ELASTIC-PLASTIC STRESS ANALYSIS IN MULTIPLE LAYERED COMPOSITE PLATES WITH HOLES

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We certify that we have read this dissertation and that in our opinion it is fully adequate in scope and in quality, as a dissertation, for the degree of Doctor of Philosophy.

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ABSTRACT

Advanced composite materials such as fiber reinforced plastic (FRP) and fiber reinforced metal (FRM) which have excellent mechanical properties have been recently developed. Before supplying them for commercial, we should perform various kinds of tests for characterizing their material properties.

If a material has high capacity of carrying load, then it is called to have high mechanical properties. A good elasticity prevents the occurrence of sudden cracking deformations. It is very difficult to find materials which are both strong and elastic.

What makes composite materials preferred over metalic materials is their having a much wider range of properties. Because of this, metals would not replace the composites where many requirements should be met.

This study based on the three main stage. The first is that, manufacture of metal reinforced thermoplastic (Polyethylene F2-12) matrix composites and improve the product methods. The second, specimen preparation and finding the machine properties with the experiment. And the last, analyzing the result with finite element method by computer.

ÖZET

Fiber metal matriks ve fiber plastik matriks gibi kompozit malzemeler gün geçtikçe gelişerek mükemmel mekanik özelliklere sahip olmaktadır. Ticari alanlara dönük uygulamalardan önce, onlara malzeme özelliklerinin karakterize edilebilmeleri için değişik testler uygulanması gerekmektedir.

Malzemelerin yüksek mekanik özelliklere sahip olmaları malzemeye daha fazla yük taşıma kapasitesi kazandırır. İyi bir süneklilik ise ani ve yıkıcı hasarların meydana gelmesini önler. Tabiatta aynı malzemede yüksek dayanım ve iyi bir süneklilik aynı anda bulunmaz. Bu farklı özelliklerin aynı malzemeye indirgenmesi gerekmektedir.

Kompozit malzemelerin metal malzemelere göre önem kazanmaları bir çok özelliklerinin metallerinkine nazaran geniş bir ölçüde olmasıdır. Bu her zaman bir üstünlük anlamına gelmez. Bu geniş özellik farkı kompozitlerin yerini metallerin dolduramayacağını gösterir.

Bu çalışma üç ana temelden oluşur. Birincisi, metal takviyeli termoplastik (polyethylene F.2-12) matriks, kompozitlerin imali, ikincisi; numune hazırlama ve deneysel çalışmalarla mekanik özelliklerin tespiti ve son olarak da elde edilen sonuçların, bilgisayar yardımıyla sonlu elemanlar metodu kullanılarak incelenmesidir.

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NOMENCLATURE

$\{\sigma\}$, $\{\varepsilon\}$	: Stress vector, Strain vector
σ_1, σ_2, σ_3	: Normal stresses in principal material directions
τ_{12}, τ_{23}, τ_{31}	: Shear stresses
γ_{23}, γ_{31}	: Shear strain
ε_1, ε_2, ε_3	: Unit strain
E_1	: Young's modulus in the 1-direction
E_2	: Young's modulus in the 2-direction
G_{12}	: Shear modulus in the 1-2 plane
υ_{12}, υ_{21}	: The Poisson's ratio
Q_{ij}	: Reduced stiffnesses
σ_x, σ_y, σ_z	: Components of normal stress
τ_{xy}, τ_{yz}, τ_{xz}	: Components of shear stress
ε_x, ε_y, ε_z	: Components of normal strains
γ_{xy}, γ_{yz}, γ_{xz}	: Components of shear strains
D_{ij}	: Elasticity matrix
θ	: Orientation angle
u_o, v_o, w	: Displacements
Ψ_x, Ψ_y	: The rotations of normal
Π, π,	: The total potential energy
U_b	: The strain energy of bending
U_s	: The strain energy of shear
V	: Potential energy of external forces

N_x, N_y, N_{xy}	: In-plane forces
M_x, M_y, M_{xy}	: Bending moments
r, s	: Natural coordinates
D	: Diameter of the hole
x, y, z	: Local coordinates
n	: The total number of nodes
N_i	: The shape function at node i
K_x, K_y, K_{xy}	: Middle surface curvatures
B_{bi}, B_s, B_g	: Bending, shear and geometric transformation matrices
f	: The yield function
σ_e	: The effective stress
σ_o	: The yield stress
D	: Elasticity matrix
I	: Unit matrix
a	: Flow vector
$\|\sigma\|$, $\|\sigma_b\|$, $\|\sigma_s\|$	: Bending and shear matrices
R	: Nodel forces
t	: Time
Δ_t	: Time interval
U	: Overal displacement vector
K	: The stiffness matrix
D_b, D_s	: Bending and shear parts of the material matrix
k_1, k_2	: Shear correction factors
A_{ij}	: The extensional coupling stiffnesses
B_{ij}	: The coupling stiffnesses
D_{ij}	: The bending twist coupling stiffnesses
LDPE	: Low density polyethylene

PE	: Polyethylene
X	: Axial or longitudinal strength (1-direction)
Y	: Transverse strength (2-direction)
S	: Shear strength (1-2 plane)
k	: Plasticity constant
n	: Strain-hardening exponent
P_{ult}	: Ultimate force
A	: Cross-sectional area
ρ_{pe}	: Specific weight
v_f	: Volume of the fiber
V	: The total volume
V_f	: The total volume fraction of the fiber
V_m	: The total volume fraction of the matrix
L_o	: Initial length
A_o	: Initial cross-section
K	: Speed factor
σ_p	: True plastic stress
ε_p	: True plastic strain
n	: Deformation hardening power

CHAPTER ONE
INTRODUCTION

1.1. Thermoplastic Composites

Fiber-reinforced composite materials and, in particular, polymer matrix composites have gained widespread applicability in high-performance air-craft and aerospace structures due to their high specific-stiffness and specific-strength (stiffness and strength per mass); thus, offering a considerable weight saving capability. The matrix, however, is the most significant limitations to broader and more efficient use of composite materials. It limits the overall performance of the composite system due to its inferior elastic/strength properties and large thermoelastic property mismatch with reinforcing fibers, as well as its high sensitivity to environmental conditions.

Among fiber-reinforced composites, thermoplastic matrix composites are gaining popularity due to many advantages. They offer, in relation to thermoset composite systems including among others, improved interlaminar fracture toughness, increased impact resistance, and higher solvent resistance.

The high specific-stiffness and specific-strength; low density and tailorability of composite materials have greatly increased the potential for designing composite structures, which are more efficient than metallic structures.

In addition, thermoplastic composites have the unique characteristic that they may be remelted, reprocessed, and reformed thereby offering a degree of post processing freedom unavailable in thermoset composites. This characteristic makes thermoplastic easily repairable as they can be remelted locally and resolidified (repair of transverse cracking and delamination), and recyclable.

In addition to their competitive mechanical properties, thermoplastic composites do not require complex chemical reactions to be processed and they can be formed without length curing process. These features offer engineers a potential for cost-effective forming processes and the capability of integrating material processing and structure design into a single step.

Jegley presented a manufacturing process of thermoforming and the results of a study of the effects of impact damage on compression loaded trapezoidal corrugation sandwich and semisandwich graphite-thermoplastic panels. Marissen et al. investigated woven fabric in $(0°/90°)_4$ composites with a thermoplastic matrix because they combine excellent mechanical performance with ease of processing.

Representative experimental investigations on the forming of advanced thermoplastic composites can be found in references. (Jegley,Chen,Cantwell,Shi,Wang.)

Owen et al. carried out an elastic-plastic analysis of anisotropic plates and shells undertaken by means of the finite element displacement method; in that solution, a thick shell formulation accounting for shear deformation is considered, which is based on a degenerate three-dimensional continuum element-tensile residual stresses in the matrix are particularly important. In that solution, plastic yielding is based on the Huber-Mises yield surface extended by Hill for anisotropic materials. Because they may represent a significant fraction of the tensile strength of the polymer and can lead to premature failure. Prediction and measurement of residual stresses are important in relation to production design and performance of the composite elements. Jeronimidis et al. obtained residual stresses in carbon fiber-thermoplastic matrix (ICI plc.) laminates. Structural polymer composite materials used for aerospace and automotive applications may exhibit time-dependent creep or relaxation behaviour of polymer matrix and/or fiber materials. It has been found that creep deformations show similarities to plastic deformations. Chung et al. used the same potential function to describe plastic flow and time-dependent creep deformation in AS4/PEEK thermoplastic composites.

Domb et al. developed a numerical model for the prediction of the process-induced thermal residual stresses in thermoplastic composite laminates. The model represents the development of the three-dimensional residual stress state in fracture-critical free-edge regions as well as through-thickness stress variations. An elastic-viscoplastic constitutive model was used to describe the inelastic and rate-dependent behaviour of APC-2 thermoplastic composite at elevated temperatures (Wang). Composite creep properties as determined from the finite element models agree very well with the properties predicted by the micromechanical models and with conventional creep test results. The close agreement between the numerical finite element models is certainly sufficiently accurate for the characterization of creep behaviour of composites (Wen). This method appears to be an attractive way of applying finite element models in the prediction of viscoelastic properties for composites.

Elastic-plastic stress analysis was carried out in a laminated composite plate (Bahei-El-Din,Karakuzu).

Finite element method gives the excellent solution to the elastic-plastic stress analysis of laminated plate (Karakuzu, Sayman).

In the present study, an elastic-plastic stress analysis in symmetric and antisymmetric cross-ply and angle-ply laminated plastic-metal fiber composite plate is carried out by using the finite element method. The composite plate reinforced with (long galvanized wire fiber) is manufactured by using moulds under the action of 30 MPa pressure and heating up to 190°C. A laminated plate consists of four plastic matrix layers bonded symmetrically or antisymmertically by applying pressure and heat. The first order shear deformation theory and nine-node Lagrangian finite element is used. In the numerical solution the transverse load is increased gradually.

The new material formed by physically bringing at the macro level and having properties different from those of the constituents is called “composite material”.

If a material has high capacity of carrying load, then it is called to have high mechanical properties. A good elasticity prevents the occurrence of sudden cracking deformations. It is very difficult to find materials, which are both strong and elastic. Advanced composite materials such as fiber-reinforced plastic (FRP) and fiber-reinforced metal (FRM) which have excellent mechanical properties have been recently developed. Before supplying them for commercial use, we should perform various kinds of tests for characterising their material properties. Informing composite materials is to bring these two properties together.

What makes composite materials preferred over metallic materials is their much wider range of properties. Because of this, metals mould not replace the composites were many requirements should not be met.

For example, composites with their specific weight of 1 to 2 gr/cm^3 are preferred for light construction over metals with specific weights of 1.6 to 13 gr/cm^3. Additionally, fiber-strengthened composites are less vulnerable to corrosition and provide good thermal, sound, and electric isolation.

CHAPTER TWO

MACROMECHANICAL PROPERTIES OF A LAMINATE

2.1. Introduction

A laminate is two or more laminae bonded together to act as an integral structural element (Fig. 2.1.). The laminae principal material directions are oriented to produce a structural element capable of resisting load in several directions. The stiffness of such a composite material configuration is obtained from the properties of the constituent laminae. The procedures enable the analysis of laminates that have individual laminae with principle material directions oriented at arbitrary angles to the chosen or natural access of the laminate. As a consequence of the arbitrary orientations, the laminate may not have definable principle directions.

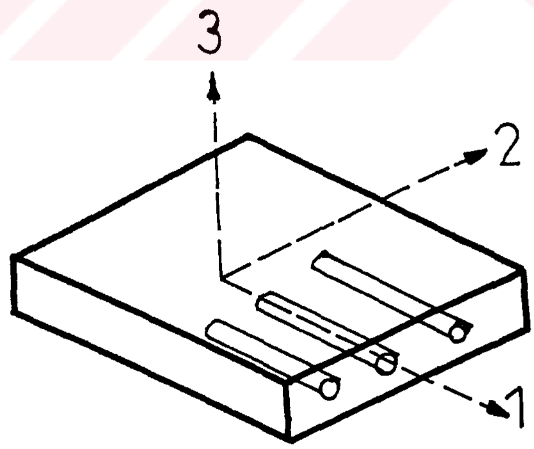


Figure 2.1. Unidirectionally reinforced laminae

2.2. Stress-Strain Relations for Plane Stress in an Orthotropic Material

For a laminae in the 1-2 plane as shown in Figure 2.1., a plane stress state is defined by setting

$$\sigma_3 = 0 \qquad \tau_{23} = 0 \qquad \tau_{31} = 0 \tag{2.1.}$$

For an orthotropic material, such a procedure results in implied strains of

$$\varepsilon_3 = S_{13}\sigma_1 - S_{23}\sigma_2 \qquad \gamma_{23} = \gamma_{31} = 0 \tag{2.2.}$$

The stress-strain relations are,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{2.3.}$$

supplemented by Eq. (2.2.) where

$$S_{11} = \frac{1}{E_1}, \qquad S_{22} = \frac{1}{E_2}, \qquad S_{66} = \frac{1}{G_{12}}, \qquad S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}. \tag{2.4.}$$

The strain-stress relations in Eq. (2.3.) can be inverted to obtain the stress-strain relations:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{2.5.}$$

where the Qij, the called reduced stiffness are (Vinson),

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \qquad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \qquad Q_{11} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}, \qquad Q_{66} = G_{12} \tag{2.6.}$$

The processing stress-strain and strain-stress relations are based for the stiffness and stress analysis and individual laminae to forces in its own plane. The relations are therefore indispensable in the analysis of laminates.

2.3. Mathematical Formulation

The laminated plate of constant thickness is composed of orthotropic layers bonded symmetrically or antisymmetrically about the middle surface of the plate. In the solution of this problem, the Cartesian coordinates are used where the middle surface of the plate coincides with the x-y plane (Figure 2.2).

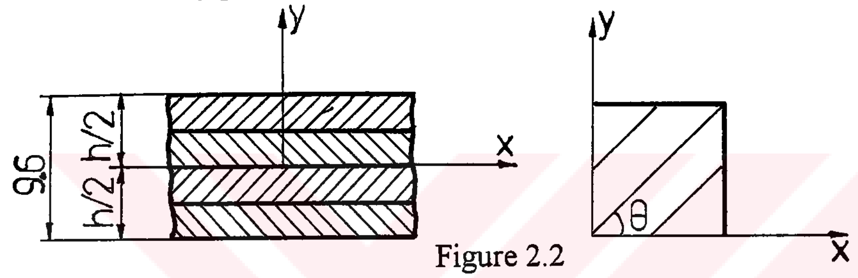


Figure 2.2

The stress-strain relations for anorthotropic layer can be written as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.7}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{54} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{2.8}$$

where Qij are reduced stiffness and can be defined in terms of elastic constants Dij and the orientation angle θ. Here, we use the theory of plates with transverse shear deformations theory which uses the assumption that particles of the plate originally on a line that is normal to the undeformed middle surface remain on the straight line during deformations, but this line is not necessarily normal to the deformed middle surface. By using this assumption, the displacement components of a point of coordinates x,y,z for small deformations are (Jones, Tsai, Gibson);

$$u(x,y,z) = u_o(x,y) + z\Psi_x(x,y)$$

$$v(x,y,z) = v_o(x,y) - z\Psi_y(x,y) \tag{2.9}$$

$$w(x,y,z) = w(x,y)$$

where u_o, v_o, and w are displacements at any point of the middle surface and Ψ_x, Ψ_y are the rotations of normals to the y and x axes, respectively.

The bending strains vary linearly through the plate thickness, whereas shear strains are assumed to be constant through the thickness as (Lin)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{vmatrix} + z \begin{vmatrix} \dfrac{\partial \psi_x}{\partial x} \\ -\dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} - \dfrac{\partial \psi_y}{\partial x} \end{vmatrix} or \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{vmatrix} = \begin{vmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{vmatrix} + z \begin{vmatrix} K_x \\ K_y \\ K_{xy} \end{vmatrix} \tag{2.10}$$

$$\begin{vmatrix} \gamma_{yz} \\ \gamma_{xz} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} + \psi_x \end{vmatrix}$$

The total potential energy of a laminated plate under static loadings is given as:

$$\Pi = U_b + U_s + V \tag{2.11}$$

where U_b is the strain energy of bending, U_s is the strain of energy shear, and V represents potential energy of external forces. They are as,

$$U_b = \frac{1}{2} \int_{-h/2}^{h/2} \left[\int_A \left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} \right) dA \right] dz$$

$$U_s = \frac{1}{2} \int_{-h/2}^{h/2} \left[\int_A \left(\tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz} \right) dA \right] dz$$

$$V = -\int_A wp dA - \int_{\partial R} \left(N_n^b u_n^o + N_s^b u_s^o \right) ds \tag{2.12}$$

where dA = dx.dy, p is the transverse loading per unit area, and N_n^b and N_s^b are the in-plane loads applied on the boundary ∂R.

In-plane forces N_x, N_y, and N_{xy} and moments M_x, M_y, and M_{xy} defined per unit length are given as:

$$\begin{Bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} (1, z) dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{2.13}$$

Equilibrium requires that π is the stationary or π is a minimum that the second variation of Π is positive at the stationary point. It may be regarded as the principle of

$$\Pi = \Pi(u_0, v_0, w, \psi_x, \psi_y) \tag{2.14}$$

virtual displacement (Bathe). By using the finite element method, the total energy is expressed as a function of displacement components as :

2.4. Finite Element Analysis

By using characteristic values found from the experiments mentioned in chapter three, we applied the finite element analysis by using computer to the laminates for variation of angles (cross-ply and angle-ply), laminate position (symmetric and antisymmetric), thickness and hole effect in the laminates and following results are found. The area of the laminates taken as 150x150 mm^2 (Figure 2.3.)

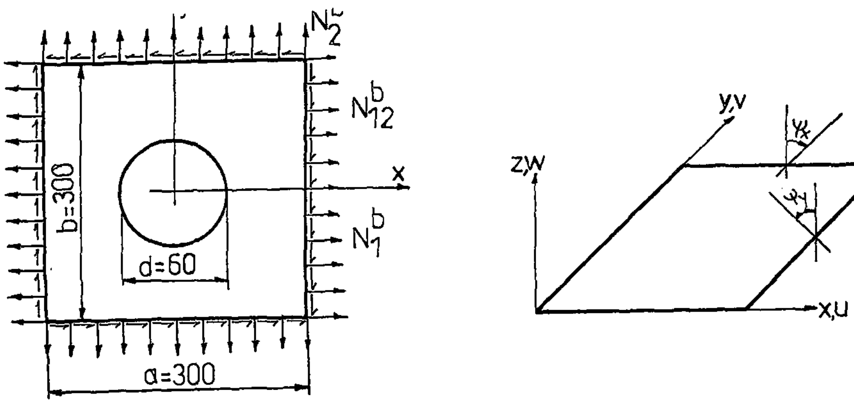


Figure 2.3. Loading laminated plate

The nine-node finite element is used in the present study. The quadratic interpolation functions for the nine-node Lagrangian finite element and mesh model are given as (Figure 2.4.a, b)

$$N_1(r,s) = \frac{1}{4}(1-r)(1-s)rs$$

$$N_2(r,s) = -\frac{1}{2}(1-r^2)(1-s)s$$

$$N_3(r,s) = -\frac{1}{4}(1+r)(1-s)rs$$

$$N_4(r,s) = \frac{1}{2}(1+r)(1-s^2)r$$

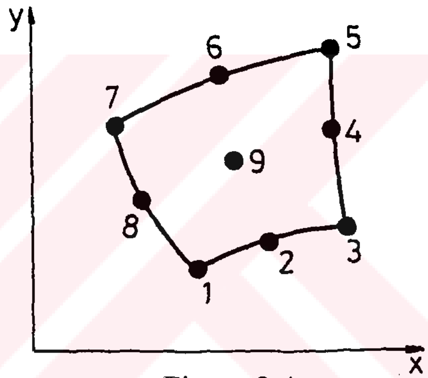


Figure 2.4.a

$$N_5(r,s) = \frac{1}{4}(1+r)(1+s)rs$$

$$N_6(r,s) = \frac{1}{2}(1-r^2)(1=s)s$$

$$N_7(r,s) = -\frac{1}{4}(1-r)(1+s)rs$$

$$N_8(r,s) = -\frac{1}{2}(1-r)(1-s^2)r$$

$$N_9(r,s) = (1-r^2)(1-s^2)$$

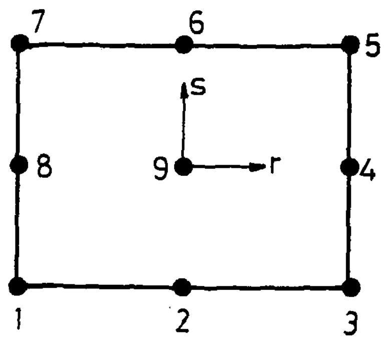


Figure 2.4.b

The displacement field can be expressed in the following matrix form as:

$$[d]=\begin{bmatrix} u_0 \\ v_0 \\ w \\ \psi_x \\ \psi_y \end{bmatrix}=\sum_{i=1}^{n}\begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}\begin{bmatrix} u_0 \\ v_0 \\ w \\ \psi_x \\ \psi_y \end{bmatrix}_i \tag{2.15}$$

in which n is the total number of nodes and Ni is the shape function at node i.

