UNIVERSITY OF ÇUKUROVA
INSTITUTE OF NATURAL AND
APPLIED SCIENCES

MSc THESIS

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Diffraction of Plane Waves by a Two-Impedance Wedge in Cold Plasma

DEPARTMENT OF ELECTRICAL AND
ELECTRONICS ENGINEERING

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FEN BİLİMLERİ ENSTİTÜSÜ

DIFFRACTION OF PLANE WAVES BY A TWO-IMPEDANCE WEDGE IN COLD PLASMA

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YÜKSEK LİSANS TEZİ

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ABSTRACT

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A wedge is an important canonical structure in diffraction theory. Practically a two-impedance wedge in a cold plasma may be used in modelling the electromagnetic scattering from a variety of large and complex objects. In this study, the scattered field is represented in the form of a Sommerfeld integral with an unknown spectral function. By using the impedance boundary conditions a nonhomogeneous functional equation is obtained. The solution of the functional equation can be represented in terms of χ-functions and S integrals. After determining the unknown spectral function in the Sommerfeld integral, the Sommerfeld contour is deformed into the steepest descent paths. Diffraction coefficients are also discussed. During the deformation process the geometrical optics wave contributions is found.

Keywords : Electromagnetic wave diffraction, Two-impedance wedge, Cold plasma, Sommerfeld integral, Steepest descent paths

ÖZ

YÜKSEK LİSANS TEZİ

SOĞUK PLAZMA İÇİNDE DÜZLEMSEL DALGALARIN İKİ EMPEDANSLI KAMADAN KIRINIMI

Filiz KARAÖMERLİOĞLU

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Kama, kırınım teorisinde önemli bir kanonik yapıdır. Pratik olarak, soğuk plazma içinde iki-empedanslı kama geniş ve kompleks nesnenin çeşitliliğinden elektromanyetik saçılma modellemesinde kullanılabilir. Bu çalışmada saçılan alan bilinmeyen spektral fonksiyon ile Sommerfeld integral formunda gösterilecektir. Homojen olmayan fonksiyonel eşitlik, empedans sınır koşulları kullanılarak bulunur. Fonksiyonel eşitliğin çözümü, χ fonksiyonları ve S integrallerine dayanarak gösterilecek ve Sommerfeld integralinde bilinmeyen spektral fonksiyon belirlendikten sonra, Sommerfeld konturu en dik iniş çizgisi içinde deforme edilecektir. Deformasyon işleminde geometrik optik dalgaları ve yüzey dalgalarının katkıları bulunarak. numerik hesaplamalar yapılacak ve difraksiyon katsayısının çeşitli parametreler için yorumu yapılacaktır.

Anahtar Kelimeler : Elektromanyetik dalga kırınımı, iki empedanslı kama, soğuk plasma, Sommerfeld integrali, Semer noktası.

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NOTATIONS

E	Electric Field intensity
H	Magnetic Field intensity
H_{dc}	DC magnetic field vector
$\hat{\varepsilon}$	Tensor of dielectric permittivity of cold plasma
ω	Operating frequency
ω_c	Cyclotron frequency
ω_p	Plasma frequency
Z_0	The free space impedance
$Z_{1,2}$	The relative surface impedances
χ	Kappa function
$s(\beta)$	The unknown Spectral function
$H_z^t(r,\varphi)$	Total magnetic field
$H_z(r,\varphi)$	Diffracted field
$D(\varphi)$	Diffraction coefficient

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1. INTRODUCTION

The ionosphere is the region of charged particles surrounding earth that is created by the ultraviolet radiation from the sun. The ionosphere begins at an altitude of about 90 km and continues upwards into space. Variations in the composition and density of the ionosphere with altitude produce regions where different physical processes are important.

The ionosphere is also a plasma. A plasma is composed of a collection of discrete ionized particles, but not every collection of charged particles qualifies as a plasma. In order to a collection of charged particles to termed a plasma, certain criteria must be met. Over large length scales, the medium must be electrically neutral.

Earth's ionosphere is divided into several regions designated by the letters D, E, F. These regions may be further divided into several regularly occurring layers, such as F1 and F2. D is below about 90 km, E is between 90 and 150 km, and F is above 150km.

Now the full set of equations to describe plasmas is composed of the above equation and Maxwell's equations:

$$\text{div}\ \vec{E}(\vec{r},t) = \rho/\varepsilon_0 \tag{1.1}$$

$$\text{div}\ \vec{B}(\vec{r},t) = 0 \tag{1.2}$$

$$\text{curl}\ \vec{E}(\vec{r},t) = \partial\vec{B}/\partial t \tag{1.3}$$

$$\text{curl}\ \vec{B}(\vec{r},t) = \mu_0\vec{J} + \mu_0\varepsilon_0\,\partial\vec{E}/\partial t \tag{1.4}$$

where μ_0 is the permeability of free space.

The relation between magnetic flux density and magnetic field intensity for the case with earth's magnetic field effect is as follows,

$$\vec{B}(\vec{r},t) = \vec{B_0} + \mathrm{Re}\left\{\vec{B}(\vec{r})e^{j\omega t}\right\} = \mu_v \vec{H}(\vec{r},t) \tag{1.5}$$

and the equation of motion is given as,

$$e\vec{E} = j\omega m(1+j\frac{v}{m})\vec{v} + e\vec{B_0}\times\vec{v} \tag{1.6}$$

with v denotes the collision frequency.

By arranging the equations appropriately, the permittivity of the plasma may be obtained in dyadic form as,

$$\nabla\times\vec{H}(\vec{r}) = j\omega\varepsilon_v\overline{K}\vec{E}(\vec{r}) \tag{1.7}$$

in which

$$\overline{K} = K'\hat{x}\hat{x}+jK''\hat{x}\hat{y}-jK''\hat{y}\hat{x}+K'\hat{y}\hat{y}+K_0\hat{z}\hat{z} \tag{1.8}$$

where $\hat{x},\hat{y}$ and $\hat{z}$ denote the unit vectors in the Cartesian coordinates system. Gauss's Law may also be rewritten using the relative permittivity dyadic $\overline{K}$. In the matrix form, this equation can be written as,

$$\begin{pmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix}\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = j\omega\varepsilon_v\begin{pmatrix} K' & jK'' & 0 \\ -jK'' & K' & 0 \\ 0 & 0 & K_0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{1.9}$$

with K', K" and K_0 being certain constants dependent on the constitutive parameters of the medium and the operating frequency.

The displacement density (electric flux density) $\vec{D}$ given by

$$\vec{D} = \varepsilon_v \overline{\overline{K}} \vec{E} \qquad (1.10)$$

is not parallel to $\vec{E}$, indicating that the magnetized plasma is an anisotropic medium.

The interactions of electromagnetic fields with materials are characterized by the constitutive parameters: complex dielectric constant ε and permeability μ. In an isotropic medium, the property of the material does not depend on the direction of electric or magnetic field polarizations. Thus ε and μ are scalar quantities.

In anisotropic media, however, the material characteristics depend on the direction of the electric or magnetic field vectors. Thus in general, the displacement vector $\vec{D}$ and the magnetic flux density vector $\vec{B}$ are not necessarily in the same direction as the electric field $\vec{E}$ and the magnetic field vector $\vec{H}$, respectively. Therefore, the dielectric constant ε must then be represented by a tensor ε_{ij}:

$$D_i = \sum_{j=1}^{3} \varepsilon_{ij} E_j, \qquad i = 1,2,3, \qquad (1.11)$$

where $i, j = 1,2,$ and 3 denote the x,y, and z components, respectively. It is written in the following form:

$$\vec{D} = \hat{\varepsilon} \vec{E} \qquad (1.12)$$

Using matrix notation in the rectangular system, eq. (1.1) is expressed by

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{1.13}$$

Similarly, the tensor permeability $\hat{\mu}$ is related to $\vec{B}$ to $\vec{H}$,

$$\vec{B} = \hat{\mu}\vec{H} \tag{1.14}$$

As will be shown shortly, in general, the reciprocity theorem does not hold for anisotropic media, and for a plane wave, $\vec{E}$ and $\vec{H}$ are not necessarily transverse to the direction of wave propagation.

In the magnetoionic theory, it is assumed that the motions of electrons are caused by the Lorentz force. In fact, the electrons are also effected by the pressure variations and the finite temperature of the electron gas. If the small effect of the finite temperature is ignored as in the case of magnetoionic theory, the plasma is called "cold plasma". The presence of a DC magnetic field makes the plasma anisotropic; therefore due to the earth magnetic field, the ionosphere is an anisotropic medium. In an anisotropic medium, the relationship between the electric and the magnetic field vectors for a cold plasma with a tensor permittivity $\hat{\varepsilon}$, is given as

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{1.15}$$

where

$$\varepsilon_1 = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \left[1 - \left(\frac{\omega_c}{\omega}\right)^2\right]^{-1} \tag{1.16}$$

$$\varepsilon_2 = \left(\frac{\omega_p}{\omega}\right)^2 \left[\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega}\right]^{-1} \tag{1.17}$$

and

$$\varepsilon_3 = 1 - \left(\frac{\omega_c}{\omega}\right)^2 \tag{1.18}$$

where ω is the operating frequency. The plasma frequency ω_p, is defined in terms of the number of free electrons per unit volume n, the charge of the electron e, and the mass of the electron m, as

$$\omega_p = \left(4\pi n e^2/m\right)^{1/2} \tag{1.19}$$

The cyclotron frequency ω_c is the frequency of circular motion of an electron in a plane perpendicular to dc magnetic field H_{dc}, and it is given by

$$\omega_c = |e|\mu_0 H_{dc}/m \tag{1.20}$$

where μ_0 is the permeability of vacuum.

A dc magnetic field is often present in plasma. The presence of a dc magnetic field makes the plasma anisotropic. The equation of motion for an electron in electromagnetic fields $\left(\vec{E},\vec{H}\right)$ in the presence of a dc magnetic field $\vec{H}_{dc}$ is given by

$$m\frac{d\vec{v}}{dt}=e\vec{E}+e\left[\vec{v}\times\left(\vec{B}+\vec{B}_{dc}\right)\right]-m\nu\vec{v} \tag{1.21}$$

where $\vec{B}=\mu_0\vec{H}$ and $\vec{B}_{dc}=\mu_0\vec{H}_{dc}$.

Since $\vec{B}=\mu_0\vec{H}$ and $\left|\vec{H}\right|$ is on the order of $\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2}\left|\vec{E}\right|$ and $\left|\vec{B}\right|$ is in the order of $\left(\frac{1}{c}\right)\left|\vec{E}\right|$ so assume that, $\left|\vec{v}\right|\langle\langle c$. So the term with $\vec{B}$ is negligible compared with $e\vec{E}$.

So, for a time-harmonic electromagnetic field with time dependence $\exp(j\omega t)$, neglecting the term with $\vec{B}$,

$$j\omega m\vec{v}=e\vec{E}+\mu_0 e\left(\vec{v}\times\vec{H}_{dc}\right)-m\nu\vec{v} \tag{1.22}$$

The polarization vector $\vec{P}$ is given by

$$\vec{P}=N_0 e\vec{r} \tag{1.23}$$

$$[P]=\varepsilon_0[\chi_e][E] \tag{1.24}$$

where the relative tensor dielectric constant $[\varepsilon_r]$ is given in terms of $[\chi_e]$ as,

$$[\varepsilon_r]=[1]+[\chi_e] \tag{1.25}$$

and

$$[\chi_e] = -\frac{X}{U(U^2 - Y^2)} \begin{pmatrix} U^2 - Y_x^2 & -jY_zU - Y_xY_y & jY_yU - Y_xY_z \\ jY_zU - Y_xY_y & U^2 - Y_y^2 & -jY_xU - Y_yY_z \\ -jY_yU - Y_xY_z & jY_xU - Y_yY_z & U^2 - Y_z^2 \end{pmatrix} \tag{1.26}$$

where $Y^2 = Y_x^2 + Y_y^2 + Y_z^2 = \omega_c^2/\omega^2$, $X = \omega_p^2/\omega^2$, $U = 1 - j(\nu/\omega)$ and $\vec{Y} = e\mu_0 \vec{H}_{dc}/m\omega$.

