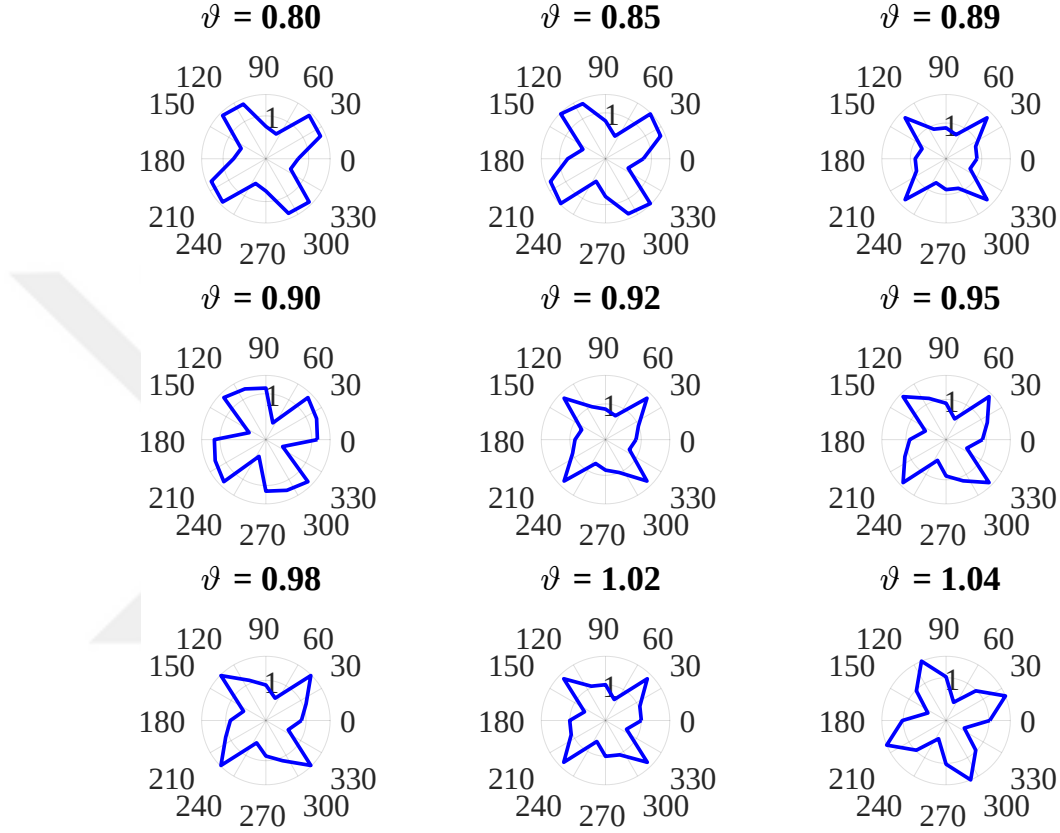


Figure 100: Comparison of the distribution of arrays with same mean and standard deviation values

However, based on the definition of ϑ , it can be seen that lower values represent more uniform sprays and higher values represent less uniform sprays. In other words, ϑ is inversely proportional to uniformity. To eliminate this, a new representation of uniformity is defined (Eq. 452) where higher values represent the uniform spray, and lower values represent non-uniform sprays. A drawback of this definition is, in the case of perfect spray distribution, where Q_{max} , Q_{min} and Q_{mid} are equal to each other,

Γ is not defined due to the division by zero.

$$\Gamma = \frac{Q_{mid}}{Q_{max} - Q_{min}} \quad (452)$$

6.2 Design of the Atomizer

Atomizer used to study uniformity have a unique design, therefore, explained in a separate section. Design of this atomizer is done at the Fluid Dynamics and Spray Laboratory at Ozyegin University [91].

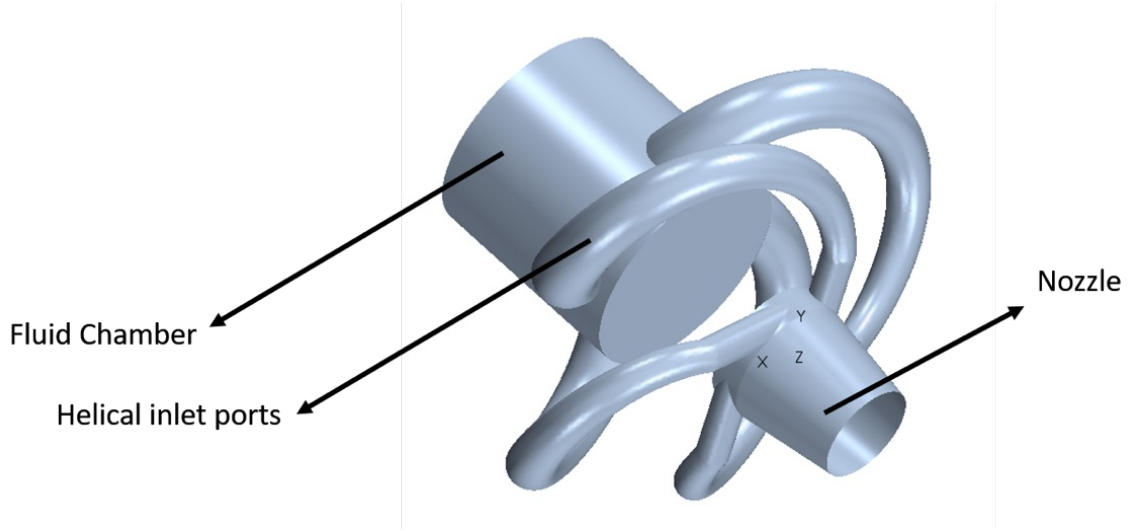


Figure 101: Surface of the fluid region of the designed atomizer

Surface of the fluid region of the designed atomizer and the image of the produced atomizer is shown in Figure 101 and Figure 102, respectively. It can be seen that atomizer consists of three regions, namely fluid chamber, helical inlet ports, and the nozzle. Liquid enters the fluid chamber from the top and then directed to the helical inlet ports, which are connected to the nozzle in tangential orientation. It should be mentioned that these ports are designed to eliminate the separation of the liquid that might occur. Also, there is no orifice section of the atomizer. Although the design of the helical inlet ports dissolves the separation, it also led to a design that cannot be manufactured with traditional methods.

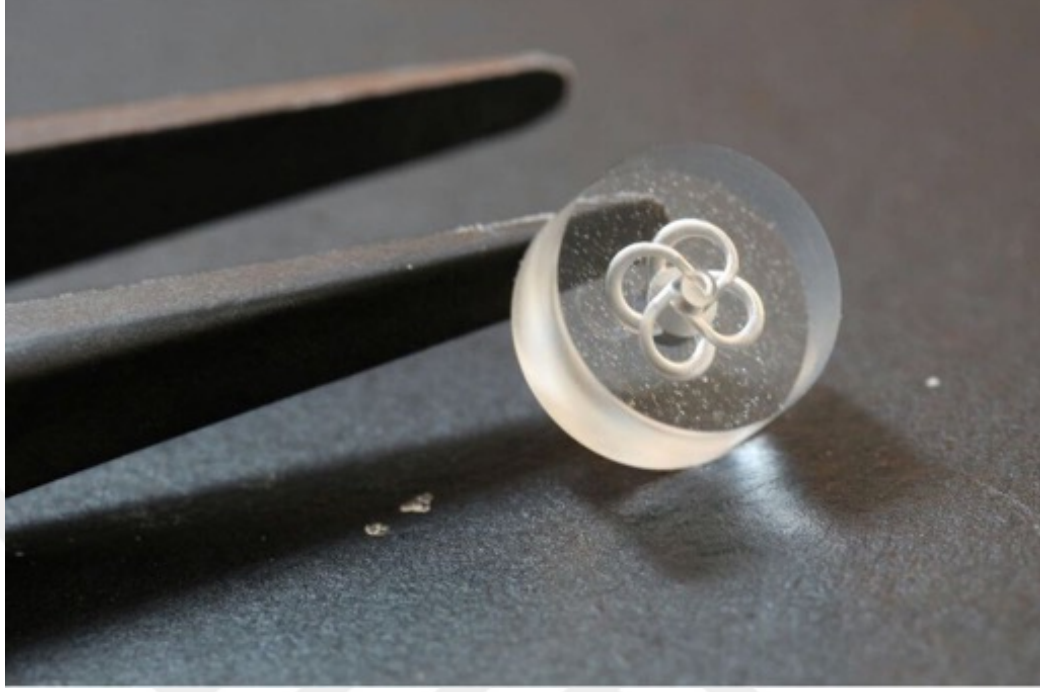


Figure 102: Image of the tested atomizer

Therefore, a non-traditional method known as laser-lithography is used for manufacturing. Detailed explanation of this method can be found in [92]. Based on the reported information from the manufacturer, maximum deviation between the designed and the manufactured dimensions were $1\ \mu\text{m}$. That is, the simulated and tested atomizer geometries have almost identical dimensions which is crucial in order to eliminate the possibility of manufacturing tolerances effecting the uniformity.

6.3 Results

First, results of the numerical studies are presented. Overall, 16 different geometries are used and details of these geometries are given in Table 26.

Table 26: Geometrical Details of the Simulated Cases

Case Number	D_p (mm)	D_s (mm)	α	D_o (mm)
Effect of Diameter of Inlet Ports				
1-2	0.30	1.20	10-13	0.84-0.71
3-4	0.28	1.20	10-13	0.84-0.71
5-6	0.26	1.20	10-13	0.84-0.71
7-8	0.24	1.20	10-13	0.84-0.71
Effect of Convergence Angle-Constant Swirl Chamber Diameter				
9	0.30	1.20	5	1.02
10	0.30	1.20	17	0.53
Effect of Convergence Angle-Constant Orifice Diameter				
11	0.30	1.50	16.70	0.90
12	0.30	1.20	8.53	0.90
13	0.30	1.00	2.86	0.90
Effect of Diameter of Swirl Chamber-Constant Convergence Angle				
14	0.30	1.50	5	1.32
15	0.30	1.00	5	0.82
16	0.30	0.90	5	0.72

It should be noted that all of the geometries have four tangential inlet ports and convergent section length of 1 mm. Length of the swirl chamber is equal to the diameter of the tangential ports for each geometry and all of the cases are studied at a constant flow rate of 3.34 g/s.

Inspection of Table 26 shows that table consists of four parts according to altered geometrical parameters. Effect of diameter of tangential inlet ports is studied through Cases 1-to-8. Effect of convergence angle is studied with Cases 1, 2, 9, and 10, where the diameter of the swirl chamber is kept constant. In other words, since the length

of the convergent section is constant, diameter of the orifice is changed. Cases 11-13 are also used for understanding the effect of convergence angle, yet, for these cases diameter of orifice is kept constant, and diameter of swirl chamber is altered. Lastly, Cases 14-16 and 9 are used to understand the effect of diameter of the swirl chamber while the convergent angle is constant.

For the visual inspection and evaluation of uniformity, two different pseudo-planes were introduced into the computational domain. These planes are located at the exit of the atomizer and 5 mm downstream of the atomizer. Calculation of the mass flux distribution at both planes is done, and obtained values at 5 mm downstream are used for assessing the uniformity. As mentioned in Chapter 3, simulations are conducted with Star-CCM+ 13.04.011, and calculation of uniformity is done with MATLAB.

Figure 103 shows the effect of convergence angle when the diameter of swirl chamber is constant. This figure refers to the results of Cases-1, 9, and 10. Inspection of the results clearly shows that both the flow at the exit of the atomizer and downstream of the atomizer is more uniform when the diameter of orifice is smaller. In other words, increasing the ratio of diameter of swirl chamber to the diameter of orifice increases the uniformity.

Increasing the Diameter of Orifice - Constant Swirl Chamber Diameter

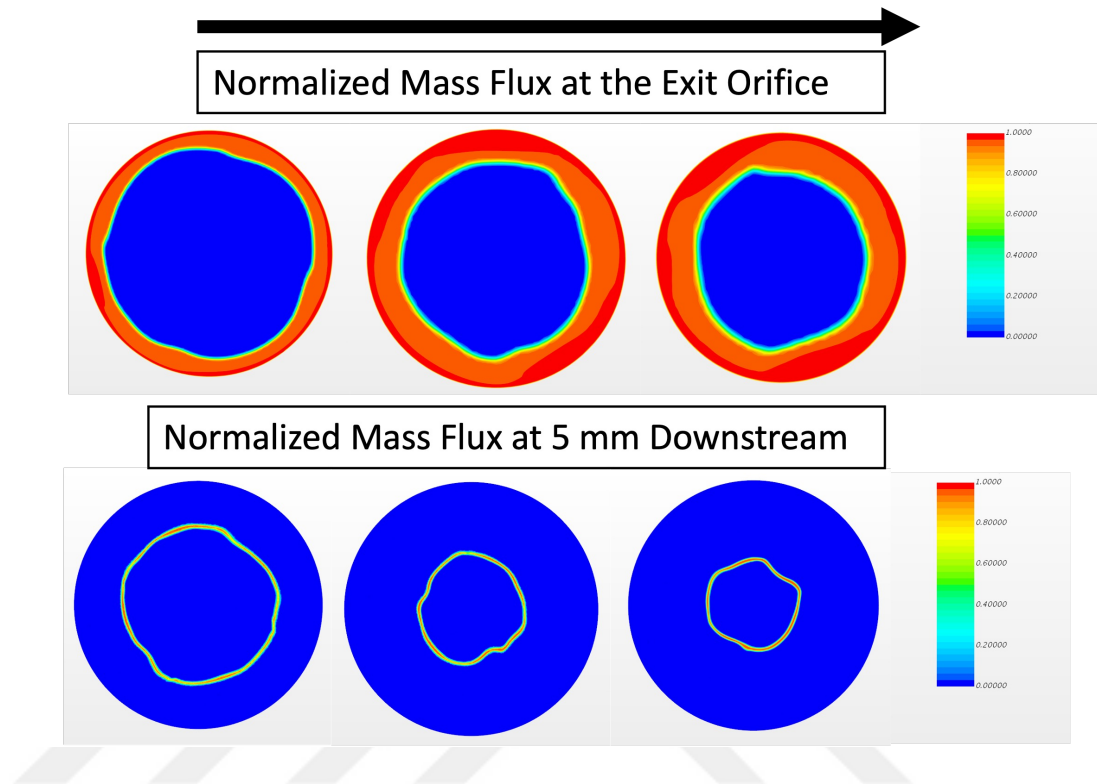


Figure 103: Effect of convergence angle on the mass flux distribution - constant swirl chamber diameter (Cases 1-9-10)

Figure 104 displays the effect of convergence angle when the diameter of orifice is constant. These are the results of Cases-11, 12, and 13. It is obvious that when the diameter of swirl chamber is smaller, the flow at the exit of the atomizer and downstream of the atomizer becomes non-uniform. In other words, again, increasing the ratio of diameter of swirl chamber to the diameter of orifice leads to more uniform internal flow and spray.

Increasing the Diameter of Swirl Chamber - Constant Orifice Diameter

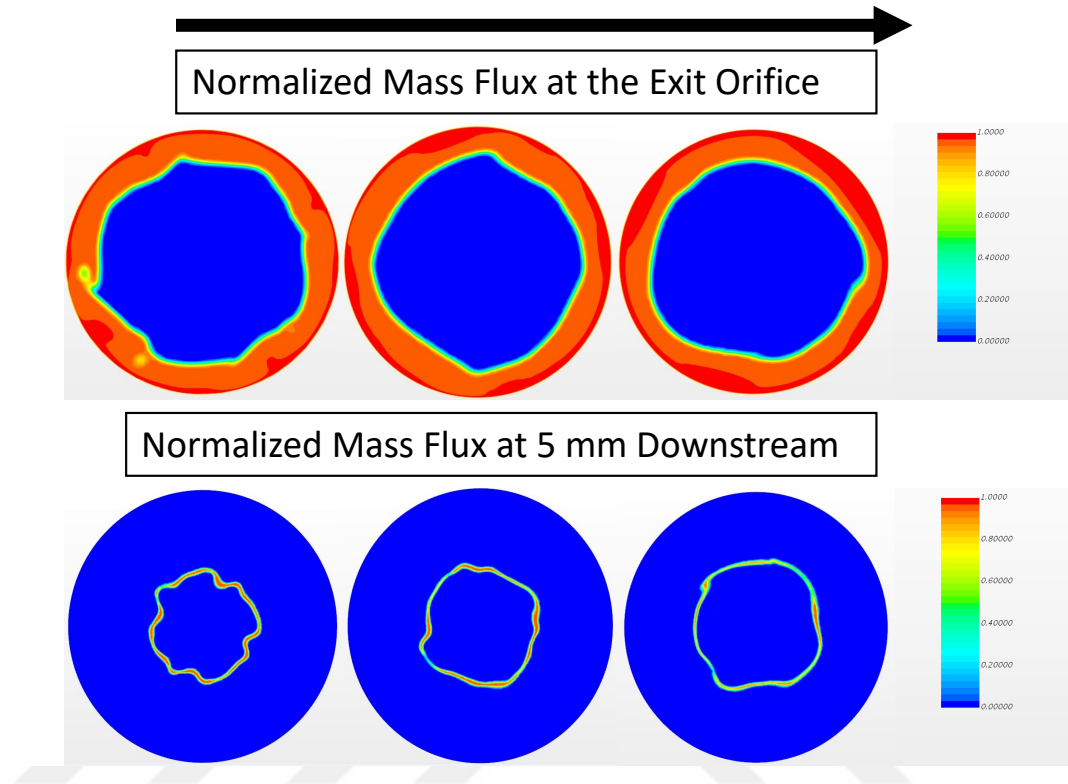


Figure 104: Effect of convergence on angle on the mass flux distribution - constant orifice diameter (Cases 11-12-13)

Figure 105 presents the results of Cases-14, 15, and 16, which are studied to analyze the effect of diameter of swirl chamber when the convergence angle is constant. In other words, as the diameter of the swirl chamber is increases, diameter of the orifice increases with the same ratio. Inspection of the results yields that spray and exit flow are more uniform when both diameters are smaller. However, it should be noted that difference in these cases, in terms of uniformity, is less obvious than the previous ones.

Increasing the Diameter of the Swirl Chamber - Constant Convergence Angle

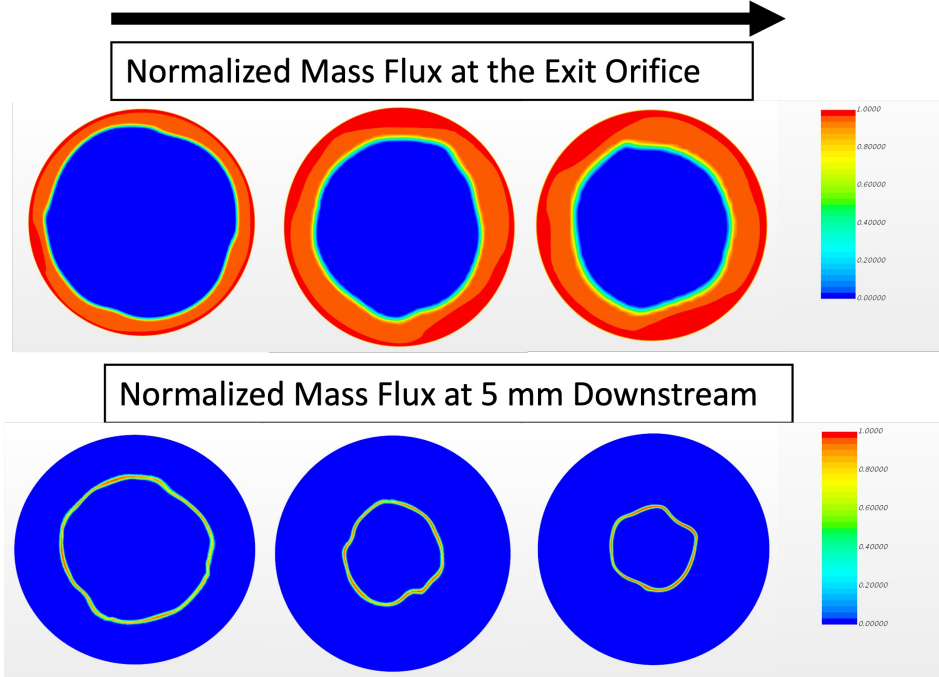


Figure 105: Effect of diameter of swirl chamber on the mass flux distribution - constant convergence angle (Cases 14-15-16)

As mentioned earlier, to quantify the uniformity, obtained mass flux values at 5 mm downstream of the atomizer are used. Γ value of each simulated case is calculated by dividing downstream mass flux into 24 pie-shaped sectors, as shown in Figure 106.

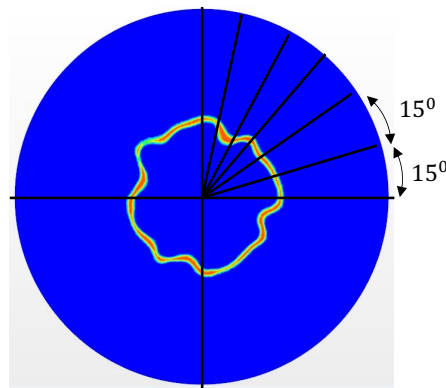


Figure 106: Calculation of uniformity

Summary of the results, based on the altered geometrical parameter, is provided in Figure 107. Results obviously show that increasing the convergence angle and diameter of the swirl chamber leads to more uniform flow. In contrast, increasing discharge orifice diameter and the diameter of tangential inlet ports lead to decreased uniformity.

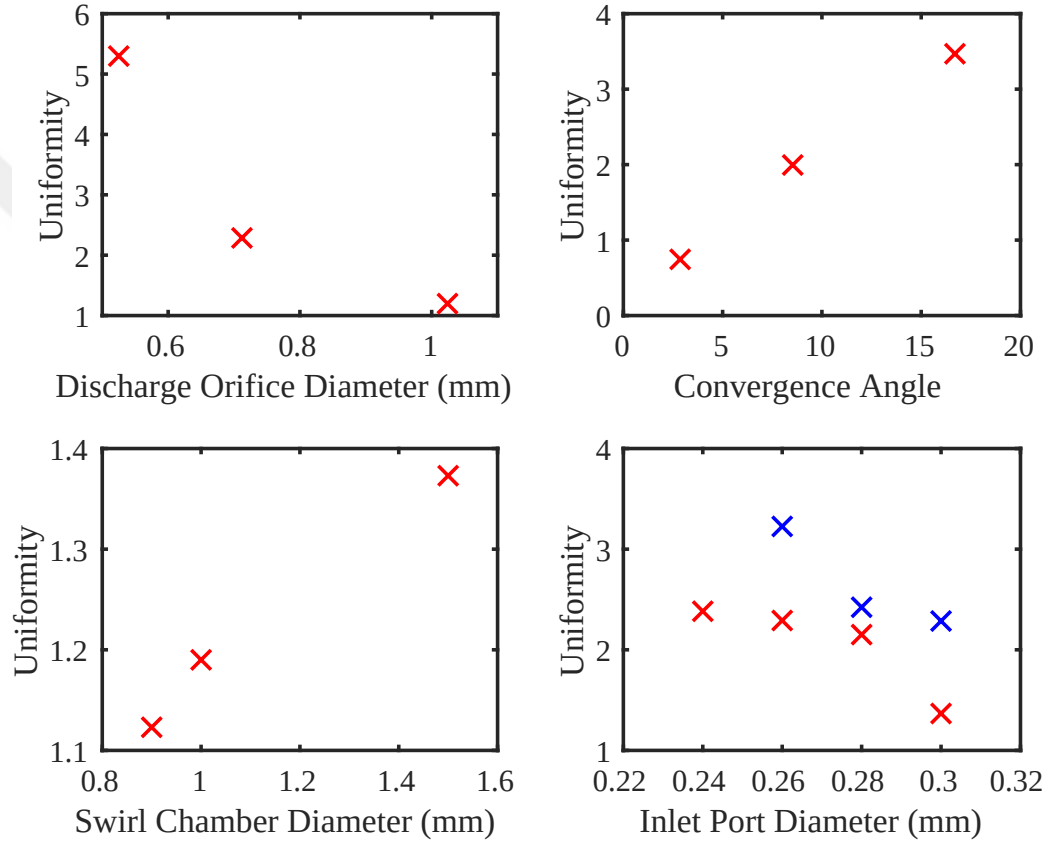


Figure 107: Effect of geometrical parameters on the uniformity

After evaluating the uniformity of the simulated cases, three of the studied atomizers, namely Cases-1, 6, and 13 are selected to be manufactured and to perform experimental evaluation. It should be noted that two of these atomizers, Case-1, and Case-6, have uniform flow while Case-13 is selected since it produced, according to the numerical results, lowest Γ value (most non-uniform spray) while having same inlet port diameter as Case-1. Moreover, it must be stated that experimental results

of these atomizers are presented in Chapter 4, and these atomizers are referred to as Atomizer - 22, 23, and 24.

Further validation of the numerical method is also conducted through the experiments, as well. Comparison of numerical and experimental results shows less than 8.5 % error in discharge coefficient and less than 3.5 degrees difference in full spray angle.

Table 27: Comparison of Numerical and Experimental Values at 3.34 g/s

Case	Num. C_D	Exp. C_D	Abs. Rel. Error of C_D
1	0.166	0.152	8.4 %
6	0.175	0.187	6.9 %
13	0.222	0.206	7.2 %
Case	Num. 2θ	Exp. 2θ	Abs. Error of 2θ
1	98.4	100.7	2.3
6	103.0	100.8	2.2
13	77.5	74.4	3.1

Furthermore, all three of the produced atomizers are inspected to evaluate the uniformity by means of high-speed imaging at axial plane and LIF/Mie method at the radial plane. An important note that has to be made is that even though uniformity values are calculated 5 mm downstream of the atomizer, in simulations, for the experiments same value is obtained at 32 mm downstream due to the optical access. Figure 108 displays the difference of flow regimes at both of the distances.

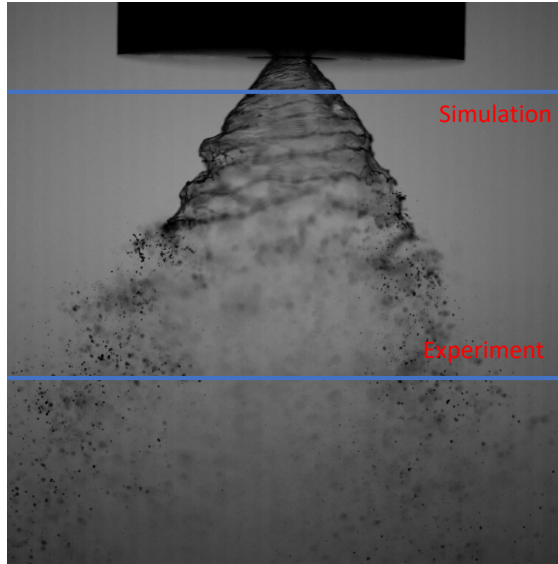


Figure 108: Location of uniformity calculations made for the simulations and experiments

Obtained instantaneous high-speed images of the spray are given in Figure 109. These images are obtained since many of the reviewed studies have related the presence of streaks and voids in the spray to the non-uniformity of the spray. Inspecting the images shows the presence of streaks for Case-13, whereas other two cases seem to have no streaks or voids.

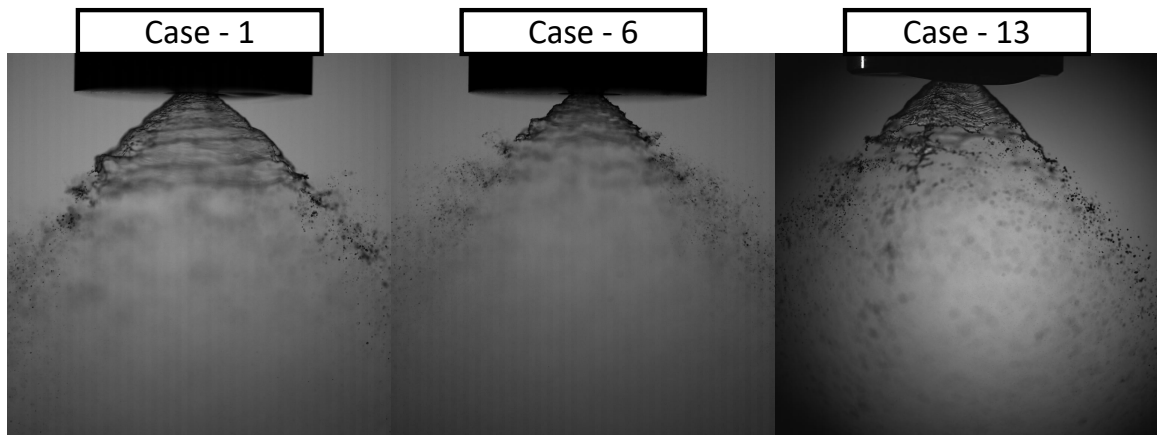


Figure 109: Instantaneous high-speed images of manufactured atomizers

For further inspection of existence of streaks or voids, mean of the obtained high-speed images is calculated and given in Figure 110. Inspection of results shoe that, Case-13 obviously has streaks and voids in the spray. Case-1, while being less obvious than Case-13, has some small voids and streaks in the spray, and Case-6 seem to have no streak or void in the spray.

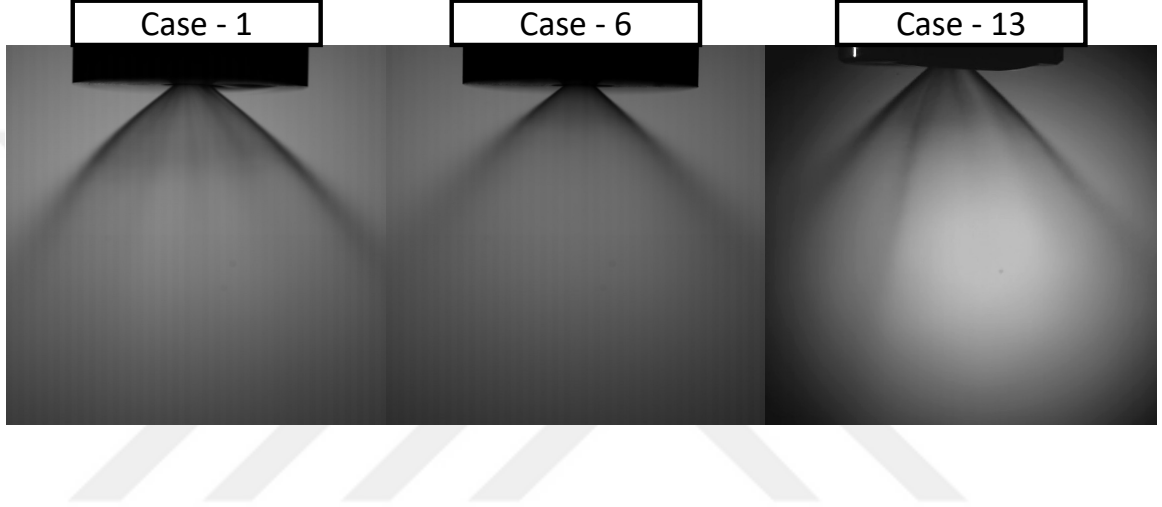


Figure 110: Mean of the high-speed images of manufactured atomizers

Comparison of LIF/Mie results with the numerical ones is given in Figure 111 for Case-1. Numerical results show that there are four thicker regions of film at 5 mm downstream, while LIF/Mie image shows some voids in the spray and a region where droplets are clustered. It should be stated again that there is a significant distance between the calculation locations of experiment and simulation. The Γ value calculated from the simulation is found to be 1.37, whereas the value of the experiment is 1.13.

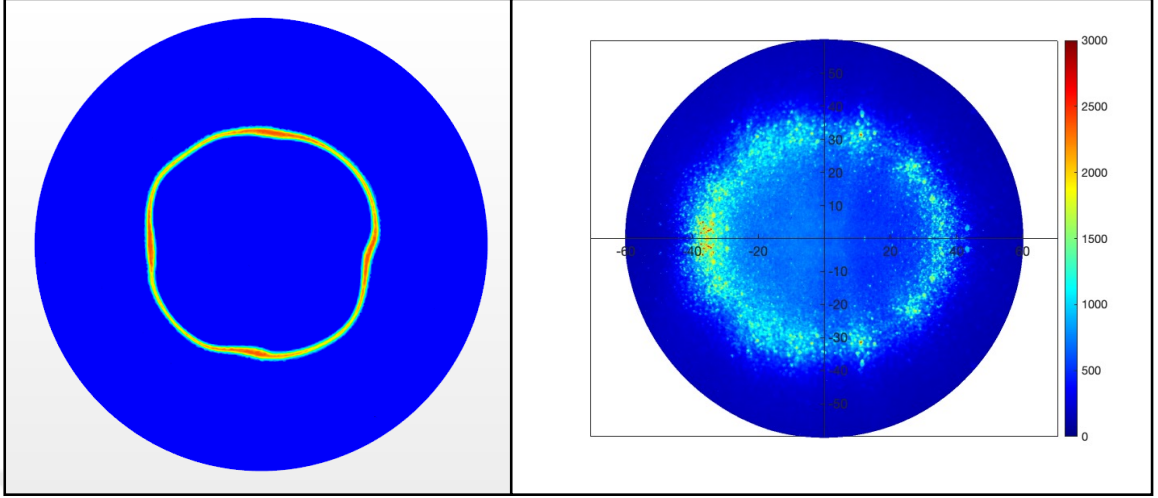


Figure 111: Comparison of LIF/Mie experiment with the simulation for Case-1

Figure 112 presents the results of simulation and LIF/Mie experiments for Case-6. For this case, Γ value is calculated as 2.97 from the simulations, and it is calculated as 2.18 from the experiments. Moreover, analyzing the images shows that Case-6 almost has a perfect spray at 5 mm downstream, and also distribution of the droplets further downstream does not have any voids or clustered regions.

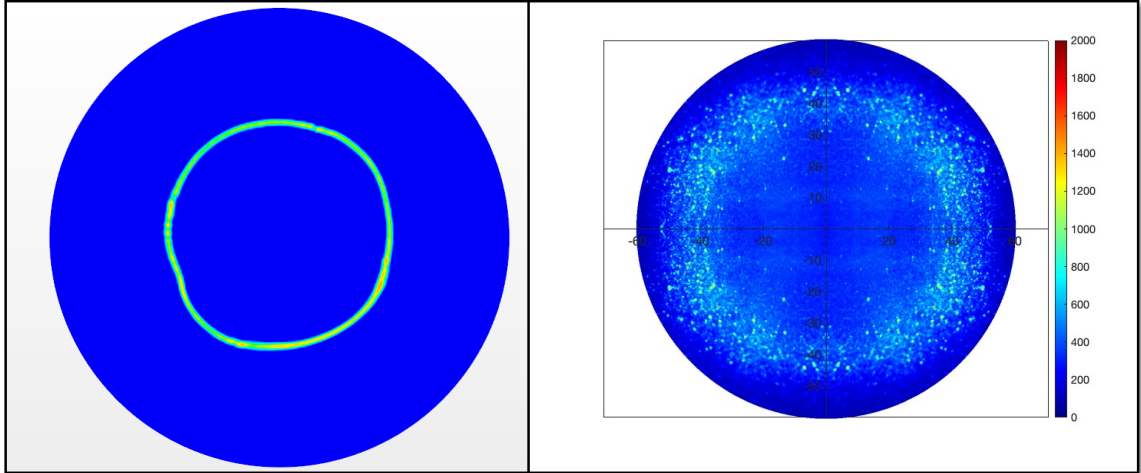


Figure 112: Comparison of LIF/Mie experiment with the simulation for Case-16

Results of the simulation and the LIF/Mie experiments for Case-13 are given in Figure 113. It is clear that the film of this atomizer displays high non-uniformity compared to other two cases. However, numerical results yield that areas with voids and high concentration are more evenly distributed compared to experimental results. LIF/Mie image shows that there are areas with very large concentration of droplets and large voids. From the experimental results, the Γ value is calculated to be 0.48, whereas the value obtained from the simulation is 0.59.

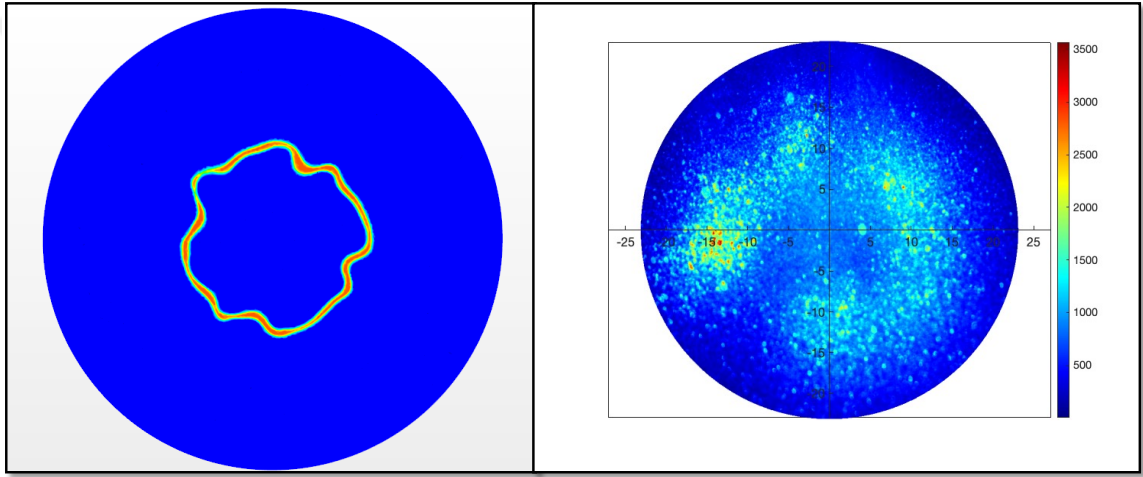


Figure 113: Comparison of LIF/Mie experiment with the simulation for Case-13

Lastly, both numerical and experimental results are presented in Figure 114. Inspection of the complete results clearly certifies the relation of streaks and voids with the non-uniformity of the flow. The most uniform spray exhibits no such behavior, whereas the least uniform spray has clear streaks and voids. A small note can be made that, in Case-1 four regions of thicker film can be seen from the simulation results. Also, for Case-13, there are four regions where the film is folded inwards and outwards. As mentioned earlier, all of these atomizers have four inlets, and film of these atomizers displays same number of regions with flow deficiency.

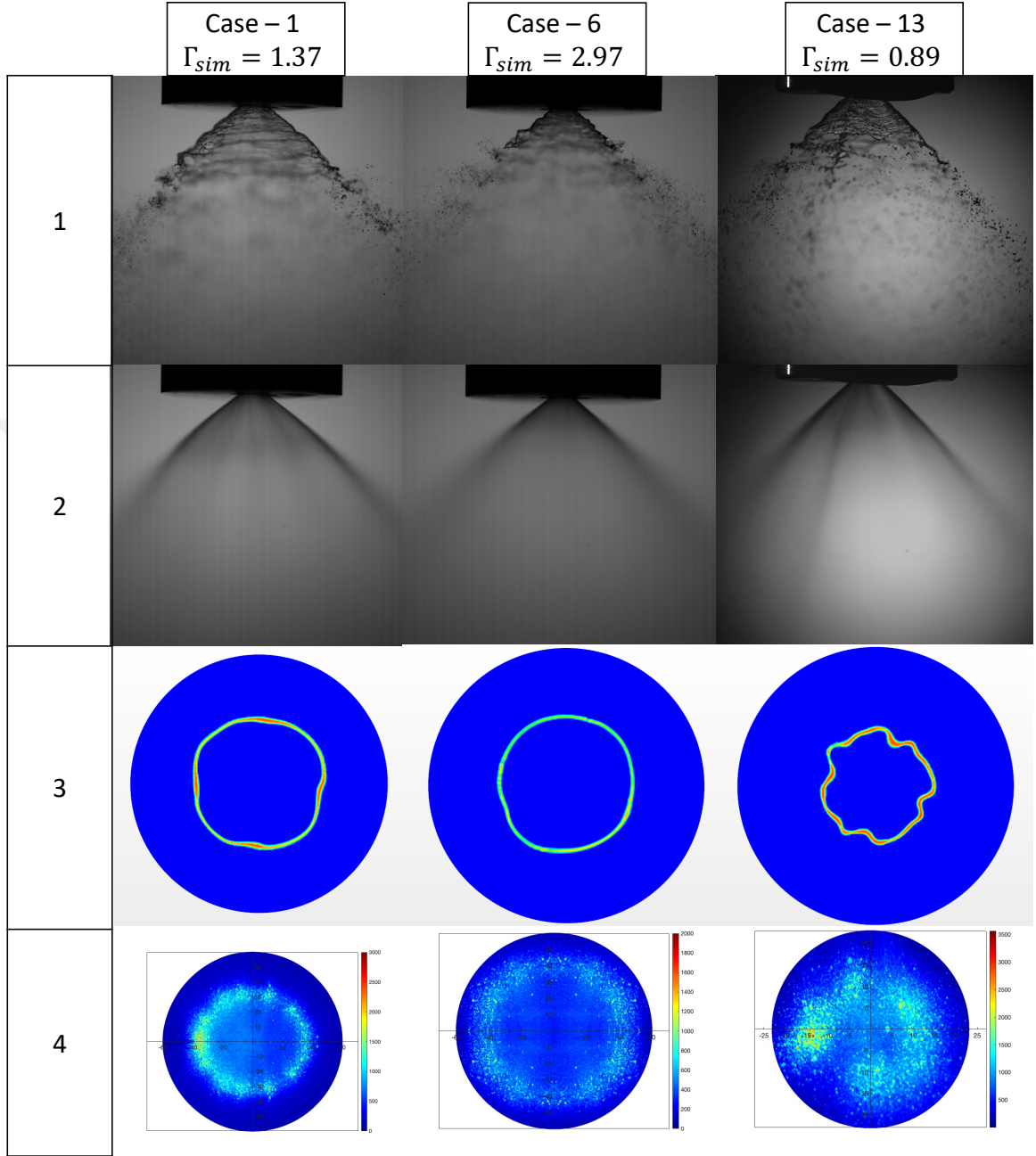


Figure 114: Obtained instantaneous high-speed images (row 1), average of high-speed images (row 2), obtained mass fluxes from the simulations at 5 mm downstream (row 3) and LIF/Mie experiment results (row 4)

Finally, to understand the reason of non-uniformity, internal flow of Case-1 and Case-13 is analyzed. Figure 115 shows the distribution of the velocity for Case-1 and

Case-13 at the center-line of the inlets. It should be reminded that for these cases, diameter and number of tangential ports were same, and since both are simulated at constant flow rate, they share the same inlet velocity. From the distribution of the velocity, it can be seen that Case-13 displays high-velocity regions close to the center axis of the atomizer, whereas Case-1 also has some regions with higher velocity, but it is not as dominant as Case-13.

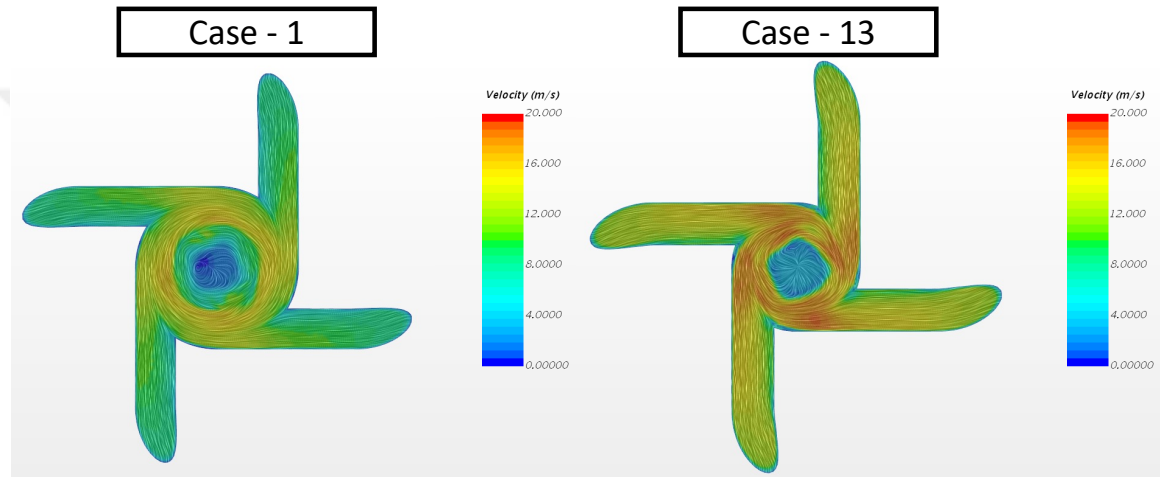


Figure 115: Velocity distribution of Case-1 and Case-13 at the centerline of the inlets

For further inspection, close-up velocity vectors at the inlet region, Figure 116, for these atomizers are inspected. Results for Case-13 show that there are regions of separation where the tangential inlet ports open up to the swirl chamber. Behavior of flow at the swirl chamber indicates that as the flow enters from one of the tangential ports, it is pushed to center axis of the atomizer via the flow from the adjacent tangential port. Case-1 displays no such behavior, and flow seems to be attached to the walls of the swirl chamber in all regions. It should be mentioned that the only difference between these two cases is the distance between the tangential inlet ports, whereas Case-13 has 20% smaller distance. It is clear that having a smaller distance causes the flow from the tangential inlet ports to interact with each other and disrupts both the internal and external flow of the atomizer.

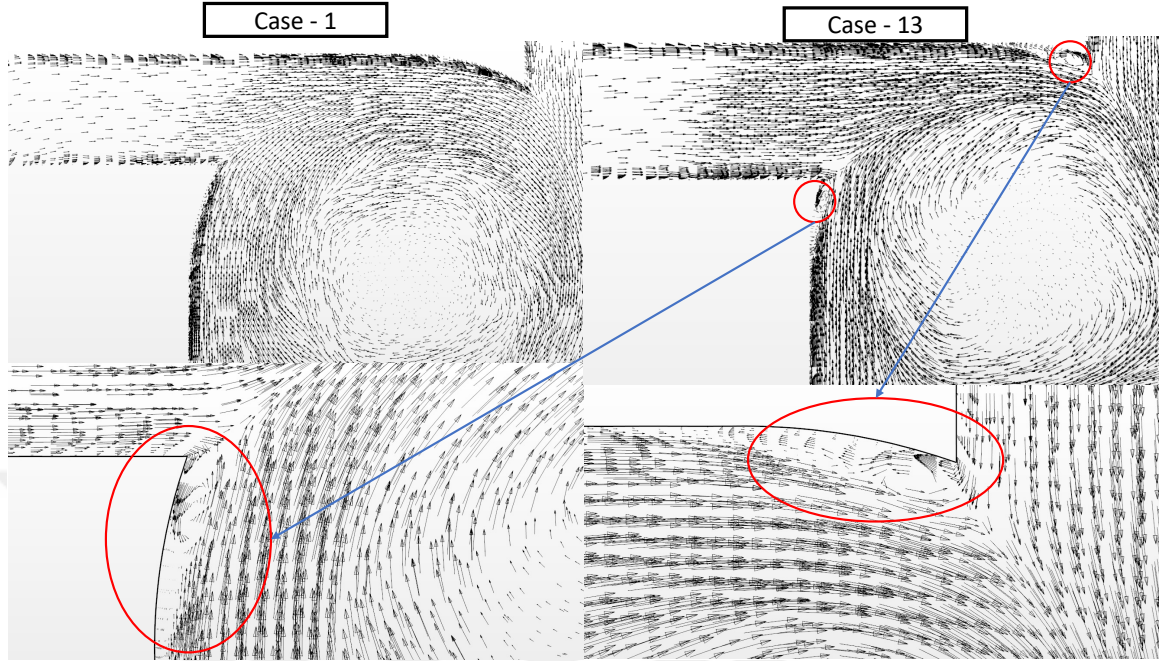


Figure 116: Velocity vectors at tangential inlet ports for Case-1 and Case-13

To emphasize this effect structure of the air core and the liquid sheet leaving the atomizer is given in Figure 117. It should be mentioned the yellow part inside the atomizer displays the structure of the air core, and red part at the atomizer exit is the structure of the liquid sheet. It can be clearly seen that there are regions with voids in the air core and regions of wrinkles present for the liquid sheet. Moreover, number of these regions is equal to the number of the tangential inlet ports, and these defects are clearly caused by the above-mentioned interaction of the flow from the tangential inlet ports.



Figure 117: Structure of air core and liquid sheet for Case-13

6.4 *Model*

A new model for describing the uniformity, based on the geometrical parameters and the size of the air core, is obtained using the Γ values of the simulations. Obtained model is given in Eq. 453, and comparison of the derived model with the model of [5] is done in Figure 118. It should be noted that, to the author's best knowledge, this

is the only correlation that exists in the literature for calculating uniformity.

$$\Gamma = 1.74A^{-0.17}\phi^{0.45} \quad (453)$$

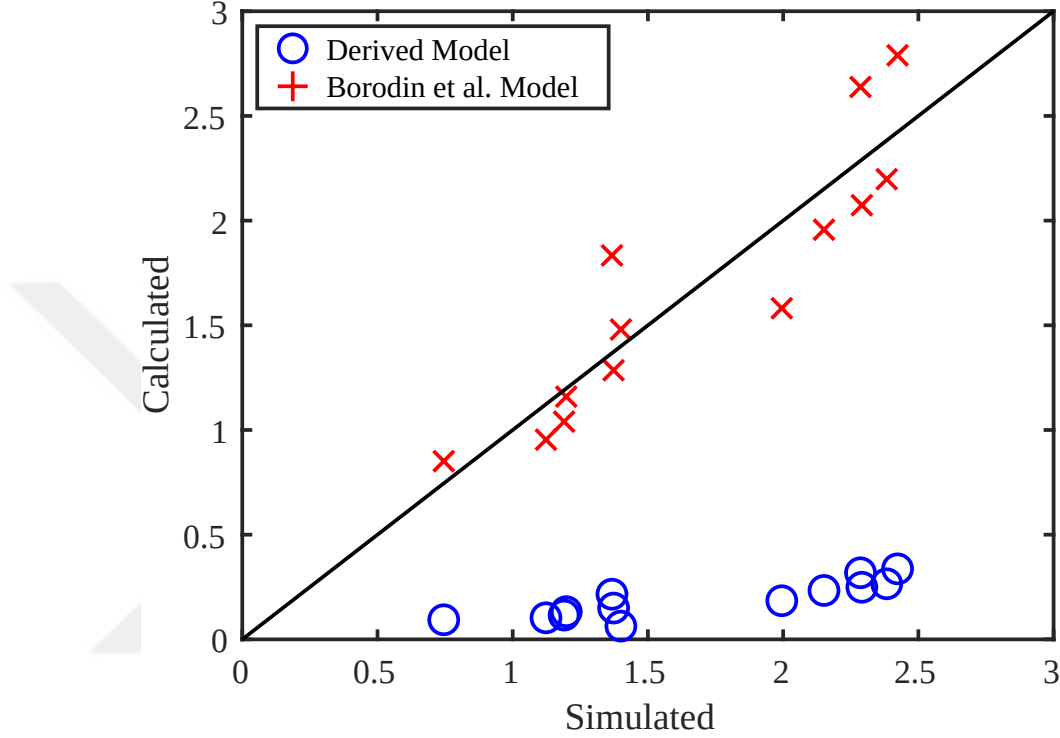


Figure 118: Comparison of semi-empirical correlations for calculating uniformity using simulations

From the results, it is clear that derived correlation can accurately predict the uniformity values obtained from the simulations. Model of the [5], Eq. 450, underpredicts the uniformity values for all cases. To evaluate the validity of derived correlation and to make further comparison with Eq. 450, experimental data of [30] is also used. Data from this study can be found in Table 53. Similar to the correlation for calculation the uniformity, to the author's best knowledge, this data is the only one that used ϑ for quantifying uniformity. However, measurement or calculation of film thickness is not available for these atomizers. Since both Eq. 453 and Eq. 28 include a term for film thickness, this parameter is calculated using Eq. 156.

Results of the comparison can be found in Figure 119. It is clear that obtained correlation can predict uniformity more accurately. Model of this study, Eq. 453, has the AMRE value of 0.200, whereas Eq. 450 has AMRE value of 0.786.

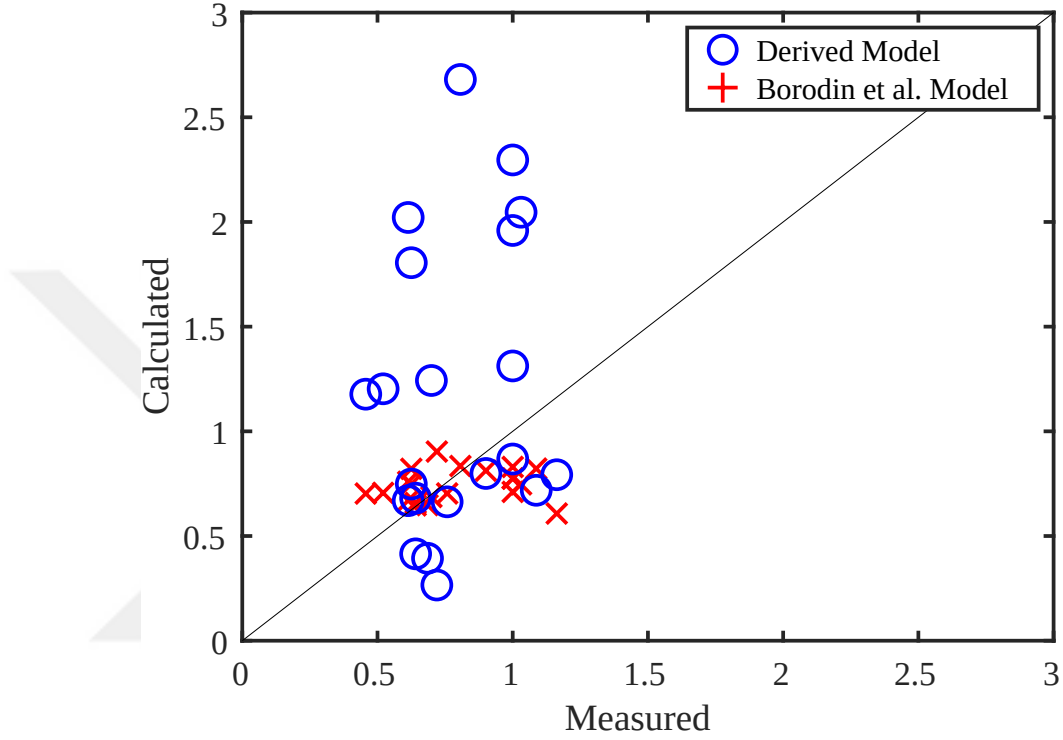


Figure 119: Comparison of semi-empirical correlations for calculating uniformity using data of [30]

An important note that has to be made is, as mentioned earlier, geometry of the atomizer used to study uniformity is different from the conventional PSA geometry. In the studied geometry, length of the swirl chamber is equal to the diameter of the tangential inlet ports. Moreover, this is also the case for the geometries of the atomizers in [30]. Here, the authors used atomizers with rectangular inlets, and the length of the swirl chamber was equal to the height of the tangential inlet port. This could be the reason Eq. 450 has high deviations since it is derived for more conventional geometries. However, this needs to be studied.

