DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

EVALUATION OF INVENTORY CONTROL POLICIES FOR PERISHABLE PRODUCTS

by

Bahar YALÇIN

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İZMİR

EVALUATION OF INVENTORY CONTROL POLICIES FOR PERISHABLE PRODUCTS

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by
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Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"EVALUATION OF INVENTORY CONTROL POLICIES FOR PERISHABLE PRODUCTS"** completed by **BAHAR YALÇIN** under supervision of **ASSOC. PROF. DR. UMAY ZEYNEP UZUNOĞLU KOÇER** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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EVALUATION OF INVENTORY CONTROL POLICIES FOR PERISHABLE PRODUCTS

ABSTRACT

For inventory management, it is very important to meet customer demands on time and exact quantity. Especially when the product is perishable, this importance is increasing even more for the companies. The studies in the perishable inventory field began to be matched to real life problems by going out of mathematical modeling. We have examined an (s, S) continuous review inventory model for perishable items, one of the most common problems in the literature. Firstly, a comprehensive literature review was conducted for perishable items. Two different demand classes for one perishable item were examined. It was assumed that the demands for both classes occur according to Poisson process and the customer demands were met according to the on-hand inventory level. Accordingly, it was assumed that when the on-hand inventory drops to s, only the priority customer demands were met whereas the demands from ordinary customers were lost. And also, it is assumed that the demand occurring stock out periods was lost. The system is characterized by Markov process, besides lifetime and lead time have an exponential distribution. The stationary probabilities were derived and the expected total cost formula was provided. The pseudo-convexity of the expected cost function was proved. A numerical study was performed with MATLAB. Differences were illustrated between the inventory management of the perishable products and non-perishable products. As a result, perishable items cause a constant cost increase, it is important to model perishable items in the most appropriate way. The novelty of this study is to provide a model and a solution to a perishable problem with two demand classes, in which both the lead time and the lifetime is stochastic. At the same time, this dissertation may lead to other studies that have a different distribution of lead time or lifetime. By characterizing the model with different distributions and different processes, the system can be even closer to the real life problem.

Keywords: Markov process, perishable items, two demand classes, random lifetime, random lead time

BOZULABİLİR ÜRÜNLER İÇİN ENVANTER KONTROL POLİTİKALARININ DEĞERLENDİRİLMESİ

ÖZ

Envanter yönetimi için müşteri taleplerini zamanında ve kesin miktarda karşılamak çok önemlidir. Özellikle ürün bozulabilir olduğunda bu önem daha da artmaktadır. Envanter alanında yapılan çalışmalar zamanla sadece matematiksel olarak modellenmenin dışına çıkarak günlük hayat problemlerine uygun hale gelmeye başlamıştır. Bu tezde sıkça karşılaşılan problemlerden biri olan (s, S) sürekli gözden geçirme politikasını inceledik. Öncelikle bozulabilir ürünler için kapsamlı bir literatür taraması yapıldı. Bir ürüne iki farklı müşteri grubundan talep olabileceği bir durum ele alındı. Taleplerin Poisson sürecinden olduğu ve eldeki ürünün stok durumuna göre müşterilerin taleplerini karşılandığı varsayıldı. Buna bağlı olarak, eldeki envanter seviyesi önceden belirlenen s düzeyinin altına düştüğünde sadece öncelikli müşteri grubunun talebinin karşılandığı, sıradan müşteri talebinin kaybolduğu varsayıldı. Ayrıca elde hiç envanter kalmadığı durumda ise her iki talep grubu için talepler karşılanamadığı varsayıldı. Ürün ömrünün ve tedarik süresinin üssel bir dağılıma sahip olduğu durum için sistem Markov süreci ile karakterize edildi. Durağan durum olasılıkları elde edilmiş ve toplam maliyet formülü çıkarılmıştır. Beklenen maliyet formülünün pseudo-konveks olma koşulunu sağladığı gösterildi. MATLAB yardımı ile nümerik bir çalışma yapıldı. Bozulabilir ürünlerin ve bozulmayan ürünlerin envanteri yönetimi arasında farklılıklar gösterildi. Sonuç olarak bozulabilir ürünler sürekli bir maliyet artışın sebep olduğu için gerçeğe en uygun şekilde modellenmesi önemlidir. Bu çalışmanın yeniliği, hem tedarik süresinin hem de ürün ömrünün stokastik olduğu iki talep sınıflı bir bozulabilir ürün problemine bir model ve bir çözüm sağlamaktır. Aynı zamanda bu çalışma tedarik süresi veya ürün ömrü gibi temel bileşenlerin daha farklı dağılımlara uyduğu başka çalışmalara yol gösterebilir. Farklı dağılımlar ve farklı süreçler ile model karakterize edilerek, sistem gerçek hayat problemine daha da yakınlaştırılabilir.

Anahtar kelimeler: Markov süreci, bozulabilir ürünler, iki talep sınıfı, rassal ürün ömrü, rassal tedarik süresi

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CHAPTER ONE
INTRODUCTION

Inventory management is to keep stocks to meet customer demand at the right time and it is a method used not to increase the total cost. Inventory control involves the procurement, care and disposition of materials. Raw materials, in process or semi-finished goods and finished goods are the kinds of inventory. Managers should protect the inventory against obsolescence, deterioration or theft. Goyal & Giri (2001) present a broad overview of the literature about perishable inventory. According to the classification given in Goyal & Giri (2001), the situations that may occur can be examined in three different categories. The products that cannot be sold due to the time-lapse fashion or the aging of the technology (obsolescence). The products that have been damaged or have passed the last consumption date (deterioration) and shelf life is endless in products where none of these two conditions are observed.

Most inventory systems assume that items can be stored indefinitely to meet future demands. Conversely, the deterioration of the product affects the unlimited storage conditions. Perishable items can rot or spoil easily and may affect other items' quality over time, and these items may become lesser in value i.e. fresh foods, chemicals, plants, blood cells, photographic films, drugs and other pharmaceuticals. Food and health products are the first products that come to mind when you think of perishable products but technological products like computer hardware parts have lifetime, too. Nowadays the technology has developed rapidly and technological products are not sold quickly. These products can be considered as perishable products even though there is no physical deterioration. Perishable or outdating items are known to require special management attention due to their high obsolescence or perishability costs. It is important for companies to maintain the true amount of perishable inventory. If companies keep too few items; they lose profit because customers would not be able to purchase the items they demanded. On the other hand, if companies store too many items, they will have to discard them after the items perish and this may result in losing profit. For this reason, inventory control of perishable products is important.

1.1 Outline of the Dissertation

In this thesis, two different customer demand classes for perishable products are discussed which have not been studied before. A new contribution has been made to the literature considering such limitations as the product life and lead time caused by the perishability of the product. The information is given in each chapter as follows:

Chapter 1 provides general information on perishable products and the information about the outline of the dissertation. Definition of perishable products and importance of controlling the perishable inventory has been discussed. Finally, the contribution of the thesis to the literature was mentioned.

To solve an inventory problem, we first have to model the problem. In Chapter 2, basic components needed for modeling of perishable inventory system, such as lifetime, demand structure, lead time and policies, are described in detailed. It is important to determine which reorder policy should be used to meet the customer demand. For these reasons, extensive literature review under various assumptions of perishable inventory modelling is provided in this section. The literature review is presented in different categories.

In Chapter 3, expressed a broad summary of the stochastic processes which some of them are used in this thesis is expressed. Since inventory models modeled as Markov process mostly in the literature, general characteristics of Markov processes are defined. Some important statistical distributions such as Poisson distribution and Exponential distribution used in this process are mentioned.

Chapter 4 provides information about a continuous review (s, Q) inventory model for perishable items with two demand classes with random lifetime and random lead time. An appropriate mathematical model established for one perishable item and proved that the model provides the necessary assumptions. Then we gave a numerical analysis for different values of all parameters. It is seen that the perishable model with two demand classes is consistent when numerical results are compared with other studies.

In Chapter 5, conclusion and suggestions for the future work are presented.

1.2 Scope of the Dissertation

In this dissertation, we examines an (*s, S*) continuous review inventory model for perishable items with two demand classes. Demands for both classes occur according to Poisson process. The items have exponential lifetimes. The lead time is an exponential random variable. When the on-hand inventory drops to *s*, only the priority customer demands are met whereas the demands from ordinary customers are lost. And also, the demand occurring stock out periods are lost.

When working with perishable products, many problems arise due to the structure of the product. One of these problems is how often the product is to be reviewed. The periodic review control is easier to deal with, but the continuous review is appropriate for real life problems. Products with a life can be kept under control before they perish or products can be ordered again without stock out. For this reason, we used continuous review policy. The fact that the lifetime is not deterministic may seem to complicate the calculation. But it gives us a lot of conveniences when we assume the lifetime is exponential. The exponential distribution is represents the lifetime of the product in the best possible way. Because the deterioration will continue to increase continuously depending on the time. This distribution brings along the Markovian feature together. At the same time, the use of the exponential distribution is a basis for the further research like lifetime distributed randomly. In addition, the same situation is true for lead time. As the lead time is different from zero, it is open the discussion for the further research and the same model can be re-examined with different distributions and characterized with different processes.

Our main improvement with this dissertation is characterize a perishable inventory system with two different customer demand classes by continuous-time Markov process. When we look at the papers in multi-customer demand, papers generally assumed that there is no perishability and lifetime is considered unlimited. Our model corresponds to some other studies in the literature. When the products in inventory are non-perishable, our model are in agreement with given by Isotupa (2006); when all customers have the same priority; our model are in agreement with the results of that of Kalpakam & Sapna (1994). In addition, of our model for are in agreement with that of Liu et.al (2014) when the inventory is perishable and the probability for customer equals to 0 for the same parameter values.

CHAPTER TWO
BASIC COMPONENTS OF A PERISHABLE INVENTORY SYSTEM

Some basic components must be taken into consideration in modelling of perishable inventory problem. Inventory costs, control policies, structure of customer demand, distribution of lead time and distribution of lifetime are some of these components. This section contains common descriptions of the components mentioned above.

2.1 Costs

The aim of the inventory control models is to determine the optimal policy that minimizes total cost and inventory costs are an important part at a company's cash flow. In this section, all items stocked have a certain cost and these costs are defined in detail.

Set-up or ordering costs (K) are the costs incurred from getting a machine ready to produce the desired good. In a manufacturing setting, this would require the use of a skilled technician who disassembles the tools that is currently in use on the machine. If the company purchases the part or raw material, then an order cost, rather than a set-up cost, is incurred. In addition, sometimes companies include the cost of shipping the purchased goods in the order cost.

Purchasing cost (p) is simply the cost of the purchased item itself. If the company purchases a finished product, the company can determine its annual purchasing cost by multiplying the cost of one purchased unit (p) by the number of finished products demanded in a year (D). The purchasing cost includes the variable labor cost, variable general expenses and raw material cost associated with purchasing or production of a single unit. If goods are ordered from an external source, the unit purchase cost must include shipping cost.

Holding cost (h) is the cost of carrying one unit of inventory for the unit time period. Inventory in excess of current demand frequently means that its holder must provide a place for its storage when not in use. Storage facilities also require heating, cooling, lighting, and water. The holding cost usually includes storage cost, insurance cost, taxes on inventory, and a costs due to the possibility theft. All of these things add cost of holding or carrying inventory.

Stockout or shortage cost (s) is occurred when customer demand is not met on time. If customer accepts the delivery even it is late, we say that demands may be back-ordered. If customer does not accept late delivery, then this is the lost sales case.

Failure or perishability cost (g) is the cost that arises from the deterioration of a unit product, for the unit time period. Failure cost may increase according to the value of the product or failure cost can be considered equal to human life in some products of vital importance for humans.

2.2 Policies

When the products in inventory are perishable products, on-hand stock level may decrease to zero rapidly. Even orders are placed immediately, stock out may be happen due to the lead time. On the other hand, new orders may arrive before the end of the inventory life. In this case, the lifetime of the products may change item to item, there is a variation of lifetime. Depending on the perishability, the manager will decide which product are removed first from stock or how many items will order at reorder point. The manager needs to evaluate system performance very well to make this decision. The system performance is controlled with four different types of policies. The first one of these is “Issuing policy”.

2.2.1 Issuing Policy

The issuing policies concern the sequence of which items are removed from the inventory of finitely many units of varying lifetimes. Most of the studies in the literature have studied first in first out (FIFO) principle as issuing policy. However, Floerkemeier (2008) described the seven issuing policies. They analyzed all seven policies using a simple supply chain and compared them. These policies as shown below:

1. Sequence In Random Order (SIRO): Products in the distribution center are selected randomly and issued to the retailer.
2. First In First Out (FIFO): Products that have been longest in the distribution center are selected first.
3. Last In First Out (LIFO): Products that have been shortest in the distribution center are selected first.
4. First Expiry First Out (FEFO): Products in the distribution center are selected by their age, the items which were manufactured earlier being the first to be issued.
5. Lowest Quality First Out (LQFO): Products are selected by their quality, the items which have the lowest quality being first to be issued.
6. Latest Expiry First Out (LEFO): Products are selected by their age, the items which were manufactured latest are issued first.
7. Highest Quality First Out (HQFO): Products are selected by their quality, the items which have the highest quality are issued first.

Floerkemeier et al. (2008) could distinguish between issuing policies based on time of arrival (FIFO, LIFO) and those based on age (FEFO, LEFO).

Depending on what issuing policy used items in inventory stays fresh. Two of these issuing policies are used in perishable inventory literature extensively: FIFO and LIFO. Another obsolescent policy named FEFO is studied too. Each one has its own benefits and disadvantages and one of them is actually preferred over the other by most companies.

i. FIFO (First In First Out) Policy is the most general approach for the inventory problems. It is assume that the first items put on the shelf (the oldest) are the first items sold, so the oldest goods are sold first. Companies generally want to sell the oldest items in inventory to prevent them from deteriorating. It is clear that for a fixed lifetime FIFO minimizes expected outdating.

ii. LIFO (Last In First Out) assumes that the last items put on the shelf are the first items sold. Older inventory is left over. LIFO issuing may provide excellent service, but it usually causes excessive outdating. Therefore, it is a good system to use when your products are not perishable. However, sometimes LIFO issuing will be the result when the utility of new units is higher. The theory in using this method is that costs will either remain the same or increase.

We have to look at the differences between FIFO and LIFO, to decide which issuing policy to use for perishable inventory. With FIFO, you get a better estimation of the value of inventory but LIFO is not a good indicator of inventory value because some of the inventory may be old or deteriorated further use. While FIFO can be used to increase net profits, it will also increase the amount of taxes that a company will have to pay. On the other hand, LIFO decreases net profits and gives you a break on taxes. When the expiration dates are known, FIFO is commonly used when the suppliers control the issuing order, while customers prefer selecting the freshest items (LIFO). Using LIFO can expect customer advantage as compared to FIFO. On the other hand, LIFO gives higher perishing rates and lower service levels.

iii. FEFO (First Expired First Out) picking of perishable items with an expiration date. Lots having oldest expiry dates (but not expired yet) are issued first. The FEFO method attempts to ensure that perishable products are sold while they are still in good condition. This method is used mostly in process manufacturing industries like medicine, chemical, paint etc. where expiry date of the lot is very critical.

2.2.2 Ordering Policy

An ordering policy focuses on '*when*' and '*how much*' to order from an external supplier. One of the most important components affecting the modeling of an inventory problems is the review methods. The ordering policy has studied widely in literature about perishable inventory system. Basically there are two options, periodic and continuous review. Some preliminary concepts about these common ordering policies and notations will be examined in this section. These models bring about some costs like holding cost, ordering cost, shortage (backlogging) cost and outdating cost.

These costs are often taken balanced to, respectively, the number of products in stock, the order quantity, the number of products short (or backlogged), and the number of obsolescence products. When the lead time is positive, the order amount depends on the on-hand inventory. When the lead time equals to zero, the stock position equal to stock level. By the help of these definitions, we will describe the some ordering policies. When the literature is examined, there are two main groups in ordering policies; periodic review policy and continuous review policy.