For example, the z axis along the direction of propagation $\vec{k}$ and the dc magnetic field $\vec{H}_{dc}$:

$$\vec{k} = k\hat{z},\ \vec{H}_{dc} = H_{dc}\hat{z} \tag{1.27}$$

Noting that $Y_z = Y$, $Y_x = Y_y = 0$, the tensor electric susceptibility $[\chi_e]$:

$$[\chi_e] = -\frac{X}{U(U^2 - Y^2)} \begin{pmatrix} U^2 & -jYU & 0 \\ jYU & U^2 & 0 \\ 0 & 0 & U^2 - Y^2 \end{pmatrix} \tag{1.28}$$

Thus, the tensor relative dielectric constant $[\varepsilon_r]$ is given by

$$[\varepsilon_r] = \begin{pmatrix} \varepsilon & ja & 0 \\ -ja & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \tag{1.29}$$

where

$$\varepsilon = 1 - \frac{XU}{U^2 - Y^2} = 1 - \frac{(\omega_p\ \omega)^2 \left[1 - j(v\ \omega)\right]}{\left[1 - j(v\ \omega)\right]^2 - (\omega_c\ \omega)^2}, \tag{1.30}$$

$$a = \frac{XY}{U^2 - Y^2} = -\frac{(\omega_p\ \omega)^2 (\omega_c\ \omega)}{\left[1 - j(v\ \omega)\right]^2 - (\omega_c\ \omega)^2}, \tag{1.31}$$

$$\varepsilon_z = 1 - \frac{X}{U} = 1 - \frac{(\omega_p\ \omega)^2}{1 - j(v\ \omega)}. \tag{1.32}$$

Rapid development of new materials and their applications in industry it necessary to study the diffraction of electromagnetic waves by bodies of complex shape with impedance surfaces. Owing to the wide use of the high frequency diffraction theory in engineering practice, it is important to study the canonical problems.

As is known, there is still interest for the impedance wedge problem with various physical properties of the wedge and the surrounding medium. Two-impedance wedge in cold plasma problem has not been studied till now. Therefore, it is believed that the results of this study have practical importance due to the effect of the ionosphere in communication between various systems.

2. REVIEW OF LITERATURE

The generalized problem with impedance boundary conditions on the faces was solved by G.D. Malyuzhinets (1950), generalization of the reflection method in the theory of diffraction of sinusoidal waves; and described in a series of classic papers: the radiation of sound by the vibrating boundaries of an arbitrary wedge (1955); radiation of sound from the vibrating faces of an arbitrary wedge (1955); inversion formula for the Sommerfeld integral (1958); concluding in the short solution outlined in his 1958 paper: excitation, reflection and emission of surface waves from a wedge with given face impedances. His solution was understood in the form of a Sommerfeld integral with an integrand involving a new special function $\Psi_{\Phi}(z)$. Malyuzhinets later gave a short review of the method in 1958.

Zavadskii and Sakharova (1961) developed the first numerical procedures for computing the Maliuzhinets special function and deduced some useful analytical representations. Corresponding solutions relevant to the boundary conditions on the wedge faces are presented in more detail in this studies; A.A. Tuzhilin (1963) solved new representations of diffraction fields in wedge-shaped regions with ideal boundaries. Application of the special function $\Psi_{\Phi}(z)$ in problems of wave diffraction in wedge-shaped regions (1967). Diffraction from a wedge is a well covered topic, dating back over a century; J.J. Bowman and T.B.A. Senior (1969) discussed electromagnetic and acoustic scattering by simple shapes.

Further contributions to the Maliuzhinets theory were made by Tuzhilin who developed a theory of related functional equations (1970). On asymptotic expansion of certain functions occurring in the wedge diffraction theory (1970). P. Ambaud and A. Bergassoli (1972) discussed the problem of the wedge in acoustics. Tuzhilin demonstrated the possibility of extending the Maliuzhinets approach to more sophisticated boundary conditions (1973). L.B. Felsen and N. Marcuvitz (1978) discussed radiation and scattering of waves.

Maliuzhinets' solution, having the form of a Sommerfeld integral, is perfectly suited for the purpose of subsequent far-field analysis. Budaev and Petrashen' (1983) found series representations of the field diffracted by a wedge with equal face impedance, and Osipov (1991) deduced the series solution for the general case of arbitrary face impedances.

Numerical calculations of the function $\Psi_{\pi}(\alpha)$, relevant to a screen, were discussed by Volakis and Senior (1985). Osipov calculated of the Maliuzhinets function in a complex region.

Many papers have appeared dealing with both electromagnetic and acoustic applications. For instance: plane wave scattering from an impedance half-plane; radiation of a line source at the tip of an absorbing wedge; Green's functions; edge waves, as: M.P. Zavadskaya and R.P. Starovoitova (1988) discussed edge waves in G.D. Maliuzhinets' solution of an impedance wedge; diffraction of surface; plane, cylindrical and transient scalar waves by an impedance wedge of arbitrary angle, for example: G.L. James (1977) studied uniform diffraction coefficients for an impedance wedge; T. Griesser and C.A. Balanis (1989) solved reflection, diffraction and surface waves for an interior impedance wedge of arbitrary angle; R. Tibero, G. Pelosi and G. Manara (1985) solved a uniform GTD formulation for the diffraction by a wedge with impedance faces.

Roberto G. Rojas (1988) studied electromagnetic diffraction of an obliquely incident plane wave field by a wedge with impedance faces. A uniform asymptotic solution is presented for the electromagnetic diffraction by a wedge with impedance faces and with included half plane (angle equal to 0), right-angled wedge ($\pi/2$ and $3\pi/2$), and two-part plane (π).

A.V. Kostyuk (1997) calculated of the Malyuzhinets function in the problem of diffraction of electromagnetic waves by a wedge with impedance faces. A numerically efficient method is offered for approximating the Malyuzhinets function.

G.Pelosi, S. Selleri and R.D. Graglia (1997) solved numerical analysis of the diffraction at an anisotropic impedance wedge. A numerical approach to the electromagnetic scattering of a plane wave impinging at skew incidence on the edge of a wedge with two different anisotropic face impedances is presented. The solution supplied is valid in the high-frequency region.

G. Pelosi, G. Manara and P. Nepa (1998) discussed electromagnetic scattering by a wedge with anisotropic impedance faces. Electromagnetic scattering from the edge of an anisotropic impedance wedge, explained at oblique incidence, is addressed. Both numerical and analytical techniques, suitable to properly account for the scattering properties of the wedge's anisotropic impedance faces, are considered.

G. Pelosi, G. Manara and P. Nepa (1998) solved UTD solution for the scattering by a wedge with anisotropic impedance faces: Skew incidence case. Asymptotic expressions for the fields scattered by an anisotropic impedance wedge at oblique incidence are derived in the context of the uniform geometrical theory of diffraction (UTD).

J.M.L. Bernard (1998) solved new class of canonical cases for the diffraction by an anisotropic impedance wedge. The diffraction of a skew incident plane wave by a passive anisotropic wedge of any angle is studied. The incident plane wave is characterized by the z-components of the magnetic and electric fields. The z-components of the field in cylindrical coordinates are searched in the form of Sommerfield-Maliuzhinets integrals.

M. A. Lyalinov and N. Y. Zhu (1999) solved diffraction of a normally incident plane wave at a wedge with identical tensor impedance faces. Diffraction of a normally incident plane wave by a wedge with identical tensor impedance faces is studied.

A.V. Osipov and A.N. Norris (1999) solved Malyuzhinets theory for scattering from wedge boundaries: A review. The Malyuzhinets technique is reviewed based on his fundamental papers of the 1950s. A detailed derivation of the well-known Malyuzhinets expressions for the wave field diffracted by an impedance wedge is presented.

M. A. Lyalinov and N. Y. Zhu (1999) solved diffraction of a skewly incident plane wave by an anisotropic impedance wedge – a class of exactly solvable cases. The Sommerfeld-Malyuzhinets' technique and the special function χ_Φ have been used to find the exact solution for the diffraction of a skewly incident.

G. Manara and P. Nepa (2000) analyzed electromagnetic diffraction of an obliquely incident plane wave by a right-angled anisotropic impedance wedge with a perfectly conducting face. The proposed solution procedure applies both to the exterior and the interior right-angled wedges. The rigorous spectral solution for the field components parallel to the edge is determined through the application of the Sommerfeld-Maliuzhinets technique. A uniform asymptotic solution is provided in the framework of the uniform geometrical theory of diffraction (UTD).

3. FORMULATION OF THE PROBLEM

In the problem under consideration, a wedge shaped region with relative surface impedances Z_1 and Z_2 and wedges opening angle Φ is placed in cylindrical coordinate system $(r,\ \phi,\ z)$ such as the edge of the wedge coincides with z-axis. The wedge is uniform along the z-axis, so the problem can be reduced to a two dimensional problem. The spaces between the surface of the wedge is filled by an homogeneous cold plasma which is subjected to a constant magnetic field of the strength $\overrightarrow{H_0}$ in z direction, and it is assumed that the time dependence of the incident fields is $\exp(j\omega t)$.

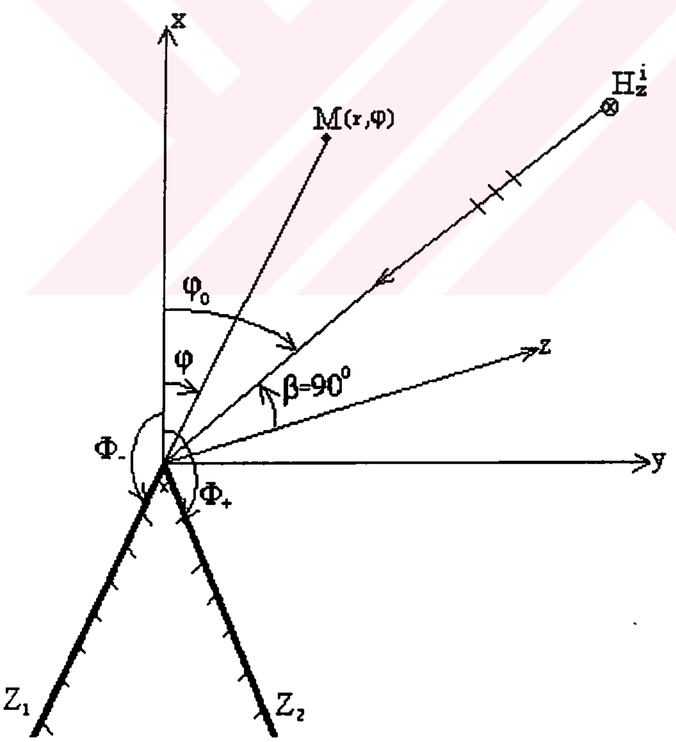


Figure 3.1. The geometry of the problem

Consider that $|\varphi| \le \Phi,\ \dfrac{\pi}{2} \le \Phi \le \pi,\ 0 \le r \le \infty$.

The incident magnetic field vector is,

$$\overline{H_z^i} = \overline{e_z} C_0 \exp\left[-ikr\cos(\varphi-\varphi_0)\right] \tag{3.1}$$

where C_0 is the amplitude of the incident magnetic field, and it will be considered as unity during the formulation.

With respect to (3.1), the electric and magnetic field components are,

$$\vec{E}(E_r, E_\varphi, 0) \qquad \vec{H}(0,0,H_z). \tag{3.2}$$

It is well known that Maxwells' equations are valid in plasma with the tensor dielectric permittivity. By using Maxwell's equation along with (1.24) the electric field components can be expressed in terms of the total magnetic field H_z, as follows:

$$\nabla\times\overline{E} = i\omega\mu_0\overline{H} \tag{3.3}$$

and

$$\nabla\times\overline{H} = -i\omega\varepsilon_0\hat{\varepsilon}\overline{E} \tag{3.4}$$

where $\vec{J} = \sigma\overline{E} = 0$ since the conductivity σ of the medium is assumed to be zero. We can obtain the expression of all field components in terms of H_z.

