

Electromechanical Analysis of Linear Compressor for Household Refrigerator

by

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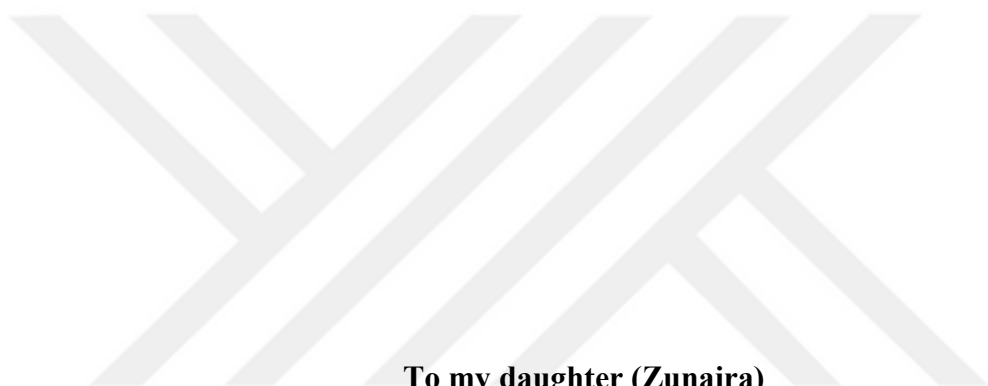
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**To my daughter (Zunaira)
and my mother**

ABSTRACT

Linear compressors provide an exquisite alternative for the conventional rotary refrigerator compressors. Linear actuators are employed in linear compressors to eradicate the need for the cumbersome rotational to linear conversion mechanisms. This research provides a detailed insight of two application specific linear actuator types: Moving iron (solenoids) and moving magnet linear actuators (MMLAs). A solenoid based linear actuator in combination with springs is proposed for linear oscillations in linear compressors. Switching control of the stator coil enables back and forth motion of the armature. Line pressure, oscillation frequency and stroke provide the baseline for the magnitude of the electromagnetic force which defines the geometric and coil parameters of the solenoid. Reluctance Method is used in this study to establish the relationship of the geometry, coil parameters and the resulting magnetic force. Considering the nonlinearity of magnetic materials, a finite element analysis is presented to estimate the magnetic force in stationary and time dependent domains for static and dynamic characteristics of the solenoid. An experimental setup is used to validate the static and dynamic characteristic responses of the actuator. Moreover, frequency response function is acquired through an impact hammer and accelerometer. The results are validated with electrically actuated solenoid at different frequencies against forces and current values to obtain optimal points of excitation frequency for better performance.

MMLAs are superior over other types for their significantly higher efficiency, high thrust density, lower losses, fast responses and smaller time constants. This research presents an analytical model to quantify the electrical as well as mechanical dynamic parameters of MMLA. In the design phase, the presented methodology provides a novel tool to analyze and conclude the design as well as the operating parameters of the actuator to achieve the desired performance. Laplace transformation is applied to obtain the transfer function of the electromechanical system. Furthermore, the equations are converted into the frequency

domain to estimate the frequency response for the velocity, stroke and coil current. The effects of the motor constant and the excitation frequency on the magnitude and phase of the dynamic parameters are estimated using Phasors. Additionally, frequency response function of the stroke to current ratio is established to find the optimal point of excitation for achieving better system efficiency. An experimental setup is designed to validate the system model. The acquired experimental results endorse the model for the effects of the motor constant and the excitation frequency.

A methodology to estimate the magnetic flux density as well as the magnetic force of MMLA is presented. Considering the simulation time of a 3D FEM software, a combination of 2D FEM with the analytical models makes this technique convenient and expeditious. With the help of the 2D FEM, the magnetic flux path is segregated into several loops. Furthermore, the loops are divided into finite zones and the mean values are represented as lines and arcs to estimate the reluctance of each magnetic flux loop. The reluctance models incorporate the effect of armature position to predict the magnetic flux density as a function of stroke positions. A numerical iterative simulation is carried out to incorporate the stator saturation with the help of Ampere's Circuit Law. Additionally, the reluctance models embody the geometric and magnetic parameters of the linear actuator including stator and armature. Consequently, the optimization of the geometric design parameters of the linear actuator is obtainable with a quick numerical simulation. Furthermore, the magnetic force is also estimated numerically with the help of work-energy method. A 3D FEM simulation is executed to compare the computation times. The presented methodology reduces the simulation time drastically.

A novel moving magnet linear actuator is proposed for linear oscillations in the linear resonant compressors for household refrigerators. This research provides stator and armature design including CAD model and geometric parameters. Furthermore, the working principle of the proposed actuator is explained. The stator assembly is composed of two reversely

wound coils, which are electrically excited with single phase AC power and oscillates the radially magnetized armature. With the help of the electromechanical analytical model, the dynamic parameters such as stroke, velocity and acceleration of the armature are derived. Additionally, the time-dependent current model of the stator winding is proposed. An experimental setup is used to validate these responses at the resonance excitation frequency with the help of sensors. The system kinetics are discussed to estimate the spring, damping, inertial and magnetic forces. A simulation is executed to estimate the time domain responses of these dynamic parameters and the effects of excitation frequency are discussed. The force models are experimentally validated at the resonance frequency excitation. In order to evaluate the performance of the proposed actuator, a comparison of the performance parameters such as efficiency, stroke, current and mass flow rate is demonstrated with the conventional rotary as well as the linear motors for linear compressor application.

ÖZET

Lineer kompresörler konvansiyonel buzdolabı kompresörlerine üstün bir alternatif sağlamaktadır. Lineer kompresörlerde lineer aktüatörler kullanılarak, dönel hareketi lineer harekete çeviren külfetli mekanizmaların gerekliliğini ortadan kaldırmaktadır. Bu araştırma iki farklı aplikasyona yönelik lineer aktüatörleri detaylı bir şekilde incelemektedir: hareketli demir (sarmal bobin) ve hareketli mıknatıs lineer aktüatörler (MMLAs). Lineer kompresördeki lineer salınım hareketi için Sarmal bobin bazlı lineer aktüatör ile birlikte yaylı bir sistem önerilmektedir. Statör bobininin aç-kapa kontrolü, armatürün ileri geri hareketini sağlamaktadır. Çıkış basıncı, salınım hareket frekansı ve strok uzunluğu elektromanyetik kuvvetin büyüklüğü için esas alınan parametreler olup, bunlar geometrik ve bobin sarım özelliklerini tanımlamaktadır. Bu özellikler ve sonucunda oluşan elektromanyetik kuvvetin arasındaki ilişki relüktans metodu kullanılarak belirlenmektedir. Sarmal bobinin statik ve dinamik özelliklerinin zamana bağlı ve zamandan bağımsız durumlardaki manyetik kuvvetlerine ulaşmak için sonlu elemanlar analizi kullanılmıştır. (kullanılan manyetik malzemelerin lineer olmayan tepkileri göz önünde bulundurulduğunda,). Aktüatörün statik ve dinamik tepkilerini doğrulamak için deneysel düzenek kurulmuş ve burada toplanan veriler kullanılmıştır. Frekansa bağlı fonksiyonu çıkarmak için darbe çekici ve ivme ölçer metodu kullanılmıştır. Elde edilen sonuçlar, elektrikle harekete geçirilmiş sarmal bobinini farklı frekanslar ile kuvvetlerin ve akım değerlerinin karşılaştırılması ile ve en iyi performans için gerekli en iyi frekans bulunmaktadır.

MMLA'lar yüksek verimlilikleri, hızlanma yoğunlukları, düşük kayıpları, hızlı tepkileri ve daha küçük zaman sabitleri ile diğer kompresörlere kıyasla daha üstündür. Bu araştırma MMLA ile ilgili dinamik elektriksel ve mekanik değerleri belirlemek için analitik bir model sunmakta. Tasarım aşamasında, aktüatörün istenen performansa ulaşması için tasarımın incelenmesi, belirlenmesi ve istenilen çalışma parametrelerinin belirlenmesi için özgün bir

yöntem sunulmaktadır. Elektromekanik sistemin transfer fonksiyonunu bulmak için Laplas dönüşü uygulanmıştır. Hız, strok ve bobin akımının frekansa bağlı değişimini incelemek için, fonksiyonlar frekans bazında incelenmiştir. Fazörler kullanılarak motor sabiti ve uyarım frekansının dinamik parametreler üzerindeki büyüklük ve faz etkileri incelenmiştir. Ek olarak, sistemin daha verimli çalışması için en iyi uyarım frekansını bulmak amacı ile frekans tepki fonksiyonunun strok ve akıma olan oranı bulunmuştur. Tüm matematiksel modeller deneysel veriler ile desteklenmiştir. Motor sabiti ve uyarım frekansının etkileri için toplanan deneysel veriler modellerin sonuçları ile örtüşmektedir.

MMLA’de manyetik akı yoğunluğu ve manyetik kuvvetleri tahmin etmek için geliştirilen bir yöntem sunulmaktadır. 3B sonlu elemanlar yöntemi ile yapılan bir analizin hesap süresi göz önünde bulundurulduğunda, 2B sonlu elemanlar ile analitik modellerin birleşimi bu yöntemi cazip kılmaktadır. 2 boyutlu sonlu elemanların yardımı ile manyetik akı yolları birkaç döngüye ayrılmaktadır. Dahası, döngüler sonlu bölgelere bölünüp ortalama değerler çizgi ve yaylarla temsil edilerek her manyetik akı döngüsünün relüktansı tahmin edilmektedir. Relüktans modeli armatür pozisyonunun etkisini de içererek, manyetik akı yoğunluğunu strok pozisyonunun fonksiyonu olarak tahmin edebilmektedir. Stator yoğunluğu da dahil etmek için Ampère yasası ile birlikte sayısal yinelemeli bir simülasyon koşulmuştur. Ek olarak, relüktans modeli hem statör hem de armatürün lineer aktüatörün geometrik ve manyetik parametrelerini içermektedir. Sonuç olarak, hızlı bir simülasyon ile lineer aktüatörün geometrik parametrelerinin optimizasyonunu sağlamak mümkün. Dahası, iş enerji metodu ile manyetik kuvvet nümerik olarak hesaplanmaktadır. 3 boyutlu sonlu elemanlar metodu ile simülasyon koşma süreleri karşılaştırılmıştır. Sunulan yöntem simülasyon süresini önemli düzeyde azaltmaktadır.

Ev tipi buzdolaplarındaki lineer rezonans kompresörlerinde lineer git-gel hareketi için hareketli mıknatıslı özgün bir lineer aktüatör sunulmaktadır. Bu çalışma statör ve armatür tasarımları ile birlikte CAD modellerini ve geometrik parametreleri sağlamaktadır. Dahası,

sunulan aktüatörün çalışma prensipleri açıklanmaktadır. Statör montajı tek fazlı AC elektrikle uyarılmış iki ters sarılmış bobinden oluşmakta ve radyal olarak manyetize olan armatürün git-gel hareketini sağlamakta. Ev Elektromekanik analitik model sayesinde, armatürün strok, hız ve akselerasyon gibi dinamik parametreleri türetilmektedir. Ek olarak, statör sarımının zamana bağlı akım modeli de sunulmuştur. Bu tepkiler rezonans frekansta, deneysel düzenek kurulup sensörlerin yardımı ile deneysel olarak onaylanmıştır. Yay, sönümleme, atalet ve manyetik kuvvetlerin tahmini için sistemin kinematikleri tartışılmaktadır. Bu dinamik parametrelerin zaman alanında tepkisini tahmin etmek için bir simülasyon koşulmuş ve uyarılma frekansı üzerindeki etkileri tartışılmıştır. Kuvvet modelleri rezonans frekansta deneysel olarak doğrulanmıştır. Lineer kompresör uygulamasında lineer aktüatörün performansını değerlendirmek için, lineer kompresör ile birlikte konvansiyonel döner kompresörün; verim, strok, akım, debi gibi performans parametrelerinin karşılaştırması sunulmuştur.

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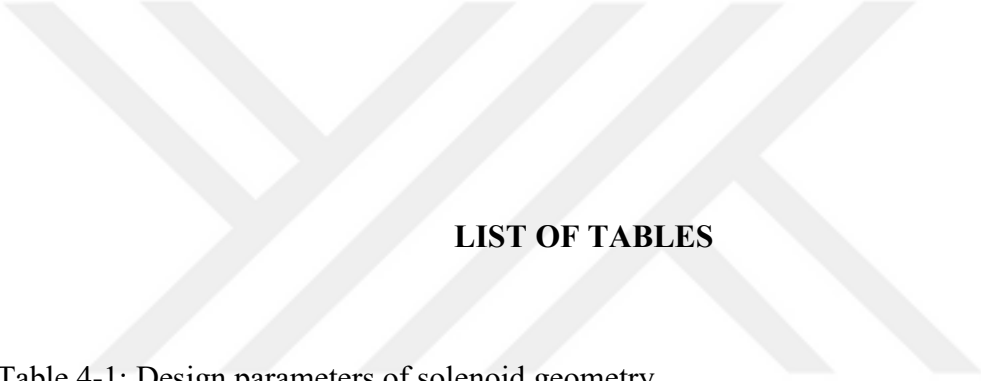
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Chapter 1

LINEAR COMPRESSORS FOR HOUSEHOLD REFRIGERATOR

1.1 Overview

The refrigerators are one of the most extensively used domestic appliances nowadays. They maintain the temperature of the substances under the environmental temperature. The most renowned refrigerant is water, ice and air. Initially, the food was preserved by freezing it with ice. With the discovery of microbes, it was realized the growth of these microbes is very lethargic at lower temperatures. The first mechanical refrigeration was built in 1860 and in early 80's the Ammonia compressor was developed [1]. With the availability of the electricity, usage of the refrigerators started commercially. Today, a refrigerator is considered as a necessity and used in almost every house. For instance, the domestic use of the compressor in Europe consumes 14-15% of the electrical energy. In Switzerland, the food cooling units are consuming 2500 GWh/A [2]. China exported to the US more than 30 Million compressors in the year 2013 [3] and the export value of 5000 Million USD [3].

1.2 Refrigeration Cycle

The refrigerator is based on the refrigeration cycle or the reverse Carnot cycle as shown in Figure 1.1. The overall process is categorized into four stages such as the evaporation, the compression, the condensation and the expansion stage [4]. As shown in the figure, the vertical dotted black lines divide the cycle into two halves: the high-pressure side and low-pressure side. The expansion and compression change the pressure state of the refrigerant as well as the temperature. Whereas, the evaporation and condensation change the material state of the refrigerant as a liquid to gas or gas to liquid.

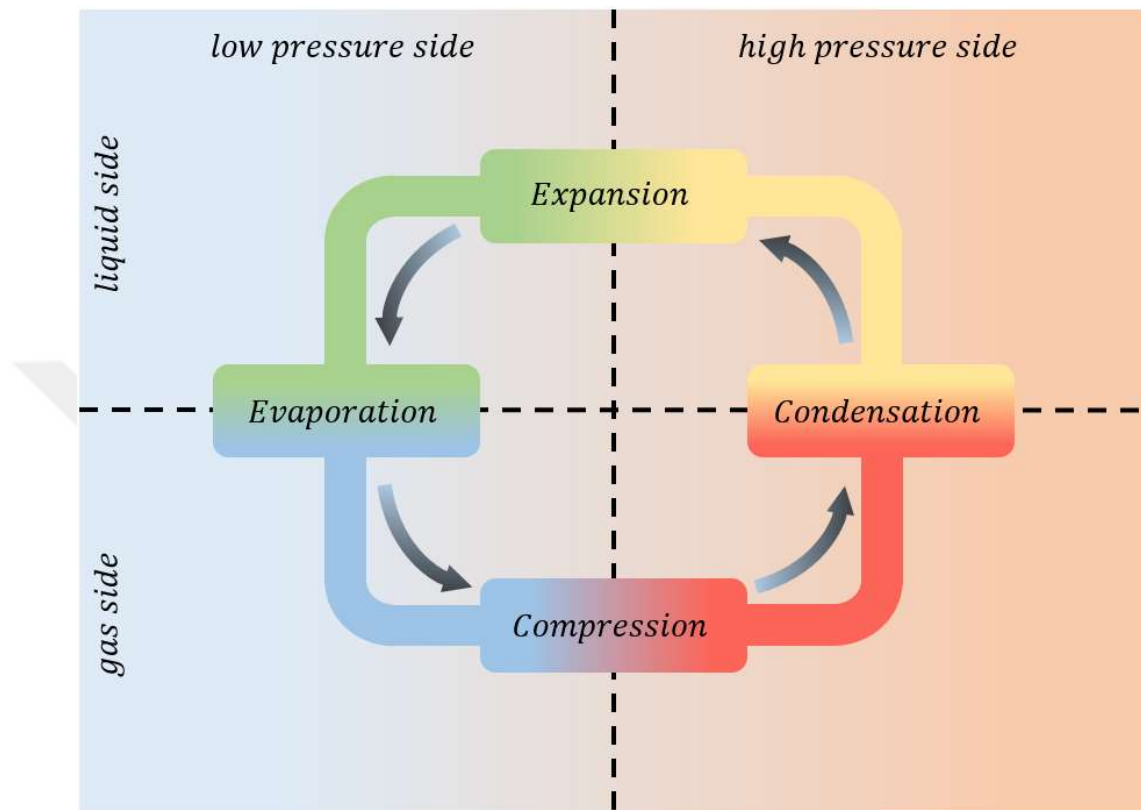


Figure 1.1: The stages of the refrigeration cycle or reverse Carnot cycle

The refrigerant after absorbing heat from the refrigerator, it evaporates to a vapor state. Consequently, the cooling is created in the refrigerator. The compressor takes the input from the output of the evaporator and compresses the gas to increase the pressure of the refrigerant. Therefore, the temperature of the refrigerant is increased. Furthermore, in the evaporation stage, the refrigerant is passed through the radiator ducts to cool it down. The heat absorbed from the refrigerant is transferred to the air or surroundings. These processes result in a high pressurized liquid phase of the refrigerant which should be at normalized pressure to go in the evaporator chamber easily. Therefore, an expansion valve is used and the refrigerant pressure is reduced and in the liquid form it goes to the evaporator again and the process continues.

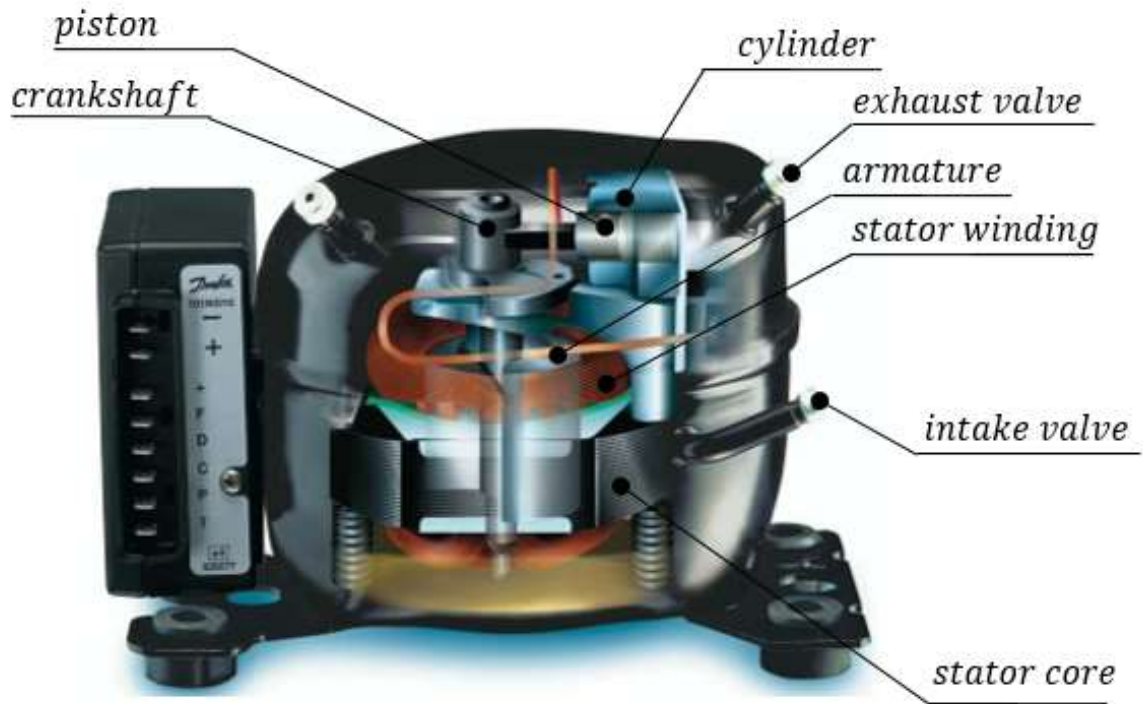


Figure 1.2: The conventional refrigerator compressor with a rotary motor and crank piston mechanism [5]

The compressor plays a vital role in the refrigeration cycle. A conventional rotary compressor is shown in Figure 1.2. It has a rotary motor which is composed of a stator and permanent magnet based armature. The motor rotates the armature which rotates the crankshaft. This mechanism transforms the rotational motion into the linear motion. The crankshaft is connected to the piston through the connecting rod. The crankshaft rotates continuously whereas the piston oscillates from the top dead center (TDC) to the bottom dead center (BDC). When the piston travels from TDC to BDC, the intake valve opens and it takes the cold low-pressurized gas inside the cylinder. Whereas, the exhaust valve is closed at this time. Conversely, when the piston is traveling from the BDC to TDC, the intake valve is closed and the pressure of the refrigerant increases, consequently the exhaust valve opens.

Therefore, a high-temperature high-pressure refrigerant in the gas form is collected in the exhaust line.

1.3 Why Linear Compressor?

Following are the main reasons of the motivation for the invention and growing interest of researchers in this area:

- Save global energy
- Less mechanical parts, lesser friction
- Reduce noise and vibration
- Eradicating the need for lubricants

Unlike conventional compressors, linear compressors use a linear motor with a free piston, instead of a rotary motor and a piston connected to the crankshaft. Consequently, there is just a linear force acting on a free piston and no force is acting in the cross direction. Hence, by using a linear compressor, the side forces are eliminated and no component of the force is wasted. Secondly, the capacity control algorithm is used to control the efficiency of the linear compressor. Instead of using a compressor at full capacity all the time, its capacity is controlled based on the environment and need. With the help of these two features, linear compressors are playing a vital role in conserving the energy. Almost 20~40% better efficiency is claimed by some linear compressor due to the advantages [6].

In linear compressors, there are less mechanical parts than the rotational compressor. There are four friction joints in the conventional compressor such as the armature bearings, the crankshaft, the connecting rod joint and the piston cylinder contact surface. These joints are the source of noise in the mechanical system. As the compressor is energized on the frequencies around 50 Hz, the parts connected with the piston oscillate and get the reaction forces. In the linear compressor, a free piston is connected to the armature of the linear motor

directly and without additional mechanisms or joints, the linear motion is transferred. Therefore, less mechanical losses are generated and more energy is conserved.

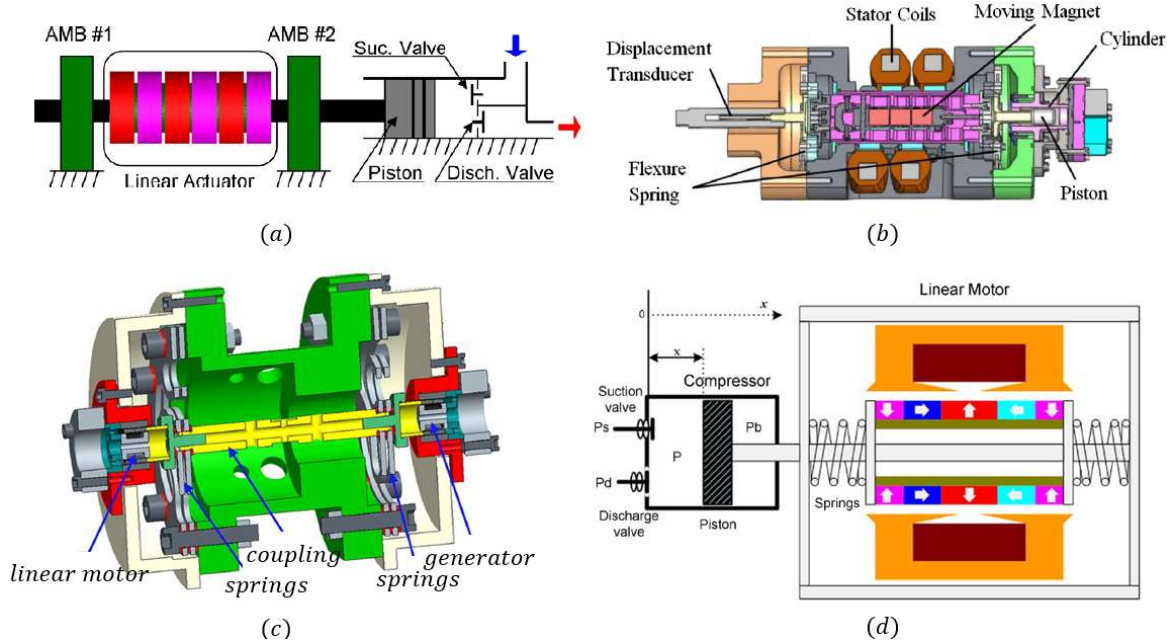


Figure 1.3: Already developed linear compressors (a) magnetically levitated linear compressor [7] (b) toroidal motor linear compressor [8] (c) small moving magnet linear actuator for high-frequency linear compressor [28] (d) Halbach magnetized linear compressor [10]

There are already developed linear compressors with different motor designs and configuration as shown in Figure 1.3. However, most of them are working on the mechanical resonance excitation frequency which will be discussed in Chapter 5. Additionally, already developed linear motor designs will be discussed in Chapter 3. Considering the large-scale utilization of the refrigerators, the power consumption of a compressor plays a vital role in energy savings. Figure 1.4 demonstrates the reduction in power consumption in different compressor configuration [6]. For instance, replacing the crank-driven compressor with the linear compressor results in 24% energy saving per month as shown in the figure. With

further modifications in the refrigeration cycle, the power consumption reduces to 47% compared to the conventional compressor. Capacity modulation algorithms even provide lesser input powers due to a controlled cooling capacity of a linear compressor.

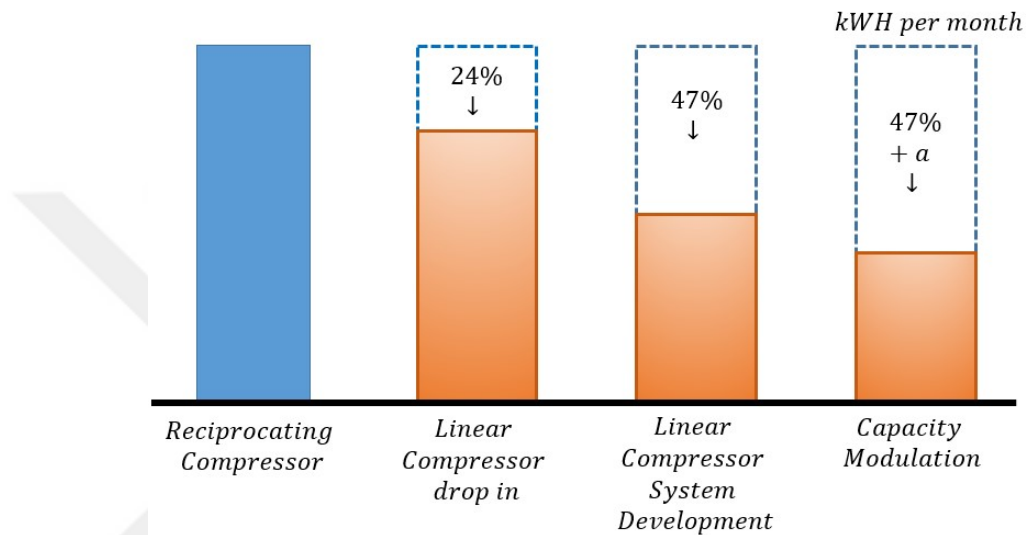


Figure 1.4: Electricity comparison of refrigerator linear compressor with conventional rotary compressors [6]

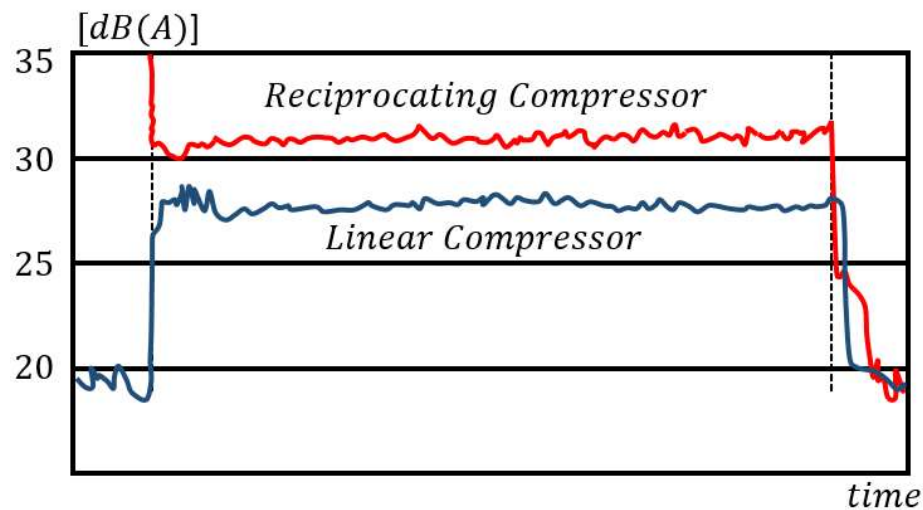


Figure 1.5: Noise level comparison of linear compressor with conventional rotary compressors [6]

Figure 1.5 presents the noise level comparison of the conventional compressor with the linear compressor [6]. The noise levels of the linear compressors are significantly lower than the conventional compressor. This is mainly because of the less number of parts in the linear compressors resulting in fewer friction points.

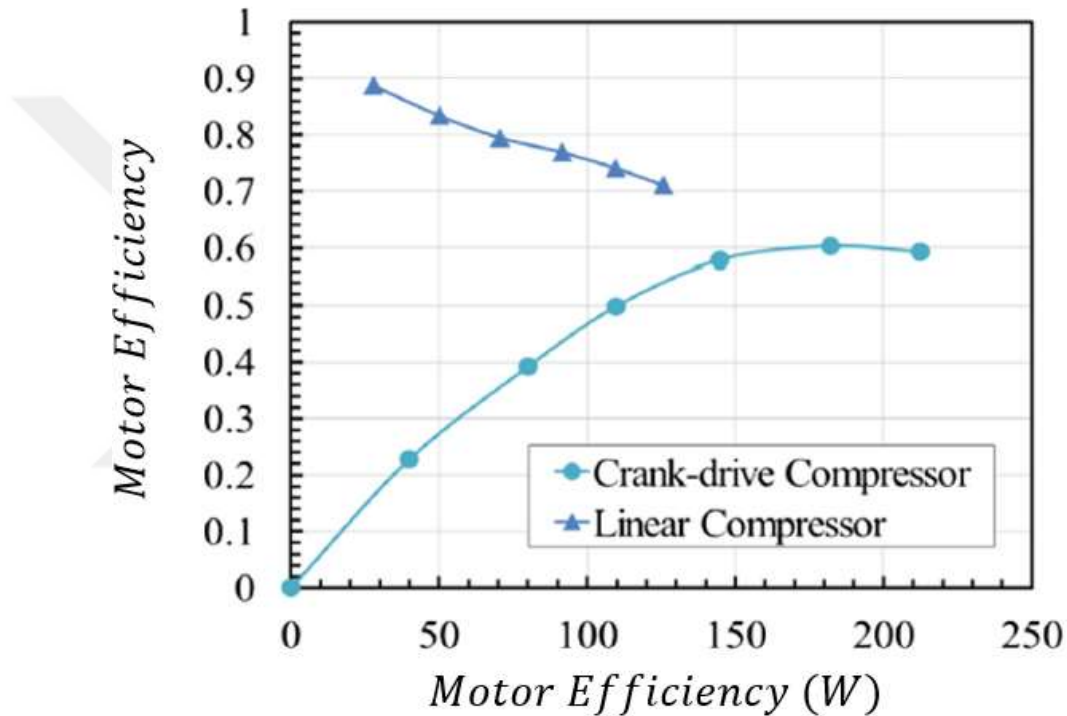


Figure 1.6: Motor efficiency comparison of the conventional crank-driven and linear compressor [11]

[11] provided a comparative study of the efficiency of a linear motor compared to the induction motor used in compressors. As shown in Figure 1.6, the efficiency of a linear compressor is higher than the conventional crank-driven compressors. The conventional compressors are using induction motors and high amount of power input is required to generate the required power output of the compressor. Whereas, linear compressors require lesser input power for the same amount of work. Generally, the efficiency of the motors

decreases as the input power increase due to higher core losses at higher power inputs. Therefore, using linear compressors not only play a vital role in energy savings, they also provide better efficiency of linear motors as seen in the figure.



Chapter 2

ELECTROMAGNETISM

2.1 Introduction

This chapter provides a general overview of the concepts related to the magnetism. Magnetism is a physical phenomenon that generates a force of attraction or repulsion between two magnetized bodies. When a body is magnetized, two poles are created commonly referred as the north pole and the south pole. There are imaginary magnetic flux lines flowing from the south pole to the north pole inside the body and the north pole to the south pole outside the body. If two magnetized bodies are brought closer to one another, a force of attraction or repulsion is created based on the facing poles of the bodies. Opposite poles attract each other and similar poles repel each other. This attraction and repulsion forces in magnetized objects are used in several applications such as motors, pumps, sensors. The amount of the magnetic flux ϕ Webers passing through a unit surface area is known as magnetic flux density B Webers/m². The expression for the magnetic flux density is given by Equation (2-1):

$$\phi = \int B \, ds \quad (2-1)$$

where s is the surface through which the flux is passing.

The magnetism is commonly categorized as temporary or permanent magnetism. When a magnetized object is brought closer to a non-magnetized object capable of being magnetized, the magnetic flux lines start flowing through the non-magnetized object and temporary poles are created on the non-magnetized object. If the magnetization source is removed, the second object is demagnetized till the time all the stored magnetic energy is

released. Faraday's law estimates the electromotive force induced by the change in magnetic flux with respect to the time as shown in Equation (2-2).

$$emf = -N \frac{d\phi}{dt} \quad (2-2)$$

where N is the number of turns of the coil. The negative sign comes from the Lenz's law. It states that the induced voltage tries to oppose the cause which created it. When the current flows through a wire, it creates a magnetic field around it. This magnetic field is estimated by Biot-Savart Law [12] as shown in Equation (2-3) and Figure 2.1(a):

$$dB = \frac{\mu_0 I dL \times \hat{r}}{4\pi r^2} \quad (2-3)$$

where μ_0 is the permeability of free space, dL is a small differential length of the wire, I is the current passing through the wire, $|r|$ is the magnitude of the radial distance of the point of measured magnetic field and $\hat{r}$ is the unit vector that represents the direction.

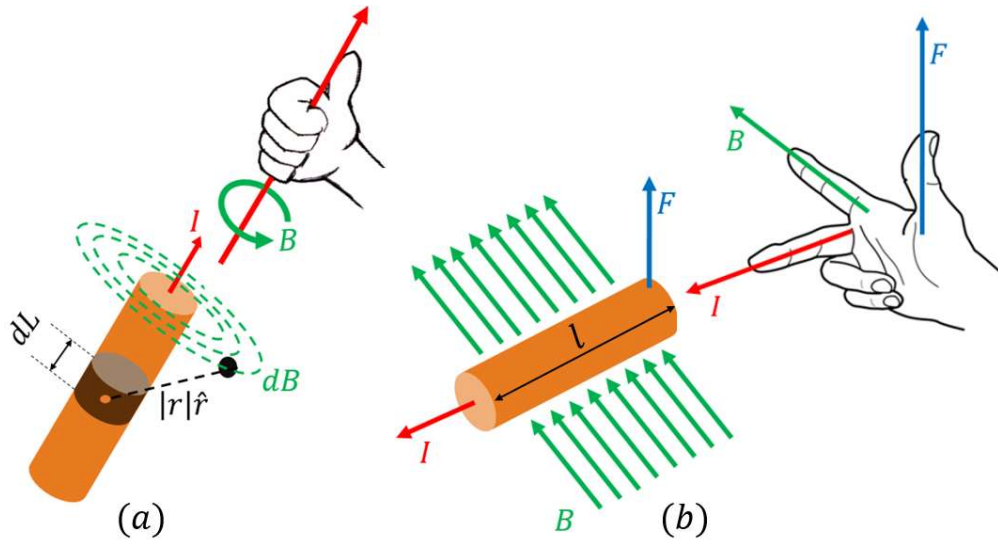


Figure 2.1: (a) Current carrying wire and magnetic field estimation by Biot-Savat Law (b) force on a current carrying wire in magnetic field

When the current flows through a conducting wire, the direction of the magnetic flux is determined by right-hand rule as shown in Figure 2.1(a). The right-hand rule states if the thumb represents the direction of the current, the curl of the fingers determines the direction of the magnetic flux lines. Furthermore, if a wire of length l is placed in the magnetic field B , conducts a current I , it experiences a force. The direction of the force is determined by the right-hand rule shown in Figure 2.1(b). Force, magnetic field and the current are perpendicular to each other as shown in the figure. The amplitude of the force is determined by Equation (2-4).

$$F = BIL\sin\theta \quad (2-4)$$

where θ is the angle between the wire and the magnetic field. If the angle between the current and the magnetic flux is 90 (Degrees), a maximum magnetic force normal to the plane of current and magnetic flux is obtained.

The reluctance is the amount of resistance that the magnetic flux lines are facing while passing through a material. More reluctance causes less magnetic flux to pass through the material. The inverse of reluctance is called permeance. The materials with less reluctance and more permeance are called permeable materials. The permeability of free space μ_0 is a constant $4\pi \times 10^{-7}$ H/m. All the materials are evaluated on the basics of their relative permeability value. The relative permeability of a material is a no unit quantity and represents the number of times the material is permeable than the air. The relative permeability of a material is expressed as shown in Equation (2-5) and the magnetic field intensity H ATurns/m is related to magnetic flux density as shown in Equation (2-6).

$$\mu_r = \frac{\mu}{\mu_0} \quad (2-5)$$

$$B = \mu H \quad (2-6)$$

2.2 Magnetic Materials

B-H curves are used to analyze the magnetic characteristics of the material. Figure 2.2 demonstrate a generalized B-H curve of a magnetic material [13]. The slope of this curve is the permeability μ of the material as shown in Equation (2-5) and (2-5). As seen in the figure, the permeability of the material is not constant and its value depends on the value of the magnetization and the magnetic flux passing through it. The B-H curve is divided into three zones: zone I, zone II and zone III as shown. Initially, when a magnetic material is provided with some magnetization potential, the value of the permeability exponentially increases, known as zone I. When the material is magnetized further, its permeability remains constant for a range of zone II. Furthermore, the material saturates and the value of the permeability becomes almost constant, known as zone III. Each material has a unique B-H curve, therefore, unique permeability values as well. The higher slope of the B-H curve corresponds to the fact that at lower magnetization, more flux passes through the material.

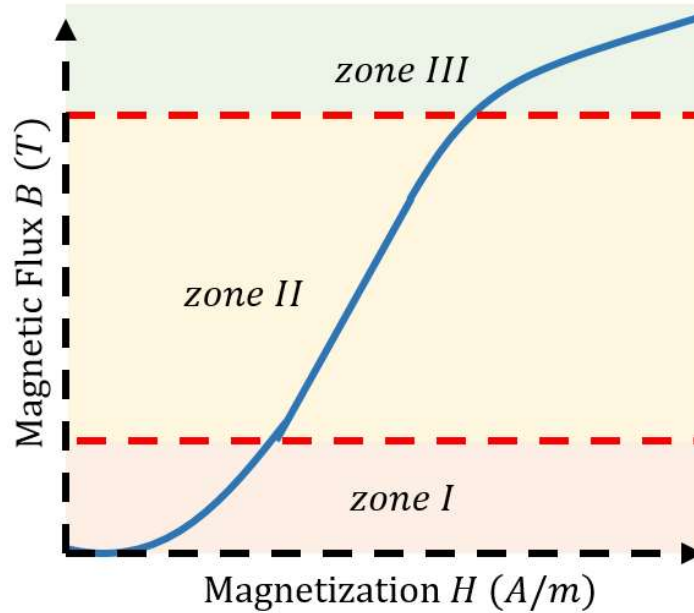


Figure 2.2: B-H curve pattern of the magnetic materials [13]

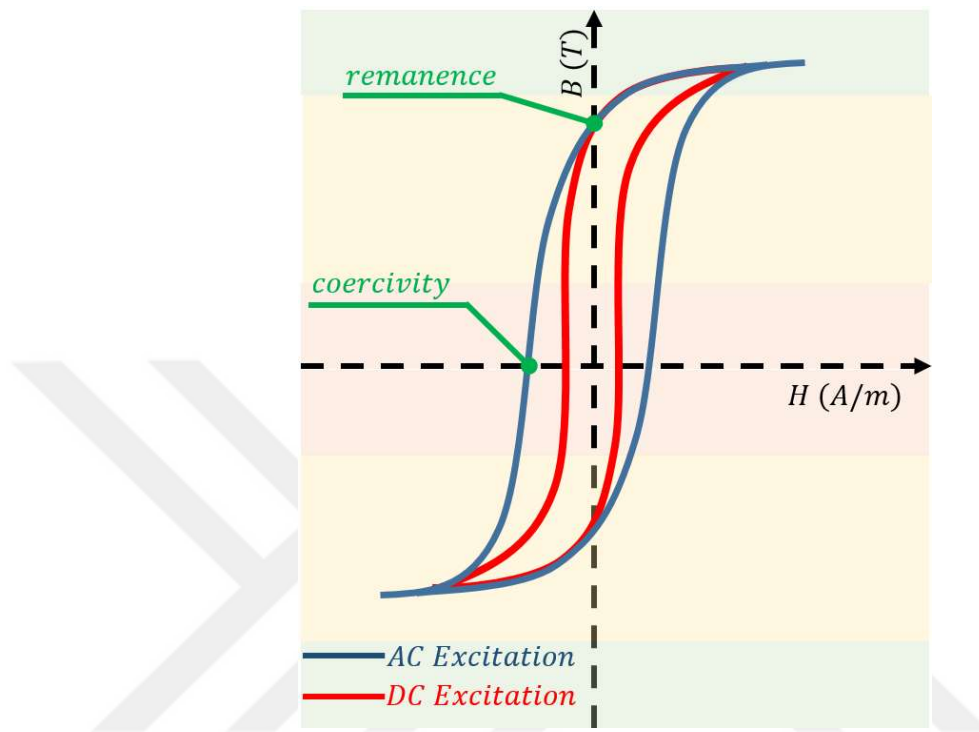


Figure 2.3: B-H curve pattern of magnetization and demagnetization (Hysteresis) of the magnetic materials [13]

If a magnetized material is provided with the reverse magnetization potential the material gets demagnetized. If the demagnetization is continued, the material gets magnetized in the reverse polarity. A graphical representation of the magnetization and demagnetization B-H curves are shown in Figure 2.3. When the material is continuously magnetized and demagnetized, the magnetization curve and the demagnetization curve do not follow the same pattern. This specific shape of the B-H curve is known as hysteresis. The width of the hysteresis is dependent upon the frequency of the magnetization and demagnetization. If the frequency of the oscillation is more, the width of the hysteresis loop is more. Whereas, low frequency or DC excitation corresponds to a thinner hysteresis loop. The hysteresis loop is

vital in the selection of the material for the actuators as the linear actuators are excited at higher frequencies and hysteresis losses affect the performance of the actuator.

Materials are classified as diamagnetic, paramagnetic and ferromagnetic materials. As the relative permeability of air is 1, materials with relative permeability less than 1 are referred as diamagnetic materials. For example, Bismuth, copper, gold, hydrogen and alcohol are diamagnetic in nature. This implies that the material is naturally repellent to magnetic flux and a very weak magnetic field is created. Materials with relative permeability slightly higher than 1 are known as paramagnetic materials. For example, aluminum, platinum, chromium and Manganese are paramagnetic in nature. The presence of these materials is considered as air. However, the relative permeability of paramagnetic as well as diamagnetic materials is considered as 1. Ferromagnetic materials demonstrate the B-H curves discussed earlier and their relative permeability is much higher than paramagnetic materials. These materials are magnetic flux loving elements and provide the least reluctance to the magnetic flux. For example, all kinds of alloys of iron, nickel and cobalt are perfect examples of ferromagnetic materials.

While designing a linear actuator, material selection is very critical. Low carbon (0.04-0.1%) stainless steel usually referred as martensitic steel, is used material as magnetic components in linear actuator. Type 410, 410S, 431 and 440 are the readily available martensitic steel in several shapes. Additionally, silicon steel is extensively used in the stator core of the actuators. It is also known as electrical steel and transformer steel. While designing an actuator, there are several components where the magnetic properties are either not required or sometimes their non-magnetic behavior is essential. For such applications, aluminum, Kestamid, carbon fiber and several kinds of polymers are used. In any case, if the magnetic properties are not required, choosing a non-magnetic material helps in extensive weight reduction due to their lower density.

2.3 Stator Windings

The stator is the stationary part of the actuator and it is the source of the magnetic field. When the externally insulated copper wires are combined in a form of a bulk, the effect of each wire is combined and depending on the direction of the current in the wires, the magnetic fields generated by each wire is added or canceled out. If all the wires have the same direction of the current, the magnetic field produced by each wire is added and larger magnetic fields are created. The direction is estimated the same way as the estimation is done for single wire with the help of right-hand rule [14]. The thumb represents the current direction and the curl of the fingers show the direction of the magnetic field as shown in Figure 2.4(a). If the coils are placed in the air, the magnetic field generated is very low. The maximum flux passes near the coil and it fades away as the distance increases. Therefore, the stronger magnetic field is present near the coil as shown in Figure 2.4(b). If the coil is placed in a closed magnetic material with high relative permeability as shown in Figure 2.4(c), the amount of the magnetic flux is increased and stronger magnetic field is created. The coil is commonly wound in a circular fashion as shown in Figure 2.4(d) around a central symmetric axis. In this scenario, the direction of the magnetic field is also estimated with the help of right-hand rule. Whereas the curl of the fingers corresponds to the direction of the current and the thumb indicates the direction of the magnetic flux. The side view and the section view are shown in Figure 2.4(e) and (f). Above the central axis, the direction of the current is out of the plane, therefore, the direction of the flux is towards right inside the coil and towards the left side outside the coil as shown in the figure. The major motivation of making a circular wound coil is the concentration of magnetic flux in the center. If the center of the coil is filled with a high permeable material, a very strong magnetic field is created. This configuration of coil winding is very common in solenoid actuators which will be discussed in Chapter 4.

The coil should be wound very tightly with one another in order to avoid the air gap between the wires. In order to standardize this, a terminology commonly referred as a filling factor is used. The formula for filling factor is shown in Equation (2-7). Generally, the filling factor of the motor winding is in the range of 35-45 (%).

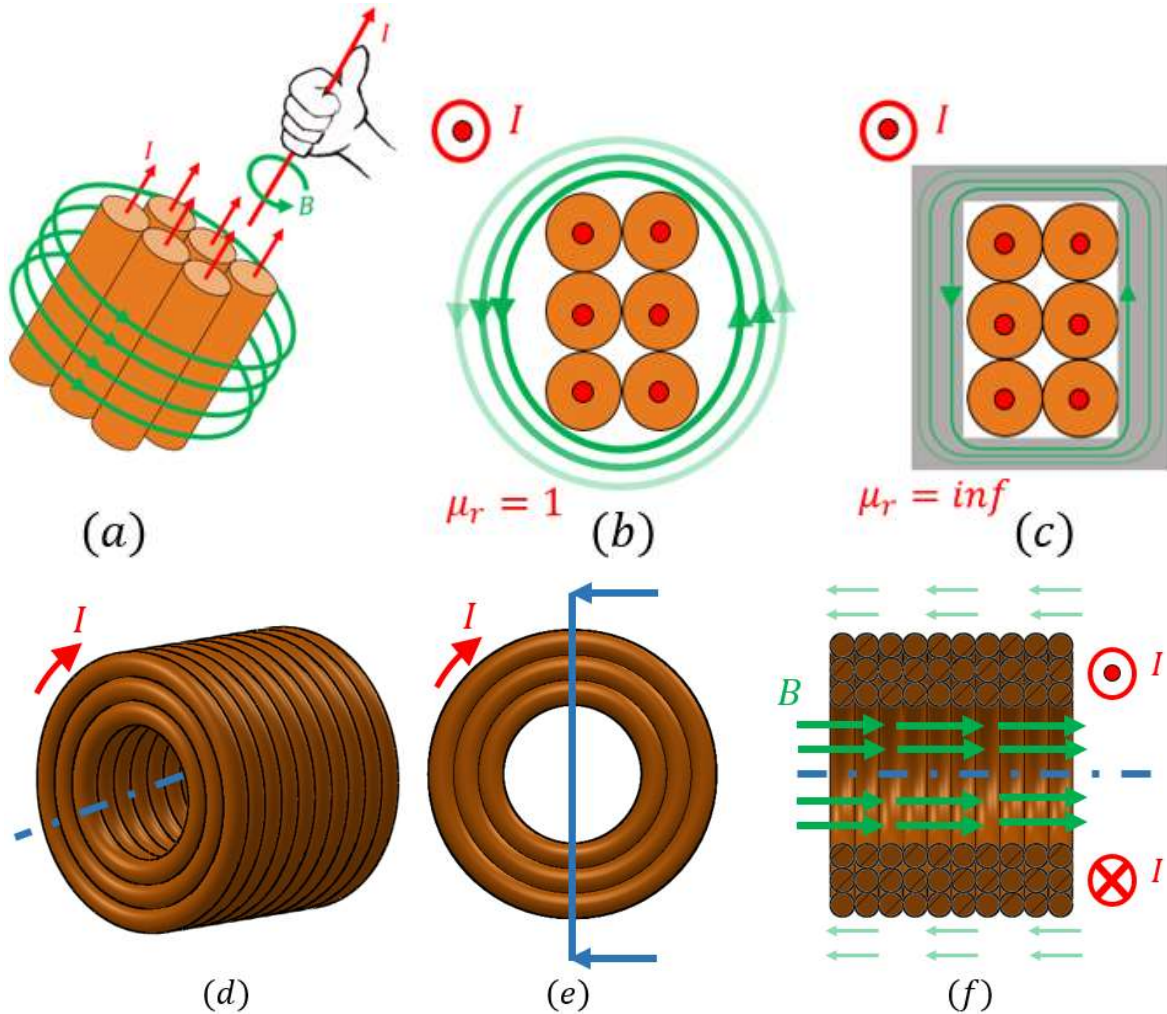


Figure 2.4: (a) Coil bulk (b) coil bulk in air (c) coil bulk enclosed in magnetic material (d) circular coil bulk (e) circular coil bulk side view (f) circular coil bulk section view

$$K_f = \frac{A_{cu}}{A_T} \times 100 \quad (2-7)$$

where K_f is the filling factor, A_{cu} is the area of the copper wires which is estimated by multiplying the cross-sectional area of the wire with the number of turns, A_T is the total area of the bulk.

For motor windings, copper wires are used due to their low-cost, good conductivity and availability. American wire gauge (AWG) is the system to standardize the wire diameters. Lower the AWG number, the higher is the wire diameter. The wire sizes are available from AWG 0000 (11.64 mm wire diameter) to AWG 40 (80 μ m wire diameter). The resistance of the wire is expressed in terms of the resistivity ρ , the length of the wire l_{wire} and the cross-sectional area A_{cs} or the diameter of the wire d_{wire} as shown in Equation (2-8). The density of Copper is $1.68 \times 10^{-8} \Omega m$. The resistance of the wire is inversely related to the wire diameter. Therefore, the lower AWG has more current carrying capacity as compared to the higher AWG numbers. Whereas, the length of the wire is directly proportional to the resistance. Therefore, longer lengths correspond to more resistive coil bulks as compared to the shorter lengths.

$$R_{wire} = \rho \frac{l_{wire}}{A_{cs}} = 4\rho \frac{l_{wire}}{\pi d_{wire}^2} \quad (2-8)$$

$$V = L_{coil} \frac{dI}{dt} \quad (2-9)$$

When a coil is energized with external electrical voltage, it resists to the change the current passing through it commonly referred as Inductance. Equation (2-9) represents the voltage and the current dependence on the coil inductance. The coil is excited either with a constant DC current or a time-varying voltage such as alternating current or square waves. The coil is modeled as a series circuit of the resistance and the inductance of the coil.

$$X_{coil} = X_R + X_L = R + j\omega L \quad (2-10)$$

where X_{coil} is the impedance of the coil, X_R is the impedance due to the internal resistance of the coil, X_L is the impedance due to the inductance and ω is the excitation frequency. As shown in Equation (2-10), the impedance of the coil consists of a real and an imaginary part. The real part of the impedance is not dependent on the excitation frequency. Therefore, at constant DC, the coil acts as a resistor only. Whereas, the imaginary part of the impedance depends on the excitation frequency. The current to the excitation voltage transfer function is shown in Equation (2-11).

$$\frac{I}{V} = \frac{1}{X_R + X_L} = \frac{1}{R + j\omega L} \quad (2-11)$$

2.4 Stator Core, Laminations and Eddy Current Losses

As shown in Figure 2.4(c), the magnetic field is enhanced when the coil bulk is surrounded by a highly permeable material. Therefore, linear motors use stator cores to take utmost advantage of the magnetic field generating capacity of the stator windings. One of the most common winding technique in AC motors is demonstrated in Figure 2.5. In the figure, in (a) it is demonstrated that the coil is wound around a letter m shaped core. The direction of the current is also shown into the plane and out of the plane on the left and right sides of the middle leg of the core respectively. With the help of the right-hand rule, the direction of the magnetic flux is estimated. The air gap between the legs is created to make magnetic poles on the stator core. These poles are the source of linear actuation that will be discussed later. As shown in the figure, the strong poles are created at the edges near to the air gap depending upon the direction of the magnetic flux. The magnetic flux exhaling edges become a strong north pole and inhaling magnetic flux edges become a strong south pole demonstrated as red and blue colors respectively. The side legs are commonly referred as tooth and the edges are named as tooth edges. Most of the stator cores are made in a tubular

design to utilize the maximum surface area of the stator core as shown in Figure 2.5(b). The number of poles depends on the number of windings and the winding technique used for the stator.

