

THEORETICAL AND EXPERIMENTAL ANALYSIS OF A SOFT AND MINIATURE QUADRUPE

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By
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AND MINIATURE QUADRUPE

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We certify that we have read this thesis and that in our opinion it is fully adequate,
in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Multi-body dynamic modeling of non-rigid and legged robots is an active research area with the goal of investigating the interaction of a robot with its environment and its resulting locomotion. The results of such studies can be used to build a simulation that will be used for the iterative process of the mechanical design of the robot, and the algorithm design of its high- and low-level controllers. The dynamics of soft robots is more challenging comparing to rigid robots because of the partial derivatives and the shape integrals existing in the dynamic models, and the literature is open to improvement.

The subject of this study is S-Quad, a soft and miniature quadrupel with c-shaped legs. To analyze the effect of the compliance of its body and legs to its locomotion, a soft-body dynamic model has been developed. The development process starts with the rigid-body dynamic analysis, which will be a base for the soft-body dynamic analysis and used to examine the rigid body - rigid legs (RBRL) version of the robot. The Newton-Euler method has been used to develop this model, in which the contact forces are estimated with the viscoelasticity theory. Scenarios of different model parameters were simulated to estimate the motion of the robot. With the obtained results the effects of the model parameters were discussed, and then appropriate parameters have been selected. The contact analysis of a curved leg inside of the robot dynamics, in which any intersection algorithm is not used, is an advantageous aspect of this study.

The compliance of the legs has been incorporated into this dynamic analysis with a non-linear viscoelastic model. The dependency of the leg compliance to the roll angle of the leg was derived from Castigliano's theorem. Then, the motion of the rigid body - soft leg (RBSL) robot was estimated accordingly. The comparison of the RBRL and the RBSL results was utilized to interpret the effect

of the leg compliance. The modeling and the integration with robot dynamics of this compliant leg model is a novel aspect of this model.

An offline Finite Element Analysis has been conducted to estimate the deflections of the soft body under the contact forces, which were exported from the RBRL simulation. The estimated deflections were put back into this simulation to obtain the simulation of soft body - rigid legs (SBRL) robot. Thus, the capability of the developed model to include body deflections has been proven.

Finally, the RBRL and the RBSL robot simulations have been verified with the experiments conducted with a motion capture system. These experimental results were also used to interpret the advantages and disadvantages of using a soft body in the robot.

Keywords: soft robots, robot dynamics, soft robot dynamics, legged robots, viscoelasticity, Castigliano's theorem.

ÖZET

DÖRT BACAĞLI VE YUMUŞAK BİR MİNYATÜR ROBOTUN KURAMSAL VE DENEYSEL ANALİZİ

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Bacaklı ve katı olmayan robotların çok gövdeli dinamik modellemesi, bir robotun çevresi ile etkileşimini ve ortaya çıkan hareketini araştırmak amacıyla çalışılan aktif bir araştırma alanıdır. Bu çalışmaların sonuçlarından, robotun mekanik tasarımının yinelemeli süreci ile, yüksek ve düşük seviyeli kontrolörlerinin algoritma tasarımı için kullanılabilecek bir simülasyon oluşturmak için faydalanılabilir. Fakat, parçalı türevler ve şekil integrallerinden ötürü yumuşak robotların dinamik modellerinin geliştirilmesi sert robotlara kıyasla daha zorludur ve bu konudaki literatür gelişmeye açıktır.

Bu çalışmanın konusu C-bacaklı yumuşak ve minyatür bir robottur (S-Quad). Gövdesinin ve bacaklarının yumuşaklığının hareketine olan etkisini analiz etmek için bir yumuşak cisim dinamik modeli geliştirilmiştir. Geliştirme süreci, yumuşak cisim dinamik modeli için bir temel olacak ve robotun rijit gövde – rijit bacaklı (RBRL) versiyonunu analiz etmek için kullanılacak rijit cisim dinamik analizi ile başlamıştır. Newton-Euler yöntemi, temas kuvvetlerinin viskoelastisite teorisi ile tahmin edildiği bu modeli geliştirmek için kullanılmıştır. Robotun hareketini tahmin etmek için farklı model parametrelerinin senaryoları simüle edilmiştir. Elde edilen sonuçlarla model parametrelerinin etkileri tartışılmış ve daha sonra uygun parametreler seçilmiştir. Robot dinamiklerinin içindeki kavisli bir bacağın temas analizinin herhangi bir kesişme algoritmasının kullanılmadan yapılması, bu çalışmanın avantajlı bir yönüdür.

Bacakların yumuşaklığı, doğrusal olmayan viskoelastik bir model kullanılarak bu dinamik analize dahil edilmiştir. Bacağın yumuşaklığının bacağın yuvarlanma açısına bağlılığı Castigliano teoremi ile türetilmiştir. Daha sonra, sert gövde – yumuşak bacaklı (RBSL) robotun hareketi bu model göre tahmin edilmiştir.

RBRL ve RBSL sonuçlarının karşılaştırılması bacak uyumunun etkisini yorumlamak için kullanılmıştır. Bu yumuşak bacak modelinin modellenmesi ve robot dinamikleri ile entegrasyonu, bu çalışmanın getirdiği yeni bir yöndür.

Yumuşak gövdenin temas kuvvetleri altındaki esnemesini tahmin edebilmek için, RBRL simülasyonundan elde edilen kuvvet değerleriyle çevrimdışı bir Sonlu Eleman Analizi yapılmıştır. Tahmini esnemeler, yumuşak gövde – sert bacaklı (SBRL) robotun simülasyonunu elde etmek için bu simülasyona geri konulmuştur. Böylece geliştirilen modelin gövde esnemelerini de içermeye kabiliyeti kanıtlanmıştır.

Son olarak, RBRL ve RBSL robot simülasyonları, bir hareket yakalama sistemi ile yapılan deneylerle doğrulanmıştır. Bu deneysel sonuçlar, robotta yumuşak bir gövde kullanmanın avantajlarını ve dezavantajlarını yorumlamak için de kullanılmıştır.

Anahtar sözcükler: yumuşak robotlar, robot dinamiği, yumuşak robot dinamiği, bacaklı robotlar, viskoelastiklik, Castigliano teoremi.

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I dedicate this thesis to my mother. Without her, none of these achievements could have been possible.

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Chapter 1

Introduction

This thesis introduces the locomotion analysis of a C-legged miniature quadruped and the effect of the compliance of its individual parts on its locomotion. This analysis is conducted theoretically by creating a dynamic model of the robot and measuring its motion. Then, this model is verified with experiments conducted with a motion capture system.

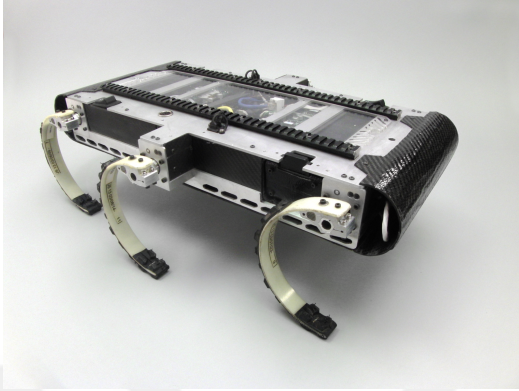
1.1 Motivation

Starting from the 1930s, robotic manipulators have an ever-increasing place in the industry as they replace human effort in repetitive manufacturing processes such as assembly, disassembly, and welding. International Federation of Robotics estimates that about 3 million industrial robots are used in 2020 [1]. And yet, they have a limited workspace as they are fixed to the ground. Concerning this matter, mobile robotics is another active research subject. These mobile robots can operate in many places out of human reach (e.g. Mars [2], nuclear reactors [3], underwater [4], earthquake debris [5], volcano [6]), as much as they can work within human environments.

The locomotion mechanisms studied by researchers include wheels and other nature-inspired mechanisms ([7], see Table 1.1 for a summary). Considering the factors like speed, stability, and complexity, legged robots are favorable for uneven terrains comparing to the other ground locomotion mechanisms. Researchers have studied many examples of legged robot designs that can be categorized into monopods (Raibert hopper [8], SALTO [9]), bipeds (Valkyrie [10], ATLAS [11]), quadrupeds (BigDog [12], ANYmal [13], HAMR [14]), hexapods (Genghis [15], RHex [16], iSprawl [17]), and octopods (Dante II [6]) according to their number of legs. There are also different geometries of legs studied in the literature. Half-circular (c-shaped) legs are a promising option among these geometries since they show the advantages of a wheel (in terms of speed) and a leg (in terms of ability to walk on rough terrain) at the same time with a simple design (see Figure 1.1)..

Locomotion	Motion Source	Animal	Robot
Crawling	Two-anchor	Caterpillar	Origami worm [18]
	Peristaltic	Earthworm	Softworm [19]
	Slithering	Snake	AmphiBot I [20]
Legged	Running/Walking	Horse	RHex [16]
	Jumping	Frog	SALTO [9]
Flying	Fixed-wing	Flying squirrel	Glider [21]
	Flapping	Bird	RoboBee [22]
Swimming	Lift-induced	Manta ray	Robotic manta ray [23]
	Undulation	Eel	TadRob [24]
	Drag-induced	Fish	Soft robotic fish [25]
	Jet propulsion	Octopus	Robojelly [26]
Others	Deformation	-	JSEL [27]
	Vibration	-	Kilobot [28]

Table 1.1: Fundamental locomotion types



(a) X-RHex [29]



(b) Flex-Foot Cheetah [30]

Figure 1.1: Example usages of C-shaped legs

Soft robotics is a recent research area that has its pioneering works especially between 2009 and 2012 [31]. They can be identified as mechanisms that inherently and structurally compliant [32]. The main advantages of soft robots are safer human-machine interaction, higher degrees of freedom, diversity of applications [33], physical robustness, and the ability to morph when necessary[31].

Soft materials have been used in the making of both manipulators and mobile robots. There are examples of grippers that utilizes granular jamming [34], and pneumatic actuators [35, 36]. Mobile soft robots have for more options for actuation, such as shape memory alloys (SMA) [37, 32, 38, 39, 40, 41, 42], pneumatic nets [43, 44, 45, 46], combustion [47], and directional adhesion [48]. Among these examples, only a few of them are fully soft and untethered. In soft robotics, it is a significant challenge to build an untethered robot, because pumps, cables, tubes, high voltage sources are heavy components. SEAQ [37] is very similar to the robot that is subject to this study (S-Quad [49]). However, it has shape memory alloys as the main actuator, and using these actuators limits its operation time.

S-Quad is an untethered miniature quadruped that can fit into the palm of a hand. It has four identical half-circular shaped legs that are independently driven by a DC motor. Although the use of soft materials to build the robot is disadvantageous for the locomotion performance (i.e., speed and balance), it provides other

advantages. This includes safety for the environment and user, adaptation to rough terrain, higher obstacles to climb, passing through openings smaller than the body (using a contraction mechanism of SMA).

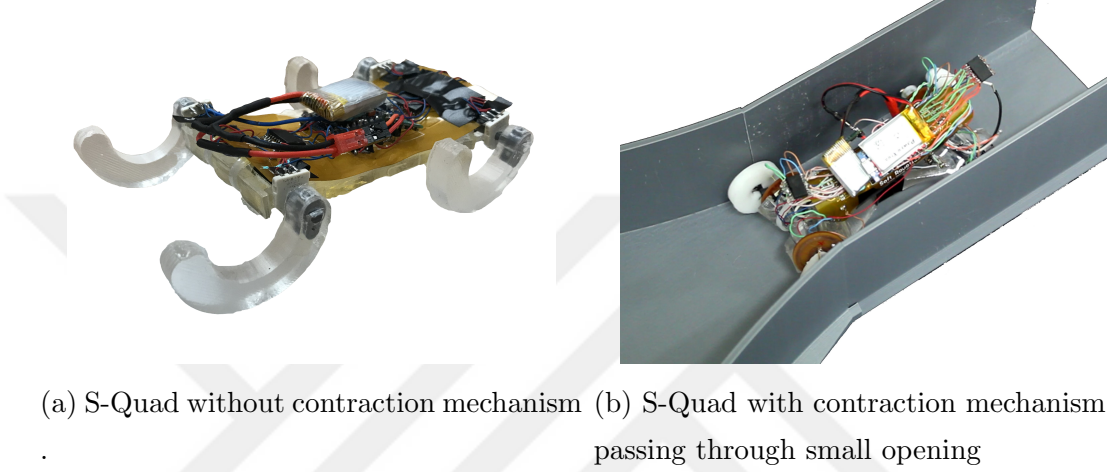


Figure 1.2: S-Quad

Modeling a robot provides valuable input for simulation and control, which is vital for the efficient design and manufacturing of the robot. With the dynamic simulation of a robot, one can investigate the effect of design parameters on the robot locomotion, even before actually manufacturing it. The design of the robot can be improved iteratively in a shorter period. Additionally, a dynamic model of a robot can be utilized to design an appropriate controller for the robot, or this dynamic model can be even used in a model-based controller.

Today, it is possible to achieve quick and accurate simulations of rigid robots with commercial software (e.g., MSC Adams[©], MATLAB Simscape[©]) or open-source libraries (e.g., Open Dynamics Engine[©], OpenSim[©]). The rigidity assumption yields this speed and accuracy by lowering the computational cost. According to this assumption, the relative position between two points fixed on a rigid body does not change with time (i.e., any possible deflection is ignored). Unfortunately, for the bodies made of soft materials (such as Polydimethylsiloxane (PDMS)), deflections reach a level that cannot be neglected, even with the gravitational loads. To accommodate these deflections into a mathematical model of the robot requires the computation of partial derivatives and shape integrals. As a result, it is more challenging to simulate soft robots.

Another challenge that is dealt with in this study is related to the modeling of legged robots. Even before including the deflections, the contact problem of legged robots is highly non-linear due to the discontinuous and very large impact forces occurring momentarily when a leg touches the ground. Additionally, the half-circular shape of the leg brings two more challenges: the change in the position contact point (i.e. rolling motion) and the change in the effective stiffness of the leg.

1.2 Literature Review

Dynamic modeling of rigid robots can be also described as dynamics of rigid body systems and therefore it has well-established methods to be applied. First of them is Newton-Euler method, which is based on the conservation of momentum. Second one is Lagrange method, which is based on the conservation of energy. And the last one is Kane method, which is based on virtual work principle. Even though they have different derivations, they are equivalent in terms of the final results. These methods are also used in the dynamic modeling of legged robots [50, 51, 52]. Together with the rise of computers, these methods turned into computational methods in 1960s. The foremost ones of these methods are the Composite-Rigid-Body Algorithm [53, 54, 55, 56, 57] and the Articulated-Body Algorithm [58, 59, 60, 61].

An important amount of methods used in the modeling soft-multi-body dynamics are based on continuum mechanics. The firstly derived and maybe most widely used one of these methods is Floating Frame of Reference Formulation (FFRF) [62]. It is firstly developed for aerospace applications in 1960s and 1970s [63, 64, 65]. In this method, deflections are defined in elastic reference frames while motions are defined in body reference frames [66]. Another continuum mechanics-based method is Absolute Nodal Coordinate Formulation (ANCF) [67]. This method is developed in 1990s by defining position vectors as nodal coordinates as a solution to the inefficiency of FFRF in large deflection problems [68, 69, 70]. A more recent method called Cosserat Rod Theory is using the Cosserat

continuum in which the rotations of the particles is also defined [71, 72]. All three methods are used in the soft robots modeling, but they generally cover robotic manipulators [73, 74, 75, 76]. Yet, there are only a few applications for soft mobile robots [77].

Because of the computational costs of these mechanics-based models, the compliance in the walking robots is modeled using simpler models called ‘template’ [78]. The most famous one of these templates is called Spring Loaded Inverted Pendulum (SLIP) [79]. In this template, the robot is represented with a point mass and a massless spring. Accordingly, it jumps forward with a loop of flight and stance phases. SLIP template has been studied frequently and it is developed into different versions so that it involves different aspects of walking mechanism [80, 81, 82, 83, 84, 85, 86, 87, 88].

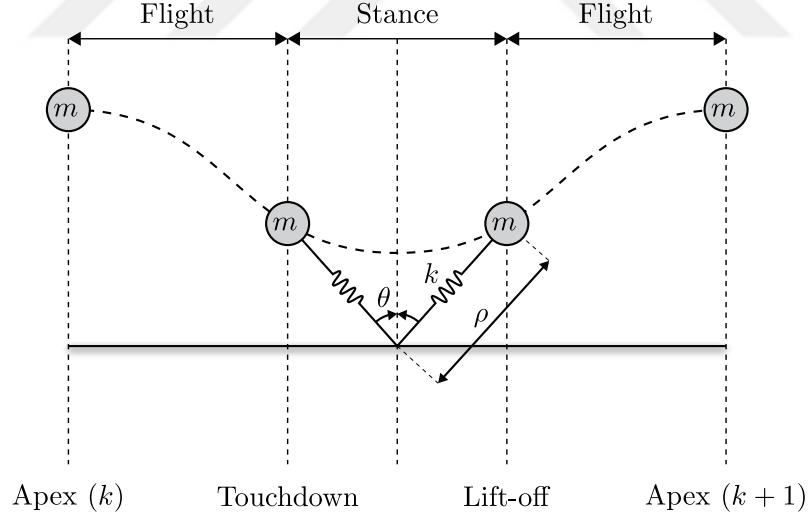


Figure 1.3: The variables and parameters of SLIP template.

SLIP template is also improved to model the walking mechanism of curved legs. First of these studies is done by Ankarali et al. [89]. In this study, the initial length of the SLIP spring is varied to accommodate the rolling and its stiffness is varied to accommodate the variable compliance of the curve. This variable stiffness is modeled by using Castigliano’s method [90]. In another study conducted

by Jun and Clark [91], the curved leg is represented by two rigid links that are connected to each other with a torsional spring and connected to the mass with a torque source. The rolling is represented with the varying angle between the links and the compliance of the curve is represented with the variable stiffness of the torsional spring. This variable stiffness is modeled by using Pseudo-Rigid Body Model (PRBM) [92]. Lastly, Huang et al. [93] represented the curved leg with a rigid link connected to the mass and a curved rigid link connected to that with a torsional spring. The variable stiffness of the torsional spring is modeled by using Castigliano's method.

1.3 Research Aims and Objectives

The subject of this study is an untethered miniature quadruped, named S-Quad [49]. It has four different body versions, which are combinations of a soft/rigid body and a soft/rigid c-shaped legs. They are identical in terms of dimensions, but their material properties are significantly different.

The main goal of this study is to create a simulation tool, which is an important instrument to investigate the effect of the compliance of the different parts on the robot's locomotion. Full dynamic analysis is chosen for this simulation instead of a pure kinematic analysis since it is desired to analyze the ground interaction and the deflection of the legs. Similarly, walking templates like Spring-Loaded Inverted Pendulum (SLIP) are not used as they lack the ground reactions and rolling of the leg. Additionally, an aerial phase does not exist in the locomotion of this, in contrast to the SLIP template. To the best of our knowledge, this is one of the first studies that investigate the locomotion of a walking robot with compliant parts. The leg deflection model (which will be described in the following chapters) is a novel aspect developed under this thesis.

The starting point of this study is the construction of the rigid-body dynamics of the robot. While this model will be enough to analyze the locomotion of the rigid-body-rigid-leg robot, it will also be used as a base that will later include leg

and body deflections. This dynamic model is a system of ordinary differential equations that are generated with the Newton-Euler method. The inputs of these equations are the weight of the robot, the impact forces (estimated with Hunt-Crossley model), and the friction forces (estimated as energy loss of a rolling body). The reasons to choose these estimation methods will be explained in their relative sections. The estimation of these ground reaction forces necessitates the calculation of the position and the velocity of the contact point. Therefore a kinematic analysis of the legs is done up to the velocity level. It is shown that the parameters of the impact force model do not have a significant effect on the estimated locomotion of the robot, while the friction parameters have a considerable effect on the estimated longitudinal speed of the robot. It is also shown that an asymmetry in the mass distribution causes an asymmetry in its locomotion at the half stride moments. Finally, the dependence of the robot's direction to its states at the first contact moment is explained.

To investigate the effect of the leg deflection on the robot's locomotion, a deflection model is incorporated into the main equations of motion mentioned in the previous paragraph. For this purpose, the deflection of the leg is modeled as a spring-damper model with variable stiffness and damping coefficients. This dependency on the leg's roll angle is determined by Castigliano's method. With this resulting dynamic model, the locomotion of the rigid body - soft leg robot is analyzed, and it is shown that using the soft legs results in faster and more balanced walking.

The deflection of the body under the contact forces is analyzed with an offline finite element analysis (FEA) using COMSOL ©. The necessary data of the forces are exported from the rigid-body-rigid-leg robot simulation, then they are used in this FEA simulation as the input. Resulting deflections of the hip joint locations are put back into the rigid-body-rigid-leg robot simulation, thus the soft-body-rigid-leg robot simulation is obtained. Consequently, it is shown that this simulation developed in this study is capable of including the effect of the body deflections on locomotion.

At the final stage, the results of this simulation are verified with the experimental results that are obtained with a motion capture system. By considering all the results, the different locomotion of the robots and their possible reasons are reflected.

1.4 Structure of the Thesis

The rest of this thesis is organized as follows.

Chapter 2 briefly introduces the general design of the robot S-Quad. That includes its locomotion design, its geometric ve material properties, how it is powered, and how it is controlled.