From (3.3) we have

$$\frac{1}{r}\left[\frac{\partial(rE_\varphi)}{\partial r} - \frac{\partial(E_r)}{\partial\varphi}\right]\overline{e_z} = i\omega\mu_0 H_z\overline{e_z} \tag{3.5}$$

and from (3.4) we have

$$\frac{1}{r}\left[\frac{\partial H_z}{\partial\varphi}\vec{e_r} - r\frac{\partial H_z}{\partial r}\vec{e_\varphi}\right] = -i\omega\varepsilon_0\hat{\varepsilon}\vec{E}. \tag{3.6}$$

By substituting (1.24) into(3.6)

$$\frac{1}{r}\left[\frac{\partial H_z}{\partial\varphi}\vec{e_r} - r\frac{\partial H_z}{\partial r}\vec{e_\varphi}\right] = -i\omega\varepsilon_0\begin{pmatrix} \varepsilon_1\overrightarrow{e_r e_r} & -i\varepsilon_2\overrightarrow{e_r e_\varphi} & 0 \\ i\varepsilon_2\overrightarrow{e_\varphi e_r} & \varepsilon_1\overrightarrow{e_\varphi e_\varphi} & 0 \\ 0 & 0 & \varepsilon_3\overrightarrow{e_z e_z} \end{pmatrix}\begin{pmatrix} E_r\vec{e_r} \\ E_\varphi\vec{e_\varphi} \\ 0 \end{pmatrix} \tag{3.7}$$

is obtained, and by rearranging the last equation one can get that;

$$\frac{1}{r}\left[\frac{\partial H_z}{\partial\varphi}\vec{e_r} - r\frac{\partial H_z}{\partial r}\vec{e_\varphi}\right] = -i\omega\varepsilon_0\left[\left(\varepsilon_1 E_r - i\varepsilon_2 E_\varphi\right)\vec{e_r} + \left(i\varepsilon_2 E_r + \varepsilon_1 E_\varphi\right)\vec{e_\varphi}\right]. \tag{3.8}$$

According to this equation the components of electric field vector are obtained in terms of the derivatives of magnetic field vector,

$$\frac{1}{r}\frac{\partial H_z}{\partial\varphi} = -i\omega\varepsilon_0\left(\varepsilon_1 E_r - i\varepsilon_2 E_\varphi\right) \tag{3.9}$$

and

$$-\frac{\partial H_z}{\partial r} = -i\omega\varepsilon_0\left(i\varepsilon_2 E_r + \varepsilon_1 E_\varphi\right). \tag{3.10}$$

From the last two equations by eliminating E_φ and E_r respectively, we can obtain that,

$$E_r = \frac{i\varepsilon_1}{\omega\varepsilon_0\left(\varepsilon_1^2-\varepsilon_2^2\right)}\frac{1}{r}\frac{\partial H_z}{\partial\varphi}+\frac{\varepsilon_2}{\omega\varepsilon_0\left(\varepsilon_1^2-\varepsilon_2^2\right)}\frac{\partial H_z}{\partial r} \quad (3.11)$$

and

$$E_\varphi = \frac{\varepsilon_2}{\omega\varepsilon_0\left(\varepsilon_1^2-\varepsilon_2^2\right)}\frac{1}{r}\frac{\partial H_z}{\partial\varphi}-\frac{i\varepsilon_1}{\omega\varepsilon_0\left(\varepsilon_1^2-\varepsilon_2^2\right)}\frac{\partial H_z}{\partial r}. \quad (3.12)$$

It is obvious that, the only unknown function is the z-component of the total magnetic field vector or the scattered component of the magnetic field, and after determining H_z the other field component may be obtained via the last two equations.

3.1. Application of the Boundary Conditions

On the wedge's faces the Leontovich impedance boundary condition are given as, (Senior T.B.A. and Volakis J.L., 1995)

$$\vec{E}-(\hat{n}.\vec{E})\hat{n} = Z_0 Z_{1,2}\left[\hat{n}\times\vec{H}\right], \quad \text{for } \varphi=\pm\Phi \quad (3.13)$$

where Z_1 and Z_2 are the relative surface impedances, and $Z_0=\sqrt{\mu_0/\varepsilon_0}$ is the free space impedance.

The electric and magnetic fields are,

$$\vec{E}=\vec{a}_r E_r+\vec{a}_\varphi E_\varphi\ , \quad \vec{H}=\vec{a}_z H_z . \quad (3.14)$$

Using Leontovich boundary condition, with $\varphi = \Phi$, and $\hat{n} = -\hat{a}_\varphi$, the electric and magnetic fields are, as follows;

$$\hat{n} \times \vec{H} = -\hat{a}_r H_z \tag{3.15}$$

and

$$\hat{n}.\vec{E} = -E_\varphi . \tag{3.16}$$

If the last two equations are substituted into (3.13), the electric field component is,

$$E_r = -Z_0 Z_1 H_z \quad \text{at } \varphi = \Phi . \tag{3.17}$$

Similarly, for $\varphi = -\Phi$, and $\hat{n} = \hat{a}_\Phi$, the electric and magnetic fields are,

$$\hat{n} \times \vec{H} = \hat{a}_r H_z \tag{3.18}$$

and

$$\hat{n}.\vec{E} = E_\varphi . \tag{3.19}$$

By substituting (3.18) and (3.19) into (3.13), the electric field component,

$$E_r = +Z_0 Z_2 H_z \quad \text{at } \varphi = -\Phi \tag{3.20}$$

is obtained for the geometry of the considered problem. The total and scattered magnetic field satisfies the Helmholtz equation,

$$\nabla^2 H_z + k^2 H_z = 0 \tag{3.21}$$

where the propagation constant is $k = k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, and $\varepsilon = \varepsilon_1^2 - \varepsilon_2^2$. By substituting (3.11) into (3.17) and (3.20), we can get that,

$$\frac{i\varepsilon_1}{\omega\varepsilon_0\varepsilon}\frac{1}{r}\frac{\partial H_z}{\partial\varphi} + \frac{\varepsilon_2}{\omega\varepsilon_0\varepsilon}\frac{\partial H_z}{\partial r} = \mp Z_0 Z_{1,2} H_z \quad \text{at} \quad \varphi = \pm\Phi \tag{3.22}$$

is obtained. If the both sides of the last equation is multiplied by $\left(-i\varepsilon_0\omega\sqrt{\varepsilon}\right)$ the following equation is derived by rearranging the coefficients,

$$\frac{\varepsilon_1}{\sqrt{\varepsilon}}\frac{1}{r}\frac{\partial H_z}{\partial\varphi} + \frac{i\varepsilon_2}{\sqrt{\varepsilon}}\frac{\partial H_z}{\partial r} = \pm ik\sqrt{\varepsilon}Z_{1,2}H_z \quad \text{at} \quad \varphi = \pm\Phi. \tag{3.23}$$

Or by considering the new constants, $\cos\theta = \frac{\varepsilon_1}{\sqrt{\varepsilon}}$, $\sin\theta = \frac{i\varepsilon_2}{\sqrt{\varepsilon}}$, and $\sin\sigma_{1,2} = \sqrt{\varepsilon}Z_{1.2}$, the boundary condition is reduced to the following form;

$$\cos\theta\frac{1}{r}\frac{\partial H_z}{\partial\varphi} - \sin\theta\frac{\partial H_z}{\partial r} \mp ik\sin\sigma_{1,2}H_z = 0 \quad \text{at} \quad \varphi = \pm\Phi. \tag{3.24}$$

If the dc components of the magnetic field is zero, from (1.20) ω_c, and from (1.17) ε_2 will take the value of zero. Finally from the definition of θ, it is obvious that θ will have the same value as ε_2. If one substitutes these values into (3.23), the standard impedance boundary condition can derived easily as follows;

$$\frac{1}{r}\frac{\partial H_z}{\partial\varphi} \mp ik\sqrt{\varepsilon}Z_{1,2}H_z = 0 \tag{3.25}$$

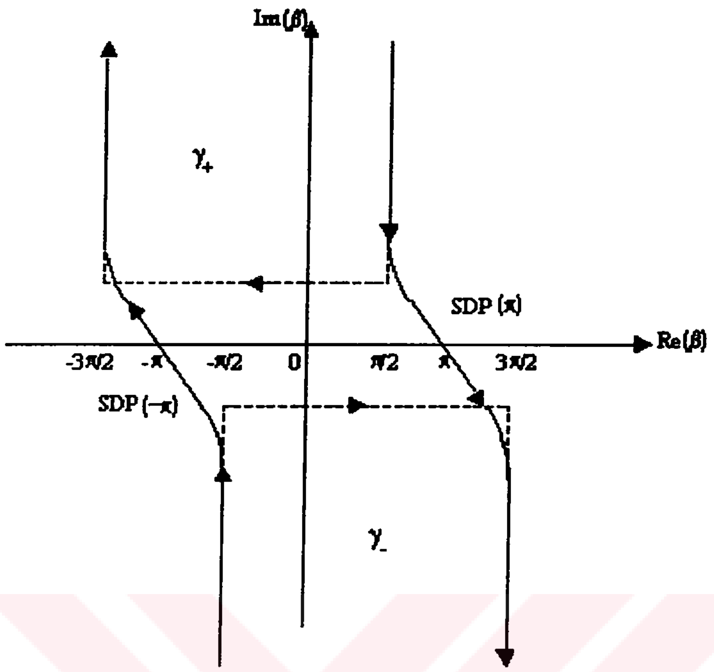


Figure 3.2. The plane for the complex variable β with Sommerfeld double loops γ and the steepest descent paths SDP(-π) and SDP(π).

For an unknown function which satisfies Helmholtz equation, the solution may be represented in terms of Sommerfeld integral (SI),

$$H_z = \frac{1}{2\pi i}\int_{\gamma} s(\beta) e^{ikr\cos(\beta-\varphi)} d\beta \tag{3.26}$$

where $s(\beta)$ is the unknown spectral function which will be determined and γ is the contour in complex β-plane, as given in fig.(3.2.). By changing the variable of the integration from β to $(\beta+\varphi)$, Sommerfeld integral is rewritten as,

$$H_z = \frac{1}{2\pi i}\int_{\gamma} s(\beta+\varphi) e^{ikr\cos\beta} d\beta . \tag{3.27}$$

Knowing the solution of H_z in terms of SI enables us to determine its derivatives in terms of SI also.

Taking the derivative of the magnetic field H_z with respect to r,

$$\frac{\partial H_z}{\partial r} = -\frac{k}{2\pi}\int_{\gamma} s(\beta+\varphi)\cos\beta e^{-ikr\cos\beta}d\beta \tag{3.28}$$

and with respect to φ,

$$\frac{\partial H_z}{\partial \varphi} = -\frac{1}{2\pi i}\int_{\gamma}\frac{\partial}{\partial\varphi}\left[s(\beta+\varphi)\right]\cos\beta e^{-ikr\cos\beta}d\beta \tag{3.29}$$

It is clear that,

$$\frac{\partial}{\partial\varphi}s(\beta+\varphi) = \frac{\partial}{\partial\beta}s(\beta+\varphi). \tag{3.30}$$

and using the partial integration into (3.29) is shown below;

$$u = e^{-ikr\cos\beta}\;, \quad v' = \frac{\partial}{\partial\beta}\left[s(\beta+\varphi)\right]. \tag{3.31}$$

The partial integral of H_z according to φ is rewritten as,

$$\frac{\partial H_z}{\partial\varphi} = \frac{s(\beta+\varphi)}{2\pi i}e^{-ikr\cos\beta}\Big|_{\gamma} - \frac{kr}{2\pi}\int_{\gamma}s(\beta+\varphi)\sin\beta e^{-ikr\cos\beta}d\beta \tag{3.32}$$

Because the Sommerfeld integral has double loops, the first term of (3.32) goes to zero. Because the spectral function is an odd function,

$$s(i\infty) = -s(-i\infty). \tag{3.33}$$

The result of this integral is,

$$\frac{\partial H_z}{\partial \varphi} = -\frac{kr}{2\pi}\int_{\gamma} s(\beta+\varphi)\sin\beta e^{-ikr\cos\beta}d\beta\,. \qquad (3.34)$$

By substituting (3.27), (3.28), and (3.34) into (3.24) to get,

$$\int_{\gamma}\left[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1\right]s(\beta+\Phi)e^{-ikr\cos\beta}d\beta=0 \text{ for } \varphi=+\Phi \qquad (3.35)$$

and

$$\int_{\gamma}\left[\cos\theta\sin\beta-\sin\theta\cos\beta-\sin\sigma_2\right]s(\beta-\Phi)e^{-ikr\cos\beta}d\beta=0 \text{ for } \varphi=-\Phi\,. \qquad (3.36)$$

3.2. Reduction of Boundary Conditions to a Set of Functional Equations

Following the procedure due to Malyuzhinets' theorem,

$$\frac{1}{2\pi i}\int_{\gamma} f(\beta)e^{-ikr\cos\beta}d\beta=0\,. \qquad (3.37)$$

Let $f(\beta)$ be a regular function inside the loop γ_+, and γ_- everywhere, besides the possible exception of infinity, and satisfy the estimate,

$$f(\beta)=\mathrm{O}\left(\exp\left[(n+1-a)\left|\mathrm{Im}\,\beta\right|\right]\right), \qquad \left|\mathrm{Im}\,\beta\right|\to\infty \qquad (3.38)$$

where $0\langle a\langle 1$ and $n\geq 0$ is an integer then,

$$f(\beta)=f_e(\beta)+\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta \tag{3.39}$$

where $f_e(\beta)$ is an arbitrary even function, and C_γ is an arbitrary constant. As a corollary, $f(\beta)$ should satisfy the functional equation of,

$$f(\beta)-f(-\beta)=\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta \tag{3.40}$$

where $f_0(\beta)=\left[f(\beta)-f(-\beta)\right]/2$ is the odd part of $f(\beta)$.

