DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

ENABLING DYNAMIC SPECTRUM ACCESS FOR COGNITIVE RADIO USING SOFTWARE DEFINED RADIO PLATFORM

by
Savaş ŞAHİN

December, 2017
İZMİR

ENABLING DYNAMIC SPECTRUM ACCESS FOR COGNITIVE RADIO USING SOFTWARE DEFINED RADIO PLATFORM

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science in Electrical and Electronics Engineering

by
Savaş ŞAHİN

December, 2017
İZMİR

M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ENABLING DYNAMIC SPECTRUM ACCESS FOR COGNITIVE RADIO USING SOFTWARE DEFINED RADIO PLATFORM"** completed by **SAVAŞ ŞAHİN** under supervision of **ASSOC. PROF. DR. DAMLA GÜRKAN KUNTALP** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

....

Assoc. Prof. Dr. Damla GÜRKAN KUNTALP

Supervisor

....

Asst. Prof. Dr. Nalan ÖZKURT

(Jury Member)

....

Assoc. Prof. Dr. Olcay Akay

(Jury Member)

....

Prof.Dr. Kadriye ERTEKİN

Director

Graduate School of Natural and Applied Sciences

ACKNOWLEDGEMENT

Firstly I am very grateful to my parents that show patience during the writing period of this thesis. My mother Fatma Şahin and my father Mehmet Şahin undoubtfully have the big impact on keeping myself motivated. My sisters Berçem and Pervin and my little brother Barış deserve special thanks for always pushing me further.

I would like to specially thank my advisor Assoc. Prof. Dr. Damla Gürkan Kuntalp for her valuable review, guidance, and contribution. For sharing all kinds of information and his valuable opinions with me, I am very grateful to Timur Düzenli. I also would like to thank Assoc. Prof. Dr. Serkan Günel for his valuable explanations and supports. Furthermore, I would like to thank all members of the Dokuz Eylul University Electrical and Electronics Faculty who have supported me at courses and other times.

Finally, of course, I am indebted to all my friends who showed me respect during my research.

Savaş ŞAHİN

ENABLING DYNAMIC SPECTRUM ACCESS FOR COGNITIVE RADIO USING SOFTWARE DEFINED RADIO PLATFORM

ABSTRACT

Spectrum usage increases in the begining of 2000s, a common belief appears that we are running out of usable radio frequencies. Cognitive Radio concept was mentioned and developed at this time with the aim of efficient spectrum usage. Cognitive Radio senses its enviroment and makes decisions about channels' states by using software defined radio platforms which allow software implementations on hardware. According to these decisions, channels are being used or not depending on the state of the channel namely idle or not. By detecting the unused spectrum opportunities, Cognitive Radio systems allow spectrum resources to be used efficiently.

Cognitive Radio systems are used by unlicensed users. Since there exist licensed users, they should be protected from possible interference from unlicensed users with strict regulatory policies. According to the primary user's activitiy, three spectrum access schemes evaluated so far. Sensing-based spectrum sharing is one of the three approaches which has low and high power configurations according to the primary user's activity.

System model using sensing-based spectrum sharing has interference constraints that prevent licensed users from possible interferences and establishes reasonable long term power budgets. In this thesis, we will investigate the optimal sensing time that maximizes the total achievable throughput under these conditions and sensing-based spectrum access scheme, since tradeoff exists between sensing time and total achievable throughput.

Keywords: Optimal sensing time, sensing-based spectrum sharing, cognitive radio, software radio

YAZILIM TABANLI RADYO KULLANARAK BİLİŞSEL RADYO İÇİN DİNAMİK SPEKTRUM KULLAMININ AKTİFLEŞTİRİLMESİ

ÖZ

Spectrum kullanımının 2000'li yılların başlarında artmasıyla birlikte, kullanılabilir radyo frekanslarını bitirmek üzere olduğumuza dair bir düşünce ortaya çıkmaya başladı. Bilişsel radyo konsepti ilk defa bu dönemde verimli spektrum kullanımını amaçlayarak ortaya atıldı. Bilişsel radyo, yazılım yüklenebilmesini mümkün kılan yazılım tabanlı radyo platformlarını kullanarak bulunduğu çevreyi dinler ve dinlediği kanalların durumu hakkında kararlar verir. Kanalın uygun olup olmamasına bağlı olarak kanalın kullanılıp kullanılmayacağına karar verilir. Bilişsel radyo sistemleri kullanılmayan spektrum imkanlarını sezerek, spektrum kaynaklarının verimli kullanılmasına yardımcı olur.

Bilişsel radyo sistemleri lisanssız kullanıcılar tarafından kullanılır. Lisanslı kullanıcılar, lisanssız kullanıcıların neden olabileceği bozucu etkilerden sıkı spektrum politikaları aracılığı ile korunmalıdırlar. Bu zamana kadar lisanslı kullanıcıların aktivite durumuna göre üç çeşit spektrum kullanımı belirlenmiştir. Bunlardan biri birincil kullanıcının aktivitesine göre değişebilen yüksek ve düşük güç ayarları olan dinlemeye bağlı spektrum paylaşımıdır.

Dinlemeye bağlı spektrum paylaşımının kullanıldığı sistem modelinde lisanslı kullanıcıları olası bozucu etkilerden koruyan ve uzun dönemde gerçeklenebilir güç stoğunun sağlanabilmesi için kıstaslar vardır. Bu tezde, bu kıstaslar altında ve bu spektrum kullanım politikasında, birbirleri arasında ödünleşme bulunan sistemin elde edilebilir çıktısını maksimum seviyeye çıkaracak optimum dinleme süresini inceleyeceğiz.

Anahtar kelimeler: Optimal dinleme süresi, dinlemeye bağlı spektrum kullanımı, bilişsel radyo, yazılımsal radyo

CONTENTS

Page

M.Sc THESIS EXAMINATION RESULT FORM ii

ACKNOWLEDGEMENTS iii

ABSTRACT iv

ÖZ v

LIST OF FIGURES viii

LIST OF TABLES ix

CHAPTER ONE – INTRODUCTION 1

1.1 Cognitive Radio 1

1.2 Dynamic Spectrum Access 2

1.3 Goals of this thesis 4

1.4 Scope of chapters 4

CHAPTER TWO – BACKGROUND 6

2.1 Basis of Cognitive Radio 6

2.1.1 Spectrum Sensing 8

2.1.2 Spectrum Sharing 12

2.1.3 Spectrum Management 12

2.2 Literature Reviews and Releated Works 13

CHAPTER THREE – SYSTEM MODEL AND FORMULATIONS 18

3.1 Spectrum Access Scheme 18

3.1.1 Wideband Sensing-based Spectrum Sharing(WSSS) 20

3.2 Optimal Sensing Time 22

3.2.1 Channel Model 22

3.2.2 Detector 23

3.2.3 Probabilities 25

3.2.4 Transmission Rates and Throughput 26
3.2.5 Constraints 29
3.2.6 Problem Formulation 30
3.3 Simulation Algorithm 34
3.4 Results 35

CHAPTER FOUR – CONCLUSION **41**

REFERENCES **43**

APPENDICES **46**

Appendix-A : Simulation Algorithm Matlab Code 46
Appendix-B : Optimization Problem Formulations 54

LIST OF FIGURES

Page

Figure 1.1 Dynamic Spectrum Acces Models 3
Figure 2.1 Basic congitive cycle tasks 7
Figure 2.2 Sensing methods in terms of accuracy vs complexity 11
Figure 2.3 Optimal spectrum access strategy 14
Figure 2.4 Ergodic capacity under peak and average power constraints 16
Figure 3.1 Frame structure of cognitive radio 18
Figure 3.2 Spectrum opportunity 19
Figure 3.3 Wideband downconversion scheme 21
Figure 3.4 Perfect sensing 21
Figure 3.5 Cognitive radio network with channel power gains 23
Figure 3.6 Energy detector block diagram 24
Figure 3.7 Non perfect sensing 25
Figure 3.8 Throughput vs sensing time – Different average power constraints 35
Figure 3.9 Throughput vs sensing time – Different interference power constraints. 37
Figure 3.10 Throughput vs sensing time – Different received worst case SNR values 39
Figure 3.11 Throughput vs sensing time – Different channel power gains 40

LIST OF TABLES

Page

Table 2.1 Comparison of single-radio and dual radio architecture 9

Table 3.1 Instantaneous transmission rate notation according to primary user activity and secondary user decision 28

Table 3.2 High power values of spectrum access strategy.. 36

Table 3.3 Low power values of spectrum access strategy 37

CHAPTER ONE

INTRODUCTION

During the wireless technology development in the last two decades, demand for multimeadia communication increases instead of voice only communication. This leads to an increase in used bandwith and consequently an increase in the usage of spectrum. As spectrum usage increases, a common belief appears that we are running out of usable radio frequencies. As a result, innovative technics that offer exploiting the use of available spectrum needed. Cognitive Radio (CR) arises in this era to offer efficient usage of resources.

1.1 Cognitive Radio

According to Federal Communications Commission (FCC)'s definition: "Cognitive radio: A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets." (FCC, 2005). This means CR is a form of wireless communication in which a transceiver can intelligently detect which communication channels are in use and which are not, and instantly move into vacant channels while avoiding occupied ones. In another words, CR is a radio that has the ability of detecting its enviroment and capability of opportunity decision according to the detection. This optimizes the use of available radio frequency (RF) spectrum while minimizing interference to other users. In its most basic form, CR is a hybrid technology involving software defined radio (SDR) as applied to spread spectrum communications. Possible functions of cognitive radio include the ability of a transceiver to determine its geographic location, identify and authorize its user, encrypt or decrypt signals, sense neighboring wireless devices in operation, and adjust output power and modulation characteristics.

The concept of cognitive radio was first proposed by Joseph Mitola III in a seminar at the Royal Institute of Technology in Stockholm (KTH), in 1998 and published in an article by Mitola and Gerald Q. Maguire, Jr. in 1999. It was a novel approach in wireless communications. Main idea is to show that SDRs are sufficiently and computationally intelligent enough to be able to understand its needs and adapt its capability according to the enviroment.

Understanding and using this concept lead to the efficient resource usage. Since it is software radio, adaptation according to signal parameters and enhancements are not required to change hardware. A fully reconfigurable wireless radio platform (WRP) can respond to the network and users' demands. Since WRP can detect the channel's availability, if the channel is idle, it can use that channel and helps efficient spectrum usage; if channel is not idle, it can either use the channel with a relatively low power or shut downs some operations to be able to save power which is a crucial operation for battery powered devices. In both cases, resources are used efficiently.

1.2 Dynamic Spectrum Access

The basic tasks of cognitive systems can be classified as spectrum sensing, spectrum analysis, and spectrum decision. It is known that spectrum sensing is a crucial task in cognitive networks, which must be performed before the cognitive user's access to the licensed spectrum, in order to limit the interference to the primary user. Dynamic Spectrum Access can be explained as finding new spectrum holes by sensing one or more channels and using these holes with associated channel parameters. Term "dynamic" means that usable channel could be changed according to new decisions. As shown in Figure 1.1, dynamic spectrum access can be categorized into three different models.

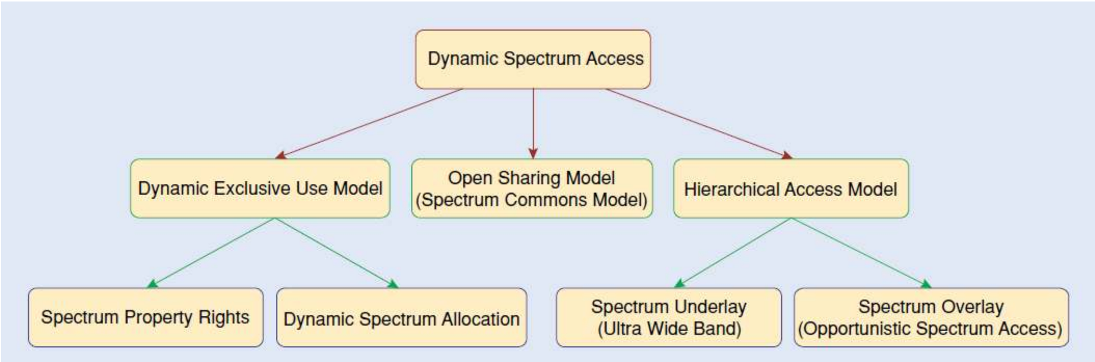

Figure 1.1 Dynamic spectrum acces models (Zhao Q. & Sadler B. , 2007)

Dynamic exclusive use model is the classical spectrum usage model which uses licensed services for specific use. In this model, there exist two approaches. Spectrum Property Rights allows licenses to sell spectrum and to freely choose technology (Zhao Q. & Sadler B., 2007). Second approach is Dynamic Spectrum Allocation which allows spectrum in a specific geographical area, in a specific time being allocated for different services and for different use.

Open sharing model, as known as spectrum commons is the sharing approach of specific spectral region. Centralized and distributed sharing models are investigated under this model.

Hierarchical access model is the strategy where hiyerarchy exists between users. Primary users could use the spectrum freely, as the secondary users have limited access because of interference and power constraints which effect primary user's quality of service (QoS). Spectrum Underlay and Spectrum Overlay are main approaches of this model. Spectrum Underlay approach transmits signal below primary users' noise level by spreading spectrum over very wide band. Spectrum Overlay as known as Opportunistic Spectrum Access (OSA) has no restrictions like power level by identifying the availability of spectrum. OSA also has two possible scenarios; first, secondary user does not transmit if channel is not detected as idle (OSA); second, secondary user detected channel as idle and transmission is done with a high power setting and if channel detected as not idle but transmission occurs with relatively low power setting. The second one is known as sensing-based spectrum sharing (SSS). In this thesis, the term dynamic spectrum access mostly refers to SSS. Otherwise, it will be specified.

