UNIVERSITY OF ÇUKUROVA
INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

Adem BİLGİLİ

DIFFRACTION OF PLANE ELECTROMAGNETIC WAVES BY A CHIRAL HALF-PLANE – WIENER-HOPF SOLUTION

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

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ÇUKUROVA ÜNİVERSİTESİ

FEN BİLİMLERİ ENSTİTÜSÜ

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YÜKSEK LİSANS TEZİ

ELEKTRİK-ELEKTRONİK MÜHENDİSLİĞİ ANA BİLİM DALI

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ÖZ

YÜKSEK LİSANS TEZİ

DÜZLEMSEL DALGALARIN CHIRAL YARIM DÜZLEMDEN KIRINIMI–WIENER-HOPF ÇÖZÜMÜ

Adem BİLGİLİ

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Bu çalışmada,yüzeyi kuplajlı sınır şartlarıyla simulize edilmiş sonsuz ince yarım düzlem chiral tabakaya gelen düzlemsel elekromagnetik dalganın kırınım problemi incelendi. Daha önce, M. A. Lyalinov, A. H. Serbest ve T. İkiz tarafından Maliuzhinetz yöntemiyle çözülen bu problem Wiener-Hopf tekniğiyle ele alınmıştır. Bu yapıya karşılık gelen sınır değer problemi Fourier dönüşüm tekniği kullanılarak bir Wiener-Hopf denklemine indirgendi. Bazı belirli dönüşümler kullanılarak skaler Wiener-Hopf denklemi elde edildi. Wiener-hopf çekirdeğinin faktorizasyonu Maliuzhinetz fonksiyonu kullanılarak yapıldı. En dik iniş çizgisi yöntemi ile analizi yapılan alan integralinden difraksiyon katsayısı elde edildi ve parametrelerin belirli değerleri için nümerik sonuçlar elde edildi.

Anahtar Kelimeler : Chiral Yarım Düzlem, Wiener-Hopf Metodu, Maliuzhinetz Fonksiyonu, Fourier Dönüşüm Tekniği, En Dik İniş Çizgisi Yöntemi

ABSTRACT

MSc THESIS

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In this thesis, the diffraction of plane electromagnetic waves from a chiral half-plane simulated by the surface coupled boundary conditions is investigated. This problem was considered by M. A. Lyalinov, A. H. Serbest and T. İkiz and the solution was obtained by Maliuzhinetz method. Here, the corresponding boundary-value problem was reduced to a Wiener-Hopf equation by using Fourier transform technique. By certain transformations, the Wiener-Hopf equation of scalar type was obtained and the factorization of the Wiener-Hopf kernel is made by using the Maliuzhinetz function. The diffraction coefficient was obtained by the steepest descent method and numerical solutions were obtained for certain values of some parameters.

Keywords: Chiral Half-Plane, Wiener-Hopf Method, Maliuzhinetz Function, Fourier Transform Technique, The Steepest Descent Method.

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NOTATIONS

B	The impedance matrix
d	Thickness of the slab
k_0	The free space wavenumber
$k_{\pm}$	The wavenumbers for right and left circulary polarized plane waves
$M_p(z)$	Maliuzhinetz function
e	Dielectric constant
m	Permeabilty
h	The impedance of the slab
h_0	The free space impedance
h_c	Chirality impedance
x_c	Chirality admittance
l_j	The eigenvalues of B
w	The circular frequency of the field

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1. INTRODUCTION

Chiral materials have been known and investigated since the beginning of last century. Due to the possibility of artificial construction of chiral materials and their applications in the design of antennas, microwave devices and waveguides, the propagation of electromagnetic waves in an environment with chiral slabs receives a considerable attention.

There are many studies about chiral media and interaction of electromagnetic waves with chiral objects. Reflection and transmission characteristics of chiral slabs, propagation in chiral waveguides, properties of chiral mirrors and similar chiral materials are investigated by various authors [A. Gökçen and İ. Derin, July 2005, C.Sabah and S.Uçkun, 2005, Quiang Chang and Tie Jun Cui, 10 March 2006, S. A. Naqvi, Q. A. Naqvi and A. Hussain, May 2006, S. Uçkun, T. Ege, 2006, and M. Yokota, Y. Yamanaka, 8 June 2006]. Because of the possibility of artificial construction of chiral materials, the investigations about chiral materials for both academic and practical purposes are improving.

In this thesis, diffraction of a plane wave incident on a semi-infinite chiral slab which was solved in an explicit form by M. A. Lyalinov, A. H. Serbest and T. İkiz via Maliuzhinetz technique is solved by Wiener-Hopf method. It is believed that the Wiener-Hopf solution of this problem will be in a better form to investigate the effects of various parameters on diffraction phenomena.

In this problem, in order to obtain the diffracted field expressions outside the slab, the slab was modelled with an infinitely thin surface where appropriate approximate boundary conditions are satisfied. Due to the coupling between the field components in the chiral medium, a similarity transform is used by introducing to auxiliary functions to decouple the field components; and then the boundary conditions are expressed in terms of this auxiliary functions. Fourier transforms are used to obtain Wiener-Hopf equation and the scattering waves from chiral slab are obtained by using Wiener-Hopf method. The diffraction coefficients are obtained by using the steepest descent method for the field analysis. Finally, some numerical solutions and results are presented.

2. FORMULATION OF THE PROBLEM

A thin semi-infinite chiral slab of thickness d is located at $x \in (-\infty, 0)$ and y=0; and the incident field is assumed to be

$$\begin{bmatrix} H_z^i h_0 \\ E_z^i \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \exp\left\{-ik_0 r \cos\left(f - f_0\right)\right\}. \tag{2.1}$$

Here (r, f) are the variables of the polar coordinates, $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ is the free space wavenumber and $h_0 = \sqrt{m_0 / e_0}$ is the free space intrinsic impedance. $\left(H_z^i, E_z^i\right)$ are the z-components of the incident electromagnetic fields with the z-axis being directed along the edge of the slab.

