

DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**MODELING AND SOLVING DYNAMIC LOT
SIZING PROBLEMS UNDER DIFFERENT
PRODUCTION ENVIRONMENT**

by
Burcu KUBUR ÖZBEL

December, 2022

İZMİR

MODELING AND SOLVING DYNAMIC LOT SIZING PROBLEMS UNDER DIFFERENT PRODUCTION ENVIRONMENT

**A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Doctor of
Philosophy in Industrial Engineering, Industrial Engineering Program**

**by
Burcu KUBUR ÖZBEL**

December, 2022

İZMİR

Ph.D. THESIS EXAMINATION RESULT FORM

We have read the thesis entitled “**MODELING AND SOLVING DYNAMIC LOT SIZING PROBLEMS UNDER DIFFERENT PRODUCTION ENVIRONMENT**” completed by **BURCU KUBUR ÖZBEL** under supervision of **PROF.DR. ADİL BAYKASOĞLU** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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ACKNOWLEDGMENTS

First, I would like to thank my supervisor, Prof. Dr. Adil BAYKASOĞLU, for his support, guidance, and understanding. It is my great pleasure and honor to be associated with him.

I would like to express my sincere thanks to the other members of my thesis committee: Prof. Dr. Efendi NASİBOĞLU and Asst. Prof. Dr. Burcu FELEKOĞLU for their constructive comments to improve the quality of this thesis.

I would like to express my deep thankfulness to my parents, who have loved and supported me whole my life, Sevim, Ali KUBUR and my sister Bahar KAYÇEN. I want to present my special thanks to my husband, Yaşar Can ÖZBEL for his endless patience, encouragement, continuous support.

Last but not least, I would like to thank TUBITAK (The Scientific and Technological Research Council of Turkey) 2211-A National Ph.D. Scholarship Program.

Burcu KUBUR ÖZBEL

MODELING AND SOLVING DYNAMIC LOT SIZING PROBLEMS UNDER DIFFERENT PRODUCTION ENVIRONMENT

ABSTRACT

This thesis addresses common lot sizing and scheduling problems that are encountered in the industries. Among these problems, General Lot Sizing and Scheduling Problem (GLSP) has received a lot of attention from researchers. The first aim of this Ph.D. study is to propose novel Simulated Annealing (SA) based hybrid approaches for solving the GLSP. First, a mixed-integer programming (MIP) model for the GLSP is given in order to solve smaller size problems. Afterwards, a matheuristic algorithm that integrates SA algorithm and MIP model is devised for solving larger size problems. The proposed matheuristic approach decomposes the GLSP into subproblems. The second aim of this Ph.D. study is to solve a practical production issue in the paper bag industry that contains the problem of capacitated lot sizing and scheduling with cutting plan assignment. A MIP model is proposed for modeling and solving the problem. To handle the industrial case, a problem-based matheuristic approach that resolves the lot sizing and assignment problems separately is provided based on this mathematical formulation. A similar approach is also proposed for the extended version which several real life applications such as existence backordered and defective items, external procurement, and sequence dependent setup times. The last aim of this Ph.D. study is to solve a practical production issue in the pool equipment manufacturer industry that contains the problem of the multi-machine capacitated lot sizing and scheduling problem with machine eligibility constraints. The considered problem is also formulated as a multi-objective MIP model. Computational experiments indicate that the constructed model meets the practical requirements for the pool equipment manufacturer industry.

Keywords: Integrated lot sizing and scheduling problem; production planning; mixed integer programming; matheuristics

FARKLI ÜRETİM ORTAMLARINDA DİNAMİK PARTİ BÜYÜKLÜĞÜ BELİRLEME PROBLEMLERİNİN MODELLENMESİ VE ÇÖZÜMÜ

ÖZ

Bu tez, endüstrilerde karşılaşılan yaygın parti büyüklüğü ve çizelgeleme problemlerini ele almaktadır. Bu problemlerden Genel Parti Büyüklüğü ve Çizelgeleme Problemi (GPBÇP) araştırmacılar tarafından oldukça ilgi görmüştür. Bu doktora çalışmasının ilk amacı GPBÇP 'ni çözmek için yeni Tavlama Benzetimi (TB) tabanlı hibrit yaklaşımlar önermektir. İlk olarak, küçük boyutlu GPBÇP'ni çözmek için Karışık Tamsayı Programlama (KTP) modeli verilmiştir. Daha sonra, büyük boyutlu problemlerin çözümü için TB algoritması ve KTP modelini entegre eden bir matsezigisel tasarlanmıştır. Önerilen matsezigisel yaklaşım, GPBÇP 'ni alt problemlere ayrıştırmaktadır. Bu doktora çalışmasının ikinci amacı, kağıt çanta endüstrisinde kapasiteye sahip parti büyüklüğü ve çizelgeleme problemini içeren pratik bir üretim sorununu kesim planı ile entegre çözmektir. Problemin modellenmesi ve çözümü için bir KTP modeli önerilmiştir. Endüstriyel boyutlu durumu ele almak için, parti büyüklüğü ve atama problemlerini ayrı ayrı çözen önerilen matematiksel formülasyona dayalı bir matsezigisel algoritma tasarlanmıştır. Benzer bir yaklaşım, birikmiş sipariş özelliği, kusurlu ürünler, dışardan satın alma ve sıraya bağlı kurulum süreleri gibi çeşitli gerçek hayat uygulamalarını içeren genişletilmiş versiyonu içinde uygulanmıştır. Bu doktora çalışmasının son amacı, çok makineli kapasiteye sahip parti büyüklüğü ve çizelgeleme problemini içeren, havuz ekipmanı imalat endüstrisindeki pratik bir üretim sorununu makine uygunluk kısıtlamaları ile çözmektir. Ele alınan problem aynı zamanda çok amaçlı KTP modeli olarak formüle edilmiştir. Hesaplamalı deneyler, oluşturulan modelin havuz ekipmanı imalat endüstrisi için pratik gereksinimleri karşıladığını göstermektedir.

Anahtar kelimeler: Entegre parti büyüklüğü ve çizelgeleme problemi; üretim planlaması; karışık tamsayı programlama; matsezigisel

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CHAPTER ONE

INTRODUCTION

1.1 Background and Motivation

Demands from the market that are always changing make manufacturing hard. Companies must make the best use of their resources to make high-quality products and meet market needs quickly. So, efficient production planning has become very important because it helps companies make more profit by making better use of their resources. In general, production planning is the proper planning of raw materials, resources, and production operations in order to transform raw materials into marketable goods while meeting consumer expectations in the most cost-effective way possible. However, there is a conflict in finding the most efficient way to use resources for product production while meeting client demands (Wolosewicz, Dauzère-Pérès & Aggoune, 2015). The hierarchical process of production planning includes long-term, medium-term, and short-term decision making (Drexl, & Kimms, 1997). Long-term planning is a strategic decision that typically includes facility location, design, and resource allocation. Medium-term planning focuses on material requirement planning and lot sizing decisions to optimize total costs while meeting demand requirements and existing capacity restrictions. Short-term planning is concerned with the day-to-day scheduling of operations (Karimi, Ghomi & Wilson, 2003). Among them, the problem of lot sizing and scheduling becomes one of the most critical and difficult (Allahverdi, Gupta & Aldowaisan, 1999; Potts & Kovalyov, 2000; Rizk & Martel, 2001; Zhu & Wilhelm, 2006; Jans & Degraeve, 2007; Allahverdi, Ng, Cheng & Kovalyov, 2008; Gicquel, Minoux & Dallery, 2008; Robinson, Narayanan & Sahin, 2009; Copil, Wörbelauer, Meyr & Tempelmeier, 2017).

The motivation behind this Ph.D. thesis has two aspects: new models and new solution techniques. New models can consider many factors while generating powerful and efficient production plans. Production of such models would not only improve overall satisfaction but also make these models more applicable in real-world systems. In terms of the motivation of the thesis, the latter is to search for new solution

techniques and develop such solution approaches and tools that can find the best-known production plans. In general, a simulated annealing algorithm can be embedded in solution frameworks, and results can be evaluated. In this thesis, mathematical programming models and metaheuristics have received a great deal of interest for several reasons. While mathematical models promise optimal solutions for small problems, it may perform poorly on large data sets. On the other hand, metaheuristics can tackle large problems and produce near-optimal results while sacrificing optimality. For this reason, the research on solution techniques has extensively been on the hybridization of mathematical models and metaheuristic based algorithms.

1.2 Research Objectives

The purpose of this thesis is twofold. The first objective is to introduce efficient solution approaches for a GLSP. The second is to formulate the most challenging real-world problems of lot sizing and scheduling that are encountered under different production environments and to propose efficient solution approaches that can effectively solve problems of any size and generate optimal or near-optimal solutions. The following paragraphs summarize the objectives in detail.

Research objective 1: *Development of new solution methodologies and frameworks capable of solving problems of any size and producing optimal or near-optimal results.*

The GLSP is known for its computational complexity. In this Ph.D. study, the GLSP with sequence dependent changeovers and backorders are investigated through mathematical modeling and computational approaches. SA and SA-based matheuristic algorithms are developed for solving the GLSP. Extensive computational tests are performed on randomly generated problem instances for comparing the computational performances of the proposed SA and SA-based matheuristic algorithms. New valid inequalities are proposed for the GLSP, and extensive micro period analysis on the randomly generated test instances is presented.

Research objective 2: *Development of new models to represent real-world paper bag industry applications and matheuristic based solution approaches.*

When the studies in the scientific literature on the subject are examined, it is observed that the researchers focus either on the most basic problems or on business-specific problems. Since the basic problem models cannot meet the practical needs of the enterprises, and the enterprises have various constraints and objectives, it is necessary to develop models specific to the business needs. But at the modeling stage, it might be helpful to look at how some special constraints and situations were dealt with in the models that were built from the scientific literature. Therefore, in the Ph.D. study, a MIP model is proposed for the integrated lot sizing and scheduling problem in a business that produces made-to-order paper bags. The proposed model differs from what has been written in literature because it includes cutting plans and external procurements. The developed model aims to minimize the total of the holding, backorder, procurement, setup, and production costs.

Research objective 3: *Development of extended new models to integrate defective items to more accurately represent real-world paper bag industry applications and matheuristic based solution approaches.*

Environmental and energy concerns are growing and industrial systems are facing increased obstacles. This is also the case in the paper bag industry, which transforms raw materials into specialized customer products. In this transformation process, there are several possible cutting plans to cut the raw materials to obtain different sets of paper bags from each plan. Before starting the production process, a cutting equipment can be configured. Although technological advancements in new cutting machines and equipment enable higher performance, defective items are inevitable in many production environments. It is crucial for manufacturers to recognize manufacturing quality issues and prevent the delivery of defective items to customers. Therefore, in the Ph.D. study, a new model of the capacitated lot sizing and scheduling problem with external procurements, defective and backordered items, setup times, and cutting plan assignment for paper bag industry is provided. Our objective is to minimize the total

cost function, including the holding cost of good quality and defective items, the backorder, the procurement, the production, and the sequence-dependent setup. In addition, a novel solution methodology is suggested for the proposed model.

Research objective 4: *Development of new models to represent the real-world pool equipment manufacturer industry problem and a multi objective solution approach.*

In this Ph.D. study, the multi-machine capacitated lot sizing and scheduling problem with machine eligibility constraints is presented for the pool equipment manufacturing industry. In this problem, not all machines in the facility are eligible for every product type, and product types can be produced on several alternative machines. The considered problem is also formulated as a multi-objective MIP model. We attempt to solve it by transforming the two objectives into a single objective via Weighted Sum Scalarization (WSS) approach.

1.3 Outline of the Thesis

The remainder of this thesis is divided into five chapters and is ordered as follows:

In Chapter 2, the GLSP in single level capacitated environments with sequence dependent item changeovers is explained in detail. Initially, a formal problem description is given along with the MIP formulation of the problem. Afterward, a matheuristic algorithm that integrates the SA algorithm and the given MIP model is presented.

In Chapter 3, introduces an integrated lot sizing and scheduling problem in the paper bag production industry. Initially, the developed MIP model is presented. Next, the proposed matheuristic algorithm's details are given and computational results are presented.

In Chapter 4, we extend our work presented in Chapter 3 by considering practical production issues in the paper bag industry that contain the problem of capacitated lot

sizing and scheduling with cutting plan assignment. After giving the MIP model, the problem-based matheuristic approach is given. Next, the computational results obtained for the extended problem are presented. In addition, we conducted sensitivity analyses on setup time, capacity, and procurement costs.

In Chapter 5, considers the multi-machine capacitated lot sizing and scheduling problem with the additional considerations of machine eligibility, sequence-dependent setups, and common server operator. Initially, the MIP model is provided. After that, multi-objective version of the presented model is given. Next, the computational results obtained from both versions are presented.

Finally, Chapter 6 summarizes the thesis and provides directions for future research.

CHAPTER TWO

A MATHEURISTIC BASED SOLUTION APPROACH FOR THE GENERAL LOT SIZING AND SCHEDULING PROBLEM WITH SEQUENCE DEPENDENT CHANGEOVERS AND BACK ORDERING

2.1 Introduction

The simultaneous lot sizing and scheduling (SLS) problem is an essential aspect of production planning. Production planning, among other things, defines the number of production lots created when inputs are transformed into final products. Production system of many companies comprises one of the stages of this problem. The main objective is to satisfy the best trade-off between the customer request and financial objectives (Pochet, 2001). Lot sizing and scheduling literature has several versions of the problem containing different models, mathematical formulations and solution approaches. Especially in the process industry, time-consuming changeovers become important when utilizing the existing capacity. The integration of lot sizing and sequencing decisions has received considerable attention in the related literature as it considerably improves usage of high-cost production resources by efficiently controlling final inventories and managing available capacity (Maravelias & Sung, 2009).

The GLSP is known for its computational complexity. Fleischmann & Meyr (1997) have shown that the GLSP with a non-zero minimum lot size is a challenging combinatorial optimization problem. Therefore, the lot sizing literature tends to create solution approaches that are computationally efficient, such as matheuristics, metaheuristics, and greedy heuristics to cope with the combinatorial complexity of the problem. Metaheuristics have received significant attention in lot sizing and scheduling problems among these techniques. Various metaheuristics techniques have been used to solve the GLSP effectively. Recently, exact methods and metaheuristic algorithms have been combined in order to form a new class of algorithms, which are known as matheuristics (Santos, Massago & Almada-Lobo, 2010). Capability of identifying the optimum balance between the efficacy of exact methods and the

efficiency of metaheuristics is the most significant benefit of matheuristics (Guimaraes, Klabjan & Almada-Lobo, 2013; Guimarães, Klabjan & Almada-Lobo, 2014). These algorithms have several advantages over classical heuristics, such as needing fewer parameters to tune and requiring only minor revisions in response to changes in the mathematical formulation. The following studies are some of the good examples that present the advantages of combining metaheuristics with mathematical programming methodologies for handling lot sizing or lot sizing and scheduling problems.

Figueira, Santos & Almada-Lobo (2013) proposed an efficient solution method that integrates variable neighborhood search (VNS) algorithm and an exact method for solving lot sizing problems in pulp and paper industry. Toledo, de Oliveira, de Freitas Pereira, Franca & Morabito (2014) successfully solved a specific two-level lot sizing and scheduling problem by integrating Genetic Algorithms (GA) with mathematical programming techniques in the soft drink industry. Furlan, Almada-Lobo, Santos & Morabito (2015) solved the lot sizing and scheduling problem of pulp and paper mills by applying a hybrid approach based on a customized GA with an embedded residual linear programming model. Goerler, Lalla-Ruiz & Voß (2020) studied the extended GLSP problem with rework and lifetime constraints by developing a late acceptance hill-climbing based matheuristic. Lee & Lee (2022) proposed a new optimization model with sequence dependent setups for the GLSP problem. They proposed a set of valid inequalities to compare the tightness of the linear programming relaxations of their model. Carvalho & Nascimento (2022) solved the lot sizing and scheduling problem (LSP) for parallel machines with non-triangular sequence dependent setups and setup carryover. They proposed a hybrid matheuristic approach that integrates local search heuristics and mathematical programming.

In line with the aforementioned studies, the following contributions are sought for the current study: (i) a matheuristic approach is proposed for solving the GLSP better than MIP and SA, (ii) micro period analysis on the randomly generated test instances is presented, (iii) new valid inequalities are proposed for the GLSP.

The rest of this paper is organized as follows. Section 2.2, related literature is reviewed. Section 2.3 presents the mathematical formulation of the GLSP. Section 2.4 describes the proposed metaheuristic and matheuristic approaches. Section 2.5 presents the computational experiments and the obtained results. Finally, Section 2.6 concludes the paper and presents advices for further work.

2.2 Literature Review

There are two types of model formulation for lot sizing and scheduling problems with big and small time buckets. Big bucket models allow for multiple setup operations in a single period, whereas small bucket models only allow one setup for each period. Big bucket models are better for problems with longer periods, whereas small bucket models are better for problems with shorter periods. The split of macro periods of big bucket problems into micro periods for dealing with small bucket problems considerably increases the mathematical complexity (Almada-Lobo, Klabjan, Carravilla & Oliveira, 2007). Small bucket model formulations are less common than big bucket versions (Copil et al., 2017). Fleischmann & Meyr (1997) developed a model for a single line based on GLSP. Meyr (2004) expanded GLSP to multiple production stages (GLSPMS). Koçlar & Süral (2005) investigated lot sizing and planning reformulations in GLSP. Multi-level GLSP with multiple machines (MLGLSP_MM) was proposed by Fandel & Stammen-Hegene (2006). Their model's objective is to minimize the total costs of sequence-dependent setup, production, inventory and maintaining the setup conditions of the machines. They proposed a non-linear programming model. Transchel, Minner, Kallrath, Löhndorf & Eberhard (2011) developed a hybrid mixed-binary optimization model for the GLSP. This model was customized to meet the two-stage production structure of a company that operates in the process industry. Ferreira, Clark, Almada-Lobo & Morabito (2012) presented several single-step formulations for solving synchronized two-stage lot size and scheduling problem in the soft drink production. Most of the medium and large size lot sizing and scheduling problems are very challenging to solve. Addition of realistic constraints into models complicates the situation further. For instance, when dealing with the limited capacity, meeting all customer requests on time may be impossible.

Backlogging is critical in this condition, especially in highly capacitated environments (Babaei, Mohammadi & Ghomi, 2014). There is a rich literature that discusses all of these issues along with possible formulations and more. Readers can refer to Drexl & Kimms (1997), Copil et al. (2017), Karimi et al. (2003), Jans & Degraeve (2008), Díaz-Madroñero, Mula & Peidro (2014) for a comprehensive review of the previously developed models. On the other hand, the need for effective solution approaches for the devised models is still in a very high demand.

2.3 Problem Statement and Mathematical Formulation

In the GLSP products $j=1,...,J$ are to be scheduled in the planning period T that consists of discrete macro periods $t=1,...,T$ with a given length. In each macro period, t is represented by a set S_t of non-overlapping micro periods r where the number $|S_t|$ of micro periods must be predetermined. In a single micro period, only one product can be produced. The micro period lengths are decision variables determined by the quantity x_{jr} produced in each micro period r multiplied by the production coefficient p_j . The following assumptions are made in the GLSP model formulation: Each item's demand is deterministic; A shortage is referred as backlogging; Backlog and inventory amounts are calculated at the end of the planning horizon; The triangle inequality is valid in terms of setup cost and setup time, i.e. $s_{ik} \leq s_{ij} + s_{jk}$ and $bs_{ik} \leq bs_{ij} + bs_{jk}$ for all products i, j , and k .

2.3.1 Notation

Table 2. 1 presents the symbols of the indices, parameters, and variables, which is used to formulate the GLSP model.

Table 2.1 Indices, parameters and variables that are used in the GLSP model.

Indices:

J : Number of products;
 T : Number of macro-periods;
 R : Number of micro-periods;

Sets:

S_t : Set of micro-periods r in each macro-period t ;

Parameters:

d_{jt} : Demand of product j in macro-period t ;
 h_j : Inventory holding cost of product j ;
 g_j : Backorder cost of product j ;
 p_j : Production time to produce one unit of product j ;
 k_j : Minimum lot size amount of product j ;
 C_t : Available capacity (time) in macro-period t ;
 s_{ij} : Sequence dependent setup cost of production changeover from product i to j ;
 bs_{ij} : Setup time of production changeover from product i to j ;
 I_{jo}^+ : Initial inventory amount of product j ;
 I_{jo}^- : Initial backorder amount of product j ;
 y_{j0} : $\begin{cases} 1, \text{ if there is an initial setup of product } j \\ 0, \text{ otherwise} \end{cases}$

Variables:

I_{jt}^+ : Inventory of product j in macro-period t ;
 I_{jt}^- : Backorder amount of product j in macro-period t ;
 x_{jr} : Production amount of product j in micro-period r ;
 z_{ijr} : $\begin{cases} 1, \text{ if production changeover occurs from product } i \text{ to } j \text{ in micro – period } r \\ 0, \text{ otherwise} \end{cases}$
 y_{jr} : $\begin{cases} 1, \text{ if machine is setup for production of product } j \text{ in micro – period } r \\ 0, \text{ otherwise} \end{cases}$

Minimize

$$Z = \sum_{j=1}^J \sum_{t=1}^T (h_j I_{jt}^+ + g_j I_{jt}^-) + \sum_{r=1}^R \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (s_{ij} z_{ijr}) \quad (2.1)$$

subject to

$$I_{j(t-1)}^+ + I_{jt}^- + \sum_{r=1}^{S_t} x_{jr} - I_{jt}^+ - I_{j(t-1)}^- = d_{jt}, \quad \forall j, t \quad (2.2)$$

$$\sum_{j=1}^J \sum_{r=1}^{S_t} p_j x_{jr} + \sum_{r=1}^{S_t} \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (b s_{ij} z_{ijr}) \leq c_t, \quad \forall t \quad (2.3)$$

$$x_{jr} \leq \frac{c_t}{p_j} y_{jr}, \quad \forall j, t, r \in S_t \quad (2.4)$$

$$\sum_{j=1}^J y_{jr} = 1, \quad \forall r \quad (2.5)$$

$$z_{ijr} \geq y_{i(r-1)} + y_{jr} - 1, \quad \forall r, j, i, j \neq i \quad (2.6)$$

$$x_{jr} \geq k_j (y_{jr} - y_{j(r-1)}), \quad \forall r, j \quad (2.7)$$

$$I_{jt}^+, I_{jt}^- \geq 0 \quad \forall j, t; x_{jr} \geq 0 \quad \forall j, r; z_{ijr}, y_{jr} \in \{0,1\}, \quad \forall i, j, r \quad (2.8)$$

The objective function (2.1) aims to minimize the sum of the cost of inventory, backorder, and setup of production changeover in periods. The inventory balance constraints are described in (2.2). Constraints (2.3) satisfy the capacity limits of each period. Constraints (2.4) ensure that the production of product j only occurs in case the setup of production is organized. According to constraints (2.5) the setup must be prepared for a single product for each micro period. Since both $y_{i(r-1)}$ and y_{jr} are binary, the objective function (2.1) and the constraints (2.6) limit the variable z_{ijr} to have value 1 if there is a switch from the product i to j . Constraints (2.7) satisfy the minimum lot sizes that prevent the incorrect evaluation of setup times. Constraints (2.8) specify the range of the variables. It is worth noting that zero-length micro periods (idle periods) can occur. The GLSP formulation (2.1) - (2.8) is only useful in real situations where the setup state is preserved. In addition, the GLSP model fulfills the setup carryover automatically.

2.3.2 Valid Inequalities

In this section, valid inequalities (cuts) that strengthen the GLSP model is presented. Wolsey (2002) presented a categorization system for several small bucket and big bucket models. The lower bounds given by the LP-relaxation of small bucket models yield poor lower bounds. Lower limit can only be improved by using customized reformulations and adding valid inequalities to the problem formulation. On the other hand, most of the big bucket models give significantly better lower limits. These valid inequalities are not necessary for the model formulation but make it tighter. Belvaux & Wolsey (2001), Wolsey (2002), Pochet & Wolsey (2006), and Kaczmarczyk (2020) extended various valid inequalities for different lot sizing problems. The following constraints (2.9) are valid for satisfying the inventory and backorder constraint for all micro periods r in macro period t .

$$I_{j(t-1)}^+ + I_{jt}^- \geq d_{jt} \left(1 - \sum_{r=1}^{S_t} y_{jr} \right), \quad \forall j, t \quad (2.9)$$

Proof.

