

İSTANBUL TEKNİK ÜNİVERSİTESİ * FEN BİLİMLERİ ENSTİTÜSÜ

YÜZEYDE ÇOĞALAN SİSTEMLERİN MODELLENMESİNDE KALICI ÜRÜN OLUŞUMUNUN ETKİSİ

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Ana Bilim Dalı : ÇEVRE MÜHENDİSLİĞİ
Programı : ÇEVRE MÜHENDİSLİĞİ

MART 1991

**THE EFFECT OF RESIDUAL MICROBIAL PRODUCT GENERATION
ON THE MODELLING OF FIXED - FILM SYSTEMS**

Ph. D. THESIS

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Date of Submission : 30 January 1991

Date of Approval : 19 March 1991

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MARCH 1991

F O R E W O R D

Many of the processes used in wastewater treatment have been introduced more than half a century ago, but the changing demands of today, requires more severe research and technology applicable to the Field of Environmental Engineering.

So far, design of fixed-film systems is based mainly on empirical design equations and not much is known about the removal mechanism of the organic substrate and generation of residual substrate and the behaviour of particulate components within the biofilm media. This study attempts to bring insight to better understand the fixed-film kinetics by the simulation model developed with the hope that it will contribute further researches to be conducted in this field.

I am deeply indebted to my thesis supervisor, Prof. Dr. Derin Orhon, who helped me throughout my study with advice, guidance and encouragement.

It is difficult to phrase an adequate expression for the assistance of Assoc.Prof.Dr. Hasan Ali San during the development of the model. It has been only through his fine cooperation and support that the model used in this thesis was developed.

I am also grateful for the generous help and advice given by Prof. Dr. I. Ethem Gönenc throughout my study.

Warm thanks are expressed to Prof.Dr. Olcay Tünay for his help and tolerance during the months I worked in the room next to his in the laboratory.

Finally, I would like to pay an affectionate tribute to my husband Mustafa Tanık and to my parents Nezahat and Orhan Baysal for the generous support and encouragement they gave throughout my study.

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NOTATION LIST

- A = Area of the biofilm surface, (m^2)
- b_H = Decay coefficient for heterotrophic biomass, (day^{-1})
- D = Effective diffusion coefficient, (m^2/day)
- f_e = Inert fraction of active biomass in death/regeneration approach
- f_i = Volume fraction of species i within the biofilm
- H_o = Hydraulic loading, ($m^3/m^2 \cdot day$)
- k_H = Maximum specific hydrolysis rate, (day^{-1})
- K_s = Half saturation coefficient for growth, (mg/lt)
- k_x = Half saturation coefficient for hydrolysis, (mg/lt)
- L_f = Thickness of the biofilm, (m)
- L_o = Organic loading, ($gr/m^2 \cdot day$)
- N_{SR} = Generation rate of soluble residual product defined in terms of flux, (gr/m^2)
- N_{SS} = Removal rate of soluble substrate defined in terms of flux ($gr/m^2 \cdot day$)
- $N_{x\pm}$ = Flux of particulate species i within biofilm, ($gr/m^2 \cdot day$)

- Q = Flow of wastewater, (m^3 / day)
- r_{ss} = Reaction rate expression of soluble substrate, ($gr/m^3 \cdot day$)
- r_{xi} = Rate expression of particulate species i within the biofilm, ($gr/m^2 \cdot day$)
- S_i = Soluble inert substrate concentration, (mg/lt)
- S_{min} = Soluble substrate concentration at the support media of the biofilm, (mg/lt)
- S_o = Influent substrate concentration, (mg/lt)
- S_R = Effluent inert soluble residual product, (mg/lt)
- S_R^* = Effluent inert soluble residual product at the bulk liquid surface, (mg/lt)
- S_S = Soluble substrate concentration, (mg/lt)
- S_S^* = Effluent soluble substrate concentration at the bulk liquid surface, (mg/lt)
- S_T = Total Effluent soluble substrate concentration, (mg/lt)
- V_B = Volume of the biofilm, (m^3)
- V_R = Volume of the reactor, (m^3)
- X_A = Active biomass concentration, (mg/lt)

X_E = Inert particulate organic matter concentration,
(mg/lt)

X_S = Particulate slowly biodegradable substrate
concentration, (mg/lt)

X_T = Sum of all particulate species concentration
within the biofilm, (mg/lt)

μ = Specific growth rate (day^{-1})

$\hat{\mu}$ = Maximum specific growth rate (day^{-1})

μ_{oi} = Observed specific growth rate (day^{-1})

Y = Yield coefficient

β = Hydrolysis associated product formation
coefficient

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S U M M A R Y

The simulation model developed in this thesis emphasizes on differentiation between readily biodegradable and slowly biodegradable substrate, incorporates the concepts of viability of active biomass and inert soluble residual product formation for attached growth systems in a similar manner as is formulated for the suspended growth systems.

The basic attempt of this study is to give more insight to the scientist, even to the designer involved in fixed-film systems and to increase the present state of knowledge on both the design and performance of such systems where till now, their design were mainly based on empirical relations and on fundamental mass transport and diffusion theory.

In Part I, the significance and the objectives of the work are summarized.

In Part II, theoretical approach to fixed-film kinetics is reviewed. An overview of various kinetic models that have gained importance from the scientific point of view as well as the significant loading criteria including flow rate, surface hydraulic loading, influent substrate concentration are mentioned in detail.

The theoretical background of the model is introduced in Part III enriched by the definition of both kinetic and stoichiometric parameters used in the two-staged modelling.

The model at two levels, (micro and macro) and the related steady-state equations together with the basic functions of the model are given in Part IV.

Part V, involves the evaluation of the model under various operating conditions. The effect of certain governing parameters on the model are analysed in Part VI, together with application of the model to anaerobic fixed-film systems.

Part VII includes the conclusions derived from this work along with recommendations.

YÜZEYDE ÇOĞALAN SİSTEMLERİN MODELLENMESİNDE KALICI ÜRÜN OLUŞUMUNUN ETKİSİ

Ö Z E T

Atıksu arıtımında uygulanan ilk yöntemlerden biri olan biyofilm sistemleri, aktif çamur sistemlerinin daha etkin gelişmesi ve geniş kullanım alanlarına sahip olması nedeni ile 1960 öncesinde popüleritesini kaybetmişti. Yıllarca, biyofilm tekniği damlatmalı filtrelerde uygulanmış, ancak, düşük akımlı evsel atıksuların ve/veya endüstriyel kaynaklı atıksuların ön arıtımında istenilen verim elde edilmiştir. 1960'lı yılların başında damlatmalı filtrelerde kullanılan dolgu malzemelerinin geliştirilmesine paralel olarak biyofilm sistemlerine olan ilgi yeniden canlanmaya başlamıştır. 1960'lı yılların sonunda ve 1970'li yılların başında döner disk sistemlerinin tasarım yöntemlerinin ve teknolojisinin gelişmesi ile birlikte, uluslararası ticari ilişkilerin hızlanması çeşitli biyofilm sistemlerinin dezavantajlarını azaltmış ve yeniden uygulanmaya başlamıştır. Özellikle, 1970'li yılların sonu ve 1980'li yılların başında akışkan yataklı biyolojik reaktörlerin laboratuvar ölçeğinden arazi ölçeğine uyarlanması, biyofilm sistemlerinin değişik reaktör tipleri ile geniş alanlarda uygulanmaya başlamasına yol açmıştır. Tasarım esaslarının sürekli gelişmesi ve biyofilm sistemlerinin aktif çamur sistemlerine oranla daha az enerji tüketmesi bu konuda çalışan uzman ve araştırmacıların ilgisini yeniden çekmiştir.

Gelişmelere paralel olarak, biyofilm sistemlerine artan ilgi, bu konudaki modelleme çalışmalarını da hızlandırmıştır. Model çalışmaları, tasarım mühendisliğinin ana yol göstericisidir. Modellemenin önderliğinde herhangi bir konunun teorisi daha iyi kavranabilmekte, hatta model bulguları deneysel yöntemlerle pekiştirildiğinde ortaya geçerliliğini yıllarca koruyabilecek çalışmalar konulmaktadır. Deneysel çalışmaların güç, hatta imkansız olduğu durumlarda ise model çalışmalarının geçerliliği çeşitli varsayımlara dayandırılarak mümkün kılınmaktadır..

Bu çalışmaların amacı, karmaşık ve kompleks bir yapıya sahip biyofilm sistemlerinde kalıcı ürün oluşum mekanizmasının modellenmesi ve tasarıma yönelik model bulgularının değerlendirilmesidir. Amaca iki aşamalı modelleme ile ulaşılabilmektedir. Birinci aşamada, spesi-

fik bir biyofilm tabakasında, biyofilmi oluşturan partiküler komponentlerin, (X_A , X_S , X_E), tabaka boyunca dağılımının, dolayısıyla giderme ve oluşma mekanizmalarının, ki, bu mekanizmalar akı cinsinden ifade edilmektedir, dağılımının modellendiği mikro model'dir. Bu aşamada, biyofilmi oluşturan komponentlerin tabaka boyunca toplam derişimleri değişmemekte, ancak her birinin, herhangi bir noktadaki hacim oranları değişmektedir. 1985'li yılların öncesinde, partiküler komponentlerin toplam konsantrasyonu Monod kinetiğinde kullanılarak sabit kalınlıkta ve sabit mikroorganizma yoğunluğunda, çözünmüş substratın giderme mekanizması, çeşitli analitik ve numerik yöntemlerle incelenmiştir. Biyofilm sistemlerinde giderme mekanizmasını kontrol eden ikinci dereceden doğrusal olmayan Monod kinetiği yaklaşımı çeşitli varsayınlarla lineerize edilmiş, 1. ve 0. derecelere indirgenerek basitleştirilmiştir. Bu çalışmaların tümünde ilgi sadece çözünmüş substrat'ın giderme mekanizmasına dayanmıştır. Ancak 1980'li yılların ortalarında, biyofilm tabakasını oluşturan mikroorganizmanın, yani, partiküler komponentlerin, tabaka boyunca davranışları dikkati çekmiş ve dolayısıyla çözünmüş kalıcı ürün oluşumunun bu partiküler komponentlerle olan ilgisi ortaya çıkmıştır. 1985 yılında bir grup bilim adamı ve araştırmacının aktif çamur sistemleri için geliştirdikleri "Task Group Model" çalışmasında sözü edilen partiküler komponentler sınıflandırılmış ve 'viabilite' kavramı ortaya çıkmıştır. Bu model çalışması, aynı mikroorganizma faaliyetlerinin ve aynı reaksiyonların gerçekleştiği biyofilm sistemlerine, biyofilm kinetiği kullanılarak uyarlanmıştır.

Çalışmanın ikinci aşaması ise "makro model" veya "reaktör modeli" diye adlandırılmıştır. Mikro model bulguları ile reaktör hidroliğinin birleştirildiği bu bölümde, atıksu karakterizasyonunun ve işletme koşullarının, örneğin atıksu debisinin, ve reaktör yüzey alanının değişimi ile modelin sonuçları, kalıcı ürün oluşumu ve toplam ürün oluşumu üzerine incelenmiş ve değerlendirilmiştir. Çözünmüş substratın sistemdeki kütle dengesinden yola çıkılarak reaktöre uygulanabilecek hidrolik yükleme hesaplanarak kalıcı ürün konsantrasyonu bulunmuştur.

Mikro model, beş temel bileşen ve üç süreçten oluşmaktadır. Model bileşenleri, kolay ayrışan substrat, S_S , çözünmüş inert ürün, S_E , aktif biyokitle, X_A , yavaş ayrışan substrat, X_S , ve inert partiküler organik madde, X_E 'dir. Biyofilm tabakasını oluşturan partiküler komponentler; X_A , X_S ve X_E 'dir. Model'de, sabit biyofilm

kalınlığı ve yoğunluğu varsayılmıştır. Dolayısıyla, $X_T = X_A + X_B + X_E$ sabittir ancak her bir bileşenin film tabakası boyunca hacimsel oranları değişmektedir. Sistemdeki üç süreç, aerobik heterotrofik çoğalma, heterotrofik bozunma ve hidroliz'dir.

Heterotrofik çoğalma sadece kolay ayrışan substratla gerçekleşebilmektedir. Çoğalma süreci Monod tipi bir reaksiyon ifadesiyle temsil edilmiştir. Bozunma aktif biyokitleye göre birinci dereceden bir ifadeyle tanımlanmış, hidroliz ise hem aktif biyokitleye, hem de yavaş ayrışan partiküler maddenin aktif biyokitleye olan oranına bağımlı olduğu varsayılmıştır.

Sistemde, biyokitle kolay ayrışabilir substratı kullanarak çoğalmaktadır. Bu süreç içinde oksijen kullanılmakta ve karbondioksit açığa çıkmaktadır. Modelde tekli substratla çalışıldığından oksijen kullanımı sisteme dahil edilmemiştir. Çoğalma ile oluşan aktif biyokitle bozunarak kısmen yavaş ayrışan partiküler substrata, kısmen de inert partiküler organik maddeye dönüşmektedir. Inert partikülür organik madde oluşumu daha başka bir reaksiyona girmemektedir. Yavaş ayrışan partiküler substrat ise hidroliz yoluyla bir kısmı kolay ayrışabilen substrata, diğer kısmı da çözünmüş inert ürüne dönüşmektedir. Çözünmüş inert ürün biyofilm kalınlığı boyunca difuzyonun hız kısıtlayıcı özelliğinden dolayı sıvı faza doğru azalarak ulaşır.

Yukarıda açıklanan çevrim proseslerinde oksijen kullanan tek süreç çoğalmadır ve çoğalma sadece kolay ayrışan substratla olmaktadır. Yavaş ayrışan substrat ise hidroliz yoluyla kolay ayrışır hale geçmesiyle dolaylı olarak oksijen kullanımına bağlıdır. Hidroliz yavaş bir süreç olduğundan oksijen kullanımı ve çoğalma, substrat giderimine nazaran gecikmektedir.

Önerilen modelde, substrat çözünmüş ve partiküler olarak ikiye ayrılmış, dolayısıyla giriş ve çıkış akımlarının etkileşimleri ortaya konulmuştur. Özetlenen reaksiyonlar uygun giriş ve çıkış özellikleri kullanılarak, çözünmüş sistem bileşenlerinin, S_B ve S_R 'in kütle denge denklemleri yazılmıştır. Partiküler madde bileşenlerinin spesifik çoğalmaları tabaka boyunca ifade edilerek, spesifik çoğalma hız sabitlerinin tüm partiküler bileşenler için sınır tabakasında (dayanma yüzeyinde) eşit ve minimum değerde olduğu varsayılmış ve biyofilm kalınlığı boyunca her partiküler komponentin spesifik

çoğalma ve azalma hızlarının dağılımından yola çıkarak tüm bileşenlerin çoğalma ve giderme eğrileri elde edilmiştir. Kütle denge denklemlerinde sistem bileşenlerinin ortalama değerleri kullanılmıştır. Elde edilen denklem takımları, üç bilinmeyenli, üç non-lineer denklem haline indirgenerek sonlu farklar ve Newton-Ralpson numerik integrasyon yöntemleri ile kararlı denge hali için çözülmüştür.

Her iki modelleme aşaması sonrasında, sistemi oluşturan belli başlı parametreler için duyarlılık analizleri yapılmış ve çeşitli çalışma koşullarında çözülmüş kalıcı ürün ve toplam çözünmüş madde konsantrasyonları saptanmıştır.

Bu çalışmalardan elde edilen sonuçlar, aşağıdaki şekilde özetlemek mümkündür.

- Biyofilm tabakası boyunca, özellikle aktif biyokitle, X_A , dağılımı substrat giderme mekanizması ile yakından ilgilidir ve giderme hızını kontrol etmektedir.

- Biyofilm tabakası boyunca, partiküler komponentlerin davranışı "aktif derinlik" veya "etkili derinlik" diye tanımlanan kalınlığa erişildikten sonra dayanma yüzeyine kadar sabitleşmektedir.

Dolayısıyla substrat giderme hızı'nda bu derinlikten itibaren dayanma yüzeyine kadar sabit olarak devam eder. Bu durum, kısmen sisteme giren oksijenin tükenmesinden ve kısmen de difuzyonun hız kısıtlayıcı etkisinden olmaktadır.

- Biyofilm tabakası boyunca, yavaş ayrışan substratın, X_B , diğer iki partiküler komponent'e oranla çok düşük konsantrasyonlara ulaştığı gözlenmektedir. Böylece, kalıcı ürün oluşum mekanizmasının yavaş hızla oluştuğu ortaya çıkmaktadır. Sistemde oluşan yavaş ayrışan substratın ortalama konsantrasyonunun düşük olmasından dolayı giderme hızına karşılık, yine akı cinsinden ifade edilen kalıcı ürün oluşum hızı çok düşüktür.

- Kalıcı ürün oluşum mekanizmasının kinetik modellenmesinde yavaş ayrışan substratın bozunması çok önemlidir. Ancak, model bulguları, bu mekanizmanın oluşan yavaş ayrışan substratın konsantrasyonunun düşük olması nedeni ile ihmal edilebilir düzeyde olduğunu ortaya çıkarmaktadır. Kalıcı ürün oluşum mekanizmasının modellenmesinden de görülebileceği gibi etkili parametreler b_H , β ve k_H 'dir.

- Çözünmüş kalıcı ürün oluşumuna etki eden en önemli dış etkenin, reaktörün toplam yüzey alanının atıksu debisine olan oranı olduğu makro model çalışmasında görülmüştür.

$$S_R^* = \frac{A}{Q} \cdot N_{SR}$$

kısacası, sıvı fazdaki kalıcı ürün konsantrasyonun sistemin hidrolik şartlarına bağımlı olduğu ortaya çıkmıştır. Akı değerinin düşük olması nedeni ile düşük hidrolik yüklemelerde bile kalıcı ürün konsantrasyonu yüksek çıkmamaktadır.

- Çeşitli kabul edilebilir hidrolik ve organik yüklemeler altında makro model bulgularının değerlendirilmesinden ortaya çıkan sonuçlara göre reaktör çıkışında düşük çözünmüş substrat ($S_E^* < 40$ mg/lt) konsantrasyonu gözleendiğinde, yine çıkıştaki kalıcı ürün konsantrasyonun, toplam çözünmüş substrata olan oranı, giriş çözünmüş substratın değişik konsantrasyonlarına bağlı kalarak 0.15 ila 0.65 arasında değişmekte olduğu saptanmıştır. Ancak, daha yüksek çıkış konsantrasyonlarında ($40 < S_E^* < 100$ mg/lt,) bu oran 0.15 ila 0.03 arasında değişken giriş substrat konsantrasyonuna bağlı kalarak değiştiği gözlenmiştir.

- Sistemin hidrolik yükünü etkileyen diğer faktörler ise; çözünmüş substrat cinsinden atıksu giriş ve çıkış konsantrasyonlarıdır. Dolayısıyla, kalıcı ürün konsantrasyonu giriş ve çıkış konsantrasyonlarının da dolaylı fonksiyonudur. Sabit çıkış konsantrasyonu düşünüldüğünde, kalıcı ürün konsantrasyonu giriş konsantrasyonunun değişimi ile doğru orantılıdır.

- Biyofilm sistemleri ile aktif çamur sistemleri, kalıcı ürün oluşum mekanizması açısından, aynı giriş ve çıkış şartlarında karşılaştırıldığında, biyofilm sistemlerinde oluşan ve sıvı faza geçen kalıcı ürün konsantrasyonu'nun aktif çamur sistemlerine oranla daha fazla aktif biyokitleyi bünyelerinde bulundurdıkları halde daha düşük olduğu ortaya çıkmıştır. Bu durumun, biyofilm kalınlığı boyunca difüzyonun hız kısıtlayıcı etkisinden olduğu gözlenmiştir. Dayanma yüzeyindeki kalıcı ürün konsantrasyonu, aktif çamur sisteminde açığa çıkan kalıcı ürün konsantrasyonundan daha yüksektir. Ancak difüzyonun hız kısıtlayıcı etkisinden kalıcı ürün konsantrasyonu, film tabakası boyunca azalarak sıvı faza geçmektedir.

- Geliştirilen modelin birkaç örnek çalışma ile anaerobik şartlara uyarlanması yapılmış, sonuçta hidroliz prosesinin ihmal edilebilir düzeyde olduğu gözlenmiştir. Dolayısıyla, sıvı fazda kalıcı ürün konsantrasyonu aerobik biyofilm sistemlerine oranla çok daha düşük olduğu ortaya çıkmıştır. Biyofilm kalınlığı boyunca, yavaş ayrılan substrat konsantrasyonunun çok düşük olması, sistem gereği etkili parametrelerin düşük olması, hidroliz prosesinin çok yavaş hızda gerçekleştiğini göstermektedir. Sistemde, hidrolik yükün çok düşük alınması bile, sıvı fazda kalıcı ürün konsantrasyonunu arttırmamaktadır.

Biyofilm konusunda yürütülen tüm çalışma ve araştırmaların ana hedefi, bilgi boşluklarını doldurarak, bilgi birikimini arttırmak ve teorisini daha açıklığa kavuşturmadır. Karmaşık yapısından dolayı kompleks halde ifade edilebilen biyofilm kinetiği ve model çalışmaları doğrultusunda önerilebilecek çalışmalar ise şöyle sıralandırılabilir;

- Benzer modelleme çalışmasına çiftli, hatta çoklu substrat kullanılarak devam edilmesi,

- Geliştirilmiş olan iki aşamalı modelin nitrifikasyon ve denitrifikasyon proseslerinin de uygulandığı durumlara uyarlanması,

- Geliştirilmiş olan iki aşamalı modelin, bir kısmı anaerobik, diğer bir kısmı aerobik çalışan bir biyofilm tabakası gözönüne alınarak uygulanabilirliğini saptamak şeklinde düşünülmektedir.

PART I

INTRODUCTION

Mankind realized the necessity of wastewater treatment after damaging the environment due to disposal of wastes generated by human activities. Today, a wide variety of treatment processes are available for use in order to prevent further deterioration of the environment. Fixed-film systems have been proved to play a significant role among alternative treatment systems in the sense that these systems have the ability to produce high-quality effluent, withstand load variations and require low power consumption to produce a given effluent quality, compared to especially activated sludge systems.

The present state of knowledge on both the design and performance of such systems is not able to predict clearly the removal mechanism based on biofilm kinetics and the corresponding kinetic expressions. So far, the design of biofilm systems mainly depend upon empirical relations and on fundamental mass transport and diffusion theory.

Although useful in some cases, the empirical relations are limited by a narrow range of reactions and operating conditions. This fact must give rise to develop models allowing design to be based on more fundamental and unified criteria.

1.1. SIGNIFICANCE OF THE WORK

Fixed-film biological processes found early application to wastewater treatment, but their use declined with the development and wide-scale application of the activated sludge systems. Among many reasons for this, the dominant result is that for many years, fixed-film processes (notably trickling filter in the early years) were relegated primarily to the treatment of low flow domestic wastewater. During the early 1960's however, with the advent of plastic media for trickling filters, there started a resurgence of interest in fixed-film systems. This interest was accelerated in the late 60's and early 70's by further development of technology and commercialization of rotating disc filters providing many of the advantages of trickling filters without inheriting some of the disadvantages. Finally, in the late 70's and early 80's fluidized bed biological reactors moved from lab-scale to field-scale thereby opening up a wholly new area for application of biological fixed-films. Because of these advances in process development and as the fixed-film reactors are generally less energy intensive than activated sludge reactors, engineers are now employing fixed-film systems with a great deal of success.

With these new developments in fixed-film reactors there comes through a renewed interest in their modelling. There are at least two reasons for this. One is that, models are the basic tools of engineering which facilitate the design process. The other is that models aid to achieve better understanding of theoretical basis of the concept mostly if it is experimentally verified. Modelling and experimentation are two interrelated phases of a proper research where one provides input, the other provides the output.

Within the past fifteen years, many models have been developed and put forth by scientists on fixed-film kinetics (Atkinson and Daoud, 1968; Harremoes 1978; Jennings et.al., 1976; Williamson and Mc Carty, 1976a; Rittmann and Mc Carty 1978, 1979a, 1980a). Most of the improvement is due to idealizing substrate utilization processes, molecular diffusion and mass transport in the form of simultaneous differential equations of variable order, such as half order, first order and even more complicated Monod Kinetics for a homogeneous layer of microorganisms. Despite of their validity, these models are seldom applied for design, due to the fact that general analytical solutions of these simultaneous differential equations are unavailable. Another important limitation of these models is that relative activity of heterotrophic microorganisms is not considered to vary significantly from the bulk interface towards the support media of the biofilm. In short, it can be concluded that many "rule of thumb" concepts have been applied to attached growth (fixed-film) systems without a good theoretical basis, which is mainly due to the complexity of the processes within and at the bulk liquid surface of the biofilm.

This is in contrast to the mathematical modelling of suspended growth systems (activated sludge systems) where the predictive models for design are now available (IAWPRC, 1987).

The present work was undertaken in an attempt to develop a similar kinetic model given in detail in IAWPRC Task Group Study on Mathematical Modelling For Design and Operation of Biological Wastewater Treatment-Scientific and Technical Report No:1 (IAWPRC, 1987). The

intention of this study is to explain the distribution of active and inert microorganisms together with the substrate utilization and residual product generation mechanisms for the attached growth systems under the light of the previous studies.

1.2. OBJECTIVES AND SCOPE

The simulation model developed in this study offers a differentiation between readily degradable and slowly degradable substrate, incorporates the concepts of viability of active biomass and inert residual product formation for attached growth systems in a similar manner as is formulated for the suspended growth systems.

The modelling of attached growth systems relate to the distribution of particulate components within the film media in connection with biofilm surface reactions.

Two levels of modelling is necessary in such systems:

- a) Microscopic level; where transport and reaction rate processes based on Fick's law of diffusion and on certain models describing the behaviour of biomass within the biofilm (advection processes) are to be considered.
- b) Macroscopic Level; where the information obtained at the microscopic level is used to describe the system as a whole taking into consideration the influence of external (environmental) conditions.

The requirements of both levels are to be fulfilled in a kinetic model describing both the internal reactions and external conditions. The model includes kinetic expressions as well stoichiometric parameters.

Five components and three processes are defined to describe the biofilm model as in the case of suspended growth system. The components are readily degradable substrate (S_s), slowly degradable substrate (X_s), inert particulate organic matter (X_E), viable biomass (X_A), soluble residual product (S_R). The three processes considered are growth, decay (loss of viability) and hydrolysis of adsorped organics.

The main objective of this study is to describe the distribution of particulate components, namely X_s , X_A and X_E (advection process) over the depth of the biofilm media with the aid of transport equations and correspondingly the degradation of soluble substrate S_s , and the formation of residual substrate product, S_R , (transport of dissolved components) taking into consideration the Fick's law of diffusion.

The model is developed for only steady-state conditions as there is a unique solution of the biofilm problem for a given single substrate at steady-state. Time, as a variable parameter in biofilms is quite important, but it is not possible to formulate a relation between external operating conditions and related biofilm performance any more if it is included in the model.

The study is carried out within the following scope to meet the general objectives of the research project.

1. Distribution of particulate components X_A , X_S and X_E within the biofilm media,
2. Distribution of concentration profiles of S_S and S_R due to removal and generation rates and to biofilm thickness,
3. Effect of particulate components, especially of active biomass, on the kinetic expressions of the system,
4. Transport of dissolved components considering the effect of mass transport (diffusion) as a rate limiting parameter,
5. Behaviour of the system under variable hydraulic conditions,
6. Generation of inert soluble residual product under variable hydraulic conditions,
7. Sensitivity of the parameters on soluble residual product in the model under aerobic conditions, and
8. Behaviour of the system under anaerobic conditions.

The study was undertaken with an equally important task to give more insight to the designer for better reactor design by fulfilling the above mentioned objectives.

PART II

LITERATURE REVIEW

Fixed-film systems, like other biological processes depend on the behaviour of the microorganisms for the removal of soluble substrates. It is possible to describe this removal phenomena by biological growth and transport mechanism. The approach considering removal in terms of biological growth is used in suspended growth systems by application of the continuous culture theory of microorganisms, and it is accepted that fixed film systems can also be described by the same theory. Two facts are important in deriving a mathematical model for a fixed-film system (Kornegay and Andrews, 1968).

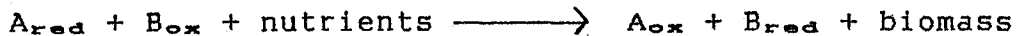
1. Organisms and substrate do not occupy a common volume as is the case for suspended growth systems. Thus, the relation between particulate and dissolved species must be established on mass basis.
2. The whole mass of attached microorganisms is not equally active in the removal of soluble substrate.

The removal mechanism is described as a process of molecular diffusion with simultaneous biochemical reactions. The difficulties of the problem is due to the complexity of the hydrodynamics and the kinetics; furthermore, the density/concentrations of microorganisms in the system is not stable; they continuously vary within a given biofilm media, in contrast to suspended growth systems.

2.1. THEORETICAL APPROACH TO BIOFILM KINETICS

All the biological processes occurring in biofilms can be simply accepted as a redox process, while one of the substrates is being oxidized, the other is reduced by utilizing certain nutrients and biomass is generated as an end product.

The general expression of the process is as follows;



The processes taking place within biofilms are being affected by a number of phenomena illustrated in Figure 2-1. It has been stated by Pasveer (1954) and

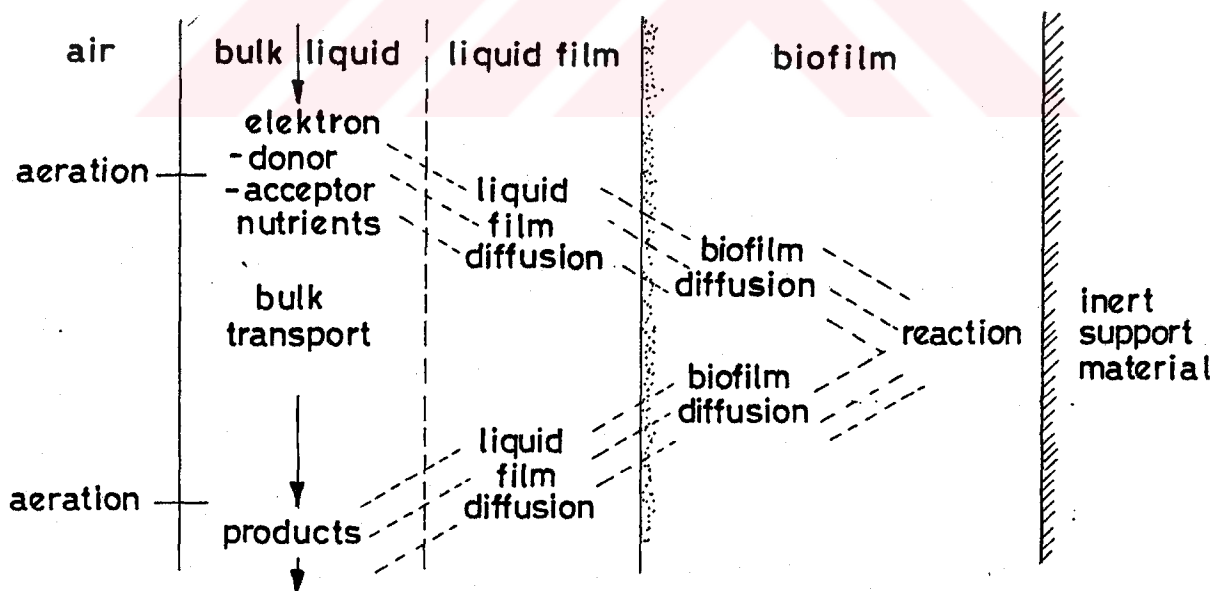


Figure 2-1. Schematic Presentation of the Potentially Rate Limiting Phenomena involved in Biofilm Reactions

Wuhrmann (1963) as referenced by Harremoes (1978) that these factors limit the rate of processes to a great extent. Similar consideration has been made by La Motta (1974), Williamson and Mc Carty (1976), Friedman et. al. (1979), Banerji (1980). In order to analyse the factors effecting the rates of processes taking place in biofilms, it is convenient to go over the processes step by step as follows;

1. Diffusion of substrates (BOD_5 , NH_3 , NO_2 , NO_3) from the bulk water to the liquid film,
2. Diffusion of substrate to the biofilm media,
3. Substrate utilization within the biofilm,
4. Outdiffusion of reaction products from the biofilm towards the bulk liquid (Gönenç, 1982).

2.1.1. The Processes Occurring in Biofilms

Any of the above mentioned transport or reaction processes may control the overall rate of the system. There may be cases where diffusion of the electron donor or the electron acceptor to and through the biofilm media can also govern the overall removal process.

The overall process, depend on second and third steps occurring simultaneously, resulting phenomenon takes place in series together with the first step under steady-state conditions, (Atkinson et. al., 1967.; La Motta, 1976a; Harremoes 1976, 1977, 1978, 1982, Williamson and Mc Carty 1976; Rittmann and Mc Carty 1980b, 1980c). La Motta (1976b) stated that external

diffusional resistances can be eliminated by proper determination of the range of fluid velocities in which the kinetic reactions occur and by finding a correlation between the transfer coefficient and fluid velocity under turbulent conditions. The rate of transport of the soluble substrate from the bulk water to the liquid film (1st step) can be represented by the following equation;

$$N = K_L (S_b - S_s^*) \quad (2.1)$$

where,

N is the flux of mass transfer per unit area (gr/m².day)

K_L is the mass transfer coefficient (m/day)

S_b is the concentration of soluble substrate for bulk liquid. (mg/lt)

S_s^* is the concentration of soluble substrate at the biofilm surface (mg/lt)

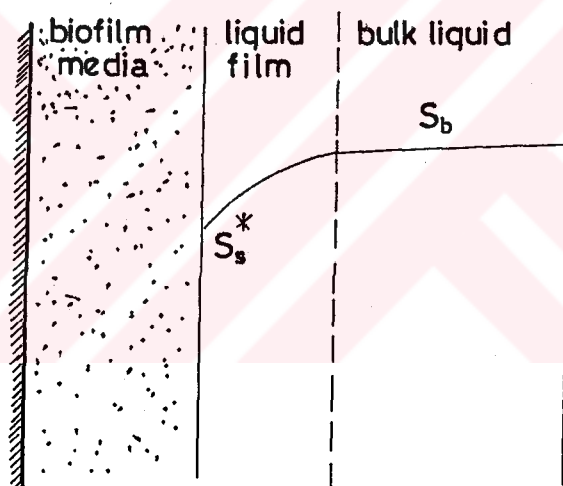
Two conditions can occur due to variations of mass transfer coefficient (Kamenetskii, 1969). If the mass transfer coefficient is high, the microorganisms will receive maximum available amount of substrate, thus overall reaction rate will depend on the kinetics of the internal-diffusion process. This situation approaching suspended growth condition is termed as "reaction controlled regime". The opposite limiting condition where low mass transfer coefficient is used, leads to a negligible concentration of substrate at the film surface; $S_s^* \approx 0$. Under this condition, the overall

process rate is determined by the rate of diffusion, being first-order,

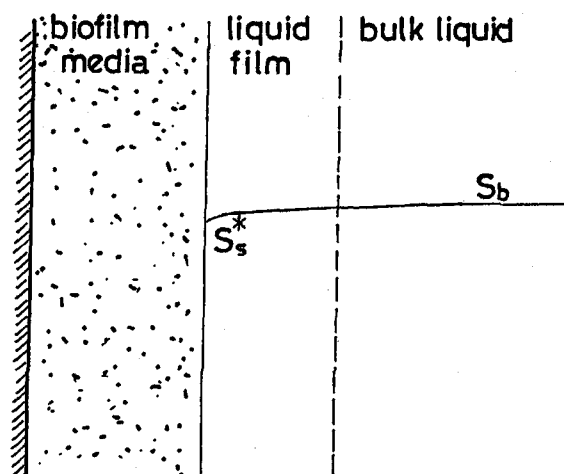
$$N = K_L \cdot S_b \quad (2.2)$$

This limit is termed as "diffusion-controlled regime" (Kamenetskii, 1969).

The transition from the "diffusion-controlled regime" to "reaction-controlled regime" can be achieved by increasing the flow velocity of the liquid. Thus, the reaction given in step (1) can be neglected. Harremoes (1981) indicated that the overall reaction rate did not



- a) Rate is controlled by external diffusion where K_L is low DIFFUSION REGIME;



b) Rate is controlled by substrate uptake rate where K_L is high KINETIC REGIME; $S_s^* \approx S_b$

Figure 2-2. External Mass Transfer in Biofilms

change by involving liquid film diffusion. Williamson and Mc Carty (1976) considered the liquid film theoretically and indicated that whether such a layer exist in all biofilms or not was currently unknown. In this study, the liquid film layer is neglected as well.

On the other hand, it is assumed that the diffusion of the reaction products from the biofilm media to the surface of the biofilm (4th step) has no effect on the overall reaction rates (2nd and 3rd steps) (La Motta, 1976). This assumption is supported by the fact that since the outdiffusion process is irreversible, its rate can not effect the overall substrate utilization rate. Harremoes (1978) indicated that production of CO_2 , acidity and alkalinity may cause

considerable variations in pH values and thus may indirectly effect the rate of reaction. However, Harremoes (1981) eliminated the outdiffusion of intermediate products. In this study, the outdiffusion process of soluble substrate as a residual product is considered in detail in the modelling stage and its variation with respect to external environmental conditions is being analysed.

2.1.2. The Reaction Surface

There are two possibilities existing to define the geometry of the reaction surface; that is to say the interface between the biofilm and the bulk liquid phase. One, referred to as the pseudo-homogenous model by Atkinson and Swilley (1963) who supposed the reaction surface consisting of a matrix of microorganisms through which the liquid phase passes as shown in Figure 2.3.a. The other, the heterogeneous model, consists of a surface of organisms over which the liquid phase flows as shown in Figure 2.3b. Both models may be understood better if they are studied as a continuous flow problem (Atkinson et.al., 1967). It is interesting to note that most equations present in the literature are based on assumptions of the pseudo-homogenous model, and Atkinson et. al., (1967), Harremoes (1976) used this model who named it pore diffusion. Atkinson et. al., (1967) stated that the effect on the fraction of the substrate removal as a result of using variable flow rate may be a useful tool to determine the applicability of both models. They concluded that mathematical models derived on a heterogeneous basis were more applicable to practice.

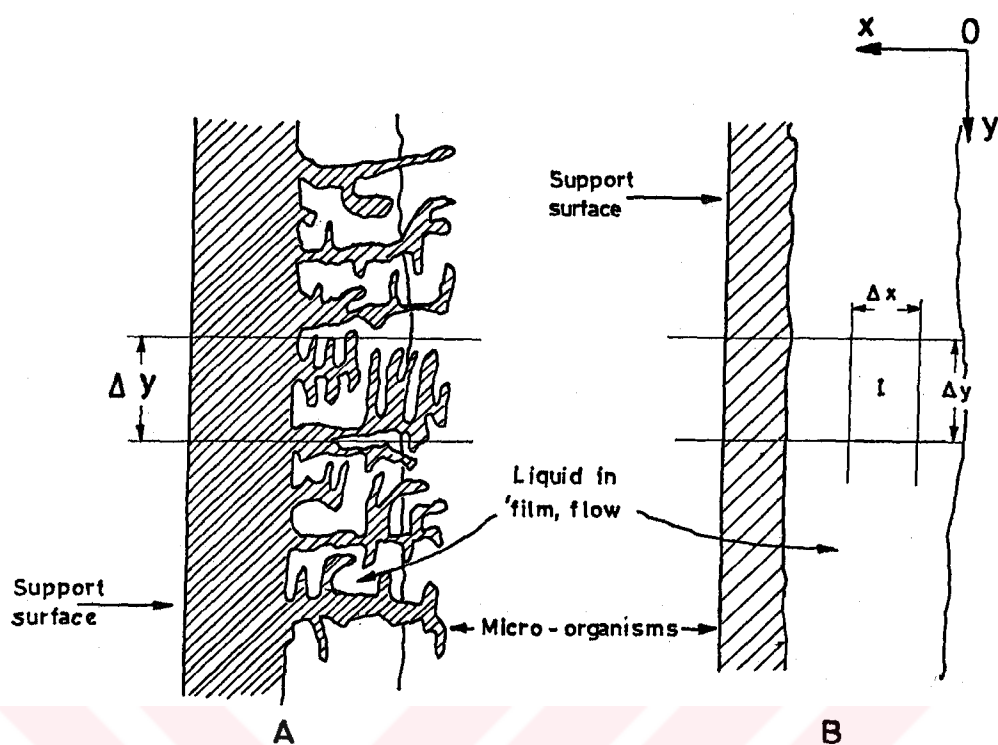


Figure. 2-3. Possible Geometries of the Reaction Surface

Harremoes (1976) idealized the pores to cylinders and assumed that the reaction took place on the walls of the cylinder. In the later report, Harremoes (1977), Kornegay and Andrews (1968), La Motta (1976) and Williamson and Mc Carty (1976) used the homogeneous model as well as the others, Rittmann and Mc Carty (1980 a, 1980 b), Watanabe et. al. (1980), Harremoes (1981), Watanabe et. al. (1984), Namkung and Rittmann (1987), Saez and Rittmann (1988), Gujer and Boller (1990); Harremoes (1990).

2.1.3. Characteristics of Biofilms

Substrate removal in a heterogenous environment is due to interaction between the rates of transport and reaction. Rates of biological reactions are generally expressed on the basis of a unit of biomass. For example, the rate of substrate removal is expressed as the mass of substrate removed per unit time per unit biomass and the rate of biomass growth is expressed as the amount of biomass produced per unit time per unit of biomass present. The available surface area for biofilm growth / development is a physical characteristic of the type of reactor being modelled. Therefore, the mass of biofilm in a reactor is a function of the thickness and density of the biofilm.

2.1.3.1 Biofilm Thickness

When considering biofilm thickness, it is important that a distinction be made between the total film thickness and the active film thickness. Atkinson and Fowler (1974) found the total film thickness to vary between 0.07 and 4.00 mm. They divided the biofilms into two groups to better reflect the growth conditions.

Group 1- When the films were subjected to mechanical or hydrodynamic control, the thickness was generally less than 0.2 mm.

Group 2- When the films were uncontrolled, they were as thick as 4.00 mm, though it has been asserted that in turbulent flow systems, thickness seldom exceeds 1 mm; (Characklis, 1981).

Thicker films are usually uncontrolled and they do not likely to have greater removal rates due to diffusional resistances. So far, observations indicated that the rate of substrate removal increased as the biofilm thickness reached up to a limiting depth of 70-100 μm ; after that the removal rate was independent of thickness, (Kornegay and Andrews, 1968; Tomlinson and Snaddon, 1966). The depth at which substrate removal reached a maximum value was defined as the active thickness (Grady, 1983). This statement is in full correlation with the interpretation that only the organisms in the active layer are contributing to substrate removal. Similar observations have been made by Hoehn and Ray (1973) and by Chen and Bungay (1981).

It is now generally accepted that the active thickness is a result of transport limitations due to diffusional resistances within the biofilm. When the film is very thin or when the rates of transport are high in relation to the reaction rates, the active film thickness will approach the total film thickness.

Prediction of the biofilm thickness within a fixed-film reactor is the least developed of all the techniques needed for adequate modelling due to the fact that little fundamental surveys have been conducted to surmise the factors governing film development.

The majority of the biofilm reactors used in wastewater treatment are of a configuration which prevents positive control of biofilm thickness. This means that the film will either reach a steady-state in which growth is compensated by decay or it will increase till sloughing occurs.

The most complete study is that reported by Trulear and Characklis (1980) who grew fixed-films in an annular reactor consisting of two concentric cylinders, one stationary and the other rotating. The rotational speed determined the shear stress developing at the biofilm: liquid interface and the biofilm detachment rate increased as the rotational speed increased (Figure 2.4). This suggests that the rate of biofilm detachment increases as the shear stress at the interface increases. Furthermore as shown in Figure 2.5. the detachment rate also increases as the biofilm mass increases. This suggests a mechanism where films of different thickness can be attained with a fixed shear stress as substrate is applied at various rates. (Figure 2-6.)

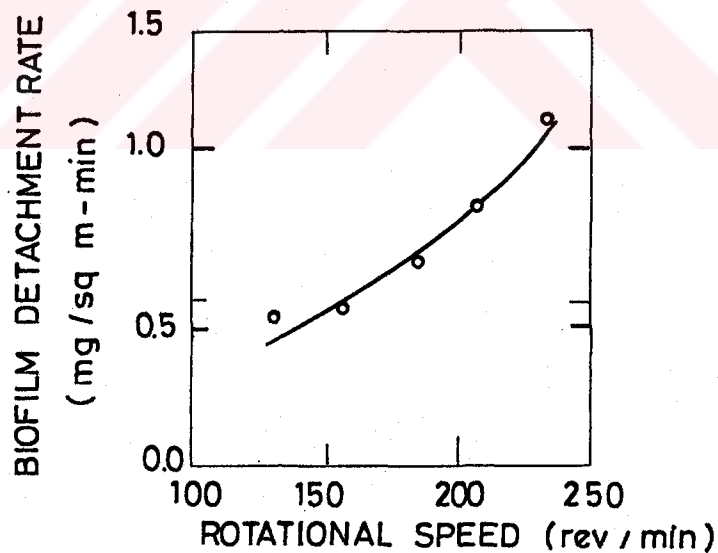


Figure 2-4. Effect of Rotational Speed in an Annular Reactor on the Detachment Rate of a Biofilm with a Mass of 150-160 mg. (Trulear and Characklis, 1980).

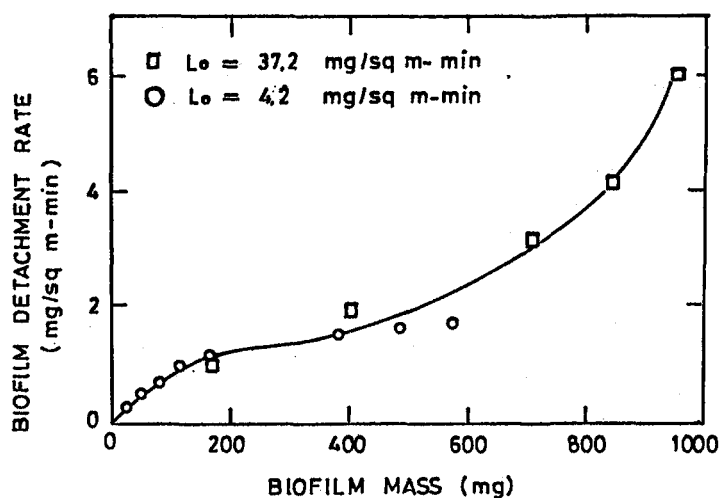


Figure 2-5. Effect of Biofilm Mass (proportional to thickness) in an Annular Reactor Rotating at Constant Speed on the Detachment Rate of Biofilm. L_o refers to the organic loading applied (Trulear and Characklis, 1980);

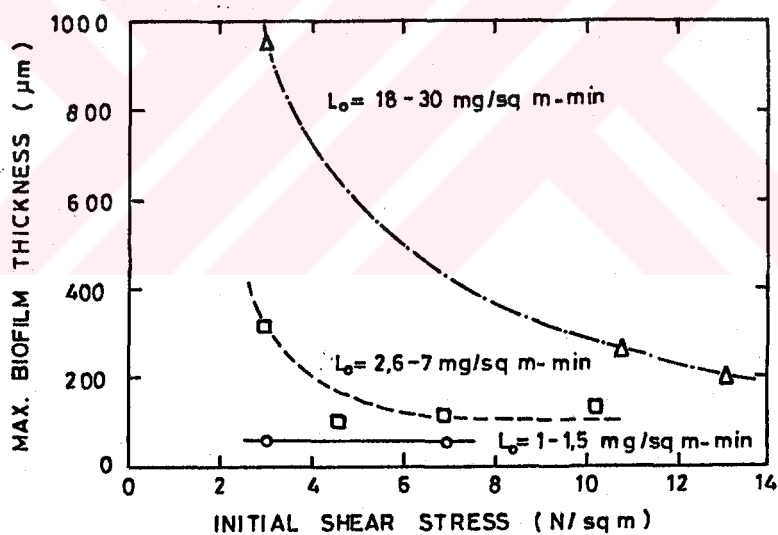


Figure 2-6. Effect of Fluid Shear Stress and Organic Loading L_o on the Maximum Biofilm Thickness attained in an Annular Reactor (Characklis, 1981).

Since the biofilm mass is a function of thickness, these results suggest the detachment rate associated with shear stress to increase as the thickness increases.

On the other hand, Trulear and Characklis (1980) have also shown the biofilm density to be a function of the applied substrate loading rate. This means that the thickness associated with a given mass is greater at a lower loading, thereby suggesting that the detachment rate cannot be expressed as only a simple function of thickness but must be expressed in terms of both thickness and density. This complexity puts forward many unanswerable questions such as the ones listed by Characklis (1980):

"What functional forms should such models take?
Why does this relationship exist?
Were the results influenced by the type of reactor employed?"

Until now, relatively, little is known about biofilm detachment, thus most of the models developed for biofilms have assumed a constant film thickness. The major attempt to model biofilm thickness is that of Rittmann and Mc Carty (1980a, 1980b) who have done so by assuming a steady-state biofilm in which growth would be balanced by decay. Harremoes in all of his studies assumed homogeneous biofilm with constant thickness and density (Harremoes and Arvin, 1989). Gujer and Wanner (1985) also depicted the biofilm thickness as constant due to an equilibrium of biomass production and shear loss at the surface.

2.1.3.2 Biofilm Density

All biofilms are experimentally analysed the according to following description by Williamson and Mc Carty (1976b);

"All biofilms were made by filtering dispersed bacteria from the culturing columns onto the support filters; as a result, the biofilm area, A, was controlled by the area of glass filter funnel used which equaled to 10.3 cm². The filtering process took approximately 30 seconds and produced biofilms with smooth even surfaces.

The biofilm thickness was determined by side examining a side view of the biofilm and its support filter with microscope having a magnification of 100 x and a calibrated ocular micrometer (Figure 2-7.). A slice approximately 4 mm. wide, was cut from the biofilm filter and mounted on its edge between two glass slides. The mean biofilm filter depth was taken as being average of about 10 measurements accross the filter. The drying and the subsequent shrinking of the biofilm depth were insignificant during measurement.

The total suspended solids (TSS) mass within the biofilm was determined at the end of each experiment as the residue remaining on the membrane support filter after drying at 103°C for at least 12 hours. The biofilm concentration, X_T, was assumed to be equal to

$$\text{TSS} / L_x \cdot A = \frac{\text{TSS}}{V_B} \quad \text{where}$$

A = area of the biofilm surface

L_x = thickness of the biofilm

V_B = volume of the biofilm

The biofilm depths had a maximum range of approximately $\pm 30 \mu\text{m}$ over the entire surface and were typically under $\pm 16 \mu\text{m}$ for biofilms less than $400 \mu\text{m}$ deep."

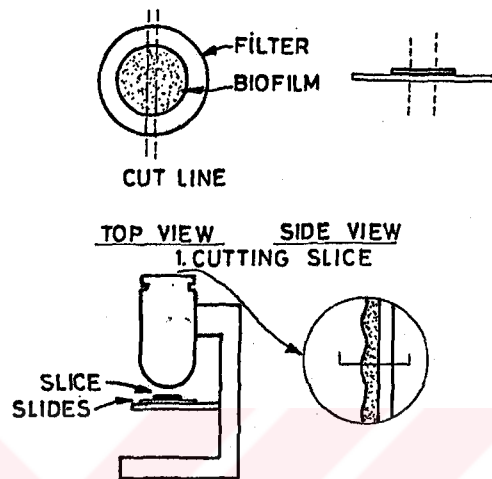


Figure 2-7. Method for Measuring Biofilm Thickness (Williamson and Mc Carty, 1976 b).

It has generally been assumed that the biofilm density is constant and independent of biofilm thickness, but there is evidence that is not the case. Many experimental studies done so far, (Tomlinson and Snaddon 1966; Hoehn and Ray, 1973) indicate that there is a dependence of dry density on the film thickness in a similar manner as mentioned above on description of how the density is being analysed. Shied et. al., (1981) have observed reductions in density with increasing thickness, although they observed no region of increasing density. They developed empirical equations the fluidized bed reactor. Trulear and Characklis (1980) have also observed changes in biofilm density, but their observations did not indicate whether thickness is the fact that, the density in their films approached a maximum value as the substrate loading rate

was increased at a constant shear stress. It was previously that the film thickness associated with a given shear stress was also a function of loading criteria that influences density.

As seen from the literature review, there are many factors responsible for changes in density, thus still more fundamental research will be required to delineate this mechanism and its importance. So far, most of the models developed in biofilm kinetics refer to constant biofilm density.

On the other hand, although biofilm density is assumed to be constant, the distribution of microbial species within the biofilm is observed to change with thickness. Mueller et. al. (1978) and Gujer and Wanner (1985) illustrated the principal aspects of species competition in biofilms.

Biofilm density determined experimentally by researchers relative to thickness are tabulated in Table 2.1.

As seen from Table 2.1, the biofilm density is high compared to suspended growth systems. High value of biofilm density is one of the advantages of the processes. In a short hydraulic detention time, even for low substrate loading, a high removal rate is achieved. The other main difference of attached growth and suspended growth systems is that; although microorganism concentration is constant in both systems; the active portion of organisms vary with thickness in the attached growth systems, whereas in suspended growth systems, it is constant throughout the reactor.

Table 2-1. Results of Various Experiments Conducted To Determine Biofilm Density

Biofilm Density	Biofilm Thickness	References
90.000 mg/lt TSS	$L_x \leq 300 \mu\text{m}$	Kornegay and Andrews (1968)
50.000 mg/lt TSS	$100 < L_x < 1100 \mu\text{m}$	Tomlinson and Snaddon (1966)
110.000 mg/lt TSS 20.000 mg/lt TSS	$L_x = 200 \mu\text{m}$ $200 \mu\text{m} < L_x < 200 \mu\text{m}$	Hoehn and Ray (1973)
40.000 mg/lt VSS		Rittmann and Mc Carty (1980 c)
42.000 - 89.000 mg/lt TSS	$L_x = 240 - 500 \mu\text{m}$	Williamson and Mc Carty (1976 a)
30.000 mg/lt VSS	$L_x = 200 \mu\text{m}$	Andrew et. al. (1980)
25.000 - 45.000 mg/lt TSS	$L_x = 100 - 1350 \mu\text{m}$	Huang et. al. (1985)
66.900 mg/lt VSS	$L_x = 92 \mu\text{m}$	Namkung et. al. (1983)
20.000 - 60.000 mg/lt TSS	$L_x = 300 - 500 \mu\text{m}$	Owenand Williamson (1976)
45.000 - 70.000 mg/lt VSS		Harremoes and Arvin (1989)
80.000 mg/lt TSS	$L_x = 500 \mu\text{m}$	Gujer and Boller (1989)

2.1.4. Transport and Reaction Within Biofilm

As discussed in Section 2.1.1., the processes in taking place in biofilms occur simultaneously (transport of the electron donor and the acceptor together with reaction processes, constituting concentration profiles in the reflecting their relative rates. The reactive capability of the film, in other words, the overall reaction rate depends on the nature of the concentration profiles. The most important fact that must be considered in biofilm modelling is the conceptualization and mathematical expression of these simultaneously occurring transport and reaction processes. The movement of reactive constituents within the film is accepted to reach the organisms by diffusion, which is characterized by Fick's law:

$$N_{ss} = D \frac{dS_s}{dL} \quad (2.3)$$

where,

N_{ss} = the flux, or the mass of substrate removed per unit area per unit time (gr/m².day)

D = the effective diffusion coefficient (m²/day)

S_s = the concentration of the soluble substrate (mg/lt)

L = the unit thickness of the biofilm (m)

If a mass balance on soluble substrate is performed around a differential element of a steady-state film of constant microbial composition, the result is an equation which is almost universally used to model reactions within biofilms:

$$V \frac{\partial S_s}{\partial t} = N \cdot A - \left[N + \frac{\partial N}{\partial L} dL \right] A - r_{ss} V \quad (2.4)$$

where A = area of the biofilm surface (m^2)

r_{ss} = the reaction rate expression ($gr/m^3 \cdot day$)

V = volume element of the biofilm; $V=A \cdot dL$, (m^3)

t = time (day)

At steady-state conditions; $V \frac{\partial S_s}{\partial t} = 0$, equation 2.4 is simplified to;

$$\frac{dN_{ss}}{dL} = - r_{ss} \quad (2.5)$$

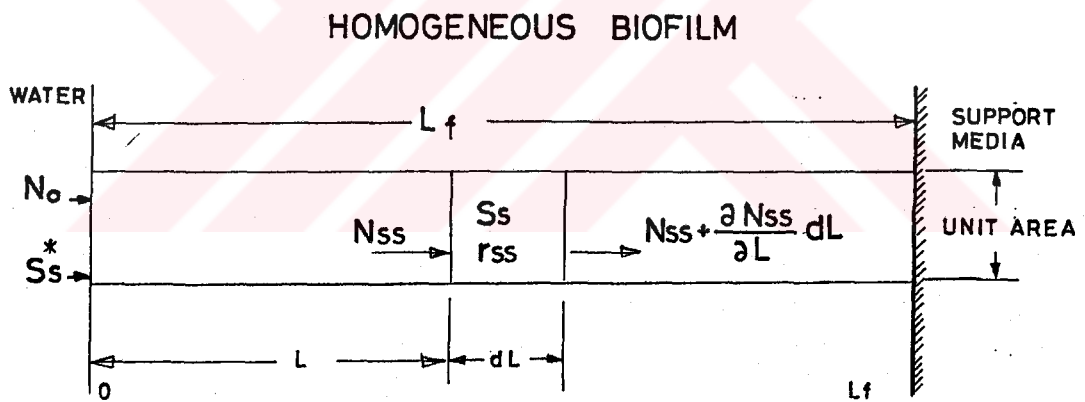


Figure 2-8. The Geometry of an Idealized Biofilm

The key elements that must be inserted into equations (2.3) and (2.5) are the effective diffusivity and the reaction rate expression r_{ss} as they determine the nature of the resulting concentration gradient and overall reaction rate.

2.1.4.1 The Concept of Diffusion

Although considerable effort has been expended on evaluations of diffusion coefficient within biofilms, there is little agreement between scientists about how the presence of the biofilm influences diffusivity. This is mostly due to the variation of the biofilm character with respect to organisms growing in it and also due to the presence of different experimental techniques applied to estimate the coefficient.

The most direct method of estimating effective diffusion coefficient is to measure a concentration gradient through a biofilm and to couple it with the flux of material into the film to allow direct computation of D , (Bungay and Harold, 1971; Chen and Bungay, 1981). Bungay and his associates have done the experiments for oxygen diffusion. The coefficient for laboratory-grown films was approximately 80 % of the value in water whereas the coefficient for field-grown films was about 35 %.

Another technique is to place a film in a special chamber which allows a component to diffuse through it, allowing measurement of the flux. Then, from knowledge of the film thickness the diffusivity may be estimated. Williamson and Mc Carty (1976 b), Matson and

Characklis (1976) followed up this technique and their values were all in excess of 80 % of the values in water.

A third technique that has been employed for estimating D is to grow biofilms in a fixed-film reactor, measure its performance under a variety of conditions and then evaluate D by curve fitting under the light of the experimental data. Mulcahy et. al. (1980) Mulcahy et. al. (1981) and Janson and Kristensen (1980) used this technique and found that the D varied from 30 to 100 % of the value in water for films grown in a rotating annular reactor depending upon the film thickness. Furthermore, even though the variation in D with thickness is consistent with the variations in biofilm density, the experiments conducted so far, illustrate the dangers in computing coefficients from assumed models.

Because of the difficulties associated with direct measurement of diffusivity, most modelling attempts have used assumed values. Table 2.2. illustrates certain D values of different compounds that are estimated experimentally.

2.1.4.2 Reaction Rate Expressions

Fixed-film processes are generally used for one of these purposes:

- i) Removal of soluble organic matter,
- ii) Nitrification,
- iii) Denitrification.

In some cases, more than one of these may be accomplished within a single reactor, but in all cases two soluble, transporting components are necessary for the reactions to occur. These are an electron donor (Organic matter) and an electron acceptor (Oxygen).

Table 2-2. Effective Diffusion Coefficients of Various Components

D (effective diffusivity coefficient)	Compound	Reference
$5-6 \times 10^{-10} \text{ m}^2/\text{sec}$	Mineralization reaction	Owen et. al. (1976) Harremoes (1981)
$1.7 \times 10^{-4} \text{ m}^2/\text{day}$ $1.03 \times 10^{-4} \text{ m}^2/\text{day}$ $1.98 \times 10^{-4} \text{ m}^2/\text{day}$ $0.6 \times 10^{-4} \text{ m}^2/\text{day}$	methanol (denitrification) acetate (COD) propionate (COD) aerobic organic oxidation (COD)	Harremoes (1982)
$0.048 \times 10^{-5} \text{ cm}^2/\text{sec}$ $0.06 - 0.6 \times 10^{-5} \text{ cm}^2/\text{sec}$	glucose glucose	Matson and Characklis (1976)
$106 \times 10^{-6} \text{ m}^2/\text{day}$ $31 \times 10^{-6} \text{ m}^2/\text{day}$	oxygen glucose	Gujer (1990)
$8.8 \times 10^{-10} \text{ m}^2/\text{sec}$ $4.3 \times 10^{-10} \text{ m}^2/\text{sec}$ $2.6 \times 10^{-10} \text{ m}^2/\text{sec}$ $2.3 \times 10^{-10} \text{ m}^2/\text{sec}$ $1 \times 10^{-10} \text{ m}^2/\text{sec}$	Oxygen (methanol) Oxygen(acetic acid) Methanol Acetic acid glucose	Rittmann and Mc Carty (1980 a, 1980 b)
$1.81 \times 10^{-2} \text{ m}^2/\text{day}$	Acetate	Namkung et. al. (1983)
$9.8 \times 10^{-5} \text{ m}^2/\text{day}$ $1.86 \times 10^{-4} \text{ m}^2/\text{day}$	Benzoate Oxygen	Kissel et. al. (1984)

It is now widely recognized that in the most general case the reactions within a biofilm may be controlled by the concentrations of both the electron donor and the electron acceptor. If the concentration of one is much higher than the other, then only one constituent controls the reactions. This situation has been assumed to exist by most modellers and thus most of the models have been developed in terms of only one constituent. In this study, the soluble substrate will be the only constituent to control the reactions. The other constituent, oxygen, is not included in the model. Nitrification and denitrification processes are also excluded.

Bacterial growth and substrate removal are generally considered to be coupled reactions, i.e., substrate removal occurs because of bacterial growth. The proportionality constant is the growth yield, Y . Furthermore, the rate of bacterial growth is proportional to the biomass concentration within the film X_T :

$$r_{XT} = \frac{dX}{dt} = \mu X_T \quad (2.6)$$

where X = concentration of bacteria (particulate matter) at time t , (mg/l)

μ = Specific growth rate, (day^{-1}).

X_T = biomass concentration within the film, (mg/l)

Likewise with the substrate removal rate:

$$-r_{ss} = q \cdot X_T \quad (2.7)$$

where q = Specific substrate removal rate (day^{-1}), which is related to the specific growth rate by the following equation;

$$q = \mu / Y \quad (2.8)$$

These definitions assume that all substrate utilization is achieved by bacterial growth and that bacterial maintenance needs are met by decay.

A variety of models are written to depict the relationship between the specific growth rate of bacteria and the concentration of a single limiting nutrient, since all such models are strictly empirical. Consequently, this review will be limited to the most widely use one. In 1940's Monod has observed the dependence of specific growth rate on the rate limiting substrate S_s as shown in Figure 2.9. and proposed the following expression to formulate the relation between the two:

$$\mu = \hat{\mu} \frac{S_s}{K_s + S_s} \quad (2.9)$$

where $\hat{\mu}$: maximum specific growth rate (day^{-1})
 K_s : half saturation coefficient (mg/lt)

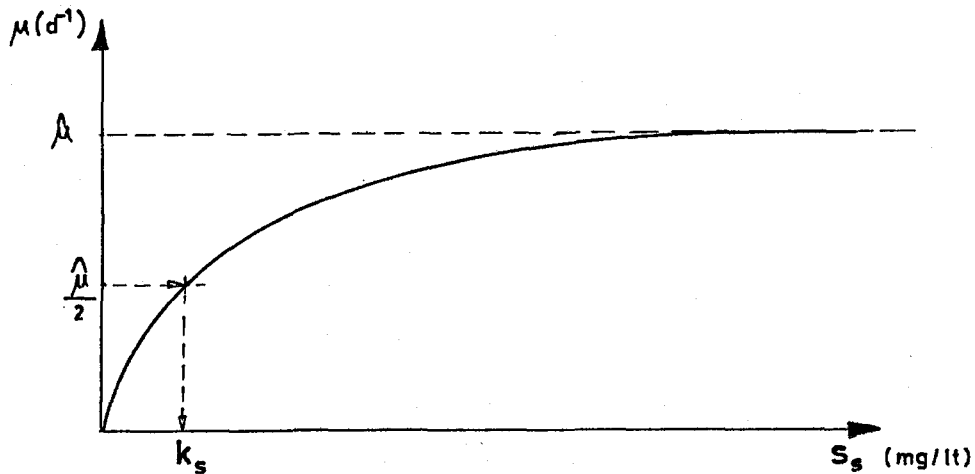


Figure 2-9. The Relationship between Specific Growth Rate and the Rate Limiting Substrate.

The classical approach in modelling of biomass growth does not include any differentiation in terms of viability and uses the total biomass as one homogeneous quantity. However, in 1960's scientists working on activated sludge systems have recognized the presence of inert organics in the sludge mass and observed that inert mass constitutes the unused and nonbiodegradable fraction of microorganisms. They reported that this portion does not undergo any reactions and that it can not be used in the growth process. It is then that the concept of viability / active biomass started to gain importance in activated sludge models. IAWPRC (1987) differentiated the concepts of viability / active biomass, (X_A), inert biomass, (X_E), and slowly degradable substrate, (X_S). The Task Group Model is successfully being used from then onwards and Gujer and Boller (1990) tried to adapt the model developed for activated sludge systems to biofilm reactors. In this study, the same basic ideas of Gujer and Boller are followed up during the development of the model.

The complete reaction rate expression for the removal of soluble substrate is obtained by combining equations (2.7) and (2.3), yielding

$$-r_{ss} = - \left[\frac{dS_s}{dt} \right]_{rx} = \frac{1}{Y} \hat{\mu} \frac{S_s}{K_s + S_s} \quad (2.10)$$

according to Monod kinetics.

Kinetic behaviour of multiple substrate has been studied by many scientists; but as this study deals with a single substrate, emphasis will not be given to multi-substrate kinetics of biofilms.

The IAWPRC Task Group on Activated Sludge Modelling (from hereon referred to as the Task Group Model) has integrated the recent progress in activated sludge research to prepare a general model, (IAWPRC, 1987) that assumes two different forms of substrate that can be used by the biomass: readily biodegradable and slowly biodegradable. A similar model is used by Gujer and Boller (1990) for biofilm kinetics. The details of this model and the adaptation of it to the biofilm concept are summarized in Sections 3.1 and 3.2. In the Task Group Model, slowly biodegradable substrate is solubilized through hydrolysis and turned into readily biodegradable substrate in the liquid phase and then used for synthesis along with the influent readily biodegradable matter. Such mechanism continuously aims to produce readily biodegradable substrate through hydrolysis. However, hydrolysis is known to be a slow mechanism thus it is assumed to be the rate limiting

step. The concept of hydrolysis is used in the same manner as in the case of suspended growth systems in this study. But, in the classical approaches, the concept of hydrolysis and the concept of viability is not considered at all; thus emphasis to these new concepts is given in the following Sections.

In general, the total loss of biomass in a biological system is called decay. Loss of biomass is assumed to be linear during endogenous respiration:

$$r_{XT} = - \frac{dX}{dt} = b \cdot X_T \quad (2.11)$$

where, X = concentration of bacteria at time t , (mg/l)
 b = decay coefficient (day^{-1})

Thus; the overall rate of bacterial growth and decay is equal to the summation of equations (2.6) and (2.11);

$$r_{XT} = \mu X_T - b \cdot X_T = (\mu - b) X_T \quad (2.12)$$

The recent terms such as viability, active biomass, inert mass, hydrolysis, particulate organic matter have not been considered till 1985, therefore the interaction of these new concepts in biofilm kinetics will be described later on after reviewing the previous model studies.