2.2.2.1 Periodic Review Policy

In this system time horizon is divided into discrete periods such as week, month, year, etc. and inventory level is reviewed periodically at fixed time intervals *T* and placed an order to return the inventory level to a predetermined order-up-to level *S*. There are two different time interval, short intervals (e.g. daily review) and longer intervals (e.g. monthly review). In short intervals, the (s, S) policy is used. However, in longer intervals always order after an inventory level review and generally used base-stock level policy. Figure 2.1 shows inventory level through time under periodic review policy with positive lead time *L*.

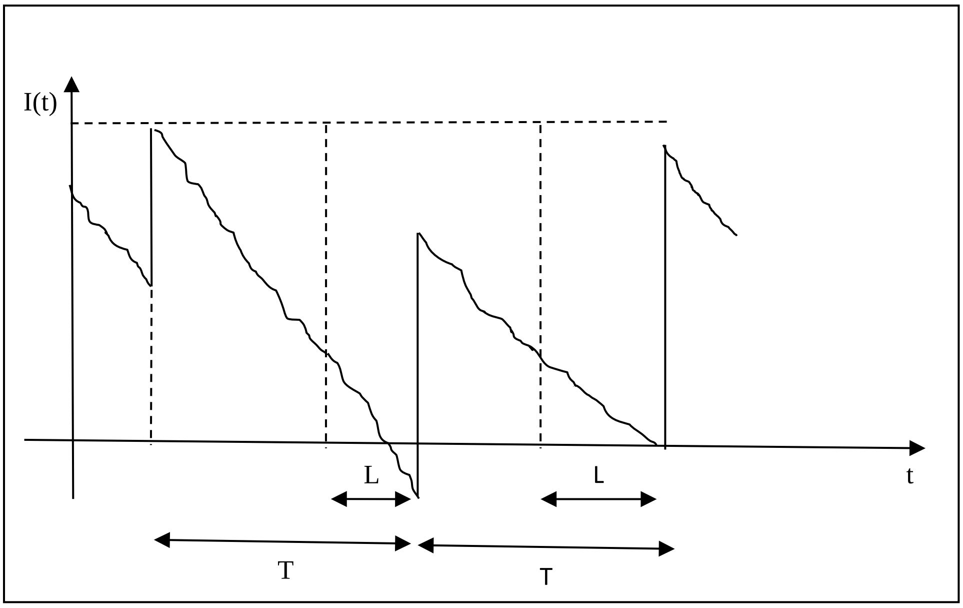

Figure 2.1 Periodic review policy

Order quantity may differ depending on the chosen policy and total inventory level at the beginning of each period. Periodic review system can be classified into three main groups according to order quantity: Order-up-to *S* policy, (*s*, *S*) policy and (*S-1, S*) policy. In Order-up-to *S* policy, the total inventory level, old plus new inventory, is brought up to *S* at the beginning of every period. The base stock level *S* is determined by calculating the quantity needed between the times of the order is placed and the time that the next period's order is received, and adding a quantity of safety stock to allow for variation in the demand. Other commonly used names for this policy are “Critical-number policy” and “Total-inventory-to-*S* (TIS) policy”. The Order-up-to-S policy is simple, common and has a long tradition in the literature. In the (*s, S*) policy, orders are placed when the stock position is at *s* or below the amount of inventory level is brought up to *S*. When the stock position is higher than *s*, no order is placed. This policy is commonly called an (*s, S*) policy. (*S-1, S*) policy is a special case of the (*s, S*) policy, when *s* equals to $(S-1)$. This policy is optimal when the demand is discrete and basically says if the inventory level is one below the maximum inventory level, place an order to bring it to maximum inventory level.

2.2.2.2 Continuous Review Policy

In this system, a company continuously reviews its inventory levels after each purchase or sale and places an order for a fixed quantity *Q* when the inventory drops below a calculated reorder point *s*. The order arrives to replenishment the inventory after a lead time *L*. If a customer demand occurs when there is no inventory on hand, the demand is lost. This model allows individual review frequencies. Figure 2.2 shows the graphical representation of continuous review inventory policy with positive lead time.

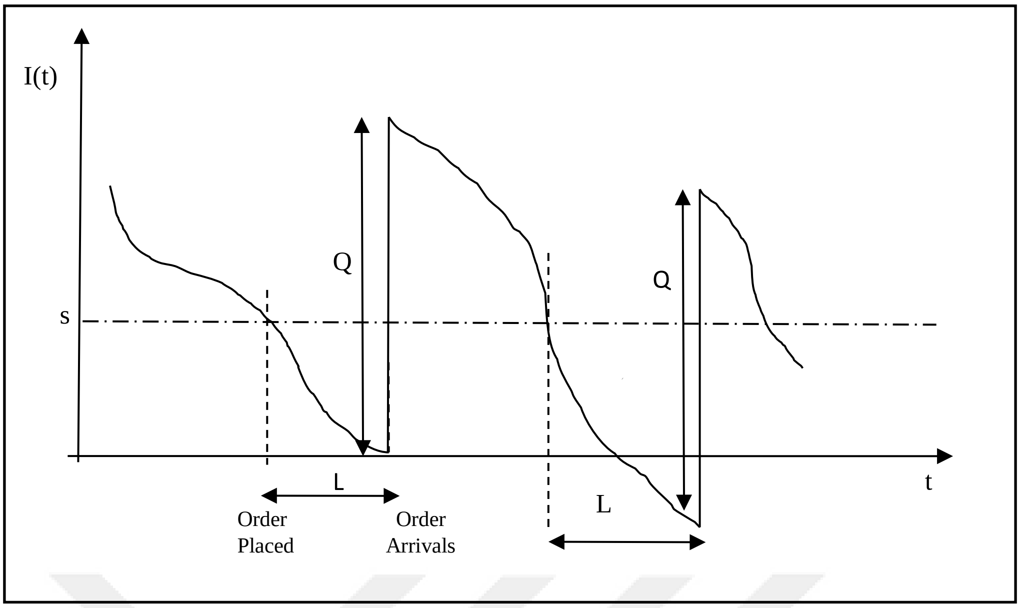

Figure 2.2 Continuous review policy

Policies providing for each type of inventory review have advantages and disadvantages. Periodic inventory review may not provide exact inventory levels for companies with high-volume sales. On the other hand continuous inventory review allows actual replacement of inventory levels, which can make it easier to know when to reorder items.

The (*s, Q*) inventory policy, alternatively called the order quantity-reorder point system, assumes that the inventory level is observed continuously. When the level declines to some specified reorder point, *r*, an order is placed for a lot size; *Q*. this policy has different special cases. The (*Q, r, T*) inventory policy is one of the special case and it provides inventory modelling both in terms of order quantity and time. A replenishment order of size *Q* is placed either when the inventory drops to r, or when T units of time elapsed since the last instance at which the inventory level hit *Q*, whichever occurs first (Tekin et. al., 2001). This model allows a fair comparison of special case policies because it obtained a unified cost model. As a second special case of (*s, Q*), in (*s, nQ*) policy, whenever the inventory level declines to *s*, a quantity *nQ* is ordered where *n* is the minimum integer required increase the inventory level above the *s*, $n \in \{1, 2, \ldots\}$. In (*s, nQ, T*) policy, each stage reviews its inventory in every *T* period and orders according to an echelon (*s, nQ*) policy.

2.2.3 Disposal Policy

The disposal policy is applied when the strategic disposal of part of the inventory is desirable, such as slow moving stock. The assumption that the product life is deterministic is easily encountered in daily life. Shelf life is written on many products purchased from grocery stores. However, the shelf life of some products, such as fresh vegetables and fruit or meat, may change according to certain conditions such as temperature. The main questions of disposal policy are ‘when and how much to dispose’ under stochastic demand and perishing. Veinott (1960) examined the disposal policy extensively. Martin (1986) investigated optimal decision models for disposal of perishable items. Rosenfield (1989) was the first researcher to study the disposal policy under stochastic demand. The demand was assumed to a Poisson distribution. Rosenfield (1989) also examined the effect of inventory perishability on the excess stock disposal decision. Moreover, Rosenfield (1992) showed the optimality of a myopic policy when disposing excess stock, assuming that the same disposal opportunity and given Poisson demand. It is generally considered that lifetime is a random variable. Thus, the system is easily modeled as a Markov process. Exponential distribution assumption is used for lifetime frequently. In addition to this, some studies assumed that the lifetime distribution is arbitrary Erlang distributions.

2.3 Other Components: Demand Structure, Lead time and Lifetime

Most of the studies conducted on perishable inventory, generally assumed that the demand distributed with Poisson process, which helps to express the system with the Markov process. Others assume discrete distribution either geometric or general discrete phase-type renewal arrivals. Oftentimes the continuous demand is assumed for convenience results appear to generalize to general demand distributions. In some studies, the demands are assumed that large size with randomly distributed instead of single demand it is called batch demand. Furthermore, it is assumed to have the time between the occurrences of the demands is arbitrary distributed and this is expressed by renewal process. Demand distributions have been modeled with Markov renewal process recently. In addition, two different demand classes are popular topics recently.

The demand for the items can continue to occur when the inventory level drops to zero due to the deterioration or lead time. In this case, there are two different cases for the modelling. Either the demand generated during this period will be met from the new incoming orders (backorder case) or the demand will not be met and the sales will be lost (lost sale case). In both cases, there is a cost for not satisfying the demand on time. These cases has been studied extensively in the literature. Excess demand is either assumed to be backlogged, or modeled as lost sales.

Lead time is defined as the time between the orders is placed and received. Positive or random distributed lead time is one of the main complication in the modelling of the perishable inventory problems. The problem becomes more complicated because of the variability in lead time. The demands during the lead time period is considered to be random. For this reason the stock level may drop to zero or below during the lead time period. In the perishable inventory literature, there are many studies that assume the lead time as zero, deterministic or random variable.

Most papers assume deterministic lifetimes of general length like zero, one period, two periods or n periods. Some paper allows for general discrete phase-type lifetimes and assumes that all items perish at the same time, in other words disaster model. Another studies permits general discrete lifetimes, but assumes items perish in the same sequence as they were ordered or that items within a lot only begin to age after all items from the previous lot have left inventory. Item lifetimes may also be continuous; exponentially distributed, generally distributed, Erlang distributed and in one case they decay.

2.4 Literature Review

The studies about the inventory management have been still continuing to improve. The studies are also shown along a timeline in the Figure 2.3. The timeline starts with the first mathematical models with the deterministic problems and continues with mathematical models with stochastic problems. Just after this problems solved, the researchers head for the perishability problems. An extensive literature study about perishable problems is given by Haijema (2008). These studies have fixed maximal shelf life and different solution methods. These solution methods were collected in four main titles. These are analytical studies, numerical studies, simulation studies and experimental or empirical studies.

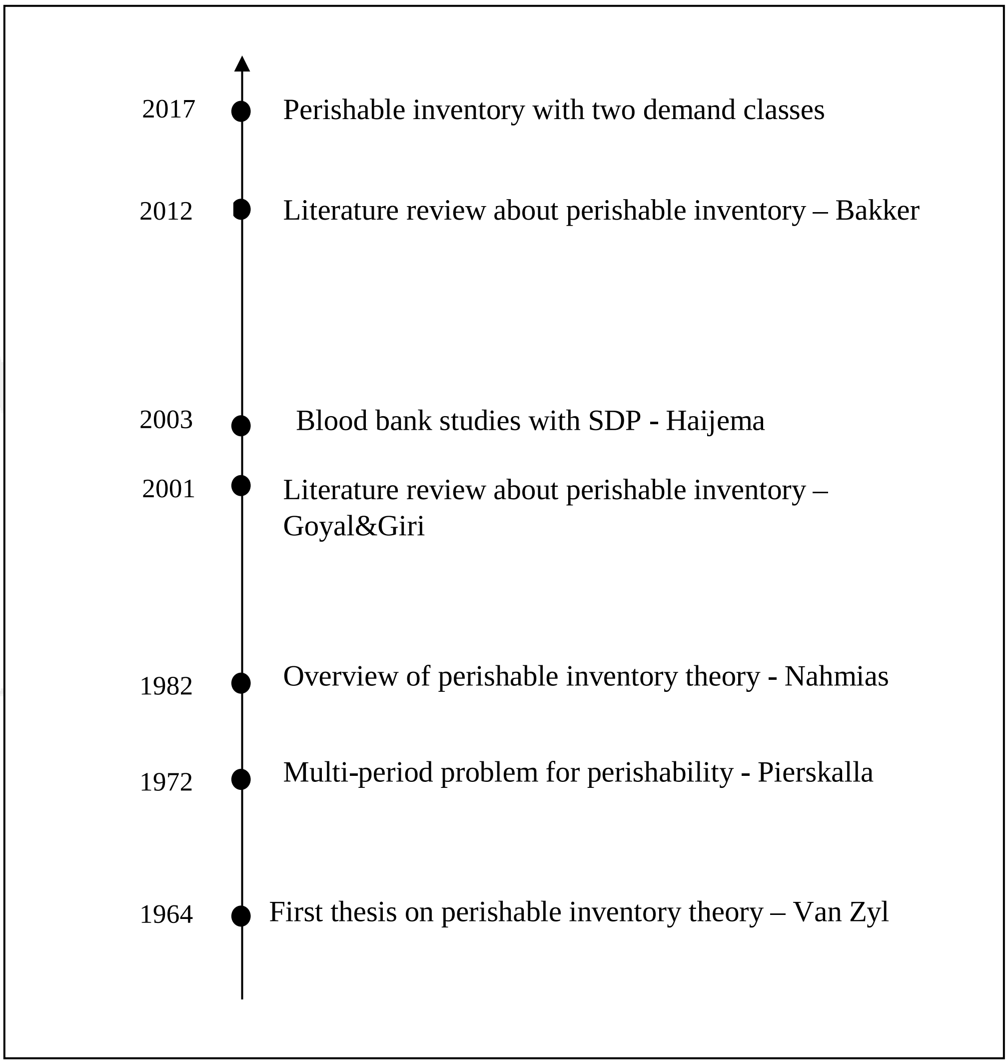

Figure 2.3 The history of perishable inventory management

In addition, there are various classifications in the literature regarding to definitions and inventory policies related to perishable products. The studies usually divided into two main issuing policy titles: FIFO and LIFO. The early work of Derman & Klein (1958), Lieberman (1958), Zehna (1962) and Eilon (1961, 1964) proved FIFO or LIFO was optimal for given conditions. Veinott (1960) discussed periodic review with deterministic demand and fixed lifetime and considered three problems; determining an optimal ordering policy, the disposal policy and issuing policies. Veinott (1960) showed that when the fixed lifetime function a FIFO issuing policy is used on optimal order policy would order an amount equal to demand. Pierskalla & Roach (1972) analyzed the issuing policies for three different objective functions and found the optimal policy for each model. Also, they give the specific assumptions for the mathematical model. Items deteriorated over time with a step function and demand occurred periodically. The quantity of items added to the inventory is assumed to be known. All excess demand assumed to be backlogged or lost. The first objective function considered is based on utility approach. If all excess demand is backlogged, they proved that FIFO is the optimal policy. However, if all excess demand is lost, LIFO is the optimal policy for this model. The second objective function considered is to minimize the total number of backlogged (lost) demands. Pierskalla & Roach (1972) showed that FIFO is optimal again for both excess demands and LIFO is the optimal for the lost sale case too. The third objective function is to minimize the total number of items in the last age category. For backlogged case, FIFO and LIFO policies are both optimal, but for the lost sale case, FIFO was the optimal policy. Parlar et al. (2011) considered an inventory model for perishable items with Poisson demand. The lifetime was finite and deterministic. They compared two issuing policies: FIFO and LIFO. They provided a sensitivity analysis of the optimal solution for both policies and observed the maximum expected profit. They concluded LIFO model is dominated FIFO model for all possible combinations of model parameters. However, the most widely used policy in the literature is the FIFO policy.

