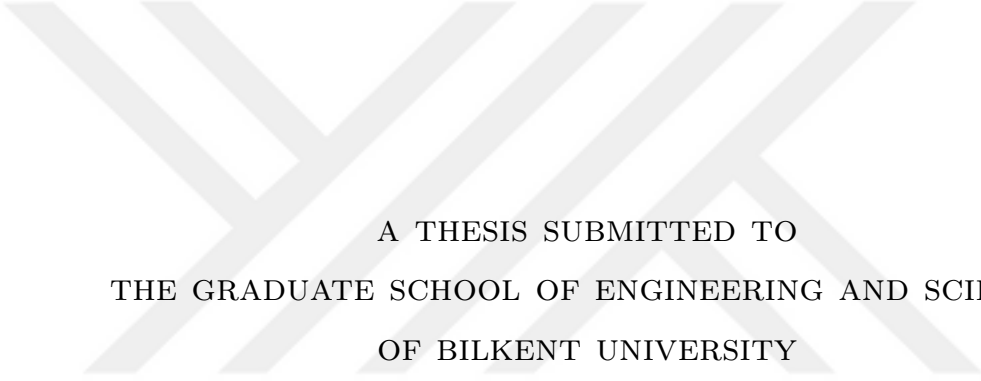


ASSORTMENT PLANNING UNDER NON-LINEAR COST STRUCTURES



A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF
MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

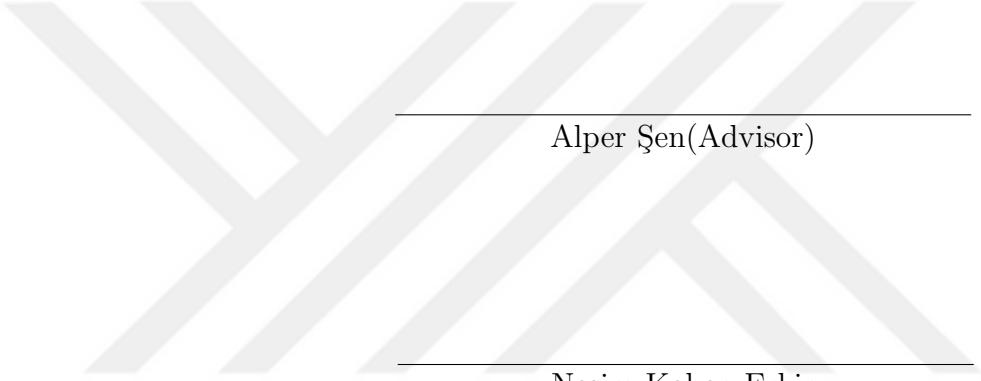
By
Farzad Shams
April 2019

Assortment Planning under Non-linear Cost Structures

By Farzad Shams

April 2019

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.



Alper Şen(Advisor)

Nesim Kohen Erkip

Sinan Gürel

Approved for the Graduate School of Engineering and Science:

Ezhan Karaşan
Director of the Graduate School

ABSTRACT

ASSORTMENT PLANNING UNDER NON-LINEAR COST STRUCTURES

Farzad Shams
M.S. in Industrial Engineering
Advisor: Alper Şen
April 2019

We first consider the assortment optimization problem with fixed product costs under the Mixtures of Multinomials (MMNL) Model. The problem is NP-hard even under the Multinomial Logit Model and the existing literature focuses on developing heuristics and bounds. We develop a conic integer programming formulation for the problem and valid inequalities to strengthen the formulation. We show that this approach can be used to solve instances that are very large – sizes beyond which it would be very difficult to accurately estimate parameters of the choice model – in a short amount of time, eliminating the need to develop and implement specialized algorithms for the problem. We also study the assortment planning problem where the inventory and replenishment costs are considered using the Economic Order Quantity model and the customers’ choice is governed by the MMNL model. We show that the problem is NP-hard and propose a conic integer program for this problem. Our numerical experiments show that moderately sized instances can be solved in reasonable times and McCormick inequalities are effective in tightening the formulation.

Keywords: assortment optimization, mixtures of multinomials model, conic integer programming, economic order quantity.

ÖZET

DOĞRUSAL OLMAYAN MALİYET YAPISI ALTINDA ÜRÜN ÇEŞİDİ PLANLAMA

Farzad Shams

Endüstri Mühendisliği, Yüksek Lisans

Tez Danışmanı: Alper Şen

Nisan 2019

Bu çalışmada, ürün çeşidi en iyilemesi problemi öncelikle sabit ürün maliyetiyle beraber karışık MNL modeli altında çalışılmıştır. Bu problem MNL modeli altında bile NP-Zordur ve varolan literatür sezgisel ve sınır bulan modellere yoğunlaşmıştır. Bu çalışmada bu problem için konik tamsayı programlama formülasyonu geliştirilmiş ve bu formülasyon geçerli eşitsizliklerle güçlendirilmiştir. Bu yaklaşım yoluyla ürün çeşidi oldukça fazla – bundan büyük boyutlu tüketici tercihi problem parametrelerinin doğru tahmini oldukça zordur – olan problemlerin, özelleşmiş bir algoritma geliştirme ihtiyacı duymadan, kısa zamanda çözülebileceği gösterilmiştir. Ayrıca ürün çeşidi planlama problemini envanter ve yenileme maliyetlerini göz önünde bulundurarak incelenmiş ve bu problem için ayrı bir matematiksel programlama modeli önerilmiştir. Bu modelde maliyetler EOQ modeli kullanılarak; tüketicilerin tercihleri ise MMNL modeli kullanılarak tanımlanmıştır. Çalışmada ürün çeşidi planlama probleminin envanter ve yenileme maliyetleri göz önünde bulundurulduğunda NP-Zor olduğu gösterilmiştir. Nümerik çalışmalarımız sonucu orta ölçekli ürün çeşidi içeren problemlerin bu konik tamsayı programı kullanılarak makul bir zaman diliminde çözülebileceği gösterilmiştir. Aynı zamanda McCormick eşitsizliklerinin problem formülasyonuna eklenmesinin formülasyonu güçlendirdiği gösterilmiştir.

Anahtar sözcükler: ürün çeşidi en iyilemesi, MMNL tüketici seçim modeli, konik tamsayı programlama, ekonomik sipariş miktarı.

Acknowledgement

I would like to express my sincere gratitude and appreciation to my advisor Assoc. Prof. Alper Şen for his guidance, attention, support, and patience during my graduate studies and preparation of this thesis.

I am indebted to Prof. Dr. Nesim Kohen Erkip and Prof. Dr. Sinan Gürel for accepting to read and review this thesis; and for their valuable feedbacks and suggestions.

I cannot thank my mother, sister and brother in law enough for their consistent support during every stage of my studies. This thesis would have not been possible without their encouragement and moral support.

I am thankful to all my friends in the IE department, especially Milad Malekipirbazari, Beyza Çelik, Ayşenur Karagöz, Utku Karaca, my assistantship partners Hale Erkan and Erman Gözü; and all my officemates in EA307. Particular thanks reserved for Beyza, for her support and impeccably translated özet, and Milad for his guidance and academic insights throughout my grad studies.

Finally, I would like to thank my close friends, Ali Farid, Mobin Alipour, Salar Rahimi, and Yashar Kardar for their support and all the fun that we had in Turkey together.

Contents

1	Introduction	1
2	Literature Review	6
3	Assortment Optimization with Fixed Costs: Problem Formulation	13
4	Numerical Study	20
4.1	Multinomial Logit Model	21
4.2	Multinomial Logit Model under Capacity Constraints	23
4.3	MMNL Model	28
4.4	MMNL Model under Capacity Constraints	31
5	Assortment Optimization with Replenishment Costs	34
5.1	Model and Formulation	34
5.2	Numerical Study	41

6 Conclusion	46
A Tables for Assortment Optimization with Fixed Costs under MNL Model	54
B Tables for Assortment Optimization with Fixed Costs under MNL Model with Capacity Constraints	64
C Tables for Assortment Optimization with Fixed Costs under MMNL Model	92
D Tables for Assortment Optimization with Fixed Costs under MMNL Model with Capacity Constraints	120
E Tables for Assortment Optimization with Replenishment Costs	139

List of Figures



List of Tables

4.1	Results for MNL model	22
4.2	Results for capacitated MNL model, $\kappa = 50$	25
4.3	Results for capacitated MNL model, $\kappa = 100$	26
4.4	Results for capacitated MNL model, $\kappa = 150$	27
4.5	Results for MMNL model, $ \mathcal{C}_i = 25$	29
4.6	Results for MMNL model, $ \mathcal{C}_i = 75$	30
4.7	Results for MMNL model, $ \mathcal{C}_i = 100$	31
4.8	Results for capacitated MMNL model, $ \mathcal{C}_i = 100, \kappa = 100$	32
4.9	Results for capacitated MMNL model, $ \mathcal{C}_i = 100, \kappa = 150$	33
5.1	Results for assortment with replenishment costs: $\nu_0 = 0.5$	43
5.2	Results for assortment with replenishment costs: $\nu_0 = 1.0$	44
5.3	Results for assortment with replenishment costs: $\nu_0 = 1.5$	45
A.1	Results for problems with 300 products, $\phi : 0.75, \gamma : 1$	55

A.2	Results for problems with 300 products, $\phi : 0.75, \gamma : 0.5$	56
A.3	Results for problems with 300 products, $\phi : 0.75, \gamma : 0.25$	57
A.4	Results for problems with 300 products, $\phi : 0.5, \gamma : 1$	58
A.5	Results for problems with 300 products, $\phi : 0.5, \gamma : 0.5$	59
A.6	Results for problems with 300 products, $\phi : 0.5, \gamma : 0.25$	60
A.7	Results for problems with 300 products, $\phi : 0.25, \gamma : 1$	61
A.8	Results for problems with 300 products, $\phi : 0.25, \gamma : 0.5$	62
A.9	Results for problems with 300 products, $\phi : 0.25, \gamma : 0.25$	63
B.1	Results for instances with 300 products for MNL model, $\phi :$ $0.75, \gamma : 1, k : 50$	65
B.2	Results for instances with 300 products for MNL model, $\phi :$ $0.75, \gamma : 0.5, k : 50$	66
B.3	Results for instances with 300 products for MNL model, $\phi :$ $0.75, \gamma : 0.25, k : 50$	67
B.4	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ $1, k : 50$	68
B.5	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ $0.5, k : 50$	69
B.6	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ $0.25, k : 50$	70
B.7	Results for instances with 300 products for MNL model, $\phi :$ $0.25, \gamma : 1, k : 50$	71

B.8	Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.5, k : 50$	72
B.9	Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.25, k : 50$	73
B.10	Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 1, k : 100$	74
B.11	Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.5, k : 100$	75
B.12	Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.25, k : 100$	76
B.13	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 1, k : 100$	77
B.14	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.5, k : 100$	78
B.15	Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.25, k : 100$	79
B.16	Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 1, k : 100$	80
B.17	Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.5, k : 100$	81
B.18	Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.25, k : 100$	82
B.19	Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 1, k : 150$	83

B.20 Results for instances with 300 products for MNL model, $\phi :$ 0.75, $\gamma : 0.5, k : 150$	84
B.21 Results for instances with 300 products for MNL model, $\phi :$ 0.75, $\gamma : 0.25, k : 150$	85
B.22 Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ 1, $k : 150$	86
B.23 Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ 0.5, $k : 150$	87
B.24 Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$ 0.25, $k : 150$	88
B.25 Results for instances with 300 products for MNL model, $\phi :$ 0.25, $\gamma : 1, k : 150$	89
B.26 Results for instances with 300 products for MNL model, $\phi :$ 0.25, $\gamma : 0.5, k : 150$	90
B.27 Results for instances with 300 products for MNL model, $\phi :$ 0.25, $\gamma : 0.25, k : 150$	91
C.1 Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 100, \mathcal{C} : 25$	93
C.2 Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 75, \mathcal{C} : 25$	94
C.3 Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 50, \mathcal{C} : 25$	95
C.4 Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 100, \mathcal{C} : 25$	96

C.5	Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 75, \mathcal{C} : 25$	97
C.6	Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 50, \mathcal{C} : 25$	98
C.7	Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 100, \mathcal{C} : 25$	99
C.8	Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 75, \mathcal{C} : 25$	100
C.9	Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 50, \mathcal{C} : 25$	101
C.10	Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 100, \mathcal{C} : 75$	102
C.11	Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 75, \mathcal{C} : 75$	103
C.12	Results for instances with 300 products for MMNL model, $\phi :$ 0.75, $ \mathcal{M} : 50, \mathcal{C} : 75$	104
C.13	Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 100, \mathcal{C} : 75$	105
C.14	Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 75, \mathcal{C} : 75$	106
C.15	Results for instances with 300 products for MMNL model, $\phi :$ 0.5, $ \mathcal{M} : 50, \mathcal{C} : 75$	107
C.16	Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 100, \mathcal{C} : 75$	108

C.17 Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 75, \mathcal{C} : 75$	109
C.18 Results for instances with 300 products for MMNL model, $\phi :$ 0.25, $ \mathcal{M} : 50, \mathcal{C} : 75$	110
C.19 Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 100, \mathcal{C} : 100$	111
C.20 Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 75, \mathcal{C} : 100$	112
C.21 Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 50, \mathcal{C} : 100$	113
C.22 Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 100, \mathcal{C} : 100$	114
C.23 Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 75, \mathcal{C} : 100$	115
C.24 Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 50, \mathcal{C} : 100$	116
C.25 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 100, \mathcal{C} : 100$	117
C.26 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 75, \mathcal{C} : 100$	118
C.27 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 50, \mathcal{C} : 100$	119
D.1 Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 100, \mathcal{C} : 100, k : 100$	121

D.2	Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 75, \mathcal{C} : 100, k : 100$	122
D.3	Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 50, \mathcal{C} : 100, k : 100$	123
D.4	Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 100, \mathcal{C} : 100, k : 100$	124
D.5	Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 75, \mathcal{C} : 100, k : 100$	125
D.6	Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 50, \mathcal{C} : 100, k : 100$	126
D.7	Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 100, \mathcal{C} : 100, k : 100$	127
D.8	Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 75, \mathcal{C} : 100, k : 100$	128
D.9	Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 50, \mathcal{C} : 100, k : 100$	129
D.10	Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 100, \mathcal{C} : 100, k : 150$	130
D.11	Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 75, \mathcal{C} : 100, k : 150$	131
D.12	Results for instances with 300 products for the MMNL model, $\phi : 0.75, \mathcal{M} : 50, \mathcal{C} : 100, k : 150$	132
D.13	Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 100, \mathcal{C} : 100, k : 150$	133

D.14 Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 75, \mathcal{C} : 100, k : 150$	134
D.15 Results for instances with 300 products for the MMNL model, $\phi : 0.5, \mathcal{M} : 50, \mathcal{C} : 100, k : 150$	135
D.16 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 100, \mathcal{C} : 100, k : 150$	136
D.17 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 75, \mathcal{C} : 100, k : 150$	137
D.18 Results for instances with 300 products for the MMNL model, $\phi : 0.25, \mathcal{M} : 50, \mathcal{C} : 100, k : 150$	138
E.1 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 50$	139
E.2 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 150$	140
E.3 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 250$	140
E.4 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 500$	140
E.5 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 1000$	140
E.6 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 2000$	141
E.7 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 50$.	141
E.8 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 150$	141
E.9 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 250$	141
E.10 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 500$	142
E.11 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 1000$	142

E.12 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 2000$	142
E.13 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 50$	142
E.14 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 150$	143
E.15 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 250$	143
E.16 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 500$	143
E.17 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 1000$	143
E.18 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 2000$	144
E.19 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 50$	144
E.20 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 150$	144
E.21 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 250$	144
E.22 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 500$	145
E.23 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 1000$	145
E.24 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 2000$	145
E.25 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 50$	145
E.26 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 150$	146
E.27 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 250$	146
E.28 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 500$	146
E.29 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 1000$	146
E.30 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 2000$	147
E.31 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 50$	147

E.32 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 150$	147
E.33 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 250$	147
E.34 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 500$	148
E.35 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 1000$	148
E.36 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 2000$	148
E.37 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 50$	148
E.38 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 150$	149
E.39 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 250$	149
E.40 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 500$	149
E.41 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 1000$	149
E.42 Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 2000$	150
E.43 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 50$	150
E.44 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 150$	150
E.45 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 250$	150
E.46 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 500$	151
E.47 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 1000$	151
E.48 Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 2000$	151
E.49 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 50$	151
E.50 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 150$	152
E.51 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 250$	153

E.52 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 500$ 153

E.53 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 1000$ 154

E.54 Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 2000$ 154



Chapter 1

Introduction

Assortment, i.e., the set of products that a firm should offer to its customers, can be decisive in short- and long-term profitability of a retailer. Offering a larger assortment may increase market share, but may also increase costs as the firm may no longer exploit economies of scale. In addition, a larger assortment may dilute the revenue of the firm as its customers are more likely to choose lower priced products. Evaluating these trade-offs requires a formal choice model which governs how consumers choose given an assortment of products offered to them. In the assortment literature, Multinomial Logit Model (MNL) and its extensions have been widely used [1, 2, 3, 4] and we will give description of these models in this section. Given a choice model, the assortment optimization problem is to select an assortment that maximizes the firm's profit subject to various constraints such as shelf and warehouse space and material handling. A firm typically selects its assortment from a very large set of potential products, resulting in a tremendous number of candidate assortments to choose from which makes the assortment optimization problem a challenging problem for retailers.

Given the importance of the problem and recent availability of relevant data, there is a growing body of assortment optimization research in Operations Research literature and this problem is getting more important for retailers, consultants and software providers [5]. Indeed, According to the 2018 Integrated

Planning and Inventory Management Survey conducted by Boston Retail Partners, “30% of retailers currently use advanced analytics for assortment planning and another 49% plan to within three years” [6].

As is mentioned above, one of the most popular choice models to capture the customer demand in assortment planning is the Multinomial Logit Model. This model is a utility based discrete choice model that has been successfully applied in economics, marketing and transportation demand studies [7, 8]. Under this model, a customer chooses the product that has the highest utility for her. The mathematical structure of the purchase probabilities will be given in Chapter 3. Although this model is convenient, as it provides a tractable model for estimations, it has some drawbacks. The major flaw of this model is the so called Independent of Irrelevant Alternatives (IIA) property. This property states that the ratio of purchase probabilities for any two products is independent of the assortment that they are offered in and this might not be true as different products cause different amounts of cannibalization on each other. Nested Logit Model is an extension of the MNL model that partly alleviates the IIA issue of the MNL model. This model was introduced by Williams [9]. In this model, there is a two step process for the customer to choose a product. First, the customer would choose a nest and then from that nest a product. For instance, nests could correspond to different brands and the products in each nest could correspond to different products that is from the same brand. Although the Nested Logit Model eliminates the IIA shortcoming of the MNL model among any two products in different nests, the IIA problem still exists among the products in the same nest. Another extension of the MNL model which completely eradicates the IIA problem is the Mixture of Multinomials (MMNL) Model. In this model there are several customer classes and each class’s demand is governed by a different MNL model. An advantage of this model is that it can approximate any random utility choice model as accurate as one needs [10]. However, the presence of multiple classes complicates the assortment problem as customers belonging to different segments can have dissimilar preferences for different products [11]. Indeed, Bront et al. [12] show that assortment optimization problem under MMNL model is NP-hard.

Most research in assortment planning assumes that either there are no costs

associated with offering a product in the assortment or costs depend linearly on the sales volume [13, 4, 3, 11, 14]. In many cases, offering an additional product increases the costs in a non-linear fashion. For example, Mahajan and van Ryzin [15] consider an assortment problem in which the retailer incurs typical holding and backorder costs in a Newsvendor setting. Assortment problems with fixed costs received considerably less attention in the literature. This problem is relevant in many practical environments. For example, some auto dealers, especially those for high-end brands, do not carry inventory except for a limited number of vehicles that are displayed in their showrooms. A customer is shown different models and trim levels in the showroom and once he/she makes a decision for a particular model and trim level, the dealer places an order with the factory for his/her desired color and accessory options. In this case, the cost of displaying a particular vehicle is a fixed cost for a given period and is independent from the (expected) sales volume of that vehicle. This fixed cost is obviously different for each vehicle. The problem for the dealer is to determine the vehicles it will showcase to its customers (i.e., its assortment) such that its profit is maximized. Another example where the fixed costs could be important is when the product assortment has to be changed frequently as in the fast fashion industry. In this case, if the fixed costs relating to the design of the items are high then these costs gain more importance as they have to be recovered in a short period of time [16, 17].

The fixed cost of including an item in the assortment can also be considered as the cost of capacity the item is consuming. In fact, if we relax the space capacity constraint into the objective function for an assortment optimization problem using a Lagrangian relaxation approach, the problem we face is exactly an assortment optimization problem with fixed costs. This approach is used, for example, in the work of Feldman and Topaloglu [18] for a constrained assortment problem under MMNL model. While Talluri and van Ryzin [1] show that the assortment problem without fixed costs is polynomially solvable, Kunnumkal et al. [19], show that introducing fixed costs into the problem formulation makes the problem NP-complete even for the case where the customer choice model is Multinomial Logit Model.

We study the assortment optimization problem with fixed costs under MMNL

model in Chapters 3 and 4. In contrast to most research on assortment optimization, we follow a mathematical programming approach. In particular, we formulate the problem as a conic integer program. We also propose McCormick inequalities to strengthen the formulation. This is similar to the approach followed in the work of Şen et al. [20] for the same problem without fixed costs but with capacity constraints. We numerically test the effectiveness of the formulation and the suggested inequalities in a variety of settings (same numerical settings used by Kunnumkal and Martínez-de-Albéniz [16] to test their relaxation and heuristic). We consider 810 different instances all with 300 products: 90 with a single class (MNL), 270 with MNL under a capacity constraint, 270 with MMNL and 180 with MMNL under a capacity constraint. All 810 instances were solved following the conic approach within 30 minutes – in many cases within a few seconds – using a commercial optimization software. Given the difficulty of accurately estimating parameters of a discrete choice model for large choice sets (see Chiong and Shum [21] where the authors use machine learning tools to study data with high number of product alternatives – average of 280 products in their application) and given that a retailer typically plans its assortment for related items at a category or a sub-category level, we believe that the problem sizes we consider in these experiments are what are or can be tackled in industry. Our numerical results show that, in contrast to the conic formulations, linearized formulations that are typically used in the literature are far less effective; optimization times are as much as 50 times higher and 110 of the instances cannot be solved within the time limit.

In addition to assortment decisions, a typical retailer also needs to determine inventory levels and replenishment frequencies of the items that are included in the assortment. Inventory and replenishment costs can be decisive and should be considered when making assortment decisions. For instance, if an item is procured from abroad at a high fixed ordering cost, then this fixed replenishment cost should be incorporated in the assortment planning decisions. Another example could be the floor space costs associated with large items such as refrigerators. Holding costs of such items cannot be neglected and must be included in the assortment planning. Essentially, there is a trade-off between enlarging the assortment and the increase in inventory costs. This is because as the variation in assortment increases the demand is split over a larger number of products and

this in turn causes more variability in demand and more inventory costs [15]. Further, as the number of products in the assortment increases, the retailer can no longer benefit from the economies of scale. Hence, assortment planning and the inventory management problems are interrelated and planning each of them separately will not result in an overall optimal solution [22].

In Chapter 5, we study the assortment optimization problem considering inventory and replenishment costs and we capture such costs with the Economic Order Quantity (EOQ) model when the customer choice model is governed by the Mixture of the Multinomials Model. The EOQ model provides the optimal order (or production) quantities that a firm should purchase in order to minimize the inventory holding and fixed ordering costs. This model assumes a constant rate of demand and that there is no uncertainty in demand. The EOQ model also assumes an instantaneous replenishment of products and constant ordering and inventory holding costs. We propose a conic formulation for the problem, and perform numerical studies that show the effectiveness of the conic formulation when accompanied with the McCormick inequalities.

The rest of the thesis is organized as follows. In Chapter 2, a review of related literature of assortment optimization is given. In Chapter 3, the mixed integer linear and conic quadratic mathematical models of the problem are presented alongside with valid McCormick inequalities for these formulations. In Chapter 4, numerical studies for the variations of the models with and without capacity constraints are provided. In Chapter 5, we consider inventory decisions by the EOQ model in our assortment problem and present a conic quadratic mathematical model and provide related numerical results. Finally, Chapter 6 concludes the thesis and presents several possible future research directions.

Chapter 2

Literature Review

In our literature review we focus on papers that model the customer choice behavior in assortment planning problem via Multinomial Logit Model and its variants, including the ones that consider capacity constraints. We refer the reader to K  k et al. [23] and Karampatsa et al. [24] for thorough reviews of the general assortment planning literature. We also provide a review of the papers related to the joint assortment planning and inventory management problems at the end of this section.

Under the Multinomial Logit Model (MNL) a product’s market share can be expressed as a ratio of its own popularity (that can be defined as a function of the product’s attributes) and the sum of popularities of all products that are placed in the assortment. When the assortment optimization problem is modeled under the MNL model, Talluri and van Ryzin [1] show that the optimal assortment is one that includes a certain number of products with the highest revenues. When there are capacity constraints, leapfrogging in product prices is possible, but Rusmevichientong et al. [3] provide a polynomial algorithm to find the optimal assortment. They further extend their model for the dynamic case where preferences of the customers are unknown and must be estimated from the past data. Gallego et al. [25] show that the unconstrained assortment optimization problem under the MNL model can be formulated as a linear program. Davis et al. [4] show that a similar linear program can be used when there are constraints

that exhibit a totally unimodular structure. The authors also show that their model can be used to solve several other assortment problems in which there are cardinality constraints, deciding on the location for the offered assortment, pricing from a finite set, and precedence constraints. Wang [26] considers the joint assortment and price optimization problem and finds the efficient assortment sets both under static and dynamic assortment planning. Under certain assumptions such as twice differentiability of the utility function and its concavity in price, he proposes efficient algorithms to find the optimal assortment. Farias et al. [27], motivated by the shortcomings of the MNL model, consider the assortment optimization under general choice models when there are constraints over the total number of products to be included in the assortment and propose an algorithm based on greedy exchange. For the MNL model, which is a special case of their proposed model, this algorithm finds the optimal assortment efficiently and is more efficient than what Rusmevichientong et al. [3] propose for this problem.

As noted in Chapter 1, the MNL model suffers from what is known as the Independence of Irrelevant Alternatives property which states that the ratio of market shares of any two products is independent of the assortment they are in. A natural alternative for the MNL model that alleviates this problem is the Nested Logit (NL) model where a consumer first selects a nest and then a product from the nest he/she has selected. Davis et al. [14] classify the unconstrained assortment optimization problem based on the nest dissimilarity and no purchase behavior of the customers. They show that the assortment optimization problem under the NL model in general is NP-hard. They also show that when the nest dissimilarity parameters are less than one and the customers always make a purchase after selecting a nest, the problem is polynomially solvable. However, relaxing any one of these conditions makes the problem NP-hard. For the NP-hard cases they propose approximation algorithms with worst-case performance guarantees. They further formulate a convex program to find upper bounds on the optimal expected revenue to test the performance of their proposed solutions. K  k and Xu [28] consider the setting where there are only two brands (nests) and customers either first choose a brand and then a product type or vice versa. They consider this problem under centralized and decentralized management and provide the characteristics of the optimal assortment that reduces the search for the optimal solution. Rusmevichientong et al. [29] show that the

problem under the investment (knapsack or space like) constraint is NP-complete using a reduction from the knapsack problem. They then formulate the problem as an optimization of sum-of-ratios and propose a polynomial-time approximation scheme to solve it. Gallego and Topaloglu [30] study the NL assortment optimization problem when there are separate constraints in each nest and show that the cardinality-constrained case can be solved using a linear program. Although the space-constrained case is NP-hard, the authors can obtain assortments with a performance guarantee of two. Feldman and Topaloglu [31] study the case when the constraints are over all nests. They consider both cardinality and space constraints in their study and develop a linear program with a polynomial running time for the cardinality constrained case and propose another linear program for the space constrained case which obtains a 4-approximation solution. Désir and Goyal [2] consider the assortment problem with capacity constraints in each nest and a capacity constraint over all nests together, and show that there is a fully polynomial time approximation scheme (FPTAS) for this problem.

A further generalization of the MNL model is the Mixtures of Multinomials (MMNL) model where there are multiple classes, in each of which the customer choice is governed by a separate MNL model. Bront et al. [12] show that the unconstrained assortment optimization problem appears as the sub-problem of their linear programming formulation for the network revenue management problem, and they also show that the problem is NP-hard when the number of customer segments is equal or greater than the number of products. They propose a mixed integer linear programming formulation and a greedy heuristic for this problem. Rusmevichientong et al. [11] show that assortment planning under MMNL is NP-complete even when there are only two customer segments. Motivated by this, they characterize the cases where the revenue-ordered assortments are optimal depending on the utilities of the products. They also provide performance guarantees for revenue-ordered assortments when they are not optimal. Méndez-Díaz et al. [5] also formulate the problem as a mixed integer linear program and propose a branch and cut algorithm based on five families of valid inequalities. In their demand model they allow consideration sets, i.e. the set of products offered to different segments, to overlap. Feldman and Topaloglu [18], motivated by the difficulty of verifying the quality of the solutions from the heuristics, propose new strategies that are based on solving the assortment problem for a single customer

type to reach tight upper bounds on the expected revenues under the MMNL model. Their numerical study shows that their proposed bounds deviate, on average, less than 0.15% from the optimal solution. Kunnumkal [32] proposes a similar approach used by Feldman and Topaloglu [18] for finding upper bounds on the expected revenue and shows that his calculation for finding the upper bound is tractable. He also shows that this procedure finds the tightest upper bound among the available methods in the literature that are tractable. A recent work by Şen et al. [20] shows that conic reformulations, together with some strengthening inequalities, are very effective in solving large size assortment optimization problems under MMNL model. They consider the assortment optimization problem with capacity constraints and utilize McCormick estimators that are based on conditional bounds of the variables.

In some problem settings there are fixed costs associated with products in assortment planning. Kunnumkal et al. [19] are the first to study the assortment planning problem with fixed costs directly under MNL model and show that the problem is NP-hard. They also develop a two-approximation algorithm and a polynomial time approximation scheme for the problem. Kunnumkal and Martínez-de-Albéniz [16] develop a continuous relaxation of the problem that is based on the assortment attractiveness. The authors show that this relaxation constitutes an upper bound for the problem and provide a theoretical guarantee for its performance. The authors also show, through a numerical study, that a heuristic based on this relaxation can be used to generate near-optimal assortments. Atamtürk and Gómez [33] show that the problem is NP-hard even when the unit revenues are same for all products and propose a $1/2$ -approximation algorithm for this setting. Schön [34] studies a general product line design problem where customer choice is governed by a generalized attraction model. There is a fixed cost associated with offering a particular product attribute. Schön [34] shows that this problem can be transformed into a mixed integer program with a concave objective function which can be solved effectively using a commercial solver.

As explained above most papers in the assortment optimization do not incorporate fixed costs. A few papers that consider fixed costs use heuristics and approximation approaches due to the NP-hardness of the problem. We, on the

other hand, follow a mathematical programming approach that delivers exact solutions for the problem. We show that by adding McCormick inequalities to the model we are able to solve large instances of the problem. Furthermore, the model easily incorporates cardinality constraints. The closest study to ours is the one done by Kunnumkal and Martínez-de-Albéniz [16].

Finally, we review some of the studies in which inventory is considered when making assortment decisions. van Ryzin and Mahajan [15] were the first to study the joint assortment and inventory planning problem. They model the customer demand with the Multinomial Logit Model. In their model, the inventory costs are captured by the Newsvendor model and they assume identical prices for the products. Their essential finding is that the optimal assortment is composed of a certain number of most popular products.

Urban [35] considers an integrated assortment, allocation and replenishment problem. He proposes a deterministic demand rate that is a function of the displayed inventory level and presents two heuristics. The first heuristic is a greedy search heuristic which can be applied to small- to moderately-sized problems. The second heuristic that the writer proposes is a genetic algorithm. The author believes that the near optimal solutions can be found by this heuristic, although there is a factor of randomness. Gaur and Honhon [36] consider the problem under the locational choice model both under static and dynamic substitutions. They find, in contrast to van Ryzin and Mahajan [15], that the optimal assortment does not need to include the most popular products for the static substitution case and propose heuristics for the dynamic substitution model. Hariga et al. [22] model the joint assortment, inventory replenishment, and space allocation problems as a non-linear mixed integer program and separate the backroom and showroom inventories in their model. They use a deterministic demand rate which is a function of displayed inventory levels and the total shelf space allocation of the products. However, their model does not seem to be able to solve even moderately sized instances.

Honhon et al. [37] propose a dynamic programming algorithm when the demand is stochastic with a known distribution and find the structure of the problem under a dynamic substitution customer behavior. They model the customer

choice by classifying customers into different customer types where each customer type has a specific sequence of products, arranged in decreasing order of attraction factor, that he/she is willing to purchase. This model is general and other choice models like the Multinomial Logit is a special case of this model. The algorithm they provide to solve their proposed dynamic program has a pseudopolynomial complexity of $O(8^n)$. Cachon et al. [38] consider several customer choice models and include the search costs, i.e., they account for the possibility that a customer may choose not to buy a product even though she might be satisfied with a product in the assortment, to search other retailers for a better product. They find that ignoring the search costs may result in considerably less lower expected profits. Furthermore, they show that when a model accounts for the consumer search, it may be profitable to include products that are not profitable to reduce the customer search effect.

Rajaram [39] models the demand as a random variable and considers the joint assortment and inventory problem in a single period setting. He formulates the problem as a mixed integer nonlinear problem and proposes two heuristics. An extension of the problem incorporating shelf space constraints is also developed. Smith and Agrawal [40] use a base stock policy for replenishment and include the impact of substitution in their demand model by choice probabilities of the customers and a substitution matrix. They assume that customers randomly choose from choice sets. These choice sets are disjoint sets that contain all products that are potential substitutes. They model the problem as a nonlinear integer program and propose an approximation for the objective function to solve the model. K  k and Fisher [41] characterize the structure for the assortment of a retailer in relation to the inventory levels of the offered products based on a proposed iterative heuristic. They consider the case when there is assortment based substitution with a MNL choice model in presence of space constraints. They find that the products with smaller demand variability and case sizes should be included in the assortment first. However, if the available space is sufficiently large, then more inventory should be allocated to products with higher demand variance and larger case sizes.

We consider the joint inventory and assortment planning problem under the MMNL customer choice model. To the best of our knowledge we are the first to

consider this problem under the MMNL choice model and capture the inventory holding and replenishment costs using the EOQ model. We show that the problem is NP-hard and we propose a conic formulation for the problem alongside the valid McCormick inequalities for strengthening the mathematical model. Our numerical study shows that moderately sized instances of the problem can be solved in reasonable times.



Chapter 3

Assortment Optimization with Fixed Costs: Problem Formulation

In this chapter we consider the assortment optimization problem when there are fixed costs associated with the products and the customer choice follows the Mixtures of Multinomials (MMNL) Model. We present the mixed integer linear program (MILP) and conic quadratic mixed integer (Conic) models for the problem and derive inequalities for strengthening these formulations.

In the MMNL choice model customers belong to different classes. The set of customer classes is denoted by $\mathcal{M}$. The probability that a customer belongs to class i is denoted by λ_i such that $\sum_{i \in \mathcal{M}} \lambda_i = 1$. In each class, the customers' choice is governed by a separate MNL model. A customer in class i gains utility $\vartheta_{ij} + \epsilon_{ij}$ from consuming product j , where ϑ_{ij} is a constant term and ϵ_{ij} is a Gumbel random variable with mean zero and variance $\mu_i^2 \pi^2 / 6$. Given an assortment $\mathcal{S}$, the probability that a customer in class i selects product j can then be written as $P_{ij}(\mathcal{S}) = \nu_{ij} / (\nu_{i0} + \sum_{k \in \mathcal{S}} \nu_{ik})$, where $\nu_{ij} = e^{\vartheta_{ij} / \mu_i}$ is the attraction factor for product j and ν_{i0} is the attraction factor for the no-purchase (outside) option in class i . The firm's assortment needs to be a subset from a universal set of products, denoted by $\mathcal{N}$. Product j 's unit revenue in class i is ρ_{ij} . Including

product j in the assortment incurs a fixed cost δ_j for the firm. We also define a constant θ as a scale parameter which scales the fixed costs per arrival.

For a given assortment $\mathcal{S}$, the firm's expected profit per arrival can be written as:

$$\sum_{i \in \mathcal{M}} \lambda_i \left[\frac{\sum_{j \in \mathcal{S}} \rho_{ij} \nu_{ij}}{\nu_{i0} + \sum_{j \in \mathcal{S}} \nu_{ij}} \right] - \sum_{j \in \mathcal{S}} \theta \delta_j. \quad (3.1)$$

This problem (finding the optimal $\mathcal{S}$) is NP-hard, even when $|\mathcal{M}| = 1$ as shown in the work of Kunnumkal et al. [19]. Our assortment optimization problem can be formulated as the following non-linear integer program:

$$\text{(FAOP1)} \quad \max \sum_{i \in \mathcal{M}} \lambda_i \left[\frac{\sum_{j \in \mathcal{N}} \rho_{ij} \nu_{ij} x_j}{\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j} \right] - \sum_{j \in \mathcal{N}} \theta \delta_j x_j \quad (3.2)$$

$$\text{s.t. } x_j \in \{0, 1\}, \quad \forall j \in \mathcal{N}. \quad (3.3)$$

where x_j is a binary decision variable which takes on the value 1 if product j is included in the assortment and 0, otherwise.