The appropriate transition boundary conditions for a thin chiral slab in terms of the total fields are [Lyalinov, M. A. and A. H. Serbest, 1998]

$$\begin{bmatrix} \dfrac{\partial}{\partial_y}(H_z \eta_0) \\ \dfrac{\partial}{\partial_y}(E_z) \end{bmatrix}_{y=\pm 0} = B. \begin{bmatrix} [H_z \eta_0]_-^+ \\ [E_z]_-^+ \end{bmatrix}, \quad \text{for} \quad x < 0 \tag{2.2}$$

in which $[f]_-^+ = f(x,+0) - f(x,-0)$ and (H_z, E_z) are the z components of the total electromagnetic fields and B is the impedance matrix given by

$$B = \frac{k_0 k^3 \eta \eta_0}{d k_+^2 k_-^2} \begin{pmatrix} \eta_0^{-2} & i x \eta_0^{-1} \\ -i x_C \eta_0^{-1} & \eta_C^{-2} \end{pmatrix}. \tag{2.3}$$

The field components $H_z h_0$ and E_z satisfy the Helmholtz equation together with Meixner's edge conditions and Sommerfeld's radiation conditions at infinity. As is seen, the components $H_z h_0$ and E_z are coupled through the boundary conditions in (2.2). In order to decouple the problem two auxiliary scalar functions $U_j\left(r,f\right)$, $j=1,2$ are introduced instead of $H_z h_0$ and E_z as was proposed in [Pelet, P. and N. Engheta, January 1990]:

$$\begin{bmatrix} H_z h_0 \\ E_z \end{bmatrix} = Q \begin{bmatrix} U_1^t \\ U_2^t \end{bmatrix} \tag{2.4}$$

where Q is a constant 2×2 matrix to be chosen. It is suggested to take this matrix as the similarity transform which diagonalizes the impedance matrix B [M. A. Lyalinov and A. H. Serbest].

The eigenvalues and the eigenvectors of B can be obtained as

$$l_j = \frac{k_0 k^3 h h_0}{2 d k_+^2 k_-^2}\left(h_0^{-2} + x_c^{-2} - \left(-1\right)^j \delta\right), \tag{2.5}$$

and

$$\vec{q}_j = \left[\frac{2 i x_c h_0^{-1}}{\sqrt{\left(h_c^{-2} - h_0^{-2} - \left(-1\right)^j \delta\right)^2 + 4 x_c^2 h_0^{-2}}}, \frac{\left(h_c^{-2} - h_0^{-2} - \left(-1\right)^j \delta\right)}{\sqrt{\left(h_c^{-2} - h_0^{-2} - \left(-1\right)^j \delta\right)^2 + 4 x_c^2 h_0^{-2}}} \right]^T, \tag{2.6}$$

for j=1,2, where $\delta = \sqrt{\left(h_0^{-2} - h_c^{-2}\right)^2 + 4 x_c^2 h_0^{-2}}$. So, the matrix Q is given as

$$Q = \begin{pmatrix} \dfrac{2 i x_c h_0^{-1}}{\sqrt{\left(h_c^{-2} - h_0^{-2} + \delta\right)^2 + 4 x_c^2 h_0^{-2}}} & \dfrac{2 i x_c h_0^{-1}}{\sqrt{\left(h_c^{-2} - h_0^{-2} - \delta\right)^2 + 4 x_c^2 h_0^{-2}}} \\ \dfrac{\left(h_c^{-2} - h_0^{-2} + \delta\right)}{\sqrt{\left(h_c^{-2} - h_0^{-2} + \delta\right)^2 + 4 x_c^2 h_0^{-2}}} & \dfrac{\left(h_c^{-2} - h_0^{-2} - \delta\right)}{\sqrt{\left(h_c^{-2} - h_0^{-2} - \delta\right)^2 + 4 x_c^2 h_0^{-2}}} \end{pmatrix}. \tag{2.7}$$

When Eq.(2.4) is substituted into Eq.(2.2), the following decoupled boundary conditions are obtained:

$$\begin{bmatrix} \frac{\partial}{\partial_y} U_1^t \\ \frac{\partial}{\partial_y} U_2^t \end{bmatrix}_{y=\pm 0} = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix} \begin{bmatrix} \left[U_1^t\right]_-^+ \\ \left[U_2^t\right]_-^+ \end{bmatrix}, \quad \text{for} \quad x<0 \tag{2.8}$$

So the problem is reduced to the solution of two scalar problems for U_1 and U_2.The incident electromagnetic waves on the chiral slab from Eq.(2.4) are expressed as

$$\begin{bmatrix} U_1^i \\ U_2^i \end{bmatrix} = Q^{-1} \begin{bmatrix} H_z^i h_0 \\ E_z^i \end{bmatrix} \tag{2.9}$$

resulting with

$$\begin{bmatrix} U_1^i \\ U_2^i \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} e^{-ik_0 r\cos(f-f_0)} \tag{2.10}$$

in which

$$T_1 = \frac{\left[i\left(h_c^{-2} - h_0^{-2} - \delta\right)A_1 + 4x_c h_0^{-1} A_2\right]}{\sqrt{\left(h_c^{-2} - h_0^{-2} - \delta\right)^2 + 4x_c^2 h_0^{-2}}} \tag{2.11}$$

and

$$T_2 = \frac{\left[-i\left(h_c^{-2} - h_0^{-2} + \delta\right)A_1 - 4x_c h_0^{-1} A_2\right]}{\sqrt{\left(h_c^{-2} - h_0^{-2} + \delta\right)^2 + 4x_c^2 h_0^{-2}}} \; . \tag{2.12}$$

When U_1 and U_2 are assumed as the scattered field, then the total field is taken as the sum of the incident and scattered field for all y: $U_{1,2}^t = U_{1,2}^i + U_{1,2}$. Thus, the scattered field components will directly satisfy the Helmholtz equation.

By making known mathematical calculations used for solving Helmholtz equation by separation of variables technique, the following equation is obtained:

$$U_1(x,y) = \begin{cases} \int_L A(a) e^{iK(a)y} e^{-iax} da, & y > 0 \\ \int_L B(a) e^{-iK(a)y} e^{-iax} da, & y < 0. \end{cases} \tag{2.13}$$

Here, $K(a) = \sqrt{k_0^2 - a^2}$ is defined in the appropriately cut complex a-plane with $K(0) = +k$ and $K(a) = i|a|$ for $a \to \pm\infty$. The integration line L is any straight line parallel to the real axis in the regularity band, $\operatorname{Im}(-k_0) < \operatorname{Im}(a) < \operatorname{Im}(k_0)$.