Let $(\hat{x}, \hat{y})$ be a feasible solution for (2.2) - (2.8). Consider the case in which $\hat{y}_{jr} = 0$ for every $j, r \in S_t$. This implies that $\hat{x}_{jr} = 0$ for every $j, r \in S_t$, and thus constraint (2.4) ensure that $I_{j(t-1)}^+ + I_{jt}^- - I_{jt}^+ - I_{j(t-1)}^- = d_{jt}$ for every j, t . If the obtained equation $I_{j(t-1)}^+ + I_{jt}^- = I_{jt}^+ + I_{j(t-1)}^- + d_{jt}$ is replaced in equation (2.9), it is obtained that the non-negativity of the summation of inventory and backorder $I_{jt}^+ + I_{j(t-1)}^- \geq 0$. Now assume that the $\hat{y}_{jr} = 1$ for at least one of the $j, r \in S_t$. Then the following equation is satisfied $I_{j(t-1)}^+ + I_{jt}^- + (k - 1) * d_{jt} \geq 0$ for every j, t . Where k shows the total number of decision variables that satisfy the $\hat{y}_{jr} = 1$. Therefore, the inequality is valid.

2.4 The Proposed Solution Approaches

2.4.1 Simulated Annealing Algorithm

Kirkpatrick, Gelatt & Vecchi (1983) developed the stochastic hill climbing-based single-solution metaheuristic method that is known as the SA algorithm. The SA algorithm starts with a randomly generated initial solution, generates a neighbor solution based on the specified neighborhood structure at each iteration, and evaluates it by using the fitness function. Better moves are always acceptable. On the other hand, accepting a worse neighbor is possible with a probability, $P = e^{-\frac{\theta}{T}}$, where θ is the difference between the current and neighbor solution's fitness, T is the temperature (the main parameter of SA) that is iteratively reduced in relation to a predefined cooling schedule. The SA algorithm continues until the termination conditions are reached. In the following subsections, solution representation, initial solution generation process, neighborhood generation procedure, the fitness (cost) function of the proposed SA algorithm is presented. The total violation minimization approach (Baykasoglu & Akpinar, 2015) is employed for handling infeasibilities related to problem constraints.

2.4.1.1 Solution Representation

In this study, Ponnambalam & Reddy (2003)'s solution representation scheme is used for both lot sizing and scheduling decisions. A solution contains paired values for each macro-period in this representation. The first number in a pair indicates the product type, while the second shows the product's lot size. The pair order specifies the order in which the corresponding lots need to be manufactured. An example is presented in Figure 2.1, where four products need to be assigned into four micro periods and four macro periods. 10 units of product 1 is produced in the first micro period, 27 units of product 2 is produced in the second micro period, 33 units of product 3 is produced in the third micro period etc.

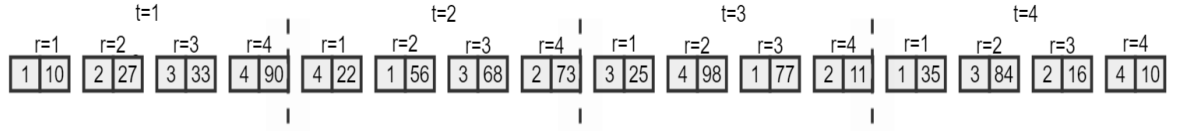


Figure 2.1 An example for SA solution representation

2.4.1.2 Initialization

The initial solution is randomly generated. Lot size for every product in the initial solution is determined by using the following formula: “*lower bound for demand*” + $\text{random}(0,1) \times (\text{“upper bound for demand”} - \text{“lower bound for demand”})$.

2.4.1.3 Neighbor Solution Generation

The current solution is tried to be improved through the neighbor generation process. Based on the solution structure as depicted in Figure 2.1, three neighborhood operators, namely arithmetic and zero operators (Ramezani & Saidi-Mehrabad, 2013) are used for altering the lot sizes and swap operator for varying the sequence at each SA iteration.

Arithmetic operator: This operator decreases the quantity of a randomly selected product by the amount Δ and then adds it to another randomly selected product at each period. The value of Δ is set to $\text{random}(0,1) \times \max(d_{jt})$.

Zero operator: This operator reduces the lot size of a randomly selected product to zero and then adds its lot size to the lot size of another randomly selected product at each period.

Swap operator: Randomly selected products from randomly selected micro periods within macro periods are swapped.

2.4.1.4 Cooling Schedule

One of main parameters of the SA algorithm is temperature, T , which steadily decreases as the algorithm progresses. T is initially set to a high level and reduced by using a cooling schedule. In this study, T is reduced with the following equation, $T_i = q * T_{i-1}$ (where q is the cooling factor that is usually set to a number between 0.9 and 1).

2.4.1.5 Termination Condition

The termination criterion is defined as ($T > T_f$). As long as the termination criterion is not satisfied, the SA algorithm repeats its steps.

The pseudocode of the proposed SA algorithm is depicted in Figure 2.2. The following abbreviations are used in the pseudocode:

$F_{(n)(o)(c)}$: Objective function value of the (neighbor) (initial) (current) solution

$X_{(n)(o)(c)}$: Solution vector of (neighbor) (initial) (current)

X_{best} : Solution vector with the minimum fitness value

F_{best} : Minimum fitness value

2.4.2 The Proposed SA Based Matheuristic Approach for Solving GLSP

The GLSP is NP-Complete (Fleischmann & Meyr, 1997). Medium and large-scale problem instances cannot be solved by using commercial solvers within a reasonable time. Therefore, a SA-based matheuristic approach is proposed for the present GLSP. Actually, the proposed SA-based matheuristic approach uses the general framework of the SA algorithm, which calls a MIP model, performs fitness assessments and operates the swap/inversion operators to generate neighbor solutions. The simplified pseudocode of the proposed matheuristic approach is depicted in Figure 2.3.

```

1. Initialization: Specify SA parameters
2. Build an initial solution and current solution
3.  $X_{best} := X_{(o)}; F_{best} := F_{(o)}; X_{(c)} := X_{(o)}; F_{(c)} := F_{(o)};$ 
4.  $T := T_0;$ 
5. While ( $T > T_f$ )
6.      $I := 0;$ 
7.     While ( $I < MaxI_{iter}$ )
8.         Generate the neighboring solutions by selecting a random number between (1, 3)
           and determine the neighboring solution  $X_{(n)}$  with the objective function value  $F_{(n)}$ 
9.         If 1, apply the Arithmetic operator
10.        If 2, apply the Zero operator
11.        If 3, apply the Swap operator
12.         $\Delta := F_{(n)} - F_{(c)};$ 
13.        If ( $\Delta < 0$ )
14.             $X_{(c)} := X_{(n)};$ 
15.             $F_{(c)} := F_{(n)};$ 
16.        Else
17.             $r_1 := \text{random}(0,1);$ 
18.            If ( $r_1 < \exp(-\Delta/T)$ );
19.                 $X_{(c)} := X_{(n)};$ 
20.                 $F_{(c)} := F_{(n)};$ 
21.            End
22.        End
23.        If ( $F_{(c)} < F_{best}$  and  $X_{(c)}$  is feasible)
24.             $X_{best} := X_{(c)}; F_{best} := F_{(c)};$ 
25.        End
26.         $I := I+1;$ 
27.    End
28.     $T := qT;$ 
29. End While
30. Report  $X_{best}; F_{best}$ 

```

Figure 2.2 Pseudocode of the SA algorithm

The matheuristic algorithm begins by initializing solution, $X_{(o)}$, afterwards it computes the decision variables, x_{jr} , I_{jt}^+ and I_{jt}^- for the corresponding solution $X_{(o)}$ via the MIP model. Then the result is used to calculate the fitness value, $F_{(o)}$. Subsequently, the matheuristic algorithm employs the inversion and swap operators to create $X_{(n)}$ from $X_{(o)}$. On the other hand, the SA process is started to produce $X_{(c)}$

from $X_{(n)}$. If a better solution is found, i.e., $F_{(c)} < F_{best}$, then X_{best} and F_{best} are updated. The matheuristic algorithm stops when it reaches to $MaxI_{iter}$.

```

1. Initialization: Specify SA parameters
2. Initialize  $X_{(o)}$  →Section 2.4.2.2
3. Compute the lot sizing and inventory decisions for  $X_{(o)}$  by LP model →Section 2.4.2.4
4. Evaluate  $F_{(o)}$ 
5.  $X_{best} := X_{(o)}$ ;  $F_{best} := F_{(o)}$ ;
6.  $T := T_0$ ;
7. While ( $T > T_f$ )
8.      $I := 0$ ;
9.     While ( $I < MaxI_{iter}$ )
10.        Generate neighboring solutions by selecting a random number between (1, 2) and
            determine the neighboring solution  $X_{(n)}$  with the objective function value  $F_{(n)}$ .
            →Section 2.4.2.3
11.        If 1, apply Inversion
12.        If 2, apply Swap
13.         $\Delta := F_{(n)} - F_{(c)}$ ;
14.        If ( $\Delta < 0$ )
15.             $X_{(c)} := X_{(n)}$ ;
16.             $F_{(c)} := F_{(n)}$ ;
17.        Else
18.             $r_1 := \text{random}(0,1)$ ;
19.            If ( $r_1 < \exp(-\Delta/T)$ );
20.                 $X_{(c)} := X_{(n)}$ ;
21.                 $F_{(c)} := F_{(n)}$ ;
22.            End
23.        End
24.        If ( $F_{(c)} < F_{best}$  and  $X_{(c)}$  is feasible)
25.             $X_{best} := X_{(c)}$ ;  $F_{best} := F_{(c)}$ ;
26.        End
27.         $I := I + 1$ ;
28.    End
29.     $T := qT$ ;
30. End While
31. Report  $X_{best}$ ;  $F_{best}$ 

```

Figure 2.3 Pseudocode of the proposed SA-based matheuristic algorithm

2.4.2.1 Solution Encoding

Since decisions to be made in the present problem entail sequencing of processes, it is logical to employ a permutation based encoding in the proposed matheuristic algorithm. In this encoding (solution representation), products are sequenced in each macro period. An example is shown in Figure 2.4, where four products are sequenced in four micro periods of four macro periods.

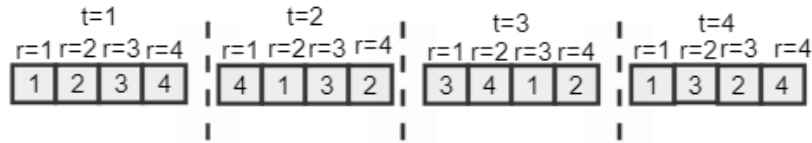


Figure 2.4 An example of solution representation

2.4.2.2 Initialization

Solutions are randomly initialized.

2.4.2.3 Neighbor Solution Generation

Two operators are used to generate neighbor solutions, namely inversion and swap. Inversion operator inverts randomly selected products from two randomly selected micro periods within macro periods. Swap operator swaps randomly selected products from randomly selected micro periods within macro periods.

2.4.2.4 Linear Programming Model

As aforementioned, the SA algorithm is responsible for determining y_{jr} and z_{ijr} , while the LP model is used to determine optimal values of decision variables related to lot sizing and inventory decisions. The LP model is given by equations (2.10) - (2.13).

Minimize

$$Z = \sum_{j=1}^J \sum_{t=1}^T (h_j I_{jt}^+ + g_j I_{jt}^-) + \sum_{r=1}^R \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (s_{ij} \overline{z_{ijr}}) \quad (2.10)$$

subject to

$$I_{j(t-1)}^+ + I_{jt}^- + \sum_{r=1}^{S_t} x_{jr} - I_{jt}^+ - I_{j(t-1)}^- = d_{jt}, \quad \forall j, t \quad (2.11)$$

$$\sum_{j=1}^J \sum_{r=1}^{S_t} p_j x_{jr} + \sum_{r=1}^{S_t} \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (b s_{ij} \overline{z_{ijr}}) \leq c_t, \quad \forall t \quad (2.12)$$

$$I_{jt}^+, I_{jt}^- \geq 0 \quad \forall j, t; \quad x_{jr} \geq 0, \quad \forall j, r \quad (2.13)$$

The algorithm, which is shown in Figure 2.5 is used for verification of model feasibility (capacity requirements, minimum lot size constraints and production changeovers) while solving the LP model.

```

forall product  $j$ , micro period  $r \in S_t$  and macro period  $t$ ; do
    if  $y_{jr} = 1$  then
        add  $x_{jr} \leq \frac{c_t}{p_j}$ 
    forall product  $j$  and micro period  $r \in S_t$ ; do
        if  $y_{jr} - y_{j(r-1)} = 1$  then
            add constraints  $x_{jr} \geq k_j$ ;
        forall product  $i, j$  and micro period  $r \in S_t$  where  $i \neq j$ ; do
            if  $y_{i(r-1)} = 1$  and  $y_{jr} = 1$  then
                 $z_{ijr} = 1$ ;
            Else
                 $z_{ijr} = 0$ ;

```

Figure 2.5 Algorithm for preserving feasibility in solving the LP model

2.4.3 Parameter Tuning

In this section, it is aimed to determine good levels of parameters for the proposed SA and SA-based matheuristic algorithm. There are three parameters to be tuned and three levels are determined for each parameter as shown in Table 2.2. L9 is selected from the standard table of orthogonal arrays. For the preliminary trials, fifteen problems are used. The problems are constructed by selecting five random problems from each classes of A, B, and C. As a result, (9x15) 135 runs are carried out in each analysis. All statistical analyses are performed with Minitab 20 software. “The Smaller, The Better” signal-to-noise ratio (S/N) is used within the Taguchi method in order to identify optimal parameter levels. The S/N ratio is computed via Eq. (2.14).

$$\frac{S}{N} = -10 * \log \left(\frac{1}{n} \sum_{i=1}^n Y_i^2 \right) \quad (2.14)$$

where n is the number of observations in each trial and Y_i is the objective function value. Since SA is a stochastic algorithm, the best values of 10 runs are selected in 10 experiments to obtain results that are more reliable. The optimal levels of the parameters are shown in Tables 2.2 and 2.3. The optimal levels of the parameters are determined as 20, 20000, 0.95 for SA and 30, 20, 0.95 for SA-based matheuristic.

Table 2.2 SA parameter levels

<i>Factors</i>	<i>Levels</i>
Initial temperature (A)	20, 30, 50
Number of iterations at each temperature level (B)	5000, 10000, 20000
Cooling rate (C)	0.9, 0.95, 0.985

Table 2.3 SA-based matheuristic parameter levels.

<i>Factors</i>	<i>Levels</i>
Initial temperature (A)	20, 30, 50
Number of iterations at each temperature level (B)	10, 20, 30
Cooling rate (C)	0.9, 0.95, 0.985

2.5 Computational Experiments

In the following subsections, performance analyses of the SA and the SA-based matheuristic algorithms are presented. Section 2.5.1 is devoted to randomly generated data instances. All computational tests are performed on the Windows 64-bit operating system and a Core (TM) i5-4590 CPU with 4 Gigabyte RAM and 3.30 Gigahertz speed. MATLAB R2020b is used for programming the SA algorithm. The SA-based matheuristic algorithm is programmed in Microsoft Visual Studio 2019 C++ by employing the callable library of ILOG CPLEX Optimization Studio Version 12.10 for solving the MIP model by the MIP solver of CPLEX.

2.5.1 Data Instances

In order to analyze computational performance of the proposed algorithms, 150 randomly generated data instances within three data classes (A, B, and C) are generated. Table 2.4 shows the parameter values for all data classes.

Table 2.4 Parameters for the data classes

<i>Parameter</i>	<i>Class A</i>	<i>Class B</i>	<i>Class C</i>
# of instances	50	50	50
# of products (j)	5	6	7
# of macro-periods (t)	4	3	2
# of micro-periods within t ($ S_t $)	5,7,10	6,8,10	7,8,10
d_{jt}	[0;40,120]	[0;40,120]	[0;40,120]
s_{ij}	[100,400]	[100,400]	[100,400]
bs_{ij}	$s_{ij}/10$	$s_{ij}/10$	$s_{ij}/10$
p_j	1	1	1
k_j	10	10	10
h_j	[10,20]	[1,5]	[1,10]
g_j	[10,20]	[1,5]	[1,10]
C_t	$\sum_{j=1}^J \sum_{t=1}^T d_{jt} * 2$	$\sum_{j=1}^J \sum_{t=1}^T d_{jt} * 2$	$\sum_{j=1}^J \sum_{t=1}^T d_{jt} * 2$

2.5.2 Analysis of Valid Inequality

The valid inequalities further reduce the computational time in class A by 61.98%, in class B by 42.59%, and in class C by 68.81% compared to the one without valid inequalities. The detailed analysis in each class A, B, and C are shown in Figures 2.6 - 2.8 respectively. As a result, it can be concluded that the valid inequalities help to improve MIP formulation and reduce the computing effort required to resolve the large problem instances.

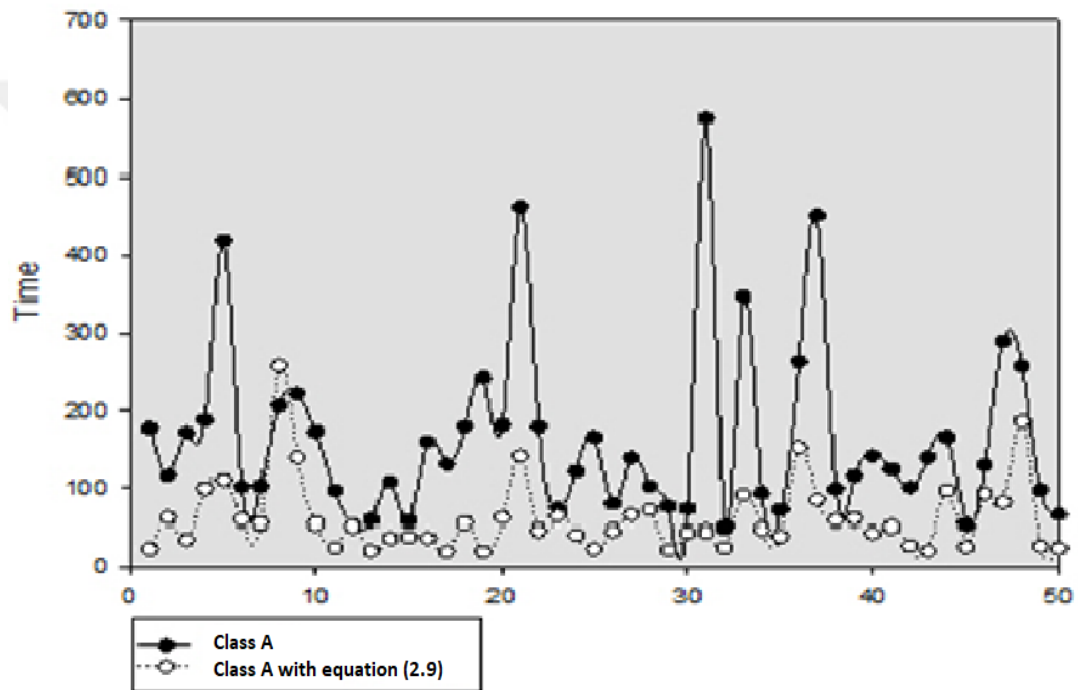


Figure 2.6 Results of class A's validity analysis

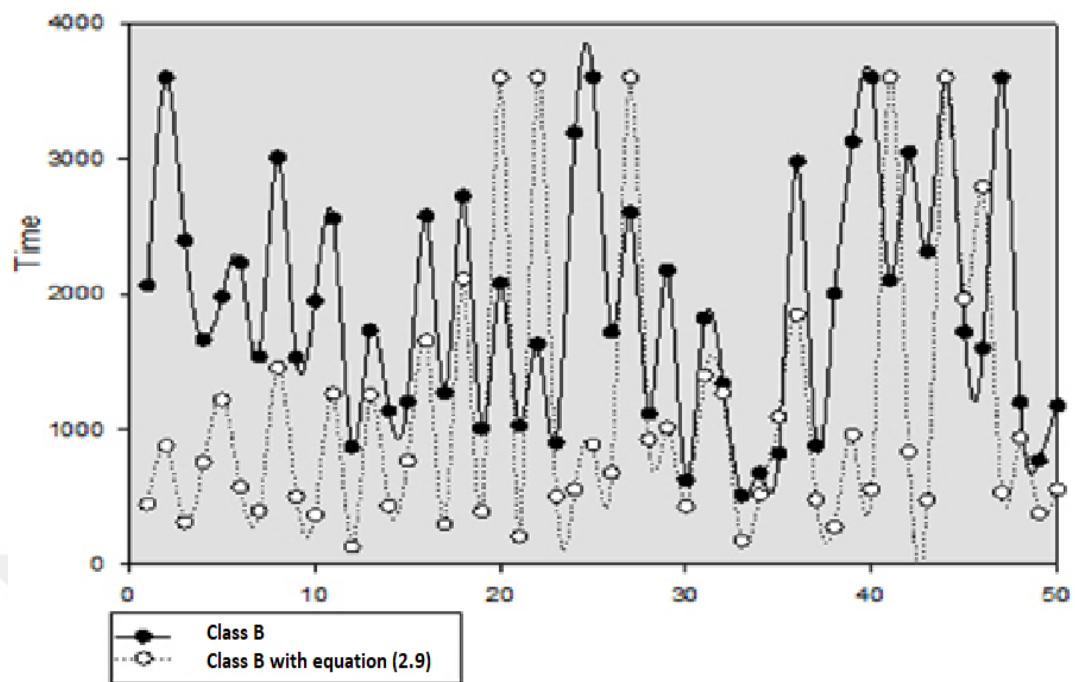


Figure 2.7 Results of class B's validity analysis

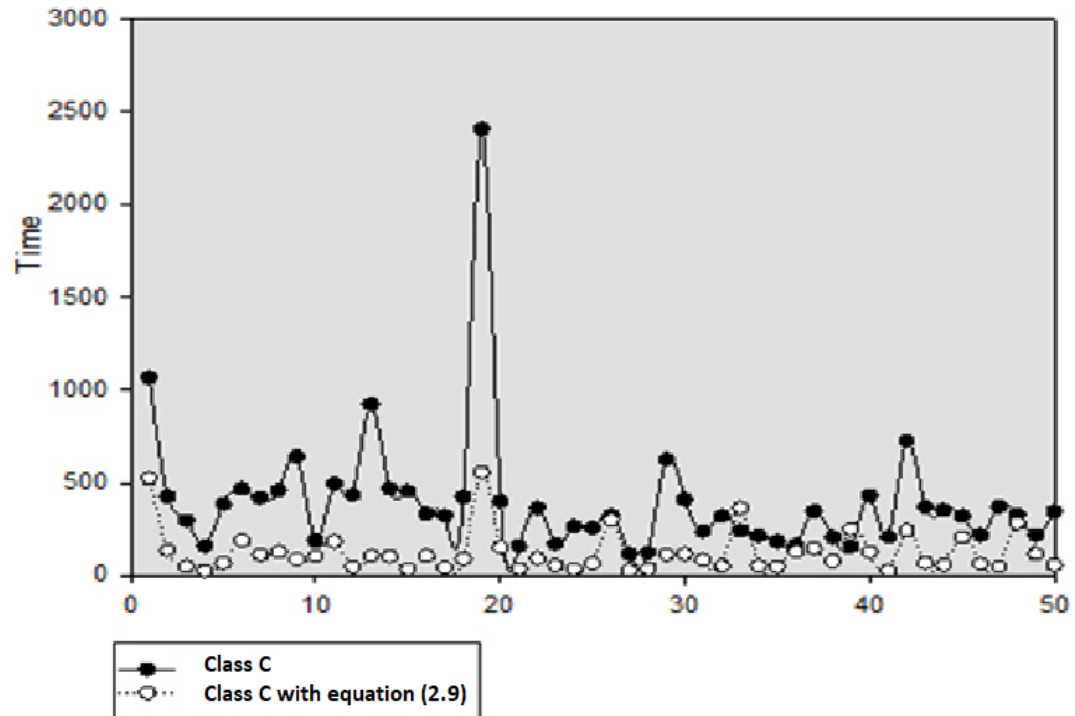


Figure 2.8 Results of class C's validity analysis

2.5.3 Computational Results for Data Instances

For testing the proposed algorithms, we set 3600s time limit for the GLSP model and the best solution obtained from the CPLEX 12.10 solver is used to compare the results. In Tables 2.5 - 2.7, all of the results for the three different data classes are shown. In Tables 2.5 - 2.7 show the lower bounds (LBs) provided by CPLEX after running for 3600s. Hence, it is essential to provide a computable lower bound near to the optimal solution for evaluating the efficacy of the algorithms proposed in this research. In Table 2.5, we analyze Class A with micro periods equal to 5, 7, and 10. In each case, the average CPLEX result is 3187.68. Overall, the average CPLEX time is equal to 165.27s in micro period 5. For class A, instances with micro periods 7 and 10, CPLEX cannot identify optimum solutions for any case within the time limit of 3600s. In Class A, the proposed matheuristic algorithm can find the optimal solutions for 24 out of the 50 problems in micro period 5, for which GLSP provided the optimal results. For the micro periods 7 and 10 in Class A, the matheuristic algorithm can find the best solutions for 11 and 3 out of 50 problems, respectively, for which CPLEX is unable to identify optimum solutions for any case within the time limit of 3600s. For analyzing the performance of the SA algorithm, the average gap increases by 28.24%, 36.40%, and 38.02% compared to CPLEX, respectively, for micro periods 5, 7, and 10. For analyzing the performance of the matheuristic the average gap increases through 0.56%, 1.42%, and 3.41% compared to CPLEX respectively for micro periods 5, 7, and 10. These results show that the average matheuristic performance outperforms the CPLEX and the SA algorithm in Class A. Furthermore, the average computation time of the SA algorithm is reduced by 91.45%, 99.60%, and 99.58% compared to CPLEX results, respectively, for micro periods 5, 7, and 10. The average computation time of the matheuristic algorithm reduces by 78.53%, 98.93%, and 98.79% compared to CPLEX results, respectively, for micro periods 5, 7, and 10. The SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class A problems.

