Disagreement and Information Acquisition with Multiple Experts

by

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[To universe]

ABSTRACT

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I examine a persuasion game between a decision maker and two advisers. The decision-making is centralized, and each adviser exerts costly effort to acquire information, which is publicly observed if generated. Crucially, the players have the same preferences but differing priors. The latter assumption implies that even if the players observe the same information, their preferred actions differ, i.e., they have differences of opinion about the optimal action. I show that each advisor has free-riding incentives, so that their effort choices are strategic substitutes. As a result, an advisor's effort choice is increasing in the degree of his own differences of opinion with the decision maker, but it is decreasing in that of the other advisor. This results in a trade-off for the decision maker when choosing advisers. By increasing her differences of opinion with an adviser, the decision maker elicits higher effort from one advisor but lower effort from the other one. Nevertheless, I show that the decision maker is better off by choosing two advisers with extreme differences of opinion instead of choosing two like-minded advisers.

Keywords: Information acquisition, public information, differences of opinion, persuasion, free-riding.

ÖZETÇE

Bahadır Ülgey
Ekonomi, Yüksek Lisans

Bu tez bir karar verici ve iki danışman arasındaki bir ikna etme oyununu incelenmektedir. Karar verme mekanizması merkezidir ve her danışman bilgi edinmek için bedeli olan bir efor sarf eder. Danışmanlar tarafından elde edilen herhangi bir bilgi tüm oyunculara açık olacak şekilde gözlemlenir. En önemlisi, oyuncuların tercihleri aynı olmasına rağmen aralarında fikir ayrılıkları mevcuttur. Son varsayım, oyuncular aynı bilgiye sahip olsalar bile farklı aksiyonları tercih ettiklerini, başka bir deyişle en uygun aksiyon konusunda fikir ayrılığına sahip olduklarını ima eder. Bu tez, danışmanların hazıra konma eğilimleri olduğunu, yani sarf ettikleri eforun birbirinin stratejik olarak yerini tuttuğunu göstermektedir. Bunun bir sonucu olarak, bir danışmanın sarf ettiği efor düzeyi, karar verici ile arasındaki fikir ayrılığı arttıkça artarken, diğer danışmanın karar verici ile fikir ayrılığı arttıkça azalır. Bunu takriben, karar verici danışmanlarını seçerken bir ödünleşmeyle karşı karşıya kalır: Danışmanlarıyla fikir ayrılığını artırarak, bir danışmanın efor düzeyini yükseltirken diğer danışmanın efor düzeyini düşürmek durumunda kalır. Buna rağmen, karar verici eğer iki ekstrem düzeyde aykırı fikirli danışman seçerse, iki benzer fikirli danışmanı seçtiği duruma kıyasla daha iyi durumda olur.

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Chapter 1

INTRODUCTION

Making most important decisions require information from various sources. Therefore, decision makers usually look for professional advice, such as a board of directors asking for opinions of marketing consultants about the next branding project, a judge requiring plantiff and defendant sides to provide evidence, or a student who consults his or her professors about which graduate schools to apply. Being motivated by such settings, I examine a setup where a decision maker consults two advisers to make a decision. The decision maker and advisers have the same payoff function, which is maximized by the true value of the state. However, they might have different opinions about the right decision, i.e., the true value of the state may be different for each player. Each adviser exerts effort to obtain a publicly observed signal. Signals are normally distributed with mean equal to the true state and variance is the same across two signals. The probability that each adviser obtains a signal is equal to his effort choice. Then the decision maker takes an action by using information provided by advisers (if they obtain any), and all players obtain the same payoff.

My setup closely follows Che and Kartik (2009), where my model includes a second adviser while excluding information disclosure. By consulting two advisers instead of one, the decision maker may observe more information and take an action closer to the true state if both advisers acquire a signal. Furthermore, since the signals are publicly observed, all players benefit from information acquired by each adviser. However, the possibility of observing two signals does not contribute to the marginal benefit of an adviser equally. In particular, an adviser's own signal contributes to his marginal effort more than the other adviser's signal does. Nevertheless, as advisers benefit from every signal acquired, they tend to free-ride over the efforts of each other. For example, suppose that two business consultants are hired by an advertising agency, and one of these consultants works more than the other. Then the other consultant relies on him and works less. In equilibrium, this result implies that advisers' effort choices are strategic substitutes.

Each adviser exerts some effort in equilibrium under mild assumptions. Their incentives to exert effort is higher if their difference of opinion with the decision maker is higher since they expect to *persuade* the decision maker. In particular, each adviser believes that if he gathers information, then he can *pull* the decision maker to take an action closer to his preferred decision. This result is parallel to what Che and Kartik (2009) found. However, the reason behind it is different than that of Che and Kartik (2009), as there is no strategic disclosure in my model. Hence, there is no prejudicial effect that punishes an adviser with a larger difference of opinion when he cannot acquire a signal. Instead, advisers have free-riding incentives, which implies that an adviser exerts less effort if the other adviser has a higher difference of opinion with the decision maker. In other words, an adviser works less if the other adviser has to work more due to his more differing opinion. As a result, the adviser who has a higher difference of opinion with the decision maker exerts more effort than the other adviser.

On the other hand, the effect of signal precision on the effort choices of advisers is ambiguous. I run a simulation to illustrate this result, showing that advisers may increase their effort with signal precision when the signal is almost perfectly precise. This is because as an adviser reduces his effort, the other adviser has a weaker incentive to free-ride and exerts more effort to compensate the *laziness* of the other adviser. However, I show in Section 7 that if each adviser acquires a signal with different precision, then this ambiguity is resolved. In particular, the effort of each adviser is increasing in the precision of his own signal and decreasing in the precision of the other adviser's signal. The intuition behind this result can be explained by the following real-world example. Suppose that there are two lawyers trying to help their client but one of them is more competent, i.e., he is able to find more convincing evidence in his cases. Then, the less-competent lawyer may not work as much as he would in the absence of a more-competent lawyer.

In the last part of my analysis, I allow the decision maker to choose the difference of opinion of advisers with herself. The decision maker considers a trade-off when she chooses advisers with a higher difference of opinion. Choosing an adviser with a higher difference of opinion increases that adviser's effort, but decreases the other adviser's effort. However, the decision maker is better off by choosing two advisers with extreme differences of opinion with herself, rather than choosing two like minded advisers, i.e., advisers with the same opinion as her.

The outline of my thesis is as follows. Section 2 discusses the related literature. Section 3 develops the model setup. Section 4 examines the optimal effort choices of advisers. Section 5 describes the equilibrium behavior and presents comparative statics analyses. Section 6 analyzes the decision maker's choice of differences of opinions with advisers. Section 7 introduces an extension to the comparative statics analysis of signal precision. Section 8 concludes the paper with discussing some possible extensions.

Chapter 2

LITERATURE REVIEW

My thesis is closely related to Che and Kartik (2009). My model is based on theirs, but I include a second adviser in my model and discard the strategic disclosure decision of advisers. As a result, there are differences in our results. Che and Kartik (2009) show that the decision maker is better off by choosing an adviser with some difference of opinion instead of a like-minded one. In my model, I specify the differences of opinion that the decision maker must choose so that she must choose two advisers with extreme differences of opinion instead of two like-minded ones.

As Che and Kartik (2009), my thesis contributes to the literature on differing priors, i.e., the principal and the agent having same preferences but different opinions. There are a number of papers in the literature that have examined the assumption of differing opinions in a principal-agent setting (e.g., Morris (2001), Van den Steen (2009), Che, Dessein and Kartik (2013), Hirsch (2016)). However, the current analysis differs from these papers in two dimensions. First, I focus on the agent's information acquisition behavior rather than information disclosure decision. In other words, I explore the agent's effort choice instead of his decision about revealing the information he acquires. Second, my model consists of two agents, hence allowing me to observe additional results that cannot be observed in a single-agent setting, such as free-riding incentives and strategic substitubility of effort choices.

Besides differing priors, the current study falls into the literature on information acquisition with multiple experts. Bhattacharya and Mukherjee (2013) examine a persuasion game with a decision maker and a panel of agents, where agents are allowed to withhold the information they acquire. They show that choosing advisers with extreme agendas is optimal for the decision maker.[1] However, the decision maker does not always prefer an agent with a higher probability of obtaining information. My result is in line with the former but the reverse of the latter, so that the decision maker always prefers advisers with higher probability of obtaining information in my setting. In addition, Bhattacharya, Goltsman and Mukherjee (2018) show that the agents with not only extreme but also identical agendas should be preferred. In my analysis, only the distance between an adviser's opinion and the decision maker's opinion matters, i.e., it does not matter that advisers have identical or opposed opinions. Furthermore, Kartik, Lee and Suen (2015) show that the efforts of two biased senders with linear and opposite preferences are strategic substitutes of each other, as in my model.[2] However, my result is due to free-riding incentives of advisers since signals are publicly observed, while Kartik, Lee and Suen (2015) show that "the strategic substitutability of efforts" in their study cannot be degraded to a free-riding phenomenon. In particular, they provide counterexamples to show that experts may not reduce the impact of each other on the DM.

[1] Instead of agendas, I attach types to advisers.

[2] "Opposite preferences" means that one sender wants to induce higher beliefs in the decision maker while the other sender wants to induce lower beliefs in the decision maker.

Chapter 3

MODEL SETUP

There are three players in the game, one decision maker (DM) and two advisers.[3] The DM needs to take an action $a \in \mathbb{R}$, and all players' payoffs depend on the state of the world $\omega \in \mathbb{R}$ and the action taken. The state of the world is unknown to all players, but each player has a prior belief about what the state is. The DM consults the two advisers before choosing action a.

Prior Beliefs: For adviser i, $i = 1, 2$, the prior distribution of the state of the world is $\omega \sim \mathcal{N}(\mu_i, \sigma_0^2)$. For simplicity, I assume $\mu_{DM} = 0$ and let the distribution of the state for the DM be $w \sim \mathcal{N}(0, \sigma_0^2)$. Throughout the paper, I refer to μ_i as the type of adviser i, where $\mu_i \in [\underline{\mu}, \overline{\mu}]$ with $\underline{\mu} < 0 < \overline{\mu}$.

Preferences: Each player has the same quadratic loss function as her/his payoff function, which depends on the state and the DM's decision, i.e.,

$$u(a, \omega) := -(a - \omega)^2$$

Information Acquisition: Advisers exert effort for information acquisition. In particular, adviser i chooses effort $e_i \in [0, \overline{e}]$, where $\overline{e} < 1$, at a cost of $c(e_i)$. I assume $c(\cdot)$ to be twice continuously differentiable, strictly increasing and convex, i.e., $c'(\cdot) > 0$ and $c''(\cdot) > 0$. Further, $c(\cdot)$ satisfies Inada conditions: $c(0) = 0$, $c'(0) = 0$ and $c'(e) \to \infty$ as $e \to \overline{e}$. With probability e_i, adviser i observes signal s_i from $\mathcal{N}(\omega, \sigma_s^2)$. With probability $1 - e_i$, he does not obtain any signal. Note that advisers acquire signals from the same distribution.