CHAPTER VII

SPRAY DEVELOPMENT ANALYSIS

7.1 *Introduction*

Development of the spray is an important topic of the performance of the PSA. However, during the literature review, it is seen that only a few studies have studied the development of the spray, and fewer have accepted the challenge of modeling it. Most of the studies are limited to reporting the stage of the spray at different flow rates or pressure differences.

Spray of the PSA goes through several stages before forming into its final form, which is usually named fully developed spray. It should be noted that there have been debates about the number and definitions of these stages. One of the studies has suggested that there are five stages during the development of the spray [2]. Authors have named them dribble stage, distorted pencil stage, onion stage, tulip stage, and fully developed spray. Schematic of these stages is given in Figure 120.

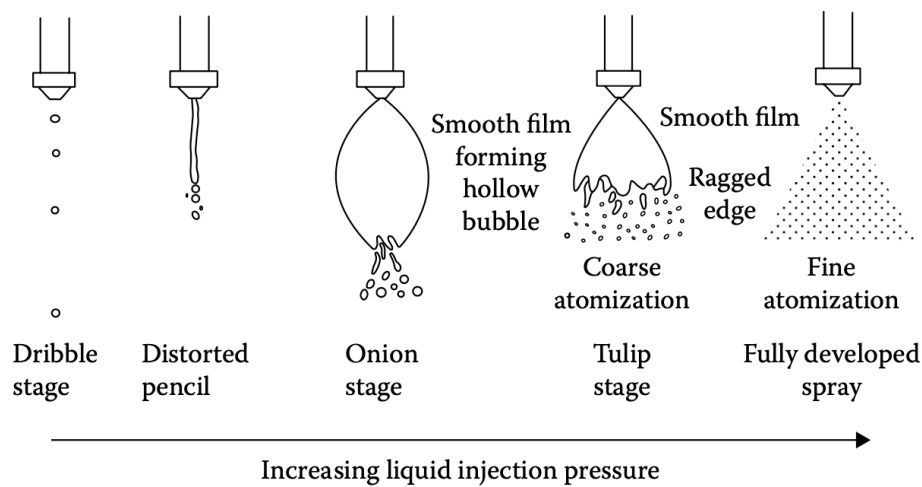


Figure 120: Schematic of spray development stages [2]

Whereas it was also stated that there are nine different stages, as shown in Figure 121 [3]. Inspection of this schematic yields that there are resemblances with the schematic in Figure 120. It is clear that Stage 1 in Figure 121 refers to the dribble and distorted pencil stage. Stage 2 is the onion stage, Stage 3 is the tulip stage, and Stage 4 is the fully developed spray. Differences between these studies are in the further stages.

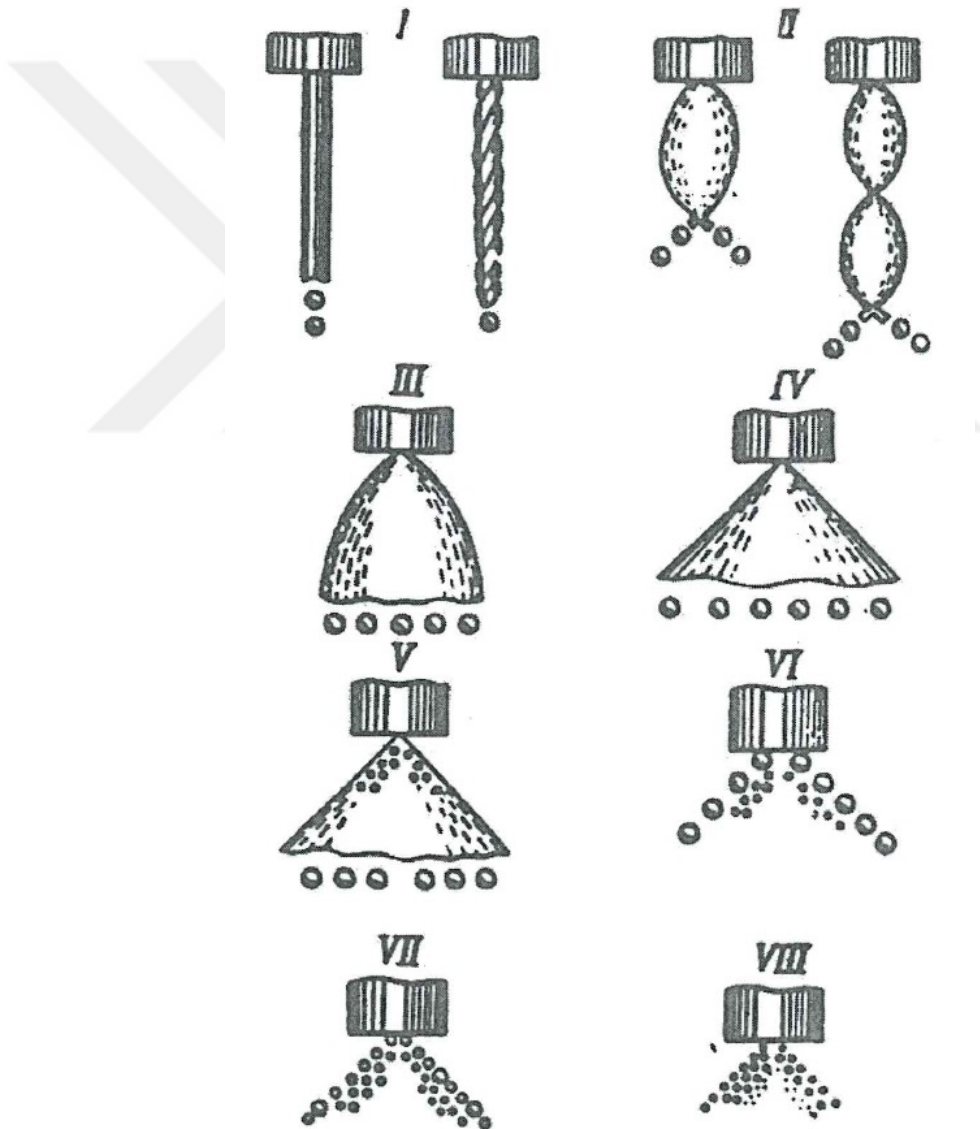


Figure 121: Schematic of spray development stages [3]

According to the author, Stages 5-to-8 are defined as:

- Stage 5: Small droplets appear in the center of the spray, whereas in Stage 4 these droplets are larger,
- Stage 6: Film leaving the atomizer disappears, yet droplet sizes are similar to the Stage 5,
- Stage 7: Droplet sizes are smaller in comparison to Stage 6,
- Stage 8: Atomizer external flow is in the form of continuous fog.

It should be noted that, during the literature survey, no other study could be found with a spray going through the stages shown in Figure 121. All of the studies seem to agree with the description shown in Figure 120, even though there might be small differences. For example, a stage described as wavy stage is observed between the tulip stage, and fully developed spray [93]. Moreover, presence of another stage between the wavy stage and tulip stage is also reported [94, 95]. However, there are also studies, in which only the five stages described in Figure 120 [96].

Experiments conducted in this study are also an example of such case. During the high-speed imaging of the sprays, images at low flow rates are also obtained to observe the development of the spray. Figure 122 shows these stages for Atomizer-17 of this study.

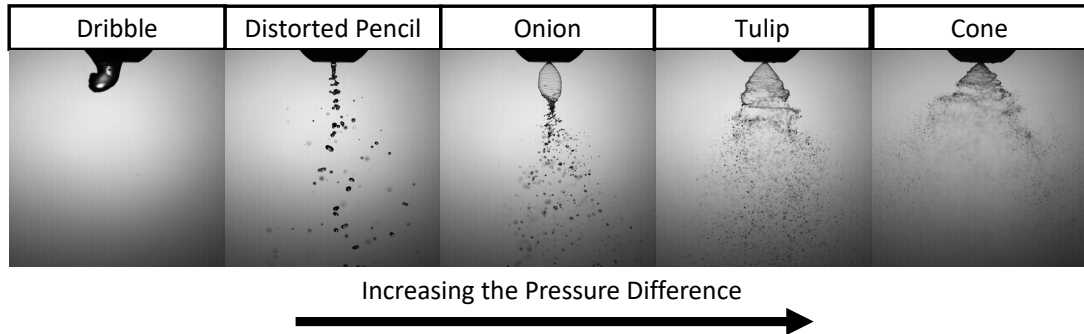


Figure 122: Obtained high-speed images of spray development stages

Figure 122 allows to detect five stages and also allows to make further observations. It is clear that during the dribble and distorted pencil stages, quality of the atomization is very low. That is, size of the resultant droplets for these stages is large in comparison to the others. In the onion stage, liquid sheet leaving the atomizer opens up at first yet later collapses and results in, again, low atomization quality. Moreover, it is clear that in onion stage, spray cannot produce desired coverage area or spray angle. Starting from the tulip stage, instead of large droplets, smaller droplets begin to form, which indicates that, atomization quality is getting better, and in the fully developed stage, spray takes its final form. Similar observations were also made earlier than the present study, [16]. Authors have defined a term called "Minimum Useful Spray Point (MUSP)" which corresponds to stage where spray opens up, i.e. tulip stage. Also, authors have stated that, from this point on, spray can be accepted as useful spray.

It should be noted that development of the spray is important since all of the models developed or evaluated in the frame of this study either assumes that spray is developed or fails to make accurate predictions at non-developed stages. For example, boundary layer model presented in Chapter 5 assumes that the spray would not collapse on itself. Moreover, it is clear that semi-empirical correlations of spray angle or SMD that are evaluated and derived in Chapter 4 will fail to make accurate predictions for the sprays at dribble, distorted pencil, or onion stage.

7.2 Previous Models

As mentioned earlier, only a small number of studies have attempted to model the development of the spray. In one of the earliest studies, modeling was done based on the results of experiments conducted with five different atomizers and fuels having three different viscosity values [16]. Based on the supplied experimental data in this study, a correlation is formed and presented in Eq. 454. It should be noted that

FN_{dim} is the dimensional flow number, defined in Eq. 76 yet here, flow rate has the unit of kg/hr and pressure difference has the unit bar. Moreover, viscosity has the unit of Pa.s, and the resultant MUSP has the unit of g/s.

$$MUSP = 0.266 \times FN_{dim}^{0.785} \times \mu_L^{0.320} \quad (454)$$

Modeling based on determining the region of each stage is also available in the literature, as shown in Figure 123 [17]. Here, authors have created separation lines to distinguish the stages distorted pencil, onion, tulip, and fully developed spray, based on the Ohnesorge number (Eq. 455) and the pressure difference.

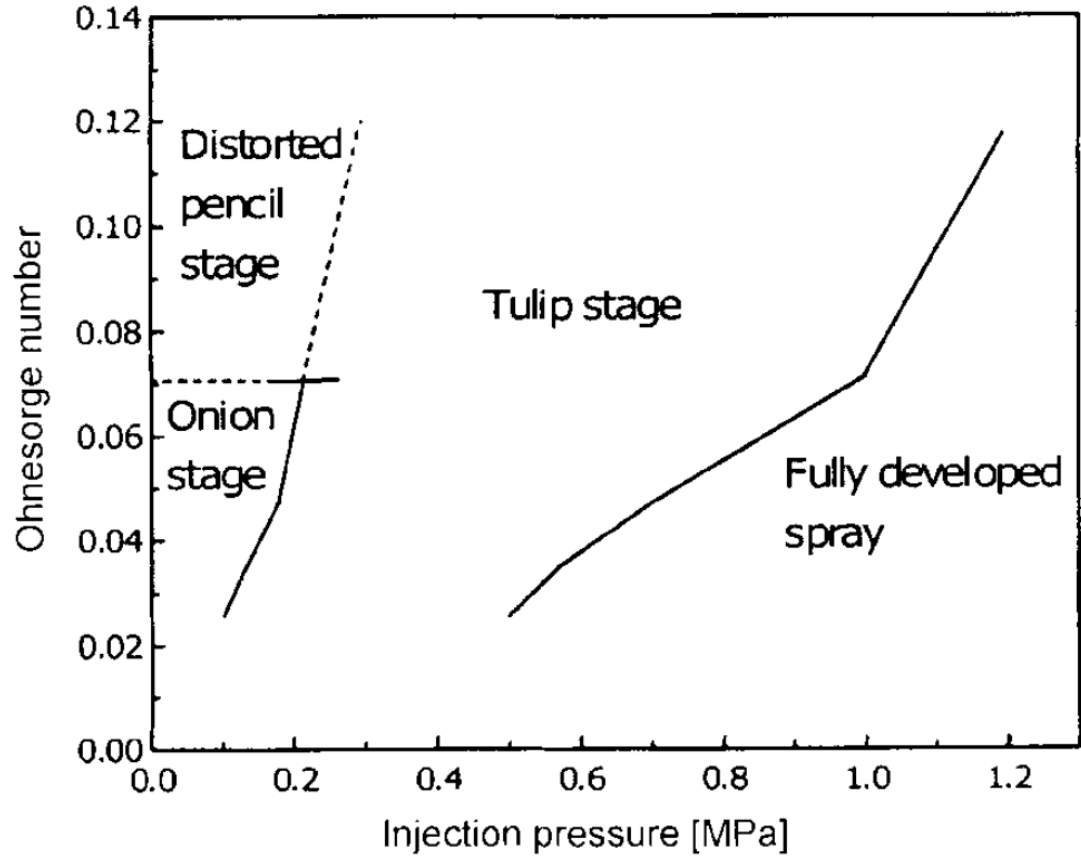


Figure 123: Stage-by-stage modeling of spray development [17]

$$Oh = \frac{\sqrt{We_B}}{Re_B} \quad (455)$$

Moreover, much simpler models are also present in the literature. It should be noted that these models are based on the variations of Reynolds and Weber numbers. A model based on the bulk Reynolds number and bulk Weber suggests that minimum useful spray point occurs at $Re_B = 500$ and $We_B = 50$ [18]. Another model is based on the Weber number calculated from Eq. 456, where U_B is the bulk velocity, and t is the film thickness [19]. In this study, authors have found that for $We_L \leq 140$ spray will be in tulip stage, and for $140 \leq We_L \leq 170$, tulip stage-to-fully developed spray will occur and finally, for $We_L \geq 170$ spray will be fully developed.

$$We_l = \frac{\rho_L \times U_B^2 \times t}{2 \times \sigma_L} \quad (456)$$

Lastly, use of the exit Reynolds number for the determination of the stage of spray is suggested [3]. Suggestions of this study can be listed as:

- at $Re_E = 1500$, spray is at onion stage,
- at $Re_E = 2100-2500$, spray is at Stage-5 of Figure 121,
- at $Re_E = 2800-3000$, spray is at Stage-6 of Figure 121,
- at $Re_E = 4500-5000$, spray is at Stage-8 of Figure 121

where,

$$Re_E = C_D \times Re_B \quad (457)$$

7.3 Experimental Data

In this section, experimental data obtained from the conducted experiments and data obtained from the literature, regarding the development of the spray, will be presented. These data will be used to evaluate models of the literature and to derive

a new model. Experimental data from the literature are included since experiments conducted in this study are done using water only, and this could lead to biased results.

Moreover, instead of dividing data into five stages, as shown in Figure 120, it is divided into two groups, namely useful and non-useful spray as defined by [16]. This method is used since the aim here is to obtain the point of useful spray and not the specific stage of the spray for the given atomizer geometry and operating condition.

First, data from the experiments conducted in this study is presented. Figure 124, shows the results of 24 atomizers, which are also mentioned in Chapter 4. It should be noted that data given in Chapter 4 only contains the results of experiments at the useful spray points. Overall, 573 data points can be found in Figure 124.

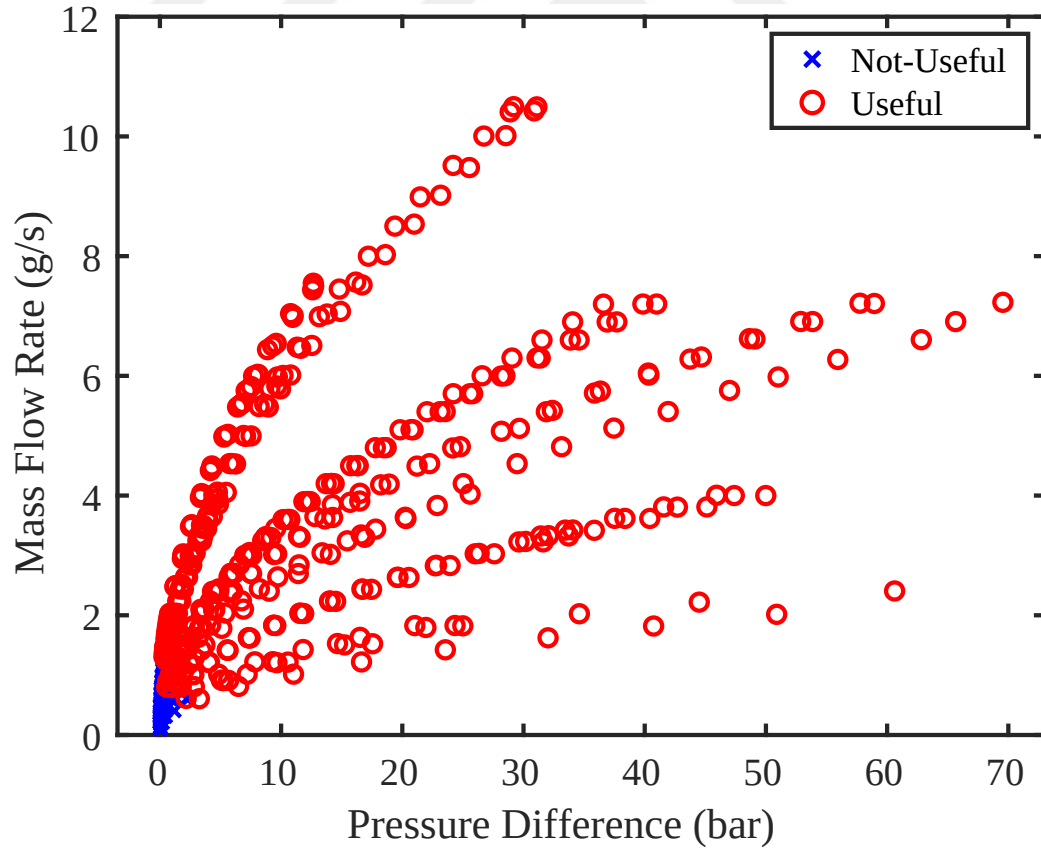


Figure 124: Useful and non-useful spray points of this study

Additional experimental data is obtained from the literature and contains 240 data points [9, 26, 29, 30, 34, 38–41, 43, 44, 52, 97–99]. Similar to data of this work, data obtained from the literature is divided into two groups, namely, useful and non-useful spray and is presented in Figure 125.

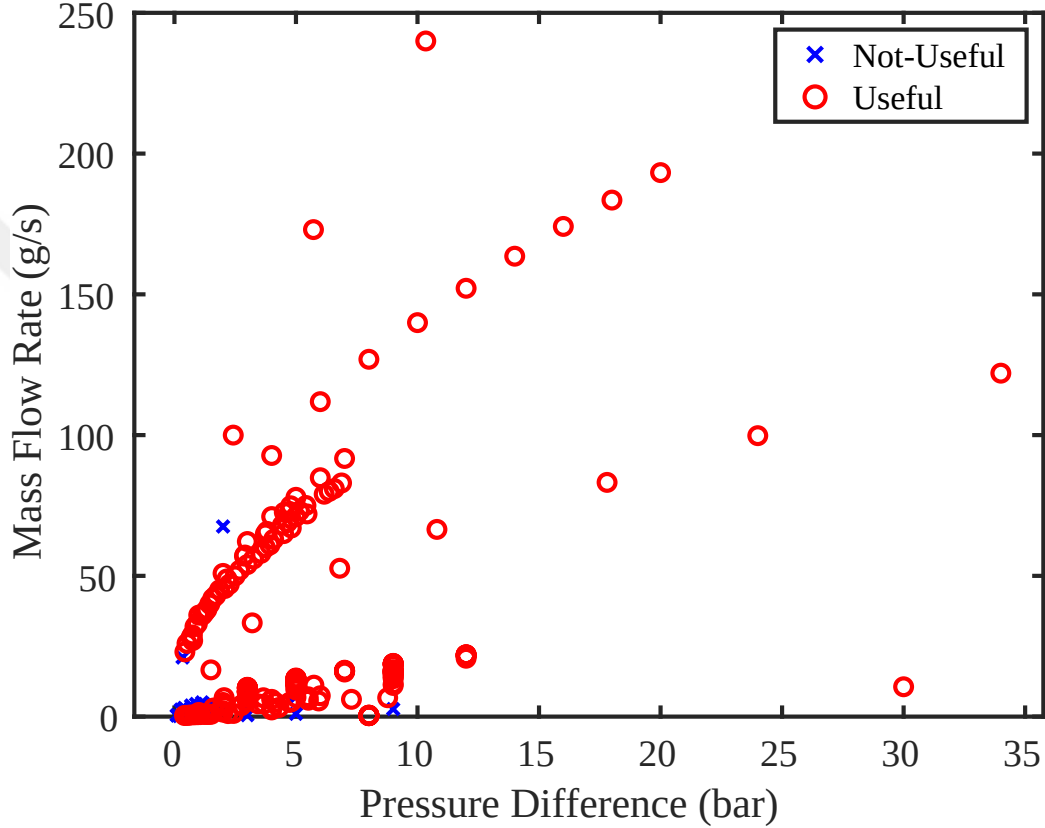


Figure 125: Useful and non-useful spray points of literature

Inspection of Figure 125 shows that obtained literature data contains a wide range of flow rates. In other words, it contains data of atomizers with various scales. Moreover, it contains data from the experiments conducted with various liquids such as methanol, water-detergent mixture, glycerol-water mixture [98], light heating oil [26], kerosene [99], and other types of aviation fuels [38, 41]. In addition, data for both open-end and close-end atomizers is present. It should be noted that some of the studies referenced here are also mentioned in Chapter 4, yet the data presented in

this chapter only contains the experimental results of the developed spray stage. Also, determination of the spray stage and hence useful or non-useful stage separation is done by either inspecting the images of the spray that are presented in these studies or through the statement of the author(s) that conducted the study.

7.4 Obtaining Spray Development Model

In this section, derivation and results of the obtained model are presented. Moreover, comparison of the obtained model with the models obtained from the literature is also done. Inspection of the models of the literature has shown that most of the correlations include a term of performance parameters such as pressure difference, discharge coefficient, or film thickness. It should be noted that aim of having a model for determining whether the spray is useful or non-useful should be determined prior to calculation of the performance parameters. As mentioned earlier, models for determining the performance either assume that the spray is developed or derived based on the experiments conducted at useful spray regime. Therefore, it can be said that using the models derived to calculate performance parameters in the non-useful spray regime will lead to erroneous results. Also, models that are not based on the performance parameter are derived using bulk Reynolds and Weber numbers. Hence, in this study same method is applied. In other words, developed model will be based on the bulk Reynolds and Weber numbers. Moreover, when all of the data is plotted in terms of bulk Reynolds and Weber numbers and using logarithmic scale, it is seen that a separation line can be obtained as shown in Figure 126.

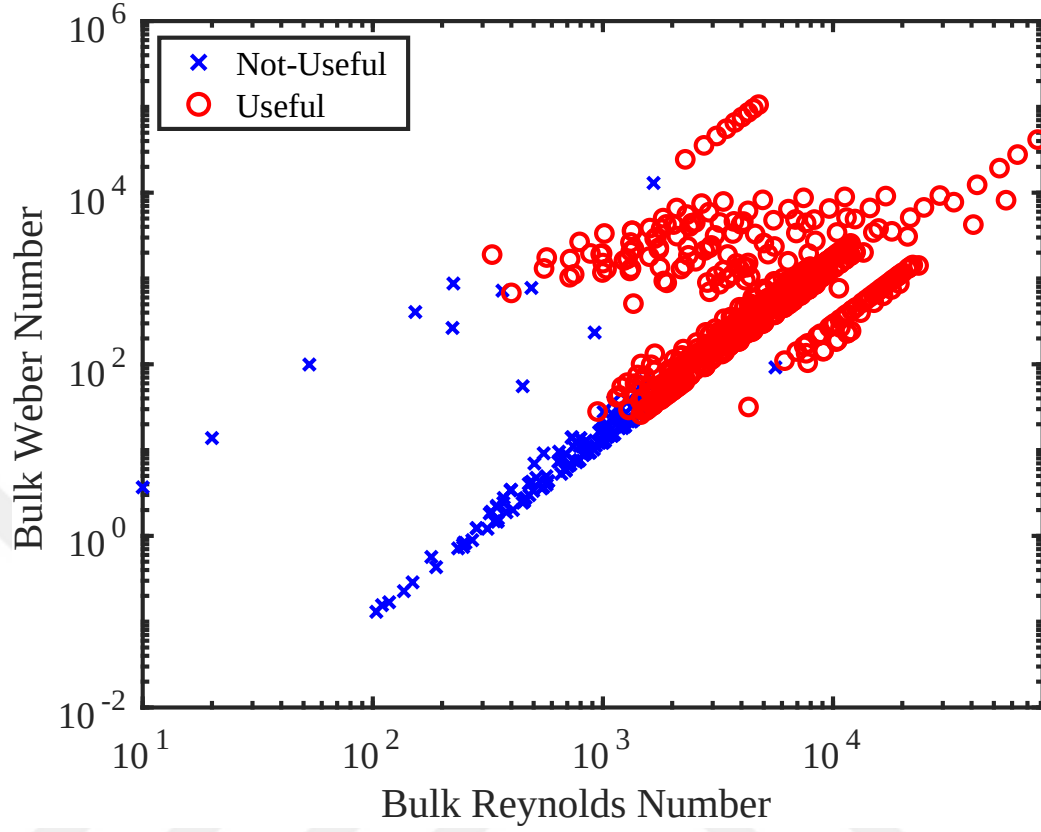


Figure 126: Useful and non-useful spray points of combined data (present study and literature)

Before, explaining the methodology of obtaining the separation line, basics of separating classes with lines are explained. For a simple case that is shown in Figure 127, there are two points on a line that has the coordinates of (p_1, p_2) and (p_1', p_2') .

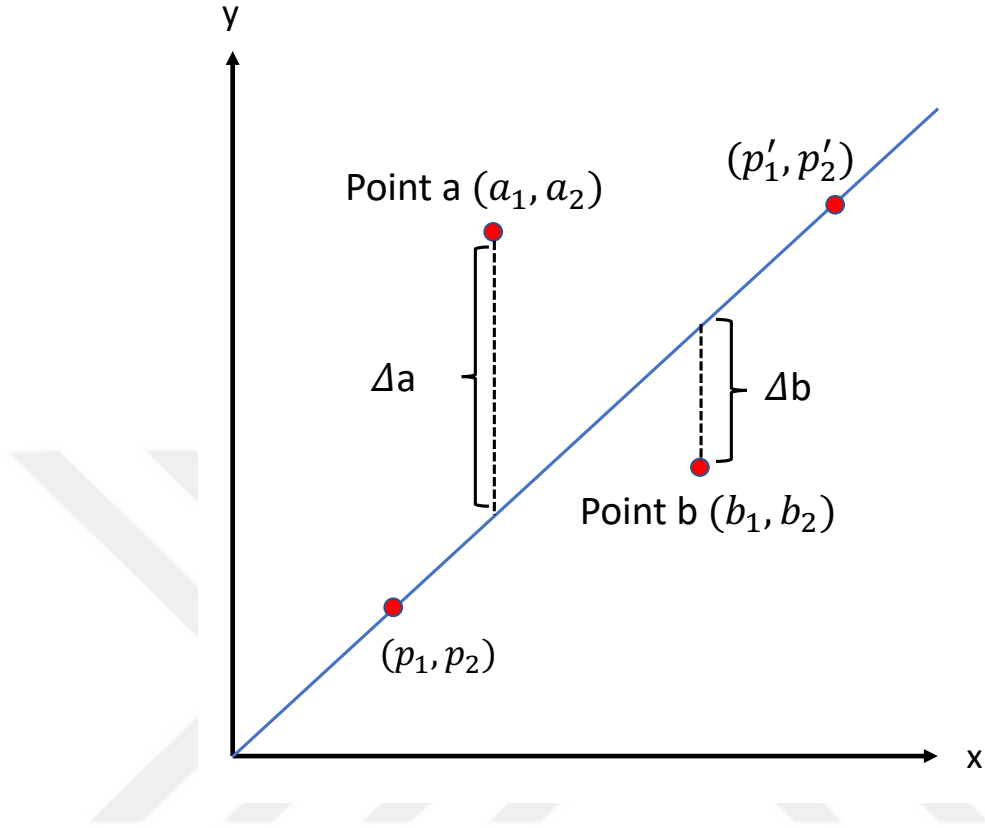


Figure 127: Separation line demonstration

This line can be defined as (since it goes through origin):

$$y = mx \quad (458)$$

where

$$m = \frac{p_1}{p_2} \quad (459)$$

Therefore for the second point on this line, it can be said that:

$$m \times p_1' - p_2' = 0 \quad (460)$$

For point b, which is located below the line, there is a distance to the line that can be denoted by Δb . It can be said that, the point with the coordinates of $(b_1, b_2 + \Delta b)$ is on the line. Hence Eq. 460 can be written as:

$$m \times b_1 - (b_2 + \Delta b) = 0 \quad (461)$$

or

$$m \times b_1 - b_2 = \Delta b \quad (462)$$

Since Δb is the distance between point b and the line, it will always have a positive value and therefore $m \times b_1 - b_2 > 0$.

Similarly, for point a, which is located above the line, with a distance of Δa , it can be said that the point with coordinates of $(a_1, a_2 - \Delta a)$ is on the line. Therefore, Eq. 461 becomes:

$$m \times a_1 - (a_2 - \Delta a) = 0 \quad (463)$$

or

$$m \times a_1 - a_2 = -\Delta a \quad (464)$$

Again, since Δa is the distance between point a and the line it will have a positive value and hence, $m \times a_1 - a_2 < 0$.

To sum up, for any point with coordinates of (x_1, x_2) and a line with the form of $y = mx$, if $mx_1 - x_2 > 0$, this point is located below the line, if $mx_1 - x_2 = 0$ it is located on the line and finally, if $mx_1 - x_2 < 0$ this point is located above the line.

Now, with the explanation above, methodology used to obtain the separation line is explained. This method is known as the linear classification method and uses the gradient-descent algorithm which is used to estimate the point where a function has its lowest value [100]. To obtain such value, negative of the gradient is followed, where for a multi-variable function $f(x,y,...)$ the gradient is defined as:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \vdots \end{bmatrix} \quad (465)$$

The gradient-descent can be represented as:

$$\begin{bmatrix} x \\ y \\ \vdots \end{bmatrix}^{n+1} = \begin{bmatrix} x \\ y \\ \vdots \end{bmatrix}^n - \lambda \times \nabla f \quad (466)$$

where, n is the iteration step and λ is the step size. In our case, f will be the error or cost function. Since the gradient-descent is used for the binary classification, sigmoid function is used as cost function. Definition of this function is given in Eq. 467, and it is used to turn a regression line into a boundary of decision for binary classification.

$$\varepsilon(x) = \frac{1}{1 + e^{-x}} \quad (467)$$

Moreover, obtained separation line will have the form shown in Eq. 468.

$$\varphi = a_1 \times \log_{10} Re_B + a_2 \times \log_{10} We_B \quad (468)$$

Above equation, using the vector notation can be written as:

$$\varphi = a^T \times z \quad (469)$$

where,

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (470)$$

and,

$$z = \begin{bmatrix} Re_B \\ We_B \end{bmatrix} \quad (471)$$

Using Eq. 467, it can be said that $\varepsilon(\varphi) < 0.5$ if $\varphi < 0$ and $\varepsilon(\varphi) \geq 0.5$ if $\varphi \geq 0$.

Hence, the following formulation can be obtained:

$$Class = \begin{cases} 0 & \text{if } \varepsilon(\varphi) < 0.5 \\ 1 & \text{if } \varepsilon(\varphi) \geq 0.5 \end{cases} \quad (472)$$

Now, the cost function is defined using the logarithmic loss, as shown in Eq. 473.

$$Cost(a) = \begin{cases} -\log(\varepsilon(\varphi)) & \text{if } Class = 1 \\ -\log(1 - \varepsilon(\varphi)) & \text{if } Class = 0 \end{cases} \quad (473)$$

Furthermore, above equation can also be written as:

$$Cost(a) = -\frac{1}{m} \times \sum_{i=1}^m [Class_i \times \log(\varepsilon(\varphi))_i + (1 - Class_i) \times \log(1 - \varepsilon(\varphi))_i] \quad (474)$$

where m is the number of data. Final form of this equation can be written as:

$$Cost(a) = -\frac{1}{m} \times \sum_{i=1}^m [Class_i \times \log(\varepsilon(a^T \times z_i))_i + (1 - Class_i) \times \log(1 - \varepsilon(a^T \times z_i))_i] \quad (475)$$

Taking the derivative with respect to each element of a (Eq. 470) will yield:

$$\frac{\partial Cost(a)}{\partial a_j} = -\frac{1}{m} \times \sum_{i=1}^m [\varepsilon(a^T \times z_i) - Class_i] \times z_{ij} \quad (476)$$

Obtained separation line, using Eqs. 465 - 476, along with the experimental data is given in Figure 128. Moreover, equation that describes the obtained line can be found in Eq. 477.

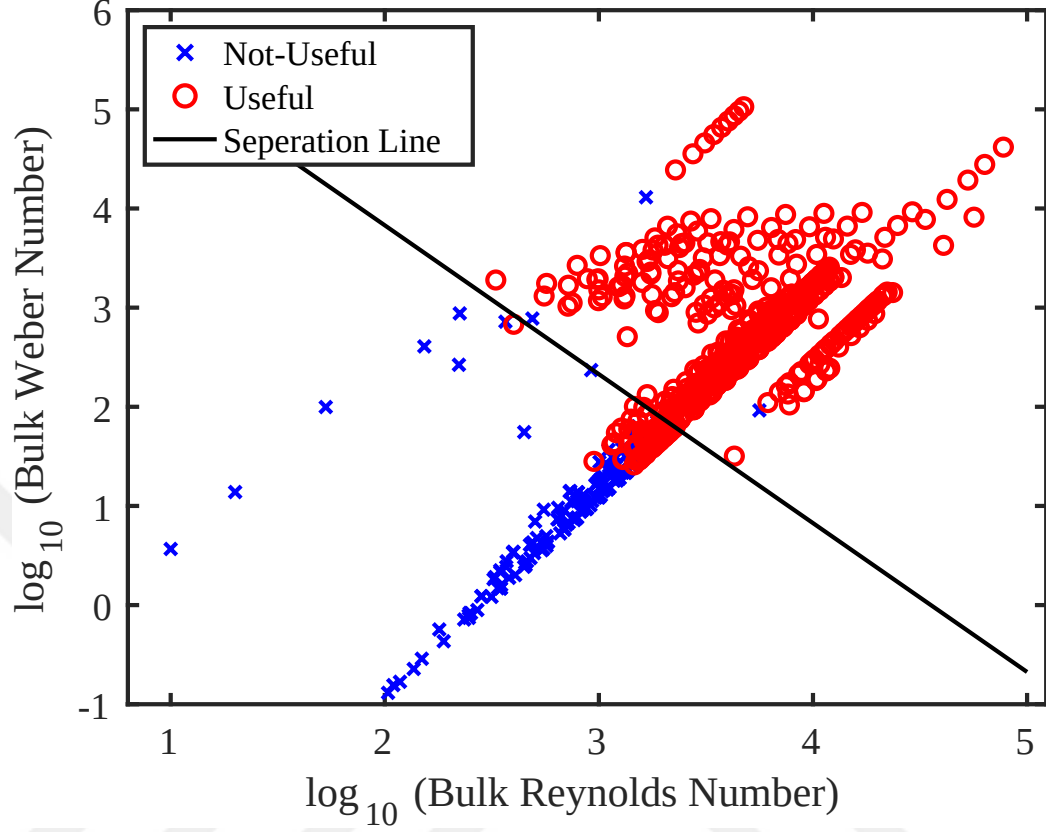


Figure 128: Obtained separation line for combined data (present study and literature)

$$3.696 \times \log_{10} Re_B + 2.461 \times \log_{10} We_B - 16.820 = 0 \quad (477)$$

Error of the obtained line is calculated as the ratio of number of miss-classified cases to the total number of cases. For the correlation given in Eq. 477 error value is 0.114. In other words, of the total of 813 cases, only 93 of them are misclassified.

To evaluate the error of the obtained model, error of the presented literature models is also calculated. First, analysis of the MUSP model is done [16]. As shown in Eq. 454, this model is derived for calculating the minimum flow rate where the spray is useful. In other words, it creates a separation line for the mass flow rate of useful stage from the FN_{dim} and liquid viscosity. Separation line obtained from this

model is shown in Figure 129, along with the experimental data. It can be seen that model deviates from the experimental data drastically. Moreover, oscillations in the separation line are present, for the cases having similar FN_{dim} and different viscosity values. Error of this correlation is calculated as 0.782 which means out of 813 cases, 636 of them are miss-classified.

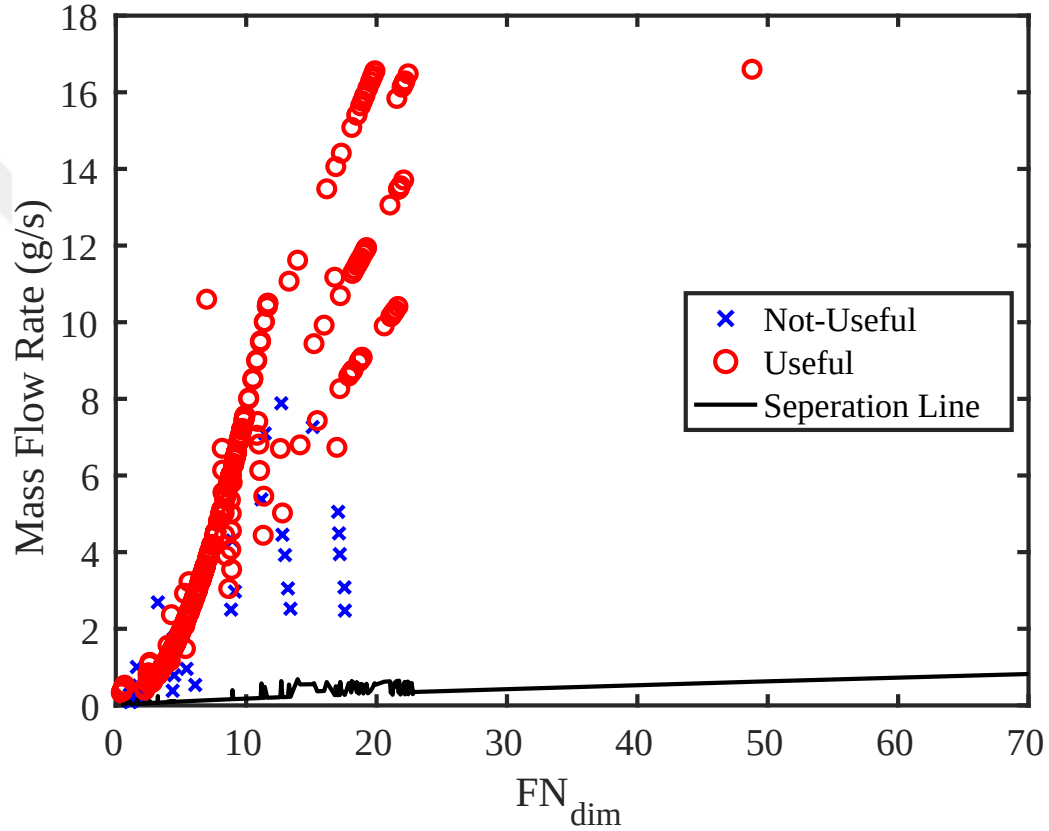


Figure 129: Comparison of literature with the combined spray development data [16]

Second, model based on the Weber number calculated from the film thickness is analyzed [19]. This study presented We_l number for separating the tulip stage and the developed stage. Since, in this study, starting from the tulip stage spray is considered as useful spray, $140 \leq We_L$ is taken as the separation line. It should be noted that the film thickness value, which is present in Eq. 456, is calculated from Eq. 62. Figure 130, shows the comparison of experimental data with the separation line. Inspection

of the results shows that this model is more accurate than the model discussed earlier and has an error value of 0.539 which means it miss-classifies 437 data points.

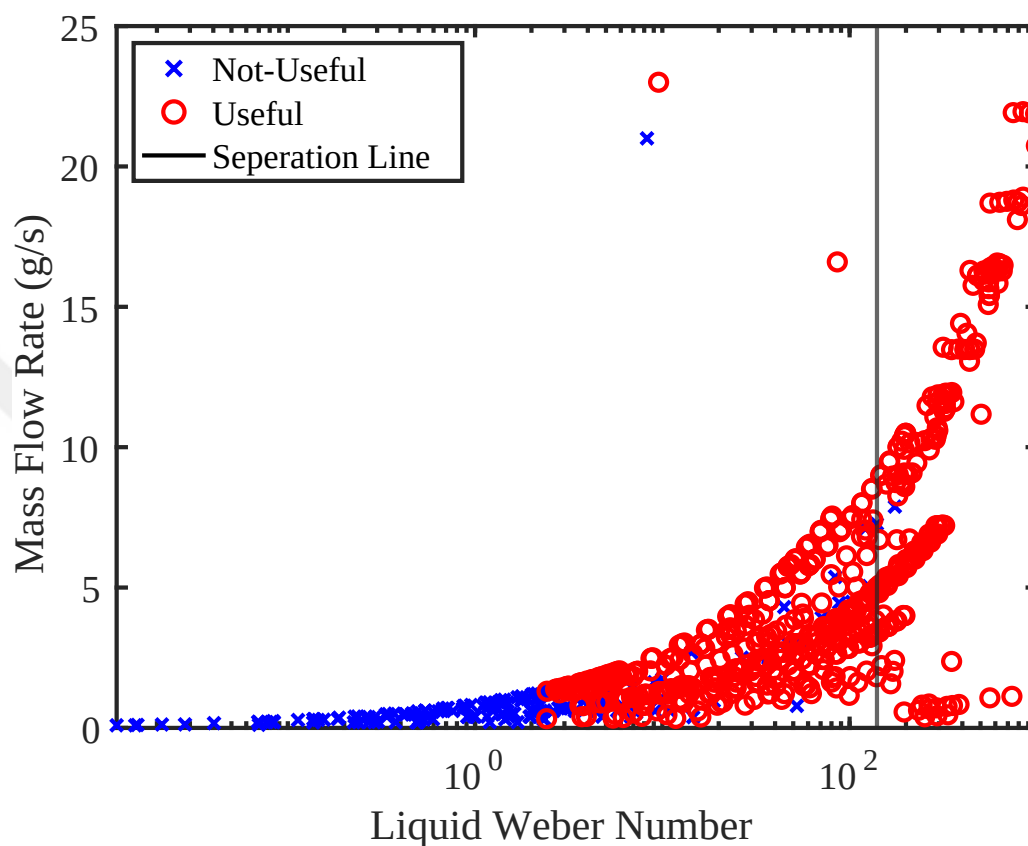


Figure 130: Comparison of literature model with the combined spray development data [19]

Third, model derived based on the Ohnesorge number and pressure difference is inspected [17]. This model is based on the pressure difference and Ohnesorge number and separates the stages of distorted pencil, onion, tulip, and fully developed spray as mentioned earlier. Similar to the above-mentioned model, only the line that separates the tulip and fully developed stage is considered here. Figure 131, presents the experimental data with the separation line. Obtained results showed that this model has similar accuracy to model above and the error value is calculated as 0.504. This means this model miss-classifies 410 data points.

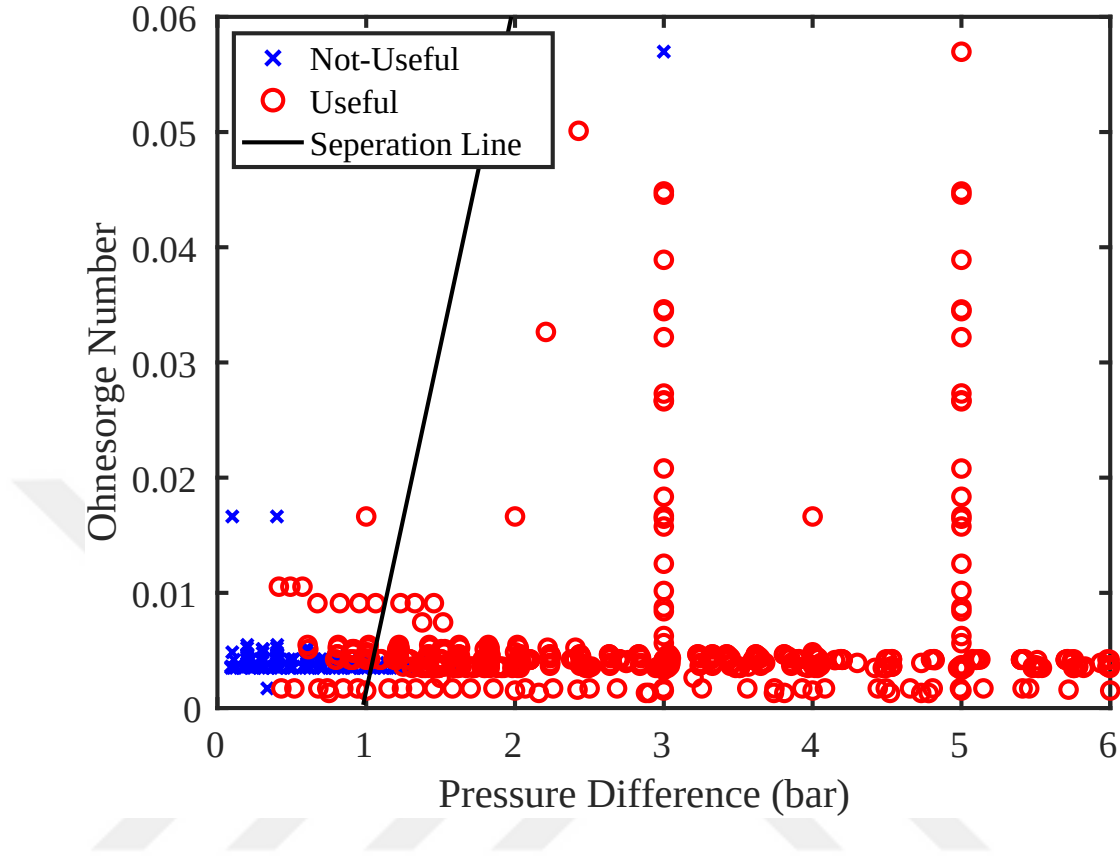


Figure 131: Comparison of literature model with the combined spray development data [17]

Figure 132, shows the comparison of model based on the exit Reynolds number with the experimental data [3]. It should be noted that this line is actually defined for separating Stage-4 and Stage-5 that are shown in Figure 121. However, since the stages in this figure are not experienced in any of the reviewed studies and conducted experiments and since the separation of the useful and non-useful spray is not presented in the referenced study, this criteria is considered as the separation line. Obtained results showed that this line, with the assumption mentioned earlier, can predict the separation of the useful and non-useful sprays more accurately than the above-reviewed models. Error of this model is calculated as 0.279 which corresponds to 227 cases being miss-classified.

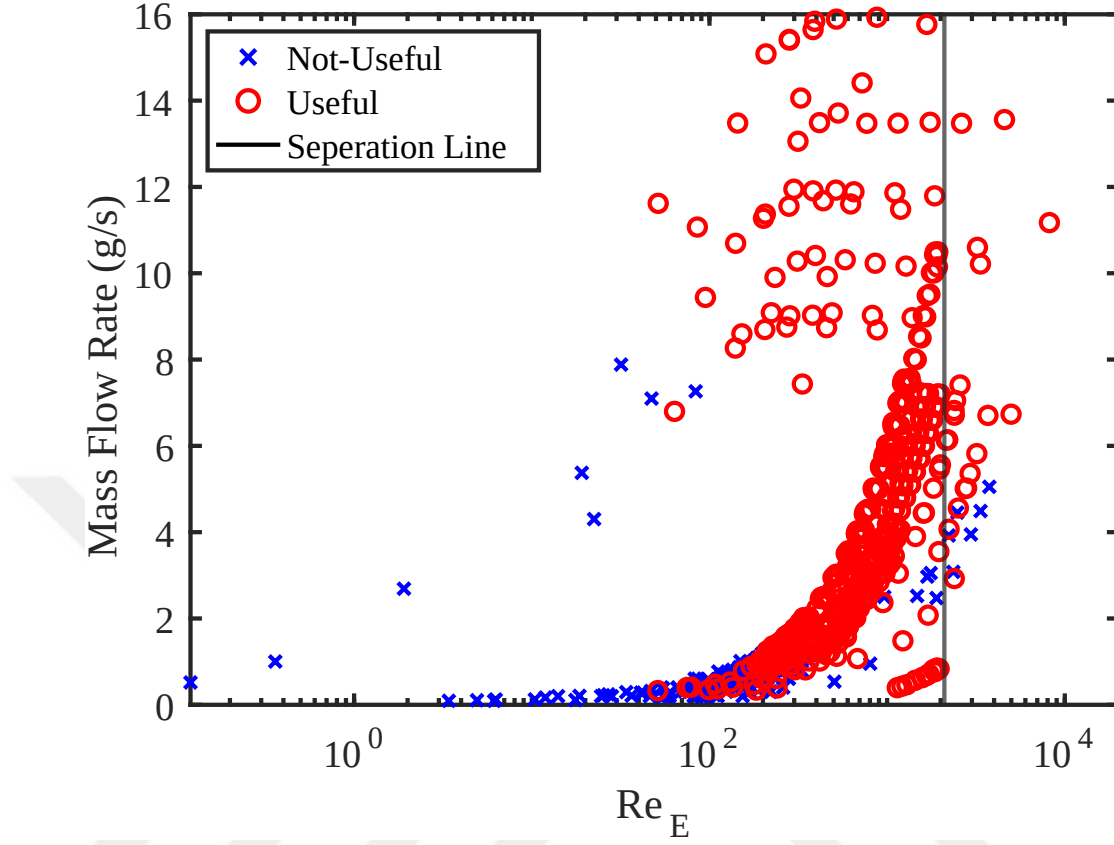


Figure 132: Comparison of literature model with the combined spray development data [3]

Finally, analysis of the model based on the bulk Reynolds and Weber numbers is done [18]. This model is very similar to the obtained model in this study, in terms of the contained parameters. However, instead of a single line, this model is created by defining a limit bulk Reynolds and Weber numbers. Below these limit values, the spray is predicted to be non-useful and above these limit values, spray is predicted as useful spray. Comparison of these limit values with the experimental data is given in Figure 133. Obtained error value of this model is 0.164 which yields that this model is the most accurate model obtained from the literature yet still has lower accuracy than the derived model in this study. This model miss-classifies, 133 data which is 40 data points higher than the derived model.