The unknown spectral functions $A(a)$ and $B(a)$ could be determined by using the boundary and continuity conditions given for the respective regions of space. In explicit form, the boundary conditions which are given by Eq.(2.8), for x<0 are

$$\left.\frac{\partial}{\partial y} U_1^t\right|_{y=\pm 0} = l_1 \left[U_1^t(x,+0) - U_1^t(x,-0) \right] \tag{2.14}$$

For x>0 and y=0, U_1 is continuous in the region, so the continuity conditions are written as;

$$U_1^t(x,+0) - U_1^t(x,-0) = 0, \qquad x > 0 \tag{2.15}$$

$$\frac{\partial}{\partial y} U_1^t(x,+0) - \frac{\partial}{\partial y} U_1^t(x,-0) = 0, \qquad x > 0 \tag{2.16}$$

and then Eqs.(2.14) and (2.16) yield:

$$\frac{\partial}{\partial y}U_1^t\left(x,+0\right)=\frac{\partial}{\partial y}U_1^t\left(x,-0\right), \qquad x\rightarrow\pm\infty\,. \tag{2.17}$$

By substituting the integral representation given in Eqs.(2.13) and (2.10) into Eqs.(2.14), (2.15) and (2.17), the following system of integral equations are obtained:

$$-T_1 ik_0\sin f_0 e^{-ik_0 x\cos f_0}+\int_L\left[\left(iK(a)-l_1\right)A(a)+l_1 B(a)\right]e^{-iax}da=0\,,\,x<0 \tag{2.18}$$

$$\int_L\left(A\left(a\right)-B\left(a\right)\right)e^{-iax}da=0, \qquad x>0 \tag{2.19}$$

and

$$\int_L\left(A(a)+B(a)\right)iK(a)e^{-iax}da=0, \qquad x\rightarrow\pm\infty \tag{2.20}$$

which yields $A(a)=-B(a)$ and then, when $A(a)$ is substituted into Eqs.(2.18) and (2.19), the equations are reduced to the following integral equations

$$\int_L\left(iK(a)-2l_1\right)A(a)e^{-iax}da=\begin{cases}T_1 ik_0\sin f_0 e^{-ik_0 x\cos f_0}, & x<0\\ g(x), & x>0\end{cases} \tag{2.21}$$

and

$$\int_L 2A(a)e^{-iax}da=\begin{cases}0, & x>0\\ h(x), & x<0.\end{cases} \tag{2.22}$$

Here, $g(x)$ and $h(x)$ are the unknown functions. Thus, application of the inverse Fourier transform to the equations yields:

$$\left(iK(a) - 2l_1\right) A(a) = \frac{1}{2p} T_1 i k_0 \sin f_0 \int_{-\infty}^{0} e^{ix\left(a - k_0 \cos f_0\right)} dx + \Phi^{+}(a) \qquad (2.23)$$

and

$$2A(a) = \Phi^{-}(a)\,. \qquad (2.24)$$

The functions $\Phi^{+}(a)$ and $\Phi^{-}(a)$ are defined as

$$\Phi^{+}(a) = \frac{1}{2p} \int_{0}^{\infty} g(x) e^{iax} dx \qquad \text{and} \qquad \Phi^{-}(a) = \frac{1}{2p} \int_{-\infty}^{0} h(x) e^{iax} dx \qquad (2.25)$$

The regularity band is obtained as $\mathrm{Im}(-k_0) < \mathrm{Im}(a) < \mathrm{Im}(k_0 \cos f_0)$. When Eq.(2.24) is substituted into Eq.(2.23), the following equation is obtained;

$$\left(\frac{iK(a) - 2l_1}{2}\right) \Phi^{-}(a) = \frac{1}{2p} T_1 \frac{k_0 \sin f_0}{\left(a - k_0 \cos f_0\right)} + \Phi^{+}(a)\,. \qquad (2.26)$$

The above equation is written as

$$G(a)\Phi^{-}(a) = F(a) + \Phi^{+}(a)\,, \qquad (2.27)$$

which is defined in the regularity band and named as Wiener- Hopf equation. Here, the source function $F(a)$ and kernel function $G(a)$ are defined as follows:

$$F(a) = \frac{1}{2p} T_1 \frac{k_0 \sin f_0}{\left(a - k_0 \cos f_0\right)} \qquad \text{and} \qquad G(a) = \left(\frac{iK(a) - 2l_1}{2}\right). \qquad (2.28)$$

3. SOLUTION OF WIENER-HOPF EQUATON

The standard procedure will be applied to solve Wiener-Hopf equation which consist of the scalar functional equations (Noble - 1958). For that reason, firstly the function $G(a)$ should be expressed as

$$G(a) = G^{-}(a)G^{+}(a), \tag{3.1}$$

where $G^{+}(a)$ and $G^{-}(a)$ and their inverses are regular functions of a with algebraic behaviour in an upper and lower half of the complex $a-$ plane, respectively.

For the factorization of $G(a)$, the expression of $G(a)$ is rewritten in the following form

$$\left(\frac{iK(a) - 2l_1}{2}\right) = \frac{1}{2} iK(a) 2iC_1 \left(\frac{\dfrac{K(a)}{2iC_1} + k_0}{K(a)}\right) \tag{3.2}$$

where $l_j = C_j k_0$, for $j = 1,2$ with

$$C_j = \frac{k^3 h h_0}{2dk_+^2 k_-^2}\left(h_0^{-2} + x_c^{-2} + d\right), \qquad \text{for } j = 1,2. \tag{3.3}$$

Now, the kernel $G(a)$ will be written terms of a function $H(a)$ as

$$G(a) = \frac{1}{2} iK(a) \frac{1}{b_1} H^{-1}(a) \tag{3.4}$$

with $b_1 = \dfrac{1}{2iC_1}$ and $H(a) = \dfrac{K(a)}{k_0 + b_1 K(a)}$.