At this stage, the general reaction rate expressions are given in a simple manner as the following section reviews various kinetic models based on these fundamental and universally accepted equations.

2.2. AN OVERVIEW OF VARIOUS KINETIC MODELS

In an idealized biofilm, it is assumed that all required nutrients are in excess concentration, except one which is called "the rate-limiting substrate" or simply the substrate. As illustrated in Figure 2.8, the idealized biofilm has a uniform biomass of X_r and a locally uniform thickness, L_r . Substrate concentration within the biofilm changes only in the $-x$ direction which is normal to the surface of the biofilm.

The substrate utilization at any point in the biofilm is assumed to follow the Monod relation as mentioned in the previous section (eq.2.10).

Molecular diffusion, which the only means of mass transport within the biofilm, is related to the substrate concentration by Fick's Second Law.

Referring back to equation (2.3) which characterizes Fick's law and equation (2.5) which is derived from a mass balance on soluble substrate at steady-state conditions;

$$N_{ss} = D \frac{dS_s}{dL} \quad (2.3)$$

$$\frac{dN_{ss}}{dL} = -r_{ss} = -\frac{dS_s}{dt} \quad (2.5)$$

yield the Fick's Second Law if both equations (2.3) and (2.5) are combined as follows;

$$\begin{aligned}\frac{dN_{ss}}{dL} &= \frac{d}{dL} \left[-D \frac{dS_s}{dL} \right] \\ \frac{dN_{ss}}{dL} &= -D \frac{d^2 S_s}{dL^2}\end{aligned}\quad (2.13)$$

In short,

$$r_{ss} = \left[\frac{dS_s}{dt} \right]_{\text{diff.}} = D \frac{d^2 S_s}{dL^2} \quad (2.14)$$

Summing up equation (2.10) which gives the complete rate expression for the removal of soluble substrate according to Monod Kinetics $\left[\frac{dS}{dt} \right]_{rx}$, and equation (2.14) which gives the mass transport within the biofilm yields the total (overall) reaction rate expression of the substrate $\left[\frac{dS_s}{dt} \right]$ for any point in the biofilm.

$$r_{ss} = \left[\frac{dS_s}{dt} \right]_{\text{total}} = D \frac{d^2 S_s}{dL^2} \left[- \frac{\hat{\mu}}{Y K_s + S_s} X_T \right]$$

Assuming steady-state conditions $\left[\frac{dS_s}{dt} = 0 \right]$, the kinetic expression puts forth the ordinary differential equation for substrate concentration within the biofilm,

$$D \frac{d^2 S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_T \quad (2.15)$$

As this equation is non-linear, it cannot be solved explicitly. Numerical workers have presented analytical solutions where the Monod relation reduces to the first and zero order approximations. The most relevant solutions seen from the engineering point of view are given below in Table 2.3.

In the following sub-sections, emphasis will be given to the analytical solutions to show how they are derived and how they differ from each other.

Table 2-3. Analytical Solution of the Monod Relation by Numerical Workers (Baykal, 1986)

First Order Reaction	Zero Order and Half Order Reaction	Monod kinetics
Atkinson (1967)	La Motta (1976)	kornegay and Andrews (1968)
Levenspiel (1972)	Riemer (1977)	
Antonie (1976)	Murphy (1977)	Atkinson (1970)
Hansford (1978)	Harremoes (1977), (1978), (1981), (1982),	Williamson and Mc Carty (1976)
	Watanabe (1980)	Mueller (1978)
	Gönenc (1983)	Rittmann and Mc Carty (1980)

2.2.1. First Order Reaction kinetics

The general differential equation for substrate concentration within the biofilm reduces to first order in cases where the substrate concentration at the biofilm surface is lower than the half saturation constant:

$$K_s \gg S_s ;$$

$$D \frac{d^2 S_s}{dL^2} = r_{ss} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_T$$

negligible

$$r_{ss} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s} X_T$$

$$\frac{d^2 S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{X_T}{D \cdot K_s} S_s = k_1 \cdot S_s$$

$$\therefore \frac{d^2 S_s}{dL^2} = k_1 \cdot S_s \quad (2.16)$$

where

$$k_1 = \frac{\hat{\mu}}{Y \cdot D \cdot K_s} X_T \quad (2.17)$$

named as the 1st order rate constant.

If the general rate term is to be expressed in dimensionless form;

$$\frac{d^2 S_s}{d\xi^2} = \frac{L_f^2}{D S_s^*} \cdot r_{ss} \quad (2.18)$$

where $S_s = \frac{S_s}{S_s^*}$, $\xi = \frac{L}{L_f}$

S_s^* is the substrate concentration at the biofilm surface.

Equation (2.16) can be rewritten in dimensionless form as;

$$\frac{d^2 S_s}{d\xi^2} = \frac{k_1 L_f^2}{D} \cdot S_s = \alpha^2 \cdot S_s \quad (2.19)$$

Here α is a characteristic coefficient for an ideal biofilm and is defined as "biofilm coefficient";

$$\alpha = \sqrt{\frac{k_1 \cdot L^2}{D}} \quad (2.20)$$

This equation is solved by Levenspiel (1972) with the following boundary conditions;

At $L=0$ (biofilm bulk surface) $\xi=0$ $S_s=1$

At $L=L_f$ (biofilm support media) $\xi=1$ $\frac{\partial S_s}{\partial \xi} = 0$ (No flux)

$$S_s(L) = \frac{\cosh \sqrt{\frac{k_1 \cdot (L_f - L)^2}{D}}}{\cosh \alpha} \quad (2.21)$$

is the resulting equation;

On the other hand, the total removal per unit area, N_{ss} is given as;

$$N_{ss}(L=0) = -D \left[\frac{dS_s}{dL} \right]_{L=0} = k_1 \cdot L_f \cdot S_s^* \cdot \epsilon \quad (2.22)$$

where,

$$\epsilon = \frac{\tanh \alpha}{\alpha} \quad (2.23)$$

Thus, a first order basic reaction occurring within a biofilm leads to a total 1st - order removal with a rate constant modified by an effectiveness factor given in equation (2.23);

Figure 2-10. shows the distribution of concentration in a biofilm for a 1st order reaction.

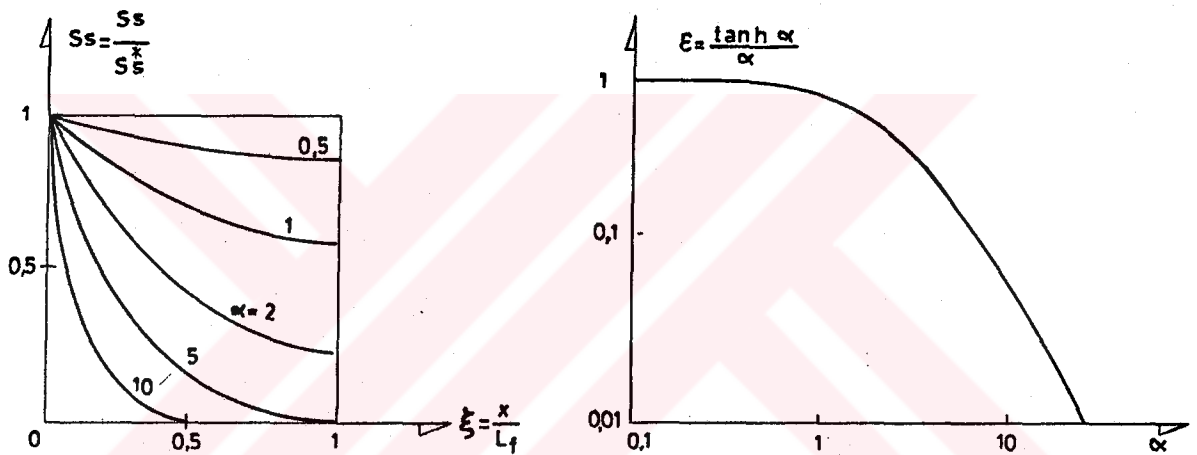


Figure 2-10. Concentration Profile in a Biofilm for a 1st - order Reaction

2.2.2. Zero Order Reaction Kinetics

The general differential equation for substrate concentration within the biofilm reduces to zero order in cases where the substrate concentration at the biofilm surface is higher than the half saturation constant:

$$\underline{k_s \ll S_s};$$

$$D \frac{d^2 S_s}{dL^2} = r_{ss} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_T$$

negligible

$$r_{ss} = \frac{\hat{\mu}}{Y} X_T$$

$$\frac{d^2 S_s}{dL^2} = \frac{\hat{\mu}}{Y} X_T = k_o$$

$$\frac{d^2 S_s}{dL^2} = k_o \quad (2.24)$$

where,

$$k_o = \frac{\hat{\mu}}{Y} X_T \quad (2.25)$$

named as the 0th order rate constant. If the general rate term is to be expressed in dimensionless form;

$$\frac{d^2 S_s}{d\xi^2} = \frac{k_o L_f^2}{DS_s^*} = \frac{2}{\beta^2} \quad (2.26)$$

where,

$$\beta = \sqrt{\frac{2DS_s^*}{k_o L_f^2}} \quad (2.27)$$

β is a dimensionless coefficient named as penetration ratio. The same boundary conditions of 1st order reaction kinetics are applied as in the ideal biofilm concept where biomass is assumed to be equally active at every point throughout the media.

$$L=0 \text{ (biofilm bulk surface)} \quad \xi=0 \quad S_s=1$$

$$L=L_f \text{ (biofilm support media)} \quad \xi=1 \quad \frac{\partial S_s}{\partial \xi} = 0 \text{ (No flux)}$$

With these boundary conditions, the resulting equation is obtained as;

$$S_s = \frac{\xi^2}{\beta^2} - 2 \frac{\xi}{\beta^2} + 1 \quad (2.28)$$

The total removal per unit area, N_{ss} is given as;

$$N_{ss} = -D \left[\frac{dS_s}{dL} \right]_{L=0} = \left[\frac{-DS_s^*}{L_f} \right] = \left[\frac{\partial S_s}{\partial \xi} \right]_{\xi=0} = \text{where } \beta > 1$$

$$N_{ss} = \frac{DS_s^*}{L_f} \cdot \frac{2}{\beta^2} = k_o \cdot L_f \quad (2.29)$$

In a fully active biofilm for the condition $\beta > 1$ substrate is considered to fully penetrate into the biofilm leading to 0 - order reaction kinetics.

For $\beta < 1$, substrate cannot fully penetrate into the biofilm; this condition is known as partial penetration of the substrate, (Levenspiel, 1972).

The dimensionless concentration S_s is shown on Figure 2-11. for full and partial penetration conditions.

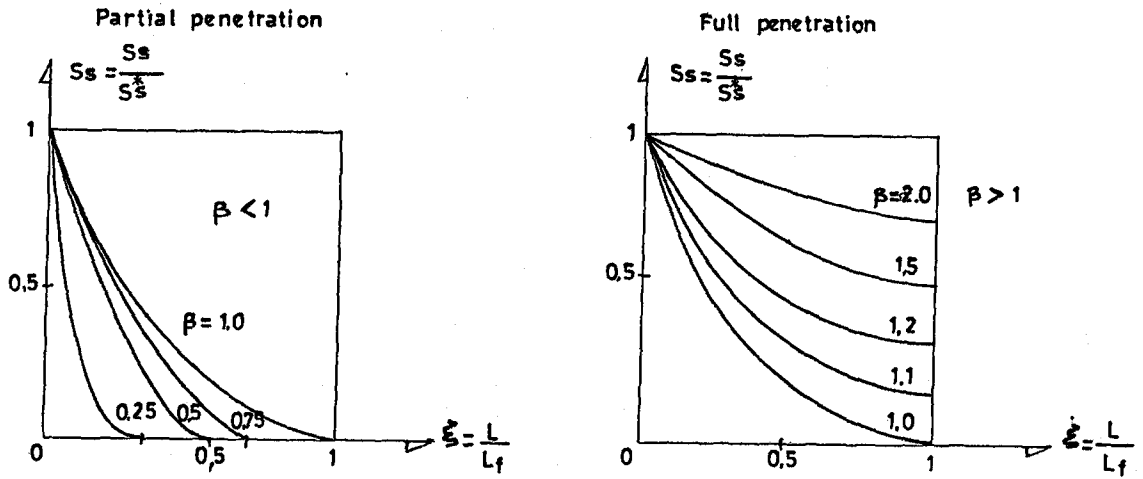


Figure 2-11. Concentration Profiles for full and Partial Penetration Conditions for a 0 order Reaction

The total removal per unit are; N_{ss} is given as;

$$N_{ss} = -D \left[\frac{dS_s}{dL} \right]_{L=0} = \left[\frac{-DS_s^*}{L_f} \right] \cdot \left[\frac{dS_s}{d\xi} \right]_{\xi=0} \quad \text{where } \beta < 1$$

$$N_{ss} = \frac{DS_s^*}{L_f} \cdot \frac{2}{\beta} = \left[\sqrt{2Dk_0} \right] \cdot S_s^{*\frac{1}{2}} \quad (2.30)$$

In a partial penetration condition, $\beta < 1$, substrate removal leads to a $\frac{1}{2}$ order reaction kinetics.

The rate constant for a $\frac{1}{2}$ order reaction is therefore equal to;

$$K_{\frac{1}{2}} = \sqrt{2Dk_0} \quad (2.31)$$

2.2.3. Transient Conditions Between Different order kinetics

Zero and first order reactions in a homogeneous biofilm lead to zero, half and first order reactions. The prevailing process depends on the biofilm geometry, the rate constants (K_m) and on the concentration of the substrate at the biofilm surface (S_s^*).

In the attached growth systems that are expressed kinetically by the Monod relation, two conditions take place;

- i) When substrate concentration is quite low; 1^{st} - order reaction prevails,
- ii) When substrate concentration is high; 0 - order reaction prevails.

This consideration is still valid for attached growth culture. During modelling stage, three different approaches can be used.

- i) All the biomass within a biofilm behave according to either 1^{st} - order or 0 - order kinetics.
- ii) An abrupt change occurs between the two kinetic behaviour due to variations of substrate concentration. This leads to solution by formulating the new kinetic process by combining both order differential equations.

iii) As illustrated in Figure 2-12, the Monod kinetics is valid at any point of the biofilm (Baykal, 1986).

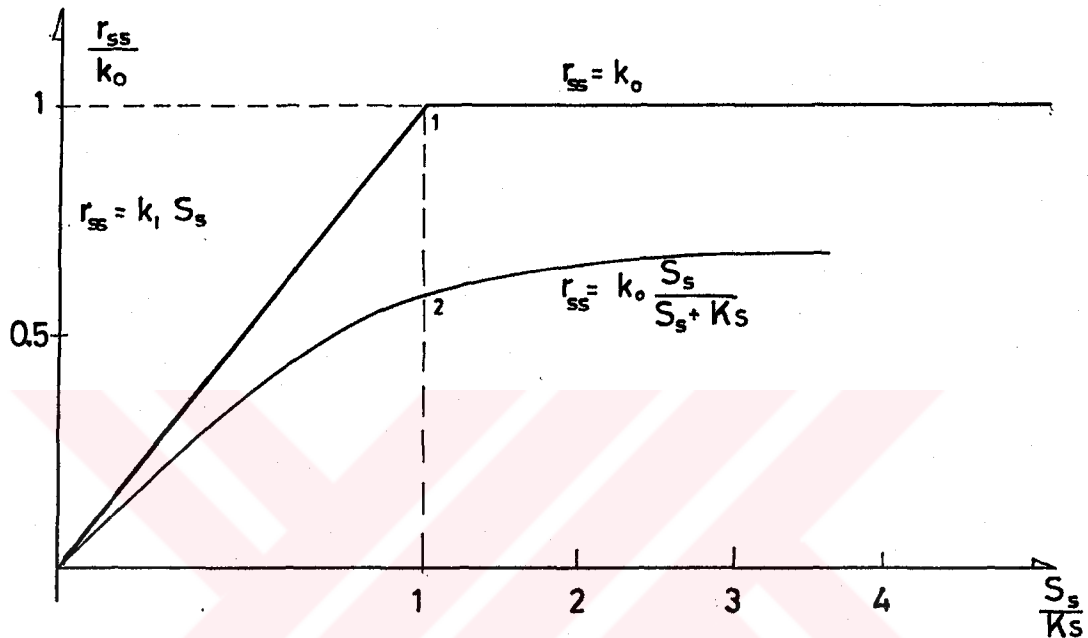


Figure 2-12. Two transition Approaches from 1st order to 0 - order

1. $S_s = K_s$ abrupt change
2. Michaelis - Menten kinetics

According to the first approximation;

When $S_s^* > S_s^*$ critical 1st - order kinetics

When $S_s^* < S_s^*$ critical 0 - order kinetics dominate

the system where, S_s^* is the substrate concentration at which both kinetic expressions give the same removal rate.

However, if the biofilm is thick, both reaction kinetics may be affected by diffusional resistance.

In order to find out the right kinetic expression for a system, it is convenient to use three dimensionless concentration value to determine the type of reaction and the transitions between different processes;

$$S_A = \frac{k_1 S_S^*}{k_o}, S_B = \frac{k_1^2 S_S^*}{k_o^2}, S_E = \frac{k_1^2 S_S^*}{k_1^2} \quad (2.32)$$

where S_S^* is the substrate concentration at the biofilm surface.

The relation between these three dimensionless quantities is given as, (Harremoes, 1978):

$$S_A^2 = S_E \cdot S_B \quad (2.33)$$

The reaction rate in a biofilm is equal to,

$$X = \frac{r_{ss}}{k_o L_f} \quad (2.34)$$

and as a function of concentration; it is defined as,

$$S_A = \frac{k_1}{k_o} S_S^* = \frac{k_1}{k_o} S_S^* \cdot \frac{\tanh \alpha}{\alpha} \quad (2.35)$$

The biofilm characteristics as a function of S_A/S_B is expressed as

$$\frac{S_A}{S_B} = \frac{k_1 \cdot k_o}{k_1^2} = \frac{\alpha}{2} \tanh \alpha \quad (2.36)$$

Figure 2-13. illustrates the dimensionless removal rates for different order reactions.

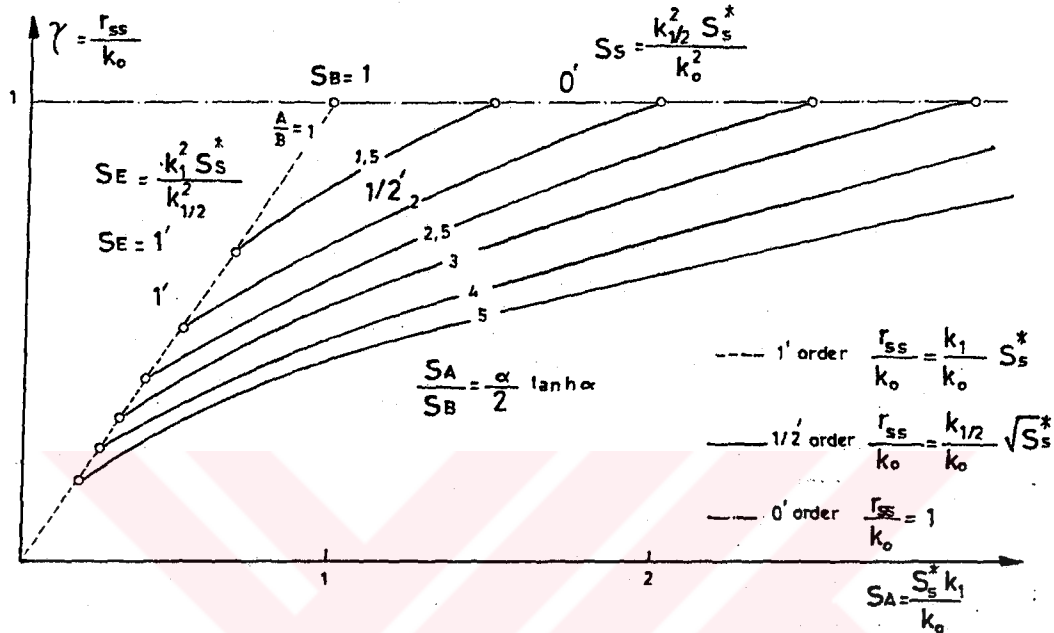


Figure 2-13. Dimensionless Removal Rate versus Dimensionless Substrate Concentration for a 0 - order and 1st - order reaction.

The following general explanation can be done with respect to the segregations illustrated in Figure 2.13.

- i) The transition from 1st - order to $\frac{1}{2}$ - order occurs at $S_E = 1$.

$$S_E = \frac{k_1^2 S_E^*}{k_+^2} = \frac{\tanh^2 \alpha}{2} \cdot \frac{k_1 S_E^*}{k_o} \quad (2.37)$$

- ii) The transition from $\frac{1}{2}$ order to 0 - order occurs at $S_E = 1$.

$$S_E = \frac{k_+^2 S_E^*}{k_o^2} = \beta^2 \quad (2.38)$$

iii) At the condition,

$$\frac{S_A}{S_B} = \frac{\alpha}{2} \quad \tanh \alpha \leq 1 \text{ or } \alpha \leq 2 \quad (2.39)$$

$\frac{1}{2}$ order reaction is impossible, thus the transition from 1^{st} - order to 0 - order occurs at $S_A=1$. All these three result are illustrated schematically in Figure 2.14.

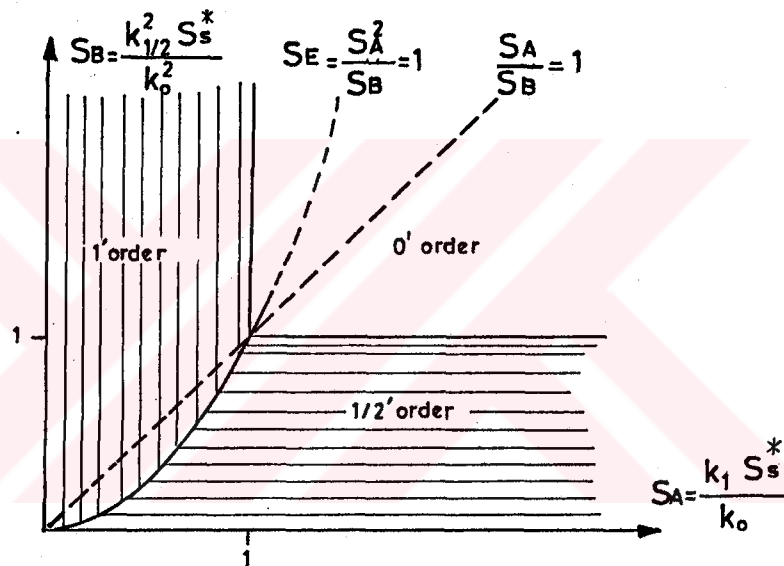


Figure 2-14. Regions for the Three Different Orders of Reactions.

On the other hand, La Motta's (1976) approach consider internal diffusional resistances to effect the rate of substrate removal in biofilms significantly. Therefore, one of his objectives during establishing the mathematical description of the rate of substrate removal is the film thickness. He used suitable boundary conditions according to complete penetration within the film or partial penetration during solving the general equation (2.14) and has found out that for all practical

substrate concentrations, the true rate of substrate removal is zero order in cases of full penetration. This true reaction rate is commonly termed as "intrinsic" reaction rate. In cases, where the depth of penetration is small compared to the entire film thickness, the effect of internal diffusion masks the true zero order kinetics yielding an apparent reaction order of $\frac{1}{2}$. This apparent or observed rate where the reaction is limited by diffusional resistance is termed as "macroscopic rate".

Therefore, where there is partial substrate penetration (in cases of thick films) the observed rate depends on the magnitude of the depth of penetration and is independent of total thickness.

2.2.4. Half Order Reaction Kinetics

Harremoes (1977) attempted to develop a new model after he has fully analysed 0 - order, 1st - order reactions as well as the Monod kinetics. In his application the definition of Monod kinetics is given as;

$$r_{ss} = k_o \frac{S_s}{K_s + S_s} \quad (2.40)$$

and used the following terms to solve the dimensionless differential equation whose boundary conditions are shown in Figure 2.15.

Biofilm constant:

$$\alpha = \frac{k_1 L_f^2}{D} = \frac{k_o L_f^2}{K_s D}$$

Concentration Ratio;

$$S_x = \frac{S_s}{K_s} \quad \text{and}$$

Dimensionless distance within biofilm;

$$\xi = \frac{L}{L_x}$$

Therefore; the general expression becomes;

$$\frac{\partial^2 S_x}{\partial \xi^2} = \alpha^2 \frac{S_f}{S_f + 1} \quad (2.41)$$

For the simple transition from 1st - order to $\frac{1}{2}$ - order

$$S_s \frac{\tanh^2 \alpha}{2} \cdot \frac{k_1 S_s^*}{k_o} = 1$$

$$S_s^* = 2 (\tanh \alpha)^{-2} \approx 2 \text{ for } (\alpha > 1)$$

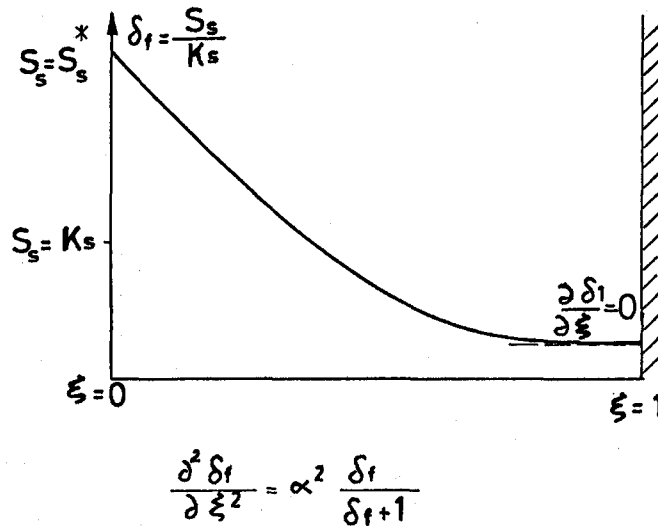


Figure 2-15. The Boundary Conditions of the Differential Equation Related to Monod Kinetics (Harremoes, 1977)

For $\alpha \leq 2$ order reaction is impossible and the transition from 1st order to 0 order occurs directly.

The maximum error occurs at $s^* = 2$ and $\alpha = 2$ when this simple model is applied. At around $\alpha > 4$ and $s^* > S_s$ the deviation is below 40 %.

The solution of both the Monod kinetics and $\frac{1}{2}$ order kinetics obtained for various α values and for dimensionless concentration in the internal $s^* = \frac{S_s^*}{K_s}$ is illustrated in Figure 2.16 (Harremoes, 1977).

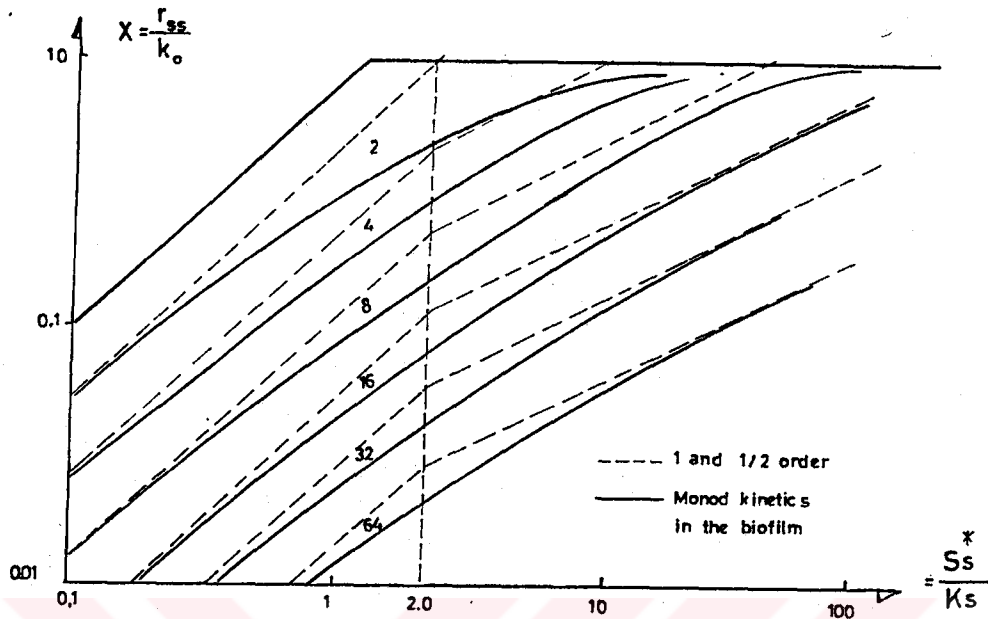


Figure 2-16. Solution of the Differential Equation based on Monod Kinetics compared with the $\frac{1}{2}$ order approximation to Biofilm Kinetics (Harremoes, 1977).

Thus, for most practical purposes the simple $\frac{1}{2}$ order theory is sufficient as it is easier to understand and apply.

2.2.5. Monod Kinetics

Biofilm with intrinsic Monod kinetics was first investigated by Atkinson and Daoud (1968). A numerical solution of equations supplied with a Monod type kinetics existed in heterogeneous biofilm concept. The solution has been adapted to fixed-film systems and has been approximated by analytical solutions. The theory covers diffusion into both bacteria and biological films. The original expressions had a limited field of

application but have been researched later to cover all combinations of concentrations and kinetic parameters, (Atkinson & Fowler 1974).

The parameters used in the developed model are k_1 , k_2 , and k_3 as follows;

$$k_1 = \frac{a \alpha_b}{K_s} \quad (2.42)$$

where k_1 = 1 order reaction constant for low concentrations

a = specific surface of bacteria (m^{-1})

α_b = rate coefficient for the single bacteria ($gr/m^2 \cdot day$)

$$k_2 = \left[\frac{a \cdot \alpha_b}{K_s D} \right]^{\dagger} = \left[\frac{k_1}{D} \right]^{\dagger} = \left[\frac{k_o}{K_s D} \right]^{\dagger} \quad (2.43)$$

which can be interpreted as the inverse of the penetration depth, when $S_s^* = \frac{1}{2} K_s$ has the unit (m^{-1})

$$k_3 = \frac{1}{K_s} \quad (2.44)$$

namely the inverse saturation constant ($gr^{-1} \cdot m^3$).

The overall rate equations based on these parameters give out well numerical solutions but these solutions are obtained from rather complex equations. Atkinson and Fowler (1974) developed more simplified set of equations but these solutions have been applicable to a restricted area.

Williamson and Mc Carty (1976a) have also solved the basic differential equation with Monod kinetics for two limiting cases shown in Table 2.4. The solution for a given biofilm is obtained by iteration using a Runge-Kutta finite difference technique.

The computation has been worked out by starting at a certain point in the interior of the biofilm assuming the concentration at this point to be of value, a small fraction of the Monod constant and also considered the concentration gradient to be zero. By working backwards towards the biofilm surface, the relations between the interfacial concentration S_s^* and the effective thickness, L_e , have been established, (Figure 2.17). This procedure accounts for the limiting cases of 1^{st} - order and $\frac{1}{2}$ - order reactions but does not reveal the existence of the 0 - order used as initial conditions. The entire procedure terminates when the substrate concentration just equals or slightly exceeds the surface concentration, S_s^* , at which point the value of effective thickness, L_e , may be determined. The biofilm surface flux, N_{ss} , is then given as the product of the biofilm surface gradient times the substrate diffusion coefficient through the biofilm and the biofilm area. The results were presented in graphs of active film thickness and limiting constituent flux as a function of S_s^* plots of different values of K_s were prepared for different values of the group $\frac{\mu_{max} D}{K_s L^2}$. These curves can then be used to solve any fixed film process. An example of plots is given in Figure 2.18. The flux, N_{ss} , and bulk substrate concentration, S_b are assumed and the value of S_s^* is determined to incorporate external mass transfer effects. Using S_s^* and the appropriate graph (Fig. 2.18), the internal flux is determined and then compared with the assumed value.

Table 2-4. Biofilm Surface Flux and Biofilm Substrate Concentration for the Limiting Cases at $S_s \gg K_s$ and $S_s \ll K_s$ (Williamson and Mc Carty, 1976a)

Comparison of S_s with K_s	Differential Equations	Biofilm surface Flux	Biofilm substrate concentration
$S_s \approx K_s$	$\frac{d^2 S_s}{dL^2} = \frac{\hat{\mu} S_s X_z}{D (K_s + S_s)}$	No Explicit Solution	No Explicit Solution
$S_s \gg K_s$	$\frac{d^2 S_s}{dL^2} = \frac{\hat{\mu} X_z}{D}$	$N_o = A \cdot \hat{\mu} \cdot X_z \cdot L_o$	$S_s = S_s^* - \frac{\hat{\mu} X_z}{D} \left[L_o \cdot L - L^2/2 \right]$
$S_s \ll K_s$	$\frac{d^2 S_s}{dL^2} = \frac{\hat{\mu} X_z S_s}{D K_s}$	$N_o = A (D \hat{\mu} X_z / K_s) \cdot S_s^*$	$S_s = S_s^* \frac{\cosh \left[(\hat{\mu} X_z / D K_s) \cdot (L_o - L) \right]}{\cosh \left[(\hat{\mu} X_z / D K_s) \cdot (L_o) \right]}$

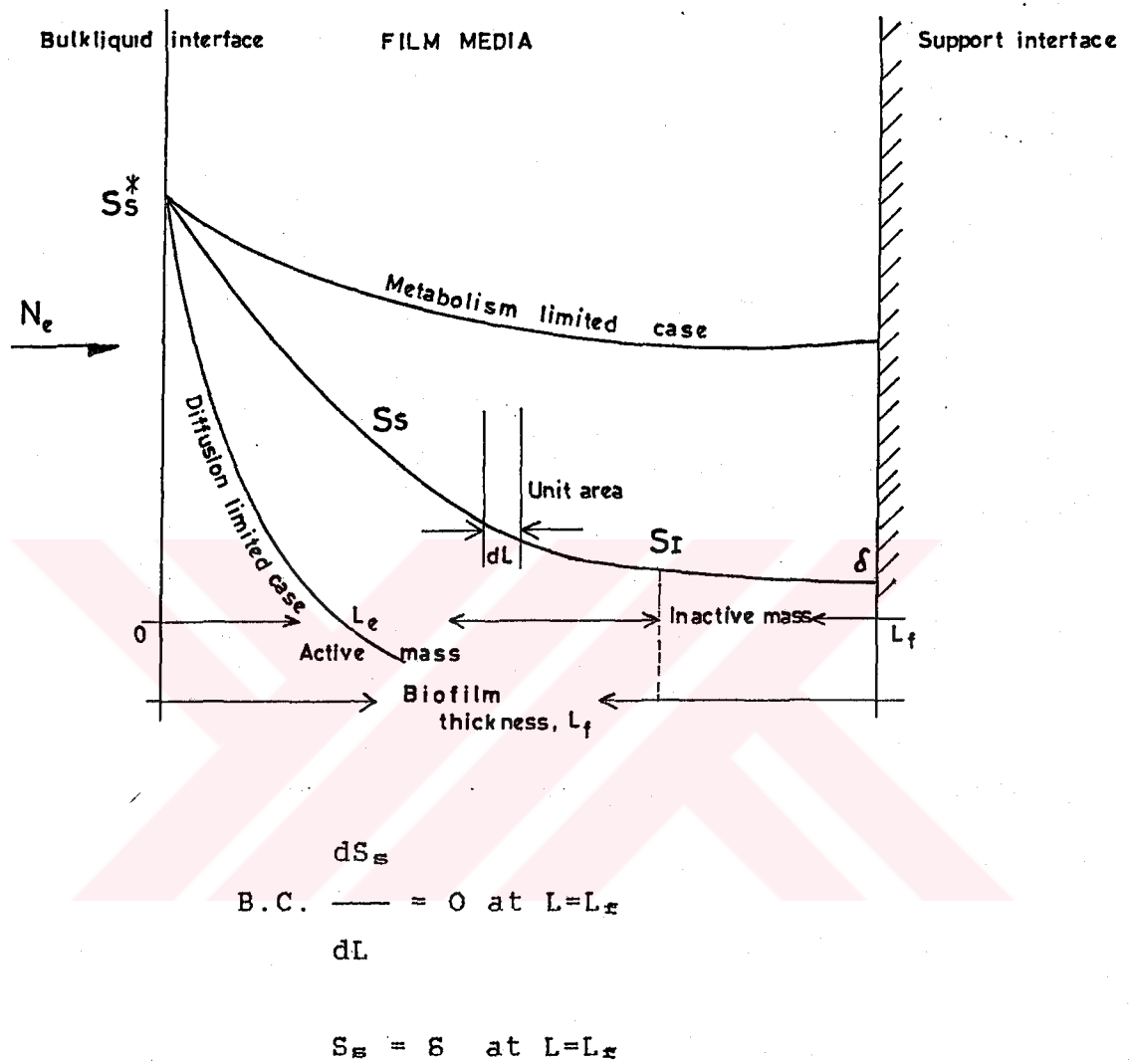


Figure 2-17. Potential Substrate Profiles within a Biofilm (Willimason and Mc Carty, 1976 a)

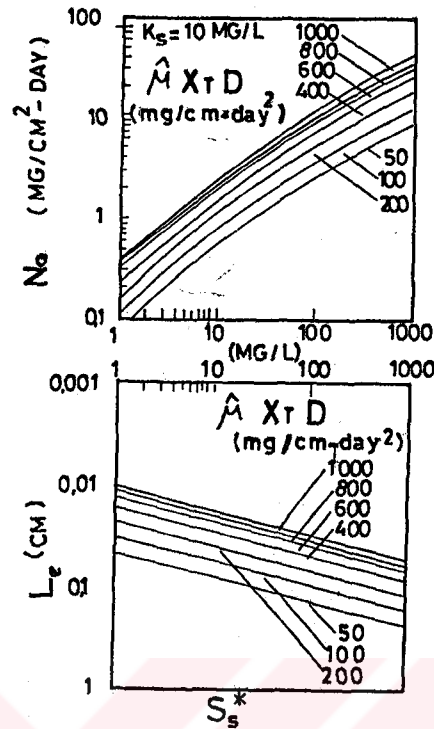


Figure 2-18. Biofilm Surface flux versus Substrate Concentration at the Biofilm Surface for $K_s = 10$ mg/lt (Williamson and Mc Carty, 1976a)

The procedure is repeated till the assumed and the determined values are equal.

This solution technique makes it possible to model processes without using complex numerical techniques, the indirect solution provided by the graphs is limited in the number of parameter values considered. Furthermore, one must also estimate beforehand whether the electron donor or the acceptor is rate limiting.

This fact limits the utility of the model. Nevertheless, this model gained a high importance in the field of biofilm kinetics as it served a useful purpose

as the starting point from which other models have been developed.

Williamson has continued to work on the second order differential equation by another solution technique. He integrated the equation once by assuming that the concentration approaches zero within the film, thereby giving an equation for the substrate concentration gradient. Multiplication of the gradient value at the biofilm; (concentration of the substrate at the surface of the biofilm, S_s^*) by the effective diffusion coefficient, D , results in the flux associated with S_s^* . This technique is only applicable when the concentration of the substrate approaches zero within the interior of the biofilm and thus the solution is limited to what is called "thick" or "deep" biofilms.

Rittmann and Mc Carty (1978) also used the integration technique to solve a single substrate Monod Model; The same general solution approach was utilized, but because only one limiting constituent was considered they presented their results in dimensionless form, thereby increasing the generality of their plot versus bulk substrate concentration. The parameters in their plot were effective diffusivity, D , and active dept, L_a , both in dimensionless form. As a consequence, their curves could be applied to any combination of kinetic and mass transfer parameters. The major limitation of this model is that they are limited to "thick" films because of the use of the integration technique. By inspecting the curves the authors developed simplified equations thereby facilitating their use in the solution of a broad range of models for processes which contain "thick" biofilms.

The majority of the biofilm models have been solved for a biofilm thickness which is either assumed or which is a coupling point with the process model. Rittmann and Mc Carty (1980), however, extended their previous solution technique to one in which cell growth is just compensated by decay for a steady-state biofilm. Here is the new contribution of a steady-state model! The net growth rate of bacterial mass in a differential section of biofilm is represented as follows:

$$\frac{AX_T dL}{t} = Y \frac{\mu_{\max} S_S}{K_S + S_S} A \cdot X_T \cdot dL - b \cdot X_T dL \quad (2.45)$$

where b = specific decay coefficient, day^{-1}

A = cross-sectional area of the biofilm section (m^2)

dL = thickness of the biofilm section.

In a steady-state situation, there is a unique film thickness associated with each bulk substrate concentration. When that thickness is "deep", the concentration of substrate reaches zero at the support media. When it is "shallow", a finite substrate concentration remains at the support (Figure 2.19). Two

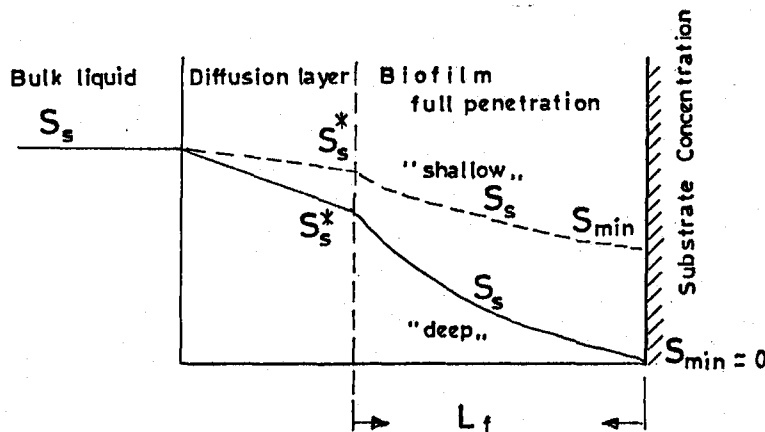


Figure 2-19. Conceptual Basis for Biofilm Model

solution techniques were utilized to generate the plot of flux versus bulk substrate concentration, depending on whether the film is "deep" or "shallow". Using the steady-state assumption, the film thickness, L_x , was calculated for an assumed flux and the deep film technique was used to get the bulk substrate concentration associated with that flux. The thickness of a biofilm is not only determined by the flux of substrate but also by considering the bacterial growth and decay rates. The net energy that becomes available to the biofilm per unit time and per unit cross-sectional area can be expressed as NY ; the total maintenance energy requirement can be expressed as $X_T L_x b$. In order to determine the steady-state biofilm mass, it is assumed that the total amount of biomass is just equal to that which can be supported by the substrate flux. The steady-state biofilm thickness can then be computed by equating the available and maintenance energy rate,

$$L_x = \frac{NY}{bX_T} \quad (2.46)$$

Equation (2.46) requires that the steady-state biofilm be a dynamic one. The decaying portion continually gives up space to the growing portion in such a manner that the net thickness remains constant. Because of the need for growth to balance decay in steady-state biofilm there will be some minimum substrate concentration, S_{min} , required to maintain a steady-state film media. When that concentration is reached, the flux into the film will be zero and no more film will be maintained. This means that film thickness will vary from zero at S_{min} to some maximum determined by the maximum substrate concentration. Furthermore, the flux into a "shallow", steady-state biofilm will vary from zero at S_{min} to the deep value at some higher bulk substrate concentration.

The S_{min} expression used by Rittmann and Mc Carty is as follows;

$$S_{min} = K_s \left[\frac{b}{(1 - Y - b)} \right] \quad (2.47)$$

The finite difference technique is also used in this model by dividing the calculated thickness into a finite difference grid and the steady-state concentration profile in the biofilm is solved for the boundary condition that there is no flux at the support media.

Following Rittmann and Mc Carty's modelling techniques several scientists studied on kinetic modelling of biofilm systems utilizing the steady-state differential equation of Monod kinetics on substrate removal rate within the biofilm. The ones that are of interest related to the modelling aspects of this thesis may be briefly summarized as follows;

- Benefield and Molz (1985) considered substrate utilization by only heterotrophic microorganisms, where increase in biofilm thickness is due to bacterial growth and decrease in bacteria concentration is due to decay and finally loss of biofilm is due to sloughing. The model consists of two partial differential equations and three ordinary differential equations. The response of the model is illustrated through a numerical solution of the system of five differential equations using specified parameter values. In its present form, the model illustrates how loading, contact time, thickness and oxygen concentration interact to effect substrate removal in a fixed-film process.

- Logan et. al. (1987) formulated a fundamental model for trickling filter design where the predictions are based on substrate transport and first order kinetics. The authors emphasized on media geometries, in other words, on specific surface area (m^2/m^3) of the reactor and also on external conditions, such as hydraulic loading and organic loading criteria which are of high importance during design of a fixed-film system.

- Suidan et. al. (1987) followed up the model developed by Rittmann and Mc Carty for shallow and deep films and tried to put forth a graphical procedure for assessing the nature of substrate penetration within a biofilm. This procedure which is based on a measure of the effectiveness of substrate utilization by microorganisms is used to determine the simple analytical solutions of biofilm models.

These analytical solutions correspond to extreme values of bulk substrate concentration and to cases where the biofilm is fully penetrated or deep.

The main difficulty in using the equations describing a steady-state biofilm is that the second-order differential equation for simultaneous substrate utilization and diffusion in the biofilm cannot be analytically integrated due to the nonlinearity of the Monod-type reaction term. As discussed so far, several researchers analytically solved special limiting cases or used numerical techniques for obtaining approximated solutions. A most significant advancement is the development of a pseudoanalytical solution for the steady-state biofilm model.

So far, review on the kinetic modelling of a biofilm system is based on the fundamental assumption that the activity of microorganisms composing a biofilm is constant and same at every point within the film media. From 1985 onwards, it is recognized by a few scientists that this is not the case. The concept of variation of microbial activity within a biofilm brought together the concept of particulate components and their removal aspects. The next section emphasize on the contribution of these new phenomena in biofilm kinetics.

2.2.6 The kinetics and Removal Mechanism of Particulate Components in a Biofilm

Särner (1986) attempted to examine different factors affecting the efficiency of a fixed-film process such as the organic and hydraulic loading of a system and pointed out the importance of the interaction between particulate and dissolved substrates. So far, emphasis has not been given to the behaviour of particulate substrates (microorganisms) in an attached growth culture. The major difference between the removal of dissolved and particulate substrates is that the dissolved portion is transported into the biofilm where the reaction takes place, while the particulate portion is adsorbed on the biofilm surface. The adsorbed particles have to be hydrolyzed to make transport into the active biofilm possible. Thus, from 1985 onwards, scientists realized the distinction between both components and considered the necessity of including the removal mechanism of particulate substrate at modelling stage.

Wanner and Gujer (1985) indicated that the relative activity of both the heterotrophic and autotrophic microorganisms within a biofilm media varies significantly from the bulk liquid surface till the support media. They assumed a biofilm with gradients only perpendicular to the supporting media, to be composed of different microbial species that may be expressed as a continuum. The densities of the different species are assumed to be constant. At steady-state biofilm thickness is kept constant due to an equilibrium of biomass net production and shear loss at the surface.

With these assumptions a mass balance for a single species i can be written for a differential volume element AdL of the biofilm as shown in Figure 2.8,

$$AdLX_i \frac{\partial f_i(t, L)}{\partial t} = AdL\mu_{oi}(t, L)X_i f_i(t, L) + AN_{xi}(t, L) - AN_{xi}(t, L+dL)$$

where X_i = volume fraction of species i
 μ_{oi} = observed specific growth rate
 N_{xi} = flux of species i through area A .

Dividing the equation by $AdLX_i$ yields

$$\frac{\partial f_i(t, L)}{\partial t} = \left[\mu_{oi}(t, L)f_i(t, L) \right] - \frac{1}{X_i} \frac{N_{xi}(t, L+dL) - N_{xi}(t, L)}{dL}$$

which for $dL \rightarrow 0$ leads to

$$\frac{\partial f_i(t, L)}{\partial t} = \left[\mu_{oi}(t, L)f_i(t, L) \right] - \frac{1}{X_i} \frac{\partial N_{xi}(t, L)}{\partial L} \quad (2.48)$$

The biomass flux may be expressed in terms of velocity, u , at which species move with respect to the support media by the concentration $X_i f_i$ of the microorganisms,

$$N_{x1} = uX_1f_1 \quad (2.49)$$

The average net or observed specific growth rate of n_x microbial species in the biofilm is defined as

$$\bar{\mu}_o = \sum_{i=1}^{n_x} \mu_{o1} f_1 \quad (2.50)$$

Because of

$$= \sum_{i=1}^{n_x} f_1 = \frac{X_1}{X_{Total}} = 1$$

(as the biofilm thickness is constant, $L_f = \text{constant}$, the total biomass composing the thickness is also constant, $X_{Total} = \text{constant}$, only the volume fraction of species is variable)

Summation of equations (2.48) and (2.49) over all microbial species yields,

$$\begin{aligned} \sum \mu_{o1} f_1 &= \sum \frac{\partial f_1}{\partial t} + \sum \frac{1}{X_1} \frac{\partial N_{x1}}{\partial L} = \frac{\partial}{\partial t} \sum f_1 + \frac{\partial}{\partial L} \sum \frac{1}{X_1} N_{x1} = \\ \frac{\partial}{\partial L} \sum \frac{N_{x1}}{X_1} &= \end{aligned} \quad (2.51)$$

and

$$\sum \frac{N_{x1}}{X_1} = u \sum f_1 = u \quad (2.52)$$

The average net or observed specific biomass growth rate is defined as,

$$\bar{\mu}_o = \frac{\partial u}{\partial L} \quad (2.52)$$

If equations (2.51) and (2.52) are substituted into equation (2.50), equation (2.53) is obtained:

$$\bar{\mu}_o = \sum \mu_{oi} f_i = \frac{\partial}{\partial L} \sum \frac{N_{xi}}{X_i} = L \frac{\partial U}{\partial L}$$

The substitution of

$$\frac{1}{X_i} \frac{\partial N_{xi}}{\partial L} = \frac{\partial}{\partial L} \frac{N_{xi}}{X_i} = \frac{\partial}{\partial L} u f_i = f_i \frac{\partial u}{\partial L} + u \frac{\partial f_i}{\partial L} = \bar{\mu}_{oi} f_i + u \frac{\partial f_i}{\partial L}$$

in equation (2.48) provides

$$\frac{\partial f_i}{\partial t} = [\mu_{oi} - \bar{\mu}_o] f_i - u \frac{\partial f_i}{\partial L} \quad (2.54)$$

which reduces at the film-support interface to an ordinary differential equation;

$$\frac{df_i}{dt} = [\mu_{oi} - \bar{\mu}_o] f_i \quad (2.55)$$

due to the no-flux condition at this interface.

For steady-state, the boundary condition at the film-support interface reduces to,

$$(\mu_{oi} - \bar{\mu}_o) f_i = 0 \quad (2.56)$$

where,

$$i=1, \dots, nx$$

The spatial distribution of the microbial species in a steady-state biofilm may be predicted qualitatively by analysis of the observed or net specific growth rates μ_o . In other words, suitable

boundary conditions should be used in solving equation (2.56) by any numerical method. As far as, this study is interrelated with the removal of particulate substrates to define the generation mechanism of soluble residual product in a fixed-film system, the above mentioned analysis of specific net growth rates developed by Gujer (1988), is used during modelling.

According to Wanner and Gujer (1986), the equations derived for microbial species (particulate components) can only be fulfilled if either;

- the specific net growth rate of a species is equal to the mean specific net growth rate of the biomass, or

- the concentration of that species is zero, i.e. if the species has been displaced from the film.

The latter is going to happen if μ_o at $\xi = 0$ of the dominant species is still higher than the value of μ_o of a species that already has been almost completely displaced from the film.

Figure 2-20. illustrates this situation for three fictitious microbial species. During modelling, the main assumption is due to the fact that the specific net growth rate of a species is equal to the mean specific net growth rate of the biomass at the film-support interface as will be seen in further sections.

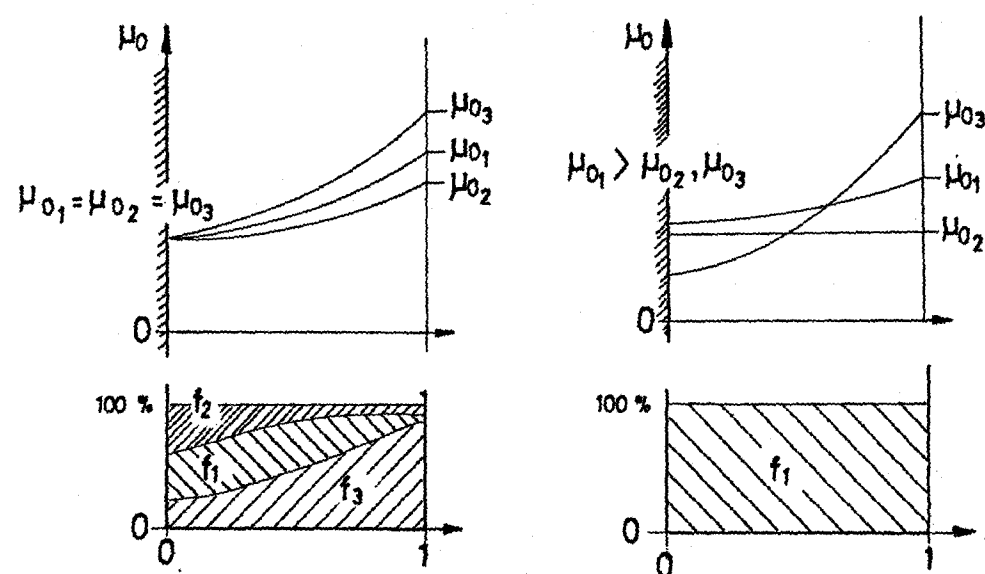


Figure 2-20. Specific Net Growth Rates μ and Volume Fractions f for three Microbial Species in Mixed (left) and Homogeneous (right) Steady state biofilms (Wanner and Gujer, 1986).

So far, literature review on kinetic modelling of fixed-film systems show that new models, in other words, recent models relate specifically to film internal distribution of different species of microorganisms which allow consideration of biofilm surface reactions. Still, little is known about the exchange reactions of particulate components at the bulk surface of a biofilm. Flocculation from the bulk liquid to the biofilm, shear of solids from the support media of the biofilm to the bulk liquid and sloughing of large pieces of biomass from the biofilm are important processes. Researchers developed simple mathematical rate equations to describe these surface reactions, such as flocculation, shear of solids and sloughing (Gujer and Boller, 1990), but, as this study does not involve these processes, no more emphasis will be given.

In short, defined in terms of a mass balance, the following partial differential equation expresses the change of biofilm particulate composition over time and thickness:

$$\frac{\partial X_i}{\partial t} = \frac{\partial N_{xi}}{\partial L} + r_{xi} \quad (2.57)$$

with

$$N_{xi}(L) = X_i(L) \cdot \int_{L_x}^L \sum_i \left[r_{xi}/X_i \right] dL \quad (2.58)$$

which replaces Fick's Law of Molecular Diffusion in the context of dissolved substrate where

X_i = concentration of particulate species i within the biofilm (mg/lt)

X_T = Sum of all particulate species concentration within the biofilm, X_i , (mg/lt)

N_{xi} = Flux of particulate species; within biofilm (gr/m². day)

t = time (day)

L = Depth coordinate into biofilm ($x=0$ at surface), (m)

L_x = Thickness of biofilm ($x= L_x$ at support), (m)

r_{xi} = Rate expression of particulate species i within biofilm (gr/m². day)

2.2.7 Elements of a Biofilm System Model

In general, mechanistic mathematical models of treatment systems are based on non steady-state balances for all dissolved and particulate (biomass and inert material) substrates that are of interest. A mass balance allows to predict the state-of-art of a system as functions of transport and reaction processes. The dominant transport process of different components is extremely specific for the location of a reactor, reaction processes, on the other hand, depend only on the environment and may be described with one common model for the entire system.

Modelling of fixed film systems depend on the activity of biofilms and must include two levels of modelling (Gujer and Boller, 1990):

- Microscopic level, where transport and reaction processes within the biofilm are considered. The predictions at this level allow to characterize the exchange of material between the biofilm and the bulk liquid, information which is required at the macroscopic level.

So far, literature review given in this study relates to microscopic level where only the concept of biofilm is described in detail.

- Macroscopic level, where the system as a whole is modelled. At this level, loading criteria, mixing conditions, exchange of substrates and biomass within the biofilm and similar other operational conditions interact.

The next section will emphasize briefly on significant loading criteria of fixed-film system; the information necessary to develop a kinetic model describing the microbiological reactions as they are influenced by the environment.

Gujer and Boller (1990) developed a kinetic model of a fixed-film system that is similar to IAWPRC (1987) Task Group Study on Activated Sludge Systems and this Study utilizes the similar approaches of this study and follow up the mathematical model developed by the two scientists.

The short description of the Task Group Model on activated sludge system and its adaptation to fixed-film systems take place in the following part.

In short, macroscopic modelling will be based on complete mixing and on advection mechanism, whereas, microscopic modelling will consider molecular diffusion for soluble substrates and displacement for particulate substrates.

2.3. AN OVERVIEW OF THE SIGNIFICANT LOADING CRITERIA IN AEROBIC FIXED FILM SYSTEMS

Modelling at microscopic level includes only kinetic information that characterizes the behaviour of particulate substrates and diffusion of soluble substrates in a fixed-film system; in other words, involves sufficient information required during macroscopic modelling. At macroscopic modelling, where the system whole is defined as a whole necessitates information on certain major factors that control the operation and performance of a system. Thus, at this

level, such a model should include stoichiometric information together with kinetic information.

The major factors controlling the operation and performance of a fixed-film system are known to be,

- i) Flow Rate or Surface hydraulic loading
- ii) Influent Wastewater Concentration
- iii) Organic Loading
- iv) Specific control parameters of a chosen system such as rotational speed, effect of disc immersion depth, disc surface area if the reactor under consideration is RBC (Rotational Biological Contactor)
- v) Oxygen Concentration

Out of these factors, (iv) will not be included in this study as modelling of a fixed-film system is not carried out on a specifically chosen reactor type. Factor (v) will also be excluded in this study as the modelling is based on a single substrate limitation.

Figure 2-21. presents the most common biofilm reactors employed in wastewater treatment just to give an idea about the types of reactors.

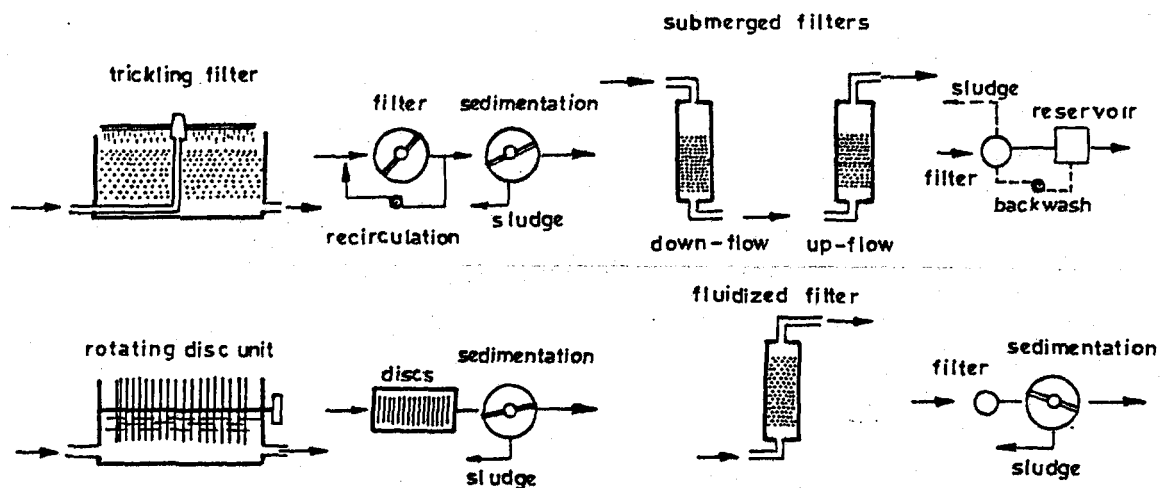


Figure 2-21. Biofilm Reactors Employed in Wastewater Treatment

2.3.1. Flow Rate and Surface Hydraulic Loading

The performance of any reactor type in fixed-film systems is highly dependent on the flow rate, Q , and on surface hydraulic loading, H_o ($H_o=Q/A$).

Extensive data on flow rate and surface hydraulic loading of rotating disc systems are present in literature. Gönenc (1982) and Baykal (1986) summarize the findings of many experimental studies conducted on disc systems, some of which are given below;

- Hartmann (1960) stated that increasing flow rate decreased the detention time of the wastewater in the rotating disc system and consequently the efficiency of the system was decreased.

- Pöpel (1964) reported that increasing the submerged disc area increased substrate utilization.

- Chittenden et.al. (1971) also used the disc system to treat anaerobic lagoon effluents. At a hydraulic loading of $0.16 \text{ m}^3/\text{m}^2.\text{day}$ produced a 79.5 % BOD removal. In other words, higher hydraulic loading caused decrease in efficiency.

- Antonie (1972) concluded that the primary criterion for domestic wastewater treatment is hydraulic loading. He showed BOD removal as high as 96 % achieved at a hydraulic loading of approximately $0.04 \text{ m}^3/\text{m}^2.\text{day}$.

- Antonie and Hynek (1973) indicated the possibility to achieve more than 90 % BOD or COD removal for treating various food and nonfood processing wastewaters of a hydraulic loading of $0.04 \text{ m}^3/\text{m}^2.\text{day}$.

- Clark et.al. (1978) recommended hydraulic loading as $0.061 \text{ m}^3/\text{m}^2.\text{day}$ for domestic wastewater using four stages rotating disc systems.

- Famularo et.al. (1978) illustrated the removal of BOD per unit area as related to hydraulic loading and as well as to applied BOD.

Many researchers have reported that increasing disc surface area or decreasing surface hydraulic loading increases substrate utilization. This increase is mainly attributed to longer detention times and to the fact that the amount of active biomass on the disc surface increases relative to the increase of substrate loading.

Table 2-5. Design Parameters of RBC systems obtained from literature (Baykal, 1986)

Type of Wastewater	Hydraulic loading	Organic loading	Removal Rate N_{ss}	
	$m^3/m^2 \cdot day$	$grBOD/m^2 \cdot day$	$grBOD/m^2 \cdot day$	Reference
Synthetic waste prepared from apple juice	0.0061 0.009 0.012 0.015		5.54 8.17 10.56 13.20	5.3 7.9 10.3 12.9
Domestic wastewater	0.027		5.67	4.8
Domestic wastewater/	0.044 0.129 0.076 0.196 0.072 0.065 0.065		2.97 8.22 2.44 17.26 3.88 4.10 2.81	2.84 4.76 2.06 12.95 3.6 3.5 2.2
Wastewater of a fish processing industry	0.0116 0.056 0.028 0.020 0.014		13.83 8.46 11.20 10.10 4.13	8.37 7.05 11.06 9.92 4.11
Domestic Wastewater	0.074		9.18	8.5

Corneill et.al.
(1975)
Sack et.al.
(1973)
Clark et.al.
(1978)
Hudson et.al.
(1976)
Torpey et.al.
(1971)

Wastewater of a	0.020		12.44		11.75	Johnson and
Meat Processing	0.014		8.12		7.92	Krill (1976)
Industry after	0.061		42.77		35.9	
Primary	0.122		87.84		56.12	
Treatment						
Synthetic	0.061		2.14		1.9	Mueller et.al.
domestic	0.036		1.45		1.34	(1978)
wastewater	0.042		2.50		2.27	
	0.050		2.02		1.8	
	0.038		1.80		1.6	
	0.055		2.09		1.87	
Synthetic waste	0.042		10		8.1	Friedman et.al.
prepared from	0.037		16.5		14.6	(1976)
sucrose	0.039		32.4		25.4	
	0.077		18.1		16.8	
	0.179		43.86		38.5	
Domestic						
Wastewater						
primarily						
treated under						
anaerobic						
conditions	0.181		39.8		12.7	Pretorius (1971)
Synthetic waste	0.0204		10.2		9.7	Stover et.al.
prepared from	0.0204		20.4		18.9	(1976)
sucrose	0.0204		40.8		35.5	
Wastewater	0.0204		28.8		22.1	Stover et.al.
of a Slaughter	0.0204		46.1		19.4	(1976)
House	0.0204		92.0		16.6	

Wastewater of a	0.116		22.4		13.25	Hudson et.al.
Meat Processing	0.056		14.2		11.98	(1976)
Industry after	0.028		16.88		15.51	
Primary	0.020		17.34		15.62	
Treatment	0.014		7.28		6.38	
Domestic	0.0737		22.11		17.61	Torpey et.al.
Wastewater						(1971)
Domestic	0.027		11.2		8.0	Sack et.al.
Wastewater						(1973)

2.3.2. Influent Wastewater Concentration

The performance of a fixed-film biological system depends mainly on the activity of biomass growing on the surface of biofilm media. That is to say, an appropriate detention time is required for high efficiency. High hydraulic loading combined with low organic concentrations in the wastewater can be treated satisfactorily as long as the hydraulic detention time is longer than 50 to 60 minutes. Conversely, a highly concentrated waste at a very low hydraulic load still requires 50 to 60 minutes for satisfactory treatment. As referred by Gönenç (1982), Poon (1979) stated that beyond 50 to 60 minutes range, organic removal is insignificant. For these reasons, hydraulic loading alone does not determine the efficiency of a system and should be combined with influent wastewater concentration and organic loading when operated under various circumstances and with different types of wastewater (Gönenç, 1982).

Various scientists referred to contradictory observations according to the results of the pilot-scale investigations. Some of these findings are referred by Göneng (1982):

- Weng (1972) investigated that the BOD remaining in the effluent was directly proportional to the influent BOD concentration, flow rate and surface area.

$$S_T = f(S_o, Q, A)$$

where,

$S_T = S_E^* + S_R^*$ = Total effluent wastewater concentration (mg/lt)

S_E^* = Effluent soluble substrate concentration (mg/lt)

S_R^* = Soluble residual product of microbial origin (mg/lt)

S_o = Influent wastewater concentration (mg/lt)

Q = Flow rate of wastewater (m³/day)

A = Total surface area of the system (e.g. disc system) (m²)

- Wu et.al. (1980) showed that the BOD remaining in the effluent was directly proportional to the hydraulic loading and was inversely proportional to the influent concentration.

$$S_T = S_E^* + S_R^* = f(Ho, \frac{1}{S_o})$$

- Murphy and Wilson (1978) concluded that the organic loading (L_o) with effluent BOD concentration are closely related to each other and no other operational criterion had a large effect on BOD removal.

2.3.3. Organic Loading

The importance of organic loading in design of a fixed-film system and in evaluating the efficiency has recently been overlooked.

It can be concluded that organic loading (L_o) combines both with hydraulic loading and influent concentration and all the three variables have equal importance during design of a system. Table 2.6 summarizes the typical ranges of organic loading depicted in literature.

Table 2-6. Typical Ranges of Organic Loading Values
Depicted in Literature for RBC Systems
(Gönenç, 1982)

Wastewater	Organic Loading (Lo)	% Removal	Reference
Synthetic domestic wastewater	6.4 kgCOD/m ³ .day	80	Antonie and Welch (1969)
Whey and combined wastewater	48 kgCOD/m ³ .day	50	"
Wastewater originating from oil and petroleum industry	10-20 grCOD/m ² .day	82-95	Bringman and Kuehn (1968)
Domestic wastewater	≤6.4 kgCOD/m ³ .day	≥90	Stover and Kincannon (1976)
Wastewater of a slaughterhouse	1.6 kgCOD/m ³ .day 6.4 kgCOD/m ³ .day	70 15	" "
Domestic wastewater	36.7 grCOD/m ² .day 19.1 grCOD/m ³ .day	Maximum Removal 52	Famularo et.al. (1978)
Domestic wastewater	18.4-26.6 grCOD/m ² .day	Maximum Removal Achieved	Poon et.al. (1979)
Industrial wastewater	8.7-18.9 grCOD/m ² .day	99-91	Padini et.al. (1979)

2.4. SUMMARY AND EVALUATION OF LITERATURE AS IT RELATES TO THE PRESENT STUDY

Having reviewed the main characteristics of a number of biofilm models, one question remains: How good are they? When used in the simulation of various fixed-film processes, all do a reasonable job for at least limited situations for which they were derived.