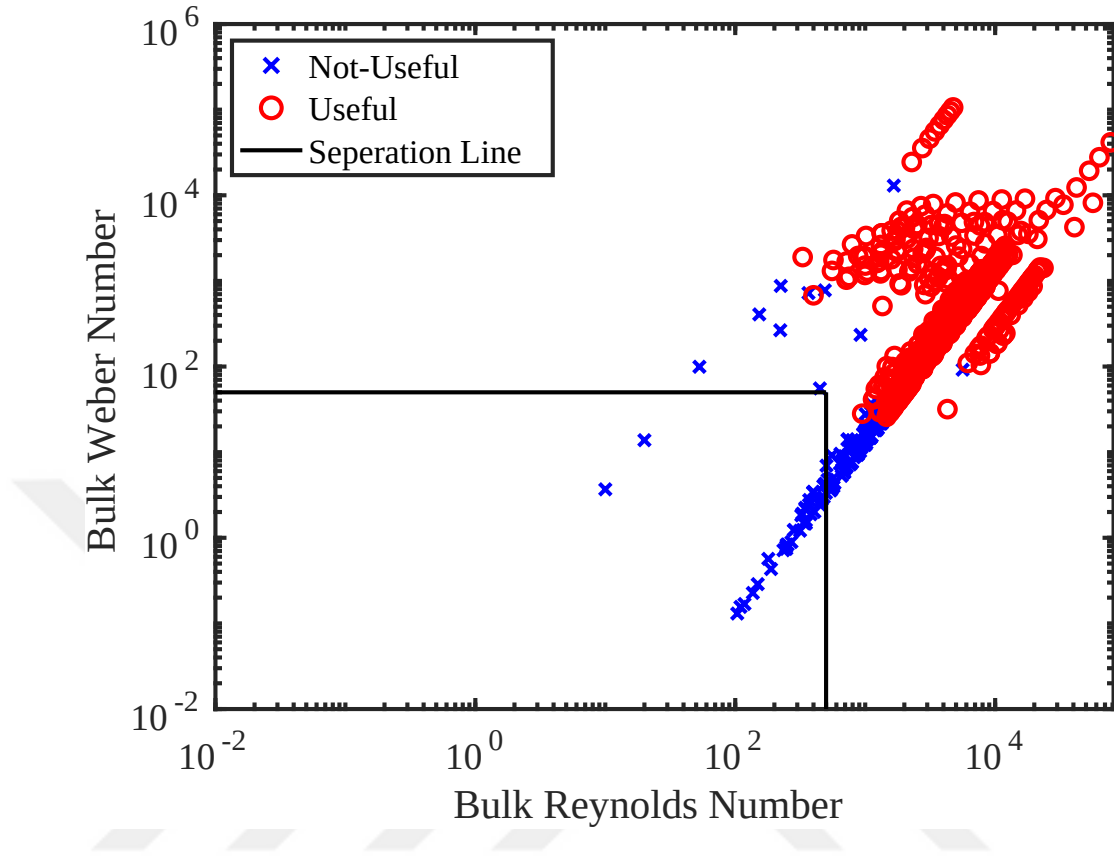


Figure 133: Comparison of literature model with the combined spray development data [18]

Obtained error values of the derived and evaluated models, along with the number of miss-classified data is given in Table 28.

Table 28: Error Values of the Derived and Literature Spray Development Models

Model	Mean Error Value	Miss-Classified Data Points / Total Data Points
This Study	0.114	93/813
[16]	0.782	636/813
[19]	0.539	437/813
[17]	0.504	410/813
[3]	0.279	227/813
[18]	0.164	133/813

CHAPTER VIII

DROPLET DIAMETER MODEL

8.1 Introduction

This chapter explains the model obtained for the calculation of the droplet size of the PSA. Similar to Chapter 5, this model is obtained since, it is seen that semi-empirical correlations, both obtained from the literature and derived in this study, have large errors especially for calculating the SMD.

8.2 LISA Model and Analysis

Atomization is a complex process including complex phenomena such as turbulence and instabilities. Literature survey have shown that there are many different models to describe the atomization, developed for the different regions of the spray [1–3]. Atomization of the PSA can be inspected by dividing it into two different regions namely, primary atomization region and secondary atomization region, as shown in Figure 134. In the case of PSA, when the liquid leaves the nozzle, hydrodynamic instabilities are augmented by aerodynamic interaction. As the sheet expands away from the nozzle its thickness declines, perforations are formed and these expand toward each other to form ligaments. When these ligaments collapse droplets are formed. This process is called primary atomization. Secondary atomization describes the region where droplets become hydro-dynamically unstable and break into smaller droplets and where droplet-droplet interactions, such as collision occurs [2].

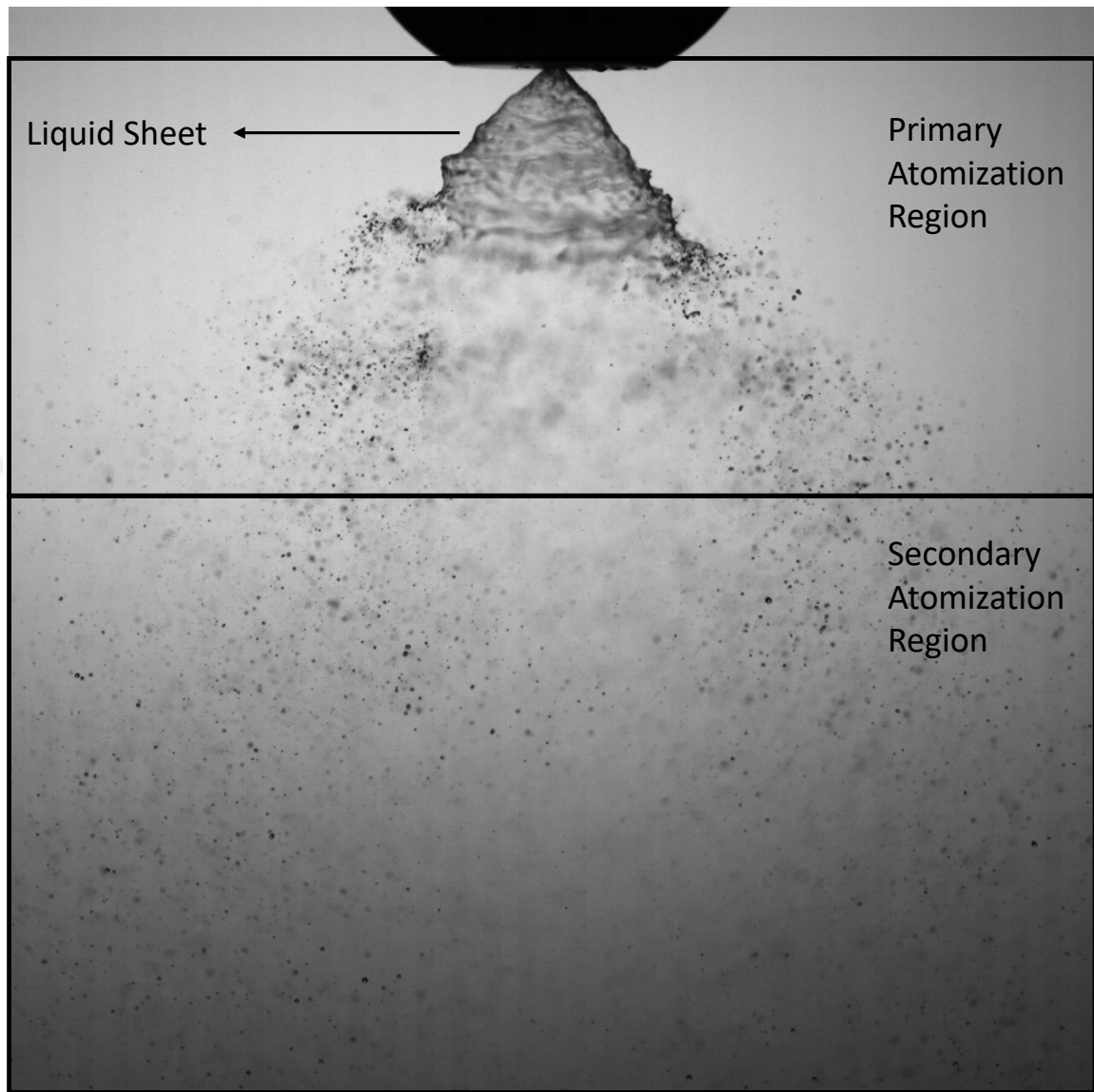


Figure 134: Primary and secondary atomization regions of the spray

It should be mentioned that in this study modeling of the primary atomization process is considered only. Since secondary atomization involves more chaotic processes such as droplet disintegration and droplets may also collide and/or coalesce, it requires a more complex model, and also modeling of the ambient flow field is also needed.

With the developments of computational algorithms and capacity, CFD has been

demonstrated to be a reliable method to perform calculations of atomization. It is seen that commercial CFD solvers, such as Star-CCM+ and ANSYS Fluent make use of the Lagrangian methods since it requires less computing power. Two different methods can be used in Lagrangian models, for simulating the sprays, namely introducing the droplets according to given distribution and the Linearized Instability Sheet Atomization (LISA) model. The former method introduces the droplets according to the provided droplet size distribution and velocities and performs calculations of secondary atomization process. Therefore it requires the experimental measurements of droplet diameter and velocity distribution in the primary break-up region.

Whereas the LISA model requires the inputs of pressure difference for the desired flow rate, exit diameter of the atomizer, spray angle, and the disturbance ratio and calculates the resultant droplet size distribution [77], [78]. It should be noted that the disturbance ratio is calculated from the break-up length of the liquid sheet and will be explained below. Moreover, LISA model used in commercial CFD codes also calculates the internal flow parameters such as air core size. The LISA model can be separated into three parts namely, internal flow, stability analysis, and breakup and droplet formation. Hence, model is inspected in three parts.

In the next section, presentation of the LISA model is done and then comparisons of the model with the performed experiments are given. As mentioned earlier, all of these experiments, in this study, are conducted in the primary atomization region. Moreover, an important point that has to be made is, aim of this study is not to perform stability analysis on the liquid sheet leaving the PSA but to apply the LISA model to calculate the droplet sizes, similar to the commercial CFD codes.

8.2.1 Presentation of LISA Model

8.2.1.1 Internal Flow

Equations that are used for calculating the internal flow are given below. Only required inputs for the internal flow calculations are the physical properties of the liquid such as density (ρ_L) and viscosity (μ_L), exit diameter of the nozzle (D_o), spray angle (θ), flow rate ($\dot{m}_L$), and the pressure difference (ΔP_L) corresponding to this flow rate. Calculation steps are given below.

- Calculation of the Velocity Coefficient

$$k_V = \max \left[0.7, \frac{4 \times \dot{m}_L}{\pi \times \rho_L \times D_o^2 \times \cos \theta} \times \sqrt{\frac{\rho_L}{2 \times \Delta P_L}} \right] \quad (478)$$

- Calculation of the Exit Velocity

$$V_L = k_V \times \sqrt{\frac{2 \times \Delta P_L}{\rho_L}} \quad (479)$$

It should be noted that these two equations are also given in Chapter 4 (Eq. 144 and Eq. 145).

- Calculation of the Velocity Components

$$u_L = V_L \times \cos \theta \quad (480)$$

$$v_L = V_L \times \sin \theta \quad (481)$$

where, u_L is the mean axial velocity and v_L is the mean tangential velocity.

- Film Thickness

$$\dot{m}_L = \pi \times \rho_L \times u_L \times \delta_o \times (D_o - \delta_o) \quad (482)$$

where, δ_o is the thickness of the film at the exit. Solving this equation to obtain the film thickness yields (following the steps below):

$$\dot{m}_L = \pi \times \rho_L \times u_L \times \delta_o \times D_o - \pi \times \rho_L \times u_L \times \delta_o^2 \quad (483)$$

$$\delta_o^2 - \delta_o \times D_o + \frac{\dot{m}_L}{\pi \times \rho_L \times u_L} = 0 \quad (484)$$

$$\Delta = b^2 - 4 \times a \times c = D_o^2 - \frac{4 \times \dot{m}_L}{\pi \times \rho_L \times u_L} \quad (485)$$

$$(\delta_o)_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2 \times a} = \frac{D_o - \sqrt{\frac{4 \times \dot{m}_L}{\pi \times \rho_L \times u_L}}}{2} \quad (486)$$

Since it is known that the thickness of the liquid film cannot be larger than the radius of the exit orifice.

$$\delta_0 = \frac{D_o - \sqrt{D_o^2 - \frac{4 \times \dot{m}_L}{\pi \times \rho_L \times u_L}}}{2} \quad (487)$$

8.2.1.2 Linear Stability Analysis

It should be noted that LISA model is derived with the assumption of two dimensional, viscous, incompressible liquid sheet moving with relative velocity of V_s through an inviscid, incompressible gas medium. For this condition, a spectrum of infinitesimal disturbances of the form (Eq. 488) is imposed on the velocity and pressure for both the liquid and the gas phases.

$$\eta = \eta_0 \times e^{(ikx+wt)} \quad (488)$$

Here η_0 is the initial wave amplitude and,

$$k = \frac{2\pi}{\lambda} \quad (489)$$

is the wave number and,

$$w = w_r + iw_i \quad (490)$$

is the complex growth rate and lastly,

The most unstable disturbance has the largest value of w_r denoted by Λ_s and is assumed to be responsible from the break-up. Thus, it is desired to obtain a dispersion relation $w = w(k)$ from which the most unstable disturbance can be found.

It is shown that two solutions exist that satisfy the governing equations subject to the boundary conditions at the upper and lower interfaces [77, 78]. First solution is

called the sinuous mode, in which the waves at the upper and lower interfaces are in phase. For the second solution, which is called varicose mode, waves are π radians out of phase. Sinuous mode and varicose mode are shown in the Figure 135 and Figure 136, respectively.

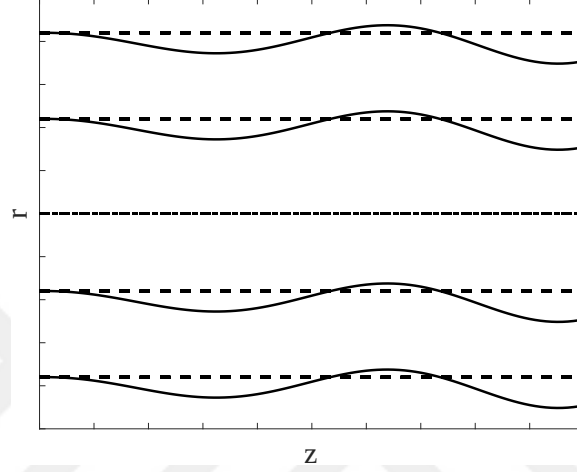


Figure 135: Sinuous mode

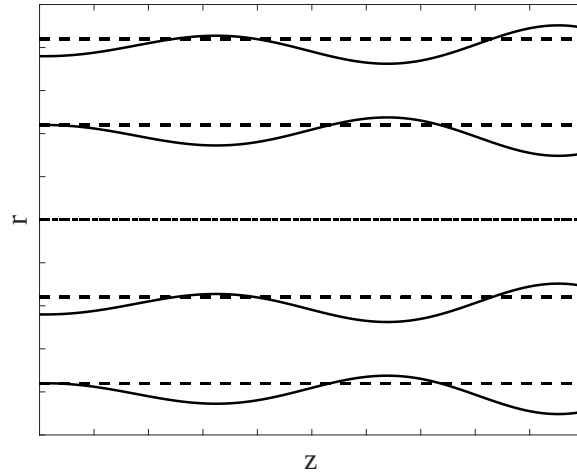


Figure 136: Varicose mode

By numerous authors, it has been shown that the sinuous mode is dominant for low velocities and low gas-to-liquid density ratios [77,78,101]. Moreover, it was reported

that modes become indistinguishable for high-velocity flows.

Dispersion relation, relating wave number to its frequency, for the sinuous mode is given in Eq. 491.

$$w^2 [\tanh(kh) + \Gamma] + w [4v_l k^2 \tanh(kh) + (2i\Gamma V_s)] + 4v_l^2 k^3 [k \tanh(kh) - \xi \tanh(\xi h)] - \Gamma V_s^2 k^2 + \frac{\sigma k^3}{\rho_l} = 0 \quad (491)$$

where,

$$\Gamma = \frac{\rho_a}{\rho_l} \quad (492)$$

$$\xi = k^2 + \frac{w}{v_f} \quad (493)$$

For the varicose mode, the relation can be obtained by replacing the $\tanh(kh)$ and $\tanh(\xi h)$ in Eq. 491 with the $\coth(kh)$ and $\coth(\xi h)$, respectively. For the varicose mode, dispersion relation can be written as:

$$w^2 [\coth(kh) + \Gamma] + w [4v_l k^2 \coth(kh) + (2i\Gamma V_s)] + 4v_l^2 k^3 [k \coth(kh) - \xi \coth(\xi h)] - \Gamma V_s^2 k^2 + \frac{\sigma k^3}{\rho_l} = 0 \quad (494)$$

As can be seen, these equations are difficult to solve. Therefore, a simplified version of these equations is obtained by neglecting the second order viscosity terms since they are small compared to other terms [77]. Resulting expression for sinuous mode is given below.

$$w_r = -\frac{2v_l k^2 \tanh(kh)}{\tanh(kh) + \Gamma} + \frac{\sqrt{4v_l^2 k^4 \tanh^2(kh) - \Gamma^2 V_s^2 k^2 - (\tanh(kh) + \Gamma) \left(-\Gamma V_s^2 k^2 + \frac{\sigma k^3}{\rho_l} \right)}}{\tanh(kh) + \Gamma} \quad (495)$$

According to the suggestion above, expression for the varicose mode is obtained

as:

$$w_r = -\frac{2v_l k^2 \coth(kh)}{\coth(kh) + \Gamma} + \frac{\sqrt{4v_l^2 k^4 \coth^2(kh) - \Gamma^2 V_s^2 k^2 - (\coth(kh) + \Gamma) \left(-\Gamma V_s^2 k^2 + \frac{\sigma k^3}{\rho_l}\right)}}{\coth(kh) + \Gamma} \quad (496)$$

It is been suggested that these equations can be simplified further depending on whether the disturbance waves are short or long. Distinction between the short or long waves is done by evaluating the gas phase Weber number which is given in Eq. 139. It should be noted that the sheet half thickness in Eq. 139 is calculated from the film thickness at the atomizer exit using Eq. 497.

$$h = \frac{1}{2} \times \delta_o \times \cos \theta \quad (497)$$

It is stated that short waves are formed for $We_{gas} > 27/16$ and correspond to high speed flows with $\Gamma \ll 1$. Otherwise, long waves are formed. In the case of long waves, it is suggested that the wavelengths $\lambda = 2\pi/k$ are large compared with the sheet thickness. This suggests that:

$$\lambda = \frac{2\pi}{k} \gg h \Rightarrow 2\pi \gg kh \quad (498)$$

For the sinuous mode following simplification can be made:

$$\tanh(kh) \cong kh. \quad (499)$$

In return for the varicose mode:

$$\coth(kh) \cong \frac{1}{kh} \quad (500)$$

For the short waves, it assumed that the term kh is large such that $\tanh(kh) \cong 1$ for the sinuous mode and for the varicose mode $\coth(kh) \cong 1$. This corresponds to the above-mentioned phenomenon that the modes become indistinguishable at high speeds.

To analyze the validity of the assumptions above, the graphs of the hyperbolic tan and cot functions are analyzed. Figure 137 shows the graph of the hyperbolic tangent function. As can be seen, the value of $\tanh(kh)$ approximates to 1 as the kh value is increased.

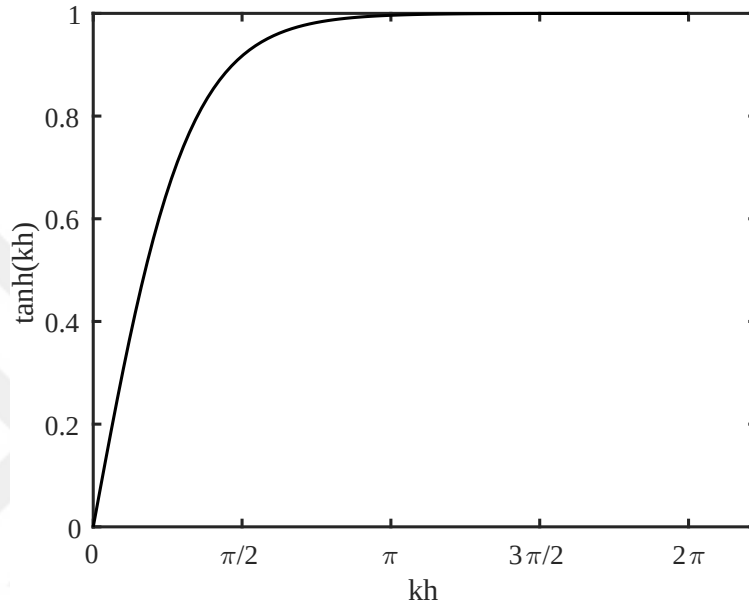


Figure 137: $\tanh(kh)$

Figure 138 presents the relative difference between the $\tanh(kh)$ values and the value of kh (left), and the difference between the $\tanh(kh)$ values and 1 (right).

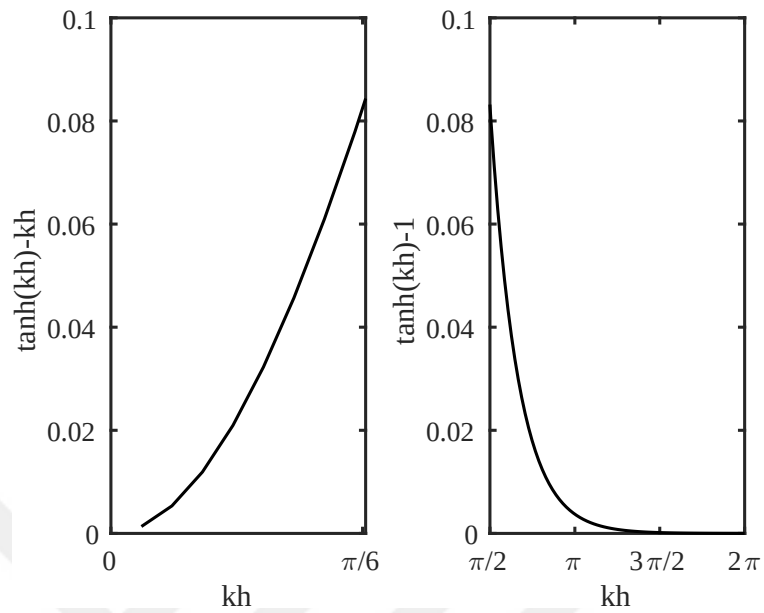


Figure 138: Difference between $\tanh(kh)$ and kh (left) and difference between the $\tanh(kh)$ and 1 (right)

Same graphs for the $\coth(kh)$ function are given in Figure 139 and Figure 140.

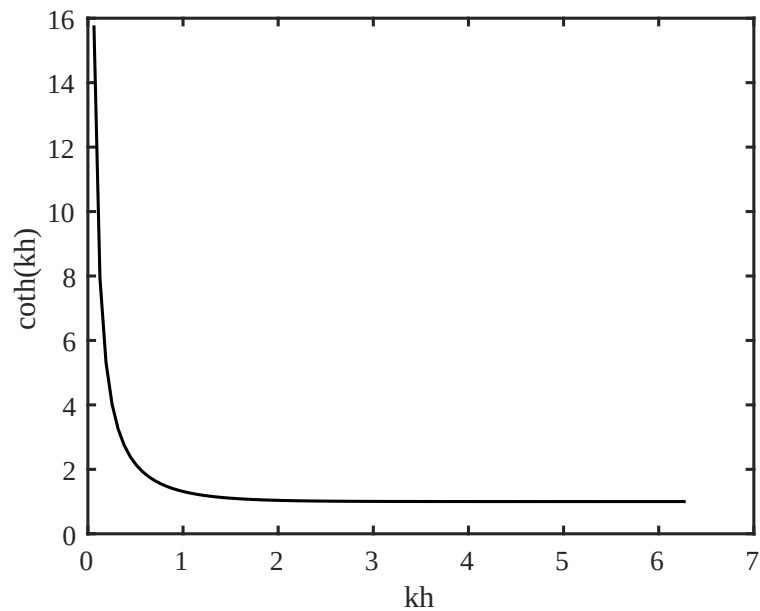


Figure 139: $\coth(kh)$

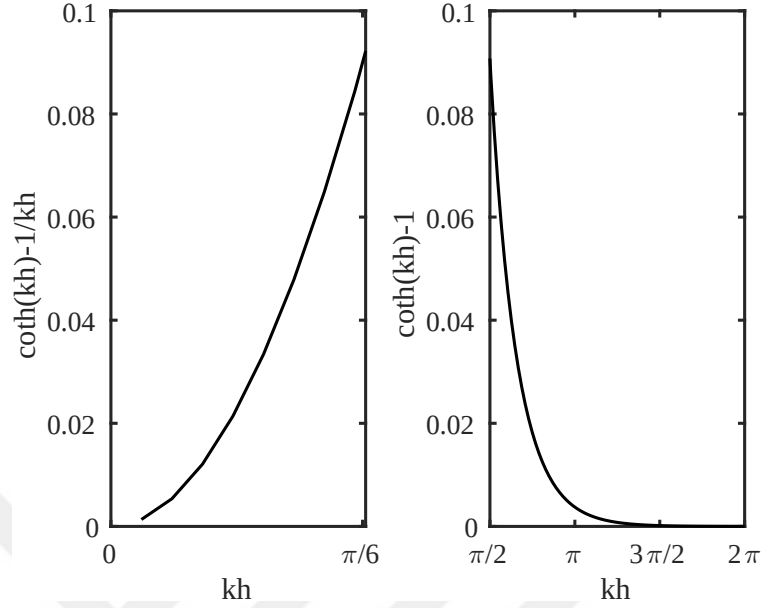


Figure 140: Difference between $\text{coth}(kh)$ and $1/kh$ (left) and difference between the $\text{coth}(kh)$ and 1 (right)

It can be seen that for values of $kh < (\pi/6) \cong 0.524$, relative difference between the $\tanh(kh)$ and kh is less than 9 percent. Similar trend can also be seen for values of $kh > (\pi/2) \cong 1.571$ for which the relative difference between the $\tanh(kh)$ and 1 is less than 9 percent.

Similarly, it can be seen that as the value of kh increases, $\text{coth}(kh)$ approximates to value of 1. Again, for the values of $kh < (\pi/6) \cong 0.524$, relative difference between the $\text{coth}(kh)$ and $1/kh$ is less than 9 percent. Similar trend can also be seen for values of $kh > (\pi/2) \cong 1.571$ for which the relative difference between the $\text{coth}(kh)$ and 1 is less than 9 percent. As a result, it can be said that the assumptions made above are reasonable.

Another assumption that is used for the simplification of the dispersion relation is the fact that the density ratio is small compared to value of $\tan(kh)$ or $\text{coth}(kh)$.

Hence, following correlations can be written.

$$\frac{1}{\tanh(kh) + \Gamma} \cong \frac{1}{\tanh(kh)} \quad (501)$$

$$\frac{1}{\coth(kh) + \Gamma} \cong \frac{1}{\coth(kh)} \quad (502)$$

This assumption is more evident for the large kh values since it approximates to value of 1 and considering that the working liquid is mostly water or fuel and ambient gas being air density ratio is in the order of 10^{-3} . Effect of ignoring the density ratio for the $1/\tanh(kh)$ function is shown in Figure 141.

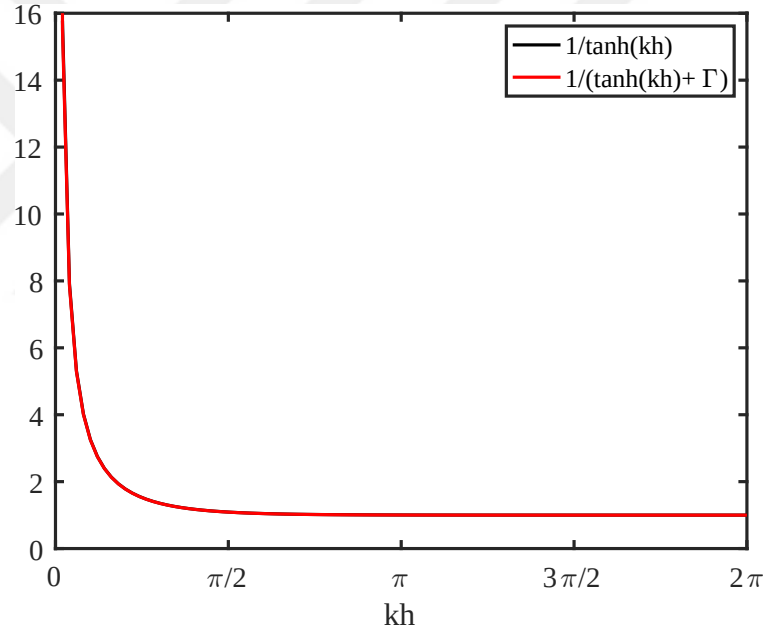


Figure 141: Effect of density ratio on hyperbolic tangent function

Results yield that ignoring the density ratio does not affect the value. Similar result is also seen for the $1/\coth(kh)$ function, as shown in Figure 142.

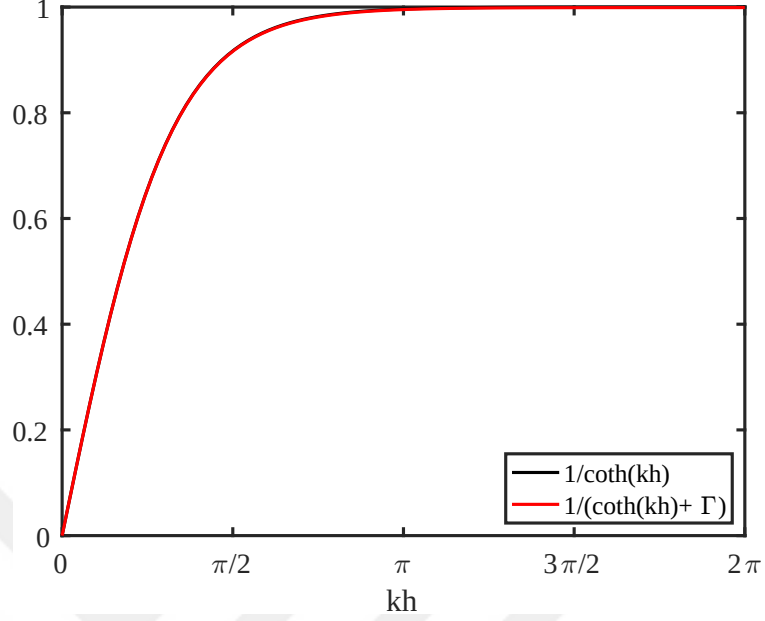


Figure 142: Effect of density ratio on hyperbolic cotangent function

First with the help of Eq. 501, Eq. 495 becomes:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 - \frac{\Gamma^2 V_s^2 k^2}{\tanh^2(kh)} + \frac{\Gamma V_s^2 k^2}{\tanh(kh)} - \frac{\sigma k^3}{\rho_l \tanh(kh)}} \quad (503)$$

Rearranging this equation:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \frac{\Gamma V_s^2 k^2}{\tanh(kh)} \left(1 - \frac{\Gamma}{\tanh(kh)}\right) - \frac{\sigma k^3}{\rho_l \tanh(kh)}} \quad (504)$$

For the term with density ratio it can be said that (since $\Gamma \ll \tanh(kh)$):

$$\frac{\Gamma}{\tanh(kh)} \ll 1 \quad (505)$$

and

$$1 - \frac{\Gamma}{\tanh(kh)} \cong 1 \quad (506)$$

Final form the equation becomes:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \frac{\Gamma V_s^2 k^2}{\tanh(kh)} - \frac{\sigma k^3}{\rho_l \tanh(kh)}} \quad (507)$$

For the varicose method same steps are repeated, using Eq. 502,

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 - \frac{\Gamma^2 V_s^2 k^2}{\coth^2(kh)} + \frac{\Gamma V_s^2 k^2}{\coth(kh)} - \frac{\sigma k^3}{\rho_l \coth(kh)}} \quad (508)$$

Rearranging the equation above:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \frac{\Gamma V_s^2 k^2}{\coth(kh)} \left(1 - \frac{\Gamma}{\coth(kh)}\right) - \frac{\sigma k^3}{\rho_l \coth(kh)}} \quad (509)$$

For the term with density ratio it can be said that (since $\Gamma \ll \coth(kh)$):

$$\frac{\Gamma}{\coth(kh)} \ll 1 \quad (510)$$

and

$$1 - \frac{\Gamma}{\coth(kh)} \cong 1 \quad (511)$$

Final form the equation becomes:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \frac{\Gamma V_s^2 k^2}{\coth(kh)} - \frac{\sigma k^3}{\rho_l \coth(kh)}} \quad (512)$$

For the short waves sinuous mode and varicose mode result in the same correlation which is given below. Using, $\tanh(kh) \cong 1$ and $\coth(kh) \cong 1$:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \Gamma V_s^2 k^2 - \frac{\sigma k^3}{\rho_l}} \quad (513)$$

For the long waves sinuous mode and varicose mode present different correlations which are given below.

Sinuous mode:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \frac{\Gamma V_s^2 k}{h} - \frac{\sigma k^2}{\rho_l h}} \quad (514)$$

Varicose mode:

$$w_r = -2v_l k^2 + \sqrt{4v_l^2 k^4 + \Gamma V_s^2 k^3 h - \frac{\sigma k^4 h}{\rho_l}} \quad (515)$$

In addition to the correlations above, existence of additional correlations must be discussed. These correlations are the long wave and short-wave correlation for the inviscid liquid sheet and can be obtained by neglecting the viscosity terms.

Inviscid Short Wave:

$$w_r = \sqrt{\Gamma V_s^2 k^2 - \frac{\sigma k^3}{\rho_l}} \quad (516)$$

Inviscid Long Wave Sinuous Mode:

$$w_r = \sqrt{\frac{\Gamma V_s^2 k}{h} - \frac{\sigma k^2}{\rho_l h}} \quad (517)$$

Inviscid Long Wave Varicose Mode:

$$w_r = \sqrt{\Gamma V_s^2 k^3 h - \frac{\sigma k^4 h}{\rho_l}} \quad (518)$$

To sum up, it can be seen that there are total of 8 correlations that can be used for calculating the disturbance of the liquid sheet. To prevent any confusion list of the correlations is given below.

- Correlation-1: sinuous mode without simplifications (Eq. 495)
- Correlation-2: varicose mode without simplifications (Eq. 496)
- Correlation-3: short wave (Eq. 513)
- Correlation-4: sinuous mode long wave (Eq. 514)
- Correlation-5: varicose mode long wave (Eq. 515)
- Correlation-6: inviscid short wave (Eq. 516)
- Correlation-7: inviscid long wave sinuous mode (Eq. 517)
- Correlation-8: inviscid long wave varicose mode (Eq. 518)

It should be noted that commercial CFD softwares use Correlation-3 and Correlation-7 based on the value of We_{gas} . As mentioned above, aim of using these correlations is to find $k=K(s)$ for which the w_r has the maximum value, which is denoted by the Λ_s and is assumed to be responsible for the breakup.

To find the value of the Λ_s we can take the derivative of the w_r with respect to k and find the values of k for which the derivative becomes zero. However, for correlations other than the inviscid ones, solution to the derivative of w_r is difficult to obtain. Therefore, for these correlations, the graphical solution is used. For the inviscid cases derivative of the correlations and their solutions are given below.

Correlation-6:

$$\frac{dw_r}{dk} = \frac{\frac{3\sigma k^2}{\rho_l} - 2\Gamma V_s^2 k}{2\sqrt{\Gamma V_s^2 k^2 - \frac{\sigma k^3}{\rho_l}}} = 0 \quad (519)$$

$$k = K(s) = \frac{2\Gamma V_s^2 \rho_l}{3\sigma} = \frac{2V_s^2 \rho_a \rho_l}{3\rho_l \sigma} = \frac{2}{3} \frac{V_s^2 \rho_a}{\sigma} \quad (520)$$

$$K(s) h = \frac{2}{3} \frac{V_s^2 h \rho_a}{\sigma} = \frac{2}{3} We_g \quad (521)$$

Correlation-7:

$$\frac{dw_r}{dk} = \frac{\frac{\Gamma V_s^2}{h} - \frac{2k\sigma}{h\rho_l}}{2\sqrt{\frac{\Gamma V_s^2 k}{h} - \frac{\sigma k^2}{h\rho_l}}} = 0 \quad (522)$$

$$k = K(s) = \frac{\Gamma V_s^2 \rho_l}{2\sigma} = \frac{V_s^2 \rho_a \rho_l}{2\rho_l \sigma} = \frac{1}{2} \frac{V_s^2 \rho_a}{\sigma} \quad (523)$$

$$K(s) h = \frac{1}{2} \frac{V_s^2 h \rho_a}{\sigma} = \frac{1}{2} We_g \quad (524)$$

Correlation-8:

$$\frac{dw_r}{dk} = \frac{\frac{3\Gamma V_s^2 h k^2}{\rho_l} - \frac{4k^3 h \sigma}{\rho_l}}{2\sqrt{\Gamma V_s^2 h k^3 - \frac{\sigma h k^4}{\rho_l}}} = 0 \quad (525)$$

$$k = K(s) = \frac{3\Gamma V_s^2 \rho_l}{4\sigma} = \frac{3V_s^2 \rho_a \rho_l}{4\rho_l \sigma} = \frac{3}{4} \frac{V_s^2 \rho_a}{\sigma} \quad (526)$$

$$K(s) h = \frac{3}{4} \frac{V_s^2 h \rho_a}{\sigma} = \frac{3}{4} We_g \quad (527)$$

8.2.1.3 Sheet Break-up and Droplet Formation

After the calculation of the wave numbers and the disturbance, calculation of the droplet sizes and the break-up parameters such as the break-up length and the break-up time is done. The break-up length is calculated from:

$$L_b = V_f \times \tau_b \quad (528)$$

where τ_b is the breakup time and calculated from Eq. 529 and Eq. 530.

$$\tau_b = \frac{\zeta_b}{\Lambda_s} \quad (529)$$

$$\zeta_b = \ln \frac{\eta_b}{\eta_0} \quad (530)$$

Here the break-up length is defined as the diagonal distance from the atomizer tip to the point of break-up and not the vertical distance.

Value of the ζ_b is the ratio of the surface disturbance at sheet break-up location (η_b) to the initial disturbance (η_0) and it is defined by the user. It should be noted that default value used in commercial CFD codes is 12. However, this parameter can be determined experimentally if the break-up length can be measured accurately. Thickness of the liquid sheet at any location is calculated from Eq. 531, where L is the distance from the nozzle.

$$h = \frac{h_0(d_0 - \delta_0)}{2L \sin \theta + d_0 - \delta_0} \quad (531)$$

Consequently, thickness at the breakup location is calculated from:

$$h_b = \frac{h_0(d_0 - \delta_0)}{2L_b \sin \theta + d_0 - \delta_0} \quad (532)$$

For the case of long waves, it is assumed that the ligaments are formed from tears in the sheet twice per wavelength, and the resulting ligament diameter is given by:

$$d_L = \sqrt{\frac{8h_b}{K_s}} \quad (533)$$

For the short waves, it is assumed that the ligaments are formed from tears in the sheet once per wavelength, and resulting diameter is given by:

$$d_L = \sqrt{\frac{16h_b}{K_s}} \quad (534)$$

Mean diameter for the Rosin-Rammler distribution is calculated from:

$$d_D = 1.88d_L(1 + 3Oh)^{1/6} \quad (535)$$

where, Oh is the Ohnesorge number based on the ligament size (Eq. 536).

$$Oh = \frac{\mu_l}{\sqrt{\rho_l \sigma d_L}} \quad (536)$$

An important point that should be made is, spreading parameter of the Rosin-Rammler distribution is not calculated with this model and is set to 3.5 by default in the commercial CFD codes. Moreover, to the author's best knowledge, this value cannot be changed by the user.

8.2.2 Analysis of LISA Model

In this section analysis and application of the above-mentioned correlations are done. First, analysis of the internal flow part is done. Here, the experimental data of the film thickness measurements, presented in Chapter 4, is used for the analysis. It should be noted that experimental data of both close-end and open-end atomizers are used. Comparison of the calculated values with the experimental values has shown that internal flow part of the LISA model has the AMRE value of 0.245 and SE value of 0.027. These values are higher than that of Eq. 62 and Eq. 68 which were the most accurate correlations that are obtained from the literature and also, derived semi-empirical correlation for film thickness (Eq. 156) have higher accuracy.

Later, linear stability analysis part of the LISA model is applied to an atomizer of this study (Atomizer-16). As stated above, there are total of eight correlations and two of them are derived with the assumption of short waves. Even though the gas phase Weber number of the selected case is lower than 27/16 which suggests the formation of the long waves, correlations of short waves are also included in the calculations. Graph of the disturbance with respect to wave number is presented in Figure 143.

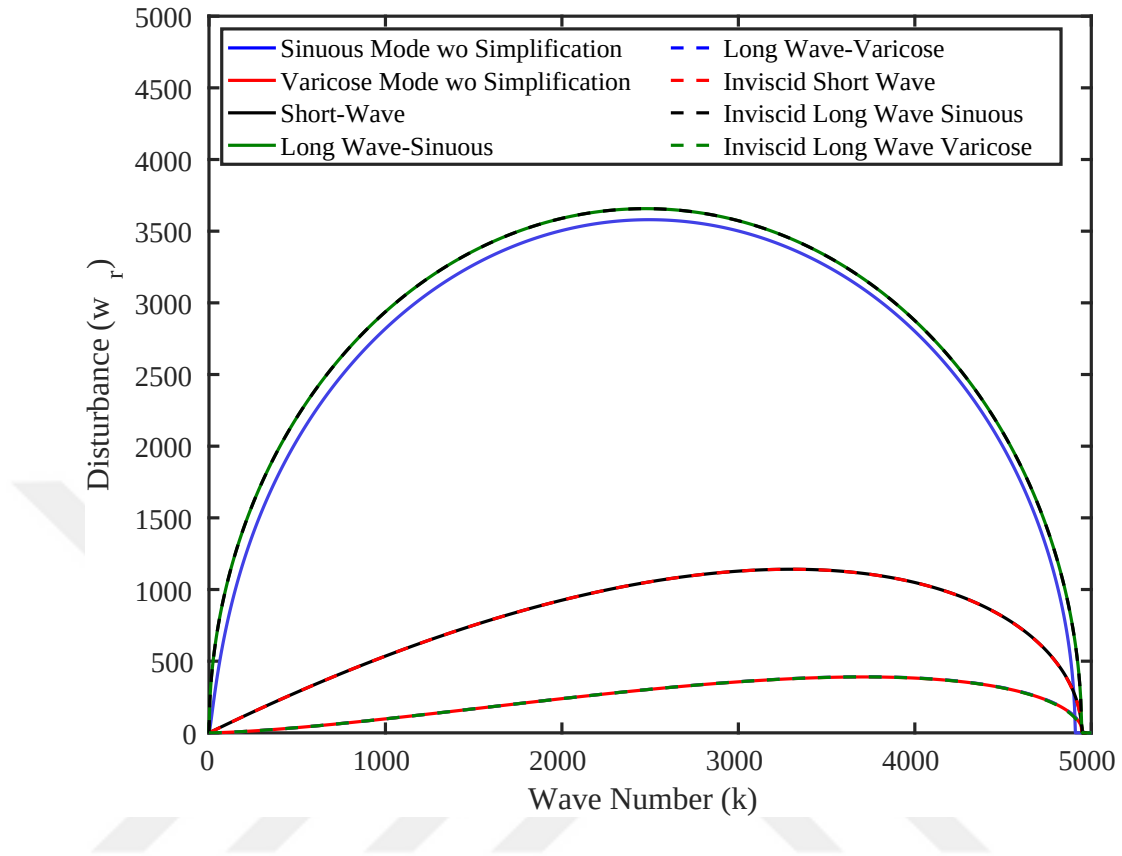


Figure 143: Calculated disturbance (w_r) for Atomizer-16

Non-dimensional form of the graph is given in Figure 144. From the inspection of the graphs, it can be seen that some of the correlations produce almost same values. These correlations can be listed as:

1. Inviscid Long Wave Sinuous Mode and Long Wave Sinuous Mode
2. Short Wave and Inviscid Short Wave
3. Varicose Mode wo Simplification, Inviscid Long Wave Varicose Mode and Long Wave Varicose Mode

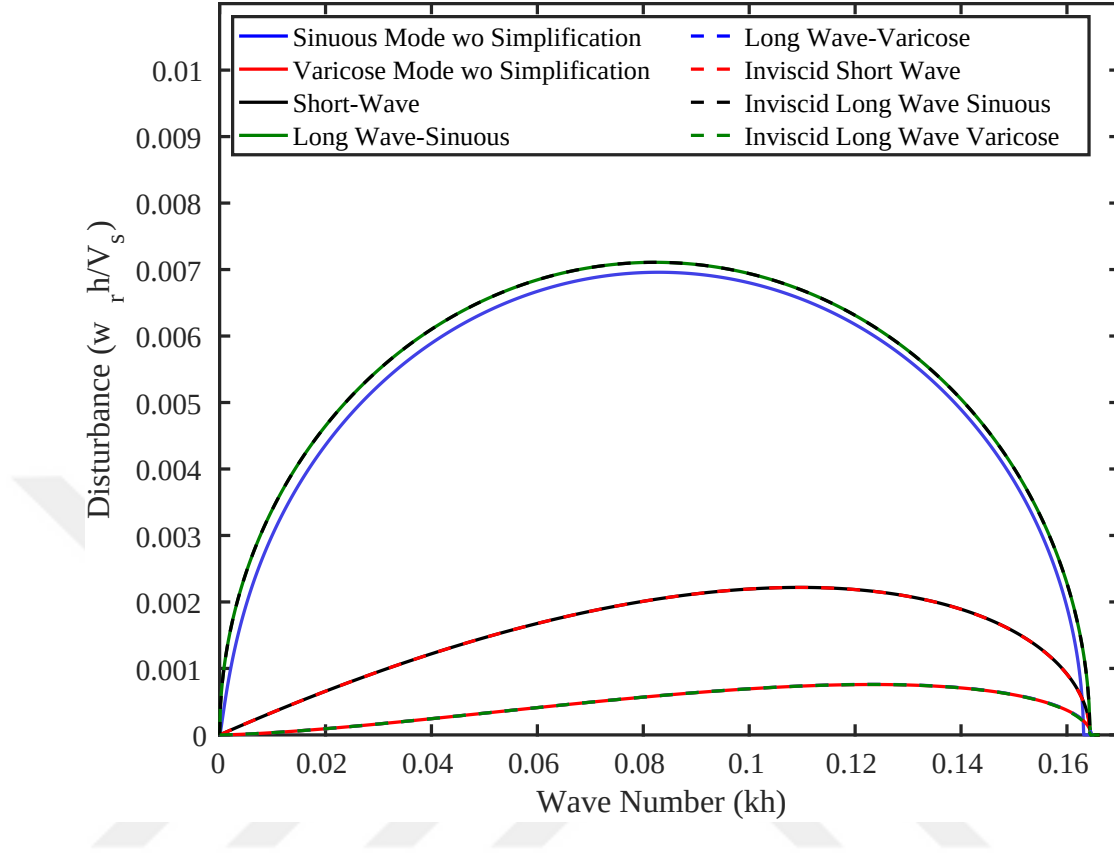


Figure 144: Calculated non-dimensional disturbance for Atomizer-16

An important deduction that can be made is, inviscid assumption does not affect the value of the disturbance for the short wave and long wave. Moreover, for the varicose mode whether there are simplifications or not same values for the long wave are obtained. As mentioned above, aim of the wave analysis is to obtain the wave number that produces the largest disturbance on the sheet. The largest disturbance value for each correlation and corresponding wave number is listed in the table below.

After calculating the wave numbers and disturbances, calculation of diameter of the ligaments and droplets is done for each correlation. It should be noted that value of the ζ_b which is the ratio of the surface disturbance at sheet break-up to the initial disturbance is not known. Therefore, different values of this term are used, and the effect of it analyzed. Calculated break-up length values for different correlations and

Table 29: Largest Disturbance Values and Corresponding Wave Numbers for Atomizer-16

Correlation	k_{max} or $K(s)$	$hK(s)$	w_{rmax} or Λ_s	$\Lambda_s h/V_s$
1	2.48E3	0.083	3.57E3	69.00E-4
2	3.63E3	0.121	0.37E3	7.12E-4
3	3.26E3	0.108	1.12E3	22.00E-4
4	2.46E3	0.082	3.65E3	71.00E-4
5	3.64E3	0.121	0.37E3	7.14E-4
6	2.30E3	0.110	1.14E3	22.00E-4
7	2.47E3	0.082	3.66E3	7.59E-4
8	3.71E3	0.123	0.39E3	7.59E-4

different ζ_b values are given in Figure 145. It can be seen that the ζ_b has significant influence on the break-up length. Increasing the ζ_b by 5 times would lead to 6 times higher break-up length values.

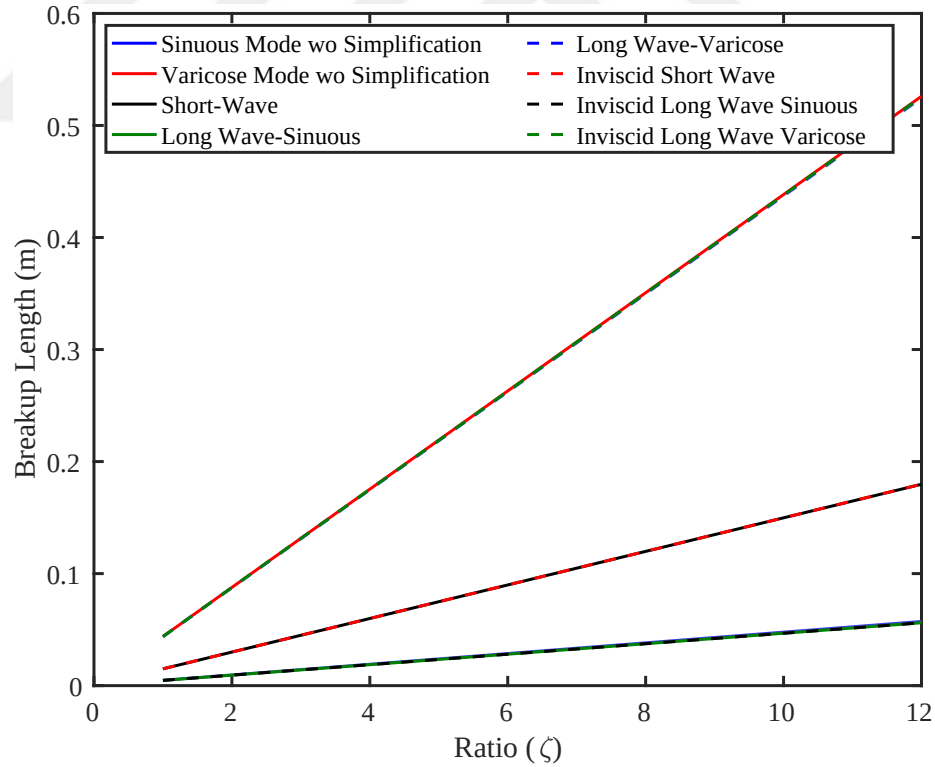


Figure 145: Calculated break-up length values w.r.t to disturbance ratio for Atomizer-16

Thickness of the liquid sheet at the breakup is shown in Figure 146. Highest value of the sheet thickness at the break-up is around $4.5 \mu\text{m}$ and decreases with increasing ζ_b and eventually approximates to a constant value.

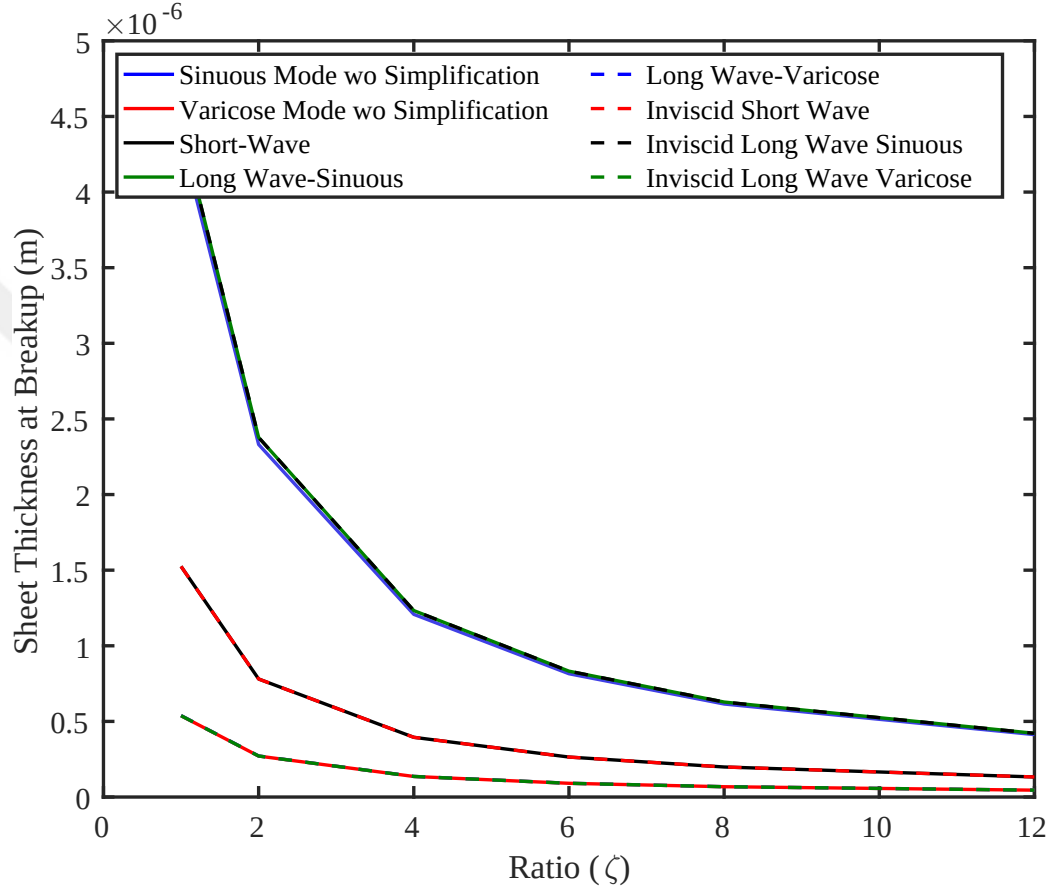


Figure 146: Calculated thickness of the sheet at break-up locations w.r.t to disturbance ratio for Atomizer-16

Finally, results for the droplet diameters are given in Figure 147. Effect of the correlation used for the calculation of the disturbance and wave number and the ζ_b can be seen more clearly from the calculated droplet diameter values. It is also clear that the diameter values approximate to a constant value as the ζ_b is increased.