The function $H(a)$ can be factorized as $H(a) = H^{+}(a)H^{-}(a)$ in terms of Maliuzhinetz function [Senior-1952, Uzgören ve ark.-1989]. So, Eq.(2.27) becomes

$$i\frac{1}{2b_1}\frac{K^{-}(a)}{H^{-}(a)}\Phi^{-}(a) = \frac{H^{+}(a)}{K^{+}(a)}F(a) + \frac{H^{+}(a)}{K^{+}(a)}\Phi^{+}(a). \tag{3.5}$$

In the above equation, the factors $H^{+}(a)$ and $H^{-}(a)$ of $H(a)$ can be obtained in terms of the Maliuzhinetz function. The following variable changes $a = -k_0 \cos f_0$ and $b_1 = \sec l$ are made with $\arcsin\left(\frac{a}{k_0}\right) = f_0 - \frac{p}{2}$ and $\sqrt{1-b_1^2} = -i\tan l$ which yield the factors as

$$H^{+}\left(-k_0 \cos f_0\right) = \sqrt{\frac{2}{b}}\sin\left(\frac{f_0}{2}\right) f\left(p - f_0 + l\right) f\left(p - f_0 - l\right). \tag{3.6}$$

The function $f(z)$ is defined as

$$f(z) = \frac{2^{3/4}}{1+\sqrt{2}\cos(z/2)}\left[\frac{M_p(z)}{M_p(p/2)}\right]^2 \tag{3.7}$$

with $M_p(z)$ being the Maliuzhinetz function.

Then, the next step is to write the additive split of the function which appear in the right-hand sides of Eq.(3.5). By using the following integral

$$Z^{\pm}(a) = \pm\frac{1}{2p\,i}\int_{C_{U,L}} \frac{Z(z)}{(z-a)}dz\ , \tag{3.8}$$

$$Z^{\pm}(a) = \pm\frac{1}{2p\,i}\int_{C_U,C_L} \frac{1}{2p}T_1\frac{k_0 \sin f_0}{\left(z - k_0 \cos f_0\right)}\frac{H^{+}(z)}{K^{+}(z)}\frac{1}{(z-a)}dz \tag{3.9}$$

is obtained which requires the application of the residue theorem and

$$Z^{+}(a)=\frac{T_1}{2p}\frac{k_0\sin f_0}{\left(a-k_0\cos f_0\right)}\left[\frac{H^{+}(a)}{K^{+}(a)}-\frac{H^{+}(k_0\cos f_0)}{K^{+}(k_0\cos f_0)}\right]. \tag{3.10}$$

Similarly for $Z^{-}\left(a\right)$, expression by residue theorem is

$$Z^{-}(a)=\frac{T_1}{2p}\frac{k_0\sin f_0}{\left(a-k_0\cos f_0\right)}\frac{H^{+}(k_0\cos f_0)}{K^{+}(k_0\cos f_0)}. \tag{3.11}$$

When $Z^{\pm}(a)$ are substituted into Eq.(3.5), the following equation

$$i\frac{1}{2b_1}\frac{K^{-}(a)}{H^{-}(a)}\Phi^{-}(a)-Z^{-}(a)=\frac{H^{+}(a)}{K^{+}(a)}\Phi^{+}(a)+Z^{+}(a) \tag{3.12}$$

is written and it gives

$$q(a)=\begin{cases} i\dfrac{1}{2b_1}\dfrac{K^{-}(a)}{H^{-}(a)}\Phi^{-}(a)-Z^{-}(a); & \operatorname{Im}(a)<\operatorname{Im}(k_0\cos f_0) \\ \dfrac{H^{+}(a)}{K^{+}(a)}\Phi^{+}(a)+Z^{+}(a); & \operatorname{Im}(a)>\operatorname{Im}(-k_0) \end{cases}, \tag{3.13}$$

in which $q(a)$ is a regular function in the complex $a-$plane and it will be determined by using the asymptotic behaviour of the function for $a\to\infty$ that appear in its expressions which correspond to the edge conditions of the field compenents .

From Liouville's theorem, it gives $q(a)=0$ and in this case, from Eq.(3.13);

$$\Phi^{+}(a)=-Z^{+}(a)\frac{K^{+}(a)}{H^{+}(a)} \quad \text{and} \quad \Phi^{-}(a)=-i2b_1Z^{-}(a)\frac{H^{-}(a)}{K^{-}(a)} \tag{3.14}$$

are obtained. By using Eqs.(2.14) and (3.14) the unknown spectral functions are determined as:

$$A(a) = -ib_1 Z^-(a)\frac{H^-(a)}{K^-(a)} \tag{3.15}$$

4. DIFFRACTION COEFFICIENT

The integral expressions given by Eq.(2.13) are used to obtain the diffraction coefficient. These integrals can be evaluated approximately for $k_0 \to \infty$ (far field expressions) by using the steepest descent path method.

Let the following variable change made,

$$x = r\cos f\ ,\ y = r\sin f\ ,\ a = -k_0 \cos t \text{ and } da = k_0 \sin t dt\ . \tag{4.1}$$

The scattered field expression $U_1(r,f)$ for $y > 0$ is

$$U_1(r,f) = ib_1 \frac{T_1}{2p}\sqrt{1-\cos f_0}\int_T \frac{H^+(k_0\cos f_0)H^-(-k_0\cos t)}{\cos t + \cos f_0}$$

$$x\ \sqrt{1-\cos t}e^{ik_0 r\cos(t-f)}dt\ . \tag{4.2}$$

The saddle point of the integrand is $t_s = f$, and by considering only the saddle point contribution, the diffracted field $U_1^d(r,f)$ is calculated as

$$U_1^d(r,f) \approx ib_1 \frac{T_1}{\sqrt{2p}}\frac{1}{\sqrt{k_0 r}}\sqrt{1-\cos f_0}\sqrt{\frac{2p}{k_0 r\left|p''(t_s)\right|}}$$

$$x\ \frac{H^+(k_0\cos f_0)H^-(-k_0\cos t_s)}{\cos t_s + \cos f_0}\sqrt{1-\cos t_s}\,e^{ik_0 r p(t_s) - i\frac{p}{4}} \tag{4.3}$$

resulting with

$$U_1^d(r,f) \approx b_1 \frac{T_1}{\sqrt{2p}} \frac{1}{\sqrt{k_0 r}} \sqrt{1-\cos f_0}\,\sqrt{1-\cos f}$$

$$x\,\frac{H^+(k_0\cos f_0)H^-(-k_0\cos f)}{\cos f+\cos f_0} e^{ik_0 r} e^{i\frac{p}{4}}. \tag{4.4}$$

