

AHMET GÖKÇEN MAHMUTOĞLU
MODELING AND ANALYSIS OF NONSTATIONARY
LOW-FREQUENCY NOISE IN ELECTRONIC
CIRCUIT SIMULATION

A Thesis Submitted to the
Graduate School of Sciences and Engineering
in Partial Fulfillment of the Requirements for
the Degree of

Doctor of Philosophy

in

Electrical and Electronics Engineering

Koç University

August 2015

KOÇ UNIVERSITY
GRADUATE SCHOOL OF SCIENCES AND ENGINEERING

This is to certify that I have examined this copy of a Ph.D. thesis
by

Ahmet Gökçen Mahmutoglu

and have found that
it is complete and satisfactory in all respects,
and that any and all revisions required by the final
examining committee have been made.

Reading Committee:

Prof. Alper Demir: _____

Assoc. Prof. Serdar Taşiran: _____

Assoc. Prof. Emre Mengi: _____

Prof. Yusuf Leblebici: _____

Asst. Prof. Önder Şuvak: _____

MODELING AND ANALYSIS OF NONSTATIONARY
LOW-FREQUENCY NOISE IN ELECTRONIC CIRCUIT
SIMULATION

AHMET GÖKÇEN MAHMUTOĞLU

Department of Electrical and Electronics Engineering
Graduate School of Sciences and Engineering
Koç University

August 2015

This work was supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under project 111E188.

Ahmet Gökçen Mahmutoglu: *Modeling and Analysis of Nonstationary Low-Frequency Noise in Electronic Circuit Simulation*, © August 2015

[N]obody really thinks who does not abstract from that which is given, who does not relate the facts to the factors which have made them, who does not—in his mind—undo the facts. Abstractness is the very life of thought, the token of its authenticity.

— Herbert Marcuse, *One Dimensional Man*

ABSTRACT

Modeling and analysis of low-frequency noise, such as $1/f$ and burst noise, under time-varying bias conditions is a long-standing open problem in circuit simulation. It has been shown, both in theory and experimentally, that analyses based on simplistic noise models with stationary or modulated stationary statistics produce erroneous results. We offer a solution to this problem and present a new computational modeling and analysis framework for low-frequency noise. This framework is based on a mathematical description of the random activity that is caused by the defects located in the atomic lattice structures of semiconductors. The merits of our noise model are two-fold: First, it is fully nonstationary. It can represent noise processes with time-varying statistics that are tightly coupled to the circuit variables in a stochastically correct manner. Second, its mathematical structure allows the utilization of well-established, non Monte Carlo noise analysis techniques which are orders of magnitude faster than their Monte Carlo counterparts. We develop our low-frequency noise modeling and analysis techniques by building on a mathematical foundation based on the theory of Markov chains and the theory of stochastic differential equations aided by the extensive literature on modeling and simulation of stochastic chemical kinetics. We present a detailed explication of our modeling method with regard to its stochastic properties and develop carefully crafted circuit simulation algorithms that are stochastically correct. We implement these algorithms in a new circuit simulation tool and establish the correctness of our methods via rigorous simulations. We examine circuits and operating conditions where the legacy noise model mispredicts the effects of low-frequency noise and demonstrate how these effects can be correctly simulated using the new noise modeling and analysis framework we propose.

ÖZET

Devre simülasyonunda, $1/f$ ve yırtılma gürültüsü gibi alçak frekanslı gürültünün, zamanla değişen öngerilim koşullarında modellenmesi ve analizi uzun süredir açık kalmış bir problemdir. Durağan ya da kiplenmiş durağan istatistiklere sahip, basit modellere dayanan analizlerin hatalı sonuçlar ürettikleri, hem kuramsal hem de deneysel olarak gösterilmiştir. Bu probleme bir çözüm öneriyor ve alçak frekanslı gürültü için yeni bir hesaplamalı modelleme ve analiz sistemi sunuyoruz. Bu sistemin temelini, yarı iletkenlerin atomik kafes yapılarındaki bozuklukların yarattığı rassal faaliyetlerin matematiksel bir tarifi oluşturmaktadır. Gürültü modelimizin marifetleri çift katmanlıdır: Öncelikle, bütünüyle durağan olmayan yapıdadır. Zamanla değişen istatistiklere sahip ve devre değişkenlerine yakın şekilde bağlı rassal süreçleri, stokastik olarak doğru biçimde temsil edebilir. İkinci olarak, gürültü modelinin matematiksel yapısı, iyice anlaşılmış, Monte Carlo olmayan gürültü analizi yöntemlerinin kullanımına olanak sağlamaktadır. Bu yöntemler, Monte Carlo karşılıklarına oranla kat be kat daha hızlıdır. Alçak frekanslı gürültü modelimizi, stokastik kimyasal kinetik hakkındaki geniş yazından yararlanarak, Markov zincirleri ve stokastik diferansiyel denklemler teorilerine dayanan bir matematiksel temel üzerine kuruyoruz. Modelleme yöntemimizin stokastik özelliklerini detaylandıran bir izahat sunuyor ve stokastik olarak doğru simülasyon algoritmaları geliştiriyoruz. Bu algoritmaları yeni bir devre simülasyonu aracı içinde gerçekleştiriyor ve yöntemlerimizin doğruluğunu titiz simülasyonlar ile sabitliyoruz. Eski modellerin alçak frekanslı gürültünün etkilerini yanlış kestirdikleri devreleri ve çalışma koşullarını inceliyor ve bu etkilerin, önerdiğimiz yeni modelleme ve analiz sistemiyle nasıl doğru olarak simüle edilebileceklerini gösteriyoruz.

PUBLICATIONS

Contents of this thesis have previously appeared in the following publications:

A. G. Mahmutoglu and A. Demir, “Analysis of Low-Frequency Noise in Switched MOSFET Circuits: Revisited and Clarified,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 4, pp. 929–937, Apr. 2015.

A. G. Mahmutoglu and A. Demir, “Modeling and Simulation of Low-Frequency Noise in Nano Devices: Stochastically Correct and Carefully Crafted Numerical Techniques,” *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 34, no. 5, pp. 794–807, May 2015.

A. G. Mahmutoglu and A. Demir, “Modeling and Analysis of Nonstationary Low-Frequency Noise in Circuit Simulators: Enabling non Monte Carlo Techniques,” in *2014 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2014, pp. 309–315.

A. G. Mahmutoglu, A. Demir, and J. Roychowdhury, “Modeling and Analysis of (Nonstationary) Low Frequency Noise in Nano Devices: A Synergistic Approach Based on Stochastic Chemical Kinetics,” in *2013 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2013, pp. 500–507.

A. G. Mahmutoglu and A. Demir, “CIRSIUM: A Circuit Simulator in MATLAB with Object Oriented Design,” in *Ph.D Research in Microelectronics and Electronics (PRIME)*, 2013 9th Conference on, Jun. 2013, pp. 173–176.

ACKNOWLEDGMENTS

The completion of this thesis would have been a much less arduous journey if I had followed the advice of my professor, Dr. Alper Demir, more carefully. This fact is reason alone—although not the only one—for my most sincere gratitude to him for offering me such thorough guidance, which, even followed ineptly, led to this satisfactory result.

I would like to thank my friends for being gentle enough as to provide me with a comfort zone and subversive enough in order to help me break out of it.

I thank my family for being there. No matter where I looked. Backward, sideward, forward, in all directions; whether there is still any up or down.

CONTENTS

1	INTRODUCTION	1
1.1	Low-Frequency Noise in Electronic Circuits	2
1.1.1	Sources of Low-Frequency Noise—Oxide Traps	2
1.1.2	RTS Noise in Transistors and Electronic Circuits	2
1.1.3	Modeling and Analysis of Low-Frequency Noise in Circuits	3
1.2	Noise in the Nervous System	4
1.2.1	Membrane Ion Channel Noise in Nerve Cells	4
1.2.2	Modeling and Analysis of Noise in the Nervous System	4
1.3	A Synergistic Approach for Modeling and Analysis	5
1.3.1	A Perfect Analogy	5
1.3.2	Approach to Modeling and Analysis of Low-Frequency Noise	6
1.4	Summary of Contributions and Outline of the Thesis	7
2	NONSTATIONARY STOCHASTIC DESCRIPTION OF LOW- FREQUENCY NOISE SOURCES	9
2.1	Introduction	9
2.2	Preliminaries	9
2.3	Chapman-Kolmogorov Equation for a Trap	10
2.4	Stochastic Differential Equation for a Trap	11
2.4.1	Gaussian Approximation Route	12
2.4.2	Fokker-Planck Equation Route	13
2.5	First and Second-Order Statistics of a Trap	15
2.5.1	Mean Behavior of a Trap	15
2.5.2	Second-Order Moment for a Trap	15
2.5.3	Auto-correlation Function for a Trap	16
2.5.4	Spectral Density for a Trap	16
3	CIRSIUM: A CIRCUIT SIMULATOR IN MATLAB [®]	18
3.1	Introduction	18
3.2	Simulator Architecture and Principles	19
3.3	RTS Noise Simulation	21
3.3.1	Oxide Traps	21
3.3.2	Embedding Traps into the PSP Compact Model	22
3.3.3	Simulation Example—Ring Oscillator with Oxide Traps	24
4	STOCHASTICALLY CORRECT AND CAREFULLY CRAFTED NUMERICAL TECHNIQUES	26
4.1	Introduction	26

4.2	Simulation Framework	26
4.2.1	Simulation Basics	27
4.3	Stochastic Simulation based on Stochastic Automaton Model (SAM)	28
4.3.1	Poisson Thinning and Uniformization Algorithms	29
4.3.2	Adaptive Uniformization with Backtracking Produces Incorrect Statistics	31
4.3.3	Summary	33
4.4	Stochastic Simulation based on Langevin Equation Model (LEM)	33
4.4.1	The Square Root Process	35
4.4.2	Numerical Treatment of the Square Root Process	36
4.4.3	Our Solution: Smooth Diffusion Algorithm	39
4.4.4	Comparison of SAM and LEM Computational Cost	41
4.5	Monte Carlo Simulation Results	42
4.5.1	Common-Source Amplifier: Constant Input, Two Traps	42
4.5.2	Common-Source Amplifier: Switched Input, Single Trap	43
4.5.3	Phase Jitter in a Ring Oscillator	45
5	NON MONTE CARLO ANALYSIS OF LOW-FREQUENCY NOISE	48
5.1	Introduction	48
5.2	Treatment of Noise Dynamics Based on Langevin Equation Model	49
5.3	Frequency-Domain, Non Monte Carlo, Semi-Analytical Noise Evaluation	52
5.4	Verification of non Monte Carlo Methods via Monte Carlo Simulations	55
5.4.1	Spectral Estimation via Monte Carlo Simulations and Average Spectral Density	56
5.4.2	Common Source Amplifier	57
5.4.3	Ring Oscillator	58
6	ANALYSIS OF LOW-FREQUENCY NOISE IN SWITCHED MOSFET CIRCUITS: REVISITED & CLARIFIED	61
6.1	Introduction	61
6.2	Idealized Trap Noise in a Switched Transistor	62
6.2.1	Model Setup and Assumptions	62
6.2.2	Derivation of Switched Trap Noise Spectrum	63
6.2.3	Switched Trap Noise Spectrum and its Meaning	67
6.2.4	Validation via Monte Carlo Simulations	68
6.2.5	$1/f$ Noise in a Switched Transistor	70
6.2.6	Limitations of the Idealized Trap Model	71
6.3	Trap Noise Simulations based on the Realistic Model	72
6.4	Related Work	78

7	EXPOSITION OF INTRICATE NONSTATIONARY BEHAVIOR OF LOW-FREQUENCY NOISE AND COMPARISON WITH THE LEGACY MODEL	80
7.1	Introduction	80
7.2	Legacy Modulated Stationary Model	80
7.3	Comparison with the Legacy Model	81
7.3.1	Noise Reduction: Switched Biasing based on Legacy Model	82
7.3.2	Noise Reduction: Legacy Model vs. Nonstationary Model	83
7.4	$1/f$ Noise Reduction in the Coupled Sawtooth Oscillator via Switched Biasing—A Case Study	85
8	SUMMARY AND CONCLUSION	89
	BIBLIOGRAPHY	91

LIST OF FIGURES

Figure 1	Class diagram of the CIRSIUM core.	19
Figure 2	Capture and emission process of an electron by a trap located close to the conduction band edge.	22
Figure 3	Output of a transient simulation sample for a 3-stage ring oscillator with oxide traps. The inset shows a time span of high trap activity.	25
Figure 4	Time-variant rate function of an inhomogeneous Poisson process.	32
Figure 5	A sample path traced by the capture and emission rates of a single trap during a simulation of a ring oscillator circuit.	37
Figure 6	Maximum ratio of the integration time-step and the mean capture time plotted for different values of k in order to keep the failure probability under 10^{-6}	39
Figure 7	Power spectral density at the output of a common source amplifier at a fixed bias point. The n-channel transistor contains two oxide traps with different transition rates.	43
Figure 8	Comparison of noise power in a common source amplifier for the constant and switched input cases via LEM simulations.	44
Figure 9	One hundred output waveforms of a ring oscillator plotted on top of each other. As the simulation time progresses, the waveforms drift apart and create a variation in the phase of the oscillation.	45
Figure 10	Phase jitter variance of a three-stage ring oscillator with a total of twelve non-stationary traps.	46
Figure 11	Time-varying mean and variance of the occupancy function of a single trap in a ring oscillator circuit.	47
Figure 12	Block diagram of a circuit coupled with a noise model that captures the internal dynamics of nonstationary noise.	50
Figure 13	The average power spectral density of the common source amplifier with a noisy transistor.	57
Figure 14	Common source amplifier.	57
Figure 15	Noise spectrum of a three-stage ring oscillator containing a single trap as the noise source.	58
Figure 16	Variance of accumulating phase jitter for a ring oscillator.	60

Figure 17	a) An n-channel MOSFET with a trap in its gate-oxide. b) the periodic input signal supplied to the gate node of the device. c) The resulting variation of the trap rates over time. λ_c^{on} and λ_e^{on} are the capture and emission rates of the trap when the transistor is in its on-state. Similarly, λ_c^{off} and λ_e^{off} are the event rates when the transistor is turned off.	62
Figure 18	Trap noise spectrum: Stationary and switched. .	67
Figure 19	Trap noise spectrum: Validation via Monte Carlo simulations.	70
Figure 20	$1/f$ noise spectrum due to a multitude of traps. .	71
Figure 21	A nano-scale transistor operated in common source configuration with a periodically switched input voltage, V_{in}	72
Figure 22	Capture and emission rates of a trap embedded in the realistic transistor model, simulated in a common source configuration.	73
Figure 23	The average trap auto-covariance function: Stationary and switched. Note the magnified y-axes in the two inset plots (light and dark gray backgrounds).	74
Figure 24	Trap noise spectrum: Simulation based on a realistic trap model.	75
Figure 25	Switched input CS amplifier. Dependence of noise reduction on the switching frequency (in comparison to the non switched case). The point corresponding to the setup in Figure 24 is marked with a circle. The simulations for the realistic trap model were conducted using RTM-NAA. . .	76
Figure 26	The “re-appearance of $1/f$ noise” in a switched transistor with 10 traps. The three curves show the PSD of the output voltage at the drain end of a transistor in common source configuration. The capture emission rates of the traps are spread out between 1 sec^{-1} and 10^8 sec^{-1} and the switching frequency is 10^5 Hz . The results for <i>constant input</i> and <i>realistic switched</i> cases were obtained by RTM-NAA simulations.	77
Figure 27	a) Modulation signal for the noise process with a 50% duty cycle. b) Modulation signal for the average ACF.	82
Figure 28	$1/f$ noise reduction in the common source amplifier via switched biasing. Comparison of the legacy modulated stationary model to our fully nonstationary noise model.	84

Figure 29	Stage 1 of a multiple stage coupled sawtooth oscillator. The colored (greyed out) circuit components are responsible for the switched biased operation. The circuit schematic given here is based on the circuit schematics given in [11] and [121].	85
Figure 30	Simulated waveforms of the six stage coupled sawtooth oscillator.	86
Figure 31	Carrier offset noise PSDs for the sawtooth oscillator.	87

LISTINGS

Listing 1	Definition of a class for a non-linear resistor. . .	20
Listing 2	Minimal code for the simulation of a 3-stage ring oscillator.	24
Listing 3	MATLAB [®] code for idealized trap simulation. . .	69

ACRONYMS

ACF	auto-covariance function
AC	alternating current
ADMS	Automatic Device Model Synthesizer
AWGN	additive white Gaussian noise
CIRSIUM	CIRcuit SIMulator Using Matlab
CKE	Chapman-Kolmogorov equation
CMOS	complementary metal-oxide semiconductor
CPU	central processing unit
CS	common source
CTMC	continuous-time Markov chain
DAE	differential-algebraic equation
DC	direct current
DFT	discrete Fourier transform
DRAM	dynamic random-access memory
HDL	hardware description language
ICTMC	inhomogeneous continuous-time Markov chain
IC	integrated circuit
IPP	inhomogeneous Poisson process

KCL	Kirchoff's current law
KVL	Kirchoff's voltage law
LEM	Langevin equation model
LHS	left-hand side
LPTV	linear periodically time-varying
LTi	linear time-invariant
LTV	linear time-varying
MNA	modified nodal analysis
MOSFET	metal-oxide semiconductor field effect transistor
MOS	metal-oxide semiconductor
NAA	noise analysis algorithm
PDF	probability density function
PLL	phase locked loop
PP	Poisson process
PSD	power spectral density
PSP	Phillips surface potential
RF	radio frequency
RTM	realistic trap model
RTS	random telegraph signal
SAM	stochastic automaton model
SDE	stochastic differential equation
SLM	symmetric linearization method
SPICE	Simulation Program with Integrated Circuit Emphasis
SRAM	static random-access memory
SRH	Shockley-Read-Hall
SSA	stochastic simulation algorithm
SUNDIALS	SUite of Nonlinear and Differential/ALgebraic equation Solvers

INTRODUCTION

Low-frequency noise is an inherent part of many physical systems [1]. For electronic devices and circuits, modeling and analysis of low-frequency noise is crucial in order to assure correct circuit operation before moving on to the production stage. With the advancing technology nodes, this has proved to be especially important in nano-scale transistors, since low-frequency random effects such as random telegraph signal (RTS) and $1/f$ noise become more prominent with shrinking device dimensions [2]. However, since the early 1990's, the circuit design community has seen a number of reports on interesting, low-frequency noise related phenomena and the failure of existing models and techniques to accurately predict them [3, 4, 5, 6, 7]. This failure has been attributed to the fact that low-frequency noise is an inherently nonstationary stochastic process [8]. In the particular case of electronic circuit components, low-frequency noise statistics depend on the voltages across the noisy device and hence exhibit a time-varying behavior especially in circuits that do not operate in small-signal mode [9]. Later, noise models were revealed as the culprit for not modeling this nonstationary behavior in a stochastically correct manner [8, 10]. It was realized in 2000's that models based on simply modulating stationary low-frequency noise sources produce inaccurate noise evaluation results for circuits that operate in the large-signal regime [11, 12, 6, 10]. Although a large body of work exists on nonstationary noise analysis of transistors and circuits with time-varying bias conditions [13, 14, 15, 16], these techniques have not been successfully applied to low-frequency noise, yet, due to the lack of a general noise model which is both truly nonstationary, correct and analytically tractable.

In this thesis, we first identify an almost perfect analogy between low-frequency noise in transistors and the so-called ion channel noise in biological nerve cells, and propose a new approach to modeling and analysis of low-frequency noise that is founded on this connection. We offer a solution to the long-standing open problem of correctly modeling low-frequency noise in electronic circuit simulators and present a noise model for the treatment of inherently nonstationary noise phenomena such as RTS, burst and $1/f$ noise. We build our model by capturing the internal dynamics of the individual noise sources inside the transistors and arrive at a representation in which these sources are handled as simple subcircuits with internal pseudo-nodes. This representation permits the utilization of efficient, semi-analytical, non Monte Carlo noise evaluation methods, and, hence, achieves a performance which is orders of magnitude faster than conventional Monte Carlo techniques.

1.1 LOW-FREQUENCY NOISE IN ELECTRONIC CIRCUITS

1.1.1 *Sources of Low-Frequency Noise—Oxide Traps*

Advances brought about by the last two decades of research on metal-oxide semiconductor (MOS) devices has pushed the boundaries of transistor scaling in integrated circuits (ICs) so far that the granular nature of the electrical current has become a factor to be accounted for in designing advanced electronic circuits. This granularity, due to the fact that electrical current is carried by discrete charged particles, manifests itself most prominently in the noise behavior of nano-scale metal-oxide semiconductor field effect transistors (MOSFETs) [17]. The trapping of single electrons and holes by the irregularities and defects (generally called traps) in the lattice structure of the gate-oxides of MOSFETs is widely accepted to be responsible for diverse low-frequency noise phenomena [18, 19, 2, 20].

Oxide traps capture/emit charge carriers from/to the conduction paths in a semiconductor and cause fluctuations in the current through the device in a random manner [21]. The trap occupancy function, indicating whether the trap is full or not, is usually modeled as a two-level RTS [22]. Hence, the noise due to the random trapping of charge carriers in semiconductors and transistors is usually referred to as RTS noise [6]. Moreover, the combined effects of a large number of traps is believed to be the cause of more complicated low-frequency noise phenomena, such as $1/f$ noise. Since the seminal paper of Kirton and Uren [9], RTS noise has received a great deal of attention. The interest in this type of noise has been increasing with shrinking device dimensions due to aggressive technology scaling. This is because the effects of individual defects are more prominent in smaller devices [2]. It were the physical properties and mathematical modeling of these traps that attracted the attention of the early researchers on the subject [9]. After it was demonstrated by multiple studies starting in the early 1990's that the trap noise can be reduced by *switching* the circuits on and off periodically, it was realized that having a stochastically correct model of trapping events was of prime importance [3, 4, 5, 11, 23].

1.1.2 *RTS Noise in Transistors and Electronic Circuits*

As explained above, defects or traps in the crystal structure of semiconductors can randomly capture and emit charge carriers, resulting in fluctuations in the number of mobile charge carriers, which can subsequently cause noise in electric fields, currents and voltages. The disturbing influence of the random fluctuations caused by oxide traps was at first a major concern in the design of analog circuits used in radio frequency (RF) applications (low-noise amplifiers, mixers, oscillators, phase locked loops (PLLs), etc.) and complementary metal-oxide

semiconductor (CMOS) image sensors [12, 8, 24]. However, due to the nano-scale device sizes in the latest technology nodes, RTS noise has become a limiting factor in the design of digital circuits, such as SRAMs and DRAMs, as well [25, 26]. Since these digital circuits and analog circuits such as oscillators and mixers do not operate in the small-signal regime, results produced by simple stationary (or modulated stationary) low-frequency noise models are inaccurate due to the bias voltage dependent capture/emission rates of traps [27]. The goal of this work is to introduce novel models and numerical techniques that can be used to conduct analyses of both the impact of individual traps and also the collective effect of multiple traps on circuit operation, enabling the analysis of diverse low-frequency noise phenomena.

1.1.3 *Modeling and Analysis of Low-Frequency Noise in Circuits*

Modeling of low-frequency and $1/f$ noise in MOSFETs, a subject with rampant speculations and a great deal of controversy, has been a preoccupation for researchers for many decades [1, 28, 29, 30, 31, 32, 13, 33]. However, almost all of this effort was directed at modeling low-frequency noise in transistors at a constant bias point, with *stationary* statistics for the noise source. Stationary, frequency domain models for $1/f$ noise was incorporated into circuit simulators such as SPICE in the very early days [34, 35] and frequency domain, AC noise analysis has been a workhorse tool in analog circuit design since then.

With the explosive growth of wireless mobile communications in the 90's, circuits for RF applications such as mixers, oscillators and PLLs have become an essential part of electronic systems and consumer electronic devices. Analysis of RF circuits posed a great challenge to circuit simulators due to their highly nonlinear nature and the wide frequency range of operation. Since these circuits did not operate in the small-signal regime, noise models for transistors based on quiescent conditions and frequency domain AC noise analysis based on linear time-invariant (LTI) system theory and stationary stochastic processes were deemed not applicable [13]. As a result, a great deal of work was done in the 90's in developing nonstationary noise models for transistors and noise analysis techniques that can handle time-varying bias conditions [13, 14, 36, 37, 15, 38, 16]. This work resulted in specialized circuit noise analysis techniques that were then implemented in RF circuit simulators. However, the modeling of (nonstationary) low-frequency and $1/f$ noise under time-varying bias conditions has remained as an open problem [13, 39, 40, 41]. Just when analog/RF electronic circuit designers started using the newly developed RF circuit noise analysis techniques, researchers reported that the traditional noise models mispredicted the impact of low-frequency noise on the performance of oscillators and CMOS image sensor circuitry under strongly time-varying bias conditions, see e.g. [11, 12, 8, 6]. Subsequently, starting with the seminal

paper by Tian *et al.* [8], substantial amount of work has been done in developing correct, truly nonstationary low-frequency noise models, see e.g. [12, 8, 6, 27, 10]. However, it seems that the results of this work have not been incorporated, to date, into previously developed circuit noise analysis techniques and circuit simulators.

1.2 NOISE IN THE NERVOUS SYSTEM

1.2.1 *Membrane Ion Channel Noise in Nerve Cells*

Nerve cells (also known as neurons), special types of cells that make up the brain and the nervous system, process and transmit information based on electrical and chemical signaling. Neurons are able to maintain a voltage difference between the cell interior and exterior by pumping charged ions (such as sodium and potassium) through *ion channels* in their membranes and hence creating an ion concentration difference. Ion channels are made up of proteins that can open and close and hence allow or block the passage of ions. The state of some ion channels, so-called *voltage-gated* channels, is a function of the (trans) membrane voltage. Other so-called *ligand-gated* channels are controlled by the binding of a substance. The operation and dynamical behavior of neuron ion channels and currents and the membrane voltage was first laid out by Hodgkin and Huxley in 1952 [42]. It was later realized that individual ion channels behave stochastically, by closing and opening at random times [43], but not in a totally arbitrary manner. For voltage-gated channels, the rates at which they close and open is a function of the membrane voltage. The stochastic ion channel behavior has been identified as a source of electrical *channel noise* and fluctuations in the membrane voltage [43]. Channel noise in neurons is believed to both influence the reliability of nerve cell responses in a detrimental manner but at the same time enable a “broader repertoire of neuronal behavior” [43]. Ion channel noise is claimed to be one of the determining factors in the size of a brain [44].

1.2.2 *Modeling and Analysis of Noise in the Nervous System*

The deterministic dynamical behavior of neurons has long been analyzed based on the celebrated Hodgkin-Huxley equations, a small set of nonlinear differential equations describing the dynamics of the membrane voltage and the ion channels, commonly interpreted as an electronic circuit composed of a nonlinear capacitor and nonlinear, voltage-controlled conductances [42, 45, 46]. Modeling of ion channel noise in neurons is typically done by a stochastic modification/extension of the Hodgkin-Huxley equations [47, 48, 49, 50, 51, 45, 52, 43]. The so-called exact method is based on modeling every ion channel as a two-state, continuous-time, discrete-space Markov chain, resulting in

a stochastic automaton model [45]. Then, the differential equations for the membrane voltage from the Hodgkin-Huxley model and the Markov chains for the individual ion channels are simulated together in a Monte Carlo manner using a hybrid variant of the stochastic simulation algorithm (SSA) [53, 54, 55, 56]. SSA was first proposed as a Monte Carlo algorithm for simulating coupled chemical reactions, where a molecule count based discrete-space model is used to represent the amount of chemical reactants. In hybrid variants of SSA [56], a continuous-space, differential equation model is used for some parts of the system while the rest is modeled in discrete-space. While the simulation algorithm keeps track of (i.e., generates a sample path for) the states of all of the ion channels, the differential equations for the membrane voltage are integrated with an appropriate numerical technique where the channel conductances are determined by the states of the ion channels. Since this is a computationally costly algorithm, simpler variants which keep track not of the states of every ion channel but only the total number of channels in each state were also used [43]. This type of hybrid Monte Carlo simulation was deemed to be still too expensive and approximate models where channel noise is represented by continuous-space, stochastic differential equations (SDEs), i.e., Langevin equations [57], were then proposed [45, 52, 43]. This Langevin approach essentially amounts to introducing stochastic noise terms into the original Hodgkin-Huxley equations, resulting in a system of nonlinear SDEs, which are then solved numerically with an appropriate technique, again in a Monte Carlo manner to generate sample paths for the noisy membrane voltage [45, 52]. Currently, there seems to be an unsettled controversy in the channel noise modeling literature [48, 49, 50, 51, 45, 52, 43] on the accuracy of the continuous Langevin channel noise model against the discrete Markov model.

1.3 A SYNERGISTIC APPROACH FOR MODELING AND ANALYSIS

1.3.1 *A Perfect Analogy*

Given the description of the charge carrier traps in semiconductors and transistors and the ion channels in nerve cell membranes, the following analogy can be formed between them. Both traps and ion channels behave stochastically and are in one of two states, empty/full for traps and open/closed for ion channels. The rates at which traps capture and emit charge carriers are in fact a function of the voltages across the MOSFET [8, 6], very similar to the case of voltage-gated ion channels. Stochastic behavior of ion channels results in a noisy ion channel current which subsequently causes noise in the membrane voltage. Similarly, the “channel” current in a MOSFET becomes noisy due to the random emission and capture of charge carriers by the traps which then results in

noisy voltages. Furthermore, both traps in transistors and ion channels in neurons behave independently of each other in a stochastic sense, but the transition rates of all of the ion channels in a particular nerve cell membrane and the traps in a particular transistor are a function of the same voltage, the membrane voltage in one case and the voltage applied across the transistor in the other. Even though the traps and ion channels are stochastically independent of each other, the same voltage that controls the transition rates creates a sort of global coupling [45].

As electronic circuit models are commonly used for neurons and neuronal networks, the almost perfect and complete analogy described above becomes very natural and can be extended to encompass the noisy behavior of networks of neurons on one side and networks of transistors on the other. A neuron is the most basic building block of a biological neuronal network or the nervous system, just like the transistor for a microelectronic system. It turns out that exactly the same mathematical model, i.e., an asymmetrical two-level RTS signal corresponding to a simple two-state Markov chain, has been used in modeling the stochastic behavior of both traps [8, 6] and ion channels [43]. This analogous nature of traps in transistors and ion channels in neurons was also identified in [58]. The voltage dependence of transition rates in the RTS model in both cases is the source of intricate nonstationary behavior, which makes the modeling and analysis of the resulting noise and its impact on the system performance a great challenge.

1.3.2 *Approach to Modeling and Analysis of Low-Frequency Noise*

Based on the similarities in noise modeling and analysis techniques in cellular biology, biochemistry, neurobiology and electronic design, a synergistic approach to noise modeling and analysis problems in these domains is not only warranted but also extremely beneficial due to complementary strengths. Quantitative and computational approaches to noise modeling and simulation that have been developed in molecular and neurobiology are mathematically sophisticated, rigorous and theoretically well-founded. On the other hand, the noise analysis techniques developed in these disciplines are usually of the stochastic, Monte Carlo simulation type, which usually become quite inefficient in analyzing large and complex systems. In the noise modeling and analysis work that has been done in electrical engineering during the past five decades, the emphasis has mostly been on developing (semi) analytical, non Monte Carlo simulation methods based on modern and efficient numerical techniques that scale well when analyzing large and complex systems. In this thesis, we adopt a synergistic approach in developing noise modeling and analysis techniques for RTS and low-frequency noise in transistors and electronic circuits based on techniques for modeling ion channel noise in neurons and neuronal networks and, in general, for noise in the nervous system.

1.4 SUMMARY OF CONTRIBUTIONS AND OUTLINE OF THE THESIS

We begin our treatment by laying out the mathematical background of our modeling technique for low-frequency noise. Using this theoretical background, we arrive at stochastic descriptions of oxide traps in transistors which immediately lend themselves to be used in conjunction with Monte Carlo type noise analysis methods. We conduct a detailed investigation of the stochastic properties of our new noise source descriptions and the Monte Carlo noise analysis algorithms associated with them. These Monte Carlo type analysis methods constitute the basis of our low-frequency noise modeling and analysis framework. However, as all Monte Carlo methods, they have a high computational cost which renders them impractical for many real-life applications. Therefore, we use them as a theoretical tool and a stepping stone towards our final goal of developing a low-frequency noise model which can be used in conjunction with fast and robust non Monte Carlo noise analysis techniques.

In Chapter 2, we derive two fully nonstationary probabilistic descriptions for traps, a *fine-grained* Markov chain model based on recent previous work and a completely novel *coarse-grained* Langevin model based on similar models for ion channels in neurons. The nonstationary trap descriptions we derive subsume and unify all of the work that has been done recently in the device modeling and circuit design literature on modeling nonstationary trap noise.

Chapter 3 is devoted to the presentation of CIRSIUM, a CIRcuit SIMulator Using MATLAB[®] and its object oriented programming language. CIRSIUM has been developed as a flexible and modular simulation tool in order to enable the rapid development of new device models and prototyping of new circuit analysis algorithms proposed in this thesis. We also present a detailed physical description of oxide traps in transistors and the calculation of their voltage and position dependent capture/emission time constants. We expand the industry standard PSP compact MOSFET model with this physical description and implement it into CIRSIUM.