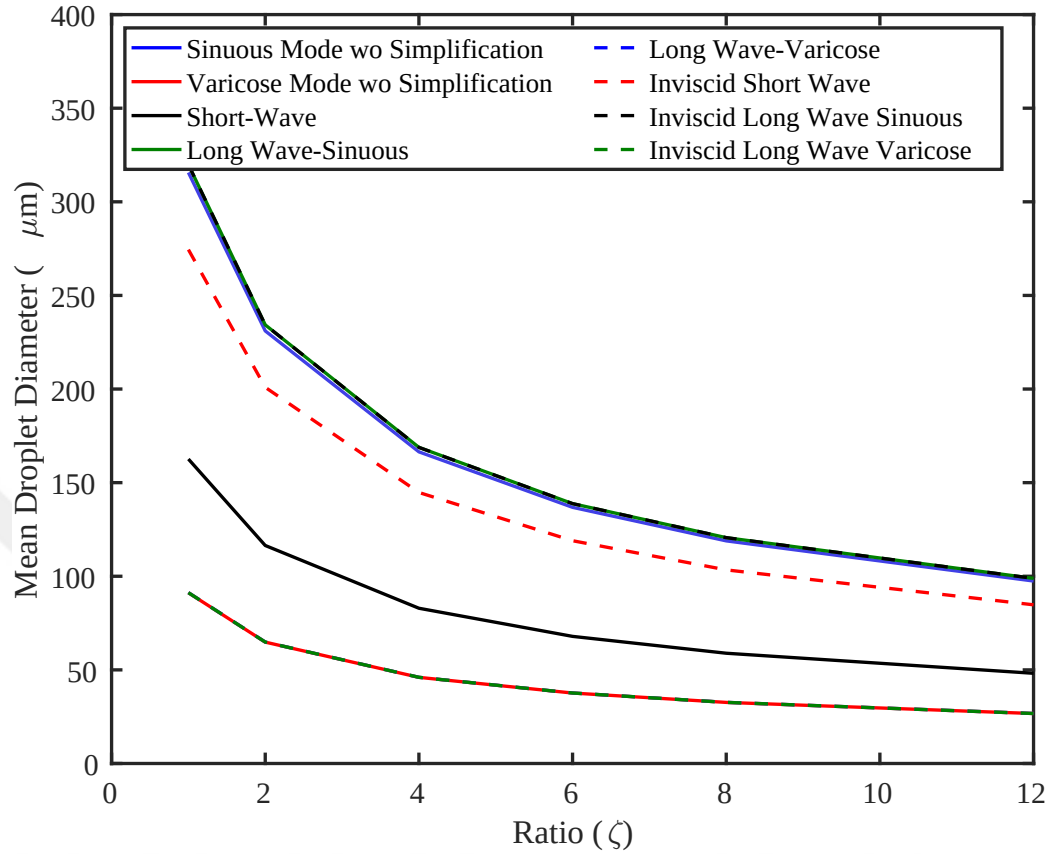


Figure 147: Calculated Rosin-Rammler droplet diameter w.r.t to disturbance ratio for Atomizer-16

In addition to the analysis of different correlations, a small computation is performed to understand the effect of spreading parameter and presented below. As mentioned earlier, calculated droplet diameter values are the Rosin-Rammler diameter values and the spreading parameter is set to 3.5 by default in the commercial CFD codes and cannot be altered by the user. Effect of the spreading parameter value on the distribution of the droplets is shown in Figure 148. Here, the Rosin-Rammler diameter value is kept constant and only the spreading parameter is altered. As can be seen, spreading parameter has a significant influence on the distribution characteristic of the droplets.

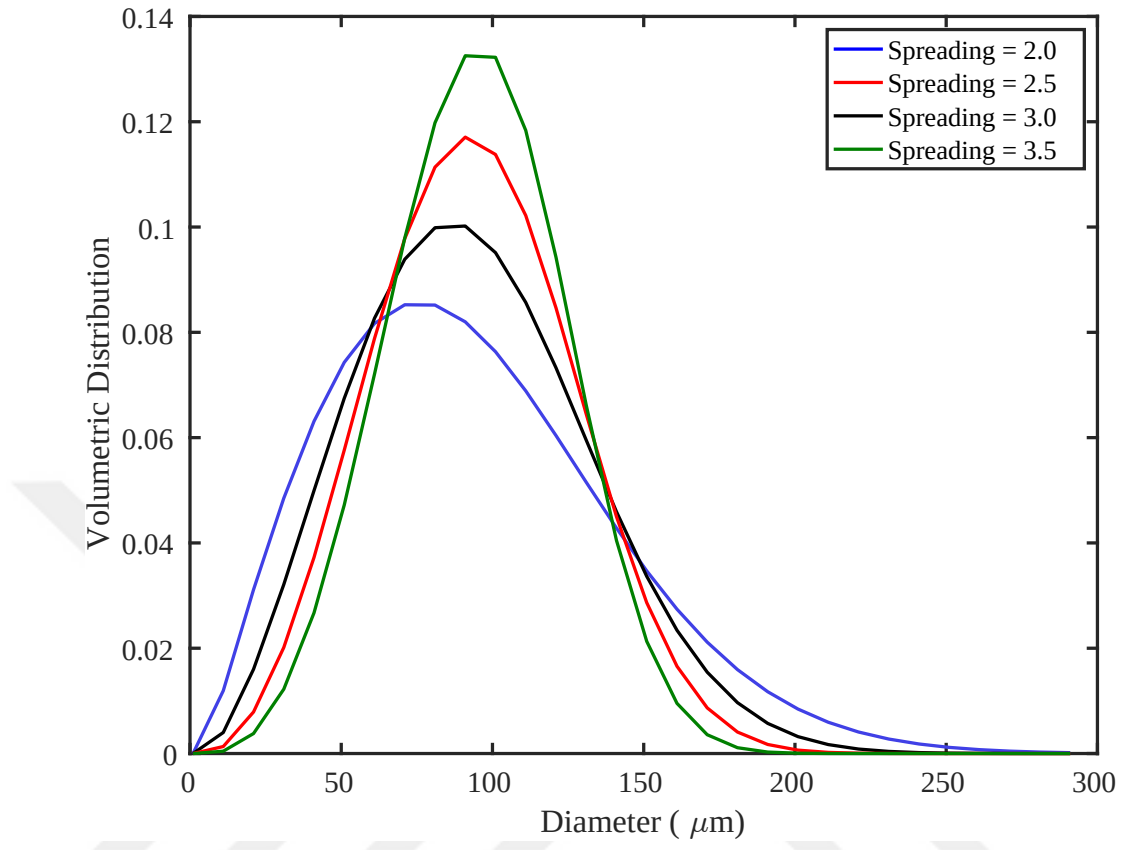


Figure 148: Effect of Rosin-Rammler spreading parameter on the droplet size distribution

When the representative droplet diameters such as Dv_{10} , Dv_{50} , Dv_{90} are calculated this effect can be seen more clearly. Table 30 displays the calculated values for same droplet diameter and different spreading factor values. It can be seen that, as the spreading factor increases the value of Dv_{10} increases and the value of Dv_{90} decreases. Also, the change in the Dv_{50} is considerably small. In other words, as the spreading factor increases, droplet size distribution becomes more uniform.

Table 30: Effect of Spreading Factor on Representative Droplet Diameters

Spreading Factor	Dv ₁₀	Dv ₅₀	Dv ₉₀
2.0	32	83	152
2.5	41	86	140
3.0	47	88	132
3.5	53	90	127

8.3 Obtaining Droplet Diameter Model

Finally, obtained droplet diameter model and its comparison with the experimental data are presented. Derived droplet diameter model is based on the LISA model. Since this model requires input parameters such as pressure difference at the given mass flow rate, spray angle, film thickness, and break-up length, use of the models derived and explained in Chapter 4 and Chapter 5 are done. As a result, obtained model is a combination of the models derived in the framework of this study. Moreover, a new model is acquired which is used for calculating the spreading parameter. As shown earlier, this parameter has a considerable effect on the resultant droplet diameter distribution.

In addition, only the experimental data obtained in the framework of this study is used to make comparisons with the obtained model. This is due to the fact that all of the presented data in Chapter 4 regarding the droplet size measurements consist of experiments conducted in the secondary atomization region. From the reviewed studies it is seen that the closest measurement of droplet size is conducted at 30 mm downstream of the atomizer. In comparison, in this study, experiments are conducted just below the location of break-up as shown in Figure 26.

As stated in Chapter 4, overall 24 atomizers are used to perform experiments in this study. Of these 24 atomizers, 12 of them are used for the Shadowgraphy

experiments. List of the atomizers and the flow rates of Shadowgraphy experiments is given in Table 31.

Table 31: List of the Atomizers used in Shadowgraphy Experiments and Flow Rates

Atomizer	Flow Rate (g/s)	Data Count
Atomizer-3	0.80, 1.00, 1.25, 1.50	4
Atomizer-5	1.50, 3.00	2
Atomizer-8	1.50, 3.00, 6.60	3
Atomizer-10	1.70, 5.80, 7.50	3
Atomizer-13	1.70, 5.80, 7.50	3
Atomizer-16	0.80, 1.00	2
Atomizer-17	0.80, 1.00, 1.50	3
Atomizer-18	1.50, 3.00	2
Atomizer-19	1.50, 3.00, 6.60	3
Atomizer-22	2.40, 3.00, 3.34	3
Atomizer-23	2.40, 3.00, 3.34, 4.00	4
Atomizer-24	2.40, 3.00, 3.34, 4.00	4

As can be seen, experiments are conducted at 36 different conditions. It should be noted that, for each of these 36 experiment conditions, 8 point measurement method described in Figure 26 is done. Obtained droplet diameters from each of these 8 locations are combined into a single data which is used to represent the diameter values of that experiment condition.

First, testing of the assumption that droplet diameters of the PSA can be represented with Rosin-Rammler distribution is done. As explained in Chapter 2, there are two versions of Rosin-Rammler distribution, namely Rosin-Rammler (Eq. 2) and Modified Rosin-Rammler (Eq. 3). Figure 149 shows the comparison of these distributions with the experimental data for a selected case. It can be seen that both

distributions can be used to represent the spray. Moreover, when this comparison is conducted for all of the experimental data, it is seen that deviation of Rosin-Rammler distribution from the experimental data is 0.13, and deviation of modified Rosin-Rammler is 0.14. Here, the deviation values are calculated by averaging the absolute difference between the experimental and calculated CDF values corresponding to the same droplet diameter.

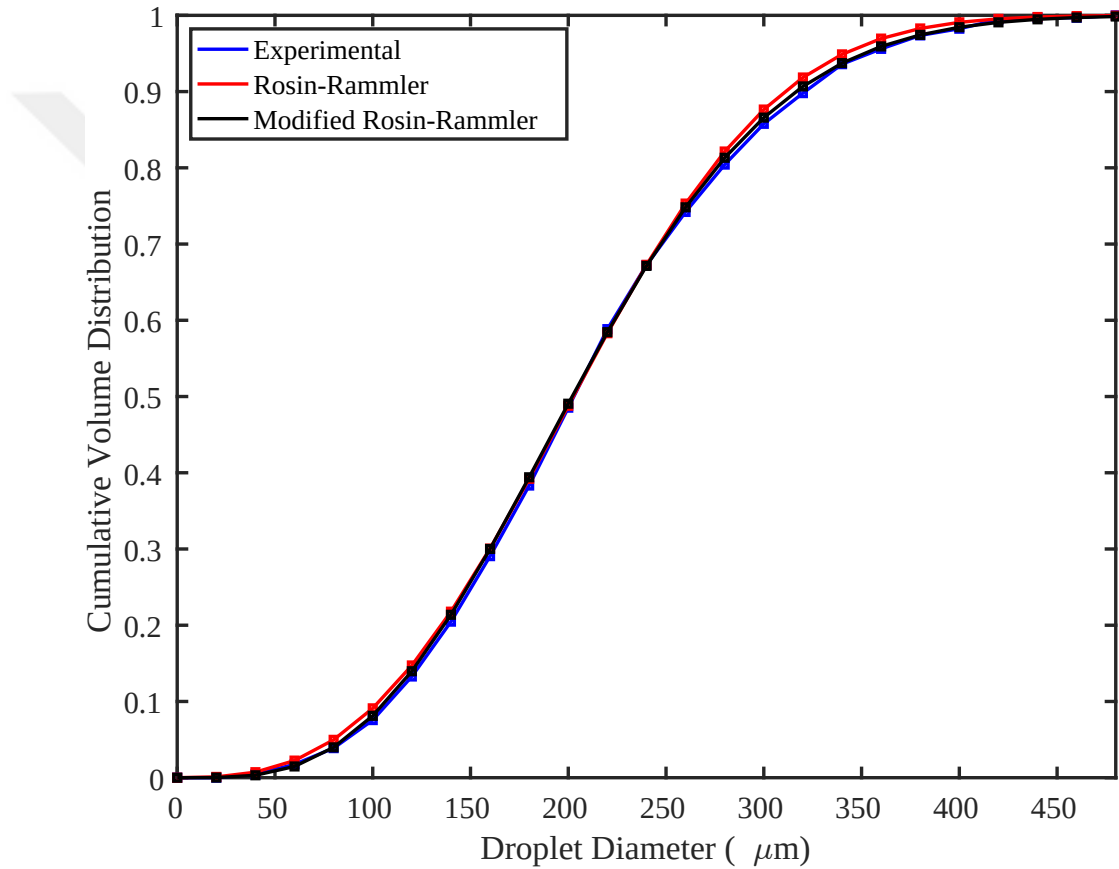


Figure 149: Comparison of Rosin-Rammler and modified Rosin-Rammler distributions with the experimental data

Second, obtained Rosin-Rammler and modified Rosin-Rammler spreading parameter and diameter values from the experiments are given in Table 32. Two deductions from the provided data can be made. First, it is seen that diameter values of Rosin-Rammler and Modified Rosin-Rammler distributions are almost identical yet

spreading parameters are significantly different from each other. Second, spreading parameter (for both Rosin-Rammler and Modified Rosin-Rammler) is not a constant value and changes with the atomizer and the flow rate.

Table 32: Results of the Shadowgraphy Experiments

Atomizer	Flow Rate (g/s)	Rosin-Rammler Diameter	Rosin-Rammler Spreading	Modified Rosin- Rammler Diameter	Modified Rosin- Rammler Spreading
Atomizer-3	0.80	118.50	2.95	118.44	13.65
Atomizer-3	1.00	77.31	2.57	77.18	10.77
Atomizer-3	1.25	70.79	2.68	70.74	10.97
Atomizer-3	1.50	66.51	2.47	66.34	10.07
Atomizer-5	1.50	102.02	2.33	101.94	10.62
Atomizer-5	3.00	63.43	2.47	63.26	9.90
Atomizer-8	1.50	150.46	3.06	150.45	14.75
Atomizer-8	3.00	98.54	3.11	98.62	13.62
Atomizer-8	6.60	29.26	1.79	29.37	6.24
Atomizer-10	1.70	246.74	3.35	246.70	17.65
Atomizer-10	5.80	75.31	3.08	75.14	12.64
Atomizer-10	7.50	56.26	2.62	56.05	10.24
Atomizer-13	1.70	325.06	2.74	324.10	15.35
Atomizer-13	5.80	102.45	4.06	102.36	17.87
Atomizer-13	7.50	77.21	2.96	77.07	12.41
Atomizer-16	0.80	90.73	3.04	90.70	13.23
Atomizer-16	1.00	73.38	3.10	73.23	12.78
Atomizer-17	0.80	80.93	2.51	80.88	10.58
Atomizer-17	1.00	73.91	2.07	73.75	8.48
Atomizer-17	1.50	68.54	2.77	68.45	11.15
Atomizer-18	1.50	172.44	2.15	172.39	10.96
Atomizer-18	3.00	120.03	2.34	120.03	11.08
Atomizer-19	1.50	143.17	3.05	143.10	14.58
Atomizer-19	3.00	135.25	2.34	135.16	10.77
Atomizer-19	6.60	60.83	2.55	60.77	10.27
Atomizer-22	2.40	166.47	2.50	166.40	12.41
Atomizer-22	3.00	131.12	2.42	131.08	11.40
Atomizer-22	3.34	126.34	1.90	126.24	8.97
Atomizer-23	2.40	126.34	2.52	126.24	11.77
Atomizer-23	3.00	125.70	2.14	125.61	10.05
Atomizer-23	3.34	113.08	1.75	112.98	8.01
Atomizer-23	4.00	61.76	2.11	61.65	8.68
Atomizer-24	2.40	285.34	2.52	285.29	13.80
Atomizer-24	3.00	230.73	2.81	230.72	14.80
Atomizer-24	3.34	217.70	2.57	217.66	13.40
Atomizer-24	4.00	186.51	2.38	186.45	12.16

Later, analysis and comparison of the LISA model when inputs (pressure difference, spray angle, film thickness, and break-up length) are calculated from the derived models are done. As stated in Chapter 4 atomizers used in this study are classified as complete data and parametric data, based on the known geometrical parameters.

For Atomizer-22, Atomizer-23, and Atomizer-24 all of the geometrical information is available since these atomizers are designed and manufactured in the framework of this study. For these atomizers, pressure difference, film thickness and spray angle are calculated from the model derived in Chapter 5 and lastly break-up length is calculated from semi-empirical correlation, derived in Chapter 4.

For the rest of the atomizers, only the exit diameter of the atomizers is known and hence, internal flow model cannot be used. Therefore, experimental pressure difference value is used for them. It should be noted that these atomizers are the product of companies such as Schlick, Steinen, and Hago. During the experiments of these atomizers, more than one copy of them is tested and the atomizer that has the closest pressure difference value to the catalog value is used in Shadowgraphy experiments. For the calculation of film thickness, Eq. 156 is used. In addition, calculation of the spray angle and break-up length values are done with Eq. 158 and Eq. 161, respectively. It should be reminded that these are the semi-empirical correlations that are derived in this study. Calculated AMRE and SE values of the derived disturbance correlations are given in Table 33.

Table 33: Error Values in Rosin-Rammler Diameter for the Obtained Model

Correlation	AMRE	SE
Correlation-1	0.194	0.022
Correlation-2	0.241	0.024
Correlation-3	0.265	0.038
Correlation-4	0.198	0.023
Correlation-5	0.241	0.024
Correlation-6	0.263	0.037
Correlation-7	0.197	0.023
Correlation-8	0.265	0.025

Inspection of the results shows that correlations obtained with sinuous mode assumption are more accurate and highest error values are seen for the correlations of short wave. It should be noted that inspection of the experimental data has yielded

that, except for one experiment condition, gas phase Weber numbers were lower than the suggested limit value of $27/16$, as shown in Figure 150.

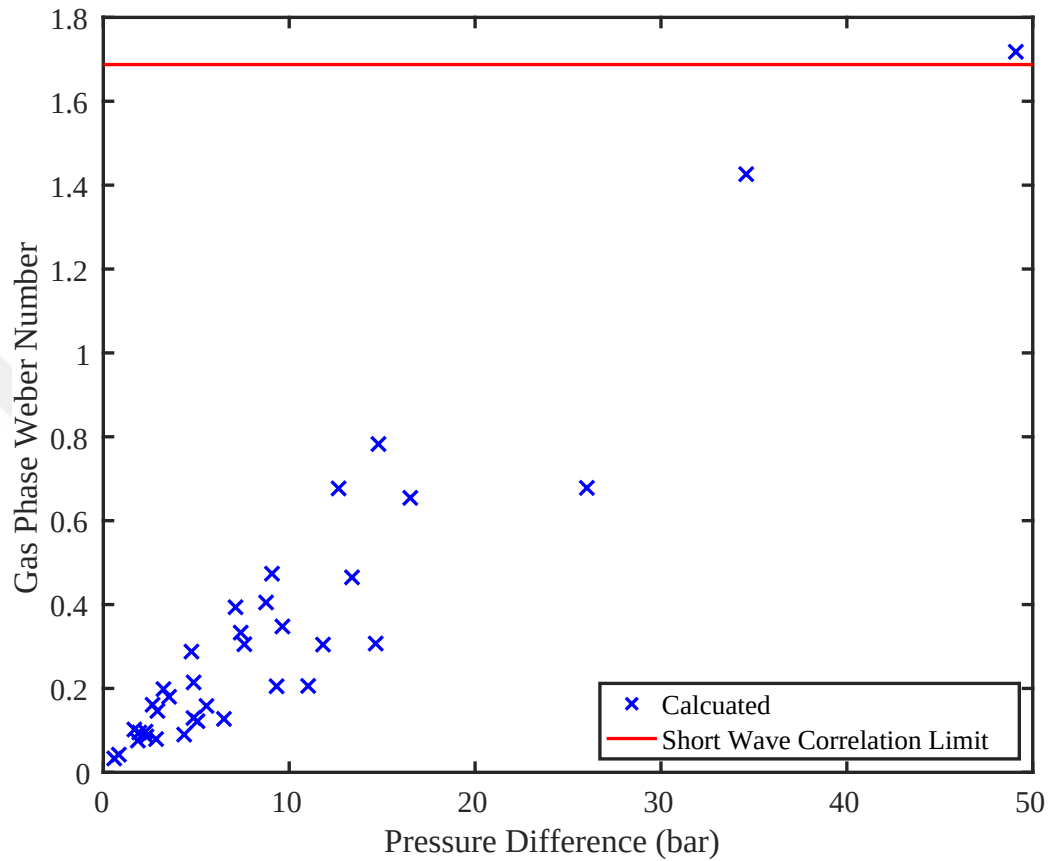


Figure 150: Calculated gas phase Weber number for Shadowgraphy experiments and comparison with limit value

Moreover, most accurate correlation is found to be Correlation-1, which is derived with sinuous mode assumption and without simplifications. This correlation is followed by the Correlation-7 and Correlation-4 which are derived with sinuous mode long wave assumption and the only difference between them is for Correlation-7 inviscid assumption is also done. It should be noted that Correlation-7 is the one that commercial CFD codes use. When the comparison of complexity between Correlation-1 and Correlation-7 is done, it can be seen that Correlation-7 is much easier to implement and only has a 0.003 higher AMRE value. Hence, this correlation is

selected to be the used correlation for droplet diameter calculation.

8.3.1 Modeling of the Spreading Parameter

After evaluating the accuracy of the correlations, derivation and comparison of the model for spreading parameter is done. As noted more than once, this parameter has a significant effect on the droplet diameter distribution yet by the commercial CFD codes is taken as constant. Figure 151 shows the values of Rosin-Rammler and modified Rosin-Rammler spreading parameters and also compares them with the value used by the commercial CFD codes.

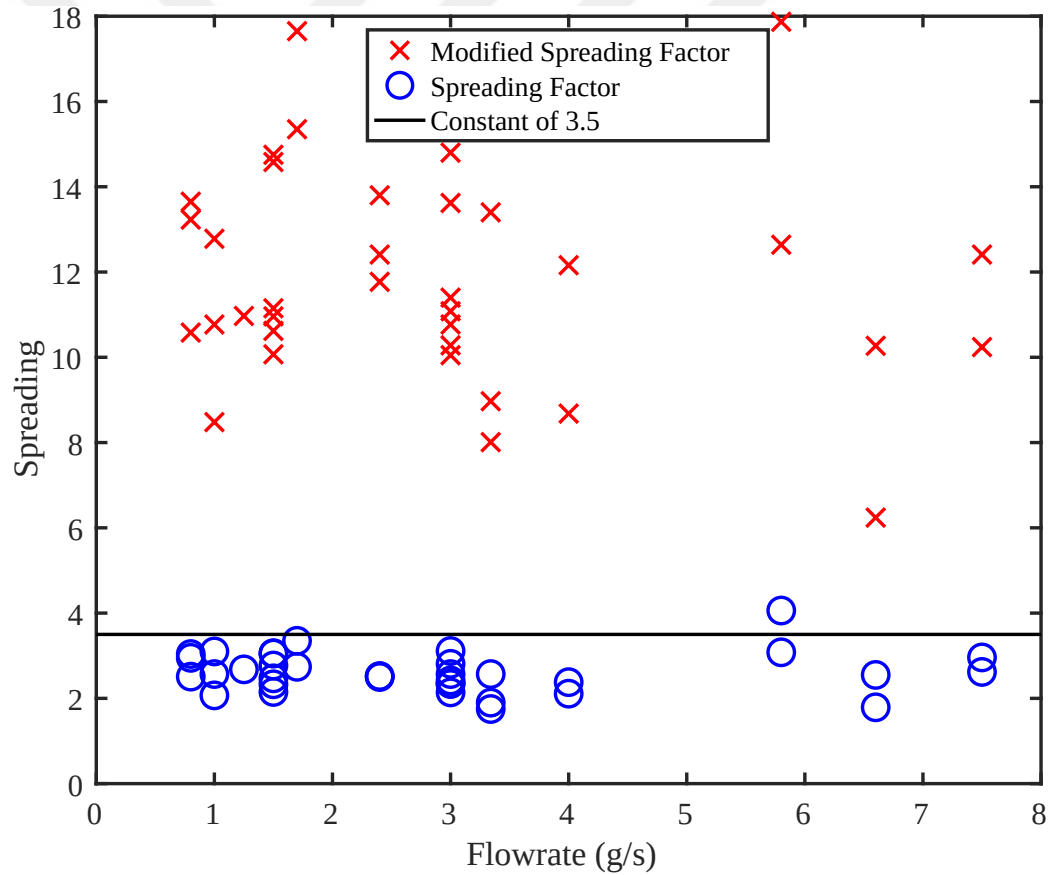


Figure 151: Comparison of Rosin-Rammler (Eq. 2) and modified Rosin-Rammler (Eq. 3) spreading parameter values with value of CFD

Inspection of the experimental results using Figure 151 and Table 32, shows that

modified Rosin-Rammler spreading parameter is more sensitive to the changes in experiment condition. For example, Table 32 clearly shows that increasing the flow rate of a given atomizer leads to lower modified Rosin-Rammler spreading values. Therefore, modified Rosin-Rammler spreading parameter is selected to be modeled. It should be reminded that also can be seen in Table 32, diameter values for Rosin-Rammler and modified Rosin-Rammler are almost identical. Parametric inspection of the modified Rosin-Rammler spreading parameter with the flow parameters such as liquid Reynolds number, gas phase Weber number, discharge coefficient, and spray angle is given in Figure 152.

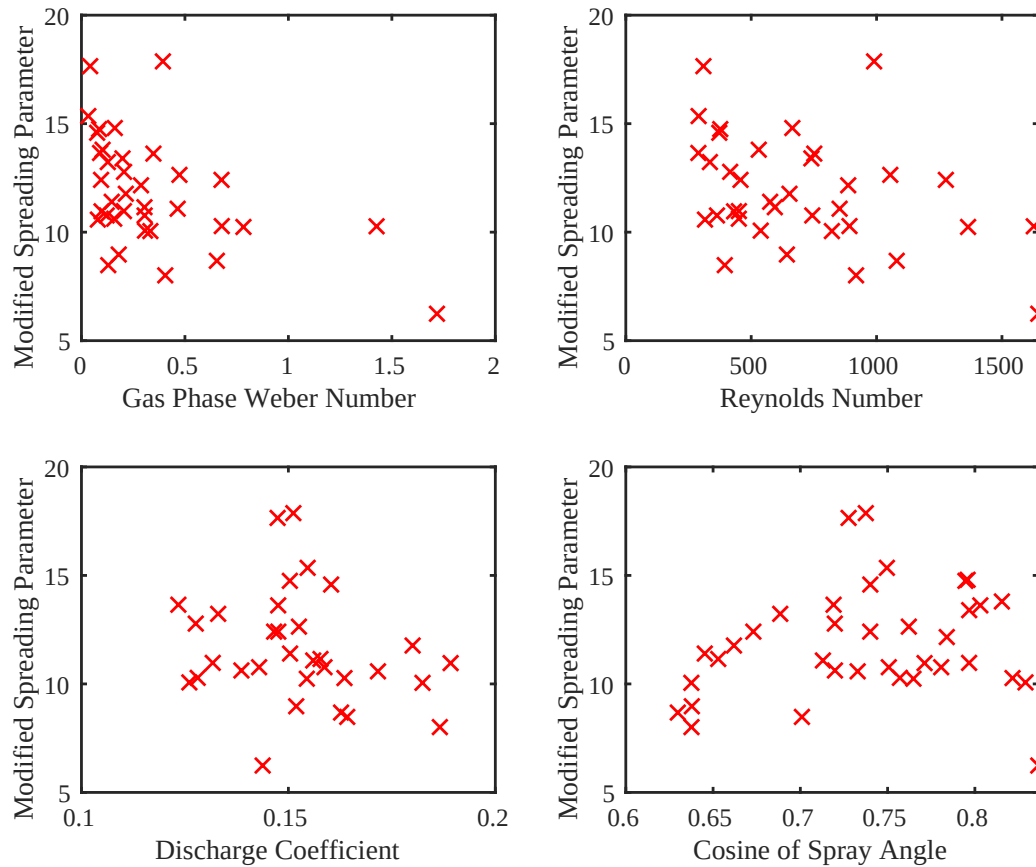


Figure 152: Parametric evaluation of modified Rosin-Rammler spreading parameter with the flow parameters

Based on these parameters, a semi-empirical correlation which has an AMRE value of 0.126 and SE value of 0.003 is obtained and given in Eq. 537.

$$q_{modified} = 0.037 \times We_{gas}^{-0.448} \times Re_L^{-0.645} \times \left(\frac{C_D}{\cos \theta} \right)^{-0.613} \quad (537)$$

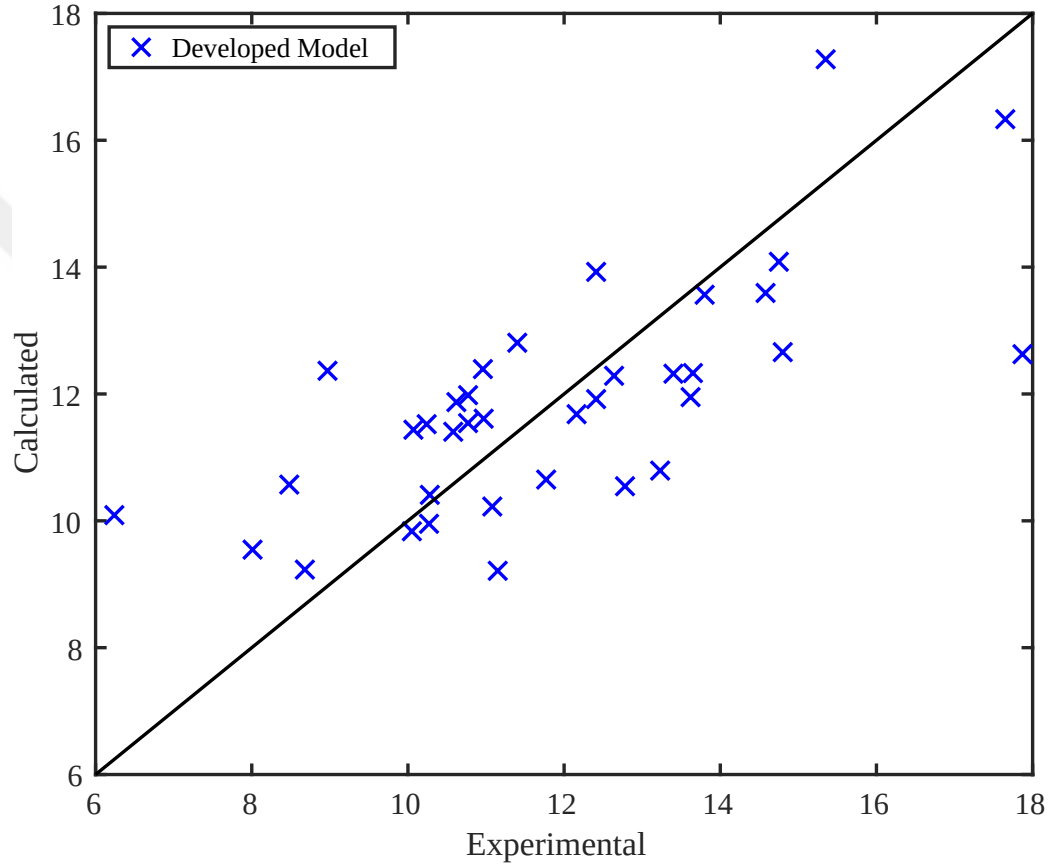


Figure 153: Comparison of calculated and experimental values of modified Rosin-Rammler spreading parameter

Finally, analysis of calculated droplet distributions is done. Comparisons with both the experimental data and the LISA model of the commercial CFD codes are done. It should be reminded that for replicating the LISA model of commercial CFD codes, disturbance ratio is taken as default value of 12 and constant value of 3.5 for the spreading parameter is used. Comparison for 9 different cases is shown

in Figure 154. It can be seen that obtained model is more accurate in terms of predicting the distribution of the droplet diameters. When this comparison is done for all of the experimental data shown in Table 32, it is seen that obtained model has the deviation of 0.21, and model of CFD codes has the deviation of 0.56. Here, as mentioned above, deviation values are calculated by averaging the absolute difference between the experimental and calculated CDF values corresponding to same droplet diameter.

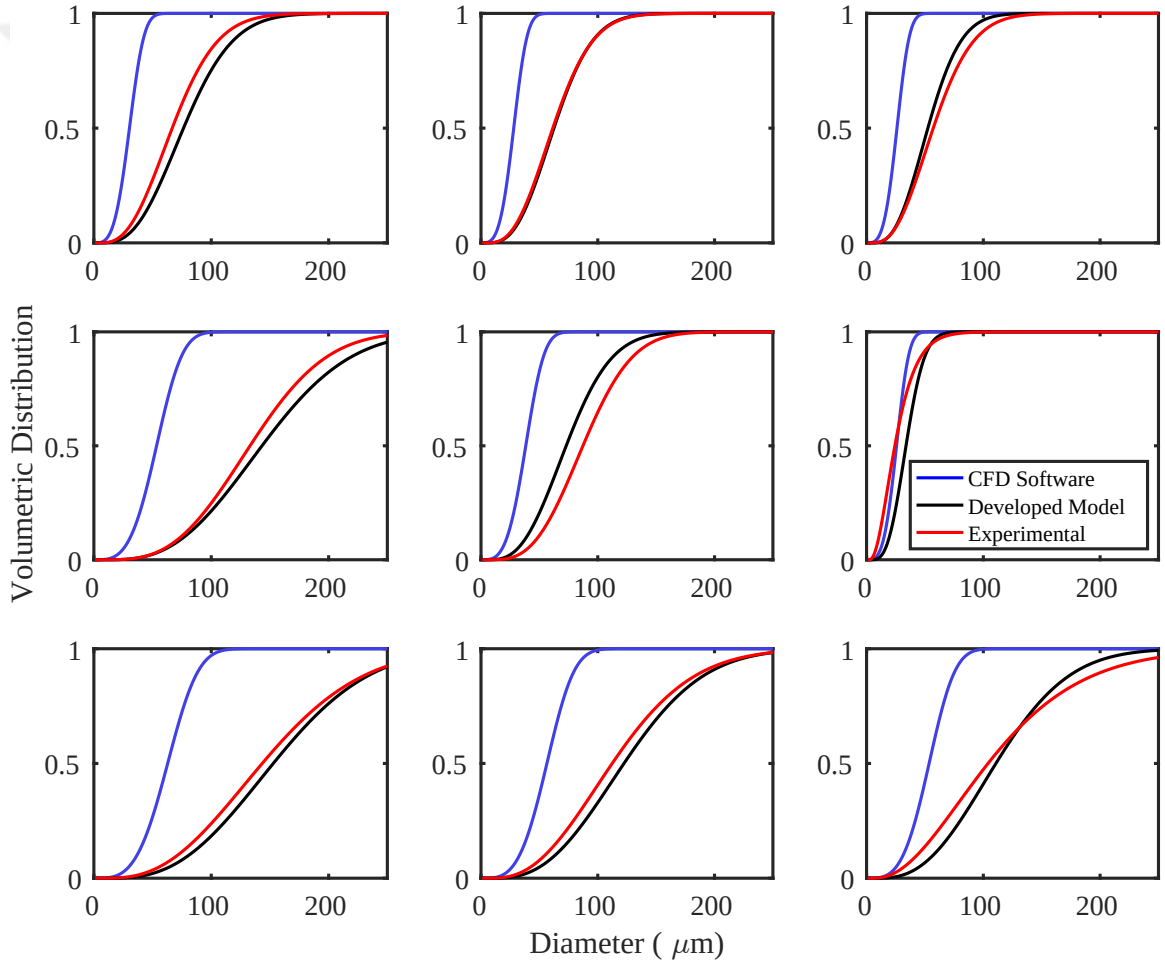


Figure 154: Comparison of obtained model with the model of CFD softwares and experimental data

CHAPTER IX

PRESENTATION OF THE COMBINED MODEL

In this section, obtained models and their use for optimization study of PSA are explained. Through Chapters 5-to-8, 4 different models for the calculations of performance parameters of PSA are obtained, which are boundary layer model, uniformity model, spray development model, and droplet size model. Also, as shown in Chapter 5 and Chapter 7, both the boundary layer model and the spray development model only need the geometry and the physical properties of the liquid and ambient to perform the calculation. Output of the boundary layer model is the pressure difference, film thickness, and the spray angle, and output of the spray development model is in binary form (0 and 1) where non-useful spray is assigned with 0, and useful spray is assigned with 1. As shown in Chapter 8, these models are obtained in a way that they can be used with each other, such as using calculated film thickness and spray angle values from the boundary layer in the droplet size calculation. It should be noted that droplet diameter model also requires the input of break-up length, which is obtained from the semi-empirical correlation derived in Chapter 4.

Before explaining the use of the derived models, first, a small explanation of the optimization of the PSA is done. In the most general form optimization of PSA can be represented with Eq. 538. For any case subject to optimization, aim is to minimize one or more performance parameters. This could be minimizing, for example, SMD or minimizing both the SMD and the pressure difference for the desired flow rate. Performance parameters to be minimized are dependent on the needs of the application and/or operation. Moreover, there could be limitations based on the dimensions or the performance parameters. To illustrate, based on the available

manufacturing methodology and the accuracy, certain limits for the dimensions of the atomizer could be given. Similarly, based on the application and/or operation certain limits for the performance parameters could be desired, such as spray angle being in a provided range.

$$\begin{aligned}
 &\textbf{minimize} \quad \textit{Performance Parameter}(s) \\
 &\textbf{subject to} \quad \textit{Limits of Dimension}(s) \\
 &\quad \quad \quad \textit{Limits of PerformanceParameter}(s)
 \end{aligned} \tag{538}$$

Derived models are used in optimization calculations starting with the spray development model as shown in Fig. 155. As mentioned earlier, only required input parameters are the physical properties of the liquid and ambient and the mass flow rate.

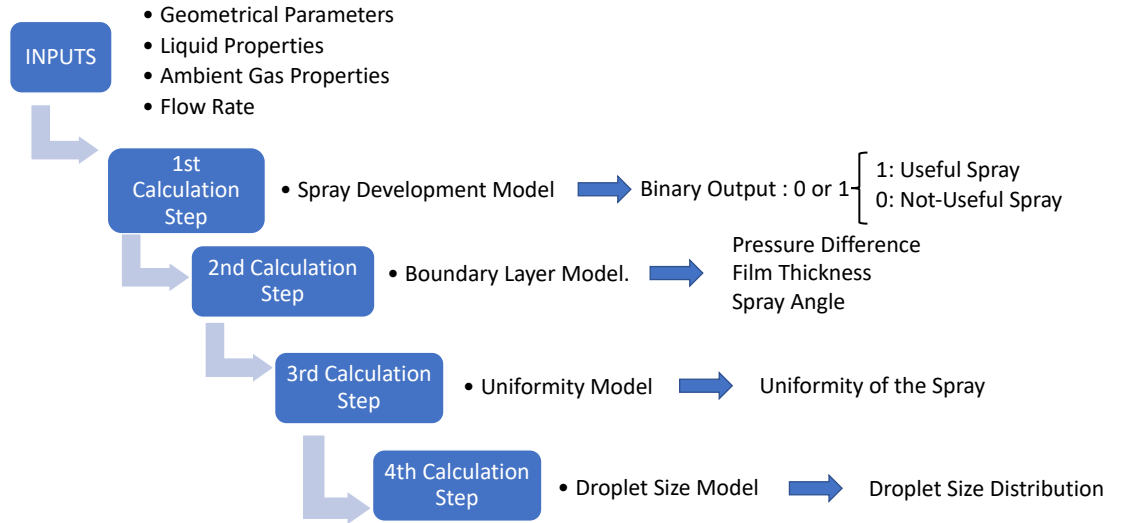


Figure 155: Order of the use of derived models

It should be noted that geometry of the atomizer is subjected to optimization based on the desired output criteria and iterated throughout the calculations. From the input parameters and the geometry obtained in each iteration, calculation of the

spray development is performed in the first step. As mentioned earlier, for any given application, it is desired to have a spray in the useful spray stage.

As stated earlier, only properties of the liquid and ambient are needed along with the geometrical parameters of the PSA. This step is the most important step of calculation since both the uniformity model and the droplet size model are dependent on the parameters calculated here. For the uniformity model, film thickness from the boundary layer is needed and for the droplet size model, pressure difference, spray angle, and film thickness values calculated from the boundary layer model and break-up length calculated from Eq. 161 is needed, as mentioned earlier.

CHAPTER X

DISCUSSION AND CONCLUSIONS

This study is conducted with the aim of obtaining a model that could be used for the optimization calculations of pressure-swirl atomizers. Large number of numerical and experimental studies are conducted, and also work of various researchers is also presented. Total of 5 different models are obtained, which are semi-empirical models, boundary layer based internal flow model, uniformity model, spray development model, and droplet diameter model.

Conducted numerical studies are divided into two groups namely, 2D axisymmetric simulations and full 3D simulations. Study of the 2D axisymmetric simulations has shown that with the adjustment of inlet velocity components, it can accurately predict the internal flow field and the spray angle of the PSA. Model for making such adjustments is provided, and comparisons with experimental data of atomizers having various sizes have shown significant improvement in accuracy. Full 3D simulations are conducted with atomizer having a unique shape factor and used for understanding the coupling of the uniformity of the spray and internal flow of the atomizer.

A large number of experimental data is obtained from the literature and presented with the experiments conducted in the framework of this study. A total of 1,777 experimental data is provided, of which 366 are obtained in this study. Moreover, data is divided into four groups based on the available geometrical data.

Experimental studies of this work consist of high-speed imaging, Shadowgraphy, and Laser Induced Fluorescence methods. High-speed imaging experiments have yielded a peculiarity in the obtained break-up length values. From the inspection of the results of Atomizers 1-to-21, it can clearly be seen that with the increasing

pressure, spray angle decreases, and break-up length decreases at first and later increases except for Atomizer-16, 17, and 18. It should also be noted that these three atomizers are the only ones in this group that do not include a divergent section after the orifice. In other words, all the atomizers that follow the trend of decreasing spray angle and at first decreasing and later increasing break-up length include a trumpet section. The reasoning behind such behavior is suggested to be performed in future studies.

Obtained experimental data is used for evaluation of the semi-empirical models obtained from the literature. Results have yielded large error values, especially for the calculation of parameters such as pressure difference and break-up length. Therefore, this experimental data is used for derivation of new semi-empirical correlations as well. Correlations are derived such that, while they had significantly lower error values, they could also be used as a system. Hence, only required inputs for these equations are the physical properties of the liquid and the dimensions of the atomizer.

Internal flow of the PSA is modeled with a boundary layer based model. Two different models, one for the straight sections and second for the convergent and divergent sections are obtained and solved with the Pohlhausen method. Different from the existing models in the literature, derived boundary layer models are obtained with acknowledging the existence of the air core inside the atomizer and also do not require any additional assumptions such as maximum flow principle. Moreover, in this study, velocity profiles having polynomial forms are employed for the boundary layer, which is used for laminar flows. In the literature swirling pipe flow is also studied with turbulent velocity profiles and this could be extended to the pressure swirl atomizer.

Moreover, during the validation of the model obtained for the straight section, it is seen that using higher initial air core radius value led to more accurate prediction of the velocity components and the air core radius at the exit plane for the simulated

case. However, when same initial value is used for available experimental data, a slight increase in the AMRE value of film thickness is found. It should be noted that, simulated open-end atomizer case have a larger air core at the exit plane, filling 94% of the orifice. Whereas for the experimental data this value ranges from 73%-to-84%. It should be mentioned that there are only 5 experimental data for the film thickness of open-end PSA. Hence, further research is suggested for obtaining a model for the initial air core values of open-end atomizers.

Validation of each boundary layer model is performed in addition to the validation of the combined model. First model is compared with the experimental data of open-end atomizers, second model is compared with the experimental data of atomizers of this study. Since three of the atomizers used in this study had small swirl chamber lengths and lacks orifice section, this comparison could be done. Validation of the combined model is done with the experimental data of close-end atomizers. Results have shown improvement in the accuracy for the calculation of pressure difference, film thickness, and spray angle in comparison to the semi-empirical correlations.

An important note that could be made here is, spray angle is calculated from the ratio of mean tangential velocity to the mean axial velocity. In the case of PSA, when the liquid leaves the atomizer, it starts to expand radially while obtaining axial and tangential velocity. Hence, the spray angle of atomizer actually is the ratio of radial velocity to axial velocity. However, used equation for the spray angle calculation suggest that, when the liquid leaves the atomizer, all of the tangential velocity will convert to radial velocity and this assumption is erroneous. Experience obtained from high-speed imaging has shown that liquid sheet continues to have a swirling motion after leaving the atomizer and modeling of this is suggested for future work. In addition, issue of using a semi-empirical correlation for the calculation of break-up length could be challenged in future works.

Uniformity of the spray and relation of it to the internal flow is studied by conducting large number of 3D simulations. Of the studied cases three of the atomizers are manufactured with highest possible accuracy using a non-conventional method named laser lithography. These geometries are tested with the LIF/Mie method, and it was shown that non-uniformity existing inside the atomizer effects the spray significantly. Moreover, it was shown that non-uniformity is formed due to phenomenon named inlet-inlet interaction, which is mainly due to the distance between the inlet ports being small. Hence, flow entering from one of the inlets interacts with flow from the adjacent port. A semi-empirical model for the description of the non-uniformity is also obtained and compared with the existing model in the literature. It should be noted that this comparison is done with experimental data of this study and also experimental study existing in the literature.

Analysis and modeling of the development of spray are also performed. Similar to the above-mentioned studies, large number of experimental data is obtained from the literature and combined with the experimental data obtained in this work. Later, this data is used for the evaluation of existing models and derivation of a new model. Results have shown that derived model has higher accuracy than the existing model and also, derived model can be used with the inputs of liquid properties and geometrical parameters.

Modeling of the droplet diameters is done with the LISA model, which is widely used by commercial CFD codes. Inspection of the dispersion relation is done, and total of 8 different correlations for describing the disturbance on the liquid sheet is obtained. Obtained results have shown that correlation obtained with long wave and sinuous mode assumption has a higher accuracy, and this is the model that is used by the commercial CFD codes as well.

Moreover, improvements of the model are done by using the boundary layer model to calculate pressure difference, film thickness, and spray angle and introducing a

model for the modified Rosin-Rammler spreading parameter, and using the semi-empirical correlation for the break-up length. It should be mentioned that parameter called disturbance ratio is calculated from the break-up length of the atomizer and have a default value of 12 in the commercial CFD codes. Similarly, spreading parameter has the default of 3.5. It was shown that these parameters have a significant effect on the results. Comparison of calculations done with the LISA model using default values and the LISA model using the values obtained from the models in this study have shown that suggested model has significantly higher accuracy in the prediction of the droplet size distribution.

Finally, use of the models obtained in this dissertation for performing an optimization study is explained. It should be mentioned that calculation time of the suggested method for a single geometry was found to be 20.0 seconds for a single atomizer geometry and single flow rate. It should be noted that implementation of the model is done with MATLAB and a processor with 2.0 GHz is used for calculation. Studies of the 2D axisymmetric and full 3D simulations have shown that calculation time for them is 208 core-hour and 2,880 core-hour with 2.3 GHz processors, respectively. This means that in the calculation time of a single 2D simulation 37,440 calculations and in the time of a single 3D simulation 518,440 calculations with the suggested model can be done.

It should be noted that almost all of the experimental data used for the development and testing of the models are obtained from the studies conducted at atmospheric pressure values. From the limited number of studies in the literature and the theoretical knowledge, it is known that increasing the ambient pressure leads to increase of the discharge coefficient and decrease of the film thickness, spray angle, and break-up length. Inspection of the derived droplet diameter model shows that having smaller spray angle and break-up length will result in larger droplet sizes

whereas having a thinner liquid film at the exit will lead to smaller droplet sizes. Similarly, contradicting results have been reported in the literature regarding the effect of ambient pressure on droplet sizes [1, 2, 25]. It should be noted that, these studies have measured the droplet sizes in the secondary atomization region where effects of droplet collision and coalescence have a significant effect.

In addition, effect of ambient pressure is not included in the derived uniformity and spray development models. From the theoretical knowledge it can be said that increasing the ambient pressure will hamper the development of the spray. The model derived for the spray development is used for separating the useful spray from non-useful spray which is essentially the separation of the onion stage from the tulip stage. Difference between these stages is that in onion stage spray collapse onto itself and in tulip stage spray opens up. With the increased ambient pressure, it is expected that spray will tend to collapse. Effect of increased ambient pressure on the uniformity of the film is expected to be small since it is caused by the internal flow dynamics. However, it was stated that increasing air density will cause the droplets to follow the air flow around which could lead to non-uniformity [1, 2]. For better understanding and modeling of the effect of ambient pressure on the performance of PSA, measurement of the droplet sizes in the primary break-up region, inspections of the development of the spray, and uniformity are suggested for future work.

