

Prioritized Routing and Scheduling for Home Healthcare Services: Static and Dynamic Optimization

by

Ahmet ÇINAR

A Dissertation Submitted to the
Graduate School of Sciences and Engineering
in Partial Fulfillment of the Requirements for
the Degree of

Doctor of Philosophy

in

Industrial Engineering and Operations Management



August, 2019

**Prioritized Routing and Scheduling for Home Healthcare Services: Static
and Dynamic Optimization**

Koç University

Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a doctoral dissertation by

Ahmet ÇINAR

and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
examining committee have been made.

Committee Members:

Prof. Dr. F. Sibel Salman

Prof. Dr. Necati Aras

Prof. Dr. Metin Türkay

Prof. Dr. Burçin Bozkaya

Assoc. Prof. Dr. Özgür M. Araz

Date: _____



Tu be her kucâ ki bâşî buved ân behîst mâ râ

ABSTRACT

Home healthcare services provide medical assistance to people at their residences. There are many companies worldwide that provide this type of service with a dedicated crew of health professionals. In the problem that motivated our research, nurses check upon the conditions of patients using nutritional products. Each nurse has a set of skills and a set of patients assigned to her. Home healthcare services require solving an optimization problem to decide the set of patients to serve each day of a planning horizon and the sequence in which patients are going to be visited during their requested time windows.

In the first part of the thesis, we study a deterministic and static problem. Due to a large number of patients and their varying conditions, a nurse must select carefully which patients to visit at home for the upcoming days. We propose assigning priorities to patients according to factors such as the last visit time and the severity of their condition so that the priorities of unvisited patients increase exponentially by day. The objective is to maximize the total priority of the visited patients primarily and to minimize the total traveling time secondarily. After having observed the computational limits of an exact formulation, we develop an Adaptive Large Neighborhood Search (ALNS) algorithm and a matheuristic to generate near-optimal solutions for realistic-sized instances. We measure the quality of both algorithms by computing the optimality gaps using upper bounds generated by Lagrangean relaxation. Tests on real-life data show that both algorithms yield high quality solutions, but the matheuristic outperforms ALNS in large instances. On the other hand, the ALNS algorithm provides very short running times, while the running times of the matheuristic increase exponentially with problem size. We provide insights by quan-

tifying the trade-offs two objectives on the real data set complemented with various realistic parameters.

In the second part of the thesis, we study an extension of the home healthcare scheduling and routing problem where urgent patient visit requests emerge during a day, while an optimized plan is implemented. We define a dynamic optimization framework to increase the effectiveness and efficiency of the visits during the entire planning horizon. At the beginning of the planning horizon, our ALNS algorithm runs to determine which patients to visit on each day of a multi-period planning horizon and in which order to visit them to maximize the total priority of visited patients and to minimize the total routing time. Then, on each day some emergent patients with urgent conditions arrive dynamically. The emergencies are accumulated until predetermined re-optimization times. When a re-optimization time comes, the current plan is re-optimized with the accumulated new arrivals in addition to the remaining existing ones by a multi-period static mixed-integer programming (MIP) model. Solutions generated with respect to different processing rules for handling the new arrivals are evaluated with respect to several performance measures, including the waiting time of urgent patients and the number of patients whose visit times are delayed. Extensive computational tests are performed on real data sets via simulation studies.

ÖZETÇE

Evde sađlık bakımı sektörü belirli sađlık sorunları olan insanlara sıhhi hizmetler sađlayan bir alıřma alanıdır. Genellikle, bu tür sađlık hizmetlerine ihtiya duyan hastalar ihtiyar veya engelli bireylerden oluşur. Dünya genelinde evde sađlık bakımı hizmetleri veren, özel olarak bu işe tahsis edilmiş hemřire ekiplerinden oluşan eřitli sađlık kuruluşları vardır. Her hemřire eřitli becerilere sahiptir ve kendisine atanan belirli sayıda hasta ile ilgilenmek zorundadır. Evde sađlık bakımı hizmetleri; her hemřire için bir planlama periyodu dahilinde hangi gün hangi hastalara gidileceğini ve bu hastaların hangi sıra ile ziyaret edileceğini belirleyen müşteri bir optimizasyon probleminin özölmesini gerektirmektedir.

Evde sađlık bakımı hizmetleri alanında alıřtığımız tezin ilk bölümünde, ilgilenmek durumunda olduğu hastaları evde ziyaret eden veya telefon ile arayan bir hemřire için deterministik ve statik bir problem incelenmektedir. Hasta sayısının fazla olması ve hasta durumlarının deęişiklik göstermesinden dolayı, her hemřire önündeki iş günleri içinde hangi hastaları ziyaret edeceğini dikkatle belirlemek zorundadır. Bu alıřma erevesinde her hastanın son ziyaret edilme tarihi, sađlık durumu gibi kriterleri göz önünde bulunduran bir önceliklendirme metrięi tanımlanmaktadır. Böylelikle herhangi bir hasta ziyaret edilmedike bu öncelik metrięi onun için artmaktadır. Bu problem için geliştirilen sistemin ürettięi özüm aynı anda hangi hastaların ziyaret için seçileceğine ve seçilen hastaların hangi sıra ile ziyaret edileceğine karar vermektedir. Bu hedefe varmak için karar verilen amaç fonksiyonu ilk olarak toplam öncelik metrięini maksimize ederken ziyaret için gerekli olan seyahat süresini minimize etmektedir. Bu erevede geliştirilen kesin özüm yaklaşımının özüm süresi açısından sınırları (computational limits) gözlemlendikten sonra, gerek boyutlu veri

ölçekleri için optimal çözüme yakın sonuçlar üreten adaptif bir algoritma (ALNS) ile bir matheuristic geliştirilmiştir. Geliştirilen iki algoritmanın ürettiği sonuçların kalitesi bir Lagrangean modeli ile üretilen üst limitler kullanılarak hesaplanmıştır. Gerçek verilerle yapılan testler iki algoritmanın da yüksek kaliteli çözümler ürettiğini göstermektedir. Bununla birlikte büyük boyutlu problem örneklemelerinde matheuristic yaklaşımın daha iyi sonuçlar verdiği gözlenmektedir. Diğer taraftan ALNS çok kısa süreler içinde iyi sonuçlar verirken, problem örnekleminin boyutu büyüdükçe matheuristic için gereken çalışma süreleri eksponansiyel olarak artmaktadır. Çalışma çerçevesinde öncelik metriği ile seyahat süresi arasındaki ilişki niceliksel bir bağlamda incelenmiş ve bu incelemeler çerçevesinde elde edilen sonuçlar da paylaşılmıştır.

Tezin ikinci bölümünde evde sağlık bakımı probleminin gerçek zamanlı dinamik versiyonu çalışılmıştır. Bu bağlamda planlama periyodu boyunca gerçekleştirilecek hasta ziyaretlerini daha verimli ve efektif yönetmek için bir optimizasyon yapısı geliştirilmiştir. Planlama sürecinin başında geliştirilen ALNS algoritması ile planlama periyodu boyunca hangi hastaların hangi gün ve ne sıra ile ziyaret edileceği belirlenmektedir. Daha sonra her gün çeşitli sağlık sorunlarından dolayı bazı hastalar acil ziyaret edilme olasılığı oluşmaktadır. Acil vakalar gün içerisinde belirli bir saate kadar için kabul edilmektedir. Daha önceden belirlenmiş acil hasta kabul edilme son vakti geldiği zaman acil hastalar ve gün içerisinde henüz ziyaret edilmemiş hastalar için optimize bir plan üretecek bir tamsayılı matematiksel model ile o günün planı yeni baştan yapılmaktadır. Bu süreç sonunda elde edilen çözümler acil hastaların bekleme süreleri, ziyareti ertelenen hasta sayısı, acil hasta sayısı gibi nicelikler göz önüne alınarak değerlendirilmektedir. Geliştirilen optimizasyon yapısı gerçek veriler kullanılarak simülasyon çalışmaları ile test edilmiştir.

ACKNOWLEDGMENTS

Firstly, I would like to express my sincere gratitude to my adviser Prof. Dr. F. Sibel Salman for the continuous support during my Ph.D study and related research, for her patience, motivation, and immense knowledge. Her guidance helped me in all the time of research and writing of this thesis.. I believe that if she was not there in times of despair, maybe it would be impossible for me to write these lines. I sincerely thank to Prof. Dr. Burçin Bozkaya for his support and invaluable ideas during the research. I also sincerely thank to the members of my thesis advisory committee: Prof. Dr. Necati Aras and Prof. Dr. Metin Türkay for their time, ideas and comments during the entire process of this thesis. I would also like to thank my other thesis committee member Assoc. Prof. Dr. Özgür M. Araz for his time and support by reading my thesis, listening to my work, and giving great feedback on it.

In the long nights when we were reading poems from earlier centuries with my father and mother, they were not aware but my darkness was getting enlightened each time and the void inside my heart was filling with their compassion. I do not know how to thank them. Yusuf is not only my brother, he is also my best friend. He is someone that even the idea of his absence breaks my heart and burst my eyes into tears. I also would like thank Ahmet Yükseltürk, Can Candemir, Çetin Şahin, Levent Erdal Aygün, Sinan Ziya Padak and Uğur Yılmaz for being such good friends.

Finally, I would like to thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for the financial support during my master years.

TABLE OF CONTENTS

List of Tables	xi
List of Figures	xii
Nomenclature	xv
Chapter 1: Introduction	1
Chapter 2: Literature Review	8
2.1 Deterministic and Static Studies in Home Healthcare Routing and Scheduling	10
2.2 Orienteering Studies	18
2.3 Dynamic Studies in Home Healthcare Routing and Scheduling	20
Chapter 3: Deterministic and Static Approach	24
3.1 Problem Formulation	24
3.1.1 Prize Calculation	24
3.1.2 MIP Model	27
3.2 An upper bound by Lagrangean Relaxation	29
3.2.1 Subgradient Optimization Method	31
3.3 Solution Methods	32
3.3.1 A Matheuristic	32
3.3.2 An ALNS Algorithm	34
3.4 Computational Results	40
3.4.1 Objective Trade-off Analysis	42

3.4.2	MIP Results	44
3.4.3	Lagrangian Relaxation Results	45
3.4.4	Comparison of the Proposed Heuristics	47
3.5	Conclusions	54
Chapter 4:	Dynamic Approach	56
4.1	Problem Definition	56
4.2	Optimization Framework	58
4.2.1	Solution Approach	60
4.3	Computational Results	64
4.3.1	An Illustrative Example with Single Re-optimization	64
4.3.2	An Illustrative Example with Multiple Re-optimization	72
4.3.3	Simulation Results	79
Chapter 5:	Conclusion	95
Chapter 6:	Appendix	98
	Bibliography	109

LIST OF TABLES

3.1	Routes with constant prizes	26
3.2	Routes with decreasing prizes	26
3.3	ALNS parameters	35
3.4	The ALNS Heuristic	36
3.5	MIP Statistics	45
3.6	Lagrangian Relaxation Results for 5-day Runs	46
3.7	Lagrangian Relaxation Results for 10-day Runs	47
3.8	MIP - SSPH - ALNS Statistics with Loose Time Windows	48
3.9	SSPH - ALNS with Tight Time Windows (TTW) 5-Days	50
3.10	ALNS with TTW 10-Days	52
4.1	Statistics for Re-Optimization Times	79
4.2	Simulation Results for DPHHP with Poisson Arrivals Rate = 1	82
4.3	Simulation Results for DPHHP with Poisson Arrivals Rate = 1/2 . .	85
4.4	Simulation Results for DPHHP with Poisson Arrivals Rate = 1/3 . .	87
4.5	Simulation Results for DPHHP with Poisson Arrivals Rate = 1/4 . .	90

LIST OF FIGURES

1.1	Published Articles in ScienceDirect	1
3.1	Patient locations	42
3.2	Total Collected Prize vs. Total Route Time	43
3.3	Removal algorithms statistics for 5 days	53
3.4	Insertion algorithms statistics for 5 days	53
3.5	Removal algorithms statistics for 10 days	53
3.6	Insertion algorithms statistics for 10 days	54
4.1	The Simulation Process for the Dynamic Solution Approach	59
4.2	Replication 1 - Poisson Rate $\lambda = 1$	65
4.3	Simulation 2 - Poisson Rate $\lambda = 1$	65
4.4	Replication 3 - Poisson Rate $\lambda = 1$	66
4.5	Replication 4 - Poisson Rate $\lambda = 1$	66
4.6	Replication 5 - Poisson Rate $\lambda = 1$	67
4.7	Day 1 Schedule	67
4.8	Day 1 Modified Schedule	68
4.9	Day 2 Schedule	68
4.10	Day 2 Modified Schedule	68
4.11	Day 3 Schedule	69
4.12	Day 3 Modified Schedule	69
4.13	Day 4 Schedule	70
4.14	Day 4 Modified Schedule	70
4.15	Day 5 Schedule	71

4.16 Day 5 Modified Schedule	71
4.17 Day 1 Schedule	72
4.18 Day 1 Modified Schedule	73
4.19 Day 2 Schedule	73
4.20 Day 2 Modified Schedule - 1	73
4.21 Day 2 Modified Schedule - 2	74
4.22 Day 3 Schedule	74
4.23 Day 3 Modified Schedule	75
4.24 Day 4 Schedule	75
4.25 Day 4 Modified Schedule - 1	76
4.26 Day 4 Modified Schedule - 2	76
4.27 Day 5 Schedule	76
4.28 Day 5 Modified Schedule - 1	77
4.29 Day 5 Modified Schedule - 2	77
4.30 Visit Statistics	78
4.31 Patients Statistics - Poisson Rate $\lambda = 1$	80
4.32 Overtime (hrs) - Poisson Rate $\lambda = 1$	81
4.33 Patients Statistics - Poisson Rate $\lambda = 1/2$	83
4.34 Overtime (hrs) - Poisson Rate $\lambda = 1/2$	83
4.35 Patients Statistics - Poisson Rate $\lambda = 1/3$	84
4.36 Overtime (hrs) - Poisson Rate $\lambda = 1/3$	86
4.37 Patients Statistics - Poisson Rate $\lambda = 1/4$	88
4.38 Overtime (hrs) - Poisson Rate $\lambda = 1/4$	88
4.39 Total Overtime with different arrival rates	91
4.40 Emergencies with different arrival rates	92
4.41 Postponements with different arrival rates	92
4.42 Total served patients with different arrival rates	93

6.1	Routes for Nurse 1 on Day 1	99
6.2	Routes for Nurse 1 on Day 2	99
6.3	Routes for Nurse 1 on Day 3	100
6.4	Routes for Nurse 1 on Day 4	100
6.5	Routes for Nurse 1 on Day 5	101
6.6	Routes for Nurse 2 on Day 1	101
6.7	Routes for Nurse 2 on Day 2	102
6.8	Routes for Nurse 2 on Day 3	102
6.9	Routes for Nurse 2 on Day 4	103
6.10	Routes for Nurse 2 on Day 5	103
6.11	Routes for Nurse 3 on Day 1	104
6.12	Routes for Nurse 3 on Day 2	104
6.13	Routes for Nurse 3 on Day 3	105
6.14	Routes for Nurse 3 on Day 4	105
6.15	Routes for Nurse 3 on Day 5	106
6.16	Routes for Nurse 4 on Day 1	106
6.17	Routes for Nurse 4 on Day 2	107
6.18	Routes for Nurse 4 on Day 3	107
6.19	Routes for Nurse 4 on Day 4	108
6.20	Routes for Nurse 4 on Day 5	108

NOMENCLATURE

PHHP: Prioritized Home Healthcare Problem

DPHHP: Dynamic Prioritized Home Healthcare Problem

MIP: Mixed Integer Programming

ALNS: Adaptive Large Neighborhood Search

CPLEX: A commercial solver for mathematical optimization

OP: Orienteering Problem

TOP: Team Orienteering Problem

SSPH: Successive Single Period Heuristic

SPM: Single Period Model

LST: Last Service Time

TW: Time Windows

Chapter 1

INTRODUCTION

As life expectancy and the percentage of the working class have risen rapidly, the need for home healthcare has increased astonishingly in European countries and other regions [22]. According to a study by Grand View Research, the need for home healthcare is anticipated to increase 3% annually.

Increasing cases of chronic diseases coupled with rise in geriatric population is anticipated to fuel the market growth. According an article by United Nations in 2017, elderly population accounted for nearly 13% of the global population and is anticipated to increase by 3% annually. Furthermore, elderly individuals are prone to chronic diseases and need more healthcare services, which is anticipated to drive the market.

Accordingly, the amount of research for effectively designing home healthcare services has increased especially within the last decade [15]. In Figure 1.1, the increase in the amount of research related to home healthcare studies is depicted.

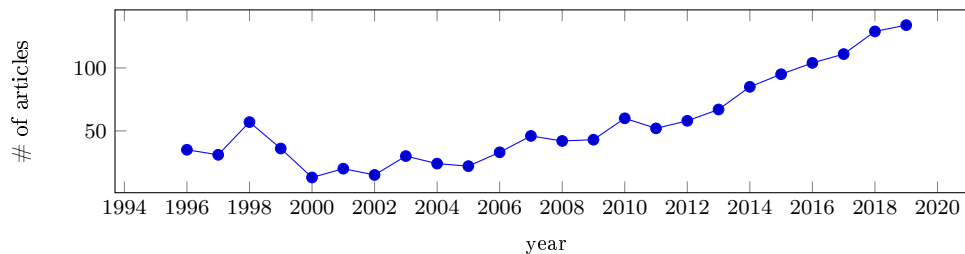


Figure (1.1) Published Articles in ScienceDirect

In [26] Nadine et al. discusses how home healthcare is conceived very differently

across countries and sectors and they define home healthcare as a professional care that is provided by caregivers and health professionals.

This study showed that the term ‘home care’ is understood very differently across countries and sectors. The services included vary considerably among countries and even ‘home’ turns out to be an elastic term. Home care can be conceived of as any care provided behind someone’s front door or, more generally, referring to services enabling people to stay living in their home environment. In some countries, ‘someone’s front door’ can include a home for the elderly. As regards the type of services, home care may refer to care given only by professionals or in combination with care given by a spouse or relative (personal care or housekeeping). The EU-RHOMAP study has defined home care as care provided by professional carers within clients’ own homes. Professional care that relieves informal caregivers (respite care) has also been taken into account. The study has been limited to care provided to adults. Furthermore, home care has been considered in its context, such as demographic developments and attitudes in society towards informal care. Although the main focus is on formal care (provided by professional carers), informal care has been taken into account as complementary to formal care and as a co-determinant in the allocation of formal care. Informal care is provided by spouses or family members, friends and volunteers who are usually not paid; and by privately hired non-professionals who are paid informally.

Home healthcare is a special service provided by health professionals. A health professional is someone who has sufficient skills for providing health-related care. Home healthcare has a very broad spectrum in terms of the types of services. The people who need home healthcare may suffer from multiple chronic conditions, such as diabetes or dementia, and functional impairments, such as difficulty with the basics of life like mobility and managing one’s household. Thus, a health professional may help for the chronic conditions as well as very basic life support.

Healthcare services are provided by a crew of health professionals. Each health professional generally have a set of patients for whom they are responsible for providing health related services. Each patient has a different need and they are assigned to health professionals according to their needs and their geographic locations. The health professionals should have schedule on each planning period to serve their patients as effectively as possible. In Turkey, there are a few companies that provide home healthcare services. Our research has been motivated by one of the healthcare companies. This company has a crew of professionals and they need to provide service to their patients. Patients are clustered in terms of their geographic locations. Therefore, each clustered area is assigned to a health professional. Health professionals are supposed to make their daily and monthly plans by considering the patients assigned to them. There appears to be an inefficiency when the plans are made by health professionals. Our research is motivated by the need to create effective and efficient plans in order to better manage the healthcare operations and increase the utilization of the nurses as well as provide better services.

The problem addressed in this study is a patient scheduling and nurse routing problem which has been introduced to us by a company that sells medical nutrition products. The company provides free follow up services to its patients who are fed orally or by tube. The company employs a group of nurses and each nurse has a pre-determined group of patients for whom he is responsible. Each nurse needs to either visit each of her patients at his/her home, within the requested time window, or call him/her once over a multiple-day planning horizon, for instance two weeks. In the current system, each nurse decides by herself how to follow up her patients. As such, the managers cannot control whether the nurses visit the right patients in the right order and at the right time. In this company, there are approximately 80 nurses who are responsible for patients all over Turkey. A set of patients is assigned to each nurse according to the proximity of home locations. When a patient is visited by a nurse,

an associated service time is incurred. The number of patients that can be visited in a day is limited due to regulated working hours. A nurse is responsible for 180 to 350 patients regularly per month. Considering these circumstances, it is clear that the number of patients that can be visited per month is limited. Hence it becomes important to determine which patients to visit in an upcoming planning period. Also, the daily routing decisions specify the visiting order and time of the patients, as well as the routing cost. The company wants to plan each nurse's schedule by considering patient priorities and distances between the patients' homes.

Patient priority for an upcoming visit is considered to be dynamic, meaning that it should be updated over time. The priority is set considering several factors such as the last visit time, severity of the patient's condition, etc. As time progresses, the priority of an unvisited patient should increase, typically in an exponential manner to account for increasing urgency. It is critical to visit patients with high priorities on earlier days of the planning horizon for effective service, while grouping nearby patients to the same day as much as possible for efficiency.

Our objective is to develop an approach to construct a schedule and the daily routes for each nurse to maximize both service effectiveness and cost/time efficiency. A schedule is valid and can be realized if it respects the daily total working hours of a nurse and the time window requests of the patients. We consider situations where the number of patients that the nurse is responsible for is too high to allow a visit to each patient within the planning horizon, as in the company under consideration. Patients who cannot be visited within the upcoming planning horizon are checked upon by phone calls. Under these circumstances, a policy is set such that a patient can be visited at most once within a planning horizon. We model this problem as a multi-period orienteering problem with time windows where the goal is to collect as much prize as possible by visiting nodes within a limited amount of travel time. Here the notion of "prize" should be defined carefully to ensure early visits to high

priority patients. As the priority of an unvisited patient increases by time, correspondingly, the prize obtained from visiting a patient should decrease so that earlier visits are preferable. This property adds an interesting dimension to the problem which we name as the Prioritized Home Healthcare Problem (PHHP). PHHP is NP-hard as its special case with a single period in the planning horizon is the orienteering problem with time windows, which is known to be NP-hard [12]. To the best of our knowledge, this is the first study where patient priorities are modeled explicitly as time-dependent parameters of a home healthcare routing and scheduling problem. Furthermore, the underlying structure of our problem is a multi-period orienteering problem with time-windows and time-dependent prizes, which has not been studied in the literature before.

In the first part of our proposed approach, we formulate a mixed integer programming (MIP) model, a matheuristic called the Successive Single Period Heuristic (SSPH) and an Adaptive Large Neighborhood Search (ALNS) algorithm to solve the described problem. The ALNS algorithm employs a variety of destroy and repair moves utilizing the time-dependent structure of the problem. We also implement a Lagrangean relaxation procedure to be able to evaluate the quality of the feasible solutions output by our algorithms.

Although the solution methods introduced in first part of the thesis perform quite effective in terms of routing and scheduling, the real problem faced by the health professionals has a potential dynamic extension. The health professionals follow up their patients and at the beginning of each planning horizon they decide the entire plan for the planning horizon. Thus, on each day they know which patients to visit and in which order. However, some patients might have chronic problems such as alzheimer or cancer so that their situations might become critical. In these situations, they become emergent patients and they might require immediate service at their homes. Therefore, while the plans created at the beginning of the planning horizon is under

execution, the health professional may have to change the plans when emergent patients appear. The patients are classified either emergent patients or regular patients. Emergent patients should be visited on the same day while regular patients might be postponed for the next days. However, as the patients are postponed their priority increase so that they will be served as soon as possible.

In the second part of the thesis, we extend our study and introduce a solution methodology to handle the dynamic arrivals of emergent patients during the day. We define an optimization framework to increase the effectiveness and efficiency of the visits during the entire planning horizon. At the beginning of the planning horizon, an Adaptive Large Neighborhood Search (ALNS) algorithm runs to determine which patients to visit on each day of a multi-period planning horizon and in which order to visit them to maximize the total priority of visited patients and to minimize the total routing time. Then, on each day some emergent patients with urgent conditions arrive dynamically. The emergencies are accumulated until predetermined re-optimization times. When a re-optimization time comes, the current plan is re-optimized with the accumulated new arrivals by a mixed-integer programming model (MIP). Solutions generated with respect to different processing rules for new arrivals are evaluated with respect to several performance measures, including the waiting time of urgent patients and the number of patients whose visit times are delayed.

In literature, majority of the studies concerning home healthcare scheduling and routing problems are modeled as different variants of the well-known vehicle routing problems. As such, in a planning horizon all patients are required to be visited. However, there is an imbalance between supply and demand for the home healthcare services. It means that there are a few health professional to provide healthcare services while there are a lot of patients who require to get services. The health professionals may be perceived as supply and patients may be perceived as demand. This imbalance between supply and demand makes it impossible to serve all the patients

in a planning horizon. Thus, a new metric is needed to differentiate the patients. While a differentiating metric is introduced, we need to model the scheduling and routing problem by a different formulation rather than as a variant of vehicle routing problem. In this thesis, we model the scheduling and routing problem as an team orienteering problem and we introduce a priority metric to differentiate the patients by considering their health situations and last service times. For the reasons that will be clear in Chapter 1, we define a time-dependent prize metric to model the priority parameter for each patient. To the best of our knowledge, we are the first who introduce a time-dependent prize function in order to determine routes and schedules for the entire planning horizon for a health professional. Also, we introduce a generic optimization-based solution methodology to solve the dynamic situations encountered in real-life problems that requires a simultaneous scheduling and routing decision where the demand points have different priorities.

The rest of this thesis is structured as follows. Chapter 2 provides a literature review on studies related to static and dynamic home healthcare scheduling and routing and the orienteering problem since the structure of this general routing and selection problem resembles the structure of the problem that we defined to solve the home healthcare problem. In Chapter 3, we present a deterministic and static approach to optimize the simultaneous scheduling and routing decisions in the provision of home healthcare services. This approach is based on a mixed-integer programming formulation and an adaptive large neighborhood search algorithm. Chapter 4 discusses a dynamic approach for the scheduling of home visits where an optimization-based solution methodology is introduced for handling emergency cases. The methodology is evaluated by a simulation study. Finally, Chapter 5 concludes the thesis and discusses some future research directions.

Chapter 2

LITERATURE REVIEW

Increased life expectancy, especially within the last three decades, has stimulated a need for home healthcare services. Correspondingly, the number of organizations providing home healthcare services has been increasing [26]. However, these organizations often have difficulties in coping with the constantly increasing demand for home healthcare. Therefore, a need to optimize the operations of such organizations under tight resources arises. Various articles concerning the daily routing and scheduling of nurses responsible for home healthcare have been published over the last years. The common characteristics in these studies are minimizing the total distance traveled by the nurses, while obeying the preferences of the nurses or their patients, such as time windows on the arrival times, and matching the skills of the nurses with patient requirements.

Routing problems with different attributes are classified under four categories in terms of data availability and uncertainty ([27], [48], [45]). Based on the information evolution, if the input is known beforehand then the problem becomes static. If the input changes over time, then the problem becomes dynamic. In terms of information quality, input can be deterministic and stochastic. Therefore, a problem can fall into one of the following four categories: static and deterministic, static and stochastic, dynamic and deterministic, dynamic and stochastic.

In static and deterministic problems, input data is known beforehand and there appears no change in the plans during the execution of the routes. The classical version of this VRP structure has been extensively studied in the literature ([60], [34]).

In static and stochastic problems, the input data is partially known and often characterized by an underlying probability distribution. The realizations of the random variables are revealed during the execution of actual plans. It is important to note that the routes and schedules are designed a-priori and only minor changes are allowed. The application is independent of technology. The uncertainty may be seen in any of the input data including travel times, stochastic service times, whether a customer is going to be served or the demand of a customer. The most studied uncertainty situations are: stochastic customers [63], stochastic times [33] and stochastic demands [30].

In dynamic and deterministic problems, either some part of the input data or all of it is unknown and dynamically revealed during the execution of the plan. For these class of problems, there are no a-priori routes which must be obeyed during the execution. The routes are subject to changes in an ongoing fashion. Hence, technological support is required and is quite useful with these kind of problems in order to ease the communication between the decision maker and the vehicle [32].

In dynamic and stochastic problems, the input data is either partially known or not known at all. However, contrary to the previous category, it has an exploitable stochastic knowledge which is revealed dynamically. The vehicle routes are not necessarily subject to an a-priori plan. The routes can be changed in an ongoing fashion. A comprehensive overview of the problem types in this category and proposed solution strategies can be found in [45].

Our problem has three main divisions which directs us to examine different studies in literature. The first part of our study focuses on a deterministic and static problem where we solve a scheduling and routing problem for home healthcare services at the beginning of each planning horizon. In the second part of the thesis, we face a real-life problem where emergent arrivals might be encountered so that the

plans under execution might require a change. Thus, we need to examine studies where deterministic problems are introduced as well as we need to examine studies in which dynamic approaches are discussed. Finally, our modeling approach is based on orienteering problem. Therefore, we also need to study the articles for orienteering problem. Hence, we divide the literature review into three main category where we discuss deterministic studies, orienteering studies and dynamic studies in home healthcare routing and scheduling.

2.1 Deterministic and Static Studies in Home Healthcare Routing and Scheduling

Home healthcare planners face difficult optimization problems including staff scheduling, staff assignment and routing decisions [43], [59]. The related studies often focus on solving real-life problems and examples of applications are seen mostly in North America and Europe. Begur et al. [4] provide a tour and roster planning model for a US-based company, whereas Eveborn et al. [20] focus on a staff planning problem arising in Sweden where people receive home healthcare from the local authorities. Maya et al. [17] study a real-life home healthcare scheduling and routing problem, where the visit frequencies and service durations are determined in a pre-admission process for each patient. They provide a solution method for a non-profit organization located in Belgium providing home healthcare services in different provinces.

In [13] Cheng et al. provide two different mixed-integer programming formulation one with double-indexed and the other is triple-indexed. Their objective is to minimize the cost associated with the amount of overtime and part-time work that is scheduled. They also present a heuristic approach to solve a home-healthcare problem. In the heuristics they devised collections of tours for each nurse are generated using a randomized greedy algorithm. The randomized greedy algorithm initially construct a solution. In the next step, the algorithm tries to remove assignments of two nurses where at least one of those nurses working overtime. The heuristics

algorithm is tested using random instances and nearly half of the test instances the optimal solutions are reached in less than seconds whereas the exact approach with triple-indexed MIP formulation the run-times varied between 3 and 20 minutes.