In Chapter 3, the dynamic analysis of the rigid body - rigid leg robot is conducted. It starts with the related assumptions, kinematic definitions of the robot and the kinematic analysis of the contact points up to the velocity level. Given the position and the velocity of a contact point, the impact force is estimated with Hunt-Crossley theory, and the friction is estimated as rolling friction caused by energy loss. Finally, Newton-Euler equations are solved to obtain the rigid body - rigid leg robot's walking trajectory.

In Chapter 4, the effect of the leg's compliance on the locomotion is investigated. Leg's deflection is calculated with a Kelvin-Voigt model-like equation. The variable stiffness in this model is estimated using Castigliano's theorem. Finally, the Newton-Euler equations together with the leg deflection equations are solved to obtain the rigid body - soft leg robot's walking trajectory.

In Chapter 5, the effect of the body's compliance on its locomotion is investigated. The body deflections are estimated with an offline Finite Element Analysis. The obtained results are put back into the Newton-Euler equations to obtain the soft body - rigid leg robot's walking trajectory.

In Chapter 6, the experimental analysis of the robot is given. Firstly, the measurement setup is explained. Then, the position and the orientation results of all four material combinations are given. The results are also compared with the dynamic model results of the previous chapters. Lastly, the effect of particular parts' compliance on the locomotion performance is discussed.

In Chapter 7, the results of this study are discussed in a broad sense. Some thoughts about how this study can be developed further and how it can be used for other purposes are given.

Chapter 2

Design of the Robot

This chapter introduces the robot's mechanic, electronic, and software components that are designed for a desired locomotion. Even though this design process is not a part of this study, the reader should have a general information about these designs in order to better understand the rest of this thesis.

2.1 Mechanical Design

The robot that is subject to this study is an untethered miniature quadruped that can fit into palm of a hand. It has four identical half-circular shaped legs that are independently driven by a DC motor. The rest of the components including the microcontroller, the battery, and the encoders are fixed onto the main body. The main body is a rectangular frame with supporting beams inside, and the leg is a half annulus with rectangular cross-section and half circular caps. Both of these parts have two different versions in terms of material. The rigid versions are made of Polylactic acid (PLA) and manufactured with 3D printing. The soft version are made of Polydimethylsiloxane (PDMS) and manufactured with molding. Other than the material, these counterparts are identical.

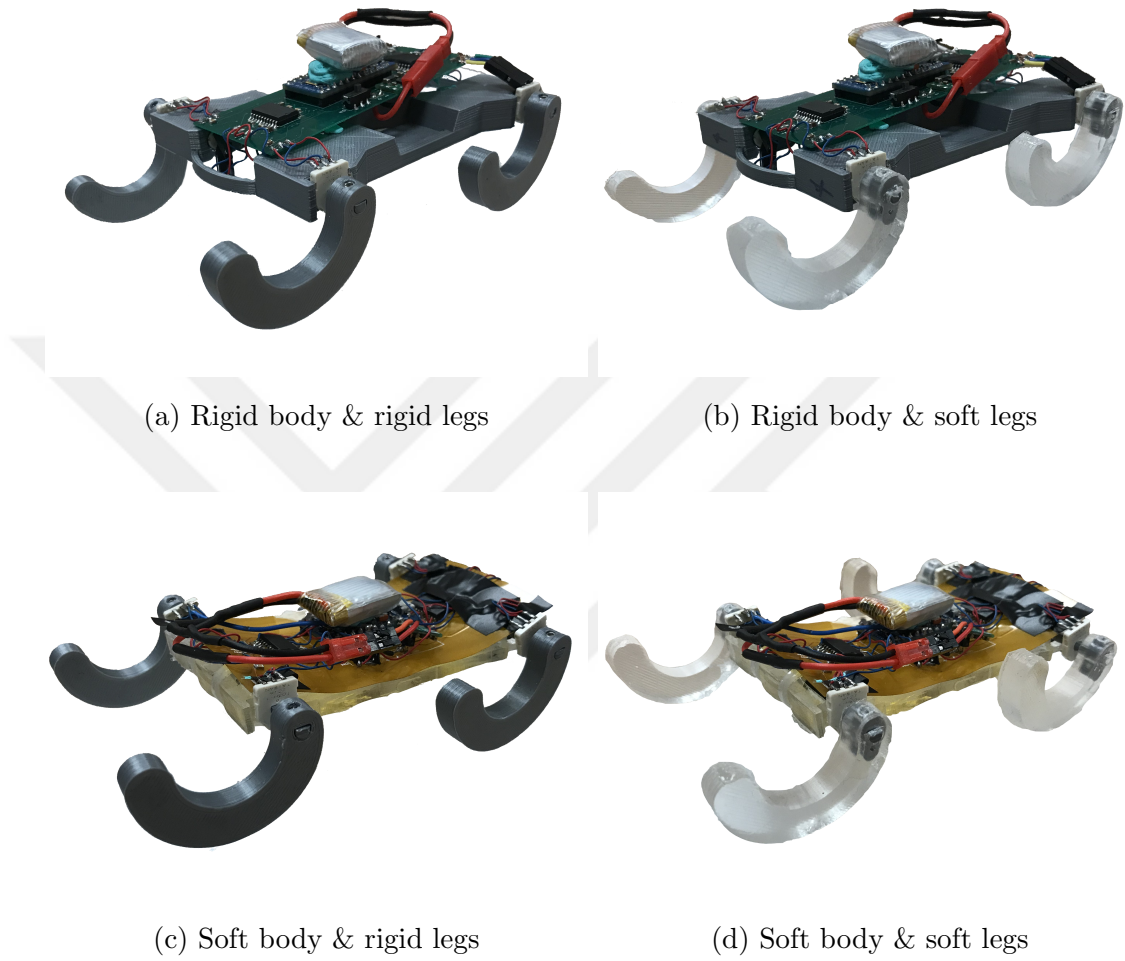


Figure 2.1: Four combinations of rigid/soft body and rigid/soft legs.

Body	Length	100 mm
	Width	75 mm
	Thickness	10 mm
	Mass	0.0529 kg
Leg	Inner Radius	12.5 mm
	Outer Radius	22.5 mm
	Width	7 mm
	Mass	0 kg
	Attachment Point	$[\pm 42.5 \pm 46 \ 0]$ mm

Table 2.1: Geometric parameters of the robot.

Part	Material	Young's modulus	Poisson ratio	Viscosity
Leg	PDMS (5:1)	1.54 MPa [49]	0.50	122 Pa s [94]
	PLA	3.15 GPa [49]	0.33	-
Body	PDMS (10:1)	1.37 MPa [49]	0.50	340 Pa s [94]

Table 2.2: Material properties of the robot

2.2 Controller Design

A low-level controller algorithm is used to control the position and the speed of the DC motor that drives each of the legs. As the state-space model of the DC motor is known, a linear quadratic regulator is used to guarantee control goal. The state vector of this model consists of the angular position of the motor and the angular velocity of the motor ($[\theta_m, \dot{\theta}_m]^T$), and it is fully controllable. The system is also fully observable, however it is not possible to measure all of the states since the rotary sensor of the motor measures only the angular position. Therefore, the angular speed is estimated using a linear quadratic estimator. The block diagram of the general system can be seen Figure 2.2.

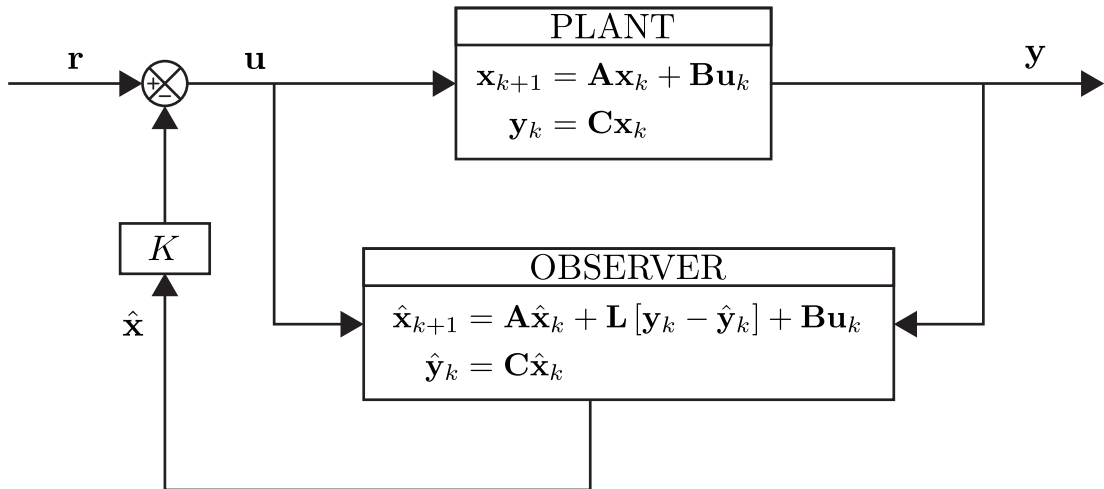


Figure 2.2: Block diagram of the controller and the estimator

All of the four legs have the same desired speed which is user-defined. However, their positions relative to each other is dictated by the desired gait types. Prominent gait types of a quadruped can be seen in Table 2.3 [95]. For the rest of this thesis, trot gait is the default gait of the robot.

Gait Type	Front Right	Front Left	Rear Left	Rear Right
Amble	0.5	0	0.75	0.25
Trot	0.5	0	0.5	0
Pace	0.5	0	0	0.5
Canter	0.3	0	0.7	0
Tranverse Gallop	0.1	0	0.5	0.6
Rotary Gallop	0.1	0	0.6	0.5
Bound	0	0	0.5	0.5
Pronk	0	0	0	0

Table 2.3: Relative phases of the feet regarding to gait type

2.3 Electronic Design

The motion control unit explained in the previous section is implemented onto the robot using a microcontroller board. It reads the position of position feedback coming from the rotary sensor connected to the motor, estimate the states, crates a control input according to the desired speed and gait, and sends this input to the motor through an H-bridge driver. It is powered by a 3.5V Li-Po whose voltage is regulated to 5V. All these components together with several passive components (capacitors, resistors etc) are placed on a double-sided printed circuit board. Similar to leg and body, this PCB has also rigid and soft/flexible versions. The rigid one is made of copper-clad FR-4 (a composite material made of woven fiberglass cloth with a flame resistant epoxy resin binder) while the flexible one is made of copper-clad polyimide (Kapton). The rigid PCB is shown in Figure 2.3 and the individual component are listed in Table 2.4.

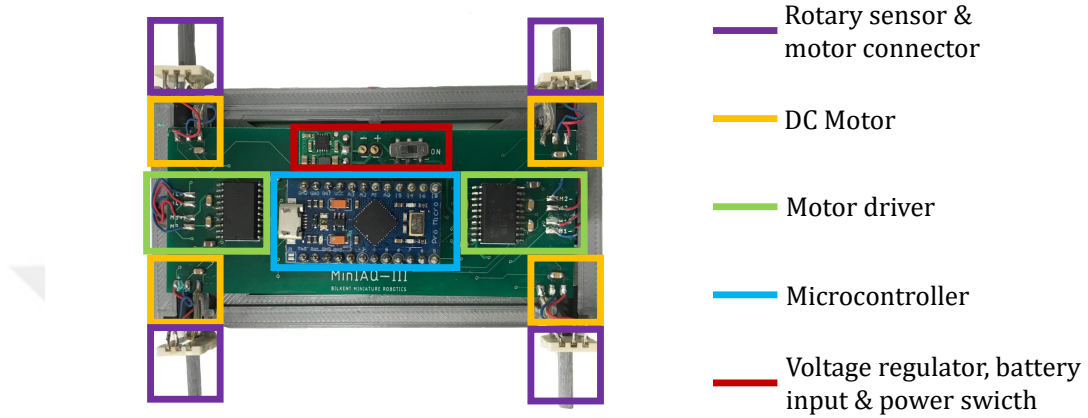


Figure 2.3: Image of the PCB carrying the electronic components of the robot

Microcontroller	Sparkfun Pro Micro - 5V / 16MHz
Motor drivers	TI L293DD Quadruple Half-H Drivers
Motors	Pololu 136:1 Sub-Micro Plastic Planetary Gearmotor
Rotary sensor	Bourns Inc. Rotary Position Sensor - Model 3382
Battery	1C, 3.7V, 150mA h LiPo
Regulator	Pololu 5V Step-Up Voltage Regulator U1V10F5

Table 2.4: List of the electronic components of the robot

This electronic design of the robot is open to modifications. The use of the encoders might not be accurate enough to measure the states of the compliant robot. Therefore, more sensors (such as an Inertial Measurement Unit) can be added to the board to enhance the state measurement capabilities. Moreover, a communication protocol can be built between the user and the robot with the addition of a receiver (such as Bluetooth) to the board.

Chapter 3

Rigid-Body Dynamics of the Robot

Recall the subject (a mobile miniature robot that has 6 degrees of freedom in space but actuated by four motors) and the goals (especially to investigate the effect of different material properties) of this study. These factors necessitate the modeling of the robot's motion with a dynamic analysis rather than sole kinematics.

This chapter introduces the dynamic modeling of the robot, whose parts are accepted to be rigid. The following assumptions were made to simplify this analysis.

1. The robot consists of a rectangular box-shaped rigid body and four c-shaped (half annulus with rectangular cross-section and half-circles at both ends) rigid legs. They do not show any deflection under gravitational or contact forces.
2. Considering the ratios of the mass of a leg to the mass of the main body, and the flight duration of a leg to its stance duration, the mass and the inertia of the legs are assumed to be zero. This assumption reduces the multi-body system analysis of the robot to the single-body analysis of the

body, and therefore significantly reduces the computational complexity.

However, the inertia of the legs might have an effect that cannot be ignored on the robot's locomotion, since they are located at the corners of the robot's total volume. Therefore, adding or ignoring the leg inertia requires further investigation since it might increase the fit of the experimental and the simulation results. For the rest of this thesis, they are also neglected.

3. Each leg is connected to the body with an actuated ideal revolute joint. They rotate parallel to the sagittal plane, and stay parallel during the locomotion.
4. The motor control algorithm works perfectly. In other words, the angular position and velocity reach the desired values dictated by the gait type of the robot, while they do not show any transitional or steady-state error. The motors are capable to supply any torque amount required for this perfect controller without being affected by the disturbances. This assumption simplifies the calculation of position and the velocity of the contact point, while it enables us to omit the torque analysis of the motor.
5. Contact between a leg and the ground occurs at a single point. This assumption simplifies the calculation of the impact force.
6. Due to the curved shape of the leg, the location of the contact point on the leg changes as it is rolling. To find the location of the contact point on the leg more easily, the periphery of the leg is separated into N_{leg} many points with constant arc angles between them. The lowest of these points represents the contact point of the leg. Then, the leg is assumed to be a rigid link between the hip joint and the contact point.

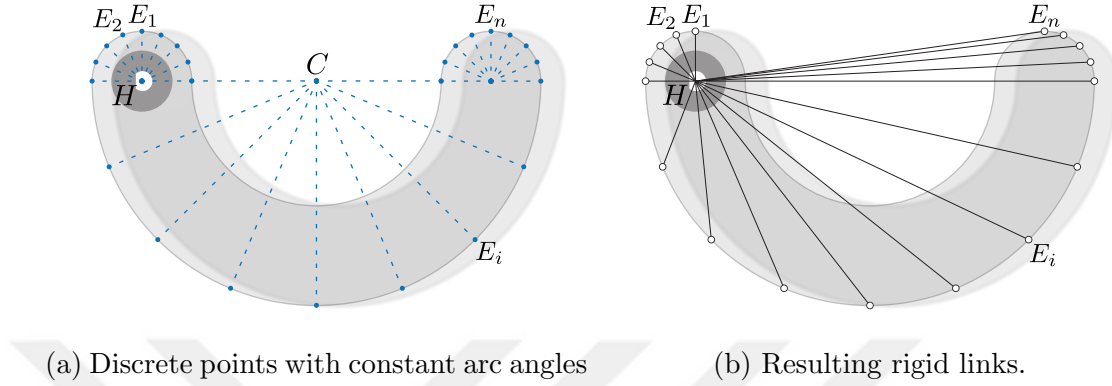


Figure 3.1: Geometric discretization of the leg.

3.1 Kinematic Analysis

As it will be described in the next section, the estimation of the ground reaction requires the information of the position and the velocity of the contact point. For this reason, kinematic analysis of the legs is completed up to the velocity level.

3.1.1 Reference Frames

The kinematic vectors are defined with respect to three different reference frames in this analysis. Fundamentally, the first of them is the inertial reference frame that is fixed in the space. The other five frames are floating bases (i.e., they undergo homogeneously transformation with respect to the inertial reference frame), a frame defined for the body and a frame defined for each leg.

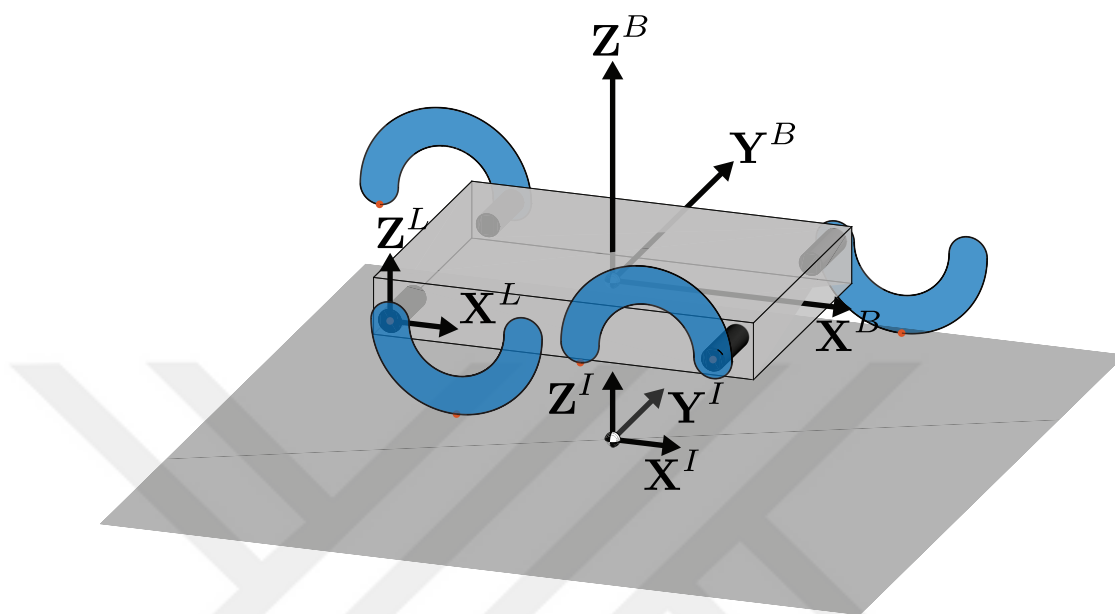


Figure 3.2: Reference frames defined for the kinematic analysis.

