

MSc Thesis

**Estimation of Price Elasticity and Optimal Pricing
Decisions for an Omni-Channel Retailer**

by

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A Dissertation Submitted to the
Graduate School of Sciences and Engineering
in Partial Fulfillment of the Requirements for
the Degree of
Master of Science
in
Industrial Engineering



June, 2019

MSc Thesis
Estimation of Price Elasticity and Optimal Pricing Decisions for an
Omni-Channel Retailer

Koç University
Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
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To my brother Canel Tellal.

ABSTRACT

Pricing decisions and high competition are two significant factors in profit maximization in the consumer electronics market. Competitors sell high valued identical products most of the time and the consumers tend to monitor price of a product and substitutes of that product before making the purchase. Because of dynamic nature of consumer electronics market, there are dozens of pricing decisions to give every day. For a pricing manager, it is not easy to find optimal prices for dozens of products each day. In addition, a product's price is not the only factor that effects its demand. Price of the substitute products and price of the product at the competitor stores effect the demand of the product. In this study, we develop a demand prediction model which can make daily sales forecasts and develop two optimization methods to maximize total profit. First, we develop a model which can identify cross-price elasticity and competitor-price elasticity as well as price elasticity of a product. Then, we propose a non-linear integer model and an Iterative Bisection Algorithm to solve multi-product price optimization problem. As an additional study, a field experiment is conducted and potentially promising results are observed.

ÖZETÇE

Fiyat kararları ve yüksek rekabet tüketici elektronik perakende sektöründe, kar en iyilemesi yaparken en önemli iki faktördendir. Rakip firmalar genellikle, birbirine benzer ve fiyat olarak değeri yüksek ürünleri satmaktadırlar ve müşteriler de bir alım işlemi yapmadan önce marketteki bütün fiyatları ve ikame ürünleri inceleme eğilimindedirler. Tüketici elektronik perakende sektörünün bu dinamik yapısı nedeniyle, bir firma her gün onlarca fiyat kararı verir. Bir firmanın fiyatlandırmadan sorumlu çalışanı için, her gün onlarca en iyi fiyat bulmak kolay değildir. Ek olarak, bir ürünün talebini etkileyen tek fiyat kararı, ürünün kendi fiyatı değildir. İkame ürünlerin fiyatları ve rakip satıcılarda bulunan ürünün fiyatı da ürünün talebini etkilemektedir. Bu çalışmada, gün ve ürün seviyesinde tahmin yapabilen bir talep tahmin modeli geliştiriyor ve iki farklı en iyileme yolu ile en büyüklenmiş karı bulmayı hedefliyoruz. Önce, ürünün kendi fiyat elastikiyeti ile birlikte, ikame ürün fiyat elastikiyeti ve rakip fiyat elastikiyetini hesaplayan bir talep tahmin modeli oluşturulmuştur. Sonrasında, doğrusal olmayan bir tam sayı modeli ve tekrarlayan ikiye bölme algoritması ile çoklu ürün fiyat en iyilemesi problemini çözmeyi önerdik. Ek olarak, bir alan deneyi yürütüp, potansiyel olarak ümit verici sonuçlar alıp raporladık.

ACKNOWLEDGMENTS

I would like to express my deep and sincere gratitude to my research supervisors, Professor Gürhan Kök and Professor Fikri Karaesmen for giving me the opportunity to do research and providing invaluable guidance throughout this research. His dynamism, vision, sincerity and motivation have deeply inspired me. He has taught me the methodology to carry out the research and to present the research works as clearly as possible. It was a great privilege and honour to work and study under his guidance. I am extremely grateful for what he has offered me.

Also, I would like to express my special thanks of gratitude to Assistant Professor Kris Johnson Ferreira from Harvard Business School who helped and guided me throughout this research. Even though, we were miles and hours away with each other, she always answered my call for help every single time.

Besides my advisors, I would like to thank the rest of my thesis committee: Asst. Prof. İhsan Yanıkoğlu, Asst. Prof. Pelin Gülşah Canbolat, and Asst. Prof. Erhun Özkan, for their encouragement, insightful comments, feedbacks and hard questions.

I am extremely grateful to my parents and brother for their love, caring and support for educating and preparing me for my future.

Finally, I want to present my thanks to my friends who were sympathetic and supportive while I was working busy for this thesis.

S.Canberk Tellal

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Chapter 1

INTRODUCTION

1.1 Introduction and Problem

This study presents a collaboration with one of the largest omni-channel consumer electronics retailer in Turkey. Consumer electronics is one of the biggest markets in retail and it is highly competitive. Most of the time, retailers sell high valued identical products. Most customers monitor all retailer prices online and make the purchase decision accordingly in consumer electronics market [Wu et al., 2014]. Customers tend to check all available prices online and try to purchase at the lowest price [Baye et al., 2003]. This nature of consumer electronics makes pricing decisions difficult for pricing managers because there are multiple moving pieces when it comes predicting demand. Some pieces are in control of the company such as price, promotions, advertisements, and inventory management. Seasonality, competitor prices, the product's life cycle, special days, macro-economic factors are not in the control of the company. Also, all these pieces change daily, so many pricing decisions should be given in daily basis. It is not easy for pricing managers to monitor all these pieces and give optimal pricing decisions. Thus, there is a huge opportunity for a consumer electronics company. A company in the electronics retailer market can generate more revenue with a better pricing method. Almost all factors the company can control require some cost, this cost may be money, effort, or time. But pricing decisions do not require a cost in terms of money, physical effort or time. They only require decision making effort. So, without making any other investment, better pricing decisions would help the company make more revenue or profit. In our case, the company in

this study is a centralized company which has electronic labels at the stores. They can change the price on the electronic label of a product any day, any time from the headquarters.

This study aims to provide a method for a consumer electronics retailer to use its past data to estimate price elasticities and develop a model to maximize its profit or revenue. Consumer electronics retailers must deal with highly perishable products, as their inventory loses value quickly because new technological products enter the market every day [Talluri and Van Ryzin, 2006]. Consumer electronics retail market is an extremely competitive market since it is easy for a consumer to monitor competitor prices. In Turkey, there are some search engines which can do online price searches for almost all products. So, we can assume most consumers can see all available prices online. In addition to competitor prices, there is another price related effect called the substitution effect. The substitution effect can be defined as, the price decision for one product affects the demand of all product in the product category [Reibstein and Gatignon, 1984]. For example, a customer might select to purchase a different product than her primary selection if she sees a discounted substitute product. A good example of study on importance of substitution effect is presented by Vulcano, Van Ryzin and Ratliff [Vulcano et al., 2012].

In this study, a parametric model is proposed to understand how demand of a product changes with price of the product, prices of substitutable products and price of same product in the competitor companies. First, demand parameters of products will be predicted from the proposed parametric demand prediction model. Then, a multi-product price optimization model which can be solved in a short time will be developed. Decision variables of the model will be the price of the products and model business constraints. Since the price of a product affects other products' demand, it will be a hard problem to be solved. To summarize, our objective is to develop an algorithm to efficiently solve the multi-product price optimization problem which will

maximize retailer's profit or revenue by using the outputs of the proposed demand estimation model as input.

1.2 Company Information

The company collaborated with this study is one of the top three companies in the consumer electronics market in Turkey. They are following the business to consumer model. The company has 59 active stores including 2 different online channels. Offline stores are located on 19 different cities across Turkey. Although online sales percentage increases in the consumer electronics market as time passes [Candemir et al., 2016], only 5% of the sales are made from online channels. It is fair to say, store experience for consumers are still important for customers in consumer electronics in Turkey.

Predicting future demand by using historical data is one of the crucial steps in the study. The company provided store-day-SKU level; inventory, revenue and sales quantity data. By using this dataset, the average sales price of a SKU is calculated at store-day level. Also, the company has provided SKU-day level promotion data and products hierarchies. Top hierarchy is department for a product and bottom hierarchy is SKU. The company has provided competitor prices of significant products. The company has three main competitors in the market. They crawl the online store prices of the competitor's identical products. Unfortunately, when a competitor is stocked-out, the competitor price or their stock-out status was not provided. Thus, there are significantly many missing values in the competitor data. Also, SKU level attribute data is provided. This attribute data can be used to identify which products are similar to each other. This similarity information is important for this study to capture substitute product effect.

The company uses a strategy to make pricing decisions. Firstly, pricing managers cluster all products into five groups manually, then they are using different pricing

strategies for each group. These strategies are listed in the following order:

- 1) Image Setters: Image Setters are most popular products in the market, maximizing profit from image setters is not the main aim of the company. Since image setters are most popular products of the market, customers are searching for each competitor in the market and choose to buy from cheapest alternative. The company use these products to attract customers into the stores and manage the price perception of the customers. The company is trying to give the message of 'lowest price of the market' using this strategy. Pricing strategy is matching the minimum competitor price.
- 2) Standard Products: Standard products are not as popular as image setters, but most of the competitors have these products and customers monitor the price of these products. The pricing strategy of this cluster is placing in a place between minimum and maximum priced competitors.
- 3) Margin Products: Margin products are mostly cheap products, customers do not usually monitor competitor prices. Priority of the company for this cluster is maximizing profit for these products. Pricing strategy for this cluster is offering a price as high as possible.
- 4) Exclusive Articles: These articles are only available on the company. So, pricing of these products does not have a strict policy.
- 5) Dead Articles: These products are at the end of their life cycle or the company has decided to end the contract with supplier. So this product's inventory will not be replenished. The company tries to offer a price as low as possible to get rid of the remaining inventory.

Chapter 2

RELATED LITERATURE

2.1 Revenue Management and Demand Functions Overview

There have been different studies about price optimization in the past decade. See Van Ryzin and McGill's study [McGill and Van Ryzin, 1999] for an excellent research overviews on the Revenue Management. In revenue management demand prediction is crucial. If someone can understand how demand function is behaving under different prices, she can build a revenue optimization models on the demand function. Bitran and Caldentey [Bitran and Caldentey, 2003] have studied dynamic pricing models for generic revenue management problem. In this problem in a price sensitive market, a seller tries to maximize its revenue with fixed and perishable goods. Talluri and Van Ryzin's book [Talluri and Van Ryzin, 2006] provides an excellent in-depth overview on Revenue Management. They list common demand function into a group of four;

- i. Linear Demand Function,
- ii. Log-Linear(Exponential) Demand Function,
- iii. Constant Elasticity Demand Function,
- iv. Logit Demand Function.

They show that, these functions show the following regularity conditions;

- i. As price increases, demand has a decreasing behaviour,
- ii. On the price vector, the demand function is differentiable,
- iii. The demand function is bounded between zero and infinity,
- iv. The demand function converges to zero as price gets higher,
- v. For all products, the revenue function is between zero and a maximum value.
- vi. The maximum revenue is satisfied by a finite number of price points.

One of the crucial things for a company to decide is the pricing approach. There are different types of pricing modalities in literature such as fixed pricing, bargain pricing, seller auction, customized pricing and contingent pricing. In consumer electronics retail market, it is common to have fixed pricing or bargain pricing. In recent years, the most important trend is to have fixed pricing. Fixed pricing modality has four elements;

- I. The price is set by the seller ,
- II. The seller announces the price of the product to market for potential customers to see,
- III. Potential customers observe the price and choose to make the purchase or not,
- IV. Transactions take place exclusively at the posted or advertised price [Özer et al., 2012].

For an excellent example of using historic sales data to capture effects of sales parameters see [Caro and Gallien, 2012]. In this paper they implement a decision support tool for a large apparel retailer's clearance period. To start with, they develop a demand prediction model which captures effect of five different parameters such that;

- I. Purchase quantity,
- II. Age of article,
- III. Previous period sales,
- IV. Broken assortment effect,
- V. Price elasticity.

This demand model, with its estimation parameters, has been able to explain the factors affecting the demand in the past and creates a structure that can predict future sales in different situations. Then, for a short discount period, they developed a model to maximize their profit. It takes the business rules of Zara as set of constraints and decides which pricing decisions maximizes profit at each period while clearing remaining inventory. For a product, price candidates are coming from a discrete set since the retailer chooses to use prices with last digit rules [Macé, 2012]. A model like

this can be solved by dynamic programming, but the curse of dimensionality makes it hard to solve [Li et al., 2017]. So, they approach this problem with an approximation, they replace future sales with by their expected values. For given price and inventory inputs, they calculate expected demand for a product at a period as inputs to the optimization model.

2.2 Price Optimization Problem Overview

In the literature, there are lots of papers about clearance pricing optimization. Clearance pricing optimization problem can be defined as a special case of pricing optimization problem. In clearance pricing optimization retailers try to clean up the remaining inventory from current season with the highest profit possible for short clearance periods. The decisions are which products get price decrease decision and when this price decrease decision will take place. Generally, there are lots of business rules applied. For example, retailers can cluster some products together to give them same price or discount rate, no price increase decisions, cost and profit margin constraints, number of price change decisions constraints are some examples.