In [10] Burke et al. give an extensive review and discussion over a wide range of nurse scheduling papers which cover a broad spectrum of models, methodologies, and approaches that tackle with the nurse scheduling problem. They classified the approaches in literature in terms of their usage. Majority of the studies are classified as not applied in practice but tested on real data. Rest of the studies are labeled as applied in practice. They further divide these studies into those approaches applied in just one hospital and approaches applied in multiple hospitals. They mainly focus on the fact that the many of the attempts to tackle the nurse scheduling have just concentrated on simplified versions. They also outline some of the important issues that need to be taken into consideration in future studies These are multi-criteria reasoning, flexibility and dynamic reasoning, robustness, ease of use, human/computer interaction, problem decomposition, exploitation of problem specific information, hybridization, and inter-disciplinarity. Prioritized home nurse scheduling problem is a combination of the vehicle routing problem with time windows (VRPTW) [9] and the nurse rostering problem (NRP). Both problems are known as NP-hard problems and difficult to solve in practice. Exact methods can be used to solve very small instances. They create an initial solution using this hybridization approach using linear programming and constraint programming techniques. Then the solutions are improved using meta-heuristics.

In [5] Bertels et al. design an optimization kernel which is a combination of linear programming, constraint programming and meta-heuristics which are simulated annealing and tabu search. The core optimization model is a hybrid of a rostering model and routing model. Due to violation of hard/soft qualification and nurses' time windows constraints, they assert that it is not possible to use a two-stage ap-

proach to first generates feasible routes and then assigns nurses to them. Hence, they propose an integrated approach that considers time scheduling, rostering and route planning simultaneously.

In [20] Eveborn et al. focus on a staff planning problem arising in Sweden where people receive home care from the local authorities. The objective is to develop visiting schedules for care providers that incorporate some restrictions and soft objectives. Each visit has a particular task to be performed, for example: cleaning, washing, personal hygiene and/or nursing activities. Each staff member has skills and each client should, if possible, be visited by the same contact person. The operational situation is continuously changing and planning is done each day. They describe the development of a decision support system Laps Care to aid the planners. The system consists of a number of components including information data bases, maps, optimization routines, and report possibilities. They formulate the problem using a set partitioning model and, for a solution method, we make use of a repeated matching algorithm. Starting from an initial matching, repeated matching algorithm iteratively produces new matchings by local improvements.

In [21] Eveborn et al. present the benefits of their optimization based planning tool LAPS CARE, a system that was developed in 2002, uses operations research modeling to eliminate the manual planning of home care unit assignments and they provide some financial and operational improvements. Based on the data collected from different healthcare companies using their planning tool it is observed that the system has increased operational efficiency by 10-15 percent; this corresponds to an annual savings of 20-30 million euros (\$30-\$45 million).

In [49] Rasmussen et al. formulate the home care crew scheduling problem as a set partitioning problem with side constraints and provide an exact branch-and-price solution algorithm. They have a multi-criteria objective. They associate a cost with

the uncovered visits and for each patient they assume a visit preference and these two costs are further incorporated with the total traveling costs. They model a daily scheduling problem with multiple nurses where temporal dependencies are observed. They model these temporal dependencies as generalized precedence constraints. We also introduce priorities for the visits. What differentiates our approach from their studies is that in our model the visit priorities dynamically change and we provide a multiple days model.

In [43] Stefan et al. concentrate on routing and scheduling problems arising in home health care industry. Their task by solving home health care problem (HCCP) is to create a service plan with nurses and patients such that the patients are served with the nurses assigned to them. To tackle this problem they propose a two stage approach. In the first stage a constraint programming based heuristic provides feasible solutions within allowed time period. Given additional time if possible, a hybrid constraint programming-adaptive large neighborhood search looks for improving the initial solution. In the second stage they apply an adaptive large neighborhood search (ALNS) to improve the solutions obtained in the first stage. They tested their approach with real world data. Their future direction is to integrate the approaches into a decision support system for home health care planning. They also pose the question of increasing demand. As demand is currently increasing the health care providing organizations have to answer which customers to include next in the waiting list. The approach we provide in our prioritized home health care model tackles this situation.

In [8] Kris et al. investigates the trade-off between two conflicting objectives which are operating costs as opposed to service level. The model they propose try to minimize operating costs while maximizing the service level offered. They propose a bi-objective model for home-care scheduling and routing. They devised a meta-heuristic algorithm based on a large neighborhood search heuristic in multi-directional local search framework. For a real life data they performed several computational experi-

ments and observed that with small instances their algorithm performs well enough. They conclude that for companies that offer health-care services there exist an immense trade-off between costs and client convenience.

In [39] Dorota et al. proposes a model that encapsulates daily planning of the health-care services provided by a home care company. There are several issues that they take into account while preparing the plans. Patients have individual service requirements and the qualifications of the staff has to be in parallel with the requirements of the patients. They handle possible interdependencies where a patient may require different services that need more than one nurse. They propose a mathematical model and solution method that includes a sophisticated heuristic approach for the home health-care and services. Their heuristic achieves reduced traveling costs for home care providers at the same time less waiting times for patients.

In [59] Felipe et al. developed a mixed integer programming model for periodic schedule of medical staff and simultaneously they present routes for patient visits. Based on the type of pathology, a patient may require a specialist with specific abilities. They categorizes the specialists into three groups that are doctors, nurses and therapists. Depending on the conditions of the patients more than one person from different categories may visit the patients. The model they propose handles the synchronization of the visits. They validated their model by a test data provided a Colombian company in which there are 16 patients requiring certain type of services such as domicile, blood anticoagulation, chronic care and palliative care. The solution they obtained determines the visit order per day to provide the necessary service each patient requires.

In [17] Maya et al. studies a real life problem observed by a non-profit organization located in Belgium providing home care services in different provinces. The problem that shed light under the supervision of the company involves two objectives, namely service level maximization and total travel distance minimization. Despite the stan-

dard approach adapted in literature as having to solve these conflicting objectives simultaneously, they prefer to solve the problem in a hierarchical order in which the service level optimization is performed first. A mathematical model based on set partitioning problem is proposed and they designed a two-stage approach to tackle the problem under consideration. In the first stage, they aim to maximize service level while the second stage minimizes the total distance traveled.

In [37] Ran et al. solves a periodic vehicle routing problem in health-care logistics. In the problem they investigate there are three types of demands of patients. There must be a need to transport medical devices or drugs between home-care provider's depot and patients' homes. Another demand is the delivery of particular drugs from hospitals to patients' homes and finally blood samples may be delivered to the lab from patients' homes. Each patient must be visited a certain number of times in a planning period and they have a predetermined set of possible visit days. There are time windows constraints for the lab, the hospital and patients. They would like to minimize the maximal routing cost within the set of all plausible routes by determining the visit days for each patient. In order to solve the problem they developed a tabu search algorithm with distinct local search strategies using feasible and infeasible moves. They reported that it is an interesting hybridization starting with the infeasible local searches with small probabilities and then with high probabilities using feasible local searches. Their solutions have been tested with benchmark data and numerical results showed that the algorithm they proposed performs well. They also performed several experiments with a real data provided by a home care company and their approach reduced the total cost while balancing the workload between caregivers.

Recently, Fikar and Hirsch [22] have provided a comprehensive overview of the studies related to home healthcare scheduling and routing. They divide home healthcare scheduling and routing problems into two main categories, single-period and multi-

period, each having different ramifications. The majority of research papers focuses on single-period optimization problems, such as [11], [1], and [3]. The studies are further divided into subcategories in terms of their objectives, constraints and solution methods. The objectives in healthcare studies are mainly minimizing travel time, travel cost, travel distance, overtime, waiting time, number of nurses, as in [31], [61], and maximizing the number of served tasks and client or nurse preferences, as in [20], [49]. The constraints of single-period models are time windows, skill requirements, working time regulations and precedence. While time windows are important in certain time-sensitive procedures such as insulin injection, the majority of studies use soft time windows (e.g. [5], [20], [61], [40]). The solution methods used in single-period home healthcare problems are Variable Neighborhood Search [40], Branch-and-Price [49], Fuzzy Simulated Evolution Algorithm, Memetic Algorithm [16], Simulated Annealing Algorithm, Scatter Search [31] and Particle Swarm Optimization [11]. Steeg and Schröder [58] propose a hybrid approach that combines the strengths of constraint programming and the large neighborhood search metaheuristic, where their main objective is to minimize the number of nurses that visit a patient during the schedule, in other words, maximizing the nurse-patient loyalty.

Shi et al. [56], solve a healthcare routing problem with stochastic travel and service times which comes from the logistics practice of a home healthcare company. They propose a stochastic programming model with recourse (SPR), a Hybrid Genetic Algorithm (HGA) and a simulation model, and they compare the results of the algorithms on an instance with 25 patients. HGA has the winning performance. Shi et al. [57], introduce time windows and fuzzy demand in their earlier models mentioned in [56]. Again, HGA performs well in both large and small size instances. Florian et al. [28] provide a set partitioning heuristic for a home healthcare routing and scheduling problem. Their method is based on a set partitioning formulation and a large neighborhood search framework (LNS). They test their algorithms using three different sets of instances with different problem sizes. According to their results, a

dramatic reduction in solutions' costs for the instances (respectively 13.76%, 20.82% and 14.39% for the small, medium and large sets) is observed.

Nasir and Dang [42] also study a home healthcare scheduling and routing problem. They propose an integrated three-phase decision support methodology for the home healthcare system, where they aim to determine well-defined decision rules to prevent a low satisfaction levels among all the stakeholders of the home healthcare system. In the first phase, they construct a mixed integer linear programming (MILP) model and develop a Variable Neighborhood Search (VNS) based algorithm to find heuristic solutions for the problem. In the second phase, they use Bender's method and Receiver Operating Characteristic (ROC) curves to determine thresholds based on the solutions obtained from MILP and VNS. In the third phase, they create a new solution for the problem with new data and validate the threshold values obtained in the second phase. The threshold values are used as a tool for the process of determining patient's selection, staffing level, and scheduling of the healthcare staff.

Multi-period home healthcare problems differ from single-period problems as service requests and availability of nurses may change from day to day. This requires taking into account constraints related to working time regulations and continuity of care considerations. Based on a real-life application, Begur et al. [4] introduce a decision support system, which includes a MIP formulation to optimize nurses' routes for a five-day planning problem. Bachouch et al. [3] also consider a planning horizon of five consecutive days. They solve a routing problem by an integer linear program to determine which nurse should be used and when to provide the service during the planning horizon. Their model is tested with different problem sizes: from 3 to 7 nurses and 7 patients to 20 patients. In the case of planning 7 nurses and 20 patients, they find a feasible solution in 34 seconds. A multi-objective approach is developed by Maya et al. [17], where a decision hierarchy in the solution approach including a two-stage solution procedure is implemented.

Manerba and Mansini [38] study a nurse routing problem. They model the problem as the multi-vehicle traveling purchaser problem and develop a branch-and-price approach based on a set covering reformulation of their original model. Based on their computational tests, the branch-and-price approach provides efficient results. Mousavi [41] et al. study a multi-period home healthcare planning problem where they assign patients to staff and staff to days. A patient requires services that vary from one day to another. They provide a generalized assignment formulation. They also introduce a matheuristic approach based on the decomposition of the formulation. Their model computes the total and daily number of staff members needed to cover all services, the services (patients) assigned to each staff member, the periods in which each staff member is assigned to each patient in each day and the routes that each staff member must travel in each day. They use a real dataset for 31 instances where the duration of the planning horizon varies between 2-14 days, number of patients varies between 10-30 and the number of staff members varies between 4 and 10. They show that for 90% of the instances, their matheuristic performs better than the MILP formulation.

2.2 Orienteering Studies

Many studies on home healthcare routing and scheduling problems base their approaches on different variants of the vehicle routing problem [60]. By considering the lack of sufficient number of nurses in a resource-constrained organization, contrary to the majority of the studies in the literature, we perceive the routing problem as a multi-period orienteering problem. The name "Orienteering Problem" (OP) originates from the sport game of orienteering [12]. In this game, individual competitors start at a specified control point, try to visit as many checkpoints as possible and return to the control point within a given time frame. Each checkpoint has a certain score (prize) and the objective is to maximize the total collected score. The OP is a combination of vertex selection and determining the shortest Hamiltonian path in-

cluding the selected vertices. As a consequence, the OP can be seen as a combination of the Knapsack Problem (KP) and the Travelling Salesperson Problem (TSP). The OP's goal is to maximize the total score collected within a time limit, while the TSP tries to minimize the total travel time or distance. Furthermore, not all vertices have to be visited in the OP [62]. A comprehensive review on orienteering problems is given in [29].

The team orienteering problem (TOP) generalizes the OP in that a given number of people, i.e. k team members, collect prizes by completing k tours within the given time limit. The total prize collected in the k tours is maximized [18], [35]. When looked closely, one sees a resemblance between TOP and the multi-period home healthcare routing and scheduling problem that we have defined. The nurse's tour on each day of the planning horizon in PHHP corresponds to the tour of each team member in TOP. As such, our approach follows the team orienteering problem structure [23], [7], [36]. However, we also need to integrate the time effect in the prizes. In general, prizes are constant in orienteering settings. However, there are few studies where the prizes are variable [19]. Our approach defines a time-dependent prize function so that visiting a patient on different days makes a difference. To the best of our knowledge, a multi-period orienteering problem with time-dependent prize function has not been studied before.

In [18], Erdoğan et al. study an attractive traveling salesman problem where they partition the vertex set into facility vertices and customer vertices. They aim to create a maximum profit tour from a subset of facility vertices. They introduce an attraction function where every visited facility attracts a portion of the profit. They formulate their problem as an integer non-linear program where they maximize the total profit. They devise a linearization scheme using the linear tangents of the concave portions of the objective function as valid inequalities, and a branch-and-cut algorithm. They also implement a tabu search heuristic. They conduct computational experiments on

randomly generated instances and on instances derived from TSPLIB for different number of vertices from 50 to 100.

Our study proposes a new modeling approach to home healthcare routing and scheduling problems. The majority of the studies seen in the literature assumes all the patients should be serviced. However, in reality a nurse is often expected to provide service to more patients than he can handle in a limited time. Therefore, it becomes practically impossible to serve the entire patient list in a planning horizon. In addition to this observation, our study also takes into consideration a differentiation of the patients according to their medical conditions. We propose a priority parameter, translated into prizes for optimization purposes, that changes dynamically and is a function of each patient's medical conditions and the last visit time. The idea of creating a time-dependent prize matrix provides us a systematic approach to assign patients to the right days.

2.3 Dynamic Studies in Home Healthcare Routing and Scheduling

In real-time routing problems, the routes assigned to the vehicles are dynamically generated in an ongoing fashion as certain parameters such as travel distances, times, customer demands and requests are revealed over time. Because of the uncertainty embedded in the problem structure, one cannot address and solve these problems using the classical methodologies employed in the transportation and logistics field.

A comprehensive study on the most important and relevant modeling and algorithmic techniques on vehicle routing problems (VRP) has been provided by Toth and Vigo [60]. In contrast to the widely accepted description of the vehicle routing problem, real world applications include two significant aspects: evolution and quality of information [48]. Evolution of information suggests that in some problems the information available for the planner may change during the execution of the routes and schedules, for example, a new customer may arrive. On the other hand, the quality of

information indicates the uncertainty on the data, for example, whether a customer will be visited or the demand of the customer is only known as a range estimate of its real demand [45]. A vehicle route can be designed statically (a-priori) or dynamically. The well-known VRP with Stochastic Demands (VRPSD) can be observed from both perspectives. From a static perspective, the problem is to design a set of routes which might undergo minor changes during the execution of the plans ([6], [25]). From a dynamic perspective, the problem becomes to create vehicle routes in an online fashion which suggests that a communication with the vehicle is required ([44], [53], [54]).

The first study in dynamic routing was conducted by Psaraftis [47] where he investigates a single-vehicle, many-to-many, immediate-request dial-a-ride problem in two parts. The first part focuses on the "static" case of the problem. In this case, the immediate requests that may appear during the execution of the route are not considered. A dynamic programming approach is developed for solving the static problem. In the second part, the immediate requests are considered instantaneously. The author applies an open-ended sequence of updates for each request. Few research was conducted between the work of Psaraftis in 1980 and the late 1990s, but since then a renewed and increasing interest has been seen for the dynamic routing problems including solution techniques ranging from linear programming to advanced metaheuristics [45].

Because of the nature of the dynamic problems, optimal solutions can only be achieved a-posteriori. Moreover, decision makers must give quick decisions whenever a new request arrives during the execution of the plans. Therefore, most dynamic approaches use heuristic algorithms to compute "good" solutions in small run times. Gendreau et al. [24] develop the first application of tabu search to a dynamic routing problem. They solve a vehicle routing problem with time-windows which is motivated by a long distance express courier service. The general idea in their approach is to keep a pool of good solutions that is used to generate an initial solution for a parallel tabu search

algorithm. Whenever a new request arrives, it is checked against all the solutions in the adaptive memory (solution pool) in order to decide whether to accept the request or to reject it.

Angelelli et al. [2] propose short term strategies for a dynamic multi-period routing problem. They consider a dynamic multi-period routing problem faced by a company which needs to address real-time pick-up requests by a fleet of uncapacitated vehicles. They associate a deadline $d \leq 2$ in days with a request which determines whether a request is postponable or not. If $d = 1$ the request cannot be postponed. Hence it has to be satisfied on the same day while if $d = 2$ the request may be served on the same day or it may be postponed to the day after. They propose different short term routing strategies and analyze their impact on the long term objective.

Our study proposes a dynamic optimization framework that utilizes the heuristic algorithm developed for the static home healthcare routing and scheduling problems addressed in [14]. The majority of the studies seen in the literature assumes all the patients should be serviced. However, in reality a nurse is often expected to provide service to more patients than she can handle in a limited time. Therefore, it becomes practically impossible to serve the entire patient list in a planning horizon. In addition to this observation, our study also takes into consideration a differentiation of the patients according to their medical conditions.

In dynamic prioritized home health-care problem (DPHHP), we introduce real-time dynamic emergent arrivals into our optimization framework. We use different performance metrics to evaluate the effectiveness and efficiency of our approach. We monitor the total priorities of the patients assigned to the a-priori routes. At the same time, we keep track of the overtime that is caused by the emergent arrivals. The number of patients whose original plan is revised is another performance metric we consider. We also count the total number of postponements. Finally, the response time to the

emergent request is also a metric in our framework. These performance metrics are used to compare the alternative policies selected to handle the emergent requests. We introduce an optimization-based solution strategy where dynamic arrivals are accumulated and the current plans are re-optimized at a predetermined re-optimization time.

To the best of our knowledge, we introduce the first study where the routing and scheduling decisions for home healthcare problems are modeled using a time-dependent prize metric which adapts itself according to the service schedules and the health conditions of the patients. We introduce exact mathematical models and complementary heuristics to solve the scheduling and routing problems for home healthcare services. We also introduce a generic optimization-based solution methodology in order to use the prize-based models to address dynamic real-life scheduling and routing problems for different sectors where a priority-induced difference might be observed between customers.

Chapter 3

DETERMINISTIC AND STATIC APPROACH

3.1 Problem Formulation

In this section, we formulate the problem that we introduced in Section 1, namely the PHHP. We use a scheme to update the prizes dynamically over time. Our problem requires a simultaneous decision for scheduling and routing. The scheduling decisions make sure when to visit the patients while routing decisions try to decrease the time spent while traveling. The reason for using a prize function which changes dynamically over time is to be able to differentiate the patients in terms of their priorities and try to assign the patients into the earlier days as much as possible. We assume that the prizes decrease over time, therefore the models introduced in this chapter will force assignments to earlier days. Here we describe a procedure to calculate the prize of each patient over the planning horizon. In the OP, prizes are collected by visiting each client and the total collected prize is maximized. We follow a similar approach, and translate patient priorities to prizes such that maximizing the total collected prize will enable the selection of patients with the highest priorities as early as possible. In the next section, a procedure is introduced to calculate the prizes for each patient.

3.1.1 Prize Calculation

At the beginning of the planning period, each patient has a different priority according to his medical condition and needs. Furthermore, the prize assigned to each patient changes with respect to the time between the current day and the last visit day. As time progresses, the urgency of visiting the patient will increase. Therefore, we would like to avoid delaying the next visit to a patient as much as possible. Hence, the prize values decrease towards the end of the planning period. While other decreasing

functions of time may be considered, we propose to decrease the prize by a factor that differs for each patient due to the patient's medical condition or possibly other factors. Let P denote the set of patients assigned to the nurse. Let us further define:

- T_{now} , current day of planning,
- LST_i , last service day of patient $i \in P$,
- u_i , time elapsed between the current day and the last service day of patient $i \in P$,
- $w_i \in [0, 1]$, prize reduction factor for patient $i \in P$,
- π_i , priority factor of patient $i \in P$ depending on his/her condition,
- p_{it} , prize of patient $i \in P$ on day t .

The prize values are calculated using the following relations:

$$u_i = T_{now} - LST_i \quad (3.1)$$

$$p_{it} = \pi_i u_i (w_i)^t \quad (3.2)$$

Here w_i reflects the urgency of service for patient i . Since $w_i \in [0, 1]$, we have $p_{it} \geq p_{i,t+1}$ for every patient i and day t . Here we note that the prize decreases exponentially by day with respect to the prize reduction factor. The prizes can be calculated a priori and input as parameters to the MIP model that we present next section.

In Table 3.1 we provide an example which shows the schedules and routes for 30 patients for a 5-day planning horizon. When the prizes are constant, we see that there are some patients on each day. The patients are represented by their IDs and arrival time indicates the time to arrive to patient's residence. Since the prizes are constant, it is not important on which day the patients are visited. In Table 3.2 we

Day 1	Arrival Time	Day 2	Arrival Time	Day 3	Arrival Time	Day 4	Arrival Time	Day 5	Arrival Time
25	9:03:36 AM	17	9:09:00 AM	6	9:09:36 AM	21	9:01:12 AM	7	9:06:00 AM
1	9:48:36 AM	4	10:01:48 AM	22	10:03:00 AM	18	9:31:48 AM	29	9:38:24 AM
28	10:19:48 AM	8	10:31:48 AM	14	10:28:12 AM	20	10:04:12 AM		
24	11:09:36 AM	11	11:17:24 AM			5	10:51:36 AM		
27	11:33:00 AM	9	11:47:24 AM			23	11:12:00 AM		
15	11:56:24 AM	30	12:07:48 PM						
		19	12:37:48 PM						
		10	12:58:12 PM						
		16	1:43:48 PM						
		3	2:04:48 PM						
		12	2:51:36 PM						
		2	3:12:00 PM						
		26	3:33:00 PM						
		13	4:03:36 PM						

Table (3.1) Routes with constant prizes

see the change in the plan when the prizes are decreasing. Although there are patients on each day when the prizes are constant, when we change the prizes dynamically we see that the patients are scheduled on the first three days. In Appendix, we provide daily routes for 4 different nurses for a 5-day planning horizon with decreasing prizes.

Day 1	Arrival Time	Day 2	Arrival Time	Day 3	Arrival Time
24	9:01:48 AM	8	9:01:12 AM	29	9:07:12 AM
14	9:30:00 AM	23	9:46:12 AM	3	9:43:12 AM
7	10:15:00 AM	25	10:19:48 AM	20	10:31:12 AM
17	10:46:48 AM	15	11:08:24 AM	19	11:18:00 AM
1	11:36:36 AM	21	11:55:48 AM		
4	12:09:36 PM	26	12:27:00 PM		
18	12:39:36 PM	11	12:57:36 PM		
2	1:10:48 PM	28	1:31:12 PM		
12	1:30:36 PM	22	2:34:12 PM		
9	1:51:36 PM	6	3:27:36 PM		
30	2:12:00 PM	27	4:22:48 PM		
16	2:42:00 PM				
13	3:03:00 PM				
10	3:49:48 PM				
5	4:34:48 PM				

Table (3.2) Routes with decreasing prizes

3.1.2 MIP Model

In PHHP, each patient has a time interval within which he/she has to be visited, i.e. a time window. When a patient is visited by a nurse, there is also an associated service time during which the nurse has to serve the patient. We assume that the service times can be estimated from past data and do not show much variability. Therefore, we assume they are deterministic.

Input Parameters and Decision Variables

Below, we provide a list of input parameters and decision variables used in the MIP formulation for our problem:

- $\{0\} \cup P$ represents the set of patients, where $P = \{1, \dots, n\}$, and the nurse's home base indexed as zero.
- T is the length of the planning horizon consisting of days $t = 1, \dots, T$.
- p_{it} is the prize of patient i on day t ; higher p_{it} values belong to more prioritized patients.
- s_i is the service time required to serve patient i .
- e_i is the earliest service start for visiting patient i on a day.
- l_i is the latest service start time for visiting patient i on a day.
- t_{ij} is the time required to travel between locations of patients i and j .
- W is total working hours for the nurse per day.
- M is a sufficiently big number.
- ϵ is a sufficiently small number.
- $x_{ijt} \in \{0, 1\}$ represents whether patient j is visited right after patient i on day t .
- $y_{it} \in \{0, 1\}$ indicates whether patient i is visited on day t .
- $a_i \geq 0$ is the arrival time to the location of patient i on any day. Note that the patient can be visited at most once during the planning horizon. In addition, a_i will be zero if the patient is not visited at all.
- a_0 is the start time of the nurse from her home location.

$$z = \max \sum_{i=1}^n \sum_{t=1}^T p_{it} y_{it} - \epsilon \left(\sum_{i=1}^n \sum_{t=1}^T s_i y_{it} + \sum_{i=0}^n \sum_{\substack{j=0 \\ j \neq i}}^n \sum_{t=1}^T t_{ij} x_{ijt} \right)$$

s.t.

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{ijt} = y_{it} \quad \forall i \in P, \quad t = 1, \dots, T \quad (3)$$

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{jit} = y_{it} \quad \forall i \in P, \quad t = 1, \dots, T \quad (4)$$

$$\sum_{j=0}^n x_{0jt} = 1 \quad t = 1, \dots, T \quad (5)$$

$$\sum_{i=0}^n x_{i0t} = 1 \quad t = 1, \dots, T \quad (6)$$

$$\sum_{t=1}^T y_{it} \leq 1 \quad i \in P \quad (7)$$

$$\sum_{i=1}^n s_i y_{it} + \sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ijt} \leq W \quad t = 1, \dots, T \quad (8)$$

$$a_i + s_i + t_{ij} - a_j \leq M(1 - x_{ijt}) \quad \forall i, j \in P, \quad t = 1, \dots, T \quad (9)$$

$$e_i \sum_{t=1}^T y_{it} \leq a_i \leq l_i \sum_{t=1}^T y_{it} \quad \forall i \in P \quad (10)$$

$$x_{ijt} \in \{0, 1\} \quad \forall i, j \in P \cup \{0\}, \quad t = 1, \dots, T \quad (11)$$

$$y_{it} \in \{0, 1\} \quad \forall i \in P, \quad t = 1, \dots, T \quad (12)$$

$$a_i \geq 0 \quad i \in P \quad (13)$$

The objective function has two components. The first one is the total prize of the visited patients and the second one is the negative of the total time needed to complete the routes over the planning horizon multiplied with a small weight ϵ . Constraints (3-4) ensure that if a patient is visited on day t , then either the patient is the first patient to be visited or there is another patient who was visited before him and simi-

larly the patient is either the last patient to be visited or there will be another patient who is going to be visited right after him on day t . Constraints (5-6) enforce that the nurse has to leave her home and return to it each day, unless no patient visit is scheduled for that day. Constraints (7) state that each patient can be visited at most once during the planning horizon. Constraints (8) take total working hours of the nurse into consideration and limits the total time of the tour each day. Constraints (9) make sure that if patient j is visited after patient i , then the time to begin service for patient j must be at least $s_i + t_{ij}$ after the nurse arrives at patient i . Constraints (10) state that if a patient is selected to be visited on some day, then the arrival time has to be within the time window of the patient. Otherwise, the arrival time should be zero. Constraints (11-13) define the decision variables.

3.2 An upper bound by Lagrangean Relaxation

We implement the Lagrangean relaxation method to the presented formulation in order to obtain a strong upper bound for the PPHP. Constraints (7), which indicate that a patient can be chosen at most once during the planning horizon, are dualized in order to decompose the relaxed problem into single day problems. Here we note that the $\sum_{t=1}^T y_{it}$ term in both the left hand side and right hand side of Constraints (10) can be removed and the arrival time to an unvisited patient can be positive without affecting the objective function value. Thus, the decomposition into single day problems works out and we can set the arrival time of an unvisited patient to zero after we obtain the solution of a single day problem.

The penalty component added to the objective function is

$$\sum_{i=1}^n \lambda_i (1 - \sum_{t=1}^T y_{it})$$

where $\lambda_i \geq 0$ are the Lagrangean multipliers. The constant term $\sum_i \lambda_i$ is omitted from the objective function of the Lagrangean relaxation problem given below.

$$\begin{aligned} \text{MIP}(\lambda) : z_\lambda = \max & \sum_{i=1}^n \sum_{t=1}^T (p_{it} - \lambda_i) y_{it} - \epsilon \left(\sum_{i=0}^n \sum_{t=1}^T s_i y_{it} + \sum_{i=0}^n \sum_{j=0}^n \sum_{t=1}^T t_{ij} x_{ijt} \right) \\ \text{s.t.} & \\ & (3) - (6), (8) - (13) \end{aligned}$$

MIP(λ) decomposes into T single day MIP models, one for each day. The model for day k is represented by MIP(λ, k). Below, we present MIP(λ, k) for the sake of completeness. Here we note that the single day problem is the Orienteering Problem with time windows, and it is therefore an NP-hard problem.

$$\begin{aligned} \text{MIP}(\lambda, k) : z_{\lambda, k} = \max & \sum_{i=1}^n (p_i^{(k)} - \lambda_i) y_i^{(k)} - \epsilon \left(\sum_{i=0}^n s_i y_i^{(k)} + \sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ij}^{(k)} \right) \\ \text{s.t.} & \end{aligned}$$

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{ij}^{(k)} = y_i^{(k)} \quad \forall i \in P \quad (14)$$

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{ji}^{(k)} = y_i^{(k)} \quad \forall i \in P \quad (15)$$

$$\sum_{j=0}^n x_{0j}^{(k)} = 1 \quad (16)$$

$$\sum_{i=0}^n x_{i0}^{(k)} = 1 \quad (17)$$

$$\sum_{i=1}^n s_i y_i^{(k)} + \sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ij}^{(k)} \leq W \quad (18)$$

$$a_i + s_i + t_{ij} - a_j \leq M(1 - x_{ij}^{(k)}) \quad \forall i, j \in P \quad (19)$$

$$e_i y_i^{(k)} \leq a_i \leq l_i y_i^{(k)} \quad \forall i \in P \quad (20)$$

$$x_{ij}^{(k)} \in \{0, 1\} \quad \forall i, j \in P \cup \{0\} \quad (21)$$

$$y_i^{(k)} \in \{0, 1\} \quad \forall i \in P \quad (22)$$

$$a_i \geq 0 \quad i \in P \quad (23)$$

After solving the sub-problem $\text{MIP}(\lambda, k)$ for each day k and fixed λ vector, we get an upper bound to our problem. The bound is expressed as

$$\text{UB}(\lambda) = \sum_k z_{\lambda,k} + \sum_i \lambda_i.$$

A subgradient optimization algorithm is implemented to solve the Lagrangean dual problem and the initial Lagrangean multipliers are set as the dual variables obtained from the linear programming relaxation of the MIP model.