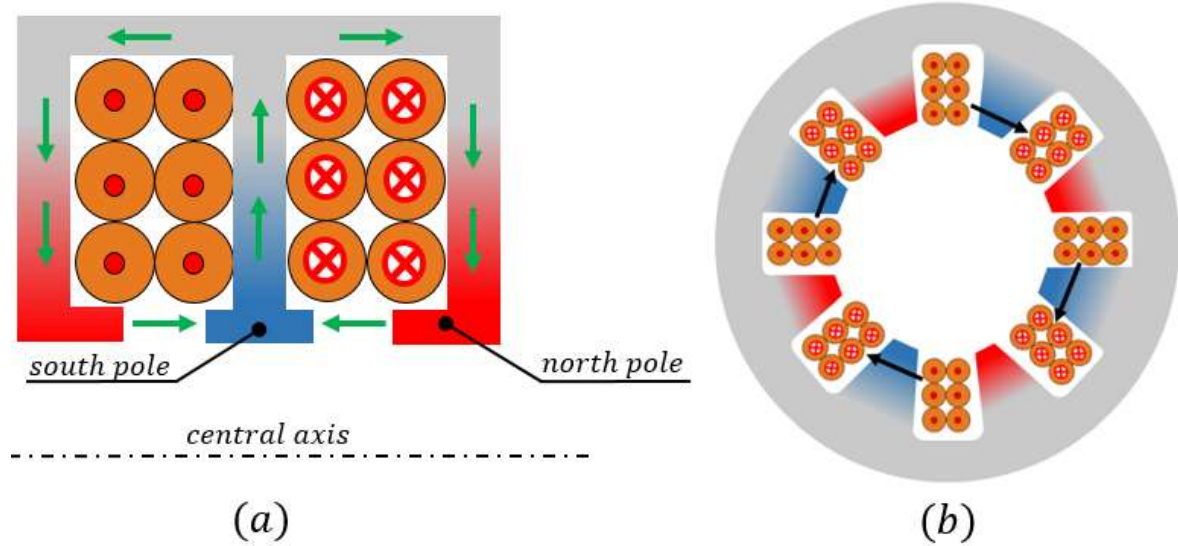


Figure 2.5: The schematic of the stator core of the AC machines

In the case of the time-variant excitations, the current is always changing in the coil. This alternating current creates an alternating magnetic field inside the stator as well. When a conductor experiences a changing magnetic field, the current loops are induced inside the conductor as shown in Figure 2.6(a). The induced current generates its own magnetic field which is in the opposite direction of the magnetic field which created it. These losses are known as Eddy Current Losses. If a solid stator core is used, the Eddy Current loops are created and oppose the direction of the desired magnetic flux direction. Therefore, the magnetic poles created on the stator core tooth are weaker. Eddy Currents not only affect the magnetic field but it also affects the temperature profile of the stator core. As the stator material is a conductor and possesses a finite resistance, the power losses in these resistances are generated as heat and eventually increase the temperature of the stator core [15]. This rise in temperature affects the B-H curve of the material. In order to overcome Eddy Current

Losses, a well-known method of using laminated core [16] is used instead of a solid unit as shown in Figure 2.6(b).

The laminates are designed by slicing the solid stator in the direction to cut the path of the induced current loop. For instance, as shown in (a), the current loops are vertical, therefore, horizontal slicing of the stator core is done as shown in (b). If the stator core is sliced vertically, laminates will not be effective as the magnitude of the current loop will be bigger and the magnetic field would still be created. By using horizontally sliced laminates, the magnitude of the induced current loop is smaller than the solid stator core. Although, this loss cannot be eliminated completely but it can be reduced immensely by using this technique. The thickness of the laminate t_l is designed as low as possible in order to weaken the induced current loops. In order to manufacture the stator core, instead of using blocks as a raw material, silicon steel metal sheets are used and are cut through laser precisely in the desired shape. After manufacturing, these laminates are joined together to construct the desired stator core geometry with mechanical restrictions, resins or adhesives.

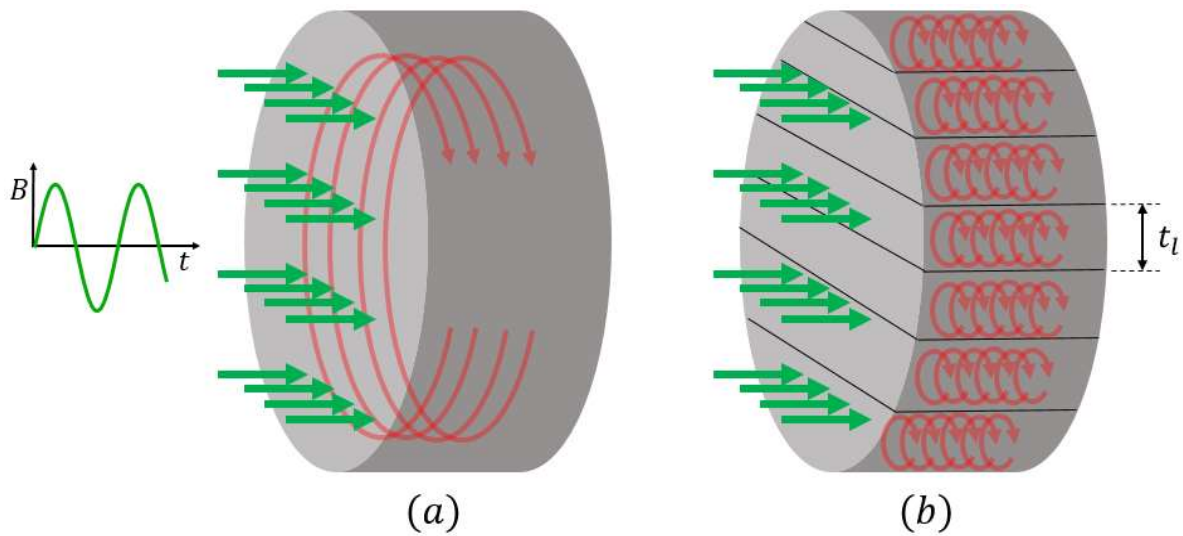


Figure 2.6: (a) Induced current loops due to changing magnetic field in a solid conductor
(b) induced current loops due to changing magnetic field in a laminated conductor

2.5 Permanent Magnets and their Types

When a ferromagnetic material is magnetized, it retains the magnetic properties even the magnetization source is removed. The B-H curve as shown in Figure 2.3 demonstrates a point known as remanence. Remanence is the amount of the magnetic flux; a material keeps on producing at zero magnetization potential. This point is different for different materials. All ferromagnetic materials demonstrate remanence phenomena. Materials with high remanence points are classified as permanent magnets. In order to bring the magnetic flux back to zero, the material should be magnetized in the reverse magnetization potential. This magnetization value is known as coercivity. Neodymium, AlNiCo, Ferrite, Ceramic and SmCo magnets are the readily used permanent magnets [17].

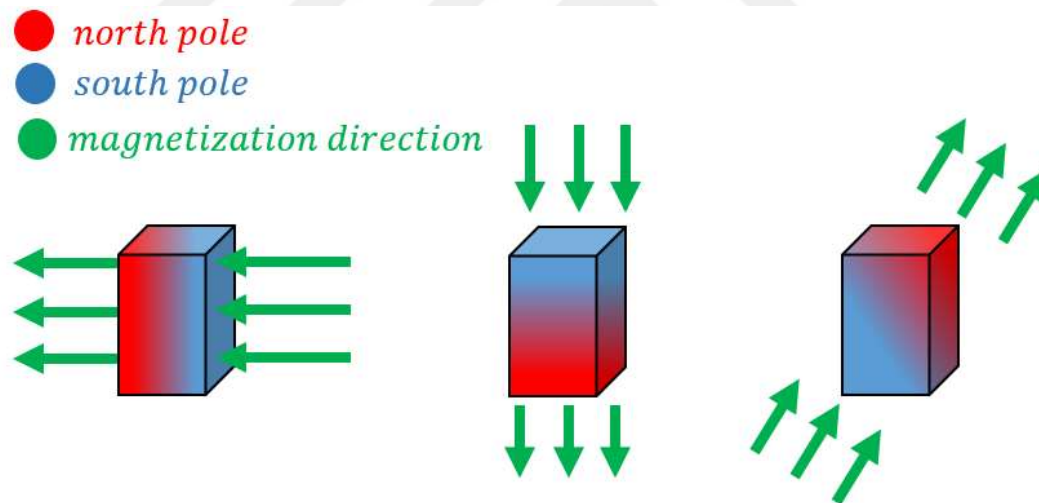


Figure 2.7: Magnetization schematic of a cube in three directions

Magnetization is a process of providing magnetization potential on permanent magnet materials. The magnitude of the magnetization is standardized as A/m and Oe. The direction of magnetization [17] defines the location of the pole on the magnetized material as shown in Figure 2.7. The area from which the magnetic flux enters the material become the south pole and the area from where the magnetic flux leaves become the north pole.

As linear actuators are mostly tubular in shape, cylindrical shaped ring magnets are required. The axially magnetized ring magnets shown in Figure 2.8(a) are easy to manufacture whereas the radial magnetization as shown in Figure 2.8(b) for small size ring magnets is not possible. Therefore, arc magnets [18] are used to make a ring as shown in Figure 2.8(c). Each arc is individually magnetized in the desired orientation as shown in Figure 2.8(d) and they are assembled later to make the radially magnetized ring magnet. Although, to some extent, radial magnetism is achieved but it is not perfectly radial throughout the ring. Therefore, usually ring magnets are made up of 9 or 12 arc magnets to achieve as much radial magnetism as possible. More arcs provide better radial magnetization profile. Depending on the motor configuration, radial magnetism in permanent magnets is usually required for the mover or armature of the linear actuators.

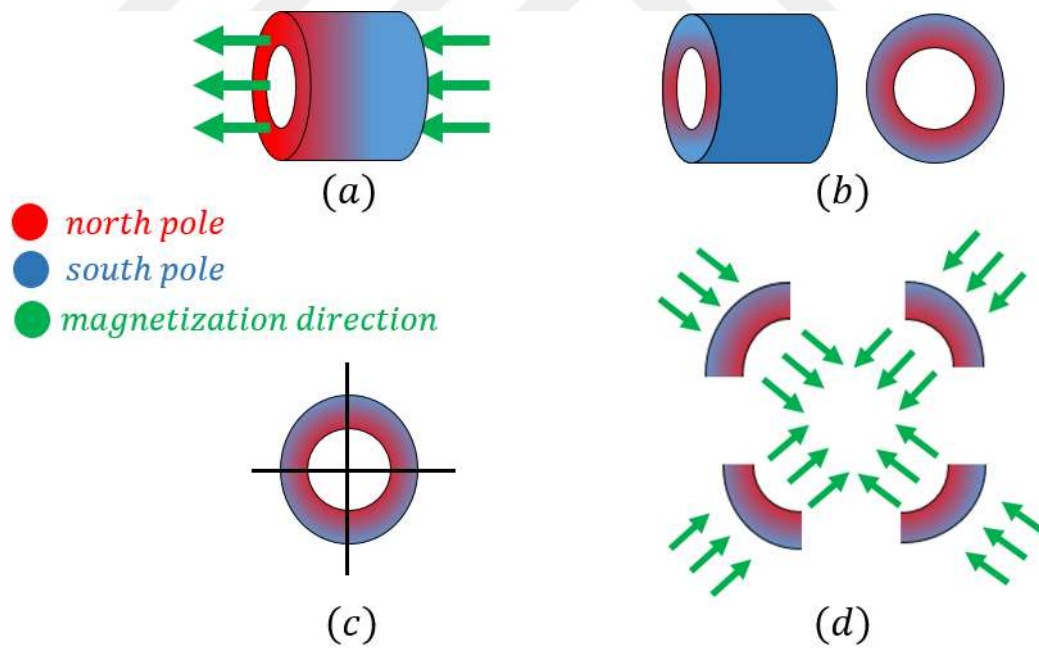


Figure 2.8: (a) Axially magnetized ring magnets (b) radially magnetized ring magnets (c) arc magnets to a ring magnet (d) magnetization of arc magnets to make a radially magnetized ring magnet

2.6 Magnetic and Electric Circuit Analogy

According to the Gauss' law [13], the magnetic flux passing through a closed surface is zero as shown in Equation (2-12) [19]. This implies that the magnetic flux is conserved and there is no source of magnetic flux, it keeps on circulating in a closed loop as shown in Figure 2.9(a).

$$\oiint \vec{B} \cdot d\vec{a} = 0 \quad (2-12)$$

where B is the magnetic flux and da is the surface area.

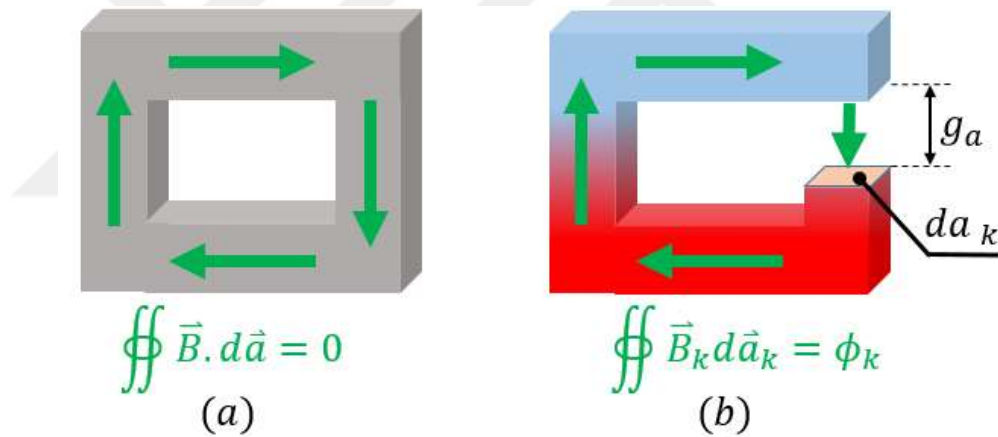


Figure 2.9: (a) The interpretation of Gauss' law for magnetism for a closed surface (b) the interpretation of Gauss's law for a small surface area of a closed surface

If the closed surface is cut as shown in Figure 2.9(b), the magnetic flux lines will start flowing through the air gap g_a and surface area da . The magnetic flux for this small area is shown in Equation (2-13). If the closed surface is divided into n number of small areas da_k , in order to satisfy the Gauss' law, the integral sum of all incremental small areas should be equal to zero as shown in Equation (2-14). This equation proves the analogy of the magnetic circuits with the Kirchhoff's current law (KCL) of the electric circuits.

$$\oint \vec{B}_k \cdot d\vec{a}_k = \phi_k \quad (2-13)$$

$$\sum_{k=1}^n \phi_k = 0 \quad (2-14)$$

According to the Ampere's law [12], the magnetic field created in the closed space by the electric current is directly proportional to the electric current of the coil as shown in Equation (2-15).

$$\oint \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{a} \quad (2-15)$$

where J is known as the current density of the coil.

The current density of the coil is also written as:

$$\iint \vec{J} \cdot d\vec{a} = NI \quad (2-16)$$

where N is the number of turns of the coil and I is the current.

The integral of the magnetomotive force (MMF) is a voltage and is defined by the integral of the magnetic field over a length as shown in Equation (2-17).

$$mmf = \oint \vec{H} \cdot d\vec{l} \quad (2-17)$$

Comparing Equation (2-15), (2-16) and (2-17),

$$mmf = NI \quad (2-18)$$

This equation proves the analogy of the magnetic circuits with the Kirchhoff's voltage law (KVL) of the electrical circuits. Hence, KVL and KCL are applicable on magnetic circuits as well [19].

As discussed in section 2.1, reluctance is the opposition faced by the magnetic flux to flow in a material, is analogous to the resistance in the electric circuits. The magnetic flux is analogous to the current in the electrical circuits. Voltage potential in an electrical circuit is analogous to the magnetomotive force in the magnetic circuits.

Chapter 3

LINEAR ACTUATOR TYPES

3.1 Introduction

Linear actuators are electromechanical devices which provide bi-directional motion in a single axis for a specific load without any screws, gears or crankshaft [20]. Like rotational motors, they transform the electrical energy (voltage and current) to the mechanical energy (force and displacement) with an intermediate magnetic energy form. Linear actuators are reversible in operation as a rotational actuator for use as a generator. These motors are composed of two main parts: the stator and the armature or mover. The stator is the stationary component which generates a controlled electromagnetic energy. That energy is transformed into mechanical work by moving the armature to the position where the electromagnetic flux faces the least reluctance. The motivation for the development in the field of the linear actuator is to eradicate the need for the cumbersome conversion mechanisms that converts the rotational motion to the linear motion. For instance, ball screw joints, crankshafts, rack and pinions are the most prevailing examples for such mechanisms. For low-speed operations, it is convenient to use these solutions whereas for high-speed motion, these mechanisms cannot be used. Moreover, friction is dominant due to many contact points. Additionally, the conversion mechanisms are noisy and need uninterrupted oil lubrication for orderly performance. Furthermore, the number of parts are more in these conventional converters increase the cost enormously. Linear actuators are the ultimate alternatives for such mechanisms.

Plenty of the oscillatory systems are using conventional rotational motors and with the help of the crankshaft, the linear oscillations about the mean position are achieved. The peak to peak amplitude is commonly referred as stroke. Depending on the size of the machine and application, the stroke is one of the most critical parameters. The frequency of oscillation is

also of vital importance. The force that the linear actuator provides to the load is referred as the thrust force. Commonly used linear oscillatory systems maintain a short stroke ranging 10-20 (mm) and the frequency is around 50-60 (Hz) as they are usually powered up with the normal AC line. Pumps, cryocoolers and compressors are the major application industries of linear oscillatory actuators. Their sizes range from micro to macro scale. Low-frequency oscillators are rowdier than the high frequency.

Linear actuators are available in planar as well as tubular configuration whereas the working principle is the same [13]. Tubular configurations are the unrolled form of the planar actuators. As discussed in the previous chapter, the gap between the stator and actuator has an enormous effect on the thrust force, tubular configurations are preferred due to their high thrust to current ratio due to the more surface area. The planar configurations are single as well as double sided configurations. This research focuses on the tubular linear oscillating actuators used for linear compressors only.

The linear actuators are classified as the moving iron, the moving magnet and the moving coil linear actuators based on their design configurations [21].

3.2 Moving Iron Linear Actuators (Solenoid)

Moving iron linear actuators usually referred as solenoid actuators are low-cost solutions for the linear actuation applications. As discussed in section 2.6, the magnetic flux always pushes the system towards the least reluctance path position. The moving iron linear actuators are designed in such a manner that the stator and the armature together complete the reluctance path. Both are adjusted in such a manner that there is a gap in the reluctance path when the stator winding is not energized. When the stator coil is energized, the magnetic flux pushes the armature to the point of least reluctance. For oscillatory applications, the armature is connected with the springs. Therefore, when the coil is de-energized, the energy stored in the springs pulls the armature back to its initial position. By repeating the energizing and de-energizing cycle, the armature oscillates linearly in the desired stroke range.

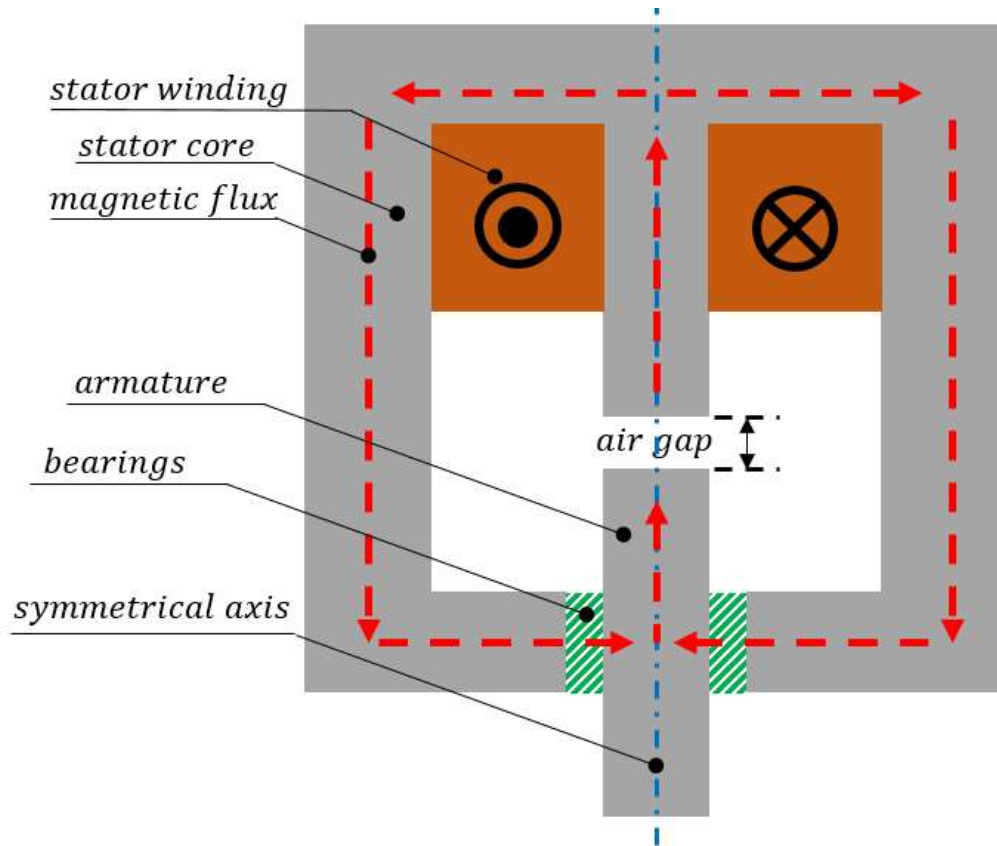


Figure 3.1: Schematic of the moving iron linear actuator [13]

Most of the tubular moving iron linear actuators are symmetric around the central axis. A schematic of a solenoid linear actuator is shown in Figure 3.1 demonstrating all the essential components. The stator comprises of the stator winding which is done on the stator core as shown. As discussed earlier, the armature is placed in such a way that there is an air gap in the reluctance path of the magnetic flux lines. Linear bearings are sometimes employed to ensure the axial motion of the armature. The material for the stator core and the armature should possess good magnetic properties. Therefore, the reluctance of the stators and the armature is negligible as compared to the air gap. As the magnetic materials are ferromagnetic, their density is higher than the non-magnetic materials. Consequently, the

actuator is heavier. Secondly, the stator winding generates a lot of heat which heats up the actuator and effects the performance. A detailed research is done on these actuators and the performance of such actuators is discussed in Chapter 4. The schematic shown is one of the configurations, any configuration that creates a closed magnetic circuit with a gap provided in the path will work for this technique. The maximum stroke of the armature is defined by the air gap in the path of the magnetic flux lines.

3.3 Moving Magnet Linear Actuators (MMLA)

Moving magnet linear actuators are superior over other actuators for high thrust density, lower losses, compact design, fast responses, higher speed of mover and smaller electric time constants [13]. The major disadvantages of them are the high cost of the permanent magnets and stray magnetic fields in its different configurations [22]. Cylindrical design is preferred in this case for compact design and better thrust to weight ratios. These actuators are composed of two major parts; stator and mover. The stator is composed of coil or coils depending upon the selected technique of actuation. Mover is composed of permanent magnets and their placement is different for different categories.

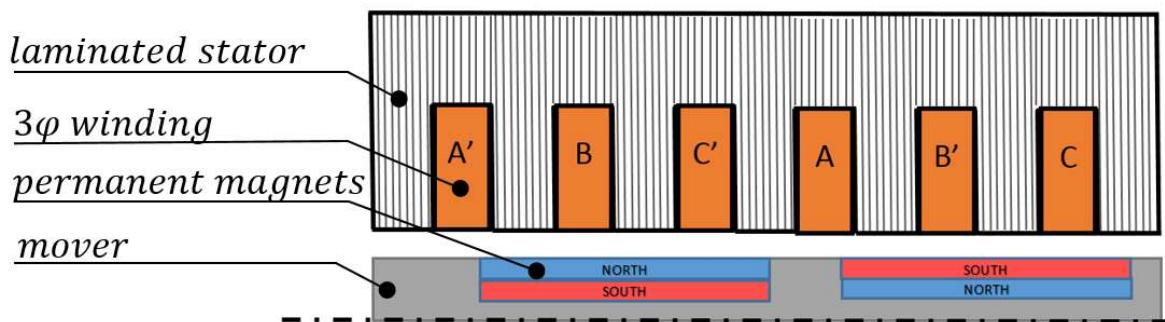


Figure 3.2: Schematic of the moving magnet synchronous linear actuator

One of the MMLA configuration is the linear permanent magnet synchronous actuators (LPMSA). A schematic diagram of a simple LPMSA is shown in Figure 3.2. In these designs,

the stator comprises of three phase winding located in the slots represented in the figure as (A), (B) and (C). These windings are wound in a circular fashion around the central axis. Mover is also axis-symmetrical and consists of embedded ring magnets in a specific orientation. A three-phase AC power is given to the coil, from the basic electromagnetic principle, the flux lines of coil surround the coil in a loop and are changing with time. On the other hand, there are permanent flux lines flowing around the permanent magnets as well. An axial thrust force is created and the direction depends on the polarity of the power input of the coil. Thus, mover oscillates in the forward and backward direction as desired.

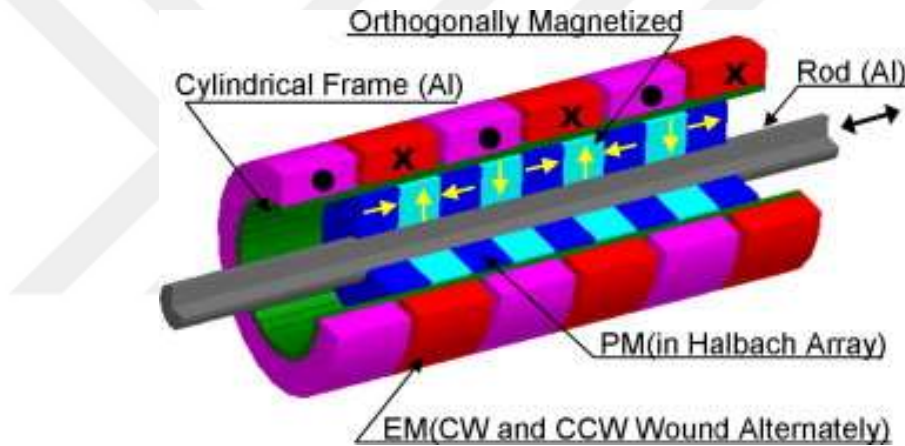


Figure 3.3: Schematic of the moving magnet linear actuator with Halbach array [7]

In 1973, one sided flux concept was introduced which has been popular among researchers for the past decade [7]. This one-sided flux is achieved by arranging the permanent magnet in a special orientation called Halbach array shown in Figure 3.3. The only difference of the Halbach array from the normal magnetic piston discussed earlier is the direction of the magnetic flux is radially enhanced and it is minimum in the center. So, there is no need of back iron to give the return path to the flux. This also reduces the weight of the piston, which helps in the quick response of the mover [23].

Instead of using a three-phase coil, [24] preferred a single phase to have a compact size of the actuator. Single phase AC powers requires only one major coil to generate an axial

thrust and using a Halbach array to improve the factors discussed earlier. Detailed analytical and mathematical models are already developed to prove the efficiency of the LPMSAs [25]. Figure 3.4 demonstrates the design of the single phase LPMSA. At 50 (Hz) excitation frequency, the stroke of 10.5 (mm) is achieved with the output power of 87 Watts.

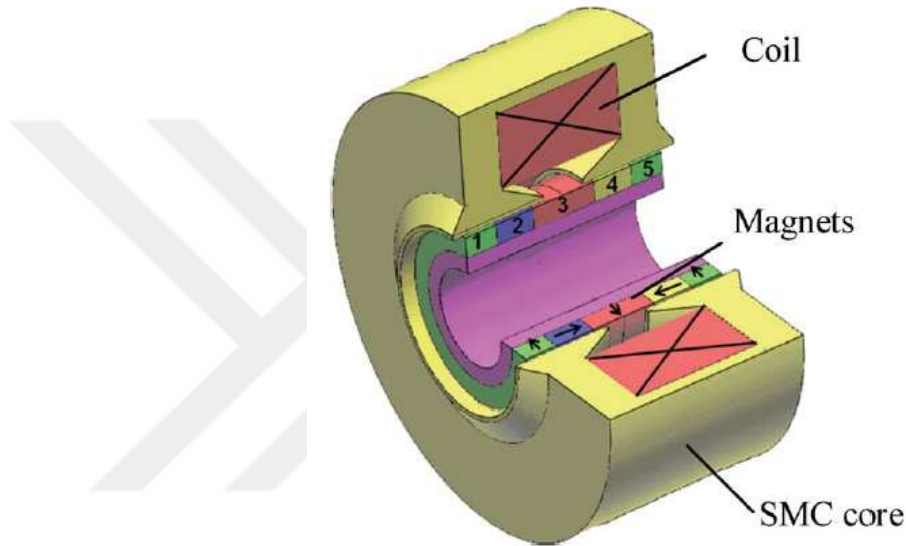


Figure 3.4: Schematic of the single phase moving magnet linear actuator with Halbach array [25]

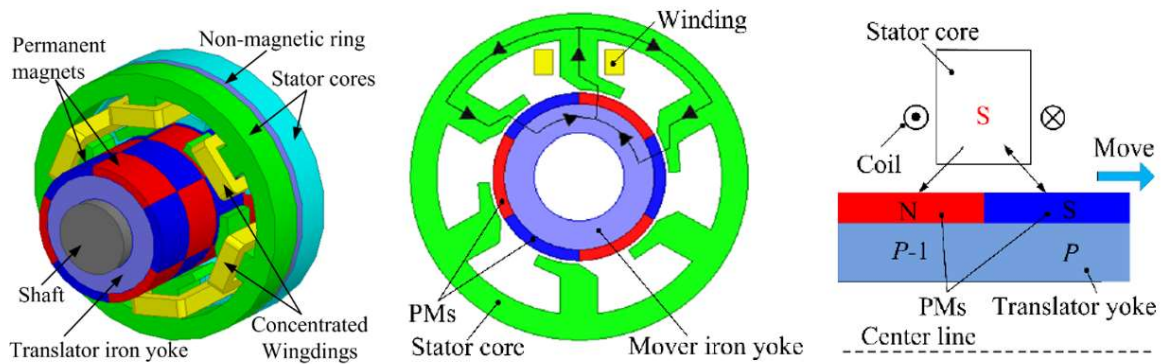


Figure 3.5: Schematic & working principle of staggered tooth transverse flux moving magnet linear actuator [26]

Furthermore, the tubular staggered-tooth transverse-flux moving magnet linear actuator is presented by [27]. The schematic is shown in Figure 3.5. This design is composed of three coils energized with a three-phase AC drive. The similar winding technique used in conventional rotary motors is used as shown. Stator cores are designed in a way to provide transverse flux to circulate in a loop as shown in the figure. As each phase of the power energizes one coil, a unique pair of permanent magnets oscillates under the stator yoke as demonstrated. The translator yoke provides the return path for the flux lines. The rated frequency of 83.33 Hz and the speed of 1 m/sec of the mover is achieved.

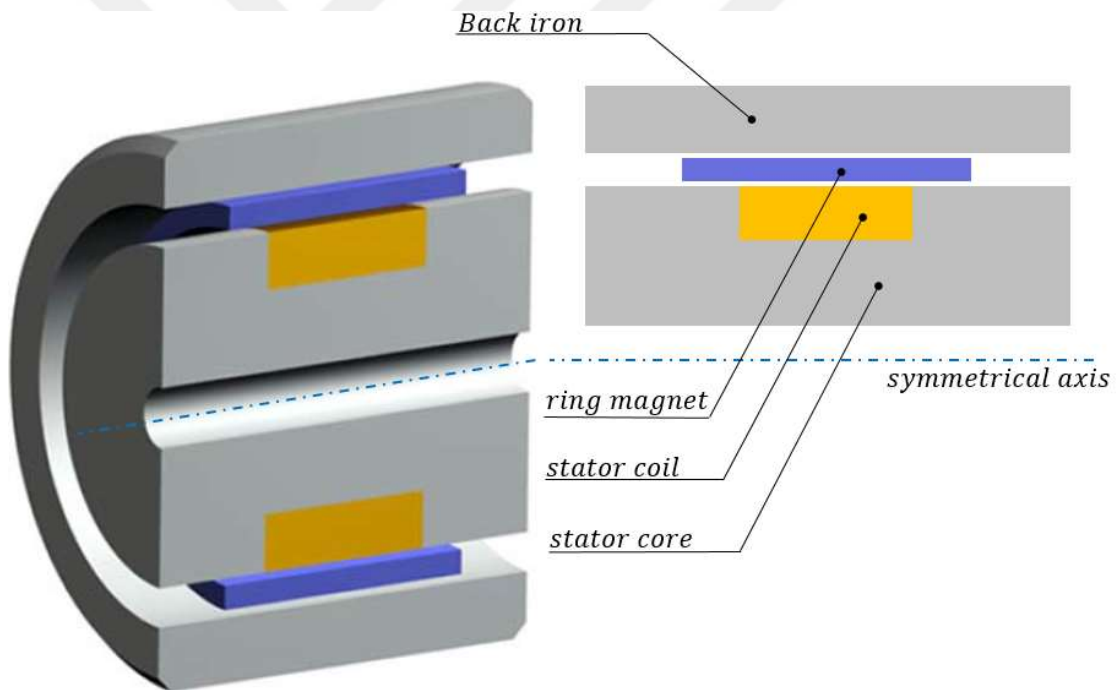


Figure 3.6: Schematic of a small, single-phase moving magnet linear actuator with ring magnet armature [28]

A simple configuration of a moving magnet linear actuator is presented by [28] as shown in Figure 3.6. The stator winding is done in a circular fashion in a slot available in the stator core as shown. Radially magnetized ring magnets (NdFeB) are used which are attached to

the moving assembly of the actuator. Back iron provides the return path for the magnetic flux lines. At excitation of 15 V and at resonance frequency excitation, 2.17 mm of stroke is achieved with an output power of 10 Watts.

A moving magnet linear motor with toroidal winding is presented by [8] and the schematic is shown in Figure 3.7. Transverse magnetic flux among the stator and mover that contains permanent magnets, is used. As shown in the figure, the toroidal winding is done on a semi-circular core. In the discontinuity region of the stator core, the mover is placed with magnets and the mover aligns itself based on the phase of the current provided to the stator winding and the polarity of the permanent magnets. A maximum stroke of 14 mm with an approximate shaft power of 50 Watts is demonstrated.

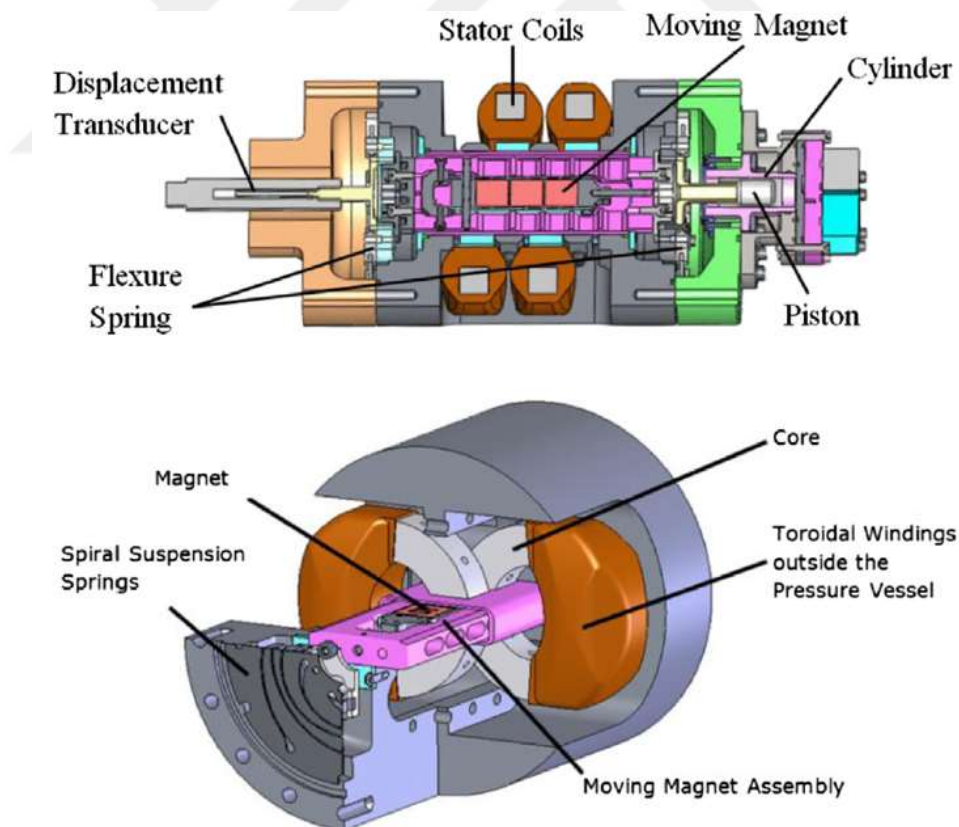


Figure 3.7: Schematic of the moving magnet linear actuator with toroidal windings [8]

3.4 Moving Coil Linear Actuators (MCLA)

Moving coil linear actuators (MCLA) are generally used in the applications for low range strokes and velocities. Like MMLA, they are also manufactured as tubular as well as planar configurations. In planar configurations, they are designed as single, as well as double sided arrangements. A schematic of the MCLA is shown in Figure 3.8. The actuator is composed of primary and secondary windings. Three phase winding is used to drive these actuators with the help of inverters. The winding of MCLA is analogous to rotary AC machines. The stator consists of a longitudinally laminated primary coil which is wound in a ring shape. The secondary core is the solid iron with slots for the secondary winding as shown in the figure.

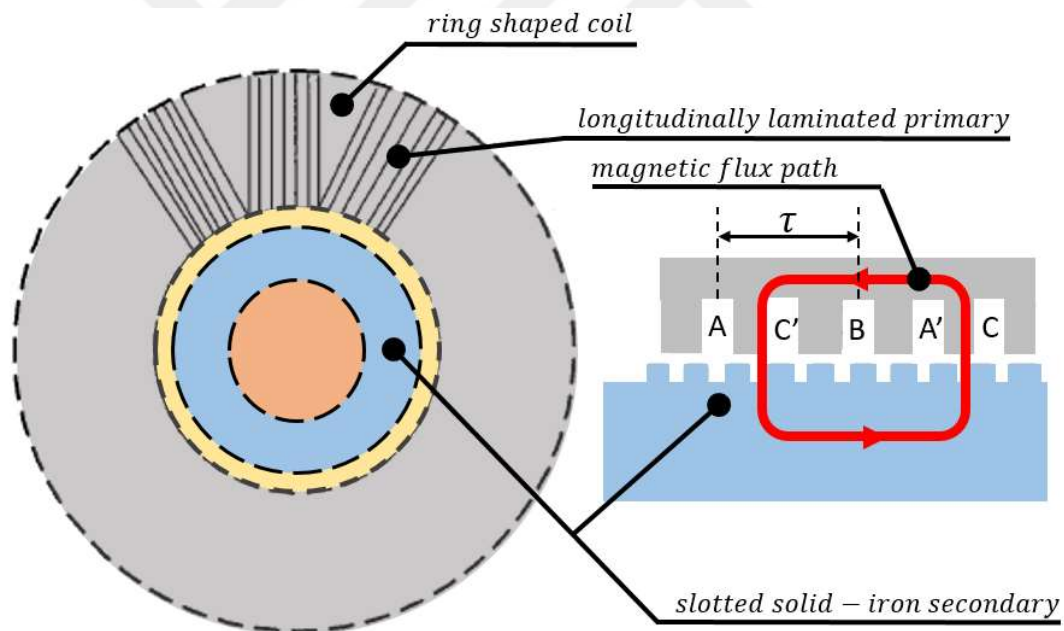


Figure 3.8: Schematic of the tubular moving coil linear actuator [13]

As the design is composed of cumbersome laminated cores, the manufacturing of these motors is difficult than the other categories. Also, arduous winding techniques in MCLA makes other configuration more superior. Furthermore, the size and weight of these actuators

are larger than the others. Therefore, researchers are not considering this configuration suitable for linear oscillatory actuators. This research is based on the studies on the moving iron and moving magnet linear actuators.

3.5 Electronic Control and Drives

3.5.1 Moving iron linear actuator driver

As discussed in section 3.2, moving iron linear actuators are used as a linear oscillator with the help of a spring. When the stator is energized, it pulls the armature and when de-energized, the springs pull the armature back to the initial position. Therefore, a switching device is used to turn on and turn off the current through the stator coil. A pulse width modulated (PWM) signal is used to excite the system through a power amplification switch. A PWM voltage signal with respect to the time is shown in Figure 3.9. The time for one complete pulse is the inverse of the frequency of the signal. T_{ON} is the time in which the coil is excited and T_{OFF} is the time the stator coil is de-energized. Duty cycle is the percentage of the ratio of the T_{ON} to the time period of the pulse. The frequency of the armature oscillation is identical with the frequency of excitation. More amplitude corresponds to more current which leads to the more electromagnetic force. Therefore, the amplitude of the signal corresponds to the amount of the enacted stroke. The time constant is the ratio of the inductance of the stator to the internal resistance of the coil. The time required for the stator current to reach the maximum value is approximately five times to the time constant. Therefore, the frequency of the coil excitation is bounded by the time constant of the linear actuator. The current of the stator coil is exponentially related to the inverse of the time constant. If T_{ON} is higher than five times the time constant, the stator current will reach its maximum value. Otherwise, based on the exponential curve, the current value starts decaying from the value achieved during the on time.

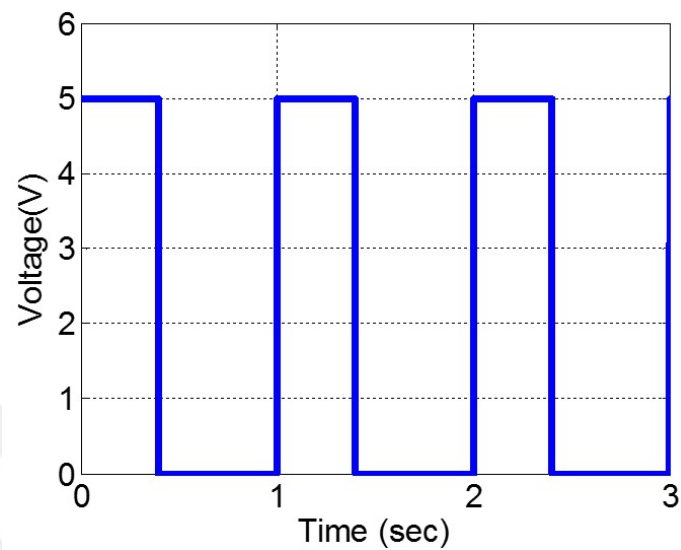


Figure 3.9: Voltage excitation with respect to time of a pulse width modulated signal

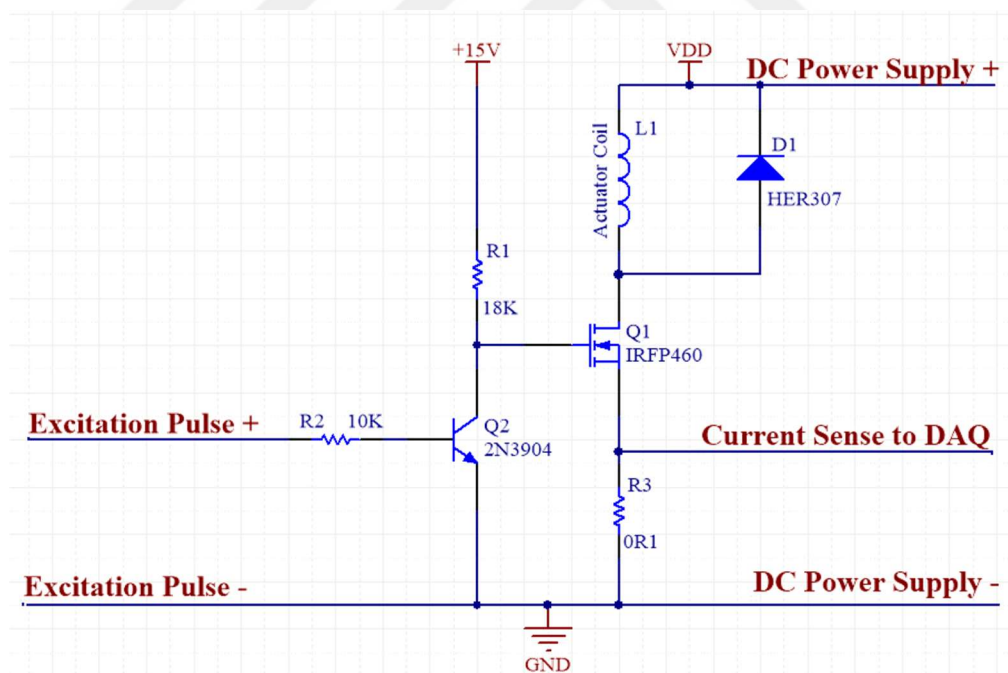


Figure 3.10: Electronic switch for stator coil of solenoid linear actuator

An electronic circuit as shown in Figure 3.10 is used to excite the coil. As the armature is required to oscillate, a simple MOSFET based switch circuit is used. A DC power source from TTI (CPX4000D) capable of variable voltage generation from 0 to 60 V each channel is used with a maximum capacity current of 20 Amperes. A digital frequency generator from GW-INSTEK (GFG-3015) is used to generate a variable frequency and duty ratio pulses to the electronic circuit. This function generator has a frequency range of 0-15 MHz, a resolution of 0.01 Hz, and adjustable duty ratio in between 20% and 80%.

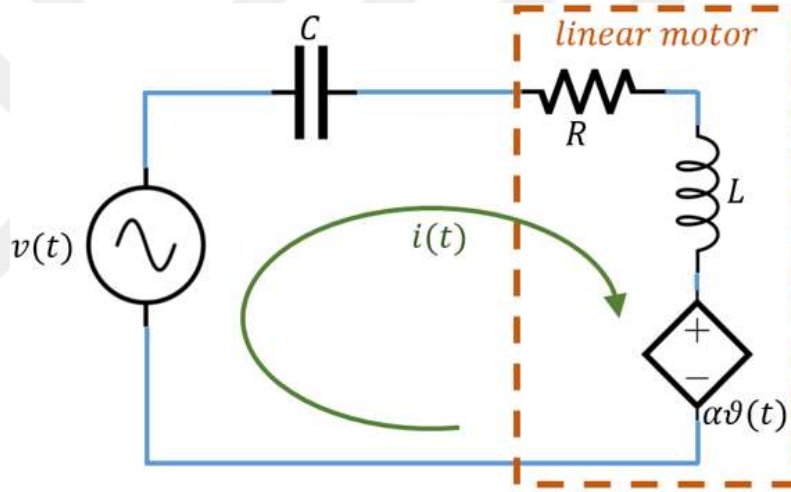


Figure 3.11: Schematic of RLC circuit for driving the moving magnet linear actuator

3.5.2 Moving magnet linear actuator driver

Due to the presence of the constant magnetic poles of the permanent magnets, MMLA is driven by the alternating current power source. During the positive cycle, the permanent magnets pull the armature towards one side. Whereas, in the negative cycle, the reverse motion is achieved. The Stator coil is modeled as a series combination of inductance L and resistance R . When an LR circuit [29] is excited, there is a phase difference between the current i and the voltage v excitation. Therefore, a series capacitance is used to achieve electrical resonance in the circuit. In an RLC circuit, an electrical field is stored in the

capacitor whereas magnetic energy is stored in the inductor. In circuit elements, impedance is the opposition to the current when excited with a finite voltage. At resonance, the impedance of the inductor is equal and opposite to the impedance of the capacitor and the system starts oscillating by transforming electrical field and magnetic field energies. Additionally, the imaginary part of the impedance is diminished whereas the real part (resistance) becomes dominant and damps the oscillations. The schematic of an RLC circuit is shown in Figure 3.11. The instantaneous voltage corresponds to the following equation:

$$v(t) = Ri(t) + L \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) dt \quad (3-1)$$

Taking Laplace of Equation (3-1):

$$\bar{V} = R\bar{I} + sL\bar{I} + \frac{\bar{I}}{sC} \quad (3-2)$$

Using the natural frequency equation of RLC circuit shown in Equation (3-3):

$$\omega_{ne} = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_{ne}^2 L} \quad (3-3)$$

Putting the value of the electrical natural frequency in Equation (3-2):

$$\bar{V} = \left[R + sL + \frac{\omega_{ne}^2 L}{s} \right] \bar{I} \quad (3-4)$$

Converting Equation (3-4) to frequency domain:

$$\bar{V} = \left[R + j\omega L + \frac{\omega_{ne}^2 L}{j\omega} \right] \bar{I} \quad (3-5)$$

$$\bar{V} = [R + j\omega L(1 - r_e^2)] \bar{I} \quad (3-6)$$

where

$$r_e = \frac{\omega_{ne}}{\omega} \quad (3-7)$$

Equation (3-7) is written as the ratio of the current to the voltage as:

$$\frac{I}{V} = \frac{1}{R + j\omega L(1 - r_e^2)} \quad (3-8)$$

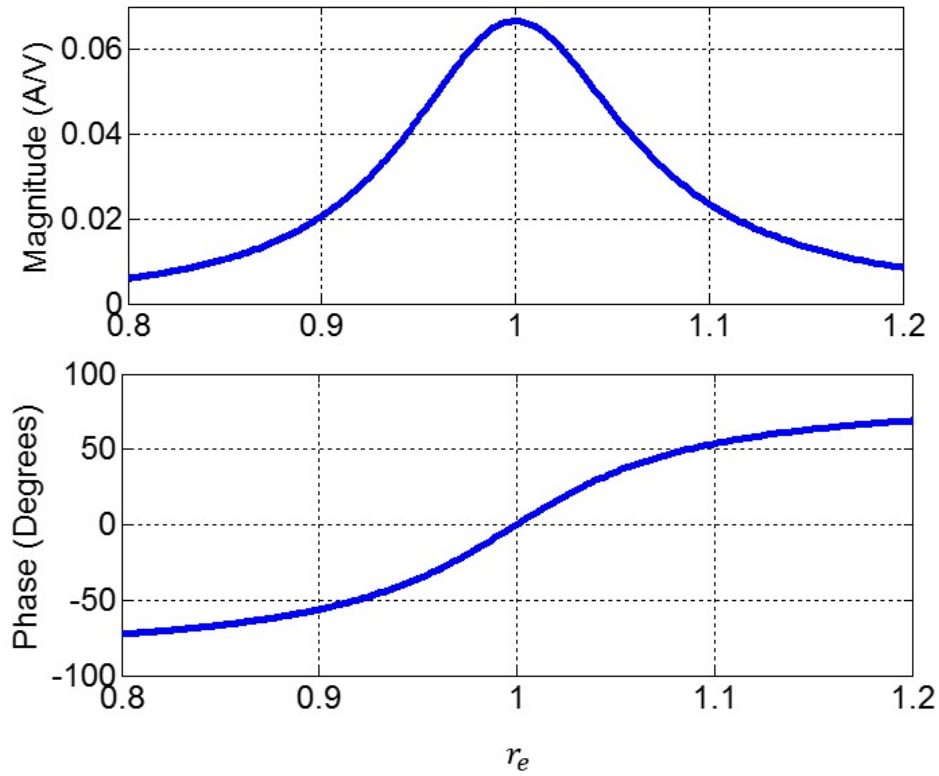


Figure 3.12: Magnitude and phase plot of an RLC circuit

The magnitude and phase plot of the inductance current to the excitation voltage is shown in Figure 3.12. It is apparent from the figure that if the ratio of the excitation frequency is

Figure 3.13, to alternate the current with the stator winding. Polar capacitors cannot be used for this application because their reverse polarity threshold voltage value is very low. The value of the capacitor is designed in such a way that the resonance frequency of the

electrical circuit is equal to the mechanical resonance frequency. The reason for resonance frequency matching is discussed in Chapter 5. As the inductance and the resonance frequency of the MMLA are known, the value of the motor capacitor is calculated by Equation (3-3). The variation in the excitation frequency affects the performance of the MMLA. Therefore, a precise AC source is required to power up the RLC circuit. The AC power source from Kikusui (PCR1000LE) shown in Figure 3.13 is used. It is variable amplitude and frequency power source with a resolution of 0.1 V and 0.01 Hz. The voltage ripple noise is less than 0.2 Vrms, whereas the error in the frequency is within $\pm 0.2\%$.

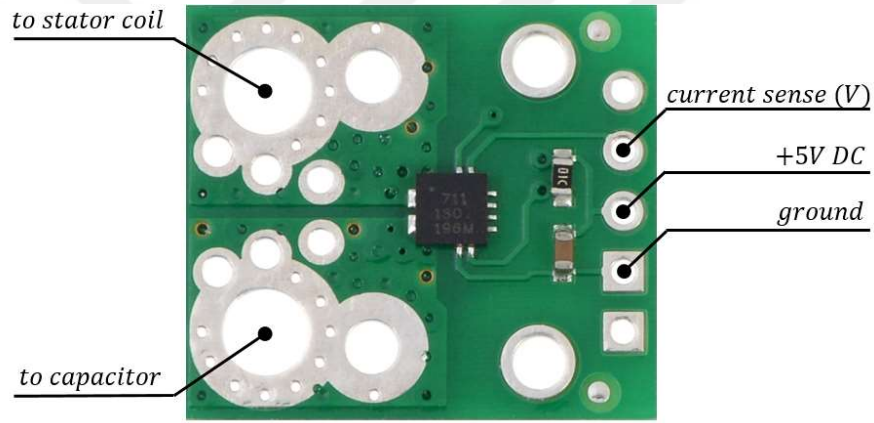


Figure 3.15: ACS714 based current sense printed circuit board from Pololu [31]

3.5.3 Current Sensing

In order to investigate the current response of the MMLA, the real-time current needs to be monitored. Therefore, Pololu® ACS714 Current Sense Carrier piggy pack shown in Figure 3.16, is used. ACS714 is a hall effect based current sensor by Allegro®. It is used in series with the stator coil and provides a very low impedance of approximately 0.0012Ω . As discussed earlier, the resistance in the RLC circuit acts as a damping parameter at resonance. The low impedance of the current sensor ensures the real-time measurement of the current without affecting the damping of the circuit.