APPENDIX A

EXPERIMENTAL DATA OF LITERATURE

A.1 Close-End Atomizers

A.1.1 Complete Data

A.1.1.1 Swirl Atomizers Operating at High Water Pressure, 1996 [8]

Table 34: Dimensions of the Atomizers [8]

No.	D_p (mm)	L_p (mm)	N_p	β	D_s (mm)	L_s (mm)	L_{es}/L_s	α	D_o (mm)	L_o (mm)
1	1.20	2.40	2	90^0	8.50	17.80	0.196	50^0	1.20	3.60
2	1.70	1.47-4.17	2	90^0	8.50	19.20	0.187	50^0	1.30	2.10
3	1.20	1.04-4.41	4	90^0	8.50	19.20	0.187	50^0	1.30	2.10
4	1.20	1.04-4.41	2	90^0	8.50	19.20	0.187	50^0	1.30	2.10
5	1.20	2.40	2	90^0	8.50	3.55-32.50	0.196	50^0	1.30	2.10
6	1.20	2.40	2	90^0	8.50	21.92	0.027-0.936	50^0	1.30	2.10
7	1.20	2.40	2	45^0	8.50	3.55-35.50	0.196	50^0	1.30	2.10
8	1.20	2.40	2	90^0	4.26-10.48	9.63-23.50	0.26	50^0	1.30	2.10
9	1.20	2.40	2	90^0	8.50	19.10	0.115	50^0	1.30	0.65-13.00
10	1.20	2.40	2	90^0	8.50	19.10	0.115	50^0	0.80-2.50	2.00-2.75

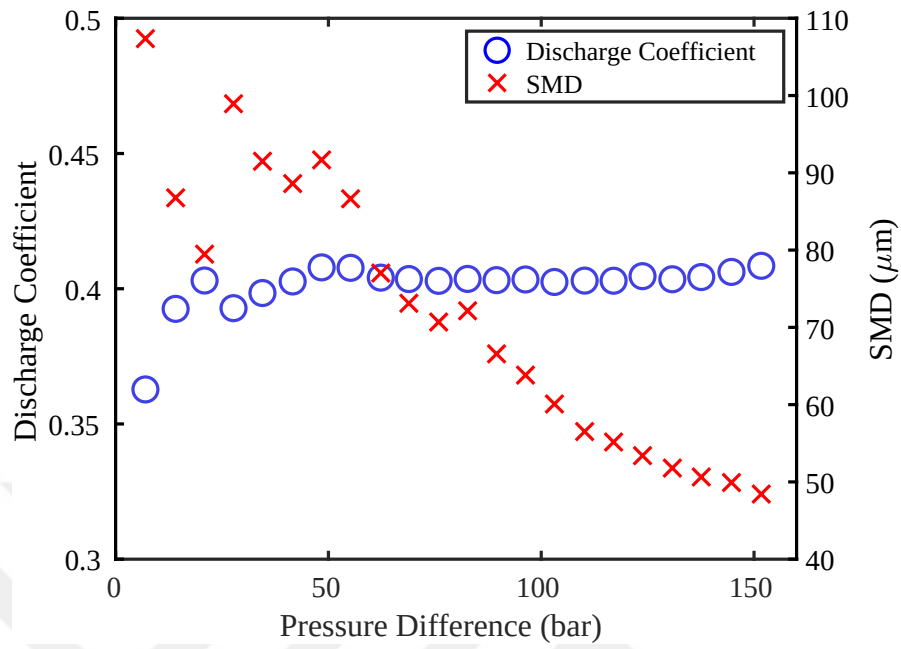


Figure 156: Experimental results - Geometry-1 [8]

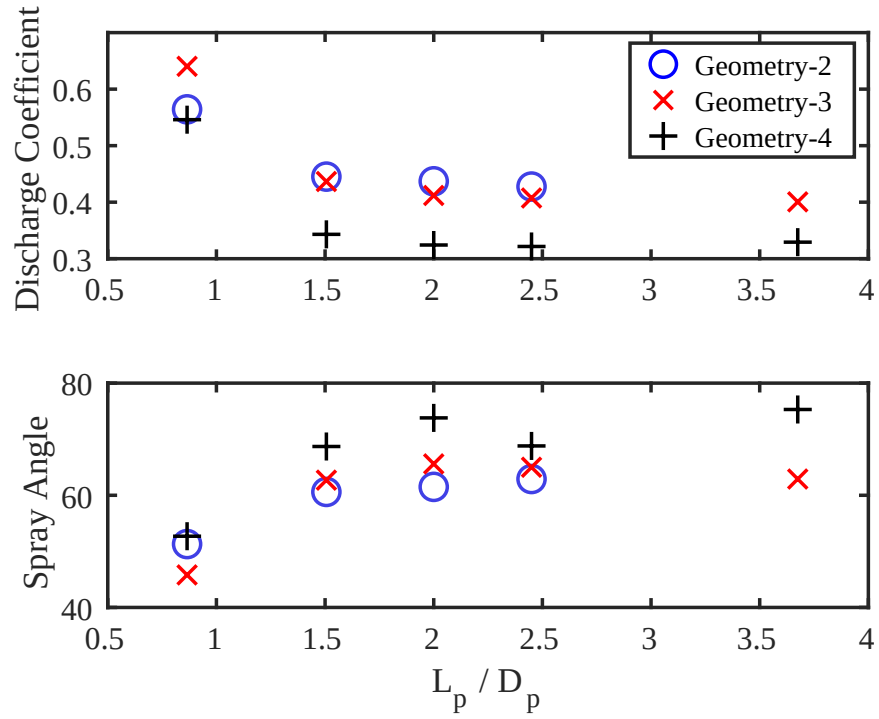


Figure 157: Experimental results - Geometry-2,3,4 [8]

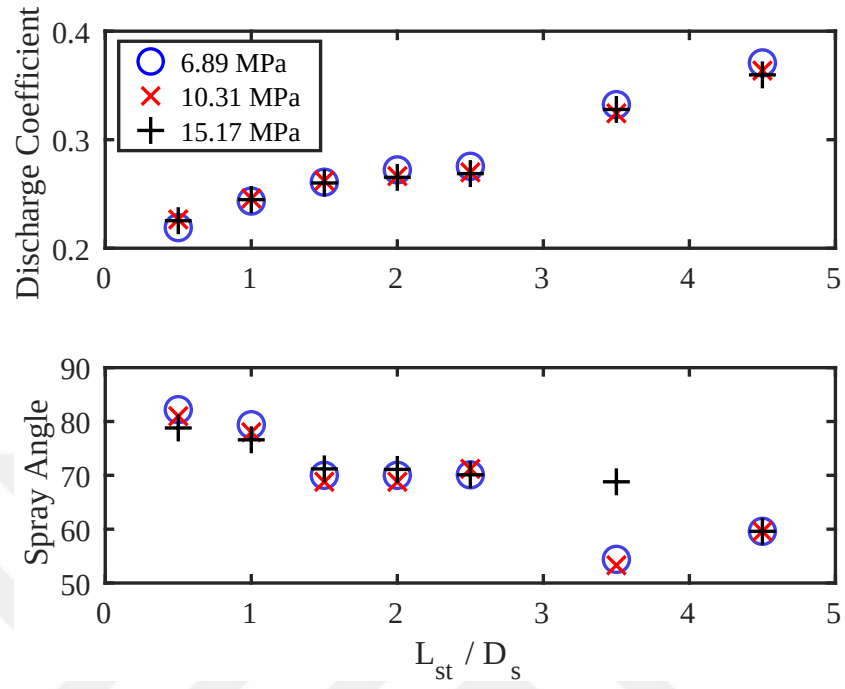


Figure 158: Experimental results - Geometry-5 [8]

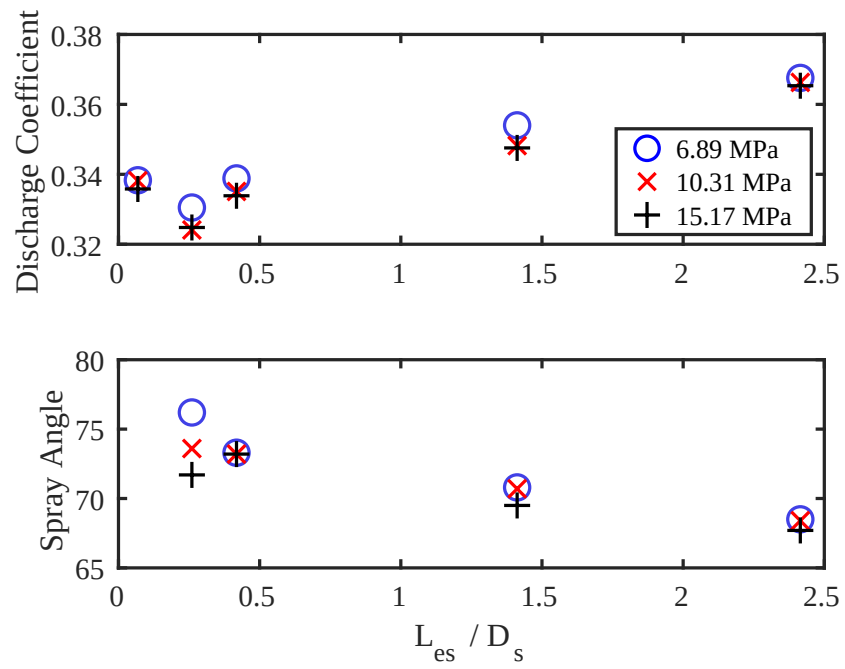


Figure 159: Experimental results - Geometry-6 [8]

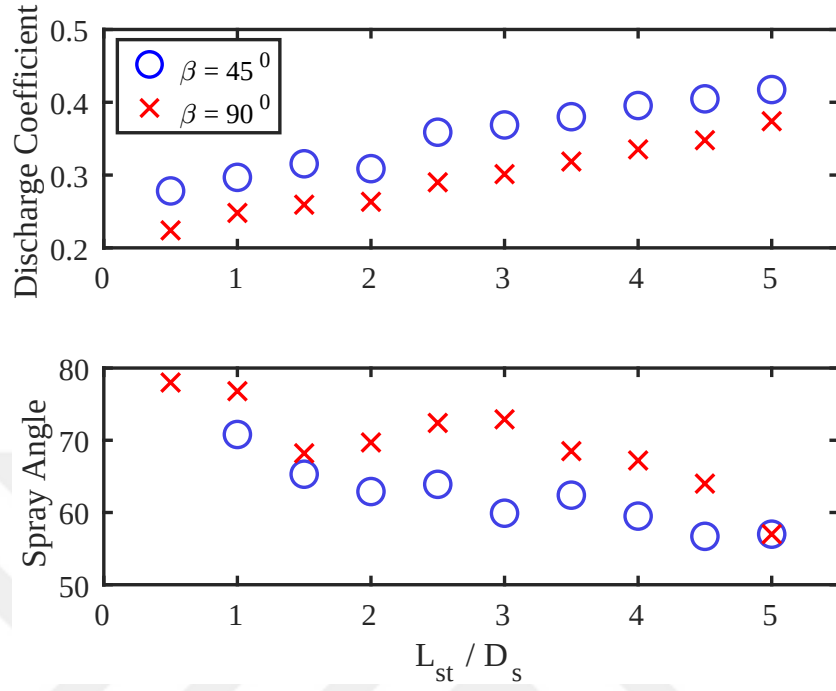


Figure 160: Experimental results - Geometry-5 and 7 [8]

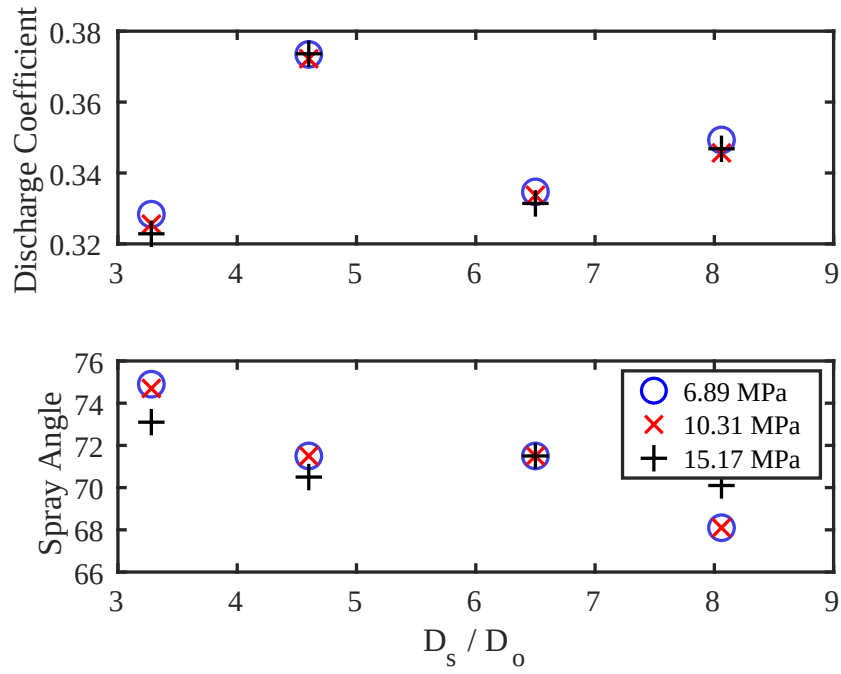


Figure 161: Experimental results - Geometry-8 [8]

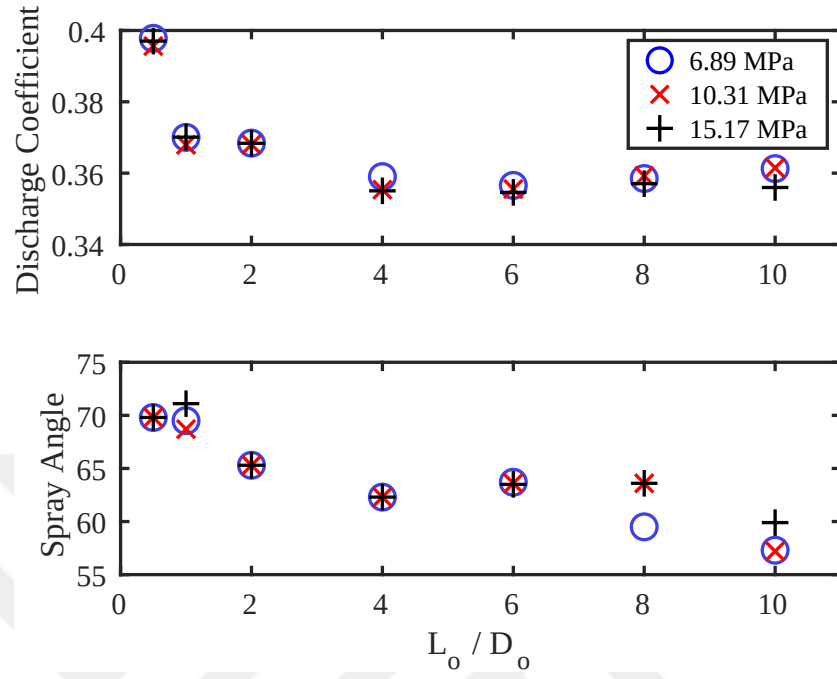


Figure 162: Experimental results - Geometry-9 [8]

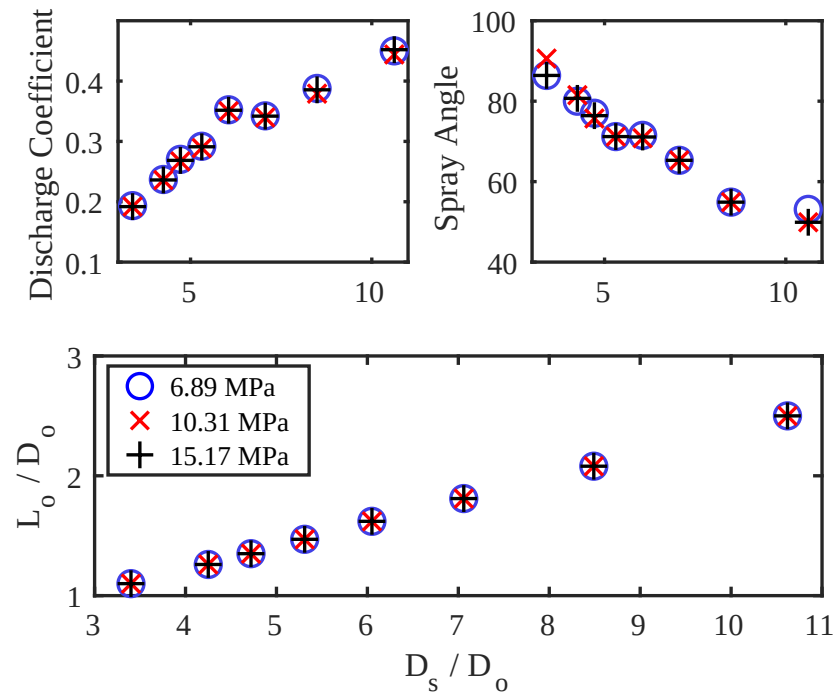


Figure 163: Experimental results - Geometry-10 [8]

Table 35: Experimental Results - 1 [3]

No.	D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)	ΔP_L (bar)	C_D
1	0.86	1.20	5	4.17	1.30	90^0	0.80	2.00	39.2	0.42
2	1.02	1.20	5	5.12	1.30	90^0	0.80	2.00	39.2	0.69
3	0.86	1.20	5	4.17	1.30	90^0	1.92	2.00	39.2	0.33
4	1.02	1.20	5	5.12	1.30	90^0	1.92	2.00	39.2	0.50

Table 36: Dimensions of the Atomizers - 1 [3]

No.	D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
5	0.85	1.20	1	4.50	1.20	30^0	0.90	0.50

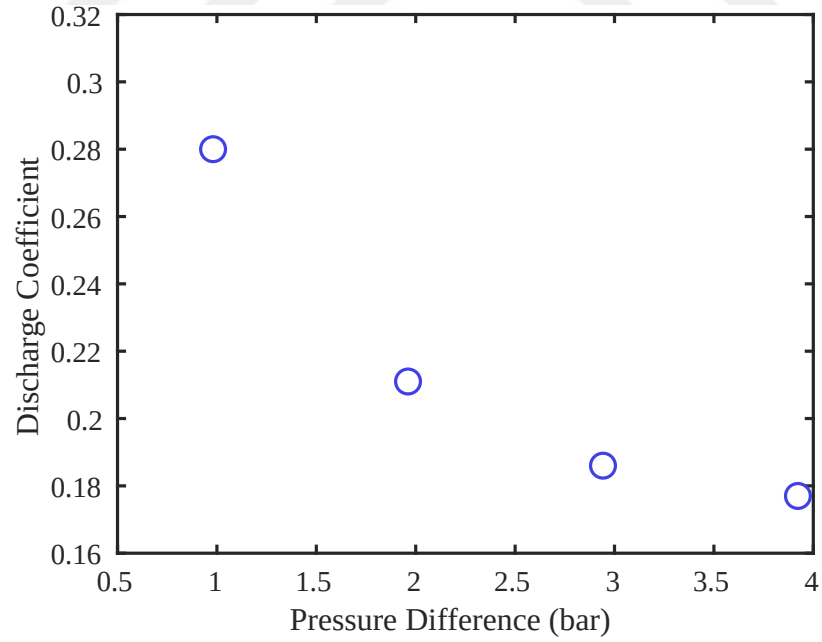
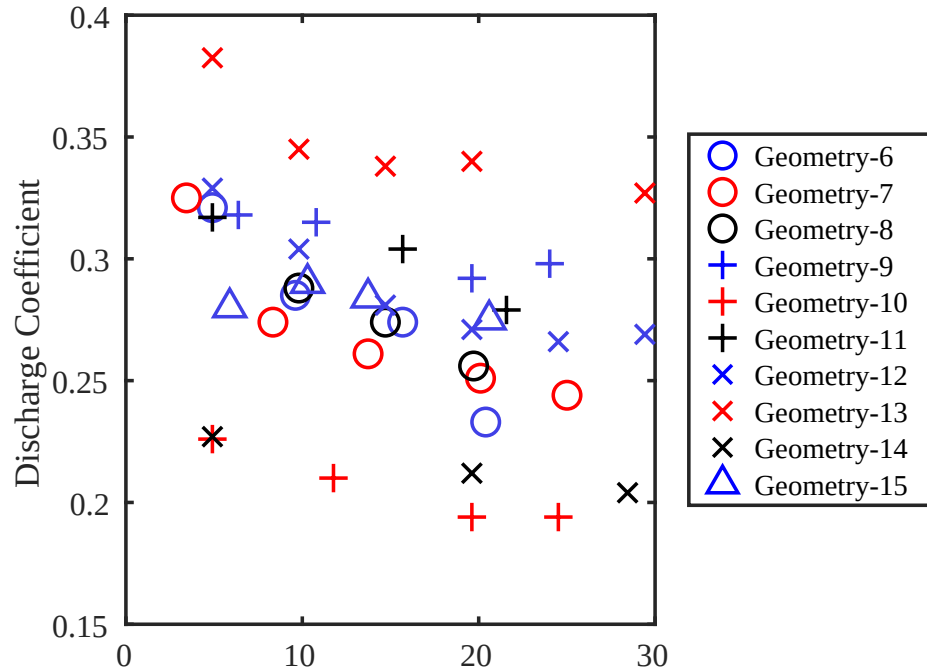


Figure 164: Experimental results - Geometry-5 [3]

Table 37: Dimensions of the Atomizers - 2 [3]

No.	D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
6	0.91	4.00	2	7.71	5.10	90^0	0.91	0.40
7	0.91	3.50	2	8.86	5.10	90^0	0.91	0.40
8	0.35	5.00	2	5.00	3.50	90^0	0.50	0.20
9	0.94	4.00	2	6.00	4.00	90^0	0.94	0.40
10	0.67	4.50	2	3.25	3.00	90^0	1.01	0.40
11	0.35	4.70	2	3.00	2.00	90^0	0.38	0.20
12	0.67	4.70	2	3.25	3.00	90^0	0.74	0.30
13	0.67	4.70	2	3.25	3.00	90^0	0.60	0.30
14	0.94	4.00	1	6.00	4.00	90^0	0.94	0.40
15	1.58	3.50	2	9.00	6.00	90^0	1.58	0.60

**Figure 165:** Experimental results - Geometry-6-15 Discharge Coefficient [3]

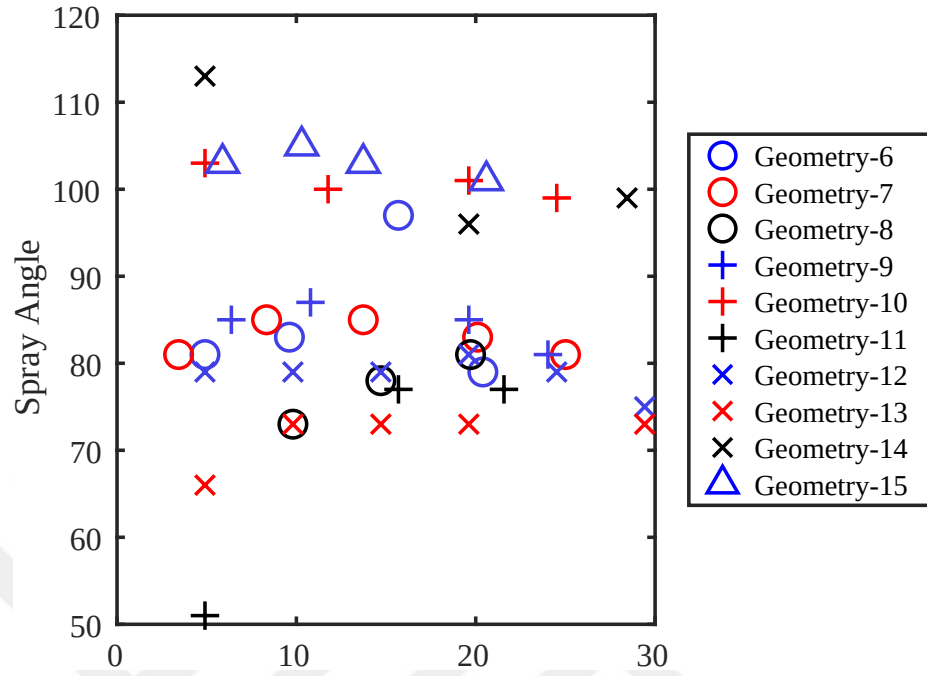


Figure 166: Experimental results - Geometry-6-15 Spray Angle [3]

Table 38: Dimensions of the Atomizers - 3 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
16	1.50	1.50	7.50	6	10.00	1.50	90°	2.30-4.50	4
17	1.50	1.50	3.50-8.00	6	8.00-18.00	1.50	90°	3.90	4

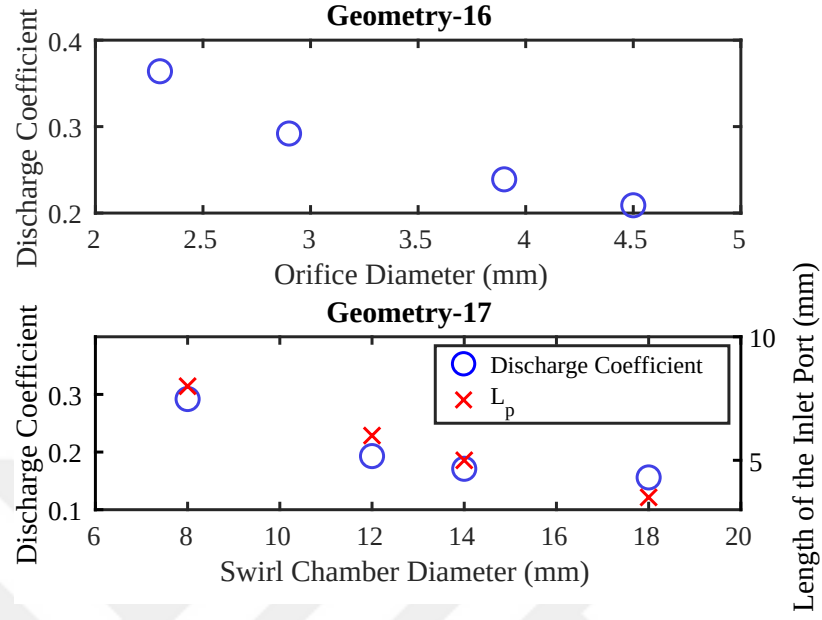


Figure 167: Experimental results - Geometry-16 and 17 [3]

Table 39: Experimental Results - 2 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)	ΔP_L (bar)	C_D
18	1.50	1.50	7.50	4	10.00	1.50	90°	3.90	4	9.80	0.16
19	1.50	1.50	7.50	2	10.00	1.50	90°	3.90	4	9.80	0.08
20	4.50	4.50	7.50	2	10.00	1.50	90°	3.90	4	9.80	0.23
21	3.00	3.50	7.50	3	10.00	1.50	90°	3.90	4	9.80	0.22

Table 40: Dimensions of the Atomizers - 4 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
22	1.50	1.50	3.50-8.00	6	8.00-18.00	1.50	90°	2.30	4.00
23	1.50	1.50	3.50-8.00	6	8.00-18.00	1.50	90°	4.50	4.00

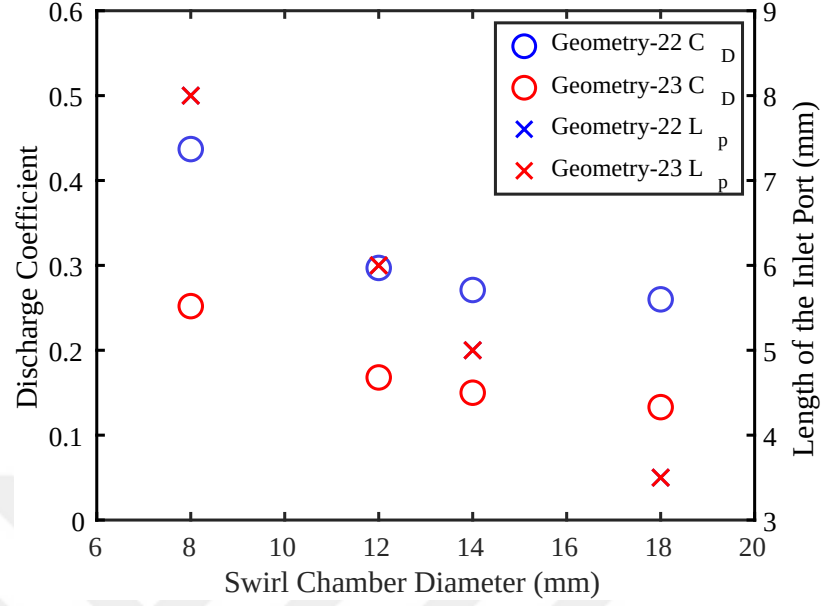


Figure 168: Experimental results - Geometry-22 and 23 [3]

Table 41: Dimensions of the Atomizers - 5 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
24	1.50	1.50	7.50	4	10.00	1.50	90^0	2.30-4.50	4.00
25	1.50	1.50	7.50	2	10.00	1.50	90^0	2.30-4.50	4.00
26	3.00	3.00	7.50	3	10.00	1.50	90^0	2.30-4.50	4.00
27	4.50	4.50	7.50	2	10.00	1.50	90^0	2.30-4.50	4.00

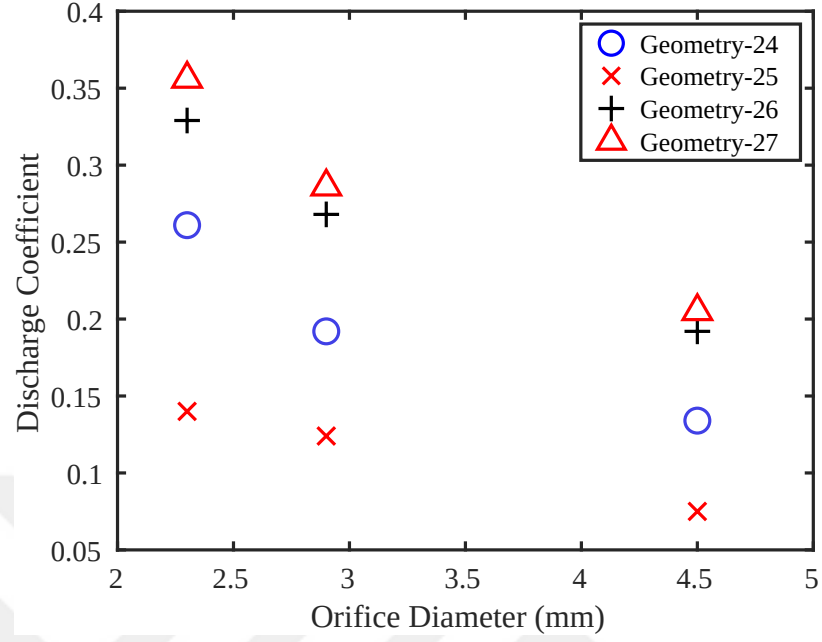


Figure 169: Experimental results - Geometry-24, 25, 26, 27 [3]

Table 42: Dimensions of the Atomizers - 6 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
28	3.18	3.18	6.30	2	19.00	7.90	45^0	1.59-12.70	1.20
29	3.18	3.18	6.30	3	19.00	7.90	45^0	1.59-12.70	1.20
30	3.18	3.18	6.30	4	19.00	7.90	45^0	1.59-12.70	1.20
31	1.59	1.59	6.30	2	19.00	7.90	45^0	1.59-12.70	1.20
32	1.59	1.59	6.30	2	19.00	7.90	45^0	1.59-12.70	1.20
33	1.59	1.59	6.30	2	19.00	7.90	45^0	1.59-12.70	1.20

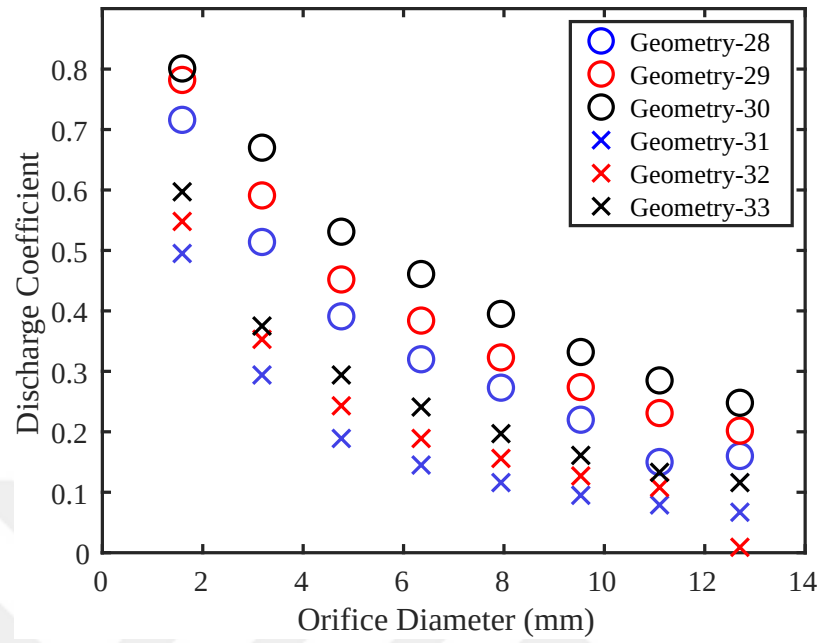


Figure 170: Experimental results - Geometry-28, 29, 30, 31, 32, 33 - Discharge Coefficient [3]

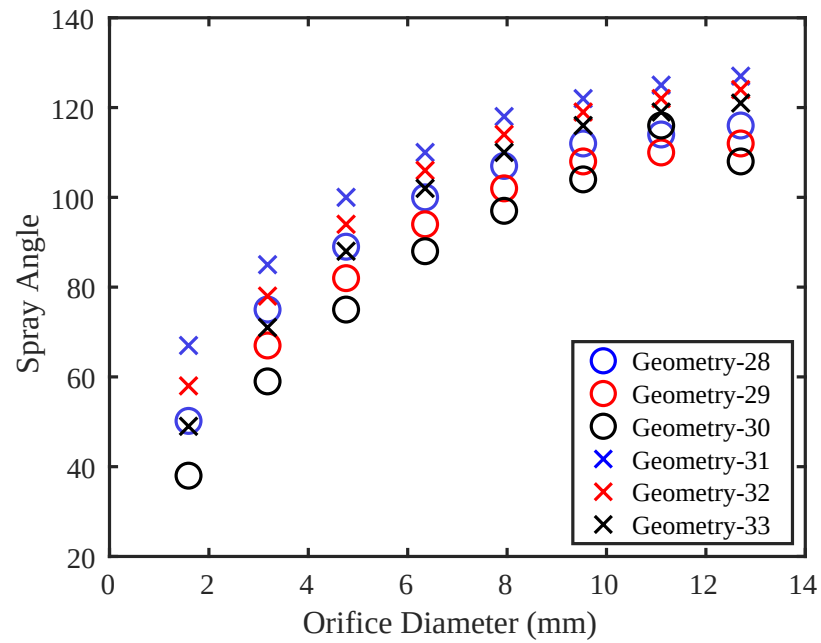
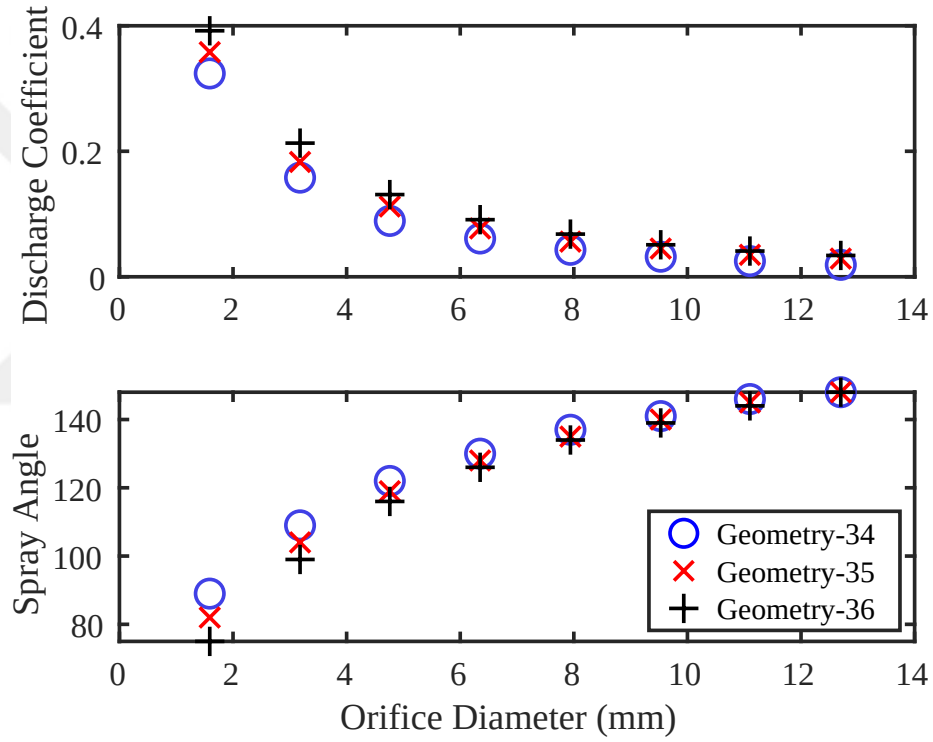


Figure 171: Experimental results - Geometry-28, 29, 30, 31, 32, 33 - Spray Angle [3]

Table 43: Dimensions of the Atomizers - 7 [3]

No.	D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
34	1.59	6.30	2	19.00	7.90	45^0	1.59-12.70	1.20
35	1.59	6.30	3	19.00	7.90	45^0	1.59-12.70	1.20
36	1.59	6.30	4	19.00	7.90	45^0	1.59-12.70	1.20

**Figure 172:** Experimental results - Geometry-34, 35, 36 [3]**Table 44:** Dimensions of the Atomizers - 8 [3]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
37	1.14	1.14	6.50	8	29.00	0.20-0.60	90^0	7.00	2.00
38	0.50	0.50	1.50	4	4.50	0.20-0.60	90^0	1.80	1.50

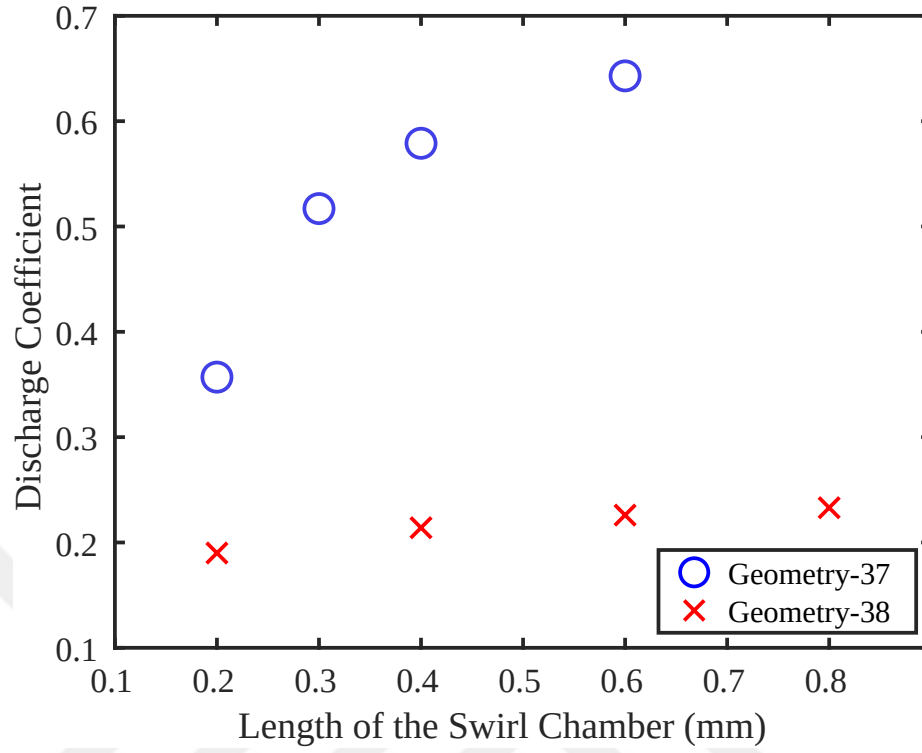


Figure 173: Experimental results - Geometry-37 and 38 [3]

A.1.1.3 Effect of Chamber Back-pressure on Swirl Injector Fluid Mechanics, 2009 [25]

Table 45: Dimensions of the Atomizers [25]

D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1.56	3.73	3	4.78	3.93	59^0	2.08	17.12

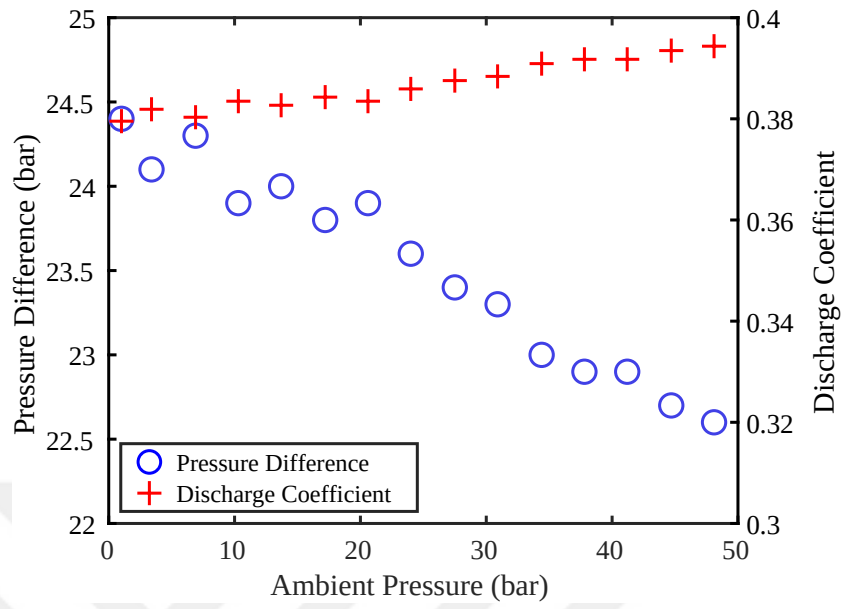


Figure 174: Experimental results - 1 [25]

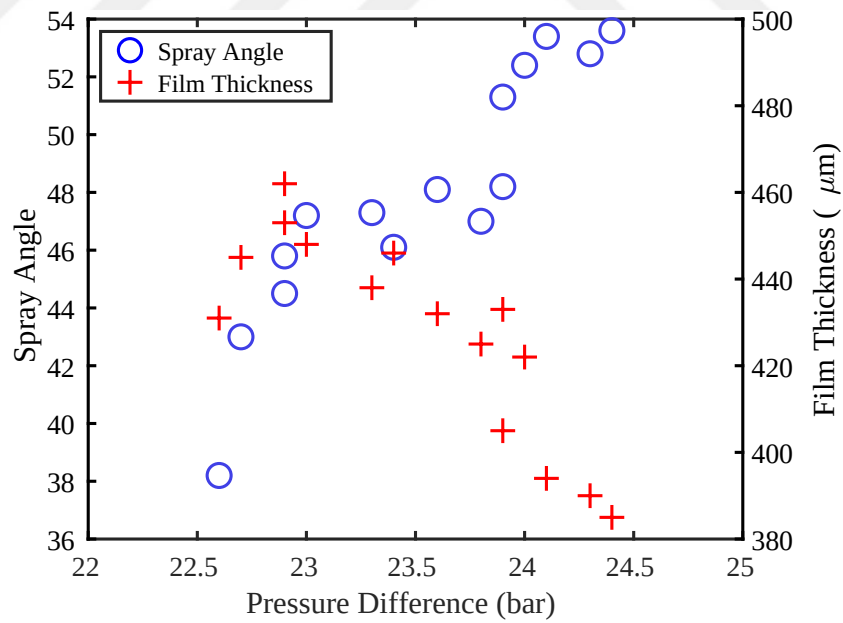


Figure 175: Experimental results - 2 [25]

A.1.1.4 Optimization of Pressure-Swirl Atomizer for a Burner Refroit, 2013 [26]

Table 46: Physical Properties of Working Liquid [26]

ρ_L (kg/m ³)	μ_L (Pa.s)	σ_L (N/m)
874	0.0185	0.0297

Table 47: Dimensions of the Atomizers [26]

No.	w_p (mm)	h_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	2.90	2.90	3.80	4	12.00	0.00	30 ⁰	2.80	1.00
2	2.90	2.90	3.80	4	8.40	0.00	30 ⁰	2.98	1.25
3	2.00	2.00	3.80	4	12.00	5.00	30 ⁰	2.96	1.25
4	2.90	2.90	3.80	4	12.00	0.00	30 ⁰	2.99	1.25
5	2.90	2.90	3.80	4	8.40	0.00	30 ⁰	2.98	0.50
6	2.00	2.00	3.80	4	12.00	5.00	30 ⁰	2.96	0.50
7	2.90	2.90	3.80	4	12.00	0.00	30 ⁰	2.99	0.50

Table 48: Experimental Results at 2 MPa [26]

No.	$\dot{m}_L$ (kg/s)	Spray Angle (⁰)	SMD (μ m)
1	0.196	56.8	88.9
2	0.305	56.6	83.0
3	0.226	54.0	87.9
4	0.152	73.8	97.8
5	0.316	54.6	83.4
6	0.232	59.8	86.3
7	0.154	74.2	93.4

A.1.1.5 *Energy Considerations in Spraying Process of a Spill-Return Pressure-Swirl Atomizer, 2014 [27]*

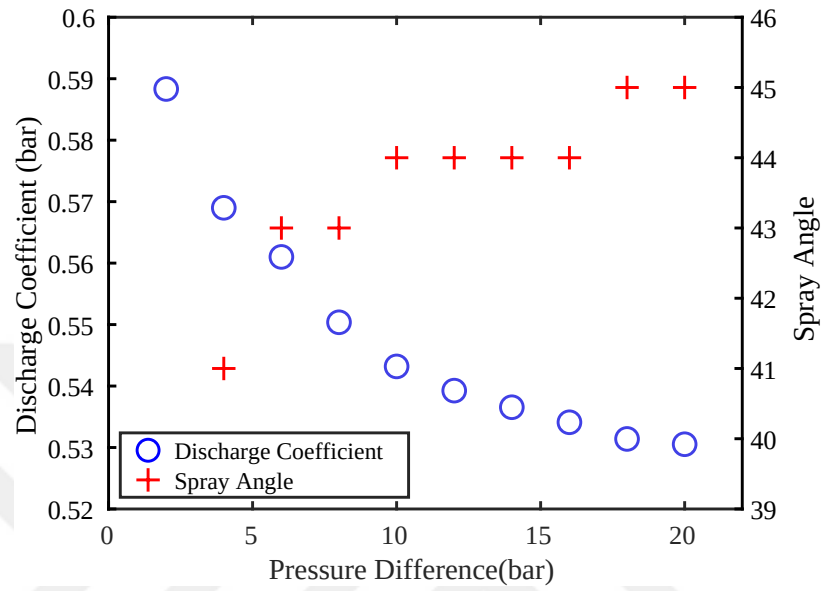


Figure 176: Experimental results - 1 [27]

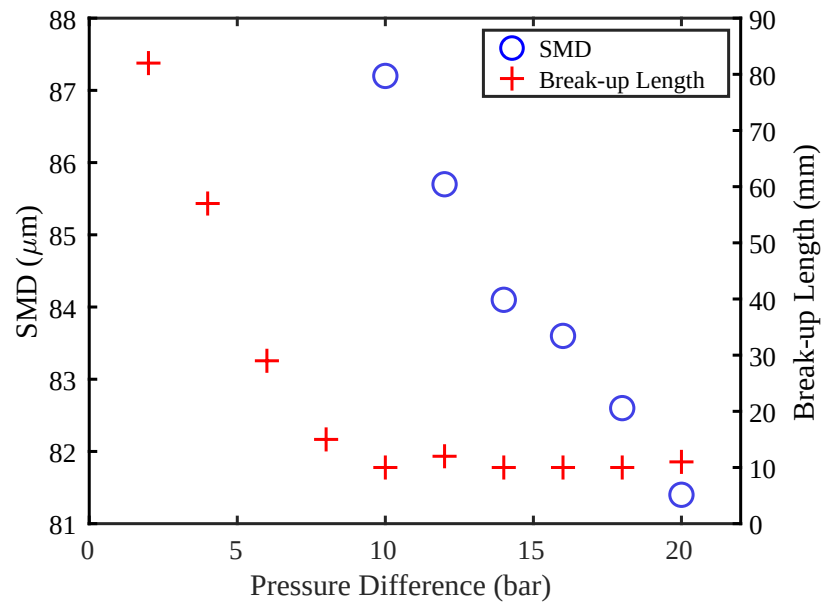


Figure 177: Experimental results - 2 [27]

A.1.1.6 *Experimental Assessment of a Pressure-Swirl Atomizer for Ethanol Operation on a Small Gas Generator, 2017 [28]*

Table 49: Physical Properties of Working Liquid [28]

ρ_L (kg/m ³)	μ_L (Pa.s)	σ_L (N/m)
812.6	0.0013	0.0268

Table 50: Dimensions of the Atomizers [28]

D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
3.00	3.15	3	5.00	5.00	59 ⁰	3.00	2.55

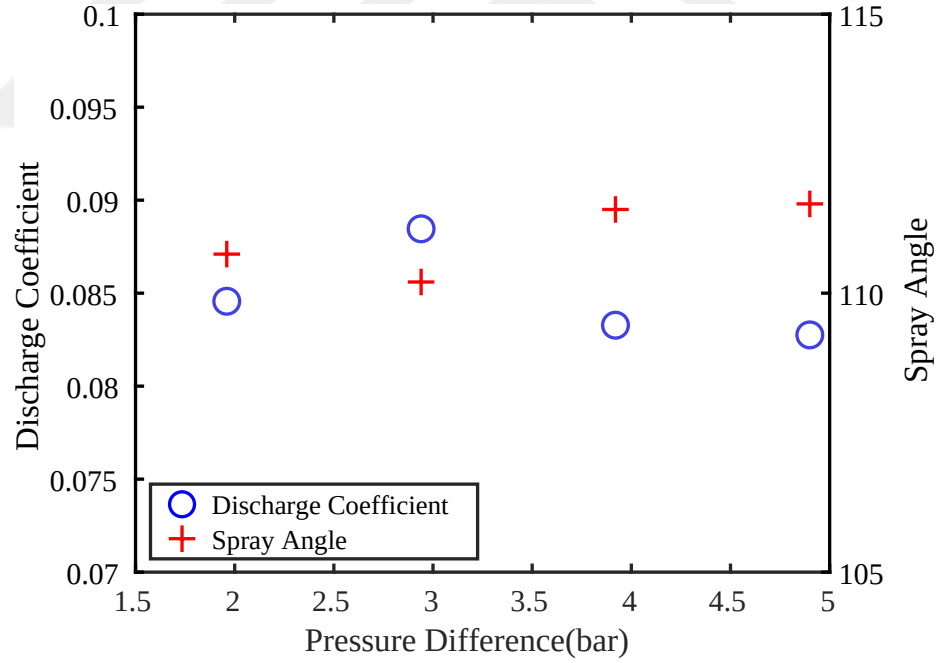


Figure 178: Experimental results - 1 [28]

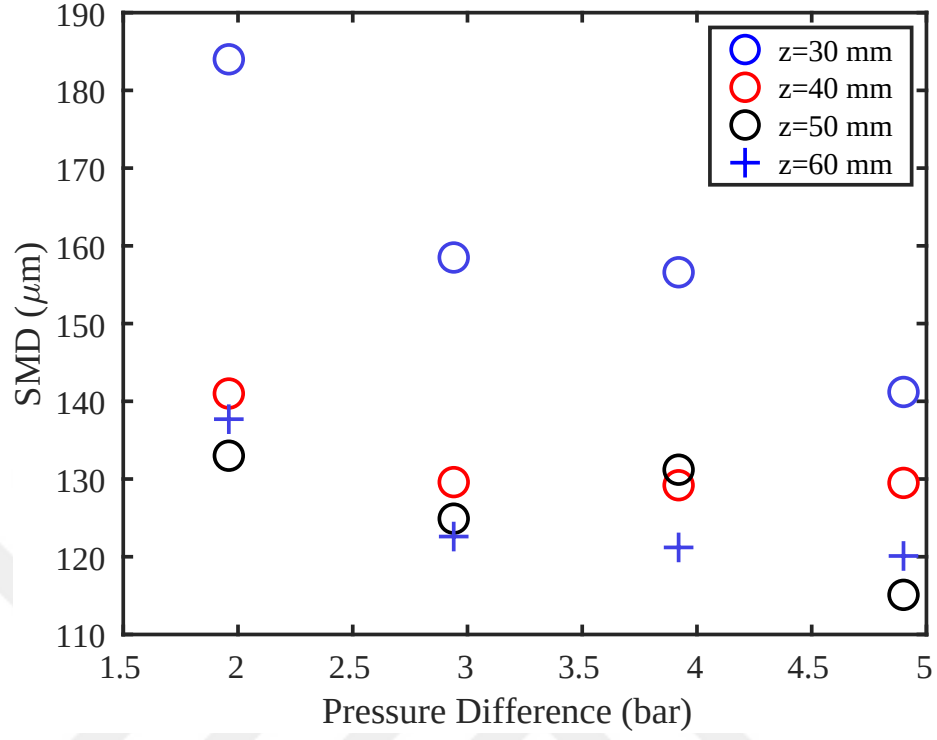


Figure 179: Experimental results - 2 [28]

A.1.1.7 Parametric Evaluation of Swirl Injector Dynamics in the High-Frequency Range, 2017 [29]

Table 51: Dimensions of the Atomizers [29]

D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1.27	2.53	4	6.96	7.80	30^0	4.75	50.00

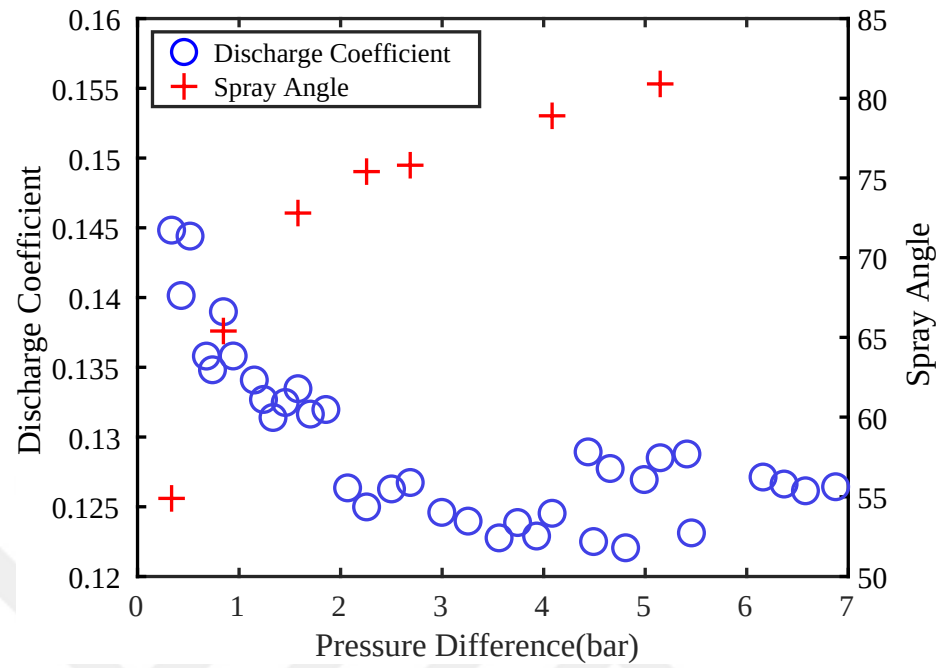


Figure 180: Experimental results [29]

A.1.1.8 A Study of Spray Performance of MEMS based Microscale Pressure Swirl Atomizer, 2019 [30]

Table 52: Dimensions of the Atomizers [30]

No.	w_p (μm)	h_p (μm)	N_p	L_p (μm)	D_s (mm)	L_s (μm)	D_o (μm)	L_o (μm)
1	60	100	2	71.5	1.2	100	294	350
2	60	100	3	71.5	1.2	100	294	350
3	70	100	2	71.5	1.2	100	294	350
4	70	100	3	71.5	1.2	100	294	350
5	80	100	3	71.5	1.2	100	294	350
6	90	100	2	71.5	1.2	100	294	350
7	90	100	3	71.5	1.2	100	294	350
8	100	100	2	71.5	1.2	100	294	350
9	100	100	3	71.5	1.2	100	294	350
10	100	100	4	71.5	1.2	100	294	350
11	60	100	2	71.5	1.2	100	368	350
12	70	100	3	71.5	1.2	100	368	350
13	80	100	2	71.5	1.2	100	368	350
14	80	100	3	71.5	1.2	100	368	350
15	90	100	2	71.5	1.2	100	368	350
16	90	100	4	71.5	1.2	100	368	350
17	100	100	3	71.5	1.2	100	368	350
18	100	100	4	71.5	1.2	100	368	350
19	60	100	2	71.5	1.2	100	442	350
20	70	100	3	71.5	1.2	100	442	350
21	80	100	2	71.5	1.2	100	442	350
22	80	100	3	71.5	1.2	100	442	350
23	90	100	2	71.5	1.2	100	442	350
24	90	100	4	71.5	1.2	100	442	350
25	100	100	4	71.5	1.2	100	442	350

Table 53: Experimental Results [30]