Another field that is studying widely in the perishable inventory systems is review policies. Periodic and continuous reviews are two common methods used to control inventory level for calculating the order time. Most of studies assume that inventory is review continuously. The periodic review system has an advantage in that, it is possible to detect the perished items and discard them immediately. Since solutions of periodic review models is not much studied, even for the systems that are not very complex, but continuous review models are more common in recent studies. Also, calculate the optimal ordering policy for both review policies becomes more difficult when lead time is positive. Nahmias (1982) have studied early studies on perishable inventory systems with periodic review extensively. Nahmias (1982) classified both perishable inventory deterioration with fixed and random lifetime. In addition, he provide a review on ordering policies of the perishable inventory. He considered both deterministic and stochastic demand for fixed and random lifetime of the single and multiple products. Later, Rafaat (1991) presented a comprehensive of inventory literature for the deteriorating inventory models where deterioration was consider as a function of the on-hand level of inventory. But he did not touch on the effect of constant decay into the variety of existing inventory models. Furthermore, deteriorating inventory models need to be developed to consider the effects of quantity discount and multiple-item stocking. Goyal & Giri (2001) studied the perishable inventoy models under these conditions. Bakker et al. (2012) presented a review of inventory system of perishable items. This study contained the literature review since Goyal & Giri (2001). In addition, Deniz (2007) presented a thesis about number of challenges in perishable inventory management in supply chains, providing both theoretical results and managerial insights.

The perishable inventory with periodic review policy can be classify into the following categories. Lifetime distribution (fixed or random), demand distribution (deterministic or stochastic) and status of the lead time (zero, positive).

2.4.1 Periodic Review Policy with Fixed Lifetime

Periodic review models fit the fixed lifetime well. If the lifetime is exactly one period, the ordering decisions in successive periods are independent and problem reduces to a sequence of simple newsboy problem (Arrow, 1958). Nahmias (1982) showed that when the lifetime is one period and there is no backlogging, the optimal policy is order up to S. When backlogging is allowed, the optimal policy is base-stock policy, as shown by Bulinskaya (1964). Olsson & Tydesjö (2010) studied periodic review problem with Poisson demand, fixed lifetime and fixed lead time. In this model backlog was allowed and replenishment policy was assumed to be an order-up-to S-policy. They considered different cases and results were compare to (s, Q) policies. The Order-up-to-S policy have been shown to have good performance compared to optimal ordering policies as Nahmias (1976), Nandakumar & Morton (1993) studies. Patil & Mishra (2017) studied an inventory model for perishable products with periodic demand and fixed lifetime. Their model considers both the cases one allowing with shortages and the other without shortages. When lifetime exceeds one period and demand is random, determining the optimal ordering policy is challenging. Van Zyl (1964) investigated a periodic review problem of a product having exactly two period lifetimes with stochastic demand and zero lead time. Van Zyl (1964) described the optimal ordering policy as order-up-to and optimal issuing policy as FIFO. Also, Van Zyl (1964) minimized expected costs contain holding costs, shortage costs and outdated cost. Nahmias & Pierskalla (1973) extended Van Zyl's model and considered the problem of computing optimal ordering policies for lifetime exactly two periods when demand is random and shortage issue allowed. Nahmias & Pierskalla (1973) minimized the expected shortage cost and ordered an infinite quantity. Williams & Patuwo (1999) studied the same model with positive lead time, lost sale policy, and computed optimal order quantities that minimize the total inventory cost for different demand distribution. Kouki & Jouini (2015) studied a periodic review (T, s, Q) policy with Erlang lifetime, Poisson demand fixed lifetime.

2.4.2 Periodic Review Policy with n Period Lifetimes

When lifetime is greater than two periods, Nahmias (1975) generalized Nahmias & Pierskalla (1973) model and computed the optimal ordering policies with lifetime exactly n periods with backlogging. The dynamic programming approach was used to extend the model to the multi-period case and showed that the base-stock policy was optimal. Nandakumar & Morton (1993) analyzed the same model with lost sale and derived the myopic upper and lower bounds on the order quantities for base-stock policy. Broekmeulen & van Dorselaar (2009) extended the same model with fixed lead time. They suggested different replenishment policies and used simulation model to compare them. In addition, van Dorselaar & Broekmeulen (2009) focused on periodic review (s, nQ) policy and proposed a combining stochastic modelling, simulation and regression. Minner & Transchel (2010) analyzed Nahmias (1975) model with positive lead time and presented a numerical approach to dynamically determine replenishment quantities for perishable items with limited lifetime, positive lead time, under FIFO and LIFO issuing policies and with multiple service level constraints. They showed that a constant order policy might provide good results under stationary demand, short lifetime and LIFO inventory depletion.

2.4.3 Periodic Review Policy with Random Lifetime

When the exact lifetime of the perishable product is not known, the random lifetime is assumed. Friedman & Hoch (1978) used periodic review policy with random lifetime and deterministic demand. Approximate solutions for random lifetime have been proposed by Cohen (1976) and Nahmias (1975, 1976). Kouki et al. (2010) considered periodic review policy with Poisson demand, positive lead time and lost sale policy. In addition, they considered exponential lifetime like Friedman & Hoch (1978). In Kouki et al. (2014) a periodic review (T, S) policy is proposed under assumption with Poisson demand, exponential lifetime and constant lead time. This model has Markov properties.

Table 2.1 shows the literature of perishable inventory system with periodic review. The studies are listed with respect to their lifetime, demand and lead time distribution.

Table 2.1 The literature of perishable inventory with periodic review policy

Article	Lifetime	Demand	Lead time	Excess
Veinott (1960)	Fixed	Deterministic	0	Lost sale
Bulinskaya (1964)	Fixed	Deterministic	Fixed	Lost sale
Olsson & Tydesjö (2010)	Fixed	Poisson	Fixed	Lost sale
van Dorselaar & Broekmeulen (2012)	Fixed	Random Demand	Fixed	Lost sale
Patil & Mishra (2017)	Fixed	Deterministic	0	Lost sale/ Backorder

Table 2.1 The literature of perishable inventory with periodic review policy

Van Zyl (1964)	Fixed (2 periods)	Random Demand	0	Lost sale
Nahmias & Pierskalla (1973)	Fixed (2 periods)	Random demand	0	Backorder
Williams & Patuwo (1999)	Fixed (2 periods)	Random demand	Fixed	Lost sale
Williams & Patuwo (2004)	Fixed (2 periods)	Random demand	0	Lost sale
Nahmias (1975, 1977)	Fixed (n periods)	Random demand	0	Backorder
Nandakumar & Morton (1993)	Fixed (n periods)	Random demand	0	Lost sale
Broekmeulen & van Dorselaar (2009)	Fixed (n periods)	Random demand	Fixed	Lost sale
Minner & Transchel (2010)	Fixed (n periods)	Random demand	Fixed	Lost sale
Friedman & Hoch (1978)	Random	Known demand	0	Lost sale
Kouki, Sahin, Jemai & Dallery (2010)	Exponential	Poisson demand	Fixed	Lost sale
Kouki, Jouini, Jemai & Dallery (2014)	Exponential	Poisson demand	Fixed	Lost sale/ Backorder
Kouki & Jouini (2015)	Erlang	Poisson demand	Fixed	Lost sale

Continuous review is the most popular and applicable inventory system in perishable inventory literature and can be classify into the following categories, too.

Lifetime distribution (fixed or random), demand distribution (deterministic or stochastic) and status of the lead time (zero, positive).

2.4.4 (s, S) Continuous Review Policy with Fixed Lifetime

Sivazlian (1974) studied a (*s, S*) continuous review system where demand is defined by a discrete-valued process with Poisson distribution. The lifetime was assumed as fixed and the lead time was zero. Weiss (1980) extended Sivazlian (1974) model for an exact n period lifetime. Weiss (1980) treated the optimal policies for perishable items with continuous review. Demands are assumed to occur according to Poisson process and the lead time is zero. The ordering policy, which minimizes the expected total costs, is determined. Another fixed lifetime research belongs to Lian & Liu (1999). They studied a continuous review (*s, S*) model with a fixed lifetime and random demand distribution. They used Laplace-Stieltjes transforms and analyzed the structure of the cost function with the random batch size. We have not found any study about (*s, S*) continuous review policy with fixed lifetime and positive lead time yet. Tekin et al. (2001) introduced a (*Q, s, T*) policy and analyzed the impact of lot size-reorder control policy for perishable items with fixed lifetime. They considered an inventory system with Poisson demand and all unmet demand were lost with positive lead time. They compared the performance of suggested policy with the classical (*s, Q*) policy. They concluded that the (*Q, s, T*) policy performed better than the (*s, Q*) policy according to the properties of the cost function, especially in cases where outdating was an issue. Chiu (1995a) proposed an approximate (*s, Q*) policy with full backorders, fixed lifetime, random demand distribution and deterministic lead time. A similar model has also been studied by Kouki et al. (2013). They investigated the impact of perishability on the total cost by comparing different policies. Berk & Gürler (2008) analyzed the optimal total cost of the (*s, Q*) policy with lost sales and positive lead time. Kouki et al. (2015) consider a perishable inventory system under stochastic demand, constant lifetime and constant lead time. The systems employs a (*s, Q*) continuous review policy.

2.4.5 (s, S) Continuous Review Policy with Random Lifetime

The literature is extensively wide about the random lifetime case. Ghare & Schrader (1963) generalized the standard EOQ model with perishability and deterministic demand. The complexity of the perishable items having stochastic demand depends on the lead time. When the lead time is zero, the model is straightforward respectively. The positive lead time assumption makes the system more complicated. Because of this complexity, the lead time assumed zero in most studies. Ravichandran (1995) analyzed continuous review model with Poisson demand model, random lifetime and positive and constant lead time. Lian & Liu (1999) studied with random demand but this method is rather complicated for the models with batch demand. So Lian & Liu (2001) used embedded Markov chain methods for the same model. Also, they studied the effect of the demand rate and the lifetime. They provided a heuristic for the positive lead time and the main analytical result hold only for some special cases such as Poisson arrivals with exponential inter arrival times. Gürler & Özkaya (2003) generalized inter demand times of Lian & Liu (2001) for the case where the arrivals follow an arbitrary renewal process. Gürler & Özkaya (2008) assumed a random shelf-life and allowing backorders for the Lian & Liu (1999) and Lian & Liu (2001) models. Also they extensively investigate the impact of the shelf-life distributions and show that the expected cost rate function is quasi-convex in (*s*, *S*) for unit demand. Gürler & Özkaya (2008) also provide a heuristic and the heuristic they proposed performs as well as Lian & Liu (2001). They represented the expected cost function for model using which can be evaluated only using a computer numerical method. Baron et al. (2010) analyzed the same model and derived closed form expressions for the expected costs. Lian & Liu (1999) consider a discrete-time (*s*, *S*) model for perishable items. This model has a discrete distribution random lifetime and PH-renewal process with batch demand. The lead time was zero and shortages were permitted. Lian et al. (2005) analyzed the same model and used Markov chain to obtain the cost function and compared the result with the special case in which the fixed lifetime.

2.4.6 (s, S) Continuous Review Policy with Exponential Lifetime

Kalpakam & Arivarignan (1988) studied a continuous review (s, S) model with Poisson demand with zero lead time and an exponential lifetime. By assuming no backorders and instantaneous delivery of orders, the steady state probability distribution of the stock level and mean time between successive reorders are derived. Liu (1990) allows backorders for the same model but used an alternative approach which gives the stationary probability distribution of the stock level and suggested that the reorder point s would be less than one. Kalpakam & Sapna (1994) extended Kalpakam & Arivarignan (1988) model with the exponential lead time. They investigate a lost-sales (s, S) system with exponential lifetimes. Liu & Shi (1999) reconsidered the model of Liu (1990) with the renewal demand through a Markov process. They assumed that the demands are from renewal process with inter-demand distribution function. Zero lead time and exponential lifetime. Lian et al. (2009) studied an (s, S) continuous review model for items with exponential lifetime and a general Markovian renewal demand process. They assumed zero lead time. By constructing Markovian renewal equations they compared the numerical results obtained by Markovian renewal process to the ones obtained by renewal process. Kalpakam & Shanthi (2006) analyzed the same model under renewal demand and exponential lead time. They formulated the system using semi-regenerative process which applied to obtain the various operating characteristics. Sivakumar (2009) considered a continuous review perishable inventory system with a finite number of homogeneous of demand (retrial demand). The lifetime of items was assumed exponential with exponential lead time. The model is analyzed within the framework of Markov processes. Ravichandran (1988) analyzed a similar continuous review model. Ravichandran (1988) assumed Erlang distribution lifetime instead of exponential distribution with Markovian demand and random lead time. Baron et al. (2017) considered a continuous review (s, S) model of perishable items with lost sales. Once items are perished the entire inventory drops instantaneously to zero. Both the lifetime and the lead times are assumed to be exponentially distributed while two cases of demand distribution are considered: Poisson and compound Poisson with general demand sizes.

Table 2.2 summarize the literature of perishable inventory system with (s, S) and (Q, s, T) continuous review policy. The studies are listed with respect to their lifetime, demand and lead time distribution.

Table 2.2 The literature of perishable inventory with (s,S) and (Q,s,T) continuous review policy

Article	Lifetime	Demand	Lead time	Excess
Sivazlian (1974)	Fixed	Poisson	0	Lost sale
Lian & Liu (1999)	Fixed	Random	0	Backorder
Chiu (1995a)	Fixed	Random	Fixed	Lost sale/ Backorder
Berk & Gürler (2008)	Fixed	Poisson	Fixed	Lost sale
Kouki, Sahin, Jamei & Dallery (2013)	Fixed	Random	Fixed	Backorder
Kouki, Jemai & Minner (2015)	Fixed	Random	Fixed	Lost sale
Lian & Liu (2001)	Random	Batch Demand	0	Lost sale
Gürler & Özkaya (2003)	Random	Batch Demand	Exponential	Lost sale
Gürler & Özkaya (2008)	Random	Batch demand	0	Backorder
Baron (2010)	Random	Batch demand	0	Lost sale
Lian & Liu (1999)	Random	Batch demand	0	Backorder
Lian, Neuts & Liu (2005)	Random	Batch demand	0	Backorder
Ghare & Schrader (1963)	Exponential	Deterministic	0	Lost sale
Kalpakam & Arivarignan (1988)	Exponential	Poisson	0	Lost sale
Liu (1990)	Exponential	Poisson	0	Backorder

Table 2.2 The literature of perishable inventory with (s,S) and (Q,s,T) continuous review policy

Kalpakam & Sapna (1994)	Exponential	Poisson	Exponential	Lost sale
Liu & Shi (1999)	Exponential	Renewal Demand	0	Lost sale
Lian, Liu & Zhao (2009)	Exponential	Renewal Demand	Random	Backorder
Kalpakam & Shanthi (2006)	Exponential	Renewal Demand	Exponential	Lost sale
Sivakumar (2009)	Exponential	Retrial demand	Exponential	Backorder
Baron, Berman & Perry (2017)	Exponential	Poisson/ Compound Poisson	Exponential	Lost sale
Ravichandran (1988)	Erlang	Markovian demand	Random	Lost sale

2.4.7 (s, S) Continuous Review Policy with Exponential Lifetime and MAP

Recent studies consider an (s, S) perishable inventory model in which demands for items arrive according to a Markovian Arrival Process (MAP). Chakravarthy & Daniel (2004) consider an (s, S) perishable inventory model with *MAP* demands, exponential lifetimes, phase type lead times, and a provision for finite backlogging was studied. Chakravarthy (2010) considered a version of (s, S) inventory system with MAP demand with exponential lead time. The lifetime of items were assumed to be phase type. Any unmet demand was considered lost. Yadavalli et al. (2007) considered a continuous review perishable inventory system at a service facility with a finite waiting capacity. The customers were assumed to arrive according to a Markov arrival process. The lifetime of each item and the service time were assumed to have independent exponential distributions. Yadavalli et al. (2011) considered multi-server inventory system. They assumed the negative customers arrived according to an independent Markovian arrival process (MAP) with exponential lead time. Yadavalli et al. (2012) studied a continuous review retrial inventory system with finite source of customers. The customers arrived according to the quasi-random process. The service time, lifetime and lead time was distributed exponentially. Sivakumar & Arivarignan (2005) considered an inventory system with service facility and negative customers. They assumed the model with MAP demand, exponential lifetime and lead time. Manuel et al (2007) assumed that the customers arrive according to MAP. The lifetime of each item was exponential and lead time was positive and fixed. Manuel et al (2008) extended this model with exponential lead time and service time was assumed to be of phase type in this model.