By using Equation (2.3), the relationship between strains and displacements can be written in the matrix form as:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_u \\ K_v \\ K_w \end{bmatrix}=\begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & -N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & -N_{i,x} \end{bmatrix}_i\begin{bmatrix} u_0 \\ v_0 \\ w \\ \psi_x \\ \psi_y \end{bmatrix}_i \tag{2.16}$$

or symbolically as

$$\{\varepsilon_{bi}\}=[B_{bi}][u_i] \tag{2.17}$$

in which i is the node number, and commas represent the partial derivation.

The relationship between the transverse shear strains and displacement components can be written as:

$$\begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}_i=\begin{bmatrix} 0 & 0 & N_{i,y} & 0 & -N \\ 0 & 0 & N_{i,x} & N & 0 \end{bmatrix}_i\begin{bmatrix} u_0 \\ v_0 \\ w \\ \psi_x \\ \psi_y \end{bmatrix} \tag{2.18}$$

or symbolically as $$\{\varepsilon_{si}\} = [B_{si}][u_i]$$ (2.19)

in the elastic-plastic solution the tangential modular matrix is used instead of the elasticity matrix (Cristfield). The yield function f is as:

$$f = \sigma_e - \sigma_0 = 0 \tag{2.20}$$

where σ_e is the effective stress and σ_0 is the yield stress. The tangential modular matrix is found as:

$$D_t = D(I - aa^T D / a^T Da) \tag{2.21}$$

where D, I and a are the elasticity matrix, unit matrix and flow vector, respectively. The flow vector is found by using the Prandtl-Reuss equations (Mandelson and Chakrabarty).

$$a = \partial f / \partial \sigma \tag{2.22}$$

Once the nodal displacements are calculated, the strain components of each layer can be found by using Equations (2.17.) and (2.19.); and the stress components can be calculated and used to check the yield state of the material.

Since the calculated stresses do not generally coincide with the true stresses in a nonlinear problem, the unbalanced nodal forces and the equivalent nodal forces must be calculated. The equivalent nodal point forces corresponding to the element stresses at each iteration can be calculated as (Hu):

$$\{R\}_{equivalent} = \int_{vol} [B]^T [\sigma]\, dA = \int_{vol} [B_b]^T [\sigma_b] dA + \int_{vol} [B_s]^T [\sigma_s] dA \tag{2.23}$$

When the equivalent nodal forces are known, the unbalanced nodal forces can be found by:

$$\{R\}_{unbalanced} = \{R\}_{applied} - \{R\}_{equivalent} \tag{2.24}$$

These unbalanced nodal forces are applied for obtaining increments in the solution and must satisfy the convergence tolerance in a nonlinear analysis. A widely used iteration procedure is the modified Newton Iteration Method as (Bathe). This iterative schema can be derived from the Newton-Raphson Method Equations. The method is quite effective in many analyses and contains the basic solution steps used in practically all incremental solution strategies but various other procedures are effective in only some individual problems.

The equations used in the modified Newton iteration are, for the number of i=1,2,3,..,

$$ {}^{t}K\Delta U^{(i)} = \{R\}_{unbalanced} = {}^{t+\Delta t}\{R\}_{applied} - {}^{t+\Delta t}\{R\}_{equivalent^{(i-1)}} \tag{2.25}$$

$$ {}^{t+\Delta t}U^{(i)} = {}^{t+\Delta t}U^{(i-1)} + \Delta U^{(i)} $$

with the initial condition

$$ {}^{t+\Delta t}U^{(0)} = {}^{t}U, \qquad {}^{t+\Delta t}F^{(0)} = {}^{t}F \tag{2.26}$$

where t, Δt, U and K are time, time interval and overall displacement vector, and stiffness matrix, respectively.

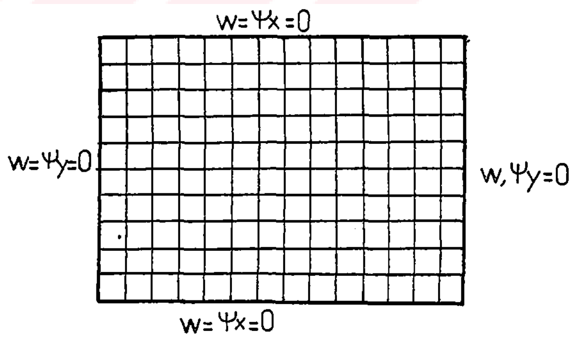


Figure 2.5.

The difference between the plastic and elastic solution gives the residual stresses. The mesh generation and boundary condition are illustrated in Figure 2.5. The residual stresses may increase the possible failure of the laminated plates. In this solution, 169 nodes and 36 elements are used.

The stiffness matrix of the plate element is obtained by using the minimum potential energy method or the principle of virtual displacement Eq.(2.19) bending and shear stiffness matrices are:
where

$$[K_b] = \int_A [B_b]^T . [D_b][B_b] dA$$

$$[K_s] = \int_A [B_b]^T . [D_s][B_s] dA$$

$$[D_b] = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \qquad [D_s] = \begin{bmatrix} k_1^2 A_{44} & 0 \\ 0 & k_2^2 A_{55} \end{bmatrix} \tag{2.27}$$

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz \qquad (i, j = 1,2,6)$$

$$(A_{44}, A_{55}) = \int_{-h/2}^{h/2} (Q_{44}, Q_{55}) dz \tag{2.28}$$

$|B_b|_{6x45}, |B_b|_{2x45}$, and D_b and D_s are the bending and shear parts of the material matrix, respectively; A_{45} is neglected in comparison with A_{44} and A_{55} and shear correction factors for rectangular cross - sections are given as (Lin, Barbero), $k_1^2 = k_2^2 = 5/6$.

Once the nodal displacements are calculated, the strain components of each layer can be found by using Eq. (2.17) and Eq. (2.19); and the stress components can be calculated and used to check the yield state of the material.

CHAPTER THREE
EXPERIMENTAL PROCEDURES

3.1. Introduction

In this study, low density polyethylene (LDPE) F.2.12 was used as a thermoplastic matrix. The preparation of composite laminates consisted of two important steps. The first step was that, melting granule, raw material of thermoplastics, as a flat plate. The second was combining these plates with fibers as composite plates. These two steps were applied successfully in the laboratory.

Thermoplastic plates were obtained by melting the granule in the rectangular mould. The material temperature was increased up to 160°C without pressure in five minutes. Then, the materials were waited five minutes under 2.5 MPa pressure. And the last, the temperature was decreased to 30°C under 15 MPa pressure in three minutes. Consequently, polyethylene plate was obtained.

3.2 Production of Laminates

Approximately 30 grams of granulized plastic material was inserted into an iron mould of 160 mm size and 1 mm thickness. To avoid sticking, acetate was put between the surfaces. The compression pressure and time date were reported for the low density. PE were as before chapter. Then, a single laminate was produced by cooling down to 30°C degrees under 15 MPa pressure within 3 minutes.

3.3 Press Apparatus

Figure 3.1 shows the schematic figure of a press apparatus, which combines fiber and thermoplastic matrix as a composite structure.

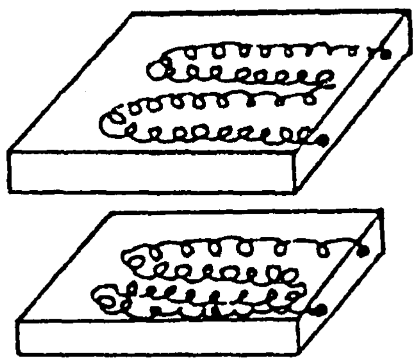


Figure 3.1.

3.3.1 Properties of Machines

This machine is a complex one formed from two machines; one for compressing and the other one for cooling.

3.3.1 a) Hydraulic press

Specifications:

Including pressure gauge, automatic temperature controller and electrical heater for platen and oil pump unit for automatic pressurizing:

Die spare (platen) dimensions	:	355x305 mm
Daylight	:	500 mm(max.)
Ram stroke	:	200 mm
Ram diameter	:	150 mm φ
Max. pressure	:	210 kgs/cm^2 G

Operating pressure	:	For PE sheet
		15-20 kgs/ cm² G at low
		40-50 kgs/cm² G at high
		For PP sheet
		50-60 kgs/cm² G
Chump force	:	37 tons at 210 kgs/ cm² G
Ram packing	:	U type
Discharge of pump	:	20 kgs/cm² G at low pressure
	:	210 kgs/cm² G at high pressure
	:	1.3 l/min at high temperature
	:	14.5 l/min at low temperature
Oil reservoir capacity	:	60 l
Pump power	:	1.5 kw, 6P
Operating temperature	:	190°C for PE sheet
	:	230°C for PP sheet
	:	(250°C) (max.)
Safety device	:	Double action switch.
Temperature control accuracy	:	±3°C at 190°C
	:	±2°C at 230°C (Within the area of heating plate)
Heater capacity	:	3.2 kw x 2
Electric source	:	380 V, 50 Hz, 3P

3.3.1. b) Cooling Press (Water Cooled)

Specifications:

Including pressure gauge, cooling platen, temperature indicator and oil pump unit for automatic pressurizing:

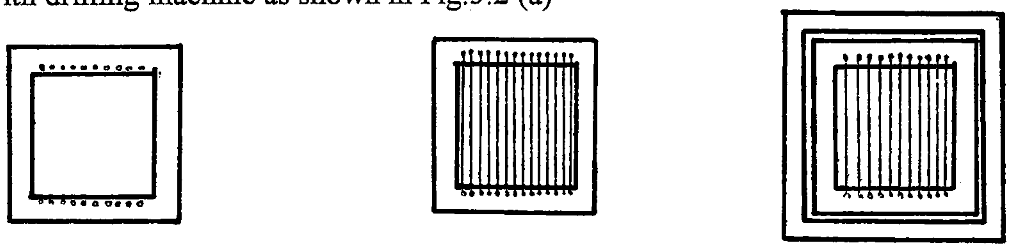
Die space	:	355x305 mm

Daylight	:	500 mm(Max.)
Ram stroke	:	200 mm
Ram diameter	:	150 mm ϕ
Max. pressure	:	210 kgs / cm^2 G
Operating pressure	:	150 kgs / cm^2 G
Clamp force	:	37 tons at 210 kgs/cm^2 G
Ram packing	:	U type
Pump pressure	:	20 kgs / cm^2 G at low pressure
	:	210 kgs / cm^2 G at high pressure
Discharge of pump	:	1.3 l / min at high pressure
	:	14 l / min at low pressure
Oil reservoir capacity	:	60 l
Operating temperature	:	130→ 40°C for PE sheet
	:	230→ 50°C for PP sheet
Pump power	:	1.5 kw, 6 P
Safety device	:	Double action switch
Water flow meter(range)	:	3.5 N 18.5 m^3/HR
Electric source	:	380V, 50Hz, 3P
Water temperature control device		
Method	:	By means of mixing water and steam which are supplied for this device.
Cooling speed	:	10-20°C / min for PE sheet within the range of 28-32°C for PP sheet

3.4 Preparation of Composite Plates

For experimental studies, two types of laminated composite materials were produced. These are unidirectional galvanized wire and one layered woven Cr-Ni fiber reinforced laminates.

At first, frame is prepared and then two cross-side of the frame square frame is pierced with drilling machine as shown in Fig.3.2 (a)



(a) (b) (c)

Figure 3.2. Preparation steps

Then galvanized wire was passed through the holes Fig. 3.2.(b). After this step, wire frame and two polyethylene plates were measured to find volume fraction. In addition, for woven Cr-Ni the same measuring operations are done. Table 3.1 shows measuring results.

Table .3.1. Measuring results

	Galvanized Wire	One Layered Woven Cr-Ni
Fiber (gr)	13.542	16.6
PE (gr)	30.68	53.4

Then the last form is put into the square mould Fig.3.2 (c). This form was placed between two plates of polyethylene and then it was placed between two papers of acetate in order for them not to stick together. The last form was placed between two steel plates and clamps of press are shown in Fig.3.3.

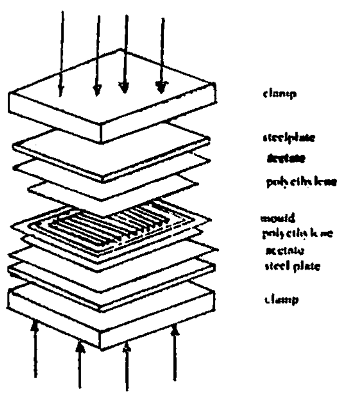
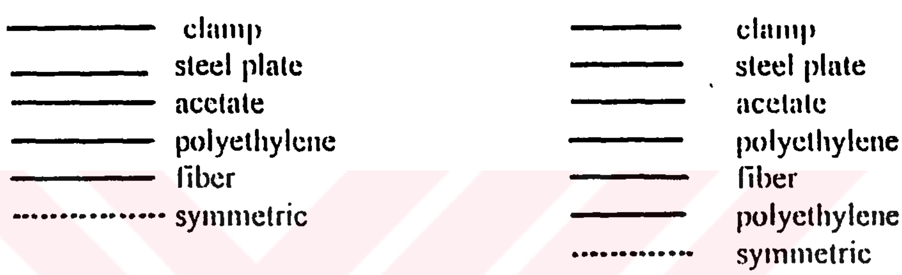


Figure 3.3. Press operation

The mould temperature is increased up to 160°C in five minutes without pressure. The hot mould is waited five minutes under 2.5 MPa pressure. The cooling process have the temperature decreased to 30°C under 15 MPa pressure in three minutes. Thus, the composite combination is obtained.

The other composite laminates can be produced in this way. But for the woven composite frame and drilling operation is not necessary. Fig. 3.4 (a). shows layered and (b) shows two layered operation.



(a) (b)

Fig. 3.4 Woven composites press operation

3.5. Type of Materials

In the present study, two different type matrix materials with different properties were produced.

1.Unidirectionally Galvanized Wire Fiber Reinforced Laminates.

2.One layered woven Cr-Ni Fiber Reinforced Laminates.

3.6. Production of These Composite Materials

3.6.1. Unidirectionally Galvanized Wire Fiber Reinforced Laminates

An iron square frame of 160 x 160 mm² area was driven by 0.5 mm diameter holes along its two facing edges every other mm. Then, 0.5 mm - diameter galvanized fiber was passed through these holes producing a single directional fibered frame.

Then, this frame was sandwiched between two polyethylene laminates and inserted in a mould suitable to its thickness. Thereafter, it was put in a 2.5 MPa pressure under 160°C degrees for 5 minutes, by a press using acetate in between the faces to avoid sticking.

Then, it was cooled down to 30°C degrees within 3 minutes under 15 MPa pressure. In this way, a matrix material was produced.

3.6.2. One Layered Woven Cr-Ni Fiber Reinforced Laminates

A woven stainless steel material of 160 x 160 mm² area was cut. Then, these materials were sandwiched between two polyethylene laminates and inserted in a mould suitable to its thickness. Thereafter, it was put in a 2.5 MPa pressure under 160°C degrees for 5 minutes, by a press using acetate in between the faces to avoid sticking.

Then, it was cooled down to 30°C degrees within 3 minutes under 15 MPa pressure. In this way, a matrix material was produced.

3.7. Specimen Preparation and Testing Procedures

Four test specimens from each mould have been prepared to find the strength in the x-direction (Figure 3.5). Similarly, to find the strength in the y-direction, samples perpendicular to the fiber direction were taken away. (Figure 3.6.)

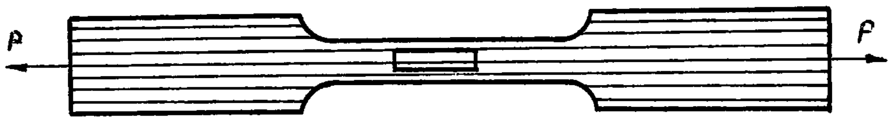


Figure 3.5.

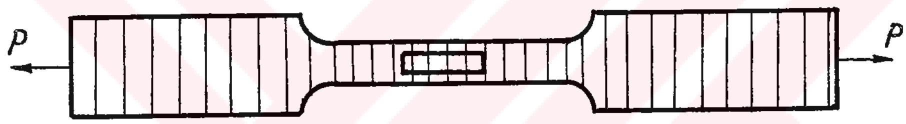


Figure 3.6

Test specimens were prepared to find shear modulus. The orientation angle of the fiber was 45° with the x axis as shown in Fig. 3.7.

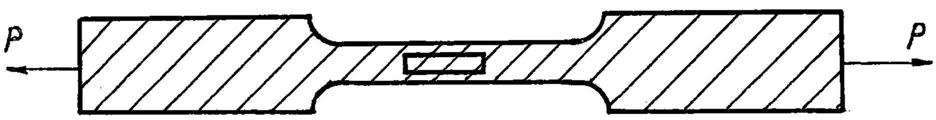


Figure 3.7.

The test specimen was prepared to find the yield point of the shear stress, as shown in Figure 3.8.

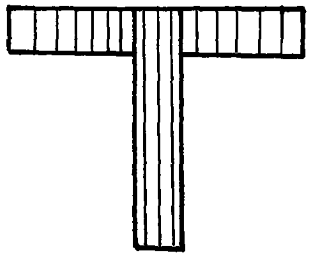


Figure 3.8

The experiments, when conducted properly, generally revealed both the strength and stiffness characteristics of the material. These stiffness characteristics were,

E_1 = Young's modulus in the 1-direction

E_2 = Young's modulus in the 2-direction

$\nu_{12} = -\dfrac{\varepsilon_2}{\varepsilon_1}$ for $\sigma_1 = -\sigma$ and all other stresses are zero.

$\nu_{21} = -\dfrac{\varepsilon_1}{\varepsilon_2}$ for $\sigma_2 = \sigma$ and all other stresses are zero.

G_{12} = Shear modulus in the 1-2 plane.

where only three of E_1, E_2, ν_{12}, ν_{21} are independent and the strength characteristics are,

X = Axial or longitudinal strength(1-direction)

Y = Transverse strength (2-direction)

S = Shear strength (1-2 plane)

In addition, the other required characteristics are,

k = Plasticity constant

n = Strain-hardening exponent (or slope of the curve)

The specimens were prepared to find required (above) characteristics. Four types of specimens were enough to find those values.

From every mould we could take out at least four specimens. The specimens must be cut off and processed carefully to achieve the experiments.

First, considered a uniaxial tension test in the 1-direction on a flat piece of unidirectionally reinforced lamina as shown in Fig.3.9.

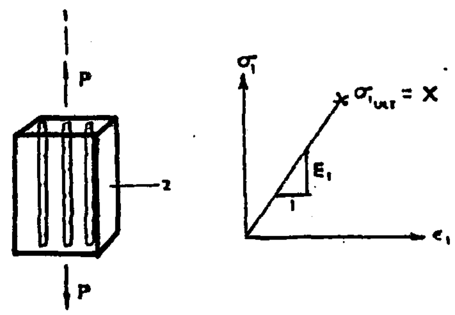


Fig. 3.9 Uniaxial loading in the 1-direction

The strains ε_1 and ε_2 are measured in the test where upon, by definition,

$$\sigma_1 = P/A$$

$$E_1 = \sigma_1 / \varepsilon_1 \tag{3.1}$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

$$X = P_{ult}/A$$

where A is the cross-sectional area of the specimen and perpendicular to the applied load.

Second, considered a uniaxial tension test in the 2-direction on a flat piece of unidirectionally reinforced lamina as in Fig. 3.10.

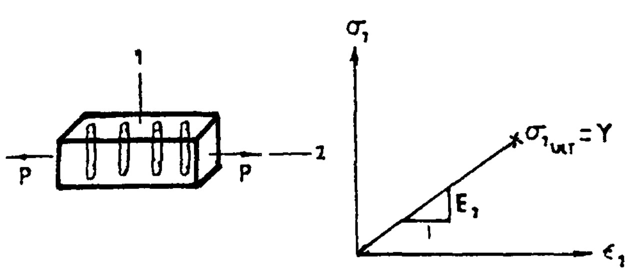


Fig. 3.10. Uniaxial loading in the 2-direction

As in the first experiment, ε_1 and ε_2 were measured so,

$$\sigma_2 = P/A$$

$$E_2 = \sigma_2 / \varepsilon_2 \tag{3.2}$$

$$\nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2}$$

$$Y = P_{ult} / A$$

where again A is the cross-sectional area of the specimen.

At this point, the stiffness properties should satisfy the reciprocal relations.