[Smith and Achabal, 1998] argues that, it is optimal to have higher prices early in the season, followed by deeper markdowns in the clearance period. This is the main difference of clearance pricing optimization problem and price optimization problem. Because in the price optimization problem, there is no fixed period like clearance period. Of course, for products which are at the end of their life-cycle, retailers try a similar approach as clearance pricing. Since each product has its own life-cycle, we cannot observe clearance period like an apparel retailer. Another main difference is in the clearance pricing optimization, price increase decision is not allowed. But in the pricing optimization both price increases and decreases are allowed. Other than the differences mentioned above, both problems and approaches are similar. In both problems the objective is maximizing profit. Also in both problems, estimating demand from historical sales and using these estimations to make pricing decisions are

main ideas. Also, a great application of markdown optimization problem is presented by Mantrala and Rao [Mantrala and Rao, 2001] and Chen, Chen, Pundoor, Acharya and Yi [Chen et al., 2015].

Another approach to this problem is Dynamic Programming. Theoretically, in the problem settings, we see multiple periods in the horizon. Objective is finding a pricing path that would maximize total profit of an item. In this approach inventory has an important impact on the sales, as every price decision effects the inventory level for upcoming periods which definitely effects the demand predictions and optimal price decisions. But such formulations are difficult to apply on multi-product problems due to curse of dimensionality. [Bertsimas and Popescu, 2003] tries to solve this problem by considering approximate formulations.

Fisher, Gallino and Li [Fisher et al., 2017] provides a paper which has concerns about the companies those use competitor based dynamic pricing. They exploit their own and competitors' stock-outs as a source of variation in addition to price fluctuations. After obtaining model estimates, they solve a constrained optimization model to define optimal price responses to competitor price changes. Firstly, in order to calculate price elasticity of products they make a field experiment in which they took top 15 selling product and randomly divide them into 2 groups. first group randomly changes prices with respect to a baseline price defined by the product's historical price at the retailer. Second group randomly changes prices with respect to the competitors' prices for that product. Also, they come up with a customer choice model which describes how consumers make choices among substitutable products that multiple competing retailers offer. In this model they are calculating the utility of a consumer at consumer, retailer, price and period levels. For a given period a consumer would choose retailer who would give biggest utility to consumer. In the model, α_j stands for the constant utility obtained from purchasing product j regardless of the retailer the product is purchased from. α_r stands for the constant utility obtained from making

the purchase at retailer j . β_j stands for price elasticity of the product j . In order to obtain utility for consumer i from purchasing product j at retailer r at price $p_{j,r,t}$ on day t they defined $u_{i,j,r,t}$ and they defined utility $u_{i,0,t}$ as purchasing from one of the R retailers,

$$u_{i,j,r,t} = \alpha_j + \alpha_r + \beta_j * \log(p_{j,r,t}) \quad u_{i,0,t} = X_{0,t} + \epsilon_{i,0,t}$$

A consumer will purchase product j from retailer r at price $p_{j,r,t}$ if

$$u_{i,j,r,t} = \max \{u_{i,j,r,t}, u_{i,0,t}; \forall j \ \& \ \forall r\}$$

and will choose the outside option if $u_{i,0,t}$

$$u_{i,0,t} = \max \{u_{i,j,r,t}, u_{i,0,t}; \forall j \ \& \ \forall r\}$$

For a customer i , utility of the shopping at a store is affected by, product's utility, retailer's utility, price of the product and utility shocks or errors. Interesting part here is $\epsilon_{i,j,r,t}$ are not independent from each other. To allow such correlation they assume $\epsilon_{i,j,r,t}$ have a joint cumulative distribution. Ideally, this approach can calculate the probability of buying a product at a specific time and retail as given price information; which can be used as market share of the retailer that day. Since sales data of the competitors are not available for a company, they can keep track of the days when a competitor stock-out. This approach is applicable in online retail, but it is not possible in omni-channel competition as it is not easy to keep track of stock-outs in brick and mortar stores.

In Turkish electronics retail, there are three major companies competing with each other, although there are hundreds of online sellers. Since collecting and tracking all of these small online competitors nearly impossible, the company we collaborated with, collect the data of other electronic retailer's online price data at SKU and day level. Li, Netessine and Koulayev [Li et al., 2017] have published a paper which focuses on competitor priced based markets. They especially try to solve the problem of high

dimensionality and simultaneity bias. High dimensionality occurs when there are too many competitors in the market. For example, in 2014, 2 million third party merchants were competing on Amazon [Li et al., 2017]. In other words, the methodology should strike a balance between a reasonable model fit and screening out the non-relevant variables. High simultaneity occurs because all competitors change their prices simultaneously. In a market with N stores, they consider all of them apply competition-based pricing, they come up with following pricing model.

$$\begin{aligned}
 P_{1,j,t} &= \alpha_1 + \beta_{1,2}P_{2,j,t} + \beta_{1,3}P_{3,j,t} + \dots + \beta_{1,N}P_{N,j,t} + X_{j,y}\gamma_1 + \epsilon_{1,j,t}, \\
 P_{2,j,t} &= \alpha_2 + \beta_{2,1}P_{1,j,t} + \beta_{2,3}P_{3,j,t} + \dots + \beta_{2,N}P_{N,j,t} + X_{j,y}\gamma_2 + \epsilon_{2,j,t}, \\
 &\dots \\
 P_{N,j,t} &= \alpha_N + \beta_{N,1}P_{1,j,t} + \beta_{N,2}P_{2,j,t} + \dots + \beta_{N,N-1}P_{N-1,j,t} + X_{j,y}\gamma_N + \epsilon_{N,j,t},
 \end{aligned}$$

$X_{j,t}$ denotes travel date booking date, γ is a firm specific parameter. In this problem every competitor seems to be affecting each other. But, with this high dimensionality it is hard to identify which shops are actually competing. To estimate the model, one can use a Two Stage Least Square approach. In the first stage they calculate the prices for each store without using price parameters of the other stores and in stage two they write the price formulas again to cross-price elasticity parameters of other store prices (β parameters). They use Two-Stage Predictor Substitution and Two-Stage Residual Insertion. Either approach gives unbiased estimates of β parameters.

Ferreira, Lee and Simchi-Levi [Ferreira et al., 2015] has a similar work which inspired this study. They worked with an online retailer for optimizing pricing decision of flash sales. Flash sales are limited-time discounts on apparel and accessories. For flash sales, prices of offered products are fixed, so a bad pricing decision cannot be changed. They approach this subject by dividing the problem into two; demand forecasting and price optimization. When it comes to predicting the demand, they

distinguish sales and demand from each other, since most of the products are sold out before flash sales period ends. The idea behind their method is using sales history of the products which did not sell out. Then, they aggregated hourly sales of those products and calculated the percent of the sales that occur at each hour and that gave them empirical distribution of the proportion of sales that occur in the first X hours of an event. With empirical distributions they identified the time that the item sold out and used the appropriate demand curve to estimate the proportion of sales that typically occur within that amount of time. By using this method, they are identifying true demand of stocked-out products and have a better insight for future predictions. Also in this paper, they use price of the competing products as a feature to demand prediction. They are considering the average price of competing products as a reference point for the customers. Despite using all price data of the competing products, they calculate the average price of competing products in the product's category and use that information as an input. We can argue that selecting competing products is a crucial step in this problem as a consumer would be effected by a substitutable product. Calculating a reference price for a product is an inspiring idea for our study as well.

Like other retailers in their project, Ferreira, Lee and Simchi-Levi [Ferreira et al., 2015] worked with price ladders which end at 4.90 or 9.90. Thus, every product has a limited and discrete price set. Still trying all combinations of the all products would be often computationally intractable. For example, if there are 2 products with 5 different price ladders, there are 25 different combinations for 2 products. If there are 3 product, combinations increases to 125. There are M^N combinations if there are M different price ladders and N products. This means, there are $O(MN)$ possible sums to consider. And this ratio increases as number products in the optimization model increases. Trying all combinations as in brute force, can be used when number of products in the optimization is small. If we want to optimize each product group one by one, it might be applicable. In the paper, they have written an integer

formulation and an efficient algorithm to solve it, since they are optimizing every product. Let's say they have an Integer Programming formulation (IP) . Consider they have created sub-problems to (IP) as a Linear Program LP_k and an another Integer Program IP_k for different reference price k . We can say that, IP_k is analogous with the multiple-choice knapsack problem (MCKP). LP_k is linear relaxation of IP_k problem. They have developed an efficient algorithm by using the solutions of IP_k and LP_k , $z_{IP_k}^*$ and $z_{LP_k}^*$ respectively. They solved sub-problems until $z_{IP_k}^*$ is bigger than the lower bound ($z_{LP_k}^*$), then they terminated the algorithm. We can say that, they could solve an $O(MN)$ problem with a sub-problem and a linear relaxation.

The results they obtained from this work are, by conducting a field experiment to evaluate their pricing decision support tool and showed an expected increase in first exposure styles' revenue in the test group of approximately 9.7% with an associated 90% confidence interval of [2.3%, 17.8%], by developing an efficient algorithm to solve a multi-product price optimization model that incorporates reference price effects which can be used by other retailers to set prices of new products, regression trees would be effective in predicting demand for (i) new products and (ii) products whose price can be considered a signal of quality.

Also, there are many other papers about this subject with different approaches. For example, [Li et al., 2017] has written a paper focusing on competitor priced based markets. Especially it tries to solve the problem of high dimensionality and simultaneity bias. They use market share based models as like [Fisher et al., 2017].

There has been considerable number of studies in the literature for price optimization. [Subrahmanyam, 2000] provides a review on the quantitative models for setting retail prices. Writer defines the problem and explain two mathematical tool to solve her problem, (i) Bayesian update of info, (ii) Dynamic Programming. [3] research optimal pricing decisions under the influence of elasticities and cross-elasticities. They

clearly state that, the price of a product affects not only its demand, but it affects other product's demand as well. Especially in retail business, a customer would shop among various alternative products and brands. In this study, in a log-linear model, they formulate how to find the optimal price for single-product and multi-product cases. They present three models; first model is for single product case, it does not allow any kind of interaction between products. The second model is actually the same as the first model, but the difference is that error terms of different products can interact with each other. The third model lets price of each product directly effect demand of other products, which is really helpful in solving multi-product case. In this study, they show that optimal price is sensitive to inclusion or exclusion to cross-price elasticities. [Gallego and Wang, 2014] studies a case which tries to solve multi-product optimization problem with competition. They approach this problem as a nested logit problem. First, a customer selects a nest of products, then customer purchases one of the products in the selected nest. Then, they define the concept of adjusted markup. Adjusted markup is defined as price minus cost minus price sensitivity. They show that, at optimality, adjusted markup is same for all products in a selected nest. By using this information, in spite of having price of all products as a variable, they can have one variable in the revenue formulation, which is easier to solve. In addition those findings, they can deal with competitor prices with some slight adjustment to previous model. They provide the conditions which would make the model uni-modal, thus they can characterize Nash Equilibrium for oligopolistic multi-product optimization problem.

2.3 Price Elasticity Overview

Price elasticity of demand is a common term discussed in Revenue management. Alfred Marshall mentions in his book [Marshall and Marshall, 1920]; The elasticity (or responsiveness) of demand in a market is great or small according as the amount demanded increases much or little for a given fall in price, and diminishes much or little for a given rise in price.. In other words, price elasticity is a response measure

to change in the price. Price elasticity can be calculated by dividing percentage of demand change by percentage of price change. In this study, there are three types of price elasticities to consider:

Own-Price Elasticity: This is the price elasticity of the focal product itself. Own-price elasticity shows, how a product's price effect its own demand. [Gillespie, 2014] For a regular product, it is expected to have a negative sign, because price of the product and demand of the product have negative correlation.

Cross-Price Elasticity: This is the price elasticity of the focal product versus each of the other products in the product category. Cross-price elasticity shows, how a product's demand is effected by a price change of a different product [Gillespie, 2014]. Say there are N products in the product category. Then there are $N-1$ candidates for being substitute products in the product category. When a substitute product's price increase, it can be expected that product to lose a part of its demand. That lost demand may switch to substitute products. If a substitute product's price decrease, it can be expected that product to gain extra demand. That extra demand may come from other substitute products. It is expected to have a positive sign, because price of the substitute product and demand of the focal product have positive correlation.

Competitor Cross-Price Elasticity: this is the cross-price elasticity of the focal product vs. the competitor's focal product for each of the competitors. Say there are M competitors that also sell the focal product. Then there will be M competitor cross-price elasticities to estimate. Competitor Cross-Price Elasticity is expected to have a positive sign, because of the same reasoning at Cross-Price Elasticity.

