ENTROPY BASED WEIGHT ASSIGNMENT APPROACH IN MULTI-CRITERIA GROUP DECISION MAKING

by

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TABLE OF CONTENTS

LIST OF SYMBOLS

DM	**:** Decision maker
TOPSIS	**:** Technique for Order of Preference by Similarity to Ideal Solution
MCDM	**:** Multi-Criteria Decision Making
AHP	**:** Analytical Hierarchy Process
DS	**:** Dempster–Shafer
VIKOR	**:** VIseKriterijumska Optimizacija I Kompromisno Resenje
BWM	**:** Best-Worst Method
GRA	**:** Grey Relational Analysis
MOORA	**:** Multi-Objective Optimization By Ratio Analysis
ELECTRE	**:** Elimination Et Choix Traduisant la Réalité
PROMETHEE	**:** Preference Ranking Organization Method for Enrichment Evaluations
DEMATEL	**:** Decision Making Trial and Evaluation Laboratory
ANP	**:** Analytic Network Process
IT2FAHP	**:** Interval Type-2 Fuzzy Hierarchy Process
BSC	**:** Balance Score Card
COPRAS	**:** Complex Proportional Assessment
SMART	**:** Simple Multi-Attribute Rating Technique
GDM	**:** Group Decision Making

LIST OF TABLES

ABSTRACT

Nowadays, decision making is one of the most popular research area with the rapid development of society and increasing number of alternatives almost in every area. As the alternatives are increasing, the number of criteria is also increasing which leads to decision making problems having multi criteria. Therefore, it becomes harder for a person to have a knowledge about every topic and select the best alternatives among multiple. That is why, a group of DMs started to play role in decision making problems rather than a single decision maker (DM). In group decision making cases, the best alternative is selected based on the DMs' opinion. However, unless the decision is made after a consensus has been reached among DMs, not each DM has the same weight when selecting the best alternative since each DM has different properties such as, coming from different backgrounds, having different expertise levels and ages etc. There are different approaches to assign weights to DMs such as entropy method, variable weight theory, analytic hierarchy process etc. The purpose of this study is to find the appropriate weights for DMs using an entropy based approach.

Therefore, for a real case group decision making problem to rank five alternatives with respect to six criteria by a group of six DMs is solved with equally important DMs approach and then using an already existing entropy based approach. However, this method contains some drawbacks. Those drawbacks have been pointed out, addressed and solved with two novel approaches, namely, constant weight method and variable weight method, proposed in this study.

Relative priorities of DMs have been found using these four methods with respect to the data collected from DMs. Afterwards aggregated performance values of the alternatives are calculated. The results also have been discussed in detail.

ÖZET

Günümüz koşullarının beraberinde getirdiği, hızla gelişen teknoloji ve toplumun ihtiyaçları doğrultusunda neredeyse her alanda artan alternatif sayısı karar verme konusunu popüler bir çalışma alanı olarak öne çıkarmaktadır. Karar verme konusunda amaç, kriterler doğrultusunda en iyi alternatifin seçilmesidir. Alternatif ve kriterlerin artması ile birlikte karar verme problemleri çok kriterli karar verme başlığı altında spesifik bir hal alıp önem kazanmıştır. Karar verme problemlerinde en iyi alternatif seçilirken karar vericilere ihtiyaç duyulur. Fakat, alternatif ve kriter sayılarının artması ile birlikte bir karar vericinin en iyi alternatifi seçmesi zorlaşabilmektedir. Bunun nedeni de bir karar vericinin her alanda yetkinlik düzeyine sahip olamaması ve dolayısıyla en iyi alternatifi her durumda seçemeyecek olmasıdır. Bu yüzden birden fazla karar vericinin olduğu grup karar verme problemleri, karar verme problemlerinde daha fazla tercih edilen bir yöntem haline gelmiştir. Grup karar verme problemlerinde bulunan birden fazla karar verici, kendi yetkinlikleri doğrultusunda karar problemi içerisindeki sübjektif değerlendirmeleri gerçekleştirirler. Karar vericilerin uzmanlıkları göz önüne alınarak sahip oldukları ağırlıklar dahilinde çeşitli karar verme yöntemleri kullanılarak en iyi alternatif seçilir.

Grup karar verme problemlerinin en zorlayıcı kısımlarından biri karar vericilere uygun ağırlıklar atayabilmektedir. Her karar vericinin farklı eğitim seviyelerine sahip olduğu, farklı kültürlerden geldiği, farklı yaşlara sahip olduğu vb. özellikler karar vericinin eldeki problem için sahip olduğu ağırlığı ve dolayısıyla sonucu değiştirebileceğinden ağırlıklara karar verilirken bu özellikler dikkate alınmalıdır. Literatürde karar vericilerin ağırlıklarını belirlemek üzerine yapılan çeşitli çalışmalar bulunmaktadır.

Yue çalışmasında entropi metodundan faydalanarak karar vericilere uygun ağırlıklar atamıştır. Fakat, bu çalışmasında karar verme sistemini bir bütün olarak ele aldığı ve kriterleri alt sistem olarak değerlendirmediği için karar vericilere entropi yönteminin temel mantığına aykırı olacak şekilde ağırlıklar belirlemiştir. Bu çalışmanın amacı Yue'nin çalışmasında bulunan zayıflığı düzelterek karar vericilere entropi yöntemi ile uygun ağırlıkların verilmesini sağlamaktır.

Bu çalışmada Yue' nin çalışmasından hareketle sabit ağırlık ve değişken ağırlık olmak üzere iki farklı yöntem önerilmiştir. Daha sonra beş alternatif, altı karar verici ve altı kriterli gerçek bir grup karar verme vaka çalışması için veriler toplanmıştır. Bu veriler Yue'nin önermiş olduğu, karar vericilerin eşit ağırlıklara sahip olduğu ve bu çalışmada önerilen iki yöntem ile işlenerek sonuçlara ulaşılmıştır.

Eşit ağırlık dışında uygulanan üç farklı yöntem sonucunda karar vericilerin ağırlıkları her karar verici için sırasıyla aşağıdaki gibi bulunmuştur:

- Yue'nin Yöntemi: 0.004, 0.012, 0.001, 0.385, 0.114 , 0.484.
- Sabit Ağırlık Yöntemi: 0.112, 0.377, 0.043, 0.182, 0.096, 0.190.
- Değişken Ağırlık Yöntemi ile her karar vericinin her bir kritere göre ağırlıkları: [0.1; 0.142; 0.096; 0.085; 0.088; 0.241], [0.388; 0.315; 0.44; 0.450; 0.343; 0.168], [0.024; 0.133; 0.023; 0.069; 0.047; 0.072], [0.192; 0.079; 0.207; 0.140; 0.140; 0.249], [0.094; 0.192; 0.051; 0.126; 0.191; 0], [0.201; 0.139; 0.183; 0.131; 0.190; 0.270].

Yukarıda belirtilmiş olan karar verici ağırlıkları kullanılarak alternatiflerin performansları hesaplandığında alternatif sıralaması en iyiden en kötüye doğru aşağıdaki gibi sıralanmıştır:

- Yue'nin Yöntemi: A_1, A_2, A_5, A_4, A_3
- Eşit Ağırlıklı Karar Verici Yöntemi: A_1, A_2, A_5, A_4, A_3
- Sabit Ağırlık Yöntemi: A_1, A_2, A_5, A_4, A_3
- Değişken Ağırlık Yöntemi: A_2, A_1, A_5, A_4, A_3

Yue, eşit ağırlıklı karar verici ve sabit ağırlık yöntemleri uygulandığında en iyi alternatif A_1 olarak hesaplanırken, değişken ağırlık yöntemi uygulandığında A_2, en iyi alternatif olarak karşımıza çıkmıştır.

1. INTRODUCTION

Decision making has become one of the most popular research area with the increasing amount of opportunity that the current century brings. Researchers prefer to study this topic because it can answer various questions in many research areas where there is most of the time no right or wrong answer because of subjective nature of process. In decision making, the aim is to find the best possible alternative with the most suitable approach to the problem at hand.

Since the number of alternatives and attributes are increasing day by day almost in all research areas and even daily life choices, the research area is specified as multi criteria decision making (MCDM). MCDM methods have been widely used in various fields such as:

- Supplier selection: Application of fuzzy Analytical Hierarchy Process (AHP) and TOPSIS method for selecting green supplier with considering environmental and economical features (Chen et al., 2016). A numerical example for supplier selection by using TOPSIS method with intuitionistic fuzzy sets (Boran et al., 2009). Determination of best supplier alternative with Dempster–Shafer-VIseKriterijumska Optimizacija I Kompromisno Resenje (DS-VIKOR method) (Fei et al., 2019). Evaluation of socially sustainable supplier alternatives with best-worst method (BWM) (Bai et al., 2019).