First, we present the formulation of this problem as a mixed integer linear program following the approach used previously in the literature [12, 5, 20]. For this purpose, first define $y_i = 1/(\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j)$ and formulate (FAOP1) as an equivalent bilinear program:

$$\max \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \lambda_i \rho_{ij} \nu_{ij} y_i x_j - \sum_{j \in \mathcal{N}} \theta \delta_j x_j \quad (3.4)$$

$$\text{(FAOP2)} \quad \text{s.t. } \nu_{i0} y_i + \sum_{j \in \mathcal{N}} \nu_{ij} y_i x_j = 1, \quad \forall i \in \mathcal{M}, \quad (3.5)$$

$$y_i \geq 0, \quad \forall i \in \mathcal{M}, \quad (3.6)$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{N}. \quad (3.7)$$

The bilinear terms $y_i x_j$ in the objective function can be linearized using the approach in Wu [42], by defining a continuous variable $z_{ij} = y_i x_j$ and adding the following inequalities to the model: $y_i - z_{ij} \leq M(1 - x_j)$, $z_{ij} \leq y_i$, $0 \leq z_{ij} \leq M x_j$ where M is a number greater than y_i . Noting that y_i takes its maximum value

when the assortment is a null set, we set $M = 1/\nu_{i0}$, and rewrite (FAOP2) as a mixed integer linear program as follows:

$$\max \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \lambda_i \rho_{ij} \nu_{ij} z_{ij} - \sum_{j \in \mathcal{N}} \theta \delta_j x_j \quad (3.8)$$

$$\text{s.t. } \nu_{i0} y_i + \sum_{j \in \mathcal{N}} \nu_{ij} z_{ij} = 1, \quad \forall i \in \mathcal{M}, \quad (3.9)$$

$$\nu_{i0}(y_i - z_{ij}) \leq 1 - x_j, \quad \forall i \in \mathcal{M}, \quad \forall j \in \mathcal{N}, \quad (3.10)$$

$$0 \leq z_{ij} \leq y_i, \quad \forall i \in \mathcal{M} \quad \forall j \in \mathcal{N}, \quad (3.11)$$

$$\text{(MILP)} \quad \nu_{i0} z_{ij} \leq x_j, \quad \forall i \in \mathcal{M}, \quad \forall j \in \mathcal{N}, \quad (3.12)$$

$$y_i \geq 0, \quad \forall i \in \mathcal{M}, \quad (3.13)$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{N}, \quad (3.14)$$

$$y_i \geq 0, \quad i \in \mathcal{M}. \quad (3.15)$$

Linear formulations are shown to be not very effective for the MMNL assortment optimization problem when there are no fixed costs. In contrast, Şen et al. [20] show that conic formulations can be used to solve very large instances. We adopt the conic mixed integer formulation proposed by Şen et al. [20] for the case of fixed costs. For this purpose, we first convert the objective function in (FAOP1) to minimization. We can write the objective function in (3.3) as:

$$\sum_{i \in \mathcal{M}} \lambda_i \bar{\rho}_i - \sum_{i \in \mathcal{M}} \lambda_i \left[\frac{\nu_{i0} \bar{\rho}_i + \sum_{j \in \mathcal{N}} \nu_{ij} (\bar{\rho}_i - \rho_{ij}) x_j}{\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j} \right] - \sum_{i \in \mathcal{N}} \theta \delta_j x_j, \quad (3.16)$$

where $\bar{\rho}_i = \max_{j \in \mathcal{N}} \rho_{ij}$. As the first term in (3.16) is constant, we can convert the problem to a minimization problem where the objective is the sum of the second and third terms. Hence the problem can be cast as:

$$\text{(FAOP3)} \quad \min \sum_{i \in \mathcal{M}} \lambda_i \left[\frac{\nu_{i0} \bar{\rho}_i + \sum_{j \in \mathcal{N}} \nu_{ij} (\bar{\rho}_i - \rho_{ij}) x_j}{\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j} \right] + \sum_{i \in \mathcal{N}} \theta \delta_j x_j \quad (3.17)$$

$$\text{s.t. } x_j \in \{0, 1\}, \quad \forall j \in \mathcal{N}. \quad (3.18)$$

As is done for the case of linear formulation, we can define variables $y_i = 1/(\nu_{i0} +$

$\sum_{j \in \mathcal{N}} \nu_{ij} x_j$) and $z_{ij} = y_i x_j$ and reformulate (FAOP3) as:

$$\min \sum_{i \in \mathcal{M}} \lambda_i \nu_{i0} \bar{\rho}_i y_i + \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \lambda_i \nu_{ij} (\bar{\rho}_i - \rho_{ij}) z_{ij} + \sum_{j \in \mathcal{N}} \theta \delta_j x_j \quad (3.19)$$

$$(FAOP4) \quad z_{ij} \geq y_i x_j, \quad i \in \mathcal{M}, \quad j \in \mathcal{N}, \quad (3.20)$$

$$y_i \geq \frac{1}{\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j}, \quad i \in \mathcal{M}, \quad (3.21)$$

$$x_j \in \{0, 1\}, \quad j \in \mathcal{N}, \quad (3.22)$$

$$z_{ij} \geq 0, \quad i \in \mathcal{M}, \quad j \in \mathcal{N}, \quad (3.23)$$

$$y_i \geq 0, \quad i \in \mathcal{M}. \quad (3.24)$$

Now, if we define $w_i = \nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j$, we can reformulate constraints (3.20) and (3.21) as conic (rotated cone) constraints as follows:

$$z_{ij} w_i \geq x_j^2, \quad i \in \mathcal{M}, \quad j \in \mathcal{N}, \quad (3.25)$$

$$y_i w_i \geq 1, \quad i \in \mathcal{M}. \quad (3.26)$$

The above hyperbolic inequalities are equivalent to conic constraints. These inequalities can be represented as conic quadratic inequalities as follows:

$$\|2x_j, z_{ij} - w_i\| \leq z_{ij} + w_i, \quad (3.27)$$

$$\|2, y_i - w_i\| \leq y_i + w_i, \quad (3.28)$$

where $\|\cdot\|$ is the $L2$ norm.

Together with a strengthening inequality $\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} z_{ij} \geq 1, i \in \mathcal{M}$ (see Şen et al. [20]), we have the conic quadratic mixed integer programming formulation

as:

$$\min \sum_{i \in \mathcal{M}} \lambda_i \nu_{i0} \bar{\rho}_i y_i + \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \lambda_i \nu_{ij} (\bar{\rho}_i - \rho_{ij}) z_{ij} + \sum_{j \in \mathcal{N}} \theta \delta_j x_j \quad (3.29)$$

$$\text{s.t. } w_i = \nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j, \quad i \in \mathcal{M}, \quad (3.30)$$

$$z_{ij} w_i \geq x_j^2, \quad i \in \mathcal{M} \quad j \in \mathcal{N}, \quad (3.31)$$

$$y_i w_i \geq 1, \quad i \in \mathcal{M}, \quad (3.32)$$

$$\text{(CONIC)} \quad \nu_{i0} y_i + \sum_{j \in \mathcal{N}} \nu_{ij} z_{ij} \geq 1, \quad \forall i \in \mathcal{M}, \quad (3.33)$$

$$x_j \in \{0, 1\}, \quad j \in \mathcal{N}, \quad (3.34)$$

$$z_{ij} \geq 0, \quad i \in \mathcal{M}, \quad j \in \mathcal{N}, \quad (3.35)$$

$$y_i \geq 0, \quad i \in \mathcal{M}, \quad (3.36)$$

$$w_i \geq 0, \quad i \in \mathcal{M}. \quad (3.37)$$

MILP and CONIC formulations can be strengthened utilizing McCormick inequalities for the bilinear terms. For the bilinear term $z_{ij} = y_i x_j$, we use global and conditional bounds on variable $y_i = 1/(\nu_{i0} + \sum_{j \in \mathcal{N}} \nu_{ij} x_j)$ to derive McCormick inequalities (as proposed in McCormick [43]). This is done in the work of Şen et al. [20] when there are constraints on the size of the assortment. In this case, we derive these inequalities when there are no capacity constraints. Let y_i^u denote the global upper bound of y . Also, let $y_{i|x_j=b}^\ell$ and $y_{i|x_j=b}^u$ denote the lower and upper bounds on y , respectively, conditional on x_j taking the value b .

It is easy to check that the global upper bound is attained when the assortment is a null set:

$$y_i^u = \frac{1}{\nu_{i0}}, \quad i \in \mathcal{M}. \quad (3.38)$$

The conditional bounds are given below:

$$y_{i|x_j=1}^u := \frac{1}{\nu_{i0} + \nu_{ij}}, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (3.39)$$

$$y_{i|x_j=0}^\ell := \frac{1}{\nu_{i0} - \nu_{ij} + \sum_{k \in \mathcal{N}} \nu_{ik}}, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (3.40)$$

$$y_{i|x_j=1}^\ell := \frac{1}{\nu_{i0} + \sum_{k \in \mathcal{N}} \nu_{ik}}, \quad i \in \mathcal{M}, j \in \mathcal{N}. \quad (3.41)$$

Then, the following McCormick inequalities are valid for the bilinear terms:

$$z_{ij} \leq y_{i|x_j=1}^u x_j, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (3.42)$$

$$(MC) \quad z_{ij} \geq y_{i|x_j=1}^\ell x_j, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (3.43)$$

$$z_{ij} \leq y_i - y_{i|x_j=0}^\ell (1 - x_j), \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (3.44)$$

$$z_{ij} \geq y_i - y_i^u (1 - x_j), \quad i \in \mathcal{M}, j \in \mathcal{N}. \quad (3.45)$$

MILP formulation strengthened with McCormick inequalities is denoted by MILP+. Similarly, CONIC+ refers to the CONIC formulation plus the McCormick inequalities.

A further strengthening of the formulations is possible when there is a single class in the MMNL model (i.e., MNL model). For this purpose, we drop the index i from unit revenues (ρ_j), product attraction factors (ν_j), and no-purchase attraction factor (ν_0). The following proposition states that some products are always preferred over others in a profit-maximizing assortment under MNL.

Proposition 3.1. *Under MNL model, for any two products j and k such that $\rho_j \nu_j > \rho_k \nu_k$, $\nu_j \leq \nu_k$ and $\delta_j \leq \delta_k$, in any optimal solution, product k is in the assortment only if product j is also in the assortment.*

Proof. Consider an assortment $\mathcal{S}$ such that $j \notin \mathcal{S}$ and $k \notin \mathcal{S}$. Denote $P(\mathcal{S}) = \sum_{j \in \mathcal{S}} \rho_j \nu_j$, $N(\mathcal{S}) = \sum_{j \in \mathcal{S}} \nu_j$ and $\Delta(\mathcal{S}) = \sum_{j \in \mathcal{S}} \delta_j$. Then one can write the expected profit for assortment $\mathcal{S} \cup \{j\}$ as

$$\frac{P(\mathcal{S}) + \rho_j \nu_j}{N(\mathcal{S}) + \nu_j} - \Delta(\mathcal{S}) - \delta_j,$$

and for assortment $\mathcal{S} \cup \{k\}$ as

$$\frac{P(\mathcal{S}) + \rho_k \nu_k}{V(\mathcal{S}) + \nu_k} - \Delta(\mathcal{S}) - \delta_k.$$

Clearly the first quantity is larger than the second quantity if $\rho_j \nu_j > \rho_k \nu_k$, $\nu_j \leq \nu_k$ and $\delta_j \leq \delta_k$. Therefore, there cannot be an optimal solution in which product k is part of the assortment, but product j is not. ■

Proposition 3.1 suggests that we can add the *dominance inequality* $x_k \leq x_j$ for any product pairs j and k that satisfy $\rho_j \nu_j > \rho_k \nu_k$, $\nu_j \leq \nu_k$ and $\delta_j \leq \delta_k$ to the formulation without eliminating any potentially optimal solution. Addition of these dominance inequalities implied by Proposition 3.1 for the proper product pairs to formulations MILP+ and CONIC+ are called formulations MILP++ and CONIC++, respectively.

Chapter 4

Numerical Study

The goal of our numerical study is to investigate whether the mathematical programming formulations presented in Chapter 3 are effective in solving large instances of the assortment optimization problem with fixed costs under MMNL model. We first consider the problem when there is a single class (MNL model). We then study the same single class model when there is a capacity constraint, defined as an upper bound on the number of products that can be offered in the assortment. Finally, we consider the uncapacitated and capacitated problems when there are multiple classes of customers (MMNL model). All of our test problems are generated following the approach in Kunnumkal and Martínez-de-Albéniz [16].

All instances are solved with Gurobi 7.5.1 solver [44] on a computer with an Intel Core i7-6700HQ 2.60GHz processor and 16 GB RAM operating on a 64-bit Windows 10. For the conic formulations, we force the solver to use linear outer-approximation approach when solving the continuous relaxations. For each instance, we set the time limit to 1800 seconds.

4.1 Multinomial Logit Model

The first set of problems are generated for the MNL model with 300 products ($|\mathcal{N}| = 300$). For each product j , we set the attraction factor to $\nu_j = X_j / \sum_{k \in \mathcal{N}} X_k$ where X_j is uniformly distributed on the interval $[0, 1]$. We set the no-purchase attraction factor to $\nu_0 = [\phi / (1 - \phi)] \sum_{j \in \mathcal{N}} \nu_j$ where $\phi \in \{0.75, 0.50, 0.25\}$. With parameter ϕ we control for the importance of the no-purchase attraction factor on the solution times. The unit revenue for each product ρ_j is drawn from a uniform distribution in the interval $[0, 2000]$ and its fixed cost δ_j from a uniform distribution in the interval $[0, \gamma \rho_j \nu_j / (\nu_0 + \nu_j)]$ where $\gamma \in \{1.00, 0.50, 0.25\}$. We set the parameter θ to 1 in all of the instances. We note that with parameter γ , we control for the fixed cost of a product in comparison to its price. For each ϕ and γ combination, we generate ten problem instances resulting in a total of 90 instances. We test the performance of the six formulations MILP, MILP+, MILP++, CONIC, CONIC+, and CONIC++. The results are reported in Table 4.1.

For each ν_0 (ϕ) and γ combination, Table 4.1 reports the average number of products in the optimal assortment (assort), root gap (rgap), end gap (egap), solution time (time), and the number of search nodes (nodes) over 10 instances. The root gap is computed as $\text{rgap} = |Z_{\text{opt}} - Z_{\text{root}}| / |Z_{\text{opt}}|$ where Z_{opt} is the objective function value of the integer optimal solution (same for all formulations) and Z_{root} is the objective value of the continuous relaxation before presolve and before root cuts of the solver are added. The egap is computed as $|Z_{\text{opt}} - Z_{\text{blb}}| / |Z_{\text{opt}}|$ where Z_{blb} is the best lower bound found by the solver at termination. Note that if an instance is solved to optimality within the time limit, $|Z_{\text{opt}} - Z_{\text{blb}}| = 0$. The table also reports the number of instances (#) that are solved to optimality within the time limit. When taking the averages for the solution time, only these instances are taken into account.

Results in Table 4.1 show that the MILP formulation performs poorly especially when there are no additional tightening cuts accompanying the model. In 60 instances, out of 90, the time limit is reached for the MILP formulation. Root gaps are large under this formulation leading to excessive branching. The performance gets poorer as the attraction factor for the no-purchase option gets smaller

Table 4.1: Results for MNL model

ν_0	ϕ	γ	assort	MILP		MILP+		MILP++		CONIC		CONIC+		CONIC++	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	1.00	172	0.95%	151.27/10	0.08%	10.82/10	0.08%	7.16/10	9.08%	8.13/10	0.04%	0.90/10	0.04%	1.35/10
				0.00%	637220	0.00%	57583	0.00%	21359	0.00%	1658	0.00%	1188	0.00%	979
		0.50	245	0.95%	0.43/10	0.00%	0.13/10	0.00%	0.33/10	18.73%	2.08/10	0.00%	0.15/10	0.00%	0.31/10
				0.00%	1	0.00%	1	0.00%	1	0.00%	1115	0.00%	1	0.00%	1
1	0.50	0.25	256	0.50%	0.21/10	0.00%	0.08/10	0.00%	0.16/10	44.27%	1.51/10	0.00%	0.11/10	0.00%	0.26/10
				0.00%	1	0.00%	1	0.00%	1	0.00%	1097	0.00%	1	0.00%	1
		1.00	112	3.24%	-	0.81%	14.53/10	0.81%	43.62/10	16.60%	496.17/3	0.31%	7.10/10	0.31%	4.69/10
				1.01%	2431718	0.00%	49271	0.00%	67681	0.08%	1216601	0.00%	22003	0.00%	2303
1/3	0.25	0.50	160	4.26%	-	0.63%	42.98/9	0.63%	43.99/9	34.01%	348.08/9	0.26%	5.55/10	0.26%	6.02/10
				2.25%	2809644	0.01%	1349301	0.01%	472301	0.01%	190282	0.00%	3409	0.00%	1924
		0.25	196	2.98%	-	0.04%	0.45/10	0.04%	0.79/10	49.20%	117.62/10	0.01%	0.24/10	0.01%	0.30/10
				1.10%	2987845	0.00%	194	0.00%	216	0.00%	43509	0.00%	1	0.00%	1
Average		1.00	66	8.25%	-	4.19%	440.69/9	4.19%	281.90/5	25.67%	1003.22/2	1.66%	14.27/10	1.66%	14.20/10
				1.65%	954891	0.01%	1662576	0.13%	1333833	0.62%	1387998	0.00%	3410	0.00%	2700
		0.50	89	11.19%	-	5.06%	-	5.06%	-	24.58%	-	1.69%	28.34/10	1.69%	120.21/10
				4.24%	1727011	0.58%	1842314	0.74%	1044299	0.97%	460892	0.00%	9029	0.00%	73787
Average		0.25	118	10.17%	-	3.01%	594.87/5	3.01%	1329.87/2	28.27%	1010.12/4	0.97%	7.81/10	0.97%	7.47/10
				2.25%	561632	0.12%	553378	0.17%	518679	0.18%	762164	0.00%	1307	0.00%	1426
		Average		4.72%	50.64/30	1.54%	138.09/73	1.54%	213.48/66	27.82%	373.37/58	0.55%	7.16/90	0.55%	17.20/90
				1.39%	1345551	0.08%	612735	0.12%	384263	0.21%	451702	0.00%	4483	0.00%	9236

and the retailer needs to be selective in determining its assortment. Adding the McCormick inequalities improves the root gaps and the solution times: MILP+ formulation is able to solve 73 instances overall and we see a decrease in the root gap from 4.72% to 1.54%. Adding the dominance inequalities does not seem to strengthen the formulation. They do not have a positive effect on the solution times. We also note that in spite of the fact that the root gaps for both MILP+ and MILP++ are the same, average of times for the MILP++ is higher. This can be due to a large number of dominance inequalities that are added to the model.

The root gaps with CONIC formulation (without valid inequalities) are larger than root gaps with MILP formulation. Nevertheless, the solution times are typically smaller: 58 instances can be solved within the time limit. Adding the McCormick inequalities to the CONIC formulation has an enormous effect. The average root gap is reduced to 0.55% (from 27.82%) and all instances can now be solved within the time limit (on the average, in less than 18 seconds) under CONIC+. CONIC+ formulation’s performance relative to MILP+ formulation seems to be especially striking for the more challenging (low ν_0) instances. Dominance inequalities do not seem to benefit the CONIC formulation either. Individual run results of the instances can be found in Appendix A.

4.2 Multinomial Logit Model under Capacity Constraints

In this part we add cardinality constraints (i.e., $\sum_{j \in \mathcal{N}} x_j \leq \kappa$) to the MNL model and generate our instances as we did for the MNL model. We choose the capacity κ from the set $\{50, 100, 150\}$. We note that dominance inequalities implied by Proposition 3.1 are no longer valid for the case of capacity constraints and are therefore not analyzed here. The results are reported in Tables 4.2, 4.3, and 4.4 for κ values 50, 100, and 150, respectively.

In general, assortment problems under capacity constraints are easier as the feasible region is smaller under capacity constraints. In addition, it is possible to

obtain tighter lower bounds for the y variables leading to stronger McCormick inequalities described in Chapter 3. The lower bounds for the y variables for the case of cardinality constraints can be written as follows:

$$y_{i|x_j=0}^\ell := \frac{1}{\nu_{i0} + \sum_{k=1}^{\kappa} \nu_{i[k]_{-j}}}, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (4.1)$$

$$y_{i|x_j=1}^\ell := \frac{1}{\nu_{i0} + \nu_{ij} + \sum_{k=1}^{\kappa-1} \nu_{i[k]_{-j}}}, \quad i \in \mathcal{M}, j \in \mathcal{N}, \quad (4.2)$$

where $\nu_{i[k]_{-j}}$ is the k th largest ν_{im} value where $m \in \mathcal{N} \setminus \{j\}$. In the analysis with capacity constraints, we use these lower bounds for MILP+ and CONIC+ formulations.

The results in Table 4.2 show that when the capacity is tight at $\kappa = 50$, the problem indeed gets easier. MILP formulation is now able to solve 54 instances and MILP+ formulation is able to solve all instances within the time limit. CONIC and CONIC+ formulations are able to solve all problems very quickly, on the average, in 1.09 and 0.31 seconds, respectively.

When the capacity is increased to $\kappa = 100$ in Table 4.3, the problem gets more difficult. The capacity constraint is binding at optimality, except when $\nu_0 = 1/3$ and $\gamma = 1$ or $\gamma = 0.5$. MILP, MILP+ and CONIC formulations are able to solve 30, 84, and 81 instances, respectively, within the time limit. CONIC+ formulation, on the other hand, can solve all instances in less than two seconds on the average.

When the capacity is further increased to $\kappa = 150$, the capacity is no longer binding in the majority of the instances. However, note that the capacity constraint is still useful in reducing the solution times as it is possible to derive strong McCormick inequalities (stronger than the uncapacitated case). The results in Table 4.4 agrees with this observation. MILP+ formulation leads to more instances (80 instead of 73 when the problem is unconstrained) being solved within the time limit. CONIC+ formulation, once again, solves all instances within the time limit; on the average in about 3 seconds. Detailed runs of individual instances can be found in Appendix B.

Table 4.2: Results for capacitated MNL model, $\kappa = 50$

ν_0	ϕ	γ	assort	MILP		MILP+		CONIC		CONIC+	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	1.00	50	0.31%	1.61/10	0.01%	0.19/10	1.31%	0.37/10	0.00%	0.19/10
				0.00%	8837	0.00%	2	0.00%	968	0.00%	1
		0.50	50	0.43%	5.32/10	0.01%	0.20/10	1.11%	0.28/10	0.00%	0.16/10
				0.00%	34961	0.00%	11	0.00%	684	0.00%	1
		0.25	50	0.46%	3.60/10	0.01%	0.18/10	0.88%	0.25/10	0.00%	0.16/10
				0.00%	21413	0.00%	12	0.00%	672	0.00%	9
1	0.50	1.00	50	2.24%	78.09/10	0.12%	0.51/10	3.39%	0.74/10	0.02%	0.30/10
				0.00%	386458	0.00%	1137	0.00%	1124	0.00%	1
		0.50	50	3.00%	516.17/8	0.13%	0.77/10	3.07%	0.6/10	0.01%	0.29/10
				0.04%	2942998	0.00%	2807	0.00%	1129	0.00%	1
		0.25	50	3.48%	497.81/6	0.09%	0.54/10	2.51%	0.40/10	0.00%	0.22/10
				0.03%	2703322	0.00%	2169	0.00%	1066	0.00%	1
1/3	0.25	1.00	50	8.52%	-	1.13%	3.74/10	8.50%	3.87/10	0.29%	0.63/10
				0.73%	1473027	0.00%	10151	0.00%	5680	0.00%	81
		0.50	50	12.18%	-	1.18%	11.70/10	7.89%	1.96/10	0.18%	0.45/10
				2.50%	1104286	0.00%	43444	0.00%	1517	0.00%	1
		0.25	50	13.88%	-	0.82%	3.87/10	6.77%	1.33/10	0.09%	0.39/10
				3.03%	1272955	0.00%	10703	0.00%	1229	0.00%	1
Average				4.94%	183.78/54	0.39%	2.41/90	3.94%	1.09/90	0.07%	0.31/90
				0.70%	1105362	0.00%	7826	0.00%	1563	0.00%	11

Table 4.3: Results for capacitated MNL model, $\kappa = 100$

ν_0	ϕ	γ	assort	MILP		MILP+		CONIC		CONIC+	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	1.00	100	0.71%	179.26/10	0.03%	0.47/10	2.09%	1.56/10	0.01%	0.21/10
				0.00%	766486	0.00%	800	0.00%	4431	0.00%	1
		0.50	100	0.96%	73.08/10	0.02%	0.43/10	1.86%	0.72/10	0.00%	0.19/10
				0.00%	313835	0.00%	920	0.00%	1113	0.00%	1
		0.25	100	0.93%	12.77/10	0.01%	0.31/10	1.57%	0.77/10	0.00%	0.18/10
				0.00%	57518	0.00%	438	0.00%	1115	0.00%	1
1	0.50	1.00	100	3.24%	-	0.32%	5.21/10	5.11%	12.13/10	0.10%	0.91/10
				0.90%	2654692	0.00%	19142	0.00%	1979	0.00%	525
		0.5	100	4.65%	-	0.25%	4.72/10	5.10%	6.33/10	0.06%	0.53/10
				1.50%	1094063	0.00%	19408	0.00%	1447	0.00%	1
		0.25	100	4.79%	-	0.12%	1.10/10	4.27%	2.14/10	0.02%	0.31/10
				1.24%	1849424	0.00%	3443	0.00%	1150	0.00%	1
1/3	0.25	1.00	66	8.25%	-	2.55%	34.87/10	9.79%	114.76/3	0.88%	2.61/10
				1.38%	2283343	0.00%	102582	0.49%	1101330	0.00%	978
		0.50	89	11.19%	-	2.73%	411.49/5	10.69%	271.76/8	0.79%	6.79/10
				4.48%	1846386	0.03%	2396740	0.03%	170868	0.00%	1611
		0.25	100	10.29%	-	1.07%	74.45/9	10.09%	264.43/10	0.30%	0.87/10
				3.89%	804978	0.01%	509457	0.00%	69261	0.00%	311
Average				5.00%	88.37/30	0.79%	59.22/84	5.62%	74.95/81	0.24%	1.40/90
				1.49%	1296747	0.01%	339214	0.06%	150299	0.00%	381

Table 4.4: Results for capacitated MNL model, $\kappa = 150$

ν_0	ϕ	γ	assort	MILP		MILP+		CONIC		CONIC+	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	1.00	149	0.92%	215.70/9	0.04%	1.13/10	2.43%	2.12/10	0.02%	0.35/10
				0.00%	1628433	0.00%	2415	0.00%	1285	0.00%	100
		0.50	150	1.16%	251.86/10	0.01%	0.72/10	2.22%	1.66/10	0.00%	0.21/10
				0.00%	1169752	0.00%	1209	0.00%	1202	0.00%	7
		0.25	150	0.99%	3.22/10	0.01%	0.34/10	2.00%	0.81/10	0.00%	0.19/10
				0.00%	10359	0.00%	168	0.00%	1097	0.00%	1
1	0.50	1.00	114	3.10%	-	0.55%	8.64/9	5.37%	54.99/7	0.20%	1.73/10
				0.94%	2612548	0.01%	682029	0.02%	660288	0.00%	1231
		0.50	150	4.28%	-	0.32%	173.49/10	5.96%	36.18/10	0.13%	1.60/10
				2.29%	2352986	0.00%	898136	0.00%	16464	0.00%	1454
		0.25	150	3.34%	-	0.06%	0.53/10	5.25%	10.13/10	0.02%	0.29/10
				1.60%	1294487	0.00%	1	0.00%	3175	0.00%	1
1/3	0.25	1.00	66	8.25%	-	3.40%	222.19/10	9.79%	-	1.24%	7.82/10
				1.24%	4347180	0.00%	634664	1.11%	1023316	0.00%	1737
		0.50	89	11.19%	-	3.95%	972.61/3	10.71%	223.23/1	1.23%	14.27/10
				4.50%	929535	0.20%	2089038	0.28%	437414	0.00%	2792
		0.25	117	10.16%	-	1.97%	298.36/8	10.71%	518.75/3	0.59%	2.26/10
				2.94%	575836	0.02%	443664	0.15%	226982	0.00%	1109.3
Average				4.82%	156.93/29	1.15%	186.45/80	6.05%	105.98/61	0.38%	3.19/90
				1.50%	1657902	0.03%	527925	0.17%	263469	0.00%	937

4.3 MMNL Model

We now consider the case of multiple customer classes. Once again there are $|\mathcal{N}| = 300$ products. The set of customer classes is denoted by $\mathcal{M}$ whose cardinality takes one of the values in $\{50, 75, 100\}$. Other parameters are generated in a way similar to what is done in the MNL model. For each problem instance, the probability that a customer belongs to class i is set as $\lambda_i = Y_i / \sum_{i \in \mathcal{M}} Y_k$, where Y_i , $i \in \mathcal{M}$ is uniformly distributed between 0 and 1. We assume that each class i has a consideration set, denoted by $\mathcal{C}_i$, from which customers in that class can choose products. The cardinality of the consideration set is same for each class, and takes one of three possible values from $\{25, 75, 100\}$. For each class, products are randomly assigned to its consideration set. The no-purchase attraction factor and price of a product are assumed to be the same for all customer classes, i.e., $\nu_{i0} = \nu_0$ for all $i \in \mathcal{M}$ and $\rho_{ij} = \rho_j$ for all $i \in \mathcal{M}$, $j \in \mathcal{N}$. We also fix the γ value to 0.5. Fixed cost of the products are also the same across all customer segments and are generated using the preference weights of the first customer class in the problem instances. In particular, δ_j is drawn from a uniform distribution in the interval $[0, \gamma \rho_j \nu_{1j} / (\nu_0 + \nu_{1j})]$ where ν_{1j} is the attraction factor of product j in class 1. Finally, for each customer class, attraction factors (ν_{ij}) are generated separately and similarly to what is done in the MNL model.

Tables 4.5, 4.6 and 4.7 report the results when the cardinality of the consideration sets is 25, 75 and 100, respectively. Table 4.5 shows that when the consideration sets are small at $|\mathcal{C}_i| = 25$, all formulations perform very well. All formulations, except CONIC, can solve the problem at the root node and in less than eight seconds on the average.

When the cardinality of the consideration sets is increased to 75, the problem gets significantly more difficult. MILP, MILP+, and CONIC formulations are now able to solve only 81, 89, and 67 instances, respectively, within the time limit. In contrast, CONIC+ formulation still solves all problems at the root node, in about 70 seconds on the average.

The preeminence of the CONIC+ formulation is more visible, when the consideration set size is increased to 100. MILP, MILP+, and CONIC formulations

Table 4.5: Results for MMNL model, $|\mathcal{C}_i| = 25$

ν_0	ϕ	$ \mathcal{M} $	assort	MILP		MILP+		CONIC		CONIC+	
				rgap	time/# egap nodes	rgap	time/# egap nodes	rgap	time/# egap nodes	rgap	time/# egap nodes
3	0.75	100	85	0.00%	2.12/10	0.00%	4.89/10	0.12%	4.43/10	0.00%	7.96/10
				0.00%	1	0.00%	1	0.00%	26	0.00%	1
		75	78	0.00%	2.13/10	0.00%	2.76/10	0.13%	3.28/10	0.00%	5.25/10
				0.00%	1	0.00%	1	0.00%	40	0.00%	1
		50	81	0.00%	1.21/10	0.00%	1.97/10	0.13%	2.58/10	0.00%	3.48/10
				0.00%	1	0.00%	1	0.00%	105	0.00%	1
1	0.50	100	79	0.01%	3.17/10	0.00%	5.08/10	0.35%	29.50/10	0.00%	11.09/10
				0.00%	1	0.00%	1	0.00%	1066	0.00%	1
		75	78	0.01%	3.55/10	0.00%	3.87/10	0.38%	24.37/10	0.00%	7.84/10
				0.00%	1	0.00%	1	0.00%	1079	0.00%	1
		50	75	0.02%	2.09/10	0.00%	3.21/10	0.38%	16.67/10	0.00%	4.84/10
				0.00%	1	0.00%	1	0.00%	1055	0.00%	1
1/3	0.25	100	70	0.09%	5.94/10	0.01%	8.50/10	0.97%	52.49/10	0.00%	14.12/10
				0.00%	1	0.00%	1	0.00%	1061	0.00%	1
		75	69	0.09%	5.12/10	0.01%	4.85/10	1.04%	50.88/10	0.00%	10.26/10
				0.00%	1	0.00%	1	0.00%	1052	0.00%	1
		50	65	0.10%	2.74/10	0.01%	3.45/10	1.01%	33.01/10	0.00%	6.36/10
				0.00%	1	0.00%	1	0.00%	1052	0.00%	1
Average				0.04%	3.12/90	0.00%	4.29/90	0.50%	24.13/90	0.00%	7.91/90
				0.00%	1	0.00%	1	0.00%	726	0.00%	1

Table 4.6: Results for MMNL model, $|\mathcal{C}_i| = 75$

ν_0	ϕ	$ \mathcal{M} $	assort	MILP		MILP+		CONIC		CONIC+	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	100	157	0.04%	9.73/10	0.00%	9.76/10	0.53%	66.77/10	0.00%	20.56/10
				0.00%	1	0.00%	1	0.01%	1091	0.00%	1
		75	153	0.04%	7.24/10	0.00%	6.56/10	0.53%	59.88/10	0.00%	13.72/10
				0.00%	1	0.00%	1	0.00%	1092	0.00%	1
		50	158	0.04%	4.032/10	0.00%	4.40/10	0.55%	47.93/10	0.00%	7.71/10
				0.00%	1	0.00%	1	0.01%	1077	0.00%	1
1	0.50	100	138	0.25%	18.59/10	0.02%	23.15/10	1.46%	376.63/10	0.00%	65.67/10
				0.00%	1	0.00%	1	0.00%	1151	0.00%	1
		75	136	0.26%	13.99/10	0.02%	14.69/10	1.42%	457.39/10	0.00%	39.27/10
				0.00%	1	0.00%	1	0.01%	2399	0.00%	1
		50	135	0.26%	8.03/10	0.02%	10.48/10	1.53%	533.34/8	0.01%	18.76/10
				0.00%	1	0.00%	1	0.01%	6923	0.00%	1
1/3	0.25	100	104	1.23%	1117.17/5	0.22%	473.12/9	3.68%	-	0.09%	271.46/10
				0.04%	7343	0.01%	5871	0.30%	2228	0.00%	1
		75	106	1.27%	630.58/7	0.21%	325.19/10	3.55%	1181.27/3	0.08%	133.42/10
				0.01%	9977	0.00%	3219	0.03%	2754	0.00%	1
		50	103	1.30%	464.80/9	0.24%	252.33/10	3.48%	1016.13/6	0.09%	70.51/10
				0.00%	11421	0.00%	3357	0.08%	5237	0.00%	1
Average				0.52%	252.68/81	0.08%	124.41/89	1.86%	467.41/67	0.03%	71.23/90
				0.01%	3194	0.00%	1384	0.05%	2661	0.00%	1

perform poorly especially when the no-purchase attraction factor is low, and can solve only 60, 64, and 57 instances, respectively, within the time limit. CONIC+ formulation, once again, solves all problems at the root node, taking, on the average, approximately 197 seconds solution time. Results for each of the instances are provided in Appendix C.

Table 4.7: Results for MMNL model, $|\mathcal{C}_i| = 100$

ν_0	ϕ	$ \mathcal{M} $	assort	MILP		MILP+		CONIC		CONIC+	
				rgap egap	time/# nodes	rgap egap	time/# nodes	rgap egap	time/# nodes	rgap egap	time/# nodes
3	0.75	100	183	0.09%	22.35/10	0.00%	17.3/10	0.70%	124.97/10	0.00%	36.6/10
				0.00%	1	0.00%	1	0.00%	1085	0.00%	1
		75	179	0.08%	17.84/10	0.00%	12.62/10	0.73%	112/10	0.00%	22.95/10
				0.00%	1	0.00%	1	0.00%	1110	0.00%	1
		50	179	0.09%	8.81/10	0.00%	8.11/10	0.74%	79.29/10	0.00%	13.54
				0.00%	1	0.00%	1	0.00%	1092	0.00%	1
1	0.50	100	151	0.51%	117.53/10	0.04%	77.99/10	1.95%	1020.47/8	0.02%	152.66/10
				0.00%	932	0.00%	1	0.00%	1755	0.00%	1
		75	149	0.50%	50.51/10	0.03%	40.67/10	1.96%	524.56/10	0.01%	84.17/10
				0.00%	1	0.00%	1	0.00%	1170	0.00%	1
		50	144	0.48%	22.67/10	0.04%	19.44/10	1.92%	160.48/9	0.02%	33.42/10
				0.00%	1	0.00%	1	0.01%	3317.2	0.00%	1
1/3	0.25	100	107	2.07%	-	0.48%	940.47/1	4.79%	-	0.21%	881.08/10
				0.48%	905	0.16%	10609	0.43%	1226	0.00%	1
		75	109	2.17%	-	0.51%	-	4.58%	-	0.20%	406.42/10
				0.41%	1728	0.16%	16535	0.20%	1314	0.00%	1
		50	105	2.16%	-	0.45%	1284.36/3	4.70%	-	0.19%	138.51/10
				0.20%	7127	0.06%	22247	0.22%	4198	0.00%	1
Average				0.91%	39.95/60	0.17%	300.12/64	2.45%	336.96/57	0.07%	196.59/90
				0.12%	1189	0.04%	5489	0.10%	1807	0.00%	1

4.4 MMNL Model under Capacity Constraints

We now include cardinality constraints ($\sum_{j \in \mathcal{N}} x_j \leq \kappa$) in the MMNL model. We analyze the case $|\mathcal{C}_i| = 100$, as it is proven to be the most challenging in

the unconstrained case. Table 4.8 reports the results for $\kappa = 100$ and Table 4.9 reports the results for $\kappa = 150$.