Let in (3.35),

$$f(\beta)=\left[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1\right]s(\beta+\Phi) \tag{3.41}$$

and let in (3.36),

$$f(\beta)=\left[\cos\theta\sin\beta-\sin\theta\cos\beta-\sin\sigma_2\right]s(\beta-\Phi). \tag{3.42}$$

So, for (3.41),

$$\begin{aligned}f(\beta)-f(-\beta)&=\left[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1\right]s(\beta+\Phi)\\&\quad-\left[\cos\theta\sin(-\beta)-\sin\theta\cos(-\beta)+\sin\sigma_1\right]s(-\beta+\Phi)\\&=\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta\end{aligned} \tag{3.43}$$

and for (3.42),

$$\begin{aligned} f(\beta)-f(-\beta) &= [\cos\theta\sin\beta-\sin\theta\cos\beta-\sin\sigma_2]s(\beta-\Phi) \\ &\quad -[\cos\theta\sin(-\beta)-\sin\theta\cos(-\beta)-\sin\sigma_2]s(-\beta-\Phi) \\ &= \sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta \end{aligned} \tag{3.44}$$

By using trigonometric identities,

$$\begin{aligned} &\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B \\ &\sin(-A)=-\sin A \end{aligned} \tag{3.45}$$

into (3.43),

$$\begin{aligned} [\sin(\beta-\theta)+\sin\sigma_1]s(\beta+\Phi) &= -[\sin(\beta+\theta)-\sin\sigma_1]s(-\beta+\Phi) \\ &\quad +\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta \end{aligned} \tag{3.46}$$

and into (3.44),

$$\begin{aligned} [\sin(\beta-\theta)-\sin\sigma_2]s(\beta-\Phi) &= -[\sin(\beta+\theta)+\sin\sigma_2]s(-\beta-\Phi) \\ &\quad +\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta \end{aligned} \tag{3.47}$$

From (3.46), the inhomogeneous functional equations given below are obtained:

$$\begin{aligned} &[\sin(\beta-\theta)+\sin\sigma_1]s(\beta+\Phi) \\ &\quad -[-\sin(\beta+\theta)+\sin\sigma_1]s(-\beta+\Phi)=C_1\sin\beta \end{aligned} \tag{3.48}$$

and from (3.47),

$$\begin{aligned}&\left[\sin(\beta-\theta)-\sin\sigma_2\right]s(\beta-\Phi)\\&\qquad-\left[-\sin(\beta+\theta)-\sin\sigma_2\right]s(-\beta-\Phi)=C_2\sin\beta\end{aligned}\tag{3.49}$$

3.3. Solution of the Functional Equation

Due to the symmetry of the Sommerfeld double loops $s(\beta)$ can be replaced by $s(\beta)+C_j/2$ without changing the value of the Sommerfeld integrals. By this change nonhomogeneous functional equations are reduced to the homogeneous functional equations. (Osipov A.V. and Norris A.N., 1999)

For (3.43).

$$\begin{aligned}s(\beta)-s(-\beta)&=\left[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1\right]\left[s(\beta+\Phi)+C_j/2\right]\\&\quad-\left[\cos\theta\sin(-\beta)-\sin\theta\cos(-\beta)+\sin\sigma_1\right]\left[s(-\beta+\Phi)+C_j/2\right]\\&=\sin\beta\sum_{\gamma=1}^{n}C_\gamma\cos^{\gamma-1}\beta\end{aligned}\tag{3.50}$$

and with some simple calculation, (3.50) is rewritten as,

$$\begin{aligned}&\left[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1\right]s(\beta+\Phi)\\&\qquad-\left[\cos\theta\sin(-\beta)-\sin\theta\cos(-\beta)+\sin\sigma_1\right]s(-\beta+\Phi)\\&\qquad+\left[\cos\theta\sin\beta\, C_j/2-\sin\theta\cos\beta\, C_j/2+\sin\sigma_1\, C_j/2\right]\\&\qquad-\left[\cos\theta\sin(-\beta)C_j/2-\sin\theta\cos(-\beta)C_j/2+\sin\sigma_1\, C_j/2\right]\\&\qquad=\sin\beta\cos\theta C_j\end{aligned}\tag{3.51}$$

Using $\cos(-A)=\cos A$, and $\sin(-A)=-\sin A$, same terms are ignored so the result of nonhomogeneous functional equations is,

$$\begin{aligned}&[\cos\theta\sin\beta-\sin\theta\cos\beta+\sin\sigma_1]s(\beta+\Phi)\\&\qquad-[\cos\theta\sin(-\beta)-\sin\theta\cos(-\beta)+\sin\sigma_1]s(-\beta+\Phi)=0.\end{aligned}\tag{3.52}$$

and using some trigonometric identities, (3.52) is rewritten as follows,

$$\begin{aligned}&[\sin(\beta-\theta)+\sin\sigma_1]s(\beta+\Phi)\\&\qquad-[-\sin(\beta+\theta)+\sin\sigma_1]s(-\beta+\Phi)=0\end{aligned}\tag{3.53}$$

Let us assume that $s_0(\beta)$ it is the solution of the homogeneous equation, which satisfies

$$\begin{aligned}&[\sin(\beta-\theta)\pm\sin\sigma_{1,2}]s_0(\beta\pm\Phi)\\&\qquad-[-\sin(\beta+\theta)\pm\sin\sigma_{1,2}]s_0(-\beta\pm\Phi)=0\end{aligned}\tag{3.54}$$

By changing the parameters, it is useful to take a close look at (3.54), a generalization of Maliuzhinets' functional equation is as follows:

Let $\beta\to(\beta+\Phi)$,

$$[\sin(\beta+\Phi-\theta)+\sin\sigma_1]s_0(\beta+2\Phi)=[-\sin(\beta+\Phi+\theta)+\sin\sigma_1]s_0(-\beta)\tag{3.55}$$

and let $\beta\to(\beta-\Phi)$,

$$[\sin(\beta-\Phi-\theta)-\sin\sigma_2]s_0(\beta-2\Phi)=[-\sin(\beta-\Phi+\theta)-\sin\sigma_2]s_0(-\beta)\tag{3.56}$$

The ratio of these two functions, (3.55) and (3.56) is,

$$\frac{\left[\sin(\beta+\Phi-\theta)+\sin\sigma_1\right]s_0(\beta+2\Phi)}{\left[\sin(\beta-\Phi-\theta)-\sin\sigma_2\right]s_0(\beta-2\Phi)}=\frac{\left[-\sin(\beta+\Phi+\theta)+\sin\sigma_1\right]s_0(-\beta)}{\left[-\sin(\beta-\Phi+\theta)-\sin\sigma_2\right]s_0(-\beta)} \tag{3.57}$$

and by using the following trigonometric identities,

$$\begin{aligned}\sin A+\sin B&=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)\\ \sin A-\sin B&=2\sin\frac{1}{2}(A-B)\cos\frac{1}{2}(A+B)\end{aligned} \tag{3.58}$$

(3.57) is written as.

$$\begin{aligned}\frac{s_0(\beta+2\Phi)}{s_0(\beta-2\Phi)}=&\frac{\sin\frac{1}{2}(\beta-\Phi-\theta-\sigma_2)\cos\frac{1}{2}(\beta-\Phi-\theta+\sigma_2)}{\sin\frac{1}{2}(\beta+\Phi-\theta+\sigma_1)\cos\frac{1}{2}(\beta+\Phi-\theta-\sigma_1)}\\ &\frac{\sin\frac{1}{2}(\beta+\Phi+\theta-\sigma_1)\cos\frac{1}{2}(\beta+\Phi+\theta+\sigma_1)}{\sin\frac{1}{2}(\beta-\Phi+\theta+\sigma_2)\cos\frac{1}{2}(\beta-\Phi+\theta-\sigma_2)}\end{aligned} \tag{3.59}$$

Since.

$$\begin{aligned}\sin A&=\cos\left(A-\frac{\pi}{2}\right)\\ -\sin A&=\cos\left(A+\frac{\pi}{2}\right)\end{aligned} \tag{3.60}$$

Using (3.60) into (3.59), it is obtained that,

$$\frac{s_0(\beta+2\Phi)}{s_0(\beta-2\Phi)}=\frac{\cos\left(\dfrac{\beta-\Phi-\theta-\sigma_2+\pi}{2}\right)\cos\left(\dfrac{\beta-\Phi-\theta+\sigma_2}{2}\right)}{\cos\left(\dfrac{\beta+\Phi-\theta+\sigma_1-\pi}{2}\right)\cos\left(\dfrac{\beta+\Phi-\theta-\sigma_1}{2}\right)}\,\frac{\cos\left(\dfrac{\beta+\Phi+\theta-\sigma_1+\pi}{2}\right)\cos\left(\dfrac{\beta+\Phi+\theta+\sigma_1}{2}\right)}{\cos\left(\dfrac{\beta-\Phi+\theta+\sigma_2-\pi}{2}\right)\cos\left(\dfrac{\beta-\Phi+\theta-\sigma_2}{2}\right)} \tag{3.61}$$

Each cosine function can be expressed in terms of χ_Φ functions such as,

$$\cos\left(\frac{z}{2}\right)=\frac{\chi_\Phi(z+2\Phi)}{\chi_\Phi(z-2\Phi)} \tag{3.62}$$

So (3.61) can be written as,

$$\frac{s_0(\beta+2\Phi)}{s_0(\beta-2\Phi)}=\frac{\dfrac{\chi_\Phi(\beta+\Phi-\theta-\sigma_2+\pi)}{\chi_\Phi(\beta-3\Phi-\theta-\sigma_2+\pi)}\,\dfrac{\chi_\Phi(\beta+\Phi-\theta+\sigma_2)}{\chi_\Phi(\beta-3\Phi-\theta+\sigma_2)}}{\dfrac{\chi_\Phi(\beta+3\Phi-\theta+\sigma_1-\pi)}{\chi_\Phi(\beta-\Phi-\theta+\sigma_1-\pi)}\,\dfrac{\chi_\Phi(\beta+3\Phi-\theta-\sigma_1)}{\chi_\Phi(\beta-\Phi-\theta-\sigma_1)}}\,\frac{\dfrac{\chi_\Phi(\beta+3\Phi+\theta-\sigma_1+\pi)}{\chi_\Phi(\beta-\Phi+\theta-\sigma_1+\pi)}\,\dfrac{\chi_\Phi(\beta+3\Phi+\theta+\sigma_1)}{\chi_\Phi(\beta-\Phi+\theta+\sigma_1)}}{\dfrac{\chi_\Phi(\beta+\Phi+\theta+\sigma_2-\pi)}{\chi_\Phi(\beta-3\Phi+\theta+\sigma_2-\pi)}\,\dfrac{\chi_\Phi(\beta+\Phi+\theta-\sigma_2)}{\chi_\Phi(\beta-3\Phi+\theta-\sigma_2)}} \tag{3.63}$$

By substituting $\beta \to (\beta \mp 2\Phi)$ into the (3.63) to obtain the expression of $s_0(\beta)$, the functions appear in numerator and the denominator of these two equations. Therefore the expression of $s_0(\beta)$ is determined easily as,

$$s_0(\beta)=\frac{\chi_\Phi(\beta+\Phi+\theta-\sigma_1+\pi)\chi_\Phi(\beta+\Phi+\theta+\sigma_1)}{\chi_\Phi(\beta+\Phi-\theta+\sigma_1-\pi)\chi_\Phi(\beta+\Phi-\theta-\sigma_1)} \frac{\chi_\Phi(\beta-\Phi-\theta-\sigma_2+\pi)\chi_\Phi(\beta-\Phi-\theta+\sigma_2)}{\chi_\Phi(\beta-\Phi+\theta+\sigma_2-\pi)\chi_\Phi(\beta-\Phi+\theta-\sigma_2)} \tag{3.64}$$

The solution of the given problem must satisfy the additional condition,

$$s(\beta)-\frac{1}{\beta-\varphi_0} \quad \text{is regular in} \quad |\mathrm{Re}\,\beta|\langle\varphi. \tag{3.65}$$

To get the solution which satisfies the additional condition, a new unknown spectral functions $\xi(\beta)$ are now introduced

$$s(\beta)=s_0(\beta)\sigma_{\varphi_0}(\beta)\xi(\beta) \tag{3.66}$$

where $\sigma_{\varphi_0}(\beta)$ is defined as (3.55) and (3.56),

$$\sigma_{\varphi_0}(\beta)=\frac{\mu\cos(\mu\varphi_0)}{\sin(\mu\beta)-\sin(\mu\varphi_0)} \tag{3.67}$$

where $\mu=\dfrac{\pi}{2\varphi}$ for $0<\varphi\leq\pi$ and satisfies the following relation,

$$\sigma_{\varphi_0}(\beta\pm\Phi)=\sigma_{\varphi_0}(-\beta\pm\Phi). \tag{3.68}$$

Clearly the known functions $\left(s_0(\beta)\sigma_{\varphi_0}(\beta)\right)$ satisfy (3.55) and (3.56) as $s_0(\beta)$.