Reibstein and Gatignon, presented a model, which solves optimal prices of all products in a product line simultaneously [Reibstein and Gatignon, 1984]. In their study, they developed three different models to understand price elasticities of the products

in the product line. First model only considered own-price elasticity and did not allow any interactions between products. Second model also only considered own-price elasticity, but allowed interaction of error terms between products. Lastly, third model allowed own-price elasticity and cross-price elasticity between products which outperformed the first two models.



Chapter 3

DEMAND PREDICTION

3.1 *The Parametric Demand Prediction Model*

We present a demand prediction model, which can capture own-price elasticity, cross-price elasticity and competitor cross-price elasticity of a product. We can take the third model of [Reibstein and Gatignon, 1984] and add competitor price elasticity as an another price indicator to the model. Also, for other factors that changes demand for the product, we add a constant C which will capture all non-price related factors.

In Equation 3.1 D stands for demand for the focal product. p_f is the price of the focal product, p_i is the price of another product (product i) in the category, p_j is the price of a competitor's product, and the β s are the associated price elasticities in the Equation 3.1. The constant elasticity demand model is as follows:

$$D = C(p_f^{\beta_f})\left(\prod_{i=1}^{N-1} p_i^{\beta_i}\right)\left(\prod_{j=1}^M p_j^{\beta_j}\right) \quad (3.1)$$

Daily historical data for sales and all three types of prices are provided by the company. The goal here is to estimate the price elasticity parameters (the β s). Since the model is non-linear and have lots of exponential components, it is hard to estimate those elasticity parameters. The easiest way to do this is to convert the demand model to a linear demand model by taking the natural logarithm of both sides. This gives us the following linear-in-parameters equation:

$$\ln(D) = C' + \beta_f \ln(p_f) + \sum_{i=1}^{N-1} \beta_i \ln(p_i) + \sum_{j=1}^M \beta_j \ln(p_j) \quad (3.2)$$

Note that, in Equation 3.2 C' is a different constant than C , but that is not a problem in this case, because the main objective is to learn price elasticities. For any given product category, the parameters of this linear regression model can be estimated via a simple linear regression, which will also give us p-values of the elasticity parameter estimates to help show us which ones are statistically significant and which parameters are not significant.

There are multiple moving pieces when it comes to predicting demand. Some pieces are in control of the company such as price, promotions, advertisements, and inventory management. Some pieces are not under the company's control such as, seasonality, competitor prices, life cycle of the product, special days, and macro-economic factors. In order to capture those effects, those type of parameters has been added to the model.

3.2 The Proposed Demand Prediction Model

With the dataset provided, the following parameters are significant for predicting demand;

- Price of the product
- Price lag of the product
- Price of the product at competitors' stores
- Price of products in the same product group (potentially substitutable)
- Promos and Advertisement
- Inventory level and Stockouts
- Previous Sales Rates

The day of the week

The demand prediction model for product i is the following:

$$\ln(D_i^t) = \alpha_0 + \alpha_1 \ln(D_i^{t-1}) + \alpha_2 \ln(D_i^{t-7}) + \alpha_3 \ln(I_i^t) + \alpha_5 \ln(p_i^{comp}) + \alpha_6 \ln(d_r^t) \\ + W_t + \beta_i \ln(P_i^t) + \gamma \ln(p_i^{ref})$$

3.2.1 Parameters of The Proposed Demand Prediction Model

Parameters for the proposed demand prediction model are discussed below:

D_i^t : Demand of product i at period t. This is the dependent variable of the proposed forecast model.

D_i^{t-1} : Demand of product i at period t-1. This is one of the parameters used to capture sales trend effect. The α_1 parameter is expected to be positive, because when there is a sales increase in the recent days, it can be expected to have larger sales in the following days if other parameters remain same.

D_i^{t-7} : Demand of product i at period t-7. This is one of the parameters used to capture sales trend effect. The α_2 parameter is expected to be positive, because it can be expected to have similar sales quantities between one day and the same day one week ago if other parameters remain same.

I_i^t : Inventory Availability for product i at period t. One of the most key factor for a retailer is inventory effect. How many stores and which stores have inventory is a key factor at retailers. In order to capture inventory effect, inventory availability of a product is used in this study rather than using total inventory amount directly. There are more than 50 stores across Turkey. The distribution of the products in the stores makes enormous difference. Let's assume we have identical demand for

a product from all stores across Turkey. Distributing all products to only one store would not be a good, but distributing them into n stores, would offer more supply, so that we will have a bigger chance to sell more products. So, the approach is finding for what percent of the demand is supplied with inventory. The inventory data have the levels of SKU, store and date. For each product group (97 PG are available) what percent of the sales occurred at each store (59 stores) is calculated. Thus, for each day, the weighted average of inventory availability for each product is calculated. It can be expected to have a positive parameter for α_4 parameter, supplying in more stores can increase the sales quantity numbers as it would decrease lost sales. Inventory availability for a product is bounded between 0 and 1. 0 denotes there are no inventory and 1 denotes there are sufficient inventory at all stores.

$p_{i,t}^{comp}$: Competitor-price elasticity. The data shows minimum competitor price for product i at time t . With this parameter, competitor price effect on the sales can be captured. In the dataset there are three competitor firm's price data. But, most of the days at most one competitor firm's price information is visible due to some data provider problems. For each day, by taking minimum competitor price out of these three competitor firms, new column named, 'Minimum of Competitor Price' can be calculated easily. So, with this parameter, the impact of minimum market price against the company's price can be captured. The α_5 parameter is expected to be positive, as when minimum of competitor price gets smaller, a negative impact on our sales can be expected.

d_i^t : Promotion rate for product i at period t . Besides regular price changes, the company sometimes makes some special promotions for their products. They can announce these types of promotions via TV advertisements, flyers, internet advertisements, and some other alternative channels. So, these types of promotions attract more attention than regular price changes. As expected these promotions have different quantities on price. To capture that promotion effect, promotion ratio is

subtracted from 1. For example, if promotion is 20 % on a product, it is used as 80% because product is being sold at 80% of its previous price. The α_6 parameter is expected to be negative as discount ratio gets smaller, promotion becomes deeper and that would have a positive impact on the sales of the product.

W_t : Weekday indicator. This is a dummy variable in the dataset, showing on which day sales data has been recorded. This indicator is necessary because, the weekday is a major effect on a customer's behavior, since most of the customers have days-off from their job on the weekends. Weekends is expected to have a positive impact if we compare them to workdays.

P_i^t : Own-price elasticity. Data shows the average sales price of the product i at all stores at period t . This is one of the most crucial parameters in demand prediction model, since optimizing the price of the product is our objective in this research. So, getting correct parameters for own-price elasticity is one of the most crucial steps here. The β parameter is expected to be negative, as the price of the product goes down, expected sales would go up. The β parameter is discussed more detailed under single product optimization section.

$P_{i,t}^{ref}$: Cross-price elasticity. Data shows weighted average sales price of the all other products rather than i at all stores at period t . In other words, for each product a reference price is calculated. A study on durable goods shows that usage of substitutable products' prices is a useful reference price for a product [Mazumdar et al., 2005]. In these weights, similarities between two product pairs is used. Attribute data of all products like price, brand, specific features of the product, or average sales quantity for each day are provided by the company. The similarity between each product pair is calculated to find which product are more likely to be a competitor to each other. The assumption here is more similar product are more likely to be each other's substitute. To be clearer on the calculation, reference price for the focal product is calculated.

Reference price is calculated by taking weighted average of the other products by using this similarity calculation. The γ parameter is expected to be positive, as when substitutable products' price get smaller, a negative impact on the focal product's sales is expected.

3.3 Profit Functions

3.3.1 Profit Functions Single Product Case

After calculating sales parameters of demand estimation model, they are ready to be implemented into the price optimization models. The first model to be proposed is the single product optimization. Objective function is maximizing the expected profit for a single product. Where,

$$\Pi_i = (p_i - c_i) D_i$$

Π_i is the profit from product i,

p_i is the price of product i,

c_i is the cost of the product i,

D_i is the demand of product i.

The non-price related attributes of the prediction demand model can be called as E for simplicity. Then by taking the derivative of the function with respect to p_i ; following solution has been reached.

$$\Pi = D_i * (p_i - c_i) \tag{3.3}$$

Calling other parameters rather than price as E_i

$$\Pi = (E_i p_i^{\beta_i}) * (p_i - c_i) \tag{3.4}$$

Rewrite

$$\Pi = (E_i p_i^{\beta_i + 1}) - (E_i c_i p_i^{\beta_i}) \tag{3.5}$$

First derivative with respect to p_i

$$\frac{\partial \Pi}{\partial p_i} = (\beta_i + 1)E_i p_i^{\beta_i} - c_i(\beta_i)E_i p_i^{\beta_i - 1} \quad (3.6)$$

Second derivative with respect to p_i

$$\frac{\partial^2 \Pi}{\partial p_i^2} = (\beta_i)(\beta_i + 1)E_i p_i^{\beta_i - 1} - (\beta_i)(\beta_i - 1)E_i c_i p_i^{\beta_i - 2} \quad (3.7)$$

Rewrite

$$\frac{\partial^2 \Pi}{\partial p_i^2} = \beta_i E_i p_i^{\beta_i - 2} [(\beta_i + 1)p_i - c_i(\beta_i - 1)] \quad (3.8)$$

Second derivative value depends on β_i , p_i and c_i

Letting first order derivative equal to zero

$$(\beta_i + 1)E_i p_i^{\beta_i} - c_i \beta_i E_i p_i^{\beta_i - 1} = 0 \quad (3.9)$$

$$(\beta_i + 1)E_i p_i^{\beta_i} = c_i \beta_i E_i p_i^{\beta_i - 1} \quad (3.10)$$

$$(\beta_i + 1)p_i = c_i \beta_i \quad (3.11)$$

We are left with:

$$p_i^* = \frac{\beta_i c_i}{\beta_i + 1} \quad (3.12)$$

For a single product solution, the second order derivative value depends on the β_i (the own price elasticity of product i) and c_i (the cost of product i). The company did not supply the cost data of the products. But under normal circumstances, we are told the cost of a product between from 60% to 80% of average sales product of the product. The cost of a product is a positive and a fixed value. So we can treat cost as a fixed, positive number. But the own price elasticity is estimated from the demand estimation model. So that, checking how the single product profit function behaves with different own price elasticity parameters is important for this study. We investigate, how this solution gives a result under different β values.

$$\beta \geq 0 : c_i > p_i^* = \frac{\beta * c_i}{1 + \beta} \geq 0 \quad (3.13)$$

For β values which are greater than or equal to 0, this formulation does not give optimal prices. If we examine extreme ends which are 0 and infinity, we can easily see that, for β is equal to 0, the solution is zero and for β is equal to infinity, solution is converging to c_i . For a middle value like β is equal to 1, optimal solution is $c_i/2$. We can say that, in this range no solution of this formulation is optimal, as all solutions are smaller than cost.

$$0 < \beta \leq -1 : 0 > p_i^* = \frac{\beta * c_i}{1 + \beta} > -\infty \quad (3.14)$$

For β values which are between zero and minus one, the formulation does not give optimal price as solution of the formulation gives a negative value. For example, if β is equal to -0.5, solution of the formulation is equal to $-c_i$. For β is equal to -1, the solution is undefined as divisor is equal to zero.

$$\beta < -1 : \infty > p_i^* = \frac{\beta * c_i}{1 + \beta} > 0 \quad (3.15)$$

For β values which are strictly less than -1, formulation gives optimal results. For example for β value is equal to -2, optimal solution is $2*c_i$, which seems to be a feasible solution. We can show that optimal price is equal to $2*c_i$ when c_i is equal to 10 and β is equal to -2 at Figure 3.1.

Specifically, the β parameter changes the shape of the graph. Below the profit graphs can be seen when cost is equal to 80% of average sales price and when profit equation has different price elasticity parameter. For the β values bigger than 0 (See Graphs 3.3.1, 3.3.2, 3.3.3), then the function is not unimodal, it is converging to infinity. But when the β parameter value is smaller than -1, then the profit functions are unimodal which can be optimized with known methods. A value smaller than -1, this function would have a maxima point with a more reasonable point. (See Graph 3.3.4)

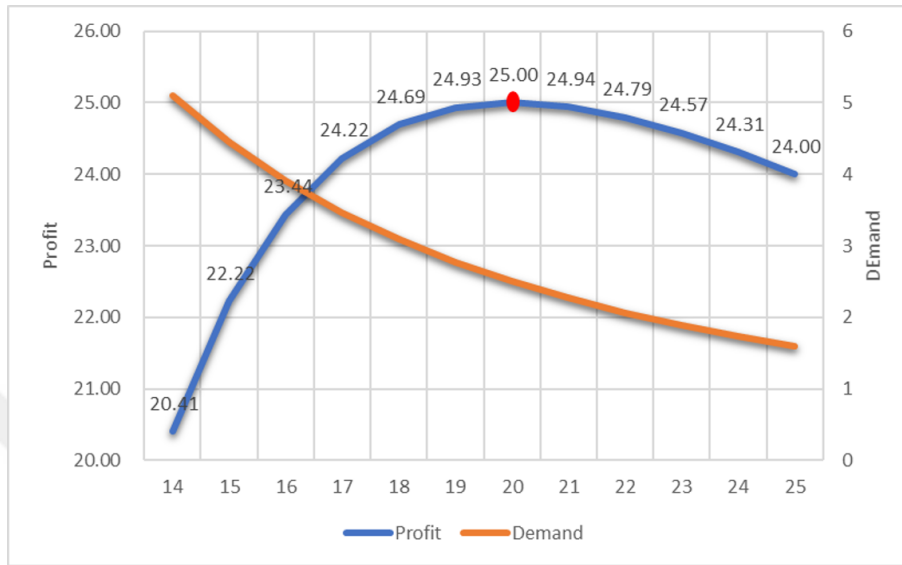


Figure 3.1: Profit curve for a product with $\beta = -2$ and $c_i=10$: Price at x axis, Profit on primary y axis and Demand at secondary y axis

Analytical interpretation of the graph and the solution found by the first derivative gives the same result. Which means, using the first derivative of the function gives optimal results if the function is unimodal.