In Chapter 4, we provide a detailed explication of the low-frequency noise source descriptions developed in Chapter 2 with regard to their stochastic properties and develop carefully crafted circuit simulation techniques of Monte Carlo type that are stochastically correct. We implement these simulation techniques in CIRSIUM and present simulation results for the low-frequency noise characterization of a common source amplifier and the phase jitter of a ring oscillator. After demonstrating the correctness of our noise modeling methods on these benchmark circuits, we proceed, to the theoretical elucidation of our nonstationary, non Monte Carlo low-frequency noise model in Chapter 5. In one embodiment of non Monte Carlo noise analysis techniques that we present in this thesis, we show how our noise model can be used

to perform frequency-domain, nonstationary, non Monte Carlo, semi-analytical noise evaluation for circuits under periodic large-signal excitations and autonomous oscillators. We verify that the noise spectral densities computed with the new non Monte Carlo noise model match the ones obtained via spectral estimation from time-domain Monte Carlo noise simulation data in Chapter 4.

In the remaining part of this work, we present applications of the new noise model we propose to various noise analysis problems. In particular, in Chapter 6, we present a detailed analysis of noise in switched MOSFET circuits, i.e., circuits that are periodically switched on and off in order to reduce the effects of low-frequency noise. We build our analysis on the seminal work of Tian *et al.* published in 2001 [8] where an idealized, nonstationary trap model was proposed. In Chapter 6, we first identify an oversight in their model derivation which had produced an incorrect expression for the single trap noise spectrum. We next derive the correct spectrum expression, verify it against detailed idealized trap simulations and discuss its implications. The idealized trap model is amenable to analytical derivations and very useful as a first stage in understanding nonstationary trap noise but it breaks down in real-life situations where its idealization assumptions are not satisfied. We then demonstrate that noise simulations based on our physics based, nonstationary low-frequency noise model are needed for an accurate characterization of low-frequency noise in switched MOSFET circuits that matches experimental results.

Finally, in Chapter 7, we contrast our nonstationary noise model with the traditional low-frequency noise model. We present a detailed analysis of the limitations of the legacy model which operates on the basis of modulating noise sources with stationary statistics in order to model nonstationary characteristics of low-frequency noise. As a case study, we examine the $1/f$ noise behavior in the coupled sawtooth oscillator and show that, whereas simulations conducted with the legacy modulated stationary model suggest that no noise reduction is obtainable with switched biasing, our nonstationary noise model predicts the correct amount of noise reduction observed in previously published experimental data.

NONSTATIONARY STOCHASTIC DESCRIPTION OF LOW-FREQUENCY NOISE SOURCES

2.1 INTRODUCTION

In this chapter, we develop stochastic, nonstationary models for charge carrier traps in semiconductors, i.e., RTS noise, by benefiting from the two decades of ion channel noise modeling and simulation work in neurobiology [43] and the extensive literature on modeling and simulation of stochastic chemical kinetics [54, 55]. In particular, we develop two models for a charge carrier trap, that are both fully nonstationary capturing the impact of arbitrary time-varying bias conditions:

- (i) Stochastic automaton model (SAM) in Section 2.3 – a continuous-time, discrete-space, fine-grained Markov chain model (previously known [26]),
- (ii) Langevin equation model (LEM) in Section 2.4 – a continuous-time, continuous-space, coarse-grained SDE model (completely new).

The Langevin equation model we develop, even though approximate and coarse-grained, correctly and *exactly* captures the first (mean behavior) and second-order (autocorrelation and spectral properties) probabilistic characteristics of a trap. In most, but possibly not all, applications in electronic design, such a probabilistic characterization is adequate.

2.2 PRELIMINARIES

Let $N(t)$ be the *occupancy function* for a certain trap, i.e., $N(t) = 1$ if the trap is full and $N(t) = 0$ if empty. $N(t)$ is modeled as a two-state, continuous-time Markov chain that is characterized by the capture rate $\lambda_c(t)$ and the emission rate $\lambda_e(t)$. The time dependence of the capture and emission rates stem from the fact that these rates depend on the voltages across the MOSFET which can considerably vary with time during large-signal operation. For notational simplicity, we do not express the capture/emission rates as an explicit function of these voltages but simply denote their time dependence. As a result, the trap occupancy function $N(t)$ must be regarded as an *inhomogeneous* Markov chain. Moreover, due to noise in the circuit, the voltages across the MOSFET must be treated as noisy signals and hence represented as stochastic

The content of this chapter has previously appeared in [59].

processes. This means that the capture/emission rates are also stochastic, making $N(t)$ a *doubly stochastic* Markov chain [60]. We will deal with the doubly stochastic nature of $N(t)$ and its implications later in our treatment.

2.3 CHAPMAN-KOLMOGOROV EQUATION FOR A TRAP

Let $\mathbb{P}[n, t]$ denote the probability that $N(t) = n$, where n is either 0 (empty) or 1 (full). Given that we know the probability that the trap is in any one of the possible states at time t , we will derive an equation for the trap to be in a certain state n at time $t + dt$, where dt is assumed to be small enough so that at most one capture or emission event can occur in the time interval $[t, t + dt)$ [55]. For the trap to be in state n at time $t + dt$, either the trap was already in state n at time t and no event has occurred in $[t, t + dt)$, or the trap was in some other state n' at time t and an event has occurred in $[t, t + dt)$ which caused the state to change from n' to n . Let A be the event that the trap is in state n at time $t + dt$. Let $H_\emptyset$, H_c and H_e be the events that the trap is in state n , $n - 1$ and $n + 1$ (at time t) respectively. In this case, the conditional probabilities $\mathbb{P}[A|H_c]$ and $\mathbb{P}[A|H_e]$ are in fact the probabilities of the capture and emission events occurring in $[t, t + dt)$ [55]. Due to the Markov property and based on the capture and emission rates $\lambda_c(t)$ and $\lambda_e(t)$ [61], these conditional probabilities can be expressed as follows

$$\mathbb{P}[A|H_c] = \lambda_c(t) dt \quad (1)$$

$$\mathbb{P}[A|H_e] = \lambda_e(t) dt \quad (2)$$

The conditional probability $\mathbb{P}[A|H_\emptyset]$, i.e., the probability that no event occurred in $[t, t + dt)$ is given by

$$\mathbb{P}[A|H_\emptyset] = 1 - [(1 - n) \lambda_c(t) dt + n \lambda_e(t) dt], \quad (3)$$

where the factors n and $(1 - n)$ are required in order for the expression to be correct. Above, the probability that a capture occurs when the trap is in state n is set as $(1 - n) \lambda_c(t) dt$. A capture event has a nonzero probability only if the trap is empty, i.e., if $n = 0$. If the trap is full, i.e., if $n = 1$, then this expression correctly yields zero capture probability. Similarly, the probability that an emission occurs when the trap is in state n is set as $n \lambda_e(t) dt$. And hence, the probability that no event occurs in $[t, t + dt)$ is given by (3).

Since the events $H_\emptyset$, H_c and H_e are disjoint and at least one of them must happen, the law of total probability [55] gives us

$$\mathbb{P}[A] = \mathbb{P}[A|H_\emptyset] \mathbb{P}[H_\emptyset] + \mathbb{P}[A|H_c] \mathbb{P}[H_c] + \mathbb{P}[A|H_e] \mathbb{P}[H_e]. \quad (4)$$

If we substitute (1), (2) and (3) in (4), we obtain

$$\begin{aligned} \mathbb{P}[n, t + dt] = & [1 - ((1 - n) \lambda_c(t) dt + n \lambda_e(t) dt)] \mathbb{P}[n, t] + \\ & \lambda_c(t) dt \mathbb{P}[n - 1, t] + \lambda_e(t) dt \mathbb{P}[n + 1, t]. \end{aligned} \quad (5)$$

We rearrange the terms above

$$\frac{\mathbb{P}[n, t + dt] - \mathbb{P}[n, t]}{dt} = -[(1 - n)\lambda_c(t) + n\lambda_e(t)]\mathbb{P}[n, t] + \lambda_c(t)\mathbb{P}[n - 1, t] + \lambda_e(t)\mathbb{P}[n + 1, t] \quad (6)$$

and take the limit as $dt \rightarrow 0$ to arrive at

$$\frac{d\mathbb{P}[n, t]}{dt} = -[(1 - n)\lambda_c(t) + n\lambda_e(t)]\mathbb{P}[n, t] + \lambda_c(t)\mathbb{P}[n - 1, t] + \lambda_e(t)\mathbb{P}[n + 1, t] \quad (7)$$

The above, called the Chapman-Kolmogorov equation (CKE) [61] in the theory of stochastic processes, known as the Master equation [57] in physics, provides the ultimate, complete stochastic characterization for the two-state Markov chain model for the trap under consideration. If we evaluate (7) for the two states $n = 0$ and $n = 1$, keeping in mind that $\mathbb{P}[n, t]$ is zero for all other n , we obtain

$$\frac{d\mathbb{P}[0, t]}{dt} = -\lambda_c(t)\mathbb{P}[0, t] + \lambda_e(t)\mathbb{P}[1, t] \quad (8)$$

$$\frac{d\mathbb{P}[1, t]}{dt} = -\lambda_e(t)\mathbb{P}[1, t] + \lambda_c(t)\mathbb{P}[0, t] \quad (9)$$

which is a system of two simple linear differential equations with time-varying coefficients that can be easily solved for the state probabilities $\mathbb{P}[0, t]$ and $\mathbb{P}[1, t]$ if the time-varying capture and emission rates $\lambda_c(t)$ and $\lambda_e(t)$ are available. We note that (8) and (9) yield

$$\frac{d}{dt}(\mathbb{P}[0, t] + \mathbb{P}[1, t]) = 0 \quad (10)$$

which also follows from the fact that the total probability of being in state 0 or 1 is always 1, i.e.,

$$\mathbb{P}[0, t] + \mathbb{P}[1, t] = 1. \quad (11)$$

2.4 STOCHASTIC DIFFERENTIAL EQUATION FOR A TRAP

The Markov chain model for a trap described above can be regarded as a continuous-time, discrete-space stochastic model. As we discuss later, this model could become too costly (computationally) when a large number of traps and a circuit are jointly analyzed. We now derive an approximate, coarse-grained, continuous-time, continuous-state stochastic model for a trap in the form of an SDE [57], which can offer considerable computational cost savings in the joint analysis of a circuit and a large number of traps. We present two different routes for the derivation which produce the same result. The first approach uses a Gaussian approximation to the trap state random variables [53] whereas the second approach uses the so-called Fokker-Planck equation [62].

2.4.1 Gaussian Approximation Route

We consider a time interval $[t, t + \tau)$ and assume that the capture and emission rates $\lambda_c(t)$ and $\lambda_e(t)$ are constant over this interval. Then, the trap occupancy function $N(t)$ can be (approximately) expressed as follows

$$N(t + \tau) \approx N(t) - \mathcal{P}_e(N(t), \tau) + \mathcal{P}_c(N(t), \tau), \quad (12)$$

where $\mathcal{P}_e(N(t), \tau)$ and $\mathcal{P}_c(N(t), \tau)$ represent the (random) number of emission and capture events that occur in the time interval $[t, t + \tau)$. With the above approximation, which is known as the *tau-leaping* condition [54, 55], we are allowing a multiple number of emission and capture events in $[t, t + \tau)$. As such, the resulting $N(t + \tau)$ is not guaranteed to be 0 or 1, even if $N(t)$ satisfies this condition. This is due to the approximate nature of the leap condition that led us to (12). In order to make (12) more concrete, we need to precisely characterize the random variables $\mathcal{P}_e(N(t), \tau)$ and $\mathcal{P}_c(N(t), \tau)$. It follows that these random variables can be approximated as simple Poisson (counting) random variables with parameters $N(t) \lambda_e(t) \tau$ and $(1 - N(t)) \lambda_c(t) \tau$ respectively. The factors $N(t)$ and $(1 - N(t))$ have been introduced into these expressions based on similar reasoning as for the ones in (3). However, with the continuous approximation we are developing here, $N(t)$ will take all values between 0 or 1 (and possibly outside), perhaps representing a *fractionally full* trap. As such, with the $N(t)$ factor appearing in the parameter for the Poisson random variable $\mathcal{P}_e(N(t), \tau)$ that represents the number of emission events, having i emissions in $[t, t + \tau)$ will have a probability of $e^{-N(t)\lambda_e(t)\tau} N(t)\lambda_e(t)\tau / i!$. When $N(t) = 0$, number of emissions will be zero with probability one, whereas for larger values of $N(t)$, higher number of emissions will become more probable.

The approximate equation in (12) can in fact be directly used in simulating (in a Monte Carlo manner) a trap with an appropriate choice (most likely, adaptive) for the time step τ [54, 55]. However, (12) is not yet a truly continuous-space model in a differential equation form. $N(t)$ in (12) is still integer valued, though not restricted to be 0 or 1 anymore. We proceed further as follows. The Poisson random variables $\mathcal{P}_e(N(t), \tau)$ and $\mathcal{P}_c(N(t), \tau)$ have means (and also variances) equal to $N(t) \lambda_e(t) \tau$ and $(1 - N(t)) \lambda_c(t) \tau$ respectively. The increment term in (12) is composed of the difference of two Poisson random variables which has a probability distribution known as the Skellam distribution [63]. This distribution has as its mean, the difference of the means of the Poisson random variables and as its variance, the sum of them. For large mean values, the Skellam distribution can be approximated by a Gaussian distribution with the same mean and variance. We approximate

$$N(t + \tau) \approx N(t) + (1 - N(t))\lambda_c(t)\tau - N(t)\lambda_e(t)\tau + \sqrt{(1 - N(t))\lambda_c(t)\tau + N(t)\lambda_e(t)\tau} \mathcal{N}(0, 1), \quad (13)$$

where $\mathcal{N}(0, 1)$ is a zero-mean, unit-variance Gaussian random variable. With the Gaussian approximation, $N(t)$ is now continuous valued. Next, we recognize (13) above as the Euler discretization of the following SDE

$$dN_t = \left[(1-N_t)\lambda_c(t) - N_t\lambda_e(t) \right] dt + \sqrt{(1-N_t)\lambda_c(t) + N_t\lambda_e(t)} dW_t \quad (14)$$

where W_t is the Wiener process [61]. The subscript “ t ” in N_t and W_t is used to indicate that these quantities are continuous-time, continuous-state stochastic processes. In physics, an SDE in the form above is known as a *Langevin equation* and a Wiener process as a Brownian motion. The differential of the Wiener processes, dW_t in (14), is simply a white noise process. SDEs are commonly expressed in differential form as in (14) with differentials of Wiener processes as opposed to a regular differential equation form with white noise forcing, since white noise is, mathematically, not well defined.

We note that the noise source (the Wiener process) in (14) is modulated with a state (N_t) dependent term. Hence, this Langevin equation serves as a nonstationary stochastic, continuous-time, continuous-space differential equation model for a trap which we call LEM. It can be solved numerically (in a Monte Carlo manner) using the Euler discretization in (13). It can also be used as an analytical tool in performing (semi) analytical joint analysis for a circuit and a number of traps. Even though the LEM was derived after a number of approximations starting with the Markov chain model and the CKE in (7), it can be used to produce the correct (and exact) first and second-order probabilistic characteristics for a trap, exactly the same as one would have obtained based on the CKE, which we show in Section 2.5. We note that in the doubly stochastic case, where there is also a coupling from the trap states to the event rates (λ_e , λ_c) the above statement no longer holds and the first and second-order statistics can only be approximated. Here, a first and second-order characterization refers to the mean behavior and all of the second-order moments, i.e., variance, autocorrelation function and spectral density.

2.4.2 Fokker-Planck Equation Route

Because the discrete valued trap state, n , assumes the role of a parameter in (7) which can only be either 1 or 0, the Chapman-Kolmogorov equation (7) can also be written as

$$\begin{aligned} \frac{d\mathbb{P}[n, t]}{dt} = & - [(1-n)\lambda_c(t) + n\lambda_e(t)] \mathbb{P}[n, t] \\ & + [1 - (n-1)]\lambda_c(t) \mathbb{P}[n-1, t] \\ & + (n+1)\lambda_e(t) \mathbb{P}[n+1, t] . \end{aligned} \quad (15)$$

We can now make the crucial move towards building the continuous state model for the traps here and assume that Equation (15) is also

valid for values of n other than zero or one. This way, we can treat n as a second, continuous valued variable and transform the CKE into a partial differential equation for the time-varying probability density function (PDF) $P(n, t)$. We note that this step involves no approximations. We subject Equation (15) only to a conceptual shift by letting the trap state vary freely between and outside of $\{0, 1\}$. However, we then use a second order *Taylor approximation* for the second and third summands in (15) around n and arrive at the so-called Fokker-Planck equation:

$$\begin{aligned} \frac{\partial P}{\partial t} = & (\lambda_e + \lambda_c)P + \left[(n+1)\lambda_e - (2-n)\lambda_c \right] \frac{\partial P}{\partial n} \\ & + \frac{1}{2} \left[n\lambda_e + (1-n)\lambda_c \right] \frac{\partial^2 P}{\partial n^2} . \end{aligned} \quad (16)$$

In (16), the explicit time dependence of the capture and emission rates was dropped for the sake of brevity.

The Fokker-Planck equation corresponds to a stochastic process N_t which has the time dependent PDF $P(n, t)$, i.e., the solution of (16). The subscript t is used to signify that N_t is a continuous time, continuous state stochastic process. We now show that this stochastic process can be described by the Itô SDE [64]

$$dN_t = \mu(N_t, t) dt + \sqrt{\nu(N_t, t)} dW_t . \quad (17)$$

Here, $\mu(N_t, t)$ and $\sqrt{\nu(N_t, t)}$ are the time-varying *drift* and *diffusion* coefficients, respectively, and W_t is the Wiener process. The corresponding Fokker-Planck equation to (17) is given by [64]

$$\begin{aligned} \frac{\partial P}{\partial t} = & -\frac{\partial}{\partial n} (\mu P) + \frac{1}{2} \frac{\partial^2}{\partial n^2} (\nu P) \\ = & \left(\frac{1}{2} \frac{\partial^2 \nu}{\partial n^2} - \frac{\partial \mu}{\partial n} \right) P + \left(\frac{\partial \nu}{\partial n} - \mu \right) \frac{\partial P}{\partial n} + \left(\frac{1}{2} \nu \right) \frac{\partial^2 P}{\partial n^2} . \end{aligned} \quad (18)$$

Equating (16) and (18), we can obtain the drift and diffusion parts for the SDE of a trap as

$$\begin{aligned} \mu(N_t, t) = & (1 - N_t)\lambda_c(t) - N_t\lambda_e(t) , \\ \nu(N_t, t) = & (1 - N_t)\lambda_c(t) + N_t\lambda_e(t) . \end{aligned} \quad (19)$$

Finally, we can write down the Langevin equation for the trap state:

$$dN_t = \left[(1 - N_t)\lambda_c(t) - N_t\lambda_e(t) \right] dt + \sqrt{(1 - N_t)\lambda_c(t) + N_t\lambda_e(t)} dW_t \quad (20)$$

This is the same equation as (14) and it constitutes the basis for the LEM for oxide traps.

2.5 FIRST AND SECOND-ORDER STATISTICS OF A TRAP

2.5.1 Mean Behavior of a Trap

If we simply drop the stochastic or “noise” terms in (14) and express it in differential equation form, we arrive at the following deterministic, continuous-time, continuous-space model for a trap

$$\frac{dN(t)}{dt} = \lambda_c(t) - [\lambda_c(t) + \lambda_e(t)]N(t) \quad (21)$$

We next determine the mean behavior of the trap occupancy function $N(t)$ based on the CKE in (7). The mean of $N(t)$ is given by

$$\mathbb{E}[N(t)] = \sum_{n=0}^1 n \mathbb{P}[n, t] = \mathbb{P}[1, t] \quad (22)$$

Hence, we use (11) in (9) and obtain

$$\frac{d\mathbb{E}[N(t)]}{dt} = \lambda_c(t) - (\lambda_c(t) + \lambda_e(t)) \mathbb{E}[N(t)] \quad (23)$$

We observe that the deterministic equation (21) derived from the LEM in (14), and (23) for mean $N(t)$ derived from the CKE in (7) are identical, despite the fact that LEM was derived employing a number of approximations. We note here that if we simply compute the expectation of both sides of the SDE in (14), we obtain (23) exactly. The mean behavior of a trap is thus precisely captured by the SDE model in (14).

2.5.2 Second-Order Moment for a Trap

A differential equation for the second-order moment of the trap occupancy function $N(t)$, i.e., $\mathbb{E}[N^2(t)]$, can be derived based on the SDE in (14) using stochastic (Itô) calculus [57]. We skip the details of this somewhat involved derivation but present the result:

$$\begin{aligned} \frac{d\mathbb{E}[N^2(t)]}{dt} = & \lambda_c(t) + (\lambda_c(t) + \lambda_e(t)) \mathbb{E}[N(t)] \\ & - 2(\lambda_c(t) + \lambda_e(t)) \mathbb{E}[N^2(t)] \end{aligned} \quad (24)$$

The above equation also involves the first-order moment, i.e., the mean $\mathbb{E}[N(t)]$, which satisfies (23). One can easily deduce that the solutions of (23) and (24) must satisfy $\mathbb{E}[N^2(t)] = \mathbb{E}[N(t)]$ provided that an initial condition satisfying the same condition is given. Thus, based on the SDE model in (14), we conclude that the mean and the second-order moment of the trap occupancy function are equal to each other. This result is in fact consistent with the CKE model in (7). With the discrete-space Markov chain model, $N(t)$ is a random process that can take only two values, 0 or 1. For any such random variable, the mean and the second-order moment are both equal to $\mathbb{P}[N(t) = 1] = \mathbb{P}[1, t]$.

2.5.3 Auto-correlation Function for a Trap

Equations for the autocorrelation function of a trap can be easily derived based on the Markov chain model and the CKE as follows:

$$\begin{aligned} R_N(t, \tau) &= \mathbb{E} [N(t) N(t + \tau)] \\ &= \mathbb{P} [N(t) = 1 \wedge N(t + \tau) = 1] \\ &= \mathbb{P} [N(t + \tau) = 1 | N(t) = 1] \mathbb{P} [N(t) = 1] \end{aligned} \quad (25)$$

$\mathbb{P} [N(t) = 1] = \mathbb{P} [1, t]$ and $\mathbb{P} [N(t + \tau) = 1 | N(t) = 1]$ can be obtained by solving (8) and (9) with appropriate initial conditions.

Based on the Langevin model, one can also derive equations for $R_N(t, \tau) = \mathbb{E} [N(t) N(t + \tau)]$. Through simple manipulations based on stochastic Itô calculus, we obtain

$$\frac{dR_N(t, \tau)}{d\tau} = \lambda_c(t) \mathbb{E} [N(t)] - (\lambda_c(t) + \lambda_e(t)) R_N(t, \tau) \quad (26)$$

In order to compute $R_N(t, \tau)$ using the above, we first compute $\mathbb{E} [N(t)]$ and $R_N(t, t) = \mathbb{E} [N^2(t)]$ by solving (23) and (24). We then use $R_N(t, t) = \mathbb{E} [N^2(t)]$ as an initial condition for (26) in computing $R_N(t, \tau)$ as a function of τ .

2.5.4 Spectral Density for a Trap

We now compute the spectral density of the trap occupancy function $N(t)$ based on the SDE model, when the capture and emission rates are time-invariant ($\lambda_c(t) = \lambda_c, \lambda_e(t) = \lambda_e$) and the trap is in stationary steady-state. We first compute the steady-state mean $\mathbb{E} [N(t)]$ by setting the LHS of (23) to zero and obtain

$$\mathbb{E} [N(t)] = \frac{\lambda_c}{\lambda_c + \lambda_e} = N_{ss} \quad (27)$$

Then, we define the fluctuation around the steady-state mean by

$$\tilde{N}(t) = N(t) - N_{ss} \quad (28)$$

We substitute $N(t) = N_{ss} + \tilde{N}(t)$ in (14) and approximate $N(t) \approx N_{ss}$ for the noise term and arrive at

$$d\tilde{N}(t) = -(\lambda_c + \lambda_e) \tilde{N}(t)dt - \sqrt{\alpha} dW_t \quad (29)$$

where

$$\alpha = \frac{2\lambda_c \lambda_e}{\lambda_c + \lambda_e} \quad (30)$$

Equation (29) is a constant-coefficient, linear SDE and hence represents an LTI system with two independent noise source inputs. The spectral density of $\tilde{N}(t)$ can be easily computed via

$$S_{\tilde{N}}(f) = |H(f)|^2 S(f) \quad (31)$$

where $S(f) = 1$ is the spectral density of the white noise source dW_t , and

$$H(f) = \frac{\sqrt{\frac{2\lambda_c \lambda_e}{\lambda_c + \lambda_e}}}{j2\pi f + (\lambda_c + \lambda_e)} \quad (32)$$

is the transfer function from the noise source to the “output” $\tilde{N}(t)$. By substituting (32) into (31) we obtain

$$S_{\tilde{N}}(f) = \frac{2 \frac{\lambda_c \lambda_e}{\lambda_c + \lambda_e}}{(2\pi f)^2 + (\lambda_c + \lambda_e)^2} \quad (33)$$

We have derived the above spectral density based on the approximate SDE model in (14). However, one can show that one obtains exactly the same spectral density expression based on the CKE model in (7) that does not involve any approximations. Spectral density, just like the second-order moment discussed above, constitutes a second-order probabilistic characterization and can be obtained from the SDE model directly. Finally, we note that even though we discussed the stationary case with time-invariant capture and emission rates, the SDE model in (14) captures the full nonstationary behavior of a trap and will produce the correct second-order probabilistic characterization for arbitrary time-varying capture and emission rates $\lambda_c(t)$ and $\lambda_e(t)$.

3.1 INTRODUCTION

Numerical simulation is an indispensable tool in the product development cycles and design processes in various engineering disciplines. This is particularly true in IC design, since the development costs due to multiple design iterations amount to the larger part of the end product's cost for low volume chip fabrication. This aspect led to an interest and intense work in circuit simulators, proportional to the rate of rapid growth in this field in early the 70's. The public domain simulation tool SPICE and its derivatives have been used and enhanced extensively since then. These enhancements were directed to extend the capabilities of SPICE and also to deal with its shortcomings regarding simulation speed and the complexity of the original code.

One shortcoming of the original SPICE code is the lack of a clean interface between device models and the numerical simulation algorithms: The device model code and the numerical simulation code are unnecessarily intertwined with each other, rendering the implementation of new device models and new numerical simulation algorithms very cumbersome. Attempts have been made in order to deal with this issue, e.g., the Verilog-A hardware description language (HDL) [66]. Our work follows the spirit of recent work on ModSpec [67]. Here we introduce an effort towards a flexible and modular circuit simulator infrastructure developed in the widely used computing environment of MATLAB® and designed to substantially reduce the complexity of expanding the simulator core with new device models and simulation algorithms. An additional tool we have developed based on ADMS [68] allows the translation of existing device models from Verilog-A HDL and automatically generates the code for the computation of Jacobians. CIRSIUM uses the IDA solver from the SUNDIALS suite [69] that is based on multi-step methods with time-step control for the efficient and robust numerical integration of differential-algebraic equations (DAEs).

In order to implement the noise simulation algorithms we develop in this thesis into CIRSIUM, we expand the translated code of the PSP MOSFET model [70] to include a physical description of gate oxide defects. The capture and emission of electrons from the channel by the traps in the oxide cause a change in the flatband voltage of the transistor and thus affect the dynamical behaviour of the circuit. Conversely,

The content of this chapter has previously appeared in [65].

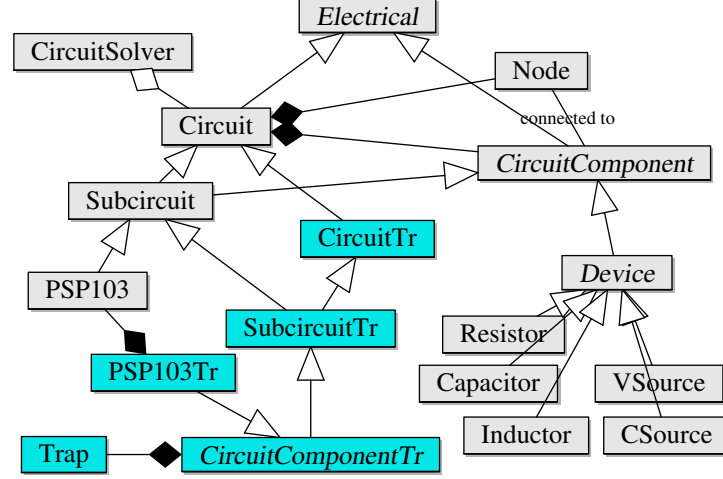


Figure 1: Class diagram of the CIRSIUM core.

the position dependent voltage in the channel changes the rate for the capture and emission events, establishing a two-way coupling between the node voltages of the circuit and the occupation states of the traps. As an example, we demonstrate the effects of these dynamics on the periodic waveforms generated by a ring oscillator circuit. Throughout this thesis, we use CIRSIUM in order to test the new modeling techniques and algorithms we develop.

3.2 SIMULATOR ARCHITECTURE AND PRINCIPLES

Electrical circuits are best represented in an hierarchical manner. At the bottom of the hierarchy, there are the devices without internal nodes, such as resistors and capacitors. The next level, subcircuits, are composed of these devices and possibly other subcircuits. At the top most level, the circuit serves as a controller for all of its components. This hierarchical structure suggests a partitioning of the simulator core into individual units, each with isolated functionalities. CIRSIUM uses MATLAB[®]'s object oriented programming language to realize this partitioning in terms of software classes which is depicted in a class diagram in Figure 1. On the other hand, these units bear some similarities to each other. For instance, during the generation of KCL/KVL equations, subcircuits and devices are handled almost in the same manner by the circuit and hence, a subcircuit can be construed as a circuit component with internal nodes. These relationships between different parts of the simulator translate into inheritance relations among classes which are shown by arrows (from sub to super-class) in Figure 1. In the same diagram, lines with filled and hollow diamond heads represent composition and aggregation relationships respectively, e.g., a circuit consists of circuit components and nodes.

This object oriented framework constitutes the basis for a modular structure that makes adding new functionality to CIRSIUM quite easy.

As an example, Listing 1 shows the definition of a non-linear resistor class with a quadratic I-V characteristic, $I = V^2$.

```

classdef resistor < device
    properties (Constant)
        numTerms = 2; numCurrentVars = 0;
    end

    methods
        function obj = resistor(varargin)
            obj = obj@device(varargin{:});
        end

        function s = I(thisResistor, x, t)
            i = (x(1) - x(2))^2;
            s = [ i; -i];
        end

        function s = dI(thisResistor, x, t)
            G1 = 2*x(1); G2 = 2*x(2);
            s = [ G1, -G2; -G1, G2];
        end
    end
    % charge and source functions are not shown.
end

```

Listing 1: Definition of a class for a non-linear resistor.

The functions $\mathbf{I}(\mathbf{x}, t)$ and $\mathbf{dI}(\mathbf{x}, t)$ define the dependence of the device current on the voltages of its two terminals and the Jacobian of the current with respect to these voltages. The definition of a device in its most general form would also include the charge storage function $\mathbf{Q}(\mathbf{x}, t)$, its Jacobian $\mathbf{dQ}(\mathbf{x}, t)$ and the source function $\mathbf{J}(\mathbf{x}, t)$. These functions collectively constitute the *stamp* of the device and are used by the **circuit** class in the solution of the dynamical KCL/KVL equations

$$\frac{d}{dt}\mathbf{q}(\mathbf{x}, t) + \mathbf{i}(\mathbf{x}, t) = \mathbf{j}(\mathbf{x}, t) \quad (34)$$

Here, the vector $\mathbf{q}$ can also include the magnetic flux (instead of charge) for inductive devices. The variable vector $\mathbf{x}$ will have entries for branch currents (instead of node voltages) in this case. The Jacobian matrices

$$\mathbf{dQ} = \frac{\partial \mathbf{q}}{\partial \mathbf{x}} \quad , \quad \mathbf{dI} = \frac{\partial \mathbf{i}}{\partial \mathbf{x}} \quad (35)$$

are usually sparse and are stored in MATLAB[®]'s sparse data structures which enables the use of fast linear system solution algorithms in the implicit numerical integration methods for Equation (34). These methods discretize the time variable t in an adaptive manner and find the vectors $\mathbf{x}_k$ at time points t_k that satisfy the discretized version of the circuit equations. The discretization of the time variable implies that we must

use a discrete approximation for the time derivatives as well. This can be done using the charge Jacobian, i.e., $\dot{\mathbf{q}} = \mathbf{dQ} \dot{\mathbf{x}}$, and then discretizing the variable vector $\mathbf{x}$. This is equivalent to linearizing the non-linear capacitors in the circuit, which introduces an additional source of numerical error in the solution that has to do with charge conservation [71]. In order to remedy this issue, we expand the DAE system (34) to include a vector of auxiliary variables, $\hat{\mathbf{q}}$, for charges [72]

$$\frac{d}{dt} \hat{\mathbf{q}} + \mathbf{i}(\mathbf{x}, t) = \mathbf{j}(\mathbf{x}, t) \quad (36)$$

$$\hat{\mathbf{q}} = \mathbf{q}(\mathbf{x}, t) \quad (37)$$

This new DAE system is then solved for the new variable vector $\hat{\mathbf{x}} = [\mathbf{x} \ \hat{\mathbf{q}}]^T$. CIRSIUM also includes an option to find a solution for a reduced set of variables. In circuits without inductive components, the variable for the current through an independent voltage source together with the voltage of one of its nodes can be eliminated. This is especially useful in cases where abrupt changes in the circuit cause large deviations in these currents.