No.	C_D	2θ ($^\circ$)	ϑ
1	0.066	-	0.855
2	0.103	55	-
3	0.095	39	-
4	0.106	53	1.000
5	0.110	64	1.031
6	0.103	61	0.457
7	0.125	59	0.613
8	0.095	54	0.521
9	0.121	59	1.000
10	0.143	62	0.806
11	0.061	52	1.163
12	0.082	64	0.699
13	0.077	66	0.641
14	0.087	78	1.000
15	0.075	65	0.613
16	0.094	53	0.625
17	0.096	77	0.901
18	0.110	73	1.000
19	0.044	33	-
20	0.065	84	0.719
21	0.054	77	0.685
22	0.067	76	0.758
23	0.059	69	0.641
24	0.078	80	1.087
25	0.086	78	0.625

A.1.1.9 Dynamical Similarity and Universality of Drop Size and Velocity Spectra in Sprays, 2019 [18]

Table 54: Dimensions of the Atomizers [18]

No.	w_p (mm)	h_p (mm)	N_p	L_p (mm)	D_s (mm)	L_s (mm)	D_o (mm)	L_o (mm)
1	0.10	0.10	4	0.35	1.20	0.10	0.368	0.35
2	0.30	0.10	4	1.05	3.59	0.30	1.100	1.05
3	0.50	0.50	4	1.75	5.98	0.50	1.840	1.75
4	1.00	1.00	4	3.5	12.00	1.00	3.680	3.5

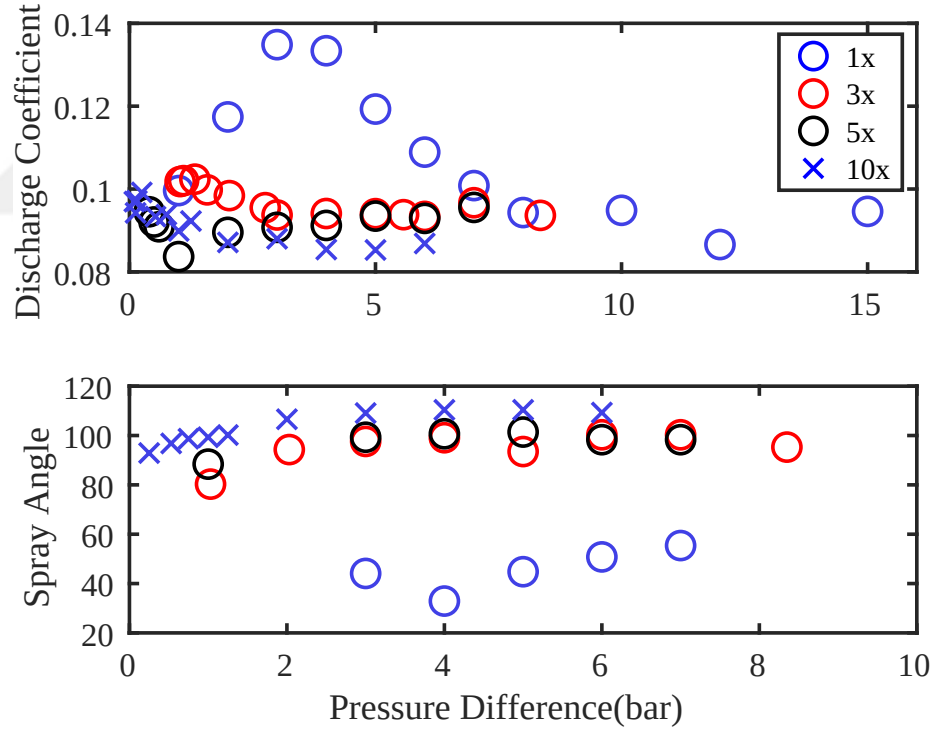


Figure 181: Experimental results [18]

A.1.2 Quasi-Complete Data

A.1.2.1 The Performance of a Type of Swirl Atomizer, 1953 [31]

Table 55: Constant Dimensions of the Atomizers [31]

N_p	L_s (m)	D_s (m)	α
2	2.86E-03	1.27E-02	45^0

Table 56: Altered Geometrical Parameters and Results
[31]

No.	D_p (m)	R_{in} (m)	D_o (m)	L_o (m)	$\dot{m}_L$ (kg/s)	C_D	θ
1	4.06E-04	3.86E-03	4.06E-04	3.81E-04	2.96E-03	6.87E-01	53
2	4.06E-04	1.27E-03	4.95E-04	5.08E-04	3.58E-03	5.59E-01	64
3	5.08E-04	1.24E-03	4.95E-04	5.08E-04	4.40E-03	6.87E-01	63
4	4.06E-04	2.44E-03	4.95E-04	5.08E-04	3.34E-03	5.22E-01	60
5	5.08E-04	2.44E-03	4.95E-04	5.08E-04	3.70E-03	5.79E-01	63
6	7.87E-04	2.45E-03	4.95E-04	5.08E-04	5.18E-03	8.09E-01	57
7	1.32E-03	1.32E-03	4.95E-04	5.08E-04	5.72E-03	8.94E-01	53
8	4.06E-04	3.86E-03	4.95E-04	5.08E-04	3.18E-03	4.97E-01	55
9	5.08E-04	3.86E-03	4.95E-04	5.08E-04	3.62E-03	5.65E-01	57
10	7.87E-04	3.85E-03	4.95E-04	5.08E-04	4.91E-03	7.68E-01	59
11	1.02E-03	3.86E-03	4.95E-04	5.08E-04	5.41E-03	8.45E-01	62
12	4.06E-04	5.03E-03	4.95E-04	5.08E-04	3.25E-03	5.08E-01	53
13	5.08E-04	5.03E-03	4.95E-04	5.08E-04	3.91E-03	6.10E-01	51
14	7.87E-04	5.03E-03	4.95E-04	5.08E-04	4.83E-03	7.54E-01	55
15	4.06E-04	1.27E-03	8.38E-04	7.75E-04	5.08E-03	2.77E-01	70

16	5.08E-04	1.24E-03	8.38E-04	7.75E-04	6.38E-03	3.48E-01	67
17	7.87E-04	1.26E-03	8.38E-04	7.75E-04	8.97E-03	4.89E-01	53
18	1.02E-03	1.26E-03	8.38E-04	7.75E-04	1.12E-02	6.08E-01	43
19	1.32E-03	1.26E-03	8.38E-04	7.75E-04	1.20E-02	6.57E-01	47
20	4.06E-04	2.44E-03	8.38E-04	7.75E-04	4.88E-03	2.66E-01	75
21	5.08E-04	2.44E-03	8.38E-04	7.75E-04	4.64E-03	2.53E-01	71
22	7.87E-04	2.45E-03	8.38E-04	7.75E-04	7.81E-03	4.26E-01	65
23	1.02E-03	2.46E-03	8.38E-04	7.75E-04	8.82E-03	4.81E-01	58
24	1.32E-03	1.32E-03	8.38E-04	7.75E-04	1.02E-02	5.56E-01	54
25	4.06E-04	3.86E-03	8.38E-04	7.75E-04	4.60E-03	2.51E-01	63
26	5.08E-04	3.86E-03	8.38E-04	7.75E-04	5.64E-03	3.08E-01	70
27	7.87E-04	3.85E-03	8.38E-04	7.75E-04	7.33E-03	4.00E-01	63
28	1.02E-03	3.86E-03	8.38E-04	7.75E-04	7.90E-03	4.31E-01	65
29	1.32E-03	3.84E-03	8.38E-04	7.75E-04	8.74E-03	4.77E-01	58
30	1.96E-03	3.82E-03	8.38E-04	7.75E-04	1.04E-02	5.67E-01	52
31	4.06E-04	5.03E-03	8.38E-04	7.75E-04	4.66E-03	2.54E-01	72
32	5.08E-04	5.03E-03	8.38E-04	7.75E-04	5.85E-03	3.19E-01	65
33	7.87E-04	5.03E-03	8.38E-04	7.75E-04	7.17E-03	3.91E-01	68
34	1.02E-03	5.04E-03	8.38E-04	7.75E-04	7.55E-03	4.12E-01	63
35	1.32E-03	5.04E-03	8.38E-04	7.75E-04	8.50E-03	4.64E-01	58
36	1.93E-03	5.04E-03	8.38E-04	7.75E-04	9.32E-03	5.09E-01	54
37	2.11E-03	5.03E-03	8.38E-04	7.75E-04	9.51E-03	5.19E-01	48
38	2.54E-03	5.00E-03	8.38E-04	7.75E-04	1.05E-02	5.71E-01	48
39	1.52E-03	1.27E-03	1.07E-03	9.91E-04	5.85E-03	1.97E-01	75
40	5.08E-04	1.24E-03	1.07E-03	9.91E-04	7.56E-03	2.55E-01	73
41	7.87E-04	1.26E-03	1.07E-03	9.91E-04	1.15E-02	3.88E-01	60

42	1.02E-03	1.26E-03	1.07E-03	9.91E-04	1.42E-02	4.80E-01	67
43	1.32E-03	1.03E-03	1.07E-03	9.91E-04	1.61E-02	5.43E-01	58
44	4.06E-04	2.44E-03	1.07E-03	9.91E-04	5.56E-03	1.87E-01	83
45	5.08E-04	2.44E-03	1.07E-03	9.91E-04	7.02E-03	2.36E-01	79
46	7.87E-04	2.45E-03	1.07E-03	9.91E-04	9.84E-03	3.31E-01	69
47	1.02E-03	2.46E-03	1.07E-03	9.91E-04	1.14E-02	3.84E-01	65
48	1.32E-03	1.32E-03	1.07E-03	9.91E-04	1.36E-02	4.58E-01	61
49	1.96E-03	2.41E-03	1.07E-03	9.91E-04	1.71E-02	5.77E-01	48
50	2.11E-03	2.36E-03	1.07E-03	9.91E-04	1.92E-02	6.45E-01	45
51	4.06E-04	3.86E-03	1.07E-03	9.91E-04	5.04E-03	1.70E-01	78
52	5.08E-04	3.86E-03	1.07E-03	9.91E-04	6.74E-03	2.27E-01	77
53	7.87E-04	3.85E-03	1.07E-03	9.91E-04	9.13E-03	3.08E-01	75
54	1.02E-03	3.86E-03	1.07E-03	9.91E-04	1.00E-02	3.38E-01	72
55	1.32E-03	3.84E-03	1.07E-03	9.91E-04	1.15E-02	3.87E-01	68
56	1.96E-03	3.82E-03	1.07E-03	9.91E-04	1.44E-02	4.84E-01	56
57	2.13E-03	3.84E-03	1.07E-03	9.91E-04	1.59E-02	5.35E-01	48
58	2.59E-03	3.76E-03	1.07E-03	9.91E-04	1.57E-02	5.31E-01	52
59	3.10E-03	3.81E-03	1.07E-03	9.91E-04	1.73E-02	5.81E-01	48
60	4.06E-04	5.03E-03	1.07E-03	9.91E-04	5.22E-03	1.76E-01	83
61	5.08E-04	5.03E-03	1.07E-03	9.91E-04	6.99E-03	2.36E-01	80
62	7.87E-04	5.03E-03	1.07E-03	9.91E-04	9.00E-03	3.03E-01	75
63	1.02E-03	5.03E-03	1.07E-03	9.91E-04	9.63E-03	3.24E-01	72
64	1.32E-03	5.04E-03	1.07E-03	9.91E-04	1.23E-02	4.14E-01	68
65	1.93E-03	5.04E-03	1.07E-03	9.91E-04	1.18E-02	3.99E-01	62
66	2.11E-03	5.03E-03	1.07E-03	9.91E-04	1.29E-02	4.33E-01	61
67	2.54E-03	5.00E-03	1.07E-03	9.91E-04	1.44E-02	4.84E-01	53

68	4.06E-04	1.27E-03	1.32E-03	1.35E-03	6.50E-03	1.43E-01	80
69	5.08E-04	1.02E-03	1.32E-03	1.35E-03	8.81E-03	1.94E-01	81
70	7.87E-04	1.26E-03	1.32E-03	1.35E-03	1.45E-02	3.18E-01	67
71	1.02E-03	1.26E-03	1.32E-03	1.35E-03	1.89E-02	4.15E-01	59
72	1.32E-03	1.26E-03	1.32E-03	1.35E-03	2.15E-02	4.73E-01	63
73	4.06E-04	2.44E-03	1.32E-03	1.35E-03	6.24E-03	1.37E-01	85
74	5.08E-04	2.44E-03	1.32E-03	1.35E-03	8.00E-03	1.76E-01	83
75	7.87E-04	2.45E-03	1.32E-03	1.35E-03	1.20E-02	2.64E-01	75
76	1.02E-03	2.46E-03	1.32E-03	1.35E-03	1.44E-02	3.16E-01	75
77	1.32E-03	1.32E-03	1.32E-03	1.35E-03	1.78E-02	3.90E-01	69
78	1.96E-03	2.41E-03	1.32E-03	1.35E-03	1.57E-02	3.46E-01	54
79	2.11E-03	2.36E-03	1.32E-03	1.35E-03	2.42E-02	5.32E-01	50
80	2.67E-03	2.41E-03	1.32E-03	1.35E-03	2.60E-02	5.70E-01	52
81	3.10E-03	2.44E-03	1.32E-03	1.35E-03	2.77E-02	6.09E-01	44
82	4.06E-04	3.86E-03	1.32E-03	1.35E-03	5.57E-03	1.22E-01	86
83	5.08E-04	3.86E-03	1.32E-03	1.35E-03	7.66E-03	1.68E-01	86
84	7.87E-04	3.85E-03	1.32E-03	1.35E-03	1.10E-02	2.42E-01	80
85	1.02E-03	3.86E-03	1.32E-03	1.35E-03	1.24E-02	2.73E-01	82
86	1.32E-03	3.84E-03	1.32E-03	1.35E-03	1.42E-02	3.13E-01	77
87	1.96E-03	3.82E-03	1.32E-03	1.35E-03	1.85E-02	4.07E-01	58
88	2.13E-03	3.84E-03	1.32E-03	1.35E-03	2.13E-02	4.68E-01	57
89	2.59E-03	3.76E-03	1.32E-03	1.35E-03	2.07E-02	4.54E-01	60
90	3.10E-03	3.81E-03	1.32E-03	1.35E-03	2.24E-02	4.93E-01	55
91	4.06E-04	5.03E-03	1.32E-03	1.35E-03	5.59E-03	1.23E-01	91
92	5.08E-04	5.03E-03	1.32E-03	1.35E-03	7.86E-03	1.73E-01	84
93	7.87E-04	5.03E-03	1.32E-03	1.35E-03	1.06E-02	2.34E-01	88

94	1.02E-03	5.04E-03	1.32E-03	1.35E-03	1.17E-02	2.56E-01	87
95	1.32E-03	5.04E-03	1.32E-03	1.35E-03	1.40E-02	3.07E-01	74
96	1.93E-03	5.04E-03	1.32E-03	1.35E-03	1.54E-02	3.38E-01	75
97	2.11E-03	5.03E-03	1.32E-03	1.35E-03	1.69E-02	3.71E-01	80
98	2.54E-03	5.00E-03	1.32E-03	1.35E-03	1.83E-02	4.01E-01	71
99	4.06E-04	1.27E-03	1.70E-03	1.02E-03	6.99E-03	9.26E-02	89
100	5.08E-04	1.24E-03	1.70E-03	1.02E-03	9.83E-03	1.30E-01	83
101	7.87E-04	1.26E-03	1.70E-03	1.02E-03	1.76E-02	2.33E-01	82
102	1.02E-03	1.26E-03	1.70E-03	1.02E-03	2.58E-02	3.42E-01	70
103	1.32E-03	1.26E-03	1.70E-03	1.02E-03	3.21E-02	4.25E-01	67
104	4.06E-04	2.44E-03	1.70E-03	1.02E-03	6.61E-03	8.76E-02	93
105	5.08E-04	2.44E-03	1.70E-03	1.02E-03	9.13E-03	1.21E-01	94
106	7.87E-04	2.45E-03	1.70E-03	1.02E-03	1.45E-02	1.92E-01	90
107	1.02E-03	2.46E-03	1.70E-03	1.02E-03	1.84E-02	2.43E-01	85
108	1.33E-03	1.32E-03	1.70E-03	1.02E-03	2.34E-02	3.10E-01	75
109	1.96E-03	2.41E-03	1.70E-03	1.02E-03	3.28E-02	4.34E-01	63
110	2.11E-03	2.36E-03	1.70E-03	1.02E-03	3.49E-02	4.62E-01	64
111	2.67E-03	2.41E-03	1.70E-03	1.02E-03	3.78E-02	5.00E-01	53
112	3.10E-03	2.44E-03	1.70E-03	1.02E-03	4.03E-02	5.34E-01	54
113	4.06E-04	3.86E-03	1.70E-03	1.02E-03	6.34E-03	8.39E-02	93
114	5.08E-04	3.86E-03	1.70E-03	1.02E-03	8.82E-03	1.17E-01	92
115	7.87E-04	3.85E-03	1.70E-03	1.02E-03	1.39E-02	1.83E-01	87
116	1.02E-03	3.86E-03	1.70E-03	1.02E-03	1.57E-02	2.08E-01	87
117	1.32E-03	3.84E-03	1.70E-03	1.02E-03	1.81E-02	2.40E-01	82
118	1.96E-03	3.82E-03	1.70E-03	1.02E-03	2.52E-02	3.34E-01	78
119	2.13E-03	3.84E-03	1.70E-03	1.02E-03	2.85E-02	3.77E-01	76

120	2.59E-03	3.76E-03	1.70E-03	1.02E-03	2.77E-02	3.67E-01	74
121	3.10E-03	3.81E-03	1.70E-03	1.02E-03	3.24E-02	4.29E-01	68
122	4.06E-04	5.03E-03	1.70E-03	1.02E-03	5.92E-03	7.84E-02	91
123	5.08E-04	5.03E-03	1.70E-03	1.02E-03	8.69E-03	1.15E-01	69
124	7.87E-04	5.03E-03	1.70E-03	1.02E-03	1.26E-02	1.67E-01	87
125	1.02E-03	5.04E-03	1.70E-03	1.02E-03	1.45E-02	1.92E-01	88
126	1.32E-03	5.04E-03	1.70E-03	1.02E-03	1.83E-02	2.42E-01	80
127	1.93E-03	5.04E-03	1.70E-03	1.02E-03	2.02E-02	2.67E-01	86
128	2.11E-03	5.03E-03	1.70E-03	1.02E-03	2.27E-02	3.00E-01	83
129	2.54E-03	5.00E-03	1.70E-03	1.02E-03	2.60E-02	3.44E-01	73
130	4.06E-04	1.27E-03	1.93E-03	1.12E-03	7.06E-03	7.26E-02	90
131	5.08E-04	1.24E-03	1.93E-03	1.12E-03	1.04E-02	1.07E-01	90
132	7.87E-04	1.26E-03	1.93E-03	1.12E-03	2.02E-02	2.07E-01	86
133	1.02E-03	1.26E-03	1.93E-03	1.12E-03	2.83E-02	2.92E-01	82
134	1.32E-03	1.26E-03	1.93E-03	1.12E-03	3.46E-02	3.56E-01	71
135	4.06E-04	2.44E-03	1.93E-03	1.12E-03	6.72E-03	6.91E-02	113
136	5.08E-04	2.44E-03	1.93E-03	1.12E-03	9.76E-03	1.00E-01	100
137	7.87E-04	2.45E-03	1.93E-03	1.12E-03	1.64E-02	1.68E-01	94
138	1.02E-03	2.46E-03	1.93E-03	1.12E-03	2.08E-02	2.14E-01	92
139	1.33E-03	1.32E-03	1.93E-03	1.12E-03	2.71E-02	2.79E-01	86
140	1.96E-03	2.41E-03	1.93E-03	1.12E-03	3.65E-02	3.76E-01	72
141	2.11E-03	2.36E-03	1.93E-03	1.12E-03	3.91E-02	4.02E-01	68
142	2.67E-03	2.41E-03	1.93E-03	1.12E-03	4.54E-02	4.67E-01	67
143	3.10E-03	2.44E-03	1.93E-03	1.12E-03	4.66E-02	4.80E-01	62
144	4.06E-04	3.86E-03	1.93E-03	1.12E-03	6.58E-03	6.77E-02	101
145	5.08E-04	3.86E-03	1.93E-03	1.12E-03	9.22E-03	9.49E-02	99

146	7.87E-04	3.85E-03	1.93E-03	1.12E-03	1.47E-02	1.52E-01	95
147	1.02E-03	3.86E-03	1.93E-03	1.12E-03	1.74E-02	1.79E-01	96
148	1.32E-03	3.84E-03	1.93E-03	1.12E-03	2.08E-02	2.14E-01	90
149	1.96E-03	3.82E-03	1.93E-03	1.12E-03	2.73E-02	2.81E-01	80
150	2.13E-03	3.84E-03	1.93E-03	1.12E-03	3.50E-02	3.60E-01	74
151	2.59E-03	3.76E-03	1.93E-03	1.12E-03	3.34E-02	3.43E-01	70
152	3.10E-03	3.81E-03	1.93E-03	1.12E-03	3.53E-02	3.63E-01	77
153	4.06E-04	5.03E-03	1.93E-03	1.12E-03	6.05E-03	6.22E-02	90
154	5.08E-04	5.03E-03	1.93E-03	1.12E-03	8.38E-03	8.62E-02	97
155	7.87E-04	5.03E-03	1.93E-03	1.12E-03	1.44E-02	1.48E-01	93
156	1.02E-03	5.04E-03	1.93E-03	1.12E-03	1.60E-02	1.65E-01	90
157	1.32E-03	5.04E-03	1.93E-03	1.12E-03	2.09E-02	2.15E-01	88
158	1.93E-03	5.04E-03	1.93E-03	1.12E-03	2.15E-02	2.22E-01	82
159	2.11E-03	5.03E-03	1.93E-03	1.12E-03	2.46E-02	2.53E-01	86
160	2.54E-03	5.00E-03	1.93E-03	1.12E-03	2.83E-02	2.92E-01	84
161	4.06E-04	1.27E-03	2.54E-03	1.60E-03	7.31E-03	4.34E-02	92
162	5.08E-04	1.24E-03	2.54E-03	1.60E-03	1.10E-02	6.55E-02	97
163	7.87E-04	1.26E-03	2.54E-03	1.60E-03	2.31E-02	1.37E-01	88
164	1.02E-03	1.26E-03	2.54E-03	1.60E-03	3.53E-02	2.10E-01	82
165	1.32E-03	1.26E-03	2.54E-03	1.60E-03	4.40E-02	2.61E-01	85
166	4.06E-04	2.44E-03	2.54E-03	1.60E-03	7.31E-03	4.34E-02	97
167	5.08E-04	2.44E-03	2.54E-03	1.60E-03	1.05E-02	6.25E-02	114
168	7.87E-04	2.45E-03	2.54E-03	1.60E-03	1.97E-02	1.17E-01	100
169	1.02E-03	2.46E-03	2.54E-03	1.60E-03	2.68E-02	1.59E-01	100
170	1.33E-03	1.32E-03	2.54E-03	1.60E-03	3.40E-02	2.02E-01	91
171	1.96E-03	2.41E-03	2.54E-03	1.60E-03	4.74E-02	2.81E-01	84

172	2.11E-03	2.36E-03	2.54E-03	1.60E-03	5.29E-02	3.14E-01	80
173	2.67E-03	2.41E-03	2.54E-03	1.60E-03	5.81E-02	3.45E-01	74
174	3.10E-03	2.44E-03	2.54E-03	1.60E-03	7.71E-02	4.58E-01	70
175	4.06E-04	3.86E-03	2.54E-03	1.60E-03	6.93E-03	4.12E-02	99
176	5.08E-04	3.86E-03	2.54E-03	1.60E-03	1.00E-02	5.96E-02	93
177	7.87E-04	3.85E-03	2.54E-03	1.60E-03	1.74E-02	1.03E-01	95
178	1.02E-03	3.86E-03	2.54E-03	1.60E-03	2.14E-02	1.27E-01	96
179	1.32E-03	3.84E-03	2.54E-03	1.60E-03	2.58E-02	1.53E-01	97
180	1.96E-03	3.82E-03	2.54E-03	1.60E-03	3.81E-02	2.26E-01	84
181	2.13E-03	3.84E-03	2.54E-03	1.60E-03	4.56E-02	2.71E-01	83
182	2.59E-03	3.76E-03	2.54E-03	1.60E-03	4.38E-02	2.61E-01	90
183	3.10E-03	3.81E-03	2.54E-03	1.60E-03	4.88E-02	2.90E-01	88
184	4.06E-04	5.03E-03	2.54E-03	1.60E-03	6.39E-03	3.80E-02	112
185	5.08E-04	5.03E-03	2.54E-03	1.60E-03	1.01E-02	6.00E-02	114
186	7.87E-04	5.03E-03	2.54E-03	1.60E-03	1.70E-02	1.01E-01	100
187	1.02E-03	5.04E-03	2.54E-03	1.60E-03	2.41E-02	1.43E-01	95
188	1.32E-03	5.04E-03	2.54E-03	1.60E-03	2.61E-02	1.55E-01	94
189	1.93E-03	5.04E-03	2.54E-03	1.60E-03	2.86E-02	1.70E-01	93
190	2.11E-03	5.03E-03	2.54E-03	1.60E-03	3.31E-02	1.97E-01	90
191	2.54E-03	5.00E-03	2.54E-03	1.60E-03	3.88E-02	2.31E-01	88
192	4.06E-04	1.27E-03	3.05E-03	1.65E-03	7.50E-03	3.09E-02	99
193	5.08E-04	1.24E-03	3.05E-03	1.65E-03	1.26E-02	5.20E-02	88
194	7.87E-04	1.26E-03	3.05E-03	1.65E-03	2.52E-02	1.04E-01	91
195	1.02E-03	1.26E-03	3.05E-03	1.65E-03	3.78E-02	1.56E-01	92
196	1.32E-03	1.26E-03	3.05E-03	1.65E-03	4.56E-02	1.88E-01	87
197	4.06E-04	2.44E-03	3.05E-03	1.65E-03	7.31E-03	3.02E-02	144

198	5.08E-04	2.44E-03	3.05E-03	1.65E-03	1.11E-02	4.58E-02	98
199	7.87E-04	2.45E-03	3.05E-03	1.65E-03	2.18E-02	8.99E-02	99
200	1.02E-03	2.46E-03	3.05E-03	1.65E-03	2.90E-02	1.20E-01	96
201	1.33E-03	1.32E-03	3.05E-03	1.65E-03	3.82E-02	1.58E-01	99
202	1.96E-03	2.41E-03	3.05E-03	1.65E-03	5.72E-02	2.36E-01	86
203	2.11E-03	2.36E-03	3.05E-03	1.65E-03	6.24E-02	2.57E-01	86
204	2.67E-03	2.41E-03	3.05E-03	1.65E-03	6.99E-02	2.89E-01	74
205	3.10E-03	2.44E-03	3.05E-03	1.65E-03	7.57E-02	3.12E-01	73
206	4.06E-04	3.86E-03	3.05E-03	1.65E-03	7.06E-03	2.91E-02	104
207	5.08E-04	3.86E-03	3.05E-03	1.65E-03	1.02E-02	4.21E-02	104
208	7.87E-04	3.85E-03	3.05E-03	1.65E-03	2.02E-02	8.32E-02	104
209	1.02E-03	3.86E-03	3.05E-03	1.65E-03	2.39E-02	9.88E-02	102
210	1.32E-03	3.84E-03	3.05E-03	1.65E-03	3.00E-02	1.24E-01	104
211	1.96E-03	3.82E-03	3.05E-03	1.65E-03	4.64E-02	1.91E-01	92
212	2.13E-03	3.84E-03	3.05E-03	1.65E-03	5.52E-02	2.28E-01	86
213	2.59E-03	3.76E-03	3.05E-03	1.65E-03	5.37E-02	2.21E-01	88
214	3.10E-03	3.81E-03	3.05E-03	1.65E-03	7.17E-02	2.96E-01	81
215	4.06E-04	5.03E-03	3.05E-03	1.65E-03	7.18E-03	2.96E-02	100
216	5.08E-04	5.03E-03	3.05E-03	1.65E-03	1.05E-02	4.32E-02	106
217	7.87E-04	5.03E-03	3.05E-03	1.65E-03	1.85E-02	7.64E-02	104
218	1.02E-03	5.04E-03	3.05E-03	1.65E-03	2.20E-02	9.10E-02	98
219	1.32E-03	5.04E-03	3.05E-03	1.65E-03	3.02E-02	1.25E-01	98
220	1.93E-03	5.04E-03	3.05E-03	1.65E-03	3.29E-02	1.36E-01	92
221	2.11E-03	5.03E-03	3.05E-03	1.65E-03	4.04E-02	1.67E-01	92
222	2.54E-03	5.00E-03	3.05E-03	1.65E-03	4.66E-02	1.92E-01	92

A.1.2.2 Discharge Coefficient and Spray Angle Measurement for Small Pressure-Swirl Nozzles, 1994 [32]

Table 57: Physical Properties of Liquid [32]

T (°C)	ρ_L (kg/m ³)	μ_L (Pa.s)	σ (N/m)
100	941.4	0.0332	0.0289
110	936.1	0.0255	0.0285
120	930.8	0.0195	0.0278
135	922.9	0.0138	0.0275

Table 58: Constant Dimension of the Atomizers [32]

h_p (mm)	N_p	D_s (mm)	L_s (mm)	L_c (mm)	L_o (mm)
0.80	3	6.00	0.80	2.40	0.25

Table 59: Dimensions of the Atomizers [32]

No.	w_p (mm)	D_o (mm)	α ($^\circ$)
1	0.35	1.10	45.6
2	0.35	0.60	48.4
3	0.35	0.55	48.6
4	0.35	0.50	48.9
5	0.35	0.45	49.1
6	0.26	1.1	45.6
7	0.26	0.80	47.3
8	0.26	0.6	48.4
9	0.26	0.55	48.6
10	0.26	0.50	48.9
11	0.26	0.45	49.1

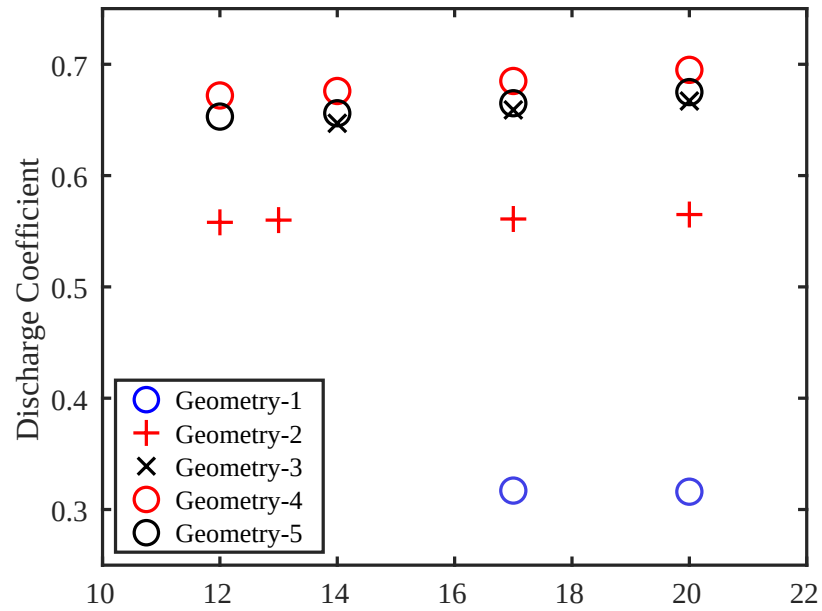


Figure 182: Experimental results - 1 [32]

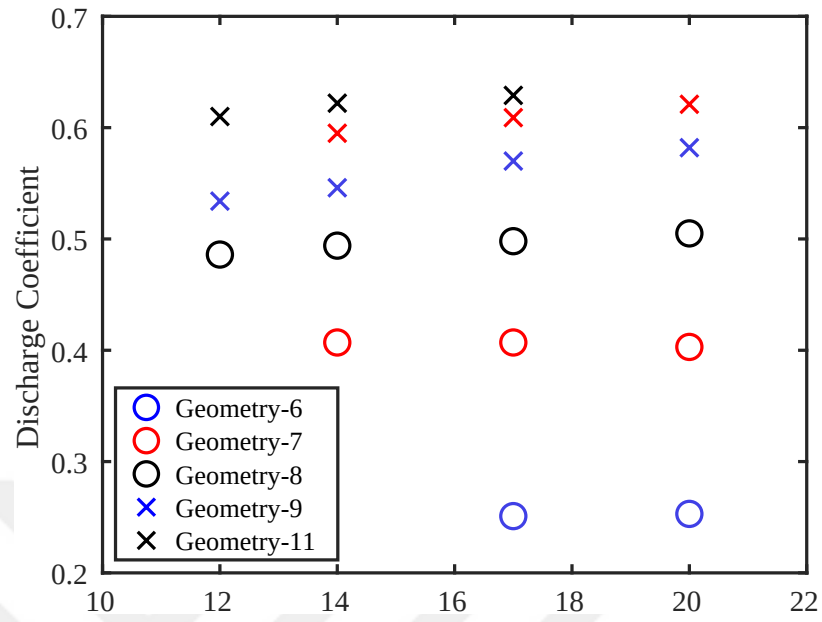


Figure 183: Experimental results - 2 [32]

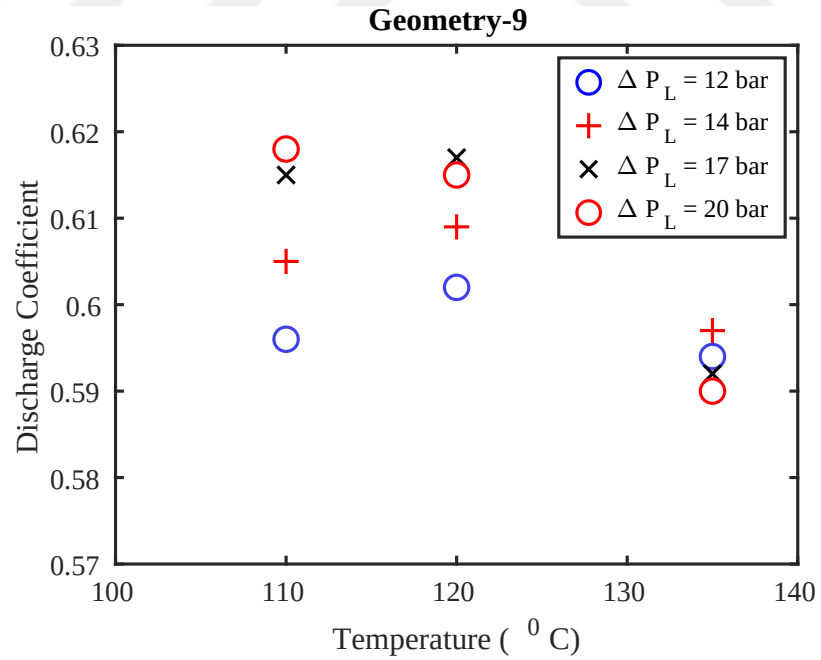


Figure 184: Experimental results - 3 [32]

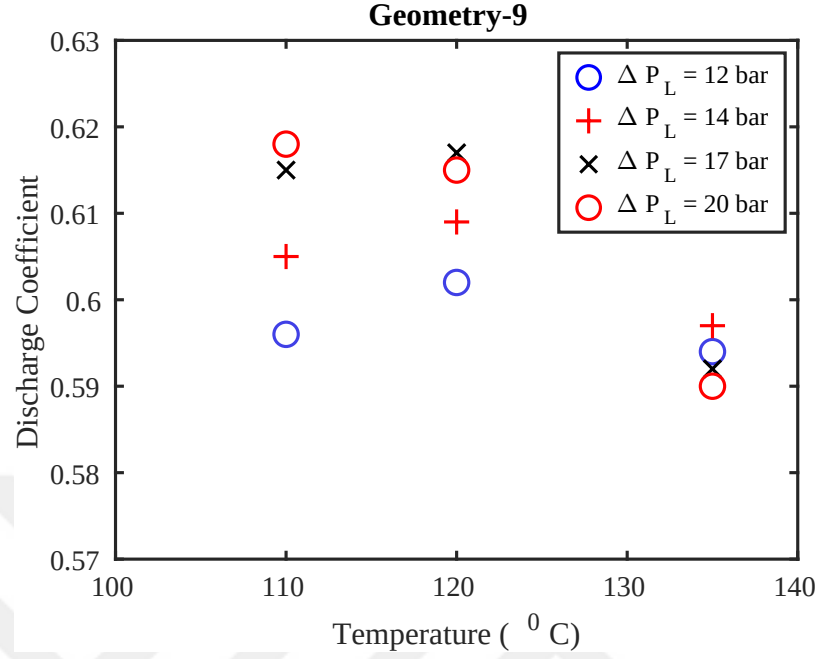


Figure 185: Experimental results - 4 [32]

A.1.2.3 Effect of Inlet Slot Number on the Spray Cone Angle and Discharge Coefficient on Swirl Atomizer, 2012 [33]

Table 60: Dimensions of the Atomizers [33]

No.	N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	2	2.00	40.00	100.00	30^0	3.00	3.00
2	3	2.00	40.00	100.00	30^0	3.00	3.00
3	4	2.00	40.00	100.00	30^0	3.00	3.00
4	5	2.00	40.00	100.00	30^0	3.00	3.00

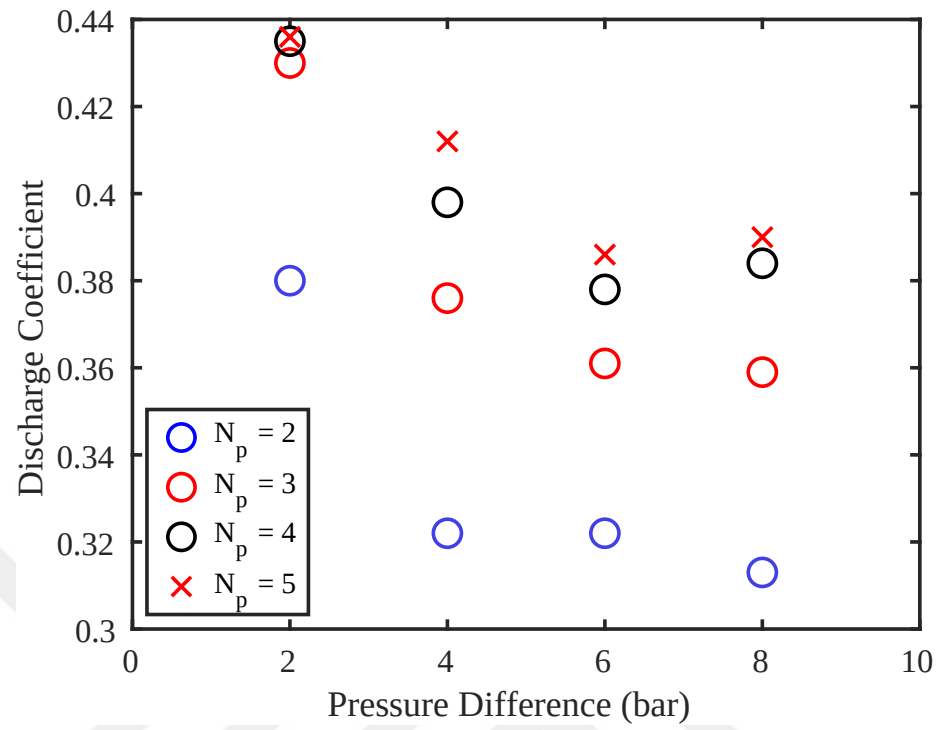


Figure 186: Experimental results - 1 [33]

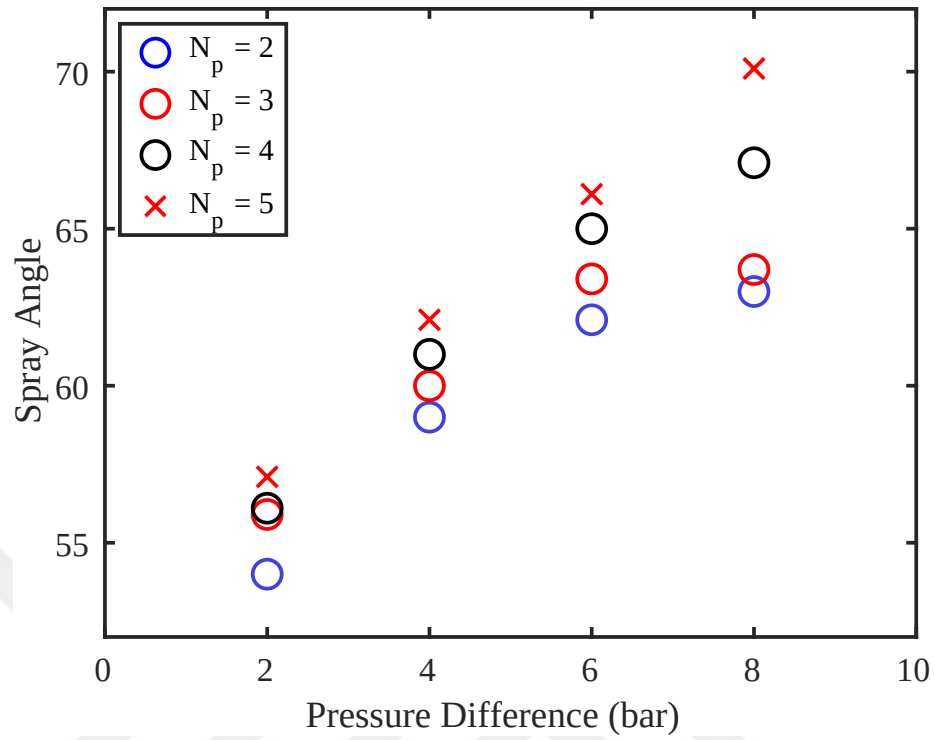


Figure 187: Experimental results - 2 [33]

A.1.2.4 Experimental Investigation of Flow Through a Pressure Swirl Atomizer and the Resulting Hollow Cone Spray at Different Operating Conditions, 2013 [34]

Table 61: Dimensions of the Atomizers [34]

N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
2	2.52	8.00	5.25	20^0	2.00	4.00

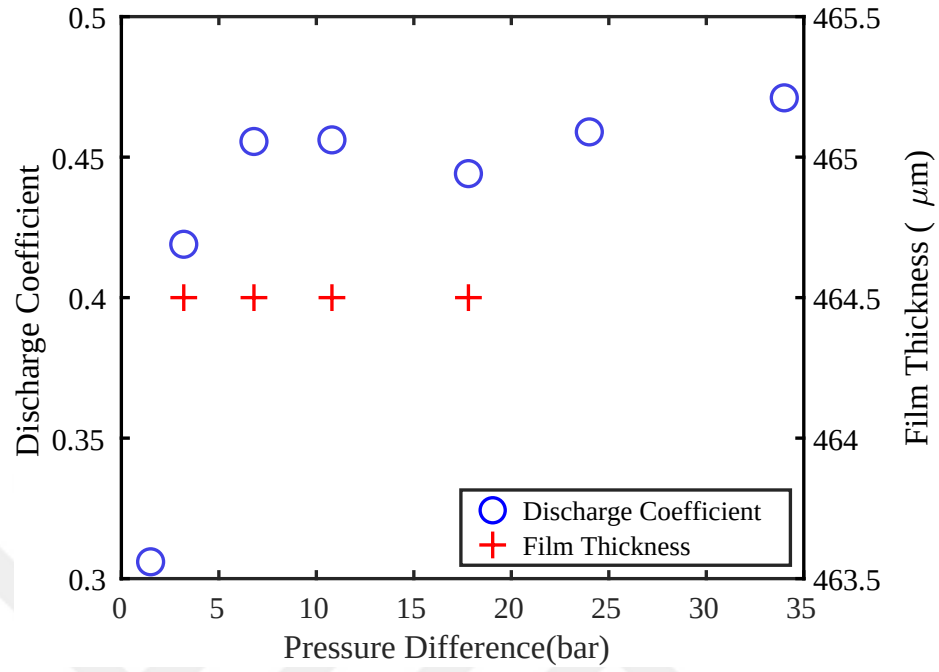


Figure 188: Experimental results [34]

A.1.2.5 Numerical and Experimental Investigation of a Pressure-Swirl Atomizer Flow Field, 2013 [35]

Table 62: Dimension of the Atomizer and Results [35]

N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)	$\dot{m}_L$ (g/s)	ΔP_L (bar)	2θ
2	0.8	10.6	10.6	59°	3.2	4.8	20.8	8.3	112.6

A.1.2.6 Spray Characteristics of Closed-Type Swirl Injectors with Varying Swirl Chamber Geometry, 2015 [9]

Table 63: Dimensions of the Atomizers [9]

No.	N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	3	1.4	30.0	60.0	45^0	6.0	15.0
2	3	1.4	40.0	60.0	45^0	6.0	15.0
3	3	1.4	50.0	60.0	45^0	6.0	15.0
4	3	1.4	60.0	60.0	45^0	6.0	15.0
5	3	1.4	30.0	80.0	45^0	6.0	15.0
6	3	1.4	30.0	100.0	45^0	6.0	15.0
7	3	1.4	30.0	120.0	45^0	6.0	15.0

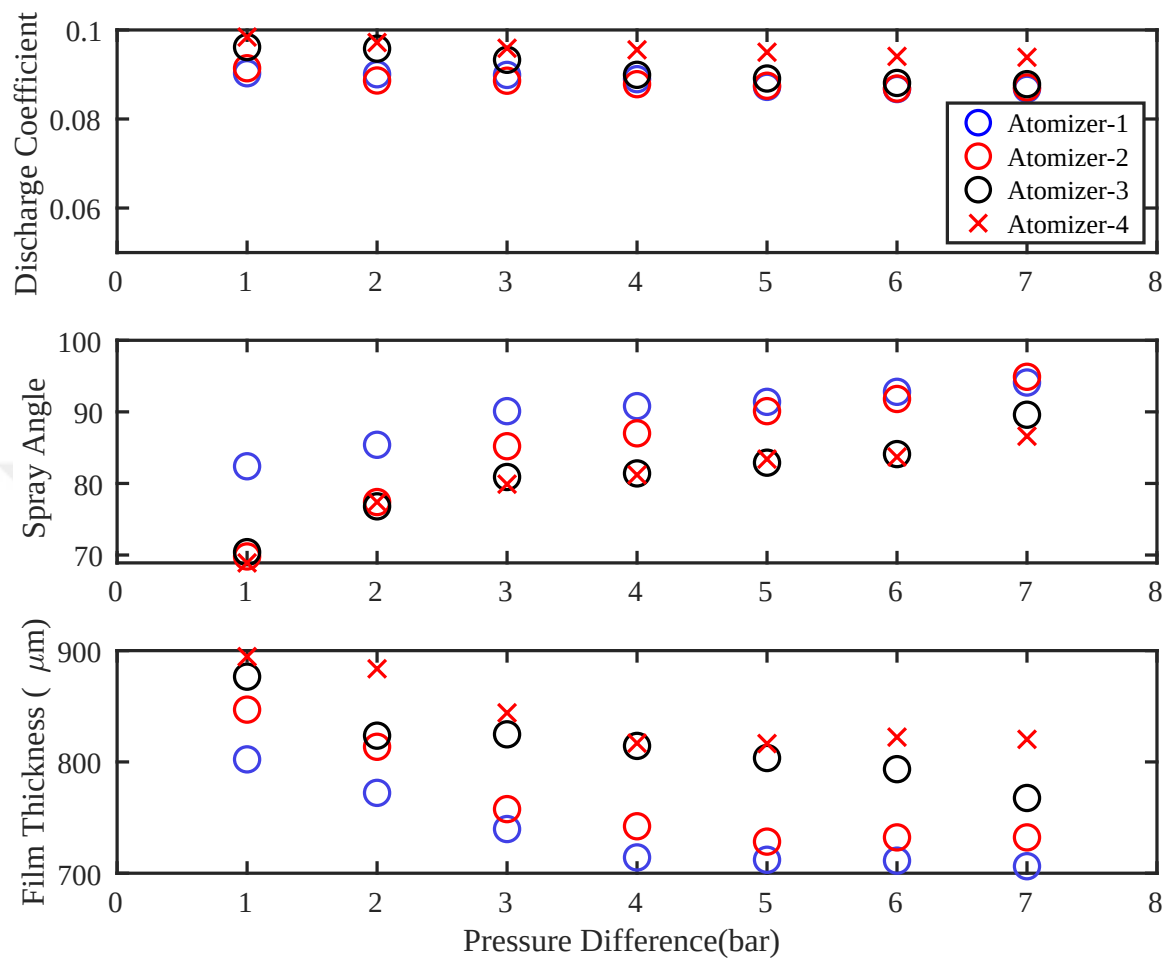


Figure 189: Experimental results - 1 [9]

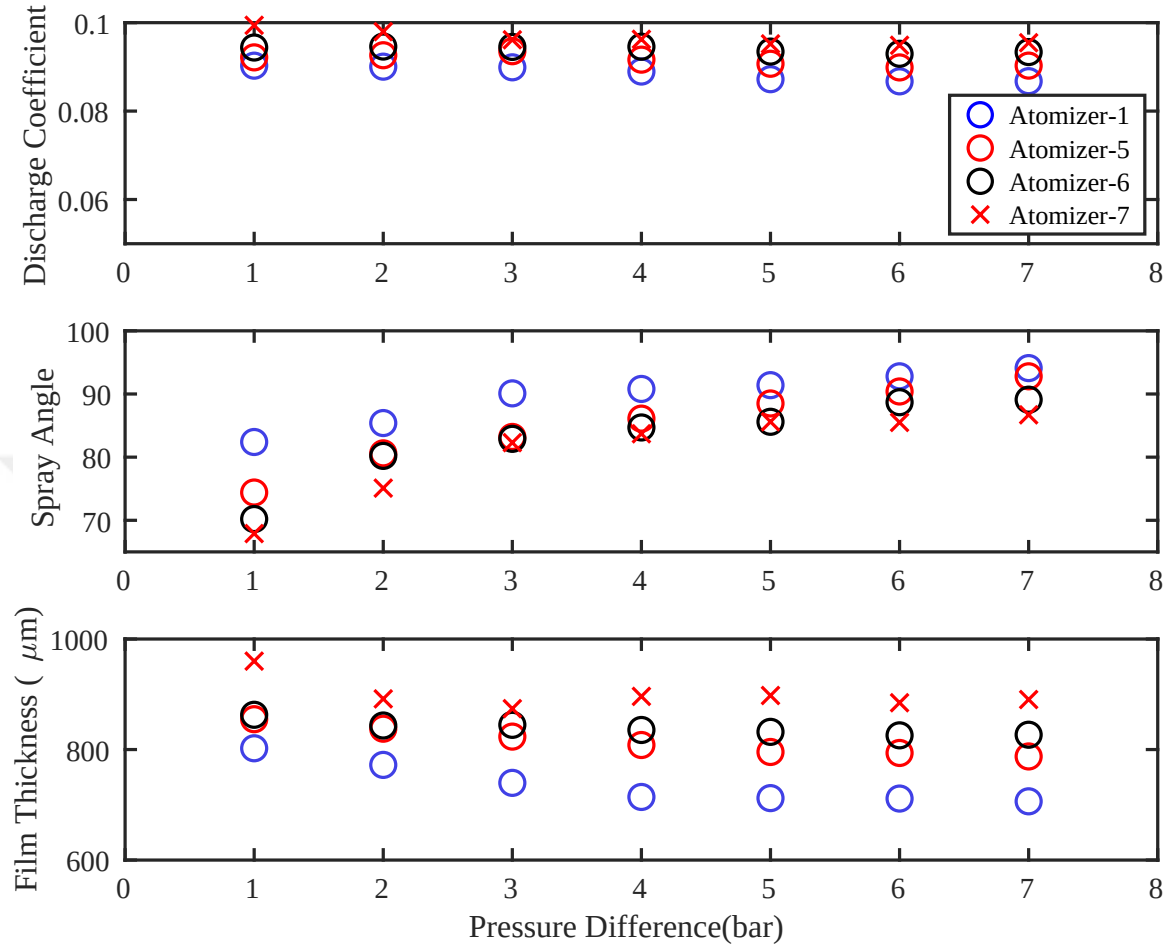


Figure 190: Experimental results - 2 [9]

A.1.2.7 Effects of Orifice Divergence on Hollow Cone Spray at Low Injection Pressures, 2018 [36]

Table 64: Dimensions of the Atomizers [36]

No.	N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)	D_t (mm)	L_t (mm)	γ
1	3	0.80	4.40	9.00	29.5^0	1.00	4.00	-	-	0
2	3	0.80	4.40	9.00	29.5^0	1.00	2.00	1.35	2.00	5
3	3	0.80	4.40	9.00	29.5^0	1.00	2.00	1.71	2.00	10
4	3	0.80	4.40	9.00	29.5^0	1.00	2.00	2.07	2.00	15
5	3	0.80	4.40	9.00	29.5^0	1.00	2.00	2.46	2.00	20

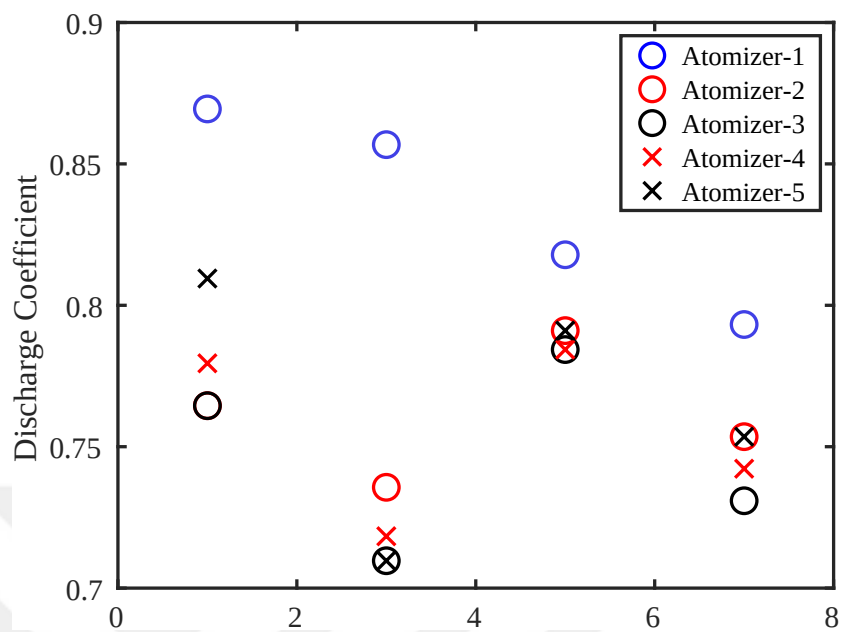


Figure 191: Experimental results - 1 [36]