The transformation of the body reference frame (denoted by $\mathcal{B}$) with respect to the inertial frame (denoted by $\mathcal{I}$) has 6 degrees of freedom. Its translation is represented with the position vector $\mathbf{r}_{OB}^{\mathcal{I}} = [x \ y \ z]^T$. Its orientation is represented with the ZYX Euler (Tait-Bryan) angles. In this convention, the total rotation described with a rotation around the local $\mathbf{Z}^{\mathcal{B}}$ axis (called yawing, angle denoted by ψ), then a rotation around the local $\mathbf{Y}^{\mathcal{B}}$ axis (called pitching, angle denoted by θ), and finally a rotation around the local $\mathbf{X}^{\mathcal{B}}$ axis (called rolling, angle denoted by ϕ). The resulting rotation can be mathematically represented with a rotation matrix as follows.

$$\begin{aligned}
\mathbf{R}_{\mathcal{B}}^{\mathcal{I}}(\phi, \theta, \psi) &= \mathbf{R}_z(\phi) \mathbf{R}_y(\theta) \mathbf{R}_x(\psi) \\
&= \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \\
&= \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}
\end{aligned} \tag{3.1}$$

The transformation of a leg reference frame (denoted by $\mathcal{L}_i$) with respect to the inertial frame has just 1 degree of freedom, which is the rotation of the motor around $\mathbf{Y}^{\mathcal{L}_i}$ axis. This rotation can be represented with the following rotation matrix, which has one free parameter, the angular position of the motor (denoted by θ_m).

$$\mathbf{R}_{\mathcal{B}}^{\mathcal{L}} = \mathbf{R}_y(\theta_m) = \begin{bmatrix} \cos \theta_m & 0 & \sin \theta_m \\ 0 & 1 & 0 \\ -\sin \theta_m & 0 & \cos \theta_m \end{bmatrix} \tag{3.2}$$

Since the rotation matrices are elements of special orthogonal matrices group ($\mathbf{R} \in SO(3)$), the reverse rotation matrices can be calculated easily using the orthonormality condition.

$$\mathbf{R}_{\mathcal{L}}^{\mathcal{B}} = (\mathbf{R}_{\mathcal{B}}^{\mathcal{L}})^{-1} = (\mathbf{R}_{\mathcal{B}}^{\mathcal{L}})^T \tag{3.3a}$$

$$\mathbf{R}_{\mathcal{B}}^{\mathcal{I}} = (\mathbf{R}_{\mathcal{I}}^{\mathcal{B}})^{-1} = (\mathbf{R}_{\mathcal{I}}^{\mathcal{B}})^T \tag{3.3b}$$

The linear velocity of the robot can be defined with respect to the body reference frame (represented by vector $\dot{\mathbf{r}}_{OB}^B = [u \ v \ w]^T$) and with respect to the inertial reference frame (represented by vector $\dot{\mathbf{r}}_{OB}^I = [\dot{x} \ \dot{y} \ \dot{z}]^T$). The relationship between these two vectors can be described using the corresponding rotation matrix.

$$\dot{\mathbf{r}}_{OB}^I = \mathbf{R}_B^I \dot{\mathbf{r}}_{OB}^B \quad (3.4)$$

Similarly, the angular velocity of the robot can be defined with respect to the body reference frame (represented by vector $\boldsymbol{\omega}^B = [p \ q \ r]^T$) and with respect to the inertial reference frame (represented by vector $\boldsymbol{\omega}^I = [w_x \ w_y \ w_z]^T$). The relationship between these two vectors can be described using the corresponding rotation matrix.

$$\boldsymbol{\omega}_{OB}^I = \mathbf{R}_B^I \boldsymbol{\omega}_{OB}^B \quad (3.5)$$

The angular velocity of the robot with respect to the inertial reference frame is related to the rotation matrix between the inertial and body frames through a skew-symmetric matrix as follows.

$$\mathbf{S}(\boldsymbol{\omega}^I) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = \dot{\mathbf{R}}\mathbf{R}^T \quad (3.6)$$

By using the last two equations, the relationship between the angular velocity of the robot with respect to the inertial frame and the derivatives of the Euler angles is derived as follows.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.7)$$

3.1.2 Position Level Kinematics

The position of any point (E_i) fixed on the leg is calculated with respect to the inertial frame as follows.

$$\begin{aligned} \mathbf{r}_{OE_i}^I &= \mathbf{r}_{OB}^I + \mathbf{r}_{BH}^I + \mathbf{r}_{HE_i}^I \\ &= \mathbf{r}_{OB}^I + \mathbf{R}_B^I \mathbf{r}_{BH}^B + \mathbf{R}_B^I \mathbf{R}_L^B \mathbf{r}_{HE_i}^L \end{aligned} \quad (3.8)$$

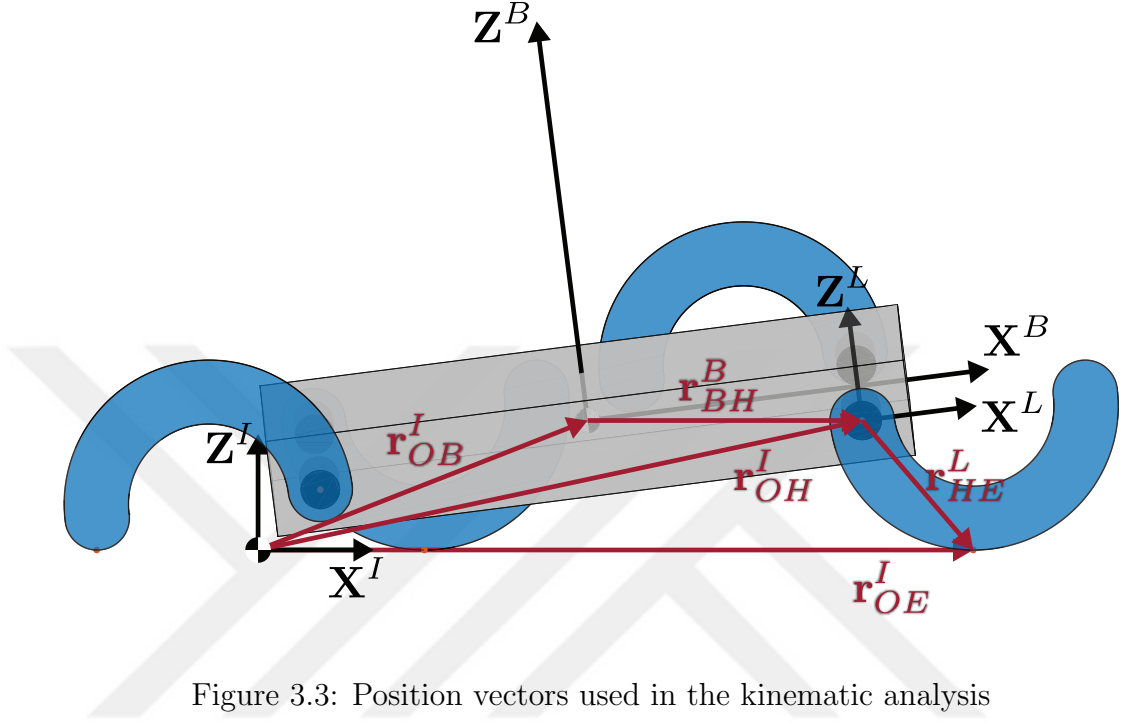


Figure 3.3: Position vectors used in the kinematic analysis

Here,

- $\mathbf{r}_{OB}^I = [x \ y \ z]^T$, is the position of the robot in the inertial frame
- $\mathbf{r}_{BH}^B$, is the location of the hip joint in the body frame.
- $\mathbf{r}_{HE_i}^L$, is the position of the point E_i in the leg frame.

After the positions of the leg points described in 3.1 are calculated, the lowest of them (i.e. the one that has the minimum $\mathbf{Z}^I$ value) is selected to be the contact point (denoted by E). Then, the variables needed for the velocity analysis are calculated as follows.

$$l = \|\mathbf{r}_{HE}^L\| \quad (3.9a)$$

$$\theta_c = \angle(\mathbf{r}_{HC}^L, \mathbf{r}_{HE}^L) \quad (3.9b)$$

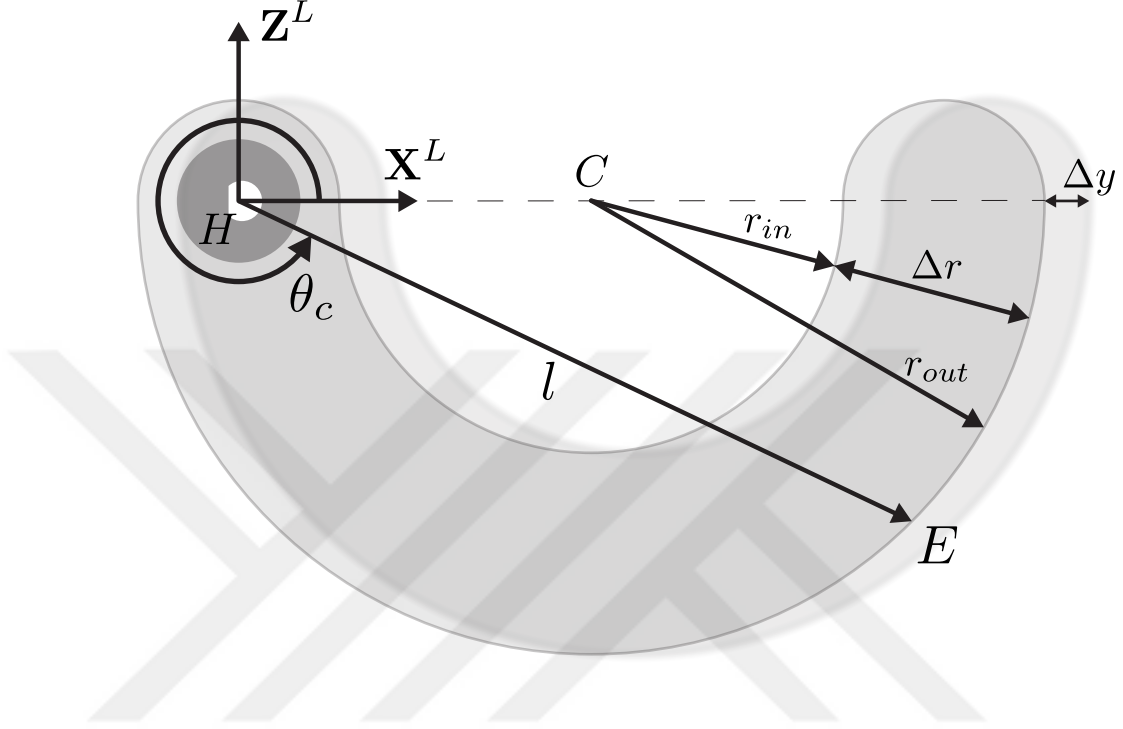


Figure 3.4: Geometric variables and parameters of a leg

3.1.3 Velocity Level Kinematics

Velocity of the point E can be calculated by simply taking the time derivative of Equation (3.8).

$$\begin{aligned}
 \mathbf{v}_{OE}^{\mathcal{I}} &= \dot{\mathbf{r}}_{OE}^{\mathcal{I}} = \dot{\mathbf{r}}_{OB}^{\mathcal{I}} + \dot{\mathbf{R}}_{\mathcal{B}}^{\mathcal{I}} \mathbf{r}_{BH}^{\mathcal{I}} + \mathbf{R}_{\mathcal{B}}^{\mathcal{I}} \dot{\mathbf{r}}_{BH}^{\mathcal{I}} + \dot{\mathbf{R}}_{\mathcal{B}}^{\mathcal{I}} \mathbf{r}_{HE}^{\mathcal{I}} + \mathbf{R}_{\mathcal{B}}^{\mathcal{I}} \dot{\mathbf{r}}_{HE}^{\mathcal{I}} \\
 &= \dot{\mathbf{r}}_{OB}^{\mathcal{I}} + \boldsymbol{\omega}_{\mathcal{I}}^{\mathcal{B}} \times (\mathbf{R}_{\mathcal{B}}^{\mathcal{I}} \mathbf{r}_{BH}^{\mathcal{B}} + \mathbf{R}_{\mathcal{B}}^{\mathcal{I}} \mathbf{r}_{HE}^{\mathcal{B}}) + \mathbf{R}_{\mathcal{B}}^{\mathcal{I}} (\dot{\mathbf{r}}_{BH}^{\mathcal{B}} + \dot{\mathbf{r}}_{HE}^{\mathcal{B}})
 \end{aligned} \tag{3.10}$$

Here,

- $\dot{\mathbf{r}}_{OB}^{\mathcal{I}} = [\dot{x} \ \dot{y} \ \dot{z}]^T$, is the velocity of the robot in the inertial frame.
- $\boldsymbol{\omega}^{\mathcal{B}} = [p \ q \ r]^T$ is the angular velocity of the robot in the body frame.
- As a result of the rigidity assumption,

$$\dot{\mathbf{r}}_{BH}^{\mathcal{B}} = 0 \tag{3.11}$$

- Considering the plane geometry,

$$\dot{\mathbf{r}}_{HE}^{\mathcal{C}} = \begin{bmatrix} -l (\sin \theta_c) \dot{\theta}_m \\ 0 \\ l (\cos \theta_c) \dot{\theta}_m \end{bmatrix} \quad (3.12)$$

- Desired motor speed,

$$\dot{\theta}_m = 2\pi f_{desired} \quad (3.13)$$

Thus, Equation (3.10) takes its final form as follows.

$$\mathbf{v}_E^{\mathcal{I}} = \mathbf{R}_B^{\mathcal{I}} \mathbf{v}_B^{\mathcal{I}} + \boldsymbol{\omega}_I^{\mathcal{B}} (\mathbf{r}_{OE}^{\mathcal{I}} - \mathbf{r}_{OB}^{\mathcal{I}}) + \mathbf{R}_B^{\mathcal{I}} \begin{bmatrix} -l (\sin \theta_c) \dot{\theta}_m \\ 0 \\ l (\cos \theta_c) \dot{\theta}_m \end{bmatrix} \quad (3.14)$$

3.2 Estimation of Ground Reaction Forces

The force term in the equations of motion of the robot is the sum of the robot's weight and the contact forces coming from each leg. This contact force consists of the impact force perpendicular to the ground and the friction force parallel to the ground. The friction force is dependent on the magnitude of the impact force. Hence, the impact force is estimated by using the position and the velocity of the leg points, then the friction force is estimated by using this impact force.

3.2.1 Impact Force

There are several viscosity models in the literature (some of which are illustrated in Figure 3.5) that are used to estimate impact forces between two bodies. However, these linear models result in some non-realistic properties [96]. Firstly, they estimate a non-zero force at the initial contact moment. Secondly, they estimate a power flow from moving body to the fixed body. To address these issues, Hunt and Crossley [97] proposed a non-linear version of Kelvin-Voigt model.

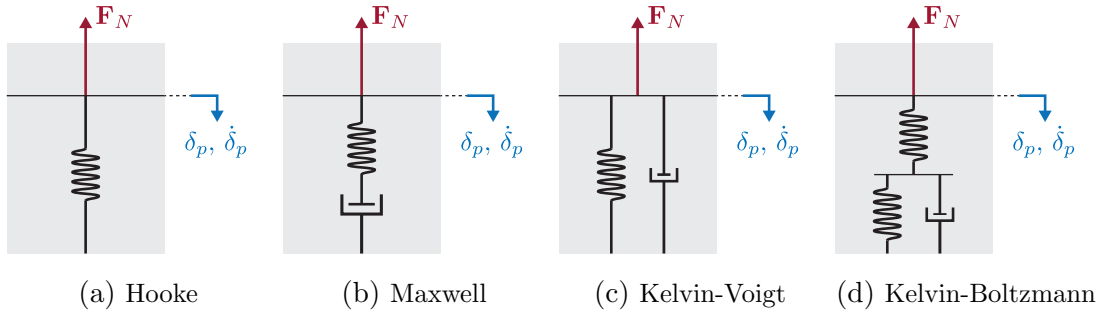


Figure 3.5: Linear viscoelastic impact models

According to Hunt-Crossley model, the impact force is dependent on the penetration depth and the velocity of the contact point as follows.

$$F_N = K(\delta_p)^n + C(\delta_p)^n(\dot{\delta}_p) \quad (3.15)$$

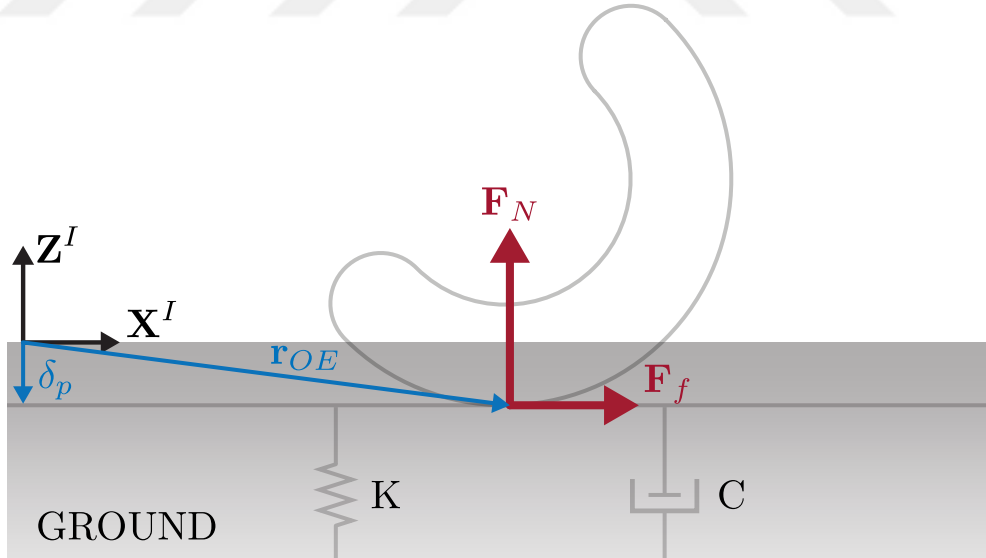


Figure 3.6: Illustration of the Hunt-Crossley model for the impact

The penetration depth and velocity can be found by using the kinematic analysis results of the leg.

$$\delta_p = |\min(0, (\mathbf{r}_{OE}^T)_z)| \quad (3.16a)$$

$$\dot{\delta}_p = (\mathbf{v}_{OE}^T)_z \quad (3.16b)$$

Two different sets of values have been investigated for the parameters K , C , and n used in Equation (3.15). They are the results of a system identification process done by Haddadi and Hashtrudi-Zaad [98], and they correspond to rigid contacts (ground contact of a material with relatively high Young's modulus, such as PLA) and soft contacts (ground contact of a material with relatively high Young's modulus, such as PDMS) respectively. This classification could be done by looking at the Tabor parameter, which is given below. If $\mu_T > 5$ it can be classified as a soft material, while it can be classified as rigid if $\mu_T < 5$.

$$\mu_T = \left(\frac{Rw^2}{E^{*2}z_0^3} \right)^{1/3} \quad (3.17)$$

Material	μ_T	Type	K	C	n
PLA	0.0477	Rigid	1500 N m ⁻¹	50 N s m ⁻¹	1.9
PDMS	7.9345	Soft	10 N m ⁻¹	2 N s m ⁻¹	1.2

Table 3.1: The parameter sets for Hunt-Crossley model

3.2.2 Friction Force

3.2.2.1 Effect of Material Properties on Friction

The following equation shows the relationship between the impact force and the friction force occurring at a contact point [99].

$$F_f = \mu F_N + \sigma A \quad (3.18)$$

The first term of the right-hand side is called the load-controlled friction, and the second one is called the adhesion-controlled friction. The adhesion-controlled term is generally at negligible levels. Whether it can be neglected or not can be checked by looking at the Tabor parameter of the contacting body. If it is sufficiently large ($\mu_T > 5$) we can ignore the adhesion-controlled term and the remaining of Equation (3.18) becomes the Amonton law ($F_f = \mu F_N$) or the

Coulomb law ($F_f = \mu(v)F_N$). This is the case for the PLA leg while the Tabor parameter of the PDMS leg shows that the adhesion-controlled term cannot be ignored.

The contact area A given in Equation (3.18) is also dependent on the normal force. For the cylindrical soft bodies, it can be estimated as follows [100]

$$w = \left(\frac{\left(\frac{3\pi K a^2 l}{8R} \right) - P}{6\pi K a l^2} \right)^2 \quad (3.19)$$

Here,

$$\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \quad (3.20)$$

$$K = \frac{4}{3}E^* \quad (3.21)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (3.22)$$

$$A = 2al \quad (3.23)$$

In these equations, w represents the common surface energy, K represents the common elastic modulus, P represents the load, l represents the contact length between the contacting bodies, while E_i represents Young's moduli, and R_i represents radii of the contacting bodies.

Equation (3.19) approaches to the Hertz mechanics as $w \rightarrow 0$, and it approaches to the JKR mechanics as $w \gg 0$.

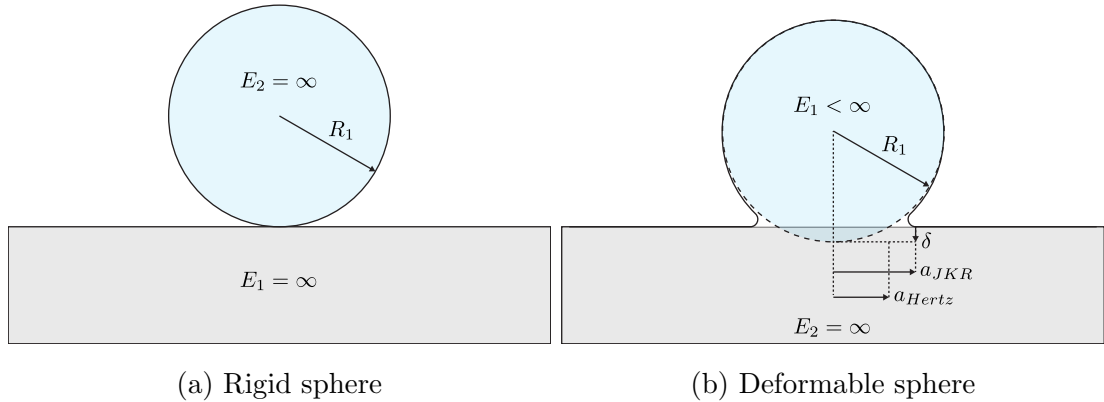


Figure 3.7: Illustration of Hertz vs JKR mechanics

Results of Equation (3.19) for two materials subject to this study and two different surface mechanisms ($w = 0$ for Hertz, and $w = 57 \text{ mJ m}^{-2}$ for JKR [101]) are presented as follows.

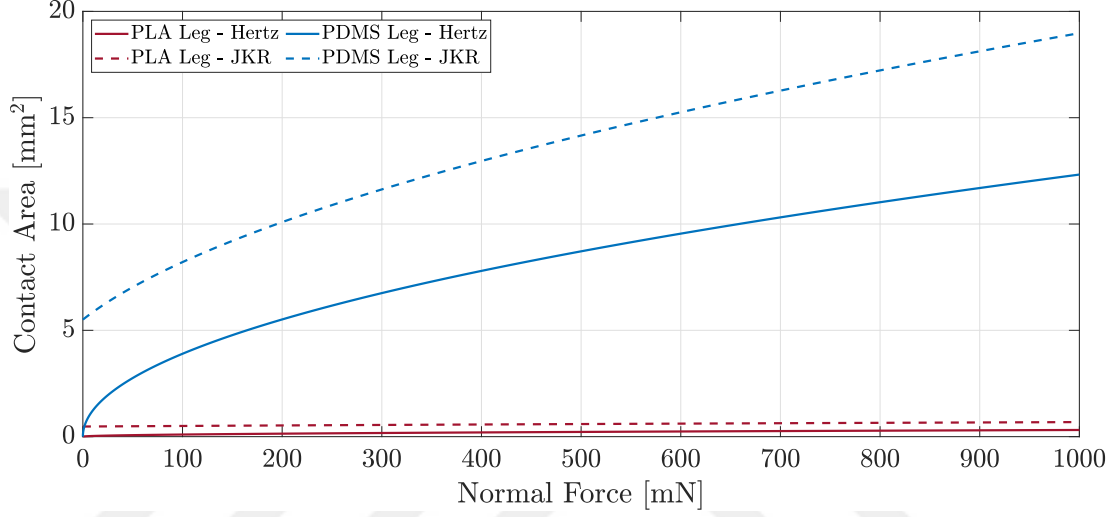


Figure 3.8: Dependence of contact area on normal force

Results of Equation (3.18) with the calculated contact area are presented as follows. They satisfy the expectations explained in this chapter, i.e. the adhesion-controlled friction term is much smaller than the load-controlled friction for PLA, and it is vice versa for PDMS.