3.2.1 Subgradient Optimization Method

The steps of the subgradient optimization algorithm are listed below:

1. Choose a starting point λ^0 ; $r = 0$.
2. Solve the single day problems $\text{MIP}(\lambda^r, k)$ for $k = 1, \dots, T$ and calculate $\text{UB}(\lambda^r)$ and find the solution x^r .
3. Choose a subgradient $s^r = b - Ax^r$ of the function z at λ^r . If $s^r = 0 \rightarrow \mathbf{STOP}$, because the optimal value has been reached.
4. Compute $\lambda^{r+1} = \max\{0, \lambda^r + \gamma^r s^r\}$, where γ^r denotes the step size.
5. Increment r and go to 2.

We use Held and Karp's step size formula for updating the step size [50]:

$$\gamma^r = \mu^r \frac{z^* - z_{\lambda}^r}{\sum_{i=1}^m (b_i - \sum_{j=1}^n a_{ij} x^r)^2}$$

where

- z^* is the value of the best solution for the original problem found so far
- z_{λ}^r is the value of the solution using the lagrangean multipliers λ^r
- μ^r is a parameter with $0 < \mu^0 \leq 2$ and

$$\mu^{r+1} = \begin{cases} \alpha\mu^r, & \text{if the dual objective did not increase in the last } S \text{ iterations} \\ \mu^r, & \text{otherwise.} \end{cases}$$

with parameters $0 < \alpha < 1$ and $S > 1$.

- the denominator is the square of the norm of the subgradient vector $b - Ax^r$.

3.3 Solution Methods

Since PHHP is an NP-hard problem, the time to find an optimal solution increases exponentially as the number of patients and the length of the planning horizon increase. Our goal is to develop algorithms that yield near-optimal solutions in a reasonable amount of time. For this purpose, we propose two different solution methods. The first method is based on an iterative mathematical programming subroutine where in each iteration an exact mathematical model is solved. The procedure iterates over the planning days in the planning horizon. This method is a matheuristic where an exact optimization algorithm is used at each iteration. The other method proposed in Section 3.3.2 is an Adaptive Large Neighborhood Search (ALNS) algorithm. The ALNS algorithm contains several destroy and repair moves specific to our problem.

3.3.1 A Matheuristic

In our problem, the nurse visits several patients each day. If a patient is visited on some day in the planning horizon, then this patient is not allowed to be visited on other days. This leads to a decomposition approach over the days. Furthermore, by the nature of the problem, we would like patients with higher priorities to be visited as early as possible. Having these points in mind, we suggest solving a single day model and iterate over subsequent days. For each day in the planning horizon, we solve the MIP model as a single period model with prizes corresponding to that day and a subset of the patients, namely the set of yet unvisited patients. Once the model of a day is solved, we remove the visited patients from the patient set and re-solve

the model for the next day. The number of iterations is the number of days in the planning horizon, T . We call this procedure the Successive Single Period Heuristic (SSPH).

SSPH: Successive Single Period Heuristic

T : Number of days
 P : Patient list

```

1: function SSPH
2:    $z \leftarrow 0$ 
3:    $R \leftarrow \emptyset$ 
4:    $k \leftarrow 1$ 
5:    $P_k \leftarrow P$ 
6:   while  $k \leq T$  do
7:      $(z_k, R_k) \leftarrow \text{SPM}(k, P_k)$ 
8:      $z \leftarrow z + z_k$ 
9:      $R \leftarrow R \cup R_k$ 
10:     $P_{k+1} \leftarrow P_k \setminus R_k$ 
11:     $k \leftarrow k + 1$ 
12: return  $z, R$ 

```

Let $\text{SPM}(k, P_k)$ be the MIP model presented in Section 3.2 for day k such that $T = 1$ and the patient list is P_k , at iteration k of the algorithm SSPH. In iteration k , the model $\text{SPM}(k, P_k)$ is solved for the corresponding day k and patient list P_k . Initially we have $|P|$ patients. At each iteration k of the algorithm SSPH, the corresponding values of the prize matrix p_{ik} is used for each patient i . $\text{SPM}(k, P_k)$ is a single period model in which the route of the nurse for day k is found. At iteration k , the solution of the model $\text{SPM}(k, P_k)$ yields the patient list R_k to be visited and the corresponding objective value z_k . After the solution is obtained for iteration k , the patient list is updated for the next iteration by removing the set of selected patients R_k from the list of patients, that is, $P_{k+1} = P_k \setminus R_k$. Once the model is solved, the route and

the associated objective value is stored. The objective value of the feasible solution found by SSPH is the sum of the objective values z_k found for each day k . The overall patient list selected to be visited is given by $R = \cup_k R_k$. Below, we present $\text{SPM}(k, P_k)$ for the sake of completeness.

$$\begin{aligned} \text{SPM}(k, P_k) : z_k = \max & \sum_{i=1}^n p_{ik} y_{ik} - \epsilon \left(\sum_{i=1}^n s_i y_{ik} + \sum_{i=0}^n \sum_{\substack{j=0 \\ j \neq i}}^n t_{ij} x_{ijk} \right) \\ \text{s.t.} & \\ & \sum_{\substack{j=0 \\ j \neq i}}^n x_{ijk} = y_{ik} \quad \forall i \in P_k \end{aligned} \quad (24)$$

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{jik} = y_{ik} \quad \forall i \in P_k \quad (25)$$

$$\sum_{j=0}^n x_{0jk} = 1 \quad (26)$$

$$\sum_{i=0}^n x_{i0k} = 1 \quad (27)$$

$$\sum_{i=1}^n s_i y_{ik} + \sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ijk} \leq W \quad (28)$$

$$a_i + s_i + t_{ij} - a_j \leq M(1 - x_{ijk}) \quad \forall i, j \in P_k \quad (29)$$

$$e_i y_{ik} \leq a_i \leq l_i y_{ik} \quad \forall i \in P_k \quad (30)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i, j \in P_k \cup \{0\} \quad (31)$$

$$y_{ik} \in \{0, 1\} \quad \forall i \in P_k \quad (32)$$

$$a_i \geq 0 \quad i \in P_k \quad (33)$$

3.3.2 An ALNS Algorithm

Adaptive Large Neighborhood Search (ALNS) was introduced by Ropke and Pisinger [51] as an algorithm that uses several large neighborhoods in an adaptive way. The authors provide a more general ALNS framework in a subsequent study [52]. The ALNS heuristic extends the idea behind the large neighborhood search heuristic in-

roduced by Shaw [55]. In the ALNS, each repair and destroy move pair i is associated with a score ϑ_i and a weight ω_i whose values depend on the past performance of the moves. By a roulette-wheel mechanism, a pair of moves j is selected out of all n pairs at each iteration with probability $\frac{\omega_j}{\sum_i \omega_i}$. Initially, all weights are set to one and all scores are set to zero. The search is divided into segments of γ iterations each, and the weights are computed by considering the performance of the moves during the last segment. At each iteration, the score of the selected pair is increased by σ_1 if a new best solution is found; by σ_2 if a solution better than the incumbent is found; by σ_3 if the solution is not better but accepted. After γ iterations, the weights are updated. The scores are reset to zero at the end of each segment. We use an acceptance criterion based on simulated annealing. In Table 3.3 we provide the values for some ALNS parameters used in our experiments. The last two parameters in Table 3.3 denote the cooling and initial temperature parameters used in simulated annealing in our ALNS implementation.

Table (3.3) ALNS parameters

γ	σ_1	σ_2	σ_3	α	T_0
200	10	5	2	0.99	1.0

We use the formula in equation (34) for updating the temperature in simulated annealing-based acceptance criterion in our ALNS.

$$T_{k+1} = T_k \alpha^k \quad (34)$$

We develop several destroy and repair heuristics taking advantage of the problem structure and the time-decreasing prizes. A list of the destroy and repair algorithms that we use in our ALNS implementation is given in Table 1 in the online supplement. In the following, we provide a brief explanation of each algorithm (move). After the selection of a pair of destroy and repair moves, we further execute a 2-OPT route improvement step.

Table (3.4) The ALNS Heuristic

Destroy Algorithms		Repair Algorithms	
BR	Best Removal	IBC	Insert Best Candidates
WR	Worst Removal	IWC	Insert Worst Candidates
RR	Random Removal	IRC	Insert Random Candidates
GR	Geographic Removal	PWI	Percentage Wise Insertion
SR	Shaw Removal	IBR	Insert to Best Route
DR	Daily Removal	OI	Outlier Insertion
LDR	Last Day Removal	RI	Random Insertion
L2DR	Last Two Days Removal	ILDR	Insert into Least Dense Route
L3DR	Last Three Days Removal	REG	Regret
FDR	First Day Removal		
RNDR	Random N Days Removal		
OR	Outlier Removal		
CSR	Combined Similarity Removal		

Destroy Algorithms

In each of the removal (destroy) algorithms below, given a list of routes for each day in the planning horizon, the algorithm chooses a set of patients to be removed from the routes according to the specified criteria.

Best Removal: The algorithm chooses a random number of patients for each day and removes the selected patients from the routes. Let r_k represent the route on day k . For each day k , let $q_k \in [1, N_k]$ be the number of patients selected to be removed from the route, where N_k is the number of patients assigned to r_k . Then, q_k patients with the highest prizes are removed from r_k .

Worst Removal: The algorithm chooses a random number of patients for each day and removes the selected patients from the routes. For each day k , let $q_k \in [1, N_k]$ be the number of patients selected. Then, q_k patients with the lowest prizes are removed from r_k .

Random Removal: The algorithm chooses a random number of patients for each day and removes $q_k \in [1, N_k]$ patients from each route r_k with equal probabilities.

Geographic Removal: Given a list of routes for each day in the planning horizon, let ξ_k be the average time on day k given by $\frac{\Gamma_k}{N_k}$, where Γ_k is the total time of the route and N_k is the number of patients assigned to this route. Let $f(i) = s_i + t_{ii^*}$ where patient i^* immediately follows patient i . The algorithm selects to remove patient i if $f(i) \geq \xi_k$.

Shaw Removal: Shaw introduced the idea of relatedness in [55]. We use the idea of relatedness in this heuristic; hence we call the heuristic as Shaw Removal. Let μ_{ij} be the relatedness measure between patients i, j . We define the measure as follows: $\mu_{ij} = \psi t_{ij} + (1 - \psi)p_{ij}$, where $\psi \in [0, 1]$ represents the relatedness between patient i and patient j . The parameter p_{ij} is defined as $p_{ij} = \frac{\max(p_i, p_j)}{\min(p_i, p_j)}$. For each successive pair of patients in the route, the relatedness measure is calculated and then an average relatedness measure is found. The pair of patients whose relatedness measure falls below the average relatedness are removed from the route. Note that other relatedness measures can also be defined. We choose to define it in this way to incorporate the prizes and the time between patients. The lower the μ_{ij} values, the similar the patients are. We remove the patients who are similar in order to diversify the search.

Combined Similarity Removal: Let θ_{ij} be the combined similarity measure between patients i and j . We define the measure as follows: $\theta_{ij} = \psi \frac{1}{t_{ij}} + (1 - \psi) \frac{1}{p_{ij}}$, where $\psi \in [0, 1]$. The parameter p_{ij} is defined as above. For each successive patient in the route, the combined similarity measure is calculated and then an average combined similarity measure is found. The patients whose combined similarity measure is above the average combined similarity are removed from the routes. This algorithm is a counter-part of the Shaw removal algorithm explained above. The higher the combined similarity measure θ_{ij} is, the similar

the patients are. We again remove the patients who are similar to diversify the search.

Daily Removal: Out of all days, a day is randomly selected with equal probability and the route for the selected day is eliminated, i.e. all patients in the route are removed.

Last Day Removal: The patients assigned to the last planning day are removed from the route they are assigned to.

Last Two Days Removal: The patients assigned to the last two days of the planning horizon are removed from the routes of these days.

Last Three Days Removal: The patients assigned to the last three days of the planning horizon are removed from the routes of these days.

First Day Removal: Patients assigned to the first day are removed from the route of this day.

Random N Days Removal: Suppose that the solution has T routes. The algorithm randomly selects $N \in [1, T]$ days with equal probability and all of the patients assigned to those days are removed from the routes.

Outlier Removal: For each day k in the planning horizon, the algorithm chooses a random number, $q_k \in [1, N_k]$. Then the q_k patients whose total distance to the patients on the same day are the highest are selected to be removed. Let $f_k(i)$ be the fairness value of the patient i on day k calculated as $f_k(i) = \sum_{j \in I(k)} t_{ij}$, where $I(k)$ is the list of patients selected to be visited on day k , excluding patient i . Among all patients assigned to day k , we select the q_k outliers i.e., those patients whose fairness values are the highest. Those selected patients are removed from the route to which they are assigned.

Repair Algorithms

Given the list of routes for each day, each of the insertion (repair) algorithms described below selects unassigned patients according to some criterion and inserts them at specified positions in the routes. While the patients are inserted into the routes, the feasibility of the positions in the routes are checked. We have a two-stage feasibility check. First, we check whether the total time constraint is violated. Then, we check if inserting the patient into a position will force other patients to violate their time-window constraints. If an insertion is not feasible, we do not insert the patient and look for another patient in the unassigned patient list. Unless stated otherwise, the patients are inserted into the best possible route and position where the objective function increases the most.

Insert Best Candidate: The algorithm sorts the unassigned patients descendingly by their initial prizes. Then starting from the patient with the highest prize, it inserts the next patient into the selected route and position.

Insert Worst Candidate: The algorithm sorts the unassigned patients ascendingly by their initial prizes in ascending order. Then starting from the patient with the lowest prize, it inserts the next patient into the selected route and position.

Insert Random Candidate: A randomly chosen unassigned patient is inserted into the selected route and position.

Percentage-wise Insertion: For each patient in the unassigned patient list, the patient is inserted into each route in the best position and the changes in total prize and total time of the routes are recorded. Then the patient is inserted into the route for which the ratio of the increase in total prize to the increase in total time is the highest.

Insert to Best Route: For each patient, the best route in terms of total objective value is found. The best route is the route into which inserting the patient will

increase the objective function value the most. The patients are inserted into the selected routes to the position where the objective function value increases the most.

Outlier Insertion: The patients from the unassigned list are sorted in terms of their fairness values, where the fairness value of patient i is calculated as $f(i) = \sum_{j \in N} t_{ij}$. Starting from the patient with the highest fairness values, the patients are inserted into the routes to the best position.

Random Insertion: A patient from the unassigned pool and a route are randomly selected and the patient is inserted into the route to the best position.

Insert into Least Dense Route: For each day k , a density parameter d_k is defined. The density parameter is the ratio of the total time of the route $\Gamma(k)$ to the total time limit W , that is $d_k = \frac{\Gamma(k)}{W}$. Patients from the unassigned patient list are inserted into the least dense route to the best position. As the patients in the route change, d_k is dynamically updated.

Regret: For each patient $i \in P$, a regret parameter γ_i is defined. Each patient i is inserted into each route r and the change in the objective function is recorded, as $reg(r, i)$ for each patient-route (i, r) pair. The higher the change, the higher the regret if the patient is not inserted to the route. Once the regret values are calculated for each patient and route pair, then the patient-route (i, r) pairs are sorted in terms of their regret values. Starting from the patient with the highest regret value, patients are inserted into the corresponding routes to the best position.

3.4 Computational Results

In this study, we use a real data set made available at <http://dx.doi.org/10.17632/j545x729m6.1> corresponding to a nurse and the set of patients assigned to her. The data set contains the coordinates of each patient and the nurse's home as well as the average service

time at each patient. We perform experiments for 48 instances using both loose time windows (essentially no time windows) and tight time windows. For the loose time windows case, we take the time window of each patient as the working hours of the nurse. For the tight time windows case, we randomly assign patients into one of the following three time windows: [9:00-12:00], [12:00-15:00] and [14:00-17:00]. Using the locations, first a distance matrix is produced by means of a road map. The patients' locations are shown in Figure 3.1, where the maximum distance between the patients is 22 km.

We assume an average urban speed of 50 km/h. Once the distance matrix is calculated, a time matrix is constructed using this information. In addition, each patient assigned to the nurse has a last visit date LST_i and an assessed priority factor, p_i . Using the last visit date, the current day and the priority of a patient, the prizes of each day are calculated by equation (3.2) to form the prize vector for each patient. The nurse starts working at 9:00 and returns to her home by 17:00. In the experiments conducted for the no time windows case, the same time interval is taken as the time window of each patient. The total working hour is $W = l_0 - e_0$ where $e_0 = 9$ and $l_0 = 17$. We also assume that $t_{00} = 0$ and the nurse stays at home if no visit is planned for a day.

We constructed the mathematical model and solved it using CPLEX 12.7. All of the codes in this section were written in Visual Studio 2017 using C# on a computer with 8GB memory (RAM) and Core(TM) i7-6500U CPU 2.59 Ghz processor Intel(R).

We test the computational limits of the MIP model on instances with patient numbers varying between 30 and 180 and 5-day and 10-day planning horizons. Then, we investigate the strength of the upper bounds obtained using the Lagrangean relaxation procedure. Lastly, we compare the performance of the two heuristics we propose, SSPH and ALNS.

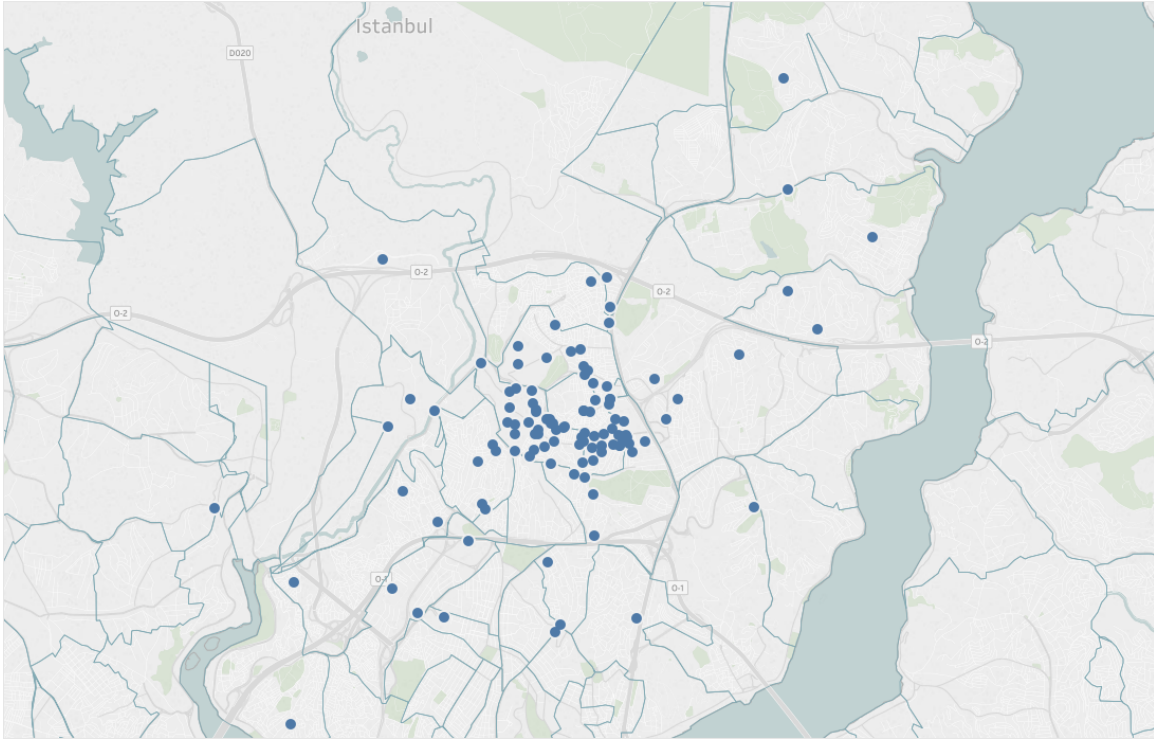


Figure (3.1) Patient locations

3.4.1 Objective Trade-off Analysis

Our MIP has two components in its objective function, where the total route time has a small weight compared to the total collected prize. While the model tries to maximize the total collected prizes primarily, it also aims to achieve it with the least possible total route time. We introduce a positive parameter, ϵ , to weigh and scale the two components of the objective function. In the real-life problem that motivated this study, servicing high priority customers at earlier days carries more importance than minimizing total route times. Hence, ϵ should be set along this goal. However, we can also observe the trade-off between total prize collected and the total route time by varying the ϵ value in order to select a preferable solution. We selected an instance with 30 patients and a 5-day planning horizon to investigate this trade-off. We set a run time limit of 30 minutes, an optimality gap of 0.01% and varied ϵ between 0.1 and 90. Figure 3.2 depicts the trade-off between route time and collected prize for

different ϵ values.

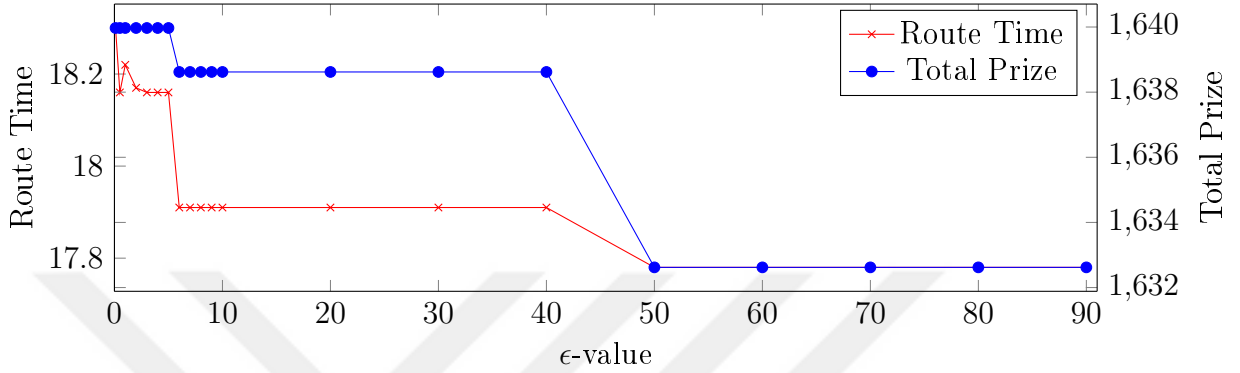


Figure (3.2) Total Collected Prize vs. Total Route Time

As ϵ values increase, the solution times generally increase. The bigger the ϵ value, the more important the route time is. It is observed that when ϵ value increases, the prize values decrease. We also observe that all the patients are visited in the first three days. Since with $\epsilon = 1$ we obtain a solution quickly in our experiment and there is no significant difference with other good options of ϵ -values, we used $\epsilon = 1$ in the rest of the experiments in order not to increase the weight of the total route time. In practice, ϵ can be set according to an acceptable trade-off level.

The route time component in the objective function plays a rather practical role in the nurse's daily operations. While the goal is to visit patients with the high prizes, we would like to visit those patients with the least possible route time. As the ϵ value increases, weight of the route time becomes sufficiently large to result in choosing less critical patients in the interest of more compact routes. Therefore, it becomes an option not to choose the patient while she is selected for smaller ϵ values. Since, the model finds solutions by respecting working hours of the nurse as long as ϵ value is small enough, that is, if it does not exceed the prize value for a patient, the total prize collected should not change. Detailed values concerning this trade-off analysis can be seen in Table 2 in the online supplement.

If we remove the prize component from the objective function and consider only minimizing the route time while visiting each patient exactly once during the planning horizon for 30 patients instance (in this instance we can serve all patients), the total routing time decreases from 18.22 to 17.45 (a small difference) and each day in the planning horizon at least one patient is served; however, when we consider the prize component, the patients are visited in the first three days and better service is provided this way. On the other hand, when we remove the prize component from the objective function and minimize the total route time, the solution time increases from 8.58 to 463.69 seconds.

3.4.2 MIP Results

In order to observe the performance of the MIP model, a patient size (30, 40, ..., 180) is determined and a set of patients of the specified size is selected randomly out of the available 180 patients. Up to 90 patients, a run time limit of 1 hour is imposed for the CPLEX solver. For 100 patients and more, the time limit is set as 2 hours. For all the test cases, an optimality gap of 0.25% is imposed as a stopping criterion. For 30, 40 and 50 patients, CPLEX stopped by reaching the optimality gap limit. As the number of patients increases, CPLEX cannot find near-optimal solutions within the time limit. From 60 patients and on, it is no longer able to find good solutions within the specified gap limit. As the problem size increases, the model cannot find a solution that serves all the patients in the planning horizon. For the tight time windows case, CPLEX cannot find a solution even for 50 patients both in 5-day runs and 10-day runs. For the 5-day run, CPLEX reports the first solution after 14,341 seconds with a gap value of 16.33%.

In the experiments where the algorithms are compared, we impose a time limit and an optimality gap. However, we are also interested in knowing how solution time would be changed if we do not impose an optimality gap for the CPLEX solver. We still assume a time limit of 2 hours and perform the analysis for instances with 30

to 60 patients. We run 5 independent experiments with randomly selected patients for each run. While for 30 and 40 patients cases, the solver returns optimal solution within 1 and 2 minutes successively, the solver is not able find the optimal solution for the instances with 50 and 60 patients.

Table (3.5) MIP Statistics

# of Patients	Avg. Time	Std. in Time	Avg. % Gap
30	65.234	3.67	0
40	123.604	0.36	0
50	7206.85	0.54	0
60	7201.455	0.82	0.155

In Table 3.5, we provide the results for the experiments conducted. According to the results, within a 2 hour solution time, CPLEX is only able to yield optimal solutions for only small sized instances. For small sized instances, the entire set of patients are visited within the earlier days in the planning horizon. However, when the size of the instances increase, routing decisions become more difficult and the complexity issues for the routing problems arise. Our model simultaneously solves a routing and scheduling problem. Since we model the problem as an orienteering problem, the model also tries to select the optimal candidates in order to route and schedule. For the small sized instances with a 5 day planning horizon, there appears no selection process because each patient can be assigned to a day. However, when the size of the instances get larger, the selection of the patients also becomes a consideration for the CPLEX which in turns result in long solution times.

3.4.3 Lagrangean Relaxation Results

In Table 3.6, the results of our experiments with the Lagrangean relaxation method and MIP results are reported for instances having different number of patients ranging from 30 to 180. For this experiment, the planning horizon is set to 5 days and 2-hours solution time limit is given to CPLEX. It is a common observation that the solvers

sometimes exceed the solution time limit when they do not yet prune the current node.

Table (3.6) Lagrangean Relaxation Results for 5-day Runs

# of Patients	MIP				Lagr.		# of Iter.	Best Iter.
	Time (s)	% Gap	Lower Bound	Upper Bound	Bound	Time (s)		
30	43.83	0.19	1620.15	1623.23	1621.80	1601.86	100	77
40	163.81	0.25	1955.97	1960.86	1963.18	3146.32	30	1
50	2062.48	0.23	2185.74	2190.85	2189.42	5157.13	30	28
60	3602.53	0.36	2365.02	2373.65	2368.47	13812.25	30	28
70	3601.08	2.32	2578.31	2638.11	2639.69	5165.34	10	1
80	3600.00	2.54	2763.73	2833.83	2836.77	9222.43	10	1
90	3601.00	2.98	2905.37	2992.06	2991.07	1262.18	1	1
100	7201.00	2.60	3057.89	3137.40	3135.09	1507.12	1	1
120	7203.00	6.93	3113.96	3329.66	3327.24	1531.83	1	1
140	7201.00	3.95	3465.78	3602.76	3598.47	2469.26	1	1
160	7270.00	46.18	2564.57	3748.91	3731.15	2965.23	1	1
180	7348.00	86.76	2040.46	3810.83	3796.95	2462.42	1	1

In subgradient optimization, the parameter μ is set to 1 and the parameter α is set to 0.4. The initial Lagrangean multipliers λ^0 are set to the dual variable values associated with the constraints (6) of the LP relaxation of the original MIP model. The number of iterations for the subgradient optimization algorithm is set to different values for each run, considering how long each iteration takes as the problem size increases. The number of iterations are listed in the column named *# of Iter.* The last column in Table 3.6 called *Best Iter.* represents the iteration number in which the minimum upper bound is found. In the 5-day runs reported in the table, in 3 out of 12 instances, the best upper bound obtained from the MIP model is stronger than the upper bound obtained by the Lagrangean relaxation procedure. When the number of patients exceeds 120, in all of the test results, the Lagrangean relaxation provides better bounds in much shorter times than CPLEX's best bounds. For 160 and 180 patients, MIP gaps are extremely high because the best feasible solutions found within the 2 hour time limit are quite poor. CPLEX stopped with near-optimal solutions

when the number of patients are between 30 and 60. Here we note that we terminated the Lagrangean relaxation procedure after a single iteration in instances with 90-180 patients due to the large run time of completing even one iteration. If the procedure is run long enough, it is likely that better bounds can be found.