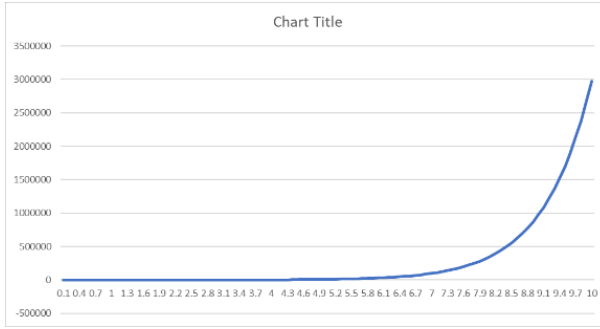


Figure 3.3.1: $\beta = 1$: Price at x axis, Profit on primary y axis

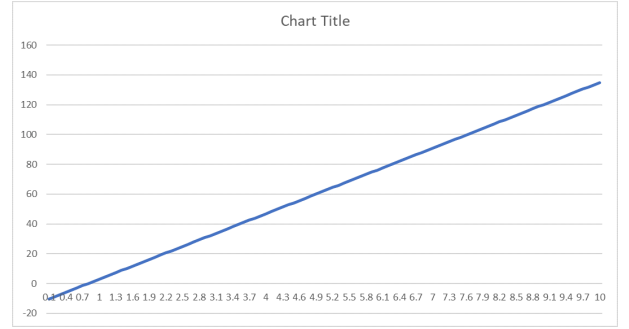


Figure 3.3.2: $\beta = 0$: Price at x axis, Profit on primary y axis

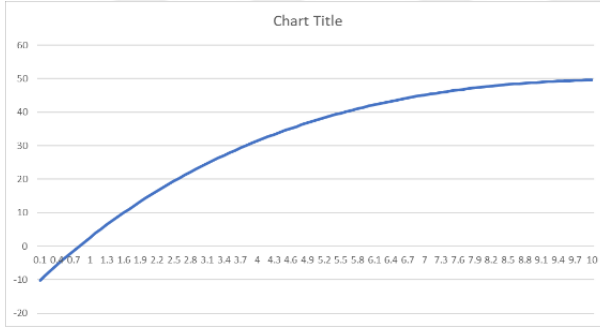


Figure 3.3.3: $\beta = -0.1$: Price at x axis, Profit on primary y axis

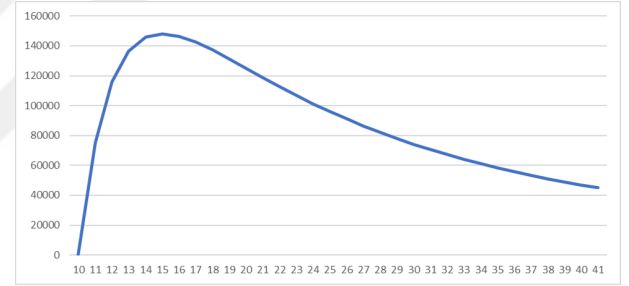


Figure 3.3.3: $\beta = -5$: Price at x axis, Profit on primary y axis

Figure 3.3: Profit curves for a product with different β values

3.3.2 Profit Functions $N=2$ Case

To get a better understanding of the multi product function, taking a closer look to the case where there are two products to optimize would be the first step. First, the profit function should be rewritten for two products. Each product's price affects other product's demand, and this is called the cross-price elasticity effect. The cross-price elasticity effect is the major difference between the single-product optimization and the two-product optimization. In addition to the β parameter, the γ parameter is introduced which stands for parameter values of the cross-price elasticity. For simplicity, we consider there are no other products in this environment. So, reference price of each product is equal to price of the other product. The non-price related attributes of the prediction demand model for each product can be called as E_1 and

E_2 for simplicity.

$$\Pi = E_1 * p_1^{\beta_1} * (p_2^\gamma) * (p_1 - c_1) + E_2 * p_2^{\beta_2} * (p_1^\gamma) * (p_2 - c_2) \quad (3.16)$$

Basically, this profit function is the summation of two unimodal functions (with some certain β values mentioned at the single product profit function section). The summation of two unimodal functions are not unimodal necessarily. Some cases are examined, so that the function behavior can be observed.

After calculating sales parameters of demand estimation model, they are ready to be implemented into the price optimization models. The first model to be proposed is the single product optimization. Objective function is maximizing the total expected profit for two products case. Where,

$$\Pi = \sum_{i=1}^2 (P_i - c_i) D_i$$

Π is the total profit,

P_i is the price of product i,

c_i is the cost of the product i,

D_i is the expected demand of product i. Then by taking the derivative of the function with respect to P_i ; following solution has been reached.

$$\Pi = D_1(P_1 - c_1) + D_2(P_2 - c_2) \quad (3.17)$$

Note that, $D_1 = E_1 P_1^{\beta_1} (P_2^\gamma)$ and $D_2 = E_2 P_2^{\beta_2} (P_1^\gamma)$

$$\Pi = E_1 P_1^{\beta_1} (P_2^\gamma) (P_1 - c_1) + E_2 P_2^{\beta_2} (P_1^\gamma) (P_2 - c_2) \quad (3.18)$$

$$\Pi = E_1 (P_2^\gamma) P_1^{\beta_1+1} - c_1 E_1 (P_2^\gamma) P_1^{\beta_1} + E_2 P_2^{\beta_2} (P_1^\gamma) (P_2 - c_2) \quad (3.19)$$

Taking derivative of the function with respect to P_1

$$\frac{\partial \Pi}{\partial P_1} = (\beta_1 + 1) E_1 (P_2^\gamma) P_1^{\beta_1} - \beta_1 c_1 E_1 (P_2^\gamma) P_1^{\beta_1-1} + \gamma E_2 P_2^{\beta_2} (P_1^{\gamma-1}) (P_2 - c_2) \quad (3.20)$$

$$\frac{\partial \Pi}{\partial P_1} = (\beta_1 + 1)D_1 - \frac{\beta_1 c_1 D_1}{P_1} + \frac{\gamma(P_2 - c_2)D_2}{P_1} \quad (3.21)$$

Setting equation equal to zero. Fixing D_1 and D_2 with current P_1 and P_2 values.

$$\frac{P_1}{P_1}(\beta_1 + 1)D_1 - \frac{\beta_1 c_1 D_1}{P_1} + \frac{\gamma(P_2 - c_2)D_2}{P_1} = 0 \quad (3.22)$$

for P_1 is not equal to zero.

$$P_1^*(\beta_1 + 1)D_1 - \beta_1 c_1 D_1 + \gamma(P_2 - c_2)D_2 = 0 \quad (3.23)$$

$$P_1^*(\beta_1 + 1)D_1 = \beta_1 c_1 D_1 - \gamma(P_2 - c_2)D_2 \quad (3.24)$$

The optimal P_1^* value is given as:

$$P_1^* = \frac{\beta_1 c_1 D_1 - \gamma(P_2 - c_2)D_2}{(\beta_1 + 1)D_1} \quad (3.25)$$

The optimal P_2^* value is given as:

$$P_2^* = \frac{\beta_2 c_2 D_2 - \gamma(P_1 - c_1)D_1}{(\beta_2 + 1)D_2} \quad (3.26)$$

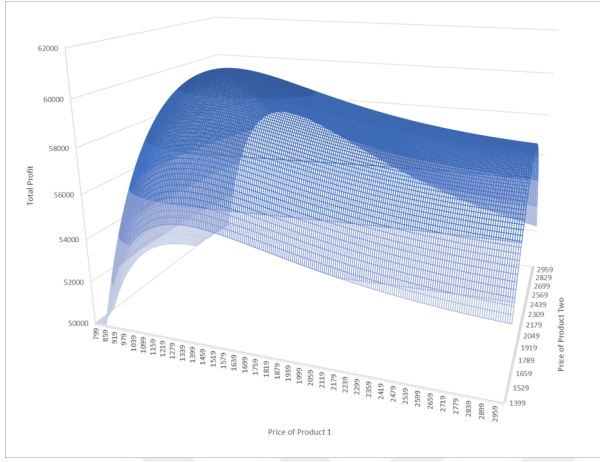
Note that, at Equation 3.22 we are fixing D_1 and D_2 values with original P_1 and P_2 values for simplicity. In order to find optimal prices with Equations 3.25 and 3.26, an iterative approach is needed, where we need to update P_1 and P_2 values with the solutions until they converge.

Table 3.1: Example Output for Two Vacuum Cleaners

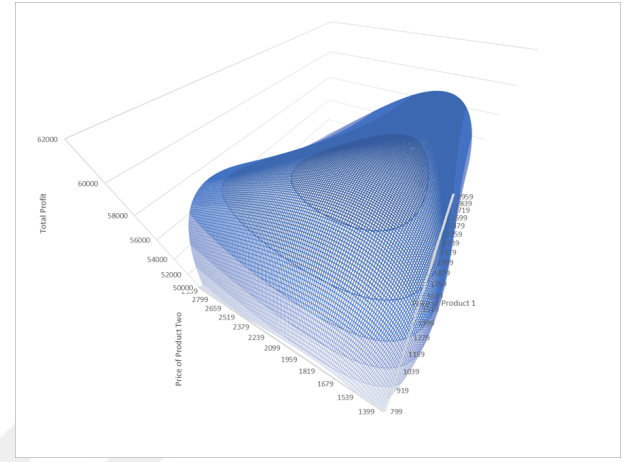
Parameter	Value
β_1	-2.14470
β_2	-2.16886
γ	0.26

Here, an output of a regression for our demand prediction model for two high quality vacuum cleaners is presented. Parameters that are coming from regression model can be seen at Table 3.1.

With these parameters it can be seen that, the own-price elasticity parameters have negative values which can be interpreted as, when the price of the product increases, the expected demand of the product decreases. The cross-price elasticity and the competitor-price elasticity parameters have positive signs, which can be interpreted as when those prices go up, expected demand of the product decreases. Also, neither cross-price elasticity nor competitor price elasticity don't have an effect as much as own-price elasticity which seems to be reasonable. When this profit function is checked analytically, there is one maxima point for this product. (Figure 3.5) It is not easy to prove, there is one maxima point for two products or multi-products case. Talluri and Van Ryzin states that, the maximum revenue of a revenue function is satisfied by a finite number of price points [Talluri and Van Ryzin, 2006]. Hanson and Martin showed that profit functions are generally nonconcave [Hanson and Martin, 1996]. But, Dong, Kouvelis and Tian showed that the profit function is concave under some circumstances [Dong et al., 2009]. So, there are different cases for different types of profit functions. But, we could observe a profit function based on a log linear demand function, would satisfy one maxima point solution under elasticity values we observed in the model.