2.4.8 (S-1, S) Continuous Review Policy with Fixed Lifetime

In the literature on (*S-1, S*) perishable systems, almost all the models deal with Poisson demand like Perry & Posner (1998) and Kalpakam & Sapna (1995). Perry & Posner (1998) studied the simple model with fixed lifetime and Poisson demand. The order lead time was constant. Kalpakam & Sapna (1995) studied a (*S-1, S*) continuous review system with exponential lifetime and a general lead time distribution. They used Markov renewal technique to analyze the level of inventory. Fries (1975) studied perishable inventory model with two period lifetimes and proved that the optimal ordering policy was (*S-1, S*). Liu & Cheung (1997) consideration of different lead times, partial backorders, lost sales for the same model with Kalpakam & Sapna (1995). Another Poisson demand article is Kalpakam & Shanthi (2005). They analyzed a lost sale (*S-1, S*) policy under exponential lifetime. They used the Markov renewal techniques to obtain the characteristics. In addition, Schultz (1990) and Schmidt & Nahmias (1985) showed in their paper (*S-1, S*) is optimal policy.

2.4.9 (S-1, S) Continuous Review Policy with Random Lifetime

The first paper that deals with (*S-1, S*) perishable system under renewal demands is due to Kalpakam & Sapna (1996). They used the Kalpakam & Sapna (1995) technique presented a lost sale (*S-1, S*) model with the renewal demand and the exponential lead time. Kalpakam & Shanthi (2005) reconsidered this model. They used the semi-regenerative techniques process to obtain the steady state operating characteristics. In Table 2.3 shows the literature of perishable inventory system with (*S-1, S*) continuous review policy. Perry & Posner (1998) studied fixed lifetime, Poisson demand and fixed lead time (*S-1*, *S*) policy. Chiu (1995b) studied the backlogged case for the fixed lifetime and random demand. The studies listed with respect to their lifetime, demand and lead time distribution.

Table 2.3 The literature of perishable inventory with (*S-1*, *S*) continuous review policy

Article	Lifetime	Demand	Lead time	Excess
Fries (1975)	Fixed	Continuous	0	Lost sale
Perry & Posner (1998)	Fixed	Poisson	Fixed	Lost sale
Schmidt & Nahmias (1985)	Fixed	Poisson	Positive	Lost sale
Chiu (1995b)	Fixed	Random	Fixed	Backorder
Kalpakam & Sapna (1995)	Exponential	Poisson	Random	Lost sale
Liu & Cheung (1997)	Exponential	Poisson	Random	Lost sale
Kalpakam & Shanthi (2001)	Exponential	Poisson	Separate Unit Lead time	Lost sale
Kalpakam & Sapna (1996)	Exponential	Renewal demand	Exponential	Lost sale
Kalpakam & Shanthi (2005)	Exponential	Renewal demand	Exponential	Lost sale

2.4.10 Two Demand Classes

In the literature, there are studies have two demand classes, but there is no deterioration in these studies. Veinott (1965) was the first to consider the several demand classes problem in inventory systems. He analyzed a periodic review with multiple demands. The model had no lead time and studied about critical level policy. Kleijn & Dekker (1999) provided an overview a comprehensive literature on inventory systems with several demand classes. They focused on single-location inventory system and critical level policy. Nahmias & Demmy (1981) studied a rationing policy with two demand classes modelling with Poisson distribution. They analyzed (*s*, *Q*) policy inventory system with positive lead time and backorders. Moon & Kang (1998) generalized the results of Nahmias & Demmy (1981) to include compound Poisson demand. Melchiors et al. (2000) analyzed the same two demand and continuous review (*s*, *Q*) model with lost sales. However, they did not show the convexity of the cost function. Isotupa (2006) studied a lost sales (*s*, *Q*) inventory system with two types of customers (ordinary and priority customers), exponentially distributed lead time. The long-run expected cost rate is derived and derived an algorithm calculating the optimal values for reorder level and reorder quantity is developed. Cohen et al. (1988) added the different lead time lengths to an (*s*, *S*) policy with two demand classes inventory model. Dekker et al. (2002) studied an inventory model with several demand classes with (*S-1*, *S*) inventory model with lost sales. For each demand class there is a critical stock level at and below which demand from that class is not satisfied from stock on hand. In this way, stock is retained to meet demand from higher priority demand classes. They constructed the cost function to derive the optimal stock level yet did not show the convexity. Sivakumar & Arivarignan (2005) analyzed an (*s*, *Q*) inventory system with Markovian arrival process and exponentially distributed lead times for the case of items which are perishable and have an exponentially distributed lifetime. Liu et.al (2014) studied a Markovian inventory system with two class customers. They examined a system with the demand of ordinary customers is met with probability *p*.

2.4.11 Blood Bank Studies

The blood products are the most studied and demanded area of the perishable products in recent years. For this reason, a literature search for blood products is also included in this section. Blood is a perishable commodity with limited lifetime. Its only source is the healthy human being so good management of blood inventories is vital. Blood can be safely stored 45 days, after while it is too old and discard. The management of blood is very important for human race. Stanger et al. (2012) described a blood supply chain very comprehensible. According to Chapman (2007) and Spens

& Bask (2002) this chain has four elements: donors, blood centers, the hospitals and the patients. Blood is collected from donors in donation centers or mobile units and transport to the blood centers. Then whole blood tested for contagious viruses, determine the blood type and separate into components (red blood cells, platelets and plasma). The different components have different lifetimes. After processing, the components are stored in centers for needs of hospitals. Every hospital delivers the blood from related blood center or sometimes supplies it from each other. Hospitals supply the blood for their patients. The supply of donor blood is irregular and the demand of blood is stochastic. Inventory shortage of blood can cause high cost and mortality rates. The excessive demand is also a waste of blood. The studies about perishable problems with MDP modeling have increased recent years. Belien & Forcé (2012) illustrate the distribution of the blood papers according to publication date (Figure 2.4) in their literature review.

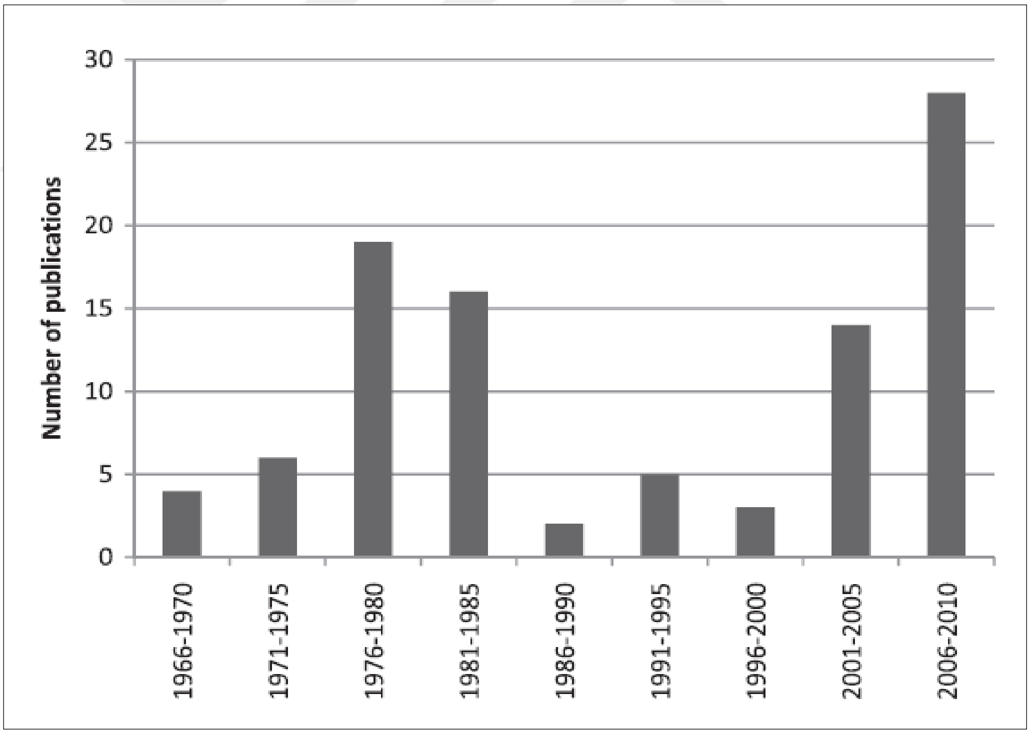

Figure 2.4 Number of publications by publication period (Belien&Forcé, 2012)

Stanger et al. (2012) review the blood inventory literature goes to back 1960s. In 1970s the major period activity was regression models were first developed to identify optimal inventory levels and order quantity. Jennings (1973) identified the three measures of performance (shortage, wastage/outdating, the cost of transportation). Brodheim et al. (1975) developed an inventory model based on the average age and wastage of blood units using a Markov chain approach. Cumming et al. (1976) developed a planning model for issuing units to hospitals. Brodheim & Prastacos (1979) published a prototype computer based regional distribution model called Programmed Blood Distribution System (PBDS). Cohen & Pierskalla (1979) improved target stock levels for hospital blood banks. Prastacos (1984) reviewed the literature looking at various models from an operations research point of view. Stanger et al. (2012) identify the major research area is which operations research techniques and simulation dominated the others in 2000s. Owens et al. (2001) analyzed the impact of the average age of varied blood groups units on the inventory performance and conclude that an extension of shelf life had reductions in wastage. The establishment of the Blood Stocks Management Scheme (BSMS) in the UK and the instigation of a large database monitoring stock levels and wastage rates in hospitals and blood centers have opened new possibilities in blood inventory management research. The availability of this new data has led to greatly improved transparency and consequently an increased understanding of blood inventory management and improved visibility of the blood supply chain (Chapman & Cook, 2002). In the first simulation paper, Rytila & Spens (2006) used discrete event simulation models to investigate the impact of changes to hospital blood bank procedures on wastage and shortage. Simulation has also been applied in studies of blood supply chain ordering policies with the objective of reducing wastage and shortages by increasing service levels (Katsaliaki & Brailsford, 2007; Katsaliaki, 2008). Kopach et al. (2008) determine an optimal policy for a blood inventory management system, applying level-crossing techniques in addition to simulation, demand rates, cost and shortage and service levels. Perera et al. (2009), is based on a survey of 265 UK hospitals to identify wastage levels, and comparing hospitals.

An optimal ordering policy in blood bank inventory papers can be formulated by MDP. The latest papers about the modeling the blood products with MDP belongs to Rene Haijema. He studied about blood platelet problems as perishable items and solved this problem with different ways, like Markov Decision Process, Stochastic

Dynamic Process and proposed a new policy (s,S,q,Q). Haijema et al. (2007) studied blood platelets which are highly perishable, limited lifetime. Under the periodic production and order-up-to replenishment policy, he split the demand age into two groups: Young platelets and any age platelets. He used the MDP and simulation approach for modeling the problem and also presented an application with a real life data of a Dutch blood bank. He derived a double-order-up-to rule (2D) with one level corresponding to young platelets and one to the total inventory. The simulation shown that 2D rule is nearly optimal. In his academic dissertation, Haijema (2008) described the perishable products, blood platelets problems. He modeled these problems with MDP, Stochastic Dynamic Programming (SDP)-simulation approach and extended SDP-simulation approach. Haijema et al. (2009) extended the combined SDP-simulation approach for the same data in Haijema et al. (2007). Vila-Parrish et al. (2008) studied the pharmaceutical items with impatient hospital pharmacy and modeled the patient type with the MDP. They considered two stages of inventory and used two-phased approach to explore policy structures. Ahiska & King (2010) investigate the remanufactured products. The system was defined by recoverable inventory level and serviceable inventory level which is modeled by MDP. Haijema (2013) focused on the perishable products with balancing outdating and shortages. It is assumed that the Demand is stochastic and periodic; the production lead time is 1 period and replenishments do not happen every day. This model includes the fixed ordering cost; as a result the optimal ordering policy is more complex. Hence, Haijema (2013) derived a new class of ordering policies (*s, S, q, Q)* and a new procedure to determine nearly optimal parameter values. The ordering problem is formulated by a MDP. The optimal policy can be approximated by a periodic (*s, S*) policy. The structure of the (*s, S*) policy suggests to restrict the order quantity by minimum q and maximum Q respectively. This new policy is called (*s, S, q, Q*) policy. Civelek et. al (2015) considered a discrete-time inventory system for a perishable product where demand exists for product of different ages. They modeled the problem as a Markov Decision Process and derived the costs of heuristic policy. Also, they introduced a critical-level policy for an age-differentiated perishable product. Haijema et al. (2017) illustrated how MDP or SDP can be used in practice for blood management at blood banks. An optimal policy is derived by SDP. The structure of optimal policies is investigated by simulation.

In Figure 2.5, a summary table is presented for the years and subjects in the perishable inventory literature which studied in this dissertation. We have been decided to contribute to the literature with present dissertation through this wide literature review.

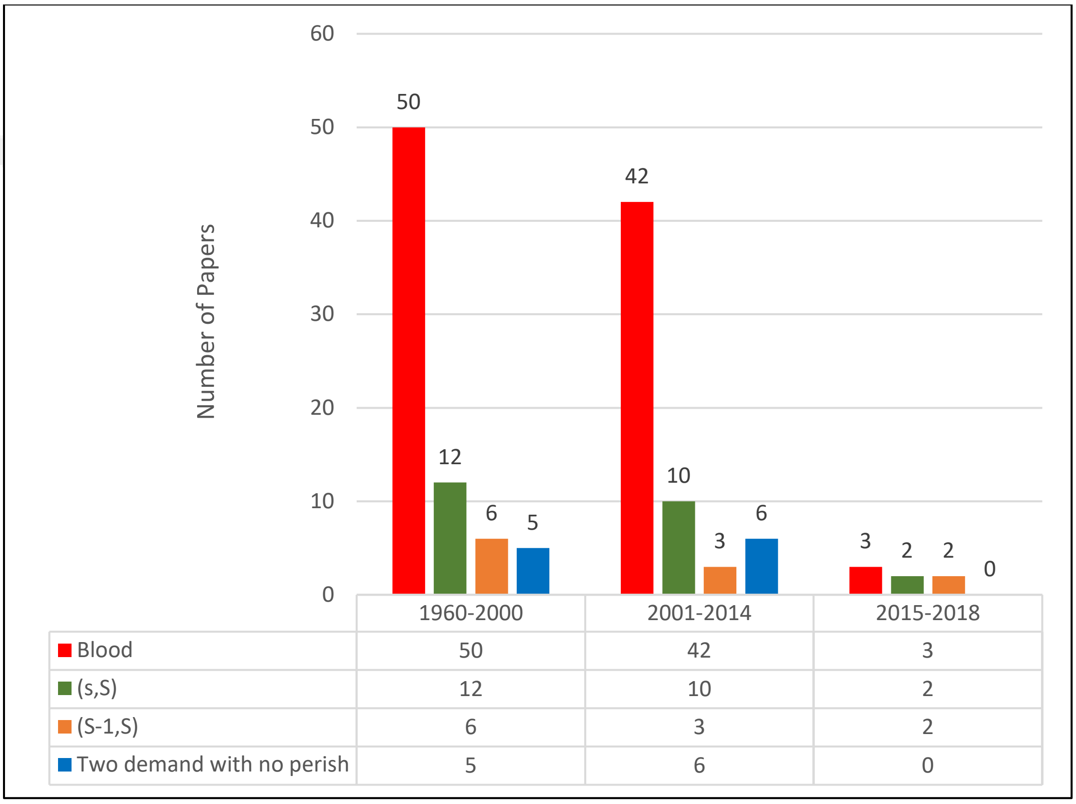

Figure 2.5 Number of papers according to years

CHAPTER THREE
STOCHASTIC PROCESSES

A stochastic process is a probabilistic process. Let {X(t), tϵT} is known as a stochastic process procedure characterized a group of irregular factors relying upon time or index parameter. X(t) represents the state of process at time *t.* The index set T may be discrete $T = \{0, 1, 2 \ldots\}$ or continuous T=[0,∞) . If T is a countable set, at that point the stochastic process is called discrete time process. If T is an interval of real line then the stochastic process is said to be a continuous time process. State space that includes the acknowledge of X(t), can also be classified as discrete and continuous.