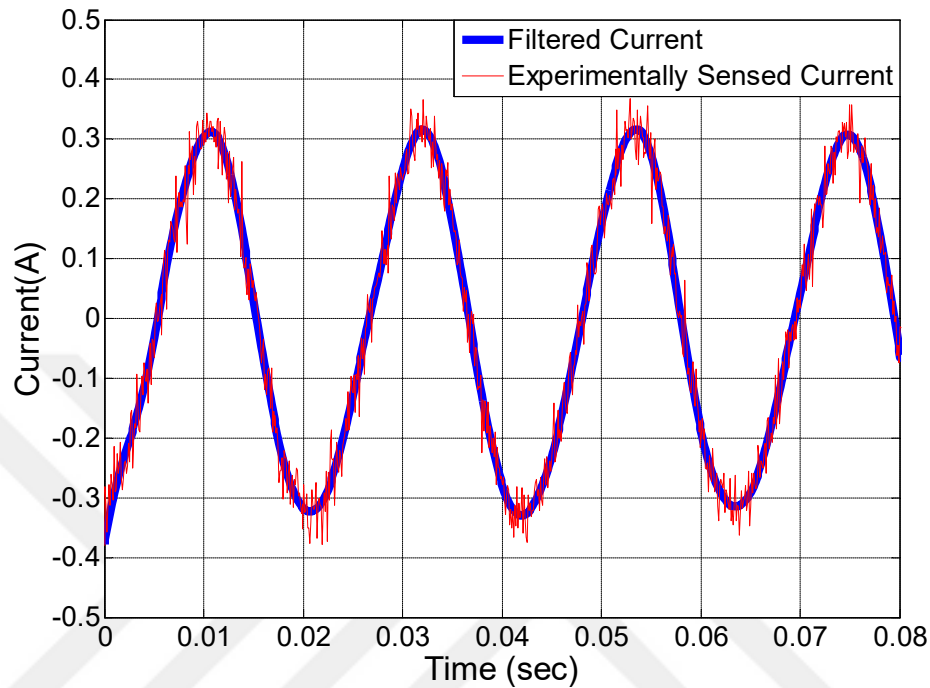


Figure 3.16: Experimentally recorded current time curve of the stator coil current

The printed circuit board of the piggy pack is shown in the figure. The board is divided into power zone (left) and the signal zone (right). The two terminals of the power zone are connected in series with the RLC circuit. One of the connections is connected to one of the stator winding terminals whereas the other is connected with one of the capacitor terminals. The board needs an external power source of 5 V and provides the analog output of the current sense that is recorded through DAQ. The analog voltage linearly corresponds to the sensed current of ± 5 A to 0-5 V with the gain of 185 mV/A. The recorded current is shown in Figure 3.16

3.5.4 Armature Position Sensing

A hall effect sensor IC A1324UA is used to sense the real-time position of the armature. This IC is used in combination with an axially magnetized permanent magnet. A small

magnet is mounted on the moving body and the IC is mounted on the stationary part. As this IC works on the principle of hall effect sensing, the amount of magnetic flux passing through the IC is unique throughout the armature positions while oscillations. Therefore, the position of the armature is calibrated with the analog output of the IC and a sense of the real-time position of the armature is estimated. The hall effect sensing IC consists of three pins as shown in

Figure 3.17. It is powered up with a 5 V DC power supply. The sensitivity of the hall effect sensing IC is the most important parameter for selection. It is provided as a voltage to magnetic field ratio mV/G. The bandwidth of the sensor is 17 kHz and the operational temperature range of -40 to 150 °C. The calibration of the sensor is done with a 20 mm diameter cylindrical Neodymium Magnet. An experiment is performed to estimate the relationship between voltage and position. Equation (3-9) provides the relationship between the selected magnet and the voltage values.

$$x = 9.88e^{-1.55v} \quad (3-9)$$

where x is the position mm and v is the voltage V. The x is considered zero when there is no gap between the IC and the permanent magnet.

A printed circuit board is designed to drive the moving magnet linear actuator as shown in Figure 3.18. It includes all the features discussed above as well as inductor and capacitor real-time voltage sensing, connections for laser displacement sensor, in-line and cylinder pressure sensor interfaces, a serial communication port for controlling the AC power source electronically. There is an Arduino® Mega board mounted on the PCB as shown for communication between the graphical user interface and display the real-time information about the moving magnet linear actuator on a single screen. Additionally, the controller is also monitoring the current of the stator coil. The RLC circuit is excited through the PCB and there is a solid-state relay in series with the circuit. If the current exceeds the maximum RMS value, the solid-state relay opens the series circuit.

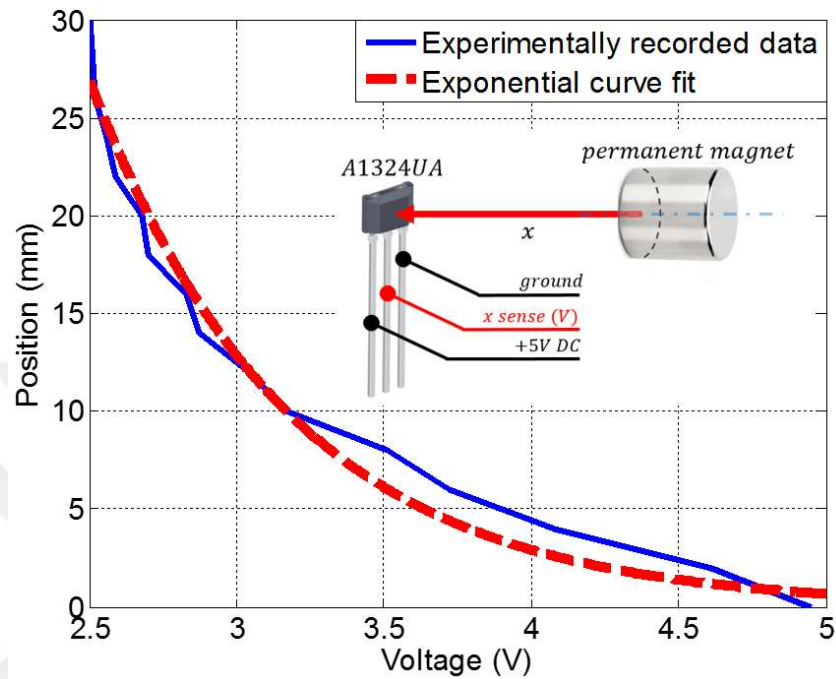


Figure 3.17: Experimentally recorded position-time curve of the armature position

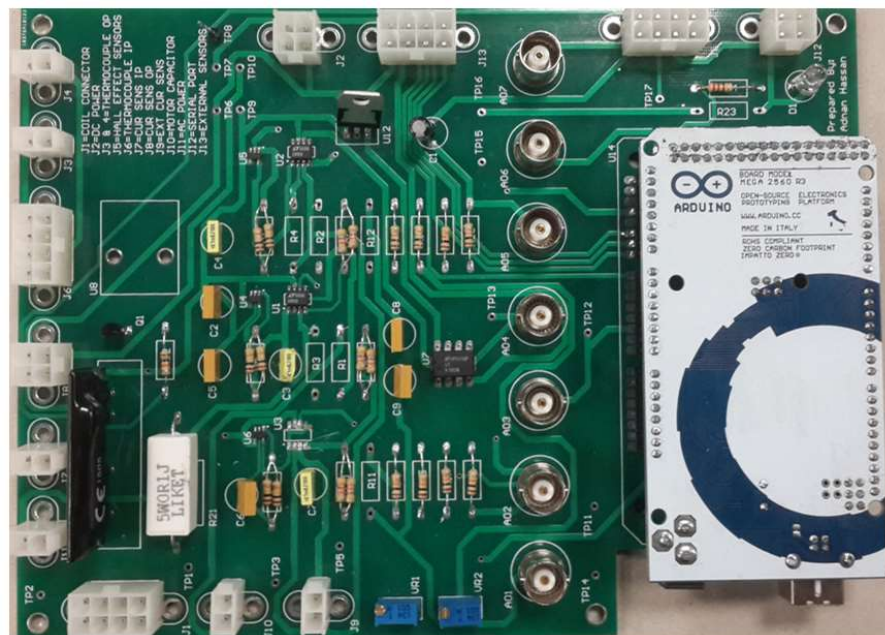


Figure 3.18: Printed circuit board for driving the moving magnet linear compressor

Chapter 4

MOVING IRON LINEAR ACTUATORS

4.1 Overview

A solenoid based linear actuator is proposed for linear oscillations in linear compressors. As discussed in section 3.5.1, switching control of the stator coil enables back and forth motion of the armature. Line pressure, oscillation frequency and stroke position provide the baseline for the magnitude of the electromagnetic force, which defines the geometric and the coil parameters of the solenoid. Reluctance Method is used in this study to establish the relationship of the actuator geometry, coil parameters and the resulting magnetic force. Considering the nonlinearity of magnetic materials, a finite element analysis is presented to estimate the magnetic force in stationary and time dependent domains for the static and the dynamic characteristics of the solenoid. An experimental setup is used to validate the static and dynamic characteristic responses of the actuator. The effect of current and stroke positions on the electromagnetic force is also simulated and verified experimentally. Finally, frequency response function is acquired through an impact hammer and accelerometer. The results are validated with electrically actuated solenoid at different frequencies against forces and current values to obtain optimal points of excitation frequency for the better performance.

4.2 Literature Review

A solenoid transforms the electrical energy (voltage and current) first into magnetic energy and then it converts it into the mechanical energy (force and displacement) plus the frictional and the heating losses. The conversion of this energy is dependent upon the geometrical parameters of the actuator. A solenoid is composed of a stator (core and coil) and an armature [32]. Both armature and stator core are magnetic materials to provide a

closed loop path for the magnetic flux lines. The details of solenoid design parameters are presented in [33] considering geometric factors, aspect ratios and filling factors. Additionally, Force-stroke and work-stroke curves are discussed. Parameters and bottlenecks necessary for scaling of the actuator are also presented. A Reluctance Method is used to obtain the magnetic force acting upon the actuator. The dynamic response of the solenoid is presented in [32], where an iterative FEA is used to compute the magnetic field to estimate the electromagnetic force. Consequently, using basic dynamics principles, the stroke of the solenoid is calculated. A new instance is created and the iterative loop is repeated to get the new forces and displacements. In the case of a fast-acting low stroke linear actuators, a transient response is necessary to understand the dynamic behavior of actuator [34]. The coupled electromechanical system is modeled and FEM is used to estimate the time-dependent behavior of the actuator and results are validated.

Magnetic materials are extremely non-linear in nature and nonlinear analytical modeling is relatively difficult. Therefore, most of the researches are performed using FEM software to evaluate the magnetic field. Additionally, the inductance changes nonlinearly as the armature moves inside the stator. A nonlinear inductance model of a solenoid actuator used in starter motor is presented by [35], where the inductance is modeled as a function of stroke and current by using the energy method. Considering the wide industrial usage of the solenoids for various applications, several models and techniques are developed to model the static and dynamic parameters of the solenoid [88]-[98].

Solenoid linear actuators are widely used in industries other than refrigerator compressors such as humanoid robots, pneumatics, aerospace industry and automotive industry. Similar models and techniques are used to evaluate the linear force. For instance, a fast switching electro-pneumatic valve is investigated in [36], [37] where the force acting on a solenoid valve is modeled as a function of valve parameters, air gaps and displacements. A nonlinear

solenoid controlled valve model is implemented by [38] in a solenoid controlled pilot operated servo valves. A nonlinear model to define a force and flux relationship is presented.

Design optimization is also vital for achieving better efficiency of the actuator. An optimal design method for deriving the equation based on empirical method is presented by [39]. With the help of the electromagnetic theories, the design parameters of a solenoid are optimized to minimize the power consumption and maximizing efficiency. FEM and numerical optimization technique are used by [40] to analyze the effect of different plunger (armature) shapes. A moving line technique to save the computation time is proposed.

This chapter is based on the studies related to the solenoids specifically designed to achieve oscillations in linear compressors. It is comprised of solenoid design and compilation of the governing equations to estimate the force on the armature in both static and dynamic states. In the beginning, the geometric and design parameters are characterized that should be considered for designing the linear actuator. Furthermore, the nonlinear stationary and time dependent FEM simulation are executed and the results are validated with experimental setups containing orthogonal force measuring dynamometer, tri-axial accelerometer, laser displacement sensors and impact hammer. Overall system dynamics are discussed and system identification is presented with the help of the conventional step response for identifying the damping coefficients, natural frequency and damping ratio. As the magnetic force changes with the stroke as well as a change in current also affect the force magnitude. Therefore, simulations and experimental validation for different current and stroke length are executed. In the end, frequency response function is presented and the optimal points of excitation frequency are suggested for achieving higher efficiency.

4.3 Solenoid Based Linear Compressor

A linear compressor is composed of three major parts: a linear actuator, spring assemblies and air/gas compression chamber as shown in Figure 4.1. The moving iron linear actuator when energized with the amount of current, pulls the armature causing the spring to extend

and compressing the gas/air, called the exhaust stroke. While the actuator is turned off, the spring pulls the armature back, creating a vacuum in the chamber and sucking air inside the chamber, called the intake stroke. By repeating the intake and exhaust strokes, the air/gas is taken in and compressed repeatedly, maintaining an exhaust line pressure.

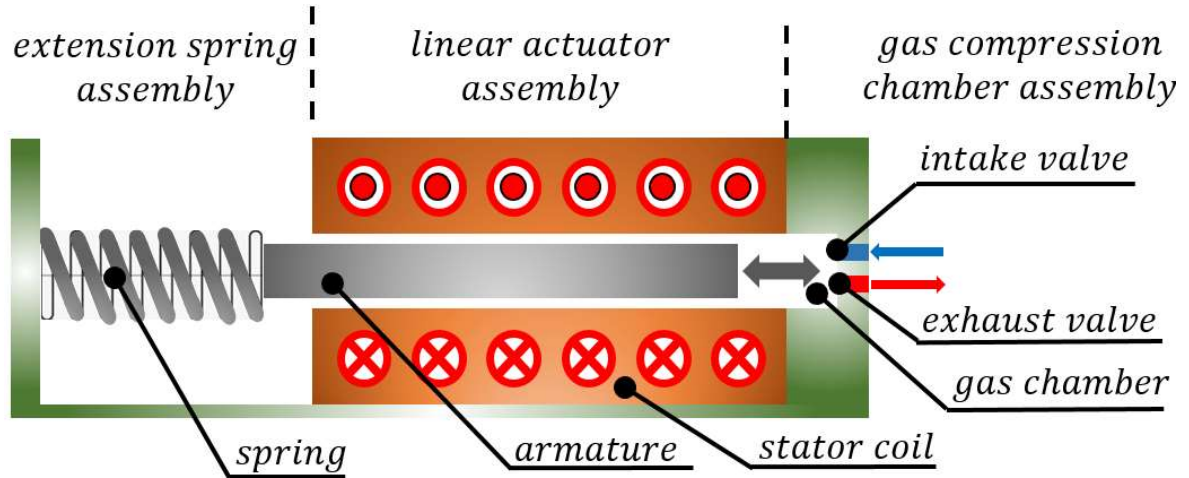


Figure 4.1: Linear compressor schematic

When the armature tries to move forward, the spring and gas in the chamber try to push it in the back direction. Therefore, the overall dynamics of the system are expressed in terms of Equation (4-1):

$$F_{mag} = F_{inertia} + F_{spring} + F_{pressure} + F_{damping} \quad (4-1)$$

4.3.1 Analytical Modeling with Energy Method

The energy method [41] uses the conservation of energy principle which states that the difference of the electrical energy consumed and the mechanical output of the system are equal to the stored energy. The electrical power input P_e to the system is written as:

$$P_e = VI = i \frac{d\lambda}{dt} \quad (4-2)$$

where V is the voltage, I is current, t is the time and λ is the flux linkage. Flux linkage is the total magnetic field passing through the surface. Similarly, the mechanical power output P_m of the system is written as:

$$P_m = F \frac{dx}{dt} \quad (4-3)$$

where F is the external force acting on the system and x is the displacement of the armature. According to the definition of the conservation of energy, the energy stored in the solenoid is equal to the difference between the electrical and mechanical energies and is written as:

$$\frac{dE}{dt} = P_m - P_e = F \frac{dx}{dt} - i \frac{d\lambda}{dt} \quad (4-4)$$

where E is the energy stored in the system. As there are two variables in the system: flux linkage and displacement, the above equation is written as:

$$dE = Id\lambda - Fdx \quad (4-5)$$

By taking the partial derivative with respect to x Equation (4-5) becomes:

$$F = \frac{-\partial E}{\partial x} = \frac{1}{2} I^2 \frac{\partial}{\partial x} L(x) \quad (4-6)$$

where L is the inductance and it is the function of the position of the armature.

4.3.2 Analytical Modeling with Reluctance Method

As discussed earlier, the magnetic flux lines always try to find the shortest reluctance path. The overall magnetic flux loop is divided into finite reluctance zones and the reluctance

of each zone is calculated. Then with the help of the magnetic circuit analysis, the resistances are added in series or parallel to find the magnetic flux.

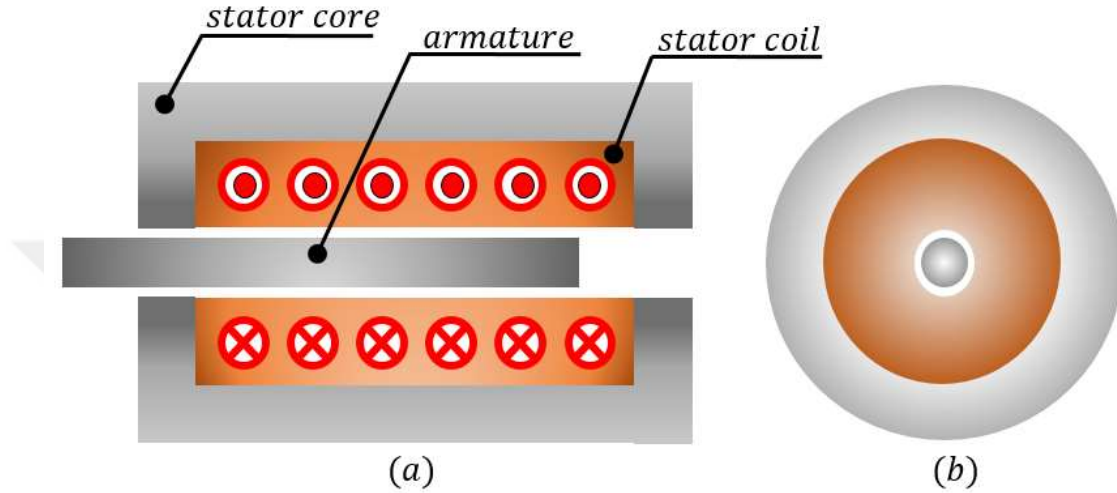


Figure 4.2: (a) Cross-section schematic view of linear solenoid actuator (b) side view cross-section schematic of moving iron linear actuator

In order to calculate the spring force, the relationship between the magnetic force, geometric parameters of the actuator and current input must be established. In this section, the linear magnetic actuator model is established incorporating all geometric and control parameters. A simple solenoid actuator is used for this purpose, which is composed of three parts: Coil, static core and armature as shown in

Figure 4.2. When the coil is energized by certain voltage, the current passing through the coil produces the magnetic field around it. Thus, magnetic flux lines are generated that make a closed loop circuit, referred as reluctance path or loop. The armature tries to move into such a position in which the flux lines encounter the least reluctance. Consequently, a linear force or magnetic force comes into existence, which is used for linear motion applications. This section establishes a direct measure of the linear force from the geometric parameters

of the actuator and the input power applied to the coil. Moreover, the effect of solenoid coil parameters is discussed.

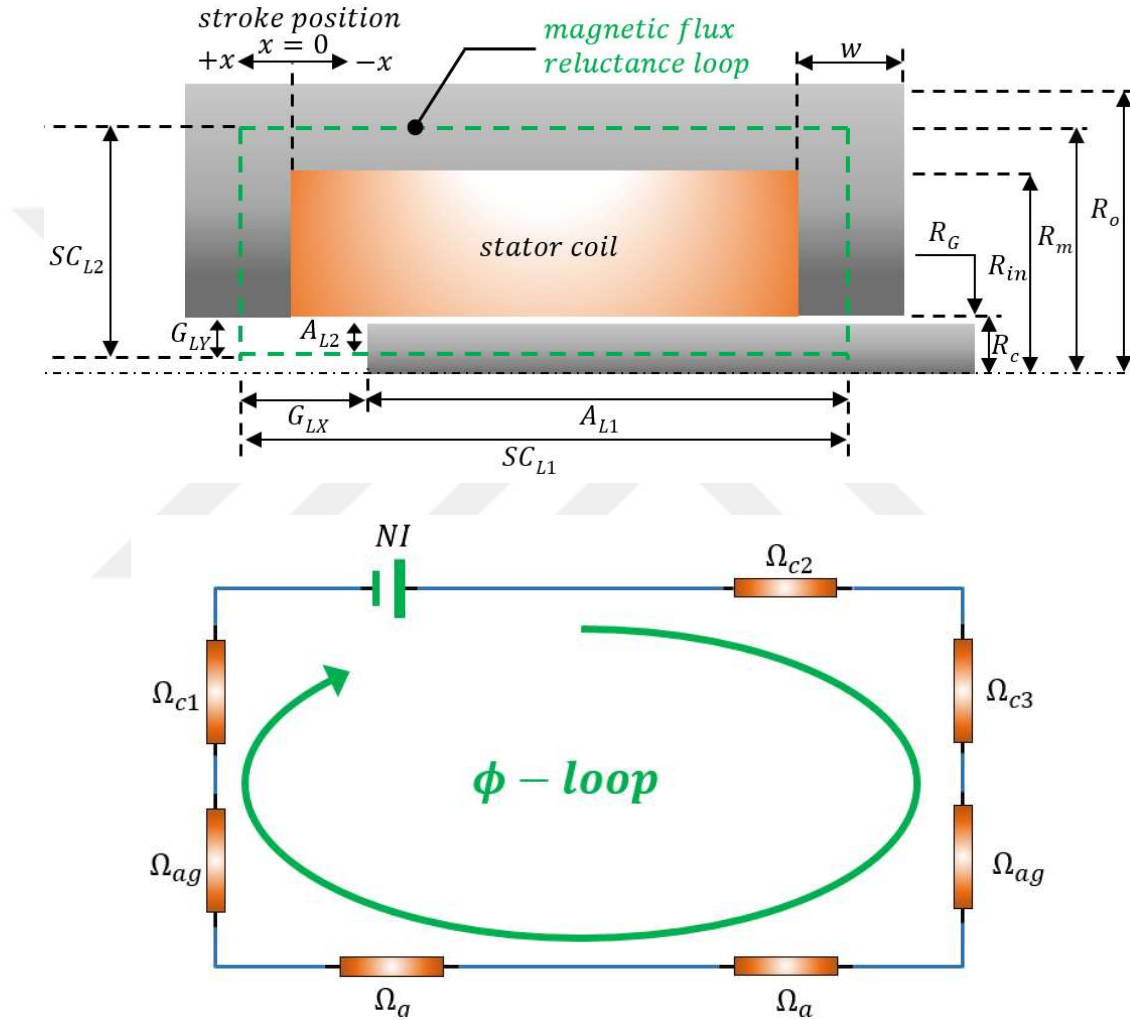


Figure 4.3: Geometric parameters in axis symmetric linear magnetic actuator and the magnetic circuit loop

Figure 4.3 describes the geometric parameters of the axis-symmetrical linear actuator. In this figure, A_{L1} is the armature length 1, A_{L2} is the armature length 2, SC_{L1} is the static core

length 1, SC_{L2} is the static core length 2, G_{LX} is the air gap length along x-axis, G_{LY} is the air gap length along y-axis, R_c is the radius of the armature chamber, R_i is the static core inner diameter, R_m is the static core middle diameter, R_o is the static core outer diameter and w is the width of static core components. The overall reluctance of the flux loop is equal to the sum of the reluctance of armature, static core, air gap and the gap between the armature and static core.

$$\Omega = \Omega_a + \Omega_c + \Omega_{ag} + \Omega_g \quad (4-7)$$

where Ω is the overall reluctance, Ω_a is the reluctance of armature, Ω_c is the reluctance of static core, Ω_{ag} is the reluctance of air gap, and Ω_g is the reluctance of gap between the armature and static core. The following expression defines the reluctance encountered by flux lines passing through any medium:

$$\Omega = \frac{\text{the length of the medium}}{\text{the permeability of vacuum} \times \text{relative permeability} \times \text{area}}$$

Therefore,

$$\Omega_a = \frac{A_{L1}}{\mu_o \mu_r \pi w (R_c^2)} + \frac{A_{L2}}{\mu_o \mu_r w \pi (R_c)} \quad (4-8)$$

$$\Omega_c = \frac{2SC_{L2}}{\mu_o \mu_r \pi w (R_c + R_i)} + \frac{SC_{L1}}{\mu_o \mu_r \pi (R_c^2 - R_i^2)} \quad (4-9)$$

$$\Omega_{ag} = \frac{R_G}{\mu_o \pi w R_c} \quad (4-10)$$

$$\Omega_g = \frac{G_{Lx}}{\mu_o \pi w R_c} + \frac{G_{Ly}}{\mu_o \pi R_c^2} \quad (4-11)$$

Table 4-1: Design parameters of solenoid geometry

<i>Symbol</i>	<i>Quantity</i>	<i>Value</i>
SC_{L1}	stator core length 1 (mm)	38.2
SC_{L2}	stator core length 2 (mm)	95.4
G_{LY}	air gap y length (mm)	6.6
G_{LX}	air gap x length (mm)	23-x
A_{L1}	armature length 1 (mm)	$SC_{L2} - G_{LX}$
A_{L2}	armature length 2 (mm)	6.25
R_O	the outer radius of the actuator (mm)	64.1
R_C	the inner radius of the cylinder (mm)	13.75
N	number of turns	1000

Depending on the geometry of the actuator, the reluctance of magnetic flux loop is established. The reluctance of Ω_a , Ω_c , Ω_{ag} are constant because they are stationary and their geometric values are not changing during oscillations as shown in Equation (4-8) to (4-10). However, once the coil is energized, due to the linear force, the armature moves. Thus, G_{LX} changes, which changes the reluctance of the air gap as shown in Equation (4-11). Flux is calculated from reluctance by Equation (4-12):

$$\varphi = \frac{NI}{\Omega} \quad (4-12)$$

where N is the number of turns of the coil, I is the current passing through the coil. Flux density is defined by Equation (4-13);

$$B = \frac{\varphi}{\pi R_c^2} \quad (4-13)$$

The linear force of the armature is estimated by Equation (4-14):

$$F = \frac{B^2 A}{2\mu_0} \quad (4-14)$$

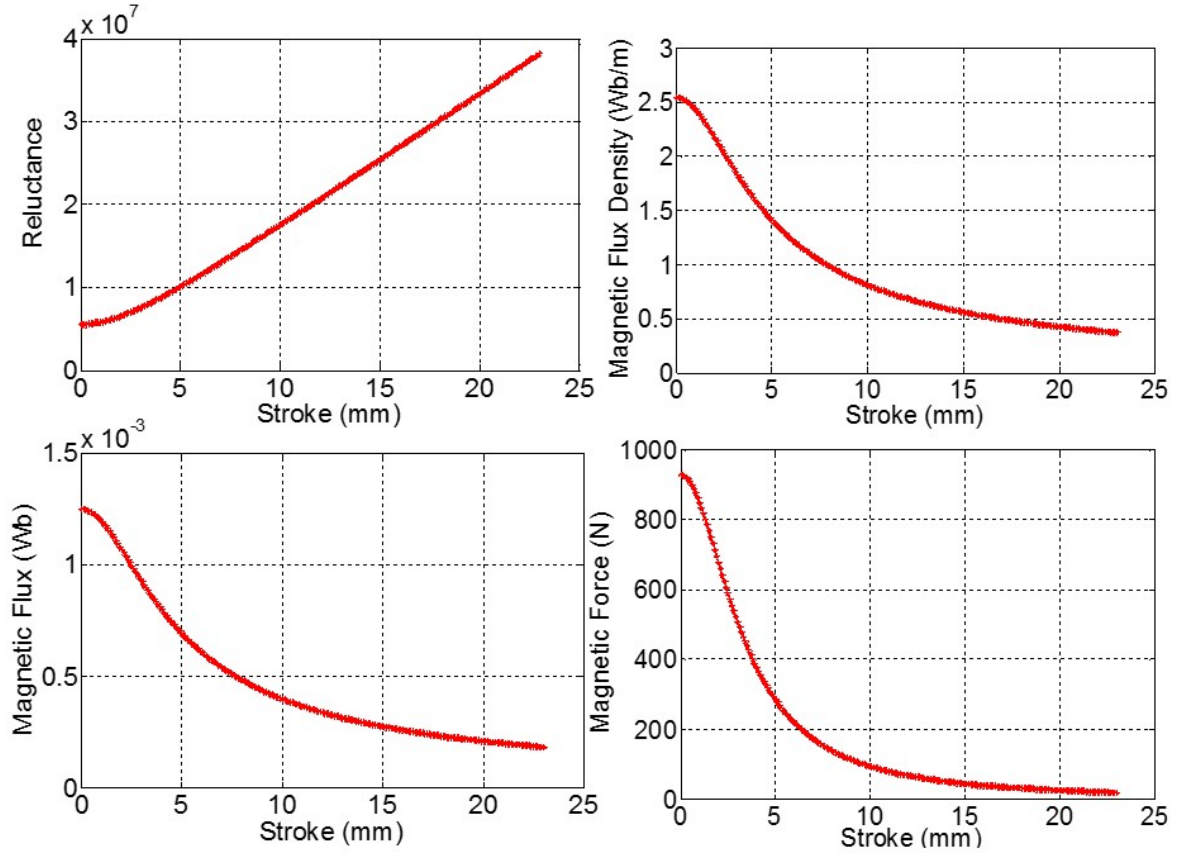


Figure 4.4: Matlab simulation results for a) change in reluctance with respect to stroke position b) change in magnetic flux density with respect to stroke position c) change in magnetic flux with respect to stroke position d) change in magnetic force with respect to stroke position

Selecting appropriate geometric parameters lead to electromagnetic force estimation for the required application. Equation (4-8) to (4-14) represent, all the geometric parameters, number of turns of the coil, current and provided electrical power are affecting the resultant

force. Such a model helps in designing an actuator. Stroke is defined as the distance traveled by the armature once it is energized. As shown in

Figure 4.3, the stroke position is zero in the middle, positive towards the left and negative towards the right. A change in stroke length changes the reluctance of the overall flux loop. Consequently, the linear or magnetic force changes.

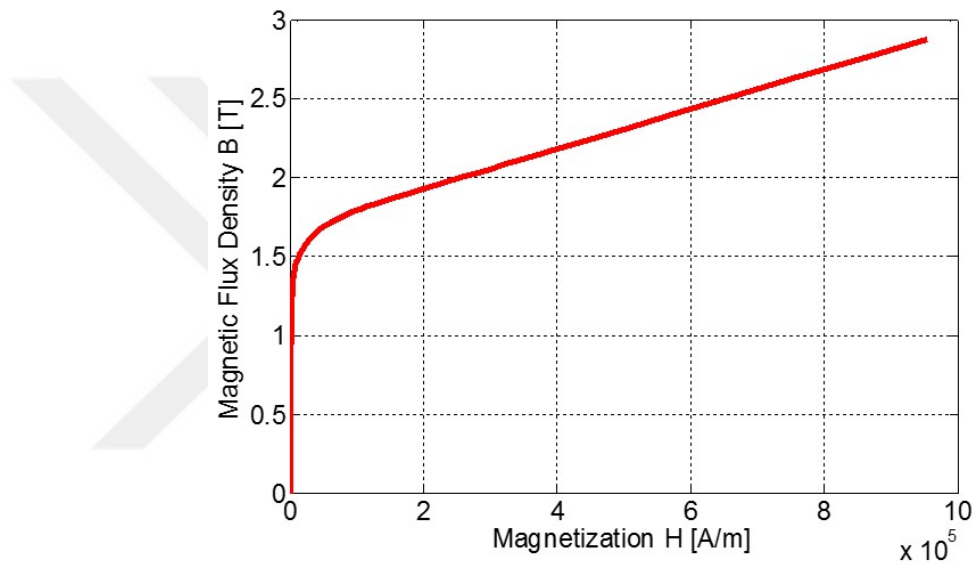


Figure 4.5: BH Curve from Comsol for stainless steel 430

The parameters of the solenoid are modeled as shown in

Table 4-1. A numerical simulation is carried out with the stroke values ranging from 2 to 23 mm and the results are shown in Figure 4.4. As the piston is near to 0 mm of stroke position, minimum reluctance and maximum magnetic flux density, magnetic flux and the electromagnetic force is estimated. The equations shown are conceptually true but there is a nonlinearity introduced into the system due to the magnetic material properties. Magnetic materials provide least resistance to the magnetic flux lines and the property which allow magnetic flux to flow through the material is called permeability. The permeability of free

space or air μ_o is 12.57×10^{-7} H/m. Materials with permeability other than air are represented by relative permeability defined as:

$$\mu_r = \frac{\mu}{\mu_o} \quad (4-15)$$

The material selected for the stator core and armature is Stainless Steel 430 and the BH curve for the selected material is implemented in finite element simulation. The BH curve is shown in Figure 4.5

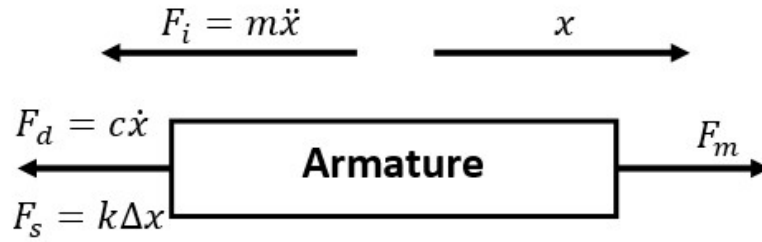


Figure 4.6: Free body diagram of the armature

4.3.3 Dynamic Analysis

In order to oscillate the armature, it is attached with a spring of stiffness k and the spring is attached to a stationary support on the other side. A free body diagram of the armature at a stroke length x is shown in Figure 4.6. At any instant, stroke length is x due to the magnetic force F_m and it should overcome the inertial force F_i , damping force F_d and spring force F_s to move the armature in the direction of x . This defines the dynamic equation of the armature as shown in Equation 4-16.

$$F_m = m\ddot{x} + c\dot{x} + k\Delta x \quad (4-16)$$

where m , c and k are the mass of the armature, the damping coefficient of the system and the spring stiffness respectively. This equation implies the sum of inertial, spring and damping

force defines the magnetic force at any instant. For any mass, spring and damper system, the damping coefficient is calculated by using Equation 4-17:

$$c = 2\zeta\sqrt{mk} \quad (4-17)$$

4.4 Mechanical Model

The 3D mechanical model is composed of the following components (all dimensions are in millimeters):

4.4.1 Coil Holding Tube

The coil holding tube is 1 mm thick with the inner diameter of 25 mm and the outer diameter of 27 mm as shown in Figure 4.7. There are 0.5 mm collars on the sides to make this component sit in the overall actuator assembly. This component performs as a holder for the coil as well as it works as a linear guide for the armature. As this part is not participating in the magnetic circuit, this component should be non-magnetic, otherwise, all the magnetic flux will pass through it. Therefore, aluminum is used as a material for this part.

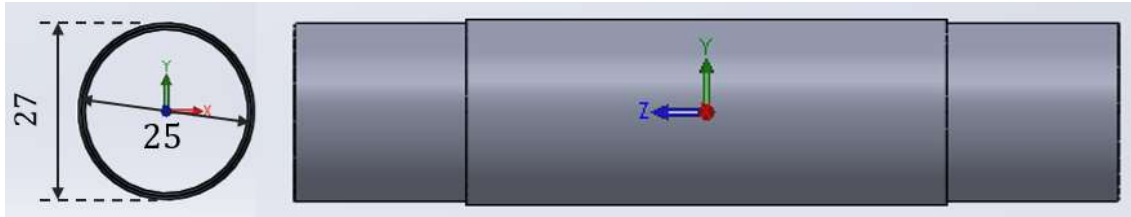


Figure 4.7: CAD model of coil holding tube of the moving iron linear actuator

4.4.2 Stator Winding

The stator winding is 18 AWG (1 mm diameter) copper wire wound around a central symmetrical axis in a circular fashion. There are 1000 number of turns and the geometric parameters are shown in Figure 4.8. The coil inductance is 25 mH and the resistance of the

coil is $3\ \Omega$. The response time of the coil is 7.5 msec. This winding is wound around the coil tube tightly to avoid the air gap between the wire for achieving better filling factor. Use of adhesive after each layer of wires is recommended to have a better binding for making the coil rigid to overcome the vibrations in the actuator.

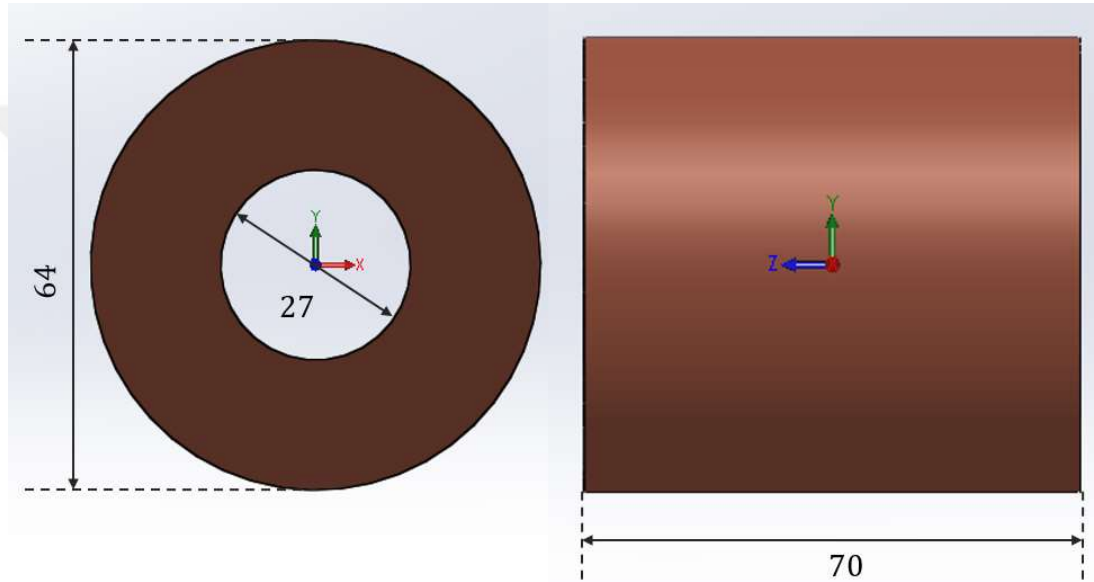


Figure 4.8: CAD model of stator winding of the moving iron linear actuator

4.4.3 Side Plates

Side plates are designed to hold the stator winding as shown in Figure 4.9. The coil holding tube sits inside the location (A) whereas the stator windings are placed on the location (B). Two side plates are used to hold the stator winding from each side. Location (C) holds the other side of the stator winding. Location (D) are the positions for the screws to hold the coil cover. The material for the side plates must possess magnetic properties. Therefore, Ferromagnetic Stainless Steel 430 is used.

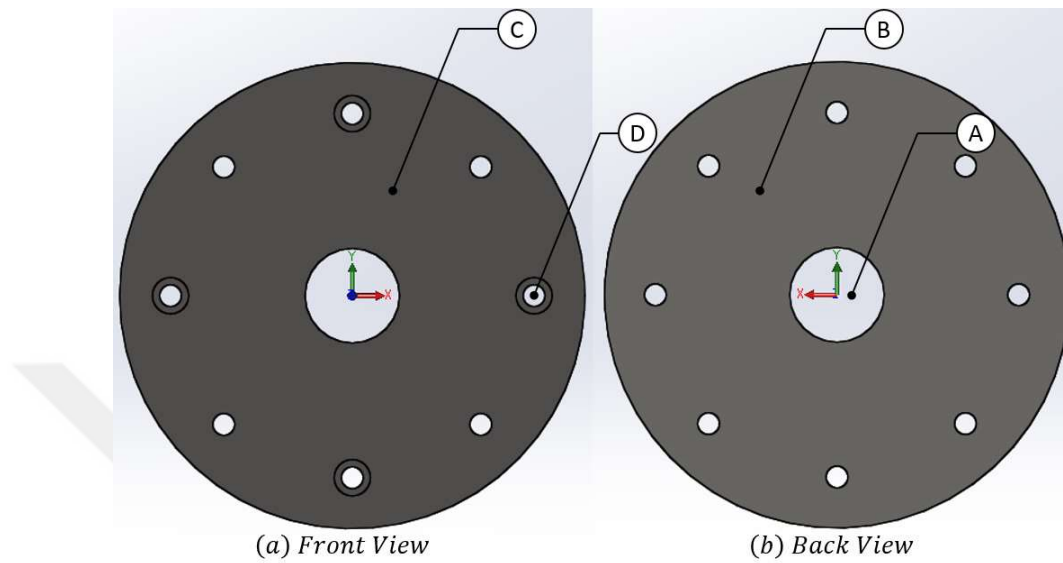


Figure 4.9: CAD model of side plates of the moving iron linear actuator

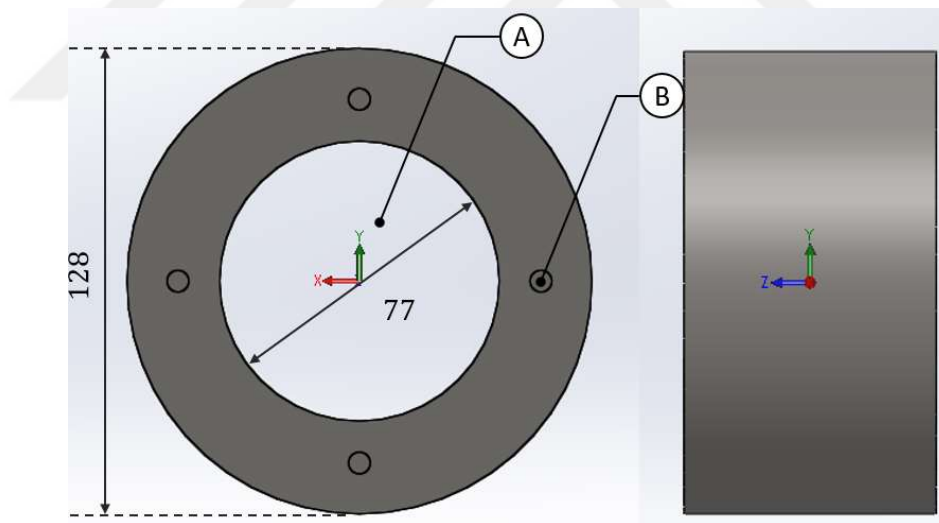


Figure 4.10: CAD model of coil cover of the moving iron linear actuator

4.4.4 Coil Cover

Coil cover is the hollow cylindrical part with the geometric parameters shown in Figure 4.10. The length of the coil cover is same as the coil. The function of the coil cover is to

provide the return path for the electromagnetic flux. Same material as the side plates is used because of the requirement of the magnetic properties for the electromagnetic flux. The overall stator assembly is shown in Figure 4.11.

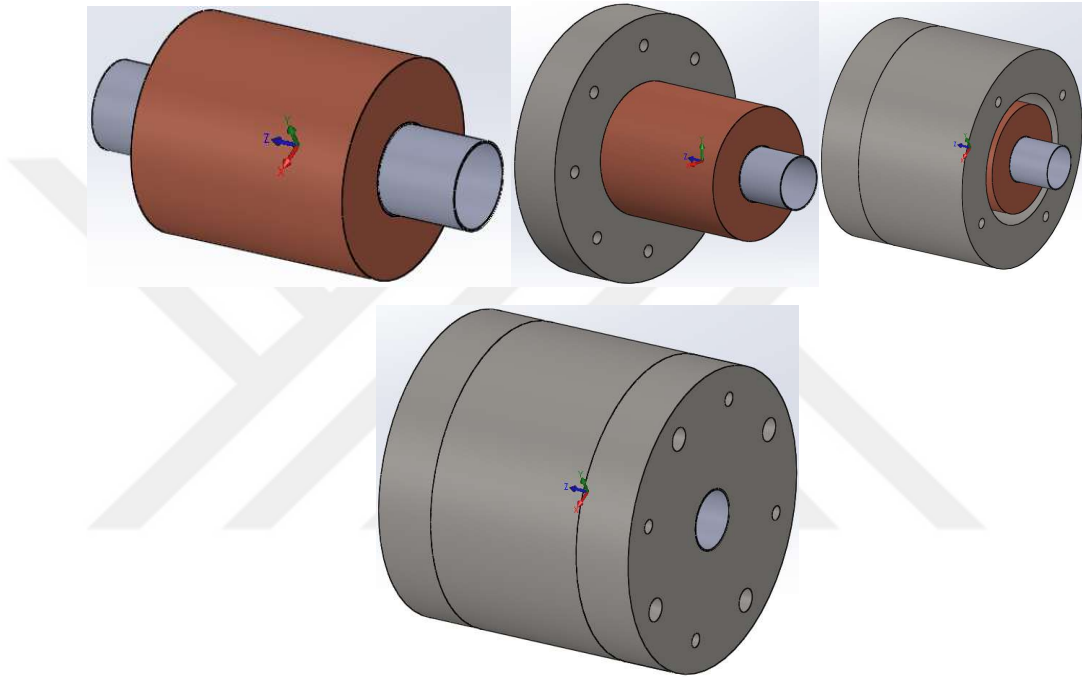


Figure 4.11: Assembly process of the stator of the moving iron linear actuator

4.4.5 Support Plates

Support plates are required to mount the stator assembly on the base plate. The design is shown in Figure 4.12. As there is no magnetic circuit involved and the purpose of these plates is to support the stator, therefore aluminum is used for these plates. The stator assembly sits at the location (A) and the screws are mounted at location (B).

The overall linear actuator assembly is shown in Figure 4.13. A 25 mm stainless steel 430 piston is used as shown in the figure. This piston is used as an armature for the actuator as well as for the compression of the gas.

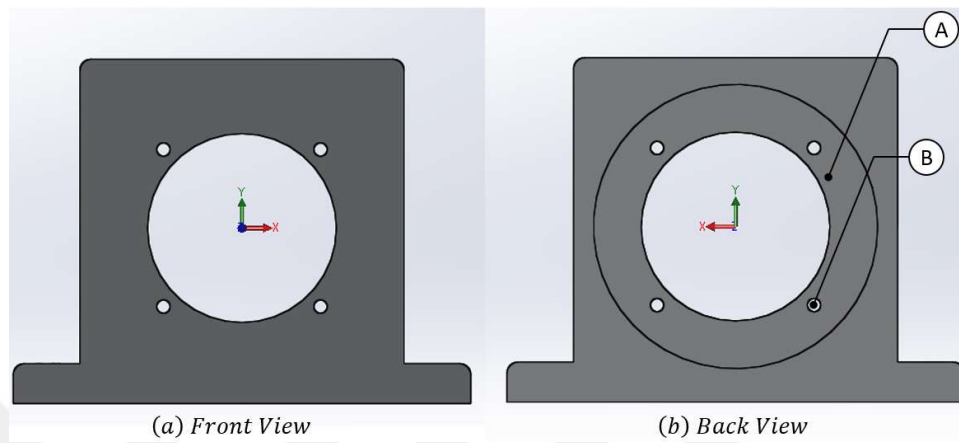


Figure 4.12: CAD model of the support plate of the solenoid actuator

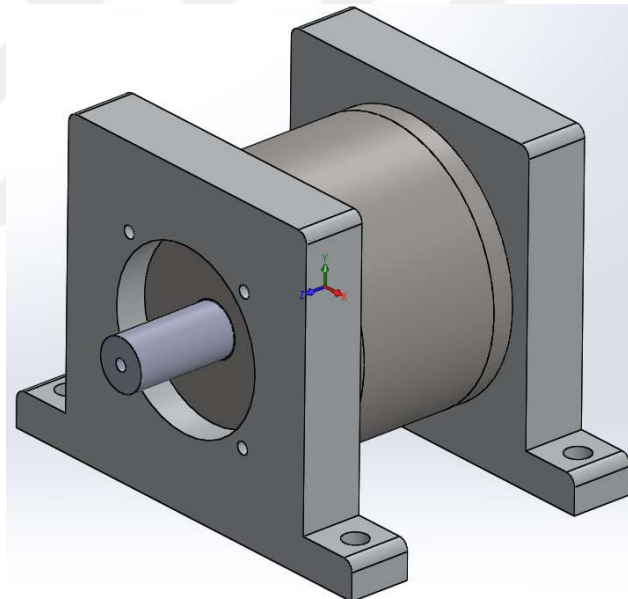


Figure 4.13: 3D CAD model of the moving iron linear actuator

4.5 Finite Element Simulation

Finite element method is used to estimate the electromagnetic force [42]. As the actuation involves electromagnetism, Magnetic field (mf) physics module is used. As the actuator is completely symmetric, simulation is done in 2-D axis-symmetric space dimension. Same

geometry is built in FEM environment with 1000 number of turns and three constant current values of 3, 4 and 5 A are simulated. Same material discussed above is used and material nonlinearity is considered by implementing BH curve. The stationary solution is computed using Maxwell-Ampere's Law and Faraday's law given in the following equations:

$$J_e = \frac{NI_{coil}}{A} e_{coil} \quad (4-18)$$

$$\nabla \times \left(\frac{B}{\mu_0 \mu_r} \right) - \sigma v \times B = J_e \quad (4-19)$$

$$B = \nabla \times A \quad (4-20)$$

where J , N , I_{coil} , A , e_{coil} , B , μ_0 , μ_r , σ and v are current density, number of turns, coil current, potential, magnetic flux density, relative permeability in vacuum, relative permeability of material, electric conductivity and velocity, respectively. The magnitude and direction of force is estimated in Finite Element Simulation using the following equation:

$$F = \int_{\partial\Omega} 2\pi r n T dS \quad (4-21)$$

where r , n and T are position vector, normal unit vector and stress tensor, respectively. Standard meshing is used as shown in Figure 4.14 with a maximum element size of 6 mm, minimum element size of 0.0225 mm, maximum element growth rate is 1.2 and curvature factor of 0.25.

For dynamic analysis, an FEM analysis is performed to estimate the magnetic force which is validated with the experimentally measured inertial, damping and spring forces. The time-dependent analysis is executed with the same configurations explained in the stationary analysis. In contrast with these settings, the solver is time dependent and instead of the

constant current, the time response of inductor over the switching voltage is used. Similar meshing is used and 200 msec time-dependent simulation is run with 1 msec time resolution.

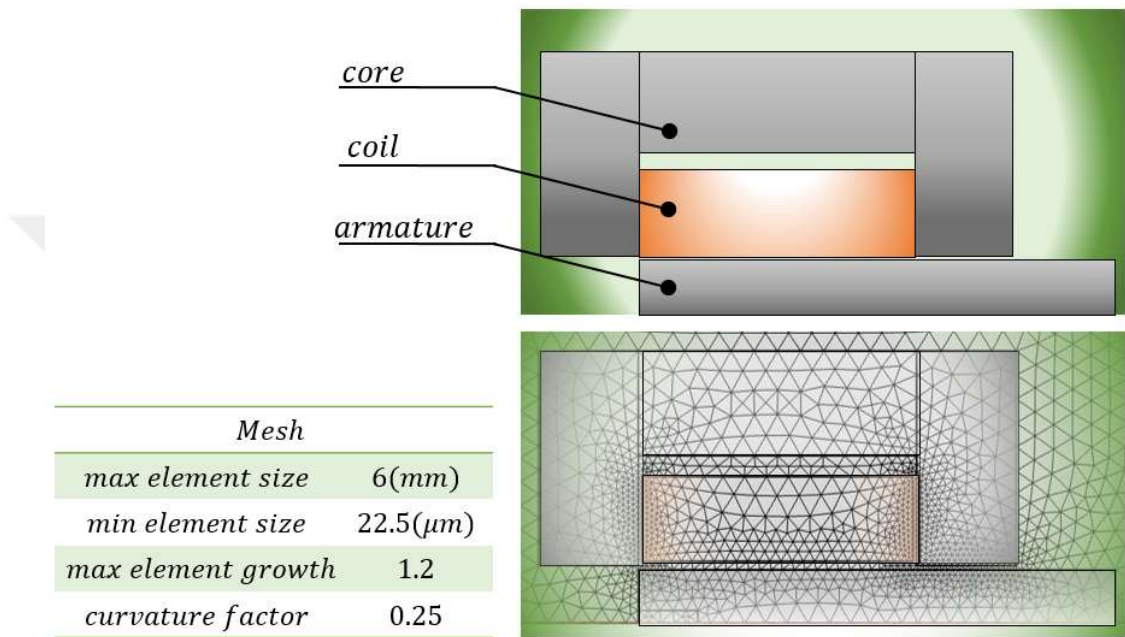


Figure 4.14: Finite element simulation for axis-symmetrical geometry and meshing sizes

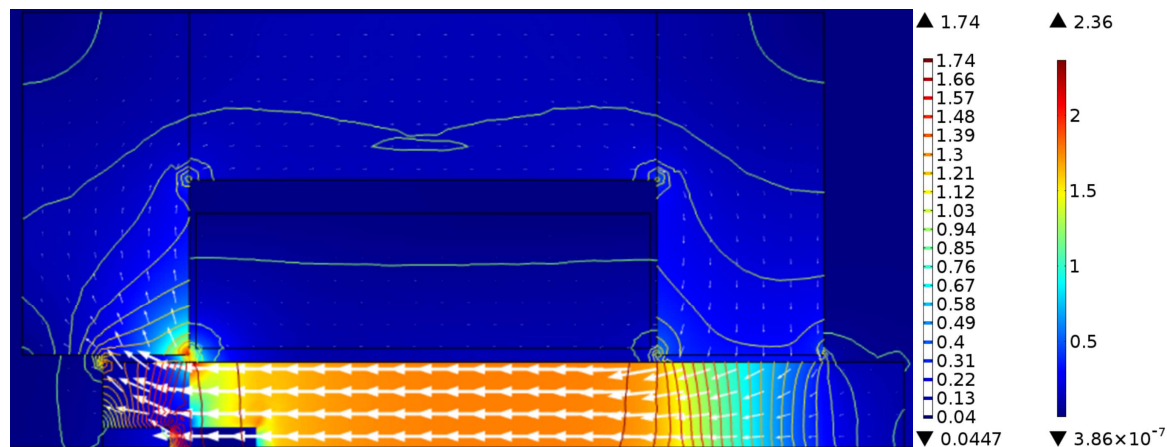


Figure 4.15: Magnetic flux density T for stroke = 0 mm for current of 3 A

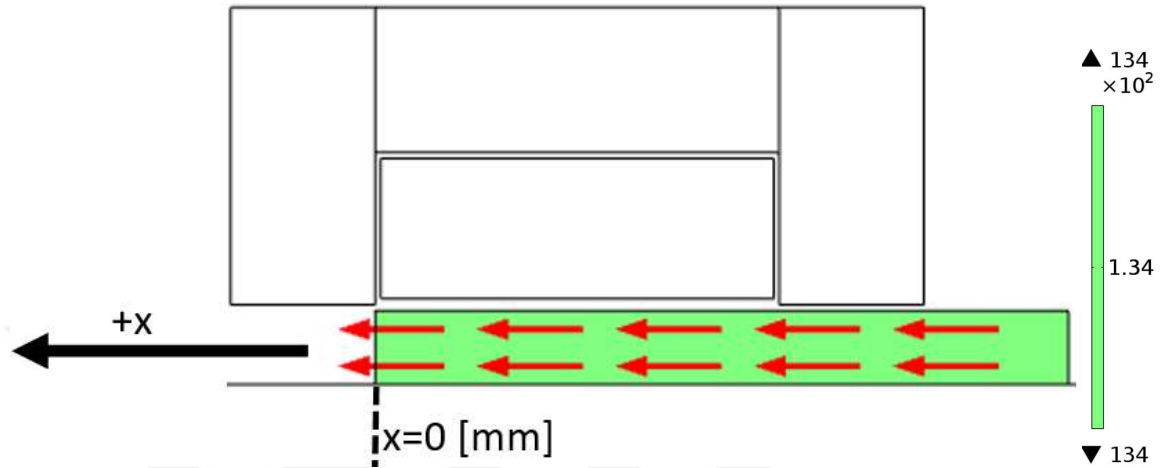


Figure 4.16: Magnetic force N against the armature position at 0 mm

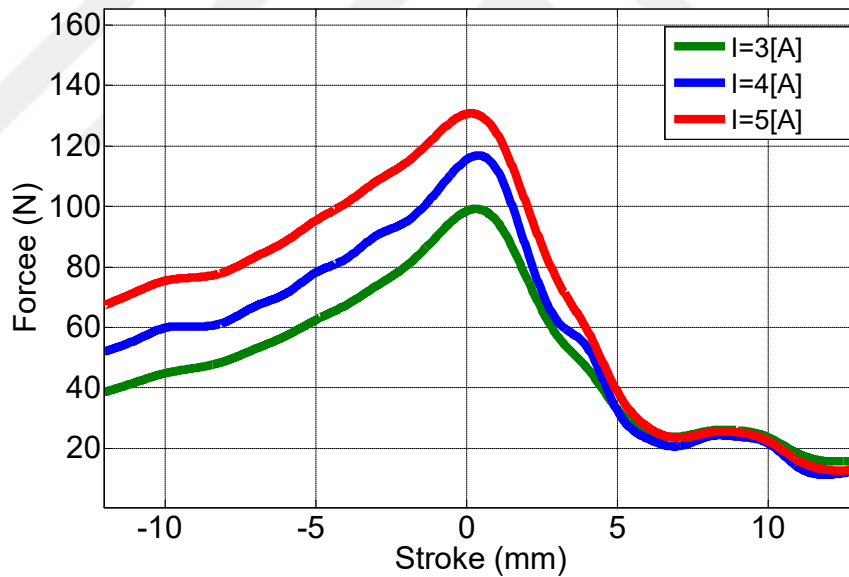


Figure 4.17: Magnetic Force at different stroke positions and currents

Simulation is performed by the settings described in Section 4.4. The result of magnetic flux density at 0 mm stroke position is shown in Figure 4.15. As the current flows through the coil, the magnetic flux lines shown follow a loop around the coil and the edges of the armature and stator become strong opposite poles as shown. Due to the strong poles

formation, a magnetic force is created that pulls the armature as shown in Figure 4.15. The effect of armature position on magnetic force is estimated by executing parametric sweep on the armature position from -13mm to +13mm at currents as 3, 4 and 5 Amperes. For each current and stroke position, FEM simulates a stationary solution and the results for magnetic force are shown in Figure 4.17: Magnetic Force at different stroke positions and currents.