Now it is certain that intrinsic Monod kinetics is the only form of differential equation that describes the concept of transport and diffusion of soluble substrates in a fixed-film process.

One of the important links between the biofilm model and the process model is the biofilm thickness as it is an important determinant of concentration profiles which develop within the biofilm. Unfortunately, it is still unclear what controls the thickness.

Rittmann and McCarty have presented the concept of a steady-state biofilm in which cell growth is just balanced by cell decay and this appeared to be a useful concept for further improvement of biofilm phenomena. It is then that the removal of particulate components and the distribution of microorganisms and their variable activeness within the biofilm has been realized. Although, surface reactions such as sloughing, flocculation and shear loss are well known concepts in biofilm phenomena, related mechanisms are still unclear. Thus, the recent models consider constant biofilm thickness due to an equilibrium of biomass net

production and sloughing (shear loss) at the surface. Almost all biofilm models assume steady-state conditions.

The variation of biofilm density with thickness has also been discussed frequently by researchers with regard to its importance during modelling. This fact is also unclear due to the difficulty of experimentation. It is therefore assumed by scientists that to work under constant density condition would simplify the models to a great extent.

It has recently been recognized by Gujer and Boller (1990) that the Task Group Study, IAWPRC, (1987) on Activated Sludge Modelling can also be adapted in a similar way to fixed-film systems in the sense that both systems either attached growth or suspended growth, bear the same reaction rates.

For the attainment of developing a more advanced mechanistic model for a fixed-film system, this study uses the similar approach of the Task Group Study together with its modified form developed by Artan (1988) and further attempts to put forth a kinetic model leading to the generation mechanism of soluble residual product (SRP) in a fixed-film system. It is important to note that such a model involving both the removal mechanism of soluble and particulate substrates due to the behaviour of microorganisms (particulate substrates) within the biofilm as well as the generation of residual soluble substrate at the film outer interface has not been developed yet.

This study puts forth the kinetic model at microscopic level and enriches it with operational

parameters at macroscopic level just to give more insight to the designer in charge of fixed-film systems. Similar study has been conducted by Artan (1988) on suspended growth systems.

In short, it is clear that without analyzing the behaviour of particulate components within the biofilm (X_A , X_B and X_E), one cannot identify or find out the generation mechanism of SRP (from hereon called S_R).

So far, it is certain that a complete and a general mechanistic model for biofilms to simulate the performance of a wide range of fixed-film processes has not been achieved yet. The important fact that should be kept in mind is that further studies and researches are needed in this field to fill in the remaining gaps of knowledge of the fixed-film theory.

The brief description of the Task Group Model and its adaptation to biofilm kinetics will be given in a separate part in the sense that it forms the theoretical background of the model that is developed in this study.

PART III

THEORETICAL BACKGROUND OF THE MODEL

The new developments in fixed-film systems has now gained a renewed interest in their modelling. There are two important reasons of this interest. One is that, models are no doubt the basic tools of engineering as they facilitate the design process. The other is that, models aim to achieve better understanding of the complex reactions that can or even cannot be experimentally verified. Modelling and experimentation are interdependent where one provides input, the other provides or correlates the output. For cases in which experimentation is difficult or impossible to handle as in the case of this study, where the generation mechanism of soluble residual product, S_R , is to be analysed, the only way to understand the mechanism is to model it out by the aid of certain assumptions.

Mathematical models may be divided into two categories: empirical and mechanistic. Empirical models simply relate input and output operating variables to each other and make pretense of representing the phenomena under observation. Such "black box" descriptions are quite useful for design purposes and in the field of environmental engineering, many of the models describing treatment systems, either activated sludge or fixed film, are based on empirical expressions.

On the other hand, mechanistic models, express the effect and relationship of mechanistic phenomena in such a way that allows the researcher to find out how

the system might respond under unexplored conditions. Thus, it can be stated that, the basic purpose of mechanistic models is to further understand the concepts that have not been made clear so far. This additional understanding will be an advantage to the designer as it is the nature of application to use knowledge in areas where no prior experience exists.

Mechanical models of biochemical processes generally are developed by application of reactor engineering principles. They combine the expression representing the intrinsic kinetic and transport phenomena together with mass balance equations describing the characteristics of a particular physical system under consideration. Consequently, simulation with such models aim to give insight to the basic events occurring within a process. The flow diagram of a mechanistic model for a fixed-film system under consideration is illustrated in Figure 3.1.

As a conclusion, it can be stated that, mechanistic models have wider utility than empirical ones and the basic attempt in this study is to develop a mechanistic model that fulfills the objectives mentioned in the first part of the study.

As mentioned previously, the developed model is similar to the one developed by the Task Group for Activated Sludge System, in the sense that, the reaction kinetics for both suspended growth and attached growth systems are identical. A brief description of this study will be given in the following section as it emphasizes on differentiation between soluble and particulate substrates, incorporates the viability concept including inert residual soluble substrate generation in suspended-growth systems and same concepts are adapted to attached-growth systems.

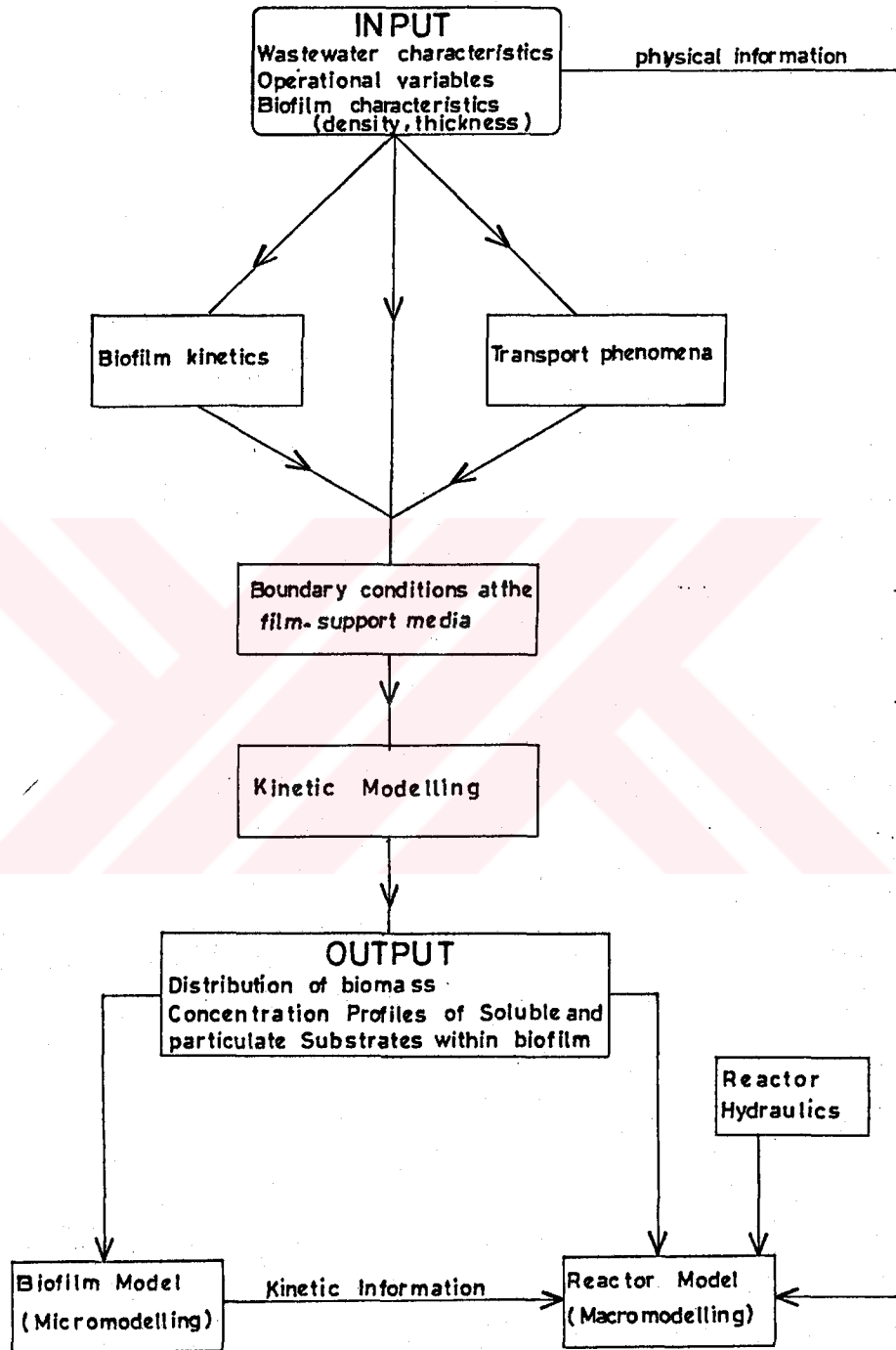


Figure 3-1. Flow Diagram of the Model Under Consideration illustrating the Biofilm Model together with Reactor Model

3.1. REACTION KINETICS

3.1.1 The Task Group Model

In 1983, IAWPRC (International Association on Water Pollution Research and Control) brought together a group of experts under the name of Task Group to review the work done on activated sludge systems and to develop a simplified model solvable by related engineers and scientists working in this field. They have chosen a matrix format for the presentation of their model in a more compact manner. The matrix presentation is assumed to be useful tool in giving the kinetic and stoichiometric characteristics of the system under consideration. The reaction matrix is also used and illustrated by Artan, (1988) and Baykal, (1988). A similar matrix for fixed-film systems is also developed.

In the Task Group model, all soluble components are symbolized by S, and all particulate components are shown by X with various subscripts as a general rule.

Organic matter in the wastewater are classified as biodegradable / nonbiodegradable and soluble /particulate. Biodegradable organics are further categorized as readily biodegradable and slowly biodegradable. Readily biodegradable substrate, consists of simple molecules that can be directly utilized by the biomass. On the other hand, slowly biodegradable substrate consists of complex molecules that have to be converted extracellularly into readily biodegradable form upon hydrolysis, before it can be indirectly used up by the microorganisms. As a simplification, all readily biodegradable substrate is assumed to be soluble and

denoted by the symbol, S_s , and all slowly biodegradable substrate is considered to be particulate and denoted by the symbol, X_s .

Within the context of this work, the heterotrophic removal of COD is concerned thus, all the components given will be in the COD units as COD can form a link between the biomass and organic matter. Soluble COD consists of the inert soluble COD entering the system, S_i , and readily biodegradable substrate S_s in the influent. Particulate COD is the sum of inert particulate matter in the influent, X_i , slowly biodegradable substrate, X_s , active heterotrophic biomass, X_A , and particulate products from biomass decay, X_E . (Figure 3.2). As in the fixed-film model, only heterotrophic microorganisms are under consideration, definition of autotrophic bacteria is not given here. Also, oxygen is included as well as various forms of soluble and particulate nitrogen in the Task Group Model, but here emphasis will only be given to components that is of interest in this study.

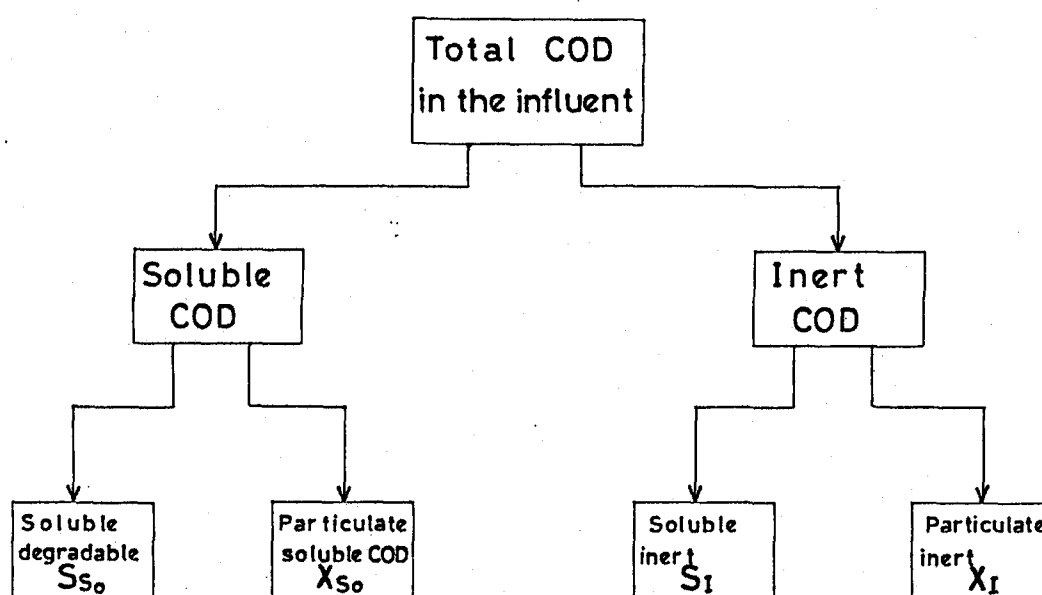


Figure 3-2. The fractions of Total COD within Influent Wastewater

The major processes involved in the Task Group Model are;

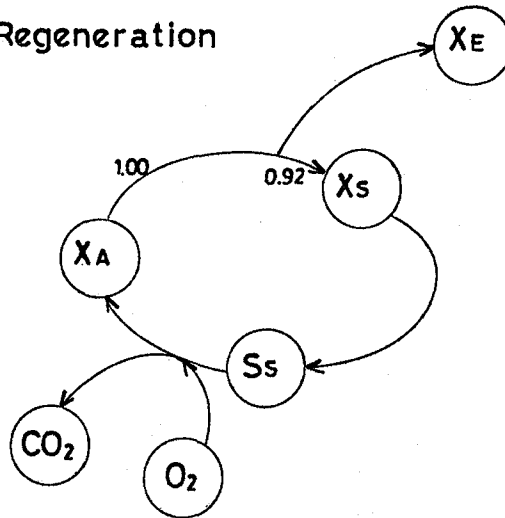
- growth,
- decay, and
- hydrolysis of entrapped organics.

The active biomass increases by growth and decreases by decay. In the model, the death/regeneration approach is adopted. Figure 3.3. illustrates the death/regeneration cycle and decay process of heterotrophic microorganisms as given by Baykal (1988).

The readily biodegradable substrate is used in the growth process to yield active biomass, which in turn undergoes decay to built up slowly biodegradable substrate and inert particulate products. Slowly biodegradable substrate is subsequently converted to the readily biodegradable form by the hydrolysis process and the cycle starts again. The only process that uses oxygen in the cycle is growth. On other hand, no oxygen is used by the decay process directly. However, the portion of S_E that is generated by hydrolysis does consume oxygen, hence there is an indirect consumption of oxygen by the end product of decay, i.e. S_E . In the Task Group approach, S_E is the only substrate utilizable by the biomass and the same approach is used in this study.

The Task Group Model make use of certain switching functions such as oxygen just to activate or deactivate the processes affected by the electron acceptor. For example, the oxygen as a switching function will be zero as its concentration leads to zero, thus aerobic conditions will no longer prevail. Hence, aerobic

Death / Regeneration



Decay

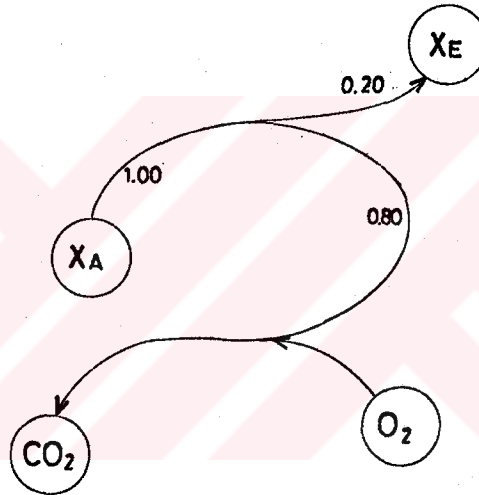


Figure 3-3. The Death / Regeneration Cycle Adopted by the Task Group and the Conventional Decay Process (Baykal, 1988).

heterotrophic growth rate will be zero and the whole process will cease. This is not the case in this study as, oxygen, as a second soluble substrate, is not included in the model.

Degradation of slowly biodegradable substrate is important in the modelling of both activated sludge and fixed-film systems as the inert soluble residual substrate generation is highly due to the degradation of slowly biodegradable substrate controlled by decay-associated mechanisms. This degradation process needs knowledge of the mechanism of particulate organic matter as briefly mentioned in section 2.2.6. whose importance has been realized a few years ago.

3.1.2. The Modified Task Group Model

Artan (1988) has made two changes in Task Group Model. First, she assumed all soluble substrate except the inert fraction S_I to be readily degradable. S_I , both at the influent and effluent bear the same concentration as it passes through the system biochemically unaltered. The second modification is a more significant one as it is claimed that the soluble residual inert product, S_R , is generated as a by-product of the system processes. Two alternatives for soluble inert matter production were considered:

- generation of S_R as a result of microbial metabolism and growth,
- generation of S_R during hydrolysis process.

Components of total particulate and soluble substrates in the effluent (bulk-liquid) are given in Figure 3.4.

As a conclusion, the modified model contains eight components and three major processes which are presented schematically in Figure 3.5. Broken lines indicate the alternative inert product generation due to growth.

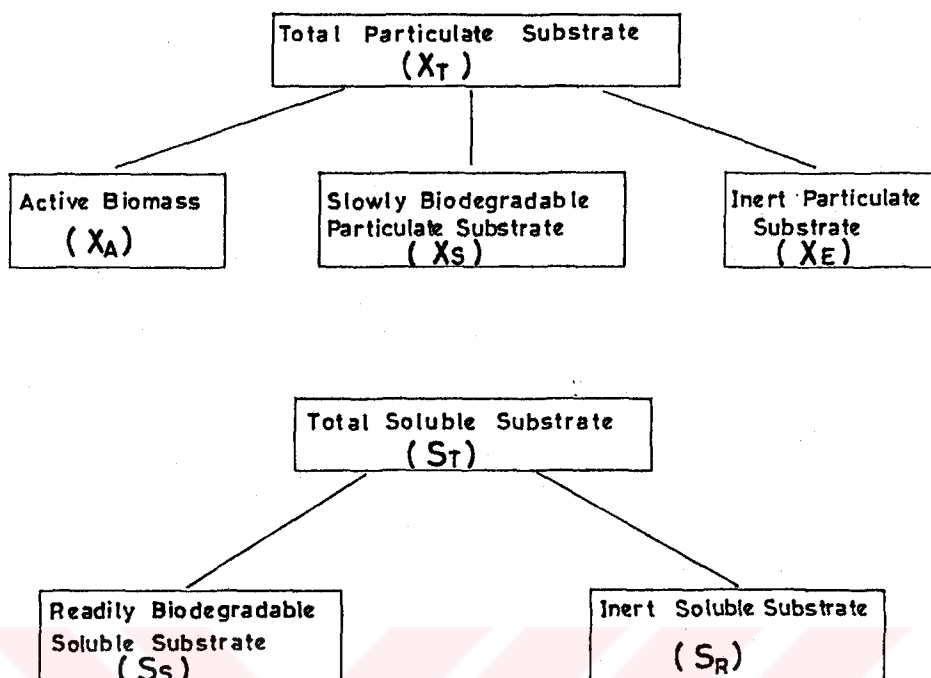


Figure 3-4. Fractions of Total Particulate and Soluble Substrates

Both alternatives of inert residual product generation has been recently analysed by many scientists either theoretically or experimentally as referred by Orhon et.al. (1989). It can be concluded that growth associated mechanism may be limited by the prensence of low concentration of S_E in the bulk liquid, thus the decay-associated mechanism predominates. Both mechanisms leading to SRP formation may be included in the model, but experimental studies done so far, indicate the mutual corespondence of both mechanism rates and there is enough experimental information that the coefficient β which indicates the portion of S_R to be generated has a value 0.1, assigning the predominance of the decay-associated mechanism. On the other hand, considering both mechanisms in the model lead to further simplification of terms that have negligible practical

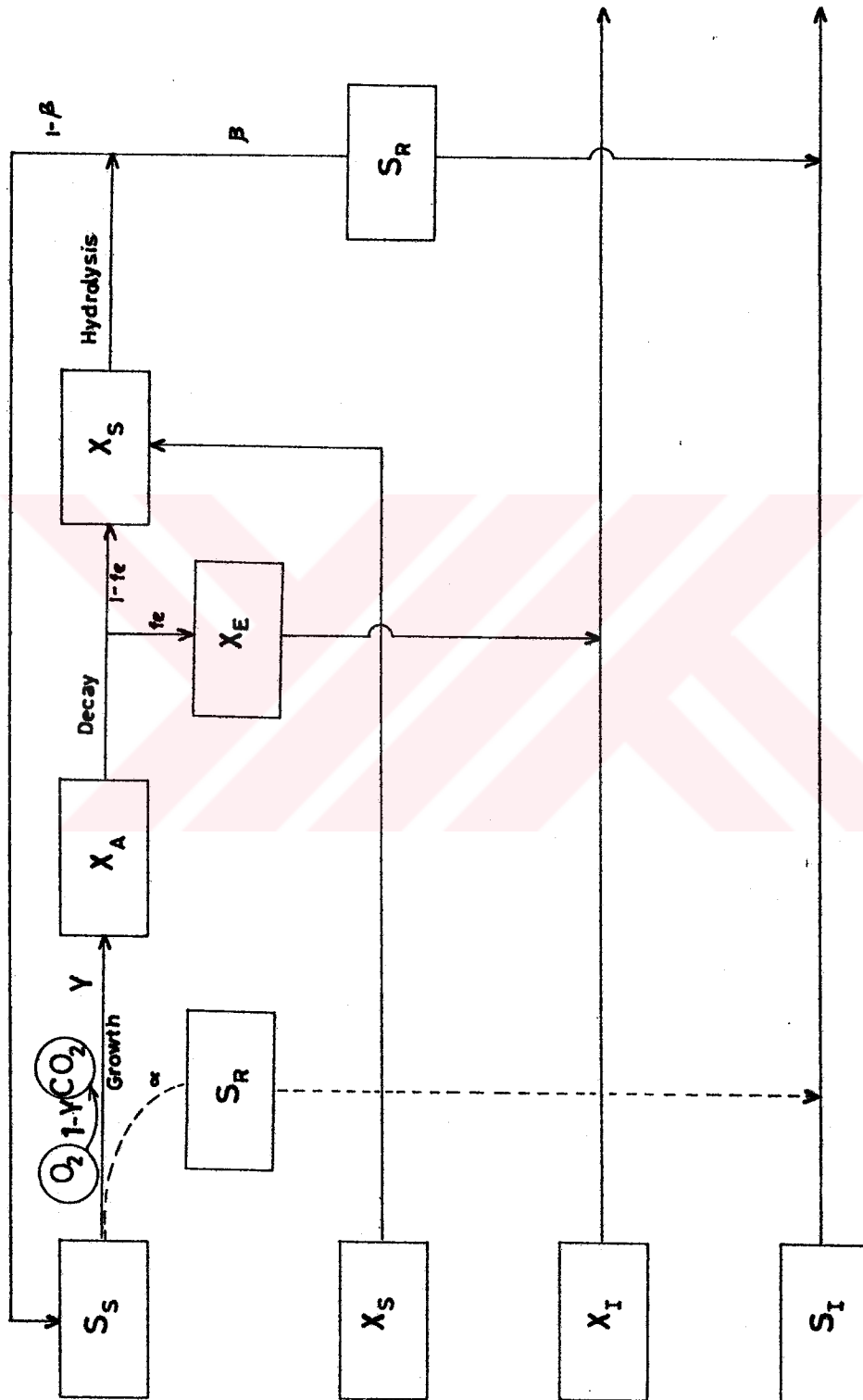


Figure 3-5. The Process Scheme for the Soluble Inert Product Generation

significance and this orientation puts forth the model where SRP generation is due to decay-associated mechanism. In other words, this formation occurs through hydrolysis of X_s . This statement is also maintained in the modelling of fixed-film systems.

3.2. THE PROCESS REACTION RATES WITH RESPECT TO MODIFIED TASK GROUP MODEL

The process reaction rates of any biological treatment system due to the modified Task Group Model may be given in a matrix form. A matrix presentation is a condensed method of presenting the model fundamentals and enables one to convey maximum amount of information in a simple way, including processes, components and the interactions between them. Such a reaction matrix is given in Table 3.1.

3.2.1. The Method of Matrix Representation

The reaction matrix for fixed-film systems is quite similar to the matrix for activated sludge system, in the sense that both systems bear the identical reaction kinetics.

The model components and processes have been described in detail in the previous part. In this section, the reaction matrix is denoted without emphasizing on components and processes.

The matrix presentation is quite is helpful during writing mass balance equations as the reaction terms in the mass balances can directly be written by summing the products of the stoichiometric

Table 3-1. The Reaction Matrix of the Model

Component		i	1	2	3	4	5	Process Rate
J	Process		S_s	S_R	X_A	X_S	X_E	ρ_j
1	Aerobic Growth of Heterotrophs		-1 $\frac{1}{Y}$		1			$\mu \frac{S_s}{K_s + S_s} X_A$
2	Death of Heterotrophs (Loss of viability)				-1	$1 - f_e$	f_e	$b_H X_A$
3	Hydrolysis of Heterotrophs (Loss of mass)		$1 - \beta$	β		-1		$k_h \frac{X_s/X_A}{k_x + X_s/X_A} X_A$
	Stoichiometric Parameters Y, f_e, β		Readily Biodegradable Substrate	Soluble Inert Organics	Active Heterotrophic Biomass	Slowly Biodegradable Organics	Particulate Inert Organics	Kinetic Parameters μ, K_s, b_H, k_h, k_x

$$r_i = \sum_j \gamma_{ij} \cdot \rho_j$$

coefficients μ_{ij} and the process rate expression r_i for each component i . The reaction terms can be written as follows from the matrix representation;

Readily Degradable Substrate

$$\left[\frac{dS_S}{dt} \right]_r = - \frac{\mu}{Y} \frac{S_S}{K_S + S_S} X_A + (1-\beta)k_h \frac{X_S/X_A}{K_X + X_S/X_A} X_A \quad (3.1)$$

Viable Biomass

$$\frac{dX_A}{dt} = \frac{\mu}{Y} \frac{S_S}{K_S + S_S} X_A - b_H X_A \quad (3.2)$$

Slowly Degradable Biomass

$$\frac{dX_S}{dt} = (1-f_e) b_H X_A - k_h \frac{X_S/X_A}{K_X + (X_S/X_A)} X_A \quad (3.3)$$

Inert Soluble Residual Product

$$\left[\frac{dS_R}{dt} \right]_r = \beta k_h X_S \quad (3.4)$$

Inert Particulate Organic Matter

$$\frac{dX_E}{dt} = f_e b_H X_A \quad (3.5)$$

For the practical utility of the Task Group Model, the model parameters (both kinetic and stoichiometric) have to be evaluated. A list of these parameters related to the model developed in this study are presented in the following table, where, only the heterotrophic microorganisms are considered to be present within the biofilm media.

Table 3-2. Stoichiometric and Kinetic Parameters in the TASK GROUP MODEL

<u>Stoichiometric Parameters</u>	
Y	=Yield of heterotrophic biomass
f_p	=Fraction of biomass leading to particulate products
β	=Fraction of soluble inert products
<u>Kinetic Parameters</u>	
$\hat{\mu}$	=Maximum specific growth rate for heterotrophs
K_s	=Half-saturation coefficient for heterotrophs
b_h	=Decay coefficient for heterotrophic biomass
k_h	=Maximum specific hydrolysis rate
K_x	=Half-saturation coefficient for hydrolysis of slowly biodegradable substrate

Typical values of those parameters suggested by the Task Group, especially for municipal wastewater are given in Table 3.3.

By combining the reaction rate expressions with suitable input/output terms, mass balance of each component can be written. In case of fixed-film theory, the way followed up to form the model equations are given in detail in the next part. However, the reaction rate equations (eqn 3.1 to eqn. 3.5) form the basis of the governing mechanisms taking place in the fixed-film systems.

Table 3-3. Typical Values of Parameters Given by the
TASK GROUP

Symbol	Unit	Value of 20°C	Value at 10°C
<u>Stoichiometric parameters</u>			
Y	<u>gr cell COD formed</u> gr cell COD oxidized	0.67	0.67
f _e	dimensionless	0.08	0.08
<u>kinetic parameters</u>			
μ	day ⁻¹	6.0	3.0
K _s	grCOD/m ³	20	20
b _H	day ⁻¹	0.62	0.20
k _H	<u>gr slowly biodegradable COD</u> gr cell COD. day	3.0	1.0
k _x	<u>gr slowly biodegradable COD</u> gr cell COD	0.03	0.01

Before getting on more involved with the mass balance and model equations, emphasis will be given to the values or even to the ranges of both the kinetic and stoichiometric model parameters that will be used in the model formation.

3.2.2. Typical Values of Parameters to be used in the Reaction Kinetics of Fixed-film Systems

In order to better evaluate a biofilm model, the related stoichiometric and kinetic parameters should be determined. So far, in section 2.1.3. the important parameters such as biofilm thickness, L_x , and biofilm density, X_T , have been mentioned in detail. In section 2.1.4.1 the diffusion coefficients of many compounds have been listed as well. In this section, emphasis will be given to all other parameters that is of importance in the developed model.

3.2.2.1 The Maximum Specific Growth Rate and The Half-Saturation Coefficient

It is assumed that the Monod maximum specific growth rate coefficient, $\hat{\mu}$, for a biofilm does not differ from its value for attached-growth systems (Williamson and Mc Carty, 1976 a). The mass transfer effects induced by the biological matrix should not change a reaction rate that is not diffusion limited. The coefficient, $\hat{\mu}$, has units of mass of substrate consumed per unit mass of microorganisms per day. Typical values range from 0 to 10 mg of substrate / day / mg of microorganisms. Experimental measurements under given conditions are required for exact determinations because environmental factors such as temperature, pH, and mean cell age can radically modify the value of $\hat{\mu}$.

On the other hand, the half saturation coefficient, K_s , can vary from 1 mg/lt up to approximately 1000 mg/lt depending on bacterial species,

substrate and on environmental conditions for aerobic fixed-film systems. Both coefficients can be determined experimentally for specific bacteria using chemostats at various dilution rates (Namkung et. al., 1983).

The $\hat{\mu}$ and K_s parameters are interrelated as given in the following equation (Namkung et. al., 1983):

$$\frac{1}{D} = \frac{K_s}{\hat{\mu}} \cdot \frac{1}{S_s} + \frac{1}{\hat{\mu}} \quad (3.6)$$

where D is the dilution rate = flow rate / reactor volume (day^{-1})

Table 3.4 indicates the experimentally determined values of these two parameters in fixed-film systems.

Table 3-4. The Experimentally Determined Values of K_s and $\hat{\mu}$ in Fixed-film Systems

Type of compound used	$\hat{\mu}$ (day^{-1})	K_s (mg/lt)	Reference
Domestic wastewater	6.72	121	Kornegay and Andrews (1967)
Domestic wastewater Biological oxidation		10-120	
Methanol		15	Rittmann and Mc Carty (1980a)
Acetate COD		930	
Propionate COD		1070	
Aerobic Organic Oxidation COD		22	
Domestic wastewater		32-431	Clark et.al. (1978)
Domestic wastewater		180	Hansford et.al (1978)
Carbon removal COD	1.87 at 5.8°C 6.20 at 16.3°C 5.95 at 20°C	61.6 at 5.8°C 262.2 at 16.3°C 276.4 at 20.8°C	Pano and Middlebrook (1983)
Carbon and Nitrogen removal (COD)	2	10	Gujer (1990)

3.2.2.2. The Yield Coefficient

The yield coefficient, Y , can also be determined experimentally in batch type reactors that initially contain 1 gr/lit of one substrate and the related biofilm bacteria. The yield coefficient is calculated from

$$Y = \frac{\Delta X_A}{\Delta S_S} \quad (3.7)$$

where

ΔX_A = increase in active biomass concentration at the time when the change in soluble substrate concentration is ΔS_S .

Table 3.5. illustrates the experimentally determined values of this parameter; depicted in literature.

3.2.2.3. The Decay Coefficient

The decay coefficient, b_H , can be measured by using cells grown in the batch reactor for determining Y . This coefficient is estimated by using the following equation:

$$b_H = \frac{\ln (X_A^*/X_A)}{t} \quad (3.8)$$

where X_A^* = the initial concentration of active biomass, (mg/lit)

X_A = the concentrariton of active biomass at time t , (mg/lit)

t = time, day

Table 3-5. The Experimentally Determined Values of the Yield Coefficient in Fixed-film Systems

Type of compound used	Yield coefficient (gr VSS/gr COD)	Reference
Biological Oxidation	0.37 -0.79 mg VSS/MgNO ₃ -N	Sharma (1977)
Domestic Wastewater Biological oxidation	0.26 gr VSS/gr.glucose	Kornegay and Andrews (1968)
Domestic Wastewater Biological Oxidation	0.69 gr VSS/gr.Carbon	Namkung and Rittmann (1987)
Denitrification by Biological oxidation	0.20 gr VSS/gr.methanol	Rittmann and Mc Carty (1980 b)
Biological oxidation		
Acetate COD	0.05 gr VSS/gr.acetate	"
Propionate COD	0.051 gr VSS/gr. propionate	"
Aerobic Organic oxidation	0.2 gr VSS/gr.glucose	"
Domestic Wastewater Biological oxidation	0.418 gr VSS/gr.phenol 0.421 gr VSS/gr.gracetate	Namkung and Rittmann (1987)
Organic Oxidation	0.54 gr VSS/gr.galactose	Rittmann and Brunner (1983)
Carbon Removal	0.66 at 5.8°C 0.8 at 16.3°C 0.68 at 20°C	Pano and Middlebrooks (1983)
Organic Oxidation	0.47 gr VSS/gr.glucose 0.86 gr VSS/gr.protein 0.98 gr VSS/gr. carbohydrate	Droste and Kennedy (1986)
Organic Oxidation	0.21-0.32 gr VSS/gr glucose	Kornegay and Andrews (1968)
Organic Oxidation	0.96 gr VSS/gr.glucose	Clark et. al. (1978)

As seen in equation (3.8) the decay coefficient, b_H , is one of the most objectionable parameter as it relates to the effect of loss of viability of the active biomass and depends indirectly on the hydrolysis of the slowly degradable particulate matter, X_E . The decay coefficient, is evaluated for X_T which is constant itself, but its components are variable with respect to increasing thickness of biofilm.

$$X_T = X_A + X_S + X_E \quad (3.9)$$

and where the term viability is equal to

$$V_{XA} = \frac{X_A}{X_A + X_S + X_E} = \frac{X_A}{X_T} \quad (3.10)$$

This parameter is also sensitive to temperature changes, the following equation indicate its temperature dependency;

$$(b_H)_T = (b_H)_{20} \cdot \sigma_d^{T-20} \quad (3.11)$$

where,

$(b_H)_T$ = decay coefficient at temperature T ,
day⁻¹

$(b_H)_{20}$ = decay coefficient at 20°C, day⁻¹

σ_d = temperature factor, and

T = temperature, °C.

By conducting similar chemostat experiments that is followed up to determine all the other kinetic parameters, the conventional values of different compounds are obtained by scientists. Table 3.6 summarizes the values determined experimentally.

Table 3-6. The Experimentally Determined Values of the Decay Coefficient in Fixed-film Systems

Type of compound used	Decay Coefficient day ⁻¹	Reference
Denitrification by biological Oxidation	0.05	Rittmann and Mc Carty (1980b)
Biological Oxidation acetate	0.011	"
propionate	0.04	"
Aerobic organic oxidation	0.20	"
Carbon Removal by biological oxidation	0.07 at 5.8 °C 0.22 at 16.3 °C 0.26 at 20.8 °C	Pano and Middelbrooks (1983)
Aerobic Organic Oxidation galactase	0.08	Rittmann and Brunner (1983)
Aerobic Organic oxidation	0.15	Namkung and Rittmann (1986)
Nitrification by biological Oxidation	0.35	Gujer and Boller (1990)

As can be realized from the literature review on b_H , the experimentally determined values are quite low compared to the b_H value used by the Task Group for the activated sludge system. The magnitude of this parameter used in the Task Group model is different from the usually encountered rate constant of a conventional system because the Model involves the recycling of

readily biodegradable S_s generated due to hydrolysis of X_s . In the usual technique, there is not such a recycling process. The loss of one unit of cell mass, S_s , leads to the utilization of one unit of oxygen leading to the formation of endogenous mass X_e . Thus, the amount of active biomass, X_A , is reduced. In the Task Group Model, the loss of one unit of cell mass, S_s , results in the final formation of one unit of S_s due to the hydrolysis process while a portion of cell mass undergo decay. When the readily biodegradable substrate is used for cell synthesis, only a fraction of a unit of oxygen will be required because of the energy incorporated into the cell mass. That cell mass must in turn undergo decay before the unit of oxygen is finally removed. In other words, due to the Task Group Model there is not a decrease in the amount of active biomass, but there is a decrease of its viability which in turn forms slowly degradable substrate and endogenous mass due to decay. Therefore, in order to give the same amount of oxygen utilization per time due to decay, b_H value is ought to be higher in the case of considering the Task Group Model where value of b_H is given as 0.62 d^{-1} at 20°C . In the usual technique, this value is around 0.24 d^{-1} . As far as the Task Group Model is adapted to attached-growth system, same high value will be used in the modelling.

3.2.2.4. The Other Parameters

According to the process flow diagram of the soluble inert product generation, decay of heterotrophic biomass occurs with a rate constant b_H and results in a fraction of the biomass, f_e , being transformed into endogenous mass. The value of f_e is not likely to vary

greatly from one wastewater to another as it is only a characteristic of the biomass. The Task Group proposes a value of 0.08 gr COD/gr COD for their model in which decay results in the recycling of substrate.

In the case of fixed-film theory, the same f_e value is used.

The maximum specific hydrolysis rate, k_h , and the half-saturation coefficient for hydrolysis of slowly biodegradable substrate, k_s , are relatively new parameters for which little information exists. Thus it is not possible to give the ranges within which the values are likely to lie. The values given by the Task Group for these two parameters are used in the modelling of SRP generation in aerobic fixed-film systems.

The last parameter, β , that is used in the modified Task Group Model (Artan 1988) in the modelling of SRP for activated-sludge systems is the key element of the SRP formation model. It indicates the ratio of soluble product that is generated as a result of hydrolysis. Artan (1988) calibrated and verified her model with a typical value of 0.1 but conducted her sensitivity analyses within a range of 0.05 - 0.3.

3.3. TYPICAL VALUES AND RANGES OF PARAMETERS USED IN THE MODELLING OF FIXED-FILM SYSTEMS

The typical values of both stoichiometric and kinetic parameters valid in fixed-film systems are listed below. These typical values are used in simulation of the model equations. The model calibration

with these typical values will be followed up by sensitivity analyses of certain parameters. All the parameters can more or less be determined by experiments and they vary within a range due to certain environmental conditions; such as pH, temperature effects etc. and on the kind of substrate and on bacteria species used. Thus, sensitivity analyses is followed up to see the effect of each parameter on the model components and on the effluent water quality and on SRP generation. Sensitivity refers to the effect of each parameter to the model results and it is of mathematical nature. The range of values used in sensitivity analyses are shown in Table 3.8.

Table 3-7. Typical Values of Parameters Used in the SRP Generation Model for Aerobic Fixed-Film Systems (for heterotropic microorganisms)

Parameter	Typical Values
μ	4.2 day ⁻¹
K_s	60 mg/l
Y	0.6
b_H	0.62 day ⁻¹
k_n	3 day ⁻¹
k_x	0.03 day ⁻¹
f_e	0.08
β	0.1
L_x	500 μ m
X_T	80 000 mg/l
D	31×10^{-6} m ² / day

Table 3.8. Ranges of Parameters Used in the Sensitivity Analyses

Paratemer		Typical Values
λ		2.0 - 6.0 day ⁻¹
K_s		20 - 250 mg/l
b_H		0.25 - 0.62 day ⁻¹
k_H		0.65 - 3 day ⁻¹
β		0.05 - 3

PART IV

THE MODEL

In general, the major processes and related appropriate expressions for both the soluble and particulate substrates constitute the most important part of modelling and have been put forth in the previous part.

The developed model for attached-growth systems under aerobic conditions bear close similarities with the inert residual soluble substrate generation model developed by Artan (1988). In reality, the most significant difference of that model from the Task Group Model is that hydrolysis process ends up by the generation of readily biodegradable substrate, S_s , and inert residual soluble substrate, S_R , in which β is a coefficient illustrating the portion of S_R generated in the model.

The model in these studies categorizes the substrate as soluble and particulate and classifies them as readily biodegradable and inert for the particulate components. It also makes use of the viability / active biomass concept and adopts the death / regeneration approach for decay.

4.1. THE STEPS OF MODELLING

The model in this work consists of two levels of modelling, one is the micro-modelling (biofilm model) and the other is the macro-modelling (reactor model).

Transport phenomena and reaction kinetics utilized together with appropriate boundary conditions at the film-support media yield the kinetic modelling of a specific biofilm media. The model will, then, be able to give a biofilm model where the biomass distribution and concentration gradients of both the soluble and particulate substrates are obtained as functions of biofilm thickness. Simply, the biofilm model will supply kinetic information necessary at macro level modelling.

The reactor model make use of the kinetic information of the biofilm model together with certain hydraulic properties of the wastewater and with certain physical properties of the reactor under consideration to solve the mass balance equations of soluble substrates on reactor basis. It is then that the objective of this study is fulfilled.

4.1.1. Micro-Modelling

The model is developed for steady-state conditions where only heterotrophic organisms are present. Surface reactions such as flocculation and shear has not been considered, thus biofilm thickness is kept constant due to an equilibrium of biomass net production and shear loss at the surface.

The model is derived in only one spatial direction. This simplification was made because changes in substrate concentrations per unit length are usually orders of magnitude smaller in other directions. In addition, all elements needed for discussion of the principles of microbial interaction are included in the one-dimensional model, thereby keeping the computational effort at a reasonable level.

The model deliberately makes an assumption that microbial kinetics applied to suspended growth systems also hold true for attached growth systems, (Gujer and Wanner, 1985; Gujer and Boller, 1990; Harremoes and Arvin, 1990). Therefore, the microbial parameters, both kinetic and stoichiometric, may change with time, thickness and with external environmental conditions except for overall biofilm density, X_T , which is assumed to be a constant property of the system.

The complexity of the system is mostly due to the variation of particulate substrates (X_A , X_S , X_E) throughout the biofilm media. That is why, there is a necessity to formulate the equations governing the distribution of microorganisms along the biofilm thickness, in other words, each particulate component must be defined as a function of biofilm thickness. Therefore, development of a biofilm model relies on the distribution of microbial species along a biofilm media.

Another important objective of the biofilm model is to obtain the fluxes of both soluble substrates (N_{SS} and N_{SR}) along thickness to be used in macro-modelling during the solution of mass balance equations of soluble substrates.

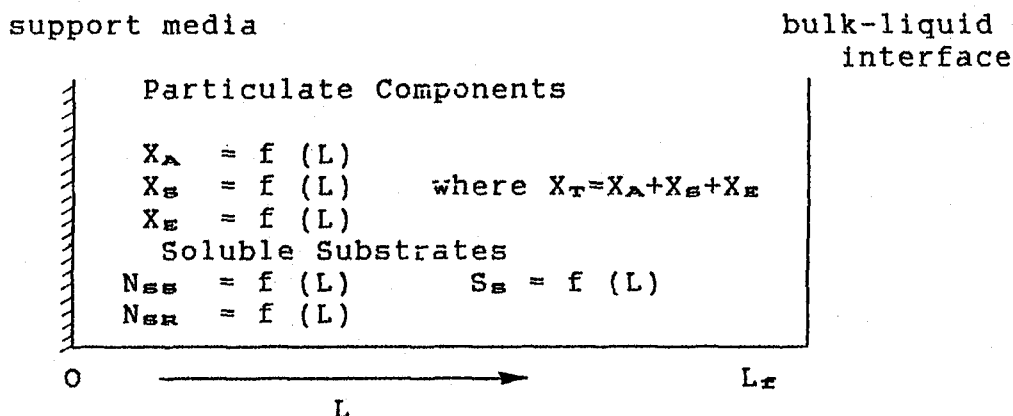


Figure 4.1. The Parameters to be Defined as Functions of Biofilm Thickness in Micro-Modelling

In developing the model, the governing process rate equations for soluble substrates has been obtained previously as,

$$r_{ss} = \left[\frac{dS_s}{dt} \right]_r = - \frac{\hat{\mu}}{Y} \cdot \frac{S_s}{K_s + S_s} X_A + (1-\beta) k_h \frac{X_s/X_A}{k_x + (X_s/X_A)} X_A \quad (3.1)$$

in part III, according to Monod kinetics as mentioned in part II. The reaction rate equation of molecular diffusion has also been derived in part II as;

$$\left[\frac{dS_s}{dt} \right]_{\text{diff.}} = \frac{D \, d^2 S_s}{dL^2} \quad (2.14)$$

Therefore; the total conversion (reaction+diffusion) rate expression of the soluble substrate,

$$\left[\frac{dS_s}{dt} \right]_{\text{total}},$$

for any point in the biofilm can be written as follows;

$$\left[\frac{dS_s}{dt} \right]_{\text{total}} = \frac{D \, d^2 S_s}{dL^2} - \frac{\hat{\mu}}{Y} \cdot \frac{S_s}{K_s + S_s} X_A + (1-\beta) k_h \frac{X_s/X_A}{k_x + (X_s/X_A)} X_A$$

Considering steady-state conditions; $\frac{dS_s}{dt} = 0$, the above equation is reduced to,

$$\frac{D \, d^2 S_s}{dL^2} = \frac{\hat{\mu}}{Y} \cdot \frac{S_s}{K_s + S_s} X_A - (1-\beta) k_h \frac{X_s/X_A}{k_x + (X_s/X_A)} X_A \quad (4.1)$$

In the same manner, for soluble residual product (SRP);

$$\left[\frac{dS_R}{dt} \right]_r = \beta k_h \frac{X_s/X_A}{k_x + (X_s/X_A)} X_A \quad (3.4)$$

$$\left[\frac{dS_R}{dt} \right]_{\text{diff.}} = \frac{D \, d^2 S_R}{dL^2} \quad (2.14)$$

Therefore, the total conversion (reaction+diffusion) rate expression becomes;

$$\left[\frac{dS_R}{dt} \right]_{\text{total}} = \frac{D}{dL^2} \frac{d^2 S_R}{dL^2} + \beta k_h \frac{X_S/X_A}{k_X + (X_S/X_A)} X_A$$

Considering steady-steady conditions where $\left[\frac{dS_R}{dt} \right]_{\text{total}} = 0$,

the above equation yields,

$$\frac{D}{dL^2} \frac{d^2 S_R}{dL^2} = -\beta k_h \frac{X_S/X_A}{k_X + (X_S/X_A)} X_A \quad (4.2)$$

where it must be kept in mind that,

$X_T = X_{\text{Total}} = X_A + X_S + X_E$, which is a constant value as far as L_f , biofilm thickness is kept constant.

On the other hand, it has also been derived in part II that,

$$N_{SS} = -D \left[\frac{dS_S}{dL} \right]_{L=L_f}$$

and

(2.3)

$$N_{SR} = -D \left[\frac{dS_R}{dL} \right]_{L=L_f}$$

where N_{ss} and N_{sr} are soluble substrate removal and residual substrate generation rates according to Fick's law respectively, defined in terms of flux, (gr/m².day).

One can now easily visualize that in order to solve the governing equations (4.1 and 4.2) to define soluble substrate (S_s and S_r) distribution along the biofilm with appropriate values for respective fluxes, the particulate components must be defined as functions of biofilm thickness. It is also known that the first derivatives of these equations (4.1 and 4.2) yield the fluxes of both soluble substrates at $L=L_f$ to be utilized in the mass balance equations at macro-level.

4.1.2. Macro-Modelling

At this stage, the mass balance equations on soluble substrates must be defined clearly:

The mass balance on S_s ;

$$Q (S_{so} - S_s^*) = A.N_{ss} \quad (4.3)$$

where,

Q = flow rate of the system, m³/day

S_{so} = influent substrate concentration, mg/lt

S_s^* = substrate concentration at the biofilm bulk surface, mg/lt.

A = Surface area of the biofilm, m².

N_{ss} = substrate removal rate according to Fick's law, defined in terms of flux, gr/m². day.

As defined previously;

$$N_{ss} = -D \left[\frac{dS_s}{dL} \right]_{L=L_f} \quad (2.3)$$

In the same way, the mass balance on S_R ;

$$Q (S_{R0} - S_R^*) = A \cdot N_{SR} \quad (4.3)$$

where;

S_{R0} = influent inert soluble residual product, mg/lt, which is equal to zero.

S_R^* = effluent inert soluble residual product at the bulk-liquid interface, mg/lt.

N_{SR} = generation rate of the soluble residual product according to Fick's law, defined in terms of flux, gr/m².day.

Therefore,

$$Q S_R^* = A \cdot N_{SR}$$

$$S_R^* = \frac{A}{Q} \cdot N_{SR} \quad (4.5)$$

As defined previously,

$$N_{SR} = -D \left[\frac{dS_R}{dL} \right]_{L=L_f} \quad (2.3)$$

As far as the operational conditions of a system are known, such as the flow rate of the wastewater, Q , and influent substrate concentration, S_{s0} , together with the surface area of the biofilm, A , which is a physical property of a selected reactor, one can easily calculate;

- the surface hydraulic loading Q/A ($m^3/m^2 \cdot day$) for a given efficiency of the system

$$\left[\eta = \text{efficiency} = \frac{S_{so} - S_{s^*}}{S_{so}} \right]$$

- the concentration of the soluble residual product, S_{s^*} , by substituting the calculated surface hydraulic loading value in equation (4.5). The corresponding flux value calculated due to equation (2.3) in the micro-level model is also used in equation (4.5) to fulfill the main objective of this study which is to obtain the soluble residual product at the bulk-liquid interface.

So far, both levels of modelling are considered in general. It is seen that the modelling approach at microlevel can be achieved unless the behaviour or distribution of particulate components are expressed as functions of biofilm thickness. In the following section, details of biofilm modelling will be given together with the boundary conditions at the biofilm support media.

4.2. BIOFILM MODELLING

Modelling at micro-level mainly depends on the kinetics and removal mechanism of particulate components in a biofilm media, as developed by Wanner and Gujer (1985). In section 2.2.6., the basic idea used by both

authors has been briefly mentioned. In developing the model on particulate components, this basic idea of advection process is followed up.

4.2.1. Equations for Particulate Components

A mass balance on particulate components (X_A, X_S, X_E) can be written generally as;

$$\frac{\partial f_i}{\partial t} \cdot X_i \cdot A \cdot dL = X_i f_i \mu_i A dL + A N_{xi} - A \left(N_{xi} + \frac{\partial N_{xi}}{\partial L} \cdot dL \right) \quad (4.6)$$

for species i, where,

$A dL$ = differential volume element of the biofilm

X_i = density of the biofilm (concentration of particulate matter)

f_i = volume fraction of species, i,

μ_i = specific growth rate of species, i,

N_{xi} = flux of species, i, through area, A,

Figure 4.2. illustrates the flux of biomass through a differential volume element of an idealized biofilm.

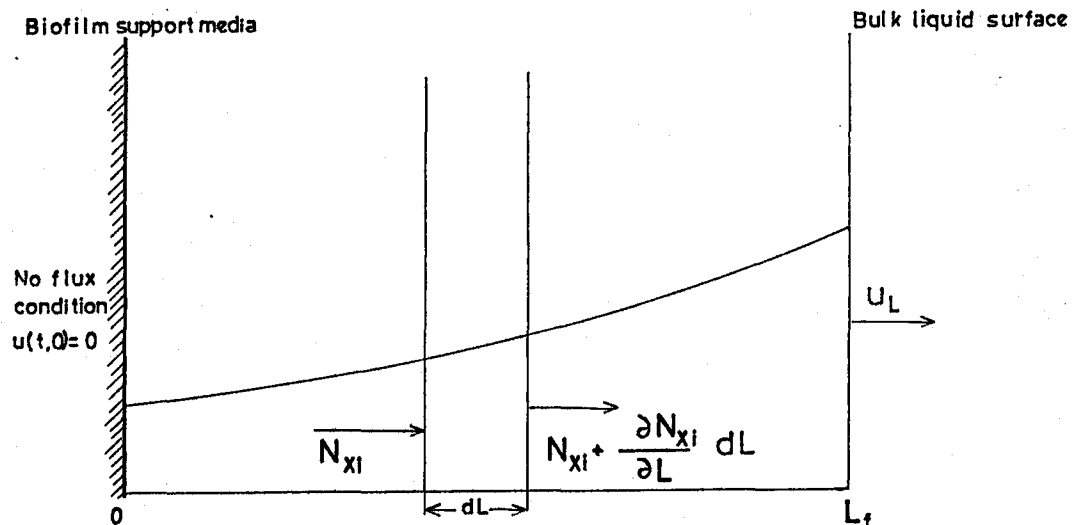


Figure 4.2. Flux of Biomass Through a Differential Volume Element of an Idealized Biofilm

Dividing both sides of the mass balance equation (equation 4.6) by $A \cdot dL$, yields

$$\frac{\partial f_1}{\partial t} = f_1 \mu_1 - \frac{1}{X_1} \frac{\partial N_{x1}}{\partial L} \quad (4.7)$$

where

$$f_1 = \frac{X_1}{X_T} = f_1(t, L) \quad (4.8)$$

If u is the velocity at which the biomass is displaced with respect to the film support media due to biofilm growth or shrinkage, the biomass flux of species, i , can be calculated as follows; (Gujer 1984, 1985, 1990)

$$N_{x1} = uX_1f_1 \quad (2.49)$$

Substituting equation (2.49) into equation (4.6) results in,

$$\begin{aligned} \frac{\partial f_1}{\partial t} &= f_1 \mu_1 - \frac{1}{X_1} \cdot X_1 \left(f_1 \frac{\partial u}{\partial L} + u \frac{\partial f_1}{\partial L} \right) \\ \frac{\partial f_1}{\partial t} &= \left(\mu_1 - \frac{\partial u}{\partial L} \right) f_1 - u \frac{\partial f_1}{\partial L} \end{aligned} \quad (4.9)$$

The average net specific growth rate of n_x microbial species in the biofilm is defined as; (Gujer(1984, 1985, 1990)

$$\bar{\mu} = \frac{\sum_{i=1}^{n_x} \mu_i f_i}{\sum_{i=1}^{n_x} f_i} \quad (2.50)$$

Summation of equation (4.9) for all the three microbial species (X_A, X_B, X_E) yields,

$$\begin{aligned}
\frac{\partial(X_A/X_T)}{\partial t} + \frac{\partial(X_S/X_T)}{\partial t} + \frac{\partial(X_E/X_T)}{\partial t} &= \frac{X_A}{X_T} \mu_{XA} + \frac{X_S}{X_T} \mu_{XS} + \frac{X_E}{X_T} \mu_{XE} \\
&- \frac{(X_A+X_S+X_E)}{X_T} \cdot \frac{\partial U}{\partial L} \\
&- U \left[\frac{\partial(X_A/X_T)}{\partial L} + \frac{\partial(X_S/X_T)}{\partial L} + \frac{\partial(X_E/X_T)}{\partial L} \right]
\end{aligned}$$

As, $X_A+X_S+X_E = X_T$, the above equation is simplified to the form,

$$\frac{\partial}{\partial t} \left(\frac{X_A+X_S+X_E}{X_T} \right) = \frac{X_A}{X_T} \mu_{XA} + \frac{X_S}{X_T} \mu_{XS} + \frac{X_E}{X_T} \mu_{XE} - \frac{\partial U}{\partial L} - U \frac{\partial}{\partial L} \left(\frac{X_A+X_S+X_E}{X_T} \right)$$

The left-hand side of the equation leads to zero, as

$$\frac{\partial}{\partial t} \left(\frac{X_A+X_S+X_E}{X_T} \right) = \frac{\partial}{\partial t} \left(\frac{X_T}{X_T} \right) = \frac{\partial}{\partial L} (1) = 0$$

Further simplification yields,

$$0 = \frac{X_A}{X_T} \mu_{XA} + \frac{X_S}{X_T} \mu_{XS} + \frac{X_E}{X_T} \mu_{XE} - \frac{\partial U}{\partial L} - 0$$

and, the final form becomes to be

$$\frac{\partial U}{\partial L} = \frac{X_A}{X_T} \mu_{XA} + \frac{X_S}{X_T} \mu_{XS} + \frac{X_E}{X_T} \mu_{XE} \quad (4.10)$$

In order to handle equation (4.10) in a simple form, the following abbreviations are used and will be referred as follows from hereon;

$$\frac{X_A}{X_T} = fX_A = x \quad (4.11)$$

$$\frac{X_S}{X_T} = fX_S = y \quad (4.12)$$

$$\frac{X_E}{X_T} = fX_E = z \quad (4.13)$$

Therefore equation (4.10) can be rewritten as;

$$\frac{\partial u(t,L)}{\partial L} = x\mu_{XA} + y\mu_{XS} + z\mu_{XE} \quad (4.14)$$

Integrating equation (4.14) yields,

$$u(t,L) = \int_0^{L_f} (x\mu_{XA} + y\mu_{XS} + z\mu_{XE}) dL \quad (4.15)$$

At the film support media, $u(t,0) = 0$ due to equation (2.49). In other words, there is no-flux condition at the support media (Figure 4.2). In developing the model, the support media ($L=0$) is assumed to gain thickness as the biofilm grows towards the bulk-liquid interface.

If the biofilm grows or shrinks, the biofilm thickness, L_f , changes and the biofilm surface moves at a velocity, u_L , where

$$u_L(t) = \frac{dL(t)}{t} \quad (4.16)$$

with respect to the film support interface.

If $\delta(t)$ is defined as the velocity at which biomass is exchanged between biofilm and bulk-liquid, the velocity at the bulk-liquid interface may be expressed as,

$$u_L(t) = \delta(t) + \int_0^{L_f} (x\mu_{xA} + y\mu_{xS} + z\mu_{xE}) dL \quad (4.17)$$

As biofilm thickness, L_f , is kept constant in developing the model, equation (4.15) is still valid.

Substituting equations (4.14) and (4.15) into equation (4.9), the very transparent form of mass balance for particulate components is obtained;

$$\frac{\partial x}{\partial t} = [\mu_{xA} - x\mu_{xA} - y\mu_{xS} - z\mu_{xE}]x - \frac{\partial x}{\partial L} \int_0^L [x\mu_{xA} + y\mu_{xS} + z\mu_{xE}] dL \quad (4.18)$$

$$\frac{\partial y}{\partial t} = [\mu_{xS} - y\mu_{xS} - x\mu_{xA} - z\mu_{xE}]y - \frac{\partial y}{\partial L} \int_0^L [x\mu_{xA} + y\mu_{xS} + z\mu_{xE}] dL \quad (4.19)$$

$$\frac{\partial z}{\partial t} = [\mu_{xE} - z\mu_{xE} - x\mu_{xA} - y\mu_{xS}]z - \frac{\partial z}{\partial L} \int_0^L [x\mu_{xA} + y\mu_{xS} + z\mu_{xE}] dL \quad (4.20)$$

At steady-state conditions, $\partial f_i / \partial t = 0$, therefore

$$\frac{\partial x}{\partial t} = \frac{\partial y}{\partial t} = \frac{\partial z}{\partial t} = 0$$

Equations (4.18), (4.19) and (4.20) are further simplified at steady-state conditions, as,

$$[\mu_{xA} - x\mu_{xA} - y\mu_{xS} - z\mu_{xE}]x = \frac{\partial x}{\partial L} \int_0^{L_f} [x\mu_{xA} + y\mu_{xS} + z\mu_{xE}] dL \quad (4.21)$$

$$\left[\mu_{xS} - Y\mu_{xE} - X\mu_{xA} - Z\mu_{xP} \right] Y = \frac{\partial Y}{\partial L} \int_0^{L_F} \left[X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} \right] dL \quad (4.22)$$

$$\left[\mu_{xE} - Z\mu_{xP} - X\mu_{xA} - Y\mu_{xS} \right] Z = \frac{\partial Z}{\partial L} \int_0^{L_F} \left[X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} \right] dL \quad (4.23)$$

As can be recognized from the above equations (4.21), (4.22) and (4.23), the specific growth rates of each particulate substrate, X_i , must be defined. The process rate equation of particulate matter are used to express the specific growth rates of each microbial species.

$$\mu_{xA} = \frac{dX_A}{dt} = \left(\mu \frac{S_S}{K_S + S_S} - b_H \right) X_A \quad (3.2)$$

$$\mu_{xS} = \frac{dX_S}{dt} = (1-f_e)b_H X_A - k_h \frac{X_S/X_A}{K_X + (X_S/X_A)} X_A \quad (3.3)$$

$$\mu_{xE} = \frac{dX_E}{dt} = f_e b_H X_A \quad (3.5)$$

Using the abbreviations denoted in equations (4.11), (4.12) and (4.13), the above equations can be expressed as follows,

$$\mu_{xA} = \frac{dX_A}{dt} = \left(\mu \frac{S_S}{K_S + S_S} - b_H \right) X X_T \quad (4.24)$$

$$\mu_{xS} = \frac{dX_S}{dt} = (1-f_e)b_H X_{TX} - k_h X X_T \frac{(Y/X)}{K_X + (Y/X)} \quad (4.25)$$

$$\mu_{xE} = \frac{dX_E}{dt} = f_e b_H X X_T \quad (4.26)$$

So far, the mass balance expressions of each particulate matter are obtained (equations 4.21, 4.22 and 4.23) together with their definitions of specific growth rates. In order to determine the distribution of each

species along biofilm thickness, suitable boundary conditions must be used, as will be mentioned in the following sub-section.

4.2.1.1. Boundary Conditions at the Support Media

At the biofilm support media ($L=0$) the boundary conditions is that there is no biomass flux ($\frac{\partial(x,y,z)}{\partial L}=0$)

Therefore, rewriting equations (4.21), (4.22) and (4.23) for the support media yields;

$$\begin{aligned} [\mu_{xA} - X\mu_{xA} - Y\mu_{xS} - Z\mu_{xE}]X &= 0; & \mu_{xA} &= X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} \\ [\mu_{xS} - Y\mu_{xS} - X\mu_{xA} - Z\mu_{xE}]Y &= 0; & \mu_{xS} &= X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} \\ [\mu_{xE} - Z\mu_{xE} - X\mu_{xA} - Y\mu_{xS}]Z &= 0; & \mu_{xE} &= X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} \end{aligned}$$

Therefore at the support media;

$$\mu_{xA} = \mu_{xS} = \mu_{xE} = X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE}$$

The above expression shows that at $L=0$, the specific growth rates of each particulate species is equal to a common value which can further be defined by any of the expressions given to define specific growth rates (equations 4.24, 4.25 and 4.26). If the expression of μ_{xE} is chosen as is more simply defined (equation 4.26) than the other two, the above equality leads to the form;

$$\mu_{xA} = \mu_{xS} = \mu_{xE} = X\mu_{xA} + Y\mu_{xS} + Z\mu_{xE} = f_{sbH}XX_T = \mu_{min}XX_T \quad (4.27)$$

In other words, the growth rates of microorganisms at the support media is equal to a common minimum value given in equation (4.27) and is schematically illustrated in Figure 4.3a.

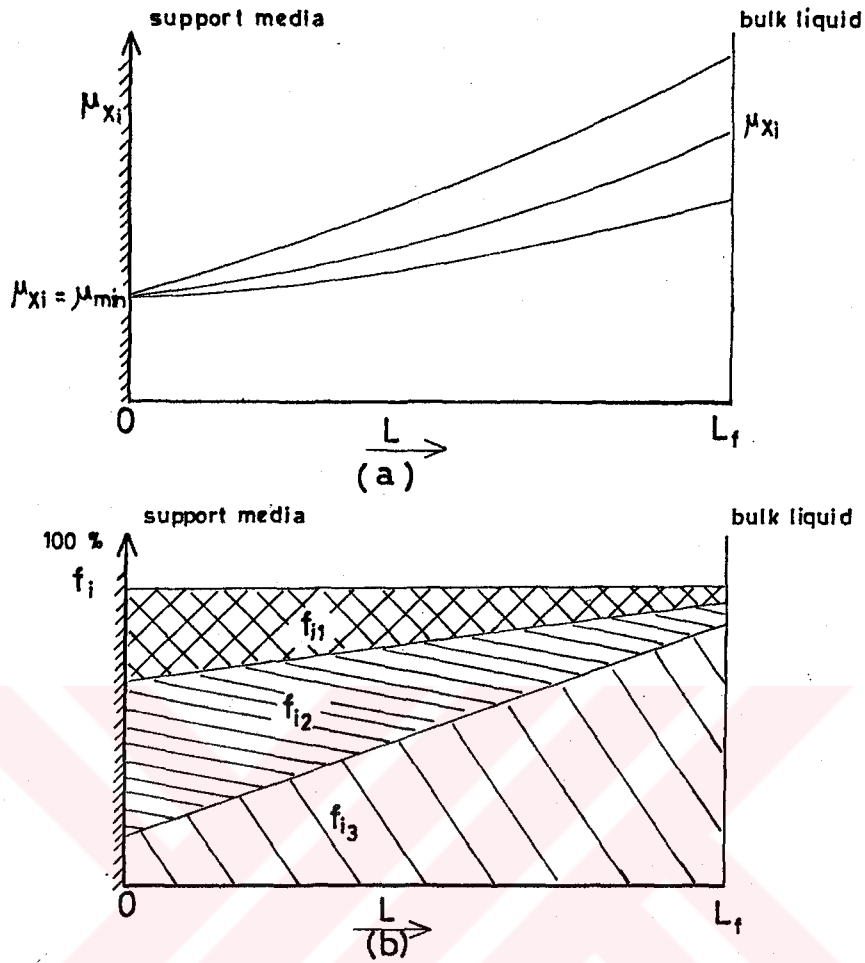


Figure 4-3. Specific Net Growth Rates (a) and Volume Fractions, f_i , for different Particulate Species (b) in Steady-State Biofilms

For following up the equations more easily, further abbreviations are done as follows;

$$\mu_{xA} = \mu_x \cdot X \cdot X_T \quad (4.28)$$

$$\mu_{xB} = \mu_y \cdot X \cdot X_T \quad (4.29)$$

$$\mu_{xE} = \mu_z \cdot X \cdot X_T \quad (4.30)$$

Considering the fact that there is no biomass flux at the support media, there exists three unknown values that must be defined, namely x_o , y_o and z_o , where $x_o + y_o + z_o = 1$.

In order to reduce the number of unknown values at the support media, y_o may be expressed as a function of x_o as follows;

$$y_o = w_o x_o \quad (4.31)$$

At the support media; it is known that

$$\mu_{xA} = \mu_{xB} = \mu_{xE}$$

Equalizing equations (4.25) and (4.26) both expressing specific growth rates of species X_B and X_E respectively, yields,

$$\left[(1-f_e)b_H - k_H \frac{(y_o/x_o)}{k_x + (y_o/x_o)} \right] \cdot X_T \cdot X_o = f_e b_H X_T X_o$$

$$k_H \frac{y_o}{k_x x_o + y_o} = (1-2f_e)b_H, \quad y_o = w_o x_o$$

$$w_o = k_x \frac{(1-2f_e)b_H}{k_H - (1-2f_e)b_H} \quad (4.32)$$

It can easily be recognized that w_o is only a function of kinetic coefficients, therefore, the only unknown at the biofilm support media is x_o .