Then $\xi(\beta)$ obeys the simple functional equations

$$\xi(\beta\pm\Phi)-\xi(-\beta\pm\Phi)=0 \qquad (3.69)$$

From the relationship

$$\operatorname{Res} s_0(\beta)\sigma_{\varphi_0}(\beta)\xi(\beta)\Big|_{\beta=\varphi_0}=1 \qquad (3.70)$$

where $\operatorname{Res} g(\beta)\big|_{\beta=\varphi_0}$ stands for the residue of a function $g(\beta)$ at a point φ_0, it follows that

$$\xi(\beta)=\frac{1}{s_0(\varphi_0)} \qquad (3.71)$$

So the spectral functions are given by

$$s(\beta)=\frac{s_0(\beta)\sigma_{\varphi_0}(\beta)}{s_0(\varphi_0)} \qquad (3.72)$$

4. ANALYSIS OF THE FIELD

For the asymptotic analysis it is assumed that $|\Phi| \geq \pi/2$. The other cases can be treated analogously.

For r>0 the Sommerfeld contour can be deformed to the steepest descent paths SDP(-π) and SDP(π) fig.(3.2). During this process, poles of distinct physical nature may be crossed.

With this in mind, the solution can be written as,

$$H_z^t(r,\varphi) = 2\pi i \sum_n \mathrm{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right] + \frac{1}{2\pi i}\int_{SDP(-\pi)-SDP(\pi)} s(\beta+\varphi)e^{-ikr\cos\beta}d\beta \quad (4.1)$$

In (4.1) the residues correspond to the geometrical optics (incident and reflected waves) and surface waves.

$$2\pi i \sum_n \mathrm{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right] = H_z^{go} + H_z^{sw} = H_z^i + H_z^r + H_z^{sw} \quad (4.2)$$

The second term in (4.1) corresponds to cylindrical waves which are diffracted from the edge of the wedge and it can be rewritten as,

$$H_z(r,\varphi) = \frac{e^{ikr-i\pi\,4}}{\sqrt{2\pi kr}}\left[s(\varphi-\pi)-s(\varphi+\pi)\right] = \frac{e^{ikr}}{\sqrt{kr}}D(\varphi). \quad (4.3)$$

The diffracted field $H_z(r,\varphi)$ which is expressed by the integral of (4.1) can be evaluated asymptotically with the method of steepest descend

$$D(\varphi)=\frac{e^{-i\pi/4}}{\sqrt{2\pi}}\left[s(\varphi-\pi)-s(\varphi+\pi)\right]. \tag{4.4}$$

The poles of the surface waves are due to the homogeneous solution of the functional equation $s_0(\beta)$.

The poles β_n of the geometrical optics waves are due to $\sigma_{\varphi_0}(\beta)$. A spectral function is,

$$s(\beta_n+\varphi)=s_0(\beta_n+\varphi)\sigma_{\varphi_0}(\beta_n+\varphi)\xi(\beta_n+\varphi) \tag{4.5}$$

and

$$\sigma_{\varphi_0}(\beta_n+\varphi)=\frac{\mu\cos(\mu\varphi_0)}{\sin\mu(\beta_n+\varphi)-\sin(\mu\varphi_0)}. \tag{4.6}$$

Since the poles of $\sigma_{\varphi_0}(\beta_n+\varphi)$ is at

$$\beta_n=-\varphi+(-1)^n\varphi_0+2n\Phi \tag{4.7}$$

with $n=-1,\ 0,\ 1$. The residue at these points,

$$\begin{aligned}\operatorname{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right]=\lim_{\beta_n\to-\varphi+(-1)^n\varphi_0+2n\Phi}&\Big\{\left(\beta_n+\varphi-(-1)^n\varphi_0-2n\Phi\right)\\&\times s_0(\beta_n+\varphi)\sigma_{\varphi_0}(\beta_n+\varphi)\xi(\beta_n+\varphi)e^{-ikr\cos\beta_n}\Big\}\end{aligned} \tag{4.8}$$

so,

$$\operatorname{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right]=s_0\left((-1)^n\varphi_0+2n\Phi\right)$$
$$\times\xi\left((-1)^n\varphi_0+2n\Phi\right)e^{-ikr\cos\left(-\varphi+(-1)^n\varphi_0+2n\Phi\right)} \qquad .(4.9)$$
$$\times\lim_{\beta_n\to-\varphi+(-1)^n\varphi_0+2n\Phi}\left\{\frac{\mu\cos(\mu\varphi_0)}{\sin\mu(\beta_n+\varphi)-\sin(\mu\varphi_0)}\left(\beta_n+\varphi-(-1)^n\varphi_0-2n\Phi\right)\right\}$$

By using the L'Hospital rule the limit is found as,

$$\frac{\mu\cos(\mu\varphi_0)}{\mu\cos\mu\left((-1)^n\varphi_0+2n\Phi\right)}=(-1)^n \qquad (4.10)$$

and the residues are,

$$\operatorname{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right]=(-1)^n s_0\left((-1)^n\varphi_0+2n\Phi\right)$$
$$\times\xi\left((-1)^n\varphi_0+2n\Phi\right)e^{-ikr\cos\left(-\varphi+(-1)^n\varphi_0+2n\Phi\right)} \qquad (4.11)$$

So,

$$\operatorname{Res}\left[s(\beta_n+\varphi)e^{-ikr\cos\beta_n}\right]=\frac{(-1)^n s_0\left((-1)^n\varphi_0+2n\Phi\right)}{s_0\left((-1)^n\varphi_0\right)}e^{-ikr\cos\left(-\varphi+(-1)^n\varphi_0+2n\Phi\right)} \qquad .(4.12)$$

The geometrical optics fields $H_z^{go}(r,\varphi)$ are given below,

$$H_z^{go}(r,\varphi)=\frac{(-1)^n s_0\left((-1)^n\varphi_0+2n\Phi\right)}{s_0\left((-1)^n\varphi_0\right)}e^{-ikr\cos\left(-\varphi+(-1)^n\varphi_0+2n\Phi\right)} \qquad (4.13)$$

so, (4.13) is rewritten as,

$$H_z^{go}(r,\varphi)=\begin{cases} e^{-ikr\cos(-\varphi+\varphi_0)}, & \beta_0=-\varphi+\varphi_0 \\ -\dfrac{s_0(-\varphi_0+2\Phi)}{s_0(-\varphi_0)}e^{-ikr\cos(-\varphi-\varphi_0+2\Phi)}, & \beta_1=-\varphi-\varphi_0+2\Phi \\ -\dfrac{s_0(-\varphi_0-2\Phi)}{s_0(-\varphi_0)}e^{-ikr\cos(-\varphi-\varphi_0-2\Phi)}, & \beta_{-1}=-\varphi-\varphi_0-2\Phi \end{cases} \quad (4.14)$$

5. NUMERICAL RESULTS

$10\log_{10}|D(\phi,\phi)|$

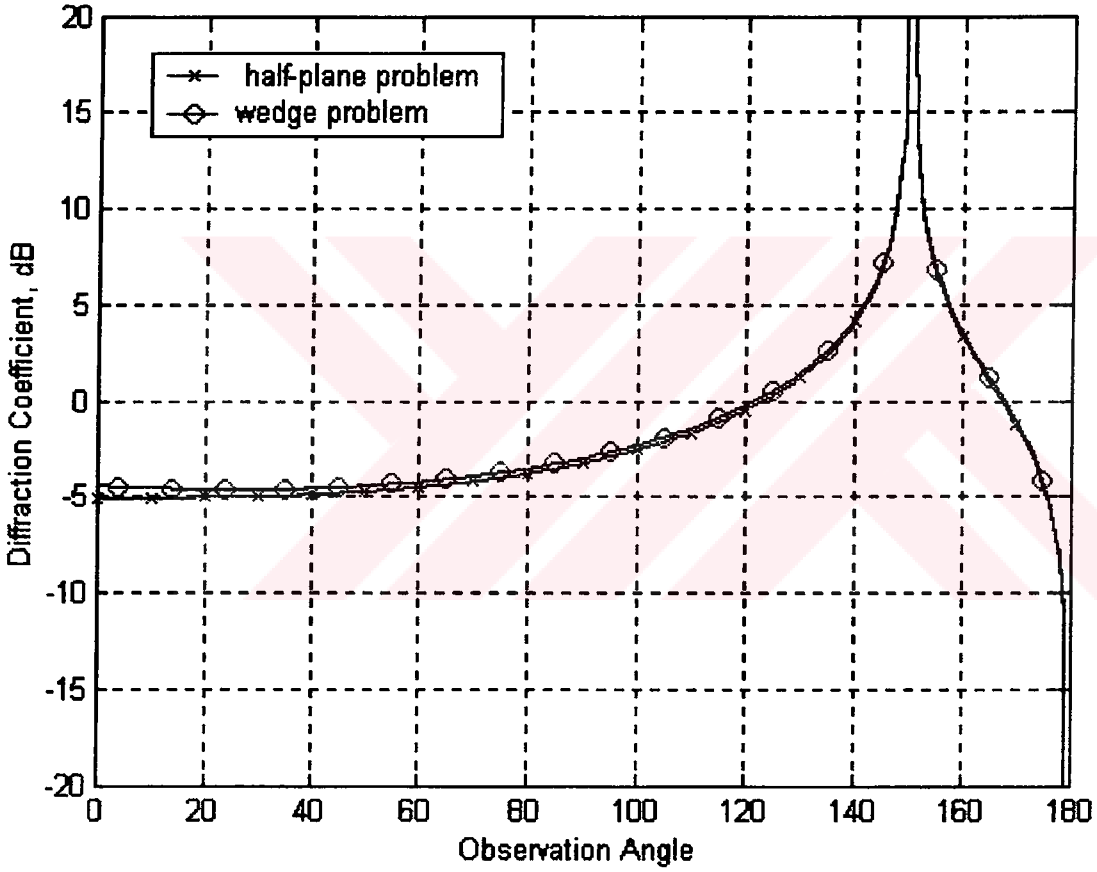


Figure 5.1. Comparison the results with Yener's results, $10\log_{10}|D(\phi_0,\phi)|$ with $\phi_0 = 30^o$, $\varepsilon_1 = 1$, $Z = 1 + i0.5$.