Table (3.7) Lagrangean Relaxation Results for 10-day Runs

# of Patients	MIP				Lagr.		# of Iter.	Best Iter.
	Time (s)	% Gap	Lower Bound	Upper Bound	Bound	Time (s)		
30	26.05	0.19	1621.75	1624.81	1621.82	913.25	100	61
40	482.08	0.25	1953.92	1958.80	1963.37	1824.11	30	1
50	3242.70	0.25	2186.27	2191.73	2194.44	6135.22	30	1
60	7205.00	0.35	2365.94	2374.19	2370.91	5306.98	10	9
70	7215.61	1.16	2606.30	2636.64	2637.10	1083.40	1	1
80	7216.16	4.54	2728.95	2852.74	2855.43	798.38	1	1
90	7202.33	1.97	2982.76	3041.62	3041.10	1357.37	1	1
100	7249.83	5.20	3060.34	3219.50	3216.45	3507.02	1	1
120	7271.98	33.42	2602.45	3472.16	3471.68	1282.35	1	1
140	7215.69	84.61	2069.65	3820.83	3812.19	4760.32	1	1
160	7205.28	128.50	1758.40	4017.19	4009.99	12355.93	1	1
180	7208.09	124.40	1831.87	4110.69	4101.22	25029.92	1	1

In Table 3.7, we report the results of a similar experiment for instances having a 10-day planning horizon. In 4 instances out of 12, CPLEX found better upper bounds than our Lagrangean relaxation implementation, whereas in 8 instances the Lagrangean relaxation provided the best upper bounds. Though the Lagrangean relaxation performs better than CPLEX in terms of finding strong upper bounds, for the cases with more than 90 patients, the solution times of the Lagrangean relaxation procedure increase dramatically as well, even when a single iteration is performed.

3.4.4 Comparison of the Proposed Heuristics

Since MIP yields solutions with poor quality as the number of patients increases, we aim to obtain good feasible solutions from the SSPH matheuristic and the ALNS algorithm introduced previously. The matheuristic SSPH iterates successively over

days and for each day, it solves an exact MIP model in the form of an OP (with or without time window restrictions).

Table (3.8) MIP - SSPH - ALNS Statistics with Loose Time Windows

# of Patients	# of Days	MIP			SSPH			ALNS			
		% Gap	Time (s)	Best Obj.	% Gap	Time (s)	Obj.	% Gap	Time (s)	Best Obj.	Best Bound
30	5	0.10	43.83	1620.15	0.00	2.55	1621.78	0.00	9.07	1621.81	1621.82
30	10	0.00	26.05	1621.75	0.00	2.66	1621.78	0.00	12.97	1621.81	1621.82
40	5	0.25	163.81	1955.97	0.20	8.28	1957.03	0.50	16.11	1951.05	1960.86
40	10	0.25	482.08	1953.92	0.09	7.94	1957.03	0.28	31.24	1953.29	1958.80
50	5	0.17	2062.48	2185.74	0.11	68.51	2187.01	0.53	14.27	2177.87	2189.42
50	10	0.25	3242.70	2186.27	0.22	70.08	2187.01	0.85	18.72	2173.32	2191.73
60	5	0.15	3602.53	2365.02	0.14	176.51	2365.27	0.69	22.22	2352.24	2368.47
60	10	0.21	7205.00	2365.94	0.24	170.36	2365.27	0.81	33.11	2351.90	2370.91
70	5	2.32	3601.08	2578.31	1.62	2678.26	2596.04	1.89	32.06	2589.14	2638.11
70	10	1.16	7215.61	2606.30	1.56	4047.5	2596.04	1.59	24.93	2595.27	2636.64
80	5	2.54	3600.00	2763.73	1.82	27.21	2783.30	1.62	18.98	2788.79	2833.83
80	10	4.54	7216.16	2728.95	1.46	33.34	2811.73	0.99	51.86	2824.65	2852.74
90	5	2.95	3601.00	2905.37	1.48	41.69	2947.32	1.87	24.46	2936.27	2991.07
90	10	1.96	7202.33	2982.76	1.19	66.62	3005.47	1.29	64.00	3002.41	3041.10
100	5	2.52	3601.00	3057.89	1.38	172.44	3092.50	2.05	34.93	3072.02	3135.09
100	10	5.10	7249.83	3060.34	1.12	339.53	3180.93	2.78	55.00	3129.35	3216.45
120	5	6.85	3603.00	3113.96	2.06	54.53	3260.03	3.01	32.97	3230.14	3327.24
120	10	33.40	7271.98	2602.45	1.80	170.05	3410.20	4.54	86.78	3320.87	3471.68
140	5	3.83	7201.00	3465.78	1.77	165.34	3535.88	3.67	23.60	3471.10	3598.47
140	10	84.19	7215.69	2069.65	1.61	456.70	3751.66	5.93	94.79	3598.71	3812.19
160	5	45.49	7270.00	2564.78	1.23	565.97	3685.79	3.18	43.10	3616.06	3731.15
160	10	128.05	7212.34	1758.40	1.48	1669.36	3951.36	3.87	104.42	3860.55	4009.99
180	5	86.08	7348.00	2040.46	1.46	891.25	3742.43	4.17	36.73	3644.96	3796.95
180	10	123.88	7208.09	1831.87	1.69	2541.16	4033.04	6.64	108.93	3845.68	4101.22
Min	-	0.00	26.05	1620.15	0.00	2.55	1621.78	0.00	9.07	1621.81	-
Max	-	128.05	7348.00	3465.78	2.06	4047.50	4033.04	6.64	108.93	3860.55	-
Avg	-	22.34	5521.79	2432.74	1.07	601.16	2860.25	2.20	41.47	2822.05	-

In Table 3.8, we provide experimental results for SSPH and ALNS, and compare their performance with the solutions obtained from the MIP within the limited run times. The best upper bound values found by Lagrangean relaxation are reported in the last column and we calculate the optimality gaps with respect to these values. Once

SSPH is solved for each day, the objective values are added and the *SPPH % Gap* is calculated using these values. The best of the upper bound values obtained by *MIP Best Bound* and Lagrangean relaxation are given in the *Best Bound* column. For all the instances, SSPH produces solutions within the given gap limit. However, as the number of patients increases, the gap values of MIP increase drastically. For the 5-day instances, SSPH performs better in all runs than MIP both in terms of gaps and solution times. When the number of days is increased to 10 days, we observe that in 2 out of 12 instances, MIP provides better feasible solutions compared to SSPH. However, in general SSPH performs better than MIP both in terms of gap and solution times.

Solution statistics of SSPH with 5 days and 10 days show that when the number of patients increases, the solution time increases as well. Ranging from 30 patients to 120 patients, with the exception of 70 patients, the solution is reached in less than 10 minutes. However, when the number of patients exceeds 120, the solution times increase significantly. ALNS algorithm overcomes this problem and provides solutions within 41.47 seconds on average, where the worst solution time is 108.93 for the case of 180 patients and 10 days. A detailed comparison of MIP, SSPH and ALNS is displayed in Table 3.8. The ALNS algorithm runs for N iterations for each instance. However, if the algorithm does not yield any progress in the last M iterations, it stops. Throughout the experiments, we use $N = 10,000$ and $M = 2,000$.

We also perform experiments to observe how our proposed algorithms perform under tight time windows. To this aim, we randomly assign patients into one of the following three time windows: [9:00-12:00], [12:00-15:00] and [14:00-17:00]. In Table 3.9, a comparison of ALNS and SSPH is provided. The best bounds are obtained using the Lagrangean relaxation procedure presented earlier. However, the bounds obtained by the Lagrangean relaxation under tight time windows are weak because the subgradient algorithm requires excessive amount of time when we increase the

number of iterations. Thus, whenever the Lagrangean bounds for tight time windows are weak, we use the bounds obtained for the loose time windows case as they are also valid bounds for our problem under tight time windows. In loose time windows experiments, we set the ALNS iterations according to $N = 10,000$ and $M = 2,000$. In tight time windows experiments for 5-day runs, we again keep $M = 2,000$, but we change N according to the number of patients.

Table (3.9) SSPH - ALNS with Tight Time Windows (TTW) 5-Days

# of Patients	SSPH			ALNS			Best Bound
	% Gap	Time (s)	Obj.	% Gap	Time (s)	Best Obj.	
30	0.39	28830.11	1606.12	0.38	8.38	1606.29	1612.41
40	1.05	36571.33	1931.59	1.49	16.04	1923.02	1952.14
50	0.71	48547.18	2156.63	0.99	34.36	2150.65	2172.05
60	0.95	55631.22	2332.55	0.96	64.68	2332.26	2354.90
70	1.13	65416.30	2592.03	1.41	60.00	2584.75	2621.78
80	1.69	70206.60	2774.29	2.55	117.13	2750.01	2822.04
90	2.00	72392.90	2931.17	3.22	148.37	2894.75	2991.07
100	2.67	90812.56	3051.28	3.77	166.32	3016.77	3135.09
120	2.41	90709.75	3247.05	4.11	340.09	3190.64	3327.24
140	2.73	91209.12	3499.69	5.07	275.48	3415.49	3597.89
160	3.09	91336.56	3615.92	5.78	362.91	3515.31	3731.15
180	3.11	91207.14	3678.95	6.91	234.32	3534.42	3796.95
Min	0.39	28830.11	-	0.38	8.38	-	-
Max	3.11	91336.56	-	6.91	362.91	-	-
Avg	1.83	67423.97	-	3.05	152.34	-	-

We set the number of iterations for ALNS as

$$N = 10,000 + 10,000 \frac{|P|}{100} \quad (35)$$

where $|P|$ represents the number of patients. Hence, as the number of patients increases, the number of iterations in ALNS also increases. The results indicate that SSPH provides better solutions in terms of gap values. However, it takes 67,423.97 seconds on average to reach the solutions. On the other hand, ALNS provides relatively good solutions in terms of gap values. The average gap is 3.05% for ALNS and

it reaches these solutions in merely 152.34 seconds on average.

In Table 3.10, we provide the best, worst and average solutions of the ALNS algorithm for 10-day runs with 10 replications. Since, the SSPH and Lagrangean relaxation runs require excessive amount of time as seen in Table 3.9, we do not perform those runs here. We only provide objective function values and optimality gaps (in percentages) for 10-day runs. We use here the upper bounds of loose time windows cases. The average gap value is 3.55% whereas the maximum gap is 6.90%. In 10-day runs, we change the values of the parameters N and M . Throughout the experiments, we use $M = 5,000$ whereas N is again determined with respect to the number of patients. We set the number of iterations for ALNS as

$$N = 50,000 + 50,000 \frac{|P|}{100} \quad (36)$$

where $|P|$ represents the number of patients. In general, for 5-day and 10-day runs, the ALNS algorithm finds near-optimal solutions within several minutes at most. We observe that as the number of iterations, N , is increased, the likelihood of finding better solutions also increases, as indicated in Table 3.10.

SSPH yields near optimal solutions but as the number of patients exceeds 100, the run time also increases considerably. The solutions are on average within 1.13% of the optimal solution when the planning horizon is 5 days and 1.01% when the planning horizon is 10 days with loose time windows. On the other hand, when we solve for tight time windows, we get 1.83% gap value for 5-day runs on the average. Since for 5-day runs it already takes an excessive amount of time to achieve the solutions, we do not solve SSPH for 10 days with tight time windows. To obtain shorter run times, we also develop an Adaptive Large Neighborhood Search algorithm. The ALNS algorithm provides near-optimal solutions within short run times. While ALNS performs nearly as good as SSPH in terms of objective values, especially in small and medium size instances, it performs better than SSPH in terms of run time. When the time horizon is 5 days, SSPH takes 404.38 seconds on the average whereas ALNS takes 25.71

seconds. Moreover, when the time horizon is 10 days, SSPH takes 797.94 seconds on the average and ALNS takes 57.23 seconds. For tight time windows cases, when the time horizon is 5 days, SSPH takes 67,423.97 seconds on the average whereas ALNS takes 152.34 seconds.

Table (3.10) ALNS with TTW 10-Days

# of Patients	ALNS - 10 Replications					
	Worst Obj.	Average Obj.	Best Obj.	Average Time (s)	%Gap	Best Bound
30	1588.51	1599.44	1607.22	7.08	0.90	1621.82
40	1899.32	1909.11	1919.07	14.52	2.03	1958.80
50	2131.47	2139.04	2145.06	29.47	2.13	2191.73
60	2311.04	2316.12	2320.81	30.33	2.11	2370.91
70	2558.59	2571.19	2580.56	34.01	2.13	2636.64
80	2746.25	2764.88	2781.92	48.29	2.48	2852.74
90	2930.10	2939.27	2942.42	49.03	3.24	3041.10
100	3021.39	3069.40	3105.74	71.59	3.44	3216.45
120	3261.57	3291.67	3301.12	126.36	4.91	3471.68
140	3514.68	3551.73	3594.49	214.62	5.71	3812.19
160	3659.24	3729.35	3745.89	402.30	6.59	4009.99
180	3741.42	3779.95	3818.13	483.15	6.90	4101.22
Min	-	-	-	8.04	0.90	-
Max	-	-	-	1113.30	6.90	-
Avg	-	-	-	259.06	3.55	-

In ALNS, 13 destroy (BR-CSR in Table 3.4) and 9 repair (IBC-REG in Table 3.4) moves are deployed. At each iteration of the algorithm, a destroy and repair move pair is selected, by a roulette wheel mechanism, based on their performance scores. The general performance scores of the moves are then updated according to their most recent performances. In Figures 3.3-3.5, the performances of the destroy and repair algorithms are portrayed. The bars correspond to the selection percentage of the corresponding algorithms. The percentages are calculated over the entire set of experiments performed. For each instance (m -days n -patients) we perform 10 independent experiments. Then, at each experiment we store the performance of the algorithms and calculate the percentages over the entire experiment results. When

we analyze the figures for destroy algorithms, clearly by destroying the routes at the last days of the planning horizon, the performance of ALNS gets better. This is in line with the structure of the problem, as it is targeted to serve as many patients with high prizes as possible within the early days.

Figure (3.3) Removal algorithms statistics for 5 days

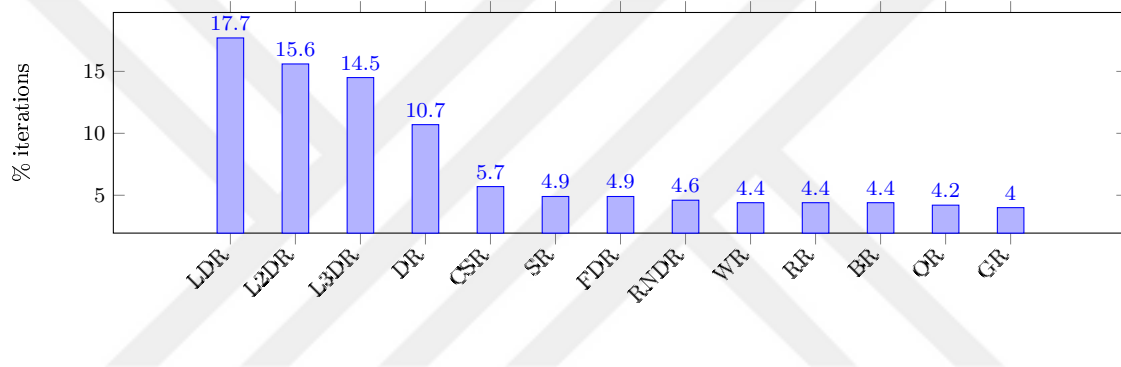


Figure (3.4) Insertion algorithms statistics for 5 days

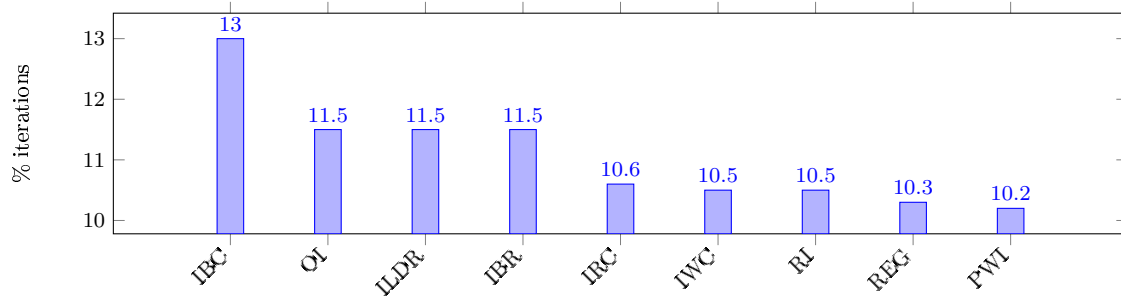


Figure (3.5) Removal algorithms statistics for 10 days

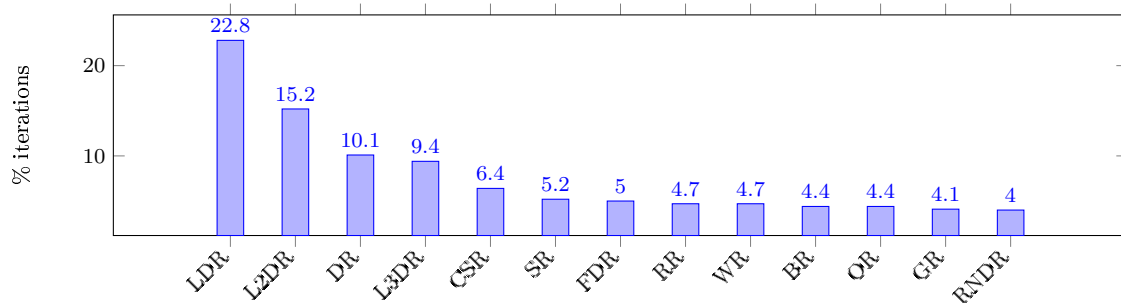
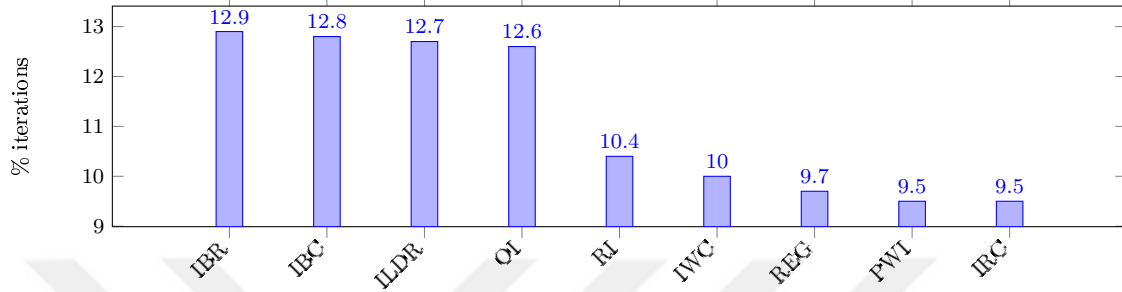


Figure (3.6) Insertion algorithms statistics for 10 days

The figures also show that it is more important to assign the patients to the right days than assigning them in the right order. This is in line with our small choice of the epsilon parameter. On the other hand, in the repair algorithms, once we destroy some routes, it is important to select the best patients in terms of their prizes and assign them into the right days. Thus, inserting best candidates to the least-dense route becomes an effective methodology as indicated in Figures 3.4 and 3.6.

3.5 Conclusions

In this chapter, we provide solution methods for a real-life problem in which a nurse is required to serve patients he is responsible for by home visits. However, due to the large number of patients, he has to be selective when planning a schedule. Therefore, priorities are assigned to patients such that the priorities of unvisited patients increase by day. The solution simultaneously specifies a visit schedule, based on patient priorities over multiple days, and the sequence of the visits to selected patients on each day. Patients that cannot be visited within the planning horizon are checked upon by phone calls. Those patients will have higher priorities compared to the visited ones when the problem is solved again for the next planning horizon since the priorities are updated dynamically.

We first develop a MIP model to solve this multi-period routing and scheduling problem. We introduce a priority parameter which is transformed into prizes that lets

us construct the MIP model as a team orienteering problem with time-dependent prizes and time windows. A Lagrangean relaxation procedure is implemented to get strong upper bounds. We present two algorithms to generate high quality solutions in reasonable time. The first algorithm is a matheuristic called SSPH. It solves the multi-period model day by day by updating the prize matrix and the unvisited patient list at each iteration. To obtain shorter run times, we also develop an Adaptive Large Neighborhood Search algorithm with many destroy and repair operators. The ALNS algorithm provides near-optimal solutions within short run times, as assessed by the upper bounds from the Lagrangean relaxation procedure.

Chapter 4

DYNAMIC APPROACH

4.1 Problem Definition

Prioritized home health-care problem (PHHP) is a multi-period routing and scheduling problem. In PHHP defined in Chapter 3, at the beginning of the planning horizon, the routes and schedules for each day are determined. We develop solution strategies including an exact mathematical model and two complementary heuristics to solve the PHHP. The algorithms provide solutions for the category of static and deterministic problems. In the real situation, every now and then a nurse encounters several on-line requests which manifest themselves as medical emergencies. Therefore, we need to develop new strategies that handle real-time requests in order to address the dynamic situation that takes place in practice.

Consider a nurse who is required to serve a number of patients assigned to her/him over a multi-period horizon. Suppose that there are T days in the horizon and at the beginning of the planning horizon the routes and the visit schedules are determined for the nurse. Let R_d denote the route for day d and N_d denote the patients assigned to be served on day d . Let $Y := \{1, 2\}$ denote the types of patients. A patient is either a Type-1 patient or a Type-2 patient. If a patient is of Type-1, then he cannot be postponed. If a patient i is of Type-2, then the patient can be postponed at most τ_i times. Suppose further that each patient has a postponement parameter $\tau_i \leq k$ for $i = 1, \dots, P$ which denotes the number of times that a patient can be postponed where $k \leq T$ is an upper bound on the postponement parameter. Thus, a patient can be postponed at most k times. Also suppose that S is a random process. Each day a number of emergency requests (Type-1 patients) is realized that is governed by the

random process S .

At the beginning of the planning horizon, the nurse has an a-priori prepared plan for the entire planning horizon. Each day a number of new requests arrive according to a random process and the nurse has to decide whether to postpone any patients while accepting the new emergency arrivals. Since the emergency patients are of Type-1, the nurse has to serve the emergent patients on the same day of the requests. Therefore, she needs to revise the plan for the current day. She might postpone some of the patients assigned to the current day a-priori or she may prefer to work over-time. However, no matter what, she must serve the patients who are of Type-1. If the patients are of Type-2, then she may prefer to serve the patients within the current day or postpone some of the patients. When the current plan is modified and an already assigned patient is removed from the route, the patient cannot be postponed more than her maximum postponement day parameter. Moreover, each day she is postponed, her priority is going to be increased exponentially.

We assume that we can estimate the service times as each patient belongs to one of the two types. Therefore, we have an estimation about the service times.

One important aspect is to decide when to re-optimize the routes if there is a new request during the day. We can specify re-optimization times during the day to re-optimize the current plan if there are any new requests before the specified time. In our problem, we introduce one re-optimization time, namely 12:30. The nurse starts her shift at 09:00 and her shift ends at 18:00. The emergencies that arrive before 12:30 are accumulated until 12:30 and at 12:30 the current plan is revised by introducing the set of emergent patients into the plan. The updated schedule also considers the lunch time for the nurse. Thus, at 12:30 the re-optimization engine is triggered but the schedule is updated by also taking the lunch time into consideration. While emergent patients are included in the plan, the nurse faces two situations i.e., either she

will work overtime or some of the patients who are of Type-2 will be postponed for the next day plan.

The objective of the dynamic home health-care scheduling and routing problem consists of several components which are as follows: routing costs in terms of total time or distance; an overtime cost caused by emergent requests; the number of patients whose original plan is changed during the planning horizon and the response time to the emergent arrivals.

Since at the beginning of each planning horizon a multi-period orienteering problem is solved, the maximization of priority and the minimization of the routing cost are taken into consideration a-priori. At this stage, the overtime cost is not considered. Therefore, total working time constraint is treated as a tight constraint. However, once the plan is under execution and new requests appear, we relax this constraint and introduce an overtime cost for working in overtime.

4.2 Optimization Framework

Our solution approach for the dynamic prioritized home healthcare scheduling and routing problem (DPHHP) has three main steps. The first step is to solve an adaptive large neighborhood search (ALNS) algorithm at the beginning of each day in the entire planning horizon. Once the ALNS provides schedules for each planning day, we select the first day schedule. Up to re-optimization time, we allow emergent arrivals to be queued in a waiting list in order to consider them for the next plan at the next re-optimization time. If there is no emergent patient, then there is no need to revise the current schedule. However, if there are emergent arrivals, then a mixed-integer programming model is triggered at the re-optimization time to include the emergent patients in the revised schedule. In Figure 4.1, a flow chart indicating the steps in the simulation for DPHHP is depicted. In our simulation, we have a total number of replications denoted by N . At each replication n , we first check whether the

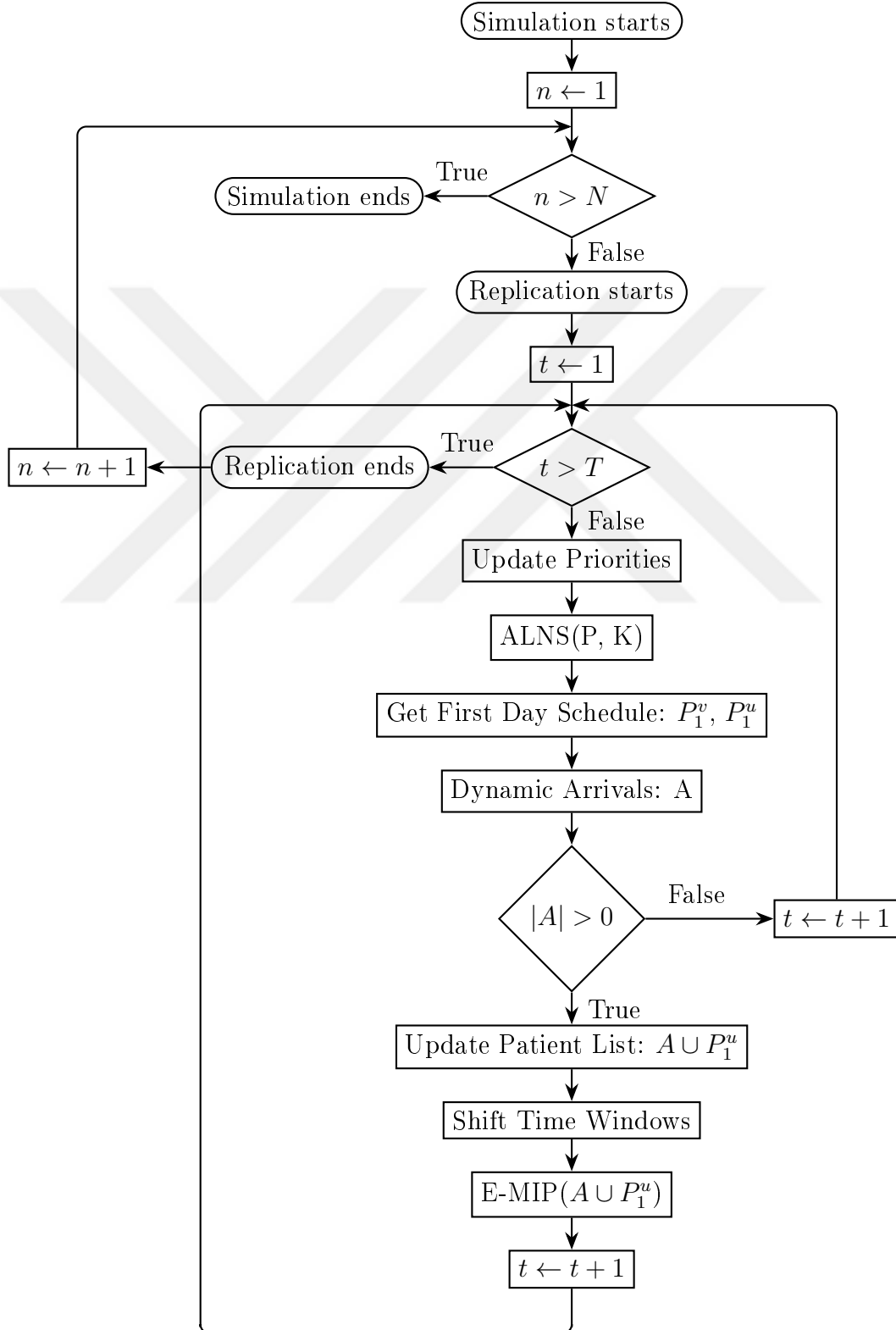


Figure (4.1) The Simulation Process for the Dynamic Solution Approach

replication counter exceeds the limit, i.e., $n \geq N$. If the total number of replications has not been exceeded yet, the n^{th} simulation starts. At each replication, we repeat the entire optimization procedure for T days. On day t , we check whether the day counter exceeds the total number of days, i.e., $t \geq T$. If it has not exceeded the day limit, then we first update the priorities for each patients on day t . Once the priorities are updated, we call the ALNS algorithm to run for a planning horizon of K days for all patients. The ALNS algorithm returns a K -day schedules and routes. Among these K days, we get the first day's schedule. We divide the patients of the first day as visited P_1^v and unvisited P_1^u patients until re-optimization time. We also accumulate the dynamic arrivals on a list A . If there is no emergent arrival, i.e., $|A| = 0$, then the plan under execution does not change and we increase the day counter t . However, if there are emergent patients, i.e., $|A| > 0$, then we update patient list of day t by joining the emergent patients A and patients P_1^u who are not visited yet. Once the patient list is updated for the current day, we allow an overtime duration by shifting the time windows. Once the time windows are shifted, we call the mixed-integer programming formulation $E - MIP(A \cup P_1^u)$ and create the plan for the rest of the day for emergent patients and patients who are planned for the first day but not visited yet. Finally, the day counter t is increased. Whenever the replication ends, i.e., $t \geq T$, we increase the replication counter n .

4.2.1 Solution Approach

At the beginning of each day in the planning horizon, an ALNS algorithm is triggered to create the schedules and routes for each day in the planning horizon. However, due to emergent arrivals during the day, the plans previously generated are subject to change because the policy is to serve the emergent patients on the day they arrive. Thus, until the predetermined re-optimization time all emergent requests are queued in a waiting list and once the re-optimization comes, a smart procedure is required in order to include the emergent patients in the plan while postponing as few patients as possible. To this aim, we introduce a mixed integer programming

formulation (MIP). The MIP model tries to maximize the prize while minimizing the routes and overtime. The model tries to serve emergent patients during the normal shift hours. However, if it is not possible to serve the emergent patients during the normal shift hours, then the nurse tries to serve as many patients as possible. Below, we provide the list of input parameters and decision variables for the MIP, which is to be solved at a re-optimization time point. We call this model as E-MIP where E stands for emergent patients. The fundamental difference between E-MIP and the exact mathematical model introduced in Chapter 3 are: a new cost component which accounts for overtime, a new set of binary variables which controls whether overtime has been made and finally a set of constraints that forces the emergent patients to be included in the current day's plan.