In Table 2.6, we analyze Class B problems with number of micro periods equal to 6, 8, and 10. In Class B, the matheuristic algorithm can find 6 optimal solutions for

micro period 6. Average CPLEX results are 2349.38, 2352.68, and 2359.76 for micro periods 6, 8, and 10. Average CPLEX time is equal to 1930.45s for micro periods 6, 8, and 10. For the class B instances with micro periods 8 and 10, CPLEX cannot identify optimum solutions for any case within the time limit of 3600s. The average gap increases to 18.54%, 23.18%, and 25.77% compared to CPLEX for micro periods 6, 8, and 10 in the SA algorithm. The average gap increases by 3.26%, 3.78%, and 5.02% compared to CPLEX, for micro periods 6, 8, and 10 in the matheuristic algorithm. This results show that the average matheuristic performance outperforms CPLEX and SA for Class B problems. Furthermore, the average computation time of SA reduced by 99.41%, 99.68%, and 99.52% compared to CPLEX for micro periods 6, 8, and 10. Average computation time of the matheuristic algorithm reduces 98.01%, 98.80%, and 98.83% in comparison to CPLEX for micro period 6, 8, and 10. Both the SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class B problems.

In Table 2.7, we analyze Class C problems with number of micro periods equal to 7, 8, and 10. In Class C, the matheuristic algorithm can find 13 optimal solutions for micro period 7. Average CPLEX results are 2086.02 and average CPLEX time equals to 398.82s and 3511.80s for micro periods 7 and 8. CPLEX cannot identify optimum solutions for any case within the time limit of 3600s for the class C instances with micro period 10. The average CPLEX results are 2086.10 for the micro period 10. For analyzing the performance of SA, the average gap increases through 15.75%, 22.99% and 22.94% compared to CPLEX, respectively, for micro periods 7, 8, and 10. For analyzing the performance of the matheuristic, the average gap increases by 2.01%, 2.32% and 2.45% compared to CPLEX, respectively, for micro periods 7, 8, and 10. These results show that the average matheuristic performance outperforms CPLEX and the SA algorithm in Class C. Furthermore, the average computation time of the SA is reduced by 97.99%, 99.77%, and 99.77% compared to CPLEX results for micro periods 7, 8, and 10. The average computation time of the matheuristic algorithm is reduced by 90.45%, 98.92% and 98.75% compared to CPLEX results for micro periods 7, 8, and 10. Both the SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class C problems.

2.5.4 Computational Results for Various Capacity Settings

Different capacity (c_t) levels are analyzed in order to observe their effect on the quality of the solution. We divided the capacity into three categories: Tight ($0.8c_t$), Medium (c_t), and Loose ($1.2c_t$). Results for the average computational time of the solution are provided in Table 2.8, with 50 instances in each class and a total of 150 instances for each solution approach. The average computational time spent on the solution approaches increases from Tight to Loose in each class.

Table 2.8 Average computational time

Classes	MIP	SA	Matheuristic
Class A (<i>T</i>)	104,49	14,07	33,19
Class A (<i>M</i>)	165,27	14,12	35,48
Class A (<i>L</i>)	181,16	14,91	36,56
Class B (<i>T</i>)	1860,06	11,02	37,11
Class B (<i>M</i>)	1930,45	11,27	38,26
Class B (<i>L</i>)	2374,07	11,84	38,49
Class C (<i>T</i>)	254,84	7,87	37,72
Class C (<i>M</i>)	398,82	8,01	38,08
Class C (<i>L</i>)	443,73	8,27	38,64

2.6 Conclusion

In this study, the GLSP with sequence dependent changeovers and backorders is investigated through mathematical modelling and computational approaches. SA and SA-based matheuristic algorithms are developed for solving the GLSP. Extensive computational tests are performed on randomly generated problem instances for comparing the computational performances of the proposed SA and SA-based matheuristic algorithms. The computational results demonstrate that the proposed SA-based matheuristic approach is superior to SA and classical MIP solver for solving the present GLSP. The computational time requirement of the proposed SA-based matheuristic algorithm is considerable less than the CPLEX solver. Moreover, in order to reveal the effect of valid inequalities on the overall time performance, all problem instances in Classes A, B, and C with micro periods 5, 6, and 7 are also solved by

including valid inequality constraints into model formulations. It is observed that valid inequality constraints can decrease average computational times by 61.98%, 42.59%, and 68.81% in problem classes A, B, and C.

Future studies will be devoted to incorporating different real life constraints into the present GLSP model and enhancing the performance of the proposed matheuristic algorithm further by employing different local search strategies within SA.



Table 2.5 Results of class A

Class A Instance	MIP/Micro Period=5				SA		Matheuristic		MIP/Micro Period=7				SA		Matheuristic		MIP/Micro Period=10				SA		Matheuristic	
	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time
1	3320	3320	177,80	0,00	4104	14,37	3320	33,26	3320	1790,81	3600	46,06	4603	14,12	3320	38,85	3320	914,33	3600	72,46	4576	14,81	3520	42,76
2	3480	3480	116,59	0,00	4249	14,59	3480	34,41	3480	1712,51	3600	50,79	4491	14,12	3484	39,51	3480	731,50	3600	78,98	4287	14,70	3499	41,41
3	3172	3172	172,02	0,00	4213	14,72	3172	33,56	3172	1511,46	3600	52,35	4686	14,36	3184	39,86	3172	633,77	3600	80,02	4615	14,96	3220	41,27
4	3784	3784	188,83	0,00	4732	14,37	3804	36,98	3784	1784,91	3600	52,83	4685	14,38	3804	36,29	3784	1014,49	3600	73,19	4986	14,88	3892	43,65
5	3816	3816	418,67	0,00	4810	14,01	3816	34,70	3816	1669,88	3600	56,24	4928	14,20	3869	36,27	3816	667,04	3600	82,52	4818	14,78	3864	42,05
6	2996	2996	101,78	0,00	3999	14,19	2996	32,10	2996	1779,32	3600	40,61	4015	14,21	2996	35,74	2996	959,32	3600	67,98	4122	14,82	3180	44,96
7	3512	3512	102,81	0,00	4391	14,01	3528	35,34	3512	1757,05	3600	49,97	4529	14,29	3528	33,91	3512	1034,64	3600	70,54	4585	14,70	3609	40,65
8	3620	3620	206,55	0,00	4681	14,00	3620	35,92	3620	1445,83	3600	60,06	4621	14,48	3620	34,16	3620	924,19	3600	74,47	4841	14,65	3684	43,58
9	2988	2988	221,34	0,00	4020	14,27	3046	32,84	2988	1105,26	3600	63,01	3749	14,44	3030	34,24	2988	812,44	3600	72,81	4174	14,79	3155	41,40
10	3512	3512	173,41	0,00	4384	14,20	3520	32,27	3512	1326,48	3600	62,23	4922	14,15	3520	39,88	3512	919,44	3600	73,82	4913	14,36	3534	46,48
11	3292	3292	96,64	0,00	4138	14,06	3320	32,82	3292	1828,05	3600	44,47	4471	14,30	3393	36,20	3292	774,61	3600	76,47	4313	14,98	3328	41,28
12	2760	2760	52,56	0,00	3477	14,47	2795	35,69	2760	1259,66	3600	54,36	4101	14,10	2820	35,38	2760	832,14	3600	69,85	4122	14,87	2825	40,12
13	2884	2884	59,56	0,00	3804	13,87	2884	33,12	2884	1660,32	3600	42,43	4118	14,23	2985	36,22	2884	775,80	3600	73,10	4253	14,65	3009	41,30
14	2964	2964	107,97	0,00	4047	14,12	2988	36,24	2964	1482,00	3600	50,00	3936	14,32	3041	36,84	2964	765,60	3600	74,17	4084	14,70	3097	41,70
15	3088	3088	58,97	0,00	4078	14,04	3164	33,21	3088	1547,09	3600	49,90	4667	14,17	3164	39,13	3088	774,47	3600	74,92	4627	14,69	3194	43,12
16	3140	3140	160,45	0,00	4158	14,15	3161	46,21	3140	1313,46	3600	58,17	4104	14,38	3185	37,21	3140	659,71	3600	78,99	4232	14,85	3168	44,01
17	3460	3460	131,38	0,00	4365	14,12	3460	37,32	3460	2013,37	3600	41,81	4707	14,50	3460	36,61	3460	864,65	3600	75,01	4675	14,71	3644	47,25
18	3344	3344	180,00	0,00	4415	14,31	3384	37,57	3344	1737,88	3600	48,03	4551	14,02	3384	36,13	3344	783,83	3600	76,56	4484	14,90	3344	48,46
19	3232	3232	241,83	0,00	3819	14,03	3232	37,74	3232	1759,82	3600	45,55	4431	14,08	3232	36,31	3232	881,04	3600	72,74	4576	14,88	3349	42,48
20	3244	3244	182,00	0,00	3961	14,17	3251	36,62	3244	1554,20	3600	52,09	4410	14,12	3285	36,13	3244	781,16	3600	75,92	4405	14,60	3303	43,17
21	3348	3348	461,98	0,00	4147	14,16	3348	41,79	3348	1178,83	3600	64,79	4489	14,28	3357	37,92	3348	756,31	3600	77,41	4598	14,95	3435	46,92
22	3404	3404	180,11	0,00	4346	14,05	3432	36,50	3404	1576,73	3600	53,68	4696	14,32	3451	34,95	3404	781,90	3600	77,03	4684	15,02	3489	47,35
23	3016	3016	74,22	0,00	3846	13,99	3049	42,90	3016	1477,84	3600	51,00	4226	14,20	3046	34,48	3016	728,06	3600	75,86	4437	14,92	3148	44,60
24	3228	3228	122,00	0,00	4284	13,99	3228	37,75	3228	1469,06	3600	54,49	4118	14,15	3316	33,81	3228	500,66	3600	84,49	4139	14,88	3363	40,63
25	3364	3364	166,27	0,00	4240	14,06	3373	42,48	3364	1702,52	3600	49,39	4779	14,41	3382	48,81	3364	878,68	3600	73,88	4737	14,95	3555	44,55
26	2712	2712	80,38	0,00	3594	14,11	2748	36,64	2712	1460,41	3600	46,15	3898	14,38	2858	47,19	2712	749,60	3600	72,36	4068	14,93	2896	43,14
27	3124	3124	139,61	0,00	3739	14,12	3188	38,46	3124	1338,95	3600	57,14	4060	14,20	3273	47,07	3124	903,46	3600	71,08	4147	14,81	3320	42,36
28	2672	2672	102,39	0,00	3842	14,07	2693	39,14	2672	1443,95	3600	45,96	3511	14,00	2825	43,74	2672	629,26	3600	76,45	3747	14,82	2693	46,22
29	2516	2516	76,39	0,00	3676	14,09	2516	36,34	2516	1446,95	3600	42,49	3783	13,99	2516	44,57	2516	776,94	3600	69,12	3699	14,93	2633	43,55
30	2940	2940	73,44	0,00	3649	13,99	2940	37,78	2940	1420,02	3600	51,70	4312	14,03	2981	38,59	2940	792,92	3600	73,03	4383	14,78	3007	41,12
31	3256	3256	574,28	0,00	3859	14,14	3287	37,15	3256	1199,51	3600	63,16	4320	14,36	3256	37,54	3256	797,39	3600	75,51	4471	14,99	3300	41,71
32	2400	2400	51,00	0,00	2891	14,01	2400	34,50	2400	1324,56	3600	44,81	3656	14,23	2402	38,69	2400	718,08	3600	70,08	3538	14,78	2458	42,72
33	2936	2936	347,11	0,00	3641	13,89	2936	32,52	2936	1675,58	3600	42,93	3708	14,41	2936	41,20	2936	552,26	3600	81,19	4072	14,92	2936	41,60
34	3684	3684	93,70	0,00	4646	14,08	3770	32,48	3684	1988,62	3600	46,02	4997	14,30	3712	40,96	3684	1067,25	3600	71,03	4945	15,01	3869	44,39
35	2760	2760	72,42	0,00	3724	14,23	2764	32,88	2760	1392,42	3600	49,55	4299	14,37	2768	37,54	2760	796,81	3600	71,13	4239	14,95	2768	44,74
36	3544	3544	262,52	0,00	4375	14,05	3558	33,03	3544	1798,23	3600	49,26	4775	14,06	3596	36,08	3544	833,55	3600	76,48	4674	14,74	3610	43,79
37	3668	3668	450,86	0,00	4408	14,15	3668	33,07	3668	1850,14	3600	49,56	4701	14,11	3712	38,54	3668	907,83	3600	75,25	4733	14,86	3735	46,16
38	3052	3052	98,98	0,00	4109	14,21	3052	38,46	3052	1769,85	3600	42,01	4259	14,25	3162	36,14	3052	993,73	3600	67,44	4435	14,92	3407	40,58
39	3516	3516	116,08	0,00	4497	14,10	3516	33,41	3516	1814,26	3600	48,40	4468	14,33	3607	39,23	3516	719,02	3600	79,55	4595	14,90	3630	43,97
40	3124	3124	141,98	0,00	3987	14,07	3160	31,76	3124	1745,38	3600	44,13	4335	14,39	3160	41,20	3124	602,93	3600	80,70	4315	14,83	3363	41,17
41	3376	3376	124,67	0,00	4413	13,96	3376	34,28	3376	1399,35	3600	58,55	4215	14,26	3376	40,59	3376	708,28	3600	79,02	4527	14,96	3545	46,99
42	2940	2940	101,28	0,00	3832	14,02	2940	34,44	2940	1795,46	3600	38,93	4303	14,02	2940	39,65	2940	422,77						

Table 2.6 Results of class B

Class B Instance	MIP/Micro Period=6				SA		Matheuristic		MIP/Micro Period=8				SA		Matheuristic		MIP/Micro Period=10				SA		Matheuristic	
	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time
1	2237	2237	2060,95	0,00	2863	11,44	2438	33,81	2237	1345,78	3600	39,84	2968	11,60	2508	38,62	2237	602,65	3600	73,06	2991	11,24	2559	41,21
2	2563	1706,44	3600	33,42	2935	11,25	2668	34,27	2563	971,38	3600	62,10	3121	11,58	2668	44,62	2563	775,56	3600	69,74	3035	11,41	2722	40,78
3	2365	2365	2395,76	0,00	2899	11,37	2365	46,77	2365	1106,35	3600	53,22	3027	11,50	2365	44,30	2365	775,48	3600	67,21	3104	11,39	2382	50,33
4	2139	2139	1664,45	0,00	2517	11,20	2242	44,64	2139	1369,60	3600	35,97	2622	11,48	2242	40,26	2197	740,61	3600	66,29	2706	11,33	2242	50,30
5	2527	2527	1979,34	0,00	2948	11,18	2636	36,87	2527	1452,52	3600	42,52	3137	11,56	2664	39,37	2527	874,85	3600	65,38	3247	11,29	2678	40,34
6	2935	2935	2229,69	0,00	3311	11,29	2935	34,85	2935	1282,60	3600	56,30	3659	11,31	3024	42,89	2978	886,55	3600	70,23	3767	11,40	2935	41,41
7	1845	1845	1532,48	0,00	2223	11,30	1966	34,28	1845	1234,31	3600	33,10	2283	11,45	1966	41,70	1845	689,29	3600	62,64	2361	11,42	2028	40,28
8	2469	2469	3010,09	0,00	2954	11,36	2516	34,27	2476	1183,03	3600	52,22	3083	11,56	2516	41,30	2476	785,39	3600	68,28	3168	11,43	2564	50,97
9	2459	2459	1528,02	0,00	2904	11,28	2532	32,81	2459	1324,42	3600	46,14	3047	11,47	2567	44,71	2459	793,03	3600	67,75	3063	11,35	2589	41,33
10	2800	2800	1950,16	0,00	3267	11,34	2901	35,07	2800	1405,60	3600	49,80	3619	11,41	2937	43,82	2800	845,32	3600	69,81	3550	11,30	2999	41,16
11	2226	2226	2558,01	0,00	2747	11,28	2271	32,01	2226	1070,71	3600	51,90	2754	11,35	2271	44,15	2279	930,97	3600	59,15	2885	11,36	2334	45,53
12	2423	2423	866,33	0,00	2816	11,31	2423	39,50	2423	1290,25	3600	46,75	2956	11,32	2423	43,00	2423	966,53	3600	60,11	2828	11,31	2439	56,57
13	3038	3038	1730,31	0,00	3490	11,20	3169	38,11	3038	1214,89	3600	60,01	3551	11,16	3169	44,38	3038	954,54	3600	68,58	3580	11,36	3196	56,62
14	1976	1976	1133,49	0,00	2548	11,18	2070	47,83	1976	1295,86	3600	34,42	2507	11,49	2138	39,85	1976	824,58	3600	58,27	2484	11,36	2243	44,59
15	2189	2189	1200,05	0,00	2624	11,36	2286	38,53	2189	1378,41	3600	37,03	2628	11,51	2317	38,69	2189	646,41	3600	70,47	2629	11,26	2329	40,52
16	2573	2573	2576,83	0,00	2976	11,15	2612	40,37	2573	1017,36	3600	60,46	3125	11,42	2612	43,49	2616	790,82	3600	69,77	3095	11,20	2656	41,98
17	2515	2515	1265,72	0,00	2859	11,39	2515	38,12	2515	1233,36	3600	50,96	2963	11,35	2515	40,77	2515	635,79	3600	74,72	3074	11,23	2528	40,40
18	2715	2715	2725,36	0,00	3128	11,23	2792	37,17	2715	1340,67	3600	50,62	3297	11,25	2792	38,42	2776	1093,19	3600	60,62	3308	11,31	2866	40,32
19	2932	2932	1005,61	0,00	3418	11,13	3027	39,74	2948	1029,15	3600	65,09	3548	11,41	3041	46,20	2932	917,13	3600	68,72	3746	11,42	3027	42,30
20	2612	2612	2079,20	0,00	3150	11,23	2713	40,99	2612	1140,92	3600	56,32	3270	11,39	2713	44,75	2612	699,49	3600	73,22	3237	11,43	2744	41,95
21	2170	2170	1025,28	0,00	2795	11,36	2279	41,15	2170	1412,89	3600	34,89	2628	11,11	2310	37,74	2170	1065,90	3600	50,88	2982	11,38	2367	36,49
22	1907	1907	1629,95	0,00	2371	11,29	1967	45,73	1907	1067,73	3600	44,01	2527	11,40	1967	42,80	1907	776,15	3600	59,30	2448	11,25	2091	51,07
23	1765	1765	897,41	0,00	2273	11,16	1936	37,87	1765	1179,90	3600	33,15	2261	11,32	1936	40,80	1777	617,33	3600	65,26	2472	11,26	1936	49,21
24	2575	2575	3189,47	0,00	3182	11,28	2715	46,61	2575	1116,52	3600	56,64	3257	11,34	2715	49,12	2575	1104,68	3600	57,10	3361	11,32	2791	54,13
25	2292	2168,46	3600	5,39	2608	11,33	2396	38,11	2292	1166,63	3600	49,10	2776	11,24	2448	40,01	2292	880,59	3600	61,58	2730	11,39	2416	36,02
26	2219	2219	1718,09	0,00	2657	11,33	2301	38,08	2219	989,45	3600	55,41	2797	11,36	2355	44,67	2219	565,40	3600	74,52	2749	11,35	2359	36,75
27	2922	2922	2605,22	0,00	3321	11,20	2931	45,26	2922	1161,50	3600	60,25	3278	11,45	2931	40,34	2922	1147,47	3600	60,73	3444	11,40	2955	47,31
28	2046	2046	1111,50	0,00	2471	11,45	2091	37,49	2046	978,60	3600	52,17	2588	11,48	2091	44,18	2046	816,35	3600	60,10	2567	11,42	2091	35,10
29	2060	2060	2177,78	0,00	2571	11,40	2129	38,78	2068	906,40	3600	56,17	2503	11,46	2129	41,04	2068	284,97	3600	86,22	2696	11,39	2186	53,47
30	2413	2413	620,75	0,00	2955	11,49	2549	38,96	2452	1083,54	3600	55,81	2981	11,49	2569	41,52	2413	801,60	3600	66,78	2952	11,35	2589	35,85
31	2044	2044	1815,14	0,00	2290	11,35	2151	38,31	2044	813,31	3600	60,21	2507	11,35	2160	40,50	2062	701,08	3600	66,00	2504	11,30	2191	36,87
32	2055	2055	1337,52	0,00	2553	11,34	2158	37,65	2074	1157,71	3600	44,18	2598	11,41	2158	42,93	2055	851,80	3600	58,55	2454	11,26	2299	37,90
33	2015	2015	505,03	0,00	2477	11,25	2070	37,86	2015	1367,58	3600	32,13	2682	11,38	2070	43,96	2015	734,67	3600	63,54	2698	11,25	2243	37,92
34	1926	1926	675,44	0,00	2227	11,27	1969	38,07	1926	906,38	3600	52,94	2648	11,29	2039	40,61	1926	670,25	3600	65,20	2731	11,26	2058	44,77
35	2016	2016	819,13	0,00	2426	11,36	2138	37,97	2016	937,04	3600	53,52	2519	11,30	2138	48,74	2024	784,10	3600	61,26	2580	11,29	2199	50,46
36	2717	2717	2981,64	0,00	3177	11,20	2766	37,96	2717	1159,89	3600	57,31	3355	11,32	2766	46,07	2717	705,33	3600	74,04	3551	11,32	2864	39,29
37	2050	2050	878,86	0,00	2508	11,29	2168	38,16	2050	1183,06	3600	42,29	2566	11,35	2168	39,65	2050	947,92	3600	53,76	2699	11,38	2168	37,57
38	2540	2540	1999,49	0,00	3012	11,24	2586	37,32	2540	1083,06	3600	57,36	3156	11,28	2586	38,70	2563	718,15	3600	71,98	3353	11,37	2611	37,81
39	2397	2397	3125,33	0,00	2817	11,19	2494	38,18	2423	1082,59	3600	55,32	2917	11,25	2520	40,01	2423	685,95	3600	71,69	2975	11,35	2503	38,83
40	1774	1362,61	3600	23,19	2185	11,30	1853	37,95	1774	380,35	3600	78,56	2171	11,29	1853	44,30	1839	521,72	3600	71,63	2264	11,30	1911	36,23
41	2292	2292	2099,48	0,00	2768	11,15	2352	37,89	2292	1399,50	3600	38,94	2563	11,36	2406	43,35	2292	946,14	3600	58,72	2830	11,39	2529	36,16
42	2621	2621	3050,95	0,00	2982	11,27	2621	38,26	2621	1058,89	3600	59,60	3308	11,39	2621	49,17	2621	821,16	3600	68,				