Following the similar procedure, the diffracted field $U_2^d(r,f)$ for $y>0$ is obtained as;

$$U_2^d(r,f) \approx b_2 \frac{T_2}{\sqrt{2p}} \frac{1}{\sqrt{k_0 r}} \sqrt{1-\cos f_0}\,\sqrt{1-\cos f}$$

$$x\,\frac{H^+(k_0\cos f_0)H^-(-k_0\cos f)}{\cos f+\cos f_0} e^{ik_0 r} e^{i\frac{p}{4}} \tag{4.5}$$

in which $b_2 = \dfrac{1}{2iC_2}$. It is shown that these expressions are also valid for $y<0$.

Then, when $U_1^d(r,f)$ and $U_2^d(r,f)$ are substituted into (2.4), $H_z^d h_0$ and E_z^d can easily obtained, but in practice, the diffraction coefficients $D_H(f_0,f)$ and $D_E(f_0,f)$ are of the most importance and they are defined by the following equations

$$\begin{pmatrix} H_z^d h_0 \\ E_z^d \end{pmatrix} \approx U^i(o) \begin{pmatrix} D_H(f_0,f) \\ D_E(f_0,f) \end{pmatrix} \frac{e^{ik_0 r+ip/4}}{\sqrt{2p\,k_0 r}} \tag{4.6}$$

with

$$\begin{pmatrix} D_H(f_0,f) \\ D_E(f_0,f) \end{pmatrix} = Q \begin{pmatrix} b_1 T_1 \sqrt{1-\cos f_0}\,\sqrt{1-\cos f}\,\dfrac{H^+(k_0\cos f_0)H^-(-k_0\cos f)}{\cos f+\cos f_0} \\ b_2 T_2 \sqrt{1-\cos f_0}\,\sqrt{1-\cos f}\,\dfrac{H^+(k_0\cos f_0)H^-(-k_0\cos f)}{\cos f+\cos f_0} \end{pmatrix}. \tag{4.7}$$

For various parameter values, numerical analysis is performed.

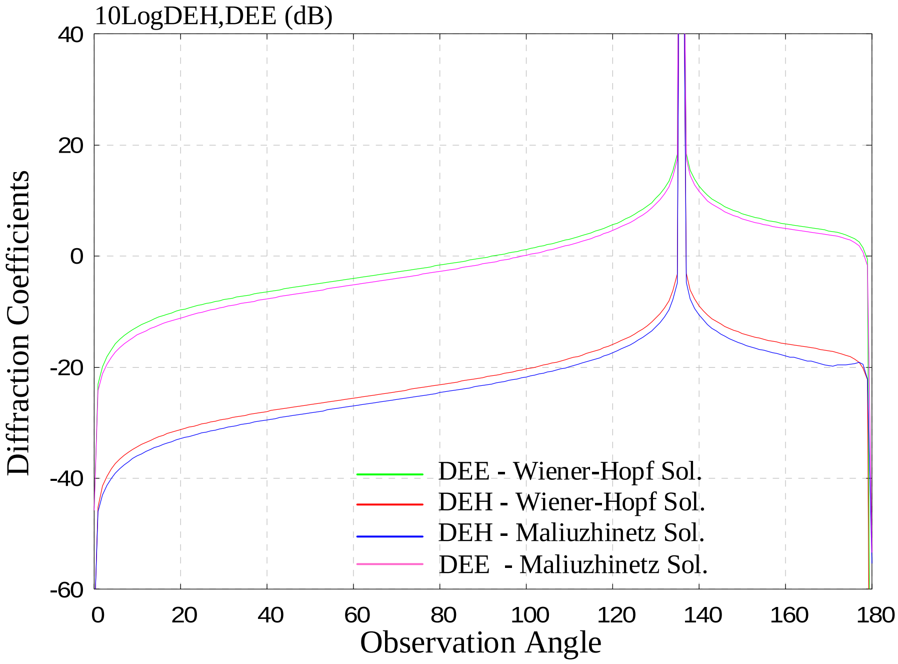

Figure 4.1.Variation of the diffraction coefficient DE together with the co polarized and cross polarized terms for μ_r=1.55+i0.165, x_c =0.35+i0.4, ε_r =6.5+i2.74, f =6GHz, $Ø_0$=45^0, d=1 mm.

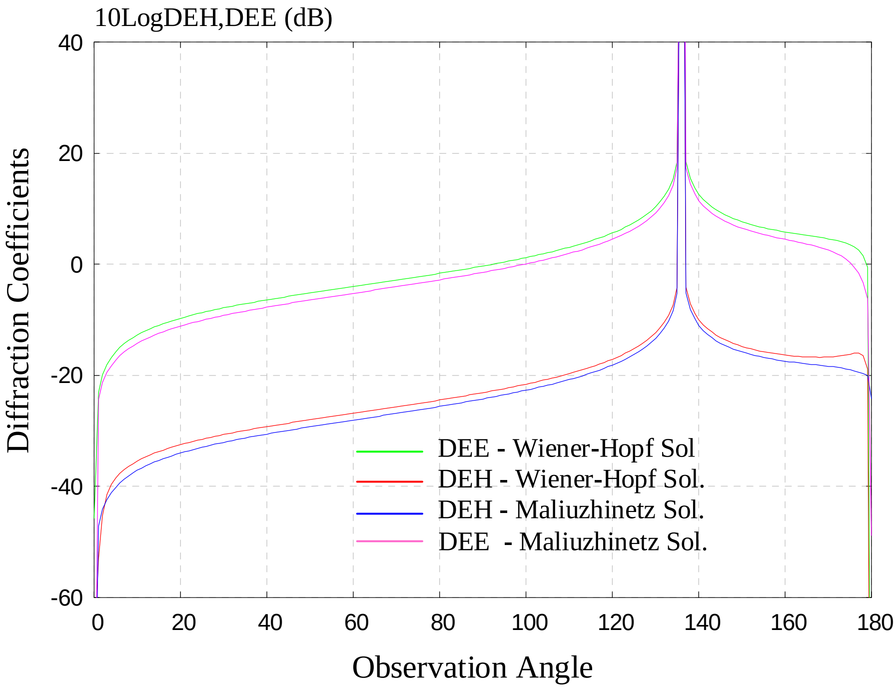

Figure 4.2.Variation of the diffraction coefficient DE together with the co polarized and cross polarized terms for μ_r =0.845+i0.225, x_c =-0.6+i0.5, ε_r=2.5+i2.25,f=9GHz, $Ø_0$=45^0, d=1 mm.