1.3 Goal of Thesis

Finding spectrum holes in spectrum and efficient usage are interest of this thesis. Since dynamic spectrum access in licensed bands by unlicensed users could be achieved by detecting the availability of channels, CR could decide the availability of this spectrum hole by listening its enviroment. The listening duration of every transmission, which maximizes the total achievable throughput of the system, is called optimal sensing time. We will investigate the optimal sensing time to be able to maximize total throughput of the system while minimizing the interference to the primary users under defined circumstances.

1.4 Scope of Thesis

This study is started by concentrating on CR systems; sensing, management and decision approaches in the following chapter, Chapter 2. In the third chapter, we will briefly focus on CR literature and releated works with our thesis. Fourth chapter will investigate the system configuration and problem formulation of optimal sensing time. To be able to find the optimal sensing time, some simulation results will also be presented in this chapter. In the final chapter, concluding remarks will be made and some future considerations will be touched on.

In the begining, we investigate the optimal sensing time while maximizing the total throughput of CR network. Optimal sensing time is the first and one of the most important sections of the spectrum sensing. The frame structure of any cognitive radio system that employs spectrum sensing studied so far, consists of a sensing time slot and a data transmission slot. Therefore, an inherent tradeoff exists between the sensing time and the data transmission time, thus the throughput of the cognitive radio network.

Some works releated to wideband sensing-based spectrum access cognitive radio network, where the problem of finding the optimal sensing time and power allocation scheme that maximizes the average achievable throughput was studied under two different power constraints (instantaneous and average transmit power constraint) and it was shown that the average transmit power constraint leads to higher average achievable throughput compared to the instantaneous transmit power constraint (Zhang, Kang and Liang, 2009).

We will start with the sensing part of the problem. As we want to process the signals, we should first listen to our enviroment. Problem now arises how much time is needed for spectrum sensing? If sensing time increases, acievable throughput will decrease. If sensing time decreases, probability of false alarm and miss detection will increase and these will lead to interference to primary user's signal. That's why there should be an optimal duration for sensing. We will try to find the optimal sensing time for the maximization of total achievable throughput.

CHAPTER TWO

BACKGROUND

Cognitive Radio concept as stated in previous section was first proposed by Joseph Mitola III in a presentation at the KTH in 1998. After this presentation, the attention to the topic increases rapidly. CR was a novel approach which developed aganist inefficient electromagnetic radio spectrum usage. CR was expected to overcome this problem. Besides, a fully reconfigurable radio concept gives freedom to the flexibility of development without any hardware change. S. Haykin (2005), stated that definiton of CR has six keywords: awareness, intelligence, learning, adaptivity, reliability, and efficiency and added another capability: cognition reconfigurability. Implementation of all of these properties in one hardware configuration was difficult when this approach was proposed. Today, it is much more easier to setup different combinations of these capabilities in one device with the help of digital signal processing technology in hardware and software platforms.

2.1 Basis of Cognitive Radio

Electromagnetic radio spectrum is a natural resource which should be used in an efficient manner. Spectrum under utilization by licensed users is not being used all the time. This means unused spectrum resources exist which could be used by secondary unlicensed users. This unused spectrum resources at any specific geographic location, any specific time and any specific frequency band could be named as spectrum holes.

Software defined radio, sometimes shortened as software radio, is a multiband radio that supports multiple air interfaces and protocols and is reconfigurable through software run on digital signal processors (DSP) or general purpose microprocessors (Mitola III J., 2000). CR capabilities implemented on SDR platforms are intelligent radios that can be aware of their enviroment and have the capabilitiy of reconfiguration by learning and adapting according to this awareness. To be able to do these adaptations, CR systems should sense radio spectrum, make some measurements, make decisions according to detection by predefined spectrum policies and accessing channels according to the decision. This basic cognitive task for one way communication is shown in Figure 2.1.

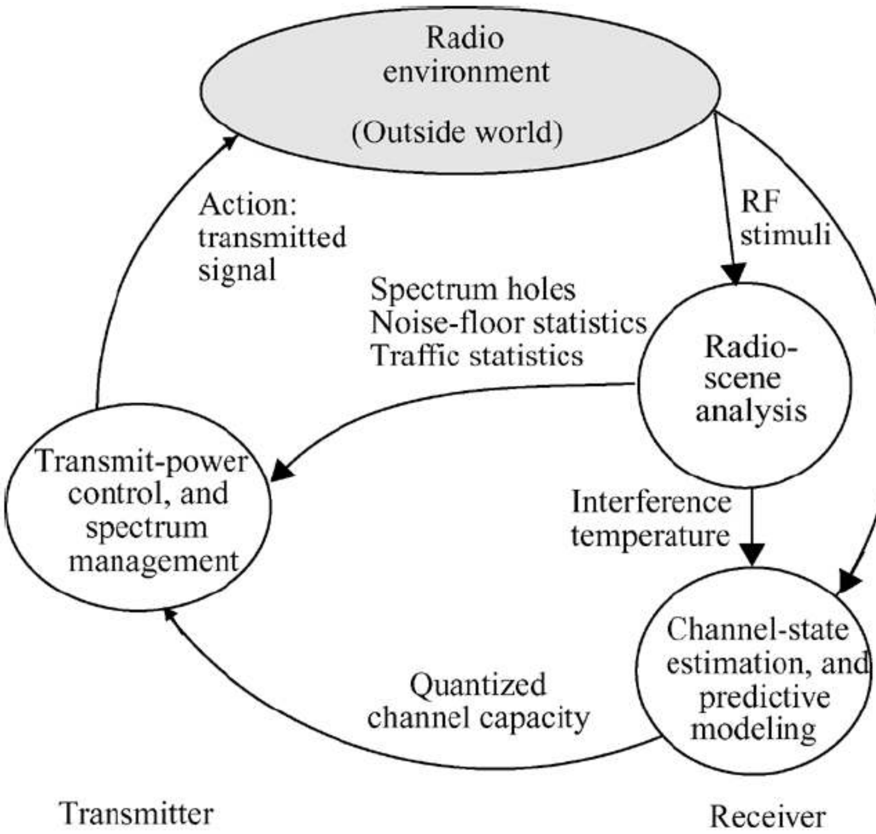

Figure 2.1 Basic congitive cycle tasks (Haykin S.,2005)

As seen in Figure 2.1, basic cognitive tasks performed in CR system are the sensing of RF stimuli and identifying the channel parameters which are performed in the receiver section; finding the spectrum holes which could be a spectrum access opportunity at given frequency band, and management of accessing the spectrum which are performed in the transmitter section. Therefore, CR system is an example of feedback communication system (Haykin S., 2005).

Spectrum sharing and management are not the only tasks of CR systems. Spectrum sharing consideration by secondary users with primary and/or secondary users is another component of CR basis.

2.1.1 Spectrum Sensing

Cognitive Radio system could be considered as an opportunity tracking system which searches for spectrum opportunities to use these spectrum holes and increase the spectrum efficiency. During the search of these spectrum opportunities, CR use spectrum sensing components to be able to aware of its enviroment. Identification of its electromagnetic enviroment in terms of time, space, energy, frequency, code, angle and modulation type is known as spectrum sensing. Since, traditional receivers are tuned for specific frequencies with specific bandwidths, CR should process over much wider band for utilizing any opportunity. Spectrum sensing components of a CR system require high sampling rate, high resolution of analog to digital converters (ADCs) which has high dynamic range and, high speed signal proccessors. In addition to high speed ADCs, to be able to overcome the high computational data rate high speed processors (DSPs or FPGAs) are also needed. Noise variance estimation, channel estimation and other statictics are also performed at these stages. While these factors increase the software design complexity, more robust detection and opportunity utilization are the good rewards in return.

Sensing of spectrum can be done in two different architectures; single radio and dual radio (Yücek & Arslan, 2009). In single radio architecture, only a specific portion of the time slot is allocated for sensing. Since specific time exists for spectrum sensing, limitation of the sensing period causes some degradation in accuracy of the results. Moreover, spectrum usage time for data transmission decreases as some portion of the time slot used for sensing, which results in decrease of spectrum efficiency. Single radio’s most obvious advantage aganist all these previous disadvantages is its simplicity and lower cost. In dual radio architecture, as seen from its name there exist two dedicated radios; one for data reception and transmission, one for spectrum sensing. The disadvantages of this architecture are the hardware cost and the power consumption. An overall comparison of single and dual radio architecture is given by Yücek and Arslan (2009) , also listed in Table 2.1.

Table 2.1 Comparision of single-radio and dual radio architecture (Yücek & Arslan, 2009)

	Single Radio	**Dual Radio**
Advantages	Simplicity	Higher spectrum efficiency
	Low cost	Better sensing accuracy
Disadvantages	Lower spectrum efficiency	Higher cost and power consumption
	Poor sensing accuracy	Higher complexity

Selection of system parameters brings tradeoffs among speed, cost, complexity, reliability etc. As explained in the previous sections, careful design of CR is very important. Total achievable throughput depends on this careful design specific to CR enviroments and its capabilities. As Haykin stated in 2005 that;

"In passively sensing the radio scene and thereby estimating the power spectra of incoming RF stimuli, we have a basis for classifying the spectra into three broadly defined types,summarized here.

1) *Black spaces*, which are occupied by high-power 'local' interferers some of the time.

2) *Grey spaces*, which are partially occupied by low-power interferers.

3) *White spaces*, which are free of RF interferers except for ambient noise, made up of natural and artificial forms of noise, namely:

- broadband thermal noise produced by external physical phenomena such as solar radiation;
- transient reflections from lightening, plasma (fluorescent) lights, and aircraft;
- impulsive noise produced by ignitions, commutators, and microwave appliances;
- thermal noise due to internal spontaneous fluctuations of electrons at the front end of individual receivers."

As seen from above, CR system should identify black and white spaces for sure and grey spaces in some circumstances according to some selected parameters.

Common spectrum sensing techniques in CR literature can be categorized under following groups: energy detector-based, waveform-based, cyclostationarity-based, matched filtering, radio identification- based, multitaper spectral estimation, wavelet transform-based estimation, Hough transform, and time-frequency analysis (Yücek & Arslan, 2009). We will briefly explain first three techniques below which are very popular in real world applications.

- **Energy Detector-Based Sensing:** This method is the most common way of spectrum sensing which is known as radiometry or periodogram. Detection is done by comparing the measured energy to a threshold value according to the noise floor. No primary signal characteristic knowledge is required. Since it only detects energy, it can not identifies primary user signals and noise. Consequently, poor performance occurs in low SNR values. Advantages of this technique are its low complexity and cost.

- **Waveform-Based Sensing:** Know patterns like preamble, end flags, pilot tones are used to correlate the received signal with a known copy. Signal is synchronised to the known pattern, that's why this technique termed as coherent sensing.

- **Cyclostationarity-Based Sensing:** This technique uses received signal's cyclostationary features caused by the periodicity in the signal or signal's statistics like mean and autocorrelation. In the sensing, cyclic correlation function is used instead of power spectral density (PSD).

A very primitive comparision of the techniques according to their complexity and accuracies are illustrated in Figure 2.2.

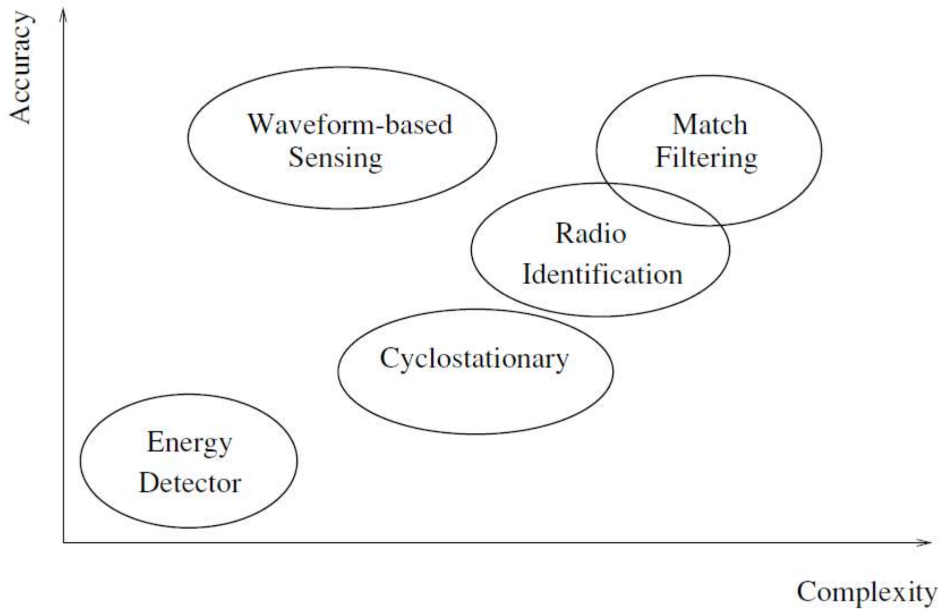

Figure 2.2 Sensing methods in terms of accuracy vs. complexity (Yücek & Arslan, 2009)

Besides these, spectrum sensing can be done cooperatively. Centralized, distributed and external type of sensing exist. Centralized sensing, as understood from the name, sensing data collected from multiple CR devices and transmitted to a central unit which directly manages the traffic or sends information to other CR devices. Main goal is to lower the fading effects and increase the detection performance. In distributed sensing, different CR devices share their sensing information while making decion on their own. This method is lower in cost and complexity since there is no need for a central unit and backbone network. These two sensing methods are done internally in CR devices. External sensing, is a method which sensing is done externally in another agent. This technique is performed to overcome some problems, like hidden primary user, which occurs when sensing is done internally.

Spectrum sensing can be considered as front end of the CR system. How well the spectrum sensing is performed depends on the awareness of CR system of its enviroment. Succesful spectrum sensing lessens the interference to the primary user and increases the accuracy in reception of the transmitted data. Spectrum sensing algorithms in CR systems are in their very earlier form and still under investigation.

It seems performance of these techniques will be enhanced in near future as hardware limitations are enhanced.