3.3 RTS NOISE SIMULATION

We now describe the implementation of RTS noise simulation capabilities into CIRSIUM. We first provide a short overview on the emergence of RTS noise through defects in the gate oxide of a transistor and how these defects can be modeled within the PSP compact model. We then present an example simulation of the transient behavior of a ring oscillator built with transistors subject to RTS noise.

3.3.1 Oxide Traps

Oxide defects and the RTS noise caused by them is a major concern in integrated circuits fabricated in modern nano-scale process technologies. These defects act as traps and capture charge carriers from the channel of the transistor resulting in a change in the currents through the transistor. The capture and the subsequent emission of the charge carriers occur randomly governed by the statistics of a non-stationary Poisson process. The classical Shockley-Read-Hall (SRH) theory [9] estimates the capture and emission time constants, τ_c and τ_e , of this random process as

$$\tau_c = \frac{1}{c\bar{v}\bar{\sigma}} \quad , \quad \tau_e = \tau_c e^{-(E_T - E_F)/kT} \quad (38)$$

Here, c is the carrier concentration at the trap location, $\bar{v}$ is the average velocity of charge carriers, $\bar{\sigma}$ is a constant called the capture cross section of the trap, E_T is the trap energy level and E_F is the Fermi level. The capture/emission process of an electron and the associated

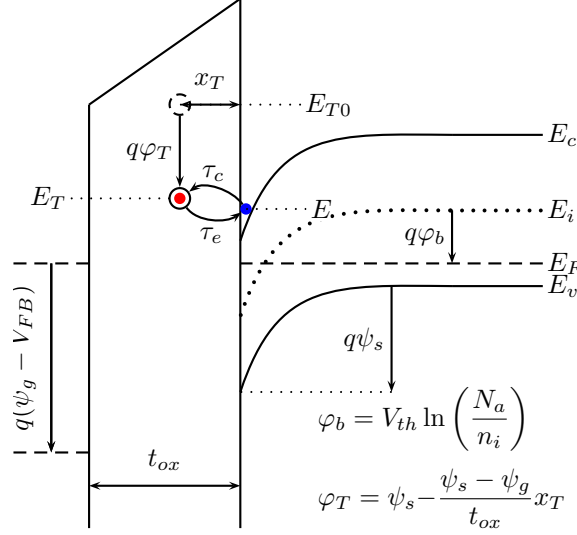


Figure 2: Capture and emission process of an electron by a trap located close to the conduction band edge.

balance among the quantities given above is shown on a band diagram in Figure 2.

3.3.2 Embedding Traps into the PSP Compact Model

PSP is a state of the art compact MOSFET model actively developed by the *Arizona State University* and *NXP Semiconductors* [73]. The physics of the PSP model is based on the surface potential calculations at the channel of the transistor. The intermediate variables, such as the position dependent sheet charge, can be used to build an extension to PSP and calculate the trap capture/emission rates using SRH statistics. The time constants of each trap will depend (among other parameters of the trap) on its position along the channel and into the oxide as well as on the voltages across the transistor. Moreover, the carrier capture and emission processes governed by these time constants will cause a change in the behavior of the whole transistor. We model this effect in a simple, straightforward manner.

According to Equation (38), the time constants τ_c and τ_e can be calculated based on the electron concentration at the location of the trap. For this purpose, we use the location dependent normalized sheet charge information, q_i , from the PSP model. This quantity is calculated using the auxiliary variable q_{im} with [70]

$$q_i(y) = q_{im} - \alpha(\psi_s(y) - \psi_m) \quad (39)$$

Here, y is the location along the channel, α is a linearization coefficient and $\psi_s(y)$ and ψ_m are the surface potential at y and the potential mid-point respectively. The position dependent surface potential is cal-

culated using a method called symmetric linearization (SLM) and is given by

$$\psi_s(y) = \psi_m + H \left[1 - \sqrt{\frac{2\Delta\psi}{HL}(y - y_m)} \right] \quad (40)$$

The definitions of the symbols used in Equation (40) can be found in [70]. These variables are all supplied by the PSP code with the following names: Normalized drain and source potentials are called **x_d** and **x_s** and they have to be multiplied with the thermal voltage ϕ_T in order to obtain the surface potentials. The mid-point potential, ψ_m , is the arithmetic mean of the two. $\Delta\psi$ is called **dps_dc** and H is simply called **H_dc**. L is the channel length and the position of the surface potential mid-point, y_m , is given by

$$y_m = \begin{cases} \frac{L}{2} \left(1 + \frac{\Delta\psi}{4H} \right) & \text{for } V_G > 0 \\ \frac{L}{2} & \text{otherwise} \end{cases} \quad (41)$$

The remaining variables in Equation (39) are named **qim_dc** and **alpha_dc** in the calculations for DC-characteristics. For the AC-characteristics of the model, the **_dc** suffixes have to be switched with **_ac**.

With the equations above, we can calculate the sheet charge at the trap location per unit area via $Q_i(y) = q_i(y)C_{ox}$, where C_{ox} is the oxide capacitance per unit area. The knowledge of Q_i enables us to determine the carrier concentration. For this purpose, we define an effective channel thickness, d_{eff} , assuming an exponential decay of the carrier numbers as a function of the distance from the oxide interface, $c(t) = n_0 \exp(-\phi(t)/\phi_T)$, and drawing the line for the end of the channel at the distance where the relationship $E_{\perp} d_{eff} = 3/2 \phi_T$ is satisfied [74]. Here, $E_{\perp}$ is the electric field component perpendicular to the oxide interface and it is found by introducing a proportionality constant η which is given by the normalized vertical electric field parameter of PSP, **Eeffm**: $E_{\perp} = \eta |\psi_s - \psi_g| / t_{ox}$. Thus, for the effective channel thickness and consequently for the volume concentration of carriers we have

$$d_{eff} = \frac{3\phi_T}{2\eta |\psi_s - \psi_g|} t_{ox}, \quad c(y) = \frac{Q_i(y)}{d_{eff}} \quad (42)$$

Finally, for the mean capture time we can write

$$\tau_c = \frac{1}{c(y_T) \bar{v} \bar{\sigma}(x_T)} \quad (43)$$

This quantity depends on the bias voltage and the trap position along the channel through $c(y_T)$ and on the position into the oxide through $\bar{\sigma}(x_T) = \bar{\sigma}_0 \exp(-x_T/\lambda)$ with a characteristic attenuation length λ [75]. For the calculation of the mean emission time through Equation (38), we need the trap energy level under the bias conditions of the transistor.

In this case, the trap energy is simply modified by the potential at the trap location as shown in Figure 2:

$$E_T = E_{T0} - q \left[\psi_s - (\psi_s - \psi_g) \frac{x_T}{t_{ox}} \right] \quad (44)$$

Finally, the effect of a captured charge carrier in the channel of the MOSFET is modeled by a change in the flatband voltage, ΔV_{FB} , with the simple relationship [76]

$$\Delta V_{FB} = \frac{q}{C_{ox} W_{eff} L_{eff}}. \quad (45)$$

This effect is easily integrated into PSP by changing the instance parameter `VFB_i`.

3.3.3 Simulation Example—Ring Oscillator with Oxide Traps

The colored (darker shaded) boxes in Figure 1 stand for classes specifically created for RTS noise simulation. The abstract class `circuitComponentTr` models a container that can hold a device, such as PSP, together with objects representing the oxide traps. It also defines the necessary functions for RTS noise simulation, e.g., time constant calculations. The `PSP103Tr` class implements these functions as described in the previous section. A minimal piece of code for the simulation of a ring oscillator built with noisy transistors is given in Listing 2. Note that this script also illustrates the use of subcircuits in CIRSIUM. Finally, Figure 3 shows the output of a sample run of this code and the effects of the traps on the waveforms which were artificially amplified for demonstration purposes.

```

nmos = psp103Tr('n', 35e-9, 100e-9);
pmos = psp103Tr('p', 35e-9, 180e-9);

inv = subcircuitTr('inverter');
inv.addComponent(nmos, 'out', 'in', 'vss', 'vss');
inv.addComponent(pmos, 'out', 'in', 'vdd', 'vdd');
inv.setExternalNodes('in', 'out', 'vdd', 'vss');

ckt = circuitTr('ring oscillator');
ckt.addComponent(inv.copy, 'in', 'out1', 'vdd', 'gnd');
ckt.addComponent(inv.copy, 'out1', 'out2', 'vdd', 'gnd');
ckt.addComponent(inv.copy, 'out2', 'in', 'vdd', 'gnd');
ckt.addComponent(voltageSourceDC(1), 'vdd', 'gnd');

ckt.setGroundNode('gnd');
ckt.seal;

solver = circuitSolver(ckt, 'y0', [1;0;0], ...
                        't0', 0, ...
                        'tend', 5e-11);

solver.solve();

```

Listing 2: Minimal code for the simulation of a 3-stage ring oscillator.

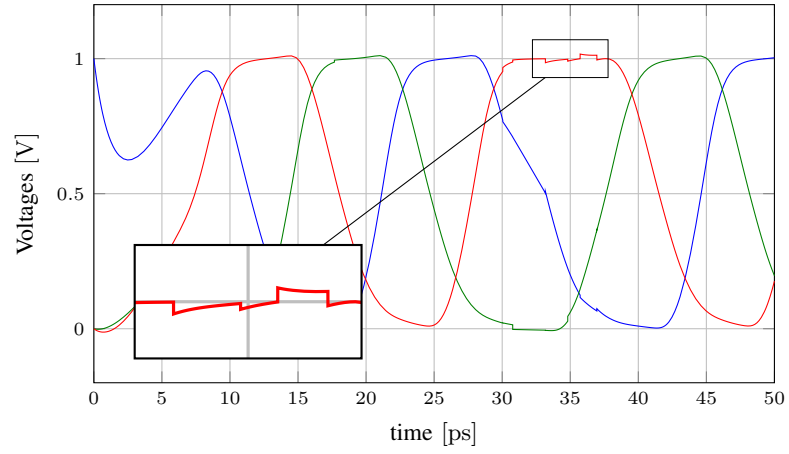


Figure 3: Output of a transient simulation sample for a 3-stage ring oscillator with oxide traps. The inset shows a time span of high trap activity.

STOCHASTICALLY CORRECT AND CAREFULLY CRAFTED NUMERICAL TECHNIQUES

4.1 INTRODUCTION

In this chapter, we incorporate and expand the content of Chapter 2 and Chapter 3 by providing a detailed explication of the proposed trap models (SAM and LEM) with regard to their stochastic properties and develop carefully crafted circuit simulation techniques that are stochastically correct. In particular, we describe

- (i) in Section 4.3, a hybrid Monte Carlo type simulation algorithm, based on the *Stochastic Automaton Model* (SAM), for the joint simulation of a circuit described by a system of nonlinear differential equations and a number of traps represented by a fine-grained *Markov chain* [26],
- (ii) in Section 4.4, a Monte Carlo type simulation algorithm, based on the *Langevin Equation Model* (LEM), for the joint simulation of a circuit described by a system of nonlinear differential equations and a number of traps represented by a set of coarse-grained *stochastic differential equations*.

In Section 4.5, we present results obtained by the proposed models and the simulation techniques for the low-frequency noise characterization of a common source amplifier and the phase jitter of a ring oscillator.

4.2 SIMULATION FRAMEWORK

The models presented in Chapter 2 provide a foundation for fully nonstationary and yet efficient simulation techniques for RTS noise. Stochastic simulation schemes for these models and their numerical properties will be analyzed in detail later in Section 4.3 and Section 4.4. However, before delving into this analysis, we need to set up a simulation framework that will combine our nonstationary noise models with classical circuit simulation approaches. In order to make use of the novel features of these models, the simulation framework, in like manner, has to have specific features. These features have to include a capability for discrete event simulation to be used with SAM and numerical SDE

The content of this chapter has previously appeared in [77].

integration algorithms for LEM. Moreover, a modern compact MOSFET model that includes a physical description of the traps and the effects of capture/emission events on the device is needed. All of these features are provided by CIRSIUM. Here, we show how the simulation framework of CIRSIUM, described in detail in Chapter 3, can be combined with the mathematical trap models developed in Chapter 2 and provide the basic principles for the joint simulation of circuits and traps.

4.2.1 *Simulation Basics*

In Chapter 2, we have discussed nonstationary stochastic models for the occupancy function for a single trap. Since the traps are assumed to be stochastically independent, the generalization of the single trap model to a collection of traps in a single transistor or many transistors is trivial. On the other hand, a two-way link between the trap occupancy function $N(t)$ and the currents and voltages of a transistor needs to be established, i.e., we need models on how the currents in a MOSFET are affected by a certain trap in the gate oxide and how the capture and emission rates are a function of the voltages across the transistor. The impact of a capture/emission event is modeled based on a change in the flatband voltage of the transistor [78] as given in Equation (45). The V_{FB} shift for a single charge carrier can be multiplied with the fractional trap occupancy function in Equation (14) and incorporated into the continuous state model as well. Moreover, the trapping events of charge carriers can be modeled with the classical SRH theory of recombination [9]. Based on this model, the capture and emission time constants, $\tau_c = 1/\lambda_c$ and $\tau_e = 1/\lambda_e$, of the random process are estimated with Equation (38).

In order to capture the impact of traps on the circuit, we expand the modified nodal analysis (MNA) equations that formalize the KCL/KVL description of a circuit as follows.

$$\frac{\partial \mathbf{q}(\mathbf{x}, \mathbf{N})}{\partial t} + \mathbf{f}(\mathbf{x}, \mathbf{N}) + \mathbf{b}(t) = 0 \quad (46)$$

In the above equation, $\mathbf{x} \in \mathbb{R}^M$ is a vector of M circuit unknowns such as node voltages, inductor and voltage source currents. The vector $\mathbf{b}(t)$ represents the external and bias inputs to the circuit and the nonlinear vector functions $\mathbf{f}, \mathbf{q} : \mathbb{R}^M \times \mathbb{R}^R \rightarrow \mathbb{R}^M$ represent the nonlinear memoryless components and the energy storing elements in the circuit, respectively. $\mathbf{N} \in \mathbb{R}^R$ is vector of R trap occupancy functions, and $\mathbf{q}(\mathbf{x}, \mathbf{N})$ and $\mathbf{f}(\mathbf{x}, \mathbf{N})$ represent the impact of the traps on the KCL and KVL equations possibly in the form of a change in the threshold voltages of the transistors.

In order to have a complete description for the circuit with the traps, we also need a model that describes the dynamics of the trap occupancy

function vector $\mathbf{N}(t)$ in addition to Equation (46). For this, we may use the Langevin model, LEM, that was given in Chapter 2 in the form

$$\begin{aligned} dN_{t,i} = & [(1 - N_{t,i})\lambda_{c,i}(\mathbf{x}, t) - N_{t,i}\lambda_{e,i}(\mathbf{x}, t)] dt \\ & + \sqrt{(1 - N_{t,i})\lambda_{c,i}(\mathbf{x}, t) + N_{t,i}\lambda_{e,i}(\mathbf{x}, t)} dW_{t,i} . \end{aligned} \quad (47)$$

Above, $dW_{t,i}$ $i = 1, \dots, R$ represent R independent Gaussian white noise sources. The Langevin equations for different traps are not directly coupled with each other, but coupled with the rest of the circuit equations in (46). The trap dynamics can also be described with the stochastic automaton, i.e., discrete-space Markov chain, model, SAM, which is the ultimate model for a trap. In this case, the dynamics of each entry $N_i(t)$ of $\mathbf{N}(t)$ is governed by an independent, two-state Markov chain fully specified by the capture and emission rates $\lambda_{c,i}(\mathbf{x}, t)$ and $\lambda_{e,i}(\mathbf{x}, t)$.

4.3 STOCHASTIC SIMULATION BASED ON STOCHASTIC AUTOMATON MODEL (SAM)

For the most detailed and accurate analysis of a nonlinear circuit with time-varying bias conditions and a number of traps associated with it, the discrete-space, two-state Markov chain models for the traps and the nonlinear DAEs of the circuit are jointly simulated in the time-domain in a Monte Carlo manner. Due to the two-way coupling between the circuit and the traps, the simulation techniques mentioned above need to be properly interfaced with each other. More specifically, the simulation of the discrete-space Markov chain trap model generates a number of events in time that result in jumps in the trap occupancy functions and hence the currents in the MOSFET transistors based on (45). The numerical DAE solver needs to be synchronized with these jump events. Moreover, the state transition rates of the Markov chains depend on the circuit variables that are being computed with the DAE solver. Thus, the jump event times with which the DAE solver needs to be synchronized are not only not known ahead of time but also depend on the outcome of the DAE solver itself. In dealing with this interfacing problem, one method is to set up integral equations involving the DAE-state dependent Markov transition rates which implicitly determine the timing of the jump events [56, 79, 80]. These equations then need to be solved in an iterative manner coupled with the time-step control mechanism of a DAE solver that can handle implicit events [69]. With this technique, however, one may incur considerable overhead in the DAE solver by iteratively solving for the jump event times.

Alternatively, based on a useful property of Poisson processes, called *Poisson thinning* [81], and its generalization to continuous-time Markov chains, called *uniformization* [82, 83], one can generate jump event times which are known ahead of time. In the thinning/uniformization technique, one uses a homogeneous Poisson process (PP) with a known,

constant rate in order to generate a number of potential jump event times for the non-homogeneous Markov chains with time-varying transition rates that model the traps. Then, these jump events are selected (sub-sampled) in a probabilistic manner and partitioned among the transitions of the Markov chains. With this technique, a small fraction of the potential jump events may eventually get wasted, but the DAE solver no longer needs to iteratively compute the timing of the implicit jump events. Instead, the time-step control of the DAE solver is simply forced to place a time point at all of the jump event times that are known ahead of time. Moreover, one may be able to avoid wasting jump events based on the use of a further generalization of Poisson thinning, called *adaptive uniformization* [83]. A similar technique based on uniformization was applied to the simulation of RTS noise in SRAMs and DRAMs in [26].

The remaining part of this section is dedicated to a detailed investigation of the stochastic simulation techniques mentioned above in order to reveal the intricacies and challenges in the practical implementation and applications of these methods. In what follows, we begin with a summary of the *uniformization* algorithm and provide some insight as to how it can be utilized in noise simulation. We also investigate, in detail, the more efficient simulation possibility offered by *adaptive uniformization* [83]. The result of this investigation shows that although it constitutes a more efficient means of computing transition probabilities, the adaptive uniformization algorithm cannot be used to generate sample paths of a nonstationary, doubly stochastic process in a straightforward manner.

4.3.1 Poisson Thinning and Uniformization Algorithms

Uniformization is the algorithm we use in order to generate sample paths (event times) for the stochastic processes describing the discrete valued trap states. To this end, we have to generate random time points for the capture and emission events of the traps, stop the DAE solver at these points and continue with the simulation after incorporating the effects of captured/emitted charge carriers on the circuit. Tackling this problem requires the simulation of an inhomogeneous continuous-time Markov chain (ICTMC).

A solution to a similar problem was proposed under the name *Poisson thinning* as a simple, efficient and stochastically precise way to sample inhomogeneous Poisson process (IPP) paths [81]. An IPP can be viewed as a special case of an ICTMC and hence, the thinning algorithm, as a special case of uniformization. This is because the random process determining the state transition (event) times in a continuous-time Markov chain (CTMC) is precisely a Poisson process with a rate equal to the sum of the transition rates of the Markov chain states.

Generating event times T_i for an IPP which is governed by a time dependent event rate $\lambda(t)$ is fundamentally different from the homogeneous case. The thinning algorithm solves this problem in a simple and elegant manner: It first generates event times according to a homogeneous PP and probabilistically subsamples these event times during the simulation. The event rate of the PP is chosen such that it exceeds the rate of the IPP, $\lambda^* \geq \lambda(t)$, at all times and it is called the *majorizing event rate*. Every event generated with this rate is accepted with probability $\lambda(t)/\lambda^*$ [81].

The uniformization algorithm is a generalization of Poisson thinning that uses the same idea for the computation of the state transition probability vector, $\boldsymbol{\pi}(t)$, of an inhomogeneous continuous-time Markov chain. Similar to Poisson thinning, the uniformization algorithm can also be used to generate sample paths of a Markov chain. This algorithm can be described as follows. At the time of the i^{th} event T_i , the time of the next event is determined by generating a sample of an exponential random variable, $\Delta T_i \sim \text{Exp}(\lambda^*)$, where the rate parameter λ^* (called the *uniformization rate*) is chosen such that

$$\lambda^* \geq \sum_{k=1}^R \lambda_k(T_i) . \quad (48)$$

In (48), $\lambda_k(t)$ is the voltage dependent event rate of the k^{th} trap. The differential equation solver is then forced to put a time point at $t = T_i + \Delta T_i$ and a uniform random variable, $r \sim U(0, 1)$, determines which trap will make a transition, where the likelihood of a transition is governed by the probabilities $\lambda_k(T_i + \Delta T_i)/\lambda^*$ [54].

Adaptive uniformization was introduced as a modification to the transition probability computation algorithm offering further reduction to its computational complexity. This modification adapts the uniformization rate during the run of the computation and it can be shown that this piecewise adaptation of λ^* still generates correct statistics [83]. One could assume that this modification can be applied just as well in the generation of actual sample paths for the random trap state variables. However, because the trap event rates are coupled with the circuit variables in a doubly stochastic manner, it is impossible to guarantee that the total event rate will not exceed λ^* until the next event time. This means that one has to keep track of the rates in order to maintain the validity of (48). This way, at the beginning, an optimistic value for λ^* can be chosen and one can implement a *backtracking* procedure if the condition in (48) is violated. We have in fact proposed such a simulation scheme based on adaptive uniformization for nonstationary RTS noise as an extension of the simulation algorithms described in [26]. However, it turns out that adaptive uniformization with backtracking produces

incorrect statistics¹. We have arrived at this very subtle conclusion after a deep and arduous investigation. In the following subsection, we demonstrate by a simple example why this technique does not create event times with correct statistics.

4.3.2 Adaptive Uniformization with Backtracking Produces Incorrect Statistics

If we consider a single trap, effectively reducing the number of possible transitions in the Markov chain to a single one, the uniformization algorithm becomes essentially the same as Poisson thinning. For the more general case, the thinning algorithm forms the central part of uniformization that determines the statistics of event times. Therefore, in our statistical analysis, we can concentrate on *adaptive thinning* for the sake of simplicity.

Algorithm 1 Adaptive Thinning with Backtracking

```

 $t \leftarrow 0, T_0 \leftarrow 0, i \leftarrow 1$ 
while  $t < t_{\text{end}}$  do
   $\lambda^* \leftarrow (1 + OH)\lambda(t)$  ▷ Rate overhead:  $OH$ 
   $\Delta T \leftarrow \sim \text{Exp}(\lambda^*)$  ▷ Generate next time increment
  if  $\lambda(\tau) \leq \lambda^* \forall \tau \in (t, t + \Delta T)$  then
     $t \leftarrow t + \Delta T$ 
     $r \leftarrow \sim U(0, 1)$  ▷ Generate uniform random number
    if  $r < \lambda(t)/\lambda^*$  then
       $T_i \leftarrow t$  ▷ Accept event
       $i \leftarrow i + 1$ 
      reset  $OH$ 
    end if
  else
     $t \leftarrow T$  ▷ Return to last event time
     $OH \leftarrow K \times OH$  ▷ Proceed with a larger  $OH$ 
  end if
end while

```

The important fact adaptive thinning builds on is that, at every trap event time T_i , we can calculate the transition rates $\lambda_k(T_i)$ and choose the majorizing rate with a predetermined overhead such as 10%, i.e. $\lambda^* = 1.1 \times \sum_{k=1}^R \lambda_k(T_i)$. This way, in adaptive thinning, λ^* becomes a time-varying quantity in a piecewise constant manner. This is of course a desirable property since it will reduce the amount of wasted events in regions where the actual event rate of the IPP is low, and therefore, result in a faster simulation. The immediate follow-up to this idea is to use a *backtracking* procedure for the cases where the total event rate exceeds

¹ All of the results presented in [26] were obtained with standard uniformization, and are correct. The simulation scheme based on adaptive uniformization was proposed only as an extension in [26].

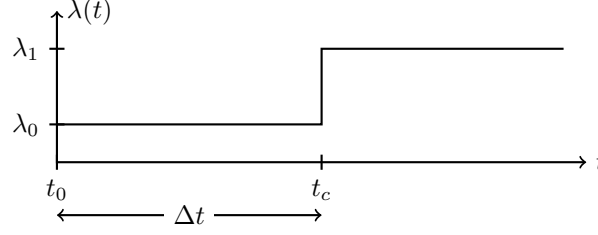


Figure 4: Time-variant rate function of an inhomogeneous Poisson process.

the majorizing rate. After the calculation of λ^* at T_i , if it is exceeded at a time point before T_{i+1} , one can go back to T_i and recalculate λ^* with a larger overhead. A pseudo code of this procedure is given in Algorithm 1. In the following analysis, we show that the *adaptive thinning with backtracking* procedure given in Algorithm 1 produces incorrect statistics for an inhomogeneous Poisson process.

Consider an IPP in $[t_0, t_1]$ with a rate function given in Figure 4 and let us investigate the statistical properties of simulated sample paths using adaptive thinning with backtracking. Here we will focus on the probability that this IPP has a first event time larger than t_c . It is clear that the number of events that occur in $[t_0, t_c]$ has a Poisson distribution with the rate parameter λ_0 . Thus, we have

$$\mathbb{P}[T_1 > t_c] = \mathbb{P}[N_{\lambda_0}(\Delta t) = 0] = P_0 = e^{-\lambda_0 \Delta t}. \quad (49)$$

Any stochastic simulation technique must produce the same probability for the above event for it to be stochastically unbiased. We next see if thinning with backtracking indeed produces the same probability.

The probability in (49) can be computed for the random process generated by the *adaptive Poisson thinning* algorithm with *backtracking* with the following probabilistic analysis. Assume we choose the majorizing event rate such that the rate overhead is zero, i.e., we start the simulation with $\lambda^* = \lambda_0$. If the first event is to be later than the rate change time, i.e. $T_1 > t_c$, no events are generated until t_c ; since otherwise the generated event would be accepted with probability one. In this case however, at the first event generated, we will be in the higher rate region and the backtracking mechanism will decide to go back because the actual rate of the IPP will now be larger than the majorizing rate $\lambda^* = \lambda_0$. Therefore, the algorithm will start over from $t = 0$, this time using the new majorizing rate $\lambda^* = \lambda_1$. Now, in the lower rate region, where $\lambda(t) = \lambda_0$, the rejection probability of an event is given by $(1 - \lambda_0/\lambda_1)$. Hence, the probability that no event is generated in Δt can be computed considering the following scenario: The first pass with majorizing rate λ_0 generates no events, because otherwise it would be accepted with probability one. We then backtrack to $t = 0$ and start over with the new majorizing rate λ_1 and all of the m events generated

are rejected in the second run until t_c . Using the law of total probability and summing over all m , we obtain

$$\mathbb{P}[T_1 > t_c] = \mathbb{P}[N_{\lambda_0}(\Delta t) = 0] \sum_{m=0}^{\infty} \mathbb{P}[N_{\lambda_1}(\Delta t) = m] \left(1 - \frac{\lambda_0}{\lambda_1}\right)^m$$

Noticing $\mathbb{P}[N_{\lambda_1}(\Delta t) = m] = e^{-\lambda_1 \Delta t} (\lambda_1 \Delta t)^m / m!$ and using the Taylor expansion for the exponential function, this expression can be simplified as

$$\mathbb{P}[T_1 > t_c] = e^{-2\lambda_0 \Delta t}. \quad (50)$$

The event probability realized by thinning with backtracking is not the same as in (49); it is in fact smaller. This means that smaller steps in the Poisson process are favored. This fact can be construed as the generation of Poisson events with a biased random number generator.

4.3.3 Summary

RTS noise simulation with the *stochastic automaton model* produces statistically exact results and is used, for our purposes, as the golden model to which all other simulation methods are compared in order to evaluate their accuracy. This property, of course, comes with a cost, high computational complexity. The main reason for this high complexity is the fact that every trap has random capture/emission times that are unknown beforehand and have to be determined during the simulation. Moreover, the DAE solver has to be forced to put an internal time point whenever a trap event occurs. In this section, we have investigated techniques that can reduce the computational complexity of the stochastic automaton model such as *uniformization* and its even lower complexity modification *adaptive uniformization*. We have drawn the conclusion that the adaptive uniformization algorithm is not suitable for our purposes. Therefore, we use the ordinary uniformization algorithm in the form given in Algorithm 2.

4.4 STOCHASTIC SIMULATION BASED ON LANGEVIN EQUATION MODEL (LEM)

The hybrid Monte Carlo simulation scheme we described in Section 4.3 that is based on the stochastic automaton model for the traps will indeed produce the most accurate results as there are no approximations involved. On the other hand, when simulating a nonlinear circuit with a large number of transistors and hence a very large number of traps, the computational cost of this scheme can become prohibitive due to very high density of jump events. In this case, the DAE solver will be forced to take very small time-steps that are essentially dictated by the need to pause at all of the jump events. This limits the use of this accurate simulation scheme with circuits that have at most tens of transistors.

Algorithm 2 Uniformization

Require: $\lambda^* > \sum_{k=1}^R \lambda_k(t)$ $\triangleright$ has to be satisfied at all times
 $t \leftarrow 0$
 $T_{\text{next}} \leftarrow \sim \text{Exp}(\lambda^*)$ $\triangleright$ Generate first trap event time
while $t < t_{\text{end}}$ **do** $\triangleright$ Begin DAE solution
 advance DAE solver for one time step, Δt
 $t \leftarrow t + \Delta t$
 calculate $\lambda_i(t)$ for $i = 1 \dots P$
 if $t = T_{\text{next}}$ **then**
 $r \leftarrow \sim U(0, 1)$ $\triangleright$ Generate uniform random number
 if $r < \sum_{k=1}^R \lambda_k(t) / \lambda^*$ **then**
 find j such that $r < \sum_{k=1}^j \lambda_k(t) / \lambda^*$
 generate trap event for trap j
 end if
 $\Delta T \leftarrow \sim \text{Exp}(\lambda^*)$
 $T_{\text{next}} \leftarrow T_{\text{next}} + \Delta T$ $\triangleright$ Generate next event time
 end if
end while

A more computationally efficient joint simulation scheme can be developed based on the approximate, continuous-space, Langevin SDE model that was described in Section 2.4, as one is no longer required to deal with discrete jump events. In this technique, which we call the Langevin equation model (LEM), the circuit DAEs in (46) are numerically solved along with one additional (stochastic) differential equation as in (47) per trap. However, due to the white noise sources (with state-dependent intensities) in (47), the joint solution of these nonlinear differential equations needs to be performed with specialized numerical integration techniques for SDEs [84, 85].

The transformation of discrete trap states into continuous variables through (14) considerably simplifies the coupled solution of a circuit and its traps because it lifts the difficulty of jointly simulating the time evolution of two qualitatively different set of state variables. However, it turns out that the numerical solution of (47) has its own intricacies. This is because the continuous state formulation for the trap states in (14) involves a *square root* expression for the time-dependent diffusion coefficient of the SDE. This immediately raises the question as to what happens if the expression in the square root becomes negative. This question should be answered from two different perspectives, namely theoretical and practical. In Section 4.4.1, we begin by introducing the so-called square root process as a formal basis for the formulation of the problem mentioned above. Moreover, we discuss some well-known results in the theory of stochastic processes which reveal the theoretically well-defined nature of (14). In Section 4.4.2, we examine the fact that, although the square root is almost surely non-negative for the exact solution paths of (14), the same does not apply to the numeri-

cal approximations to these sample paths [86]. Finally, in Section 4.4.3, we propose a numerical algorithm for the simulation of the square root process that is suitable for circuit simulation purposes.

4.4.1 The Square Root Process

Stochastic processes in form of (14) are encountered in various fields such as biology and finance [87, 88]. Oftentimes, a stochastic process with the structure

$$dS_t = \lambda S_t dt + \sigma \sqrt{S_t} dW_t \quad (51)$$

is called a *square root process*. Here, λ is the so-called *drift coefficient* and σ , the *diffusion coefficient*. The related process

$$dS_t = \lambda(\mu - S_t)dt + \sigma \sqrt{S_t} dW_t \quad (52)$$

is known as the *mean reverting square root process* because it oscillates around a mean value, μ .

Equation (14) can in fact be rewritten in the same form as in (52). To this end, we have to apply a transformation to the trap state variable N_t . A general transformation, $S_t = f(N_t)$, requires the utilization of the so-called *Itô calculus*. Itô's lemma dictates the inclusion of additional terms (Itô terms) to the stochastic differential dS_t because of the non-zero *quadratic variation* of the Wiener process [64].

$$dS_t = \frac{\partial f}{\partial N_t} dN_t + \frac{1}{2} \frac{\partial^2 f}{\partial N_t^2} dt \quad (53)$$

However, if we define $S_t = (1 - N_t)\lambda_c + N_t\lambda_e$, we can rewrite (14) without any additional Itô terms (because S_t is linear in N_t) in the same form as in (52)

$$dS_t = (\lambda_e + \lambda_c) \left(\frac{2\lambda_e\lambda_c}{\lambda_e + \lambda_c} - S_t \right) dt + (\lambda_e - \lambda_c) \sqrt{S_t} dW_t \quad (54)$$

Equation (54) has a diffusion part that does not satisfy a Lipschitz condition around $S_t = 0$ and thus, the usual existence and uniqueness theorem for SDEs [64, Theorem 5.5] does not guarantee a solution for it. Nevertheless, it can be shown that a unique solution to (54) exists [89].