Table 4.8: Results for capacitated MMNL model, $|\mathcal{C}_i| = 100$, $\kappa = 100$

ν_0	ϕ	$ \mathcal{M} $	assort	MILP		MILP+		CONIC		CONIC+	
				rgap egap	time/# nodes	rgap egap	time/# nodes	rgap egap	time/# nodes	rgap egap	time/# nodes
3	0.75	100	100	0.06%	26.75/10	0.00%	30.28/10	0.53%	653.91/9	0.00%	43.37/10
				0.00%	1	0.00%	1	0.00%	4626	0.00%	1
		75	100	0.06%	18.53/10	0.00%	16.02/10	0.55%	174.67/10	0.00%	26.21/10
				0.00%	1	0.00%	1	0.00%	1260	0.00%	1
		50	100	0.06%	10.08/10	0.00%	8.68/10	0.57%	108.25/10	0.00%	14.05/10
				0.00%	1	0.00%	1	0.00%	1107	0.00%	1
1	0.50	100	100	0.44%	98.07/10	0.05%	89.61/10	1.89%	1378.35/5	0.01%	165.92/10
				0.00%	1	0.00%	1	0.01%	1791	0.00%	1
		75	100	0.46%	69.22/10	0.05%	68/10	1.65%	994.95/10	0.01%	112.57/10
				0.00%	1	0.00%	1	0.00%	2262	0.00%	10
		50	100	0.45%	30.70/10	0.04%	36.91/10	1.61%	470.23/10	0.01%	39.53/10
				0.01%	1	0.01%	1	0.00%	1848	0.00%	1
1/3	0.25	100	100	2.17%	-	0.46%	-	6.25%	-	0.14%	468.85/10
				0.46%	983	0.17%	5869	1.09%	1398	0.00%	1
		75	100	2.05%	-	0.45%	738.58/3	6.08%	-	0.16%	297.84/10
				0.30%	1712	0.09%	12382	0.63%	1386	0.00%	1
		50	100	2.14%	-	0.47%	971.68/2	7.73%	-	0.18%	128.88/10
				0.20%	6068	0.09%	19429	0.34%	1411	0.00%	1
Average				0.88%	42.23/60	0.17%	244.97/65	2.98%	630.06/54	0.06%	144.14/90
				0.11%	974	0.04%	4187	0.23%	1899	0.00%	2

When $\kappa = 100$, the capacity constraint is binding in all instances. Due to stronger McCormick inequalities, the solution times are improved for MILP+ and CONIC+ formulations in comparison to the unconstrained case. The effect of other parameters are similar to what we observe in the unconstrained case. MILP, MILP+, and CONIC formulations can solve 60, 65, and 54 instances, respectively, within the time limit. CONIC+ formulation solves all instances within the time limit; on the average in 144 seconds. With CONIC+ formulation, root gaps are very small and most instances are solved at the root node. When the capacity is increased to $\kappa = 150$, MILP, MILP+ and CONIC formulations can

solve 61, 65, and 52 instances, respectively. The solution times under CONIC+ also gets slightly larger. Nevertheless, all instances are solved at the root node with 170 seconds on the average. All results for each of the instances are provided in Appendix D.

Table 4.9: Results for capacitated MMNL model, $|\mathcal{C}_i| = 100$, $\kappa = 150$

ν_0	ϕ	$ \mathcal{M} $	assort	MILP		MILP+		CONIC		CONIC	
				rgap	time/#	rgap	time/#	rgap	time/#	rgap	time/#
				egap	nodes	egap	nodes	egap	nodes	egap	nodes
3	0.75	100	149	0.07%	19.82/10	0.00%	17.83/10	0.69%	305.84/10	0.00%	40.45/10
				0.00%	1	0.00%	1	0.00%	1222	0.00%	1
		75	149	0.08%	17.38/10	0.00%	17.83/10	0.72%	192.62/10	0.00%	27.32/10
				0.00%	1	0.00%	1	0.00%	1130	0.00%	1
		50	149	0.08%	9.08/10	0.00%	11.29/10	0.73%	105.36/10	0.00%	13.26/10
				0.00%	1	0.00%	1	0.00%	1121	0.00%	1
1	0.50	100	147	0.50%	77.24/10	0.04%	76.53/10	2.10%	1171.09/5	0.02%	152.79/10
				0.00%	1	0.00%	1	0.01%	2096	0.00%	1
		75	149	0.53%	58.03/10	0.04%	64.91/10	2.10%	999.19/8	0.02%	93.29/10
				0.00%	1	0.00%	1	0.02%	3317	0.00%	1
		50	147	0.52%	32.91/10	0.04%	40.79/10	2.01%	534.76/9	0.02%	47.40/10
				0.00%	1	0.00%	1	0.00%	3823	0.00%	1
1/3	0.25	100	109	2.11%	-	0.44%	1040.47/2	4.93%	-	0.18%	637.99/10
				0.47%	914	0.13%	8752	1.67%	1320	0.00%	1
		75	107	2.01%	-	0.43%	460.77/2	4.75%	-	0.18%	361.35/10
				0.31%	1906	0.08%	14484	0.40%	1374	0.00%	1
		50	105	2.13%	836.07/1	0.46%	264.35/1	5.65%	-	0.19%	156.34/10
				0.19%	7171	0.08%	22690	0.18%	1338	0.00%	1
Average				0.89%	150.08/61	0.16%	221.64/65	2.63%	551.48/52	0.07%	170.02/90
				0.11%	1111	0.03%	5104	0.25%	1860	0.00%	1

The numerical results in this section show that the conic formulation together with McCormick inequalities can be effectively used to solve large instances of the fixed cost assortment optimization problem when the customer choice is governed by a very flexible MMNL model. We believe that this approach should be a viable option in practice as the assortment decisions are usually made at a category or sub-category level with a limited number of product alternatives and accurately estimating the parameters of a choice model when the number of alternatives is large is very difficult.

Chapter 5

Assortment Optimization with Replenishment Costs

5.1 Model and Formulation

In this part of the thesis we focus on the assortment optimization problem with replenishment and inventory holding costs, using the well-known Economic Order Quantity (EOQ) model which was first proposed by Harris [45]. Considering the importance of inventory costs in the operations management one needs to incorporate such costs when planning assortments [22].

To be able to balance the trade-off between more market share (and possibly revenue) by enlarging the assortment and the increased inventory and replenishment costs we need a customer choice model. We use the Mixtures of Multinomials (MMNL) Model to capture the customer choice behavior as this model is a general choice model as mentioned earlier [10]. The inventory holding and replenishment costs are captured by the EOQ model. This model assumes that the demand is deterministic and constant over time, inventory holding costs are linear, there is a fixed cost for replenishment, and that there is no lead time for replenishment. Although the EOQ model has some simplifying assumptions such as the constant rate of demand, we believe that it would be a good starting

point to include such inventory replenishment costs in the assortment optimization problem under the MMNL choice model and gain insights and implications about the optimal assortments and their stocking levels under this model.

As in Chapter 3, the retailer has a set of products $\mathcal{N}$. Products are sourced from a set of suppliers $\mathcal{K}$. The set of products sourced from supplier k is denoted by $\mathcal{N}_k$ where $\bigcup_{k \in \mathcal{K}} \mathcal{N}_k = \mathcal{N}$ and $\mathcal{N}_{k_1} \cap \mathcal{N}_{k_2} = \emptyset$; for all $k_1, k_2 \in \mathcal{K}$. The set of customer segments is denoted by $\mathcal{M}$. The arrival rate in segment i is denoted by λ_i per unit of time. Product j 's unit price is denoted by ρ_j and inventory holding cost per unit per unit of time is denoted by η_j . The attraction factor for product j in segment i is denoted by ν_{ij} . No-purchase attraction factor in segment i is denoted by ν_{i0} . The fixed replenishment cost for supplier k is denoted by κ_k .

The ASSORTMENT OPTIMIZATION PROBLEM WITH REPLENISHMENT COSTS is defined as selecting an assortment $\mathcal{S} \subseteq \mathcal{N}$ such that the profit per unit of time given by,

$$\sum_{i \in \mathcal{M}} \lambda_i \frac{\sum_{j \in \mathcal{S}} \rho_j \nu_{ij}}{\nu_{i0} + \sum_{l \in \mathcal{S}} \nu_{il}} - \sum_{k \in \mathcal{K}} \sqrt{2\kappa_k \sum_{j \in \mathcal{N}_k \cap \mathcal{S}} \eta_j \sum_{i \in \mathcal{M}} \frac{\lambda_i \nu_{ij}}{\nu_{i0} + \sum_{l \in \mathcal{S}} \nu_{il}}}, \quad (5.1)$$

is maximized. The first term of this function is the expected profit as given in the MMNL model, and the second term is the incurred inventory and replenishment cost. This cost is coming from the total cost when the optimal ordering amount is used in the EOQ model. Indeed, in the classical EOQ model with a single product, the cost function is given as the summation of ordering and inventory holding costs as follows:

$$G(T) = \frac{\kappa}{T} + \frac{1}{2} \eta \beta T, \quad (5.2)$$

where G is the cost function in terms of replenishment cycle time T , κ is the fixed replenishment cost, η is the inventory holding cost per unit time, and β is the demand rate per unit time. If there is a set $\mathcal{S}$ of products that are replenished

jointly with a common order cycle time T and joint ordering cost κ , then the total cost for the products replenished from supplier k is given by,

$$G(T) = \frac{\kappa}{T} + \frac{1}{2} \sum_{j \in \mathcal{S}} \eta_j \beta_j T. \quad (5.3)$$

The optimal cycle time T^* can be found as,

$$T^* = \sqrt{\frac{2\kappa}{\sum_{j \in \mathcal{S}} \eta_j \beta_j}}. \quad (5.4)$$

Inserting the value of T^* back in the cost function G , we will obtain the optimal cost as,

$$G(T^*) = \sqrt{2\kappa \sum_{j \in \mathcal{S}} \eta_j \beta_j}. \quad (5.5)$$

The second term in the profit function in (5.1) is the summation of optimal ordering and inventory holding cost coming from equation (5.5) summed over all the suppliers in $\mathcal{K}$. The demand for a product j is the summation of the expected demand from customer segments, expressed as the probability of arrival from a segment times the market share of that product in that segment.

The assortment problem under the MMNL model is known to be NP-hard in general as is mentioned in the introduction of this study [11]. The assortment optimization problem with replenishment costs is also NP-hard and this can be shown using a reduction from the set partitioning problem. We next present the proof for the NP-hardness of the problem.

Theorem 1. *The ASSORTMENT OPTIMIZATION PROBLEM WITH REPLENISHMENT COSTS is NP-hard.*

Proof. The proof uses a reduction from the set partitioning problem (see [46]). The set partitioning problem is given a set $\mathcal{N}$ of positive integers s_j is there a

subset $\mathcal{S}$ such that $\sum_{j \in \mathcal{S}} s_j = \sum_{j \in \mathcal{N} \setminus \mathcal{S}} s_j$?

We construct an instance of the feasibility problem of the assortment problem from the set partitioning problem as follows. The set of products is $\mathcal{N}$. There is only one customer segment $|\mathcal{M}| = 1$ (therefore, we drop the index i). Each product is sourced from a separate supplier leading to $\mathcal{K} = \mathcal{N}$ (we drop the index k). Then, the net profit of a given assortment $\mathcal{S}$ is given as follows:

$$\pi(\mathcal{S}) = \lambda \frac{\sum_{j \in \mathcal{S}} \rho_j \nu_j}{\nu_0 + \sum_{j \in \mathcal{S}} \nu_j} - \frac{\sum_{j \in \mathcal{S}} \sqrt{2\lambda \kappa_j \eta_j \nu_j}}{\sqrt{\nu_0 + \sum_{j \in \mathcal{S}} \nu_j}}. \quad (5.6)$$

The attraction factor for product j is $\nu_j = s_j$. The attraction factor for no-purchase option is $\nu_0 = 1$. The revenue for product j is $\rho_j = 1$. The inventory holding cost for product j is $\eta_j = 1$. The setup cost for product j is $k_j = s_j$. The arrival rate is $\lambda = \frac{1}{2}(L+1)(L+2)^2$ where $L = \frac{1}{2} \sum_{j \in \mathcal{N}} s_j$. The threshold for the feasibility problem is given as $\frac{1}{2}L^3 + L^2$. The feasibility problem is stated as follows:

Is there an assortment $\mathcal{S}$ such that:

$$\pi(\mathcal{S}) = \frac{1}{2}(L+1)(L+2)^2 \frac{\sum_{j \in \mathcal{S}} s_j}{1 + \sum_{j \in \mathcal{S}} s_j} - \sqrt{(L+1)(L+2)^2} \frac{\sum_{j \in \mathcal{S}} s_j}{\sqrt{1 + \sum_{j \in \mathcal{S}} s_j}} \geq \frac{1}{2}L^3 + L^2. \quad (5.7)$$

Now define a function

$$f(x) = \frac{1}{2}(L+1)(L+2) \frac{x}{1+x} - \sqrt{L+1} \frac{x}{\sqrt{1+x}}. \quad (5.8)$$

Then clearly,

$$\max_{\mathcal{S}} \pi(\mathcal{S}) = (L+2) \max_{\mathcal{S}} f\left(\sum_{j \in \mathcal{S}} s_j\right) = (L+2) \max_x f(x) \geq \frac{1}{2}L^3 + L^2. \quad (5.9)$$

We can show that:

$$\begin{aligned} f'(x) &= \frac{1}{2} \frac{(L+1)(L+2)}{(1+x)^2} - \frac{1}{2} \sqrt{L+1} \frac{x+2}{(1+x)^{\frac{3}{2}}}, \\ f''(x) &= -\frac{(L+1)(L+2)}{(1+x)^3} + \frac{1}{4} \sqrt{L+1} \frac{\sqrt{1+x}(4+x)}{(1+x)^3}. \end{aligned} \quad (5.10)$$

Clearly $f''(x)$ is negative and hence $f(x)$ is concave for $x \in [0, 2L]$. Solving $f'(x^*) = 0$ we get $x^* = L$ and $f(x^*) = \frac{1}{2}L^2$. Therefore, the feasibility problem has an affirmative answer if and only if we have a subset $\mathcal{S}$ of $\mathcal{N}$ such that $\sum_{j \in \mathcal{S}} s_j = \sum_{j \in \mathcal{N}/\mathcal{S}} s_j = L$. $\square$

We also note that when there is one customer segment (MNL choice model) and when inventory and replenishment costs are not considered, the problem reduces to the basic assortment optimization without costs which has been shown to be polynomially solvable (Talluri and van Ryzin [1]). However, as shown in the above proof, when the inventory and replenishment costs are present, the problem is NP-hard even when there is only one customer segment.

Motivated by the NP-hardness result, we formulate the problem as a mathematical program. We reformulate the problem as a second order cone program by first converting the objective function in (5.1) to minimization. We let $\bar{\rho} = \max_j \rho_j$ and write the first term in (5.1) objective function as follows:

$$\sum_{i \in \mathcal{M}} \lambda_i \bar{\rho} - \sum_{i \in \mathcal{M}} \lambda_i \frac{\bar{\rho} \nu_{i0}}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il}} - \sum_{i \in \mathcal{M}} \lambda_i \frac{\sum_{j \in \mathcal{N}} (\bar{\rho} - \rho_j) \nu_{ij}}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il}}. \quad (5.11)$$

The first term in (5.11) is constant and can be omitted from the formulation. Hence, the objective function can be written as the minimization of the second

and the third term in (5.11) and the second term in (5.1) as:

$$\begin{aligned} \min \sum_{i \in \mathcal{M}} \lambda_i \frac{\bar{\rho} \nu_{i0}}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l} &+ \sum_{i \in \mathcal{M}} \lambda_i \frac{\sum_{j \in \mathcal{N}} (\bar{\rho} - \rho_j) \nu_{ij} x_j}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l} \\ &+ \sum_{k \in \mathcal{K}} \sqrt{2\kappa_k \sum_{j \in \mathcal{N}_k} \eta_j \sum_{i \in \mathcal{M}} \frac{\lambda_i \nu_{ij} x_j}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l}}. \end{aligned} \quad (5.12)$$

Next, we use the following change of variables as we defined in Chapter 3,

$$w_i = \nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l, \quad \forall i \in \mathcal{M}, \quad (5.13)$$

$$y_i = \frac{1}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l}, \quad \forall i \in \mathcal{M}, \quad (5.14)$$

$$z_{ij} = \frac{x_j}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l}, \quad \forall i \in \mathcal{M}, \forall j \in \mathcal{N}, \quad (5.15)$$

so that we can add the following conic quadratic constraints to the model:

$$w_i y_i \geq 1, \quad \forall i \in \mathcal{M}, \quad (5.16)$$

$$z_{ij} w_i \geq x_j^2, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}. \quad (5.17)$$

Furthermore, we define the following change of variables:

$$\hat{w}_i = \sqrt{\nu_{i0}} + \sum_{l \in \mathcal{N}} \sqrt{\nu_{il}} x_l, \quad \forall i \in \mathcal{M}. \quad (5.18)$$

Then the following inequality holds,

$$w_i = \hat{w}_i^2 - 2 \sum_{j, l \in \mathcal{N}: j \neq l} \sqrt{\nu_{ij} \nu_{il}} x_j x_l - 2 \sum_{l \in \mathcal{N}} \sqrt{\nu_{i0} \nu_{il}} x_l = q_i^2, \quad \forall i \in \mathcal{M}. \quad (5.19)$$

If we now denote $o_{jl} = x_j x_l$, then we can linearize the product of two binary

variables in (5.19) by adding the following inequalities:

$$o_{jl} \leq x_j, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.20)$$

$$o_{jl} \leq x_l, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.21)$$

$$o_{jl} \geq x_j + x_l - 1, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.22)$$

$$o_{jl} \geq 0, \quad \forall j, l \in \mathcal{N} : j < l. \quad (5.23)$$

In equation in (5.19) we defined variable q_i to be able to write the following conic quadratic constraint:

$$\hat{w}_i^2 \geq q_i^2 + 2 \sum_{j,l \in \mathcal{N}: j < l} \sqrt{\nu_{ij}\nu_{il}} o_{jl}^2 + 2 \sum_{l \in \mathcal{N}} \sqrt{\nu_{i0}\nu_{il}} x_l^2, \quad \forall i \in \mathcal{M}, \quad (5.24)$$

Finally, we replace $\sqrt{\frac{x_j}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l}}$ with t_{ij} :

$$t_{ij} = \sqrt{\frac{x_j}{\nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l}} = \sqrt{\frac{x_j}{w_i}}, \quad \forall i \in \mathcal{M}, \forall j \in \mathcal{N}. \quad (5.25)$$

Noting that x_j s are binary, we can write the above equality as:

$$t_{ij} = \sqrt{\frac{x_j}{w_i}} = \sqrt{\frac{x_j^2}{w_i}} = \frac{x_j}{\sqrt{w_i}} = \frac{x_j}{q_i}, \quad \forall i \in \mathcal{M}, \forall j \in \mathcal{N}. \quad (5.26)$$

Then, we can write this as a conic constrain as,

$$t_{ij} q_i \geq x_j^2, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}, \quad (5.27)$$

since the objective function involves minimizing t_{ij} 's. Using the above mentioned change of variables we can formulate the problem as second order cone program as follows:

$$\min \sum_{i \in \mathcal{M}} \lambda_i \nu_{i0} \bar{\rho} y_i + \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{N}} \lambda_i (\bar{\rho} - \rho_j) \nu_{ij} z_{ij} + \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_k} \sum_{i \in \mathcal{M}} \sqrt{2\kappa_k \eta_j \lambda_i \nu_{ij}} t_{ij} \quad (5.28)$$

$$\text{s.t. } w_i = \nu_{i0} + \sum_{l \in \mathcal{N}} \nu_{il} x_l, \quad \forall i \in \mathcal{M}, \quad (5.29)$$

$$\hat{w}_i = \sqrt{\nu_{i0}} + \sum_{l \in \mathcal{N}} \sqrt{\nu_{il}} x_l, \quad \forall i \in \mathcal{M}, \quad (5.30)$$

$$w_i y_i \geq 1, \quad \forall i \in \mathcal{M}, \quad (5.31)$$

$$z_{ij} w_i \geq x_j^2, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}, \quad (5.32)$$

$$t_{ij} q_i \geq x_j^2, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}, \quad (5.33)$$

$$(\text{AREP}) \quad \hat{w}_i^2 \geq q_i^2 + 2 \sum_{j, l \in \mathcal{N}: j < l} \sqrt{\nu_{ij} \nu_{il}} o_{jl}^2 + 2 \sum_{l \in \mathcal{N}} \sqrt{\nu_{i0} \nu_{il}} x_l^2, \quad \forall i \in \mathcal{M}, \quad (5.34)$$

$$o_{jl} \leq x_j, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.35)$$

$$o_{jl} \leq x_l, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.36)$$

$$o_{jl} \geq x_j + x_l - 1, \quad \forall j, l \in \mathcal{N} : j < l, \quad (5.37)$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{N}, \quad (5.38)$$

$$w_i, \hat{w}_i, q_i, y_i \geq 0, \quad \forall i \in \mathcal{M}, \quad (5.39)$$

$$z_{ij}, t_{ij} \geq 0, \quad \forall i \in \mathcal{M}, j \in \mathcal{N}, \quad (5.40)$$

$$o_{jl} \geq 0, \quad \forall j, l \in \mathcal{N} : j < l. \quad (5.41)$$

5.2 Numerical Study

In this section we generate random instances for the problem of assortment planning with replenishment costs to test the performance of the proposed formulation. We consider the case when there is only one customer segment with 50 products ($\mathcal{N} = 50$). We set the value of the no-purchase attraction factor, ν_0 , to one of the values in the set $\{0.5, 1.0, 1.5\}$ and generate attraction factors, ν_j s, from uniform distribution on interval $[0, 1]$. The unit revenues of each product j is generated from a uniform distribution on $[0, 400]$. Inventory holding costs, η_j s, are dependent on the revenues of the products and we control this dependency by a multiplier ι . We pick ι from the set $\{0.025, 0.050, 0.075\}$ and we set

the inventory holding cost for product j as $\eta_j = \rho_j \iota$. The fixed replenishment cost for supplier k is drawn from a uniform distribution on $[0, 2000]$. We set the arrival rate values to one of the values in the set $\{50, 150, 250, 500, 1000, 2000\}$. We generate five instances for each combination of ν_0 , ι , and λ resulting in a total of 270 instances. Instances are solved with the proposed conic formulation with and without McCormick inequalities. McCormick inequalities are added for the bilinear term z as in Chapter 3 (inequalities (3.42-3.45)). The computer and solver settings are the same as in Chapter 4.

We report the average number of optimal assortment (assort), solution times, nodes explored, and the end gap (egap) of the solver over five instances in Tables 5.1, 5.2, and 5.3. It should be noted that the reported end gaps (egap) in these tables are computed as $|Z_{blb} - Z_{iso}| / |Z_{iso}|$ where Z_{blb} and Z_{iso} are the MIP objective bound and incumbent solution objective, respectively, found by the solver at termination. The tables also report the number of instances that are solved to optimality (#) by the solver within the time limit of 1800 seconds. CONIC denotes the proposed formulation for the problem and CONIC+ denotes the same formulation when the McCormick inequalities are added to the formulation. Each table corresponds to a single value of the no-purchase attraction factor (ν_0), while the values of the ι and λ change.

Results indicate that the conic formulation when accompanied by the McCormick estimators can solve most of the instances, 189 out of 270 overall. We see that when the value of no-purchase parameter or the demand rate increases the number of products in the optimal assortment also increases and the problem gets harder to solve. The relationship of the revenue and inventory costs indicates that when the inventory holding costs are relatively low compared to the revenue generated by a product, instances become easier to solve. This could be due to the fact that in such a setting there are some products that dominate other candidate products to be put in the assortment in terms of both revenue and costs, hence, reducing the search effort of the solver.

In Table 5.1 when the value of the no-purchase attraction is the smallest, we see that the CONIC formulation can solve only 37 instances, out of 90, and that the average end gap is quite high (9.8%) when compared to that of CONIC+

Table 5.1: Results for assortment with replenishment costs: $\nu_0 = 0.5$

ι	λ	assort	CONIC			CONIC+		
			time\#	nodes	egap	time\#	nodes	egap
0.075	50	2.2	-	133762	24.8%	403.5/4	107959	2.7%
	150	2.6	-	121650	21.8%	407.9/5	84567	0.0%
	250	3.0	-	131696	17.5%	339.7/5	65493	0.0%
	500	3.6	864.5/1	132789	9.1%	121.8/5	21069	0.0%
	1000	4.0	357.3/2	123124	4.2%	56.8/5	9074	0.0%
	2000	4.8	467.2/5	66773	0.0%	44.5/5	4259	0.0%
0.050	50	2.4	-	136403	24.4%	314.3/4	117318	1.5%
	150	3.0	-	148631	18.3%	37.3/4	76141	0.6%
	250	3.6	-	132824	12.9%	167.1/5	40682	0.0%
	500	4.0		118527	6.3%	70.4/5	12917	0.0%
	1000	4.4	692.2/5	86854	0.0%	48.0/5	6169	0.0%
	2000	5.2	276.3/5	32594	0.0%	40.1/5	3776	0.0%
0.025	50	2.6	-	126593	21.9%	176.1/3	80580	2.4%
	150	3.6	1225.1/2	141233	9.8%	135.3/5	23086	0.0%
	250	4.0	535.9/2	116356	6.2%	73.1/5	13359	0.0%
	500	4.4	835.4/5	90475	0.0%	37.7/5	6396	0.0%
	1000	5.2	272.9/5	36047	0.0%	38.1/5	4243	0.0%
	2000	5.8	159.5/5	19589	0.0%	30.9/5	2638	0.0%
Average		3.8	568.6/37	105329	9.8%	147.5/85	37763	0.4%

formulation (0.4%). When the no-purchase attraction factor increases to 1, in Table 5.2, we see that the problem gets harder and CONIC formulation can only solve 13 instances to optimality and the end gap difference between the CONIC and CONIC+ increases from 9.4 % to 12.2 %. When the no-purchase attraction factor further increases to 1.5, in Table 5.3, we see similar results and now CONIC formulation can only solve 4 instances to optimality, while CONIC+ solves 42 instances with an average end gap of 6.5% as compared to end gap of CONIC formulation (18.9%). Individual results for the averaged instances in Tables 5.1, 5.2, and 5.3 are provided in Appendix E.

Table 5.2: Results for assortment with replenishment costs: $\nu_0 = 1.0$

ι	λ	assort	CONIC			CONIC+		
			time\#	nodes	egap	time\#	nodes	egap
0.075	50	2.6	-	93590	29.9%	846.3/1	208293	11.9%
	150	4.0	-	84209	27.2%	720.3/1	223585	9.4%
	250	4.0	-	85402	23.9%	720.3/3	223585	9.4%
	500	5.2	-	86828	18.8%	551.8/4	134566	2.5%
	1000	6.4	-	89816	13.2%	177.2/4	58181	1.9%
	2000	6.4	768.8/1	119413	5.2%	177.2/4	58181	1.9%
0.050	50	3.2	-	86701	29.1%	1041.8/3	188532	6.6%
	150	4.6	-	87020	23.2%	600.1/2	228132	6.4%
	250	5.2	-	82706	21.3%	573.3/3	174748	2.8%
	500	5.6	-	100823	15.6%	222.4/4	90031	2.5%
	1000	6.8	-	95424	8.9%	100.5/4	51090	1.5%
	2000	7.6	1100.2/4	91441	2.1%	229.6/5	42373	0.0%
0.025	50	4.0	-	89089	26.2%	790.9/2	234662	6.6%
	150	5.2	-	92559	19.1%	548.4/4	122224	2.2%
	250	5.6	-	107031	15.6%	219.1/4	58984	2.7%
	500	6.8	-	126838	8.0%	94.6/4	50165	1.4%
	1000	7.6	768.3/4	68134	2.1%	232.4/5	43940	0.0%
	2000	8.0	245.6/4	40970	1.2%	102.4/5	11510	0.0%
Average		5.5	720.7/13	90444	16.1%	441.6/62	122377	3.9%

Table 5.3: Results for assortment with replenishment costs: $\nu_0 = 1.5$

ι	λ	assort	CONIC			CONIC+		
			time\#	nodes	egap	time\#	nodes	egap
0.075	50	3.2	-	73805	31.9%	-	200744	15.4%
	150	4.2	-	69814	28.2%	-	219370	13.4%
	250	5.2	-	71824	25.9%	1142.1/1	164009	10.2%
	500	6.2	-	66711	21.6%	372.7/1	198388	7.1%
	1000	7.4	-	81485	14.9%	500.4/4	92398	2.6%
	2000	8.2	-	91501	9.0%	205.0/4	37963	1.9%
0.050	50	3.6	-	64105	31.1%	-	228344	15.7%
	150	4.8	-	66012	26.0%	1333.7/1	211347	10.5%
	250	5.6	-	75566	23.4%	525.6/2	110907	6.3%
	500	7.0	-	89220	17.1%	841.7/3	145552	3.9%
	1000	8.0	-	88092	11.5%	276.1/4	52690	2.1%
	2000	9.2	844.6/1	81522	6.5%	129.2/4	59210	1.2%
0.025	50	4.2	-	66905	28.8%	1604.2/1	194716	11.8%
	150	6.0	-	75011	21.6%	523.6/1	174238	7.3%
	250	7.0	-	75011	21.6%	636.2/3	142694	3.9%
	500	8.0	-	88534	12.4%	267.1/4	56438	2.0%
	1000	9.2	-	79395	6.9%	149.2/4	46971	1.4%
	2000	10.6	748.4/3	63120	2.2%	251.9/5	36772	0.0%
Average		6.5	796.5/4	75980	18.9%	583.9/42	131819	6.5%

Chapter 6

Conclusion

The assortment planning problem is of essential importance for many retailers. It has direct impact on the profitability and a retailer faces a large possible assortment to offer to its customers. The problem is not only important by itself in practice, but may also appear as a subproblem in other assortment planning problems or in some specific revenue management problems. While the assortment planning has been an active area of research, most studies do not consider the fixed costs of including the products in the assortment. In many settings the fixed costs are relevant and must be considered while planning the assortment. We have studied the assortment optimization problem incorporating fixed, inventory, and replenishment costs into the problem.

We first consider the assortment optimization problem with fixed costs under the Mixtures of Multinomials customer choice model and propose a conic quadratic mixed-integer formulation. This problem is known to be NP-hard and it is challenging to solve even for moderately sized problems. The limited literature on this problem is devoted to developing specialized algorithms and heuristics to obtain upper bounds and approximate solutions. In contrast, we study mathematical programming formulations of the problem to obtain exact solutions and analyze their effectiveness by conducting a comprehensive numerical study. We show that when the McCormick inequalities are added to the formulation, large

instances of the problem can be solved efficiently as opposed to the poor performance of the available MILP formulation. The performance of the formulation is observed to be robust to the attractiveness of the no-purchase (outside) option, the relative magnitude of the fixed costs, the number of classes, the number of products that a customer considers when making purchasing decisions and the level of capacity. In contrast, the traditional linear formulations for the problem perform poorly. The size of the instances we successfully solve in the numerical study can be considered large noting the fact that firms typically plan their assortments at the category or sub-category level. In addition, it will be very difficult to accurately estimate the parameters of a choice model with a large number of alternatives. Finally, the choice model we use in this thesis (MMNL) is a very general model as it can be used to approximate any random utility model to a desired level of accuracy. Therefore, we believe that the conic programming approach that we follow in this thesis is a viable option to find profit-maximizing assortments in practice.

The analysis with fixed costs can be extended in a number of ways. First, one can consider capacity constraints that are more general than the simple cardinality constraints. The study by Şen et al. [20] for the case of no fixed costs shows that the conic formulation remains to be effective for generalized capacity constraints. We expect the same to be true for the case of fixed costs. Second, other valid inequalities similar to lifting inequalities for the knapsack problem can be investigated for further tightening the feasible region.

We also consider the inventory and replenishment costs in the assortment optimization problem. We capture such costs with the Economic Order Quantity model and present a conic quadratic formulation for this problem when the customers' choice behavior is modeled by the MMNL model. We provide a proof for the NP-hardness of the problem and conduct numerical studies that reveal the effectiveness of the McCormick valid inequalities in tightening the formulation and hence reducing the solution times of the problem instances. As a future work for this part of the study it could be helpful to analyze the structure of the optimal solution and attempt to find upper bounds on the number of products that should be included in the optimal assortment. This upper bound would help strengthening the valid McCormick inequalities. This is a promising approach

as strong McCormick inequalities has been shown to be very effective in solving large instances of the problem [20].



Bibliography

- [1] K. Talluri and G. van Ryzin, “Revenue management under a general discrete choice model of consumer behavior,” *Management Science*, vol. 50, pp. 15–33, 2004.
- [2] A. Désir and V. Goyal, “Near-optimal algorithms for capacity constrained assortment optimization,” tech. rep., Columbia University, 2014.
- [3] P. Rusmevichientong, Z.-J. M. Shen, and D. B. Shmoys, “Dynamic assortment optimization with a multinomial logit choice model and capacity constraint,” *Operations Research*, vol. 58, pp. 1666–1680, 2010.
- [4] J. Davis, G. Gallego, and H. Topaloglu, “Assortment planning under the multinomial logit model with totally unimodular constraint structures,” tech. rep., Cornell University, 2013.
- [5] I. Méndez-Díaz, J. J. Miranda-Bront, G. Vulcano, and P. Zabala, “A branch-and-cut algorithm for the latent-class logit assortment problem,” *Discrete Applied Mathematics*, vol. 164, pp. 246–263, 2014.
- [6] Boston Retail Partners, *2018 Integrated Planning Survey*. 2018. Accessed: 2018-03-08.
- [7] N. Ashford and M. Benchemam, *Passengers’ choice of airport: an application of the multinomial logit model*. Loughborough University of Technology Department of Transport Technology, 1987.
- [8] P. M. Guadagni and J. D. Little, “A logit model of brand choice calibrated on scanner data,” *Marketing science*, vol. 2, no. 3, pp. 203–238, 1983.

- [9] H. C. Williams, “On the formation of travel demand models and economic evaluation measures of user benefit,” *Environment and planning A*, vol. 9, no. 3, pp. 285–344, 1977.
- [10] D. McFadden and K. Train, “Mixed mnl models for discrete response,” *Journal of Applied Econometrics*, vol. 15, pp. 447–470, 2000.
- [11] P. Rusmevichientong, D. Shmoys, C. Tong, and H. Topaloglu, “Assortment optimization under the multinomial logit model with random choice parameters,” *Production and Operations Management*, vol. 23, pp. 2023–2039, 2014.
- [12] J. J. M. Bront, I. Méndez-Díaz, and G. Vulcano, “A column generation algorithm for choice-based network revenue management,” *Operations Research*, vol. 57, pp. 769–784, 2009.
- [13] R. Wang, “Assortment management under the generalized attraction model with a capacity constraint,” *Journal of Revenue and Pricing Management*, vol. 12, no. 3, pp. 254–270, 2013.
- [14] J. M. Davis, G. Gallego, and H. Topaloglu, “Assortment optimization under variants of the nested logit model,” *Operations Research*, vol. 62, pp. 250–273, 2014.
- [15] G. v. Ryzin and S. Mahajan, “On the relationship between inventory costs and variety benefits in retail assortments,” *Management Science*, vol. 45, no. 11, pp. 1496–1509, 1999.
- [16] S. Kunnumkal and V. Martínez de Albéniz, “Tractable models and algorithms for assortment planning with product costs,” *Operations Research, Articles in Advance*, 2019.
- [17] F. Caro and V. Martínez-de Albéniz, “Fast fashion: business model overview and research opportunities,” in *Retail supply chain management*, pp. 237–264, Springer, 2015.
- [18] J. Feldman and H. Topaloglu, “Bounding optimal expected revenues for assortment optimization under mixtures of multinomial logits,” *Production and Operations Management*, vol. 24, pp. 1598–1620, 2015.

- [19] S. Kunnumkal, P. Rusmevichientong, and H. Topaloglu, “Assortment optimization under multinomial logit model with product costs,” tech. rep., Cornell University, 2010.
- [20] A. Şen, A. Atamtürk, and P. Kaminsky, “A conic integer optimization approach to the constrained assortment problem under the mixed multinomial logit model,” *Operations Research*, vol. 66, no. 4, pp. 994–1003, 2018.
- [21] K. X. Chiong and M. Shum, “Random projection estimation of discrete-choice models with large choice sets,” *Management Science*, no. forthcoming, 2018.
- [22] M. A. Hariga, A. Al-Ahmari, and A.-R. A. Mohamed, “A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions,” *European Journal of Operational Research*, vol. 181, no. 1, pp. 239–251, 2007.
- [23] A. G. Kök, M. L. Fisher, and R. Vaidyanathan, “Assortment planning: Review of literature and industry practice,” in *Retail Supply Chain Management* (N. Agrawal and S. A. Smith, eds.), vol. 122 of *International Series in Operations Research & Management Science*, pp. 99–153, Springer US, 2009.
- [24] M. Karampatsa, E. Grigoroudis, and N. F. Matsatsinis, “Retail category management: A review on assortment and shelf-space planning models,” in *Operational Research in Business and Economics: 4th International Symposium and 26th National Conference on Operational Research, Chania, Greece, June 2015* (E. Grigoroudis and M. Doumpos, eds.), pp. 35–67, Cham: Springer International Publishing, 2017.
- [25] G. Gallego, R. Ratliff, and S. Shebalov, “A general attraction model and an efficient formulation for the network revenue management problem,” tech. rep., Columbia University, 2011.
- [26] R. Wang, “Assortment management under the generalized attraction model with a capacity constraint,” *Journal of Revenue and Pricing Management*, vol. 12, no. 3, p. 254?270, 2012.
- [27] V. Farias, S. Jagabathula, and D. Shah, “Assortment optimization under general choice,” *arXiv preprint arXiv:1108.3596*, 2011.