2.1.2 Spectrum Sharing

Spectrum opportunities detected by the secondary users are indeed dedicated to primary users who are also named as licensed users. Since the secondary users detect a specific spectrum hole without any knowledge about the existence of each other, they might collide with each other, and as a result the overall performance of secondary network and spectrum efficiency degrade. All the scenarios previously mentioned are focusing on noncooperative secondary users. As this problem arises, spectrum sharing among the secondary users has a significant role on secondary network performance. To be able to overcome this problem, one approach suggests a procedure based on request to transmit (RTS) and clear to send(CTS) signals based sharing. Possible opportunity detector devices sends RTS signals, where a central unit like secondary base station, or receiver directly make decision and sends a CTS signal that defines who is going to make a transmission. Since transmission and reception of these signals are problematic, main problem of this technique is the degradation of network's capability of data transmission since this communication takes time from data transmission portion in a time slot.

Other approaches to overcome this problem are based on stochastic game theory and outside the scope of this thesis. Interested reader can start with Section VIII of "Haykin S. , 2005, Cognitive Radio : Brain Enpowered Wireless Communications".

2.1.3 Spectrum Management

Spectrum access strategy of the CR system is controlled by the dynamic spectrum access manager. Mostly actions take part in the transmitter side. According to the spectrum opportunity decision, spectrum management on transmitter power level control and spectrum access are done by a policy or predefined strategy. Traffic considerations of these managers are releated to data patterns of wireless environment. Deterministic patterns are which primary users assigned to fixed time slots for transmission and where CR system can transmit with maximum power level with a fixed spectrum access policy when the primary user is switched off. Stochastic patterns are which traffic data expressed in statistical terms where CR system use predictive model to improve accuracy of data transmission and spectrum access (Haykin, 2005).

As seen before, spectrum management agents or managers also have a significant role in spectrum efficieny since they are the decision makers for data transmission. A good spectrum access policy and management will give rise to achievable throughput and decrease the possible interference on licensed users.

2.2 Literature Review and Related Works

After CR system first mentioned by Joseph Mitola III (1999) in KTH, a detailed survey was published by Haykin in 2005. CR concept, system considerations, opportunity and spectrum hole terms are defined in philosophical manner. Sensing problems for opportunity trackers, channel estimation considerations are also listed in. Statistical processes that fit to problem formulation and design considerations are explained briefly. Furthermore, secondary networks' total achieavable throughput limitations depending on secondary users' collision on each other that takes place in same geographical area for same spectrum holes discussed in stochastic manner. Game theory approaches and limitations for secondary users' collision, further considerations of CR designs in the near future are given briefly as well. Another survey for CR system scenarios' limitations and possible solutions by using regulatory policies for these scenarios are discussed by Zhao and Sandler (2007). As tradeoffs exist between system parameters, Zhao and Sandler briefly examined interference constraints that can be tolerable by licensed users. Additionally, optimal spectrum access strategy is investigated in terms of false alarm, miss detection and maximum allowable collison probabilities. We will investigate these terms in the problem formulation section in detail; however, optimal spectrum access strategy of CR system depending on these terms is shown in Figure 2.3.

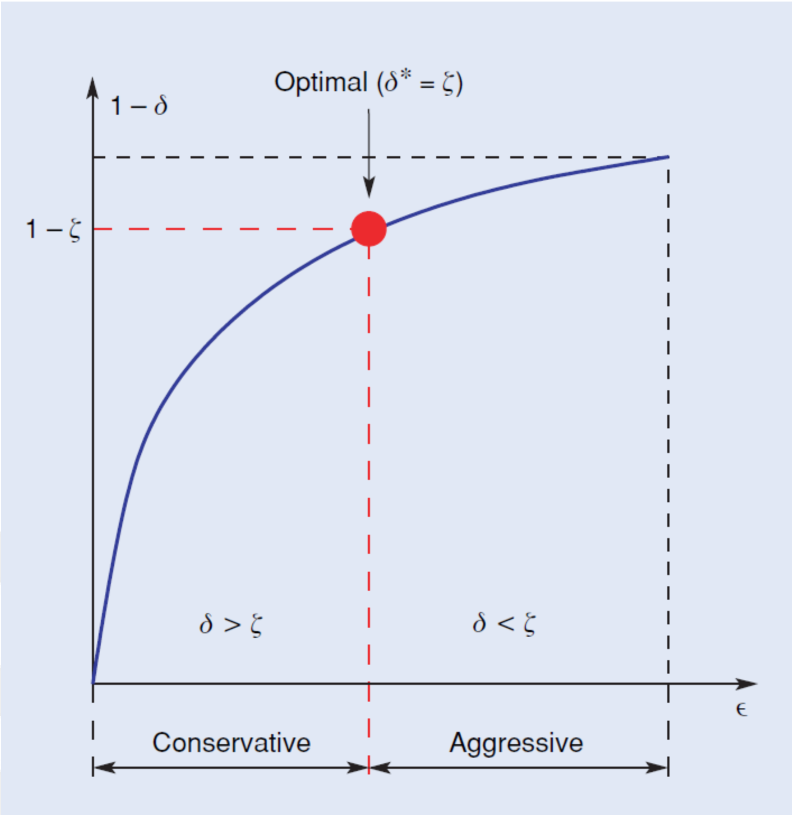

Figure 2.3 Optimal spectrum access strategy, δ: probability of miss detection, ζ**:** maximum allowable collision probability (Zhao & Sandler ,2007)

As receiver operation characteristic (ROC) seen from the figure; where detection probability is less than maximum allowable collision probability, a busy channel can be identified as idle and interference to primary user occurs. According to the graph, on the upper side or lower side of the optimal spectrum strategy point, radio behaves conservative or agressive. Both situations may cause problems or poor performance for spectrum efficiency. Optimal strategy is a must for spectrum efficiency.

Besides these surveys, only spectrum sensing considerations, limitations and comparisions are investigated by Yücek and Arslan (2009). Formulated sensing algorithms, their limitations, performances, advantages, and disadvantages over each other are given in detail.

Main goal of CR system is increasing spectrum efficiency by exploiting the unused spectrum holes. Achievable throughput is investigated in various papers with their limitation problems of power control, interference constraints, fading enviroments, modulation characteristics under both constant or time varying adaptive parameters. Statistical processes are also observed in different access schemes. Short term and long term effects of instantenous or average interference constraints are investigated and results asisted with carefully designed experiments. Optimal sensing time with careful sensing strategies for single band and wideband spectrum opportunities, centralized and distributed detection are also investigated.

We will focus on the first component of CR system. Spectrum sensing is a crucial task that should be performed before any action takes place for CR communication. The real world parameters have infinite precision. But in simulation studies assumptions are made and assumptions have limitations, like backgorund noise's being neither perfectly Gaussian nor perfectly stationary, channel fadings' being neither flat nor constant over time. Thandra and Sahai (2008) investigated these limitations and introduce the term "SNR Wall". Classical information theory says that ROC degradation in low SNR values can be countered by increasing the sensing time. Thandra and Sahai shows that in real world applications SNR below a certain threshold can not be countered no matter how much sensing time increased and this threshold named as SNR wall. For real world applications, most receivers suffer from this SNR wall. The signal below this level causes detectors to give wrong results which lead to false alarms or miss detections. This research is enhanced by Alink, Kokkeler and friends (2011) on how this wall can be lowered. They published in 2011 a sensing algorithm using cross correlation approach to be able to lower the SNR wall.

Other important issues in CR systems are interference and average power constraints which limit the throughput. Zhang, Kang and Liang (2009) focused on these subjects. Comparison of peak or average interference constraints' effect on throughput was investigated in detail. Research shows that more throughput is achieved by using avearage interference constarints compared to peak interference constraints in long term.

Research on problem formulation limitations leads researchers to start to define problems and solutions for sensing period and consequencely throughput. Liang, Zang and friends (2008) searched for tradoffs between sensing and throughputs. Effects of false alarm and detection probabilities are investigated. How they can effect the optimal sensing time is showed. An improvement in sensing time is mentioned by using distributed spectrum sensing. Liang, Teh et. al. (2009) also investigated these tradeoffs for wideband spectrum access. Throughput for the instantenous and average power constraints are compared. However, in this investigation, CR system does not transmit data unless the spectrum is idle. Primary users existence forbids the seconday users to transmit any signal.

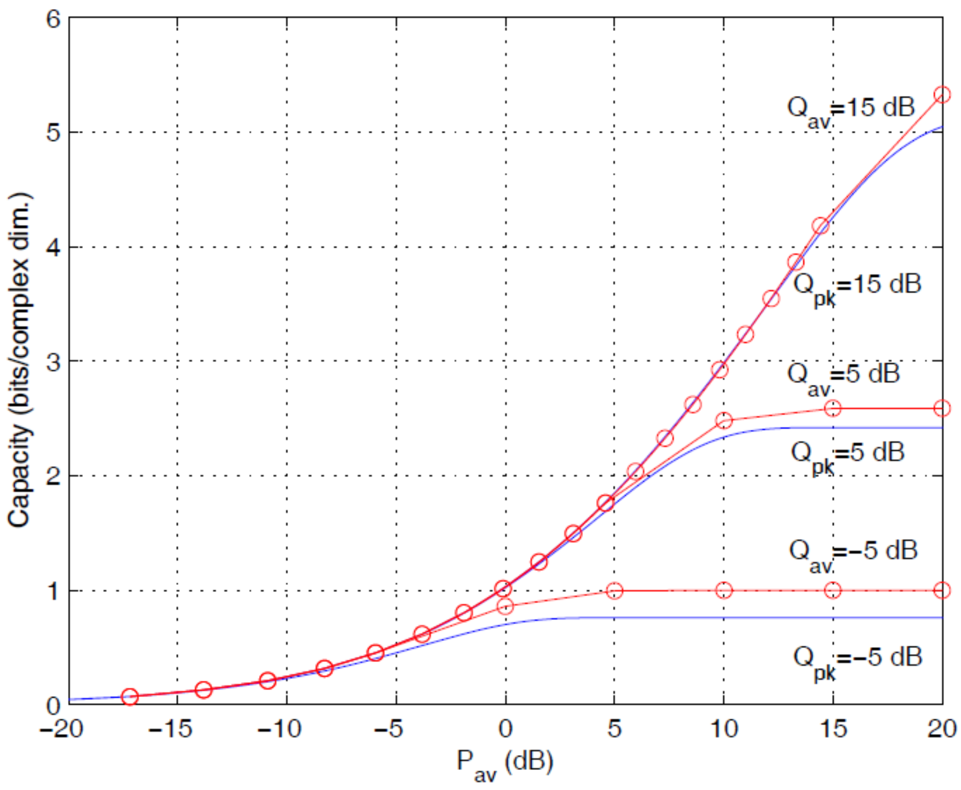

Figure 2.4 Ergodic capacity under peak and average power constraints (Liang, Teh et al, 2009)

According to Figure 2.4 for low levels of total transmit power constraint, interference constraints have not different outputs for the CR network. However, when the total transmit power started to be relaxed, it is obvious that average interference constraint defined CR network has more achieavable throughput over peak interference constraint defined CR network.

Nallathan and Stotas (2011) introduced another comparison of access scheme with transmission on the busy channel with a relatively low power configuration that does not violate the primary user's interference constraints. We will focus on this type of access scheme as Wideband Sensing-based Spectrum Sharing (WSSS) in the following sections.

Future considerations in CR network might be in mobility problems. All previously mentioned scenarios are using stationary CR nodes. Mobility is a major concern for future devices. Win et. al. (2011) investigated spectrum access strategy for mobile CRs. Some reseraches show that mobility increases the coverage of CR network and in this way, total throughput might increase. Problem is still under investigation and will be very popular in the near future.

CHAPTER THREE

SYSTEM MODEL AND FORMULATIONS

General concepts of CR systems are examined in the previous sections. As seen before, CR system works as a spectrum opportunity tracker. When it finds an opportunity, it tends to increase efficieny of electromagnetic resources. CR system as previously discussed has two parts: sensing part where enviromental awareness done and management part where decision makers according to enviroment take roles. This section will give detailed information about these components. We will discuss spectrum access schemes, try to model a scenario, set up model parameters and try to maximize the total achievable throughput of CR network by finding an optimal sensing time with respect to interference to primary user (PU) and total average transmit power constraint (P_{av}).

Every CR network has frame structure that consists of slots where sensing and data transmission portions exist as shown in Figure 3.1.

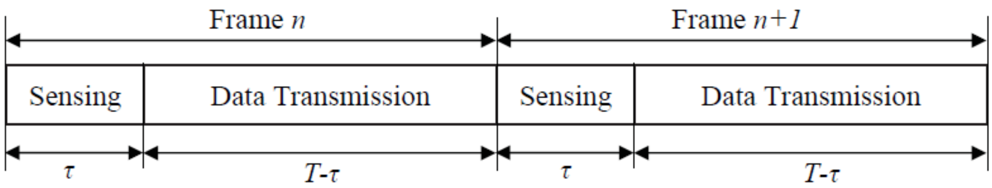

Figure 3.1 Frame structure of cognitive radio

In sensing portion of the slot, CR listens to the enviroment, and determines the channel state and uses data transmission portion for accessing the channel and starting the communication.

3.1 Spectrum Access Scheme

A possible scenario for CR system is shown in Figure 3.2. Spectrum opportunity and access terms depend on real world examination. Before explaining spectrum access strategies, let us give a definition of *spectrum opportunity*: “A channel can be considered as an *opportunity* if it is not currently used by primary users” (Zhao & Sandler, 2005).