For time-invariant λ_e and λ_c , the S_t process is always non-negative but if the capture/emission rates are time-varying, additional boundedness and differentiability constraints have to be imposed on them to ensure that the stochastic process never becomes negative [89]. The proof of this fact given in [89] is a constructive one, i.e., it shows that the square root process can be written as a sum of squares of some other stochastic processes and, therefore, it is fairly involved. A layman's version of this proof can be constructed by assuming the existence of pathwise solutions to (54) and examining the behavior of such a solution at the

critical point $S_t = 0$. For $S_t = 0$, the diffusion part of the SDE disappears and we are left with the deterministic drift part $dS_t = 2\lambda_e\lambda_c dt$. This equation tells us that the derivative of S_t evaluated at a zero-crossing is always positive, and hence, the stochastic process bounces off the boundary with probability one. This fact guarantees that $S_t \geq 0$, and hence, in theoretical treatments of the square root process, we do not have to deal with the contingency of the square root becoming imaginary.

4.4.2 Numerical Treatment of the Square Root Process

Contrary to the theoretical case, the numerical treatment of the square root process requires us to take extra measures in order to obtain meaningful results from computer simulations. We have to take into account that numerical integration techniques always involve approximation errors. The finite time-step sizes of numerical SDE algorithms cause discretization errors in the solution paths which can in turn result in the violation of the non-negativity of S_t in (54). In such a case, we either have to stop the simulation and start over with a smaller time-step size or we have to apply a fix that prevents the square root from becoming imaginary. In this section, we show that, for the first option, step sizes can get unreasonably small.

For the numerical solutions of (54), there is a relationship, called the *Feller condition*, that determines how well-behaved the solutions are [90]. This condition for (52) reads $2\lambda\mu > \sigma^2$ and it was proved by Feller that the square root process attains the critical value, $S_t = 0$, in finite time with probability one if this condition is violated. This inequality takes the following form for (54) in terms of trap capture/emission rates

$$\frac{4\lambda_e\lambda_c}{(\lambda_e - \lambda_c)^2} > 1. \quad (55)$$

Although seemingly innocent, there is no reason for this condition to hold in a physics based model for capture/emission rates of oxide traps, and indeed it is often violated. This causes stability and convergence issues for numerical simulations [91].

The Feller condition imposes a constraint on the possible values of the event rates. If we examine the ratio of λ_e and λ_c , and set $k = \lambda_e/\lambda_c$, we realize that even a small value such as $k = 6$ violates (55). This means that, for a numerically stable solution, we need to have similar values for the capture and emission rates. This is already a severe restriction for relatively simple stationary trap models where the event rates do not change with time. For a physics based device model, where the event rates are determined based on the voltages across the transistor, this condition is rarely satisfied. Figure 5 shows the evolution of λ_c and λ_e of a certain trap in a simulation of a ring oscillator circuit. In this circuit, the *gate-source* and *drain-source* voltages across the transistor change from the ground voltage all the way up to the supply voltage. This

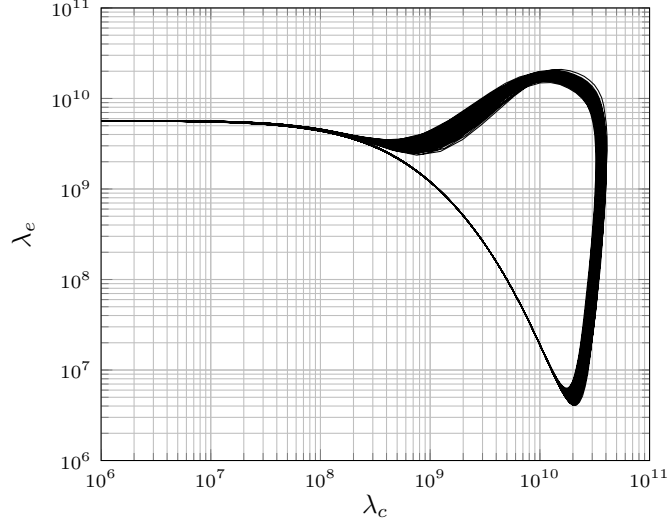


Figure 5: A sample path traced by the capture and emission rates of a single trap during a simulation of a ring oscillator circuit.

results in large variations in the event rates. Since, in extreme points in Figure 5, k can take values as large as 10^4 , a numerical stability analysis based on the Feller condition is of no use to us.

For the expression in the square root in (14) to remain non-negative, the occupancy state of the trap should satisfy

$$N \geq -\frac{\lambda_c}{\lambda_e - \lambda_c} \quad \text{if } \lambda_e > \lambda_c \quad (56)$$

and

$$N \leq \frac{\lambda_c}{\lambda_c - \lambda_e} \quad \text{if } \lambda_c > \lambda_e \quad (57)$$

Inequalities (56) and (57) are restrictions imposed on the individual sample paths of the trap occupancy state process N_t . With numerical integration methods, one can ensure that these inequalities are satisfied by using a “small enough” time-step size. However, the time-step size required is too small to be practical, much smaller than the step size imposed by the truncation error control mechanism of the numerical integration technique. In what follows, we perform a basic analysis of how small the time-steps of the DAE solver have to be chosen in order to have a non-negative expression in the square root in (14).

In the stationary case, the mean of the N_t process is time invariant and is given by

$$\mathbb{E}[N_t] = N_m = \frac{\lambda_c}{\lambda_e + \lambda_c} . \quad (58)$$

The distance from this value to the boundary values given in (56) and (57) is

$$|N_b - N_m| = \frac{2\lambda_e\lambda_c}{|\lambda_e^2 - \lambda_c^2|} . \quad (59)$$

If the SDE (14) is integrated with the Euler-Maruyama method, we have the discretization scheme [64]

$$N_{i+1} = N_i + ((1 - N_i)\lambda_c - N_i\lambda_e)\Delta t + \sqrt{(1 - N_i)\lambda_c + N_i\lambda_e}\Delta W_i, \quad (60)$$

where Δt is the time-step and ΔW_i for $i \in \mathbb{N}$ are independent and identically distributed zero-mean Gaussian random variables with variance Δt , i.e., $\Delta W_i \sim \mathcal{N}(0, \Delta t)$. Using (60), for a time point, t_i , at which the process is in its mean state, $N_i = N_m$, we can calculate the probability that the expression in the square root remains positive at the next time point $t_{i+1} = t_i + \Delta t$. For $N_i = N_m$, the drift part of the SDE disappears and we are left with

$$N_{i+1} = N_i + \sqrt{\frac{2\lambda_e\lambda_c}{\lambda_e + \lambda_c}}\Delta W_i. \quad (61)$$

The maximum value of the increment in the above equation is restricted by the boundary distance given in (59). We set $S_i = (1 - N_i)\lambda_c + N_i\lambda_e$ and consider only the case where N_i is near the upper boundary given in (57) (for the other case, the probabilities are the same). In order for the condition $S_{i+1} \geq 0$ to be satisfied, we obtain the inequality

$$\sqrt{\frac{2\lambda_e\lambda_c}{\lambda_e + \lambda_c}}\Delta W_i \leq \frac{2\lambda_e\lambda_c}{|\lambda_e^2 - \lambda_c^2|}. \quad (62)$$

Therefore, if we denote a zero-mean and unit-variance Gaussian random variable with $Z \sim \mathcal{N}(0, 1)$, the probability of S_{i+1} remaining non-negative is given by

$$\mathbb{P}[S_{i+1} \geq 0] = \Phi\left(\frac{k\sqrt{2}}{|k^2 - 1|}\left(\frac{\Delta t}{\tau}\right)^{-1/2}\right), \quad (63)$$

where $k = \lambda_e/\lambda_c$, $\tau = (\tau_c + \tau_e) = (1/\lambda_c + 1/\lambda_e)$ and Φ is the cumulative distribution function of Z .

Now we consider that at any given time point t_i , the event that S_{i+1} becomes negative can happen for only one direction of the next increment ΔW_i . For instance, if N_i is near the upper boundary in (57), for S_{i+1} to become negative, we have to draw a positive ΔW_i . So, we define a conditional *success probability* $\mathbb{P}[\text{success}] = \mathbb{P}[S_{i+1} \geq 0 | \Delta W_i \geq 0] = 2\mathbb{P}[S_{i+1} \geq 0] - 1$. Hence, we obtain

$$\mathbb{P}[\text{success}] = \text{erf}\left(\frac{k}{|k^2 - 1|}\left(\frac{\Delta t}{\tau}\right)^{-1/2}\right), \quad (64)$$

where erf denotes the so-called *error function*. Figure 6 depicts how small the time-step Δt must be in proportion to the mean transition time τ in order to keep the probability of an *unsuccessful step*, i.e., $1 - \mathbb{P}[\text{success}]$, under 10^{-6} . If we now consider that the number of total time-steps increases with decreasing Δt , we can deduce that the efficiency of such a simulation diminishes quickly with increasing k .

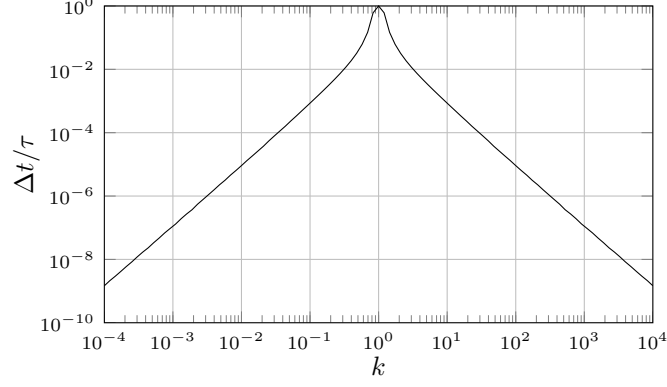


Figure 6: Maximum ratio of the integration time-step and the mean capture time plotted for different values of k in order to keep the failure probability under 10^{-6} .

The stability of Monte Carlo simulations of the square root process was the subject of some fairly recondite research in the second half of the last decade and various algorithms were proposed in order to remedy this issue [92, 91, 93, 94, 95, 96, 97, 98, 99]. A large subset of these algorithms have been implemented into CIRSIUM and simulation results were produced for simple test cases as well as the examples presented in Section 4.5. However, based on the outcome of these experiments, we concluded that none of the methods proposed in the literature work well, for some reason or another, in the numerical integration of Equation (46) together with (47) where the event rates change rapidly with the internal state of the circuit. Nevertheless, in order to conduct Monte Carlo type simulations, we need a robust and efficient numerical integration method. In the next section, we propose a simple, physically motivated technique we call the *smooth diffusion algorithm* that solves the problems regarding the numerical integration of circuit equations with traps.

4.4.3 Our Solution: Smooth Diffusion Algorithm

Instead of modeling noise sources as simple additive terms (usually in the form of current sources) in the MNA equations of the circuit as it is commonly done, the Langevin equation model augments these equations with a set of additional equations that describe the dynamics of the trap occupancy variables, i.e., the internal dynamics of the noise sources. These variables, in turn, are inserted into the original set of circuit equations as representations of the noise sources with a nonlinear, nonstationary behavior. This method of noise modeling has two distinct characteristics: While it handles the modulation of noise exclusively by internal dynamics, it also crystallizes this complicated dynamical behavior into distinct deterministic and stochastic components. The drift part of the SDE (14) describes the general deterministic trend of the

trap states, whereas the diffusion part determines their random behavior. If one removes the diffusion component from the trap equations, one is left with a deterministic dynamical system which still includes the traps and their *mean* effects on the circuit.

$$\frac{dN_m}{dt} = (1 - N_m(t))\lambda_c(t) - N_m(t)\lambda_e(t) \quad (65)$$

The time-varying mean state, $N_m(t)$, in turn, can be used in the computation of the diffusion coefficients of the trap SDEs, such that instead of (14) we have

$$\begin{aligned} dN_t &= [(1 - N_t)\lambda_c(t) - N_t\lambda_e(t)] dt \\ &\quad + \sqrt{(1 - N_m(t))\lambda_c(t) + N_m(t)\lambda_e(t)} dW_t \end{aligned} \quad (66)$$

The difference between using $N_m(t)$ instead of the actual stochastic process, N_t , in the diffusion coefficient of the SDE amounts to ignoring *modulation of noise by noise* which is a small, higher order effect. Indeed, our experiments show that this method of handling the square root problem produces correct second order statistics. A justification for this is given below.

It can be shown that the autocorrelation function of the stationary trap state process, i.e., when λ_c and λ_e are time-invariant, remains the same if the expression in the square root is replaced by its mean value. We substitute $N_t = N_m = \lambda_c/(\lambda_c + \lambda_e)$ in the square root and obtain

$$dN_t = [(1 - N_t)\lambda_c - N_t\lambda_e] dt + \sqrt{\frac{2\lambda_c\lambda_e}{\lambda_c + \lambda_e}} dW_t. \quad (67)$$

This implies that simulating this random process using the above equation will still produce correct second order statistics. Since the diffusion coefficient in (67) is time-invariant and always non-negative, the numerical issues created by the square root are automatically resolved. The same approach can be applied to the nonstationary case as well. In this case, however, the diffusion coefficient will not be constant because of the time varying event rates. The dynamics of the equilibrium value of the trap state, $N_m(t)$, need to be determined by the “noiseless” part of the SDE for N_t , which is given in (65).

Using this approach, we can integrate the coupled circuit and trap equations (46) and (47) for a circuit that contains M electrical and R trap variables.

$$\begin{aligned} \frac{\partial \mathbf{q}(\mathbf{x}, \mathbf{N})}{\partial t} + \mathbf{f}(\mathbf{x}, \mathbf{N}) + \mathbf{b} &= 0 \\ d\mathbf{N} &= [(1 - \mathbf{N})\boldsymbol{\lambda}_c - \mathbf{N}\boldsymbol{\lambda}_e] dt + \sqrt{(1 - \mathbf{N}_m)\boldsymbol{\lambda}_c + \mathbf{N}_m\boldsymbol{\lambda}_e} d\mathbf{W} \\ \frac{\partial \mathbf{N}_m}{\partial t} &= (1 - \mathbf{N}_m)\boldsymbol{\lambda}_c - \mathbf{N}_m\boldsymbol{\lambda}_e \end{aligned} \quad (68)$$

The explicit time dependence was omitted for clarity. In (68), $\mathbf{x}$, $\mathbf{q}$, $\mathbf{f}$ and $\mathbf{b}$ are $M \times 1$ while $\mathbf{N}$, $\mathbf{N}_m$, $\mathbf{W}$, $\boldsymbol{\lambda}_c$ and $\boldsymbol{\lambda}_e$ are $R \times 1$. We call this numerical integration technique the *smooth diffusion algorithm*, because

the diffusion coefficient is determined by the smooth process, $\mathbf{N}_m(t)$. It is straightforward to show that in this algorithm the expression in the square root remains always non-negative. The similar problem of guaranteeing the existence of matrix square roots in SDEs was encountered in [100] where the authors mention the use of a similar technique to smooth diffusion. We have been made aware of this technique in [100] after the publication of [77].

In the rest of this chapter, all LEM simulations were conducted using the smooth diffusion algorithm. As a final remark, we note that appending R additional equations to (46) and (47) increases the computational complexity of the LEM simulations. However, the efficiency of this model is not substantially reduced by this fact because the new set of equations require no device (e.g. transistor) evaluations which constitute by far the bigger part of the CPU-time usage. The main concern about the efficiency of the stochastic automaton model was based also on this observation since at every integration time-step added by the DAE solver, the states of all the devices in the circuit have to be evaluated anew.

4.4.4 Comparison of SAM and LEM Computational Cost

The Langevin equation model is proposed as a more efficient alternative to the stochastic automaton model and can be used to analyze the effects of RTS noise created by interface traps in nano-scale devices. The computational complexity of the Monte Carlo type simulations are reduced by the LEM technique because the effects of discrete trap events are modeled with continuous trap state variables. This in turn enables us to circumvent the necessity to use small time-steps in the DAE solver to account for the effects of rapidly occurring jump events.

More precisely, the average time gap between the break points of the DAE solver in SAM simulations (i.e., the density of jump events) is determined by the inverse of the *sum* of the rates of all the traps in the circuit, i.e., the uniformization rate. In SAM simulations based on uniformization, the time steps taken by the DAE solver are almost always dictated by the uniformization rate for the traps, as opposed to other high frequency circuit dynamics not related to noise. As such, as the number of traps in a circuit increases, the smallest time step taken by the DAE solver decreases and becomes much smaller than the smallest time step needed when traps are not included in the simulation. On the other hand, the smallest time step for the SDE solver for accurate LEM simulations needs to be on the order of the inverse of the rate of the trap with the largest rate in the circuit. For most circuits and typical LEM simulations, the largest trap rate is smaller than the frequencies of other circuit dynamics not related to noise. As such, the smallest time step needed in LEM simulations is usually not dictated by the traps but by other circuit dynamics. Since SAM time step is dictated by the *sum*

of all trap rates in the circuit, the efficiency advantage of LEM over SAM will become much more prominent for larger circuits containing a large number of traps.

The advantage of LEM described above comes together with the intricacies regarding the numerical treatment of the stochastic differential equations. However, we have provided a detailed analysis of these numerical difficulties and offered a solution which we call the *smooth diffusion algorithm*.

4.5 MONTE CARLO SIMULATION RESULTS

In this section we present noise analysis results using our nonstationary RTS noise models in Monte Carlo type simulations. We treat the discrete state stochastic automaton model as the golden model that captures the noise statistics exactly, and compare the continuous state Langevin equation model to this norm. We present results for two circuits: a common source amplifier and a ring oscillator. The common source amplifier is considered in two different setups, the stationary constant input case and the nonstationary switched input case.

4.5.1 Common-Source Amplifier: Constant Input, Two Traps

As a first example, we present noise analysis results for a common source amplifier with a time-invariant input. This example is to serve as a demonstration of the accuracy of our noise models and simulation techniques, since this simple setup allows us to compare the simulation output with well-known analytical results. The common-source amplifier circuit consists of a single n-channel transistor and a resistor (inlay of Figure 7). The transistor is biased with a fixed gate voltage, V_{in} , and hence, the only dynamical behavior in the circuit is caused by the activity of oxide traps. A single such trap is expected to generate a noise voltage component at the output of the circuit with a Lorentzian power spectral density (PSD) [101]

$$S(f) = \frac{2\Delta V^2\lambda}{4\lambda^2 + (2\pi f)^2}, \quad (69)$$

where ΔV is the change in the output voltage due to a capture/emission event and λ is the capture and emission rate of a single trap which, in this case, are set to be equal to each other. Multiple traps with different event rates result in a PSD in the shape of a superposition of multiple Lorentzians as confirmed by measurements [102].

Using CIRSIUM, this common source amplifier circuit was simulated with SAM as well as LEM. Two interface traps were placed in the transistor with transition rates adjusted to be approximately two decades apart from each other at the bias point. Figure 7 compares the results obtained with the two simulation methods for the noise voltage power

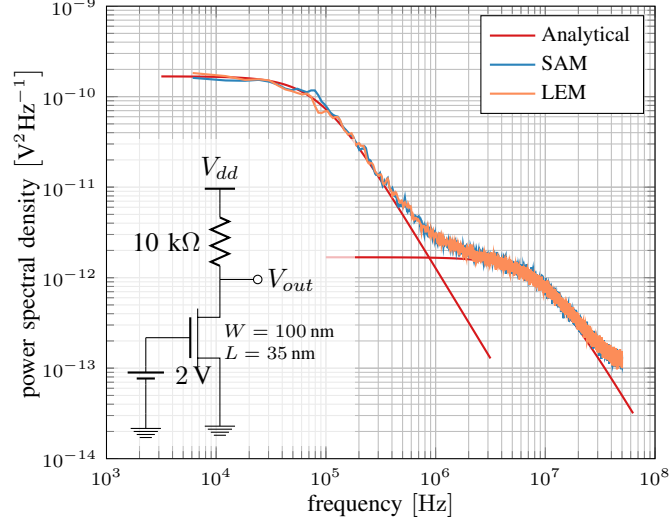


Figure 7: Power spectral density at the output of a common source amplifier at a fixed bias point. The n-channel transistor contains two oxide traps with different transition rates.

spectra at the output of the circuit. The power spectra were estimated from the time series data accumulated by running the SAM and LEM simulations long enough to capture the low-frequency spectral components. As expected, we obtain a monotonically decreasing power spectrum with two “corners”. It is not surprising that the SAM algorithm which relies on discrete switching events as noise sources closely matches the analytical results in (69) (smooth, red curves in Figure 7). The noise model in the LEM algorithm, on the other hand, has a structure that does not exhibit any resemblance to a random telegraph signal but, nevertheless, its accuracy in predicting the impact of the oxide traps on the output voltage noise spectrum is as good as the discrete algorithm based on stochastic automata.

4.5.2 Common-Source Amplifier: Switched Input, Single Trap

When transistors are cycled from inversion to accumulation, i.e., when they are periodically turned on and off, the measured low-frequency noise power spectral density is reduced compared with the case when the transistor is on all the time [3, 4]. This is of course expected, however, interestingly this reduction is more than the 3 dB one would expect solely as a result of the 50% duty cycle of switching [11]. The reason for this behavior is the inherent nonstationary character of low-frequency noise [8]. In this application of our noise models, we conduct a fully nonstationary noise analysis of a common source amplifier under switched input conditions and show that the amount of noise power spectral density reduction in our simulations is in agreement with the experimental results [3, 4, 23].

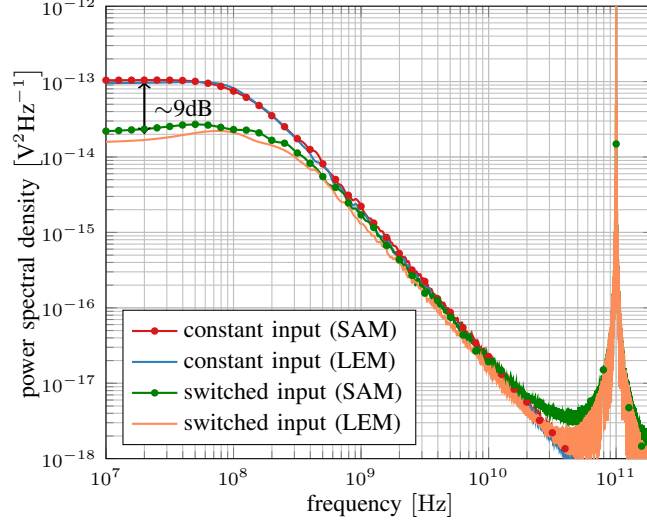


Figure 8: Comparison of noise power in a common source amplifier for the constant and switched input cases via LEM simulations.

For the simulation setup, we use the same common source amplifier circuit as in Section 4.5.1 and compare the noise analysis results for two different modes of operation. A single trap with voltage dependent capture/emission rates is placed in the transistor. The circuit is first simulated with a time-invariant input at the gate of the transistor. The noise PSD obtained from this simulation is then compared with the switched input case. The switched input simulation is conducted by replacing the DC voltage source at the gate of the transistor with a square wave input. Since the noisy transistor under test has a nano-scale length of 35 nm and a width of 100 nm, the switching frequency of the square wave is chosen accordingly high as 100 GHz.

Figure 8 shows the SAM and LEM simulation results for both the constant and the switched input cases. The PSD in the switched input case corresponds to the Fourier transform of the time average of the periodically time-varying autocorrelation function for the *cyclostationary* output voltage. It can be clearly seen that the reduction in noise power at low frequencies is higher than 3 dB. Moreover, the noise power reduction diminishes gradually at higher frequencies and, at the switching frequency, the switched input circuit exhibits higher values of noise power in comparison to the constant input case. This behavior is similar to the results obtained from measurements, e.g., in [23], whereas it stands in a disagreement with the analytical results obtained for this circuit in [8], where a frequency-independent power reduction is predicted. We note that our simulations produce more accurate results in comparison to the results reported in [8] and offer a closer match to the measurement data, which also exhibit a frequency-dependent, gradually diminishing reduction in noise power spectral density [23]. The reason for this disagreement will be discussed in detail in Chapter 6. For now,

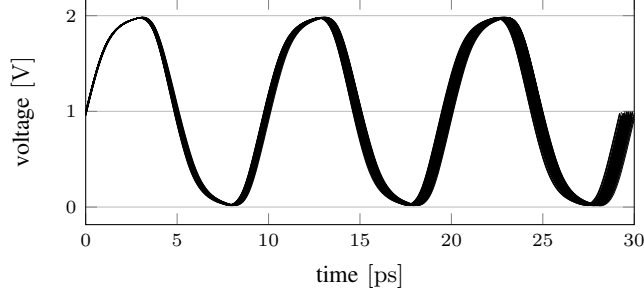


Figure 9: One hundred output waveforms of a ring oscillator plotted on top of each other. As the simulation time progresses, the waveforms drift apart and create a variation in the phase of the oscillation.

we confine ourselves to mentioning two separate factors: First, only the trap state is analyzed in [8] and not the drain voltage of the transistor. This approach neglects the nonlinear dynamics of the transistor. Second, instead of the idealized trap model considered in [8], which jumps to the empty state instantaneously with probability one when the transistor turns off, we use our fully nonstationary trap model with its compact transistor model implementation described in Section 4.2. Hence, our simulations take the noise contributions into account that stem from the non-trivial stochastic dynamics of the traps at the switching instances of the gate voltage.

4.5.3 Phase Jitter in a Ring Oscillator

As a third application of our nonstationary noise analysis framework, we analyze the effects of RTS noise on the phase evolution of a ring oscillator. The ring oscillator is an autonomous oscillator, and hence, it exhibits increasing phase fluctuations if the circuit is subject to noise. Because the oscillation is autonomous, these random fluctuations accumulate and their variance increases over time. This effect is known as *phase jitter* and it is illustrated in Figure 9. Low-frequency noise has a particular importance on the phase jitter of oscillators because it creates transient effects [36].

In an n-channel transistor, the threshold voltage increases due to the capture of an electron by a trap, resulting in a delay in the discharging process of the next stage in the ring oscillator which subsequently causes timing jitter. Because the node voltages in the gate nodes of the transistors swing from the ground to the supply voltage, the capture and emission rates of traps change in a wide range as shown in Figure 5. Hence, the ring oscillator circuit exhibits truly nonstationary noise statistics due to this change in event rates.

Here, we analyze a ring oscillator consisting of three CMOS inverter stages connected in a closed loop chain. Each transistor ($W = 100$ nm, $L = 35$ nm) in the inverters contains two oxide traps with different

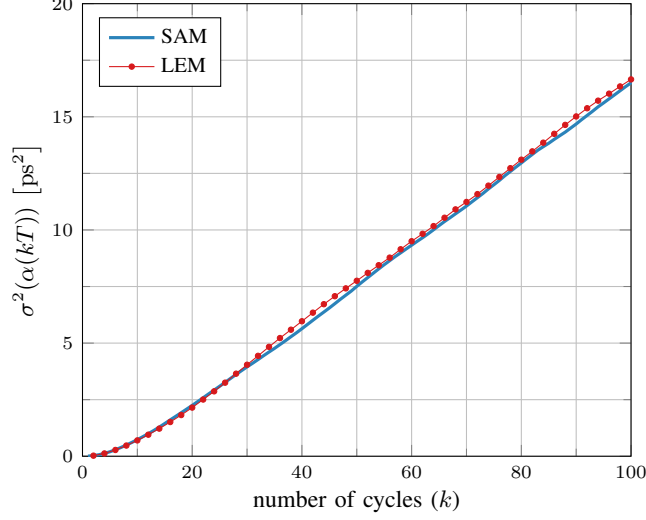


Figure 10: Phase jitter variance of a three-stage ring oscillator with a total of twelve non-stationary traps.

capture cross sections and therefore with differing voltage dependent transition rates. In order to characterize the variation in the phase evolution of the circuit, we consider the output voltage of the first inverter

$$V(t) = x(t + \alpha(t)) + y(t) , \quad (70)$$

where $x(t)$ is the periodic waveform of the unperturbed oscillator, $y(t)$ represents the variation in the amplitude of this signal and $\alpha(t)$ is the phase deviation [36]. We determine $\alpha(t)$ for each multiple of the oscillation period, kT , by calculating the crossing times of the waveforms through the level corresponding to the half of the power supply voltage. In a Monte Carlo manner, we proceed to build ensembles of a large number of simulations and determine the variances of the random variables $\alpha(kT)$.

Figure 10 compares the results obtained from simulations with SAM and LEM. As in the previous example, the Langevin model closely matches the discrete event model. Moreover, the initial exponential growth of the phase jitter and its eventual asymptotic behavior, both predicted by theory, can be clearly observed [41].

The nonstationary nature of the noise can be observed clearly by considering the statistics of the trap occupancy states. Figure 11 shows the evolution of the mean and the variance of the occupancy state of a single trap in the same ring oscillator as in Figure 10. Again, the SAM results are closely matched by the LEM simulations.

As a final remark, we note that the RTS and low-frequency noise simulation techniques we have described in this chapter are of the stochastic, Monte Carlo, simulation type, which usually become quite inefficient in analyzing large and complex systems. However, the Langevin trap model proposed in this chapter can be incorporated into the *non*

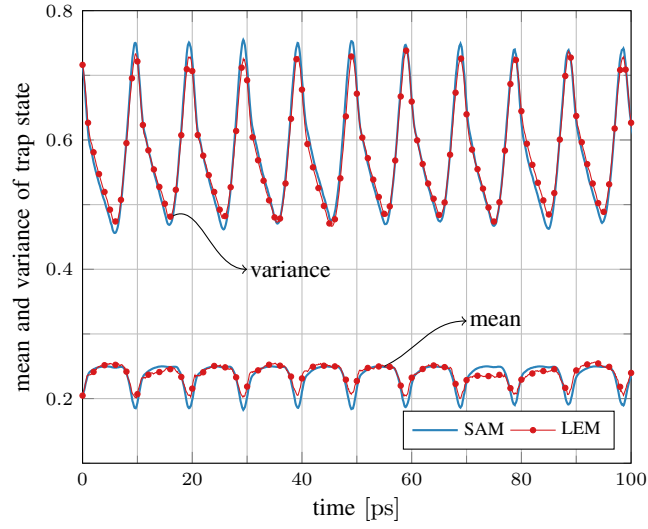


Figure 11: Time-varying mean and variance of the occupancy function of a single trap in a ring oscillator circuit.

Monte Carlo noise analysis algorithms that are currently implemented in circuit simulators for RF circuits and phase noise analysis of oscillators [13, 16, 38, 36]. This forms the content of the remaining part of this thesis.

NON MONTE CARLO ANALYSIS OF LOW-FREQUENCY NOISE

5.1 INTRODUCTION

In Chapter 4, our nonstationary low-frequency noise models were used in a Monte Carlo simulation framework which requires a simulated ensemble of the stochastic noise process in order to calculate expectations for estimating correlations and spectral densities. In this chapter, we expand this work and view the models introduced in Chapter 2 in a new light. We show how they can be used to analyze nonstationary low-frequency noise in a manner that transcends the conventional Monte Carlo simulation methods by eliminating the need for the estimation of noise characteristics through repeated, long simulations. Our method captures the internal, stochastic dynamics of the individual oxide traps via dedicated subcircuits that are coupled with the rest of the circuit. Moreover, it admits a probabilistic treatment through which expectations can be calculated in a semi-analytical manner. Hence, instead of repeatedly simulating the circuit, we can use semi-analytical noise evaluation methods which are orders of magnitude faster than brute-force Monte Carlo.

Our modeling work emanates from a nonstationary probabilistic description of the oxide traps as regards to their switching behavior. These switching events occur when a charge carrier is captured or emitted and are inherently discrete in nature, i.e., a trap is either *full* or *empty*. When an electron is captured from the channel of a transistor, and the trap changes its state, we observe a drop in the current through the device. Consequently, a jump occurs in some of the state variables of the circuit computed by the simulator. The probability of such a jump in turn is dependent on the same circuit variables. Therefore, in the SAM modeling paradigm, the noise source is represented in an *implicit* manner, in the sense that the dynamics of noise generation is inseparable from the dynamics of the circuit itself. The inseparability of the noise sources from the circuit requires the use of special stochastic simulation techniques of the Monte Carlo type, which are computationally expensive. This difficulty can be overcome by using the LEM as discussed in Section 4.4. The canonical discrete state characterization of traps as full or empty necessitates the coupled simulation of discrete state Markov chains intertwined with the continuous circuit variables. On the other

Some of the content of this chapter has previously appeared in [103].

hand, when the trap states are mapped to continuous valued random processes which can assume fractional values (between and outside of $\{0, 1\}$), the emerging dynamical system can be described by SDEs. The MNA equations for the circuit itself can then be augmented with an SDE for each trap in the circuit to obtain a system that can be analyzed with methods that are similar to standard numerical integration techniques.