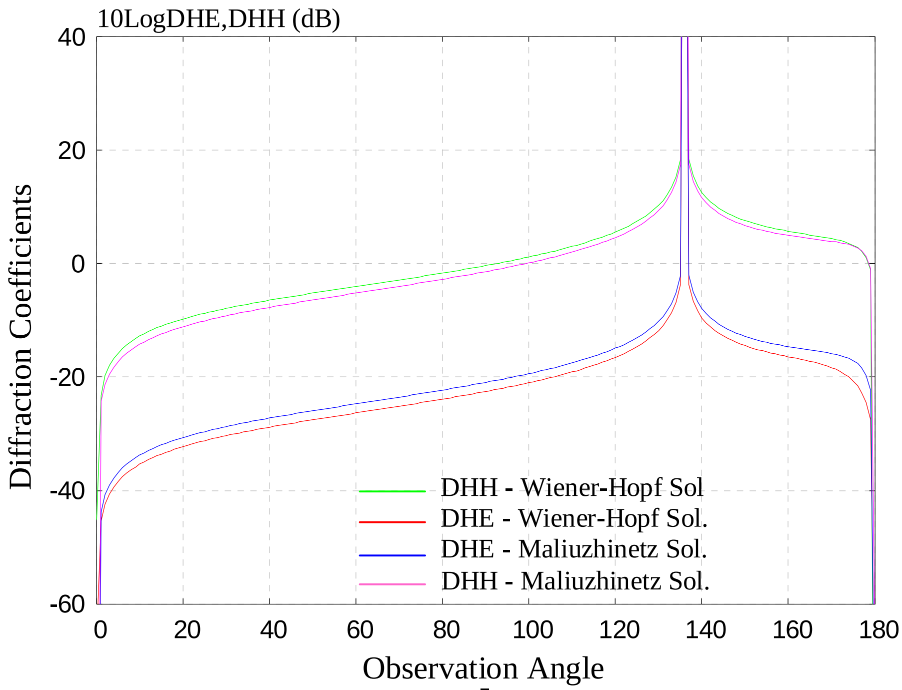

Figure 4.3.Variation of the diffraction coefficient DH together with the co polarized and cross polarized terms for μ_r=1.55+i0.165, x_c =0.35+i0.4, ε_r =6.5+i2.74, f =6GHz, $Ø_0$=45^0, d=1 mm.

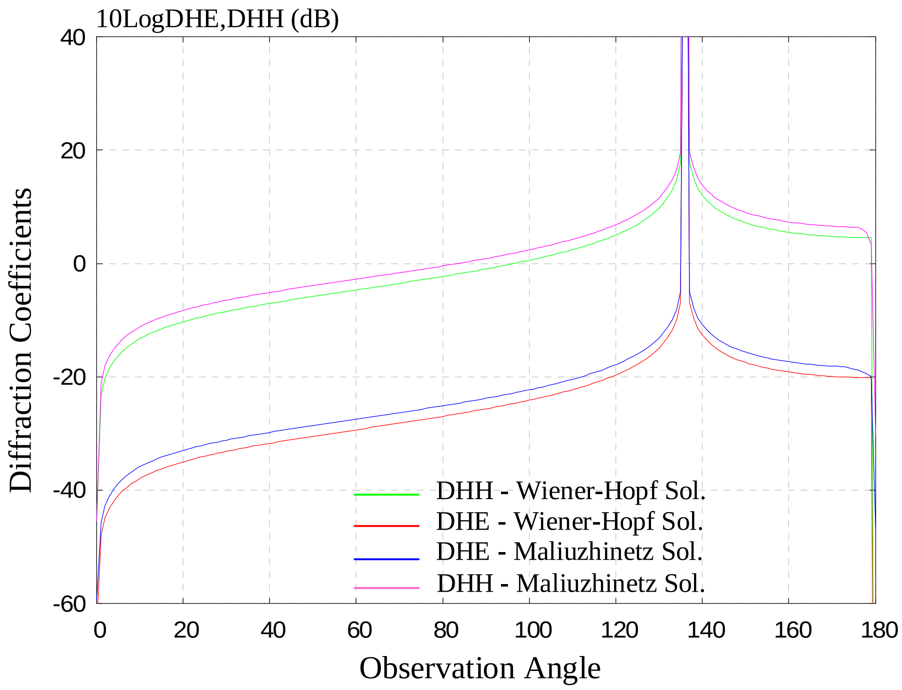

Figure 4.4.Variation of the diffraction coefficient DH together with the co polarized and cross polarized terms for μ_r =0.845+i0.225, x_c =-0.6+i0.5, ε_r=2.5+i2.25,f=9GHz, $Ø_0$=45^0, d=1 mm.