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \tag{3.3}$$

Third, considered a uniaxial tension test at 45° to the 1-direction on a flat place of lamina as shown in Fig. 3.11., by measurement of ε_x alone, obviously

$$E_x = \frac{P/A}{\varepsilon_x} \tag{3.4}$$

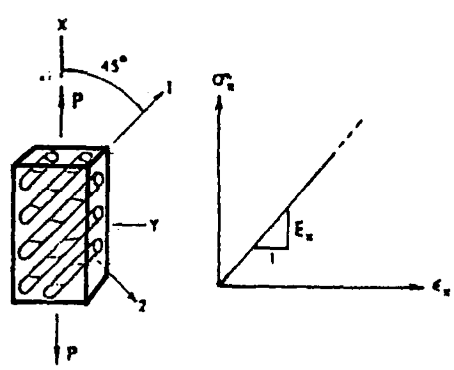


Fig. 3.11. Uniaxial loading at 45° to the 1-direction

Then by the use of the transformation relations in following equation,

$$\frac{1}{E_x} = \frac{1}{4}\left(\frac{1}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}} + \frac{1}{E_2}\right) \tag{3.5}$$

wherein G_{12} is the only unknown, thus,

$$G_{12} = \frac{1}{\left(\frac{4}{E_x} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}\right)} \tag{3.6}$$

This experiment could not be related upon to determines, the ultimate shear stress since a pure shear deformation mode has not been excited with accompanying failure in shear. Accordingly, the following approach to obtaining S must be used.

Material flow strength (σ_1), elasticity modulus (E_1) by using strain gauge, Poisson's Ratio (ν_{12}) and k and n plasticity constants from the diagram produced by Instron machine are found by using the sample.

3.7.1. Finding V_m and V_f

Before building composite material the weight of fiber and matrix materials were measured. Then, the volume of fiber could be found by the following way:

Galvanized wire is weighted

$$\rho_i = 7.8 \text{ gr/cm}^2$$

$$\rho_{pe} = 0.9 \text{ gr/cm}^2$$

$$v_f = \frac{weight}{\rho_f} \qquad V_f = \frac{v_f}{\rho_f} \qquad V_m = 1 - V_f \qquad (3.7)$$

where, v_f is the volume of the fiber,

V is the total volume,

V_f is the volume fraction of the fiber,

V_m is the volume fraction of the matrix.

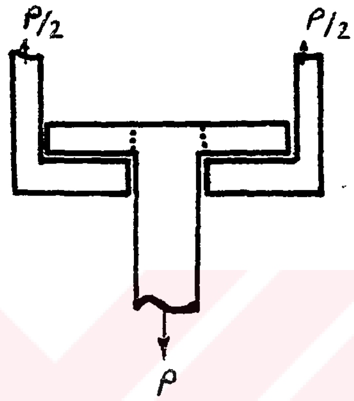


Fig. 3.12. Uniaxial loading to the 1-direction.

To achieve the experiment, cross-section of the specimen is prepared closely to square otherwise during the loading operation. The specimen may be twisted. By defining P_{ult}, S was found by the following equation.

$$S = \tau = P_{ult} \;/\, 2A \qquad (3.8)$$

The other characteristics (k, n) can be found from above designs.

3.7.2. Finding the values of k and n by tensile experiment

Tensile experiment is conducted to find how the force on the bar changes the form of the bar and how the force is distributed along accross the cross-section of the bar.

When a composite sample hanged from the top with the values of the initial length is L_o and initial cross-section, A_o, is gradually loaded by P from the bottom then the extension will be proportional to the tensile force applied by the measurement device. After a while, after passing the yield point, larger extensions will be observed even with a little force increment.

Before the sample break out, the experiment is stopped and the force (P) and extension (L) diagrams are obtained. The amount of force applied is read from vertical axis and extension amount is read from horizontal axis.

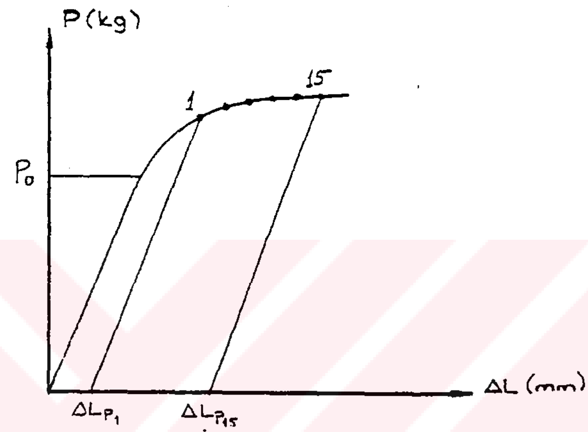


Figure 3.13. Force-extension diagram

It is known that force and extension is linear up to the yield point. After the point, linear part and permanent part can be found from the diagram. Yield stress is found by:

With the assumption that the volume of the material will be constant during deformation:

$A_o.L_o=A.L$

By taking out A from the equation; then true stress can be written:

$\sigma=P/A=P/A_oL_o/L=PL/A_oL_o$

This equation is written in plastic part.

Each point is this area can be expressed by "i" index, then the equation becomes:

$\sigma i = P_i L_i / A_o L_o$ (i=1,2,3,.....,n)

Loads (P_i) corresponding to plastic extensions (ΔL_{pi}) are found from the diagram.

K= Speed measure = Paper speed/clamp speed

According to this; the new length of the bar is,

$L_{pi} \cong L_o + \Delta L_{pi}/K$

σi stresses that correspond to each point are calculated. Ludwik equation is used to find plasticity constant (k) and deformation hardening power n:

$\sigma = \sigma_o + k.\varepsilon^n_p$

where σ: True plastic stress

εp : True plastic deformation

$\varepsilon p = \text{In } L_{pi}/L_o$

Ludwik equation:

$\sigma = \sigma_o + k.\varepsilon^n_p$

$\text{In } (\sigma - \sigma_o) = \text{In } k + n \text{ In } \varepsilon_p$

Also it can be written in this form:

$y = a + n.x$

In this equation, deformation hardening power n gives slope of the curve.

$n = \tan\theta$

$0<n<1$ and $n=0$ is ideal plastic material, $n=1$ is elastic material.

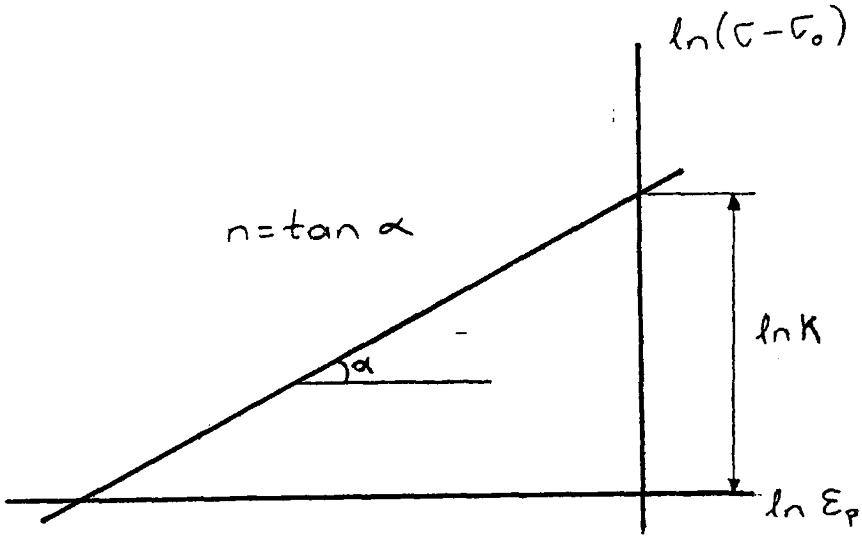


Figure 3.14. Finding k and n values

3.8. Experimental Results

The major focus of the experiments was to characterize the elastic-plastic behaviour of metal reinforced thermoplastic matrix composite laminated plates, and to evaluate the results with finite element solution. The experiment was also characterized by the hole effect in laminated plates.

3.8.1. Unidirectionally Galvanized Wire Reinforced Laminate

First of all, galvanized wire and thermoplastic plates, which are components of the composite, were weighted. Volume fractions were calculated in the following way:

$$V_f = \frac{Volume\ of\ Fibers}{Total\ Volume\ of\ Composite} \quad \text{and} \quad V_m = 1 - V_f$$

Table 3.2 Volume fractions

	Weight (gr.)	Specific weight (gr/cm^3)	Volume fraction
Fiber	13.764	7.8	0.046
PE (poliethylene)	32.680	0.9	0.953

After specimen preparation, the tensile tests were carried out. From these tests, both tensile strength and σ - ε diagrams were found. (k, n) plastic coefficients were calculated from this diagram by using elastic-plastic region as shown in Table 3.3 and Fig. 3.13.

Table 3.3 Values of tensile test

Data		Tensile increasing		Strain inc.	Stress	X-Axis	Y-Axis
Lo=	55	P1	112	4	20.98875	-4.92725	-1.23416
Po=	89	P2	113	6	21.2526	-4.52359	-0.58892
Ao=	43	P3	120	9.5	22.71121	-4.0672	-0.69989
Ao*Lo=	2365	P4	124	11.5	23.55214	-3.87794	1.048883
So=	20.69767	P5	130	15	24.84567	-3.61535	1.422624
M=	8	P6	135	17.5	25.91543	-3.46342	1.652068
Paper velo.	5	P7	139.5	21.5	26.96803	-3.2611	1.835834
Tensile velo.	0.5	P8	146	27	28.49624	-3.03814	2.053939
V=	10	P9	149.5	30.5	29.35636	-2.9193	2.158563
		P10	152.5	34	30.126	-28137	2.243719
		P11	154.5	36.5	30.65175	-2.74491	2.297983
		P12	157	40.5	31.36017	-2.64435	2.366732
		P13	160	45.5	32.23002	-2.53221	2.445156
		P14	162.5	51.7	33.07442	-2.40972	2.515819
		P15	164	57.7	33.71258	-2.30496	2.566095

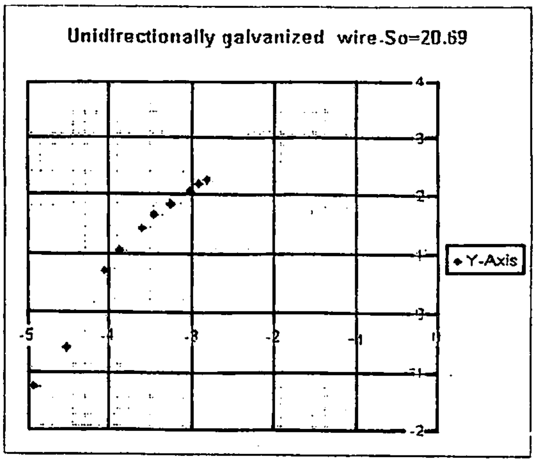


Fig. 3.15. Diagram of tensile test.

From above diagram, following values were found.

Table 3.4. Tensile test of 1-direction.

Number of tests	A_o (mm²)	X (MPa)	k (MPa)	n
20	23.700	21.010	47.183	0.713

Similarly, loading process applied to 2-direction of specimen the results were in the following table:

Table 3.5. Tensile test of 2-direction.

Number of tests	A_o (mm²)	Y (MPa)
5	23.50	5.22

Then to find shear stress in 1-2 plane, the specimen whose design shown in Figure 3.11 was investigated. The results are given in Table 3.6.

Table 3.6. Results of shear tests

Number of tests	A_o (mm²)	S (MPa)
5	10.75	5.85

To find E_1 and ν_{12} , two strain gauges bounded on the specimen one of which is parallel to loading direction and the other perpendicular to loading direction. The test results are in Table 3.7.

Table 3.7. E_1 and ν_{12} test results.

P (kg)	Ao (mm^2)	σ_1 (MPa)	ε_1	ε_2	E_1(MPa)	ν_{12}
10	33.75	2.976	692.E-6	278.5.E-6	4300	0.4

Similarly, to find E_2 , the loading operation was applied to 2-direction and E_2 was calculated then ν_{21} was calculated from (Eq. 3.3.)

The results are in Table 3.8.:

Table 3.8. E_2 and ν_{21} test results.

P (kg)	Ao (mm^2)	σ_2 (MPa)	ε_1	E_2(MPa)	ν_{21}
5	27.50	18.18	1899 .E-6	957	0.0898

Shear modulus is found by applying the loading to the x-direction.

Table 3.9. Shear modulus test results.

P (kg)	Ao (mm^2)	ε_x	E_x(MPa)	G_{12}(MPa)
4	28.405	1838.72.E-6	765.86	241.48

3.8.2. One layered Woven Cr-Ni Fiber-Reinforced Laminates

The characteristic values were found as explained in Section 3.8.1. similarly, the results found respectively.

Table 3.10 Volume fractions

	Weight (gr.)	Specific weight (gr/cm³)	Volume fraction
Fiber	16.6	7.8	0.035
PE (poliethylene)	53.4	0.9	0.965

Table 3.11. Tensile test of 1-direction.

Number of tests	A_o (mm²)	X (MPa)	k (MPa)	n
10	19.50	17.5	94.63	0.669

Table 3.12. Results of shear tests.

Number of tests	A_o (mm²)	S (MPa)
5	19	5.92

Table 3.13. Elasticity modulus and poisson's ratio test results

P (kg)	A_o (mm²)	τ (MPa)	ε_1	ε_2	$E_1 = E_2$	$\nu_{12}=\nu_{21}$
40	29.988	11.671	3699.10E-6	1150.4 x 10E-6	3155.268	0.311

Here, also because of the woven structure $E_1 = E_2$ and $\nu_{12}=\nu_{21}$

Table 3.14. Shear modulus test results.

P (kg)	A_o (mm²)	ε_x	E_x(MPa)	G_{12}(MPa)
6	28.5	1971 . 10E-6	1071.69	303.43

3.9. Plates With a Hole

In this case elastic-plastic stress analysis is carried out in plates with a hole under the in plane loading greatest are found as shown in Figure 3.16.

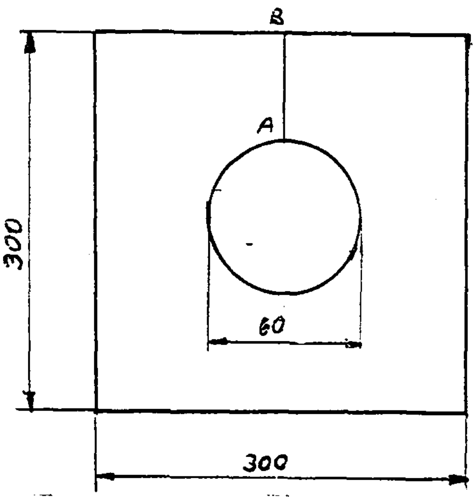


Figure 3.16.

The greatest stress components are found around the hole. Elastic-plastic, elastic and residual stress components are given in the symmetric cross-ply laminated plate at point A for 100, 200 and 300 iteration numbers, at Table 4.3.1., 4.3.2.

Table 3.15 Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plates with a hole (0°), for 100, 200 and 300 iterations.

Stresses	Iteration Number	σ_x (MPa)	σ_y (MPa)	τ_x (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	21.205	0.533	-0.205	0.000	0.000
	200	21.435	0.470	-0.213	0.000	0.000
	300	21.616	0.437	-0.087	0.000	0.000
Elastic Stresses	100	24.169	1.804	-0.214	0.000	0.000
	200	29.307	2.187	-0.260	0.000	0.000
	300	34.446	2.571	-0.360	0.000	0.000
Residual Stresses	100	-2.964	-1.271	0.010	0.000	0.000
	200	-7.873	-1.718	0.047	0.000	0.000
	300	-12.831	-2.134	0.219	0.000	0.000

Table 3.16. Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plates with a hole (0°), for 100, 200 and 300 iterations.

Stresses	Iteration Number	σ_x (MPa)	σ_y (MPa)	τ_x (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	4.199	1.493	-0.201	0.000	0.000
	200	4.209	2.262	-.0205	0.000	0.000
	300	4.210	3.265	-0.072	0.000	0.000
Elastic Stresses	100	5.311	0.486	-0.214	0.000	0.000
	200	6.440	0.590	-0.260	0.000	0.000
	300	7.570	0.693	-0.306	0.000	0.000
Residual Stresses	100	-1.112	1.007	0.014	0.000	0.000
	200	-2.231	1.673	0.055	0.000	0.000
	300	-3.360	2.572	0.234	0.000	0.000

As seen from these Tables when we increase the iteration numbers, the intensity of residual stress become higher. The intensity of the residual stresses in the layer of 0° orientation angle is greather than in the layer of 90°orientation angle, because of the higher elastic contants.

CHAPTER FOUR
RESULTS AND DISCUSSIONS

4.1. Unidirectionally Galvanized Wire Reinforced Laminates

The mechanical properties of the laminated plate are found by using strain-gauges and Instron test machine.

Table 4.1.1. Mechanical properties and yield points of a layer.

Mechanical Properties:

E_1	4300 MPa
E_2	957 MPa
G_{12}	242 MPa
ν_{12}	0.4

Yield points and parameters:

Axial Strength	X	21.01 MPa
Transverse Strength	Y	5.22 MPa
Shear Strength	S	5.85 MPa
Hardening parameter	k	47.18 Mpa
Strain hardening parameter	n	0.71

When the laminated plates composed of four layers are loaded transversely, yield points for transverse load are given in Table 4.1.2.

Table 4.1.2. The yield points of the laminated plates loaded transversely.
(S: Symmetric, A: Antisymmetric)

	$(0°/90°)_2$ S and A	$(30°/-30°)_2$ S and A	$(45°/-45°)_2$ S and A	$(60°/-60°)_2$ S and A
Yield Point (MPa)	S: 0.0412 A:0.0384	S: 0,0438 A: 0,0551	S: 0,0438 A: 0,0550	S: 0,0396 A: 0,0537

As seen from this Table the yield point in the symmetric cross-ply laminated plate is higher than in the antisymmetric cross-ply laminated plate. The yield points in the antisymmetric angle-ply laminated plates are higher than the yield points in angle-ply symmetric laminated plates.

In symmetric cross-ply and angle-ply symmetric laminated plates the maximum stress components are at the upper and lower surfaces, and they are equal absolutely. Therefore, in symmetric laminated plates, stress components are given at the upper surface only at node 85, at the mid-point. But in antisymmetic laminated plates, stress components are given at the upper and lower surfaces, because they are different at these surfaces.

Table 4.1.3.1. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply symmetric laminated plate for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	17.346	3.530	-0.005	-0.014	-0.030
Elastic Stresses	0°	18.757	5.144	-0.007	-0.007	-0.032
Residual Stresses	0°	-1.412	-1.613	-0.007	-0.007	-0.002

Table 4.1.3.2. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply symmetric laminated plate for 150 iterations.

	Layer	σ_x (Mpa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	18.437	3.137	-0.005	-0.023	-0.037
Elastic Stresses	0°	20.589	5.646	-0.004	-0.008	-0.035
Residual Stresses	0°	-2.152	-2.508	-0.002	-0.015	-0.001

Table 4.1.3.3. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply symmetric laminated plate for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	19.384	2.701	-0.007	-0.025	-0.042
Elastic Stresses	0°	22.421	6.148	-0.004	-0.009	-0.039
Residual Stresses	0°	-3.037	-3.447	-0.003	-0.017	-0.003

As seen from these Tables, when we increase the iteration numbers the residual stresses become greater.

Table 4.1.3.4. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (Mpa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	15.750	3.976	-0.001	-0.053	-0.053
	90°	-3.998	-15.615	0.016	-0.056	-0.056
Elastic Stresses	0°	17.223	6.438	0.002	-0.041	-0.042
	90°	-6.392	-17.009	0.016	-0.041	-0.042
Residual Stresses	0°	-1.473	-2.462	-0.003	-0.012	-0.011
	90°	2.394	1.394	0.000	-0.015	-0.014

Table 4.1.3.5. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	16.250	3.824	-0.001	-0.055	-0.055
	90°	-3.298	-15.915	0.012	-0.066	-0.066
Elastic Stresses	0°	17.823	6.838	0.008	-0.043	-0.043
	90°	-4.526	-5.084	0.011	-0.039	-0.039
Residual Stresses	0°	-1.127	-2.825	-0.002	-0.011	-0.011
	90°	1.824	2.185	0.000	-0.026	-0.026

As seen from these Tables, residual stress components are different at the upper and lower surfaces in antisymmetric cross-ply laminated plates.

