

BEYOND ACCURACY: EXAMINING STRATEGY USE ACROSS DIFFERENT
TASK FORMATS AND TYPES IN EARLY PATTERNING ABSTRACTION

by

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ABSTRACT

BEYOND ACCURACY: EXAMINING STRATEGY USE ACROSS DIFFERENT TASK FORMATS AND TYPES IN EARLY PATTERNING ABSTRACTION

This study aimed to investigate 4-to 6-year-old children's strategies in repeating pattern abstraction across different task formats and types. A convergent parallel mixed-methods design was used, so qualitative and quantitative data were collected in parallel from 56 participants in two public preschools in Mugla/Turkiye during the Spring 2025 semester. Data collection instruments were the Repeating Patterning Abstraction Task (RPAT) and the Corsi Block Tapping task. RPAT was developed in three different model patterns: vertical, horizontal, and hiding, as well as two different formats: multiple choice and drag and drop. Hybrid thematic analysis was used for qualitative analysis, while multiple linear regression and mixed-effect logistic regression were performed for the quantitative part. Results showed that there are five strategy categories similar to the Lüken and Sauzet's (2021) findings, but the indicators of the strategies had differences. Findings revealed that children's RPAT strategy scores were quadratically related to RPAT scores. Also, RPAT strategy scores, age, and Corsi Forward explained 76% of the variance in RPAT scores. Children's use of relational strategies did not vary across different models and formats; however, sequence awareness and comparison strategies were significantly more prevalent in the horizontal model. These findings suggest that the spatial alignment plays an important role in children's strategy use. Thus, educators and researchers should consider spatial features when designing the early repeating pattern abstraction tasks.

ÖZET

DOĞRULUĞUN ÖTESİNDE: ERKEN DÖNEM ÖRÜNTÜ SOYUTLAMA GÖREVLERİNDE FARKLI GÖREV MODELLERİ VE FORMATLARINDA STRATEJİ KULLANIMININ İNCELENMESİ

Bu çalışma 4-6 yaş arası çocukların farklı görev formatları ve türleri arasında tekrarlayan örüntü soyutlama görevlerinde kullandıkları stratejileri araştırmayı amaçlamaktadır. Yakınsak paralel karma yöntem tasarımı kullanılmış, 2025 bahar döneminde Muğla/ Türkiye'deki iki devlet anaokulunda 56 katılımcıdan nitel ve nicel veriler paralel olarak toplanmıştır. Veri toplama araçları, Tekrarlayan Örüntü Soyutlama Görevi (RPAT) ve Corsi dokunma görevleridir. RPAT, dikey, yatay ve gizleme olmak üzere üç farklı modelde ve çoktan seçmeli ve sürükle ve bırak olmak üzere iki farklı formatla geliştirilmiştir ve bilgisayar aracılığıyla uygulanmıştır. Nitel analiz için hibrit tematik analiz kullanılırken, nicel kısım için çoklu doğrusal regresyon ve karışık etkili lojistik regresyon uygulanmıştır. Sonuçlar, Lüken ve Sauzet (2021) ile benzer beş strateji kategorisi olduğunu, ancak stratejilerin göstergelerinde farklılıklar olduğunu göstermiştir. Bulgular, çocukların RPAT strateji puanlarının ve RPAT puanlarının kuadratik olarak ilişkili olduğunu göstermiştir. RPAT strateji puanları, yaş ve Corsi ileri değişkenleri RPAT puanlarındaki varyansın %76'sını açıklamaktadır. Çocukların ilişkisel strateji kullanımı farklı modeller ve formatlarda farklılık göstermemiştir, ancak sıra farkındalığı ve karşılaştırma stratejilerinin kullanımı yatay modelde önemli ölçüde daha yaygındır. Bu bulgular, uzamsal hizalamanın çocukların strateji kullanımında önemli bir rol oynadığını göstermektedir. Bu nedenle, eğitimciler ve araştırmacıların erken tekrarlayan örüntü soyutlama görevlerini tasarlarlarken uzamsal özellikleri dikkate almaları önerilmektedir.

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LIST OF ACRONYMS/ABBREVIATIONS

DD	Drag and Drop
GLMM	Generalized Linear Mixed Model
GLIMMPSE	General Linear Mixed Model and Sample Size
MC	Multiple Choice
MLR	Multiple Linear Regression
MoNE	Ministry of National Education
OR	Odds Ratio
RPAT	Repeating Pattern Abstraction Task
UI	Unit Identification
VIF	Variance Inflation Factor
WM	Working Memory

1. INTRODUCTION

Early mathematics education is one of the essential components of early childhood education because mathematics is beyond numbers; it is a way of thinking and perceiving the world. Also, early mathematics achievement contributes to future academic achievement in not only mathematics but also reading and science (Claessens and Engel, 2013). Therefore, researchers examined the early mathematical competencies for several years and mainly focused on the arithmetic side. However, recent studies showed the importance of the patterning in mathematics education (e.g. Borriello *et al.*, 2023; Kidd *et al.*, 2013; Rittle-Johnson *et al.*, 2017; Wijns *et al.*, 2021) and unpack how the patterning can be useful for the development of algebraic thinking (e.g., Mulligan *et al.*, 2020; Pittalis, 2023; Radford, 2018).

Patterning is a central competence that has short-term and long-term contributions to the mathematics achievement of children (Aunio and Niemivirta, 2010). It can be defined as understanding the unit of the pattern—the smallest portion that repeats (Rittle-Johnson *et al.*, 2013). Operationally, it can be described as a body of knowledge encompassing tasks such as duplicating, extending, abstracting, and identifying the unit of a pattern (Rittle-Johnson; Wijns, Torbeyns, Bakker, *et al.*, 2019). It involves understanding the pattern's structure, determining relationships, and generalizing these relationships to solve new problems. Patterning is seen as a building block of mathematics education (Clemson and Clemson, 2001; National Council of Teachers of Mathematics, 2000) because it plays a crucial role in the development of mathematical thinking, especially algebraic thinking (Lee *et al.*, 2011; Papic *et al.*, 2011; Sarama and Clements, 2004; Warren and Cooper, 2006) as well as inductive and deductive reasoning (Sen and Guler, 2022). Several studies revealed the relationship between early patterning knowledge and future mathematics achievement (Borriello *et al.*, 2023; Burgoyne *et al.*, 2019; Pasnak *et al.*, 2015; Rittle-Johnson *et al.*, 2017). Therefore, early patterning knowledge plays a vital role in the development of mathematical thinking, and it is the first step to algebra.

This study particularly focused on repeating patterns, as preschool children are generally exposed to repeating patterns (Wijns, Torbeyns, Bakker, *et al.*, 2019). There are different types of repeating patterning tasks. While some of them can be successfully completed by preschoolers, some are claimed to be more advanced, and few children can complete them (Lüken and Sauzet, 2021; Rittle-Johnson *et al.*, 2013, 2015). Duplication -making an exact copy of the pattern- and extension -identifying what comes next in the pattern- activities are claimed as more basic, and these are claimed as less related to mathematical thinking (Economopoulos, 1998). However, abstraction -recreating the pattern with new materials- and unit identification -identifying the smallest portion that repeats- are claimed to be more advanced patterning tasks (Rittle-Johnson *et al.*, 2013). These tasks require comprehending the structure of the pattern, determining the relationships, and generalizing it into the new contexts (Economopoulos, 1998; Rittle-Johnson *et al.*, 2015).

There are different repeating pattern assessments in the literature. In abstraction knowledge, some studies use the word transfer or translate to refer to the same concept as abstraction (e.g., Wijns, Torbeyns, De Smedt *et al.*, 2019). These studies similarly assess abstraction knowledge by giving a pattern and asking children to recreate the pattern with new materials (e.g., Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, De Smedt *et al.*, 2019). Nevertheless, there are different outcomes regarding the level of difficulty in abstraction items. While some of them indicated abstraction items are difficult than the extension (Collins and Laski, 2015; Rittle-Johnson *et al.*, 2013; Sen and Guler, 2022), Junker *et al.* (2025) revealed abstraction is easier than extension. Furthermore, some of them showed extension and abstraction have the same level of difficulty (Msall *et al.*, 2025; Wijns, Torbeyns, Bakker, *et al.*, 2019).

However, there are format-related differences in these studies. Generally, the format of the abstraction items was constructed response in which children create their pattern using different physical materials (Collins and Laski, 2015; Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, Bakker, *et al.*, 2019). The only exception was the study by Msall *et al.* (2025), which employed multiple-choice items for abstraction tasks. Also,

some studies used multiple-choice (Wijns, Torbeyns, Bakker, *et al.*, 2019; Msall *et al.*, 2025) and the constructed response format in extension tasks (Collins and Laski, 2015; Junker *et al.*, 2025; Rittle-Johnson *et al.*, 2013). These different assessment formats may cause different outcomes in the literature. Therefore, further studies should examine the format-related differences in repeating pattern assessments.

Additionally, most of the studies assess children’s repeating patterning knowledge, considering the correct and incorrect responses. However, correct responses may not mean children attend to the pattern’s structure and use the unit of repeat. Therefore, examining the children’s strategies for repeating patterning tasks is crucial to determine their understanding and the cognitive demand of tasks. There are a limited number of studies focusing on children’s strategy use in repeating patterns. Some studies found that abstraction tasks can be completed only by understanding the pattern’s structure. Thus, children use more relational strategies, which involve comprehending the structure of the pattern in these tasks (Collins and Laski, 2015). However, some of them explored that these activities are not required to comprehend the structure of a pattern and can be solved using surface features of the pattern without awareness of the unit of it. For this reason, children use recursive strategies such as one-to-one appearance matching (Junker *et al.*, 2025; Lüken and Sauzet, 2021).

The variability in results may be linked to the methodologies employed in the studies, especially in terms of data collection and analysis. Collins and Laski (2015) analyzed children’s errors in their responses to assess their strategy use. Junker *et al.* (2025) analyzed children’s speech and gestures while completing tasks, in addition to their responses. However, both studies did not prompt children to explain their thinking and strategies also used only incorrect responses. To the best of our knowledge, only one study explored children’s thinking and asked about how they reached the solution via task-based interviews and provided the results of the strategy used in abstraction tasks (Lüken and Sauzet, 2021). They analyzed the children’s responses, actions, and gestures as well.

Moreover, the coding procedure of the studies differs. For example, Lüken and Sauzet (2021) accepted creating an AB pattern instead of ABBABB as a recursive strategy, but other studies (Collins and Laski, 2015; Junker *et al.*, 2025) accepted it as a relational strategy use. These methodological differences make it difficult to compare the findings of studies. Considering children’s errors, actions, and gestures without asking how they found the solution might not fully capture their understanding. Therefore, there could be three possible explanations for these contradictory findings. Firstly, abstraction items might not require thinking algebraically, as some researchers claimed, since the children use more basic strategies to solve these activities (Junker *et al.*, 2025; Lüken and Sauzet, 2021). Secondly, abstraction items’ spatial alignment might not be designed to prevent recursive strategy use. Therefore, the parallel horizontal model and answer part might allow recursive strategy, and this lowered the cognitive demand of abstraction tasks. Thirdly, the methodological differences in the type of data and coding procedure might create differences between the results.

Taking into consideration the aforementioned studies, the current study has three purposes. The first one is to investigate children’s strategy use in different kinds of abstraction items and how the strategies differ for 4-, 5-, and 6-year-old children. Second is to examine the relationship between abstraction scores and strategy use while controlling for age and visuospatial working memory. The final, but nonetheless critical, purpose is to develop a measurement method that minimizes recursive strategies such as one-to-one matching and leads children to focus on the pattern’s structure, as some researchers claimed (e.g., Economopoulos, 1998; Rittle-Johnson *et al.*, 2015).

2. LITERATURE REVIEW

This chapter first examines the link between repeating patterns and mathematics, then examines the repeating pattern tasks and children's strategies in these tasks. Then the chapter continues with assessment of abstraction tasks and children's strategy use in these tasks.

2.1. Repeating Patterns and Mathematics

A pattern is defined as “any replicable regularity” (Papic *et al.*, 2011, p. 238). The most common patterns during the early childhood period are repeating patterns, but also children can encounter growing patterns that are not as common as repeating patterns. Repeating patterns can be defined as “a cyclic structure that can be generated by the repeated application of a smaller portion of the pattern” (Liljedahl, 2004, p.26). For example, ABBABBABB is a repeating pattern, and the smallest portion that repeats or the unit of repeat ABB. Growing patterns can be defined as decreasing and increasing sequences such as ABABBABBB (Papic *et al.*, 2011). This study focused on repeating patterns since they are more common during the early childhood period (Wijns, Torbeyns, Bakker, *et al.*, 2019) due to the Turkish Ministry of National Education's preschool curriculum's objectives related to the repeating patterns (MoNE, 2024).

Patterns are considered a core topic in mathematics, and some mathematicians describe mathematics as “a science of pattern” (Steen, 1988). Identifying patterns, generalizing them into new contexts, and expressing the generalization are essential for different branches of mathematics, especially for algebra. Some researchers claimed that the fundamental principle of algebraic thinking is making generalizations and expressing it using symbolic language like letters (Blanton and Kaput., 2011). Radford (2011) extend this idea by algebraic thinking can be supported not only symbolic ways but also non symbolic actions. These actions could be gestures, movement or language

(Radford, 2011; Pittalis, 2023). Therefore, algebraic reasoning of young children can be supported through various tasks to go beyond symbolic notations, and early patterning tasks may play a crucial role for the development of algebraic thinking. The development of algebraic thinking is significant for other subjects in mathematics such as arithmetic (Pittalis, 2023).

One way to support transition from numeracy to algebra is the development of relational thinking which can be defined as “a two-pronged construct of identifying relationships between quantities (known, unknown or variable) and deducing logically new information based on the known relationships” (e.g. relationships between unknown or numerical values) (Polotskaia *et al.*, 2022, p.343). For example, when children count twos, they identify the rule that each subsequent number increases by two and generalize this pattern to continue counting. Another example is arithmetic calculation principles. Children typically engaged in rote memorization for calculation, such as addition with carrying ($17+7$); nevertheless, recognizing patterns in the base-ten systems-such as the alteration of the tens digit every ten numbers like 10, 20, 30, etc.-could enhance their arithmetic calculation skills conceptually. For instance, while children are calculating with rote memorization, they cannot make sense of why they are increasing the tens place and finding the answer 24. However, a child applying relational thinking might reason, “I know the pattern that 17 needs 3 to get to number 20, so I’ll take 3 from the 7. That leaves 4. So, 20 plus 4 is 24”. This reasoning requires thinking about relationships between numbers and easily transfers to new cases.

Algebraic thinking consists of analytic, structural, and functional thinking (Kieran, 2022). Each dimension is related to broader mathematics concepts. An early form of algebraic thinking can be developed using patterns (Warren and Cooper, 2006). Analytic thinking is deductively recognizing the relationship between knowns and unknowns and coming up with a solution, acting unknowns as knowns (Kieran, 2022). Analytic thinking can be used in completing-ABABAB or extending tasks ABABAB?. The knowns are the letters, and the unknowns are question marks. Children can find the answer by identifying the relationship between letters A and B. A is in the odd positions and

B is in the even positions. Structural thinking is identifying the relationship, defining the rule of this relationship, and extending it into new contexts (Mason *et al.*, 2009). Repeating pattern abstraction tasks might support structural thinking. For instance, a blue-red-red-blue-red-red pattern is given, and children identify the rule -the unit of repeat- as blue-red-red and recreate or express the pattern using letters as ABBABB.

Another important aspect of algebraic thinking is functional thinking. It can be defined as “entails generalizing relationships between co-varying quantities, expressing those relationships in words, symbols, tables, or graphs, and reasoning with these various representations to analyze function behavior” (Blanton *et al.*, 2011, p.13). If we consider the red-blue-blue pattern, functional thinking can be supported by analyzing how many red and blue are in different numbers of units. As Warren and Cooper (2006, 2007) suggested, asking children to create a table of how the number of red and blue tiles changes for different numbers of units supports comprehending how two colors covariate. This reveals the relationship between the number of blue tiles and twice the number of red tiles, such as in one unit; one red and two blue, in two unit; two red and four blue. The input is the unit number, and the outputs are the number of red tiles $f(x)=x$ or blue tiles $f(x)=2x$. The input might be the number of red tiles, and the output is the number of blue tiles $f(x)=2x$. Also, it can be used to reveal the modular relationship, like the 30th place should be blue because the unit is repeated in every three places, so 30 can be divisible by 3, and the 30th place should be blue.

Therefore, these examples show how early repeating patterning can be used to scaffold algebraic thinking. In addition to algebraic thinking, several studies explored repeating patterning as essential for the development of different mathematical competencies such as inductive reasoning and multiplicative thinking (Sen and Guler, 2022; Warren and Cooper, 2007). Furthermore, some of them revealed the association between repeating patterns and numerical ability (Junker *et al.*, 2025; Wijns, Torbeyns, Bakker, *et al.*, 2019). Rittle-Johnson and colleagues (2017) indicated that early repeating pattern knowledge predicts even fifth-grade math achievement. The early development of repeating pattern knowledge might have an equalizing effect on future

mathematics achievement. Since repeating patterning is related to broader mathematics concepts it contributes to mathematics achievement both in the short term and long-term.

2.2. Repeating Patterning Tasks

There are different kinds of repeating patterning tasks. The most common repeating patterning tasks in preschools are creating, duplicating, and extending a pattern (Economopoulos, 1998). Duplicating a pattern involves children making an exact copy of the given pattern (Wijns, Torbeyns, De Smedt, *et al.*, 2019), while extending a pattern entails continuing the pattern by adding objects that follow the rule. For example, if there is a blue-red-blue-red pattern, children extend the pattern by adding other blue and red objects in order. However, Threlfall (1999) claimed that these tasks might not encourage comprehending the underlying structures of a pattern. Therefore, they may not contribute to the understanding of mathematical relationships and may be completed without identifying the unit of repeat (Rittle-Johnson *et al.*, 2015). Most 4-year-olds are able to duplicate and extend patterns, making these relatively simple tasks for young children (Clements *et al.*, 2008).

However, abstraction and unit identification are considered as more comprehensive tasks, and children may need to understand the underlying structure of a pattern, so a limited number of children successfully complete these tasks (Rittle-Johnson *et al.*, 2015). When children abstract a pattern, they recreate the given pattern using different objects (Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, De Smedt, *et al.*, 2019). For example, a circle-square-square-circle-square-square pattern may be recreated as triangle-rectangle-rectangle-triangle-rectangle-rectangle.

To successfully recreate the pattern in this way, a child must first mentally grasp its underlying structure by identifying the core repeating part, known as unit identification. For instance, in an ABCABC pattern, the unit is ABC. Few preschool children are capable of successfully identifying the unit, making it one of the most challenging

patterning activities (Lüken and Sauzet, 2021). These activities encourage children to focus on the elements of a pattern and comprehend the structure of its repeating part.

Some studies have claimed that abstraction tasks support going beyond surface features to focus on understanding the underlying structure and relationships within patterns (Baroody *et al.*, 2021; Collins and Laski, 2015; Economopoulos, 1998; Rittle-Johnson *et al.*, 2015). Abstracting a pattern encourages understanding the relationships between elements of a pattern, abstracting these relationships, and generalizing them to new cases (Baroody *et al.*, 2021). Thus, abstraction tasks foster deeper cognitive skills such as relational and algebraic thinking. This role is distinct from duplication and extension tasks.

Even though there are contradictory findings about abstraction activities in the literature, some of the studies explored that abstraction activities can be correctly answered using surface features without awareness of repeating units and the relationship between pattern elements (Junker *et al.*, 2025; Lüken and Sauzet, 2021). Some studies supported this claim with empirical evidence that abstraction items are easier and have the same difficulty as extension items (Msall *et al.*, 2025; Wijns, Torbeyns, Bakker, *et al.*, 2019). Contrary to these findings, according to the developmental evidence from Rittle-Johnson *et al.* (2013), abstraction tasks (level 3) harder than extension (level 2) but easier than unit identification (level 4). Baroody *et al.* (2021) argue that translating patterns into abstract representations, such as letters (e.g., ABABAB), has only a small effect on fostering broader generalization skills, such as translating patterns into new objects. Furthermore, they suggest revising the developmental sequence of patterning skills, proposing that core identification (recognizing the smallest repeating unit) should precede pattern abstraction (translating the pattern into new forms), as abstraction tasks appear to be more cognitively demanding. This proposal challenges earlier developmental models (e.g., Rittle-Johnson *et al.*, 2013, 2015) that positioned abstraction as an earlier step. Therefore, more research is required to understand how children complete abstraction tasks and whether they are encouraged to focus on the pattern's structure.

2.3. Repeating Patterning Strategies

Patterning strategies provide insights into children's thinking and decision-making procedures. Patterning tasks can be completed using different strategies, and some of the strategies facilitate relational thinking which is identifying the similarities and differences in the pattern elements and comprehending the underlying structure of the pattern or unit of repeat (Collins and Laski, 2015; Dumas *et al.*, 2013). Relational thinking encourages algebraic thinking (Polotskaia *et al.*, 2022). However, children do not always use relational strategies, sometimes they may complete pattern tasks by looking at the superficial features of the pattern without recognizing the unit which is called recursive strategies (Papic *et al.*, 2011; Collins and Laski, 2015; Lüken and Sauzet, 2021; Junker *et al.*, 2025; Pitta-Pantazi *et al.*, 2024). There are limited studies about children's strategy use (see Table 2.1), and they provided various strategy categories; the strategies can be grouped into three broad categories: recursive strategy mainly focuses on surface features; relational strategy focuses on structure and off-task strategy is random strategies not related to patterning.

Table 2.1. Studies on repeating patterning strategies.

Source	Country	Age (year)	Patterning activity	Data Collection Method	Strategy Findings
Papic <i>et al.</i> -2011	Australia	3-5 years old	Repeating Patterns: Towers Borders Hopscotch Numbers Spatial Structure Patterns	Semi-structured task-based interviews	1. Random arrangement 2. Direct comparison 3. Alternation 4. Basic unit of the repeat 5. Advanced unit of the repeat

Table 2.1 Studies on repeating patterning strategies. (cont.)

Source	Country	Age (year)	Patterning activity	Data Collection Method	Strategy Findings
Collins and Laski -2015	USA	3-5 years old	Duplicate, extend, transfer and unit isolation	Children's accuracy, Item-level Error Analysis Relations accuracy and errors on tasks and WM	1. Off-task strategy 2. One-to-one appearance matching 3. Relational similarity
Lüken and Sauzet -2021	Germany	3-5 years old	Explain, copy, repair, extend, name the last element, translate, and identify the repeating unit	Task-based interviews	1. No reference to the pattern 2. Attention the singular characteristics 3. Comparison and classification 4. Focus on sequence 5. View the unit of repeat
Junker <i>et al.</i> -2025	Norway	First-grade students	Duplicate, extend, transfer and unit isolation	Task-based interviews Incorrect Responses (not prompted to explain children's thinking)	1. Different strategies 2. Recursive strategy 3. Relational strategy

Table 2.1 Studies on repeating patterning strategies. (cont.)