[3] Throughout the paper, DM is referred as "she/her" and the advisers are referred as "he/his".

Timing: The sequence of events is as follows. First, the types of advisers are publicly observed. Then, each adviser chooses an effort level, which is privately observed by himself. If a signal is received by any adviser, it becomes available to all players as public information. In the last stage, the DM chooses an action, after which the state and the payoffs are realized.

3.1 Posterior Distribution of the State

As a preparation to my analysis, I derive players' optimal actions, conditional on whether a signal is acquired. The joint distribution of the state and the signals is given by[4]

$$\begin{pmatrix} \omega \\ s_1 \\ s_2 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_i \\ \mu_i \\ \mu_i \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2+\sigma_s^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2+\sigma_s^2 \end{pmatrix}\right] \tag{3.1}$$

For each player, the optimal action is their own expectation of the state conditional on the observed signals. If neither of the advisers acquires any signal, then the posterior distribution of ω is the same as its prior distribution for each player. To derive the posterior, suppose first that adviser j, $j \in \{1,2\}$, acquires a signal. Then the posterior distribution of ω for adviser $i \in \{1,2\}$ is given by $\omega|s_j \sim \mathcal{N}(\rho_1 s_j + (1-\rho_1)\mu_i, \tilde{\sigma}_1^2)$, where $\rho_1 = \dfrac{\sigma_0^2}{\sigma_0^2+\sigma_s^2}$ is the *weight* of signal s_j on the posterior mean and $\tilde{\sigma}_1^2 = \dfrac{\sigma_0^2\sigma_s^2}{\sigma_0^2+\sigma_s^2} = (1-\rho_1)\sigma_0^2$ is the residual variance (DeGroot, 1970). For the DM, the posterior distribution of ω is given by $\omega|s_j \sim \mathcal{N}(\rho_1 s_j, \tilde{\sigma}_1^2)$ since her prior mean is normalized to 0. Next, suppose that each adviser acquires a signal. Then the posterior distribution of the state for adviser i is given by

$$\omega|s_1, s_2 \sim \mathcal{N}(\rho_2(s_1+s_2) + (1-2\rho_2)\mu_i, \tilde{\sigma}_2^2),$$

where $\rho_2 = \dfrac{\sigma_0^2}{2\sigma_0^2+\sigma_s^2}$ is the weight assigned to each signal on the posterior mean

[4]The derivation of the joint distribution of the state and the signals and the posterior distribution of ω are presented in Appendix A.

and $\tilde{\sigma}_2^2 = (1-2\rho_2)\sigma_0^2$.[5] Similarly, the posterior distribution of ω for the DM is given by $\omega|s_1, s_2 \sim \mathcal{N}(\rho_2(s_1+s_2), \tilde{\sigma}_2^2)$.

3.2 Optimal Actions of the Players

After deriving the posterior distribution of the state conditional on the observed signals, the expected utility maximization becomes a simple problem. For each player, the optimal action is the posterior mean of the state. Let $\alpha(S|\mu_i)$ denote the preferred action of adviser i with S denoting the set of observed signals. Then the preferred action of adviser i for each set of signals is the following:

$$\begin{aligned}
\alpha(\varnothing|\mu_i) &= \mu_i \\
\alpha(s_j|\mu_i) &= \rho_1 s_j + (1-\rho_1)\mu_i \\
\alpha(s_1, s_2|\mu_i) &= \rho_2(s_1+s_2) + (1-2\rho_2)\mu_i
\end{aligned}$$

Similarly, the optimal action for the DM is $\alpha(\varnothing|0) = 0$ if both advisers failed to receive a signal, $\alpha(s_i|0) = \rho_1 s_i$ if only adviser i receives a signal, and $\alpha(\{s_1, s_2\}|0) = \rho_2(s_1+s_2)$ if each adviser receives a signal.

Note that the preferred action of the advisers and the optimal action of the DM differ unless at least one of the signals is perfectly informative. To capture the difference of opinion between an adviser and the DM, I define the terms *prior bias* and *posterior bias* as follows. The prior bias is the difference between the preferred actions of adviser i and the DM before any additional information is acquired, and is given by μ_i. The posterior bias is the difference between their preferred actions, after some information is acquired. Thus, the posterior bias is given by $B(S, \mu_i) = \alpha(S|\mu_i) - \alpha(S|0)$, where S is the set of observed signals. In particular, adviser i's posterior bias is $B(\emptyset, \mu_i) = \mu_i$ with no signals, $B(s_j, \mu_i) = (1-\rho_1)\mu_i$ with only s_j, $j = 1, 2$, and $B(\{s_1, s_2\}, \mu_i) = (1-2\rho_2)\mu_i$ with two signals.

[5] Since both advisers draw a signal from the same distribution, their signals are given the same weight.

Using the prior bias and posterior bias defined above, I describe the effect of signals on the differences of opinion between an adviser and the DM. Since the prior bias is μ_i and $B(S, \mu_i) < \mu_i$ for all $S \neq \emptyset$, and since $B(\{s_1, s_2\}, \mu_i) < B(s_j, \mu_i)$, $i, j = 1, 2$, any additional information reduces the difference of opinion between the DM and an adviser.[6] Hence, every time a new signal is observed, the DM and advisers have their opinions become closer to each other. This is because each signal is public, so that both the advisers and the DM update their expectations about the state accordingly. In addition, the sign of the posterior bias of adviser i is the same as his prior bias. To put it differently, if an adviser has a positive (negative) prior bias, then he has positive (negative) posterior bias for any $S \neq \emptyset$.

Finally, the magnitude of the posterior bias becomes smaller not only with each additional signal but also with the precision of the signals. That is, the higher the precision of signals, the lower the magnitude of the posterior bias. If signal s_j is perfectly precise, i.e., $\sigma_s^2 \to 0$, then $\rho_1 \to 1$ and $\rho_2 \to \frac{1}{2}$. That is, a perfectly precise signal eliminates the differences of opinion between the DM and an adviser. On the other hand, as $\sigma_s^2 \to \infty$, both $\rho_1 \to 0$ and $\rho_2 \to 0$, in which case the imprecise signal is ignored by all players.

[6] Note that this result does not depend on which adviser generates the first signal.

Chapter 4

OPTIMAL EFFORT CHOICES OF ADVISERS

In this section, I analyze the advisers' optimal effort choices. Because the acquired signals are public information, the DM does not need to observe advisers' effort choices before taking action a.

4.1 Advisers' Utility Maximization Problem

In the previous section, I derive the DM's optimal action conditional on information acquired by the advisers. To derive the optimal effort choices of the advisers, I express adviser i's expected payoff by plugging the DM's optimal action in for all these cases (using an approach similar to backward induction). Suppose adviser i believes that adviser j exerts effort $\hat{e}_j$. Then, if there are two signals, which happens with probability $e_i\hat{e}_j$, adviser i receives payoff $-[(1-2\rho_2)\mu_i]^2 - \tilde{\sigma}_2^2$. If there is only one signal, which is observed with probability $e_i(1-\hat{e}_j)+(1-e_i)\hat{e}_j$, adviser i receives payoff $-[(1-\rho_1)\mu_i]^2 - \tilde{\sigma}_1^2$. Finally, if there is no signal, which happens with probability $(1-e_i)(1-\hat{e}_j)$, adviser i receives payoff $-\mu_i - \sigma_0^2$. Note that the value of signals is not a part of the payoffs because a signal is a revealed piece of information about the state; thus, the exact value of the signal does not affect the distance between the DM's action and the adviser's preferred action. Then, we can write adviser i's expected utility maximization problem as follows:

$$\begin{aligned}\max_{e_i} \quad & e_i\hat{e}_j\{-[(1-2\rho_2)\mu_i]^2 - \tilde{\sigma}_2^2\} \\ &+[e_i(1-\hat{e}_j)+(1-e_i)\hat{e}_j]\{-[(1-\rho_1)\mu_i]^2 - \tilde{\sigma}_1^2\} + (1-e_i)(1-\hat{e}_j)[-\mu_i^2 - \sigma_0^2] - c(e_i)\end{aligned} \tag{4.1}$$

Writing the first-order condition with respect to e_i yields

$$-\hat{e}_j\{[(1-2\rho_2)^2-(1-\rho_1)^2]\mu_i^2+\tilde{\sigma}_2^2-\tilde{\sigma}_1^2\}$$
$$-(1-\hat{e}_j)\{[(1-\rho_1)^2-1]\mu_i^2+\tilde{\sigma}_1^2-\sigma_0^2\}-c'(e_i)=0.$$

By re-arranging terms, the following equation is obtained:

$$c'(e_i)=\hat{e}_j[\tilde{\sigma}_1^2-\tilde{\sigma}_2^2]+(1-\hat{e}_j)[\sigma_0^2-\tilde{\sigma}_1^2]$$
$$+\hat{e}_j\{[B(s_j,\mu_i)]^2-[B(\{s_1,s_2\}\mu_i)]^2\}+(1-\hat{e}_j)\{[B_0(\emptyset,\mu_i)]^2-[B(s_i,\mu_i)]^2\},$$

where the right hand side of the above equation is the marginal benefit of adviser i. The marginal benefit consists of the following two effects :

- By obtaining a signal, adviser i reduces the uncertainty about the state. $(1-\hat{e}_j)[\sigma_0^2-\tilde{\sigma}_1^2]+\hat{e}_j[\tilde{\sigma}_1^2-\tilde{\sigma}_2^2]$ is the *uncertainty reduction effect*, where the first term corresponds to the case where adviser j does not acquire any signal and the second term corresponds to the case where adviser j obtains a signal. Since $\sigma_0^2>\tilde{\sigma}_{1j}^2>\tilde{\sigma}_2^2$ for $j=1,2$, the uncertainty reduction effect is always positive.

- Beyond reducing the uncertainty about the state, adviser i persuades the DM to take an action closer to his preferred action by obtaining a signal. $(1-\hat{e}_j)\{[B_0(\emptyset,\mu_i)]^2-[B(s_i,\mu_i)]^2\}+\hat{e}_j\{[B(s_j,\mu_i)]^2-[B(\{s_1,s_2\},\mu_i)]^2\}$ is the *persuasion effect*, where the first term corresponds to the case where adviser j does not acquire any signal and the second term corresponds to the case where adviser j obtains a signal. Since $B(\emptyset,\mu_i)>B(s_j,\mu_i)>B(\{s_1,s_2\}\mu_i)$ for $i,j=1,2$, the persuasion effect is always positive as well.