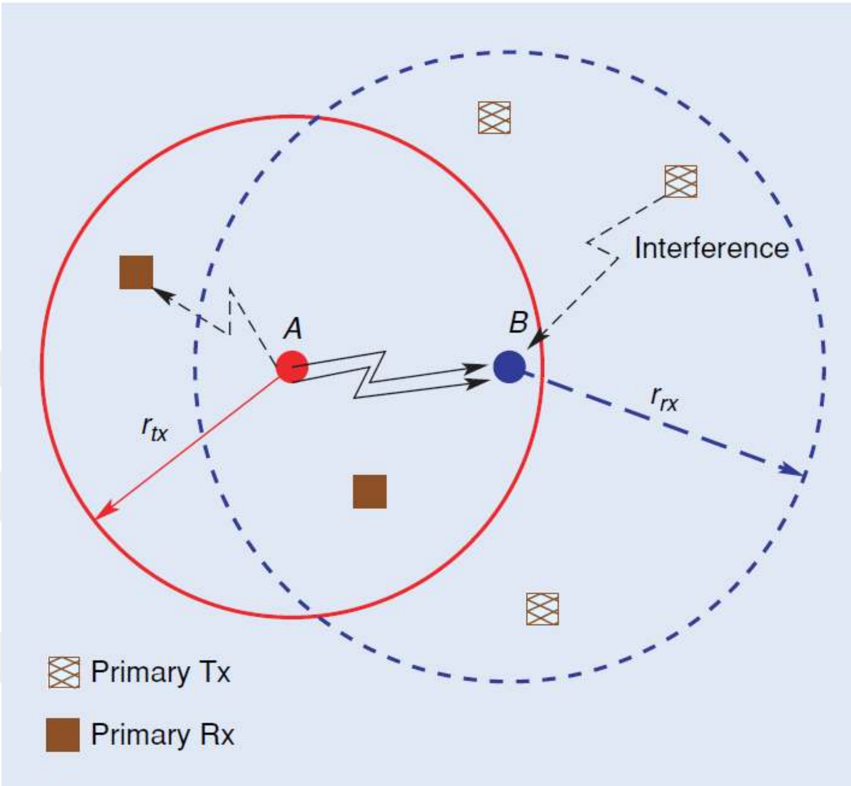

Figure 3.2 Spectrum opportunity (Zhao & Sandler, 2005)

As seen from Figure 3.2, secondary users (SUs) A and B wish to communicate with each other where they should concern about interference to primary users. That’s why they should carefully listen to the channel and if a spectrum opportunity is catched then they transmit data to each other.

Spectrum access scheme can be identified as SU’s reaction aganist spectrum opportunity decision. In today’s CR networks there exist three types of spectrum access. First one is opportunistic spectrum access (OSA) in which SU transmits data whenever the channel is detected as idle. Second one is spectrum sharing in which SU always share the channel with PU. In this type access scheme SU always uses a low power configuration not to cause any interference that can not be tolerable by PU receivers. Third one is a hybrid of the first two methods, sensing-based spectrum sharing (SSS). In this type of access scheme SU first senses the spectrum, and adapts its transmission power according to the decision.

All access schemes introduced previously concern about PUs' quality of service (QoS). Since they are licensed users, a true freedom is introduced in their spectrum access strategy. They can transmit whenever they want with the maximum transmit power configuration. This forces secondary networks to introduce maximum allowable interference constraints that can be tolerable by PUs. If we look for many standards, co-channel interference margin in many applications are 10 decibel (dB) which corresponds to signal to noise plus interferer ratio (SINR). For example, in IEC 61193-Ed.2 receiver co-channel performance test is done with a wanted signal level of -104 dBm where unwanted interferer signal level is -114 dBm. Test results expected for maximum packet error rate of 20 % which corresponds to bit error rate (BER) of 1 % which is quite good. So, SINR value of 10 dB or above can be accepted as a good approximation for SU's interference constraint.

Another constraint of SU is about total transmit power. If secondary user could transmit with no restriction in power level, with a wrong decision on PU's existence it may cause interference. Not only primary user interference would be introduced, but also adjacent channel interference for SUs would probably occur. P_{av} could help CR to prevent this type of collision in the network.

3.1.1 Wideband Sensing-based Spectrum Sharing (WSSS) Scheme

Wideband refers to using multiple channels instead of single channel operation. As understood CR network performs spectrum sensing in different non-overlapping narrowband channels and finds the opportunity for data transmission. If a channel is not occupied by PU than it is decided that the channel is idle for transmission. If a PU is doing transmission, the channel is said to be busy. Since there are only two situations, spectrum sensing for spectrum access is a binary hyphotesis. Probability of j^{th} channel state is represented with $H_{0,j}$, if it is idle and $H_{1,j}$, if it is busy.

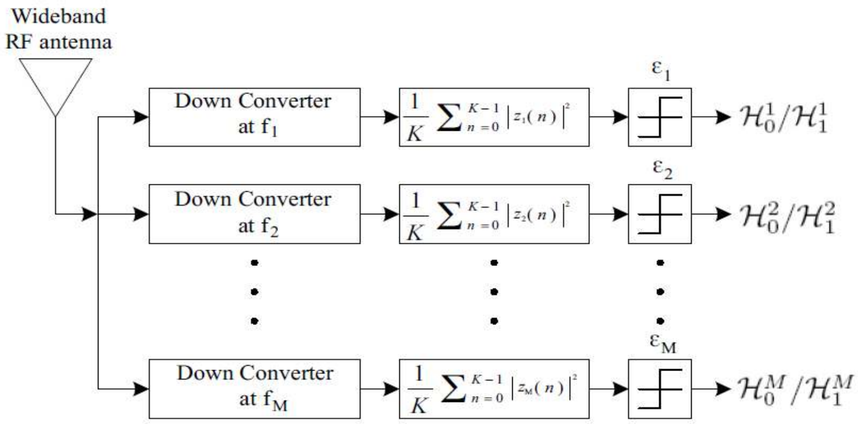

Figure 3.3 Wideband downconversion scheme (Liang, Teh & friends ,2009)

In Figure 3.3, received signals are downconverted to intermediate or baseband frequencies to be able to provide suffient sampling rate by ADCs. As Nyquist theorem states that the sampling frequency should be at least twice the desired frequency. After downconversion, sampled data is used in an energy detector scheme and decision is made by the CR. If j^{th} channel detected as idle, data transmission is done by SU with high power configuration ($P_{s,j}^{(0)}$), if the channel is detected as busy, transmission is done with a relatively low power configuration ($P_{s,j}^{(1)}$) where *s* refers secondary.

To sum up, WSSS scheme is a wideband spectrum access strategy where channel access mechanism is defined according to detected energy level by the detector regardless of the channel's actual status. It adapts its transmission power according to decision made by detector.

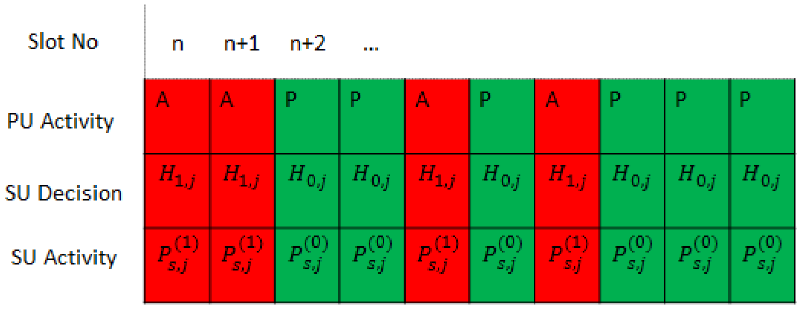

Figure 3.4 Perfect sensing, A: Active, P: Passive

In Figure 3.4, it is seen that secondary user always active in the channel. Since WSSS scheme always transmit data, it is expected to achieve high throughput over OSA, if detection is done perfectly and QoS of PU is not affected by SU's activity. Otherwise, violation of primary user interference constraints is unavoidable.

3.2 Optimal Sensing Time

Figure 3.4 shows us that spectrum opportunity tracking is done in a time division manner. We assume that all primary users transmit data in the beginning of predefined time divisioned slots. Very small amount of delay beacuse of porpagation is omitted. Let's assume that one slot's length equals to T miliseconds (ms). This time divisioned T length slots are also divided into two parts by SU. Sensing time (τ) and data transmission portion ($T - \tau$). Sensing time is very significant in total throughtput since it defines also data transmission portion. As it is clearly seen, the less the sensing time, the more data would be transmitted. So why don't we make it very small? Because some real world limitations affecting detector mechanisms force us to find an optimal value. Uncertainties in noise, fading effects etc. bound detectors robustly detect the PU activity in low SNR values. In order to prevent interference, robust detection should be investigated and interference constraints should not be violated by the CR users. That's why a tradeoff between sensing time and wrong decisions exists. To be able to minimize the number of these wrong decisions as low as that can be tolerable by primary network, we should optimize sensing period (τ).

3.2.1 Channel Model

CR network consists of PU transmitter (Tx), receiver (Rx) and SU Tx and Rx exist in the same geographical area as seen in Figure 3.5.

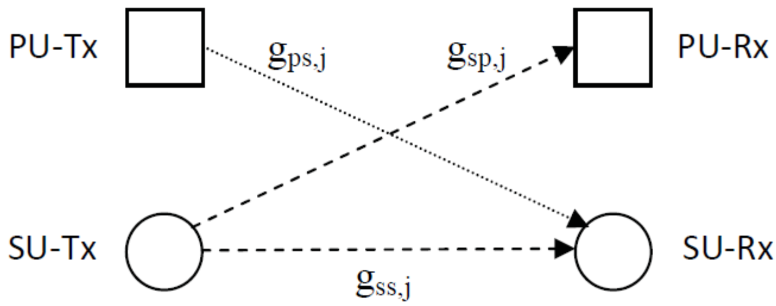

Figure 3.5 Cognitive radio network with channel power gains

Figure 3.5 also shows us the instantenous channel power gains between PU-Tx and SU-Rx, SU-Tx and PU-Rx , SU-Tx and SU-Rx for the j^{th} channel respectively, $g_{ps,j}$, $g_{sp,j}$, $g_{ss,j}$.The channels are assumed to be flat fading and channel gains are assumed to be ergodic stationary and known at the secondary users. Channel power gain's effects on total throughput will be investigated in simulations.

3.2.2 Detector

As mentioned in the spectrum sensing section, we will use well known energy detector because of its simplicity and freedom of signal parameters.

Assume that we are interested in center frequency of f_c and bandwith of B and received signal sampled at sampling frequency of f_s. Under the hypothesis of $H_{0,j}$ which means that j^{th} channel is idle, received discrete signal at the secondary receiver ($y_j(n)$) is represented as;

$$y_j(n) = w_j(n) \tag{3.1}$$

where $w_j(n)$ is the independent and identically distributed circularly symmetric complex gaussian (CSCG) noise with zero mean and N_0 variance, respectively $CN(0, N_0)$. If the channel is occupied by PU i.e., hypothesis $H_{1,j}$, received discrete signal at the secondary receiver is as follow:

$$y_j(n) = s_j(n) + w_j(n) \tag{3.2}$$

where $s_j(n)$ is the received primary signal, independent of noise. Under this condition, j^{th} channel received signal in the energy detector is represented by test static $T_j(y)$ and given by Equaiton (3.3).

$$T_j(y) = \frac{1}{N}\sum_{n=1}^{N}\left|y_j(n)\right|^2 \tag{3.3}$$

N represents total sample number which is the maximum integer that fits $N \leq \tau f_s$ criteria. Energy detector of the SU will acquire the samples and make a decision according to a predefined threshold ϵ_j for the selected channel j. If the test statistic is below the threshold, the channel assumed to be idle and transmission is done in high power configuration. If test statistic is higher than the threshold than channel assumed to be busy and low power setting is applied in transmission. A block diagram of energy detector is given in Figure 3.6.

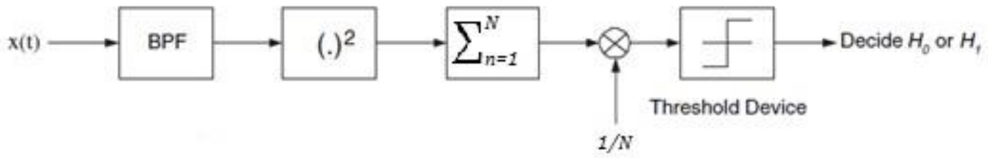

Figure 3.6 Energy detector block diagram

All the scenarios mentioned in this section assume that the detection is done perfectly. What will happen if the sensing is not perfect? A channel assumed busy can actually be idle which means a false alarm made by the detector. Now we need to introduce two terms: false alarm and miss detection.

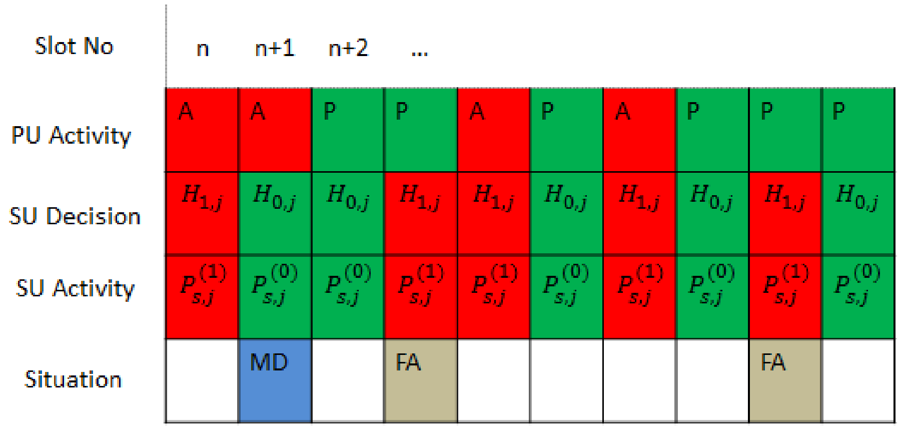

Figure 3.7 Non perfect sensing, A : Active, P :Passive, MD : Miss Detection and FA : False Alarm

As described previously, if channel's actual status is idle and detector's decision is busy, than a false alarm occurs. If channel's actual status is busy and detector's decision is idle, that means miss detection occured. The false alarm probability can be formulated as below.

$$P_f(\tau, \epsilon) = \Pr(T(y) > \epsilon | H_0) \tag{3.4}$$

It is seen that the probability of false alarm depends on test statistic which releated directly to the sensing time.