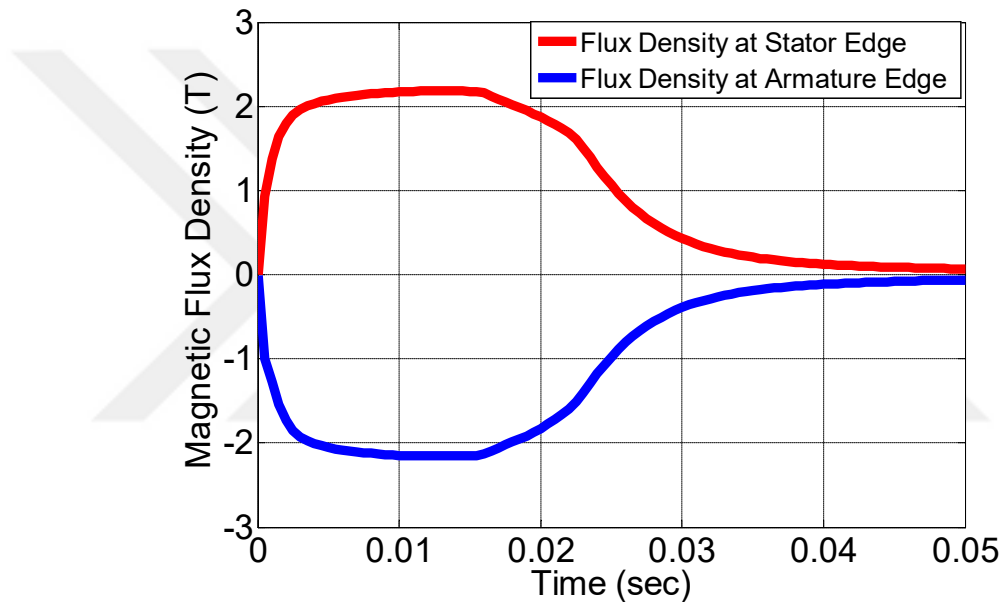


Figure 4.18: Magnetic Poles at different time instants

Time-dependent simulation is performed by the settings described previously for dynamic analysis. As discussed in the stationary solution, at each stroke position, due to the magnetic flux, strong opposite poles are created on the stator and armature. Figure 4.18 shows the time response of magnetic flux density at these points for 0 mm stroke position. The time response of the force is estimated by putting the transient current instead of the constant current. The time response of magnetic force is shown in Figure 4.19.

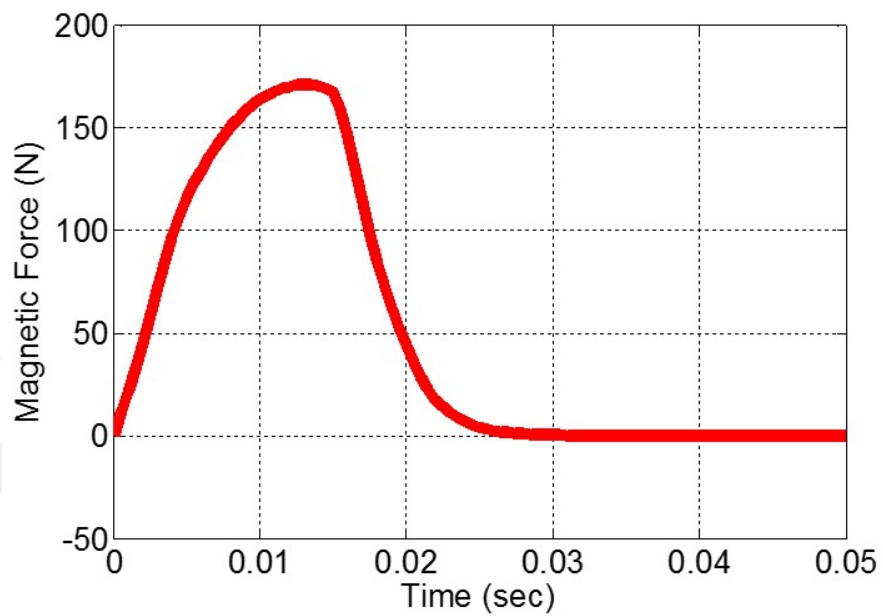


Figure 4.19: Instantaneous magnetic force (time-dependent FEM solution)

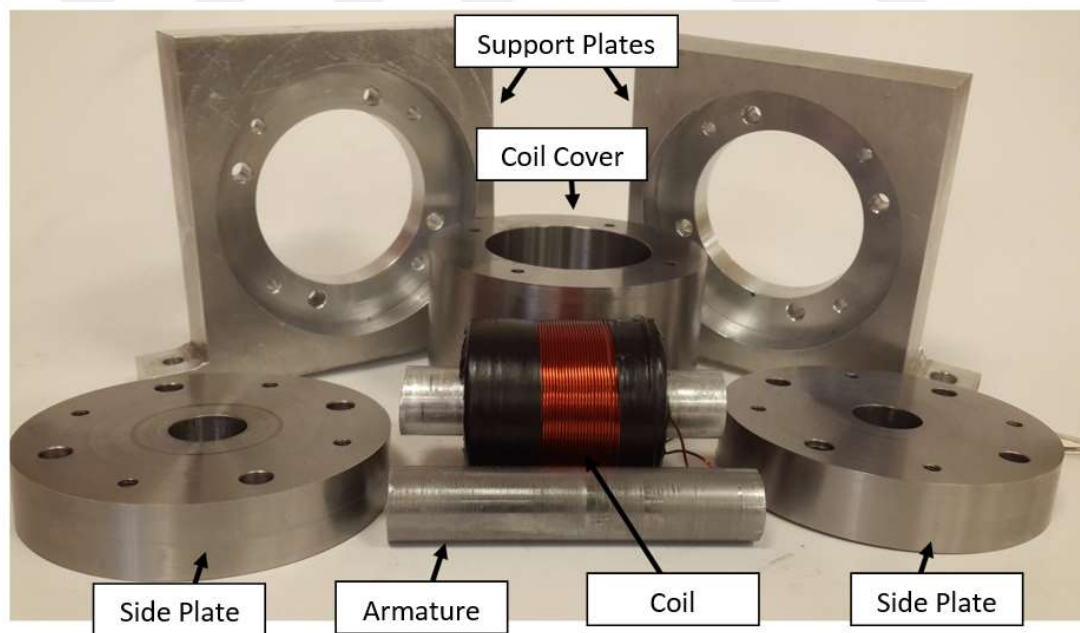


Figure 4.20: Solenoid linear actuator manufactured parts

4.6 Experimental Setups

4.6.1 Linear Actuator

The geometry discussed earlier is manufactured as shown in Figure 4.20. Armature diameter is 12.5 mm, 1000 number of turns with a wire of American Wire Gauge 18, Coil length is 70 mm, Coil Resistance is 3.3Ω and Inductance is 30 mH with armature at 0 mm stroke position.

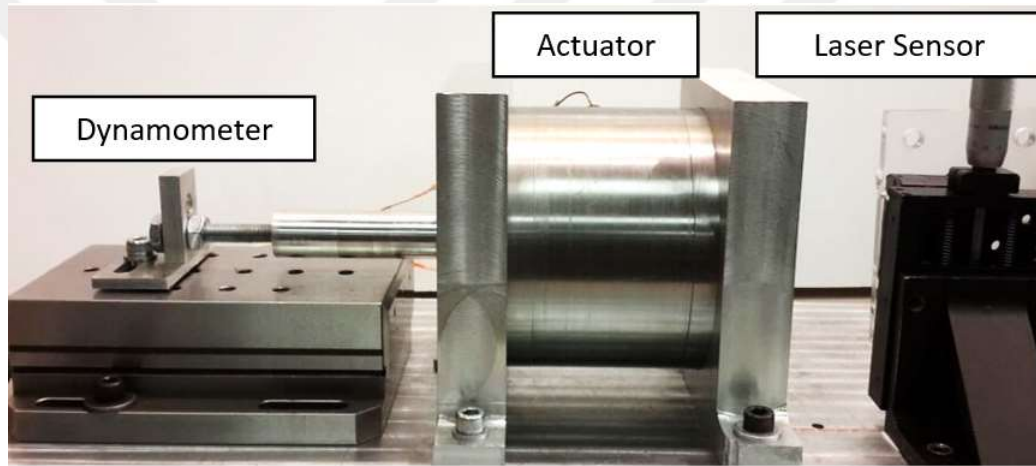


Figure 4.21: Experimental setup for measurement of the static force

4.6.2 Static Force Measurement Setup

In order to calculate the static magnetic force, a quartz tri-axial dynamometer for measuring the three orthogonal force components from Kistler (Type 9257B) with charge amplifier (Type 5070A) is used. A laser sensor is used to calculate the exact location of the armature against each force value. The laser sensor from Keyence (IL-100) with a 50 mm measuring range, 4 μm resolution and, 1 ms update rate is used. An aluminum adapter is used to change the position of the armature linearly in the stator. The magnetic force is measured at every 1 mm in the range of -13 mm to +13 mm at constant currents of 3, 4 and 5 A. A total

set of 78 test cases are obtained to estimate the effect of current on the static magnetic force at different stroke positions. Figure 4.21 shows the static force measurement setup.

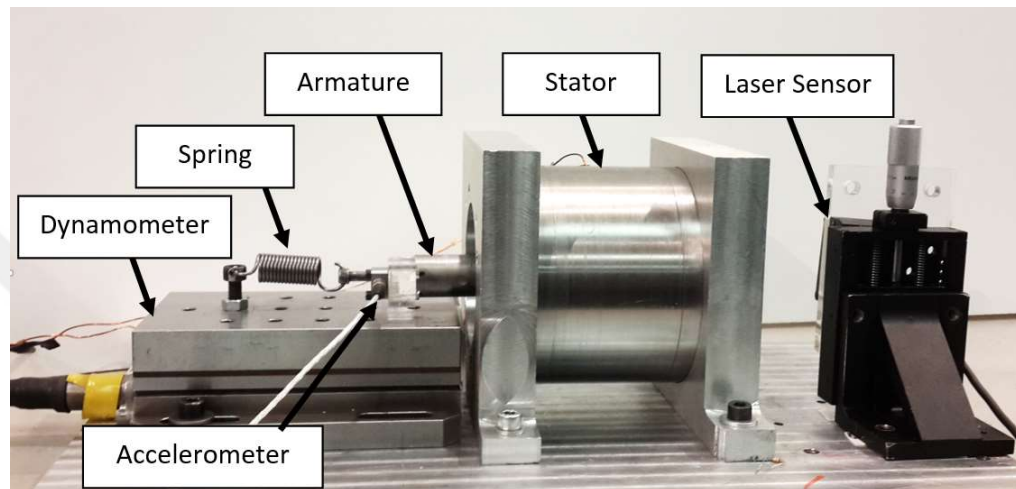


Figure 4.22: Experimental setup for measurement of the dynamic force

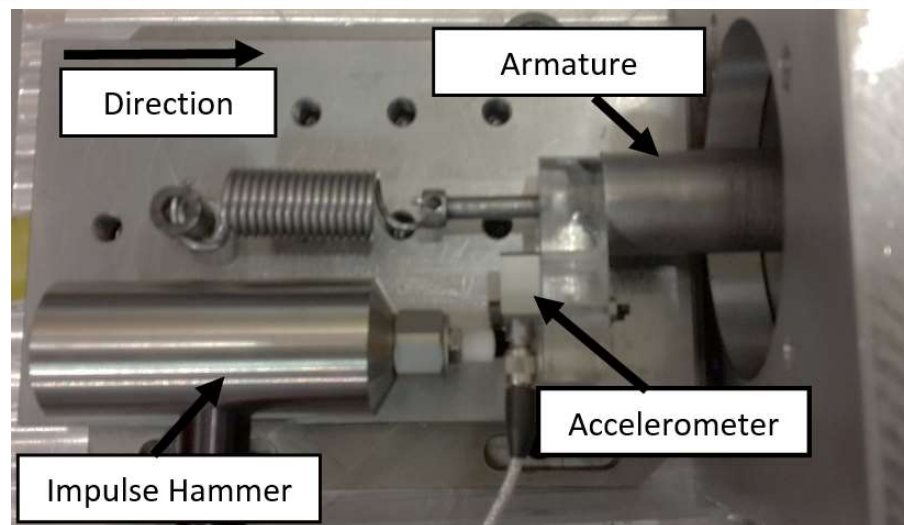


Figure 4.23: Top view of the sensors locations for experimental setup of frequency response function

4.6.3 Dynamic Force Measurement Setup

The setup shown in Figure 4.22 is developed to measure the dynamic response of the actuator. This setup comprises of the solenoid design shown in section 4.6.1. A helical extension spring of 3350 N/m stiffness which is attached to the armature, is used to create oscillations in the setup. Laser displacement sensor and dynamometer are used as described in section 4.6.2. A tri-axial accelerometer from Kistler Type 8793A is used.

4.6.4 System Identification with Impulse Hammer and Accelerometer Test Setup

The frequency response function of the system is obtained from the impact hammer from Kistler (9722A2000) to provide impulse input to the system and the response of the system is obtained using an accelerometer from Kistler (8793A). The setup shown in Figure 4.22 is used. The armature is hit with a hammer in the direction of the motion of armature as shown in Figure 4.23.

4.6.5 Frequency Response Function with Coil Excitation and Magnetic Force Test Setup

These series of experiments are performed on the setup explained in section 4.6.3. Voltage and duty ratio are kept constant at 40 V and 30% respectively. Whereas, a frequency sweep is applied from 1 Hz to 40 Hz and the forces are experimentally logged in DAQ system.

4.7 Results and Discussion

4.7.1 Static Force

To estimate the static force at different stroke positions, a setup described in section 4.6.2 is used. The experiments are conducted at three values of currents as 3, 4 and 5 A and results

are shown in Figure 4.24. The results are compared with the simulation results and the results are complementing each other.

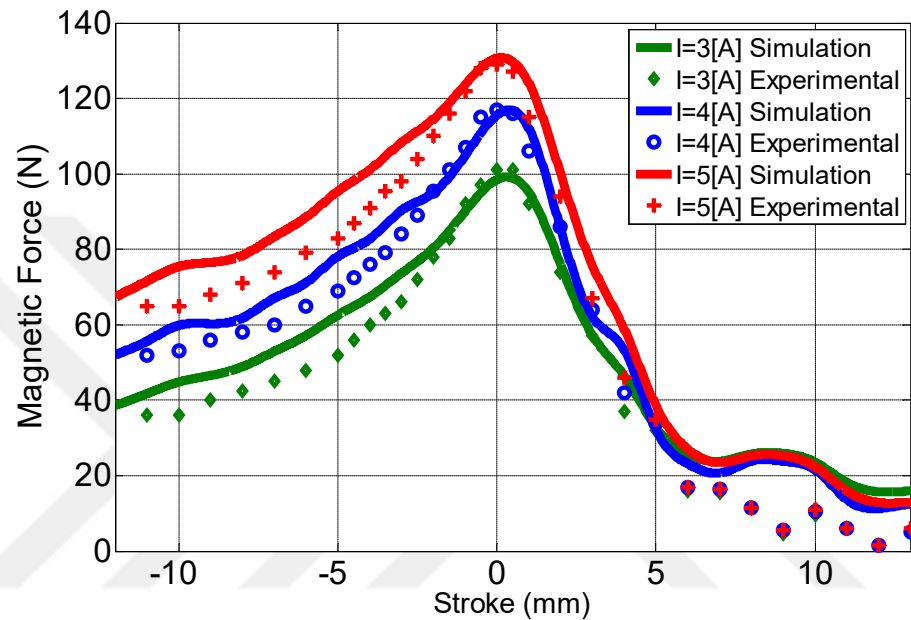


Figure 4.24: Static response experimental and simulation results comparison

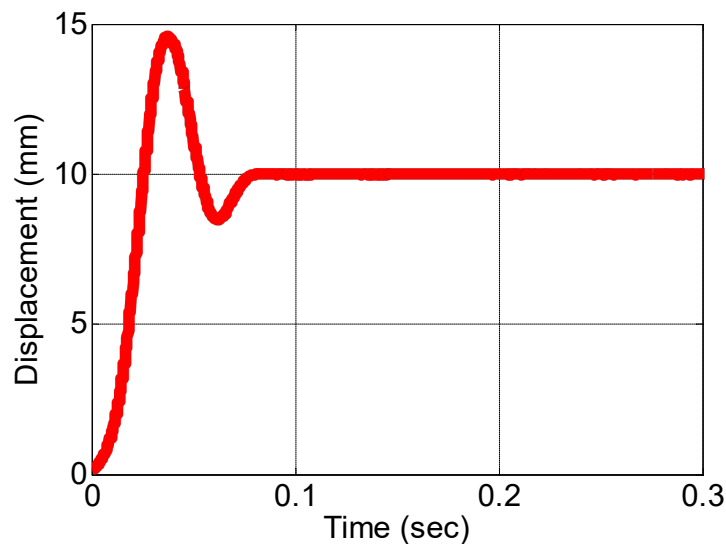


Figure 4.25: Step response of the armature displacement

4.7.2 Step Response

The setup shown in section 4.6.3 is used for determining the damping ratio ζ and damping coefficient c . The coil is energized with a step current of 3 A and the displacement results are recorded. The system is underdamped as shown in Figure 4.25 and the damping parameter is estimated by using Equation (4-17). The values of ζ and c are determined as 0.246 and 11.1 Ns/m respectively.

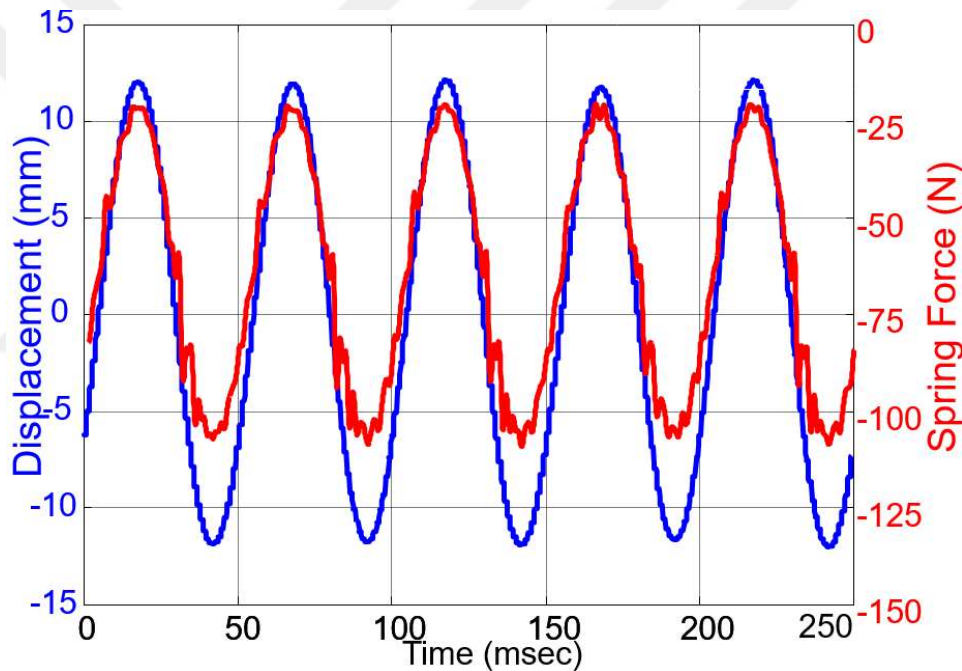


Figure 4.26: Time response of the armature displacement and the spring force

4.7.3 Dynamic Response

The experiment is performed by using the setup presented in section 4.6.3. The measured stroke position and the spring force are shown in Figure 4.26. The armature oscillates with a peak to peak amplitude of 26 mm about the zero position. The stroke position of -13 mm corresponds to the top dead center (TDC) and the stroke position of +13 mm corresponds to the bottom dead center (BDC). The maximum spring force of 110 N is measured at top dead

center and the minimum spring force of 22 N is measured at the bottom dead center. As shown in the figure, there is always a spring force acting on the armature because extension spring is always in extension mode and never comes to the initial position.

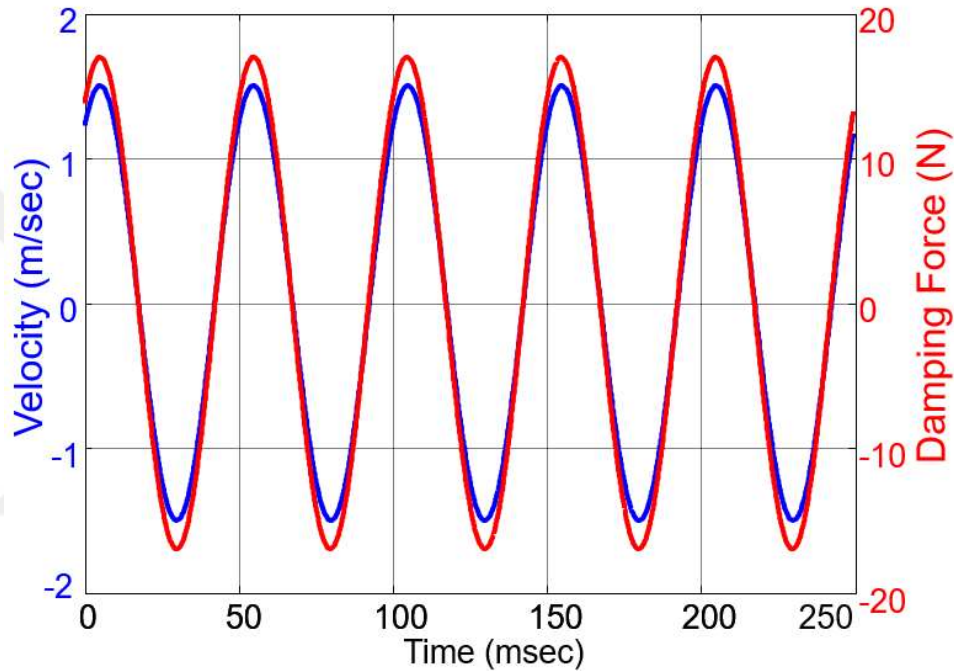


Figure 4.27: Time response of the armature velocity and the damping force

The velocity is estimated by the first-time derivative of the stroke position. As mentioned in section 4.7.2, the damping coefficient is 11.1 Ns/m. The product of the damping coefficient with the velocity values corresponds to the damping force of the linear actuator. Figure 4.27 represents the velocity and damping force of the armature. As shown in the figure, the maximum velocity of 1.5 m/s is measured at the zero-stroke position and it corresponds to a damping force of 16.6 N. The positive and negative values of the velocity correspond to the direction. When the armature is traveling from the TDC to the BDC the velocity is positive and it is negative while it's traveling from the BDC to the TDC. There is a phase difference

of 90° between the velocity and the stroke whereas the damping force and velocity are in phase. Therefore, the maximum damping force is also at the zero-stroke position.

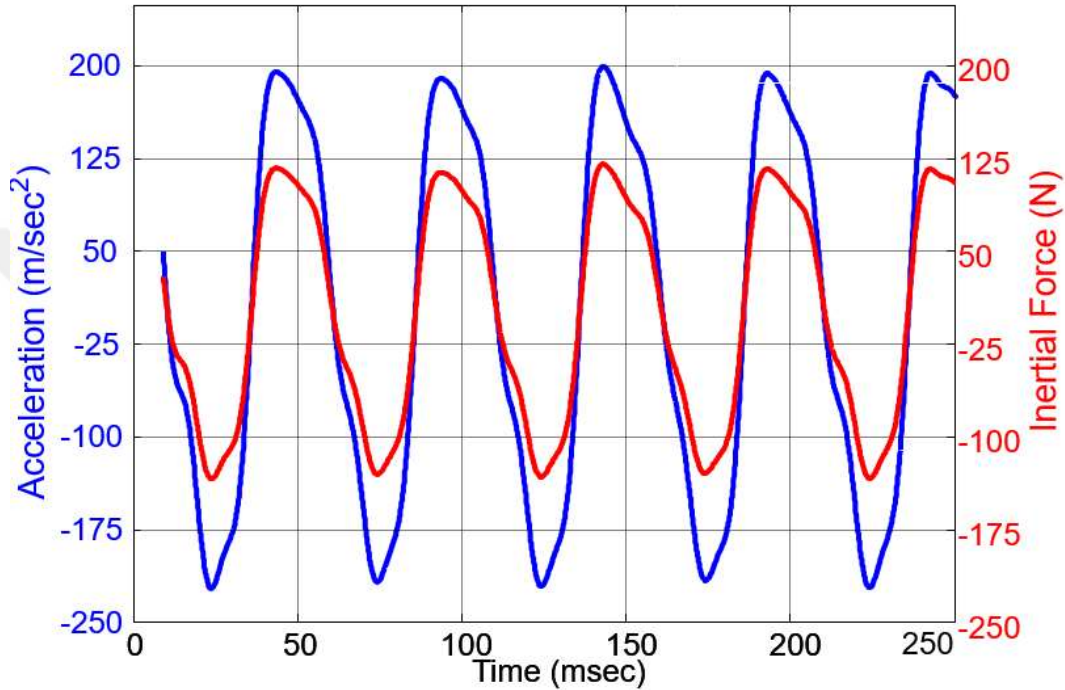


Figure 4.28: Time response of the armature acceleration and the inertial force

With the help of the tri-axial accelerometer, the oscillations in the linear actuation axis as well as in the cross-directional axis are measured. The acceleration in the axial direction and the inertial force is measured as shown in Figure 4.28. The maximum acceleration of $20g \text{ m/s}^2$ is measured at the TDC and the BDC. The inertia is measured as 123 N. As shown in the figure, the inertial force is 180° out of phase with the stroke position. As shown in Figure 4.29, the oscillations are very low in the radial direction as compared to the axial oscillations. There is a very small gap between the armature and the stator. The radial oscillations are not desired. These oscillations cause more friction and wear on the armature. As seen in the figure, there is no regular pattern in these oscillations. These oscillations occur

due to random disturbances in the system. These oscillations can be reduced by using linear bearings.

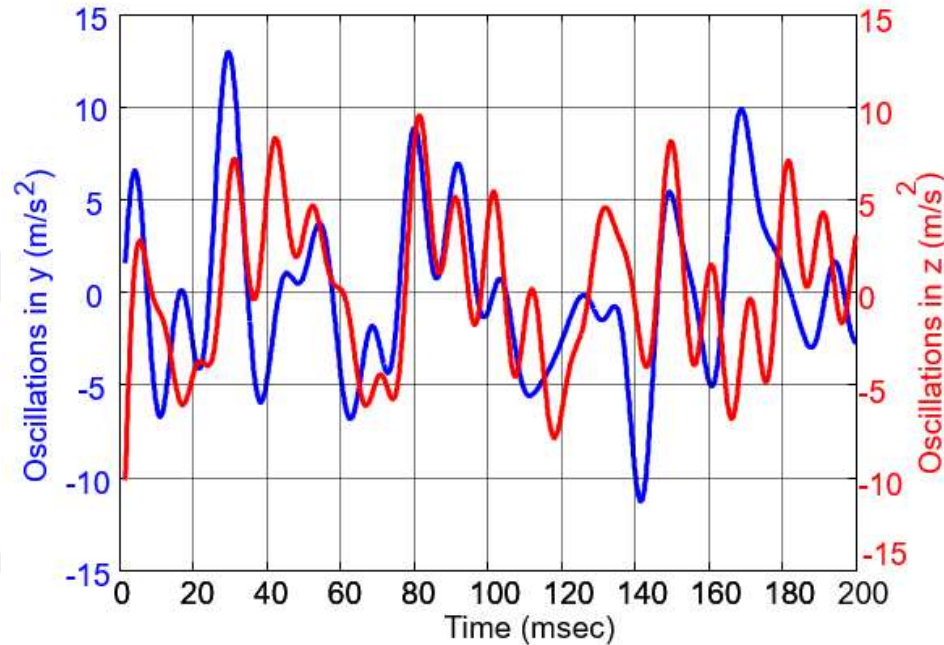


Figure 4.29: Time response of the armature acceleration in radial directions

The results of the FEM simulation are compared with the experimental results as shown in Figure 4.30. The experimental magnetic force is estimated from the sum of the inertial, the spring and the damping force. The results of the experimental magnetic force are in good agreement with the simulated magnetic force. It is seen that simulation force is based on the amount of current flowing through the coil and the magnitude of simulated force increases with the current, whereas when the current is turned off, due to residual magnetism which is not considered in the simulation, a magnetic force exists as shown. Moreover, there are Eddy current losses which are not modeled could be the reason for these errors.

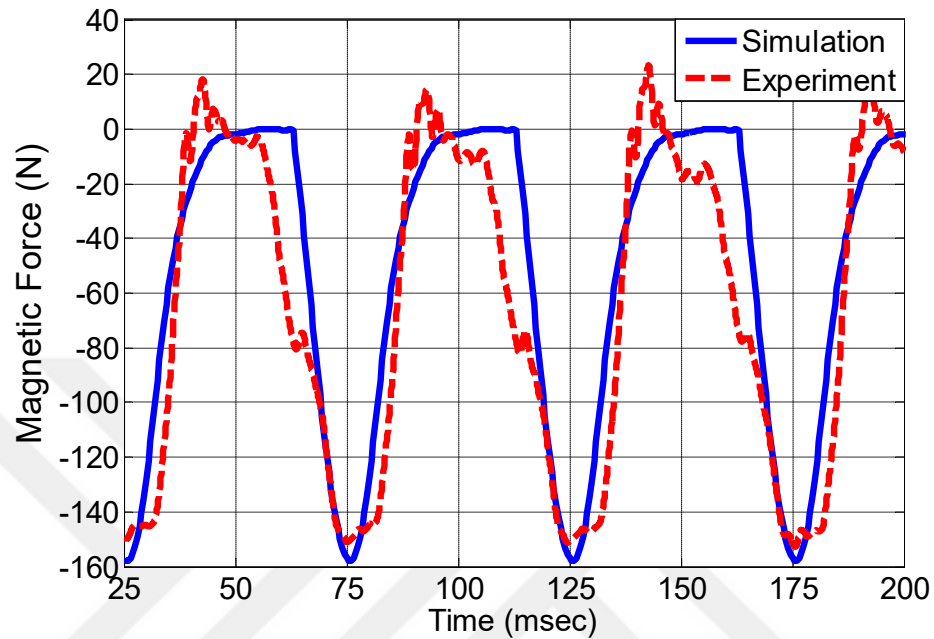


Figure 4.30: Comparison of the time response of the finite element simulation and the measured magnetic force

In order to observe the phase of all the forces, Figure 4.31 shows all the dynamic forces acting on the armature, stroke and the current in a 200 msec time period in one graph. As seen in the figure, when the current rises in the coil, the armature starts moving towards the TDC due to the increase in magnetic force. As the current is switched off, due to the time constant of the inductor, the magnetic field slowly decays and finally reaches to zero and remains zero in the off cycle. This process continues at 20 Hz period. The inertia and spring force (as well as stroke position) are 180° out of phase. The damping force is leading the spring force by 90° and lagging the inertial for by 90° .

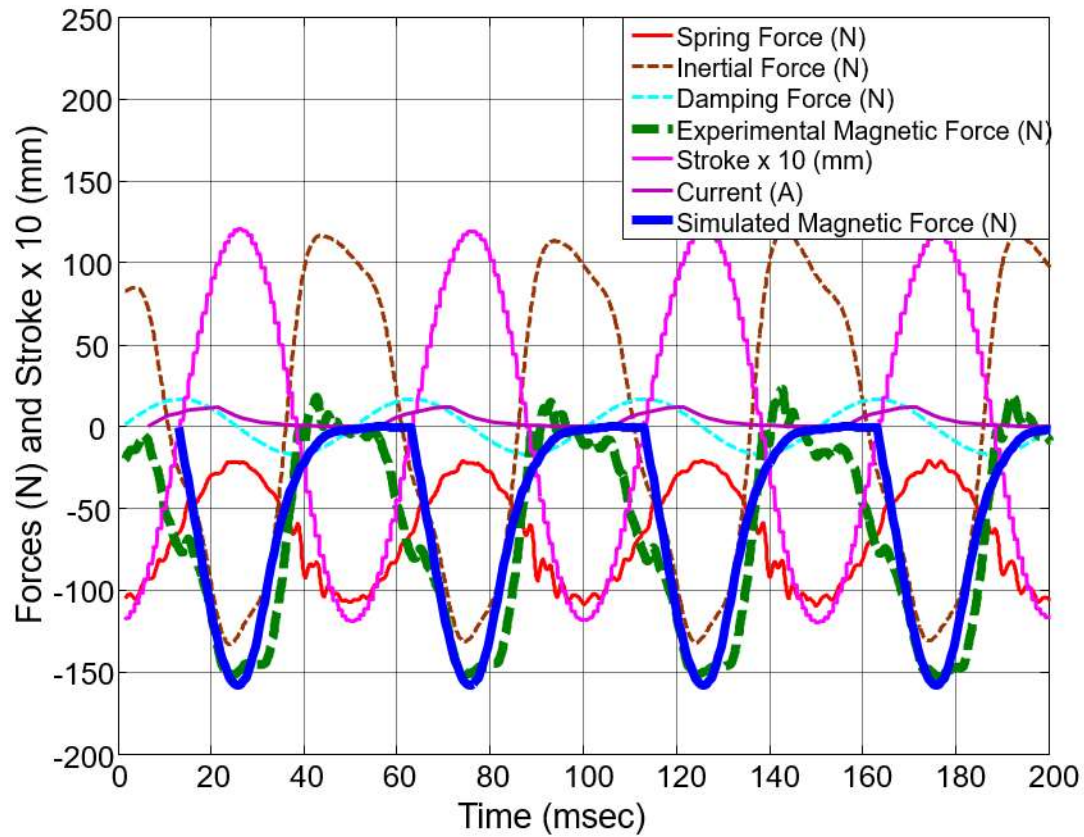


Figure 4.31: Dynamic response experimental and simulation results comparison

4.7.4 Modal Analysis

By using the experimental setup described in section 4.6.4, frequency response function is conducted. Figure 4.32 shows the real and imaginary part of the transfer function. The modal analysis of the system shows the natural frequencies of 6 Hz and 38 Hz for the first and second modes. These frequencies are also shown in the modal analysis results shown in the figure. The y-axis shows the magnitude of as impulse force to acceleration ratio Ns^2/m whereas the x-axis is the frequency Hz.

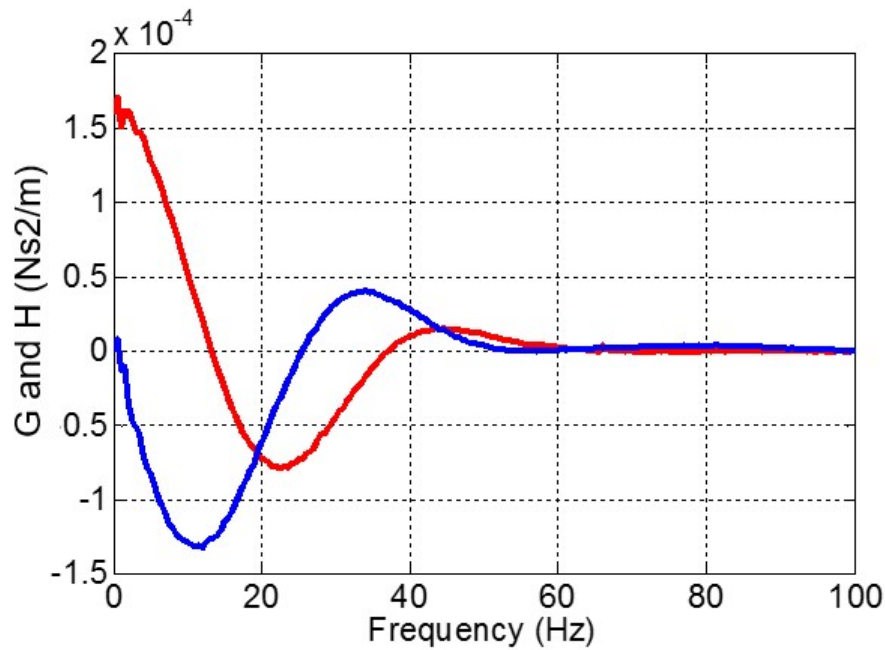


Figure 4.32: Real G and imaginary I parts of the transfer function (impact hammer test) of the solenoid based linear actuator.

4.7.5 Frequency Response Function

By using the experimental methodology explained in section 4.6.5, the frequency response function is obtained. The magnetic force and the coil current are estimated by using the same method explained in section 4.6.3. Forty test cases (0 Hz to 40 Hz) are performed to measure the magnetic force and current for each frequency. The results are shown in Figure 4.33 and they are in very good agreement with the hammer test frequency response function.

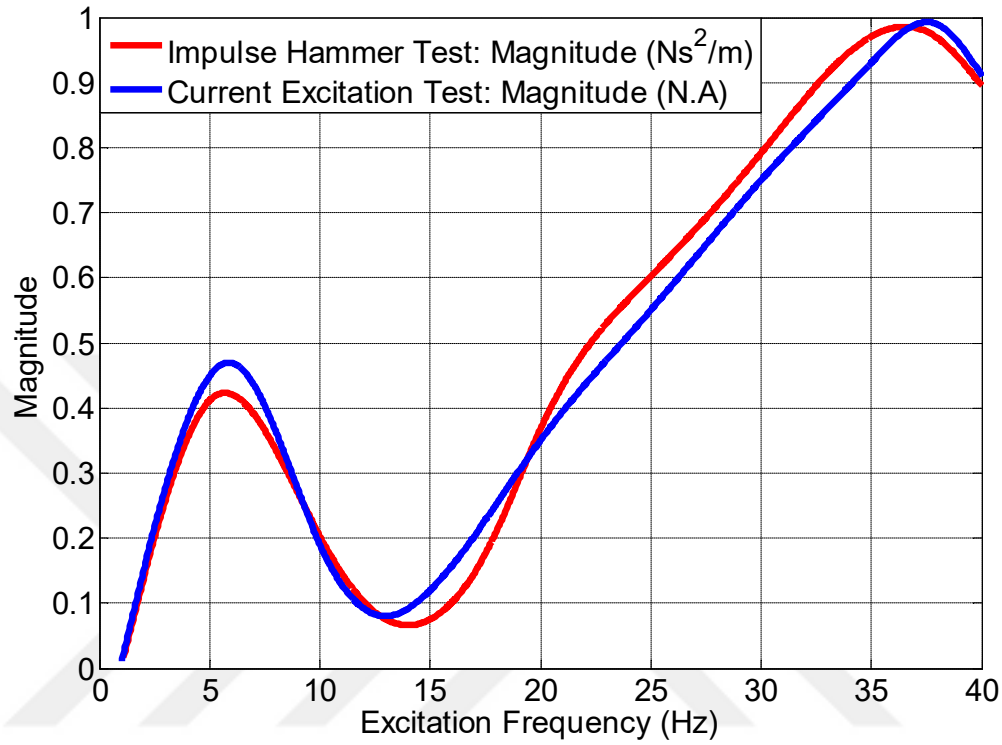


Figure 4.33: Frequency response function through the impact hammer test and the electrical current excitation of coil

4.8 Conclusion

This chapter provides an approach to design a solenoid based linear actuator for the linear compressor. With the given performance parameters, such as force, stroke and oscillation frequency, the geometric parameters of the linear actuator are estimated by using the reluctance method. A nonlinear simulation is executed to verify the forces in stationary as well as time-dependent domains. Experimental setups are built to experimentally validate the simulation results and the values of force stroke and force-time curves are in good agreement with each other. Overall static and dynamic characteristic responses are presented in this research. The time domain analysis provides the real-time behavior of current, stroke, spring, damping, inertial and magnetic force. Using impact hammer and accelerometer, modal

analysis is performed to find the natural frequencies and its modes. Frequency response function is experimentally validated with coil excitation and results are validated for better efficiencies at these excitation frequencies.



Chapter 5

RESONANCE EXCITATION OF MOVING MAGNET LINEAR ACTUATOR

5.1 Overview

This section provides an analytical methodology for evaluating the effect of the excitation frequency and linear actuator parameters on the overall dynamics of a moving magnet linear actuator used in the linear oscillatory systems especially linear compressor. The time domain electrical and mechanical equations are transformed into s-domain to acquire the transfer function between the velocity and the applied voltage as well as stroke and current. The equations are converted into the frequency domain to estimate the frequency response for the velocity, the stroke of the armature and the current passing through the stator coil. The effect of the motor constant and the excitation frequency on the magnitude and phase of the dynamic parameters are estimated using Phasors. With the help of root locus plots, the effect of these parameters on the stability of the system is also discussed. Additionally, frequency response function of the stroke to current ratio is established to find the optimal point of excitation for achieving better system efficiency. An experimental setup is designed to validate the transfer functions. The acquired results endorse the simulation results for the effect of the motor constant and the excitation frequency on the dynamics of the linear actuator.

5.2 Literature Review

Most linear compressors are operating on the mechanical resonance and when the oscillation frequency becomes equal to the resonance frequency, a minimal electromagnetic force is required to make the system oscillate [43], [44]. The resonance frequency is dependent upon the stiffness and mass of the moving body. Therefore, moving mass sets a strong criterion for selecting the motor's design configuration. The electrical motor drive

also plays a vital role in controlling the efficiency of the linear compressor. To achieve the best performance efficiency, mechanical resonance frequency should be equal to the electrical resonance frequency [45].

Single phase AC power is used to electrically excite the stator coil which is in series with a known value capacitor. The inductance of the overall actuator is known and value of capacitance is calculated through the RLC circuit resonance frequency equation. On the resonance frequency, maximum efficiency of the RLC circuit is achieved [46]. Alternatively, PWM inverter can also be used for a linear compressor drive. As the mechanical resonance frequency is changing due to gas stiffness, an inverter is used to match the excitation frequency according to the instantaneous mechanical resonance frequency. Consequently, better efficiency and power factor of the linear compressor is achieved [45].

Considering the global energy conservation and the enormous domestic use of the compressors, a thorough research is required to optimize the performance of the linear compressors. Abounding research is conducted for estimating the effect of the excitation frequency on the linear compressor, although other parameters provide a vital role in optimizing the performance of the linear compressor as well. For instance, motor constant, damping coefficient, spring stiffness, coil inductance, the effect of the difference between mechanical and electrical resonance frequencies. Other applications of the resonance based actuators, such as fluid jet actuators [47] and micro pumps [48], a detailed parametric study is available. These systems perform best when excited on their mechanical resonance frequency. Such an excitation requires a minimal electromagnetic force for armature oscillations [10][43][45][64][74][75][99][100]. In order to drive these motors, single phase AC power is usually used. The motor is modeled as a resistor and an inductor, in series with the back emf source and the mechanical system is modeled as a mass-spring-damper (MSD) system. Previously, the coupled electromechanical system is modeled and the effect of the excitation frequency on the performance of a linear actuator is presented by [43]. Similar

models are established by [74] to analyze the effect of the electrical and motional impedances on the stroke and current of the actuator. The stroke and axial force estimation of a linear actuator driven through a single-phase AC are presented by [66][75]. Although the stroke and current dynamic models of the linear actuators without a series capacitance are presented [9][63][68][77]-[87], an overall transfer function of the dynamic parameters with respect to the voltage input supply are required. Additionally, such a model will provide a tool to select the design parameters of the linear oscillating actuator in the design phase such as motor constant, spring stiffness, damping, moving mass, capacitance and coil inductance.

This research is based on providing an overall model incorporating major parameters of the linear actuator. Therefore, before designing a linear actuator, the effect of these parameters will provide an estimate of the efficiency with the help of a rapid numerical simulation. The system transfer function is written by using the time domain models of the electrical and mechanical parts. Furthermore, with the help of phasors, the frequency domain analysis is executed to estimate the effect of the design parameters as well as excitation frequency on the linear actuator's velocity, stroke, current and stroke to current ratio. Additionally, the effect of these parameters on the stability of the compressor is discussed. Finally, the simulation results are validated with the help of an experimental setup. These results provide the optimal point of excitation to achieve better dynamic characteristics of the linear actuator.