Table 4.1.3.6. Elastic-plastic, elastic and residual stress components in $(0°/90°)_2$ cross-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	0°	16.907	3.666	0.000	-0.057	-0.057
	90°	-2.736	-16.218	0.006	-0.082	-0.082
Elastic Stresses	0°	18.836	7.041	0.002	-0.045	-0.045
	90°	-2.233	-13.594	-0.006	-0.045	-0.045
Residual Stresses	0°	-0929	-3.376	-0.002	-0.012	-0.012
	90°	0.504	2.624	-0.001	-0.037	-0.037

Table 4.1.3.7. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	11.296	6.579	3.821	-0.013	-0.033
	-30°	-11.296	-6.579	-3.821	-0.013	-0.033
Elastic Stresses	30°	11.320	6.606	3.819	-0.012	-0.033
	-30°	-11.320	-6.606	-3.819	-0.012	-0.033
Residual Stresses	30°	-0.023	-0.027	0.002	-0.001	0.000
	-30°	0.023	0.027	-0.002	-0.001	0.000

Table 4.1.3.8. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	11.863	6.651	4.210	-0.023	-0.038
	-30°	-11.863	-6.651	-4.210	-0.023	-0.038
Elastic Stresses	30°	12.372	7.220	4.174	-0.013	-0.036
	-30°	-12.372	-7.220	-4.174	-0.013	-0.036
Residual Stresses	30°	-0.508	-0.569	0.036	-0.010	-0.002
	-30°	0.508	0.569	-0.036	-0.010	-0.002

Table 4.1.3.9. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	12.432	6.711	4.612	-0.022	-0.039
	-30°	-12.432	-6.711	-4.612	-0.022	-0.039
Elastic Stresses	30°	13.424	7.834	4.529	-0.014	-0.039
	-30°	-13.424	-7.834	-4.529	-0.014	-0.039
Residual Stresses	30°	-0.991	-1.123	0.083	-0.008	0.001
	-30°	0.991	1.123	-0.083	-0.008	0.001

Table 4.1.3.10. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	10.563	5.405	3.532	-0.019	-0.039
	-30°	-10.609	-5.426	3.574	-0.017	-0.043
Elastic Stresses	30°	11.455	6.442	3.410	-0.016	-0.040
	-30°	-11.487	-6.452	3.449	-0.016	-0.040
Residual Stresses	30°	-0.892	-1.037	0.122	-0.003	0.001
	-30°	0.878	1.026	0.126	-0.001	-0.004

Table 4.1.3.11. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	12.424	6.617	4.493	-0.020	-0.045
	-30°	-12.473	-6.622	4.541	-0.019	-0.052
Elastic Stresses	30°	13.704	8.119	4.279	-0.016	-0.046
	-30°	-13.743	-8.158	4.320	-0.016	-0.046
Residual Stresses	30°	-1.279	-1.542	0.215	-0.004	0.001
	-30°	1.269	1.536	0.221	-0.003	-0.006

As seen from these Tables, residual stress components in symmetric laminated plates are less than the stresses in antisymmetric laminated plates.

Table 4.1.3.12. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σx (MPa)	σy (MPa)	τxy (MPa)	τyz (MPa)	τxz (MPa)
Elastic-Plastic Stresses	30°	12.981	6.660	4.888	-0.029	-0.052
	-30°	-13.037	-6.665	4.945	-0.024	-0.062
Elastic Stresses	30°	14.682	8.742	4.584	-0.017	-0.049
	-30°	-14.724	-8.741	4.629	-0.017	-0.049
Residual Stresses	30°	-1.702	-2.082	0.304	-0.011	-0.003
	-30°	1.688	2.076	0.316	-0.007	-0.013

Table 4.1.3.13. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σx (MPa)	σy (MPa)	τxy (MPa)	τyz (MPa)	τxz (MPa)
Elastic-Plastic Stresses	45°	6.965	6.965	3.343	-0.020	-0.020
	-45°	-6.965	-6.965	-3.343	-0.020	-0.020
Elastic Stresses	45°	6.960	6.960	3.343	-0.020	-0.020
	-45°	-6.960	-6.960	-3.343	-0.020	-0.020
Residual Stresses	45°	0.005	0.005	-0.001	0.000	0.000
	-45°	-0.005	-0.005	0.001	0.000	0.000

Table 4.1.3.14. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σx (MPa)	σy (MPa)	τxy (MPa)	τyz (MPa)	τxz (MPa)
Elastic-Plastic Stresses	45°	7.485	7.485	3.725	-0.026	-0.026
	-45°	-7.485	-7.485	-3.725	-0.026	-0.026
Elastic Stresses	45°	7.661	7.661	3.680	-0.022	-0.022
	-45°	-7.661	-7.661	-3.680	-0.022	-0.022
Residual Stresses	45°	-0.176	-0.176	0.045	-0.004	-0.004
	-45°	0.176	0.176	-0.045	-0.004	-0.004

Table 4.1.3.15. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	7.825	7.824	4.148	-0.028	-0.028
	-45°	-7.825	-7.824	-4.148	-0.028	-0.028
Elastic Stresses	45°	8.363	8.363	4.017	-0.024	-0.024
	-45°	-8.363	-8.363	-4.017	-0.024	-0.024
Residual Stresses	45°	-0.538	-0.539	0.131	-0.004	-0.004
	-45°	0.538	0.539	-0.131	-0.004	-0.004

Table 4.1.3.16. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	7.635	7.633	3.908	-0.028	-0.028
	-45°	-7.665	-7.662	3.952	-0.030	-0.030
Elastic Stresses	45°	8.285	8.285	3.693	-0.026	-0.026
	-45°	-8.309	-8.308	3.734	-0.026	-0.026
Residual Stresses	45°	-0.650	-0.652	0.215	-0.002	-0.002
	-45°	0.645	0.647	0.218	-0.004	-0.004

Table 4.1.3.17. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	7.960	7.958	4.318	-0.029	-0.029
	-45°	-7.992	-7.988	4.366	-0.031	-0.031
Elastic Stresses	45°	8.935	8.936	3.983	-0.028	-0.028
	-45°	-8.962	-8.961	4.027	-0.028	-0.028
Residual Stresses	45°	-0.975	-0.978	0.335	-0.001	-0.001
	-45°	0.970	0.972	0.338	-0.003	-0.003

Table 4.1.3.18. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	8.298	8.295	4.775	-0.034	-0.034
	-45°	-8.336	-8.332	4.813	-0.036	-0.036
Elastic Stresses	45°	9.586	9.586	4.273	-0.030	-0.030
	-45°	-9.614	-9.613	4.320	-0.030	-0.030
Residual Stresses	45°	-1.287	-1.291	0.483	-0.004	-0.004
	-45°	1.278	1.281	0.492	-0.006	-0.006

Table 4.1.3.19. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	6.579	11.296	3.821	-0.033	-0.013
	-60°	-6.579	-11.296	-3.821	-0.033	-0.033
Elastic Stresses	60°	6.606	11.320	3.819	-0.033	-0.012
	-60°	-6.606	-11.320	-3.819	-0.033	-0.012
Residual Stresses	60°	-0.027	-0.023	0.002	0.000	-0.001
	-60°	0.027	0.023	-0.002	0.000	-0.001

Table 4.1.3.20. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	41.110	41.047	-0.063	-0.176	-0.152
	-60°	-41.110	-41.047	0.063	-0.176	-0.192
Elastic Stresses	60°	43.897	43.829	-0.065	-0.150	-0.110
	-60°	-43.897	-43.829	0.065	-0.150	-0.110
Residual Stresses	60°	-2.787	-2.782	0.002	-0.027	-0.082
	-60°	2.787	2.782	-0.002	0.027	-0.082

Table 4.1.3.21. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	6.712	12.431	4.612	-0.039	-0.022
	-60°	-6.712	-12.431	-4.612	-0.039	-0.022
Elastic Stresses	60°	7.834	13.424	4.529	-0.039	-0.014
	-60°	-7.834	-13.424	-4.529	-0.039	-0.014
Residual Stresses	60°	-1.121	-0.993	0.083	0.000	-0.008
	-60°	1.121	0.993	-0.083	0.000	-0.008

Table 4.1.3.22. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	6.554	11.864	4.099	-0.042	-0.020
	-60°	-6.554	-11.864	4.142	-0.047	-0.018
Elastic Stresses	60°	7.567	12.703	3.951	-0.041	-0.015
	-60°	-7.567	-12.703	3.989	-0.041	-0.015
Residual Stresses	60°	-1.014	-0.840	0.148	-0.001	-0.006
	-60°	1.014	0.840	0.153	-0.006	-0.003

Table 4.1.3.23. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	6.610	12.402	4.472	-0.044	-0.020
	-60°	-6.625	-12.452	4.520	-0.050	-0.017
Elastic Stresses	60°	8.148	13.679	4.255	-0.045	-0.016
	-60°	-8.159	-13.719	4.296	-0.045	-0.016
Residual Stresses	60°	-1.538	-1.277	0.217	-0.001	-0.004
	-60°	1.534	1.268	0.224	-0.005	-0.002

Table 4.1.3.24. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	6.654	12.961	4.868	-0.051	-0.028
	-60°	-6.670	-13.017	4.924	-0.060	-0.023
Elastic Stresses	60°	8.729	14.655	4.558	-0.048	-0.017
	-60°	-8.741	-14.698	4.602	-0.048	-0.017
Residual Stresses	60°	-2.075	-1.694	0.310	-0.003	-0.011
	-60°	2.071	1.681	0.322	-0.012	-0.006

When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller. Residual stress components in cross-ply laminated plates are greater than in angle-ply laminated plates because in cross-ply laminated plates mechanical properties are more different than angle-ply laminated plates.

Table 4.1.4.1. The residual stresses in the angle-ply $(30°/-30°)_2$, $(45°/-45°)_2$ and $(60°/-60°)_2$ symmetric laminated plates at node 85 for 100, 150 and 200 iterations.

Iteration Numbers	Orientation Angles	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
100	30°	-0.023	-0.027	0.002	-0.001	-0.001
	45°	0.005	0.005	-0.001	0.000	0.000
	60°	-0.027	-0.023	0.002	0.000	-0.001
150	30°	-0.508	-0.569	0.036	-0.010	-0.002
	45°	-0.176	-0.176	0.045	-0.004	-0.004
	60°	-0.569	-0.569	0.036	-0.002	-0.010
200	30°	-0.991	-1.123	0.083	-0.008	0.001
	45°	-0.538	-0.539	0.131	-0.004	-0.004
	60°	-1.121	-0.993	0.083	0.000	-0.008

Table 4.1.4.2. The residual stresses in the angle-ply $(30°/-30°)_2$, $(45°/-45°)_2$ and $(60°/-60°)_2$ antisymmetric laminated plates at node 85 for 100, 150 and 200 iterations.

Iteration Numbers	Orientation Angles	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
100	30°	-0.892	-1.037	0.122	-0.003	0.001
	45°	-0.650	-0.652	0.215	-0.002	-0.002
	60°	-1.014	-0.840	0.148	-0.001	-0.006
150	30°	-1.279	-1.542	0.215	-0.004	0.001
	45°	-0.975	-0.978	0.335	-0.001	-0.001
	60°	-1.538	-1.277	0.217	0.001	-0.004
200	30°	-1.702	-2.082	0.304	-0.011	-0.003
	45°	-1.287	-1.291	0.483	-0.004	-0.004
	60°	-2.075	-1.694	0.310	-0.003	-0.011

As seen from these Tables, residual stresses in $(30°/-30°)_2$, and $(60°/-60°)_2$ angle-ply laminated plates are higher than in $(45°/-45°)_2$ angle-ply laminated plate for the same iterations.

Table 4.1.5.1. Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plate $(0°/90°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.346	3.530	-0.005	-0.014	-0.030
	150	18.437	3.137	-0.024	-0.024	-0.030
	200	19.384	2.701	-0.007	-0.025	-0.042
Elastic Stresses	100	18.757	5.144	-0.004	-0.007	-0.032
	150	20.589	5.646	-0.004	-0.008	-0.035
	200	24.421	-6.148	-0.004	-0.009	-0.039
Residual Stresses	100	-1.412	-1.613	-0.001	-0.007	-0.002
	150	-2.152	-2.508	-0.002	-0.015	-0.001
	200	-3.037	-3.447	-0.003	-0.017	-0.003

Table 4.1.5.2. Elastic-plastic, elastic and residual stress components for antisymmetric cross-ply laminated plate $(0°/90°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	15.750	3.976	-0.001	-0.053	-0.053
	150	15.750	3.976	-0.001	-0.053	-0.053
	200	16.907	3.666	0.000	-0.057	-0.057
Elastic Stresses	100	17.223	6.438	0.002	-0.041	-0.042
	150	17.223	6.438	0.002	-0.041	-0.042
	200	18.836	7.041	0.002	-0.045	-0.045
Residual Stresses	100	-1.473	-2.462	-0.003	-0.012	-0.011
	150	-1.473	-2.462	-0.003	-0.012	-0.011
	200	-1.929	-3.376	-0.002	-0.012	-0.012

As seen from these Tables, residual stresses in symmetric and antisymmetric laminated plates are higher than in angle-ply laminated plates for the same iterations.

Table 4.1.5.3. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(30°/-30°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	11.296	6.579	3.821	-0.013	-0.033
	150	11.863	6.651	4.210	-0.023	-0.038
	200	12.432	6.711	4.612	-0.022	-0.039
Elastic Stresses	100	11.320	6.606	3.819	-0.012	-0.033
	150	12.372	7.220	4.174	-0.013	-0.036
	200	13.424	7.834	4.529	-0.014	-0.039
Residual Stresses	100	-0.023	-0.027	0.002	-0.001	0.000
	150	-0.508	-0.569	0.036	-0.010	-0.002
	200	-0.991	-1.123	0.083	-0.008	0.001

Table 4.1.5.4. Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate $(30°/-30°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	10.563	5.045	3.532	-0.019	-0.039
	150	12.424	6.617	4.493	-0.020	-0.045
	200	12.981	6.660	4.888	-0.029	-0.052
Elastic Stresses	100	11.455	6.642	3.410	-0.016	-0.040
	150	13.704	8.159	4.279	-0.016	-0.046
	200	14.682	8.742	4.584	-0.017	-0.049
Residual Stresses	100	-0.892	-1.037	0.112	-0.003	-0.001
	150	-1.279	-1.542	0.215	-0.004	0.001
	200	-1.702	-2.082	0.304	-0.011	-0.003

Table 4.1.5.5. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(45°/-45°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	6.965	6.965	3.343	-0.020	-0.020
	150	7.485	7.485	3.725	-0.026	-0.026
	200	7.825	7.824	4.148	-0.028	-0.028
Elastic Stresses	100	6.960	6.960	3.343	-0.020	-0.020
	150	7.661	7.661	3.680	-0.022	-0.022
	200	8.363	8.363	4.017	-0.024	-0.024
Residual Stresses	100	0.005	0.005	-0.001	0.000	0.000
	150	-0.176	-0.176	0.045	-0.004	-0.004
	200	-0.538	-0.539	0.131	-0.004	-0.004

Table 4.1.5.6. Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate $(45°/-45°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	7.635	7.633	3.908	-0.028	-0.028
	150	7.960	7.958	4.318	-0.029	-0.029
	200	8.298	8.295	4.755	-0.034	-0.034
Elastic Stresses	100	8.285	8.285	3.693	-0.026	-0.026
	150	8.935	8.936	3.983	-0.028	-0.028
	200	9.536	9.586	4.273	-0.030	-0.030
Residual Stresses	100	-0.650	-0.652	0.215	0.002	0.002
	150	-0.975	-0.978	0.335	-0.001	-0.001
	200	-1.287	-1.291	0.483	-0.006	-0.004

Table 4.1.5.7. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(60°/-60°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	6.579	11.296	3.821	-0.033	-0.013
	150	6.651	11.863	4.211	-0.038	-0.023
	200	6.712	12.431	4.612	-0.039	-0.022
Elastic Stresses	100	6.606	11.320	3.819	-0.033	-0.012
	150	7.220	12.372	4.174	-0.036	-0.013
	200	7.834	13.424	4.529	-0.039	-0.014
Residual Stresses	100	0.027	-0.023	0.002	0.000	-0.001
	150	-0.569	-0.569	0.036	-0.002	-0.010
	200	-1.121	-0.993	0.083	0.000	-0.008

Table 4.1.5.8. Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate $(60°/-60°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	6.554	11.864	4.099	-0.042	-0.030
	150	6.610	12.402	4.472	-0.044	-0.020
	200	6.654	12.961	4.868	-0.051	-0.028
Elastic Stresses	100	7.567	12.703	3.951	-0.041	-0.015
	150	8.148	13.679	4.255	-0.045	-0.016
	200	8.729	14.655	4.558	-0.048	-0.017
Residual Stresses	100	-1.014	-0.840	-0.148	-0.001	-0.006
	150	-1.538	-1.277	0.217	0.001	-0.004
	200	-2.075	-1.694	0.310	-0.003	-0.011

As seen from these Tables, residual stresses in antisymmetric angle-ply $(30°/-30°)_2$, $(45°/-45°)_2$, and $(60°/-60°)_2$ laminated plates are higher than in symmetric, $(30°/-30°)_2$, $(45°/-45°)_2$, and $(60°/-60°)_2$ angle-ply laminated plates for the same iterations. Because in antisymmetric laminated case whose mechanical properties and layer orientations are more different from in symmetric case. The differency in mechanical properties, layer orientations and in the structure of the plate produces the higher residual stresses.

Table 4.1.6.1. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.346	3.530	-0.005	-0.014	-0.030
	$(30°/-30°)_2$	11.296	6.579	3.821	-0.013	-0.033
	$(45°/-45°)_2$	6.965	6.965	3.343	-0.020	-0.020
	$(60°/-60°)_2$	6.579	11.296	3.821	-0.033	-0.033
At the Lower Surface	$(0°/90°)_2$	-17.346	-3.530	0.005	-0.015	-0.045
	$(30°/-30°)_2$	-11.296	-6.579	-3.821	-0.013	-0.033
	$(45°/-45°)_2$	-6.965	-6.965	-3.343	-0.020	-0.020
	$(60°/-60°)_2$	-6.579	-11.296	-3.821	-0.033	-0.033

Table 4.1.6.2. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	15.750	3.976	-0.001	-0.053	-0.053
	$(30°/-30°)_2$	10.563	5.405	3.532	-0.019	-0.039
	$(45°/-45°)_2$	7.635	7.633	3.908	-0.028	-0.028
	$(60°/-60°)_2$	6.554	11.864	4.099	-0.042	-0.030
At the Lower Surface	$(0°/90°)_2$	-3.998	-15.615	0.016	-0.056	-0.056
	$(30°/-30°)_2$	-10.609	-5.426	3.574	-0.017	-0.043
	$(45°/-45°)_2$	-7.665	-7.662	3.952	-0.080	-0.030
	$(60°/-60°)_2$	-6.568	-11.911	4.142	-0.047	-0.018

Table 4.1.6.3. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	18.437	3.137	-0.024	-0.024	-0.030
	$(30°/-30°)_2$	11.863	6.651	4.210	-0.023	-0.038
	$(45°/-45°)_2$	7.485	7.485	3.725	-0.026	-0.026
	$(60°/-60°)_2$	6.651	11.863	4.211	-0.038	-0.023
At the Lower Surface	$(0°/90°)_2$	-18.437	-3.137	0.024	-0.024	-0.030
	$(30°/-30°)_2$	-11.863	-6.651	-4.210	-0.023	-0.038
	$(45°/-45°)_2$	-7.485	-7.485	-3.725	-0.026	-0.026
	$(60°/-60°)_2$	-6.651	-11.863	-4.211	-0.038	-0.023

Table 4.1.6.4. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	15.750	3.976	-0.001	-0.053	-0.053
	$(30°/-30°)_2$	12.424	6.617	4.493	-0.020	-0.045
	$(45°/-45°)_2$	7.960	7.958	4.318	-0.029	-0.029
	$(60°/-60°)_2$	6.610	12.402	4.472	-0.044	-0.020
At the Lower Surface	$(0°/90°)_2$	-3.998	-15.615	0.016	-0.056	-0.056
	$(30°/-30°)_2$	-12.473	-6.622	4.541	-0.019	-0.052
	$(45°/-45°)_2$	-7.992	-7.988	4.366	-0.031	-0.031
	$(60°/-60°)_2$	-6.625	-12.452	-4.520	-0.050	-0.017

Table 4.1.6.5. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	19.384	2.701	-0.007	-0.025	-0.042
	$(30°/-30°)_2$	12.432	6.711	4.612	-0.022	-0.039
	$(45°/-45°)_2$	7.825	7.824	4.148	-0.028	-0.028
	$(60°/-60°)_2$	6.712	12.431	4.612	-0.039	-0.022
At the Lower Surface	$(0°/90°)_2$	3.495	11.949	0.003	-0.029	-0.078
	$(30°/-30°)_2$	-12.432	-6.711	-4.612	-0.022	-0.039
	$(45°/-45°)_2$	-7.825	-7.824	-4.148	-0.028	-0.028
	$(60°/-60°)_2$	-6.712	-12.431	-4.612	-0.039	-0.022

Table 4.1.6.6. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	16.967	3.666	0.000	-0.057	-0.053
	$(30°/-30°)_2$	12.981	6.660	4.888	-0.029	-0.052
	$(45°/-45°)_2$	8.298	8.295	4.755	-0.034	-0.034
	$(60°/-60°)_2$	6.654	12.961	4.868	-0.051	-0.028
At the Lower Surface	$(0°/90°)_2$	2.736	16.218	0.006	-0.082	-0.082
	$(30°/-30°)_2$	-13.037	-6.665	4.965	-0.024	-0.062
	$(45°/-45°)_2$	-8.336	-8.323	4.813	-0.036	-0.036
	$(60°/-60°)_2$	-6.670	-13.017	4.924	-0.060	-0.023

Table 4.1.6.7. Residual stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (Mpa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-1.412	-1.613	-0.001	-0.007	0.002
	$(30°/-30°)_2$	-0.023	-0.027	0.002	-0.001	0.000
	$(45°/-45°)_2$	-0.005	-0.005	-0.001	0.000	0.000
	$(60°/-60°)_2$	-0.027	-0.023	0.002	0.000	-0.001
At the Lower Surface	$(0°/90°)_2$	1.412	1.613	0.001	-0.008	-0.002
	$(30°/-30°)_2$	0.023	0.027	-0.002	-0.001	-0.001
	$(45°/-45°)_2$	-0.005	-0.005	0.001	0.000	0.000
	$(60°/-60°)_2$	0.027	0.023	-0.002	0.000	-0.001

The absolute value of the stress components in the symmetric laminated plates at the upper and lower surfaces are the same for σ_x, σ_y, and τ_{xy} stress components. It is different at the lower and upper surfaces in the antisymmetric laminated plates.