Table 2.7 Results of class C

Class C Instance	MIP/Micro Period=7				SA		Matheuristic		MIP/Micro Period=8				SA		Matheuristic		MIP/Micro Period=10				SA		Matheuristic	
	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time	Obj	LB	Time	Gap	Obj	Time	Obj	Time
1	2145	2145	1065,98	0,00	2523	8,02	2219	41,61	2145	1788,93	3600	16,60	2636	7,98	2219	33,61	2145	1170,53	3600	45,43	2762	8,06	2219	42,25
2	2181	2181	426,81	0,00	2467	7,94	2222	38,18	2181	1707,72	3600	21,70	2666	7,88	2222	33,22	2181	1003,04	3600	54,01	2544	8,00	2222	41,67
3	1739	1739	295,30	0,00	2039	8,01	1773	42,83	1739	1739	1047,61	0,00	2097	8,00	1773	33,87	1739	1031,75	3600	40,67	2149	8,12	1833	47,14
4	1889	1889	156,75	0,00	2167	7,99	1912	40,33	1889	1889	2457,20	0,00	2484	8,03	1927	32,62	1889	1083,34	3600	42,65	2556	8,02	1942	46,84
5	2363	2363	386,19	0,00	2802	8,01	2450	49,64	2363	2363	1450,44	0,00	3053	8,06	2450	32,79	2363	1099,03	3600	53,49	3054	8,03	2473	41,20
6	2452	2452	467,64	0,00	3047	8,07	2452	45,77	2452	2183,51	3600	10,95	3114	8,10	2452	33,11	2452	995,27	3600	59,41	3019	8,04	2452	41,48
7	2023	2023	420,42	0,00	2313	8,01	2133	38,06	2023	1647,53	3600	18,56	2446	7,89	2133	38,21	2023	999,97	3600	50,57	2577	7,96	2133	45,06
8	2033	2033	458,20	0,00	2217	8,03	2044	33,89	2033	1371,26	3600	32,55	2525	8,02	2044	33,26	2033	926,84	3600	54,41	2683	7,98	2044	41,86
9	2161	2161	638,92	0,00	2447	8,04	2252	32,31	2161	1748,03	3600	19,11	2660	7,88	2252	38,61	2161	1007,46	3600	53,38	2724	7,99	2252	41,74
10	2023	2023	185,58	0,00	2473	8,06	2143	31,95	2023	1567,02	3600	22,54	2620	7,96	2143	41,35	2023	803,94	3600	60,26	2572	7,99	2143	44,97
11	2297	2297	492,94	0,00	2650	7,99	2362	33,24	2297	2249,91	3600	2,05	2742	7,98	2362	32,84	2297	1082,58	3600	52,87	2779	8,00	2362	42,57
12	2077	2077	436,63	0,00	2330	7,98	2093	38,95	2077	1612,38	3600	22,37	2407	8,08	2093	33,09	2077	1014,41	3600	51,16	2457	8,04	2093	49,60
13	2049	2049	922,95	0,00	2339	7,89	2062	38,06	2049	2049	2127,77	0,00	2346	8,01	2062	44,18	2049	1331,85	3600	35,00	2465	8,12	2062	43,86
14	2401	2401	467,64	0,00	2579	7,99	2401	35,13	2401	1483,10	3600	38,23	2702	8,10	2401	42,48	2401	1012,98	3600	57,81	2857	8,13	2401	41,65
15	2178	2178	455,05	0,00	2445	8,00	2178	38,24	2178	1867,20	3600	14,27	2755	8,09	2182	40,26	2182	1085,76	3600	50,24	2579	8,10	2182	43,20
16	1868	1868	334,11	0,00	2270	7,89	1909	45,94	1868	1466,75	3600	21,48	2209	7,99	1909	42,54	1868	884,12	3600	52,67	2252	8,15	1909	46,53
17	1985	1985	321,94	0,00	2138	8,06	2008	41,97	1985	1985	1802,16	0,00	2484	7,90	2008	41,46	1985	883,72	3600	55,48	2272	8,03	2010	45,66
18	2269	2269	422,61	0,00	2582	8,02	2321	34,13	2269	1678,61	3600	26,02	2674	7,89	2321	40,54	2269	874,25	3600	61,47	2818	8,03	2321	42,64
19	2345	2345	2406,05	0,00	2667	8,03	2345	41,32	2345	1962,77	3600	16,30	2748	8,09	2407	38,33	2345	1006,94	3600	57,06	2848	8,09	2407	40,59
20	2025	2025	401,73	0,00	2425	8,22	2115	46,90	2025	2025	947,59	0,00	2438	7,88	2115	37,40	2025	999,95	3600	50,62	2335	8,10	2148	45,79
21	2474	2474	158,70	0,00	2847	8,02	2474	47,65	2474	2146,44	3600	13,24	3006	7,90	2474	37,96	2474	1399,54	3600	43,43	3088	8,09	2474	46,01
22	2155	2155	365,06	0,00	2556	8,11	2155	43,38	2155	1759,13	3600	18,37	2600	7,91	2203	38,15	2155	1165,21	3600	45,93	2789	8,07	2207	43,15
23	2015	2015	171,91	0,00	2264	8,03	2015	47,28	2015	2015	2125,81	0,00	2440	7,93	2016	38,74	2015	1120,14	3600	44,41	2378	8,05	2016	43,60
24	1957	1957	266,67	0,00	2285	8,09	1957	36,09	1957	1503,95	3600	23,15	2520	7,95	1957	37,67	1957	992,00	3600	49,31	2456	8,17	1957	49,10
25	1771	1771	258,94	0,00	2198	8,08	1876	36,40	1771	1279,55	3600	27,75	2149	7,90	1876	38,46	1771	959,35	3600	45,83	2333	8,10	1876	41,36
26	2184	2184	323,92	0,00	2595	8,05	2276	33,64	2184	1627,08	3600	25,50	2805	8,01	2293	38,11	2184	1224,13	3600	43,95	2643	7,93	2293	41,45
27	1568	1568	116,38	0,00	1962	8,11	1653	33,11	1568	1568	1286,42	0,00	2057	8,03	1653	38,36	1568	929,20	3600	40,74	1957	8,11	1653	45,42
28	2398	2398	122,80	0,00	2847	8,03	2477	33,48	2398	2398	1302,09	0,00	2971	8,17	2477	38,46	2398	1370,46	3600	42,85	2852	8,16	2477	44,23
29	2186	2186	626,05	0,00	2511	8,00	2195	38,87	2186	1930,46	3600	11,69	2741	8,15	2195	37,79	2186	1171,70	3600	46,40	2716	8,12	2217	46,55
30	2164	2164	410,33	0,00	2435	7,99	2164	40,36	2164	1402,27	3600	35,20	2546	8,12	2164	37,71	2164	1133,94	3600	47,60	2536	7,98	2164	42,02
31	2067	2067	238,69	0,00	2452	8,01	2100	33,11	2067	2067	2549,03	0,00	2635	8,06	2100	38,72	2067	1071,74	3600	48,15	2783	7,99	2100	45,22
32	2144	2144	321,01	0,00	2500	8,07	2144	33,64	2144	1579,27	3600	26,34	2774	8,09	2144	38,20	2144	1036,62	3600	51,65	2568	8,15	2144	46,00
33	1912	1912	241,71	0,00	2227	8,06	1996	32,68	1912	1912	1341,98	0,00	2389	8,09	1996	38,41	1912	864,99	3600	54,76	2507	8,01	1996	46,26
34	1725	1725	213,08	0,00	2069	8,04	1830	32,75	1725	1725	3543,11	0,00	2072	8,04	1830	37,90	1725	701,56	3600	59,33	2109	8,06	1836	45,73
35	1970	1970	183,39	0,00	2163	8,02	2051	34,94	1970	1401,06	3600	28,88	2276	8,10	2051	39,84	1970	993,67	3600	49,56	2346	8,10	2051	45,50
36	1879	1879	163,16	0,00	2320	8,03	2008	36,29	1879	1879	1305,44	0,00	2468	8,01	2008	38,48	1879	977,46	3600	47,98	2453	8,18	2008	48,45
37	2155	2155	347,47	0,00	2467	8,05	2276	33,23	2155	2155	3113,03	0,00	2742	8,03	2276	37,74	2155	1102,93	3600	48,82	2712	8,08	2283	46,30
38	2001	2001	206,16	0,00	2347	8,01	2021	33,83	2001	1648,42	3600	17,62	2225	8,05	2021	38,21	2001	981,49	3600	50,95	2553	8,01	2021	42,20
39	1957	1957	155,95	0,00	2179	8,06	1979	33,55	1957	1957	2351,20	0,00	2236	8,15	1979	38,78	1957	1157,57	3600	40,85	2145	8,04	1979	47,85
40	2215	2215	430,16	0,00	2442	8,05	2240	33,17	2215	2215	2256,08	0,00	2500	8,01	2240	37,79	2215	1298,88	3600	41,36	2584	8,18	2240	48,05
41	2113	2113	209,14	0,00	2457	8,04	2113	38,75	2113	2113	957,61	0,00	2574	8,12	2113	38,01	2113	1149,68	3600	45,59	2648	8,22	2113	48,65
42	2239	2239	725,19	0,00	2524	8,01	2330	39,07	2239	2239	2568,23	0,00	2548	8,01	2330	38,35	2239	1126,44	3600	49,69	2581	8,01	2345	47,96
43	1956	1956	369,97	0,00	2166	8,00	1956	41,39	1956	1956	2													

CHAPTER THREE

DEVELOPING A MATHEURISTIC FOR DYNAMIC LOT SIZING AND SCHEDULING PROBLEM WITH AN APPLICATION

3.1 Introduction

In today's highly competitive environment, production planning and scheduling continues to be one of the important problems that researchers and businesses try to solve effectively. Production planning; It is the activities of determining the time to be produced by determining the amount and method of the product produced according to these tools and possibilities by the efficient use of production tools, possibilities and workforce (Tatar, 1982). Businesses want the lowest cost production plan that will maximize their profits. In order to achieve this goal, it is necessary to plan the production by taking into account the production factors such as stock quantities, capacity of the machines, setup times, purchasing costs, and update the obtained plans in parallel with the dynamic structure of the production system. Various decisions related to production planning such as capacity planning, ordering, lot sizing and scheduling need to be handled in an integrated and effective manner (Stadtler, Kilger & Meyr, 2015). The present study deals with the complex production planning problem of a firm that manufactures paper bags. Production plans in the factory are created intuitively based on the past knowledge and experience of the employees. The validation and effectiveness of the current production planning approach, which is not based on a scientific basis, is controversial. The production planning problem of the firm is similar to the integrated/capacitated lot sizing and sequence dependent scheduling problem in the scientific literature in its simplest form. The basic problem is an NP-hard structure (Bitran & Yanasse, 1982). In this study, a mixed integer programming model has been developed by considering new constraints and parameters that can meet the production planning needs of the business. The proposed model has a flexible structure that can be applied to other businesses with similar production characteristics and processes.

The following sections of the study are organized as follows: In Section 3.2, there is the production system analysis of the firm and the description of the problem. In Section 3.3, scientific literature review of the related problem. In Section 3.4, a matheuristic method based on a simulated annealing algorithm is introduced to solve the problem. In Section 3.5 presents the data used for testing, computational experiments on instances. Finally, Section 3.6 presents some conclusions and identifies further research lines.

3.2 Current System

The company manufactures paper bags based on demand. The business was established in 1995 on 650 m² and has 16 employees. An average of 10 thousand paper bags are produced annually at the facility for international and local markets such as automotive, cosmetics, jewelry, textile, footwear, and health. Paper rolls delivered from the warehouse to the production unit are first cut to the desired number and dimensions using machines in the sizing section. The rolls that are not the appropriate size or worn are brought to the guillotine machine through pallets, and then cutting and trimming operations in appropriate sizes are performed. The papers produced by the guillotine machine are delivered to the location of the offset machines. Printing operation is performed on two types of offset machines (offset four colors and offset five colors), and the printed papers are covered with one of the matte, glossy, or metallic cellophanes depending on the requirement in the cellophane machine. The print on the cellophane papers is then solidified using a UV-Lacquer machine. Depending on the requirement, gilding, embossing, or gilding and embossing might be performed in the subsequent stage. The papers are then transferred to the glue machine area and the necessary gluing (box gluing, base gluing, side gluing, double gluing) is performed. Then, the carrier transports the glued items to the hole creating, hand hollowing, and rope operations region (rope splicing, rope knitting, rope cutting). The present workflow diagram is depicted in Figure 3.1.

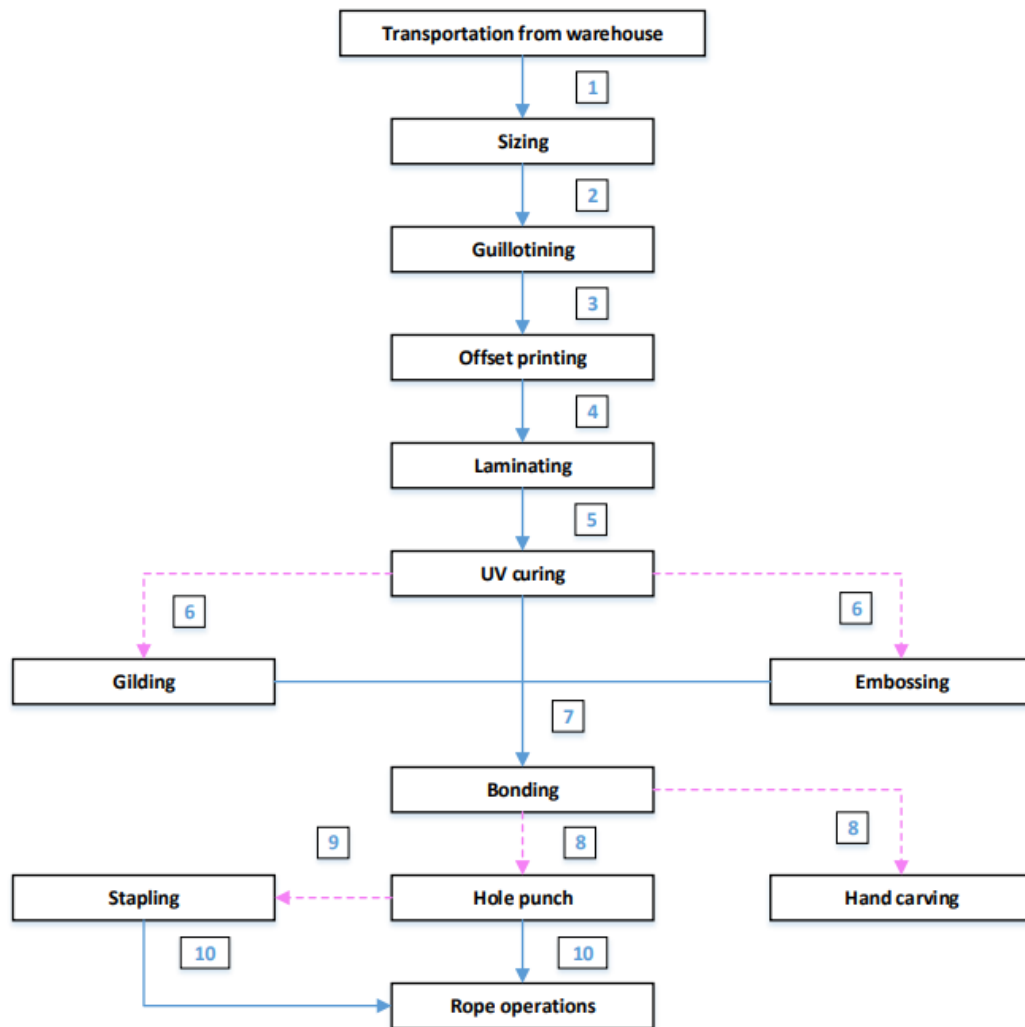


Figure 3.1 Current workflow

According to the demand, hand hollowing, hole drilling and rope operations are carried out in the bags. These operations are performed manually by the operators. Finally, if it is desired to have staples in the holes according to the demand, the bags are stapled with a stapler. Paper rolls delivered from the warehouse to the production unit are first cut to the desired number and dimensions using machines in the sizing section. There may be many cutting plans at this stage and the cutting plans need to be integrated with the remaining production planning decisions. This integration can allow for significant reductions in inventory and material waste. The dimensions of the main material are taken into account when creating cutting plans. Figure 3.2. shows a sample cutting plan for a product. In Figure 3.2 (a), the product is created using a

one-piece cutting plan, whereas Figure 3.2 (b), it is created using a combination of two pieces in the cutting plan.

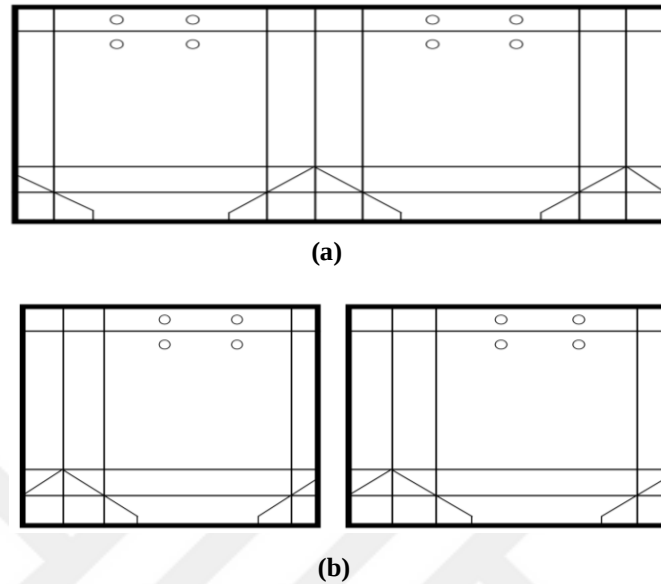


Figure 3.2 Example of cutting pattern

Product types are formed according to the process applied after the cutting process. The first type of product is standard, while the others are produced specifically for the customer. For example, while the second type bag is only gilded, the third type bag is embossed, the fourth type bag is embossed with gilding, the fifth type bag is corded, the sixth type bag is corded and stapled, and the seventh type bag is hand carved. Different product types are shown in Figure 3.3.



Figure 3.3 Product type: Standard, Gilded, Embossed, Gilded + Embossed, Corded, Corded + Stapled, Hand-carved

The following cases were determined as a result of the company's investigations.

- Regardless of the product type, the machine adjustment must be made before each product type to produce the product.
- Setup time varies according to the transitions between products.
- The transition between products is planned only once during the transitions between cutting plans.
- If the produced product changes between periods, setup is required for the product to be produced in the next period. Because the setup process is transferred from the previous period to the next period.
- Cutting plans are predetermined and only one is selected in each period.
- Processing times for each product type vary depending on the selected cutting plan.
- Cutting plans change when products change.
- If the same products are assigned to different cutting plans, the sequence dependent setup process between them is neglected because it is almost nonexistent.

In this study, it is aimed to create a production plan focused on providing solutions to the identified problems. With this plan, it is aimed to determine the production, inventory, amount of products that cannot be met and products purchased, product transitions for each product in each period.

3.3 Literature Review

In the scientific literature, there are many studies directly and indirectly related to the considered problem. In this section, the main studies related to the developed model are mentioned. Carravilla & de Sousa (1995) analyzed an integrated production planning problem with production planning, facility layout and line balancing decisions in a make-to-order company. Zhu & Wilhelm (2006) studied the integrated lot sizing and sequence dependent scheduling problem. In their studies, different machine arrangements were included, and the analysis method was examined.

Neureuther, Polak & Sanders (2004) developed a hierarchical/integrated production planning model based on linear programming for product models in a steel-to-order company. Clark, Neto & Toso (2006) studied the production lot size and scheduling problem in an animal feeding facility. Based on the mathematical model of the asymmetric traveling salesman problem, they propose two different multi-period models for the case where the setup is zero between periods and the setup is maintained from one period to the next. The problem was solved by developing an iterative solution method. Wolsey (2006) worked on two different lot size problems involving a time window. He proposed dynamic programming algorithms for both fixed capacity and non-capacity cases for the production time windows of the order that must be produced specifically for each customer in a certain time interval. Almada-Lobo et al. (2007) studied the multi-product capacitated lot size problem with sequence-dependent setups in a firm in the glass container industry. They developed a two linear mixed integer mathematical programming model containing all the necessary features for this problem. Li, Tao & Wang (2012) modeled the multi-product capacitated dynamic lot size problem by considering single-level and multi-level situations. Unlike many models in the scientific literature, their models include the cost of joint setup. Chen, Mestry, Damodaran & Wang (2009) addressed the problem of short-term capacity planning in a make-to-order environment. They presented a mathematical model to help the operations manager select a set of orders in order to maximize operational profit, provided that all selected orders are completed by the deadline. The proposed mathematical model includes regular working time, overtime, and outsourcing. Ehrenberg & Zimmerman (2012) worked on determining the production plans of companies that produce glass-to-order. They proposed a simulation-based optimization approach for the solution of the problem they dealt with. Gansterer, Almeder & Hartl (2014) proposed simulation-based optimization methods to determine planned delivery time, confidence stock and lot sizes in made-to-order systems. Lee (2013) proposed a nonlinear programming model to enable efficient and fast decision making in a multi-line production facility. Bruno, Genovese & Piccolo (2014) showed that capacitated lot size formulations can be used effectively in practical logistics problems outside the classical production planning area. Ramezani, Saidi-Mehrabad & Teimoury (2013) addressed the problem of

capacitated lot size and scheduling in flow-type production. They proposed an efficient mathematical model for the problem involving sequence-dependent setup. Rangel & Maldonado (2014) proposed three mathematical models for the integrated lot size and scheduling problem, considering the single machine and single level production process, where there are sequence dependent machine setup times and costs. To examine the impact of integrated planning, Gansterer (2015) presented a comprehensive hierarchical production planning model in a make-to-order business. The performance of the model was evaluated with the data obtained from the automotive supplier industry. Li, Guo, Liu, Du & Wang (2017) proposed a mathematical model for make-to-order planning that maximizes total profit with capacity constraint, taking into account four types of cost factors (material cost, transaction cost, delay cost, and plant occupancy cost). Ji, Wang & Hu (2016) examined the best production planning for an assembly line system with n components under uncertain capacity and random demands in a single period. Mula, Díaz-Madroñero, Andres, Poler & Sanchis (2021) examined a multi-machine capacity lot sizing and scheduling model involving a two-part injection molding process and proposed a new mixed integer mathematical model. The proposed model has been successfully applied in problem classes based on the automotive industry. In addition, the model was compared with a single machine.

When the studies in the scientific literature on the subject are examined, it is observed that the researchers focus either on the most basic problems or on business-specific problems. Since the basic problem models cannot meet the practical needs of the enterprises and the enterprises have various constraints and objectives, it is necessary to develop models specific to the business needs. However, it may be useful to examine the handling of some special constraints and situations in the models developed in the scientific literature at the modeling stage. Therefore, In the present study, a MIP model is proposed for the integrated lot sizing and scheduling problem in a business that produces made-to-order paper bags. The proposed model differs from the literature in that it integrates cutting plans and includes outsourcing. In the proposed model, the approaches suggested by Gupta & Magnusson (2005) are used in

the mathematical expression of the product produced at the beginning and end of the period.