4.2.2. Equations for Soluble Substrates

The overall reaction rate expressions for both soluble substrates at steady-state conditions has been defined in equations (4.1) and (4.2). Using the abbreviations given in equations (4.11) and (4.12), they lead to the following forms;

$$\frac{d^2 S_E}{dL^2} = \frac{\hat{\mu}}{D \cdot Y} \frac{S_E}{K_E + S_E} x \cdot X_T - \frac{(1-\beta)k_H \cdot X_T}{D} \cdot \frac{x(y/x)}{k_x + (y/x)} \quad (4.33)$$

$$\frac{d^2 S_R}{dL^2} = \frac{-\beta k_H X_T}{D} \cdot \frac{x(y/x)}{k_x + (y/x)} \quad (4.34)$$

4.2.2.1. Boundary Conditions at the Support Media

As there is a unique μ_{min} value at the support media, a minimum value for soluble substrate, S_E , can be determined by equalizing the expressions defining the specific growth rates of each particulate component.

$$i) \mu_{XS} = \mu_{XA}$$

$$\left[(1-f_e)b_H - k_H \frac{y}{k_x \cdot x + y} \right] x \cdot X_T = \left(\hat{\mu} \frac{S_E}{K_E + S_E} - b_H \right) x \cdot X_T$$

In other words, due to equations (4.28) and (4.29)

$$\mu_x = \mu_y$$

$$\left[(1-f_e)b_H - k_H \frac{y}{k_x \cdot x + y} \right] = \left(\hat{\mu} \frac{S_E}{K_E + S_E} - b_H \right) \quad (4.35)$$

$$ii) \mu_{XA} = \mu_{XE}$$

$$\left(\hat{\mu} \frac{S_E}{K_E + S_E} - b_H \right) x \cdot X_T = f_e b_H x \cdot X_T$$

Again, due to equations (4.28) and (4.30)

$$\mu_x = \mu_z$$

$$\left(\hat{\mu} \frac{S_E}{K_E + S_E} - b_H \right) = f_e b_H \quad (4.36)$$

By using any of the two expressions (4.35) or (4.36) or even a third one that may be written for the case $\mu_{XE} = \mu_{XS}$ may be used in defining S_{min} value at the support media.

From equation (4.36) S_{min} can be derived as follows;

$$S_{min} = \frac{K_S(1+f_e)b_H}{\mu - (1+f_e)b_H} \quad (4.37)$$

A similar approach has been followed up by Rittmann and Mc Carty (1980) for "deep" biofilms as mentioned in section 2.2.5.

The S_{min} value obtained by Rittmann and Mc Carty is;

$$S_{min} = K_S \left[\frac{b_H}{(\mu Y - b_H)} \right] \quad (2.47)$$

By using same values for the kinetic parameters, it is seen that both approaches yield comparable S_{min} values (Table 4.1).

Table 4-1. S_{min} Values at the Support Media

Typical values of Parameters	S_{min} Values	Reference
$K_S = 60 \text{ mg/l}$ $b_H = 0.62 \text{ d}^{-1}$ $f_e = 0.08$ $\mu = 4.2 \text{ d}^{-1}$ $Y = 0.60$	10.39 mg/l 11.38 mg/l	Rittmann and McCarty (1980 a) Author's Model

4.2.3. Final Form of Equations

In section 4.2.1 where equations for particulate components are derived, the specific growth are equations have been further abbreviated as given in equations (4.28), (4.29) and (4.30). Equalizing the other basic expression of specific growth rates in equations (4.24), (4.25) and (4.26) further simplifies the set of equations on growth rates which are the major equations indicating the distribution of microorganisms in a biofilm media.

By equating expressions (4.24) and (4.28), both defining μ_{XA} ;

$$\mu_{XA} = \left(\mu \frac{S_S}{K_S + S_S} - b_H \right) X \cdot X_T = \mu_{X \cdot X \cdot X_T}$$

yields,

$$\mu_{X \cdot} = \mu \quad (4.38)$$

By equating expressions (4.25) and (4.29), both defining μ_{XS} ;

$$\mu_{XS} = (1-f_e)b_H \cdot X \cdot X_T - k_H X_T \frac{xy}{k_X X + y} = \mu_{Y \cdot X \cdot X_T}$$

If further abbreviations are done for the constant terms as;

$$(1-f_e)b_H = q \quad (4.39)$$

$$f_e b_H = p \quad (4.40)$$

and if they are substituted in the above equalization on μ_{XS} ;

$$\mu_{xs} = (q - k_h \frac{xy}{k_{xx} + y}) x \cdot X_T = \mu_y \cdot x \cdot X_T$$

One can obtain the following expression for μ_y

$$\mu_y = q - k_h \frac{y}{k_{xx} + y} \quad (4.41)$$

By equating expressions (4.26) and (4.30), both defining μ_{xe} ;

$$\mu_{xe} = f_{eb} k_h X_T = \mu_z x X_T$$

yields,

$$\mu_z = p \quad (4.42)$$

Therefore; the behaviour of particulate components can be described in terms equations (4.38), (4.41) and (4.42).

On the other hand, one has to determine $\bar{\mu} = \mu_A = \mu_{average}$ for all the three particulate species as this average value will be used in the governing equations to be written for particulate components.

$$\mu \cdot x \cdot X_T = x \mu_{xA} + y \mu_{xs} + z \mu_{xe} \quad (4.43)$$

$$\mu \cdot x \cdot X_T = \left[x \mu + y \left(q - k_h \frac{y}{k_{xx} + y} \right) + z p \right] X_T \cdot x$$

$$\mu = \mu_{x+y} \left(q - k_h \frac{y}{k_{xx} + y} \right) + pz \quad (4.44)$$

and it is known that, $x+y+z=1$, therefore

$$z = (1 - x - y) \quad (4.45)$$

Figure 4-4. summarizes the notations to be determined at microlevel together with the boundary conditions.

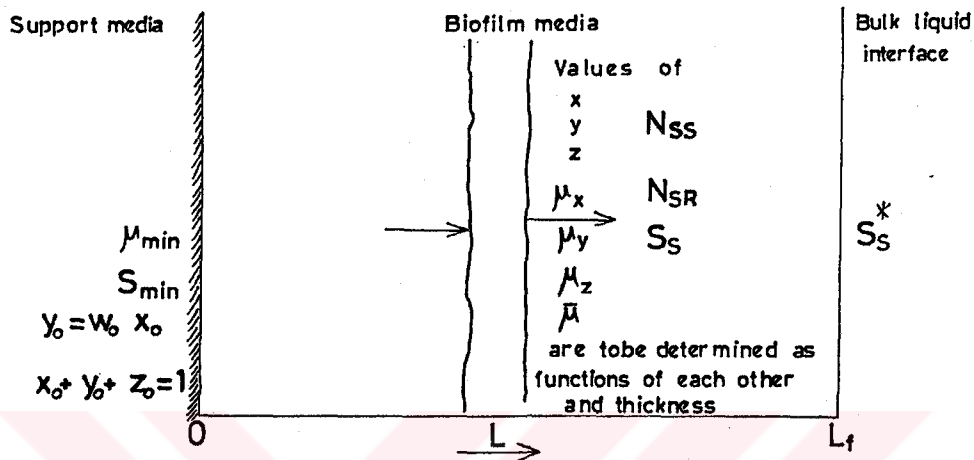


Figure 4-4. Schematic Presentation of the Model at Micro Level

All the equations related to each unknown value within a biofilm media are obtained. Now, the problem is to define the equations in such a way so as to be able to solve them numerically by utilizing the best solution technique available.

Referring back to the mass balance equations of each particulate substrate at steady-state conditions given in equations (4.21), (4.22) and (4.23) take the following forms by substituting the expression of $\bar{\mu} = \mu_A$ given in equation (4.43);

$$[\mu_{xA} - x\mu_{xA} + y\mu_{xS} + z\mu_{xE}]x = \frac{\partial x}{\partial L} \int_0^{L_f} [x\mu_{xA} + y\mu_{xS} + z\mu_{xE}]dL \quad (4.21)$$

$$(\mu - \mu_A)X_T \cdot x \cdot x = \frac{\partial x}{\partial L} \int_0^{L_f} \mu_A \cdot x \cdot X_T \cdot dL$$

$$(\mu - \mu_A)x^2 = \frac{\partial x}{\partial L} \int_0^{L_f} (\mu_A \cdot x) dL \quad (4.46)$$

$$[\mu_{xs} - y\mu_{xs} - x\mu_{xA} - z\mu_{xE}]y = \frac{\partial y}{\partial L} \int_0^{L_F} [x\mu_{xA} + y\mu_{xs} + z\mu_{xE}]dL \quad (4.22)$$

$$(\mu_Y - \mu_A)X_T \cdot x \cdot y = \frac{\partial y}{\partial L} \int_0^{L_F} \mu_A \cdot x \cdot X_T \cdot dL$$

$$(\mu_Y - \mu_A)x \cdot y = \frac{\partial y}{\partial L} \int_0^{L_F} (\mu_A \cdot x) dL \quad (4.47)$$

$$[\mu_{xE} - z\mu_{xE} - x\mu_{xA} - y\mu_{xs}]z = \frac{\partial z}{\partial L} \int_0^{L_F} [x\mu_{xA} + y\mu_{xs} + z\mu_{xE}]dL \quad (4.23)$$

$$(p - \mu_A)X_T \cdot x \cdot z = \frac{\partial z}{\partial L} \int_0^{L_F} \mu_A \cdot x \cdot X_T \cdot dL$$

$$\frac{\partial z}{\partial L} = - \left(\frac{\partial x}{\partial L} + \frac{\partial y}{\partial L} \right)$$

$$(p - \mu_A)x \cdot (1 - x - y) = - \left(\frac{\partial x}{\partial L} + \frac{\partial y}{\partial L} \right) \int_0^{L_F} (\mu_A \cdot x) dL \quad (4.48)$$

On the other hand, by substituting the μ_A value in the overall reaction rate expression for the soluble substrate S_E , (equation 4.33) and further rearrangement of the equation becomes;

$$\frac{Dd^2S_E}{dL^2} = \frac{\mu_A \cdot X_T}{Y} \cdot \frac{S_E}{K_E + S_E} \cdot x - (1 - \beta)X_Tk_h \frac{Y}{k_{xx}x + y} \cdot x$$

$$\frac{d^2S_E}{dL^2} = \frac{X_T}{Y \cdot D} \cdot x \cdot \left[\hat{\mu} \frac{S_E}{K_E + S_E} - (1 - \beta)Yk_h \frac{Y}{k_{xx}x + y} \right]$$

If further simplifications are done such as,

$$\hat{\mu} \frac{S_E}{K_E + S_E} = \mu + b_H, \quad -k_h \frac{Y}{k_{xx}x + y} = \mu_Y - q$$

The above equation can be rewritten as;

$$\frac{d^2S_E}{dL^2} = \frac{X_T}{Y \cdot D} \cdot x \cdot [\hat{\mu} + b_H + (1 - \beta)Y(\mu_Y - q)] \quad (4.49)$$

Therefore, necessary equations to be solved at the support media and along the biofilm media are completely set.

As a summary; there are four equations, one equation (4.49) written for soluble substrate S_s , and the other three relate to the behaviour of particulate substrates along the biofilm media (equations 4.46, 4.47 and 4.48).

Since all the final equations are non-linear with respect to their variables, they can best be formulated in finite difference form.

It is then that they are linearized by expanding them into Taylor's Series considering only their first derivatives.

4.2.4. Expression of Equations in Finite Difference Form

In order to write the equations in finite difference form, the biofilm thickness is divided into N equal intervals, thus, the integral operations in equation (4.46) and (4.47) are reduced to summation procedures throughout the biofilm thickness, and equation (4.49) is directly written in the finite difference form.

Equation (4.48) is not used in further solution of the equations as it is known that $x+y+z=1$ and that $z=(1-x-y)$. Therefore as variables x and y are obtained, z value can easily be determined.

The integral terms containing the function $f=\mu_{\Delta}x$ in equations (4.46) and (4.47) are simplified as follows, for any number of increments (n. L) in the biofilm media (Figure 4.5);

$$\int_0^L (\mu_{\Delta}x) dL = \sum_{i=1}^n \mu_{\Delta i} x_i \Delta L$$

Since the values of variables in the 1st increment are functions of initial values, governing equations in this increment must be considered separately.

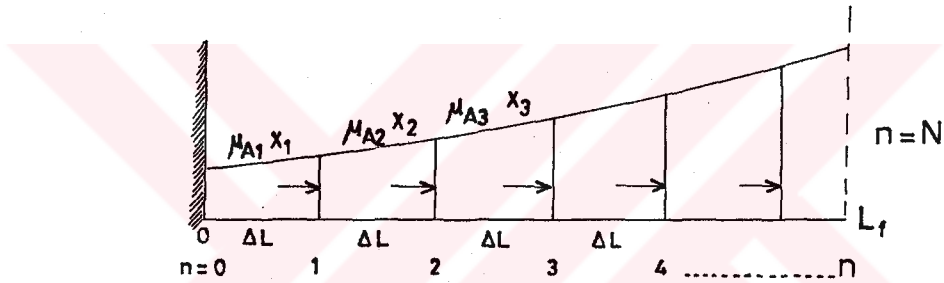


Figure 4-5. Schematic Presentation of the Solution of Equations (4.46) and (4.47).

For $n=1$ or $L=\Delta L$, this sum is equal to; $\mu_{\Delta 1} x_1 \Delta L$

For $n=2$ or $L=2\Delta L$, it becomes $(\mu_{\Delta 1} x_1 + \mu_{\Delta 2} x_2) \Delta L$

Therefore, for the 1st increment equation (4.46) becomes;

$$\frac{\mu x}{2x - x_0} = \mu_{\Delta} \quad (4.50)$$

where x_0 is the initial value of the term 'x' at the support media.

In the same way, equation (4.47) yields;

$$\frac{\mu_y \cdot y}{2y - y_0} = \mu_A \quad (4.51)$$

where y_0 is the initial value of the term 'y' at the support media.

Furthermore, the third equation for the soluble substrate (4.49) takes the following form if it is rewritten in finite difference form;

$$\frac{(\mu + b_H)}{(-\mu + \mu_N)} \cdot \frac{(\mu_{min} + b_H)}{(-\mu_{min} + \mu_N)} \cdot \frac{X_T \Delta L^2}{K_S Y_D} \cdot x \cdot \left[\mu + b_H - (1 - \beta) Y k_h \frac{y}{k_x x + y} \right] \quad (4.52)$$

where, $\mu_N = \mu_{net} = \hat{\mu} - b_H$

Both expression of μ_A (equations 4.50 and 4.51) yield,

$$\frac{\mu x}{2x - x_0} = \frac{wx}{2wx - y_0} \cdot \left(q - k_h \frac{wx}{k_x + wx} \right)$$

By cancelling the term 'x' from both sides, the expression reduces to,

$$\frac{\mu}{2x - x_0} = \frac{w}{(2wx - y_0)} \cdot \left(q - k_h \frac{w}{k_x + w} \right) = \frac{w}{(2wx - y_0)} \cdot \frac{[(q - k_h)w + k_x q]}{(k_x + w)}$$

Further simplification yields,

$$\mu(k_x + w)(2wx - y_0) = w(2x - x_0)[(q - k_h)w + k_x q]$$

or; in the even more general form for the support media, ($L=0$);

$$F(w, x, \mu) = (2wx - y_0)(k_x + w)\mu + (2x - x_0)[(k_h - q)w^2 - k_x q w] = 0 \quad (4.53)$$

As seen in this new equation (4.53) written for the support media, there are three unknowns, namely w , x and μ which are to be determined. Therefore, the two remaining relationships can be obtained by applying similar procedure on equation (4.50);

$$\frac{\mu x}{2x - x_0} = \mu x + y\mu_y + p(1 - x - y) = \mu x + wx \frac{[(q - k_h)w + k_x q]}{(k_x + w)} + p(1 - x - y)$$

$$\frac{\{\mu x + p(1 - x - wx)\} (k_x + w + wx[(q - k_h)w + k_x q])}{(k_x + w)} = \frac{\mu x}{2x - x_0}$$

$$G(w, x, \mu) = [p + \mu x - p(1 + w)x](k_x + w) - x[(k_h - q)w^2 - k_x q w](2x - x_0) - \mu x (k_x + w) = 0 \quad (4.54)$$

By considering equation (4.52), the last equation of the model is written in the following form, again for the biofilm support media ($L=0$);

$$(\mu + b_H)(-\mu_{min} + \mu_N) - (-\mu + \mu_N)(\mu_{min} + b_H) = \frac{X_T L^2}{K_S Y D} (-\mu_{min} + \mu_N)(-\mu + \mu_N)x.$$

$$\left[(\mu + b_H) - (1 - \beta) Y k_h \frac{w}{k_x + w} \right]$$

Simplifying the equation yields the third relationship;

$$H(w, x, \mu) = \frac{\hat{\mu}}{(\mu_N - p)} \cdot \frac{K_S \cdot Y \cdot D}{X_T \cdot \Delta L^2} (\mu - p)(w + k_x) + (\mu - \mu_N)[(\mu + b_H)(w + k_x) - (1 - \beta) Y k_h w] x = 0 \quad (4.55)$$

The three simultaneous non-linear equations (4.53), (4.54) and (4.55) involving three unknown variables (μ , x , w) must be solved to obtain the corresponding variables with respect to biofilm thickness.

Solution of these equations necessitates the determination of the initial values ($L=0$) of the three variables w , x and μ .

For $L=0$,

From equation (4.27);

$$\mu = \mu_y = \mu_A = p = f_e \cdot b_H \cdot \mu_{min} \quad (4.27)$$

where,

$$\mu_{min} = \hat{\mu} \frac{S_{min}}{K_S + S_{min}} - b_H = f_e b_H \quad (4.56)$$

On the other hand, S_{min} has been determined previously as;

$$S_{min} = \frac{\hat{\mu} K_S (1+f_e) b_H}{\hat{\mu} - (1+f_e) b_H} \dots\dots\dots (4.37)$$

In the simulation program, all the initial values at the biofilm support media, ($L=0$) are denoted by subscript "0".

Therefore;

$$\mu_{min} = \mu_0$$

$$S_{min} = S_0$$

The other initial values w_0 and x_0 are determined according to correlation;

$$y = wx, \quad y_0 = w_0 x_0$$

where w_0 has been determined in equation (4.32)

$$w_0 = \frac{k_x (1-2f_e) b_H}{k_H - (1-2f_e) b_H} \quad (4.32)$$

Due to its dependence on only kinetic parameters, w_0 is a constant value for a given set of kinetic coefficients.

Again for $L=0$, substituting $y=y_0$, $x=x_0$ in equation (4.41) and equating it to μ_{min} , reduce to;

$$k_H \frac{y_0}{k_x x_0 + y_0} = q - p = (1-f_e) b_H - f_e b_H$$

Solving this expression for y_0 , yield;

$$k_H y_0 = (1-2f_e) b_H (k_x x_0 + y_0)$$

$$y_0 = \frac{(1-2f_e) b_H k_x}{k_H - (1-2f_e) b_H} x_0; \quad y_0 = w_0 x_0$$

where,

$$w_0 = \frac{(1-2f_e) b_H k_x}{k_H - (1-2f_e) b_H} \quad (4.32)$$

is once more obtained. Since the sum of the three fractions x_0 , y_0 and z_0 are equal to unity;

$$x_0 + y_0 + z_0 = 1; \quad x_0 + w_0 x_0 + z_0 = 1, \quad z_0 + (1+w_0) x_0 = 1$$

$$z_0 = 1 - (1+w_0) x_0 \quad (4.57)$$

In this way, y_0 and z_0 can be determined as far as x_0 is known.

In the simultaneous solution of the equations, x_0 value is assumed and the stability of the bulk substrate concentration, S_s^* , is checked by equal stepwise increment of biofilm thickness, defined as $\Delta L = \frac{L_r}{N}$. For every N values chosen, the equations are solved at every increment and the one that has given the most stable S_s^* value are supplied at the bulk-liquid interface by observing the approach of S_s^* values to each other with respectively chosen N values. This trial and error produce has given the stability of S_s^* values at $N=200$.

Therefore, it can be stated that the simulation of the micro-model equations is conducted with $L=2.5\mu\text{m}$, as L_r chosen for the model is $500\mu\text{m}$.

4.2.5. The Method of Solution

Since the function $F(w,x,\mu)$, $G(w,x,\mu)$ and $H(w,x,\mu)$ are non-linear with respect to their variables, they are linearized using the Newton-Ralpson Method. Further rearrangement of these governing non-linear functions and the way they are derived due to each of the unknown variables according to the solution method are summarized in Appendix A.

The real roots of simultaneous algebraic equations where several unknown variables are present can be found either by the Newton-Ralpson Method or by the method of iteration. In this study, the

Newton-Ralpson Method is used, where there are three governing equations with three unknown variables.

As known; the related functions are,

$$F(w, x, \mu) = 0$$

$$G(w, x, \mu) = 0$$

$$H(w, x, \mu) = 0$$

If x , w and μ denote correction to the approximate values x_0 , w_0 and μ_0 respectively, the unkonwn variables can be written in the following forms;

$$x = x_0 + \Delta x$$

$$w = w_0 + \Delta w$$

$$\mu = \mu_0 + \Delta \mu$$

The governing functions, then, become,

$$F(x_0 + \Delta x, w_0 + \Delta w, \mu_0 + \Delta \mu) = 0$$

$$G(x_0 + \Delta x, w_0 + \Delta w, \mu_0 + \Delta \mu) = 0$$

$$H(x_0 + \Delta x, w_0 + \Delta w, \mu_0 + \Delta \mu) = 0$$

Expanding these functions by Taylor's Theorem for a function of three variables yield,

$$F(x_0, w_0, \mu_0) + \Delta x \left[\frac{\partial F}{\partial x} \right]_0 + \Delta w \left[\frac{\partial F}{\partial w} \right]_0 + \Delta \mu \left[\frac{\partial F}{\partial \mu} \right]_0 = 0$$

$$G(x_0, w_0, \mu_0) + \Delta x \left[\frac{\partial G}{\partial x} \right]_0 + \Delta w \left[\frac{\partial G}{\partial w} \right]_0 + \Delta \mu \left[\frac{\partial G}{\partial \mu} \right]_0 = 0$$

$$H(x_0, w_0, \mu_0) + \Delta x \left[\frac{\partial H}{\partial x} \right]_0 + \Delta w \left[\frac{\partial H}{\partial w} \right]_0 + \Delta \mu \left[\frac{\partial H}{\partial \mu} \right]_0 = 0$$

for determining the first corrections x_1 , w_1 , and μ_1 .

The notation $\left[\frac{\partial F}{\partial x} \right]_0$ means the value of $\frac{\partial F}{\partial x}$ when

x_0 , w_0 and μ_0 are substituted for x , w and μ for finding the first corrections. The functions can best be solved by means of determinants. In Appendix A the solution of the functions by Newton-Ralpson method is emphasized in great detail.

For increment greater than 1 (e.g. $n \geq 2$); the general equations are reduced to the following forms;

$$H(w, x, \mu) = a(\mu - r)(w + k_x) + (\mu - \mu_N)[(\mu + b_H)(w + k_x) - bw]x = 0 \quad (4.58)$$

$$F(w, x, \mu) = (\mu x^2 - tx + tx_{n-1})(2wx - y_{n-1})(w + k_x) + (2x - x_{n-1})[(cw^2 - dw)x^2 + t(w + k_x)(wx - y_{n-1})] = 0 \quad (4.59)$$

$$G(w, x, \mu) = \{[p + \mu x - p(1 + w)x](w + k_x) - x[(cw^2 - dw)]\}(2x^2 - x_{n-1} \cdot x) \cdot (\mu x^2 - tx + tx_{n-1}) \cdot (w + k_x) = 0 \quad (4.60)$$

which are derived in detail in Appendix A.

The main program involves the solution procedure of these equations (4.58), (4.59) and (4.60). The simulation program of the micromodel is given in Appendix C.

As mentioned previously, the viability of active biomass varies throughout biofilm thickness, thus, the distribution of particulate components along thickness can easily be obtained from the output of the program. A few examples of the print outs are given in Appendix D.

Another information supplied by the micro model is the average (mean) values of viability along biofilm thickness. The average values of $\Sigma \frac{X_A}{X_T}$ (viability) together with $\Sigma \frac{X_S}{X_T}$ and $\Sigma \frac{X_E}{X_T}$ are also obtained both at the bulk-liquid interface and along biofilm thickness, for a given set of input values. This problem of determining the average values of all the three particulate components is solved by adding a subroutine program to the main program. The subroutine program involves the Trapezoidal Rule to find the average values along thickness where the number of subintervals, N , is equal to 200.

The Trapezoidal Rule is a numerical integration method used to find the area under a curve, which can be applied to any number of subintervals, whether even or odd, (Scarborough, 1966). The Trapezoidal Rule is mentioned in Appendix B.

Thus the average values of the three particulate components are also checked by Simpson's Rule at the bulk-liquid interface. The comparison of results obtained at the bulk-liquid surface by both methods, did not show any noticeable deviations, correlating the application of the Trapezoidal Rule.

4.3. MACRO MODELLING

The output of the program on micro-modelling illustrates the flux values of both the soluble substrate and inert soluble residual product, N_{SS} and N_{SR} respectively along biofilm thickness. At any L_x value, the value of N_{SS} can be used in the mass balance equation of the soluble substrate, S_s^* , as follows;

$$Q (S_o - S_E^*) = A.N_{SE} \quad (4.3)$$

In other words, the soluble substrate concentration of $L_r=500\mu\text{m}$ which resembles the bulk-liquid interface is S_E^* and the corresponding flux value is the data to be used in equation (4.3) for variable x_o values indicating variable efficiency of a given system where the influent substrate concentration, S_o , is fixed.

As far as the influent substrate concentration, S_o , is known one can easily calculate the hydraulic loading, Q/A , for a given system. Therefore, the concentration of the inert soluble residual product at the bulk-liquid interface can easily be determined from equation (4.5);

$$S_R^* = \frac{A}{Q} . N_{SR} \quad (4.5)$$

where N_{SR} value is the flux value of inert residual soluble substrate at $L_r=500\mu\text{m}$ tabulated in the output of the micro-model program.

The main objective of the study is fulfilled by determining S_R^* values under certain operating conditions. One can easily conduct sensitivity analysis on individual parameter basis to observe the effect of basic kinetic parameters, $(\beta, k_n, K_E, \hat{\mu}, Y)$ on the soluble residual product.

It is then that the total effluent soluble substrate concentration, S_T , is determined for a given set of kinetic parameters and hydraulic conditions of a given reactor, as;

$$S_T = S_E^* + S_R^*$$

The next part of the study will be mainly based on the evaluation of the model both at micro and macrolevels.



PART V

EVALUATION OF THE MODEL

Once the computer program is prepared, it is possible to apply and evaluate the model under various operating conditions. As previously mentioned, the modelling of fixed-film systems closely relate to the distribution of particulate components within the biofilm media. Therefore, in the first section of this part, the model is evaluated at micro level where the behaviour of particulate components (X_A , X_S and X_E) are illustrated graphically within a given biofilm media. The distribution of soluble substrate, S_S , is also demonstrated along fixed biofilm thickness. The relationship between average viability ($\frac{\sum X_A}{X_T}$) together with the average volume fractions of the other two particulate components ($\frac{\sum X_S}{X_T}$ and $\frac{\sum X_E}{X_T}$) and bulk substrate concentration, S_S^* , are put forth. The variation of fluxes of both soluble substrates S_S and S_R are determined with respect to average viabilities.

In the second section of this part, the model is evaluated at macro level. The most important objective of this study is fulfilled within the work done at this stage which is the generation of SRP (soluble residual product) in fixed-film systems. Therefore, more insight is given to the designer on fixed-film phenomena by constituting concentration profiles on especially, S_S^* , S_R^* and on S_T which is the total effluent substrate concentration.

It is then that, the reactor model is evaluated under various operating conditions.

The SRP generated in fixed-film systems is also compared with SRP generation model developed for suspended-growth systems on a few numerical examples where the basic operating conditions and the kinetic coefficients are kept same in both systems.

It must be noted that further experimental work is needed to verify and refine the model at both levels.

5.1. EVALUATION OF THE MICRO MODEL

As seen from Appendix D, the output of the micro model yields the variation of the model components (X_A, X_E, X_B, S_S) together with the fluxes of S_S and S_R along a fixed-film media. The typical kinetic parameters and coefficients are also written in the print-outs, which have been previously mentioned.

5.1.1 Distribution of Particulate Components at Fixed-biofilm Thickness

The volume fraction of particulate components, fX_A, fX_E , and fX_B are shown as functions of biofilm thickness in Figures 5.1, 5.2 and 5.3 respectively for five typical bulk-liquid substrate concentrations, S_S^* ; tabulated in Table 5.1.

Table 5-1. The Typical Bulk Substrate Concentrations used in Model Evaluation

S_s^* ; Bulk Substrate concentration	Values (mg/lt)
S_s^{*1}	16.33
S_s^{*2}	20.34
S_s^{*3}	35.39
S_s^{*4}	45.45
S_s^{*5}	80.89

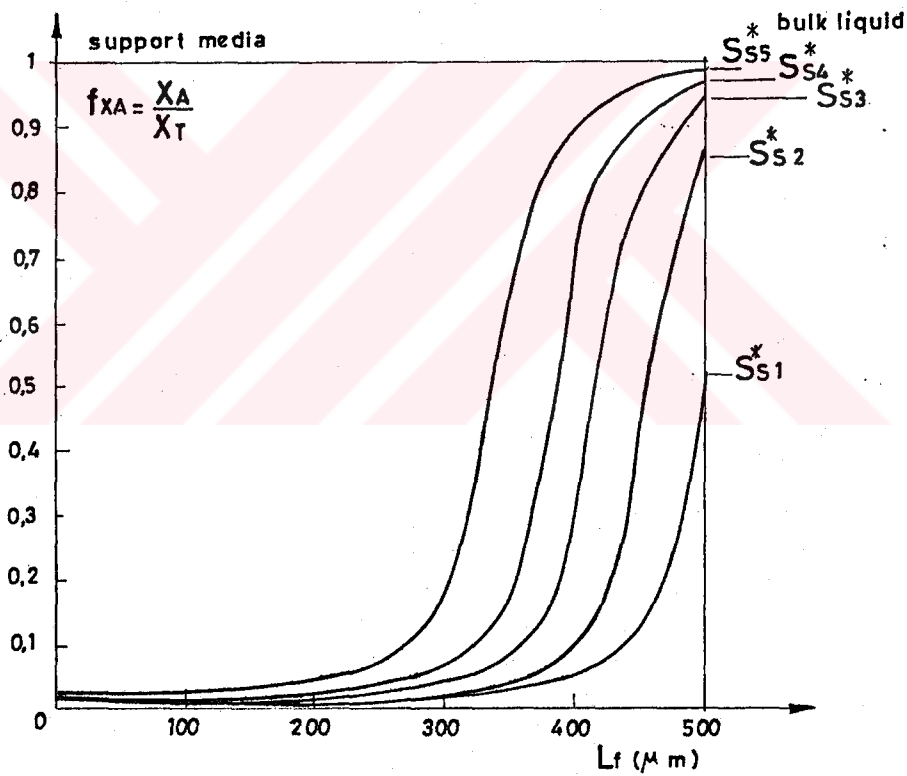


Figure 5-1. The Distribution of Volume Fraction of X_A along the Fixed-film Media

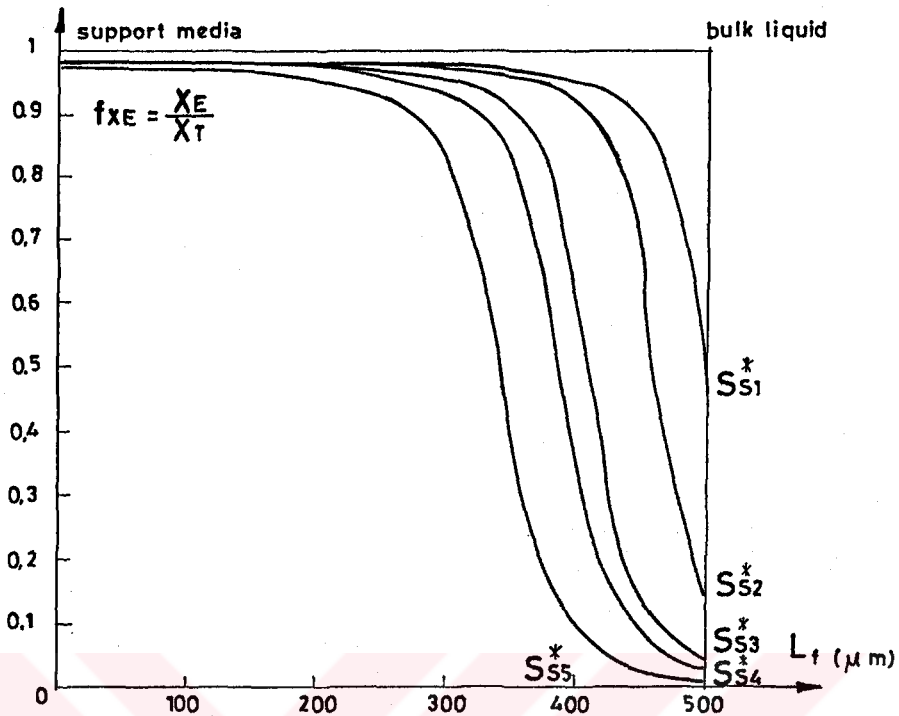


Figure 5-2. The Distribution of Volume Fraction of X_E along the Fixed-film Media

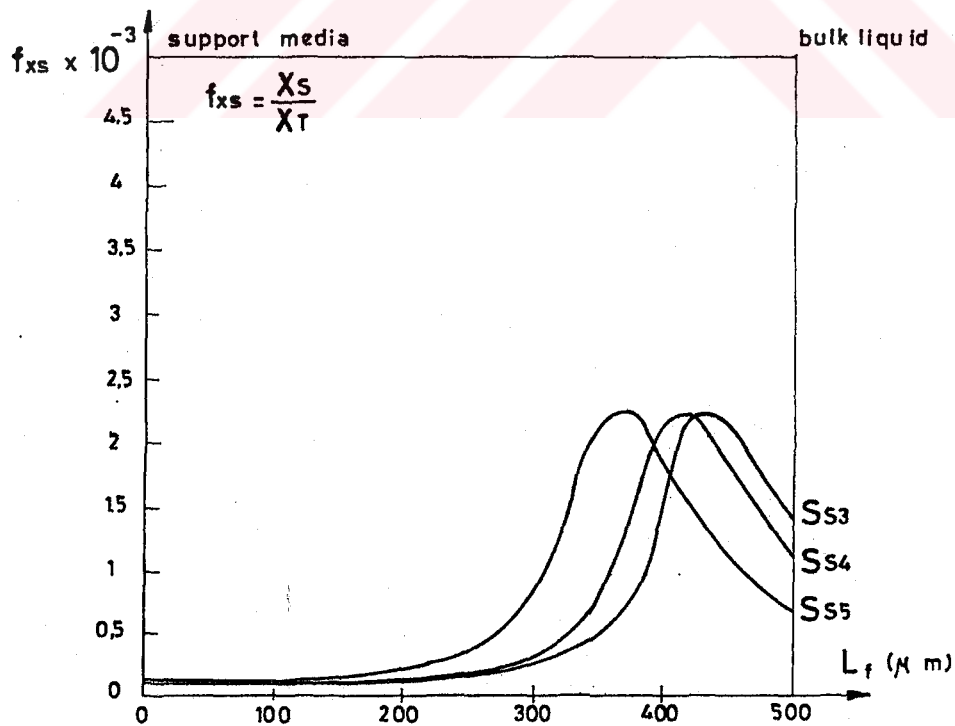


Figure 5-3. The Distribution of Volume Fraction of X_S along the Fixed-film Media

The volume fraction of active biomass having highest values at the bulk-liquid interface descends steeply towards the support media. At the mid-depth of the biofilm, stability in active biomass is observed which continues till the support media. The bulk-substrate concentration is also an important component in affecting the variation of active biomass concentration. Viability is seen to be directly proportional to S_E^* ; it can be concluded that higher the effluent substrate concentration, higher the viability (Figure 5.1). But this statement holds true for the first half of the thickness from the bulk-liquid interface, and from the mid-depth onwards towards the support media, the active biomass concentration does not vary significantly with respect to different S_E^* values.

The distribution of X_E , particulate component generated due to biomass decay surely shows an increase towards the support media from the bulk-liquid interface till the mid-depth. From thereon, a stability is again observed as in the case of active biomass distribution. The X_E concentration is inversely proportional with S_E^* values, in other words, higher the S_E^* value, lower the volume fraction of X_E , which is, no doubt, the natural result of the process reaction rates (Figure 5.2).

On the other hand, the volume fraction of X_E is shown to have a negligible distribution along thickness compared with the other two particulate components X_A and X_E . The volume fraction X_E shows a sharp increase from the bulkliquid interface for a few hundred microns and descends steeply till the mid-depth and approaches a stable value from thereon till the support media. On the whole, it can be stated that the amount of X_E , generated throughout biofilm thickness shows an insignificant variation.

In short, the distribution of volume fractions of all the particulate components indicate that, as a common fact, the distribution profiles reach to stable values from the mid-depth towards the support media verifying the previously mentioned fact that, not all of the fixed biofilm thickness involve in either generation or removal reactions. Part of the biofilm thickness is named as "active depth" where the reactions occur, and the rest is of negligible importance.

The average volume fractions are also plotted against biofilm thickness just to supply more helpful information on the behaviour of particulate components. These profiles are given in Figures 5.4 and 5.5.

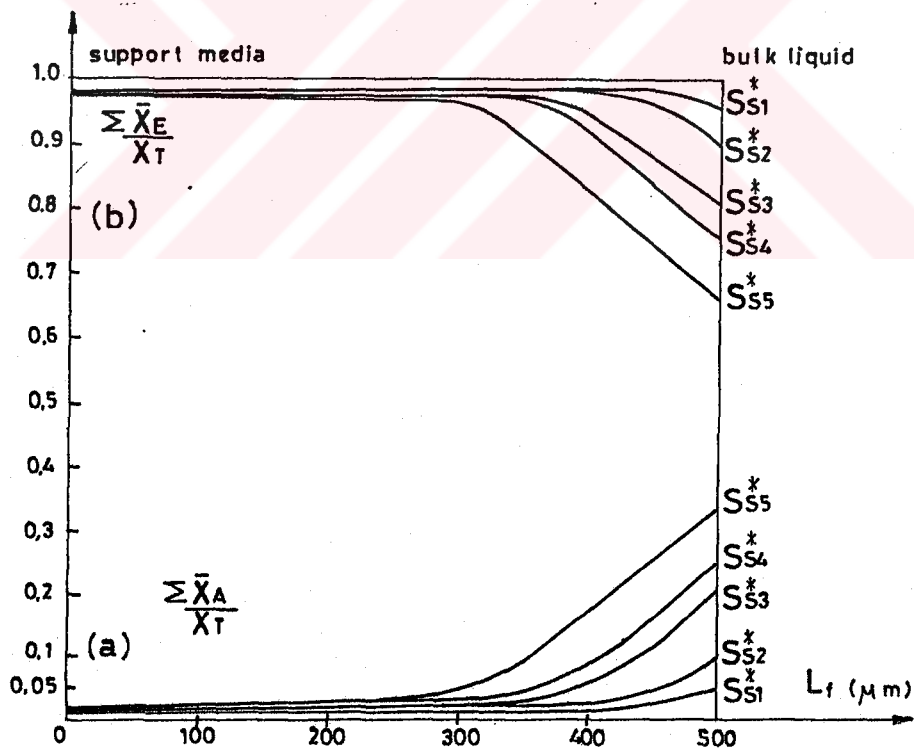


Figure 5-4. The Distribution of Average Volume Fractions of X_A (a) and X_E , (b) respectively, along the Fixedfilm Media

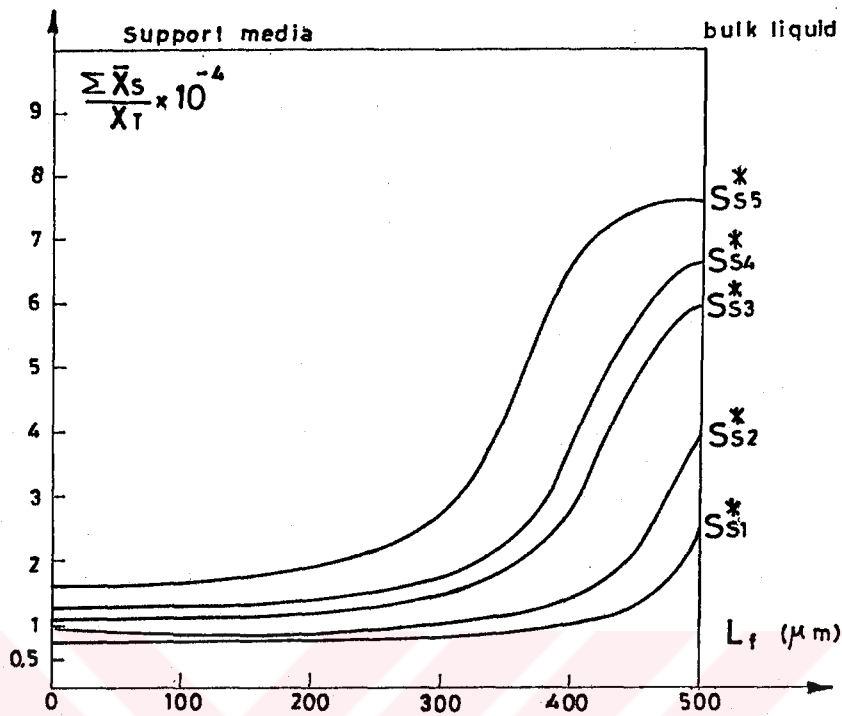


Figure 5-5. The Distribution of Average Volume Fraction of X_s along the Fixed-film Media

Figure 5.4 (a) illustrates the average viabilities of different S_s^* values due to biofilm thickness. Figure 5.4 (b) shows the average volume fraction of X_s and finally the insignificance of the component X_s is observed from Figure 5.5.

From all the average volume fractions of the three components where one is assumed to have negligible effect, shows that the variations in distribution occurs along the active depth, which is, in this study only $\approx 250 \mu m$.

5.1.2 Distribution of Soluble Substrate, S_s , at Fixed Biofilm Thickness

The most important fact that is easily observed in the behaviour of particulate components is the stability of them from the mid-depth of the biofilm media towards the support media. Due to this stability, same observations hold true for the distribution of soluble substrate as shown in Figure 5.6.

As previously mentioned during the development of the micro-model, the distribution of particulate components is of high importance, as all the process rates, either generation, or removal, are functions of them. Therefore, the behaviour of particulate components affect the soluble substrate removal in the biofilm media as observed in Figure 5.6. The S_{min} value for all

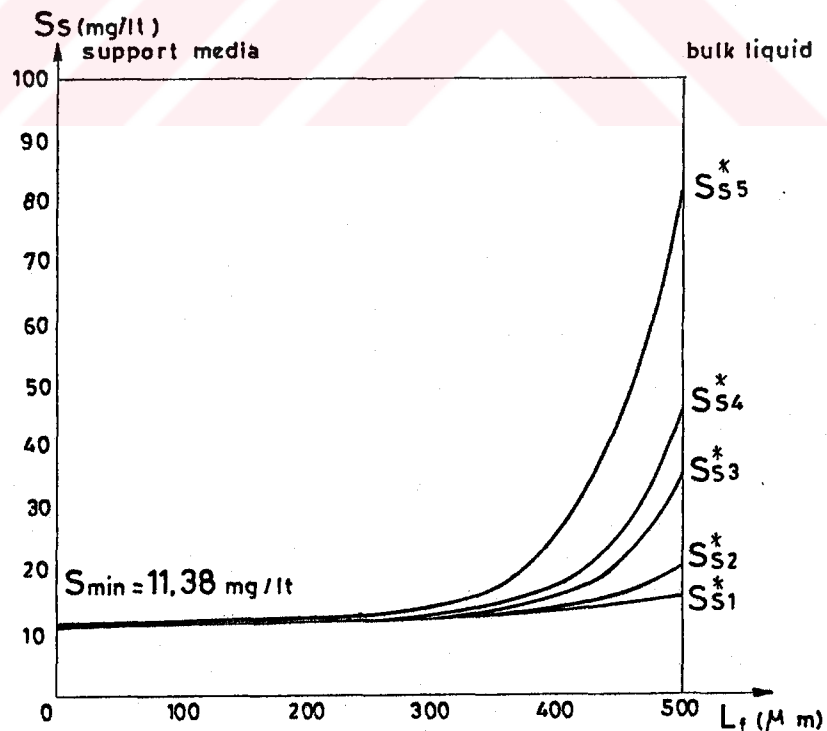


Figure 5-6. The Distribution of Soluble Substrate along the Fixed-film Media

the S_s^* values are common due to the modelling approach and it is observed that the "active depth" of the biofilm media is again 250 μm as in the case of particulate components distribution.

5.1.3. The Effect of Average Viability on the Model Findings

Active biomass, as in the case of all biological treatment processes, is the dominant component that is responsible for substrate removal. This component does not bear a constant value in fixed-film phenomena as has been recognized till now. It's behaviour closely relate the soluble substrate distribution in a fixed-film media which can be observed in the figures given in the previous sub-section.

The affect of average viability of a given fixed-film media on the model findings are of high

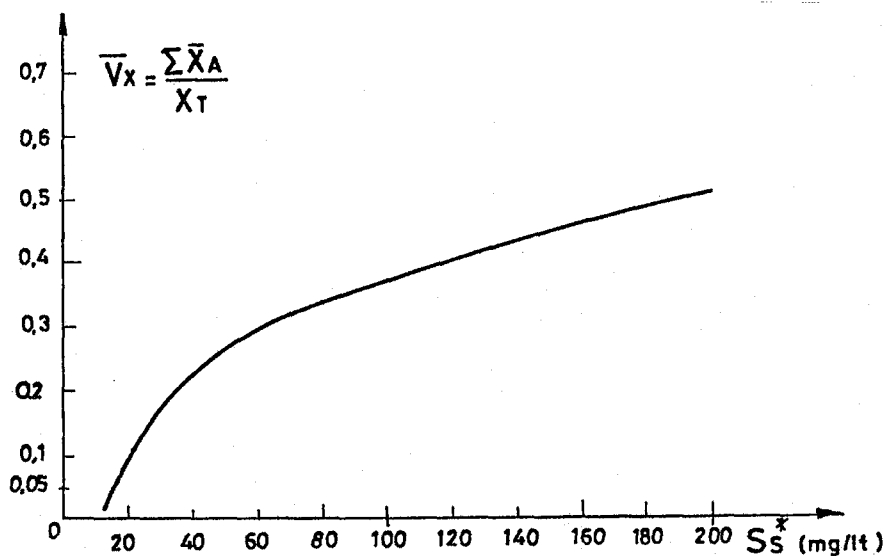


Figure 5-7. The Relationship Between Average Viability of a given Fixed-film Media and Bulk-Substrate Concentration

importance. Figure 5.7. shows how the average viability of a biofilm media increases as the bulk-substrate concentration increases.

The proportionality between S_s^* and average viability $\frac{\sum X_A}{X_T}$; of a given biofilm media is obvious due to the fact that more active biomass is necessary to take part in the substrate removal process. Another point that is observed from the model findings is that the average viability, on the whole, is only 10 % - 35 % of all the particulate components constituting the biofilm media where S_s^* varies between 20 - 100 mg/lt which is a commonly accepted range in aerobic systems, either suspended or attached growth. Even for $S_s^* = 200$ mg/lt, the viability is about 50 % of all the total components.

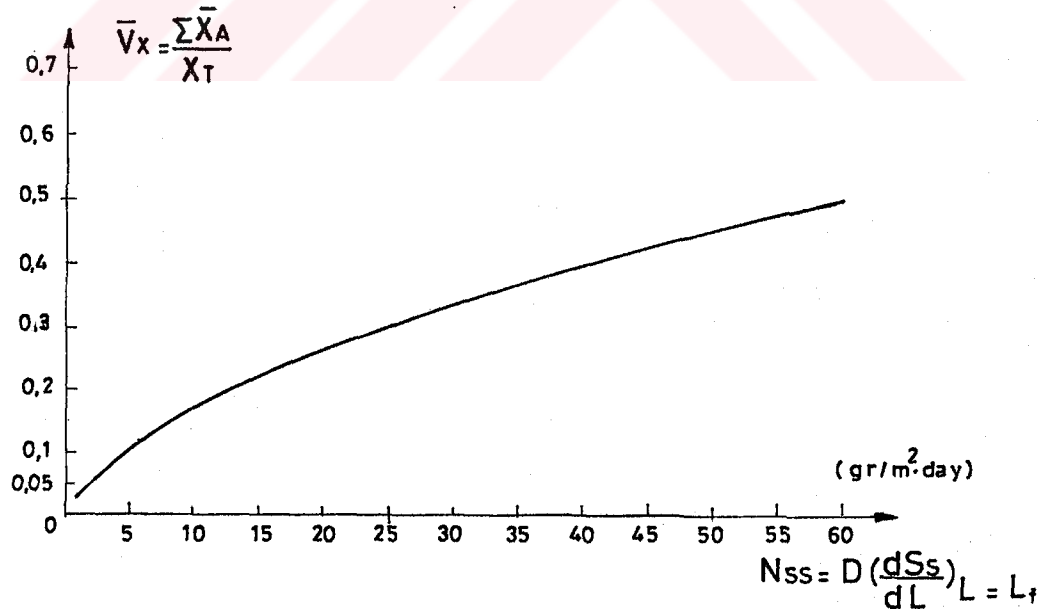


Figure 5-8. The Relationship Between Average Viability of a given Fixed-film Media and Flux of S_s^* , N_{SS}^*

The flux of S_s^* , indicates the mass of soluble substrate removed per unit area per time. Therefore, as more substrate is removed, the amount of active biomass responsible of substrate removal increases.

On the other hand, Figure 5.9 illustrates the generation of SRP profile with respect to average viability of a given biofilm media.

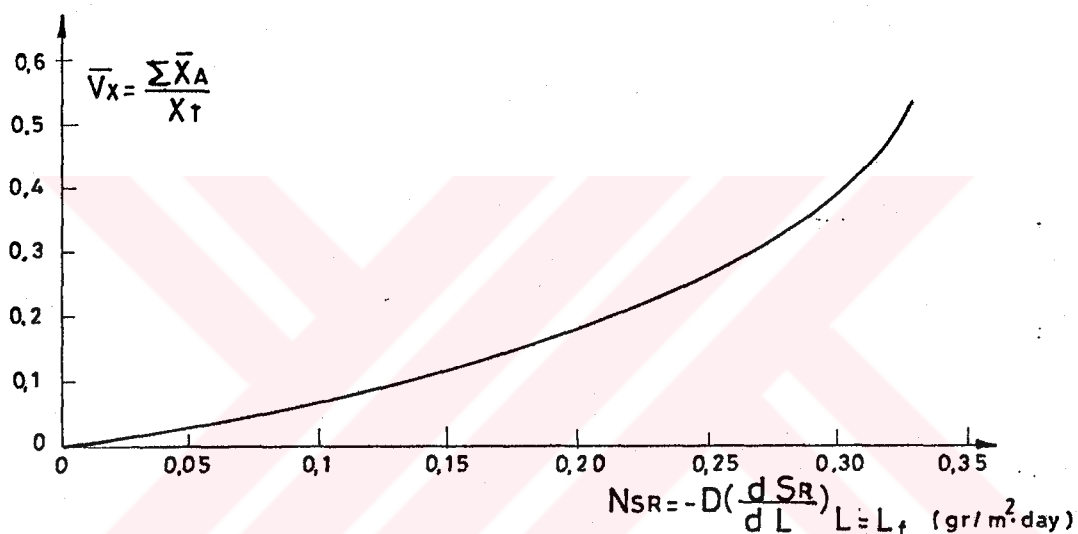


Figure 5-9. The Relationship Between Average Viability of a given Fixed-film Media and Flux of S_R^* , N_{SR}^* .

The flux of S_R^* , indicates the mass of inert soluble residual product generated per unit area per time. It is observed that the generation of SRP increases, as average viability of a given biofilm media increases. But, referring to the flux values, one can easily visualize that N_{SR} varies in very narrow range compared with the N_{ss} values depicted in Figure 5.8.

The substrate removal per unit area and time varies in a broader range, such as 5-60 gr/m² day, where the corresponding viability range is 0.1-0.5. The

residual soluble substrate generation per unit area per time varies between 0.13 and 0.325 gr/m² day for the same viability range. The substrate removal in a biofilm media is quite high compared the generation mechanism of SRP. Table 5.2 covers the N_{ss} and N_{sr} values for a biofilm media with respect to viability values for equal bulk-substrate concentrations.

Table 5-2. N_{ss} and N_{sr} Values for a Given Set of Average Viability Values

Average Viability $\bar{X}_A/X_T$	N_{ss} (gr/m ² ·day)	N_{sr} (gr/m ² ·day)
0.1	5	0.130
0.2	13	0.220
0.3	25	0.265
0.4	40	0.30
0.5	60	0.325

As a conclusion, it can be stated that the generation mechanism of SRP increases with viability and corresponding substrate removal rate expressed in terms of flux, but it is a slow mechanism and does not introduce significant generation rates compared to soluble substrate removal rates.

5.1.4. Evaluation of the Model Findings

The presentation of both the particulate components and soluble substrate behaviour along a given fixed biofilm media indicates that only a fraction of the biofilm thickness is active, in other words, is responsible of both the removal and generation mechanisms. The rest of the biofilm media is accepted to approach stability. This fact, that is observed in fixed-film systems is due to diffusional resistances of both the substrate and oxygen.

In all aerobic systems, oxygen is one of the soluble substrates that take place in the removal mechanism. Although, in this study, oxygen, as a second soluble substrate is not involved in the development of the model, it's effects can closely be recognized from the model findings. The soluble substrate and oxygen penetrates to the biofilm media from the bulk-liquid interface towards the support media. But, the diffusional resistances and even the lack of oxygen supplied to the system cause the process reaction rates to reach stability towards the support media. As a conclusion, it can be stated that, in fixed-film systems, not all of the biofilm media is equally active as in the case of suspended-growth systems.

Another noticeable fact that is observed from the model findings is that, the behaviour of slowly degradable substrate, X_s , is insignificant compared to the other two governing particulate components, X_A and X_E .

The amount of active biomass is of high importance in biological treatment. All the mechanisms occurring in a biofilm media are governed by the term viability.

The next part will be mainly based on the generation mechanism of SRP in a fixed-film system where the properties of the wastewater and the reactor are set.

5.2. EVALUATION OF THE MACRO MODEL

The aim of expanding the model at macro level is to supply more design information to the designer dealing with fixed-film reactors.

The macro model utilizes the information gathered at micro level together with data on reactor hydraulics. For a specific reactor and hydraulic conditions, one can also determine the total effluent substrate concentration, S_T , by making use of mass balance equations on both soluble substrates, S_S and S_R .

Further investigations can be followed due to hydraulic conditions of a system and due to certain operating conditions.

The first section of this part is devoted to the generation mechanism of SRP in fixed-film systems, where the concentration profiles of S_R are illustrated within a biofilm media together with the reaction rates under certain influent substrate concentrations S_0 . It is, then, that the reactor model is presented for a given set of hydraulic conditions.

This part ends with a sub-section that compares to generation mechanisms of both suspended-growth and attached-growth systems.

5.2.1. The Soluble Residual Product Generation in Fixed-Film Systems

In the micro-modelling; the flux values of S_s^* and S_R^* are given for a bulk-substrate concentration, S_s^* . At the macro-modelling, one can easily determine the hydraulic loading of a system for a given influent substrate concentration, S_o , by making use of the mass balance equation on soluble substrate S_s . Since the hydraulic loading of a system is known, the concentration of inert soluble residual product at the bulk-liquid interface can be determined from the mass balance equation on S_R , which has been previously given as;

$$S_R^* = \frac{A}{Q} \cdot N_{SR} \quad (4.5)$$

The typical influent substrate concentrations, S_o , used in model presentation are tabulated in Table 5.3.

Table 5-3. The Typical Influent Substrate Concentrations used in Model Presentation

S_o , Influent Substrate Concentration	Values (mg/lt)
$S_o(1)$	250
$S_o(2)$	500
$S_o(3)$	750
$S_o(4)$	1000
$S_o(5)$	1500

TABLE 5-4. S_R^* Values for Variable Influent and Bulk Substrate Concentration

S_S^* (mg/lt)	S_o (1) 250 (mg/lt)		S_o (2) 500 (mg/lt)		S_o (3) 750 (mg/lt)		S_o (4) 1000 (mg/lt)	
	Q/A ($m^3/m^2 \cdot d$)	S_R^* (mg/lt)	Q/A ($m^3/m^2 \cdot d$)	S_R^* (mg/lt)	Q/A ($m^3/m^2 \cdot d$)	S_R^* (mg/lt)	Q/A ($m^3/m^2 \cdot d$)	S_R^* (mg/lt)
S_S^{*1}	0.0085	9.359	0.0041	19.37	0.00271	29.387	0.00202	39.44
S_S^{*2}	0.0216	7.695	0.0103	13.61	0.00681	20.70	0.00507	27.79
S_S^{*3}	0.0597	3.649	0.0275	7.90	0.0179	12.17	0.0132	16.515
S_S^{*4}	0.0836	2.863	0.0376	6.349	0.0242	9.84	0.0179	13.33
S_S^{*5}	0.176	1.573	0.0710	3.88	0.0445	6.219	0.0324	8.518

Table 5.4. shows the S_R^* values determined with respect to variable influent and bulk substrate concentrations. In other words, S_R^* values are depicted for reactors of known efficiency and hydraulic conditions.

There are three parameters effecting concentration of soluble residual product at the bulkliquid surface as can easily be seen in equation (4.5). One of the parameters is the generation rate expressed in terms of flux, which has been mentioned in the previous part and it has been stated that the generation is a slower mechanism compared to substrate removal mechanism. The other two parameters; the area of the biofilm reactor A , and the flow rate of the wastewater, Q , namely the surface hydraulic loading (Q/A) is a dominant factor that controls the residual bulk-substrate concentration. It can easily be recognized from the data given in Table 5.4; that as surface hydraulic loading decrease, the S_R^* value increases. If the reactor area is fixed, one has to vary the flow rate of the wastewater to obtain the calculated hydraulic loading; in this case, S_R^* increases as the flow rate of the system decrease or, if the flow rate of the wastewater is fixed, in other words, if the designer seeks an appropriate reactor area, one can recognize that as the reactor area increase, S_R^* also increase.

On the other hand; the influent substrate concentration is also another factor that effects the value of S_R^* . As the influent substrate concentration increases, in other words, as the strength of the wastewater is high, the resulting S_R^* is also higher for a fixed S_R^* value.

As a conclusion, it can be stated that S_R^* is a parameter that is mostly effected by the hydraulic conditions of a system rather than the specific generation rate of a given biofilm media.

$$S_R^* = f \left(A, \frac{1}{Q}, S_o \right) = S_R^* = f \left[\frac{A}{Q}, S_o \right]$$

The negligible effect of the specific generation rate expressed in terms of flux is due to low and narrow range of the flux values. As has been set previously, the generation rate of S_R is a function of slowly degradable substrate, X_E , whose distribution profile within a given fixed-film media has been shown to have insignificant effect compared to the distribution profile of the other two particulate components, X_A and X_E .

So far, the bulk liquid concentration of SRP is calculated by utilizing the mass balance equations of both soluble substrates. As soon as, the bulk concentration is determined, one can now, plot the SRP concentration profile along a given biofilm media.

5.2.1.1 Distribution of Soluble Residual Product at Fixed-Biofilm Thickness

It has been stated that;

$$S_R^* = \frac{A}{Q} \cdot N_{SR} \quad (4.5)$$

and that the hydraulic loading of a system at macro level is an important factor in determining the SRP generated.

At this stage, the minimum value of SRP is obtained at the bulk-liquid interface and the basic idea is to draw its concentration profile along a given biofilm thickness from the bulk-liquid interface towards the support media where the maximum concentration is generated.

The only way to formulate the concentration gradient of SRP where one of the boundary conditions is known, the following procedure is conducted;

$$S_{R^*} = \frac{A}{Q} \cdot N_{SR} \quad (4.5)$$

$$S_{R^*} = \frac{A}{Q} \cdot D \cdot \left[\frac{dS_R}{dL} \right]$$

$$S_{R^*} = \frac{A}{Q} \cdot D \cdot \left[\frac{\Delta S_R}{\Delta L} \right]$$

if L is assumed as a fixed increment, the above expression yields;

$$S_{R^*} = \frac{A}{Q} \cdot \frac{D}{\Delta L} (\Delta S_R)$$

$$S_R = S_{R^*} \left[\frac{Q}{A} \cdot \frac{\Delta L}{D} \right]$$

For the first increment towards the support media;

$$\Delta L = \Delta L_1 = \Delta L_n$$

$$S_{R1} - S_{R^*} = S_{R^*} \left[\frac{Q}{A} \cdot \frac{\Delta L_1}{D} \right]$$

$$S_{R1} = S_R^* + S_R^* \left[\frac{Q}{A} \cdot \frac{\Delta L}{D} \right]$$

$$S_{R1} = S_R^* \left[1 + \frac{Q}{A} \cdot \frac{\Delta L}{D} \right]$$

$$S_{R2} = S_{R1}^* + S_R^* \left[\frac{Q}{A} \cdot \frac{\Delta L}{D} \right]$$

Therefore; for S_{Rn}

$$S_{Rn} = S_{Rn-1} + S_R^* \left[\frac{Q}{A} \cdot \frac{\Delta L}{D} \right] \quad (5.1)$$

where;

$S_R^* \left[\frac{Q}{A} \cdot \frac{\Delta L}{D} \right]$ is a constant value for a given system.

Figure 5.10. 5.11 and 5.12 illustrate the SRP profiles along a given biofilm thickness for systems where the bulk-substrate and influent substrate concentrations are known.

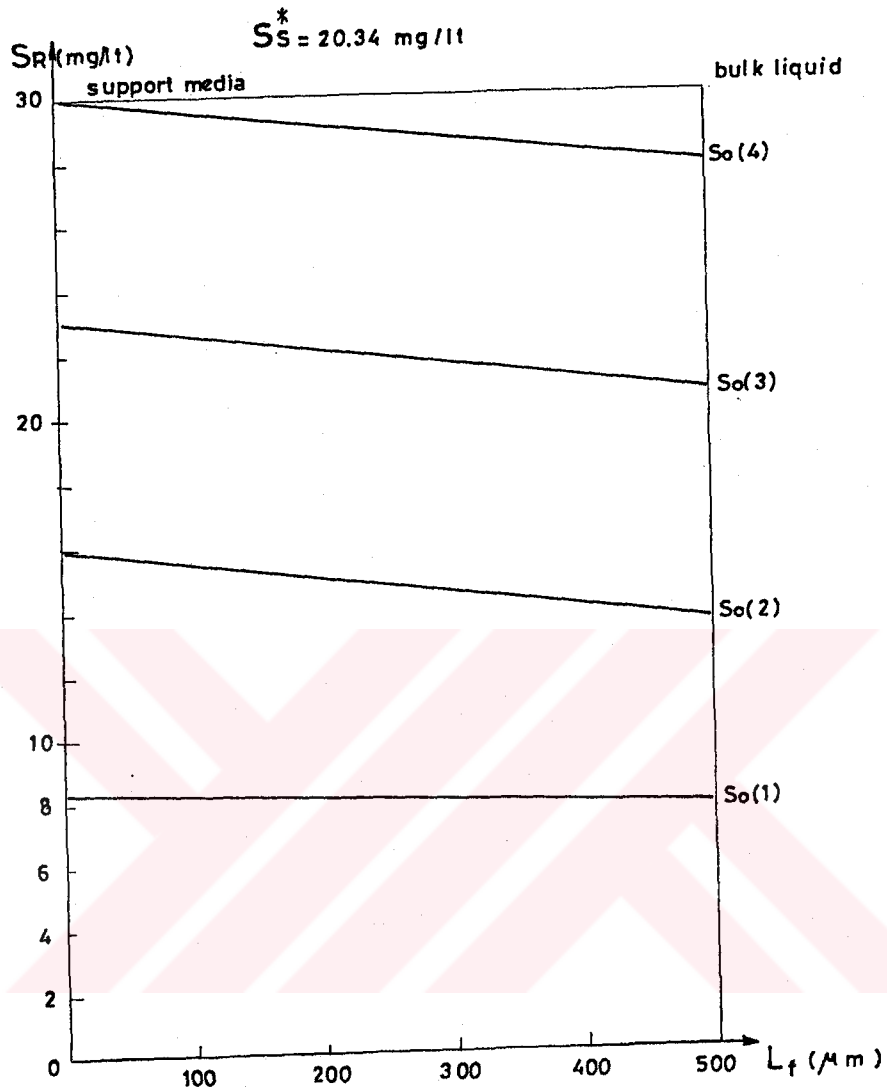


Figure 5-10. The Distribution of Soluble Residual Product along the Fixed film Media for $S_s^* = 20.34$ mg/lt and for Variable S_o Values

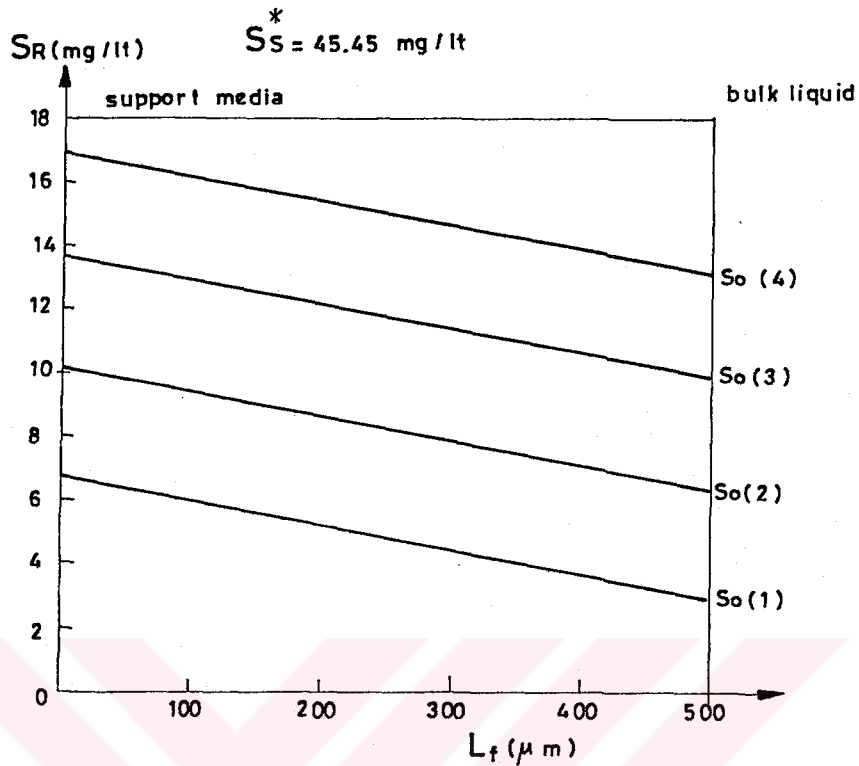


Figure 5-11. The Distribution of Soluble Residual Product along the Fixed-film Media for $S_s^* = 45.45 \text{ mg/lit}$ and for Variable S_o Values.