Source	Country	Age (year)	Patterning activity	Data Collection Method	Strategy Findings
Pitta-Pantazi <i>et al.</i> , -2024	Cyprus	First-grade students (M= 6.5 years)	Extend	Eye tracking and Verbal responses	1. Serial strategies 2. Strategies employing structure sense
Note. Adapted from Patterning strategies in grade 1 students with low and high number sense proficiency by A.Junker, G. A. Nortvedt, and D. Farsani, 2025, Educational Studies in Mathematics.					

Collins and Laski (2015) and Junker *et al.* (2025) used four pattern tasks (duplicate, extend, transfer, and unit isolation) to investigate children's strategy use. While Collins and Laski (2015) focused on children's accuracy in tasks, item-level error analysis, the relationship between children's accuracy and errors on tasks, as well as memory and inhibitory control, Junker *et al.* (2025) conducted task-based interviews and coded children's verbal, spatial, and gestural responses. However, in both studies, children were not prompted on how they thought while answering and only examined children's incorrect answers were examined. In both studies, the results indicated that children employed three strategies, ranging from the easiest to the most advanced. The most advanced strategy is relational similarity in both studies. The second strategy is labeled as one-to-one appearance matching in Collins and Laski (2015) and recursive in Junker *et al.* (2025). The basic strategy is labeled as off-task in Collins and Laski (2015) and different in Junker *et al.* (2025). Although the strategies have different labels, their indicators are similar.

Relational similarity strategy is defined as comprehending the underlying structure of a pattern (Collins and Laski, 2015; Junker *et al.*, 2025). For example, determining that AB forms a unit in the ABAB pattern and that blue-red-blue-red-blue-red is

an AB pattern. Since the studies performed error analysis, creating wrong but viable patterns is considered a relational similarity strategy such as creating an AB pattern instead of an ABC pattern. However, creating an AB pattern in the wrong contexts might not facilitate relational thinking because children may have matched the patterns, they had formed by looking only at the first two elements and repeating this structure. This might be a transition process from recursive to relational but still they did not comprehend the structure of the pattern. The recursive (including one-to-one appearance matching) strategy focuses on the surface features of the pattern without awareness of the pattern unit or structure (Collins and Laski, 2015; Junker *et al.*, 2025). For example, the square-square-circle pattern is given, and children match the square with a triangle and the circle with a trapezoid without comprehending the AAB unit structure. In the off task/different strategy, children did not attend the pattern's elements. They gave unrelated answers or sorting (AAABBB rather than ABABAB) the objects according to their color or shape (Collins and Laski, 2015; Junker *et al.*, 2025).

Papic (2011) and her colleagues determined five strategy categories in repeating patterns. Their task categories are different from aforementioned studies. There are four task categories: towers, borders, hopscotch, and numbers. The first category was random arrangement, i.e., children lack pattern awareness and form objects randomly without any repeating unit. This strategy is similar to the off-task category in Collins and Laski (2015). The second category was a direct comparison, the same as the one-to-one matching strategy defined above. The third category is alternation, and children focus on how the objects follow each other, such as blue, then red, then blue, then red, without comprehending the blue-red repeating unit. The fourth one was the basic unit of repeat, and in this category, children could define and understand the pattern unit and explicitly show the unit of repeat.

The fifth category was the advanced unit of repeat, in which children not only identified the unit but also generalized it using different materials. The fifth category is very similar to the definition of abstraction tasks and these results align with Baroody

and his colleague's (2021) recommendations that the abstraction tasks might be difficult to the unit identification tasks since implicitly identifying the unit might facilitate the abstraction tasks.

The five strategy categories of Lüken and Sauzet (2021) were listed similarly to those of Papic *et al.* (2011) in increasing order of complexity. Strategies that randomly form the elements without awareness of the pattern structure are called no reference to the pattern. Strategies that focus on one dimension of a pattern, such as shape or color, call attention to singular characteristics, and in this category, children did not regularly form correct patterns. Comparison and classification categories are similar to the one-to-one matching strategy. Children compare and match the elements one by one or create an AB pattern when it is not appropriate. A key difference in the literature lies in the classification of “viable but wrong” patterns. For example, some studies categorized creating an AB pattern instead of an ABB pattern as a relational strategy (Collins and Laski, 2015; Junker *et al.*, 2025). In direct contrast, Lüken and Sauzet (2021) considered this same error to be an example of a recursive strategy. Therefore, the evidence taken from children's responses is categorized differently, and this might be one reason for the different findings of the studies. The focus on sequence strategy is similar to the alternation strategy of Papic *et al.* (2011). Children focus on the consecutive elements and consider the alternation of elements, such as next to blue is red, next to red is yellow, without comprehending the pattern's structure. Lastly, the most sophisticated strategy is the view of the unit of repeat, and this one is the same as Papic and her colleague's (2011) basic unit of repeat strategy and children explicitly identify the unit by “chanting rhythmically, taking all elements of the unit at once out of the box, arranging all elements of the unit at once, naming the unit of repeat, naming and actively showing the unit of repeat” p. 34. The most advanced strategies differ between Papic *et al.* (2011) and Lüken and Sauzet (2021) since Papic (2011) differentiates the unit identification as basic and advanced, while Lüken and Sauzet (2021) only used the category view of the unit of repeat.

Identifying the children’s strategies in different repeating patterning tasks is essential to understanding their contribution to mathematics achievement. In the previous section, we examined the contradictory findings about the abstraction tasks and the cognitive demand of them might be lower than claimed. The reason for the different strategies used in abstraction tasks might be related to their methodology and how to categorize the responses as relational and recursive. For example, Collins and Laski (2015) and Junker *et al.* (2025) categorized creating a viable but wrong pattern as relational, such as creating an AB pattern rather than ABB. However, Lüken and Sauzet (2021) categorized this response as a recursive strategy. Therefore, they indicated that abstraction tasks might not demand the use of relational strategies, contrary to Collins and Laski (2015). Junker *et al.*’s (2025) findings have supported these findings even though they used and coded strategies similar to those of Collins and Laski (2015). The only difference between these two studies was that Collins and Laski (2015) performed error analysis while Junker used multimodal responses such as children’s responses, speech, and gestures. However, neither prompted children to explain their thinking and strategies, and only incorrect answers were analyzed to reveal children’s strategies.

The methodological differences in data collection and analysis make it difficult to compare the findings of the studies. The most important difference is whether creating a different but viable pattern, such as creating AB when the task unit was ABB, is relational or recursive. Given that the children were still unable to identify the structure of the model pattern (ABB unit), we asserted that they possessed a partial understanding of the pattern’s structure. Furthermore, children’s errors or responses without explaining their thinking might not fully capture their understanding, so there is a need for further studies about children’s strategies for abstraction tasks.

2.4. Repeating Patterning Abstraction Tasks

There are different assessment instruments to measure repeating pattern knowledge in the literature but most of them have similar tasks. This section provides a closer look at the abstraction tasks used in the literature. Before examining the tasks,

we want to remember that abstraction is defined as “recreating a pattern with new objects”. Also, some researchers use different names such as translate, transfer, or extra variable transfer rather than abstraction. We prefer to use abstraction during this study.

When we examined the assessment instruments, two of them were commonly used in the literature. The first one is an interview-based Early Mathematical Patterning Assessment (EMPA) developed by Papic *et al.* (2011). The researchers used different names for the tasks, which include towers, borders, hopscotch, and numbers, to measure knowledge of repeating patterns. The tower tasks consist of copying, continuing (similar to extension), and finding missing elements (fix) with units of AB, ABC, and ABBC using blocks and drawing. Even though there is no name of abstraction in the tasks, drawing the hiding pattern from memory tasks is similar since it requires recreating the model pattern by drawing. In this task, a model pattern created by unit cubes with different colors was shown for 5 seconds, then the model pattern was hidden, and children drew the same pattern from memory. They explained the reasoning for hiding the pattern in the assessment with the Mulligan *et al.* (2004) study, which indicated that children fail to develop structured images, and supporting them to “visualize and record a simple pattern” (p. 399) might contribute to producing structured images at an early age. Shortly, hiding the model pattern may support and assess children’s structural thinking, which is a dimension of algebraic thinking. However, there are some limitations of the study. Firstly, the study did not provide a rationale for limiting the time to 5 seconds. Secondly, controlling visuospatial memory might be important to determine whether the differences are related to pattern knowledge or working memory capacity. Lastly, the nature of the assessment poses challenges for large-scale research. As an interview-based, constructed-response tool, it requires one-on-one administration and individual coding for each child’s constructed pattern, making data collection and analysis difficult to scale for larger groups.

The second commonly used constructed response instrument measuring repeating patterning was developed by Rittle-Johnson *et al.* (2013). There are four main tasks in

this instrument: duplicate, extend, abstract, and unit tower (unit identification), with AAB, ABB, and AABB units. Similar to Papic *et al.* (2011), there are unit memory tasks that solely assess the ability to copy patterns from memory and identify units from memory that are dropped due to low item-total correlation. However, the design of these memory items did not aim to measure abstraction knowledge since it wasn't required to recreate the pattern with new materials. In the abstraction tasks, there are two types of abstraction items: abstracting the color and abstracting the shape. Abstracting the color task is provided by creating the model pattern with a uniform three-dimensional shape, and children recreate the model using different color shapes. The only difference in abstracting shape tasks is that the materials are non-colored wooden cubes and small flat shapes of unpainted wood. The alignment is horizontal in these items, i.e., the model pattern is given horizontally, and after some space, children recreate the pattern with different materials below the model pattern.

When we examined the repeating patterning studies, the repeating patterning assessment developed by Rittle-Johnson *et al.* (2013) was commonly used. Some of the instruments use the same task designs with different units (Wijns, Torbeyns, De Smedt, *et al.*, 2019). However, horizontal alignment was used in the tasks, and all the studies used constructed response items (Collins and Laski, 2015; Junker *et al.*, 2025; Lüken and Sauzet, 2021; Wijns, Torbeyns, Bakker, *et al.*, 2019). A current measurement instrument, EPA-repeating (Rittle-Johnson *et al.*, 2020), was designed with MC items in completion, extension, abstraction, and pattern identification. The study was adapted to Turkish, but it is revealed that abstraction, extension, and pattern identification items have the same level of difficulty in Türkiye and there are slight differences in the U.S. (Msall *et al.*, 2025). Although abstraction is often framed as a complex skill requiring a focus on underlying patterns, the literature presents contradictory findings regarding children's actual strategy use on these tasks. These inconsistencies have led researchers to propose different developmental levels for abstraction knowledge (Barody *et al.*, 2021; Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, Bakker, *et al.*, 2019).

Previous findings lead us to a new question: “Is the abstraction task’s spatial alignment designed to measure the intended relational knowledge and prevent the recursive strategy use, such as one-to-one appearance matching”? This question is linked with the spatial alignment principle (Matlen *et al.*, 2020). According to the spatial alignment principle (Matlen *et al.*, 2020), visual comparison efficiency increases when the tasks are presented directly, which means providing tasks in parallel rather than impeded. In other words, it is to present tasks one below the other or side by side. Therefore, the direct design of the abstraction tasks might affect the strategy used by children because it might allow for a one-to-one match. Since the spatial aspects of task design may affect the participants’ attention and engagement with tasks (Björklund *et al.*, 2020), this type of task design may lower the targeted difficulty and cognitive demand. There is a need to investigate how children engage with different spatial arrangements and explore whether horizontal direct alignment can cause them to use recursive strategies.

3. SIGNIFICANCE OF THE STUDY

Early childhood educators are increasingly interested in patterning due to its critical role in mathematics and its connection to algebraic reasoning (Warren and Cooper, 2006). Therefore, various preschool curricula, including the Turkish National Preschool Curriculum, have objectives related to repeating patterns (Ministry of National Education [MoNE], 2024). Providing children with a range of patterning activities-such as duplication, extension, abstraction, and unit identification-may support the development of their early algebraic thinking. Some researchers claim that abstraction and unit identification tasks are more related to mathematical thinking (Clements and Sarama, 2009; Rittle-Johnson *et al.*, 2015) since the demand of these tasks goes beyond recursive strategies -using surface features- and encourages relational strategies such as identifying the unit (Clements and Sarama, 2009; Economopoulos, 1998; Rittle-Johnson *et al.*, 2015).

Nevertheless, current findings about children’s strategies in repeating patterning tasks (Papic *et al.*, 2011; Collins and Laski, 2015; Lüken and Sauzet, 2021; Junker *et al.*, 2025) and repeating patterning assessments (Wijns, Torbeyns, Bakker, *et al.*, 2019) argue the claim of relational strategy use in abstraction knowledge. These tasks might not encourage the comprehension of the unit of repeat and can be completed through recursive strategies (Junker *et al.*, 2025; Lüken and Sauzet, 2021; Wijns, Torbeyns, Bakker, *et al.*, 2019). The recursive strategy use of these tasks might reduce its relationship with mathematics since children do not comprehend the structure and generalize them. For instance, a study found that all pattern tasks contribute to numerical ability, except for repeating patterning abstraction tasks (Wijns, Torbeyns, Bakker, *et al.*, 2019). Another study supported this claim in the context of number sense and the only significant difference between children who have low and high number sense is in extension tasks (Junker *et al.*, 2025). Additionally, the abstraction performance of students with low and high number sense was very similar, at 73% and 78%, respectively (Junker *et al.*, 2025).

Notably, few studies have investigated children’s strategies for repeating patterning abstraction tasks (see Table 2.1). In these studies, although the assessment tasks for abstraction were similar, the methodologies used for data collection and analysis varied. Two studies did not prompt children to explain their thinking but rather used error analysis (Collins and Laski) and multimodal responses such as speech and gestures in addition to their responses (Junker *et al.*, 2025). Also, these studies accepted creating wrong but viable patterns as relational, creating AB in a different pattern. To the best of our knowledge, only two studies have asked children to explain their thinking (Lüken and Sauzet, 2021; Papic *et al.*, 2011). Nonetheless, Papic *et al.* (2011) used different pattern tasks and did not specify the strategies employed in repeating patterns. Therefore, only Lüken and Sauzet (2021) provided a strategy framework for repeating patterning, and they categorized creating a different pattern to model a pattern as a recursive strategy. The differences in methodologies led to varying findings regarding whether abstraction tasks encourage relational thinking. Therefore, more research should investigate the children’s strategies for abstraction tasks by prompting their reasoning.

The use of recursive strategies in abstraction tasks may be related to how the questions are asked as well, since the task’s spatial format might affect children’s engagement with tasks (Björklund *et al.*, 2020). According to the spatial alignment principle, the direct (parallel) placement increases children’s visual comparison efficiency (Matlen *et al.*, 2020). This may allow children to use recursive strategy methods and decrease the cognitive demand of the tasks. Therefore, different spatial alignment models should be tested to understand the effect of task types on children’s strategies.

Furthermore, there are format-related differences in the literature. Most of the assessment instruments used a constructed response format in abstraction items that children use physical materials to construct a pattern (e.g., Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, Bakker, *et al.*, 2019). However, Msall *et al.*’s (2025) study used the multiple-choice format, and there are no strong differences across task types. Since the multiple-choice format provided choices with the whole unit, it might be easier than

the constructed response format. Further studies should explore whether the format of the tasks has an impact on children’s strategy use.

Briefly, our study aims to deepen the current understanding of repeating pattern abstraction tasks. We investigate children’s strategies using task-based interviews to comprehend their cognitive mechanisms, including age-based differences. Also, we test various abstraction items to establish a task design that minimizes the need for recursive strategies like one-to-one appearance matching as well as exploring format-related differences in strategy use.

3.1. Research Questions of the Study

There are three main research questions of the current study as follows.

- (i) What are the recursive and relational strategies used by 4-6-year-old children when solving repeating patterning abstraction tasks (RPAT)?
- (ii) To what extent do RPAT strategies predict RPAT scores when controlling for age and visuospatial working memory?
- (iii) What task design features promote the use of relational strategies and minimize the use of recursive strategies in repeating pattern abstraction tasks?

4. METHOD

This chapter presents the research design, participants' characteristics, data collection instruments and procedures, and data analysis of the study.

4.1. Research Design

A convergent parallel mixed-methods design was used since we aim to suggest a task design for abstraction items. Therefore, both the strategy uses and the answers to the tasks are important in determining it. In the convergent parallel mixed-methods design, quantitative and qualitative data are separately collected and analyzed. Then, data is interpreted together to determine whether the qualitative and quantitative data support each other (Creswell and Creswell, 2018). The abstraction task design should eliminate the recursive strategy compared to the other task designs. Therefore, a convergent parallel mixed-methods design is useful for deepening the results of quantitative data and generalizing the findings of qualitative data.

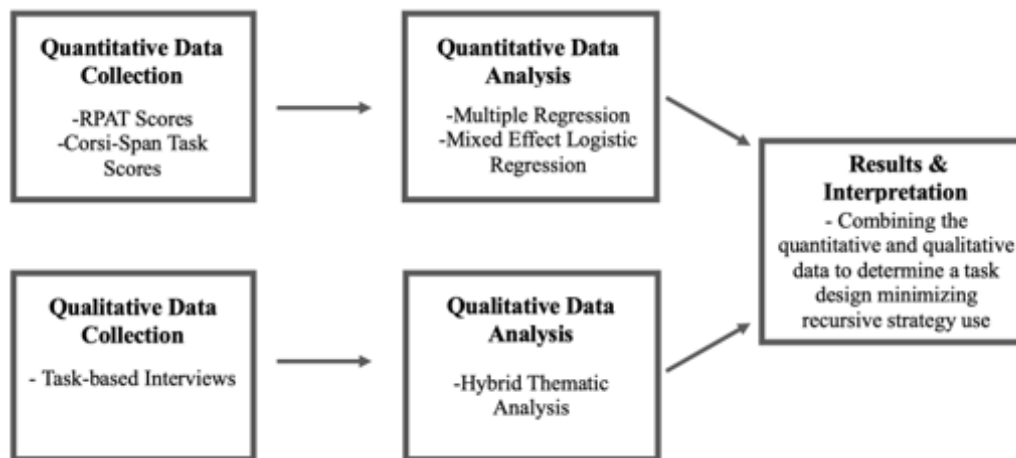


Figure 4.1. Research design.

The qualitative data was used to reveal the children's strategies using task-based interviews and which repeating patterning abstraction task model and format encourages the relational strategy use (see Figure 4.1). The quantitative part will be used

to support and generalize the findings of qualitative data, and RPAT scores, RPAT strategy and visuospatial working memory was the source of the quantitative data.

4.2. Participants

Participants were preschool children aged 4-6 years, as the development of early repeating pattern knowledge is essential for the preschool period (Sarama and Clements, 2004). Also, children's repeating pattern knowledge varies across these ages (Lüken and Sauzet, 2021), allowing us to identify different strategies used. GLIMMPSE (General Linear Mixed Model and Sample Size) software was used to determine the number of participants. This analysis investigated the interaction effect between age groups (4, 5, 6 years) and question formats (horizontal, vertical, hiding) on the abstraction score. The analysis used a Hotelling Lawley Trace test with a Type I error rate of 0.05 and a target power of 0.80. The power analysis suggested 57 participants in total to provide sufficient statistical power to detect the interaction effect between age groups and question formats on the abstraction scores. Therefore, data were collected from 60 children from two public preschools in Muğla/Türkiye by convenience sampling method. Three of them did not want to continue the study, and one of them, whose teacher confirmed that he had attention problems in the classroom, was removed due to inattentive answers.

Thus, sample of this study is 56 participants ($M_{month} = 65.22$, $SD = 8.58$, 57% female) and 19 of them were 4-years-old ($M_{month} = 55.11$, $SD = 2.97$, 59% female), 18 of them 5-years-old ($M_{month} = 65.89$, $SD = 3.03$, 61% female), 19 of them 6-years-old ($M_{month} = 75.00$, $SD = 1.88$, 56% female). However, five of the participants did not complete the visuospatial working memory tasks; therefore, they were removed from the analysis of the second research question. Therefore, the sample is 51 participants ($M_{month} = 66.22$, $SD = 8.26$) for the analysis of the second research question. Teachers of 4-year-olds reported that they generally did AB pattern extension tasks in the classrooms, and they did not do abstraction activities. Teachers of mixed-age classes of 5- and 6-year-old children reported that they primarily used extension activities with

different units, along with infrequent use of unit identification and abstraction activities. Also, children’s socioeconomic status was reported by their teachers as middle to upper middle.

4.3. Data Collection Instruments

Qualitative and quantitative data collection instruments were used. The qualitative data collection instrument is task-based interviews in Repeating Pattern Abstraction Task (RPAT). The quantitative data collection instruments are RPAT to measure abstraction knowledge and Corsi-span task to measure visuospatial working memory.

4.3.1. Repeating Pattern Abstraction Task

To determine how task design might reduce reliance on recursive strategies, such as one-to-one matching, three distinct abstraction task models were developed online. This study began with the horizontal model, which reflects the traditional format used in previous research (e.g., Clements and Sarama, 2009; Rittle-Johnson *et al.*, 2013). We included this model specifically to test the hypothesis that its direct parallel alignment encourages recursive strategy use. In response to this potential issue, we developed and tested two novel alternatives: the vertical model and the hiding model (see Figure 4.2).

The vertical model, a unique contribution to the literature, alters the alignment to specifically disrupt horizontal matching, a design informed by spatial comparison principles (Matlen *et al.*, 2020). The hiding model, adapted from unit identification tasks used in prior studies (Papic *et al.*, 2011; Wijns, Torbeyns, Bakker, *et al.*, 2019), was designed to prevent direct visual matching altogether. In this model, children view the pattern, press a button to hide it, and then complete the task from memory. We did not set a time limit for viewing, as there is no consensus in the literature; for instance, suggested timings have ranged from 5 seconds (Rittle-Johnson *et al.*, 2013) to 10 seconds (Papic *et al.*, 2011).

In our study, the average time to hide the pattern was recorded as 15.7 seconds. However, approximately 3 seconds of those were used by the researcher for prompts in Appendix D, so children spent 12-13 seconds viewing the model pattern.

In addition to the three different models, there are 2 different formats of the tasks: multiple-choice (MC) and drag-and-drop (DD). MC formats facilitate the administration and scoring of the tasks. Since children would rather spend time constructing the pattern, they choose the correct option, and we assume the MC format reduces the assessment time. Three choices were provided for the MC tasks because some studies indicated that having three options offers advantages, including higher discriminative power (Costin, 1970; Tversky, 1964 as cited in Gierl *et al.*, 2017, p. 1099) and greater consistency in writing effective distractors (Gierl *et al.*, 2017; Rodriguez, 2005). Further, we selected the distractors based on common errors found in previous studies (e.g., Collins and Laski, 2015). The units of options for the MC tasks were AB, ABB, and ABC. Creating AB patterns when the units were different is a very common error. Other common errors include matching only the first two elements, failing to look further, or selecting the same unit length.