- Location Selection: Selection of best location for a factory by using some of the MCDM methods in such as TOPSIS, simple additive weighting (SAW), Grey relational analysis (GRA), Multi-Objective Optimization By Ratio Analysis (MOORA) and Limination Et Choix Traduisant la REalité–I (ELECTRE)

- (Ray et al.,2015). Evaluation of potential location of city logistics centers by using linguistic 2-tuple approach of a fuzzy multi attribute group decision making (Rao et al. 2015). Application of an interval Type-2 Fuzzy sets for location problem of a cement factory (Çebi et al., 2015). Application of bifuzzy technique for TOPSIS method to rank the alternatives and determine the best one (Chaube et al., 2017). Application of BWM to select the best location for bioethanol production (Kheybari et al., 2019).

- Military Applications: Selection of weapon systems with hierarchical fuzzy TOPSIS method (Karadayi et al., 2019). Application of military tanks evaluation with ELECTRE III and Preference Ranking Organization Method for Enrichment Evaluations II (PROMETHEE) methods (Genc, 2015). Evaluation of military training aircrafts with AHP and TOPSIS methods in the Spanish Air force Academy (Lozano et al., 2015).

- Sports: Application of position weighting and volleyball sport skills with AHP technique (Budak et al., 2017). Examining the growth of professional Volleyball sport with Decision making trial and evaluation laboratory (DEMATEL), analytic network process (ANP) and fuzzy Delphi method in Taiwan sport business (Hu et al., 2018).

- Performance Evaluation: Evaluation of environmental performance of grey based hybrid framework with MCDM methods of VIKOR and ELECTRE (Chithambaranathan et al., 2015). Evaluation of emergency department performance in a university hospital with interval type-2 fuzzy hierarchy process (IT2FAHP) (Gul et al. 2016). Assessment of medical device producers with fuzzy AHP (Lee et al., 2017). Performance evaluation based on Balance Score Card (BSC) method with the help of ANP, Complex Proportional Assessment (COPRAS), TOPSIS, MOORA for research and technology companies.

- Project Management: Developing a hybrid fuzzy MCDM method with the help of AHP and VIKOR methods for a project selection problem (Salehi 2015).

Investigating a relation of organizational performance and project management with the VIKOR and DEMATEL based ANP methods (Lin et al., 2016). Application of AHP for a real case of construction management (Erdogan et al., 2017). Improving organizational performance of a project management to decrease the project risk with DEMATEL technique (Chen et al. 2018).

- Marketing: Strategy formulation by using ANP and TOPSIS methods for an automobile company (Gowri et al., 2018). Determination of 4P strategies in marketing using hybrid AHP (Gürbüz, et al., 2014). Determination of marketing factor affects to estimate the best targeting policy by using fuzzy ANP technique.

- Health: Application of fuzzy Evaluation Based on Distance from Average Solution (EDAS) method for hospital selection for organ transplantation (Kutlu Gündoğdu, et al., 2018). Real life application of service quality evaluation in Iranian health care services with AHP and TOPSIS methods (Afkham, et al., 2012). Developing a model for electronic health record with ANP and VIKOR methods (Liou et al. 2017).

- Energy: Comparison of Weighted Sum (WSM), VIKOR, TOPSIS, and ELECTRE techniques to rank renewable resources in Taiwan (Lee et al., 2018). Ordering renewable energy supply systems in Turkey with fuzzy TOPSIS method (Şengül et al., 2015). Assessment of energy sources with AHP method (Erol et al., 2012)

Complexity of the MCDM problem increases with increasing number of criteria and alternatives. To be able to deal with this complexity, a group of DMs may be preferable to a single DM in the decision process as various criteria would necessitate different expertise. However, this time the challenging part becomes assigning weights to DMs. The relative priority(weight) of a DM in a group decision making process has a significant importance when determining the performance of an alternative. The weights of DMs can be determined by considering their expertise levels. Because each DM comes from a different education background, has different experiences, ages, knowledge etc. all of the properties should be considered. Therefore, one of the most important issue in the group decision making is to decide the weights of the DMs.

There are various methods to determine the DMs' weights. Distance based measure is one of the most used methods to assign weight to DMs which can be Euclidean, Manhattan and Hamming. With these approaches the purpose is to find the weights of DMs by calculating the distance of each DM to the aggregated decision. For example, Yue (2011) applied a distance based measure TOPSIS to determine the weights of DMs in a group decision making case. Similarity evaluation is also one of the most preferred methods to assign weights to DMs. In this method, pairwise comparisons between assessment of each DM and the difference between each decision is calculated. As the difference increases, the weight of DMs decreases. Wang et al. (2015) provided a detailed example by computing the support degree with pairwise comparison.

With the help of several approaches to assign DMs' weight, researches have the chance of doing comparisons between approaches by applying different methods. For example, Liu et al. (2019) used variable weight theory to determine the weight of DMs where the valuation of information is identified with intuitionistic fuzzy sets. Liu et al.(2019) also used DMs' weights as a constant vector rather than a variable vector like previous researches. Zhang & Li (2006) also studied on variable weights theory. They utilized variable weighted synthesis method in factor spaces method to fuzzy implication. Yue (2011) used Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method to assign weights to DMs. Besides TOPSIS, other MCDM methods are also used to distribute weights to DMs. For instance, Dong & Cooper (2016) decided the importance weight of DMs' with the help of AHP method.

There are two techniques to determine the DMs' weights which are subjective methods and objective methods (Koksalmis & Kabak, 2019). Saaty (1980) mentioned in his study that in subjective methods DMs' weights are determined by a superior by looking at their expertise level, age, experiences etc. (Ramanathan & Ganesh, 1994). Conversely, in objective methods quantitative techniques are used to assign the weights of DMs. Objective methods aim to escape subjectivity as much as possible. Thus, they use mathematical models to assign weights to DMs with the information gathered from individual or joint decision matrices. Honert (2001) assigned the weights of DMs in a group with the help of multiplicative AHP and Simple Multi-Attribute Rating Technique (SMART) which also known as REMBRANDT systems.

Yue (2012) studied to determine the DMs' weight by using extended projection method where attribute values are given as interval number and weight of attributes are established properly.

Entropy measure method is one of the most popular method which is also applied in this study. The method which is also known as Shannon method helps to find the disorder level of the system and it is introduced by Shannon (1948). Entropy method is an important method to find the uncertainty in a given system. The perception of entropy comes from thermodynamics and according to this method as the uncertainty in the system increases, entropy of the system also increases (Rasberry, 2009). In previous studies the entropy method is used to find attributes' weight rather than DMs' weights (Ji et al., 2015). For example, Shemshadi et al. (2011) used VIKOR method to calculate the weights of the criteria and then extended the study by using entropy method to develop the objective weights of the criteria. Tian et al., (2018) combined VIKOR method, relative entropy method and fuzzy best worst scenario with the objective of increasing the performance of failure mode and effect analysis. Ghorbani et al. (2012) worked on supplier selection first using SWOT analysis to judge qualitative and quantitative attributes of suppliers, then used Shannon entropy method to determine the weight of criteria. Seker et al. (2020), integrated entropy and TOPSIS methods to select a hydrogen energy production facility in Turkey. Wood (2016) categorized 30 criteria for a supplier selection in petroleum industry by comparing eight different methods including entropy, intuitionistic fuzzy TOPSIS for three alternatives. Zhang et al. (2011) firstly used information entropy weighting to interpret 35 attributes then used TOPSIS method to rank the alternatives to evaluate tourism destination competitiveness in China.

Yue (2017) used entropy method to find a way to assign weights to DMs departing from the decision matrices in a MCDM problem. However, even though Yue's (2017) model is applicable for most cases to find the weight of DMs, there is a weakness in the technique which does not assign accurate weights in every case. The focus of this study is to deal with this weakness- which will also be the originality of the study- and develop the given model to assign weights to DMs more effectively for every possible case consistent with the basic intuition of entropy method.

The study will continue as follows: in the next section, Shannon's entropy method followed by Yue's weight assignment method based on entropy method will be explained. Third section will state the proposed approaches based on Yue's method's drawback which will also be pointed out. Section four presents a detailed numerical application stating the use of four different weight assignment approach and further, the final results obtained from application of the mentioned methods will be presented. Finally, in the fifth section the conclusion will be given.