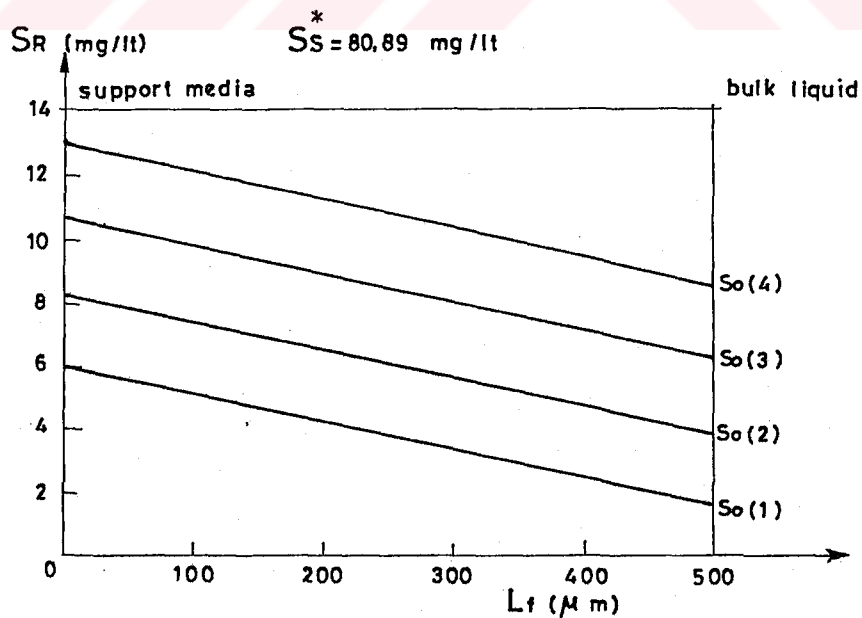


Figure 5-12. The Distribution of SRP along the Fixed-Film Media for $S_s^* = 80.89 \text{ mg/lit}$ and for Variable S_o Values

The SRP profiles along a given fixed-film media, indicate that, the SRP values at the support media reach their maximum values and descent more or less linearly towards the bulk-liquid interface. For lower values of S_R^* and for lower values of S_o , the linear profile does not demonstrate significant variations along the film media as seen in Figure 5.10 where the profile of S_o (1) reinforces the fact that S_o and S_S^* of a system are really the dominant parameters in determining S_R^* values. Observing the other two figures, where S_S^* chosen are higher, indicate quite significant variations in S_{Rmax} and S_R^* (S_{Rmin}) values for identical S_o values. The variation between S_{Rmax} and S_R^* diminish for lower S_S^* values.

Observing these figures together with their respective hydraulic loading values tabulated in Table 5.4. determined as a function of S_S^* , S_o and corresponding flux values obtained at microlevel, lead to a conclusion statement that;

- For a given S_S^* value,

 Q/A decreases as S_o increases, thus, S_R^* increases.
- For a given S_o value,

 Q/A increases as S_S^* increases, thus, S_R^* decreases.

In reality, the second statement supplies more practical information to the designer. Usually, the influent wastewater characteristics are known and the related scientist seeks an appropriate reactor to treat his water mass. Thus, by the information supplied to him, he is aware of the fact that as the removal efficiency of the system decrease, in other words, as S_S^* increase, the hydraulic loading will increase and S_R^*

values decrease. But one thing that is important to note is that, the total effluent concentration, S_T , of a system is much more important rather than separate observation of S_S^* and S_R^* due to the fact that at laboratory scale, one can only determine S_T values experimentally.

Therefore; the analyses of S_T values for various cases where both the operational conditions are considered together with the set values of S_0 and S_S^* , take place in the following section.

Meanwhile, the following figures demonstrate the removal and generation rate distribution of a typically chosen $S_S^* = 30.9$ mg/lit at various S_0 values together with S_R and S_S profiles just to make the following statements more clear;

- The concentration distribution of S_S in a biofilm media is a function of particulate component behaviour and of the removal rate equation expressed in terms of flux units,

- The concentration distribution of S_R is a function of the particulate component, X_S , whose distribution profile is negligible along a film media, thus, it is governed by S_0 , S_S^* and Q/A values of a given water mass and of given operational conditions. It is not a function of S_S and N_{SS} .

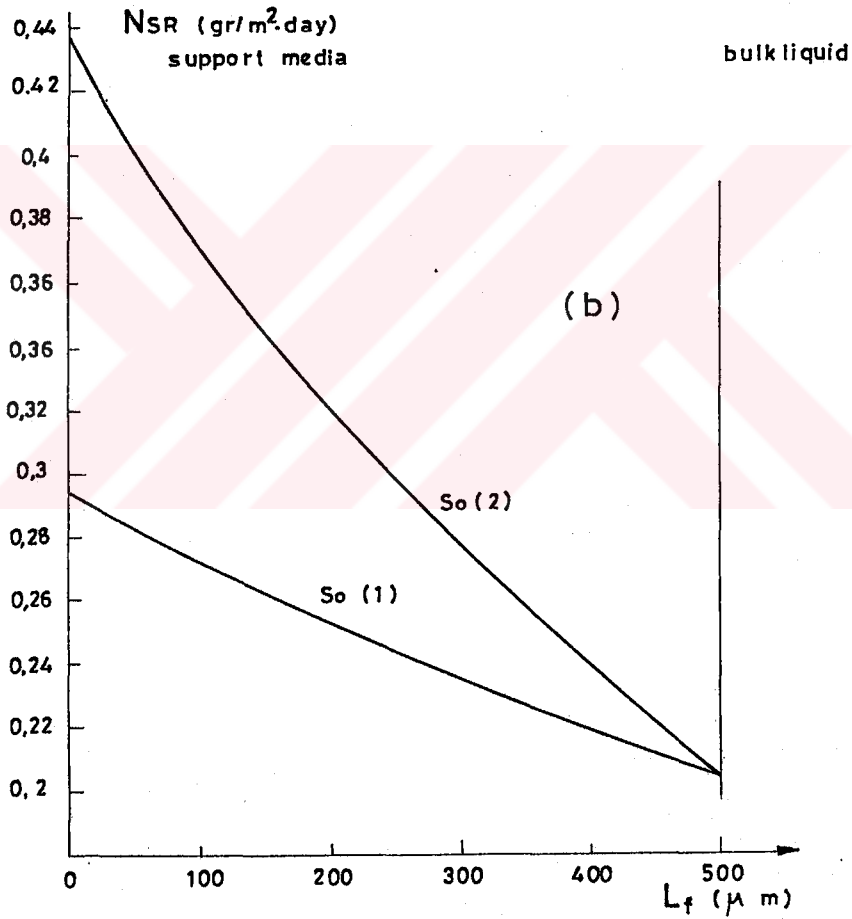
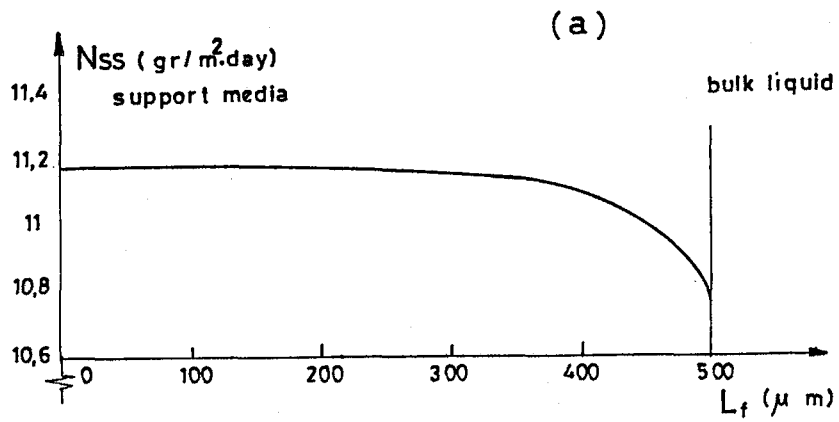


Figure 5-13. The Removal (a) and Generation Rate (b) Distribution along a Fixed-film Media for a Typical S_B^* Value of 30.9 mg/lt

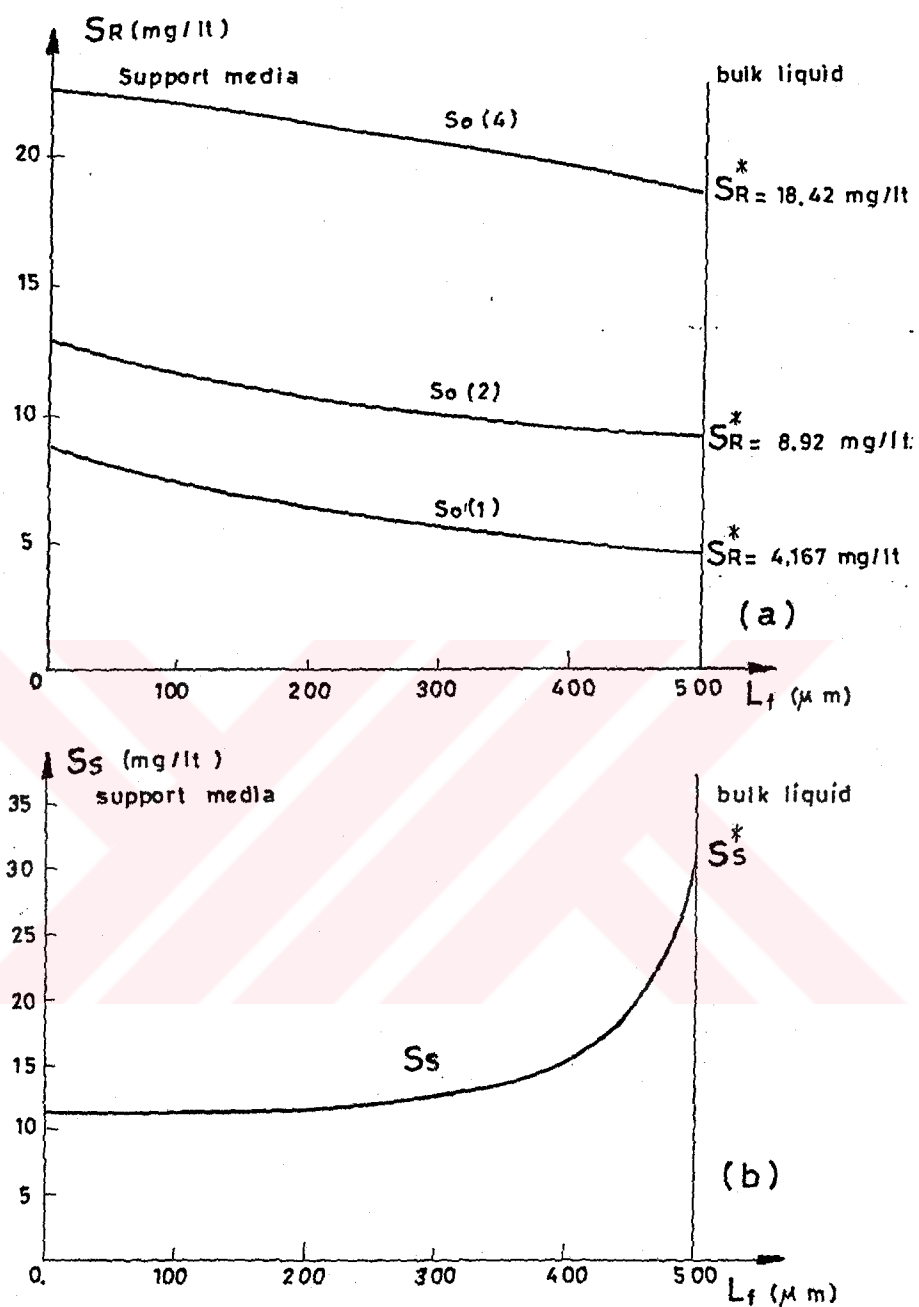


Figure 5-14. The Concentration Profiles of S_R (a) S_S (b) for a Typical Value of $S^* = 30.9$ mg/l along a Fixed-film Media.

5.2.2. Performance of the Model Under Various Operating Conditions

It is clear enough that S_R^* is a function of S_E^* , S_0 and the hydraulic loading imposed on the system. As previously stated, the average viability of a given fixed-film media effects the system's bulk-liquid substrate concentration, S_E^* . The viability of active biomass has been plotted versus increasing S_E^* values to observe the variation of active biomass with S_E^* . Figure 5.15. shows the similar relationship with respect to total effluent concentrations, S_T , of a system where various S_0 values are considered. The total effluent concentration, S_T , of a system is more meaningful practically as it can be determined experimentally.

By inspecting Figure 5.15. it is observed that as the influent substrate concentration, S_0 , increases; the total effluent substrate concentration, S_T , present similar increase with respect to S_E^* profile. Beyond a certain value of S_E^* ($S_E^* > 100$ mg/lt), S_T profiles approach each other together with the S_E^* profile, indicating the obvious fact that, the effect of S_R^* is negligible as S_E^* increases. In other words, $S_T \approx S_E^*$ beyond S_E^* value of 100 mg/lt.

This occurrence certifies the fact that diffusional resistances within a biofilm media retards the generation mechanism of SRP compared with the generated values at the support media. Thus S_R^* values beyond a certain value of S_E^* indicate insignificant effect on S_T . In short, S_T is mainly governed by S_E^* value of a given system which is in turn a function of S_0 .

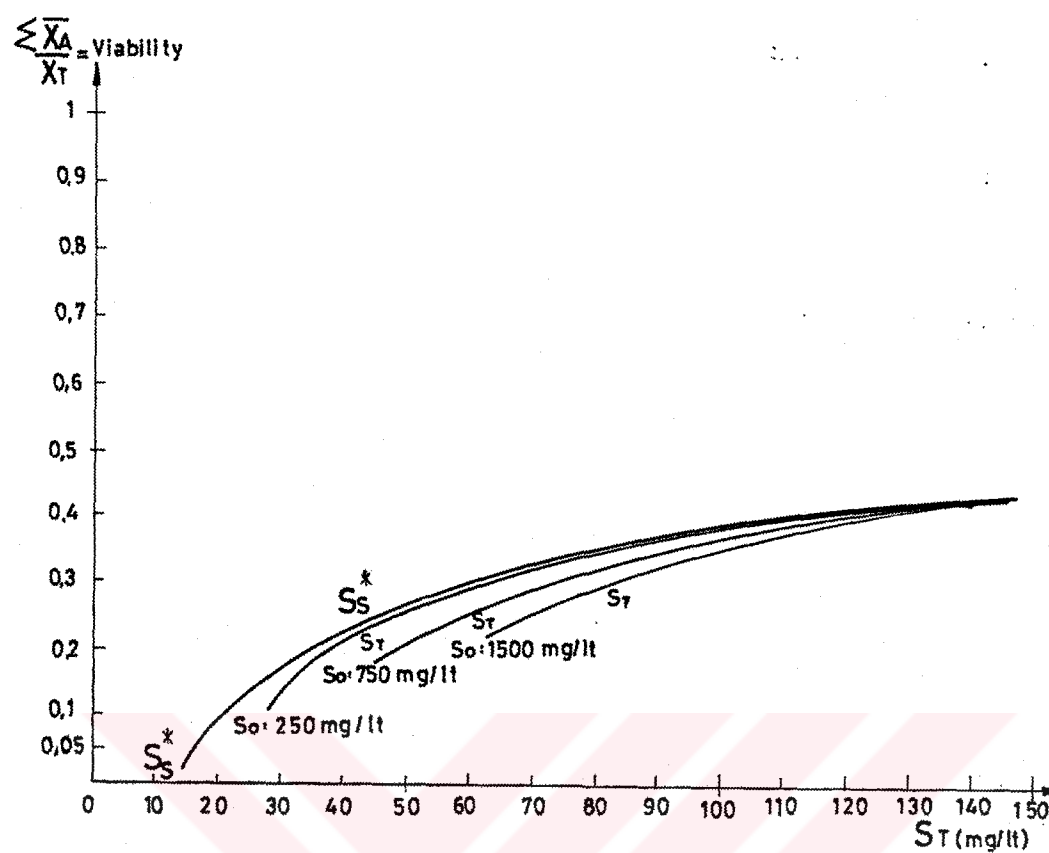


Figure 5-15. The Relationship Between Average Viability and Total Effluent Substrate Concentration for Various S_o Values

Simply; it can be stated that.;

$$S_S^* = f \left[\frac{\sum X_A}{X_T} \right]$$

$$S_R^* = f (S_S^*, S_o)$$

$$S_T = S_S^* + S_R^*$$

$$\therefore S_T = f \left[\frac{\sum X_A}{X_T}, S_S^*, S_o, S_R^* \right]$$

The above statements can be better visualized by examining the following figures (Figures 5.16, 5.17, 5.18, 5.19, 5.20) where S_E^* , S_R^* and S_T values are plotted against active biomass at various S_0 values.

The amount of active biomass in a biofilm media is equal to $\frac{A}{Q} \cdot L_f \cdot X_A$ (gr/m³.day).

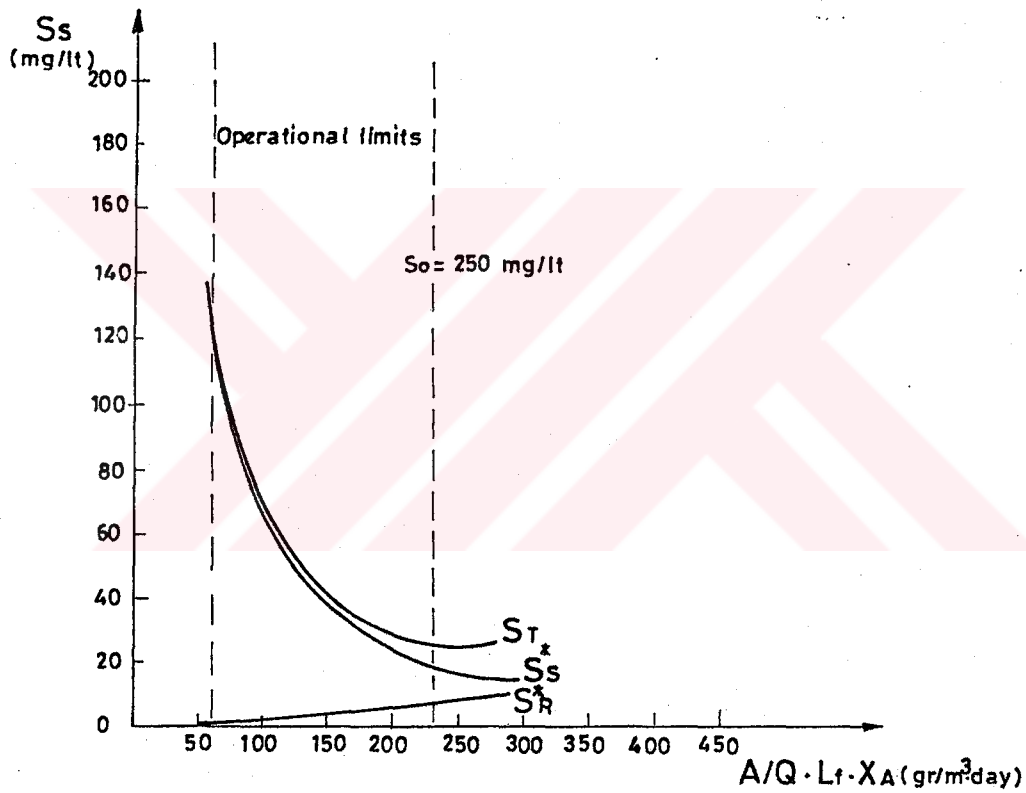


Figure 5-16. S_E^* , S_R^* and S_T values at $S_0 = 250$ mg/l.

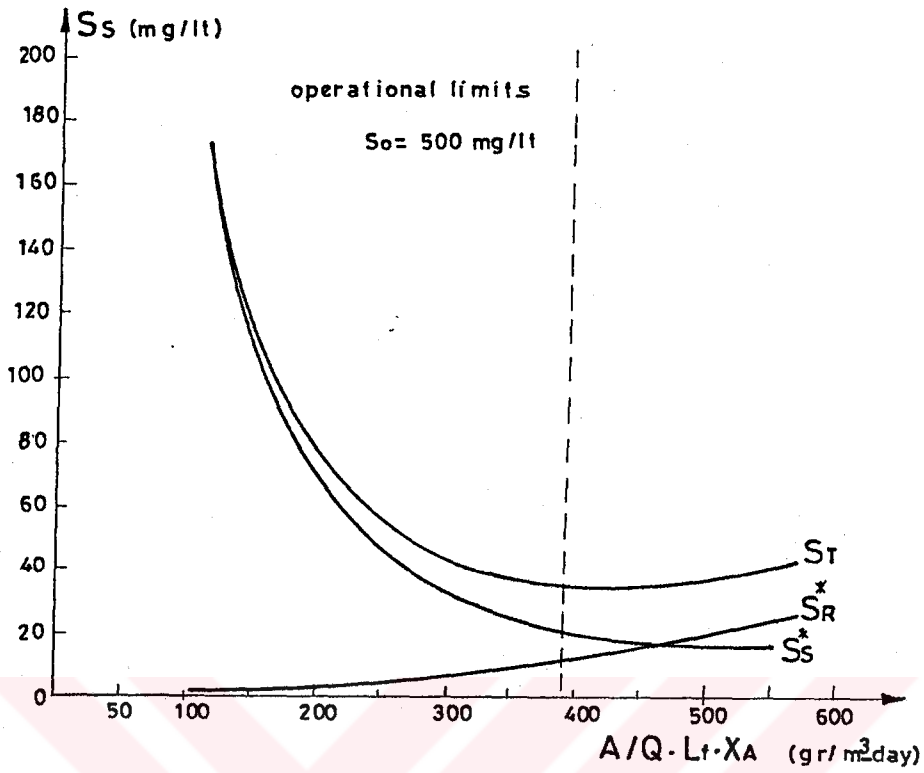


Figure 5-17. S_S^* , S_R^* and S_T values at $S_o = 500$ mg/l.

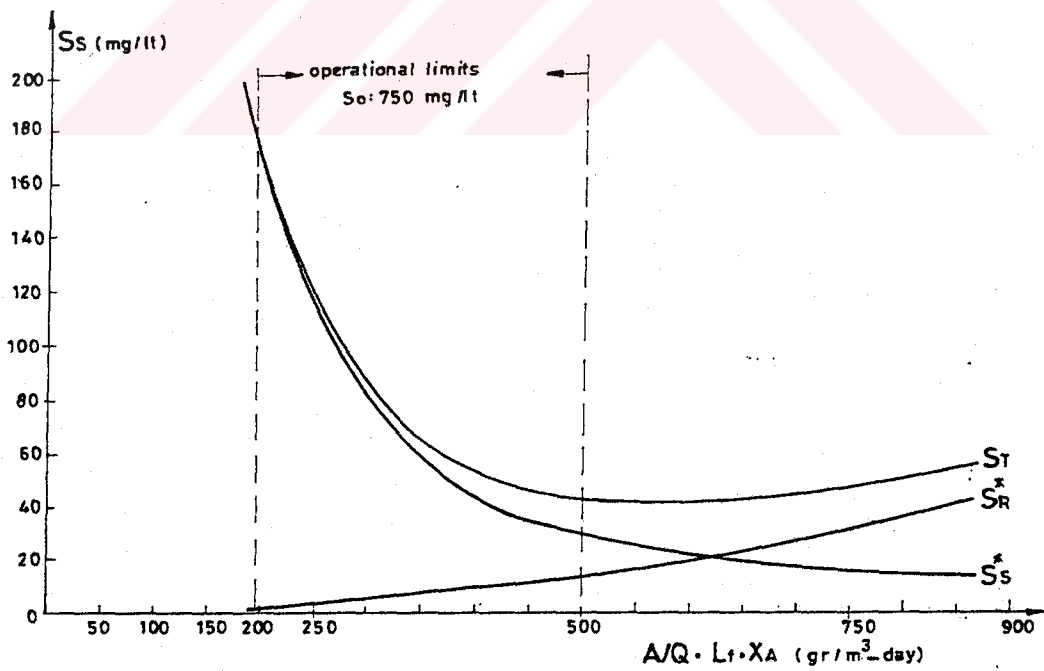


Figure 5-18. S_S^* , S_R^* and S_T values at $S_o = 750$ mg/l.

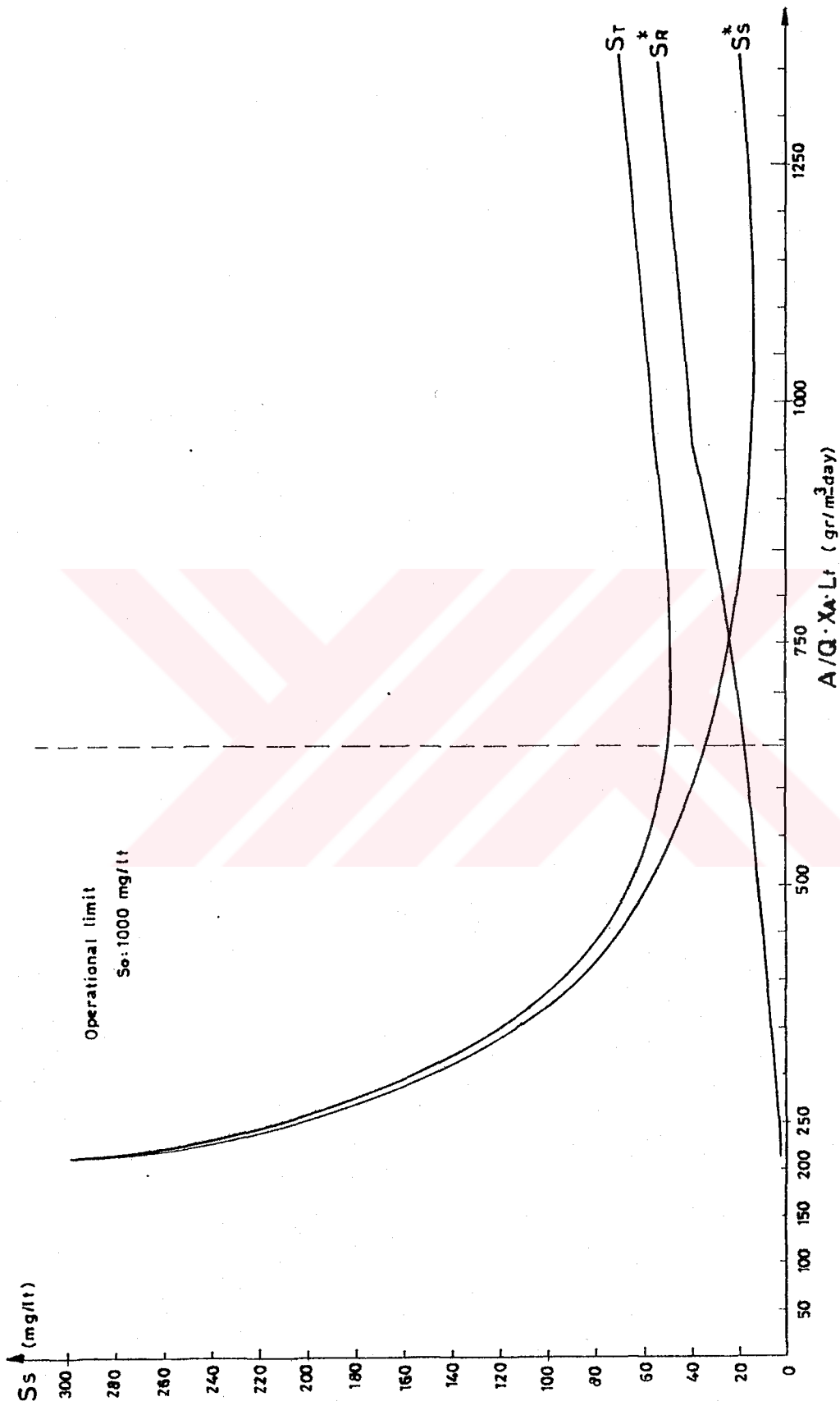


Figure 5-19. S_s^* , S_r^* and S_r values at $S_0 = 1000 \text{ mg/lit}$.

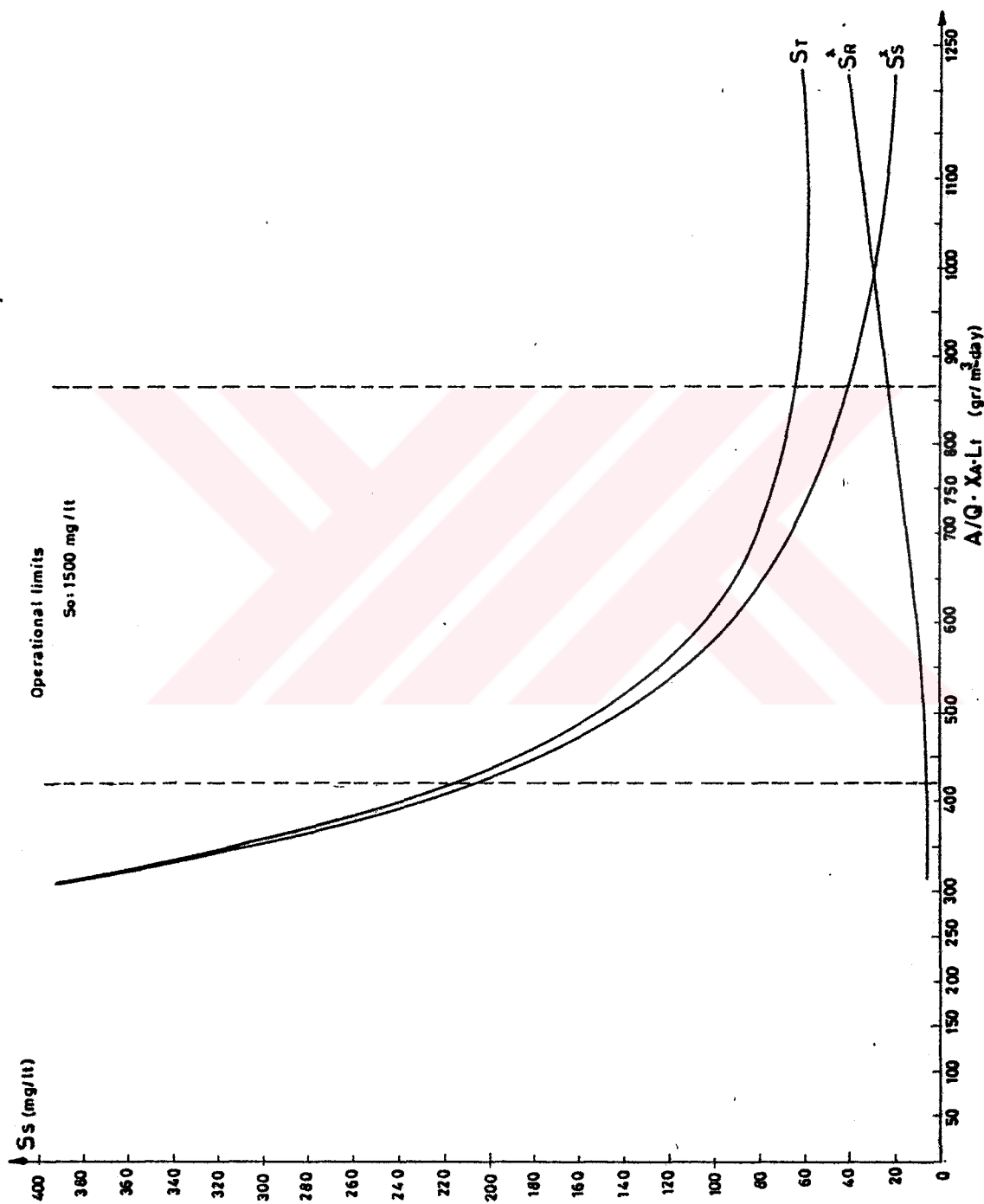


Figure 5-20. S_s^* , S_R^* and S_T values at $S_0 = 1500$ mg/lit.

The operational limits indicated on these figures are given due to the hydraulic and organic loading ranges referred in literature review part. These profiles are drawn according to the calculations given in Tables 5.5 to 5.9 where all the necessary data is available for the designer.

In these figures, the model is applied under such circumstances where the flow rate, Q , and the influent substrate concentration S_0 , of a wastewater is known and kept constant. The S_R^* and S_T values are calculated for such given conditions, at various removal efficiencies. The various efficiencies are obtained by using different reactors of variable surface areas, A .

Restrictions on proper design arise from particular ranges of hydraulic and organic loading criteria, faced in literature. Illustration of the operational limits on the concentration profiles aim to give more insight to the designer on the theory and practice of the fixed-film concept.

Better evaluation of the model findings can be done by observing the data attained in Tables 5.5. to 5.9.

From inspection of both the figures and tables, (Figures 5.16 to 5.20 and Tables 5.5 to 5.9), it can be stated that as the influent substrate concentration increase, in other words, as the wastewater is stronger, more active biomass is to be attained in a film media to realize suitable removal. In such a case, the soluble residual product, S_R^* , generated is reduced with respect to S_E^* values. In the meantime, S_T , the total effluent substrate concentration of the system, approach approximately to the value of S_E^* ($S_T \approx S_E^*$), as S_R^* generated is insignificant to take into account.

Table 5-5. Macroscopic Analysis of the Model at $S_o=250$ mg/lt

S_s^*	A/Q	S_R^*	S_T	S_R/S_T	A/Q.L Σ	X_A (avg.)	A/Q.L Σ .X A	H_o	L_o
mg/lt	m ² /m ³ .d.	mg/lt	mg/lt		day	mg/lt	gr/m ³ .day	m ³ /m ² .day	gr/m ² .day
14.56	333.3 —(*)	12.88	27.44	0.47	0.166	1717	285	0.003 —(*)	0.75
16.33	117.6	9.36	25.7	0.36	0.0588	3918.4	230.4	0.0085	2.125
20.34	47.61	6.533	26.87	0.24	0.0238	8207.2	195.33	0.021	5.25
30.90	20.4	4.166	35.06	0.118	0.0102	14341.2	146.28	0.049	12.25
40.26	14	3.22	43.5	0.07	0.0701	17776	124.6	0.071	17.75
56.61	9.09	2.30	58.91	0.039	0.0045	22120	99.54	0.11	27.5
74.66	7.09	1.895	70.4	0.026	0.003546	24576	87.14	0.141	35.2
68.51	6.329	1.72	76.38	0.022	0.00316	25702.4	81.22	0.158	39.5
80.89	5.68	1.57	82.46	0.019	0.00284	26764	76.01	0.176	44
93.52	4.65	1.32	94.84	0.013	0.00232	28720	66.63	0.215	53.75
106.24	3.84	1.12	107.40	0.010	0.00192	30480	58.52	0.26	65
118.94	3.19	0.95	119.89	0.0079	0.00159	32088	51.02	0.31	78.37
131.54	2.887	0.8575	132.4	0.00647	0.00144	33557.6	48.322	0.3463	86.57

(*) The lines under certain values indicate that operational limits are being exceeded.

Table 5-6. Macroscopic Analysis of the Model At $S_o=500$ mg/lt

S_s^*	A/Q	S_R^*	S_T	S_R/S_T	A/Q.L ϵ	X_A (avg.)	A/Q.L ϵ . X_A	H_o	L_o
mg/lt	$m^2/m^3.d.$	mg/lt	mg/lt		day	mg/lt	$gr/m^3.day$	$m^3/m^2.day$	$gr/m^2.day$
14.56	666.7	26.33	40.89	0.644	0.333	1716.88	571.72	0.00149	0.745
16.33	243.19	19.40	35.736	0.5428	0.1215	3918.9	476.149	0.0041	2.05
20.34	96.5	13.64	33.98	0.40	0.048	8207.2	393.94	0.010	5.2
30.9	43.85	8.92	39.82	0.224	0.0219	14341.2	314.07	0.022	11.4
40.26	31.25	7.1763	47.43	0.15	0.0156	17776	277.3	0.032	16
56.61	20.68	5.275	61.885	0.085	0.010	22120	221.2	0.048	24.17
68.51	16.8	4.486	72.996	0.061	0.0084	24576	206.44	0.059	29.76
74.66	15.327	4.173	78.83	0.0529	0.00766	25702.4	196.88	0.065	32.62
50.89	14.07	3.894	84.784	0.046	0.007	26764	187.348	0.071	35.5
93.52	12.05	3.43	96.95	0.035	0.006	28720	172.32	0.083	41.5
106.24	10.5	3.06	109.30	0.0279	0.00525	30480	160	0.095	47.61
118.94	9.288	2.75	121.69	0.0225	0.00464	32000	148.48	0.107	53.8
131.54	8.30	2.50	134	0.0186	0.00415	33520	139.10	0.12	60.2
143.96	7.48	2.29	146.25	0.0156	0.00374	34904	130.54	0.13	66.84
168.13	6.21	1.95	170.08	0.01146	0.003105	37228	115.75	0.16	80.5

Table 5-7. Macroscopic Analysis of the model At $S_o=750$ mg/lt

S_s^*	A/Q	S_R^*	S_T	S_R/S_T	A/Q.L _z	X_A (avg.)	A/Q.L _z .X _A	H_o	L_o
mg/lt	m ² /m ³ .d.	mg/lt	mg/lt		\day	mg/lt	gr/m ³ .day	m ³ /m ² .day	gr/m ² .day
16.33	367.84	29.35	45.68	0.6424	0.1839	3918.4	720.59	0.0027	2.03
20.34	155.27	21.89	42.23	0.518	0.0776	8200	637.69	0.0064	4.83
30.9	67	13.67	44.57	0.306	0.0335	14341.2	480.43	0.0149	11.19
40.25	47.46	11.37	51.63	0.22	0.023	17776	408.848	0.021	15.8
56.25	32.35	8.255	64.86	0.127	0.0161	22120	356.132	0.0309	23.175
68.51	26.54	7.09	75.60	0.093	0.0132	24576	324.40	0.0376	28.25
74.66	24.33	6.62	81.28	0.081	0.0121	25702.4	310.99	0.041	30.826
80.89	22.46	6.2	87	0.071	0.0112	26764	299.75	0.0445	33.39
93.52	19.465	5.54	99	0.055	0.0097	28720	278.58	0.051	38.53
106.24	17.18	4.99	111.2	0.0045	0.00859	30480	261.82	0.058	43.65
118.94	15.38	4.57	123.5	0.037	0.00769	32000	243.20	0.065	48.76
131.54	13.93	4.20	135.75	0.030	0.00696	33520	233.3	0.0717	53.84
143.96	12.74	3.91	147.87	0.026	0.00637	34904	222.34	0.078	58.5
168.13	10.89	3.43	171.56	0.019	0.00544	37280	202.80	0.092	68.87

Table 5-8. Macroscopic Analysis of the Model at $S_o=1000$ mg/lt

S_s^*	A/Q	S_R^*	S_T	S_R/S_T	A/Q.L π	X_A (avg.)	A/Q.L π .X $_A$	H_o	L $_o$
mg/lt	m ² /m ³ .d.	mg/lt	mg/lt		day	mg/lt	gr/m ³ .day	m ³ /m ² .day	gr/m ² .day
16.33	493	39.38	55.71	0.7068	0.2465	3918.4	965.88	0.002	2
20.34	197	27.77	48.117	0.577	0.0985	8200	807.7	0.00507	5.07
30.9	90	18.36	49.26	0.37	0.045	14341.2	645.35	0.0111	11.11
40.25	64.18	14.73	54.99	0.268	0.032	17776	568.83	0.01558	15.58
56.61	44	11.224	67.83	0.165	0.022	22120	486.64	0.0227	22.727
68.51	36.3	9.685	78.2	0.124	0.0181	24576	444.82	0.0275	27.548
74.66	33.34	9.07	83.73	0.108	0.0167	25702.4	429.23	0.0299	29.99
80.89	30.85	8.539	89.43	0.095	0.0154	26764	412.16	0.0324	32.40
93.52	26.87	7.63	101.15	0.075	0.0134	28720	384.85	0.03721	37.21
106.24	23.85	6.952	113.19	0.0614	0.0119	30480	362.71	0.0419	41.92
118.94	21.489	6.38	125.32	0.051	0.0107	32000	342.4	0.0465	46.53
131.54	19.56	5.90	137.44	0.043	0.00978	33520	327.825	0.05112	51.12
143.96	17.99	5.52	149.48	0.0369	0.00899	34904	313.786	0.05558	55.58
294.91	13.40	4.33	201.19	0.021	0.0067	39848	266.98	0.0746	74.62

Table 5-9. Macroscopic Analysis of the Model at $S_o=1500$ mg/lt

S_s^*	A/Q	S_R^*	S_T	S_R/S_T	A/Q.L _F	X_A (avg.)	A/Q.L _F .X _A	H _o	L _o
mg/lt	m ² /m ³ .d.	mg/lt	mg/lt		day	mg/lt	gr/m ³ .day	m ³ /m ² .day	gr/m ² .day
15.26	1314.16	71.22	66.487	0.8234	0.657	2464	1618.85	7.6X10 ⁻⁴	1.14
20.34	297.77	41.98	62.325	0.6735	0.1488	8207.2	1221.23	0.0033	4.95
30.9	136.92	27.93	58.83	0.4747	0.068	14320	973.76	0.0073	10.95
40.25	97.61	22.35	62.61	0.357	0.0488	17776	867.46	0.010	15
56.61	67.35	17.17	73.78	0.2327	0.03367	22120	744.78	0.0148	22.27
68.51	55.74	14.88	63.4	0.1784	0.02787	24576	684.93	0.01734	26.91
74.66	51.36	13.97	68.63	0.1576	0.02568	25702.4	660.03	0.0194	29.2
80.89	47.639	13.186	94.07	0.140	0.0238	26764	636.98	0.0209	31.48
93.52	41.70	11.844	105.36	0.1124	0.02085	28720	598.81	0.0239	35.97
106.24	37.196	10.824	117	0.0924	0.01859	30480	566.62	0.0268	40.32
118.94	33.684	10	128.94	0.077	0.0168	32000	537.6	0.0296	44.53
131.54	30.828	9.31	140	0.066	0.0154	33520	516.208	0.0324	48.65
143.96	28.50	8.749	152.709	0.057	0.01425	34904	497.382	0.035	52.63
179.83	23.52	7.48	187.31	0.0399	0.01176	38320	450.64	0.0425	63.77
213.26	20.3	6.65	219.91	0.030	0.01015	41199	418.17	0.049	73.9
294.91	15.22	5.288	300	0.0176	0.00761	47040	357.97	0.065	98.55

If the flow rate, Q , is assumed to be constant, the variation in hydraulic loading, Q/A , will only be due to the reactor dimensions, in other words, the only variable term will be the surface area, A , of the reactor.

The performance of the system is observed under conditions where;

- the influent substrate concentration, S_0 , is fixed,
- the flow rate, Q , is fixed,
- the surface area of the reactor is variable.

On the other hand, the graphical distribution of S_r under variable S_0 values which is another important function of S_r^* is illustrated in Figures 5.21 and 5.22 with respect to the removal and generation rate mechanisms, expressed in terms of flux.

As a summary, the following statements can be written for a wastewater of known, influent substrate concentration, S_0 ;

- The removal efficiency of the system tends to decrease, as S_r^* increases,
- The hydraulic loading of the system, (Q/A) , increases, as S_r^* increases,

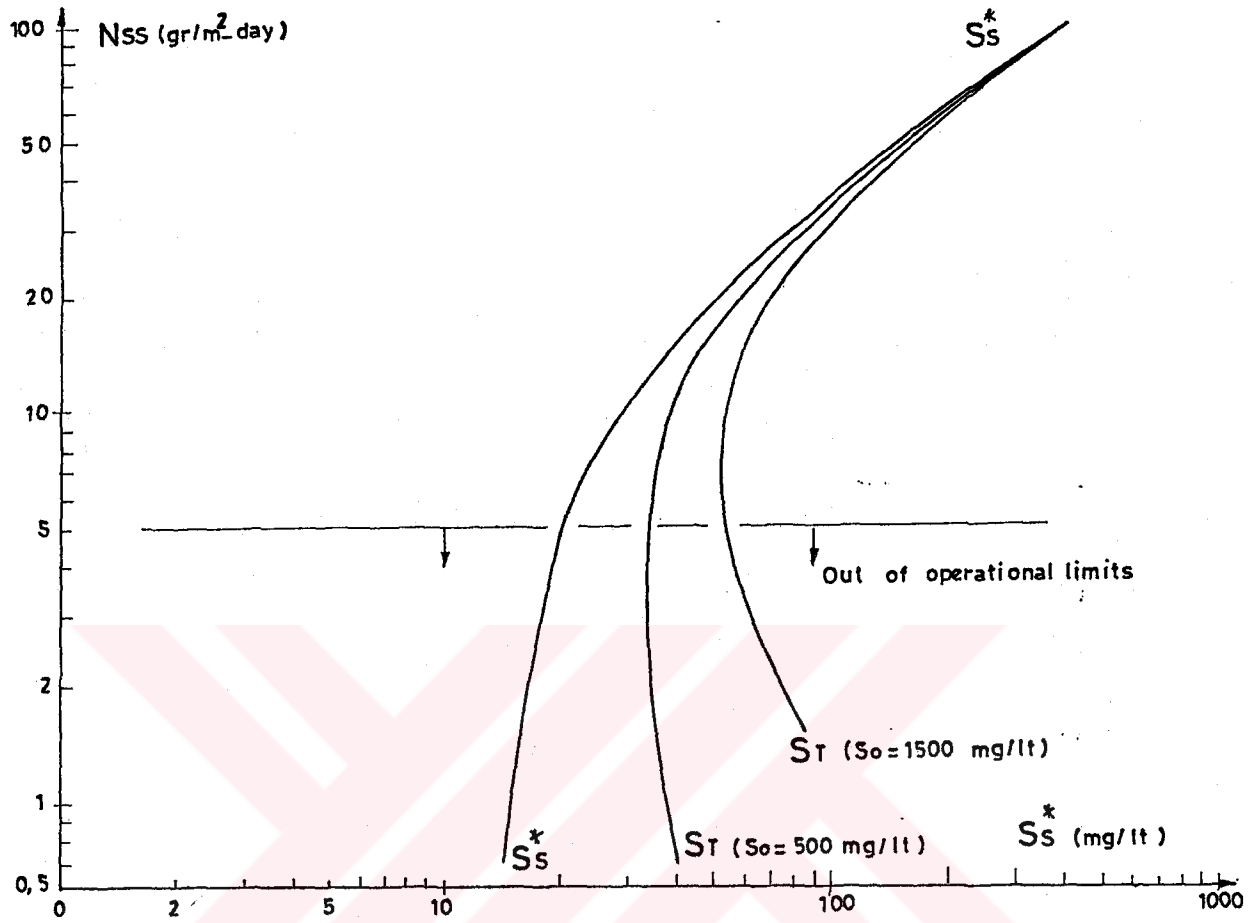


Figure 5-21. The S_s^* and S_T Values observed as compared to the Removal Rate of Soluble Substrate at Variable S_o Values

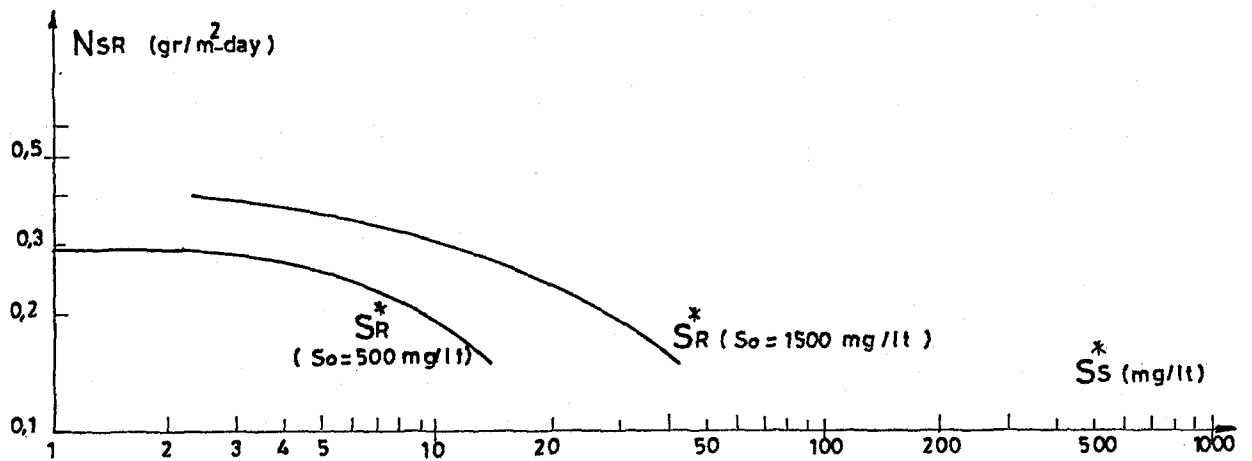


Figure 5-22. The S_R^* Values observed as compared to the Generation Rate of SRP at Variable S_o Values

- The average concentration of active microorganisms of the film media, (X_A) , increases, as S_S^* increases,

- The total amount of active biomass in the system, $(A/Q.L_f.X_A)$, decreases, as S_S^* increases,

Therefore;

- The total effluent concentration, S_T , of the system increases,

- The soluble residual product, S_R^* , of the system decreases,

- The ratio of S_R^*/S_T decreases,

- The surface area of the system should decrease as far as the flow rate of the wastewater is kept constant,

- The flow rate of the wastewater should increase as far as the surface area of the system is kept constant.

In order to make these statements more clear, a few more graphical distributions are illustrated.

Figure 5.24 shows the variation of S_T values obtained at variable S_0 values with respect to the total amount of active biomass present in a system.

On the other hand, the behaviour of S_T with respect to hydraulic and organic loading are illustrated in Figures 5.24 and 5.25 respectively.

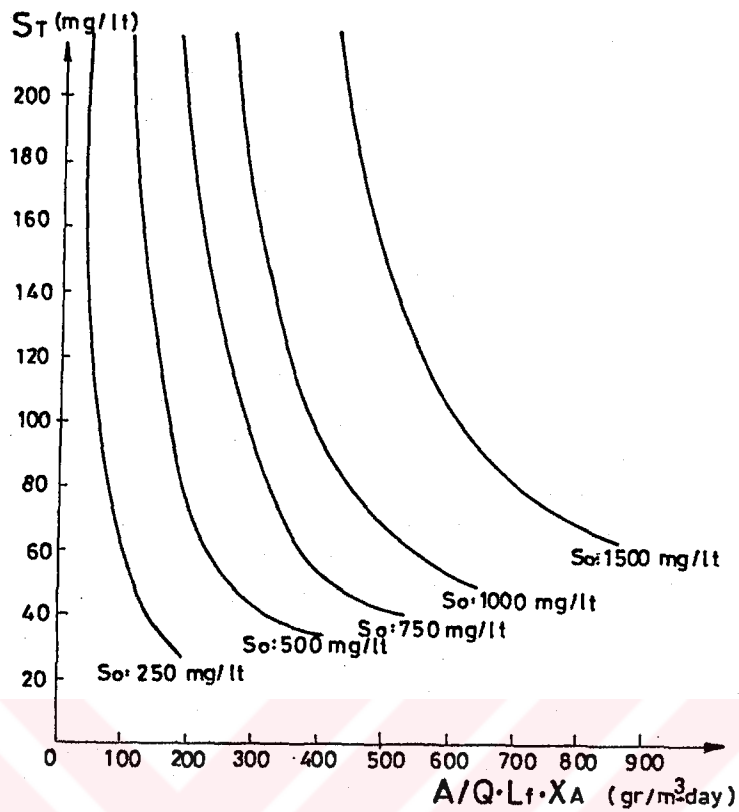


Figure 5-23. Distribution of S_T Values due to the Total Amount of Active Biomass in a System at Variable S_o Values.

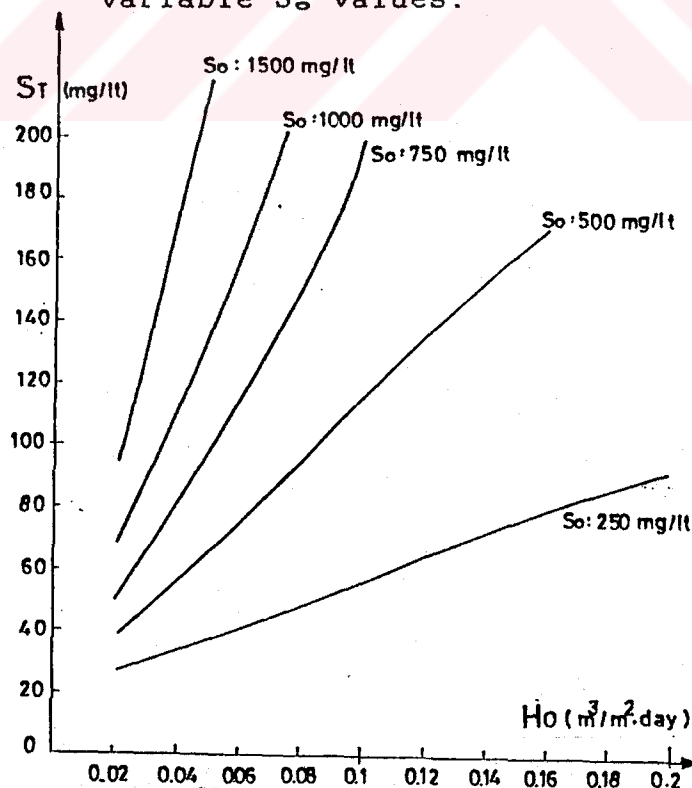


Figure 5-24. Distribution of S_T due to Hydraulic Loading at Variable S_o Values

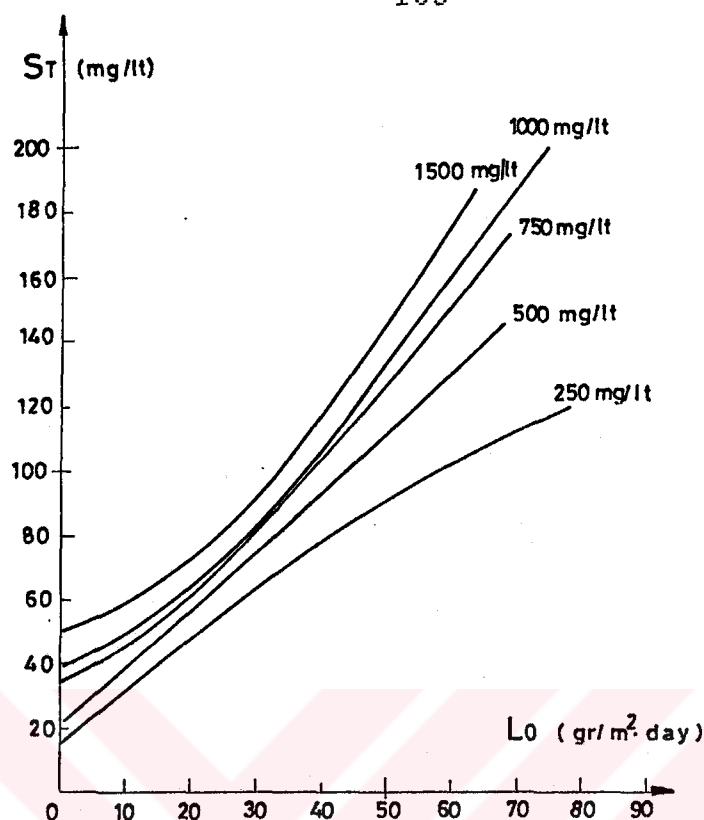


Figure 5-25. Distribution of S_T due to Organic Loading at Variable S_0 Values.

Observation of these figures (Figures 5.24 and 5.25) correlate the previously stated conclusions, that S_T , the summation of S_E^* and S_R^* of the system mainly depend on the initial characteristics of the wastewater such as, the strength of the wastewater and hydraulic conditions.

As influent wastewater concentration in terms of COD increases, S_T arising from the removal and generation mechanisms, increase as well.

On the other hand, increase of hydraulic loading cause a proportional increase of S_T , but the increase is more steeper as S_0 values increase. The increase in S_T is more smoother at S_0 value of 250 mg/lt whereas, the increase gets more steeper at S_0 values of 1500 mg/lt due to hydraulic loading.

5.2.2.1. A Design Example

Assuming a wastewater of the following initial characteristics;

- $S_o = 1000 \text{ mg/lt}$
- $Q = 500 \text{ m}^3/\text{day}$

where the required effluent substrate concentration is;

- $S_r \leq 50 \text{ mg/lt.}$

The solution procedure will be as follows for a fixed-film reactor where the biofilm thickness, $L_f=500 \mu\text{m}$, $X_f=80000 \text{ mg/lt}$, and all the other kinetic and stoichiometric coefficients are the typical values used in the modelling;

$$i) Q(S_o - S_s^*) = A \cdot N_{ss}$$

Assuming $S_s^*=35 \text{ mg/lt}$, the corresponding flux value, N_{ss} , of the system is either obtained from the output of the micromodel or can be obtained from Figure 5.26 which is a practical chart prepared for design purposes. It indicates the corresponding S_R^* and S_T values for a given S_s^* value, under variable influent substrate concentrations.

For a given S_s^* value of 35 mg/lt , the corresponding flux value is $N_{ss}=13 \text{ gr/m}^2.\text{day}$.

$$ii) \text{ Surface area of the system; } A = \frac{Q(S_o - S_s^*)}{N_{ss}}$$

$$A = 37115 \text{ m}^2$$

iii) The hydraulic loading of the system;

$$Q/A = 0.0135 \text{ m}^3/\text{m}^2\cdot\text{day}.$$

iv) The corresponding flux value of SRP is obtained from the micromodel,

$$N_{SR} = 0.218 \text{ gr/m}^2\cdot\text{day}$$

$$v) S_R^* = \frac{A}{Q} \cdot N_{SR} = 16 \text{ mg/lt.}$$

$$vi) S_T = S_S^* + S_R^* = 35 + 16 = 51 \text{ mg/lt}$$

Therefore the given effluent conditions is fulfilled.

More practically, in order to determine the A/Q and S_R^* value of a system without referring back to find N_{SS} and N_{SR} values from the output of the micro model, two charts are being prepared (Figures 4.27 and 4.28). From Figure 5.27, one can easily determine the A/Q value of a system at a given S_o value, and the corresponding S_R^* value can be determined from Figure 5.28.

Using the charts; the A/Q value of the system is determined as $74 \text{ m}^2/\text{m}^3\cdot\text{day}$, for $S_o:1000 \text{ mg/lt}$. Then; the related S_R^* value to this A/Q value at $S_o:1000 \text{ mg/lt}$ is obtained as 16 mg/lt .

If one refers to Figure 5.26, no information about the hydraulic loading of the system is enclosed, whereas using charts based on A/Q values are practically more useful, in the sense that they refer to A/Q values of a system, which is an important design parameter.

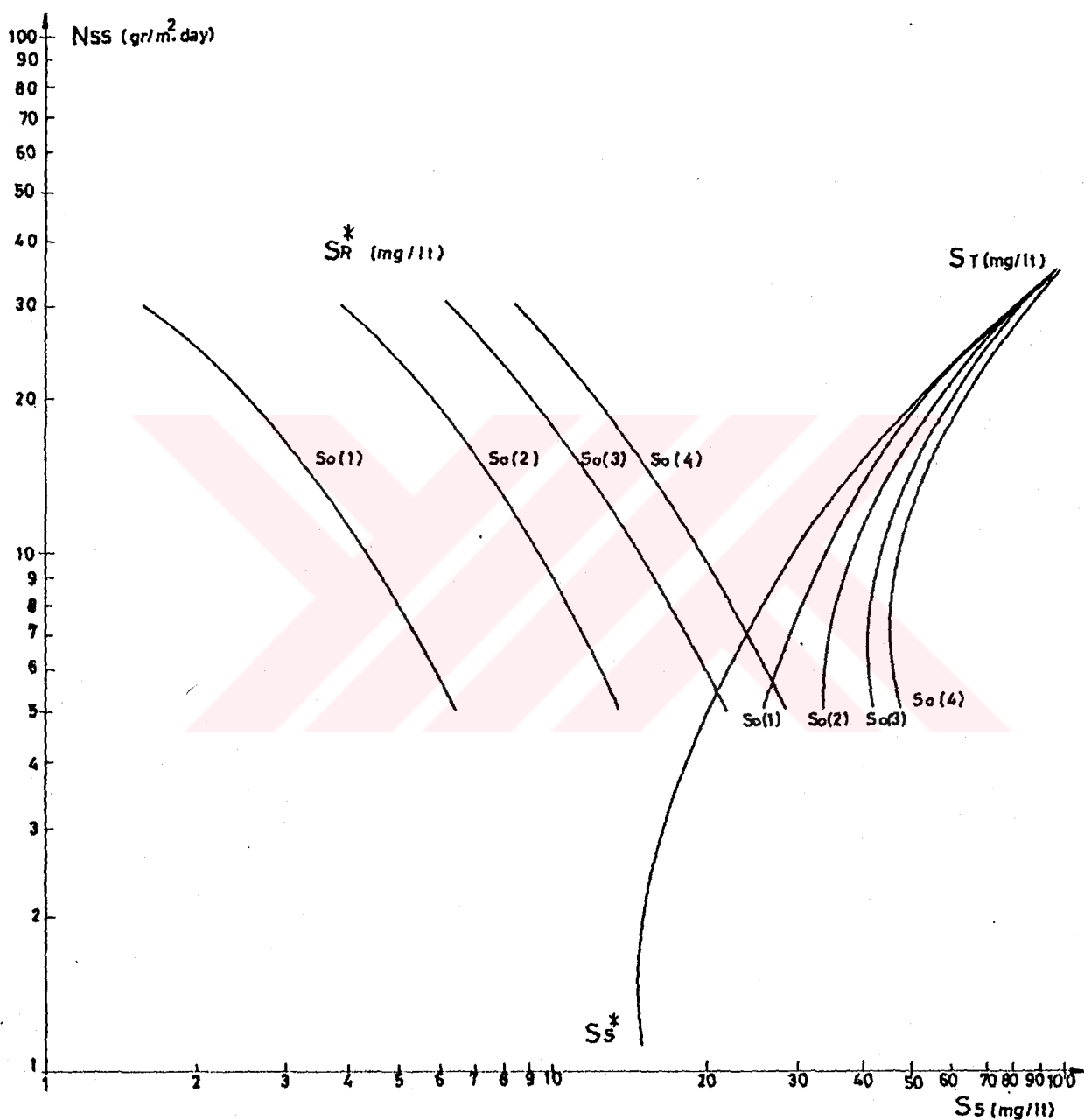


Figure 5-26. A Chart prepared for Design Purposes used to determine S_R^* and S_T Values for Variable S_o Values

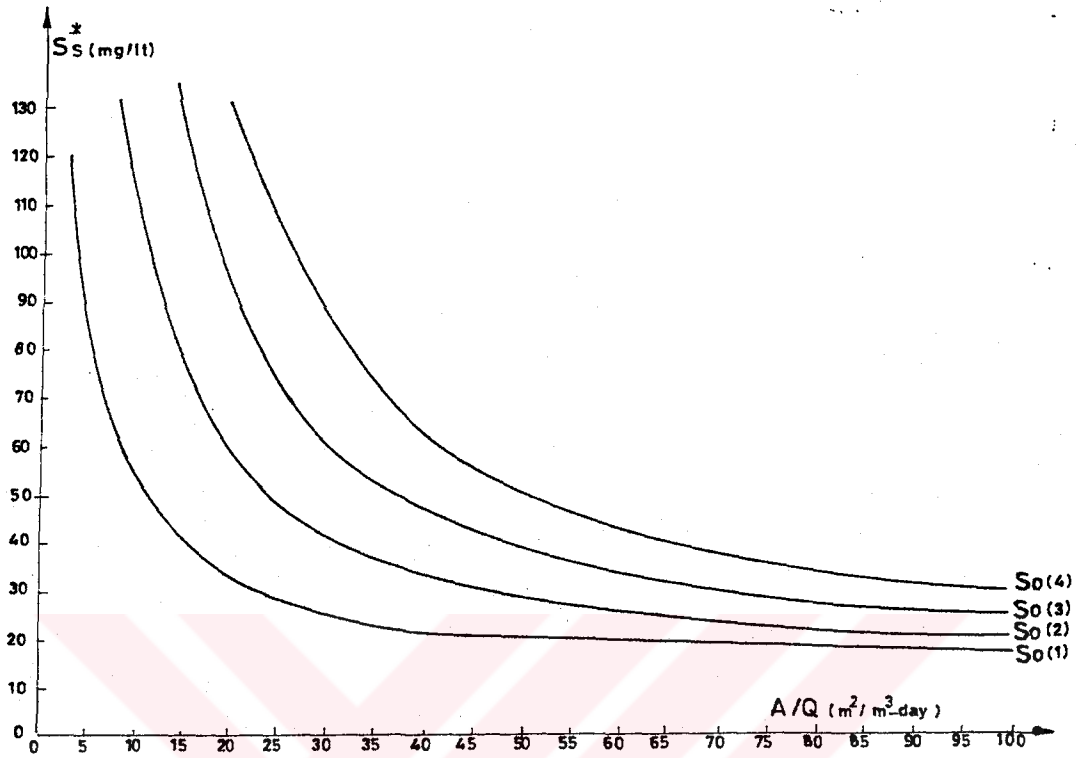


Figure 5-27. The S_s^* Values related to A/Q Values under Variable S_o Values

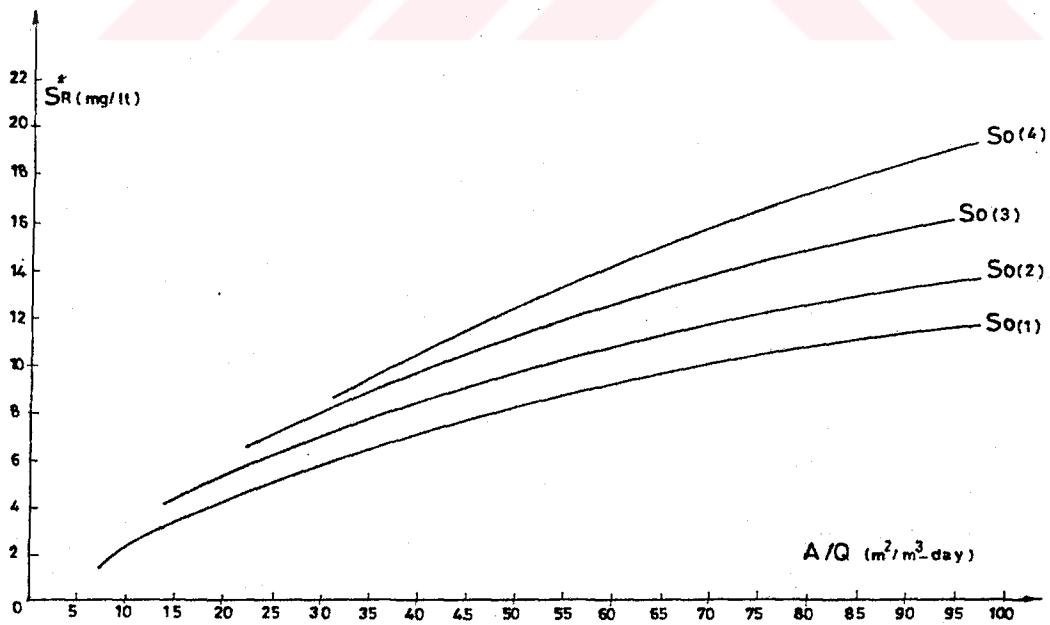


Figure 5-28. The S_R^* Values related to A/Q Values under Variable S_o Values

vii) The volume of the reactor appropriate to achieve the given effluent characteristics with the given initial conditions and specific surface of $100 \text{ m}^2/\text{m}^3$ (manufacturer's typical data) is,

$$100 \frac{\text{m}^2}{\text{m}^3} \cdot V_R : 37115 \text{ m}^3$$

$$V_R = 37 \text{ m}^3.$$

5.2.3. Comparison of Different Order Reaction kinetics of Soluble Substrate in Fixed-Film Systems

The Monod kinetics, as previously mentioned, is followed to express the soluble substrate removal at any point in a biofilm media, where, the molecular diffusion, which is the only means of mass transport within the biofilm, is related to the substrate concentration by Fick's Second Law.

In short, the total conversion rate of the substrate for any point in the biofilm at steady-state conditions puts forth the ordinary differential equation,

$$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_T \quad (2.15)$$

As this equation is non-linear, it cannot be solved explicitly. Numerical workers have presented analytical solutions where the Monod Relationship reduces to 1^{st} , $1/2$ and 0 -order approximations.

On the other hand, this study expresses the total conversion rate by the concept of viability and by utilizing the Modified Task Group Model as follows;

$$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_A - (1-\beta) k_h \frac{(X_s/X_A)}{k_x + (X_s/X_A)} X_A \quad (4.1)$$

As a summary, the different order reaction kinetics used in literature can be stated together with the reaction kinetics developed in this study in Table 5.10.