The SDE modeling strategy offers a considerable reduction in computational complexity. More importantly, however, we deal with a fundamentally different dynamical description of the combined system of a circuit and its traps. In this new system representation, the random effects generated by the trap crystallize as *separate additive components*. Therefore, the description of the circuit and noise assumes a form which is compatible with the previously proposed non Monte Carlo techniques for nonstationary noise analysis, due to the incorporation of randomness into the system as an additive term. Moreover, this random part can then be removed from the combined system and, in this case, what we are left with is a set of deterministic equations describing the *time-varying mean behavior* of the noisy circuit. In this chapter, we exploit this special structure of the continuous state model and take the next step towards a more efficient low-frequency noise simulation framework. We describe the behavior of traps as a combination of a deterministic drift component and an additive noise component that is coupled with the rest of the device and the circuit. We then show how this point of view can lead to the utilization of efficient *non Monte Carlo* noise analysis techniques for RTS and other types of low-frequency noise. After describing the new non Monte Carlo noise analysis framework, we verify the results produced by this new method by comparing them against Monte Carlo simulations.

5.2 TREATMENT OF NOISE DYNAMICS BASED ON LANGEVIN EQUATION MODEL

The stochastic automaton model, which is based on discrete valued random trap state processes, offers the most accurate description of trap dynamics since it models the physical charge carrier capture/emission events without approximations. However, this accuracy comes at the cost of a complicated mathematical structure. This complicated structure, in turn, gives rise to circuit simulator implementations with high computational complexity: Aside from the need to handle discrete events during simulations, SAM is not amenable to any straightforward analytical treatment. This is because the stochasticity in SAM is implicitly buried in the jump events in circuit variables which occur at random times. These jumps, in turn, affect the event rates of the traps and, consequently, the probabilistic timing of the subsequent jump events. This creates an inseparable structure that intertwines noise with the deterministic circuit. Contrary to the common noise models, which involve

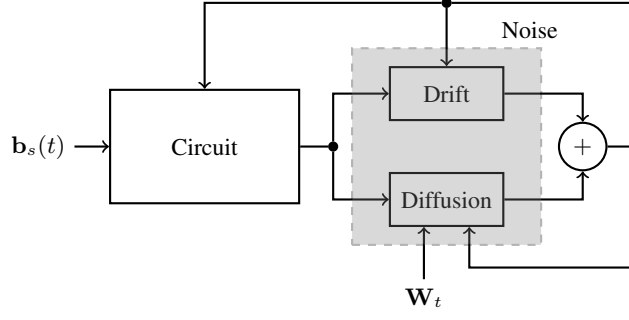


Figure 12: Block diagram of a circuit coupled with a noise model that captures the internal dynamics of nonstationary noise.

continuous valued, additive random components, such as additive white Gaussian noise (AWGN), SAM cannot be treated with non Monte Carlo methods, since it does not permit the computation of expectations analytically.

On the other hand, the inherent nonstationary nature of low-frequency noise makes it impossible to use simple AWGN models or models that distribute noise power into different frequency components in a stationary manner. The special mathematical structure of LEM which crystallizes the stochastic parts as additive components offers a method to remedy this problem. A pictorial description of this structure is given in Figure 12. One can view the noise component in Figure 12 as a sub-circuit that is coupled with the original (noiseless) circuit. In this model, the drift component is responsible for the deterministic, time-varying mean behavior of noise while the diffusion component shapes its random input, $\mathbf{W}_t$. This shaping is done in a coupled manner to the rest of the circuit and allows the model to capture the internal nonstationary dynamics of individual noise sources. The combination of the deterministic drift part and the stochastic diffusion part captures the total effect of trap noise on the circuit. This combined circuit and noise structure is mathematically expressed with the extended MNA equations.

$$\begin{aligned} \frac{\partial}{\partial t} \mathbf{q}(\mathbf{v}(t), \mathbf{N}_t) + \mathbf{f}(\mathbf{v}(t), \mathbf{N}_t) + \mathbf{b}_s(t) &= \mathbf{0} \\ d\mathbf{N}_t &= \boldsymbol{\mu}(\mathbf{v}(t), \mathbf{N}_t) dt + \boldsymbol{\sigma}(\mathbf{v}(t), \mathbf{N}_t) d\mathbf{W}_t, \end{aligned} \quad (71)$$

where $\mathbf{v}$, $\mathbf{q}$, $\mathbf{f}$, $\mathbf{b}_s$ are $M \times 1$ and $\mathbf{N}_t$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\lambda}_c$, $\boldsymbol{\lambda}_e$, $\mathbf{W}_t$ are $R \times 1$ column vectors. In (71), $\mathbf{q}$, $\mathbf{f}$ and $\mathbf{b}_s$ are the charge, current and source vectors, respectively.

We can now exploit the structure of LEM and rewrite (71) as a single set of MNA equations with R extra variables for trap nodes

$$\frac{\partial}{\partial t} \mathbf{q}(\mathbf{x}(t)) + \mathbf{f}(\mathbf{x}(t)) + \mathbf{b}_s(t) + \mathbb{B}(\mathbf{x}(t)) \boldsymbol{\eta}_t = \mathbf{0}, \quad (72)$$

where $\mathbf{q}$, $\mathbf{f}$, $\mathbf{b}_s$ are $(M + R) \times 1$ column vectors, $\mathbb{B}$ is a $(M + R) \times R$ matrix with entries as given below

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} \mathbf{v}(t) \\ \mathbf{N} \end{bmatrix} & \mathbf{q}(\mathbf{x}) &= \begin{bmatrix} \mathbf{q}(\mathbf{v}, \mathbf{N}) \\ \mathbf{N} \end{bmatrix} & \mathbf{f}(\mathbf{x}) &= \begin{bmatrix} \mathbf{f}(\mathbf{v}, \mathbf{N}) \\ -\boldsymbol{\mu}(\mathbf{v}, \mathbf{N}) \end{bmatrix} \\ \mathbf{b}_s &= \begin{bmatrix} \mathbf{b}_s \\ \mathbf{0}_R \end{bmatrix} & \mathbb{B}(\mathbf{x}) &= \begin{bmatrix} & \mathbf{0}_{M \times R} & \\ -\sigma_1(\mathbf{x}) & & \\ & \ddots & \\ & & -\sigma_R(\mathbf{x}) \end{bmatrix} \end{aligned} \quad (73)$$

and $\boldsymbol{\eta}_t$ is a column vector of R independent white noise processes. It is important to note that in (73) we have dropped the subscript t from $\mathbf{N}_t$ which was used to signify that $\mathbf{N}_t$ is a stochastic process. The entries of $\mathbf{N}$ represent now quantities equivalent to other circuit variables such as node voltages, branch currents, etc. Therefore, we can treat oxide traps as internal pseudo nodes in transistors.

We now assume that stochastic components in (72) cause small perturbations around $\mathbf{x}_s$, as it is routinely done in formulations of semi-analytical noise analysis techniques [13, 16, 104],

$$\mathbf{x}(t) = \mathbf{x}_s(t) + \mathbf{x}_n(t) \quad (74)$$

where $\mathbf{x}_s(t)$ is the solution of (72) without its stochastic terms, i.e.,

$$\frac{\partial}{\partial t} \mathbf{q}(\mathbf{x}_s(t)) + \mathbf{f}(\mathbf{x}_s(t)) + \mathbf{b}_s(t) = \mathbf{0}. \quad (75)$$

We note that this “noiseless” equation still includes the effect of mean trap noise. However, this component is deterministic and it is treated as a part of the circuit. We can now linearize (72) around $\mathbf{x}_s(t)$ to obtain

$$\frac{d}{dt} (\mathbf{C}(t) \mathbf{x}_n(t)) + \mathbf{G}(t) \mathbf{x}_n(t) + \mathbf{B}_s(t) \boldsymbol{\eta}_t = \mathbf{0} \quad (76)$$

where $\mathbf{C}$ and $\mathbf{G}$ are $(M + R) \times (M + R)$ matrices with entries given as follows:

$$\mathbf{C}(t) = \left. \frac{\partial}{\partial \mathbf{x}} \mathbf{q}(\mathbf{x}) \right|_{\mathbf{x}=\mathbf{x}_s(t)} = \begin{bmatrix} \frac{\partial \mathbf{q}}{\partial \mathbf{v}} & \frac{\partial \mathbf{q}}{\partial \mathbf{N}} \\ \mathbf{0}_{R \times M} & \mathbf{I}_R \end{bmatrix} \quad (77)$$

$$\mathbf{G}(t) = \left. \frac{\partial}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) \right|_{\mathbf{x}=\mathbf{x}_s(t)} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{v}} & \frac{\partial \mathbf{f}}{\partial \mathbf{N}} \\ \frac{\partial \boldsymbol{\mu}}{\partial \mathbf{v}} & \frac{\partial \boldsymbol{\mu}}{\partial \mathbf{N}} \end{bmatrix} \quad (78)$$

and $\mathbf{B}_s(t) = \mathbb{B}(\mathbf{x}_s(t))$, where $\mathbf{0}_{R \times M}$ is the $R \times M$ matrix consisting of all zeros and $\mathbf{I}_R$ is the $R \times R$ identity matrix.

Although the linear time-varying (LTV) system in (76) was obtained with the assumption that the effect of the random component in (72) is small in comparison to the signal levels in the circuit, it still pertains to a most general structure. Hence, the problem of nonstationary noise analysis of circuits can be tackled from different angles using this LTV representation. Time-domain examples of non Monte Carlo techniques include setting up differential equations for autocorrelation functions of the noisy circuit [13] and computing transfer functions by using the Fourier transforms of time-domain solutions [104]. Using (76), nonlinear circuits with periodic excitations can be transformed into linear periodically time-varying (LPTV) systems and their non Monte Carlo noise analyses can be conducted via frequency-domain methods [16, 38, 105]. Moreover, our new formulation for nonstationary low-frequency noise allows the analysis of noise in *autonomous oscillators* with efficient semi-analytical methods [36, 105, 106]. However, for autonomous oscillators, the linearization discussed above is not valid and a modified perturbation analysis (that takes into account the freely diffusing nature of the oscillator phase deviations) needs to be employed [107].

Non Monte Carlo techniques have been in use for the analysis of thermal, shot and also low-frequency noise. However, low-frequency noise models employed so far (usually obtained by modulating stationary noise sources) are phenomenological in nature and have been shown to produce incorrect results [11, 8]. While the models in Chapter 2 provided the foundation for stochastically correct treatment of nonstationary low-frequency noise, they were implemented only in a Monte Carlo simulation framework. In this chapter, we show how the models proposed Chapter 2 can be used in conjunction with non Monte Carlo techniques. In the next section, we demonstrate the efficacy of the proposed low-frequency noise analysis method using a tool from the arsenal of these techniques as an example. We consider non-autonomous circuits under periodic large-signal excitations and analyze the effects of oxide traps on their outputs. In particular, we investigate the *sidebands* created by nonstationary noise in their power spectra.

5.3 FREQUENCY-DOMAIN, NON MONTE CARLO, SEMI-ANALYTICAL NOISE EVALUATION

In order to conduct noise analysis on circuits under periodic large-signal excitations, we combine our nonstationary low-frequency noise model with a semi-analytical noise evaluation method as described in [16]. A non-autonomous circuit with a periodic input, which consists of a single tone and its harmonics, will have a periodic steady state solution with the same carrier frequency, f_c , and the noise sources together with the circuit variables will be *cyclostationary*. We first compute the

periodic steady state, $\mathbf{x}_s(t)$, of the deterministic circuit in (75) using the pseudospectral harmonic balance method [108]. We then linearize the circuit around $\mathbf{x}_s(t)$ to obtain the LPTV representation in (76) and its frequency-domain counterpart. Finally, we compute the *harmonic transfer functions* from the noise sources to the outputs of the circuit which can be utilized to calculate expectations and obtain the so-called *harmonic power spectral densities* of the cyclostationary noise in the circuit.

The periodic steady state solution to (75) is obtained by introducing L collocation points in time, that are uniformly spaced over one period, $T = 1/f_c$, and transforming the equation into the frequency-domain.

$$\mathbf{\Omega}_M \mathbf{\Gamma}_M \mathbf{Q}_L (\mathbf{\Gamma}_M^* \mathbf{X}_s) + \mathbf{\Gamma}_M \mathbf{F}_L (\mathbf{\Gamma}_M^* \mathbf{X}_s) + \mathbf{\Gamma}_M \mathbf{B}_{sL} = \mathbf{0} \quad (79)$$

Since in equation (79), the time-domain representations of M circuit variables were discretized with L collocation points, we can calculate the Fourier series coefficients for $\mathbf{x}_s$ starting from the $(-K)^{\text{th}}$ harmonic up to the K^{th} one, where $L = 2K + 1$. Hence, (79) becomes a non-linear equation for $M \times L$ unknowns where $\mathbf{\Omega}_M$ is the $ML \times ML$ diagonal frequency-domain differentiation matrix and $\mathbf{\Gamma}_M$ is the $ML \times ML$ K -harmonic discrete Fourier transform (DFT) matrix. The entries of these two matrices are given as

$$\mathbf{\Omega}_M = \begin{bmatrix} j2\pi(-K)f_c \mathbf{I}_M & & & \\ & j2\pi(-K+1)f_c \mathbf{I}_M & & \\ & & \ddots & \\ & & & j2\pi K f_c \mathbf{I}_M \end{bmatrix}$$

$$\mathbf{\Gamma}_M = \frac{1}{L} \begin{bmatrix} e^{-j2\pi K f_c t_1} \mathbf{I}_M & \dots & e^{-j2\pi K f_c t_L} \mathbf{I}_M \\ \vdots & \dots & \vdots \\ e^{j2\pi K f_c t_1} \mathbf{I}_M & \dots & e^{j2\pi K f_c t_L} \mathbf{I}_M \end{bmatrix}$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix. Furthermore, $\mathbf{X}_s$, $\mathbf{Q}_L$, $\mathbf{F}_L$ are $ML \times 1$ column vectors and $\mathbf{B}_{sL}$ is an $ML \times ML$ matrix defined as follows:

$$\mathbf{X}_s = \begin{bmatrix} \mathbf{X}_s^{(-K)} \\ \vdots \\ \mathbf{X}_s^{(K)} \end{bmatrix}, \quad \mathbf{B}_{sL} = \begin{bmatrix} \mathbf{B}_s(t_1) & & \\ & \ddots & \\ & & \mathbf{B}_s(t_L) \end{bmatrix}$$

$$\mathbf{Q}_L (\mathbf{\Gamma}_M^* \mathbf{X}_s) = \begin{bmatrix} \mathbf{q}(\mathbf{x}(t_1)) \\ \vdots \\ \mathbf{q}(\mathbf{x}(t_L)) \end{bmatrix}, \quad \mathbf{F}_L (\mathbf{\Gamma}_M^* \mathbf{X}_s) = \begin{bmatrix} \mathbf{f}(\mathbf{x}(t_1)) \\ \vdots \\ \mathbf{f}(\mathbf{x}(t_L)) \end{bmatrix},$$

where $\mathbf{X}_s^k$ is the k^{th} Fourier series coefficient of $\mathbf{x}_s$.

Once we obtain the solution of (79), we continue by assuming an input and solution pair to (76) with the following time-varying *frequency decomposed* representation:

$$\boldsymbol{\eta}_t = \mathbf{b}_p(t, f)e^{j2\pi ft}, \quad \mathbf{x}_n(t) = \mathbf{x}_p(t, f)e^{j2\pi ft}, \quad (80)$$

where f is the offset frequency from the carrier frequency f_c . We also assume that the noise/input representation $\mathbf{b}_p(t, f)$ and the output $\mathbf{x}_p(t, f)$ are periodic in t with the same carrier frequency. Next, by substituting (80) into (76) we obtain

$$\begin{aligned} \frac{d}{dt}(\mathbf{C}(t)\mathbf{x}_p(t, f)) + \mathbf{G}(t)\mathbf{x}_p(t, f) + \\ j2\pi f\mathbf{C}(t)\mathbf{x}_p(t, f) + \mathbf{B}_s(t)\mathbf{b}_p(t, f) = \mathbf{0}. \end{aligned} \quad (81)$$

The corresponding frequency domain equation to (81) is now given by

$$\begin{aligned} \boldsymbol{\Omega}_M \boldsymbol{\Gamma}_M \mathbf{C}_L \boldsymbol{\Gamma}_M^* \mathbf{X}_p(f) + \boldsymbol{\Gamma}_M \mathbf{G}_L \boldsymbol{\Gamma}_M^* \mathbf{X}_p(f) + \\ j2\pi f \boldsymbol{\Gamma}_M \mathbf{C}_L \boldsymbol{\Gamma}_M^* \mathbf{X}_p(f) + \boldsymbol{\Gamma}_M \mathbf{B}_{sL} \boldsymbol{\Gamma}_R^* \mathbf{B}_p(f) = \mathbf{0} \end{aligned} \quad (82)$$

with the two $ML \times ML$ block-diagonal matrices $\mathbf{C}_L$ and $\mathbf{G}_L$ defined as

$$\mathbf{C}_L = \begin{bmatrix} \mathbf{C}(t_1) & & \\ & \ddots & \\ & & \mathbf{C}(t_L) \end{bmatrix}, \quad \mathbf{G}_L = \begin{bmatrix} \mathbf{G}(t_1) & & \\ & \ddots & \\ & & \mathbf{G}(t_L) \end{bmatrix}.$$

We now rewrite (82) as

$$[\mathbb{G} + j2\pi f\mathbb{C}] \mathbf{X}_p(f) + \mathbb{B}_s \mathbf{B}_p(f) = \mathbf{0}, \quad (83)$$

with

$$\begin{aligned} \mathbb{G} &= \boldsymbol{\Omega}_M \boldsymbol{\Gamma}_M \mathbf{C}_L \boldsymbol{\Gamma}_M^* + \boldsymbol{\Gamma}_M \mathbf{G}_L \boldsymbol{\Gamma}_M^* \\ \mathbb{C} &= \boldsymbol{\Gamma}_M \mathbf{C}_L \boldsymbol{\Gamma}_M^*, \quad \mathbb{B}_s = \boldsymbol{\Gamma}_M \mathbf{B}_{sL} \boldsymbol{\Gamma}_R^*. \end{aligned} \quad (84)$$

Considering the representation in (83), we can define the harmonic transfer functions from the noise sources to the circuit variables

$$\mathbf{X}_p(f) = -[\mathbb{G} + j2\pi f\mathbb{C}]^{-1} \mathbb{B}_s \mathbf{B}_p(f) = \mathbb{H}(f) \mathbf{B}_p(f). \quad (85)$$

A fundamental property of LPTV systems is that if a sinusoidal noise signal at a frequency that is f offset from the k^{th} harmonic of the carrier frequency is applied to the system, a sinusoidal response appears at all of the output harmonics with the same offset. The individual entries of the matrix transfer function $\mathbb{H}(f)$ quantify the relationship between these input and output signals. The entry in the i^{th} row and the k^{th} column of $\mathbb{H}(f)$ determines how large the “folding” amplitude will be at the i^{th} sideband of the output if the noise is a unit magnitude sinusoidal signal at the frequency $kf_c + f$. Since $\mathbf{B}_p$ represents a stochastic process, we can compute a second order stochastic characterization for $\mathbf{X}_p$

$$\mathbb{E} [\mathbf{X}_p \mathbf{X}_p^*] = \mathbb{H}(f) \mathbb{E} [\mathbf{B}_p \mathbf{B}_p^*] \mathbb{H}^*(f) \quad (86)$$

where $\mathbb{E}[\cdot]$ is the expectation operator. Equation (86) can be rewritten in a compact form as

$$\mathbb{S}_X(f) = \mathbb{H}(f)\mathbb{S}_B(f)\mathbb{H}^*(f) . \quad (87)$$

The entries of $\mathbb{S}_X(f)$ capture the spectral correlations and cross correlations of the stochastic process $\mathbf{X}_p(f)$. A direct consequence of $\mathbf{X}_p$ being nonstationary is that there is non-zero correlation between its different frequencies. A general nonstationary stochastic process exhibits correlations between all frequencies, whereas in a cyclostationary process, as treated above, correlations are limited to the spectral components that are separated from each other by an integer multiple of the carrier frequency.

If one considers that in our model, $\mathbf{B}_p$ represents the spectral decomposition of independent white noise processes and, hence, $\mathbb{S}_B$ is the identity matrix, Equation (87) reduces to

$$\mathbb{S}_X(f) = \mathbb{H}(f)\mathbb{H}^*(f) . \quad (88)$$

Moreover, one can show that the entries of $\mathbb{S}_X$ are closely related to the *time-varying probabilistic spectral density*, $S_x(t, f)$, for the nonstationary stochastic process $x(t)$ [109]. For the cyclostationary case, $S_x(t, f)$ is periodic in t and can be expanded in terms of its Fourier series coefficients $S_x^{(k)}(f)$. These quantities are called *harmonic power spectral densities* and they can in turn be related to the spectral correlation matrix as follows [109].

$$\mathbb{E} \left[\mathbf{X}^{k_1}(f) \mathbf{X}^{*k_2}(f) \right] = S_x^{(k_1-k_2)} \left(f + \frac{k_1+k_2}{2} f_c \right) \quad (89)$$

The harmonic power spectral density calculation technique we discussed above can easily be combined with the nonstationary low-frequency noise model proposed in Section 5.2. Moreover, models for other types of noise such as thermal and shot noise can also be incorporated with ease along with low-frequency noise as described in [16]. In the next section, we present simulation results obtained using this technique and compare them with brute-force Monte Carlo.

5.4 VERIFICATION OF NON MONTE CARLO METHODS VIA MONTE CARLO SIMULATIONS

Non Monte Carlo analysis techniques are efficient but they rely on certain approximations and assumptions about the underlying system. The semi-analytical non Monte Carlo noise analysis framework discussed in the previous sections constructs a linear time-varying system from nonlinear circuits by linearizing them around a deterministic, large-signal solution (Equation (76)). This system is then solved in the presence of noise in order to analyze its effects assuming that the impact of the additional noise signal is small enough so that the LTV approximation

remains valid. Both the LTV approximation and the small noise assumption are justified in the context of noise analysis of nonautonomous electronic circuits. For autonomous oscillators, the LTV approximation is not valid, however, one can in this case use a modified perturbation analysis and still make use of a linearization for the amplitude noise component [36, 107, 110]. Nevertheless, a verification of the accuracy of non Monte Carlo simulations is in order. This can be done by comparing results of non Monte Carlo analyses to the data obtained from detailed Monte Carlo type simulations in Chapter 4—since the latter are conducted using a fully non-linear system and correctly capture the statistics of noise produced by this system. In this Section, we perform this comparison first on a single transistor circuit, a common source amplifier, we then demonstrate the validity of our model via the noise analysis of a ring oscillator circuit by investigating its carrier offset sideband noise spectrum and the variance of its phase jitter [36].

5.4.1 *Spectral Estimation via Monte Carlo Simulations and Average Spectral Density*

The PSD of a stationary stochastic process is defined as the Fourier transform of its auto-covariance function (ACF) [22]. However, nonstationary stochastic processes have a time-dependent ACF, $R(t, \tau)$, which quantifies the correlation between two samples of the stochastic process that are separated from each other by a time difference of τ but also depends on the time instance t relative to which these two samples are observed. For cyclostationary processes, the time dependence of the ACF can be expressed in a Fourier series representation as follows.

$$R_x(t, \tau) = \mathbb{E}[x(t)x^*(t + \tau)] = \sum_{k=-\infty}^{\infty} R_x^{(k)}(\tau)e^{j2\pi k f_c t}$$

Usually, when it is referred to “the PSD” of a cyclostationary stochastic process without further specification, what is really meant is the average PSD that is given by the Fourier transform of the average value of $R(t, \tau)$ with respect to t over a full period of length T .

$$\frac{1}{T} \int_0^T R_x(t, \tau) dt = R_x^{(0)}(\tau) .$$

The average noise PSD for circuits, as well as higher-order harmonic PSDs, can be computed in a semi-analytical manner by setting $k_1 = k_2$ in Equation (89) and extracting the diagonal elements of the spectral correlation matrix given in Equation (86). The average PSD can also be calculated by employing a spectral estimation method and running it over an ensemble of simulated, noisy signal waveforms obtained in a Monte Carlo manner. The spectral estimation method we use in this work in order to obtain the average PSDs from Monte Carlo simulations

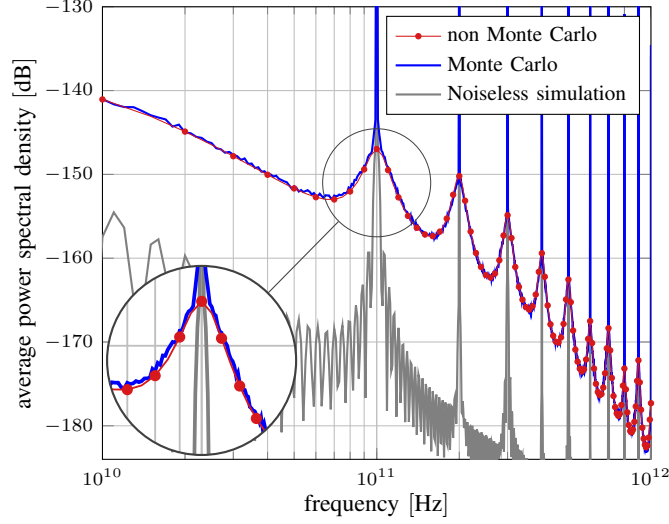


Figure 13: The average power spectral density of the common source amplifier with a noisy transistor.

is Welch's method [111]. Welch's method estimates the power spectrum by computing the magnitude square of the Fourier transforms of overlapping sections and by averaging them. This estimate, $S_x^0(f)$, converges exactly to the Fourier transform of $R_x^{(0)}(\tau)$. We compare PSD estimates generated by Welch's method from Monte Carlo simulations to their semi-analytically computed counterparts in order to verify the correctness of our newly developed non Monte Carlo noise model and analysis techniques.

5.4.2 Common Source Amplifier

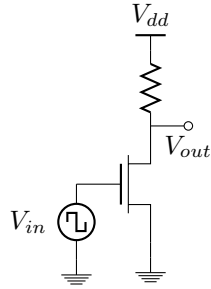


Figure 14: Common source amplifier.

The simple circuit in Figure 14 can be used to conduct a basic analysis of low-frequency noise upconversion in a periodically switched circuit. The time-varying bias in the circuit is introduced at the gate of the transistor as a square wave input. Since the only time-varying excitation to the circuit is periodic with a frequency f_c , we would expect a perfectly periodic output with the same frequency if the circuit was noise-free. This periodicity translates into discrete frequency components in the power spectrum,

i.e., spikes at the integer multiples of f_c in the PSD. However, if we place an oxide trap into the transistor, the resulting noise will be apparent as additional spectral components around the carrier frequency, called *sidebands*. Moreover, due to the cyclostationary nature of the noise and noise upconversion, further sidebands appear around integer

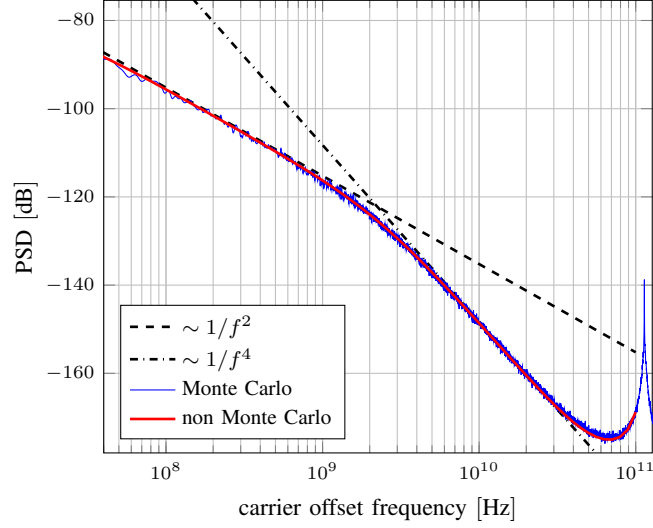


Figure 15: Noise spectrum of a three-stage ring oscillator containing a single trap as the noise source.

multiples of f_c . In order to capture this behavior, we first simulated the common source amplifier circuit with the Monte Carlo technique using the stochastically correct numerical methods described in Chapter 4. One hundred parallel simulations were conducted for a duration of 100 ns and a time averaged power spectrum was computed for all of them. The blue (solid) curve in Figure 13 is the ensemble average of these PSDs. The spikes at multiples of $f_c = 100$ GHz are the discrete harmonic components of the output and the skirts around these components are the sidebands. The PSD computed in a non Monte Carlo manner as described in Section 5.3 is shown with the red curve (dot marks) in Figure 13. As it can be seen in the inlay of the same figure, the two curves are in near perfect agreement. A third curve in solid gray is shown in Figure 13 that depicts the accuracy limit of Welch's method. This curve was obtained by simulating a noise-free circuit under the same setup as the noisy one and it is intended to indicate that the estimation error for the spectral density is well below the level of the noise generated by the trap.

5.4.3 Ring Oscillator

As a first representative of autonomous oscillators we employ the ubiquitous ring oscillator circuit as a test case. Autonomous oscillators are used among other purposes as clock references and hence are expected to be accurate producers of a constant oscillation frequency. However, because of different sources of randomness, these circuits are subject to noise which causes random fluctuations in their frequency of oscillation [112, 113, 114]. Two common measures for the effect of noise in oscillators are the carrier offset, *sideband noise spectrum* and the *phase*

jitter variance. The sideband noise spectrum describes oscillator noise in the frequency-domain through the power spectral density of the oscillation signal at a carrier offset frequency. The phase jitter variance, on the other hand, is a time-domain quantity which characterizes the drift of the oscillator phase through the variance of the random zero-crossing times of the oscillation waveform [115]. In order to verify the correctness and accuracy of our non Monte Carlo low-frequency noise model and the associated noise evaluation methods, we first compute the noise spectrum of a noisy ring oscillator with both Monte Carlo and non Monte Carlo methods and compare the two outcomes. We then repeat the same procedure for the variance of the phase jitter.

The power spectrum of RTS noise that is generated by a single trap has a Lorentzian form, i.e., it is flat for low-frequencies and falls off with a 20 dB/decade slope after a corner frequency which is determined by the average transition rate of the trap [101]. The upconversion of low-frequency noise into sideband noise in an oscillator is equivalent to an integration operation from the noise source to the phase of the oscillator in the time-domain [41]. Therefore, we would expect to see a carrier offset spectrum at the output node of the oscillator that is similar in form to the multiplication of the RTS noise Lorentzian with a $1/f^2$ roll-off that corresponds to integration. The resulting spectrum is expected to have a $1/f^2$ shape at low offset frequencies and a $1/f^4$ slope at frequencies higher than the average transition rate of the trap. The single trap setup was chosen in order to demonstrate that our methods are capable of capturing this behavior. Figure 15 shows the result of simulations conducted with a ring oscillator consisting of three equal sized CMOS inverter stages. The first of these stages has a single trap located in its n-type transistor. The transistor sizes chosen for this simulation result in an oscillation frequency around 100 GHz. As the output node of the circuit, we examine the voltage at the output of the first inverter. It can be seen in Figure 15 that the result of the non Monte Carlo simulation is in perfect agreement with the spectrum obtained with computationally laborious Monte Carlo simulations. We note that the corner frequency for the RTS noise generated by the single trap in the circuit is approximately at 2 GHz.

The phase jitter of a ring oscillator containing several traps was investigated in Section 4.5.3 based on Monte Carlo simulations. Employing our non Monte Carlo low-frequency noise modeling and analysis framework, we can compute the rate of increase for the variance of the accumulating phase jitter via the method described in [36]. This is illustrated in Figure 16. The red line (no marks) in Figure 16 has a constant slope computed with the non Monte Carlo method. The blue curve (dot marks) is the same one given in Section 4.5.3 and was computed through the ensemble average of the phase jitter in several thousand sample paths of the noisy oscillator output. As it can be seen

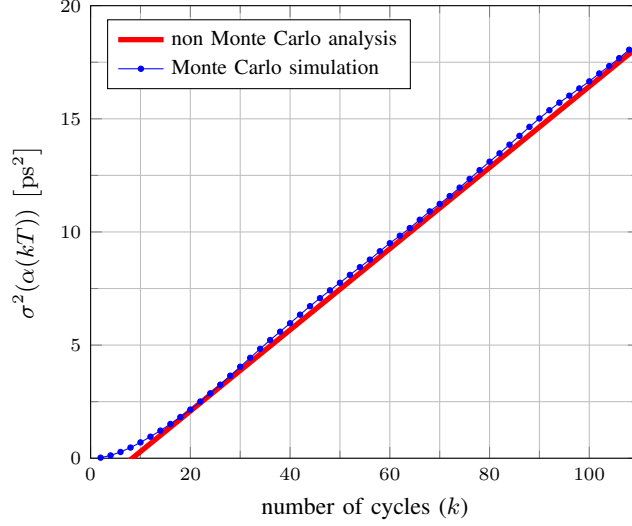


Figure 16: Variance of accumulating phase jitter for a ring oscillator.

from Figure 16, this curve settles down to the same slope as the non Monte Carlo analysis predicts.