Table 4.1.6.8. Residual stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-1.473	-2.462	-0.003	-0.012	-0.011
	$(30°/-30°)_2$	-0.892	-1.037	0.122	-0.003	0.001
	$(45°/-45°)_2$	-0.650	-0.652	0.215	-0.002	-0.002
	$(60°/-60°)_2$	-1.014	-0.840	-0.143	-0.001	-0.006
At the Lower Surface	$(0°/90°)_2$	-2.394	-1.394	0.000	-0.015	-0.014
	$(30°/-30°)_2$	0.878	1.206	0.126	-0.001	-0.004
	$(45°/-45°)_2$	0.645	0.647	0.218	-0.004	-0.004
	$(60°/-60°)_2$	1.009	0.830	0.152	-0.006	-0.003

Table 4.1.6.9. Residual stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-3.037	-3.447	-0.003	-0.017	-0.003
	$(30°/-30°)_2$	-0.991	-1.123	0.085	-0.008	-0.001
	$(45°/-45°)_2$	-0.538	-0.539	0.131	-0.004	-0.004
	$(60°/-60°)_2$	-1.121	-0.993	0.083	0.000	-0.008
At the Lower Surface	$(0°/90°)_2$	3.037	3.447	-0.003	-0.017	-0.003
	$(30°/-30°)_2$	0.991	1.123	-0.085	-0.008	-0.001
	$(45°/-45°)_2$	0.538	0.539	-0.131	-0.004	-0.004
	$(60°/-60°)_2$	1.121	0.993	-0.083	0.000	-0.008

Table 4.1.6.10. Residual stress components at upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-1.929	-3.376	-0.002	-0.012	-0.012
	$(30°/-30°)_2$	-1.702	-2.082	-0.304	-0.011	-0.003
	$(45°/-45°)_2$	-1.287	-1.291	0.483	-0.004	-0.004
	$(60°/-60°)_2$	-2.075	-1.694	0.310	-0.003	-0.011
At the Lower Surface	$(0°/90°)_2$	3.297	1.828	-0.001	-0.016	-0.015
	$(30°/-30°)_2$	1.688	2.076	0.316	-0.007	-0.013
	$(45°/-45°)_2$	1.278	1.281	0.492	-0.006	-0.006
	$(60°/-60°)_2$	2.071	1.681	0.322	-0.012	-0.006

As seen from these Tables, the intensity of the residual stress component in cross-ply, $(0°/90°)_2$, laminated plate is greater than in the angle-ply laminated plates.

FIGURES

4.1. Unidirectionally Galvanized Wire Reinforced Laminate

Figure 4.1.1. The expansion of plastic zone in symmetric, $(0°/90°)_2$, laminated plates.

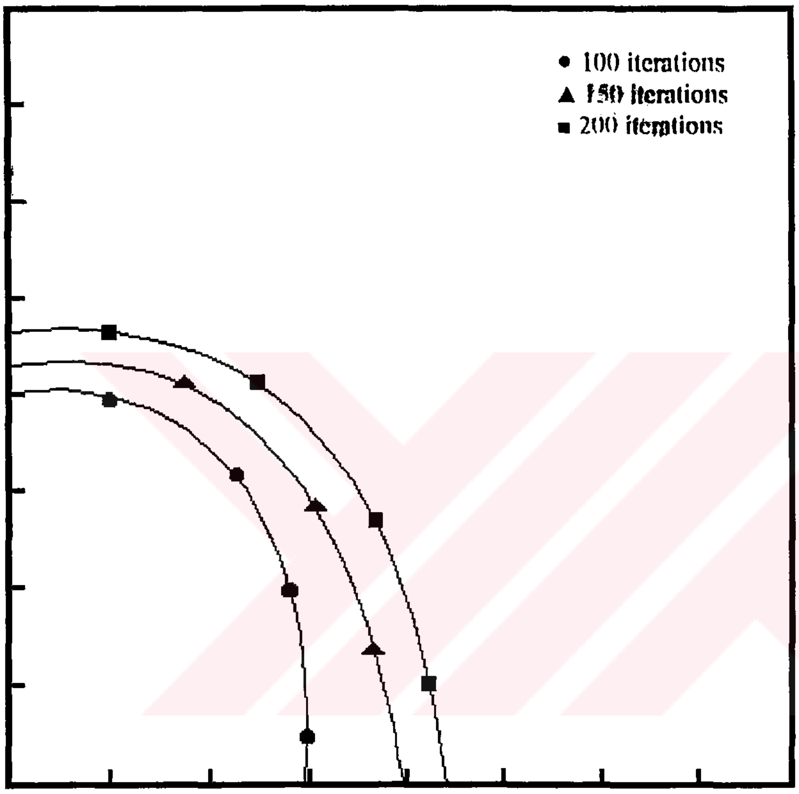


Figure 4.1.2. The expansion of plastic zone in symmetric, $(0°/90°)_2$,laminated plates at the upper surface.

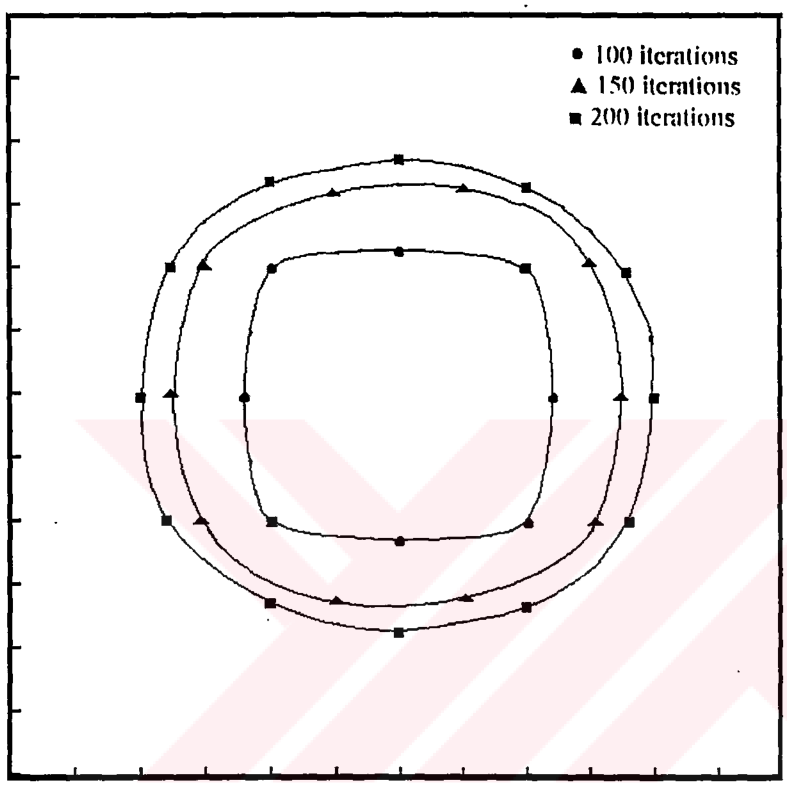
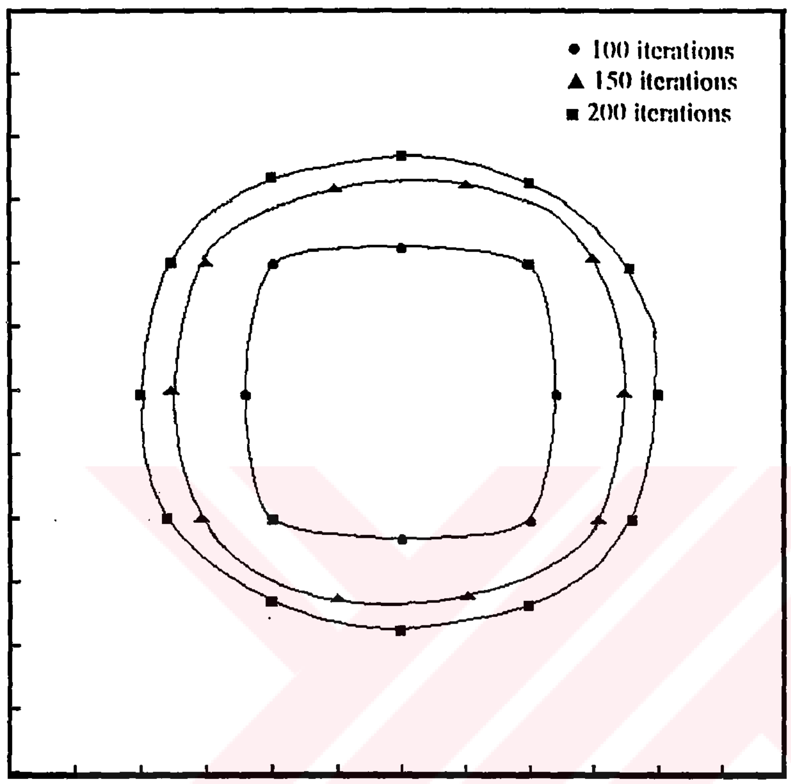


Figure 4.1.3. The expansion of plastic zone in symmetric, $(0°/90°)_2$, laminated plates at the lower surface.



The expansion of plastic zone is the similar at the lower and upper surfaces for cross-ply symmetric, $(0°/90°)_2$, and antisymmetric, $(0°/90°)_2$, laminated plates.

Figure 4.1.4. The expansion of plastic zone in symmetric $(30°/-30°)_2$, laminated plates

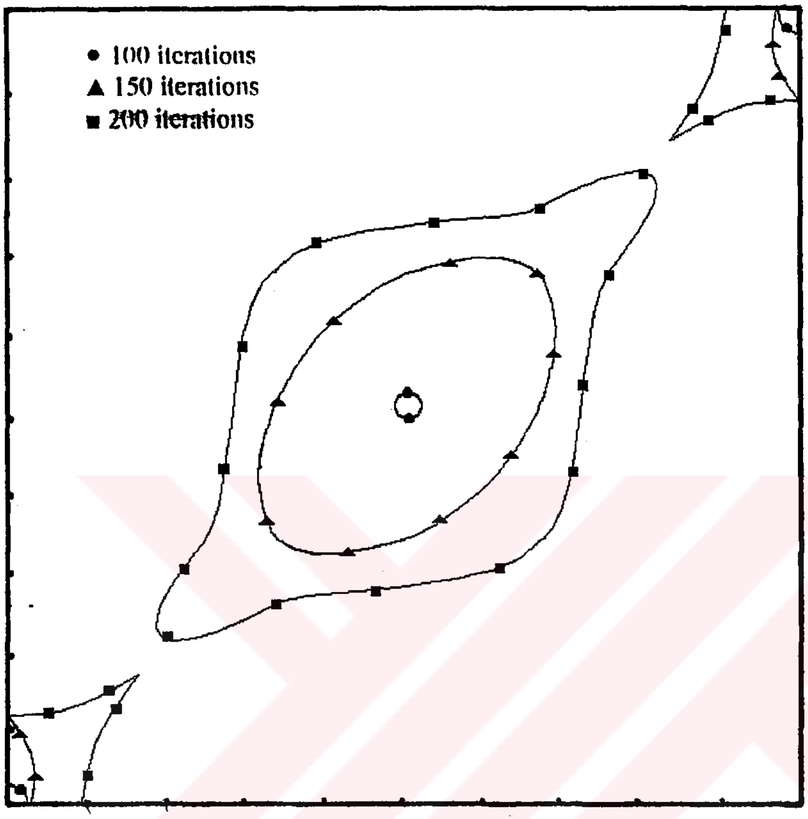


Figure 4.1.5. The expansion of plastic zone in antisymmetric, $(30°/-30°)_2$, laminated plates at the upper surface.

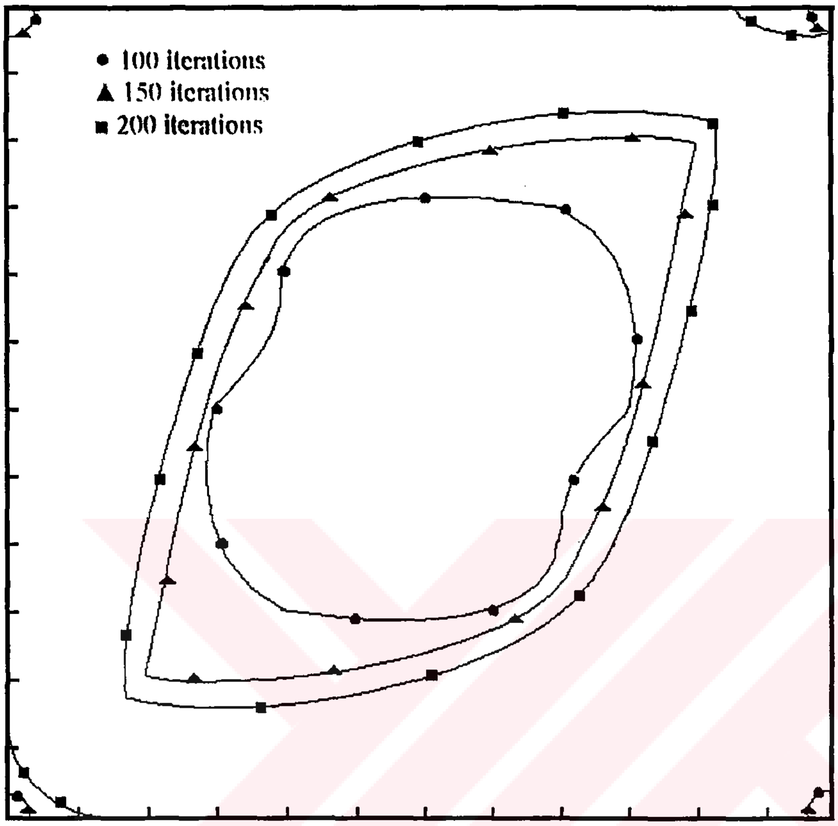


Figure 4.1.6. The expansion of plastic zone in antisymmetric, $(30°/-30°)_2$, laminated plates at the lower surface.

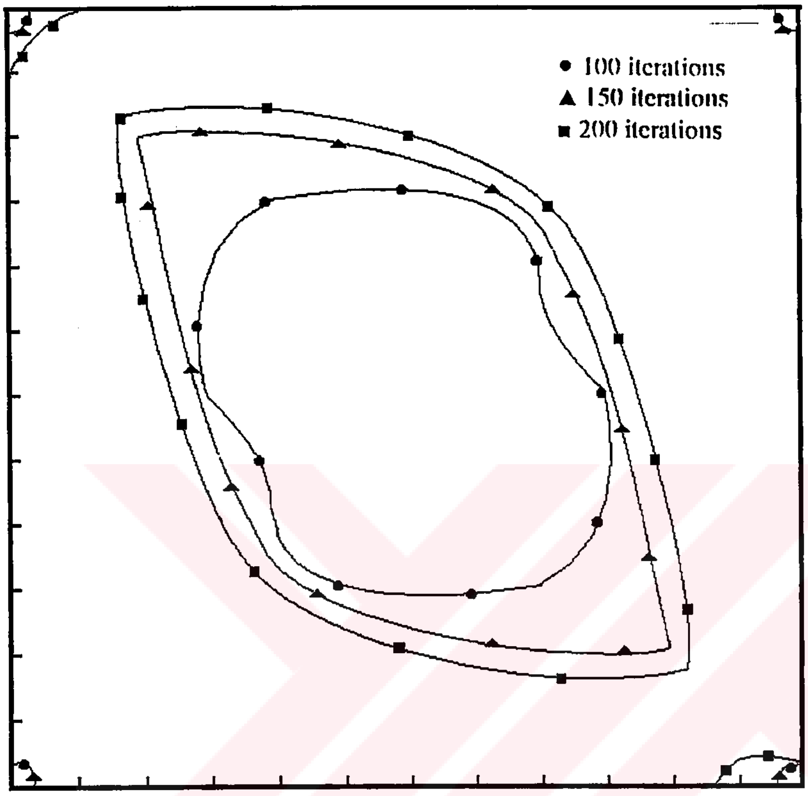


Figure 4.1.7. The expansion of plastic zone in symmetric, $(45°/-45°)_2$, laminated plates.

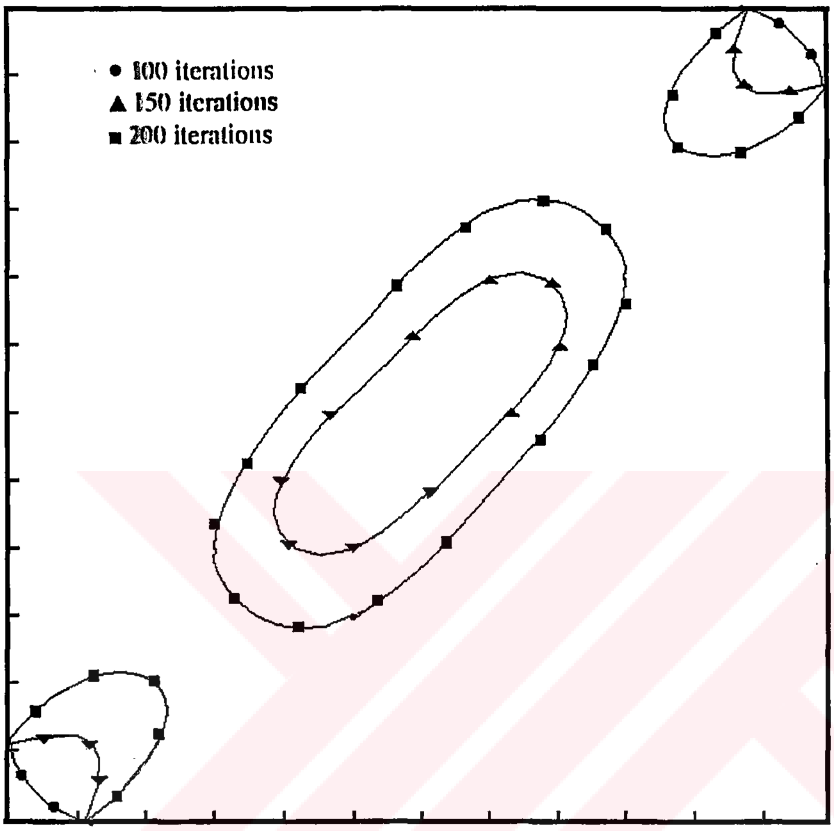


Figure 4.1.8. The expansion of plastic zone in antisymmetric, $(30°/-30°)_2$, laminated plates at the upper surface.

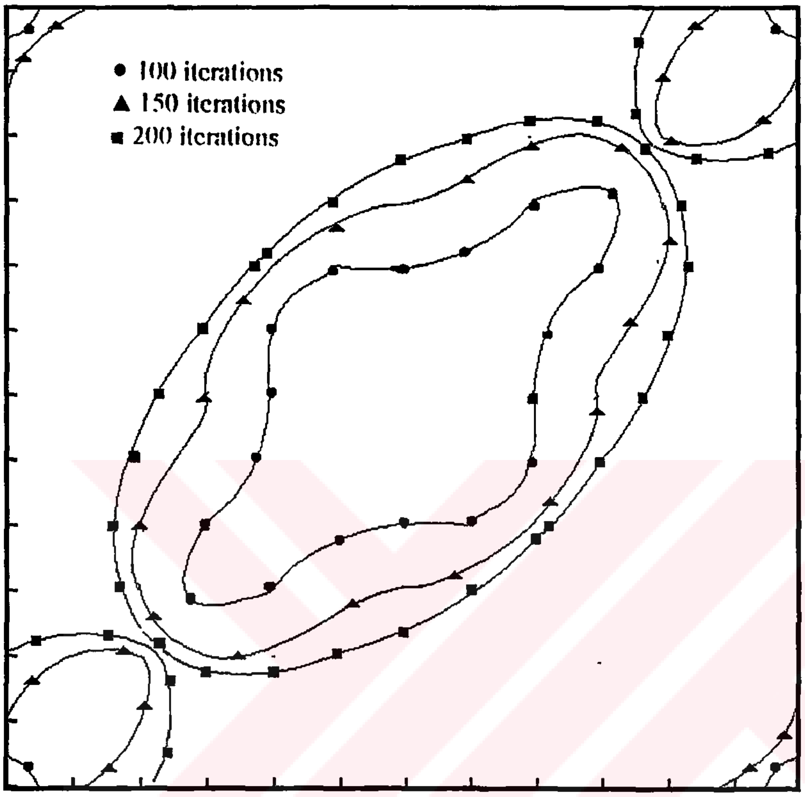


Figure 4.1.9. The expansion of plastic zone in antisymmetric, $(30°/-30°)_2$, laminated plates at the lower surface.