3.4 Solution Approach

3.4.1 Mixed-Integer Mathematical Programming Model

A mixed integer mathematical model is developed for the mathematical expression and solution of the present problem. The symbols, objective function and constraints of the model are as follows:

Sets & Indices:

i, j	: Paper bag index	
t	: Period index	
m	: Cutting plan index	
N	: Set of paper bags	$i=1,...,N; j=1,...,N$
M	: Set of cutting plans	$m=1,...,M$
T	: Set of time periods	$t=1,...,T$
F_t	: First cutting plan of the period t	

Parameters:

p_{im}	: Process time per unit of paper bag i in cutting plan m
bs_{ij}	: Sequence dependent setup time required for the transition from the paper bag i to the paper bag j
d_{it}	: Demand of paper bag i in period t
ds_i	: Procurement cost of one unit of paper bag i
g_i	: Backlogging cost of one unit of paper bag i
h_i	: Inventory holding cost of one unit of paper bag i
I_{io}^+	: Initial inventory of paper bag i
I_{io}^-	: Initial backorder of paper bag i
c_t	: Total machine capacity in period t
cp_{im}	: Production cost of one unit paper bag i in m cutting plan

s_{ij} : Sequence dependent setup cost required for the transition from the paper bag i to the paper bag j
 I_i^{max} : Maximum inventory amount of paper bag i
 e_i^{max} : The maximum amount that can be procured from type of paper bag i
 yo_{im} : 1 if setup is made for the paper bag i in the cutting plan m during the initial period, 0 otherwise
 $BIGM$: A large positive number

Decision variables:

I_{it}^+ : Inventory amount of paper bag i at the end of period t
 I_{it}^- : Backorder amount of paper bag i at the end of period t
 x_{itm} : Production amount of paper bag i in cutting plan m in period t
 od_{it} : Procurement amount of paper bag i in period t
 ω_t : Strictly positive when at least one paper bag is produced during time period t , 0 otherwise
 δ_t : 0 if exactly one paper bag is produced during time period t , an unrestricted non-negative number otherwise
 z_{ijtm} : 1 if paper bag i is changed from paper bag j in cutting plan m in period t , 0 otherwise
 y_{itm} : 1 if paper bag i is set up for production in cutting plan m in period t , 0 otherwise
 w_{it} : 1 if paper bag i is produced in period t , 0 otherwise
 a_{it} : 1 if paper bag i is produced first in period t , 0 otherwise
 b_{it} : 1 if paper bag i is produced last in period t , 0 otherwise

Mathematical programming model:

$$\begin{aligned}
 Min Z = & \sum_{i=1}^N \sum_{t=1}^T (h_i I_{it}^+ + g_i I_{it}^- + ds_i od_{it}) + \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N (s_{ij} z_{ijtm}) \\
 & + \sum_{t=1}^T \sum_{i=1}^N \sum_{m=1}^M (cp_{im} x_{itm})
 \end{aligned} \tag{3.1}$$

The objective function (3.1) expresses the minimization of the sum of the holding, backorder, procurement, setup, and production costs.

Constraints:

$$od_{it} + I_{i(t-1)}^+ + I_{it}^- + \sum_{m=1}^M x_{itm} - I_{it}^+ - I_{i(t-1)}^- = d_{it}, \quad \forall i, \forall t > 1 \quad (3.2)$$

$$od_{it} + I_{i0}^+ + I_{it}^- + \sum_{m=1}^M x_{itm} - I_{it}^+ - I_{i0}^- = d_{it}, \quad \forall i, \forall t = 1 \quad (3.3)$$

Constraints (3.2) - (3.3) represent the inventory balance constraint.

$$\sum_{m=1}^M \sum_{i=1}^N p_{im} x_{itm} + \sum_{m=1}^M \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N b_{s_{ij}} z_{ijtm} \leq c_t, \quad \forall t \quad (3.4)$$

Constraint (3.4) ensures that the sequence-dependent setup time and total production of all produced paper bags in each period do not exceed the usable machine capacity of the relevant period.

$$x_{itm} \leq \frac{c_t}{p_{im}} y_{itm}, \quad \forall i, \forall t, \forall m \quad (3.5)$$

Constraint (3.5) refers to the setup of the cutting plan for the production of each period to produce each paper bag type. With constraint (3.8), if a cutting plan is not setup for production, it is ensured that no product is produced.

$$\sum_{i=1}^N y_{itm} = 1, \quad \forall t, \forall m \quad (3.6)$$

Constraint (3.6) assures that a single paper bag is cut from a single cutting plan in each period.

$$z_{ijtm} \geq y_{it(m-1)} + y_{jtm} - 1, \quad \forall m | t > 1, \forall t, \forall j, \forall i | i \neq j \quad (3.7)$$

$$z_{ijtm} \geq y_{oim} + y_{jtm} - 1, \quad \forall m, \forall t, \forall j, \forall i | i \neq j \quad (3.8)$$

When the setup state of the paper bag types differs from the cutting plans, constraints (3.7) - (3.8) force a setup during periods.

$$I_{it}^+ \leq I_i^{max}, \quad \forall i, \forall t \quad (3.9)$$

Constraint (3.9) restricts the number of paper bags that can be stored in each period.

$$od_{it} \leq e^{max}, \quad \forall i, \forall t \quad (3.10)$$

Constraint (3.10) limits the number of bags that can be procured from outside in each period. The constraint sets (3.11) - (3.21) determines which paper bag is produced first and last in the given periods. If the produced paper bag type changes between periods, setup is necessary for the new paper bag type to be produced in the next period. Because the setup state is transferred from the previous period to the next (as described in (Gupta & Magnusson, 2005)). It should be noted that the same paper bag type can be assigned to different cutting plans in the period. In this situation, we assume that setup is negligible if they are produced in consecutive order.

$$w_{it} \leq \omega_t, \quad \forall i, \forall t \quad (3.11)$$

Constraint (3.11), ω_t is set to 1 if any paper bag is produced during the period t .

$$\sum_{i=1}^N w_{it} - 1 \leq (J - 1)\delta_t, \quad \forall t \quad (3.12)$$

Constraint (3.12) ensures that if more than one bag type is produced in the given period, then the variable δ_t takes a positive value.

$$\omega_t \leq \sum_{i=1}^N a_{it} , \quad \forall t \quad (3.13)$$

$$\sum_{i=1}^N a_{it} \leq 1 , \quad \forall t \quad (3.14)$$

Constraints (3.13) - (3.14) provides an assignment to the a_{it} variable if there is production in the given period and if exactly one bag type has been produced.

$$\omega_t \leq \sum_{i=1}^N b_{it} , \quad \forall t \quad (3.15)$$

$$\sum_{i=1}^N b_{it} \leq 1 , \quad \forall t \quad (3.16)$$

Constraints (3.15) - (3.16) provides an assignment to the b_{it} variable if there is production in the given period and if exactly one bag type has been produced.

$$a_{it} + b_{it} \leq 2 - \delta_t , \quad \forall t, \forall i \quad (3.17)$$

Constraint (3.17) ensures that if there is production in the given period and exactly one bag type has been produced, then a_{it} and b_{it} variables are assigned one. This constraint ensures that the variable δ_t takes the value of zero.

$$\sum_{m=1}^M \sum_{\substack{j=1 \\ j \neq i}}^N z_{jitm} \geq w_{it} - a_{it} , \quad \forall t, \forall i \quad (3.18)$$

$$\sum_{m=1}^M \sum_{\substack{j=1 \\ j \neq i}}^N z_{ijtm} \geq w_{it} - b_{it} , \quad \forall t, \forall i \quad (3.19)$$

Constraints (3.18) - (3.19) applies when more than one bag type is produced in a period. Thanks to these constraints, the value of at least one z_{ijtm} variable is guaranteed to be one if the bag type produced is first or last in the given period.

$$a_{it} \leq w_{it}, \quad \forall t, \forall i \quad (3.20)$$

$$b_{it} \leq w_{it}, \quad \forall t, \forall i \quad (3.21)$$

Constraints (3.20) - (3.21) expresses the condition of being first or last in that period if the type i bag is produced in period t .

$$\sum_{m=1}^M x_{itm} \leq BIGM w_{it}, \quad \forall t, \forall i \quad (3.22)$$

Constraint (3.22) ensures that the w_{it} variable takes a value if the paper bag i is created in the cutting plan m in period t ($x_{itm} > 0$).

$$z_{jitF_t} \geq y_{j(t-1)m} + a_{it} - 1, \quad \forall j, \forall i, \forall t, \forall m \quad (3.23)$$

$$z_{ij(t+1)F_t} \geq y_{j(t+1)F_t} + b_{it} - 1, \quad \forall t, \forall i, \forall j \quad (3.24)$$

Constraints (3.23) - (3.24) ensures that the variable z_{ijtm} takes a value in case different products are produced during the transition from the period $t-1$ to the period t with no idle period.

$$0 \leq \omega_t \leq 1, \quad \forall t \quad (3.25)$$

Constraint (3.25) provides the limitation of the value range of the variable ω_t .

$$\omega_t, \delta_t \in .R, \quad \forall t \quad (3.26)$$

$$od_{it}, x_{itm}, I_{jt}^+, I_{jt}^- \in .\mathbb{N}, \quad \forall i, t, m \quad (3.27)$$

$$z_{ijtm}, y_{itm}, w_{it}, a_{it}, b_{it} \in \{0,1\}, \quad \forall j, i, t, m \quad (3.28)$$

3.4.2 Illustrative Example

This section presents an illustrative example of an optimal solution obtained by the MIP model. The illustrative example consists of seven products, five periods and seven

cutting plans. The data of the sample problem resulting from the developed mixed integer mathematical programming model are as follows:

$$h_i=[12, 16, 13, 13, 17, 11, 19];$$

$$g_i=[10, 14, 18, 15, 14, 19, 16];$$

$$ds_i=[16, 12, 11, 15, 11, 13, 16];$$

$$I_i^{max}=[232, 232, 216, 226, 230, 228, 222];$$

$$e_i^{max}=[58, 58, 54, 57, 58, 57, 56];$$

$$d_{it}=[[52, 99, 86, 61, 116]$$

$$, [60, 72, 116, 103, 47]$$

$$, [76, 88, 74, 81, 108]$$

$$, [83, 91, 42, 85, 113]$$

$$, [68, 50, 115, 95, 40]$$

$$, [102, 80, 114, 47, 82]$$

$$, [111, 65, 69, 110, 92]];$$

$$cp_{im}=[[25, 27, 13, 47, 35, 20, 42]$$

$$, [36, 35, 40, 25, 33, 36, 16]$$

$$, [41, 44, 11, 46, 46, 22, 35]$$

$$, [33, 21, 27, 20, 18, 43, 38]$$

$$, [35, 26, 11, 47, 10, 46, 13]$$

$$, [21, 32, 50, 46, 46, 48, 31]$$

$$, [44, 28, 31, 26, 19, 26, 46]];$$

$$c_t=[1104, 1090, 1232, 1164, 1196];$$

$$s_{ij}=[[10000, 381, 337, 323, 251, 264, 340]$$

$$, [381, 10000, 116, 209, 165, 248, 330]$$

$$, [337, 116, 10000, 230, 367, 249, 214]$$

$$, [323, 209, 230, 10000, 332, 309, 231]$$

$$, [251, 165, 367, 332, 10000, 229, 235]$$

$$, [264, 248, 249, 309, 229, 10000, 168]$$

$$, [340, 330, 214, 231, 235, 168, 10000]];$$

When the mathematical programming model developed using these data is solved, the production plan in Figure 3.4 is obtained. The production plan can be used to determine how many and which products will be produced in each cutting plan. The total cost was found to be 45333. The proportions of all other costs that affect the total cost are given in Figure 3.5. The cost item that has the greatest effect on the total cost is the sequence dependent setup cost. This is followed by the procurement cost.

Production Plan	Period 1	Period 2	Period 3	Period 4	Period 5
Cutting plan 1	$x_{611} = 68$		$x_{631} = 57$		
Cutting plan 2		$x_{722} = 9$			
Cutting plan 3	$x_{313} = 22$	$x_{323} = 54$	$x_{133} = 127$	$x_{543} = 37$	$x_{353} = 54$
Cutting plan 4	$x_{414} = 26$				
Cutting plan 5	$x_{715} = 55$	$x_{425} = 34$	$x_{535} = 117$	$x_{745} = 67$	
Cutting plan 6				$x_{346} = 27$	
Cutting plan 7		$x_{227} = 16$	$x_{237} = 58$	$x_{247} = 45$	



Type 1 Type 2 Type 3 Type 4 Type 5 Type 6 Type 7

Figure 3.4 Production plan

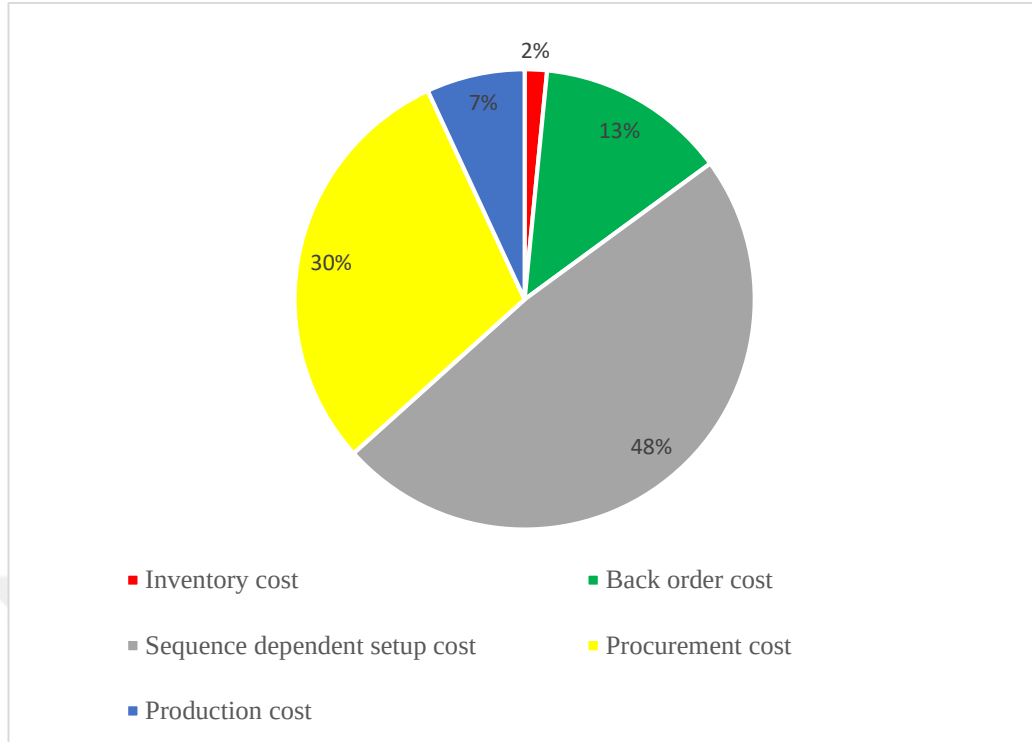


Figure 3.5 Rates of all costs affecting the total cost

3.4.3 Proposed Matheuristic Algorithm

Matheuristic methods are optimization methods that combine metaheuristic and mathematical programming methods (Boschetti, Maniezzo, Roffilli & Bolufé Röhlér, 2009). This class of methods is expressed as the use of metaheuristic algorithms and mathematical programming techniques together to provide higher performance in solving NP-hard problems (Della, Grosso & Salassa, 2013). Koch, Arbaoui, Ouazene, Yalaoui, De Brunier, Jaunet & De Wulf (2022) proposed lot sizing and scheduling problem in the tire industry that takes customer priority into account and employs a matheuristic algorithm to solve large-scale problems. Cunha, Kramer & Melo (2019) addressed the multi-part lot size issue in the reproduction condition. A matheuristic algorithm based on the column generation method was used to solve the problem. Carvalho & Nascimento (2022) proposed a matheuristic method to solve the sequence dependent lot sizing and scheduling problem involving non-triangular setup on parallel machines. This method is specially designed for problems involving non-triangular setup. Therefore, it performed better in most of the sample problems tested. The

problem is divided into two sub-decision problems for the matheuristic approach: (i) The decision to assign products to the cutting plan, (ii) Integrated lot sizing and scheduling decision. In the considered problem, the decision to assign the products to the cutting plan is obtained by using the simulated annealing algorithm. The obtained decision is provided as an input to the second subproblem and the sub-model derived from the main model is solved to provide the lot sizing and scheduling decision. The sub-model is as stated below:

$$\begin{aligned}
 \text{Min } Z = & \sum_{i=1}^N \sum_{t=1}^T (h_i I_{it}^+ + g_i I_{it}^- + ds_i od_{it}) + \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N (s_{ij} \overline{y_{itm-1}} * \overline{y_{jtm}}) \\
 & + \sum_{t=1}^T \sum_{i=1}^N \sum_{m=1}^M (cp_{im} x_{itm})
 \end{aligned} \tag{3.29}$$

Subject to; Constraint sets in Equations (3.2) - (3.4) and (3.9) - (3.10).

$$x_{itm} \leq \frac{c_t}{p_{im}} \overline{y_{itm}}, \quad \forall i, \forall t, \forall m \tag{3.30}$$

$\overline{y_{itm}}$; It shows the value of the decision variable indicating the assignment of the i paper bag type to the m cutting plan in the t period which is determined by the SA algorithm.

In the proposed SA algorithm, permutation coding approach is used in the candidate solution expression for assignment to the cutting plan. The array length of the candidate solution is $M*T$. The solution representation example for 3 bag types, 3 cutting plans and 3 periods is given in Figure 3.6 (a). Here each paper bag is randomly assigned to each cutting plan. Since the number of cutting plans in each period is known, the sample distribution is given in Figure 3.6 (b) according to 3 cutting plans and in Figure 3.6 (c) according to 2 cutting plans.

In Figure 3.6 (b), it is assumed that in Period 1, the 2nd Bag type is assigned to the 1st Cutting plan, the 3rd Bag type to the 2nd Cutting plan, and the 1st Bag type to the

3rd Cutting plan ($y_{211} = 1$, $y_{312} = 1$ and $y_{113} = 1$). From this solution representation, the information that each period starts and ends with the production of which bag type is obtained directly. As a result, the constraints in the main model were removed in the sub model, and the size of the sub model was reduced. The initial solution is obtained randomly. A neighborhood solution is generated by selecting two different operators at random (swap or inversion). The swap operator selects two different types of paper bags at random from the solution and swaps their positions. The inversion operator selects a two paper bag type. Then new solution is obtained by reversed the solution display from back to front places in the same place. The flow chart of the matheuristic algorithm proposed in this study is given in Figure 3.7.

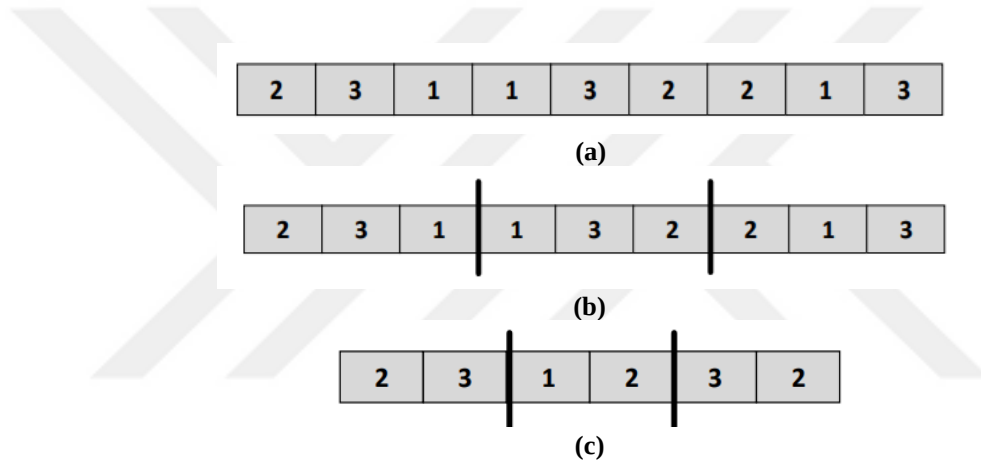


Figure 3.6 (a) An example illustration for 3 bag types, 3 periods (b) Solution representation of assigned paper bag types according to 3 cutting plans (c) Solution representation of assigned paper bag types according to 2 cutting plans

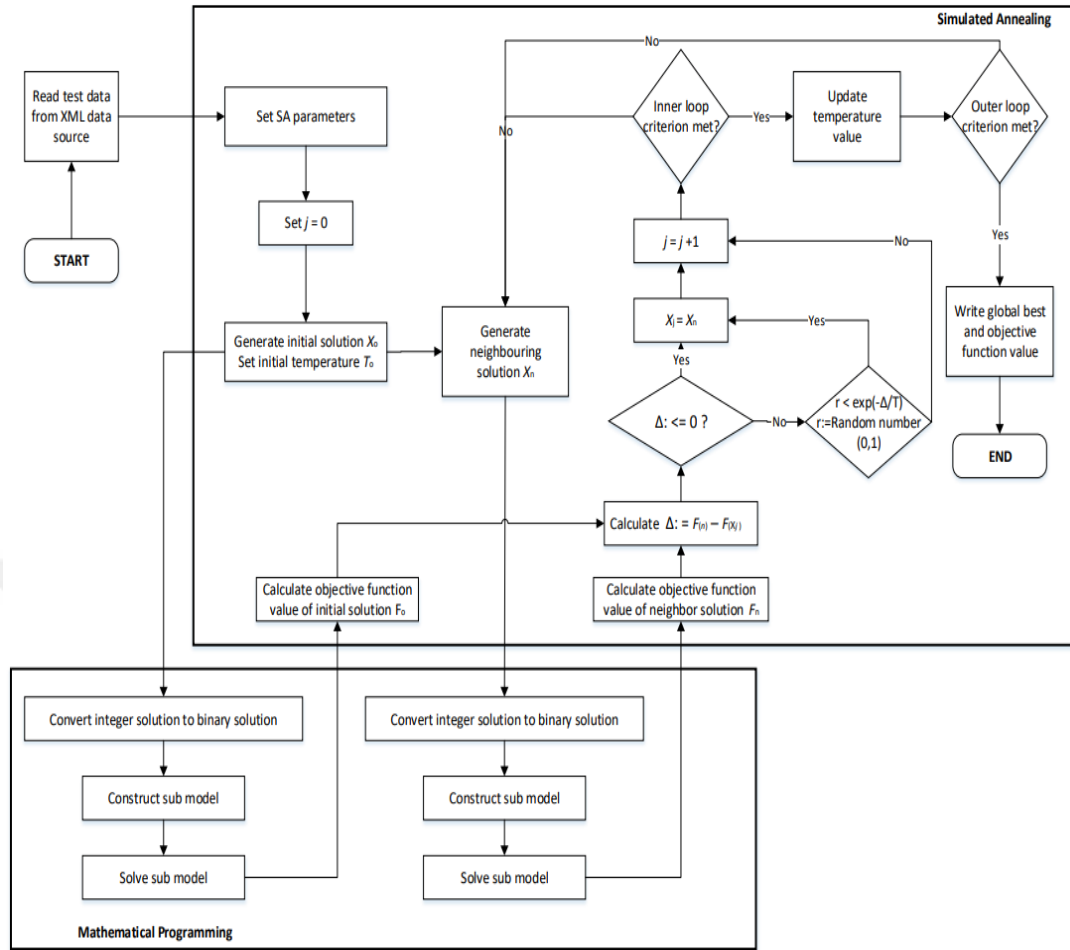


Figure 3.7 Flow chart of the matheuristic algorithm

Kirkpatrick et al. (1983) developed SA method to efficiently address optimization problems. SA has been applied to many complex optimization problems (Tsang & Wiese, 2015; Xing & Wang, 2018). The pseudocode of the proposed SA algorithm is given in Figure 3.8. Before describing the SA algorithm, the following abbreviations are used:

$F_{(n)(o)(c)}$: Objective function value of the (neighbor) (initial) (current) solution

$X_{(n)(o)(c)}$: Solution of (neighbor) (initial) (current)

X_{best} : Solution with minimum fitness value

F_{best} : Minimum fitness value

I : Iteration number

T_0 : Initial temperature

T_f : Final temperature

T : Temperature value

q : Cooling rate

$MaxI_{iter}(T)$: Maximum number of iterations at each temperature

```
1. Initialization: Specify SA parameters
2. Build an initial solution and current solution
3.  $X_{best} := X_{(o)}$ ;  $F_{best} := F_{(o)}$ ;  $X_{(c)} := X_{(o)}$ ;  $F_{(c)} := F_{(o)}$ ;
4.  $T := T_0$ ;
5. While ( $T > T_f$ )
6.      $I := 0$ ;
7.     While ( $I < MaxI_{iter}(T)$ )
8.         Generate neighboring solutions by selecting a random number between (1,2) and
            determine the neighboring solution  $X_{(n)}$  with the objective function value  $F_{(n)}$ 
            If 1 is taken apply Swap
            If 2 is taken apply Inversion
9.          $\Delta := F_{(n)} - F_{(c)}$ ;
10.        If ( $\Delta < 0$ )
11.             $X_{(c)} := X_{(n)}$ ;
12.             $F_{(c)} := F_{(n)}$ ;
13.        Else
14.             $r_1 := \text{random}(0,1)$ ;
15.            If ( $r_1 < \exp(-\Delta/T)$ );
16.                 $X_{(c)} := X_{(n)}$ ;
17.                 $F_{(c)} := F_{(n)}$ ;
18.            End if
19.        End if
20.        If ( $F_{(c)} < F_{best}$ )
21.             $X_{best} := X_{(c)}$ ;  $F_{best} := F_{(c)}$ ;
22.        End if
23.         $I := I + 1$ ;
24.    End While
25.     $T := qT$ ;
26. End While
27. Report  $X_{best}$ ;  $F_{best}$ 
```

Figure 3.8 Pseudo code of proposed simulated annealing algorithm

3.5 Computational Studies

In this part, problem classes were created in order to make comparisons in mathematical model and matheuristic method. From the provided and predicted data, lower and upper limits for each parameter were determined, and a data generator that followed a uniform distribution within the observed limits was created. Each problem class is defined by P-T-M. Here P is the paper bag type, T is the period and M is the cutting plan. Each problem class created contains 10 separate problem instances. A total of 70 problems were solved and comparisons were made. Problem classes are created using following data: d_{it} (uniform (0;40,120)), s_{ij} (uniform (100,400)), cp_{im} (uniform (10,50)), ds_i (uniform(10,25)), h_i and g_i (uniform (10,20)), I_i^{max} (uniform (0, $\max\{d_{it}\} * 2$)), e_i^{max} (uniform (0, $\max\{d_{it}\}/2$)), c_t ($= \sum_{i=1}^I \sum_{t=1}^T d_{it} * 2$), bs_{ij} ($= \lfloor s_{ij}/10 \rfloor$), and p_{im} ($= \lfloor cp_{im}/10 \rfloor$). The studies were collected in two parts. In the first part, three lower bounds will be calculated for the problem discussed in the article. In the second part, the performances of the proposed mathematical model and the matheuristic algorithm will be tested.