The Monte Carlo results in Figure 15 and Figure 16 are epitomes of dread in doing stochastic simulations. For a PSD estimation, the circuit has to be simulated for a long enough duration such that information about the lowest frequency of interest can be obtained. The oscillation frequency of the ring oscillator discussed above is around 100 GHz and the frequency axis in Figure 15 goes down to 1 MHz. These numbers dictate a simulation time at least as long as 10^5 oscillation cycles. Even if we were not interested in frequencies that low, we still had to run simulations that are comparably long in order to reduce the level of the estimation noise (an artifact from the spectral estimation method). The phase jitter variance computations on the other hand, do not require very long simulations. As seen in Figure 15, a simulation for 100 cycles is sufficient to estimate the slope of the phase jitter accumulation curve. However, in order to reduce the estimation error in this case, a large number of independent simulated sample paths (in the order of thousands) are necessary. The non Monte Carlo, semi-analytical analysis methods do not suffer from these impediments and, therefore, their run-times are orders of magnitude shorter in comparison to Monte Carlo techniques.

ANALYSIS OF LOW-FREQUENCY NOISE IN SWITCHED MOSFET CIRCUITS: REVISITED & CLARIFIED

6.1 INTRODUCTION

The seminal paper by Tian *et al.* [8], published in 2001, elucidated that the *surplus noise power spectrum reduction*, observed when a periodically switching input or power supply is present in the circuit, has its roots in the inherent nonstationarity of random trap activity. The non-stationary stochastic behavior is caused by the dependence of the charge carrier *capture* and *emission rates* of traps on the voltages across the transistor. For instance, for an n-channel MOSFET in inversion, when the electron concentration in the channel is high, the traps located in the device will capture more electrons per unit time. In other words, the *capture rate* of traps will be high. Similarly, the capture rate will decrease while the emission rate increases, as the device is pushed from inversion towards cut-off.

The simple but illuminating analysis of random trap activity by Tian *et al.* [8] is set out to explain the *power spectrum reduction* of trap noise which was experimentally observed in transistors that are periodically cycled between inversion and accumulation. Although a mere 3dB reduction is expected as a result of a switched input to the circuit with a 50% duty cycle, the observed noise power reduction is oftentimes much higher [3, 4, 23]. In [8], this surplus noise reduction phenomenon is analyzed by building an idealized trap model tailored to reflect the behavior of traps under square-wave voltage excitation. The basic idea of *diminished trap activity* due to the off-cycles of the input is articulated mathematically and quantified by performing an auto-covariance analysis. This influential work, as a seminal approach to analyze trap noise based on a nonstationary stochastic framework, inspired further research in the area such as [10, 26].

In this chapter, we start, in Section 6.2, with the same idealization assumptions for traps as in [8] and follow the same analysis path. Along this path, we identify an oversight in the derivation presented in [8] for the average auto-covariance function of a single trap and its power spectral density. We then derive the correct power spectrum expression, verify it against detailed idealized trap simulations and discuss its implications. The idealized trap model is amenable to analytical

The content of this chapter has previously appeared in [116].

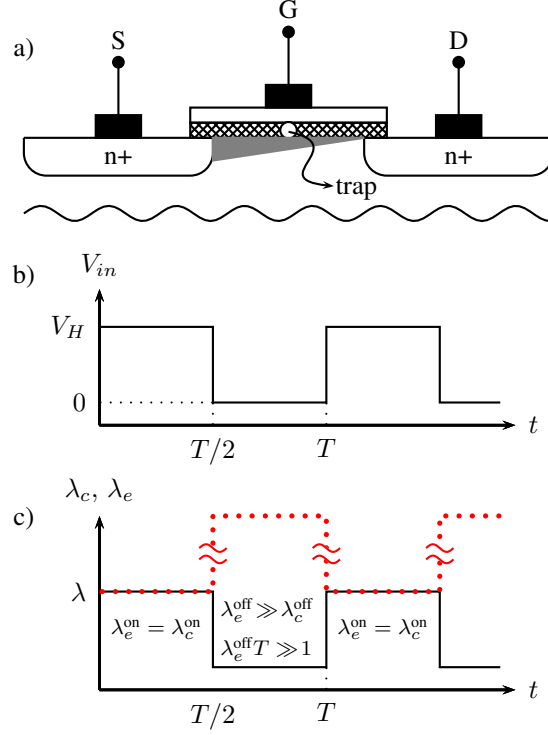


Figure 17: a) An n-channel MOSFET with a trap in its gate-oxide. b) the periodic input signal supplied to the gate node of the device. c) The resulting variation of the trap rates over time. λ_c^{on} and λ_e^{on} are the capture and emission rates of the trap when the transistor is in its on-state. Similarly, λ_c^{off} and λ_e^{off} are the event rates when the transistor is turned off.

derivations and very useful as a first stage in understanding nonstationary trap noise. However, it has inherent limitations because of its highly idealized assumptions on trap rates. This issue is discussed in Section 6.2.6 in a detailed manner. Next, in Section 6.3, we present a more accurate examination of nonstationary trap noise using realistic (non-idealized) physics based trap models. We demonstrate that noise simulations based on a detailed trap description implemented in a modern compact MOSFET model in a circuit simulator (such as the one described in Chapter 3) are needed for an accurate characterization of low-frequency noise in switched MOSFET circuits that matches experimental results. Finally, in Section 6.4, we review related work from the literature.

6.2 IDEALIZED TRAP NOISE IN A SWITCHED TRANSISTOR

6.2.1 Model Setup and Assumptions

In order to unveil the principles lying behind the inherent nonstationarity of trap noise, we will focus our attention on the noise signal gen-

erated by a single trap located in a transistor operating in the large signal regime. In doing so, we will retrace the steps taken by the authors in [8], correcting an oversight in their derivation along the way. We will present an account of the statistics of trap noise using the same setup as in [8]. This setup consists of a single transistor containing a single trap in its gate oxide as shown in Figure 17-a. A square wave voltage with a 50% duty cycle is applied to the gate node of the transistor (in a common-source configuration). This input voltage turns the transistor on and off periodically. The characterization of trap noise is achieved by computing the *time-averaged* PSD of the randomly changing state of the trap, which is represented by the cyclostationary stochastic process $N(t)$ that can assume only two values: full (1) or empty (0).

The following analysis employs an idealized trap model using the same idealization assumptions made in [8]. As depicted in Figure 17-c, this model assumes equal capture/emission rates ($\lambda = \lambda_c^{\text{on}} = \lambda_e^{\text{on}}$) when the transistor is on, i.e., when $V_{in} = V_H$. When the gate voltage goes to zero, it is assumed that the emission rate becomes much larger than the capture rate, large enough so that the trap goes to the empty state instantaneously and remains empty until the device turns on again. In the off state: $\lambda_e^{\text{off}} \gg \lambda_c^{\text{off}}$ and $\lambda_e^{\text{off}} T \gg 1$. The capture and emission rates are assumed to be memoryless functions of the gate voltage. With a square wave voltage input at the gate, the capture/emission rates are piecewise constant functions of time as shown in Figure 17-b. A further assumption in [8] is that there is a linear mapping between the noise current through the transistor (caused by the activity of a trap) and the random trap state process, $N(t)$, e.g., as given in [117, 17]. Therefore, we base our analysis on the statistics of the trap state alone. In what follows, we first compute the time-averaged ACF of cyclostationary $N(t)$ and calculate its Fourier transform to arrive at its time-averaged PSD.

6.2.2 Derivation of Switched Trap Noise Spectrum

In order to keep the notation simple, we first do the derivation below for one period of the switching input, i.e., when $0 \leq t \leq T$. We then obtain the general result by periodic replication.

We first define $p_1(t)$ as the probability that the trap is full at time t , i.e.,

$$p_1(t) = \mathbb{P}[N(t) = 1] . \quad (90)$$

Hence for a small time interval Δt , we have

$$\begin{aligned} p_1(t + \Delta t) &= p_1(t) \mathbb{P}[\text{no emission in } \Delta t] \\ &\quad + [1 - p_1(t)] \mathbb{P}[\text{capture in } \Delta t] . \end{aligned} \quad (91)$$

Since Δt is small and the inter-event times of $N(t)$ are *exponentially distributed*, the probability of an event in Δt equals to $\lambda \Delta t$ with $\lambda =$

$\lambda_e^{\text{on}} = \lambda_c^{\text{on}}$, i.e., the event rate of the trap when the transistor is switched on. Hence,

$$\frac{p_1(t + \Delta t) - p_1(t)}{\Delta t} = -2p_1(t)\lambda + \lambda, \quad (92)$$

and in the limit $\Delta t \rightarrow 0$,

$$\frac{dp_1(t)}{dt} + 2p_1(t)\lambda = \lambda. \quad (93)$$

The solution of (93) with the initial condition $p_1(0) = 0$ (meaning that the trap is empty initially) is given by

$$p_1(t) = \frac{1}{2}(1 - e^{-2\lambda t})u(t), \quad 0 \leq t < \frac{T}{2} \quad (94)$$

where $u(t)$ is the Heaviside step function. We note that the above expression is valid only for $0 \leq t < \frac{T}{2}$, since for $\frac{T}{2} \leq t < T$ the trap state assumes the value zero with probability one, i.e., $p_1(t) = 0$.

Now we compute the conditional probability that the trap is full at time $(t + \tau)$, given that it is full at t . For a non-negative τ , we have

$$\begin{aligned} p_{11}(t, \tau) &= \mathbb{P}[N(t + \tau) = 1 | N(t) = 1] \\ &= \mathbb{P}[\text{even number of events in } \tau] \\ &= \sum_{k=0}^{\infty} e^{-\lambda\tau} \frac{(\lambda\tau)^{2k}}{k!} = e^{-\lambda\tau} \frac{(e^{\lambda\tau} + e^{-\lambda\tau})}{2} \\ &= \frac{1}{2}(1 + e^{-2\lambda\tau}). \end{aligned} \quad (95)$$

Similarly,

$$\begin{aligned} p_{11}(t, -\tau) &= \mathbb{P}[N(t - \tau) = 1 | N(t) = 1] \\ &= \mathbb{P}[\text{even number of events in } \tau] \\ &= p_{11}(t, \tau) \end{aligned} \quad (96)$$

As such, p_{11} is an even function of τ (and independent of t), for positive as well as negative values of τ , we can write:

$$p_{11}(\tau) = \frac{1}{2}(1 + e^{-2\lambda|\tau|}), \quad 0 \leq t + \tau \leq \frac{T}{2}, t \geq 0. \quad (97)$$

The possible values the parameter τ can assume are restricted to a finite region. This is because both t and $t + \tau$ must lie in the *same* on-cycle of the input voltage, in $[0, T/2]$, for the derivation above. This was also pointed out in [10], where it is also mentioned that this makes the correlation of $N(t + \tau)$ and $N(t)$ undefined in other cases. On the contrary, this correlation is very well defined everywhere. When t and $t + \tau$ belong to different on-cycles, $N(t + \tau)$ is *uncorrelated* with $N(t)$, because the trap state is always reset to zero somewhere between the two time points in an off-cycle.

The symmetric auto-covariance function of $N(t)$ is given by

$$C_\lambda(t, \tau) = \mathbb{E}[N(t - \tau/2)N(t + \tau/2)] - \mathbb{E}[N(t - \tau/2)]\mathbb{E}[N(t + \tau/2)] . \quad (98)$$

Since, for a specific time point, $N(t)$ is a Bernoulli random variable, its expected value is the same as its probability of it being one. Based on (90) and (95), we can hence write

$$C_\lambda(t, \tau) = p_1(t - \tau/2)p_{11}(\tau) - p_1(t - \tau/2)p_1(t + \tau/2) \quad (99)$$

Moreover, because the ACF in (98) is symmetric in τ , we can evaluate it for the absolute value of the time difference. Using (94) and (97), we obtain

$$C_\lambda(t, \tau) = \frac{1}{4}(1 - e^{-2\lambda(t - |\tau/2|)})(1 + e^{-2\lambda|\tau|}) - \frac{1}{4}(1 - e^{-2\lambda(t - |\tau/2|)})(1 - e^{-2\lambda(t + |\tau/2|)}) \quad (100)$$

with the following condition on τ

$$\left| \frac{\tau}{2} \right| < t < \frac{T}{2} - \left| \frac{\tau}{2} \right| . \quad (101)$$

The expression in (100) simplifies to

$$C_\lambda(t, \tau) = \begin{cases} \frac{1}{4}(e^{-2\lambda|\tau|} - e^{-4\lambda t}) & \left| \frac{\tau}{2} \right| < t < \frac{T}{2} - \left| \frac{\tau}{2} \right| \\ 0 & \text{otherwise} \end{cases} \quad (102)$$

We note that the above ACF assumes the same form as in Equation (8) in [8] if one considers the case where the time difference is oriented only in the forward direction, i.e., $C_\lambda(t + \tau/2, \tau)$.

If $N(t)$ were a stationary stochastic process, its ACF would be independent of t . However, C_λ depends both on t and τ because of the nonstationary character of $N(t)$ in the switched input case. Moreover, at the beginning of the n^{th} on-cycle, i.e., $t = nT$, the trap is reset to its original unoccupied state and therefore, the ACF in (102) needs to be replicated periodically in t with period T . In other words, $N(t)$ is a (wide-sense) *cyclostationary* process.

The PSD of a stochastic process is given by the Fourier transform of its ACF with respect to the time difference τ . However, since C_λ in (102) is also a function of t , after the Fourier transform, we would get a time-varying frequency domain representation of the PSD. As mentioned in [8], if one were to put the noise signal $N(t)$ through a spectrum analyzer, one would get a time-averaged version of the time-varying PSD. The same effect can be achieved computationally by calculating the average value of $C_\lambda(t, \tau)$ over t and then Fourier transforming the resulting “stationary” ACF $C_\lambda^s(\tau)$. Although this is a straightforward operation, one has to be careful about the integration limits, because the

original ACF, $C_\lambda(t, \tau)$, is not non-zero everywhere for t in $[0, T]$ or for all values of τ . The time-varying ACF is non-zero only on a subset of $[0, T]$, the boundaries of which depend on τ as given in (101). In computing the time-averaged ACF in [8], these boundaries were overlooked and $C_\lambda(t, \tau)$ was integrated over the whole domain of a switching period. Hence, the resulting stationary ACF given in equation (10) in [8] is incorrect, which invalidates the rest of the analysis that is directly based on this result. The correct integration limits and the ensuing expression for $C_\lambda^s(\tau)$ are given as

$$C_\lambda^s(\tau) = \frac{1}{T} \int_{|\frac{\tau}{2}|}^{\frac{T}{2}-|\frac{\tau}{2}|} C_\lambda(t, \tau) dt \quad (103)$$

$$= \frac{1}{4T} \left[\left(\frac{T}{2} - \frac{1}{4\lambda} - |\tau| \right) e^{-2\lambda|\tau|} + \frac{e^{-2\lambda T}}{4\lambda} e^{2\lambda|\tau|} \right], \quad (104)$$

with the restriction on τ

$$|\tau| \leq \frac{T}{2}, \quad (105)$$

due to the fact that the covariance expression above is valid only if the two time points $t - \tau/2$ and $t + \tau/2$ are in the same on-cycle with width $T/2$. For $|\tau| > T/2$, $C_\lambda^s(\tau) = 0$ as explained before.

Now we can compute the Fourier transform of the time-averaged ACF $C_\lambda^s(\tau)$ to arrive at the PSD of the noise signal,

$$S_\lambda(\omega) = 2 \mathcal{F} \left\{ C_\lambda^s(\tau) \right\} = 2 \int_{-T/2}^{T/2} C_\lambda^s(\tau) e^{-j\omega\tau} d\tau, \quad (106)$$

where the factor 2 in front of the integral is there in order to compute the one-sided PSD. All PSD expressions and plots are for one-sided PSDs, i.e., the total power is distributed across non-negative frequencies.

The evaluation of the Fourier integral above results in

$$S_\lambda(\omega) = \frac{1}{2T(4\lambda^2 + \omega^2)} \left\{ 2\lambda T - e^{-2\lambda T} - \frac{12\lambda^2 - \omega^2}{4\lambda^2 + \omega^2} + \frac{8\lambda e^{-\lambda T}}{4\lambda^2 + \omega^2} \left[2\lambda \cos\left(\frac{T\omega}{2}\right) - \omega \sin\left(\frac{T\omega}{2}\right) \right] \right\} \quad (107)$$

which in turn can be partitioned as

$$S_\lambda(\omega) = \frac{S_\lambda^L(\omega)}{2} - S_\lambda^+(\omega). \quad (108)$$

Here,

$$S_\lambda^L(\omega) = \frac{2\lambda}{4\lambda^2 + \omega^2} \quad (109)$$

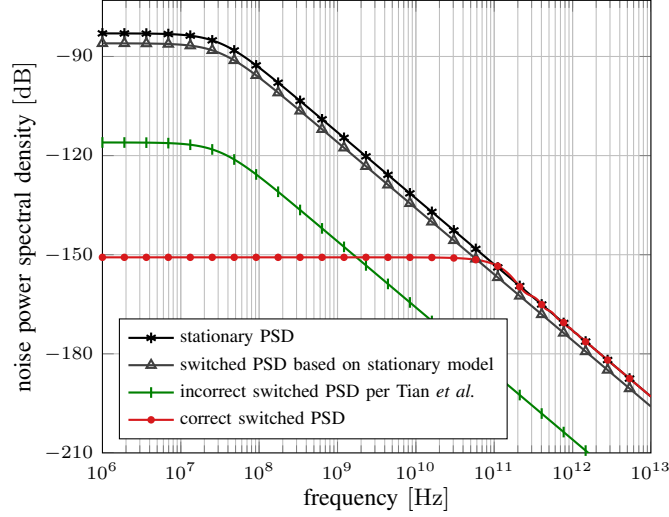


Figure 18: Trap noise spectrum: Stationary and switched.

is the *Lorentzian* PSD of a noise signal one would obtain for the *non-switched* case, i.e., when the input to the transistor in Figure 17 is a time-invariant voltage ($V_{in} = V_H$). Hence, the first term in (108) represents the 3 dB reduction in trap noise power predicted by the *stationary noise model*. The second term in (108) is the *surplus noise power reduction* given by

$$S_{\lambda}^{+}(\omega) = \frac{1}{2T(4\lambda^2 + \omega^2)} \left\{ e^{-2\lambda T} + \frac{12\lambda^2 - \omega^2}{4\lambda^2 + \omega^2} - \frac{8\lambda e^{-\lambda T}}{4\lambda^2 + \omega^2} \left[2\lambda \cos\left(\frac{T\omega}{2}\right) - \omega \sin\left(\frac{T\omega}{2}\right) \right] \right\} \quad (110)$$

6.2.3 Switched Trap Noise Spectrum and its Meaning

Figure 18 shows four different PSD curves computed for $\lambda = 10^8 \text{ sec}^{-1}$ and $T = 10^{-11} \text{ secs}$. These four PSDs, in the same order as in the legend in the figure, are (i) PSD for the non-switching, stationary case as given in (109), (ii) PSD for the switching case based on the simplistic approach where a modulated version of the stationary model is used, with a frequency independent 3 dB reduction, (iii) incorrect PSD for the switching case from Tian *et al.* [8, Eq. (11)], with a larger than 3 dB but still frequency independent reduction, (iv) correct PSD for the switching case as derived in (107). For $\lambda = 10^8 \text{ sec}^{-1}$ and $T = 10^{-11} \text{ secs}$ chosen here, we have $\lambda T = 10^{-3} \ll 1$, i.e., the switching frequency is much larger than the trap rates in the on-cycle. As such, the observations we make below for Figure 18 are valid only for this case, and do not apply to cases where $\lambda T \approx 1$ or $\lambda T \gg 1$, i.e., when the switching frequency is comparable to or smaller than the trap capture/emission rate. However,

the derivation that was carried out in Section 6.2.2 is completely general and captures these cases as well.

There are three important observations about the trap noise power reduction produced by the periodic switching of the input voltage as revealed by Figure 18. First of all, we observe a frequency dependent noise power reduction, which is mispredicted by the simplistic modulated stationary model as well as by Tian *et al.* [8] as frequency independent. The second important characteristic seen in Figure 18 is the location of the “corner” frequency. As opposed to a corner frequency at $f = \lambda/\pi$ as predicted by Tian *et al.* [8], the corner frequency of the actual PSD for trap noise is around $f = 1/T$, i.e., the switching frequency.

A little bit of tinkering in fact shows that the shift in the corner frequency makes a lot of sense. A statistically stationary trap in a non-switching transistor exhibits long time correlations, i.e., there is considerable correlation between the sampled states of a trap at two time points that are separated from each other by an amount that is less than the inverse of the capture/emission rate λ . This is reflected in the stationary spectrum in Figure 18, which decays rapidly above the corner frequency dictated by the capture/emission rates. When the transistor is switched on and off at a frequency $1/T$ much higher than the capture/emission rate, according to our ideal trap model, the trap state is perfectly reset in every switching period. As a result, the trap states sampled at two time points that are separated from each other by an amount that is larger than T become totally uncorrelated. In other words, the trap noise turns into white noise for time scales above the switching period T . This is reflected in the PSD given in Figure 18 for the switched case, which becomes flat, i.e., white, for frequencies below the switching frequency. This behavior is consistent with previously reported measurement results [6].

In summary, the switched bias for the transistor has a *whitening* effect on the trap noise, as opposed to a frequency independent power reduction as predicted by Tian *et al.* [8]. Finally, we should note that the whitening and also any noise reduction effect of switched biasing disappears altogether at frequencies above the switching frequency. Indeed, as seen in Figure 18, above the corner frequency, the time-averaged PSD for switched bias falls on top of the PSD for the non-switching case, and *not* on the one that is 3 dB below. Thus, even though the surplus noise reduction $S_\lambda^+(\omega)$ in (110) takes large positive values at low frequencies below the switching frequency, it becomes *negative* at high frequencies, i.e., $S_\lambda^+(\omega) \approx -\lambda/\omega^2$ for $\omega \gg 1/T$ for the case where $\lambda T \ll 1$ considered above.

6.2.4 Validation via Monte Carlo Simulations

We now validate the analytical derivations in Section 6.2.2 for the trap noise spectrum in a switching transistor via carefully run simulations.

In doing so, we first run extensive Monte Carlo simulations based on the idealized trap model to generate sample paths for the trap state, then estimate the trap noise spectrum based on these sample paths, and finally compare the estimated spectrum against the full expression in (107). Listing 3 shows the MATLAB[®] code for the main loop of a computer program that simulates one realization of a trap state time series. The Monte Carlo simulation is based on the ideal trap model described in Section 6.2.1, and at this stage, does not incorporate any computations related to other transistor dynamics. Here, we are interested in the evolution of the occupancy state, $N(t)$, of a single trap which has piecewise constant capture/emission rates.

```

% initialize variables
for i=1:nCycle % nCycle: number of input cycles
    t = (i-1)*T; % t: time, T: period of input

    while t - (i-1)*T - T/2 < -t*eps
        ieTime = exprnd(1/lambda); % inter-event time

        t = t + ieTime; % stop at next event
        if t - (i-1)*T >= T/2 % is the transistor off?
            t = (i-1)*T + T/2; % push time forward
            N = 0; % set trap state to empty
        else % if the transistor is on
            N = 1 - N; % flip trap state
        end

        tVec = [tVec, t]; % keep record of time
        NVec = [NVec, N]; % keep record of trap state
    end
end
end

```

Listing 3: MATLAB[®] code for idealized trap simulation.

Figure 19 shows the time-averaged trap noise PSD estimated from the simulated sample paths of the trap occupancy process, $N(t)$, generated using the idealized trap model. In order to estimate the PSD efficiently (in a Monte Carlo manner), we have simulated one thousand sample paths in parallel, each of them long enough to capture the low-frequency spectral components. We then computed the PSDs of individual sample paths using Welch's method [111]. Applying Welch's method on a sample path of a *cyclostationary* stochastic process estimates the *time-averaged* PSD, which converges to the actual time-averaged PSD as the sample path duration is increased. We then computed an ensemble average over the set of one thousand independently computed PSDs in order to obtain the smooth curve shown in Figure 19 along with a plot of the analytical PSD expression in (107). At low frequencies, the two curves fall almost exactly on top of each other. At frequencies around the corner and beyond, the match is still excellent apart from some "estimation noise" in the PSD curve computed from the simulation data.

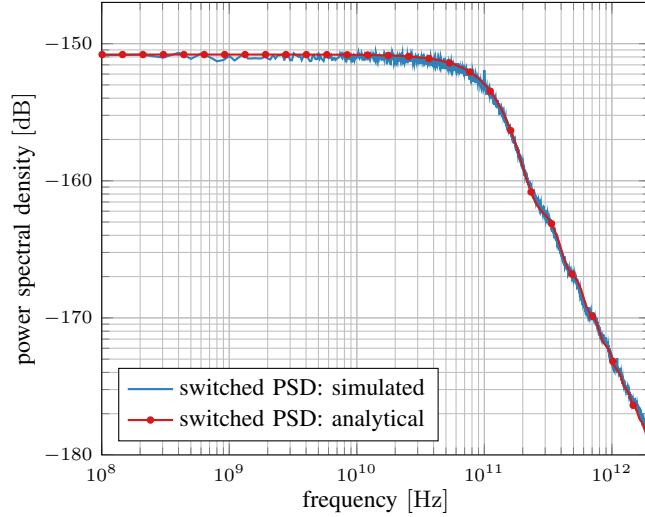


Figure 19: Trap noise spectrum: Validation via Monte Carlo simulations.

One can even discern the slight “wiggles” in the estimated PSD curve that perfectly match the ones due to the $\cos(\cdot)$ and $\sin(\cdot)$ in (107).

6.2.5 $1/f$ Noise in a Switched Transistor

We now turn our attention to the analysis of the periodically switched operation of a noisy transistor which contains not only one but a multitude of traps. As shown in [9], $1/f$ noise can be construed as a superposition of trap noise generated by individual defects in the same device with widely varying capture/emission rates. This procedure can be formalized by introducing a *distribution function*, $g(\lambda)$, for trap rates as it is done in [8] and then integrating over the contributions of a continuum of traps weighted with this distribution function. We use the same rate distribution function given in [8, Eq. (1)] and integrate over the power spectral density in (107).

$$S_{1/f}(\omega) = \int_{\lambda_L}^{\lambda_H} g(\lambda) S_{\lambda}(\omega) d\lambda \quad (111)$$

Figure 20 shows the result of this computation, again for $T = 10^{-11}$ secs. The integration limits, λ_L and λ_H , were chosen such that the whole frequency range of interest in Figure 20 was covered.

The first of the four curves in Figure 20 (with asterisk marks) is the $1/f$ noise spectrum one would obtain without periodic switching of the input voltage. In other words, if one replaces $S_{\lambda}(\omega)$ in (111) with the Lorentzian PSD $S_{\lambda}^L(\omega)$ in (109), the result of the integration is a curve that decays with a constant 10 dB/decade slope. The second curve (with triangular marks) is 3 dB below the first curve, obtained with the stationary model for the switched input case with a 50% duty

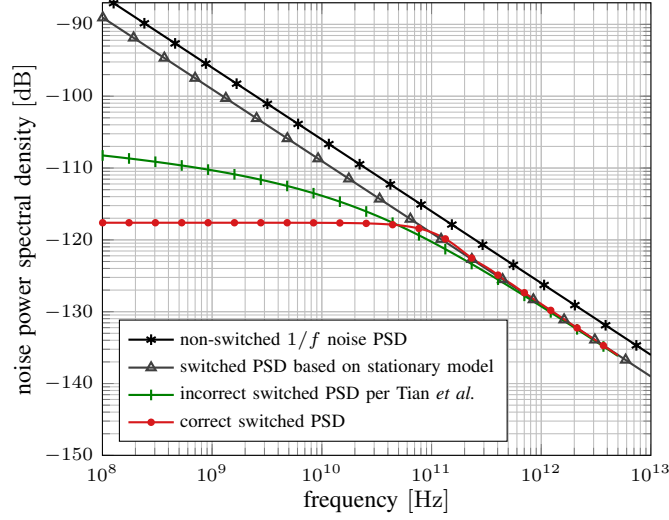


Figure 20: $1/f$ noise spectrum due to a multitude of traps.

cycle. This curve in fact represents the asymptote for the remaining two PSD curves for high frequencies. That is to say, the stationary model, the idealized switching model per Tian *et al.* (green, straight marks) and the idealized switching model (red, dot marks) developed in this chapter all produce the same result (3 dB reduction) for high frequencies. This behavior is expected, because the noise power in this frequency range is predominantly generated by traps with high capture/emission rates. If the trap rate λ is much higher than the switching frequency, the trap operates in a *quasi stationary* mode, i.e., reaches stationary state very quickly, within the on-cycle, and the surplus noise reduction effects disappear. On the other hand, the real noise reduction effect is observed at frequencies lower than the switching frequency, and this is the region where the incorrect PSD computed in [8] differs from the actual, corrected result which shows larger noise reduction at lower frequencies. We note that, since the corner frequency of the correct switched PSD in (107) is independent of λ , the integration operation in (111) does not change the location of the corner. The resulting $1/f$ noise time-averaged PSD retains the same form as the PSD in the single trap case, representing the maximum achievable noise reduction due to bias switching under idealized conditions.

6.2.6 Limitations of the Idealized Trap Model

In a physically accurate description of traps, it is hard to justify the conditions imposed on the trap capture/emission rates in the idealized model. The assumption that a trap immediately jumps to the empty state when the transistor is switched off is clearly violated in physical systems. For this condition to hold, the time span in which the transistor remains in its off-state has to be much larger than the average

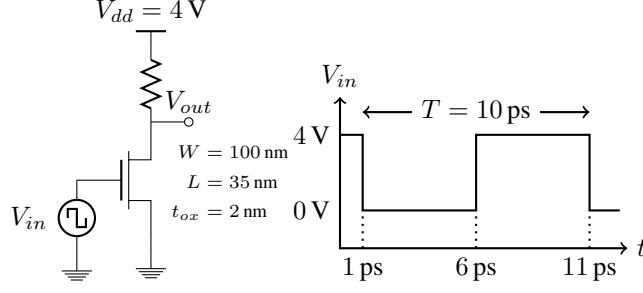


Figure 21: A nano-scale transistor operated in common source configuration with a periodically switched input voltage, V_{in} .

emission time of the trap. This corresponds to the relationship between the switching period of the input signal and the emission rate of the trap given in Figure 17: $\lambda_e^{\text{off}} T \gg 1$. However, for a nano-scale transistor with a fast switching input at GHz frequencies, this inequality generally does not hold because traps with an accordingly large *capture cross section* and hence transition rate are rare [75]. Additionally, the required relationships of λ_c and λ_e , both in the on and off-state, are not guaranteed to be satisfied in realistic systems. The equality assumption of the transition rates in the on-state, $\lambda_c^{\text{on}} = \lambda_e^{\text{on}}$, has no sound physical motivation and the condition that $\lambda_e^{\text{off}} \gg \lambda_c^{\text{off}}$ is only partially backed up by measurements results [7, 118].

Results obtained with the idealized trap model constitute a baseline for the analysis of noise reduction phenomena in switched circuits since the idealization assumptions of this model produce the maximum achievable noise reduction. Moreover, the analytical form of these results enables one to easily compare the outcome of more realistic models with the established baseline. However, it is clear that these results need further refinement in order to be used in a practical setting. To this end, in Section 6.3, we offer a comparison of the idealized trap model with the physically accurate and realistic trap model we have developed in this thesis.

6.3 TRAP NOISE SIMULATIONS BASED ON THE REALISTIC MODEL

In order to conduct a more detailed investigation of the case at hand, we will now use the realistic trap model (RTM) we have developed in this thesis. We next present results obtained by the non-ideal, realistic trap model and the noise analysis techniques mentioned above. In order to make a one-to-one comparison between the simulation results and the analytically derived results for the ideal trap case, we report noise analysis results for the trap variable $N(t)$. The circuit setup that was simulated is the same as the one considered in Section 6.2.1 for the idealized trap model as shown in Figure 21.

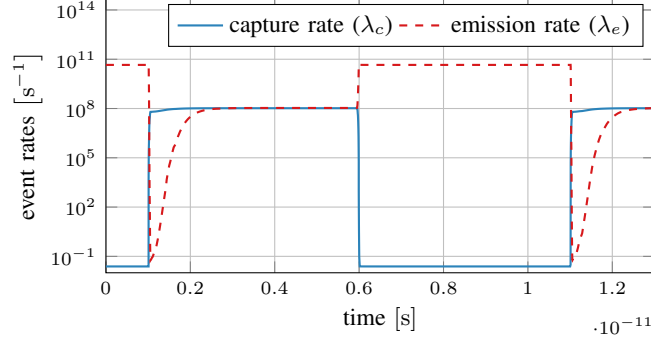


Figure 22: Capture and emission rates of a trap embedded in the realistic transistor model, simulated in a common source configuration.

Figure 22 shows a plot of the capture and emission rates, periodic functions of time, for the single trap in the MOSFET. The capture/emission rates are determined by the trap/device parameters and the gate/power supply voltages. In order for the comparison with the idealized model to be meaningful, for the simulations presented in this section, the trap parameters were chosen in such a way so that the capture and emission rates become equal (with a value 10^8 sec^{-1} as in the ideal trap model) at DC steady-state when the gate voltage is fixed at the high value. The specific values of these parameters are as follows: $\bar{\sigma} = 3.37 \times 10^{-23}$, $E_{T0} = 2.45 \times 10^{-19}$, $x_T = 0$, $y_T = 0.5L$. The non Monte Carlo, harmonic balance based noise analysis technique described in Chapter 5 can perform noise analysis on circuits with periodic excitations. This method can be applied to the switched circuit setup treated in this chapter and, as such, internal trap dynamics in the circuit can be computed and observed.