2. PRELIMINARIES

As it is mentioned in the previous section, there are various methods which helps to find the appropriate weights of the DMs in a decision-making problem. In this section, firstly Shannon's Entropy method will be explained. Afterwards, Yue's technique which is based on Entropy method will be given in detail with the necessary formulas to be able to understand the following sections which includes the proposed method.

2.1. Shannon's Entropy Method

Entropy method is introduced by Shannon (1948) to measure the uncertainty in a system. It also can be defined as the information quantity in a variable. The uncertainty of the systems is correlated with the given discrete probability distribution, p_i for $\forall i = 1, \ldots, n$. When the probability distribution is uniform, the uncertainty of the system is maximized. For example, assuming there is a class with five students who has the same exam result. The probability of selecting a paper with that said result would be equal to 1. That would mean that there is no uncertainty in the system. However, if each student had a different result, the probability of selecting the desired result would decrease to 0.2 which would increase the uncertainty and also the entropy of the system.

When it comes to assigning weights to criteria of an MCDM problem in terms of entropy method, a given criterion's relative priority will decrease if alternatives show similar values to one another. The extreme case of that would be all the alternatives having the same value with respect to a given criterion. In that case, that criterion will automatically be out of consideration as it will not contribute to the final outcome.

The formulation of entropy measure by Shannon (1948) is given as follows:

$$S(p_1, p_2, \dots, p_n) = -k \sum_{j=1}^{n} p_j \, ln \, p_j \tag{1}$$

Let *D* be the decision matrix of a decision-making problem with *m* alternatives and *n* criteria.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array} \tag{2}$$

From this decision matrix *D*, the entropy of j^{th} criterion is calculated using the following formula:

$$E_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij} \, , \forall \, j = 1, \dots, n \tag{3}$$

where

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \, , \forall \, j = 1, \dots, n \tag{4}$$

where

$$k = \frac{1}{\ln m} \tag{5}$$

Diversification degree of information can be provided with the calculated entropy value of criterion *j* as follows:

$$d_j = 1 - E_j, \forall\, j = 1, \dots, n \tag{6}$$

After calculating the diversification degree of information, if the DM has no preference order, the weight of each criterion can be calculated with the following formula:

$$w_j = \frac{d_j}{\sum_{k=1}^{n} d_k}, \forall\, j = 1, \dots, n \tag{7}$$

Example: Let's have a decision matrix *D* with three criteria and three alternatives as shown below:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \\ \begin{bmatrix} 15 & 150 & 60 \\ 25 & 220 & 59 \\ 30 & 175 & 58 \end{bmatrix} \end{array} \tag{8}$$

Then after using formula (3) to (7), the weight of each criterion will be 0.748, 0.25 0.002 respectively. As it can be seen, the third criterion has the minimum weight as the values of alternatives with respect to that criterion are minimally dispersed compared to other criteria. In the same manner, as the evaluation values given to first criterion based on three alternatives are more separated than each other compared to other criteria, the weight of first criterion becomes the highest.

2.2. Yue's Method

Entropy method which is defined above can be used as an approach to assign weights to a group of DMs in MCDM problems. Yue (2017) proposed a method with the help of entropy measure method to find the relative importance of each DM with respect to his/her decision matrix's (D_k) uncertainty amount. According to the basic logic of Entropy method, while assigning weights to DMs, if the DM assigns similar values to alternatives with respect to a given criterion, then that DM's relative priority should decrease as his/her judgments will contribute less or not contribute at all to the final result. However, in Yue's study there is a drawback that does not quite fit this intuition. Even though the values given to each criterion for each alternative by a DM is exactly the same, a weight is assigned to the DM when the DM should have a zero weight because of the basic entropy intuition.

From the same decision matrix D as before, in Yue's approach firstly the weighted normalized matrix is found. Using weighted normalized values, the entropy value of each DM can be calculated with the following formula (Yue 2017):

$$E_k = -\frac{1}{\ln(m.n)} \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} \ln p_{ij} \text{ , } \forall \qquad (9)$$

Finally using (6) and (7) the DMs' weights are calculated.

However, to see the aforementioned drawback of this method, let's assume that there is a MCDM problem which has three alternatives with three DMs as shown in the matrices below. The matrices for the first two DMs have identical rows which does not make any difference on the final result. However, the method which is provided by Yue will assign zero weight to only the first DM and a non-zero weight to the second DM which does not fit the basic intuition of entropy method.

$$D_1 = \begin{bmatrix} 70 & 70 \\ 70 & 70 \\ 70 & 70 \end{bmatrix} \quad D_2 = \begin{bmatrix} 70 & 15 \\ 70 & 15 \\ 70 & 15 \end{bmatrix} \quad D_3 = \begin{bmatrix} 70 & 90 \\ 70 & 55 \\ 70 & 17 \end{bmatrix} \tag{10}$$

When Yue's method is applied to the matrices given in (10), the weight of DMs are obtained as 0, 0.735, 0.265 respectively for D_1, D_2, D_3. Even though the third decision matrix has the highest disorder, with Yue's method it has the lowest weight. D_2, has the highest weight where it should have a zero weight. Since the decision matrix is taken as a whole system, assigning weights to the second DM even though each alternative has the identical rows becomes a challenge. However, each column of a decision matrix stands for a different sub-system.

3. PROPOSED APPROACH

In the previous section, the drawback in Yue's method is presented and a numerical example is given to show the effect of the drawback in the weight assignment. Thus, two methods are proposed to assign weights to a group of DMs.

3.1. Constant Weight Approach

In this study, the aim is to deal with the drawback mentioned above and correct the entropy formula as following:

$$E_k = \left[\sum_{j=1}^{n}\left(-\frac{1}{\ln(m)}\sum_{i=1}^{m} p_{ij}\ln p_{ij}\right)\right], \forall\, k \in \{1, \dots, r\} \tag{11}$$

After finding the entropy value of each system, the following formula is used to find the diversification degree which will help to calculate the weight of DMs' weight.

$$d_k = n - \left[\sum_{j=1}^{n}\left(-\frac{1}{\ln(m)}\sum_{i=1}^{m} p_{ij}\ln p_{ij}\right)\right], \forall\, k \in \{1, \dots, r\} \tag{12}$$

Subsequently, by using diversification values found with (12), the weights of DMs are found with (7). The correction made in (12) corresponds to calculating each sub-system's entropy separately then aggregate to achieve an entropy value of a system as a whole.

With the importance value of each DM, the aggregated performance of i^{th} alternative will be calculated with the following formula:

$$P_{agg}(A_i) = \sum_{j=1}^{n} w_{C_j} \sum_{k=1}^{K} w_k . a_{ij}^k \quad \forall \ i = 1, \ldots, m \tag{13}$$

Where w_{C_j} represents the weight of criterion j for $j = 1, \ldots, n$, w_k represents the weight of DM$_k$ for $k = 1, \ldots, r$ and a_{ij}^k represents the normalized value of the evaluation value given by k^{th} DM for alternative i criterion j. When constant weight method is applied to the matrices given in (10), the weights of DMs are found as 0. 0. and 1 respectively for D_1, D_2, D_3. As it can be seen, D_3 has all the importance because of the disorder level in the system and the weights of D_1 and D_2 have become 0 because each value given to alternatives based on the given criteria is exactly the same and there is no difference between alternatives based on the given criterion. In Yue's method D_3 is assigned the lowest weight where D_1 and D_2 shared the rest of the weight. Thus, in the constant weight method, the weights assigned to these DMs will not affect the selection of the best alternative.