- [28] A. G. Kök and Y. Xu, “Optimal and competitive assortments with endogenous pricing under hierarchical consumer choice models,” *Management Science*, vol. 57, no. 9, pp. 1546–1563, 2011.
- [29] P. Rusmevichientong, Z.-J. M. Shen, and D. B. Shmoys, “Algorithms for capacitated sum-of-ratios optimization,” *Operations Research Letters*, vol. 37, no. 4, pp. 230–238, 2009.
- [30] G. Gallego and H. Topaloglu, “Constrained assortment optimization for the nested logit model,” *Management Science*, vol. 60, pp. 2583–2601, 2014.
- [31] J. B. Feldman and H. Topaloglu, “Capacity constraints across nests in assortment optimization under the nested logit model,” *Operations Research*, vol. 63, pp. 812–822, 2015.
- [32] S. Kunnumkal, “On upper bounds for assortment optimization under the mixture of multinomial logit models,” *Operations Research Letters*, vol. 43, pp. 189–194, 2015.
- [33] A. Atamtürk and A. Gómez, “Maximizing a class of utility functions over the vertices of a polytope,” *Operations Research*, vol. 65, pp. 433–445, 2017.
- [34] C. Schön, “Optimal dynamic price selection under attraction choice models,” *European Journal of Operational Research*, vol. 205, pp. 650–660, 2010.
- [35] T. L. Urban, “An inventory-theoretic approach to product assortment and shelf-space allocation,” *Journal of Retailing*, vol. 74, no. 1, p. 15, 1998.
- [36] V. Gaur and D. Honhon, “Assortment planning and inventory decisions under a locational choice model,” *Management Science*, vol. 52, no. 10, pp. 1528–1543, 2006.
- [37] D. Honhon, V. Gaur, and S. Seshadri, “Assortment planning and inventory decisions under stockout-based substitution,” *Operations research*, vol. 58, no. 5, pp. 1364–1379, 2010.
- [38] G. P. Cachon, C. Terwiesch, and Y. Xu, “Retail assortment planning in the presence of consumer search,” *Manufacturing & Service Operations Management*, vol. 7, no. 4, pp. 330–346, 2005.

- [39] K. Rajaram, “Assortment planning in fashion retailing: methodology, application and analysis,” *European Journal of Operational Research*, vol. 129, no. 1, pp. 186–208, 2001.
- [40] S. A. Smith and N. Agrawal, “Management of multi-item retail inventory systems with demand substitution,” *Operations Research*, vol. 48, no. 1, pp. 50–64, 2000.
- [41] A. G. Kök and M. L. Fisher, “Demand estimation and assortment optimization under substitution: Methodology and application,” *Operations Research*, vol. 55, no. 6, pp. 1001–1021, 2007.
- [42] T.-H. Wu, “A note on a global approach for general 0–1 fractional programming,” *European Journal of Operational Research*, vol. 101, pp. 220–223, 1997.
- [43] G. P. McCormick, “Computability of global solutions to factorable nonconvex programs: Part i?convex underestimating problems,” *Mathematical Programming*, vol. 10, pp. 147–175, 1976.
- [44] L. Gurobi Optimization, “Gurobi optimizer reference manual,” 2018.
- [45] F. W. Harris, “How many parts to make at once,” *Operations research*, vol. 38, no. 6, pp. 947–950, 1990.
- [46] D. S. Johnson and M. R. Garey, *Computers and intractability: A guide to the theory of NP-completeness*. WH Freeman, 1979.

Appendix A

Tables for Assortment Optimization with Fixed Costs under MNL Model

Table A.1: Results for problems with 300 products, $\phi : 0.75, \gamma : 1$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	169	0.83%	52.63		0.08%	3.56		0.08%	3.57		5.97%	15.7		0.05%	0.66		0.05%	1.28	
		0.00%	235953		0.00%	19406		0.00%	7980		0.00%	4622		0.00%	738		0.00%	10	
2	171	0.92%	83.89		0.09%	12.06		0.09%	3.93		5.86%	9.19		0.04%	0.61		0.04%	1.39	
		0.00%	350064		0.00%	55199		0.00%	1124		0.00%	1104		0.00%	977		0.00%	1142	
3	178	1.04%	330.73		0.09%	22.84		0.09%	16.94		7.27%	10.35		0.05%	1.66		0.05%	2.43	
		0.00%	1478752		0.00%	135769		0.00%	55002		0.00%	1786		0.00%	3573		0.00%	2500	
4	171	0.95%	55.79		0.09%	10.93		0.09%	6.86		10.25%	5.8		0.04%	1.01		0.04%	0.96	
		0.00%	233874		0.00%	46633		0.00%	21906		0.00%	1090		0.00%	1101		0.00%	1190	
5	184	0.92%	100.86		0.10%	13.72		0.10%	11.89		12.52%	6.26		0.04%	0.76		0.04%	1.48	
		0.00%	453536		0.00%	59000		0.00%	38866		0.00%	1231		0.00%	171		0.00%	334	
6	178	0.99%	530.1		0.09%	9.16		0.09%	4.7		10.52%	9.32		0.04%	0.98		0.05%	1.05	
		0.00%	2134401		0.00%	57280		0.00%	14179		0.00%	2139		0.00%	1084		0.00%	1595	
7	164	1.01%	270.25		0.09%	18.25		0.09%	13.77		6.76%	9.55		0.04%	1.16		0.04%	2.06	
		0.00%	1076660		0.00%	109735		0.00%	46117		0.00%	1158		0.00%	1878		0.00%	1824	
8	175	1.01%	6.3		0.06%	1.96		0.06%	1.73		7.94%	3.92		0.02%	0.43		0.02%	0.95	
		0.00%	22389		0.00%	8836		0.00%	2849		0.00%	1090		0.00%	701		0.00%	1	
9	166	0.85%	2.29		0.06%	1.78		0.06%	2.01		6.46%	4.32		0.02%	0.47		0.02%	0.86	
		0.00%	1074		0.00%	8277		0.00%	3758		0.00%	1110		0.00%	1		0.00%	1	
10	161	1.01%	79.85		0.09%	13.89		0.09%	6.16		17.21%	6.89		0.04%	1.23		0.04%	1.06	
		0.00%	385492		0.00%	75694		0.00%	21807		0.00%	1247		0.00%	1660		0.00%	1196	
Average		0.95%	151.27		0.08%	10.86		0.08%	7.16		9.08%	8.13		0.04%	0.90		0.04%	1.35	
		0.00%	637220		0.00%	57582.9		0.00%	21358.8		0.00%	1657.7		0.00%	1188.4		0.00%	979.3	

Table A.2: Results for problems with 300 products, $\phi : 0.75, \gamma : 0.5$

instance	assort	MILP		MILP+		MILP++		CONIC		CONIC+		CONIC++	
		rgap	time	rgap	time	rgap	time	rgap	time	rgap	time	rgap	time
1	236	1.05%	0.49	0.00%	0.11	0.00%	0.17	21.17%	1.03	0.00%	0.14	0.00%	0.23
		0.00%	1	0.00%	1	0.00%	1	0.00%	1099	0.00%	1	0.00%	1
2	251	0.94%	0.26	0.00%	0.12	0.00%	0.23	7.81%	1.25	0.00%	0.14	0.00%	0.26
		0.00%	1	0.00%	1	0.00%	1	0.00%	1065	0.00%	1	0.00%	1
3	253	0.97%	0.47	0.00%	0.14	0.00%	0.44	16.46%	1.92	0.00%	0.14	0.00%	0.36
		0.00%	1	0.00%	1	0.00%	1	0.00%	1066	0.00%	1	0.00%	1
4	247	0.95%	1	0.00%	0.14	0.00%	0.39	11.12%	1.94	0.00%	0.14	0.00%	0.3
		0.00%	1	0.00%	1	0.00%	1	0.00%	1098	0.00%	1	0.00%	1
5	239	0.90%	0.53	0.00%	0.11	0.00%	0.33	8.43%	1.13	0.00%	0.19	0.00%	0.41
		0.00%	1	0.00%	1	0.00%	1	0.00%	1133	0.00%	1	0.00%	1
6	246	0.92%	0.28	0.00%	0.11	0.00%	0.26	7.14%	1.45	0.00%	0.16	0.00%	0.3
		0.00%	1	0.00%	1	0.00%	1	0.00%	1113	0.00%	1	0.00%	1
7	240	0.94%	0.3	0.00%	0.15	0.00%	0.41	11.61%	2.42	0.00%	0.16	0.00%	0.28
		0.00%	1	0.00%	1	0.00%	1	0.00%	1127	0.00%	1	0.00%	1
8	243	0.93%	0.3	0.00%	0.15	0.00%	0.45	69.86%	3.31	0.00%	0.17	0.00%	0.27
		0.00%	1	0.00%	1	0.00%	1	0.00%	1163	0.00%	1	0.00%	1
9	243	0.99%	0.32	0.00%	0.13	0.00%	0.32	21.83%	5.08	0.00%	0.14	0.00%	0.37
		0.00%	1	0.00%	1	0.00%	1	0.00%	1160	0.00%	1	0.00%	1
10	250	0.94%	0.34	0.00%	0.16	0.00%	0.33	11.85%	1.27	0.00%	0.14	0.00%	0.31
		0.00%	1	0.00%	1	0.00%	1	0.00%	1121	0.00%	1	0.00%	1
Average		0.95%	0.429	0.00%	0.132	0.00%	0.333	18.73%	2.08	0.00%	0.152	0.00%	0.309
		0.00%	1	0.00%	1	0.00%	1	0.00%	1114.5	0.00%	1	0.00%	1

Table A.3: Results for problems with 300 products, $\phi : 0.75, \gamma : 0.25$

instance	assort	MILP		MILP+		MILP++		CONIC		CONIC+		CONIC++	
		rgap	time	rgap	time	rgap	time	rgap	time	rgap	time	rgap	time
1	252	0.52%	0.16	0.00%	0.08	0.00%	0.16	39.82%	1.31	0.00%	0.09	0.00%	0.14
2	256	0.51%	0.27	0.00%	0.07	0.00%	0.14	31.61%	1.08	0.00%	0.14	0.00%	0.22
3	259	0.52%	0.23	0.00%	0.08	0.00%	0.17	25.38%	1.11	0.00%	0.09	0.00%	0.23
4	255	0.49%	0.22	0.00%	0.07	0.00%	0.15	29.25%	2.75	0.00%	0.13	0.00%	0.31
5	257	0.50%	0.3	0.00%	0.09	0.00%	0.15	70.89%	1.64	0.00%	0.13	0.00%	0.35
6	252	0.48%	0.16	0.00%	0.08	0.00%	0.16	58.55%	1.08	0.00%	0.09	0.00%	0.28
7	256	0.47%	0.27	0.00%	0.08	0.00%	0.16	65.72%	1.14	0.00%	0.09	0.00%	0.28
8	251	0.49%	0.16	0.00%	0.09	0.00%	0.14	29.09%	1.22	0.00%	0.14	0.00%	0.33
9	261	0.50%	0.2	0.00%	0.08	0.00%	0.17	73.85%	2.55	0.00%	0.14	0.00%	0.25
10	260	0.57%	0.14	0.00%	0.08	0.00%	0.15	18.49%	1.21	0.00%	0.09	0.00%	0.24
Average		0.50%	0.21	0.00%	0.08	0.00%	0.16	44.27%	1.51	0.00%	0.113	0.00%	0.263

Table A.4: Results for problems with 300 products, $\phi : 0.5, \gamma : 1$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	113	2.89%	-	-	0.76%	3.37	3.37	0.76%	7.41	7.41	7.66%	23.95	23.95	0.22%	2.27	2.27	0.22%	2.85	2.85
		0.81%	6158150	6158150	0.00%	2694	2694	0.00%	2153	2153	0.00%	20072	20072	0.00%	1082	1082	0.00%	1049	1049
2	118	3.33%	-	-	0.85%	34.09	34.09	0.85%	33.67	33.67	9.98%	-	-	0.38%	39.15	39.15	0.38%	6.03	6.03
		0.64%	3630382	3630382	0.00%	98392	98392	0.00%	55355	55355	0.18%	1548368	1548368	0.00%	192158	192158	0.00%	8933	8933
3	107	3.56%	-	-	0.94%	17.71	17.71	0.94%	23.78	23.78	13.81%	-	-	0.38%	4.28	4.28	0.38%	5.62	5.62
		1.55%	2688047	2688047	0.00%	74775	74775	0.00%	43773	43773	0.02%	1179657	1179657	0.00%	3043	3043	0.00%	4570	4570
4	126	3.21%	-	-	0.81%	7.24	7.24	0.81%	8.49	8.49	42.19%	753.14	753.14	0.34%	3.16	3.16	0.34%	5.74	5.74
		1.25%	2200729	2200729	0.00%	35102	35102	0.00%	10905	10905	0.00%	454029	454029	0.00%	1122	1122	0.00%	1091	1091
5	118	3.03%	-	-	0.66%	9.41	9.41	0.66%	6.4	6.4	33.76%	711.43	711.43	0.28%	2.89	2.89	0.28%	3.85	3.85
		1.18%	2054866	2054866	0.00%	44417	44417	0.00%	6474	6474	0.00%	294029	294029	0.00%	1133	1133	0.00%	1102	1102
6	110	3.44%	-	-	0.70%	11.62	11.62	0.70%	31.07	31.07	14.26%	-	-	0.26%	4.17	4.17	0.26%	6.08	6.08
		1.42%	1961818	1961818	0.00%	31669	31669	0.00%	57606	57606	0.11%	1428538	1428538	0.00%	1095	1095	0.00%	1110	1110
7	105	3.28%	-	-	0.82%	13.01	13.01	0.82%	41.01	41.01	9.65%	-	-	0.32%	3.81	3.81	0.32%	4.83	4.83
		0.47%	1720027	1720027	0.00%	55792	55792	0.00%	78452	78452	0.02%	2205627	2205627	0.00%	1149	1149	0.00%	1089	1089
8	104	3.72%	-	-	1.14%	38.28	38.28	1.14%	269.39	269.39	14.61%	-	-	0.36%	4.2	4.2	0.36%	5.44	5.44
		1.67%	1457033	1457033	0.00%	109472	109472	0.00%	413117	413117	0.19%	2191466	2191466	0.00%	1120	1120	0.00%	1109	1109
9	107	2.71%	-	-	0.66%	4.53	4.53	0.66%	5.84	5.84	8.86%	-	-	0.29%	4.98	4.98	0.29%	3.63	3.63
		0.54%	852733	852733	0.00%	6688	6688	0.00%	5476	5476	0.21%	1633645	1633645	0.00%	17036	17036	0.00%	1914	1914
10	113	3.19%	-	-	0.70%	6.01	6.01	0.70%	9.15	9.15	11.18%	-	-	0.25%	2.08	2.08	0.25%	2.82	2.82
		0.53%	1593395	1593395	0.00%	33712	33712	0.00%	3497	3497	0.04%	1210575	1210575	0.00%	1095	1095	0.00%	1063	1063
Average		3.24%	-	-	0.81%	14.53	14.53	0.81%	43.62	43.62	16.60%	496.17	496.17	0.31%	7.10	7.10	0.31%	4.69	4.69
		1.01%	2431718	2431718	0.00%	49271.3	49271.3	0.00%	67680.8	67680.8	0.08%	1216601	1216601	0.00%	22003.3	22003.3	0.00%	2303	2303

Table A.5: Results for problems with 300 products, $\phi : 0.5, \gamma : 0.5$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	161	4.30%	-		0.57%	133.83		0.57%	8.92		55.60%	158.27		0.28%	7.77		0.28%	7.73	
		2.40%	1162732		0.00%	504042		0.00%	10883		0.00%	22477		0.00%	9589		0.00%	1098	
2	163	4.87%	-		0.75%	27.45		0.75%	44.86		14.68%	101.52		0.30%	4.04		0.30%	5.58	
		2.68%	5050165		0.00%	77104		0.00%	74169		0.00%	12721		0.00%	1094		0.00%	1076	
3	161	4.24%	-		0.57%	14.08		0.57%	20.54		9.97%	442.53		0.26%	4.53		0.26%	5.88	
		2.10%	5535566		0.00%	44928		0.00%	42363		0.00%	168395		0.00%	5067		0.00%	1082	
4	158	4.02%	-		0.64%	37.21		0.64%	32.01		27.44%	915.75		0.22%	2.81		0.22%	4.81	
		1.78%	3722735		0.00%	109579		0.00%	71992		0.00%	410423		0.00%	1090		0.00%	4120	
5	153	4.52%	-		0.74%	34.93		0.74%	99.8		42.87%	606.04		0.27%	5.05		0.27%	5.58	
		2.51%	3035444		0.00%	102063		0.00%	175007		0.00%	300090		0.00%	1086		0.00%	3269	
6	163	4.12%	-		0.59%	14.2		0.59%	7.41		37.77%	54.18		0.25%	4.39		0.25%	5.52	
		2.04%	2221875		0.00%	42564		0.00%	9492		0.00%	8358		0.00%	1095		0.00%	3119	
7	160	4.22%	-		0.59%	12.53		0.59%	9.04		23.02%	188.48		0.26%	6.74		0.26%	6.2	
		2.20%	2085824		0.00%	40172		0.00%	9085		0.00%	59934		0.00%	11790		0.00%	2124	
8	162	4.19%	-		0.56%	-		0.56%	-		60.63%	-		0.21%	4.08		0.21%	5.36	
		2.32%	2390981		0.10%	12347096		0.10%	4046674		0.07%	730273		0.00%	1098		0.00%	1110	
9	162	4.08%	-		0.68%	22.65		0.68%	40.52		38.35%	454.45		0.31%	4.43		0.31%	7.7	
		2.25%	1595779		0.00%	30683		0.00%	47968		0.00%	133939		0.00%	1058		0.00%	1159	
10	161	4.08%	-		0.66%	89.97		0.66%	132.81		29.75%	211.53		0.27%	11.63		0.27%	5.85	
		2.17%	1295342		0.00%	194782		0.00%	235380		0.00%	56205		0.00%	1118		0.00%	1085	
Average		4.26%	-		0.63%	38.69		0.63%	43.99		34.01%	348.08		0.26%	5.55		0.26%	6.021	
		2.25%	2809644		0.01%	1349301		0.01%	472301		0.01%	190282		0.00%	3408.5		0.00%	1924.2	

Table A.6: Results for problems with 300 products, $\phi : 0.5, \gamma : 0.25$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	194	2.89%	-	-	0.03%	0.38	0.03%	0.7	0.03%	0.7	81.75%	15.03	0.02%	0.23	0.01%	0.31	0.01%	0.31	0.01%
		1.05%	1701128		0.00%	1		0.00%	1		0.00%	3027		0.00%	1	0.00%	1		
2	187	2.83%	-	-	0.04%	0.41	0.04%	0.68	0.04%	0.68	61.79%	36.55	0.00%	0.23	0.01%	0.3	0.01%	0.3	0.01%
		1.12%	1452641		0.00%	1		0.00%	1		0.00%	7215		0.00%	1	0.00%	1		
3	193	2.68%	-	-	0.02%	0.31	0.02%	0.48	0.02%	0.48	51.45%	12.63	0.01%	0.23	0.01%	0.27	0.01%	0.27	0.01%
		0.70%	6072022		0.00%	1		0.00%	1		0.00%	1106		0.00%	1	0.00%	1		
4	196	3.07%	-	-	0.05%	0.41	0.05%	0.69	0.05%	0.69	57.40%	21.68	0.01%	0.23	0.01%	0.27	0.01%	0.27	0.01%
		1.05%	4729364		0.00%	1		0.00%	1		0.00%	1124		0.00%	1	0.00%	1		
5	205	2.89%	-	-	0.05%	0.48	0.05%	0.8	0.05%	0.8	32.13%	25.8	0.02%	0.31	0.02%	0.33	0.02%	0.33	0.02%
		1.21%	3759296		0.00%	1		0.00%	1		0.00%	1360		0.00%	1	0.00%	1		
6	200	3.03%	-	-	0.03%	0.36	0.03%	0.53	0.03%	0.53	60.60%	31.88	0.01%	0.22	0.01%	0.25	0.01%	0.25	0.01%
		1.27%	2764961		0.00%	1		0.00%	1		0.00%	2058		0.00%	1	0.00%	1		
7	198	3.20%	-	-	0.03%	0.3	0.03%	0.53	0.03%	0.53	34.61%	20.73	0.01%	0.22	0.01%	0.38	0.01%	0.38	0.01%
		1.27%	2689284		0.00%	1		0.00%	1		0.00%	3068		0.00%	1	0.00%	1		
8	199	3.19%	-	-	0.07%	0.48	0.07%	0.88	0.07%	0.88	20.75%	17.44	0.01%	0.22	0.01%	0.31	0.01%	0.31	0.01%
		1.24%	2472954		0.00%	1		0.00%	1		0.00%	1364		0.00%	1	0.00%	1		
9	190	3.07%	-	-	0.06%	0.91	0.06%	1.8	0.06%	1.8	45.38%	23.04	0.01%	0.28	0.02%	0.31	0.02%	0.31	0.02%
		1.00%	2217433		0.00%	1927		0.00%	2146		0.00%	1509		0.00%	1	0.00%	1		
10	196	2.91%	-	-	0.05%	0.47	0.05%	0.8	0.05%	0.8	46.14%	971.39	0.01%	0.2	0.01%	0.28	0.01%	0.28	0.01%
		1.06%	2019364		0.00%	1		0.00%	1		0.00%	413261		0.00%	1	0.00%	1		
Average		2.98%	-	-	0.04%	0.45	0.04%	0.79	0.04%	0.79	49.20%	117.62	0.01%	0.24	0.01%	0.30	0.01%	0.30	0.01%
		1.10%	2987845		0.00%	193.6		0.00%	215.5		0.00%	43509.2		0.00%	1	0.00%	1		

Table A.7: Results for problems with 300 products, $\phi : 0.25, \gamma : 1$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	58	7.26%	-	-	3.49%	9.43	9.43	3.49%	28.63	28.63	14.50%	1057.69	1.33%	6.02	6.02	1.33%	7.03	7.03	7.03
		0.64%	930231	930231	0.00%	38994	38994	0.00%	49027	49027	0.00%	1744338	0.00%	2084	2084	0.00%	4361	4361	4361
2	67	8.87%	-	-	4.78%	695.86	695.86	4.78%	-	-	45.23%	-	2.03%	32.57	32.57	2.03%	24.73	24.73	24.73
		1.64%	1187779	1187779	0.00%	1874501	1874501	0.28%	2739795	2739795	0.36%	2263382	0.00%	1125	1125	0.00%	1177	1177	1177
3	66	7.15%	-	-	3.77%	51.52	51.52	3.77%	133.31	133.31	16.61%	-	1.79%	24.56	24.56	1.79%	25.3	25.3	25.3
		0.94%	1038956	1038956	0.00%	131653	131653	0.00%	223745	223745	0.76%	1388901	0.00%	1147	1147	0.00%	1126	1126	1126
4	62	8.35%	-	-	4.67%	-	-	4.67%	-	-	15.45%	-	2.10%	16.37	16.37	2.10%	23.16	23.16	23.16
		2.32%	659539	659539	0.09%	4462110	4462110	0.37%	2428324	2428324	1.47%	2057457	0.00%	5478	5478	0.00%	10084	10084	10084
5	70	7.26%	-	-	3.47%	764.39	764.39	3.47%	84.73	84.73	19.20%	-	1.58%	1.45	1.45	1.58%	2.08	2.08	2.08
		1.19%	990373	990373	0.00%	3070991	3070991	0.00%	96715	96715	1.89%	1013231	0.00%	2352	2352	0.00%	1600	1600	1600
6	64	8.97%	-	-	4.53%	435.64	435.64	4.53%	-	-	12.39%	-	1.59%	9.62	9.62	1.59%	8.58	8.58	8.58
		1.77%	1000014	1000014	0.00%	1219130	1219130	0.23%	2453016	2453016	0.20%	1872651	0.00%	1083	1083	0.00%	1114	1114	1114
7	74	9.36%	-	-	4.81%	807.45	807.45	4.81%	-	-	83.77%	-	1.61%	18.94	18.94	1.60%	15.55	15.55	15.55
		3.53%	835123	835123	0.00%	2260204	2260204	0.27%	1966218	1966218	0.23%	933081	0.00%	1087	1087	0.00%	1107	1107	1107
8	67	8.63%	-	-	4.22%	1099.93	1099.93	4.22%	-	-	15.77%	-	1.56%	21.03	21.03	1.56%	30.06	30.06	30.06
		1.79%	920943	920943	0.00%	3333422	3333422	0.15%	2268029	2268029	0.85%	1065732	0.00%	13942	13942	0.00%	1128	1128	1128
9	68	8.49%	-	-	4.09%	78.3	78.3	4.09%	912.7	912.7	10.67%	948.75	1.29%	10	10	1.29%	2.36	2.36	2.36
		1.51%	1055999	1055999	0.00%	181378	181378	0.00%	850958	850958	0.00%	554680	0.00%	1110	1110	0.00%	2344	2344	2344
10	63	8.13%	-	-	4.05%	23.67	23.67	4.05%	250.14	250.14	23.09%	-	1.74%	2.1	2.1	1.74%	3.14	3.14	3.14
		1.22%	929952	929952	0.00%	53374	53374	0.00%	262499	262499	0.44%	986529	0.00%	4690	4690	0.00%	2958	2958	2958
Average		8.25%	-	-	4.19%	440.69	440.69	4.19%	281.90	281.90	25.67%	1003.22	1.66%	14.27	14.27	1.66%	14.12	14.12	14.12
		1.65%	954891	954891	0.01%	1662576	1662576	0.13%	1333833	1333833	0.62%	1387998	0.00%	3409.8	3409.8	0.00%	2699.9	2699.9	2699.9

Table A.8: Results for problems with 300 products, $\phi : 0.25, \gamma : 0.5$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes	rgap	time	nodes
1	89	10.73%	-	-	5.08%	-	-	5.08%	-	-	21.54%	-	-	1.99%	34.71	-	1.99%	58.2	-
2	86	3.46%	4859779	-	0.43%	3820907	-	0.51%	1415820	-	0.79%	440329	-	0.00%	5740	-	0.00%	1159	-
		12.11%	-	-	5.49%	-	-	5.49%	-	-	16.86%	-	-	1.60%	18.15	-	1.60%	22.2	-
		5.39%	2205312	-	0.89%	3105764	-	0.87%	1557703	-	1.16%	507054	-	0.00%	5883	-	0.00%	1107	-
3	97	11.14%	-	-	5.49%	-	-	5.49%	-	-	25.94%	-	-	2.08%	80.73	-	2.08%	954.83	-
		3.48%	1379445	-	0.74%	1941263	-	1.46%	1507140	-	1.19%	450299	-	0.00%	66037	-	0.00%	718403	-
4	88	11.78%	-	-	5.02%	-	-	5.02%	-	-	32.49%	-	-	1.34%	16.35	-	1.34%	11.3	-
		4.50%	1935127	-	0.32%	1674350	-	0.61%	1026913	-	0.58%	445877	-	0.00%	1095	-	0.00%	1092	-
5	84	12.02%	-	-	5.31%	-	-	5.31%	-	-	14.98%	-	-	1.74%	26.56	-	1.74%	46.42	-
		4.58%	1063801	-	1.06%	1603629	-	0.82%	1046398	-	1.21%	426760	-	0.00%	3368	-	0.00%	1138	-
6	94	10.05%	-	-	4.37%	-	-	4.37%	-	-	24.52%	-	-	1.38%	20.56	-	1.38%	26.54	-
		3.70%	1598856	-	0.09%	1375309	-	0.50%	869219	-	1.24%	532570	-	0.00%	1080	-	0.00%	2196	-
7	85	11.18%	-	-	5.14%	-	-	5.13%	-	-	12.79%	-	-	1.81%	32.05	-	1.81%	36.46	-
		4.58%	1406323	-	0.84%	1450037	-	0.85%	896703	-	1.44%	457846	-	0.00%	1148	-	0.00%	1145	-
8	93	10.92%	-	-	4.43%	-	-	4.43%	-	-	23.47%	-	-	1.78%	15	-	1.78%	14.22	-
		4.37%	1000951	-	0.24%	1278688	-	0.35%	714649	-	0.87%	401506	-	0.00%	2488	-	0.00%	7227	-
9	84	11.52%	-	-	5.42%	-	-	5.41%	-	-	55.54%	-	-	1.60%	26.61	-	1.59%	19.16	-
		5.18%	1147401	-	0.75%	1174524	-	0.78%	762531	-	0.52%	523249	-	0.00%	1124	-	0.00%	1069	-
10	93	10.42%	-	-	4.88%	-	-	4.88%	-	-	17.65%	-	-	1.63%	12.65	-	1.63%	12.81	-
		3.17%	673115	-	0.47%	998667	-	0.59%	645912	-	0.66%	423429	-	0.00%	2325	-	0.00%	3331	-
Average		11.19%	-	-	5.06%	-	-	5.06%	-	-	24.58%	-	-	1.69%	28.34	-	1.70%	120.21	-
		4.24%	1727011	-	0.58%	1842314	-	0.74%	1044299	-	0.97%	460892	-	0.00%	9028.8	-	0.00%	73786.7	-

Table A.9: Results for problems with 300 products, $\phi : 0.25, \gamma : 0.25$

instance	assort	MILP			MILP+			MILP++			CONIC			CONIC+			CONIC++		
		rgap	time	nodes	rgap	time	nodes	egap	time	nodes	rgap	time	nodes	egap	time	nodes	rgap	time	nodes
1	120	9.60%	-	-	2.66%	-	-	2.66%	-	-	26.75%	-	-	1.06%	10.01	1.06%	1.06%	9.97	1063
2	109	11.20%	-	-	3.73%	-	-	3.73%	2312.83	-	34.47%	-	-	1.15%	23.11	1.15%	1.15%	9.55	1850
3	125	2.04%	598476	-	0.32%	902696	-	0.67%	263581	-	0.09%	1017913	-	0.00%	1073	0.00%	0.00%	2.17	1609
4	127	10.33%	-	-	2.81%	1020.47	-	2.81%	-	-	25.68%	1442.22	-	0.78%	2.33	0.78%	0.78%	1.37	905
5	113	1.65%	572029	-	0.01%	441044	-	0.15%	694449	-	0.01%	913313	-	0.00%	1095	0.00%	0.00%	2.2	1746
6	116	10.01%	-	-	2.60%	144.26	-	2.60%	-	-	16.02%	-	-	0.70%	0.86	0.70%	0.70%	2.34	1469
7	115	2.02%	533940	-	0.00%	63458	-	0.03%	595018	-	0.31%	1101356	-	0.00%	693	0.00%	0.00%	9.88	1126
8	118	9.67%	-	-	2.73%	1210.78	-	2.73%	-	-	25.46%	-	-	0.92%	1.78	0.92%	0.92%	2.22	2251
9	121	1.41%	420099	-	0.01%	451164	-	0.04%	613477	-	0.94%	902901	-	0.00%	2293	0.00%	0.00%	7.47	1425.9
10	112	9.99%	-	-	2.78%	250.3	-	2.78%	838.33	-	23.48%	125.35	-	0.95%	5.5	0.95%	0.95%	2.22	1162
Average		1.73%	548757	-	0.00%	103032	-	0.01%	285949	-	0.01%	26075	-	0.00%	1057	0.00%	0.00%	7.81	1306.7
		10.22%	-	-	3.31%	-	-	3.31%	-	-	19.05%	-	-	1.09%	11.84	1.09%	1.09%	7.47	1425.9
		5.03%	489426	-	0.02%	812791	-	0.15%	562532	-	0.04%	974546	-	0.00%	1085	0.00%	0.00%	7.47	1425.9
		9.34%	-	-	2.70%	348.56	-	2.70%	838.45	-	23.57%	1068.44	-	0.84%	2.11	0.84%	0.84%	7.47	1425.9
		1.63%	462400	-	0.00%	144620	-	0.00%	254090	-	0.01%	409264	-	0.00%	2537	0.00%	0.00%	7.47	1425.9
		11.05%	-	-	3.91%	-	-	3.91%	-	-	46.67%	-	-	1.21%	12.38	1.21%	1.21%	7.47	1425.9
		1.91%	541122	-	0.26%	876590	-	0.37%	572058	-	0.19%	704202	-	0.00%	1067	0.00%	0.00%	7.47	1425.9
		10.24%	-	-	2.88%	-	-	2.88%	-	-	41.53%	1404.47	-	1.02%	8.16	1.02%	1.02%	7.47	1425.9
		1.58%	558350	-	0.01%	836045	-	0.21%	589194	-	0.01%	860466	-	0.00%	1064	0.00%	0.00%	7.47	1425.9
		10.17%	-	-	3.01%	594.87	-	3.01%	1329.87	-	28.27%	1010.12	-	0.97%	7.81	0.97%	0.97%	7.47	1425.9
		2.25%	561632	-	0.12%	553378	-	0.17%	518679	-	0.18%	762164	-	0.00%	1306.7	0.00%	0.00%	7.47	1425.9

Appendix B

Tables for Assortment Optimization with Fixed Costs under MNL Model with Capacity Constraints

Table B.1: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma :$
 $1, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	0.24%	0.32	0.01%	0.16	1.58%	0.47	0.00%	0.26
		0.00%	1	0.00%	1	0.00%	1182	0.00%	1
2	50	0.31%	0.42	0.01%	0.19	1.39%	0.29	0.00%	0.22
		0.00%	1	0.00%	1	0.00%	763	0.00%	1
3	50	0.30%	1.47	0.01%	0.16	1.31%	0.49	0.00%	0.17
		0.00%	11856	0.00%	1	0.00%	1032	0.00%	1
4	50	0.30%	0.31	0.01%	0.17	1.24%	0.38	0.00%	0.21
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
5	50	0.33%	0.63	0.01%	0.17	1.35%	0.38	0.00%	0.17
		0.00%	1050	0.00%	1	0.00%	1108	0.00%	1
6	50	0.30%	0.8	0.01%	0.16	1.52%	0.38	0.00%	0.2
		0.00%	5576	0.00%	1	0.00%	1082	0.00%	1
7	50	0.34%	0.74	0.01%	0.22	1.11%	0.25	0.00%	0.15
		0.00%	1110	0.00%	3	0.00%	450	0.00%	1
8	50	0.33%	4.96	0.01%	0.22	1.31%	0.4	0.00%	0.18
		0.00%	29682	0.00%	1	0.00%	1038	0.00%	1
9	50	0.34%	1.57	0.00%	0.16	1.06%	0.26	0.00%	0.16
		0.00%	9477	0.00%	1	0.00%	900	0.00%	1
10	50	0.35%	4.86	0.02%	0.26	1.26%	0.43	0.00%	0.16
		0.00%	29615	0.00%	6	0.00%	1041	0.00%	1
Average		0.31%	1.61	0.01%	0.19	1.31%	0.37	0.00%	0.19
		0.00%	8836.9	0.00%	1.7	0.00%	968	0.00%	1

Table B.2: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.5, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	0.46%	9.66	0.01%	0.25	1.20%	0.2	0.00%	0.15
		0.00%	60842	0.00%	1	0.00%	399	0.00%	1
2	50	0.39%	5.28	0.01%	0.22	1.36%	0.37	0.01%	0.17
		0.00%	33305	0.00%	94	0.00%	1131	0.00%	1
3	50	0.43%	5.42	0.01%	0.14	1.04%	0.4	0.00%	0.16
		0.00%	32731	0.00%	1	0.00%	1103	0.00%	1
4	50	0.45%	5.01	0.01%	0.16	1.10%	0.35	0.00%	0.15
		0.00%	29632	0.00%	1	0.00%	1180	0.00%	1
5	50	0.43%	10.81	0.01%	0.2	1.09%	0.19	0.00%	0.16
		0.00%	81096	0.00%	4	0.00%	254	0.00%	1
6	50	0.42%	0.87	0.01%	0.25	1.03%	0.21	0.00%	0.18
		0.00%	1041	0.00%	1	0.00%	578	0.00%	1
7	50	0.43%	5.27	0.01%	0.2	0.98%	0.16	0.00%	0.14
		0.00%	33712	0.00%	1	0.00%	82	0.00%	1
8	50	0.39%	1.02	0.01%	0.18	1.16%	0.38	0.00%	0.16
		0.00%	1064	0.00%	1	0.00%	1087	0.00%	1
9	50	0.42%	5.11	0.01%	0.2	1.09%	0.43	0.00%	0.18
		0.00%	32663	0.00%	1	0.00%	1026	0.00%	1
10	50	0.44%	4.74	0.01%	0.16	1.04%	0.13	0.00%	0.17
		0.00%	43521	0.00%	1	0.00%	1	0.00%	1
Average		0.43%	5.32	0.01%	0.20	1.11%	0.28	0.00%	0.16
		0.00%	34960.7	0.00%	10.6	0.00%	684.1	0.00%	1

Table B.3: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.25, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	0.47%	1.51	0.01%	0.18	0.84%	0.33	0.00%	0.18
		0.00%	1111	0.00%	108	0.00%	1081	0.00%	1
2	50	0.42%	8.57	0.01%	0.23	0.91%	0.35	0.00%	0.16
		0.00%	63318	0.00%	1	0.00%	1076	0.00%	1
3	50	0.46%	4.4	0.00%	0.13	0.86%	0.41	0.00%	0.18
		0.00%	29535	0.00%	1	0.00%	1119	0.00%	79
4	50	0.43%	0.8	0.01%	0.2	0.91%	0.12	0.00%	0.14
		0.00%	1122	0.00%	1	0.00%	1	0.00%	1
5	50	0.48%	1.05	0.00%	0.15	0.71%	0.14	0.00%	0.18
		0.00%	1119	0.00%	1	0.00%	31	0.00%	1
6	50	0.48%	6.24	0.01%	0.25	0.94%	0.28	0.00%	0.17
		0.00%	43883	0.00%	1	0.00%	1011	0.00%	1
7	50	0.42%	0.82	0.01%	0.14	0.91%	0.23	0.00%	0.12
		0.00%	1145	0.00%	1	0.00%	545	0.00%	1
8	50	0.48%	5.57	0.01%	0.17	0.80%	0.17	0.00%	0.14
		0.00%	34209	0.00%	1	0.00%	175	0.00%	1
9	50	0.43%	1.15	0.01%	0.21	1.09%	0.23	0.00%	0.16
		0.00%	1068	0.00%	1	0.00%	569	0.00%	1
10	50	0.52%	5.85	0.01%	0.17	0.80%	0.27	0.00%	0.14
		0.00%	37615	0.00%	1	0.00%	1108	0.00%	1
Average		0.46%	3.60	0.01%	0.18	0.88%	0.25	0.00%	0.16
		0.00%	21412.5	0.00%	11.7	0.00%	671.6	0.00%	8.8