The indexing parameter is discrete and denoted by *n*. $\{X_n, n=0,1,2\ldots\}$ represents the discrete time process. When index parameter is continuous it is indicated by *t* and {X(t), t≥0} represents the continuous time process. If for all choices $\mathrm{t}_1 < \mathrm{t}_2 < \cdots < \mathrm{t}_\mathrm{n}$ random variables $\mathrm{X}(\mathrm{t}_1)$- $\mathrm{X}(\mathrm{t}_0)$, $\mathrm{X}(\mathrm{t}_2)$- $\mathrm{X}(\mathrm{t}_1)$, … $\mathrm{X}(\mathrm{t}_\mathrm{n})$- $\mathrm{X}(\mathrm{t}_{\mathrm{n}-1})$ are independent and all $X_i = \mathrm{X}(\mathrm{t}_\mathrm{n})$- $\mathrm{X}(\mathrm{t}_{\mathrm{n}-1})$, $\mathrm{i} = 1, 2, \ldots$ has the same distribution, then a continuous time stochastic process is said to have independent and stationary increments.

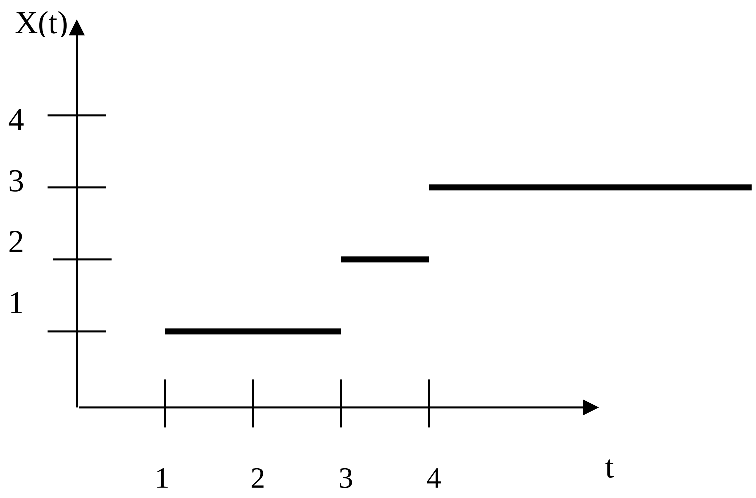

Figure 3.1 A sample path of X(t)

The possible classification of stochastic process is that the one according to the random variable characteristics. In this case, there are two major categories of stochastic processes: Arrival type processes and Markov Processes. For arrival type processes, if the arrivals occur in discrete time, it is called Bernoulli process. If arrivals occur in continuous time, this process is called Poisson process. The main feature of Markov process is the dependence on one-step before.

3.1 Arrival Type Processes

In arrival type processes, we interested in occurrences that have the character of an "arrival". These processes can be classified into two groups:

i. Bernoulli Process: Arrivals occur in discrete time and interarrival times are geometrically distributed: Bernoulli Process. Let {Xn} is a finite or infinite i.i.d random sequence of independent Bernoulli (p) random variables. All trials are independent and have the memorylessness property. The random variables in this process take only two values, 0 and 1. Let Zn be the number of success at time *n*. The probability of k success in *n* trials is given in Equation (3.1)

$$P\{Z_n = k\} = \binom{n}{k} p^k (1-p)^{n-k},\ k = 0, 1, 2 \dots n \quad (3.1)$$

where p is the probability of success. Several random variables may be derived from the Bernoulli process:

- The number of successes in first *n* trials, has a binomial distribution B(n, p).
- The number of trials needed to get *r* successes, has a negative binomial distribution NB(r, p).
- The number of trials needed to get the first success, has a geometric distribution Geo(p)

ii. Poisson Process: Arrivals occur in continuous time and interarrival times are exponentially distributed: Poisson Process. This process is widely explained in the Markov Process section because of the Markovian structure.

3.2 Markov Process

When constructing a stochastic model, there are two main concerns. Model must be analytically tamed and sufficiently flexible for mathematical modelling. A Markov processes frequently balance these two main concerns nicely. Consider a finite or infinite sequence of random variables $\{X_n, n \in T\}$, where T is a subset of the integers. For any $n \in T$, if the future process (X_m, m > n, m ∈ T) is independent of the past process (X_m, m < n, m ∈ T), this sequence has the Markov property or memorylessness, conditionally on X_n. If any process have the Markov property, then they are called the Markov process. Markov processes with discrete index set and countable or finite state space is called Markov Chain. Depending on the nature of the state space and the parameter space, we can divide Markov processes into four classes which shown as Table 3.1.

Table 3.1 Markov processes classified by parameter space and state space

Parameter Space	State Space	
	Discrete	Continuous
Discrete	Discrete parameter Markov Chain	Discrete parameter Markov Process
Continuous	Continuous parameter Markov Chain	Continuous parameter Markov Process

3.2.1 Discrete-time Markov Chain

A stochastic Markov process with discrete time and discrete parameter space is called Markov Chain. The state space *S* is referred to as finite or countable set. Let $\{X_n, n=0, 1, \dots\}$ be a Markov chain with S={0, 1, 2, …} or discussing a finite state space S= {1, 2, …m}. Let $X_n=i$ be the state of the process is *i* at time *n* and p_{ij}^n be the probability that the process is in state *j* after *n* transition, given that the process is currently in state *i*. When the process has discrete parameter and state space, we may defined the *n*-step transition probability as

$$p_{ij}^n = P\{X_n = j | X_0 = i_0\} \tag{3.2}$$

n-step refers to the time between observations. When n=1, the probability that when the process is in the state *i* it will next make a transition into state *j* is called one step transition probability and denoted as p_{ij}. For all states $i_0, i_1, \dots\ i_{n-1}, i, j\ and\ n \geq 0$

$$p_{ij} = P\{X_{n+1} = j | X_0 = i_0, \dots X_n = i\} = P\{X_{n+1} = j | X_n = i\} \qquad (3.3)$$

We also need a transition matrix to use one-step transition probabilities from state to state. Let P denote the matrix of one-step transition probabilities p_{ij}, so that

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots \\ p_{10} & p_{11} & p_{12} & \cdots \\ \vdots & & & \\ p_{n0} & p_{n1} & p_{n2} & \dots \end{bmatrix}$$

Since probabilities are nonnegative and since the process must make a transition into some state, we have following facts:

i. $p_{ij} \geq 0$, $i,j \geq 0$
ii. $\sum_{j=0}^{\infty} p_{ij} = 1,\ i = 0,1 \dots$

At the beginning, the probability that the system is in state *i* is defined as initial probability $P\{X_0=i\}=q_i$ and represented as a vector $q=(q_0, q_1, \dots, q_n)$.

Let *P* be the transition matrix and *q* be the initial probabilities of a Markov chain. We can find the matrix powers of P as P^2=P.P. In other words, with help of Chapman-Kolmogorov equations two-step transition probabilities denoted as follow

$$p_{ij}^2 = \sum_k p_{ik} p_{kj} \qquad (3.4)$$

Chapman-Kolmogorov equations enable us to build relationship for the transition probabilities between any points in T. These equations provide a method for computing these *n*-step transition probabilities. These equations are expressed in its most general form as follows.

$$p_{ij}^{n+m} = \sum_{k=0}^{\infty} p_{ik}^n\, p_{kj}^m \qquad (3.5)$$

This equations express the fact that a transition from i to j in $n+m$ steps can be achieved by moving from i to an intermediate state k in n steps with probability p_{ik}^{n}. Then given that the process is in state k, the Markov property allows indifference to the arrival route, a transition from k to j in m-steps must be achieved with probability p_{kj}^{m}. (Resnick, 2005, p.74).

3.2.1.1 Classification of States

States of Markov chain need to classify according to basic properties. Let $\{X_n, n=0,1,2\ldots\}$ be a finite homogenous Markov chain with state space S. According to Bhat & Miller (2002) the definitions are given as follows;

i. Accessible: State j is said to be accessible from state i if j can be reached from i in a finite number of steps. If two states i and j are accessible to each other, then they are said to communicate. Probabilistically this definitions imply:
   $i \rightarrow j$ (j accessible from i)
   $j \rightarrow i$ (i accessible from j)
ii. Communicate: If two states i and j are accessible to each other, then they are said to communicate.
   $i \leftrightarrow j$ (i and j communicate)
   $j \leftrightarrow i$ (i and j communicate)

If all states are communicating in a class, these states are called as equivalence class. And equivalence relation satisfies;

- ($i \leftrightarrow j$)
- If ($i \leftrightarrow j$), then ($j \leftrightarrow i$)
- If ($i \leftrightarrow j$) and ($j \leftrightarrow k$), then ($i \leftrightarrow k$)

iii. Irreducible: If a Markov chain has all its states belonging to one equivalence class, it is said to be irreducible. There is only one equivalence class and both states are communicate.

The following definitions identify the states with respect to their internal nature:

Let

$$f_{ii}^{n} = P\{\{X_n = i, X_r \neq i | X_0 = i\} \tag{3.6}$$

and

$$f_{ii}^{*} = \sum_{n=1}^{\infty} f_{ii}^{n} \tag{3.7}$$

In other words, f_{ii}^{n} is the probability that starting from *i*, the process returns to *i* for the first time in *n* steps and f_{ii}^{*} is the probability that starting from *i* the return to the initial state occurs in finite time.

iv. Recurrent States: A state *i* is recurrent if and only if, starting from state *i*, eventual return to this state is certain. $f_{ii}^{*}=1$, if and only if state *i* is recurrent.

v. Transient States: A state *i* is called transient when the process starting from state *i* and if it is not possible to return to same state. This implies that $f_{ii}^{*}<1$.

vi. Absorbing States: A state *i* is called absorbing if and only if $P_{ii}=1$.

vii. Periodic state: The period of state *i* is defined as the greatest common divisor of all integers $n \geq 1$, for which $P_{ii}^{n}>0$, for $n \geq 1$. When the period is 1, the state *i* is called aperiodic.

viii. Ergodic Markov Chain: A state *i* is said to be ergodic if all states of the chain is aperiodic and positive recurrent. Ergodic Markov chains are also called irreducible.

3.2.1.2 Canonical representation of the transition probability matrix

Classification of states is the first step for analysis a Markov chain. The classes and the states can be arranged in such a manner is known as the canonical representation. This is done by placing recurrent classes before transient classes. Let $C_1, C_2, \dots C_k$ be the equivalence classes with recurrent states, $C_{k+1}, C_{k+2}, \dots C_n$ be the equivalence classes with transient states, and $P_1, P_2, \dots P_k$ and $Q_{k+1}, Q_{k+2}, \dots Q_n$ be the corresponding transition probability submatrices within equivalence classes. Also let $[R_s] = [R_{s1}, R_{s2}, \dots R_{s,s-1}]$ be the submatrix of transition probabilities *s*th equivalence class. With these definitions the transition matrix P is given by

$$P = \begin{bmatrix} P & 0 \\ R & Q \end{bmatrix}$$

3.2.2 Continuous-time Markov Chain

A continuous time Markov chain is a Markov process with discrete state space and continuous parameter space. In this process the time expressed by exponential distribution. Let $\{X(t), t \geq 0\}$ be a Markov process with countable state space. Then the process $\{X(t), t \geq 0\}$ satisfies the Markov property which can be expressed as following for all j and $t_0 \leq t$:

$$\mathrm{P}\{\mathrm{X}(\mathrm{t}) = \mathrm{j} | \mathrm{X}(\mathrm{s}), \mathrm{X}(\mathrm{t}_0) = \mathrm{i}, \mathrm{s} < \mathrm{t}_0\} = \mathrm{P}\{\mathrm{X}(\mathrm{t}) = \mathrm{j} | \mathrm{X}(\mathrm{t}_0) = \mathrm{i}\} \tag{3.8}$$

Let τ_i be the amount of time spent in state *i*. Each time the process enters state *i*, it spends the *s* amount of time in state *i* before making a transition to a different state *j* in a time period *t*,

$$P\{\tau_i > s + t \,| \tau_i > s\} = P\{\tau_i > t\} \tag{3.9}$$

The probability that a Markov chain currently in state *i*, will be in state *j* after an additional time *t*. That is for all *s*,

$$\mathrm{p_{ij}}(\mathrm{t}) = \mathrm{P}\{\mathrm{X}(\mathrm{s}+\mathrm{t}) = \mathrm{j} | \mathrm{X}(\mathrm{s}) = \mathrm{i}\} \tag{3.10}$$

The transition probabilities provide the following properties:

i. $p_{ij}(t) \geq 0$, $t>0$

ii. $\sum_{j=0}^{\infty} p_{ij}(t) = 1$, $t>0$

iii. $p_{ij}(s+t) = \sum_k p_{ik}(t) p_{kj}(s)$, $t,s>0$

iv. $p_{ij}(t)$ is continuous, $t>0$

v. $\lim_{t \to 0} p_{ij}(t) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

Since the process has the continuous parameter *t*, we define a generator matrix with transition rates λ_{ii} and λ_{ij}

$$p'_{ij}(0) = \lambda_{ij}$$
$$p'_{ii}(0) = -\lambda_{ii}$$

$$A = \begin{bmatrix} -\lambda_{00} & \lambda_{01} & \lambda_{02} & \cdots \\ \lambda_{10} & -\lambda_{11} & \lambda_{12} & \cdots \\ \lambda_{20} & \lambda_{21} & -\lambda_{22} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

For all i, j the derivative $p'_{ij}(t)$ exists and is continuous in t, and satisfy the Kolmogorov differential equations. As backward equation

$$p'_{ij}(t) = -\lambda_{ii}p_{ij}(t) + \sum_{k\neq i}\lambda_{ik}p_{kj}(t) \qquad (3.11)$$

and forward equation

$$p'_{ij}(t) = -\lambda_{jj}p_{ij}(t) + \sum_{k\neq j}\lambda_{kj}p_{ik}(t) \qquad (3.12)$$

The Poisson process is an excellent example of a Markov Process.

3.3 Poisson Process

Poisson process is one of the most important process used in stochastic models for modeling the random points in time, such as arrival times of customers at a system and the positions of flaws in a piece of material. Several important probability distributions originated from the Poisson process such as Poisson distribution, exponential distribution, and gamma distribution.

Poisson process is a counting process in which the interarrival times Z_1, Z_2, … are i.i.d. random sequence. A counting process {N(t), t≥0} is said to be a Poisson process having rate λ, (λ>0), if

i. N(0)=0
ii. The process has independent increments property. The numbers of occurrences in non-overlapping time intervals (t_0, t_1], (t'_0, t'_1,] are independent from each other and the number of arrivals in each interval; N(t_1)- N(t_0) and N(t'_1,)- N(t'_0,) are independent random variables .

iii. The process has the stationary increments property. The number of arrivals in any interval of t has Poisson distribution with mean λt.

$$P\{N(t+s)-N(s)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}, \quad n=0,1,2\ldots \tag{3.13}$$

From (iii), it follows E[N(t)]= λt = V(N(t)).

A system based on the Poisson process assumes exponential distribution of time between arrivals. The time intervals Z_1, Z_2 … are independent and identical. Exponential distribution has the Markovian property. Then mathematical representation of the memorylessness property can be represented by

$$\mathrm{P}\{\mathrm{Z_n} > \mathrm{t}+\mathrm{s}|\mathrm{Z_n} > \mathrm{t}\} = \mathrm{P}\{\mathrm{Z_n} > \mathrm{s}\},\ \mathrm{s},\mathrm{t} > 0 \tag{3.14}$$

3.3.1 Birth and Death Process

In the previous section we considered the Poisson process and describe the arrivals of service requests in many cases of practical interest. But, an important subclass of Markov chains with continuous time parameter space which embedded discrete time Markov chain is birth and death processes. These processes are characterized by the property that if a transition occurs, then this transition leads to a neighboring state. The population size is always increase in pure birth process always and always decrease in pure death process. But in real life both increase and decrease occur in the same time. The birth and death process is frequently used as a mathematical model of Queueing systems and inventory system. A useful visualization of the birth and death process is given in Figure 3.2. Let X_n be the number of the system at time t. The number of circles indicates the X_n. λ_n indicates the birth rate and μ_n indicates the death rate. The state diagram allows only the nearest neighbor transition and only the birth transition is allowed from state zero.