3.2. Variable Weight Approach

The formula given in (12) provides a constant weight for each DM. Although this approach provides a valid and viable correction to Yue's approach, a more intricate approach is also developed within the spectrum of this study. A variable weight approach will be presented in this sub-section. Thus, another formula is proposed where the variable weight scheme is aimed. Each DM's weight is calculated with respect to each criterion one by one. Therefore, a weight vector will be formed for each DM. The formula below shows the calculation of entropy value of each criterion.

$$E_{kj} = -\frac{1}{\ln(m)}\sum_{i=1}^{m} p_{ij} \ln p_{ij}, \forall\ k = 1, \ldots, r, j = 1, \ldots, n \tag{14}$$

After calculating the entropy value of each criterion, the diversification degree of each criterion j is calculated with the equation:

$$d_{kj} = 1 - \left[\sum_{j=1}^{n}\left(-\frac{1}{\ln(m)}\sum_{i=1}^{m} p_{ij} \ln p_{ij}\right)\right], \forall\ k = 1, \ldots, r \tag{15}$$

After finding the diversification degree for $k = 1, \ldots, r$ and $j = 1, \ldots, n$ weight of DM_k for the j^{th} criterion is calculated as shown below:

$$w_{kt} = \frac{d_{kj}}{\sum_{t=1}^{n} d_t}, \forall\ k \in \{1, \ldots, r\} \tag{16}$$

Finally, the following formula calculates the aggregated performance of each alternative.

$$P_{agg}(A_i) = \sum_{j=1}^{n} w_{C_j} \sum_{t=1}^{n}\sum_{k=1}^{K} w_{kt}.\, a_{ij}^{k} \quad \forall i = 1, \ldots, m \tag{17}$$

Where w_{C_j} represents the weight of j^{th} criterion for $j = 1, \dots, n$, w_{kt} represents the weight of k^{th} DM for the t^{th} criterion for $k = 1, \dots, r$ and $t = 1, \dots, n$ and a_{ij}^{k} represents the normalized value of the value given by k^{th} DM for alternative i with respect to criterion j. When the formulas given above are applied to the matrices given in (10) the weight of each criterion is calculated separately. As a result, it is seen that only the second criterion evaluated by D_3, has a disorder in the system, thus all the other criteria in each system will get a 0 weight. Which means that the values given to the alternatives based on the given criteria is not important and does not affect the selection of the best alternative.

4. NUMERICAL APPLICATION

4.1. Decision Problem's Data

In order to analyze the drawback in Yue's method a real case with six DMs (DM_k where $k = 1, \ldots, 6$) and five alternatives (A_i where $i = 1, \ldots, 5$) is considered. In the real case, it is asked to two Professor Doctors and four Assistant Professor Doctors who works in the same department in a university in Istanbul to evaluate five research assistants (who are all master students and works in the same department). It is also known that the professors include the dean of the faculty, the head of the department and vice chairman of the department. The aim is to collect all the subjective values from each DM and for each alternative based on six criteria (C_j where $j =1, \ldots, 6$) which are given below.

- C_1: RELATIONSHIP WITH STUDENTS: This criterion has been evaluated by considering the relationship of the research assistant with the students during the classes, outside of the classes and in the office hours.

- C_2: RELATIONSHIP WITH THE PROFESSORS: This criterion has been evaluated by considering the relationship of the research assistant with the professors.

- C_3: ACADEMIC GOALS: This criterion has been evaluated by looking at the number of studies done by the research assistants, the willingness of research and making new projects.

- C_4: RELATIONSHIP WITH COLLEAGUES: This criterion has been evaluated by observing the research assistants during their launch hours, meetings etc.

- C_5: PUNCTUALITY: This criterion has been evaluated by considering whether or not various tasks given to research assistants has been done in a timely manner. For example, making the hard copies of exams, preparing the examples for the practice classes, reporting etc.

- C_6: ACCOMPLISHMENT: This criterion has been evaluated similar to the criterion 5. It is evaluated if the given task is finished completely without any mistake rather than looking if it is finished on time. For example: Taking the right amount of exam paper to the exam, sending the report of the number of students registered during a semester etc.

The matrices for each alternative given by each DM based on the criteria given above are shown as follows:

$$DM_1 = \begin{bmatrix} 90 & 95 & 85 & 90 & 100 & 100 \\ 87 & 93 & 85 & 90 & 90 & 90 \\ 80 & 85 & 70 & 85 & 90 & 85 \\ 85 & 90 & 80 & 90 & 90 & 85 \\ 95 & 92 & 87 & 85 & 85 & 85 \end{bmatrix} \tag{18}$$

$$DM_2 = \begin{bmatrix} 80 & 95 & 75 & 85 & 95 & 100 \\ 90 & 91 & 85 & 95 & 85 & 95 \\ 70 & 80 & 60 & 80 & 75 & 85 \\ 75 & 89 & 80 & 75 & 70 & 90 \\ 65 & 90 & 100 & 80 & 80 & 93 \end{bmatrix} \tag{19}$$

$$DM_3 = \begin{bmatrix} 90 & 95 & 96 & 92 & 95 & 95 \\ 85 & 92 & 92 & 96 & 87 & 90 \\ 90 & 90 & 90 & 99 & 91 & 91 \\ 90 & 96 & 88 & 100 & 85 & 87 \\ 85 & 100 & 97 & 100 & 88 & 86 \end{bmatrix} \tag{20}$$

$$DM_4 = \begin{bmatrix} 80 & 85 & 75 & 90 & 95 & 95 \\ 80 & 90 & 90 & 85 & 80 & 85 \\ 80 & 85 & 70 & 85 & 90 & 90 \\ 90 & 90 & 85 & 95 & 85 & 95 \\ 70 & 85 & 95 & 85 & 80 & 80 \end{bmatrix} \tag{21}$$

$$DM_5 = \begin{bmatrix} 90 & 85 & 90 & 90 & 85 & 90 \\ 90 & 95 & 90 & 95 & 90 & 90 \\ 90 & 95 & 80 & 100 & 95 & 90 \\ 95 & 95 & 90 & 90 & 90 & 90 \\ 80 & 90 & 95 & 90 & 75 & 90 \end{bmatrix} \tag{22}$$

$$DM_6 = \begin{bmatrix} 85 & 90 & 80 & 85 & 95 & 95 \\ 95 & 85 & 85 & 90 & 85 & 80 \\ 75 & 80 & 70 & 85 & 85 & 85 \\ 80 & 85 & 75 & 80 & 80 & 80 \\ 80 & 85 & 95 & 90 & 75 & 80 \end{bmatrix} \tag{23}$$

4.2. Weight Assignment Process

As it is mentioned previously, the aim in this study is to assign weights to a group of DMs. Thus, four different method is applied to find the weights of DMs shown detailed as follows. However, in the case where each DM has the same equal weight, the total weight 1 is divided by the number of DMs and the weight is found without applying any method.

4.2.1. Yue's Method

Firstly, the evaluation values given by each DM which are shown in (18) to (23) is normalized and the weights of DMs are calculated with Yue's method. With the normalized values of each matrix, the entropy values are calculated. In (24), the calculation of the entropy value with (9) is applied for the matrix in (18).

$$DM_1 = \begin{bmatrix} -0.116 & -0.117 & -0.117 & -0.113 & -0.121 & -0.123 \\ -0.113 & -0.115 & -0.117 & -0.113 & -0.112 & -0.114 \\ -0.106 & -0.108 & -0.102 & -0.109 & -0.112 & -0.110 \\ -0.111 & -0.112 & -0.112 & -0.113 & -0.112 & -0.110 \\ -0.120 & -0.114 & -0.119 & -0.118 & -0.108 & -0.110 \end{bmatrix} \tag{24}$$

With the help of the matrix given in (24), the entropy value of DM_1 is calculated as 0.9995. The other entropy values of the other DMs are calculated similarly, and the detailed calculations are given in Appendix A. After finding the entropy values, the diversification degree is calculated with the formula shown in (6) and it is calculated as 0.0005. The weights of DMs are calculated with the formula given in (7) and they are equal to [0.004 0.012 0.001 0.385 0.114 0.484] for k =1, …, 6 respectively However, as it is mentioned before, the weights assigned with Yue's method are not appropriate since the systems are not considered separately.

4.2.2. Constant Weight Method

After applying the Yue's method, the proposed constant weight method is applied to the data given in (18) to (23). The entropy calculation for (18) with constant weight method is shown as follows:

$$DM_1 = \begin{bmatrix} -0.325 & -0.327 & -0.327 & -0.322 & -0.333 & -0.335 \\ -0.321 & -0.325 & -0.327 & -0.322 & -0.321 & -0.323 \\ -0.311 & -0.313 & -0.303 & -0.315 & -0.321 & -0.316 \\ -0.318 & -0.321 & -0.320 & -0.322 & -0.321 & -0.316 \\ -0.332 & -0.323 & -0.330 & -0.328 & -0.313 & -0.316 \end{bmatrix} \quad (25)$$

The entropy value of DM_1 is obtained as 5.994 and the rest of the detailed calculations are given in Appendix C. After finding the entropy values for each DM, diversification degree values are calculated with (12) and they are calculated as [0.006, 0.020, 0.002, 0.010, 0.005, 0.010] respectively for $k = 1, \ldots, 6$. The weights of DMs are calculated with (7) and they are equal to [0.112 0.377 0.043 0.182 0.096 0.190] for $k = 1, \ldots, 6$.