5.3 Analytical Modeling

5.3.1 Transfer function of velocity to voltage

The electrical part of the linear actuator is composed of an RLC circuit [29] and an AC power source that excites the system. The electrical model of the linear actuator is shown in Equation (5-1):

$$v(t) = Ri(t) + L \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) dt + \alpha \vartheta \quad (5-1)$$

where R is the Resistance, L is the Inductance, C is the capacitance, v is the voltage supply, i represents the current, α is the back-emf constant Ns/m and ϑ is the velocity of the armature. Taking Laplace transform of Equation (5-1):

$$\bar{V} = R\bar{I} + sL\bar{I} + \frac{\bar{I}}{sC} + \alpha\bar{\vartheta} \quad (5-2)$$

$$\omega_{ne} = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_{ne}^2 L} \quad (5-3)$$

where ω_{ne} is the electrical resonance frequency of the linear actuator. Equation (5-1) and (5-3) are written for current as shown in Equation (5-4):

$$\bar{I} = \frac{s\bar{V} - \alpha s\bar{\vartheta}}{s^2 L + sR + \omega_{ne}^2 L} \quad (5-4)$$

The mechanical model of the linear actuator is given by Equation (5-5):

$$F_m = \alpha I = m\ddot{x} + b\dot{x} + kx \quad (5-5)$$

where m is the mass of the armature, b is the damping coefficient, α is the motor constant N/A and k is the spring constant. It can be noticed that the back emf constant and motor constant are same in electrical and mechanical equations. The equivalence of both constants will be discussed in Chapter 7. Taking Laplace Transform of the Equation (5-5) and putting the displacement x in terms of velocity $\bar{\vartheta}$:

$$\alpha\bar{I} = ms\bar{\vartheta} + b\bar{\vartheta} + k\frac{\bar{\vartheta}}{s} \quad (5-6)$$

$$\omega_{nm} = \sqrt{\frac{k}{m}} \rightarrow k = m\omega_{nm}^2 \quad (5-7)$$

where ω_{nm} is the mechanical resonance frequency of the linear actuator. Equation (5-6) and (5-7) are written as:

$$\bar{I} = \frac{\bar{\vartheta}}{\alpha s} [ms^2 + bs + m\omega_{nm}^2] \quad (5-8)$$

Comparing Equation (5-4) and (5-8)

$$\frac{\bar{\vartheta}}{\alpha s} [ms^2 + bs + m\omega_{nm}^2] = \frac{s\bar{V} - \alpha s\bar{\vartheta}}{s^2L + sR + \omega_{ne}^2L} \quad (5-9)$$

$$\frac{\bar{\vartheta}}{\bar{V}} = \frac{\alpha s^2}{[A_4s^4 + A_3s^3 + A_2s^2 + A_1s + A_0]} \quad (5-10)$$

where

$$A_0 = mL\omega_{nm}^2\omega_{ne}^2$$

$$A_1 = bL\omega_{ne}^2 + mR\omega_{nm}^2$$

$$A_2 = bR + mL\omega_{ne}^2 + mL\omega_{nm}^2 + \alpha^2$$

$$A_3 = mR + bL \text{ and}$$

$$A_4 = mL$$

5.3.1 Velocity of the armature

Equation (5-10) represents the transfer function of the linear actuator between the velocity and the applied voltage in s-domain. In order to obtain the effect of the excitation frequency, Equation (5-10) is written in the frequency domain by putting $s = j\omega$, the equation becomes:

$$\frac{\bar{\vartheta}}{\bar{V}_s} = \frac{\alpha s^2}{[A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0]}$$

$$\frac{\bar{\vartheta}}{\bar{V}_s} = - \frac{\alpha \omega^2}{[A_4 (j\omega)^4 + A_3 (j\omega)^3 + A_2 (j\omega)^2 + A_1 (j\omega) + A_0]}$$

$$\frac{\bar{\vartheta}}{\bar{V}_s} = - \frac{\alpha \omega^2}{[A_4 \omega^4 - j A_3 \omega^3 - A_2 \omega^2 + j A_1 \omega + A_0]}$$

$$= - \frac{\alpha \omega^2}{(A_4 \omega^4 - A_2 \omega^2 + A_0) - j(A_3 \omega^3 - A_1 \omega)}$$

$$\frac{\bar{\vartheta}}{\bar{V}_s} = - \frac{\alpha \omega^2}{Y + j\psi}$$

Replacing the real and imaginary part as:

$$\frac{\bar{\vartheta}}{\bar{V}_s} = - \frac{\omega^2 \alpha}{Y + j\psi} \times \frac{Y - j\psi}{Y - j\psi} = - \frac{\omega^2 \alpha (Y - j\psi)}{Y^2 + \psi^2} = - \frac{\alpha \omega^2 Y}{Y^2 + \psi^2} + j \frac{\alpha \omega^2 \psi}{Y^2 + \psi^2}$$

$$Y = -bR\omega^2 - \alpha^2 \omega^2 + mL\omega^4 - mL\omega_{ne}^2 \omega^2 - mL\omega_{nm}^2 \omega^2 - mL\omega_{nm}^2 \omega_{ne}^2$$

$$Y = -bR\omega^2 - \alpha^2 \omega^2 + mL\omega^4 \left(1 - \frac{\omega_{ne}^2}{\omega^2}\right) \left(1 - \frac{\omega_{nm}^2}{\omega^2}\right)$$

$$Y = mL\omega^4 (1 - r_e^2)(1 - r_m^2) - \omega^2 (bR + \alpha^2) \quad (5-11)$$

$$\psi = b\omega\omega_{ne}^2 L + m\omega\omega_{nm}^2 R - mR\omega^3 + bL\omega^3$$

$$\psi = -mR\omega^3 \left(1 - \frac{\omega_{nm}^2}{\omega^2}\right) - bL\omega^3 \left(1 - \frac{\omega_{ne}^2}{\omega^2}\right)$$

$$\psi = -mR\omega^3 (1 - r_m^2) - bL\omega^3 (1 - r_e^2) \quad (5-12)$$

wherein Equation (5-11) and (5-12), r_m and r_e are the ratio of the mechanical and the electrical resonance frequencies to the excitation frequency. The transfer function in frequency domain is written as:

$$\frac{\bar{\vartheta}}{\bar{V}_s} = -\frac{\alpha\omega^2\Upsilon}{\Upsilon^2 + \psi^2} + j\frac{\alpha\omega^2\psi}{\Upsilon^2 + \psi^2} = \left[\frac{\bar{\vartheta}}{\bar{V}_s}\right]_{real} + j\left[\frac{\bar{\vartheta}}{\bar{V}_s}\right]_{imag} \quad (5-13)$$

where

$$\left[\frac{\bar{\vartheta}}{\bar{V}}\right]_{real} = -\frac{\alpha\omega^2\Upsilon}{\Upsilon^2 + \psi^2} \quad (5-14)$$

$$\left[\frac{\bar{\vartheta}}{\bar{V}}\right]_{imag} = \frac{\alpha\omega^2\psi}{\Upsilon^2 + \psi^2} \quad (5-15)$$

The transfer function of the linear actuator between velocity and applied voltage is represented in the form of phasors as shown in Equation (5-16), (5-17) and (5-18):

$$\frac{\vec{\vartheta}}{\vec{V}} = \left|\frac{\vec{\vartheta}}{\vec{V}}\right| \angle \varphi_{\frac{\vec{\vartheta}}{\vec{V}}} \quad (5-16)$$

$$\left|\frac{\vec{\vartheta}}{\vec{V}}\right| = \alpha\omega^2 \sqrt{\frac{1}{\Upsilon^2 + \psi^2}} \quad (5-17)$$

$$\varphi_{\frac{\vec{\vartheta}}{\vec{V}}} = -\tan^{-1}\left(\frac{\psi}{\Upsilon}\right) \quad (5-18)$$

The applied AC Voltage supply in Phasors is written as shown in Equation (5-19):

$$\vec{V} = |\vec{V}| \angle \varphi_V \quad (5-19)$$

The frequency response of the armature velocity is estimated by multiplying Equation (5-16) and (5-19):

$$\vec{\vartheta} = \left| \frac{\vec{\vartheta}}{\vec{V}} \right| \times |\vec{V}| \angle \left(\varphi_{\frac{\vec{\vartheta}}{\vec{V}}} + \varphi_V \right) \quad (5-20)$$

$$|\vec{\vartheta}| = \frac{|\vec{V}| \alpha \omega^2}{\sqrt{\gamma^2 + \psi^2}} \quad (5-21)$$

$$\angle \varphi_{\vartheta} = \varphi_V - \tan^{-1} \left(\frac{\psi}{\gamma} \right) \quad (5-22)$$

5.3.2 Current of the stator coil

In order to find the current, Equation (5-8) is written as (5-23), (5-24) and (5-25):

$$\vec{I} = |\vec{I}| \angle \varphi_I \quad (5-23)$$

$$|\vec{I}| = \frac{|\vec{\vartheta}|}{\alpha} \sqrt{b^2 + \varsigma^2} = \frac{|\vec{V}| \omega^2}{\sqrt{\gamma^2 + \psi^2}} \sqrt{b^2 + \varsigma^2} \quad (5-24)$$

$$\angle \varphi_I = \tan^{-1} \left(\frac{\varsigma}{b} \right) \quad (5-25)$$

$$\varsigma = \omega m (1 - r_m^2)$$

5.3.3 Stroke of the armature

By replacing velocity with displacement in Equation (5-10), the transfer function becomes:

$$\begin{aligned}
\frac{\vec{\vartheta}}{\vec{V}} &= s \frac{\vec{x}}{\vec{V}} = -\frac{\alpha Y \omega^2}{Y^2 + \psi^2} + j \frac{\alpha \psi \omega^2}{Y^2 + \psi^2} \\
(j\omega) \frac{\vec{x}}{\vec{V}} &= -\frac{\alpha Y \omega^2}{Y^2 + \psi^2} + j \frac{\alpha \psi \omega^2}{Y^2 + \psi^2} \\
\frac{\vec{x}}{\vec{V}} &= j \frac{\alpha Y \omega}{Y^2 + \psi^2} + \frac{\alpha \psi \omega}{Y^2 + \psi^2} = \frac{\alpha \psi \omega}{Y^2 + \psi^2} + j \frac{\alpha Y \omega}{Y^2 + \psi^2} = \left[\frac{\vec{x}}{\vec{V}} \right]_{real} + \left[\frac{\vec{x}}{\vec{V}} \right]_{imag} \\
\vec{x} &= \frac{\alpha \psi \omega \vec{V}}{Y^2 + \psi^2} + j \frac{\alpha Y \omega \vec{V}}{Y^2 + \psi^2} = [\vec{x}]_{real} + [\vec{x}]_{imag}
\end{aligned} \tag{5-26}$$

where

$$[\vec{x}]_{real} = \frac{\alpha \psi \omega \vec{V}}{Y^2 + \psi^2} \tag{5-27}$$

$$[\vec{x}]_{imag} = \frac{\alpha Y \omega \vec{V}}{Y^2 + \psi^2} \tag{5-28}$$

$$|\vec{x}| = \frac{\alpha \omega |\vec{V}|}{\sqrt{Y^2 + \psi^2}} \tag{5-29}$$

$$\angle \varphi_x = \varphi_V + \tan^{-1} \left(\frac{Y}{\psi} \right) \tag{5-30}$$

5.3.4 Transfer function of the stroke with respect to current

The transfer function of stroke to current is written from Equation (5-24) and (5-27) and is shown in Equation (5-31):

$$\frac{\bar{x}}{\bar{I}} = \frac{\alpha}{\omega \sqrt{b^2 + \zeta^2}} \quad (5-31)$$

Table 5-1: Parameters of the linear actuator

<i>Symbol</i>	<i>Quantity</i>	<i>Value</i>
L	inductance (mH)	384
R	coil resistance (Ω)	15
m	moving mass (g)	684
b	damping coefficient (Ns/m)	9.7 Ns/m
V	AC voltage supply (V)	40

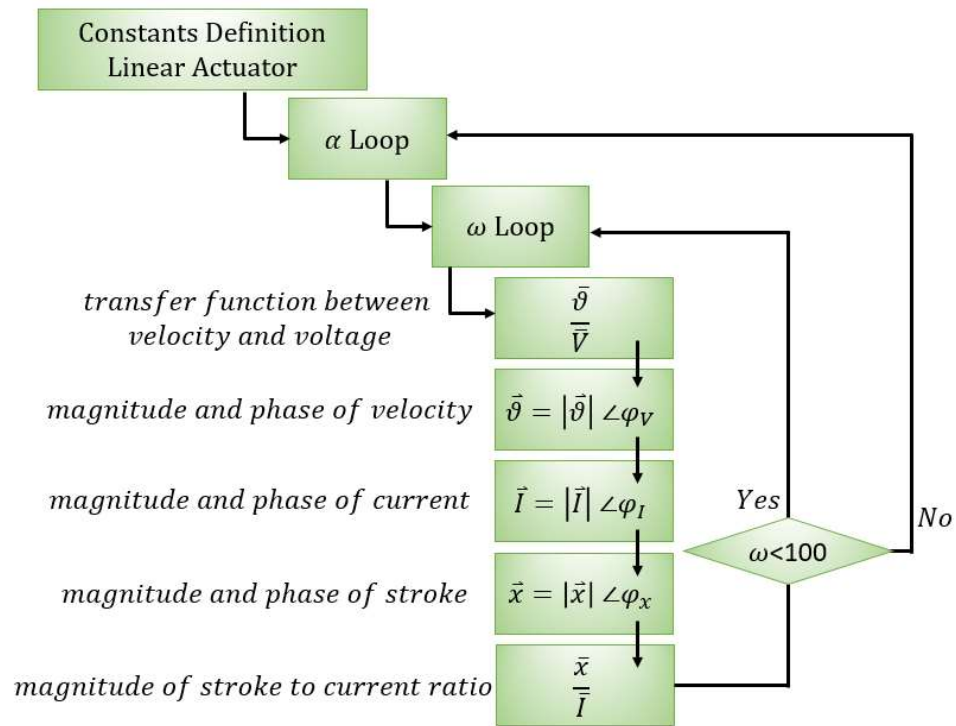


Figure 5.1: Simulation flow chart for system dynamics analysis

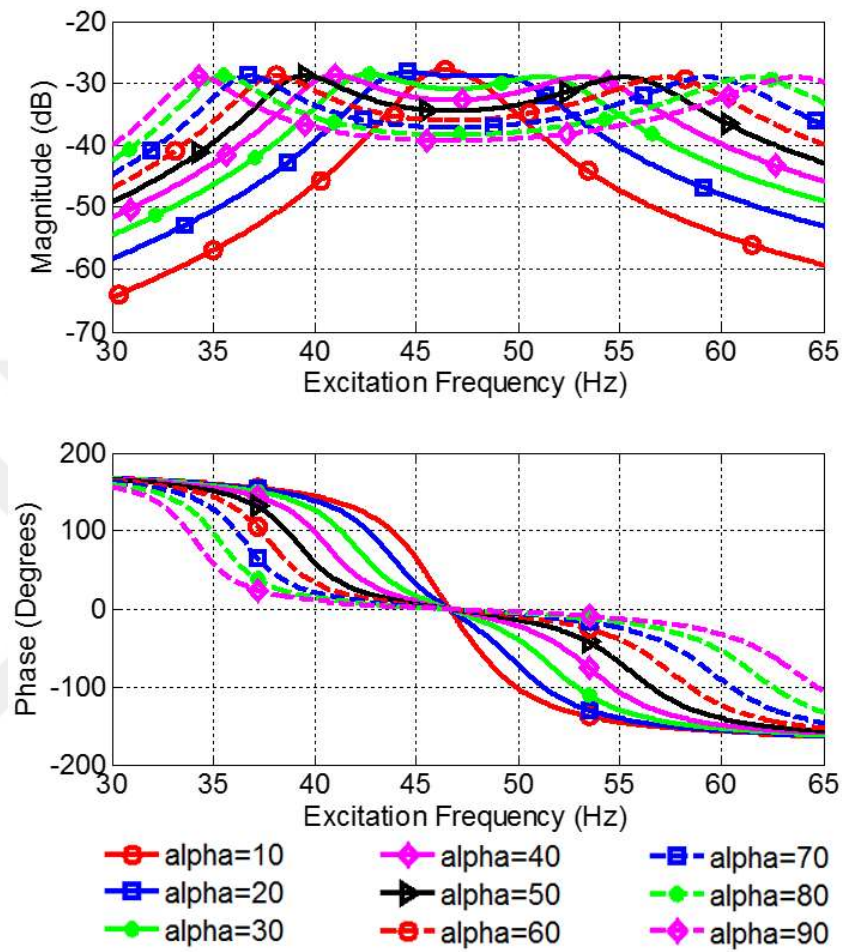


Figure 5.2: Bode plot of the velocity to voltage transfer function of the linear actuator

5.4 Simulation

A simulation is carried out to estimate the system dynamics of the linear actuator. Figure 5.1 represents the algorithm flow chart. The simulation starts with the initialization of the known variables of the system such as the mass of the armature, the spring stiffness, the damping coefficient, the inductance of the stator coil and the capacitance used in the RLC circuit to excite the linear actuator. The parameters used for simulating the system dynamics of the linear actuator are shown in Table 5-1. As the aim of this simulation is to evaluate the

effect of motor constant and exciting frequency on the system, the first loop is the motor constant loop that increments for α from 10 to 90 and the second loop increments for ω from $30 \times 2\pi$ radians/sec to $65 \times 2\pi$ radians/sec. Firstly, the transfer function between velocity and voltage is estimated using Equation (5-10). The bode plot provides the measure of magnitude and phase response of this transfer function. Later, Equation (5-20) is used to estimate the velocity magnitude and phase response by using the phasors. Current and stroke are also estimated by using the same technique with the help of Equation (5-24) and (5-27) respectively. In the end, the stroke to current ratio is calculated to understand the optimal point of excitation for the linear actuator by using Equation (5-31).

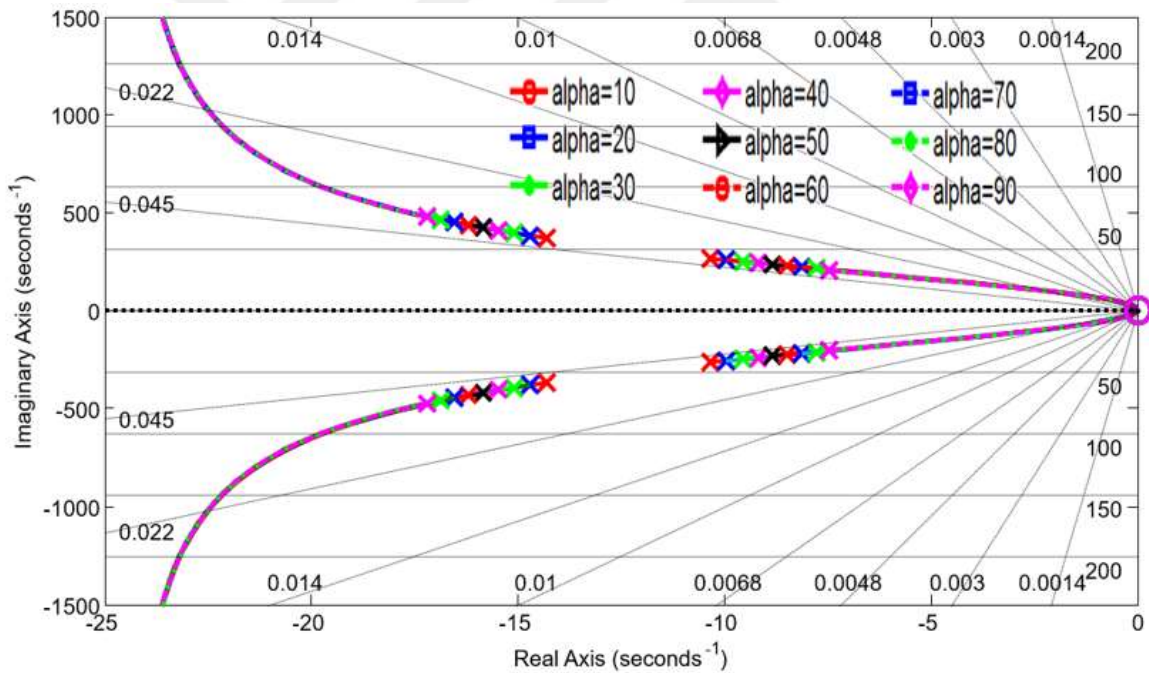


Figure 5.3: Root locus plot of the velocity to voltage transfer function of the linear actuator

5.5 Simulation Results

The bode plot of a transfer function between the velocity and the voltage is shown in Figure 5.2. For one value of the motor constant, the magnitude of velocity has two distinct peaks around the resonance frequency. These peaks are due to the presence of the square of the motor constant in the second order term of the transfer function. It is also clear that the gap between these two peaks increases with the increase in the motor constant. Additionally, the magnitude of the velocity at resonance frequency decreases as the motor constant increases. Whereas, the difference between the velocity magnitudes at two adjacent motor constants is lower for higher values of motor constants. For instance, if the motor constant is improved from 10 to 20 N/A, the velocity to voltage ratio decrements by 0.32 dB. Whereas, if the motor constant is improved from 80 to 90 N/A, velocity to voltage ratio decreases by 0.031 dB only. For any value of the motor constant, when the excitation frequency is equal to the resonance frequency, the phase difference between the velocity and the voltage is zero. Whereas, it goes towards $\pm 180^\circ$ below and above 30 and 65 Hz excitation frequency respectively. For higher values of the motor constants, the phase shift is away from the resonance frequency. Figure 5.3 provides the effect of the motor constant on the stability of the system as the closed loop poles are in the left half of the plane. Motor constant does not affect the stability of the linear actuator.

The results for the frequency response of velocity magnitude and phase are shown in Figure 5.4. The magnitude of the velocity is consistent with the bode plot discussed before. For the resonance frequency, lowest the motor constant α , the more is the magnitude of velocity at the resonance frequency. Whereas, the change in the velocity amplitudes between different α values reduces as it becomes higher. Figure 5.4 also represents the phase difference between velocity and applied voltage is zero at the resonance frequency, whereas the 180° phase change is observed as the excitation frequency crosses the peak velocity magnitude points.

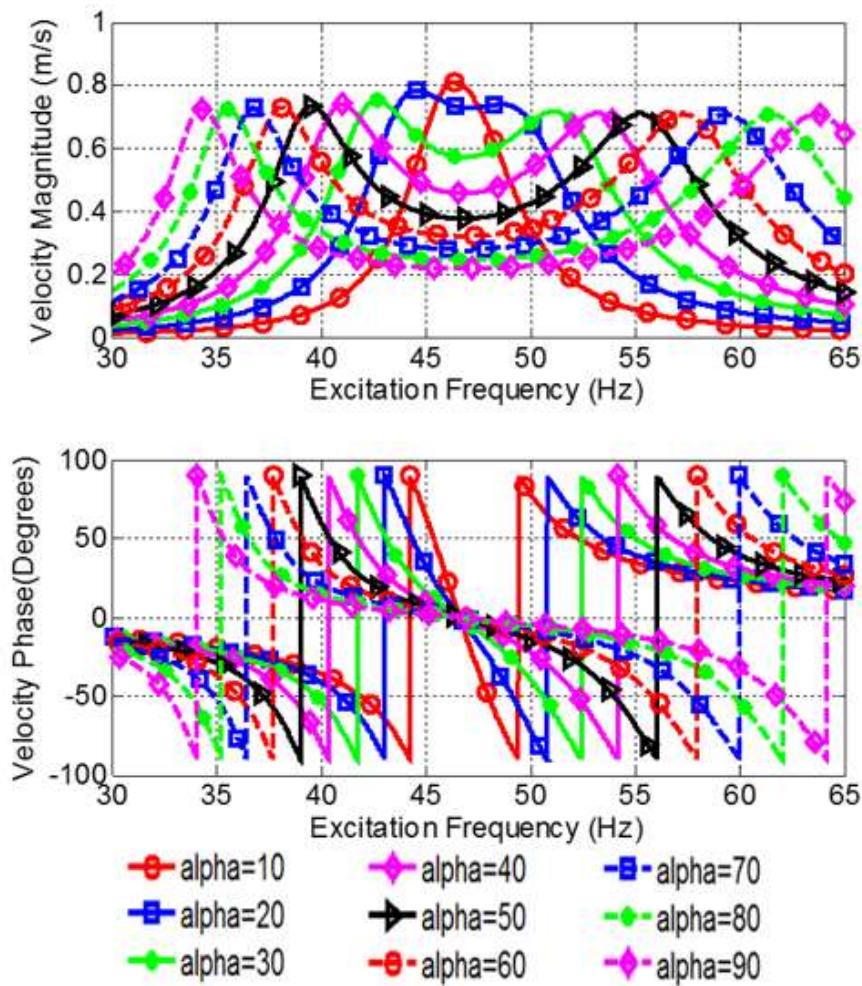


Figure 5.4: Frequency response of the magnitude and phase of the velocity with respect to the applied voltage

The frequency response of the magnitude of the current is shown in Figure 5.5. The magnitude of the current is minimum at the resonance frequency and it starts increasing when the excitation frequency is other than resonance frequency and the rate of change of current is lesser for higher alpha values. For instance, the current reaches its peak value at 44 Hz excitation frequency for 10 N/A motor constant, whereas, it reaches to its peak value at 33 Hz for the motor constant of 90 N/A. As the mechanical resonance frequency is changing due the gas stiffness, higher alpha values provide a wider range of operating frequencies to

achieve better performance than the lower α values. Although, the velocity is higher at some points other than the resonance frequency but current at those points is high as well. The frequency response of the phase of the current is also shown in the figure. The phase difference of current with respect to velocity is zero at the resonance frequency. Whereas, there is an 180° phase shift of the current with respect to the velocity for the excitation before 40 Hz and after 55 Hz. Additionally, there is no effect of motor constant α on the current phase.

The frequency response of the magnitude of the stroke is shown in Figure 5.6. At the resonance frequency, lowest the motor constant α , the higher is the magnitude of the stroke at the resonance frequency. Whereas, the change in the stroke magnitude at the resonance frequency between different α values reduces as it becomes higher. For instance, improving the motor constant from 10 to 30 N/A, 30% reduction of stroke is observed. Whereas, modifying the motor constant from 70 to 90 N/A reduces the stroke to 20%. The frequency response of the stroke phase is also shown in Figure 5.6. The phase difference between the stroke and applied voltage is zero at the resonance frequency, whereas the 180° phase change is observed as the excitation frequency crosses the resonance frequency.

In order to identify the overall performance of the motor, stroke to current ratio provides a better picture. The magnitude plot of the transfer function between the stroke and the current is shown in Figure 5.7. Excitation at the resonance frequency that is 46 Hz, results in the best performance of the linear actuator considering stroke to current ratio. However, the motor constant has an impact on stroke to the current ratio as well. Bigger the motor constant, the higher is the stroke to current ratio. It is obvious from this figure, that the optimal point of excitation frequency is the resonance frequency because the stroke to current ratio is highest at this point.

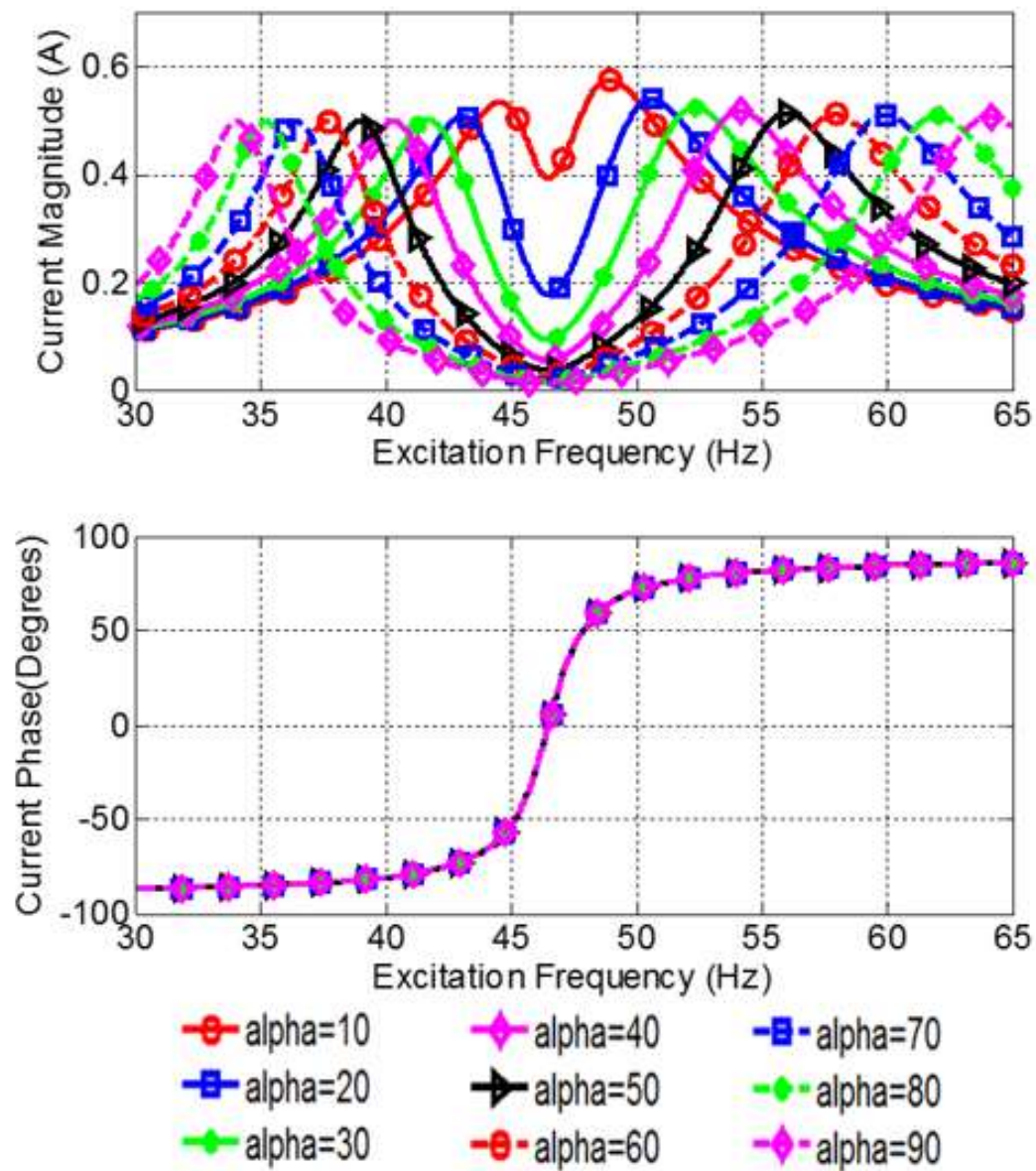


Figure 5.5: Frequency response of the magnitude and phase of the current with respect to the applied voltage

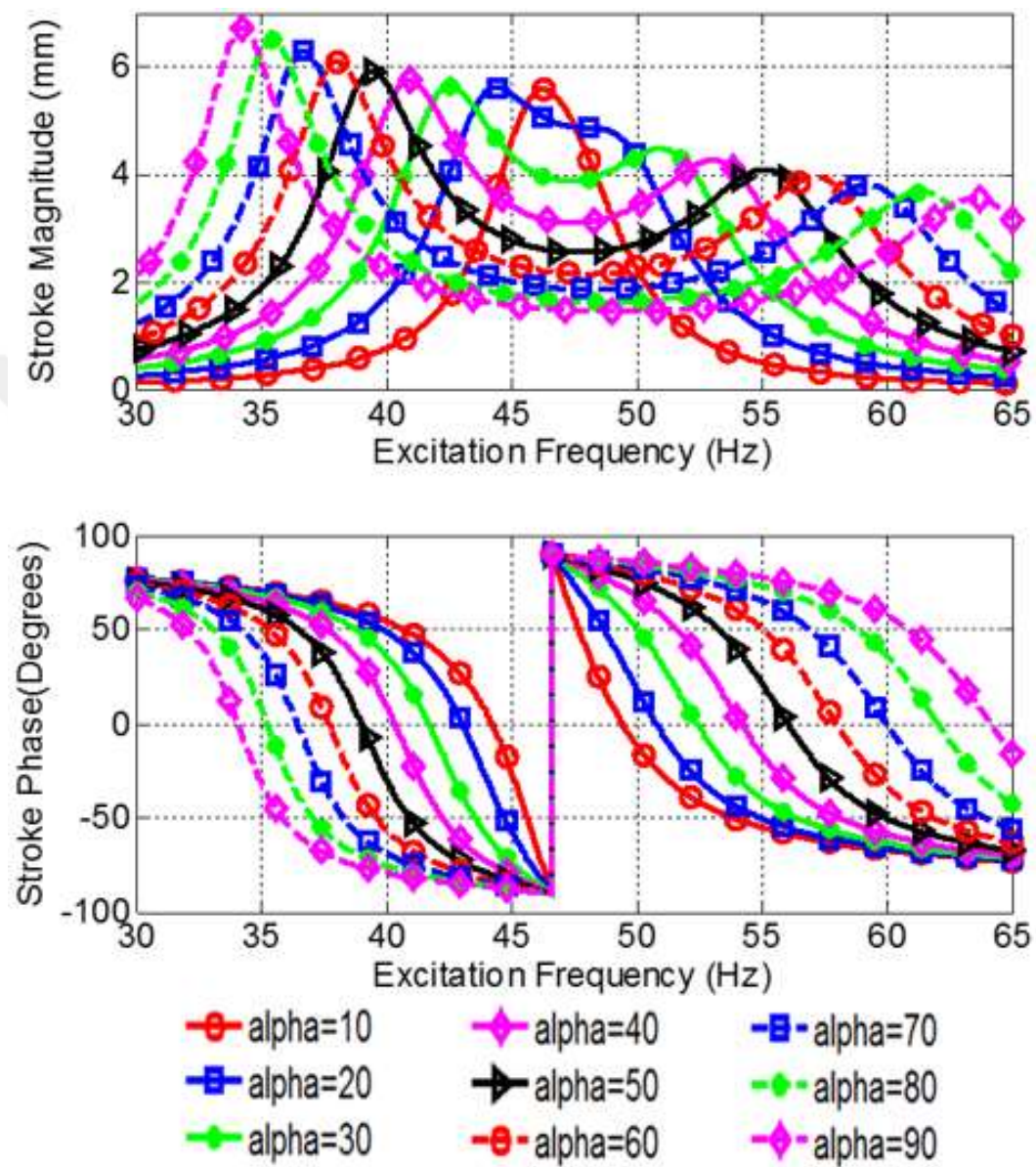


Figure 5.6: Frequency response of the magnitude and phase of the stroke with respect to the applied voltage

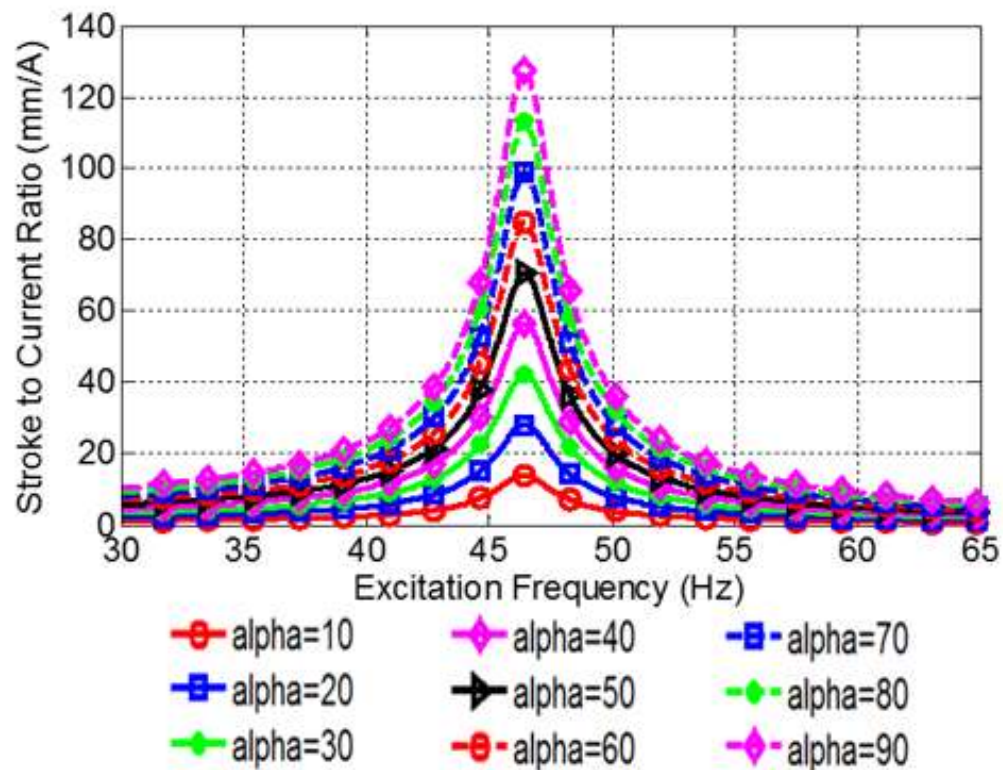


Figure 5.7: Frequency response of the magnitude of the stroke to current ratio

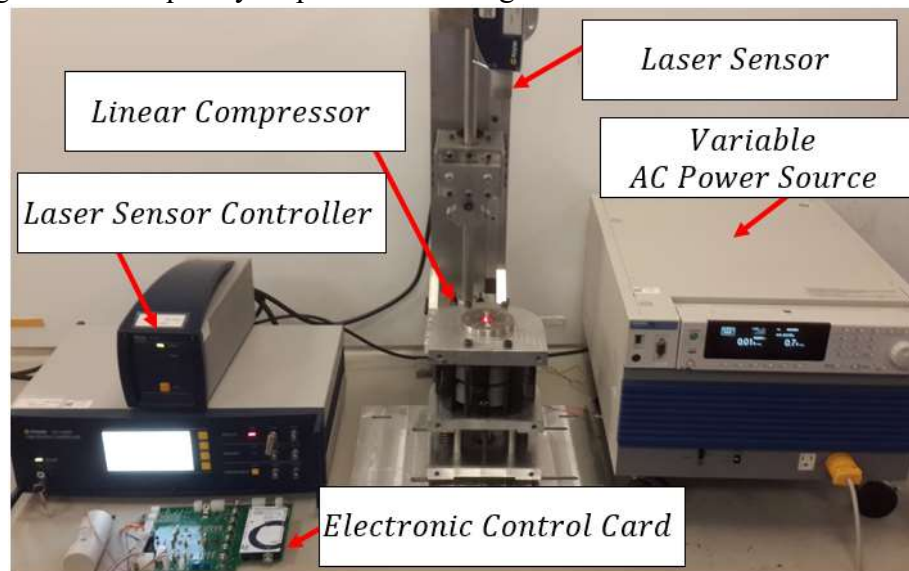


Figure 5.8: Experimental setup the system dynamics of the moving magnet linear actuator

5.6 Experimental Setup

A moving magnet linear actuator is manufactured as shown in Figure 5.8. The actuator is composed of two coils of 640 number of turns each. The inductance of the stator is 384 mH, coil resistance of $15\ \Omega$ and the series capacitance is 30 μF . Eight springs of approximately 7600 N/m stiffness are used to adjust the resonance frequency of the moving mass. The motor constant is 33 N/A and damping coefficient is 9.7 Ns/m. A variable voltage and frequency AC power supply from Kikusui (PCR1000LE) is used to provide the controlled amplitude and frequency of the voltage input. A vibrometer from Polytech (OFV-534 sensor head, OFV5000 controller) is used to measure the real-time stroke and velocity of the armature. A series of experiments with the voltage excitation frequency ranging 35 Hz to 55 Hz and an amplitude of 20 V are executed.

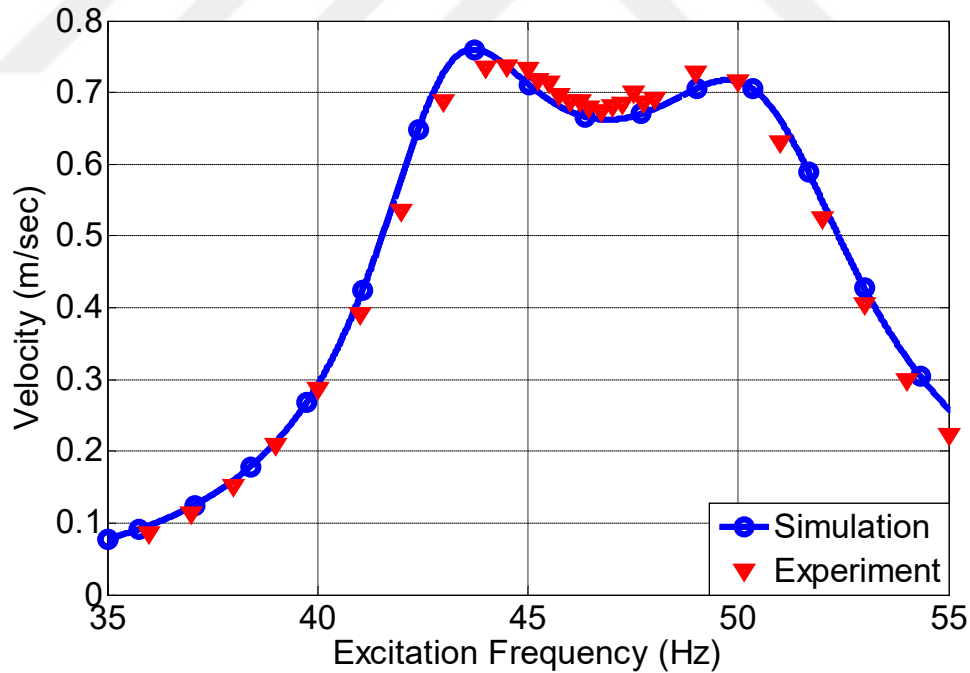


Figure 5.9: Comparison of the simulation and experimental results of the velocity of the armature at different excitation frequencies

5.7 Results

The simulation is executed with the parameters of the manufactured actuator and the results of the velocity of the armature are shown in Figure 5.9. The experimental results are in good agreement with the simulation results. At the excitation frequencies far lower and higher than the resonance frequency of the system, the amplitude of the velocity is lower. Whereas, same response as expected around resonance frequency as discussed earlier is obtained. The maximum of 3% error between the simulation and the experimental results of velocity is estimated.

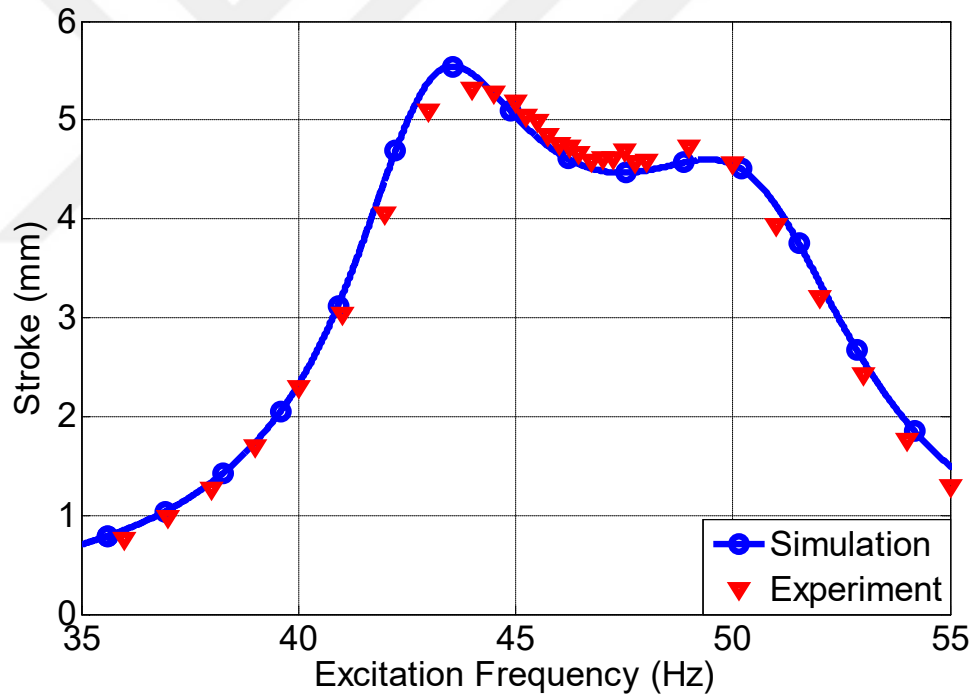


Figure 5.10: Comparison of the simulation and experimental results of the stroke position of the armature at different excitation frequencies

In linear compressors, the stroke position of the moving mass is the most critical. The experimental results of the stroke position with respect to the excitation frequency is shown

in Figure 5.10. The experimental results are coherent with the simulation results and the maximum of 5.6% error is noticed in the simulations. There are two peaks around the resonance frequency as seen in the figure. The amplitude difference is due to the difference between the electrical and mechanical resonance frequencies. The difference between the resonance frequencies is due to the limitation of the capacitor values. Additionally, the stiffness of the identical geometry springs is slightly different as well.

The coil current is the measure of the input power consumed by the linear compressor. The results of the stator coil current are shown in Figure 5.11. The experimental results of the stator coil current are complementing the simulation results. Exploiting the resonance frequency excitation phenomenon, the stator coil current as low as 80% of the maximum value is obtained in contrast with the excitation on other frequencies.

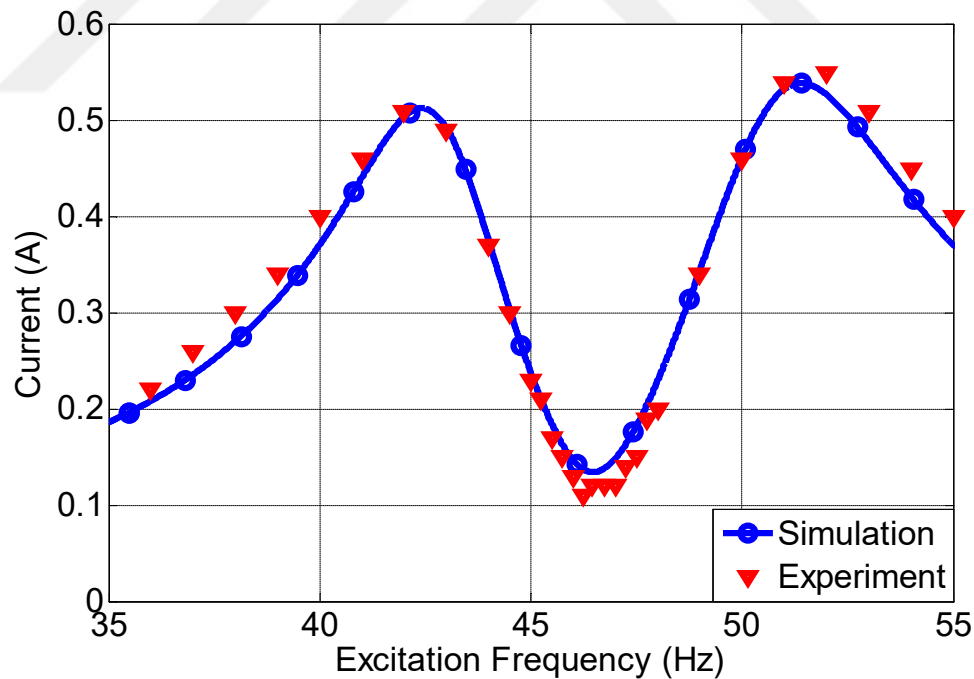


Figure 5.11: Comparison of the simulation and experimental results of the stator coil current at different excitation frequencies

Figure 5.10 and Figure 5.11 represents the effect of the excitation frequency on the stroke and current individually. Whereas, to conclude the optimal excitation frequency to achieve the best efficiency of the actuator, a frequency response function of stroke to current ratio is shown in Figure 5.12. The experimental results are in good agreement with the simulation results. As shown in Figure 5.10, the stroke value at the resonance frequency is lower than the excitation around resonance frequency, but due to the low current at resonance frequency provides higher stroke to current ratio. Therefore, the actuator excitation at 46 Hz is approximately 8 times better than the excitation at 35 Hz.

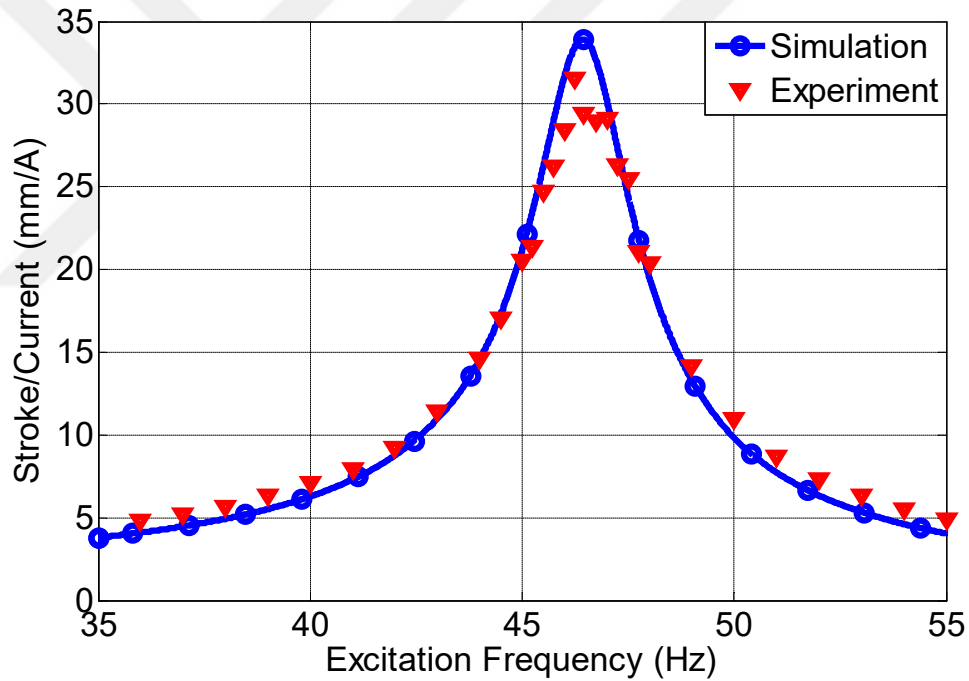


Figure 5.12: Comparison of the simulation and experimental results of the stroke of the armature to current of the stator coil ratio at different excitation frequencies

5.8 Conclusion

This chapter provides analytical models of the electromechanical dynamic parameters of the linear oscillating actuator. With the help of the developed model, the effects of the

mechanical resonance frequency, the electrical resonance frequency and the excitation frequency are estimated. The models incorporate the design parameters such as the moving mass, the stiffness of the springs, the damping coefficient, the inductance of the coil, the coil resistance, the capacitance and the motor constant of the linear actuator. The presented methodology presents a tool for the first time to conclude the design and operating parameters to acquire the desired performance of the linear oscillating actuator. With the help of a simulation in the design phase, the parameters can be engineered based on the operational requirements of the actuator.

The overall transfer function of the system is derived by transforming the mechanical and the electrical circuit models to the Laplace domain. With the help of the phasors, system dynamic parametric models are developed and a numerical simulation is executed. The system dynamic response includes the frequency response functions of the velocity, stroke position, stator coil current and the stroke to current ratio. The simulation results are experimentally validated with the experimental setup. It is concluded that the excitation at the resonance frequency provides the optimal point to achieve better stroke to current ratios.

Chapter 6

MAGNETIC FLUX DENSITY MODELING OF MOVING MAGNET LINEAR ACTUATOR

6.1 Overview

This chapter presents a methodology to estimate the magnetic flux density as well as the magnetic force of a moving magnet linear actuator. Considering the simulation time of a 3D FEM software, a combination of 2D FEM with the analytical models makes this technique convenient and expeditious. A numerical iterative simulation is carried out to incorporate the stator saturation with the help of Ampere's Circuit Law. Additionally, the optimization of the geometric design parameters of the linear actuator is obtainable with a quick simulation. Furthermore, the magnetic force is also estimated numerically with the help of work-energy method. A 3D FEM simulation is executed to compare the computation times. The presented methodology reduces the simulation time drastically.

6.2 Literature Review

Reluctance method [49], [50] is commonly used to identify the influence of each parameter on the magnetic field density, stroke length and resultant magnetic force. In order to calculate the magnetic flux, the equivalent magnetic circuit is established by using circuit analogies [13]. The magnetic flux path [51] is estimated with the 2-D FEM into circular loops and each loop is divided into the reluctance zones. Magnetic flux is estimated by using electric circuit fundamentals for magnetic circuits [7], [8], [54], [55]. Whereas, the stator saturation should be considered to find the realistic value of the magnetic flux density as used in rotary machines [56]. The saturation effect is incorporated to find the reluctance by executing an iterative simulation along the magnetic flux path to balance the magnetomotive force [28], [57].

This article is based on the same modeling technique on a moving magnet linear actuator used in the linear compressors. Initially, the design of a novel linear actuator is discussed and distinct reluctance loops are identified with the help of the 2-D FEM. Each magnetic flux loop is divided into finite reluctance zones in which the average magnetic flux is assumed to travel in the form of lines and arcs. The analytical model for each reluctance zone is established based on geometric parameters. The reluctance models also incorporate the effect of armature position. An iterative simulation is executed to find the reluctance of the magnetic circuit loop and magnetic flux density on the stator poles with the help of magnetomotive force balancing using the Ampere's Law. Consequently, the magnetic flux density in each stator pole is estimated through the algebraic sum of the flux values in each loop. Additionally, a nonlinear FEM simulation is executed to validate the magnetic flux density values. Additionally, using the work-energy principle, the magnetic force with respect to the stroke position is estimated and validated with the FEM results. Furthermore, a numerical simulation is executed to see the effect of the geometric parameters on the magnetic flux densities.

6.3 Magnetic Circuit Analysis

The cross-sectional schematic of the proposed linear actuator is shown in Figure 6.1. Considering the planar view, a three-legged stator encapsulates two coils. These coils are wound around the symmetrical axis in a circular fashion. The direction of current is opposite to each other in the coils which cause the opposite magnetic flux direction in the outer legs of the stator and same direction magnetic flux in the central leg. Consequently, the lower edges of the stator become temporarily magnetized and strong magnetic poles are created. The outer poles are opposite to the central leg magnetic poles. The armature consists of three cavities for the radially magnetized permanent magnets. The polarity of the magnets is similar in outer cavities whereas opposite in the central cavity of the armature for interaction with each stator leg. Back Iron is a stationary part and it provides the return path for the

magnetic flux lines. The geometric parameters of linear actuator are mentioned in Table 6-1.

Table 6-1: Parameters of the linear actuator

<i>Symbol</i>	<i>Quantity</i>	<i>Value</i>
N	number of turns	640
I	coil current (A)	1
w_s	stator width (mm)	15
w_{se}	the width of stator pole edge (mm)	5
h_{se}	the height of stator pole edge (mm)	3
t_b	bobbin thickness (mm)	2
C_w	coil width (mm)	20
r_b	the radius of the back iron (mm)	25.5
r_a	the radius of the armature (mm)	32
r_m	the inner radius of permanent magnets (mm)	29
r_s	inner radius of stator poles (mm)	33.5
g_{sa}	the air gap between stator and armature (mm)	1.5
g_{mb}	the air gap between the armature and back iron (mm)	3.5
h_b	the height of the back iron (mm)	3

6.4 Analytical Modeling

In order to find the path of the magnetic flux lines, an FEM simulation is carried out. The streamline plot is represented in Figure 6.2. The magnetic flux lines following the same flux path and approximate length are segregated into zones. Additionally, another assumption of the magnetic flux lines facing the same cross-sectional area in each zone is considered. There are 6 zones labeled as 1, 2, 3, 4, 5 and 6 as shown for the designed linear actuator. The streamline plot is representing one armature location. Whereas, it varies with the armature position. Therefore, the length and perpendicular area of the cross-section of the magnetic

flux lines are changing with the stroke position. The effect of armature position is incorporated in the presented models.

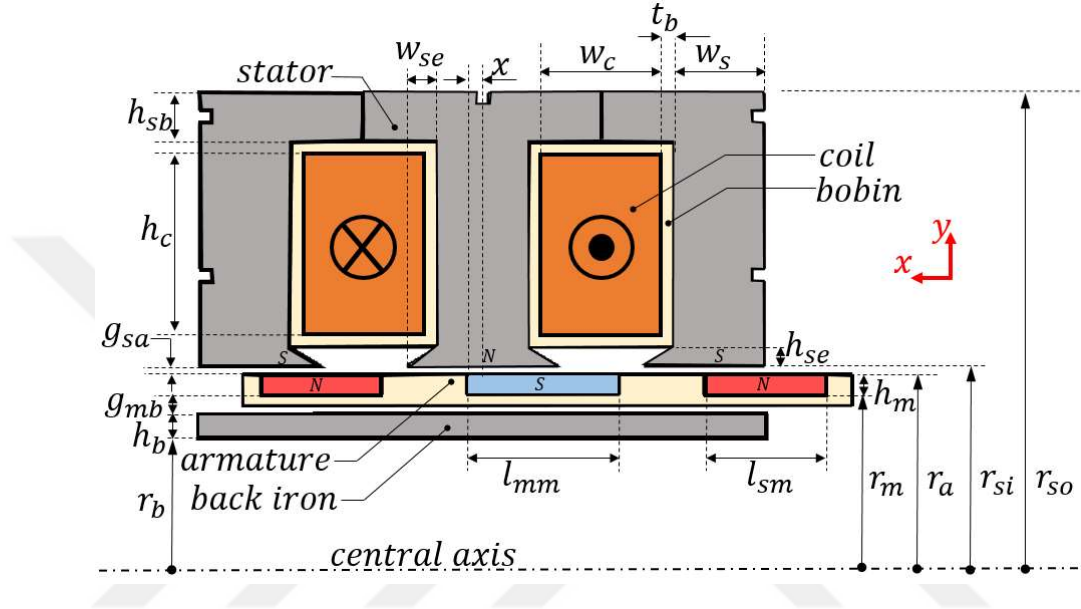


Figure 6.1: Cross-sectional view of axis-symmetrical moving magnet linear actuator

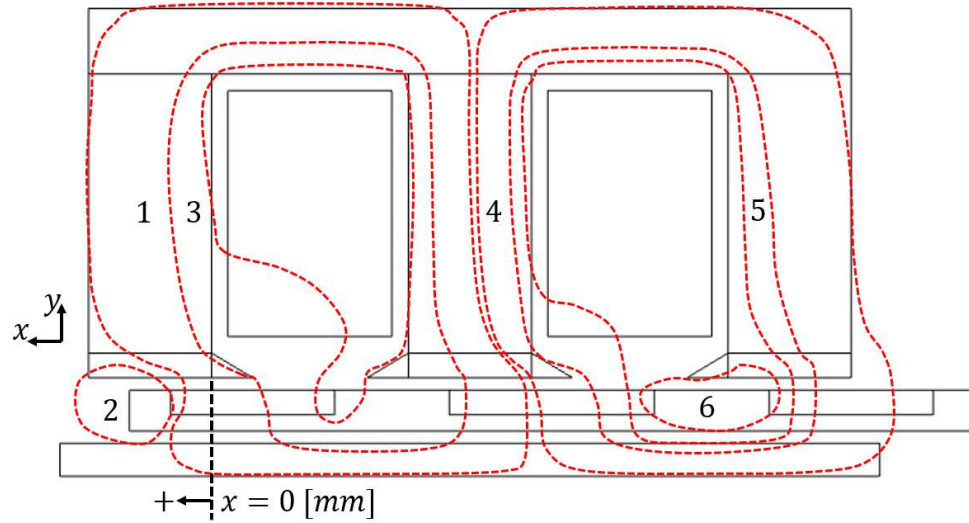


Figure 6.2: Streamline plot from finite element simulation for the identification of the magnetic flux loops

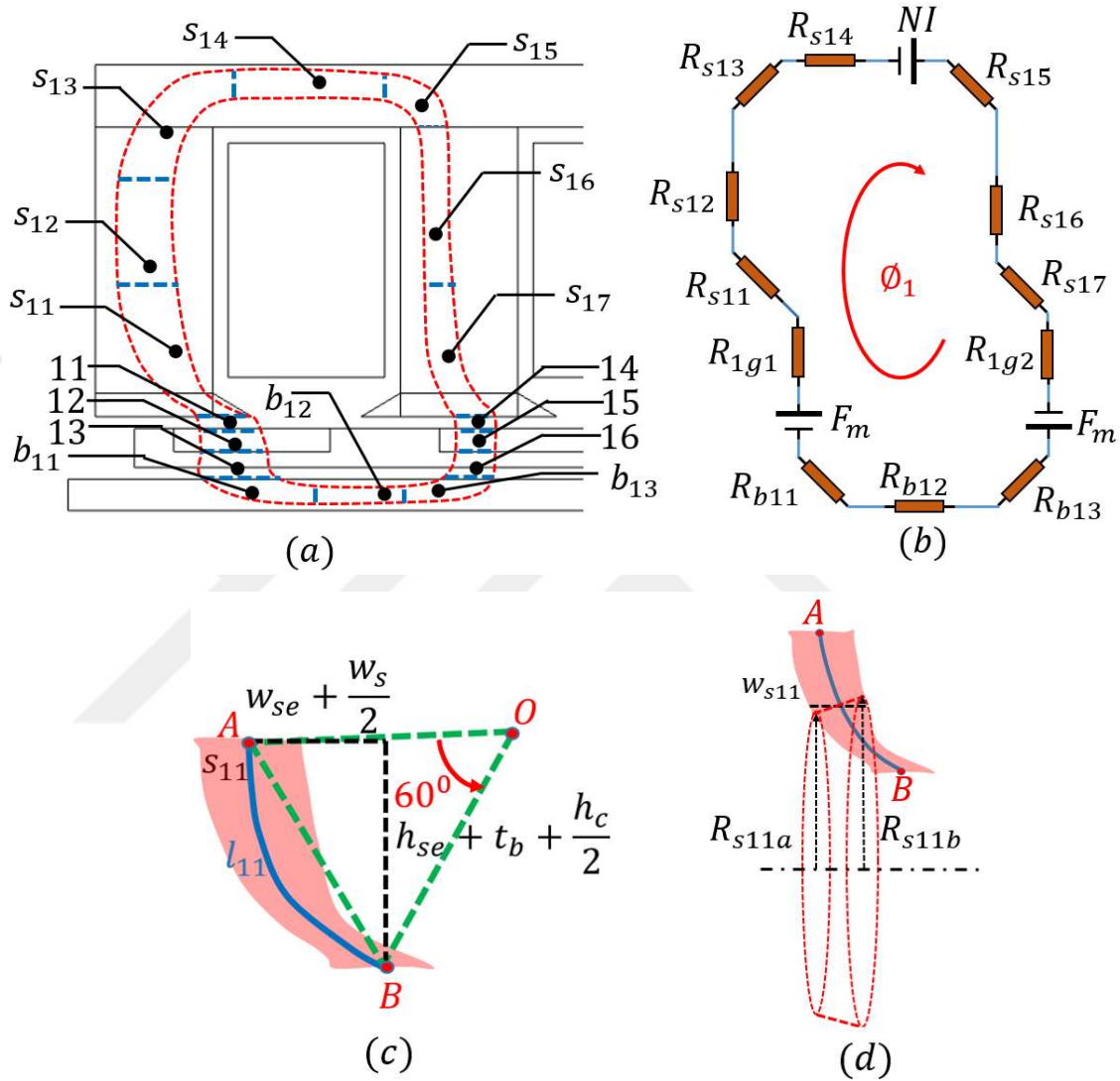


Figure 6.3: (a) Magnetic flux loop 1 representing the reluctance zones (b) the equivalent magnetic circuit for magnetic flux loop 1 (c) length of reluctance zone s_{11} (d) cross-sectional area faced by magnetic flux in zone s_{11}

The magnetic flux loops are approximated as the lengths and arcs as shown in Figure 6.3(a). The stator zones are numbered starting with s , back iron with b and zones with relative permeability equals to 1 are represented by numbers only. The equivalent magnetic circuit is

shown in Figure 6.3(b). Each permanent magnet is modeled as a series circuit of reluctance and a magnetomotive force F_m . Stators are modeled as finite reluctance zones and the relative permeability is not considered as infinity. The coil is modeled as a source of magnetomotive force which is the product of the number of turns and the current passing through the coil. The length of the magnetic circuit in zone s_{11} is estimated geometrically as shown in Figure 6.3(c). The surface area faced by the magnetic flux lines in zone s_{11} is represented in Figure 6.3(d). The geometric parameters of the moving magnet linear actuator are shown in Figure 6.1.