Hence, adviser i's incentive to obtain a signal stems from two sources. By obtaining a signal, adviser i reduces the uncertainty about the state and persuades the DM to take an action closer to his preferred action.

4.2 Free-Riding Incentives

One of the key results is that both uncertainty reduction and persuasion effects are decreasing with adviser j's effort e_j. There are two reasons for this. First, the second signal, which is adviser j's signal provides less uncertainty reduction about the state compared to the first signal, which is adviser i's own signal.[7] This is because the first signal already reduces some of the uncertainty about the state. Furthermore, the second signal is less persuasive than the first signal since the DM attaches a higher weight to a single signal in her optimal action than each of the two signals, conditional on that the second signal is acquired by the other adviser. These two reasons are summarized by the following lemma.[8]

Lemma 4.1: The contribution of the second signal to both the uncertainty reduction effect and persuasion effect is lower than that of the first signal, i.e., $\tilde{\sigma}_1^2 - \tilde{\sigma}_2^2 < \sigma_0^2 - \tilde{\sigma}_1^2$ and $[B(s_j, \mu_i)]^2 - [B(\{s_1, s_2\}, \mu_i)]^2 < [B(\emptyset, \mu_i)]^2 - [B(s_i, \mu_i)]^2$ for $i, j = 1, 2$, $i \neq j$.

As a result of this lemma, one can deduce that an adviser's marginal benefit from exerting effort is lower if the other adviser exerts effort as well. To demonstrate this result, I derive the following decomposition of the marginal benefit of the adviser i by rearranging its terms:

$$c'(e_i) = \{\Phi(\mu_i^2) - O(\mu_i^2)\}\hat{e}_j + O(\mu_i^2),$$

where $O(\mu_i^2) = \sigma_0^2 - \tilde{\sigma}_1^2 + [B(\emptyset, \mu_i)]^2 - [B(s_i, \mu_i)]^2$ is the marginal benefit of adviser i if adviser j does not exert any effort, and $\Phi(\mu_i^2) = \tilde{\sigma}_1^2 - \tilde{\sigma}_2^2 + [B(s_j, \mu_i)]^2 - [B(\{s_1, s_2\}, \mu_i)]^2$ is the marginal benefit that adviser i obtains through the effort of adviser j. Since $\Phi(\mu_i^2) < O(\mu_i^2)$ by Lemma 1, $\{\Phi(\mu_i^2) - O(\mu_i^2)\}\hat{e}_j$ can be viewed as the loss in the marginal benefit of exerting effort for adviser i if he expects adviser j

[7] I use the terms "first signal" and "second signal" even though it is not a sequential game of information acquisition. In my paper, adviser i's own signal is the first signal to him and adviser j's signal is the second.

[8] All proofs are presented in Appendix C.

to exert effort $\hat{e}_j$. Hence, as an adviser's belief about the other adviser's effort level increases, his marginal benefit from increased effort decreases. In other words, if the DM hires a second adviser, then the first adviser exerts less effort as a response. This result shows that as long as the signals are public information, each adviser has an incentive to free ride over the other adviser's effort.

Chapter 5

EQUILIBRIUM AND COMPARATIVE STATICS ANALYSES

In this section, I characterize the equilibrium and perform comparative statics analyses on signal precision and adviser types.

5.1 Equilibrium Effort Choices

In equilibrium, each adviser i chooses e_i to maximize (4.1). Inada conditions on the cost function ensure that each adviser's equilibrium effort level is in the interior of $[0, \bar{e}]$. Also, the belief of each adviser about the other adviser's effort must be correct in equilibrium. Therefore, equilibrium effort levels $(e_1^*, e_2^*) \in (0, \bar{e})$ should satisfy

$$c'(e_i^*) = (\Phi(\mu_i^2) - O(\mu_i^2))e_j^* + O(\mu_i^2) \tag{5.1}$$

for $i, j = 1, 2$, $i \neq j$. Then, the equilibrium can be characterized as follows:

Proposition 5.1: For all finite and non-zero σ_0^2, σ_1^2 and σ_2^2, and for all finite μ_1 and μ_2, the equilibrium effort levels e_1^* and e_2^* satisfy (5.1), where $e_1^*, e_2^* \in (0, \bar{e})$ are unique for $i, j = 1, 2$, $i \neq j$.

The equilibrium shows that both advisers exert some effort to acquire a signal, provided that their differences of opinion with the DM are not extremely large and the signals they collect are only partially precise. Before examining the effects of signal precision and adviser types on the equilibrium effort choices, I show that adviser i's effort choice decreases with adviser j's effort level. Differentiating (5.1) with respect to e_j^*, where $i, j = 1, 2$, $i \neq j$, yields

$$\frac{\partial e_i}{\partial e_j} = \frac{\Phi(\mu_i^2) - O(\mu_i^2)}{c''(e_i^*)} < 0. \tag{5.2}$$

I show in the previous section that as adviser j exerts more effort, the effect of s_j on the marginal benefit of adviser i increases, and the effect of s_i on adviser i's marginal benefit decreases. By Lemma 1, the effect of adviser i's signal on his own marginal benefit is larger than the effect of adviser j's signal, i.e., $\Phi(\mu_i^2) - O(\mu_i^2) < 0$. These results yield (5.2), which implies that adviser i's marginal benefit from effort decreases with adviser j's effort. In other words, advisers' effort choices are strategic substitutes, as in Kartik, Lee and Suen (2015). The reason is that, as explained in the previous section, advisers tend to free ride over the efforts of each other since the signals are publicly observed.

5.2 *Comparative Statics Analyses*

In this section, I analyze the effects of two parameters on the equilibrium effort choices, namely adviser types, μ_i, and signal precision, σ_s^2.

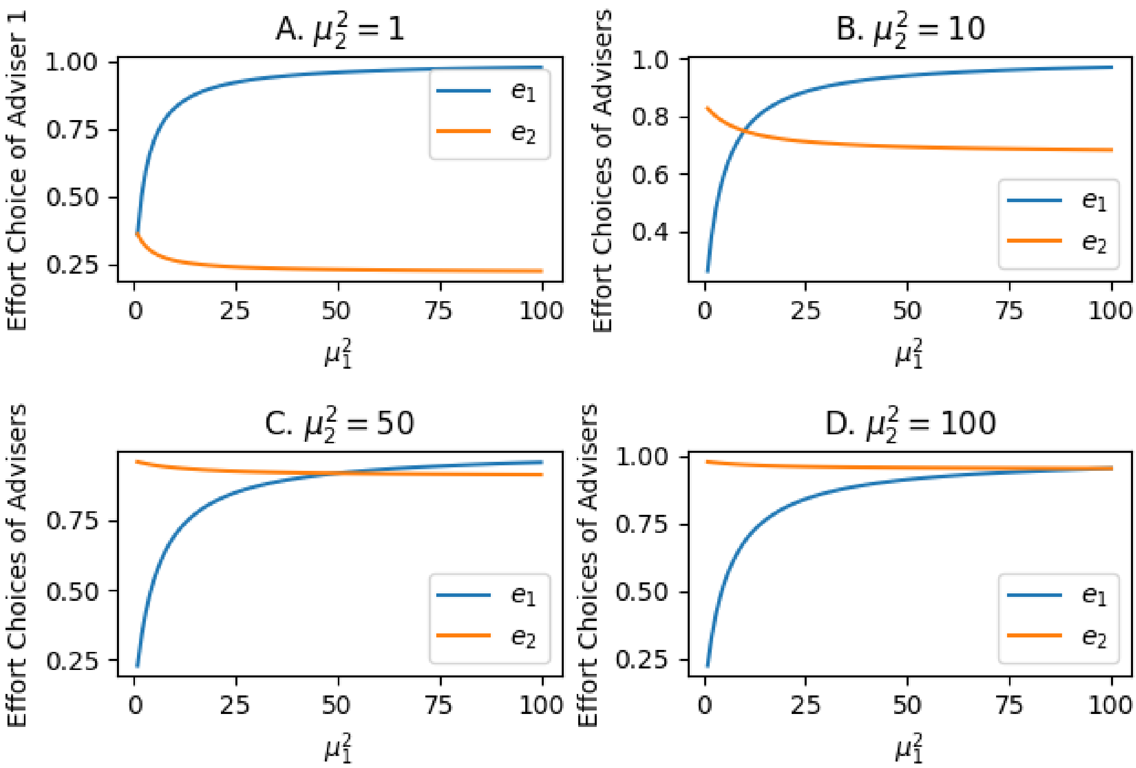
5.2.1 Adviser Types

Che and Kartik (2009) show that if the signal acquired by the adviser is public information, then an adviser with a higher difference of opinion with the DM has higher persuasion incentive and hence exerts higher effort. I examine the extent to which this result applies to the case of two advisers. First, adviser i's effort e_i increases with $|\mu_i|$, while adviser j's effort e_j decreases with $|\mu_i|$. To put it differently, the higher the difference of opinion between adviser i and the DM is, the higher adviser i's effort is and the lower adviser j's effort is. The following corollary formally states this result.

Corollary 5.1: e_i^* increases with $|\mu_i|$ while e_j^* decreases with it.

The first part of the result follows from that adviser i with higher $|\mu_i|$ has a higher persuasion incentive, just as in Che and Kartik (2009). On the other hand, the effect of $|\mu_i|$ on e_j is negative due to the strategic substitubility of efforts. To illustrate this result, I run a simulation by using cost function $c(e) = -e - log(1-e)$ and the parameter values $\sigma_0^2 = 0.5$ and $\sigma_s^2 = 1$[9]. Then, the following four-way graph illustrates the effect of adviser 1's type on the effort levels of each adviser:

Figure 5.1: The Effect of Adviser 1's Type on Equilibrium Effort Choices



From Figure 5.1, one can observe that even though e_2 decreases with $|\mu_1|$, it remains above a certain level of effort determined by $|\mu_2|$. This is because $|\mu_1|$ affects e_2 only through e_1 and e_1 is bounded above. In particular, that threshold e_2 approaches as $|\mu_1|$ increases is increasing in $|\mu_2|$. Note also that the initial value of e_1 becomes smaller as $|\mu_2|$ becomes higher. Furthermore, Figure 1 demonstrates that $e_1 > e_2$ whenever $|\mu_1| > |\mu_2|$, which can be presented as a formal result:

[9]Since the cost function $\dfrac{e^2}{(1-e)}$ used by Che and Kartik (2009) yields complex values for $e_i \in (0,1)$, $i = 1, 2$, I use $c(e) = -e - log(e)$ instead.

Corollary 5.2: Adviser with a higher difference of opinion with the DM always exerts higher effort, i.e., if $e_1^* > e_2^*$, then $|\mu_1| > |\mu_2|$.