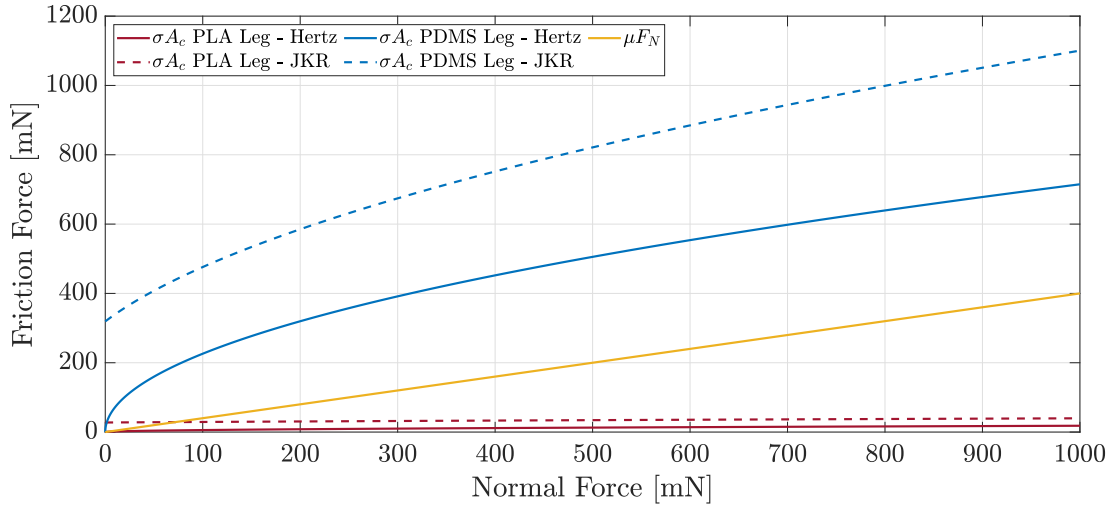


Figure 3.9: Dependence of friction force terms on normal force

3.2.2.2 Estimation of Friction Force

When the walking of the robot is visually observed during experiments, it can be assumed that there is almost no sliding between the leg and the ground. Also considering the shape of the leg, the friction between a leg and the ground can be identified as rolling friction instead of sliding friction. To simplify the estimation of this rolling friction, the legs are considered as spheres with Hertzian properties. Accordingly, the elastic energy stored during the contact is calculated as follows [99].

$$E_{contact} = \int F_N d\delta \quad (3.24)$$

When the relations between the normal force, the contact area, and penetration depth are integrated to the previous equation, we can obtain the following.

$$E_{contact} = \frac{2}{5} \left(\frac{(F_N)^5}{K^2 R} \right)^{1/3} \quad (3.25)$$

If we assume that ε ($0 \leq \varepsilon \leq 1$) of this energy dissipates into heat, the friction force can be estimated as follows.

$$F_f = \frac{\varepsilon E_{contact}}{2a} = \frac{1}{5} \varepsilon \left(\frac{(F_N)^4}{K R^2} \right)^{1/3} \quad (3.26)$$

This equation can be simplified as follows.

$$F_f = \mu_r (F_N)^{4/3} \quad (3.27a)$$

$$\mu_r = \frac{1}{5} \varepsilon \left(\frac{1}{K R^2} \right)^{4/3} \quad (3.27b)$$

After estimating the magnitude of the friction, the $\mathbf{Z}^I$ and $\mathbf{Y}^I$ components of the friction force vector can be found by decomposing the magnitude of the friction force along $\mathbf{Z}^I$ and $\mathbf{Y}^I$ components of the tangential velocity of the contact point.

$$(\mathbf{F}_f)_y = F_f \cdot \frac{(\mathbf{v}_{OE}^I)_y}{\sqrt{((\mathbf{v}_{OE}^I)_y)^2 + ((\mathbf{v}_{OE}^I)_z)^2}} \quad (3.28a)$$

$$(\mathbf{F}_f)_z = F_f \cdot \frac{(\mathbf{v}_{OE}^I)_z}{\sqrt{((\mathbf{v}_{OE}^I)_y)^2 + ((\mathbf{v}_{OE}^I)_z)^2}} \quad (3.28b)$$

Finally, the contact force acting on a leg- j can be found with the unification of the impact and the friction forces as follows.

$$\mathbf{F}_j^{\mathcal{I}} = \begin{bmatrix} F_N \\ (\mathbf{F}_f)_y \\ (\mathbf{F}_f)_z \end{bmatrix} \quad (3.29)$$

3.3 Equations of Motion

Newton-Euler method is selected to construct the equations of motion of the robot since we are modeling a single body's motion under the effect of contact forces. When it is applied in the body reference frame of the robot, the equations of motion can be as follows. Please notice that the body reference frame is a non-inertial reference frame, hence the following equation must include Coriolis terms.

$$\mathbf{F}_{total}^B = m (\dot{\mathbf{r}}_{OB}^B + \boldsymbol{\omega}^B \times \mathbf{r}_{OB}^B) \quad (3.30a)$$

$$\mathbf{M}_{total}^B = \mathbf{I}^B (\dot{\boldsymbol{\omega}}^B + \boldsymbol{\omega}^B \times \boldsymbol{\omega}^B) \quad (3.30b)$$

The force and moment terms on the left-hand side comes from the weight of the robot and the contact forces.

$$\mathbf{F}_{total}^B = \mathbf{R}_I^B \mathbf{W}^I + \sum_{j=1}^4 \mathbf{F}_j^B \quad (3.31a)$$

$$\mathbf{M}_{total}^B = \mathbf{r}_{com}^I \times (\mathbf{R}_I^B \mathbf{W}^I) + \sum_{j=1}^4 (\mathbf{r}_{OE}^I \times \mathbf{F}_j^B) \quad (3.31b)$$

Equations (3.30), (3.4), (3.7), and (3.13) together creates a system of ordinary differential equations that completely describes the robot's motion in both the inertial and the body frames.

$$\dot{\boldsymbol{\Gamma}} = f(\boldsymbol{\Gamma}) \quad (3.32)$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & u & v & w & p & q & r & \theta_{m1} & \theta_{m2} & \theta_{m3} & \theta_{m4} \end{bmatrix}^T \quad (3.33)$$

3.4 Results of Rigid-Body Dynamics

The motion of the robot having a rigid body, 4 rigid legs, 1 Hz of stride frequency, and trot gait is obtained with the solution of Equation (3.32) using MATLAB's ode15s function. The required parameters are given in the Tables 2.1, 2.2, and 3.1. The initial value is given as follows.

$$\mathbf{\Gamma}_0 = \begin{bmatrix} 0 & 0 & 42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.4708 & 1.6708 & -1.4708 & 1.6708 \end{bmatrix}^T \quad (3.34)$$

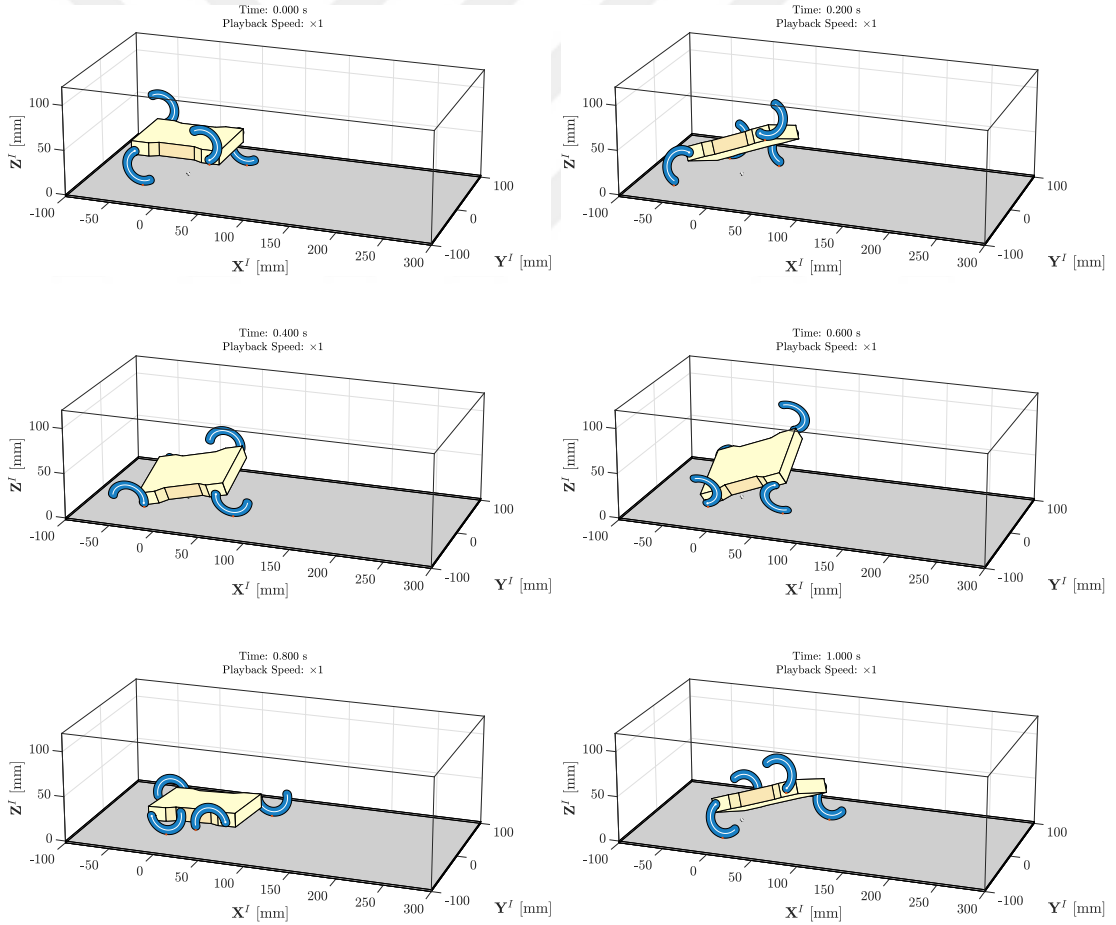


Figure 3.10: Snapshots from the simulation of the RBRL robot

3.4.1 Effect of the Impact Parameters

Two sets of impact parameters are used to estimate the impact forces for the RBRL and RBSL robots correspondingly. When the deflections are ignored, the effect of the impact parameters on the estimated locomotion is negligible despite the difference in the magnitude of the impact. This also shows the strong dependence of the developed analysis on the robot geometry. Nonetheless, it is more accurate to use soft contact parameters that result in lower forces for the leg deflection analysis.

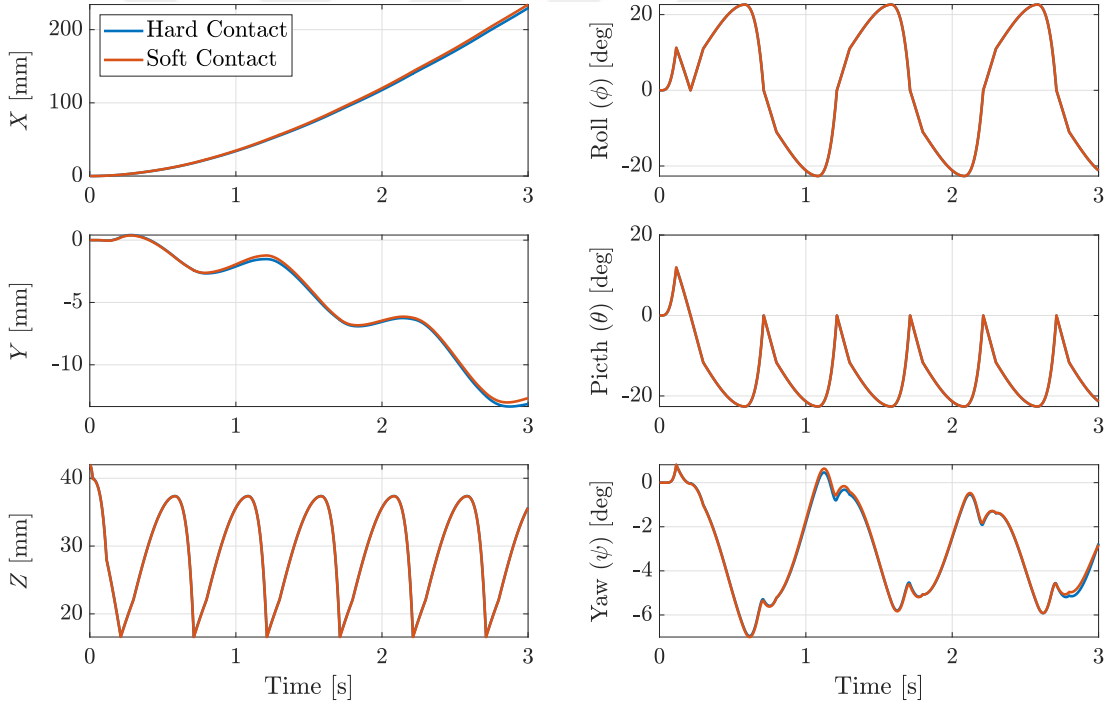


Figure 3.11: Simulation results of the RBRL robot with a set of impact parameters K , C , and n

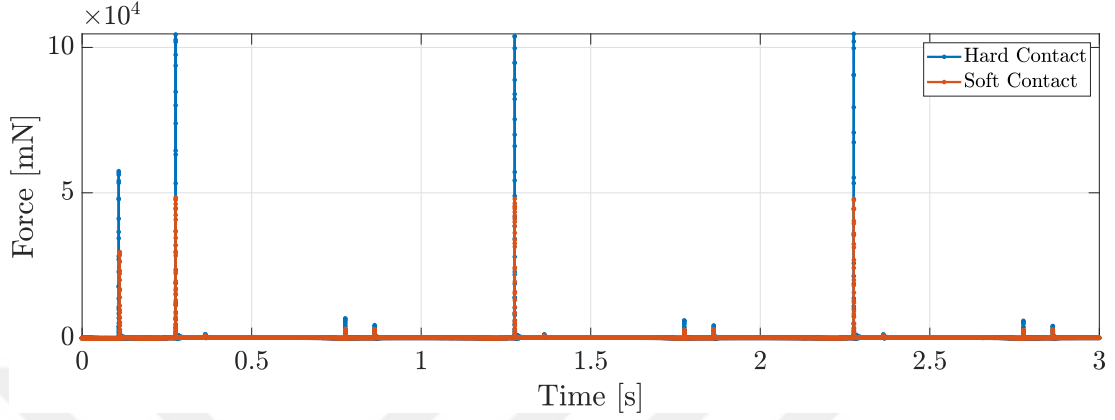


Figure 3.12: Effect of contact parameters on the impact force estimation

3.4.2 Effect of the Friction Coefficient

This time, the simulation of the RBRL robot is repeated with a set of different values of the friction parameter ε . When the $\varepsilon = 0$, the forward motion of the robot is almost zero (i.e., it only slips in its place). This demonstrates that the source of the locomotion is the friction between the legs and the surface. Furthermore, the longitudinal speed increases with increased friction, but its slope is decreasing to zero. Therefore, it can be argued that the speed is reaching its kinematic limit. Lastly, the effect of the increased friction on the pitch and roll angles is negligible. This supports the previous comment on the impact parameters: the dependence of the developed analysis to the robot geometry.

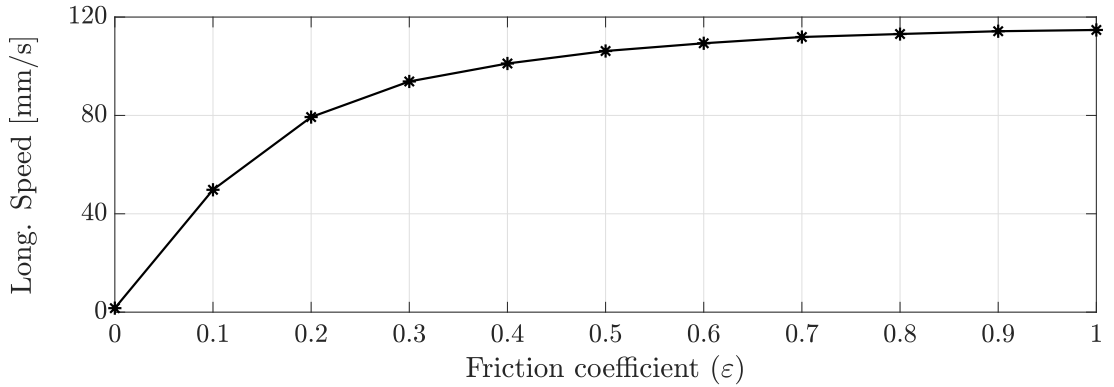
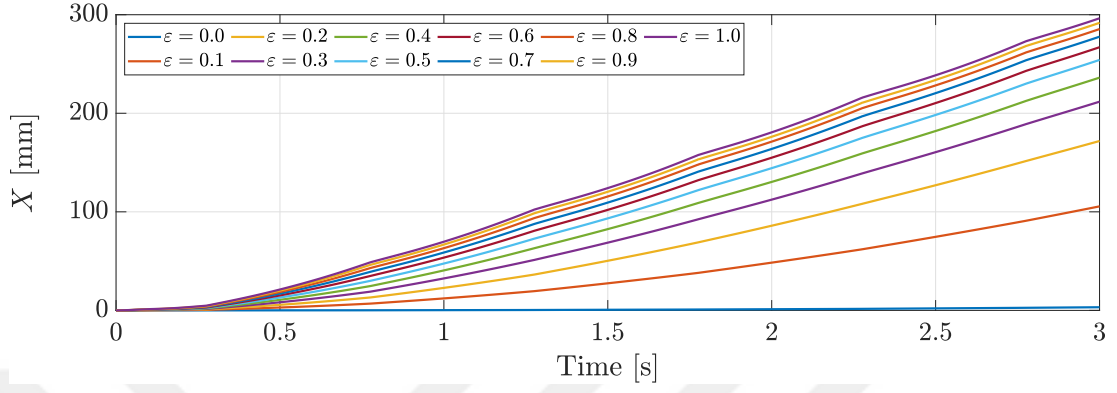
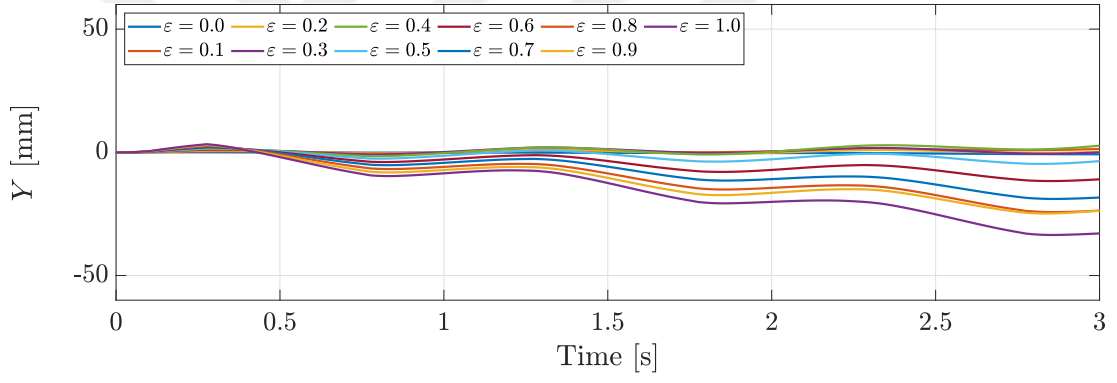


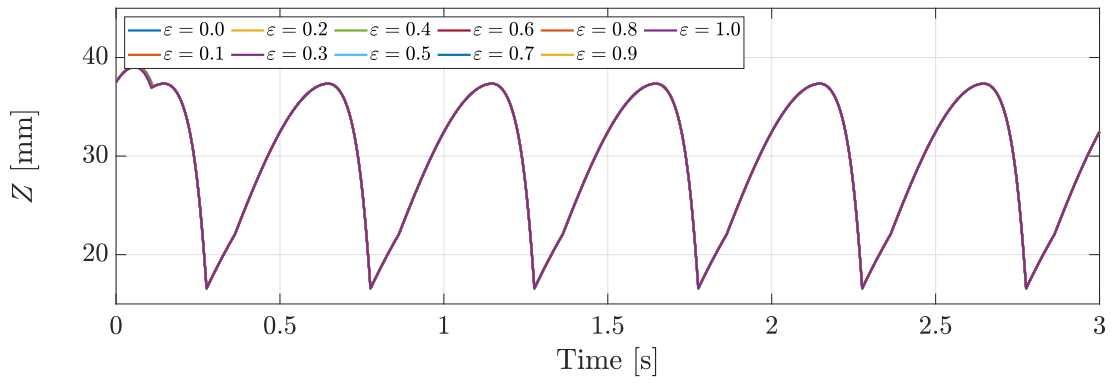
Figure 3.13: Dependence of the longitudinal speed onto friction