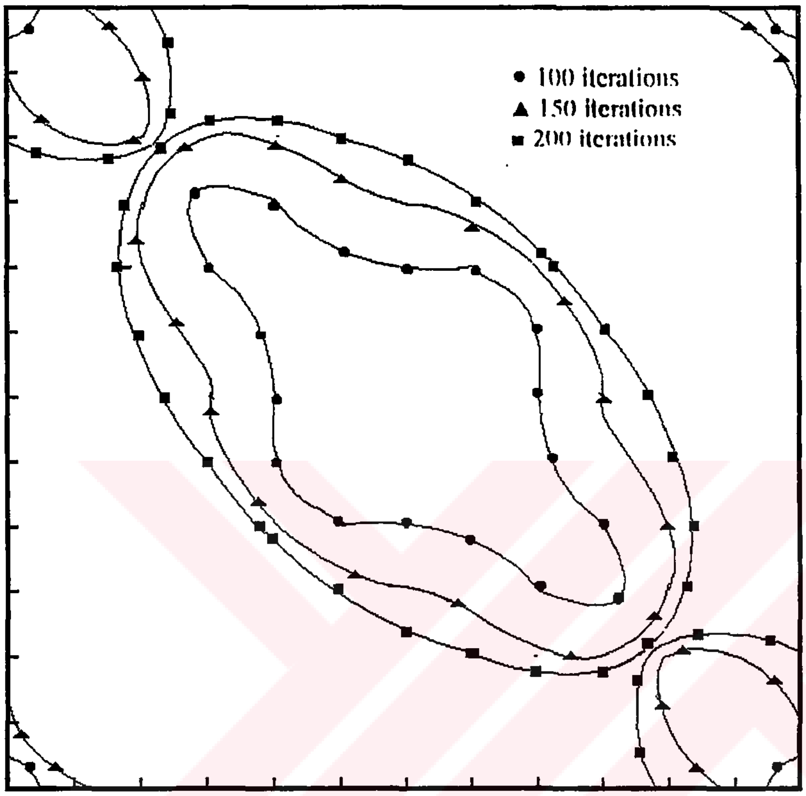


Figure 4.1.10. The expansion of plastic zone in symmetric, $(60°/-60°)_2$, laminated plates.

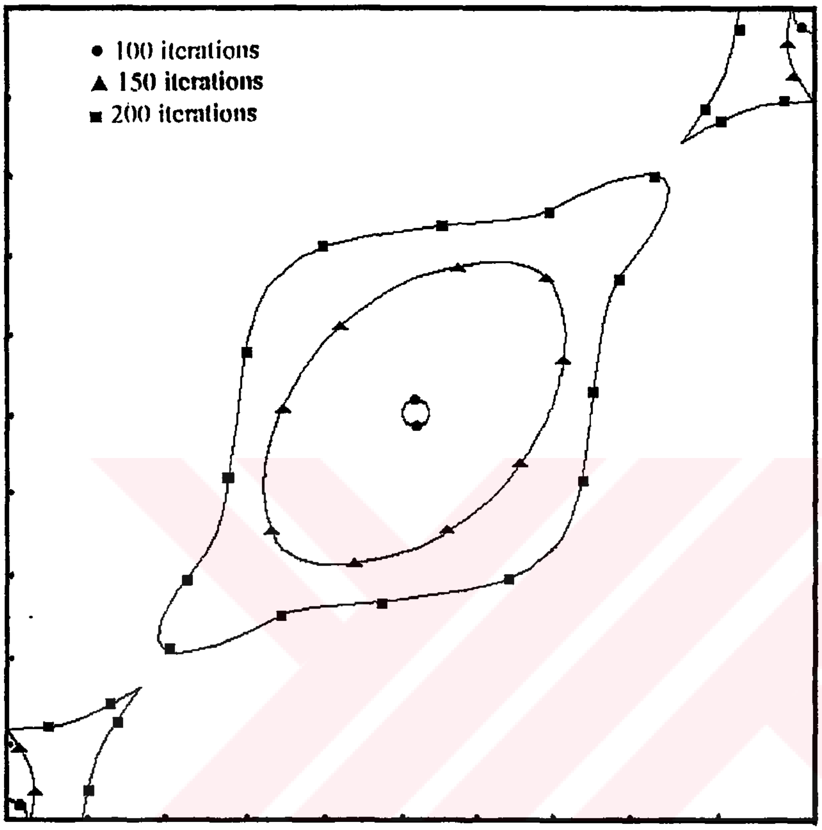


Figure 4.1.11. The expansion of plastic zone in antisymmetric, $(60°/-60°)_2$, laminated plates at the upper surface.

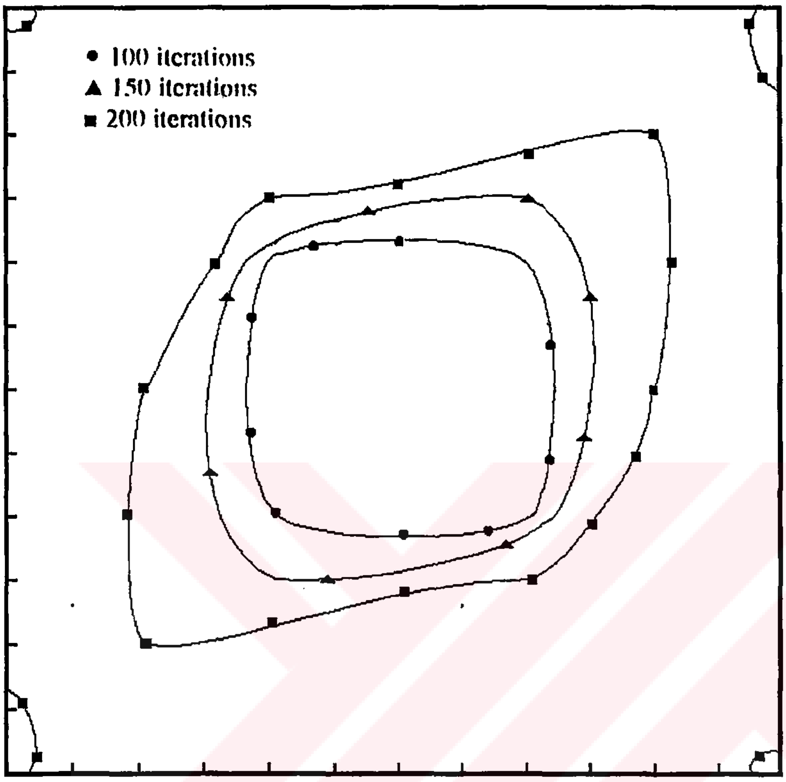
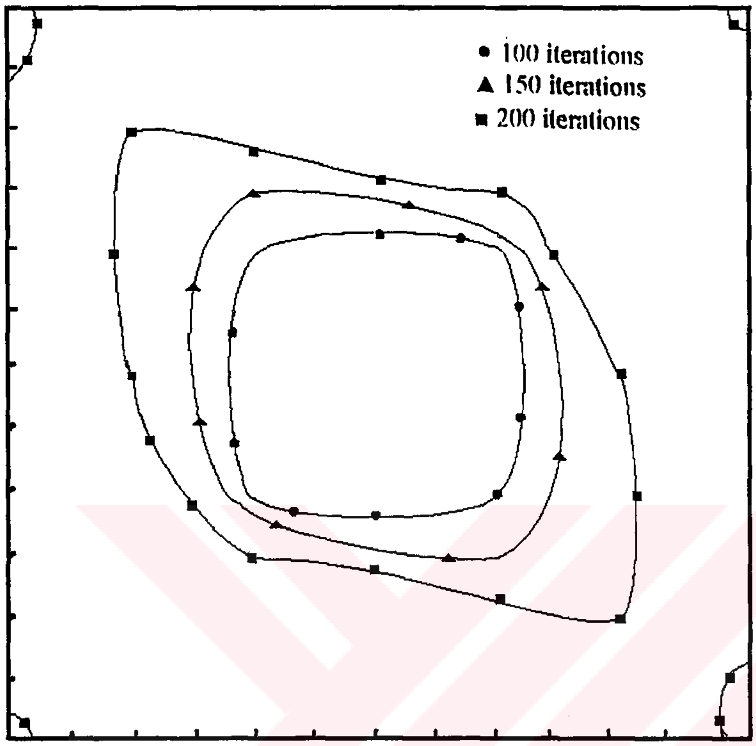


Figure 4.1.12. The expansion of plastic zone in symmetric, $(60°/-60°)_2$, laminated plates at the lower surface.



It is seen that the expansion of plastic zone is the same at the upper and lower surfaces of the symmetric angle-ply laminated plates. It spreads in different directions at the upper and lower surfaces of the antisymmetric angle-ply laminated plates. Also, plastic zone spreads at the corners of the plates because of the shear stresses in angle-ply laminated plates.

4.2.One Layered Woven Cr-Ni Fiber Reinforced Laminates

The mechanical properties of the laminated plate are found by using strain-gauges and Instron test machine.

Table 4.2.1. Mechanical properties and yield points of a layer

Mechanical Properties

E_1	3155 Mpa
E_2	3155 Mpa
G_{12}	303 MPa
ν_{12}	0.31

Yield stress and parameters

Axial Strength	X	17.5 MPa
Transverse Strength	Y	17.5 MPa
Shear Strength	S	5.92 MPa
Hardening parameter	k	94.63 MPa
Strain hardening parameter	n	0.67

As seen from these Tables, the mechanical properties are the same in 1 and 2 principal material directions because of the symmetry in the woven Cr-Ni fiber.

Table 4.2.2. The yield points of the laminated plates.

(S: Symmetric, A: Antisymmetric)

	$(0°/90°)_2$	$(30°/-30°)_2$ S and A	$(45°/-45°)_2$ S and A	$(60°/-60°)_2$ S and A
Yield Point(MPa)	S:0.0743	S:0.0935 A: 0.0951	S: 0.0878 A: 0.0878	S:0.0935 A: 0.0951

It is seen that the maximum yield point is found for $(30°/-30°)_2$ and $(60°/-60°)_2$ angle-ply laminated plates.

Table 4.2.3.1. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.468	17.494	-0.029	-0.085	-0.077
	-30°	-17.468	-17.494	0.029	-0.085	-0.077
Elastic Stresses	30°	18.694	18.723	-0.029	-0.050	-0.066
	-30°	-18.694	-18.723	0.029	-0.050	-0.066
Residual Stresses	30°	-1.226	-1.228	0.001	-0.035	-0.011
	-30°	1.226	1.228	-0.001	-0.035	-0.011

Table 4.2.3.2. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.469	17.493	-0.027	-0.085	-0.078
	-30°	-17.469	-17.493	0.027	-0.085	-0.078
Elastic Stresses	30°	19.597	19.627	-0.031	-0.052	-0.069
	-30°	-19.597	-19.627	0.031	-0.052	-0.069
Residual Stresses	30°	-2.128	-2.134	0.003	-0.033	-0.009
	-30°	2.128	2.134	-0.003	-0.033	-0.009

Table 4.2.3.3. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.469	17.493	-0.029	-0.086	-0.079
	-30°	-17.469	-17.493	0.029	-0.086	-0.079
Elastic Stresses	30°	20.501	20.531	-0.032	-0.055	-0.072
	-30°	-20.501	-20.531	0.032	-0.055	-0.072
Residual Stresses	30°	-3.031	-3.038	0.004	-0.032	-0.007
	-30°	3.031	3.038	-0.004	-0.032	-0.007

If we increase the iteration numbers the residual stress becomes greater.

Table 4.2.3.4. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.462	17.505	-0.022	-0.096	-0.073
	-30°	-17.462	-17.491	0.027	-0.073	-0.095
Elastic Stresses	30°	19.337	19.351	-0.023	-0.062	-0.062
	-30°	-19.337	-19.333	0.030	-0.062	-0.062
Residual Stresses	30°	-1.845	-1.846	0.000	-0.034	-0.011
	-30°	1.846	1.841	-0.003	-0.012	-0.033

Table 4.2.3.5. Elastic-Plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.491	17.505	-0.024	-0.097	-0.074
	-30°	-17.505	-17.492	0.028	-0.074	-0.097
Elastic Stresses	30°	20.257	20.272	-0.024	-0.065	-0.065
	-30°	-20.272	-20.253	0.031	-0.065	-0.065
Residual Stresses	30°	-2.765	-2.767	-0.001	-0.032	-0.009
	-30°	2.767	2.761	-0.003	-0.009	-0.032

Table 4.2.3.6. Elastic-plastic, elastic and residual stress components in $(30°/-30°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	30°	17.491	17.505	-0.026	-0.099	-0.075
	-30°	-17.505	-17.492	0.028	-0.075	-0.098
Elastic Stresses	30°	21.177	21.193	-0.025	-0.068	-0.068
	-30°	-21.192	-21.172	0.032	-0.068	-0.068
Residual Stresses	30°	-3.685	-3.687	-0.002	-0.031	-0.007
	-30°	3.688	3.680	-0.004	-0.007	-0.031

The residual stress in antisymmetric angle-ply, $(30°/-30°)_2$, laminated plate is greater than in symmetric, $(30°/-30°)_2$, plate.

Table 4.2.3.7. Elastic-Plastic, elastic and residual stress components $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	16.392	16.392	-0.029	-0.048	-0.048
	-45°	-16.392	-16.392	0.029	-0.048	-0.048
Elastic Stresses	45°	16.378	16.378	-0.029	-0.048	-0.048
	-45°	-16.378	-16.378	0.029	-0.048	-0.048
Residual Stresses	45°	0.014	0.014	0.000	0.000	0.000
	-45°	-0.014	-0.014	0.000	0.000	0.000

Table 4.2.3.8. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	17.247	17.247	-0.031	-0.050	-0.050
	-45°	-17.247	-17.247	0.031	-0.050	-0.050
Elastic Stresses	45°	17.215	17.215	-0.030	-0.050	-0.050
	-45°	-17.215	-17.215	0030	-0.050	-0.050
Residual Stresses	45°	0.032	0.032	0.000	0.001	0.001
	-45°	-0.032	-0.032	0.000	0.001	0.001

Table 4.2.3.9. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	17.489	17.489	-0.038	-0.066	-0.066
	-45°	-17.489	-17.489	0.038	-0.066	-0.066
Elastic Stresses	45°	18.052	18.052	-0.032	-0.053	-0.053
	-45°	-18.052	-18.052	0032	-0.053	-0.053
Residual Stresses	45°	0.564	0.564	-0.006	-0.013	-0.013
	-45°	-0.564	-0.564	0.006	-0.013	-0.013

Table 4.2.3.10. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	16.392	16.352	-0.029	-0.048	-0.048
	-45°	-16.392	-16.392	0.029	-0.048	-0.048
Elastic Stresses	45°	16.378	16.378	-0.029	-0.048	-0.048
	-45°	-16.378	-16.378	0.029	-0.048	-0.048
Residual Stresses	45°	0.014	0.014	0.000	0.000	0.000
	-45°	-0.014	-0.014	0.000	0.000	0.000

Table 4.2.3.11. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	17.247	17.247	-0.031	-0.050	-0.050
	-45°	-17.247	-17.247	0.031	-0.050	-0.050
Elastic Stresses	45°	17.215	17.215	-0.030	-0.050	-0.050
	-45°	-17.215	-17.215	0.030	-0.050	-0.050
Residual Stresses	45°	0.032	0.032	0.000	0.001	0.001
	-45°	-0.032	-0.032	0.000	0.001	0.001

Table 4.2.3.12. Elastic-plastic, elastic and residual stress components in $(45°/-45°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	45°	17.489	17.489	-0.038	-0.066	-0.066
	-45°	-17.489	-17.489	0.038	-0.066	-0.066
Elastic Stresses	45°	18.052	18.052	-0.032	-0.053	-0.053
	-45°	-18.052	-18.052	0.032	-0.053	-0.053
Residual Stresses	45°	-0.564	-0.564	-0.006	-0.013	-0.013
	-45°	0.564	0.564	0.006	-0.013	-0.013

The structure of $(45°/-45°)_2$, symmetric or antisymmetric laminated plates is the same therefore elastic-plastic, elastic and residual stresses are the same.

Table 4.2.3.13. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 100 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	17.494	17.468	-0.029	-0.077	-0.085
	-60°	18.723	18.694	-0.029	-0.066	-0.050
Elastic Stresses	60°	18.723	18.694	-0.029	-0.066	-0.050
	-60°	-18.723	-18694	0.029	-0.066	-0.050
Residual Stresses	60°	-1.228	-1.226	0.001	-0.011	-0.035
	-60°	1.228	1.226	-0.001	-0.011	-0.035

Table 4.2.3.14. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	17.493	17.469	-0.027	-0.078	-0.085
	-60°	-17.493	-17.469	0.027	-0.078	-0.085
Elastic Stresses	60°	19.627	19.597	-0.031	-0.069	-0.052
	-60°	-19.627	-19.597	0.031	-0.069	-0.052
Residual Stresses	60°	-2.134	-2.128	0.003	-0.009	-0.033
	-60°	2.134	2.128	-0.003	-0.009	-0.033

Table 4.2.3.15. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply symmetric laminated plate at node 85 for 200 iterations.

	Layer	σx (MPa)	σy (MPa)	τxy (MPa)	τyz (MPa)	τxz (MPa)
Elastic-Plastic Stresses	60°	17.494	17.470	-0.029	-0.080	-0.087
	-60°	-17.494	-17.470	0.029	0.080	-0.087
Elastic Stresses	60°	20.821	20.790	-0.033	-0.073	-0.056
	-60°	-20.821	-20.790	0.033	-0.073	-0.056
Residual Stresses	60°	-3.327	-3.320	0.004	-0.006	-0.031
	-60°	3.327	3.320	-0.004	-0.006	-0.031

Table 4.2.3.16. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σx (MPa)	σy (MPa)	τxy (MPa)	τyz (MPa)	τxz (MPa)
Elastic-Plastic Stresses	60°	17.505	17.491	-0.027	-0.073	-0.095
	-60°	-17.492	-17.505	0.022	-0.096	-0.073
Elastic Stresses	60°	19.351	19.333	-0.030	-0.062	-0.062
	-60°	-19.337	-19.351	0.023	-0.062	-0.062
Residual Stresses	60°	-1.846	-1.841	0.003	-0.012	-0.033
	-60°	1.845	1.846	0.000	-0.034	-0.011

Table 4.2.3.17. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	17.505	17.492	-0.028	-0.074	-0.097
	-60°	-17.491	-17.505	0.024	-0.097	-0.074
Elastic Stresses	60°	20.272	20.253	-0.031	-0.065	-0.065
	-60°	-20.257	-20.272	0.024	-0.065	-0.065
Residual Stresses	60°	-2.767	-2.761	0.003	-0.009	-0.032
	-60°	2.765	2.767	0.001	-0.032	-0.009

Table 4.2.3.18. Elastic-plastic, elastic and residual stress components in $(60°/-60°)_2$ angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	60°	17.505	17.492	-0.028	-0.075	-0.098
	-60°	-17.491	-17.505	0.026	-0.099	-0.075
Elastic Stresses	60°	21.192	21.172	-0.032	-0.068	-0.068
	-60°	-21.177	-21.193	0.025	-0.068	-0.068
Residual Stresses	60°	-3.688	-3.680	0.004	-0.007	-0.031
	-60°	3.685	3.687	0.002	-0.031	-0.007

The residual stress components in antisymmetric angle-ply, $(60°/-60°)_2$, laminated plate are higher than in symmetric, $(60°/-60°)_2$, laminated plate.

Table 4.2.4.1. The residual stresses in the angle-ply $(30°/-30°)_2$, $(45°/-45°)_2$ and $(60°/-60°)_2$ symmetric laminated plates at node 85 for 100, 150 and 200 iterations.

Iteration Numbers	Orientation Angles	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
100	30°	-1.226	-1.228	0.001	-0.035	-0.011
	45°	0.014	0.014	0.000	0.000	0.000
	60°	-1.228	-1.226	0.001	-0.011	-0.035
150	30°	-2.128	-2.134	0.003	-0.033	-0.009
	45°	0.032	0.032	0.000	0.001	0.001
	60°	-2.134	-2.128	0.003	-0.009	-0.033
200	30°	-3.031	-3.038	0.004	-0.032	-0.007
	45°	-0.564	-0.564	-0.006	-0.013	-0.013
	60°	-3.327	-3.320	0.004	-0.006	-0.031

It is seen that in this Table, residual stress components in angle-ply, $(60°/-60°)_2$, laminated plate are higher than in the other angle-ply laminated plates.

Table 4.2.4.2. The residual stresses in the angle-ply $(30°/-30°)_2$, $(45°/-45°)_2$ and $(60°/-60°)_2$ antisymmetric laminated plates at node 85 for 100, 150 and 200 iterations.

Iteration Numbers	Orientation Angles	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
100	30°	-1.845	-1.846	0.000	-0.634	-0.011
	45°	0.014	0.014	0.000	0.000	0.000
	60°	-1.846	-1.841	0.009	-0.012	-0.033
150	30°	-2.765	-2.767	-0.001	-0.032	-0.009
	45°	0.032	0.032	0.000	0.001	-0.001
	60°	-2.767	-2.761	0.003	-0.009	-0.032
200	30°	-3.685	-3.687	-0.002	-0.031	-0.007
	45°	-0.564	-0.564	-0.006	-0.013	-0.013
	60°	-3.688	-3.680	0.004	-0.007	-0.031

Residual stresses in antisymmetric angle-ply, $(60°/-60°)_2$ and $(30°/-30°)_2$, laminated plate are greater than in the angle-ply, $(45°/-45°)_2$, laminated plate. Also they are higher than in symmetric laminated plates for the same iterations.