The reluctance zone s_{11} is considered as a circular arc. The x and y components between A and B are written in the form of the geometric parameters as shown in Figure 6.3(c) and the linear distance l_{AB} is estimated as shown in Equation (6-1). Assuming point O as the vertex of the isosceles triangle, the magnetic flux length l_{s11} is approximated by considering point O as the circle center as shown in Equation (6-2). Equation (6-3) represents the change in the width of the stator zone w_{s11} with the change in armature position. This equation is derived by linear fitting of the streamline plots of the armature locations from 0 to 10 mm. Figure 6.3(d) represents the average surface area faced by the magnetic flux lines. This surface area is modeled as a conical frustum [58] as shown in Equation (6-4).

$$l_{AB} = \sqrt{\left(w_{se} + \frac{w_s}{2} - w_{s11}\right)^2 + \left(h_{se} + \frac{h_c}{4} + t_b\right)^2} \quad (6-1)$$

$$l_{s11} = \frac{60^\circ \pi}{180} l_{AB} \quad (6-2)$$

$$w_{s11}(x) = 0.56x + 4.7 \quad (6-3)$$

$$A_{s11}(x) = \pi(R_{s11b} + R_{s11b})w_{s11} \quad (6-4)$$

The reluctance zone s_{12} is considered as a rectangle. The approximate length l_{s12} of the zone in the form of actuator geometry is shown in Figure 6.4(a) and analytically represented as shown in Equation (6-5). The surface area faced by the magnetic flux lines in this zone is estimated as shown in Equation (6-6).

$$l_{s12} = \frac{2}{3} h_c \quad (6-5)$$

$$A_{s12} = 2\pi(r_{si} + t_b + \frac{4}{9} h_c) w_{s12} \quad (6-6)$$

The reluctance zone s_{13} is considered as a circular arc. The approximate length l_{s1} of the zone in the form of actuator geometry is shown in Figure 6.4(b) and analytically represented as shown in Equation (6-8). The length CD Equation (6-7) is the linear distance between two points as shown in the figure and l_{s13} represents the circular arc length of the zone. As seen in the figure, the surface area faced by the magnetic flux lines is varying throughout the zone, the average surface area is considered in between the entrance Equation (6-9) and the exiting Equation (6-10) magnetic flux zones [59]. The average surface area faced by the magnetic flux for this zone is estimated as shown in Equation (6-11).

$$|CD| = \sqrt{\left(\frac{w_s}{2} + t_b\right)^2 + \left(\frac{h_c}{10} + t_b + \frac{h_{sb}}{10}\right)^2} \quad (6-7)$$

$$l_{s13} = \frac{60^\circ \pi}{180} |CD| \quad (6-8)$$

$$A_{s13_ent} = 2\pi(r_{so} - h_{sb} - \frac{9}{10} h_c) w_{s13} \quad (6-9)$$

$$A_{s13_exit} = \pi \left(r_{so}^2 - \left(r_{so} - \frac{4}{9} h_{sb} \right)^2 \right) \quad (6-10)$$

$$A_{s13_ent} = \frac{A_{s13_ent} + A_{s13_exit}}{2} \quad (6-11)$$

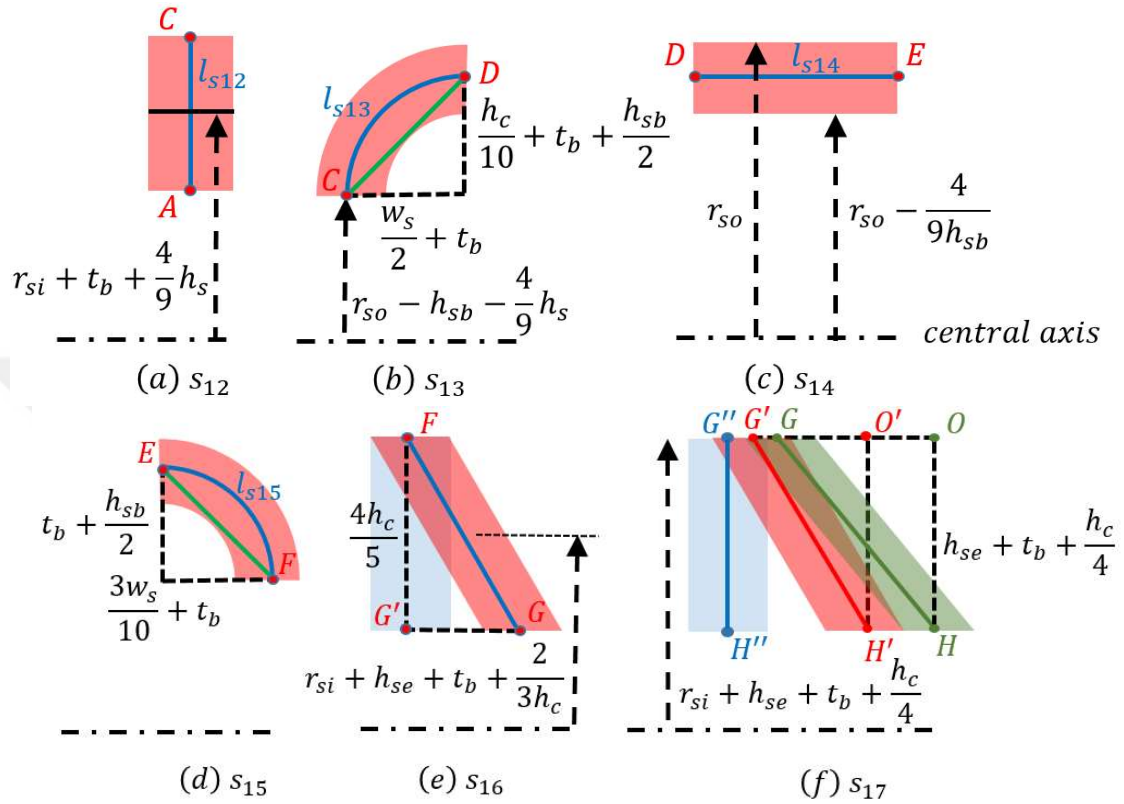


Figure 6.4: (a) Stator zone s_{12} (b) stator zone s_{13} (c) stator zone s_{14} (d) stator zone s_{15} (e) stator zone s_{16} (f) stator zone s_{17}

The reluctance zone s_{14} is considered as a rectangle. The approximate length l_{s14} of the zone is equal to the width of the coil as shown in Figure 6.3(a) and analytically shown in Equation (6-12). The radial distances from the central axis are shown in Figure 6.4(c) in the form of geometric parameters and the surface area faced by the magnetic flux lines is shown in Equation (6-13) which is equal to the area faced by the exiting magnetic flux for zone s_{13} .

$$l_{s14} = w_c \quad (6-12)$$

$$A_{s14} = \pi \left(r_{so}^2 - \left(r_{so} - \frac{4}{9} h_{sb} \right)^2 \right) = A_{s13_exit} \quad (6-13)$$

The reluctance zone s_{15} is considered as a circular arc. The approximate length l_{s15} of the

zone in the form of actuator geometry is shown in Figure 6.4(d) and analytically represented as shown in Equation (6-15). The length EF Equation (6-16) is the linear distance between two points as shown in the figure and l_{s15} represents the circular arc length of the zone. The surface area faced by the magnetic flux for this zone is estimated as shown in Equation (6-16).

$$|EF| = \sqrt{\left(\frac{h_{sb}}{2} + t_b\right)^2 + \left(\frac{3w_s}{10} + t_b\right)^2} \quad (6-14)$$

$$l_{s15} = \frac{60^\circ \pi}{180} |EF| \quad (6-15)$$

$$A_{s15} = \pi \left(r_{so}^2 - \left(r_{so} - \frac{4}{9} h_{sb} \right)^2 \right) = A_{s13_exit} \quad (6-16)$$

The reluctance zone s_{16} is considered as a parallelogram which transforms into a rectangle as the armature moves in the positive x -axis direction as demonstrated in Figure 6.4(e). The center point G travels to G' as the armature moves and the travel length of this segment is approximated the same way with the help of 2-D FEM as discussed earlier for Equation (6-3). The length GG' is analytically written as Equation (6-17) and the length of the magnetic circuit for this zone is shown is Equation (6-18). The surface area faced by the magnetic flux lines is analytically represented in Equation (6-19) in the form of the actuator geometry.

$$|GG'| = -0.25x + 2.5 \quad (6-17)$$

$$l_{s16} = \sqrt{\left(\frac{4}{5} h_c\right)^2 + GG'^2} \quad (6-18)$$

$$A_{s16} = \frac{3}{5}\pi \left(r_{si} + h_{se} + t_b + \frac{2}{3}h_c \right) w_s \quad (6-19)$$

The reluctance zone s_{17} is considered as a circular arc which transforms into a rectangle as the armature moves in the positive x -axis direction as demonstrated in Figure 6.4(f). The upper segment G , as well as the lower segment H , are moving with the armature. The linear distances GH , $G'H'$ and $G''H''$ represents the varying lengths of the magnetic circuit in this zone as shown in Equation (6-20) and Equation (6-21). The arc length l_{s17} is written as Equation (6-22). The area faced by the magnetic flux lines is analytically represented in Equation (6-23) in the form of the actuator geometry.

$$|OG| = -0.75x + 7.4 \quad (6-20)$$

$$|GH| = \sqrt{(OG)^2 + \left(h_{se} + t_b + \frac{h_c}{4} \right)^2} \quad (6-21)$$

$$l_{s17} = \frac{60^\circ\pi}{180} |GH| \quad (6-22)$$

$$A_{s17} = \frac{3}{5}w_s\pi \left(r_{si} + h_{se} + t_b + \frac{h_c}{4} \right) \quad (6-23)$$

The reluctance zone 11 to 16 are approximated as relative permeability 1 zones as they are either air gaps or the non-magnetic material or permanent magnets. The reluctance zone 11 is considered as a rectangle and the length of this zone is equal to the air gap between the armature and the stator poles as shown in Equation (6-24). The width of the zone is estimated the same way as Equation (6-3) and Equation (6-20) with the help of FEM. The surface area of the zone is calculated of shown in Equation (6-26). Zone 14 is similar to zone 11 except the width of the zone. The length, width and area of zone 14 are written as shown in Equation

(6-24), (6-27) and (6-28).

$$l_{11} = g_a = l_{14} \quad (6-24)$$

$$w_{11} = x + 7 \quad (6-25)$$

$$A_{11} = \pi w_{11}(r_{si} + r_a) \quad (6-26)$$

$$w_{14} = 0.6x + 7 \quad (6-27)$$

$$A_{14} = \pi w_{14}(r_{si} + r_a) \quad (6-28)$$

The reluctance zone 12 and 15 are rectangular zones made while the magnetic flux is passing through the permanent magnets. The lengths of these zones are approximated as the height of the permanent magnets h_m as shown in Equation (6-29). The width is geometrically written as Equation (6-30) and (6-32) with the help of the 2-D FEM. The surface area for these zones are expressed as Equation (6-31) and (6-33).

$$l_{12} = h_m = l_{15} \quad (6-29)$$

$$w_{12} = 0.9x + 9 \quad (6-30)$$

$$A_{12} = \pi w_{12}(r_m + r_a) \quad (6-31)$$

$$w_{15} = 0.6x + 7 = w_{14} \quad (6-32)$$

$$A_{15} = \pi w_{15}(r_m + r_a) \quad (6-33)$$

The reluctance zone 13 and 16 are rectangular zones created while the magnetic flux is passing through the permanent magnets. The lengths of these zones are considered as the air gap between the inner radius of the permanent magnets and the outer radius of the back iron g_{mb} as shown in Equation (6-34). The width is geometrically written as Equation (6-35) and (6-37) with the help of the 2-D FEM. The surface area faced by the magnetic flux lines in

these zones are approximated as shown in Equation (6-36) and (6-38).

$$l_{13} = g_{mb} = l_{16} \quad (6-34)$$

$$w_{13} = 0.63x + 6.9 \quad (6-35)$$

$$A_{13} = \pi w_{13}(r_m + r_b) \quad (6-36)$$

$$w_{16} = 0.85x + 7.2 \quad (6-37)$$

$$A_{16} = \pi w_{16}(r_m + r_b) \quad (6-38)$$

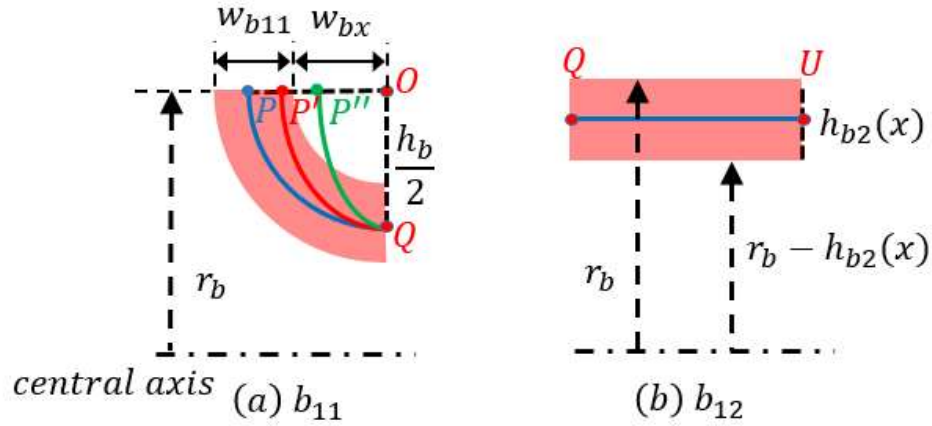


Figure 6.5: (a) Back iron zone b11 (b) back iron zone b12

The reluctance zone b_{11} , b_{12} and b_{13} are the back-iron zones for magnetic flux. The reluctance zone b_{11} is approximated as a circular arc as shown in Figure 6.5(a). The point P , P' and P'' corresponds to the variation of the exiting flux zone of b_{11} . The linear distance w_{b1} shown in the figure represents the width the exiting flux zone. Whereas, w_{bx} corresponds to the distance between point O and the starting point of the width of the exiting zone b_{11} . The entrance region of zone b_{11} and the height of the zone b_{22} shown in Figure 6.5(b) are same as shown in Equation (6-41) written as a function of x . The length PQ is the

linear distance between two points as shown in the figure and l_{b11} represents the circular arc length of the zone. The average surface area faced by the magnetic flux for this zone is estimated as shown in Equation (6-44).

$$w_{b11} = 1.1x + 11 \quad (6-39)$$

$$w_{bx} = -0.75x + 8.1 \quad (6-40)$$

$$h_{b2} = 0.16x + 1.9 \quad (6-41)$$

$$|PQ| = \sqrt{w_{bx}^2 + h_{b2}^2} \quad (6-42)$$

$$l_{b11} = \frac{60^\circ \pi}{180} |PQ| \quad (6-43)$$

$$A_{b11} = \pi w_{b11} r_b \quad (6-44)$$

The reluctance zone b_{12} is considered as a rectangle. The approximate length l_{b1} of the zone is equal to half of the length of the permanent magnets as shown in Equation (6-45). The radial distances for this zone are written in the form of geometric parameters of the actuator and are shown in Figure 6.5(b). The surface area faced by the magnetic flux lines in this zone is estimated as shown in Equation (6-46).

$$l_{b12} = \frac{l_m}{2} \quad (6-45)$$

$$A_{b12} = \pi(r_b^2 - (r_b - h_{b2})^2) \quad (6-46)$$

Equation (6-1) to (6-46) describes the methodology of defining the lengths and the surface areas of the magnetic circuit and magnetic flux density for loop 1. The same methodology is used to define the lengths and areas of the loop 2-6 mentioned in Figure 6.2.

Thus, all the geometric actuator parameters are included in defining the reluctance zones for each loop.

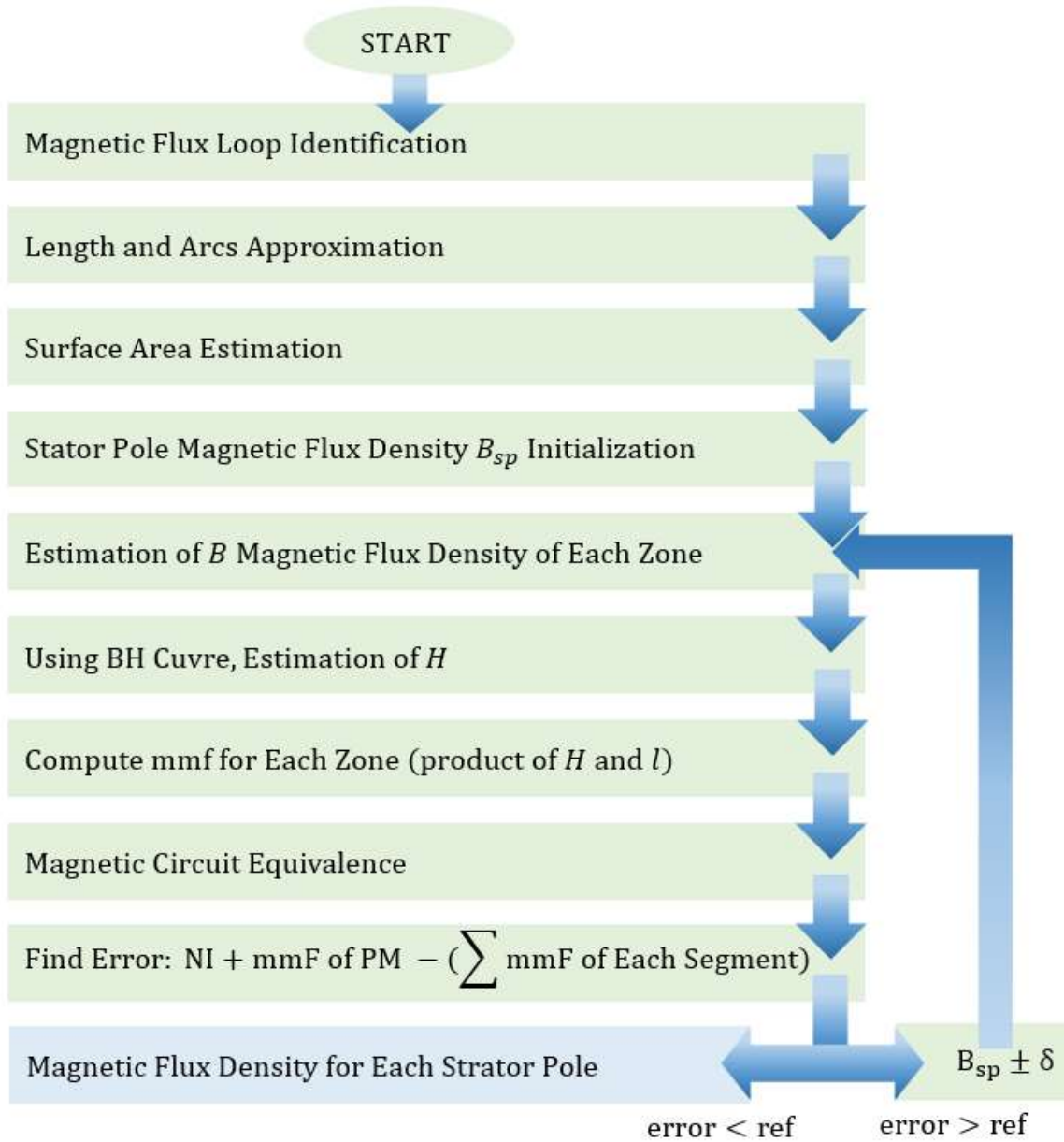


Figure 6.6: Block diagram for the numerical simulation

6.5 Numerical Simulation

After finding the geometric relationship of lengths and arc of all the magnetic flux loops defined earlier, a numerical simulation is carried out. The block diagram is shown in Figure 6.6 that represents the algorithm for each magnetic flux loop. Initially, a value of the magnetic flux density of a stator pole $B_{sp}[i]$ is assumed, where i is the number of the magnetic flux loop under iteration and its value ranges from 1 to 6 [59]. Then, for each magnetic flux loop, the stator pole area $A_{sp}[i]$ is estimated. For each i loop, following steps are carried out:

$$B[i, j] = B_{sp}[i] \times A_{sp}[i] / A_{s/b, i}[j] \quad (6-47)$$

where j is the zone number for each loop considered in section 6.4, s in the subscript represents the stator zones, b represents the back-iron zones and only i in the subscript corresponds to the zones with relative permeability equals to 1. Equation (6-47) estimates the magnetic flux density B for each zone in the magnetic flux loop under iteration. Furthermore, for the zones with relative permeability equals to 1, the magnetization H for each zone is estimated as shown in Equation (6-48).

$$H_{s/b, i}[j] = B[i, j] / \mu_0 \quad (6-48)$$

where μ_0 is the permeability of free space and its value is 12.57×10^{-7} H/m. In order to find the magnetization of magnetic material zones, BH curve is used. The analytical model must consider the saturation of the magnetic materials used in the linear actuator [57]. In order to achieve this, the magneto motive force $Hl_{s/b, i}$ for each zone is estimated by using Equation (6-49). The error Δ is obtained by subtracting the magnetomotive force of the sources and sinks respectively as shown in Equation (6-50). If the error is positive, a small increment in $B_{sp}[i]$ is made and vice versa as shown in Figure 6.6. This iterative loop keeps on

incrementing and decrementing the magnetic flux density of the stator until the error is less than the desired value.

$$Hl_{s/b,i}[j] = H_{s/b,i} \times l_{s/b,i} \quad (6-49)$$

$$\Delta = NI + F_m - \sum_{j=1}^n Hl_{s/b,i}[j] \quad (6-50)$$

where $l_{s/b,i}$ is the length of the zones in the i^{th} loop, N is the number of turns of the stator coil, I is the current, F_m is the magneto motive force of the permanent magnets, n is the zone number. With the help of this methodology, the magneto motive force balance between the sources and the sinks is achieved [59]. The reluctance of each magnetic flux loop is estimated by using Equation (6-51) [28], [12].

$$\mathfrak{R}[i] = \frac{\sum_{j=1}^n (H_{s/b,i}[j] \times l_{s/b,i}[j])}{B_{spi}[i] A_{sp}[i]} \quad (6-51)$$

The magnetic energy is estimated by using Equation (6-52) [52]. The magnetic energy provides the direct estimation of the magnetic force $\mathcal{F}_m$ with the help of work-energy principle as shown in Equation (6-53) for each magnetic flux loop. The partial differentiation of the magnetic energy with respect to the stroke x is calculated numerically. The overall magnetic force $\mathcal{F}_M$ is estimated by taking the sum of the magnetic force created by all the magnetic flux loops as shown in Equation (6-54).

$$W[i] = \frac{1}{2} (B_{spi}[i] A_{sp}[i])^2 \mathfrak{R}[i] \quad (6-52)$$

$$\mathcal{F}_m[i] = \frac{\partial W[i]}{\partial x} \quad (6-53)$$

$$\mathcal{F}_M = \sum_{i=1}^n \mathcal{F}_m[i] \quad (6-54)$$

6.6 Finite Element Simulation

Finite element method is used to estimate the electromagnetic force. As the actuation involves electromagnetism, Magnetic field (mf) physics module is used. FEM simulation is done in 3-D space dimension. Same geometry discussed in earlier is built in the FEM environment with 640 number of turns for each coil and constant current value of 1 A is simulated. Considering nonlinearity of the system, BH curve is implemented in the stator blocks as well as on the back iron. The stationary solution is computed using Maxwell-Ampere's Law and Faraday's law given in the following equations:

$$J_e = \frac{NI_{coil}}{A} \quad (6-55)$$

$$\nabla \times \left(\frac{B}{\mu_0 \mu_r} \right) - \sigma v \times B = J_e \quad (6-56)$$

$$B = \nabla \times A \quad (6-57)$$

where J , N , I_{coil} , A , B , μ_0 , μ_r , σ and v are current density, number of turns, coil current, magnetic flux density, relative permeability in vacuum, relative permeability of material, electric conductivity and velocity, respectively. Standard meshing is used with a maximum element size of 9.54 mm, minimum element size of 0.054 mm, maximum element growth rate is 1.3 and curvature factor of 0.3. A parametric sweep is executed on different stroke positions ranging from 0 mm to 10 mm to find the magnetic flux density at each stroke position of 1 mm. The magnitude and Direction of force is estimated in Finite Element Simulation using Equation (6-58):

$$F = \int_{\partial\Omega} 2\pi r q T dS \quad (6-58)$$

where r , q and T are position vector, normal unit vector and stress tensor, respectively.

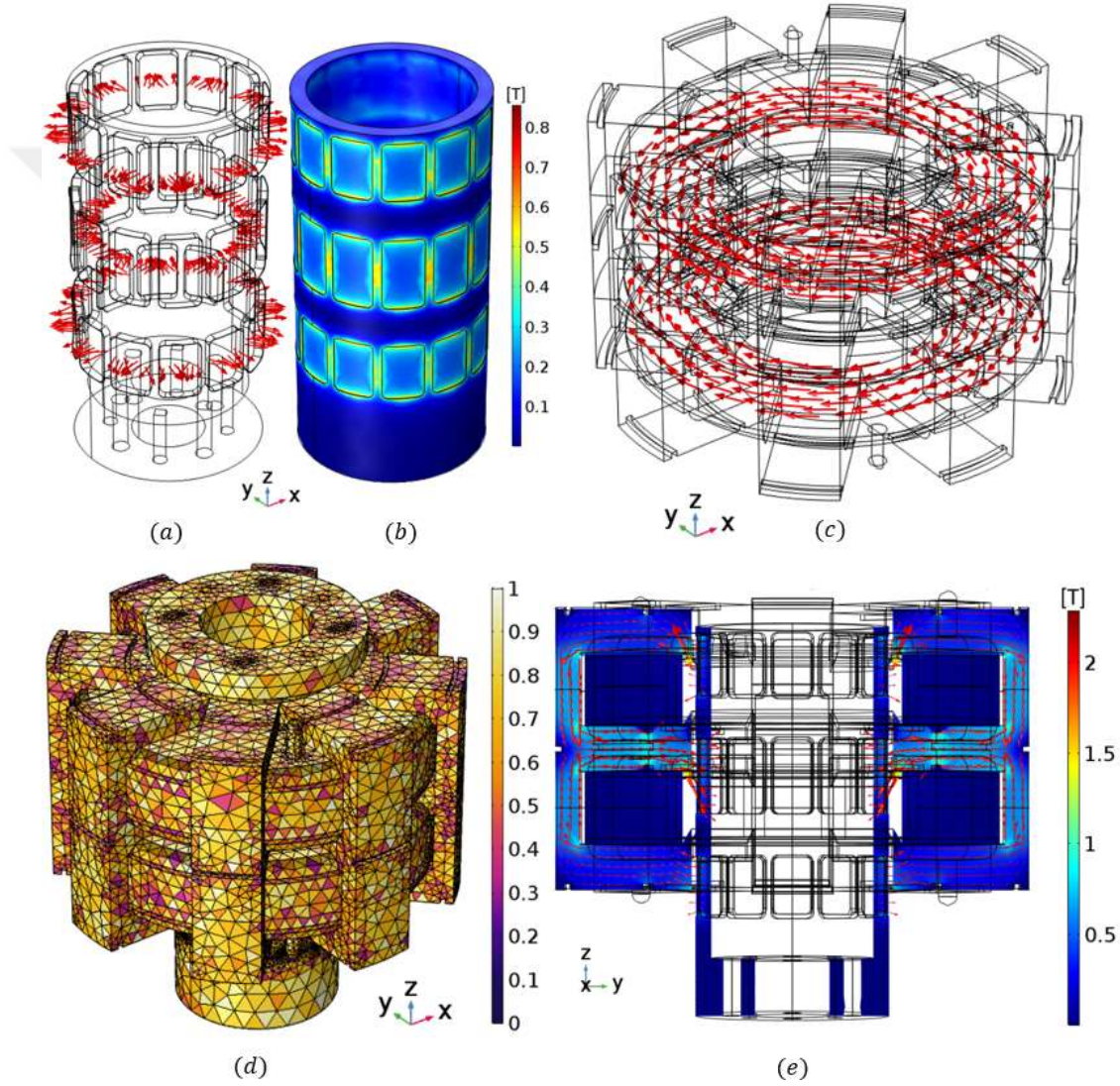


Figure 6.7: (a) Radial magnetization of permanent magnets (b) magnetic flux density results of armature (c) current density arrow volume plot (d) mesh diagram of the moving magnet linear actuator (e) magnetic flux density results of the finite element simulation

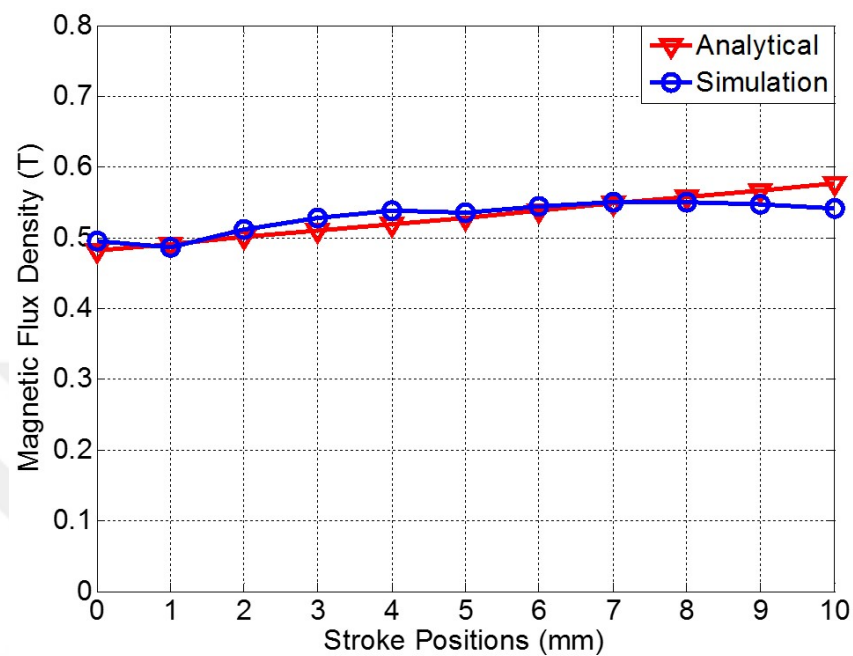


Figure 6.8: Flux density comparison of the finite element method and analytical simulation for the left stator

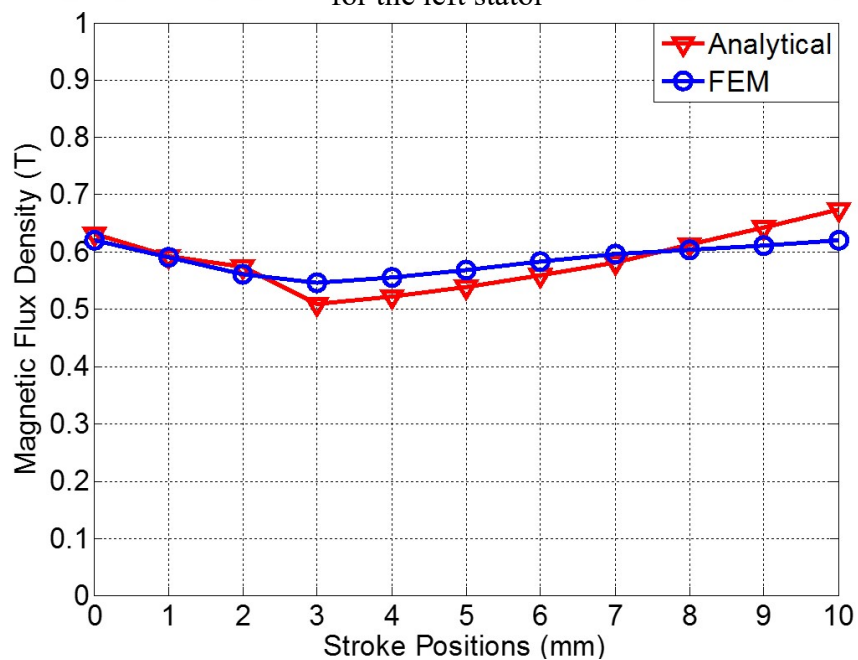


Figure 6.9: Middle stator flux density comparison of the finite element method and analytical simulation

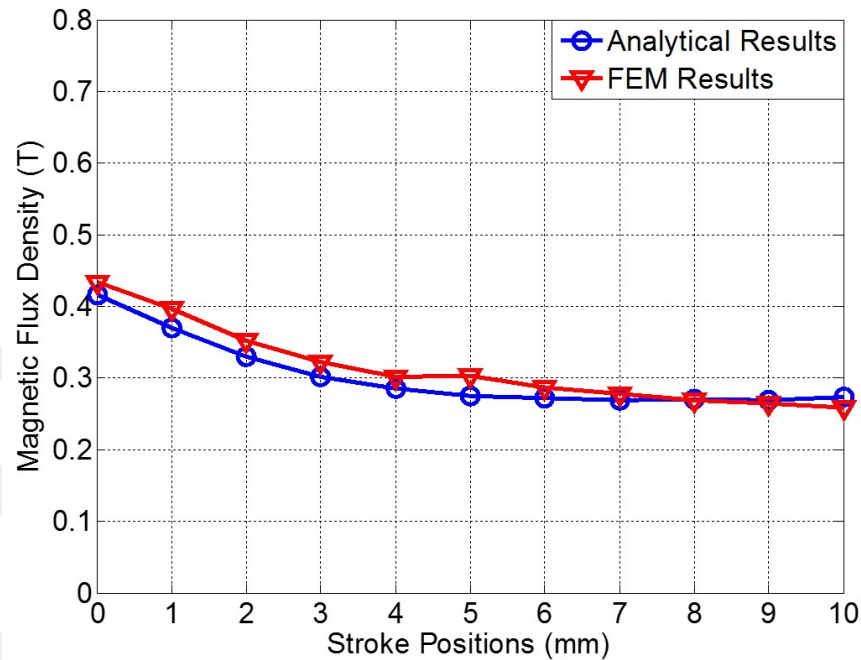


Figure 6.10: Right stator flux density comparison of the finite element method and analytical simulation

Figure 6.7 demonstrates the results of the finite element simulation executed in Comsol software. As shown in Figure 6.7(a) and (b), the three rows of permanent magnets are radially magnetized. The current density is represented by an arrow volume plot as shown in Figure 6.7(c). As discussed earlier, the direction of currents in both coils are equal in magnitude and opposite in direction. Figure 6.7(d) represents the comparative magnitude of the mesh sizes of the moving magnet linear actuator geometry. The finite element simulation results of the magnetic flux density are shown in Figure 6.7(e). The direction of the magnetic flux is demonstrated as arrow surface plot. It is seen that the stator pole edges become strong magnetic poles. Consequently, pulling or pushing the armature with embedded permanent magnets. The simulation is performed on a workstation on a dual core 3.33 GHz processor with a 32 GB RAM. The simulation time is recorded as 8.5 hours for a parametric sweep of 10 stroke positions of the armature.

6.7 Results

Based on the analytical estimation of the reluctance zones mentioned in section 6.4, the simulation based on the algorithm explained in section 6.5 is executed. The time for this simulation is recorded as 3 min. Additionally, an FEM simulation is executed based on the settings described in section 6.6. As discussed in section 6.6, the simulation time recorded for the FEM simulation is 8.5 Hours. Both simulations are executed with the same current excitation and geometric parameters discussed in the design section. Both simulations are considering the stator and back iron saturation through the Ampere's Law as discussed earlier. For each stroke position from 0 to 10 mm, the comparison of both simulations for the magnetic flux density results are shown in Figure 6.8 to Figure 6.10.

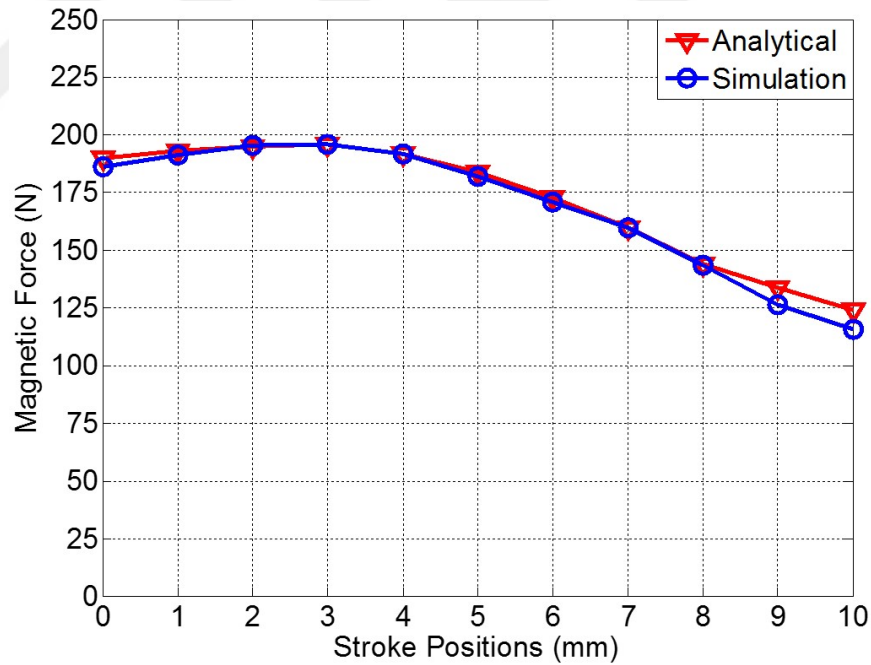


Figure 6.11: Magnetic force comparison of the finite element method and simulation for current of 1 A

The analytical simulation results are in good agreement with the FEM solution. It is clear from these figures the magnetic flux density is between 0.3 T to 0.65 T depending on the stroke position. A maximum of 8% error is seen in these plots because of the assumptions of the analytical solution. For instance, in one reluctance loop, the magnetic flux is assumed to travel in the same direction. Additionally, the geometric lengths and areas are approximated for each zone and for the zones varying with the stroke position, a linear fit is used to simplify the system. Secondly, the analytical simulation is divided into a limited number of reluctance zones whereas FEM uses finer meshes for estimation of magnetic flux. Lastly, flux leakage is not considered in analytical solution. The analytical simulation for the force-stroke relationship is also executed based on Equation (6-52), (6-53) and (6-54). The results are complementing the simulation results as shown in Figure 6.11. A maximum of 10% error is observed in between the analytical and FEM simulation.

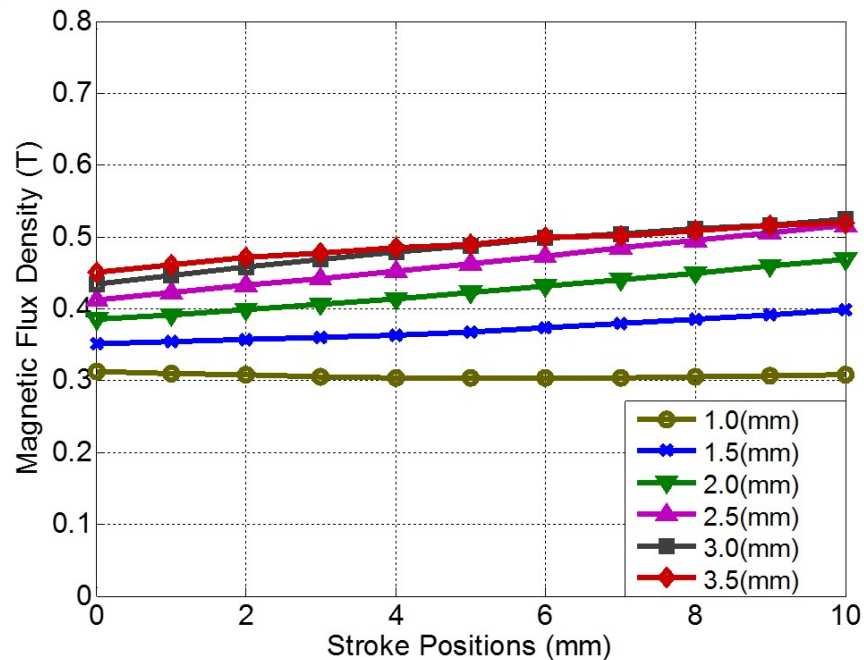


Figure 6.12: The effect of the height of the permanent magnets on the magnetic flux density with respect to the stroke position

The stator is one of a high-density material and contributes to the weight of the linear actuator effectively. By using this model, the stator and mover geometry can be optimized very quickly. Figure 6.12 and Figure 6.13 represents the effect of the two geometric parameters (height of the permanent magnet and air gap between the stator pole and the armature) on the magnetic flux density of the left stator of the linear actuator. Permanent magnets are expensive and to optimize the cost of the linear actuator such a simulation can be very helpful. Figure 6.12 provides an overview of the effects of the height of the permanent magnets on the magnetic flux density. Increasing the height from 1 to 1.5 mm provides a considerable increase (16%) in the magnetic flux density. Whereas, increasing the height from 3 to 3.5 mm provides an insignificant increase (4%) in the magnetic flux density.

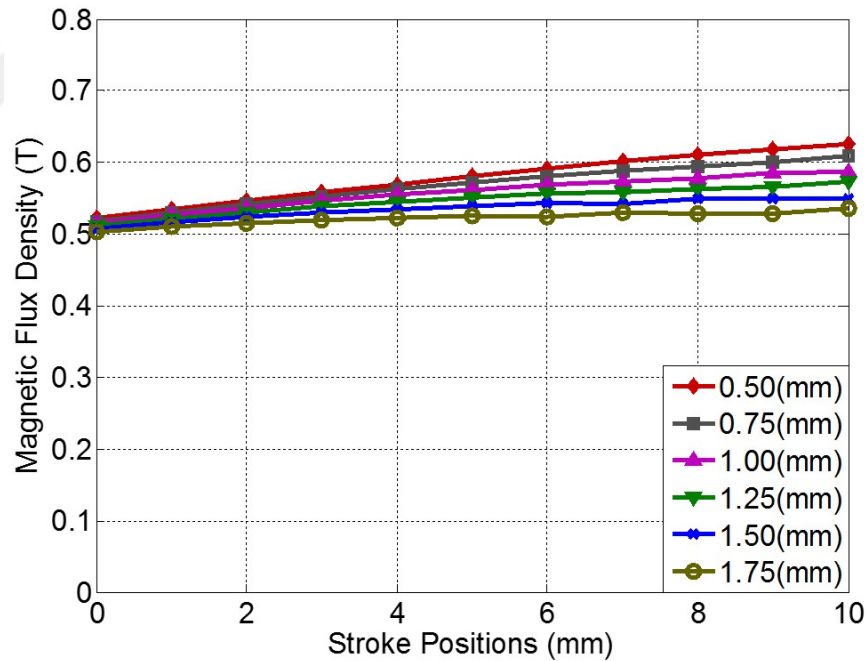


Figure 6.13: The effect of the air gap between the armature and stator poles on the magnetic flux density with respect to the stroke position

Similarly, the effects of the air gap between the stator poles and the armature are shown

in Figure 6.13. The air gap between the stator and the armature plays a vital role in the assembly process as well as in the operational phase. Therefore, this figure is useful in making a decision for the air gap considering the assembly constraints and the effect on the magnetic flux density can be estimated. For low stroke applications, the larger air gaps will decrease the magnetic flux density by 8% approximately. Whereas, for longer stroke applications, increasing the air gap will decrease the magnetic flux density by 16%. By using the same methodology, all the geometric parameters can be optimized according to the design requirements, performance and cost constraints.

6.8 Conclusion

This section presents a methodology to find the magnetic flux density as well as the electromagnetic force of a moving magnet linear actuator used in the linear compressor. This technique involves analytical modeling in combination with a 2-D FEM simulation to reduce the computation time effectively. This method can be used to find the magnetic flux density for any desired shape. Additionally, the analytical model for magnetic force of the moving magnet linear actuator with the help of work-energy relationship is also presented. In order to generate the realistic results, the BH curve is implemented to incorporate the stator saturation instead of considering a constant value of relative permeability of magnetic materials. The reluctance values are estimated with the help of the actuator geometry as well as the magnetomotive force balance with the help of the Ampere's Law. A numerical simulation is executed based on these reluctance values and with the help of magnetic circuit analogy, the magnetic flux values are estimated.

This method provides the effects of each geometric parameter of the actuator on the magnetic flux density. With the help of this technique, building a very compact and efficient actuator is possible. An FEM simulation is executed to validate the results of the analytical simulation. The results for the magnetic flux density as well as the electromagnetic forces are less than 10%. Using the simulation output, motor performance parameters like efficiency

and motor constant can also be estimated.

The major advantage of the presented methodology drastically reduces the simulation time compared to the FEM simulation. Additionally, the optimization of the geometrical design parameters of the moving magnet linear actuator in the conferred simulation is comparatively easy and rapid. Additionally, to see the result of scaling the actuator to bigger or smaller sizes can be done instantly.



Chapter 7

MMLA BASED LINEAR COMPRESSOR

7.1 Overview

A novel moving magnet linear actuator is proposed for linear oscillations in the linear resonant compressors for household refrigerators. This section provides stator and armature design including CAD model and geometric parameters. Furthermore, the working principle of the proposed actuator is explained. With the help of the electromechanical analytical model, the dynamic parameters such as stroke, velocity and acceleration of the armature are derived. Additionally, the time-dependent current model of the stator winding is proposed. An experimental setup is used to validate these responses at the resonance excitation frequency with the help of sensors. The system kinetics are discussed to estimate the spring, damping, inertial and magnetic forces. The force models are experimentally validated at the resonance frequency excitation. In order to evaluate the performance of the proposed actuator, a comparison of the performance parameters such as efficiency, stroke, current and mass flow rate is demonstrated with the conventional rotary as well as the linear motors for linear compressor application.

7.1 Literature Review

As discussed in Chapter 3, a three-phase excited linear permanent magnet synchronous actuator is presented by [13]. The mover is axis-symmetrical with embedded ring magnets. When the coil is energized, an axial thrust force is created in the direction depending on coil current. Consequently, mover oscillates between forward and backward directions. In order to have a compact size of the actuator, [24] preferred a single-phase Redlich motor with outer and inner stator cores. A similar design with a Halbach magnetized armature is proposed by [25] which eliminates the need of back iron by concentrating one-sided flux. Similar

armature with seven poles and corresponding DC excited wound coils are proposed by [10]. Furthermore, tubular staggered-tooth transverse-flux moving magnet linear actuator is developed by [26], [45], [65]. The winding technique similar to the conventional motors is used with three-phase excitation. Stator cores are designed in a way to provide transverse flux to circulate in a loop. As each phase of the power energizes one coil, a unique pair of permanent magnets oscillates under the stator yoke. A very simple configuration of MMLA is developed by [28]. A circular stator winding on the outer core along with the inner core (back iron) oscillates radially magnetized armature assembly. Furthermore, a moving magnet linear motor with toroidal winding done on a semi-circular core is developed by [8]. In the discontinuity region of the stator core, the mover with embedded magnets is placed, which aligns itself based on the phase of the current and the polarity of the permanent magnets.

Single phase AC power is used to drive the single coil linear motors for linear compressors. The motor is modeled as a resistor and an inductor, in series with the back emf source and the mechanical system is modeled as a mass-spring-damper (MSD) system. The dynamics of the MMLA modeled as LR circuit is discussed by many researchers to estimate the amount of actuator force and stroke. A coupled electromechanical model to estimate the electrical and motional impedances and the effect of mechanical resonance on the stroke and current is presented by [63]. Using the similar method, [45] discussed the effect of electrical and mechanical natural frequencies on the efficiency of a linear actuator. The stroke and axial force estimation of a linear actuator driven through a single-phase AC are presented by [66-68]. Although, the stroke and current dynamic models of the linear motors without a series capacitance are presented [66-69], the similar models for the overall dynamic and kinetics of the coupled RLC and MSD modeled systems are required.

This research provides the design of a novel moving magnet linear actuator. The geometric parameters, CAD model and working principle of the proposed actuator are presented. The linear motor is modeled as an LR series circuit and driven with single phase

AC in series with a capacitor. The electromechanically coupled system is modeled analytically to estimate the dynamic parameters: stroke, current, velocity and acceleration of the armature. With the help of numerical simulation, the effect of excitation frequency and the mechanical impedance on the magnitude and phase of the system dynamics is discussed in detail. The simulation results are validated with the help of experimental setup at the resonance excitation frequency. Furthermore, a comparison of the performance parameters is demonstrated with the conventional rotary as well as linear motors in linear compressor application.

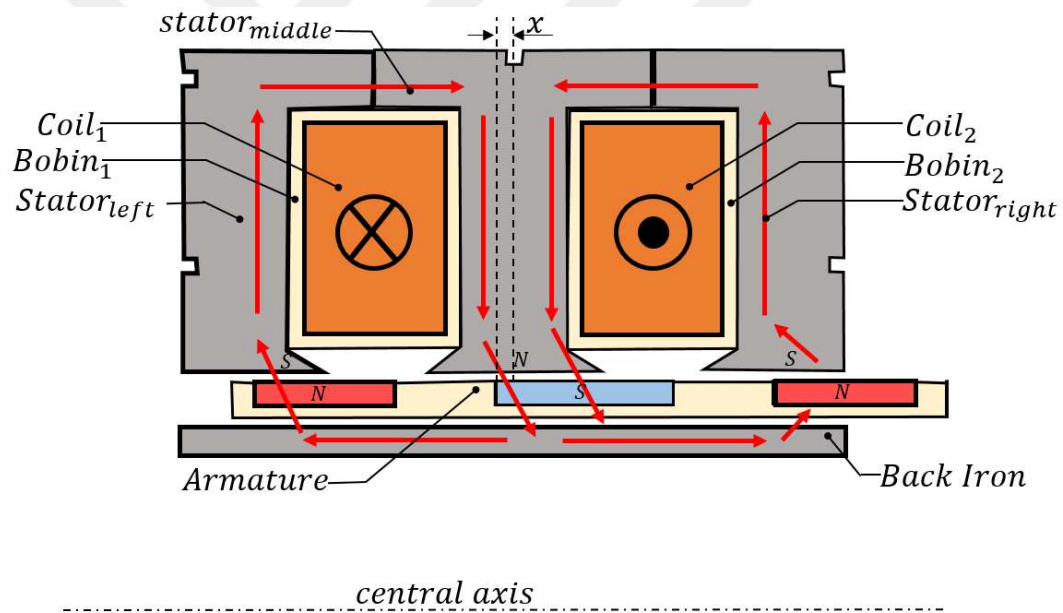


Figure 7.1: Cross-section of the schematic of proposed moving magnet linear actuator

7.2 Theory

A Linear compressor is composed of three major parts: a linear actuator, spring assemblies and air/gas compression chamber. This chapter focuses on the design of a moving magnet linear actuator. The cross-sectional schematic of the proposed linear actuator is

shown in Figure 7.1. Considering the planar view, a three-legged stator encapsulates two coils. These coils are wound around the symmetrical axis in a circular fashion. The direction of current is opposite in these coils which causes the same magnetic flux direction in the side legs of the stator and opposite direction magnetic flux in the middle leg. Consequently, the lower edges of the stator become temporarily magnetized and strong magnetic poles are formed. The side poles are opposite to the middle leg magnetic poles. The armature consists of three slots for permanent magnets. The polarity of the magnets is similar in side slots whereas opposite in the middle slot of the armature for interaction with each stator leg. Back Iron is a stationary part and it provides the return path for the magnetic flux lines.

At one instant, when the opposite polarity current is flowing through the coils, the corresponding temporary magnetic poles are established on the stator legs. The magnetic poles pull or push the armature axially depending on the direction of the current. Reversing the coil current will cause the armature to behave oppositely.

7.3 Mechanical Design

Considering the required oscillatory motion, the stator is mounted on the stationary support plates and mover is connected to the stationary plates with the help of eight springs as shown in Figure 7.2. Springs are used to provide the support for the moving assembly to stay in the center as well as to define the stiffness to the moving armature. The armature is also connected to the piston for the gas compression chamber and piston oscillates with the armature. Consequently, the intake and exhaust stroke happen and maintain a line pressure. The exploded view of the linear compressor is represented in Figure 7.3.