Input Parameters

- $\{0, n + 1\} \cup P$ represents the set of patients on the current day, where $P = \{1, \dots, n\}$, and $\{0\}$ represents the starting patient location and the nurse's home base is indexed as $\{n + 1\}$.
- E represents the set of emergent patients.
- p_i is the prize of patient i ; higher p_i values belong to more prioritized patients.
- s_i is the service time required to serve patient i .
- e_i is the earliest service start time for visiting patient i on a day.
- l_i is the latest service start time for visiting patient i on a day.
- t_{ij} is the time required to travel between locations of patients i and j .
- W is total working hours for the nurse per day.
- M is a sufficiently big number.
- ϵ is a sufficiently small number.

Decision Variables

- $x_{ij} \in \{0, 1\}$ represents whether patient j is visited right after patient i .
- $y_i \in \{0, 1\}$ indicates whether patient i is selected.

- $a_i \geq 0$ is the arrival time to the location of patient i .
- a_0 is the start time from the last patient visited.
- $u \in \{0, 1\}$ represents whether overtime needs to be done.

$$z = \max \sum_{i=1}^n p_i y_i - \epsilon \left(\sum_{i=1}^n s_i y_i + \sum_{i=0}^n \sum_{\substack{j=0 \\ j \neq i}}^n t_{ij} x_{ij} \right) - Mu$$

s.t.

$$\sum_{\substack{j=1 \\ j \neq i}}^{n+1} x_{ij} = y_i \quad \forall i \in P \quad (37)$$

$$\sum_{\substack{j=0 \\ j \neq i}}^n x_{ji} = y_i \quad \forall i \in P \quad (38)$$

$$\sum_{j=1}^{n+1} x_{0j} = 1 \quad (39)$$

$$\sum_{i=0}^n x_{i,n+1} = 1 \quad (40)$$

$$y_i \leq 1 \quad i \in P \quad (41)$$

$$y_i = 1 \quad i \in E \quad (42)$$

$$t_{n+1} - l_{n+1} \leq Mu \quad (43)$$

$$\sum_{i=0}^{n+1} s_i y_i + \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} t_{ij} x_{ij} \leq W \quad (44)$$

$$a_i + s_i + t_{ij} - a_j \leq M(1 - x_{ij}) \quad \forall i, j \in P \quad (45)$$

$$e_i y_i \leq a_i \leq l_i y_i \quad \forall i \in P \quad (46)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in P \cup \{0, n+1\} \quad (47)$$

$$y_i \in \{0, 1\} \quad \forall i \in P \quad (48)$$

$$a_i \geq 0 \quad i \in P \quad (49)$$

$$u \in \{0, 1\} \quad (50)$$

The objective function has three components. The first one is the total prize of

the visited patients, the second one is the negative of the total time needed to complete the routes over the planning horizon multiplied with a small weight ϵ and the third one is charging the overtime with a big cost. The overtime is not wanted as long as the emergent arrivals are served within normal shift hours. However, if it is not possible to serve the emergent patients within normal shift hours, then we allow an overtime of 2 hours. Once the overtime is to be made, then we ask the nurse to use the overtime duration as much as possible. Constraints (37)-(38) ensure that if a patient is visited, then either the patient is the first patient to be visited or there is another patient who was visited before him and similarly the patient is either the last patient to be visited or there will be another patient who is going to be visited right after him. Constraints (39)-(40) enforce that the nurse has to start from a starting location and return to her home at the end of the day, unless no patient visit is scheduled for that day. Constraints (41) state that not each patient has to be visited necessarily. Constraints (42) state that the nurse has to visit each emergent patient. Constraint (43) controls whether an overtime is required. Constraint (44) takes total working hours of the nurse into consideration and limits the total time of the tour each day. Constraints (45) make sure that if patient j is visited after patient i , then the time to begin service for patient j must be at least $s_i + t_{ij}$ after the nurse arrives at patient i . Constraints (46) state that if a patient is selected to be visited, then the arrival time has to be within the time window of the patient. Otherwise, the arrival time should be zero. Constraints (47)-(50) define the decision variables.

The mathematical models introduced in Chapter 3 solves a multi-period deterministic problem for home healthcare scheduling and routing problem. At the beginning of a planning horizon, the model is triggered and it provides the schedules and routes for the entire planning horizon. In E-MIP, we have a couple of new decision variables and a set of new constraints because of emergency situations. We have to make sure that the emergent patients will definitely be visited. Therefore, we introduce the new set of constraints in (41). While the emergent patients are visited, we might encounter

an overtime situation. Therefore, we also introduce a new constraint in (43) which controls whether an overtime has been made. Also, we introduce a new objective component that accounts for the overtime. The remaining constraints have similar functionalities as the counterparts in the optimization models introduced earlier.

4.3 Computational Results

In order to evaluate our solution approach with respect to some metrics, we conduct a simulation study. We consider a nurse who is responsible for 196 patients. The nurse works between 09:00 AM and 18:00 PM. We allow an overtime of 2 hours if necessary. The number of days in the simulation is 10 and at the beginning of each day an adaptive large neighborhood search (ALNS) algorithm with several destroy and repair heuristics runs in order to create visit schedules for each day. Then, at the beginning of the planning horizon emergent arrivals occur. These arrivals are mandatory to visit. By a mixed integer mathematical model we construct a modified schedule for the new arrivals and patients yet to be visited on the same day. Since the schedules are subject to change because of emergent arrivals, at the beginning of the second day we run ALNS again and repeat the same procedure for 10 days. For the initial computational results, we assume that there is only one re-optimization time and we only accept the emergent patients alerted before the re-optimization time. In the experiment, the re-optimization time is assumed to be 13:30 and dynamic arrivals come from a Poisson process where the number of arrivals denoted by Q is bounded from below and above, $Q \leq 4$.

4.3.1 An Illustrative Example with Single Re-optimization

We provide an illustrative example in order to show how the methodology depicted in Figure 4.1 works. We have a nurse who is responsible for 196 patients and the emergent arrivals occur by a Poisson process with different arrival rates λ . We provide the plans and modifications for the first five days.

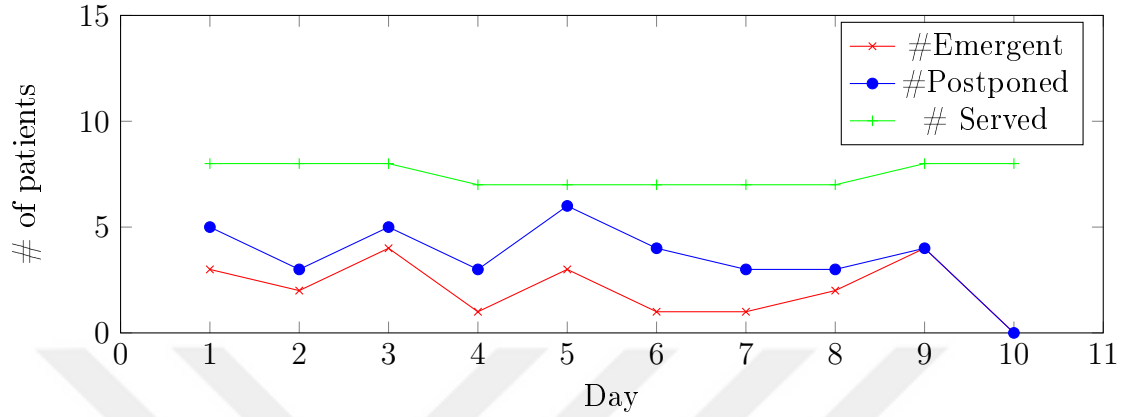


Figure (4.2) Replication 1 - Poisson Rate $\lambda = 1$

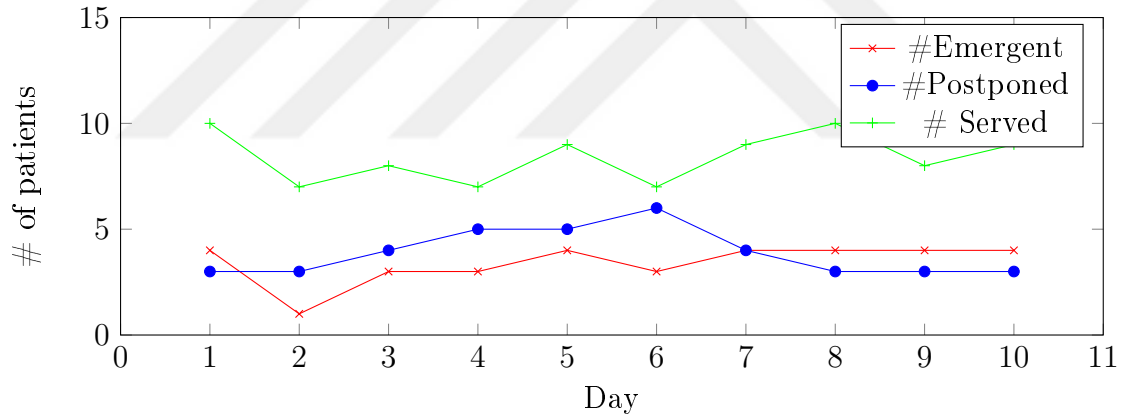


Figure (4.3) Simulation 2 - Poisson Rate $\lambda = 1$

The number of emergent patients, postponed patients and total served patients are depicted for these days in Figures 4.2-4.6. The horizontal axis denotes the day and the vertical axis denotes the number of patients.

In Figure 4.7, we see the schedule at the beginning of the planning horizon. During the day, four emergent arrivals (patients P_{64} , P_{185} , P_{123} , P_{187}) occur. It is mandatory to visit the emergent patients. At the re-optimization time, the MIP model runs and updates the schedules by taking emergent arrivals into consideration. The circle with a blue color denotes a patient who is visited and the circle with a red color indicates a

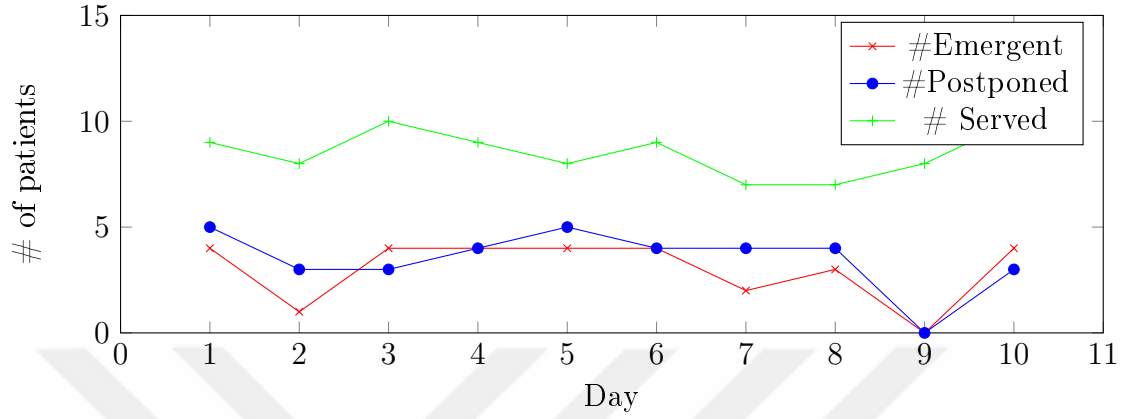


Figure (4.4) Replication 3 - Poisson Rate $\lambda = 1$

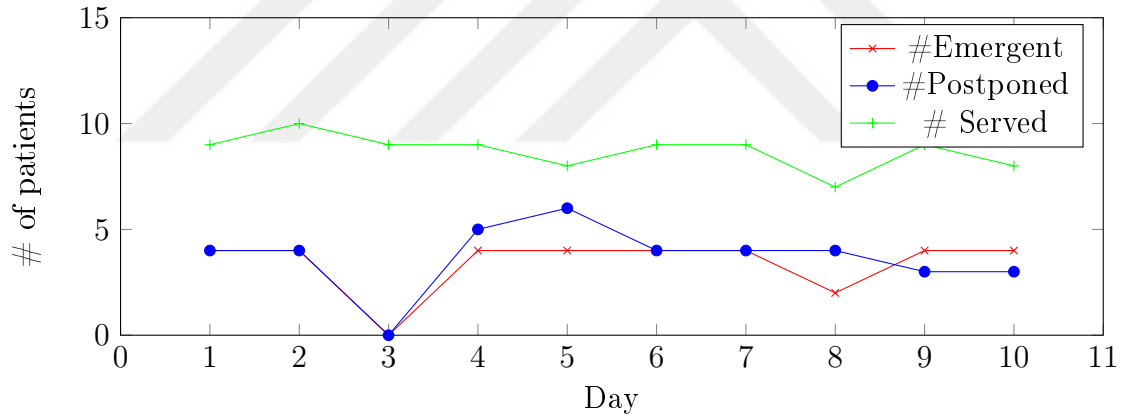


Figure (4.5) Replication 4 - Poisson Rate $\lambda = 1$

patient who is postponed therefore is not visited on the current day. The green color indicates the nurse's home location.

In Figure 4.8, the modified schedule for the first day is depicted. The route starts from patient P_{14} and only visits the mandatory emergent arrivals and returns to nurse's home location. Several patients who were in the schedule at the beginning of the day are postponed because of the emergent arrivals.

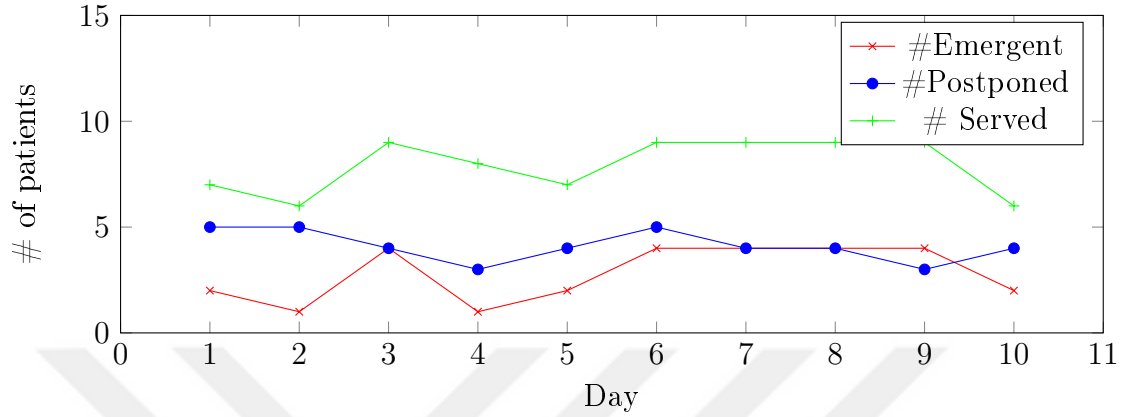
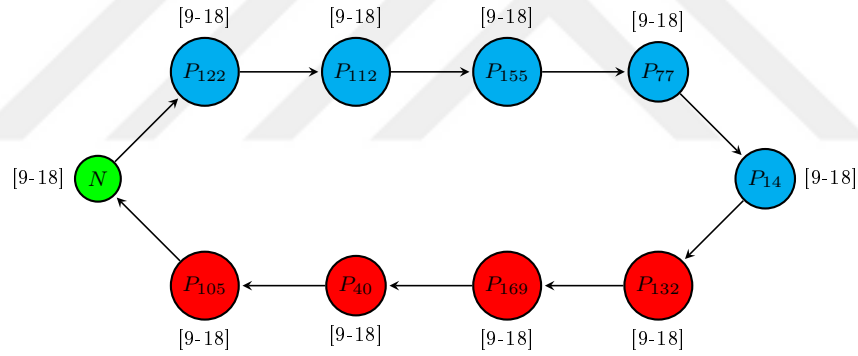
Figure (4.6) Replication 5 - Poisson Rate $\lambda = 1$ 

Figure (4.7) Day 1 Schedule

Since the emergent arrivals are mandatory to visit, the schedule after re-optimization changes. The nurse may work overtime if the emergent patients cannot be served during the shift hours, otherwise the patients are postponed for the next planning days. On the first day, the emergent patients are served during the normal shift hours but patients P_{132} , P_{169} , P_{40} and P_{105} are postponed. At the beginning of the second day, ALNS provides a 5-day plan which provides schedules for days 2-6. The visit schedule at the beginning of day 2 is provided in Figure 4.9.

It is important to note that patients P_{105} , P_{132} and P_{169} were in the original schedules of day 1. However, due to emergent arrivals they were postponed. As the patients

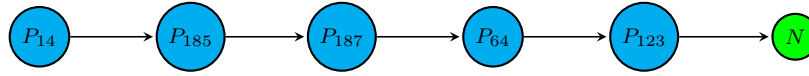


Figure (4.8) Day 1 Modified Schedule

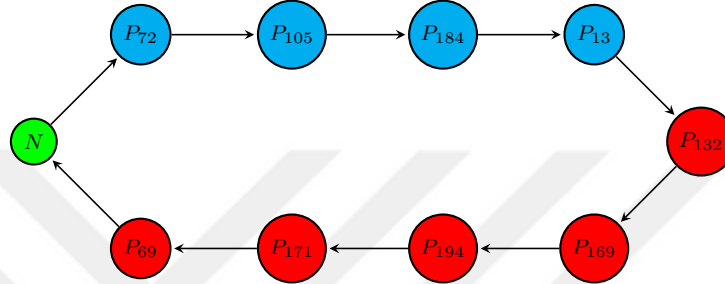


Figure (4.9) Day 2 Schedule

are postponed, their priorities increase. Therefore, it is not surprising to see these patients in the second day's plan. During the second day, we observe four emergent arrivals, patients P_{39} , P_{58} , P_{117} and P_{48} . Thus, we need to modify the original plan using the MIP.

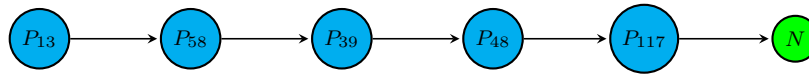


Figure (4.10) Day 2 Modified Schedule

After re-optimizing the plan, the emergent patients are included in the modified plan, but patients P_{132} , P_{169} , P_{194} , P_{171} , and P_{69} are postponed for the next days.

For the third day, the ALNS again provides a 5-day schedule, namely for the upcoming days 3-7. The visit schedule of the third day is depicted in Figure 4.11. Up to re-optimization time, patients P_{62} , P_{171} , P_{84} , and P_{132} are served. During the third day, we observe only four emergent arrivals, patient P_{26} , P_{122} , P_{179} and P_{33} who will be served. Modifying the current schedule because of the emergent arrival causes changes in the plan and some of the patients are necessarily left to be served in the

upcoming days. Since we allow postponement for the patients who are not emergent, patients P_{26} , P_{122} , P_{179} and P_{33} are postponed for the next days even though they were in the schedule at the beginning of the third day.

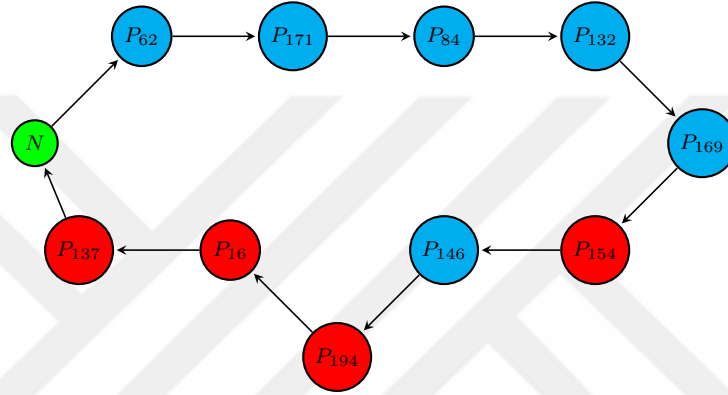


Figure (4.11) Day 3 Schedule

After re-optimization time where we force emergent patients P_{154} , P_{194} , P_{16} and P_{137} to be included in the schedule, the plan for the third day is modified. The remaining partial plan for the third day is depicted in Figure 4.12.

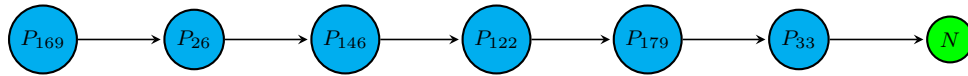


Figure (4.12) Day 3 Modified Schedule

The simulation repeats for 10 days. At the beginning of each day, the ALNS algorithm runs for 5-days, then emergent arrivals occur by a stochastic process and a mixed integer mathematical model runs for the current day in which emergent patients are served even if it requires an overtime. If within the working time hours, it is possible to serve all emergent patients, then the model prefers to postpone some of the patients who are previously selected to be visited. Of course, it depends on the overtime cost. If we decrease overtime cost, this may not happen. At the beginning of the fourth

day, the ALNS algorithm assigns patients who are postponed from earlier days to the first day. The schedule for the fourth day at the beginning of the 5-day planning horizon is depicted in Figure 4.13.

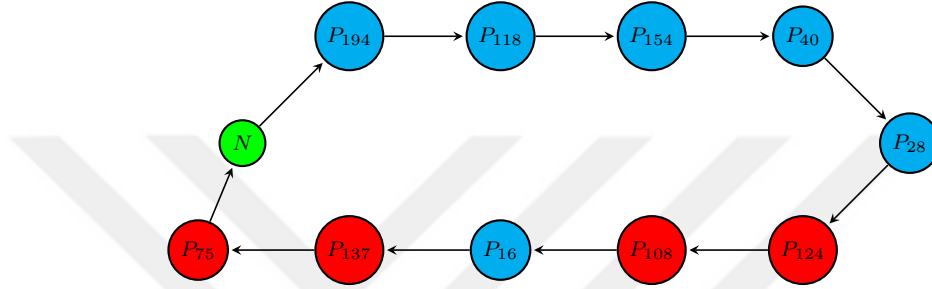


Figure (4.13) Day 4 Schedule

During the fourth day, three emergent arrivals occur, patients P_{121} , P_{74} and P_{192} . At the re-optimization time, the MIP model runs and according to its solution, the route is modified by adding the emergent patients and postponing patients P_{124} , P_{108} , P_{137} and P_{75} for the upcoming days. The modified route starts at the location of patient P_{40} and returns to the nurse's home location. The modified schedule is depicted in Figure 4.14.

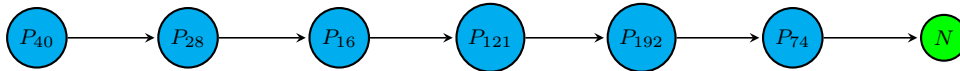


Figure (4.14) Day 4 Modified Schedule

The schedules for the remaining days are generated by applying the same flow. The plan at the beginning of the fifth day is depicted in Figure 4.15.

After the emergent arrivals, patients P_{113} , P_{48} , P_{53} , and P_{188} , the schedule is modified as in Figure 4.16.

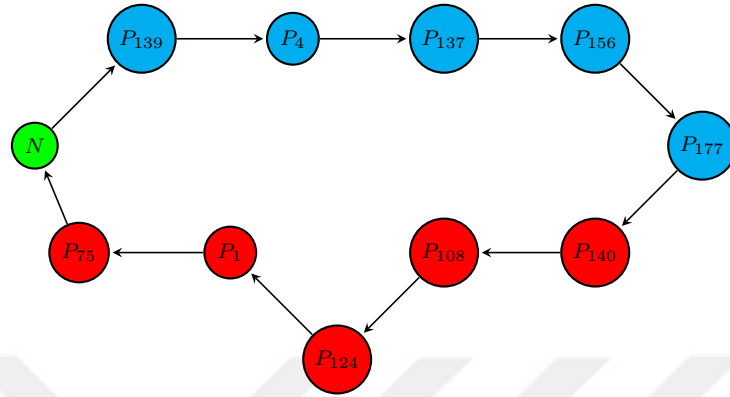


Figure (4.15) Day 5 Schedule



Figure (4.16) Day 5 Modified Schedule

This illustrative example indicates how the optimization framework we introduce works. At the beginning of the planning horizon, schedules and routes for the planning horizon is created. Then, until a re-optimization time the emergent arrivals are accumulated and at the re-optimization time the plan is revised according to the emergent patients. In this illustrative example, we assumed an arrival rate of $\lambda = 1$ in order to have several emergencies to see how the emergent arrivals are going to be included in the plan. As observed from the graphs for each day, when we have many emergent arrivals before re-optimization time the model forces many of the planned patients to be postponed to provide service for the emergent patients. While emergent patients are included in the plan, if it is possible to serve non-emergent patients within updated shift hours the model ensures that at least one of the patients are also included in the plan. However, if the emergent patients use the entire overtime duration, then the model postpones the non-emergent patients. If a non-emergent patients, of Type-2, is postponed, then we observe that the patient is immediately included in the next day's plan. It is expected to be included in the next day's plan because as we postpone the patient, his or her priority increases which makes sure

that the patient will be included in the plan as earlier as possible.

4.3.2 An Illustrative Example with Multiple Re-optimization

In order to see the performance of the optimization framework with more than one re-optimization time during the day, we test the system with 2, 3 and 4 re-optimization times during the day. The results for 2 re-optimization times are displayed in the following figures. The nurse is responsible for 196 patients as in the case with a single re-optimization time. In this case, we assume two re-optimization times, 12:00 and 15:00. The emergencies that occurred before 12:00 are accumulated until the first re-optimization time and then the mathematical model is triggered to reschedule the plan under execution. Then, once the plan is modified after the first re-optimization time, the emergencies are accumulated till the second re-optimization time. We provide the plans and modifications for the first five days.

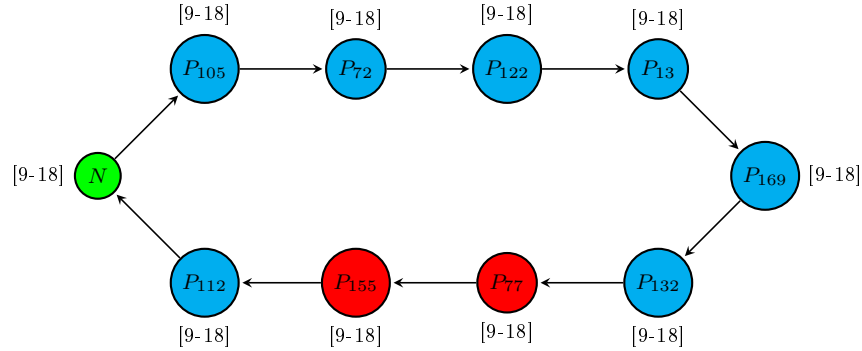


Figure (4.17) Day 1 Schedule

In Figure 4.17, we see the schedule at the beginning of the day. Before the first re-optimization time an emergent patient P_{177} arrives. Therefore, the E-MIP model is triggered for a rescheduling the plan under execution. In Figure 4.18, the modified route after the first re-optimization time is depicted.

For the first day, we observe only an emergency before 12:00. Thus, the route is mod-

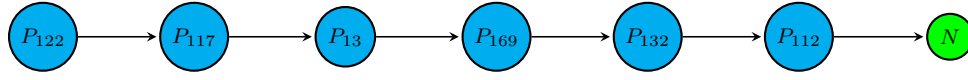


Figure (4.18) Day 1 Modified Schedule

ified only once. Two patients P_{155} and P_{77} are postponed while the emergent patient P_{177} are included the modified plan. The nurse does work overtime for the first day.

In Figure 4.19, we see the schedule generated for the second day. The postponed patients from previous day are included in the plan. Since their priorities get higher each day they are postponed, it is obvious to understand why the model prefers to visit them on the very next day after they are postponed.

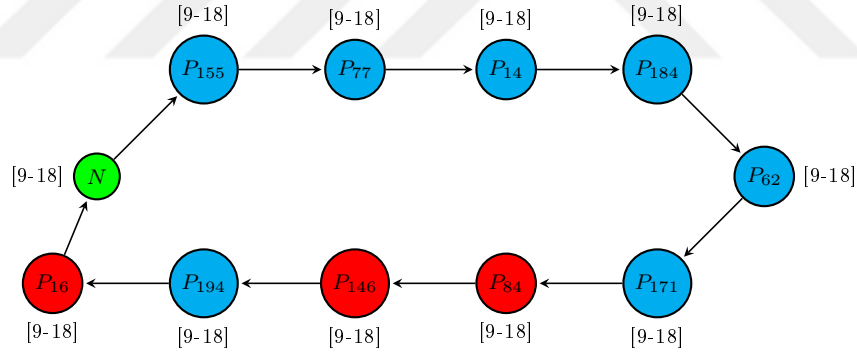


Figure (4.19) Day 2 Schedule

Before the first re-optimization time two emergent patients P_{57} and P_{167} arrive. Therefore, E-MIP is triggered at the first re-optimization time and the schedule under execution is changed. In Figure 4.20 we see the updated plan.

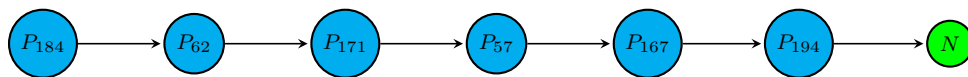


Figure (4.20) Day 2 Modified Schedule - 1

While some patients are included in the plan because of their emergency situation,

some other patients of Type 2 are postponed. Between first re-optimization time 12:00 and second re-optimization time 15:00, we observe another emergency, patient P_{99} , which also create a change in the plan and results in an overtime. In Figure 4.21, we see the reschedule after the second re-optimization time.

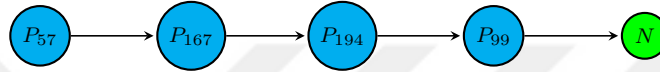


Figure (4.21) Day 2 Modified Schedule - 2

At the beginning of each day, the ALNS algorithm is called to yield a 5 day plan. Thus, each day we get a different set of patients to visit which might have slight changes compared to the previous days. However, the algorithm always tries to maximize the total prize collected while minimizing the route cost. In Figure 4.22, the plan for the third day is depicted. As usual, patients postponed from previous days because emergencies are selected to be served.