3.2.3 Probabilities

We talked in the previous sections about term SNR wall. Thandra and Sahai (2008) stated that probability of detection for a certain threshold can be formulated as;

$$P_{d,j}(\tau, \epsilon_j) = Q\left(\left(\frac{\epsilon_j}{N_0} - \gamma_j - 1\right)\sqrt{\frac{\tau f_s}{2\gamma_j + 1}}\right) \tag{3.5}$$

where ϵ_j is the detection threshold of the detector for j^{th} channel, N_0 is the noise variance, γ_j is received SNR from PU at the SU detector and f_s is the sampling frequency. Q is complementary Gaussian cumulative distribution function (cdf):

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt \tag{3.6}$$

In the same manner, we write the probability of false alarm;

$$P_{fa,j}(\tau, \epsilon_j) = Q\left(\left(\frac{\epsilon_j}{N_0} - 1\right)\sqrt{\tau f_s}\right) \tag{3.7}$$

If one looks these last two Equations (3.5) and (3.7), similarity will be seen easily. In Equation (3.5) if we select γ_j= 0, than Equation (3.7) is achieved. In CR designs, mostly a certain detection probability chosen and other design parameters fitted for the evaluation of this detection probability. In this manner, we need to formulate the false alarm probability in terms of detection probability.

$$P_{fa,j}(\tau, \epsilon_j) = Q\left(\sqrt{2\gamma_j + 1}Q^{-1}(P_{d,j}) + \sqrt{\tau f_s}\gamma_j\right) \tag{3.8}$$

Miss detection probability of the system is the case that detection could not be performed by the energy detector properly and it is formulated as in Equation (3.9).

$$P_M(\tau, \epsilon_j) = 1 - P_{d,j}(\tau, \epsilon_j) \tag{3.9}$$

$$= 1 - Q\left(\left(\frac{\epsilon_j}{N_0} - \gamma_j - 1\right)\sqrt{\frac{\tau f_s}{2\gamma_j + 1}}\right) \tag{3.10}$$

3.2.4 Transmission Rates and Throughput

We are trying to find optimal sensing time that maximizes the SU ergodic capacity. CR network's ergodic capacity maximization depends on the optimization of the sensing time. Instantaneous transmission rate regardless of bandwidth is given by Shannon theorem (1948). It is;

$$R = log_2(1 + SNR) \tag{3.11}$$

To be able to formalize the throughput we first give the instantaneous transmission rates of secondary network. Transmission rate of j^{th} channel under hypothesis $H_{0,j}$, primary user is absent, denoted as $r_{0,j}$. Transmission rate of j^{th} channel under hypothesis $H_{1,j}$, primary user is present, denoted as $r_{1,j}$. Equations (3.12) and (3.13) respectively give these transmission rates.

$$r_{0,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{N_0}\right) \tag{3.12}$$

$$r_{1,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{g_{ps,j}P_{p,j}+N_0}\right) \tag{3.13}$$

In Equations (3.12) and (3.13), if we look at the terms inside the logarithm numerator, terms represent received signal power at the SU Rx and denominator term in the first equation is noise, in the second equation interferer power plus noise power. In both equations, received SNR values for energy detector are evaluated in the formula.

However; in real world cases, things are little bit confusing, since the nature of spectrum sensing is not perfect. Due to the limitations in techniques and some wireless communication phenomena like shadowing and fading cause detectors to give false alarms or possible miss detections. If we count these in transmission rate scheme, we will have two more transmission rates under these conditions as given below.

Table 3.1 Instantaneous transmission rate notation according to primary user activity and secondary user decision

PU Activity	**SU Decision**	**Insta. Trans. Rate**
passive	idle	$r_{00,j}$
passive	busy	$r_{01,j}$
active	idle	$r_{10,j}$
active	busy	$r_{11,j}$

According to the given notations, all transmission rates are given below.

$$r_{00,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{N_0}\right) \tag{3.14}$$

$$r_{01,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{N_0}\right) \tag{3.15}$$

$$r_{10,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(0)}}{g_{ps,j}P_{p,j}+N_0}\right) \tag{3.16}$$

$$r_{11,j} = \log_2\left(1 + \frac{g_{ss,j}P_{s,j}^{(1)}}{g_{ps,j}P_{p,j}+N_0}\right) \tag{3.17}$$

Thus, the average throughput of the j^{th} channel for SU under WSSS can be formulated as follow.

$$C_j = P(H_{0,j})\left(1 - P_{fa,j}(\tau, \epsilon_j)\right) r_{00,j} + P(H_{0,j})P_{fa,j}(\tau, \epsilon_j)r_{01,j}$$

$$+ P(H_{1,j})\left(1 - P_{d,j}(\tau, \epsilon_j)\right) r_{10,j} + P(H_{1,j})P_{d,j}(\tau, \epsilon_j)r_{11,j} \tag{3.18}$$

In Equation (3.18), sensing time ignored for the time being. $P(H_{0,j})$ and $P(H_{1,j})$ are probabilities that j^{th} channel are idle and busy respectively.

3.2.5 Constraints

Constraint terms are introduced for the QoS of primary and secondary networks by preventing interferences and keeping long term power budgets. To be able to keep long term power budget of secondary users, an average power constraint is introduced.

$$\frac{T-\tau}{T}\mathbb{E}\Big\{\sum_{j=1}^{M}[P(H_{0,j})P_{s,j}^{(0)}\left(1-P_{fa,j}(\tau,\epsilon_j)\right)+P(H_{0,j})P_{s,j}^{(1)}P_{fa,j}(\tau,\epsilon_j)$$

$$+P(H_{1,j})P_{s,j}^{(0)}\left(1-P_{d,j}(\tau,\epsilon_j)\right)+P(H_{1,j})P_{s,j}^{(1)}P_{d,j}(\tau,\epsilon_j)]\}\leq P_{av} \quad (3.19)$$

P_{av} in Equation (3.19) represents the average power constraint, $\mathbb{E}$ is the expectation operator. As seen from the Equation (3.19), transmission powers of all the states are averaged over data transmission portion.

Additional constraint is for protecting primary users from interference. Interference occurs when primary user active in the channel. This happens when the primary user is detected and low power configuration is used and additionally; when miss detection occurs, interference to primary user happens with high power configuration. It is given by;

$$\frac{T-\epsilon}{T}\mathbb{E}\Big\{g_{sp,j}P_{s,j}^{(0)}P(H_{1,j})\left(1-P_{d,j}(\tau,\epsilon_j)\right)$$

$$+g_{sp,j}P_{s,j}^{(1)}P(H_{1,j})P_{d,j}(\tau,\epsilon_j)\}\leq\Gamma, j=1,..,M \quad (3.20)$$

where j represents channel number, Γ is the interference power constraint. By introducing these two average constraint terms as investigated by Zhang, Kang and

Liang (2009), research proved that introducing average constraints instead of instantenous constraints increases the ergodic capacity and at the same time primary users are better protected.

3.2.6 Problem Formulation

After giving the components of our CR system, let us examine briefly again overall system. Our CR system is a wideband system where multiple narrowband frequencies with B bandwidth are being sensed at the same time. Sensing-based spectrum access scheme is used which has two transmission power configurations. According to the channel state, if it is idle H_0, P_0 is used, if it is busy H_1, P_1 is used. Energy detector senses and takes the test statistic of each channel and makes a decision according to this test statistic. We calculate the detector's false alarm and miss detection probabilities. To be able to find ergodic capacity we first introduce instantaneous transmission rates in all transmission states. According to the probabilities of these states, channel throughput is averaged which is given by Equation (3.18). Since CR aims to prevent interference to primary user with a tolerable rate, average interference constraint is introduced. Also for long term power budget and to prevent possible interference among secondary users, average total transmit power constraint is introduced. This parameter also has a significant role for battery used operations. Now, the problem can be formulated to maximize throughput under these conditions as:

$$\underset{\left\{\tau, P_s^{(0)}, P_s^{(1)}\right\}}{maximize} \mathbb{E}\left\{\sum_{j=1}^{m} \frac{T-\tau}{T} C_j\left(\tau, P_s^{(0)}, P_s^{(1)}\right)\right\} \tag{3.21}$$

subject to Equation (3.19) and (3.20), $P_s^{(0)} > 0$, $P_s^{(1)} > 0$, $T > \tau > 0$

Above optimization problem is convex with respect to the transmit powers $P_s^{(0)}, P_s^{(1)}$ but not with respect to sensing time τ. Because; according to the sensing time, probability of false alarm changes and according to Equation (3.7) that affects the overall throughput. Since the optimization problem (3.21) is not convex according to the sensing time, evaluation of the problem can not be done by using convex optimization methods. However, knowing that sensing time τ relies on the slot duration *T*, makes it easy to find by using an exhaustive search. In this manner, above optimization problem can be maximized according to transmit powers and then optimal sensing time can be evaluated by using an exhaustive search algorithm.

Optimal power allocation strategy of the CR system that maximizes ergodic capacity can be solved by using Lagrangian form of optimization problem given in Equation (3.21). The Lagrangian with respect to $P_s^{(0)}$, $P_s^{(1)}$ is given below.

$$\begin{aligned} L\left(P_s^{(0)}, P_s^{(1)}, \lambda, \mu\right) \\ &= \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\tau}{T}\left(a_{0,j} r_{00,j} + \beta_{0,j} r_{10,j} + a_{1,j} r_{11,j} + \beta_{1,j} r_{01,j}\right)\right\} \\ &-\lambda\left[\frac{T-\tau}{T}\mathbb{E}\left\{\sum_{j=1}^{M}\left[\left(a_{0,j}+\beta_{0,j}\right).P_{s,j}^{(0)} + \left(a_{1,j}+\beta_{1,j}\right)\right]\right\} - P_{av}\right] \\ &-\sum_{j=1}^{M}\mu_j\left[\frac{T-\tau}{T}.\mathbb{E}\left\{g_{sp,j}\beta_{0,j}P_{s,j}^{(0)} + g_{sp,j}\beta_{1,j}P_{s,j}^{(1)}\right\} - \Gamma\right] \end{aligned} \tag{3.22}$$

where parameters $a_{0,j}$, $\beta_{0,j}$, $a_{1,j}$, and $\beta_{1,j}$ are;

$$a_{0,j} = P\left(H_{0,j}\right)\left(1 - P_{fa,j}(\tau)\right) \tag{3.23}$$

$$\beta_{0,j} = P\left(H_{1,j}\right)\left(1 - P_{d,j}\right) \tag{3.24}$$

$$a_{1,j} = P\left(H_{0,j}\right)P_{fa,j}(\tau) \tag{3.25}$$

$$\beta_{1,j} = P\left(H_{1,j}\right)P_{d,j} \tag{3.26}$$

j is the channel number respectively. Solution for this problem can be countered by using dual optimization problem and which is given below by Equation (3.27).

$$\underset{\lambda \geq 0, \mu \geq}{minimize} \; g(\lambda, \mu) \tag{3.27}$$

Optimization problem (3.27) represents Lagrange dual function and can be evaluated by taking supremum of Equation (3.22) with respect to transmit powers $P_s^{(0)}$, $P_s^{(1)}$.

$$g(\lambda, \mu) = \underset{P_s^{(0)} P_s^{(1)}}{sup} L\left(P_s^{(0)}, P_s^{(1)}, \lambda, \mu\right) \tag{3.28}$$

In Equation (3.21), it can be seen that optimization problem with respect to transmit powers $P_s^{(0)}$, $P_s^{(1)}$ is convex with affine inequality constraints that Slather's condition holds. Since Slather's conditions hold duality gap between primal function and dual function is zero which means solution for the dual function is also solution for the primal optimization function. A very small brief about used optimization terms are given in Appendix-B.

As given by Stotas and Nallanathan (2011), given dual problem (3.28) can be solved in two subproblems. By finding supremum of Lagrangian with respect to $P_s^{(0)}$, $P_s^{(1)}$, subproblems are given as follows:

Subproblem 1 (SP1) :

$$\underset{P_s^{(0)} \geq 0}{maximize} \; \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\tau}{T}\left(a_{0,j} r_{00,j} + \beta_{0,j} r_{10,j}\right)\right\}$$

$$-\lambda \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\tau}{T}\left(a_{0,j} + \beta_{0,j}\right) P_{s,j}^{(0)}\right\}$$

$$-\sum_{j=1}^{M} \mu_j \frac{T-\tau}{T} \mathbb{E}\left\{g_{sp,j} \beta_{0,j} P_{s,j}^{(0)}\right\} \tag{3.29}$$

Subproblem 2 (SP2) :

$$\underset{P_s^{(1)} \geq 0}{maximize} \ \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\tau}{T}\left(a_{1,j} r_{11,j} + \beta_{1,j} r_{01,j}\right)\right\}$$

$$-\lambda \mathbb{E}\left\{\sum_{j=1}^{M} \frac{T-\tau}{T}\left(a_{1,j} + \beta_{1,j}\right) P_{s,j}^{(1)}\right\}$$

$$-\sum_{j=1}^{M} \mu_j \frac{T-\tau}{T} \mathbb{E}\left\{g_{sp,j} \beta_{1,j} P_{s,j}^{(1)}\right\} \tag{3.30}$$

Above subproblems SP1 and SP2 are convex optimization problems. If Lagrangians are written and Karush-Kuhn-Tucker conditions are applied, the optimal power when the channel detected to be idle $H_{0,j}$ can be evaluated as:

$$P_{s,j}^{(0)} = \left[\frac{A_{0,j} + \sqrt{\Delta_{0,j}}}{2}\right]^{+} \tag{3.31}$$

where $[x]^{+}$ denotes max $(x,0)$ and;

$$A_{0,j} = \frac{\log_2(e)\left(a_{0,j} + \beta_{0,j}\right)}{\lambda\left(a_{0,j} + \beta_{0,j}\right) + \mu_j \beta_{0,j} h} - \frac{2N_0 + S_j}{g_{ss,j}} \tag{3.32}$$

$$\Delta_{0,j} = A_{0,j}^{2} - \frac{4}{g_{ss,j}}\left\{\frac{N_0^2 + S_j N_0}{g_{ss,j}} - \frac{\log_2(e)\left[a_{0,j}\left(N_0 + S_j\right) + \beta_{0,j} N_0\right]}{\lambda\left(a_{0,j} + \beta_{0,j}\right) + \mu_j \beta_{0,j} h}\right\} \tag{3.33}$$

$S_j = g_{ps,j} P_{p,j}$, $h = g_{ps,j}$

When the channel is detected as busy relatively low power configuration is used and it is given as;

$$P_{s,j}^{(1)} = \left[\frac{A_{1,j} + \sqrt{\Delta_{1,j}}}{2}\right]^{+} \tag{3.34}$$

where:

$$A_{1,j} = \frac{\log_2(e)(a_{1,j}+\beta_{1,j})}{\lambda(a_{1,j}+\beta_{1,j})+\mu_j\,\beta_{1,j}h} - \frac{2N_0+S_j}{g_{ss,j}} \tag{3.36}$$

$$\Delta_{1,j} = A_{1,j}^2 - \frac{4}{g_{ss,j}}\left\{\frac{N_0^2+S_jN_0}{g_{ss,j}} - \frac{\log_2(e)[a_{1,j}(N_0+S_j)+\beta_{1,j}N_0]}{\lambda(a_{1,j}+\beta_{1,j})+\mu_j\,\beta_{1,j}h}\right\} \tag{3.37}$$

Optimal power strategy is defined with respect to interference and total average power constraints while maximizing the ergodic capacity. As seen from the formula optimal values of λ and μ should be evaluated.