Intuitively, if adviser i's difference of opinion with the DM is higher than that of adviser j, then adviser i has a higher persuasion incentive compared to adviser j, and therefore, he exerts higher effort.

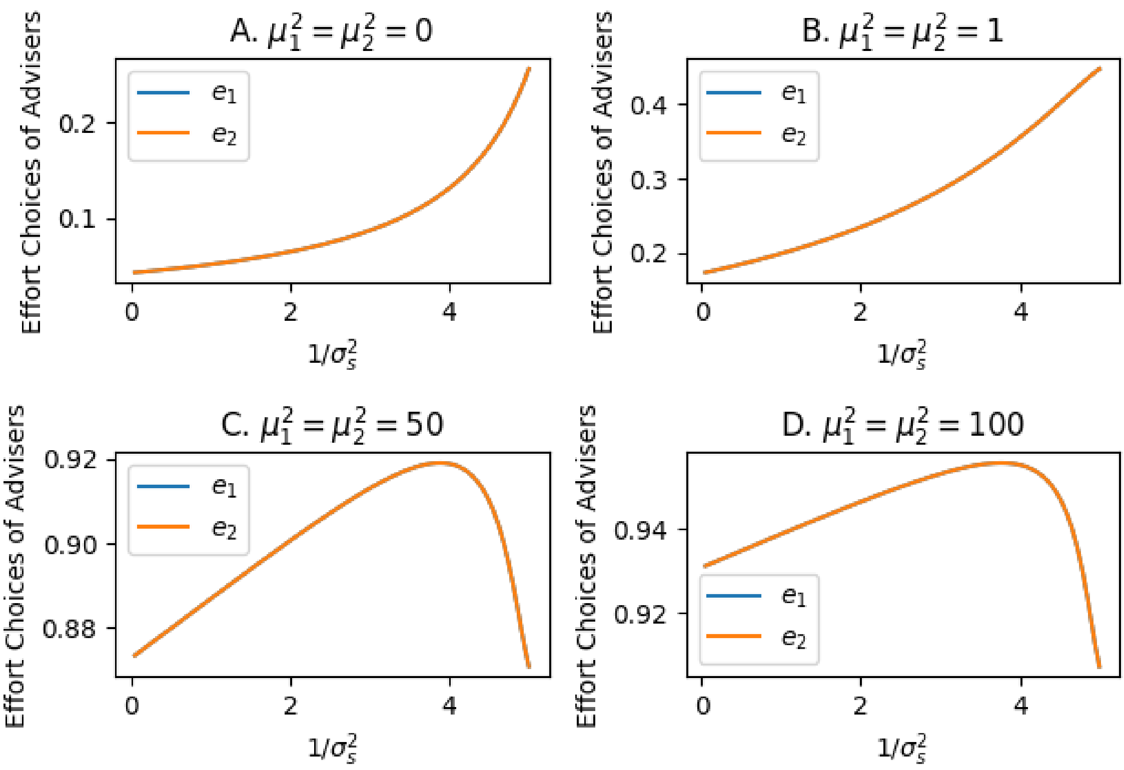
5.2.2 Signal Precision

In the one adviser case, a less precise signal implies a lower equilibrium effort level for the adviser (Che and Kartik, 2009). The reason is that the marginal benefit of obtaining a less precise signal for an adviser is lower since it provides less uncertainty reduction and it is harder to persuade the DM with a less precise signal. However, the effect of signal precision on the equilibrium effort choices is ambiguous when there are two advisers. This is because an adviser's effort is negatively affected by the other adviser's effort and the magnitude of this effect becomes smaller as the precision of the signal decreases. In this case, the adviser has an incentive to increase his effort because his incentive to free ride over the other adviser's effort decreases. Therefore, there are two opposite effects of signal precision on the equilibrium effort choices, and it is not certain which one of these effects is dominant. The following corollary formally states this result:

Corollary 5.3: The effect of signal precision on the equilibrium effort levels is ambiguous.

To illustrate the above relationship, I first consider the case of identical types, i.e., $\mu_1 = \mu_2$. The following figure shows how the optimal effort choices change with signal precision $1/\sigma_s^2$.

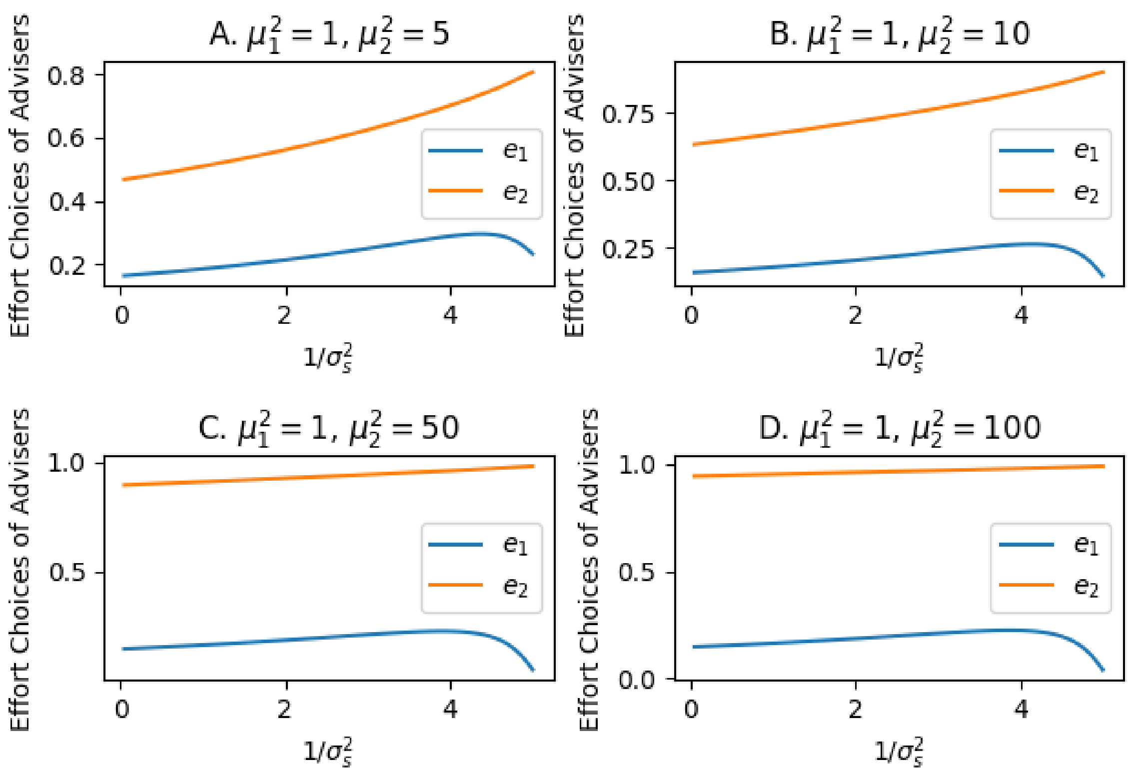
Figure 5.2: The Effect of Signal Precision on Equilibrium Effort Choices



First, note that since the focus is on the symmetric equilibrium, the graphs of e_1 and e_2 overlap. As it can be seen from Figure 5.2.A and 5.2.B, the effort levels of both advisers decrease with signal variance when the advisers have no disagreement with the DM or their disagreement is "low". However, one can observe from Figure 5.2.C and 5.2.D that the effort of each adviser is higher for higher signal precision up to a point. Furthermore, as the advisers' differences of opinion with the DM increase, their effort increases for even larger values of σ_s^2. The reason is that an adviser's incentive to free ride over the other adviser's effort is lower in this case. If the signal precision is high enough to begin with, a decline in signal precision reduces an adviser's free riding incentive, which surpasses the increment in his incentive to persuade the DM and reducing the uncertainty about the state. In other words, an adviser may find it optimal to increase his effort in response to a decline in signal precision if the other adviser reduces his effort. As long as the signal precision is high enough, each adviser exerts more effort, so that they can compensate the decline in each other's persuasion and uncertainty reduction incentives. This behavior persists even longer for advisers with higher difference of opinion with the DM since they benefit from information acquisition more, compared to advisers with lower differences of opinion with the DM.

To observe what happens in a non-symmetric equilibrium, I run a simulation by fixing μ_1^2 at 1 and assigning four different values to μ_2^2. The following figure shows the result of the simulation.

Figure 5.3: The Effect of Signal Precision on Equilibrium Effort Choices



First of all, one can observe that in Figure 5.3, only the adviser with a lower difference of opinion with the DM increases his effort when the signal precision is very high. This is because the adviser with a lower difference of opinion with the DM has a lower incentive to persuade the DM (Corollary 1). Hence, the decline in his incentive to free ride over the other adviser's effort surpasses his persuasion incentive, and his effort is higher as a result. Furthermore, Adviser 2 with higher $|\mu_2|$ is less responsive to a decline in signal precision. The reason is that his effort is higher due to his larger difference of opinion and the effect of signal precision on the persuasion incentive of an adviser is bounded. In particular, as $\sigma_s^2 \to \infty$, $\Phi(\mu_i^2) = \mu_i^2$ and $O(\mu_i^2) = \sigma_0^4 + \mu_i^2$ so that the marginal benefit of Adviser 2 is $\sigma_0^4(1 - e_1) + \mu_2^2$. For higher $|\mu_2|$, his marginal benefit is higher since e_1 is lower and μ_2^2 is higher in this case.

As a final note to this section, observe that the sign of μ_i is not important for advisers' effort choices. Even if the advisers have opposite opinions, what matters is their distances with the DM's opinion. That is, adviser i with type μ_i is treated the same as adviser i with type $-\mu_i$.

Chapter 6

THE OPTIMAL DIFFERENCES OF OPINION FOR THE DM

In this chapter, I allow the DM to choose the type of the advisers, $|\mu_1|$ and $|\mu_2|$, and show that she prefers to work with advisers with as high difference of opinion as possible. To perform this analysis, I begin with defining the DM's welfare by using the equilibrium effort choices of the advisers.

The DM's expected utility can be derived for each set of signals S by plugging her optimal action $\alpha(S|0)$ into her expected utility. By using the mean-variance decomposition, the DM's expected utility is $-\sigma_0^2$ if there is no signal, $-\tilde{\sigma}_1^2$ if there is one signal and $-\tilde{\sigma}_2^2$ if there are two signals. Note that in equilibrium, the DM observes no signal with probability $(1-e_1^*)(1-e_2^*)$, one signal with probability $e_1^*(1-e_2^*)+e_2^*(1-e_1^*)$, and two signals with probability $e_1^*e_2^*$. Then, the DM's expected utility can be expressed as follows.

$$\mathbb{E}[U_{DM}] = e_1^*e_2^*(-\tilde{\sigma}_2^2) + [e_1^*(1-e_2^*) + e_2^*(1-e_1^*)](-\tilde{\sigma}_1^2) + (1-e_1^*)(1-e_2^*)(-\sigma_0^2)$$

The first observation I obtain from this expression is that the DM's welfare is increasing with advisers' equilibrium effort choices.