We now make some key observations for the results presented in Figure 22. The capture/emission rates exhibit transient behavior after the gate voltage switches instantaneously from the off to the on value, which were assumed instantaneous in the idealized trap model. The emission rate increases by almost three orders of magnitude when the transistor turns off while the capture rate decreases dramatically by more than nine orders of magnitude. The transient behavior observed in λ_e is due to the dependence of E_T in Equation (38) on the drain voltage of the transistor. This dependence causes the device characteristics (resistance, capacitance) to come into play and the emission rate to exhibit a complex dynamical behavior.

The settled value of the emission rate in the off-state is $\lambda_e^{\text{off}} = 4.52 \times 10^{10} \text{ sec}^{-1}$. While it is extremely unlikely (with probability $1 - e^{-\lambda_e^{\text{off}} T/2} \approx 10^{-13}$) that an empty trap captures a charge carrier in the off-state, it is quite likely (with probability $e^{-\lambda_e^{\text{off}} T/2} \approx 0.8$) that a full trap stays full through the off-state. In other words, the trap state is not always reset to the empty state when the gate voltage switches instantaneously from the on to the off-state. The trap emission rate in the off-state is

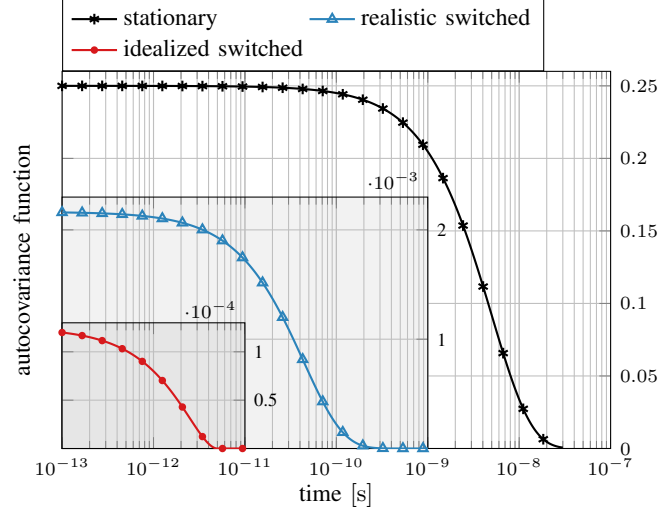


Figure 23: The average trap auto-covariance function: Stationary and switched. Note the magnified y-axes in the two inset plots (light and dark gray backgrounds).

not large enough to guarantee that, at the beginning of the next on-cycle, the trap is empty. In fact, a full trap at the end of an on-cycle may stay as full through the subsequent off-cycle and the beginning of the next on-cycle. As a result, the trap states sampled at two time points that are separated from each other by more than a switching period become correlated, as opposed to the zero correlation one gets in the simple, idealized model. However, the degree of correlation for these samples is less than the case when the transistor is always on. These observations are confirmed by the results presented in Figure 23, which shows the time-averaged auto-covariance functions for these three cases: (i) For a stationary trap when the gate voltage is fixed at the high value, corresponding to the PSD in (109), (ii) for a switched trap, based on the non-ideal realistic model, obtained by RTM-noise analysis algorithm (NAA), (iii) for a switched trap based on the idealized model, as given in (104). The observations and results above are consistent with a similar qualitative discussion that was given in [6] on the correlation characteristics of cyclo-stationary RTS noise.

Figure 24 shows four different PSD curves computed for $\lambda = 10^8 \text{ sec}^{-1}$ and $T = 10^{-11} \text{ secs}$: (i) PSD when the gate voltage is fixed at the high value, resulting in a stationary trap. (ii) Time-averaged PSD computed by RTM-NAA. (iii) Correct PSD for the switching case as derived in (107) based on the idealized trap model. (iv) Incorrect PSD for the switching case from Tian. *et al.* [8, Eq. (11)]. As seen in Figure 24, with the realistic, non-ideal trap model, the whitening and noise reduction effect of switched biasing at low frequencies is still there, but it is not as pronounced as the one predicted by the idealized trap model considered in Section 6.2. The corner frequency for the switched trap spectrum with the realistic trap model is *lower* compared with the corner at the

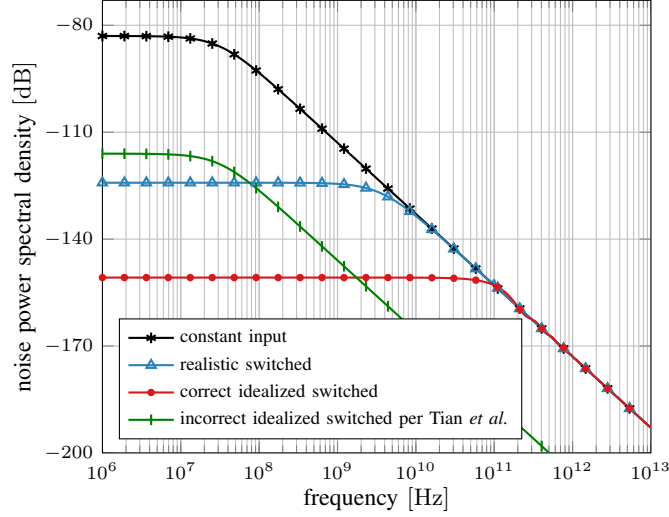


Figure 24: Trap noise spectrum: Simulation based on a realistic trap model.

switching frequency predicted by the ideal model. In fact, the lower corner frequency predicted by the realistic model is determined essentially by λ_e^{off} , i.e., the emission rate in the off cycle. This result is consistent with an analytical expression derived in [10]. If we rewrite Eqn. 43 in [10] keeping in mind that the average emission rate is much larger than the average capture rate, we obtain

$$S_\lambda(\omega) = \frac{2\lambda}{(\lambda_e^{\text{off}}/2)^2 + \omega^2}, \quad (112)$$

where $\lambda = \lambda_c^{\text{on}} = \lambda_e^{\text{on}}$. In the non-idealized, realistic case, the corner frequency of the reduced PSD and hence the amount of noise power reduction is not determined solely by the switching frequency anymore. This is because if a physically realistic λ_e^{off} is used, the trap is not guaranteed to be reset to its empty state when the transistor is turned off. The resetting process assumes a probabilistic nature and the probability of the trap getting reset is dependent on the value of λ_e^{off} . The above suggests that the whitening and noise reduction effect of switched biasing can be enhanced if the emission rate in the off cycle, λ_e^{off} , can be made larger while keeping the capture rate small and the transistor in the off condition.

Another way of controlling the amount of noise reduction is to keep the capture/emission rates constant and change the switching frequency of the input voltage. Again, we examine the realistic trap model with rates given in Figure 22. Figure 25 depicts the effect of changing the switching frequency on the observed reduction of the noise power. The amount of noise reduction was computed by comparing the values of the noise PSD at the low-frequency asymptotes of both the switched and constant input cases. The three curves in Figure 25 provide the comparison of the noise reduction predicted with the realistic model to the idealized model developed in this chapter and the analytical expression derived

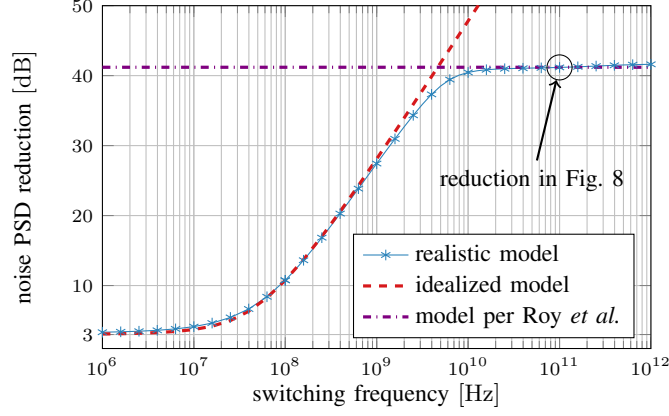


Figure 25: Switched input CS amplifier. Dependence of noise reduction on the switching frequency (in comparison to the non switched case). The point corresponding to the setup in Figure 24 is marked with a circle. The simulations for the realistic trap model were conducted using RTM-NAA.

in [10]. Figure 25 shows that, for values of the switching frequency much lower than the trap rates in the on-state, $\lambda_c^{\text{on}} = \lambda_e^{\text{on}} = 10^8$ Hz, the noise reduction is limited to the 3 dB expected due to the 50% duty cycle of the input voltage. As the switching frequency is increased, the reduction becomes more prominent and, finally, saturates at around a switching frequency equal to λ_e^{off} . The idealized model developed in this chapter can capture the amount of noise reduction up to those switching frequencies where the assumption $\lambda_e^{\text{off}} T \gg 1$ is still valid. On the other hand, the result according to the model of Roy *et al.* [10, Eqn. 43]) is valid at higher switching frequency values because it assumes that event rates for all t are much lower than the switching frequency, i.e., $\lambda(t)T \ll 1$. Finally, Figure 25 shows that the realistic model together with RTM-NAA captures both low and high switching frequency regimes. The idealized model developed in this chapter and the trap model of [10, Eqn. 43]) serve as low and high switching frequency approximations, respectively, to the more accurate realistic trap model.

The realistic model can be used in analyzing the noise produced by not only one trap but also by a multitude of traps. Using CIRSIUM, one can place several traps in one transistor. If the rates of the traps span multiple orders of magnitude on the frequency axis, the resulting noise will exhibit a $1/f$ type behavior [1]. Figure 26 shows the results of such a simulation using RTM-NAA. The simulation was conducted for a transistor in switched input common source configuration that contains 10 traps. In this simulation, the trap parameters used to produce Figure 22 were reused except for the capture cross section parameter, $\bar{\sigma}$. The capture cross section of each trap was adjusted so that the capture/emission rates in the on-state span 10 orders of magnitude on the frequency axis (one decade for each trap). This way, the on-state rates of the traps vary between 10^8 sec^{-1} to 10^{-1} sec^{-1} . This configuration of

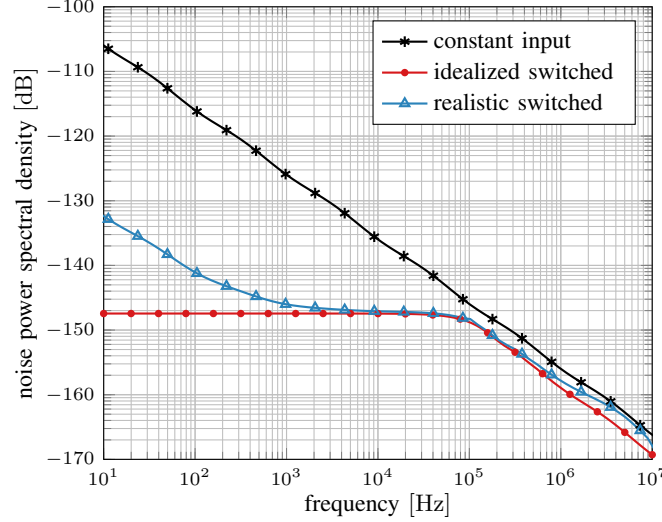


Figure 26: The “re-appearance of $1/f$ noise” in a switched transistor with 10 traps. The three curves show the PSD of the output voltage at the drain end of a transistor in common source configuration. The capture emission rates of the traps are spread out between 1 sec^{-1} and 10^8 sec^{-1} and the switching frequency is 10^5 Hz . The results for *constant input* and *realistic switched* cases were obtained by RTM-NAA simulations.

one trap corner for each decade of frequency was found to be sufficient to produce $1/f$ noise [1]. Indeed, the black curve (asterisk marks) in Figure 26, which is the output voltage noise PSD in the constant input case, exhibits the expected $1/f$ characteristic. The blue curve (triangle marks), on the other hand, is the reduced noise PSD of the output voltage in the switched input case. The switching frequency was set to 10^5 Hz where the corner in the figure appears. In this case, we observe a whitening effect below the switching frequency just like the single trap setup. However, at lower frequencies, we observe “a re-appearance of $1/f$ noise” which is consistent with the measurements and simulations reported in the literature [119, 23]. This behavior of the noise PSD at very low frequencies is called *resurgence of $1/f$ noise* and investigated in detail in [119].

In fact, the idealized model of Section 6.2 and equation (112) together provide an intuitive explanation for this behavior. If we assume that for all traps $\lambda_e^{\text{off}} > \lambda = \lambda_c^{\text{on}} = \lambda_e^{\text{on}}$, the set of 10 traps located in the transistor can be partitioned into two subsets. First subset is constituted by those traps that have large enough capture cross sections as to result in λ_e^{off} values that are higher than the switching frequency. The remaining traps with lower λ_e^{off} values belong to the second subset. According to the idealized model (red curve, dot marks in Figure 26), the traps in the first group exhibit a flat noise PSD up to the switching frequency. On the other hand, according to (112), the PSDs of the traps in the second group are only flat up to their respective emission rates. Therefore,

while the first group contributes to the “whitened” part of the noise PSD in Figure 26, the second group is responsible for the re-appearance of $1/f$ noise at very low frequencies.

6.4 RELATED WORK

Throughout our treatment in this chapter, we have described in detail how the results we have obtained are related to the ones obtained by Tian *et al.* [8]. We next review and comment on other related work, even though we certainly do not claim to be performing a comprehensive literature survey in doing so, which is beyond the scope of this work. We include work that we believe is closely related to our treatment here.

Roy *et al.* [10] consider a single trap with periodic capture/emission rates that are much smaller than the fundamental frequency of periodicity, and derive a simple analytical result for the time-averaged PSD in the form of a Lorentzian-like spectrum that depends on the time averages of capture and emission rates. This result that was obtained by a rigorous derivation in [10] is quite valuable and matches the results obtained by RTM-NAA in the restricted setting considered. However, even though the simple result in [10, Eqn. 43] for arbitrary periodic capture/emission rates is naturally valid for piecewise constant rates due to a square-wave excitation, it can not be used to arrive at the results we have obtained in Section 6.2 with the ideal trap model. In the ideal trap model, it is assumed that the emission rate in the off state is very large, i.e., $\lambda_e^{\text{off}} T \gg 1$, whereas the simple result in [10] is valid when $\lambda_e(t) T \ll 1$ and $\lambda_c(t) T \ll 1$ for all t . On the other hand, Roy *et al.* [10] do first analyze the general case before assuming $\lambda(t) T \ll 1$. However, the result they obtain in the general setting does not seem to be easily manipulable to arrive at a simple expression for the ideal trap model with $\lambda_e^{\text{off}} \rightarrow \infty$.

The two contemporaneous Ph.D. dissertations by Kolhatkar [6] and van der Wel [27], on the other hand, have their strength in the valuable empirical measurement work performed in various settings. The most interesting results of van der Wel, related to our findings, are the outcomes of numerical calculations conducted for cyclostationary RTS noise. These results are summarized in [27, Fig. 4.8] where three different bias switching setups are simulated and compared with their analytical counterparts. The last one of these curves [27, Fig. 4.8 (E)], corresponds to the case where, in the off state of the transistor, the capture rate becomes zero and the emission rate becomes infinite. This is exactly the configuration we call the *idealized model* in this chapter and the missing analytical result in [27] for this case is derived in our work, given by equation (107). The Ph.D. dissertation by Kolhatkar [6] also makes valuable contributions by observing the dependence of RTS noise reduction on the amplitude of the periodically switched input signal and the non-instantaneous nature of the time evolution of capture/emission

rates. The semi-empirical RTS noise model developed by Kolhatkar [6] can be used to conduct a basic transient simulation of nonstationary trap behavior with time-varying rates.

EXPOSITION OF INTRICATE NONSTATIONARY BEHAVIOR OF LOW-FREQUENCY NOISE AND COMPARISON WITH THE LEGACY MODEL

7.1 INTRODUCTION

In Chapter 5, we have described a noise analysis framework which will relieve designers from the requirement of conducting laborious Monte Carlo simulations and implemented fully nonstationary, non Monte Carlo techniques for low-frequency noise. We have shown how trap SDEs can be construed as augmentations to the usual circuit equations and how noise sources can be represented as pseudo-nodes inside the transistors that are coupled to the rest of the circuit nodes. In this manner, we were able to mold our formulations into a mathematical form that conforms with the existing, well-established non Monte Carlo simulation techniques which are orders of magnitude faster than their Monte Carlo counterparts. In this chapter, we bring our research program into its full fruition. We present a case study where results of non Monte Carlo noise analyses conducted with our new noise model are contrasted to the analyses based on the legacy modulated stationary model for $1/f$ noise. As our test setup, we use the application of the switched biasing technique to the coupled sawtooth oscillator circuit and compare our results to the previously reported measurement data for this circuit [11]. We show that while the modulated stationary model fails to predict the noise reduction obtained by switched biasing, the analyses based on our noise model and techniques match the measured data quite well.

7.2 LEGACY MODULATED STATIONARY MODEL

The low-frequency noise model discussed in Chapter 5 derives its nonstationary representation capability through the two-way coupling of noise sources to the rest of the circuit and by capturing the internal dynamics of the noise source. The same is not true for low-frequency noise models that have been in use [13]. One commonly used such model is the *modulated stationary noise model*. This model can be realized using a frequency domain characterization for low-frequency noise through its PSD, i.e., by generating a noise signal with the desired spectrum. This characterization is by definition stationary. In this case, the nonstationarity is introduced to the system by modulating the stationary noise source with a modulation signal $m(t)$ that is usually a function of the internal circuit variables such as the gate voltage of a transistor. The modulation is supposed to capture the effect of the time-varying bias on

the statistics of the noise source. However, this technique does not capture the internal, nonstationary dynamics of the noise sources and for this reason fails to predict effects that arise from the true nonstationary nature of low-frequency noise [8].

The modulated stationary model can be implemented into simulators with or without introducing extra equations (and pseudo-nodes) to the MNA structure of the circuit. For the former case, the noise signal is generated directly in the noisy device and has a preset PSD. For instance for a noisy resistor, the constitutive equations of the device assume the form

$$I = R(v_1 - v_2) + m(t)\gamma_t, \quad (113)$$

where R is the resistance value of the component, I , the current flowing through it and v_1 and v_2 are the voltages at its first and second terminals. The noise process γ_t can for example be a “pink noise” signal with a $1/f$ spectral density and the modulation signal $m(t)$ is a function of the terminal voltages. If one refrains from introducing extra noise variables and generates a noise signal through prescribing a PSD, this model can be directly used in conjunction with non Monte Carlo analysis methods that operate in the frequency-domain. In order to employ time-domain methods as well, a similar equation set as in (71) has to be constructed. The general structure of this set would be

$$\begin{aligned} \frac{\partial}{\partial t} \mathbf{q}(\mathbf{v}(t), \mathbf{m}(t)\mathbf{N}_t) + \mathbf{f}(\mathbf{v}(t), \mathbf{m}(t)\mathbf{N}_t) + \mathbf{b}_s(t) &= \mathbf{0} \\ d\mathbf{N}_t &= \boldsymbol{\alpha} dt + \boldsymbol{\beta} \boldsymbol{\eta}_t, \end{aligned} \quad (114)$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are constant vectors and $\boldsymbol{\eta}_t$ is a vector of independent white noise processes. For comparison purposes, the modulated stationary model was also implemented in our simulation framework in the form of equation (114). In Section 7.3, we will conduct a detailed mathematical analysis that exposes the properties and limitations of this model. We will also investigate a case where the simulation results produced by the new nonstationary model and the legacy modulated stationary model differ significantly.

7.3 COMPARISON WITH THE LEGACY MODEL

In this section, we illustrate how the legacy modulated stationary model fails to capture effects created by the true nonstationary nature of low-frequency noise. To this end, we examine the influence of the switched biasing technique on the low-frequency noise generated by a single transistor. Switched biasing has been the setup of choice in the literature to demonstrate the consequences of the nonstationary behavior of low-frequency noise [3, 4, 11, 23, 120]. As explained in detail in Chapter 6, in this technique, the noisy transistor is switched off for a period of time during circuit operation. Since the behavior of low-frequency noise is

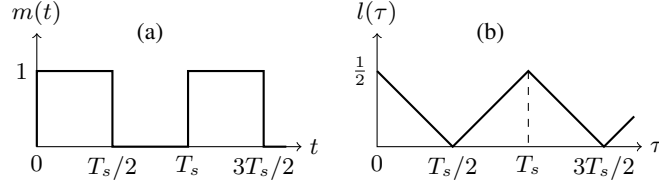


Figure 27: a) Modulation signal for the noise process with a 50% duty cycle.
b) Modulation signal for the average ACF.

highly dependent on the operating point of the transistor, with switched biasing, a certain amount of noise power reduction is expected. However, the legacy model mispredicts this amount [23, 6, 8].

In [8] and in Chapter 6, the amount of noise power reduction predicted by the modulated stationary model was simplistically assumed to be 3 dB at all frequencies for the case when the transistor is held in the off-state half of the time, since the noise signal is switched off in that period. In this section, we present a mathematically precise analysis of the modulated stationary model and show that, in fact, the amount of noise power reduction predicted by this model is frequency dependent and equals to 6 dB (instead of 3 dB) at frequencies much lower than the switching frequency. We then show that the actual reduction is, in fact, more than 6 dB, not predicted by the modulated stationary model for reasons explained in detail in Chapter 6. We present a $1/f$ noise simulation result based on the new, correct model for a single transistor in a common source configuration which is turned on and off periodically with a 50% duty cycle.

7.3.1 Noise Reduction: Switched Biasing based on Legacy Model

In order to arrive at an analytical expression for the amount of noise power reduction predicted by the modulated stationary model we conduct a similar treatment to the one in Section 6.2.2. In particular, we assume that the stationary noise employed by this model has the general PSD $S(f)$ and compute the following quantities:

1. The ACF, $C_m(t, \tau)$, of the noise generated by the modulated stationary model.
2. The average value of $C_m(t, \tau)$ over t , i.e., $\bar{C}_m(\tau)$.
3. The Fourier transform of the average ACF, $\bar{C}_m(\tau)$.

The symmetric autocorrelation function of a stochastic process $n(t)$ is defined by

$$C(\tau) = \mathbb{E} [n(t - \tau/2)n(t + \tau/2)]. \quad (115)$$

If the noise signal $n(t)$ is modulated, i.e., multiplied, with the signal given in Figure 27-a, with the switching period T_s and the switching

frequency f_s , we obtain the ACF for the modulated noise source $m(t)n(t)$ as follows

$$\begin{aligned} C_m(t, \tau) &= \mathbb{E}[n(t - \tau/2)m(t - \tau/2)n(t + \tau/2)m(t + \tau/2)] \\ &= \mathbb{E}[n(t - \tau/2)n(t + \tau/2)] m(t - \tau/2)m(t + \tau/2) \\ &= C(\tau)l(t, \tau) \end{aligned}$$

$C_m(t, \tau)$ does not only depend on the time difference between the correlated samples, i.e., τ , but also on the point of time t . In order to arrive at a PSD as a spectrum analyzer device would measure, we compute the average ACF over a switching period T_s .

$$\bar{C}(\tau) = \frac{1}{T_s} \int_0^{T_s} C(\tau)l(t, \tau)dt = C(\tau)\bar{l}(\tau) \quad (116)$$

Equation (116) shows that the modulation of the noise signal with $m(t)$ gives rise to the modulation of its time averaged ACF with a different signal $\bar{l}(\tau)$ depicted in Figure 27-b.

The Fourier transform of $\bar{C}(\tau)$ is the convolution of the Fourier transform of $C(\tau)$, i.e., $S(f)$ and the Fourier transform of $\bar{l}(\tau)$. The modulation signal $\bar{l}(\tau)$ can be represented with its Fourier coefficients as follows.

$$\bar{l}(\tau) = \frac{1}{4} + \frac{2}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos\left(\frac{2\pi n}{T_s}\tau\right) \quad (117)$$

Therefore, for its Fourier transform, $\bar{L}(f)$, we have

$$\bar{L}(f) = \frac{1}{4}\delta(f) + \frac{1}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} [\delta(f - nf_s) + \delta(f + nf_s)]$$

where δ stands for the Dirac delta function. Finally the convolution of the original PSD $S(f)$ and $\bar{L}(f)$ gives us the time averaged PSD of the noise signal generated by the modulated stationary noise model.

$$S_m(f) = \frac{1}{4}S(f) + \frac{1}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} [S(f - nf_s) + S(f + nf_s)]$$

In frequencies much lower than f_s , the first term in $S_m(f)$ dominates, considering the fact that $S(f)$ represents a noise source with power concentrated at low frequencies. Thus, we have a 6 dB ($10\log(1/4)$) reduction in noise power. However, for frequencies greater than f_s , the influence of the remaining terms becomes significant and they add up to a PSD that is approximately 3 dB below $S_m(f)$.

7.3.2 Noise Reduction: Legacy Model vs. Nonstationary Model

In order to verify the mathematical analysis presented in the previous section, we use the implementation of the modulated stationary model

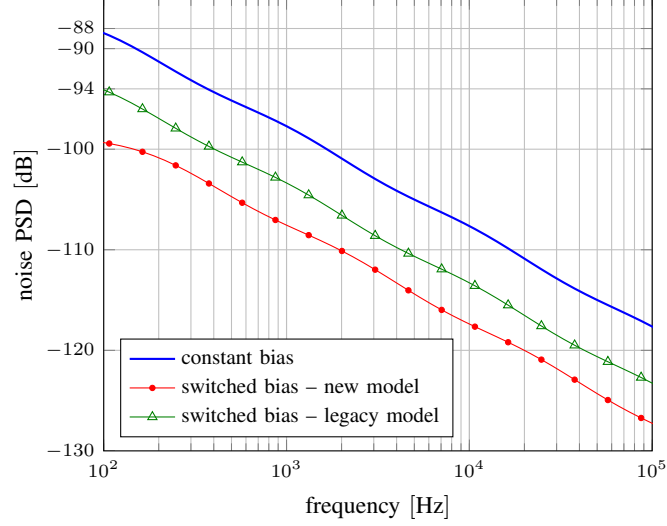


Figure 28: $1/f$ noise reduction in the common source amplifier via switched biasing. Comparison of the legacy modulated stationary model to our fully nonstationary noise model.

in our simulation framework, CIRSIUM. As the test circuit, we examine the single transistor in common source configuration depicted in Figure 14. The circuit is operated with a periodically switched input signal as before (switched biasing) with a frequency of $f_s \approx 16$ MHz. The transistor contains a multitude of traps with transition rates spread out to several decades in frequency. This configuration of traps is known to produce a $1/f$ spectrum [1]. Indeed, the blue curve (no marks) in Figure 28 exhibits a 10 dB/decade roll-off. This curve is the simulated noise PSD (time averaged) for the constant bias case, when the gate voltage is held fixed at the high value. The other two curves are results of simulations under switched biasing with the two competing noise models. The green curve (triangle marks) is the noise PSD generated using the legacy model and lies, as expected, 6 dB below the constant bias PSD. The red curve (dot marks) is simulated using our fully nonstationary model and exhibits a greater noise reduction than the one predicted by the modulated stationary model. This is in accordance with numerous measurement results reported in the literature [11, 23, 120]. The underlying mechanism for this behavior is rooted in the internal, individually different, nonstationary dynamics of trap activities that sum up to $1/f$ noise and is explained in detail in Chapter 6. As a final remark, we note that a general analytical treatment for the truly nonstationary model is intractable. In the case of a single transistor circuit, a revealing analytical derivation was presented in Chapter 6, for more complicated circuits however, one has to resort to numerical methods as it is done in Section 7.4.

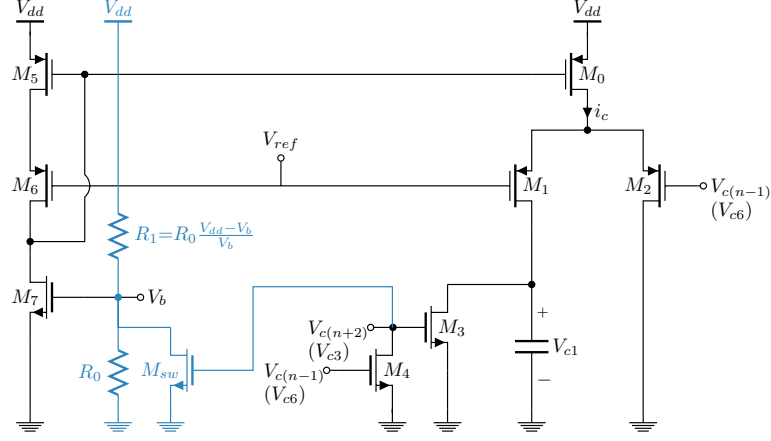


Figure 29: Stage 1 of a multiple stage coupled sawtooth oscillator. The colored (greyed out) circuit components are responsible for the switched biased operation. The circuit schematic given here is based on the circuit schematics given in [11] and [121].

7.4 $1/f$ NOISE REDUCTION IN THE COUPLED SAWTOOTH OSCILLATOR VIA SWITCHED BIASING—A CASE STUDY

In Chapter 5, we have shown that the new non Monte Carlo low-frequency noise modeling and analysis framework we propose produces the same results when compared with computationally much more demanding Monte Carlo simulations, and that, it can correctly capture the impact of nonstationarity where the traditional noise model falls short. In order to demonstrate these capabilities, we have used simple circuits with predictable behaviors for which Monte Carlo type simulations were still possible. We now set out to examine a real-life circuit, the coupled sawtooth oscillator [121], and conduct a case study on the $1/f$ noise reduction observed under switched biasing. There are two reasons why the coupled sawtooth oscillator was selected for this case study. First, there are measurement results for this circuit reported in [11], with which simulation results can be compared. Second, the switched biasing technique does not interfere with the functional mechanism of this circuit and therefore offers a sensible and interesting comparison possibility with the constant bias case.

The coupled sawtooth oscillator is an autonomous oscillator which consists of a number of identical stages [121]. In this case study, we use a setup with six stages and Figure 29 shows the circuit schematic for the first stage and its bias circuitry. The colored (greyed out) part of this schematic (M_{sw} , R_0 , R_1) is required for the switched bias operation and will be discussed later. In the remaining part, the transistors M_0 to M_4 make up the functional unit of the circuit whereas M_5 , M_6 and M_7 serve as bias circuitry. We now summarize the operation mechanism of this circuit from [121] which is based on charging up the capacitor and discharging it periodically with a frequency that is deter-

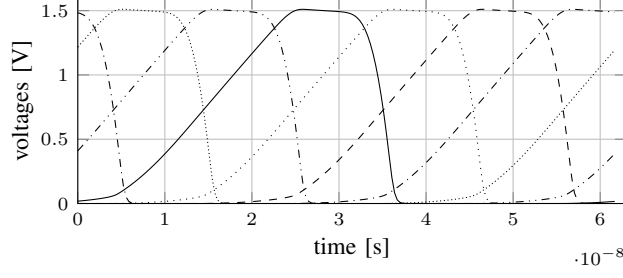


Figure 30: Simulated waveforms of the six stage coupled sawtooth oscillator.

mined by the bias current. M_0 is part of the current mirror along with M_5 and supplies the current, i_c , that charges the capacitor. This current is routed either to the capacitor or to the ground depending on the voltages across the differential stage constituted by M_1 and M_2 . Whenever M_2 has a higher gate voltage than V_{ref} , the capacitor is charged. This gate node is connected to the capacitor voltage of the previous stage (sixth stage in this case, i.e. V_{c6}). This means that as the capacitor voltage in the stage $(n-1)$ reaches V_{ref} , the capacitor in the n th stage starts charging. When $V_{c(n-1)}$ drops under V_{ref} , i_c is diverted to the ground through M_2 and the discharging transistor M_3 is activated. This mechanism, combined with multiple concatenated stages like the one in Figure 29, results in the sawtooth shaped periodic waveforms depicted in Figure 30. For further details about the operation of the coupled sawtooth oscillator, we refer the interested reader to [121].

The noise behavior of the coupled sawtooth oscillator was examined in [11] where measurement results were reported for two modes of operation, i.e., constant bias and switched bias. It was found that the switched biasing scheme reduces the noise power spectral density by 8 dB at low frequencies. In our case study, we recreate the exact same setup used in [11] within our simulation environment and conduct a non Monte Carlo noise analysis. In this setup, the noisy transistor under investigation is M_7 in stage 1 (other 5 stages as well as other transistors in this stage and the bias circuitry are noise-free). The noise in M_7 translates to noise in i_c , which affects the time required to charge the capacitor. The result of this chain of effects appears as noise in the phase and amplitude of the oscillator waveforms.