4.2.3. Variable Weight Method

In this method, different than other methods entropy value is calculated with (14) for each criterion separately rather than for a system as shown below for DM_1:

$$DM_1 = \begin{bmatrix} -0.325 & -0.327 & -0.327 & -0.322 & -0.333 & -0.335 \\ -0.321 & -0.325 & -0.327 & -0.322 & -0.321 & -0.323 \\ -0.311 & -0.313 & -0.303 & -0.315 & -0.321 & -0.316 \\ -0.318 & -0.321 & -0.320 & -0.322 & -0.321 & -0.316 \\ -0.332 & -0.323 & -0.330 & -0.328 & -0.313 & -0.316 \end{bmatrix} \quad (26)$$

And for each criterion the entropy values are obtained as [0.999 1.000 0.998 1.000 0.999 0.999] for j = 1, .., 6. Subsequently, the diversification degree values for k = 1, …, 6 and j = 1, …, 6 are obtained with (15) and they are equal to [0.001 0.000 0.002 0.000 0.001 0.001]. The weight of DM_1 for criterion t is calculated with (16) as [0.100 0.142 0.096 0.085 0.088 0.241] where t = 1, …, 6. The rest of the calculations are shown detailed in Appendix D.

4.3. Aggregation

In the previous section, the weight calculation is presented for six DMs by applying four different methods. In this section, by using the found weight of DMs, aggregated performance values are calculated.

4.3.1. Yue's Method

Performances of the alternatives are calculated by multiplying the weights of DMs with the normalized values of (18) to (23). The performance calculation of alternatives for

DM_1 is obtained as shown below and the rest of the calculations are shown in Appendix A.

$$DM_1 = \begin{bmatrix} 0.00012 & 0.00012 & 0.00012 & 0.00012 & 0.00013 & 0.00013 \\ 0.00012 & 0.00012 & 0.00012 & 0.00012 & 0.00012 & 0.00012 \\ 0.00011 & 0.00011 & 0.00010 & 0.00011 & 0.00012 & 0.00011 \\ 0.00011 & 0.00012 & 0.00012 & 0.00012 & 0.00012 & 0.00011 \\ 0.00013 & 0.00012 & 0.00013 & 0.00012 & 0.00011 & 0.00011 \end{bmatrix} \quad (27)$$

After calculation of all performance values for all the DMs, the aggregated performance values are calculated with (13) and it is obtained as shown below for $i = 1, \ldots, 5$:

$$P_{agg}(A_i) = \begin{bmatrix} 0.032 & 0.031 & 0.029 & 0.030 & 0.035 & 0.035 \\ 0.034 & 0.031 & 0.032 & 0.030 & 0.031 & 0.031 \\ 0.030 & 0.030 & 0.026 & 0.030 & 0.033 & 0.032 \\ 0.032 & 0.031 & 0.029 & 0.030 & 0.031 & 0.032 \\ 0.029 & 0.030 & 0.034 & 0.030 & 0.029 & 0.030 \end{bmatrix} = \begin{bmatrix} 0.191 \\ 0.189 \\ 0.180 \\ 0.186 \\ 0.183 \end{bmatrix} \quad (28)$$

4.3.2. Same Weight for DMs

The normalized versions of matrixes given in (18) to (23) are multiplied with the weights of DMs where each DM has the equal weight and the performance values of the alternatives are calculated as shown below for DM_1. The detailed calculations for the rest of the DMs are shown detailed in Appendix B.

$$DM_1 = \begin{bmatrix} 0.034 & 0.033 & 0.031 & 0.032 & 0.037 & 0.037 \\ 0.033 & 0.033 & 0.033 & 0.033 & 0.033 & 0.034 \\ 0.031 & 0.031 & 0.028 & 0.032 & 0.033 & 0.032 \\ 0.032 & 0.033 & 0.031 & 0.031 & 0.033 & 0.032 \\ 0.036 & 0.032 & 0.036 & 0.032 & 0.031 & 0.032 \end{bmatrix} \qquad (29)$$

After finding all performance values for all DMS and alternatives, the aggregated performance values are calculated with (13) as follows for $i = 1, \ldots, 5$:

$$P_{agg}(A_i) = \begin{bmatrix} 0.033 & 0.033 & 0.031 & 0.032 & 0.036 & 0.035 \\ 0.034 & 0.033 & 0.033 & 0.033 & 0.033 & 0.033 \\ 0.031 & 0.031 & 0.028 & 0.032 & 0.033 & 0.032 \\ 0.033 & 0.033 & 0.031 & 0.031 & 0.031 & 0.032 \\ 0.030 & 0.032 & 0.036 & 0.032 & 0.030 & 0.032 \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.198 \\ 0.187 \\ 0.192 \\ 0.193 \end{bmatrix} \quad (30)$$

4.3.3. Constant Weight Method

After finding the weights of DMs with the constant weight method, aggregated performances of the alternatives are calculated by multiplying these weights with the normalized values of (18) to (23). The performance calculation of alternatives for DM_1 is obtained as shown below.

$$DM_1 = \begin{bmatrix} 0.023 & 0.023 & 0.023 & 0.022 & 0.025 & 0.025 \\ 0.022 & 0.023 & 0.023 & 0.022 & 0.022 & 0.023 \\ 0.021 & 0.021 & 0.019 & 0.021 & 0.022 & 0.021 \\ 0.022 & 0.022 & 0.022 & 0.022 & 0.022 & 0.021 \\ 0.024 & 0.023 & 0.024 & 0.024 & 0.021 & 0.021 \end{bmatrix} \tag{31}$$

The aggregated performance values for i = 1, …, 5 are found as shown below:

$$P_{agg}(A_i) = \begin{bmatrix} 0.034 & 0.034 & 0.032 & 0.034 & 0.037 & 0.036 \\ 0.037 & 0.034 & 0.035 & 0.035 & 0.034 & 0.033 \\ 0.031 & 0.031 & 0.027 & 0.033 & 0.033 & 0.032 \\ 0.034 & 0.034 & 0.033 & 0.032 & 0.031 & 0.033 \\ 0.030 & 0.033 & 0.039 & 0.033 & 0.031 & 0.032 \end{bmatrix} = \begin{bmatrix} 0.208 \\ 0.208 \\ 0.188 \\ 0.197 \\ 0.200 \end{bmatrix} \tag{32}$$

4.3.4. Variable Weight Method

Finally, the performance values are calculated with the found weights by variable weight method. Normalized values of the data given in (18) to (23) is multiplied with those weights and the following values are obtained for DM_1.

$$DM_1 = \begin{bmatrix} 0.021 & 0.030 & 0.020 & 0.017 & 0.019 & 0.054 \\ 0.020 & 0.029 & 0.020 & 0.017 & 0.017 & 0.049 \\ 0.018 & 0.027 & 0.017 & 0.016 & 0.017 & 0.046 \\ 0.020 & 0.028 & 0.019 & 0.017 & 0.017 & 0.046 \\ 0.022 & 0.029 & 0.021 & 0.018 & 0.016 & 0.046 \end{bmatrix} \tag{33}$$

The aggregated performance values for i = 1, …, 5 are found as shown below:

$$P_{agg}(A_i) = \begin{bmatrix} 0.034 & 0.034 & 0.032 & 0.033 & 0.037 & 0.037 \\ 0.037 & 0.034 & 0.035 & 0.036 & 0.034 & 0.033 \\ 0.031 & 0.032 & 0.027 & 0.033 & 0.033 & 0.033 \\ 0.034 & 0.034 & 0.033 & 0.032 & 0.032 & 0.033 \\ 0.030 & 0.034 & 0.039 & 0.033 & 0.031 & 0.032 \end{bmatrix} = \begin{bmatrix} 0.207 \\ 0.208 \\ 0.189 \\ 0.197 \\ 0.199 \end{bmatrix} \quad (34)$$

4.4. Analysis of Results

After applying four different methods that are mentioned above, the following results for weights of DMs are obtained as shown in the table 4.1 below.