3D Graph of N=2 Case, x-axis: Price of Product 1, y-axis: Price of Product 2, z-axis: Total Profit



3D Graph of N=2 Case, x-axis: Price of Product 1, y-axis: Price of Product 2, z-axis: Total Profit

Figure 3.5: 3D Graph of N=2 Case, x-axis: Price of Product 1, y-axis: Price of Product 2, z-axis: Total Profit

3.3.3 Profit Functions $N > 2$ Case

To extend this model to a multi-product optimization model, the formulation from [Reibstein and Gatignon, 1984] can be used. For n products the model can be written as

$$\Pi = \sum_{i=1}^n E_i * p_i^{\beta_i} * \left(\prod_{j=1, j \neq i}^n p_j^{\gamma_j} \right) * (p_i - c_i) \quad (3.27)$$

This model theoretically captures all interactions between all products. But in real life, it is not easy to learn effects of all products pair by looking at their sales history. In order to capture the effect of the cross prices, we use the idea from [Ferreira et al., 2015]. In this study, they use a reference price for each product which is the average price of the all products in a group. So, we calculate R_i as reference price for product i and rewrite the formulation.

$$\Pi = \sum_{i=1}^n E_i p_i^{\beta_i} R_i^{\gamma} (p_i - c_i) \quad (3.28)$$

The profit function can be written as:

$$\Pi = \sum_{i=1}^n [E_i R_i^{\gamma} P_i^{\beta_i+1} - c_i E_i R_i^{\gamma} P_i^{\beta_i}] + \sum_{j \neq i, j=1}^n E_j p_j^{\beta_j} R_j^{\gamma} (p_j - c_j) \quad (3.29)$$

Note that; $R_j = \sum_{k \neq j, k=1}^n \frac{P_k}{n-1}$, derivative with respect to P_i : $\frac{\partial R_j^{\gamma}}{\partial P_i} = \gamma \frac{\sum_{k \neq j, k=1}^n P_k^{\gamma-1}}{n-1}$

Taking derivative of the expression in (3.30) with respect to P_i :

$$\frac{\partial \Pi}{\partial P_i} = (\beta_i + 1) E_i (R_i^{\gamma}) P_i^{\beta_i} - \beta_i c_i E_i (R_i^{\gamma}) P_i^{\beta_i-1} + \gamma \sum_{j=1, j \neq i}^n E_j P_j^{\beta_j} (p_j - c_j) \frac{\sum_{k \neq j, k=1}^n P_k^{\gamma-1}}{n-1} \quad (3.30)$$

Also, note that $E_i (R_i^{\gamma}) P_i^{\beta_i} = E[D_i]$

$$\frac{\partial \Pi}{\partial P_i} = (\beta_i + 1) E[D_i] - \frac{\beta_i c_i E[D_i]}{P_i} + \gamma \sum_{j=1, j \neq i}^n \frac{E[D_j] (p_j - c_j)}{\frac{\sum_{k \neq j, k=1}^n P_k}{n-1}} \quad (3.31)$$

Also note that $\frac{\sum_{k \neq j, k=1}^n P_k^{\gamma-1}}{n-1} = R_j$

Setting the derivative equal to zero.

$$P_i (\beta_i + 1) E[D_i] - \beta_i c_i E[D_i] = -\gamma P_i \sum_{j=1, j \neq i}^n \frac{E[D_j] (p_j - c_j)}{R_j} \quad (3.32)$$

$$P_i (\beta_i + 1) E[D_i] = \beta_i c_i E[D_i] - \gamma P_i \sum_{j=1, j \neq i}^n \frac{E[D_j] (p_j - c_j)}{R_j} \quad (3.33)$$

$$P_i = \frac{\beta_i c_i E[D_i]}{(\beta_i + 1) E[D_i]} - \frac{\gamma P_i}{(\beta_i + 1) E[D_i]} \sum_{j=1, j \neq i}^n \frac{E[D_j] (p_j - c_j)}{R_j} \quad (3.34)$$

$$P_i^* = \frac{\beta_i c_i}{(\beta_i + 1)} - \frac{\gamma P_i}{(\beta_i + 1)E[D_i]} \sum_{j=1, j \neq i}^n \frac{E[D_j](p_j - c_j)}{R_j} \quad (3.35)$$

Unfortunately, we cannot simplify the first derivative of $N > 2$ case further than this. The solution of derivative would not help us finding the optimal solution.

3.4 Proposed Optimization Models and Iterative Algorithm

The function 3.29 has N dimensions which makes it hard to optimize. Here we propose two different approach to solve this problem. First idea is using a nonlinear integer programming. We precalculate E_i which stands for all parameters from log-linear model except own and relative price of the product. Then, we can solve the problem by finding best prices for each product. Difficulty of this formulation is, as dimension gets bigger; this formulation will be hard to solve. For example, if there are 2 products with 5 different price ladders, there are 25 different combinations for 2 products. If there are 3 product, the combinations increase to 125. There are M^N combinations if there are M different price ladders and N products. This means, there are $O(MN)$ possible combinations to consider. The second idea to solve this multi-product optimization problem is a Bisection Method method. The idea behind this algorithm is, it takes all parameters from the log-linear model as its inputs. Then for each product, the proposed algorithm optimizes its own-price by using Bisection Method. Since the single-product profit function is unimodal for the cases where $\beta < -1$ cases (own-price elasticity) as we showed on single product optimization part (3.3.1.), it is easy to find an optimal price point on the search space. So, we find optimal price for each product separately by using Bisection Method, then the relative price for each product is calculated with the results of Bisection Method. If the prices have not been changed, then it can be concluded this algorithm has converged. The convergence criteria for this algorithm is 0.01 TL. Otherwise, the algorithm is going to the previous step to re-run Bisection Method with updated prices. Convergence is not an issue for this algorithm as it searches for optimal price points for each product. So, at this step we

would not have any convergence problem. We might end up with local optimal points, but it would converge to a solution for each product. After updating the prices, the shape of single product profit function curve does not change; but the single product profit function curve shifts with different reference prices. So, in couple of steps, we observed that the algorithm always converges to a solution.

3.4.1 Nonlinear Integer Model

Here we present the nonlinear integer model to solve multi-product price optimization problem. Sets, parameters and decision variables in the model are explained below.

Sets:

I: Product Set

J: Price Ladder Set for a Product

Parameters:

$W_{i,k}$: Similarity weight of products i and k . 1 denotes full similarity and 0 denotes no similarity between two products. $\forall i \in I, \forall k \in I$ where $i \neq k$.

$P_{i,j}$: Denotes price of product i at price ladder j .

C_i : Denotes cost of product i .

E_i : Representation of other parameters of product i from the regression model.

β_i : Own price elasticity parameter of product i from the regression model.

γ : Cross price elasticity parameter for all products from the regression model.

Decision Variables:

$x_{i,j}$: $x_{i,j} \in \{1, 0\}$ indicates whether product i should be sold at price j . 1 denotes product i should be sold at price j . 0 denotes it should not be sold at price j .

$d_{i,j}$: Expected demand quantity of product i at price ladder j

R_i : Relative price of product i .

Model:

$$\max \sum_{j \in J} \sum_{i \in I} x_{i,j} * d_{i,j} * (P_{i,j} - C_i) \quad (3.36)$$

s.t.

$$d_{i,j} = E_i * (x_{i,j} * P_{i,j})^{\beta_i} * (R_i)^\gamma, \quad \forall i \in I, \quad \forall j \in J \quad (3.37)$$

$$\sum_{j \in J} x_{i,j} = 1, \quad \forall i \in I \quad (3.38)$$

$$R_k = \frac{\sum_{i \in I} (\sum_{j \in J} x_{i,j} * P_{i,j}) * W_{i,k}}{\sum_{i \in I} W_{i,k}}, \quad \forall k \in I \quad (3.39)$$

$$x_{i,j} \in \{1, 0\} \quad \forall i \in I, \quad \forall j \in J, \quad (3.40)$$

Constraints:

Equation 3.37 is the objective function of the model. It calculates the total profit made from all products.

Equation 3.38 is the expected demand equation of the model. $d_{i,j}$ is a free variable and calculates expected demand of a product under different own-prices and relative prices.

Equation 3.39 is a constraint, which ensures that each product can take only one price.

Equation 3.40 calculates relative price of each product. R_i is a free variable and shows relative price of each product.

Equation 3.40 ensures $x_{i,j}$ is a binary variable.

3.4.2 The Iterative Bisection Algorithm:

Here we present the Iterative Bisection Algorithm to solve multi-product price optimization problem. The algorithm has three steps and they are explained below.

Step 1

Initialize parameters from the proposed regression model,

Parameters:

$W_{i,k}$: Similarity weight of products i and k . 1 denotes full similarity and 0 denotes no similarity between two products. $\forall i \in I, \forall k \in I$ where $i \neq k$.

$P_{i,0}$: Denotes price of product i at initial price.

C_i : Denotes cost of product i .

E_i : Representation of other parameters of product i from the regression model.

β_i : Own price elasticity parameter of product i from the regression model.

γ : Cross price elasticity parameter for all products from the regression model.

Step 2 (For all products in the group separately)

Use Bisection Method [Burden and Faires, 1985] to obtain P_i' , calculate the new relative price R_i .

$$R_i = \frac{\sum_{k \in I} P_k' * W_{i,k}}{\sum_{k \in I} W_{i,k}}, \quad \forall i \in I$$

Step 3

By using updated P_i' values, calculated new cross-prices, re-run step 2 until prices converge.

Chapter 4

RESULTS

4.1 The Proposed Log-Linear Demand Prediction Model Results

In several categories in the company, the proposed log-linear demand prediction model is applied with the data the company has supplied. Data is SKU-day level historical sales data of all company for almost one year. Results are validated with K-fold cross validation method with K=5. This validation is made by dividing time series data into five equal data parts for each product. Further detail on this validation method can be found on Kohavi's study [Kohavi et al., 1995]. In the Table 4.1, results of k-fold validation can be found. Also in appendix Table 6.2 you can see detailed results of the proposed log-linear demand prediction model.

Table 4.1: Average R^2 and MAPE Values of K-Fold Cross Validation

Category	Average R^2 of the Train Data	Average R^2 of the Test Data	Average MAPE of Train Data	Average MAPE of Test Data
Bagless Vacuum Cleaner	81%	77%	23%	27%
Coffee Makers	70%	66%	33%	38%
External Hard Disks	77%	71%	25%	34%
Lazer IPL Epilation	68%	69%	34%	34%
Male Grooming	79%	73%	25%	30%
Portable Chargers	74%	69%	31%	35%
Steam Irons	80%	67%	25%	36%

In the Table 4.1, we can see that, training and test errors of all categories except steam irons are similar. This shows, we are not suffering from overfitting.

Here we take a closer look for vacuum cleaners product group data. It has 288 days and 18 products in the data. 80% of the dataset is used as training, and remaining

data is used as test. All signs of the parameters pass the eye test, which means the proposed demand prediction's are in the direction of our expectation.

We can argue about those coefficients as follows:

LagSales1Day: It has a positive sign which means an increase in the previous day would mean an increase in the prediction in the output of this model. So this parameter would capture the recent sales trend.

LagSales7Days: It has a positive sign which means an increase in the same day of previous week would mean an increase in the prediction in the output of this model. Note that coefficient for this predictor is smaller than LagSales1Day, which means previous day is more effective than same day of previous week.

InventoryAvailability: It has a positive sign which means an increase in the product availability across stores would mean an increase in the prediction in the output of this model.

DiscountRate: It has a negative sign which means a decrease in the discount rate would mean an increase in the prediction in the output of this model. This makes sense as we used this parameter as 1 as no discount and 0 as 100% discount in product price.

ReferancePrice: It has a positive sign which means an increase in the substitute products' price would mean an increase in the prediction in the output of this model. This makes sense as we can expect demand shifting from products which have a price increase to products which have constant prices.

MinofComp: It stands for 'Minimum of Competitor Prices'. It has a positive sign

which means an increase in the cheapest priced competitor's price would mean an increase in the prediction in the output of this model. Which makes sense as we can expect demand shifting from cheapest priced competitor to our partner.

WeekDays: All weekdays are compared to WeekDay1 which is Monday in our partner's calendar. Weekday2 is Tuesday and so on. In the appendix you can see that, the day with lowest sales number is Monday and days with highest sales is Saturday and Sunday. You can see total sales quantities by weekdays in appendix Table 6.1.

OwnPriceElasticities: All products are represented by its own unique product ID. For example first product is represented by 'PriceOfProduct1'. All products have negative sign, which mean an increase in the price would mean a decrease in the output of the model, which makes sense as we expect this behaviour from customers. If we check quantities of the coefficients they are placed between -1.94 and -3.06 which can be interpreted as, 5% price change means 9.73% and 14.89% sales change in the output of the log-linear regression model (Table 6.6).

At the Table 4.2, output of the log-linear regression model can be seen for 18 Vacuum Cleaners.