Proposition 6.1: The DM's welfare, measured by $\mathbb{E}[U_{DM}]$, increases with both e_1^* and e_2^*.

This result can be interpreted easily. As an adviser exerts more effort, the probability of acquiring a signal increases. So, the DM takes an action with more information, i.e., an action that is closer to the true state, with a higher probability. Hence, the DM is better-off as advisers exert higher effort.

Che and Kartik (2009) show that if the DM can choose the type of the adviser, she is better-off by choosing an adviser with at least some difference of opinion instead of choosing a like-minded one. In their setup, the DM faces the following trade-off. While an adviser with a higher difference of opinion has higher persuasion incentive, he withholds more information as the adviser can choose to conceal the information he obtains. In my setup, the DM's trade-off is a different one. If she chooses adviser i with higher $|\mu_i|$, the adviser exerts higher effort but adviser j exerts lower effort.

Next, I show that the DM is better-off by choosing $|\mu_1| = |\mu_2| > 0$ instead of choosing $|\mu_1| = |\mu_2| = 0$, and if she does, she must choose $|\mu_i|$ as high as possible. I start by showing that if $|\mu_1| = |\mu_2| = |\mu|$, then the effort choices of both advisers increase as $|\mu|$ increases. The following lemma formally states this result.

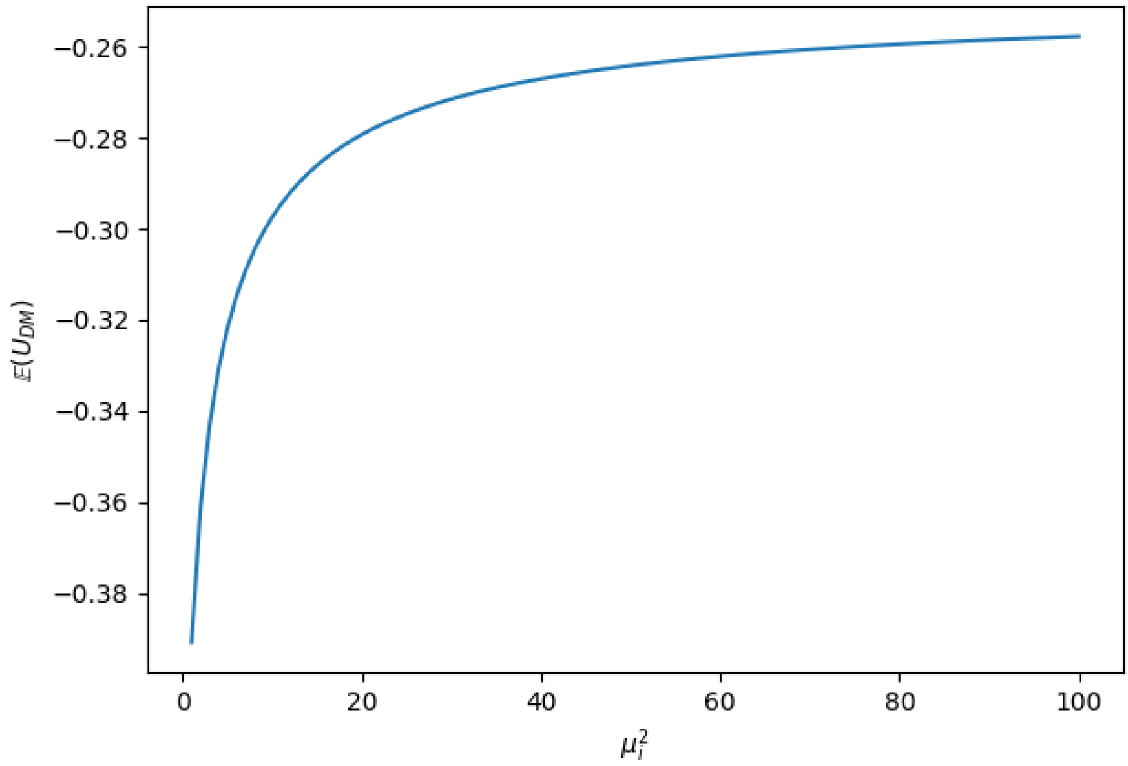
Lemma 6.1: Let $|\mu_1| = |\mu_2| = |\mu|$. Then $\dfrac{\partial e_i}{\partial \mu^2} > 0$ for $i = 1, 2$.

Lemma 6.1 implies that the DM can increase the effort choices of both advisers by choosing advisers with a higher difference of opinion with her, but the advisers must be equally distant to the DM's opinion. Since the expected utility of the DM increases with both e_1 and e_2, choosing advisers with higher $|\mu|$ increases the expected utility of the DM. In other words, the DM must choose $|\mu|$ as high as possible, which is formally stated by the following proposition.

Proposition 6.2: If the DM is allowed to choose the types of each adviser, she is strictly better off by choosing $\mu_i = max\{|\underline{\mu}|, \overline{\mu}\}$ instead of choosing $\mu_i = 0$, $i = 1, 2$.

Proposition 6.2 demonstrates that it is better for the DM to choose advisers with extreme differences of opinions rather than choosing two like-minded advisers. The following figure shows the expected utility of the DM as $|\mu_i|$ goes from 0 to 100, $i = 1, 2$.

Figure 6.1: The Expected Utility of the DM



By increasing $|\mu_i|$ for each adviser i, the DM increases the persuasion incentive of both advisers and induce each one of them to exert more effort. However, Proposition 6.2 does not state that $\mu_i = max\{|\underline{\mu}, \overline{\mu}\}$, $i = 1, 2$, maximizes the expected utility of the DM. Instead, it shows that for the DM, choosing $\mu_i = max\{|\underline{\mu}, \overline{\mu}\}$ is strictly dominant to choosing $|\mu_i| = 0$, $i = 1, 2$. In other words, the DM is better off by hiring two advisers with identical and extreme differences of opinion instead of hiring two like-minded advisers.

Chapter 7

EXTENSION: EQUILIBRIUM WITH A HETEROGENEOUS SIGNAL STRUCTURE

In this section, I add another layer of heterogeneity to the model by allowing each adviser to obtain signals with different variances, i.e., $s_i \sim \mathcal{N}(\omega, \sigma_i^2)$ for $i = 1, 2$. With this difference, the signal precision is no longer the same across advisers, and this additional layer of heterogeneity enables me to show some additional results.

Since the variances of the signals are different, their precision are different too. Hence, the DM attaches different weights to each signal in her optimal action. If only adviser i acquires a signal, then the DM's optimal action is given by $\alpha(s_i, 0) = \rho_{1i} s_i$, where $\rho_{1i} = \dfrac{\sigma_0^2}{\sigma_0^2 + \sigma_i^2}$ is the weight that the DM attaches to signal s_i on her posterior. If there are two signals, the DM takes action $\alpha(\{s_1, s_2\}, 0) = \rho_{21} s_1 + \rho_{22} s_2$, where $\rho_{2i} = \dfrac{\sigma_0^2 \sigma_j^2}{\sigma_0^2 \sigma_1^2 + \sigma_0^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2}$ is the weight assigned to signal s_i. The posterior variance of the state is $\tilde{\sigma}_{1i}^2 = \dfrac{\sigma_0^2 \sigma_i^2}{\sigma_0^2 + \sigma_i^2}$ if only adviser i acquires a signal and $\tilde{\sigma}_2^2 = \dfrac{\sigma_0^2 \sigma_1^2 \sigma_2^2}{\sigma_0^2 \sigma_1^2 + \sigma_0^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2}$ if each adviser acquires a signal.

As the main model, the preferred action of each adviser differs from the DM's action by their posterior bias. If only adviser i acquires a signal, then the posterior bias of adviser i is given by $B(s_i, \mu_i) = (1 - \rho_{1i})\mu_i$ while the other adviser has posterior bias $B(s_i, \mu_j) = (1 - \rho_{1i})\mu_j$. So, if the signals have different variances, then the posterior bias depends on which adviser acquires the signal in the one-signal case. If both advisers acquire a signal, then each one has posterior bias $B(\{s_1, s_2\}, \mu_i) = (1 - \rho_{21} - \rho_{22})\mu_i$. Note that $\rho_{21} + \rho_{22} > \rho_{1i}$ for $i = 1, 2$, i.e., the posterior bias of adviser i is lower if there are two signals instead of one, as in the main model. To sum up, heterogeneity in signal variances changes the posterior bias, so that if only one signal is acquired, then the adviser who obtains the signal matters. If two signals are acquired, however, only the magnitude of posterior bias changes since the DM attaches different weights to each signal in posterior.

In equilibrium, each adviser exerts effort $e_i^* \in (0, \overline{e})$, which satisfies the following first order conditions:

$$c'(e_i^*) = (\Phi_i(\mu_i^2) - O_i(\mu_i^2))e_j^* + O_i(\mu_i^2)$$

for $i = 1, 2$, where

$$\Phi_i(\mu_i^2) = \tilde{\sigma}_{1j}^2 - \tilde{\sigma}_2^2 + [B(s_j, \mu_i)]^2 - [B(\{s_1, s_2\}, \mu_i)]^2,$$
$$O_i(\mu_i^2) = \sigma_0^2 - \tilde{\sigma}_{1i}^2 + [B(\emptyset, \mu_i)]^2 - [B(s_i, \mu_i)]^2$$

for $j = 1, 2$, $i \neq j$. Note that since there is a second layer of heterogeneity in the model with the introduction of different signal variances, the functions $\Phi(\cdot)$ and $O(\cdot)$ are defined such that they are specific to each adviser.

Next, by using the first order conditions, I analyze the effect of signal precision on the equilibrium effort choices. In the main analysis, I show that if the signals have the same variance, then how the equilibrium effort choices of advisers depend on signal precision is ambiguous. Having signals with different variances resolves the ambiguity as follows.[10]

Corollary 7.1: As σ_i^2 increases (i.e., the precision of s_i decreases), e_i decreases and e_j increases.

Corollary 7.1 implies that adviser i exerts more effort to obtain a more precise signal, while adviser j exerts less effort if the precision of s_i is smaller. In other words, adviser j's effort is decreasing in the precision of s_i. So the effects of signal precision and adviser types on advisers' effort choices are similar if signals have heterogeneous

[10]Note that the previous results, except Corollary 5.3, hold automatically under heterogeneous signal variances.

variance. To illustrate how the ambiguity disappears, I replicate Figure 5.2 by fixing $\sigma_2^2 = 1$.