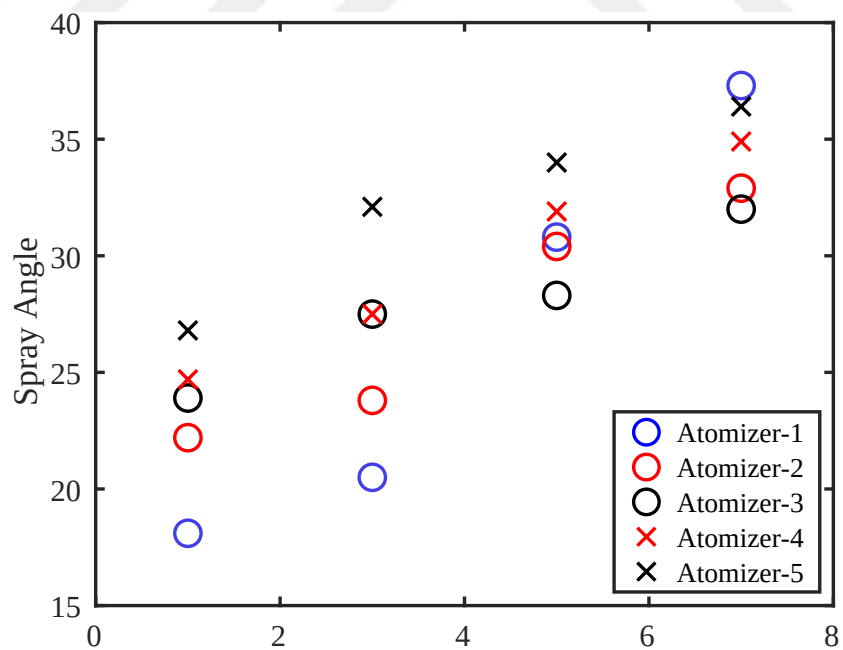


Figure 192: Experimental results - 2 [36]

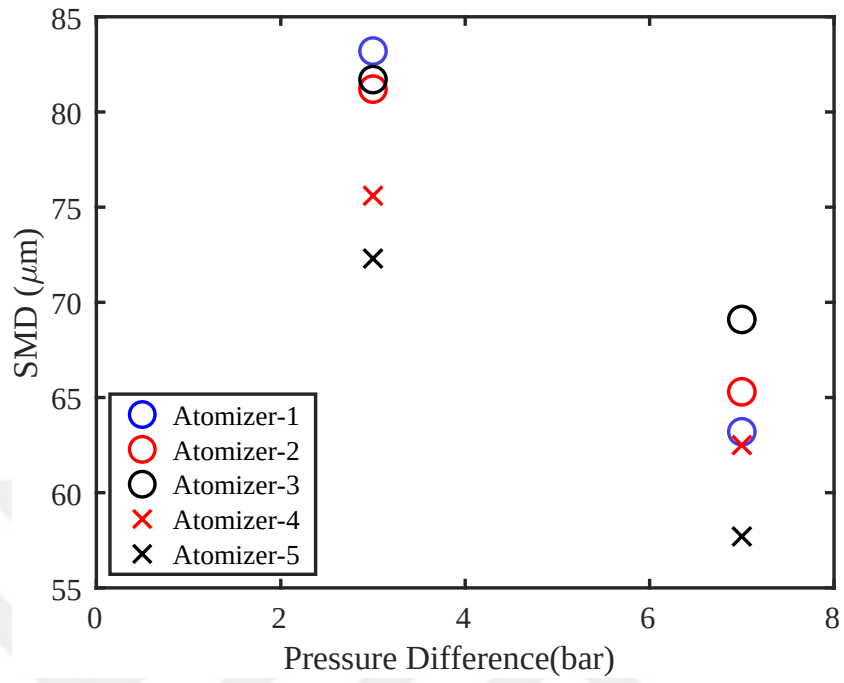


Figure 193: Experimental results - 3 [36]

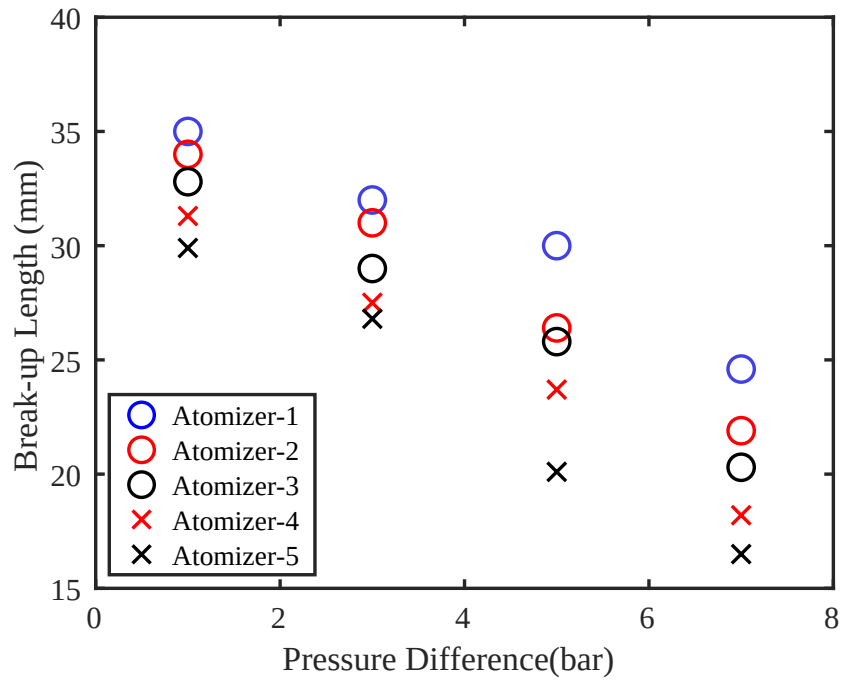


Figure 194: Experimental results - 4 [36]

A.1.2.8 *Experimental Study of Spray Break-up Phenomena in Small-Scale Simplex Atomizers with and without Air Swirl, 2018 [37]*

Table 65: Dimensions of the Atomizers [37]

No.	N_p	D_p (mm)	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	3	0.52	4.00	8.00	45°	0.90	0.50
2	3	0.90	4.00	8.00	45°	0.90	0.50
3	3	1.00	4.00	8.00	45°	0.90	0.50

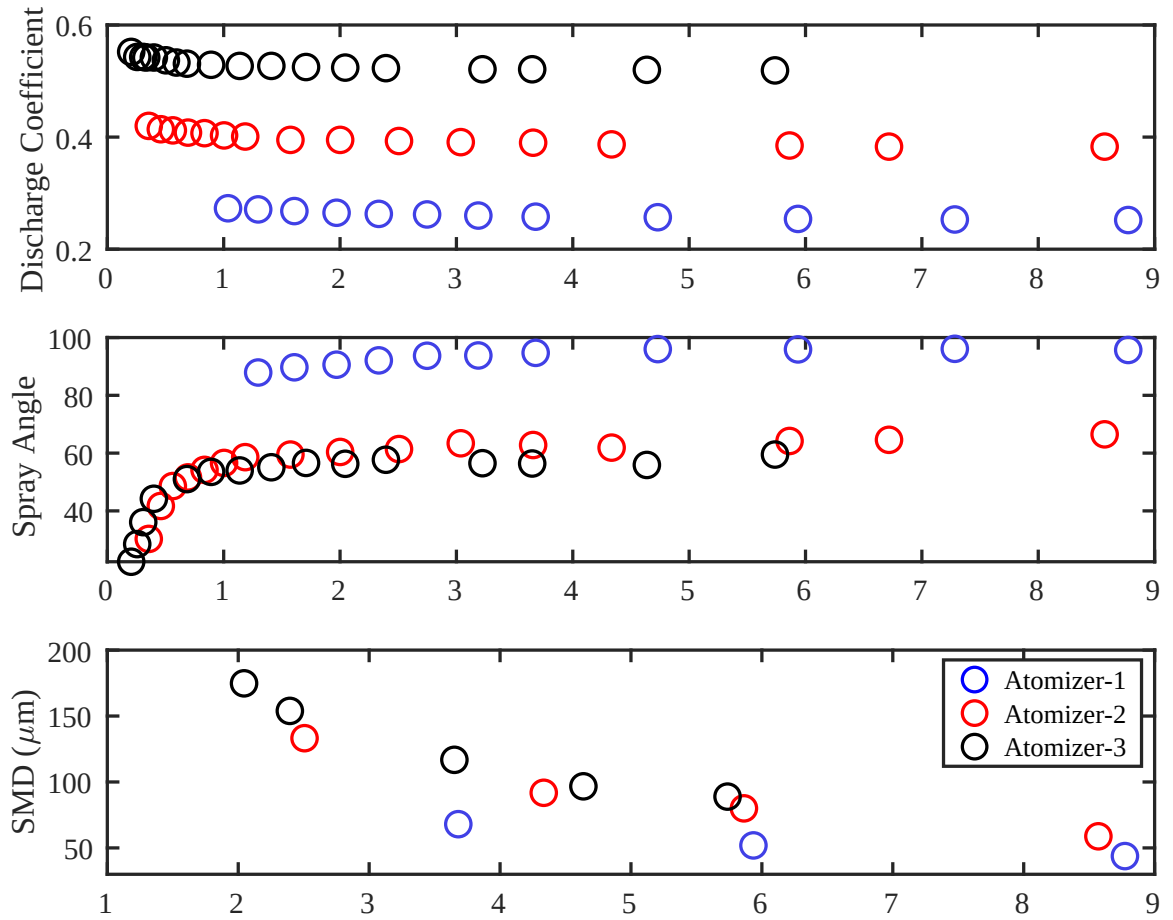


Figure 195: Experimental results - 1 [37]

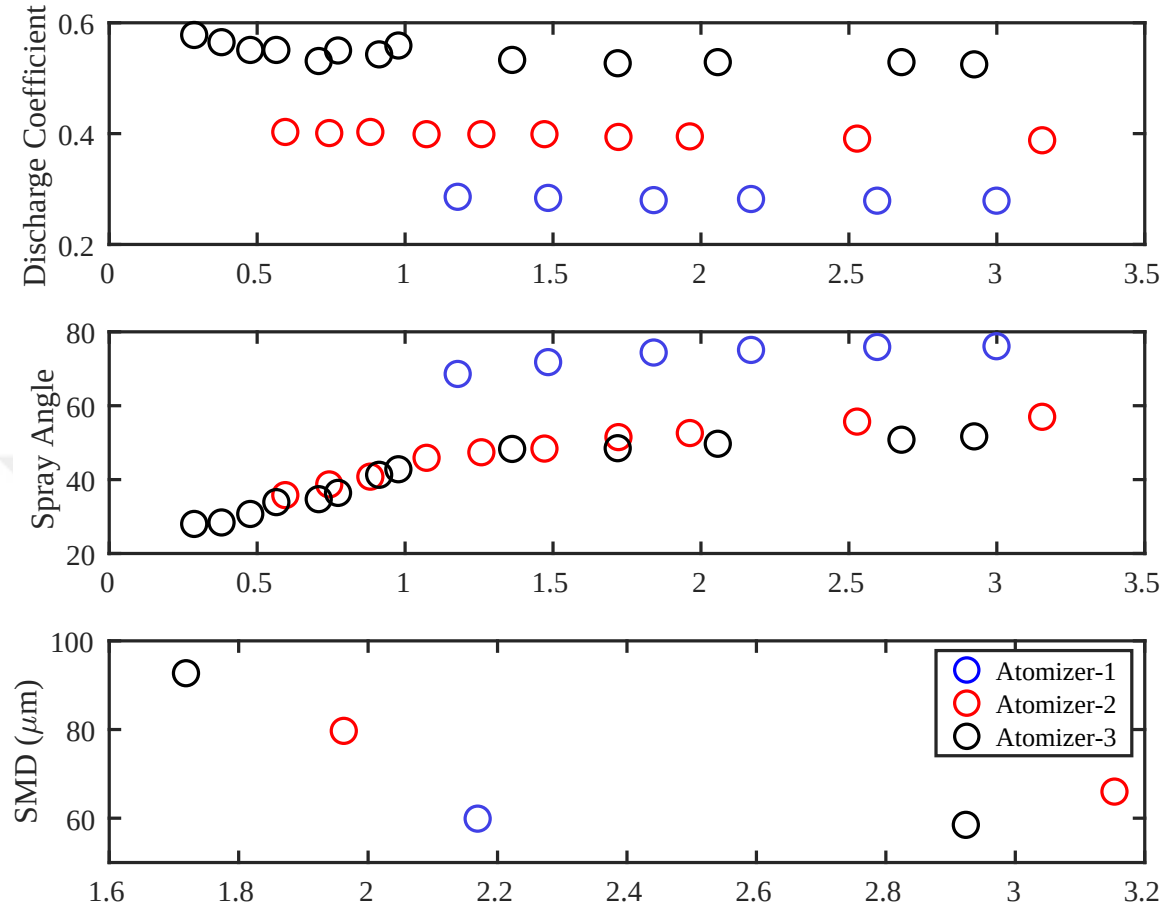


Figure 196: Experimental results - 2 [37]

A.1.2.9 Evaluation of the Atomization Characteristics of Aviation Fuels with Different Viscosities using a Pressure Swirl Atomizer, 2019 [38]

Table 66: Dimensions of the Atomizers [38]

w_p (mm)	h_p (mm)	N_p	β	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
0.5	0.56	6	45°	3.00	0.00	60°	1.50	1.70

Table 67: Physical Properties of the Liquid [38]

Temperature	ρ_l (kg/m ³)	ν_L (mm ² /s)	σ_L (N/m)	ρ_l (kg/m ³)	ν_L (mm ² /s)	σ_L (N/m)
	Jet A-1			Fuel-B		
−20	845	4.20	0.0265	969	17.00	0.0346
−10	837	3.24	0.0257	963	13.10	0.0339
0	830	2.51	0.0249	957	10.17	0.0332
10	822	1.70	0.0241	951	7.80	0.0325
20	814	1.50	0.0233	945	6.00	0.0318
40	799	0.80	0.0218	933	3.64	0.0305
60	784	0.53	0.0202	922	1.90	0.0291
	Fuel-A			Fuel-C		
−20	959	10.20	0.0331	-	-	-
−10	953	7.80	0.0323	993	43.10	0.0352
0	947	6.10	0.0315	987	30.25	0.0346
10	941	4.60	0.0307	981	21.16	0.0341
20	935	3.65	0.0299	976	14.80	0.0335
40	923	2.18	0.0283	964	7.24	0.0324
60	912	1.31	0.0267	952	3.50	0.0313

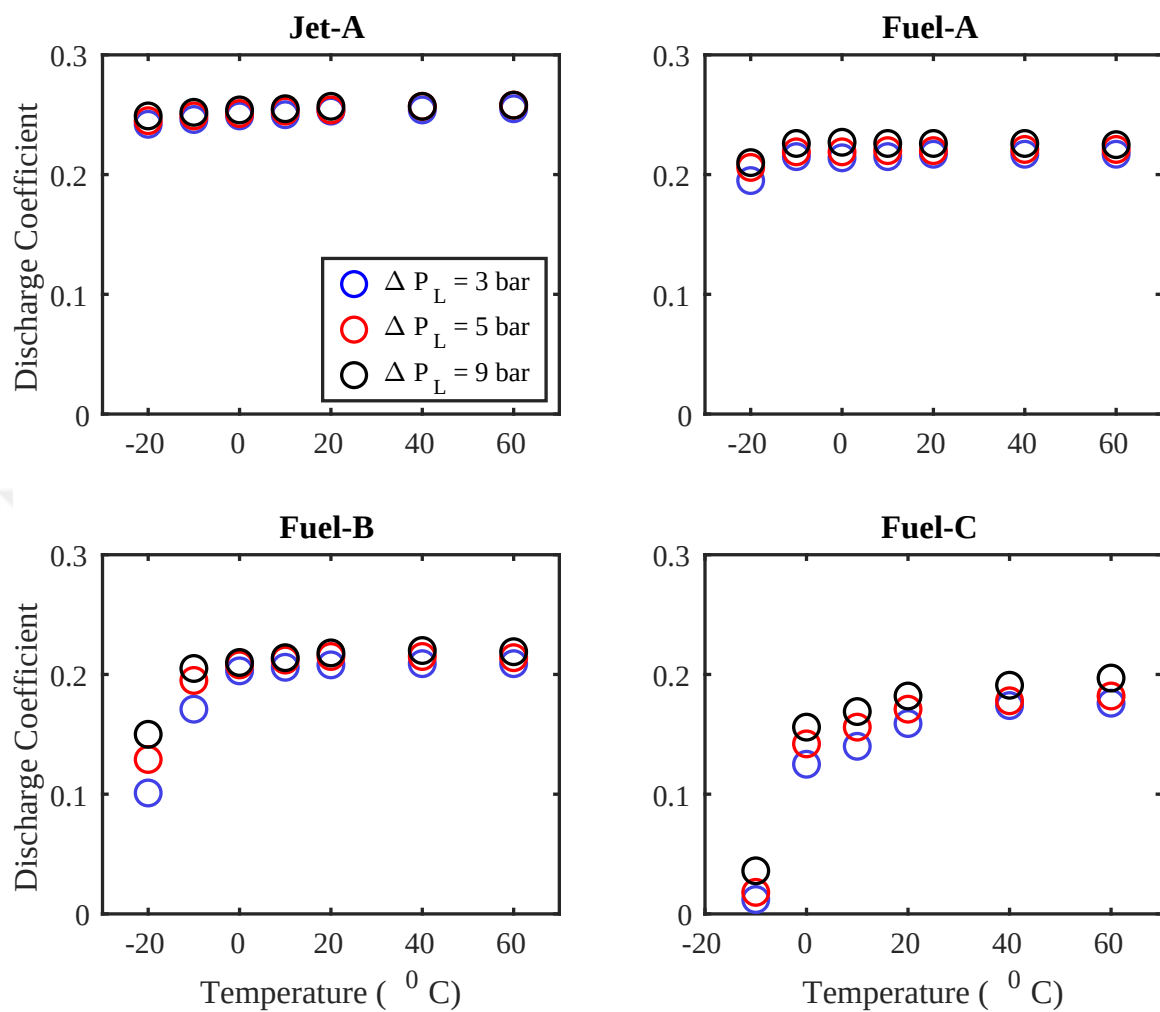


Figure 197: Experimental results - 1 [38]

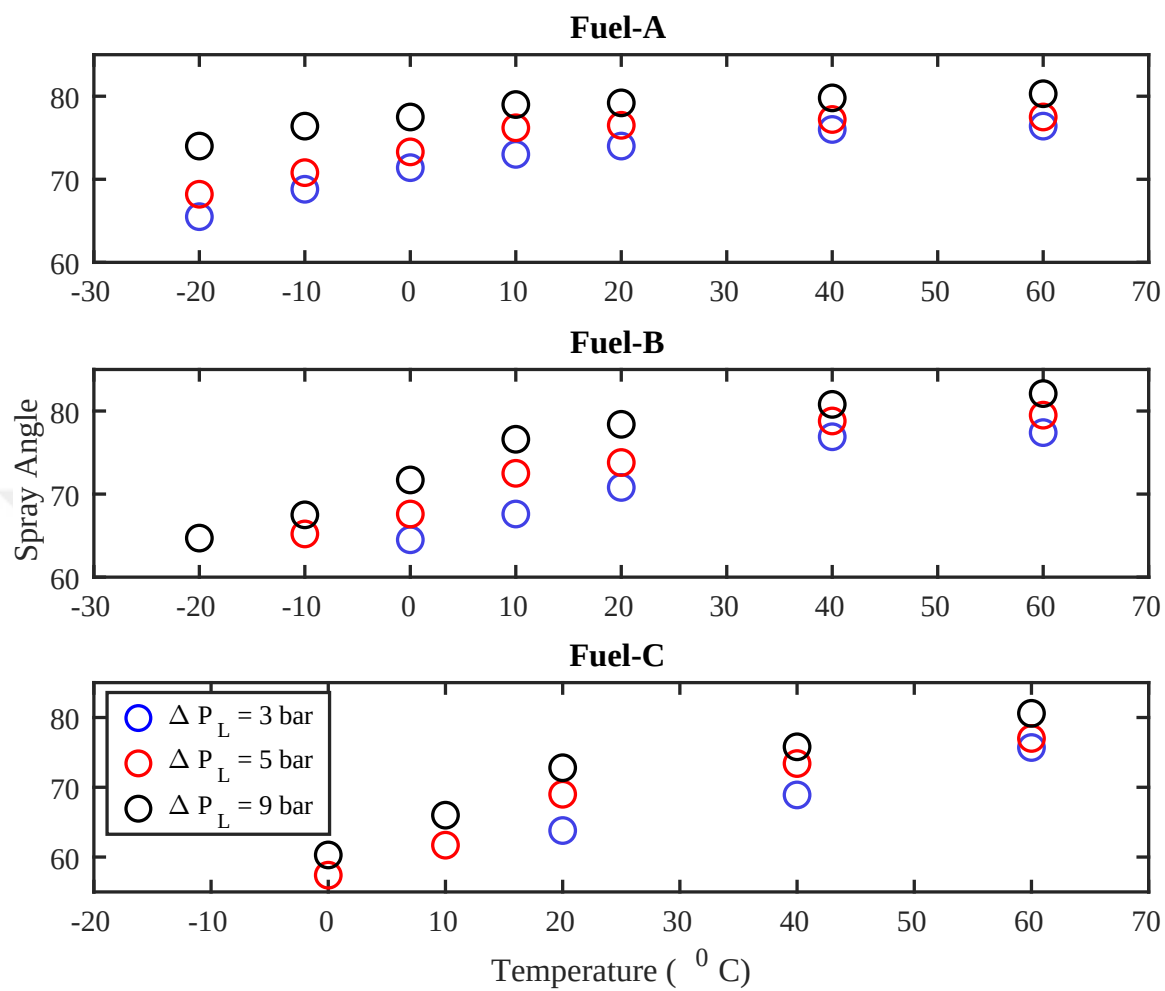


Figure 198: Experimental results - 2 [38]

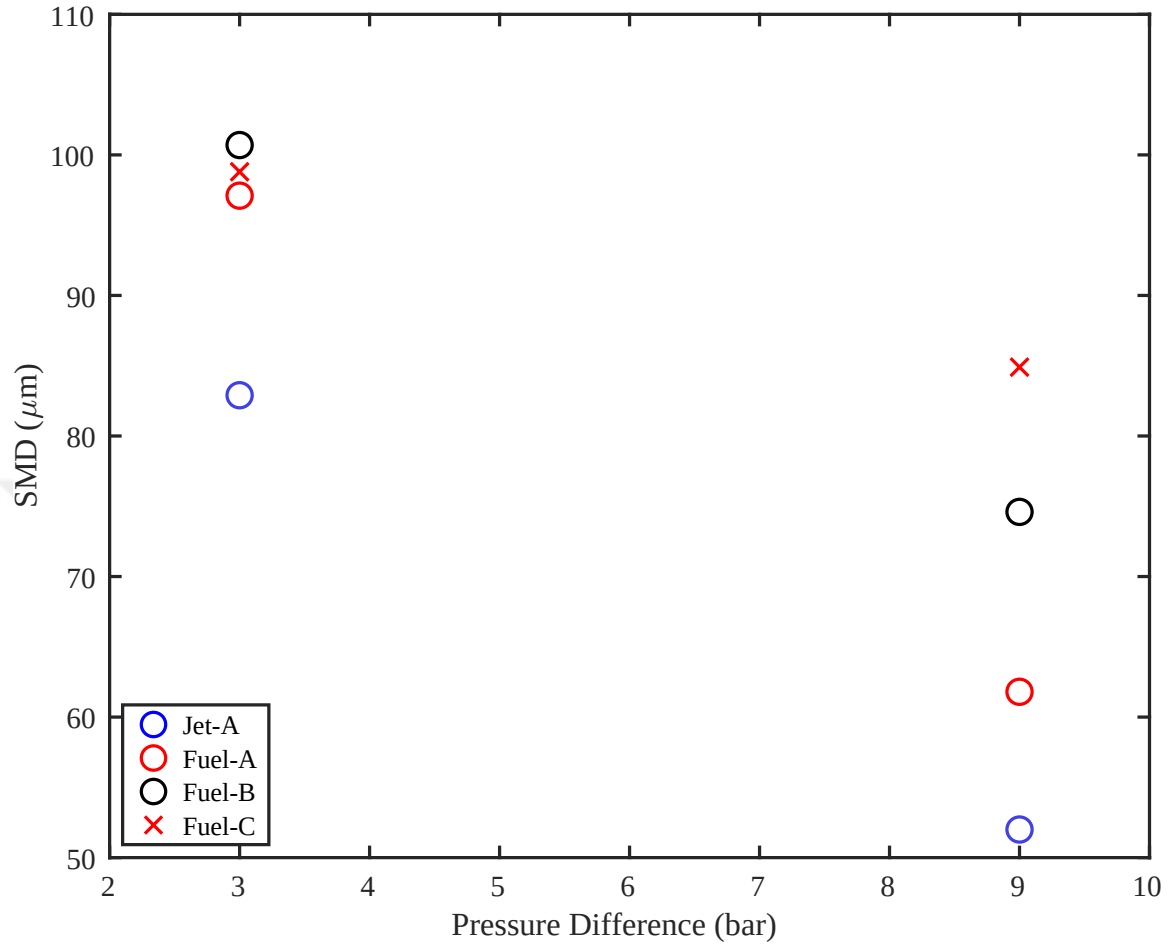


Figure 199: Experimental results - 3 [38]

A.1.2.10 Experimental Investigations of Spray Characteristics of a Pressure Swirl Atomizer, 2019 [39]

Table 68: Dimensions of the Atomizers [39]

w_p (mm)	h_p (mm)	N_p	β	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
0.20	0.30	3	45°	3.40	0.00	36°	0.40	0.25

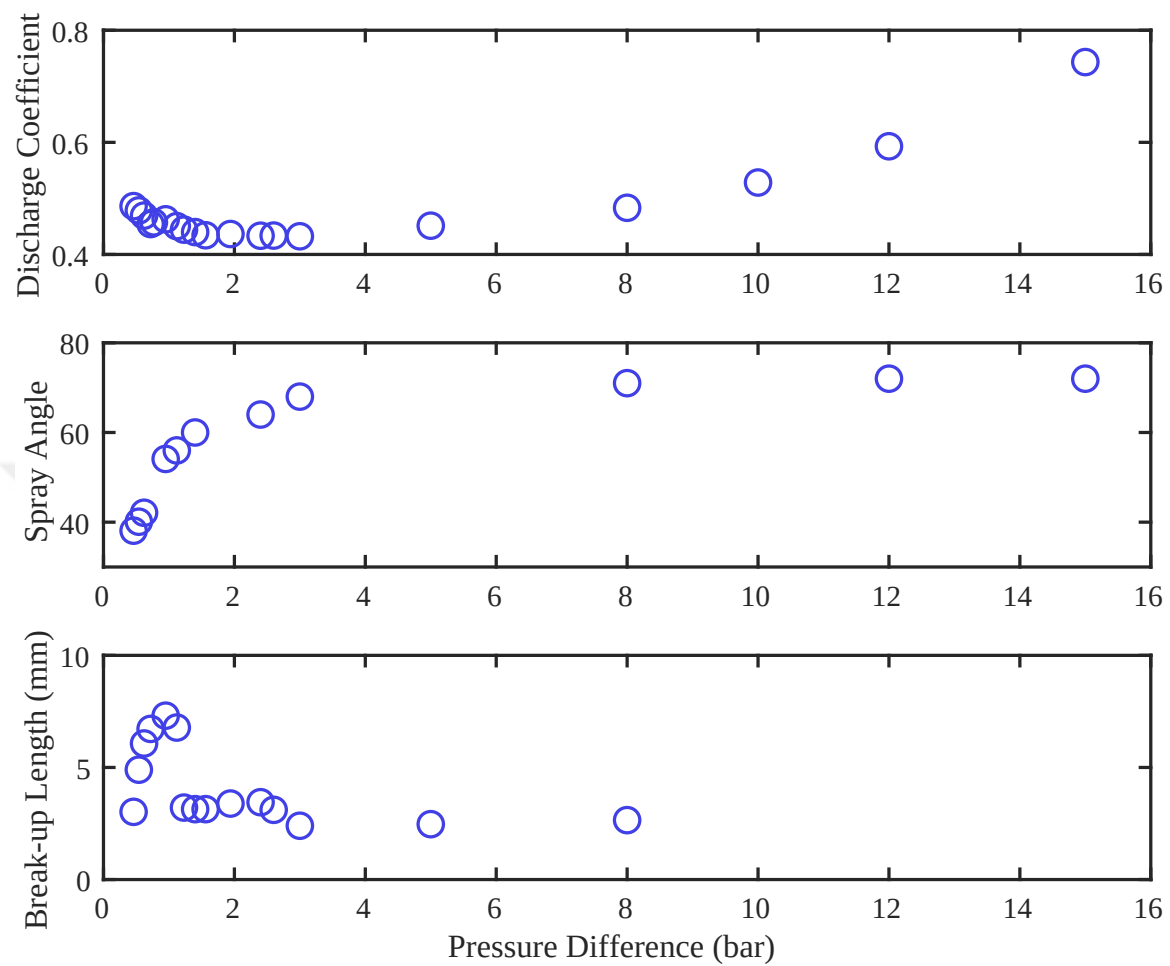


Figure 200: Experimental results [39]

A.1.2.11 Investigation of the Effect of Geometrical Parameters of Pressure Swirl Atomizer on the Hollow Cone, 2019 [40]

Table 69: Dimensions of the Atomizers [40]

No.	D_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	1.7	6	9.0	10.4	40^0	5.4	10.2
2	1.7	6	9.0	10.4	40^0	4.1	10.2
3	1.7	6	9.0	10.4	40^0	6.8	10.2
4	1.7	6	9.0	10.4	40^0	5.4	10.2
5	1.7	6	9.0	10.4	40^0	5.4	10.2
6	1.7	6	9.0	10.4	40^0	5.4	8.1
7	1.7	6	9.0	10.4	40^0	5.4	13.5

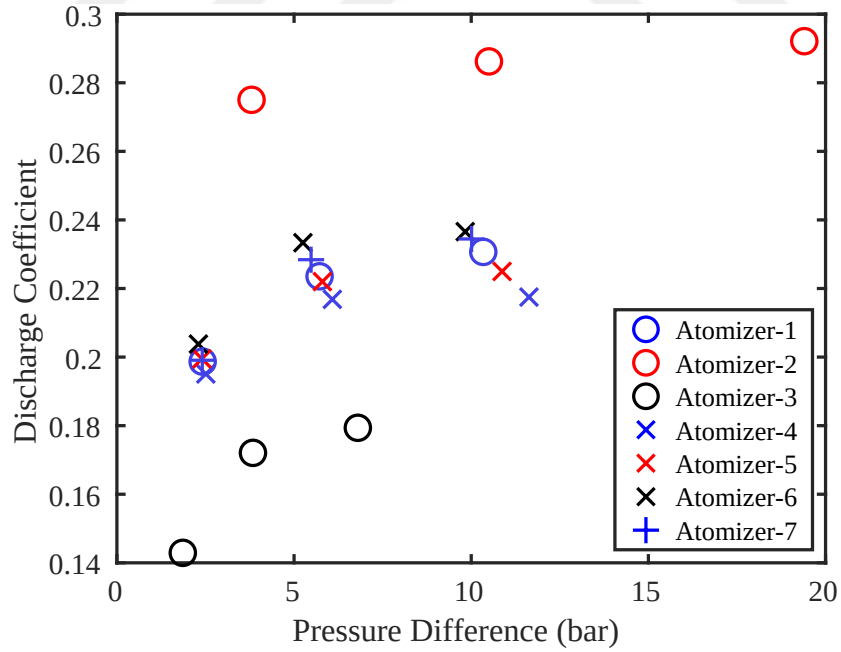


Figure 201: Experimental results - 1 [40]

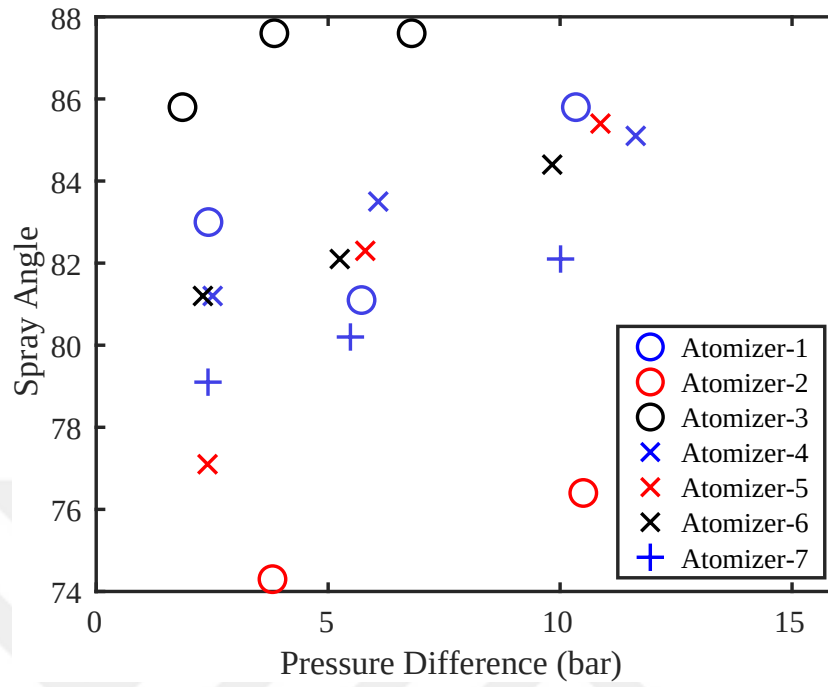


Figure 202: Experimental results - 2 [40]

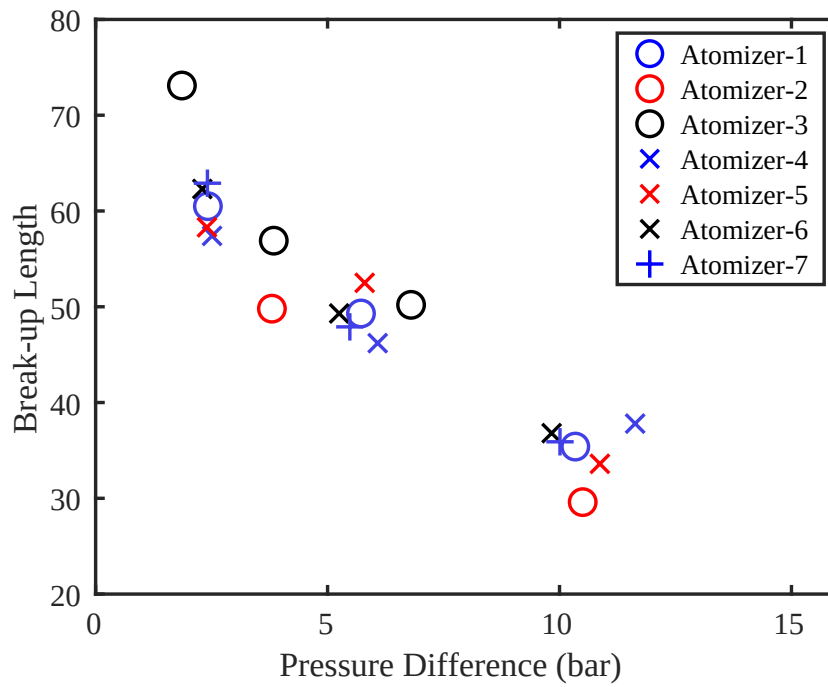


Figure 203: Experimental results - 3 [40]

Table 70: Physical Properties of the Liquid [41]

Temperature	ρ_l (kg/m ³)	ν_L (cSt)	σ_L (N/m)
-20	956.0	8.8	0.0326
-15	947.5	7.2	0.0316
-10	940.7	5.9	0.0308
0	930.3	4.0	0.0295
10	926.5	2.7	0.0286
20	918.8	1.8	0.0281
30	915.6	1.2	0.0277
45	912.7	0.7	0.0273

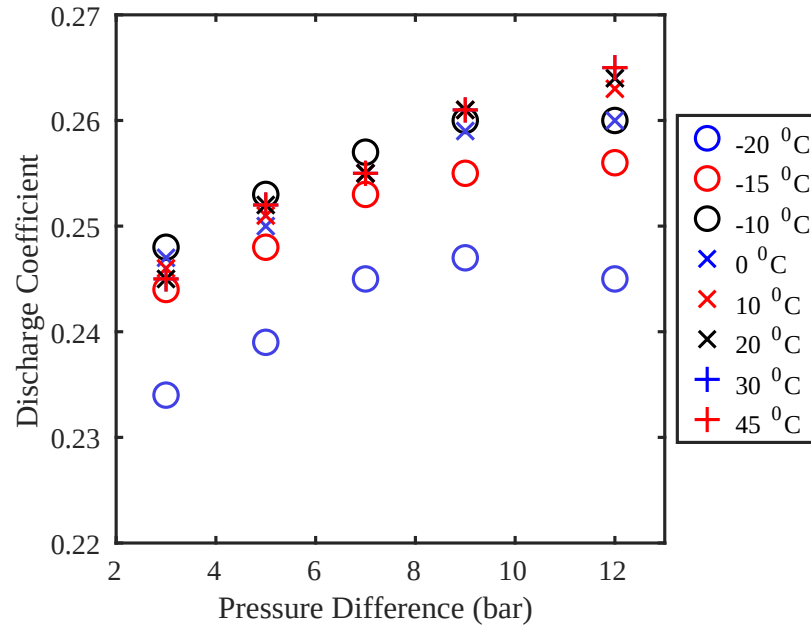


Figure 204: Experimental results [41]

A.1.2.13 Importance of Geometrical Factors on Spray Characteristics of Spill-Return Atomizers, 2021 [42]

Table 71: Dimensions of the Atomizers [42]

No.	w_p (mm)	h_p (mm)	N_p	D_s (mm)	L_s (mm)	α	D_o (mm)	L_o (mm)
1	0.60	0.60	2	3.00	0.00	47°	0.42	0.20
2	0.60	0.60	2	3.00	0.00	69°	0.42	0.20

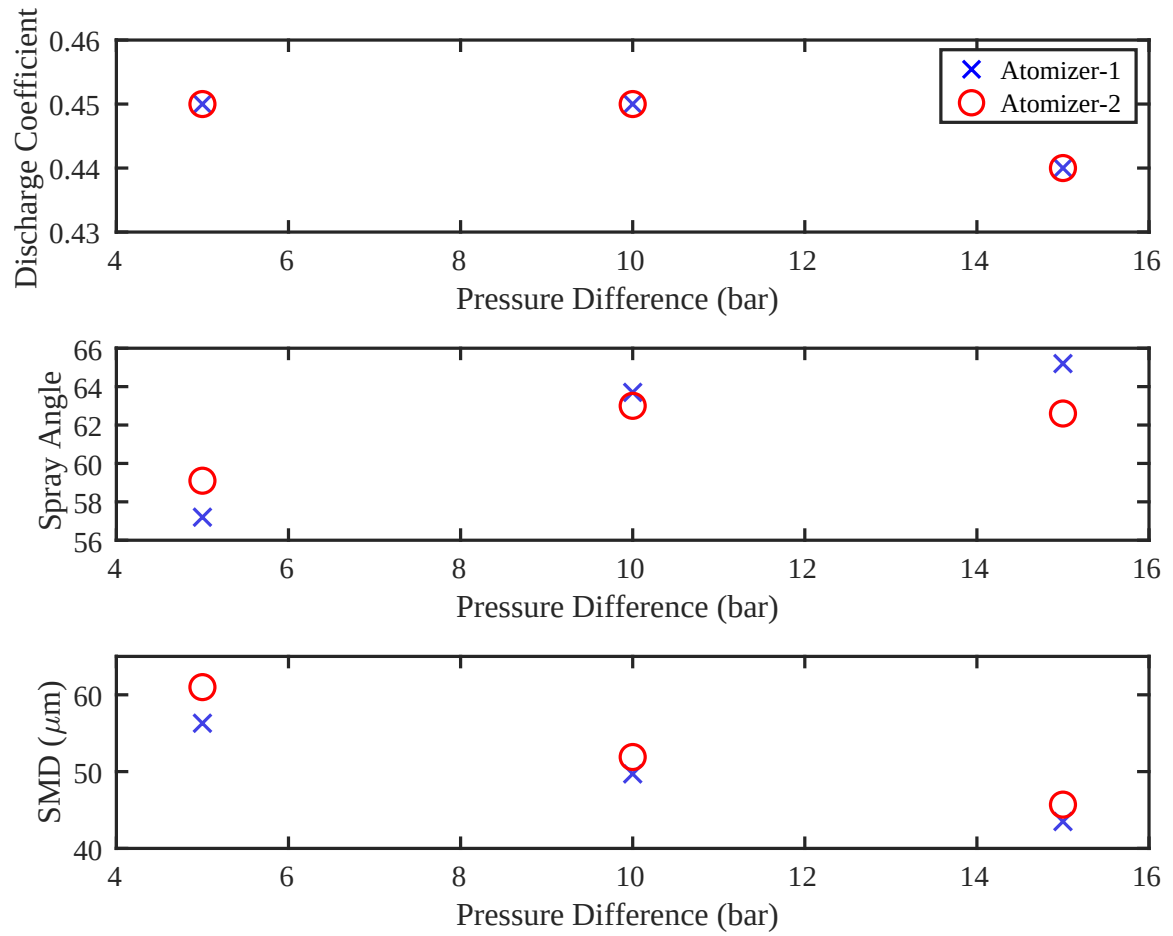


Figure 205: Experimental results [42]

A.1.3 Psuedo-Complete Data

A.1.3.1 Studies on Spray Behavior of a Pressure-Swirl Atomizer in Transition Regime, 2008 [43]

Table 72: Dimensions of the Atomizers [43]

w_p (mm)	h_p (mm)	N_p	β	D_s (mm)	L_s (mm)	D_o (mm)	L_o (mm)
0.80	0.80	2	23°	5.00	0.00	0.80	1.00

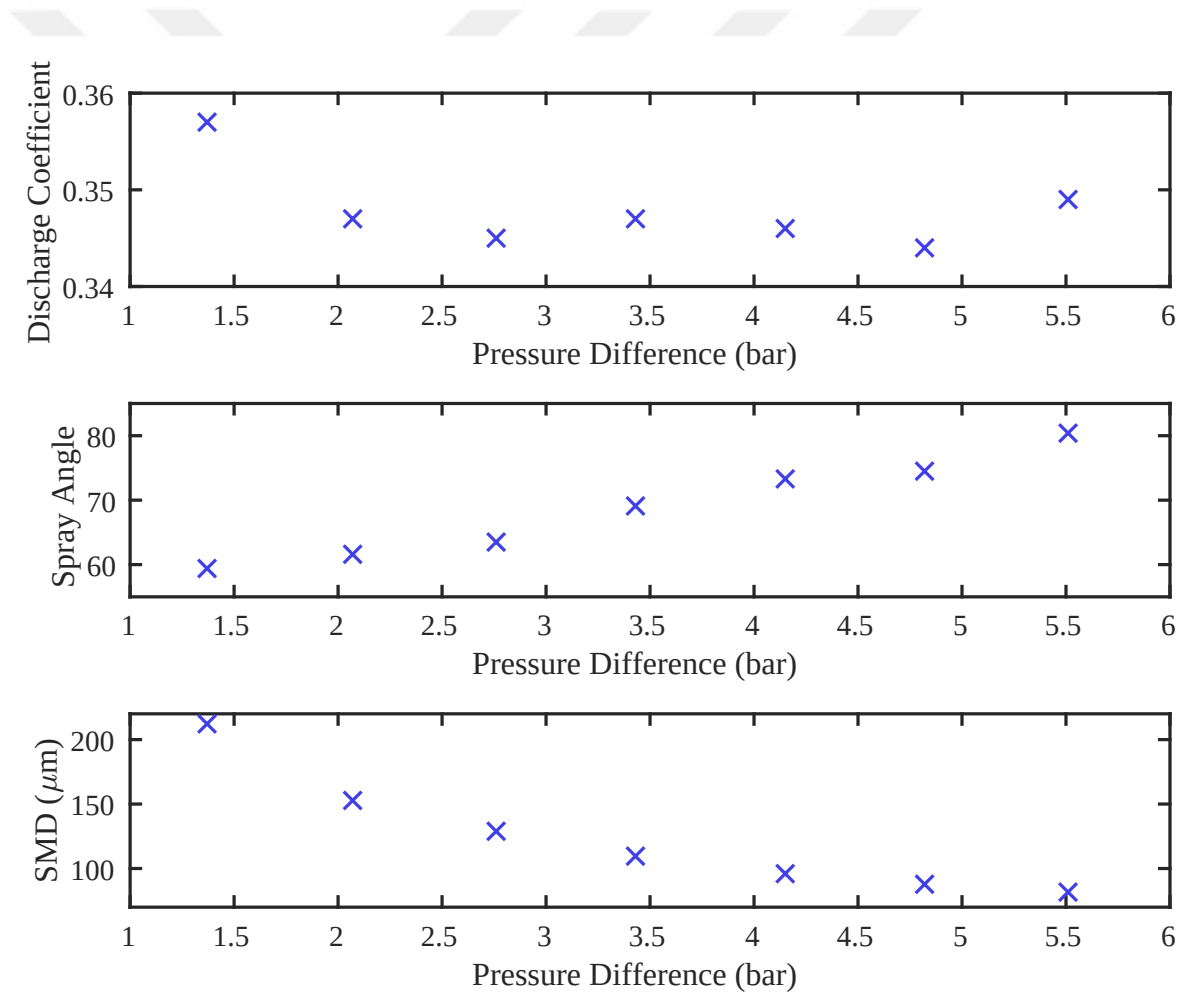


Figure 206: Experimental results [43]

A.1.3.2 Experimental Evaluation of Low Pressure Swirl Atomizer Applied Engineering Design Process, 2009 [44]

Table 73: Dimensions of the Atomizers [44]

D_p (mm)	L_p (mm)	N_p	D_s (mm)	L_s (mm)	D_o (mm)	L_o (mm)
0.60	1.00	4	3.00	3.00	1.00	1.00

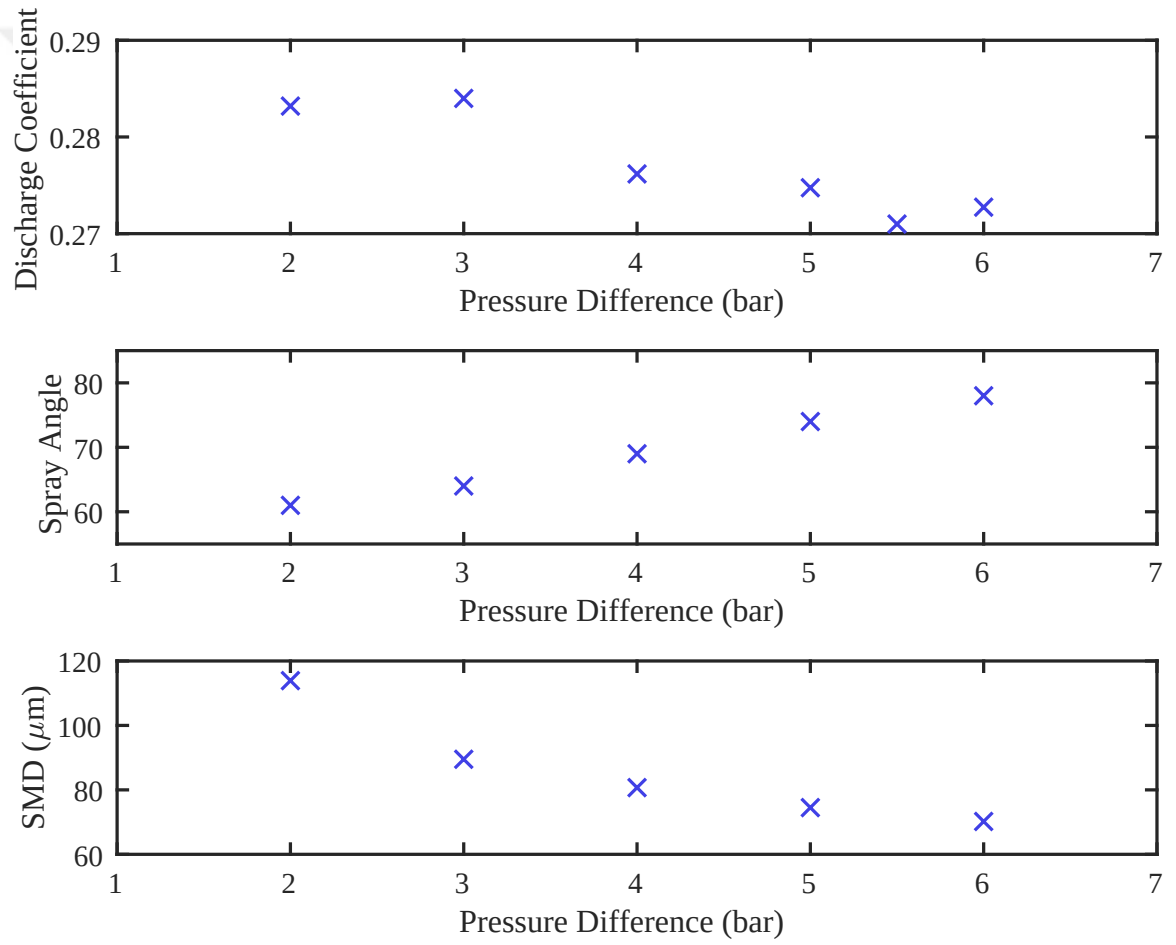


Figure 207: Experimental results [44]

A.1.3.3 Effect of Inlet Tangential Port Area on the Performance of Small-Scale Simplex Atomizer, 2012 [45]

Table 74: Dimensions of the Atomizers [45]

No.	w_p (mm)	h_p (mm)	N_p	D_s (mm)	L_s (mm)	D_o (mm)	L_o (mm)
1	0.70	0.90	2	4.00	8.00	0.90	0.50
2	0.90	0.90	3	4.00	8.00	0.90	0.50
3	1.00	0.90	4	4.00	8.00	0.90	0.50

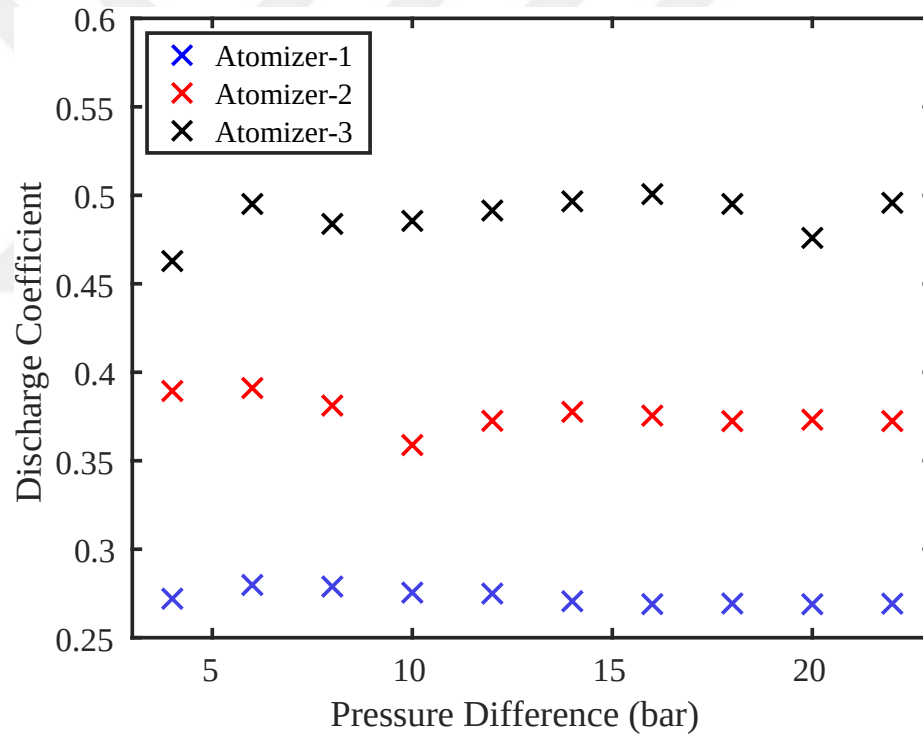


Figure 208: Experimental results - 1 [45]