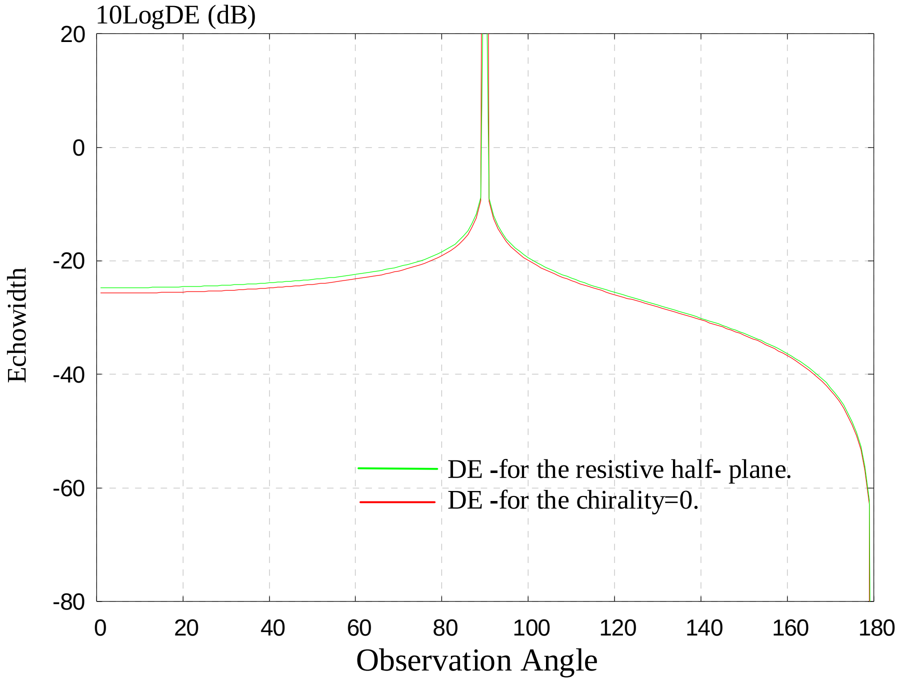

Figure 4.5.Variation of the diffraction coefficient DE obtained when chirality equals to zero together with the diffraction coefficient DE obtained for the resistive half-plane in the literature for μ_r=1.55+i0.165, ε_r =7.4-i1.11, f =6GHz, d=0.02 l .

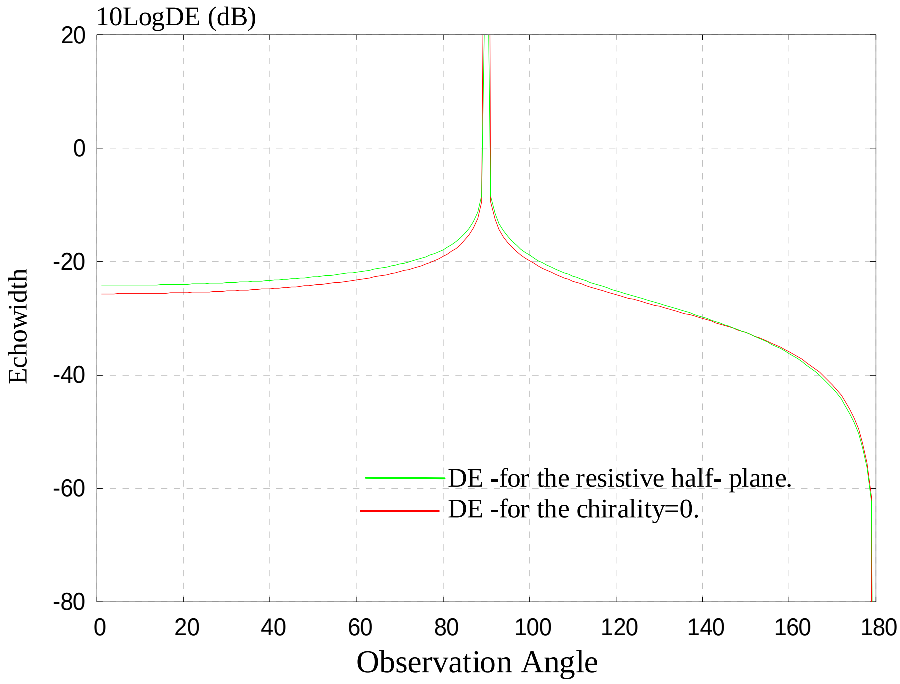

Figure 4.6.Variation of the diffraction coefficient DE obtained when chirality equals to zero together with the diffraction coefficient DE obtained for the resistive half-plane in the literature for μ_r=1.55+i0.165, ε_r =6.5+i2.74, f =6GHz, d=0.02 l .

5. CONCLUSION

In the present thesis, the diffraction of a plane wave by a thin semi-infinite chiral slab is considered by Wiener-Hopf technique and the diffraction coefficient is obtained.

The boundary conditions applied for modeling the isotropic chiral slab can give the depolarization effect of chirality:

$$D_H(f_0,f) = D_{HE} + D_{HH} = \Delta^{-1}\{A_1[q_{11}q_{22}D_1(f_0,f) - q_{12}q_{21}D_2(f_0,f)]$$

$$+A_2[q_{12}q_{11}(D_2(f_0,f) - D_1(f_0,f))]\} \qquad (5.1)$$

and

$$D_E(f_0,f) = D_{EE} + D_{EH} = \Delta^{-1}\{A_1[q_{21}q_{22}(D_1(f_0,f) - D_2(f_0,f))]$$

$$+A_2[q_{11}q_{22}D_2(f_0,f) - q_{12}q_{21}D_1(f_0,f)]\} \qquad (5.2)$$

In the above equations, $D_{1,2}(f_0,f)$ are defined as

$$D_{1,2}(f_0,f) = b_{1,2}\sqrt{1-\cos f_0}\,\sqrt{1-\cos f}\,\frac{H^+(k_0\cos f_0)H^-(-k_0\cos f)}{\cos f + \cos f_0} \qquad (5.3)$$

and Δ is the determinant of the matrix Q. The amplitudes of the auxiliary functions are expressed as $T_1 = \Delta^{-1}[q_{22}A_1 - q_{12}A_2]$ and $T_2 = \Delta^{-1}[-q_{21}A_1 + q_{11}A_2]$. D_{HE} and D_{EH} are the cross-polarized components and D_{HH} and D_{EE} are co-polarized components.

As a verification of the solution, the results obtained by Maliuzhinetz method are used. Fig.(4.1-4.4) illustrate the numerical results obtained by Maliuzhinetz method together with the results obtained by Wiener- Hopf method. As is seen from the figures same results were obtained by these of two methods.

For a second way of verification; we may use the fact that if chirality equals to zero, the half-plane would be reduced to a resistive half-plane. From this point of view, the numerical results obtained when chirality equals to zero are compared with the results for the resistive half-plane in the literature. As is seen from fig.(4.5 and 4.6), the same results have been obtained.