3.5.1 Lower Bound for Proposed Mathematical Model

Three lower bound values for the proposed mathematical model are calculated in this subsection. It will be computed using the Ramezani et al. (2013) method for the capacitated lot sizing and scheduling problem. Basically, the first lower bound is calculated by relaxing all binary variables in the proposed model. The second lower bound is calculated by relaxing the decision variables z_{ijtm} and y_{itm} . Finally, the third lower bound is calculated by relaxing the decision variable z_{ijtm} only. All class of problems are considered in order to evaluate and compare the performance of the three lower bounds obtained to the mathematical model given. The results are shown in Table 3.1. The values in the gap column were calculated by dividing the difference between the mathematical model's objective value and the lower bounds' objective value by the mathematical model's objective value. Table 3.1 shows that the third lower bound has an advantage over the other two lower bounds.

Table 3.1 Computation times and comparison results of lower bounds

	Problem No	Lower Bound 1			Lower Bound 2			Lower Bound 3		
		Result	Time	Gap(%)	Result	Time	Gap(%)	Result	Time	Gap (%)
(4P-3T-4M)	1	13531	0,01	15,16	14669,11	0,19	8,02	15949	0,08	0,00
	2	13398	0,00	9,69	14476,14	0,05	2,43	14836	0,23	0,00
	3	12360	0,00	10,94	13389,32	0,06	3,53	13879	0,08	0,00
	4	13838	0,00	8,03	14906,53	0,08	0,93	15046	0,08	0,00
	5	16430	0,01	15,22	17562,02	0,14	9,38	19380	0,09	0,00
	6	15202	0,02	7,27	16093,57	0,05	1,83	16393	0,11	0,00
	7	13158	0,02	16,65	14752,54	0,08	6,55	15787	0,14	0,00
	8	14093	0,02	20,77	16282,16	0,13	8,46	17787	0,09	0,00
	9	14730	0,00	7,92	15736,07	0,06	1,63	15997	0,14	0,00
	10	15173	0,00	13,40	16360,31	0,06	6,62	17521	0,08	0,00
(5P-4T-6M)	1	22780	0,02	14,15	24877,35	0,19	6,25	26536	0,55	0,00
	2	23644	0,02	9,57	25677,04	0,22	1,79	26145	0,33	0,00
	3	18764	0,02	14,37	21162,73	0,20	3,42	21913	2,17	0,00
	4	23285	0,02	20,31	25864,86	0,17	11,48	29220	0,47	0,00
	5	26349	0,02	11,68	28625,86	0,39	4,05	29833	0,38	0,00
	6	30444	0,02	8,96	32651,32	0,17	2,36	33441	0,33	0,00
	7	16845	0,03	19,43	19480,11	0,31	6,82	20907	0,38	0,00
	8	19780	0,02	12,00	21630,27	0,22	3,76	22476	0,45	0,00
	9	28606	0,02	11,56	30340,45	0,27	6,20	32346	0,42	0,00
	10	20427	0,02	11,37	22397	0,20	2,82	23047	0,58	0,00
(7P-5T-7M)	1	35513	0,03	17,79	40696,35	0,69	5,79	43199	2,34	0,00
	2	36472	0,03	18,86	41453,44	1,89	7,78	44950	11,38	0,00
	3	37184	0,03	18,31	41412,32	0,78	9,03	45521	2,05	0,00
	4	39031	0,03	15,78	43340,05	1,28	6,48	46345	15,91	0,00
	5	41522	0,03	14,02	46304,34	2,70	4,12	48293	2,91	0,00
	6	41958	0,03	11,31	45754,73	0,55	3,28	47308	2,80	0,00
	7	33577	0,02	13,22	37079,19	0,56	4,17	38692	28,52	0,00
	8	38356	0,01	17,91	43679,09	0,64	6,51	46723	23,52	0,00
	9	34041	0,03	17,00	38841,26	0,77	5,30	41015	19,81	0,00
	10	33320	0,03	17,33	37931,72	0,73	5,89	40304	33,64	0,00
(9P-6T-8M)	1	48362	0,06	17,53	55048,92	1,84	6,13	58641	69,88	0,00
	2	52427	0,06	21,10	59660,04	0,97	10,22	66451	118,59	0,00
	3	57602	0,06	22,28	65471,87	7,98	11,66	74116	81,61	0,00
	4	56446	0,05	17,20	64023,65	8,72	6,08	68171	43,61	0,00
	5	56476	0,05	14,60	62707,97	2,20	5,18	66131	68,55	0,00
	6	63776	0,08	15,29	70213,71	2,92	6,74	75291	83,19	0,00
	7	49238	0,06	20,80	56721,84	3,75	8,76	62166	63,19	0,00
	8	45674	0,06	21,72	52857,01	1,67	9,41	58349	69,91	0,00
	9	55450	0,06	19,76	61978,26	2,47	10,32	69108	129,27	0,00
	10	51395	0,06	21,68	58003,28	2,69	11,61	65620	260,19	0,00
(10P-6T-10M)	1	65600	0,09	16,95	72621,61	2,41	8,06	78988	186,08	0,00
	2	61162	0,09	17,91	69639,59	15,45	6,53	74502	3600	0,00
	3	64150	0,11	16,88	71548,25	4,44	7,29	77177	553,52	0,00
	4	62368	0,09	18,23	70365,3	5,84	7,74	76269	3593,67	0,00
	5	59613	0,08	19,96	67075,92	2,83	9,94	74476	3600	0,00
	6	57523	0,08	17,61	65550,73	3,63	6,11	69820	80,17	0,00
	7	56535	0,09	24,95	64449,35	2,89	14,45	75334	1799,69	0,00
	8	53850	0,09	22,33	60928,7	3,23	12,12	69332	1294,09	0,00
	9	60538	0,08	15,80	67505,36	12,56	6,10	71894	3421,12	0,00
	10	64254	0,09	17,80	72304,78	6,05	7,50	78165	3600	0,00
(12P-8T-8M)	1	87511	0,16	25,45	99149,38	20,53	15,53	117383	331,86	0,00
	2	98237	0,14	22,56	110494,8	29,81	12,90	126860	879,09	0,00
	3	100436	0,16	16,29	112275,4	25,06	6,42	119980	3600	0,00
	4	94498	0,14	19,36	106271,6	42,39	9,31	117179	3600	0,00
	5	93908	0,16	15,09	103850,2	18,55	6,10	110594	383,77	0,00
	6	96482	0,16	20,41	109255,1	31,03	9,87	121220	843,84	0,00
	7	92097	0,16	20,64	103566,7	31,34	10,76	116052	805,08	0,00
	8	98808	0,16	17,39	110132,1	27,21	7,92	119607	3600	0,00
	9	94368	0,14	27,08	107662,1	28,69	16,81	129415	3600	0,00
	10	110821	0,14	17,31	122199,2	27,19	8,82	134022	320,77	0,00
(15P-10T-7M)	1	164064	0,27	20,31	182504,6	42,77	11,36	205884	3600	0,00
	2	153662	0,25	24,60	172284,5	34,30	15,47	203809	3600	0,00
	3	184775	0,30	17,45	202456,6	115,58	9,55	223839	3600	0,00
	4	148790	0,27	22,42	165853,4	54,77	13,52	191784	3600	0,00
	5	173226	0,23	21,47	193506,3	35,14	12,28	220593	3600	0,00
	6	156361	0,27	21,40	173711,7	32,36	12,67	198922	3600	0,00
	7	177893	0,25	20,21	196431	180,42	11,89	222945	3600	0,00
	8	144718	0,27	21,53	162820,3	61,36	11,71	184422	3600	0,00
	9	168542	0,28	20,35	187510,5	44,06	11,38	211593	3600	0,00
	10	165366	0,27	21,66	183416,2	41,48	13,11	211088	3600	0,00

3.5.2 Performance Analysis of the Proposed Mathematical Model and Matheuristic Algorithm

The computer used in the current study has an Intel i7 Core 2.4 GHz processor and 16 GB RAM and has Windows 10 operating system. ILOG CPLEX Optimization Studio Version 12.10 program was used to solve the proposed mathematical model. The proposed heuristic algorithm is coded in Microsoft Visual Studio 2019 C++ by integrating CPLEX's MIP solver. Model run time is limited to 3600 CPU seconds. Since the simulated annealing algorithm contains randomness, it was run 20 times. The parameters of SA are taken as follows: initial temperature (T_0) = 100, the final temperature (T_f) = 1, the cooling rate (q) = 0.985 and the total number of iterations ($MaxI_{iter}(T)$) = 10.

Table 3.3 provides the best and average results as well as the times required to achieve the best and average results. As shown in Table 3.3, the mathematical model's solution times increase as the problem size grows. Some instances in the 15P-10T-7M size problem class could not be solved in 3600 CPU seconds, resulting in a difference (gap) of less than 0.28%. The results found with the mathematical model are compared with the results found by the matheuristic algorithm and the deviation (result found with the math model - the best result found by the matheuristic algorithm / Result with the matheuristic x 100%) graph is shown in Figure 3.9.

The matheuristic algorithm was able to optimally solve 40 problems out of 70 problem instances. As it can be seen in Table 3.3, the solution times and the optimum found by the mathematical model or the deviation from the best value found in the specified time period is less than 2%, which shows that the proposed algorithm can always obtain high quality solutions and the results are quite satisfactory. As a result, the proposed matheuristic method is effective in terms of solution quality and solution time of the integrated lot sizing and scheduling problem in a paper bag manufacturing industry.

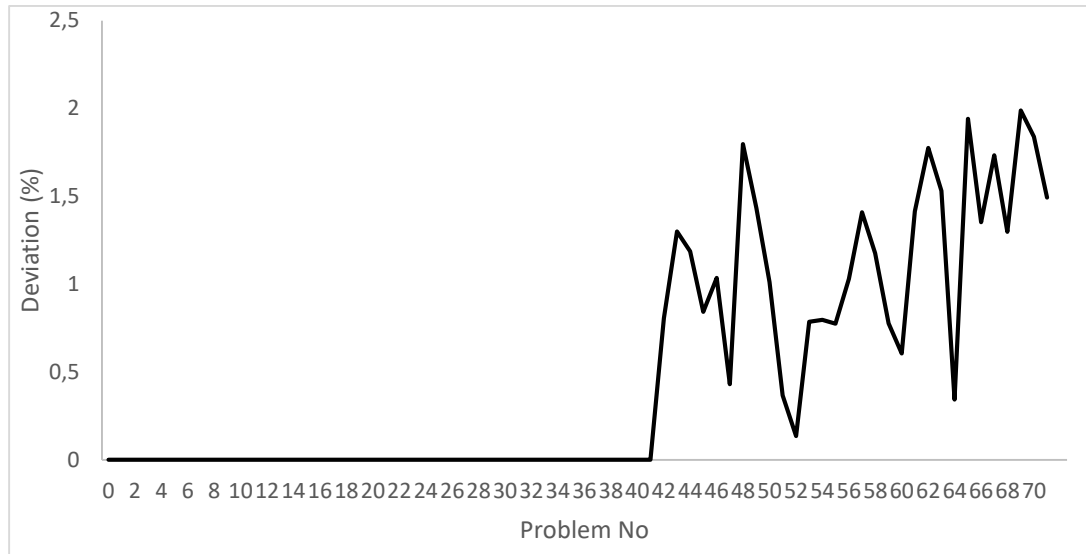


Figure 3.9 Deviation graph of computation results of matheuristic and mathematical model

A nonparametric Friedman test implemented by Derrac, García, Molina & Herrera (2011) is considered in this work to find the significance of algorithm by comparing mathematical model results optimal in 70 problem instances.

It is observed from the Friedman ANOVA Table 3.2 that the value of probability $p < 0.05$ indicates that the optimal response values obtained in matheuristic algorithm differ from each other, and there is a significant effect of an algorithm in obtaining the optimal parameter values. Hence, it is accepted that the results obtained using the matheuristic algorithm is near-optimal.

Table 3.2 Friedman's ANOVA table

Friedman's ANOVA Table					
Source	SS	df	MS	Chi-sq	Prob>Chi-sq
Columns	1.21	1	1.21	11	0.0009
Error	4.29	49	0.08755		
Total	5.5	99			

Table 3.3 Results of problem instances

	Problem No	MIP Solution			Proposed SA-based Matheuristic approach			
		Best	Time	Gap	Best	Time (Best)	Average	Time (Average)
(4P-3T-4M)	1	15949	1,11	0,00	15949	40,16	15949	40,68
	2	14836	1,45	0,00	14846	41,35	14846	41,73
	3	13879	1,64	0,00	13879	39,87	13879	40,07
	4	15046	1,43	0,00	15046	36,37	15046	36,99
	5	19380	1,87	0,00	19380	35,12	19380	35,52
	6	16393	1,09	0,00	16393	36,99	16393	37,10
	7	15787	1,37	0,00	15787	37,12	15787	37,59
	8	17787	1,28	0,00	17787	38,85	17787	39,07
	9	15997	1,01	0,00	15997	39,83	15997	40,01
	10	17521	0,87	0,00	17521	40,10	17521	40,36
(5P-4T-6M)	1	26536	3,56	0,00	26536	42,12	26536	42,65
	2	26145	4,84	0,00	26145	42,17	26145	42,57
	3	21913	3,85	0,00	21913	43,74	21913	43,90
	4	29220	3,09	0,00	29220	41,12	29220	41,67
	5	29833	4,01	0,00	29833	40,16	29833	40,75
	6	33441	5,47	0,00	33441	39,19	33441	39,64
	7	20907	4,86	0,00	20907	41,83	20907	42,07
	8	22476	3,12	0,00	22476	40,89	22476	41,00
	9	32346	5,84	0,00	32346	42,90	32346	43,21
	10	23047	5,21	0,00	23047	41,27	23047	41,67
(7P-5T-7M)	1	43199	15,24	0,00	43199	51,13	43199	51,45
	2	44950	15,78	0,00	44950	50,45	44950	50,96
	3	45521	14,78	0,00	45521	49,02	45521	49,47
	4	46345	16,21	0,00	46345	49,57	46345	49,89
	5	48293	16,09	0,00	48293	52,71	48293	53,02
	6	47308	15,55	0,00	47308	49,03	47308	49,43
	7	38692	14,01	0,00	38692	49,33	38692	49,75
	8	46723	12,11	0,00	46723	48,17	46723	48,47
	9	41015	12,54	0,00	41015	49,99	41015	50,12
	10	40304	13,67	0,00	40304	50,40	40304	50,81
(9P-6T-8M)	1	58641	35,73	0,00	58641	43,76	58641	43,96
	2	66451	36,35	0,00	66451	42,36	66451	43,63
	3	74116	43,45	0,00	74116	42,01	74116	42,34
	4	68171	34,22	0,00	68171	44,25	68171	44,66
	5	66131	45,31	0,00	66131	45,86	66131	46,00
	6	75291	48,78	0,00	75291	41,56	75291	41,86
	7	62166	51,18	0,00	62166	42,70	62166	42,85
	8	58349	52,22	0,00	58349	41,12	58349	41,33
	9	69108	44,25	0,00	69108	43,18	69108	43,53
	10	65620	38,01	0,00	65620	42,10	65620	42,39
(10P-6T-10M)	1	78988	331,56	0,00	79625	86,66	81654	88,86
	2	74502	532,89	0,00	75470	94,15	76431	96,54
	3	77177	206,73	0,00	78009	88,91	79356	90,97
	4	76269	413,99	0,00	76912	96,25	78236	98,44
	5	74476	304,47	0,00	75247	92,10	76532	94,59
	6	69820	71,16	0,00	70120	90,57	71354	92,56
	7	75334	373,41	0,00	76687	87,52	78334	88,97
	8	69332	364,84	0,00	70325	89,99	71831	91,58
	9	71894	265,34	0,00	72621	83,73	74157	85,66
	10	78165	614,07	0,00	78451	94,51	79920	96,85
(12P-8T-8M)	1	117383	278,83	0,00	117541	109,90	119782	112,86
	2	126860	462,80	0,00	127856	106,27	130454	108,54
	3	119980	797,59	0,00	120936	108,79	122146	112,00
	4	117179	641,22	0,00	118086	110,90	120104	113,88
	5	110594	285,34	0,00	111733	113,72	113541	115,55
	6	121220	439,03	0,00	122928	104,56	125033	107,86
	7	116052	367,00	0,00	117416	109,04	119339	111,36
	8	119607	563,95	0,00	120536	108,10	122321	110,25
	9	129415	448,84	0,00	130199	107,35	131824	109,69
	10	134022	297,41	0,00	135918	106,73	137547	109,00
(15P-10T-7M)	1	205884	3600,48	0,22	209537	147,31	212569	150,54
	2	203809	3361,84	0,00	206929	146,79	209785	152,00
	3	223839	629,27	0,00	224605	144,82	227034	150,53
	4	191784	1538,81	0,00	195507	145,06	197941	149,16
	5	220593	1074,63	0,00	223578	146,22	226035	149,33
	6	198922	3601,38	0,15	202369	147,01	204964	150,31
	7	222945	3600,78	0,18	225839	145,94	228135	148,89
	8	184422	3607,64	0,28	188089	146,83	190357	150,00
	9	211593	1015,06	0,00	215482	147,59	217835	149,86
	10	211088	728,86	0,00	214238	149,28	216885	152,55

3.6 Conclusion

In this study, the integrated lot sizing and scheduling problem in a company that produces made-to-order paper bags is investigated. A MIP model is proposed to solve the problem. The developed model aims to minimize the sum of the holding, backorder, procurement, setup, and production costs. The proposed model has been tested and verified with test data and it has been seen that it can create realistic and feasible optimal production plans that can meet all constraints. In future studies, a production system where a part of the planned lot size is defective can also be included in the problem, and a heuristic solution approach can be presented by developing heuristic methods in this problem.

CHAPTER FOUR

A HYBRID SIMULATED ANNEALING ALGORITHM AND MATHEMATICAL PROGRAMMING-BASED SOLUTION APPROACH FOR PAPER BAG PRODUCTION PLANNING

4.1 Introduction

Environmental and energy concerns are growing and putting pressures on industrial systems all around the world. This is also the case in the paper bag industry, which transforms raw materials into specialized customer products. In this transformation process, there are several possible cutting plans to be cut from the raw materials in order to obtain different sets of paper bags from each plan. Before starting the production process, a cutting equipment need to be configured. Although technological advancements in new cutting machines and equipment enable higher performance, nevertheless, defective items are inevitable in many production environments. It is also essential for manufacturers to recognize manufacturing quality issues and prevent the delivery of defective items to customers. Manufacturers generally employ expert personnel to conduct quality inspections that compare one or more product qualities to product requirements. To avoid releasing low-quality items, products that do not satisfy the criteria are rejected or returned for improvement. Additionally, some bags must satisfy extra requirements concerning their use. Production procedures generally ensures high level of product quality, yet defective items may be produced occasionally. Each paper bag is visually inspected in order to prevent any defective paper bags from being supplied to a client. In many situations, the paper bag production process consumes costly raw materials, in addition to requiring use of energy, water and resultant CO₂ emissions (Darghouth & Abdel-Aal, 2021). Therefore it is critical for manufacturers to rework defective items wherever possible rather than disposing them.

This paper aims to develop a model for analyzing the capacitated lot sizing model with cutting plan assignment, sequence-dependent setups, external procurements, defective items, and backlogging. This study offers numerous advances and contributions, chiefly:

1. Existing models are extended to include new parameters such as external procurements, setup times, defective items and cutting plan assignment in order to better reflect the real life paper bag production environment. To the best of our knowledge, first time in the related literature all of these parameters are taken into account simultaneously.
2. A new MIP model for the capacitated lot sizing and scheduling problem is devised. The MIP model can assign cutting plans and scheduling of products according to the inventory balancing constraints to meet demands and capacity constraints. The assignment of cutting plan configurations can change the total cost and time. Noteworthy inventory and waste material reductions may be possible due to the flexibility of these configurations' selection and their interaction with scheduling and lot sizing decisions (Boas, Camargo & Morabito, 2021).
3. A problem-based matheuristic approach that incorporates simulated annealing and MIP is proposed to solve the problem efficiently. Some managerial insights are also provided after a comprehensive computational study.

The structure of this paper is as follows. In Section 4.2 briefly provides a brief literature review of the considered problem. The problem description for this study is presented in Section 4.3. In Section 4.4 describes the mathematical model formulation for the concerned problem. In Section 4.5, a matheuristic method based on a simulated annealing algorithm is introduced to solve the problem. In Section 4.6 presents the data used for testing, computational experiments on instances, and sensitivity analysis. Finally, Section 4.7 presents some conclusions and identifies further research lines.

4.2 Literature Review

Over the past several decades, lot sizing issues have received extensive research in the literature. In Section 4.2.1, we briefly review the integrated problem of lot sizing and scheduling. In Section 4.2.2, we review lot sizing and scheduling problems with process configuration/ cutting pattern selection. In Section 4.2.3, we review lot sizing and scheduling problems with rework/ remanufacturing.

4.2.1 Lot Sizing and Scheduling

Synchronized lot sizing and scheduling problem has attracted many researcher's attention in the literature due to its relevance to real life problems. The scientific literature on lot sizing and scheduling models is rather comprehensive. A detailed overview of the formerly published models can be found in (Drexel & Kimms, 1997; Zhu & Wilhelm, 2006; Copil et al., 2017; Wörbelauer, Meyr & Almada-Lobo, 2019). Some examples of lot sizing and scheduling studies in different industries are those of foundries (de Araujo, Arenales & Clark, 2008; Furtado, Camargo & Toledo, 2019), yogurt production (Kopanos, Puigjaner & Georgiadis, 2010), spinning (Camargo, Toledo & Almada-Lobo, 2014), food (Claassen, Gerdessen, Hendrix & van der Vorst, 2016), and beverages (Toscano, Ferreira & Morabito, 2019, 2020), among others.