Table 4.2: Log-Linear Regression Output for 18 Vacuum Cleaners

Coefficient	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	11.21	61.15647	-1069	<2.00E-16
LagSales1Day	0.42	0.02208	19.304	<2.00E-16
LagSales7Days	0.21	0.02165	12.073	<2.00E-16
InventoryAvailability	1.41	0.30627	3.455	0.000556
DiscountRate	-5.36	0.68173	-3.847	0.000121
ReferancePrice	0.26	0.11307	42.135	<2.00E-16
MinOfComp	0.22	0.10309	35.938	<2.00E-16
weekday2	0.29	0.05742	5.018	0.000000543
weekday3	0.26	0.05737	4.569	0.00000503
weekday4	0.34	0.0585	141	8.9E-10
weekday5	0.32	0.05834	7.047	2.12E-12
weekday6	0.76	0.06089	13.758	<2.00E-16
weekday7	0.69	0.05773	8.924	<2.00E-16
PriceOfProduct1	-2.79	0.17345	-15.255	<2.00E-16
PriceOfProduct2	-2.89	0.17347	-15.416	<2.00E-16
PriceOfProduct3	-2.38	0.17347	-15.416	<2.00E-16
PriceOfProduct4	-2.14	0.17347	-15.416	<2.00E-16
PriceOfProduct5	-2.97	0.17347	-15.416	<2.00E-16
PriceOfProduct6	-2.84	0.17347	-15.416	<2.00E-16
PriceOfProduct7	-2.50	0.17347	-15.416	<2.00E-16
PriceOfProduct8	-1.94	0.17347	-15.416	<2.00E-16
PriceOfProduct9	-2.14	0.17347	-15.416	<2.00E-16
PriceOfProduct10	-2.32	0.17347	-15.416	<2.00E-16
PriceOfProduct11	-2.02	0.17347	-15.416	<2.00E-16
PriceOfProduct12	-2.68	0.17347	-15.416	<2.00E-16
PriceOfProduct13	-2.87	0.17347	-15.416	<2.00E-16
PriceOfProduct14	-3.06	0.17347	-15.416	<2.00E-16
PriceOfProduct15	-2.71	0.17347	-15.416	<2.00E-16
PriceOfProduct16	-3.23	0.17347	-15.416	<2.00E-16
PriceOfProduct17	-2.03	0.17347	-15.416	<2.00E-16
PriceOfProduct18	-2.51	0.17347	-15.416	<2.00E-16

4.2 The Nonlinear Integer Model Results

By using the parameters of the log-linear regression model of vacuum cleaners data, the proposed nonlinear integer model is tried to be solved by using MINLP solver of LINDOGlobal. LINDOGlobal guarantees to find global optimal solutions to nonlinear problems with discrete sets. The LINDOGlobal uses branch-and-cut methods to break a nonlinear problem into a list of sub-problems. By analyzing every sub-problem, LINDOGlobal shows if it found an optimal solution or not. In the dataset, the real cost of the products were not supplied; so by the insights from the the company, we took cost of the products as 60% of the current price of themselves. Minimum of the Competitor price was assumed to be same with the previous day's prices. At Table 6.3, price minimum of competitor price, Cost and other parameters' effects from the log-linear demand prediction model can be seen. For each product a discrete price set is used. Each product has two price alternatives lower than initial price, two price alternatives higher than initial product and initial price as its discrete price set. For example, if a product's initial price is 1999 TL, we use the price set 1799, 1899, 1999, 2099 and 2199 as price candidates.

The novelty of this model is, it captures the cross-price elasticity effect of each product pair separately. We calculated similarities between products pairs using the attribute data of the products. For vacuum cleaners, we took brand, price, power, average customer star points on the website and total sales quantities in the data. In the appendix you can see attribute data (Table 6.4) and calculated similarity weights of the product set (Table 6.5).

Rather than optimizing all products separately, we optimize total profit of the products. By comparing with results of grid search algorithm, we tried to ensure our algorithm converges to optimal. Limitation of this model is, when number of the products gets bigger, finding optimal solution gets harder. Thus, we could examined results until $N=5$ case. When N gets bigger than 5, in some cases the solver ends

up ending before compilation. For example, for N=18 case, the solver reaches the 10 hour time limit before compilation.

4.2.1 The Nonlinear Integer Model Results N=2 Case

For N=2 case, we tried all product pairs and the solutions of the nonlinear integer model has converged to optimal points. Here we report the most similar product pair (Product 1 and Product 2, see Table 4.3) and most selling product pair (Product 4 and Product 9, see Table 4.4).

Table 4.3: The Nonlinear Integer Model Results for N=2 Case for Product 1 and Product 2

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	Grid Search Result P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 1	2199	1.037	2199	1.031	2199	1.031	912.15	906.87	906.87
Product 2	2199	1.023	2149	1.093	2149	1.093	899.83	906.75	906.75
Total							1,811.98	1813.62	1813.62

For two similar and low selling products, model did not change the prices too much. Only Product 2's price has decreased by 50 TL, which effected 2 TL change in the profit expectation. But with this example we can see that, even a price change increase which is lower than 2%, effected the expected demand of substitutable product by 0.5%. Also model has converged to same results with grid search which tries every price alternative for each product.

For high selling products, model did change the prices of each product. Each product's price is increased by 200 TL by the optimization model solution. We can see, despite losing almost one third of the demand, model managed to find two price points which is expected to generate 6.40% more profit than initial prices. Also, the model has converged to same results with grid search which tries every price alternative for each

Table 4.4: The Nonlinear Integer Model Results for N=2 Case for Product 4 and Product 9

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	Grid Search Result P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 4	1599	50.51	1799	41.14	1799	41.14	32,078.50	34,541.14	34,541.14
Product 9	999	58.15	1199	40.55	1199	40.55	23,235.54	24,313.78	24,313.78
Total							55,314.04	58,854.92	58,854.92

product.

4.2.2 The Nonlinear Integer Model Results N=3 Case

For N=3 case, we tried with all product trio and the solutions of the nonlinear integer model has converged to optimal points. Here we report the most selling product trio (Product 4, Product 9 and Product 17, see Table 4.5).

Table 4.5: The Nonlinear Integer Model Results for N=3 Case for Product 4, Product 9 and Product 17

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	Grid Search Result P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 4	1599	48.92	1799	50.00	1799	50.00	31,291.79	33,261.59	33,261.59
Product 9	999	53.46	1199	43.83	1199	43.83	21,364.21	22,367.48	22,367.48
Product 17	749	57.88	849	57.88	849	57.88	17,342.05	18,642.54	18,642.54
Total							69,998.05	74,271.61	74,271.61

The model solution implies increasing prices of all products as much as possible is the optimal solution. Despite losing 22.90% demand, solution's expected profit is 6.11% higher than initial solution. Also, the model has converged to same results with grid search which tries every price alternative for each product.

4.2.3 The Nonlinear Integer Model Results N=5 Case

For N=5 case, the solutions of the nonlinear integer model has converged to optimal points. Here we report the top 5 selling products (Product 4, Product 9, Product 12, Product 15 and Product 17, see Table 4.6).

Table 4.6: The Nonlinear Integer Model (NLIM) Results for N=5 Case for Product 4, Product 9, Product 12, Product 15 and Product 17

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	Grid Search Result $\hat{P}$	Expected Demand with $\hat{P}$	Profit P_0	Profit P^*	Profit $\hat{P}$
Product 4	1599	49.44	1799	39.32	1799	39.32	31,619.91	33,009.71	33,009.71
Product 9	999	52.29	1199	36.02	1199	36.02	20,896.28	21,596.39	21,596.39
Product 12	1099	42.46	1099	43.87	1099	43.87	18,663.66	19,283.49	19,283.49
Product 15	699	31.47	699	32.34	699	32.34	8,800.13	9,041.43	9,041.43
Product 17	749	55.70	849	44.22	849	44.22	16,688.32	17,668.31	17,668.31
Total							96,668.30	100,599.23	100,599.23

The model solution implies, keeping prices of Product 12 and 15 same and increasing prices of the remaining products is optimal. Note that, despite losing 15.39% demand, solution's expected profit is 4.07% higher than initial solution. Also model has converged to same results with grid search which tries every price alternative for each product in the same price points.

4.3 The Iterative Bisection Algorithm Results

By using the parameters of the log-linear regression model of vacuum cleaners data, the proposed Iterative Bisection Algorithm is tried to be solved by. The dataset we used is same with we used at the nonlinear integer model's solution. The real cost of the products were not supplied; so by the insights from the the company, we took cost of the products as 60% of the current price of themselves. Minimum of the Competitor price was assumed to be same with the previous day's prices. At Table 6.3, price minimum of competitor price, cost and other parameters effects from the log-linear demand prediction model can be seen. Different than the nonlinear integer model,

each product has a continuous price set which is bounded by maximum and minimum values of the discrete sets we mentioned at 4.2.

Again this model captures the cross-price elasticity effect of each product pair separately. We calculated similarities between product pairs using the attribute data of the products. For vacuum cleaners, we took brand, price, power, average customer star points on the website and total sales quantities in the data. In the appendix you can see attribute data (Table 6.4) and calculated similarity weights of the product set (Table 6.5).

The main difference between nonlinear integer model and this iterative algorithm is the objective of the model. The nonlinear model maximizes the summation of profits of each products. On the other hand, this iterative approach maximizes each product's profit separately by using Bisection Method; then updates all products' relative price until summation of total profit converges.

4.3.1 The Iterative Bisection Algorithm Results for N=2 Case

First, N=2 case is solved. Same vacuum cleaner pairs from optimization model are presented here. Difference is, our proposed algorithm makes a continuous search between maximum and minimum allowed ranges. Solutions can be seen at Table 4.7 and Table 4.8. As we can from see both tables, both the proposed algorithm and the nonlinear interger model converged to similar points. It can be said that the proposed algorithm has converged at least a near-optimal solution. Also, it can be seen that; the proposed algorithm gives a slightly better solution than the NLIM (Nonlinear Integer Model). The reason behind this result is; the NLIM is choosing from a discrete price set as mentioned before.

From Table 4.7, we can see that nonlinear interger model results are not same with IBA (The Iterative Bisection Algorithm) results, since IBA makes Bisection Method

Table 4.7: The Iterative Bisection Algorithm (IBA) Results for Product 1 and Product 2

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	IBA P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 1	2199	1.037	2199	1.031	2184.53	1.07	912.15	906.87	925.70
Product 2	2199	1.023	2149	1.093	2129.21	1.15	899.83	906.75	931.28
Total							1,811.98	1813.62	1,856.98

Table 4.8: The Iterative Bisection Algorithm (IBA) Results for N=2 Case for Product 4 and Product 9

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	IBA P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 4	1599	50.51	1799	41.14	1799	41.14	32,078.50	34,541.14	34,541.14
Product 9	999	58.15	1199	40.55	1199	40.55	23,235.54	24,313.78	24,313.78
Total							55,314.04	58,854.92	58,854.92

on a continuous price set. Nevertheless, optimal results from both solutions are somehow similar with each other. Solution of IBA for Product 1 is 2184.53. In the discrete price set, the closest candidate is 2199 which is optimal solution for nonlinear integer model. Solution of IBA for Product 2 is 2129.21. In the discrete price set, the closest candidate is 2149 which is optimal solution for nonlinear integer model. Also total profit from IBA is slightly higher than the nonlinear integer model solution, since on a continuous price set, it is expected to find a solution which has a better objective value.

From Table 4.8, we can see that, both solutions are exactly same as both solution gives the highest allowed price for each product. We can say that, IBA gives optimal solution for this case.

4.3.2 The Iterative Bisection Algorithm Results $N=3$ Case

Then, we report the results for $N=3$ case we mentioned at 4.2.2. Results can be seen at Table 4.9.

From Table 4.9, we can see that, both solutions gives similar solutions. We can see

Table 4.9: The Iterative Bisection Algorithm (IBA) Results for $N=3$ Case for Product 4, Product 9 and Product 17

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	IBA Result P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 4	1599	48.92	1799	39.62	1,797.52	39.47	31,291.79	33,261.59	33,080.64
Product 9	999	53.46	1199	37.30	1,174.65	40.63	21,364.21	22,367.48	23,372.41
Product 17	749	57.88	849	46.65	849	46.30	17,342.05	18,642.54	18,500.53
Total							69,998.05	74,271.61	74,953.58

that, result of IBA is slightly better than NLIM solutions, a Product 4 and Product 9 have solutions which are not in the discrete price set.

4.3.3 The Iterative Bisection Algorithm Results $N=5$ Case

Here, we report the results for $N=5$ case we mentioned at 4.2.3. Results can be seen at Table 4.10.

As we can from Table 4.10, both the proposed algorithm and the nonlinear interger model converged to similar points. It can be said that the proposed algorithm has converged at least a near-optimal solution. For this product set, we can see that, IBA converged to a point with 0.04% higher profit value than NLIM solution. Again, the reasoning behind this is, NLIM is allowed to use price points from a discrete price set.

Table 4.10: The Iterative Bisection Algorithm (IBA) Results for N=5 Case for Product 4, Product 9, Product 12, Product 15 and Product 17

	P_0	Expected Demand with P_0	NLIM Result P^*	Expected Demand with P^*	IBA P^{**}	Expected Demand with P^{**}	Profit P_0	Profit P^*	Profit P^{**}
Product 4	1599	49.44	1799	39.32	1799	39.42	31,619.91	33,009.71	33,097.87
Product 9	999	52.29	1199	36.02	1199	36.11	20,896.28	21,596.39	21,649.76
Product 12	1099	42.46	1099	43.87	1139.97	39.76	18,663.66	19,283.49	19,105.06
Product 15	699	31.47	699	32.34	688.85	33.72	8,800.13	9,041.43	9,086.93
Product 17	749	55.70	849	44.22	849	44.30	16,688.32	17,668.31	17,701.88
Total							96,668.30	100,599.23	100,641.50

4.3.4 The Iterative Bisection Algorithm Results N=18 Case

In this section we report for the solutions of the Iterative Bisection Algorithm for 18 products. With the parameters from the log-linear regression, the proposed algorithm converges quickly. After 4 iterations any of 18 products' prices did not change more than 0.0001. Unfortunately, we cannot compare results with Grid Search or Nonlinear Integer Model, because problem is too hard at a N=18 case. If we have 5 different price candidate for each product, there are 5^{18} price alternative since this problem is a N^M where N is number of products and M is number of price alternative for each product. But we can compare our results with the solution with initial prices. In the Table 4.11, we can see the results of IBA for this case.