Table 5-10. Different Order Reaction kinetics and The Related Expressions of the Total Conversion Rates at Steady-State Conditions

Type of Reaction kinetics	Expression of the Total Conversion Rate of Soluble Substrate at Steady-State Conditions
Monod Relationship	$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} \cdot X_T$
First Order Approximation of the Monod kinetics	$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \cdot S_s \cdot X_T$
Zero Order Approximation for cases of full penetration	$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \cdot X_T$
Zero Order Approximation Leading to Half-Order for Cases of Partial Penetration	$\frac{Dd^2S_s}{dL^2} = K_{1/2a} \cdot S_s^{1/2}$
The Modified Task Group Model Relationship	$\frac{Dd^2S_s}{dL^2} = \frac{\hat{\mu}}{Y} \frac{S_s}{K_s + S_s} X_A - (1-\beta) k_h \frac{X_s/X_A}{k_x + (X_s/X_A)} X_A$

The total conversion rate expression at steady-state conditions for soluble substrate developed and used in this study resembles the intrinsic Monod Relationship.

The model differentiates the particulate matter as, active biomass, X_A , slowly degradable substrate, X_S , and as particulate matter due to decay, X_E . The active biomass, instead of total particulate matter is responsible for the removal mechanism in all biological treatment systems. Although, the amount of active biomass is constant in a suspended-growth culture, its distribution in an attached-growth culture varies within the biofilm media.

The second term in the expression is due to recycling of soluble substrate, S_S , generated as an end-product of hydrolysis process where, a portion of it exists as soluble residual product. The negligible effect of X_S , with respect to active biomass, X_A , and to inert mass, X_E , in a biofilm media, causes the second term to have an insignificant effect in the expression. Neglecting the second term yields;

$$\frac{Dd^2S_S}{dL^2} = \frac{\hat{\mu}}{Y} \cdot \frac{S_S}{K_S + S_S} X_A \quad (5.2)$$

The only difference of this expression from the Monod Relationship is that, Monod Utilizes the total mass of microorganisms, where, the above expression (Eqn. 5.2) utilizes only the active portion of the microorganisms present in a biofilm media.

Figure 5-28 shows the S_S^* and S_T values observed as compared to removal rate of soluble substrate,

expressed in terms of flux. The Monod Relationship reduces to an order of zero in cases of full penetration and to halforder, in cases of partial penetration if certain assumptions are done initially. In this study, partial penetration is valid as the S_{min} value of the model never approach to zero. The profile, N_{ss} versus S_s^* shows that the similar Monod profile is obtained as reductions due to certain assumptions are not considered. One can easily see that, as S_s^* exceeds 200 mg/lt, the order of the removal rate mechanism approach to an order of 0.66.

It is seen that the S_T profiles of the two different reactors having different surface areas, follow parallel paths with respect to S_s^* profile till S_T exceeds the value of 100 mg/lt. It is, then, that the S_T profiles approach each other for both reactors and even collide as S_T exceeds the value of 200 mg/lt. This collision is mainly due to the previously mentioned and illustrated fact that, as S_s^* exceeds a certain value, S_R^* generated becomes more insignificant compared to higher S_s^* and thus, $S_T \approx S_s^*$. This fact can be better visualized in Figure 5.29.

5.2.4. Comparison of Soluble Residual Product (SRP) in Activated Sludge and in Fixed-Film Systems

Artan (1988) proposed a model for SRP generation in activated sludge system where S_R^* is defined as,

$$S_R^* = \beta \cdot K \cdot X_B Q \quad (5.3)$$

where;

$$K = \beta \cdot k_h \cdot \frac{X_B/X_A}{k_x + (X_B/X_A)} \cdot X_A \quad (5.4)$$

Q = hydraulic detention time, day.

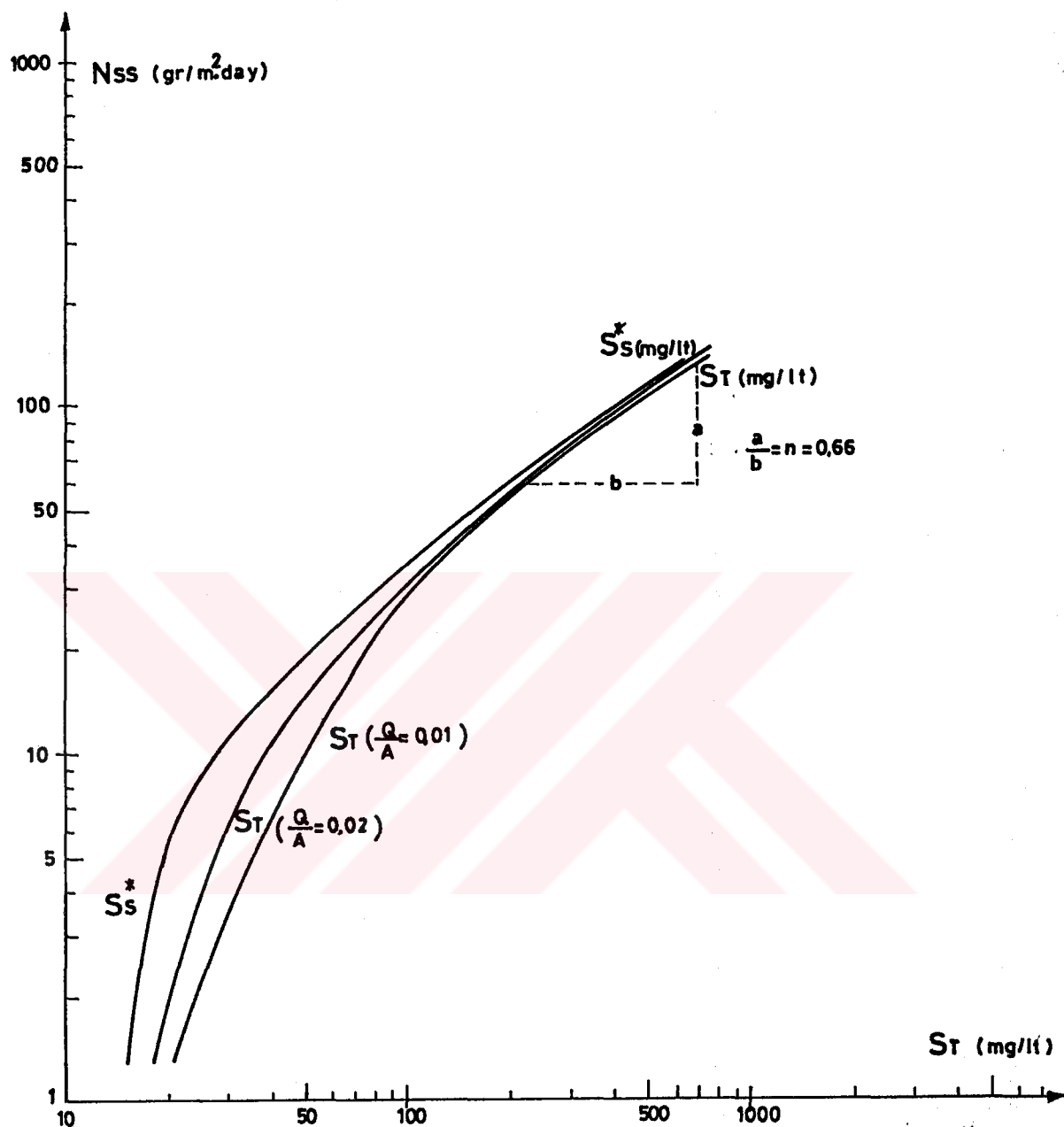


Figure 5-29. The S_r Values observed as compared to the Removal Rate of Soluble Substrate

In the proposed model, continuously mixed aeration tank is under consideration where the values of the particulate matter, X_A , X_B and X_E are constant everywhere in the reactor. They do not show any variations as in the case of fixed-film systems. Therefore, X_B is a fraction of X_A and is denoted as follows;

$$X_B = \alpha \cdot X_A \quad (5.5)$$

where,

α is a constant value.

Therefore X_B is defined as;

$$X_B = \frac{(1-f_e)b_H}{\mu - b_H + K} \cdot X_A \quad (5.6)$$

where,

$$\mu - b_H = \frac{1}{Q_c} \quad (5.7)$$

Q_c = sludge age, day.

The expression for S_R^* in a suspended-growth system can be written as follows;

$$S_R^* = \frac{\beta K(1-f_e)b_H}{\mu - b_H + K} \cdot X_A \cdot Q \quad (5.8)$$

In this equation, $X_A \cdot Q$, yields the total amount of active biomass in a suspended-growth system.

In an activated sludge system, it can be concluded that SRP generation is a function of the total amount of active biomass present in the reactor, $X_A \cdot Q$, which in turn, is a function of S_0 and Q_c .

$$S_R^* = f(X_A Q)$$

$$X_A Q = f(S_0, Q_c)$$

In fixed-film systems, the total amount of active biomass responsible for the reactions is a function of hydraulic loading and biofilm thickness.

$$S_R^* = f\left[\frac{A}{Q}, L_f, X_A\right]$$

The following table compares the results of both systems on an example where the flow rate of the wastewater and the volume of the reactors are equal.

Table 5-11. shows, that, both systems of equal reactor volume and of equal flowrate may hold equal amount of active biomass, however, in activated sludge system, the entire volume of the reactor is used, whereas, the effective volume in the fixed-film system is the volume of the biofilm which is only 5 m³.

The next example aims to calculate SRP generated from both systems where the influent and effluent substrate concentrations are equal. In other words, comparing both systems where equal removal conditions prevail (Table 5.12).

Table 5-11. Comparison of Activated Sludge and Fixed-Film Systems where the Flowrate of the Wastewater and the Volume of Reactor are equal.

Parameter	Activated Sludge System	Fixed-Film System
Flow Rate of the wastewater Q (m^3/day)	100	100
Volume of the reactor V_R (m^3)	100	100
Active Biomass (X_A) Concentration (mg/lt)	800-5000	40.000 - 110.000
Typically chosen, X_A (mg/lt)	3000	60.000
The total amount of active biomass ($kg/m^3 \cdot day$)	$X_A Q = 3 \text{ Kg}/m^3 \cdot day$	$\frac{A}{Q} \cdot L_f \cdot X_A = 3 \text{ kg}/m^3 \cdot day$ $L_f = 500 \mu m$ $A = 10.000 \text{ m}^2 \text{ using}$ Manufacturer's typical data of specific surface = $100 \text{ m}^2/m^3$
Effective Volume of the reactor (m^3)	$V_R : 100$	$V_B = 5$ (Volume of the Biofilm Media)

Table 5.12. Comparison of SRP generation in Activated Sludge and in Fixed-Film System where S_o and S_s^* Values are Equal

Parameter	Activated Sludge System	Fixed-Film System
Influent Substrate Concentration S_o , mg/lt	250	250
Effluent Substrate Concentration S_s^* , mg/lt	23.5	23.5
Active Biomass (X_A) Concentration (mg/lt)	800-5000	40.000 - 110.000
The total amount of active biomass, gr/m ³ .day	$X_A Q = 118$	$\frac{A}{Q} \cdot L_f \cdot X_A = 173$
The residual soluble product S_R^* , mg/lt	6.6	5.57
The total Effluent Concentration $S_T = S_s^* + S_R^*$, mg/lt	30.1	29.07

It is seen that more active biomass is needed in fixed-film systems to reach equal removal efficiency in both systems, but, fixed-film systems generate less amount of SRP at the bulk-liquid surface compared to activated sludge systems. The SRP concentration within an aeration tank of an activated sludge system is constant and same everywhere within the reactor. But, this fact is not valid in the case of fixed-film system. SRP generation perform a concentration profile throughout biofilm thickness as mentioned previously. Figure 5.30. shows the concentration profile of SRP in a biofilm media using the given initial and effluent conditions used in Table. 5.12.

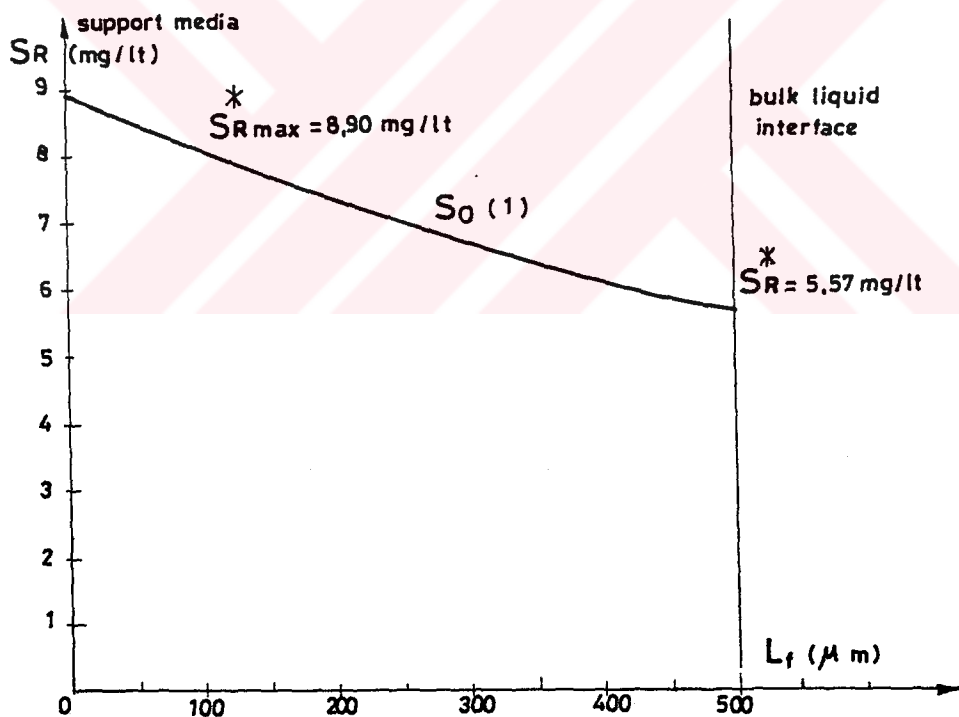


Figure 5-30. SRP Generation Profile within a Biofilm Media

Figure 5.29 indicates that S_R value at the support media is equal to 8.90 mg/lt and this maximum value generated tends to decrease towards the bulk interface due to diffusion limitation. In the case of attached-growth systems, diffusion is rate limiting, therefore, for equal influent and effluent substrate concentration;

$S_{Rmax} = 8.90 \text{ mg/lt}$ at the support media

$S_R^* = 5.57 \text{ mg/lt}$ at the bulk-liquid interface

for a fixed-film system, and

$S_R^* = 6.6 \text{ mg/lt}$ in an activated sludge system.

Actually, the SRP generation is higher in fixed-film systems, but, according to diffusional resistance, the S_R^* value is lower compared to activated sludge system.



PART VI

SENSITIVITY OF PARAMETERS ON SRP GENERATION IN THE MODEL

The only way that the model parameters are analysed happens to be by means of sensitivity analyses, which refer to the effect of individual parameters on the model findings and is of mathematical nature.

The most important parameters of the model are; b_H , K_S , $\hat{\mu}$, β and k_H . Each governing parameter is analysed individually and their effect on S_R^* and on S_T are, in turn, evaluated.

6.1. EFFECT OF PARAMETERS ON THE MODEL FINDINGS

6.1.1. Decay Coefficient

In order to determine the effect of the decay coefficient on the model, two other b_H values; 0.25 d^{-1} and 0.40 d^{-1} are selected apart from the typical value of 0.62 d^{-1} , and the analyses are done using two different S_0 values; 250 mg/lt and 750 mg/lt , so as to evaluate the results in terms of variable b_H and S_0 values. The following tables (Tables 6.1 to 6.4) summarize the results of calculations at macro level, and the figures (Figures 6.1 to 6.4) attempt to achieve better understanding of the effect of b_H on the model findings.

Table 6-1. Analysis of the Model at $S_0=250$ mg/lt, $b_H=0.25$ d⁻¹ $S_{min}=4.12$ mg/lt

S_E^* mg/lt	$A/Q(^*)$ m ² /m ³ .d.	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L\#$ day	X_A (avg.) mg/lt	$A/Q.L\# \cdot X_A$ gr/m ³ .day	H_2 m ³ /m ² .day	$L_0(^*)$ gr/m ² .day
$S_E^*=6.11$	224.20	8.05	14.16	0.568	0.1121	4789.6	536.91	0.00446	1.115
$S_E^*=8.82$	79.24	4.9258	13.74	0.358	0.03962	10518.4	416.74	0.0126	3.154
$S_E^*=13.33$	42.37	3.234	16.56	0.195	0.021	15466.4	324.8	0.0236	5.90
$S_E^*=19.33$	26.93	2.2675	21.597	0.105	0.01346	19520	262.739	0.037	9.28
$S_E^*=26.53$	18.89	1.6812	28.21	0.0595	0.009445	23010	217.33	0.0529	13.234
$S_E^*=34.63$	14.12	1.305	35.93	0.0363	0.00706	25943	183.158	0.0708	17.70
$S_E^*=57.27$	8.10	0.79	58.06	0.0136	0.00405	31712	128.43	0.123	30.86
$S_E^*=66.79$	6.77	0.67	67.46	0.0099	0.00338	33520	113.3	0.1477	36.927
$S_E^*=81.19$	5.328	0.538	81.72	0.00658	0.00266	36000	95.76	0.1876	46.92
$S_E^*=95.49$	4.29	0.438	95.92	0.0045	0.00214	38128	81.6	0.233	58.27
$S_E^*=104.87$	3.75	0.389	105.26	0.00369	0.001875	39392	73.86	0.267	66.67
$S_E^*=118.64$	3.09	0.32	118.96	0.002689	0.001545	41096	63.49	0.3236	80.90
$S_E^*=127.58$	2.73	0.289	127.869	0.00226	0.001365	42128	57.50	0.3663	91.575
$S_E^*=140.62$	2.27	0.24	140.86	0.0017	0.001135	43550	49.43	0.44	110.132

The lines under certain values of A/Q and L_0 denote that operational limits are exceeding.

Table 6-2. Analysis of the Model at $S_o=750$ mg/lt, $b_H=0.25$ d⁻¹, $S_{min}=4.12$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_F$ day	X_A (avg.) mg/lt	$A/Q \cdot L_F \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 (*) gr/ $m^2 \cdot day$
$S_E^*=6.11$	683.84	24.56	30.67	0.80	0.3419	4790	1637.72	0.00146	1.096
$S_E^*=8.82$	243.568	15.125	23.94	0.63	0.12178	10518.4	1280.93	0.00410	3.079
$S_E^*=13.33$	131.89	10.067	23.397	0.43	0.0659	15466.4	1019.23	0.00758	5.6865
$S_E^*=19.33$	85.30	7.1827	26.51	0.27	0.04265	19520	832.528	0.01172	8.7924
$S_E^*=26.53$	61.13	5.44	31.97	0.170	0.0305	23010	701.805	0.01635	12.268
$S_E^*=34.63$	46.909	4.33	38.964	0.111	0.0234	25943	608.480	0.0213	15.988
$S_E^*=57.27$	29.12	2.845	60.115	0.047	0.01456	31712	461.726	0.0343	25.755
$S_E^*=66.79$	25.263	2.508	69.298	0.036	0.0126	33520	423.407	0.03958	29.687
$S_E^*=81.19$	21.1	2.1456	83.33	0.0257	0.0105	36000	378	0.0474	35.545
$S_E^*=95.49$	18.186	1.870	97.36	0.0192	0.00909	38128	346.58	0.0549	41.2505
$S_E^*=104.87$	16.675	1.7309	106.60	0.0162	0.00833	39392	328.43	0.0599	44.9775
$S_E^*=118.64$	14.866	1.56	120.20	0.01297	0.00743	41096	305.466	0.06726	50.45
$S_E^*=127.58$	13.88	1.4699	129.05	0.01139	0.00694	42128	292.368	0.080	60.53
$S_E^*=140.62$	12.65	1.3536	141.97	0.0095	0.00632	43550	275.45	0.079	59.288
$S_E^*=149.06$	11.959	1.289	150.35	0.00857	0.005979	44400	265.46	0.0836	62.714
$S_E^*=169.22$	10.554	1.15	170.37	0.006749	0.005277	46320	244.43	0.09475	71.06
$S_E^*=205.77$	8.626	0.9687	206.738	0.00464	0.00431	49520	213.43	0.1159	86.946
$S_E^*=266.20$	6.44	0.754	266.95	0.00282	0.00322	54056	175.68	0.1552	116.459

TABLE 6-3. Analysis of the Model at $S_o=250$ mg/lt, $b_H=0.40$ d⁻¹, $S_{min}=6.8789$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_s^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_{\infty}$ day	$X_A(avg.)$ mg/lt	$A/Q \cdot L_{\infty} \cdot X_A$ gr/ $m^3 \cdot day$	H_o $m^3/m^2 \cdot day$	L_o^* gr/ $m^2 \cdot day$
$S_s^*=9.34$	269.40	10.326	19.667	0.525	0.1347	2821.08	380	0.00371	0.9279
$S_s^*=10.35$	132.476	8.081	18.431	0.4384	0.06628	5037.76	333.49	0.007548	1.887
$S_s^*=14.19$	52.369	5.295	19.485	0.2717	0.02618	10213.6	267.39	0.01909	4.774
$S_s^*=20.25$	29.345	3.677	23.927	0.1536	0.01467	14792	216.998	0.034	8.519
$S_s^*=28.08$	19.1988	2.686	30.766	0.087	0.00960	18660.4	179.139	0.05208	13.021
$S_s^*=42.24$	11.8652	1.818	44.06	0.041	0.00593	23415.2	138.850	0.08428	21.07
$S_s^*=58.24$	8.1645	1.3188	59.558	0.022	0.00408	27200	110.976	0.12248	30.62
$S_s^*=69.47$	6.60	1.0935	70.56	0.0155	0.0033	29429.6	97.1176	0.1515	37.878
$S_s^*=80.91$	5.44	0.9206	81.83	0.01125	0.00272	31368	85.32	0.1838	45.955
$S_s^*=109.60$	3.55	0.62	110.22	0.00562	0.001775	35423.2	62.876	0.28169	70.42
$S_s^*=120.88$	3.03	0.538	121.418	0.00443	0.001515	36792	55.7398	0.33	82.508
$S_s^*=142.85$	2.219	0.4017	143.251	0.0028	0.001109	39221.6	43.4967	0.45	112.66

Table 6-4. Analysis of the Model at $S_0=750$ mg/lt, $b_H=0.40$ d⁻¹, $S_{min}=6.8789$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3.d.$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q.L_x$ day	$X_A(avg.)$ mg/lt	$A/Q.L_x.X_A$ gr/ $m^3.day$	H_0 $m^3/m^2.day$	L_0^* gr/ $m^2.day$
$S_E^*=9.34$	661.41	25.35	34.69	0.73	0.3307	2821.08	932.08	0.00151	1.1339
$S_E^*=10.35$	408.78	24.93	35.285	0.7065	0.20439	5037.76	1029.667	0.00244	1.83
$S_E^*=14.19$	163.51	16.53	30.72	0.5380	0.08175	10160	830.58	0.0061158	4.5868
$S_E^*=20.25$	93.21	11.65	31.90	0.36518	0.0466	14792	689.38	0.01073	8.046
$S_E^*=28.08$	62.455	8.737	36.81	0.2274	0.03122	18660.4	582.72	0.0160	12
$S_E^*=42.24$	40.42	6.1936	48.43	0.1278	0.02021	23415.2	473.22	0.0247	18.55
$S_E^*=58.24$	29.45	4.74	62.98	0.075	0.01472	27200	400.384	0.0339	25.466
$S_E^*=69.47$	24.889	4.10	73.576	0.05572	0.0124	29429.6	366.236	0.033	25.09
$S_E^*=80.91$	21.55	3.64	84.55	0.043	0.010775	31368	337.99	0.0464	34.80
$S_E^*=109.60$	16.20	2.835	112.435	0.0252	0.0081	35423.2	286.927	0.0617	46.3
$S_E^*=120.88$	14.76	2.623	123.5	0.0212	0.00738	36792	271.52	0.0677	50.81
$S_E^*=142.85$	12.57	2.276	145.126	0.01568	0.00628	39221.6	246.5	0.0795	59.66
$S_E^*=178.93$	10.04	1.878	180.80	0.0103	0.00502	42640	214.05	0.099	74.70
$S_E^*=265.19$	6.50	1.287	266.477	0.0048	0.00325	49405.6	160.568	0.1538	115.38

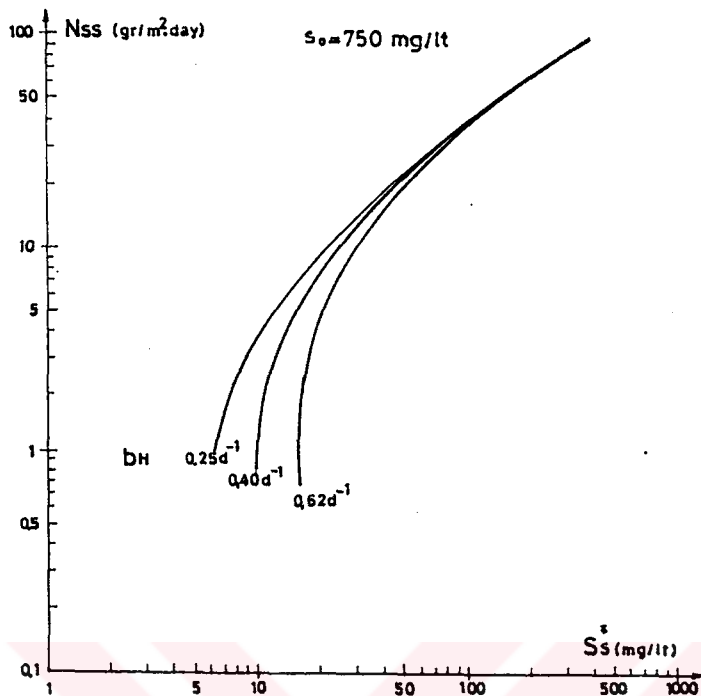


Figure 6-1. The Effect of b_H on N_{SS} versus S_{S^*} Profile

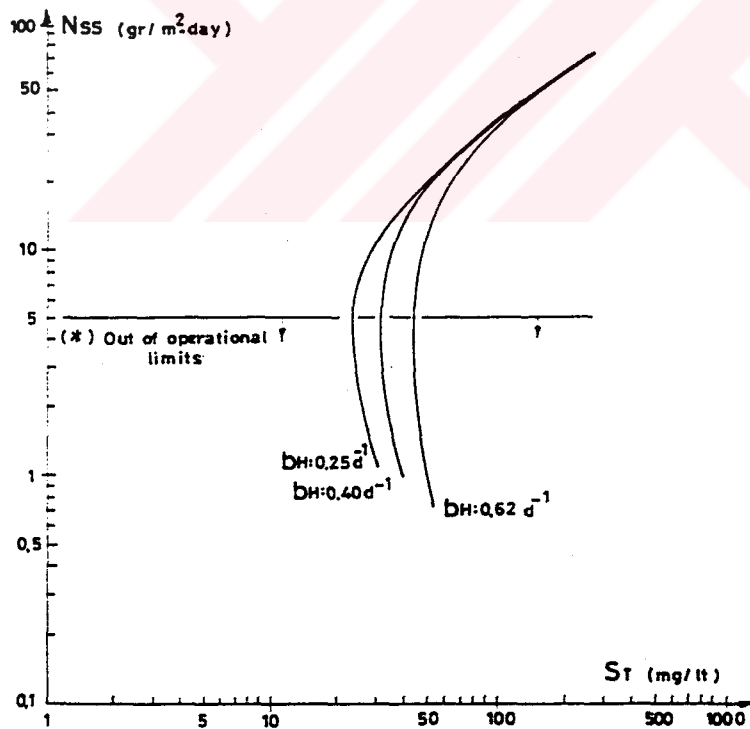


Figure 6-2. The Effect of b_H on N_{SS} versus S_T Profile at $S_0: 750 \text{ mg/lt}$.

(*) The direction indicated designates the fact that the acceptable ranges of Q/A and L_0 are being exceeded.

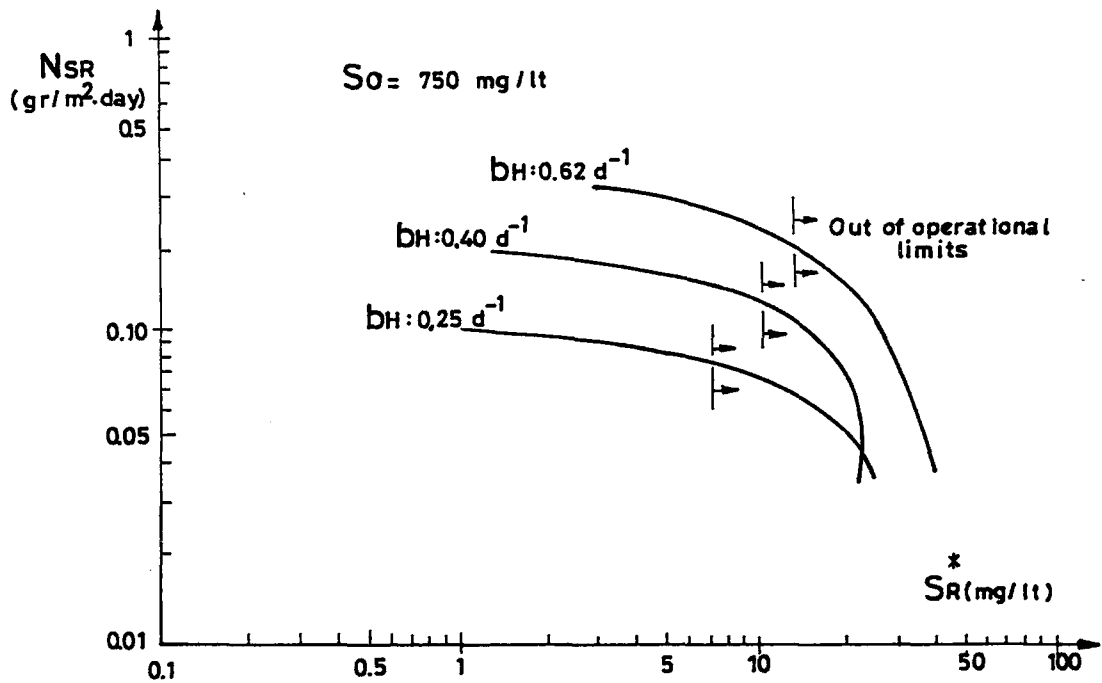


Figure 6-3. The Effect of b_H on N_{SR} versus S_R^* Profile at $S_0 = 750 \text{ mg/lt}$.

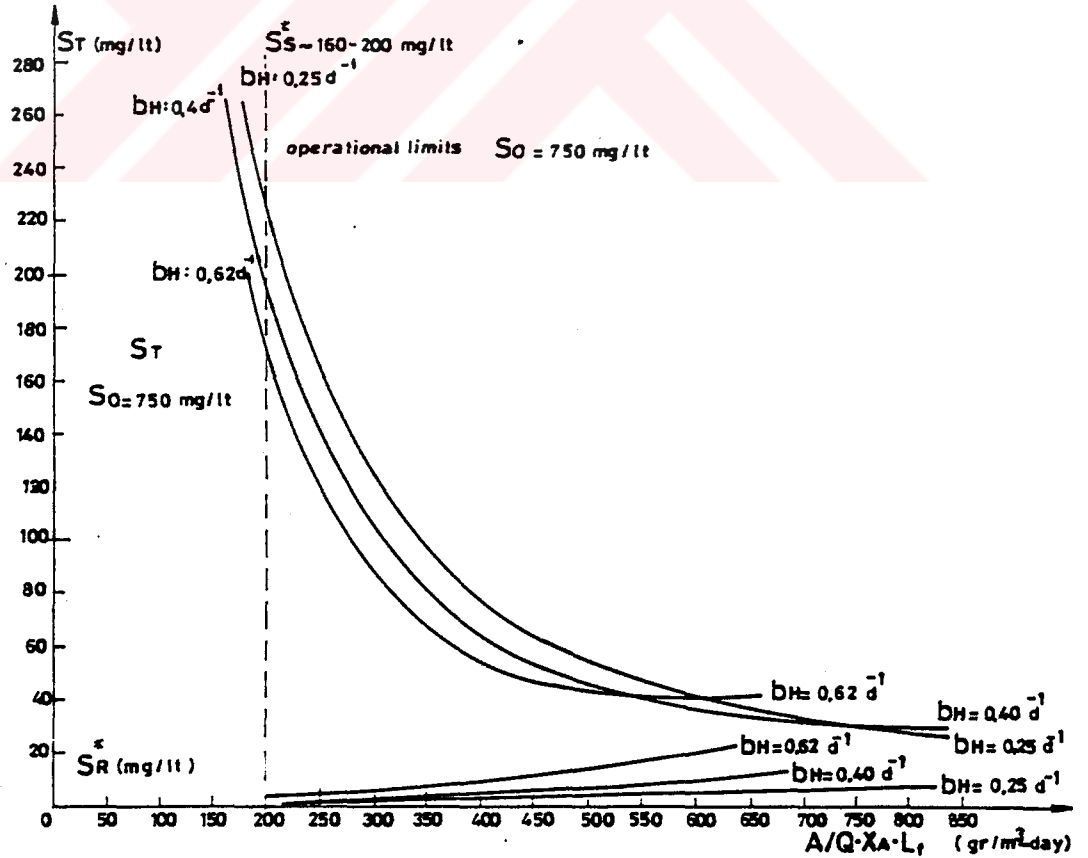


Figure 6-4. S_S^* , S_R^* and S_T Values at Variable b_H Values and at $S_0 = 750 \text{ mg/lt}$

By going over the figures referring to the effect of b_H on the model, one can easily visualize that for S_S^* values in the range of 20-100 mg/lt, decrease in b_H causes increase in N_{SS} , whereas at $S_S^* > 100$ mg/lt N_{SS} is seen to be independent on variations of b_H . Therefore, S_T values overlap S_S^* values ($S_T \approx S_S^*$) indicating the insignificance S_R^* at higher values of S_S^* . Within the range of $S_S^* = 20-100$ mg/lt, the positive effect of b_H on N_{SR} is observed. In other words, as the value of b_H increases, the generation rate of SRP increases as well. The increase in b_H causes the average viability of the system to decrease, thus, the total amount of active biomass decrease respectively, for a given set of S_0 and S_S^* values.

In short; for a given value of S_0 , the increase in b_H value causes;

- N_{SS} to decrease, thus S_S^* increases,
- N_{SR} to increase, thus S_R^* increases,
- Average viability of the system to decrease, thus the total amount of active biomass decreases,
- the hydraulic loading of the system to decrease, insignificantly.

As X_T of the biofilm media is equal to unity throughout the thickness, the decrease in the average viability of the system as b_H increases, cause increase of X_E and X_S of the system to keep the unity of X_T . The increase in S_R^* in proportion b_H is due to the increase of X_S value as S_R^* is a function of X_S . Nevertheless, the increase of X_E is much more pronounced compared to increase in X_S .

For cases, where the effect of b_H variation is surged for conditions where equal amount of active biomass is considered (Figure 6.4), one can observe that as b_H increases, S_R^* and therefore S_T of the system increase.

It can be concluded that;

$$S_R^* = f(b_H)$$

and that this coefficient is the most objectionable parameter of the SRP generation model.

6.1.2. Half-Saturation Constant

The K_S values selected to be used in sensitivity analyses are 100 mg/lt and 250 mg/lt apart from the typical value of 60 mg/lt. In the same manner, typical influent substrate concentrations used are 250 and 750 mg/lt. Summary of the macro level calculations are given in Tables 6.5 to 6.8. This parameter is also one of the parameters effecting S_{min} value as the previously mentioned parameter, b_H . Similar concentration profiles are also given in Figures 6.5 to 6.8.

The profiles illustrating the effect of K_S on either S_S^* , S_R^* and S_T indicate that, increase in K_S , increases S_S^* while N_{SS} decrease. Each time K_S value change, S_{min} value differ as it is one of the parameters responsible for the determination of S_{min} . S_{min} value is directly proportional to K_S and therefore as S_{min} increases, S_S^* and S_T increase giving rise to lower removal rates. In other words, increase in K_S effects the removal mechanism of the system and the overall removal efficiency decrease.

Table 6-5. Analysis of the Model at $S_0=250$ mg/lt, $K_S=100$ mg/lt, $S_{min}=18.699$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^3 \cdot d$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_x$ day	X_A (avg.) mg/lt	$A/Q \cdot L_x \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_2 gr/ $m^2 \cdot day$
$S_E^*=21.49$	593.53	15.82	37.31	0.4239	0.2967	1070.93	314.74	0.00168	0.4212
$S_E^*=26.65$	91.387	9.851	36.50	0.2698	0.04569	5095.84	232.82	0.0109	2.7356
$S_E^*=33.83$	33.46	6.5587	40.388	0.1623	0.01673	11048	184.83	0.02988	7.4716
$S_E^*=44.91$	18.08	4.70	49.61	0.0947	0.00904	16639.2	150.4188	0.055	13.827
$S_E^*=58.46$	11.68	3.50	61.96	0.0564	0.00584	21259.2	124.153	0.0856	21.40
$S_E^*=173.41$	8.26	2.718	76.128	0.0357	0.00413	25088	103.61	0.121	30.26
$S_E^*=89.04$	6.15	2.148	91.188	0.02355	0.00307	28320	87.084	0.1626	40.65
$S_E^*=104.90$	4.73	1.732	106.63	0.0162	0.00236	31088	73.367	0.211	52.85
$S_E^*=166.08$	1.826	0.745	166.82	0.00446	0.000913	39192	35.78	0.5476	136.91

Table 6-6. Analysis of the Model at $S_o=750$ mg/lt, $K_s=100$ mg/lt $S_{min}=4.12$ mg/lt

S_B^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_B^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L^*$ day	X_A (avg.) mg/lt	$A/Q \cdot L^* \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L^2 gr/ $m^2 \cdot day$
$S_B^*=21.49$	1891.74	50.43	71.92	0.70	0.9458	1070.93	1012.96	0.000528	0.3964
$S_B^*=26.65$	295.969	31.917	58.567	0.5449	0.1479	5095.84	754.10	0.003378	2.534
$S_B^*=33.83$	110.817	21.72	55.55	0.39	0.0554	11048	612.15	0.00902	6.7679
$S_B^*=44.91$	62.177	16.10	61.01	0.264	0.031	16639.2	517.287	0.0160	12.06
$S_B^*=58.46$	42.177	12.65	71.11	0.1778	0.0210	21259.2	448.32	0.0237	17.78
$S_B^*=73.41$	31.651	10.41	83.82	0.12419	0.0158	25088	397.82	0.03159	23.696
$S_B^*=89.04$	25.27	8.84	97.88	0.090	0.0126	28320	357.82	0.0395	29.679
$S_B^*=104.90$	21.04	7.70	112.6	0.068	0.01052	31088	327.04	0.04752	35.646
$S_B^*=166.08$	12.71	5.186	171.26	0.030	0.00635	39192	249.06	0.07864	59
$S_B^*=220.18$	9.20	4.01	224.19	0.0718	0.0046	44592	205.12	0.1086	81.52
$S_B^*=343.39$	5.08	2.476	345.86	0.00715	0.00254	54112	137.44	0.1968	147.637

Table 6-7. Analysis of the Model at $S_o=250$ mg/lt, $K_s=250$ mg/lt, $S_{min}=47.4167$ mg/lt

S_s^* mg/lt	$A/Q(^*)$ $m^3/m^3.d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L^*$ day	$X_A(avg.)$ mg/lt	$A/Q.L^*:X_A$ gr/ $m^3.day$	H_g $m^3/m^2.day$	L_g gr/ $m^2.day$
$S_s^*=49.65$	681.46	15.33	64.98	0.2359	0.3407	878.72	299.379	0.001467	0.366
$S_s^*=51.15$	385.36	14.68	65.83	0.2229	0.19268	1503.12	289.62	0.00259	0.648
$S_s^*=55.34$	152.20	12.90	68.247	0.1890	0.0761	3453.84	262.83	0.00657	1.6425
$S_s^*=64.64$	45.96	9.325	73.965	0.126	0.02298	9122.4	209.63	0.02175	5.439
$S_s^*=80.61$	17.83	6.23	86.848	0.0717	0.00891	17920	159.667	0.056	14
$S_s^*=111.45$	7.667	3.74	115.19	0.03246	0.00383	28472	109.147	0.130	32.60
$S_s^*=163.71$	2.87	1.746	165.456	0.0105	0.001435	39656	56.906	0.348	87.108

Table 6-8. Analysis of the Model at $S_o=750$ mg/lt, $K_s=250$ mg/lt, $S_{min}=47.4167$ mg/lt

S_s^* mg/lt	$A/Q(^*)$ $m^2/m^3.d.$	S_s^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L^*$ day	$X_A(avg.)$ mg/lt	$A/Q.L^*.X_A$ gr/ $m^3.day$	H^* $m^3/m^2.day$	L^* gr/ $m^2.day$
$S_s^*=49.65$	<u>2382.14</u>	53.59	103.24	0.519	1.191	878.72	1046.61	0.000419	0.3148
$S_s^*=51.15$	<u>1354.36</u>	51.60	102.75	0.50	0.677	1503.12	1017.88	0.000738	0.5537
$S_s^*=55.34$	<u>543.169</u>	46.06	101.40	0.45	0.2715	3453.84	938	0.00184	1.38
$S_s^*=64.64$	<u>169.93</u>	34.48	99.12	0.3478	0.08496	9122.4	775.08	0.00588	4.41
$S_s^*=80.61$	78.108	27.32	107.93	0.2531	0.039	17920	699.84	0.0128	9.60
$S_s^*=111.45$	35.323	17.238	128.688	0.1339	0.01766	28472	502.858	0.0283	21.23
$S_s^*=163.71$	19.543	11.868	175.578	0.0675	0.00977	39656	387.49	0.05116	38.37
$S_s^*=210.27$	13.709	9.28	219.55	0.04226	0.00685	46880	321.33	0.0729	54.708
$S_s^*=286.18$	8.75	6.72	292.9	0.0229	0.00437	56168	245.735	0.114	85.71
$S_s^*=393.61$	<u>5.08</u>	4.4758	398.08	0.01122	0.00254	66720	169.468	0.1968	<u>147.63</u>

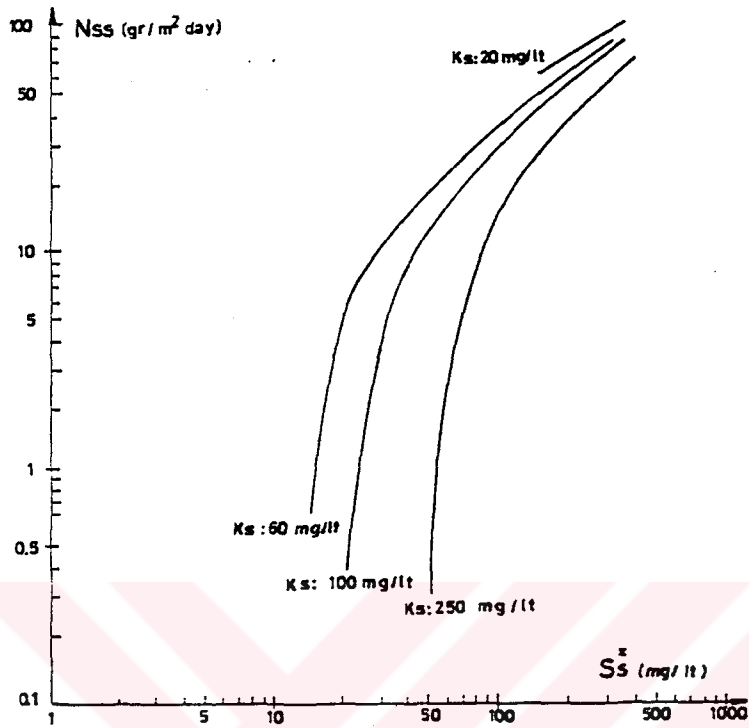


Figure 6-5. The Effect of K_s on N_{ss} versus S_s^* Profile at $S_o=750$ mg/l

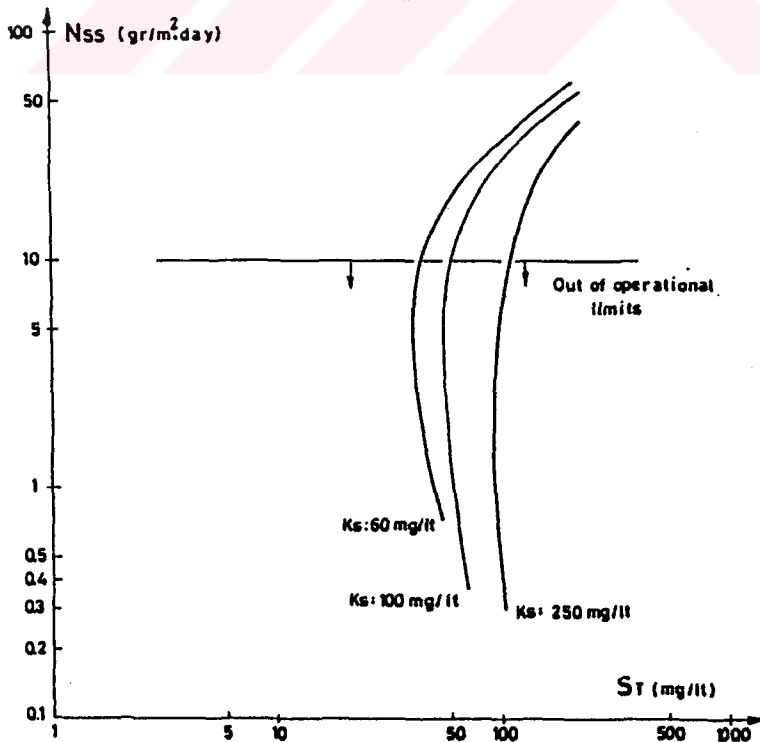


Figure 6-6. The Effect of K_s on N_{ss} versus S_T Profile at $S_o=750$ mg/l

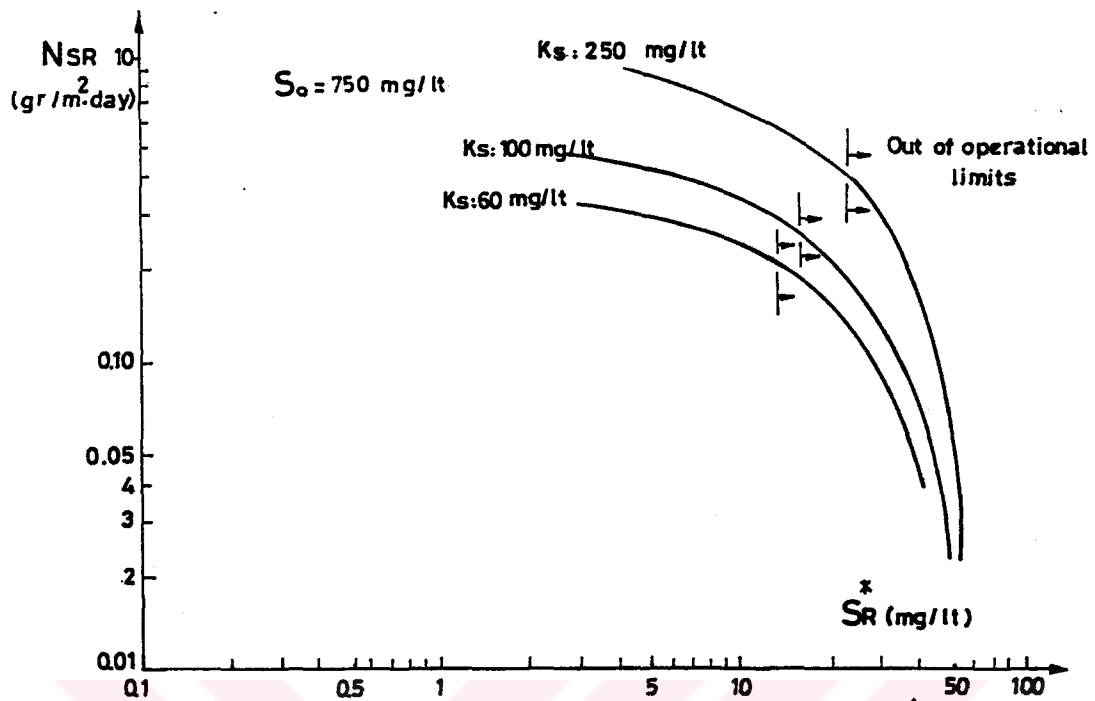


Figure 6-7. The Effect of K_s on NSR versus SR^* Profile at $S_o = 750$ mg/lt

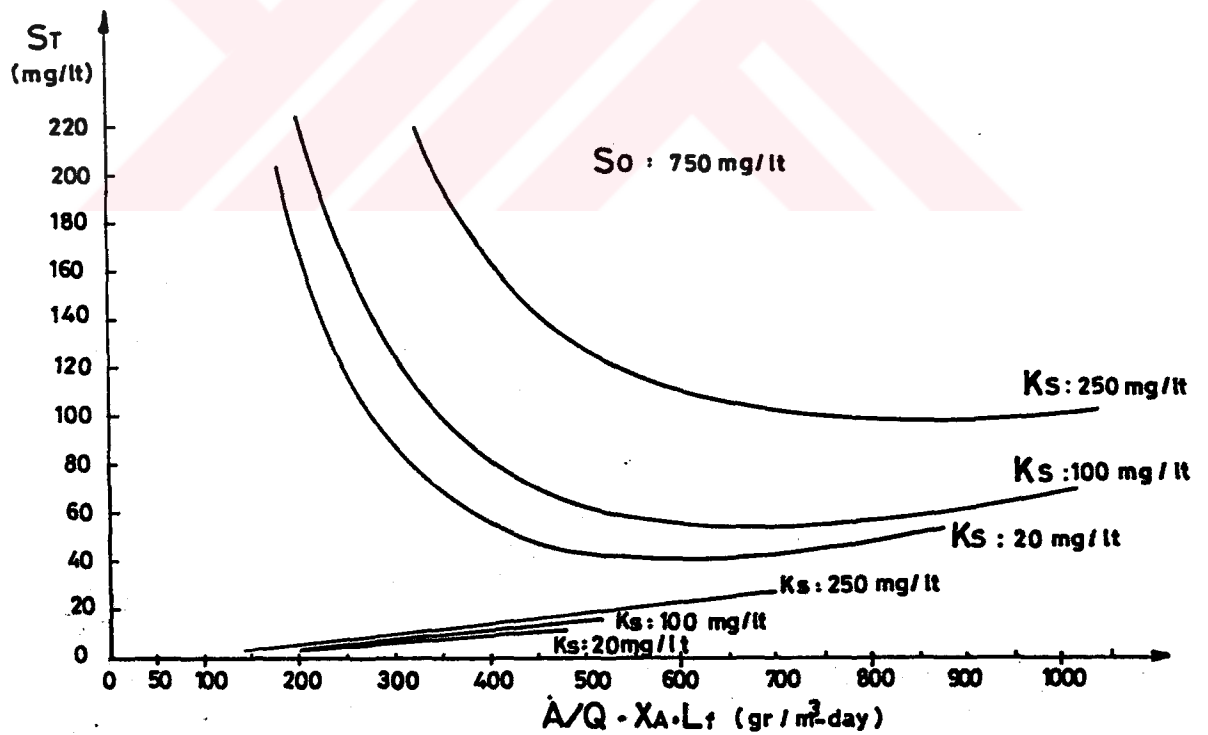


Figure 6-8. S_B^* , SR^* and ST Values at Variable K_s Values and at $S_o = 750$ mg/lt

On the other hand, although K_s is also directly proportional to S_R^* and N_{SR} causing higher values of S_R^* at higher K_s values as illustrated in Figure 6.8, this increase is negligible compared to more higher increase in S_S^* and S_T .

It can be stated that S_R^* is not too sensitive to variations in K_s . This fact is in agreement with the findings of similar study followed on activated sludge systems, (Orhon et.al., 1989).

The average viability of the system as well as the amount of active biomass is also directly proportional with K_s variations and thus the following statements summarize the effect of K_s on the model.

For a given value of S_o , the increase in K_s value causes;

- N_{SS} to decrease, thus S_S^* increases,
 - N_{SR} to decrease, thus S_R^* increases,
 - the total amount of active biomass increase,
- as the hydraulic loading of the system decrease significantly.

6.1.3. Maximum Specific Growth Rate

The $\hat{\mu}$ values selected to be used in sensitivity analyses are $2d^{-1}$ and $6d^{-1}$ apart from the typical value of $4.2d^{-1}$. Summary of the macrolevel calculations are given in Tables 6.9 to 6.12. Similar concentration profiles are also shown in Figures 6.9 to 6.12. This coefficient is also one of the parameters effecting S_{min} value. S_{min} value is inversely proportional to $\hat{\mu}$, therefore, as $\hat{\mu}$ increase, S_{min} value decrease causing S_S^* to be lower whereas N_{SS} is higher leading to high removal efficiencies for a given value of S_o .

On the other hand, observations of the related figures indicate that the effect of $\hat{\mu}$ on S_R^* is negative; in other words, both terms are inversely proportional to each other.

As in the sensitivity analyses of k_s which is one of the parameters responsible for bacterial growth, the effect of k_s on S_R^* has been determined to be negligible. The other parameter, $\hat{\mu}$, which is another parameter responsible for growth is also seen to be insensitive to soluble residual product formation.

For a given value of S_0 , the increase in $\hat{\mu}$ causes;

- N_{ss} to increase, thus S_s^* decreases,
- N_{sr} to decrease, thus S_R^* decreases,
- Average viability of the system to increase
- the total amount of active biomass decrease significantly, as the hydraulic loading of the system increase.

In addition, for a given amount of total active biomass, S_R^* , is insensitive to variations in $\hat{\mu}$, even though S_T varies significantly, as S_s^* of the system is effected by variations in $\hat{\mu}$.

Table 6-9. Analysis of the Model at $S_0=250$ mg/lt, $\hat{\mu}=2d^{-1}$, $S_{min}=30.1984$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3.d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L\tau$ day	$X_A(avg.)$ mg/lt	$A/Q.L\tau.X_A$ gr/ $m^3.day$	H_2 $m^3/m^2.day$	L_0 gr/ $m^2.day$
$S_s^*=32.47$	715.559	16.45	48.927	0.336	0.3577	901.592	322.5	0.00139	0.3475
$S_s^*=34.05$	392.62	15.615	49.665	0.314	0.1963	1576	309.368	0.00254	0.6367
$S_s^*=36.86$	195.95	14	50.9	0.274	0.0979	2928	286.87	0.0051	1.276
$S_s^*=41.08$	94.1	11.857	52.9	0.224	0.047	5420.56	254.766	0.010	2.656
$S_s^*=51.62$	32.415	8.32	59.94	0.138	0.0162	12464	202	0.0308	7.712
$S_s^*=61.63$	19.828	6.692	68.32	0.0979	0.00991	17600	174.416	0.0504	12.6
$S_s^*=72.95$	13.83	5.53	78.48	0.929	0.00691	22152	153.07	0.0723	18.076
$S_s^*=84.82$	10.36	4.66	89.48	0.052	0.00518	26000	134.68	0.0965	24.13
$S_s^*=96.80$	8.14	3.988	100.78	0.03957	0.00407	29352	119.462	0.122	30.71
$S_s^*=108.61$	6.59	3.4549	112	0.0307	0.00329	32248	106.09	0.1517	37.93
$S_s^*=131.25$	4.55	2.6278	133.87	0.01957	0.00227	37064	84.135	0.2197	54.94

Table 6-10. Analysis of the Model at $S_o=750$ mg/lt, $\hat{\mu}=2d^{-1}$, $S_{min}=30.1984$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^2 \cdot d$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L \cdot \tau$ day	X_A (avg.) mg/lt	$A/Q \cdot L \cdot \tau \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_2 gr/ $m^2 \cdot day$
$S_E^*=32.47$	2360.3	54.28	86.75	0.625	1.180	901.592	1064	0.00042	0.317
$S_E^*=34.05$	1301.72	51.76	85.82	0.603	0.65	1576	1025.75	0.000768	0.576
$S_E^*=36.86$	655.64	47.06	83.92	0.56	0.3278	2928	959.85	0.00152	1.143
$S_E^*=41.08$	319	40.199	81.279	0.494	0.1595	5420.56	864.579	0.00313	2.35
$S_E^*=51.62$	114	29.21	80.83	0.36	0.057	12464	710.448	0.00877	6.579
$S_E^*=61.63$	72.46	24.41	86	0.283	0.036	17600	637.64	0.0138	10.35
$S_E^*=72.95$	52.77	21.10	94	0.224	0.026	22152	584.48	0.0189	14.21
$S_E^*=84.82$	41.73	18.77	103.6	0.181	0.020	26000	542.49	0.0239	17.97
$S_E^*=96.80$	34.7	17	113.8	0.149	0.017	29352	509.257	0.0288	21.61
$S_E^*=108.61$	29.91	15.67	124.28	0.126	0.0149	32248	482.26	0.0334	25
$S_E^*=131.25$	23.79	13.7	144.95	0.0945	0.01189	37064	440.87	0.042	31.5
$S_E^*=171.63$	17.33	11.28	182.91	0.0616	0.00866	44000	381.26	0.057	43.277
$S_E^*=189.45$	15.43	10.497	199.947	0.052	0.0077	46880	361.679	0.0648	48.6

Table 6-11. Analysis of the Model at $S_0=250$ mg/lt, $\hat{\mu}=6d^{-1}$, $S_{min}=7.537$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^3.d.$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q.Lx$ day	$X_A(avq.)$ mg/lt	$A/Q.Lx.X_A$ gr/ $m^3.day$	H_2 $m^3/m^2.day$	L_2 gr/ $m^2.day$
$S_E^*=22.76$	21.73	3.499	26.26	0.133	0.01086	12512	135.94	0.046	11.50
$S_E^*=40.72$	10.5	2	42.72	0.0468	0.00525	18248	95.80	0.095	23.80
$S_E^*=64.03$	6.147	1.27	65.30	0.01944	0.00307	22788	69.959	0.162	40.67
$S_E^*=77.01$	4.88	1.03	78.04	0.0128	0.00244	24719.2	60.3148	0.2049	51.22
$S_E^*=90.59$	3.94	0.85	91.44	0.00929	0.00197	26464	52.134	0.2538	63.45
$S_E^*=104.56$	3.35	0.738	105.29	0.007	0.001675	28000	46.9	0.298	74.5
$S_E^*=133.14$	2.14	0.48	133.62	0.00359	0.00107	30880	33	0.467	116.82

Table 6-12. Analysis of the Model at $S_o=750$ mg/lt, $\hat{\mu}=6d^{-1}$, $S_{min}=7.537$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_x$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L_x \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_s^*=22.76$	69.59	11.2	33.96	0.329	0.0347	12512	435	0.014	10.77
$S_s^*=40.72$	35.64	6.807	47.52	0.143	0.0178	18248	325.179	0.028	21.04
$S_s^*=64.03$	22.767	4.685	68.715	0.06818	0.0113	22788	258.37	0.044	39.07
$S_s^*=77.01$	19	4.175	81.18	0.0514	0.0095	24719.2	234.83	0.052	39.47
$S_s^*=90.59$	16.3	3.52	94.11	0.0374	0.00815	26464	215.68	0.061	46
$S_s^*=104.56$	14.235	3.13	107.69	0.029	0.00711	28000	199.29	0.070	52.68
$S_s^*=133.14$	11.30	2.54	135.68	0.0187	0.00565	30880	174.47	0.088	66.37
$S_s^*=147.52$	10.22	2.33	149.85	0.0155	0.00511	32144	164.25	0.0978	73.38
$S_s^*=217.77$	6.84	1.6289	219.39	0.00742	0.00342	37256	127.41	0.1461	109.65

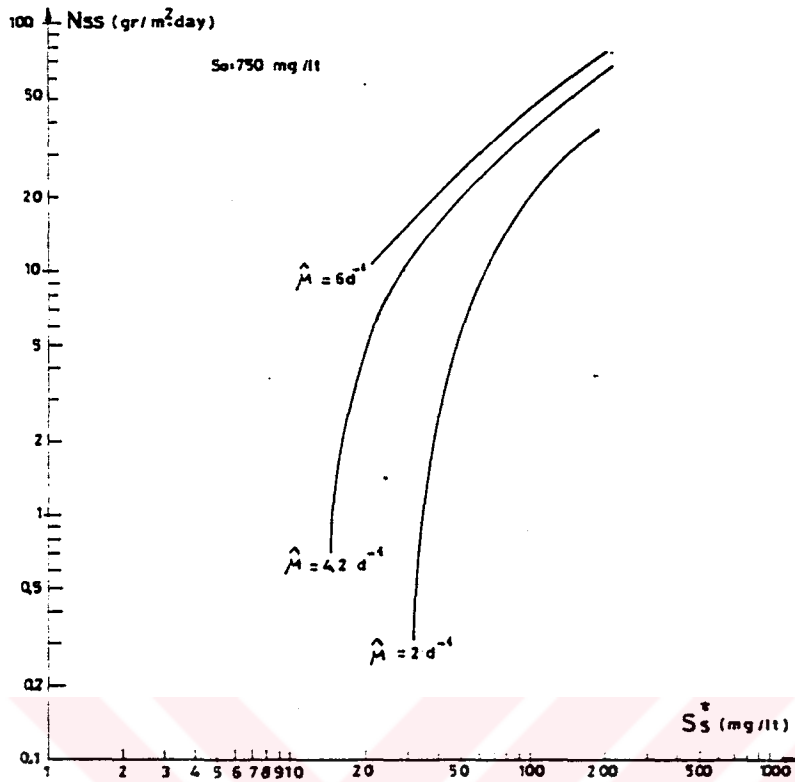


Figure 6-9. The Effect of $\hat{\mu}$ on N_{ss} versus S_{s^*} profile at $S_o = 750 \text{ mg/lit}$.

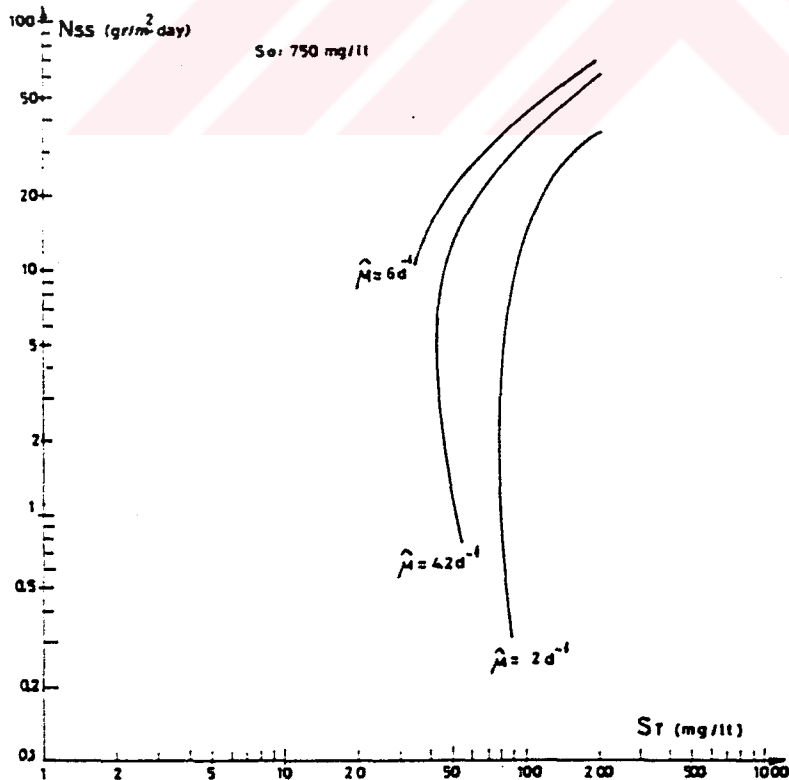


Figure 6-10. The Effect of $\hat{\mu}$ on N_{ss} versus S_T Profile at $S_o = 750 \text{ mg/lit}$

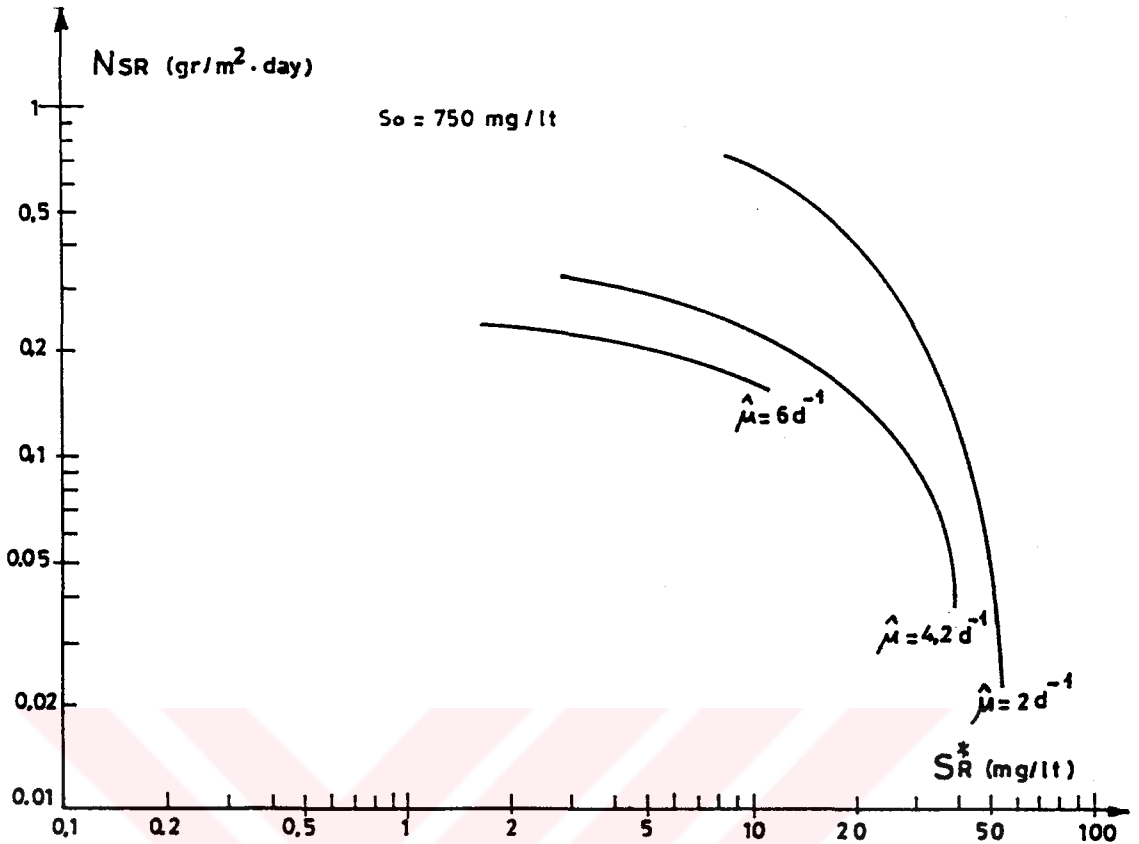


Figure 6-11. The Effect of $\hat{\mu}$ on NSR versus S_R^* Profile at S_o : 750 mg/lit

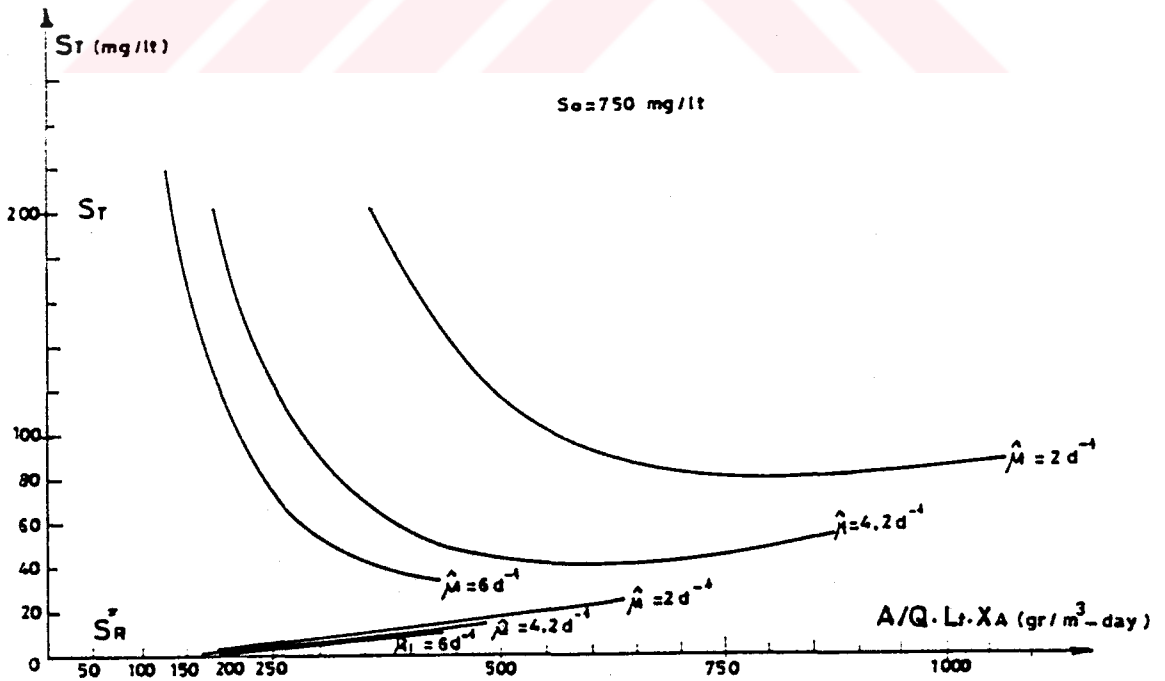


Figure 6-12. S_R^* , S_R^* and S_T Values at Variable $\hat{\mu}$ Values and at S_o : 750 mg/lit

6.1.4. The Coefficient β

The β values used in the analysis are 0.05, 0.2 and 0.3 apart from the typical value of 0.1. Summary of the macrolevel calculations are given in Tables 6.13 to 6.18. Similar concentration profiles are also illustrated in Figures 6.13 to 6.16.