Table 4.2.5.1. Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plate $(0°/90°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.499	17.499	-0.006	-0.064	-0.064
	150	17.499	17.499	-0.007	-0.066	-0.066
	200	17.499	17.499	-0.008	-0.092	-0.092
Elastic Stresses	100	19.873	19.873	-0.005	-0.048	-0.048
	150	21.052	21.054	-0.005	-0.051	-0.051
	200	22.230	22.230	-0.006	-0.054	-0.054
Residual Stresses	100	-2.374	-2.374	-0.001	-0.015	-0.015
	150	-3.553	-3.553	-0.002	-0.015	-0.015
	200	-4.731	-4.731	-0.002	-0.038	-0.038

If we increase the iteration numbers the residual stress components become greater in symmetric cross-ply, $(0°/90°)_2$, laminated plate.

Table 4.2.5.2. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(30°/-30°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.468	17.494	-0.029	-0.085	-0.077
	150	17.469	17.493	-0.027	-0.085	-0.078
	200	17.469	17.493	-0.029	-0.086	-0.079
Elastic Stresses	100	18.694	18.723	-0.029	-0.050	-0.066
	150	19.597	19.627	-0.031	-0.052	-0.069
	200	20.501	20.531	-0.032	-0.055	-0.072
Residual Stresses	100	-1.226	-1.228	0.001	-0.035	-0.011
	150	2.128	-2.134	0.003	-0.033	-0.009
	200	3.031	-3.038	0.004	-0.032	-0.007

Table 4.2.5.3. Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate $(30°/-30°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.492	17.505	-0.022	-0.096	-0.073
	150	17.491	17.505	-0.024	-0.097	-0.074
	200	13.491	17.505	-0.026	-0.099	-0.075
Elastic Stresses	100	19.337	19.351	-0.023	-0.062	-0.062
	150	20.257	20.272	-0.024	-0.045	-0.065
	200	21.177	21.193	-0.025	-0.068	-0.068
Residual Stresses	100	-1.845	-1.846	0.000	-0.034	-0.011
	150	-2.765	-2.767	-0.001	-0.032	-0.009
	200	-3.685	-3.687	-0.002	-0.031	-0.031

The stress components of elastic-plastic, elastic and residual stresses in antisymmetric angle-ply, $(30°/-30°)_2$, laminated plate are higher than in symmetric, $(30°/-30°)_2$, laminated plate for all the iteration numbers.

Table 4.2.5.4. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(45°/-45°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	16.392	16.392	-0.029	-0.048	-0.048
	150	17.247	17.247	-0.031	-0.050	-0.050
	200	17.489	17.489	-0.038	-0.066	-0.066
Elastic Stresses	100	16.378	16.378	-0.029	-0.048	-0.048
	150	17.215	17.215	-0.030	-0.050	-0.050
	200	18.052	18.052	-0.032	-0.053	-0.053
Residual Stresses	100	0.014	0.014	0.000	0.000	0.000
	150	0.032	0.032	0.000	0.001	0.001
	200	-0.564	-0.564	-0.006	-0.013	-0.013

If we increase the iteration numbers stress components become greater in, $(45°/-45°)_2$, laminated plate.

Table 4.2.5.5. Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate $(60°/-60°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.494	17.468	-0.029	-0.077	-0.085
	150	17.493	17.469	-0.027	-0.078	-0.085
	200	17.494	17.470	-0.029	-0.080	-0.087
Elastic Stresses	100	18.723	18.694	-0.029	-0.066	-0.050
	150	19.627	19.597	-0.031	-0.069	-0.052
	200	-20.821	-20.790	0.033	-0.073	-0.056
Residual Stresses	100	-1.228	-1.226	0.001	-0.011	-0.035
	150	-2.134	-2.128	0.003	-0.009	-0.033
	200	-3.327	-3.320	0.004	-0.006	-0.031

Table 4.2.5.6. Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate $(60°/-60°)_2$, at node 85 for 100, 150 and 200 iterations.

	Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Elastic-Plastic Stresses	100	17.505	17.491	-0.027	-0.073	-0.095
	150	17.505	17.492	-0.028	-0.074	-0.097
	200	17.505	17.492	-0.028	-0.075	-0.075
Elastic Stresses	100	19.351	19.333	-0.030	-0.062	-0.062
	150	20.272	20.253	-0.031	-0.065	-0.065
	200	21.192	21.172	-0.032	-0.068	-0.068
Residual Stresses	100	-1.846	-1.841	-0.003	-0.012	-0.023
	150	-2.767	-2.761	0.003	-0.009	-0.032
	200	-3.688	-3.680	0.004	-0.007	-0.031

The intensity of the elastic-plastic, elastic and residual stress components in angle-ply, $(60°/-60°)_2$, laminated plate are higher than in symmetric ones.

Table 4.2.6.1. Elastic-plastic stress components at the upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.499	17.499	-0.006	-0.064	-0.064
	$(30°/-30°)_2$	17.468	17.494	-0.029	-0.085	-0.077
	$(45°/-45°)_2$	16.392	16.392	-0.029	-0.048	-0.048
	$(60°/-60°)_2$	17.494	17.468	-0.029	-0.077	-0.085
At the Lower Surface	$(0°/90°)_2$	17.499	17.499	-0.006	-0.064	-0.064
	$(30°/-30°)_2$	-17.468	-17.494	-0.029	-0.085	-0.077
	$(45°/-45°)_2$	-16.392	-16.392	-0.029	-0.048	-0.048
	$(60°/-60°)_2$	-17.494	-17.468	-0.029	-0.077	-0.085

The intensity of the elastic-plastic stress components in symmetric angle-ply laminated plates at the upper surface are the same at the lower surface.

Table 4.2.6.2. Elastic-plastic stress components at the upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.497	17.498	-0.010	-0.103	-0.103
	$(30°/-30°)_2$	17.492	17.505	-0.022	-0.096	-0.072
	$(45°/-45°)_2$	16.392	16.392	-0.029	-0.048	-0.048
	(60°/-60°)2	17.505	17.491	-0.027	-0.073	-0.095
At the Lower Surface	$(0°/90°)_2$	-17.497	-17.498	-0.010	-0.103	-0.103
	(30°/-30°)2	-17.505	-17.491	-0.027	-0.073	-0.095
	$(45°/-45°)_2$	-16.392	-16.392	-0.029	-0.048	-0.048
	(60°/-60°)2	-17.492	-17.505	-0.022	-0.096	-0.073

Table 4.2.6.3. Elastic-plastic stress components at the upper and lower surfaces of the symmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.499	17.499	-0.007	-0.064	-0.066
	$(30°/-30°)_2$	17.469	17.493	-0.027	-0.085	-0.078
	$(45°/-45°)_2$	17.247	17.247	-0.031	-0.050	-0.050
	$(60°/-60°)_2$	17.493	17.469	-0.027	-0.078	-0.085
At the Lower Surface	$(0°/90°)_2$	-17.499	-17.499	-0.007	-0.064	-0.066
	$(30°/-30°)_2$	-17.469	-17.493	-0.027	-0.085	-0.078
	$(45°/-45°)_2$	-17.247	-17.247	-0.031	-0.050	-0.050
	$(60°/-60°)_2$	-17.493	-17.469	0.027	-0.078	-0.085

Table 4.2.6.4. Elastic-plastic stress components at the upper and lower surfaces of the antisymmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.498	17.498	-0.011	-0.106	-0.106
	$(30°/-30°)_2$	17.491	17.505	-0.027	-0.057	-0.074
	$(45°/-45°)_2$	17.247	17.247	-0.031	-0.050	-0.050
	$(60°/-60°)_2$	17.505	17.492	-0.028	-0.074	-0.097
At the Lower Surface	$(0°/90°)_2$	-17.498	-17.498	-0.011	-0.106	-0.106
	$(30°/-30°)_2$	-17.505	-17.492	-0.028	-0.074	-0.097
	$(45°/-45°)_2$	-17.247	-17.247	-0.031	-0.050	-0.050
	$(60°/-60°)_2$	-17.491	-17.505	-0.024	-0.097	-0.097

Table 4.2.6.5. Elastic-plastic stress components at the upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.499	17.499	-0.008	-0.092	-0.092
	$(30°/-30°)_2$	17.469	17.493	-0.029	-0.086	-0.079
	$(45°/-45°)_2$	17.489	17.489	-0.038	-0.066	-0.066
	$(60°/-60°)_2$	17.494	17.470	-0.029	-0.080	-0.087
At the Lower Surface	$(0°/90°)_2$	-17.499	-17.499	-0.008	-0.092	-0.092
	$(30°/-30°)_2$	-17.469	-17.493	-0.029	-0.086	-0.079
	$(45°/-45°)_2$	-17.489	-17.489	-0.038	-0.066	-0.066
	$(60°/-60°)_2$	-17.494	-17.470	-0.029	-0.080	-0.087

Table 4.2.6.6. Elastic-plastic stress components at the upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	17.498	17.498	-0.016	-0.133	-0.133
	$(30°/-30°)_2$	17.469	17.493	-0.029	-0.086	-0.079
	$(45°/-45°)_2$	17.489	17.489	-0.038	-0.066	-0.066
	$(60°/-60°)_2$	17.494	17.470	-0.029	-0.080	-0.087
At the Lower Surface	$(0°/90°)_2$	-17.498	-17.498	-0.016	-0.133	-0.133
	$(30°/-30°)_2$	-17.469	-17.493	-0.029	-0.086	-0.079
	$(45°/-45°)_2$	-17.489	-17.489	-0.038	-0.066	-0.066
	$(60°/-60°)_2$	-17.494	-17.470	-0.029	-0.080	-0.087

Table 4.2.6.7. Residual stress components at the upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-2.374	-2.374	-0.001	-0.015	-0.015
	$(30°/-30°)_2$	-1.226	-1.228	0.001	-0.035	-0.011
	$(45°/-45°)_2$	0.014	0.014	0.000	0.000	0.000
	$(60°/-60°)_2$	-1.228	-1.226	0.001	-0.011	-0.035
At the Lower Surface	$(0°/90°)_2$	2.374	2.374	-0.001	-0.015	-0.015
	$(30°/-30°)_2$	1.226	1.228	-0.001	-0.035	-0.011
	$(45°/-45°)_2$	0.014	0.014	0.000	0.000	0.000
	$(60°/-60°)_2$	1.228	1.226	-0.001	-0.011	-0.035

Table 4.2.6.8. Residual stress components at the upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-2.374	-2.374	-0.001	-0.015	-0.015
	$(30°/-30°)_2$	-1.845	-1.846	0.000	-0.034	-0.011
	$(45°/-45°)_2$	0.014	0.014	0.000	0.000	0.000
	$(60°/-60°)_2$	-1.846	-1.841	-0.003	-0.012	-0.023
At the Lower Surface	$(0°/90°)_2$	2.374	2.374	-0.001	-0.015	-0.015
	$(30°/-30°)_2$	1.846	1.841	0.003	-0.012	-0.013
	$(45°/-45°)_2$	0.014	0.014	0.000	0.000	0.000
	$(60°/-60°)_2$	1.845	1.846	0.000	-0.034	-0.011

Table 4.2.6.9. Residual stress components at the upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-4.731	-4.731	-0.002	-0.038	-0.038
	$(30°/-30°)_2$	-3.031	-3.038	0.004	-0.032	-0.007
	$(45°/-45°)_2$	-0.564	-0.564	-0.006	-0.013	-0.013
	$(60°/-60°)_2$	-3.327	-3.320	0.004	-0.006	-0.031
At the Lower Surface	$(0°/90°)_2$	4.731	4.731	-0.002	-0.038	-0.038
	$(30°/-30°)_2$	3.031	3.038	-0.004	-0.032	-0.007
	$(45°/-45°)_2$	0.564	0.564	0.006	-0.013	-0.013
	$(60°/-60°)_2$	3.327	3.320	-0.004	-0.006	-0.031

Table 4.2.6.10. Residual stress components at the upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
At the Upper Surface	$(0°/90°)_2$	-4.731	-4.731	-0.002	-0.044	-0.044
	$(30°/-30°)_2$	-3.685	-3.687	0.002	-0.031	-0.007
	$(45°/-45°)_2$	-0.564	-0.564	-0.006	-0.013	-0.013
	$(60°/-60°)_2$	-3.688	-3.680	0.004	-0.007	-0.031
At the Lower Surface	$(0°/90°)_2$	4.731	4.731	-0.006	-0.038	-0.038
	$(30°/-30°)_2$	3.688	3.680	-0.004	-0.007	-0.031
	$(45°/-45°)_2$	0.564	-0.564	0.006	-0.013	-0.013
	$(60°/-60°)_2$	3.685	3.687	0.002	-0.031	-0.007

As seen from these Tables, the intensity of the residual stress components in cross-ply, $(0°/90°)_2$, laminated plate is higher than in the other angle-ply laminated plates.

FIGURES

4.2. One-Layered Woven Cr-Ni Fiber-Reinforced Laminates

Figure 4.2.1. The expansion of plastic zone in $(0°/90°)_2$, symmetric laminated plates at the upper surface.

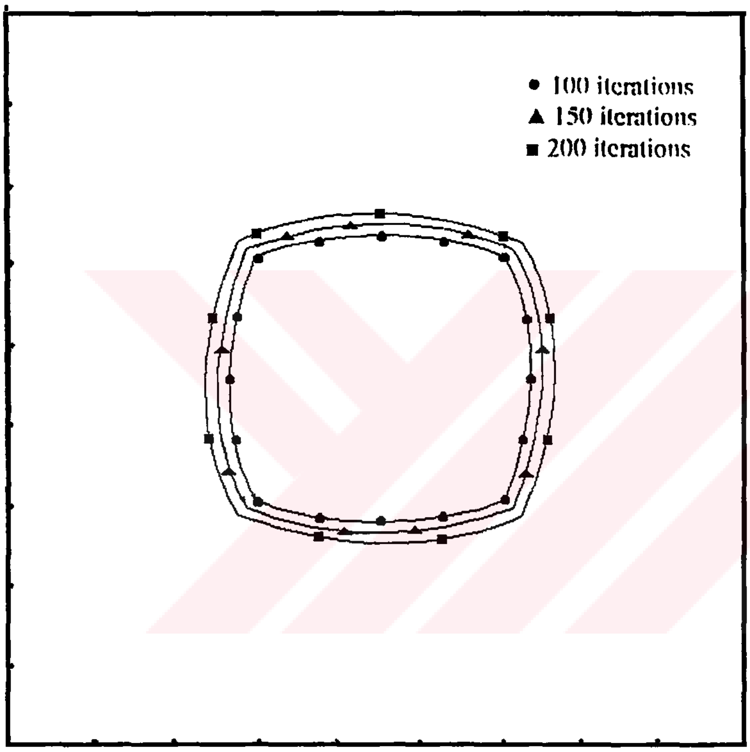
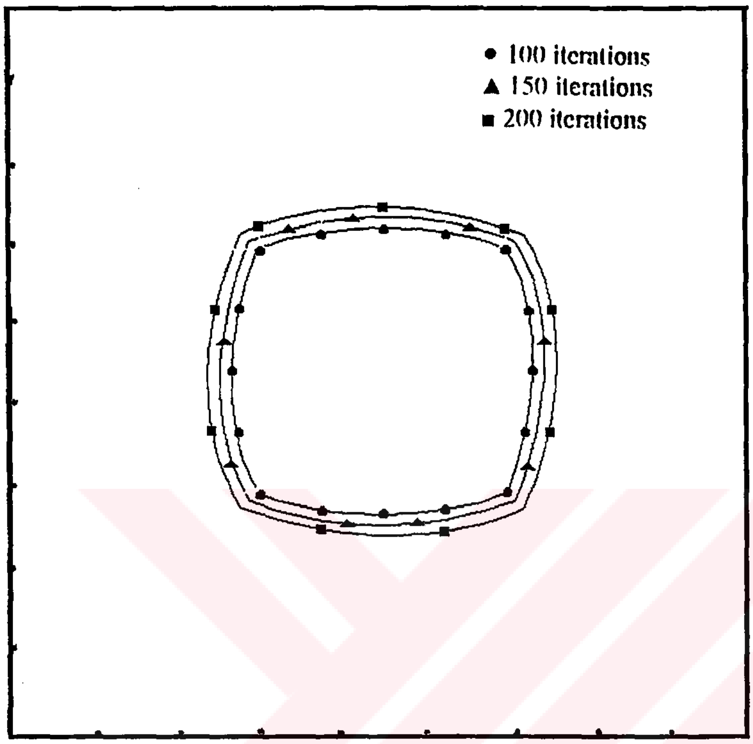
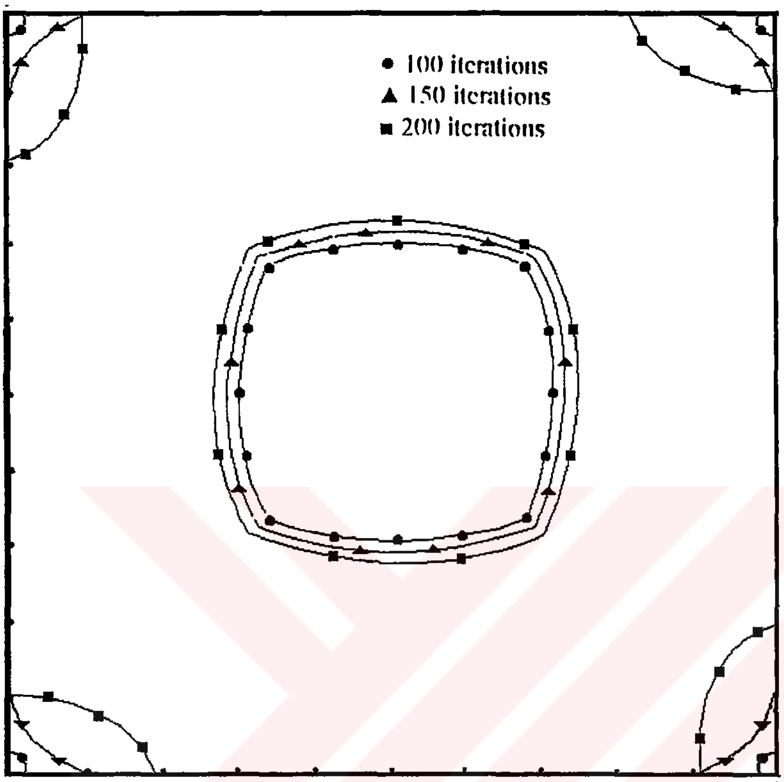


Figure 4.2.2. The expansion of plastic zone in $(0°/90°)_2$, symmetric laminated plates at the lower surface.



As seen from these Tables, the expansion of plastic zone in cross-ply $(0°/90°)_2$, laminated plates is the same at the upper and lower surfaces.

Figure 4.2.3. The expansion of plastic zone in $(30°/-30°)_2$, symmetric laminated plates.



It is seen that the plastic zone also expands at the corners in angle-ply, $(30°/-30°)_2$, laminated plates.

Figure 4.2.4. The expansion of plastic zone in $(30°/-30°)_2$, antisymmetric laminated plates at the upper surface.