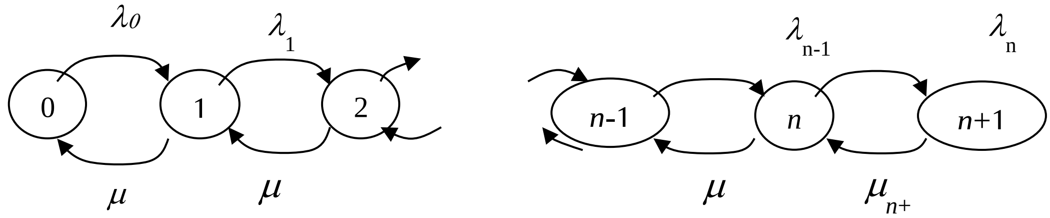

Figure 3.2 The birth and death process

If a birth and death process is in state j at time t, then the following laws of motion conduct the motion of the process:

Law 1. A birth occurs between t and $t+\triangle t$ with probability $\lambda_j \triangle t + o(\triangle t)$

Law 2. A death occurs between t and $t+\triangle t$ with probability $\mu_j \triangle t + o(\triangle t)$

Law 3. Births and deaths are independent of each other.

To compute the transition probability at time $t+\triangle t$ there are four ways. From state j to state j the probability equals to $p_{ij}(t)(-\mu_j - \lambda_j)$; from state $j-1$ to state j the probability equals to $p_{ij-1}(t)(\lambda_j)$; from state $j+1$ to state j the probability equals to $p_{ij+1}(t)(\mu_j)$; from any state to state j the probability equals to $p_{ij}(t)(0)$. For example if there exist nonnegative birth rate in state j is λ_j and nonnegative death rate in state j is μ_j, the rate matrix of the process defined as

$$A = \begin{bmatrix} \lambda_0 & -\lambda_0 & 0 & 0 \ldots & \\ \mu_1 & -(\lambda_1 + \mu_1) & \lambda_1 & 0 \ldots & \\ 0 & \mu_2 & -(\lambda_2 + \mu_2) & \lambda_2 & 0 \ldots \\ \cdots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$

CHAPTER FOUR
CONTINUOUS REVIEW INVENTORY SYSTEM WITH RANDOM LIFETIME AND TWO DEMAND CLASSES

Many of the studies in the literature assumed that the lifetime is fixed and single customer demand. We examined the continuous review inventory system with random lifetime and two demand classes. In real-life usually the customer demands with different priorities are met from the inventory. To our knowledge, no literature has been found that studied a perishable inventory model with multiple demand classes. In this study, we have focused on (s, Q) continuous review inventory model of a perishable product with exponentially distributed lead time and two types of customer demands. Our study is a generalization of Kalpakam & Sapna (1994) to demands with two customer classes. Also, our paper is an extension of Isotupa (2006). We added the perishability for Isotupa's inventory model with two demand classes. The system modeled as a Markov process. The steady-state probability distribution and the performance measures of the inventory system derived. The long-run average inventory cost function set up and the pseudo-convexity of the expected cost function has been proved. The optimization algorithm used for presented and numerical study provided with different rates.

4.1 Model definition

In this study, we consider an inventory system with perishability and two types of demands: priority customer demand and ordinary customer demand. Each customer generates demand for same products. Priority customer demand arrive according to a Poisson process with rate λ_1 (>0) and ordinary customer demand arrive according to a Poisson process with rate λ_2 (>0). We review the system with continuously. In addition, when the on-hand inventory drops to a prefixed level s, an order for Q units is placed. Hence, the maximum on-hand inventory is (Q + s). The condition $Q > s$ ensures that there are no continual shortages. The lead time for the order is exponentially distributed with rate μ (>0). When the on-hand inventory is below the reorder level s, only priority customers are served. No shortages are allowed for all demand types. The lifetime of the inventory is exponentially distributed with rate γ (>0). Figure 4.1 shows the structure of Markov process of the inventory system with perishability and two demand classes.

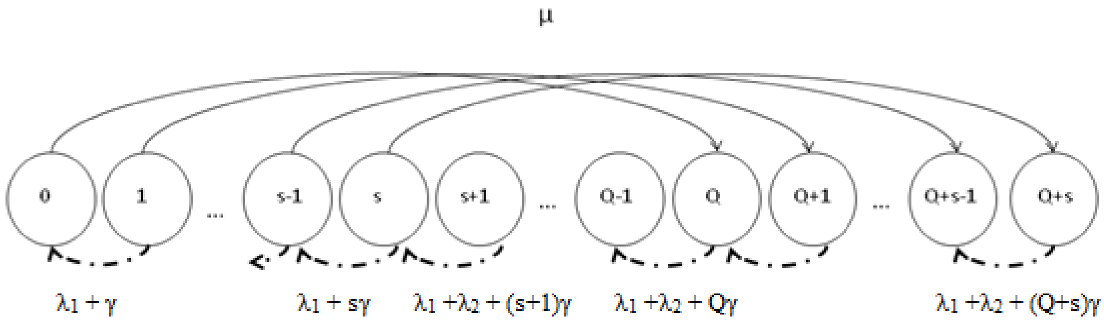

Figure 4.1 Markov Process of the inventory system with perishability and two demand classes

Let I(t) denote the on-hand inventory level at time *t*. The transition probability from state *i* at time 0 to state *j* at time *t* is shown as below:

$$P_{ij}(t) = P[I(t) = j \mid I(0) = i], \quad i,j \in S \tag{4.1}$$

On-hand inventory level $\{I(t), t \geq 0\}$ establish an irreducible ergodic Markov process with state space $S = \{0, 1, 2, \dots Q + s\}$. The distribution of $P_{ij}(t)$ as $t \to \infty$ exists. Then steady-state probability $P_j(t)$ satisfies the following balance equations:

$$\begin{aligned}
&\mu P_0 = (\lambda_1+\gamma)P_1 \text{ , n=0} \\
&(\mu+ \lambda_1 + n\gamma)P_n = (\lambda_1+(n+1)\gamma)P_{n+1} \text{ , } 1\leq n\leq s-1 \\
&(\mu+ \lambda_1 + n\gamma)P_n = (\lambda_1+\lambda_2+(n+1)\gamma)P_{n+1} \text{ , n=s} \\
&(\lambda_1 + \lambda_2 + n\gamma)P_n = (\lambda_1+\lambda_2+(n+1)\gamma)P_{n+1} \text{ , } s+1\leq n\leq Q-1 \\
&(\lambda_1 + \lambda_2 + n\gamma)P_n = (\lambda_1+\lambda_2+(n+1)\gamma)P_{n+1}+ \mu P_{n-Q} \text{ , } Q\leq n\leq Q+s-1 \\
&(\lambda_1 + \lambda_2 + n\gamma)P_n = \mu P_{n-Q} \text{ , n=Q+s}
\end{aligned} \tag{4.2}$$

By solving the above equations recursively, we have the steady state probabilities as;

$$P_n = \frac{\mu}{(\lambda_1+n\gamma)} A_n P_0 \qquad , \qquad 1 \le n \le s$$
$$P_n = \left(\frac{\mu+\lambda_1+s\gamma}{(\lambda_1+\lambda_2+n\gamma)}\right)\frac{\mu}{(\lambda_1+s\gamma)} A_s P_0 \qquad , \qquad s+1 \le n \le Q \tag{4.3}$$
$$P_n = \left(\frac{\mu+\lambda_1+s\gamma}{(\lambda_1+\lambda_2+n\gamma)}\right)\frac{\mu}{(\lambda_1+\alpha)} A_s P_0 - \frac{\mu}{(\lambda_1+\lambda_2+n\gamma)} A_{n-Q} P_0 \quad , \quad Q+1 \le n \le Q+s$$

where

$$A_n = \begin{cases} 1 & n \le 1 \\ \prod_{i=1}^{n-1}\left(\frac{\mu}{(\lambda_1+i\gamma)}+1\right) & n \ge 2 \end{cases} \tag{4.4}$$

Using the normalizing condition $\sum_{n=0}^{Q+s} P_n = 1$, from (4.2) we have

$$P_0 = \left[1+\mu(\lambda_2+Q\gamma)\sum_{n=1}^{s}\frac{A_n}{(\lambda_1+n\gamma)(\lambda_1+\lambda_2+(Q+n)\gamma)} + \frac{\mu(\mu+\lambda_1+s\gamma)}{\lambda_1+s\gamma}A_s\sum_{n=s+1}^{Q+s}\frac{1}{(\lambda_1+\lambda_2+n\gamma)}\right]^{-1} \tag{4.5}$$

Inserting Equation (4.5) in (4.3), we have the analytical steady-state probability distributions of the inventory level. Let $\bar{I}$ denote the average inventory level. Using $\bar{I} = \sum_{j=1}^{s+Q} j\, P_j(t)$, we have

$$\bar{I} = \lambda_2\mu\sum_{n=1}^{s}\frac{nA_n}{(\lambda_1+n\gamma)(\lambda_1+\lambda_2+(Q+n)\gamma)}P_0 - \sum_{n=1}^{s}\frac{Q\lambda_1\mu A_n}{(\lambda_1+n\gamma)(\lambda_1+\lambda_2+(Q+n)\gamma)}P_0$$
$$+\frac{\mu+\lambda_1+s\gamma}{\lambda_1+s\gamma}A_s\sum_{n=s+1}^{Q+s}\frac{n\mu}{(\lambda_1+\lambda_2+n\gamma)}P_0 \tag{4.6}$$

Let us denote

Γ_1= the mean failure rate

Γ_2= the mean rate of lost sales for priority customers

Γ_3= the mean rate of lost sales for ordinary customers

Γ_4= the mean reorder rate

The mean reorder rate, the mean shortage rates for prioritized customers and the mean shortage rates for the ordinary customers, respectively, are written by

$$\Gamma_1 = \sum_{n=1}^{Q+s} \gamma n P_n = \gamma \sum_{n=1}^{Q+s} n P_n = \gamma \bar{I} \qquad (4.7)$$

$$\Gamma_2 = \lambda_1 P_0 \qquad (4.8)$$

$$\Gamma_3 = \lambda_2 \sum_{n=0}^{s} P_n = \lambda_2 \left(P_0 + \sum_{n=1}^{s} \frac{\mu}{\lambda_1 + n\gamma} A_n P_0 \right) = \lambda_2 A_{s+1} P_0 \qquad (4.9)$$

$$\Gamma_4 = (\lambda_1 + \lambda_2) P_{s+1} + (s+1)\gamma P_{s+1} \qquad (4.10)$$

Remark 1. The results for the case $\gamma \rightarrow 0$ corresponds the model which is given by Isotupa (2006)

Remark 2. For the case $\lambda = \lambda_1 + \lambda_2$, corresponds the model which is given by Kalpakam & Sapna (1994)

Remark 3. The results of our model for $\gamma \to 0$ are in agreement with that of Liu et.al (2014) when $p=0$ for the same parameter values.

Remark 4. For the case $\lambda = \lambda_1 + \lambda_2$ and $\gamma \rightarrow 0$, the results corresponds to the classical Markovian inventory system with (s, Q) continuous review inventory policy.

4.2 Formulation of Total Cost Function and Analysis

In this section, we defined an inventory cost function to obtain the mean rates and discussed the analytical property. We define the unit cost parameters as follows:

K: fixed ordering cost per order

h: the unit inventory holding cost per unit time

c: the purchase price per unit

g_1: the failure cost per unit

g_2: the lost sale cost per unit for the lost priority customers

g_3: the lost sale cost per unit for the lost ordinary customers

The long run expected cost function TC(s, Q) has four components such as inventory holding cost, cost of failure, costs due to the lost sales for the priority and ordinary customers and finally ordering cost. The expected cost function is given as follows:

$$TC(s,Q) = h\bar{I} + g_1\Gamma_1 + g_2\Gamma_2 + g_3\Gamma_3 + (K + cQ)\Gamma_4$$

$$= (h + g_1\gamma)\frac{\mu + \lambda_1 + s\gamma}{\lambda_1 + s\gamma}A_s \sum_{n=s+1}^{Q+s} \frac{n\mu}{\lambda_1 + \lambda_2 + n\gamma} P_0$$

$$-(h + g_1\gamma)\sum_{n=1}^{s} \frac{Q\lambda_1\mu}{(\lambda_1 + n\gamma)(\lambda_1 + \lambda_2 + (n + Q)\gamma)} A_n P_0$$

$$+(h + g_1\gamma)\sum_{n=1}^{s} \frac{n\lambda_2\mu}{(\lambda_1 + n\gamma)(\lambda_1 + \lambda_2 + (n + Q)\gamma)} A_n P_0$$

$$+(g_2\lambda_1 P_0) + (g_3\lambda_2 A_{s+1} P_0) + (K + cQ)\frac{\mu+\lambda_1+s\gamma}{\lambda_1+s\gamma}\mu A_s P_0 \quad (4.11)$$

where P_0 is given by Equation (4.5).

The goal is to determine the optimal reorder level s, and the optimal order quantity Q that minimizes the expected total cost. For finding this optimal level we must show that the function has a local minima at least one point. Many of the theoretical results and computational algorithms of nonlinear programming require that the functions entering the problem be convex, pseudo-convex or quasi-convex. As our total cost function is complex and difficult to show the convexity. In this case, it is sufficient to show that the pseudo-convexity of the function. A pseudo-convex function is a function that behaves like a convex function with respect to finding its local minima, but need not actually be convex. Informally, a differentiable function is pseudo-convex if it is increasing in any direction where it has a positive directional derivative. Every convex function is pseudo-convex, but the converse is not true. Any pseudo-convex function is quasi convex, but the converse is not true. The hierarchy between differentiable functions is shown in Figure 4.2. Pseudo-convexity is primarily of interest because a point x^* is a local minimum of a pseudo-convex function f if and only if it is a stationary point of f, which is to say that the gradient of f vanishes at x*: $\nabla f(x^*) = 0$. Therefore, we are looking for quasi-convex functions. The pseudo-convex functions have many properties of convex functions, particularly about derivative properties and finding local minima. The function f is said pseudo-convex if for every $x, y \in E$, $y < x$ implies $f(y) - f(x) < 0$. If f(y)-f(x) does not cross zero-axis, f(y)-f(x)>0, the function TC(s,Q) is increasing in s. If f(y)-f(x) crosses zero-axis once, there is an integer s_0. The definition states that;

when any point $s_0 < s$ if the total cost function $TC(s+1,Q) - TC(s,Q) > 0$ is monotonically increasing function of s.

when any point $s_0 < s$ if the total cost function $TC(s+1,Q) - TC(s,Q) > 0$ is monotonically decreasing function of s;

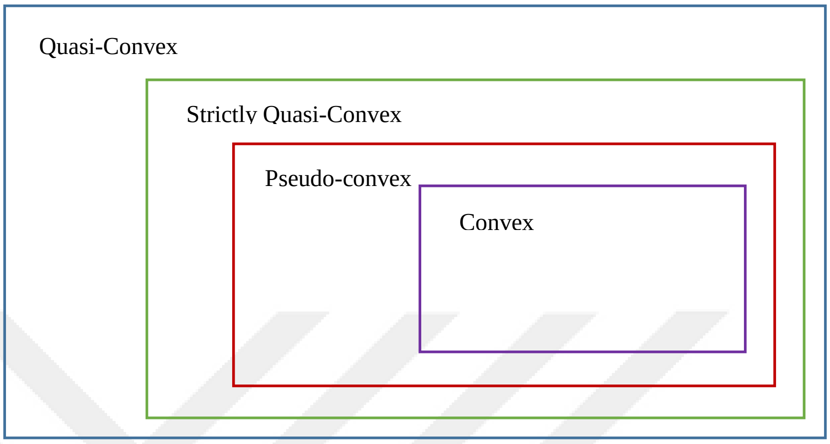

Figure 4.2 Hierarchy among differentiable function

Theorem 1. The long-run average cost function given by Equation (3) is pseudo-convex in s and has a unique minimum.