Figure 7.1: The Effect of Signal 1's Precision on Equilibrium Effort Choices

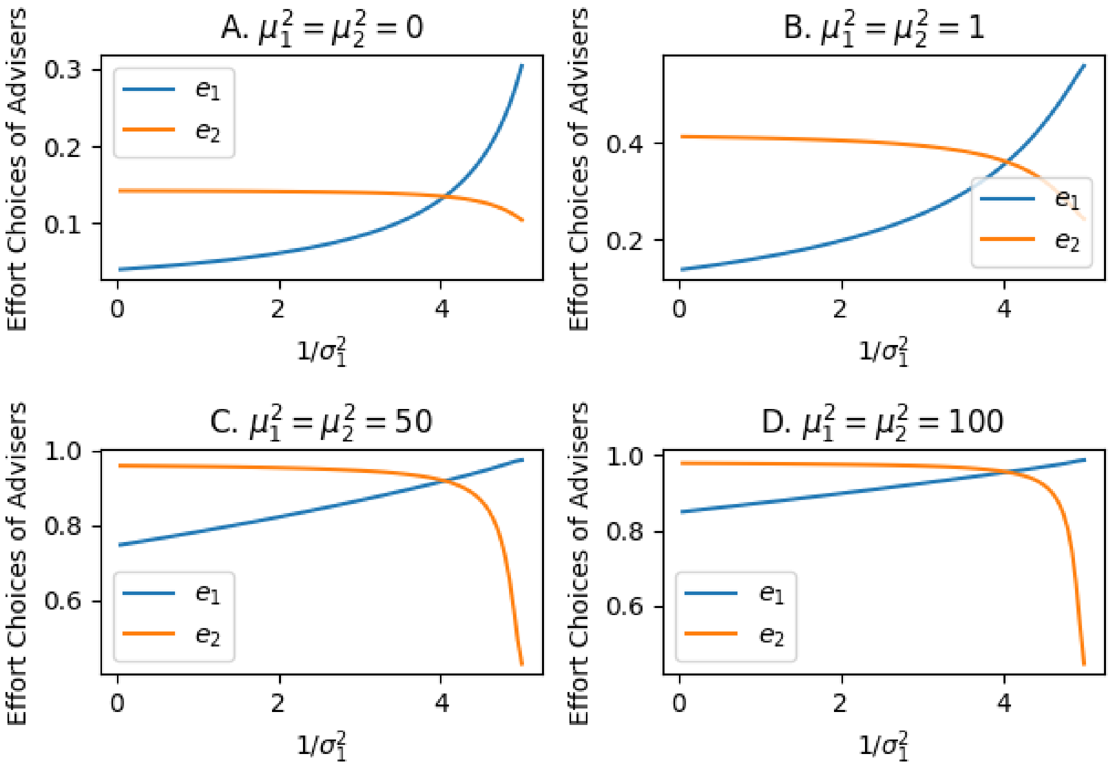


Figure 7.1 shows that Adviser 1's effort decreases as σ_1^2 increases, i.e., the precision of s_1 decreases, while Adviser 2's effort increases. These results apply regardless of how large the difference of opinion between the advisers and the DM is. Hence the ambiguity shown in Figure 5.2 disappears in Figure 7.1 when the signals have different precision.

Chapter 8

CONCLUSION

In this study, I analyzed a persuasion game with a decision maker and two advisers, where advisers and the DM have the same preferences but their opinion about the optimal action are different. Advisers acquire costly signals, which are publicly observed. I first show that from the perspective of each adviser i, his signal s_i provides more uncertainty reduction and more persuasive. Therefore, the marginal benefit of exerting effort for adviser i decreases as adviser j increases his effort e_j, leading to advisers free-ride over the efforts of each other.

In equilibrium, each adviser exerts effort to acquire information. Since they have the incentive to free-ride over each other's efforts, their effort choices are strategic substitutes. Then, I perform comparative statics analyses on adviser types and signal precision. I find that the effort choice of each adviser increases with his own difference of opinion with the DM and decreases with the other adviser's difference of opinion with the DM. This contrast is due to the strategic substitubility of effort choices. In addition, when each adviser obtains a signal with the same precision, a higher precision may not always elicit higher effort choice from an adviser. However, if advisers obtain signals with different precision, then an adviser increases his effort if his signal is more precise, and he reduces his effort if the other adviser's signal is more precise. The latter result is again due to the strategic substitubility of effort choices. In other words, making signal precision heterogeneous resolves the ambiguity in the effect of signal precision on advisers' effort choices.

Next, I allow the DM to choose the types of each adviser, and prove that the DM is better-off by choosing two advisers with extreme differences of opinion instead of choosing two like-minded advisers. It is not certain, however, that choosing advisers with extreme differences of opinion with the DM maximizes the expected utility of the DM. For example, the DM might be better off by choosing a like-minded adviser and an adviser with extreme difference of opinion with her, or she might choose two advisers with a moderate differences of opinion with her. The complete solution of the DM's optimal type choice problem is left for future research.

In the current study, I analyze a model where signals are publicly observed. As a result, the sign of the type of an adviser is not important in the analysis. However, it becomes important if the advisers are allowed to withhold information they acquire. Hence, examining the model with strategic information disclosure can be an interesting question to answer. Another possible direction to follow is to incorporate the DM's delegation decision. Even though the adviser who is given the decision rights loses the persuasion incentive, the other adviser, who does not have decision rights, may increase his effort. If the types of the two advisers are not both positive (or negative), then the adviser with no decision rights has a higher persuasion incentive under delegation. The implications of these observations on equilibrium effort choices and the expected utility of the DM awaits further research.

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Appendix A

DERIVATION OF THE POSTERIOR DISTRIBUTION OF THE STATE

Let Z be an arbitrary $(n+m)$-vector of random variables, where $(X)_n$ and $(Y)_m$ are two sets of random variables constituting $Z = \begin{pmatrix} X \\ Y \end{pmatrix}$.

Let the mean vector of X be μ_X and the mean vector of Y be μ_Y. Let $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{21} \end{pmatrix}$ be the covariance matrix of these two sets of variables, where Σ_{11} gives variances and covariances for X vector, Σ_{22} gives variances and covariances for Y vector, Σ_{12} and Σ_{21} give covariances between the variables in X and the variables in Y. The mean vector of Z can be written as $\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}$.

Then, for the following distribution;

$$Z = \begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$$

the conditional distribution of X given known values of $Y = y$ is given by

$$X|Y \sim \mathcal{N}(\mu_X + \Sigma_{12}\Sigma_{22}^{-1}(y - \mu_Y), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}) \tag{A.1}$$

Means: From its distribution, it is clear that for adviser $i = 1, 2$, $\mathbb{E}[\omega] = \mu_i$ and for the DM, $\mathbb{E}[\omega] = 0$. For $s_i \in \{s_1, s_2\}$, I use the law of iterated expectations (LIE):

$$\mathbb{E}[s_i] = \mathbb{E}\{E[s_i|\omega]\} = \mathbb{E}[\omega] = \mu_i.$$

Variances: Again, it is clear that $\mathbb{V}(\omega) = \sigma_0^2$. For the variance of s_1 and s_2, I apply the law of iterated variances (LIV),

$$\mathbb{V}(s_i) = \mathbb{E}[\mathbb{V}(s_i|\omega)] + \mathbb{V}[\mathbb{E}(s_i|\omega)] = \mathbb{E}[\sigma_s^2] + \mathbb{V}[\omega] = \sigma_s^2 + \sigma_0^2,$$

Covariances: First, note that

$$\omega = \mu_i + \sigma_0 \epsilon_0$$
$$s_1 = \omega + \sigma_s \epsilon_1$$
$$s_2 = \omega + \sigma_s \epsilon_2$$

where $\epsilon_0, \epsilon_1, \epsilon_2 \sim \mathcal{N}(0,1)$ i.i.d. Then $cov(s_1, s_2) = cov(\omega + \sigma_s\epsilon_1, \omega + \sigma_s\epsilon_2) = cov(\omega, \omega) + \sigma_s^2 cov(\epsilon_1, \epsilon_2) = \sigma_0^2$.

To derive $cov(\omega, s_i)$, I use the LIE again,

$$cov(\omega, s_i) = \mathbb{E}[(s - \mu_i)(\omega - \mu_i)] = E[s\omega] - \mu_i^2$$
$$= \mathbb{E}\{\mathbb{E}[s\omega|\omega]\} - \mu_i^2 = \mathbb{E}\{\omega\mathbb{E}[s|\omega]\} = \mathbb{E}[\omega^2] - \mu_i^2 = \sigma_0^2.$$

For our model, if only agent $i \in \{1, 2\}$ acquired a signal, then $X = \omega$, $Y = s_i$, and their joint distribution is written as

$$\begin{pmatrix} \omega \\ s_i \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_i \\ \mu_i \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 + \sigma_i^2 \end{pmatrix}\right]$$

so that $\mu_X = \mu_Y = \mu_i$, $\Sigma_{11} = \sigma_0^2$, $\Sigma_{12} = \Sigma_{21} = \sigma_0^2$ and $\Sigma_{22} = \sigma_0^2 + \sigma_i^2$. Then a simple calculation based on (A.1) yields

$$\omega|s_i \sim \mathcal{N}(\rho_{1i}s_1 + (1 - \rho_{1i})\mu_i, \tilde{\sigma}_{1i}^2)$$

where $\rho_{1i} = \dfrac{\sigma_0^2}{\sigma_0^2 + \sigma_i^2}$ and $\tilde{\sigma}_{1i}^2 = \dfrac{\sigma_0^2\sigma_i^2}{\sigma_0^2 + \sigma_i^2}$.

If both agents were able to acquire a signal, then $X = \omega$, $Y = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$ and their joint distribution is given by (3.1). Given that, $\mu_X = \mu_i$, $\mu_Y = \begin{pmatrix} \mu_i \\ \mu_i \end{pmatrix}$, $\Sigma_{11} = \sigma_0^2$, $\Sigma_{12} = \begin{pmatrix} \sigma_0^2 & \sigma_0^2 \end{pmatrix}$, $\Sigma_{21} = \begin{pmatrix} \sigma_0^2 \\ \sigma_0^2 \end{pmatrix}$ and $\Sigma_{22} = \begin{pmatrix} \sigma_0^2 + \sigma_1^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 + \sigma_2^2 \end{pmatrix}$, a simple calculation based on (A.1) yields

$$\omega | s_1, s_2 \sim \mathcal{N}(\rho_{21} s_1 + \rho_{22} s_2 + (1 - \rho_{21} - \rho_{22})\mu_i, \tilde{\sigma}_2^2).$$

where $\rho_{2i} = \dfrac{\sigma_0^2 \sigma_j^2}{\sigma_0^2\sigma_1^2 + \sigma_0^2\sigma_2^2 + \sigma_1^2\sigma_2^2}$, $i \neq j$, and $\tilde{\sigma}_2^2 = (1 - \rho_{21} - \rho_{22})\sigma_0^2$.