Table B.4: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$
 $1, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	2.09%	3.64	0.12%	0.35	3.21%	0.7	0.01%	0.25
		0.00%	5405	0.00%	1	0.00%	1236	0.00%	1
2	50	2.01%	86.72	0.14%	0.47	3.75%	1.3	0.02%	0.25
		0.00%	449258	0.00%	1	0.00%	1187	0.00%	1
3	50	2.41%	41.25	0.10%	0.3	2.96%	0.7	0.02%	0.3
		0.00%	194314	0.00%	1	0.00%	1107	0.00%	1
4	50	2.17%	11.84	0.14%	0.32	3.42%	0.63	0.03%	0.28
		0.00%	63989	0.00%	1	0.00%	1171	0.00%	1
5	50	2.16%	4.09	0.10%	0.28	3.31%	0.61	0.01%	0.47
		0.00%	5521	0.00%	1	0.00%	1062	0.00%	1
6	50	2.43%	69.76	0.06%	0.28	3.28%	0.58	0.00%	0.26
		0.00%	490071	0.00%	1	0.00%	1151	0.00%	1
7	50	2.24%	14.47	0.12%	0.4	3.11%	0.3	0.01%	0.24
		0.00%	69976	0.00%	1	0.00%	992	0.00%	1
8	50	2.60%	471.95	0.19%	1.96	3.55%	0.6	0.03%	0.26
		0.00%	2219285	0.00%	11357	0.00%	1080	0.00%	1
9	50	2.09%	2.11	0.07%	0.22	3.72%	0.91	0.01%	0.18
		0.00%	2729	0.00%	1	0.00%	1085	0.00%	1
10	50	2.19%	75.05	0.12%	0.49	3.64%	1.06	0.03%	0.47
		0.00%	364034	0.00%	1	0.00%	1166	0.00%	1
Average		2.24%	78.09	0.12%	0.51	3.39%	0.74	0.02%	0.30
		0.00%	386458	0.00%	1136.6	0.00%	1123.7	0.00%	1

Table B.5: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.5, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	2.97%	302.73	0.16%	0.62	3.26%	0.55	0.02%	0.28
		0.00%	1483361	0.00%	1	0.00%	1063	0.00%	1
2	50	3.20%	99.29	0.08%	0.32	2.57%	0.44	0.01%	0.22
		0.00%	357544	0.00%	636	0.00%	1193	0.00%	1
3	50	3.09%	-	0.13%	0.84	2.71%	0.48	0.01%	0.57
		0.39%	9273313	0.00%	3241	0.00%	1152	0.00%	1
4	50	3.04%	401.75	0.13%	0.57	3.11%	0.49	0.02%	0.37
		0.00%	1088379	0.00%	1559	0.00%	1060	0.00%	1
5	50	3.08%	398.3	0.12%	1.78	3.13%	0.5	0.01%	0.23
		0.00%	1109028	0.00%	7732	0.00%	1032	0.00%	1
6	50	3.08%	18.98	0.09%	0.4	3.19%	0.5	0.01%	0.27
		0.00%	57061	0.00%	1014	0.00%	1169	0.00%	1
7	50	3.02%	1696.24	0.13%	1.26	2.85%	0.7	0.02%	0.22
		0.00%	4812115	0.00%	4774	0.00%	1205	0.00%	1
8	50	2.76%	-	0.14%	1.05	3.42%	0.85	0.02%	0.3
		0.04%	5491442	0.00%	9106	0.00%	1203	0.00%	1
9	50	2.86%	21.98	0.14%	0.34	3.33%	0.99	0.02%	0.27
		0.00%	54572	0.00%	1	0.00%	1101	0.00%	1
10	50	2.92%	1190.08	0.14%	0.48	3.13%	0.5	0.01%	0.21
		0.00%	5703169	0.00%	1	0.00%	1110	0.00%	1
Average		3.00%	516.17	0.13%	0.77	3.07%	0.60	0.01%	0.29
		0.04%	2942998	0.00%	2806.5	0.00%	1128.8	0.00%	1

Table B.6: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.25, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	3.43%	309.47	0.04%	0.2	2.22%	0.41	0.00%	0.19
		0.00%	1403359	0.00%	1	0.00%	1075	0.00%	1
2	50	3.41%	-	0.08%	0.92	2.54%	0.57	0.00%	0.18
		0.08%	7578526	0.00%	4212	0.00%	1175	0.00%	1
3	50	3.67%	863.24	0.11%	0.56	2.64%	0.34	0.00%	0.32
		0.00%	2009917	0.00%	3167	0.00%	1157	0.00%	1
4	50	3.55%	-	0.11%	0.5	2.28%	0.46	0.01%	0.23
		0.16%	4264812	0.00%	2386	0.00%	1073	0.00%	1
5	50	3.06%	-	0.12%	0.53	2.68%	0.45	0.00%	0.22
		0.01%	3723343	0.00%	2050	0.00%	1029	0.00%	1
6	50	3.24%	28.86	0.08%	0.5	2.61%	0.41	0.00%	0.12
		0.00%	65378	0.00%	1152	0.00%	1149	0.00%	1
7	50	3.54%	108.81	0.10%	0.52	2.94%	0.46	0.00%	0.28
		0.00%	230180	0.00%	1645	0.00%	1150	0.00%	1
8	50	3.39%	1580.87	0.10%	0.76	2.58%	0.43	0.01%	0.18
		0.00%	3299952	0.00%	3293	0.00%	1032	0.00%	1
9	50	4.06%	95.59	0.09%	0.35	1.94%	0.2	0.00%	0.32
		0.00%	200879	0.00%	167	0.00%	794	0.00%	1
10	50	3.46%	-	0.11%	0.59	2.71%	0.31	0.01%	0.2
		0.05%	4256874	0.00%	3613	0.00%	1027	0.00%	1
Average		3.48%	497.81	0.09%	0.54	2.51%	0.40	0.00%	0.22
		0.03%	2703322	0.00%	2168.6	0.00%	1066.1	0.00%	1

Table B.7: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 1, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap egap	time nodes	rgap egap	time nodes	rgap egap	time nodes	rgap egap	time nodes
1	50	7.37% 0.20%	- 1164692	0.77% 0.00%	0.64 1	7.54% 0.00%	1.77 1603	0.18% 0.00%	0.47 1
2	50	8.96% 0.78%	- 1034273	1.22% 0.00%	2.76 2947	8.58% 0.00%	3.62 1150	0.24% 0.00%	0.47 1
3	50	7.39% 0.95%	- 2292125	1.44% 0.00%	3.25 4691	9.15% 0.00%	3.76 5303	0.49% 0.00%	0.82 1
4	50	8.41% 0.65%	- 2004252	1.36% 0.00%	6.43 18385	10.72% 0.00%	11.44 30105	0.25% 0.00%	0.62 1
5	50	7.74% 0.15%	- 983204	0.98% 0.00%	1.09 2480	9.73% 0.00%	3.28 1320	0.27% 0.00%	0.42 1
6	50	9.21% 0.76%	- 2239776	0.97% 0.00%	1.56 5027	7.13% 0.00%	2.89 1116	0.28% 0.00%	0.5 1
7	50	9.95% 1.52%	- 1856023	1.47% 0.00%	11.19 37495	8.20% 0.00%	3.97 7639	0.38% 0.00%	1.24 1
8	50	8.82% 1.10%	- 1443392	1.36% 0.00%	8.74 28573	8.46% 0.00%	4.19 6203	0.34% 0.00%	1 801
9	50	8.93% 0.62%	- 1167988	0.93% 0.00%	0.96 1912	7.52% 0.00%	1.89 1214	0.17% 0.00%	0.33 1
10	50	8.41% 0.55%	- 544545	0.79% 0.00%	0.77 1	7.92% 0.00%	1.87 1147	0.28% 0.00%	0.45 1
Average		8.52% 0.73%	- 1473027	1.13% 0.00%	3.74 10151.2	8.50% 0.00%	3.87 5680	0.29% 0.00%	0.63 81

Table B.8: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma :$
 $0.5, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	11.73%	-	1.28%	5.89	7.86%	4.15	0.22%	0.45
		3.12%	1342224	0.00%	2116	0.00%	1395	0.00%	1
2	50	12.67%	-	1.17%	4.72	7.71%	1.44	0.13%	0.42
		1.97%	1449344	0.00%	11525	0.00%	1187	0.00%	1
3	50	11.65%	-	1.39%	67.12	9.16%	1.42	0.19%	0.57
		1.81%	1459993	0.00%	343643	0.00%	2984	0.00%	1
4	50	12.78%	-	1.00%	4.37	6.74%	1.27	0.19%	0.4
		3.71%	1085743	0.00%	1089	0.00%	1503	0.00%	1
5	50	12.64%	-	1.25%	3.13	8.17%	1.26	0.15%	0.37
		2.75%	1001320	0.00%	2861	0.00%	1530	0.00%	1
6	50	11.51%	-	0.91%	2.92	7.75%	1.97	0.16%	0.33
		2.41%	1050910	0.00%	2097	0.00%	1083	0.00%	1
7	50	11.73%	-	1.06%	4.33	8.68%	1.68	0.23%	0.47
		2.73%	870735	0.00%	5998	0.00%	1880	0.00%	1
8	50	12.39%	-	1.08%	2.73	7.29%	1.45	0.16%	0.41
		1.78%	927796	0.00%	1116	0.00%	1124	0.00%	1
9	50	12.94%	-	1.39%	8.13	6.93%	2.13	0.19%	0.54
		2.67%	1009804	0.00%	20844	0.00%	1386	0.00%	1
10	50	11.79%	-	1.26%	13.63	8.60%	2.79	0.21%	0.53
		2.10%	844990	0.00%	43153	0.00%	1097	0.00%	1
Average		12.18%	-	1.18%	11.70	7.89%	1.96	0.18%	0.45
		2.50%	1104286	0.00%	43444.2	0.00%	1516.9	0.00%	1

Table B.9: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma :$
 $0.25, k : 50$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	50	13.31%	-	0.65%	1.19	7.08%	1.79	0.09%	0.65
		2.07%	795003	0.00%	1088	0.00%	1118	0.00%	1
2	50	13.96%	-	0.75%	1.44	6.28%	0.83	0.08%	0.34
		3.45%	624828	0.00%	2121	0.00%	1150	0.00%	1
3	50	14.99%	-	0.91%	20.94	6.84%	1.65	0.08%	0.49
		3.05%	610839	0.00%	91818	0.00%	1163	0.00%	1
4	50	13.75%	-	0.72%	1.25	6.99%	1.09	0.04%	0.23
		2.28%	510445	0.00%	1082	0.00%	1202	0.00%	1
5	50	13.74%	-	0.67%	1.1	6.46%	0.74	0.09%	0.28
		3.78%	466316	0.00%	1066	0.00%	1101	0.00%	1
6	50	14.14%	-	0.84%	1.35	6.10%	0.94	0.11%	0.29
		3.81%	447122	0.00%	1055	0.00%	1222	0.00%	1
7	50	14.27%	-	1.02%	4	6.80%	2.18	0.15%	0.6
		3.30%	590953	0.00%	5517	0.00%	1135	0.00%	1
8	50	12.96%	-	0.93%	1.67	7.17%	0.84	0.11%	0.33
		2.87%	624759	0.00%	1115	0.00%	1736	0.00%	1
9	50	13.64%	-	1.01%	4.55	7.94%	2.6	0.09%	0.44
		4.23%	518883	0.00%	1105	0.00%	1250	0.00%	1
10	50	13.99%	-	0.73%	1.22	6.07%	0.6	0.03%	0.23
		1.47%	7540404	0.00%	1059	0.00%	1208	0.00%	1
Average		13.88%	-	0.82%	3.87	6.77%	1.33	0.09%	0.39
		3.03%	1272955	0.00%	10702.6	0.00%	1228.5	0.00%	1

Table B.10: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma :$
 $1, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.61%	614.04	0.04%	0.66	2.35%	0.91	0.01%	0.25
		0.00%	2900930	0.00%	1067	0.00%	1116	0.00%	1
2	100	0.66%	8.39	0.03%	0.41	2.22%	0.92	0.01%	0.2
		0.00%	20153	0.00%	1	0.00%	1069	0.00%	1
3	100	0.73%	144.31	0.03%	0.41	2.20%	1.82	0.01%	0.19
		0.00%	519741	0.00%	720	0.00%	1160	0.00%	1
4	100	0.70%	270.32	0.03%	0.59	1.99%	1.47	0.01%	0.28
		0.00%	1045661	0.00%	1289	0.00%	1099	0.00%	1
5	100	0.68%	1.67	0.03%	0.31	1.99%	0.93	0.00%	0.2
		0.00%	1121	0.00%	1	0.00%	1047	0.00%	1
6	100	0.72%	732.15	0.04%	1.07	2.36%	5.11	0.01%	0.21
		0.00%	3108302	0.00%	4918	0.00%	34271	0.00%	1
7	100	0.72%	2.64	0.02%	0.27	1.83%	0.69	0.01%	0.18
		0.00%	1094	0.00%	1	0.00%	1154	0.00%	1
8	100	0.79%	8	0.02%	0.29	2.07%	0.86	0.00%	0.22
		0.00%	26028	0.00%	1	0.00%	1120	0.00%	1
9	100	0.69%	1.47	0.01%	0.28	1.96%	1.38	0.00%	0.23
		0.00%	1064	0.00%	1	0.00%	1184	0.00%	1
10	100	0.78%	9.6	0.02%	0.4	1.99%	1.52	0.01%	0.18
		0.00%	40766	0.00%	1	0.00%	1087	0.00%	1
Average		0.71%	179.26	0.03%	0.47	2.09%	1.56	0.01%	0.21
		0.00%	766486	0.00%	800	0.00%	4430.7	0.00%	1

Table B.11: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.5, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	1.06%	268.67	0.02%	0.4	1.87%	0.83	0.00%	0.19
		0.00%	1324337	0.00%	830	0.00%	1192	0.00%	1
2	100	0.96%	39.5	0.02%	0.48	2.10%	0.7	0.00%	0.15
		0.00%	29430	0.00%	1488	0.00%	1145	0.00%	1
3	100	0.93%	19.28	0.01%	0.33	1.84%	0.62	0.00%	0.2
		0.00%	79971	0.00%	1	0.00%	1053	0.00%	1
4	100	0.92%	103.73	0.02%	0.59	1.87%	0.63	0.00%	0.23
		0.00%	431703	0.00%	1574	0.00%	1112	0.00%	1
5	100	0.94%	20.48	0.01%	0.36	1.80%	0.71	0.00%	0.21
		0.00%	75678	0.00%	843	0.00%	1049	0.00%	1
6	100	0.96%	6.81	0.02%	0.27	1.76%	0.69	0.00%	0.19
		0.00%	18947	0.00%	1	0.00%	1117	0.00%	1
7	100	0.99%	17.59	0.01%	0.25	1.75%	0.78	0.00%	0.24
		0.00%	62952	0.00%	326	0.00%	1091	0.00%	1
8	100	0.89%	109.09	0.02%	0.55	1.89%	0.63	0.00%	0.17
		0.00%	482765	0.00%	1553	0.00%	1062	0.00%	1
9	100	0.99%	108.81	0.01%	0.38	1.92%	0.89	0.00%	0.18
		0.00%	476349	0.00%	786	0.00%	1160	0.00%	1
10	100	0.95%	36.87	0.02%	0.65	1.78%	0.72	0.00%	0.18
		0.00%	156222	0.00%	1794	0.00%	1148	0.00%	1
Average		0.96%	73.08	0.02%	0.43	1.86%	0.72	0.00%	0.19
		0.00%	313835	0.00%	919.6	0.00%	1112.9	0.00%	1

Table B.12: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma :$
 $0.25, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.96%	1.28	0.01%	0.27	1.48%	0.61	0.00%	0.17
		0.00%	2894	0.00%	363	0.00%	1182	0.00%	1
2	100	0.98%	2.38	0.01%	0.28	1.48%	0.57	0.00%	0.2
		0.00%	1127	0.00%	340	0.00%	1150	0.00%	1
3	100	0.89%	1.52	0.01%	0.38	1.70%	0.58	0.00%	0.2
		0.00%	1102	0.00%	797	0.00%	1124	0.00%	1
4	100	0.89%	1	0.01%	0.35	1.65%	0.6	0.00%	0.19
		0.00%	1098	0.00%	570	0.00%	1037	0.00%	1
5	100	1.00%	33.21	0.01%	0.29	1.50%	1.17	0.00%	0.2
		0.00%	135371	0.00%	515	0.00%	1070	0.00%	1
6	100	0.93%	2.67	0.01%	0.28	1.60%	1.06	0.00%	0.22
		0.00%	7392	0.00%	1	0.00%	1084	0.00%	1
7	100	0.93%	82.33	0.01%	0.46	1.67%	0.57	0.00%	0.14
		0.00%	422986	0.00%	1284	0.00%	1194	0.00%	1
8	100	0.93%	0.92	0.01%	0.28	1.43%	0.58	0.00%	0.16
		0.00%	1084	0.00%	1	0.00%	1068	0.00%	1
9	100	0.86%	1.28	0.01%	0.17	1.77%	1.38	0.00%	0.18
		0.00%	1076	0.00%	1	0.00%	1183	0.00%	1
10	100	0.98%	1.14	0.01%	0.33	1.44%	0.59	0.00%	0.18
		0.00%	1045	0.00%	505	0.00%	1061	0.00%	1
Average		0.93%	12.77	0.01%	0.31	1.57%	0.77	0.00%	0.18
		0.00%	57517.5	0.00%	437.7	0.00%	1115.3	0.00%	1

Table B.13: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$
 $1, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	2.94%	-	0.30%	2.78	4.66%	2.47	0.08%	0.65
		0.99%	8099892	0.00%	11538	0.00%	1089	0.00%	1
2	100	3.30%	-	0.36%	10.45	5.39%	25.7	0.13%	0.95
		1.36%	4827448	0.00%	36328	0.00%	4138	0.00%	1483
3	98	3.54%	-	0.32%	1.69	4.82%	14.45	0.11%	0.96
		0.20%	1561401	0.00%	1064	0.00%	1155	0.00%	1
4	100	3.26%	-	0.34%	6.58	5.05%	9.11	0.12%	0.92
		0.51%	870088	0.00%	39277	0.00%	1471	0.00%	1468
5	100	3.04%	-	0.26%	3.09	5.13%	7.98	0.09%	0.8
		0.45%	1458882	0.00%	13494	0.00%	1236	0.00%	1
6	100	3.43%	-	0.28%	8.08	5.02%	10.67	0.10%	0.85
		0.40%	1354109	0.00%	29678	0.00%	2807	0.00%	1
7	100	3.26%	-	0.34%	7.74	4.70%	15.1	0.12%	1.39
		1.33%	2489905	0.00%	29554	0.00%	1645	0.00%	1079
8	100	3.73%	-	0.48%	9.47	5.35%	12.98	0.11%	0.79
		1.57%	1913561	0.00%	30487	0.00%	1261	0.00%	1063
9	100	2.71%	-	0.28%	1.15	5.62%	15.27	0.10%	0.88
		0.88%	2186787	0.00%	1	0.00%	3813	0.00%	148
10	100	3.19%	-	0.23%	1.03	5.32%	7.61	0.07%	0.93
		1.28%	1784848	0.00%	1	0.00%	1173	0.00%	1
Average		3.24%	-	0.32%	5.21	5.11%	12.13	0.10%	0.91
		0.90%	2654692	0.00%	19142.2	0.00%	1978.8	0.00%	524.6

Table B.14: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.5, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	4.82%	-	0.28%	2.02	5.28%	17.39	0.09%	0.73
		1.10%	993544	0.00%	1136	0.00%	3136	0.00%	1
2	100	5.29%	-	0.30%	1.95	4.64%	6.48	0.08%	0.74
		2.91%	1327930	0.00%	1418	0.00%	1228	0.00%	1
3	100	4.71%	-	0.21%	6.86	4.80%	2.21	0.07%	0.56
		0.90%	1267530	0.00%	29897	0.00%	1131	0.00%	1
4	100	4.29%	-	0.28%	7.13	5.26%	8.04	0.06%	0.67
		0.58%	872892	0.00%	29447	0.00%	1144	0.00%	1
5	100	4.84%	-	0.24%	7.08	5.17%	4.6	0.06%	0.5
		0.87%	898150	0.00%	29781	0.00%	1237	0.00%	1
6	100	4.58%	-	0.22%	6.09	5.13%	7.19	0.06%	0.34
		2.34%	1505025	0.00%	41164	0.00%	1199	0.00%	1
7	100	4.59%	-	0.20%	1.41	4.74%	2.5	0.06%	0.5
		1.07%	768669	0.00%	1078	0.00%	1197	0.00%	1
8	100	4.47%	-	0.24%	6.95	5.29%	4.94	0.04%	0.49
		2.24%	1249285	0.00%	29375	0.00%	1222	0.00%	1
9	100	4.45%	-	0.28%	6.24	5.28%	7.96	0.06%	0.47
		2.25%	1341644	0.00%	29720	0.00%	1844	0.00%	1
10	100	4.49%	-	0.23%	1.46	5.38%	2.01	0.04%	0.34
		0.74%	715965	0.00%	1063	0.00%	1135	0.00%	1
Average		4.65%	-	0.25%	4.72	5.10%	6.33	0.06%	0.53
		1.50%	1094063	0.00%	19407.9	0.00%	1447.3	0.00%	1

Table B.15: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$
 $0.25, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	4.61%	-	0.12%	0.8	4.27%	4.63	0.02%	0.26
		0.31%	249290	0.00%	1	0.00%	1241	0.00%	1
2	100	4.53%	-	0.09%	0.52	4.44%	1.22	0.01%	0.28
		0.74%	286389	0.00%	1	0.00%	1224	0.00%	1
3	100	4.82%	-	0.13%	0.8	4.20%	3.38	0.02%	0.27
		0.75%	381953	0.00%	1	0.00%	1207	0.00%	1
4	100	4.93%	-	0.12%	0.73	4.04%	2.94	0.01%	0.51
		1.40%	706092	0.00%	1	0.00%	1086	0.00%	1
5	100	5.02%	-	0.17%	1.17	4.52%	1.35	0.02%	0.31
		0.75%	557805	0.00%	1109	0.00%	1196	0.00%	1
6	100	4.92%	-	0.15%	4.58	4.31%	1.16	0.04%	0.36
		2.88%	923361	0.00%	31095	0.00%	1042	0.00%	1
7	100	4.78%	-	0.10%	0.51	4.65%	1.62	0.01%	0.31
		0.56%	676366	0.00%	1	0.00%	1116	0.00%	1
8	100	4.91%	-	0.12%	0.69	4.15%	0.96	0.01%	0.29
		2.57%	8289844	0.00%	1098	0.00%	1055	0.00%	1
9	100	4.86%	-	0.09%	0.59	3.56%	1.93	0.01%	0.26
		0.30%	2908548	0.00%	1120	0.00%	1139	0.00%	1
10	100	4.48%	-	0.11%	0.56	4.57%	2.25	0.02%	0.29
		2.10%	3514589	0.00%	1	0.00%	1190	0.00%	1
Average		4.79%	-	0.12%	1.10	4.27%	2.14	0.02%	0.31
		1.24%	1849424	0.00%	3442.8	0.00%	1149.6	0.00%	1

Table B.16: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 1, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	57	7.26%	-	2.04%	3.94	8.87%	-	0.72%	1.33
		0.48%	1289697	0.00%	5361	0.24%	3473854	0.00%	1
2	66	8.87%	-	3.03%	57.09	9.61%	-	1.00%	6.06
		1.38%	2037096	0.00%	148342	1.00%	2495141	0.00%	1078
3	66	7.15%	-	2.57%	9.49	10.51%	-	1.06%	1.41
		0.72%	2081759	0.00%	22945	1.64%	1240080	0.00%	1950
4	62	8.35%	-	2.93%	110.57	11.68%	-	1.02%	9.91
		2.06%	1226677	0.00%	320715	0.20%	899881	0.00%	1126
5	69	7.26%	-	2.17%	27	11.35%	139.22	0.89%	0.76
		0.97%	1712569	0.00%	96452	0.01%	23332	0.00%	412
6	65	8.96%	-	2.61%	15.73	8.47%	104.69	0.79%	1.06
		1.39%	5480345	0.00%	45856	0.01%	19632	0.00%	1275
7	74	9.36%	-	2.90%	50.13	9.35%	-	0.88%	1.24
		3.10%	2188548	0.00%	142493	0.28%	936640	0.00%	1499
8	67	8.63%	-	2.70%	63.74	9.83%	-	0.93%	1.73
		1.43%	2726289	0.00%	223787	1.33%	1045130	0.00%	2208
9	68	8.49%	-	2.33%	6.62	8.94%	100.37	0.53%	0.84
		1.26%	2564282	0.00%	13827	0.01%	1552	0.00%	230
10	63	8.13%	-	2.25%	4.05	9.26%	-	0.94%	1.79
		1.00%	1526170	0.00%	6041	0.20%	878057	0.00%	1
Average		8.25%	-	2.55%	34.84	9.79%	114.76	0.88%	2.61
		1.38%	2283343	0.00%	102582	0.49%	1101330	0.00%	978

Table B.17: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.5, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	90	10.73%	-	2.67%	694.58	10.96%	240.56	0.91%	18.65
		4.07%	4317376	0.00%	2238087	0.00%	26471	0.00%	1169
2	83	12.11%	-	3.09%	-	10.44%	862.63	0.84%	8.26
		4.98%	2881476	0.06%	5961734	0.00%	327534	0.00%	3133
3	95	11.15%	-	3.25%	-	11.88%	-	0.90%	15.01
		5.11%	1667608	0.07%	4812856	0.13%	658178	0.00%	1131
4	88	11.78%	-	2.70%	76.95	9.44%	78.95	0.69%	1.15
		4.21%	1972152	0.00%	162704	0.00%	5548	0.00%	1012
5	85	12.02%	-	2.99%	-	11.28%	555.54	0.90%	7.92
		4.88%	1035469	0.02%	3084102	0.00%	145754	0.00%	1133
6	95	10.05%	-	2.03%	28.06	10.72%	96.52	0.56%	1.34
		3.42%	1894926	0.00%	38201	0.00%	5959	0.00%	1
7	84	11.18%	-	2.70%	-	11.17%	-	0.98%	10.86
		4.53%	1457150	0.02%	2525484	0.20%	502297	0.00%	1120
8	92	10.92%	-	2.30%	48.31	10.11%	134.58	0.86%	2.5
		4.06%	889054	0.00%	63072	0.00%	15430	0.00%	4978
9	84	11.51%	-	3.10%	1209.54	9.74%	121.94	0.62%	1.23
		4.89%	1322040	0.00%	1646465	0.00%	11990	0.00%	1824
10	93	10.42%	-	2.45%	-	11.13%	83	0.70%	0.94
		4.66%	1026612	0.15%	3434693	0.00%	9518	0.00%	607
Average		11.19%	-	2.73%	411.49	10.69%	271.72	0.79%	6.79
		4.48%	1846386	0.03%	2396740	0.03%	170868	0.00%	1610.8

Table B.18: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.25, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	9.70%	-	0.93%	9.88	10.45%	511.49	0.31%	1.12
		3.04%	1116393	0.00%	3724	0.00%	161217	0.00%	1
2	100	11.24%	-	1.31%	219.13	9.89%	286.61	0.33%	0.72
		4.66%	1005548	0.00%	263579	0.00%	90811	0.00%	1
3	100	10.63%	-	1.20%	309.14	9.18%	53.08	0.21%	0.76
		5.28%	850646	0.01%	636516	0.00%	2358	0.00%	1
4	100	10.23%	-	0.79%	27.7	10.28%	78.16	0.23%	0.64
		4.83%	915971	0.00%	45186	0.00%	6755	0.00%	1
5	100	9.76%	-	0.82%	6.9	10.02%	59.13	0.29%	0.81
		2.49%	773155	0.00%	2385	0.00%	2777	0.00%	1
6	100	10.06%	-	0.95%	17.21	9.43%	37.78	0.28%	0.74
		3.80%	782125	0.00%	19478	0.00%	2097	0.00%	1
7	100	10.28%	-	1.10%	60.99	10.25%	65.89	0.27%	0.5
		4.23%	778169	0.00%	78039	0.00%	7601	0.00%	1
8	100	9.46%	-	0.89%	7.62	10.53%	68.96	0.26%	0.81
		2.80%	780206	0.00%	6161	0.00%	4228	0.00%	1
9	100	11.25%	-	1.83%	-	10.82%	1408.67	0.46%	1.62
		4.85%	475699	0.11%	4030282	0.00%	412748	0.00%	3104
10	100	10.28%	-	0.89%	11.49	10.00%	74.51	0.33%	0.93
		2.94%	571864	0.00%	9217	0.00%	2020	0.00%	1
Average		10.29%	-	1.07%	74.45	10.09%	264.43	0.30%	0.87
		3.89%	804978	0.01%	509457	0.00%	69261.2	0.00%	311.3

Table B.19: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 1, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	0.80%	79.11	0.05%	0.82	2.60%	2.4	0.02%	0.52
		0.00%	292112	0.00%	1812	0.00%	1676	0.00%	200
2	149	0.89%	73.65	0.04%	0.68	2.57%	2.49	0.02%	0.34
		0.00%	264216	0.00%	425	0.00%	1115	0.00%	1
3	149	0.99%	1022.53	0.04%	1.1	2.55%	2.07	0.02%	0.39
		0.00%	4056571	0.00%	2427	0.00%	1062	0.00%	1
4	150	0.92%	80.87	0.05%	0.87	2.26%	0.78	0.02%	0.33
		0.00%	342664	0.00%	1678	0.00%	1094	0.00%	259
5	150	0.89%	563.02	0.05%	1.45	2.35%	2.83	0.02%	0.4
		0.00%	2688161	0.00%	5196	0.00%	1086	0.00%	1
6	150	0.96%	-	0.05%	2.3	2.61%	3.8	0.03%	0.38
		0.01%	8124216	0.00%	6495	0.00%	2099	0.00%	533
7	150	0.94%	55.92	0.05%	1.06	2.24%	1.4	0.02%	0.38
		0.00%	222909	0.00%	2263	0.00%	1134	0.00%	1
8	150	0.98%	10.03	0.03%	0.58	2.39%	1.27	0.01%	0.18
		0.00%	49082	0.00%	1	0.00%	1077	0.00%	1
9	150	0.84%	1.6	0.03%	0.4	2.39%	2.02	0.01%	0.16
		0.00%	1067	0.00%	1	0.00%	1410	0.00%	1
10	146	0.98%	54.55	0.05%	2.06	2.33%	2.18	0.02%	0.37
		0.00%	243327	0.00%	3847	0.00%	1094	0.00%	1
Average		0.92%	215.70	0.04%	1.13	2.43%	2.12	0.02%	0.35
		0.00%	1628433	0.00%	2414.5	0.00%	1284.7	0.00%	99.9

Table B.20: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.5, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	1.22%	3.88	0.01%	0.55	2.22%	1.6	0.00%	0.14
		0.00%	12641	0.00%	605	0.00%	1078	0.00%	1
2	150	1.18%	120.88	0.01%	0.57	2.30%	1.67	0.00%	0.15
		0.00%	596624	0.00%	878	0.00%	1181	0.00%	1
3	150	1.20%	1595.04	0.02%	1.46	2.26%	1.96	0.00%	0.3
		0.00%	7508413	0.00%	3155	0.00%	1465	0.00%	1
4	150	1.18%	32.21	0.02%	1.34	2.22%	1.7	0.00%	0.28
		0.00%	120113	0.00%	2964	0.00%	1160	0.00%	1
5	150	1.09%	1.43	0.01%	0.31	2.20%	0.92	0.00%	0.24
		0.00%	1099	0.00%	1	0.00%	1090	0.00%	1
6	150	1.17%	588.07	0.02%	0.95	2.19%	2.53	0.00%	0.16
		0.00%	2600161	0.00%	2142	0.00%	1186	0.00%	1
7	150	1.13%	2.33	0.01%	0.34	2.16%	1.2	0.00%	0.15
		0.00%	5810	0.00%	1	0.00%	1189	0.00%	1
8	150	1.12%	138.41	0.02%	0.86	2.23%	1.86	0.00%	0.38
		0.00%	706278	0.00%	1949	0.00%	1235	0.00%	56
9	150	1.16%	34.09	0.01%	0.39	2.28%	2.34	0.00%	0.15
		0.00%	141176	0.00%	393	0.00%	1266	0.00%	1
10	150	1.11%	2.27	0.01%	0.45	2.15%	0.77	0.00%	0.17
		0.00%	5202	0.00%	1	0.00%	1171	0.00%	1
Average		1.16%	251.86	0.01%	0.72	2.22%	1.66	0.00%	0.21
		0.00%	1169752	0.00%	1208.9	0.00%	1202.1	0.00%	6.5

Table B.21: Results for instances with 300 products for MNL model, $\phi : 0.75, \gamma : 0.25, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	0.99%	0.71	0.01%	0.29	1.95%	0.75	0.00%	0.25
		0.00%	1065	0.00%	1	0.00%	1109	0.00%	1
2	150	0.99%	1.1	0.01%	0.3	1.90%	0.54	0.00%	0.16
		0.00%	1116	0.00%	1	0.00%	1074	0.00%	1
3	150	1.03%	1.28	0.01%	0.35	2.14%	0.91	0.00%	0.14
		0.00%	1173	0.00%	1	0.00%	1058	0.00%	1
4	150	0.97%	0.91	0.01%	0.29	2.05%	0.54	0.00%	0.15
		0.00%	1039	0.00%	1	0.00%	1162	0.00%	1
5	150	1.03%	0.72	0.01%	0.32	1.89%	0.58	0.00%	0.14
		0.00%	1063	0.00%	1	0.00%	1089	0.00%	1
6	150	0.95%	0.84	0.01%	0.3	2.08%	0.62	0.00%	0.15
		0.00%	1036	0.00%	1	0.00%	1104	0.00%	1
7	150	0.91%	0.5	0.01%	0.3	2.08%	1.03	0.00%	0.16
		0.00%	1076	0.00%	1	0.00%	1050	0.00%	1
8	150	1.00%	1.19	0.01%	0.32	1.90%	1.29	0.00%	0.12
		0.00%	1089	0.00%	299	0.00%	1138	0.00%	1
9	150	1.04%	24.35	0.01%	0.63	2.15%	0.76	0.00%	0.33
		0.00%	93879	0.00%	1376	0.00%	1075	0.00%	1
10	150	1.00%	0.58	0.01%	0.26	1.89%	1.08	0.00%	0.29
		0.00%	1053	0.00%	1	0.00%	1107	0.00%	1
Average		0.99%	3.22	0.01%	0.34	2.00%	0.81	0.00%	0.19
		0.00%	10358.9	0.00%	168.3	0.00%	1096.6	0.00%	1

Table B.22: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$
 $1, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	113	2.89%	-	0.48%	1.61	4.91%	14.68	0.13%	0.54
		0.24%	5468816	0.00%	2502	0.01%	2803	0.00%	779
2	118	3.33%	-	0.59%	-	5.62%	-	0.25%	2.15
		0.68%	4513463	0.05%	6633040	0.10%	1715889	0.00%	1064
3	111	3.57%	-	0.61%	11.8	5.10%	34.9	0.23%	2.49
		1.50%	3221272	0.00%	33408	0.01%	3504	0.00%	1080
4	129	3.22%	-	0.55%	5.26	5.29%	29.97	0.22%	1.46
		1.33%	2748120	0.00%	1888	0.01%	12328	0.00%	2791
5	119	3.03%	-	0.46%	12.64	5.44%	44.23	0.18%	1.53
		1.11%	2316051	0.00%	29860	0.01%	6023	0.00%	1151
6	119	2.03%	-	0.49%	11.61	5.35%	54.51	0.18%	1.53
		0.58%	1931631	0.00%	30084	0.01%	5501	0.00%	1151
7	102	3.28%	-	0.57%	10.7	4.97%	60.47	0.22%	2.62
		0.86%	1565009	0.00%	34853	0.01%	27815	0.00%	1088
8	104	3.72%	-	0.80%	11.45	5.61%	-	0.22%	1.83
		1.65%	1347645	0.00%	20723	0.02%	2253184	0.00%	1070
9	107	2.71%	-	0.48%	4.77	5.81%	-	0.18%	1.92
		0.85%	1500519	0.00%	4135	0.01%	2437962	0.00%	1093
10	113	3.19%	-	0.44%	7.91	5.61%	146.15	0.14%	1.27
		0.63%	1512957	0.00%	29796	0.01%	137871	0.00%	1042
Average		3.10%	-	0.55%	8.64	5.37%	54.99	0.20%	1.73
		0.94%	2612548	0.01%	682029	0.02%	660288	0.00%	1230.9

Table B.23: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma : 0.5, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	4.31%	-	0.29%	2.86	6.02%	61.33	0.14%	1.33
		2.39%	1068350	0.00%	1082	0.00%	29550	0.00%	2275
2	150	4.89%	-	0.38%	17.52	5.41%	12.15	0.16%	1.55
		2.89%	1061789	0.00%	31358	0.00%	1138	0.00%	2184
3	150	4.25%	-	0.27%	1.86	5.85%	12.5	0.12%	1.16
		2.32%	985953	0.00%	1087	0.00%	1129	0.00%	1482
4	150	4.03%	-	0.35%	4.85	6.14%	57.91	0.12%	1.65
		1.98%	1072271	0.00%	1594	0.00%	29410	0.00%	1117
5	150	4.52%	-	0.35%	5.16	5.89%	24.77	0.12%	0.86
		2.44%	3877897	0.00%	3625	0.00%	3678	0.00%	1079
6	150	4.14%	-	0.29%	2.54	6.07%	38.24	0.13%	1.25
		2.13%	4166979	0.00%	1104	0.00%	15702	0.00%	1049
7	150	4.24%	-	0.27%	2.64	5.58%	11.48	0.10%	1.74
		2.16%	3265699	0.00%	1100	0.00%	1115	0.00%	1087
8	150	4.21%	-	0.30%	1687.48	6.19%	57.58	0.10%	2.08
		2.23%	4454708	0.00%	8937667	0.00%	23573	0.00%	1131
9	150	4.10%	-	0.35%	5.18	6.23%	58.86	0.15%	2.31
		2.23%	2005441	0.00%	1610	0.00%	50569	0.00%	1056
10	150	4.09%	-	0.31%	4.79	6.17%	26.99	0.13%	2.05
		2.18%	1570769	0.00%	1128	0.00%	8780	0.00%	2079
Average		4.28%	-	0.32%	173.49	5.96%	36.18	0.13%	1.60
		2.29%	2352986	0.00%	898136	0.00%	16464.4	0.00%	1453.9