3.3 Simulation Algorithm

Optimal sensing strategy evaluation will be done by using a software program in Matlab R2013a. According to problem formulation section a small algorithm routine is given below.

Algorithm :

1. Initialize channel parameters
2. Initialize λ and μ matrices with Δ steps until $P_s^{(0)}\ or\ P_s^{(1)} < 0$
3. For $\overline{\tau} = 0 : T$
   - Calculate $P_s^{(0)},\ P_s^{(1)}$ matrix
   - Find C_j matrix
   - Take Maximum of C_j matrix and keep R matrix

   End
4. Optimal sensing time and power allocation evaluation.

$$\overline{\tau}_{opt} = argmax\ R\ (\overline{\tau}, P_s^{(0)}, P_s^{(1)})$$

$$\left[P_{s,opt}^{(0)},\ P_{s,opt}^{(1)}\right] = \left[P_s^{(0)},\ P_s^{(1)}\right]_{\overline{\tau}=\overline{\tau}_{opt}}$$

One interested in given algorithm can find source code written in Matlab in Appendix A.

3.4 Results

Simulations are done for one narrowband channel; however it is easy to extend them for multiple channels according to given formulas. Frame duration is 100 milliseconds, primary user transmission power is $P_{p,j} = 10\ dB$, target detection probability is 0.9 and channel power gains are ergodic, stationary and known at secondary user's detector for the following simulations. These choosen parameters are identical to previous works to be able to compare the results.

We start with analyzing optimal sensing time value that maximizes throughput under different average transmit power constraints.

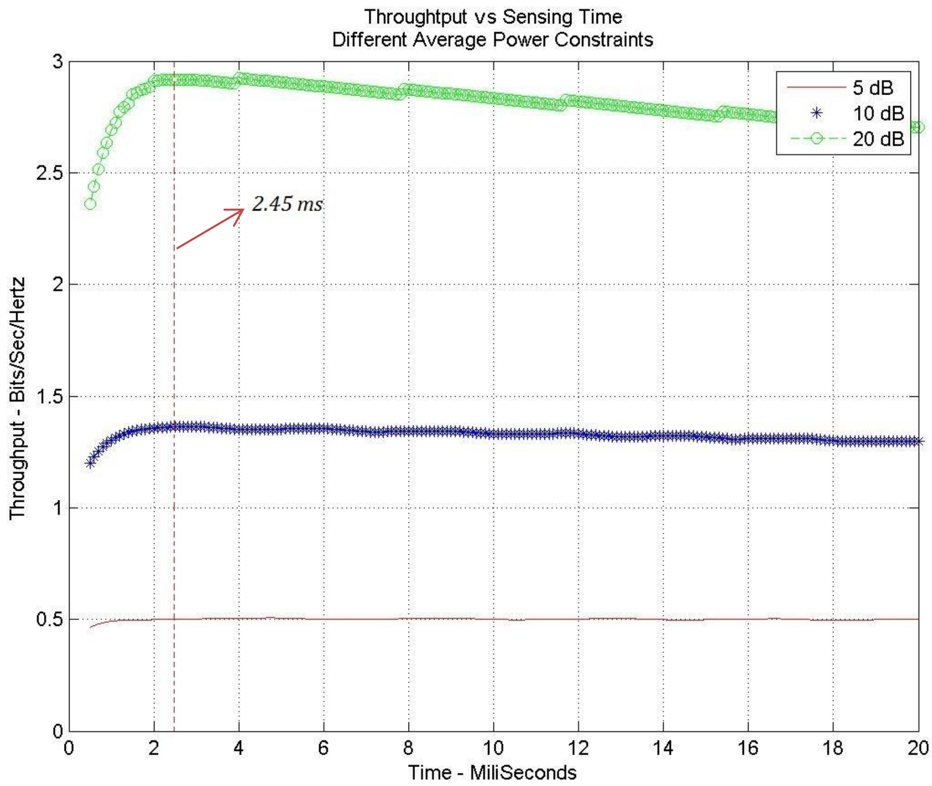

Figure 3.8 Throughput vs sensing time – Different average power constraints, $\Gamma = -10\ dB$, $g_{ps,j} = 0.01$, $g_{sp,j} = 0.01$, $g_{ss,j} = 0.09$, $SNR = -15\ dB$

Figure 3.8 gives us the maximum achievable throughput versus sensing time results under $P_{av} = 5\ dB,\ P_{av} = 10\ dB,\ P_{av} = 20\ dB$. It is seen that for almost all of the three configurations optimal sensing time $\overline{\tau}_{opt}$ is around $2.45\ ms$ which is identical to findings by Liang, Peh and friends (2008) with given SNR and interference parameters. Another conclusion can be said for the average transmit constraint and throughput tradeoff. It is seen from Figure 3.8 that if constraint is high which means SU can transmit with high average power, a SU's total achievable throughput increases. According to given average transmit power constraints optimal power setting for idle channel state and for busy channel state for a specific portion around optimal sensing time is given in Table 3.2 and Table 3.3 respectively.

Table 3.2 Idle channel transmit power values of spectrum access strategy, according to results in Figure 3.8 for different average transmit power constraints

	$P_{av} = 5\ dB,$	$P_{av} = 10\ dB$	$P_{av} = 20\ dB$
$\overline{\tau}$ (ms)	$\boldsymbol{P_s^{(0)}}$		
2	3.6838	10.6716	50.9463
2.1	3.6845	10.6734	50.9639
2.2	3.6851	10.6751	50.9790
2.3	3.6857	10.6768	50.9918
2.4	3.6863	10.6786	51.0040
2.5	3.6869	10.6802	***51.0141***
2.6	3.6875	***10.6813***	51.0038
2.7	***3.6881***	10.6813	51.0016
2.8	3.6876	10.6804	51.0009
2.9	3.6871	10.6804	51.0083
3	3.6876	10.6804	51.0056

Table 3.3 Busy channel transmit power values of spectrum access strategy, according to results in Figure 3.8 for different average transmit power constraints

	$P_{av} = 5\ dB,$	$P_{av} = 10\ dB$	$P_{av} = 20\ dB$
$\overline{\tau}$ (ms)	$P_s^{(1)}$		
2	2.4637	9.5001	22.7940
2.1	2.4695	9.5231	22.8230
2.2	2.4745	9.5469	22.8512
2.3	2.4815	9.5712	22.8772
2.4	2.4889	9.5961	22.9082
2.5	2.4951	9.6213	22.9348
2.6	2.5031	***9.6369***	22.9647
2.7	2.5013	9.6362	22.9887
2.8	***2.5082***	9.6356	***23.0033***
2.9	2.5067	9.6352	23.0012
3	2.5038	9.6348	23.0004

It is seen that SU transmits with higher power values when the channel is idle. Also it can be seen that according to power constraints, transmit power optimization can be done very well since high constraint values means high transmission power values. To be able to achieve desired long term power budget, one can define average transmit power constraint.

Another simulation shown below is done for understanding the interference constraint's effect on total achievable throughput. Total achievable throughput versus sensing time under $\Gamma = -5\ dB$, $\Gamma = -10\ dB$, $\Gamma = -20\ dB$ is given.

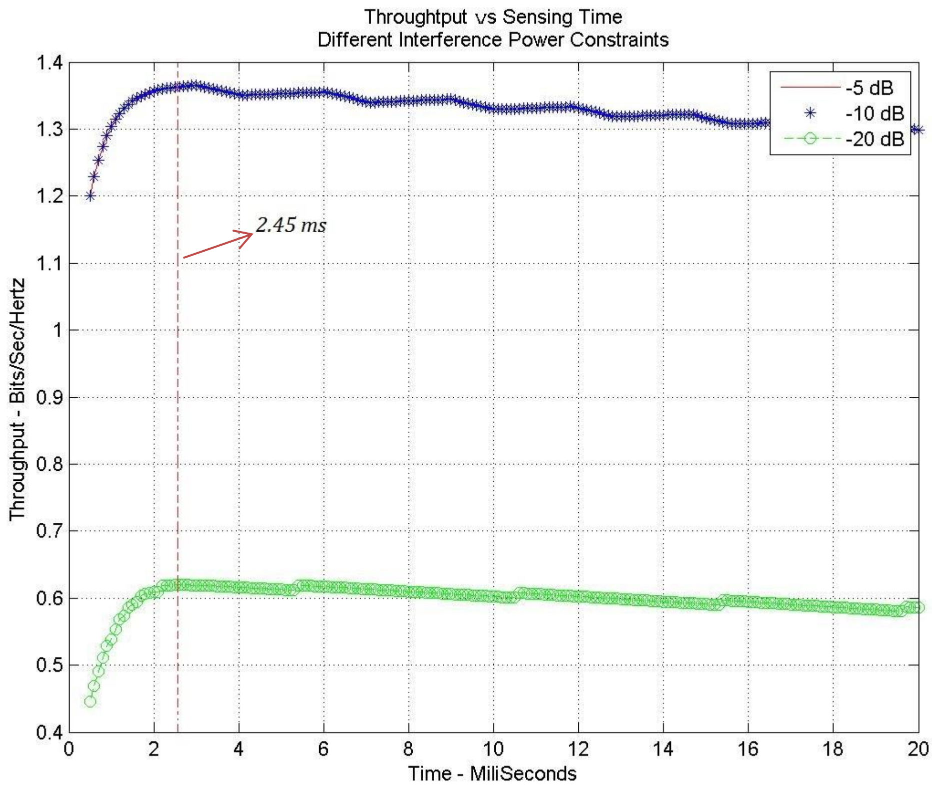

Figure 3.9 Throughput vs sensing time – Different interference power constraints, $P_{av} = 10\ dB$, $g_{ps,j} = 0.01$, $g_{sp,j} = 0.01$, $g_{ss,j} = 0.09$, $SNR = -15\ dB$

As we optimize sensing time under average interference and transmit power constraints simultaneously, Figure 3.9 tells us that in different values they can be dominant over other. It is seen that for $\Gamma = -5\ dB$ and $\Gamma = -10\ dB$ SU's total throughput is the same which means average power constraint is dominant over interference constraint which should be defined by the CR network designer. Designer can define importance of the issues, by taking into consideration of PU's QoS. When interference constraint become $\Gamma = -20\ dB$, it is seen that they change roles and interference constraint becomes more dominant over average transmit power constraint. Total achievable throughput of SU decreases, however protection of PU is increased. At last, it is seen that optimal sensing time $\bar{\tau}_{opt}$ is around $2.45\ ms$.

Another important issue is about worst case SNR value of primary signal at the secondary receiver because it affects the detector's decisions. For $SNR = -12\ dB$,

$SNR = -15\ dB$, and $SNR = -20\ dB$ values, simulation results are given in Figure 3.10.

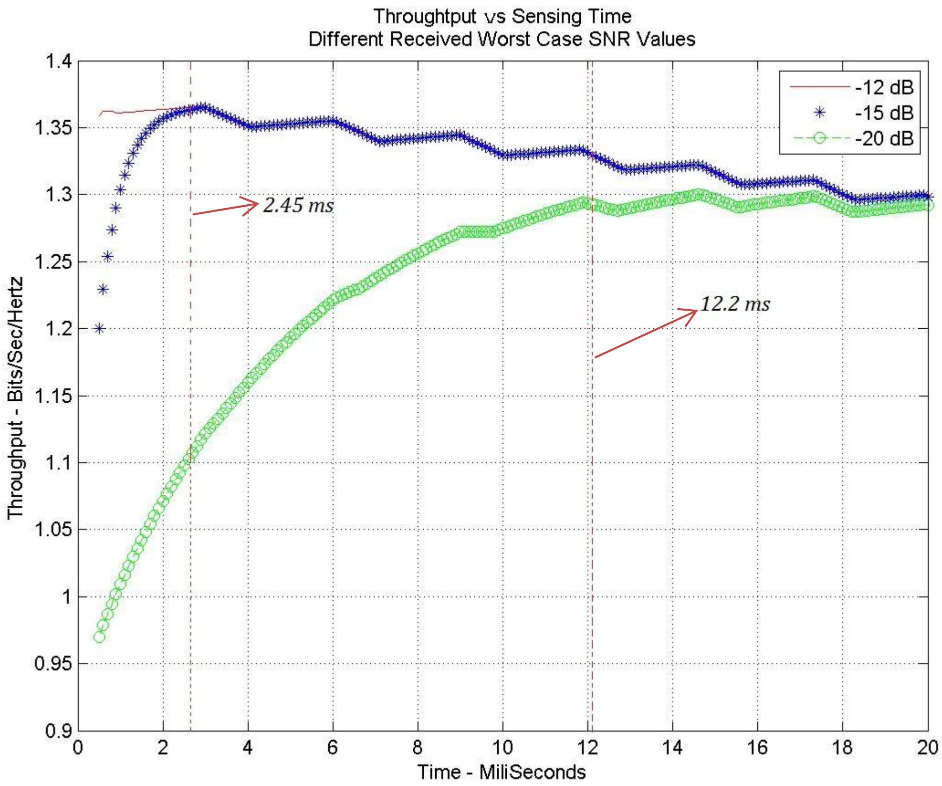

Figure 3.10 Throughput vs sensing time – Different received worst case SNR values, $P_{av} = 10\ dB$, $\Gamma = -10\ dB$, $g_{ps,j} = 0.01$, $g_{sp,j} = 0.01$, $g_{ss,j} = 0.09$,

As seen from Figure 3.10 as SNR decreases, optimal sensing time increases which is expected. Because secondary users's detector should detect signals with very low values which might cause possible miss detections and false alarms. To be able to decrease the false alarm and miss detection rates, sensing time increases as much as 12.2 miliseconds for $SNR = -20\ dB$ case.