Appendix B

PROOFS

Proof of Lemma 1

Throughout the proof, without loss of generality let $i = 1, j = 2$. I will first prove $\sigma_0^2 - \tilde{\sigma}_1^2 > \tilde{\sigma}_1^2 - \tilde{\sigma}_2^2$.
By using the definitions of posterior variances, $\sigma_0^2 - \tilde{\sigma}_1^2 = \sigma_0^2 - \dfrac{\sigma_0^2\sigma_s^2}{\sigma_0^2+\sigma_s^2} = \dfrac{\sigma_0^4}{\sigma_0^2+\sigma_s^2}$.
On the other hand,

$$\tilde{\sigma}_1^2 - \tilde{\sigma}_2^2 = \frac{\sigma_0^2\sigma_s^2}{\sigma_0^2+\sigma_s^2} - \frac{\sigma_0^2\sigma_s^2}{2\sigma_0^2+\sigma_s^2} = \frac{\sigma_0^4\sigma_s^2}{(\sigma_0^2+\sigma_s^2)(2\sigma_0^2+\sigma_s^2)}$$

Comparing these two terms and performing some algebraic manipulations yield the result.

To show $[B(\emptyset,\mu_i)]^2 - [B(s_i,\mu_i)]^2 > [B(s_i,\mu_i)]^2 - [B(\{s_1,s_2\},\mu_i)]^2$, first note that if $\mu_i = 0$ for adviser i, then all posterior biases will be equal to 0 and there will not be any persuasion incentive for adviser i. Next, suppose $\mu_i \neq 0$. I will show $[B(\emptyset,\mu_i)]^2 + B(\{s_1,s_2\},\mu_i)]^2 > 2[B(s_i,\mu_i)]^2$.

$$[B(\emptyset,\mu_i)]^2 + [B(\{s_1,s_2\},\mu_i)]^2 = \left[1 + \frac{\sigma_s^4}{(2\sigma_0^2+\sigma_s^2)^2}\right]\mu_i^2$$
$$2[B_1(\mu_1)]^2 = 2\left[\left(\frac{\sigma_s^2}{\sigma_0^2+\sigma_s^2}\right)^2\right]\mu_i^2$$

Comparing these two terms, $[B(\emptyset,\mu_i)]^2 + B(\{s_1,s_2\},\mu_i)]^2 > 2[B(s_i,\mu_i)]^2$, Q.E.D.

Proof of Corollary 1

First, I take $i = 1$ and prove that if e_1 increases with $|\mu_1|$, then e_2 decreases with $|\mu_1|$ (and if e_1 decreases with $|\mu_1|$, then e_2 increases with $|\mu_1|$). Write the first order conditions,

$$c'(e_1) = (\Phi(\mu_1^2) - O(\mu_1^2))e_2 + O(\mu_1^2) \tag{B.1}$$

$$c'(e_2) = (\Phi(\mu_2^2) - O(\mu_2^2))e_1 + O(\mu_2^2). \tag{B.2}$$

Take the derivative of both equations with respect to μ_1^2 by using the Implicit Function Theorem,

$$\begin{aligned} c''(e_1)\frac{\partial e_1}{\partial \mu_1^2} =& e_2\frac{\partial \Phi(\mu_1^2)}{\partial \mu_1^2} + (1-e_2)\frac{\partial O(\mu_1^2)}{\partial \mu_1^2} \\ &+ (\Phi(\mu_1^2) - O(\mu_1^2))\frac{\partial e_2}{\partial \mu_1^2} \end{aligned} \tag{B.3}$$

$$c''(e_2)\frac{\partial e_2}{\partial \mu_1^2} = (\Phi(\mu_2^2) - O(\mu_2^2))\frac{\partial e_1}{\partial \mu_1^2} \tag{B.4}$$

Since $\Phi(\mu_2^2) - O(\mu_2^2) < 0$ by Lemma 1 and $c''(\cdot) > 0$ by the convexity of the cost function, equation (4) implies that $\frac{\partial e_1}{\partial \mu_1^2}$ has the opposite sign with $\frac{\partial e_2}{\partial \mu_1^2}$. Hence, if e_1 increases (decreases) with $|\mu_1|$, then e_2 decreases (increases) with $|\mu_1|$.

Next, I show that $\frac{\partial e_1}{\partial \mu_1^2} > 0$ (and $\frac{\partial e_2}{\partial \mu_1^2} < 0$). Plugging $\frac{\partial e_2}{\partial \mu_1^2}$ into (3) gives

$$\frac{\partial e_1}{\partial \mu_1^2} = \frac{c''(e_2)\left\{e_2\frac{\partial \Phi(\mu_1^2)}{\partial \mu_1^2} + (1-e_2)\frac{\partial O(\mu_1^2)}{\partial \mu_1^2}\right\}}{c''(e_1)c''(e_2) - [\Phi(\mu_1^2) - O(\mu_1^2)][\Phi(\mu_2^2) - O(\mu_2^2)]} \tag{B.5}$$

The term in the numerator, $\frac{\partial \Phi(\mu_1^2)}{\partial \mu_1^2} + (1-e_2)\frac{\partial O(\mu_1^2)}{\partial \mu_1^2}$ is always strictly positive, since $\frac{\partial \Phi(\mu_1^2)}{\partial \mu_1^2} = (1-\rho_1^2) - (1-2\rho_2)^2 > 0$ and $\frac{\partial O(\mu_1^2)}{\partial \mu_1^2} = 1 - (1-\rho_1)^2 > 0$. Thus, $\frac{\partial e_1}{\partial \mu_1^2}$ is negative if and only if $c''(e_1)c''(e_2) - [\Phi(\mu_1^2) - O(\mu_1^2)][\Phi(\mu_2^2) - O(\mu_2^2)] < 0$. Next, I claim that it is not possible to have $\frac{\partial e_1}{\partial \mu_1^2} < 0$.

First, I show that e_1 must either always increase or always decrease with $|\mu_1|$. Suppose there exists $\mu_1^2 = x$ such that $\frac{dc'(e_1)}{d\mu_1^2}\Big|_{\mu_1^2 = x} = c''(e_1)\frac{\partial e_1}{\partial \mu_1^2}\Big|_{\mu_1^2 = x} < 0$ and $\mu_1^2 = y$

such that $\left.\frac{dc'(e_1)}{d\mu_1^2}\right|_{\mu_1^2=y} = c''(e_1)\left.\frac{\partial e_1}{\partial \mu_1^2}\right|_{\mu_1^2=y} > 0$, where $x < y$ without loss of generality. Then, by Darboux's Theorem, there exists $\mu_1^2 = z \in [x, y]$ such that $\left.\frac{dc'(e_1)}{d\mu_1^2}\right|_{\mu_1^2=z} = 0$. But it is impossible since the right hand side of equation (5) is never zero. Therefore, it is not possible to have both $\left.\frac{dc'(e_1)}{d\mu_1^2}\right|_{\mu_1^2=x} < 0$ and $\left.\frac{dc'(e_1)}{d\mu_1^2}\right|_{\mu_1^2=y} > 0$. Hence, it is either $\frac{\partial e_1}{\partial \mu_1^2} > 0$ for all $\mu_1^2 \in [0, \infty)$ or $\frac{\partial e_1}{\partial \mu_1^2} < 0$ for all $\mu_1^2 \in [0, \infty)$.

Second, I prove that $\frac{\partial e_1}{\partial \mu_1^2} < 0$ is not possible. By plugging $\mu_1^2 = 0$ into equation (1), I obtain that

$$c'(e_1) = (\tilde{\sigma}_1^2 - \tilde{\sigma}_2^2)e_2 + (\sigma_0^2 - \tilde{\sigma}_1^2)(1 - e_2) = K > 0 \tag{B.6}$$

and $K < \infty$, and

$$\lim_{\mu_1^2 \to \infty} c'(e_1) = \lim_{\mu_1^2 \to \infty} \Phi(\mu_1^2)e_2 + O(\mu_1^2)(1 - e_2) = \infty. \tag{B.7}$$

Hence, it is not possible that $c'(e_1)$ is decreasing for all $\mu_1^2 \in [0, \infty)$. Then $c'(e_1)$, i.e. e_1, must increase with all $\mu_1^2 \in [0, \infty)$ by the convexity of $c(\cdot)$, and hence it must increase with all $\mu_1^2 \in [0, \max\{\underline{\mu}^2, \overline{\mu}^2\}]$. Q.E.D. [11]

Proof of Corollary 2

Assume that $|\mu_1| > |\mu_2|$ but $e_1 < e_2$. Then increasing $|\mu_2|$ to $|\mu_1|$ should increase e_2 and decrease e_1 by Corollary 1, hence $e_1 < e_2$ still holds, but it contradicts with equilibrium being symmetric when $|\mu_1| = |\mu_2|$. Hence, it must hold that if $|\mu_1| > |\mu_2|$ then $e_1 > e_2$. Q.E.D.

[11]Note that this proof implies that $c''(e_1)c''(e_2) - [\Phi(\mu_1^2) - O(\mu_1^2)][\Phi(\mu_2^2) - O(\mu_2^2)] > 0$ always holds.