At this point, we would like to draw attention to the fact that the noise in i_c is only important during the charging of the capacitor. In the discharging period, i_c is sunk to the ground and has no effect on the capacitor voltage V_c . In fact, if we turned off M_7 during the discharging period, so that we have $i_c = 0$, essentially nothing would change in the operation of the circuit. The colored (greyed out) part of the circuit in Figure 29 (M_{sw} , R_0 , R_1) accomplishes precisely this, as discussed in [11]. M_{sw} switches on at the same time when the discharge transistor M_3 is turned on and pulls the gate voltage of M_7 down to ground level, i.e., turns this transistor off. As mentioned above, we would ex-

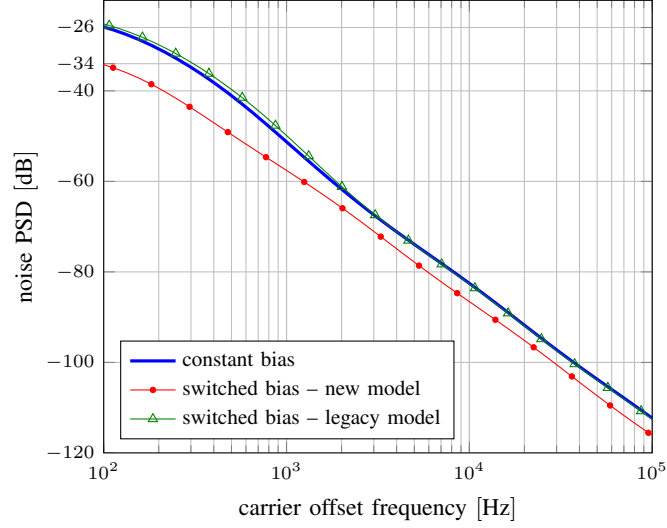


Figure 31: Carrier offset noise PSDs for the sawtooth oscillator.

pect that this additional mechanism has no effect on the behavior of the circuit. However, experiments have shown that this is not true for noise effects [11]. This is because the noise sources (traps) in M_7 exhibit their own, internal bias dependent behavior. Moreover, this behavior is governed by nonlinear, nonstationary dynamics with memory. Therefore, what happens to a noise source during the discharging period has an effect on its dynamics later on in the charging period.

The legacy model tries to characterize the nonstationary behavior of low-frequency noise by modulating it with a bias dependent signal (e.g., $m(t)$ in Figure 27) which does not take the internal dynamics of the noise sources into account. For this reason, it cannot predict the $1/f$ noise reduction brought about by switched biasing. The legacy model switches off the noise contribution of M_7 whenever this transistor is turned off. However, because the current produced by M_7 is sunk to the ground during this period (discharging phase), its noise does not register anyway. Therefore, we would expect the legacy modulated stationary model to predict no change in the noise spectrum due to switched biasing.

This expectation is confirmed by Figure 31. This figure plots three different noise PSDs for the coupled sawtooth oscillator against a carrier offset frequency axis. The first curve (blue, no marks) depicts the upconverted $1/f$ noise that is produced by M_7 and appears in V_{c1} , with the constant biasing scheme. Because of the upconversion, and the integration property of the oscillator, this spectrum rolls off with a 30 dB/decade slope, i.e., it is proportional to $1/f^3$. We should note that, under the constant biasing scheme, the legacy model and the new nonstationary model produce the same result. The second curve (green, triangle marks) is the noise PSD computed using the legacy model in the switched biasing operation modus. As expected, this curve overlaps

with the one for the constant bias case. On the other hand, the red curve (dot marks) represents the noise PSD computed with the new nonstationary model, again with switched biasing. The new model predicts a noise reduction of around 8 dB at frequencies close to the oscillation frequency, which matches the amount observed in [11].

SUMMARY AND CONCLUSION

We have presented a solution to the long-standing open problem of modeling of low-frequency noise in circuit simulation. We have developed mathematical descriptions of low-frequency noise sources which can capture the nonstationary statistics of noise due to time-varying bias conditions. Drawing upon these descriptions, we have shown how a noise model can be constructed which lends itself to be used in conjunction with semi-analytical, non Monte Carlo noise analysis techniques. This allowed us to correctly capture intricate effects of nonstationarity of low-frequency noise without having to conduct Monte Carlo type simulations which can require immense amounts of computation power. We have demonstrated the capabilities of our modeling and simulation framework through carefully chosen and rigorous simulation examples.

In Chapter 2, we have focused on how one can benefit from the ion channel noise modeling and stochastic chemical kinetics literature in developing models for RTS and low-frequency noise in electronic circuits. The description of oxide traps achieved in this manner has a mathematical form which is based on SDEs. Therefore, we could augment the set of standard MNA equations describing the dynamics of the noiseless circuit with a second set of equations for the internal dynamics of the noise sources. Once coupled to each other, the combination of these two systems results in a fully nonlinear, nonstationary description of low-frequency noise in an electrical circuit which could not be attained via conventional noise models.

In Chapter 3, we have presented CIRSIUM, a new circuit simulator developed in the object oriented programming language of MATLAB®. The modular and flexible structure of CIRSIUM enables the rapid development of new device models as well as the translation of existing ones from Verilog-A HDL. Other features of CIRSIUM include subcircuit capabilities and integration with SUNDIALS. CIRSIUM was used throughout this work in order to implement various noise models and test novel simulation algorithms.

Chapter 4 contains a rigorous treatment of the numerical integration methods for the extended MNA equations for a circuit and traps. We have shown how stochastically correct Monte Carlo type simulations can be conducted using SAM and LEM without introducing any statistical bias to the solutions. The *smooth diffusion* algorithm we have developed in this chapter constitutes the basis for the transition from computationally expensive Monte Carlo type simulation algorithms to efficient, semi-analytical non Monte Carlo analysis techniques.

In Chapter 5, we have presented our noise model that facilitates the use of non Monte Carlo simulation methods for the analysis of nonsta-

tionary low-frequency noise in integrated circuits. In this chapter, via comparison with Monte Carlo simulations, we have established that our non Monte Carlo noise modeling and simulation methods are valid and accurate. We have supported our assertions with noise analysis results for various circuits and setups.

The content of Chapter 6 was concentrated on the noise produced by a single trap in one MOSFET in a common-source configuration with a square-wave excitation. In this manner, we have investigated the *switched biasing* technique which is used in order to reduce the effects of low-frequency noise in electronic circuits. By means of this investigation, simple analytical results on the nature, amount and extent of noise reduction one can achieve with switched biasing have been derived. These results can be used to attain an intuitive understanding of the nonstationary behavior of noise in switched circuits and can serve as a baseline for more advanced analyses. We have also highlighted the shortcomings of the idealized trap model rooted in its assumptions on trap capture/emission rates and drew the conclusion that a physically more accurate trap model is required in order to attain a better understanding of low-frequency noise in switched circuits. To this end, we have used our nonstationary low-frequency trap model.

In Chapter 7, we have brought our research program to full fruition and presented a non Monte Carlo $1/f$ noise analysis for a real-life circuit with many transistors and a complicated dynamical behavior. We have established that our noise model can correctly capture the noise reduction effect in the coupled sawtooth oscillator when operated under a switched biasing scheme and demonstrated that while the legacy modulated stationary model cannot predict this effect, our new model produces results that match the measurement data surprisingly well.

For the development of our modeling technique for low-frequency noise in electronic circuits, we have used ideas from stochastic chemical kinetics and neurobiology literature. Similar models are used for the assessment of noise effects on the nervous system. On the other hand, the noise analysis methods employed in this area are usually of a Monte Carlo nature. Therefore, we recognize that a great opportunity presents itself for a cross-fertilization of the two disciplines and we believe that the non Monte Carlo noise analysis techniques developed in this thesis for electronic circuits can be put to use in large-scale analysis of noise in the nervous system [122].

BIBLIOGRAPHY

- [1] M. Keshner, “1/f Noise,” *Proceedings of the IEEE*, vol. 70, no. 3, pp. 212–218, Mar. 1982.
- [2] M. V. Haartman and M. Ostling, *Low-Frequency Noise in Advanced MOS Devices*, 1st ed. Springer Publishing Company, Incorporated, 2007.
- [3] I. Bloom and Y. Nemirovsky, “1/f noise reduction of metal-oxide-semiconductor transistors by cycling from inversion to accumulation,” *Applied Physics Letters*, vol. 58, no. 15, pp. 1664–1666, Apr. 1991.
- [4] B. Dierickx and E. Simoen, “The decrease of “random telegraph signal” noise in metaloxide noise in metal-oxide-semiconductor field-effect transistors when cycled from inversion to accumulation,” *Journal of Applied Physics*, vol. 71, no. 4, pp. 2028–2029, Feb. 1992.
- [5] S. L. J. Gierink, E. Klumperink, A. van der Wel, G. Hoogzaad, E. Van Tuijl, and B. Nauta, “Intrinsic 1/f Device Noise Reduction and Its Effect on Phase Noise in Cmos Ring Oscillators,” *IEEE Journal of Solid-State Circuits*, vol. 34, no. 7, pp. 1022–1025, Jul. 1999.
- [6] J. S. Kolhatkar, “Steady-state and Cyclo-Stationary RTS Noise in MOSFETs,” Ph.D. dissertation, University of Twente, 2005.
- [7] A. van der Wel, E. Klumperink, J. Kolhatkar, E. Hoekstra, M. Snoei, C. Salm, H. Wallinga, and B. Nauta, “Low-Frequency Noise Phenomena in Switched MOSFETs,” *IEEE Journal of Solid-State Circuits*, vol. 42, no. 3, pp. 540–550, Mar. 2007.
- [8] H. Tian and A. El Gamal, “Analysis of 1/f Noise in Switched MOSFET Circuits,” *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 48, no. 2, pp. 151–157, Feb. 2001.
- [9] M. J. Kirton and M. J. Uren, “Noise in solid-state microstructures: A new perspective on individual defects, interface states and low-frequency (1/f) noise,” *Advances in Physics*, vol. 38, no. 4, pp. 367–468, 1989.
- [10] A. Roy and C. Enz, “Analytical Modeling of Large-Signal Cyclo-Stationary Low-Frequency Noise With Arbitrary Periodic Input,” *IEEE Transactions on Electron Devices*, vol. 54, no. 9, pp. 2537–2545, Sep. 2007.

- [11] E. Klumperink, S. Gierink, A. van der Wel, and B. Nauta, "Reducing MOSFET 1/f Noise and Power Consumption by Switched Biasing," *IEEE Journal of Solid-State Circuits*, vol. 35, no. 7, pp. 994–1001, Jul. 2000.
- [12] H. Tian, "Noise Analysis in CMOS Image Sensors," Ph.D. dissertation, Stanford University, 2000.
- [13] A. Demir and A. Sangiovanni-Vincentelli, *Analysis and Simulation of Noise in Nonlinear Electronic Circuits and Systems*. Boston, MA, USA: Kluwer, 1998.
- [14] J. L. Wyatt and G. J. Coram, "Thermodynamic Properties of Simple Noise Models for Solid State Devices," NEC Technical Report 95-198, NEC Research Institute, Inc., Princeton, NJ, Tech. Rep., 1995.
- [15] M. Okumura, H. Tanimoto, T. Itakura, and T. Sugawara, "Numerical Noise Analysis for Nonlinear Circuits with a Periodic Large Signal Excitation Including Cyclostationary Noise Sources," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 40, no. 9, pp. 581–590, Sep. 1993.
- [16] J. Roychowdhury, D. Long, and P. Feldmann, "Cyclostationary Noise Analysis of Large RF Circuits with Multitone Excitations," *IEEE Journal of Solid-State Circuits*, vol. 33, no. 3, pp. 324–336, Mar. 1998.
- [17] M. V. Dunga, *Nanoscale CMOS Modeling*. Univ. California, Berkeley, CA, USA, Tech. Rep. UCB/EECS-2008-20, 2008.
- [18] T. Grasser, "Stochastic charge trapping in oxides: From random telegraph noise to bias temperature instabilities," *Microelectronics Reliability*, vol. 52, no. 1, pp. 39–70, Jan. 2012.
- [19] G. Ghibaudo and T. Boutchacha, "Electrical noise and RTS fluctuations in advanced CMOS devices," *Microelectronics Reliability*, vol. 42, no. 4-5, pp. 573–582, Apr. 2002.
- [20] Y. Nemirovsky, D. Corcos, I. Brouk, A. Nemirovsky, and S. Chaudhry, "1/f Noise in Advanced CMOS Transistors," *IEEE Instrumentation Measurement Magazine*, vol. 14, no. 1, pp. 14–22, Feb. 2011.
- [21] K. S. Ralls, W. J. Skocpol, L. D. Jackel, R. E. Howard, L. A. Fetter, R. W. Epworth, and D. M. Tennant, "Discrete Resistance Switching in Submicrometer Silicon Inversion Layers: Individual Interface Traps and Low-Frequency (1/f?) Noise," *Physical Review Letters*, vol. 52, no. 3, pp. 228–231, Jan. 1984.

- [22] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*. Tata McGraw-Hill Education, 2002.
- [23] A. van Der Wel, E. Klumperink, S. L. J. Gierkink, R. Wassenaar, and H. Wallinga, "MOSFET 1/f Noise Measurement Under Switched Bias Conditions," *IEEE Electron Device Letters*, vol. 21, no. 1, pp. 43–46, Jan. 2000.
- [24] B. Fowler, M. Godfrey, and S. Mims, "Reset Noise Reduction in Capacitive Sensors," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 8, pp. 1658–1669, 2006.
- [25] M. Agostinelli, J. Hicks, J. Xu, B. Woolery, K. Mistry, K. Zhang, S. Jacobs, J. Jopling, W. Yang, B. Lee, T. Raz, M. Mehalel, P. Kolar, Y. Wang, J. Sandford, D. Pivin, C. Peterson, M. DiBattista, S. Pae, M. Jones, S. Johnson, and G. Subramanian, "Erratic Fluctuations of SRAM Cache Vmin at the 90nm Process Technology Node," in *Electron Devices Meeting, 2005. IEDM Technical Digest. IEEE International*, 2005, pp. 655–658.
- [26] K. Aadithya, A. Demir, S. Venugopalan, and J. Roychowdhury, "Accurate Prediction of Random Telegraph Noise Effects in SRAMs and DRAMs," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 32, no. 1, pp. 73–86, Jan. 2013.
- [27] A. P. van der Wel, "MOSFET LF Noise Under Large Signal Excitation: Measurement, Modelling and Application," Ph.D. dissertation, University of Twente, Apr. 2005.
- [28] A. Ambrozy, *Electronic Noise*. McGraw-Hill International Book Company, 1982.
- [29] P. R. Gray and R. G. Meyer, *Analysis and Design of Integrated Circuits*, 3rd ed. New York, NY, USA: John Wiley & Sons, Inc., 1992.
- [30] F. N. H. Robinson, "Noise and Fluctuations in Electronic Devices and Circuits," *Oxford, Clarendon Press, 1974. 252 p.*, vol. 1, 1974.
- [31] Y. Tsididis and C. McAndrew, *Operation and Modeling of the MOS Transistor*. Oxford University Press New York, 1999, vol. 2.
- [32] A. van der Ziel, *Noise in Solid State Devices and Circuits*. Wiley New York, 1986.
- [33] L. Vandamme and F. Hooge, "What Do We Certainly Know About 1/f Noise in MOSTs?" *IEEE Transactions on Electron Devices*, vol. 55, no. 11, pp. 3070–3085, Nov. 2008.

- [34] R. Rohrer, L. Nagel, R. Meyer, and L. Weber, "Computationally Efficient Electronic-Circuit Noise Calculations," *IEEE Journal of Solid-State Circuits*, vol. 6, no. 4, pp. 204–213, Aug. 1971.
- [35] R. Meyer and L. Nagel, "Computer Simulation of 1/f Noise Performance of Electronic Circuits," *IEEE Journal of Solid-State Circuits*, vol. 8, no. 3, pp. 237–240, Jun. 1973.
- [36] A. Demir, A. Mehrotra, and J. Roychowdhury, "Phase Noise in Oscillators: A Unifying Theory and Numerical Methods for Characterization," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 5, pp. 655–674, 2000.
- [37] C. Hull and R. Meyer, "A Systematic Approach to the Analysis of Noise in Mixers," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 40, no. 12, pp. 909–919, Dec. 1993.
- [38] R. Telichevesky, K. Kundert, and J. White, "Efficient AC and Noise Analysis of Two-Tone RF Circuits," in *Design Automation Conference Proceedings 1996, 33rd*, Jun. 1996, pp. 292–297.
- [39] J. Wyatt, J.L. and G. Coram, "Nonlinear Device Noise Models: Satisfying the Thermodynamic Requirements," *IEEE Transactions on Electron Devices*, vol. 46, no. 1, pp. 184–193, Jan. 1999.
- [40] G. Coram, B. Anderson, and J. Wyatt, J.L., "Limits to the Fluctuation-Dissipation Theorem for Nonlinear Circuits," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 9, pp. 1323–1329, Sep. 2000.
- [41] A. Demir, "Phase Noise and Timing Jitter in Oscillators with Colored-Noise Sources," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, no. 12, pp. 1782–1791, Dec. 2002.
- [42] A. L. Hodgkin and A. F. Huxley, "A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve," *The Journal of Physiology*, vol. 117, no. 4, pp. 500–544, Aug. 1952.
- [43] J. A. White, J. T. Rubinstein, and A. R. Kay, "Channel noise in neurons," *Trends in Neurosciences*, vol. 23, no. 3, pp. 131–137, Mar. 2000.
- [44] A. A. Faisal, J. A. White, and S. B. Laughlin, "Ion-Channel Noise Places Limits on the Miniaturization of the Brain's Wiring," *Current Biology*, vol. 15, no. 12, pp. 1143–1149, Jun. 2005.

- [45] R. F. Fox and Y. Lu, “Emergent collective behavior in large numbers of globally coupled independently stochastic ion channels,” *Physical Review E*, vol. 49, no. 4, pp. 3421–3431, Apr. 1994.
- [46] E. M. Izhikevich, *Dynamical Systems in Neuroscience*. MIT Press, 2007.
- [47] R. Fitzhugh, “Statistical properties of the asymmetric random telegraph signal, with applications to single-channel analysis,” *Mathematical Biosciences*, vol. 64, no. 1, pp. 75–89, May 1983.
- [48] I. Bruce, “Evaluation of Stochastic Differential Equation Approximation of Ion Channel Gating Models,” *Annals of Biomedical Engineering*, vol. 37, no. 4, pp. 824–838, 2009.
- [49] —, “Implementation Issues in Approximate Methods for Stochastic Hodgkin–Huxley Models,” *Annals of Biomedical Engineering*, vol. 35, no. 2, pp. 315–318, 2007.
- [50] H. Mino, J. T. Rubinstein, and J. A. White, “Comparison of Algorithms for the Simulation of Action Potentials with Stochastic Sodium Channels,” *Annals of Biomedical Engineering*, vol. 30, no. 4, pp. 578–587, 2002.
- [51] B. Sengupta, S. B. Laughlin, and J. E. Niven, “Comparison of Langevin and Markov channel noise models for neuronal signal generation,” *Physical Review E*, vol. 81, no. 1, p. 011918, Jan. 2010.
- [52] R. Fox, “Stochastic Versions of the Hodgkin-Huxley Equations,” *Biophysical Journal*, vol. 72, no. 5, pp. 2068–2074, May 1997.
- [53] D. T. Gillespie, “The chemical Langevin equation,” *The Journal of Chemical Physics*, vol. 113, no. 1, p. 297, Jul. 2000.
- [54] —, “Stochastic Simulation of Chemical Kinetics,” *Annual Review of Physical Chemistry*, vol. 58, no. 1, pp. 35–55, 2007.
- [55] D. J. Higham, “Modeling and Simulating Chemical Reactions,” *SIAM Review*, vol. 50, no. 2, pp. 347–368, Jan. 2008.
- [56] J. Pahle, “Biochemical simulations: stochastic, approximate stochastic and hybrid approaches,” *Briefings in Bioinformatics*, vol. 10, no. 1, pp. 53–64, Jan. 2009.
- [57] C. W. Gardiner, “Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences, Second Edition,” *Applied Optics*, vol. 25, p. 3145, Sep. 1986.
- [58] S. B. Lowen and M. C. Teich, “Fractal renewal processes generate $1/f$ noise,” *Physical Review E*, vol. 47, no. 2, pp. 992–1001, 1993.

- [59] A. G. Mahmutoglu, A. Demir, and J. Roychowdhury, "Modeling and Analysis of (Nonstationary) Low Frequency Noise in Nano Devices: A Synergistic Approach Based on Stochastic Chemical Kinetics," in *2013 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2013, pp. 500–507.
- [60] H. Frydman, "A Structure of Doubly Stochastic Markov Chains," *Linear Algebra and its Applications*, vol. 67, pp. 51–66, Jun. 1985.
- [61] G. Grimmett and D. Stirzaker, *Probability and Random Processes*. Oxford University Press, May 2001.
- [62] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry*. Elsevier, 1992, vol. 1.
- [63] J. G. Skellam, "The frequency distribution of the difference between two Poisson variates belonging to different populations," *Journal of the Royal Statistical Society. Series A (General)*, vol. 109, no. Pt 3, p. 296, 1946.
- [64] B. Oksendal, *Stochastic Differential Equations: An Introduction with Applications*. Springer, Jan. 2003.
- [65] A. G. Mahmutoglu and A. Demir, "CIRSIUM: A Circuit Simulator in MATLAB with Object Oriented Design," in *Ph.D. Research in Microelectronics and Electronics (PRIME), 2013 9th Conference on*, Jun. 2013, pp. 173–176.
- [66] D. FitzPatrick and I. Miller, *Analog Behavioral Modeling with the Verilog-A Language*. Springer, 1998.
- [67] D. Amsallem and J. Roychowdhury, "ModSpec: An Open, Flexible Specification Framework for Multi-Domain Device Modelling," in *2011 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2011, pp. 367–374.
- [68] L. Lemaitre, C. McAndrew, and S. Hamm, "ADMS-Automatic Device Model Synthesizer," in *Custom Integrated Circuits Conference, 2002. Proceedings of the IEEE 2002*, 2002, pp. 27 – 30.
- [69] A. C. Hindmarsh, P. N. Brown, K. E. Grant, S. L. Lee, R. Serban, D. E. Shumaker, and C. S. Woodward, "SUNDIALS: Suite of Nonlinear and Differential/Algebraic Equation Solvers," *ACM Trans. Math. Softw.*, vol. 31, no. 3, pp. 363–396, Sep. 2005.
- [70] G. Gildenblat, X. Li, W. Wu, H. Wang, A. Jha, R. van Langevelde, G. Smit, A. Scholten, and D. Klaassen, "PSP: An Advanced Surface-Potential-Based MOSFET Model for Circuit Simulation," *IEEE Transactions on Electron Devices*, vol. 53, no. 9, pp. 1979–1993, Sep. 2006.

- [71] K. S. Kundert, *The Designer's Guide to Spice and Spectre*. Norwell, MA, USA: Kluwer Academic Publishers, 1995.
- [72] J. Fijnvandraat, S. Houben, E. ter Maten, and J. Peters, "Time domain analog circuit simulation," *Journal of Computational and Applied Mathematics*, vol. 185, no. 2, pp. 441–459, Jan. 2006.
- [73] G. Gildenblat, X. Li, H. Wang, W. Wu, R. Van Langevelde, A. J. Scholten, G. D. J. Smit, and D. B. M. Klaassen, "Introduction to PSP MOSFET model," in *Proc. the MSM 2005 Int. Conf., Nanotech 2005*, 2005.
- [74] S. Schwarz and S. Russek, "Semi-Empirical Equations for Electron Velocity in Silicon: Part II—MOS Inversion Layer," *IEEE Transactions on Electron Devices*, vol. 30, no. 12, pp. 1634–1639, Dec. 1983.
- [75] Y. Manégli, F. Rahmoune, and D. Bauza, "On the Si-SiO₂ interface trap time constant distribution in metal-oxide-semiconductor transistors," *Journal of Applied Physics*, vol. 97, no. 1, p. 014502, Dec. 2004.
- [76] S. Martin, G. Li, E. Worley, and J. White, "The Gate Bias and Geometry Dependence of Random Telegraph Signal Amplitudes," *IEEE Electron Device Letters*, vol. 18, no. 9, pp. 444–446, Sep. 1997.
- [77] A. G. Mahmutoglu and A. Demir, "Modeling and Simulation of Low-Frequency Noise in Nano Devices: Stochastically Correct and Carefully Crafted Numerical Techniques," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 34, no. 5, pp. 794–807, May 2015.
- [78] K. Sonoda, K. Ishikawa, T. Eimori, and O. Tsuchiya, "Discrete Dopant Effects on Statistical Variation of Random Telegraph Signal Magnitude," *IEEE Transactions on Electron Devices*, vol. 54, no. 8, pp. 1918–1925, Aug. 2007.
- [79] A. Alfonsi, E. Cancès, G. Turinici, B. Di Ventura, and W. Huisinga, "Adaptive Simulation of Hybrid Stochastic and Deterministic Models for Biochemical Systems," in *ESAIM: Proceedings*, vol. 14, 2005, pp. 1–13.
- [80] H. Salis and Y. Kaznessis, "Accurate hybrid stochastic simulation of a system of coupled chemical or biochemical reactions," *The Journal of Chemical Physics*, vol. 122, no. 5, p. 054103, Jan. 2005.
- [81] P. a. W. Lewis and G. S. Shedler, "Simulation of Nonhomogeneous Poisson Processes by Thinning," *Naval Research Logistics Quarterly*, vol. 26, no. 3, pp. 403–413, 1979.

- [82] J. G. Shanthikumar, “Uniformization and Hybrid Simulation/-Analytic Models of Renewal Processes,” *Operations Research*, vol. 34, no. 4, pp. 573–580, Jul. 1986.
- [83] A. P. A. van Moorsel and W. H. Sanders, “Adaptive Uniformization,” *Communications in Statistics. Stochastic Models*, vol. 10, no. 3, pp. 619–647, 1994.
- [84] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*. Springer, Jan. 1992.
- [85] D. J. Higham, “An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations,” *SIAM review*, pp. 525–546, 2001.
- [86] L. B. Andersen, P. Jäckel, and C. Kahl, “Simulation of Square-Root Processes,” in *Encyclopedia of Quantitative Finance*. John Wiley & Sons, Ltd, 2010.
- [87] J. C. Cox, J. E. Ingersoll, Jr., and S. A. Ross, “A Theory of the Term Structure of Interest Rates,” *Econometrica*, vol. 53, no. 2, pp. 385–407, Mar. 1985.
- [88] C. Dangerfield, D. Kay, and K. Burrage, “Stochastic Models and Simulation of Ion Channel Dynamics,” *Procedia Computer Science*, vol. 1, no. 1, pp. 1587–1596, May 2010.
- [89] Y. Maghsoodi, “Solution of the Extended CIR Term Structure and Bond Option Valuation,” *Mathematical Finance*, vol. 6, no. 1, pp. 89–109, 1996.
- [90] W. Feller, “Two Singular Diffusion Problems,” *Annals of Mathematics*, vol. 54, no. 1, pp. 173–182, Jul. 1951.
- [91] D. J. Higham and X. Mao, “Convergence of Monte Carlo Simulations Involving the Mean-Reverting Square Root Process,” *Journal of Computational Finance*, vol. 8, no. 3, pp. 35–61, 2005.
- [92] R. Lord, R. Koekkoek, and D. V. Dijk, “A comparison of biased simulation schemes for stochastic volatility models,” *Quantitative Finance*, vol. 10, no. 2, pp. 177–194, 2010.
- [93] G. Deelstra and F. Delbaen, “Convergence of Discretized Stochastic (Interest Rate) Processes with Stochastic Drift Term,” *Applied Stochastic Models and Data Analysis*, vol. 14, no. 1, pp. 77–84, 1998.
- [94] G. N. Milstein, E. Platen, and H. Schurz, “Balanced Implicit Methods for Stiff Stochastic Systems,” *SIAM Journal on Numerical Analysis*, vol. 35, no. 3, pp. 1010–1019, Jun. 1998.

- [95] C. Kahl, “Positive Numerical Integration of Stochastic Differential Equations,” *University of Wuppertal, Research Group Numerical Analysis*, 2004.
- [96] A. Neuenkirch and L. Szpruch, “First order strong approximations of scalar SDEs with values in a domain,” arXiv e-print 1209.0390, Sep. 2012.
- [97] E. Moro and H. Schurz, “Boundary Preserving Semianalytic Numerical Algorithms for Stochastic Differential Equations,” *SIAM Journal on Scientific Computing*, vol. 29, no. 4, pp. 1525–1549, Jan. 2007.
- [98] C. E. Dangerfield, D. Kay, S. MacNamara, and K. Burrage, “A boundary preserving numerical algorithm for the Wright-Fisher model with mutation,” *BIT Numerical Mathematics*, vol. 52, no. 2, pp. 283–304, Jun. 2012.
- [99] J. H. Goldwyn, N. S. Imennov, M. Famulare, and E. Shea-Brown, “Stochastic differential equation models for ion channel noise in Hodgkin-Huxley neurons,” *Physical Review E*, vol. 83, no. 4, p. 041908, Apr. 2011.
- [100] J. H. Goldwyn and E. Shea-Brown, “The What and Where of Adding Channel Noise to the Hodgkin-Huxley Equations,” *PLoS Comput Biol*, vol. 7, no. 11, p. e1002247, Nov. 2011.
- [101] M. J. Kirton, M. J. Uren, S. Collins, M. Schulz, A. Karmann, and K. Scheffer, “Individual defects at the Si:SiO₂ interface,” *Semiconductor Science and Technology*, vol. 4, no. 12, pp. 1116–1126, Dec. 1989.
- [102] M. J. Uren, D. J. Day, and M. J. Kirton, “1/f and random telegraph noise in silicon metal-oxide-semiconductor field-effect transistors,” *Applied Physics Letters*, vol. 47, no. 11, pp. 1195–1197, Dec. 1985.
- [103] A. G. Mahmutoglu and A. Demir, “Modeling and Analysis of Non-stationary Low-Frequency Noise in Circuit Simulators: Enabling non Monte Carlo Techniques,” in *2014 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2014, pp. 309–315.
- [104] M. Okumura and H. Tanimoto, “A Time-Domain Method for Numerical Noise Analysis of Oscillators,” in *Design Automation Conference, 1997. Proceedings of the ASP-DAC '97 Asia and South Pacific*, Jan. 1997, pp. 477–482.
- [105] W. Anzill and P. Russer, “A General Method to Simulate Noise in Oscillators Based on Frequency Domain Techniques,” *IEEE*

- transactions on microwave theory and techniques*, vol. 41, no. 12, pp. 2256–2263, 1993.
- [106] F. X. Kaertner, “Analysis of White and $f^{-\alpha}$ in Oscillators,” *International Journal of Circuit Theory and Applications*, vol. 18, no. 5, pp. 485–519, Sep. 1990.
 - [107] A. Demir, “Floquet Theory and Non-Linear Perturbation Analysis for Oscillators with Differential-Algebraic Equations,” *International Journal of Circuit Theory and Applications*, vol. 28, no. 2, pp. 163–185, Mar. 2000.
 - [108] O. J. Nastov, “Spectral Methods for Circuit Analysis,” Thesis, Massachusetts Institute of Technology, 1999.
 - [109] W. A. Gardner, *Introduction to Random Processes: With Applications to Signals and Systems*. McGraw-Hill New York, 1990.
 - [110] O. Suvak and A. Demir, “On Phase Models for Oscillators,” *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 30, no. 7, pp. 972–985, Jul. 2011.
 - [111] P. D. Welch, “The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms,” *IEEE Transactions on Audio and Electroacoustics*, vol. 15, no. 2, pp. 70–73, Jun. 1967.
 - [112] B. Razavi, “A Study of Phase Noise in CMOS Oscillators,” *IEEE Journal of Solid-State Circuits*, vol. 31, no. 3, pp. 331–343, Mar. 1996.
 - [113] A. Hajimiri and T. Lee, “A General Theory of Phase Noise in Electrical Oscillators,” *IEEE Journal of Solid-State Circuits*, vol. 33, no. 2, pp. 179–194, Feb. 1998.
 - [114] J. McNeill, “Jitter in Ring Oscillators,” *IEEE Journal of Solid-State Circuits*, vol. 32, no. 6, pp. 870–879, Jun. 1997.
 - [115] A. Demir, “Computing Timing Jitter From Phase Noise Spectra for Oscillators and Phase-Locked Loops With White and $1/f$ Noise,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 9, pp. 1869–1884, Sep. 2006.
 - [116] A. G. Mahmutoglu and A. Demir, “Analysis of Low-Frequency Noise in Switched MOSFET Circuits: Revisited and Clarified,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 4, pp. 929–937, Apr. 2015.
 - [117] A. van der Ziel, “Unified Presentation of $1/f$ Noise in Electron Devices: Fundamental $1/f$ Noise Sources,” *Proceedings of the IEEE*, vol. 76, no. 3, pp. 233–258, Mar. 1988.

- [118] N. Zanolla, D. Siprak, M. Tiebout, P. Baumgartner, E. Sangiorgi, and C. Fiegna, "Reduction of RTS Noise in Small-Area MOSFETs Under Switched Bias Conditions and Forward Substrate Bias," *IEEE Transactions on Electron Devices*, vol. 57, no. 5, pp. 1119–1128, May 2010.
- [119] A. Arnaud and M. Miguez, "Very Low Frequency Cyclostationary 1/f Noise in MOS Transistors," in *2013 22nd International Conference on Noise and Fluctuations (ICNF)*, Jun. 2013, pp. 1–4.
- [120] M. Miguez and A. Arnaud, "A Study of Flicker Noise in MOS Transistor Under Switched Bias Condition," *Journal Integrated Circuits and Systems*, vol. 3, no. 2, pp. 63–68, 2008.
- [121] S. Gierkink and E. van Tuij, "A Coupled Sawtooth Oscillator Combining Low Jitter with High Control Linearity," *IEEE Journal of Solid-State Circuits*, vol. 37, no. 6, pp. 702–710, Jun. 2002.
- [122] D. Kilinc and A. Demir, "Modeling and Simulation of Noise in Neurons and Neuronal Circuits," in *2015 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, Nov. 2015.