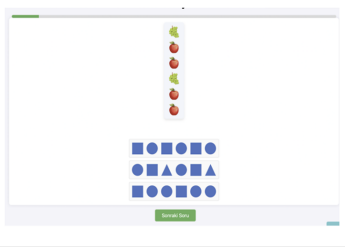
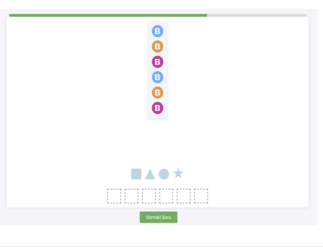
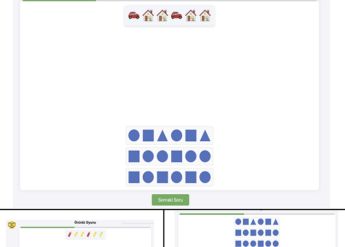
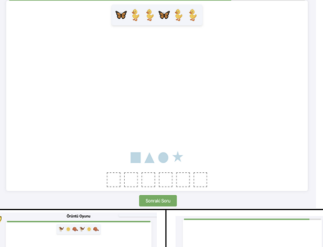
Model Name	Multiple Choice Format ABB, ABC	Drag and Drop Format ABB, ABC
Vertical Model		
Horizontal Model		
Hiding Model		

Figure 4.2. Examples of repeating pattern abstraction tasks.

However, generally constructed response items were used in the previous assessments (e.g., Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, Bakker *et al.*, 2019). Except Msall *et al.* (2025) used MC format in abstraction items and revealed that the proportion correct was relatively higher compared to the previous studies (Rittle-Johnson *et al.*, 2013 and Wijns, Torbeyns, Bakker *et al.*, 2019). Therefore, we want to test whether the question format affects the strategy choice and performance of the children, so we added the DD format as well. We provided more materials than needed to complete the pattern to avoid giving a clue about how many types of materials they should use. Moreover, we minimized the likelihood of correct responses by sequencing each element without comprehending the structure. Therefore, four different materials were provided for each DD format task. Also, we used two units, ABB and ABC, which were commonly used in previous research as well (Rittle-Johnson *et al.*, 2013; Wijns, Torbeyns, Bakker *et al.*, 2019). The RPAT was composed of 12 items, resulting from a design crossing 3 task models, 2 question formats, and 2 pattern units. A reliability analysis of the instrument yielded a Cronbach's alpha of 0.88, which indicates good reliability (George and Mallery, 2019).

Furthermore, the items were designed as color abstraction, shape abstraction, and color-shape abstraction. Shapes were the same, but the colors differed in the four-color abstraction items, and the colors were the same, but the shapes differed in the four shape abstraction items. Lastly, in the four object-color abstraction items, both the objects and colors differed, but the materials were the same for each type of task (See Appendix B). Thus, each type was distributed equally, and we tried to control the differences due to the model pattern.

Lastly, all the items were administered via computer rather than using physical materials to reduce the assessment time as well as make both implementation and scoring easier. The physical version requires different materials, such as pictures of model patterns and shapes, for children to construct their responses, but in the online version, a computer or a tablet is necessary to administer the assessment. Furthermore, the physical version is considerably more labor-intensive, requiring researchers to manually

score each child's response. This process of tracking or visually recording responses for subsequent coding is both time-consuming and effort-intensive. In contrast, the online version automates this process. At the end of the assessment, it generates a complete record of children's responses, with each item already coded as correct or incorrect. If necessary, the platform can also record reaction times for each task. This automation facilitates a more efficient and accurate method of data collection. Moreover, Larkin *et al.* (2024) demonstrated that even younger children, between the ages of 3.5 and 4.5, are capable of successfully engaging with app-based patterning tasks. Therefore, the online version of the assessment can be considered both researcher-friendly and developmentally appropriate for young children.

4.3.2. Visuospatial Working Memory

The Corsi block tapping task, which was developed by Corsi (1972), was used to measure visuospatial working memory. The task was executed using the PsyToolkit platform, which is a free website that allows designing, implementing, and analyzing experiments (Stoet, 2010, 2017). The existing Corsi task was used as a base, and we translated it into Turkish, as well as removing the feedback after each trial and adding practice questions. Children completed the task using an iPad. The task consists of nine pink squares in a fixed position for each child, and squares flash yellow for 1 second in random order and then disappear. Also, tasks consist of two parts; children first completed the forward task, then the backward Corsi.

A practice item was provided, and children were asked to copy the same pattern with flashing yellow squares, and feedback was given as correct or incorrect. Feedback was not provided after the practice question. After three consecutive wrong answers, the experiment was terminated. Even though various scoring methods have been used for the Corsi task, most studies have relied on span length for scoring (Berch *et al.*, 1998). Furthermore, recent evidence suggests that the longest sequence score, compared to the total correct score, is more sensitive in distinguishing the storage and manipulation components of working memory (Heled, 2024). Therefore, the scoring

of Corsi block tapping tasks is based on span length, which is the maximum sequence length the children were able to tap. For instance, if a child correctly completed the two-square and three-square sequences but failed at the four-square level, the child's score is three.

4.3.3. Task-based Interviews

Clinical task-based interviews were conducted to explore children's strategies in RPAT (Ginsburg, 1997). After the children answered the question, the researcher asked questions such as "How did you decide your answer?" or "Why did you choose this answer?", "How do you create your pattern?" or "Why does this one have the same rule as the one above?". When it is necessary, researchers ask some follow-up questions, such as "Do you think it is a pattern? Why?" or "Isn't the one below also a triple? Then, why did you choose the one above and not that one?". There are two reasons to prefer this method to think-aloud. Firstly, participants tend to change their answers while explaining their ideas, and previous studies indicated that self-explanation improves their performance (Rittle-Johnson *et al.*, 2015). Secondly, since children are young, explaining themselves while solving tasks might increase the cognitive demand and cause a burden.

There are two qualitative data sources that are generated by task-based interviews: audio recording and field notes. While participants answered the questions, audio was recorded, and researchers took notes about the gestures of the participants. The gestures were noted in three categories: (1) unit identification, if pointing to the unit of repeat with their finger, (2) matching, if children pointing up and down with their finger, and (3) not specifically pointing anywhere, such as sweeping their own pattern or model pattern.

4.4. Data Collection Procedure

The data was collected in the Spring 2025 semester. Following approval of ethical permission (see Appendix F) and MoNE research application permissions (see Appendix G), school principals disseminated the consent form to parents. Even though data was collected by a single researcher, a research protocol was created to avoid differences in data collection procedure and replicability of the study (see Appendix C and Appendix D).

Before the data collection, a pilot study with each age group was conducted to test the online version of the task, and children's reactions to the items, and to set a required time for children. After the pilot study, some items were revised for clarity, a scenario for the children's attention was added, and practice items were included. Additionally, computer-based applications and tablet-based applications were tested, and it worked better on a computer-based version, so we decided to administer the RPAT on a computer.

Prior to data collection, the researcher conducted a one-day observation at each preschool to build rapport with the children and acclimate them to her presence. Following parental consent, the researcher met with each child individually in a quiet room. To create a child-friendly context and motivate participation, each session with a brief narrative involving two finger puppets: Momo and Dodo. After a child agreed to participate, they completed four practice items with an AB unit designed to familiarize them with the test, the question format, and the computer interface (see Appendix A). Standardized, corrective feedback was provided after each practice items; this involved explicitly identifying the pattern's rule and unit (e.g., "the repeating part is star-moon") and explaining the structure of the correct response (see Appendix C and D).

Following the practice items, the main assessment began. The researcher used the finger puppets narrative and an on-screen progress bar to maintain the child's

engagement. Participants were informed that they would be asked to explain their reasoning after each question, and they were encouraged to try their best regardless of the outcome to reduce performance anxiety. For each item, the standard prompt asked the child to create or select a pattern that followed the same rule as the model pattern. After each response, the researcher prompted the child for an explanation using the task-based interview protocol (see Appendix C and D).

The tasks were administered to all children in fixed, counterbalanced order, beginning with the vertical, horizontal, and then hiding models; within each model, MC formats were presented before DD formats. A brief two-minute break was offered between the MC and DD sections if a child showed signs of fatigue or boredom. Through the RPAT, verbal responses were audio-recorded, and the researcher took field notes on the children's gestures. Upon completion of the RPAT, participants were administered the Corsi-Span task on a tablet, completing the forward version followed by the backward version. Each child's entire lasted approximately 30 minutes.

4.5. Data Analysis

Due to the convergent parallel mixed-methods design of the study, qualitative and quantitative data analyses were applied separately. The analysis plan was aligned directly with the study's research questions. The first research question was addressed using qualitative analysis, and the second research question was addressed with quantitative analysis. The third research question were addressed by integrating both quantitative and qualitative findings to provide a comprehensive answer.

4.5.1. Qualitative Data Analysis

The current study examined young children's strategies for different types of abstraction tasks, therefore an in-depth analysis of participants' responses was required. The data sources were generated by task-based interviews in RPAT. Audio recordings and observation notes about children's gestures were used. A hybrid process of

inductive and deductive thematic analysis was conducted to reveal strategy use. Thematic analysis is a useful qualitative data analysis method to reveal patterns in the data (Braun and Clarke, 2006). During thematic analysis, we followed the 6 phases generated by Nowell et al (2017) to conduct a trustworthy thematic analysis. As a first step, the audio recordings were transcribed and merged with observation notes for each question in RPAT. Then, the coding procedure started with deductive thematic analysis using the patterning strategy coding schema developed by Lüken and Sauzet (2021). After a priori analysis, we observed some different indicators, so we inductively generated new codes. For example, matching has different categories, and one-to-one matching is different from matching elements without order or partial matching. Also, creating AB patterns might serve different strategy uses, such as partial matching or due to a lack of explanation off-task. MAXQDA 24.8.0 was used to code data. During the initial coding phase, I engaged in weekly peer debriefing sessions with my supervisor. The purpose of these meetings was to discuss and validate the relationship between the analytical codes and their corresponding indicators. After the initial phase of coding, the main strategy categories were generated with the help of Papic *et al.* (2011) and Lüken and Sauzet (2021) strategy categories.

Five strategy categories were generated, similar to the Lüken and Sauzet (2021) themes, and examples of the codes were provided in Table 4.1. Also, we holistically considered the gestures and verbal responses. In cases where children did not provide verbal explanations, their strategic approach was often conveyed through gestures. For example, up-and-down finger movements and showing matching each element were coded as “one-to-one matching” strategy. If a child used more than one strategy, we took the most sophisticated strategy category. For example, if a child used both shape similarity and partial matching strategy, we coded it as partial matching because it is a more sophisticated strategy, and the child reached that level with explanations. Eight percent of the data were not able to be coded since the indicators were unclear and categorized as “Not-categorized”. The codes were checked in two phases: checking the within-subjects, which involves assessing codes for each participant, and checking the between-subjects, which involves recalling the code and checking that the codes

are compatible with the responses. Afterwards, the second round of coding was completed. Then, 27% of the data were chosen with stratified random sampling, which is 5 participants for each age group, and a graduate student who is familiar with patterning literature was blindly coded. Although there is no firm consensus on the percentage of data required for double-coding, a range of 10-25% is common practice (O'Connor and Joffe, 2020). In this study, 27% of data was independently coded by a second rater to establish interrater reliability. Cohen's kappa was used to determine interrater agreement. The results significantly showed almost perfect agreement, $k = 0.97$, $p < 0.001$.

Table 4.1. Strategy coding schema .

Strategies	Sub-strategies	Examples
1) Off task Not referring the to the pattern's elements		- I don't know - Because you thought - Pattern is AB - I choose the same pattern with previous one
2) Not Categorized Explanations don't have enough indicators		- Saying whole elements with slight pauses - Using the same / different or complex words
3) Attention to the Singular characteristics "Reference to singular aspects of the pattern (e.g., color, shape, number)" (Lüken and Sauzet, p. 34)	3.1: Shapes similarity	- Focusing on similarities between the shapes in the pattern or the shapes they create. "Apple looks like circle"
	3.2: Focusing only shapes	- Focusing only the shape changes on color-abstraction patterns "It is all-star"
	3.3: Counting one element	- Counting only one element type "there are two here, here too"

Table 4.1 Strategy coding schema. (cont.)

Strategies	Sub-strategies	Examples
4) Comparison “Comparing the elements between patterns”. (Lüken and Sauzet, p. 35)	4.1: Matching elements without order	- Matching elements without considering their order such as creating AAB rather than ABB pattern.
	4.2: Same unit length	-Counting the unit without considering elements such as saying it is three and choosing ABC rather than ABB pattern.
	4.3: Counting each element	- Counting without considering the location of elements in the pattern saying, “There are four of these, two of these”, in ABB pattern.
	4.4: Partial Matching	- Matching only different elements such as creating AB in ABB pattern. - Matching only two elements in ABC pattern and creating AB pattern. -Matching one element to multiple elements “I thought like this cherry (pointing square). Like this star (pointing circle). Like this apple (pointing circle). This one is like a cherry (pointing square). This one is like a star (pointing circle). This one is like an apple (pointing circle)”,

Table 4.1 Strategy coding schema. (cont.)

Strategies	Sub-strategies	Examples
5) Sequence Awareness Attention to the sequence without considering the unit	5.1: Matching one-to-one	- Matching elements one-to-one “The butterfly, of course, in the square. The ducks are 2, 2 triangles next to the square. Now the butterfly is a square, of course. There are two ducks, so put two triangles”.
	5.2: Focus on Sequence	- “Awareness of the succession of element” “Yellow pencils side by side, red pencils not side by side” or “Because here we have blue, then orange, then pink next to pink there is blue, then orange next to orange pink”
6) Unit Identification Identifying the repeating unit explicitly	6.1: Identifying the repeating unit	- Explicitly saying or showing the repeating unit “It is a yellow green, green” - Naming the unit numerically “This is a 1-2 pattern” or “This a three pattern was like the one above”
	6.2: Matching only the first unit	- Matching only the first unit and completing the pattern without matching in the second unit “this is the star, this is the triangle and this is the square, now I’ll do the same here, I don’t need that pattern”
<i>Note. Strategy coding schema adapted from Lüken and and Sauzet’s (2021) study.</i>		

4.5.2. Quantitative Data Analysis

The quantitative data analysis was conducted in three phases. The primary variables derived from the data collection were the RPAT scores, RPAT strategy scores, Corsi block tapping scores, and the children's age.

The RPAT scores for each child was calculated as the sum of correct answers (coded as 1), with a maximum possible score of 12. The RPAT strategy scores represented the mean of strategy codes (1 through 6) used by each child, and the Corsi-block tapping score was the longest correct sequence they performed.

Initially, descriptive statistics were used to summarize these key variables. Subsequently, to address the second research question regarding the relationship between strategy use and RPAT scores while controlling for age and working memory, a multiple linear regression (MLR) was performed in JMP Student Edition 18.2.1. The assumptions of the MLR model were checked and met prior to conducting the analysis.

To answer the third research question examining the influence of task type on strategy choice, a generalized linear mixed model (GLMM) was employed. Firstly, we generated an accuracy table to address how many attempts of a specific strategy use were successful. To address the first part of this research question, the analysis focused specifically on predicting the use of the unit identification strategy, as this was the study's primary indicator of relational thinking. Consequently, the dependent variable for the model was a binary outcome representing whether a child used the unit identification strategy on a given task (coded as 1 for used and 0 for not used). The model's independent variables (i.e., fixed effects) were task format (MC vs. DD), task model (vertical, horizontal, vs. hiding), and the child's age in months. If a specific task model or format yielded a higher log-odds estimate than the horizontal model in predicting the use of relational strategies, we interpreted this as evidence that it supported or promoted the use of relational strategies. A GLMM was specifically chosen because it is well-suited for analyzing binary, repeated-measures data where

each child responds to multiple items (Agresti, 2012). All analyses were conducted in R (version 4.5.1) using the “lme4” package.

Similarly, for the second part of the research question three, we conducted another mixed-effects logistic regression to examine whether specific task features predicted the use of recursive strategies. This model investigated if the task format, task model, or age influence the use of either the sequence awareness or comparison strategies. For this analysis, the dependent variable was transformed to the binary data: the use of one of these two strategies (coded as 1) versus any other strategy (coded as 0). The independent variables (i.e., fixed effects) were identical to those in the previous model: task format (MC vs DD), task model (vertical, horizontal, vs. hiding), and age. If a specific task model or format showed a lower log-odds (logit) estimate than the horizontal model in predicting the use of recursive strategies, we interpreted it as having a minimizing effect on promoting recursive strategy use.

5. RESULTS

In this chapter, qualitative and quantitative results are presented. First, qualitative analysis results were presented to address the first research question. Then the second research question was examined through quantitative analysis. Lastly, the third research question was explored through both qualitative and quantitative results.

5.1. Repeating Patterning Abstraction Strategies

This section addresses the first research question, which examined 4-to-6-year-old children’s strategies in RPAT.

Overall, there are five strategy categories similar to the previous findings (Papic *et al.*, 2011; Lüken and Sauzet, 2021). During the coding process, we used Lüken and Sauzet’s (2021) strategies as our starting point. However, we observed four key dimensions of the children’s strategic approach: shape, number, order, and repeating unit. While grouping the codes and generating strategies, we considered these dimensions. The categories are presented below from basic “off-task” to most sophisticated “unit identification”, similar to Lüken and Sauzet (2021). Figure 5.1 illustrates the distribution of strategies across ages, indicating that strategy trends are age dependent. The basic strategies, as off-task attention to the singular characteristics and comparison, were more frequently observed in younger children. On the other hand, sophisticated strategies such as sequence awareness and unit identification are more commonly observed in older children. The following section introduced each main strategy category, followed by its sub-strategies with examples of an excerpt or quotation.

5.1.1. Off-task

The Off-task strategy consists of answers that do not refer to the pattern and provide random answers. This strategy is equivalent to the term “no reference to pat-

tern besides reproduction of the pattern’s gestalt”, as defined by Lüken and Sauzet (2021), and it is also referred to as off-task in Collins and Laski (2015). We prefer to use the off-task label since the children’s explanations were not related to the pattern elements and did not demonstrate engagement with the tasks. Typical verbal explanations included phrases such as “I don’t know”, “I don’t remember”, “Because you thought so”. “The pattern is AB”, or “I chose the same pattern as the previous one”. Therefore, these explanations did not attend to any elements of the pattern.

Additionally, children did not answer 6% of the tasks, and half of them were four years old. The reason these responses are classified as off-task is that the children used off-task strategies in at least two tasks and showed signs of fatigue and boredom. Even though more than half of the four-year-olds (54%) used the off-task strategy, use of this strategy was decreasing while age was increasing (26% and 20%, respectively) (see Figure 5.1). This decrease reflects a developmental shift-children gradually stop relying on this strategy as they grow older.

5.1.2. Not Categorized

Not Categorized was assigned to 8% of the total where children’s explanations and gestures lacked sufficient indicators for definitive coding. These responses were typically characterized by a child listing the pattern’s elements (sometimes with slight pauses) or using vague comparative words without providing a clear rationale.

It was crucial to classify these responses as ambiguous because an action like pausing between units could be misinterpreted as an awareness of the pattern’s structure, when further questioning revealed this was not the case. The following excerpt illustrates this ambiguity. While the child identifies the correct elements of the model pattern, their justification reveals they are simply naming the elements of their incorrect choice rather than identifying the pattern’s structures:

C-35: Because here it says house, house, car (pause) house, house, car.
I: Yes, here there is house, house, car. But why did you choose that?

C-35: Because here it says triangle, square, circle, triangle, square, circle. I got it right.

This category is distinct from the off-task strategy because the child did refer to the pattern. However, it remained unclear whether this was a nascent form of unit identification or another emergent strategy type. The frequency of these responses was 4% for four-year-olds, 12% for five-year-olds, and 8% for six-year-olds (see Figure 5.1) so it seems age-independent.

5.1.3. Attention to the Singular Characteristics

Attention to the Singular Characteristics strategy is the same with Lüken and Sauzet (2021), and we used the same definition: “reference to singular aspects of the pattern” (e.g., color, shape, number) (p.34). Children employing this strategy generally produced responses with little regularity; their attempts either resulted in a non-pattern answer or the creation of AB pattern. This observation aligns with previous findings by Lüken and Sauzet (2021). This category consists of three sub-strategy categories.

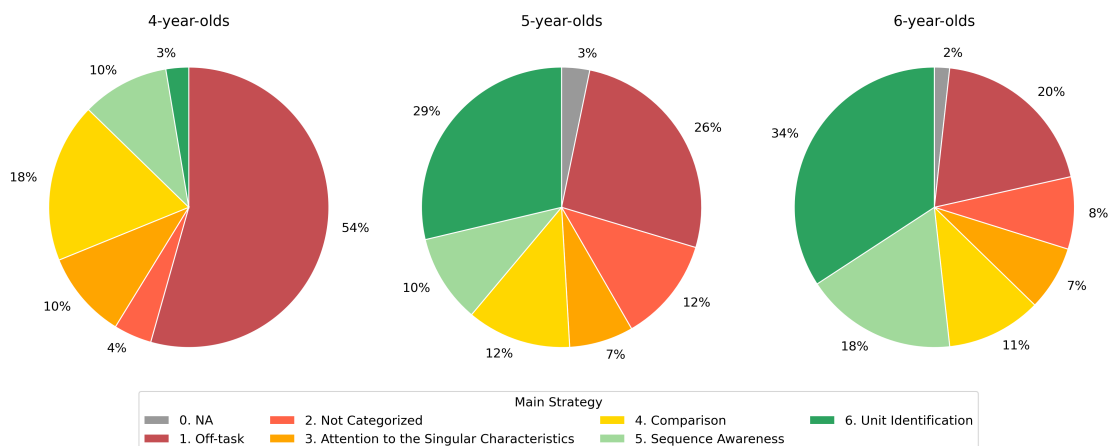


Figure 5.1. Relative frequencies of RPAT strategy per age group.

The first one is Shape Similarity, which focuses on similarities between the shapes in the pattern or the shapes they create, such as “a car that looks like a circle”, or “we can create a circle from a square”. The following excerpt exemplifies both aspects

of the sub-strategy for Item 7 (DD vertical leaf-flower-flower pattern), and the child responded to this item as triangle-square-circle-star-circle-square.

I: Why does this one have the same rule as the one above?

C-13: Because... because there is a circle inside the sunflower, it becomes a circle. It looks like a star around it.

I: And why did you choose the square and the triangle?

C-13: Triangle... the tip of the leaf looks like a triangle, so it is a triangle

I: why did you choose square?

C-13: the square is a little bit like the circle.

Children only attend to the similarity of the shapes in this category and construct triangle-square-circle-star-circle-square. He prefers to use a triangle because of the leaf's shape, a circle and a star because of the flower's shape, and lastly, a square because it is similar to the circle. Therefore, the child did not see the whole pattern and regularity in the pattern; rather, he saw the elements individually and created a non-pattern answer.