Proof. It is difficult to change both the *Q* and *s* at the same time; we first fix parameter *Q*, the cost function can be written as

$$TC(s+1,Q) - TC(s,Q) = \frac{N(s)}{D(s)} \tag{4.12}$$

$$\begin{aligned} N(s) = {} & (h + g_1\gamma)\{\mu Y + \mu^2(\lambda_2 + Q\gamma)VY + C\mu VQ\lambda_1 \\ & + \mu C(s+1)[V(Q\gamma+1) + A_{s+2}X] + \mu^2 X(\lambda_1 QV - \lambda_2 W) \\ & - C\mu[A_{s+2}Y + \lambda_2 W] + C\} + (K + cQ)\mu[1 + \mu(\lambda_2 + Q\gamma)V - CA_{s+2}] \\ & - g_2\lambda_1(\mu X + C) \\ & + g_3\lambda_2[1 + \mu(\lambda_2 + Q\gamma)V + \mu A_{s+2}X - (1 + \mu Z)(\mu X + C)] \end{aligned} \tag{4.13}$$

where

$$X = \sum_{n=s+2}^{Q+s+1} \frac{1}{\lambda_1 + \lambda_2 + n\gamma} \tag{4.14}$$

$$V = \sum_{n=1}^{s+1} \frac{1}{(\lambda_1+n\gamma)[\lambda_1+\lambda_2+(Q+n)\gamma]} A_n \tag{4.15}$$

$$W = \sum_{n=1}^{s+1} \frac{n}{(\lambda_1+n\gamma)[\lambda_1+\lambda_2+(Q+n)\gamma]} A_n \tag{4.16}$$

$$Z = \sum_{n=1}^{s+1} \frac{1}{\lambda_1+n\gamma} A_n \tag{4.17}$$

$$C = \frac{\lambda_2}{\lambda_1+\lambda_2+(s+1)\gamma} \tag{4.18}$$

$$Y = \sum_{n=s+2}^{Q+s+1} \frac{n}{\lambda_1+\lambda_2+n\gamma} \tag{4.19}$$

Obviously, the denominator D(s) is always positive. To prove that TC(s,Q) is pseudo-convex, it is enough to show that N(s) is an increasing function.

Let f(s) =ΔN(s) =N(s+1)-N(s). When f(s) is an increasing function of s, so f(s) is always positive or crosses zero-axis at most once. Since cost function has four components and hence complicated, f(s) is given for each component separately. Let $\Delta N_1(s)$ is the coefficient of the first part of the cost function. From Equation (11) the coefficient of (h+g_1) is

$$\begin{aligned}
\Delta N_1(s) = {} & \frac{\mu A_{s+2}}{\lambda_1+(s+2)\gamma} X\big[\lambda_2 Q\gamma(\lambda_1+\lambda_2)[\mu+\lambda_1+(s+2)\gamma]\big] \\
& + \Big\{\frac{\mu A_{s+2}}{\lambda_1+(s+2)\gamma} X\big[[\lambda_1+\lambda_2+(s+2)\gamma][\lambda_2 Q\mu\gamma(s+1)+\lambda_2(\lambda_1+\lambda_2)[\lambda_1+(s+2)\gamma] \\
& \qquad + \lambda_1 Q\mu[\lambda_1+\lambda_2+(s+1)\gamma]]\big]\Big\} \\
& + \mu\gamma A_{s+2} Y\{Q\mu[\lambda_1+\lambda_2+(s+1)\gamma]+\lambda_2[\lambda_1+\lambda_2+(Q+s+2)\gamma]\} \\
& + \frac{\mu}{[\lambda_1+\lambda_2+(s+2)\gamma]} V\left\{\frac{Q\mu(\lambda_2+(Q)\gamma)(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+(Q+s+2)\gamma} + \frac{\lambda_2^2[\lambda_1+\lambda_2+Q\gamma]}{\lambda_1+\lambda_2+(s+1)\gamma}\right\} \\
& + \mu\lambda_2\gamma W\left[\frac{Q\mu}{M1}+\frac{\lambda_2}{M3}\right] + (\lambda_1+\lambda_2)\left[\frac{Q}{M1}+\frac{\lambda_2}{M3}\right]
\end{aligned} \tag{4.20}$$

Let $\Delta N_2(s)$ is the coefficient of the second part of the cost function. From Equation (4.13), the coefficient of $(K + cQ)\mu$ is

$$\Delta N_2(s) = \gamma A_{s+2} \left[\frac{Q\mu + \lambda_2[\lambda_1 + \lambda_2 + (Q + s + 2)\gamma]}{M1}\right] \quad (4.21)$$

Let $\Delta N_3(s)$ is the coefficient of the third part of the cost function. Although the third term is negative in Equation (4.13), the difference $\Delta N_3(s)$ is positive and found as below:

$$\Delta N_3(s) = \frac{Q\mu\gamma}{M1} + \frac{\gamma}{M3} \quad (4.22)$$

Let $\Delta N_4(s)$ is the coefficient of the last part of the cost function. From Equation (4.13), the coefficient of $g_3\lambda_2$ is

$$\Delta N_4(s) = \mu\gamma\left(\frac{Q\mu}{M1} + \frac{1}{M3}\right) Z + \frac{Q\mu\gamma(A_{s+2} + 1)}{M1} + \frac{\gamma}{M3} \quad (4.23)$$

where

$$M1 = [\lambda_1 + \lambda_2 + (s + 2)\gamma][\lambda_1 + \lambda_2 + (Q + s + 2)\gamma]$$
$$M2 = [\lambda_1 + (s + 2)\gamma][\lambda_1 + \lambda_2 + (Q + s + 2)\gamma]$$
$$M3 = [\lambda_1 + \lambda_2 + (s + 1)\gamma][\lambda_1 + \lambda_2 + (s + 2)\gamma]$$
$$M4 = [\lambda_1 + (s + 2)\gamma][\lambda_1 + \lambda_2 + (s + 2)\gamma]$$

Then

$$f(s) = \Delta N(s) = (h + g_1)\Delta N_1(s) + (K + cQ)\mu\Delta N_2(s) + g_2\lambda_1\Delta N_3(s) + g_3\lambda_2\Delta N_4(s) \quad (4.24)$$

Since $\Delta N(s) > 0$, $N(s)$ is an increasing function of s. Therefore $TC(s,Q)$ is pseudo-convex in s, and has the minimum at s_0.

Theorem 2. As a result, of Theorem 1, we can prove that the cost function is pseudo-convex with respect to Q and has a unique minimum.

4.3 Numerical study and results

A nonlinear model will constructed here to obtain the optimal inventory control policy, which consists of priority rule, and order policy. The model is:

$$\min TC(s,Q) = \left\{h\bar{I} + g_1\Gamma_1 + g_2\Gamma_2 + g_3\Gamma_3 + (K + cQ)\Gamma_4\right\}$$

Subject to

$$s - Q \leq -1$$
$$s \geq 0$$
$$Q \geq 0$$

The objective function minimizes the total inventory cost and finds the optimal *s* and *Q* values. To minimize this formula, we use an algorithm with help of MATLAB. In this section, the sensitivity of the optimal policy to the model parameters is examined. We review the system with continuously and time unit is considered as day. Each customer generates demand for same products and customers have two types of demands: priority customer demand arrives according to a Poisson process with parameter λ_1=30 per day and ordinary customer demand arrives according to a Poisson process with parameter λ_2=20 per day. In addition, when the on-hand inventory drops to a prefixed level *s*, an order for *Q* units is placed. Hence, the maximum on-hand inventory equals to 150. The lead time for the order is exponentially distributed with parameter μ=2 (1/μ =0.5 days). No shortages are allowed for the demand types. The lifetime of the items in inventory is exponentially distributed with parameter γ=0.2 (1/γ =5 days). To ensure that products do not perish before they arrive, the condition $\boldsymbol{\gamma > \mu}$ must be held in numerical computations. In the first part, the cost parameters are fixed. The failure cost per unit is assumed as $\mathbf{g_1 = 10}$. The lost sale cost of prioritized customers is assumed as twice over that of ordinary customers; that is $\mathbf{g_2 = 50}$ and $\mathbf{g_3 = 25}$. The other cost parameters are assumed as fixed ordering cost per order per day $\mathbf{K = 100}$, the purchase price per unit $\mathbf{c = 10}$, and the unit inventory holding cost per unit time per day $\mathbf{h = 5}$.

Let us denote

$\bar{I}$ = the mean inventory level

Γ_1= the mean failure rate for one unit

Γ_2= the mean rate of lost sales for priority customers

Γ_3= the mean rate of lost sales for ordinary customers

Γ_4= the mean reorder rate

Firstly, we will show the convexity of the total cost formula. In addition, later, we will find the optimal order quantity and reorder point, which minimizes the total cost, mean inventory level, mean failure rate, mean lost sale for priority and ordinary customers and mean reorder rate for eight different case.

Case 1. Let us consider the parameter λ_1=30, λ_2=20, K=100, c=10, h=5, g_1=10, g_2=50, g_3=25 and γ=0.2. The effect of lead time rate (μ) on total cost and optimal reorder point is examined with the given parameter values above. The results are summarized in Table 4.3.

Table 4.1 The optimal inventory policy for different values of the lead time rate (γ=0.2)

μ	(s^*,Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
4	(11, 48)	984.4331	29.4034	5.8807	1.9927	5.0099	1.0020
3	(14, 52)	1027.5473	32.1715	6.4343	2.5234	6.0166	0.9025
2	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
1	(28, 76)	1264.7588	40.6014	8.1203	6.7058	10.3592	0.5180
0.5	(38, 97)	1454.6943	40.1956	8.0391	11.2428	13.1045	0.3276
0.33	(43, 111)	1567.3562	36.9049	7.3810	14.4164	14.5207	0.2396
0.25	(47, 119)	1637.2323	33.7080	6.7416	16.5325	15.4615	0.1933

Table 4.1 shows the system performance sensitivity according to the different values in the mean lead time. When μ decreases, the average lead time increases. From Table 4.1, when lead time decreases, the reorder point increases, and depending on the cost parameters, the amount of the order quantity also changes. For lower costs, the on hand inventory level and mean inventory level increase, while the lead time increases. As the costs increase, the order quantity *Q* decreases. That is when the average lead

time increases the orders are issued earlier and order quantities are increasing. The mean number of items that perish, which is mean failure rate, is also increasing when the average lead time is increasing. When average lead time is moving closer to mean lifetime, average inventory is decreasing while lost sale rates are increasing. This may be an evidence of some items perish before delivery. The lost sale rate for ordinary customers is always greater than that of prioritized customers. The last column gives the mean reorder rate; that is the average number of orders in a period. The mean reorder rate is also decreasing when the mean lead time is increasing. When the perishability cost is higher than the lost sales cost for both customer the on-hand inventory level increases and the order quantity decreases. When lead time decreases, the reorder quantity increases for all cases. As Γ_1 increases, the system keeps less inventory and perishability decreases. As Γ_2 increases, the system wants to reduce lost sales for both customers so keeps more inventory on hand and lost sale cost for priority customer decreases.

Figure 4.3 and Figure 4.4 show the convexity for *Q* and *s* over the total cost for value μ=2. In addition, convexity can be shown for all values of μ.

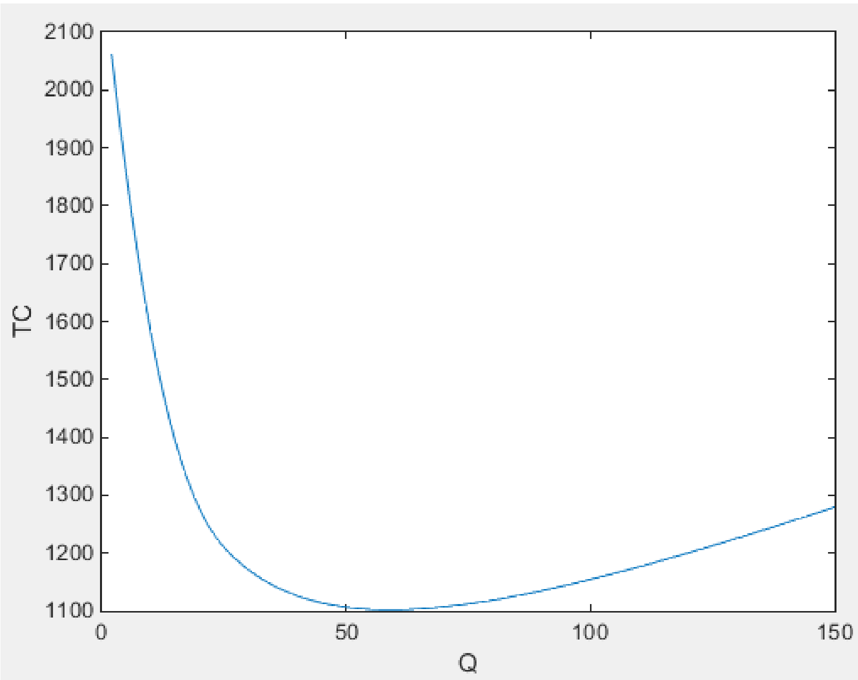

Figure 4.3 Convexity for Q over the total cost for value μ=2

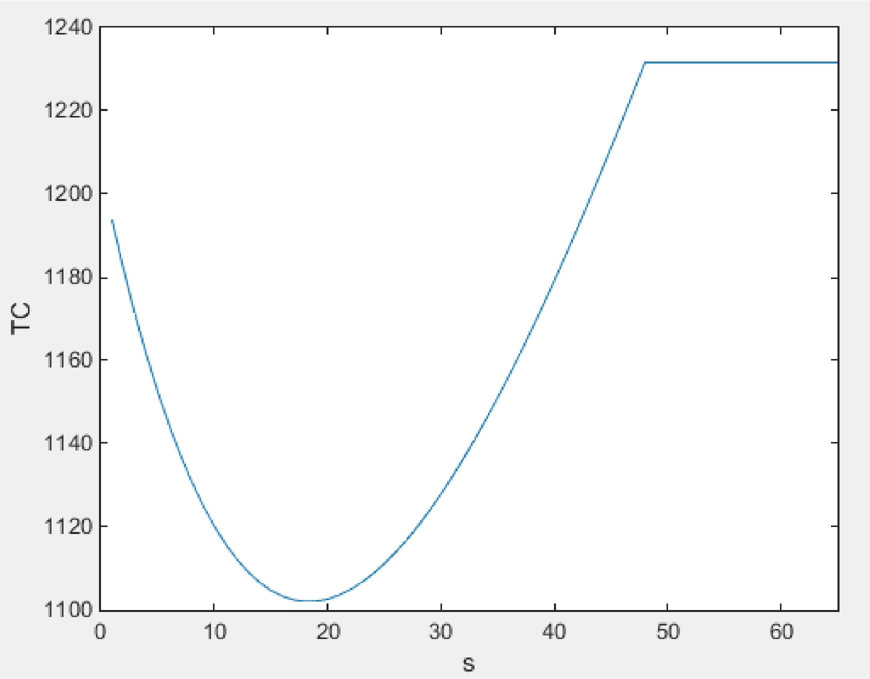

Figure 4.4 Convexity for s over the total cost for value μ=2

Case 2. Let us consider the parameters λ_1=30, λ_2=20, K=100, c=10, h=5, g_1=10, g_2=50, g_3=25 and μ=2. The effect of expected decay rate (γ) on total cost and optimal reorder point is examined with the given parameter values above. The results are summarized in Table 4.2.