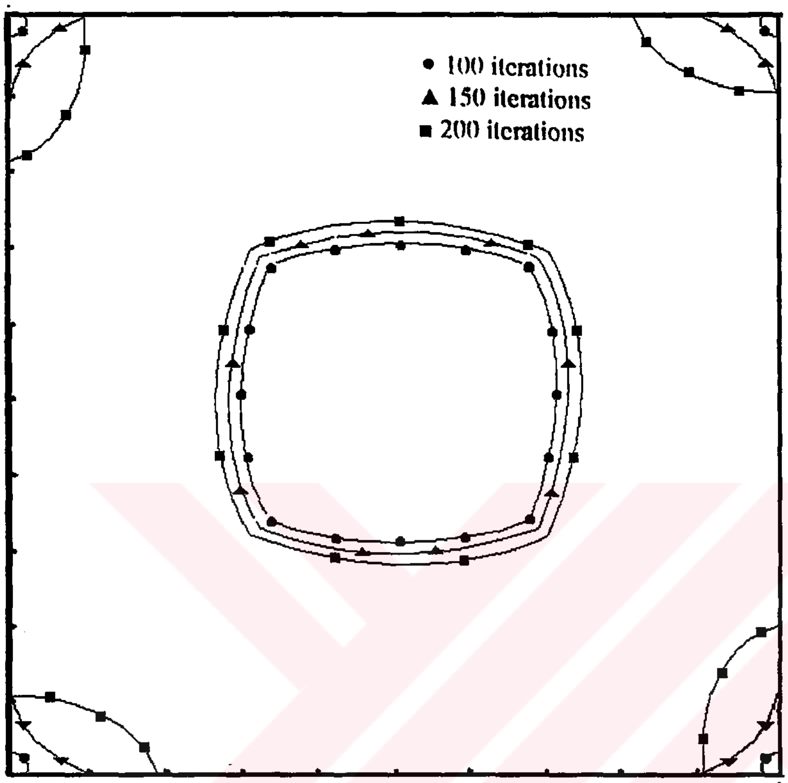
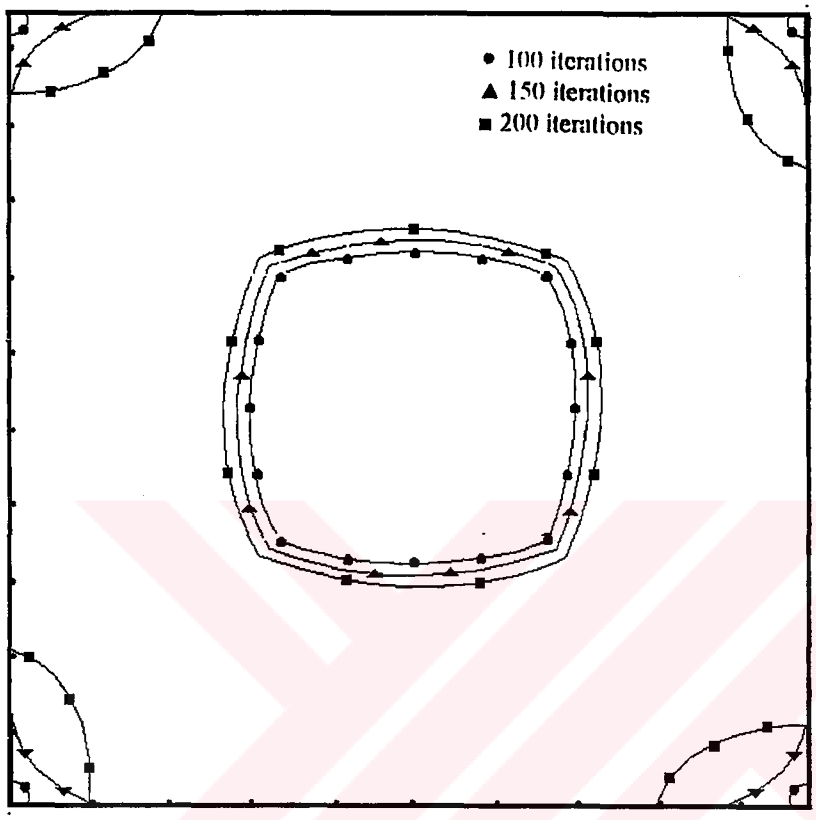


Figure 4.2.5. The expansion of plastic zone in $(30°/-30°)_2$, antisymmetric laminated plates at the lower surface.



As seen from these Tables that the expansion of plastic zone is symmetric with respect to its coordinate axes of the plate in symmetric or antisymmetric angle-ply, $(30°/-30°)_2$, laminated plates at the upper and lower surfaces.

Figure 4.2.6. The expansion of plastic zone in, $(45°/-45°)_2$, symmetric laminated plates.

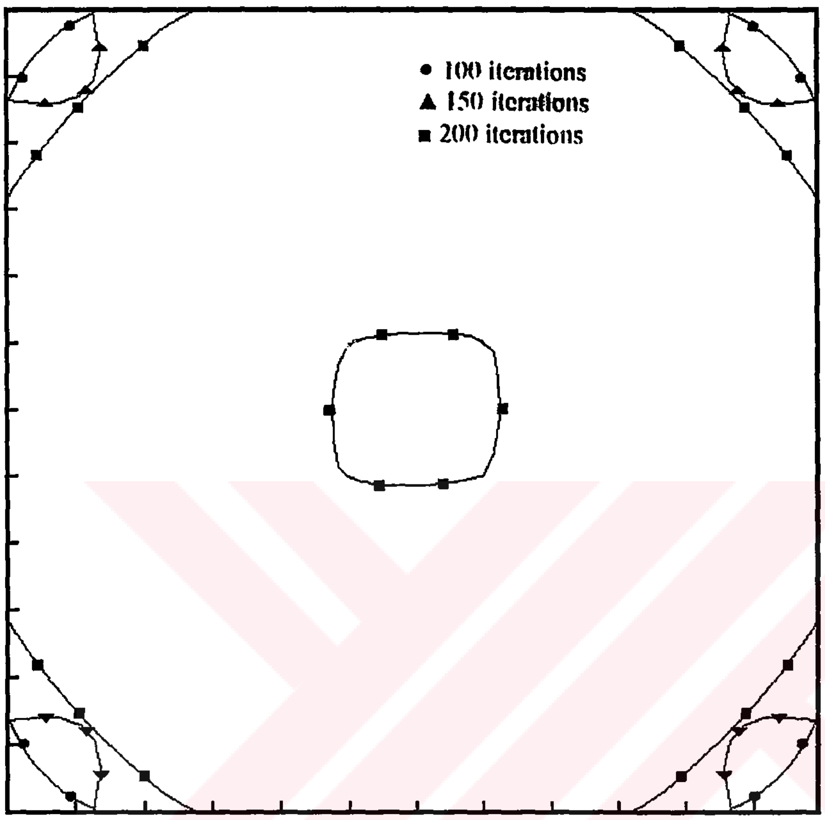


Figure 4.2.7. The expansion of plastic zone in, $(45°/-45°)_2$, antisymmetric laminated plates at the lower surface.

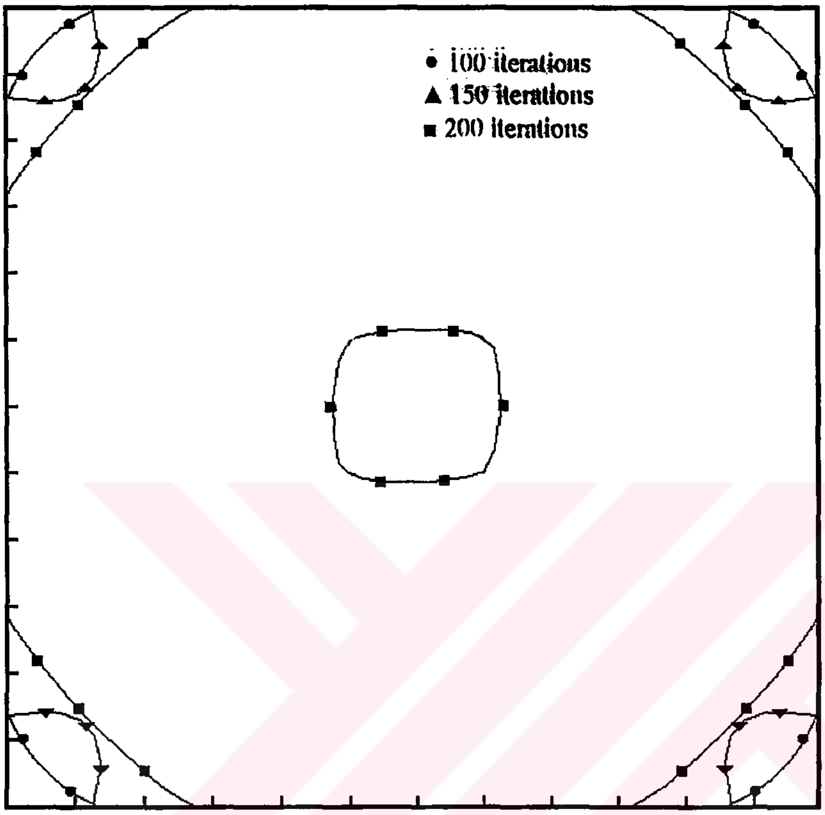


Figure 4.2.8. The expansion of plastic zone in, $(60°/-60°)_2$, symmetric laminated plates.

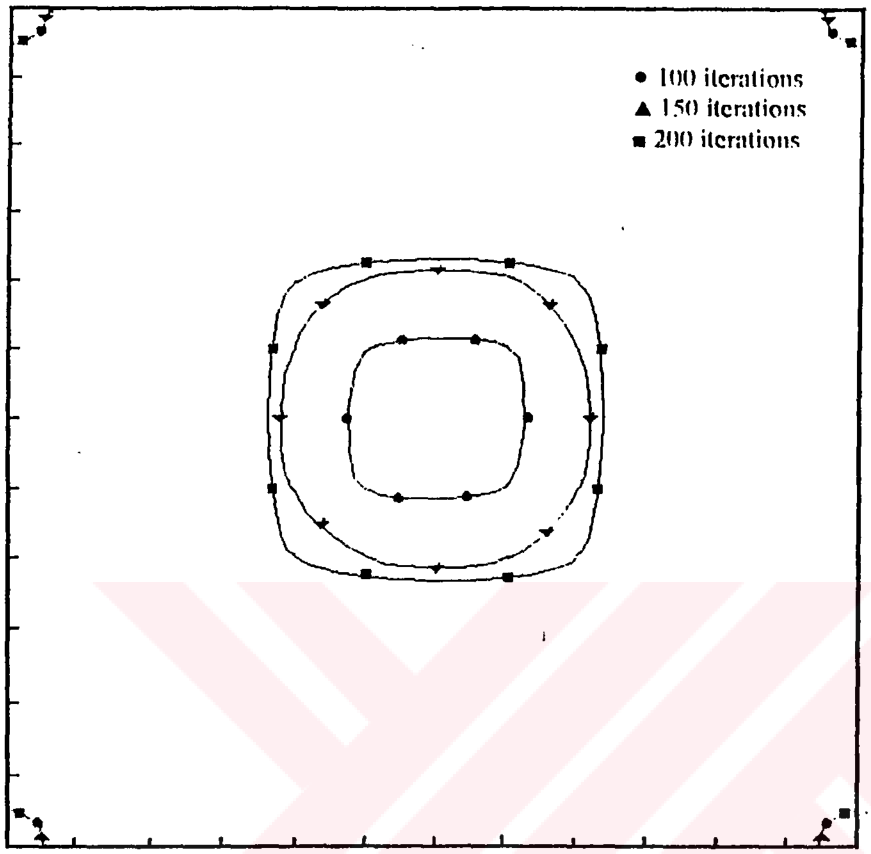


Figure 4.2.9. The expansion of plastic zone in, $(60°/-60°)_2$, antisymmetric laminated plates at the upper surface.

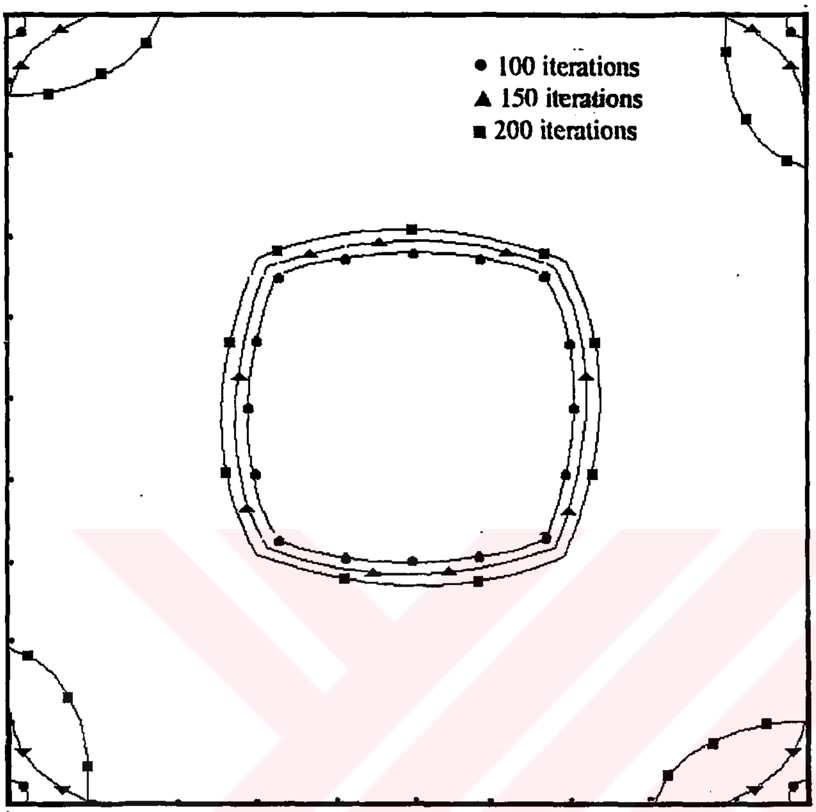
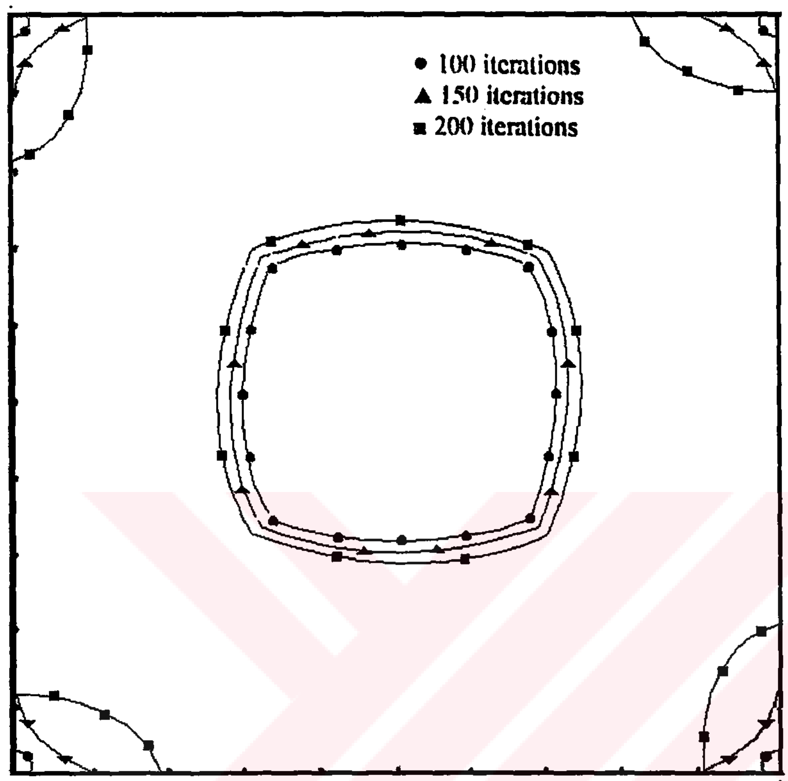


Figure 4.2.10. The expansion of plastic zone in, $(60°/-60°)_2$, antisymmetric laminated plates at the lower surface.



As seen from these Tables, the expansion of plastic zone in symmetric and antisymmetric cross-ply or angle-ply laminated plates are the same at the upper and lower surfaces, but they are different out corners. Also plastic zone expands at the corners of the angle-ply laminated plates.

CHAPTER FIVE
CONCLUSIONS

Elastic-plastic stress analysis has been carried out by using the first order shear deformation theory in thermoplastic composite laminated plates under transverse loading. The expansion of plastic zone and residual stresses are obtained in symmetric and antisymmetric cross-ply and angle-ply laminated plates. The following results of elastic-plastic stress analysis are found.

For Unidirectionally Galvanized Wire Reinforced Composite Laminated Plates:

a) The yield point in the symmetric cross-ply laminated plate is higher than in the antisymmetric cross-ply laminated plate.

b) The yield points in the antisymmetric angle-ply laminated plates are higher than the yield points in angle-ply symmetric laminated plates.

c) The yield points in the antisymmetric angle-ply laminated plates are higher than in symmetric and antisymmetric cross-ply laminated plates.

d) When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller. Residual stress components in cross-ply laminated plates are greater than in angle-ply laminated plates because in cross-ply laminated plates mechanical properties are more different than angle-ply laminated plates.

e) The residual stresses in, $(30°/-30°)_2$ and $(60°/-60°)_2$, angle-ply laminated plates are higher than in, $(45°/-45°)_2$, angle-ply laminated plate for the same iterations.

f) The residual stresses in symmetric and antisymmetric cross-ply laminated plates are higher than in angle-ply laminated plates for the same iterations.

For One Layered Woven Cr-Ni Fiber Reinforced Laminates:

g) The yield point for, $(30°/-30°)_2$ and $(60°/-60°)_2$, angle-ply woven laminated plates is higher than for, $(45°/-45°)_2$, angle-ply and $(0°/90°)_2$, cross-ply woven laminated plates.

h) The intensity of the residual stress components in symmetric angle-ply, $(60°/-60°)_2$, woven laminated plate is higher than in the others symmetric angle-ply woven laminated plates at 85 node.

i) The intensity of the residual stress components in the symmetric or antisymmetric cross-ply, $(0°/90°)_2$, woven laminated plates is higher than in the others angle-ply woven laminated plates at 85 node.

j) The expansion of plastic zone in symmetric cross-ply, $(0°/90°)_2$, woven laminated plates is the same at the upper and lower surfaces of the plates.

k) The plastic zone also expands at the corners in the angle-ply woven laminated plates.

l) The intensity of the residual stresses in the layer of 0° orientation angle is higher than in the layer of 90° orientation angle for symmetric cross-ply, $(0°/90°)_2$ laminated plate with a hole for in-plane loading.

REFERENCES

Bathe, K. J. (1982). Finite Element Procedures in Engineering Analysis. New Jersey: Prentice-Hall, Inc., Englewood Cliffs.

Chen, A. W. L., Miyase, A., Geil, P. H. And Wang, S. S. (1993). Anelastic Deformation of a Thermoplastic-Matrix Fiber Composite at Elevated Temperature; Part I: Neat Resin Structure Characterization. Journal of Composite Materials, 27, 862-885

Chung, I., Sun, C. T., Chang, I. Y. (1993).Modelling Creep in Thermoplastic Composites. Journal of Composite Materials, 27, 1009-1028

Jones, R. M. (1975). Mechanics of Composite Materials. McGraw-Hill Kogokusha, Ltd.

Lin, C. C. And Kuo, C. S. (1989). Buckling of Laminated Plates with Holes. Journal of Composite Materials, 23, 536-553

Marissen, R., Brouwer, H. R. And Linsen, J. (1995). Notched Strenght of Thermoplastic Woven Fabric Composites. Journal of Composite Materials, 29, 1544-1564

Tsai, S. W. And hahn, H. T. (1980). Introduction of Composite Materials, Technomic Publishing Co., Inc.

Wang, H. S., Hung, C. L. And Chang, F. K. (1996). Bearing Failure of Bolted Composite Joints. Part I: Expenmental Characterization. Journal of Composite Materials, 30, 1284-1358

Wang, J. Z. And Soice, D.F. (1993). Failure Strenght and Damage Mechanisms of E-Glass/Epoxy Laminates under In-Plane Biaxial Compressive Deformation. Journal of Composite Materials, 27, 40-57

Shi, F. F. The mechanical properties and Deformation of Shear-Induced Polymer Liquid Crystalline Fibers in an Engineering Thermoplastic. Journal of Composite Materials, Vol. 30. No:14/1996

Jegley, D. Impact-Damaged Graphite-Thermoplastic Trapezoidal-Corrugation Sandwich and Semi-Sandwich Panels. Journal of Composite Materials. Vol.27 NO:5/1993

Cantwell, W. J. The Influence of Stamping Temperature on the Properties of a Glass-Mat Thermoplastic Composite. Journal of Composite Materials. Vol.30 No:11/1996

Domb, M. M. The effect of Cooling Rate on Free-Edge Stress Development in Semi-Crystalline Thermoplastic Laminates. Journal of Composite Materials. Vol.32 No:4/1998

Wen, Y. F. Prediction of Momentary transverse Creep Behavior of Thermoplastic Polymer Matrix Composites Using Micromechanical Models. Journal of Composite Materials. Vol.31 No:21/1997

Sayman O., Elasto-Plastic stress analysis in stainless steel fiber reinforced aluminum metal matrix laminated plates loaded transversely. Journal of Composite Structures. Vol.43 No:2/1998 PP 147-154

Mendelson, A., Plasticity Theory and Application. The macmillan Company, Newyork, 1968

Chakrabarty, J. Theory of Plasticity. McGraw-Hill International Editions. 1987

Vinson, J. R. and T. W. Chou, Composite Materrials and Their Use in Structures, Applied Science Publishers. 1975

Bahei-El-Din, Y. A., and Dvorak, G. J., Plasticity Analysis of laminated Composite Plates,Transsections of the Asme, V.49, P.740-746, 1982

Jeronimidis, G. and Parkyn, A. T., Residual Stresses in Carbon Fibre-Thermoplastic Matrix Laminates, Journual of Composite Materials, V.22-5, 1988.

Karakuzu, R. And Sayman, O., Elasto-Plastic Finite Element Analysis of Metal Matrix Plates with Edge Notches, Computers & Structures, V.63, No.3, PP.551-558, 1997.

Hu, H.T. and Schnobrich, W.C., Nonlinear Finite Element Analysis of Reinforced Concrete Plates and Shells under Monotonic Loading, Computers and Structures, V.38, No.5/6, PP.637-651, 1991

Barbero, E. J., Anido, L. A. And Davalos, J. F. 1993. On the Mechanics of Thin Walled Laminated Composite Beam, Journual of Composite Materials, 27.806-829

Crisfield, M. A. 1991. Non-Linear Finite Element Analysis of Solids and Structures John Wiley & Sons.

Owen, D. R. J., Figueiras, J. A. Anisotropic Elasto-Plastic Finite Element Analysis of Thick and Thin Plates and Shells, International Journal for numerical methods in Enginering, Vol.19, 541-566 (1983)

Gibson, R. F. Principles of Composite Material Mechanics. McGraw-Hill, Inc., 1994