In the Table 4.11, we can see that total expected profit has increased by 5.04% as total expected demand decreased by 9.16%. IBA solution implies that, 8 product should decrease their prices and 10 products should increase. Note that, since this method works in continuous set, we do not expect no price change decisions. But the suggestions which does not end with '#9.00' are price suggestions which are within maximum and minimum allowed range and price suggestion which ends with '#9.00' are at the boundaries of the allowed ranges. An interesting point is, for some products optimal price recommendation has lower profit expectation than the initial profit

expectation. The reason behind this phenomenon is, when relative price of a product has changed even if you keep the same price, your expected profit would change. This means, even the product would have the same price, expected profit of it would be smaller than expected profit of the suggested price.



Table 4.11: The Iterative Bisection Algorithm (IBA) Results for N=18 Case

	P_0	Expected Demand with P_0	IBA Result P^*	Expected Demand with P^*	Profit P_0	Profit P^*
Product 1	2,199.00	0.92	2,058.19	0.96	805.84	709.24
Product 2	2,199.00	0.77	2,018.06	1.03	673.05	719.62
Product 3	289.00	2.56	299.49	1.18	296.28	148.79
Product 4	1,599.00	54.95	1,797.52	41.81	35,146.77	35,041.80
Product 5	799.00	1.44	724.00	1.53	460.03	374.24
Product 6	999.00	6.69	924.67	9.21	2,671.92	2,995.74
Product 7	999.00	24.54	988.32	24.50	9,804.37	9,528.54
Product 8	359.00	37.36	399.00	28.00	5,365.60	5,140.80
Product 9	999.00	49.55	1,124.00	40.52	19,800.00	21,256.79
Product 10	1,199.00	10.75	1,265.69	9.58	5,155.62	5,233.46
Product 11	1,249.00	31.05	1,449.00	21.83	15,510.45	15,272.27
Product 12	1,099.00	36.92	1,052.54	49.64	16,230.03	19,515.47
Product 13	1,299.00	2.08	1,199.00	2.94	1,080.88	1,233.62
Product 14	649.00	4.97	654.71	4.90	1,289.40	1,300.02
Product 15	699.00	27.60	712.86	27.25	7,716.17	7,996.79
Product 16	1,449.00	1.95	1,259.48	2.90	1,127.40	1,131.23
Product 17	749.00	45.86	849.00	40.93	13,740.89	16,355.63
Product 18	599.00	7.41	604.82	6.83	1,774.30	1,676.22
Total		347.37		315.54	138,649.00	145,630.25

4.4 Field Experiment

4.4.1 Experimental Design

Motivated by our results from both log-linear demand prediction model and optimization models, we wanted to test if our results can be implemented in the real life. In November 2018 we and our partner wanted to have a controlled experiment where we can change price of a group coffee makers and man groomers at some specific stores. Remaining stores would be managed business as usual. As mentioned in the company information section (1.1), our partner has 55 active stores all around Turkey. We agreed that, we would change prices of 6 stores. From quantity perspective, those 6 stores makes 18.48% of all sales for the company. From revenue perspective, those 6 stores makes 18.74% of all revenue for the company.

The goal of this experiment was testing if our assumptions on price elasticities are true or not. So, in this experiment, we used the results of demand prediction model and tried to identify if a product is elastic or inelastic. For products, we identified as elastic, we suggested pricing 10% lower at 6 experiment stores than other stores at all time. For products, we identified as inelastic, we suggested pricing 10% higher at 6 experiment stores than other stores at all time.

4.4.2 Experiment Results

Unfortunately, the experiment was neither a success nor fail. 13 of our suggestions were removed from the test list because they were labelled as 'Image Setters'. As we mentioned on section 1.1. our partner's strategy was offering minimum price at the market and this type pricing at highly competitive products would make some customers unhappy. 11 of our suggestions stocked-out and made no significant sales at the test period. So we could not make any analysis on those 24 products.

There were 7 products for which we could make significant analysis. At appendix you can find the detailed results (Table 6.7). We will call 4 weeks for test as Test Period and previous 4 weeks as PreTest period from now on. Also note that, since we do not

know the cost information of the products; we will consider cost of a product is equal to 60% of the average sales price of the control group for profit calculations . We can discuss the results as follows:

Product 1: We suggested pricing 10% lower than regular price. In test, we observed 8.46% price difference between test and control group, which is in acceptable range. At the end of the test, we observed 38% sales quantity, 25.8% revenue and 3.4% profit increase for the product. Gillespie [Gillespie, 2014] defines elasticity as, percentage change in quantity demanded divided by percentage change in the price of product. So for this product we can calculate point price elasticity as -4.43 which is defined as elastic for Gillespie's definition. We should not forget, cross-price elasticity or competitor price elasticity were not taken into consideration. This product can be counted as a successful pricing decision.

Product 2: We suggested pricing 10% higher than regular price for this product. In test we observed 9.71% lower pricing than the control group. But still, we observed 36% sales decrease for this product. There is a high chance of wholesale for this product. Unfortunately, this product is out of our analysis.

Product 3: We suggested pricing 10% higher than regular price for this product. In test we observed 26.60% lower pricing than the control group. But still, we observed 364% sales increase for this product. There is a high chance of wholesale for this product. Unfortunately, this product is out of our analysis.

Product 4: We suggested pricing 10% higher than regular price for this product. In test, we observed 9.49% difference between test and control group, which is in acceptable range. At the end of the test, we observed 4% sales quantity, 14% revenue and 77% profit increase in the product. Customer seems like, they did not noticed the price increase for the product. So for this product we can calculate point price

elasticity as 0.43 which is defined as inelastic for Gillespie's definition. We should not forget, cross-price elasticity or competitor price elasticity were not taken into consideration. This product can be counted as a successful pricing decision.

Product 5: We suggested pricing 10% higher than regular price for this product. In test, we observed 7.38% difference between test and control group, which is in acceptable range. At the end of the test, we observed 45% sales quantity, 55% revenue and 53% profit increase in the product. Customer seems like, they did not noticed the price increase for the product. So for this product we can calculate point price elasticity as 10.78 which is defined as inelastic for Gillespie's definition. This product can be counted as a successful pricing decision.

Product 6: We suggested pricing 10% lower than regular price for this product. In test, we observed 1.88% difference between test and control group but in other direction. This is not in acceptable range. At the end of the test, we observed 87% sales quantity, 684% revenue and 95% profit decrease in the product. Customer seems like, they noticed there is a price increase for the product. So for this product we can calculate point price elasticity as -21.99 which is defined as elastic for Gillespie's definition. But still, pricing action is in the wrong direction and the reaction is extreme. Unfortunately, this product is out of our analysis.

Product 7: We suggested pricing 10% higher than regular price for this product. In test, we observed 10.48% difference between test and control group, which is in acceptable range. At the end of the test, we observed 38% sales quantity, 31% revenue and 30% profit decrease in the product. Customer seems like, they noticed the price increase for the product. So for this product we can calculate point price elasticity as -3.53 which is defined as elastic for Gillespie's definition. This product can be counted as an unsuccessful pricing decision, but note that sales of test group is 9 and 5 for preTest and Test periods respectively. The sample size is not ideal for this product.

There are only 4 analyzable results. At 3 of them, we seem to successfully observe elasticity of the products and at 1 of them, we were unsuccessful to predict elasticity of the product. For those 4 products we observed 18% revenue increase, 26% sales increase and 30% profit increase. But from this result, it is not fair to call this experiment as a success or fail.



Chapter 5

CONCLUSION

Retail is one of the major industries all around the world and dynamics of the retail is changing with the recent technological developments. Especially, consumer electronics retail is one of the most dynamic market. Competition between manufacturers and retailers is high and with the help of internet search, it is easier for a consumer to track the competition among products and retailer. Every seller tries to catch opportunity of their own, so in a day hundreds of price changes in a store may be possible. This makes a product manager's job hard, as it is not easy to find the correct pricing decisions every day. So, a reliable demand forecasting is a crucial factor in their life. The ability of understanding sales behaviour of each product and making pricing decisions accordingly would be a important asset in profit making and competitive position in the consumer electronics market.

In this study, we develop a methodology to find optimal pricing decisions for a consumer electronics retailer. First we develop a demand prediction model which can understand a product's behaviour under different prices from its sales history data. This demand prediction model can identify cross-price elasticity and competitor-price elasticity as well as price elasticity of a product. First, we examine the demand function of the model and discuss if the optimal price point of this function can be found. In Section 4.1 we show that, for seven product categories, demand prediction model gives acceptable results. As this is a parametric model, we can see the coefficients of the own-price elasticity, cross-price elasticity and competitor-price elasticity and make a sense check if the coefficients are in the direction of our expectation or not.

In the second step, we propose two different methods to optimize total profit of multi-product case. We show that, in multi-product case there are M^N combinations if there are M products and N price ladder for each product. For small product sets, it can be optimized with the proposed nonlinear integer model. We show that, this proposed nonlinear integer model gives optimal solutions as we compare the results with a brute force solution, grid search. But, for bigger product sets, we developed an Iterative Bisection Algorithm which can find solutions near to the proposed nonlinear integer model in a short time. This supports the applicability of the methodology in the real life.

Last, we test our finding in the real world as well. In Section 4.4 we show that, by using only elasticity coefficients we had promising results. Unfortunately, the experiment was neither a success or fail. Because of some external factors we only had chance to perform an analysis on 4 products. For 3 of them, we seem to successfully observe elasticity of the products and at 1 of them, we were unsuccessful to predict elasticity of the product. For those 4 products we observed 18% revenue increase, 26% sales increase and 30% profit increase.

5.1 Future Work

There are couple of things we can consider in the future. For example, we assume that price of the product at competitors would remain the same. But the reality can be different than this case. For example a competitor may react to our price change and that can effect our demand predictions. So, predicting competitor price for different prices is a future work for us.

Second, the proposed nonlinear mixed integer model does not work fast when number of the products gets large. A linearized model can work better theoretically. But it is not possible to linearize the model without losing some precision as every product's price effects the demand of each product. So, we can use the idea from

[Ferreira et al., 2015]. Where they pre-calculated some reference prices for each product and solved sub-problems. In the appendix a draft of the model can be found (6.1).



Chapter 6

APPENDIX

6.1 Tables

Table 6.1: Total Sales Quantities of All Products by Weekdays

Weekday	Total Sales Quantity
Monday	5,270,718
Tuesday	5,020,297
Wednesday	4,960,335
Thursday	5,136,797
Friday	5,349,735
Saturday	8,203,701
Sunday	9,260,088

Table 6.2: Detailed Proposed Log-Linear Demand Prediction Model Results

K Fold	Category	R2 of the Train Data	R2 of the Test Data	MAPE of Train Data	MAPE of Test Data
1	Bagless Vacuum Cleaner	83%	74%	22%	28%
2	Bagless Vacuum Cleaner	82%	79%	19%	26%
3	Bagless Vacuum Cleaner	81%	79%	24%	24%
4	Bagless Vacuum Cleaner	81%	79%	24%	25%
5	Bagless Vacuum Cleaner	81%	74%	24%	31%
1	Coffee Makers	72%	75%	29%	31%
2	Coffee Makers	70%	70%	36%	34%
3	Coffee Makers	70%	65%	33%	41%
4	Coffee Makers	69%	59%	35%	46%
5	Coffee Makers	69%	63%	34%	38%
1	External Hard Disks	76%	79%	29%	25%
2	External Hard Disks	77%	78%	26%	25%
3	External Hard Disks	78%	57%	24%	45%
4	External Hard Disks	78%	58%	23%	43%
5	External Hard Disks	77%	71%	24%	32%
1	Lazer Ipl Epilation	66%	72%	34%	29%
2	Lazer Ipl Epilation	69%	71%	32%	34%
3	Lazer Ipl Epilation	69%	63%	34%	38%
4	Lazer Ipl Epilation	67%	74%	36%	32%
5	Lazer Ipl Epilation	68%	63%	34%	38%
1	Male Grooming	80%	80%	24%	24%
2	Male Grooming	79%	63%	24%	38%
3	Male Grooming	79%	72%	26%	30%
4	Male Grooming	80%	77%	25%	28%
5	Male Grooming	79%	74%	25%	28%
1	Portable Chargers	77%	71%	29%	34%
2	Portable Chargers	73%	62%	33%	43%
3	Portable Chargers	74%	71%	29%	34%
4	Portable Chargers	74%	73%	30%	31%
5	Portable Chargers	73%	69%	33%	35%
1	Steam Irons	80%	77%	24%	28%
2	Steam Irons	80%	77%	25%	27%
3	Steam Irons	80%	72%	24%	31%
4	Steam Irons	79%	57%	26%	47%
5	Steam Irons	79%	54%	26%	49%