There are two types of formulations (big and small time buckets) for simultaneous lot sizing and scheduling problems according to the characteristics of the problem. Big bucket models allow for multiple setup operations in a single period, whereas small bucket models only allow one setup for each period. The discrete lot sizing and scheduling problem (DLSP), the proportional lot sizing and scheduling problem (PLSP) and the continuous setup lot sizing problem (CSLP) used micro periods in the respective mathematical formulations. Whereas the general lot sizing and scheduling problem (GLSP) and the capacitated lot sizing problem with sequence-dependent setups (CLSD) are the most studied models that use macro periods in the related mathematical formulations as outlined in Copil et al. (2017). In GLSP models (Fleischmann & Meyr, 1997) planning horizon is divided into sub-periods, which enable production a single item only. In CLSD models (Haase, 1996), the sequence of lots is described by numbering the products manufactured during the period. These models are based on the tour of a traveling salesman problem (TSP). The GLSP is extremely similar to the CLSD, but it is more flexible because it eliminates the CLSD's constraints.

Gupta & Magnusson (2005) investigated a single-machine CLSD model by adding new variables and developing a heuristic solution combining row aggregations and

linear relaxations for a sandpaper firm. Almeder & Almada-Lobo (2011) introduce novel constraints to Gupta & Magnusson's (2005) formulation to avoid disconnected subtours. Mula et al. (2021) addressed the bi-part injection molding multi-machine CLSD.

4.2.2 Process Configuration / Cutting Pattern Selection

Many industries, including the paper, textile, glass, and steel industries, confront cutting problems. Integrated lot sizing and cutting stock problems were recently reviewed in Melega, de Araujo & Jans (2018). There are some studies that integrates process configuration selection into the lot sizing and/or scheduling model formulation in different industries. Some examples of this studies are those of chemical (Sahinidis & Grossmann, 1992), paper (Poltroniere, Poldi, Toledo & Arenales, 2008), lumber (Gaudreault, Frayret, Rousseau & D'Amours, 2011), oil refineries (Persson, Göthe-Lundgren, Lundgren & Gendron, 2004), and pulp packaging (Martínez, Morabito & Toso, 2018; Martinez, Adulyasak, Jans, Morabito & Toso, 2019), among others. The production process may be affected by changing cutting patterns, e.g., switching between different positions of the knives (Wuttke & Heese, 2018).

Before choosing the cutting plans for production, the set of feasible patterns by which products are produced can be either predetermined or developed simultaneously with lot sizing and scheduling decisions. In this study, expert planners consider possible cutting plans by considering machine capacity, paper dimensions and sequence-dependent setup and changeover times. Before cutting an object, knives are put in their assigned positions according to the cutting configuration, which might differ from the previous one. When the cutting machine is set up for a particular configuration, the cutting status must last for some time, corresponding to this configuration. Therefore, the cutting process time is configuration-dependent. The setup with the sequence of the cutting plans, which consists of the removal/insertion process of knives in different positions of the cutting machine, is neglected. Since, the manufacturing software can easily adjust the changeover from one cutting configuration to another in a short time. However, to process a specific type of product,

a setup is needed, which also depends on the previously processed products. Therefore, if the same product type is assigned to different cutting plans in consecutive orders, there is no setup count. As a result, the proposed model allow to produce paper bag type more than once in a period. It can be said that there is no need to add sub-tour elimination restrictions.

4.2.3 Rework / Remanufacturing

Concerns about the environment, society and the economy drive the search for more ecologically friendly products and industrial processes. Even though there is a considerable amount of studies in the literature on lot sizing with rework alternatives in a static manufacturing environment, studies on the dynamic lot sizing with rework is quite limited. This problem is often considered in remanufacturing. Remanufacturing can be considered as the recovery of defective products since it is a kind of rework (Goerler & Voß, 2016).

Teunter, Bayindir & Den Heuvel (2006) investigated the joint and separate setups of the single-item economic lot sizing problem with remanufacturing. The authors developed a dynamic programming approach for the solution of the problem. Schulz (2011) developed a generalization of the Silver Meal heuristic by applying methods elaborated for the relevant static problem and this method outperformed Teunter et al's. (2006) work. Sahling (2013) introduced and modeled the multi-item capacitated lot sizing problem with remanufacturing as a MIP formulation. They employed column-generation and truncated branch-bound approaches in order to find both tight dual bounds and good solutions. Mehdizadeh & Kivi (2014) developed a MIP formulation for the multi-item capacitated lot sizing problem in closed-loop supply chains with setup times, safety stock, and demand shortages. Due to difficulties in solving their MIP model, they employed three metaheuristic procedures, namely simulated annealing, vibration damping optimization and harmony search algorithms for finding near optimal solutions.

Goerler & Voß (2016) proposed a MIP model for the multi-item capacitated lot sizing problem, which required the rework of defective items and the satisfaction of minimum lot size criteria. A scenario-based, two-stage stochastic programming technique was developed by Macedo, Alem, Santos, Junior & Moreno (2016) for a multi-item lot sizing problem with remanufacturing, backloging, and stochastic demand. The multi-item lot sizing problem with remanufacturing has an uncapacitated variant, which was studied in Cunha, Konstantaras, Melo & Sifaleras (2017). Cunha et al. (2019) suggested effective matheuristics based on MIP for the multi-item capacitated lot sizing problem with remanufacturing. Goerler et al. (2020) evaluated GLSP under rework and lifetime constraints for defective items. For the solution of the problem, a metaheuristic approach that is known as late acceptance hill-climbing is utilized.

4.3 Problem Description

In the paper bag industry, the production process starts with carrying paper rolls from the warehouse to the production area in the factory. Firstly, these paper rolls are being cut according to the customer's order size and amount using a sizing machine. Demands are supposed to be deterministic and dynamic. After this operation, papers that are not being cut according to the desired size and/or are damaged are trimmed by a guillotine machine to get the same type of crisp, clean, and consistent cut. Next, papers are conveyed via conveyer to the offset printing machine area. Offset printing involves the transfer of an inked image from a metal plate to a rubber blanket, which is then transferred to the printing surface. Then, laminating operations can be done using a laminating machine. The major function of such a machine is to beautify or protect printed materials. Then, an UV curing machine is used to cure or dry inks. At the next stage, gilding, embossing, or a combined gilding and embossing process can be done according to demand. Then, the papers are transported to the area of the bonding machines where the necessary bonding operation is done. After this step, optional works such as cording, hand carving, and stapling can have performed. Finally, all products are checked according to the quality standards, and if there are existing planned lot sizes that are not serviceable, they are stored in a separate rework

inventory until they are reworked and then shipped to consumers, as shown in Figure 4.1. Backlogging is allowed with a penalty since deadlines can be renegotiated with customers. We consider this process as a single production line environment. The manufacturing section of the system processes manufacturing items, while the remanufacturing section processes remanufactured products. This study will not distinguish between the quality of high-grade and reworked products. Furthermore, we assume that the reworking process is always successful.

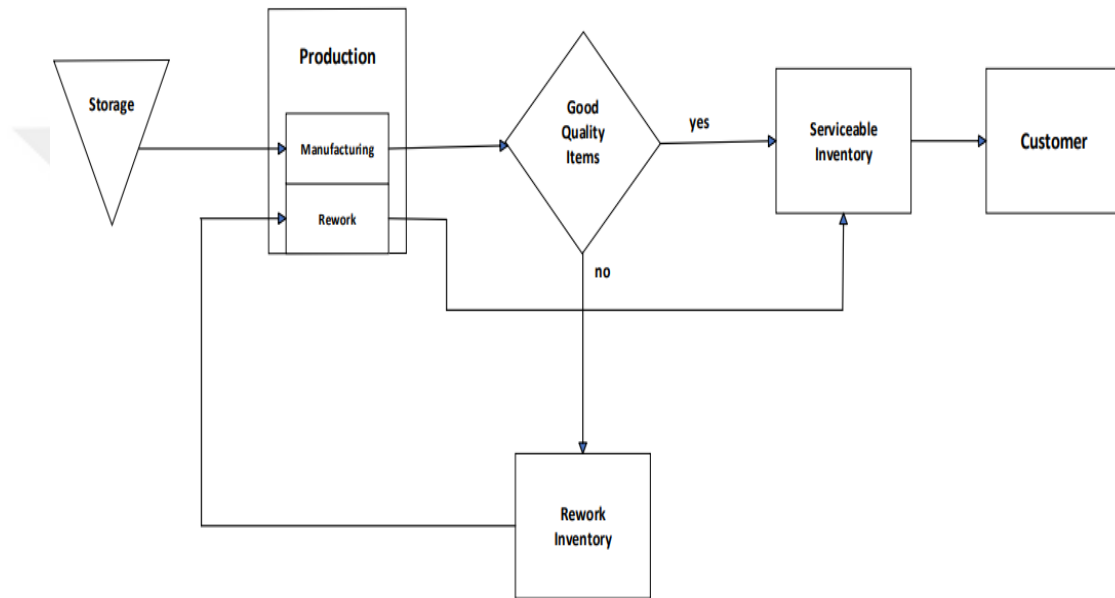


Figure 4.1 Representation of the defective production process (see (Inderfurth, Lindner & Rachaniotis, 2005; Goerler & Voß, 2016))

4.4 Problem Formulation

The mathematical model (MM) is described in line with the terminology used in the paper bag manufacturing company where the case study is performed.

4.4.1 Mixed-Integer Mathematical Programming Model

In order to formally define the problem, the following symbols, objective and constraint functions are defined:

Sets & Indices:

I, J	: Set of paper bags	$i=1, \dots, I ; j=1, \dots, J$
T	: Set of time periods	$t=1, \dots, T$
M	: Set of cutting plans	$m=1, \dots, M$
F_t	: First cutting plan of the period t	
L_t	: Last cutting plan of the period t	

Parameters:

d_{it}	: Demand of paper bag i in period t
p_{im}	: Process time per unit of paper bag i in cutting plan m
tp_{im}^R	: Process time for reworking a defective item per unit of paper bag i in cutting plan m
I_{i0}^+	: Initial inventory of paper bag i
I_{i0}^-	: Initial backorder of paper bag i
c_t	: Total machine capacity in period t
ds_i	: Procurement cost of one unit of paper bag i
g_i	: Backlogging cost of one unit of paper bag i
h_i	: Inventory holding cost of one unit of paper bag i
h_i^R	: Rework inventory holding cost of one unit of paper bag i
cp_{im}	: Production cost of one unit paper bag i in m cutting plan
bs_{ij}	: Sequence dependent setup time required for the transition from the paper bag i to the paper bag j
s_{ij}	: Sequence dependent setup cost required for the transition from the paper bag i to the paper bag j
I_i^{max}	: Maximum inventory amount of paper bag i
e_i^{max}	: The maximum amount that can be procured from type of paper bag i
Θ_{it}	: Proportion of defective items $0 \leq \Theta_{it} < 1$ of paper bag i in t
yo_{im}	: 1 if setup is made for the paper bag i in the cutting plan m during the initial period, 0 otherwise
$BIGM$	: A large positive number

Decision variables:

I_{it}^+ : Inventory amount of paper bag i at the end of period t

I_{it}^- : Backorder amount of paper bag i at the end of period t

x_{itm} : Production amount of paper bag i in cutting plan m in period t

od_{it} : Procurement amount of paper bag i in period t

ω_t : Strictly positive when at least one paper bag is produced during time period t , 0 otherwise

δ_t : 0 if exactly one paper bag is produced during time period t , an unrestricted non-negative number otherwise

R_{itm} : Number of defective items of paper bag i produced in cutting plan m in period t

s_{it}^R : Rework inventory amount of paper bag i in period t

x_{itm}^R : Quantity of rework items of paper bag i reworked in cutting plan m in period t

x_{itm}^S : Production amount of good-quality items of product i in cutting plan m in period t

y_{itm} : 1 if paper bag i is set up for production and/or rework in cutting plan m in period t , 0 otherwise

z_{ijtm} : 1 if paper bag i is changed from paper bag j in cutting plan m in period t , 0 otherwise

w_{it} : 1 if paper bag i is produced in period t , 0 otherwise

a_{it} : 1 if paper bag i is produced first in period t , 0 otherwise

b_{it} : 1 if paper bag i is produced last in period t , 0 otherwise

The problem is formulated as follows:

The objective function (4.1) expresses the minimization of the sum of the holding, backorder, procurement, setup, production, and rework inventory holding costs.

$$\begin{aligned}
 [\mathbf{MM}] \text{ Min } Z = & \sum_{i=1}^I \sum_{t=1}^T (h_i I_{it}^+ + g_i I_{it}^- + d s_i od_{it}) + \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (s_{ij} z_{ijtm}) \\
 & + \sum_{t=1}^T \sum_{i=1}^I \sum_{m=1}^M (c p_{im} (x_{itm}^R + x_{itm}^S)) + \sum_{t=1}^T \sum_{i=1}^I (h_i^R s_{it}^R) \quad (4.1)
 \end{aligned}$$

Constraints (4.2) - (4.3) represent the defective production process (as described in (Goerler & Voß, 2016)).

$$x_{itm}^S = \lfloor [x_{itm} \cdot (1 - \theta_{it})] \rfloor, \quad \forall i, t, m \quad (4.2)$$

$$R_{itm} = \lceil (x_{itm} \cdot \theta_{it}) \rceil, \quad \forall i, t, m \quad (4.3)$$

Constraints (4.4) - (4.5) represents the inventory balance constraints.

$$od_{it} + I_{i(t-1)}^+ + I_{it}^- + \sum_{m=1}^M x_{itm}^R + \sum_{m=1}^M x_{itm}^S - I_{it}^+ - I_{i(t-1)}^- = d_{it}, \quad \forall i, t > 1 \quad (4.4)$$

$$od_{it} + I_{i0}^+ + I_{it}^- + \sum_{m=1}^M x_{itm}^R + \sum_{m=1}^M x_{itm}^S - I_{it}^+ - I_{i0}^- = d_{it}, \quad \forall i, t = 1 \quad (4.5)$$

The rework inventory balancing constraint for serviceable items is shown in constraint (4.6).

$$s_{i(t-1)}^R + \sum_{m=1}^M R_{itm} - \sum_{m=1}^M x_{itm}^R = s_{it}^R, \quad \forall i, t \quad (4.6)$$

Constraint (4.7) ensures that the sequence-dependent setup time and total production (good quality and rework) time of all produced paper bags in each period do not exceed the usable machine capacity of the relevant period.

$$\sum_{m=1}^M \sum_{i=1}^I p_{im} x_{itm} + \sum_{m=1}^M \sum_{i=1}^I t p_{im}^R x_{itm}^R + \sum_{m=1}^M \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J b s_{ij} z_{ijtm} \leq c_t, \quad \forall t \quad (4.7)$$

Constraint (4.8) refers to the preparation of the cutting plan for this production or rework of each period to produce or rework each paper bag type. With constraint (4.8), if a cutting plan is not setup for production or rework, it is ensured that no product is produced or reworked. In this constraint, *BIGM* symbolizes a large number that does not restrict production volume.

$$x_{itm} + x_{itm}^R \leq BIGM y_{itm}, \quad \forall i, t, m \quad (4.8)$$

Constraint (4.9) assures that a single paper bag is cut from a single cutting plan in each period.

$$\sum_{i=1}^I y_{itm} = 1, \quad \forall m, t \quad (4.9)$$

When the setup state of the paper bag types differs from the cutting plans, constraints (4.10) - (4.11) force a setup during periods.

$$z_{ijtm} \geq y_{it(m-1)} + y_{jtm} - 1, \quad \forall m \mid m > 1, t, j, i \mid i \neq j \quad (4.10)$$

$$z_{ijtm} \geq y_{oim} + y_{jtm} - 1, \quad \forall m \mid m = 1, t \mid t = 1, j, i \mid i \neq j \quad (4.11)$$

Constraint (4.12) restricts the number of paper bags that can be stored in each period.

$$I_{it}^+ \leq I_i^{max}, \quad \forall i, t \quad (4.12)$$

Constraint (4.13) limits the number of bags that can be procured from outside in each period.

$$od_{it} \leq e_i^{max}, \quad \forall i, t \quad (4.13)$$

The constraint set (4.14) - (4.24) determines which paper bag is produced first and last in the given periods. If the produced paper bag type changes between periods, setup is necessary for the new paper bag type to be produced in the next period. Because the setup state is transferred from the previous period to the next (as described in (Gupta & Magnusson, 2005)). It should be noted that the same paper bag type can be assigned to different cutting plans in the period. In this situation, we assume that setup is negligible if they are produced in consecutive order. An example for the scheduling of paper bags is provided in Figure 4.2.

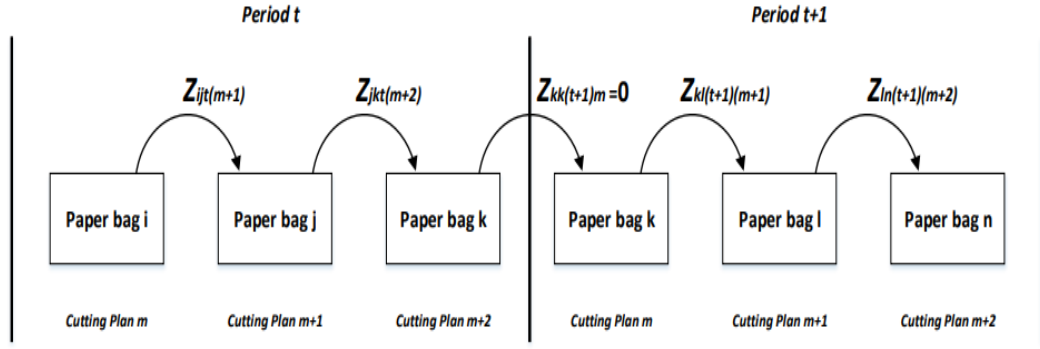


Figure 4.2 An example for scheduling of paper bag types during the cutting process

Constraint (4.14), ω_t is set to 1 if any paper bag is produced during the period t .

$$w_{it} \leq \omega_t, \quad \forall i, t \quad (4.14)$$

Constraint (4.15) - (4.24) expresses the condition of being first or last in that period if the paper bag i is produced in period t .

$$\sum_{i=1}^I w_{it} - 1 \leq (J - 1)\delta_t, \quad \forall t \quad (4.15)$$

$$\omega_t \leq \sum_{i=1}^I a_{it}, \quad \forall t \quad (4.16)$$

$$\sum_{i=1}^I a_{it} \leq 1, \quad \forall t \quad (4.17)$$

$$\omega_t \leq \sum_{i=1}^I b_{it}, \quad \forall t \quad (4.18)$$

$$\sum_{i=1}^I b_{it} \leq 1, \quad \forall t \quad (4.19)$$

$$a_{it} + b_{it} \leq 2 - \delta_t, \quad \forall t, i \quad (4.20)$$

$$\sum_{m=1}^M \sum_{\substack{j=1 \\ j \neq i}}^J z_{jitm} \geq w_{it} - a_{it}, \quad \forall t, i \quad (4.21)$$

$$\sum_{m=1}^M \sum_{\substack{j=1 \\ j \neq i}}^J z_{ijtm} \geq w_{it} - b_{it}, \quad \forall t, i \quad (4.22)$$

$$a_{it} \leq w_{it}, \quad \forall t, i \quad (4.23)$$

$$b_{it} \leq w_{it}, \quad \forall t, i \quad (4.24)$$

Constraint (4.25) ensures that the w_{it} variable takes a value if the paper bag i is created in the cutting plan m in period t ($x_{itm} > 0$).

$$\sum_{m=1}^M x_{itm} \leq BIGM w_{it}, \quad \forall t, i \quad (4.25)$$

Constraint (4.26) - (4.27) ensures that the variable z_{ijtm} takes a value in case different products are produced during the transition from the $t-1$ period to the t period with no idle period.

$$z_{jitF_t} \geq y_{j(t-1)m} + a_{it} - 1, \quad \forall t, i, j \quad (4.26)$$

$$z_{ij(t+1)F_t} \geq y_{j(t+1)F_t} + b_{it} - 1, \quad \forall t, i, j \quad (4.27)$$

Constraint (4.28) provides the limitation of the value range of the variable ω_t .

$$0 \leq \omega_t \leq 1, \quad \forall t, m \quad (4.28)$$

$$\omega_t, \delta_t \in .R, \quad \forall t \quad (4.29)$$

$$od_{it}, x_{itm}, R_{im}, s_{im}^R, x_{im}^R, x_{im}^S, I_{jt}^+, I_{jt}^- \in .\mathbb{N}, \quad \forall i, t, m \quad (4.30)$$

$$z_{ijtm}, y_{itm}, w_{it}, a_{it}, b_{it} \in \{0,1\}, \quad \forall j, i, t, m \quad (4.31)$$

4.5 A Matheuristic Based Approach for Solving the MIP Model

The proposed MIP model (equations 4.1 - 4.31) is difficult to solve for larger size problems. Therefore, a solution approach that is based on model decomposition and hybridization of simulated annealing and mathematical programming is considered for solving the proposed model in line with the literature. Several recent contributions

have successfully addressed the hybridization of mathematical programming and heuristic algorithms (Cunha et al., 2019; Goerler et al., 2020; Carvalho & Nascimento, 2022; Koch et al., 2022) for handling integrated lot sizing and/or scheduling problems. These approaches are usually named as matheuristic approaches in the related literature (Baykasoglu, Subulan & Hamzadayi, 2022).

The aim of this section is to devise a matheuristic based solution approach for solving the proposed model effectively, particularly for larger-size problems. To do so, the proposed model (equations 4.1 - 4.31) is decomposed into two sub-problems, as cutting plan assignment and lot sizing sub-problems. The goal is to solve both problems concurrently. The SA is developed to generate cutting plan-paper bag assignments as an input to the LSSP (Lot Sizing Sub Problem) model that is optimally solved by using an integer programming solver. LSSP feedbacks the objective function value back to the SA algorithm and the solution process iterates in this manner until convergence. This approach reduces the solution effort considerably as the problematic sequencing constraints are automatically satisfied by the SA solution encoding procedure. The flowchart of the matheuristic based solution approach is shown in Figure 4.3. The LSSP formulation is explained below.

4.5.1 LSSP Model

This subsection describes the mathematical model used to solve the LSSP subproblem. This subproblem determines when and how much to produce or rework, as well as how much inventory to keep for both rework and good quality products. The subproblem can be formulated as follows:

$$\begin{aligned}
 \text{[LSSP] Min } Z = & \sum_{i=1}^I \sum_{t=1}^T (h_i I_{it}^+ + g_i I_{it}^- + ds_i od_{it}) + \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^I \sum_{\substack{j=1 \\ j \neq i}}^J (s_{ij} \overline{y_{itm-1}} * \overline{y_{jtm}}) \\
 & + \sum_{t=1}^T \sum_{i=1}^I \sum_{m=1}^M (cp_{im}(x_{itm}^R + x_{itm}^S)) + \sum_{t=1}^T \sum_{i=1}^I (h_i^R s_{it}^R) \quad (4.32)
 \end{aligned}$$

subject to

$$x_{itm}^S = \lfloor [x_{itm} \cdot (1 - \theta_{it})] \rfloor, \quad \forall i, t, m \quad (4.33)$$

$$R_{itm} = \lfloor (x_{itm} \cdot \theta_{it}) \rfloor, \quad \forall i, t, m \quad (4.34)$$

$$od_{it} + I_{i(t-1)}^+ + I_{it}^- + \sum_{m=1}^M x_{itm}^R + \sum_{m=1}^M x_{itm}^S - I_{it}^+ - I_{i(t-1)}^- = d_{it}, \forall i, t | t > 1 \quad (4.35)$$

$$od_{it} + I_{i0}^+ + I_{it}^- + \sum_{m=1}^M x_{itm}^R + \sum_{m=1}^M x_{itm}^S - I_{it}^+ - I_{i0}^- = d_{it}, \quad \forall i, t | t = 1 \quad (4.36)$$

$$s_{i(t-1)}^R + \sum_{m=1}^M R_{itm} - \sum_{m=1}^M x_{itm}^R = s_{it}^R, \quad \forall i, t \quad (4.37)$$

$$\sum_{m=1}^M \sum_{i=1}^I p_{im} x_{itm} + \sum_{m=1}^M \sum_{i=1}^I t p_{im}^R x_{itm}^R + \sum_{m=1}^M \sum_{i=1}^I \sum_{j=1}^J b s_{ij} \overline{y_{itm-1}} * \overline{y_{jtm}} \leq c_t, \forall t \quad (4.38)$$

$$x_{itm} + x_{itm}^R \leq BIGM \overline{y_{itm}}, \quad \forall i, t, m \quad (4.39)$$

$$I_{it}^+ \leq I_i^{max}, \quad \forall i, t \quad (4.40)$$

$$od_{it} \leq e^{max}, \quad \forall i, t \quad (4.41)$$

$$od_{it}, x_{itm}, R_{im}, s_{im}^R, x_{im}^R, x_{im}^S, I_{jt}^+, I_{jt}^- \in \mathbb{N}, \quad \forall i, t, m \quad (4.42)$$

Where binary variables $\overline{y_{itm}}$ and $\overline{y_{jtm}}$ are determined and feed into the model by the SA algorithm in the LSSP model. In other words, cutting plans of paper bags are inputs for LSSP models which are gathered from the SA algorithm.