Focusing Only Shapes is the second sub-strategy, and it is observed in color abstraction tasks, which involve only color changing patterns. Children only focus on the shape, so they did not answer the tasks, or they used the same shape while creating. However, there is no such indicator in previous studies. The following explanations were provided for Item 8. Even though the child observed that the colors were changing, he did not find it different because only different shapes were provided for abstraction.

C-11: I don't understand. I: Why?

C-11: The shapes are the same, but the colors are different.

I: The shapes are the same, but the colors are different. So how can I do this with the shapes below?

C-11: I don't know.

I: Why did you use different shapes just before?

C-11: It was different then.

I: Don't these look different?

C-11: No, they don't.

Not every child observed what was changing, but even though they realized colors were changed, they either did not answer the question or answered by creating the same

shape, such as AAAAAA. Especially in Item 8, children recreate the pattern of only circles and argue that they are all the same shape. The color-abstraction model items and their percentages were given in Figure 5.2. Interestingly, although the tasks were designed similarly, children's strategy use differs. While none of the children used this sub-strategy in Item 5, 18% of the children used it in Item 8. They tended to use this strategy in vertical models rather than hiding models.

	
Item 2 (MC-Vertical): 11%	Item 5 (MC-Hiding): 0%
	
Item 8 (DD-Vertical): 18%	Item 11 (DD-Horizontal): 7%

Figure 5.2. Color-abstraction item models and use of focusing only shapes sub-strategy percentages.

The last sub-strategy is Counting One Element, which is children only counting one element in the pattern. For example, there are two orange, two green, two pink and children only count the pinks or in a butterfly-duck-duck pattern only count the ducks next to each other and say two ducks. This strategy might generate correct responses in ABB unit multiple choice items since children only count B's and there is one option with four B's. The following quotation is an example for Item 3 which is car-house-house pattern. "Because there are two here, here too". While making this explanation, the child showed the two houses in the first unit and then showed two

circles next to each other. Even though some of the children created correct responses using this strategy, children attended only one characteristic, number of the pattern.

All the sub-strategies only attend one aspect of the pattern so grouped as attention to the singular characteristics. This strategy was used more commonly by 4-years-old (10%) and used equally (7%) by 5 and 6-years old (see Figure 5.1). Still younger children are more likely to rely on this strategy in abstraction tasks similar to the off-task strategy.

5.1.4. Comparison

The third strategy category is comparison and can be defined as “children compare the pattern’s elements and highlight sameness within and between patterns on a basic, very concrete level” (Lüken and Sauzet, 2021, p. 35). However, our conceptualization of this strategy differs from that of Lüken and Sauzet (2021). While they include broad indicators such as ‘comparing and matching elements,’ we apply a more specific lens. In our strategy framework, we observed that generally children used two dimensions of the pattern: shape, number, or sequence. Therefore, we did not include the one-to-one matching indicator in this category since it requires attending to more than two dimensions.

There are four sub-strategy categories in the comparison category. One of them is Matching Elements Without Order, which is creating AAB instead of ABB, but this sub-strategy is observed only three times, so it is very rare. For example, Child 58 constructed a square-square-star pattern when the model was green-red-red in Item 11 (DD hiding ABB pattern, see Appendix B) and the child explanation was as follows:

There was something here... Green balloon, green balloon... Green balloon, red balloon... Green balloon, red balloon... Square square star square square star. Because the green balloons were two, the red ones became one..

The reason for this might be children being confused when the model pattern was hidden, but also two more children did this explanation in Item 3 (MC horizontal ABB pattern) and Item 7 (DD vertical ABB pattern). Even though it was observed rarely, we categorized it separately from other categories since children showed different behavior in the pattern. They attend the number and one-to-one correspondence with shapes without considering order.

The second is the Same Unit Length; children were able to determine that the repeating part consists of three items but randomly chose ABB or ABC. In other words, there is an intuition that there are three units, but the elements within the unit are being ignored. Although observed in only one child (C-21), this strategy is included as a distinct category because it provides a theoretically important and unambiguous example of a child focusing on unit length while ignoring internal structure. The following excerpt highlights the child's understanding of Item 3, which is an MC horizontal question.

C-21: One, two, three. I chose that one.

I: Why did you choose that one?

C-21: That was also a triple

I: Now I'm going to tell you something here. Isn't the one below also a triple?

C-21: Yes

I: Why did you choose the one above and not that one? there should be a reason

C: Yes. I chose that one just before, and I couldn't choose that one this time.

I: Why did you think it had the same rule as this one?

C: Because this one is also three. One, two, three.

I: Doesn't it make a difference which one you choose for the three? I'm only asking you because I am curious.

C: I know. I think it doesn't matter.

This child was aware that the pattern unit consists of three elements, but the inside of the elements did not differ for her, and she chose randomly among patterns that have three repeating units. In our options, there were ABB, ABC, and AB units so she chose randomly ABB or ABC.

The third sub-strategy category is Counting Each Element, and children count the different elements without considering the place of the elements. For example, in an ABBABB pattern, children explained their thinking as there are four B's and two A's. Especially in multiple-choice questions, children who use this sub-strategy answered correctly. The following excerpt provides an insight into this sub-strategy for the MC horizontal ABC pattern (Item 4).

C-51: There are two stars, two cherries, two apples. Then I think it's the bottom one.

I: Why did you choose that one?

C-51: Because there are two triangles. There are two circles. There are two squares.

This excerpt showed the child attending to the two dimensions of the pattern as shape and number without considering the sequence. She also used this strategy in five items. She correctly answered the questions in MC format, but in the DD format questions, she sorted the items as AABBC or DCCDCD, which were incorrect. Therefore, the accuracy of this sub-strategy depends on the question format. The fourth sub-strategy is Partial Matching. Children who use this sub-strategy compare the patterns and partially match them, either engaging with a subset of the elements or incorrect correspondence between elements. Creating an AB pattern is very common in this sub-strategy. Three indicators were identified for this sub-strategy: (1) They match some of the elements, such as creating AB rather than ABC, (2) they match only the different elements, such as creating AB rather than ABB, and (3) they match one element with multiple elements so they do not have an understanding that one shape should correspond to one shape. An example from Child 39 was provided with the following excerpt. He created an AB (star-square) pattern rather than ABB in Item 7 and explained his thinking as follows:

I: Why do you think it has the same rule as above?

C-39: For example, let's call this a star, let's call this a leaf. Let's call this a sunflower. Call this a sunflower if you want. Let's call this a leaf. Let's call these one two flowers.

I: Then my flowers can be different, right? It can be both a star and a square.

C-39: (Nodded his head yes)

In this case, he started the pattern with a star, then he named the square as a sunflower, and called the next sunflower as a star, then called the square as a leaf, and continued like this. Even though he was aware of the number of elements as “two flowers”, he misprioritized it and the one-to-one correspondence of the shapes.

Children who used the comparison strategy did not fully grasp the one-to-one correspondence, number, and sequence of elements. However, they attend to some characteristics of the pattern and make comparisons between the patterns. Trends of comparison strategy use were similar with off-task and attention to the singular characteristics but different from unit identification and sequence awareness. Similar to the aforementioned strategies, while the age increased, the use of comparison strategies decreased. The percentages were provided in Figure 5.1 as 18% of 4-year-olds, 12% of 5-year-olds, and 11% of 6-year-olds used it.

5.1.5. Sequence Awareness

It can be defined as attending to the sequence without grasping the unit, similar to the focus on sequence category in Lüken and Sauzet (2021) and alternation category in Papic *et al.* (2011). Children who use this strategy consider not only the number or shape of elements but also use the sequence between elements. There are two different sub-strategies of this category: one-to-one matching and focus on sequence. One-to-one Matching is included in this category differently from previous studies since we observed that there are different types of matching categories. In one-to-one matching, children used three different dimensions of the model pattern: number, shape, and sequence. Therefore, it is more complex than the comparison strategy we will describe above since they used these three dimensions and importantly attended to the sequence. In one-to-one matching, children are aware that the one-to-one correspondence which is the same shape should match with the same shape and different shapes match with different shapes. Children using this strategy match items one by one, either verbally or with gestures. While matching with gestures children did an up-down movement for each element with their fingers or computer’s cursor. The following quotation is

a verbal one-to-one matching example for Item 9 which is DD and horizontal model ABB pattern (see Appendix B).

C-8: The butterfly, of course, in the square. The ducks are 2, 2 triangles next to the square. Now the butterfly is of course a square. There are two ducks so put two triangles.

This excerpt showed that Child 8 attended to three dimensions of the pattern: (1) how many elements in the pattern as one butterfly and two ducks, (2) different shapes should correspond to different shapes as butterfly square and duck triangle, and (3) the order as two triangles next to the square. Overall, 10% of children used the one-to-one matching sub-strategy, so most of the sequence awareness strategy consists of this sub-strategy.

The second sub-strategy is Focusing on a Sequence, which is the same strategy defined by Lüken and Sauzet (2021) as “focusing on succession of elements” (p. 34). In this category, children alternate the consecutive elements, such as next to red is yellow. Children tended to use this strategy, especially the ABC pattern, rather than the ABB unit, since the elements in the ABC unit are different from each other. For example, Child 46 did the following explanation in Item 2, which is the MC vertical ABC pattern (see Appendix B), by referring to the pattern she created:

Because there is an orange one here, and a green one. There is a purple one, an orange one, a green one and a purple one. Because here we have orange, green, purple. Here, starts from orange. Green, then it becomes green in the center, then purple.

This explanation showed that she did not directly use the unit but focused on the place of the elements as starting from orange, then green, then purple.

She is aware that consecutive elements are different from each other, so she chooses her answer considering this. Another example similar to this is used by Child 58 in Item 6 and explained, “Because they are not side by side. The two are not side by side”. While C-58 was making this explanation, he showed with his finger that the

first and third elements of the option she chose are not next to each other, and the second and fourth are not next to each other. Overall, only 3% of the participants used this sub-strategy.

In light of these explanations, the sequence awareness category consists of one-to-one matching and focusing on sequence sub-categories, since both of them are required to attend to the sequence. 4-year-olds and 5-year-olds used this strategy equally often, at 10%, while 6-year-olds more frequently used it at 18% (see Figure 5.1). The trends of sequence awareness strategy across ages were different from the previous strategies and increased in 6-year-olds. Therefore, older children tended to consider different dimensions of the pattern.

5.1.6. Unit Identification

It is the most advanced category, similar to the previous findings (Collins and Laski, 2015; Papic *et al.*, 2011; Lüken and Sauzet, 2021) and the only relational thinking strategy since children who use this strategy focus on the structure of the pattern. In other words, children used the repeating unit rather than focusing individual elements in the pattern. Children who use this strategy showed an explicit understanding of the repeating unit. There are two different indicators of this strategy category. The first one is identifying the repeating unit, during their explanations, either they showed the repeating unit by circling with their hands or a cursor, or they verbally indicated the unit. Children also used numerical explanations such as “Because of here it’s three three. Here it’s three three”, or “Because it is starting with one-two, one-two here. It is also one-two”. These quotations showed that children not only have an explicit understanding of the repeating unit but also abstract the unit with numerical expressions such as one-two patterns. Therefore, children who use the unit identification strategy attempt to generalize the pattern and express it using different verbal notations. This shows that children did not approach the elements as constant, but they showed a flexible approach while naming them.

The second is Matching Only the First Unit, which is not indicated in previous studies. Children only match the first unit and copy the first unit to construct the second unit. Since children in this category showed an explicit understanding of the structure, it is categorized as unit identification. This is different and more sophisticated than other matching categories because it provides a structural sense. The following excerpt reflects a good example since the Child 10 explained he used the first unit and copied it to construct the second unit.

C-10: I just had a very good idea. If this happens, I'll make a star for this one. I'll put this one like this. I'll make this a triangle. Here comes the square, here comes the triangle. I did it without looking at it. Because I looked at what I did from the beginning to the end

I: Why did you do it like that, by the way?

C-10: Because what I did myself was like this (showed the first unit). So, I matched it with this and that. Then I looked at myself from the beginning to the end. I never looked at this. I would do it even if it was deleted. (Item 10)

Even though he did not name the unit, he showed that he focused on the underlying structure and viewed the unit of repeat. After this observation, children who match the only first repeating unit are categorized in this category, since we used a hybrid thematic approach. A similar approach was observed in other children. For example, Child 50 explained their thinking in Item 7 as:

There's a leaf. And here is a triangle. There are also two daisies. And there are two stars here.

Children who used this strategy limited their explanations to the first unit of the pattern and did not need to mention other elements in the pattern. This suggests that she was aware of the unit; she only referred to the elements in the first unit of the pattern. These consistent explanations support that children in matching the first unit category showed explicit use of units, so we grouped it into the unit identification strategy.

The use of the unit identification strategy showed a clear developmental trend, with its frequency increasing with age (see Figure 5.1). Only 3% of 4-year-olds used this

strategy-a finding consistent with that of Lüken and Sauzet (2021)-compared to 29% of 5-year-olds and 34% of 6-year-olds. As a result, the strategy frequencies by age were shown in Figure 5.1. 6-year-old children more frequently used sophisticated categories such as unit identification and sequence awareness. Comparison and attention to the singular characteristics strategies were more commonly used by 4-years old, while 5- and 6-year-old used similarly. Off-task strategy use was more common in 4-year-olds and decreased while the age increases.

5.2. Predictors of Repeating Pattern Abstraction Performance

This section addresses the second research question, which examined the extent to which RPAT strategies predict RPAT scores after controlling for age and visuospatial working memory.

Preliminary analyses were conducted first. Descriptive statistics for all participant variables, including RPAT scores, strategy scores, age, and Corsi task scores, are presented in Table 5.1. An examination of the skewness and kurtosis values indicated that the distribution of all variables approximated a normal distribution (skewness values between -1 and +1; kurtosis values between -2 and +2). Following the assumptions check, a linear multiple regression analysis was performed to address the research question.

Table 5.1. Descriptive statistics of dependent and independent variables.

Variables	<i>N</i>	<i>M</i>	<i>SD</i>	<i>Mdn</i>	<i>Min</i>	<i>Max</i>	<i>Skewness</i>	<i>Kurtosis</i>
RPAT Scores	56	5.43	3.83	4	0	12	0.44	-1.13
<i>MC Format Scores*</i>	56	3.11	1.87	3	0	6	0.12	-1.05
<i>DD Format Scores*</i>	56	2.32	2.21	2	0	6	0.48	-1.23
<i>Vertical Model Scores*</i>	56	1.55	1.34	1	0	4	0.65	-0.77

Table 5.1 Descriptive statistics of dependent and independent variables. (cont.)

Variables	N	M	SD	Mdn	Min	Max	Skewness	Kurtosis
<i>Horizontal Model Scores*</i>	56	2.11	1.46	2	0	4	-0.05	-1.38
<i>Hiding Model Scores*</i>	56	1.77	1.48	1	0	4	0.42	-1.20
RPAT Strategy Scores	56	3.32	1.47	3.5	1	5.92	-0.06	-1.05
<i>MC Format Strategy Scores**</i>	56	3.30	1.52	3.33	1	6	0.12	-1.05
<i>DD Format Strategy Scores**</i>	56	3.33	1.58	3.58	1	6	-0.15	-1.10
<i>Vertical Model Strategy Scores**</i>	56	3.26	1.53	3.25	1	6	0.05	-0.96
<i>Horizontal Model Strategy Scores**</i>	56	3.85	1.60	4.25	1	6	-0.50	-0.95
<i>Hiding Model Strategy Scores**</i>	56	2.84	1.70	2.38	1	6	0.49	-1.18
Age (month)	56	65.34	8.55	66	48	79	-0.18	-1.26
Corsi Forward	51	2.71	1.32	3	0	5	-0.69	-0.17
Corsi Backward	51	1.71	1.42	2	0	4	-0.15	-1.56
Note: *Five variables are subcategories of RPAT scores. **Five variables are subcategories of RPAT strategy scores.								

Prior to performing the linear regression, the assumptions for the model were tested. Linearity, independence of errors, homoscedasticity, and normality of the errors are four main assumptions in linear regression (Keith, 2019). The first and most

important assumption is linearity (Keith, 2019). An initial scatterplot of the data revealed a violation of this assumption; specifically, it showed a clear quadratic, rather than linear, relationship between RPAT strategy scores and RPAT performance scores. This quadratic relationship indicated that as children's strategy use became more sophisticated, their performance improved at an accelerating rate. For instance, the gain in performance between higher-level strategies was more substantial than the gain between lower-level strategies. To correct for this non-linearity, a power transformation was applied by squaring the RPAT strategy scores. As shown in Figure 5.3 this transformation was successful, and the relationship between the squared RPAT strategy and RPAT scores subsequently was met the assumption of linearity.

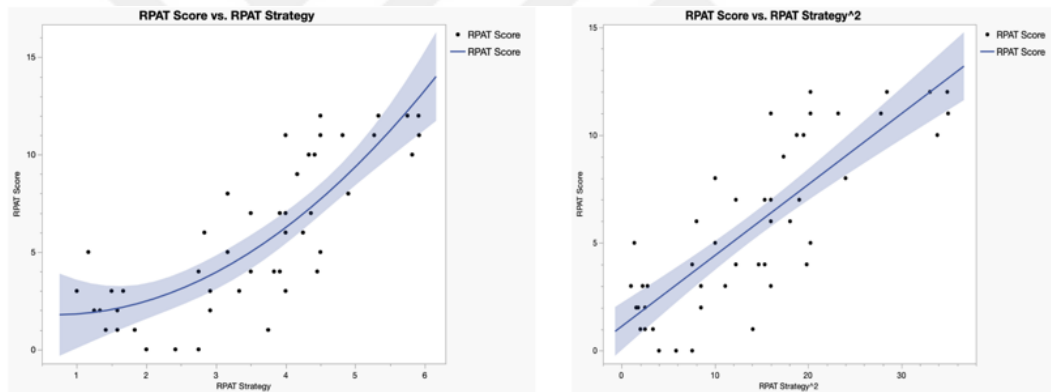


Figure 5.3. Scatter plot of RPAT scores and RPAT strategy scores.

The remaining model assumptions were also tested and met. A Durbin Watson statistic of 2.22 confirmed the independence of errors. The assumption of homoscedasticity was supported by the examination of the residual scatterplot, which revealed no discernible pattern. Furthermore, visual inspection of a histogram and Q-Q plot indicated that the normality of residuals assumption was also met.

Finally, the potential for multicollinearity was checked with a bivariate correlation matrix of the independent variables (see Table 5.2). While a moderate correlation was found between the squared RPAT strategy scores and age ($r = 0.483, p < 0.001$), this level is consistent with the literature and not considered a risk for multicollinearity.

The other inter-correlations among predictors were weak to moderate, confirming that multicollinearity was not a concern for the model.

When we examined the bivariate correlation matrix, there is a very strong and statistically significant correlation between the RPAT scores and the squared RPAT strategy scores ($r = 0.837$, $p < 0.001$), indicating that students who have higher scores tended to use sophisticated strategies. Additionally, the relationship between RPAT scores and age is strong ($r = 0.582$, $p < 0.001$) which means as children get older, their RPAT scores increase.

Table 5.2. Bivariate correlation matrix.

	RPAT scores	RPAT strategy scores	RPAT strategy scores square	Age	Corsi Forward
RPAT strategy scores	0.807***	1			
RPAT strategy scores square	0.837***	0.978***	1		
Age	0.582***	0.485**	0.483**	1	
Corsi Forward	0.110	0.366**	0.321*	0.163	1
Corsi Backward	0.369**	0.362**	0.410**	0.110	0.111

Following the assumptions, we performed a sequential multiple regression analysis to predict RPAT scores. The analysis was conducted in three steps, with the results for each model presented in Table 5.3.

In the first step, Model A was set to establish a baseline, using only the RPAT strategy score and its squared term as predictors. This model confirmed a significant

quadratic relationship between strategy and performance, explaining 69% of the variance in RPAT scores, $F(2, 48) = 52.335$, $p < 0.001$. Then, the Model B was built by adding the control variables of age and visuospatial working memory (Corsi forward and backward scores). These variables were included as they are known predictors of early mathematics performance (Kyttälä *et al.*, 2003; Li and Geary, 2013). This expanded model was also significant, $F(5, 45) = 29.267$, $p < 0.001$, and its explanatory power increased, accounting for 76% of the variance. As shown in Table 5.3 while age and Corsi Forward were significant predictors, the Corsi Backward score was not. With the inclusion of these controls, the squared RPAT strategy term was no longer statistically significant ($\beta = 0.275$, $p = 0.070$). While 12% of the participants had zero span score in Corsi Forward, 37% of the participants have zero span score in Corsi Backward.

Table 5.3. Multiple regression models to predict RPAT scores.

Predictor	Model A	Model B	Model C
Intercept	-1.975 (1.731) 1.14	-3.458 (2.428) -1.42	-3.084 (2.325) -1.33
RPAT strategy	-567 (1.097) -0.52	0.164 (1.027) 0.16	0.046 (1.00) 0.05
RPAT strategy $\times$ RPAT strategy	0.409* (0.158) 2.58	0.275 (0.151) 1.82	0.301* (0.143) 2.10
Age		1.169* (0.374) 3.12	1.147* (0.370) 3.10
Corsi Forward		-0.516* (0.226) -2.28	-0.511* (0.224) -2.28
Corsi Backward		0.130 (0.223) 0.58	
R^2	0.69	0.76	0.76
Cell entries are unstandardized β coefficients, (standard errors), and t-statistics * $p < 0.05$			

Given that Corsi Backward was not a significant predictor and 37% of participants took zero and it might lead a floor effect for this measure, a more parsimonious Model C was specified by removing it from the analysis. This final model remained significant, $F(4, 46) = 37.030$, $p < 0.001$, and maintained the same explanatory power as Model B ($R^2 = 0.76$). In this more efficient model, all remaining predictors became statistically significant. The removal of the non-significant variable also increased the model's precision, as indicated by the decrease in standard errors for all remaining predictor estimates (see Table 5.3). Therefore, Model C was retained as the final model for interpretation.

The final model (Model C) demonstrated that strategy use was a significant predictor of abstraction performance, even after controlling for the effects of age and visuospatial working memory. The main predictor of interest, the squared RPAT strategy, remained a significant positive predictor of RPAT scores ($\beta = 0.301$, $p = 0.041$). This significant quadratic effect indicates that as children used more sophisticated strategies, their scores were predicted to increase at an accelerating rate, independent of the control variables. The control variables also significantly predicted performance. Age played a positive role ($\beta = 1.147$, $p = 0.003$), with each one-year increase in age associated with a predicted 1.15-point increase in RPAT scores. Lastly, Corsi forward was a significant negative predictor ($\beta = -0.511$, $p = 0.027$) when included in the model with the RPAT strategy score, its squared term, and age. Although there is a positive correlation between RPAT scores and Corsi forward, this negative contribution to the model suggests a more complex relationship: the positive impact of more advanced strategy use on RPAT performance may be dampened for children with higher visuospatial working memory capacity.