Table 4.1: Weights of DMs

	Weights of DMs					
Methods	DM_1	DM_2	DM_3	DM_4	DM_5	DM_6
Yue's approach	0.004	0.012	0.001	0.385	0.114	0.484
Equal Weight	0.167	0.167	0.167	0.167	0.167	0.167
Constant Weight	0.112	0.377	0.043	0.182	0.096	0.190
Variable Weight	[0.1; 0.142; 0.096; 0.085; 0.088; 0.241]	[0.388; 0.315; 0.44; 0.450; 0.343; 0.168]	[0.024; 0.133; 0.023; 0.069; ;0.047; 0.072]	[0.192; 0.079; 0.207; 0.140; 0.140; 0.249]	[0.094; 0.192; 0.051; 0.126; 0.191; 0]	[0.201; 0.139; 0.183; 0.131; 0.190; 0.270]

Consequently, the aggregated performance value of alternatives are found as shown in table 4.2 with the weights of DMs shown in table 4.1. The aggregated performance values of alternatives are sorted from the highest $P_{agg}(A_i)$ to lowest for $\forall i$ = 1, …, 5 as shown in table 4.3

Table 4.2: Aggregated Performance Value of Alternatives

	Aggregated Performance of Alternatives				
Methods	A_1	A_2	A_3	A_4	A_5
Yue's approach	0.191	0.189	0.18	0.186	0.183
Equal Weight	0.206	0.204	0.192	0.198	0.199
Constant Weight	0.2078	0.2077	0.188	0.197	0.2
Variable Weight	0.207	0.208	0.189	0.197	0.199

Table 4.3: Ranking of the Alternatives

Methods	Order of the Alternatives based on the Aggregated Performance Values
Yue's approach	A_1, A_2, A_5, A_4, A_3
Equal Weight	A_1, A_2, A_5, A_4, A_3
Constant Weight	A_1, A_2, A_5, A_4, A_3
Variable Weight	A_2, A_1, A_5, A_4, A_3

After analyzing the tables given above, it can be seen that the alternatives have different performance values based on the DMs' weight calculated with the four mentioned methods. In the Yue's method, equal weight method and constant weight approach A_1 has the highest aggregated performance value. However, in the variable weight approach A_2 has the highest aggregated performance value with a slight difference to A_1. Also, it can be seen that A_3 has the lowest aggregated performance value in all four methods.

5. CONCLUSION

In this study, the focus has been one of the most important aspects of group decision making. The goal of the study is to assign appropriate weights to DMs in a group decision making situation as it affects the final choice.

Yue (2017) used entropy method to assign weights to DMs. Drawback of this approach comes from considering all decision criteria together as one system rather than considering them individually and it leads some DMs to have non-zero weights where they should not have.

To be able to correct the drawback in Yue's method, two models are proposed in this study. In the first part of the proposed approach, decision criteria are considered separately, and to each DM a constant weight has been assigned without ignoring the fact that each criterion is a sub-system of the whole decision criteria set. Although this was overcoming the drawback of Yue's approach, it has been noticed that it could be improved further. Because, with this approach the weight of a criterion that has a higher entropy value is distributed to other criteria for which the DM's judgment does not cause any effect on the final solution. In order to inhibit that conflict, another approach has been developed. Second approach assigns variable weights to DMs with respect to each criterion in a system rather than assigning constant weights.

In this study, a real case of group decision making problem which has six DMs, six criteria and five alternatives has been studied. Based on the results of four applied approaches, different weights for DMs are observed which was the expected result that comes from the variance in the methods. However, since the values given to the alternatives based on the criteria given by DMs are close to each other, the weights of DMs have not differed dramatically. For example, in Yue's method, the weights of DMs are ordered from the highest to the lowest weight for $\forall k = 1, \ldots, 6$ as DM_6, DM_4, DM_5, DM_2, DM_1, DM_3 with the values 0.484, 0.385, 0.114, 0.012, 0.004, 0.001 respectively. In the equal weight method, each DM had the same weight equals to 0.167. This method has been applied to see the effect of assuming the DMs' weights are the same. In the proposed constant weight method, the DMs are sorted from highest weight to lowest as DM_2, DM_6, DM_4, DM_1, DM_5, DM_3 for $\forall\ k = 1, \ldots, 6$ and the weights are equal to 0.377, 0.190. 0.182, 0.112, 0.096, 0.043 respectively. Lastly, in the variable weight method, six DMs are assigned a set of weights each, making their importance variable based on their judgments with respect to each decision criterion.

Thus, in this study, a correction has been done to Yue's proposed method. The DMs' weights are calculated with corrected proposed approaches that reflects the intuitionistic attributes of entropy method. A more precise way of assigning weights to DMs' by using entropy method is in our future study research area.

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APPENDIX – A: Yue's Method

Normalized Decision Matrices:

DM_1=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

DM_2=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

DM_3=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

DM_4=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

$DM_5=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

$DM_6=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

Entropy Calculation:

$DM_1=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.116	-0.117	-0.117	-0.113	-0.121	-0.123
A_2	-0.113	-0.115	-0.117	-0.113	-0.112	-0.114
A_3	-0.106	-0.108	-0.102	-0.109	-0.112	-0.110
A_4	-0.111	-0.112	-0.112	-0.113	-0.112	-0.110
A_5	-0.120	-0.114	-0.119	-0.118	-0.108	-0.110
					SUM:	-3.400
					E_1	1,000
					d_1	0.0005
					w_1	0.004

$DM_2=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.118	-0.119	-0.108	-0.115	-0.127	-0.120
A_2	-0.128	-0.115	-0.118	-0.125	-0.117	-0.115
A_3	-0.107	-0.105	-0.092	-0.110	-0.107	-0.107
A_4	-0.112	-0.113	-0.113	-0.105	-0.102	-0.111
A_5	-0.101	-0.114	-0.132	-0.110	-0.112	-0.114
					SUM:	-3.396
					E_2	0.998
					d_2	0.002
					w_2	0.012

DM_3=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.115	-0.114	-0.116	-0.109	-0.119	-0.118
A_2	-0.111	-0.111	-0.113	-0.112	-0.111	-0.114
A_3	-0.115	-0.109	-0.111	-0.115	-0.115	-0.114
A_4	-0.115	-0.115	-0.109	-0.115	-0.110	-0.111
A_5	-0.111	-0.118	-0.117	-0.115	-0.112	-0.110
					SUM:	-3.401
					E_3	1,000
					d_3	0.0002
					w_3	0.001

DM_4=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.106	-0.105	-0.098	-0.107	-0.119	-0.118
A_2	-0.106	-0.109	-0.111	-0.103	-0.105	-0.109
A_3	-0.106	-0.105	-0.093	-0.103	-0.114	-0.114
A_4	-0.115	-0.109	-0.107	-0.111	-0.110	-0.118
A_5	-0.096	-0.105	-0.115	-0.103	-0.105	-0.104
					SUM:	-3.229
					E_4	0.949
					d_4	0.051
					w_4	0.385

DM_5=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.115	-0.105	-0.111	-0.107	-0.110	-0.114
A_2	-0.115	-0.114	-0.111	-0.111	-0.114	-0.114
A_3	-0.115	-0.114	-0.102	-0.115	-0.119	-0.114
A_4	-0.120	-0.114	-0.111	-0.107	-0.114	-0.114
A_5	-0.106	-0.109	-0.115	-0.107	-0.100	-0.114
					SUM:	-3.350
					E_5	0.985
					d_5	0.015
					w_5	0.012

DM_6=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.111	-0.109	-0.102	-0.103	-0.119	-0.118
A_2	-0.120	-0.105	-0.107	-0.107	-0.110	-0.104
A_3	-0.101	-0.101	-0.093	-0.103	-0.110	-0.109
A_4	-0.106	-0.105	-0.098	-0.099	-0.105	-0.104
A_5	-0.106	-0.105	-0.115	-0.107	-0.100	-0.104
					SUM:	-3.185
					E_6	0.936
					d_6	0.064
					w_6	0.484

APPENDIX – B: Equal Weight Approach

Normalized Decision Matrices:

$DM_1=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

$DM_2=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.211	0.213	0.188	0.205	0.235	0.216
A_2	0.237	0.204	0.213	0.229	0.210	0.205
A_3	0.184	0.180	0.150	0.193	0.185	0.184
A_4	0.197	0.200	0.200	0.181	0.173	0.194
A_5	0.171	0.202	0.250	0.193	0.198	0.201

$DM_3=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.205	0.201	0.207	0.189	0.213	0.212
A_2	0.193	0.195	0.199	0.197	0.195	0.200
A_3	0.205	0.190	0.194	0.203	0.204	0.203
A_4	0.205	0.203	0.190	0.205	0.191	0.194
A_5	0.193	0.211	0.210	0.205	0.197	0.192