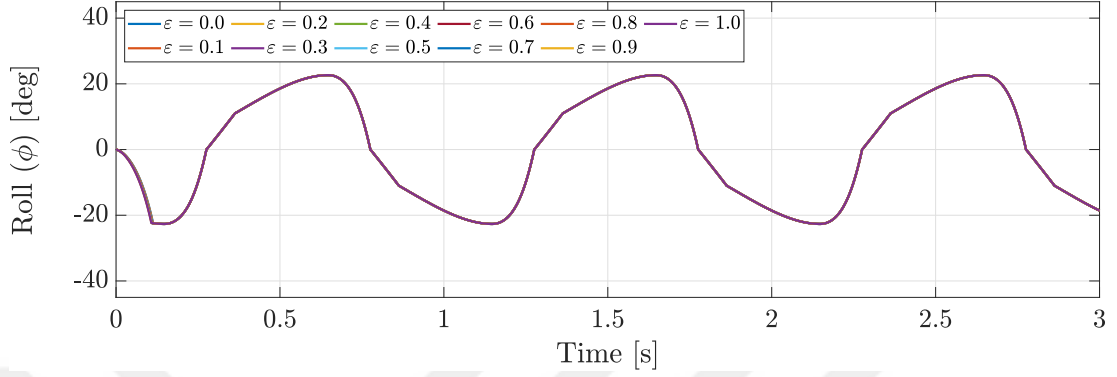
(a) Position in $\mathbf{X}^I$ direction



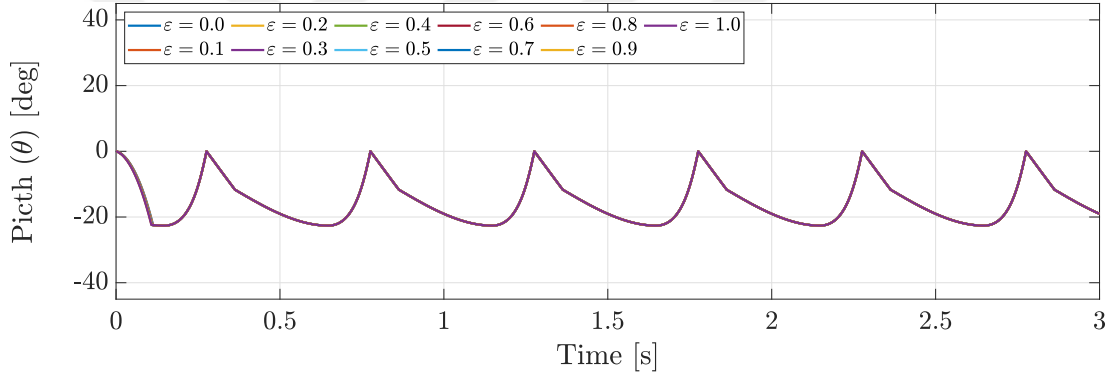
(b) Position in $\mathbf{Y}^I$ direction



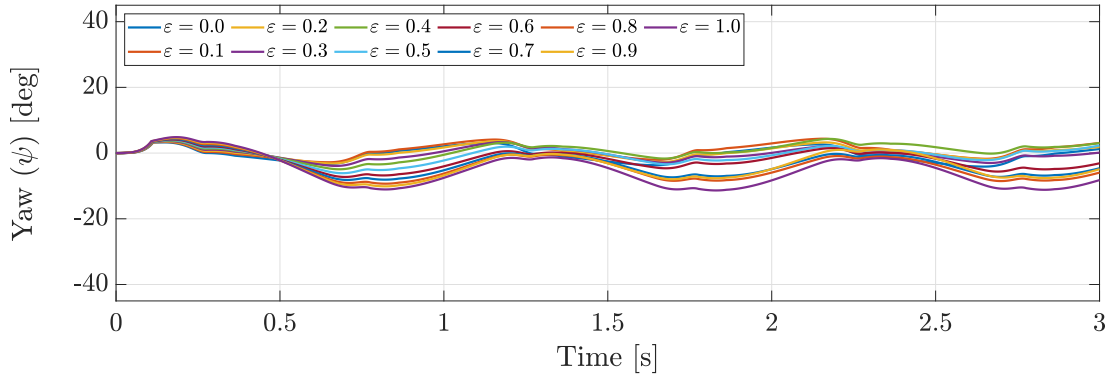
(c) Position in $\mathbf{Z}^I$ direction



(d) Orientation as roll angle



(e) Orientation as pitch angle



(f) Orientation as yaw angle

Figure 3.14: Simulation results of the RBRL robot with a set of friction coefficients

3.4.3 Effect of the Unbalanced Center of Mass

Experimental results display a deviation at the half periods of a stride. This indicates an asymmetry in terms of the mass distribution of the robot (i.e., a difference between the center of mass of the robot and the geometric center of the robot). Even though it is hard to estimate this difference for the robot subjected to the experiment, an arbitrary value of the center of mass location ($r_{com} = [5.2545 \ 1.9704 \ 0]^T$) incorporated into the dynamic analysis results in a similar behavior. This undesired effect of the unbalanced mass distribution could be prevented with a careful design of the robot (particularly the electronic parts since the mechanic parts are already designed to be symmetric). This unbalance can also be compensated after the manufacturing of the robot using an auxiliary mass.

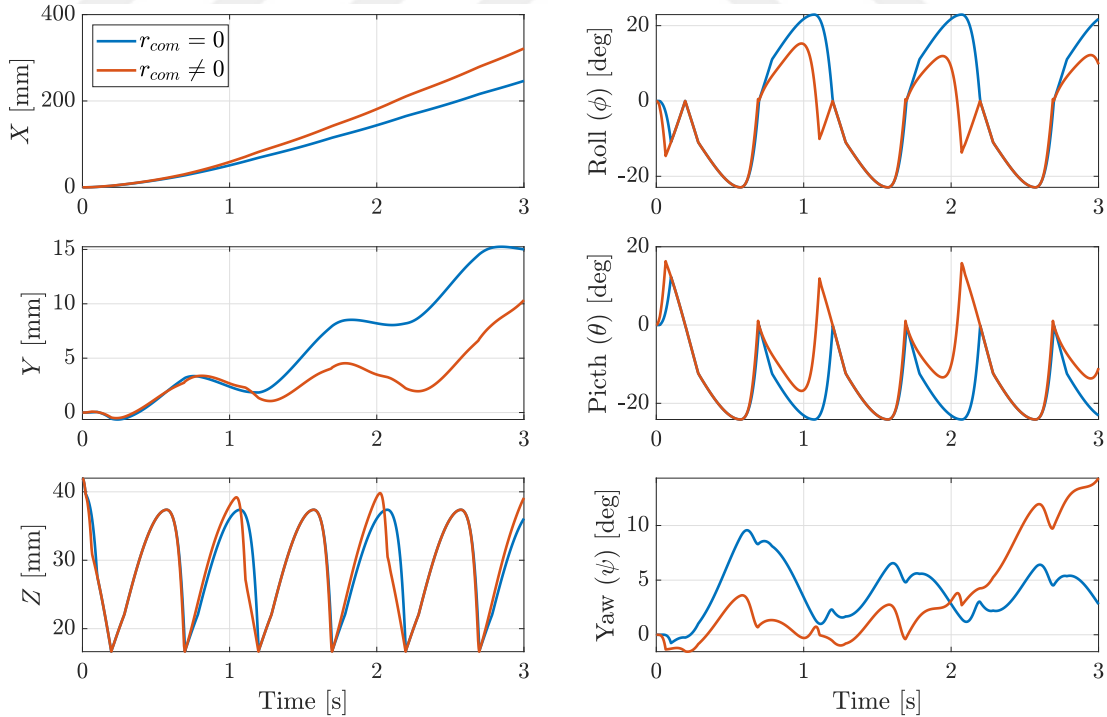
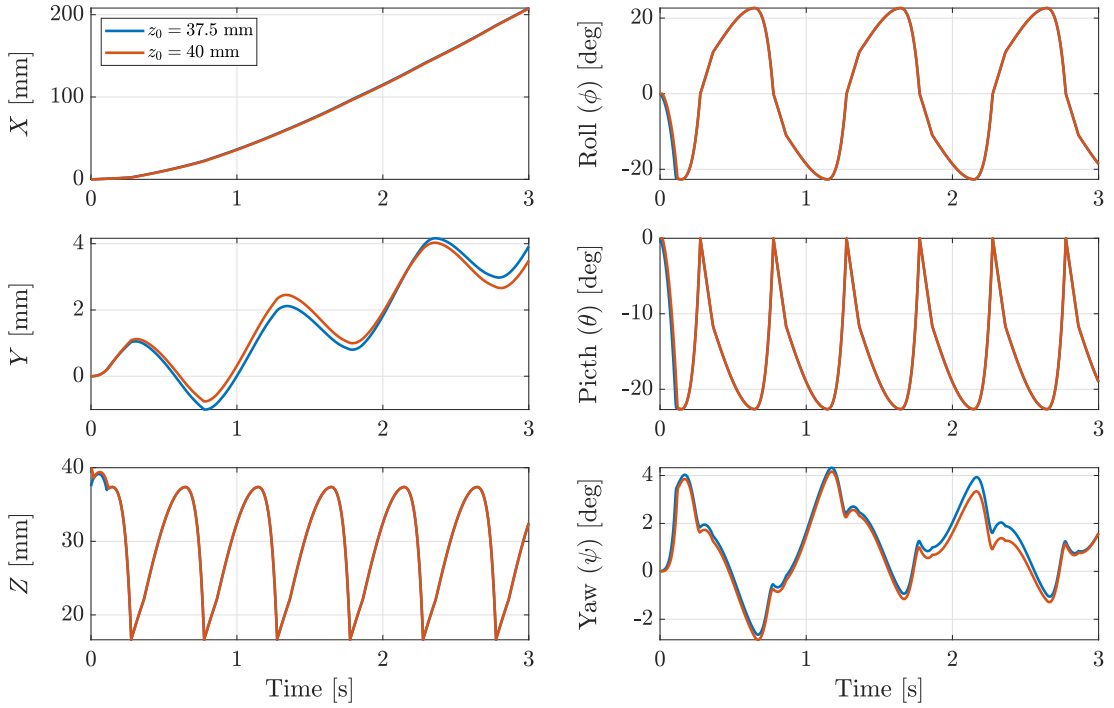


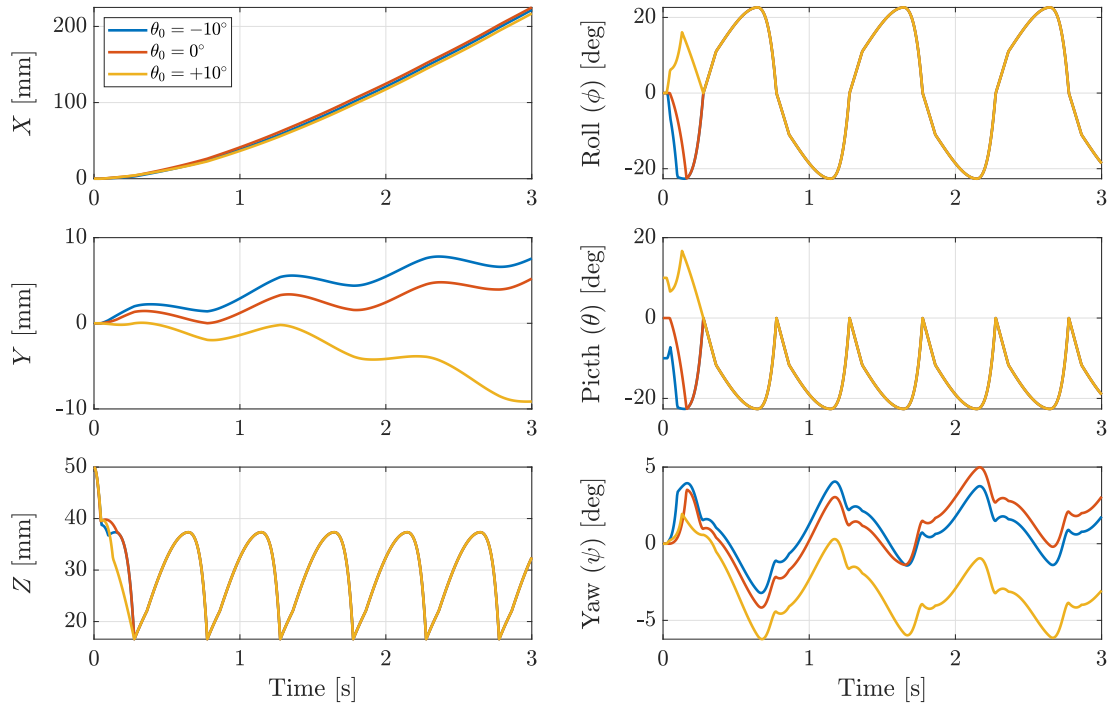
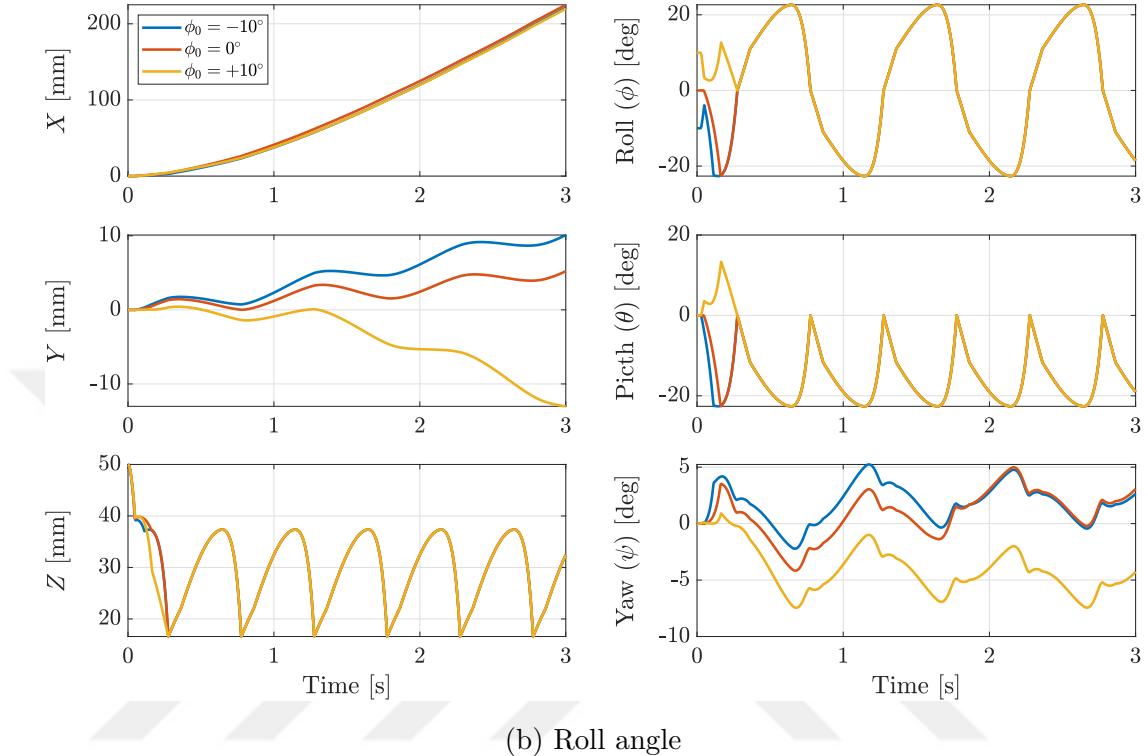
Figure 3.15: Effect of unbalance on the locomotion estimation of the RBRL robot

3.4.4 Effect of the Initial Conditions

During the experiments of all four robots, it has been observed that the walking direction of the robot is different at each repeat of the experiment. This might be because of the initial conditions (i.e., the states at the first contact moment). This hypothesis is verified with the developed simulation by solving the ODE system with different sets of possible initial conditions. They cannot be set or measured precisely during the experiments: we power on the robot while holding it up in the air, then we simply let it walk on the ground. It should be also noted that there isn't any motion planning algorithm on the robot that can control its direction. Even though it is tough to use the exact initial conditions of the experiment (as in the case of mass unbalance), the results show that the walking direction is easily changed with initial conditions while the speed, pitch, and roll remain unaffected.



(a) Height



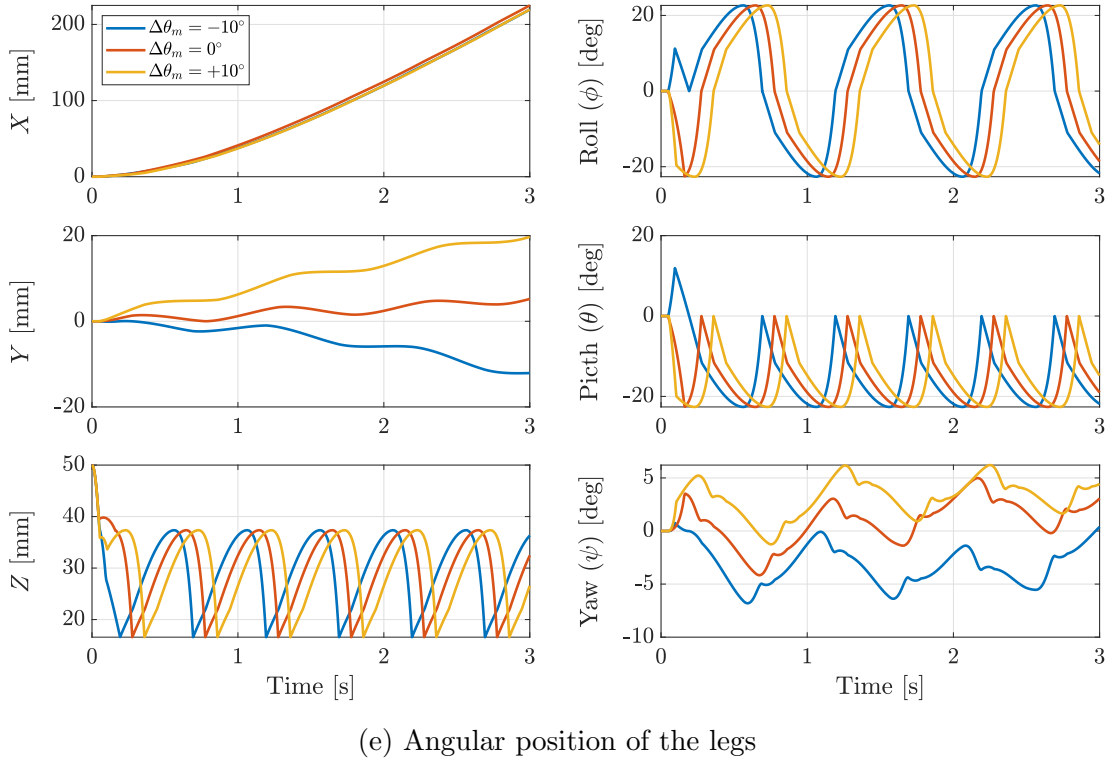
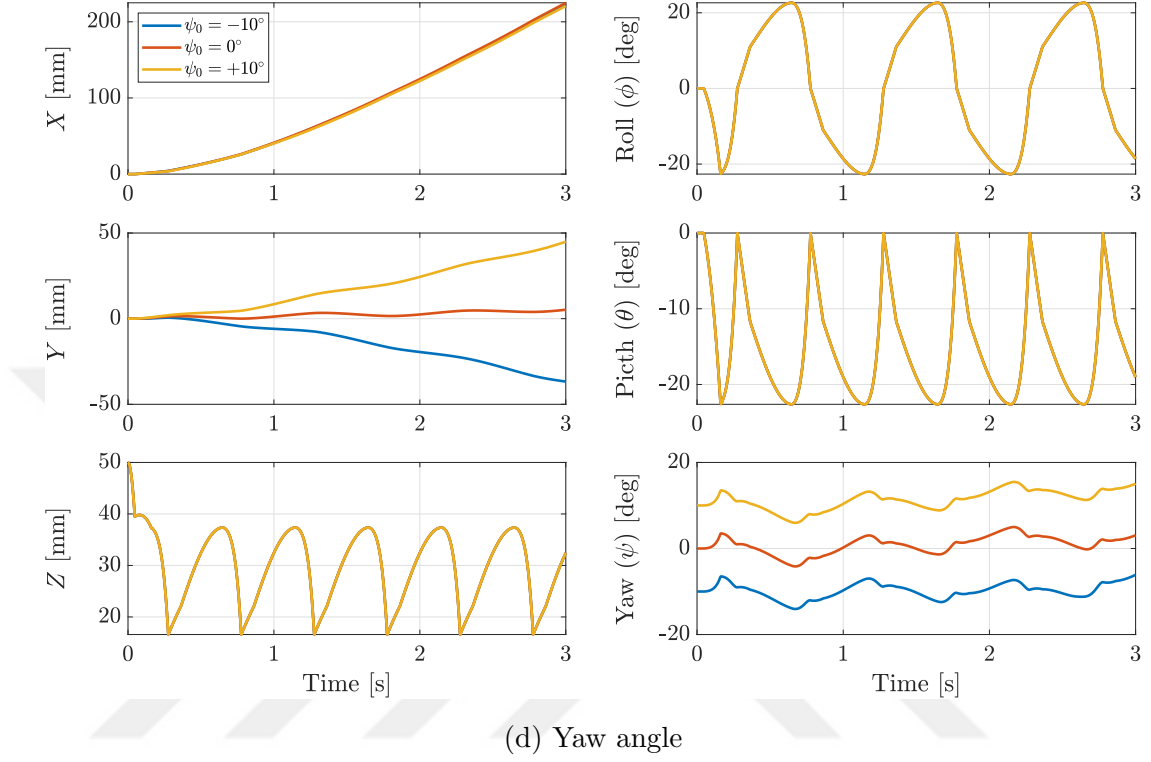