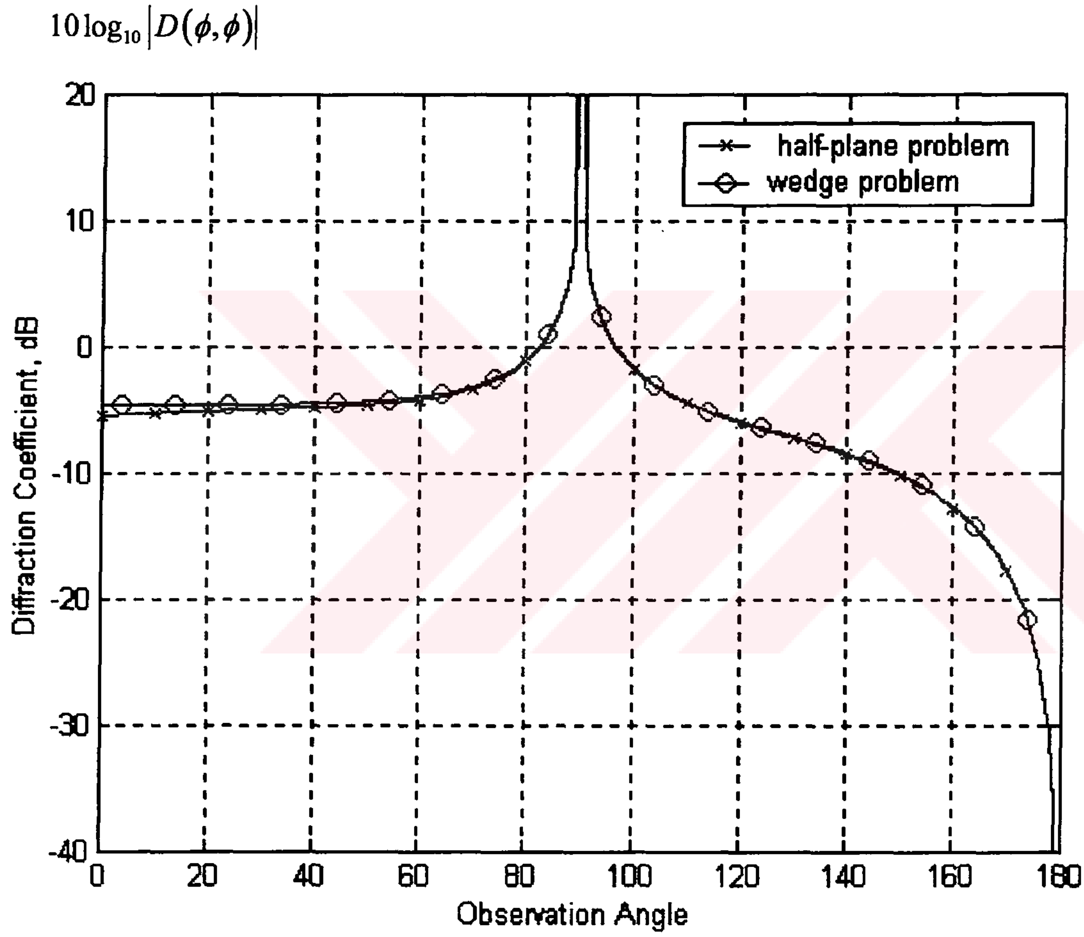
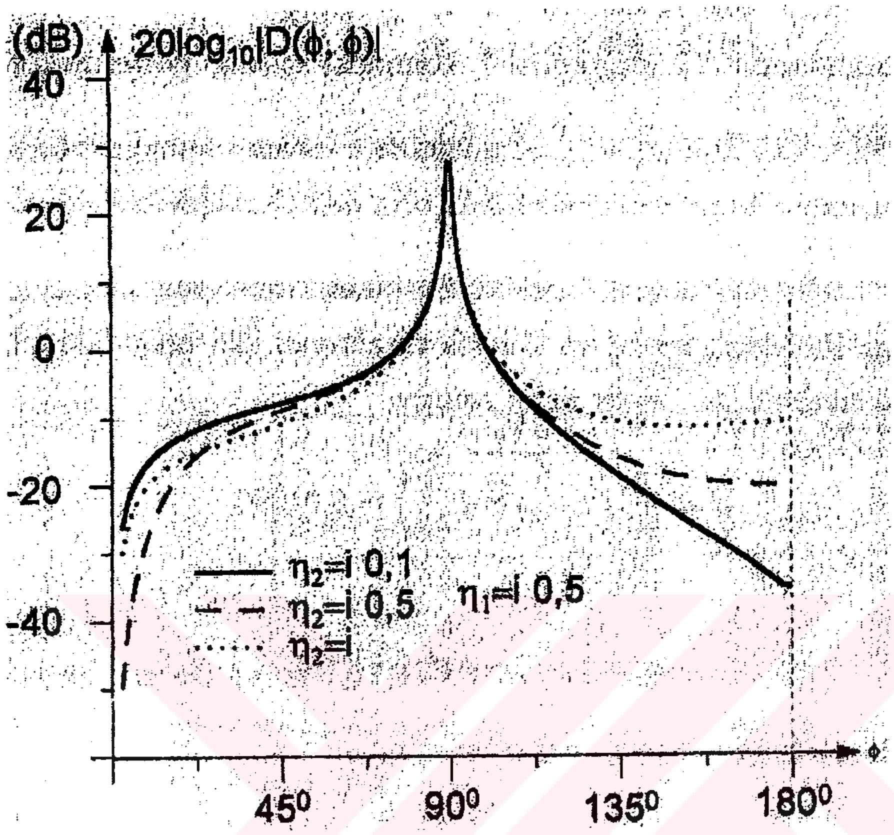


Figure 5.2. Comparison the results with Yener's results, $10\log_{10}|D(\phi,\phi)|$ in monostatic case for $\varepsilon_1 = 1,\ Z_1 = 1 + i0.5$.



Half plane in monostatic case for different values of surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 0$, $Z_1 = i0.5$ (A. Büyükaksoy and G. Uzgören, 1999).

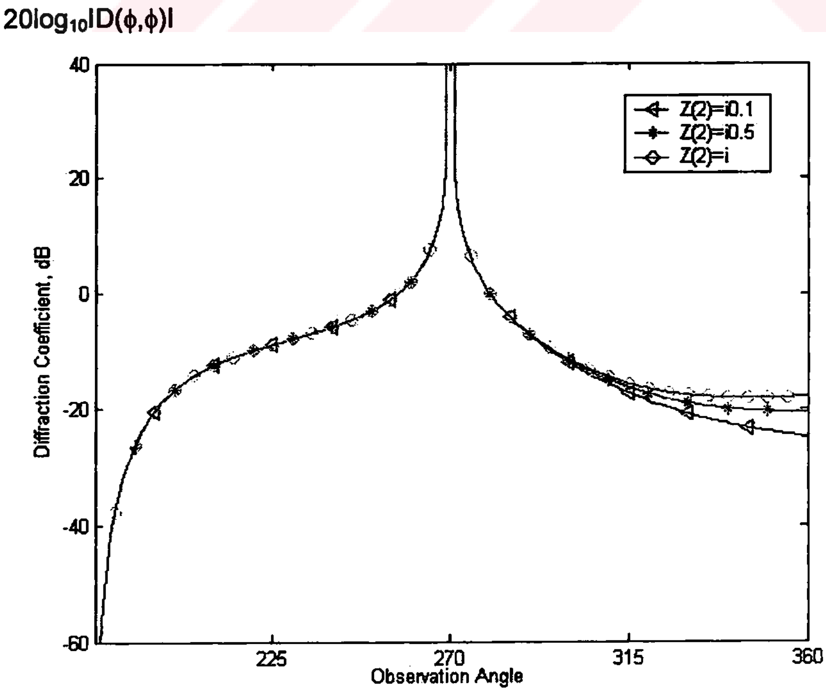
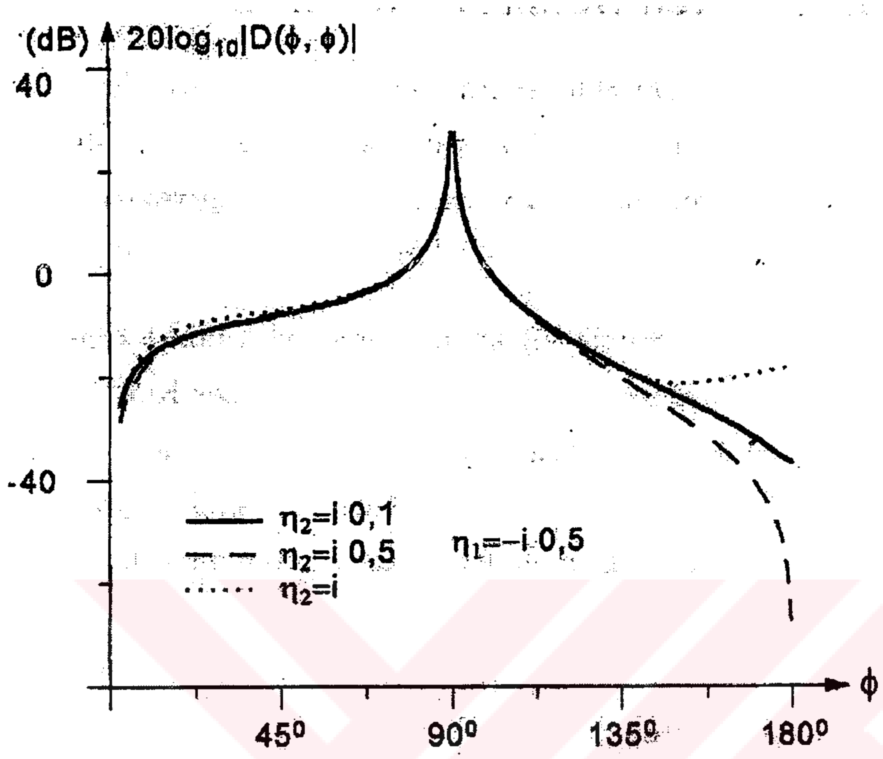


Figure 5.3. Half plane in monostatic case for different values of surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 0$, $Z_1 = i0.5$.



Half plane in monostatic case for different values of surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1,\ \varepsilon_2 = 0,\ Z_1 = -i0.5$ (A. Büyükaksoy and G. Uzgören, 1999).

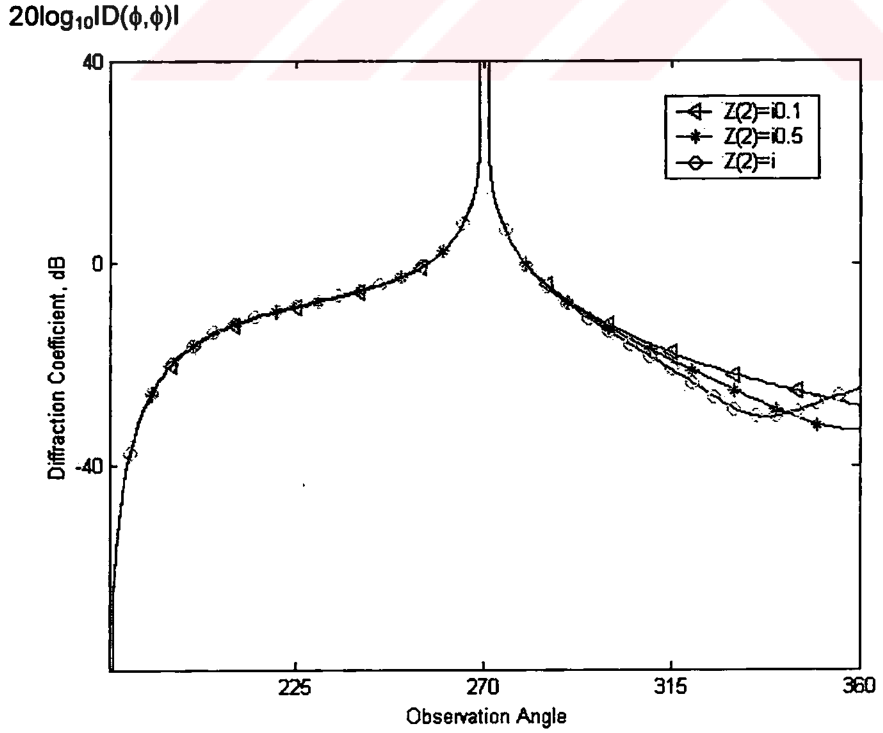
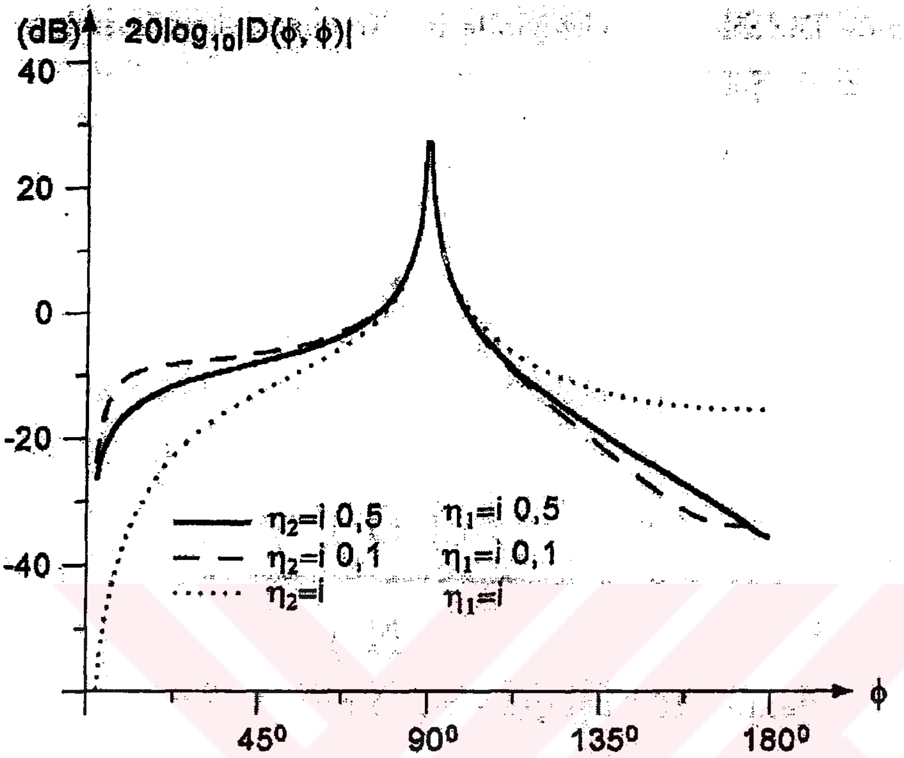


Figure 5.4. Half plane in monostatic case for different values of surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1,\ \varepsilon_2 = 0,\ Z_1 = -i0.5$.



Half plane in monostatic case for the same values of lower and upper surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1,\ \varepsilon_2 = 0$ (A. Büyükaksoy and G. Uzgören, 1999).