$DM_4=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.200	0.195	0.181	0.205	0.221	0.213
A_2	0.200	0.207	0.217	0.193	0.186	0.191
A_3	0.200	0.195	0.169	0.193	0.209	0.202
A_4	0.225	0.207	0.205	0.216	0.198	0.213
A_5	0.175	0.195	0.229	0.193	0.186	0.180

$DM_5=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.202	0.185	0.202	0.194	0.195	0.200
A_2	0.202	0.207	0.202	0.204	0.207	0.200
A_3	0.202	0.207	0.180	0.215	0.218	0.200
A_4	0.213	0.207	0.202	0.194	0.207	0.200
A_5	0.180	0.196	0.213	0.194	0.172	0.200

$DM_6=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.205	0.212	0.198	0.198	0.226	0.226
A_2	0.229	0.200	0.210	0.209	0.202	0.190
A_3	0.181	0.188	0.173	0.198	0.202	0.202
A_4	0.193	0.200	0.185	0.186	0.190	0.190
A_5	0.193	0.200	0.235	0.209	0.179	0.190

Entropy Calculation:

$DM_1=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.325	-0.327	-0.327	-0.322	-0.333	-0.335
A_2	-0.321	-0.325	-0.327	-0.322	-0.321	-0.323
A_3	-0.311	-0.313	-0.303	-0.315	-0.321	-0.316
A_4	-0.318	-0.321	-0.320	-0.322	-0.321	-0.316
A_5	-0.332	-0.323	-0.330	-0.328	-0.313	-0.316
	0.999	1.000	0.998	1.000	0.999	0.999
					E_1	5.994
					d_1	0.006
					w_1	0.167

$DM_2=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.118	-0.119	-0.108	-0.115	-0.127	-0.120
A_2	-0.128	-0.115	-0.118	-0.125	-0.117	-0.115
A_3	-0.107	-0.105	-0.092	-0.110	-0.107	-0.107
A_4	-0.112	-0.113	-0.113	-0.105	-0.102	-0.111
A_5	-0.101	-0.114	-0.132	-0.110	-0.112	-0.114
	0.996	0.999	0.992	0.998	0.997	0.999
					E_2	5.980
					d_2	0.020
					w_2	0.167

$DM_3=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.115	-0.114	-0.116	-0.109	-0.119	-0.118
A_2	-0.111	-0.111	-0.113	-0.112	-0.111	-0.114
A_3	-0.115	-0.109	-0.111	-0.115	-0.115	-0.114
A_4	-0.115	-0.115	-0.109	-0.115	-0.110	-0.111
A_5	-0.111	-0.118	-0.117	-0.115	-0.112	-0.110
	0.9998	0.9996	0.9996	0.9997	0.9995	0.9996
					E_3	5.998
					d_3	0.002
					w_3	0.167

$DM_4=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.106	-0.105	-0.098	-0.107	-0.119	-0.118
A_2	-0.106	-0.109	-0.111	-0.103	-0.105	-0.109
A_3	-0.106	-0.105	-0.093	-0.103	-0.114	-0.114
A_4	-0.115	-0.109	-0.107	-0.111	-0.110	-0.118
A_5	-0.096	-0.105	-0.115	-0.103	-0.105	-0.104
	0.9981	0.9998	0.9961	0.9994	0.9986	0.9987
					E_4	5.990
					d_4	0.010
					w_4	0.167

$DM_5=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.115	-0.105	-0.111	-0.107	-0.110	-0.114
A_2	-0.115	-0.114	-0.111	-0.111	-0.114	-0.114
A_3	-0.115	-0.114	-0.102	-0.115	-0.119	-0.114
A_4	-0.120	-0.114	-0.111	-0.107	-0.114	-0.114
A_5	-0.106	-0.109	-0.115	-0.107	-0.100	-0.114
	0.9990	0.9994	0.9990	0.9994	0.9981	1.0000
					E_5	5.995
					d_5	0.005
					w_5	0.167

$DM_6=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.111	-0.109	-0.102	-0.103	-0.119	-0.118
A_2	-0.120	-0.105	-0.107	-0.107	-0.110	-0.104
A_3	-0.101	-0.101	-0.093	-0.103	-0.110	-0.109
A_4	-0.106	-0.105	-0.098	-0.099	-0.105	-0.104
A_5	-0.106	-0.105	-0.115	-0.107	-0.100	-0.104
	0.9980	0.9996	0.9965	0.9994	0.9981	0.9985
					E_6	5.990
					d_6	0.010
					w_6	0.167

APPENDIX – C: Constant Weight Approach

Normalized Decision Matrices:

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.206	0.209	0.209	0.200	0.220	0.225
	A_2	0.199	0.204	0.209	0.200	0.198	0.202
DM_1=	A_3	0.183	0.187	0.172	0.189	0.198	0.191
	A_4	0.195	0.198	0.197	0.200	0.198	0.191
	A_5	0.217	0.202	0.214	0.211	0.187	0.191

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.211	0.213	0.188	0.205	0.235	0.216
	A_2	0.237	0.204	0.213	0.229	0.210	0.205
DM_2=	A_3	0.184	0.180	0.150	0.193	0.185	0.184
	A_4	0.197	0.200	0.200	0.181	0.173	0.194
	A_5	0.171	0.202	0.250	0.193	0.198	0.201

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.205	0.201	0.207	0.189	0.213	0.212
	A_2	0.193	0.195	0.199	0.197	0.195	0.200
DM_3=	A_3	0.205	0.190	0.194	0.203	0.204	0.203
	A_4	0.205	0.203	0.190	0.205	0.191	0.194
	A_5	0.193	0.211	0.210	0.205	0.197	0.192

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.200	0.195	0.181	0.205	0.221	0.213
	A_2	0.200	0.207	0.217	0.193	0.186	0.191
DM_4=	A_3	0.200	0.195	0.169	0.193	0.209	0.202
	A_4	0.225	0.207	0.205	0.216	0.198	0.213
	A_5	0.175	0.195	0.229	0.193	0.186	0.180

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.202	0.185	0.202	0.194	0.195	0.200
	A_2	0.202	0.207	0.202	0.204	0.207	0.200
DM_5=	A_3	0.202	0.207	0.180	0.215	0.218	0.200
	A_4	0.213	0.207	0.202	0.194	0.207	0.200
	A_5	0.180	0.196	0.213	0.194	0.172	0.200

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	0.205	0.212	0.198	0.198	0.226	0.226
	A_2	0.229	0.200	0.210	0.209	0.202	0.190
DM_6=	A_3	0.181	0.188	0.173	0.198	0.202	0.202
	A_4	0.193	0.200	0.185	0.186	0.190	0.190
	A_5	0.193	0.200	0.235	0.209	0.179	0.190

Entropy Calculation:

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.325	-0.327	-0.327	-0.322	-0.333	-0.335
	A_2	-0.321	-0.325	-0.327	-0.322	-0.321	-0.323
DM_1=	A_3	-0.311	-0.313	-0.303	-0.315	-0.321	-0.316
	A_4	-0.318	-0.321	-0.320	-0.322	-0.321	-0.316
	A_5	-0.332	-0.323	-0.330	-0.328	-0.313	-0.316
		0.999	1,000	0.998	1,000	0.999	0.999
						E_1	5.994
						d_1	0.006
						w_1	0.112

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.328	-0.330	-0.314	-0.325	-0.340	-0.331
	A_2	-0.341	-0.325	-0.329	-0.338	-0.328	-0.325
DM_2=	A_3	-0.312	-0.309	-0.285	-0.317	-0.312	-0.311
	A_4	-0.320	-0.322	-0.322	-0.309	-0.303	-0.318
	A_5	-0.302	-0.323	-0.347	-0.317	-0.320	-0.322
		0.996	0.999	0.992	0.998	0.997	0.999
						E_2	5.980
						d_2	0.020
						w_2	0.377

$DM_3=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.325	-0.322	-0.326	-0.315	-0.329	-0.329
A_2	-0.318	-0.318	-0.321	-0.320	-0.319	-0.322
A_3	-0.325	-0.316	-0.318	-0.324	-0.324	-0.323
A_4	-0.325	-0.324	-0.316	-0.325	-0.316	-0.318
A_5	-0.318	-0.329	-0.327	-0.325	-0.320	-0.317
	0.9998	0.9996	0.9996	0.9997	0.9995	0.9996
					E_3	5.998
					d_3	0.002
					w_3	0.043