5.3. Repeating Patterning Abstraction Strategies by Task Type

This section addressed the third research question, which explored the task design features that promote relational thinking strategies and help minimize the use of recursive strategies in repeating pattern abstraction tasks. Firstly, how children's

strategies differ for the different task models and formats was examined based on qualitative findings. There were three different task models: vertical, horizontal, and hiding. Each task model had four questions: two MC and two DD formats, with two different units, ABB and ABC. The strategy uses percentages for task models, and task formats were examined. Secondly, to offer a more comprehensive understanding, quantitative results were provided.

5.3.1. Strategies by Task Models

This section explored qualitatively children's strategy use on different task models. As illustrated in Figure 5.4 the different task models had a distinct influence on children's strategy use. Notably, the use of the most sophisticated strategy, unit identification, remained remarkably consistent across all three models, accounting for 21% of strategies in the vertical model and 22% in both horizontal and hiding models. This suggest that the use of relational strategy was not significantly affected by the task's spatial arrangement.

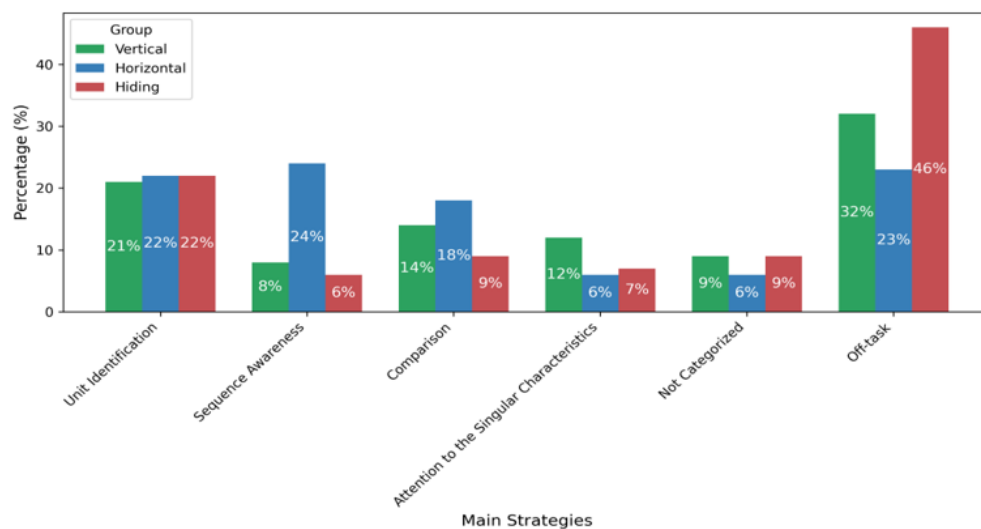


Figure 5.4. Participants strategy percentages by task model.

In contrast, the horizontal model uniquely facilitated the use of recursive strategies. The sequence awareness strategy was used far more frequently in the horizontal model (24%) compared to the vertical (8%) and hiding (6%) models. A similar pat-

tern was observed for the comparison strategy, which was also most common in the horizontal model (18%) versus the vertical (14%) and hiding (9%) models. One-to-one matching strategy use was common in the horizontal model.

The vertical model appeared to elicit a distinct pattern of recursive strategy use when compared to the other models. While the horizontal model facilitated strategies dependent on parallel visual alignment, such as sequence awareness (24%) and comparison (18%), the use of these strategies decreased in the vertical model to 8% and 14%, respectively. Conversely, the vertical model prompted the highest use of the “Attention to the Singular Characteristics” strategy at 12%, nearly double the rate of the other formats. This pattern suggests that when the vertical alignment disrupts the more common sequencing and matching strategies, some children may revert to a more basic recursive strategy of focusing on the features of individual items rather than the relationships between them.

The most significant finding regarding the task disengagement involved the hiding model. This model resulted in a substantially higher rate of off-task responses (46%) compared to both the vertical (32%) and horizontal (23%) models, suggesting it may have been the most cognitively demanding for the children. This created a polarizing effect for the hiding model: it minimized the use of “middle-ground” recursive strategies (like sequence awareness and comparison) and instead tended to elicit either a successful relational approach (unit identification) or complete task disengagement (off-task).

5.3.2. Strategies by Task Formats

To investigate how task formats (MC and DD) affect strategy use, the distribution of strategies for MC and DD items was compared (see Figure 5.5). The analysis revealed several distinct patterns. The use of the most sophisticated strategy, Unit Identification, was identical across both formats. Slight differences were observed for the Sequence Awareness (12% vs 15%, respectively) and Comparison strategies (14% vs 15%, respectively).

Both strategies were used slightly more in the DD format than in the MC format. A more notable difference was found for the Attention to the Singular Characteristics strategy, which was used twice as often in the MC format (12%) as in the DD format (6%). Furthermore, the largest shift, in terms of absolute difference, was seen in Off-task behavior, with the DD format (36%) eliciting more Off-task responses than the MC format (29%).

In summary, while the use of the relational strategy (Unit Identification) appeared unaffected by the response format, the DD format was linked to slightly higher rates of some recursive strategies and more frequent off-task behavior. Conversely, the MC format prompted a greater use of the basic “Attention to the Singular Characteristics” strategy.

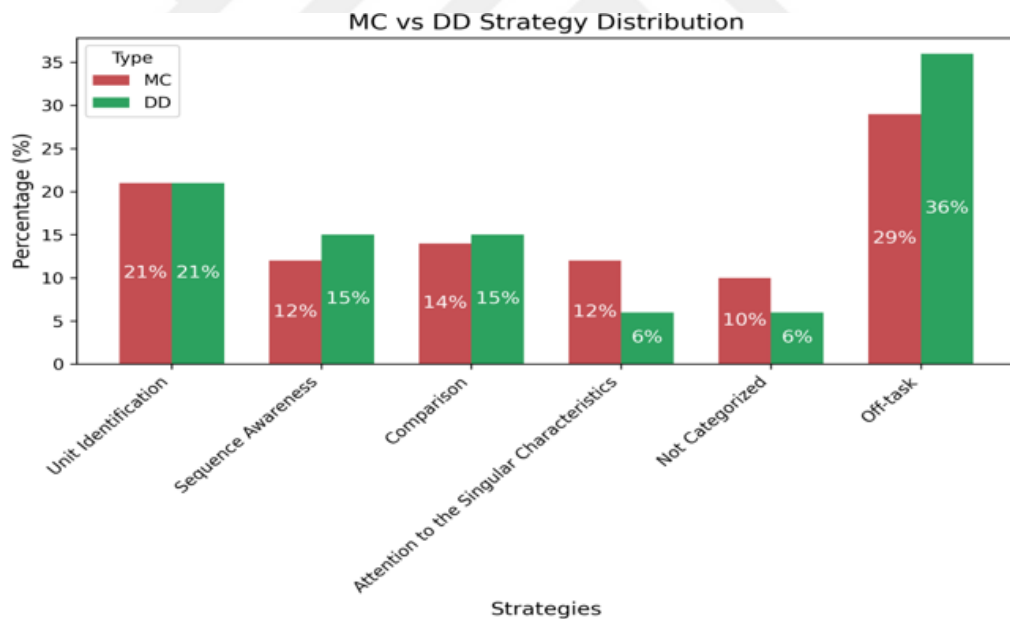


Figure 5.5. Participants strategy percentages by task format.

5.3.3. Strategy Accuracy

After examining children’s use of strategies in different task types and formats, it is important to investigate what percentage of their attempts at these strategies are successful. Table 5.4 shows how many observations there were for each strategy in each

Finally, Table 5.4 reinforces the unique role of the horizontal model for certain recursive strategies. The accuracy of the Sequence Awareness strategy (Strategy 5) use was highest in the Horizontal MC (85.7%) and Horizontal DD (84.4%) tasks. Even though fewer attempts were made to use this strategy in the vertical model, the accuracy is lower than the horizontal alignment. This may indicate that even if children attempt to use this strategy in Vertical MC, Vertical DD, and Hiding DD, their accuracy is lower than in the horizontal format, and therefore it is more difficult to use this strategy in vertical and hiding models and to give the correct answer. In the following part, we will examine whether the use of the unit identification and sequence awareness or comparison strategy depends on task format and type.

5.3.4. Predictors of Unit Identification Strategy Use

This part examined the first part of the third research question which examined how format, model, and age impact children's relational strategy use. We applied a mixed-effect logistic regression to predict the use of the unit identification strategy. This model was selected because the study's repeated-measures design violates the assumption of independent errors required by standard logistic regression. Before finalizing the model, core assumptions of logistic regression were checked (Stoltzfus, 2011). The assumption of linearity between the independent variable and the logit-transformed dependent variable was assessed using the Box-Tidwell approach. The result was a statistically significant interaction between age and its logarithm ($p = 0.016$), indicating this assumption was violated, as visualized by the non-linear trend in Figure 5.6.

To address the non-linear relationship and resolve the assumption violation, we transformed the age variable. First, we centered the age by subtracting the sample mean, a standard procedure to improve model convergence and reduce multicollinearity between the linear and quadratic terms. We, then, included both the centered age and its squared term (age^2) as predictors in the regression. In the resulting model, both terms were statistically significant, confirming that the relationship between the use of

the unit identification strategy and age is indeed quadratic. Multicollinearity was not a concern, as the Variance Inflation Factor (VIF) values were approximately 1. Finally, an analysis of Cook's distance revealed no strongly influential outliers (Cook's $d < 1$), satisfying the final assumption.

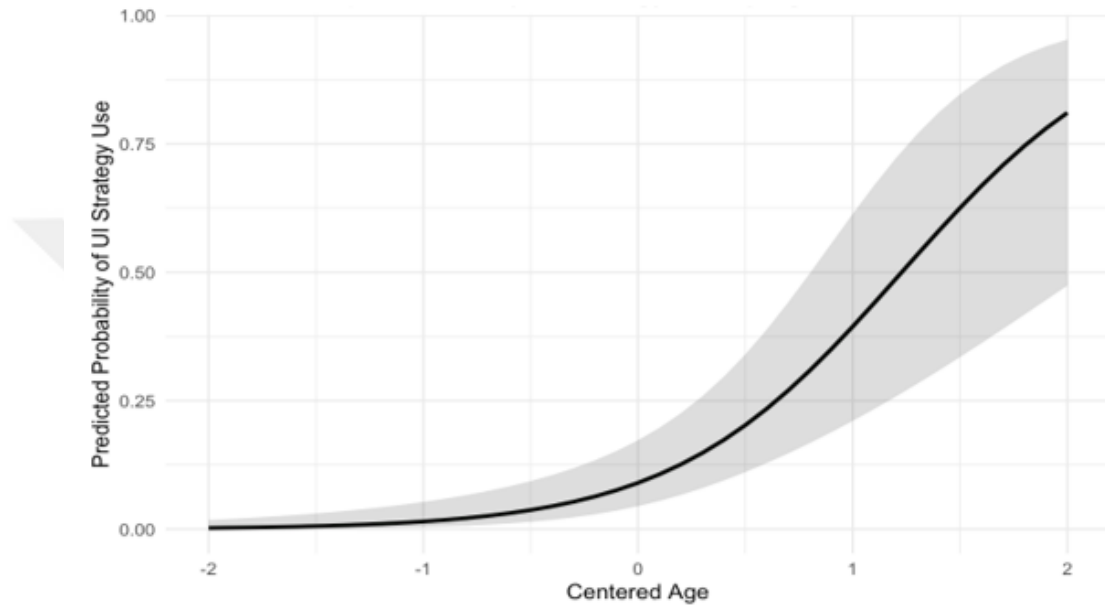


Figure 5.6. Probability of unit identification strategy use by age.

Mixed-effects logistic regression results showed that question format and visual type were not statistically significant predictors of students' use of the unit identification strategy (see Table 5.5). However, students' age was a strong and significant predictor. Also, the relationship between age and the use of the unit identification strategy was nonlinear. Specifically, for each year of age, the odds of using the strategy increased by approximately 558% ($OR = 6.58$, $p < 0.001$), though the trend flattened out near the top at older ages due to a significant negative quadratic term square of age ($OR = 0.23$, $p = 0.016$).

Table 5.5. Fixed effects from mixed-effects logistic regression predicting use of the unit identification strategy.

Predictor	Estimate (β)	Odds Ratio (OR)	95% CI for OR	p-value
Intercept	-1.322**	-	-	0.010*
Format (MC vs DD)	-0.109	0.90	[0.55, 1.46]	0.662
Type: Hiding	-0.070	0.93	[0.52, 1.67]	0.815
Type: Vertical	-0.129	0.88	[0.50, 1.56]	0.660
Age (center)	1.883***	6.58	[2.96, 14.61]	0.000***
Age (center) $\times$ Age (center)	-1.460*	0.23	[0.07, 0.76]	0.016*
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$				

Although the effect of format was not statistically significant ($p = .66$), the estimated odds ratio for multiple-choice questions was 0.90, suggesting students had 10% lower odds of using the strategy compared to drag-and-drop format tasks. Similarly, the task type did not significantly predict strategy use. Compared to horizontal questions, students had slightly lower odds of using the strategy in vertical ($OR = 0.88$, $p = 0.66$) and hiding formats ($OR = 0.93$, $p = 0.82$), though these differences were small and non-significant.

Predicted probabilities visualized these trends more clearly: the estimated probability of using the strategy was 9% for MC format and 8% for DD format. As well as the predicted probabilities for different age groups were visualized in Figure 5.7 to reveal the differences across age groups, and there are similar trends for both format types. Predicted probabilities by type were similar across formats, with all three question types yielding probabilities in the 8-9% range. Although not statistically significant, these differences may reflect subtle effects of format and layout that merit further investigation in larger or more targeted samples. Also, there are similar trends of predicted values across age for different task types as shown in Figure 5.8.

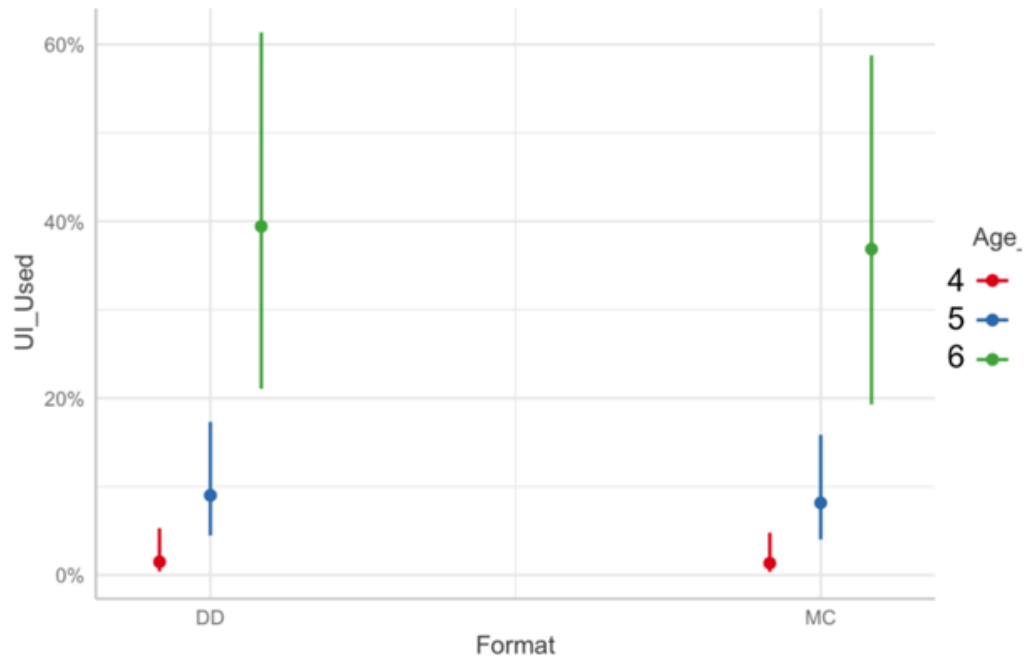


Figure 5.7. Predicted probabilities of unit identification strategy use (UI_Used) in task format across age.

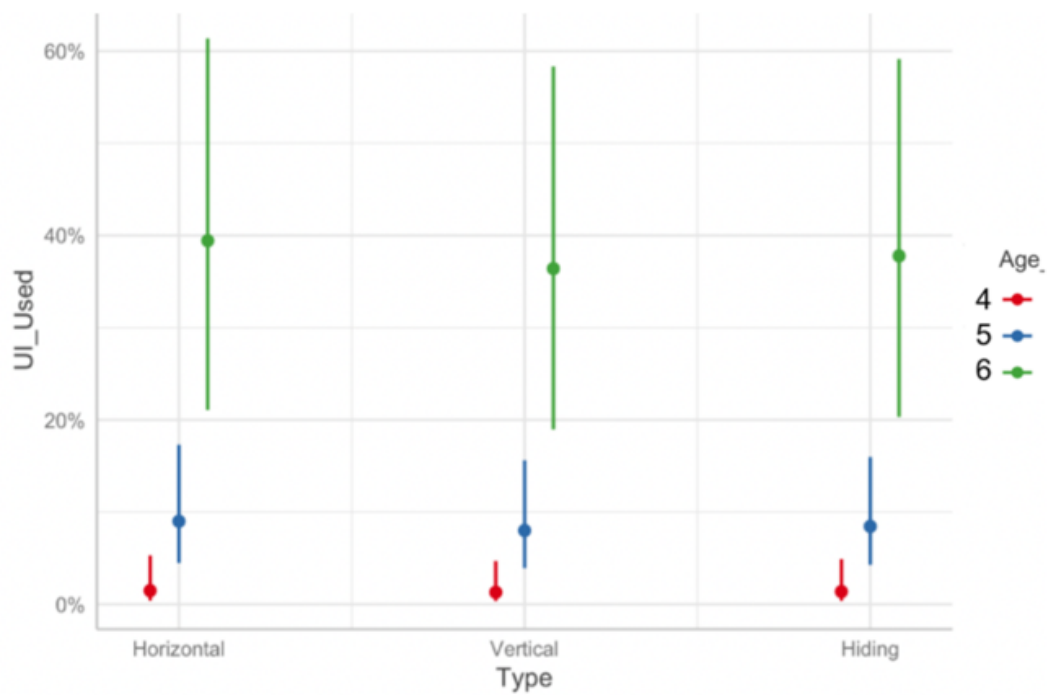


Figure 5.8. Predicted probabilities of unit identification strategy use (UI_Used) in task type across age.

5.3.5. Predictors of Comparison or Sequence Awareness Strategy Use

To address the second part of third research question quantitatively examined how task format, model, and age impact children's recursive strategy use. We applied a mixed-effect logistic regression to predict the use of sequence awareness and comparison. These recursive strategies were specifically selected because their accuracy was higher compared to the other recursive strategies. Similar to the previous model, this analysis model was selected because the study's repeated-measures design violates the assumption of independent errors required by standard logistic regression. Before finalizing the model, core assumptions of logistic regression were checked (Stoltzfus, 2011).

The assumption of linearity between the independent variable and the logit-transformed dependent variable was assessed using the Box-Tidwell approach, and it is violated. To resolve the non-linear relationship, we centered the age by subtracting the sample term similar to the previous model. Then, include both age and its squared terms as predictors of the regression. However, centered age ($\beta = 0.110$, $p = 0.690$) and its squared term ($\beta = 0.48$, $p = 0.330$) were not significant predictors for the overall model, so they were removed from the model. Since the other predictors examined the difference between two predictors, there was no need to confirm the linearity assumption. Multicollinearity was not a concern, as Variance Inflation Factor (VIF) values were approximately 1. Finally, an analysis of Cook's distance revealed no strongly influential outliers (Cook's $d < 1$), satisfying the final assumption.

Results of mixed effects logistic regression revealed that model type was a significant predictor of unit identification strategy use (see Table 5.6). Compared to the horizontal model, the odds of children using this strategy were significantly lower for both the hiding model ($OR = 0.09$, $p < 0.001$) and the vertical model ($OR = 0.29$, $p < 0.001$). The question format also showed a trend, with the odds of strategy use being 46% lower in the MC format tasks compared to the DD format, a result that approached statistical significance ($\beta = -0.625$, $OR = 0.54$, $p = 0.055$).

Table 5.6. Fixed effects from mixed-effects logistic regression predicting use of the comparison or sequence awareness strategy by fixing horizontal type.

Predictor	Estimate (β)	Odds Ratio (OR)	95% CI for OR	p-value
Intercept	-0.146	-	-	0.629
Format (MC vs DD)	-0.624	0.54	[0.28, 1.01]	0.055
Type: Hiding	-2.422***	0.09	[0.04, 0.18]	0.000***
Type: Vertical	-1.255***	0.29	[0.15, 0.56]	0.000***
Format [MC]* Type [Hiding]	1.396*	4.04	[1.34, 12.16]	0.013*
Format [MC]* Type [Vertical]	-0.071	0.93	[0.35, 2.47]	0.887
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$				

Furthermore, the analysis revealed a significant interaction between model type and question format. As well, there is a significant interaction between hiding MC and horizontal MC questions, so children have significantly higher odds in hiding MC questions compared to the Horizontal MC questions ($OR = 4.04$, $p = 0.013$). However, there is not a statistically significant difference in vertical MC tasks ($OR = 0.93$, $p = 0.887$). These findings suggest that use of comparison and sequence awareness strategies differs across different task types and children tend to use these strategies in horizontal tasks.

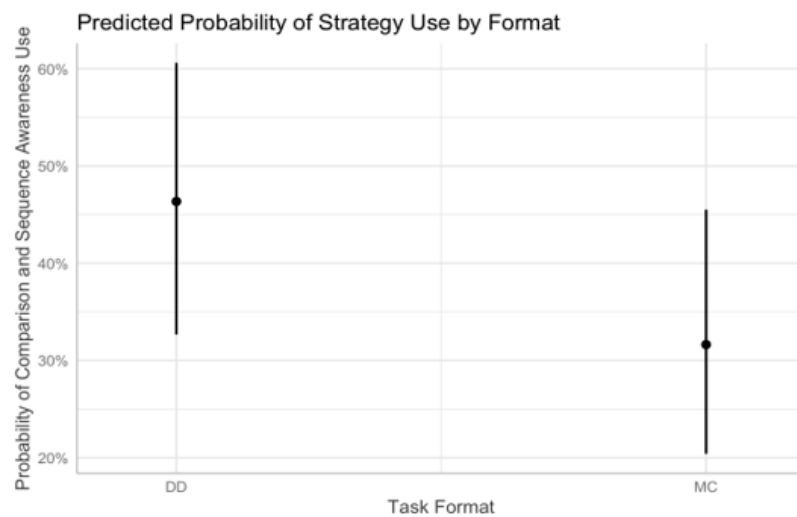


Figure 5.9. Predicted probabilities of comparison and sequence awareness strategy use in task format.