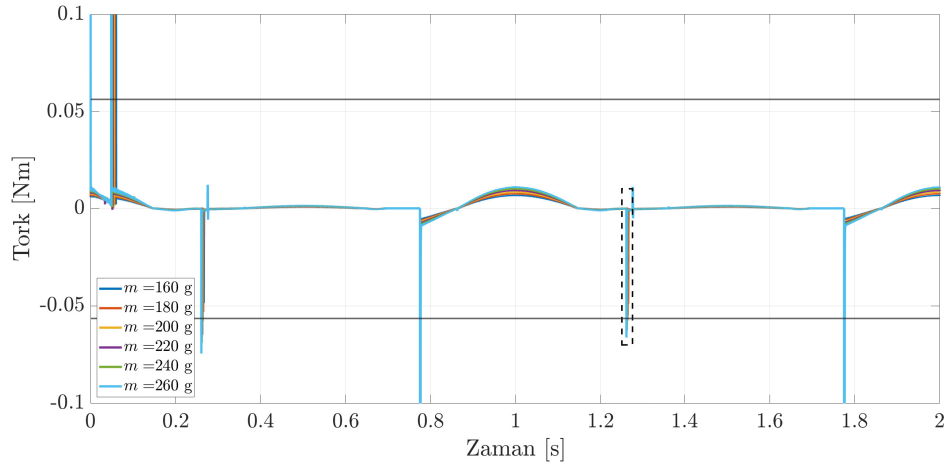
Figure 3.16: Effect of the initial conditions on the locomotion estimation of the RBRL robot

3.4.5 Payload Estimation

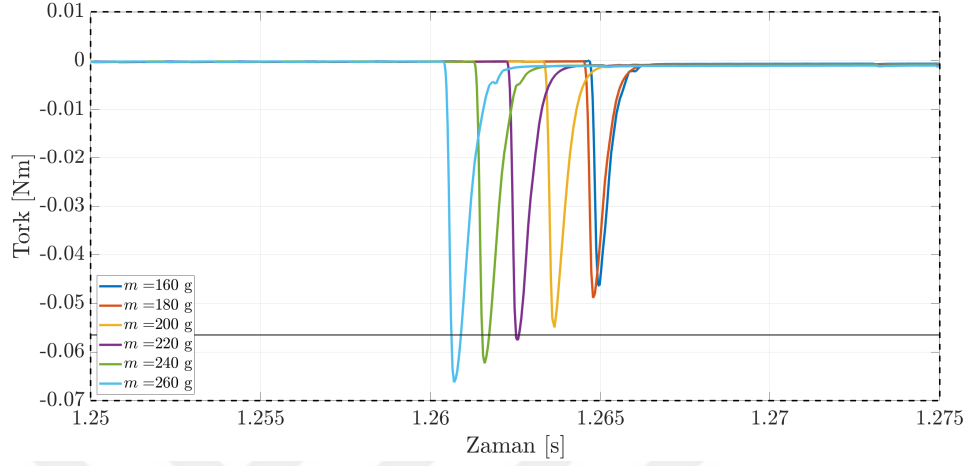
In order to estimate the maximum mass that can the robot can carry, a payload estimation has been done by comparing the stall torque of the motor and the torque caused by the friction. The stall torque of the motor is found from the datasheet. The torque caused by the friction is estimated as follows.

$$\tau_f = \mathbf{r}_{HE}^{\mathcal{L}} \times \left[(\mathbf{F}_j^{\mathcal{B}})_x \quad 0 \quad 0 \right]^T \quad (3.35)$$

This torque is calculated for different mass values of the robot. Negative values of this torque occur in the moments of contact and take-off of a leg. In order for the robot to keep walking, it is necessary to overcome the braking, which is the effect of negative torque at the moment of takeoff. Looking at the graph below, the torque caused by the friction exceeds the stall torque for the mass value of 200 g grams. Given the mass of the robot, it leaves about 150 g in the capacity of the load.



(a) Total simulation



(b) Zoomed view

Figure 3.17: Torque caused by the friction on the Rear Left Leg, estimated for different mass values.

3.4.6 Effect of Other Parameters

Other parameters that might affect the robot's locomotion performance include, but are not limited to temperature, adhesion, and terrain roughness. Furthermore, these effects might be estimated with the dynamic model developed in this study. Firstly, even though it does not have an explicit existence in the dynamic model, the effect of the friction on the motion can be seen in other parameters (e.g., Young's modulus). It is known that the mechanical properties of PDMS, in particular, have a strong dependence on the manufacturing and experimental conditions [102]. One might expect that it will mainly affect the friction of the legs.

Secondly, the adhesion between the leg and the ground can be explicitly included in the dynamic model, even though some sort of adhesion is indicated with the negative values of the impact estimation. This modification can increase the accuracy for the PDMS leg in particular, which has a Tabor parameter value higher than 5 and is more suitable for the JKR assumptions. An effective impact

force can be estimated using these assumptions as follows [103].

$$F_{Neff} = F_N + 3\pi R w + \sqrt{(3\pi R w)^2 + 6\pi R w F_N} \quad (3.36)$$

Lastly, the effect of terrain properties on the motion can be estimated as well. The inclination of a flat surface can be included by using the corresponding rotation matrix. On the other hand, the inclusion of the terrain roughness might require an intersection algorithm to solve for the contact point. In either case, the locomotion performance of the robot would likely deteriorate.

Chapter 4

Modeling of the Soft C-Leg

The experimental results of the RBRL and the RBSL legs indicate that the effect of the deflection on the robot locomotion is not negligible. The dynamic model developed in the previous chapter is not capable to show this difference, it should be extended in such a way that it includes the leg deflection. Accordingly, this chapter introduces the modeling the leg deflection dependent to the rolling angle of the leg, and its integration with the equations of motion of the robot.

As usual, there are some assumptions made to simplify the modeling process. They are listed as follows.

1. The legs are solid, homogeneous, and isotropic.
2. The legs do not deflect in the normal direction of their sagittal plane (i.e., $\mathbf{Y}^L$).
3. The experiments conducted by Lin [104] show that the leg deflection has 2 degrees of freedom.
4. Legs preserve their circular shape after deflection.

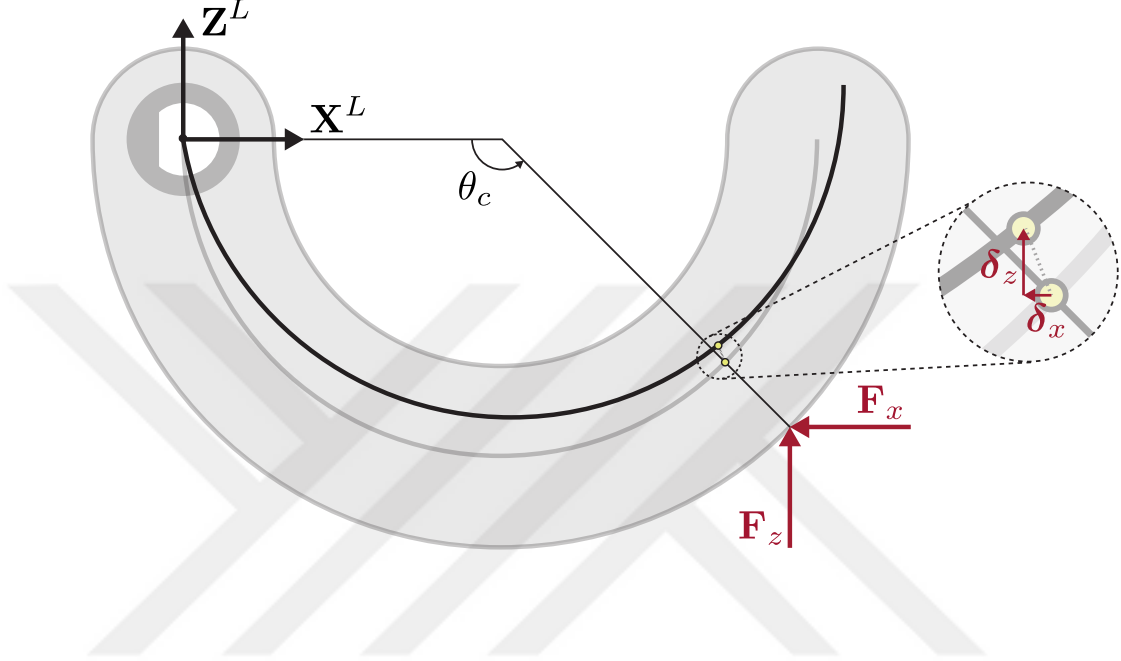


Figure 4.1: Illustration of the planar leg deflection

4.1 Variable Stiffness of the Leg

When considered with Hooke's law ($F = kx$) in a general sense, the amount of deflection will be dependent to the contact forces. However, due to the nature of leg geometry, the amount of deflection will be dependent the location the force on the leg as well. Therefore, Castigliano's method [90] is used to model the deflection of a such circular beam. According to this method, the deflection produced by a contact force $\mathbf{F}^{\mathcal{L}} = [F_x, F_y, F_z]^T$ can be estimated as follows.

$$\delta_i = \frac{\partial U}{\partial F_i} \quad (4.1)$$

In this equation, $i \in \{x, y, z\}$ represents the direction of the force and the corresponding deflection while U represents the strain energy caused by the force. This energy consists of four terms that are due to bending moment, normal force,

tangential force and shear. They are estimated in the same order as follows.

$$U_1 = \int_0^{\theta_c} \frac{M^2}{2AeE} d\theta \approx \int_0^{\theta_c} \frac{M^2 R}{2EI} d\theta \quad (4.2a)$$

$$U_2 = \int_0^{\theta_c} \frac{F_\theta^2 R}{2AE} d\theta \quad (4.2b)$$

$$U_3 = \int_0^{\theta_c} \frac{-MF_\theta}{AE} d\theta \quad (4.2c)$$

$$U_4 = \int_0^{\theta_c} \frac{CF_r^2 R}{2AG} d\theta \quad (4.2d)$$

The last three terms are negligible for this application.

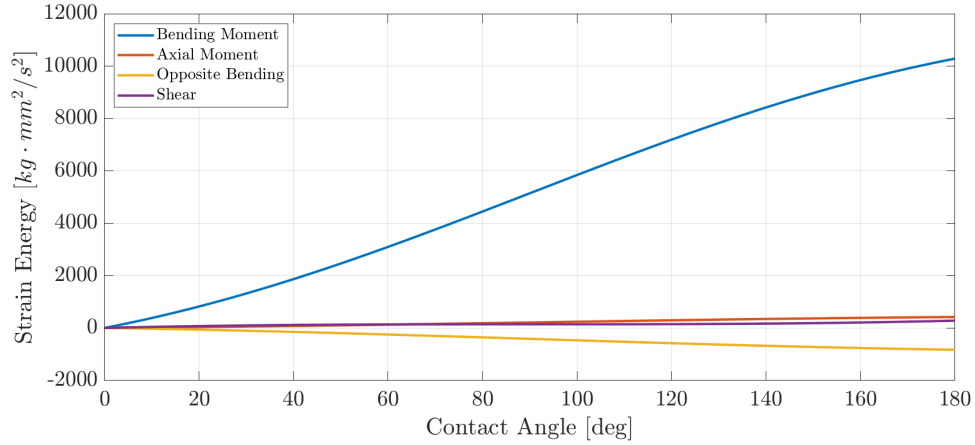


Figure 4.2: Strain energy terms (under 1N tangential and 1N radial force) of the soft leg with respect to the roll angle of the leg

The moment M used in these equations can be calculated as follows.

$$M = R(F_z \sin \theta_c + F_x (\cos \theta_c - 1)) \quad (4.3)$$

By putting Equation (4.3) into Equation (4.2) and taking the integral, we obtain the deflections in the $\mathbf{X}^{\mathcal{L}}$ and $\mathbf{Z}^{\mathcal{L}}$ directions as follows.

$$\delta_x = \frac{R^3}{2EI} \left((3\theta_c - 3\sin \theta_c - 2\sin^2(\theta_c/2) \sin \theta_c) F_x + (-4\sin^2(\theta_c/2) + \sin^2 \theta_c) F_z \right) \quad (4.4a)$$

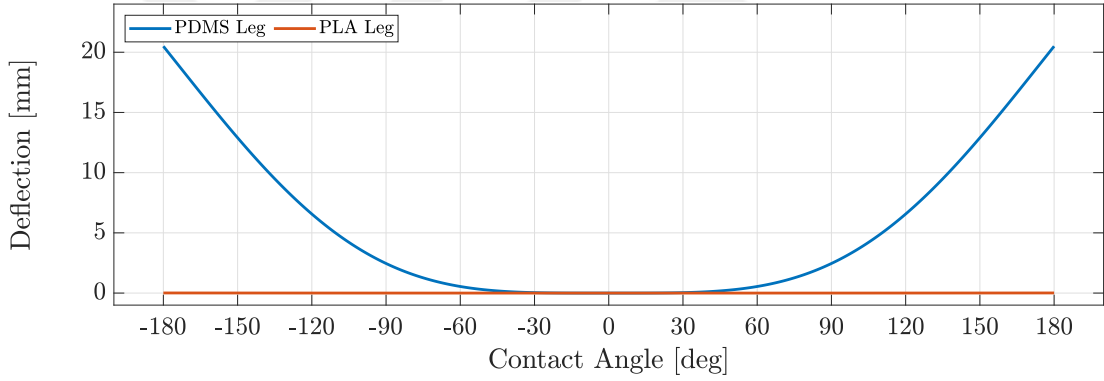
$$\delta_z = \frac{R^3}{2EI} \left((-1 + 2\cos \theta_c - \cos^2 \theta_c) F_x + (\theta_c - 0.5\sin(2\theta_c)) F_z \right) \quad (4.4b)$$

Since Equation (4.4) is linear, it can be written in the matrix format as follows.

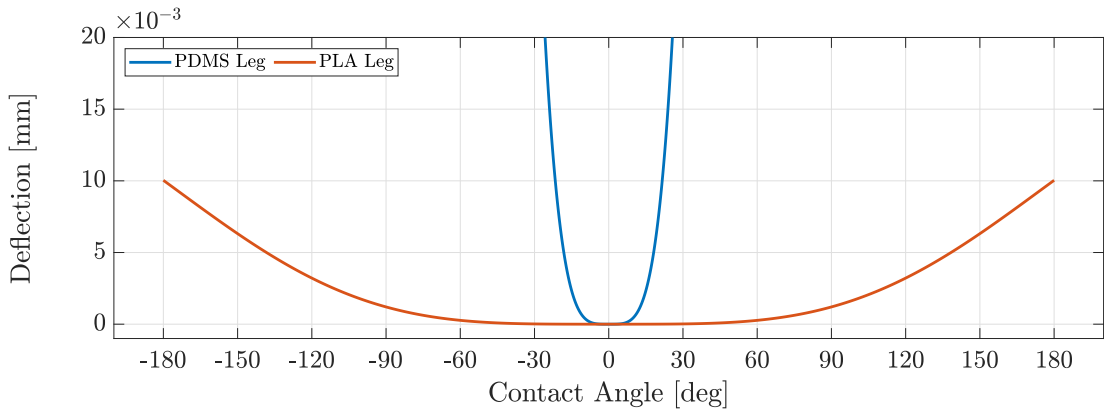
$$\begin{bmatrix} \delta_x \\ \delta_z \end{bmatrix} = \mathbf{C} \begin{bmatrix} F_x \\ F_z \end{bmatrix} \quad (4.5a)$$

$$\mathbf{C} = \frac{R^3}{2EI} \begin{bmatrix} 3\theta_c - 3\sin\theta_c - 2\sin^2(\theta_c/2)\sin\theta_c & -4\sin^2(\theta_c/2) + \sin^2\theta_c \\ -1 + 2\cos\theta_c - \cos^2\theta_c & \theta_c - 0.5\sin(2\theta_c) \end{bmatrix} \quad (4.5b)$$

In accordance with the the empirical observations, the variable coefficients of Equation (4.5) shows that the deflection amount increases as the application point of the force moves from the fixed end to the free end of the leg even the magnitude of the force remains constant.



(a) Difference between two legs' deflection



(b) Zoomed in to show the smaller values of PLA leg's deflection

Figure 4.3: Deflection amount of the robot leg with respect to contact angle under 1 N horizontal force

4.2 Dynamic Analysis of the Leg Deflection

Equation (4.5) does not include any time-dependent term explicitly, hence it can be identified as a static equation. When the sudden and very large changes in the magnitude of the impact forces are considered (see Figure 3.12), it can be argued that such an analysis may not be realistic. Therefore, a damping term is added to equation (4.5) considering time amount for the leg to deflect and then turn back to its original shape.

A mass-spring-damper system has the following equation of motion.

$$m\ddot{x} + b\dot{x} + kx = f \quad (4.6)$$

This equation can be rewritten as follows, when the compliance of the spring is used instead of its stiffness.

$$mc\ddot{x} + bc\dot{x} + x = cf \quad (4.7)$$

Analogously, a variable damping term can be added to Equation (4.5) as follows.

$$\begin{bmatrix} \delta_x \\ \delta_z \end{bmatrix} + c_d \mathbf{C} \begin{bmatrix} \dot{\delta}_x \\ \dot{\delta}_z \end{bmatrix} = \mathbf{C} \begin{bmatrix} F_x \\ F_z \end{bmatrix} \quad (4.8)$$

Therefore, the system of ordinary equations that describes the robot's motion can be extended with addition of the previous equation, so that it also describes the legs' deflection at the same time.

$$\begin{bmatrix} \mathbf{I}_{16 \times 16} & & & \\ & c_d \mathbf{C}_1 & & \\ & & c_d \mathbf{C}_2 & \\ & & & c_d \mathbf{C}_3 \\ & & & & c_d \mathbf{C}_4 \end{bmatrix} \begin{bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\Gamma, \delta_1, \delta_2, \delta_3, \delta_4) \\ \mathbf{C}_1 \mathbf{F}_1 - \delta_1 \\ \mathbf{C}_2 \mathbf{F}_2 - \delta_2 \\ \mathbf{C}_3 \mathbf{F}_3 - \delta_3 \\ \mathbf{C}_4 \mathbf{F}_4 - \delta_4 \end{bmatrix} \quad (4.9)$$

With the calculated deflection, the radius of the robot's leg changes accordingly. This calculation is explained in Appendix A.

4.3 Effect of the Soft Leg on the Locomotion

The following figures successfully repeats the differences observed in the experiments of the RBRL and the RBSL robots. The soft legged robot has higher speed in the longitudinal direction, its body is closer to the ground, and its walking is more balanced in terms of roll and pitch angles. Therefore, it can be argued that the soft legs are more advantageous than the rigid legs when they are used with the rigid body.