$DM_4=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.322	-0.319	-0.309	-0.325	-0.334	-0.330
A_2	-0.322	-0.326	-0.331	-0.318	-0.313	-0.316
A_3	-0.322	-0.319	-0.300	-0.318	-0.327	-0.323
A_4	-0.336	-0.326	-0.325	-0.331	-0.320	-0.330
A_5	-0.305	-0.319	-0.338	-0.318	-0.313	-0.309
	0.998	1.000	0.996	0.999	0.999	0.999
					E_4	5.990
					d_4	0.010
					w_4	0.182

$DM_5=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.323	-0.312	-0.323	-0.318	-0.319	-0.322
A_2	-0.323	-0.326	-0.323	-0.324	-0.326	-0.322
A_3	-0.323	-0.326	-0.309	-0.331	-0.332	-0.322
A_4	-0.330	-0.326	-0.323	-0.318	-0.326	-0.322
A_5	-0.309	-0.319	-0.330	-0.318	-0.303	-0.322
	0.999	0.999	0.999	0.999	0.998	1.000
					E_5	5.995
					d_5	0.005
					w_5	0.096

$DM_6=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.325	-0.329	-0.320	-0.320	-0.336	-0.336
A_2	-0.338	-0.322	-0.328	-0.327	-0.323	-0.316
A_3	-0.309	-0.314	-0.303	-0.320	-0.323	-0.323
A_4	-0.317	-0.322	-0.312	-0.313	-0.316	-0.316
A_5	-0.317	-0.322	-0.340	-0.327	-0.308	-0.316
	0.998	1.000	0.997	0.999	0.998	0.999
					E_6	5.990
					d_6	0.010
					w_6	0.190

APPENDIX – D: Variable Weight Approach

Normalized Decision Matrices:

DM_1=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.206	0.209	0.209	0.200	0.220	0.225
A_2	0.199	0.204	0.209	0.200	0.198	0.202
A_3	0.183	0.187	0.172	0.189	0.198	0.191
A_4	0.195	0.198	0.197	0.200	0.198	0.191
A_5	0.217	0.202	0.214	0.211	0.187	0.191

DM_2=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.211	0.213	0.188	0.205	0.235	0.216
A_2	0.237	0.204	0.213	0.229	0.210	0.205
A_3	0.184	0.180	0.150	0.193	0.185	0.184
A_4	0.197	0.200	0.200	0.181	0.173	0.194
A_5	0.171	0.202	0.250	0.193	0.198	0.201

DM_3=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.205	0.201	0.207	0.189	0.213	0.212
A_2	0.193	0.195	0.199	0.197	0.195	0.200
A_3	0.205	0.190	0.194	0.203	0.204	0.203
A_4	0.205	0.203	0.190	0.205	0.191	0.194
A_5	0.193	0.211	0.210	0.205	0.197	0.192

DM_4=

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.200	0.195	0.181	0.205	0.221	0.213
A_2	0.200	0.207	0.217	0.193	0.186	0.191
A_3	0.200	0.195	0.169	0.193	0.209	0.202
A_4	0.225	0.207	0.205	0.216	0.198	0.213
A_5	0.175	0.195	0.229	0.193	0.186	0.180

$DM_5=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.202	0.185	0.202	0.194	0.195	0.200
A_2	0.202	0.207	0.202	0.204	0.207	0.200
A_3	0.202	0.207	0.180	0.215	0.218	0.200
A_4	0.213	0.207	0.202	0.194	0.207	0.200
A_5	0.180	0.196	0.213	0.194	0.172	0.200

$DM_{6=}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.205	0.212	0.198	0.198	0.226	0.226
A_2	0.229	0.200	0.210	0.209	0.202	0.190
A_3	0.181	0.188	0.173	0.198	0.202	0.202
A_4	0.193	0.200	0.185	0.186	0.190	0.190
A_5	0.193	0.200	0.235	0.209	0.179	0.190

Entropy Calculation:

$DM_1=$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.325	-0.327	-0.327	-0.322	-0.333	-0.335
A_2	-0.321	-0.325	-0.327	-0.322	-0.321	-0.323
A_3	-0.311	-0.313	-0.303	-0.315	-0.321	-0.316
A_4	-0.318	-0.321	-0.320	-0.322	-0.321	-0.316
A_5	-0.332	-0.323	-0.330	-0.328	-0.313	-0.316
	0.999	1.000	0.998	1,000	0.999	0.999
					E_1	5.994
	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}
	0.001	0.0004	0.002	0.0004	0.001	0.001
	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}
	0.100	0.142	0.096	0.085	0.088	0.241

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.328	-0.330	-0.314	-0.325	-0.340	-0.331
	A_2	-0.341	-0.325	-0.329	-0.338	-0.328	-0.325
DM_2=	A_3	-0.312	-0.309	-0.285	-0.317	-0.312	-0.311
	A_4	-0.320	-0.322	-0.322	-0.309	-0.303	-0.318
	A_5	-0.302	-0.323	-0.347	-0.317	-0.320	-0.322
		0.996	0.999	0.992	0.998	0.997	0.999
						E_2	5.980
		d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}
		0.004	0.001	0.008	0.002	0.003	0.001
		w_{21}	w_{22}	w_{23}	w_{24}	w_{25}	w_{26}
		0.388	0.315	0.440	0.450	0.343	0.168

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.325	-0.322	-0.326	-0.315	-0.329	-0.329
	A_2	-0.318	-0.318	-0.321	-0.320	-0.319	-0.322
DM_3=	A_3	-0.325	-0.316	-0.318	-0.324	-0.324	-0.323
	A_4	-0.325	-0.324	-0.316	-0.325	-0.316	-0.318
	A_5	-0.318	-0.329	-0.327	-0.325	-0.320	-0.317
		0.9998	0.9996	0.9996	0.9997	0.9995	0.9996
						E_3	5.998
		d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}
		0.0002	0.0004	0.0004	0.0003	0.0005	0.0004
		w_{31}	w_{32}	w_{33}	w_{34}	w_{35}	w_{36}
		0.024	0.133	0.023	0.069	0.047	0.072

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.322	-0.319	-0.309	-0.325	-0.334	-0.330
	A_2	-0.322	-0.326	-0.331	-0.318	-0.313	-0.316
DM_4=	A_3	-0.322	-0.319	-0.300	-0.318	-0.327	-0.323
	A_4	-0.336	-0.326	-0.325	-0.331	-0.320	-0.330
	A_5	-0.305	-0.319	-0.338	-0.318	-0.313	-0.309
		0.998	1.000	0.996	0.999	0.999	0.999
						E_4	5.990
		d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}
		0.002	0.0002	0.004	0.001	0.001	0.001
		w_{41}	w_{42}	w_{43}	w_{44}	w_{45}	w_{46}
		0.192	0.079	0.207	0.140	0.140	0.249

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.323	-0.312	-0.323	-0.318	-0.319	-0.322
	A_2	-0.323	-0.326	-0.323	-0.324	-0.326	-0.322
DM_5=	A_3	-0.323	-0.326	-0.309	-0.331	-0.332	-0.322
	A_4	-0.330	-0.326	-0.323	-0.318	-0.326	-0.322
	A_5	-0.309	-0.319	-0.330	-0.318	-0.303	-0.322
		0.999	0.999	0.999	0.999	0.998	1.000
						E_5	5.995
		d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}
		0.001	0.001	0.001	0.001	0.002	0.000
		w_{51}	w_{52}	w_{53}	w_{54}	w_{55}	w_{56}
		0.094	0.192	0.051	0.126	0.191	0.000

		C_1	C_2	C_3	C_4	C_5	C_6
	A_1	-0.325	-0.329	-0.320	-0.320	-0.336	-0.336
	A_2	-0.338	-0.322	-0.328	-0.327	-0.323	-0.316
DM_6=	A_3	-0.309	-0.314	-0.303	-0.320	-0.323	-0.323
	A_4	-0.317	-0.322	-0.312	-0.313	-0.316	-0.316
	A_5	-0.317	-0.322	-0.340	-0.327	-0.308	-0.316
		0.998	1.000	0.997	0.999	0.998	0.999
						E_6	5.990
		d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}
		0.002	0.0004	0.003	0.001	0.002	0.001
		w_{61}	w_{62}	w_{63}	w_{64}	w_{65}	w_{66}
		0.201	0.139	0.183	0.131	0.190	0.270