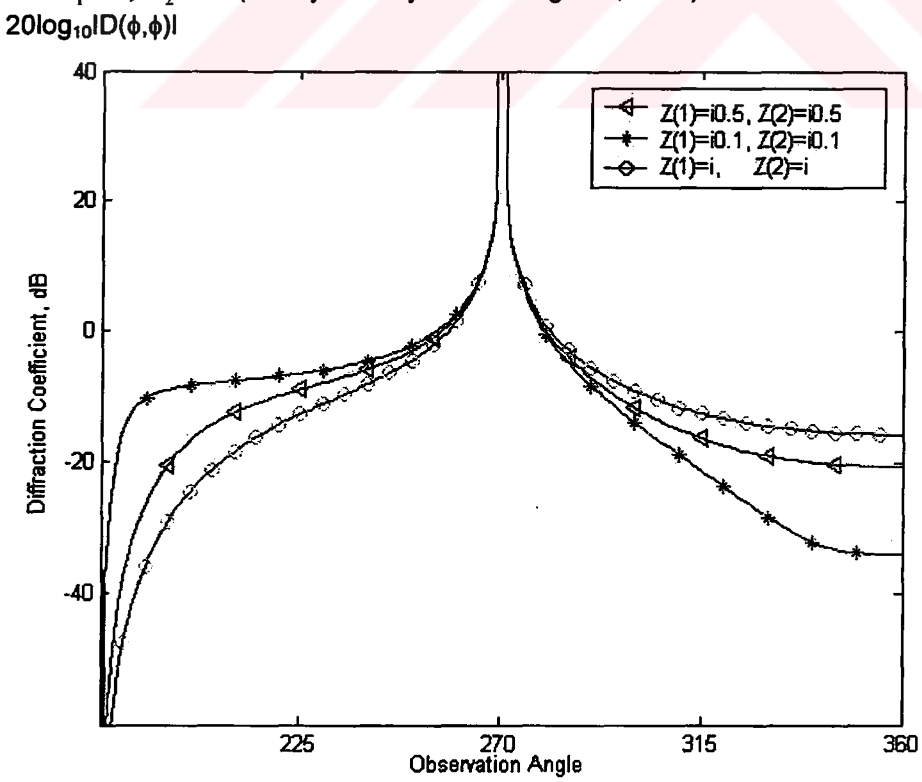
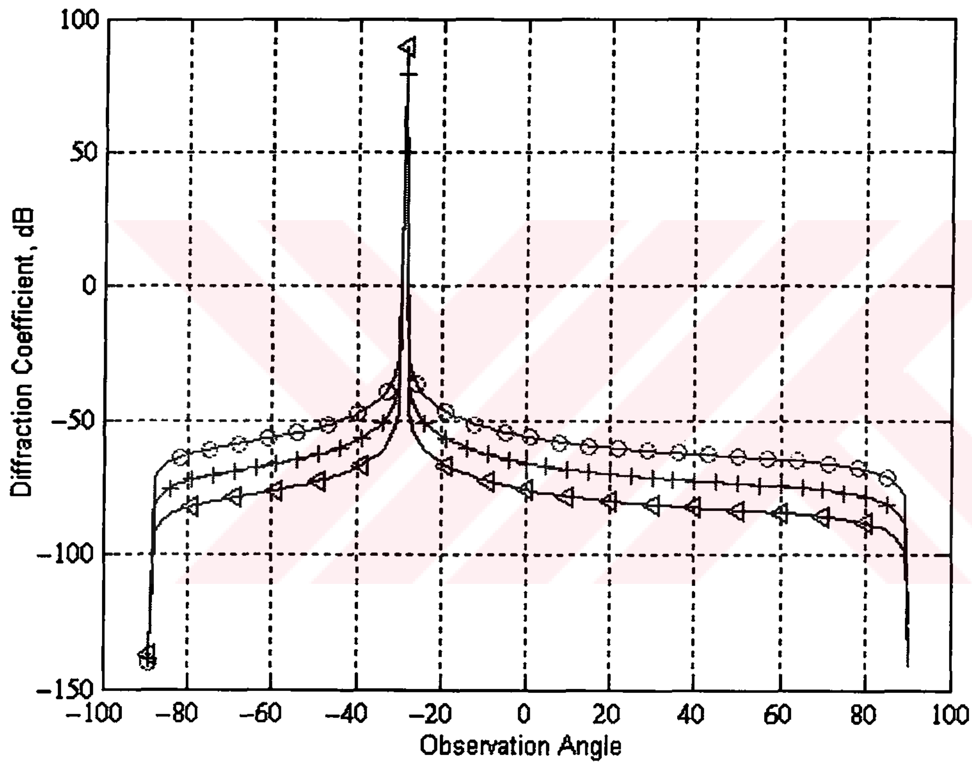


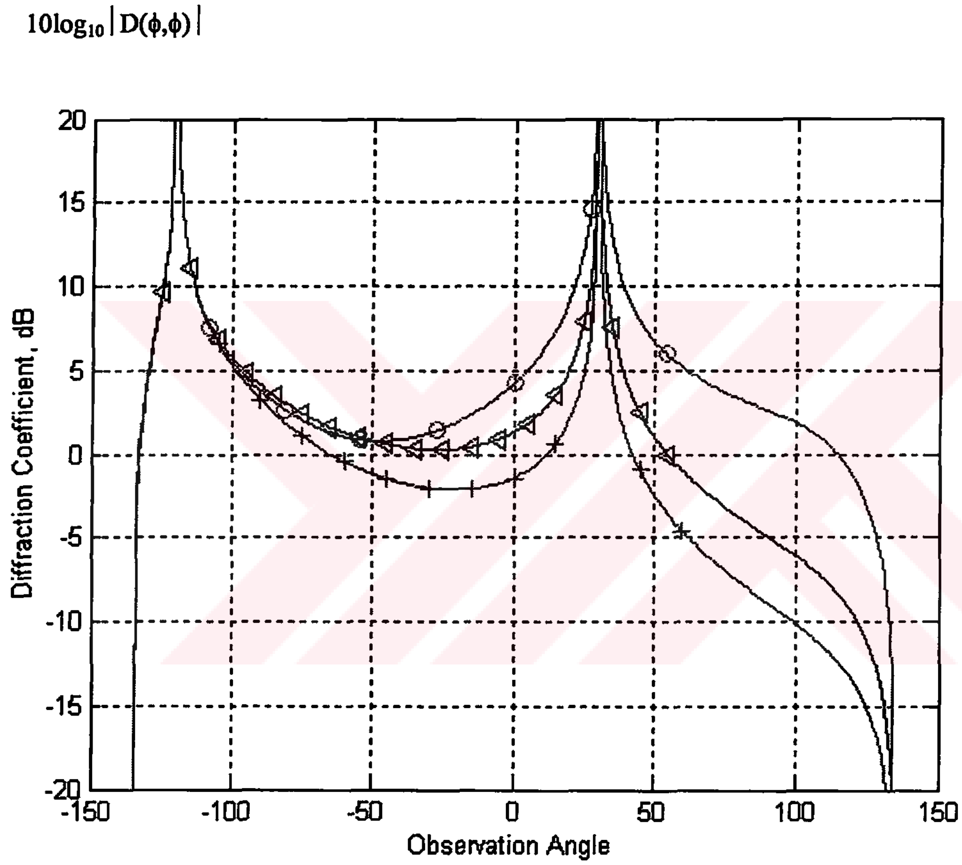
Figure 5.5. Half plane in monostatic case for the same values of lower and upper surface impedance $\tilde{\sigma}/\lambda$ with $\varepsilon_1 = 1,\ \varepsilon_2 = 0$.

$10\log_{10}|D(\phi,\phi)|$



$\Phi_{\triangleleft} = 89{,}99999991°$, $\Phi_{+} = 89{,}9999991°$, $\Phi_{o} = 89{,}999991°$

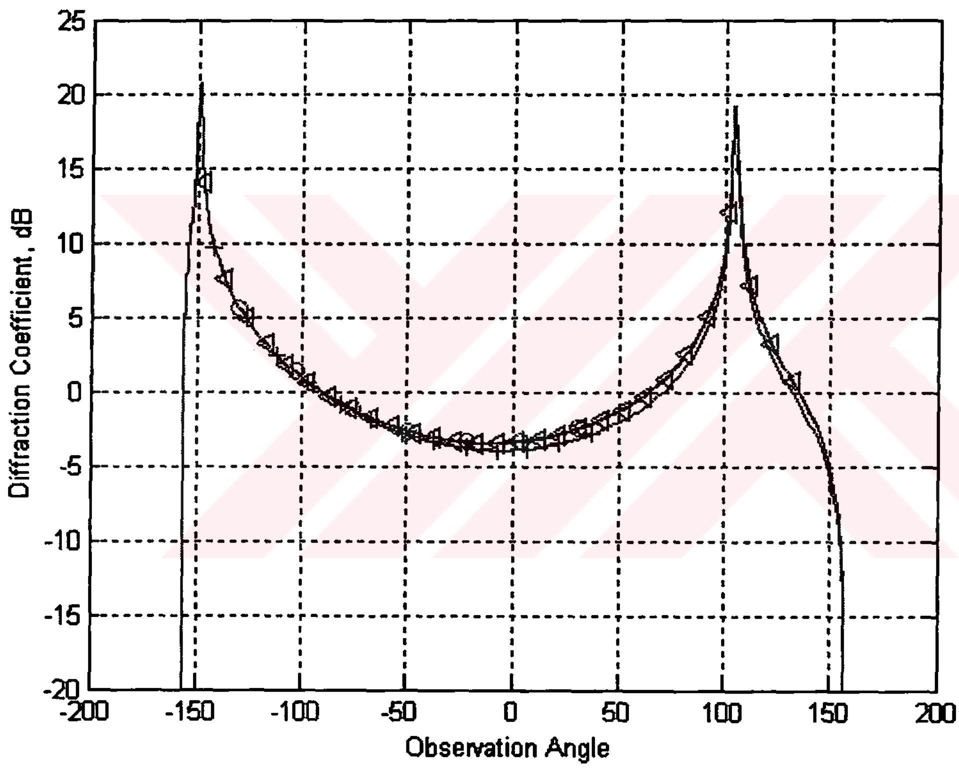
Figure 5.6. $10\log_{10}|D(\phi,\phi)|$ in bistatic case with $\varepsilon_1 = 1$, $\varepsilon_2 = 0$, $Z_1 = i0.5$, $Z_2 = i0.5$, $\Phi \sim 90^o$, $\phi_0 = 30^o$.



◄ : Z_1=1+i0.5, Z_2=0.5+i; + : Z_1=1, Z_2=i0.5; o : Z_1=i0.5, Z_2=1+i

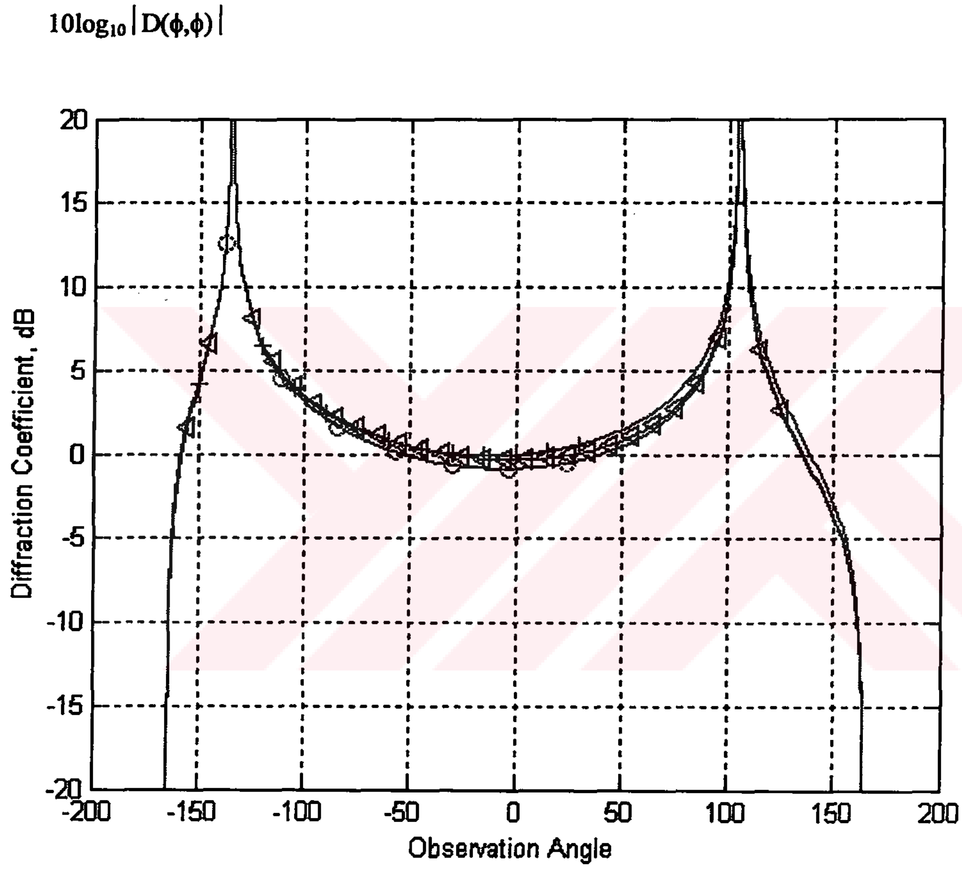
Figure 5.7. $10\log_{10}\left|D(\phi,\phi)\right|$ in bistatic case with $\varepsilon_1 = 1+i0.5$, $\varepsilon_2 = 0.1-i0.2$, $\phi_o = 60^o$, $NPP = 3$, $NQQ = 2$, $\Phi = 135^o$.