When we examined the predicted probabilities, the estimated probability of using comparison or sequence awareness strategy is 46% for DD format questions, and 32% for the MC format questions, as shown in Figure 5.9. Predicted probabilities by type were 46% for horizontal, 20% for vertical, and 7% for hiding type questions (see Figure 5.10). Since we fixed the horizontal type in the previous model above, we also want to check whether there is a difference between vertical and hiding models. Therefore, a second model was created by fixing the vertical model. The results were provided in Table 5.7. According to the results, there are no significant differences between format types, similar to the previous findings. However, there is a significant difference between vertical and hiding model tasks ($\beta = -1.166$, $p = 0.001$). These findings show that children used these strategies in hiding tasks 69% ($OR = 0.31$) less than vertical tasks.

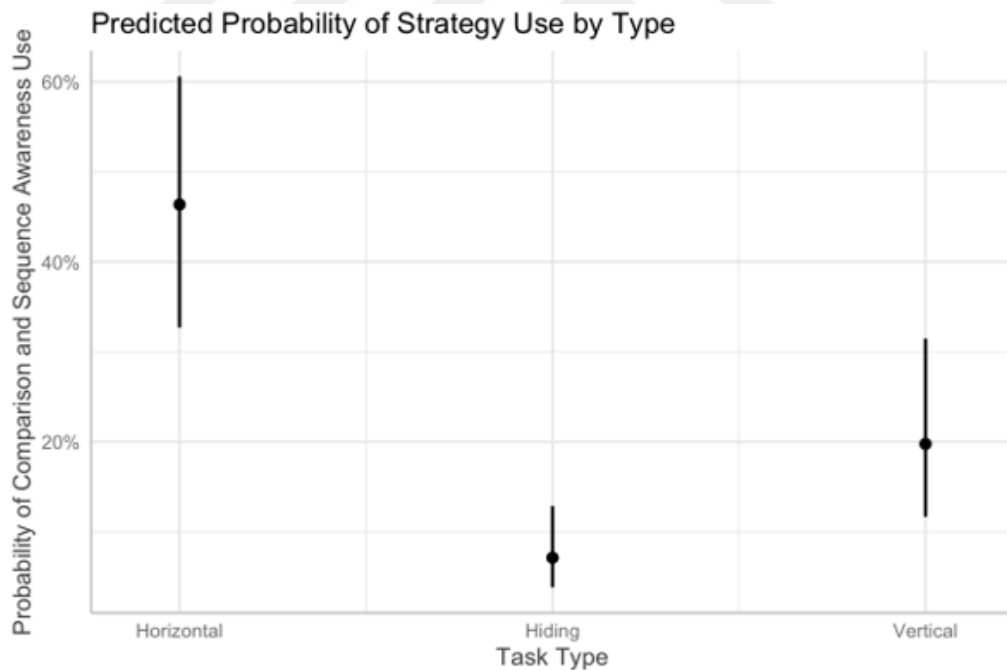


Figure 5.10. Predicted probabilities of comparison and sequence awareness strategy use in task type.

As a result, these two analyses showed that there are no significant format differences for predicting use of comparison or sequence awareness strategy. However, task types are an important factor that affects children's use of this strategy. Specifically,

children tended to use this strategy mostly in the horizontal model rather than the vertical model and lastly the hiding model. Therefore, the use of comparison or sequence awareness strategies is model dependent.

Table 5.7. Fixed effects from mixed-effects logistic regression predicting use of the comparison or sequence awareness strategy by fixing vertical type.

Predictor	Estimate (β)	Odds Ratio (OR)	95% CI for OR	p-value
Intercept	-1.401***	-	-	0.000***
Format (MC vs DD)	-0.695	0.50	[0.24, 1.05]	0.066
Type: Hiding	-1.166**	0.31	[0.15, 0.63]	0.001***
Type: Horizontal	1.255***	3.51	[1.8, 6.86]	0.000***
Format [MC]* Type [Hiding]	1.465*	4.33	[1.35, 13.89]	0.014*
Format [MC]* Type [<i>Horizontal</i>]	0.700	1.07	[0.41, 2.84]	0.888
*p < 0.05, **p < 0.01, ***p < 0.001				

6. DISCUSSION

The findings of the current study are discussed in three sections, each addressing a research question. First, we examine the children’s strategies based on the results from qualitative data. Then, the relationship between strategies and repeating pattern abstraction scores is discussed based on the results of quantitative data. Finally, how task designs influence strategy use is explored by suggesting a task design that minimizes recursive strategy use using the results of qualitative and quantitative findings.

6.1. What are the Recursive and Relational Strategies Used by 4-6-year-old Children When Solving Repeating Patterning Abstraction Tasks?

Not only answers but also strategies have importance in mathematics education since they provide children’s conceptual understanding as well as aid the researchers and educators to diagnose the misconceptions. This study deepens the findings of previous studies in the scope of repeating patterning abstraction strategies (Collins and Laski, 2015; Junker *et al.*, 2025; Lüken and Sazuet, 2021; Papic *et al.*, 2011) by encouraging children to explain their reasoning in the abstraction tasks. The qualitative data collected from 56 participants, along with the results of a hybrid thematic analysis, revealed five strategy categories: off-task, attention to singular characteristics, comparison, sequence awareness, and unit identification, which are similar to those identified in the study by Lüken and Sauzet (2021). Although the main strategy categories align with the findings of Lüken and Sauzet (2021), our study makes a critical conceptual contribution by considering different strategy indicators.

As Lüken and Sauzet (2021) suggested, it is important to differentiate basic strategies to differentiate children’s knowledge. This aligns with our study findings and our basic strategies consists of off-task and attention to the singular characteristics.

Even though children were less accurate responses in these basic strategies their attention and engagement with tasks differs.

Children who use off-task strategy do not attend to the elements of the pattern; rather, they provide random or unrelated answers. More than half of the four-year-olds (54%) used the off-task strategy, which is aligned with previous findings (Collins and Laski, 2015; Lüken and Sauzet, 2021). These results showed that most of the four-year-olds did not engage with tasks. The reason might be the less exposure to pattern abstraction activities, as their teachers reported four-year-olds did not do any abstraction activity in their classrooms. This highlights the importance of providing differentiated abstraction activities for four-year-olds. The dependency on this strategy is decreased while the age increases show a developmental shift and are compatible with the findings of Lüken and Sauzet's (2021) study.

The second basic strategy is attention to the singular characteristics, and children referenced only one aspect of the pattern. This strategy is mostly used by four-year-olds, similar to the off-task strategy. In this strategy, we observe some intriguing explanations in the shape's similarity sub-strategy. Some of the children refer to a game called the Dalgona Candy Game, which is from a popular TV series named Squid Game, explaining that they can create a square/star inside the circle. Since the shapes we provided to create the pattern consist of a square, triangle, circle, and star, it is the same as these candy game shapes. Additionally, in this category, children used the similarity between the shapes in the model pattern and provided shapes such as flowers that looked like stars and circles. The findings highlight to the potential impact of shape selection in early patterning assessments, emphasizing its importance for both researchers and educators. When children are not aware of a pattern's structure, they are investigating its surface features. The first thing they consider is whether there are any similar shapes in the response section.

Even though we try to balance the abstraction types as shape-abstraction, color-abstraction, and shape-color-abstraction, children behave differently in color abstrac-

tion (see items in Appendix B). Interestingly, some children focus only on the shape changes and use of attention to the singular characteristics' strategy. One reason for this tendency might be that we provided color-changing in the model pattern, but when children recreate the pattern, only the different shapes were provided, so this might be confusing for some of the children. Also, this might increase the demand for the tasks since they need to consider the color changes in the model pattern, as well as shape changes, while creating the pattern.

When we examined the frequencies in terms of task type, children tended to use this strategy more in vertical alignment rather than horizontal alignment. One possible explanation is to increase in cognitive load: the vertical spatial alignment is less familiar as well as children abstract the color changes in the pattern and represent them with shapes. Therefore, considering all the aspects of the pattern might add another complexity to tasks.

Interestingly, 18% of children especially used this strategy for Item 8. This tendency might be due to the shape of the item, as it incorporates both a circle and the letter B. This increases the difficulty to identify the unit and neglect the unnecessary dimensions, such as shape. On the other hand, none of the children used this strategy in Item 5, which consists of colored pencils. Since the shapes do not resemble those in the options, children may be less dependent on shape similarity. When we examined the other three items, the shapes were similar to the answer parts. For example, since both items 8 and 11 resemble circles, this similarity might lead children to focus solely on shapes. This suggests the shapes play an important role in use of strategy.

Furthermore, children who use the counting only one element sub-strategy in attention to the singular characteristics are able to correctly solve the MC format ABB tasks. For example, children only count the B's in the ABB pattern, so there are four B's, and the only option that includes four B's is the ABB pattern in the options. Therefore, it is important to add additional distractors to avoid the use of this strategy and correctly respond to the tasks. These indicators showed that children who pay

attention to the singular characteristics generally focus on the shapes or numbers of the pattern.

The comparison strategy consists of strategies in which children compare the pattern's elements. In line with previous research, our findings confirm that using a comparison strategy is linked to higher response accuracy (Lüken and Sauzet, 2021). However, our study makes a critical conceptual contribution by distinguishing between true one-to-one matching and other, less sophisticated matching heuristics. While prior studies often grouped all direct comparison types into a single category (Papic *et al.*, 2011; Lüken and Sauzet, 2021), we observed four distinct sub-strategies (matching elements without order, same unit length, counting each element, and partial matching) that are qualitatively different from one-to-one matching.

These matching categories differ from one-to-one matching as they concentrate solely on two dimensions of the pattern: shape, number, or order. For example, children who use matching elements without an order strategy consider the shape and number of elements but not the order. Additionally, in partial matching, children either match one element with multiple elements or only match different elements without considering the number of elements, creating an AB pattern. However, in a one-to-one matching strategy, children consider both shape, number, and order and are aware of one-to-one correspondence. This distinction is crucial because one-to-one correspondence is one of the key components of counting (Gelman and Gallistel, 1978, as cited in McGuire *et al.*, 2012) and is essential for the development of number sense (Jordan *et al.*, 2006). Therefore, comprehending this concept in the early years contributes to children's early math knowledge. This characteristic makes one-to-one matching more sophisticated than other comparison-based strategies.

Children are able to comprehend the sequence and consecutive elements in the sequence awareness strategy. There are two different indicators of this strategy: one-to-one matching and focusing on sequence. This category is used by 13% of the children, and 10% of them use a one-to-one matching sub-strategy. Our tasks, which solely in-

involve abstraction, may account for this distribution. Many children rely on one-to-one matching between the model pattern and the answer pattern instead of analyzing the consecutive elements with a focus on the sequence sub-strategy. The trends of this strategy differ across age, and it is mostly (18%) used by six-year-olds. Although this strategy is more sophisticated and results in more accurate answers, it remains a recursive strategy because children do not fully understand the structure of the pattern. However, children are better at analyzing the elements of the pattern by considering different shapes, numbers, and the order of elements in the pattern. Therefore, we propose that sequence awareness acts as a crucial bridge between recursive and relational since children are more engaged with structure compared to the aforementioned strategies.

Final and only relational strategy is unit identification, it requires the awareness of the pattern's structure align with previous findings (Collins and Laski, 2015; Junker et al, 2025; Lüken and Sauzet, 2021). Our findings revealed that the unit identification strategy consists of two sub-strategies: identifying the repeating unit and matching only the first unit. To the best of our knowledge, previous studies did not mention matching only the first unit sub-strategy. Therefore, this type of strategy use might be observed only in our sample, or other studies might categorize this sub-strategy into different categories. There is a clear developmental trend among 4-5- and 6-years old (respectively 3%, 29% and 34%) consistent with the findings of Lüken and Sauzet (2021). However, in our sample, 5-year-olds more commonly (29%) used unit identification compared to Lüken and Sauzet (2021) (8%).

There could be four potential explanations for this difference: education system differences, methodological differences, the sample of the current study or task differences. Regarding educational differences, participants in both studies lacked a specialized patterning curriculum. However, participants of Lüken and Sauzet's (2021) did not engage in any repeating patterning activities, whereas the teachers of participants in the current reported that they conducted some patterning activities, primarily focusing on extension activities and rarely on abstraction activities. However, we did

not have exact data about the frequencies of these activities during the semester. The second explanation could be methodological differences, particularly categorizing the matching only the first unit sub-strategy. They might be included in the comparison since it is based on matching, but only 3.5% of the children used a matching only first unit sub-strategy, so the differences might not be independent from that. The third explanation could be the socioeconomic status of the current study's sample, which is classified as upper-middle. The final explanation could be subtle task differences. While our quantitative results showed no direct effect of task format, it is possible that our task presentation reduced the overall cognitive load, allowing more children to access and apply their fragile relational knowledge.

Ultimately, our findings highlight the critical need to investigate children's underlying strategies rather than focusing solely on their accuracy. Children can correctly respond to a task using unit identification, sequence awareness, or even off-task strategies; additionally, the reasoning behind their incorrect responses may involve off-task strategy use or comparison. These strategies reflect different conceptual understandings and treating the responses as correct and incorrect suppresses the instructional needs of children. For example, children who use off-task strategies are unaware of the dimensions of the pattern (shape, number, and order) and need more scaffolded activities compared to children who use sequence awareness. In this regard, differentiated instructional approaches may be necessary to more effectively address the diverse needs of learners.

This study not only supports some of Lüken and Sauzet's (2021) findings but also transforms the strategy categories and provides a deeper understanding of the strategy use of repeating pattern abstraction. Previous studies categorized the strategies as off-task/different, recursive, and relational (Collins and Laski, 2015; Junker *et al.*, 2025), but it is also essential which recursive strategy was used. Even though we called recursive "attention to the singular characteristics", "comparison", and "sequence awareness" strategies, each level reflects another aspect of their cognitive mechanisms. However, we don't know whether each child followed these strategies while de-

veloping pattern knowledge, since our study does not longitudinally measure children’s abstraction strategies.

Our findings align with previous studies, and abstraction tasks can be responded to correctly by recursive strategies (Junker *et al.*, 2025; Lüken and Sauzet, 2021; Wijns, Torbeyns, Bakker *et al.*, 2019). Also, the findings of Lüken and Sauzet (2021) show that even the use of the unit identification strategy in identifying the unit tasks that isolate the unit of the pattern is limited. Therefore, the reason for the limited use of this strategy could not be attributed only to tasks, but rather, children are incapable of applying this sophisticated strategy due to their developmental level.

In light of these findings, we suggest that the assessment of abstraction knowledge should limit the use of the recursive strategy, as we discuss in section 6.3. Current findings challenge the findings of Collins and Laski (2015) and claims of other researchers (e.g., Economopoulos, 1998; Rittle-Johnson, 2015), since they suggest that abstraction tasks elicit relational strategies. As a result, our study revealed that abstraction items can be completed correctly using recursive strategies. Nevertheless, we only examined the strategies in abstraction, so we cannot know the strategies in the other types of tasks, such as extension and identifying the unit. Therefore, we cannot make direct comparisons about children’s relational strategy use across different tasks, such as extension.

Additionally, the methodological differences might provide a different categorization of the strategies. For example, Collins and Laski (2015) categorized a viable but wrong pattern as a relational strategy, such as creating AB rather than ABB. However, according to our findings, children can create AB patterns using either an off-task strategy or a comparison strategy. Some of the children are only exposed to the AB pattern; therefore, they misconceptualize the pattern as AB, and this explanation showed that they did not attend to any aspects of the given pattern. Furthermore, in the comparison strategy, children engage with the elements of the pattern but do not fully grasp or analyze its structure, which leads them to create an AB pattern. Therefore, error

analysis is not enough to reveal strategies. Eliciting children’s reasoning through verbal explanations or employing alternative methodologies that reveal cognitive processes may offer researchers a more accurate understanding of strategy use.

Additionally, 8% of the data could not be categorized, so we label it as “not categorized”. We code this category as the second category because even though children referred to the pattern, their explanations do not fit any category. For example, saying whole elements in the model pattern or the response pattern are not aligned with our categories. Despite the researcher’s encouragement through follow-up questions, children still struggle to elaborate further. Most of them do not have the willingness to explain it. Moreover, 1.6% of the participants changed their answers while explaining their reasoning and correctly answered, and we don’t know why they changed their answers or which strategy they employed before the explanation. The evidence indicates that conducting interviews with young children under age 7 presents challenges, particularly because they have limited vocabulary and struggle to verbally express themselves (Garbarino and Stott, 1992, as cited in Vogl, 2015). Therefore, there is a need to capture children’s cognitive mechanisms in addition to the interviews. One possible method to achieve it might be using eye-tracking methodology. Some of the current studies attempted to use it in extension tasks (Pitta-Pantazi *et al.*, 2024), but there is no study exploring children’s abstraction knowledge.

6.2. To What Extent do RPAT Strategies Predict RPAT Scores When Controlling Age and Visuospatial Working Memory?

The second research question aims to investigate how RPAT strategies are related to RPAT scores, as well as whether this relationship remains after adding age and visuospatial working memory to the model. The results of multiple linear regression show that there is a significant and quadratic relationship between strategy and RPAT score, as shown in Table 5.3. A previous study indicates that there is a linear relationship between patterning scores and strategy scores, and this model explained 94% of the variance (Lüken and Sauzet, 2021). While our results indicate a quadratic

relationship, this means that when children used higher-level strategies, their scores increased at an accelerating rate. In other words, the change of strategy score between attention to the singular characteristics and comparison is not the same as the change between sequence awareness and unit identification. When children use sophisticated strategies frequently, their scores will increase more. Also, our Model A, which includes only the RPAT strategy and the squared term of the RPAT strategy, explained 69% of the variance in RPAT scores.

Furthermore, we added the model age and the visuospatial working memory tasks, specifically Corsi forward and Corsi backward, because previous research has shown that both age and visuospatial working memory are key predictors of patterning knowledge (Collins and Laski, 2015). Our final model (Model C) showed that while we added age and Corsi forward for the visuospatial working memory, strategy scores contribute to the model quadratically, and this model explained 76% of the variance. These findings are crucial even though integrating model age and visuospatial working memory, the squared term of strategy scores is still significant, and this shows that children's strategies have a crucial role in abstraction performance. However, this study only focused on abstraction tasks, so we cannot generalize the results to other types of patterning tasks. This could potentially explain the differences observed in Lüken and Sauzet's (2021) study. Therefore, whether this relationship is quadratic or linear, as suggested in Lüken and Sauzet (2021), should be investigated using a variety of tasks.

Previous studies preferred to use scoring Corsi backward and forward together (Collins and Laski, 2015; Rittle-Johnson *et al.*, 2015). Hence, the performance in the Corsi backward was lower than the Corsi forward, as shown in Table 5.1 similar to the previous findings (Cornoldi and Mammarella, 2008; Heled, 2024). Thus, we preferred to enter these two tasks separately. The initial weak correlation between Corsi backward and RPAT scores lost significance after controlling for age. This suggests that the variance these two measures share is largely attributable to the general developmental maturation captured by age, rather than a unique contribution of backward span itself.

Another plausible explanation could be the skewed distribution of Corsi backward scores, as 37% of the children were unable to complete any trial successfully and thus received a score of zero. A similar floor effect has been reported in previous research (Bull *et al.*, 2008). Both the current study and prior studies have shown that some children do not fully understand the task and instead continue to tap forward rather than backward (Bull *et al.*, 2008; Reindl *et al.*, 2021). These findings question the suitability of the Corsi backward task for this age group.

Interestingly, Corsi forward has a significant but negative contribution to predicting scores when the RPAT strategy, its squared term and age is in the model, contrary to a previous study (Collins and Laski, 2015). One possible explanation might be that Corsi forward and backward might require different aspects of visuospatial working memory (Cornoldi and Mammarella, 2008) so adding the model separately may have led to the different results and lack of significance in Corsi backward and negative contribution of Corsi forward into the model.

The unexpected negative contribution of Corsi forward when age and RPAT strategy scores included can be understood as a potential statistical artifact related to inhibitory control. We observed that some children with poor inhibition perseverated on the forward task, even during backward trials. These children might achieve a high Corsi forward score not because of superior visuospatial memory, but due to this lack of cognitive flexibility. This same underlying difficulty with inhibitory control would then logically hinder their performance on the complex, multi-step RPAT tasks. Therefore, in our model, the Corsi forward score may be inadvertently acting as an inverse proxy for inhibitory control.

6.3. What Task Design Features Promote the Use of Relational Strategies and Minimize the Use of Recursive Strategies in Repeating Pattern Abstraction Tasks?

A mixed method approach was used to investigate whether task format and type influence children's strategy. Since the assessment tools should be designed to lead children to flexible and adaptive strategy use (Heinze *et al.*, 2009), our aim is to suggest a task design that promote relational strategy use and minimizes the recursive strategies such as attention to the singular characteristics, comparison or sequence awareness.

6.3.1. Task Type

To examine the children's strategy use across different task types, we created a bar chart that shows percentages of strategy use for each task model. In terms of the relational strategy use, which is the unit identification strategy, children do not show large differences, and their percentages are 21% in vertical, 22% in horizontal, and hiding type, as shown in Figure 5.4. The findings of logistic mixed effects regression support qualitative findings and the use of unit identification independent from the task model, as the accuracy of this strategy is very similar across task types and over 90%, as shown in Table 5.5. Therefore, spatial alignment does not have an impact on the use of relational strategy in repeating pattern abstraction tasks.

This finding suggests that once children have a robust understanding of a pattern's unit, their ability to apply this relational knowledge is flexible and not easily disrupted by superficial changes like spatial orientation. It implies that to truly assess the limits of this knowledge, more fundamental task modifications may be needed. This finding suggests that once children have a robust understanding of a pattern's unit, their ability to apply this relational knowledge is flexible and not easily disrupted by superficial changes like spatial orientation. It implies that to truly assess the limits of this skill, more fundamental task modifications may be needed. These results revealed that the use of a unit identification strategy is essential, and it increases the perfor-

mance in patterning abstraction tasks. A possible explanation might be the use of the unit identification strategy to comprehend the structure of the pattern, and children are able to use this knowledge in different situations, similar to relational and algebraic thinking. Relational thinking requires not only identifying the relationships but also deducing them in a new context (Polotskaia *et al.*, 2022). Similarly, algebraic thinking is based on generalizing the relationships with symbolic (Blanton and Kaput, 2011) or non-symbolic ways (Radford, 2011). Therefore, children who are able to use the unit identification strategy are able to find the rule and, independent of the context, apply it. This procedure shows a non-symbolic way for the development of early algebra.

Furthermore, this suggests that fostering the use of the unit identification strategy in early years provides a critical foundation for algebra. Educators and researchers should encourage the use of this strategy by designing instructions that explicitly emphasize the unit. As well as providing learning environments and task designs that children engage with unit identification tasks. Moreover, they should enhance children's reasoning abilities through classroom discussions to augment strategies focused on unit identification.