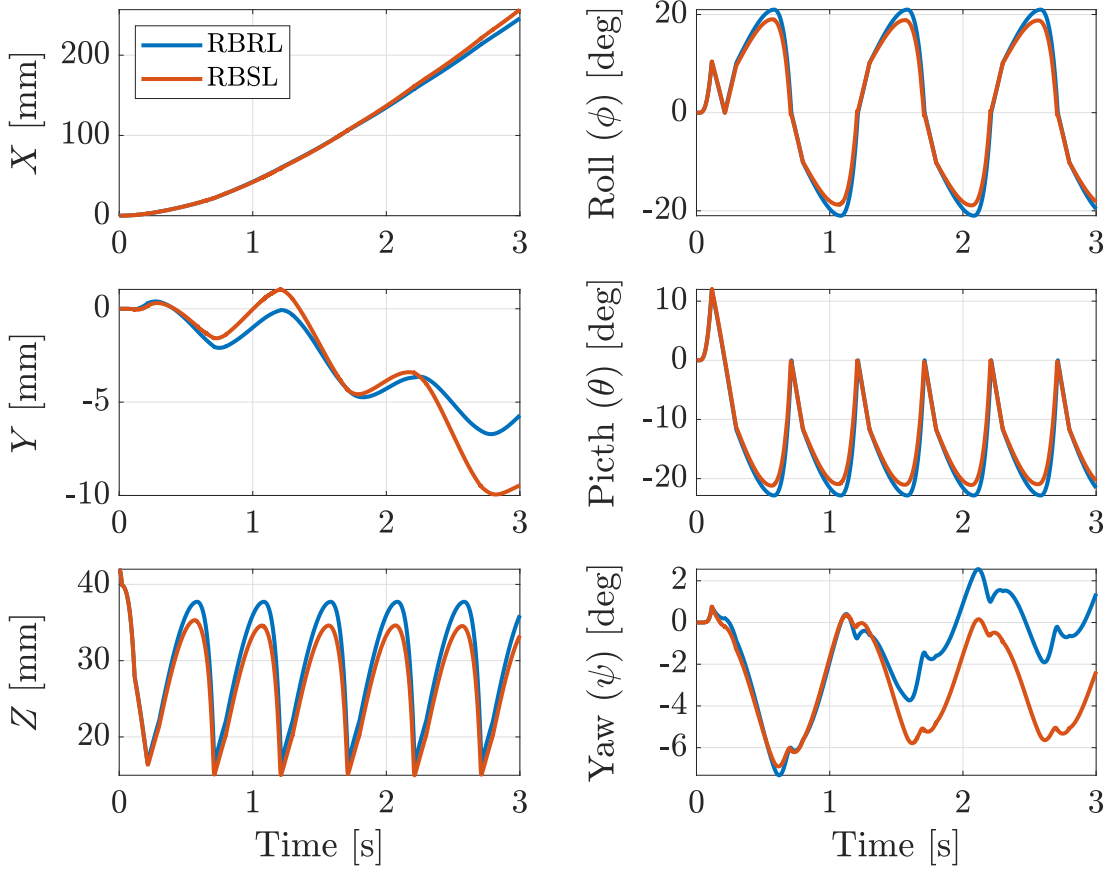


Figure 4.4: Simulation results of the RBSL robot

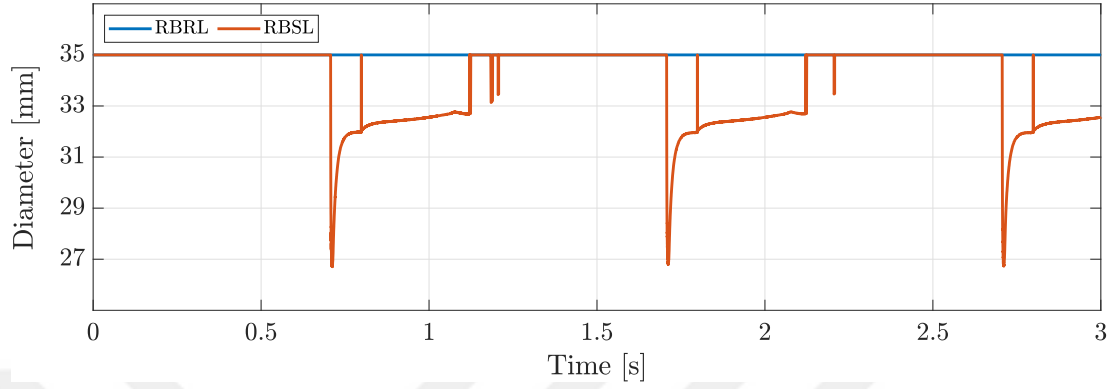


Figure 4.5: Diameter of the front right leg during 1 Hz trotting

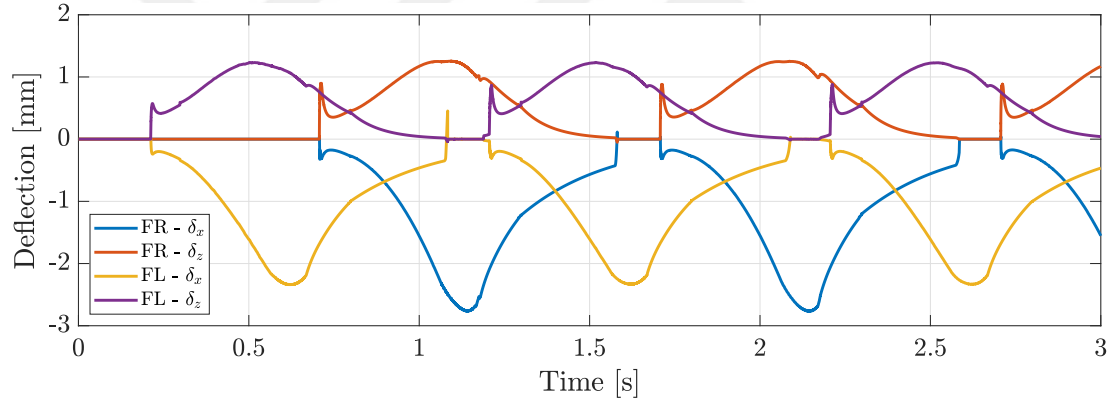


Figure 4.6: Diameter of the front right leg and the front left leg during 1 Hz trotting

Chapter 5

Modeling of the Soft Body

Unlike the soft leg model described in the previous, there is no method to model the soft body without the use of continuum mechanics and finite element methods. Therefore, the deflection of the soft body is not modeled during the simulation of the equations of motion of the robot, but by the subsequent use of the contact forces obtained from the results of this simulation.

5.1 Sinusoidal Deflection of the Soft Body

As the first step of the SBRL simulation, the deflections of the motor locations are represented with a sinusoidal function as follows.

$$\bar{\mathbf{r}}_{BH}^B = \mathbf{r}_{BH}^B + \begin{bmatrix} \pm 3 \cos(\pi t) & 0 & \mp 3 \cos(\pi t) \end{bmatrix}^T \quad (5.1)$$

where plus and minus signs changes according to the gait type. The results are as follows.

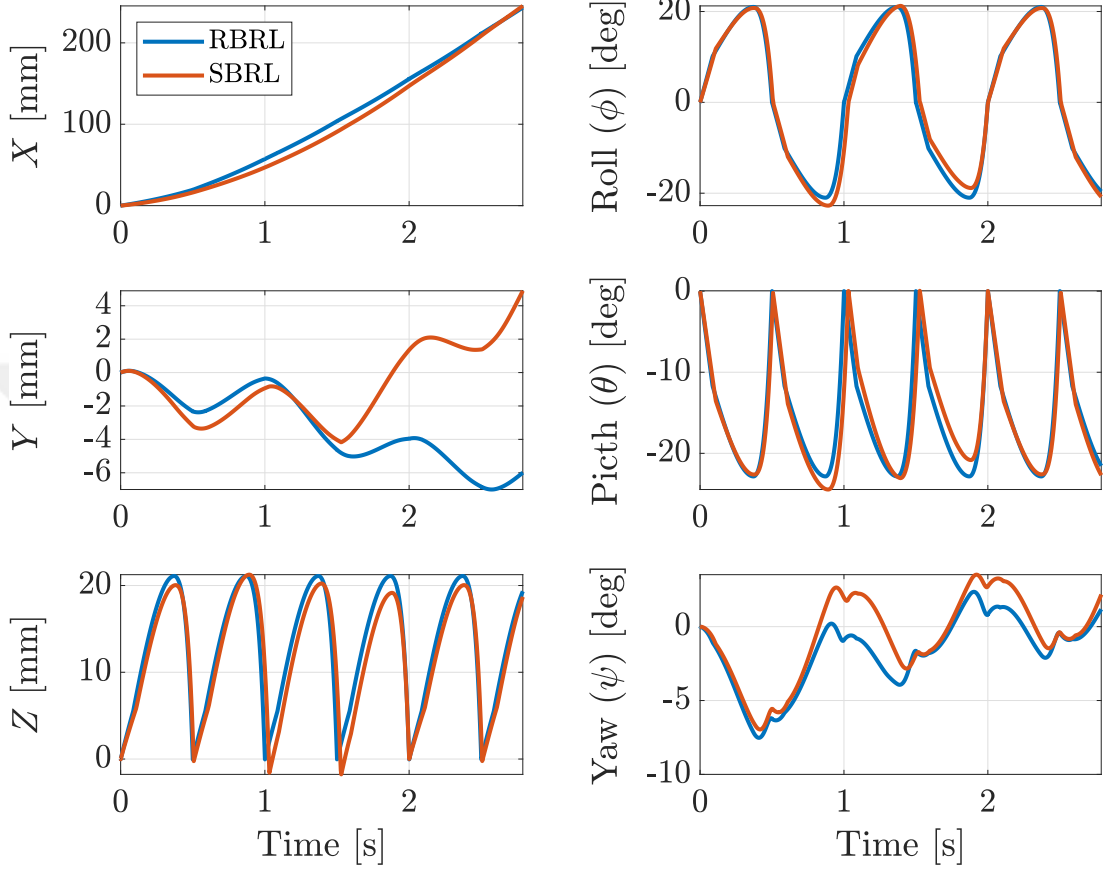


Figure 5.1: Simulation results of the SBRL robot

5.2 FEM Analysis of the Soft Body

Before starting the analysis, the geometry of the robot body is simplified for computational efficiency. Simplified 3D drawing of the robot body has been transferred to the COMSOL model via the COMSOL Multiphysics LiveLink mapping feature. Then, using the COMSOL material library, PDMS and 1050 solid Aluminum materials were defined regionally on 3D drawing. Material properties, differing with experimental data, were replaced by experimental data. The whole body is assumed to have Kelvin-Voigt flexible material properties. During the Structural Analysis, it is necessary to assume a surface as a fixed point as a reference point for the efficiency and proximity of the analysis. This is due to the fact

that the extensions that may occur on the body as a result of the applied forces use these fixed surfaces as a reference. To serve this purpose, a hole is drilled in the form of a cylinder at the very middle point of the robot body, the inner surface of this hole is considered to be a fixed surface throughout the analysis.

The time-dependent working part under the structural analysis module was selected on COMSOL to determine the sizes and areas of the extension (displacement) in the robot body for a period of 3 seconds. This is due to the fact that during the structural analysis, the strength directions and magnitude will change depending on the time. Time-dependent force data obtained as a result of dynamic analysis also contains position information. However, during the structural analysis, the delivery of forces from a fixed point for each leg connection was found appropriate in terms of efficiency and feasibility of the analysis. Therefore, force position data was not used throughout the analysis.

Forces acting on each leg connection are divided into separate coordinates of x , y , and z for a total of 12 different data sets. A different.xlsx file is created for the forces acting on each leg, the x , y , z coordinates are divided into different working tabs in the same .xlsx file. Then, these data were transferred as data to the model using 12 different intermediate value calculation functions on COMSOL. When defining boundary loads, these 12 intermediate value calculation functions are used, which are defined in the model. Time, in seconds, is defined as input to functions. Forces in mN are obtained as a function output. Thus, force data can be applied on the body by using the desired time step and time interval. The time interval is limited to 0-3 seconds because the data at hand contains 3-second force data, but the frequency of force applied to the body by changing the time step can be changed. While choosing a large time step may shorten the analysis time, it negatively affects the reliability of the data obtained. While choosing small time steps increases the analysis time, it makes the data obtained healthier and more reliable. For this reason, the time step in the analysis is determined as 1ms.

5.3 Effect of the Soft Body on the Locomotion

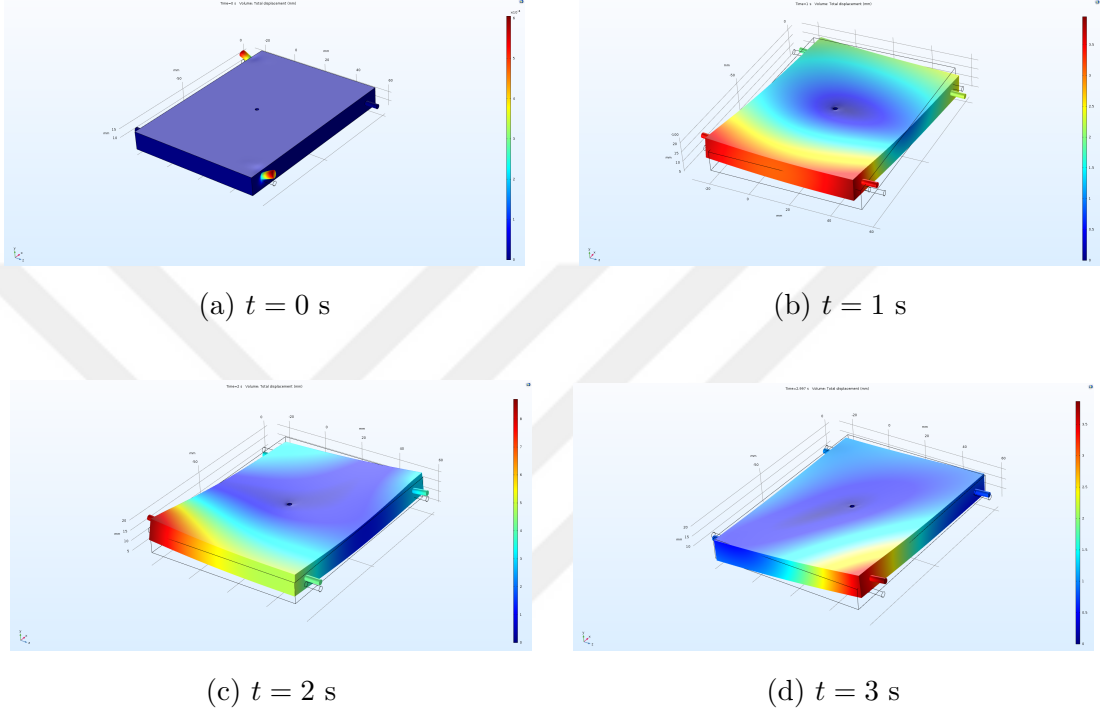


Figure 5.2: Screenshots from the FEA simulation of the soft body

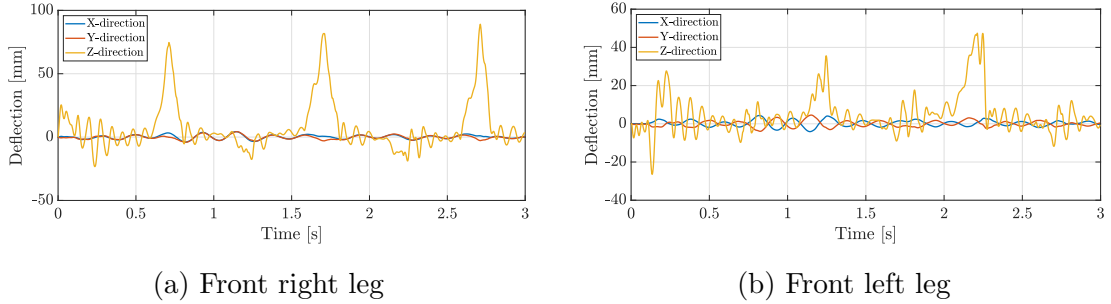


Figure 5.3: FEA estimation of the deflections of the hip joint locations

In the model where the body is represented as a solid body, the vector $\mathbf{r}_{BH}^B$ is considered constant. In order to solve the effect of body deflections on walking, the deflection results from the COMSOL simulation will change this vector depending on the time as follows.

$$\mathbf{r}_{BH}^B = \mathbf{r}_{BH}^B + \begin{bmatrix} \Delta x & \Delta y & \Delta z \end{bmatrix}^T \quad (5.2)$$

As a result of the experiments, it can be said that the movement of the soft-body robot is more complex and variable than the solid body robot. Therefore, although the finite elements described above are not expected with the use of the method, a high degree of accuracy of the simulation results according to the results of the experiment, The following results are important to demonstrate that the simulation may also include body deflections.

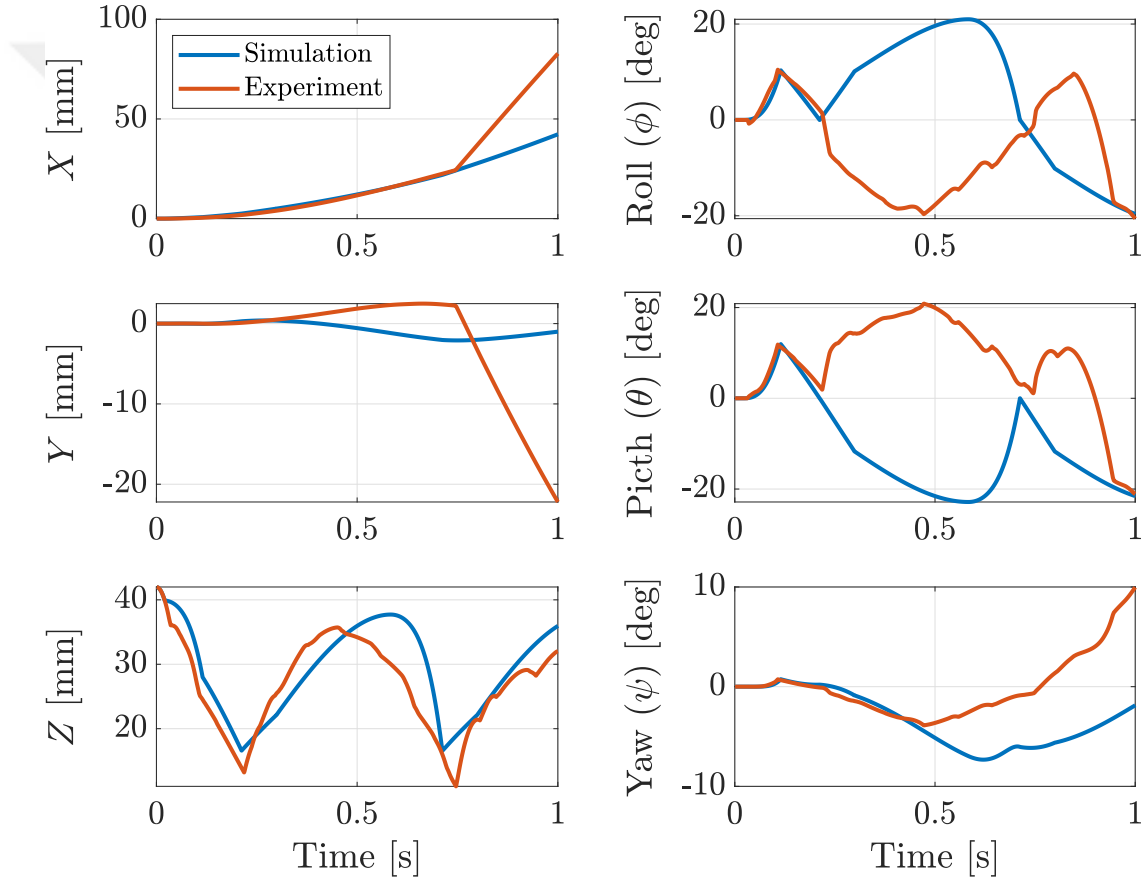


Figure 5.4: Simulation results of the SBRL robot

Chapter 6

Experimental Analysis

This chapter presents the experimental analysis of the walking mechanism of the different versions of S-Quad. Position and orientation variables of the robot are measured while it is walking with a motion capture system. The measurement system is described in Section 6.1 while the results are shown and discussed in Section 6.3.

6.1 Motion Capture System

The position and orientation measurements of the robot is conducted by using a motion capture system. This system consists of six OptiTrack Flex13 cameras with a hub between the cameras and a computer. They consist of an infrared light emitter and an infrared light sensor that has a data frequency of 120 FPS. The position of an infrared reflector is estimated through built-in image process and 3D reconstruction algorithms.

A set of three or more infrared reflector markers placed on a rigid body enables the computation of the position and the orientation of a rigid body. For the rigid body of the robot, they are randomly placed on a flat surface attached to the robot. This random placement enables the motion capture software to correctly

identify the orientation of the robot during the experiment. In addition to these random markers, one marker is placed at the geometric center to identify the origin of the reference frame.



Figure 6.1: Motion capture system

For the soft version of the robot body, the marker placement give rise to an issue. When the markers are separated, the relative positions will change during the experiment. This situation contradicts the rigidity assumption and complicates the definition of the orientation angles. In this study, the markers placed on top of the microcontroller board, which is certain to be rigid, and its orientation angles are accepted to be the robot's orientation. For the future experiments, they can be placed on the body as a mesh.

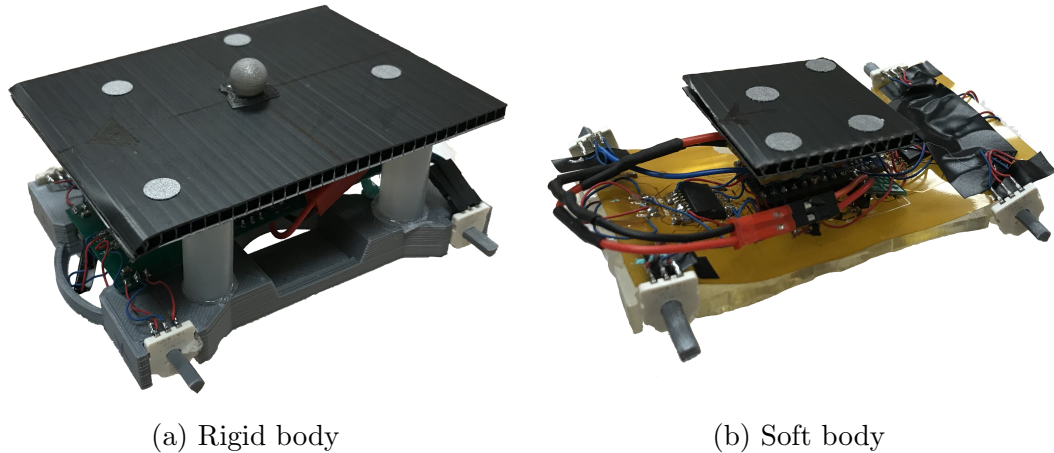


Figure 6.2: Placement of the infrared reflective markers on the robot body

6.2 Verification of the Dynamic Model

The simulation results of the RBRL robot displays a good fit with the experimental results. The small difference in the longitudinal position can be explained with the kinematic analysis of the leg while the the difference in the height can be explained with the unbalance in the experiment setup, and the difference in the pitch angle can be explained with the unbalance in the robot body.