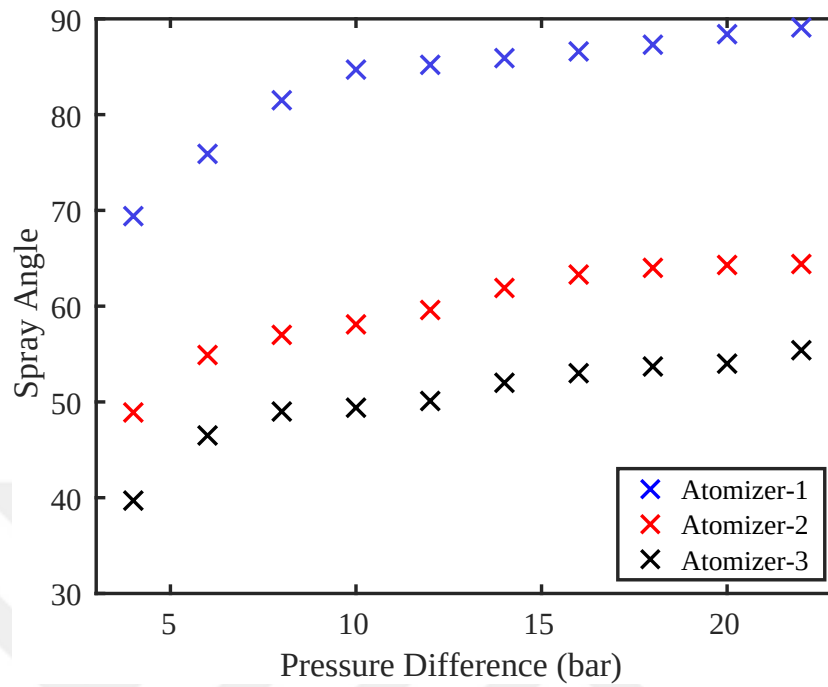


Figure 209: Experimental results - 2 [45]

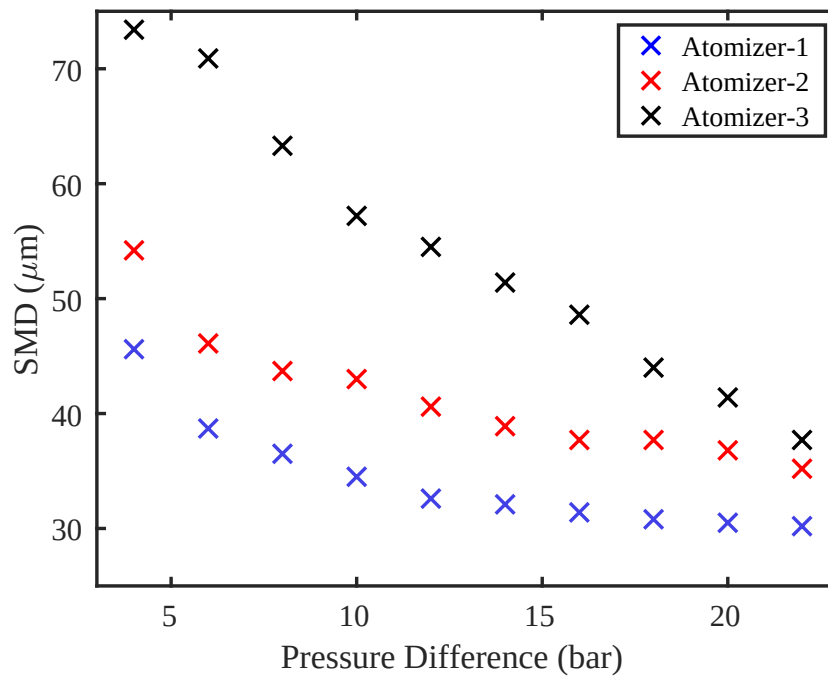


Figure 210: Experimental results - 3 [45]

A.1.3.4 Experimental Study of Spray Characteristics of Kerosene-Ethanol Blends from a Pressure-Swirl Nozzle, 2018 [46]

Table 75: Names and Physical Properties of Liquids [46]

Name	ρ_l (kg/m ³)	ν_L (mm ² /s)	σ_L (N/m)
Kerosene	802.0	2.30	0.026
10% Ethanol (E-10)	801.1	2.18	0.026
20% Ethanol (E-20)	799.2	2.08	0.026
30% Ethanol (E-30)	798.1	1.89	0.026

Table 76: Dimensions of the Atomizers [46]

D_p (mm)	N_p	β	D_s (mm)	L_s (mm)	D_o (mm)	L_o (mm)
0.20	3	30 ⁰	5.00	5.20	0.80	1.50

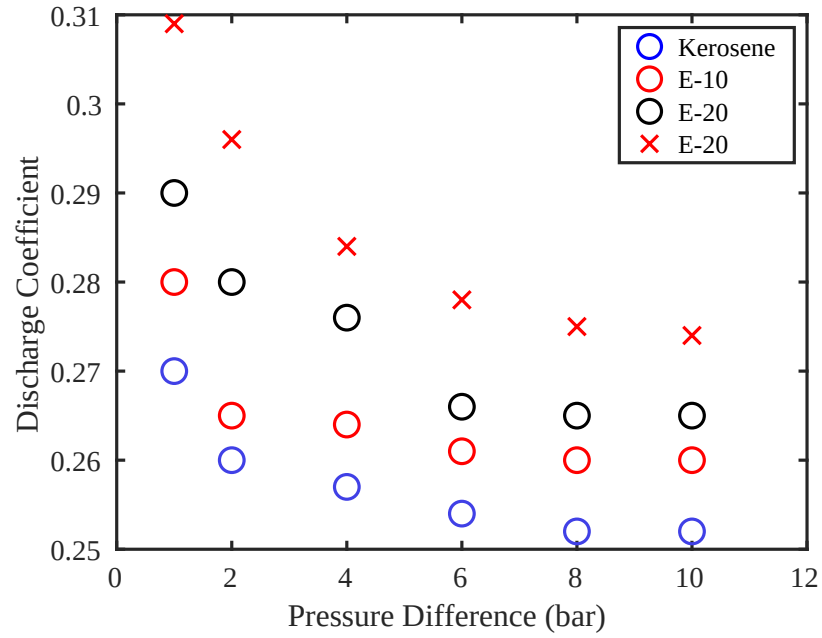


Figure 211: Experimental results - 1 [46]

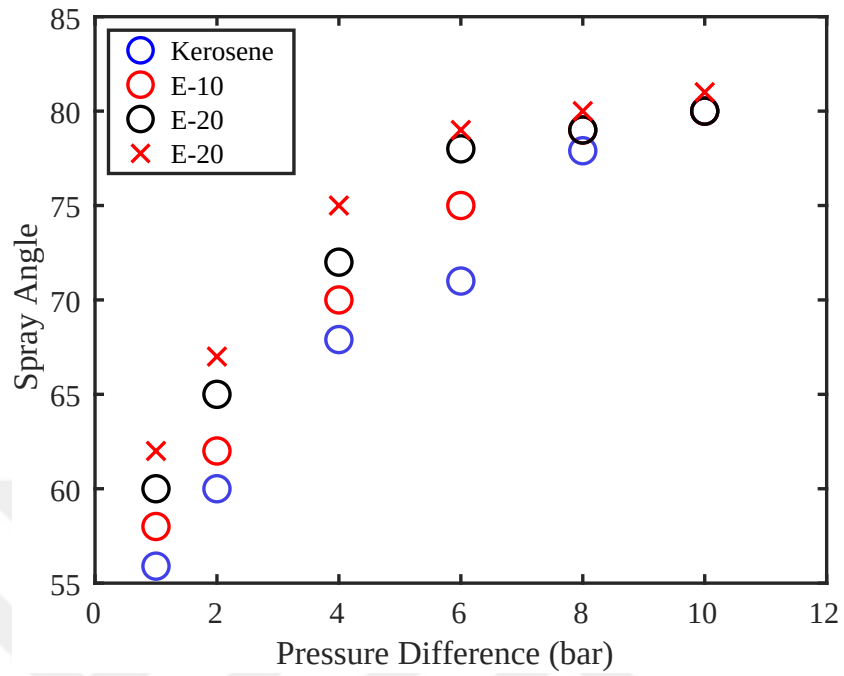


Figure 212: Experimental results - 2 [46]

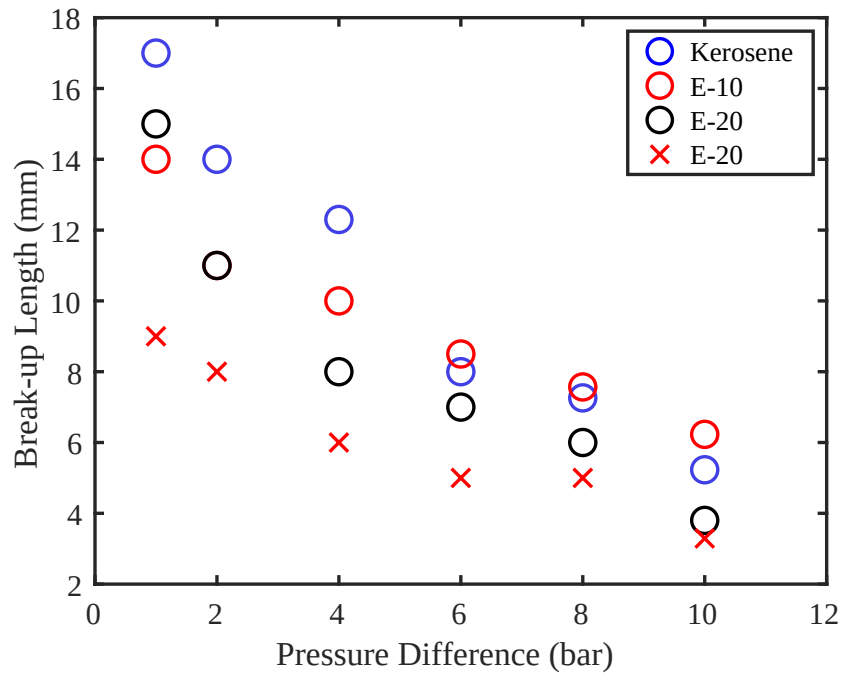


Figure 213: Experimental results - 3 [46]

A.1.4 Parametric Data

A.1.4.1 Design of Swirl Chamber Atomizers, 1987 [47]

Table 77: Dimensions of the Atomizers [47]

No.	D_p (mm)	N_p	D_s (mm)	α	D_o (mm)	L_o (mm)
1	1.15	3	8.00	45^0	2.00	2.00
2	0.77	3	9.00	45^0	0.90	0.90
3	1.65	3	11.60	45^0	1.50	1.50
4	1.24	3	8.00	45^0	1.00	1.00

Table 78: Experimental Results [47]

No.	C_D (mm)	θ	SMD (μm)
1	0.216	90^0	68
2	0.222	77^0	30
3	0.365	77^0	66
4	0.253	78^0	101

Table 79: Dimensions of the Atomizers [48]

No.	w_p (m)	h_p (m)	L_s (m)	D_s (m)	D_o (m)
1	1.61E-03	1.23E-03	1.23E-03	1.18E-02	7.62E-04
2	1.61E-03	1.23E-03	1.23E-03	1.18E-02	1.78E-03
3	2.45E-03	1.88E-03	1.88E-03	1.07E-02	1.78E-03
4	2.45E-03	3.77E-03	3.77E-03	1.07E-02	7.62E-04
5	2.45E-03	3.77E-03	3.77E-03	1.07E-02	1.27E-03
6	2.45E-03	3.77E-03	3.77E-03	1.07E-02	1.78E-03
7	2.45E-03	1.36E-03	1.36E-03	1.07E-02	1.78E-03
8	2.45E-03	1.88E-03	1.88E-03	1.07E-02	1.27E-03
9	2.45E-03	1.36E-03	1.36E-03	1.07E-02	1.02E-03

Table 80: Experimental Results [48]

ρ_L (kg/m ³)	μ_L (Pa.s)	σ (N/m)	Atomizer	ΔP_L (bar)	C_D	θ (deg)	SMD (μ m)	Diagonal L_b (m)
1240	0.01626	0.072	1	123	0.527	55	58.86	10
1247	0.01867	0.072	1	152	0.541	50.5	52.92	10
1220	0.01213	0.072	2	19	0.232	80	130.63	11
1213	0.0106	0.072	3	22	0.286	72	113.06	9
1232	0.01424	0.072	4	84	0.847	60	58.17	10
1226	0.01324	0.072	5	38	0.532	55	92.62	13
1201	0.00869	0.072	6	8	0.403	63	171.49	17
1276	0.04025	0.072	2	22	0.227	61	112.61	22
1282	0.04866	0.072	7	46	0.287	59	90.74	18
1277	0.04156	0.072	8	71	0.508	50	72.74	15
1285	0.05285	0.072	3	30	0.386	52	110.97	24
1288	0.054	0.072	4	80	0.849	15	66.46	18
1267	0.0325	0.072	5	35	0.566	49	77.54	21
1281	0.04679	0.072	6	29	0.456	47	100.02	31
1315	0.12783	0.072	2	87	0.328	43.4	89.09	33
1310	0.17141	0.072	7	89	0.310	42	73.6	33
1314	0.14	0.072	8	139	0.567	33	61.74	24
1306	0.11	0.072	3	46	0.396	45	91.62	30
1307	0.11616	0.072	5	80	0.622	25	80.05	23
1304	0.1018	0.072	6	58	0.489	42	80.91	30
1277	0.06345	0.0514	1	105	0.744	45	73.6	11
1269	0.056	0.0485	2	30	0.271	54	103.58	21
1275	0.05784	0.0579	7	40	0.309	63	89.31	17
1257	0.0435	0.052	8	73	0.505	64	63.88	12.5
1280	0.06557	0.049	5	72	0.614	38	71	23
1250	0.0521	0.0521	6	39	0.443	56	79.01	23
1300	0.1669	0.0531	2	69	0.334	48	90.05	37
1290	0.146	0.0497	7	54	0.379	47	91.43	30
1290	0.1576	0.0465	6	40	0.529	42	88.76	30
1297	0.1521	0.0515	9	65	0.660	23	88.04	16

A.1.1.4.3 Improved Semi-empirical Correlation to Predict Sauter Mean Diameter for Pressure-Swirl Atomizers, 2014 [49]

Table 81: Dimensions of the Atomizers [49]

No.	D_p (mm)	N_p	D_s (mm)	D_o (mm)	L_o (mm)
1	1.84	3	18.00	1.00-3.00	1.00
2	2.26	3	18.00	1.00-3.00	1.00
3	2.61	3	18.00	1.00-3.00	1.00
4	2.26	3	12.00	1.00-3.00	1.00
5	0.50	3	2.00	0.40	0.20-1.00
6	0.50	3	2.00	0.60	3.00-5.00

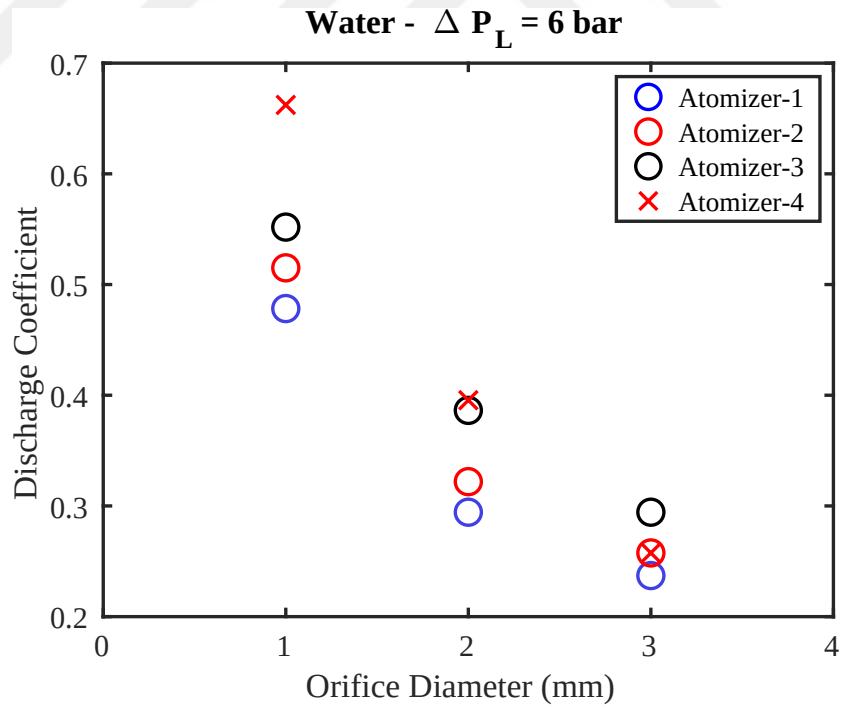


Figure 214: Experimental results - 1 [49]

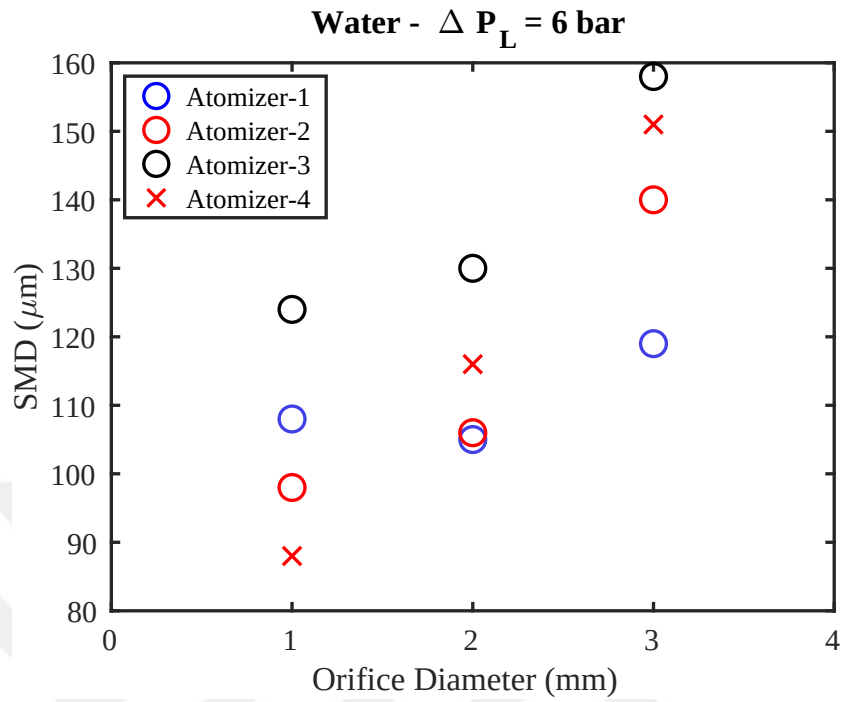


Figure 215: Experimental results - 2 [49]

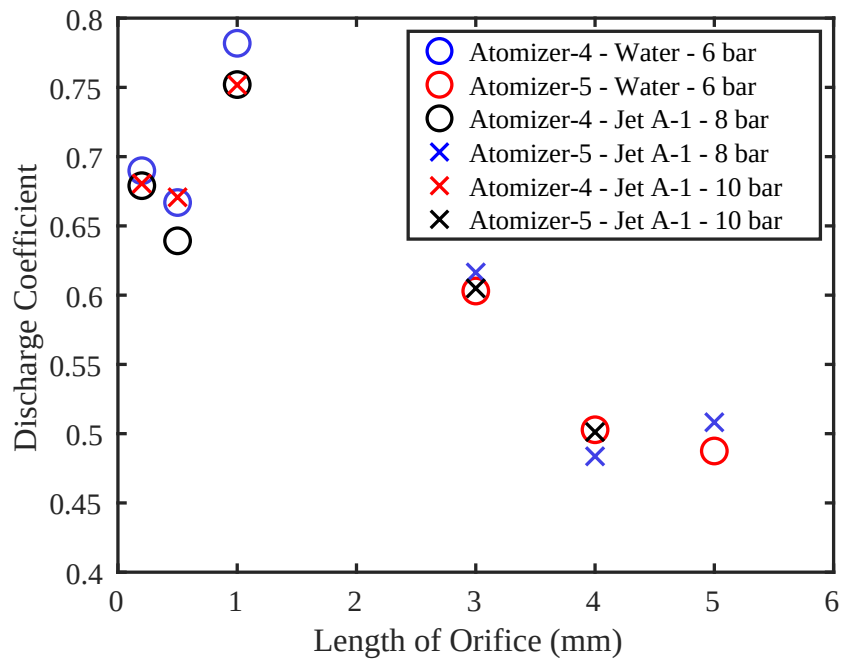


Figure 216: Experimental results - 3 [49]

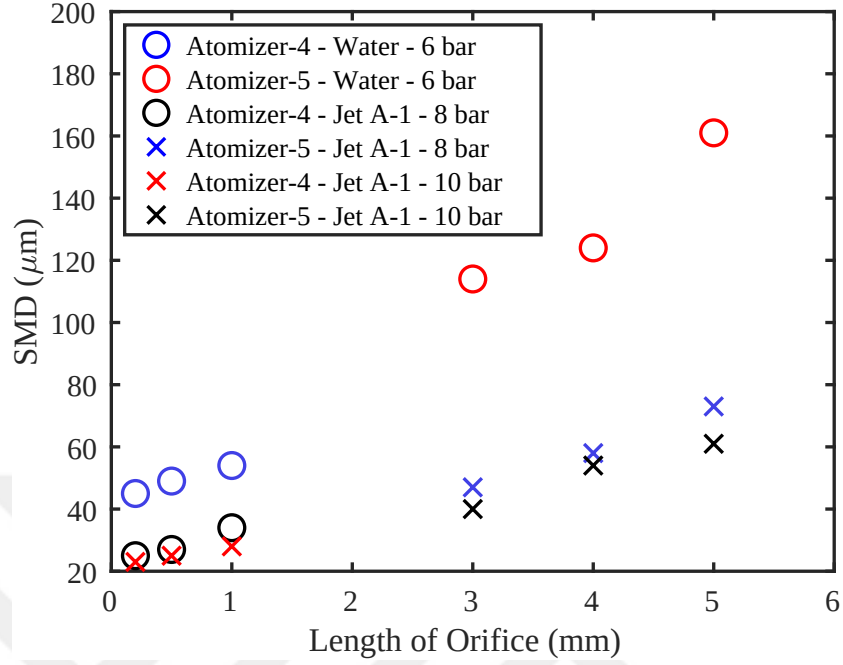


Figure 217: Experimental results - 4 [49]

A.1.4.4 Influence of Convergence Angle on Hollow Cone Spray Characteristics, 2019 [50]

Table 82: Dimensions of the Atomizers [50]

No.	D_p (mm)	N_p	D_s (mm)	α	D_o (mm)
1	0.80	3	4.40	30^0	1.00
2	0.80	3	4.40	45^0	1.00
3	0.80	3	4.40	60^0	1.00
4	0.80	3	4.40	75^0	1.00

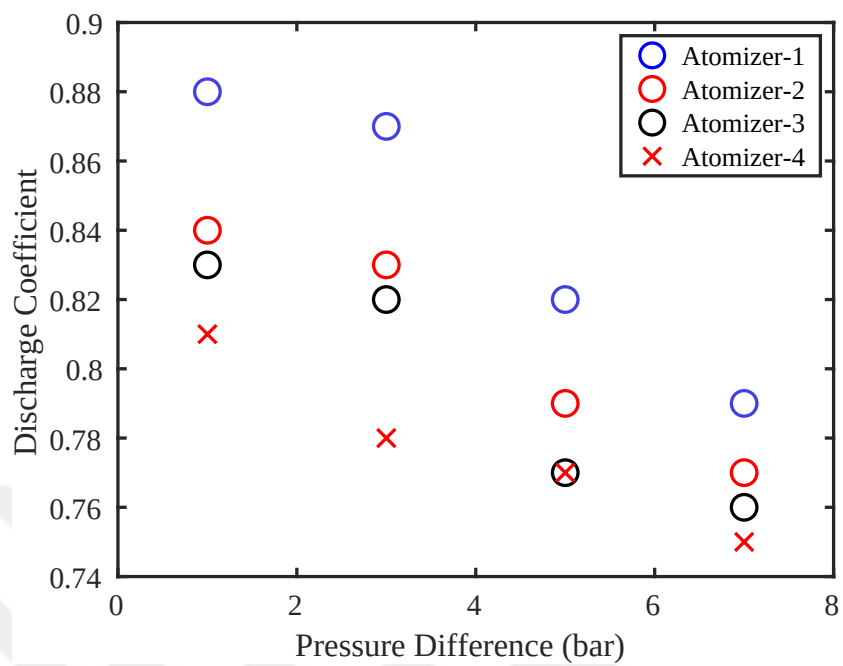


Figure 218: Experimental results - 1 [50]

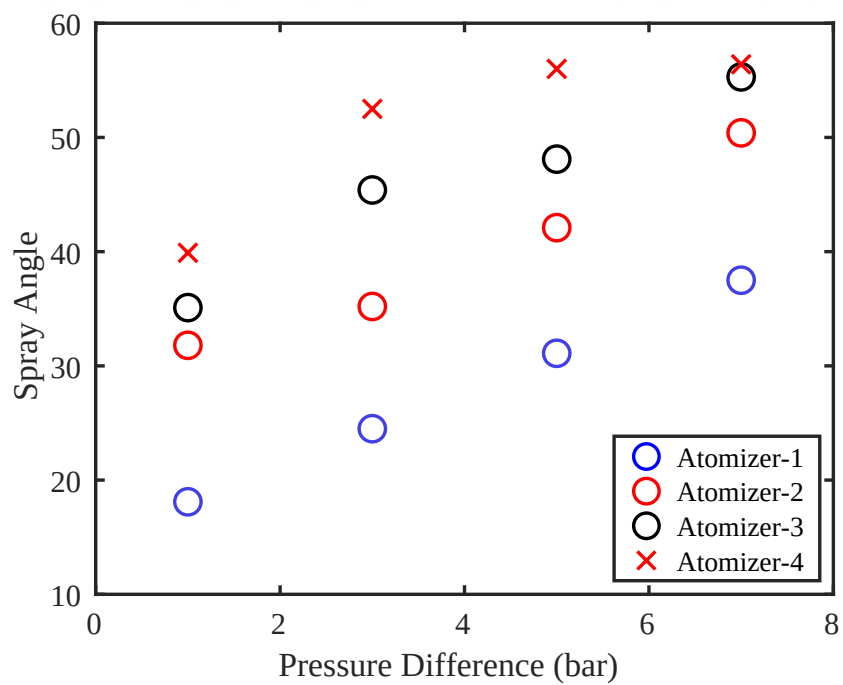


Figure 219: Experimental results - 2 [50]

A.2 Open-End Atomizers

A.2.1 Quasi-Complete Data

A.2.1.1 Geometrical Effects on the Fluid Dynamics of an Open-End Swirl Injector, 2011 [51]

Table 83: Dimensions of the Atomizers [51]

No.	D_p (mm)	N_p	D_o (mm)	L_o (mm)
1	1.60	1	8.00	38.50
2	1.80	1	8.00	38.50
3	2.00	1	8.00	38.50
4	2.20	1	8.00	38.50

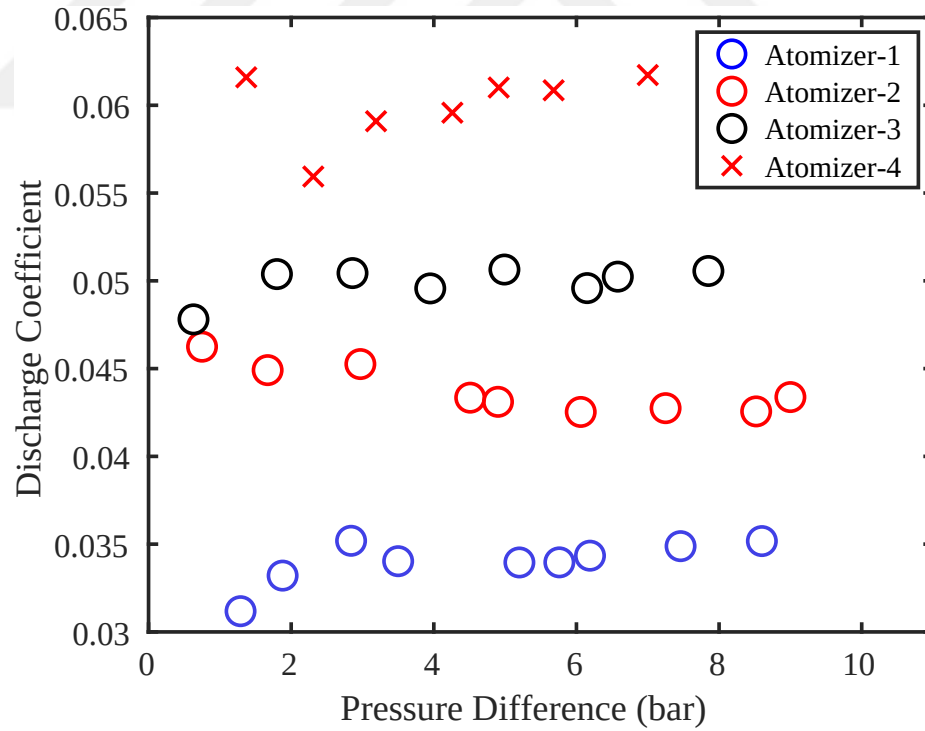


Figure 220: Experimental results - 1 [51]

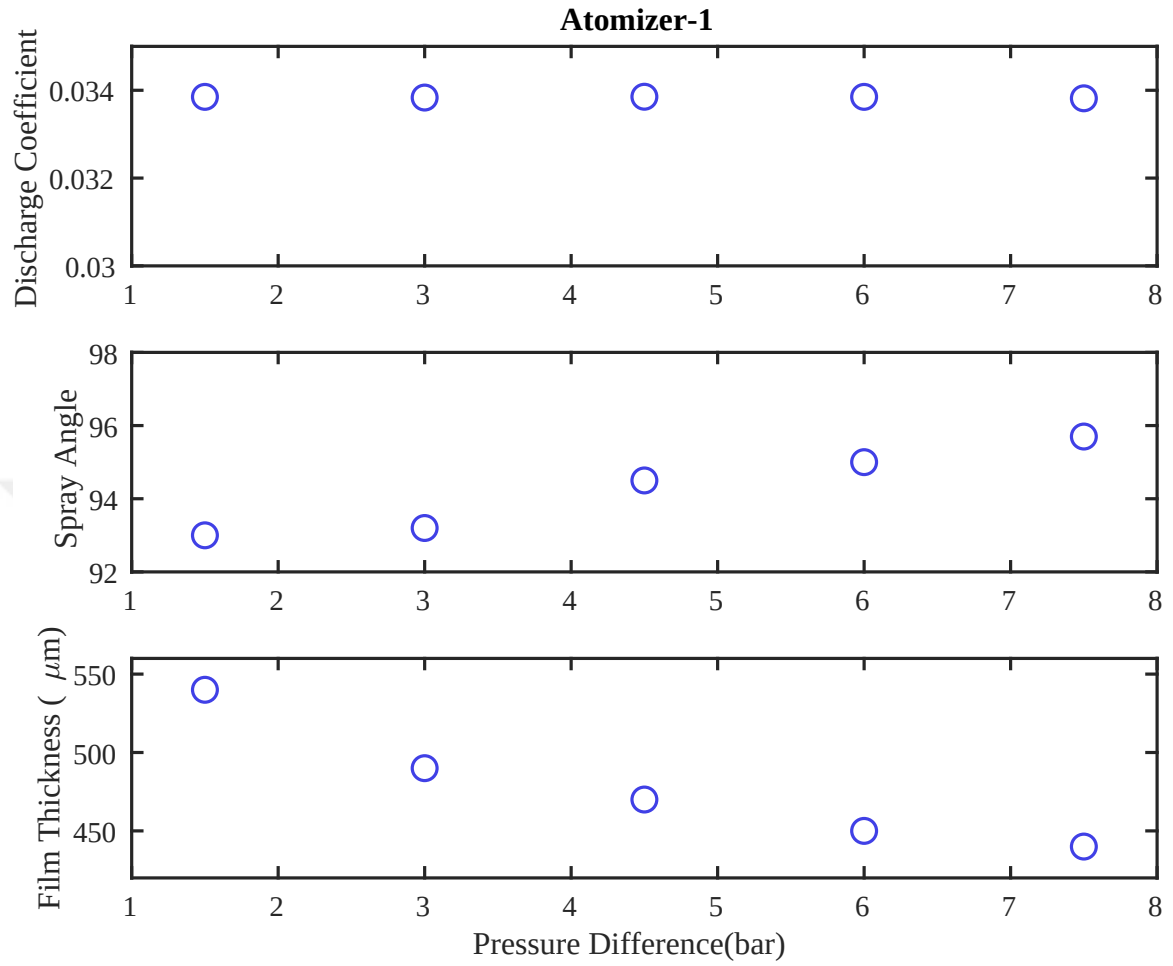


Figure 221: Experimental results - 2 [51]

A.2.1.2 Spray Characteristics of an Open-End Swirl Injector, 2012 [52]

Table 84: Dimensions of the Atomizers [52]

No.	D_p (mm)	N_p	D_o (mm)	L_o (mm)
1	1.60	1	8.00	38.50
2	1.90	1	8.00	38.50
3	1.98	1	8.00	38.50
4	2.20	1	8.00	38.50
5	2.70	1	8.00	38.50

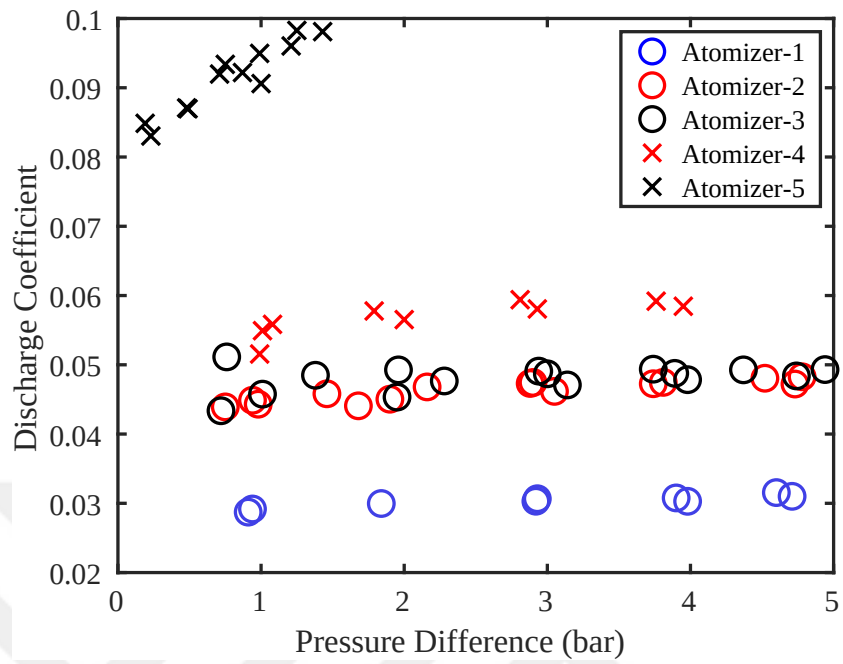


Figure 222: Experimental results - 1 [52]

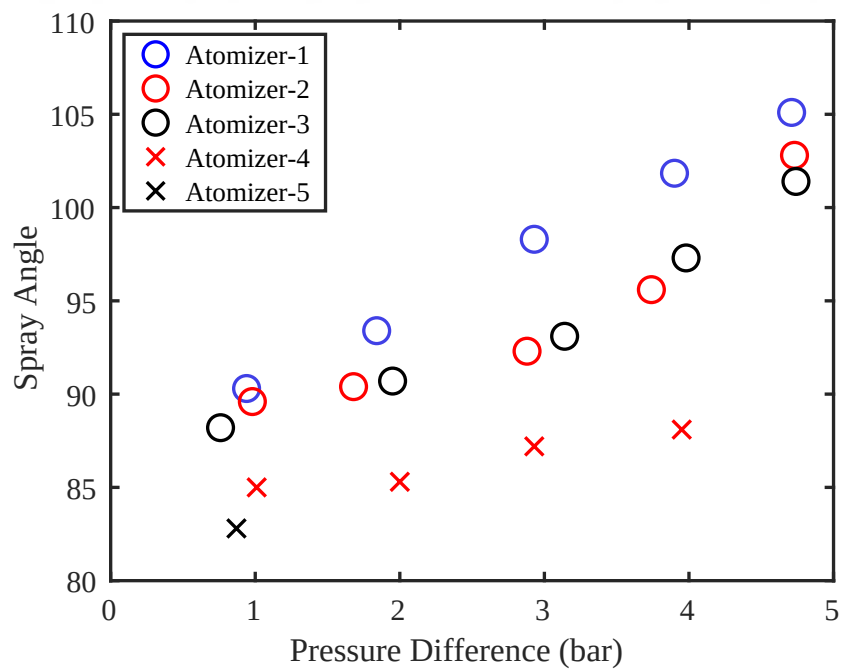


Figure 223: Experimental results - 2 [52]

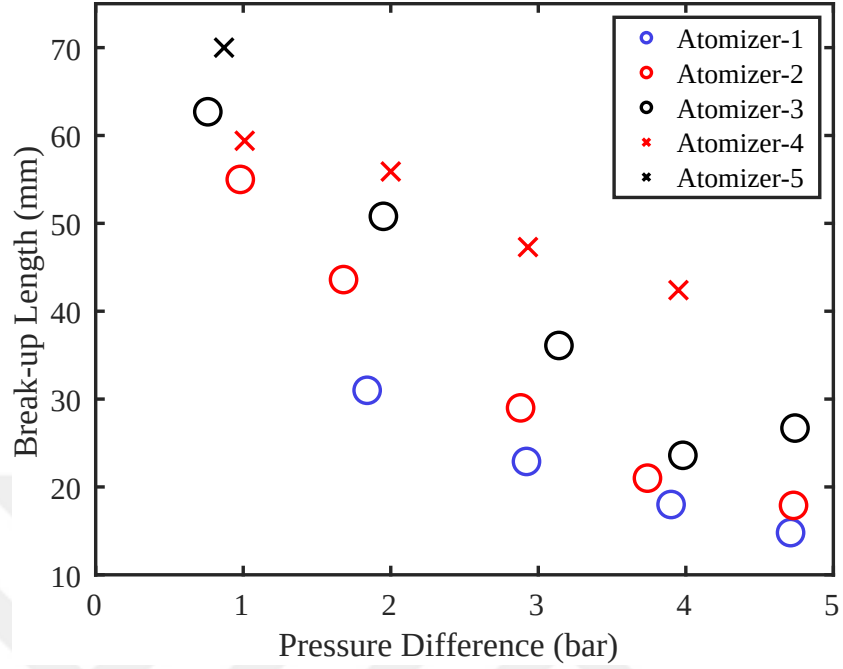


Figure 224: Experimental results - 3 [52]

A.2.1.3 The Spray Characteristics of an Open-End Swirl Injector at Ambient Pressure, 2017 [53]

Table 85: Dimensions of the Atomizers [53]

No.	D_p (mm)	N_p	D_o (mm)	L_o (mm)
1	0.80	4	5.00	50.00
2	1.00	4	5.00	50.00
3	1.40	4	5.00	50.00
4	1.20	4	5.00	50.00
5	1.20	4	5.00	40.00
6	1.20	4	8.00	67.00
7	1.20	4	8.00	72.00
8	1.20	4	8.00	77.00

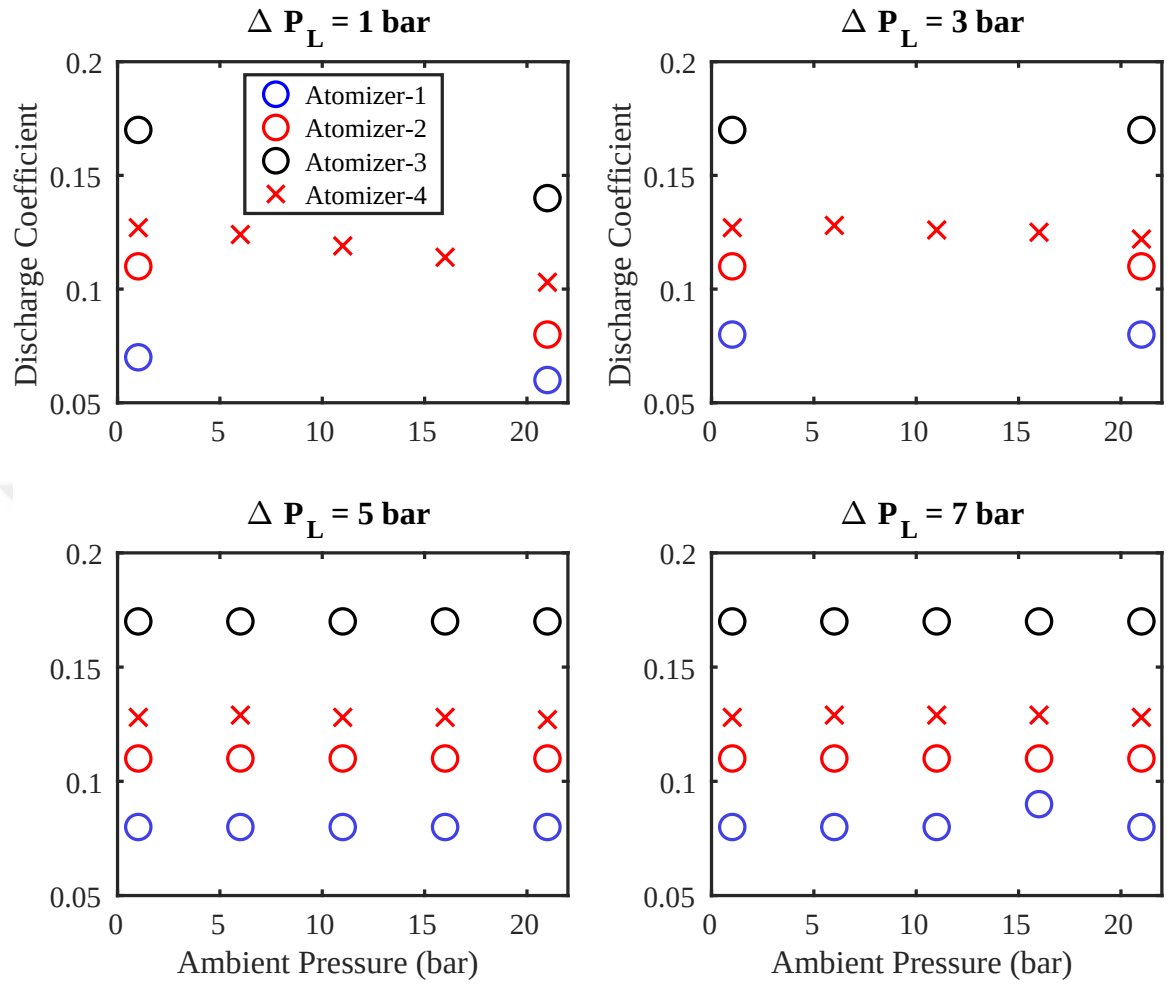


Figure 225: Experimental results - 1 [53]

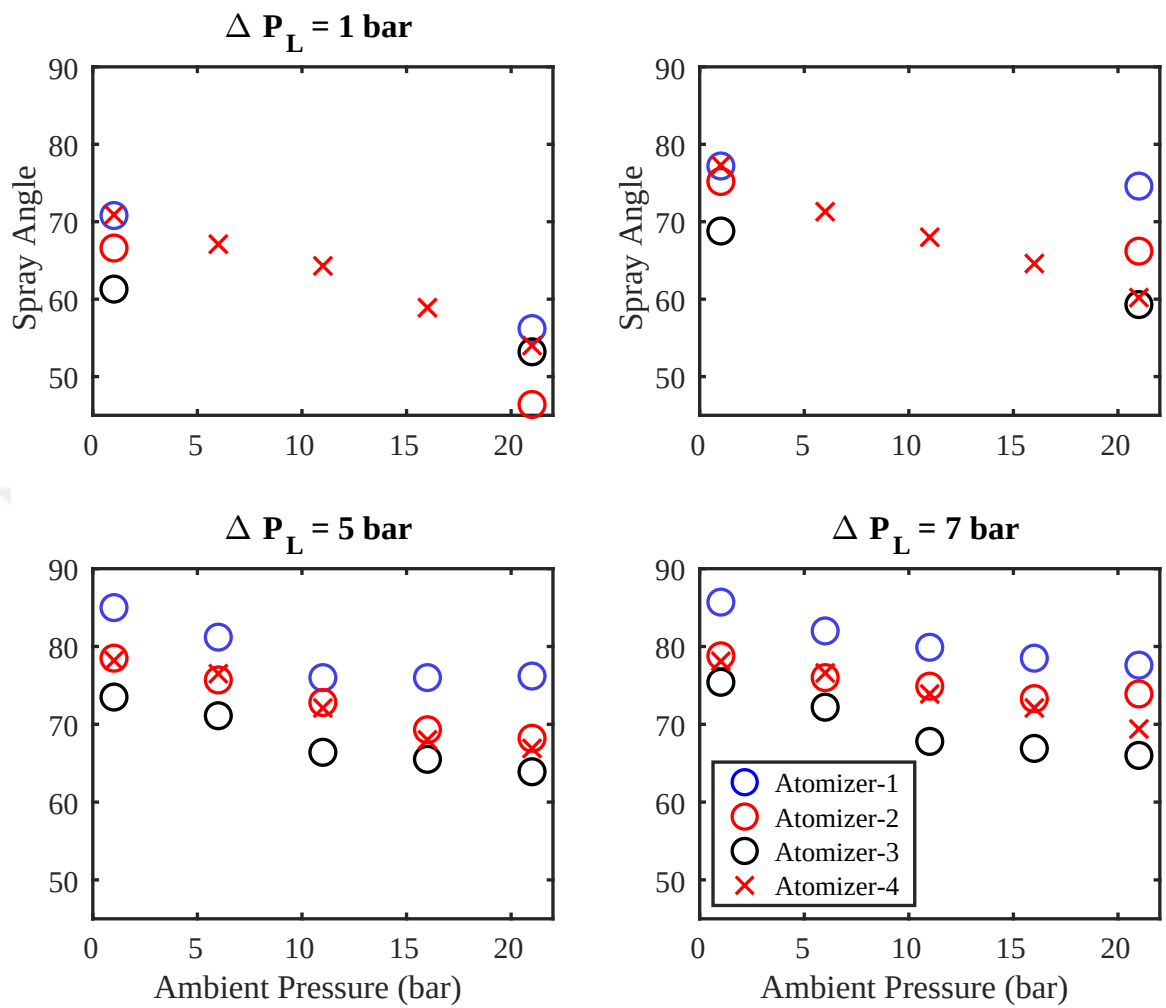


Figure 226: Experimental results - 2 [53]

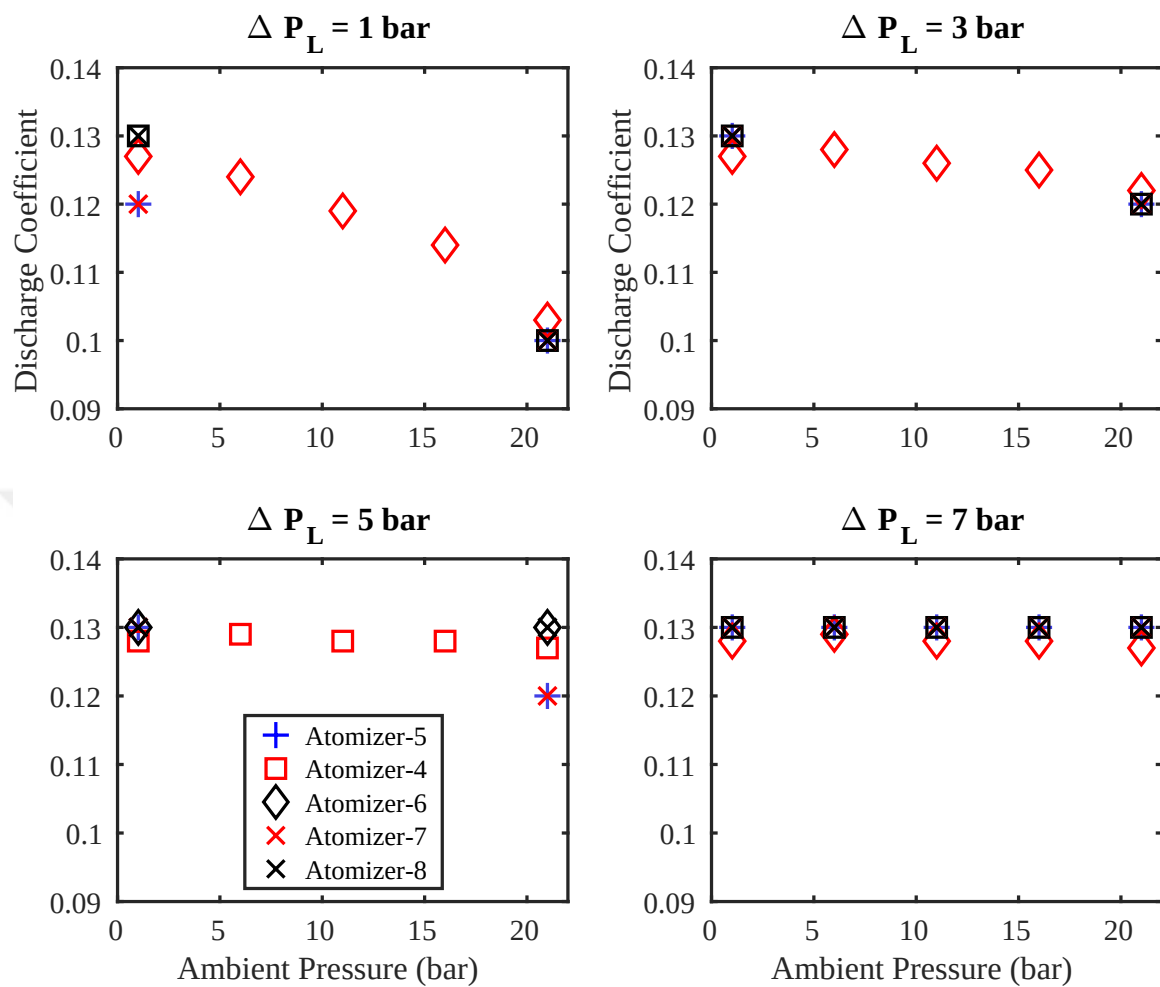


Figure 227: Experimental results - 3 [53]

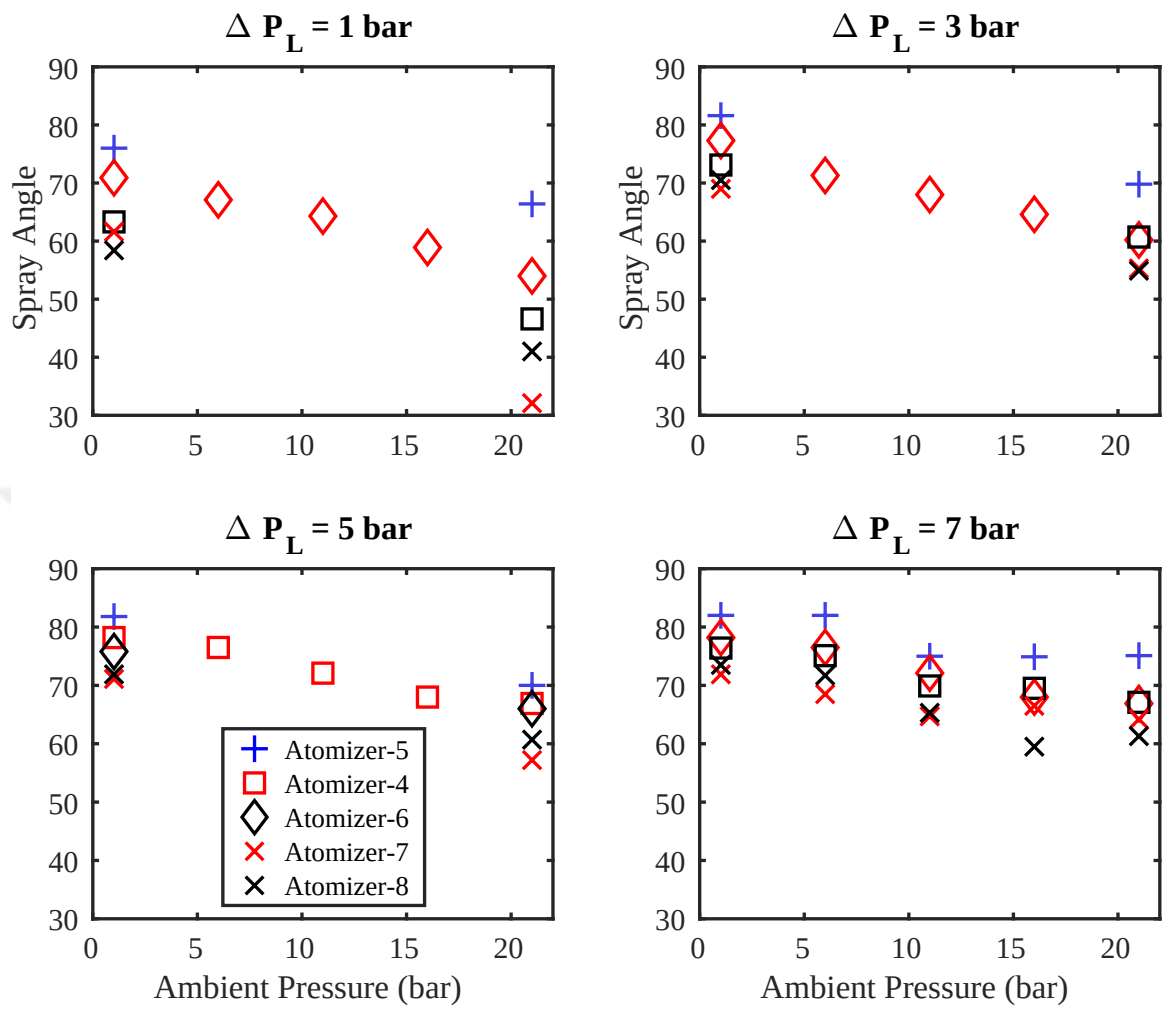


Figure 228: Experimental results - 4 [53]

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