Table 6.3: Price, Competitor Price, Cost and Ei of the Products

ID	Price	MinOfComp	Cost	Ei (Other Parameters Effect From the Log-Linear Prediction Model)
1	2199	2799	1319.4	290,909,372.15
2	2199	2449	1319.4	632,045,409.45
3	289	417	173.4	162,912.92
4	1599	1578	959.4	66,388,157.02
5	799	789	479.4	78,361,664.99
6	999	819	599.4	410,025,815.92
7	999	898.99	599.4	127,585,651.85
8	359	388	215.4	542,761.71
9	999	888.01	599.4	24,225,879.82
10	1199	1129.6	719.4	24,508,909.28
11	1249	1248.99	749.4	8,941,545.94
12	1099	889	659.4	985,316,302.72
13	1299	1099	779.4	328,234,774.39
14	649	599	389.4	366,321,698.61
15	699	599	419.4	263,623,751.71
16	1449	1258	869.4	5,476,504,962.36
17	749	549.9	449.4	6,103,969.13
18	599	659	359.4	10,374,372.65

Table 6.4: Attribute Dataset of Vacuum Cleaners

ID	Product Name	Average Customer Star Points(out of 5)	Brand	Power	Average Price	Total Sales Quantity
1	DYSON DC52 Total Animal 1300W 263 AW Bagless Vacuum Cleaner	4	DYSON	1300	1,711.12	994
2	DYSON DC52 Allergy 1300W 263 AW Bagless Vacuum Cleaner	4.4	DYSON	1300	1,460.08	918
3	ROWENTA RO6221 2100 W Bagless Vacuum Cleaner	4.6	ROWENTA	2100	416.27	2199
4	DYSON DC33C Origin 750 W Bagless Vacuum Cleaner	4.5	DYSON	750	1,555.94	4619
5	ROWENTA RO8324 Silence Force MultiCyclonic Yellow Bagless Vacuum Cleaner	4.3	ROWENTA	750	603.26	1340
6	ROWENTA RO8341 Silence Force MultiCyclonic Blue Bagless Vacuum Cleaner	3.5	ROWENTA	750	691.81	1869
7	ROWENTA RO8366 Silence Force MultiCyclonic Bagless Vacuum Cleaner	4.3	ROWENTA	750	886.38	2894
8	PHILIPS PowerPro Compact FC9323/07 B Class Bagless Vacuum Cleaner	3.9	PHILIPS	750	356.68	5090
9	PHILIPS Marathon Ultimate FC9919/07 A Class Bagless Vacuum Cleaner	4.8	PHILIPS	750	841.82	4364
10	PHILIPS FC9923 07 New Eco Bagless Vacuum Cleaner Titanium	4.4	PHILIPS	650	945.15	2890
11	PHILIPS Marathon Ultimate FC9924/07 A Energy Class Bagless Vacuum Cleaner	4.7	PHILIPS	650	1,077.10	5067
12	ELECTROLUX UltraCaptic ZUCANIMATR Animal Plus A Energy Class Bagless Vacuum Cleaner	4.8	ELECTROLUX	700	877.52	2319
13	ELECTROLUX UltraCaptic Zucdeluxetr 700 W Bagless Vacuum Cleaner	3.7	ELECTROLUX	700	661.19	707
14	FAKIR Veyron Turbo ko Power Bagless Vacuum Cleaner	3.4	FAKIR	850	409.27	809
15	FAKIR Veyron Turbo ko Power Bagless Vacuum Cleaner	3.4	FAKIR	850	554.76	1848
16	BOSCH BGS6PRO4 900 W Bagless Vacuum Cleaner	3.4	BOSCH	900	687.96	678
17	ARNICA Tesla Premium ET14320 Bagless Vacuum Cleaner Silver	4.7	ARNICA	750	582.22	3583
18	PHILIPS FC9532/09 PowerPro Active A Energy Class Bagless Vacuum Cleaner	4.1	PHILIPS	750	481.84	1835

Table 6.5: Similarity Weights Between Products

ProductID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	-	93%	21%	75%	28%	28%	30%	19%	27%	31%	31%	30%	29%	25%	25%	29%	23%	25%
2	93%	-	23%	76%	30%	29%	33%	19%	30%	35%	36%	33%	31%	26%	25%	31%	25%	25%
3	21%	23%	-	23%	72%	69%	66%	41%	31%	32%	27%	35%	32%	44%	41%	31%	40%	46%
4	75%	76%	23%	-	39%	40%	48%	39%	50%	36%	42%	47%	37%	18%	23%	24%	43%	37%
5	28%	30%	72%	39%	-	91%	84%	45%	49%	36%	30%	51%	58%	37%	46%	42%	58%	57%
6	28%	29%	69%	40%	91%	-	87%	43%	52%	39%	33%	55%	58%	34%	46%	46%	54%	55%
7	30%	33%	66%	48%	84%	87%	-	42%	61%	51%	42%	63%	49%	26%	35%	36%	52%	47%
8	19%	19%	41%	39%	45%	43%	42%	-	77%	60%	63%	40%	42%	39%	34%	27%	49%	84%
9	27%	30%	31%	50%	49%	52%	61%	77%	-	79%	78%	61%	49%	25%	33%	36%	54%	78%
10	31%	35%	32%	36%	36%	39%	51%	60%	79%	-	91%	61%	48%	25%	34%	34%	38%	66%
11	31%	36%	27%	42%	30%	33%	42%	63%	78%	91%	-	55%	43%	21%	28%	30%	35%	61%
12	30%	33%	35%	47%	51%	55%	63%	40%	61%	61%	55%	-	83%	26%	36%	36%	52%	48%
13	29%	31%	32%	37%	58%	58%	49%	42%	49%	48%	43%	83%	-	39%	41%	51%	53%	51%
14	25%	26%	44%	18%	37%	34%	26%	39%	25%	25%	21%	26%	39%	-	86%	52%	33%	41%
15	25%	25%	41%	23%	46%	46%	35%	34%	33%	34%	28%	36%	41%	86%	-	54%	44%	47%
16	29%	31%	31%	24%	42%	46%	36%	27%	36%	34%	30%	36%	51%	52%	54%	-	37%	35%
17	23%	25%	40%	43%	58%	54%	52%	49%	54%	38%	35%	52%	53%	33%	44%	37%	-	55%
18	25%	25%	46%	37%	57%	55%	47%	84%	78%	66%	61%	48%	51%	41%	47%	35%	55%	-

Table 6.6: Sales Change Prediction of Log Linear Model for Elasticities -1.94 and -3.06

Price	-1.94	Sales Change	-3.06	Sales Change
0.6	269.39		477.37	
0.65	230.65	-14.38%	373.67	-21.72%
0.7	199.76	-13.39%	297.85	-20.29%
0.75	174.74	-12.53%	241.16	-19.03%
0.8	154.17	-11.77%	197.95	-17.92%
0.85	137.07	-11.10%	164.43	-16.93%
0.9	122.68	-10.50%	138.04	-16.05%
0.95	110.46	-9.96%	116.99	-15.25%
1	100.00	-9.47%	100.00	-14.53%
1.05	90.97	-9.03%	86.13	-13.87%
1.1	83.12	-8.63%	74.70	-13.27%
1.15	76.25	-8.26%	65.20	-12.72%
1.2	70.21	-7.92%	57.24	-12.21%
1.25	64.86	-7.61%	50.52	-11.74%
1.3	60.11	-7.33%	44.81	-11.31%
1.35	55.87	-7.06%	39.92	-10.91%
1.4	52.06	-6.81%	35.71	-10.53%
Average	-9.73%		-14.89%	

6.2 Linear Integer model

The idea is fixing value of the relative prices for each price ladder. With fixed own and relative prices of each product, we precalculate expected demand for a product for different own and relative price levels. Then, with precalculated expected demand values, we try to solve linear integer problem.

$$\max \sum_{k \in K} \sum_{j \in J} \sum_{i \in I} x_{i,j,k} * D_{i,j,k} * (P_{i,j} - C_i) \quad (6.1)$$

s.t.

$$\sum_{k \in K} \sum_{j \in J} x_{i,j,k} = 1, \quad \forall i \in I \quad (6.2)$$

$$\sum_{k \in K} y_{i,k} = 1, \quad \forall i \in I \quad (6.3)$$

$$\sum_{k \in K} R_{i,k} * y_{i,k} \approx \frac{\sum_{k \in K} \sum_{j \in J} \sum_{r \in I, r \neq i} x_{r,j,k} * P_{r,j} * W_{r,i}}{|I| - 1}, \quad \forall i \in I \quad (6.4)$$

$$x_{i,j,k} \in \{1, 0\}, \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (6.5)$$

$$y_{i,k} \in \{1, 0\}, \quad \forall i \in I, \quad \forall k \in K \quad (6.6)$$

Sets:

I: Product Set

J: Price Ladder Set for a Product

K: Relative Price Set of a Product

Parameters:

$W_{i,r}$: Similarity weight between products i and r . 1 denotes full similarity and 0 denotes no similarity between two products. $\forall i \in I, \forall r \in I$ where $i \neq r$.

$P_{i,j}$: Denotes price of product i at price ladder j .

C_i : Denotes cost of product i .

$R_{i,k}$: Relative price of product i at relative price k .

$D_{i,j,k}$: Demand quantity of product i at price ladder j and relative price k .

β_i : Own price elasticity parameter of product i from the regression model.

γ : Cross price elasticity parameter from the regression model.

Decision Variables:

$x_{i,j,k}$: $x_{i,j,k} \in \{1, 0\}$ indicates whether product i should be sold at price j and relative price k .

$y_{i,k}$: $y_{i,k} \in \{1, 0\}$ indicates whether relative price k of is selected for product i .

Table 6.7: Field Experiment Results

Product	Price Suggestion	TEST ASP Control	TEST ASP Treatment	TEST Sales Control	TEST Sales Treatment
1	10% Lower	101.37	92.79	537	80
2	10% Higher	67.46	60.91	283	54
3	10% Higher	124.81	91.60	147	44
4	10% Higher	214.47	234.82	85	33
5	10% Higher	59.31	63.69	66	31
6	10% Lower	90.66	92.37	84	1
7	10% Higher	175.66	194.07	29	5

Product	TEST Normalized Sales Control	TEST Normalized Sales Treatment	TEST Sales Total	TEST Sales Control%	TEST Sales Treatment%
1	558.62	83.22	617	87%	13%
2	449.69	85.81	337	84%	16%
3	28.86	8.64	191	77%	23%
4	62.38	24.22	118	72%	28%
5	74.48	34.98	97	68%	32%
6	264.41	3.15	85	99%	1%
7	33.52	5.78	34	85%	15%

Product	PreTest 4 Weeks Sales Control	PreTest 4 Weeks Sales Treatment	PreTest 4 Weeks Sales Total	PreTest 4 Weeks Sales% Control	PreTest 4 Weeks Sales% Treatment
1	559	61	619	90%	10%
2	450	133	583	77%	23%
3	29	2	31	94%	6%
4	62	23	86	73%	27%
5	74	24	99	75%	25%
6	264	25	290	91%	9%
7	34	9	43	78%	22%

Product	PreTest Revenue	Test Revenue	Delta Sales Change to Treatment	Sales Change %	Revenue Change %
1	6,134.93	7,722.38	22.70	38%	25.88%
2	8,982.05	5,226.89	- 47.33	-36%	-41.81%
3	232.40	791.36	6.78	364%	240.52%
4	4,991.94	5,686.88	0.94	4%	13.92%
5	1,435.71	2,228.04	10.78	45%	55.19%
6	2,279.11	290.76	- 21.99	-87%	-87.24%
7	1,635.44	1,121.50	- 3.53	-38%	-31.43%

Product	Test Control ASP	Cost	Pre Test Treatment Profit	Test Treatment Profit	Profit Change %
1	101.37	60.82	2,473.55	2,557.51	3.39%
2	67.46	40.48	3,589.10	1,103.56	-69.25%
3	124.81	74.88	99.84	735.69	636.83%
4	214.47	128.68	1,973.11	3,502.64	77.52%
5	59.31	35.59	569.38	871.12	53.00%
6	90.66	54.40	906.64	37.97	-95.81%
7	175.66	105.40	632.37	443.37	-29.89%

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