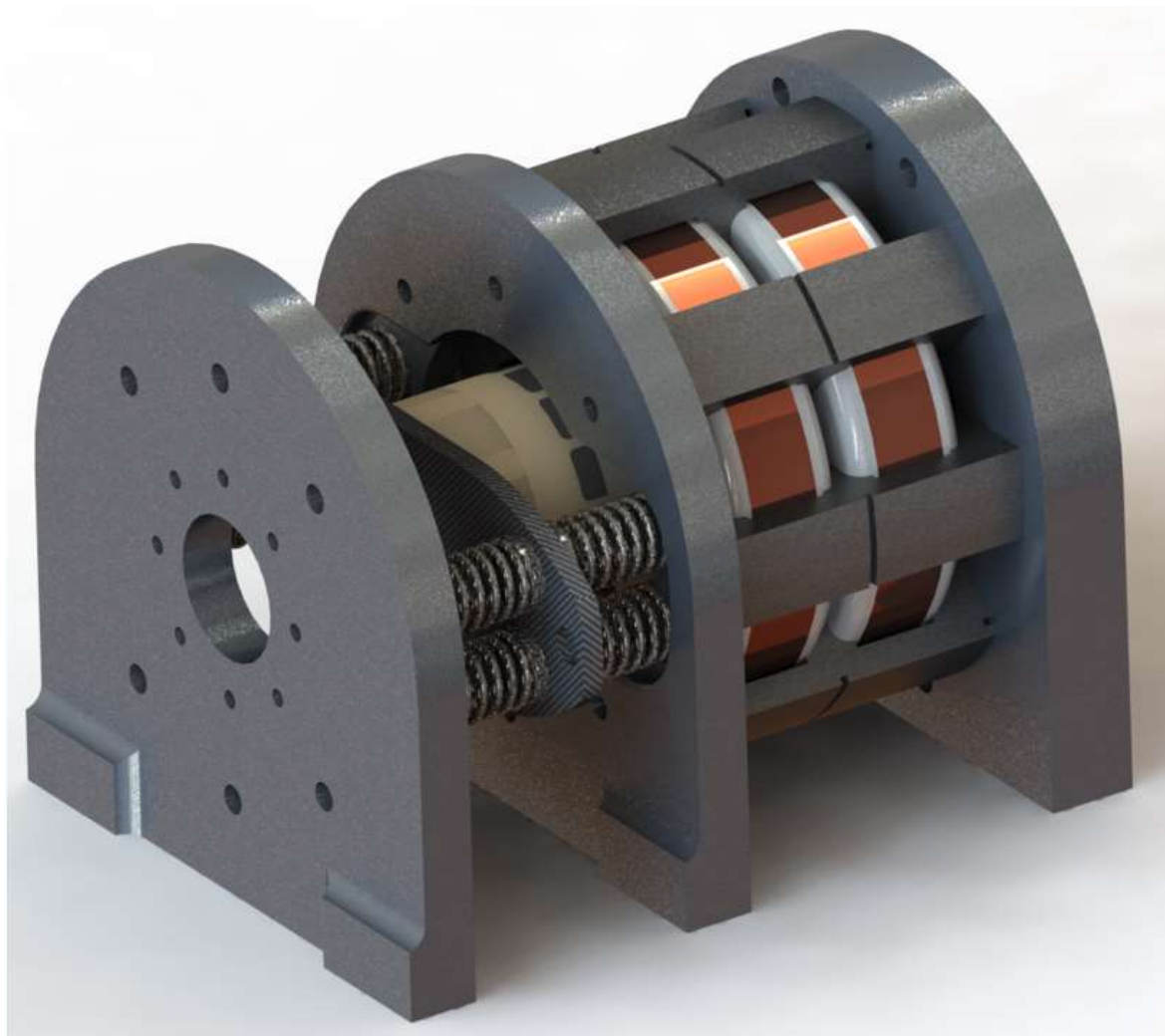


Figure 7.2: CAD model of the linear compressor

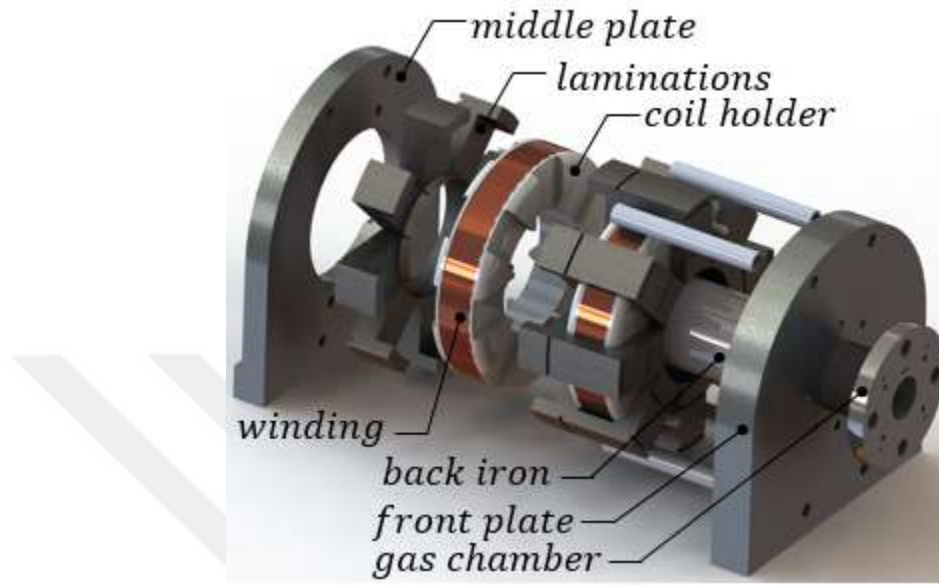


Figure 7.3: Exploded view of the linear compressor

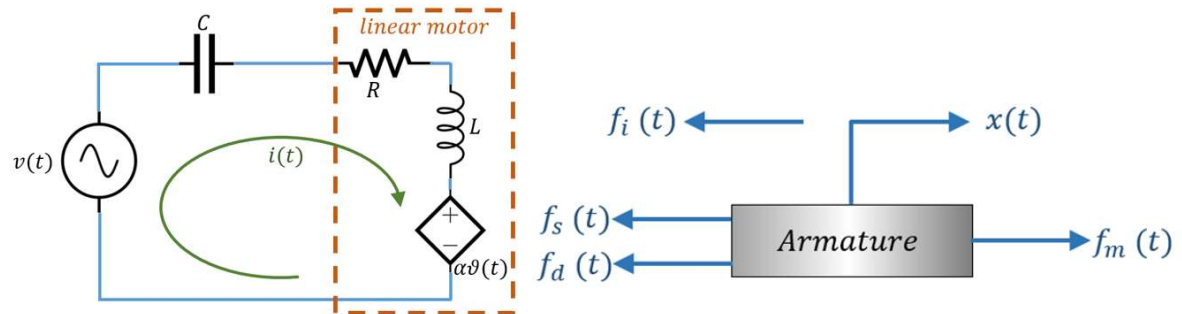


Figure 7.4: (a) Electrical circuit model of the linear actuator (b) free body diagram of the armature

7.4 Analytical Modeling

A series RLC circuit as shown in Figure 7.4(a), with single phase AC excitation is used and the time response is estimated by using Equation (7-1), where R is the coil resistance, L is the coil inductance, C is the capacitance, α is the back emf constant, v is the instantaneous voltage input and ϑ is the armature velocity. Adjustable AC power source is used to excite

the system on any desired frequency and amplitude. Figure 7.4(a) demonstrates the passive circuit elements in the current loop including the back emf.

$$v(t) = Ri(t) + L \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) dt + \alpha \vartheta \quad (7-1)$$

By taking the Laplace transform:

$$\bar{V} = R\bar{I} + sL\bar{I} + \frac{\bar{I}}{sC} + \alpha s\bar{X} \gg \bar{V} - \alpha \bar{\vartheta} = \bar{I} \left[R + sL + \frac{1}{sC} \right] \quad (7-2)$$

Converting into the frequency domain by putting $s = j\omega$ where ω is the excitation frequency of the system:

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I} \left[R + j\omega L + \frac{1}{j\omega C} \right]$$

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I} \left[R + j\omega L - j\frac{1}{\omega C} \right]$$

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I} \left[R + j\omega L \left(1 - \frac{1}{\omega^2 LC} \right) \right]$$

Introducing the natural frequency of the electrical system $\omega_{ne} = 1/\sqrt{LC} \gg LC = 1/\omega_{ne}^2$:

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I} \left[R + j\omega L \left(1 - \frac{\omega_{ne}^2}{\omega^2} \right) \right]$$

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I} [R + j\omega L(1 - r_e^2)]$$

$$\therefore r_e^2 = \frac{\omega_{ne}^2}{\omega^2}$$

$$Z_e = R + j\omega L(1 - r_e^2)$$

$$\bar{V} - j\alpha\omega\bar{X} = \bar{I}Z_e$$

where $r_e = \omega_{ne}/\omega$ defined as the ratio of the electrical natural frequency and the excitation frequency and $Z_e = R + j\omega L(1 - r_e^2)$ is electrical impedance. The current of the system is written as shown in Equation (7-1):

$$\bar{I} = \frac{\bar{V} - \alpha\bar{\vartheta}}{Z_e} \quad (7-3)$$

The free body diagram of the armature is shown in Figure 7.4(b) and the mechanical model of the actuator is derived as Equation (7-4):

$$ma(t) + b\vartheta(t) + Kx(t) = \alpha i(t) \quad (7-4)$$

where m is the moving mass, b is the damping coefficient, K is the spring constant, x and a are the armature stroke and acceleration. According to the Faraday's law, the back emf shown in Equation (7-1) is the reverse voltage generated in the coil. It is defined as the product of the back emf constant Vs/m and the armature velocity. Whereas, the motor constant N/A in Equation (7-4) represents the amount of magnetic force developed per unit of the applied current. It can be noticed that the back emf and the magnetic force, include same constant α and both units satisfy the equivalence of electrical and mechanical work done [63], [68]. The experimental demonstration of the equivalence is presented in results section.

Using the same methodology, the current from the mechanical model is estimated as shown below:

$$ms^2\bar{X} + bs\bar{X} + K\bar{X} = \alpha\bar{I}$$

$$\alpha \bar{I} = s \bar{X} \left[ms + b + \frac{K}{s} \right]$$

Putting $s = j\omega$:

$$\alpha \bar{I} = j\omega \bar{X} \left[b + j\omega m + \frac{K}{j\omega} \right]$$

$$\alpha \bar{I} = j\omega \bar{X} \left[b + j\omega m - j \frac{K}{\omega} \right]$$

$$\alpha \bar{I} = j\omega \bar{X} \left[b + j\omega m \left(1 - \frac{K}{\omega^2 m} \right) \right]$$

$$\omega_{nm} = \sqrt{\frac{K}{m}} \gg \frac{K}{m} = \omega_{nm}^2$$

$$\alpha \bar{I} = j\omega \bar{X} \left[b + j\omega m \left(1 - \frac{\omega_{nm}^2}{\omega^2} \right) \right]$$

$$\alpha \bar{I} = j\omega \bar{X} [b + j\omega m(1 - r_m^2)]$$

$$\therefore r_m^2 = \frac{\omega_{nm}^2}{\omega^2}$$

$$Z_m = b + j\omega m(1 - r_m^2)$$

$$\alpha \bar{I} = j\omega \bar{X} Z_m$$

$$\bar{I} = j\omega \frac{\bar{X}}{\alpha} Z_m = \frac{\bar{\theta}}{\alpha} Z_m \quad (7-5)$$

where $Z_m = b + j\omega m(1 - r_m^2)$ is the impedance associated with the mechanical system. When the excitation frequency is equal to the mechanical resonance ($r_m = 1$), the imaginary part of Z_m becomes zero. Therefore, least amount of current is required for armature oscillations.

Comparing Equation (7-3) and (7-5), the velocity of the armature is derived as shown below:

$$\frac{\bar{V} - \alpha\bar{\vartheta}}{Z_e} = \frac{\bar{\vartheta}}{\alpha} Z_m$$

$$\bar{V} = \frac{\bar{\vartheta}}{\alpha} Z_m Z_e + \alpha\bar{\vartheta}$$

$$\bar{V} = \left[\frac{1}{\alpha} Z_m Z_e + \alpha \right] \bar{\vartheta}$$

$$\alpha\bar{V} = [Z_m Z_e + \alpha^2] \bar{\vartheta}$$

$$\bar{\vartheta} = \frac{\alpha\bar{V}}{[Z_m Z_e + \alpha^2]}$$

Simplifying Z_m and Z_e :

$$Z_m Z_e = [b + j\omega m(1 - r_m^2)][R + j\omega L(1 - r_e^2)]$$

$$Z_m Z_e = bR - \omega^2 mL(1 - r_m^2)(1 - r_e^2) + j(\omega bL(1 - r_e^2) + \omega Rm(1 - r_m^2))$$

$$\bar{\vartheta} = \frac{\alpha\bar{V}}{[bR - \omega^2 mL(1 - r_m^2)(1 - r_e^2) + \alpha^2] + j[(\omega bL(1 - r_e^2) + \omega Rm(1 - r_m^2))]}$$

$$Y = bR - \omega^2 mL(1 - r_m^2)(1 - r_e^2) + \alpha^2$$

$$\psi = \omega bL(1 - r_e^2) + \omega Rm(1 - r_m^2)$$

$$\bar{\vartheta} = \frac{\alpha\bar{V}}{Y + j\psi} \times \frac{Y - j\psi}{Y - j\psi} = \frac{\alpha\bar{V}Y}{Y^2 + \psi^2} - j \frac{\alpha\bar{V}\psi}{Y^2 + \psi^2}$$

$$|\bar{\vartheta}| = \sqrt{\left(\frac{\alpha \bar{V} Y}{Y^2 + \psi^2}\right)^2 + \left(\frac{\alpha \bar{V} \psi}{Y^2 + \psi^2}\right)^2} = \frac{\alpha \bar{V}}{\sqrt{Y^2 + \psi^2}}$$

$$\angle \bar{\vartheta} = \tan^{-1} \left(\frac{-\frac{\alpha \bar{V} \psi}{Y^2 + \psi^2}}{\frac{\alpha \bar{V} Y}{Y^2 + \psi^2}} \right) = -\tan^{-1} \left(\frac{\psi}{Y} \right)$$

$$\vartheta(t) = \frac{\alpha \bar{V}}{\sqrt{Y^2 + \psi^2}} \sin \left(\omega t - \tan^{-1} \left(\frac{\psi}{Y} \right) \right)$$

The acceleration and stroke of the armature are estimated by taking the time derivative and integral of velocity as shown below:

$$x(t) = \frac{\alpha V}{\omega \sqrt{Y^2 + \psi^2}} \sin \left(\omega t - \tan^{-1} \left(\frac{\psi}{Y} \right) \right) \quad (7-6)$$

$$a(t) = -\frac{\alpha \omega V}{\sqrt{Y^2 + \psi^2}} \sin \left(\omega t - \tan^{-1} \left(\frac{\psi}{Y} \right) \right) \quad (7-7)$$

The current of the actuator is modeled by using the using the mechanical model equation as shown below:

$$\bar{I} = \frac{\bar{\vartheta}}{\alpha} Z_m = \frac{\bar{\vartheta}}{\alpha} [b + j\omega m(1 - r_m^2)] = \frac{\bar{\vartheta}}{\alpha} [b + j\varsigma]$$

$$\varsigma = \omega m(1 - r_m^2)$$

$$\angle \bar{I} = \angle \bar{\vartheta} + \tan^{-1} \left(\frac{\varsigma}{b} \right)$$

$$\bar{I} = \frac{1}{\alpha} [b + j\varsigma] \left[\frac{\alpha V Y}{Y^2 + \psi^2} - j \frac{\alpha V \psi}{Y^2 + \psi^2} \right]$$

$$\bar{I} = \left[\frac{b}{\alpha} \frac{\alpha V Y}{Y^2 + \psi^2} - j \frac{b}{\alpha} \frac{\alpha V \psi}{Y^2 + \psi^2} + j \frac{\varsigma}{\alpha} \frac{\alpha V Y}{Y^2 + \psi^2} + \frac{\varsigma}{\alpha} \frac{\alpha V \psi}{Y^2 + \psi^2} \right]$$

$$\bar{I} = \left[V \left(\frac{bY}{Y^2 + \psi^2} + \frac{\varsigma\psi}{Y^2 + \psi^2} \right) - jV \left(\frac{b\psi}{Y^2 + \psi^2} + j \frac{\varsigma Y}{Y^2 + \psi^2} \right) \right]$$

$$\bar{I} = \left[V \left(\frac{bY + \varsigma\psi}{Y^2 + \psi^2} \right) + jV \left(\frac{\varsigma Y - b\psi}{Y^2 + \psi^2} \right) \right]$$

$$|\bar{I}| = \sqrt{V^2 \frac{(bY + \varsigma\psi)^2}{(Y^2 + \psi^2)^2} + V^2 \frac{(\varsigma Y - b\psi)^2}{(Y^2 + \psi^2)^2}}$$

$$|\bar{I}| = \frac{V}{Y^2 + \psi^2} \sqrt{(bY + \varsigma\psi)^2 + (\varsigma Y - b\psi)^2}$$

$$|\bar{I}| = \frac{V}{Y^2 + \psi^2} \sqrt{b^2 Y^2 + \varsigma^2 \psi^2 + 2bY\varsigma\psi + \varsigma^2 Y^2 + b^2 \psi^2 - 2\varsigma Y b\psi}$$

$$|\bar{I}| = \frac{V}{\sqrt{Y^2 + \psi^2}} \sqrt{b^2 + \varsigma^2}$$

$$\angle \bar{I} = \tan^{-1} \left(\frac{V \left(\frac{\varsigma Y - b\psi}{Y^2 + \psi^2} \right)}{V \left(\frac{bY + \varsigma\psi}{Y^2 + \psi^2} \right)} \right) = \tan^{-1} \left(\frac{\varsigma Y - b\psi}{bY + \varsigma\psi} \right)$$

$$i(t) = \frac{V \sqrt{b^2 + \varsigma^2}}{\sqrt{Y^2 + \psi^2}} \sin \left(\omega t + \tan^{-1} \left(\frac{\varsigma Y - b\psi}{bY + \varsigma\psi} \right) \right) \quad (7-8)$$

From Figure 7.4(b), the kinetics of the system is written in the form of the spring force f_s , the damping force f_d , the inertial force f_i and the magnetic force f_m as shown in Equation (7-9):

$$f_i(t) + f_d(t) + f_s(t) = f_m(t) \quad (7-9)$$

When the armature moves, it pulls the springs and the spring force is created which opposes the motion of the armature. The spring force is the product of the spring constant K with the stroke and is written as shown in Equation (7-10).

$$f_s(t) = \frac{K}{\omega} \frac{\alpha V}{\sqrt{Y^2 + \psi^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{\psi}{Y}\right)\right) \quad (7-10)$$

The damping force is the product of velocity and the damping coefficient b . Its time domain response is shown in Equation (7-11).

$$f_d(t) = b \frac{\alpha V}{\sqrt{Y^2 + \psi^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\psi}{Y}\right)\right) \quad (7-11)$$

The magnetic force is the product of the motor constant α and the amount of the current flowing through the coil. The dynamic response of the magnetic force is represented by Equation (7-12).

$$f_m(t) = \alpha \frac{V\sqrt{b^2 + \zeta^2}}{\sqrt{Y^2 + \psi^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{\zeta Y - b\psi}{bY + \zeta\psi}\right)\right) \quad (7-12)$$

The inertia acting on the armature is the product of the mass and acceleration of the armature as shown in Equation (7-13).

$$f_i(t) = -m \frac{\alpha \omega V}{\sqrt{Y^2 + \psi^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{\psi}{Y}\right)\right) \quad (7-13)$$

When the excitation frequency matches the electrical and mechanical natural frequencies, resulting in $r_e = r_m = 1$, $\psi = 0$, $Y = bR + \alpha^2$, $\zeta = 0$, the system kinetics simplify as:

$$f_s(t) = \frac{K}{\omega} \frac{\alpha V}{Y} \sin(\omega t) \quad (7-14)$$

$$f_d(t) = b \frac{\alpha V}{Y} \cos(\omega t) \quad (7-15)$$

$$f_i(t) = -m\omega \frac{\alpha V}{Y} \sin(\omega t) \quad (7-16)$$

$$f_m(t) = \alpha \frac{Vb}{Y} \cos(\omega t) \quad (7-17)$$

At resonance, as $\omega = \omega_{nm} = \sqrt{K/m}$ and $K/\omega = m\omega$, therefore, the spring force Equation (7-14) and the inertial force Equation (7-16) cancel out each other due to the same magnitude and opposite phase. Additionally, the damping force Equation (7-15) and the magnetic force Equation (7-17) are identical in magnitude ($b\alpha V/Y$) and phase. This implies that at the resonance excitation, minimum magnetic force is required to oscillate the armature.

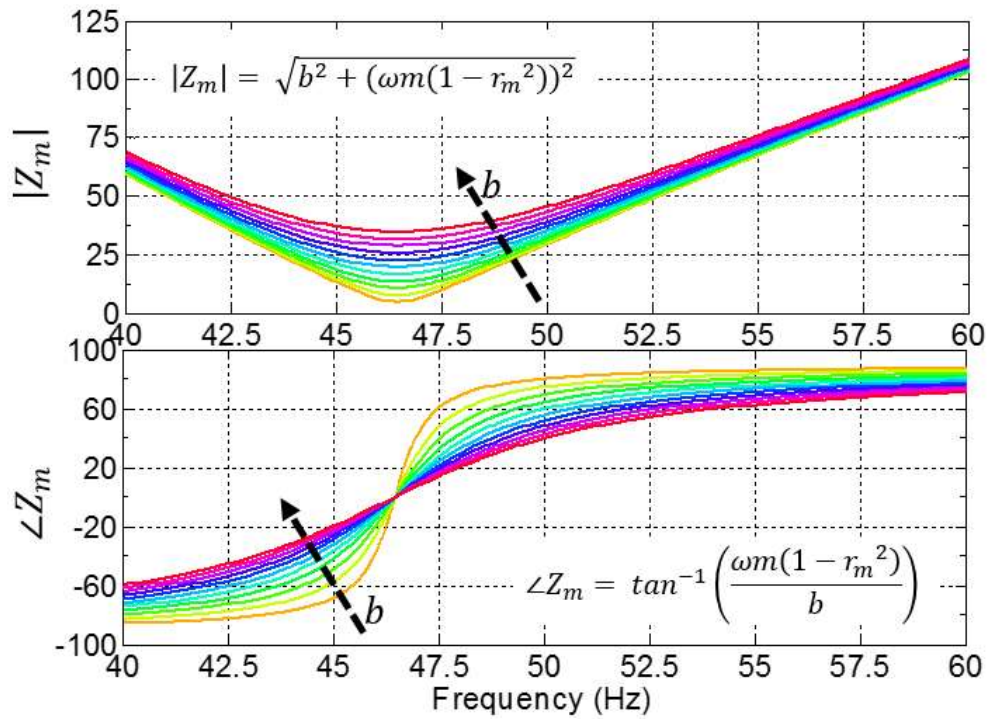


Figure 7.5: Magnitude and phase response of mechanical impedance of the moving magnet linear actuator for $b = 5$ to 45 Ns/m

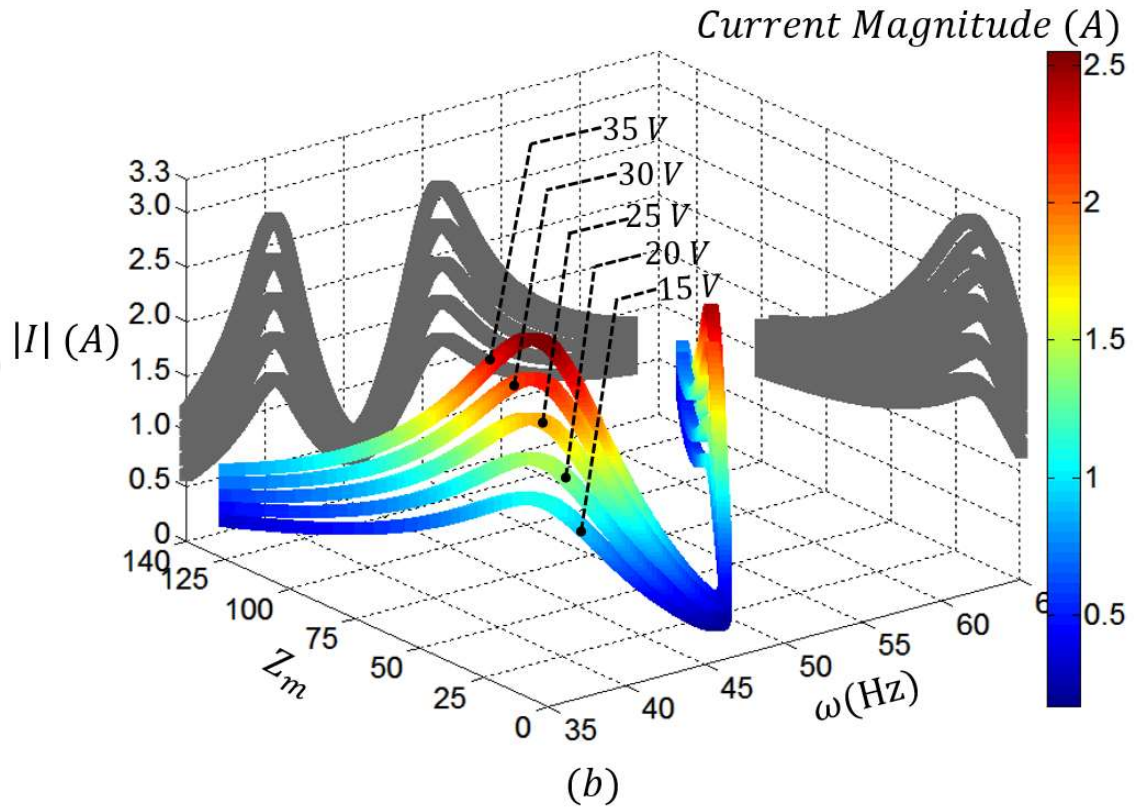


Figure 7.6: Effect of mechanical impedance on the instantaneous current for 15-35 Vrms

7.5 Simulation Results

By using the presented analytical model, a simulation is executed with the similar design parameters of the prototype to investigate the actuator dynamics. In the simulation, the motor constant is considered as 33 N/A, armature mass is 0.68 kg, damping coefficient is 9.7 Ns/m, coil inductance and resistance are 384 mH and 15 Ω respectively. As discussed in earlier, the mechanical impedance Z_m immensely influences the amount of the actuator current. Its magnitude and phase depend on the damping coefficient b and the mechanical resonance frequency ratio r_m . Figure 7.5 represents the magnitude and phase response of Z_m . For a certain value of b , the Z_m magnitude is minimum at the resonance excitation. However, an increase in b corresponds to a significant increase in Z_m magnitude. Additionally, the

imaginary part of the mechanical impedance vector becomes less effective at higher values of b . In linear compressors, the moving mass, gas damping and the friction between piston and cylinder define the damping coefficient. Consequently, there is a limited control over b due to the design specifications and constraints. Figure 7.6 demonstrates the effect of Z_m on the actuator current at different voltages and frequency excitations. The magnitude of current is minimum at the resonance due to least value of Z_m at all applied voltages.

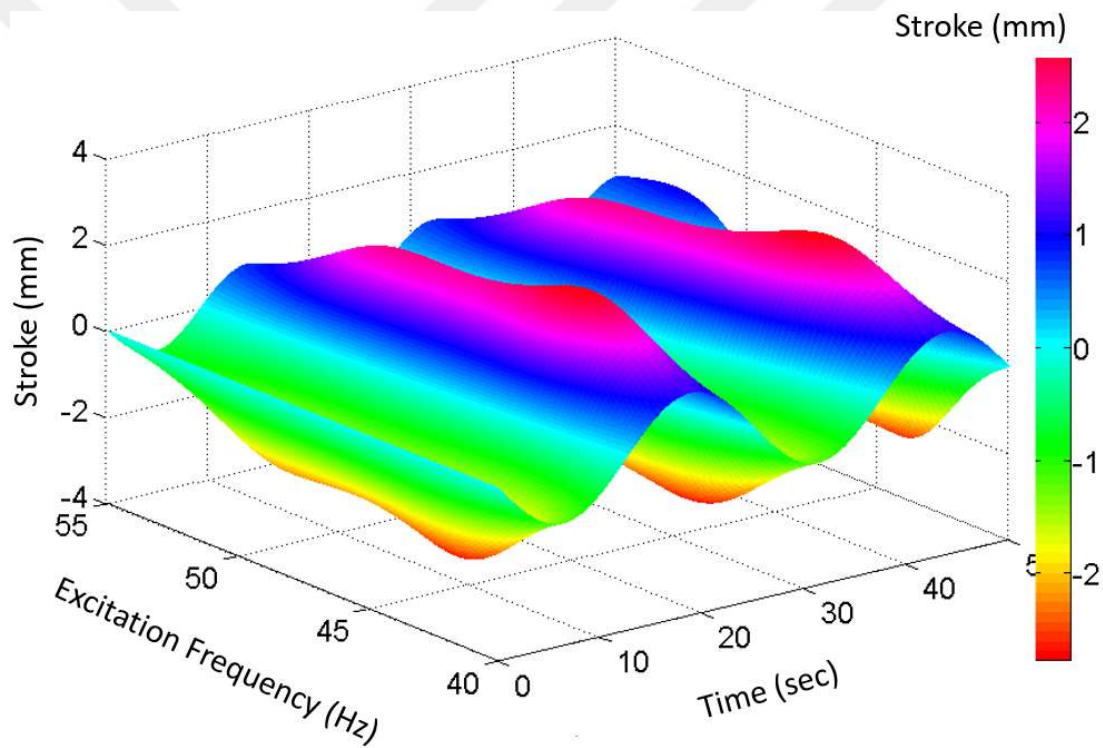


Figure 7.7: Stroke-time curves at different excitation frequencies

With the help of the Equation (7-6) and (7-8), a simulation is executed for the time response of stroke and current against excitation frequencies ranging 40-50 (Hz) and 20 (V) voltage. The force-time and the current-time surface plots are shown in Figure 7.7 and Figure 7.8. As discussed in Chapter 5, the excitation at resonance provides the best performance of the linear actuator. The Same behavior is obtained in the time domain analysis. A change in

phase of the stroke is also noticeable. The time domain behavior of the current is similar to the frequency domain analysis as well. Least amount of the current is passing through the coil at the resonance. At resonance, a 90 Degree lead in the phase of current to stroke is also visible which is explained in the frequency domain analysis as well.

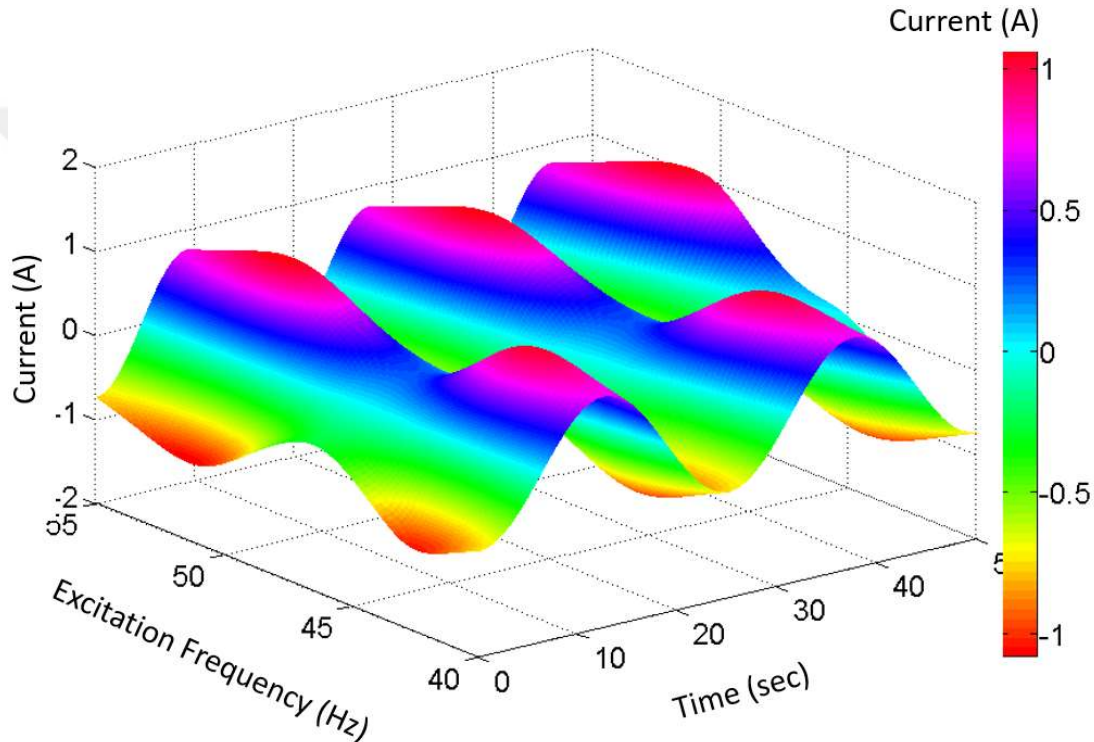


Figure 7.8: Current-time curves at different excitation frequencies

By using Equation (7-10) to (7-13), a numerical simulation is executed to find the spring force, damping force, inertial force and the magnetic force- time curves against a frequency range of 40-55 (Hz) and 20 (V) excitation. Figure 7.9 demonstrates the time domain analysis for the spring force and Figure 7.10 represents the inertial force-time curves acting on the mover. It is clear from these figures that both forces are out of phase all the time. At the resonance, the inertial force and the spring force are canceling out each other as they are 180 Degrees out of phase and have the same magnitude.

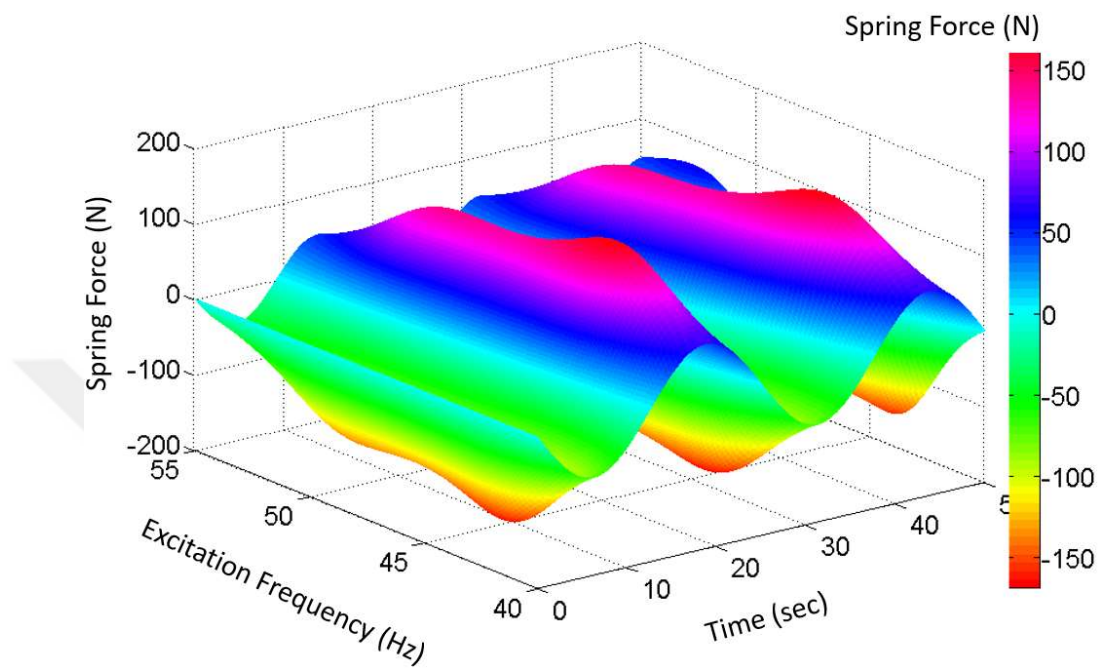


Figure 7.9: Spring force-time curves at different excitation frequencies

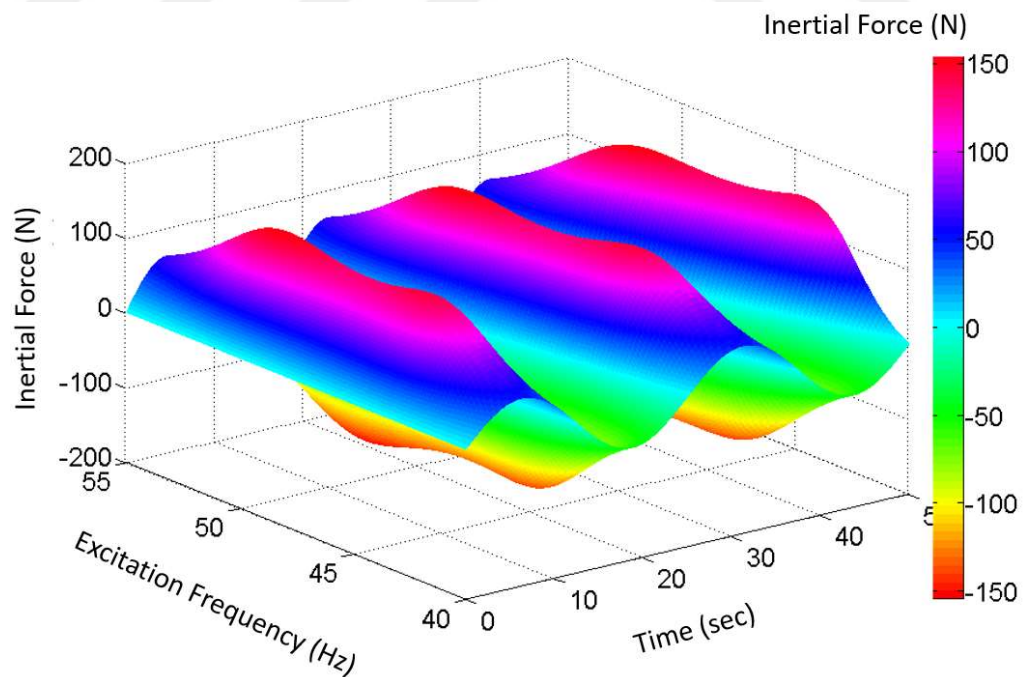


Figure 7.10: Inertial force-time curves at different excitation frequencies

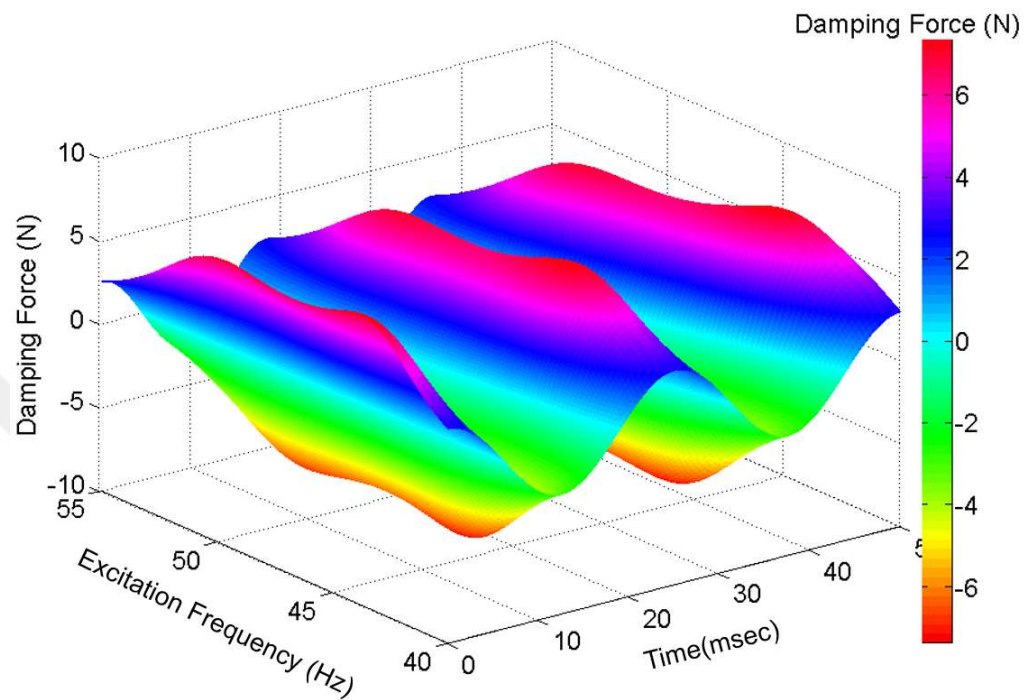


Figure 7.11: Damping force-time curves at different excitation frequencies

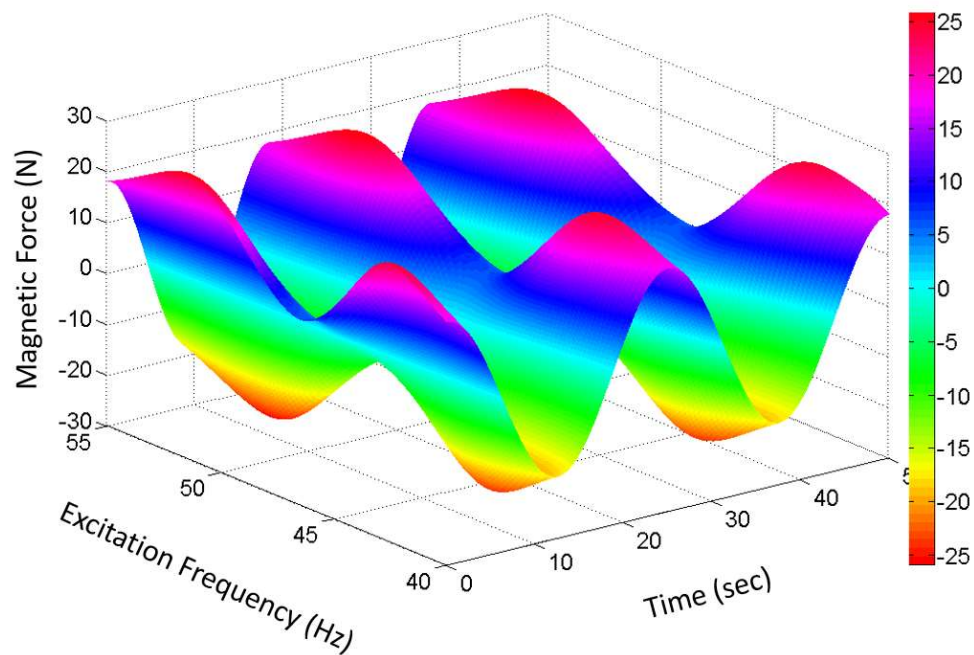


Figure 7.12: Magnetic force-time curves at different excitation frequencies.

Figure 7.11 demonstrates the behavior of the damping force acting on the armature. The magnitude of the damping force is relatively lower than the magnitude of the rest of the forces. Figure 7.12 represents the response time curve of the magnetic force and as discussed earlier, the amount of magnetic force is very low at the natural frequency. This implies a very small magnetic force is required to oscillate the linear actuator at the resonance. Moreover, the damping force and magnetic force are out of phase all the time at all frequencies. At resonance, the magnetic force is equal in magnitude as compared to the damping force.

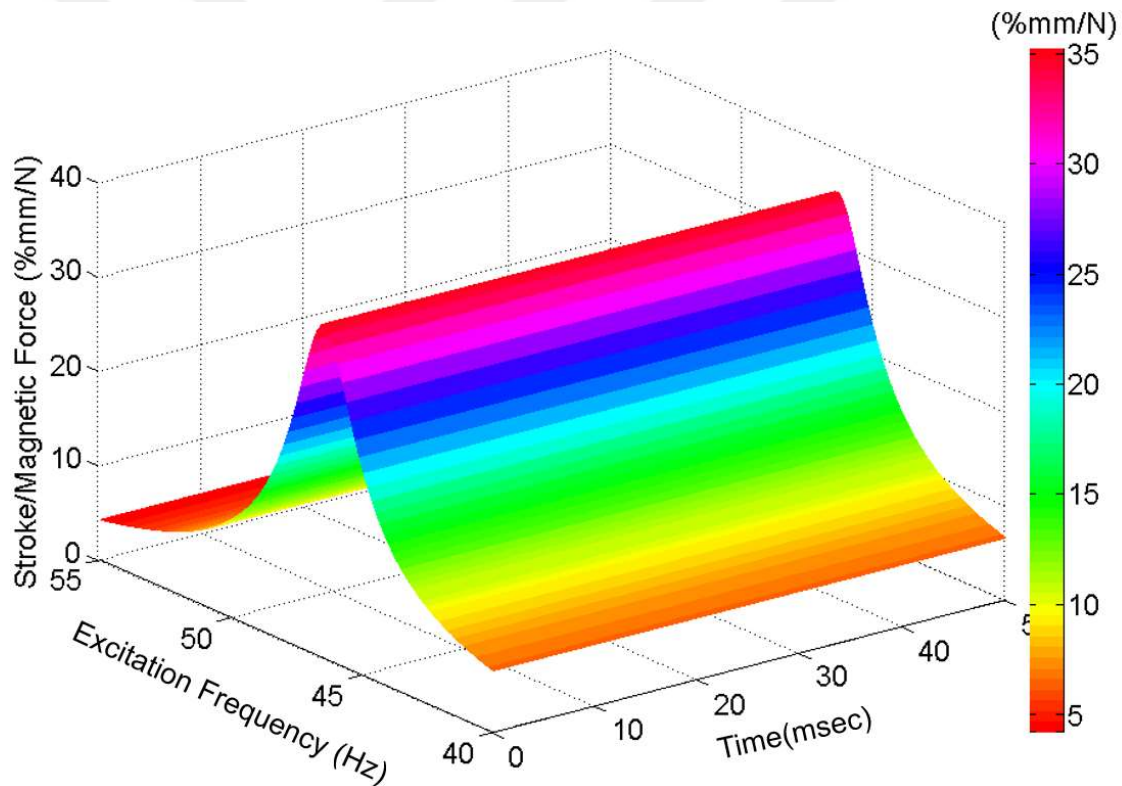


Figure 7.13: Stroke to magnetic force ratio response at different excitation frequencies

In order to find the effect of the magnetic force on the stroke of the armature, a simulation is executed to find the ratio of the stroke to the magnetic force. The result of this ratio is

shown in Figure 7.13. As expected the ratio is 35% more than the non-resonance excitation. This value remains constant as shown below.



Figure 7.14: Disassembled view of the manufactured parts of the moving magnet linear actuator

7.6 Experimental Results and Comparison

Moving magnet linear compressor parts are manufactured as discussed in the design section. The disassembled view of the actuator is shown in Figure 7.14. The same experimental setup shown in Figure 5.8 is used to perform the set of experiments. A moving magnet linear actuator with the same geometric parameters discussed in the mechanical design section is manufactured as shown in Figure 7.14. The actuator is composed of two coils of 640 number of turns each. The inductance of the stator is 384 mH, coil resistance of $15\ \Omega$ and the series capacitance is 30 μF . Eight springs of approximately 7600 N/m stiffness are used. A variable voltage and frequency AC power supply from Kikusui (PCR1000LE) provides the controlled amplitude and frequency of the voltage input. The real-time stroke and velocity of the armature are monitored by a laser vibrometer (Polytech, OFV-534 sensor head, OFV5000 controller). Five tests are performed for 15-35 Vrms at the resonance

frequency and the instantaneous current is recorded by the electronic control card. The acceleration of the armature is estimated by taking the time derivative of the velocity. The damping coefficient b is measured by using $b = 2\zeta\omega_{nm}$, where ζ is the damping ratio and is found experimentally from the damped natural frequency ω_d by using $\zeta = \sqrt{1 - \omega_d^2/\omega_{nm}^2}$. The damped natural frequency is experimentally measured as 46.46 Hz which corresponds to the least amount of actuator current. The damping ratio and damping coefficient are estimated as 0.17 and 9.7 Ns/m. The time signals of the stroke, velocity, acceleration and current provide a measure of the spring, damping, inertial and magnetic forces respectively by multiplying them with their corresponding coefficients.

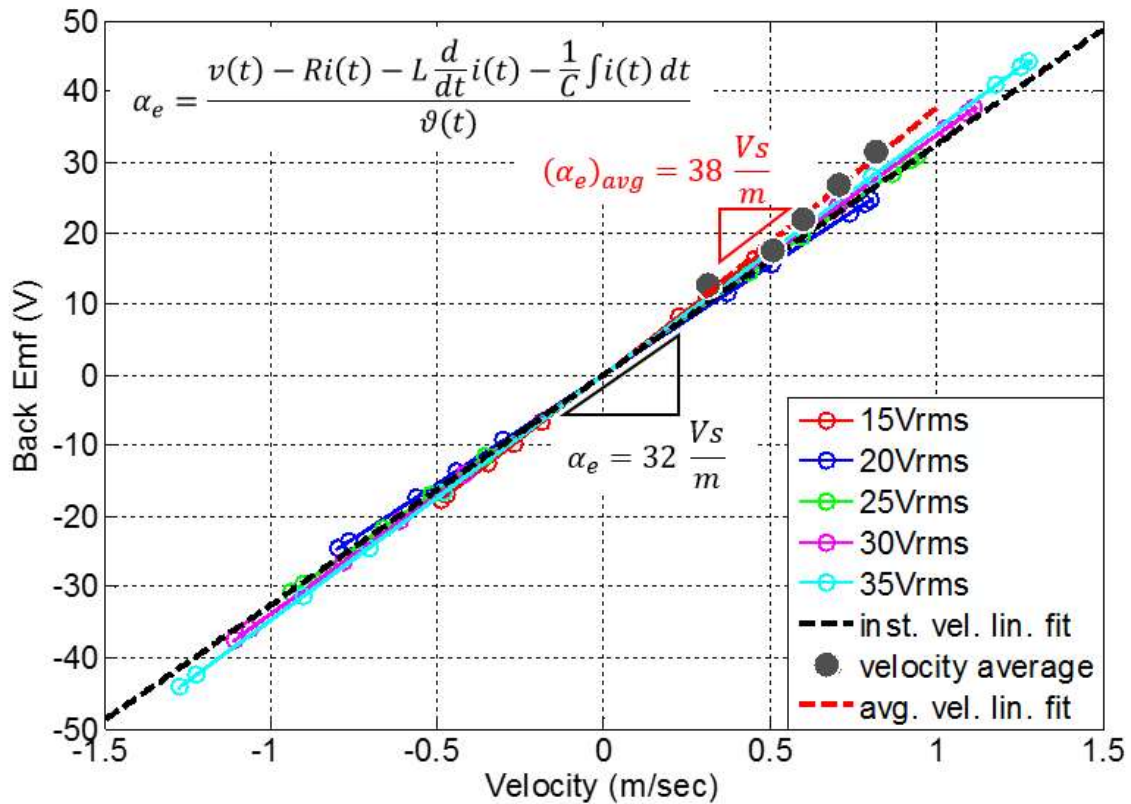


Figure 7.15: Estimation of back emf constant of the moving magnet linear actuator with instantaneous and average approximation

The back emf constant is estimated experimentally by using (7-1) as shown in Figure 7.15. The linear fit slope α_e of the instantaneous velocity and corresponding back emf for a voltage range of 15-35 Vrms is shown. Additionally, considering the average velocity and RMS current, the average back emf constant is obtained. Approximately, a difference of 15% is noticed between these two methods. However, the constant obtained by instantaneous velocity provides a more realistic estimate.

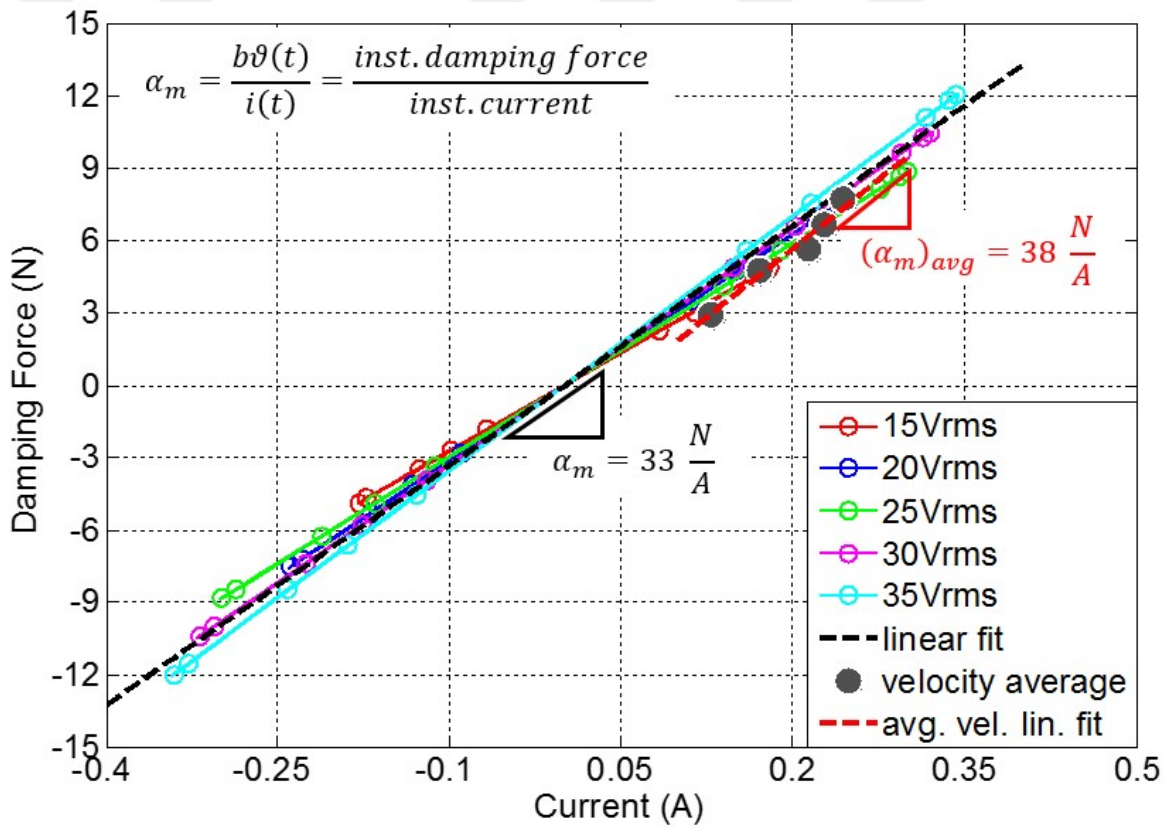


Figure 7.16: Estimation of motor constant of the moving magnet linear actuator with instantaneous and average approximation

A similar methodology is used to obtain the motor constant α_m by considering the magnetic force and current [20]. As discussed earlier, the damping and magnetic force are equal at resonance. Therefore, the linear fit slope of the instantaneous damping force and

current provides the motor constant as shown in Figure 7.16. Additionally, the average data are also plotted to compare the estimated motor constant. The difference in the motor constants through average estimation is also presented by [63]. The estimates of the instantaneous back emf and motor constant are very close to one another. Therefore, the back emf and motor constants are approximated as 33 N/A.