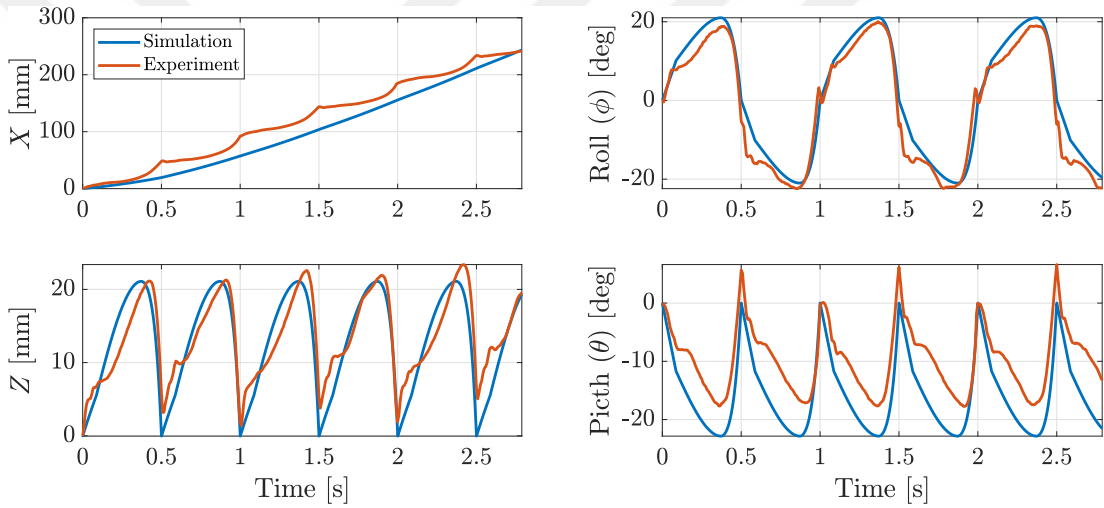


Figure 6.3: Verification of the RBRL robot simulation

Although the difference between the simulation and the experimental results of the RBSL robot is larger, it still displays the behavior shown with the comparison of the RBRL and the RBSL simulations (Figure 4.4): the RBSL robot walks lower with smaller rolling and pitching angles. Therefore, it can be claimed that the fit between the simulation and the experimental results of the RBSL robot can be increased with the increase in the leg deflections.

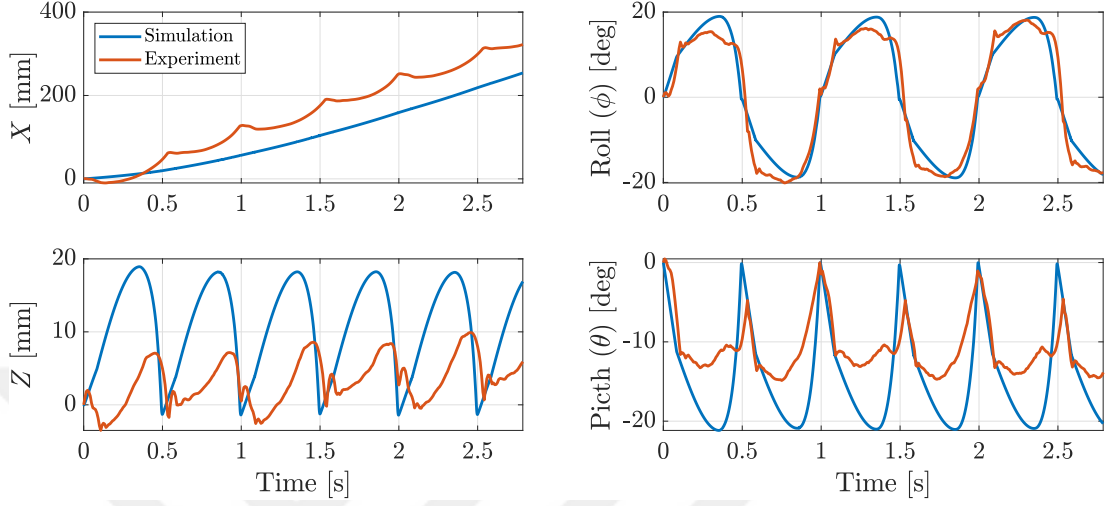
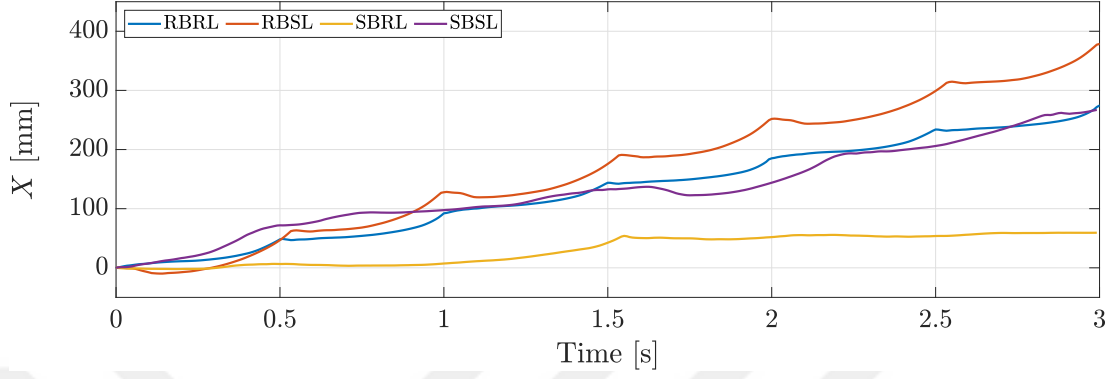


Figure 6.4: Verification of the RBSL robot simulation

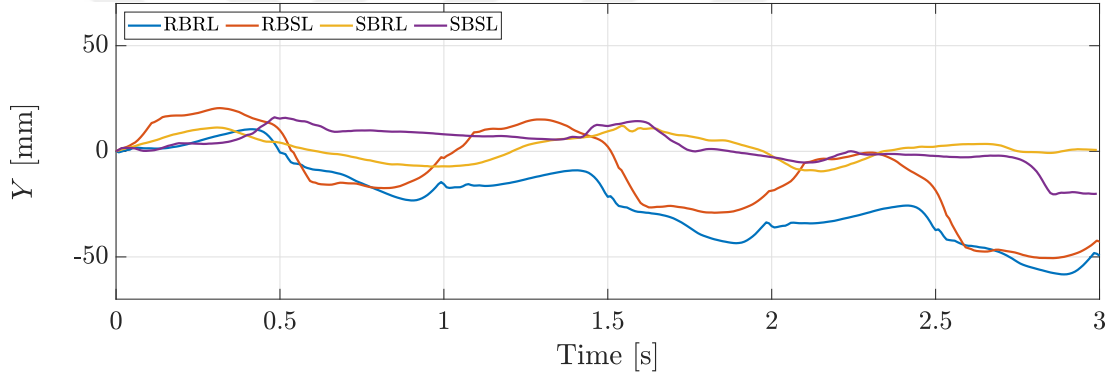
6.3 Comparison of the Robots

The first outcomes arrive from the leg deflection. In accordance with the experimental results (Figure 6.5), the RBSL robot walks faster. This can be explained with the higher friction of the soft material, and the increased contact time of the legs due to the deflection. Furthermore, the RBSL robot has a better balance in terms of pitch and roll angles. It can be argued that the reason is the higher friction in lateral direction.

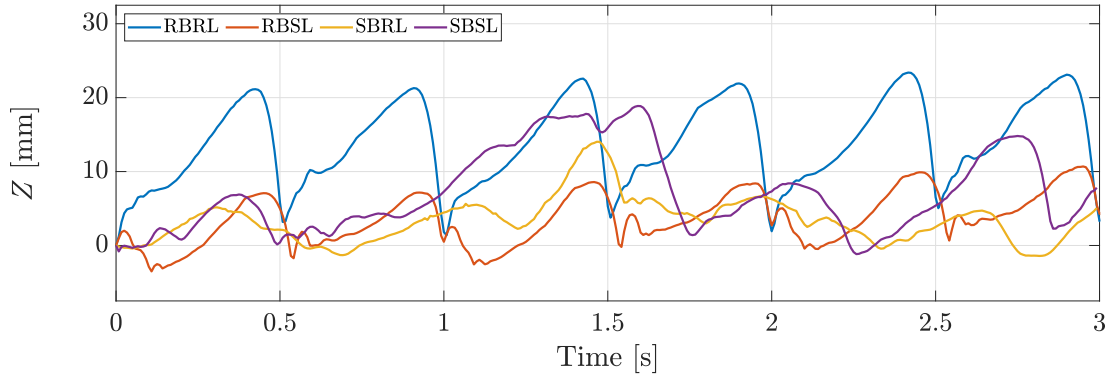
The body compliance is the next comparison aspect. According to the experimental results (Figure 6.5), the SBRL robot is the slower than the other versions by an order of magnitude. It can be argued that the friction at the legs cannot overcome the stiction at the body. In general, rigid bodied robots are faster and more balanced than the soft bodied robots. This is a trade-off between the rigid and the soft body. In return, the soft bodied robots has an advantage of better locomotion on rough terrains and obstacle climbing.



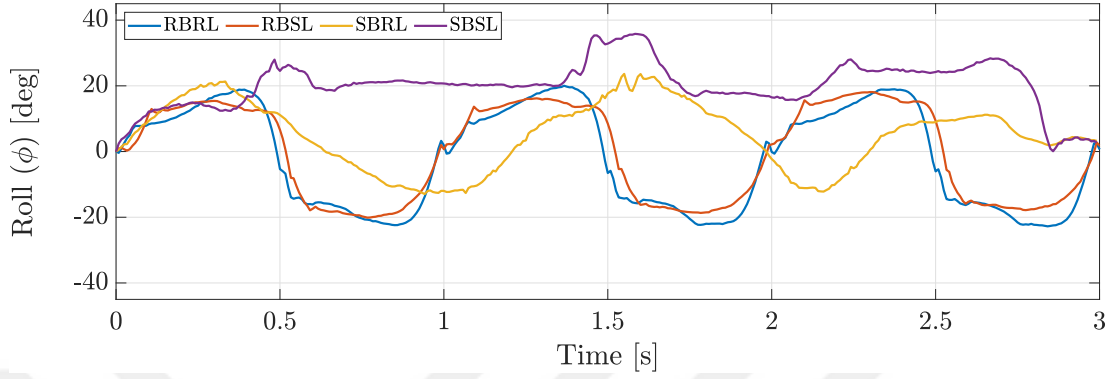
(a) Position in $\mathbf{X}^{\mathcal{I}}$ direction



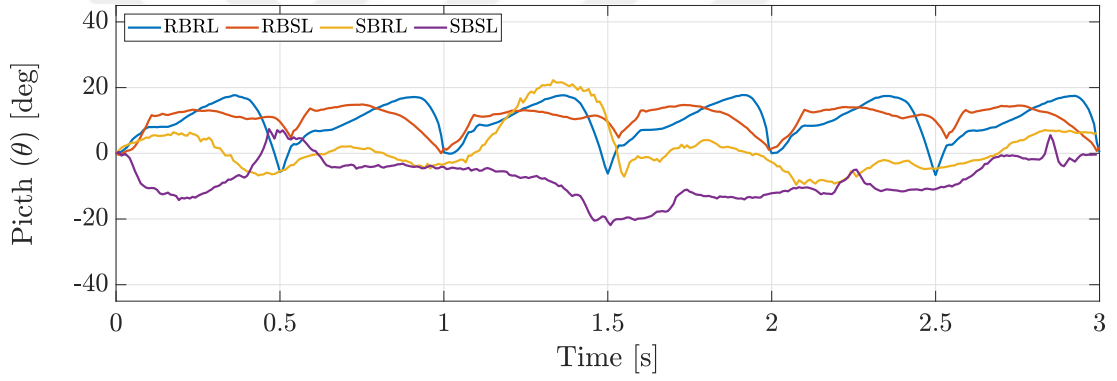
(b) Position in $\mathbf{Y}^{\mathcal{I}}$ direction



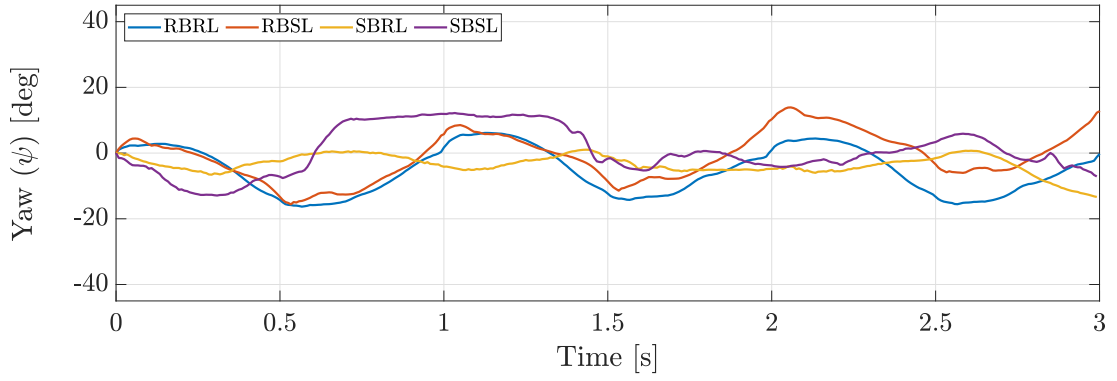
(c) Position in $\mathbf{Z}^{\mathcal{I}}$ direction



(d) Orientation as rolling angle



(e) Orientation as pitching angle



(f) Orientation as yawing angle

Figure 6.5: Experimental results for the four different versions of the robot

Chapter 7

Conclusion

To sum up, a dynamic model has been produced with the Newton-Euler method to investigate the relation of a miniature robot with its environment. The ground reaction forces are estimated with a non-linear viscoelastic model. Then, this model is extended such that it includes the compliance of the legs. The deflection of these is estimated with a viscoelastic circular beam model. Afterward, the compliance of the body is added to this dynamic model using an offline finite element analysis. Finally, the locomotion of all versions of the robot is measured with a motion capture system to verify this dynamic model. To the best of our knowledge, this thesis is one of the first studies that develop a full-body dynamic analysis of a walking robot that has compliant legs without the calculation of partial derivatives and shape integral (i.e., without complex numerical methods such as Finite Elements).

On the other hand, body compliance is included in the dynamic model using an offline FEA in contrast to leg compliance. The integration of body compliance with this dynamic model is open to development. Possible methods include the modeling of the body as a system of spring and dampers (as in the case of the leg model) or using a simplified version of the Kirchhoff-Love plate equation. Furthermore, the contraction mechanism of the S-Quad can also be similarly integrated into this dynamic model by including the forces exerted by the SMA actuators.

Additionally, the accuracy of the simulation can be increased (if needed) with the changes in the parameters described in Section 3.4. Those parameters can be estimated with experimental methods and system identification tools. A library of some primary environments can be built this way. Another way to increase the accuracy is related to the leg deflection modeling. It is observed that the deflected leg shape is actually elliptical, while it is assumed to be circular in this study. Such a modification in the kinematic analysis might give better results for the RBSL simulation, as discussed for Figure 6.4. Finally, a different numerical method (such as [105]) for the solution of the ODE system of this model can be considered.

The simulation tool developed under this study can be used in the iterative design process of the robot. The effect of the changes in its design parameters (such as dimensions, leg number, mass distribution, mechanical properties of materials, gait type, gait speed, ground inclination) can be analyzed before manufacturing. The different leg shapes can also be examined if the kinematics is known. Additionally, this simulation can also be used as a plant model in the development of an appropriate motion planning algorithm. With a small modification in the kinematic analysis (i.e., the inclusion of the motor torque & speed model), it can be utilized in the low-level controller design as well.

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Appendix A

Leg Radius after Deflection

Since the deflection of the leg changes its radius and center, the rolling angle of the leg is kept independent of its radius using the arc length of the contact point instead of it. Given the position of the contact point $\mathbf{r}_{H\bar{E}}^{\mathcal{L}}$ and the deflected radius $\bar{R}$, the arc length of the contact point can be calculated as follows.

$$\widehat{H\bar{E}} = \widehat{HE} = 2 \sin^{-1} \left(\frac{|H\bar{E}|}{2\bar{R}} \right) \quad (\text{A.1})$$

Then, the corresponding arc angle of the contact point in the original leg can be calculated as follows.

$$\theta_c = \frac{\widehat{HE}}{R} \quad (\text{A.2})$$

The location of the contact point in the original leg can be calculated as follows. Note that the center location is fixed and known.

$$\mathbf{r}_{HE}^{\mathcal{L}} = \mathbf{r}_{HC}^{\mathcal{L}} + \begin{bmatrix} -R \cos \theta_c \\ 0 \\ R \sin \theta_c \end{bmatrix} \quad (\text{A.3})$$

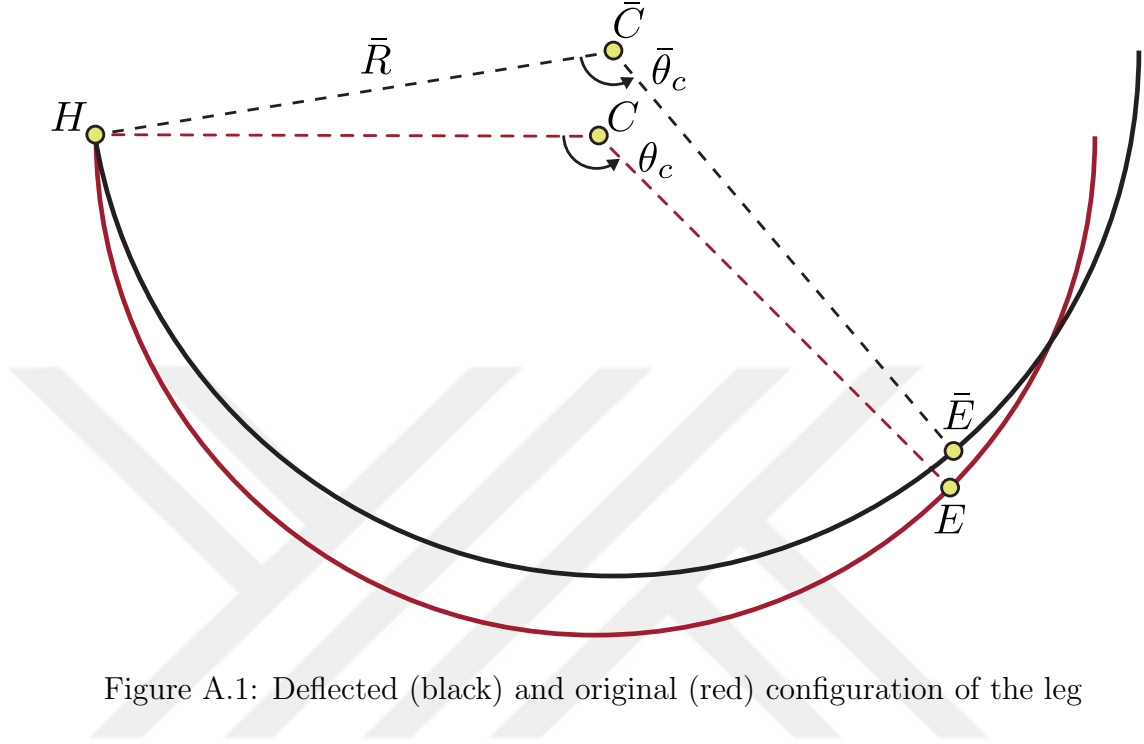


Figure A.1: Deflected (black) and original (red) configuration of the leg

The location of the contact point after the deflection can be calculated as follows.

$$\mathbf{r}_{HE'}^{\mathcal{L}} = \mathbf{r}_{HE}^{\mathcal{L}} + \begin{bmatrix} \delta_x & 0 & \delta_z \end{bmatrix}^T \quad (\text{A.4})$$

Arc length and the chord length of the point are easy to calculate, thus the new rolling angle can be calculated by numerically solving (with Newton-Raphson method) the following transcendental equation.

$$c = |HE'| \quad (\text{A.5a})$$

$$a = \widehat{HE'} \quad (\text{A.5b})$$

$$c\theta'_c = 2a \sin(\theta'_c/2) \quad (\text{A.5c})$$

The new radius of the leg can be calculated as follows.

$$R' = \frac{a}{\theta'_c} \quad (\text{A.6})$$

The center of the new leg can be calculated as a circle passing through two points ($\mathbf{r}_{HH}^{\mathcal{L}}$ and $\mathbf{r}_{HE'}^{\mathcal{L}}$).

$$mp = (\mathbf{r}_{HH}^{\mathcal{L}} + \mathbf{r}_{HE'}^{\mathcal{L}})/2 \quad (\text{A.7a})$$

$$dm = |\mathbf{r}_{HH}^{\mathcal{L}} - \mathbf{r}_{HE'}^{\mathcal{L}}| \quad (\text{A.7b})$$

$$dd = \sqrt{R'^2 - (dm/2)^2} \quad (\text{A.7c})$$

$$c_x = mp_x + \frac{dd}{dm} \left((\mathbf{r}_{HH}^{\mathcal{L}})_x \pm (\mathbf{r}_{HE'}^{\mathcal{L}})_x \right) \quad (\text{A.7d})$$

$$c_z = mp_z + \frac{dd}{dm} \left((\mathbf{r}_{HH}^{\mathcal{L}})_z \mp (\mathbf{r}_{HE'}^{\mathcal{L}})_z \right) \quad (\text{A.7e})$$

$$\mathbf{r}_{HC'}^{\mathcal{L}} = \begin{bmatrix} c_x & 0 & c_z \end{bmatrix}^T \quad (\text{A.7f})$$

Finally, the total arc length of the leg deflected leg can be found by the ratio of R'/R .

Appendix B

Code

The related codes can be provided upon request.