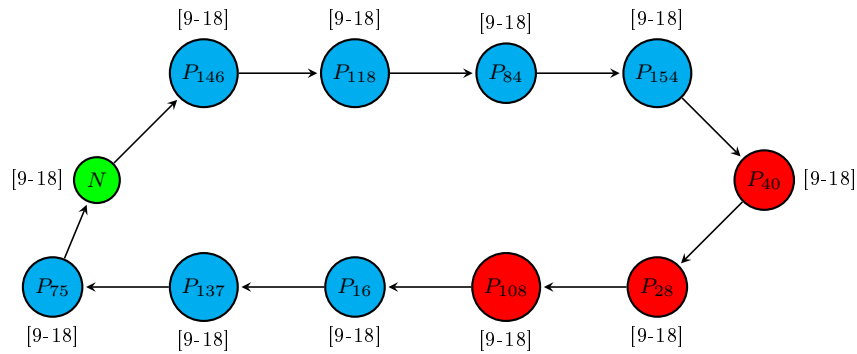


Figure (4.22) Day 3 Schedule

We observe two emergencies before first re-optimization and between first and second re-optimization times, there are no emergent patients arrival. Therefore, the E-MIP model is called only once at the re-optimization time and modified the plan under execution. In Figure 4.23, we see the modified plan. After the reschedule, the patients

P_{28} , P_{40} and P_{108} are postponed.



Figure (4.23) Day 3 Modified Schedule

In Figure 4.24, we see the plans for the fourth day produced by the ALNS algorithm. The algorithm is called to provide plan for the next 5 days and we are interested in the current day's plan. As in the previous days, we see patients who are postponed in earlier days because of emergency arrivals. The algorithm understands that their priority has increased and they must be included in the plan.

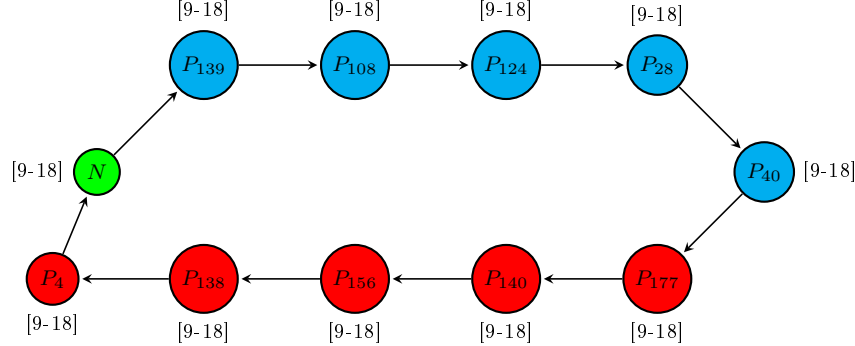


Figure (4.24) Day 4 Schedule

In fourth day, we see two emergency arrivals before the first re-optimization time and two emergency arrivals between the first and second re-optimization times. Thus, the plans are revised twice during the day which causes to postpone a few patients for the next days. In Figure 4.25, we see the updated plan after the first re-optimization time.

While the emergency arrivals before the first re-optimization time forces the E-MIP to postpone patients in order not to have an overtime, the emergencies between the first and second re-optimization time often result in an overtime for the nurse. In

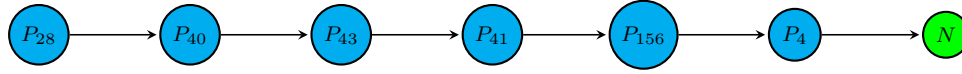


Figure (4.25) Day 4 Modified Schedule - 1

Figure 4.26, we see the modified plan for the fourth day and the nurse works overtime.



Figure (4.26) Day 4 Modified Schedule - 2

On the fifth day, we observe two emergent arrivals, patients P_{46} and P_{145} , before first re-optimization time and two emergent arrivals, patients P_{32} and P_{100} , between first and second re-optimization times. As the number of emergencies increase, the possibility for an overtime increase as well. In figure 4.27, we see the plan for the fifth day at the beginning of the day.

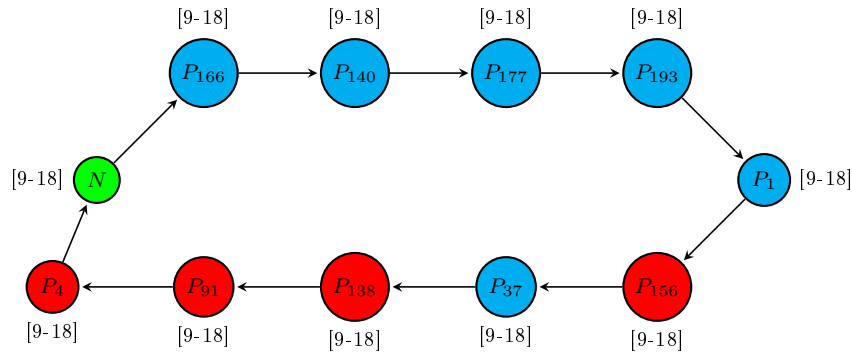


Figure (4.27) Day 5 Schedule

Because of the nature of the mathematical model E-MIP, we always want the nurse not working in overtime. Thus, for the emergencies before the first re-optimization time the model tries to postpone as many patients as possible in order to include the

emergencies as well as not forcing the nurse to work in overtime.



Figure (4.28) Day 5 Modified Schedule - 1

However, when we see emergent arrivals later on during the day, the model needs to serve the emergent arrivals no matter what the situation is by respecting the maximum available overtime duration. Therefore, for the emergent arrivals between first and second re-optimization times, we often see that the nurse works in overtime to be able to serve all the emergencies occurred during the day.

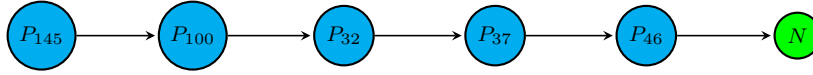


Figure (4.29) Day 5 Modified Schedule - 2

As a result, the optimization framework introduced in this thesis is able to incorporate several re-optimization times and provide reschedules for the days in which emergency arrivals are observed. The strategy chosen in this thesis is to define predetermined re-optimization times and accumulate the emergency arrivals until re-optimization times. At the re-optimization time, if there exists any emergency then the E-MIP model is called and the plan under execution for the current day is modified. However, this approach might result in a potential issue where the emergencies are not served immediately. Of course, the emergencies in home healthcare sector is not life threatening or putting in another way the nurses for the home healthcare services are not responsible for the cases which requires an emergency room attention. Thus, our goal in emergency case is to visit the patients on the same day. This is why we define predetermined re-optimization times and accumulate the emergent patients.

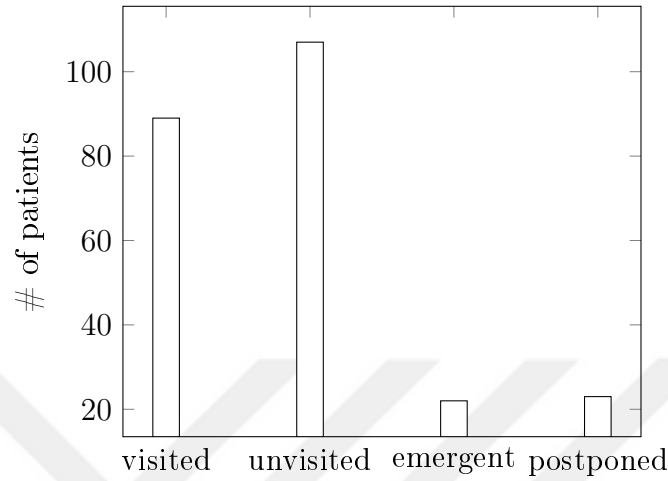


Figure (4.30) Visit Statistics

During the day, we might re-optimize our plans several times according to the number of re-optimization times. But the model and the optimization framework can be modified in such a way that whenever an emergency is observed, the model may be triggered and the plan under execution can be modified. As long as the data communication is done appropriately, our approach is robust enough to answer instant re-optimization. Another potential discussion point is what if emergencies occur after the last re-optimization time. In theory, an emergency can be observed any time during the day. However, a nurse has a shift and a maximum allowable overtime during the day. If emergencies occur frequently and some of them are not served on the day they become emergent, one solution might be to assign another nurse for the emergency cases. While the emergencies after the last re-optimization time are not considered by the model, if such a situation arises they might also be considered in the next day for the first re-optimization time so that they might be visited within a 24 hour time frame.

In Figure 4.30, we provide the visit frequencies for a 10 day planning horizon with 2 re-optimization time. Each day different number of emergencies arrive. Depending on the emergency times, the model either decides to postpone some patients or prefers

to visit in overtime. For this particular case, we have a nurse who is responsible for 196 patients. Within 10 day planning horizon, 89 of these patients are served where 22 patients are emergent. To serve the emergent patients on the days they arrive, 23 patients are postponed during the planning horizon. The number of unvisited patients in the 10 day planning horizon is 107. These patients most likely will be visited within the next planning horizon because those who are already served get a relatively less priority when compared to those who are not served yet.

In Table 4.1, we provide the number of visited, unvisited, emergent and postponed patients. In general, the postponed patients are visited within the next day. On the other hand, emergent patients are visited on the same day.

Table (4.1) Statistics for Re-Optimization Times

# of Re-Opt Time	Visited	Unvisited	Emergent	Postponed
2	89	107	22	23
3	86	110	26	31
4	81	115	32	39

4.3.3 Simulation Results

We perform a simulation experiment in order to see how the number of emergent patients, the number of postponed patients and the number of served patients change when we assume different arrival rates for a re-optimization time. We use a Poisson process with arrival rates $\lambda = 1$, $\lambda = 1/2$, $\lambda = 1/3$, and $\lambda = 1/4$ which denotes the average number of arrivals per hour. For all simulation results, we assume an overtime of 2 hours.

In Figure 4.31, the number of emergent patients, the number of postponed patients and the number of served patients throughout 10 days are depicted for each replication in our simulation where the emergent arrivals come from an underlying Poisson process with rate $\lambda = 1$. In Figure 4.32, the amount of overtime the nurse is required

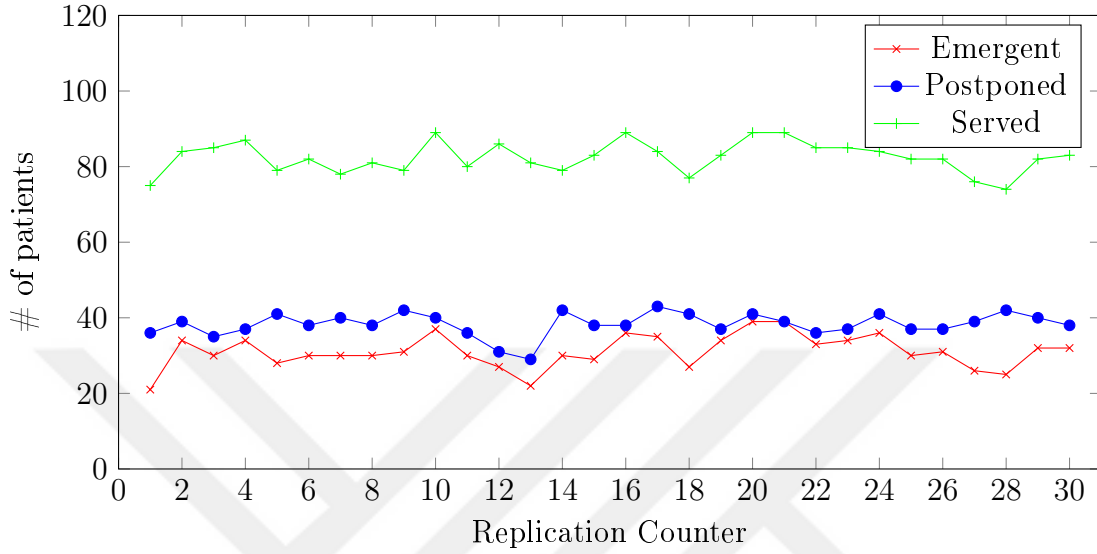


Figure (4.31) Patients Statistics - Poisson Rate $\lambda = 1$

to work in order to serve emergent arrivals are shown.

In Table 4.2, we provide the statistics concerning the total number of emergencies, postponements, overtime and patients served for each simulation run for a 10 day planning horizon. In this analysis, we provide the results for 30 independent simulation where the emergent patients arrive by a Poisson process with arrival rate 1 and we allow an overtime duration of 2 hours if the nurse cannot serve the emergent patients during the normal shift hours. The total number of emergent patients allowed to be served during a day is limited by 4. So, each day a nurse can serve up to 4 emergent patients.

For the results in Table 4.2, we have only 1 re-optimization time which is 12:30. The emergent arrivals are accepted until this hour. Then, the re-optimization procedure is triggered and the schedule for the rest of the day is modified accordingly. In Table 4.2, we observe an average of 31.07 emergent arrivals, whereas the maximum number of emergent arrivals seen is 39 during the 10 day planning horizon. The average number of postponed patients is 38.27 and 82.40 patients are served on the

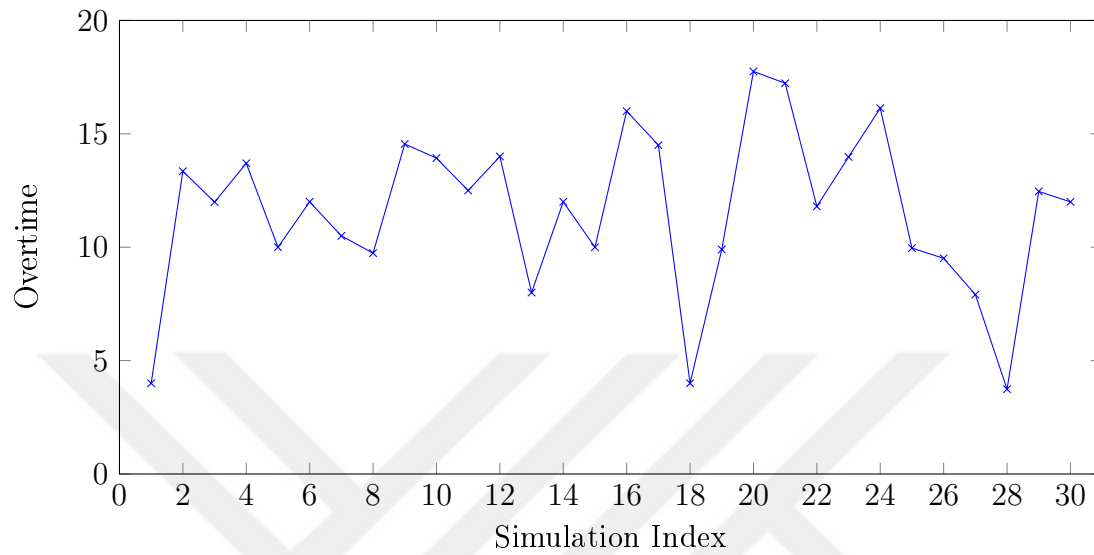


Figure (4.32) Overtime (hrs) - Poisson Rate $\lambda = 1$

average during the 10 day planning horizon. As the number of emergent patients gets higher, more patients are postponed in order to be able to serve the emergent patients.

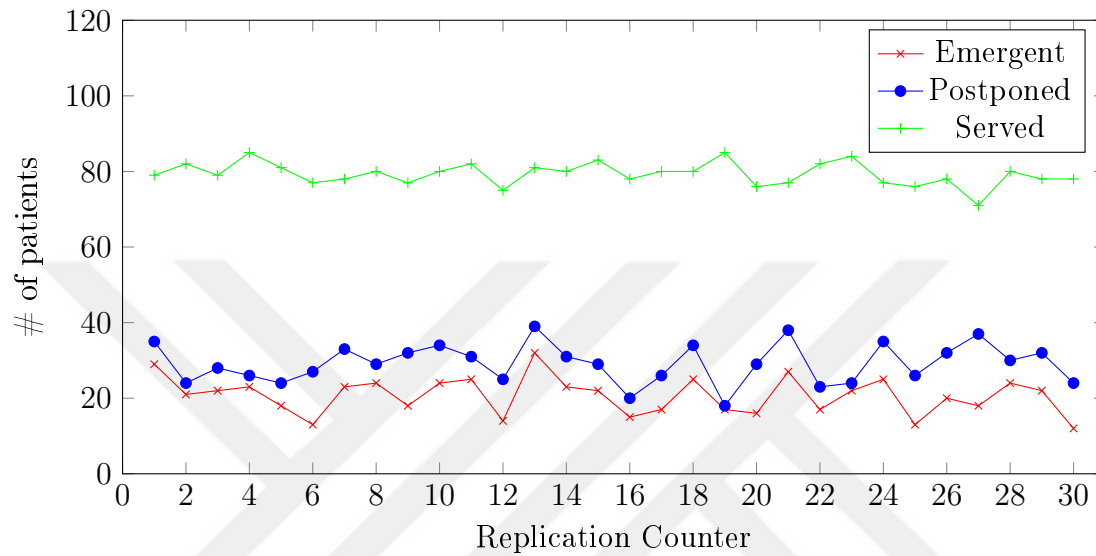
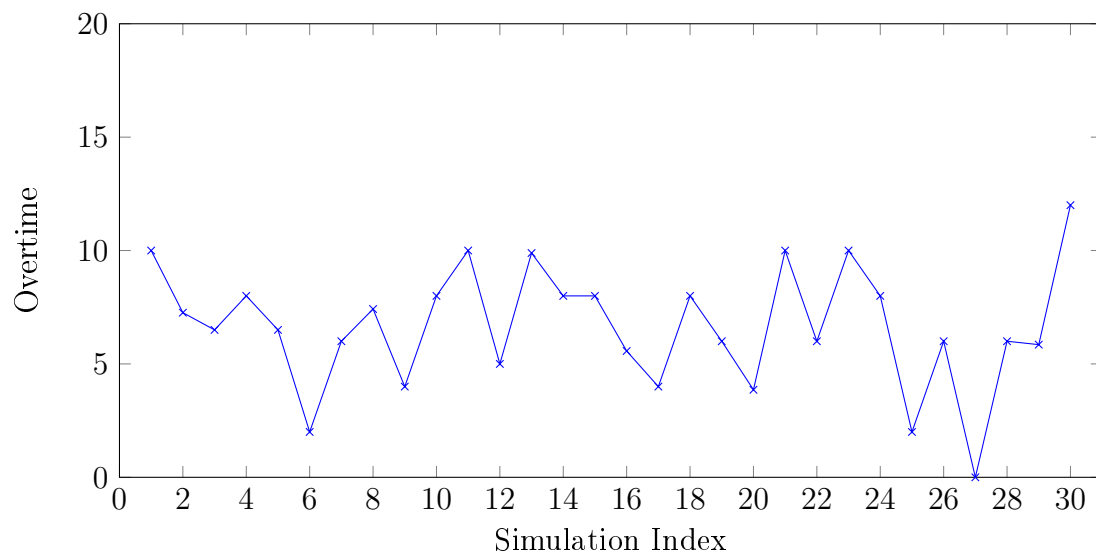
In Figure 4.33, the number of emergent patients, the number of postponed patients and the number of served patients throughout 10 days are displayed for each simulation for emergent arrivals by an underlying Poisson process with rate $\lambda = 1/2$ and an overtime of 2 hours. In Figure 4.34, the amount of overtime the nurse is required to work in order to serve emergent arrivals are shown.

In Table 4.3, we provide the statistics concerning the total number of emergencies, postponements, overtime and patients served for each simulation run for a 10 day planning horizon. In this analysis, we provide the results for 30 independent simulation runs, where the emergent patients arrive by a Poisson process with arrival rate $1/2$ and we allow an overtime duration of 2 hours if the nurse cannot serve the emergent patients during the normal shift hours. The total number of emergent patients allowed to be served during a day is limited by 4. So, each day a nurse can serve up to 4 emergent patients. For the results in Table 4.3, we have only 1 re-optimization

Table (4.2) Simulation Results for DPHHP with Poisson Arrivals Rate = 1

Simulation	# of Emergencies	# of Postponement	# of Patients	Total Overtime
1	21	36	75	4
2	34	39	84	13.35
3	30	35	85	11.99
4	34	37	87	13.7
5	28	41	79	10
6	30	38	82	12
7	30	40	78	10.5
8	30	38	81	9.74
9	31	42	79	14.55
10	37	40	89	13.93
11	30	36	80	12.5
12	27	31	86	14
13	22	29	81	8
14	30	42	79	12
15	29	38	83	10
16	36	38	89	16
17	35	43	84	14.5
18	27	41	77	4
19	34	37	83	9.9
20	39	41	89	17.75
21	39	39	89	17.23
22	33	36	85	11.8
23	34	37	85	13.98
24	36	41	84	16.13
25	30	37	82	9.96
26	31	37	82	9.51
27	26	39	76	7.91
28	25	42	74	3.74
29	32	40	82	12.46
30	32	38	83	12
Max	39	43	89	17.75
Min	21	29	74	3.74
Avg	31.07	38.27	82.40	11.57
Std	4.34	3.04	4.07	3.54

Overtime = 2hrs, ReOpt time = 12:30, Max. Accepted Emergency = 4

**Figure (4.33)** Patients Statistics - Poisson Rate $\lambda = 1/2$ **Figure (4.34)** Overtime (hrs) - Poisson Rate $\lambda = 1/2$

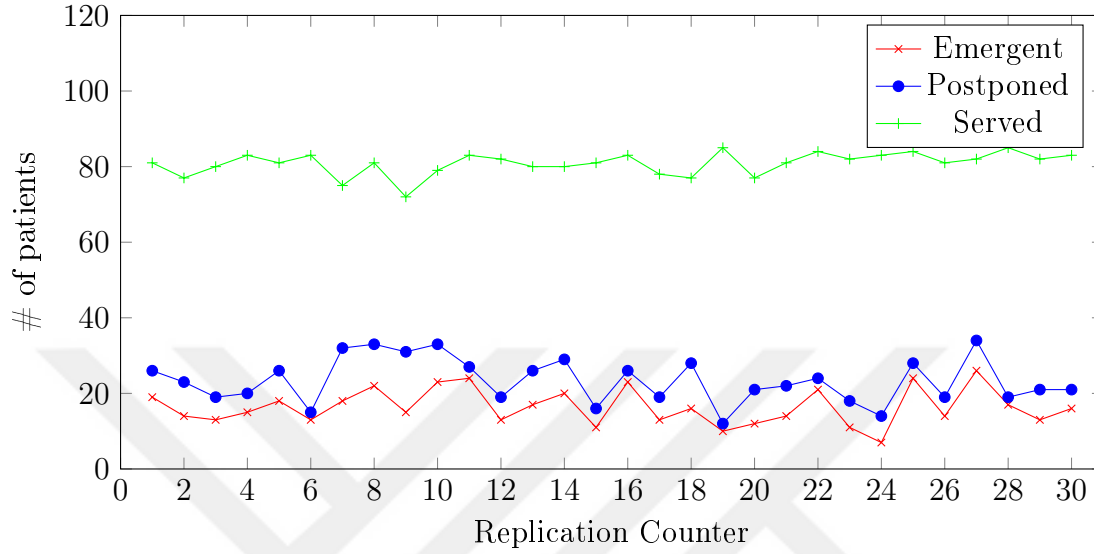


Figure (4.35) Patients Statistics - Poisson Rate $\lambda = 1/3$

time which is 12:30. The emergent arrivals are accepted until this hour. Then, the re-optimization procedure is triggered and the schedule for the rest of the day is modified accordingly. In Table 4.3, we observe an average 20.7 emergent arrivals whereas the maximum emergent arrival seen is 32 during the 10-days planning horizon. The number of average postponed patients is 29.17 and 79.30 patients are served on the average during the 10-days planning horizon. As the number of emergent patients are high, a slightly more patients are postponed in order to be able to serve the emergent patients.

In Figure 4.35, the number of emergent patients, the number of postponed patients and the number of served patients throughout 10 days are displayed for each simulation run for emergent arrivals by an underlying Poisson process with rate $\lambda = 1/3$ and an overtime of 2 hours. In Figure 4.36, the amount of overtime the nurse is required to work in order to serve emergent arrivals are shown.

In Table 4.4, we provide the statistics concerning the total number of emergencies, postponements, overtime and the number of patients served for each simulation for

Table (4.3) Simulation Results for DPHHP with Poisson Arrivals Rate = 1/2

Simulation	# of Emergencies	# of Postponement	# of Patients	Total Overtime
1	29	35	79	10
2	21	24	82	7.26
3	22	28	79	6.5
4	23	26	85	8
5	18	24	81	6.5
6	13	27	77	2
7	23	33	78	6
8	24	29	80	7.42
9	18	32	77	4
10	24	34	80	8
11	25	31	82	10
12	14	25	75	5
13	32	39	81	9.89
14	23	31	80	8
15	22	29	83	8
16	15	20	78	5.57
17	17	26	80	4
18	25	34	80	8
19	17	18	85	6
20	16	29	76	3.86
21	27	38	77	10
22	17	23	82	6
23	22	24	84	10
24	25	35	77	8
25	13	26	76	2
26	20	32	78	6
27	18	37	71	0
28	24	30	80	6
29	22	32	78	5.85
30	12	24	78	2
Max	32	39	85	10
Min	12	18	71	0
Avg	20.7	29.17	79.30	6.33
Std	4.85	5.18	3.00	2.59

Overtime = 2hrs, ReOpt time = 12:30, Max. Accepted Emergency = 4

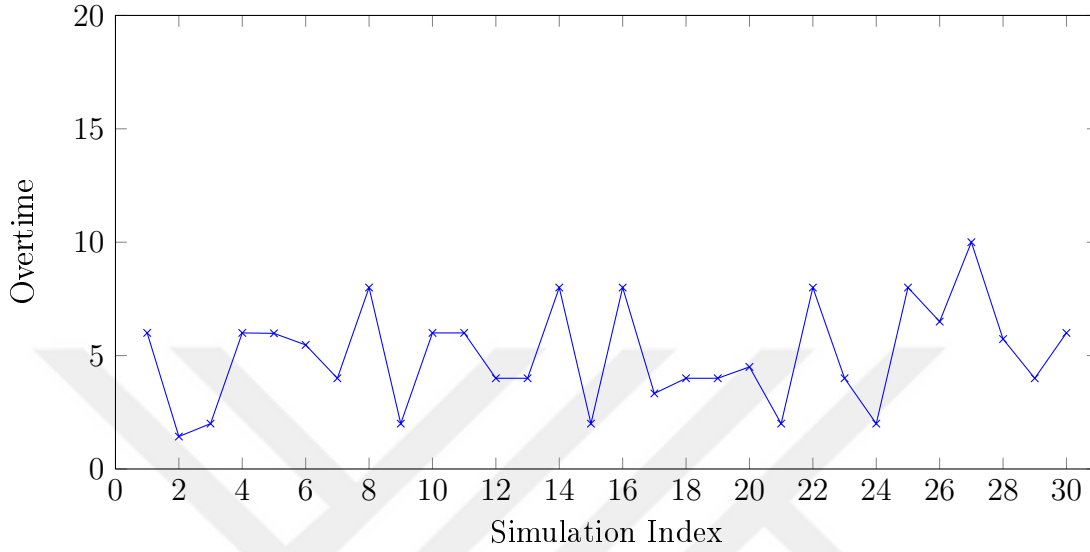


Figure (4.36) Overtime (hrs) - Poisson Rate $\lambda = 1/3$

a 10 day planning horizon. In this analysis, we provide the results for 30 independent simulation runs where the emergent patients arrive by a Poisson process with arrival rate $1/3$ and we allow an overtime duration of 2 hours if the nurse cannot serve the emergent patients during the normal shift hours. The total number of emergent patients allowed to be served during a day is limited by 4. Therefore, each day a nurse can serve up to 4 emergent patients. For the results in Table 4.4, we have only 1 re-optimization time which is 12:30. The emergent arrivals are accumulated until this hour. Then, the re-optimization procedure is triggered and the schedule for the rest of the day is modified accordingly. In Table 4.4, we observe an average of 16.40 emergent arrivals whereas the maximum emergent arrival seen is 26 during the 10-days planning horizon. The number of average postponed patients is 23.37 and 80.83 patients are served on the average during the 10-days planning horizon. As the number of emergent patients are high, a slightly more number of patients are postponed in order to be able to serve the emergent patients.