$10\log_{10}|D(\phi,\phi)|$



◄ : ε_1=0.99+i0.1, ε_2=0.1+i0.1; + : ε_1=0.99-i0.1, ε_2=0.1-i0.1; o : ε_1=0.99+i0.1, ε_2=0.1-i0.1

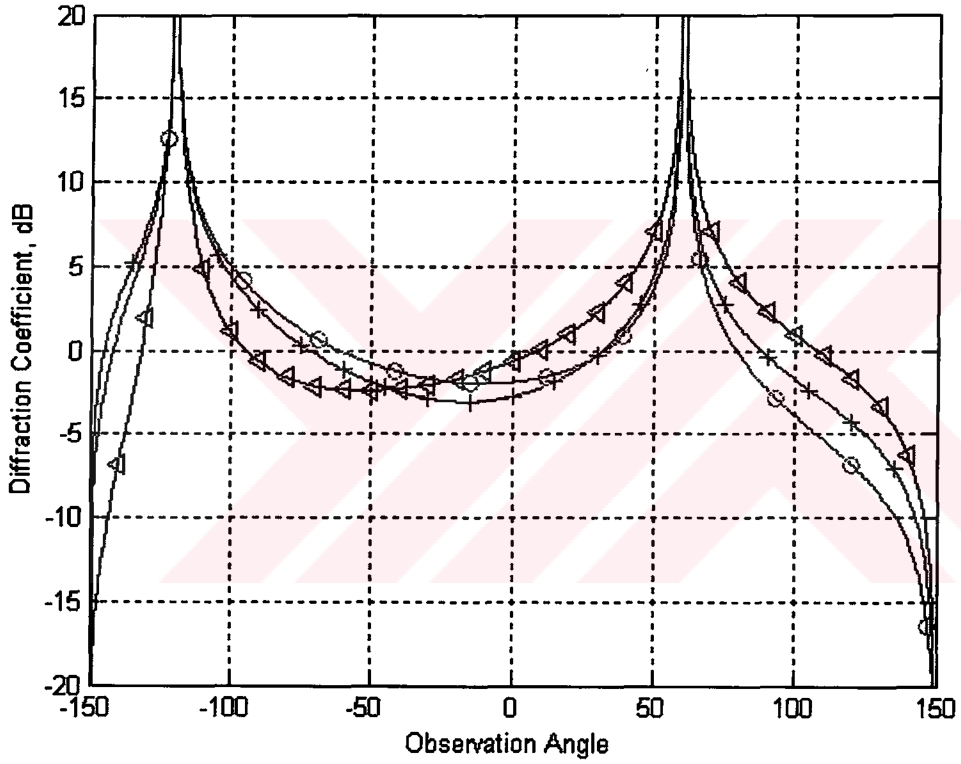
Figure 5.8. $10\log_{10}|D(\phi,\phi)|$ in bistatic case with $Z_1 = 0.5 + i$, $Z_2 = 1 + i0.5$, $NPP = 7$, $NQQ = 4$, $\phi_o = 30^o$, $\Phi = 157.5^o$.



◄ : Z_1=1+i0.5, Z_2=0.5+i; + : Z_1=0.5+i, Z_2=1+i0.5; o : Z_1=1-i, Z_2=0.5-i0.5

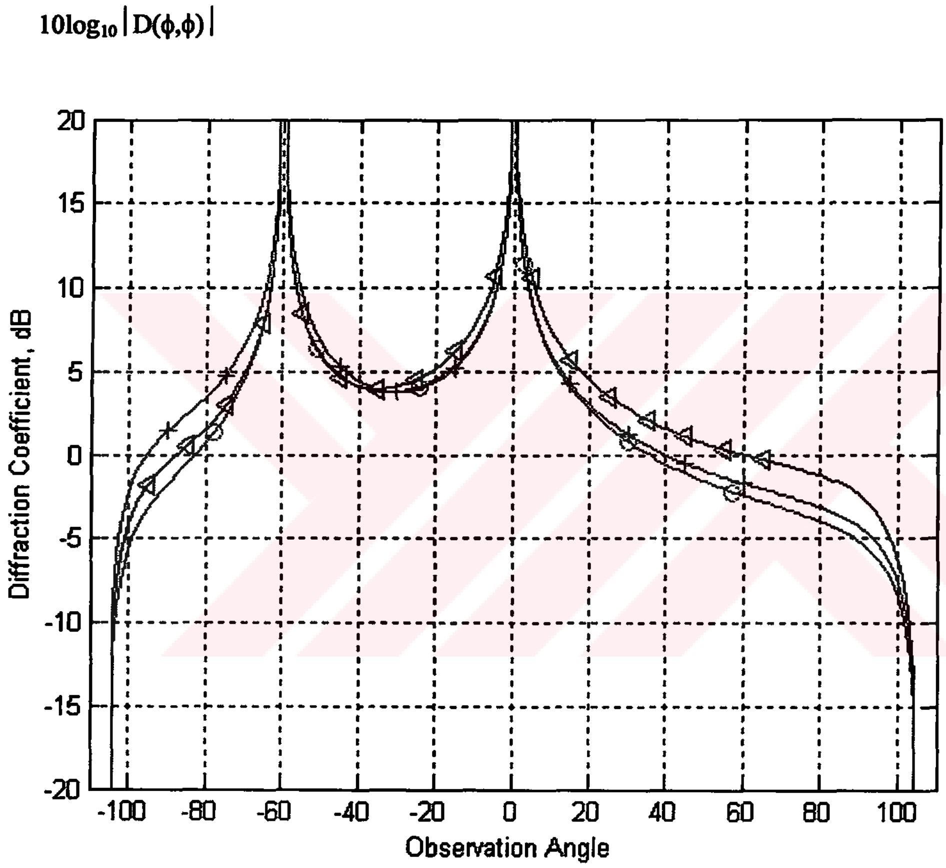
Figure 5.9. $10\log_{10}\left|D(\phi,\phi)\right|$ in bistatic case with $\varepsilon_1 = 5+i2$, $\varepsilon_2 = 3+i, \phi_o = 45^o$, $NPP = 11$, $NQQ = 6$, $\Phi = 165^o$.

$10\log_{10}|D(\phi,\phi)|$



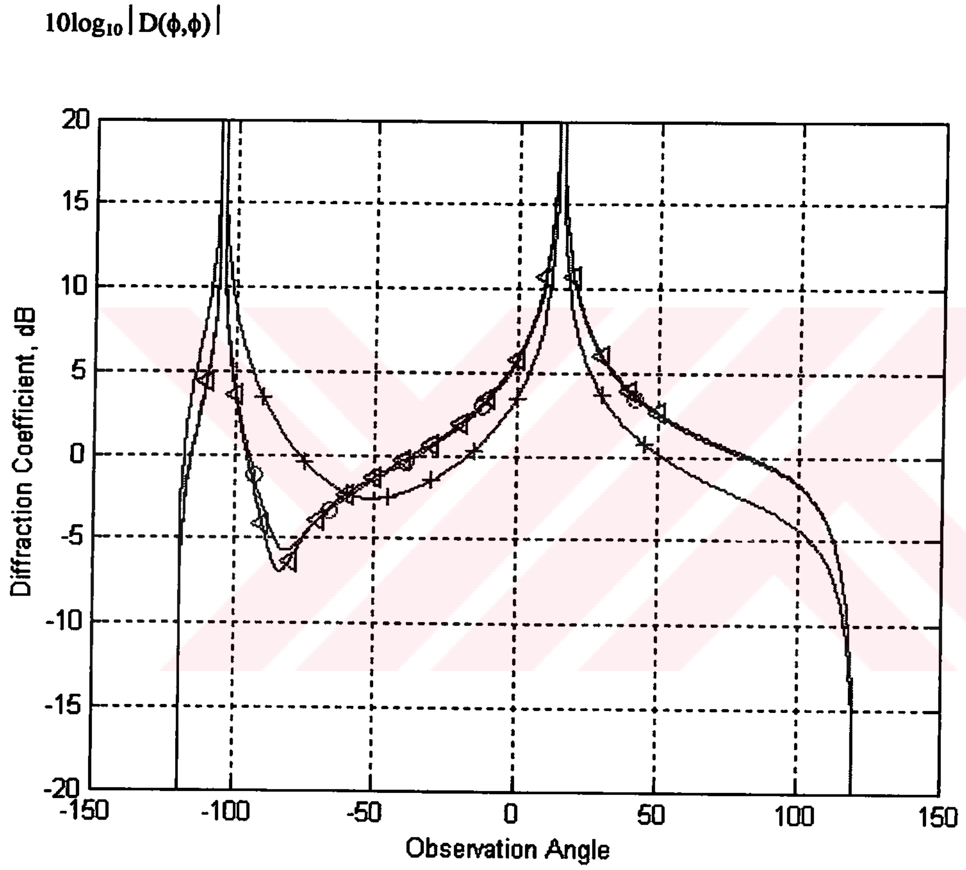
◄ : ε_1=0.1+i0.2, ε_2=0.99-i0.1; + : ε_1=0.2-i, ε_2=1+i0.8; o : ε_1=1+i0.99, ε_2=0.2-i0.99

Figure 5.10. $10\log_{10}|D(\phi,\phi)|$ in bistatic case with $Z_1=1,\ Z_2=i0.5,\quad \phi_o=60^o$, $\Phi=150^o,\ NPP=5,\ NQQ=3$.



◄ : Z_1=0.1-i0.4, Z_2=0.2+i0.3; + : Z_1=0.1+i0.4, Z_2=0.2-i0.3; o : Z_1=0.1+i0.4, Z_2=0.2+i0.3

Figure 5.11. $10\log_{10}\left|D(\phi,\phi)\right|$ in bistatic case with $\varepsilon_1 = 0.2 - i0.3$, $\varepsilon_2 = 0.1 + i0.7$, $NPP = 7$, $NQQ = 6$, $\phi_o = 30^o$, $\Phi = 105^o$.



◄ : ε_1=0.7-i0.3, ε_2=0.1+i0.2; + : ε_1=0.7+i0.3, ε_2=0.1+i0.2; o : ε_1=0.7-i0.3, ε_2=0.1-i0.2

Figure 5.12. $10\log_{10}|D(\phi,\phi)|$ in bistatic case with $Z_1 = 0.2 - i0.5$, $Z_2 = 0.3 + i0.3$, $NPP = 4$, $NQQ = 3$, $\phi_o = 45^o$, $\Phi = 120^o$.

6. CONCLUSION

As known wedge is an important canonical structure in the theory of diffraction. In this study, an impedance wedge in cold plasma is considered. Because the ionosphere can be modeled by a cold plasma and many parts of flying objects such as airplanes' or satellites' wings may be considered as wedge shaped structures.

In the present study, the nonhomogeneous functional equations obtained by using impedance boundary condition was reduced to homogeneous functional equations by using the symmetry of the Sommerfeld integral. So, a relatively simple solution for the diffraction coefficient is obtained in terms of χ_ϕ functions. The numerical calculation of χ_ϕ functions is accomplished by Zhu.

It is obvious that, the wedge can be reduced to a half plane, which is solved by using Wiener-Hopf Method, by taking the wedge opening angle equal to π as Yener. The figures from (5.1) to (5.2) are drawn for comparison of the results with the ones given by Yener. As shown almost the same results are obtained for both monostatic and bistatic diffraction coefficients. In this comparison it is considered that $Z_1 = Z_2$ and $\varepsilon_2 = 0$ as in that study.

The half plane problem with different surface impedances $(Z_1 \neq Z_2)$ in free space has been solved by Büyükaksoy. To compare the results with them, again the wedge opening angle was taken as π, and to get the standard boundary condition (3.25) in free space ε_2 was considered as zero while ε_1 takes the value of unity. As shown in figures (5.3), (5.4), and (5.5) almost the same diffraction coefficient are obtained. The small differences may be due to the reduction of the parameters ε_1 and ε_2.

Finally, the wedge with same surface impedance may be deformed to a full plane by taking the wedge opening angle as $\pi/2$. In that case, the diffraction coefficient must be equal to zero. For the opening angle very close to $\pi/2$, the diffraction coefficient must be very small. In the figure (5.6), the value of diffraction coefficients for different opening angles very close to $\pi/2$ are drawn. It is clear that as opening angle approaches to $\pi/2$ the value of diffraction coefficient reduces.

Besides the comparisons with known solutions, the value of diffraction coefficients are given in figures from (5.7) to (5.12) for different values of surface impedances and parameters of cold plasma.

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