Table B.24: Results for instances with 300 products for MNL model, $\phi : 0.5, \gamma :$
 $0.25, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	3.19%	-	0.06%	0.41	5.36%	12.34	0.02%	0.2
		1.37%	1781982	0.00%	1	0.00%	1647	0.00%	1
2	150	3.05%	-	0.04%	0.38	5.60%	10.71	0.01%	0.2
		1.34%	1676946	0.00%	1	0.00%	1578	0.00%	1
3	150	3.04%	-	0.04%	0.34	5.01%	3.29	0.02%	0.3
		1.28%	1715952	0.00%	1	0.00%	1168	0.00%	1
4	150	3.45%	-	0.07%	0.48	5.14%	10.13	0.02%	0.27
		1.63%	1410882	0.00%	1	0.00%	1236	0.00%	1
5	150	3.42%	-	0.08%	0.55	5.32%	7.91	0.01%	0.35
		1.82%	1264925	0.00%	1	0.00%	1148	0.00%	1
6	150	3.51%	-	0.08%	0.64	5.25%	12.93	0.03%	0.34
		1.86%	1237515	0.00%	1	0.00%	1807	0.00%	1
7	150	3.53%	-	0.05%	0.49	5.47%	8.54	0.01%	0.34
		1.71%	1223492	0.00%	1	0.00%	1189	0.00%	1
8	150	3.65%	-	0.08%	0.73	5.04%	9.88	0.02%	0.31
		1.77%	1095963	0.00%	1	0.00%	1259	0.00%	1
9	150	3.33%	-	0.08%	0.71	4.63%	8.32	0.02%	0.35
		1.54%	1014211	0.00%	1	0.00%	1397	0.00%	1
10	150	3.18%	-	0.06%	0.57	5.68%	17.21	0.02%	0.26
		1.48%	1010500	0.00%	1	0.00%	19320	0.00%	1
Average		3.34%	-	0.06%	0.53	5.25%	10.13	0.02%	0.29
		1.60%	1294487	0.00%	1	0.00%	3174.9	0.00%	1

Table B.25: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma :$
 $1, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	57	7.26%	-	2.80%	5.21	8.87%	-	0.98%	1.09
		0.74%	542724	0.00%	6899	1.01%	1719197	0.00%	1076
2	67	8.87%	-	3.96%	208.28	9.61%	-	1.49%	14.68
		1.11%	6872060	0.00%	618877	1.45%	798713	0.00%	1125
3	65	7.16%	-	3.18%	22.89	10.52%	-	1.41%	16.54
		0.48%	8218482	0.00%	79597	1.48%	1206391	0.00%	1117
4	62	8.35%	-	3.84%	1033.14	11.68%	-	1.55%	14.58
		1.59%	5749056	0.00%	3003398	0.94%	1132878	0.00%	3143
5	70	7.26%	-	2.84%	103.11	11.35%	-	1.25%	0.95
		0.58%	8051937	0.00%	358004	0.95%	1235954	0.00%	713
6	65	8.96%	-	3.61%	108.74	8.47%	-	1.12%	2.7
		1.28%	5355086	0.00%	277209	1.19%	1039237	0.00%	6147
7	75	9.36%	-	3.92%	215.65	9.35%	-	1.16%	7.56
		2.99%	2381976	0.00%	641632	1.27%	859352	0.00%	1109
8	66	8.63%	-	3.44%	392.32	9.83%	-	1.23%	18.06
		1.41%	2190172	0.00%	957471	1.77%	546628	0.00%	1124
9	68	8.49%	-	3.21%	29.09	8.94%	-	0.87%	1.19
		1.26%	2595052	0.00%	58393	0.82%	1013480	0.00%	1217
10	63	8.13%	-	3.19%	103.51	9.26%	-	1.34%	0.89
		0.99%	1515252	0.00%	345156	0.25%	681328	0.00%	598
Average		8.25%	-	3.40%	222.19	9.79%	-	1.24%	7.82
		1.24%	4347180	0.00%	634664	1.11%	1023316	0.00%	1736.9

Table B.26: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.5, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	90	10.73%	-	3.87%	-	10.99%	-	1.42%	33.23
		4.84%	1316900	0.14%	4688914	0.37%	704066	0.00%	1721
2	83	12.11%	-	4.36%	-	10.45%	-	1.19%	15.92
		4.64%	1284374	0.33%	3751129	0.28%	391425	0.00%	1162
3	97	11.14%	-	4.42%	-	11.89%	-	1.45%	21.79
		3.62%	697529	0.41%	2111836	0.39%	422030	0.00%	13537
4	88	11.78%	-	3.95%	1510.19	9.46%	223.23	1.01%	6.53
		5.42%	180191	0.00%	1849049	0.01%	22753	0.00%	1091
5	86	12.02%	-	4.17%	-	11.32%	-	1.33%	21.73
		3.08%	1166542	0.33%	2367408	0.14%	437752	0.00%	1171
6	95	10.05%	-	3.21%	404.63	10.76%	-	0.94%	2.12
		3.84%	1251967	0.00%	418712	0.55%	551322	0.00%	3163
7	84	11.18%	-	3.97%	-	11.17%	-	1.38%	11.44
		5.30%	935181	0.33%	1768746	0.73%	394947	0.00%	2449
8	92	10.92%	-	3.37%	1003.01	10.15%	-	1.35%	11.72
		4.49%	583992	0.00%	858353	0.15%	446704	0.00%	1135
9	84	11.51%	-	4.38%	-	9.78%	-	1.11%	9.54
		5.09%	965631	0.36%	1658338	0.06%	472880	0.00%	1081
10	93	10.42%	-	3.78%	-	11.14%	-	1.11%	8.72
		4.68%	913047	0.11%	1417899	0.14%	530264	0.00%	1410
Average		11.19%	-	3.95%	972.61	10.71%	223.23	1.23%	14.27
		4.50%	929535	0.20%	2089038	0.28%	437414	0.00%	2792

Table B.27: Results for instances with 300 products for MNL model, $\phi : 0.25, \gamma : 0.25, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	120	9.60%	-	1.79%	73.32	11.08%	-	0.68%	1.74
		2.50%	636463	0.00%	54454	0.48%	338151	0.00%	2104
2	109	11.21%	-	2.54%	-	10.47%	-	0.69%	4.54
		3.06%	814182	0.03%	1382796	0.23%	307848	0.00%	1100
3	125	10.32%	-	1.77%	1751.62	9.84%	232.97	0.40%	0.95
		1.80%	606776	0.01%	1356789	0.01%	13450	0.00%	1
4	125	10.01%	-	1.44%	14.83	10.90%	1068.53	0.37%	1
		2.09%	852385	0.00%	8616	0.01%	143148	0.00%	1
5	114	9.67%	-	1.69%	62.19	10.76%	-	0.57%	1.15
		4.91%	499482	0.00%	40585	0.20%	291346	0.00%	774
6	114	9.98%	-	1.83%	34	10.07%	254.74	0.56%	1.27
		2.33%	418828	0.00%	21830	0.01%	15283	0.00%	567
7	116	10.22%	-	2.23%	207.48	10.88%	-	0.69%	3.55
		5.00%	379182	0.00%	135825	0.10%	222209	0.00%	1084
8	117	9.34%	-	1.73%	75.88	11.21%	-	0.51%	0.96
		2.09%	511267	0.00%	50553	0.06%	259438	0.00%	1011
9	121	11.06%	-	2.85%	-	11.28%	-	0.81%	5.3
		2.53%	454199	0.09%	1274058	0.38%	289850	0.00%	1068
10	111	10.23%	-	1.82%	167.59	10.63%	-	0.64%	2.09
		3.05%	585600	0.00%	111130	0.05%	389098	0.00%	3383
Average		10.16%	-	1.97%	298.36	10.71%	518.75	0.59%	2.26
		2.94%	575836	0.02%	443664	0.15%	226982	0.00%	1109.3

Appendix C

Tables for Assortment Optimization with Fixed Costs under MMNL Model

Table C.1: Results for instances with 300 products for MMNL model, ϕ : 0.75, $|\mathcal{M}|$: 100, $|\mathcal{C}|$: 25

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	93	0.00%	2.54	0.00%	5.63	0.14%	4.23	0.00%	8.75
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
2	91	0.00%	2.4	0.00%	5.27	0.14%	3.97	0.00%	8.43
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
3	73	0.00%	3.05	0.00%	6.81	0.10%	3.75	0.00%	8.29
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
4	89	0.00%	1.89	0.00%	4.64	0.12%	7.37	0.00%	9.42
		0.00%	1	0.00%	1	0.00%	249	0.00%	1
5	92	0.00%	1.95	0.00%	4.7	0.13%	4.56	0.00%	9.11
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
6	74	0.00%	1.38	0.00%	3.66	0.11%	4.15	0.00%	5.36
		0.00%	0	0.00%	0	0.00%	1	0.00%	0
7	87	0.00%	1.95	0.00%	4.66	0.11%	4.12	0.00%	9.55
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
8	91	0.00%	1.47	0.00%	3.73	0.13%	4.14	0.00%	5.71
		0.00%	0	0.00%	0	0.00%	1	0.00%	0
9	74	0.00%	2.47	0.00%	5.11	0.10%	4.01	0.00%	7.78
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
10	84	0.00%	2.06	0.00%	4.66	0.12%	4.01	0.00%	7.18
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
Average		0.00%	2.12	0.00%	4.89	0.12%	4.43	0.00%	7.96
		0.00%	0.8	0.00%	0.8	0.00%	25.8	0.00%	0.8

Table C.2: Results for instances with 300 products for MMNL model, ϕ : 0.75, $|\mathcal{M}|$: 75, $|\mathcal{C}|$: 25

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	76	0.00%	1.58	0.00%	2.7	0.15%	2.94	0.00%	5.44
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
2	76	0.00%	1.21	0.00%	2.05	0.11%	4.08	0.00%	3.67
		0.00%	0	0.00%	0	0.00%	136	0.00%	0
3	76	0.00%	2.91	0.00%	3.15	0.10%	3.01	0.00%	5.96
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
4	73	0.00%	1.59	0.00%	2.68	0.13%	3	0.00%	5.22
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
5	83	0.00%	3.26	0.00%	3.96	0.13%	4.53	0.00%	6.61
		0.00%	1	0.00%	1	0.00%	251	0.00%	1
6	75	0.00%	1.56	0.00%	1.94	0.15%	3.13	0.00%	3.94
		0.00%	1	0.00%	0	0.00%	1	0.00%	0
7	77	0.00%	3.17	0.00%	2.62	0.13%	3.05	0.00%	6.42
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
8	73	0.00%	1.53	0.00%	2.65	0.13%	2.96	0.00%	4.97
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
9	87	0.00%	3.31	0.00%	3.84	0.14%	2.95	0.00%	6.23
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
10	79	0.00%	1.18	0.00%	1.97	0.11%	3.1	0.00%	4.01
		0.00%	0	0.00%	0	0.00%	1	0.00%	0
Average		0.00%	2.13	0.00%	2.76	0.13%	3.28	0.00%	5.25
		0.00%	0.8	0.00%	0.7	0.00%	39.5	0.00%	0.7

Table C.3: Results for instances with 300 products for MMNL model, ϕ : 0.75, $|\mathcal{M}|$: 50, $|\mathcal{C}|$: 25

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	76	0.00%	0.68	0.00%	1.32	0.13%	1.89	0.00%	2.5
		0.00%	0	0.00%	0	0.00%	1	0.00%	0
2	81	0.00%	1.92	0.00%	3	0.14%	3.13	0.00%	3.71
		0.00%	1	0.00%	1	0.00%	242	0.00%	1
3	85	0.00%	0.99	0.00%	2	0.13%	1.93	0.00%	3.58
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
4	85	0.00%	1.7	0.00%	3	0.15%	2.02	0.00%	4.89
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
5	86	0.00%	0.94	0.00%	2	0.13%	2.19	0.00%	3.53
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
6	75	0.00%	0.97	0.00%	2	0.11%	1.99	0.00%	2.68
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
7	75	0.00%	0.99	0.00%	2	0.13%	3.2	0.00%	2.87
		0.00%	1	0.00%	1	0.00%	301	0.00%	1
8	84	0.00%	1	0.00%	1.72	0.13%	1.95	0.00%	4.95
		0.00%	1	0.00%	1	0.00%	1	0.00%	1
9	74	0.00%	2.17	0.00%	2.96	0.14%	3.36	0.00%	3.52
		0.00%	1	0.00%	1	0.00%	164	0.00%	1
10	88	0.00%	0.77	0.00%	1.34	0.16%	4.12	0.00%	2.59
		0.00%	0	0.00%	0	0.00%	338	0.00%	0
Average		0.00%	1.21	0.00%	1.97	0.13%	2.58	0.00%	3.48
		0.00%	0.8	0.00%	0.8	0.00%	105.1	0.00%	0.8

Table C.4: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 100, |\mathcal{C}| : 25$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	87	0.01%	2.81	0.00%	4.64	0.36%	27.33	0.00%	11.13
		0.00%	1	0.00%	1	0.00%	1083	0.00%	1
2	83	0.01%	2.84	0.00%	4.8	0.37%	31.13	0.00%	11.83
		0.00%	1	0.00%	1	0.00%	1025	0.00%	1
3	67	0.01%	2.93	0.00%	4.81	0.32%	27.84	0.00%	10.37
		0.00%	1	0.00%	1	0.00%	1059	0.00%	1
4	70	0.01%	3.03	0.00%	4.76	0.32%	27.84	0.00%	10.78
		0.00%	1	0.00%	1	0.00%	1043	0.00%	1
5	78	0.01%	3.04	0.00%	4.89	0.34%	27.44	0.00%	11.31
		0.00%	1	0.00%	1	0.00%	1077	0.00%	1
6	70	0.01%	2.96	0.00%	4.54	0.36%	29.23	0.00%	8.88
		0.00%	1	0.00%	1	0.00%	1059	0.00%	1
7	80	0.01%	3.74	0.00%	5.96	0.35%	29.49	0.00%	10.45
		0.00%	1	0.00%	1	0.00%	1078	0.00%	1
8	88	0.01%	3.56	0.00%	5.62	0.33%	31.57	0.00%	12.15
		0.00%	1	0.00%	1	0.00%	1103	0.00%	1
9	81	0.01%	3.72	0.00%	6.01	0.34%	29.25	0.00%	11.76
		0.00%	1	0.00%	1	0.00%	1086	0.00%	1
10	87	0.02%	3.03	0.00%	4.8	0.37%	33.92	0.00%	12.25
		0.00%	1	0.00%	1	0.00%	1051	0.00%	1
Average		0.01%	3.17	0.00%	5.08	0.35%	29.50	0.00%	11.09
		0.00%	1	0.00%	1	0.00%	1066.4	0.00%	1

Table C.5: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 75, |\mathcal{C}| : 25$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	81	0.01%	4.06	0.00%	3.43	0.36%	20.82	0.00%	8.41
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
2	81	0.02%	3.19	0.00%	3.69	0.43%	28.92	0.00%	5.73
		0.00%	1	0.00%	1	0.00%	1069	0.00%	1
3	75	0.02%	4.38	0.00%	4.75	0.35%	21.22	0.00%	8.93
		0.00%	1	0.00%	1	0.00%	1087	0.00%	1
4	83	0.02%	4.15	0.00%	4.83	0.38%	25.66	0.00%	9.07
		0.00%	1	0.00%	1	0.00%	1121	0.00%	1
5	74	0.01%	2.98	0.00%	3.38	0.38%	22.85	0.00%	7.18
		0.00%	1	0.00%	1	0.00%	1087	0.00%	1
6	74	0.01%	3.76	0.00%	4.31	0.40%	24.85	0.00%	7.23
		0.00%	1	0.00%	1	0.00%	1054	0.00%	1
7	75	0.01%	3.36	0.00%	3.94	0.41%	29	0.00%	7.22
		0.00%	1	0.00%	1	0.00%	1072	0.00%	1
8	86	0.02%	3	0.00%	3.69	0.36%	25.52	0.00%	9.5
		0.00%	1	0.00%	1	0.00%	1040	0.00%	1
9	79	0.02%	2.92	0.00%	3.33	0.33%	23.27	0.00%	7.14
		0.00%	1	0.00%	1	0.00%	1076	0.00%	1
10	72	0.01%	3.67	0.00%	3.39	0.35%	21.59	0.00%	7.96
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
Average		0.01%	3.55	0.00%	3.87	0.38%	24.37	0.00%	7.84
		0.00%	1	0.00%	1	0.00%	1079	0.00%	1

Table C.6: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 50, |\mathcal{C}| : 25$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	67	0.01%	2.26	0.00%	3.32	0.41%	14.73	0.00%	4.54
		0.00%	1	0.00%	1	0.00%	1033	0.00%	1
2	77	0.02%	1.75	0.00%	2.77	0.40%	17.73	0.00%	5.01
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
3	78	0.02%	2.36	0.00%	3.66	0.39%	16.41	0.00%	4.76
		0.00%	1	0.00%	1	0.00%	1050	0.00%	1
4	84	0.02%	2.02	0.00%	3.21	0.32%	14.91	0.00%	3.88
		0.00%	1	0.00%	1	0.00%	1042	0.00%	1
5	79	0.02%	2.23	0.00%	3.61	0.43%	16.85	0.00%	4.7
		0.00%	1	0.00%	1	0.00%	1081	0.00%	1
6	70	0.01%	2.15	0.00%	3.23	0.30%	17.07	0.00%	5.28
		0.00%	1	0.00%	1	0.00%	1032	0.00%	1
7	74	0.01%	1.83	0.00%	2.73	0.27%	20.99	0.00%	4.14
		0.00%	1	0.00%	1	0.00%	1086	0.00%	1
8	76	0.02%	1.92	0.00%	2.9	0.39%	16.55	0.00%	5.44
		0.00%	1	0.00%	1	0.00%	1082	0.00%	1
9	83	0.01%	1.92	0.00%	3.01	0.43%	15.9	0.00%	5.34
		0.00%	1	0.00%	1	0.00%	1034	0.00%	1
10	66	0.01%	2.47	0.00%	3.63	0.42%	15.55	0.00%	5.32
		0.00%	1	0.00%	1	0.00%	1064	0.00%	1
Average		0.02%	2.09	0.00%	3.21	0.38%	16.67	0.00%	4.84
		0.00%	1	0.00%	1	0.00%	1054.9	0.00%	1

Table C.7: Results for instances with 300 products for MMNL model, ϕ : 0.25, $|\mathcal{M}|$: 100, $|\mathcal{C}|$: 25

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	72	0.07%	4.41	0.00%	7.15	1.05%	45.36	0.00%	12.75
		0.00%	1	0.00%	1	0.00%	1069	0.00%	1
2	69	0.10%	6.40	0.01%	9.61	0.89%	41.76	0.00%	15.22
		0.00%	1	0.00%	1	0.00%	1089	0.00%	1
3	64	0.07%	5.40	0.01%	7.72	0.84%	49.34	0.00%	11.21
		0.00%	1	0.00%	1	0.00%	1046	0.00%	1
4	74	0.11%	6.11	0.01%	8.65	0.99%	52.97	0.00%	14.4
		0.00%	1	0.00%	1	0.00%	1088	0.00%	1
5	74	0.09%	5.86	0.01%	9.08	0.94%	54.31	0.00%	14.37
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
6	83	0.13%	6.81	0.01%	9.35	0.93%	53.32	0.00%	14.41
		0.00%	1	0.00%	1	0.00%	1090	0.00%	1
7	56	0.08%	4.86	0.00%	7.94	0.91%	46.25	0.00%	12.45
		0.00%	1	0.00%	1	0.00%	1039	0.00%	1
8	76	0.09%	4.80	0.00%	7.96	0.89%	50.54	0.00%	13.49
		0.00%	1	0.00%	1	0.00%	1064	0.00%	1
9	64	0.09%	6.60	0.01%	8.22	1.14%	62.13	0.00%	16.29
		0.00%	1	0.00%	1	0.00%	1043	0.00%	1
10	72	0.10%	8.11	0.01%	9.3	1.12%	68.92	0.00%	16.63
		0.00%	1	0.00%	1	0.00%	1032	0.00%	1
Average		0.09%	5.94	0.01%	8.50	0.97%	52.49	0.00%	14.12
		0.00%	1	0.00%	1	0.00%	1060.5	0.00%	1

Table C.8: Results for instances with 300 products for MMNL model, $\phi : 0.25, |\mathcal{M}| : 75, |\mathcal{C}| : 25$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	66	0.09%	6.82	0.01%	5.8	1.14%	54.35	0.00%	11.61
		0.00%	1	0.00%	1	0.00%	1079	0.00%	1
2	68	0.08%	4.85	0.01%	4.49	0.94%	39.92	0.00%	9.35
		0.00%	1	0.00%	1	0.00%	1072	0.00%	1
3	83	0.16%	7.66	0.02%	6.62	1.07%	51	0.00%	11.27
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
4	78	0.09%	4.92	0.01%	4.61	0.98%	50.92	0.00%	10.14
		0.00%	1	0.00%	1	0.00%	1038	0.00%	1
5	67	0.09%	5.1	0.01%	4.83	1.04%	56.02	0.00%	9.88
		0.00%	1	0.00%	1	0.00%	1061	0.00%	1
6	68	0.07%	4.01	0.00%	4.27	1.29%	57.09	0.00%	10.3
		0.00%	1	0.00%	1	0.00%	1038	0.00%	1
7	70	0.07%	3.99	0.00%	4.29	0.98%	51.24	0.00%	8.29
		0.00%	1	0.00%	1	0.00%	1076	0.00%	1
8	54	0.05%	4.09	0.01%	4.04	0.89%	43.4	0.00%	9.35
		0.00%	1	0.00%	1	0.00%	1036	0.00%	1
9	74	0.12%	5.29	0.01%	5.07	1.16%	56.55	0.00%	11.39
		0.00%	1	0.00%	1	0.00%	1036	0.00%	1
10	65	0.08%	4.5	0.00%	4.5	0.89%	48.29	0.00%	11.03
		0.00%	1	0.00%	1	0.00%	1035	0.00%	1
Average		0.09%	5.12	0.01%	4.85	1.04%	50.88	0.00%	10.26
		0.00%	1	0.00%	1	0.00%	1051.6	0.00%	1

Table C.9: Results for instances with 300 products for MMNL model, ϕ : 0.25, $|\mathcal{M}|$: 50, $|\mathcal{C}|$: 25

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	64	0.09%	2.57	0.01%	4.08	0.96%	28.23	0.00%	5.41
		0.00%	1	0.00%	1	0.00%	1030	0.00%	1
2	61	0.11%	3.15	0.01%	3.74	1.05%	33.35	0.00%	6.83
		0.00%	1	0.00%	1	0.00%	1080	0.00%	1
3	57	0.07%	2.87	0.01%	3.58	1.09%	31.16	0.00%	6.7
		0.00%	1	0.00%	1	0.00%	1054	0.00%	1
4	65	0.13%	3.35	0.01%	3.2	0.99%	32.66	0.00%	6.89
		0.00%	1	0.00%	1	0.00%	1036	0.00%	1
5	60	0.10%	2.64	0.01%	3.3	0.94%	31.68	0.00%	6.83
		0.00%	1	0.00%	1	0.00%	1053	0.00%	1
6	68	0.13%	2.79	0.01%	3.41	1.01%	34.11	0.00%	6.36
		0.00%	1	0.00%	1	0.00%	1032	0.00%	1
7	62	0.07%	2.73	0.01%	3.39	0.88%	30.27	0.00%	6.22
		0.00%	1	0.00%	1	0.00%	1052	0.00%	1
8	74	0.14%	2.38	0.01%	3.11	0.91%	31.57	0.00%	6
		0.00%	1	0.00%	1	0.00%	1068	0.00%	1
9	62	0.10%	2.55	0.01%	3.34	1.14%	34.66	0.00%	6.9
		0.00%	1	0.00%	1	0.00%	1055	0.00%	1
10	75	0.09%	2.41	0.00%	3.34	1.17%	42.42	0.00%	5.45
		0.00%	1	0.00%	1	0.00%	1058	0.00%	1
Average		0.10%	2.74	0.01%	3.45	1.01%	33.01	0.00%	6.36
		0.00%	1	0.00%	1	0.00%	1051.8	0.00%	1

Table C.10: Results for instances with 300 products for MMNL model, $\phi : 0.75, |\mathcal{M}| : 100, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	165	0.04%	8.33	0.00%	9.52	0.58%	84.16	0.00%	20.22
		0.00%	1	0.00%	1	0.00%	1036	0.00%	1
2	161	0.05%	11.96	0.00%	14.14	0.49%	59.06	0.00%	21.19
		0.00%	1	0.00%	1	0.00%	1097	0.00%	1
3	156	0.04%	10.85	0.00%	10.17	0.56%	56.08	0.00%	20.68
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
4	162	0.04%	9.56	0.00%	8.34	0.55%	66.02	0.00%	20.46
		0.00%	1	0.00%	1	0.00%	1073	0.00%	1
5	152	0.04%	11.15	0.00%	9.75	0.53%	68.5	0.00%	21.53
		0.00%	1	0.00%	1	0.00%	1087	0.00%	1
6	152	0.04%	9.8	0.00%	8.09	0.55%	55.56	0.00%	20.03
		0.00%	1	0.00%	1	0.00%	1095	0.00%	1
7	163	0.04%	5.87	0.00%	7.49	0.50%	65.55	0.00%	20.87
		0.00%	1	0.00%	1	0.00%	1083	0.00%	1
8	146	0.03%	13.3	0.00%	13.58	0.56%	80.18	0.00%	20.9
		0.00%	1	0.00%	1	0.00%	1179	0.00%	1
9	157	0.04%	6.72	0.00%	7.17	0.48%	58.78	0.00%	20.51
		0.00%	1	0.00%	1	0.00%	1129	0.00%	1
10	153	0.04%	9.8	0.00%	9.3	0.48%	73.79	0.00%	19.25
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
Average		0.04%	9.73	0.00%	9.76	0.53%	66.77	0.00%	20.56
		0.00%	1	0.00%	1	0.00%	1090.8	0.00%	1

Table C.11: Results for instances with 300 products for MMNL model, $\phi : 0.75, |\mathcal{M}| : 75, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	149	0.04%	8.19	0.00%	6.61	0.51%	46.8	0.00%	13.43
		0.00%	1	0.00%	1	0.00%	1042	0.00%	1
2	159	0.04%	6.63	0.00%	6.36	0.54%	52.44	0.00%	13.69
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
3	154	0.04%	7.95	0.00%	6.73	0.56%	51.53	0.00%	12.99
		0.00%	1	0.00%	1	0.00%	1064	0.00%	1
4	149	0.04%	7.08	0.00%	6.8	0.50%	58.99	0.00%	13.89
		0.00%	1	0.00%	1	0.00%	1028	0.00%	1
5	157	0.04%	8.42	0.00%	7.51	0.56%	62.93	0.00%	13.88
		0.00%	1	0.00%	1	0.00%	1180	0.00%	1
6	149	0.03%	6.05	0.00%	4.95	0.62%	69.63	0.00%	12.97
		0.00%	1	0.00%	1	0.00%	1101	0.00%	1
7	154	0.03%	6.22	0.00%	6.35	0.49%	64.54	0.00%	13.43
		0.00%	1	0.00%	1	0.00%	1051	0.00%	1
8	135	0.03%	6.47	0.00%	6.18	0.47%	61.44	0.00%	15.95
		0.00%	1	0.00%	1	0.00%	1165	0.00%	1
9	160	0.04%	7.33	0.00%	7.26	0.52%	64.32	0.00%	13.49
		0.00%	1	0.00%	1	0.00%	1113	0.00%	1
10	164	0.05%	8.06	0.00%	6.84	0.50%	66.18	0.00%	13.49
		0.00%	1	0.00%	1	0.00%	1090	0.00%	1
Average		0.04%	7.24	0.00%	6.56	0.53%	59.88	0.00%	13.72
		0.00%	1	0.00%	1	0.00%	1091.8	0.00%	1

Table C.12: Results for instances with 300 products for MMNL model, $\phi : 0.75, |\mathcal{M}| : 50, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	157	0.05%	4	0.00%	5.03	0.54%	42.95	0.00%	7.27
		0.00%	1	0.00%	1	0.00%	1044	0.00%	1
2	151	0.03%	2.29	0.00%	4.26	0.54%	42.74	0.00%	7.16
		0.00%	1	0.00%	1	0.00%	1054	0.00%	1
3	163	0.04%	4.99	0.00%	5.13	0.63%	47.29	0.00%	7.36
		0.00%	1	0.00%	1	0.00%	1114	0.00%	1
4	153	0.03%	4.54	0.00%	4.59	0.52%	47.91	0.00%	8.1
		0.00%	1	0.00%	1	0.00%	1113	0.00%	1
5	162	0.04%	4.23	0.00%	3.53	0.58%	52.46	0.00%	8.41
		0.00%	1	0.00%	1	0.00%	1074	0.00%	1
6	145	0.03%	2.41	0.00%	2.45	0.50%	41.84	0.00%	5.59
		0.00%	1	0.00%	1	0.00%	1078	0.00%	1
7	159	0.04%	3.97	0.00%	4.49	0.49%	48.81	0.00%	7.44
		0.00%	1	0.00%	1	0.00%	1053	0.00%	1
8	166	0.05%	4.82	0.00%	5.25	0.58%	54.88	0.00%	8.31
		0.00%	1	0.00%	1	0.00%	1126	0.00%	1
9	157	0.03%	4.75	0.00%	4.82	0.59%	49.41	0.00%	7.64
		0.00%	1	0.00%	1	0.00%	1054	0.00%	1
10	164	0.04%	4.32	0.00%	4.49	0.54%	51.05	0.00%	9.78
		0.00%	1	0.00%	1	0.00%	1062	0.00%	1
Average		0.04%	4.03	0.00%	4.40	0.55%	47.93	0.00%	7.71
		0.00%	1	0.00%	1	0.00%	1077.2	0.00%	1

Table C.13: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 100, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	126	0.21%	20.45	0.02%	23.23	1.62%	272.86	0.00%	100.26
		0.00%	1	0.00%	1	0.00%	1057	0.00%	1
2	136	0.24%	16.05	0.01%	15.4	1.49%	312.34	0.00%	44.02
		0.00%	1	0.00%	1	0.00%	1078	0.00%	1
3	145	0.26%	19.7	0.01%	22.24	1.45%	456.14	0.00%	59.19
		0.00%	1	0.00%	1	0.00%	1687	0.00%	1
4	152	0.31%	14.32	0.02%	24.67	1.24%	272.22	0.01%	56.14
		0.00%	1	0.00%	1	0.00%	1091	0.00%	1
5	133	0.28%	27.01	0.02%	30	1.51%	512.21	0.01%	78.54
		0.00%	1	0.00%	1	0.00%	1099	0.00%	1
6	141	0.24%	11.81	0.01%	20.14	1.48%	304.59	0.00%	41.93
		0.00%	1	0.00%	1	0.00%	1090	0.00%	1
7	129	0.24%	19.58	0.02%	28.52	1.35%	420.67	0.01%	77.86
		0.00%	1	0.00%	1	0.00%	1097	0.00%	1
8	139	0.26%	16.81	0.02%	21.19	1.47%	391.13	0.01%	85.06
		0.00%	1	0.00%	1	0.00%	1087	0.00%	1
9	149	0.30%	23.8	0.02%	27.27	1.42%	483.56	0.01%	54.9
		0.00%	1	0.00%	1	0.00%	1116	0.00%	1
10	127	0.21%	16.4	0.01%	18.81	1.61%	340.6	0.00%	58.82
		0.00%	1	0.00%	1	0.00%	1103	0.00%	1
Average		0.25%	18.59	0.02%	23.15	1.46%	376.63	0.00%	65.67
		0.00%	1	0.00%	1	0.00%	1150.5	0.00%	1

Table C.14: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 75, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	146	0.29%	14.36	0.02%	16.99	1.39%	293	0.01%	42.89
		0.00%	1	0.00%	1	0.00%	1082	0.00%	1
2	134	0.24%	12.54	0.01%	11.09	1.44%	327.12	0.00%	26.87
		0.00%	1	0.00%	1	0.00%	1120	0.00%	1
3	130	0.28%	13.14	0.02%	12.44	1.33%	265.48	0.00%	40.53
		0.00%	1	0.00%	1	0.00%	1061	0.00%	1
4	124	0.21%	16.23	0.02%	15.78	1.42%	285.93	0.01%	44.94
		0.00%	1	0.00%	1	0.00%	1073	0.00%	1
5	145	0.30%	11.14	0.02%	15.14	1.42%	301.32	0.00%	30.76
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
6	147	0.28%	6.94	0.01%	8.62	1.34%	288.21	0.00%	20.07
		0.00%	1	0.00%	1	0.00%	1106	0.00%	1
7	131	0.28%	14.39	0.02%	17.14	1.49%	355.4	0.00%	43.45
		0.00%	1	0.00%	1	0.00%	1108	0.00%	1
8	137	0.28%	15.72	0.02%	15.37	1.37%	326.59	0.01%	50.07
		0.00%	1	0.00%	1	0.00%	1162	0.00%	1
9	131	0.24%	13.83	0.02%	17.55	1.46%	330.71	0.01%	47.01
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
10	135	0.23%	21.6	0.03%	16.74	1.50%	-	0.01%	46.09
		0.00%	1	0.00%	1	0.03%	14090	0.00%	1
Average		0.26%	13.99	0.02%	14.69	1.42%	457.39	0.00%	39.27
		0.00%	1	0.00%	1	0.00%	2398.6	0.00%	1

Table C.15: Results for instances with 300 products for MMNL model, $\phi : 0.5, |\mathcal{M}| : 50, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	125	0.20%	7.42	0.02%	9.29	1.38%	-	0.00%	20.11
		0.00%	1	0.00%	1	0.01%	39510	0.00%	1
2	140	0.27%	6.71	0.02%	9	1.51%	213.98	0.01%	16.29
		0.00%	1	0.00%	1	0.00%	1073	0.00%	1
3	124	0.24%	9.24	0.03%	12.65	1.49%	197.58	0.01%	23.76
		0.00%	1	0.00%	1	0.00%	1063	0.00%	1
4	139	0.26%	11.7	0.03%	15.55	1.67%	-	0.01%	17.77
		0.00%	1	0.00%	1	0.06%	21131	0.00%	1
5	136	0.25%	7.8	0.01%	8.46	1.42%	202.14	0.00%	15.33
		0.00%	1	0.00%	1	0.00%	1088	0.00%	1
6	143	0.26%	6.15	0.01%	8.25	1.55%	224.88	0.00%	19.45
		0.00%	1	0.00%	1	0.00%	1098	0.00%	1
7	126	0.26%	8.88	0.02%	11.44	1.51%	250.45	0.01%	19.43
		0.00%	1	0.00%	1	0.00%	1058	0.00%	1
8	139	0.31%	7.26	0.02%	10.1	1.54%	210.41	0.01%	19.64
		0.00%	1	0.00%	1	0.00%	1074	0.00%	1
9	135	0.26%	8	0.02%	10.13	1.57%	201.72	0.00%	19.46
		0.00%	1	0.00%	1	0.00%	1067	0.00%	1
10	138	0.24%	7.09	0.01%	9.97	1.68%	231.56	0.01%	16.34
		0.00%	1	0.00%	1	0.00%	1065	0.00%	1
Average		0.26%	8.03	0.02%	10.48	1.53%	533.34	0.01%	18.76
		0.00%	1	0.00%	1	0.01%	6922.7	0.00%	1

Table C.16: Results for instances with 300 products for MMNL model, ϕ : 0.25, $|\mathcal{M}|$: 100, $|\mathcal{C}|$: 75

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	103	1.17%	1422.31	0.19%	202.84	3.91%	-	0.07%	226.45
		0.00%	9236	0.00%	1	0.12%	1204	0.00%	1
2	100	1.29%	1785.45	0.22%	495.54	3.79%	-	0.09%	289.3
		0.00%	14347	0.00%	6836	0.16%	1100	0.00%	1
3	107	1.50%	-	0.30%	-	3.45%	-	0.13%	378.86
		0.15%	6094	0.05%	18768	0.09%	1126	0.00%	1
4	99	1.08%	687.83	0.19%	152.5	3.56%	-	0.07%	267.5
		0.00%	3970	0.00%	1	0.93%	8206	0.00%	1
5	107	1.26%	-	0.20%	297.89	4.01%	-	0.07%	197.85
		0.04%	7502	0.00%	1	0.14%	1117	0.00%	1
6	106	1.29%	-	0.28%	1495.34	3.65%	-	0.12%	350.36
		0.14%	6890	0.00%	23975	0.13%	1124	0.00%	1
7	107	1.12%	613.18	0.16%	106.31	3.20%	-	0.05%	157.04
		0.00%	2598	0.00%	1	0.05%	1235	0.00%	1
8	105	1.27%	-	0.24%	378.02	3.26%	-	0.10%	302.07
		0.06%	8520	0.00%	1	0.07%	1113	0.00%	1
9	104	1.24%	-	0.24%	869.94	4.20%	-	0.10%	276.45
		0.06%	8403	0.00%	6539	1.22%	4911	0.00%	1
10	102	1.09%	1077.07	0.19%	259.7	3.75%	-	0.09%	268.68
		0.00%	5865	0.00%	2582	0.11%	1139	0.00%	1
Average		1.23%	1117.17	0.22%	473.12	3.68%	-	0.09%	271.46
		0.04%	7342.5	0.01%	5870.5	0.30%	2227.5	0.00%	1