In Table 4.5, we provide the statistics concerning the total number of emergencies, postponements, overtime and number of patients served for each simulation for a 10

Table (4.4) Simulation Results for DPHHP with Poisson Arrivals Rate = $1/3$

Simulation	# of Emergencies	# of Postponement	# of Patients	Total Overtime
1	19	26	81	6
2	14	23	77	1.43
3	13	19	80	2
4	15	20	83	6
5	18	26	81	5.98
6	13	15	83	5.47
7	18	32	75	4
8	22	33	81	8
9	15	31	72	2
10	23	33	79	6
11	24	27	83	6
12	13	19	82	4
13	17	26	80	4
14	20	29	80	8
15	11	16	81	2
16	23	26	83	8
17	13	19	78	3.33
18	16	28	77	4
19	10	12	85	4
20	12	21	77	4.5
21	14	22	81	2
22	21	24	84	8
23	11	18	82	4
24	7	14	83	2
25	24	28	84	8
26	14	19	81	6.5
27	26	34	82	10
28	17	19	85	5.73
29	13	21	82	4
30	16	21	83	6
Max	26	34	85	10
Min	7	12	72	1.43
Avg	16.40	23.37	80.83	5.03
Std	4.69	5.90	2.93	2.21

Overtime = 2hrs, ReOpt time = 12:30, Max. Accepted Emergency = 4

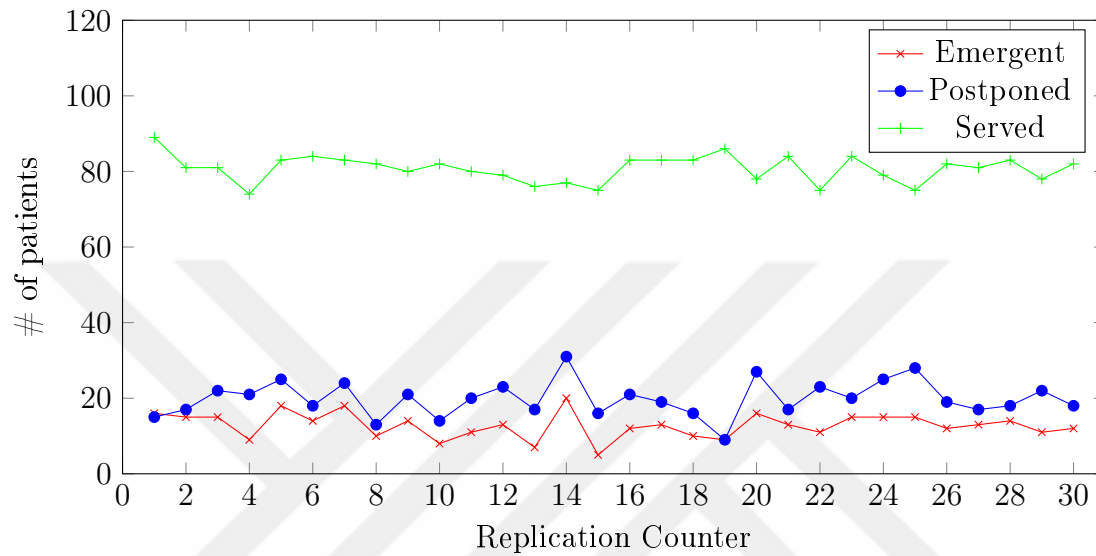


Figure (4.37) Patients Statistics - Poisson Rate $\lambda = 1/4$

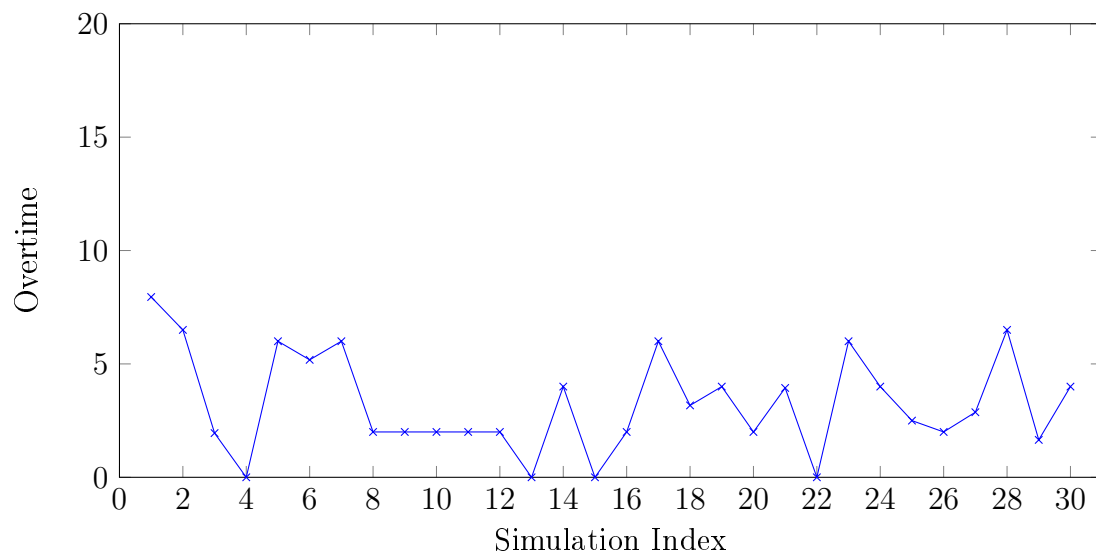


Figure (4.38) Overtime (hrs) - Poisson Rate $\lambda = 1/4$

day planning horizon. In this analysis, we provide the results for 30 independent simulation runs where the emergent patients arrive by a Poisson process with arrival rate $1/4$ and we allow an overtime duration of 2 hours if the nurse cannot serve the emergent patients during the normal shift hours. The total number of emergent patients allowed to be served during a day is limited by 4. So, each day a nurse can serve up to 4 emergent patients. For the results in Table 4.5, we have only 1 re-optimization time which is 12:30. The emergent arrivals are accumulated until this hour. Then, the re-optimization procedure is triggered and the schedule for the rest of the day is modified accordingly. In Table 4.5, we observe an average of 12.80 emergent arrivals whereas the maximum emergent arrival seen is 20 during the 10-days planning horizon. The number of average postponed patients is 19.87 and 80.73 patients are served on the average during the 10-days planning horizon. As the number of emergent patients are high, a slightly more number of patients are postponed in order to be able to serve the emergent patients.

In Figure 4.39, a comparison of the total overtime for different arrival rates are displayed. As the rate of the arrival increases, the amount spent in overtime increases as well. The increase in the rate of the arrival implies that the number of emergent arrivals per unit time will increase. Therefore, by the policy assumed in this study, the entire emergent patients will be necessarily served which will cause a mandatory work in overtime. When the emergency arrival rate is 1, on average the nurse spends 11.57 hours in overtime. However, when the emergency arrival rate is $\frac{1}{4}$, the time spent in overtime reduces to 3.27 hours on average.

In Figure 4.40, a comparison of the total emergencies for different arrival rates are displayed. As the rate of the arrival increases, the number of emergent patients also increase as expected. While the number of emergent arrivals fluctuate between 20 and 40 when the arrival rate is 1, the number of emergent patients always falls below 20 when the arrival rate is $\frac{1}{4}$.

Table (4.5) Simulation Results for DPHHP with Poisson Arrivals Rate = 1/4

Simulation	# of Emergencies	# of Postponement	# of Patients	Total Overtime
1	16	15	89	7.95
2	15	17	81	6.5
3	15	22	81	1.95
4	9	21	74	0
5	18	25	83	6
6	14	18	84	5.18
7	18	24	83	6
8	10	13	82	2
9	14	21	80	2
10	8	14	82	2
11	11	20	80	2
12	13	23	79	2
13	7	17	76	0
14	20	31	77	4
15	5	16	75	0
16	12	21	83	2
17	13	19	83	6
18	10	16	83	3.17
19	9	9	86	4
20	16	27	78	2
21	13	17	84	3.94
22	11	23	75	0
23	15	20	84	6
24	15	25	79	4
25	15	28	75	2.5
26	12	19	82	2
27	13	17	81	2.87
28	14	18	83	6.5
29	11	22	78	1.65
30	12	18	82	4
Max	20.00	31.00	89.00	7.95
Min	5.00	9.00	74.00	0.00
Avg	12.80	19.87	80.73	3.27
Std	3.31	4.62	3.50	2.16

Overtime = 2hrs, ReOpt time = 12:30, Max. Accepted Emergency = 4

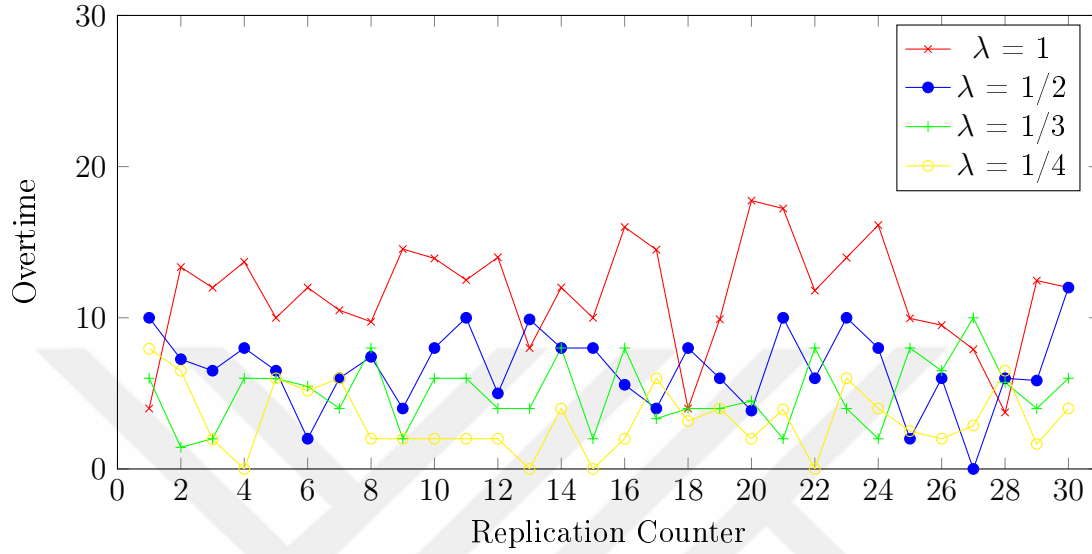


Figure (4.39) Total Overtime with different arrival rates

In Figure 4.41, a comparison of the total postponements for different arrival rates are displayed. We assume that the financial burden caused by the overtime is taken by the company. Therefore, if it is possible to serve emergent patients during the normal shift hours, then the policy forces as many patients as possible to postpone. However, if it is not possible to serve the emergent patients during the normal shift hours and an overtime seems necessary, then the nurse is expected to utilize the overtime as much as possible by serving more patients. Hence, it is possible to observe that when there is only one emergent patient and the model postpones 3 or 4 patients. Similarly, when there is 3 or 4 emergent patients and the model postpones 1 or 2 patients.

In Figure 4.42, a comparison of the total served patients for different arrival rates are displayed. Although the change in arrival rate significantly affects the number of emergent patients, the number of postponed patients and overtimes, it seems that the effect on the total number of patients served during the 10-days planning horizon is relatively less when compared the aforementioned parameters. When the the emergent arrival rate increases, i.e., the number of emergent patients arrive per unit

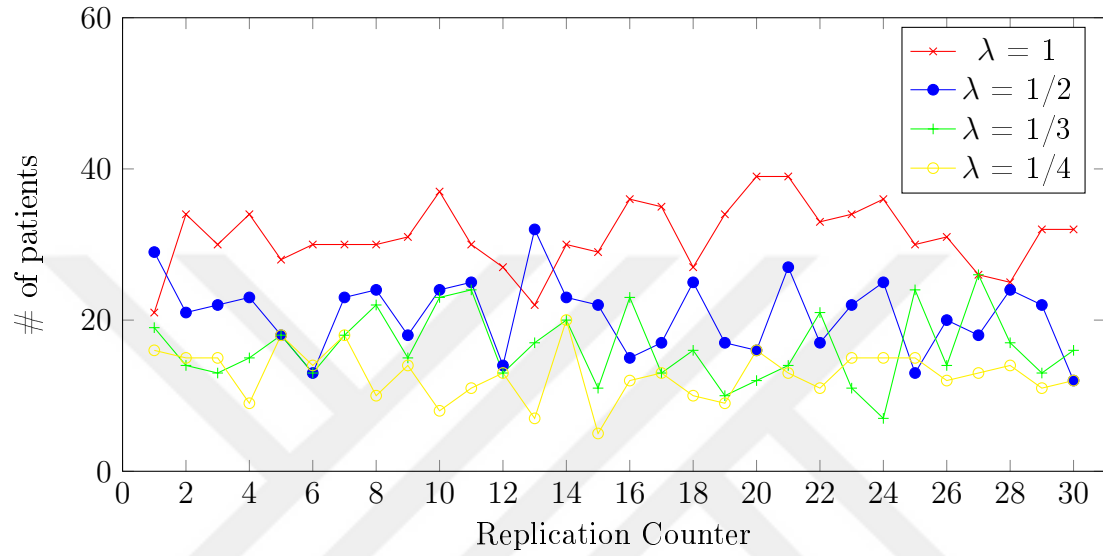


Figure (4.40) Emergencies with different arrival rates

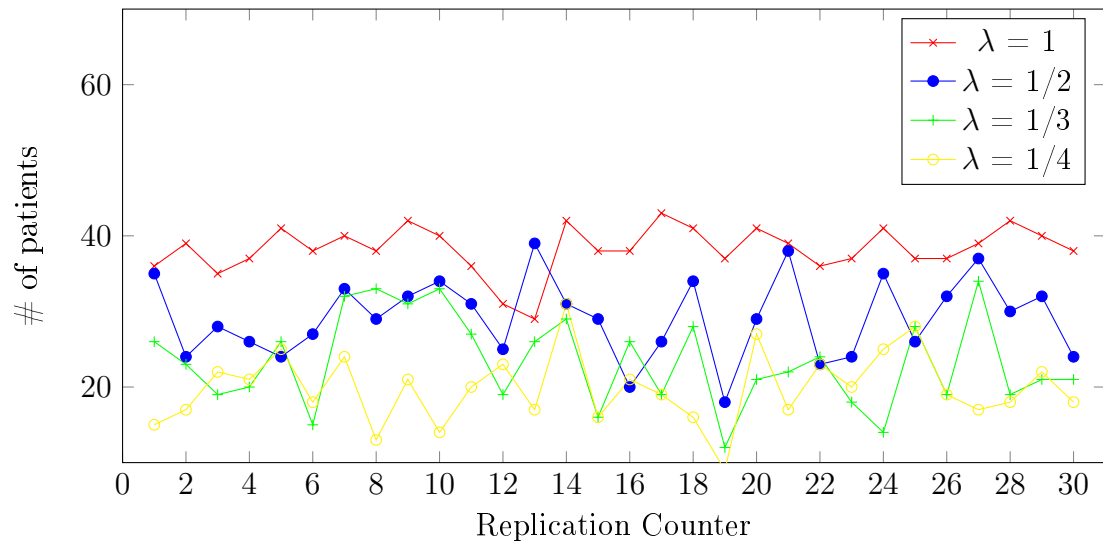


Figure (4.41) Postponements with different arrival rates

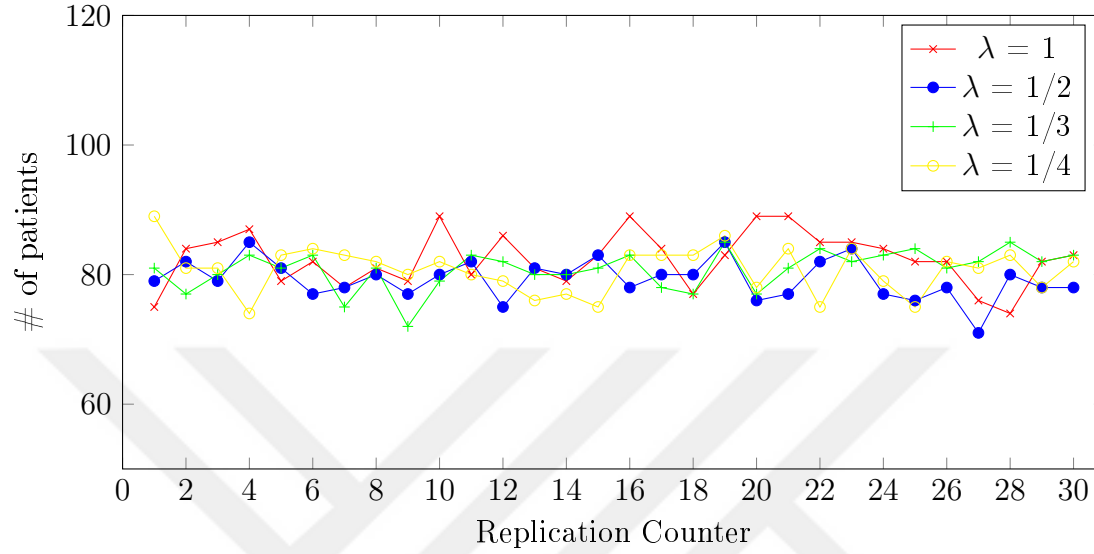


Figure (4.42) Total served patients with different arrival rates

time increases, the likelihood for postponing a high number of patients also increases. Similarly, when the number of emergent arrivals per unit time decreases, the number of postponed patients also decreases. Therefore, on the average the number of served patients looks similar for different emergent arrival rates.

We perform several experiments via a simulation study to see how our optimization framework reacts to a real-life problem. We assume a Poisson process with different arrival rates that governs the emergent arrivals. At the beginning of the planning horizon, our ALNS algorithm runs and provides schedules and routes for the entire planning horizon. Then, on each day emergent arrivals occur. We accumulate emergent arrivals until a prespecified re-optimization time and at re-optimization time we run the E-MIP model to generate a new optimized plan for the rest of the day. We allow 2 hours overtime for the nurse if overtime is made. We have an implicit assumption that the overtime cost is carried by the company. Therefore, whenever an emergency arrival is realized, the model tries to serve the emergent patient within normal shift hours. However, if it is not possible to serve the emergent patients within normal shift hours and an overtime is necessary, then the nurse is supposed to serve

as many patients as possible within overtime duration. While it is mandatory to serve emergent patients on the same day, some patients of Type-2 might be postponed. One would expect to serve the postponed patients as early as possible, namely the next day if a postponement becomes mandatory. Our optimization framework makes sure that if a patient is postponed, it is very likely that the patient will be served on the next day. According to the simulation studies, we can conclude that our approach works well enough for a real-life problem where dynamic arrivals occur. We keep track of the number of emergent patients, the number of postponed patients and the number of served patients. We observe that postponed patients are served within the earliest possible planning days. While dynamic arrivals occur, the number of served patients do not change dramatically. The study shows that the optimization framework has a solid and applicable structure.

Chapter 5

CONCLUSION

In this thesis, we provide solution methods for a real-life problem in which a nurse is required to serve patients he is responsible for by home visits. However, due to the large number of patients, he has to be selective when planning a schedule. Therefore, priorities are assigned to patients such that the priorities of unvisited patients increase by day. The solution simultaneously specifies a visit schedule, based on patient priorities over multiple days, and the sequence of the visits to selected patients on each day. Patients that cannot be visited within the planning horizon are checked upon by phone calls. Those patients will have higher priorities compared to the visited ones when the problem is solved again for the next planning horizon since the priorities are updated dynamically.

For the first part of the thesis, we first develop a MIP model to solve this multi-period routing and scheduling problem. We introduce a priority parameter which is transformed into prizes that lets us construct the MIP model as a team orienteering problem with time-dependent prizes and time windows. A Lagrangean relaxation procedure is implemented to get strong upper bounds. We present two algorithms to generate high quality solutions in reasonable time. The first algorithm is a matheuristic called SSPH. It solves the multi-period model day by day by updating the prize matrix and the unvisited patient list at each iteration. To obtain shorter run times, we also develop an Adaptive Large Neighborhood Search algorithm with many destroy and repair operators. The ALNS algorithm provides near-optimal solutions within short run times, as assessed by the upper bounds from the Lagrangean relaxation procedure.

For the second part of the thesis, we introduce an optimization-based solution methodology by which we try to solve a dynamic home healthcare problem. We modify the mathematical model introduced in Chapter 3 in order to incorporate emergent arrivals during the day. We first solve the problem for the entire planning horizon using the ALNS algorithm. Then, we consider the first day. While the plan is being executed, we observe emergent arrivals and by employing the modified mathematical model we update the daily plans at a prespecified re-optimization time. If there is no emergent arrivals, then no change is done. We devise a simulation where emergent arrivals are assumed to come from a Poisson process. We run the simulation model for different arrival rates and report different performance indicators such as number of emergent arrivals, number of postponed patients, number of served patients and the total overtime duration. The methodology developed for the dynamic prioritized home healthcare problem is generic and can be used to solve a class of problems where the underlying structure assumes a priority among the service points and a dynamic arrival pattern. Hence, the methodology developed in this thesis can be used for a number of different sectors. The typical applications include maintenance operations and staff routing in telecoms and public utilities. For example, our approach may be applied to technician scheduling and routing problems. The Technician Routing and Scheduling Problem (TRSP) consists in routing staff to serve requests for service, taking into account time windows, skills, tools, and spare parts [46]. When a new request occurs, the technician should consider to revise the plan under execution. If the technician considers the new request a mandatory visit point, then the technician might use our approach by considering the new request as an emergent arrival.

For future research, it remains a challenge to apply the ideas presented in this study to a multi-nurse and multi-period problem, where the assignment of patients to nurses is not specified a priori, but is optimized instead. Patients can be assigned to nurses not only by geographic proximity but also according to an ongoing relationship be-

tween nurses and patients. The patients might require certain medical needs that can be provided only by those nurses who have the required skill set. Another future research direction could be consideration of sharing the financial burden caused by the emergency situations between companies that provide home healthcare services and patients who require these services. Sharing the financial burden suggests that when an emergency request occurs, the methodology introduced in the thesis will be applied and if an overtime is inevitable, then the patient will be offered to share the financial burden because of forcing the nurse to work in overtime. Finally, an interesting research direction is to let other nurses to serve the emergency arrivals of another nurse. This suggests that if the emergency arrivals for a nurse creates a revised plan where some patients are postponed, then another nurse might be asked to help if she has a slightly flexible schedule for the day. This problem requires a central planner to coordinate the schedules of multiple nurses.