Figure 6.13 clearly indicates that the coefficient β is not a function of the removal rate mechanism and thus not related with S_s^* . This conclusion is in full agreement with the Task Group Model on activated sludge. It is seen that β is a determinant of the generation mechanism of SRP and is directly proportional with S_R^* . As S_R^* is a function of β , S_T is also effected as by variations of β .

Figure 6.14 shows that S_T value increases as β increases as β increases till the value of S_T reaches 150 mg/lt. Beyond this value of S_T , the increase in S_R^* does not change S_T significantly as S_R^* value diminishes at higher values of S_s^* .

Therefore, it can be stated that;

$$S_T = f(\beta) \quad \text{for} \quad S_s^* \leq 150 \text{ mg/lt}$$

Another interesting observation is the linearity of S_R^* with respect to the total amount of active biomass of the system at lower values of S_R^* and that linearity turns out to be a curve at higher values of S_R^* .

Increase in β causes the viability of the system to decrease, but not very significantly beyond $S_T \geq 120$ mg/lt. In the some manner, as β is not a function of S_s^* and N_{ss} , the total amount of active biomass is more or less identical due to variations of β .

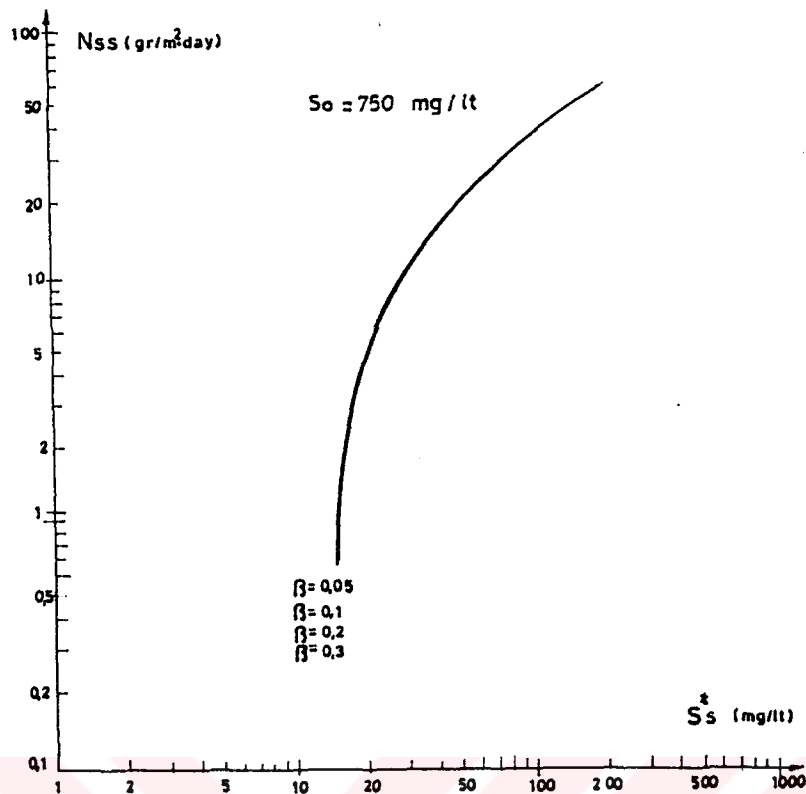


Figure 6-13. The Effect of β on N_{ss} versus S_{ss}^* profile at S_o : 750 mg/lit

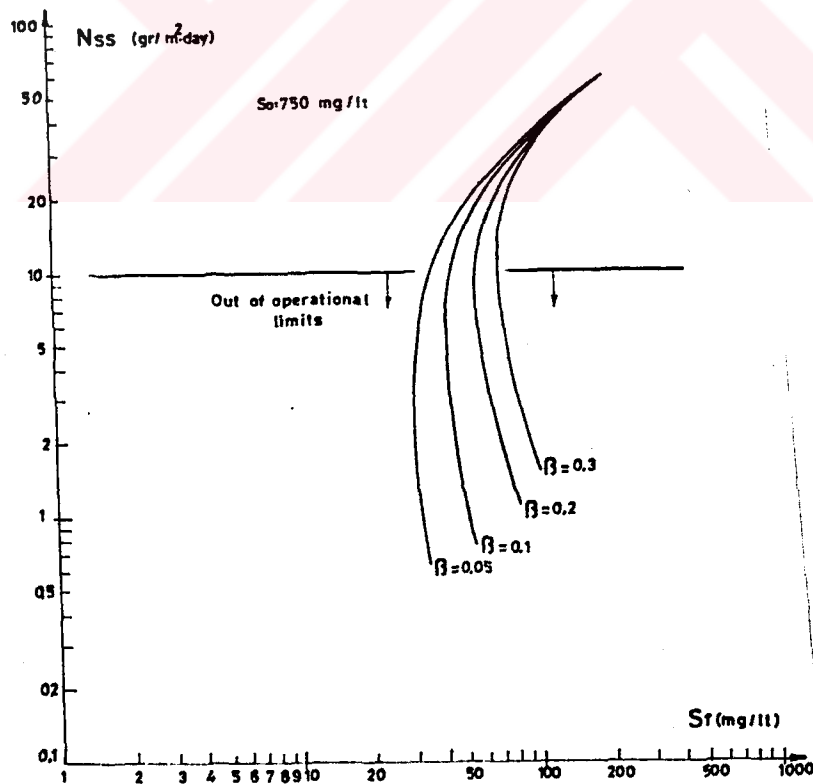


Figure 6-14. The Effect of β on N_{ss} versus S_r profile at S_o : 750 mg/lit

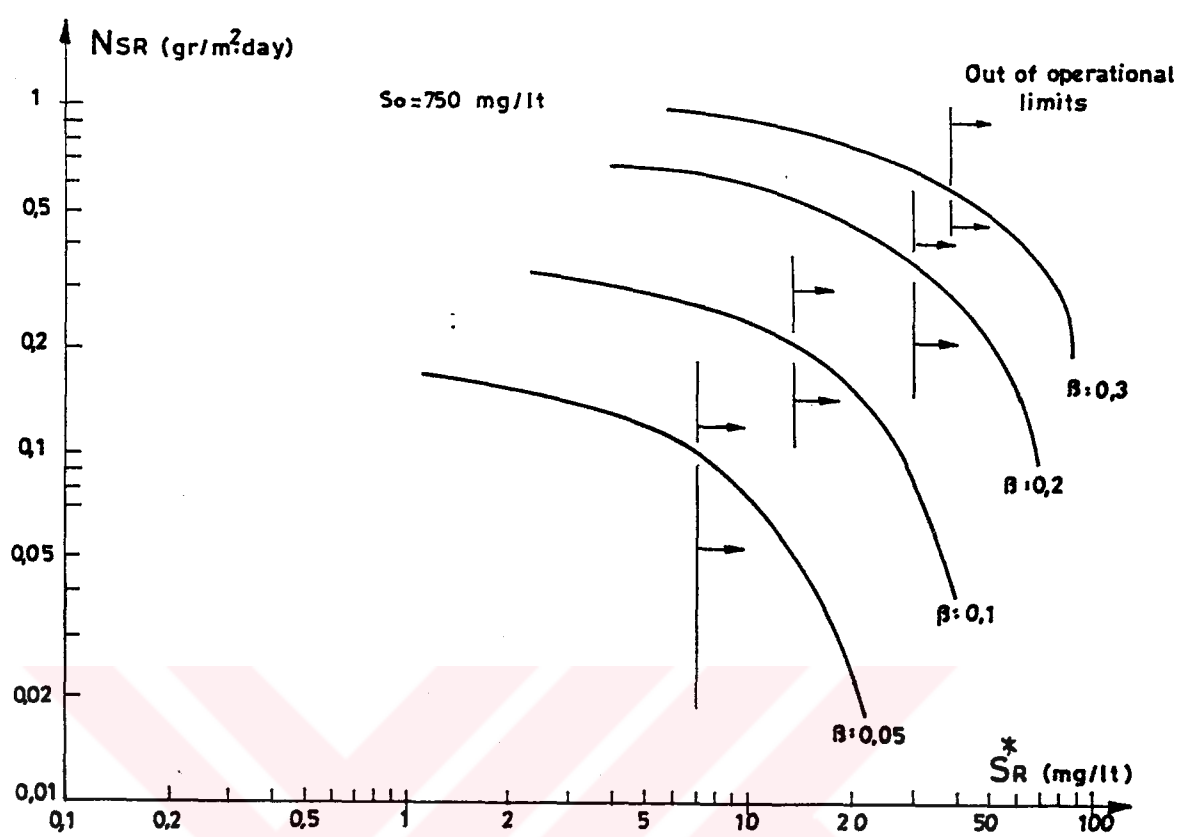


Figure 6-15. The Effect of β on N_{SR} versus S_R^* profile at S_o : 750 mg/lit

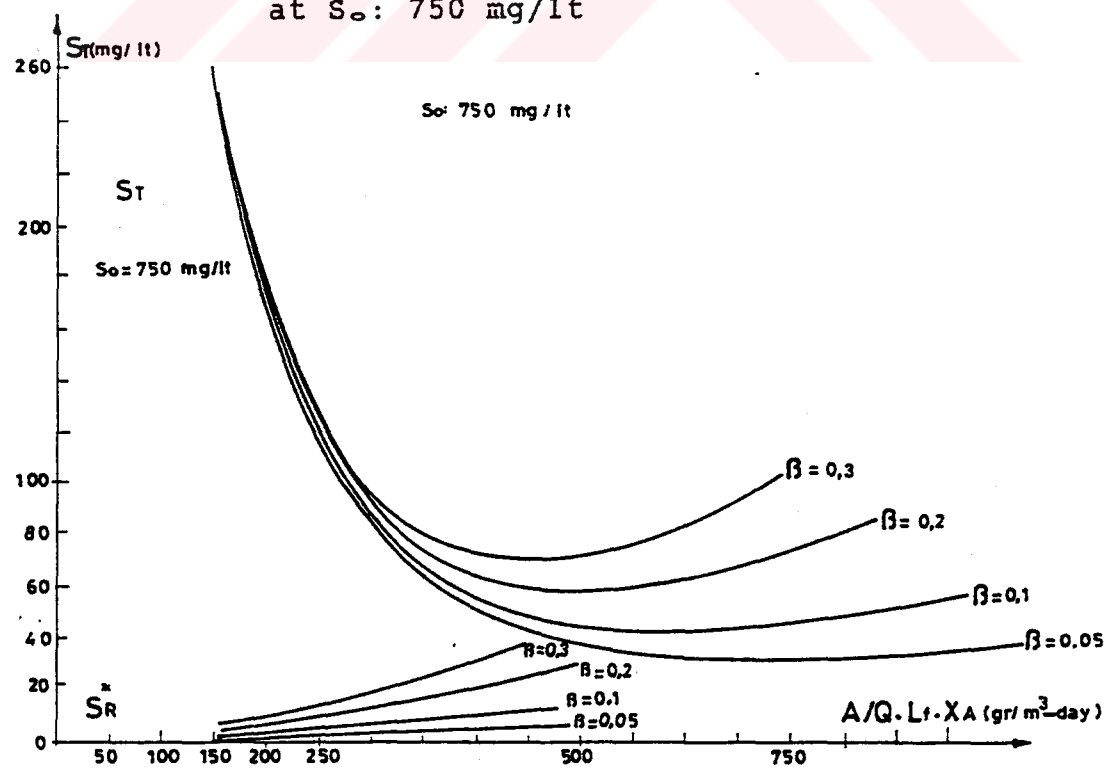


Figure 6-16. S_S^* , S_R^* and S_T values at Variable β Values an at S_o : 750 mg/lit

Table 6-13. Analysis of the Model at $S_o=250$ mg/lt, $\beta=0.05$, $S_{m+n}=11.38$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L^*$ day	X_A (avg.) mg/lt	$A/Q \cdot L^* \cdot X_A$ gr/ $m^3 \cdot day$	H_o $m^3/m^2 \cdot day$	L_o gr/ $m^2 \cdot day$
$S_s^*=14.35$	371.68	6.89	21.24	0.324	0.1858	1590.4	295.49	0.00269	0.67
$S_s^*=17.11$	89	4.345	21.45	0.202	0.0445	4998.22	222.42	0.011	2.80
$S_s^*=21.52$	40.29	3.11	24.63	0.126	0.020	9220.4	185.74	0.0248	6.20
$S_s^*=30.09$	21.37	2.158	32.24	0.0669	0.0106	14080	150.44	0.0467	11.69
$S_s^*=41.26$	13.616	1.593	42.85	0.037	0.0068	18221.6	124	0.073	18.25
$S_s^*=56.98$	8.965	1.156	58.13	0.0198	0.00448	22320	100	0.111	27.88
$S_s^*=68.37$	7.097	0.95	69.32	0.0137	0.00354	24640	87.22	0.1409	35.22
$S_s^*=83.19$	5.47	0.766	83.95	0.00912	0.002735	27288	74.63	0.1828	45.70
$S_s^*=98.32$	4.318	0.626	98.946	0.00632	0.002159	29520	63.73	0.231	57.89
$S_s^*=128.6$	2.785	0.42	129	0.00325	0.00139	33360	46.45	0.359	89.76
$S_s^*=158.01$	1.804	0.28	158.3	0.001768	0.000902	36480	32.90	0.5543	138.58

Table 6-14. Analysis of the Model at $S_o=250$ mg/lt, $\beta=0.05$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_T^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L \cdot \frac{X_A}{day}$	X_A (avg.) mg/lt	$A/Q \cdot L \cdot \frac{X_A}{m^3 \cdot day}$	H_2 $m^3/m^2 \cdot day$	L_2 $gr/m^2 \cdot day$
$S_E^*=14.35$	1160	21.524	35.87	0.559	0.58	1590.4	922.432	0.00086	0.646
$S_E^*=17.11$	280.2	13.676	30.786	0.444	0.140	4998.22	700.25	0.00356	2.676
$S_E^*=21.52$	128.47	9.91	31.438	0.3148	0.064	9220.4	592.24	0.00778	5.837
$S_E^*=30.09$	69.96	7.066	37.156	0.190	0.0349	14080	492.518	0.01429	10.72
$S_E^*=41.26$	46.23	5.409	46.669	0.1159	0.023	18221.6	421.19	0.0216	16.22
$S_E^*=56.98$	32.18	4.152	61.132	0.0679	0.01609	22320	359.12	0.031	23.3
$S_E^*=68.37$	26.63	3.59	71.96	0.0498	0.0133	24640	328.08	0.03755	28.16
$S_E^*=83.19$	21.876	3.06	86.25	0.035	0.0109	27288	298.476	0.0457	34.28
$S_E^*=98.32$	18.55	2.69	101	0.0266	0.00927	29520	273.79	0.053	40.43
$S_E^*=128.6$	14.258	2.167	130.767	0.0165	0.00712	33360	237.82	0.070	52.60
$S_E^*=158.01$	11.60	1.822	159.83	0.0113	0.0058	36480	211.584	0.086	64.66
$S_E^*=212.45$	8.505	1.408	213.858	0.00658	0.00425	41324	175.73	0.11757	88.18
$S_E^*=260.59$	6.72	1.153	261.74	0.0044	0.00336	44932.8	150.97	0.1488	111.607

Table 6-15. Analysis of the Model at $S_o=250$ mg/lt, $\beta=0.2$, $S_{min}=11.38$ mg/lt

S_B^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_B^* mg/lt	S_T mg/lt	S_B/S_T	$A/Q \cdot L_z$ day	X_A (avg.) mg/lf	$A/Q \cdot L_z \cdot X_A$ gr/ $m^3 \cdot day$	H_g $m^3/m^2 \cdot day$	L_g gr/ $m^2 \cdot day$
$S_B^*=15.04$	237.62	22.26	37.30	0.5967	0.1188	2102.76	249.80	0.00420	1.052
$S_B^*=20.35$	45.56	12.575	32.92	0.3812	0.02278	8000	182.24	0.0219	5.487
$S_B^*=27.58$	24.29	9.057	36.637	0.2472	0.0121	12594.4	152.958	0.0411	10.292
$S_B^*=37.14$	15.55	6.7358	43.876	0.153	0.00777	16512	128.29	0.0643	16.077
$S_B^*=54.52$	9.4279	4.66	59.186	0.0787	0.00471	21360	100.6	0.106	26.517
$S_B^*=74.08$	6.3646	3.386	77.466	0.0437	0.00318	25317.6	80.50	0.1572	39.30
$S_B^*=87.73$	5.07	2.794	90.524	0.03086	0.00253	27552	69.84	0.1972	49.30
$S_B^*=101.58$	4.1	2.32	103.9	0.0223	0.00205	29520	60.516	0.244	60.97
$S_B^*=122.37$	3.038	1.774	124.14	0.0142	0.001519	32184.8	48.88	0.329	82.29

Table 6-16. Analysis of the Model at $S_o=750$ mg/lt, $\beta=0.2$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3.d.$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L^*$ day	X_A (avg.) mg/lt	$A/Q.L^*.X_A$ gr/ $m^3.day$	H_2 $m^3/m^2.day$	L_2 gr/ $m^2.day$
$S_E^*=15.04$	743.88	69.70	84.74	0.82	0.3719	2102.76	782.10	0.00134	1.0
$S_E^*=20.35$	144.77	39.95	60.30	0.66	0.0723	8000	579.08	0.00690	5.18
$S_E^*=27.58$	78.95	29.44	57	0.5162	0.0394	12594.4	497.163	0.0126	9.49
$S_E^*=37.14$	52.10	22.71	59.85	0.379	0.026	16512	430.13	0.0192	14.39
$S_E^*=54.52$	33.55	16.60	71.12	0.233	0.0167	21360	358.314	0.0298	22.35
$S_E^*=74.08$	24.45	13	87.089	0.149	0.0122	25317.6	309.50	0.0408	30.67
$S_E^*=87.73$	20.677	11.39	99.12	0.1149	0.0103	27552	284.84	0.0483	36.27
$S_E^*=101.58$	17.91	10.137	111.717	0.0907	0.00895	29520	264.35	0.0558	41.876
$S_E^*=122.37$	14.94	8.727	131.09	0.0665	0.00747	32184.8	240.42	0.0669	50.20
$S_E^*=188.23$	9.67	6.064	194.29	0.0312	0.004836	38768	187.48	0.1034	77.56
$S_E^*=246.23$	7.18	4.71	250.94	0.0187	0.00359	43335.12	155.57	0.139	104.45
$S_E^*=296.5$	5.70	3.867	300.36	0.0128	0.00285	46752	133.24	0.175	131.57

Table 6-17. Analysis of the Model at $S_o=250$ mg/lt, $\beta=0.3$, $S_{min}=11.38$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3.d.$	S_s^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L\epsilon$ day	$X_A(avg.)$ mg/lt	$A/Q.L\epsilon.X_A$ gr/ $m^3.day$	H_2 $m^3/m^2.day$	L_2 gr/ $m^2.day$
$S_s^*=15.71$	<u>156.19</u>	28.114	43.82	0.6415	0.078	2835.2	221.14	0.0064	<u>1.60</u>
$S_s^*=23.44$	32.41	15.557	38.99	0.3989	0.0162	10112	163.86	0.0308	7.71
$S_s^*=32.68$	18.517	11.20	43.88	0.255	0.00925	14656.8	135.57	0.054	13.5
$S_s^*=44.23$	12.26	8.387	52.617	0.159	0.00613	18504	113.429	0.0815	20.39
$S_s^*=64.36$	7.61	5.78	70.14	0.082	0.0038	23200	88.16	0.1314	32.85
$S_s^*=86.29$	5.17	4.18	90.47	0.046	0.00258	27040	69.76	0.193	48.35
$S_s^*=101.31$	<u>4.10</u>	3.42	104.73	0.0326	0.00205	29232	59.92	0.244	60.97
$S_s^*=116.37$	3.30	2.817	119.18	0.0194	0.00165	31120	51.348	0.303	75.75
$S_s^*=138.69$	<u>2.39</u>	2.107	140.797	0.0149	0.00119	33720	40.29	0.418	<u>104.6</u>

Table 6-18. Analysis of the Model at $S_o=750$ mg/lt, $\beta=0.3$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L \cdot z$ day	X_A (avg.) mg/lt	$A/Q \cdot L \cdot z \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_2 gr/ $m^2 \cdot day$
$S_E^*=15.71$	489.52	88.114	103.82	0.848	0.244	2835.2	693.94	0.00204	1.53
$S_E^*=23.44$	103.94	49.89	73.33	0.680	0.05197	10112	525.52	0.00962	7.21
$S_E^*=32.68$	61.12	36.978	69.658	0.530	0.03056	14656.8	447.91	0.0163	12.27
$S_E^*=44.23$	42	28.769	72.99	0.394	0.021	18504	388.58	0.0238	17.857
$S_E^*=64.36$	28.11	21.36	85.72	0.249	0.0140	23200	326	0.0355	26.68
$S_E^*=86.29$	20.97	16.96	103.25	0.164	0.0104	27040	283.5	0.0476	35.76
$S_E^*=101.31$	17.92	14.93	116.24	0.128	0.00896	29232	261.918	0.0558	41.85
$S_E^*=116.37$	15.66	13.358	129.729	0.103	0.00783	31120	243.66	0.0638	47.89
$S_E^*=138.69$	13.18	11.57	150.26	0.0769	0.00659	33720	222.21	0.0758	56.90
$S_E^*=174.50$	10.46	9.53	184	0.0517	0.00523	37200	194.55	0.0956	71.7
$S_E^*=239.19$	7.42	7.116	246.306	0.0288	0.00371	42400	157.3	0.1347	101
$S_E^*=294.63$	5.74	5.705	300	0.0189	0.00287	46240	132.70	0.174	130.66

6.1.5. Maximum Specific Hydrolysis Rate

The k_h values used in the analysis are 0.65 d^{-1} and 2 d^{-1} whereas the typical value is 3 d^{-1} . Summary of the macrolevel calculations are given in Tables 6.19 to 6.24. Concentration profiles are illustrated in Figures 6.17 to 6.20.

It is recognized that the hydrolysis rate constant is independent on the removal rate mechanism, N_{ss} , and actually the same condition holds true for $S_T \geq 70 \text{ mg/l}$. This fact indicates that, even S_R^* is a function of k_h , it seems to be that variations in k_h has insignificant effect on S_R^* .

Variations in k_h values does not effect viability of the system significantly as can be seen in the tabulated values given in the Tables. Meanwhile, although X_A of the system does not change very much, X_S increase significantly while X_E diminish to keep the unity of X_T . This conclusion is in correlation with the study done on activated sludge, (Baykal, 1988). Even more, this low rate constant, k_h , increases the concentration of slowly degradable substrate, X_S , thus S_R^* generated is higher, but as the distribution of X_S within the biofilm media or in other words, the X_S concentration profile in the biofilm media is very low compared to the concentration profiles of X_A and X_E ; it's major increase due to variations in k_h causes increase in S_R^* on the whole.

Table 6-19. Analysis of the Model at $S_0=250$ mg/lt, $k_n=0.65$ d⁻¹, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^3 \cdot d$	S_T^* mg/lt	S_R/S_T	$A/Q \cdot L \cdot z$ day	X_A (avg.) mg/lt	$A/Q \cdot L \cdot z \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=14.52$	335.92	13.537	0.482	0.1679	1697.6	285	0.00297	0.744
$S_E^*=16.20$	125.76	10.09	0.3836	0.0628	3753.6	236	0.00795	1.988
$S_E^*=22.70$	36.08	6.24	0.2156	0.01804	9968	179.82	0.0277	6.929
$S_E^*=29.79$	21.955	4.7862	0.1384	0.0109	13864	151.1176	0.0455	11.386
$S_E^*=43.46$	12.386	3.388	0.0723	0.006418	18800	120.658	0.0779	19.476
$S_E^*=59.54$	8.55	2.514	0.0405	0.00427	22840	97.526	0.1169	29.24
$S_E^*=71.07$	6.80	2.10	0.0286	0.0034	25144	85.489	0.147	36.76
$S_E^*=82.97$	5.53	1.7756	0.0209	0.002765	27200	75.208	0.1808	45.20
$S_E^*=95.06$	4.567	1.515	0.01568	0.00228	29040	66.31	0.218	54.74
$S_E^*=101.13$	4.1676	1.4017	0.0136	0.00208	29912	62.216	0.2399	59.98
$S_E^*=119.29$	3.20	1.11	0.009218	0.0016	32263.92	51.62	0.3125	78.12
$S_E^*=131.26$	2.698	0.959	0.00725	0.001349	33664	45.41	0.37	92.66
$S_E^*=160.37$	1.747	0.64	0.00397	0.0008725	36712	32.067	0.57	143.10

Table 6-20. Analysis of the Model at $S_o=750$ mg/lt, $k_n=0.65$ d⁻¹, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^3 \cdot d$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q \cdot L_F$ day	X_A (avg.) mg/lt	$A/Q \cdot L_F \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=14.52$	1049.18	42.376	56.896	0.744	0.5245	1697.6	890.543	0.000953	0.7148
$S_E^*=16.20$	394.728	31.734	47.93	0.662	0.19736	3753.6	740.82	0.00253	1.90
$S_E^*=22.70$	115.44	20.041	42.74	0.4688	0.0577	9968	575.35	0.00866	6.496
$S_E^*=29.79$	71.805	15.701	45.49	0.3451	0.0359	13864	497.75	0.0139	10.44
$S_E^*=43.46$	43.91	11.59	55.05	0.210	0.0219	18800	412.754	0.0227	17.08
$S_E^*=59.54$	31	9.115	68.655	0.1327	0.0155	22840	354.02	0.03225	24.1935
$S_E^*=71.07$	25.81	7.975	79.04	0.100	0.0129	25144	324.48	0.0387	29.058
$S_E^*=82.97$	21.5	6.902	89.87	0.0767	0.01075	27200	292.4	0.0465	34.88
$S_E^*=95.06$	19.3	6.40	101.46	0.063	0.00965	29040	280.236	0.0518	38.86
$S_E^*=101.13$	18.16	6.10	107.239	0.0568	0.00908	29912	271.60	0.055	41.299
$S_E^*=119.29$	15.44	5.374	124.66	0.0431	0.00772	32263.92	249.07	0.0647	48.575
$S_E^*=131.26$	14.05	4.99	136.25	0.0366	0.00702	33664	236.489	0.07117	53.38
$S_E^*=160.37$	11.49	4.252	164.62	0.0258	0.00574	36712	210.91	0.087	65.274
$S_E^*=213.98$	8.474	3.329	217.30	0.0153	0.00423	41440	175.29	0.118	88.50
$S_E^*=261.17$	6.729	2.759	263.92	0.0104	0.00336	44982.16	151.34	0.1486	111.45

Table 6-21. Analysis of the Model at $S_0=250$ mg/lt, $k_n=1d^{-1}$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_x$ day	$X_A(avg.)$ mg/lt	$A/Q \cdot L_x \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=14.54$	329.0	13.09	27.63	0.4736	0.1645	1710.8	281.42	0.0030	0.759
$S_E^*=16.29$	119.99	9.599	25.889	0.3707	0.0599	3861.6	231.30	0.00833	2.08
$S_E^*=20.20$	47.479	6.788	26.988	0.2515	0.02373	8080	191.738	0.0210	5.265
$S_E^*=26.55$	26.3255	5.028	31.57	0.159	0.01316	12264	161.39	0.0379	9.496
$S_E^*=34.88$	17.1958	3.8809	38.76	0.10	0.00859	15920	136.754	0.058	14.538
$S_E^*=44.71$	12.275	3.066	47.77	0.0641	0.00663	19120	126.765	0.0814	20.36
$S_E^*=55.63$	9.268	2.4748	58.10	0.04259	0.00463	21920	101.577	0.1078	26.97
$S_E^*=67.29$	7.263	2.040	69.33	0.0294	0.00363	24368	88.455	0.1376	34.421
$S_E^*=79.43$	5.839	1.709	81.139	0.0210	0.002919	26552	77.505	0.171	42.81
$S_E^*=91.823$	4.778	1.442	93.26	0.01546	0.002398	28509.6	68.109	0.239	52.32
$S_E^*=104.32$	3.958	1.226	105.546	0.00214	0.001979	30272.92	59.90	0.252	63.16
$S_E^*=116.81$	3.30	1.0455	117.85	0.00887	0.00165	31876.8	52.59	0.303	75.75
$S_E^*=141.41$	2.32	0.76	142.17	0.00534	0.00116	34688	40.238	0.431	107.75

Table 6-22. Analysis of the Model at $S_0=750$ mg/lt, $k_n=1d^{-1}$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^3/m^3 \cdot d.$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L^*$ day	X_A (avg.) mg/lt	$A/Q \cdot L^* \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^3 \cdot day$	L^* gr/ $m^3 \cdot day$
$S_E^*=14.54$	1028.61	40.938	55.478	0.7379	0.5143	1710.8	879.872	0.000972	0.729
$S_E^*=16.29$	376.70	30.136	46.42	0.649	0.1883	3861.6	727.33	0.00265	1.99
$S_E^*=20.20$	150.78	21.547	41.747	0.516	0.0754	8080	609.15	0.00663	4.974
$S_E^*=26.55$	85.23	16.279	42.829	0.38	0.0426	12264	522.63	0.0117	8.799
$S_E^*=34.88$	57.14	12.897	47.77	0.2699	0.02857	15920	454.834	0.0175	13.125
$S_E^*=44.71$	42.18	10.537	55.247	0.190	0.02109	19120	403.24	0.0237	17.78
$S_E^*=55.63$	33.11	8.86	64.49	0.137	0.0165	21920	362.88	0.0302	22.65
$S_E^*=67.29$	27.145	7.627	74.91	0.1018	0.01357	24368	330.73	0.0368	27.629
$S_E^*=79.42$	22.95	6.719	86.139	0.078	0.01147	26552	304.68	0.04357	32.679
$S_E^*=91.82$	19.88	6	97.82	0.061	0.00994	28509.6	283.385	0.05	37.72
$S_E^*=104.32$	17.54	5.43	109.75	0.049	0.00877	30272	265.485	0.057	42.76
$S_E^*=116.81$	15.70	4.96	121.779	0.040	0.00785	31876.8	250.23	0.0636	47.77
$S_E^*=141.41$	13	4.26	145.67	0.029	0.0065	34640	225.16	0.0769	57.68
$S_E^*=220.14$	8.188	2.9068	223.04	0.013	0.00409	41808	170.994	0.122	91.59
$S_E^*=268.41$	6.49	2.395	270.8	0.00884	0.00324	45352	147.167	0.154	115.56

Table 6-23. Analysis of the Model at $S_0=250$ mg/lt, $k_H=2d^{-1}$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q \cdot L_z$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L_z \cdot X_A$ gr/ $m^3 \cdot day$	H_0 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=14.56$	326.54	12.93	27.49	0.47	0.16327	1715.808	280.14	0.00306	0.765
$S_E^*=16.32$	117.72	9.406	25.726	0.3656	0.05886	3904.8	229.8	0.00849	2.123
$S_E^*=20.31$	46.49	6.55	26.865	0.2438	0.02324	8176	190	0.0215	5.377
$S_E^*=26.80$	25.766	4.8469	31.647	0.1531	0.01288	12392	159.64	0.0388	9.70
$S_E^*=35.29$	16.826	3.69	38.98	0.0946	0.008413	16088	135.348	0.0594	14.857
$S_E^*=45.31$	12	2.90	48.218	0.060	0.006	19288	115.728	0.0833	20.83
$S_E^*=56.42$	9.07	2.331	58.75	0.03967	0.004536	22080	100.15	0.11025	27.56
$S_E^*=68.27$	7.105	1.9184	70.188	0.027	0.00355	24539.3	87.11	0.140	35.18
$S_E^*=74.39$	6.353	1.747	76.137	0.0229	0.00317	25661.6	81.347	0.157	39.35
$S_E^*=80.60$	5.708	1.5978	82.197	0.0194	0.00285	26720	76.152	0.175	43.79
$S_E^*=93.18$	4.66	1.306	94.486	0.0138	0.00233	28640	66.73	0.2145	53.64
$S_E^*=105.87$	3.859	1.138	107	0.0106	0.001929	30400	58.64	0.2591	64.78
$S_E^*=137.29$	2.4579	0.759	138	0.00549	0.001228	34200	41.99	0.408	102

Table 6-24. Analysis of the Model at $S_0=750$ mg/lt, $K_h=2d^{-1}$, $S_{min}=11.38$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L \cdot x$ day	X_A (avg.) mg/lt	$A/Q \cdot L \cdot x \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=14.56$	1020	40.39	54.95	0.73	0.51	1715.808	875	0.00098	0.735
$S_E^*=16.32$	369.48	29.52	45.84	0.644	0.1847	3904.8	721.7	0.0027	2.029
$S_E^*=20.31$	147.6	20.81	41.12	0.506	0.0738	8176	603.4	0.00677	5.08
$S_E^*=26.80$	83.51	15.699	42.499	0.369	0.04175	12392	517.42	0.0121	9.08
$S_E^*=35.29$	56.01	12.299	47.589	0.258	0.028	16088	450.5	0.0178	13.39
$S_E^*=45.31$	41.367	9.99	55.3	0.1806	0.020	19288	398.94	0.02417	18.13
$S_E^*=56.42$	32.49	8.37	64.79	0.129	0.01624	22080	358.689	0.0307	23.08
$S_E^*=68.27$	26.66	7.198	75.468	0.095	0.0133	24539.3	327.10	0.0375	28.132
$S_E^*=74.39$	24.438	6.72	81.11	0.0828	0.0122	25661.6	313.559	0.0409	30.689
$S_E^*=80.60$	22.557	6.31	86.91	0.0726	0.01127	26720	301.36	0.04433	33.25
$S_E^*=93.18$	19.548	5.629	98.80	0.05696	0.00977	28640	279.92	0.05115	38.367
$S_E^*=105.87$	17.25	5.088	110.958	0.0458	0.00862	30400	262.2	0.0579	43.478
$S_E^*=137.29$	13.36	4.115	141.40	0.029	0.00668	34200	228.45	0.07485	56.137
$S_E^*=196.21$	9.27	3.039	199.249	0.0152	0.00463	39811.2	184.32	0.1078	80.90
$S_E^*=248.33$	7.12	2.433	250.76	0.0097	0.00356	43840	156	0.1404	105.33

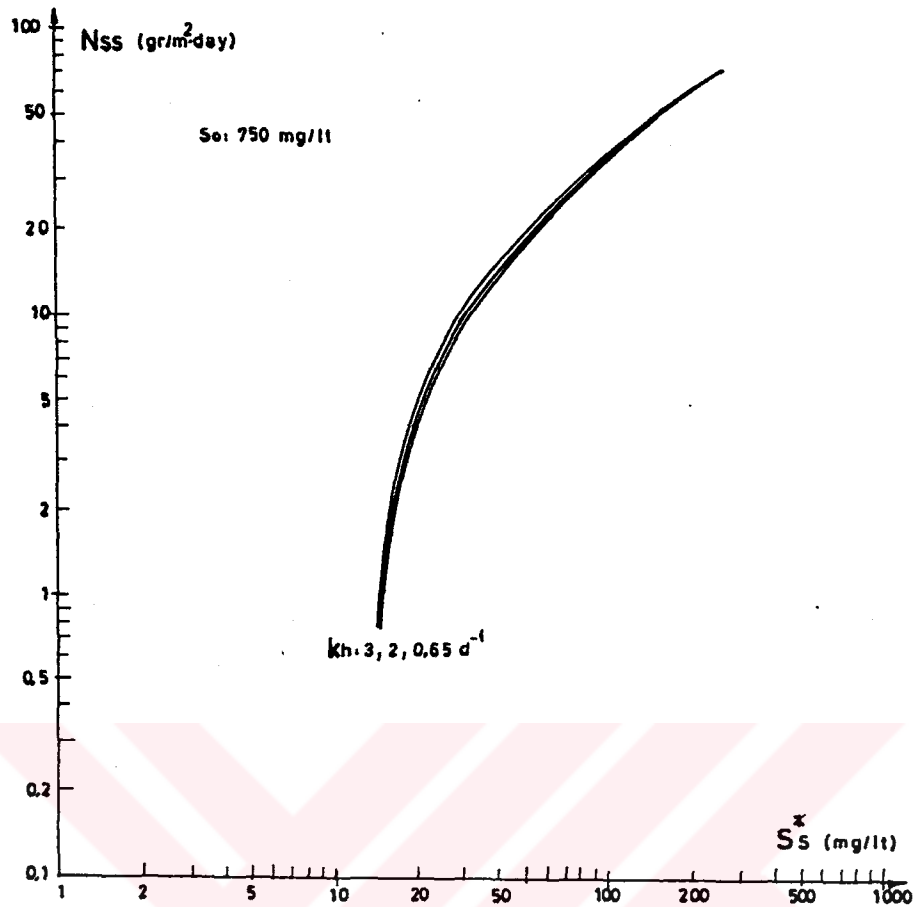


Figure 6-17. The Effect of k_h on N_{SS} versus S_S^*

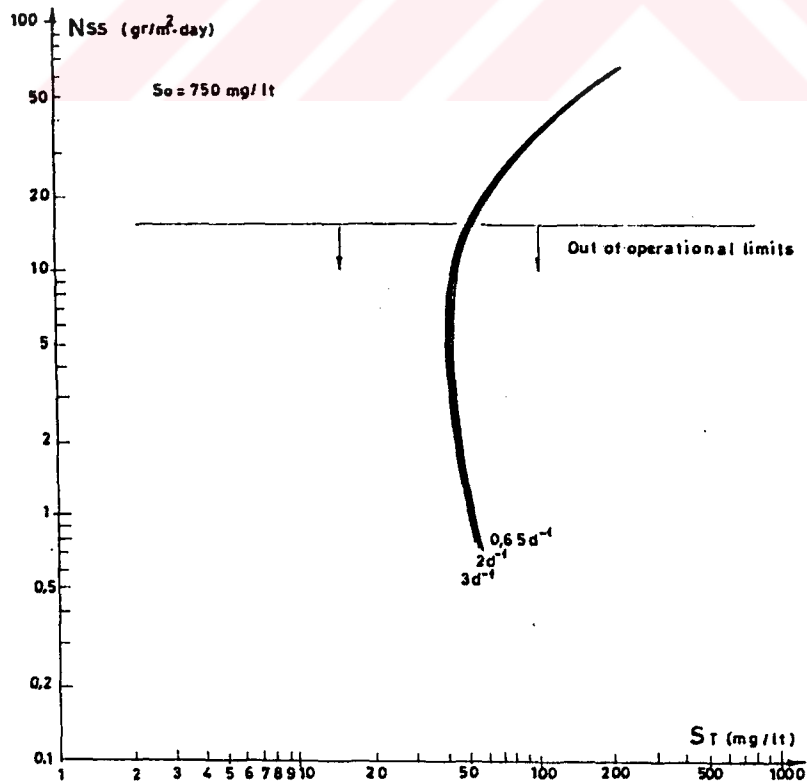


Figure 6-18 The Effect of k_h on N_{SS} versus S_T

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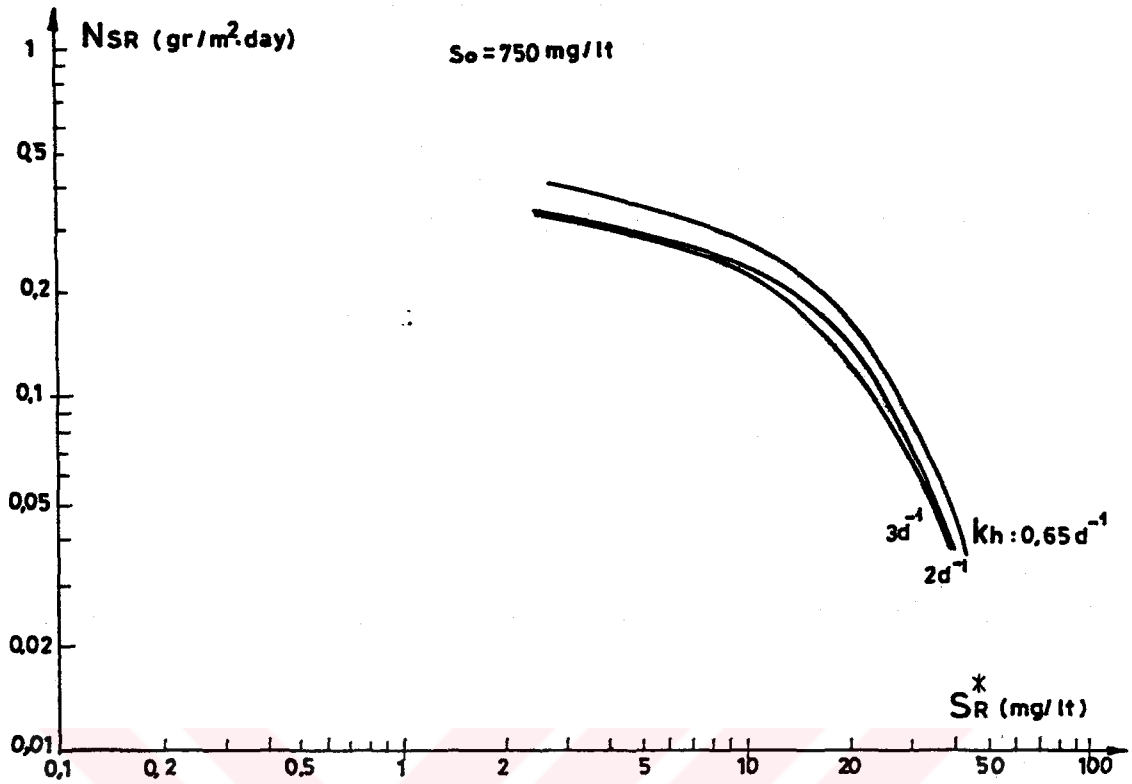


Figure 6-19. The Effect of k_h on N_{SR} versus S_R^* Profile at $S_0: 750$ mg/lit

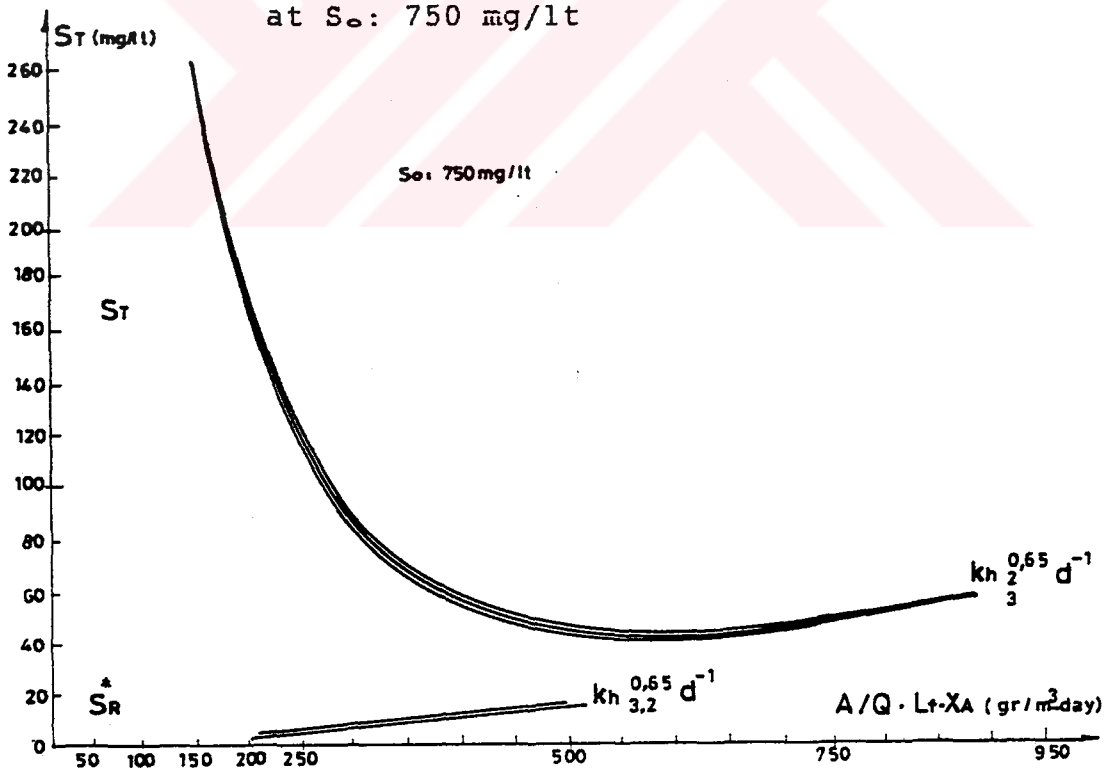


Figure 6-20 S_{SR}^* , S_R^* and S_T Values at Variable k_h Values and at $S_0: 750$ mg/lit

The analytical expression of S_R^* is;

$$S_R^* = \frac{A}{Q} \cdot N_{SR}$$

or, even,

$$S_R^* = \frac{A}{Q} \cdot \beta \cdot \frac{k_h (X_B/X_A)}{k_x + (X_B/X_A)} \cdot X_A$$

Mathematically, the decrease in the hydrolysis rate constant and increase in X_B compensate each other causing S_R^* to be insensitive to significant changes in k_h . In order to make this statement more clear; the following table consisting of a few examples is done.

Table 6-25. SRP Generated at Variable Maximum Specific Hydrolysis Rates

Parameters	$k_h = 0.65d^{-1}$	$k_h = 2 d^{-1}$	$k_h = 3 d^{-1}$
S_B^* (mg/lt)	82.97	80.60	80.89
X_A (mg/lt)	27200	26720	26764
X_B (mg/lt)	628.312	97.76	60.96
X_E (mg/lt)	52160	53178.4	53174
Active Biomass (gr/m ³)day	292.4	301.36	299.75
A/Q (m ² /m ³)day	21.5	22.55	22.46
S_R^* (mg/lt)	6.9	6.31	6.20

Table 6.25 clearly indicates that decrease in k_h , increase the X_s values, which is in correlation with the findings of similar study conducted on activated sludge system (Baykal, 1988). However, this increase in X_s is not sufficient enough to give rise to higher S_R^* values. Actually, even to those variations in k_h and X_s , the amount of active biomass does not change significantly, resulting to more or less identical S_R^* values. The volume fraction of X_s in a biofilm media is very low compared to the fractions of X_A and X_E ; thus, even for great deviations of X_s , it is not possible to reach significantly higher values of S_R^* .

6.2. APPLICATION OF THE MODEL TO ANAEROBIC FIXED-FILM SYSTEMS

The proposed model serves for mainly aerobic fixed-film systems. The studies done so far, aimed to evaluate the model under certain hydraulic conditions at the macro-level, where oxygen limitation was out of question. There arises a great interest whether the proposed model may be applicable to anaerobic fixed-film systems.

Same assumptions are applied as in aerobic systems where the kinetic process rates are the ones used for suspended-growth systems. Thus, in order to evaluate the model under anaerobic conditions the values of the model parameters need to be changed. For that reason, a literature review is done to find out the ranges of the governing parameters. Table 6.26 illustrates these ranges.

Table 6-26. Ranges of Parameters to be Used in Anaerobic Fixed-film Systems (Henze et. al., 1983; Şahinoğlu, 1990)

Parameters	Range of Parameters
K_s (mg/lt)	250 - 2000
μ (day ⁻¹)	0.3 - 0.8
Y	0.01 - 0.04
b_H (day ⁻¹)	0.01 - 0.4
k_h (day ⁻¹)	0.1 - 0.5

The typical parameters chosen to be used in the proposed model are;

$$\mu = 0.5 \text{ day}^{-1}$$

$$Y = 0.02$$

$$b_H = 0.2 \text{ day}^{-1}$$

$$k_h = 0.3 \text{ day}^{-1}$$

K_s values used are, 250, 600, 1000 and 2000 mg/lt

The influent substrate concentration, S_o , is assumed to be within a range of 2000 - 10000 mg/lt as anaerobic systems are designed to treat more stronger wastes. Similar macro-level calculations done in the previous sub-section of this part are followed in the same manner and the findings are tabulated in Tables 6.27 to 6.36. The graphical illustrations of the results are given in Figures 6.21 to 6.24. The typical S_o values used are 2000, 5000 and 10000 mg/lt. The sensitivity analysis is done only on parameter, K_s .

Table 6-27. Analysis of the Model at $S_o=2000$ mg/lt, $K_s=250$ mg/lt, $S_{min}=190.140$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_s^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_s$ day	X_A (avg.) mg/lt	$A/Q \cdot L_s \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_2 gr/ $m^2 \cdot day$
$S_s^*=231.22$	285.9	2.54	233.76	0.0108	0.1429	1104	157.76	0.00349	6.99
$S_s^*=254.02$	155.06	2.347	256.367	0.00915	0.07753	1944	150.718	0.006449	12.898
$S_s^*=286.14$	76.24	2.054	288.194	0.007127	0.03812	3673.44	140	0.0131	26.23
$S_s^*=309.36$	50.376	1.8679	311.22	0.0060	0.0251	5294	132.87	0.0198	39.70
$S_s^*=357.35$	28.94	1.6104	358.96	0.00448	0.01447	8548.8	123.70	0.03455	69.10
$S_s^*=397.55$	21.68	1.4678	399.01	0.00367	0.01084	10847.2	117.58	0.0461	92.25
$S_s^*=466.86$	15.40	1.2897	468.149	0.00275	0.0077	14120.8	108.73	0.0649	129.87
$S_s^*=517.23$	12.78	1.1887	518.418	0.00229	0.00639	16132.8	103.08	0.0782	156.49
$S_s^*=650.84$	8.76	0.98	651.82	0.0015	0.00438	20560	90.05	0.1141	228.31
$S_s^*=787.98$	6.45	0.818	788.797	0.00103	0.00322	24000	77.4	0.155	310.07

Table 6-28. Analysis of the Model at $S_o=2000$ mg/lt, $K_s=600$ mg/lt, $S_{min}=456.338$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_s^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_s$ day	X_A (avg.) mg/lt	$A/Q \cdot L_s \cdot X_A$ gr/ $m^3 \cdot day$	H_2 $m^3/m^2 \cdot day$	L_2 gr/ $m^2 \cdot day$
$S_s^*=493.08$	311.99	2.305	495.385	0.00464	0.1559	888	138.439	0.0032	6.410
$S_s^*=513.42$	190.587	2.229	515.669	0.00432	0.09529	1424	135.69	0.00524	10.488
$S_s^*=535.44$	129.26	2.150	537.6	0.00399	0.06463	2047.12	132.3	0.00773	15.47
$S_s^*=586.20$	67.5	1.964	588.164	0.00333	0.03375	3679	124.167	0.0148	29.63
$S_s^*=650.35$	36.685	1.7348	652.08	0.00265	0.0183	6221.6	113.85	0.02725	54.518
$S_s^*=734.27$	20.425	1.4767	735.746	0.002007	0.01021	10032	102.426	0.0489	97.919
$S_s^*=838.30$	12.509	1.238	839.538	0.00147	0.00625	14480	90.5	0.0799	159.88

Table 6-29. Analysis of the Model at $S_0=5000$ mg/lt, $K_S=250$ mg/lt, $S_{min}=190.140$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L \cdot \tau$ day	X_A (avg.) mg/lt	$A/Q \cdot L \cdot \tau \cdot X_A$ gr/ $m^3 \cdot day$	H_0 $m^3/m^2 \cdot day$	L_0 gr/ $m^2 \cdot day$
$S_E^*=231.22$	770.81	6.857	238.07	0.0288	0.3854	1104	425.487	0.00129	6.486
$S_E^*=254.02$	421.49	6.3838	260.40	0.0245	0.2107	1944	409.60	0.00237	11.86
$S_E^*=286.14$	209.78	5.64	291.78	0.01932	0.10489	3673.44	385.307	0.004766	23.83
$S_E^*=309.36$	139.768	5.17	314.53	0.0164	0.0698	5294	369.96	0.007154	35.77
$S_E^*=357.35$	81.78	4.55	361.9	0.01257	0.04089	8548.8	349.56	0.0122	61.139
$S_E^*=397.55$	62.27	4.215	401.76	0.01049	0.03113	10847.2	337.727	0.016	80.29
$S_E^*=466.86$	45.53	3.813	470.67	0.00810	0.02276	14120.8	321.46	0.0219	109.81
$S_E^*=517.23$	38.64	3.59	520.82	0.00689	0.01932	16132.8	311.685	0.0258	129.4
$S_E^*=650.84$	28.24	3.1628	654	0.00483	0.01412	20560	290.3	0.035	177
$S_E^*=787.98$	22.41	2.8423	790.82	0.00359	0.011205	24264	271.878	0.0446	223.11
$S_E^*=928.33$	18.726	2.5959	925.925	0.00280	0.009363	27403.2	256.576	0.0534	267
$S_E^*=1054.27$	16.17	2.3933	1056.66	0.00226	0.00808	29862.4	241.288	0.0618	309
$S_E^*=1299.1$	12.85	2.1048	1301.2	0.001617	0.006425	34534.4	221.88	0.0778	389

Table 6-30. Analysis of the Model at $S_o=5000$ mg/lt, $K_s=600$ mg/lt, $S_{min}=456.338$ mg/lt

S_B^* mg/lt	A/Q $m^2/m^3.d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L_f$ day	$X_A(avg.)$ mg/lt	$A/Q.L_f.X_A$ gr/ $m^3.day$	H_2 $m^3/m^2.day$	L_0 gr/ $m^2.day$
$S_B^*=493.08$	933	6.8863	499.96	0.01376	0.4665	888	414.25	0.00107	5.36
$S_B^*=535.44$	394	6.5411	541.98	0.0120	0.197	2047.12	403.3	0.00252	12.6
$S_B^*=586.20$	210.73	6.132	592.33	0.010	0.105	3679	387.63	0.0047	23.72
$S_B^*=650.35$	118.22	5.59	655.94	0.0085	0.059	6221.6	367.72	0.00845	42.29
$S_B^*=734.27$	68.83	4.97	739.24	0.00672	0.0344	10032	345.25	0.0145	72.64
$S_B^*=838.30$	44.81	4.44	842.74	0.005268	0.0224	14480	324.42	0.0223	111.58
$S_B^*=955.24$	32.64	4.01	959.25	0.00417	0.0163	18720	305.13	0.030	153.18
$S_B^*=1077.78$	25.668	3.67	1081.45	0.00339	0.0128	22700	286.72	0.0389	194.8
$S_B^*=1322.72$	18.114	3.152	1325.87	0.00237	0.009057	28792	260.769	0.055	276.02

Table 6-31. Analysis of the Model at $S_o=5000$ mg/lt, $K_B=1000$ mg/lt, $S_{min}=760.5633$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_x$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L_x \cdot X_A$ gr/ $m^3 \cdot \text{day}$	H_o $m^3/m^2 \cdot \text{day}$	L_o gr/ $m^2 \cdot \text{day}$
$S_E^*=796.35$	917.82	6.50	802.85	0.00809	0.4589	848	389.147	0.001089	5.447
$S_E^*=835.01$	419.4	6.29	841.3	0.00747	0.2097	1824	382.49	0.00238	11.92
$S_E^*=923.44$	168.8	5.874	929.31	0.00631	0.0844	4320	364.608	0.00592	29.62
$S_E^*=1032.57$	85.9	5.32	1037.89	0.00512	0.0429	7933.5	340.35	0.0116	58.207
$S_E^*=1168.43$	48.87	4.726	1173.15	0.004028	0.0244	13016	317.6	0.0204	102.31
$S_E^*=1326.87$	31.747	4.19	1331	0.00314	0.01587	18559.2	294.53	0.0315	157.49

Table 6-32. Analysis of the Model at $S_o=5000$ mg/lt, $K_B=2000$ mg/lt, $S_{min}=1521.1264$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_x$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L_x \cdot X_A$ gr/ $m^3 \cdot \text{day}$	H_o $m^3/m^2 \cdot \text{day}$	L_o gr/ $m^2 \cdot \text{day}$
$S_E^*=1556.26$	780.89	5.38	1561.64	0.00344	0.390	824	321.36	0.00128	6.4
$S_E^*=1710.58$	127.69	4.94	1715.52	0.002879	0.0638	4704	300.1152	0.00783	39.15
$S_E^*=1891.26$	56.327	4.43	1895.69	0.002336	0.02816	9760	274.84	0.0177	88.767
$S_E^*=2504.74$	14.94	3.06	2507.8	0.00747	0.00747	27631.2	206.4	0.0669	334.67

Table 6-33. Analysis of the Model at $S_o=10000$ mg/lt, $K_s=250$ mg/lt, $S_{min}=190.140$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d$	S_E^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q \cdot L_z$ day	$X_A(avq.)$ mg/lt	$A/Q \cdot L_z \cdot X_A$ gr/ $m^3 \cdot day$	H_o $m^3/m^2 \cdot day$	L_o gr/ $m^2 \cdot day$
$S_E^*=231.22$	1578.9	14.04	245.267	0.057	0.7894	1104	871.55	0.000633	6.33
$S_E^*=254.02$	865.53	13.069	267.089	0.04867	0.4327	1944	841.29	0.001155	11.55
$S_E^*=286.14$	432.13	11.64	297.78	0.039	0.2160	3673.44	793.70	0.00231	23.141
$S_E^*=309.36$	288.7	10.68	320	0.0333	0.144	5294	764.188	0.00346	34.638
$S_E^*=357.35$	169.88	9.45	366.8	0.0257	0.0849	8548.8	726.135	0.00588	58.86
$S_E^*=397.55$	129.91	8.80	406.35	0.0216	0.0649	10847.2	704.579	0.00769	76.97
$S_E^*=466.86$	95.76	8.015	474.87	0.01684	0.04788	14120.8	676.10	0.0104	104.42
$S_E^*=517.23$	81.748	7.60	524.83	0.01448	0.0408	16132.8	659.41	0.0122	122.32
$S_E^*=656.84$	60.70	6.799	657.639	0.0103	0.0303	20560	623.996	0.0164	164.74
$S_E^*=787.98$	49.015	6.215	794.195	0.00782	0.0245	24264	594.6499	0.0204	204
$S_E^*=923.33$	41.687	5.753	929	0.00619	0.0208	27403.2	571.178	0.0239	239.88
$S_E^*=1054.27$	36.66	5.426	1059.69	0.00512	0.0183	29862.4	547.377	0.0272	272.77
$S_E^*=1299.10$	30.211	4.9486	1304	0.00378	0.0151	34534.4	521.469	0.0331	331
$S_E^*=1520.31$	26.09	4.566	1524.87	0.00299	0.0130	38000	494	0.0383	383.3
$S_E^*=1719.76$	23.32	4.315	1724	0.00250	0.01166	40880	476.66	0.04288	428.8
$S_E^*=2064.19$	19.66	3.933	2068.123	0.0019	0.00983	45440	446.67	0.0508	508.64
$S_E^*=2707.01$	15.09	3.397	2710.4	0.00125	0.00754	52800	398.376	0.066	662.7
$S_E^*=3164.35$	12.80	3.097	3167.44	0.000947	0.0064	57520	368.128	0.0781	781

Table 6-34. Analysis of the Model at $S_o=10000$ mg/lt, $K_s=600$ mg/lt, $S_{min}=456.338$ mg/lt

S_s^* mg/lt	A/Q $m^2/m^3.d.$	S_R^* mg/lt	S_T mg/lt	S_R/S_T	$A/Q.L_x$ day	$X_A(avg.)$ mg/lt	$A/Q.L_x.X_A$ gr/ $m^3.day$	H_g $m^3/m^2.day$	L_g gr/ $m^2.day$
$S_s^*=493.08$	1968.3	14.526	507.60	0.0286	0.98415	888	873.92	0.000508	5.08
$S_s^*=586.20$	449.56	13.08	599.28	0.0218	0.22478	3679	826.965	0.00222	22.24
$S_s^*=650.35$	254.13	12.018	662.368	0.0181	0.127	6221.6	790.547	0.00393	39.34
$S_s^*=734.27$	149.52	10.81	745.33	0.0145	0.07476	10032	749.998	0.006688	66.88
$S_s^*=838.3$	98.65	9.77	848	0.0115	0.04932	14480	714.22	0.0101	101.368
$S_s^*=955.24$	73	8.979	964.219	0.0093	0.0365	18720	683.28	0.01369	136.98
$S_s^*=1077.78$	58.39	8.349	1086.129	0.00768	0.0291	22400	653.968	0.0171	171.26
$S_s^*=1322.72$	42.665	7.457	1330	0.0056	0.0213	28792	614.2	0.0234	234.38
$S_s^*=2319.45$	21.33	5.483	2324.93	0.00235	0.01066	46496	464.96	0.0468	468.82
$S_s^*=3001.94$	15.697	4.69	3006.63	0.001559	0.00784	55560	435.59	0.0637	637.06

Table 6-35. Analysis of the Model at $S_o=10000$ mg/lt, $K_S=1000$ mg/lt, $S_{min}=760.56338$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q \cdot L^*$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L^* \cdot X_A$ $gr/m^3 \cdot day$	H_o $m^3/m^2 \cdot day$	L_o $gr/m^2 \cdot day$
$S_E^*=796.35$	2013	14.278	810.628	0.0176	1.00	848	848	0.000496	4.967
$S_E^*=835.01$	922.95	13.844	848.85	0.0163	0.4614	1824	841.73	0.00108	10.83
$S_E^*=923.44$	375.84	13.079	936.519	0.0139	0.1879	4320	811.814	0.00266	26.60
$S_E^*=1032.57$	194.18	12.03	1044.6	0.0115	0.097	7933.5	770.26	0.005149	51.49
$S_E^*=1168.43$	112.63	10.89	1179.32	0.00923	0.0563	13016	732.99	0.008878	88.78
$S_E^*=1326.87$	74.96	9.895	1336.76	0.0074	0.03748	18559.2	707.96	0.0133	133.4
$S_E^*=2143.7$	29.649	7.264	2150.96	0.00337	0.0148	38480	570.44	0.0337	337.27
$S_E^*=2799.09$	20.17	6.1318	2805.22	0.00218	0.010	49592	495.92	0.04957	495.78

Table 6-36. Analysis of the Model at $S_o=10000$ mg/lt, $K_S=2000$ mg/lt, $S_{min}=1521.1267$ mg/lt

S_E^* mg/lt	A/Q $m^2/m^3 \cdot d.$	S_E^* mg/lt	S_T mg/lt	S_E/S_T	$A/Q \cdot L^*$ day	$X_A(\text{avg.})$ mg/lt	$A/Q \cdot L^* \cdot X_A$ $gr/m^3 \cdot day$	H_o $m^3/m^2 \cdot day$	L_o $gr/m^2 \cdot day$
$S_E^*=1556.26$	1914.68	13.209	1569.46	0.00841	0.9573	824	788.81	0.00052	5.22
$S_E^*=1710.58$	321.70	12.44	1723	0.007219	0.16085	4704	756.63	0.00310	31.08
$S_E^*=1891.26$	146.92	11.54	1902.8	0.00606	0.07346	9750	716.96	0.0068	68.06
$S_E^*=1940.97$	125.92	11.33	1952.30	0.0058	0.06296	11312	712.2	0.00794	79.415
$S_E^*=2212.86$	67.915	10.119	2222.97	0.00455	0.0339	19520	661.728	0.0147	147.24
$S_E^*=2504.74$	44.88	9.19	2513.9	0.00365	0.02244	27624	619.88	0.02228	222.81
$S_E^*=3067.71$	27.09	7.80	3075.51	0.00253	0.01354	40552	549	0.0369	369.13

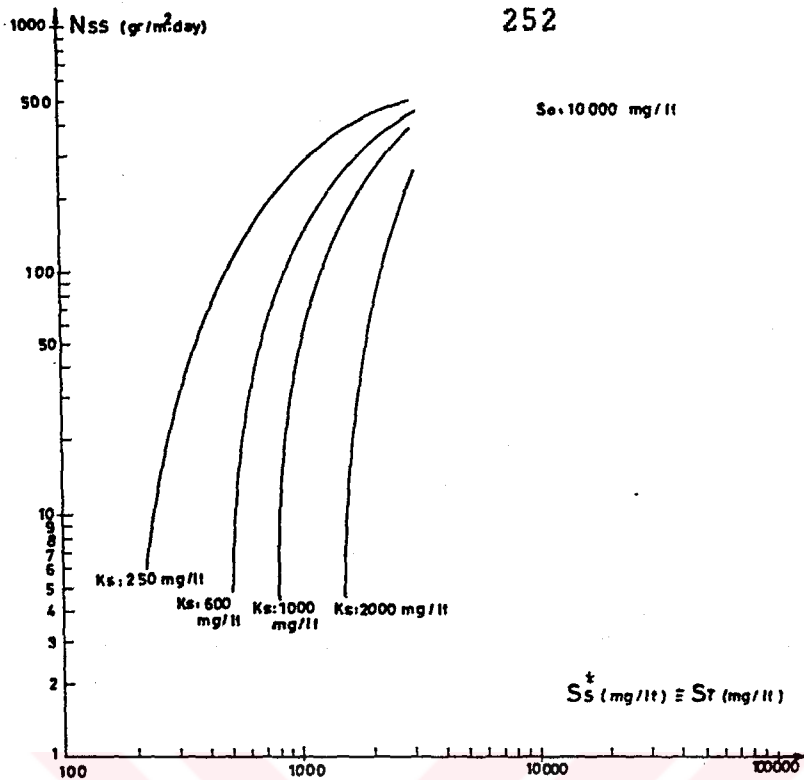


Figure 6-21. The Effect of K_s on N_{ss} versus S_s^* Profile under Anaerobic Conditions at S_o : 10000 mg/l

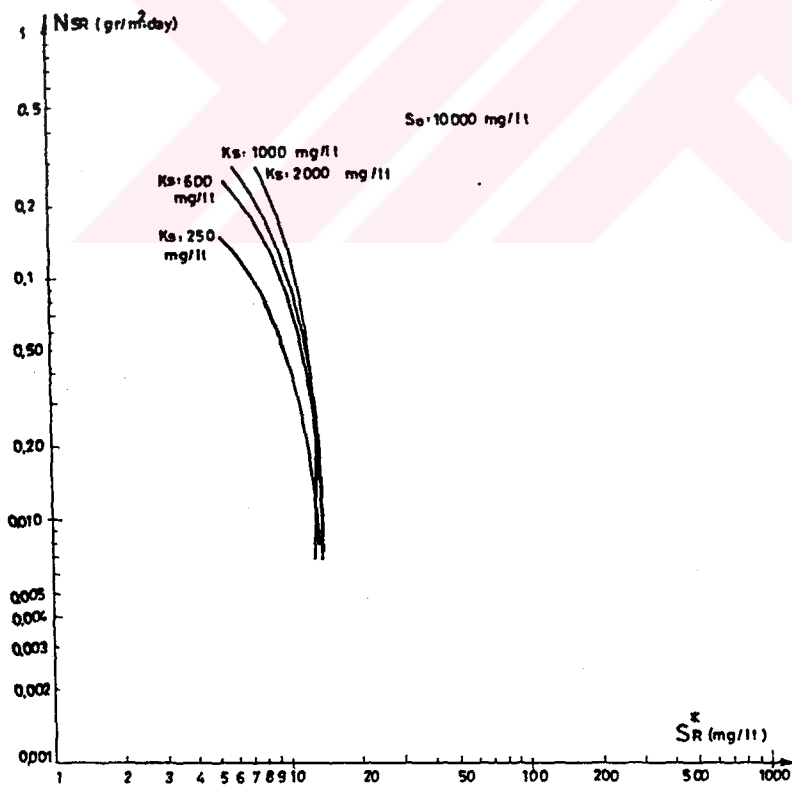


Figure 6-22. The Effect of K_s on N_{BR} versus S_R^* Profile under Anaerobic Conditions at S_o : 10000 mg/l

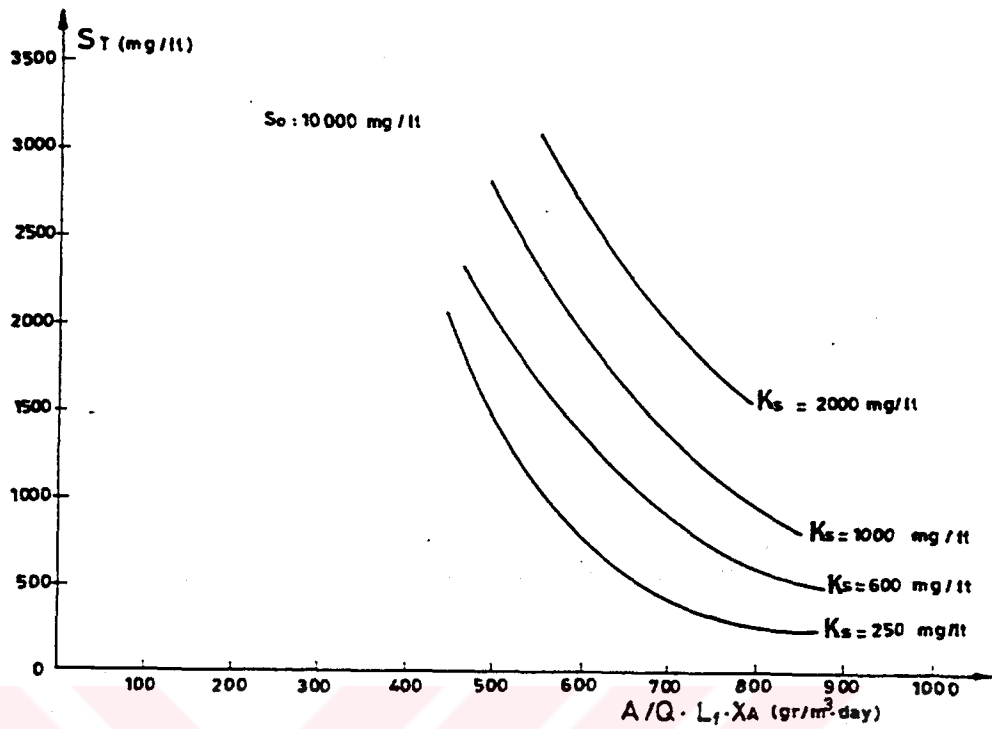


Figure 6-23. S_T Values under Anaerobic Conditions for Variable K_S Values at $S_0 : 10000$ mg/l.