One last simulation investigated channel power gains' ratio between each other. Since power gain of PU-SU affects the SINR value of the secondary receiver and directly the total achievable throughput, SU-SU proportional to PU-SU channel power gains are investigated.

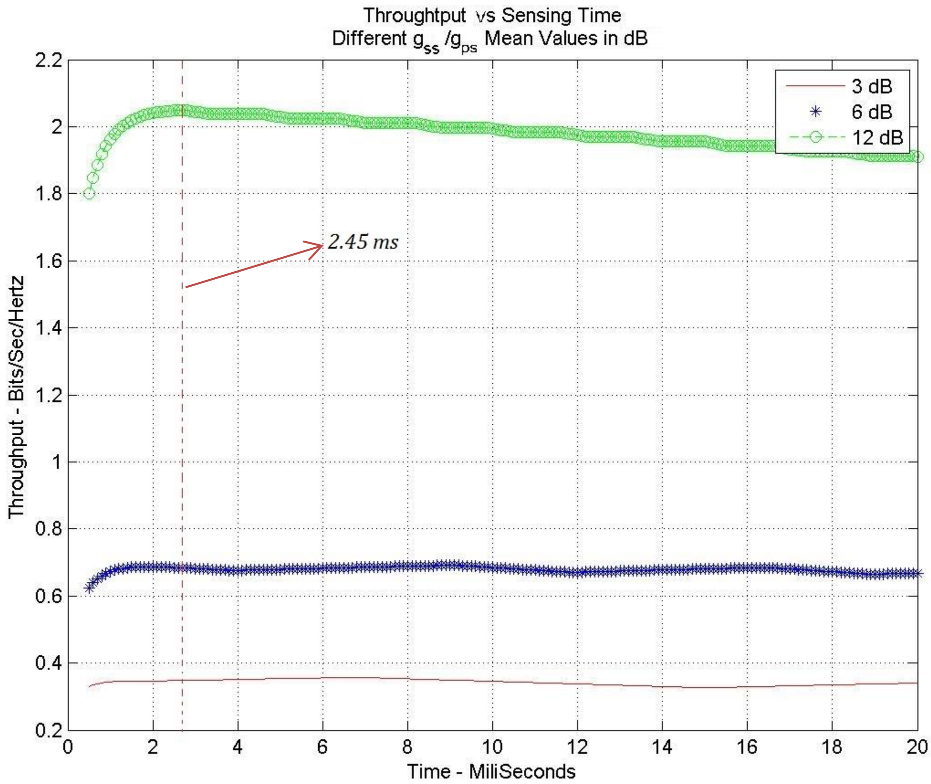

Figure 3.11 Throughput vs sensing time – Different channel power gains, $P_{av} = 10\ dB$, $\Gamma = -10\ dB$, $g_{ps,j} = 0.01$, $g_{sp,j} = 0.01$, $SNR = -15\ dB$

Different channel gains' impacts on overall throughput are obvious as SU-SU over PU-SU gains increase as expected as the total achievable throughput increases. As a very primitive assumption is made here shows us identifying channel parameters would help designer to adapt its environment more sophisticatedly and find an optimal access strategy easier to increase the spectrum efficiency. It is also seen that the optimal sensing time is still around $\overline{\tau}_{opt} = 2.45\ ms$.

CHAPTER FOUR

CONCLUSION

In this thesis, we examine CR network's basic components and dynamic spectrum access schemes. Understanding basic components and dynamic spectrum access strategies help us to analyze design steps. First, we understand spectrum opportunity and how CR increases spectrum efficiency by using these opportunities. We choose sensing-based spectrum access scheme which is a hybrid version of opportunistic access scheme and spectrum sharing scheme for investigation. In these schemes one frame is divided in two portions: sensing and data transmission. It is seen that a tradeoff exists between sensing time and throughput of the network. CR users are unlicensed users that use licensed users' channel, so the first design criteria becomes the protection of primary user's QoS by preventing possible interference. Since SSS access is keeping transmission even primary user is detected, to be able to establish tolerable interference by primary user average interference constraint introduced. For battery powered devices' long term power budgets and possible interference among secondary users are also taken into consideration and average transmision power constraint is introduced. Instead of perfect detection, we consider real world scenarios where detection is not perfect and some detector faults occur. We investigate the false alarm and miss detection probabilites for a given detection probability and sensing time. If sensing time decreases, false alarms and miss detections increase; if sensing time increases, total achievable throughput decreases. Under these circumstances we try to find optimal sensing time for the maximization of total ergodic throughput of the network.

Formulations given for WSSS access scheme investigated by simulations. Simulation is done for one channel, as said before, one can easily expand it for multiple channels using source codes given in Appendix A. For target detection probability of $P_D = 0.9,$ frame duration of $T = 100\, miliseconds,$ primary user transmission power of $P_p = 10\, dB$ simulation parameters under different interference constraints, average transmit power constraint, worst case SNR values at the secondary detector and channel power gain scenarios investigated. According to the obtained results, optimal sensing time for one channel scenario evaluated which is around $\bar{\tau}_{opt} = 2.45\ ms$ and compared to the previous investigations which is almost the same. Interference constraints, average transmit power constraints and channel power gains are significantly affecting total achievable throughput however have very little effect on optimal sensing time. Most significant effect on optimal sensing time is about detector performance which is related to detection probability and desired primary user signal's worst case received signal strength at the detector. The more strong the primary signal, the easier to detect. If not, to be able to overcome possible false alarms and miss detections optimal sensing time increases for the evaluation of target detection probability.

All in all, CR design is a very challenging optimization problem since primary users' QoS protection is very crucial. Detection should be robust and to be able to give reasonable spectrum efficiency. Some investigated distributed spectrum sensing can be considered for future investigations. Besides, mobility is a very important issue since every object in near future might have portable functionalities. On the other hand, some researches showed that mobility of SU increases total ergodic capacity, however topic is still under investigation. We see what happens in near future as demand on technology development rapidly increases.

REFERENCES

Boyd S. & Vandenberghe L. (2004) *Convex Optimization.* Cambridge University.

Federal Communications Commission (2005). "Notice of proposed rule making and order: Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," ET Docket No. 03-108.

Ghaoui L. E. (February 9, 2012). Strong Duality. Retrieved February 12, 2017, from https://people.eecs.berkeley.edu/~elghaoui/Teaching/EE227A/lecture8.pdf

Ghasemi A. & Sousa E. S. (2005). Collaborative spectrum sensing for opportunistic access in fading environments. *Proc. IEEE Dyspan*, 131-136.

Ghasemi A. & Sousa E. S. (2007). Optimization of spectrum sensing for opportunistic spectrum access in cognitive radio networks. in *Proc. 4th IEEE Consumer Commun. Networking Conf. (CCNC)*, 1022-1026.

Gordon G. & Tibshirani R. Karush-Kuhn-Tucker conditions. Retrieved February 12, 2017, from https://www.cs.cmu.edu/~ggordon/10725-F12/slides/16-kkt.pdf

Haykin S. (2005). Cognitive radio: Brain-empowered wireless communications. *IEEE J. Sel. Areas Commun., 23*(2), 201-220.

Kang X., Liang Y.-C., Garg H. K. & Zhang L. (2009). Sensing-based spectrum sharing in cognitive radio networks. *IEEE Trans. Veh. Technol., 58*(8), 4649-4654.

Kang X., Liang Y.-C., Nallanathan A., Garg H. K. & Zhang R. (2009). Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity. *IEEE Trans. Wireless Commun., 8*(2), 940-950.

Lagrangian Methods for Constrained Optimization. (n.d.). Retrieved February 12, 2017 from http://www.cmi.ac.in/~madhavan/courses/datamining13-aug/literature/Lagrangian_Methods_for_Constrained_Optimization.pdf

Liang Y.-C., Zeng Y., Peh E. C. Y. & Hoang A. T. (2008). Sensing throughput tradeoff for cognitive radio networks. *IEEE Trans. Wireless Commun.*, *7*(4), 1326-1337.

Mitola III J. & Maguire Jr. G. Q. (1999). Cognitive radios: Making software radio more personal. *IEEE Personal Commun.*, *6*(4), 13-18.

Pei Y., Liang Y.-C., Teh K. C. & Li K. H. (2009). How much time is needed for wideband spectrum sensing? *IEEE Trans. Wireless Commun., 8*(11), 5466-5471.

Stotas S. & Nallathan A. (2010). Optimal sensing time and power allocation in multiband cognitive radio networks? *IEEE Trans. On Commun., 59*(1), 226-235.

Tandra R. & Sahai A. (2008). SNR walls for signal detection. *IEEE J. Sel. Topics Signal Process., 2*(1), 4-17.

Yücek T. and Arslan H. (2009). A survey of spectrum sensing algorithms for cognitive radio applications. *IEEE Communications Surveys & Tutorials, 11*(1), 116-130.

Zhang R., Kang X. & Liang Y.-C. (2009). Protecting primary users in cognitive radio networks: peak or average interference power constraint? in *Proc. IEEE International Conf. Commun. (ICC)*, Dresden, Germany.

Zhao Q. (2007). Spectrum opportunity and interference constraint in opportunistic spectrum access. in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing (ICASSP).*

Zhao Q. & Sadler B. M. (2007). A survey of dynamic spectrum access. *IEEE Signal Processing Magazine*, *24*(3), 79-89.

Zhao Q. & Swami A. (2007). A decision-theoretic framework for opportunistic spectrum access. *IEEE Wireless Commun. Mag., 14*(4), 14-20.