Chapter 6

APPENDIX



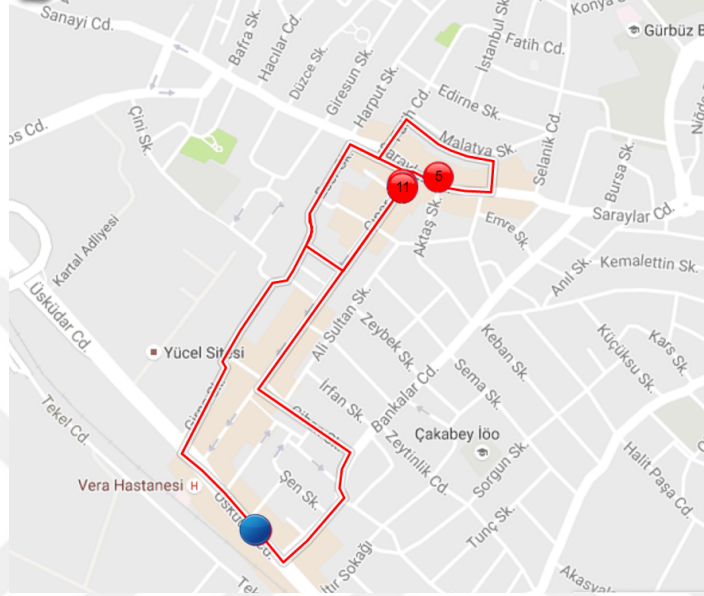


Figure (6.1) Routes for Nurse 1 on Day 1

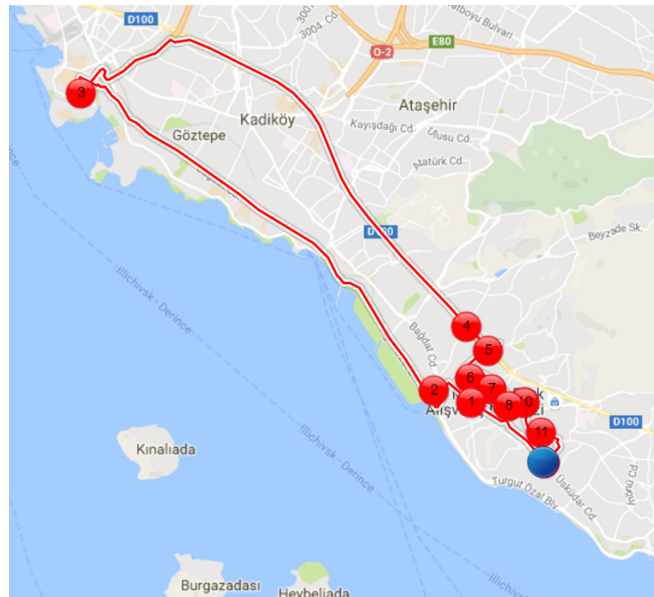


Figure (6.2) Routes for Nurse 1 on Day 2

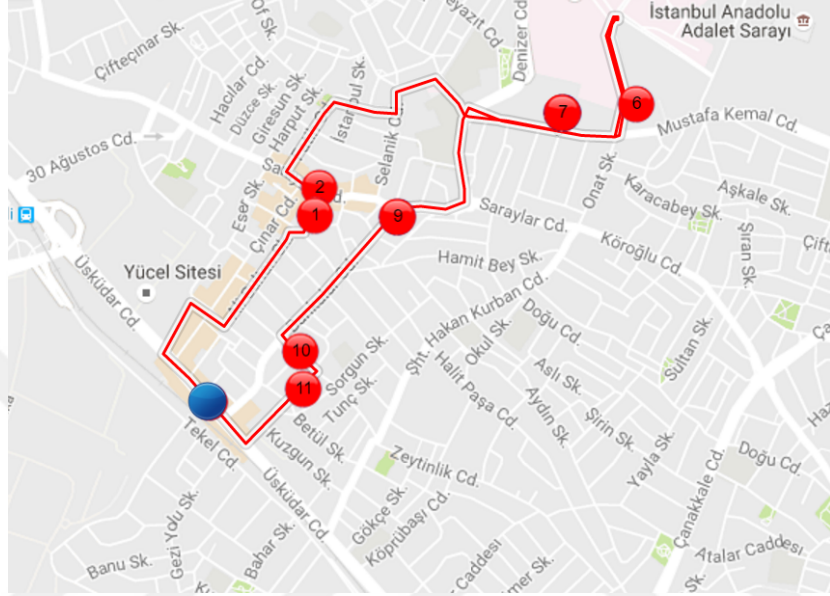


Figure (6.3) Routes for Nurse 1 on Day 3

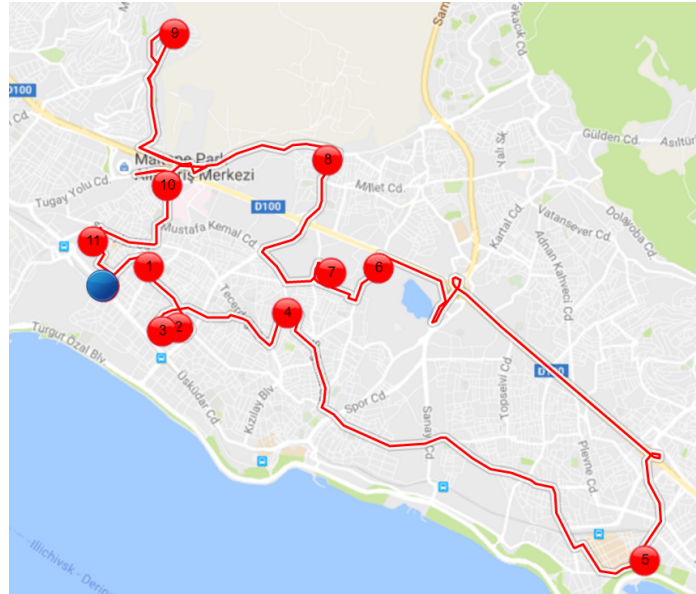


Figure (6.4) Routes for Nurse 1 on Day 4

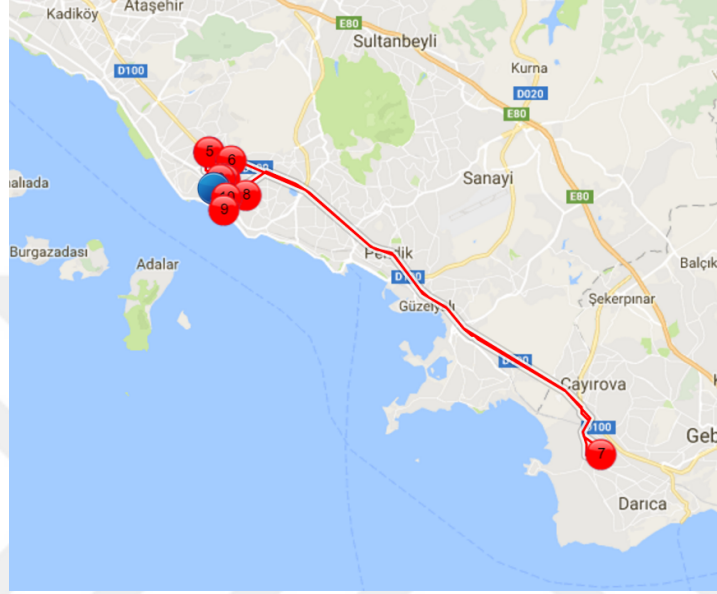


Figure (6.5) Routes for Nurse 1 on Day 5

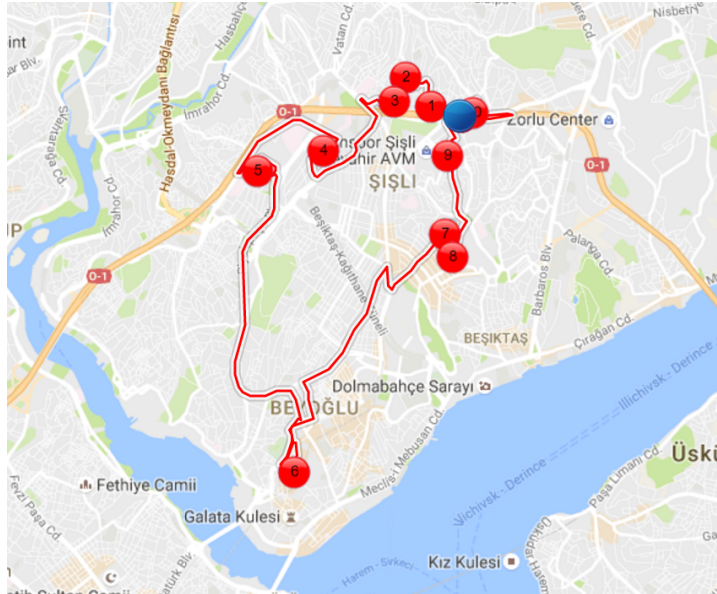


Figure (6.6) Routes for Nurse 2 on Day 1

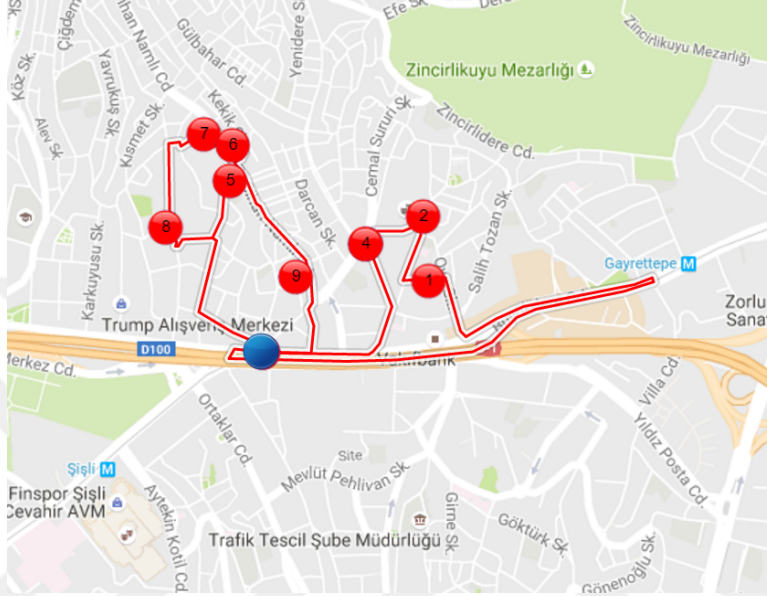


Figure (6.7) Routes for Nurse 2 on Day 2

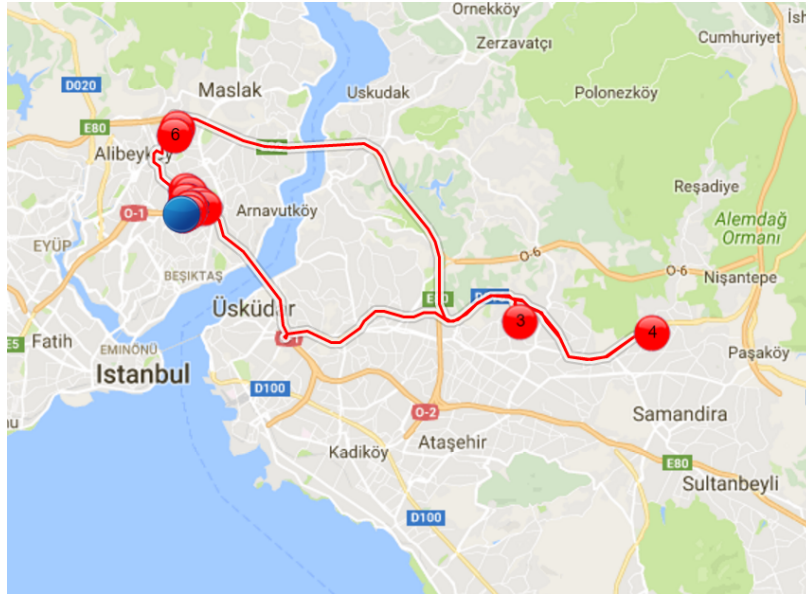


Figure (6.8) Routes for Nurse 2 on Day 3

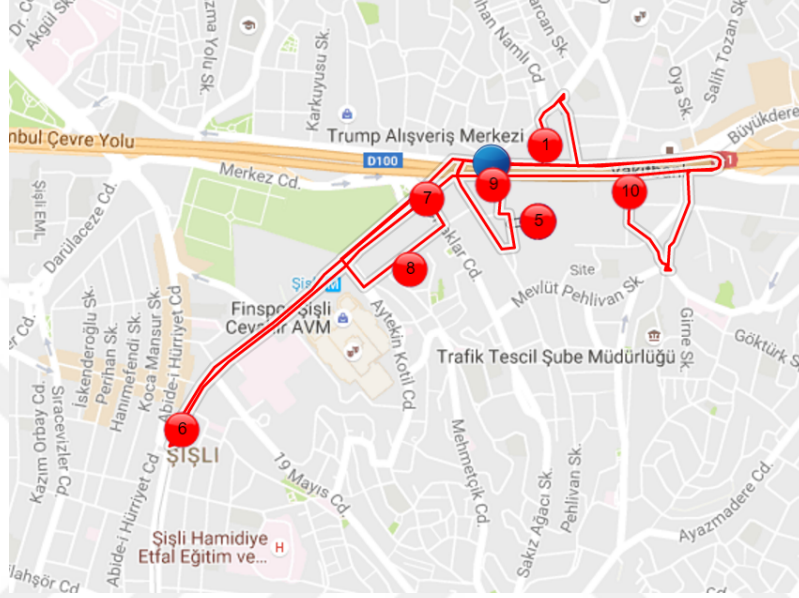


Figure (6.9) Routes for Nurse 2 on Day 4

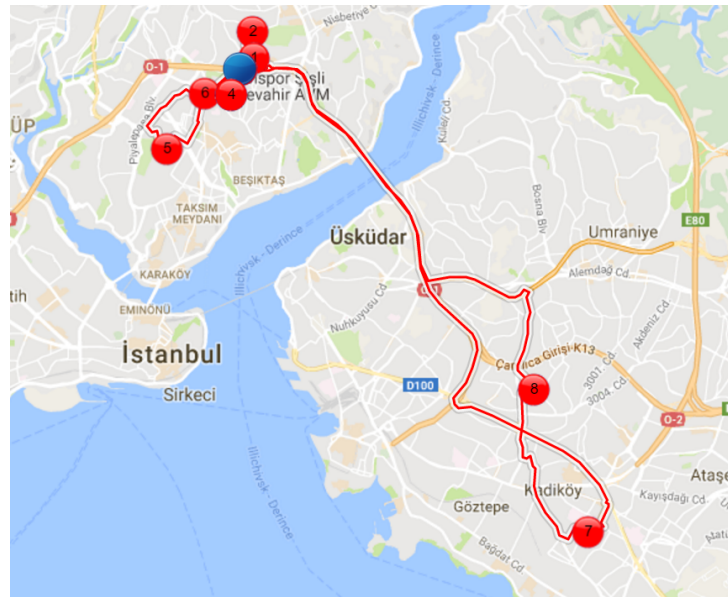


Figure (6.10) Routes for Nurse 2 on Day 5

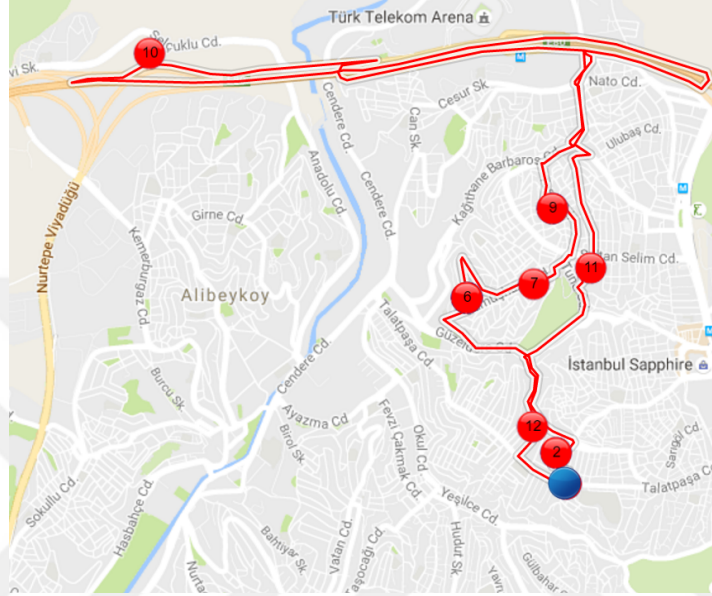


Figure (6.11) Routes for Nurse 3 on Day 1

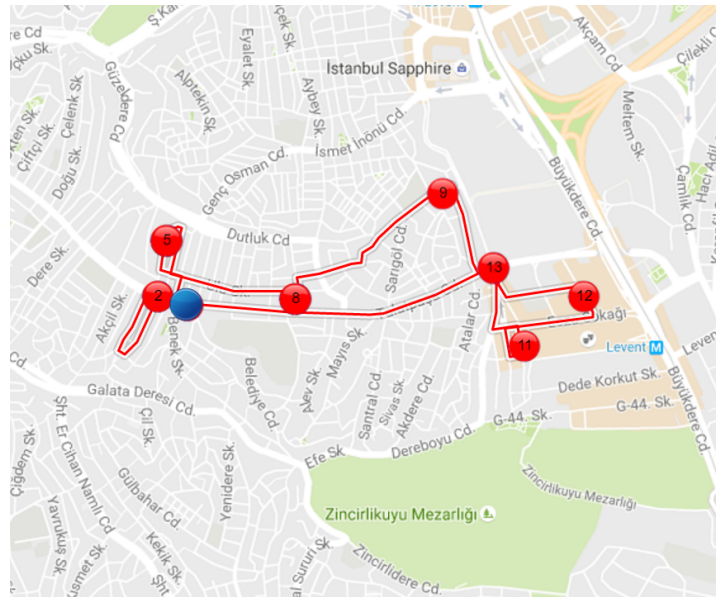


Figure (6.12) Routes for Nurse 3 on Day 2

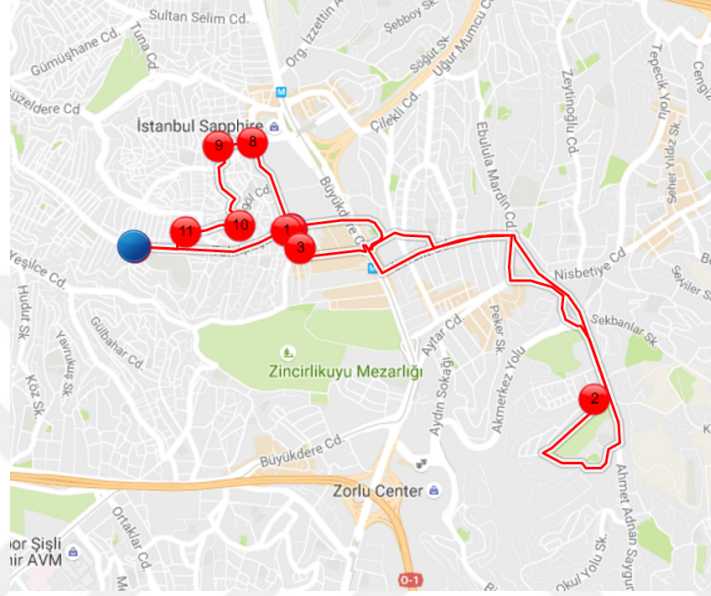


Figure (6.13) Routes for Nurse 3 on Day 3

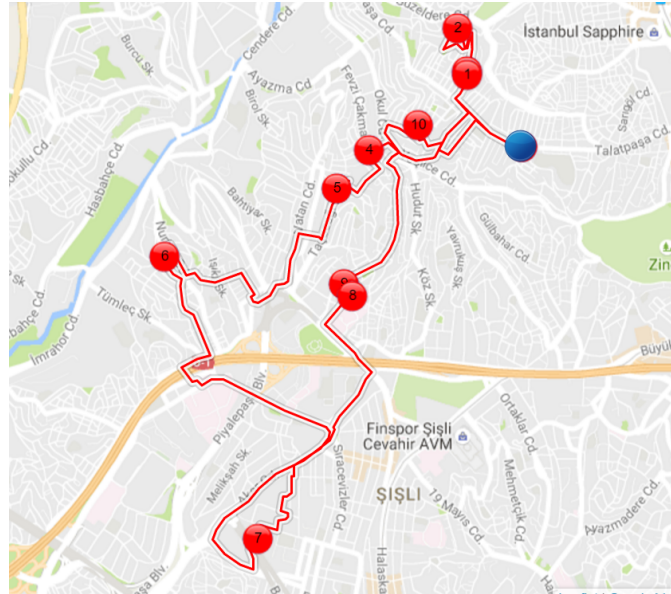


Figure (6.14) Routes for Nurse 3 on Day 4

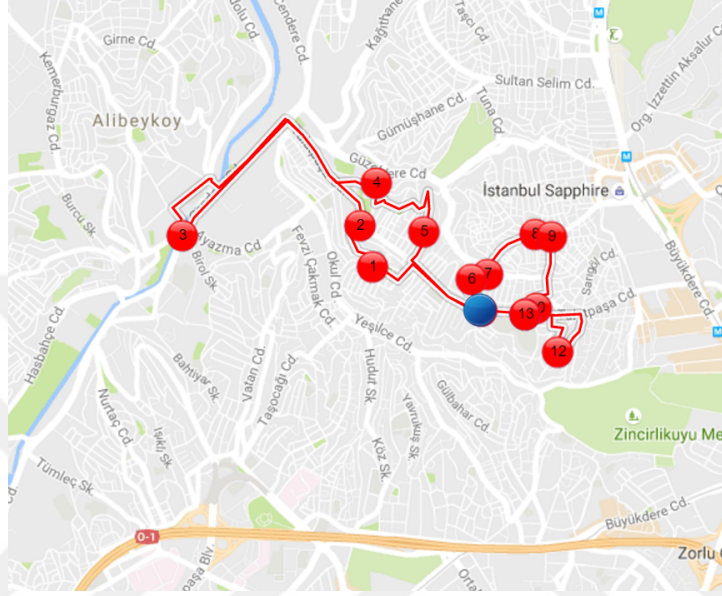


Figure (6.15) Routes for Nurse 3 on Day 5

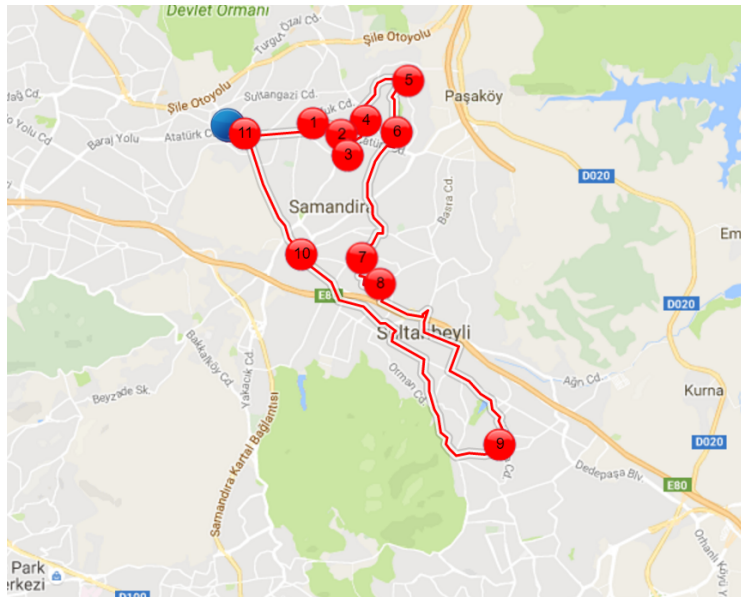


Figure (6.16) Routes for Nurse 4 on Day 1

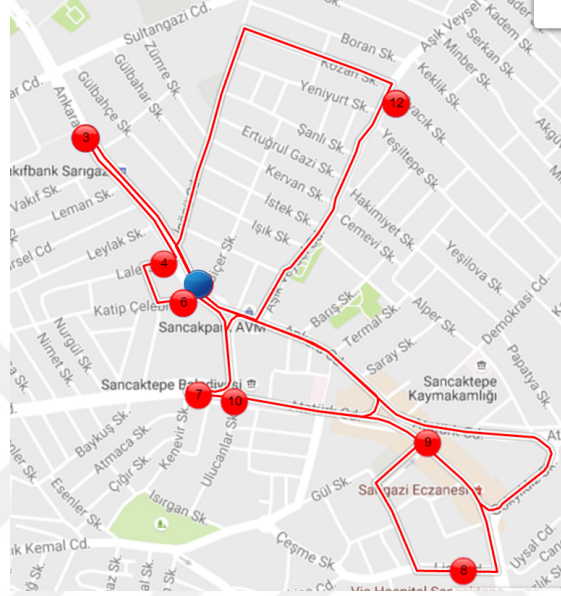


Figure (6.17) Routes for Nurse 4 on Day 2

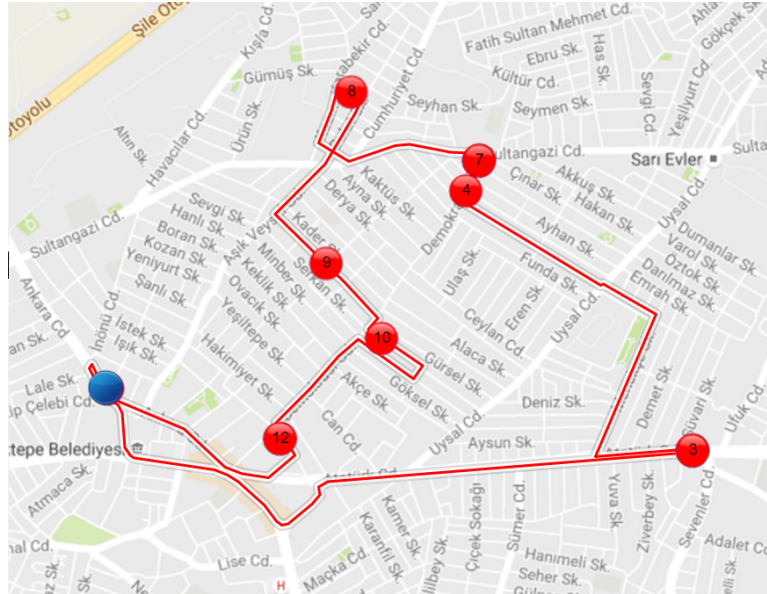


Figure (6.18) Routes for Nurse 4 on Day 3

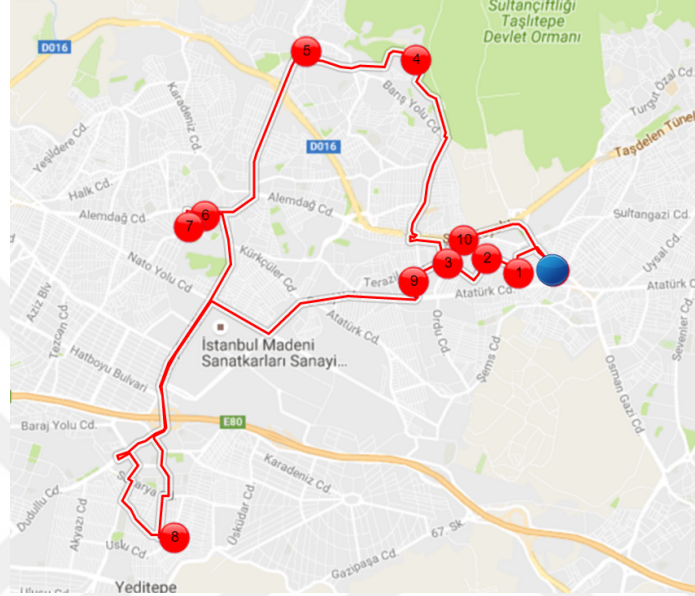


Figure (6.19) Routes for Nurse 4 on Day 4

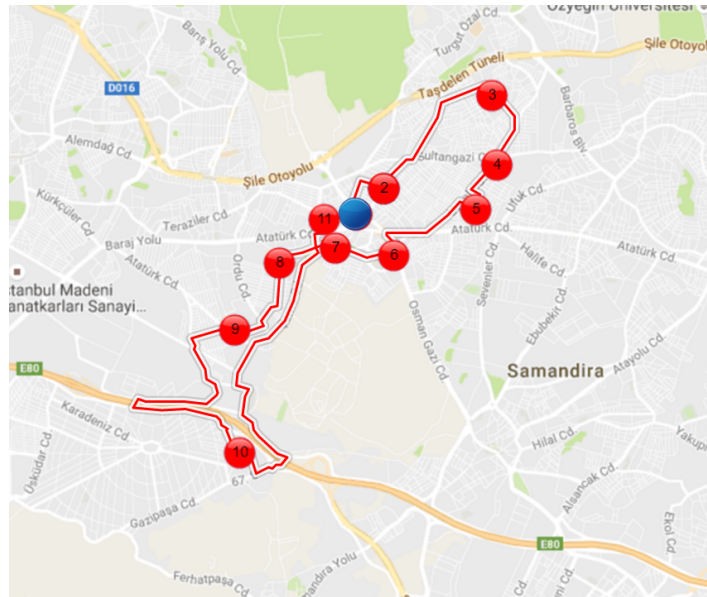


Figure (6.20) Routes for Nurse 4 on Day 5

BIBLIOGRAPHY

- [1] H. Allaoua, S. Borne, L. Letocart, and R. W. Calvo. A matheuristic approach for solving a home health care problem. *Electronic Notes in Discrete Mathematics*, 41:471–478, 2013.
- [2] E. Angelelli, N. Bianchessi, R. Mansini, and M. Speranza. Short term strategies for a dynamic multi-period routing problem. *Transportation Research Part C: Emerging Technologies*, 17(2):106–119, 2009.
- [3] R. B. Bachouch, A. Guinet, and S. Hajri-Gabouj. A decision-making tool for home health care nurses’ planning. *Supply Chain Forum: Int J*, 12(1):14–20, 2011.
- [4] S. V. Begur, D. M. Miller, and J. R. Weaver. An integrated spatial dss for scheduling and routing home-health-care nurses. *Interfaces*, 27(4):35–48, 1997.
- [5] S. Bertels and T. Fahle. A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem. *Computational Operations Research*, 33:2866–2890, 2006.
- [6] D. J. Bertsimas and D. Simchi-Levi. A new generation of vehicle routing research: Robust algorithms, addressing uncertainty. *Operations Research*, 44(2):286–304, 1996.
- [7] N. Bianchessi, R. Mansini, and M. G. Speranza. A branch-and-cut algorithm for the team orienteering problem. *International Transaction in Operational Research*, 235(1):73–87, 2017.

- [8] K. Braekers, R. F. Hartl, S. N. Parragh, and F. Tricoire. A bi-objective home care scheduling problem: Analyzing the trade-off between costs and client inconvenience. *European Journal of Operations Research*, 248:428–443, 2015.
- [9] O. Braysy and M. Gendreau. Vehicle routing problem with time windows, part i: route construction and local search algorithms. *Transportation Science*, 39(1):104–118, 2005.
- [10] E. K. Burke, P. D. Causmaecker, G. V. Berghe, and H. V. Landeghem. The state of the art of nurse rostering. *Journal of Scheduling*, 7(6):441–499, 2004.
- [11] A. Chananis, Y. Pinal, and P. R. Drake. Pso-based algorithm for home care worker scheduling in the uk. *Comput Ind Eng*, 53(4):559–83, 2007.
- [12] M. Chao, B. L. Golden, and E. A. Wail. Theory and methodology - the team orienteering problem. *European Journal of Operational Research*, 88:464–474, 1996.
- [13] E. Cheng and J. L. Rich. A home health care routing and scheduling problem. *Technical Report CAAM TR98-049, Rice University*.
- [14] A. Cinar, F. Salman, and B. Bozkaya. Prioritized single nurse routing and scheduling for home healthcare services. *European Journal of Operational Research*, 2019.
- [15] M. Cisse, S. Yalçındağ, Y. Kergosien, E. Şahin, C. Lente, and A. Matta. Or problems related to home health care: A review of relevant routing and scheduling problems. *Operations Research for Health Care*, 13-14:1–22, 2017.
- [16] J. Decerle, O. Grunder, A. H. E. Hassani, and O. Barakat. A memetic algorithm for a home health care routing and scheduling. *Journal of Cleaner Production*, 16:59–71, 2018.

- [17] P. M. Duque, M. Castro, K. Sorensen, and P. Goos. Home care service planning. the case of landelijke thuiszorg. *European Journal of Operations Research*, 243:292–301, 2015.
- [18] G. Erdoğan, J.-F. Cordeau, and G. Laporte. The attractive traveling salesman problem. *European Journal of Operational Research*, 203(1):59–69, 2010.
- [19] G. Erdoğan and G. Laporte. The orienting problem with variable profits. *Networks*, 61(2):104–116, 2013.
- [20] P. Eveborn, P. Flisberg, and M. Ronnqvist. Laps care - an operational system for staff planning. *European Journal of Operations Research*, 171:962–976, 2006.
- [21] P. Eveborn, M. Ronnqvist, H. Einarsdottir, M. Eklund, K. Liden, and M. Almqvist. Operations research improves quality and efficiency in home care. *Interfaces*, 39(1):18–34, 2009.
- [22] C. Fikar and P. Hirsch. Home health care routing and scheduling: A review. *Computers and Operations Research*, 77:86–95, 2017.
- [23] R. Gedik, E. Kirac, A. B. Milburn, and C. Rainwater. A constraint programming approach for the team orienteering problem with time windows. *Computers and Industrial Engineering*, 107:178–195, 2017.
- [24] M. Gendreau, F. Guertin, J.-Y. Potvin, and Éric Taillard. Parallel tabu search for real-time vehicle routing and dispatching. *Transportation Science*, 33(4):381–390, 1999.
- [25] M. Gendreau, G. Laporte, and R. Séguinad. Stochastic vehicle routing. *European Journal of Operational Research*, 88(1):3–12, 1996.
- [26] N. Genet, W. Boerma, M. Kroneman, A. Hutchinson, and R. B. Saltman. Home care across europe - current structure and future challenges. *European Observa-*

- tory on Health Systems and Policies, World Health Organization, Oslo, Norway, (2012).*, 2012.
- [27] G. Ghiana, F. Guerriero, G. Laporte, and R. Musmanno. Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies. *European Journal of Operational Research*, 151:1–11, 2003.
- [28] F. Grenouilleau, A. Legrain, N. Lahrichi, and L.-M. Rousseau. A set partitioning heuristic for the home health care routing and scheduling problem. *European Journal of Operational Research*, 275:295–303, 2018.
- [29] A. Gunawan, H. C. Lau, and P. Vansteenwegen. Orienteering problem: A survey of recent variants, solution approaches and applications. *Health Care Manag Sci*, 255:315–332, 2016.
- [30] C. H. Christiansen and J. Lysgaard. A branch-and-price algorithm for the capacitated vehicle routing problem with stochastic demands. *Operations Research Letters*, 35(6):773–781, 2007.
- [31] G. Hiermann, M. Prandtstetter, A. Rendl, J. Puchinger, and G. R. Raidl. Metaheuristics for solving a multimodal home-healthcare scheduling problem. *Cent Eur J Oper Res*, 23(1):89–113, 2015.
- [32] P. Jaillet and M. R. Wagner. Generalized online routing: New competitive ratios, resource augmentation, and asymptotic analyses. *Operations Research*, 56(3):745–757, 2008.
- [33] A. S. Kenyon and D. P. Morton. Stochastic vehicle routing with random travel times. *Transportation Science*, 37(1):69–82, 2003.
- [34] G. Laporte. Fifty years of vehicle routing. *Transportation Science*, 26(3):408–416, 2009.

- [35] J. Li. Research on team orienteering problem with dynamic travel times. *Journal of Software*, 7(2):249–255, 2012.
- [36] S.-W. Lin and V. F. Yu. Solving the team orienteering problem with time windows and mandatory visits by multi-start simulated annealing. *Computers and Industrial Engineering*, 114:195–205, 2017.
- [37] R. Liu, X. Xie, and T. Garaix. Hybridization of tabu search with feasible and infeasible local searches for periodic home health care logistics. *Omega*, 47:17–32, 2014.
- [38] D. Manerba and R. Mansini. The nurse routing problem with workload constraints and incompatible services. *IFAC-PapersOnLine*, 49-12:1192–1197, 2016.
- [39] D. S. Mankowska, F. Meisel, and C. Bierwirth. The home health care routing and scheduling problem with interdependent services. *Health Care Management Science*, 17:15–30, 2014.
- [40] D. S. Mankowska, F. Meisel, and C. Bierwirth. The home health care routing and scheduling problem with interdependent services. *Health Care Manag Sci*, 17(1):15–30, 2014.
- [41] E. Moussavi, M. Mahdjoub, and O. Grunder. A matheuristic approach to the integration of worker assignment and vehicle routing problems: Application to home healthcare scheduling. *European Journal of Operational Research*, 125:317–332, 2019.
- [42] J. A. Nasir and C. Dang. Quantitative thresholds based decision support approach for the home health care scheduling and routing problem. *Health Care Management Science*, 2019.

- [43] S. Nickel, M. Schroder, and J. Steeg. Mid-term and short-term planning support for home health care services. *European Journal of Operations Research*, 219:574–587, 2011.
- [44] C. Novoa and R. Storer. An approximate dynamic programming approach for the vehicle routing problem with stochastic demands. *European Journal of Operational Research*, 196(2):742–749, 2009.
- [45] V. Pillac, c. Gueret, and A. Medaglia. A parallel matheuristic for the technician routing and scheduling problem. *Optimization Letters*, 7(7):1525–1535, 2013.
- [46] V. Pillac, C. Gueret, and A. L. Medaglia. A fast reoptimization approach for the dynamic technician routing and scheduling problem. *Operations Research/Computer Science Interfaces Series*, 62:347–367, 2018.
- [47] H. N. Psaraftis. A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. *Transportation Science*, 14(2):130–154, 1980.
- [48] H. N. Psaraftis. Dynamic vehicle routing: Status and prospects. *Annals of Operations Research*, 61:143–164, 1995.
- [49] M. S. Rasmussen, T. Justesen, A. Dohn, and J. Larsen. The home care crew scheduling problem: Preference-based visit clustering and temporal dependencies. *European Journal of Operations Research*, 219(3):598–610, 2012.
- [50] K. Reinert and G. Klau. Lagrangean relaxation: An overview. *Discrete Mathematics for Bioinformatics WS 07/08*, pages 1–4, 2007.
- [51] S. Ropke and D. Pisinger. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science*, 40(4):455–472, 2006.

- [52] S. Ropke and D. Pisinger. A general heuristic for vehicle routing problems. *Computers and Operations Research*, 34(8):2403–2435, 2007.
- [53] N. Secomandi. A rollout policy for the vehicle routing problem with stochastic demands. *Operations Research*, 49(5):796–802, 2001.
- [54] N. Secomandi and F. Margot. Reoptimization approaches for the vehicle-routing problem with stochastic demands. *Operations Research*, 57(1):3214–230, 2009.
- [55] P. Shaw. A new local search algorithm providing high quality solutions to vehicle routing problems. *Technical report*, pages 1–12, 1997.
- [56] Y. Shi, T. Boudouh, and O. Grunder. A home health care routing problem with stochastic travel and service time. *IFAC PapersOnLine*, 50(1):13987–13992, 2017.
- [57] Y. Shi, T. Boudouh, and O. Grunder. A hybrid genetic algorithm for a home health care routing problem with time window and fuzzy demand. *Expert Systems With Applications*, 72:160–176, 2017.
- [58] J. Steeg and M. Schröder. A hybrid approach to solve the periodic home health care problem. *Operations Research Proceedings*, pages 297–302, 2007.
- [59] A. F. Torres-Ramos, E. H. Alfonso-Lizarazo, L. S. Reyes-Rubiano, and C. L. Quintero-Araujo. Mathematical model for the home health care routing and scheduling problem with multiple treatments and time windows. *Mathematical Methods in Science and Engineering*, 8:140–145, 2014.
- [60] P. Toth and D. Vigo. *The Vehicle Routing: problems, methods and applications*. In: *MOS-SIAM series on optimization, second edition*. SIAM, 2014.
- [61] A. Trautsamwieser and P. Hirsch. Optimization of daily scheduling for home health care services. *Journal of Applied Operational Research*, 3(3):124–136, 2011.

- [62] P. Vansteenwegen, W. Souffriau, and D. V. Oudheusden. The orienteering problem: A survey. *European Journal of Operational Research*, 209:1–10, 2011.
- [63] D. Waters. Vehicle-scheduling problems with uncertainty and omitted customers. *Journal of the Operational Research Society*, 40(12):1099–1108, 1989.