The trend of comparison and sequence awareness strategy use is distinct from the unit identification strategy use. The qualitative study results showed that there is a tendency to use these strategies in horizontal alignment. Logistic mixed-effect regression results supported these findings. Thus, task type influences the comparison and sequence awareness strategy use. These strategies were significantly more used in horizontal models, and this supported the spatial alignment principle (Matlen *et al.*, 2020; Zheng *et al.*, 2022). The parallel alignment between the model pattern and the response part encourages children to use comparison-based strategies, analyze the sequence of elements, and apply one-to-one correspondence. Therefore, spatial aspects of the tasks have a role in children's reasoning, as suggested (Björklund *et al.*, 2020). This conclusion suggests that educators and researchers should consider spatial designs of the tasks.

6.3.2. Task Format

This section addresses how task format (multiple choice and drag-and-drop) affects the strategy use of 4-6-year-old children. Multiple choice items are not very common in early childhood assessment. However, multiple choice questions are easy to administer and assess, so they might be useful for the early patterning assessment as well. Therefore, RPAT consists of MC and DD items. Qualitative results reveal that there is no difference in the use of the relational strategy use, and mixed-effect logistic regression results confirm these findings. However, there are slight differences between other strategy uses, as shown in Figure 5.5. The mixed effect logistic regression results reveal slightly no statistically significant differences in format between the use of sequence awareness or comparison strategies ($p = 0.055$). This lack of main effect in the format of the tasks aligns with findings of Katz *et al.* (2000). Their study consists of high school students, and the current study extended these findings for the early childhood period.

While our results indicate that task type have significant main effects on children's use of comparison and sequence awareness strategies, the most insightful finding is the significant interaction between these two factors. The data reveals that the effect of the question format is not uniform; rather, it depends on the cognitive demands of the task model.

Specifically, the Multiple Choice (MC) format significantly mitigates the difficulty of the Hiding task. A plausible explanation for this is the role of cognitive load. The Hiding task is uniquely demanding as it requires children to both remember the hidden elements and abstract the pattern's rule. The MC format, by presenting complete visual answer choices, likely serves to offload the working memory component of the task. This may free up crucial cognitive resources, allowing children to better focus on comparing the options and analyzing the sequence. In contrast, for tasks with a lower memory load like Vertical or Horizontal, this cognitive support is less critical, which explains why the MC format provided no significant benefit in those contexts. This

finding suggests that for complex abstraction tasks that tax working memory, an MC format may be a more effective tool for assessing a child’s underlying patterning ability than a construction-based format like drag-and-drop.

When we examine the accuracy of strategies across different task types and formats, MC items create more accurate responses. These findings align with the previous findings that MC items are easier than constructed response items (Bonner, 2013; Woodcock *et al.*, 2020). Previous studies used multiple-choice format items in patterning context (Msall *et al.*, 2025; Yildirim *et al.*, 2024), which shows that items are easier compared to the previous studies (Rittle-Johnson *et al.*, 2013). As previously suggested, this might be related to the chance factor since the accuracy of off-task MC tasks was higher than DD tasks. This shows that children are able to correctly answer even though they don’t attend to the elements of the pattern. Although there is no main effect of the MC format, it leads to more accurate answers. This may result in errors when identifying the participants’ ability levels.

6.3.3. Task Design Suggestions

The findings of this study reveals that the type and format of the tasks do not affect the use of the unit identification strategy, and children’s accuracy in this strategy is very similar to each other. This indicated that use of this strategy is independent of spatial aspects of the tasks as well as response format. This suggests that expert children who use the unit identification strategy are able to adaptively apply their strategy in various situations, as indicated by Heinze *et al.* (2009).

However, the use of comparison and sequence awareness strategies differs across task types and is more frequently used in horizontal models. The existing instruments generally used the horizontal model. Current findings support the study of Lüken and Sauzet (2021) and Junker *et al.* (2025) and questioned the claims of other researchers (Clements and Sarama, 2009; Economopoulos, 1998; Rittle-Johnson *et al.*, 2015), since abstraction tasks can be completed without understanding of the pat-

tern's structure. Therefore, the researcher who wants to measure patterning abstraction knowledge should ask themselves, "Do we want to diagnose whether children are attending to the structure of the pattern or not?" If the answer is yes, then the spatial alignment of the task should be designed to minimize the use of the recursive strategy and reduce the correct responses without engaging the structure of the pattern.

According to the findings of the current study, we suggest that hiding and vertical model tasks are better at eliminating the recursive strategies. The use of sequence awareness and comparison strategy is less frequent, and the accuracy of these strategies is less than the horizontal type. When we compare the vertical and hiding models, hiding minimizes the strategies between unit identification and off-task, as shown in Figure 5.4. It encourages either the use of unit identification (22%) or off-task (46%) strategies. Therefore, we suggest that hiding items minimize the use of the recursive strategies. However, it is more difficult for the four-year-old children, and these types of items might cause boredom and dropping out of the test. Therefore, using both vertical and hiding items increases the variability of the items as well as provides opportunities to apply their knowledge in different scenarios. Variability in the learning materials supported abstract concepts as well as led to analyzing the structure rather than surface features of the problems (Atkinson *et al.*, 2000). This might also improve the accuracy of ability estimates and help differentiate levels of knowledge in the abstraction.

In terms of the format suggestions, there is not a significant difference between MC and DD in strategy use, but the accuracy is higher in MC items. This could be related to the distractors of the questions, or our sample size is not enough to detect the format-related effect since the p-value is slightly higher ($p = 0.055$; see Table 5.6 and $p = 0.066$; see Table 5.7) for comparison or sequence awareness strategy use. As well as there is a format and type interaction in hiding tasks. This interaction shows that MC format reduces the cognitive load so more complex pattern tasks use of MC format might be effective.

Therefore, this study is unable to find format-related differences, but if researchers want to eliminate the chance factor, DD items might be better. Since some of the children created the first unit but made mistakes in the second unit, in MC items, if they correctly choose some part of the pattern, the rest is automatically chosen. For example, for an ABBABB pattern, if children determine that there is ABBA, the rest of the choices are negligible. It may be beneficial to design distractors that go beyond simply requiring the counting of a specific element. For instance, in our questions, there is only one option that has four same shapes; therefore, children only count the B's and choose the option that has four same shapes in ABB pattern. Shortly, this study is unable to suggest a certain format using the current findings, so further studies are required to examine whether there is a format-related difference.

Lastly, variability in the pattern elements has a crucial role, as suggested by Zhang and Fyfe (2024). Variability might reduce reliance on surface features of the pattern, such as shape similarity or number, and aid in comprehending the structure of the pattern (Zhang and Fyfe, 2024). Additionally, independent of the format, the choices or the materials should be designed with dissimilar shapes. For instance, if there is a leaf-leaf-flower pattern, the options should not have shapes like triangles since they are similar to the leaves or stars and circles since they are similar to flowers. These results suggest that not only educators and curriculum designers but also researchers should consider spatial and format-related design features when developing patterning instruction and assessment tools for early childhood education. Instructional and evaluative materials should be informed by such empirical evidence to more accurately measure or support the development of abstraction strategies in early mathematics.

6.3.4. Role of Age

Results of both qualitative and quantitative analyses show that age plays a crucial role in abstraction. There are age-dependent differences in children's strategy use, and one of the biggest differences is in the unit identification strategy. Children's unit identification strategy use depends on age, similar to the previous finding (Lüken

and Sauzet, 2021). Additionally, age is a significant and quadratic predictor of the use of unit identification strategies. This finding means that children's use of unit identification strategies develops more rapidly between the ages of 5 and 6 than between the ages of 4 and 5. This study supports the previous findings that children's cognitive development is faster during the early childhood period (Gualtieri and Finn, 2022). As well as 3% of four-year-olds used this strategy, which aligned with previous findings (Lüken and Sauzet, 2021; Rittle-Johnson *et al.*, 2013). This is expected since their teachers reported that they did not do abstraction tasks, and generally, they did AB pattern activities. Therefore, particularly younger children, such as 4-year-olds, may benefit from engaging in abstraction tasks that include a variety of units.

However, age-related differences decreased in comparison and sequence awareness strategy use. Age is not a significant predictor in comparison or sequence awareness strategy use. These results might be due to the merging of these two strategies in the model, but the detailed frequency percentage was provided in Figure 5.1. According to this, there is no difference between 4- and 5-year-olds' sequence awareness strategy use, and 6-year-olds used it 8% more than they did. The use of the comparison strategy was almost the same in 5-year-olds (12%) and 6-year-olds (11%), and 4-year-olds more frequently (18%) used it.

Furthermore, age significantly predicts RPAT scores; similar to the findings of Collins and Laski (2015), a one-year increase in age results in a 1.15-point increase in addition to the contribution from RPAT strategy scores. These results showed that age is an important predictor but not the sole one, so the use of unit identification strategies should be encouraged by educators.

6.4. Limitations and Future Directions

This mixed-method study design reveals strategy use of children across different task types and formats and suggests a task design minimizing recursive strategy use. However, there are certain limitations of the current study. The first one is a sample-

related issue. Data were collected from public schools, but participants were chosen by convenience sampling, and they have upper-middle socioeconomic status. Also, even though the sample size is detected by power analysis, a larger sample size that consists of mixed socioeconomic status might increase generalizability since we have a variety of variables.

Another limitation concerns the order in which the items were presented. All children received the tasks in the same order, which means we were not able to control whether there is order related effects. Since we observed some variation in strategy use depending on the task format and model, it might be important to explore whether the order of the items effect the strategy use. Future studies could benefit from using a counterbalanced or randomized order to explore whether the sequence of tasks plays a role in the use of strategies.

We tried to balance color-abstraction, shape-abstraction, and color-shape abstraction in the RPAT. Nevertheless, the color abstraction tasks behave differently from each other. When we examined the children's responses, they only focused on the shapes of Items 2, 8, and 11. For Items 2 and 11, the model pattern shapes look like circles, so children used only circles to construct answers. Interestingly, none of the children used this strategy in Item 5 (consisting of pens). Therefore, shapes have importance and providing different colors rather than shapes might avoid this situation and should be tested in further studies. Also, whether the difficulty of tasks changes in color-abstraction, shape-abstraction, and color-shape abstraction items should be explored to determine research-based task designs. In addition, some of the shapes, such as leaf or flower, are very similar to the materials we provided. This might create a tendency to use surface features of the pattern. This kind of similarity between shapes should be minimized in future tasks and assessment items.

In the MC items, the distractors consisted of AB, ABB, and ABC options, and some of the children correctly answered, counting only one element. As I mentioned earlier in the discussion, children counted the B's of the ABB pattern, and the only

option that included four of the same elements was the ABB pattern in the options. Future studies and measurement instruments should be designed accordingly.

Although this study provides valuable insights about strategy use, it is limited by qualitative findings generated by task-based interviews' transcripts and observation notes. Children were prompted after they completed the tasks not to interfere with their reasoning. However, this method might not capture children's cognitive mechanisms fully because young children might forget to mention or be incapable of mentioning some of the strategies they applied, since their verbal skills are still developing. Therefore, new methodologies that capture participants' cognitive mechanisms without interfering with them should be used in future studies. One method might be eye-tracking methodology. Eye trackers record eye movement data such as where, when, for how long, and how many times a participant looks while performing tasks (Rodrigues and Rosa, 2018). It provides objective data about participants' cognitive mechanisms without interfering with them. Therefore, this methodology might provide a clear capture of cognitive mechanisms as well as aid in interpreting data in the "Not categorized" category.

Moreover, due to the permissions, we only recorded the audio of the task-based interview, but it is also crucial to have video recordings of the data since sometimes children prefer to point out some parts of the pattern in addition to the verbal explanations. Even though the researcher took the observation notes, this might be subjective, as some gestures or movements might be missed. Even though 27% of the task-based interviews were double coded, we cannot double code the gestures of the children, and there could be some researcher biases. However, we tried to reduce it by triangulating the audio recordings and observation notes. It is essential to capture children's gestures; therefore, future studies should be recorded in the videos as well.

Although this study provides valuable insights about strategy use, it did not investigate children's strategy development longitudinally. Thus, findings lack a developmental trajectory of strategies, so we don't know whether each child follows the

strategies orderly or not to reach the unit identification level. Longitudinal studies are required to examine the developmental stages of strategy use.

Last but not least, this study only explored the strategies in repeating pattern abstraction tasks, so further studies are required to investigate the other tasks in repeating and growing patterns. Additionally, RPAT items should be integrated into the repeating pattern assessment instruments and should provide psychometric evidence in conjunction with other sub-constructs, such as extension, by examining item fit and difficulty.



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APPENDIX A: RPAT PRACTICE ITEMS

The four practice items are provided below.

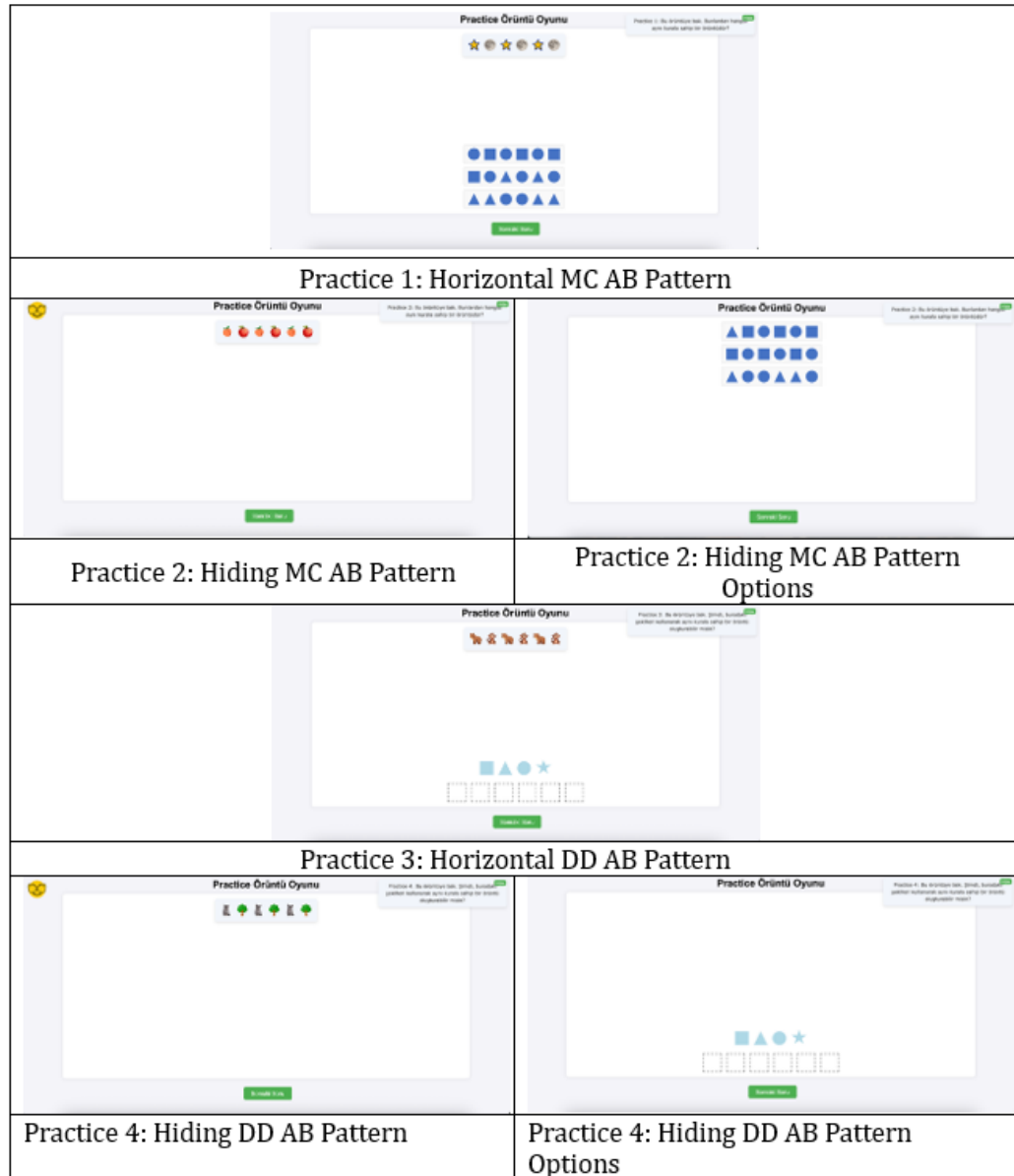


Figure A.1. Practice items.

APPENDIX B: RPAT ITEMS

The RPAT items are provided below.

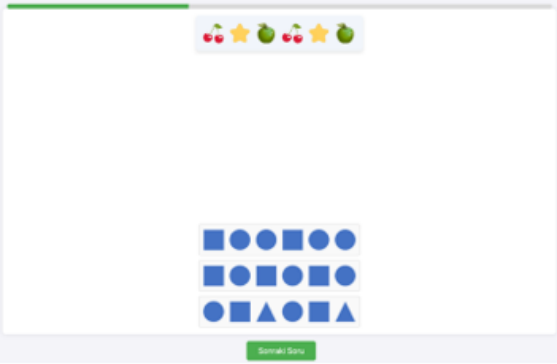
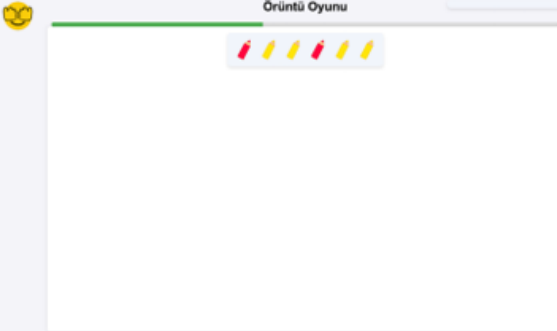
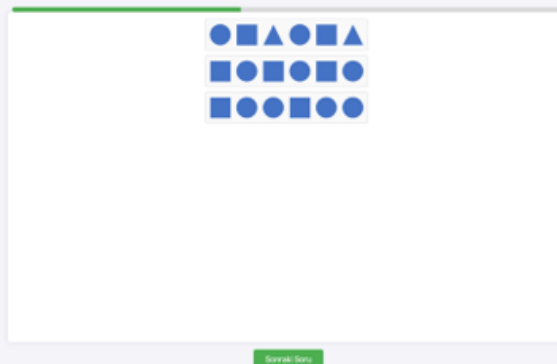
Item & Item Label	Proposed Pattern Changed
 Item 4: Horizontal MC ABC Pattern	Color-Shape Abstraction
 Item 5: Hiding MC ABB Pattern	Color Abstraction
 Item 5: Hiding MC ABB Pattern Options	

Figure B.1. RPAT items 1.

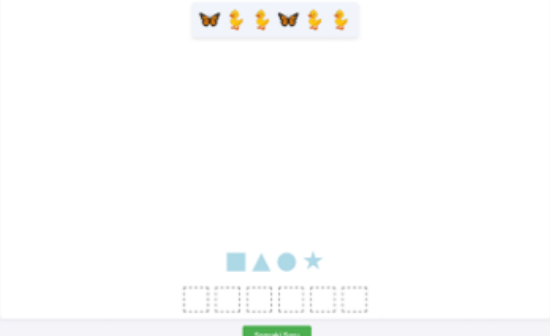
Item & Item Label	Proposed Pattern Changed
	Color Abstraction
Item 8: Vertical DD ABC Pattern	
	Shape Abstraction
Item 9: Horizontal DD ABB Pattern	
	Color-Shape Abstraction
Item 10: Horizontal DD ABC Pattern	

Figure B.3. RPAT items 3.

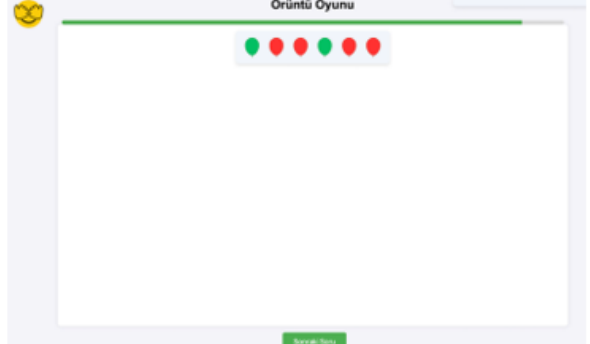
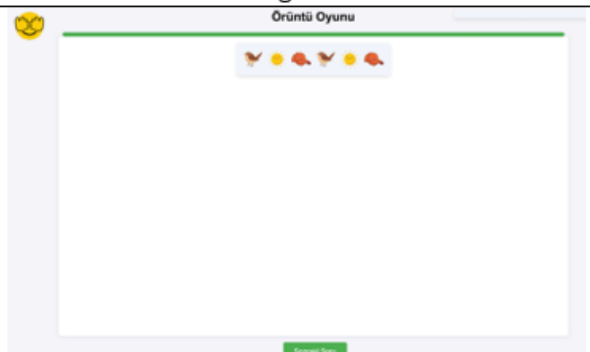
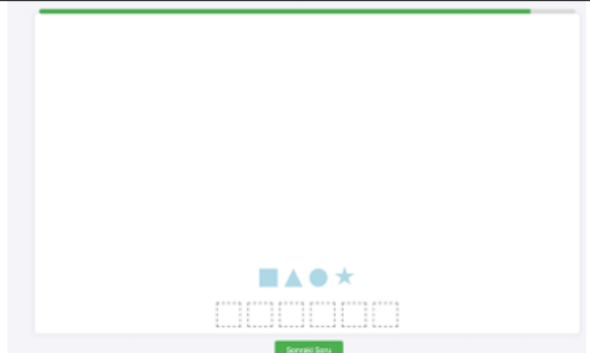
Item & Item Label	Proposed Pattern Changed
	Color Abstraction
Item 11: Hiding DD ABB Pattern 	
Item 12: Hiding DD ABC Pattern 	
Item 11 & 12: Response Part	

Figure B.4. RPAT items 4.

APPENDIX C: RESEARCH PROTOCOL TURKISH

The Turkish version of the research protocol is provided below.

Merhaba [çocuğun adı]! Örüntülerle ilgili seninle bir oyun oynamak istiyorum. Daha sonra bakabilmek için yaptıklarımızın ses kaydını almam gerekiyor. Soracağım soruları cevaplayarak benimle bu oyunu oynar mısın? (Çocuk başını sallamalı, evet demeli veya aktif olarak onay vermeli.)

(Hiçbir şey yapmazsa) “Soracağım soruları yanıtlayarak benimle oyun oynamayı kabul ediyorsan lütfen başını sallarmısın ya da evet der misin”?

“Harika! Teşekkür ederim”!

Şimdi sana karakterlerimi tanıtaacağım bu Momo bu da Dodo (kuklalar gösterilir). Momo ve Dodo çok yakın iki arkadaşlar ormanda saklambaç oynarken bir anda birbirlerini kaybetmişler. Birbirlerini bulmalarına yardım etmek ister misin?