Table C.17: Results for instances with 300 products for MMNL model, $\phi : 0.25, |\mathcal{M}| : 75, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	109	1.08%	364.48	0.21%	203.24	3.90%	-	0.09%	135.08
		0.00%	3436	0.00%	1556	0.06%	2126	0.00%	1
2	103	1.20%	514.78	0.22%	317.21	3.79%	-	0.08%	138.22
		0.00%	4934	0.00%	1	0.07%	2911	0.00%	1
3	106	1.32%	842.45	0.20%	258.32	3.21%	636.78	0.07%	106.34
		0.00%	8950	0.00%	1	0.00%	1185	0.00%	1
4	102	1.28%	404.62	0.17%	148.8	3.64%	-	0.06%	116.06
		0.00%	2148	0.00%	1	0.07%	4902	0.00%	1
5	111	1.39%	-	0.26%	713.66	3.37%	-	0.08%	115.8
		0.06%	20844	0.00%	13312	0.02%	3306	0.00%	1
6	108	1.18%	617.39	0.17%	249.65	3.58%	-	0.06%	136.32
		0.00%	4215	0.00%	1	0.03%	2582	0.00%	1
7	98	1.07%	267.46	0.15%	122.56	3.42%	-	0.07%	134.75
		0.00%	1655	0.00%	1	0.03%	4204	0.00%	1
8	109	1.42%	-	0.23%	219.1	3.22%	1641.96	0.08%	149.33
		0.01%	17712	0.00%	2713	0.00%	3280	0.00%	1
9	101	1.33%	1402.91	0.26%	235.15	3.52%	1265.06	0.07%	138.38
		0.00%	14959	0.00%	1722	0.00%	1167	0.00%	1
10	112	1.44%	-	0.28%	784.25	3.87%	-	0.10%	163.87
		0.05%	20916	0.00%	12881	0.04%	1881	0.00%	1
Average		1.27%	630.58	0.21%	325.19	3.55%	1181.27	0.08%	133.42
		0.01%	9976.9	0.00%	3218.9	0.03%	2754.4	0.00%	1

Table C.18: Results for instances with 300 products for MMNL model, $\phi : 0.25, |\mathcal{M}| : 50, |\mathcal{C}| : 75$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	1.42%	1357.59	0.28%	236.14	3.18%	-	0.10%	64.65
		0.00%	36210	0.00%	5535	0.02%	8045	0.00%	1
2	98	1.19%	249.1	0.23%	124.06	3.83%	-	0.09%	60.93
		0.00%	2985	0.00%	1	0.74%	18669	0.00%	1
3	110	1.24%	226.53	0.21%	138.17	3.75%	1625.33	0.06%	58.62
		0.00%	2793	0.00%	690	0.00%	4970	0.00%	1
4	95	1.27%	852.54	0.25%	404.04	3.78%	-	0.13%	66.31
		0.00%	20793	0.00%	11188	0.06%	3074	0.00%	1
5	114	1.23%	137.36	0.16%	116.63	3.24%	513.04	0.05%	68.8
		0.00%	1322	0.00%	1	0.00%	1285	0.00%	1
6	89	1.23%	208.79	0.22%	137.82	3.49%	-	0.09%	60.14
		0.00%	2993	0.00%	1	0.03%	5611	0.00%	1
7	101	1.50%	621.01	0.32%	214.59	3.26%	952.33	0.13%	103.32
		0.00%	8498	0.00%	2732	0.00%	3484	0.00%	1
8	100	1.20%	221.12	0.23%	184.18	3.39%	736.6	0.09%	65.6
		0.00%	3686	0.00%	1	0.00%	1825	0.00%	1
9	117	1.52%	-	0.31%	827.13	3.33%	1433.42	0.12%	95.82
		0.04%	30416	0.00%	13194	0.00%	3435	0.00%	1
10	102	1.18%	309.18	0.23%	140.54	3.50%	836.07	0.09%	60.94
		0.00%	4509	0.00%	225	0.00%	1974	0.00%	1
Average		1.30%	464.80	0.24%	252.33	3.48%	1016.13	0.09%	70.51
		0.00%	11420.5	0.00%	3356.8	0.08%	5237.2	0.00%	1

Table C.19: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 100, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	177	0.08%	21.91	0.00%	17.64	0.71%	99.48	0.00%	38.74
		0.00%	1	0.00%	1	0.00%	1115	0.00%	1
2	197	0.12%	18.82	0.00%	15	0.66%	120.41	0.00%	34.82
		0.00%	1	0.00%	1	0.00%	1046	0.00%	1
3	190	0.09%	22.5	0.00%	17.14	0.71%	79.64	0.00%	35.98
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
4	192	0.10%	27.75	0.00%	18.47	0.68%	111.66	0.00%	37.51
		0.00%	1	0.00%	1	0.00%	1031	0.00%	1
5	192	0.08%	20.72	0.00%	16.82	0.72%	127.37	0.00%	34.34
		0.00%	1	0.00%	1	0.00%	1046	0.00%	1
6	182	0.09%	25.8	0.00%	15.34	0.72%	138.55	0.00%	36.47
		0.00%	1	0.00%	1	0.00%	1182	0.00%	1
7	178	0.09%	21.73	0.00%	19.43	0.70%	167.83	0.00%	40.52
		0.00%	1	0.00%	1	0.00%	1201	0.00%	1
8	166	0.07%	24.49	0.00%	17.39	0.70%	126.12	0.00%	36.92
		0.00%	1	0.00%	1	0.00%	1033	0.00%	1
9	173	0.09%	20.16	0.00%	18.88	0.70%	166.08	0.00%	36.89
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
10	184	0.09%	19.63	0.00%	16.89	0.74%	112.58	0.00%	33.81
		0.00%	1	0.00%	1	0.00%	1068	0.00%	1
Average		0.09%	22.35	0.00%	17.3	0.70%	124.97	0.00%	36.6
		0.00%	1	0.00%	1	0.00%	1085.1	0.00%	1

Table C.20: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 75, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	178	0.08%	14.27	0.00%	10.61	0.66%	101.6	0.00%	23.4
		0.00%	1	0.00%	1	0.00%	1086	0.00%	1
2	190	0.09%	23.49	0.00%	13.18	0.79%	93.62	0.00%	23.62
		0.00%	1	0.00%	1	0.00%	1149	0.00%	1
3	179	0.09%	15.42	0.00%	11.28	0.75%	109.42	0.00%	22.8
		0.00%	1	0.00%	1	0.00%	1031	0.00%	1
4	182	0.08%	16.89	0.00%	13.18	0.76%	111.45	0.00%	22.53
		0.00%	1	0.00%	1	0.00%	1055	0.00%	1
5	181	0.08%	16.5	0.00%	11.25	0.74%	100.03	0.00%	20.36
		0.00%	1	0.00%	1	0.00%	1184	0.00%	1
6	175	0.08%	16.77	0.00%	9.7	0.65%	114.55	0.00%	22.59
		0.00%	1	0.00%	1	0.00%	1135	0.00%	1
7	180	0.09%	18.79	0.00%	14.33	0.75%	124.69	0.00%	22.22
		0.00%	1	0.00%	1	0.00%	1134	0.00%	1
8	174	0.08%	18.05	0.00%	12.63	0.72%	150.98	0.00%	21.33
		0.00%	1	0.00%	1	0.00%	1137	0.00%	1
9	174	0.08%	20.75	0.00%	18.73	0.75%	95.39	0.00%	25.44
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
10	177	0.08%	17.51	0.00%	11.29	0.73%	118.27	0.00%	25.24
		0.00%	1	0.00%	1	0.00%	1085	0.00%	1
Average		0.08%	17.84	0.00%	12.62	0.73%	112	0.00%	22.95
		0.00%	1	0.00%	1	0.00%	1109.6	0.00%	1

Table C.21: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 50, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	176	0.08%	9.2	0.00%	8.1	0.73%	63.29	0.00%	12.58
		0.00%	1	0.00%	1	0.00%	1111	0.00%	1
2	166	0.07%	7.23	0.00%	5.86	0.79%	69.94	0.00%	11.47
		0.00%	1	0.00%	1	0.00%	1045	0.00%	1
3	178	0.09%	8.92	0.00%	8.08	0.82%	86.92	0.00%	12.94
		0.00%	1	0.00%	1	0.00%	1098	0.00%	1
4	176	0.08%	9.14	0.00%	9.5	0.70%	95.41	0.00%	13.81
		0.00%	1	0.00%	1	0.00%	1153	0.00%	1
5	184	0.10%	9.78	0.00%	9.16	0.75%	72.76	0.00%	14.38
		0.00%	1	0.00%	1	0.00%	1115	0.00%	1
6	189	0.09%	9.46	0.00%	6.65	0.77%	75.42	0.00%	12.65
		0.00%	1	0.00%	1	0.00%	1025	0.00%	1
7	186	0.09%	7.01	0.00%	7	0.71%	70.14	0.00%	12.26
		0.00%	1	0.00%	1	0.00%	1052	0.00%	1
8	177	0.08%	9.65	0.00%	9.25	0.76%	96.32	0.00%	15.49
		0.00%	1	0.00%	1	0.00%	1189	0.00%	1
9	181	0.09%	8.48	0.00%	8.71	0.66%	76.14	0.00%	14.46
		0.00%	1	0.00%	1	0.00%	1079	0.00%	1
10	178	0.09%	9.19	0.00%	8.78	0.75%	86.56	0.00%	15.36
		0.00%	1	0.00%	1	0.00%	1052	0.00%	1
Average		0.09%	8.81	0.00%	8.11	0.74%	79.29	0.00%	13.54
		0.00%	1	0.00%	1	0.00%	1091.9	0.00%	1

Table C.22: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 100, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	153	0.56%	77.12	0.04%	82.53	1.85%	569.17	0.01%	140.94
		0.00%	1	0.00%	1	0.00%	1089	0.00%	1
2	144	0.47%	85.46	0.05%	78.93	1.84%	1010.71	0.02%	116.66
		0.00%	1	0.00%	1	0.00%	1854	0.00%	1
3	158	0.56%	93.11	0.04%	81.86	1.85%	832.69	0.02%	117.72
		0.00%	1	0.00%	1	0.00%	1107	0.00%	1
4	139	0.49%	84.41	0.04%	67.36	2.10%	-	0.02%	166.2
		0.00%	1	0.00%	1	0.02%	2530	0.00%	1
5	156	0.53%	51.73	0.03%	47.41	2.01%	1337.12	0.01%	177.22
		0.00%	1	0.00%	1	0.00%	1746	0.00%	1
6	153	0.55%	222.9	0.06%	167.44	2.13%	1415.74	0.03%	169.12
		0.00%	1068	0.00%	1	0.00%	1644	0.00%	1
7	145	0.53%	377.91	0.06%	97.53	1.79%	1098.89	0.02%	182.18
		0.00%	8247	0.00%	1	0.00%	1799	0.00%	1
8	157	0.52%	64.17	0.03%	58.05	1.96%	1364.35	0.01%	136.69
		0.00%	1	0.00%	1	0.00%	1847	0.00%	1
9	147	0.46%	69.83	0.04%	62.7	1.96%	535.08	0.01%	199.56
		0.00%	1	0.00%	1	0.00%	1104	0.00%	1
10	148	0.39%	48.61	0.02%	36.08	2.03%	-	0.01%	120.33
		0.00%	1	0.00%	1	0.01%	2829	0.00%	1
Average		0.51%	117.53	0.04%	77.99	1.95%	1020.47	0.02%	152.66
		0.00%	932.3	0.00%	1	0.00%	1754.9	0.00%	1

Table C.23: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 75, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	152	0.55%	50.35	0.03%	50.09	1.95%	345.04	0.01%	78.59
		0.00%	1	0.00%	1	0.00%	1083	0.00%	1
2	149	0.52%	74.47	0.05%	69.7	1.91%	578.62	0.03%	116.97
		0.00%	1	0.00%	1	0.00%	1137	0.00%	1
3	149	0.50%	50.98	0.04%	46.73	1.93%	397.14	0.01%	90.55
		0.00%	1	0.00%	1	0.00%	1084	0.00%	1
4	156	0.52%	42.31	0.03%	26.99	1.91%	380.85	0.01%	68.68
		0.00%	1	0.00%	1	0.00%	1110	0.00%	1
5	149	0.41%	38.19	0.01%	20.72	2.13%	566.82	0.00%	50.23
		0.00%	1	0.00%	1	0.00%	1539	0.00%	1
6	148	0.53%	50.09	0.04%	46.05	1.66%	526.92	0.01%	70.59
		0.00%	1	0.00%	1	0.00%	1117	0.00%	1
7	140	0.44%	41.13	0.03%	24.75	2.04%	468.88	0.01%	70.93
		0.00%	1	0.00%	1	0.00%	1080	0.00%	1
8	143	0.50%	53.77	0.03%	37.14	1.98%	447.31	0.02%	73.64
		0.00%	1	0.00%	1	0.00%	1093	0.00%	1
9	162	0.54%	59.43	0.04%	47.52	2.13%	985.46	0.01%	156.53
		0.00%	1	0.00%	1	0.00%	1317	0.00%	1
10	147	0.46%	44.42	0.03%	36.96	1.91%	548.58	0.01%	64.96
		0.00%	1	0.00%	1	0.00%	1142	0.00%	1
Average		0.50%	50.51	0.03%	40.67	1.96%	524.56	0.01%	84.17
		0.00%	1	0.00%	1	0.00%	1170.2	0.00%	1

Table C.24: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 50, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	137	0.43%	24.15	0.04%	21.99	2.03%	-	0.03%	37.79
		0.00%	1	0.00%	1	0.14%	20601	0.00%	1
2	144	0.43%	12.15	0.02%	9.38	1.85%	87.54	0.00%	24.14
		0.00%	1	0.00%	1	0.00%	1095	0.00%	1
3	149	0.50%	21.46	0.04%	22.5	1.67%	131.62	0.02%	24.73
		0.00%	1	0.00%	1	0.00%	1115	0.00%	1
4	146	0.48%	25.11	0.04%	22.2	1.89%	104.41	0.02%	28.16
		0.00%	1	0.00%	1	0.00%	1091	0.00%	1
5	139	0.43%	22.35	0.03%	18.5	2.03%	165.17	0.02%	23.46
		0.00%	1	0.00%	1	0.00%	1484	0.00%	1
6	145	0.50%	18.11	0.03%	14.58	1.80%	107.22	0.01%	27.95
		0.00%	1	0.00%	1	0.00%	1083	0.00%	1
7	151	0.51%	25.58	0.04%	22.97	2.06%	145.36	0.01%	60.09
		0.00%	1	0.00%	1	0.00%	1078	0.00%	1
8	144	0.54%	38.54	0.05%	33.15	2.05%	419.98	0.03%	45.92
		0.00%	1	0.00%	1	0.00%	3268	0.00%	1
9	145	0.52%	15.45	0.03%	13.23	1.70%	69.96	0.01%	33.41
		0.00%	1	0.00%	1	0.00%	1056	0.00%	1
10	141	0.43%	23.83	0.02%	15.94	2.09%	213.06	0.01%	28.56
		0.00%	1	0.00%	1	0.00%	1301	0.00%	1
Average		0.48%	22.67	0.04%	19.44	1.92%	160.48	0.02%	33.42
		0.00%	1	0.00%	1	0.01%	3317.2	0.00%	1

Table C.25: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 100, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	97	1.75%	-	0.45%	-	5.20%	-	0.21%	700.85
		0.34%	1129	0.11%	12940	0.73%	1389	0.00%	1
2	105	2.06%	-	0.49%	-	4.58%	-	0.19%	729.57
		0.40%	1079	0.16%	12037	0.32%	1118	0.00%	1
3	102	2.32%	-	0.61%	-	4.30%	-	0.26%	1592.33
		0.64%	312	0.28%	10047	0.29%	1121	0.00%	1
4	106	2.27%	-	0.50%	-	4.86%	-	0.23%	1177.8
		0.57%	1028	0.19%	8580	0.37%	1119	0.00%	1
5	117	2.17%	-	0.47%	-	4.40%	-	0.21%	544.79
		0.65%	1033	0.17%	17052	0.25%	1426	0.00%	1
6	108	2.04%	-	0.43%	-	4.66%	-	0.16%	484.78
		0.45%	900	0.12%	13314	0.26%	1127	0.00%	1
7	108	2.19%	-	0.50%	-	4.68%	-	0.20%	914.86
		0.52%	544	0.17%	11372	0.35%	1124	0.00%	1
8	114	2.27%	-	0.52%	-	4.96%	-	0.25%	1327.98
		0.52%	913	0.20%	4960	0.61%	1574	0.00%	1
9	96	1.58%	-	0.30%	940.47	5.47%	-	0.13%	427.63
		0.23%	1061	0.00%	6980	0.64%	1155	0.00%	1
10	117	2.00%	-	0.49%	-	4.82%	-	0.23%	910.25
		0.50%	1046	0.21%	8805	0.46%	1111	0.00%	1
Average		2.07%	-	0.48%	940.47	4.79%	-	0.21%	881.08
		0.48%	904.5	0.16%	10608.7	0.43%	1226.4	0.00%	1

Table C.26: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 75, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	94	1.90%	-	0.49%	-	5.15%	-	0.19%	513.43
		0.28%	1612	0.11%	18272	0.41%	1557	0.00%	1
2	113	2.16%	-	0.41%	-	4.30%	-	0.15%	266.35
		0.24%	2419	0.00%	21836	0.05%	1305	0.00%	1
3	104	2.49%	-	0.63%	-	4.53%	-	0.23%	447.44
		0.63%	1094	0.28%	7133	0.20%	1234	0.00%	1
4	113	2.12%	-	0.38%	-	4.28%	-	0.14%	254.2
		0.22%	2306	0.02%	35037	0.06%	1383	0.00%	1
5	109	1.92%	-	0.48%	-	4.94%	-	0.24%	530.63
		0.33%	1916	0.13%	20667	0.25%	1172	0.00%	1
6	111	2.05%	-	0.62%	-	4.61%	-	0.21%	265.15
		0.28%	3586	0.37%	1	0.23%	1257	0.00%	1
7	117	2.17%	-	0.46%	-	4.68%	-	0.20%	339.01
		0.46%	1089	0.11%	19917	0.26%	1393	0.00%	1
8	112	2.44%	-	0.66%	-	4.06%	-	0.26%	559.72
		0.68%	1077	0.24%	11957	0.14%	1161	0.00%	1
9	113	2.21%	-	0.44%	-	4.27%	-	0.14%	245.04
		0.40%	1098	0.09%	20749	0.07%	1252	0.00%	1
10	108	2.30%	-	0.58%	-	4.93%	-	0.26%	643.2
		0.60%	1083	0.23%	9782	0.36%	1423	0.00%	1
Average		2.17%	-	0.51%	-	4.58%	-	0.20%	406.42
		0.41%	1728	0.16%	16535.1	0.20%	1313.7	0.00%	1

Table C.27: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 50, |\mathcal{C}| : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	103	2.10%	-	0.49%	-	4.82%	-	0.20%	164.95
		0.17%	6466	0.06%	20958	0.09%	3803	0.00%	1
2	111	2.36%	-	0.50%	-	3.87%	-	0.16%	116.1
		0.22%	6586	0.09%	24237	0.01%	3650	0.00%	1
3	107	2.30%	-	0.39%	-	4.36%	-	0.16%	93.03
		0.16%	6097	0.04%	22428	0.02%	1826	0.00%	1
4	104	1.96%	-	0.38%	1530.12	4.79%	-	0.18%	106.21
		0.15%	7212	0.00%	26702	0.11%	1357	0.00%	1
5	108	2.34%	-	0.50%	-	4.96%	-	0.23%	192.12
		0.35%	3488	0.14%	21062	0.21%	1457	0.00%	1
6	98	2.14%	-	0.48%	-	4.80%	-	0.23%	183.35
		0.23%	6953	0.10%	21030	1.15%	19186	0.00%	1
7	108	2.11%	-	0.42%	1144.83	4.60%	-	0.17%	138.05
		0.06%	13354	0.00%	24749	0.01%	1436	0.00%	1
8	101	1.96%	-	0.40%	1178.13	4.87%	-	0.20%	115.81
		0.07%	8951	0.00%	19920	0.20%	5581	0.00%	1
9	107	2.37%	-	0.49%	-	4.32%	-	0.20%	114.37
		0.34%	6057	0.13%	20552	0.05%	2075	0.00%	1
10	101	1.98%	-	0.45%	-	5.58%	-	0.21%	161.11
		0.22%	6104	0.07%	20836	0.36%	1606	0.00%	1
Average		2.16%	-	0.45%	1284.36	4.70%	-	0.19%	138.51
		0.20%	7126.8	0.06%	22247.4	0.22%	4197.7	0.00%	1

Appendix D

Tables for Assortment Optimization with Fixed Costs under MMNL Model with Capacity Constraints

Table D.1: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.06%	32.5	0.00%	31.63	0.59%	718.94	0.00%	48.9
		0.00%	1	0.00%	1	0.00%	2352	0.00%	1
2	100	0.06%	34.22	0.00%	34.57	0.55%	509.57	0.00%	51.62
		0.00%	1	0.00%	1	0.00%	2691	0.00%	1
3	100	0.05%	28.04	0.00%	30.49	0.46%	461	0.00%	35.42
		0.00%	1	0.00%	1	0.00%	2797	0.00%	1
4	100	0.06%	30.75	0.00%	30.31	0.52%	578.32	0.00%	36.7
		0.00%	1	0.00%	1	0.00%	2207	0.00%	1
5	100	0.06%	27.63	0.00%	29.77	0.56%	511.88	0.00%	38.66
		0.00%	1	0.00%	1	0.00%	2056	0.00%	1
6	100	0.06%	22.72	0.00%	27.79	0.59%	521.88	0.00%	59.89
		0.00%	1	0.00%	1	0.00%	1672	0.00%	1
7	100	0.05%	20.32	0.00%	28.02	0.52%	-	0.00%	41.37
		0.00%	1	0.00%	1	0.00%	25220	0.00%	1
8	100	0.05%	28.47	0.00%	30.17	0.52%	430.88	0.00%	36
		0.00%	1	0.00%	1	0.00%	2511	0.00%	1
9	100	0.07%	23.28	0.00%	30.93	0.46%	476.46	0.00%	51.73
		0.00%	1	0.00%	1	0.00%	2918	0.00%	1
10	100	0.06%	19.58	0.00%	29.14	0.56%	529.97	0.00%	33.36
		0.00%	1	0.00%	1	0.00%	1837	0.00%	1
Average		0.06%	26.751	0.00%	30.282	0.53%	653.905	0.00%	43.365
		0.00%	1	0.00%	1	0.00%	4626.1	0.00%	1

Table D.2: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.06%	20.63	0.00%	15.14	0.55%	174.33	0.00%	21.55
		0.00%	1	0.00%	1	0.00%	1571	0.00%	1
2	100	0.07%	17.92	0.00%	17.29	0.53%	161.97	0.00%	28.31
		0.00%	1	0.00%	1	0.00%	1148	0.00%	1
3	100	0.06%	18.6	0.00%	16.9	0.51%	138.89	0.00%	24.36
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
4	100	0.05%	21.76	0.00%	16.36	0.57%	169.66	0.00%	30.13
		0.00%	1	0.00%	1	0.00%	1149	0.00%	1
5	99	0.05%	14.07	0.00%	16.62	0.54%	191.07	0.00%	24.97
		0.00%	1	0.00%	1	0.00%	1518	0.00%	1
6	99	0.06%	15.32	0.00%	14.61	0.63%	186.63	0.00%	24.78
		0.00%	1	0.00%	1	0.00%	1156	0.00%	1
7	100	0.06%	21.03	0.00%	16.97	0.58%	205.24	0.00%	24.23
		0.00%	1	0.00%	1	0.00%	1503	0.00%	1
8	100	0.06%	20.08	0.00%	15.89	0.64%	180.75	0.00%	31.2
		0.00%	1	0.00%	1	0.00%	1141	0.00%	1
9	100	0.06%	13.24	0.00%	14.62	0.56%	162.63	0.00%	22.44
		0.00%	1	0.00%	1	0.00%	1221	0.00%	1
10	100	0.06%	22.61	0.00%	15.77	0.43%	175.54	0.00%	30.16
		0.00%	1	0.00%	1	0.00%	1096	0.00%	1
Average		0.06%	18.526	0.00%	16.017	0.55%	174.671	0.00%	26.213
		0.00%	1	0.00%	1	0.00%	1260.3	0.00%	1

Table D.3: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.06%	9.63	0.00%	9.01	0.56%	102.28	0.00%	15.16
		0.00%	1	0.00%	1	0.00%	1047	0.00%	1
2	99	0.05%	6.84	0.00%	8.77	0.53%	93.05	0.01%	13.64
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
3	100	0.06%	9.92	0.00%	8.41	0.54%	109.42	0.00%	11.5
		0.00%	1	0.00%	1	0.00%	1100	0.00%	1
4	100	0.07%	9.88	0.01%	8.29	0.57%	139.28	0.00%	14.55
		0.00%	1	0.00%	1	0.00%	1085	0.00%	1
5	100	0.05%	7.77	0.00%	8.81	0.57%	104.92	0.00%	13.67
		0.00%	1	0.00%	1	0.00%	1092	0.00%	1
6	100	0.07%	14.1	0.01%	8.98	0.52%	94.83	0.00%	14.94
		0.00%	1	0.00%	1	0.00%	1053	0.00%	1
7	100	0.06%	11.64	0.01%	8.32	0.55%	103.16	0.00%	14.87
		0.00%	1	0.00%	1	0.00%	1260	0.00%	1
8	100	0.07%	13.63	0.00%	9	0.57%	109.31	0.00%	14.25
		0.00%	1	0.00%	1	0.00%	1157	0.00%	1
9	100	0.06%	7.3	0.00%	8.28	0.63%	100.02	0.00%	13.2
		0.00%	1	0.00%	1	0.00%	1051	0.00%	1
10	100	0.06%	10.08	0.00%	8.96	0.62%	126.23	0.00%	14.71
		0.00%	1	0.00%	1	0.00%	1124	0.00%	1
Average		0.06%	10.079	0.00%	8.683	0.57%	108.25	0.00%	14.049
		0.00%	1	0.00%	1	0.00%	1106.9	0.00%	1

Table D.4: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.43%	77.45	0.05%	66.38	1.54%	1549.88	0.01%	174.5
		0.00%	1	0.00%	1	0.01%	3183	0.00%	1
2	100	0.36%	70.66	0.04%	52.76	1.63%	1370.86	0.01%	156.47
		0.00%	1	0.00%	1	0.01%	3100	0.00%	1
3	100	0.43%	70.98	0.04%	57.61	1.51%	727.83	0.00%	146.1
		0.00%	1	0.00%	1	0.00%	1236	0.00%	1
4	100	0.49%	119.34	0.06%	105.91	2.73%	-	0.01%	129.04
		0.00%	1	0.00%	1	0.00%	1474	0.00%	1
5	100	0.47%	95.87	0.05%	88.96	1.47%	1498.88	0.01%	153.26
		0.00%	1	0.00%	1	0.00%	1895	0.00%	1
6	100	0.45%	130.52	0.07%	131.69	1.75%	1744.32	0.01%	199.94
		0.00%	1	0.00%	1	0.01%	1196	0.00%	1
7	100	0.43%	111.72	0.05%	106.4	2.00%	-	0.01%	205.67
		0.00%	1	0.00%	1	0.01%	1473	0.00%	1
8	100	0.47%	141.57	0.06%	125.92	1.61%	-	0.01%	206.8
		0.00%	1	0.00%	1	0.00%	1405	0.00%	1
9	100	0.50%	82.71	0.05%	82.66	1.54%	-	0.00%	118.17
		0.00%	1	0.00%	1	0.00%	1594	0.00%	1
10	100	0.40%	79.89	0.04%	77.84	3.14%	-	0.01%	169.25
		0.00%	1	0.00%	1	0.00%	1349	0.00%	1
Average		0.44%	98.071	0.05%	89.613	1.89%	1378.354	0.01%	165.92
		0.00%	1	0.00%	1	0.01%	1790.5	0.00%	1

Table D.5: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.51%	83.87	0.06%	77.29	1.46%	417.85	0.01%	101.64
		0.00%	1	0.00%	1	0.00%	1251	0.00%	1
2	100	0.45%	77.54	0.05%	78.18	1.85%	1398.8	0.01%	114.78
		0.00%	1	0.00%	1	0.00%	2251	0.00%	1
3	100	0.47%	55.99	0.05%	64.09	1.52%	917.9	0.01%	107.25
		0.00%	1	0.00%	1	0.00%	2952	0.00%	1
4	100	0.46%	79.13	0.06%	82.94	1.84%	1115.03	0.01%	120.45
		0.00%	1	0.00%	1	0.00%	1388	0.00%	1
5	100	0.44%	68.83	0.05%	59.94	1.72%	696.7	0.01%	111.61
		0.00%	1	0.00%	1	0.00%	1345	0.00%	1
6	100	0.45%	57.9	0.04%	46.72	1.74%	812.37	0.01%	97.93
		0.00%	1	0.00%	1	0.00%	2622	0.00%	1
7	100	0.38%	57.04	0.05%	49.97	1.73%	1371.26	0.01%	105.83
		0.00%	1	0.00%	1	0.00%	2654	0.00%	1
8	100	0.44%	76.82	0.06%	85.87	1.59%	1165.97	0.01%	96.72
		0.00%	1	0.00%	1	0.00%	3932	0.00%	1
9	100	0.50%	88.55	0.06%	90.43	1.45%	785.71	0.01%	180.94
		0.00%	1	0.00%	1	0.00%	2021	0.00%	94
10	100	0.47%	46.57	0.04%	44.57	1.56%	1267.88	0.01%	88.58
		0.00%	1	0.00%	1	0.00%	2204	0.00%	1
Average		0.46%	69.224	0.05%	68	1.65%	994.947	0.01%	112.573
		0.00%	1	0.00%	1	0.00%	2262	0.00%	10.3

Table D.6: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	0.46%	35.47	0.05%	43.44	1.83%	542.59	0.01%	36.75
		0.00%	1	0.00%	1	0.00%	1511	0.00%	1
2	100	0.46%	25.11	0.04%	27.19	1.57%	353.7	0.01%	34.74
		0.00%	1	0.00%	1	0.00%	1690	0.00%	1
3	100	0.46%	27.9	0.05%	35.73	1.54%	356	0.01%	46.56
		0.01%	1	0.01%	1	0.00%	1995	0.00%	1
4	100	0.42%	21.03	0.04%	31.02	1.34%	418.43	0.01%	29.62
		0.00%	1	0.00%	1	0.00%	1208	0.00%	1
5	100	0.50%	33.23	0.05%	32.7	1.49%	395.94	0.00%	43.48
		0.01%	1	0.00%	1	0.00%	2912	0.00%	1
6	100	0.44%	27.35	0.04%	30.89	1.51%	303.73	0.00%	35.98
		0.00%	1	0.00%	1	0.00%	1631	0.00%	1
7	100	0.51%	36.17	0.06%	49.46	1.31%	475.31	0.00%	41.33
		0.00%	1	0.01%	1	0.00%	1494	0.00%	1
8	100	0.41%	21.8	0.03%	31.94	1.64%	571.43	0.00%	33.82
		0.01%	1	0.01%	1	0.00%	2421	0.00%	1
9	100	0.37%	25.53	0.04%	30.8	1.85%	612.38	0.01%	40.82
		0.01%	1	0.01%	1	0.00%	1997	0.00%	1
10	100	0.47%	53.41	0.05%	55.97	2.02%	672.79	0.01%	52.15
		0.01%	1	0.01%	1	0.00%	1624	0.00%	1
Average		0.45%	30.7	0.04%	36.914	1.61%	470.23	0.01%	39.525
		0.01%	1	0.01%	1	0.00%	1848.3	0.00%	1

Table D.7: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	2.03%	-	0.49%	-	5.67%	-	0.13%	364.44
		0.45%	1064	0.19%	5873	1.37%	1471	0.00%	1
2	100	2.06%	-	0.49%	-	8.75%	-	0.19%	412.16
		0.34%	1250	0.20%	5121	0.81%	1520	0.00%	1
3	100	2.08%	-	0.40%	-	5.88%	-	0.13%	375.48
		0.45%	1030	0.14%	4131	1.88%	1155	0.00%	1
4	100	1.94%	-	0.30%	-	5.72%	-	0.08%	270
		0.33%	1065	0.01%	10768	0.63%	1327	0.00%	1
5	100	2.02%	-	0.44%	-	5.73%	-	0.16%	495.82
		0.47%	1035	0.15%	5948	1.51%	1451	0.00%	1
6	100	2.15%	-	0.38%	-	5.34%	-	0.12%	267.7
		0.43%	1065	0.06%	9195	0.83%	1317	0.00%	1
7	100	2.30%	-	0.55%	-	5.24%	-	0.18%	882.2
		0.52%	618	0.25%	5092	0.89%	1458	0.00%	1
8	100	2.31%	-	0.50%	-	8.53%	-	0.12%	464.62
		0.50%	1072	0.13%	6199	0.45%	1337	0.00%	1
9	100	2.39%	-	0.53%	-	5.80%	-	0.18%	624.87
		0.50%	966	0.23%	3673	1.31%	1528	0.00%	1
10	100	2.39%	-	0.53%	-	5.79%	-	0.16%	531.21
		0.56%	665	0.30%	2689	1.25%	1414	0.00%	1
Average		2.17%	-	0.46%	-	6.25%	-	0.14%	468.85
		0.46%	983	0.17%	5868.9	1.09%	1397.8	0.00%	1

Table D.8: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	99	1.81%	-	0.42%	-	5.00%	-	0.17%	394.27
		0.17%	2794	0.02%	18241	0.63%	1403	0.00%	1
2	100	2.02%	-	0.49%	-	7.84%	-	0.22%	303.16
		0.24%	2219	0.11%	15096	0.49%	1502	0.00%	1
3	100	1.90%	-	0.34%	316.2	5.49%	-	0.09%	209.55
		0.19%	1103	0.00%	2518	0.75%	1451	0.00%	1
4	100	2.31%	-	0.52%	-	8.70%	-	0.13%	208.88
		0.30%	2855	0.17%	11336	0.32%	1468	0.00%	1
5	100	2.41%	-	0.62%	-	5.08%	-	0.22%	584.48
		0.63%	1047	0.24%	9333	0.71%	1524	0.00%	1
6	100	1.80%	-	0.34%	990.32	5.63%	-	0.14%	267.54
		0.23%	1064	0.00%	9587	1.03%	1561	0.00%	1
7	100	2.22%	-	0.47%	-	5.56%	-	0.19%	298.95
		0.41%	1095	0.15%	7762	0.89%	1194	0.00%	1
8	99	2.07%	-	0.44%	-	7.52%	-	0.15%	261.01
		0.29%	1477	0.06%	20943	0.49%	1393	0.00%	1
9	100	1.88%	-	0.29%	909.21	5.41%	-	0.11%	204.64
		0.31%	1051	0.00%	8181	0.77%	1247	0.00%	1
10	100	2.09%	-	0.52%	-	4.57%	-	0.17%	245.95
		0.25%	2411	0.14%	20822	0.22%	1112	0.00%	1
Average		2.05%	-	0.45%	738.58	6.08%	-	0.16%	297.843
		0.30%	1711.6	0.09%	12381.9	0.63%	1385.5	0.00%	1

Table D.9: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 100$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	2.34%	-	0.52%	-	7.62%	-	0.19%	108.6
		0.29%	6677	0.12%	21206	0.26%	1664	0.00%	1
2	100	2.28%	-	0.46%	-	7.75%	-	0.18%	115.36
		0.20%	3738	0.08%	20840	0.21%	1222	0.00%	1
3	98	1.89%	-	0.59%	-	7.64%	-	0.31%	356.06
		0.28%	6240	0.16%	20592	0.72%	1189	0.00%	1
4	100	2.40%	-	0.47%	-	8.59%	-	0.12%	73.5
		0.26%	5117	0.10%	20680	0.20%	1419	0.00%	1
5	100	2.18%	-	0.53%	-	7.23%	-	0.17%	104.77
		0.17%	8376	0.10%	21263	0.25%	1345	0.00%	1
6	100	2.09%	-	0.39%	1548.39	8.11%	-	0.13%	94.11
		0.09%	7831	0.00%	31242	0.50%	1540	0.00%	1
7	100	2.01%	-	0.46%	-	6.93%	-	0.17%	107.13
		0.14%	9347	0.08%	20773	0.17%	1365	0.00%	1
8	100	1.78%	-	0.30%	394.97	7.60%	-	0.10%	80.36
		0.01%	6308	0.00%	6082	0.06%	1197	0.00%	1
9	100	2.37%	-	0.46%	-	8.07%	-	0.19%	113.7
		0.29%	2518	0.13%	20866	0.52%	1536	0.00%	1
10	100	2.12%	-	0.55%	-	7.81%	-	0.20%	135.2
		0.28%	4524	0.15%	10741	0.49%	1631	0.00%	1
Average		2.14%	-	0.47%	971.68	7.73%	-	0.18%	128.879
		0.20%	6067.6	0.09%	19428.5	0.34%	1410.8	0.00%	1

Table D.10: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	149	0.08%	15.16	0.00%	10.94	0.75%	226.08	0.00%	35.4
		0.00%	1	0.00%	1	0.00%	1229	0.00%	1
2	148	0.08%	16.93	0.00%	25.34	0.71%	219.16	0.01%	37.05
		0.00%	1	0.00%	1	0.00%	1483	0.00%	1
3	148	0.07%	20.16	0.00%	13.71	0.63%	398.5	0.00%	39.69
		0.00%	1	0.00%	1	0.00%	1115	0.00%	1
4	150	0.08%	17.56	0.01%	17.28	0.65%	295.84	0.00%	42.22
		0.00%	1	0.00%	1	0.00%	1312	0.00%	1
5	150	0.07%	17.42	0.00%	23.4	0.75%	348.79	0.00%	33.04
		0.00%	1	0.00%	1	0.00%	1171	0.00%	1
6	149	0.07%	17.3	0.00%	12.52	0.77%	387	0.00%	41.73
		0.00%	1	0.00%	1	0.00%	1111	0.00%	1
7	148	0.07%	26.13	0.00%	26.23	0.71%	524.82	0.01%	45.72
		0.00%	1	0.00%	1	0.00%	1543	0.00%	1
8	147	0.07%	27.51	0.00%	18.27	0.69%	199.21	0.01%	50.26
		0.00%	1	0.00%	1	0.00%	1132	0.00%	1
9	149	0.09%	20.85	0.00%	17.34	0.62%	262.86	0.00%	43.11
		0.00%	1	0.00%	1	0.00%	1037	0.00%	1
10	149	0.07%	19.18	0.00%	13.31	0.66%	196.1	0.00%	36.26
		0.00%	1	0.00%	1	0.00%	1082	0.00%	1
Average		0.07%	19.82	0.00%	17.83	0.69%	305.84	0.00%	40.45
		0.00%	1	0.00%	1	0.00%	1221.5	0.00%	1