APPENDIX –A

Simulation Algorithm Matlab Code

Simulation_WSSS_mf.m (Main Function)

```
% Summary of this function goes here
%   This function is the main routine of the given optimization problem
%   Author : Savas Sahin
%   Date: 02.11.2016
%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all;
close all;

recordfolderCreated = false;
myFileFolder = char( mfilename('fullpath'));

if ((exist((myFileFolder),'dir'))~= 7)
   mkdir  (myFileFolder);
end

recordTime = datestr(now);
recordTime = [recordTime(8:11),recordTime(4:6),recordTime(1:2),'-
',recordTime(13:14),recordTime(16:17),recordTime(19:20)];

if ((exist([(myFileFolder),'\',(recordTime)],'dir'))~= 7)
   mkdir  ([(myFileFolder),'\',(recordTime)]);
end

% Simulation Parameters
sp_gss = [0.9;0.6;0.4];     % Different Mean Value of g_ss fading coefficient
sp_P_av = [5;10; 20];   % Different Average Total Transmit Power
sp_P_int = [-5; -10; -20]; % Different  Average Interference Power Constraints
sp_SNR = [-12; -15; -20];   %Different  SNR of Primary User @Secondary Rx
sp_ProbIB = [0.6; 0.8];    % Different Channel Idle Busy Probabilities

% Simulation Parameters Len
sp_gss_len = length( sp_gss );
sp_P_av_len = length(sp_P_av  );
sp_P_int_len = length(sp_P_int  );
sp_SNR_len = length( sp_SNR );
sp_ProbIB_len = length(sp_ProbIB  );

%Detection Probability
detection_prob = double(0.90);

% Noise Parameters
noise_mean = double(0);
%noise_var_db = double(-100); % rand(1,1);
```

```
noise_var = double(1) ; % 10.^(noise_var_db./10);

%Primary Users Power Value
primary_pow_dB = double(10);
primary_pow = 10.^(primary_pow_dB./10);

% Decision Theresold
dec_theresold_dB = double(-70);
dec_theresold = 10.^(dec_theresold_dB./10);


start_condition = logical(0);
value_const = double(0);

%delta_max = double(0.01);
step_size = double(0.1);
temp_step_size = double (0.01);
%n_update = 2;

% time values in miliseconds
frame_duration = double(100);
frame_start = double(0.5);
frame_stop = double(20);

%lambda & nu update parameters
nu_start_d = 1e-3;
lambda_start_d = 1e-3;
nu_temp_step_size_d = 1e-3;
lambda_temp_step_size_d = 1e-3;
update_coef_d = 1e-1;
thr_calculated_d = false;

%lambda & nu update parameters
nu_start = 1e-3;
nu_end = 2;
lambda_start = 0.01;
lambda_end = 0.4;
nu_temp_step_size = 1e-3;
lambda_temp_step_size = 1e-3;
update_coef = 1e-1;
thr_calculated = false;

throughput_max = double(0);

lambda = double(0);
nu = double(0);

tho = double(0);
f_s = double (6e6);

%tho initialization
tho= frame_start : step_size:frame_stop;
```

```
tho_len = length(tho);

%tho in seconds
tho_s = tho./1000;

%Parameter Initialize
lambda_m = lambda_start:lambda_temp_step_size:lambda_end;
nu_m= transpose(nu_start :nu_temp_step_size:nu_end) ; %(av_int_pow/r_max);

% nxm lambda and nu matrix initialization
lambda= repmat(lambda_m,length(nu_m),1);
nu = repmat(nu_m,1,length(lambda_m));

power1 = 0.*lambda;
power2 = 0.*lambda;

frame_parameters = zeros(sp_ProbIB_len,sp_P_av_len,sp_P_int_len,sp_SNR,sp_gss_len,tho_len,6);

%Simulation starts
for ib=1:sp_ProbIB_len
  probs_iB = [sp_ProbIB(ib),1-sp_ProbIB(ib)];  %% Channel's  ıdle busy probabilities

  for avP = 1: sp_P_av_len
    %total average power
    av_pow_dB = sp_P_av(avP);
    av_pow = 10.^(av_pow_dB./10);

    for avInt = 1: sp_P_int_len
      %average interference power constraint
      av_int_pow_dB = sp_P_int(avInt);
      av_int_pow = 10.^(av_int_pow_dB./10);

      for s=1:3     %loop for differenet values of SNR
      % Primary Transmitters Received Signal to Noise Ratio at Secondary Receiver
      SNR_dB = double (sp_SNR(s));  %% Signal to Noise Ratio in Decibel
      SNR = 10.^(SNR_dB./10);

      % Probability of FalseAlarm, Detection, and MissDetection
      % Calculation Respectively
      probs_FA= probability_calc_mf(dec_theresold,f_s,tho_s,noise_var,SNR,detection_prob);
      probs_det=detection_prob;
      probs_m= 1-detection_prob;

      FA_probs(s, :) = probs_FA;

      % % alfa and beta parameters calculation
      [alfa, beta] = alfaBeta_calc_mf(probs_iB,probs_FA,probs_det);

        for f =1:3 %loop for differenet values of Fadings

          fadings = 0.4*ones(1,3); %% g_ss ; g_sp; g_ps  Accordingly
```

```
                fadings(1) = sp_gss(f);

                  for i= 1 : tho_len
                     alfa_t = alfa(i,:);
                     probs_fa = probs_FA(i);
                     counter1 = double(0);

                     throughput_max = 0;

                            [power1, power2,cond]= power_calc_mf(fadings, noise_var, primary_pow,
lambda, nu, alfa_t, beta);
                            IsConstOK = constraint_check_mf(
tho(i),frame_duration,fadings,power1,power2,probs_det,probs_fa,probs_iB,av_pow,av_int_pow);

                            %Check all conditions for
                            %optimization problem P0>0, P1>0,
                            %Interference<Interfer.Constraint, AveragePower<AveragePowerConstraint
                            IsConstOK = IsConstOK&cond;

                            % Average Throughput Calculation
                             throughput = insta_transRate_mf(
fadings,noise_var,primary_pow,power1,power2,probs_iB,probs_det,probs_fa);

                             throughput = ((frame_duration-
tho(i))/frame_duration).*throughput.*IsConstOK;

                             %Search for maximum throughput and
                             %corresponding values
                             [max_val_m, ind_m] = max(throughput);
                             [max_val, ind] = max(max_val_m);

                             row = ind_m(1,ind);
                             col = ind;

                             %Save frame parameters
                            throughput_max = throughput(row,col);
                            frame_parameters(ib,avP,avInt,s,f,i,1) = tho(i);
                            frame_parameters(ib,avP,avInt,s,f,i,2) = lambda_m(col);
                            frame_parameters(ib,avP,avInt,s,f,i,3) = nu_m(row);
                            frame_parameters(ib,avP,avInt,s,f,i,4) = power1(row,col);
                            frame_parameters(ib,avP,avInt,s,f,i,5) = power2(row,col);
                            frame_parameters(ib,avP,avInt,s,f,i,6) = throughput(row,col);

                   end

%               figure
%                plot(frame_start:step_size:(frame_stop-step_size),frame_parameters(1:((frame_stop-
frame_start)/step_size)+1,6))
%                plot(frame_start:step_size:frame_stop,frame_parameters(:,6))
%                max_value = max(frame_parameters(:,6));
%                min_value = min(frame_parameters(:,6));
%                if(max_value==0)
```

```
%                       ylim([-0.5 0.5])
%                    else
%                       ylim([0.5*min_value 2*max_value])
%                    end
%                    title('Throughtput Vs Sensing Time')
%                    xlabel('Time in miliSeconds')
%                    ylabel('Throughput')
%                    grid on
%
%                    save the current fig as specified filetype : 'jpeg'
%                    saveas(gcf,[myFileFolder,'\',recordTime,'\Throughtput Vs Sensing Time-
SNR',num2str(SNR_dB),'fadings',num2str(f)],'jpeg')
%                    saveas(gcf,[myFileFolder,'\',recordTime,'\Throughtput Vs Sensing Time-
SNR',num2str(SNR_dB),'fadings',num2str(f)],'fig')

                 % save([myFileFolder,'\',recordTime,'\Throughtput Vs Sensing Time-FullWorkspace '])

             end
           end
       end
    end
end

endTime = datestr(now);
```

probability_calc_mf.m

```
function [probability ] = probability_calc_mf(dec_thereshold, f_s, time, noise_var, SNR,detection_prob )
%UNTİTLED4 Summary of this function goes here
% Function calculates False alarm probability  according to given parameters,can be single value or
matrix
%Parameters;
%           -dec_threshold : decision threshold for the detector, not used
%           - f_s : sampling frequency of detector
%           - time : given detection time
%           - noise_var : noise variance
%           - SNR :  Worst case received Signal to Noise Ratio @detector
%           - detection_prob: desired signal detection probability
%   Author : Savas Sahin
%   Date: 02.11.2016
%%%%%%%%%%%%%%%%%%%%%%%
temp_probability = double (0);
temp = double (0) ;
temp_probability = detection_prob;

      %%%Probability of DETECTION


      %%%Probability of FALSE ALARM
         temp = (sqrt(2*SNR+1)*(norminv(1-temp_probability)))+SNR*sqrt(time.*f_s);
         probability =1- transpose(normcdf(temp))
      %%%Probability of  MISS DETECTION

end
```

alfaBeta_calc_mf.m

```
function [alfa , beta ] = alfaBeta_calc_mf( probability_idleBusy, probability_fa,probability_Det )
%UNTİTLED4 Summary of this function goes here
%   This function calculates alfa and beta parameters used in optimization
%   problem
%Parameters :
%           -probability_idleBusy : 1x2 channel's idle busy parameters
%           matrix, 1: idle and 2: busy
%           -probability_fa : false alarm probabilitiy matrix or single
%           value
%           -probability_Det : desired detection probability of system
%   Author : Savas Sahin
%   Date: 02.11.2016
%%%%%%%%%%%%%%%%%%%%%%%

    alfa(:,1) = probability_idleBusy(1).*(1-probability_fa);

    beta(:,1) = probability_idleBusy(2)*(1-probability_Det);

    alfa(:,2) = probability_idleBusy(1).*probability_fa;

    beta(:,2) = probability_idleBusy(2)*probability_Det;

end
```

power_calc_mf.m

```
%UNTİTLED7 Summary of this function goes here
%    Dynamic Spectrum Access Strategy power configuration calculation,
%    power1: High power setting power2: low power setting ,cond: Non negative power pairs indice
matrix
%    Parameters:
%            -fading_coefs: 1x3 channel power gain matrix,
%               1: g_ss, 2:g_sp , 3: g_ps
%            -noise_var: noise variance
%            -primary_pow: licensed user transmission power
%            -lambda : lambda matrix of Lagrangian optimization
%            -Nu : nu matirx of Lagrangian optimization
%            -alfa :  alfa coeffiecinets respect to probabilities
%            -beta:  beta coeffiecinets respect to probabilities
%   Author : Savas Sahin
%   Date: 03.11.2016
%%%%%%%%%%%%%%%%%%%%%%%
  cond = false;
  temp = double(0);
  Cons  = double(0);
  Cons2 = double(0);
  Cons3 = double(0);

  for i=1:1:2
    %A_calculation log/lambda constant
    Cons  = log2(exp(1)) ./((lambda.*(alfa(i)+beta(i)))+(Nu.*(beta(i)*fading_coefs(2))));
```

```
        % A_calculation 2nd constant
        Cons2 = ((2*noise_var+(fading_coefs(3)*primary_pow))/fading_coefs(1));

        %Delta_calculation 1st constant in the paranthesis
        Cons3 = (noise_var*(noise_var+(fading_coefs(3)*primary_pow)))/fading_coefs(1);

        A= (Cons.*(alfa(i)+beta(i)))-Cons2;

        delta = (A.^2) - ((4/fading_coefs(1)).*(Cons3-(Cons.*((alfa(i)*(noise_var
+(fading_coefs(3)*primary_pow)))+(beta(i)*noise_var)))));

        if(i ==1)
           power1 = (A + sqrt(delta))/2;
        end

         if(i ==2)
           power2 = (A + sqrt(delta))/2;
        end
    end
        cond = (power1>0)&(power2>0);
end
```

constraint_check_mf.m

```
function [ IsConsOK] = constraint_check_mf(
time,frame_duration,fadings,power1,power2,probs_det,probs_FA,probs_iB,p_average,interference_constr
aint)
%UNTİTLED Summary of this function goes here
% Function calculates interference values and total used transmission values
%       according to given parameters and checks violation of constraints
%Parameters:
%          -time: given sensing time
%          -frame_duration : frame duration T
%          -fadings : 1x3 channel power gain matrix,
%                      1: g_ss, 2:g_sp , 3: g_ps
%          -power1 : high power matrix of SpectrumAccess Strategy
%          -power2 : low power matrix of SpectrumAccess Strategy
%          -probs_det : desired detection probability
%          -probs_FA : False alarm probability
%          -probs_iB : 1x2 channel's idle busy parameters
%             matrix, 1: idle and 2: busy
%          -p_average : average power constraint
%          -interference_constraint : average interference constraint
%   Author : Savas Sahin
%   Date: 02.11.2016
%%%%%%%%%%%%%%%%%%%%%%%
IsConsOK = false;
interference_value = double (0);
average_value = double (0);
a_matrix = zeros(2,1);
```

```
interference_value = ((frame_duration-time)/frame_duration).*((fadings(2).*power1.*probs_iB(2).*(1-probs_det))+((fadings(2).*power2.*probs_iB(2).*probs_det)));

average_value = ((frame_duration-time)/frame_duration)*((probs_iB(1).*power1.*(1-probs_FA))+
(probs_iB(1).*power2.*probs_FA) + (probs_iB(2).*power1.*(1-probs_det))+
(probs_iB(2).*power2.*probs_det));



IsConsOK =  (interference_value < interference_constraint)&(average_value < p_average);
end
```

insta_transRate_mf.m

```
function [average_trans_rate ] = insta_transRate_mf( fading_coefs,Noise_var,primary_pow,trans_pow1,trans_pow2,probs_iB,probs_det,probs_FA)
%UNTİTLED3 Summary of this function goes here
%Function calculates instantaneous transmission rates for all states
%      and calculates average transmission rate for given parameters
%Parameters:
%          -fading_coefs : 1x3 channel power gain matrix,
%                    1: g_ss, 2:g_sp , 3: g_ps
%          -Noise_var : noise variance
%          -primary_pow : primary user transmission power
%          -trans_pow1 : high power matrix of SpectrumAccess Strategy
%          -trans_pow2 : low power matrix of SpectrumAccess Strategy
%          -probs_iB : 1x2 channel's idle busy parameters
%             matrix, 1: idle and 2: busy
%          -probs_det : desired detection probability
%          -probs_FA : false alarm probability
%   Author : Savas Sahin
%   Date: 04.11.2016
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
temp = double (0);
average_trans_rate = double(0);

   %       case  1 , %% r_00
        temp = (fading_coefs(1).*trans_pow1)./Noise_var;
        transRate1 = log2(1+temp);

   %       case 2 , %% r_01
        temp = (fading_coefs(1).*trans_pow2)./Noise_var;
        transRate2 = log2(1+temp);

   %       case 3 , %% r_10
        temp = (fading_coefs(1).*trans_pow1)./((fading_coefs(3)*primary_pow)+Noise_var);
        transRate3 = log2(1+temp);

   %       case 4 , %% r_11
        temp = (fading_coefs(1).*trans_pow2)./((fading_coefs(3)*primary_pow)+Noise_var);
        transRate4 = log2(1+temp);
```

```
%Average transmission Rate calculation
    average_trans_rate = (probs_iB(1).*(1-probs_FA).*transRate1)+ (probs_iB(1).*probs_det.*transRate2)
+ (probs_iB(2).*(1-probs_det).*transRate3) + (probs_iB(2)*probs_det.*transRate4);

end
```

APPENDIX –B

Optimization Problem Formulations

Lagragian Optimization and Dual Function

$$Primal\ problem:$$
$$P = \max_{x \in R} f(x) \quad subject\ to\ g(x) \geq 0, h(x) \geq 0$$

$$Lagrangian\ of\ P:$$
$$L(x, \lambda, \mu) = f(x) - \lambda g(x) - \mu h(x)$$

$$Dual\ function\ of\ L:$$
$$z(\lambda, \mu) = \max_{x \in R} L(x, \lambda, \mu)$$

$$Associated\ dual\ problem\ D:$$
$$D = \min_{\lambda \geq 0, \mu \geq 0} z(\lambda, \mu)$$
$$Strong\ Duality:$$
$$P = D$$

$$Duality\ gap:$$
$$f(x) - z(\lambda, \mu)$$

Karush-Kunh-Tucker Conditions

1) $0 \in f'(x) + \lambda g'(x) + \mu h'(x)$ (stationarity)
2) $\lambda g(x) = 0$ (complementary slackness)
3) $g(x) \geq 0, h(x) \geq 0$ (primal feasibility)
4) $\lambda \geq 0,\ \mu \geq 0$ (dual feasibility)

Slather's Conditions

If there exists at least one feasible point then Slather's conditions hold. For primal problem given previously, Slather's conditions hold if creteria given below holds.

$$\exists x \in R: \ g(x) \geq 0, \ h(x) \geq 0$$