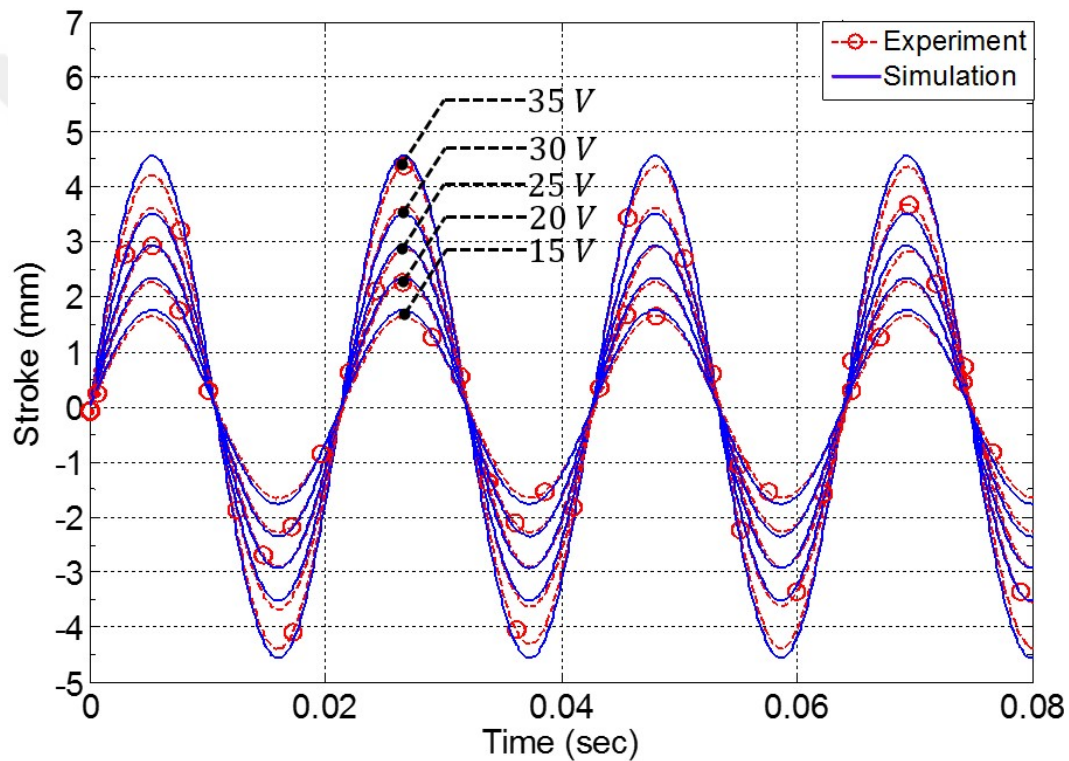


Figure 7.17: Instantaneous stroke curves at 15-35 Vrms and 46.46 Hz

The simulation and experimental results of the armature stroke are shown in Figure 7.17. The results are complementing each other. Higher magnitudes of the stroke are observed for the higher voltage inputs. Additionally, the increase in the voltage does not affect the phase of the stroke. Figure 7.18 validates the simulated and experimental results for the armature current. As seen in the figure, high voltage inputs correspond to the higher current amplitudes. Additionally, there is a 90° lag between the stroke and the coil current at

resonance. Analogous to the stroke response, the increase in the magnitude of the voltage does not affect the phase of the current.

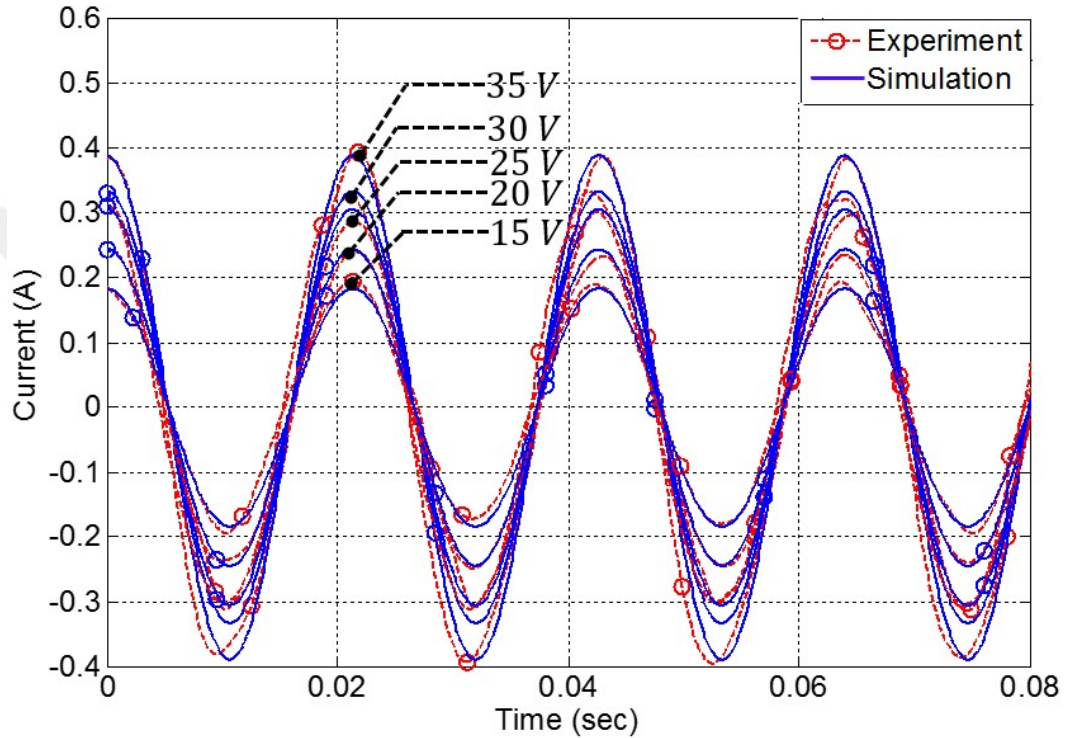


Figure 7.18: Instantaneous current curves at 15-35 Vrms and 46.46 Hz

Figure 7.19 demonstrates the kinetic analysis of the proposed linear actuator at 20 Vrms and the resonance frequency excitation. Moreover, the equilibrium at resonance in between the inertial and spring forces is demonstrated in the figure. Additionally, the results also complement the equality of the damping and magnetic forces as discussed in the analytical and simulation sections. A difference of 8% is noticed in the simulation and experimental values of forces. It is mainly because of the noise in the current sensing which is removed by using software filtering option.

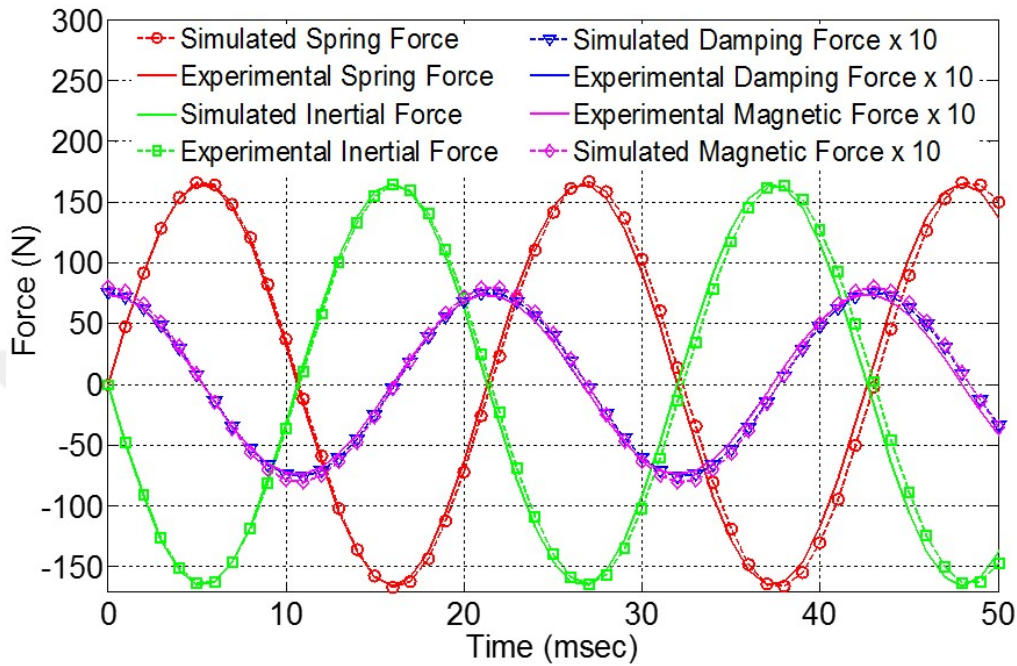


Figure 7.19: Comparison of all the forces acting on the armature at 20V and the resonance frequency excitation.

Figure 7.20 provides the performance analysis of the proposed MMLA for an applied voltage range of 15-35 Vrms. As seen in the figure, the motor efficiency is plotted against armature stroke for different power inputs. The motor efficiency is the ratio of the shaft power to the power input. The shaft power is the product of the magnetic force and armature velocity. The motor efficiency of the proposed actuator is experimentally recorded as 88-94% corresponding to the stroke values between 3-9 mm. In order to achieve higher stroke at resonance, more power input is required. As shown in the figure, more power inputs cause a reduction in motor efficiency. Similar efficiency trend is demonstrated by [8], [28]. This is mainly because of the copper losses ($i(t)^2 R$), as higher current is required for higher strokes [11]. Additionally, the predicted mass flow rate for the refrigerant R600a is also presented. The mass flow rate is estimated as the product of the piston cross-sectional area, stroke,

oscillation frequency and the density of the refrigerant (2.57 Kg/m^3). As seen in the figure, the flow rate linearly increases with the increase in the stroke value.

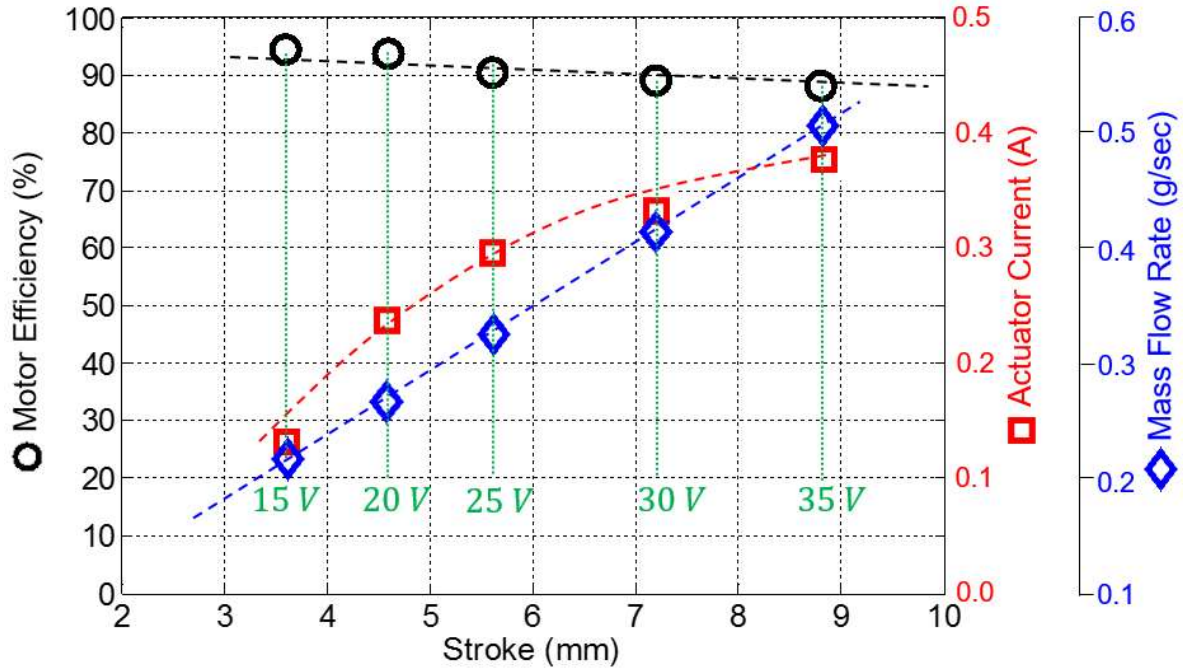


Figure 7.20: Performance analysis of proposed moving magnet linear actuator at different power inputs.

In order to analyze the advantages of the proposed MMLA, the performance comparison with the conventional rotary and already developed linear motors used in linear compressor applications is conducted. The authors referred to the linear motor articles; [8], [25], [28], [7], [63] and for the rotary motor [11] is used. Figure 7.21 represents the efficiency plots of four of the referred articles. As seen in these figures, the efficiency of the rotary motor is around 60% (Figure 7.21(a)) and the linear motors are in between 60 to 90% approximately (Figure 7.21(b), (c) and (d)). It should be noticed that the test conditions are not same for all the linear motors. For instance, [25] tested the motor by connecting it to a generator with 440Ω resistive load, [63] provided test results with no load, [28] tested with a generator (the

mover mass is 13 g) and [8] tested against gas compression of different pressure ratios. The performance comparison of the linear motors for linear compressor application should be done under the same loading conditions. Therefore, the author compared the proposed actuator with reference article [8], [25] and [63] on their corresponding loading conditions with the help of the presented simulation in section III.

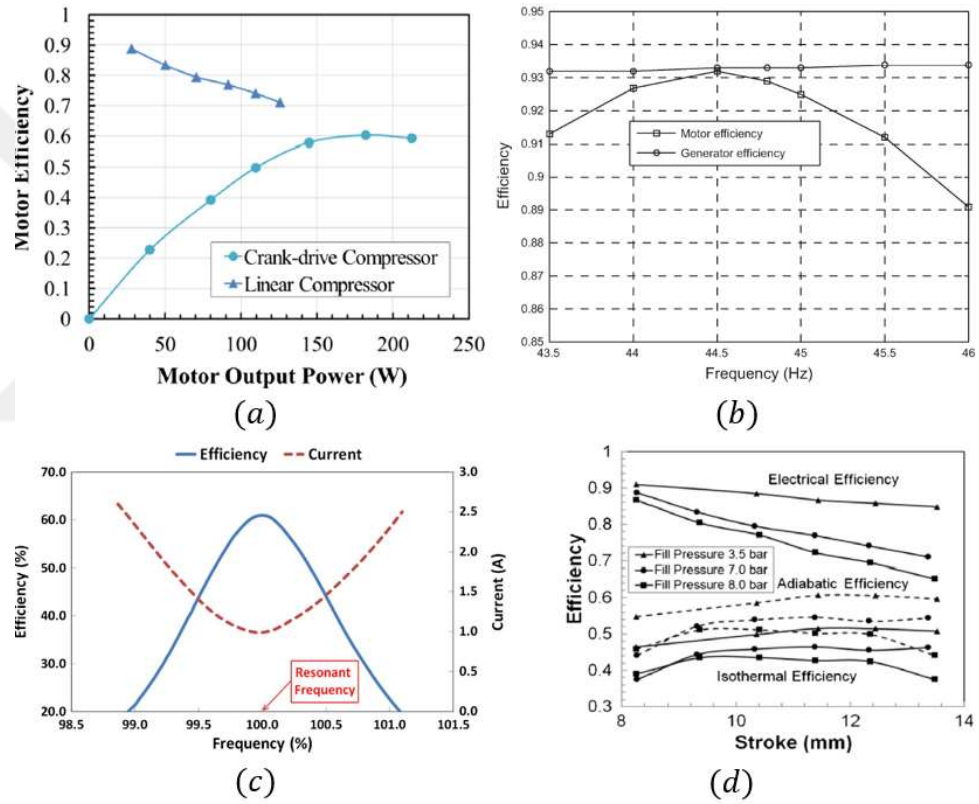


Figure 7.21: (a) Efficiency plot with respect to motor output power [11] (b) efficiency plot with respect to frequency [25] (c) efficiency plot with respect to frequency [28] (d) efficiency plot with respect to stroke [8]

- **Comparison with cited article [8] (Load Condition)**

In the presence of the refrigerant, the gas force affects the overall stiffness ($K+K_g$) and damping ($b+b_g$) of the system. The gas stiffness K_g increases the mechanical resonance

frequency [72] and the gas damping b_g adds up to the effective damping force of the system. In order to simulate the loading effect, the gas damping and stiffness coefficients are found from [8] at resonance and the efficiency of the proposed actuator is estimated. The work done in an isentropic compression W provides an average value of gas damping as shown [73]:

$$W_{[8]} = \int P dV = 2.01 \text{ J/cycle}$$

$$b_{g[8]} = \frac{W_{[8]}}{\pi \omega_{[8]} x_{[8]}^2} = \frac{2.01}{\pi (2\pi \times 36.5) 0.0134^2} = 15.5 \text{ Ns/m}$$

The gas stiffness is calculated by subtracting the contribution of the mechanical springs and moving mass from the mechanical resonance frequency as shown below:

$$\omega_{res[8]} = \sqrt{\frac{(K_m + K_g)_{[8]}}{m_{[8]}}} = 36.5(2\pi) \gg K_{g[8]} = \omega_{n[8]}^2 m_{[8]} - K_{m[8]}$$

$$K_{g[8]} = (36.5 \times 2\pi)^2 \times 0.66 - 17000 = K_g = 17712 \text{ N/m}$$

The increase in the mechanical resonance frequency ω_{res} of the proposed actuator on the same loading conditions is estimated by:

$$\omega_{res} = \sqrt{\frac{(K + K_g)}{m}} = \sqrt{\frac{60800 + 17712}{0.68}} = 54.1 \text{ Hz}$$

Considering the rated stroke of the proposed actuator, the velocity of the piston at the new resonance frequency is estimated as:

$$\vartheta = (0.0044 \times 2\pi \times 54.1) = 1.5 \text{ m/s}$$

Using the coefficient of gas damping, the max current i_{max} is estimated as:

$$f_d = f_m \gg (b + b_g)\vartheta = \alpha i_{max}$$

$$i_{max} = \frac{(b + b_g)\vartheta}{\alpha} = \frac{(9.7 + 15.5)(1.5)}{33} = 1.14 \text{ A}$$

The efficiency η of the proposed actuator at the same loading is estimated as:

$$\eta = \frac{\alpha i_{max}\vartheta}{(i_{max}R + \alpha\vartheta)i_{max}} \cong 75\%$$

- **Comparison with cited article [25] (Load Condition)**

The load conditions of the linear actuator cited in the article [10] and [25] are found. The velocity and gas damping coefficient are estimated at resonance:

$$\vartheta = (0.005155 \times 2\pi \times 50) = 1.62 \text{ m/s}$$

$$b_{g[25]} = \frac{\alpha_{[25]}i_{max[25]}}{v_{max[25]}} - b_{d[6]} = \frac{46}{1.62} - 11 = 17.4 \text{ Ns/m}$$

The gas stiffness is calculated by subtracting the contribution of the mechanical springs and moving mass from the mechanical resonance frequency as shown below:

$$\omega_{res[25]} = \sqrt{\frac{(K_m + K_g)_{[25]}}{m_{[25]}}} = 36.5(2\pi) \gg K_{g[25]} = \omega_{n[25]}^2 m_{[25]} - K_{m[25]}$$

$$K_{g[25]} = (50 \times 2\pi)^2 \times 0.43 - 29800 = K_g = 12639 \text{ N/m}$$

The increase in the mechanical resonance frequency of the proposed actuator on the same loading conditions is estimated by:

$$\omega_{res} = \sqrt{\frac{(K + K_g)}{m}} = \sqrt{\frac{60800 + 12639}{0.68}} = 52.3 \text{ Hz}$$

Considering the rated stroke of the proposed actuator, the velocity of the piston at the new resonance frequency is estimated as:

$$\vartheta = (0.0044 \times 2\pi \times 52.3) = 1.45 \text{ m/s}$$

Using the coefficient of gas damping from [25], the max current at resonance is estimated as:

$$f_d = f_m \gg (b + b_g)\vartheta = \alpha i_{max}$$

$$i_{max} = \frac{(b + b_g)\vartheta}{\alpha} = \frac{(9.7 + 17.4)(1.45)}{33} = 1.19 \text{ A}$$

The efficiency of the proposed actuator at the same loading is estimated as:

$$\eta = \frac{\alpha i_{max}\vartheta}{(i_{max}R + \alpha\vartheta)i_{max}} \cong 73\%$$

- **Comparison with cited article [63] (No Load Condition)**

The resonance frequency of the cited article [63] is estimated as:

$$\omega_{res[63]} = \sqrt{\frac{K_{[63]}}{m_{[63]}}} = \sqrt{\frac{2200}{3.8}} = 3.83 \text{ Hz}$$

The velocity for the rated stroke is calculated as:

$$\vartheta_{[63]} = (0.01 \times 2\pi \times 3.83) = 0.25 \text{ m/s}$$

The efficiency of the linear motor provided in [63] is estimated as:

$$\eta_{[63]} = \frac{\alpha_{[63]} i_{\max[63]} \vartheta_{[63]}}{(i_{\max[63]} R_{[63]} + \alpha_{[63]} \vartheta_{[63]}) i_{\max[63]}} = \frac{48 \times 0.4 \times 0.25}{(0.4 \times 6 + 48 \times 0.25) 0.4}$$

$$\eta_{[63]} = \frac{4.8}{5.76} \cong 83\%$$

Since the authors of the cited article [63] didn't provide the motor efficiency, the value is estimated from the graphs available in the article and there can be errors in the estimation method. Therefore, in the table, the authors are marking it as unavailable.

The efficiency of the proposed actuator at the same power input is estimated as:

$$\vartheta = (0.0044 \times 2\pi \times 46.46) = 1.28 \text{ m/s}$$

$$P_{ip} = 5.76 \text{ W} = (i_{\max} R + \alpha \vartheta) i_{\max} = 15 i_{\max}^2 + 42.24 i_{\max}$$

$$i_{\max} = 0.13 \text{ A}$$

$$\eta = \frac{\alpha i_{\max} \vartheta}{(i_{\max} R + \alpha \vartheta) i_{\max}} = \frac{5.49}{5.76} \cong 94\%$$

The performance parameter data of the linear motors estimated earlier is shown in Table 7-1. The performance of the MMLA prototype is compared with some already developed linear motors; [25], [7], [28], [8], [63]. The referred articles present the motor performance results on different test and loading conditions. [7] is not employing a mechanical spring and [28] is designed for very short stroke. Therefore, the comparison of [8], [25] and [63] is conducted analytically on their corresponding load conditions. In the presence of the refrigerant, the gas force affects the overall stiffness and damping of the system. The gas stiffness increases the mechanical resonance frequency and the gas damping adds up to the

effective damping force of the system [72]. The stiffness and damping gas coefficients are estimated based on the methodology developed by [73]. The comparison with [63] is established on no load condition with the same power input. For loaded comparisons, the pressure ratio of 3.0 with the discharge and suction pressure of 11.6 and 3.86 bar of the design point of [8] is taken as a benchmark. Furthermore, the comparison with [25] is conducted at the exhaust pressure of 8 bar with atmospheric suction pressure. With the help of the estimated coefficients, the dynamic parameters of the MMLA with these linear motors are shown in the table.

Table 7-1: Simulation results of the linear motors

Conditions	No Load		Load 1		Load 2	
Linear Motors	MMLA	[64]	MMLA	[8]	MMLA	[25]
<i>Efficiency (%)</i>	94	-	75	74	73	93
<i>Spring Stiffness (kN/m)</i>	60.8	2.2	60.8	17	60.8	29.8
<i>Frequency (Hz)</i>	46.46	3.83	54.1	36.5	52.3	50
<i>Max Stroke (mm)</i>	8.8	20	8.8	13.4	8.8	10.31
<i>Max Current (A)</i>	0.13	0.4	1.14	1.29	1.19	0.58
<i>Mass Flow Rate (g/s)</i>	-	-	0.59	0.35	0.57	-
<i>Moving Mass (Kg)</i>	0.68	3.8	0.68	0.66	0.68	0.43

The conventional compressors are mainly using induction motors at higher power inputs. Whereas, in linear compressors, the resonance excitation causes lower power inputs. Therefore, their motor efficiency is more than conventional compressors. The efficiency of the induction motor used in the conventional compressor reported by [11] is 60%. As seen in the table, the efficiency of the proposed actuator is more than 73%. Whereas, compared to the linear motors, at lower power inputs, the efficiency of the proposed actuator is more than 90%. At the pressure ratio of 3.0, the efficiency of the proposed actuator is similar to [8] even with the lower stroke values. The performance of the proposed actuator is lower than [25] due to their higher value of the motor constant. The shaft power is the function of motor

constant and for a specific shaft power, higher motor constant corresponds to a lower current. Therefore, the motor efficiency becomes higher due to the lower input power. Further investigation on the motor constant can improve the motor efficiency of the prototype [25]. Additionally, the efficiency of the proposed actuator can also be increased by using the design optimization techniques provided by [25]. Moreover, the oscillation frequency of the proposed actuator is the highest as shown in the table. Therefore, even with low stroke, the mass flow rate is slightly more. Higher mass flow rate results in higher cooling capacity and consequently higher coefficient of performance (COP). However, further investigation is required through calorimetric testing with refrigerant.

7.7 Conclusion

This article provides a design of a novel moving magnet linear actuator for use in refrigerator compressors. The stator and armature CAD model assemblies, as well as the principle of operation, are provided. The dynamics and kinetics of a single phase MMLA driven with a series capacitance are analytically modeled. With the help of these models, a simulation is executed to analyze the effect of excitation frequency in time-domain. Additionally, the effect of mechanical impedance and viscous damping on the current magnitude is discussed. Moreover, the back emf and motor constant estimations are experimentally demonstrated with the instantaneous and average datasets. At resonance excitation frequency, the system dynamics are analytically presented and experimentally validated. The relationship between the inertial and spring forces as well as the damping and electromagnetic forces are discussed in detail. In order to analyze the performance of the proposed linear actuator, a comparison of the performance parameters such as efficiency, stroke, current and mass flow rate is demonstrated with the conventional rotary as well as linear motors in linear compressor application.

Chapter 8

CONCLUSION

Linear compressors are proven to be the ultimate alternatives for use in refrigerators instead of the crank-driven rotary compressor due to better overall efficiency, less mechanical parts and lower noise levels. Additionally, as the linear compressors are operating at natural frequency of the armature assembly, a very less input is required for the refrigeration cycle. Consequently, better motor efficiency and lower core losses make linear compressors superior to the conventional compressors.

Linear compressors are divided into three parts: the linear actuator, resonance spring assembly and the gas/refrigerant compression assembly. This research focuses on the design and development of a linear actuator for use in linear compressors. There are three kinds of linear actuators: moving iron, moving magnet and moving coil linear actuators. This research provides a detailed overview of the mechanical design, mathematical models, FEM analysis, static and dynamic analysis of the moving iron and moving magnet linear actuators.

A solenoid based linear actuator for the linear compressor is presented. The magnetic force model is proposed based on the geometric and design parameters of the actuator. Based on the benchmark force requirements of the linear compressor, this model provides an approximate size and geometry of the linear actuator. Due to the non-linearity of the system, an FEM simulation is executed to predict the realistic magnetic force in the stationary as well as time-dependent domains. Experimental setups are built to experimentally validate the simulation results and the values of force stroke and force-time curves are in good agreement with each other. Using impact hammer and accelerometer, modal analysis is performed to find the natural frequencies and its modes. Frequency response function is experimentally validated with coil excitation and results are validated for better efficiencies at these excitation frequencies.

Moving magnet linear actuators are considered as the best linear motors for linear compressor application. This research also provided a transfer function of the coupled electromechanical system to evaluate the effect of excitation frequency and motor constant on the overall system dynamics. The transfer function incorporates the design parameters such as the moving mass, the stiffness of the springs, the damping coefficient, the inductance of the coil, the coil resistance, the capacitance and the motor constant of the linear actuator. The presented methodology presents a tool for the first time to conclude the design and operating parameters to acquire the desired performance of the linear oscillating actuator. With the help of a simulation in the design phase, the parameters can be engineered based on the operational requirements of the actuator. The presented methodology is validated experimentally and the experimental results are complementing the simulation. It is concluded that the excitation at the resonance frequency provides the optimal point to achieve better stroke to current ratios.

Furthermore, a methodology to find the magnetic flux density, as well as the electromagnetic force of a moving magnet linear actuator used in the linear compressor, is presented. In the design phase, 3D simulation provides approximated performance analysis of a motor. As these simulations are time-consuming, an analytical modeling in combination with a 2-D FEM simulation is proposed to reduce the computation time effectively. This method can be used to find the magnetic flux density for any desired shape. Additionally, the analytical model for magnetic force of the moving magnet linear actuator with the help of work-energy relationship is also presented. In order to generate the realistic results, the BH curve is implemented to incorporate the stator saturation instead of considering a constant value of relative permeability of magnetic materials. An FEM simulation is executed to validate the results of the analytical simulation. The results for the magnetic flux density as well as the electromagnetic forces are less than 10%. Using the simulation output, motor performance parameters like efficiency and motor constant can also be estimated. The major

advantage of the presented methodology drastically reduces the simulation time compared to the FEM simulation. Additionally, the optimization of the geometrical design parameters of the moving magnet linear actuator in the conferred simulation is comparatively easy and rapid. Moreover, to see the result of scaling the actuator to bigger or smaller sizes can be done instantly.

A design of a novel moving magnet linear actuator is presented with the mechanical CAD including all geometric and design parameters. The working principle and electronic drive to control the operation of a linear motor are demonstrated. The analytical models of the overall system dynamics and kinetics are proposed and experimentally validated. These models predict the stroke, velocity, acceleration and current of the actuator based on the applied voltage supply. Moreover, the back emf and motor constant estimations are experimentally demonstrated with the instantaneous and average datasets. Additionally, the effect of mechanical impedance and viscous damping on the current magnitude is discussed. At resonance excitation frequency, the system dynamics are analytically presented and experimentally validated. The relationship between the inertial and spring forces as well as the damping and electromagnetic forces are discussed in detail. In order to analyze the performance of the proposed linear actuator, a comparison of the performance parameters such as efficiency, stroke, current and mass flow rate is demonstrated with the conventional rotary as well as linear motors in linear compressor application.

BIBLIOGRAPHY

- [1] N. B. Christiansen, "Refrigeration - an introduction to the basics," *History of Cooling Systems*, vol. 6, no. 3, pp. 1–18, 2007.
- [2] C.- Bern, "Modified A++- standard refrigerator with 30 % reduction of energy consumption Higher efficiency by compressor modification," [Online]. Available: <http://www.bfe.admin.ch/>. [Accessed: 19-Nov-2016].
- [3] © Statista, "2009-2013 China Export Value of the Refrigerators to the US in USD," March 2014. [Online]. Available: <http://www.statista.com/statistics/408973/china-refrigerator-export-value/>. [Accessed: 06-Jun-2016].
- [4] M. Bahrami and R. Cycle, "The Ideal Vapor - Compression Refrigeration Cycle," *SFU ENSC 461*, vol. 461, pp. 1–8.
- [5] C. C. Control, "Variable Speed Refrigeration and Air Conditioning Compressors," [Online]. Available: <https://www.coastalclimatecontrol.com/index.php/blog/156-variable-speed-refrigeration-and-air-conditioning-compressors.html>. [Accessed: 06-Jun-2016].
- [6] H. Lee, G. Song, J. Park, and E. Hong, "Development of the linear compressor for a household refrigerator," *International Compressor Engineering Conference*, Purdue University, 2000.
- [7] N. C. Tsai and C. W. Chiang, "Design and analysis of magnetically-drive actuator applied for linear compressor," *Mechatronics*, vol. 20, Issue 5, August 2010, pp. 596–603, <http://dx.doi.org/10.1016/j.mechatronics.2010.06.001>.
- [8] K. Liang, R. Stone, G. Davies, M. Dadd, and P. Bailey, "Modelling and measurement of a moving magnet linear compressor performance," *Energy*, vol. 66, pp. 487–495, 2014.
- [9] A. Yoshitake, K. Harada, T. Todaka, Y. Ishihara and K. Hirata, "Dynamic analysis of a linear oscillatory actuator under feedback control," *IEEE Transactions on Magnetism*, vol. 33, no. 2, pp. 1662–1665, Mar 1997.
- [10] Z. Lin, J. Wang, and D. Howe, "A learning feed-forward current controller for linear reciprocating vapor compressors," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 8, pp. 3383–3390, 2011.

-
- [11] K. Liang, R. Stone, W. Hancock, M. Dadd, P. Bailey, "Comparison between a crank-drive reciprocating compressor and a novel oil-free linear compressor," *International Journal of Refrigeration*, vol. 45, 2014, pp. 25-34.
 - [12] W. Greiner, *Classical Electrodynamics*, Springer, p. 569, 1998. ISBN 0-387-94799-X.
 - [13] I. Boldea, Syed A. Nasar, *Linear electric Actuators and Generators*. Cambridge University Press, New York, 1997. pp 91-92.
 - [14] A. Von Meier, *Electric Power Systems: A Conceptual Introduction*, Wiley-IEEE Press, 2006, ISBN: 978-0-471-17859-0.
 - [15] T. J. E. Eng and C. Sci, "The effect of heat on the metallurgical structure and B-H characteristic of (stator) armature with iron-silicon," *Turkish Journal of Electrical Engineering and Computer Sciences*, vol. 18, no. 2, pp. 299–312, 2010.
 - [16] "Why is a Stator Core Made of Laminated Steel" [Online]. Available: <http://www.turbinegenerator.com/index.php/en/tips/generator-tips/item/why-is-a-stator-core-made-of-laminated-steel>, [Accessed: 06-Jun-2016].
 - [17] "Magnetization Directions of permanent magnets" [Online]. Available: <http://www.permanentmagnet.com/magnetization-direction.html>. [Accessed: 06-Jun-2016].
 - [18] R. E. Clark, D. Howe, and G. W. Jewell, "The influence of magnetization pattern on the performance of a cylindrical moving magnet linear actuator," *IEEE International Magnetism Conference*, vol. 36, no. 5, pp. 3571–3574, 2000.
 - [19] J. L. K. Jr, "Introduction to Power Systems Chapter 6 Magnetic Circuit Analog to Electric Circuits", *MIT Open Course Ware*, pp. 1–9, 2003.
 - [20] J. F. Gieras, Z. J. Piech, and B. Z. Tomczuk, *Linear Synchronous Motors*, CRC Press Taylor & Francis Group, 2011.
 - [21] J. Wang, D. Howe and Z. Lin, "Comparative Studies on Linear Motor Topologies for Reciprocating Vapor Compressors," *2007 IEEE International Electric Machines & Drives Conference*, Antalya, 2007, pp. 364-369.
 - [22] M. Norhisam, K. Alias, R. N. Firdaus, S. Mahmud, N. Mariun, and J. Abdul Razak, "Comparison of thrust characteristic of linear oscillatory actuators," *First International Power Energy Conference (PECon 2006)*, pp. 470–475, 2006.

-
- [23] J. Wang, Z. Lin and D. Howe, "Analysis of a short-stroke, single-phase, quasi-Halbach magnetized tubular permanent magnet motor for linear compressor applications," *IET Electric Power Applications*, vol. 2, no. 3, pp. 193-200, May 2008.
- [24] Park, K.; Hong, E.; and Lee, H. K., "Linear Motor For Linear Compressor," *International Compressor Engineering Conference*, 2013, Paper 1544.
- [25] J. Wang, D. Howe and Z. Lin, "Design Optimization of Short-Stroke Single-Phase Tubular Permanent-Magnet Motor for Refrigeration Applications," in *IEEE Transactions on Industrial Electronics*, vol. 57, no. 1, pp. 327-334, Jan. 2010.
- [26] S. Zhu, P. Zheng, B. Yu, L. Cheng and W. Wang, "Performance Analysis and Modeling of a Tubular Staggered-Tooth Transverse-Flux PM Linear Machine," *Energies* 2016, vol. 9(3), pp. 163-171.
- [27] B. Yu, S. Zhang, J. Yan, L. Cheng, and P. Zheng, "Thermal analysis of a novel cylindrical transverse-flux permanent-magnet linear machine," *Energies* 2015, vol. 8(8), pp. 7874–7896.
- [28] C. Pompermaier, K. Kalluf, A. Zambonetti, M. V. Ferreira da Luz and I. Boldea, "Small Linear PM Oscillatory Motor: Magnetic Circuit Modeling Corrected by Axisymmetric 2-D FEM and Experimental Characterization," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 3, pp. 1389-1396, 2012.
- [29] S. Franco, *Electric Circuit Fundamentals*, Saunders College Publishing, 1995.
- [30] "Kikusui High Performance Multifunctional AC Power Supplies PCR-LE Series". [Online]. Available: <http://www.kikusui.co.jp/en/product/detail.php?IdFamily=0117>. [Accessed: 06-Jun-2016].
- [31] "Pololu - ACS709 Current Sensor Carrier -5 to +5A" [Online]. Available: <https://www.pololu.com/product/1185>. [Accessed: 06-Jun-2016].
- [32] Y. Mitsutake, K. Hirata and Y. Ishihara, "Dynamic response analysis of a linear solenoid actuator," *IEEE Transactions on Magnetics*, vol. 33, no. 2, pp. 1634-1637, Mar 1997.
- [33] O. G. Bellmunt, S. G. Arellano, A. S. Andreu, D. M. Miracle, L. F. Campanile, "Linear electromagnetic actuator modeling for optimization of mechatronic and adaptive systems," *Mechatronics*, vol. 17, Issues 2, Mar. 2007, pp. 153-163.

-
- [34] K. Harmer, G. W. Jewell and D. Howe, "Transient performance of a short-stroke linear solenoid actuator," *IEE Proceedings - Electric Power Applications*, vol. 149, no. 5, pp. 379-384, Sep 2002.
 - [35] L. Nagy, T. Szabó, E. Jakab, "Electro-Dynamical Modeling of a Solenoid Switch of Starter Motors," *Procedia Engineering*, vol. 48, 2012, pp. 445-452, ISSN 1877-7058.
 - [36] E. E. Topçu, I. Yüksel, and Z. Kemiş, "Development of electro-pneumatic fast switching valve and investigation of its characteristics," *Mechatronics*, vol. 16, no. 6, pp. 365–378, 2006.
 - [37] S. Hodgson, M. Tavakoli, M. T. Pham, and A. Leleve, "Nonlinear discontinuous dynamics averaging and PWM-based sliding control of solenoid-valve pneumatic actuators," *IEEE/ASME Transaction on Mechatronics*, vol. 20, no. 2, pp. 876–888, 2015.
 - [38] M. Khoshzaban Zavarehi, P. D. Lawrence and F. Sassani, "Nonlinear modeling and validation of solenoid-controlled pilot-operated servo valves," *IEEE/ASME Transactions on Mechatronics*, vol. 4, no. 3, pp. 324-334, Sep 1999.
 - [39] B. J. Sung, E. W. Lee and J. G. Lee, "A design method of solenoid actuator using empirical design coefficients and optimization technique," *2007 IEEE International Electric Machines & Drives Conference*, Antalya, 2007, pp. 279-284.
 - [40] S.-B. Yoon, J. Hur, Y.-D. Chun, and D. Hyun, "Shape optimization of solenoid actuator using the finite element method and numerical optimization technique," *IEEE Transactions on Magnetics*, vol. 33, no. 5, pp. 4140–4142, 1997.
 - [41] J. L. K. Jr, "Electromagnetic Energy" *MIT Open Course Ware*, pp. 1–9, 2010.
 - [42] O. Vogel and J. Ulm, "Theory of Proportional Solenoids and Magnetic Force Calculation Using Comsol Multiphysics," *Comsol Conference*, Stuttgart, 2011.
 - [43] Z. Zhang, K. W. E. Cheng and X. D. Xue, "Study on the performance and control of linear compressor for household refrigerators," *Power Electronics Systems and Applications (PESA), 5th International Conference on*, Hong Kong, 2013, pp. 1-4.
 - [44] T. Wang, L. Yan, Z. Jiao, and P. He, "Analytical modeling of linear oscillating motor with a mixed method considering saturation effect," *Sensors Actuators, A Physical*, vol. 234, pp. 375–383, 2015.

-
- [45] T. W. Chun, J. R. Ahn, H. H. Lee, H. G. Kim and E. C. Nho, "A Novel Strategy for Efficiency Control for a Linear Compressor System Driven by a PWM Inverter," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 1, pp. 296-301, Jan. 2008.
 - [46] J. K. Kim, C. G. Roh, H. Kim, and J. H. Jeong, "An experimental and numerical study on an inherent capacity modulated linear compressor for home refrigerators," *International Journal of Refrigeration*, vol. 34, no. 6, pp. 1415–1423, 2011.
 - [47] Y. W. Lv, J. Z. Zhang, Y. Shan, and X. M. Tan, "Numerical investigation of effects of actuator parameters and excitation frequencies on synthetic jet fluidic characteristics," *Sensors and Actuators: A Physical*, vol. 219, pp. 100–111, 2014.
 - [48] S. Singh, N. Kumar, D. George, and a. K. Sen, "Analytical modeling, simulations and experimental studies of a PZT actuated planar valveless PDMS micropump," *Sensors and Actuators, A Physical*, vol. 225, pp. 81–94, 2015.
 - [49] D. Pyo, T.-H. Yang, S. Ryu, and D.-S. Kwon, "Novel linear impact-resonant actuator for mobile applications," *Sensors and Actuators, A Physical.*, vol. 233, pp. 460–471, 2015.
 - [50] E. O. Ranft, G. Van Schoor, and C. P. Du Rand, "Self-sensing for electromagnetic actuators. Part I: A coupled reluctance network model approach," *Sensors and Actuators, A Physical*, vol. 172, no. 2, pp. 400–409, 2011.
 - [51] Q. Liu, J. Fang, and B. Han, "Novel electromagnetic repeated launch locking/unlocking device (RLLUD) based on self-locking for magnetic bearing flywheel," *Sensors and Actuators, A Physical*, vol. 175, pp. 116–126, 2012.
 - [52] T. Mizuno, M. Kawai, F. Tsuchiya, M. Kosugi, and H. Yamada, "An examination for increasing the motor constant of a cylindrical moving magnet type linear actuator," *IEEE Transactions on Magnetics*, vol. 41, no. 10, pp. 3976–3978, 2005.
 - [53] T. Mizuno, M. Kosugi, and S. Fukuzawa, "Examination of Reduction in Electrical Time Constant of a Moving Magnet Linear Actuator," *Journal of Magnetic Society Japan*, vol. 31, no. 6, pp. 445–450, 2007.
 - [54] J. Chen, Y. Liao, C. Zhang and P. Sun, "Design of a cylindrical moving core small linear PM oscillatory actuator," *17th International Conference on Electrical Machines and Systems (ICEMS)*, Hangzhou, 2014, pp. 2211-2215.

-
- [55] H. Liang, Z. Jiao, L. Yan, L. Zhao, S. Wu, and Y. Li, "Design and analysis of a tubular linear oscillating motor for directly-driven EHA pump," *Sensors and Actuators, A Physical*, vol. 210, pp. 107–118, 2014.
- [56] P. Rafajdus, I. Zrak, and V. Hrabovcova, "Analysis of the Switched Reluctance Motor (SRM) Parameters," *Journal of Electrical Engineering*, vol. 55, no. 7, pp. 195–200, 2004.
- [57] C. Pompermaier, F. J. H. Kalluf, M. V. Ferreira da Luz and N. Sadowski, "Analytical and 3D FEM modeling of a tubular linear motor taking into account radial forces due to eccentricity," *Electric Machines and Drives Conference, 2009. IEMDC '09. IEEE International*, Miami, FL, 2009, pp. 413-418.
- [58] E. W. Weisstein, "Conical Frustum," From Mathworld--A wolfram Web Resource. [Online]. Available: <http://mathworld.wolfram.com/ConicalFrustum.html>. [Accessed: 28-Jun-2016].
- [59] R. Krishnan, *Switched reluctance motor drives: modeling, simulation, analysis, design, and applications*, CRC Press, 2001.
- [60] M. G. Lee, S. Q. Lee, and D. G. Gweon, "Analysis of Halbach magnet array and its application to linear motor," *Mechatronics*, vol. 14, no. 1, pp. 115–128, 2004.
- [61] J. Seok-Myeong, C. Jang-Young, L. Sung-Ho, C. Han-Wook, and J. Won-Bum, "Analysis and experimental verification of moving-magnet linear actuator with cylindrical Halbach array," *IEEE Transactions on Magnetics*, vol. 40, no. 4, pp. 2068–2070, 2004.
- [62] Z. Q. Zhu, X. Chen, D. Howe, and S. Iwasaki, "Electromagnetic Modeling of a Novel Linear Oscillating Actuator," *IEEE Transactions on Magnetics*, vol. 44, no. 11, pp. 3855–3858, 2008.
- [63] S. M. Jang, J. Y. Choi, H. W. Cho, W. B. Jang and B. H. Kim, "The influence of mechanical resonance on the dynamic performance of a tubular linear actuator with Halbach array," *IEEE International Conference on Electrical Machines and Systems*, 2005, pp. 264-269.
- [64] J. K. Son, T. W. Chun, H. H. Lee, H. G. Kim and E. C. Nho, "Method of estimating precise piston stroke of linear compressor driven by PWM inverter," *Power Electronics and Motion Control Conference and Exposition (PEMC), 2014 16th International*, Antalya, 2014, pp. 673-678.

-
- [65] P. Zheng, S. Zhu, B. Yu, L. Cheng and Y. Fan, "Analysis and Optimization of a Novel Tubular Staggered-Tooth Transverse-Flux PM Linear Machine," in *IEEE Transactions on Magnetics*, vol. 51, no. 11, pp. 1-4, Nov. 2015.
- [66] T. W. Chun, J. R. Ahn, Q. V. Tran, H. H. Lee and H. G. Kim, "Method of Estimating the Stroke of LPMSM Driven by PWM Inverter in a Linear Compressor," *APEC 07 – 22nd Annual IEEE Applied Power Electronics Conference and Exposition*, Anaheim, CA, USA, 2007, pp. 403-406.
- [67] N. Bianchi, "Analytical computation of magnetic fields and thrusts in a tubular PM linear servo motor," *Industry Applications Conference, 2000. Conference Record of the 2000 IEEE*, Rome, 2000, pp. 21-28.
- [68] S. M. Jang, J. Y. Choi, Han-Wook Cho and Sung-Ho Lee, "Dynamic characteristic analysis and experiments of moving-magnet linear actuator with cylindrical Halbach array," *IEEE Transactions on Magnetics*, vol. 41, no. 10, pp. 3814-3816, Oct. 2005.
- [69] D. G. Park, J. H. Jang, S. A. Kim and Y. H. Cho, "Modeling of Non-Linear Analysis of Dynamic Characteristics of Linear Compressor," *2012 Sixth International Conference on Electromagnetic Field Problems and Applications (ICEF)*, Liaoning, 2012, pp. 1-4.
- [70] B. Kou, J. Luo, X. Yang and L. Zhang, "Modeling and Analysis of a Novel Transverse-Flux Flux-Reversal Linear Motor for Long-Stroke Application," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 10, pp. 6238-6248, Oct. 2016.
- [71] K. Liang, R. Stone, M. Dadd, P. Bailey, "A novel linear electromagnetic drive oil-free refrigeration compressor using R134a," *International Journal of Refrigeration*, vol. 40, 2014, pp. 450-459.
- [72] M. Xia, X. Chen, "Analysis of resonant frequency of moving magnet linear compressor of stirling cryocooler," *International Journal of Refrigeration*, vol. 33, Issue 4, 2010, pp. 739-744.
- [73] C. R. Bradshaw, E. A. Groll, S. V. Garimella, "Sensitivity analysis of a comprehensive model for a miniature-scale linear compressor for electronics cooling," *International Journal of Refrigeration*, vol. 36, Issue 7, November 2013, pp. 1998-2006.

-
- [74] A. Hassan, A. Bijanzad, I. Lazoglu, "Dynamic Analysis of a Novel Moving Magnet Linear Actuator," *IEEE Transactions on Industrial Electronics*, vol. PP, no.99, pp.1-1. December, 2016.
- [75] A. Bijanzad, A. Hassan, I. Lazoglu, "Analysis of solenoid based linear compressor for household refrigerator," *International Journal of Refrigeration*, vol. 74, Feb. 2017, pp. 114-126.
- [76] M. Utsuno, M. Takai, T., Yaegashi, T. Mizuno, H. Yamamoto, K. Shibuya, and H. Yamada, "Efficiency characteristics of a linear oscillatory actuator under simulated compressor load", *Proceedings of 2001 Linear Drives and Industrial Applications (LDIA'2001)*, Nagano, Japan, pp. 264-267, 2001.
- [77] R. E. Clark, D. S. Smith, P. H. Mellor and D. Howe, "Design optimization of moving-magnet actuators for reciprocating electro-mechanical systems," in *IEEE Transactions on Magnetics*, vol. 31, no. 6, pp. 3746-3748, Nov 1995.
- [78] M. Sanada, S. Morimoto and Y. Takeda, "Analyses for sensor less linear compressor using linear pulse motor," *Conference Record of the 1999 IEEE Ind. App. Conf. Thirty-Forth IAS Annual Meeting (Cat. No.99CH36370)*, Phoenix, AZ, 1999, vol.4, pp. 2298-2304.
- [79] J. Wang, W. Wang, G. W. Jewell and D. Howe, "A low-power, linear, permanent-magnet generator/energy storage system," *IEEE Transactions on Industrial Electronics*, vol. 49, no. 3, pp. 640-648, Jun 2002.
- [80] L. Tutelea et al., "A set of experiments to more fully characterize linear PM oscillatory machines," *IEEE Transactions on Magnetics*, vol. 41, no. 10, pp. 4009-4011, Oct. 2005.
- [81] K. Hirata, T. Yamamoto, T. Yamaguchi, Y. Kawase and Y. Hasegawa, "Dynamic Analysis Method of Two-Dimensional Linear Oscillatory Actuator Employing Finite

-
- Element Method,” *IEEE Transactions on Magnetics*, vol. 43, no. 4, pp. 1441-1444, April 2007.
- [82] Z. Lin, J. Wang and D. Howe, “A Resonant Frequency Tracking Technique for Linear Vapor Compressors,” *2007 IEEE International Electrical Machines & Drives Conference*, Antalya, 2007, pp. 370-375.
- [83] L. N. Tutelea, M. C. Kim, M. Topor, J. Lee and I. Boldea, “Linear Permanent Magnet Oscillatory Machine: Comprehensive Modeling for Transients With Validation by Experiments,” *IEEE Transactions on Industrial Electronics*, vol. 55, no. 2, pp. 492-500, Feb. 2008.
- [84] T. Yamaguchi et al., “Trajectory Analysis of 2-D Magnetic Resonant Actuator,” *IEEE Transactions on Magnetics*, vol. 45, no. 3, pp. 1732-1735, March 2009.
- [85] C. Pompermaier, A. Zambonetti, F. J. H. Kalluf, M. V. Ferreira da Luz and N. Sadowski, “Performance analysis of a tubular linear motor applied in compressors,” *5th IET International Conference on Power Electronics, Machines and Drives (PEMD 2010)*, Brighton, UK, 2010, pp. 1-4.
- [86] Y. Asai, K. Hirata and T. Ota, “Amplitude Control Method of Linear Resonant Actuator by Load Estimation From the Back-EMF,” *IEEE Transactions on Magnetics*, vol. 49, no. 5, pp. 2253-2256, May 2013.
- [87] Y. Asai; T. Ota; T. Yamamoto; K. Hirata, “Proposed of Novel Linear Oscillating Actuator's Structure using Topology Optimization,” *IEEE Transactions on Magnetics*, vol. PP, no.99, pp.1-1.
- [88] H. Vahid Alizadeh and B. Boulet, “Analytical Calculation of the Magnetic Vector Potential of an Axisymmetric Solenoid in the Presence of Iron Parts,” *IEEE Transactions on Magnetics*, vol. 52, no. 3, pp. 1-4, March 2016.

-
- [89] S. Fizek, M. Reisinger, S. Silber and W. Amrhein, "A hybrid solenoid model comprising eddy currents," *2014 International Symposium on Power Electronics, Electrical Drives, Automation and Motion*, Ischia, 2014, pp. 1219-1224.
 - [90] L. Li, C. Zhang, B. Yang and X. Li, "Finite element analysis of the uniformity of magnetic field for on-off Giant Magnetostrictive Actuators," *2008 International Conference on Electrical Machines and Systems*, Wuhan, 2008, pp. 3758-3761.
 - [91] H. Zhou, J. Sun, B. S. Yeow and H. Ren, "Multi-objective parameter optimization design of a magnetically actuated intravitreal injection device," *2016 IEEE 14th International Workshop on Advanced Motion Control (AMC)*, Auckland, 2016, pp. 430-435.
 - [92] F. Mach and T. Kaminský, "Novel monostable fail-safe electromagnetic actuator for straight valve in high-speed operation," *2016 ELEKTRO*, 2016, pp. 186-191.
 - [93] Z. J.-dong and G. Y. Ouyang, "Optimization design of solenoid valve for electrically-controlled high pressure pump," *2009 International Conference on Mechatronics and Automation*, Changchun, 2009, pp. 4122-4126.
 - [94] I. Dülk and T. Kováčsházy, "Parameter Estimation in Linear Electromagnetic Devices," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3619-3628, June 2015.
 - [95] S. Nagai, T. Nozaki and A. Kawamura, "Real-time position sensorless estimation of position and force of solenoid actuator for haptic devices," *2015 17th European Conference on Power Electronics and Applications (EPE'15 ECCE-Europe)*, Geneva, 2015, pp. 1-9.
 - [96] I. Hadzhiev, D. Malamov, I. Balabozov and I. Yatchev, "Study of electromagnetic field distribution and force characteristics of a DC electromagnetic actuator with T-shaped armature," *2016 19th International Symposium on Electrical Apparatus and Technology (SIELA)*, Bourgas, 2016, pp. 1-4.

-
- [97] F. Meng, H. Zhang, D. Cao and H. Chen, "System Modeling, Coupling Analysis, and Experimental Validation of a Proportional Pressure Valve With Pulsewidth Modulation Control," *IEEE/ASME Transactions on Mechatronics*, vol. 21, no. 3, pp. 1742-1753, June 2016.
- [98] M. K. Zavarehi, P. D. Lawrence, and F. Sassani, "Nonlinear Modeling and Validation of Solenoid Controlled Pilot Operated Servovalves," *IEEE Transactions on Mechatronics*, vol. 4, no. 3, pp. 324–334, 1999.
- [99] H. Dang, L. Zhang, J. Tan, "Dynamic and thermodynamic characteristics of the moving-coil linear compressor for the pulse tube cryocooler. Part A: Theoretical analyses and modeling," *International Journal of Refrigeration*, vol. 69, September 2016, pp. 480-496.
- [100] H. Dang, L. Zhang, J. Tan, "Dynamic and thermodynamic characteristics of the moving-coil linear compressor for the pulse tube cryocooler: Part B – Experimental verifications," *International Journal of Refrigeration*, vol. 69, September 2016, pp. 497-504.