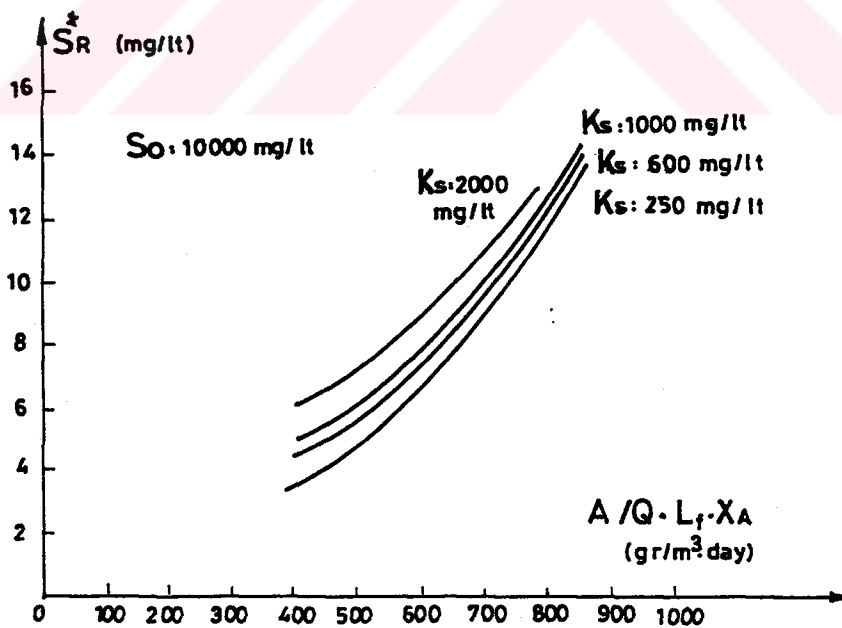


Figure 6-24. S_R^* Values under Anaerobic Conditions for Variable K_S Values at $S_0 : 10000$ mg/l.

It is seen that K_s is one of the parameters that is closely related to the removal mechanism of the system. As K_s value increase, N_{ss} of the system decreases thus causing higher S_s^* values. On the other hand, SRP generation mechanism is not too sensitive to variations in K_s , as this parameter is not related to SRP generation mechanism. Conceptually, S_T and K_s are directly proportional to each other, as $S_s^* \approx S_T$ due to the insignificance of S_R^* with respect to high S_s^* values faced under anaerobic conditions. On the whole, the fraction of S_R^* due to S_T is quite negligible as $S_R^*/S_T = \text{maximum } 1\%$.

Figure 6.25 shows S_T values at variable S_o values under anaerobic conditions where $K_s = 600$ mg/lt.

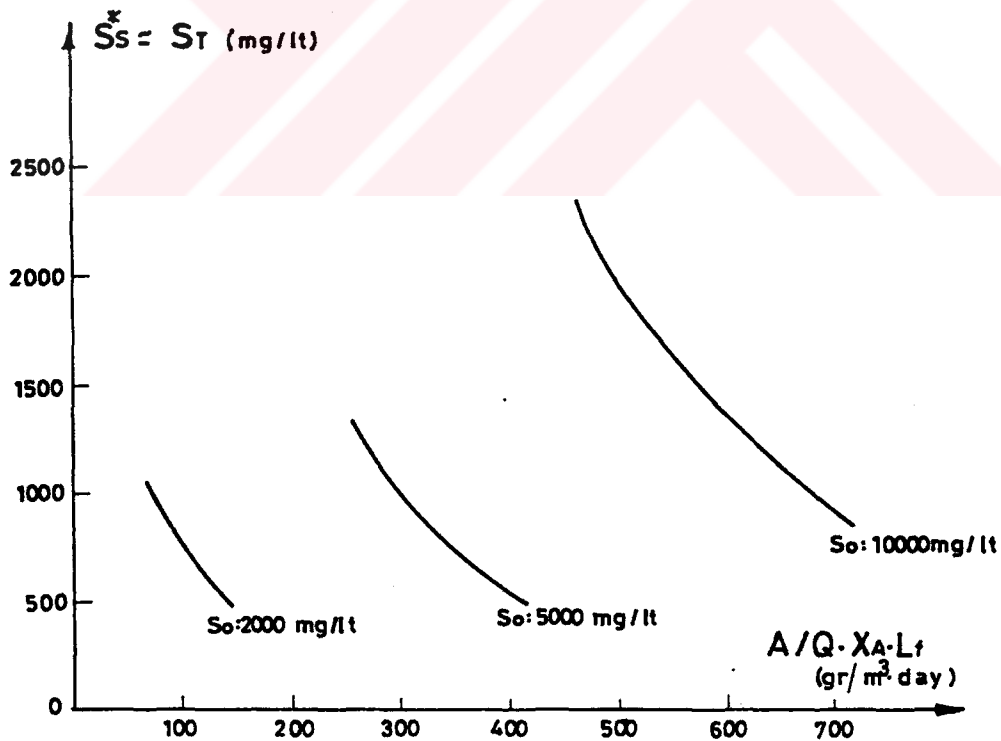


Figure 6.25 S_T Values at Variable S_o Values under Anaerobic Conditions at $K_s : 600$ mg/lt.

Observing Figure 6.25 indicates that as S_o increases, S_s^* thus, S_T of the system increases even though, S_R^* has negligible effect on S_T . The total amount of active biomass of the system increases as S_o increases, due to the decrease in the hydraulic loading.

6.3. COMPARISON OF THE MODEL FINDINGS OF BOTH AEROBIC AND ANAEROBIC FIXED-FILM SYSTEMS

Although, not much effort is given on the application of the model to anaerobic fixed-film systems, certain conclusions may still be stated as follows;

- Under anaerobic conditions, many of the parameters lie within different ranges compared to aerobic conditions. The most important parameter in the process rate equations that govern the whole of the reaction matrix is the yield coefficient, Y , which is quite low compared to aerobic systems. Thus, the governing parameters effecting SRP generation in fixed film systems, namely b_H , k_H and X_S , are lower compared to aerobic systems.
- As b_H value is low, -which is a parameter responsible for production of X_S -, not much X_S is produced.
- The hydrolysis mechanism takes place in the presence of X_S with a hydrolysis rate constant, k_H . In anaerobic systems, this hydrolysis constant is also low with respect to its value under aerobic conditions.
- Both parameters, b_H and k_H , closely related to the hydrolysis mechanism are quite low. One can state that

in anaerobic systems, the hydrolysis mechanism occurs at such a low rate, that, its consequence is seen in the very low S_R^* values generated compared to S_R^* values obtained under aerobic conditions. The maximum ratio of S_R^*/S_T of a system is about 1%.

- As has been stated previously, S_R^* is also a function of hydraulic loading. But as N_{SR} values are very low (in the order of 10^{-2} and 10^{-3}), the resulting S_R^* values are quite low respectively, even though, the A/Q values are quite high. The N_{SR} values are in the order of 10^{-1} in aerobic fixed-film systems.

- Finally, it can be stated that, the hydrolysis mechanism occurs at very low rates in anaerobic systems, thus, the S_R^* generated as an end product of hydrolysis, has insignificant effect under such conditions, compared to S_E^* of the system. Therefore, in cases where anaerobic conditions prevail; S_T of a system is equal to S_E^* ($S_T \approx S_E^*$).

PART VII

CONCLUSIONS AND RECOMMENDATIONS

In this study, the proposed model differentiates between soluble and particulate substrates, incorporates the viability concept, adopts the death/regeneration approach, and includes inert soluble substrate generation in fixed-film systems.

In the first part of the modelling stage, micro-scale analysis of a specific biofilm media is done. The behaviour of particulate components, X_A , X_B and X_E are investigated in detail. It is then that the removal mechanism of soluble substrate, S_B and the generation mechanism of inert soluble residual product, S_R , are examined.

The effect of operational parameters, such as, hydraulic and organic loading, flow rate and the influent substrate concentration of the wastewater together with the total surface area of the reactor are evaluated at macroscopic level consisting the second part of the modelling study.

Finally, the effect of certain governing kinetic and stoichiometric parameters on the model are analysed.

Observations based on this study may be summarized as follows:

- 1- Although the density/concentration of a specific biofilm media is assumed to be constant, the particulate matter composing the biofilm have variable volume fractions along thickness. Their average values

are used in calculations at macro level and it is recognized that the variation of volume fractions of each component cease as soon as the "effective" or the "active" thickness is reached. From thereon towards the support media, a stability condition within the film media is observed. That is partly due to oxygen shortage and partly due to limitations such as diffusional resistance. In this study, oxygen as a second soluble substrate is not included in modelling as single substrate is considered.

2- The distribution and the behaviour of particulate components (active biomass, slowly degradable substrate and inert particulate organic matter) closely relate and control the substrate removal mechanism. The removal rate approach stability as soon as the "active" thickness is reached.

3- The volume fraction of slowly degradable substrate is highly lower compared to the volume fractions of the other two particulate components along a specific biofilm media. Therefore, the generation rate mechanism is observed to be a slow mechanism, as the total amount of slowly degradable substrate generated within the biofilm media is of negligible amount. The resulting rates of both the removal and generation mechanisms expressed in terms of flux designate the fact that the flux of soluble substrate is quite higher compared to the flux of soluble residual product.

4- The removal rate mechanism of soluble substrate within a biofilm media basically depends on the slower growth rate as the soluble substrate within a biofilm media is always lower than the bulk liquid concentration. On the other hand, the generation rate

mechanism is basically controlled by diffusional resistances within a biofilm media, as the concentration of soluble residual product decreases towards the bulk-liquid interface.

5- Degradation of slowly biodegradable substrate is an important component in the modelling of inert soluble residual product generation mechanism, however, the resulting flux value indicate the insignificance of that mechanism. The generation mechanism is highly due to the degradation of slowly biodegradable substrate controlled by decay-associated mechanisms. As the decay-associated mechanisms control the generation mechanism of soluble residual product, the related parameters the decay coefficient, hydrolysis associated product formation coefficient and maximum specific hydrolysis rate-directly effect the resulting flux values of soluble residual product.

6- Soluble residual product is found to be a function of external conditions, in other words, a function of the flow rate of the wastewater and the total surface area of the reactor.

$$S_R^* = \frac{A}{Q} \cdot N_{SR}$$

In short, the hydraulic conditions of the system control the bulk-liquid concentration of soluble residual product. As stated above, the resulting bulk-liquid concentration of soluble residual product is also low, even at low hydraulic loading values.

7- The performance of the macro model is analysed within acceptable operational limits of hydraulic and organic loading values, indicated that for lower values of soluble substrate at the bulk-liquid interface; $S_s^* < 40$ mg/lit, the ratio of soluble residual product over total effluent concentration at the bulk-liquid interface varied within a range of 0.15 to 0.65 depending on variable influent substrate concentration. This ratio decreased to a range of 0.15 to 0.03 for $40 < S_s^* < 100$ mg/lit depending on variable S_0 values.

8- The influent substrate concentration also effects the soluble residual product at the bulk-liquid interface as the hydraulic loading of a system is a function of influent substrate concentration, soluble substrate concentration at the bulk-liquid interface and of the flux of soluble substrate.

For a specific soluble substrate concentration at the bulk-liquid interface, the ratio of soluble residual product to total effluent concentration is observed to increase directly with increase in influent substrate concentration.

9- The sensitivity analysis done on certain model parameters show that the two growth coefficients, half saturation coefficient for growth and maximum specific growth rate are not effective on soluble residual product formation, whereas the decay coefficient is observed to be an objectionable parameter of the model on soluble residual product generation. Hydrolysis associated product formation coefficient is a determinant of the generation mechanism of soluble residual product and is directly proportional to soluble residual product at the bulk-liquid interface. Even

though, the soluble residual product at the bulk-liquid interface is a function of the maximum specific hydrolysis rate, it seems to be that variations of this parameter has insignificant effect on soluble residual product at the bulk-liquid interface.

10- In cases, where equal influent and effluent conditions are considered both in an activated sludge and in a fixed-film system, the soluble residual product at the bulk-liquid interface is found to be lower in fixed-film systems, even the total amount of active biomass is higher. This fact is due to diffusional resistances in a film media, as the corresponding soluble residual product concentration at the support media slightly exceeds the soluble residual product at the bulk-liquid of an activated sludge system.

11- Although not much effort is given to the application of the proposed model under anaerobic conditions; a few trials of application put forth the fact that as the yield coefficient is quite low compared to aerobic systems, the hydrolysis process is a much more slower process due to the very low concentration of slowly degradable substrate and also due to the lower values of the governing parameters; the decay coefficient and the maximum specific hydrolysis rate, even the hydraulic loading values are quite lower compared to aerobic fixed-film systems. The maximum ratio of soluble residual product to the total effluent concentration is around 0.01.

Further studies in fixed-film systems may be recommended as follows;

- Similar modelling study considering the presence of dual substrate where one is organic matter and the other is oxygen,

- Similar modelling study considering transient order conditions of soluble substrate removal,

- Adaptation of the two-staged modelling to cases where nitrification and denitrification processes occur,

- Adaptation of the two-staged modelling to cases where part of the specific film media is under aerobic conditions while the other part is under anaerobic conditions.

As a concluding remark, it must be stated that a complete and a general mechanistic model for biofilms to simulate the performance of a wide range of fixed-film systems has not been achieved yet. The important fact that must be kept in mind is that further studies are needed to fill in the remaining gaps of knowledge of fixed-film theory.

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APPENDIX A

REARRANGEMENT OF THE EQUATIONS OF THE MICRO MODEL
IN FINITE DIFFERENCE FORM AND THE SOLUTION PROCEDURE

BY NEWTON - RALPSON METHOD

The first function, $G(w, x, \mu)$ has been expressed in finite difference form for the biofilm support media, ($L=0$) as follows;

$$G(w, x, \mu) = \{[p + \mu x - p(1+w)x](k_x + w) - x[(k_h - q)w^2 - k_x q w](2x - x_0) - \mu x(k_x + w)\} = 0 \quad (4.54)$$

The constant terms in the equation can be abbreviated accordingly, as, $k_h - q = c$, $k_x q = d$, and therefore, further simplification yields,

$$\{[p + \mu x - p(1+w)x](k_x + w) - x[cw^2 - dw]\} = G_1$$

Finally, the function $G(w, x, \mu)$ becomes,

$$G(w, x, \mu) = G_1(2x - x_0) - \mu x(k_x + w) = 0 \quad (4.54a)$$

The second function, $H(w, x, \mu)$ has been expressed in finite difference form for the biofilm support media, ($L=0$) as follows;

$$H(w, x, \mu) = \frac{\hat{\mu}}{(\mu_N - p)} \cdot \frac{K_s Y D}{X_T \Delta L^2} (\mu - p)(w + k_x) + (\mu - \mu_N)$$

$$[(\mu + b_h)(w + k_x) - (1 - \beta)Yk_h w] x = 0 \quad (4.55)$$

The constant terms can be abbreviated as given below,

$$\frac{\hat{\mu}}{(\mu_N - p)} \cdot \frac{K_s Y D}{X_T \Delta L^2} = a, \quad (1 - \beta)Yk_h w = b$$

Thus, the function $H(w, x, \mu)$ is simplified to;

$$H(w, x, \mu) = a(\mu - p)(w + k_x) + (\mu - \mu_N)[(\mu + b_H)(w + k_x) - bw]x = 0 \quad (4.55a)$$

and, the last function has been given as;

$$F(w, x, \mu) = a(2wx - y_o)(k_x + w)\mu + (2x - x_o)[(k_H + q)w^2 - k_x qw] = 0 \quad (4.53)$$

In order to solve these functions according to the numerical integration procedure of Newton-Ralpson Method, the derivatives of each function with respect to their variables are required.

$$\frac{\partial F}{\partial x} \Delta x + \frac{\partial F}{\partial w} \Delta w + \frac{\partial F}{\partial \mu} \Delta \mu = -F(x, w, \mu) \quad (A.1)$$

$$\frac{\partial G}{\partial x} \Delta x + \frac{\partial G}{\partial w} \Delta w + \frac{\partial G}{\partial \mu} \Delta \mu = -G(x, w, \mu) \quad (A.2)$$

$$\frac{\partial H}{\partial x} \Delta x + \frac{\partial H}{\partial w} \Delta w + \frac{\partial H}{\partial \mu} \Delta \mu = -H(x, w, \mu) \quad (A.3)$$

$$F_x \Delta x + F_w \Delta w + F_\mu \Delta \mu = -F \quad (A.4)$$

$$G_x \Delta x + G_w \Delta w + G_\mu \Delta \mu = -G \quad (A.5)$$

$$H_x \Delta x + H_w \Delta w + H_\mu \Delta \mu = -H \quad (A.6)$$

$$\pm \Delta x \pm \frac{F_w}{F_x} \Delta w \pm \frac{F_\mu}{F_x} \Delta \mu = \pm \frac{F}{F_x} \quad (A.7)$$

$$\Delta x + \frac{G_w}{G_x} \Delta w + \frac{G_\mu}{G_x} \Delta \mu = - \frac{G}{G_x} \quad (A.8)$$

$$\Delta x + \frac{H_w}{H_x} \Delta w + \frac{H_\mu}{H_x} \Delta \mu = - \frac{H}{H_x} \quad (A.9)$$

Subtracting (A.8) from (A.7) leads to the form;

$$\left(\frac{G_w}{G_x} - \frac{F_w}{F_x}\right) \Delta w + \left(\frac{G_\mu}{G_x} - \frac{F_\mu}{F_x}\right) \Delta \mu = \frac{F}{F_x} - \frac{G}{G_x}$$

Subtracting (A.9) from (A.7) leads,

$$\left(\frac{H_w}{H_x} - \frac{F_w}{F_x}\right) \Delta w + \left(\frac{H_\mu}{H_x} - \frac{F_\mu}{F_x}\right) \Delta \mu = \frac{F}{F_x} - \frac{H}{H_x}$$

$$\pm \Delta w \pm \frac{\left(\frac{G_\mu}{G_x} - \frac{F_\mu}{F_x}\right)}{\left(\frac{G_w}{G_x} - \frac{F_w}{F_x}\right)} \Delta \mu = \pm \frac{\left(\frac{F}{F_x} - \frac{G}{G_x}\right)}{\left(\frac{G_w}{G_x} - \frac{F_w}{F_x}\right)}$$

$$\Delta w + \frac{\left(\frac{H_\mu}{H_x} - \frac{F_\mu}{F_x}\right)}{\left(\frac{H_w}{H_x} - \frac{F_w}{F_x}\right)} \Delta \mu = \frac{\left(\frac{F}{F_x} - \frac{H}{H_x}\right)}{\left(\frac{H_w}{H_x} - \frac{F_w}{F_x}\right)}$$

$$\Delta \mu = \frac{\left(\frac{F}{F_x} - \frac{H}{H_x}\right) / / \left(\frac{H_w}{H_x} - \frac{F_w}{F_x}\right) - \left(\frac{F}{F_x} - \frac{G}{G_x}\right) / /}{\left(\frac{H_\mu}{H_x} - \frac{F_\mu}{F_x}\right) / / \left(\frac{H_w}{H_x} - \frac{F_w}{F_x}\right) - \left[\frac{G_\mu}{G_x} - \frac{F_\mu}{F_x}\right] / /}$$

$$\frac{\left(\frac{G_w}{G_x} - \frac{F_w}{F_x}\right)}{\left(\frac{G_w}{G_x} - \frac{F_w}{F_x}\right)} = \frac{\text{PAY}}{\text{PDA}}$$

where, the partial derivatives are,

$$F_x = 2w(k_x + w)\mu \quad (A.10)$$

$$F_w = \mu[2x + (k_x + w) + (2wx - y_0)] + (2x - x_0)[2(k_h - q)w - k_x q] \quad (A.11)$$

$$F_\mu = (2wx - y_0)(k_x + w) \quad (A.12)$$

$$G_x = 2G_1 + (2x - x_0) \{ (k_x + w)[\mu - p(1 + w)] + [(q - k_h)w^2 + k_x q w] \} - \mu(k_x + w) \quad (A.13)$$

$$G_w = (2x - x_0) \{ [p + \mu x - p(1 + w)x] - (k_x + w)px + x[2(q - k_h)w + k_x q] \} - \mu x \quad (A.14)$$

$$G_\mu = (k_x + w)x(2x - x_0) - (k_x + w)x = [2x - x_0^{-1}](k_x + w)x \quad (A.15)$$

$$H_x = (\mu - \mu_N)[(\mu + b_H)(w + k_x) - bw] \quad (A.16)$$

$$H_w = a(\mu - p)(\mu - \mu_N)x[(\mu + b_H - b)] \quad (A.17)$$

$$H_\mu = a(k_x + w) + x\{[(\mu + b_H)(w + k_x) - bw] + (\mu - \mu_N)(x + k_x)\} \quad (A.18)$$

For increments greater than 1 ($n \geq 2$) equation (4.46) can be expressed as follows,

$$(\mu_n - \mu_{An})x_n^2 = (x_n - x_{n-1}) \sum_{i=1}^n \mu_{Ai} x_i, \text{ where}$$

$$\sum_{i=1}^n \mu_{Ai} x_i = \frac{(\mu_N - \mu_{An}) x_n^2}{(x_n - x_{n-1})}$$

Therefore; the expression becomes,

$$(\mu_n - \mu_{An})x_n^2 = (x_n - x_{n-1}) \left[\mu_{An}x_n + \sum_{i=1}^{n-1} \mu_{Ai}x_i \right] \quad (A.19)$$

Replacing (n-1) instead of n in the above summation yields,

$$\sum_{i=1}^{n-1} \mu_{Ai}x_i = \frac{(\mu_{n-1} - \mu - \mu_{An-1})x_{n-1}^2}{(x_{n-1} - x_{n-2})}$$

Simplifying certain terms as,

$\mu_n = \mu$, $x_n = x$, $\mu_{An} = \mu_A$, and

by substituting them in equation (4.46) gives,

$$(\mu - \mu_A)x^2 = (x - x_{n-1}) \left[\mu_Ax + \frac{(\mu_{n-1} - \mu_{An-1})x_{n-1}^2}{(x_{n-1} - x_{n-2})} \right] \quad (4.46a)$$

In the same way, equation (4.47) can be written in finite difference for increment $n \geq 2$ as follows;

$$(\mu_y - \mu_A)xy = (y - y_{n-1}) \left[\mu_Ax + \frac{(\mu_{n-1} - \mu_{An-1})x_{n-1}^2}{(x_{n-1} - x_{n-2})} \right] \quad (4.47a)$$

If same procedure is applied to equation (4.49);

$$(S_n - S_{n-1}) - (S_{n-1} - S_{n-2}) = \frac{X_T L^2}{Y.D.} \left[\hat{\mu} \frac{S_n}{K_S + S_n} - (1-\beta) Y k_h \frac{Y_n}{k_x x_n + y_n} \right] x_n$$

where,

$$\hat{\mu} = \hat{\mu} \frac{S_s}{K_S + S_s} - b_H, \quad \hat{\mu} \frac{S_s}{K_S + S_s} = \mu + b_H, \quad S_s = K_S \frac{\mu + b_H}{\mu - \mu - b_H}, \text{ or}$$

$$S_s = K_S \frac{(\mu + b_H)}{(-\mu + \mu_N)}$$

Substituting $\mu_N = \mu - b_H$ in equation (4.49) yields,

$$\left(\frac{\mu + b_H}{-\mu + \mu_N} - \frac{\mu_{n-1} + b_H}{-\mu_{n-1} + \mu_N} \right) - \left(\frac{\mu_{n-1} + b_H}{\mu_{n-1} + \mu_N} - \frac{\mu_{n-2} + b_H}{-\mu_{n-2} + \mu_N} \right) = \frac{X_T L^2}{K_S Y D} \times$$

$$\left[\mu + b_H - (1-\beta) Y k_h \frac{y}{k_x x + y} \right]$$

The left-hand side of the above expression;

$$\frac{(\mu + b_H)(-\mu_{n-1} + \mu_N)(-\mu + \mu_N)(\mu_{n-1} + b_H)}{(-\mu_{n-1} + \mu_N)(-\mu + \mu_N)} -$$

I

$$\frac{[(\mu_{n-1} + b_H) - (\mu_{n-2} + b_H) - (\mu_{n-2} + b_H) - (\mu_{n-1} + \mu_N)]}{(-\mu_{n-2} + \mu_N)(-\mu_{n-1} + \mu_N)}$$

II

$$I = \mu(-\mu_{n-1} + \mu_N + \mu_{n-1} + b_H) - b_H\mu_{n-1} + b_H\mu_N - \mu_N\mu_{n-1} - \mu_N b_H$$

$$I = \mu(\mu_N + b_H) - \mu_{n-1}(\mu_N + b_H) - \mu(\mu - \mu_{n-1})$$

where,

$$\mu_N + b_H = \mu$$

$$II = \mu_{n-1}(-\mu_{n-2} + \mu_N + \mu_{n-2} + b_H) - b_H\mu_{n-2} + b_H\mu_N - \mu_N\mu_{n-2} - b_H\mu_N$$

$$II = \mu_{n-1}(\mu_N + b_H) - \mu_{n-2}(\mu_N + b_H) = \mu(\mu_{n-1} - \mu_{n-2})$$

The final terms (I) and (II) are substituted in equation (4.49) yielding,

$$\frac{\hat{\mu}(\mu - \mu_{n-1})}{(\mu_N - \mu_{n-1})(-\mu + \mu_N)} - \frac{\hat{\mu}(\mu_{n-1} - \mu_{n-2})}{(\mu_N - \mu_{n-2})(\mu_N - \mu_{n-1})} = \frac{X_T \Delta L^2}{K_S Y D} x$$

$$[\mu + b_H(1 - \beta)Yk_h \frac{Y}{k_x x + y}] \quad (4.49a)$$

where, $b = (1 - \beta)Yk_h$

$$\frac{\hat{\mu}}{\mu_N - \mu_{n-1}} \cdot \frac{K_S Y D}{X_T \Delta L^2} \left[\frac{(\mu - \mu_{n-1})}{(-\mu + \mu_N)} - \frac{(\mu_{n-1} - \mu_{n-2})}{(\mu_N + \mu_{n-2})} \right] = x(\mu + b_H - b \frac{Y}{k_x x + y})$$

where,

$$\mu_A = \frac{\mu x^2 - tx - tx_{n-1}}{(2x - x_{n-1})} \quad (A.21)$$

If, equation (4.47a) is rewritten in the same manner,

$$(\mu_Y - \mu_A)xy = (y - y_{n-1})(\mu_A x + t) \quad (4.47b)$$

where,

$$\mu_A = \frac{\mu_Y x_Y - ty + ty_{n-1}}{(2y - y_{n-1})x} \quad (A.22)$$

is obtained.

Equations (A.21) and (A.22), both yield expressions for μ_A . Equalizing both expressions yield,

$$\frac{\mu x^2 - tx - tx_{n-1}}{2x - x_{n-1}} = \frac{\mu_y xy - ty + ty_{n-1}}{2y - y_{n-1}}$$

Rearranging the above equality by writing the notation 'y' in terms of x (as; $y=wx$) takes the form;

$$(\mu x^2 - tx + tx_{n-1})(2wx - y_{n-1}) = (2x - x_{n-1}) \left[(q - k_h \frac{w}{w+k_x}) wx^2 - t(wx - y_{n-1}) \right]$$

$$(\mu x^2 - tx + tx_{n-1})(2wx - y_{n-1})(w+k_x) + (2x - x_{n-1})$$

$$\{ [(k_h - q)w^2 - k_x q w] x^2 + t(w+k_x)(wx - y_{n-1}) \} = 0$$

The constant terms can be abbreviated as follows;

$$k_h - q = c, \quad k_x q = d$$

Further rearrangement of the above expression yields,

$$\frac{\hat{\mu}}{(\mu_N - \mu_{n-1})} \cdot \frac{K_B Y D}{X_T L^2} \left[(\mu - \mu_{n-1}) + \frac{(\mu_{n-1} - \mu_{n-2})}{(\mu_N - \mu_{n-2})} (\mu - \mu_N) \right] =$$

$$x(-\mu + \mu_N)(\mu + b_H - b) \frac{y}{k_x x + y}$$

Assuming,

$$\alpha_n = \frac{\hat{\mu}}{(\mu_N - \mu_{n-1})} \cdot \frac{K_B Y D}{X_T L^2}$$

Then, the above expression becomes;

$$\alpha_n \left(1 + \frac{\mu_{n-1} - \mu_{n-2}}{\mu_N - \mu_{n-2}} \right) \left[\mu - \frac{\mu_{n-1} + \mu_N \frac{(\mu_{n-1} - \mu_{n-2})}{(\mu_N - \mu_{n-2})}}{1 + \frac{(\mu_{n-1} - \mu_{n-2})}{(\mu_N - \mu_{n-2})}} \right]$$

If the term in [] is abbreviated as $[\mu-r]$,

The final form will be;

$$a_n(\mu-r) = x (-\mu+\mu_N)(\mu+b_H-b \cdot \frac{y}{k_x x+y}) \quad (A.20)$$

Substituting $y=wx$ in equation (A.20) leads to;

$$a(\mu-r)(w+k_x) + (\mu-\mu_N) [(\mu+b_H)(w+k_x)-bw]x=0=H(w,x,\mu) = 0$$

(4.58)

The other two functions (4.46a) and (4.47a) are further rearranged.

Assuming,

$$t = \frac{(\mu_{n-1}-\mu_{An-1})}{(x_{n-1}-x_{n-2})} \cdot x_{n-1}^2$$

Therefore, equation (4.46a) can be rewritten considering the above abbreviation;

$$(\mu-\mu_A)x^2 = (x-x_{n-1}) (\mu_A x+t) \quad (4.46b)$$

Therefore;

$$(\mu x^2-tx+tx_{n-1})(2wx-y_{n-1})(w+k_x) + (2x-x_{n-1}) [(cw^2-dw)x^2 +$$

$$t(w+k_x)(wx-y_{n-1})] = 0 = F(w,x,\mu) \quad (4.59)$$

where,

$$(\mu x^2 - tx + tx_{n-1}) = \mu(x)$$

$$(2wx - y_{n-1}) = F1$$

$$(w + k_x) = wk$$

$$(2x - x_{n-1}) = F2$$

$$cw^2 - dw = cw$$

$$w + k_x = wk$$

$$(wx - y_{n-1}) = F3$$

Considering the equality of μ_A given in equation (A.21);

$$\frac{\mu x^2 - tx + tx_{n-1}}{(2x - x_{n-1})x} = \mu x + y \left[q - k_n \frac{y}{k_x x + y} \right] + P(1 - x - y)$$

as $y = wx$, the above expression becomes,

$$\frac{\mu x^2 - tx + tx_{n-1}}{(2x - x_{n-1})x} = \frac{[p + \mu x - p(1 + w)x](w + k_x) + wx[(q - k_n)w + k_x q]}{w + k_x}$$

Therefore,

$$\{[p + \mu x - p(1 + w)x](w + k_x) - x[cw^2 - dw]\} (2x^2 - x_{n-1}x)$$

$$-(\mu x^2 - tx + tx_{n-1})(w + k_x) = 0 = G(w, x, \mu) = 0 \quad (4.60)$$

where,

$$\{[p + \mu x - p(1 + w)x](w + k_x) - x(cw^2 - dw)\} = G1$$

$$(2x^2 - x_{n-1}x) = G2$$

$$(\mu x^2 - tx + tx_{n-1}) = \mu x$$

$$(w + k_x) = wk$$

In the computer program of the micro model, the derivatives of each function with respect to each variable are used.

APPENDIX B

THE TRAPEZOIDAL RULE

Trapezoidal Rule is a formula applicable to any number of subintervals, whether even or odd used to find an area under a curve. In this study, it is applied to obtain the average volume fractions of particulate components, X_A , X_B and X_E , along biofilm thickness where the number of subintervals is 200.

The formula can be derived by adding the areas of all trapezoids between $L=0$ and $L_x=500 \mu\text{m}$ and evaluating over single subintervals. Thus, for the several subintervals, one has;

$$\int_{L_0}^{L_0+L} y dL = L(y_0 + \frac{1}{2} y_0) = L(y_0 + \frac{y_1 - y_0}{2}) = L(\frac{y_0 + y_1}{2})$$

$$\int_{L_1}^{L_1+L} y dL = L(y_1 + \frac{1}{2} y_1) = L(y_1 + \frac{y_2 - y_1}{2}) = L(\frac{y_1 + y_2}{2})$$

$$\int_{L_2}^{L_2+L} y dL = L(y_2 + \frac{y_3 - y_2}{2}) = L(\frac{y_2 + y_3}{2}) \quad \text{etc.}$$

Adding these integrals, yield

$$\begin{aligned} \int_{L_0}^{L_x} y dL &= L(\frac{y_0 + y_1}{2} + \frac{y_1 + y_2}{2} + \dots + \frac{y_{n-1} + y_n}{2}) \\ &= L(\frac{1}{2}y_0 + y_1 + y_2 + \dots + y_{n-1} + \frac{1}{2}y_n) \end{aligned}$$

The trapezoidal rule is the simplest of the formulas for numerical integration. Comparison of results at the bulk-liquid interface by the Simpson's Rule has shown close similarities, correlating the application of trapezoidal rule to determine the average volume fractions of particulate components.

A large, stylized red 'X' watermark is centered on the page, spanning across the middle section where the title is located.

APPENDIX C

THE SIMULATION PROGRAM

```

5 WIDTH "LPT1:",255
10 DEFDBL A-H,D-Z
20 DEFINT I-N
30 DIM X(500),W(500),UY(500),UA(500),U(500),S(500),SL(500),FSR(500)
40 GOTO 90
50 INPUT "Mumax = ";UM: INPUT "Ks = ";SK: INPUT "Yg = ";YG
60 INPUT "bh = ";BH: INPUT "Beta = ";BET: INPUT "Kh = ";HK
70 INPUT "Kx = ";XK: INPUT "f = ";F: INPUT "Lf = ";FL
80 INPUT "Diffusion coef = ";DC: INPUT "Xt = ";XT
90 UM=4.2#:BH=.62#:SK=60#: YG=.6#: FL=.0005#: XT=80000#
100 HK=10#: XK=.03#: F=.08#: BET=.1#:DC=.000031#
110 INPUT "Number of divisions = ";N: INPUT "Xo = ";XO
115 INPUT "Do you want printing (y=1, n=0) = ";NP
116 IF NP=0 THEN 120
117 INPUT "Do you want coefficients to be printing (y=1, n=0) = ";NC
118 INPUT "Printing interval for biofilm as micron = ";LI
120 SO=(1+F)*BH*SK/(UM-(1+F)*BH): UN=UM-BH: EN=N
130 DL=FL/N: P=F*BH: Q=(1-F)*BH: UO=P: C=HK-Q: D=XK*Q
140 ALF=UM*YG*SK*DC/(XT*DL*DL): B=(1-BET)*YG*HK
150 WO=XK*(Q-P)/(HK+P-Q)
152 IF NP=0 THEN 158
155 LPRINT CHR$(15)
157 LPRINT CHR$(27);"3";CHR$(22);
158 IF NC=0 THEN 180
160 LPRINT UM;SK;YG;BH;BET;HK;XK;F;FL;XT;DC
170 LPRINT SO;UO;DL;P;Q;A;B;C;D
180 X=1.003*XO: Y=1.003*WO: U=1.003: YO=WO*XO
182 IF NP=0 THEN 185
184 LPRINT "Xo =";XO;" yo =";WO*XO;" N =";N: LPRINT
185 A=ALF/(UN-P): X(0)=XO: W(0)=WO: U(0)=UO: S(0)=SO
190 IF NP=0 THEN 200
195 LPRINT "      L          fx          fi          fs/fx          fs          Muy          Mua ";
198 LPRINT "      Mu          S          fluxS          fluxSr          ": LPRINT
200 FOR J=1 TO N
205 PRINT
210 IF J=1 THEN 238
220 X=1.003*X(J-1): XE=X(J-1)
230 Y=1.003*W(J-1): YE=W(J-1)*XE
235 U=1.003*U(J-1)
238 GOSUB 310
240 UY=Q-HK*W/(W+XK): S(J)=SK*(U+BH)/(UN-U): SL(J)=(S(J)-S(J-1))/DL
242 UA=U*X+W*X*UY+P*(1-(1+W)*X)
243 UY(J)=UY: UA(J)=UA: U(J)=U
245 IF J>1 THEN 253
250 FSR(J)=-BET*DL*HK*W*X*XT/(W+XK)
252 GOTO 255
253 FSR(J)=FSR(J-1)-BET*DL*HK*W*X*XT/(W+XK)
255 T=(U-UA)*X*X/(X-X(J-1)): R1=(U-U(J-1))/(UN-U(J-1))
260 R=(U+UN*R1)/(1+R1): A=ALF*(1+R1)/(UN-U)
265 X(J)=X: W(J)=W: U(J)=U: PRINT
266 PRINT X;XT;DC;FSR(J)
267 NEXT J
268 IF NP=0 THEN 308
269 STX=0: STY=0: STZ=0
270 FOR J=1 TO N-1 STEP 2: STX=STX+X(J): STY=STY+X(J)*W(J)
271 STZ=STZ+1-X(J)*(1+W(J)): NEXT J
272 STX=2*STX: STY=2*STY: STZ=2*STZ: SXA=0: SXS=0: SXI=0
273 FOR J=0 TO N
274 IF J=0 THEN 278
275 IF J=N THEN 278
277 SXA=SXA+X(J): SXS=SXS+X(J)*W(J): SXI=SXI+1-X(J)*(1+W(J))
278 L=J*DL*1000000#
280 IPR=L/LI: IF (LI*IPR)<>L THEN 295
282 LPRINT USING "###.##";L:; LPRINT USING "###.#####";X(J);(1-(1+W(J))*X(J));
285 LPRINT USING "###.#####";W(J);W(J)*X(J);
286 LPRINT USING "###.#####";UY(J);UA(J);U(J);
288 LPRINT USING "###.##";S(J);; LPRINT USING "###.#####";DC*SL(J);FSR(J)
295 NEXT J
296 LPRINT
297 VZ1=(1-X(0)*(1+W(0))+SXI)/EN: VZ2=(1-X(N)*(1+W(N))+SXI)/EN
298 VX1=(X(0)+SXA)/EN: VY1=(X(0)*W(0)+SXS)/EN
299 VX2=(X(N)+SXA)/EN: VY2=(X(N)*W(N)+SXS)/EN
300 GOTO 305
301 LPRINT "Vx1 =";VX1;" Vy1 =";VY1;" Vz1 =";VZ1: LPRINT
302 LPRINT "Vx2 =";VX2;" Vy2 =";VY2;" Vz2 =";VZ2: LPRINT
303 LPRINT "Vxt =";(VX1+VX2)/2;" Vyt =";(VY1+VY2)/2;" Vz1 =";(VZ1+VZ2)/2
304 LPRINT
305 LPRINT "Vxs =";(VX1+VX2+STX/EN)/3;" Vys =";(VY1+VY2+STY/EN)/3;
306 LPRINT "Vzs =";(VZ1+VZ2+STZ/EN)/3
308 END

```

```

310 FOR I=1 TO 100
315 GOSUB 480
320 PRINT USING"###.####":X;XN;Y;YN;U;U2;
330 PRINT USING"###.####":F;G;H
340 IF ABS(XN/X-1)<.0000001 THEN 370
350 X=XN
360 GOTO 380
370 X=XN: K=1
380 IF ABS(YN/Y-1)<.0000001 THEN 410
390 Y=YN
400 GOTO 420
410 Y=YN: K=2
420 IF ABS(U2/U-1)>.0000001 THEN 450
430 U=U2
440 IF K=2 THEN 470
450 U=U2
460 NEXT I
470 RETURN
480 PRINT USING"###":J;I;
490 W=Y
500 IF J>1 THEN 610
510 F=(2*W*X-YO)*(XK+W)*U+(2*X-XO)*(C*W^2-D*W)
520 G1=(P+U*X-P*(1+W)*X)*(XK+W)-X*(C*W^2-D*W)
530 G=G1*(2*X-XO)-U*X*(XK+W)
540 FX=2*W*(XK+W)*U+2*(C*W^2-D*W)
550 FU=(2*W*X-YO)*(XK+W)
560 FW=U*(2*X*(XK+W)+(2*W*X-YO))+(2*X-XO)*(2*C*W-D)
570 GX=2*G1+(2*X-XO)*(XK+W)*(U-P*(1+W))-(C*W^2-D*W))-U*(XK+W)
580 GU=(2*X-XO-1)*(XK+W)*X
590 GW=(2*X-XO)*((P+U*X-P*(1+W)*X)-(XK+W)*P*X-X*(2*C*W-D))-U*X
600 R=P: GOTO 700
610 UX=U*X*X-T*X+T*X*E: F1=2*W*X-YE: WK=W+XK: F2=2*X-XE: CW=C*W*W-D*W
620 F3=W*X-YE: G1=(P+U*X-P*(1+W)*X)*WK-X*CW: G2=F2*X
630 F=UX*F1*WK+F2*(CW*X*X+T*WK*F3): G=G1*G2-UX*WK
640 FX=WK*(F1*(2*U*X-T)+2*W*UX)+2*(CW*X*X+T*WK*F3)+F2*(2*CW*X+T*WK*W)
650 FU=F1*WK*X*X
660 FW=UX*(2*X*WK+F1)+F2*(X*X*(2*C*W-D)+T*(F3+X*WK))
670 GX=G1*(4*X-XE)+G2*(WK*(U-P*(1+W))-CW)-WK*(2*U*X-T)
680 GU=(G2-X)*WK*X
690 GW=G2*(P+U*X-P*(1+W)*X-WK*P*X-X*(2*C*W-D))-UX
700 H=A*(U-R)*(XK+W)+(U-UN)*((U+BH)*(XK+W)-B*W)*X
710 HX=(U-UN)*((U+BH)*(XK+W)-B*W)
720 HU=A*(XK+W)+X*((U+BH)*(XK+W)-B*W)+(U-UN)*(XK+W))
730 HW=A*(U-R)+(U-UN)*(U+BH-B)*X
740 PAY=(F/FX-H/HX)/(HW/HX-FW/FX)-(F/FX-G/GX)/(GW/GX-FW/FX)
750 PDA=(HU/HX-FU/FX)/(HW/HX-FW/FX)-(GU/GX-FU/FX)/(GW/GX-FW/FX)
760 DU=PAY/PDA
770 DW=(F/FX-H/HX)/(HW/HX-FW/FX)-DU*(HU/HX-FU/FX)/(HW/HX-FW/FX)
780 DX=-F/FX-DU*FU/FX-DW*FW/FX
790 XN=X+DX: YN=Y+DW: U2=U+DU
800 RETURN

```

APPENDIX D

A FEW EXAMPLES OF THE PRINT-OUTS OF THE SIMULATION
PROGRAM

4.2 60 .6 .62 .1 3 .03 .08 .0005 80000 .000031
 11.38001359619307 .0496 .0000025 .0496 .5704 0 1.62 2.4296 .017112
 Xo = .014 yo = 8.822846079380445D-05 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.01400	0.98591	6.3020D-03	8.8228D-05	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.01407	0.98585	6.2972D-03	8.8575D-05	0.04993	0.04961	0.05000	11.39	1.8171D-02	-1.4608D-03
50.0	0.01424	0.98568	6.2838D-03	8.9451D-05	0.05085	0.04962	0.05112	11.41	3.6554D-02	-2.9324D-03
75.0	0.01452	0.98539	6.2614D-03	9.0888D-05	0.05238	0.04965	0.05299	11.45	5.5342D-02	-4.4243D-03
100.0	0.01492	0.98499	6.2300D-03	9.2947D-05	0.05453	0.04969	0.05561	11.50	7.4745D-02	-5.9468D-03
125.0	0.01546	0.98444	6.1892D-03	9.5714D-05	0.05733	0.04975	0.05902	11.57	9.5003D-02	-7.5111D-03
150.0	0.01618	0.98372	6.1355D-03	9.9318D-05	0.06082	0.04982	0.06325	11.66	1.1640D-01	-9.1308D-03
175.0	0.01710	0.98279	6.0773D-03	1.0394D-04	0.06504	0.04992	0.06834	11.76	1.3930D-01	-1.0822D-02
200.0	0.01829	0.98160	6.0048D-03	1.0985D-04	0.07006	0.05006	0.07436	11.88	1.6413D-01	-1.2605D-02
225.0	0.01984	0.98005	5.9200D-03	1.1743D-04	0.07597	0.05023	0.08138	12.03	1.9151D-01	-1.4504D-02
250.0	0.02187	0.97801	5.8213D-03	1.2729D-04	0.08287	0.05048	0.08952	12.20	2.2223D-01	-1.6556D-02
275.0	0.02459	0.97527	5.7070D-03	1.4035D-04	0.09091	0.05082	0.09889	12.39	2.5746D-01	-1.8807D-02
300.0	0.02838	0.97146	5.5744D-03	1.5819D-04	0.10031	0.05131	0.10971	12.62	2.9897D-01	-2.1327D-02
325.0	0.03388	0.96594	5.4198D-03	1.8360D-04	0.11135	0.05207	0.12222	12.88	3.4955D-01	-2.4225D-02
350.0	0.04239	0.95738	5.2376D-03	2.2204D-04	0.12449	0.05331	0.13684	13.19	4.1408D-01	-2.7679D-02
375.0	0.05691	0.94281	5.0188D-03	2.8561D-04	0.14045	0.05558	0.15420	13.56	5.0193D-01	-3.2020D-02
400.0	0.08552	0.91408	4.7476D-03	4.0601D-04	0.16050	0.06041	0.17545	14.02	6.3423D-01	-3.7953D-02
425.0	0.15605	0.84326	4.3940D-03	6.8570D-04	0.18714	0.07365	0.20308	14.62	8.7047D-01	-4.7300D-02
450.0	0.36265	0.63594	3.9090D-03	1.4176D-03	0.22456	0.12028	0.24381	15.54	1.4196D+00	-6.5551D-02
475.0	0.67494	0.32284	3.2975D-03	2.2256D-03	0.27331	0.22979	0.31584	17.20	2.7384D+00	-9.9865D-02
500.0	0.84269	0.15509	2.6368D-03	2.2220D-03	0.32802	0.38208	0.44342	20.34	4.9691D+00	-1.4135D-01

Vxs = .1025908506233756 Vys = 3.960027146700547D-04 Vzs = .8970131466619543

Xo = .017 yo = 1.07134559535334D-04 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.01700	0.98289	6.3020D-03	1.0713D-04	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.01710	0.98280	6.2962D-03	1.0764D-04	0.05000	0.04961	0.05008	11.39	2.2076D-02	-1.7744D-03
50.0	0.01735	0.98254	6.2798D-03	1.0894D-04	0.05112	0.04963	0.05145	11.42	4.4467D-02	-3.5649D-03
75.0	0.01776	0.98213	6.2526D-03	1.1107D-04	0.05298	0.04967	0.05372	11.46	6.7457D-02	-5.3854D-03
100.0	0.01837	0.98152	6.2143D-03	1.1415D-04	0.05561	0.04974	0.05693	11.53	9.1368D-02	-7.2513D-03
125.0	0.01920	0.98068	6.1643D-03	1.1834D-04	0.05904	0.04982	0.06110	11.61	1.1658D-01	-9.1805D-03
150.0	0.02030	0.97957	6.1019D-03	1.2389D-04	0.06334	0.04994	0.06629	11.72	1.4354D-01	-1.1194D-02
175.0	0.02176	0.97810	6.0263D-03	1.3116D-04	0.06858	0.05010	0.07258	11.85	1.7286D-01	-1.3319D-02
200.0	0.02370	0.97616	5.9360D-03	1.4069D-04	0.07485	0.05033	0.08006	12.00	2.0533D-01	-1.5591D-02
225.0	0.02631	0.97354	5.8293D-03	1.5337D-04	0.08231	0.05064	0.08886	12.18	2.4206D-01	-1.8055D-02
250.0	0.02991	0.96992	5.7038D-03	1.7059D-04	0.09114	0.05109	0.09916	12.40	2.8467D-01	-2.0780D-02
275.0	0.03507	0.96474	5.5560D-03	1.9483D-04	0.10162	0.05177	0.11122	12.65	3.3573D-01	-2.3865D-02
300.0	0.04288	0.95689	5.3809D-03	2.3071D-04	0.11414	0.05286	0.12539	12.95	3.9951D-01	-2.7475D-02
325.0	0.05569	0.94402	5.1706D-03	2.8796D-04	0.12936	0.05478	0.14225	13.30	4.8389D-01	-3.1895D-02
350.0	0.07945	0.92016	4.9115D-03	3.9020D-04	0.14834	0.05863	0.16280	13.74	6.0555D-01	-3.7698D-02
375.0	0.13249	0.86690	4.5790D-03	6.0667D-04	0.17314	0.06814	0.18900	14.31	8.0708D-01	-4.6240D-02
400.0	0.27934	0.71950	4.1282D-03	1.1532D-03	0.20751	0.09898	0.22570	15.13	1.2261D+00	-6.1369D-02
425.0	0.58039	0.41756	3.5346D-03	2.0515D-03	0.25419	0.18749	0.28646	16.51	2.2488D+00	-9.0820D-02
450.0	0.79834	0.19937	2.8658D-03	2.2879D-03	0.30881	0.32597	0.39504	19.12	4.1758D+00	-1.3154D-01
475.0	0.89464	0.10338	2.2153D-03	1.9819D-03	0.36410	0.51434	0.56837	23.68	6.9831D+00	-1.7106D-01
500.0	0.93920	0.05923	1.6651D-03	1.5638D-03	0.41265	0.76227	0.80780	30.90	1.0730D+01	-2.0399D-01

Vxs = .1792752338402753 Vys = 5.666920381894519D-04 Vzs = .8201580741215353

4.2 60 .6 .62 .1 3 .03 .08 .0005 80000 .000031
 11.38001359619307 .0496 .0000025 .0496 .5704 0 1.62 2.4296 .017112
 Xo = .019 yo = 1.197386253630203D-04 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.01900	0.98088	6.3020D-03	1.1974D-04	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.01912	0.98076	6.2955D-03	1.2037D-04	0.05005	0.04961	0.05014	11.39	2.4682D-02	-1.9837D-03
50.0	0.01943	0.98044	6.2771D-03	1.2199D-04	0.05130	0.04964	0.05167	11.42	4.9758D-02	-3.9873D-03
75.0	0.01996	0.97992	6.2467D-03	1.2466D-04	0.05339	0.04969	0.05422	11.47	7.5585D-02	-6.0286D-03
100.0	0.02072	0.97915	6.2037D-03	1.2855D-04	0.05633	0.04977	0.05780	11.55	1.0257D-01	-8.1271D-03
125.0	0.02178	0.97809	6.1475D-03	1.3388D-04	0.06020	0.04988	0.06249	11.64	1.3121D-01	-1.0306D-02
150.0	0.02320	0.97666	6.0773D-03	1.4102D-04	0.06505	0.05004	0.06834	11.76	1.6212D-01	-1.2593D-02
175.0	0.02512	0.97473	5.9916D-03	1.5049D-04	0.07098	0.05025	0.07545	11.91	1.9609D-01	-1.5024D-02
200.0	0.02771	0.97213	5.8889D-03	1.6317D-04	0.07814	0.05056	0.08395	12.08	2.3426D-01	-1.7648D-02
225.0	0.03129	0.96853	5.7667D-03	1.8044D-04	0.08670	0.05100	0.09400	12.29	2.7825D-01	-2.0532D-02
250.0	0.03641	0.96338	5.6218D-03	2.0469D-04	0.09695	0.05166	0.10587	12.54	3.3057D-01	-2.3778D-02
275.0	0.04412	0.95564	5.4490D-03	2.4040D-04	0.10926	0.05272	0.11991	12.83	3.9538D-01	-2.7546D-02
300.0	0.05662	0.94308	5.2408D-03	2.9675D-04	0.12426	0.05455	0.13668	13.19	4.8025D-01	-3.2115D-02
325.0	0.07935	0.92025	4.9842D-03	3.9551D-04	0.14299	0.05817	0.15715	13.62	6.0084D-01	-3.8031D-02
350.0	0.12845	0.87095	4.6559D-03	5.9806D-04	0.16736	0.06683	0.18316	14.19	7.9584D-01	-4.6541D-02
375.0	0.25936	0.73955	4.2133D-03	1.0927D-03	0.20096	0.09373	0.21912	14.98	1.1852D+00	-6.1065D-02
400.0	0.54527	0.45275	3.6244D-03	1.9763D-03	0.24703	0.17406	0.27714	16.30	2.1189D+00	-8.3970D-02
425.0	0.77923	0.21847	2.9506D-03	2.2992D-03	0.30176	0.30773	0.38012	18.75	3.9376D+00	-1.2912D-01
450.0	0.88582	0.11215	2.2877D-03	2.0265D-03	0.35784	0.49026	0.54636	23.07	6.6406D+00	-1.6918D-01
475.0	0.93478	0.06361	1.7210D-03	1.6088D-03	0.40764	0.73207	0.77907	29.97	1.0275D+01	-2.0291D-01
500.0	0.95960	0.03916	1.2850D-03	1.2331D-03	0.44718	1.02592	1.06650	40.26	1.4955D+01	-2.2951D-01

Vxs = .2222683687538265 Vys = 6.353446180896319D-04 Vzs = .7770962866280838

Xo = .022 yo = 1.386447241045499D-04 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.02200	0.97786	6.3020D-03	1.3864D-04	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.02216	0.97770	6.2944D-03	1.3949D-04	0.05012	0.04961	0.05023	11.39	2.8595D-02	-2.2977D-03
50.0	0.02258	0.97728	6.2732D-03	1.4166D-04	0.05157	0.04965	0.05200	11.43	5.7717D-02	-4.6222D-03
75.0	0.02329	0.97657	6.2378D-03	1.4526D-04	0.05399	0.04973	0.05495	11.49	8.7855D-02	-6.9973D-03
100.0	0.02433	0.97552	6.1878D-03	1.5055D-04	0.05743	0.04983	0.05913	11.57	1.1957D-01	-9.4499D-03
125.0	0.02579	0.97405	6.1222D-03	1.5790D-04	0.06194	0.04999	0.06460	11.68	1.5356D-01	-1.2012D-02
150.0	0.02780	0.97203	6.0397D-03	1.6789D-04	0.06764	0.05021	0.07145	11.82	1.9073D-01	-1.4726D-02
175.0	0.03056	0.96926	5.9387D-03	1.8146D-04	0.07467	0.05053	0.07983	12.00	2.3231D-01	-1.7644D-02
200.0	0.03441	0.96539	5.8165D-03	2.0016D-04	0.08321	0.05099	0.08992	12.20	2.8009D-01	-2.0842D-02
225.0	0.03998	0.95980	5.6696D-03	2.2666D-04	0.09356	0.05170	0.10198	12.45	3.3684D-01	-2.4434D-02
250.0	0.04842	0.95131	5.4926D-03	2.6596D-04	0.10614	0.05285	0.11640	12.76	4.0717D-01	-2.8599D-02
275.0	0.06224	0.93743	5.2774D-03	3.2846D-04	0.12161	0.05486	0.13380	13.12	4.9952D-01	-3.3649D-02
300.0	0.08759	0.91197	5.0099D-03	4.3882D-04	0.14110	0.05889	0.15524	13.58	6.3147D-01	-4.0198D-02
325.0	0.14283	0.85651	4.6647D-03	6.6623D-04	0.16670	0.06870	0.18279	14.18	8.4697D-01	-4.9658D-02
350.0	0.28761	0.71119	4.1976D-03	1.2073D-03	0.20216	0.09919	0.22138	15.03	1.2812D+00	-6.5804D-02
375.0	0.57399	0.42395	3.5841D-03	2.0572D-03	0.25024	0.18465	0.28417	16.46	2.2959D+00	-9.5675D-02
400.0	0.79120	0.20650	2.8952D-03	2.2907D-03	0.30636	0.32262	0.39392	19.09	4.1943D+00	-1.3637D-01
425.0	0.89064	0.10737	2.2314D-03	1.9874D-03	0.36271	0.51151	0.56753	23.65	6.9791D+00	-1.7596D-01
450.0	0.93699	0.06144	1.6736D-03	1.5681D-03	0.41189	0.75962	0.80676	30.87	1.0709D+01	-2.0898D-01
475.0	0.96071	0.03809	1.2489D-03	1.1999D-03	0.45050	1.05787	1.09860	41.56	1.5500D+01	-2.3491D-01
500.0	0.97382	0.02526	9.4317D-04	9.1848D-04	0.47896	1.38351	1.41895	56.61	2.1434D+01	-2.5495D-01

Vxs = .2765057795523569 Vys = 7.034694744542386D-04 Vzs = .7227907509731888

4.2 60 .6 .62 .1 3 .03 .08 .0005 80000 .000031
 11.38001359619307 .0496 .0000025 .0496 .5704 0 1.62 2.4296 .017112
 Xo = .024 yo = 1.512487899322362D-04 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.02400	0.97585	6.3020D-03	1.5125D-04	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.02419	0.97566	6.2937D-03	1.5226D-04	0.05017	0.04962	0.05028	11.39	3.1206D-02	-2.5072D-03
50.0	0.02469	0.97515	6.2705D-03	1.5483D-04	0.05175	0.04967	0.05222	11.43	6.3039D-02	-5.0462D-03
75.0	0.02554	0.97430	6.2319D-03	1.5914D-04	0.05440	0.04975	0.05545	11.50	9.6086D-02	-7.6456D-03
100.0	0.02679	0.97304	6.1771D-03	1.6550D-04	0.05816	0.04988	0.06002	11.59	1.3103D-01	-1.0338D-02
125.0	0.02857	0.97126	6.1051D-03	1.7440D-04	0.06312	0.05007	0.06602	11.71	1.6874D-01	-1.3163D-02
150.0	0.03103	0.96878	6.0144D-03	1.8664D-04	0.06940	0.05035	0.07356	11.87	2.1033D-01	-1.6172D-02
175.0	0.03448	0.96532	5.9027D-03	2.0353D-04	0.07718	0.05075	0.08281	12.06	2.5743D-01	-1.9434D-02
200.0	0.03941	0.96036	5.7669D-03	2.2728D-04	0.08669	0.05136	0.09400	12.29	3.1242D-01	-2.3050D-02
225.0	0.04675	0.95299	5.6024D-03	2.6190D-04	0.09832	0.05232	0.10748	12.57	3.7917D-01	-2.7173D-02
250.0	0.05837	0.94131	5.4021D-03	3.1533D-04	0.11263	0.05395	0.12377	12.91	4.6445D-01	-3.2062D-02
275.0	0.07864	0.92096	5.1545D-03	4.0533D-04	0.13053	0.05704	0.14374	13.34	5.8161D-01	-3.8198D-02
300.0	0.11945	0.87997	4.8393D-03	5.7805D-04	0.15369	0.06392	0.16899	13.88	7.6131D-01	-4.6604D-02
325.0	0.21906	0.77997	4.4197D-03	9.6819D-04	0.18518	0.08334	0.20301	14.62	1.0903D+00	-5.9866D-02
350.0	0.45852	0.53971	3.8522D-03	1.7663D-03	0.22901	0.14408	0.25497	15.79	1.8317D+00	-8.4150D-02
375.0	0.72399	0.27371	3.1730D-03	2.2972D-03	0.28345	0.26383	0.34475	17.89	3.3791D+00	-1.2223D-01
400.0	0.86022	0.13764	2.4823D-03	2.1354D-03	0.34114	0.43189	0.49329	21.64	5.8181D+00	-1.6334D-01
425.0	0.92219	0.07608	1.8744D-03	1.7286D-03	0.39398	0.65772	0.70838	27.76	9.1703D+00	-1.9912D-01
450.0	0.95286	0.04581	1.3974D-03	1.3315D-03	0.43688	0.93907	0.98253	37.02	1.3541D+01	-2.2768D-01
475.0	0.96935	0.02964	1.0485D-03	1.0164D-03	0.46909	1.25748	1.29523	50.30	1.9031D+01	-2.4981D-01
500.0	0.97884	0.02037	8.0161D-04	7.8465D-04	0.49232	1.58629	1.61915	68.52	2.5681D+01	-2.6695D-01

Vxs = .3072450746009101 Vys = 7.355878765352708D-04 Vzs = .6920193375225547

Xo = .025 yo = 1.575508228460794D-04 N = 200

L	fx	fi	fs/fx	fs	Muy	Mua	Mu	S	fluxS	fluxSr
0.0	0.02500	0.97484	6.3020D-03	1.5755D-04	0.00000	0.00000	0.04960	11.38	0.0000D+00	0.0000D+00
25.0	0.02521	0.97463	6.2934D-03	1.5864D-04	0.05019	0.04962	0.05031	11.39	3.2512D-02	-2.6119D-03
50.0	0.02575	0.97409	6.2692D-03	1.6144D-04	0.05185	0.04967	0.05233	11.44	6.5705D-02	-5.2585D-03
75.0	0.02667	0.97316	6.2289D-03	1.6612D-04	0.05460	0.04976	0.05570	11.50	1.0022D-01	-7.9706D-03
100.0	0.02804	0.97179	6.1718D-03	1.7306D-04	0.05853	0.04991	0.06046	11.60	1.3680D-01	-1.0784D-02
125.0	0.02998	0.96983	6.0966D-03	1.8280D-04	0.06371	0.05012	0.06673	11.73	1.7640D-01	-1.3742D-02
150.0	0.03271	0.96710	6.0016D-03	1.9628D-04	0.07029	0.05042	0.07462	11.89	2.2030D-01	-1.6903D-02
175.0	0.03654	0.96324	5.8845D-03	2.1502D-04	0.07845	0.05087	0.08431	12.09	2.7029D-01	-2.0343D-02
200.0	0.04209	0.95767	5.7417D-03	2.4166D-04	0.08847	0.05157	0.09607	12.33	3.2916D-01	-2.4178D-02
225.0	0.05048	0.94924	5.5680D-03	2.8105D-04	0.10077	0.05268	0.11029	12.63	4.0143D-01	-2.8588D-02
250.0	0.06408	0.93558	5.3553D-03	3.4315D-04	0.11599	0.05462	0.12757	12.99	4.9528D-01	-3.3880D-02
275.0	0.08859	0.91095	5.0901D-03	4.5096D-04	0.13522	0.05844	0.14894	13.45	6.2743D-01	-4.0646D-02
300.0	0.14040	0.85893	4.7483D-03	6.6667D-04	0.16045	0.06747	0.17637	14.04	8.3832D-01	-5.0205D-02
325.0	0.27145	0.72738	4.2876D-03	1.1639D-03	0.19526	0.09451	0.21441	14.88	1.2477D+00	-6.5963D-02
350.0	0.54322	0.45478	3.6769D-03	1.9974D-03	0.24286	0.17240	0.27494	16.25	2.1846D+00	-9.4626D-02
375.0	0.77300	0.22470	2.9806D-03	2.3040D-03	0.29928	0.30545	0.37984	18.75	3.9821D+00	-1.3488D-01
400.0	0.88200	0.11597	2.3026D-03	2.0309D-03	0.35655	0.48904	0.54712	23.09	6.6679D+00	-1.7500D-01
425.0	0.93262	0.06577	1.7279D-03	1.6115D-03	0.40702	0.73161	0.78026	30.01	1.0293D+01	-2.0879D-01
450.0	0.95831	0.04045	1.2880D-03	1.2343D-03	0.44690	1.02581	1.06777	40.31	1.4968D+01	-2.3542D-01
475.0	0.97241	0.02664	9.7050D-04	9.4373D-04	0.47639	1.34997	1.38645	54.88	2.0783D+01	-2.5600D-01
500.0	0.98067	0.01860	7.4678D-04	7.3234D-04	0.49754	1.67687	1.70861	74.66	2.7756D+01	-2.7199D-01

Vxs = .3212867791350753 Vys = 7.490934301528902D-04 Vzs = .6779641274347718

CURRICULUM VITAE

Ayşegül Baysal Tanık was born in London, England in 1958. Upon her graduation from English High School (İngiliz Lisesi) in 1977, she entered the Engineering School of Boğaziçi University. She received her BS degree in Chemical Engineering in 1981 and MSc. degree in Environmental Engineering, Civil Engineering Department in 1984. At the same year, she started her Ph.D. Studies at the Environmental Engineering Department of Istanbul Technical University. During the years 1981-1983, she worked in the Industrial Development Bank of Turkey (TSKB) as a research engineer in the research project called "The Up to Date Position of the Chemical Sector in Turkey." She, then, entered the Boğaziçi University as a research assistant (1983-1984) in the Civil Engineering Department. From thereon, she worked in Sistem Yapı A.Ş. (1984-1985) and in Polkon A.Ş. (1985-1989). In these construction and trading companies, she worked as a project and design Engineer.