Table 4.2 The optimal inventory policy for different values of the failure rate (μ=2)

γ	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
0	(23, 75)	934.3241	51.7536	0.0000	1.9229	5.6561	0.5656
0.04	(22, 71)	972.8878	47.6846	1.9074	2.2548	6.0907	0.6091
0.06	(22, 68)	991.0806	45.6945	2.7417	2.3917	6.3958	0.6396
0.08	(21, 67)	1008.5638	43.8360	3.5069	2.6219	6.5314	0.6531
0.20	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
0.40	(15, 50)	1225.8679	26.4785	10.5914	5.5186	8.8671	0.8867
0.50	(14, 47)	1277.1456	23.6505	11.8253	6.2671	9.3798	0.9380
0.95	(10, 37)	1452.4912	14.8435	14.1014	9.5398	11.0831	1.108
1.00	(10, 36)	1467.9253	14.3085	14.3085	9.7699	11.3063	1.130

The mean lifetime decreases as failure rate (γ) increases. From Table 4.4, both reorder point and optimal order quantity is decreasing when mean failure rate is increasing. We can say that when failure rate equals to lead time ($\gamma = \mu$) all units in inventory are perished**.** In other words, when lifetime decreases, the system tends to store less inventory. Average inventory level is decreasing and lost sale rates are increasing at the same time. Ordinary customers' lost sale rate is always greater than prioritized customers' lost sale rate. Because of high perishability rate, priority customer demands are met firstly and ordinary customer demand assume the lost sale, so lost sale quantity for priority and normal customer increases. The total expected cost tends to increase since the lost sale rates for both customer classes increase. When the lost sale cost for priority customer is more than other costs, the system does not allow the perishability because of the mean perishability inventory level decreasing. While the mean inventory level decreases, the mean perishability inventory level increases. When mean failure rate equals to zero, average inventory level is high relative to the other cases in which mean failure rate greater than zero, on the other hand, the lowest expected cost is obtained in this case. As lifetime decreases, reorder point also decreases whereas the total expected cost increases. This result reveals the importance of the management of perishable inventory. When the reorder point drops, the system orders more often and the mean reorder rate increase.

Figure 4.5 and Figure 4.6 show the convexity for Q and s over the total cost for value γ=0.5. In addition, convexity can be shown for all value of γ.

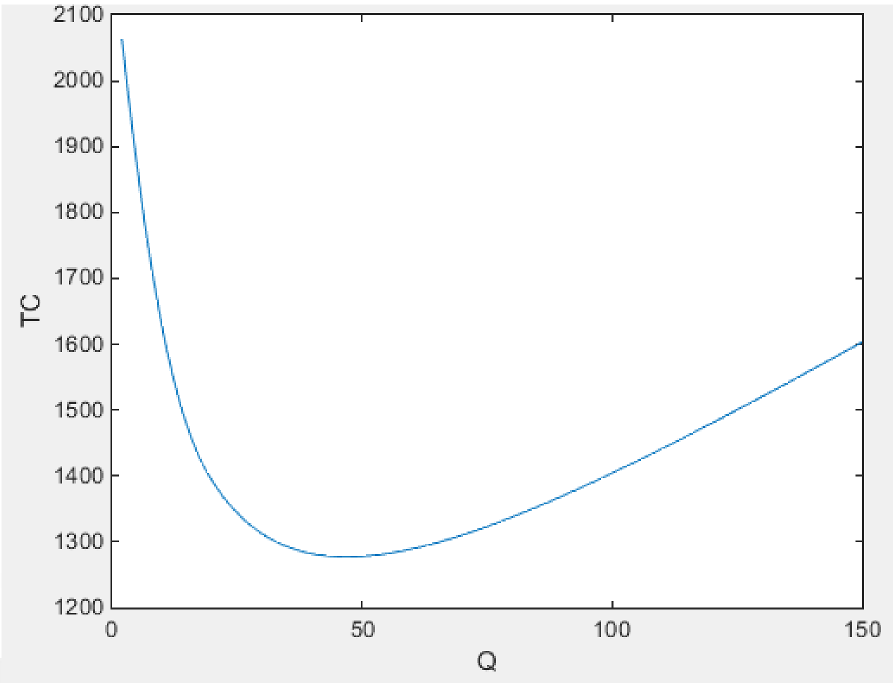

Figure 4.5 Convexity for Q over the total cost for value γ=0.5

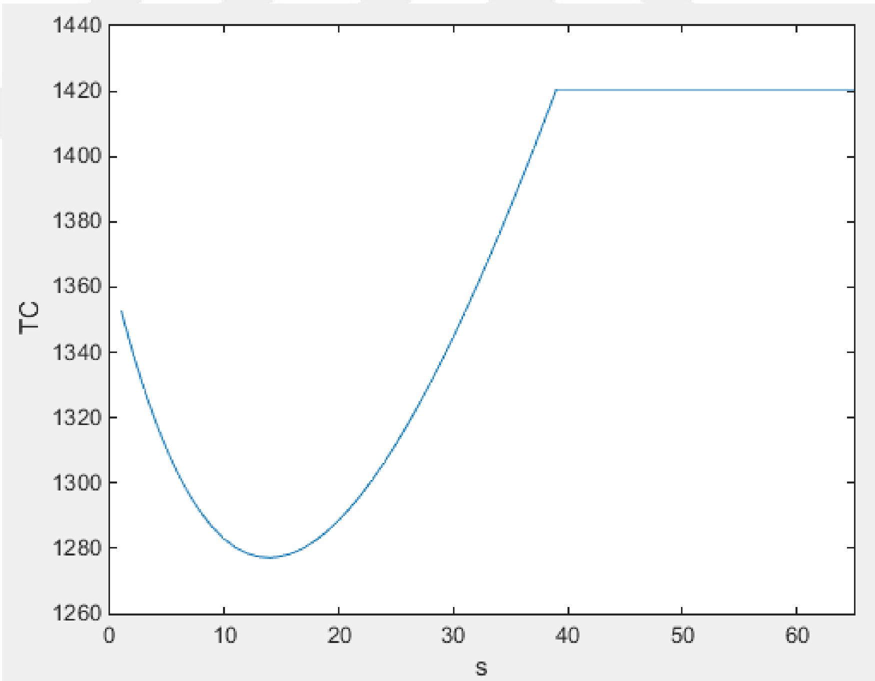

Figure 4.6 Convexity for s over the total cost for value γ=0.5

Case 3. Let us consider the parameter λ_2=20, K=100, c=10, h=5, g_1=10, g_2=50, g_3=25, γ=0.2 and μ=2. The effect of the mean rate of the priority customer (λ_1) on total cost and optimal reorder point is examined with the given parameter values above. The results are summarized in Table 4.3.

Table 4.3 The optimal inventory policy for different values of the λ_1

λ_1	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
20	(12, 48)	882.7327	26.3407	5.2681	2.4665	7.2485	0.7249
30	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
60	(40, 89)	1754.4738	64.3103	12.8621	7.3241	8.3569	0.8357
100	(70, 127)	2618.1279	103.1481	20.6296	12.2996	8.9987	0.8999

Case 4. Let us consider the parameter λ_1=30, K=100, c=10, h=5, g_1=10, g_2=50, g_3=25, γ=0.2 and μ=2. The effect of the mean rate of the ordinary customer (λ_2) on total cost and optimal reorder point is examined with the given parameter values above. The results are summarized in Table 4.4.

Table 4.4 The optimal inventory policy for different values of the λ_2

λ_2	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
20	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
30	(18, 68)	1282.7485	37.5022	7.5004	3.7813	11.3104	0.7540
50	(18, 86)	1644.3677	42.3751	8.4750	3.7800	18.8440	0.7538
100	(19, 130)	2548.9831	55.6158	11.1232	3.5938	37.9525	0.7591

The inventory level of the system varies according to the customer demand increase. When the priority customer demand is more than the normal customer demand and the cost values are not too high, then there is a constant increase for demand rates. As cost values are increased, to meet the priority customer demand becomes very important and the inventory level is reduced to the minimum. When the priority customer demand is higher than the ordinary customer demand, a small amount of the priority demand and almost all of the ordinary customer demand are assume lost sale. If the ordinary customer demand is high, close to half of the priority customer demand are assume lost sale.

From Table 4.3, it can be seen the amount of inventory increases as the demand of the priority customers increases. Because the system tries to please the priority customer. For this reason, in high numbers of quantities places at higher reorder points. So, the perishability of the product is ignored because the cost of priority customers lost sales is higher than the failure cost. From Table 4.6, it can be seen that, when mean demand rates are equal or even if mean demand rate of ordinary customers is less than the other, the lost sale rate of the ordinary customers is greater. The lost sale rate of prioritized customers seems like fixed whereas the lost sale rate of ordinary customers increases with the mean demand rate. When the ordinary customer demand increases compared to the priority customer, the amount of inventory held is increasing depending on the lost sale costs for each customer types. Order is given less frequently and higher quantities. Accordingly, the mean reorder rate remains fixed. As the amount of on hand inventory increases, the mean failure rate increases.

Case 5. As a second part, the sensitivity analysis on the cost parameters has been performed. Table 4.5 represents the changes in the optimal policy with respect to the failure cost changes with parameter λ_1=30, λ_2=20, K=100, c=10, h=5, g_2=50, g_3=25, γ=0.2 and μ=2.

Table 4.5 The optimal inventory policy for different values of failure cost

g_1	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
5	(19, 63)	1071.4808	37.9775	7.5955	3.4308	7.2461	0.7246
10	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
50	(13, 41)	1285.4053	22.4888	4.4978	6.2364	9.2821	0.9282
100	(10, 31)	1432.9432	15.4399	3.0880	8.5900	10.6823	1.0682

As failure cost increases, the system tends to operate at a lower inventory level. Average inventory level tends to decrease and since the lost sales increases, the total expected cost also increases, the order quantity and reorder point decrease. The average inventory level decreases. This leads to an increase in the lost sales for both customer classes. The total expected cost increases. The system reduces the order quantity and pulls down the reorder point. This increases the mean reorder rate because it causes more frequent ordering.

Case 6. The long-run average inventory cost with the changes in lost sale cost for the priority customer and ordinary customer classes are presented in Table 4.6 and Table 4.7, respectively.

Table 4.6 represents the changes in the optimal policy with respect to the failure cost changes with parameter λ_1=30, λ_2=20, K=100, c=10, h=5, g_1=10, γ=0.2 and μ=2.

Table 4.6 The optimal inventory policy for different values of priority customer lost sale cost for

g_2	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
10	(1, 34)	811.9430	10.2917	2.0583	12.5886	8.9482	0.8948
20	(2, 47)	926.3681	16.3419	3.2684	9.9768	7.5582	0.7558
50	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
100	(31, 64)	1230.8749	48.7516	9.7503	1.8565	7.6264	0.7626

Table 4.7 The optimal inventory policy for different values of ordinary customer lost sale cost

g_3	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
15	(21, 48)	1020.5111	32.4265	6.4853	3.7524	8.8824	0.8882
25	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
50	(14, 80)	1267.8594	41.3298	8.2660	3.7559	5.9310	0.5931
100	(9, 111)	1525.6963	50.1911	10.0382	3.9027	4.5678	0.4568

The ordinary customer lost sale rate decreases with the increment in g_3 whereas the priority customer lost sale rate remains at a certain level. When lost sale cost increases, the reorder point decreases in Table 4.7, unlike the results in Table 4.6. For this reason, the mean reorder rate decreases in Table 4.6 and increases in Table 4.7. Recall that only priority customer demand is met when on-hand inventory level drops under a certain level. Hence when the ordinary customer lost sale cost increases, the reorder point decreases.

Case 7. The changes in the optimal inventory policy with the change of inventory holding cost are presented in Table 4.8 with parameter λ_1=30, λ_2=20, K=100, c=10, g_1=10, g_2=50, g_3=25, γ=0.2 and μ=2.

Table 4.8 The optimal inventory policy for different values of holding cost

h	(s^*,Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
1	(23, 81)	961.0809	49.8699	9.9740	2.3312	6.1765	0.6176
5	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
20	(11, 34)	1394.7212	17.5984	3.5197	7.7433	10.2275	1.0227
50	(6, 20)	1657.5214	7.8122	1.5624	13.0525	12.7081	1.2708

As holding cost increases, the system tends to work without inventory. In this case, the system orders less and reduces the average inventory level. The mean reorder rate increases as the system orders more often. The mean reorder rate is increased. As the average inventory level approaches zero, stock out situation arises for two demand classes.

Case 8. The numerical results for the effect of changing the order cost and purchase price are presented in Table 4.9 and Table 4.10, respectively, with parameter λ_1=30, λ_2=20, g_1=10, g_2=50, g_3=25, γ=0.2 and μ=2.

Table 4.9 The optimal inventory policy for different values of order cost

K	(s^*,Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
10	(21, 49)	1029.3020	32.9234	6.5847	3.6991	8.7562	0.8756
20	(20, 51)	1037.9670	33.0768	6.6154	3.7834	8.4610	0.8461
50	(20, 54)	1062.9422	34.5377	6.9075	3.6353	8.1297	0.8130
100	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
1000	(8, 119)	1592.5577	52.6271	10.5254	3.9284	4.3259	0.4326

Table 4.10 The optimal inventory policy for different values of purchase price

c	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
1	(21, 78)	680.4428	46.7119	9.3424	2.6551	6.2849	0.6285
5	(20, 69)	872.7372	41.6609	8.3322	3.0541	6.8299	0.6830
10	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543
20	(16, 41)	1521.0780	24.8871	4.9774	5.3648	9.5265	0.9526
40	(9, 10)	2114.9586	4.8581	0.9716	15.1739	17.7599	1.7760

Optimal reorder point decreases when K and c are decreasing. The lost sale rate of the prioritized customer is always lower than the lost sale rate of the ordinary customer. As the setup cost increases, the order quantity, the mean inventory level and perishability cost increase. Although the priority customer lost sale does not change but the ordinary customer demand is being lost more than before and lost sale for priority customer increase. As the lot size increase, the number of reorder decrease. Lost sale for priority customer does not change much in addition to this ordinary customer lost sale is decreasing. The number of reorder is decreasing as lot size increase. However, as the cost of deterioration increases, the amount of order given decreases independently of the increase in K.

Table 4.11 The optimal inventory policy for with and without perishability

	(s^*, Q^*)	TC	$\bar{I}$	Γ_1	Γ_2	Γ_3	Γ_4
No perish	(23, 75)	934.3241	51.7536	0.0000	1.9229	5.6561	0.5656
Perish	(18, 59)	1102.0701	35.2270	7.0454	3.7825	7.5426	0.7543

Table 4.11 gives two different results for the same values, with and without perishability.

For the perishability case, the system tries to decrease total costs without compromising by keeping much less inventory. Because the failure cost is higher than all other costs, the inventory level is continuously reduce. As a result of the numerical study, the sensitivity of optimum inventory policy to the model parameters is examined. Perishability of the items in inventory makes the inventory decisions more critical. The results show the significance of the perishable inventory management.

CHAPTER FIVE
CONCLUSION

To summarize briefly what we have done in this dissertation, we examine a continuous review (s,Q) inventory model for perishable products with two demand classes. The system is characterized by Markov Process characterized and the stationary probabilities are derived. System performance measures such as average inventory level, mean failure rate, mean lost sale rates for both demand classes and mean reorder rate are formulated by using stationary probabilities and computed by an MATLAB algorithm. The expected total cost function is formulated and pseudo-convexity of the cost function is proved.

As a result of these numerical studies, it is seen that the mathematical characterization is consistent when compared to the existing studies. For some selected values, the total cost function is shown as convex. In addition, it is observed that some costs have increased as a result of failure. The total cost may increase significantly depending on the life of the products. When the lifetime of the product is over, it will deteriorate and cannot be used. Because the failure cost due to perishability is usually large, the main aim of the system is met the customer demand with low inventory level. So, it tends to hold fewer items. But in this case, the demands of some customer classes are not met and the stock out situation arises. This leads to an increase in total cost. In other words, the life of the product should be managed with great care as it will affect the total cost in any case. Sensitivity analysis were calculated and interpreted for different parameters and costs. The parameters of the demand distribution, lead time and lifetime distribution were compared as being proportional to each other. Lifetime was always considered longer than lead time. We also examined how the total cost affected by stock out cost for both customer classes, failure cost, setup cost, holding cost and unit purchasing cost.

For the further research, this dissertation is a stepping stone. The model can be more suitable for the real life with assumed the random lead time or random lifetime. With the use of different distributions, different processes like semi-Markov process or Markov decision process can characterize the system.

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