Harika. Şimdi sana bazı sorular soracağım ve sen cevap verdikçe yukarıdaki yeşil kısım yavaş yavaş ilerleyecek ve Momo Dodo'ya yaklaşacak. Ancak onlara yardım etmeden önce biraz alıştırma yapmamız lazım hazırsan başlayalım

Alıştırma Maddesi 1 (çoktan seçmeli)-yıldız-ay: “Bu örüntüye bak. Bu tekrarlayan bir örüntü çünkü tekrarlayan bir bölüm var. Yani kuralı var. Yıldız-ay tekrar eden bölüm” (Örüntüye işaret et ve tekrar eden birimi elinle göster). “Şimdi sıra sende! Bunlardan hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür” (Şıklara işaret edilir)?

Cevap veremezse – > “bu örüntü hangi şekillerden oluşuyor”? şıklar işaret edilerek “bunlardan hangisi yukarıda verilen örüntü ile aynı kurala sahip? “Sana en doğru geleni söyle”.

- Cevabın doğru çünkü her ikisinde de şekiller sırayla ilerlemiş yukarıda örüntümün kuralı yıldız ay yani bir iki bir iki tekrar ediyor. Burada da (örüntü işaret edilir) yuvarlak kare kuralında örüntüm var yani bir-iki örüntüsü.
- Cevabın doğru değil çünkü yukarıda örüntümün kuralı yıldız ay yani bir iki bir iki tekrar ediyor. Burada da (örüntü işaret edilir) yuvarlak kare kuralında yani bir-iki örüntümü seçmem gerekirdi.

“Devam etmek için aşağıdaki yeşil kutucuğa tıklayabilirsin”.

Alıştırma Maddesi 2 (Çoktan seçmeli)-kayısı-elma: “Burada da meyvelerle bir örüntü oluşturdum şimdi örüntüyü dikkatlice inceleyelim. Hazır olduğunda gözlerini kapatan emojiye basarsan örüntü kaybolacak ve bu örüntüyle aynı kurala sahip örüntüyü bulmanı isteyeceğim. Hazır olduğunda emojiğin üzerine tıkla. Bunlardan hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür”?

- Cevabın doğru az önce örüntümüzün kuralı kayısı-elmaydı yani bir iki örüntüsü. Şimdi de kare-yuvarlak yani yine bir iki şeklinde ilerliyor.
- Cevabın doğru değil çünkü az önce örüntümüzün kuralı kayısı-elmaydı yani bir iki örüntüsü. Aynı kuralda olması için kare-yuvarlak yani bir iki örüntüsü olmalı.

Alıştırma Maddesi 3 (Sürükle Bırak)-ayı-maymun: “Burada hayvanlarla bir örüntü oluşturdum. Burada verilen şekilleri kullanarak bu örüntüyle aynı kurala sahip örüntüyü oluşturmanı isteyeceğim. Bunlardan hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür”?

“Öncesinde seninle biraz alıştırma yapmamız iyi olabilir. Buradaki şekli taşımak istediğimde üzerine biraz uzun basacağım elimi hiç kaldırmadan buraya bırakabilirim. Buraya başka şekil getirmek istersem yeni bir şekli yukarıdan alacağım az evvel yaptığım gibi buraya getireceğim. Şimdi sıra sende hadi bakalım. Yukarıda verilen örüntüyle aynı kurala sahip bir örüntü oluşturabilir misin”.

- Cevabın doğru az önce tekrar eden kural kayısı-elma yani bir-iki örüntüsü vardı şimdi de kare-yuvarlak şeklinde ilerliyor. Yani bir iki.
- Cevabın doğru değil çünkü az önce örüntümüzün kuralı kayısı-elma yani bir-iki şeklindeydi aynı kuralda olması için kare-yuvarlak yani bir-iki örüntüsü olmalı

“Devam etmek için yeşil butona tıklayalım”.

Alıştırma maddesi 4 (Sürükle Bırak)-kedi-ağaç: “Burada da bir örüntü oluşturdum. Şimdi örüntüyü dikkatlice inceleyelim Sen hazır olduğunda örüntü kaybolacak ve bu örüntüyle aynı kurala sahip örüntüyü bulmanı isteyeceğim. Hazır olduğunda emoji'nin üzerine tıkla şimdi bir önceki soruda yaptığımız gibi nesnelerin üzerine uzun basarak sürükleyip bırakalım”. Çocuk tıkladıktan sonra: “Kaybolan örüntüyle aynı kurala sahip bir örüntü oluşturabilir misin”?

- Cevabın doğru az önce tekrar eden kural kedi-ağaç yani bir iki örüntüsü vardı şimdi de kare-yuvarlak şeklinde ilerliyor. Yani bir iki.
- Cevabın doğru değil çünkü az önce örüntümüzün kuralı kedi ağaç şeklindeydi aynı kuralda olması için kare-yuvarlak örüntüsü olmalı.

“Artık alıştırmalarımız bitti. Bundan sonra sana cevabının doğru ya da doğru olmadığını söylemeyeceğim. Her bir sorudan sonra sana neden o cevabı seçtiğini ya da oluşturduğunu soracağım. Şimdi Momo'yu ekranın bir tarafına Dodo'yu da karşısına koyalım sen her bir soruya cevap verdikçe yukarıdaki yeşil kısım ilerleyecek ve Momo Dodo'ya kavuşacak”.

RPAT 1-4 çoktan seçmeli maddeler: “Burada nesnelerle bir örüntü oluşturdum. Bunlardan (aşağıda verilenlerden) hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür”?

“Neden bu cevabı seçtin? Neden bu cevap yukarıdaki örüntüyle aynı kurala sahip”?

“Yeşil butona basarak devam edebilirsin”.

RPAT 5-6 çoktan seçmeli kaybolan maddeler: “Burada da nesnelerle bir örüntü oluşturdum. Gözlerini kapatan emojiye tıkladığımızda örüntü kaybolacak ve senden aynı kurala sahip örüntüyü bulmanı isteyeceğim. Hazır olduğunda tıklayabilirsin”. Tıkladıktan sonra: “Bunlardan (aşağıda verilenlerden) hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür”?

“Neden bu cevabı seçtin? Neden bu cevap yukarıdaki örüntüyle aynı kurala sahip”?

“Yeşil butona basarak devam edebilirsin”?

RPAT 7-10 sürükle bırak maddeler: “Burada nesnelerle bir örüntü oluşturdum. Burada verilen şekilleri kullanarak bu örüntüyle aynı kurala sahip örüntüyü oluşturmanı isteyeceğim. Alıştırmalarda yaptığımız gibi kutucuklara getirmek istediğimiz şekle uzun bas ve elini kaldırmadan kutucuğa sürükley. Buradaki örüntüyle aynı kurala sahip bir örüntü oluşturur musun”?

“Neden bu cevabı seçtin? / Örüntünü nasıl oluşturdu? / Neden bu cevap yukarıdaki örüntüyle aynı kurala sahip”?

“Yeşil butona basarak devam edebilirsin”?

RPAT 11-12 sürükle bırak kaybolan maddeler: “Burada da nesnelerle bir örüntü oluşturdum. Gözlerini kapatan emojiye tıkladığımızda örüntü kaybolacak ve senden aynı kurala sahip örüntüyü oluşturmanı isteyeceğim. Hazır olduğunda tıklayabilirsin”. Tıkladıktan sonra: “Bunlardan (aşağıda verilenlerden) hangisi benim oluşturduğumla aynı kurala sahip bir örüntüdür”?

“Neden bu cevabı seçtin? Neden bu cevap yukarıdaki örüntüyle aynı kurala sahip”?

“Yeşil butona basarak devam edebilirsin”?

Oyunumuz bu kadardı. Benimle bu oyunu oynadığın için teşekkür ederim. Momo ile Dodo birbirlerine kavuştular ve şimdi çok mutlular.



APPENDIX D: RESEARCH PROTOCOL ENGLISH

The English version of the research protocol is provided below.

Hello [child's name]! I want to play a game with you about patterns. I need to record what we do so I can look at it later. Will you play this game with me by answering my questions? (The child should nod, say yes, or actively agree.)

(If they don't do anything) "If you agree to play the game with me by answering the questions I'm going to ask, please nod your head or say yes"? "Great! Thank you"!

Now I'll introduce you to my characters. This is Momo, and this is Dodo (the puppets are shown). Momo and Dodo are very close friends. While playing hide-and-seek in the forest, they suddenly lost each other. Would you like to help them find each other? Great. Now I'll ask you some questions, and as you answer, the green section above will slowly move forward, and Momo will get closer to Dodo. But before we help them, we need to do a little practice. Are you ready? Let's begin.

Practice Item 1 (multiple choice) - star-moon: "Look at this pattern. It's a repeating pattern because there's a repeating part. So, there's a rule. The star-moon is the repeating part" (Point to the pattern and show the repeating unit with your hand). "Now it's your turn! Which of these has the same rule as the pattern I created" (Point to the options)?

If they can't answer — > "What shapes does this pattern consist of?" Point to the options and ask, "Which of these has the same rule as the pattern given above? Tell me the one that seems most correct to you".

- Your answer is correct because in both cases the shapes progress in sequence, and the rule of my pattern above is star-moon, i.e., one-two, one-two, repeating. Here too (point to the pattern), I have a pattern with the circle-square rule, i.e., a one-two pattern.
- Your answer is not correct because the rule of my pattern above is star-moon, i.e., one-two, one-two, repeating. Here too (pattern indicated), I should have chosen the round square rule, i.e., the one-two pattern.

To continue, you can click on the green box below.

Practice Item 2 (Multiple Choice) - Apricot-Apple: “Here, I have created a pattern with fruits. Now let’s examine the pattern carefully. When you are ready, click on the emoji with closed eyes, and the pattern will disappear. I will then ask you to find a pattern with the same rule as this one. When you are ready, click on the pattern. Which of these has the same rule as the pattern I created”?

- Your answer is correct. The rule of our pattern was apricot-apple, which is a one-two pattern. Now it is square-round, which is also a one-two pattern.
- Your answer is not correct because the rule of our pattern was apricot-apple, which is a one-two pattern. To have the same rule, it should be square-circle, which is also a one-two pattern.

“Let’s click the green button to continue”.

Practice Item 3 (Drag and Drop) - bear-monkey: “Here, I have created a pattern with animals. I would like you to use the shapes provided here to create a pattern with the same rule as this pattern. Which of these has the same rule as the pattern I created”?

“It might be a good idea to do some practice with you first. When I want to move this shape, I will press on it for a little longer and drop it here without lifting

my hand. If I want to bring another shape here, I will take a new shape from above and bring it here as I did before. Now it's your turn. Let's create a pattern that follows the same rule as the one given above. Can you create a pattern that follows the same rule as the one above?"

- Your answer is correct. The rule that was repeated earlier was apricot-apple, meaning there was a one-two pattern. Now it is continuing with square-circle, meaning one-two.
- The answer is incorrect because the rule of our pattern was apricot-apple, meaning it is one-two pattern. For it to have the same rule, it should be consists of two objects such as square-circle, i.e., one-two pattern.

"Let's click the green button to continue".

Practice Item 4 (Drag and Drop) - cat-tree: "I created a pattern here as well. Now let's examine the pattern carefully. When you are ready, the pattern will disappear, and I will ask you to find a pattern with the same rule as this one. When you are ready, click on the emoji and, as we did in the previous question, drag and drop the objects by pressing and holding them. Can you create a pattern that follows the same rule as the hiding one?"

- Your answer is correct. The rule that just repeated was cat-tree, meaning there were two patterns, and now it is continuing with square-circle. So, one-two.
- Your answer is not correct because the rule of our pattern just now was cat-tree, meaning it is one-two pattern. For it to have the same rule, it should be consists of two objects such as square-circle, i.e., one-two pattern.

"Now our exercises are over. From now on, I won't tell you whether your answer is correct or not".

After each question, I will ask you why you chose or created that answer. Now let's put Momo on one side of the screen and Dodo on the other. As you answer each question, the green section above will progress and Momo will meet with Dodo".

RPAT 1-4 multiple-choice tasks: "Here, I have created a pattern with objects. Which of the following (listed below) has the same rule as the pattern I created"?

After answering, before moving on to the next question: "Why did you choose this answer? Why does this answer have the same rule as the pattern above"?

"You can continue by pressing the green button".

RPAT 5-6 multiple-choice hiding tasks: "Here, I have created a pattern with fruits. Now let's examine the pattern carefully. When you are ready, click on the emoji with closed eyes, and the pattern will disappear. I will then ask you to find a pattern with the same rule as this one. When you are ready, click on the pattern". After clicking the emoji: "Which of these has the same rule as the pattern I created"?

After answering, before moving on to the next question: "Why did you choose this answer? Why does this answer have the same rule as the pattern above"?

"You can continue by pressing the green button".

RPAT 7-10 drag-and-drop questions: "Here, I have created a pattern with objects. Using the shapes provided here, I would like you to create a pattern that follows the same rule as this one. As we did in the exercises, press and hold the shape you want to bring to the box and drag it to the box without lifting your finger. Can you create a pattern that follows the same rule as the one above / hiding one"?

After answering, before moving on to the next question: “Why did you choose this answer? / How did you create your pattern? / Why does this answer follow the same rule as the pattern above”?

“You can continue by pressing the green button”.

RPAT 11-12 drag-and-drop hiding questions: “I created a pattern here as well. Now let’s examine the pattern carefully. When you are ready, the pattern will disappear, and I will ask you to find a pattern with the same rule as this one. When you are ready, click on the pattern and, as we did in the previous question, drag and drop the objects by pressing and holding them”. After child clicked the emoji: “Can you create a pattern that follows the same rule as the hiding one”?

After answering, before moving on to the next question: “Why did you choose this answer? / How did you create your pattern? / Why does this answer follow the same rule as the pattern above”?

“You can continue by pressing the green button”.

That’s all for our game. Thank you for playing with me. Momo and Dodo are reunited and very happy now.

APPENDIX E: RPAT SCORES and RPAT STRATEGY SCORES ACROSS AGE

The proportion correct of the items and strategy use distributions are provided below.

Table E.1. RPAT items proportion correct (SD) across age.

Items	Total	4-years-old	5-years-old	6-years-old
I1 (Vertical MC)	0.46 (0.50)	0.16 (0.37)	0.56 (0.51)	0.68 (0.48)
I2 (Vertical MC)	0.52 (0.50)	0.37 (0.50)	0.50 (0.51)	0.68 (0.48)
I3 (Horizontal MC)	0.63 (0.49)	0.32 (0.48)	0.72 (0.46)	0.84 (0.37)
I4 (Horizontal MC)	0.43 (0.50)	0.32 (0.48)	0.39 (0.50)	0.58 (0.51)
I5 (Hiding MC)	0.46 (0.50)	0.26 (0.45)	0.50 (0.51)	0.63 (0.50)
I6 (Hiding MC)	0.61 (0.49)	0.37 (0.50)	0.72 (0.46)	0.74 (0.45)
I7 (Vertical DD)	0.34 (0.48)	0.11 (0.32)	0.39 (0.50)	0.53 (0.21)
I8 (Vertical DD)	0.23 (0.43)	0.05 (0.23)	0.17 (0.38)	0.47 (0.51)
I9 (Horizontal DD)	0.61 (0.49)	0.26 (0.45)	0.67 (0.49)	0.89 (0.32)
I10 (Horizontal DD)	0.45 (0.50)	0.11 (0.32)	0.56 (0.51)	0.68 (0.48)
I11 (Hiding DD)	0.32 (0.47)	0.11 (0.32)	0.28 (0.46)	0.58 (0.51)
I12 (Hiding DD)	0.38 (0.49)	0.05 (0.23)	0.56 (0.51)	0.53 (0.51)

Table E.2. Distribution of frequencies (%) of strategy categories in different task types across age (4/5/6).

Strategy	Vertical MC	Vertical DD	Horizontal MC	Horizontal DD	Hiding MC	Hiding DD
Off-task	55/25/13	53/25/18	37/11/13	50/17/8	63/33/32	68/ 47/34
Not Categorized	5/11/24	5/6/5	3/17/8	3/6/0	5/19/5	5/14/8
Attention to the Singular Characteristics	21/ 8/11	3/22/8	18/6/8	3/0/0	8/6/11	8/3/8
Comparison	11/ 14/5	32/11/11	21/17/16	26/17/13	11/8/11	11/6/11
Sequence Awareness	5/8/11	5/3/18	21/14/21	16/28/42	8/3/5	5/6/8
Unit Identification	3/ 28/32	3/25/37	0/36/34	3/28/34	5/31/37	3/25/32
Not Answered	0/6/5	0/8/3	00/0	0/6/3	0/0/0	0/0/0

APPENDIX H: CONSENT FORM

An example of consent form is shown below.

T.C.

BOĞAZİÇİ ÜNİVERSİTESİ

SOSYAL VE BEŞERİ BİLİMLER İNSAN ARAŞTIRMALARI ETİK
KURULU

KATILIMCI BİLGİ ve ONAM FORMU

Araştırmayı destekleyen kurum: TÜBİTAK

Araştırmanın adı: 4-6 Yaş Arası Çocuklar için Tekrarlayan Örüntüde
Soyutlama Bilgisinin İncelenmesi ve Güncel Çevrimiçi Tekrarlayan
Örüntü Testi Geliştirme

Proje Yürütücüsü/Araştırmacının adı: Serkan Özel

E-mail adresi: ozels@bogazici.edu.tr

Telefonu: 212-359-6592

Sayın Veli,

Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Bölümü öğretim üyesi Dr. Serkan Özel “4-6 Yaş Arası Çocuklar için Tekrarlayan Örüntüde Soyutlama Bilgisinin İncelenmesi ve Güncel Çevrimiçi Tekrarlayan Örüntü Testi Geliştirme” adı altında bilimsel bir araştırma projesi yürütmektedir. Bu çalışmanın amacı ileri yaşlardaki matematik becerisinin tahmin edilmesinde önemli bir değişken olan örüntüleme becerisinin erken yaşta ölçülebilmesini sağlayacak bir çevrimiçi ölçme aracı geliştirmektir. 4-6 yaş arası çocuklarda erken matematik becerilerini inceleyen bu araştırmaya çocuğunuzla birlikte katılmak üzere sizi davet ediyoruz. Çocukların anaokuluna başladıklarında sahip oldukları matematik bilgisi, onların ilerleyen okul yıllarındaki başarıları ile ilişkilidir. Bu nedenle, okul öncesi dönemde matematik bilgisini geliştirmek için hangi becerilerin gerekli olduğunun belirlenmesi önemlidir. Kararınızdan önce araştırma hakkında sizi bilgilendirmek istiyoruz. Bu bilgileri okuduktan sonra araştırmaya katılmak isterseniz lütfen bu formu doldurup ozels@bogazici.edu.tr adresine e-posta ile bize ulaştırınız.

Bu araştırmaya katılmayı kabul ettiğiniz takdirde eğitimli bir araştırmacı eşliğinde çocuğunuza çocuklar için tasarlanmış göz hareketlerini takip eden bir gözlük takılacak (gözlük temin edilmesi durumunda kullanılacaktır temin edilemezse gözlük olmadan diğer araştırma adımları uygulanacaktır) ve çocuğunuzdan ekranda gösterilen örüntü sorularını çözmeleri istenecektir. Çocuklar soruları cevaplarken araştırmacı da çözüm yöntemlerine dair sorular soracaktır (örneğin, Örüntünüzü nasıl oluşturduunuz /Neden bu örüntüyü seçtin?). Bu esnada çocuğunuzun göz hareketleri kaydedilecektir. Çalışmanın yaklaşık olarak 25-30 dakika sürmesi beklenmektedir. Kaydedilen veriler kodlanarak anonimleştirilecek ve Boğaziçi Üniversitesi sunucularında güvenli bir şekilde saklanacaktır. Çalışma esnasında ses ve görüntü kaydı onay vermeniz halinde alınacaktır.

Katılımcı çocuklara bu çalışmaya katıldıklarını belgeleyen bir katılım belgesi ve oyuncak/sticker/hediye çeki verilecektir. Size ise örüntüleme becerileri ile ilgili bir broşür verilecektir. Bunların dışında herhangi bir ücret veya başka bir ödül verilmeyecektir.

Bu araştırmaya katılmak tamamen isteğe bağlıdır. Katıldığınız takdirde çalışmanın herhangi bir aşamasında herhangi bir sebep göstermeden onayınızı çekmek hakkına da sahipsiniz. Çalışmaya devam etmeyeceğiniz takdirde toplanmış olan veriler ve ses kayıtları laboratuvar bilgisayarından silinerek imha edilecektir. Araştırma projesi hakkında ek bilgi almak istediğiniz takdirde lütfen Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölümü Öğretim Üyesi Dr. Serkan Özel ile temasa geçiniz (Telefon: 0-212-359-6592, Adres: Boğaziçi Üniversitesi, Eğitim Fakültesi ET 412, 34342 Bebek, İstanbul). Araştırmayla ilgili haklarınız konusunda Boğaziçi Üniversitesi Sosyal ve Beşeri Bilimler İnsan Araştırmaları Etik Kurulu'na (sbinarek@bogazici.edu.tr) danışabilirsiniz.

Araştırmamızın size ya da çocuğunuza herhangi bir risk getirmesi beklenmemektedir. Çoğu çocuk eğlenirken bazı çocuklar sıkılabilir ya da 'yanlış' cevap vermekten endişelenebilir. Çocuğunuz istediğinde çalışmayı durdurarak sorularını ce-

vaplayacağız. Çocukların kimlikleri gizli tutulacak ve güvenli Boğaziçi sunucularında saklanacaktır. Anonim hale getirdiğimiz veriler diğer araştırmacılarla paylaşılacaktır. Bu verilerde sizinle ya da çocuğunuzla ilgili herhangi bir kimlik bilgisi bulunmayacaktır.

Ben, (katılımcının adı), yukarıdaki metni okudum ve çocuğumun ve benim katılmamız istenen çalışmanın kapsamını ve amacını, gönüllü olarak üzerine düşen sorumlulukları tamamen anladım. Çalışma hakkında soru sorma imkânı buldum. Bu çalışmayı istediğim zaman ve herhangi bir neden belirtmek zorunda kalmadan bırakabileceğimi ve bıraktığım takdirde herhangi bir ters tutum ile karşılaşmayacağımı anladım.

Bu koşullarda söz konusu araştırmaya kendi isteğimle, hiçbir baskı ve zorlama olmaksızın katılmayı kabul ediyorum.

☐ Ses kaydı alınmasına onay veriyorum.

Katılımcının Adı-Soyadı:

Adresi (varsa Telefon No, Faks No):

Tarih (gün/ay/yıl):...../...../.....

Katılımcının Velisinin Adı-Soyadı:

İmzası:

Tarih (gün/ay/yıl):...../...../.....

Araştırmacının Adı-Soyadı:

İmzası:.....

Tarih (gün/ay/yıl):...../...../.....