Proof of Corollary 3

First, I write the first order conditions

$$c'(e_1) = (\Phi(\mu_1^2) - O(\mu_1^2))e_2 + O(\mu_1^2) \tag{B.8}$$

$$c'(e_2) = (\Phi(\mu_2^2) - O(\mu_2^2))e_1 + O(\mu_2^2). \tag{B.9}$$

Take the derivative of the right hand side of adviser i's FOC with respect to σ_s^2, $i = 1, 2$:

$$c''(e_i)\frac{\partial e_i}{\partial \sigma_s^2} = (\Phi(\mu_i^2) - O(\mu_i^2))\frac{\partial e_j}{\partial \sigma_s^2} + \left(\frac{\partial \Phi(\mu_i^2)}{\partial \sigma_s^2} - \frac{\partial O(\mu_i^2)}{\partial \sigma_s^2}\right)e_j + \frac{\partial O(\mu_i^2)}{\partial \sigma_s^2} \tag{B.10}$$

After some calculations, I obtain that

$$\frac{\partial(\Phi(\mu_i^2) - O(\mu_i^2))}{\partial \sigma_s^2} = \frac{2\sigma_0^6(3\sigma_0^2 + 2\sigma_s^2)}{(\sigma_0^2 + \sigma_s^2)^2(2\sigma_0^2 + \sigma_s^2)^2} + \frac{4\sigma_0^4\sigma_s^2(7\sigma_0^4 + 9\sigma_0^2\sigma_s^2 + 3\sigma_s^4)}{(\sigma_0^2\sigma_s^2)^3(2\sigma_0^2 + \sigma_s^2)^3}\mu_i^2 > 0 \tag{B.11}$$

$$\frac{\partial O(\mu_i^2)}{\partial \sigma_s^2} = -\frac{\sigma_0^2(\sigma_0^4 + \sigma_0^2\sigma_s^2)}{(\sigma_0^2 + \sigma_s^2)^3} - \frac{2\sigma_0^4\mu_i^2}{(\sigma_0^2 + \sigma_s^2)^3} < 0 \tag{B.12}$$

However, since

$$\frac{\partial \Phi(\mu_i^2)}{\partial \sigma_s^2} = \frac{\sigma_0^4(2\sigma_0^4 - \sigma_s^4)}{(\sigma_0^2 + \sigma_s^2)^2(2\sigma_0^2 + \sigma_s^2)^2} + \frac{2\sigma_0^2\sigma_s^2(6\sigma_0^6 + 6\sigma_0^4\sigma_s^2 - \sigma_s^6)}{(\sigma_0^2 + \sigma_s^2)^3(2\sigma_0^2 + \sigma_s^2)^3}\mu_i^2 \tag{B.13}$$

is negative if $\sigma_s^6 > 6\sigma_0^6 + 6\sigma_0^4\sigma_s^2$, and positive if $2\sigma_0^4 > \sigma_s^4$, the sum $\left(\frac{\partial \Phi(\mu_1^2)}{\partial \sigma_s^2} - \frac{\partial O(\mu_1^2)}{\partial \sigma_s^2}\right)e_2 + \frac{\partial O(\mu_1^2)}{\partial \sigma_s^2}$ is not always positive or negative[12]. By plugging

[12]Note that $\sigma_s^6 > 6\sigma_0^6 + 6\sigma_0^4\sigma_s^2$ implies $\sigma_s^4 > 2\sigma_0^4$ and $2\sigma_0^4 > \sigma_s^4$ implies $6\sigma_0^6 + 6\sigma_0^4\sigma_s^2 > \sigma_s^6$

$\dfrac{\partial e_j}{\partial \sigma_s^2}$ into (C.10), I obtain

$$\frac{\partial e_1}{\partial \sigma_s^2} = \frac{c''(e_2)}{c''(e_1)c''(e_2) - (\Phi(\mu_1^2) - O(\mu_1^2))(\Phi(\mu_2^2) - O(\mu_2^2))}$$
$$\left\{ \frac{(\Phi(\mu_1^2) - O(\mu_1^2))}{c''(e_1)} \left(\frac{\partial \Phi(\mu_2^2)}{\partial \sigma_s^2} e_1 + \frac{\partial O(\mu_2^2)}{\partial \sigma_s^2}(1 - e_1) \right) + \frac{\partial \Phi(\mu_1^2)}{\partial \sigma_s^2} e_2 + \frac{\partial O(\mu_1^2)}{\partial \sigma_s^2}(1 - e_2) \right\} \tag{B.14}$$

From the proof of Corollary 1, $c''(e_1)c''(e_2) - (\Phi(\mu_1^2) - O(\mu_1^2))(\Phi(\mu_2^2) - O(\mu_2^2)) > 0$, but the term in the brackets is not always positive or negative, hence the sign of $\dfrac{\partial e_1}{\partial \sigma_s^2}$ is ambiguous. Q.E.D.

Proof of Corollary 4

Write the first order conditions

$$c'(e_1) = [\Phi_1(\mu_1^2) - O_1(\mu_1^2)]e_2 + O_1(\mu_1^2) \tag{B.15}$$
$$c'(e_2) = [\Phi_2(\mu_2^2) - O_2(\mu_2^2)]e_1 + O_1(\mu_2^2) \tag{B.16}$$

and take derivative of each equation with respect to σ_1^2.

$$c''(e_1)\frac{\partial e_1}{\partial \sigma_1^2} = e_2 \frac{\partial \Phi_1(\mu_1^2)}{\partial \sigma_1^2} + (1 - e_2)\frac{\partial O_1(\mu_1^2)}{\partial \sigma_1^2} + [\Phi_1(\mu_1^2) - O_1(\mu_1^2)]\frac{\partial e_2}{\partial \sigma_1^2} \tag{B.17}$$
$$c''(e_2)\frac{\partial e_2}{\partial \sigma_1^2} = \frac{\partial \Phi_2(\mu_2^2)}{\partial \sigma_1^2} e_1 + [\Phi_2(\mu_2^2) - O_2(\mu_2^2)]\frac{\partial e_1}{\partial \sigma_1^2} \tag{B.18}$$

since $\dfrac{\partial O_2(\mu_2^2)}{\partial \sigma_1^2} = 0$. For the other variables,

$$\frac{\partial \Phi_1(\mu_1^2)}{\partial \sigma_1^2} = -\frac{\sigma_0^4 \sigma_2^4}{(\sigma_0^2\sigma_1^2 + \sigma_0^2\sigma_2^2 + \sigma_1^2\sigma_2^2)^2} - \frac{2\sigma_0^2\sigma_1^2\sigma_2^2}{(\sigma_0^2\sigma_1^2 + \sigma_0^2\sigma_2^2 + \sigma_1^2\sigma_2^2)^3}\mu_1^2 < 0$$
$$\frac{\partial \Phi_2(\mu_2^2)}{\partial \sigma_1^2} = \frac{\sigma_0^4}{(\sigma_0^2 + \sigma_1^2)^2} - \frac{\sigma_0^4\sigma_2^4}{(\sigma_0^2\sigma_1^2 + \sigma_0^2\sigma_2^2 + \sigma_1^2\sigma_2^2)^2}$$
$$+ \left\{ \frac{2\sigma_0^2\sigma_1^2}{(\sigma_0^2 + \sigma_1^2)^3} - \frac{2\sigma_0^2\sigma_1^2\sigma_2^6}{(\sigma_0^2\sigma_1^2 + \sigma_0^2\sigma_2^2 + \sigma_1^2\sigma_2^2)^3} \right\} \mu_2^2 > 0$$
$$\frac{\partial O_1(\mu_1^2)}{\partial \sigma_1^2} = -\frac{\sigma_0^4}{(\sigma_0^2 + \sigma_1^2)^2} - \frac{2\sigma_0^2\sigma_1^2}{(\sigma_0^2 + \sigma_1^2)^3}\mu_1^2 < 0$$

Then if $\frac{\partial e_2}{\partial \sigma_1^2} > 0$, then (C.4) implies $\frac{\partial e_1}{\partial \sigma_1^2} < 0$. If $\frac{\partial e_2}{\partial \sigma_1^2} < 0$, then (C.5) implies $\frac{\partial e_1}{\partial \sigma_1^2} > 0$. Plugging $\frac{\partial e_2}{\partial \sigma_1^2}$ from (C.5) into (C.4) yields

$$\begin{aligned}
&\frac{\partial e_1}{\partial \sigma_1^2} \frac{\{c''(e_1)c''(e_2) - [\Phi(\mu_1^2) - O(\mu_1^2)][\Phi(\mu_2^2) - O(\mu_2^2)]\}}{c''(e_2)} \\
&= e_2 \frac{\partial \Phi_1(\mu_1^2)}{\partial \sigma_1^2} + (1-e_2)\frac{\partial O_1(\mu_1^2)}{\partial \sigma_1^2} + \frac{[\Phi_1(\mu_1^2) - O_1(\mu_1^2)]}{c''(e_2)} \frac{\partial \Phi_2(\mu_2^2)}{\partial \sigma_1^2} e_1
\end{aligned} \tag{B.19}$$

The right hand side of (C.6) is negative. From the proof of Corollary 1, $\{c''(e_1)c''(e_2) - [\Phi(\mu_1^2) - O(\mu_1^2)][\Phi(\mu_2^2) - O(\mu_2^2)]\} > 0$, hence $\frac{\partial e_1}{\partial \sigma_1^2} < 0$. Q.E.D.

B.1 Proof of Proposition 2

Take the derivative of $\mathbb{E}[U_{DM}]$ with respect to e_1^* to obtain

$$\frac{\partial \mathbb{E}[U_{DM}]}{\partial e_1^*} = e_2^*(\tilde{\sigma}_1^2 - \tilde{\sigma}_2^2) + (1-e_2^*)(\sigma_0^2 - \tilde{\sigma}_1^2) > 0.$$

Similar for e_2^*. Q.E.D.

Proof of Lemma 2

First, let $\mu_1^2 = \mu_2^2 = \mu^2$ and take the derivative of each first order condition with respect to μ^2:

$$\begin{aligned}
c''(e_1)\frac{\partial e_1}{\partial \mu^2} &= [\Phi(\mu^2) - O(\mu^2)]\frac{\partial e_2}{\partial \mu^2} + e_2 \frac{\partial \Phi(\mu^2)}{\partial \mu^2} + (1-e_2)\frac{\partial O(\mu^2)}{\partial \mu^2} \\
c''(e_2)\frac{\partial e_2}{\partial \mu^2} &= [\Phi(\mu^2) - O(\mu^2)]\frac{\partial e_1}{\partial \mu^2} + e_1 \frac{\partial \Phi(\mu^2)}{\partial \mu^2} + (1-e_1)\frac{\partial O(\mu^2)}{\partial \mu^2}
\end{aligned}$$

Since $e_i \frac{\partial \Phi(\mu^2)}{\partial \mu^2} + (1-e_i)\frac{\partial O(\mu^2)}{\partial \mu^2} > 0$, if $\frac{\partial e_2}{\partial \mu^2} < 0$, then $\frac{\partial e_1}{\partial \mu^2} > 0$ since $\Phi(\mu^2) - O(\mu^2) < 0$ by Lemma 1. In other words, both $\frac{\partial e_1}{\partial \mu^2} < 0$ and $\frac{\partial e_2}{\partial \mu^2} < 0$ are not possible.

Since $\mu_1^2 = \mu_2^2 = \mu^2$, $e_1 = e_2$ and hence we cannot have $\frac{\partial e_1}{\partial \mu^2} > 0$ and $\frac{\partial e_2}{\partial \mu^2} < 0$, and vice-versa. Then the only possibility remains is that $\frac{\partial e_1}{\partial \mu^2} > 0$ and $\frac{\partial e_2}{\partial \mu^2} > 0$.

Q.E.D.

Proof of Proposition 3

Lemma 2 implies that if $y > x$, then $e_i|_{|\mu_i|=y} > e_i|_{|\mu_i|=x}$, for $i = 1, 2$. By Proposition 2, $\mathbb{E}(U_{DM})$ increases with e_i, $i = 1, 2$. Combining these two results, I obtain that $\mathbb{E}(U_{DM})|_{|\mu_i|=max\{|\underline{\mu}|,\overline{\mu}\}} > \mathbb{E}(U_{DM})|_{|\mu_i|=0}$. Q.E.D.