Table D.11: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	0.06%	11.8	0.00%	10.94	0.72%	97.55	0.00%	22.12
		0.00%	1	0.00%	1	0.00%	1048	0.00%	1
2	149	0.09%	21.31	0.00%	25.34	0.69%	130.01	0.00%	31.92
		0.00%	1	0.00%	1	0.00%	1073	0.00%	1
3	150	0.09%	16.8	0.00%	13.71	0.64%	171.64	0.00%	26.96
		0.00%	1	0.00%	1	0.00%	1079	0.00%	1
4	148	0.08%	18.21	0.01%	17.28	0.72%	278.14	0.01%	27.49
		0.00%	1	0.00%	1	0.00%	1328	0.00%	1
5	149	0.09%	19.27	0.00%	23.4	0.72%	316.59	0.00%	30.02
		0.00%	1	0.00%	1	0.00%	1348	0.00%	1
6	148	0.08%	15.7	0.00%	12.52	0.81%	121.87	0.01%	24.82
		0.00%	1	0.00%	1	0.00%	1046	0.00%	1
7	150	0.09%	20.28	0.00%	26.23	0.76%	298.68	0.00%	32.34
		0.00%	1	0.00%	1	0.00%	1195	0.00%	1
8	150	0.08%	16.5	0.00%	18.27	0.79%	161.83	0.00%	29.79
		0.00%	1	0.00%	1	0.00%	1069	0.00%	1
9	150	0.08%	18.2	0.00%	17.34	0.74%	139.87	0.00%	26
		0.00%	1	0.00%	1	0.00%	1047	0.00%	1
10	149	0.08%	15.74	0.00%	13.31	0.62%	210.04	0.00%	21.74
		0.00%	1	0.00%	1	0.00%	1071	0.00%	1
Average		0.08%	17.38	0.00%	17.83	0.72%	192.62	0.00%	27.32
		0.00%	1	0.00%	1	0.00%	1130.4	0.00%	1

Table D.12: Results for instances with 300 products for the MMNL model, $\phi : 0.75, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	148	0.08%	10.27	0.00%	10.72	0.72%	71.84	0.01%	13.54
		0.00%	1	0.00%	1	0.00%	1052	0.00%	1
2	150	0.07%	8.64	0.00%	10.85	0.69%	77.38	0.00%	13.38
		0.00%	1	0.00%	1	0.00%	1086	0.00%	1
3	150	0.08%	10.81	0.00%	15.28	0.75%	79.15	0.00%	14.61
		0.00%	1	0.00%	1	0.00%	1101	0.00%	1
4	150	0.10%	10.92	0.00%	12.02	0.72%	74.6	0.00%	14.78
		0.00%	1	0.00%	1	0.00%	1059	0.00%	1
5	149	0.08%	8.59	0.00%	10.83	0.75%	121.49	0.00%	13.94
		0.00%	1	0.00%	1	0.00%	1066	0.00%	1
6	149	0.08%	6.96	0.00%	8.18	0.66%	147.62	0.00%	12.52
		0.00%	1	0.00%	1	0.00%	1266	0.00%	1
7	149	0.08%	6.81	0.00%	10.78	0.71%	145.94	0.00%	11.5
		0.00%	1	0.00%	1	0.00%	1251	0.00%	1
8	150	0.09%	8.57	0.00%	10.06	0.71%	131.52	0.00%	13.19
		0.00%	1	0.00%	1	0.00%	1153	0.00%	1
9	147	0.08%	9.12	0.00%	12.69	0.75%	89.65	0.00%	11.62
		0.00%	1	0.00%	1	0.00%	1068	0.00%	1
10	150	0.09%	10.08	0.00%	11.51	0.82%	114.45	0.00%	13.53
		0.00%	1	0.00%	1	0.00%	1107	0.00%	1
Average		0.08%	9.08	0.00%	11.29	0.73%	105.36	0.00%	13.26
		0.00%	1	0.00%	1	0.00%	1120.9	0.00%	1

Table D.13: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	149	0.46%	45.43	0.03%	53.88	2.00%	-	0.01%	139.52
		0.00%	1	0.00%	1	0.01%	2837	0.00%	1
2	136	0.44%	167.78	0.06%	96.92	2.63%	-	0.02%	176.65
		0.00%	1	0.00%	1	0.02%	1145	0.00%	1
3	140	0.44%	68.74	0.03%	53.38	1.92%	949.54	0.01%	200.26
		0.00%	1	0.00%	1	0.00%	1715	0.00%	1
4	150	0.58%	89.47	0.05%	108.12	2.61%	-	0.03%	184.66
		0.00%	1	0.00%	1	0.01%	1362	0.00%	1
5	150	0.51%	57.09	0.03%	56.38	1.83%	792.1	0.02%	89.49
		0.00%	1	0.00%	1	0.01%	1111	0.00%	1
6	150	0.54%	91.06	0.04%	83.66	2.09%	1586.74	0.02%	198.09
		0.00%	1	0.00%	1	0.01%	2542	0.00%	1
7	150	0.48%	62.21	0.04%	73.16	2.00%	1550.4	0.02%	118.42
		0.00%	1	0.00%	1	0.01%	2317	0.00%	1
8	150	0.54%	79.35	0.04%	96.5	1.87%	976.67	0.01%	162.31
		0.00%	1	0.00%	1	0.00%	1121	0.00%	1
9	147	0.53%	54.11	0.03%	62.19	1.94%	-	0.01%	133.2
		0.00%	1	0.00%	1	0.01%	3666	0.00%	1
10	150	0.44%	57.14	0.03%	81.13	2.13%	-	0.01%	125.3
		0.00%	1	0.00%	1	0.01%	3148	0.00%	1
Average		0.50%	77.24	0.04%	76.53	2.10%	1171.09	0.02%	152.79
		0.00%	1	0.00%	1	0.01%	2096.4	0.00%	1

Table D.14: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	150	0.56%	40.01	0.02%	51.8	1.90%	356.93	0.01%	89.25
		0.00%	1	0.00%	1	0.00%	1095	0.00%	1
2	150	0.52%	66.3	0.04%	69.98	2.22%	1321.39	0.02%	87.9
		0.00%	1	0.00%	1	0.00%	4911	0.00%	1
3	149	0.55%	56.48	0.04%	66.46	2.03%	1371.47	0.02%	85.81
		0.00%	1	0.00%	1	0.00%	2886	0.00%	1
4	150	0.51%	66.47	0.03%	78.87	2.17%	1427.95	0.01%	89.7
		0.00%	1	0.00%	1	0.00%	1939	0.00%	1
5	150	0.54%	51.59	0.04%	61.64	2.07%	980.77	0.02%	121.8
		0.00%	1	0.00%	1	0.01%	1717	0.00%	1
6	143	0.49%	51.49	0.03%	48.65	2.07%	-	0.02%	49.97
		0.00%	1	0.00%	1	0.21%	11875	0.00%	1
7	144	0.44%	59.68	0.04%	45.84	2.68%	-	0.02%	84.69
		0.00%	1	0.00%	1	0.01%	1827	0.00%	1
8	150	0.53%	64.6	0.05%	74.16	1.97%	1055.62	0.01%	115.24
		0.00%	1	0.00%	1	0.00%	2126	0.00%	1
9	150	0.57%	53.01	0.04%	67.07	1.90%	518.31	0.01%	103.14
		0.00%	1	0.00%	1	0.00%	1549	0.00%	1
10	150	0.60%	70.66	0.05%	84.58	1.94%	961.04	0.02%	105.42
		0.00%	1	0.00%	1	0.00%	3240	0.00%	1
Average		0.53%	58.03	0.04%	64.91	2.10%	999.19	0.02%	93.29
		0.00%	1	0.00%	1	0.02%	3316.5	0.00%	1

Table D.15: Results for instances with 300 products for the MMNL model, $\phi : 0.5, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	147	0.49%	22.14	0.03%	31.11	2.15%	256.17	0.01%	35.68
		0.00%	1	0.00%	1	0.00%	1693	0.00%	1
2	150	0.51%	23.27	0.03%	28.74	2.02%	580.06	0.01%	29.92
		0.00%	1	0.00%	1	0.00%	1926	0.00%	1
3	150	0.54%	34.67	0.05%	47.31	1.98%	628.66	0.02%	49.91
		0.00%	1	0.00%	1	0.00%	1422	0.00%	1
4	147	0.47%	26.08	0.04%	36.53	1.77%	124.9	0.03%	41.25
		0.00%	1	0.00%	1	0.00%	1067	0.00%	1
5	150	0.55%	26.31	0.03%	33.65	1.96%	214.03	0.01%	33.54
		0.00%	1	0.00%	1	0.00%	1205	0.00%	1
6	147	0.58%	49.92	0.06%	53.03	1.86%	708.05	0.03%	73.47
		0.00%	1	0.00%	1	0.00%	1799	0.00%	1
7	150	0.57%	34.38	0.05%	49.45	1.67%	287.03	0.02%	42.33
		0.00%	1	0.00%	1	0.00%	1197	0.00%	1
8	144	0.49%	36.9	0.06%	50.63	2.01%	587.05	0.03%	49.56
		0.00%	1	0.00%	1	0.00%	5319	0.00%	1
9	139	0.41%	24.62	0.04%	24.29	2.24%	1426.86	0.02%	37.94
		0.00%	1	0.00%	1	0.00%	12010	0.00%	1
10	145	0.60%	50.84	0.06%	53.15	2.43%	-	0.03%	80.4
		0.00%	1	0.00%	1	0.02%	10593	0.00%	1
Average		0.52%	32.91	0.04%	40.79	2.01%	534.76	0.02%	47.4
		0.00%	1	0.00%	1	0.00%	3823.1	0.00%	1

Table D.16: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 100, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	107	2.00%	-	0.49%	-	4.82%	-	0.16%	676.02
		0.43%	1065	0.18%	11473	3.71%	1501	0.00%	1
2	112	1.97%	-	0.43%	-	5.30%	-	0.18%	670.06
		0.35%	1083	0.09%	12229	0.62%	1376	0.00%	1
3	111	2.03%	-	0.37%	-	5.23%	-	0.15%	540.64
		0.39%	1047	0.12%	5126	2.92%	1670	0.00%	1
4	109	1.87%	-	0.29%	932.68	4.71%	-	0.11%	376.43
		0.29%	1060	0.00%	9074	0.43%	1081	0.00%	1
5	108	2.01%	-	0.44%	-	5.17%	-	0.19%	619.93
		0.54%	1079	0.15%	8448	3.79%	1357	0.00%	1
6	111	2.03%	-	0.35%	1148.26	4.55%	-	0.13%	452.79
		0.38%	1025	0.00%	11624	0.73%	1280	0.00%	1
7	107	2.28%	-	0.53%	-	4.60%	-	0.21%	845.84
		0.59%	415	0.23%	5845	0.50%	1121	0.00%	1
8	110	2.21%	-	0.48%	-	4.69%	-	0.20%	742.53
		0.53%	1080	0.13%	12389	0.34%	1244	0.00%	1
9	106	2.39%	-	0.51%	-	5.22%	-	0.23%	840.27
		0.56%	376	0.20%	4574	2.92%	1484	0.00%	1
10	110	2.32%	-	0.51%	-	5.04%	-	0.19%	615.39
		0.68%	905	0.23%	6741	0.77%	1086	0.00%	1
Average		2.11%	-	0.44%	1040.47	4.93%	-	0.18%	637.99
		0.47%	913.5	0.13%	8752.3	1.67%	1320	0.00%	1

Table D.17: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 75, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	100	1.81%	-	0.42%	-	4.72%	-	0.18%	395.42
		0.18%	3267	0.03%	17980	0.31%	1448	0.00%	1
2	110	1.96%	-	0.46%	-	4.37%	-	0.23%	360.67
		0.20%	3287	0.07%	20615	0.25%	1420	0.00%	1
3	107	1.89%	-	0.33%	-	4.92%	-	0.16%	392.2
		0.27%	1243	0.00%	3633	0.53%	1564	0.00%	1
4	118	2.18%	-	0.49%	-	4.37%	-	0.17%	357.89
		0.27%	3065	0.15%	20638	0.15%	1299	0.00%	1
5	105	2.39%	-	0.61%	-	4.64%	-	0.26%	642.79
		0.68%	384	0.22%	12524	0.56%	1568	0.00%	1
6	105	1.76%	-	0.32%	643.49	5.40%	-	0.15%	309.82
		0.24%	1076	0.00%	10689	0.67%	1145	0.00%	1
7	107	2.18%	-	0.45%	-	5.22%	-	0.20%	364.98
		0.45%	1084	0.12%	13124	0.42%	1108	0.00%	1
8	101	2.07%	-	0.44%	-	4.38%	-	0.16%	238.03
		0.31%	2040	0.07%	20751	0.33%	1553	0.00%	1
9	108	1.83%	-	0.26%	278.04	5.10%	-	0.11%	246.32
		0.25%	1066	0.00%	3979	0.69%	1460	0.00%	1
10	112	2.02%	-	0.47%	-	4.36%	-	0.17%	305.35
		0.25%	2547	0.11%	20911	0.07%	1175	0.00%	1
Average		2.01%	-	0.43%	460.77	4.75%	-	0.18%	361.35
		0.31%	1905.9	0.08%	14484.4	0.40%	1374	0.00%	1

Table D.18: Results for instances with 300 products for the MMNL model, $\phi : 0.25, |\mathcal{M}| : 50, |\mathcal{C}| : 100, k : 150$

instance	assort	MILP		MILP+		CONIC		CONIC+	
		rgap	time	rgap	time	rgap	time	rgap	time
		egap	nodes	egap	nodes	egap	nodes	egap	nodes
1	108	2.31%	-	0.50%	-	4.89%	-	0.23%	135.46
		0.25%	8318	0.10%	25339	0.10%	1172	0.00%	1
2	107	2.26%	-	0.44%	-	5.19%	-	0.18%	205.73
		0.18%	3987	0.05%	28907	0.11%	1705	0.00%	1
3	97	1.89%	-	0.59%	-	6.77%	-	0.31%	386.86
		0.27%	6935	0.15%	20578	0.40%	1192	0.00%	1
4	103	2.38%	-	0.48%	-	4.50%	-	0.13%	73.49
		0.24%	8191	0.10%	25994	0.07%	1145	0.00%	1
5	107	2.14%	-	0.50%	-	5.82%	-	0.18%	214.77
		0.12%	11858	0.06%	29551	0.07%	1147	0.00%	1
6	101	2.09%	-	0.39%	-	7.55%	-	0.14%	102.45
		0.09%	9168	0.00%	32416	0.22%	1636	0.00%	1
7	102	2.01%	-	0.47%	-	4.55%	-	0.19%	98.47
		0.17%	9111	0.07%	22248	0.11%	1151	0.00%	1
8	106	1.74%	836.07	0.27%	264.35	5.28%	-	0.10%	89.35
		0.00%	5134	0.00%	4132	0.02%	1345	0.00%	1
9	103	2.36%	-	0.46%	-	5.42%	-	0.20%	109.78
		0.30%	3156	0.12%	20578	0.31%	1455	0.00%	1
10	112	2.08%	-	0.52%	-	6.54%	-	0.24%	147.06
		0.31%	5855	0.16%	17155	0.37%	1430	0.00%	1
Average		2.13%	836.07	0.46%	264.35	5.65%	-	0.19%	156.34
		0.19%	7171.3	0.08%	22689.8	0.18%	1337.8	0.00%	1

Appendix E

Tables for Assortment Optimization with Replenishment Costs

Table E.1: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	152216	21.26%	195.7	46189	0.00%
2	2	-	104666	36.66%	1137.6	340511	0.00%
3	2	-	145560	20.80%	194.8	30882	0.00%
4	2	-	128547	29.76%	-	242070	13.71%
5	2	-	137820	15.66%	86.1	14253	0.00%
Average	2	-	133761.8	24.83%	403.5	107958.8	2.74%

Table E.2: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	113762	23.70%	230.6	53069	0.00%
2	2	-	111467	25.61%	221.0	48583	0.00%
3	3	-	129695	14.78%	124.5	24365	0.00%
4	2	-	103841	29.84%	1369.6	281090	0.00%
5	3	-	149483	14.86%	93.6	15730	0.00%
Average	3	-	121649.6	21.76%	407.846	84567.4	0.00%

Table E.3: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	119199	23.50%	214.9	47045	0.00%
2	3	-	155712	20.20%	137.0	25153	0.00%
3	3	-	113464	10.45%	86.8	14264	0.00%
4	3	-	120327	21.13%	1195.3	228526	0.00%
5	3	-	149778	12.27%	64.5	12479	0.00%
Average	3	-	131696	17.51%	339.682	65493.4	0.00%

Table E.4: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	128387	13.82%	114.4	18877	0.00%
2	3	-	117343	14.75%	105.3	13202	0.00%
3	4	864.5	83310	0.00%	67.6	7073	0.00%
4	3	-	141969	14.23%	262.5	58785	0.00%
5	4	-	192934	2.79%	59.2	7407	0.00%
Average	4	864.5	132788.6	9.12%	121.8	21068.8	0.00%

Table E.5: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	158359	5.83%	31.6	6064	0.00%
2	4	-	188310	6.49%	54.8	6897	0.00%
3	4	296.7	35759	0.00%	41.5	5272	0.00%
4	3	-	159359	8.49%	113.6	23399	0.00%
5	4	418.0	73834	0.00%	42.4	3739	0.00%
Average	4	357.3	123124.2	4.16%	56.776	9074.2	0.00%

Table E.6: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.075, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	6	445.8	71454	0.00%	42.5	1897	0.00%
2	5	575.3	71755	0.00%	22.2	3322	0.00%
3	5	175.9	27415	0.00%	40.5	3442	0.00%
4	3	939.6	135474	0.00%	86.2	10546	0.00%
5	5	199.5	27766	0.00%	30.9	2089	0.00%
Average	5	467.22	66772.8	0.00%	44.464	4259.2	0.00%

Table E.7: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	82443	31.84%	-	287906	9.09%
2	2	-	98036	36.40%	-	201330	17.59%
3	3	-	78720	30.91%	-	235120	11.90%
4	3	-	93474	31.06%	-	184387	20.80%
5	2	-	115279	19.17%	846.3	132723	0.00%
Average	3	-	93590.4	29.88%	846.3	208293.2	11.87%

Table E.8: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	59233	33.72%	-	208868	8.18%
2	4	-	83221	26.75%	-	279766	11.63%
3	4	-	81280	23.54%	-	257161	7.48%
4	4	-	90956	30.08%	-	240188	19.57%
5	4	-	106356	21.99%	720.3	131941	0.00%
Average	4	-	84209.2	27.22%	720.3	223584.8	9.37%

Table E.9: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	57924	29.65%	-	208868	8.18%
2	4	-	78807	23.26%	-	279766	11.63%
3	4	-	95395	21.94%	-	257161	7.48%
4	4	-	88181	27.55%	-	240188	19.57%
5	4	-	106703	17.15%	720.3	131941	0.00%
Average	4	-	85402	23.91%	720.3	223584.8	9.37%

Table E.10: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	73815	22.23%	751.2	147565	0.00%
2	5	-	93605	20.22%	877.7	191342	0.00%
3	5	-	76040	15.97%	378.1	77717	0.00%
4	5	-	85014	21.84%	-	224005	12.53%
5	6	-	105666	13.96%	200.2	32199	0.00%
Average	5	-	86828	18.84%	551.8	134565.6	2.51%

Table E.11: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	87958	13.52%	242.0	27690	0.00%
2	6	-	89987	12.88%	170.0	31981	0.00%
3	6	-	65886	12.11%	184.9	21884	0.00%
4	5	-	95372	18.12%	-	196887	9.30%
5	7	-	109876	9.38%	112.1	12461	0.00%
Average	6	-	89815.8	13.21%	177.2	58180.6	1.86%

Table E.12: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.075, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	139976	3.01%	242.0	27690	0.00%
2	6	-	159672	3.57%	170.0	31981	0.00%
3	6	-	91490	5.85%	184.9	21884	0.00%
4	5	-	127463	13.35%	-	196887	9.30%
5	7	768.8	78463	0.00%	112.1	12461	0.00%
Average	6	768.8	119412.8	5.16%	177.2	58180.6	1.86%

Table E.13: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	70859	32.28%	-	246408	14.48%
2	3	-	69746	34.33%	-	213099	19.25%
3	3	-	62989	31.28%	-	173682	14.55%
4	4	-	81062	35.93%	-	116848	22.26%
5	3	-	84367	25.86%	-	253683	6.38%
Average	3	-	73804.6	31.94%	-	200744	15.38%

Table E.14: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	71299	33.32%	-	213805	15.14%
2	4	-	66069	29.02%	-	279452	14.49%
3	5	-	64214	26.01%	-	196212	14.18%
4	4	-	61341	30.23%	-	161650	19.43%
5	4	-	86145	22.62%	-	245733	3.97%
Average	4	-	69813.6	28.24%	-	219370.4	13.44%

Table E.15: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	69687	31.74%	-	220892	11.68%
2	5	-	100310	24.27%	-	139668	11.10%
3	5	-	48466	24.37%	-	128177	9.82%
4	5	-	57003	28.79%	-	139622	18.42%
5	6	-	83652	20.16%	1142.1	191687	0.00%
Average	5	-	71823.6	25.86%	1142.1	164009.2	10.21%

Table E.16: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	65498	26.66%	-	268385	7.00%
2	5	-	64566	19.92%	-	243070	6.51%
3	6	-	58506	20.55%	-	248770	7.02%
4	6	-	61716	24.74%	-	172272	14.75%
5	6	-	83269	16.11%	372.7	59442	0.00%
Average	6	-	66711	21.60%	372.7	198387.8	7.06%

Table E.17: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	86895	17.60%	326.2	55492	0.00%
2	7	-	82607	13.16%	494.8	89358	0.00%
3	7	-	66947	14.16%	1012.4	91684	0.00%
4	8	-	75970	20.29%	-	208150	13.01%
5	7	-	95008	9.40%	168.4	17308	0.00%
Average	7	-	81485.4	14.92%	500.4	92398.4	2.60%

Table E.18: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.075, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	106353	10.83%	188.0	22069	0.00%
2	8	-	77294	8.13%	191.1	22065	0.00%
3	8	-	81874	9.94%	351.6	41607	0.00%
4	9	-	76317	13.46%	-	95706	9.58%
5	8	-	115668	2.72%	89.4	8366	0.00%
Average	8	-	91501.2	9.01%	205.0	37962.6	1.92%

Table E.19: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	165806	26.15%	253.7	51994	0.00%
2	2	-	117650	33.63%	706.8	195268	0.00%
3	3	-	119235	22.79%	197.0	30115	0.00%
4	2	-	119880	24.25%	-	291706	7.48%
5	2	-	159445	15.04%	99.7	17509	0.00%
Average	2	-	136403.2	24.37%	314.3	117318.4	1.50%

Table E.20: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	139520	25.20%	269.6	60252	0.00%
2	3	-	161945	20.48%	172.7	35089	0.00%
3	3	-	140895	10.21%	70.4	12309	0.00%
4	3	-	149975	21.74%	-	258417	3.05%
5	3	-	150820	13.88%	83.3	14640	0.00%
Average	3	-	148631	18.30%	149.0	76141.4	0.61%

Table E.21: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	125958	20.51%	125.0	27991	0.00%
2	3	-	129690	15.36%	99.4	17647	0.00%
3	4	-	107844	0.00%	62.3	8977	0.00%
4	3	-	137274	20.42%	488.2	138188	0.00%
5	4	-	163354	8.25%	60.5	10605	0.00%
Average	4	-	132824	12.91%	167.1	40681.6	0.00%

Table E.22: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	132900	9.37%	86.6	9080	0.00%
2	4	-	134831	14.40%	62.5	9060	0.00%
3	4	472.3	50863	0.00%	58.5	5225	0.00%
4	3	-	184276	7.66%	114.3	35518	0.00%
5	4	881.5	89765	0.00%	30.2	5703	0.00%
Average	4		118527	6.29%	70.4	12917.2	0.00%

Table E.23: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	1026.2	118078	0.00%	51.4	2950	0.00%
2	4	742.3	99913	0.00%	24.6	4117	0.00%
3	5	349.4	32677	0.00%	23.7	3847	0.00%
4	3	964.0	136796	0.00%	101.2	17003	0.00%
5	5	379.0	46804	0.00%	39.2	2928	0.00%
Average	4	692.2	86853.6	0.00%	48.0	6169	0.00%

Table E.24: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.05, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	180.5	24784	0.00%	43.1	1583	0.00%
2	6	331.8	29319	0.00%	41.6	1916	0.00%
3	5	170.5	13999	0.00%	22.8	2660	0.00%
4	3	588.6	85248	0.00%	57.7	11208	0.00%
5	5	110.4	9621	0.00%	35.5	1515	0.00%
Average	5	276.3	32594.2	0.00%	40.1	3776.4	0.00%

Table E.25: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	71418	34.00%	1387.4	158409	0.00%
2	3	-	84137	33.87%	-	282542	14.78%
3	3	-	74190	27.41%	1161.8	174562	0.00%
4	4	-	85950	28.76%	-	206400	17.99%
5	3	-	117809	21.65%	576.3	120749	0.00%
Average	3	-	86700.8	29.14%	1041.8	188532.4	6.55%

Table E.26: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	58433	28.26%	-	280786	6.06%
2	4	-	82012	25.01%	-	297688	7.38%
3	4	-	90343	20.32%	869.8	191391	0.00%
4	5	-	102229	24.33%	-	291455	18.33%
5	5	-	102081	18.11%	330.5	79342	0.00%
Average	5	-	87019.6	23.21%	600.1	228132.4	6.35%

Table E.27: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	69733	24.78%	-	308235	1.83%
2	5	-	102233	19.70%	839.4	195179	0.00%
3	5	-	78677	18.03%	672.8	131252	0.00%
4	5	-	82013	26.21%	-	195064	12.11%
5	6	-	80876	17.56%	207.6	44009	0.00%
Average	5	-	82706.4	21.26%	573.3	174747.8	2.79%

Table E.28: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	-	96800	18.19%	282.8	56045	0.00%
2	5	-	113986	14.20%	236.3	35018	0.00%
3	5	-	86116	13.49%	279.9	63353	0.00%
4	5	-	91261	21.33%	-	278813	12.27%
5	6	-	115950	10.54%	90.8	16928	0.00%
Average	6	-	100822.6	15.55%	222.4	90031.4	2.45%

Table E.29: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	111189	6.58%	83.9	11507	0.00%
2	7	-	121752	7.45%	121.5	12251	0.00%
3	6	-	87678	7.72%	157.4	21690	0.00%
4	6	-	100199	14.84%	-	201699	7.31%
5	7	-	56303	7.79%	39.3	8302	0.00%
Average	7	-	95424.2	8.88%	100.5	51089.8	1.46%

Table E.30: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.05, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	827.7	72985	0.00%	50.6	3094	0.00%
2	8	1672.2	138778	0.00%	41.1	3555	0.00%
3	8	1303.2	91559	0.00%	47.9	6851	0.00%
4	7	-	108136	10.53%	953.9	192420	0.00%
5	7	597.6	45748	0.00%	54.7	5947	0.00%
Average	8	1100.2	91441.2	2.11%	229.6	42373.4	0.00%

Table E.31: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	65694	33.29%	-	243930	11.57%
2	3	-	54239	33.70%	-	274838	17.63%
3	3	-	68395	30.30%	-	178249	15.63%
4	4	-	64415	32.15%	-	231577	23.29%
5	4	-	67783	25.82%	-	213127	10.16%
Average	4	-	64105.2	31.05%	-	228344.2	15.65%

Table E.32: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	62403	32.33%	-	182539	12.20%
2	4	-	87781	25.01%	-	240545	10.65%
3	5	-	48689	24.26%	-	221887	12.41%
4	5	-	48093	26.45%	-	175963	17.28%
5	5	-	83093	22.13%	1333.7	235799	0.00%
Average	5	-	66011.8	26.04%	1333.7	211346.6	10.51%

Table E.33: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	6	-	57332	29.99%	535.3	38735	0.00%
2	5	-	96211	20.83%	-	195810	7.19%
3	6	-	59970	22.10%	-	162691	8.25%
4	5	-	74513	26.94%	-	118562	16.26%
5	6	-	89802	17.14%	515.9	38735	0.00%
Average	6	-	75565.6	23.40%	525.6	110906.6	6.34%

Table E.34: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	87736	20.82%	982.0	185711	0.00%
2	6	-	90938	17.75%	1301.9	247582	0.00%
3	7	-	83611	14.92%	-	100493	5.30%
4	7	-	76115	20.88%	-	153054	14.37%
5	7	-	107699	11.02%	241.2	40919	0.00%
Average	7	-	89219.8	17.08%	841.7	145551.8	3.93%

Table E.35: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	104957	12.83%	197.8	21838	0.00%
2	8	-	88681	10.57%	250.0	38106	0.00%
3	8	-	70807	12.01%	527.8	47065	0.00%
4	9	-	61520	16.57%	-	145157	10.50%
5	7	-	114497	5.63%	128.8	11283	0.00%
Average	8	-	88092.4	11.52%	276.1	52689.8	2.10%

Table E.36: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.05, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	9	-	100800	7.77%	108.1	8730	0.00%
2	9	-	85437	4.19%	106.2	14558	0.00%
3	10	-	66965	8.21%	235.2	23114	0.00%
4	10	-	82592	12.51%	-	243621	6.20%
5	8	844.6	71818	0.00%	67.3	6025	0.00%
Average	9	844.6	81522.4	6.53%	129.2	59209.6	1.24%

Table E.37: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	3	-	116572	23.24%	260.9	44180	0.00%
2	2	-	121761	24.59%	220.2	60738	0.01%
3	3	-	135831	16.53%	90.7	12675	0.00%
4	2	-	110443	28.80%	-	268996	12.09%
5	3	-	148360	16.37%	132.5	16311	0.00%
Average	3	-	126593.4	21.91%	176.1	80580	2.42%

Table E.38: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	138045	18.71%	131.5	26888	0.00%
2	3	-	164348	15.52%	102.2	15606	0.00%
3	4	1065.1	101611	0.00%	31.9	6282	0.00%
4	3	-	130605	14.94%	339.5	57484	0.00%
5	4	1385.1	171554	0.00%	71.6	9169	0.00%
Average	4	1225.1	141232.6	9.83%	135.3	23085.8	0.00%

Table E.39: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	148798	10.15%	90.2	16839	0.00%
2	4	-	141403	11.96%	77.2	8917	0.00%
3	4	344.0	43434	0.00%	30.0	5463	0.00%
4	3	-	165707	9.03%	120.2	29673	0.00%
5	4	727.7	82440	0.00%	47.7	5901	0.00%
Average	4	535.9	116356.4	6.23%	73.1	13358.6	0.00%

Table E.40: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	672.6	69686	0.00%	22.9	2546	0.00%
2	4	1502.0	150520	0.00%	26.8	3590	0.00%
3	5	289.1	30202	0.00%	28.6	4552	0.00%
4	3	1360.7	167948	0.00%	87.7	18346	0.00%
5	5	352.5	34021	0.00%	22.7	2948	0.00%
Average	4	835.4	90475.4	0.00%	37.7	6396.4	0.00%

Table E.41: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	171.2	22258	0.00%	21.8	1195	0.00%
2	6	248.9	31475	0.00%	40.7	2454	0.00%
3	5	205.1	18314	0.00%	35.0	2683	0.00%
4	3	638.8	96959	0.00%	58.0	12951	0.00%
5	5	100.5	11230	0.00%	34.9	1933	0.00%
Average	5	272.9	36047.2	0.00%	38.1	4243.2	0.00%

Table E.42: Assortment with Replenishment Costs: $\nu_0 = 0.5, \iota = 0.025, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	115.9	8326	0.00%	44.0	617	0.00%
2	6	106.0	7952	0.00%	17.3	1164	0.00%
3	7	158.1	13539	0.00%	18.7	1369	0.00%
4	4	337.7	61869	0.00%	55.9	8783	0.00%
5	5	79.7	6257	0.00%	18.6	1256	0.00%
Average	6	159.5	19588.6	0.00%	30.9	2637.8	0.00%

Table E.43: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	73042	29.33%	-	259532	7.71%
2	4	-	105710	25.19%	-	365059	9.09%
3	4	-	87020	23.34%	881.3	128057	0.00%
4	4	-	84256	31.97%	-	251564	16.45%
5	4	-	95419	21.07%	700.5	169100	0.00%
Average	4	-	89089.4	26.18%	790.9	234662.4	6.65%

Table E.44: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	5	-	76565	23.29%	698.3	92532	0.00%
2	5	-	113654	18.06%	715.5	150047	0.00%
3	5	-	83680	16.97%	597.5	126137	0.00%
4	5	-	97009	22.69%	-	213250	11.15%
5	6	-	91889	14.48%	182.3	29153	0.00%
Average	5	-	92559.4	19.10%	548.4	122223.8	2.23%

Table E.45: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	-	86571	18.55%	261.8	41294	0.00%
2	5	-	115022	14.36%	265.0	51805	0.00%
3	5	-	97538	13.20%	241.4	53049	0.00%
4	5	-	130966	11.18%	-	134812	13.42%
5	6	-	105058	20.73%	108.2	13961	0.00%
Average	6	-	107031	15.61%	219.1	58984.2	2.68%

Table E.46: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	127120	7.28%	102.4	13205	0.00%
2	7	-	134162	7.39%	96.6	14630	0.00%
3	6	-	113666	7.43%	133.6	13921	0.00%
4	6	-	115163	15.23%	-	201251	6.88%
5	7	-	144077	2.55%	46.0	7816	0.00%
Average	7	-	126837.6	7.97%	94.6	50164.6	1.38%

Table E.47: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	551.2	55339	0.00%	28.1	3714	0.00%
2	8	740.5	54414	0.00%	29.3	4624	0.00%
3	8	1243.1	87222	0.00%	55.6	7170	0.00%
4	7	-	99209	10.29%	967.7	199323	0.00%
5	7	538.5	44488	0.00%	81.2	4871	0.00%
Average	8	768.3	68134.4	2.06%	232.4	43940.4	0.00%

Table E.48: Assortment with Replenishment Costs: $\nu_0 = 1, \iota = 0.025, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	210.7	13650	0.00%	46.7	1751	0.00%
2	8	192.5	17360	0.00%	34.5	1669	0.00%
3	9	352.4	34388	0.00%	45.9	4008	0.00%
4	7	-	119865	6.11%	359.1	48241	0.00%
5	8	226.7	19589	0.00%	25.7	1881	0.00%
Average	8	245.6	40970.4	1.22%	102.4	11510	0.00%

Table E.49: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 50$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	4	-	70896	36.54%	-	204941	14.90%
2	4	-	57352	29.49%	-	200836	10.96%
3	5	-	67197	26.39%	-	139822	11.59%
4	4	-	57027	29.25%	-	190889	21.44%
5	4	-	82051	22.55%	1604.2	237091	0.00%
Average	4	-	66904.6	28.84%	1604.2	194715.8	11.78%

Table E.50: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 150$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	7	-	62337	24.54%	-	283423	7.88%
2	5	-	88638	21.67%	-	261782	8.05%
3	6	-	65832	21.38%	-	103786	6.22%
4	6	-	49066	24.79%	-	157488	14.51%
5	6	-	109184	15.49%	523.6	64709	0.00%
Average	6	-	75011.4	21.57%	523.6	174237.6	7.33%

Table E.51: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 250$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	62337	24.54%	801.9	181650	0.00%
2	6	-	88638	21.67%	922.4	207821	0.00%
3	7	-	65832	21.38%	-	97289	5.05%
4	7	-	49066	24.79%	-	191200	14.25%
5	7	-	109184	15.49%	184.3	35512	0.00%
Average	7	-	75011.4	21.57%	636.2	142694.4	3.86%

Table E.52: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 500$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	8	-	104546	13.21%	199.6	21095	0.00%
2	8	-	78827	11.76%	263.5	38419	0.00%
3	8	-	62354	13.38%	493.1	58844	0.00%
4	9	-	72658	17.95%	-	154650	10.02%
5	7	-	124285	5.52%	112.1	9180	0.00%
Average	8	-	88534	12.36%	267.1	56437.6	2.00%

Table E.53: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 1000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	9	-	122140	7.95%	104.9	10094	0.00%
2	9	-	83179	5.78%	132.7	12055	0.00%
3	10	-	69947	8.65%	320.2	28708	0.00%
4	10	-	77847	12.10%	-	178793	6.81%
5	8	-	43864	0.00%	38.8	5203	0.00%
Average	9	-	79395.4	6.90%	149.2	46970.6	1.36%

Table E.54: Assortment with Replenishment Costs: $\nu_0 = 1.5, \iota = 0.025, \lambda = 2000$

instance	assort	CONIC			CONIC+		
		time	nodes	egap	time	nodes	egap
1	12	1024.6	83097	0.00%	50.0	4444	0.00%
2	9	1025.1	47501	0.00%	68.6	6794	0.00%
3	12	-	83819	2.98%	147.9	9763	0.00%
4	11	-	85218	8.27%	956.3	160648	0.00%
5	9	195.5	15966	0.00%	37.0	2210	0.00%
Average	11	748.4	63120.2	2.25%	252.0	36771.8	0.00%