It is hoped that the results given in this thesis will be a guideline for the future academic works related with the chiral objects. Also, it is believed that this thesis would be useful for understanding the behaviour of electromagnetic waves resulting with the interaction of chiral materials or objects.

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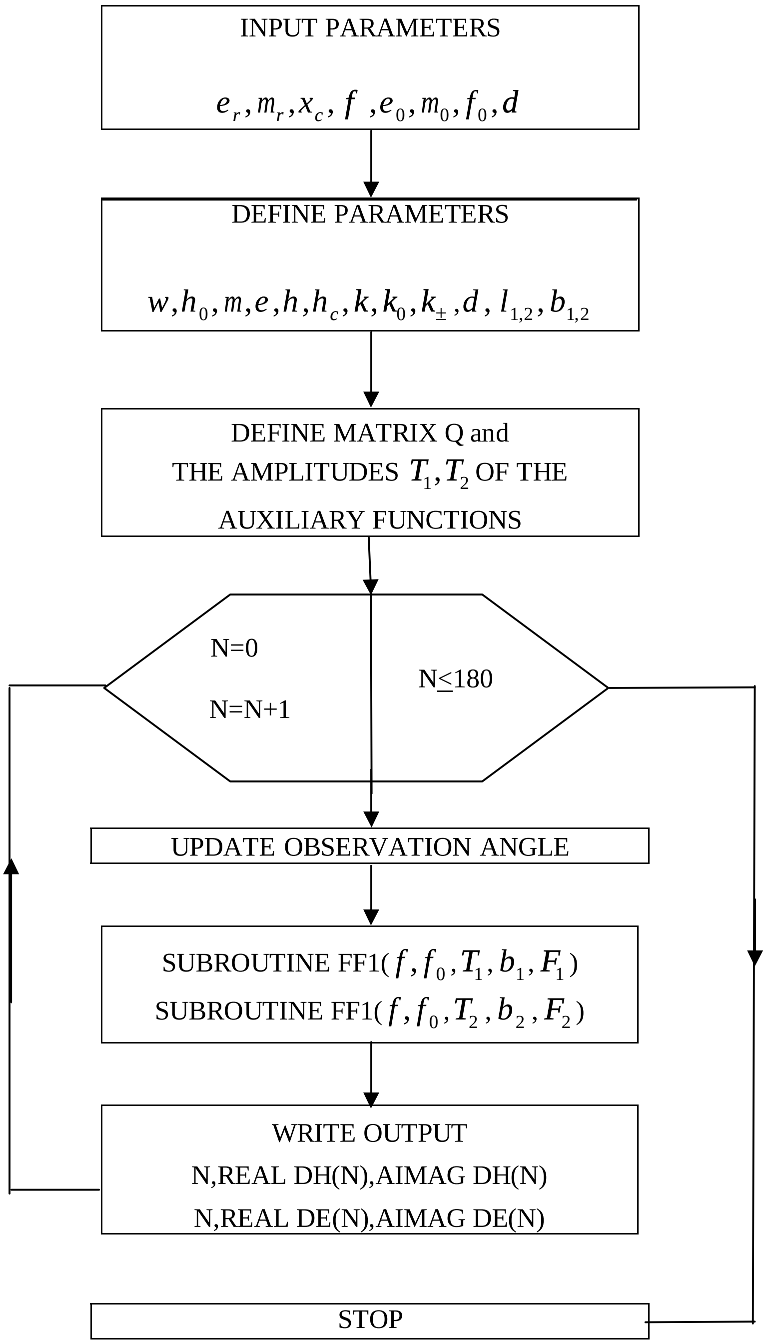
BIOGRAPHY

I was born in Erzurum, Turkey, on April 10, 1977. I graduated from high school in 1994 and entered to Electrical and Elecronics Engineering Department of Fırat University in 1997, Elazığ, Turkey. After I graduated from Electrical and Elecronics Engineering Department in 2001, I worked for a company about for one year. I have been studying towards MS degree student at Electrical and Elecronics Engineering Department of Çukurova University since 2002.

APPENDIX

1. The Flowchart of The Program That Calculates The Diffraction Coefficients



2. The Flowchart of Subroutine FF1 That Calculates Angular Parameters Related to Maliuzhinetz Function

SUBROUTINE FF1

↓

INPUT PARAMETERS
p and I

↓

X1=SQRT(1-COS(f))*SQRT(1- COS(f_0))/(COS(f)+COS(f_0))
SUBROUTINE MALIUZBS(p - f -ARCCOS(1/ b) , ZMPI1)
SUBROUTINE MALIUZBS(p - f +ARCCOS(1/ b) , ZMPI2)

↓

SUBROUTINE MALIUZBS(p /2 , ZMPI3)
ZMPI4=ZMP3^4/(ZMPI1*ZMPI2)^2
X2=COS(f /2)/(2*(1+ b *SIN(f)))

↓

X3=1+SQRT(2)*COS((p - f +ARCCOS(1/ b))/2)
X4=1+SQRT(2)*COS((p - f -ARCCOS(1/ b))/2)
SONUCA=X1*X2*X3*X4*ZMPI4

↓

SUBROUTINE MALIUZBS(p - f_0 -ARCCOS(1/ b) , ZMPI1)
SUBROUTINE MALIUZBS(p - f_0 +ARCCOS(1/ b) , ZMPI2)
SUBROUTINE MALIUZBS(p /2 , ZMPI3)

↓

ZMPI4=ZMP3^4/(ZMPI1*ZMPI2)^2
X2=COS(f_0 /2)/(2*(1+ b *SIN(f_0)))
X3=1+SQRT(2)*COS((p - f_0 +ARCCOS(1/ b))/2)

↓

X4=1+SQRT(2)*COS((p - f_0 -ARCCOS(1/ b))/2)
SONUCB=SONUCA*X2*X3*X4*ZMPI4
SONUCB=SONUCB*EXP(I * p /4)/SQRT(2* p)

↓

FF=SONUCB* T * b ^2

↓

STOP

SUBROUTINE MALIUZBS that the program computes Maliuzhinetz function.