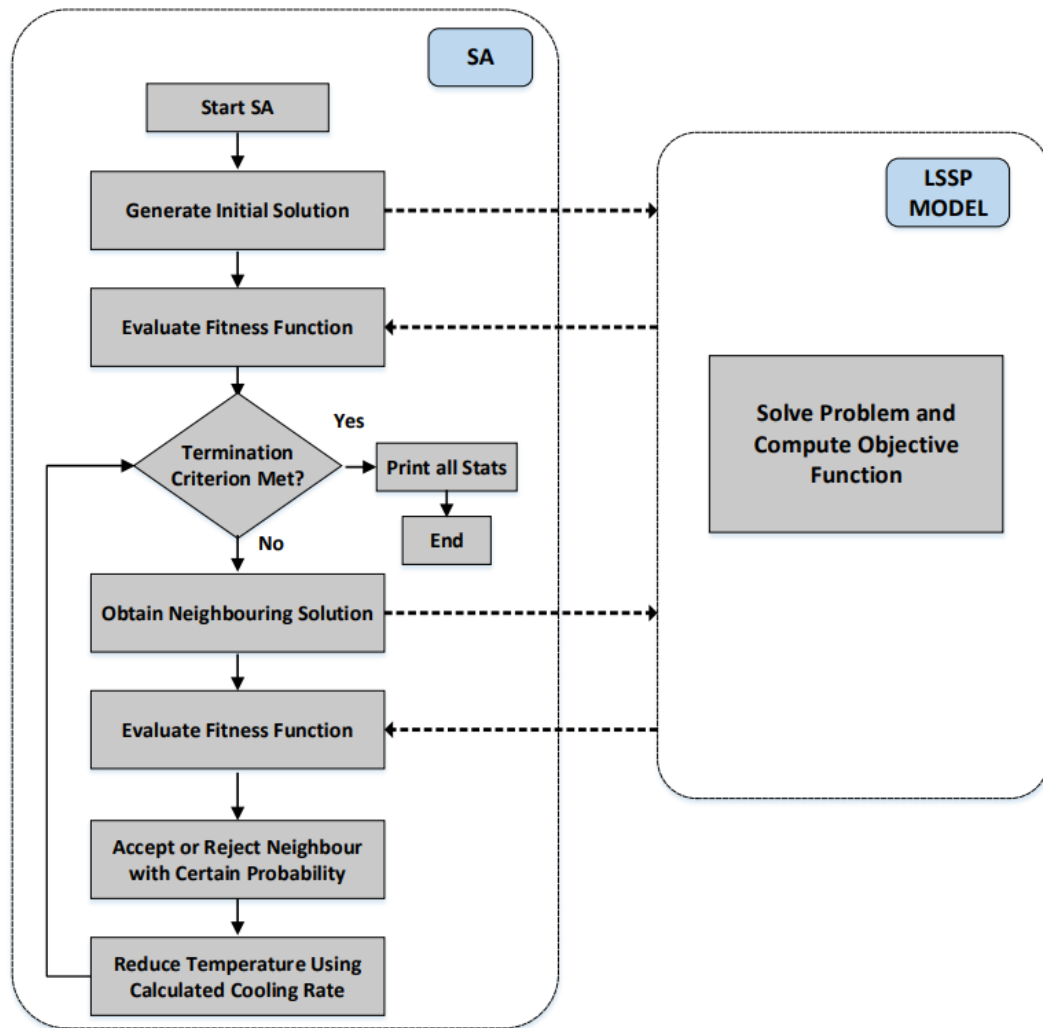


Figure 4.3 A simplified flow chart of the metaheuristic algorithm

4.5.2 Overview of the SA Algorithm

SA is a single-solution based metaheuristic algorithm that was first developed by Kirkpatrick et al. (1983). SA algorithm accepts worse solutions with a probability that is controlled by a parameter called as temperature (T) (T is reduced during the iterations) in order to escape from the local optima. SA algorithm has several parameters that all together are called as cooling schedule, such as; initial temperature (T_0), final temperature (T_f), cooling rate (q) and number of iterations at each temperature (I_{iter}). All parameters have effect on the SA's performance. SA begins with the initial solution and, at each iteration, generates a neighbor solution based on

the specified neighborhood structure and evaluates it via fitness function. Better moves are always acceptable. As previously mentioned, a worse neighbor can also be accepted with a certain probability for escaping from local optimal solutions. SA algorithm continues until the termination conditions are reached. The proposed SA algorithm is briefly explained in the following paragraphs. For detailed explanations about SA algorithm readers can refer to Van Laarhoven & Aarts (1987).

4.5.2.1 Solution Representation, Initial Solution and Neighbor Structure

A permutation-based solution representation (encoding) is used in the proposed SA algorithm. The solution length is comprised of $M*T$ number of elements. Some solution encoding examples are demonstrated in Figure 4.4. In Figure 4.4 (a), there are three paper bag types, three cutting plans, and three periods. In Figure 4.4 (b), three paper bag types, two cutting plans, and three periods. It is clear from this representation that, in Figure 4.4 (a), in the first period, paper bag type 2 is assigned to cutting plan 1, paper bag type 3 is assigned to cutting plan 2, and paper bag type 1 is assigned to cutting plan 3 ($y_{211} = 1, y_{312} = 1$, and $y_{113} = 1$).

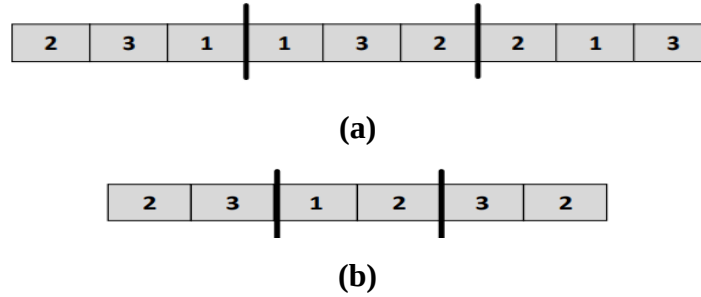


Figure 4.4 Solution representation of the SA algorithm

Based on this solution encoding, it is clear with which paper bag type production starts and ends in each period. Therefore, the related constraints (4.9) - (4.11) and (4.14) - (4.28) are automatically satisfied. Thus, these constraints are removed from the LSSP model and the size of the LSSP model is considerably reduced. In this study, an initial solution is either generated randomly or obtained from the proposed MIP model (MM) within a preset computational time limit and the associated cutting plan-paper bag assignments are fed into the LSSP model. Then the LSSP model is resolved.

As discussed and presented in the later sub-sections (see Figure 4.10) there is not much difference between two approaches. The proposed SA algorithm use two commonly used neighborhood structures, such as swap and inversion (Baykasoglu & Senol, 2019).

4.5.2.2 Fitness Function Evaluation

The value of the decision variable “ y_{itm} ”, which specifies the assignment, cutting plans, and time frame for a single paper bag is determined by using the SA algorithm as a solution and passed to LSSP model as $\overline{y_{itm}}$. After utilizing the SA to determine the decision variables' values y_{itm} , determining the values of the remaining decision variables stay challenging and remains a decision problem. The fitness value of the SA solutions are determined by solving the LSSP and feedback into SA algorithm as depicted in Figure 4.3.

4.5.2.3 Acceptance Criterion

SA algorithm always accepts improving solutions. Worse solutions are accepted with the following probability: $e^{\frac{F(n)-F(c)}{T}}$, where T is the current temperature value. The algorithm performs a predefined number of iterations at each temperature. The temperature is subsequently reduced from T to qT in order to reduce the acceptance probability of worse solutions in later iterations. Figure 4.5 shows pseudo-code of the proposed matheuristic algorithm. The following abbreviations are used in Figure 4.5:

$F_{(n)(o)(c)}$: Objective function value of the (neighbor) (initial) (current) solution

$X_{(n)(o)(c)}$: Solution of (neighbor) (initial) (current)

X_{best} : Solution with minimum fitness value

F_{best} : Minimum fitness value

```

1. Initialization: Specify SA parameters
2. Initialize  $X_{(o)}$  either randomly or by MIP model (MM)
3. Compute objective function value  $F_{(o)}$  by using LSSP Model [LSSP]
4.  $X_{best} := X_{(o)}$ ;  $F_{best} := F_{(o)}$ ;
5.  $T := T_0$ ;
6. While ( $T > T_f$ )
7.      $I := 0$ ;
8.     While ( $I < MaxI_{iter}$ )
9.         Generate neighboring solution  $X_{(n)}$  by swap or inversion operator and
           compute the fitness value  $F_{(n)}$  by solving the LSSP model [LSSP];
10.         $\Delta := F_{(n)} - F_{(c)}$ ;
11.        If ( $\Delta < 0$ )
12.             $X_{(c)} := X_{(n)}$ ;
13.             $F_{(c)} := F_{(n)}$ ;
14.        Else
15.             $r_1 := \text{random}(0,1)$ ;
16.            If ( $r_1 < \exp(-\Delta/T)$ );
17.                 $X_{(c)} := X_{(n)}$ ;
18.                 $F_{(c)} := F_{(n)}$ ;
19.            End
20.        End
21.        If ( $F_{(c)} < F_{best}$  and  $X_{(c)}$  is feasible)
22.             $X_{best} := X_{(c)}$ ;  $F_{best} := F_{(c)}$ ;
23.        End
24.         $I := I + 1$ ;
25.    End
26.     $T := qT$ ;
27. End While
28. Report  $X_{best}$ ;  $F_{best}$ 

```

Figure 4.5 Pseudo code of the SA based matheuristic approach

4.6 Numerical Analysis

The experimental design is described in this section. In Section 4.6.1 describes the test instances. The design of the experiment (DOE) method is employed in order to calibrate parameters of the proposed solution approach as explained in Section 4.6.2. Finally, the computational findings for the matheuristic are provided in Section 4.6.3.

The suggested matheuristic approach is coded in Microsoft Visual Studio 2019 C++ by utilizing the callable library of IBM ILOG CPLEX 12.10 for solving the LSSP model by the CPLEX MIP solver.

4.6.1 Test Instances

For the computational experiments, we use the following parameters: number of paper bag type (P), number of period (T), and number of cutting plan (M). In this section, problem classes are created according to real life data in order to make comparisons in mathematical model and matheuristic methods. The total number of instance classes is 7. In order to minimize sampling error, we randomly generate 10 test instances for each class, resulting in a total of 70 test instances. From the provided and predicted data, lower and upper limits for each parameter were determined, and a data generator that followed a uniform distribution within the observed boundaries is developed. Table 4.1 displays the input parameters.

Table 4.1 Values of input parameters

Input parameter	Value
d_{it}	$U [0;40,120]$
s_{ij}	$U [100,120]$
cp_{im}	$U [10,50]$
ds_i	$U [10,25]$
h_i	$U [10,20]$
h_i^R	$U [10,20]$
g_i	$U [10,20]$
p_{im}	$U [1,5]$
I_i^{max}	$U [0, \max\{d_{it}\} * 2]$
e_i^{max}	$U [0, \max\{d_{it}\}/2]$
bs_{ij}	$(= \lfloor s_{ij}/10 \rfloor)$
p_{im}	$(= \lfloor cp_{im}/10 \rfloor)$
c_t	$(= \sum_{i=1}^I \sum_{t=1}^T d_{it} * 2)$
tp_{im}^R	1

4.6.2 Parameter Calibration

The DOE method is utilized to identify the matheuristic's parameters. To determine the effects of the SA approach's control parameters, the data class 4P-3T-4M with ten problem instances is selected from among all previously generated data sets for the analysis. The parameters of the SA are: initial temperature, cooling rate and iteration number at each temperature. Each factor contains three values, such as {20, 30, 50} for the initial temperature (T), {0.85, 0.90, 0.95} for the cooling rate (q) and {10, 20, 30} for the number of iterations at each temperature ($MaxI_{iter}$). Therefore, there were a total of 27 treatments. There are 10 instances and each instance is ran 10 times for each parameter combination. As a result, consequently for a single instance $10 \times 27 = 270$ results are acquired, and for all instances, $10 \times 270 = 2700$ results are obtained. In order to evaluate the experimental results, MINITAB 21.1.0 software is used to perform analysis of means graphs and analysis of variance (ANOVA) tests. We compute relative percent deviation (RPD) value, which is calculated based on equation (4.43), for each instance-treatment pair as follows:

$$RPD(z_i) = \left(\frac{z_i - z_{best}}{z_{best}} \right) * 100 \quad (4.43)$$

In equation (4.43), z_i indicates the objective function value obtained for each treatment and z_{best} indicates the minimum objective function value found among 270 results. Therefore, the response value is calculated as the mean RPD value for each instance and treatment for ten replications. Table 4.2 displays the ANOVA results.

The ANOVA results show that the cooling rate and iteration are highly significant factors with zero p-values. Iteration has the largest F-ratio, so it is the most important factor. On the other hand, as the p-value is lower than the significance level ($\alpha=0.05$) different levels for the temperature adjustment parameter lead to statistically significant changes in the RPD value. Instead, the interaction effects between the temperature with cooling rate and temperature with iteration are also not statistically significant because their p-values are higher than the significance level.

Table 4.2 ANOVA results for SA parameters

Source	DF	Seq SS	Adj SS	Adj MS	F-Ratio	p-Value
Temperature	2	7,8887	7,8887	3,9437	13,67	0,003
Cooling Rate	2	109,323	109,323	54,6616	189,51	0,000
Iteration	2	128,567	128,567	64,2835	222,87	0,000
Temperature* Cooling Rate	4	0,621	0,621	0,1552	0,54	0,712
Temperature* Iteration	4	0,387	0,387	0,0968	0,34	0,847
Cooling Rate* Iteration	4	6,386	6,386	1,5964	5,53	0,020
Error	8	2,307	2,307	0,2884		
Total	26	255,479				

In Figure 4.6 depicts the main effect plot, which displays the differences in RPD values based on the various levels of the factors. As it can be observed in Figure 4.6, there are only minimal changes in the RPD values associated with the temperature. However, as the number of cooling rates and iterations rises, the RPD values for the cooling rate and iteration fall substantially. In addition to the main effect plot, the interaction plots are shown in Figure 4.7. Since significant factors are the cooling rates and the number of iterations, the interactions of the temperature with these factors are analyzed.

Based on the results of the DOE analyses the best combination of the parameter values for the temperature value, cooling rate and iterations are set to 50, 0.95, and 30.

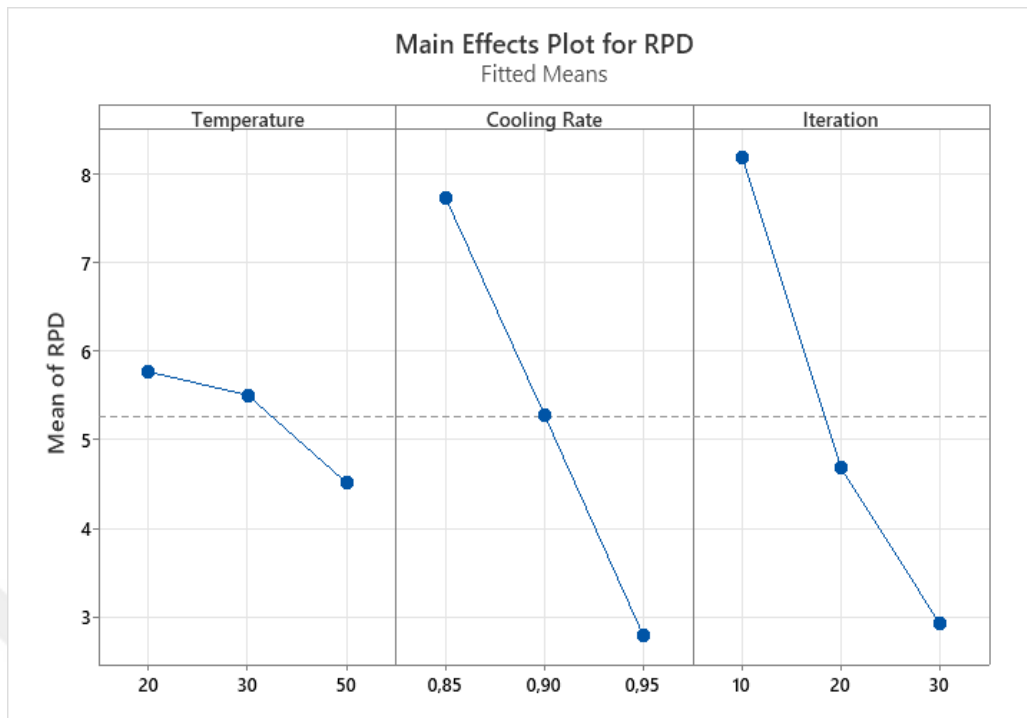


Figure 4.6 Main effect plot for average RPD.

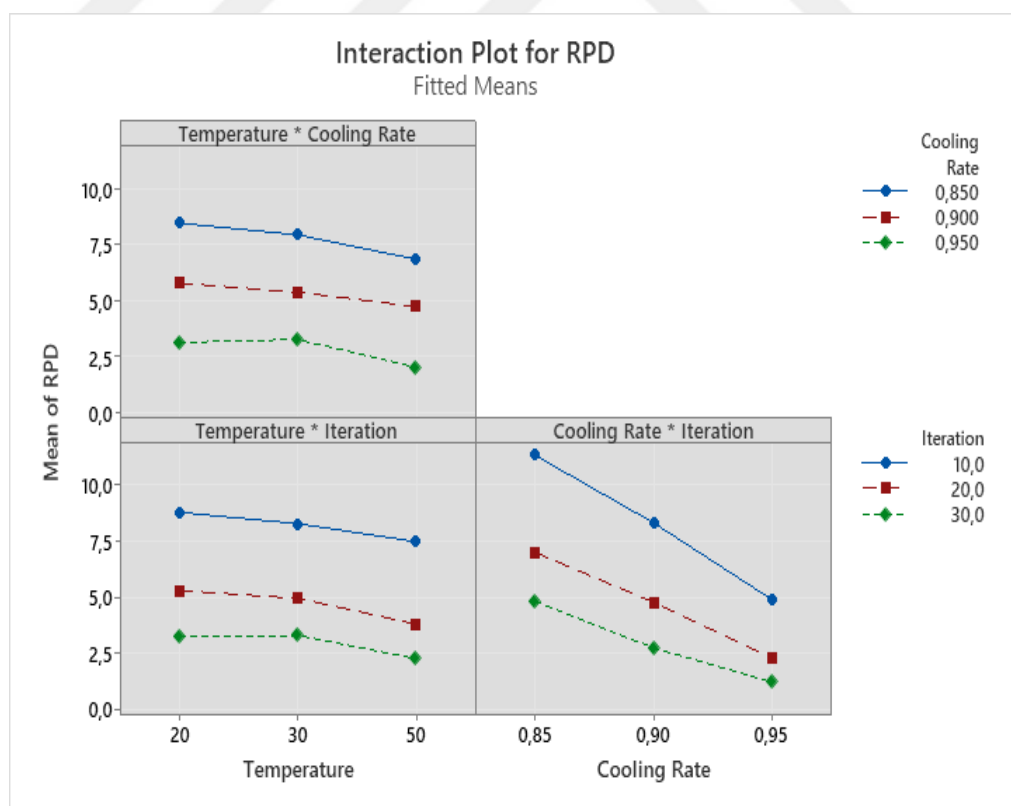


Figure 4.7 Interaction plot for average RPD.

4.6.3 Computational Results

4.6.3.1 Instances Based on Real Data

To achieve a consistent evaluation of the proposed matheuristic approach, instances are solved ten times using the ideal parameters. Computational results of the matheuristic approach, including the best, worst, and average, are reported in Table 4.3.

For the problem instances (4P-3T-4M, 5P-4T-6M, and 7P-5T-7M), the proposed matheuristic approach found optimal solutions in 30 out of 30 problem instances. For the remaining 40 problem instances, the proposed matheuristic method can produce near-optimal solutions with extremely small optimality gaps and reduced run time. Figure 4.8 depicts the optimality gap for all problem instances.

The optimality gap is determined as $100 * (z^* - LB) / (z^*)$, where z^* is the best solution found by the proposed mathematical model and LB represents the obtained lower bound provided by CPLEX after one hour of execution.

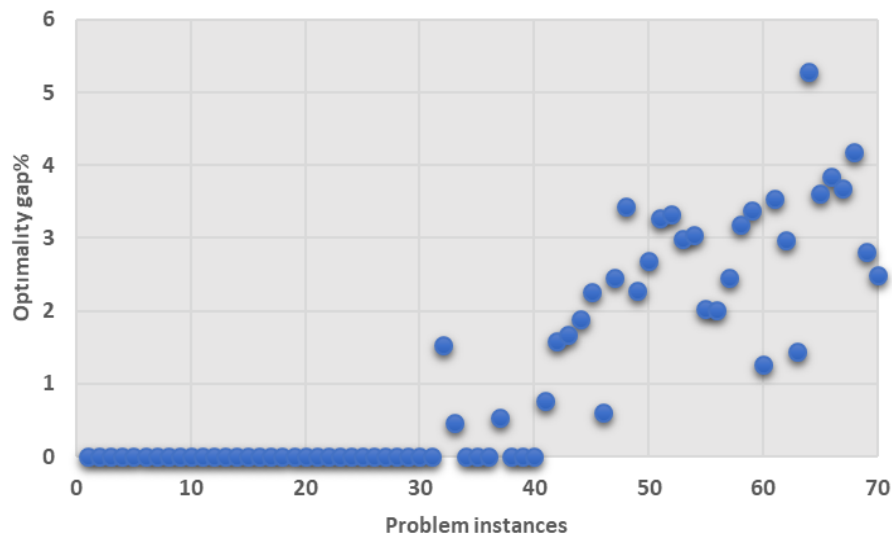


Figure 4.8 Optimality gap for the problem instances

The Gap (Obj)% indicates the percentage of deviation between the best objective values found by the proposed matheuristic approach. The Gap (Obj)% is calculated as $100 * (z_{mat}^* - z^*) / (z_{mat}^*)$, where z^* is the best solution found by the proposed mathematical model and z_{mat}^* is the best solution found by the proposed matheuristic approach. Figure 4.9 displays a plot of the percentage deviation for all problem instances. As it can be seen from Figure 4.9 the Gap (Obj)% is less than 3, namely 2.78.

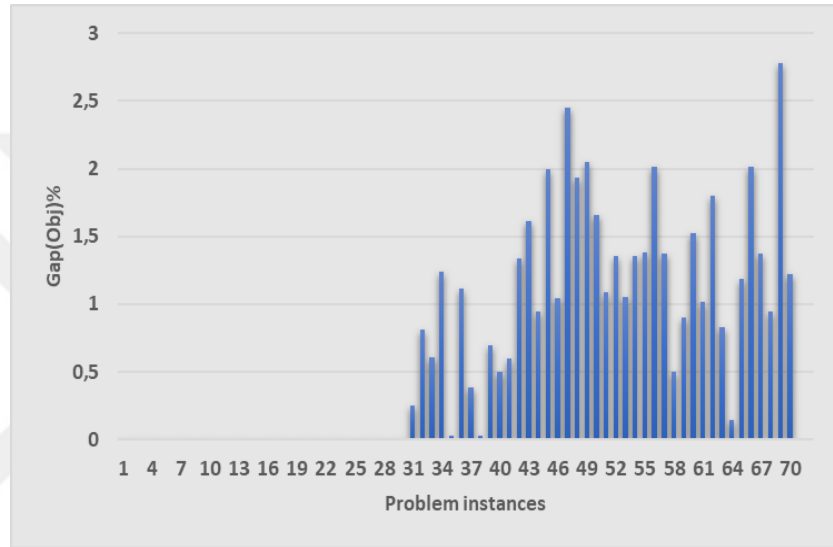


Figure 4.9 The percentage deviation between the best objective values found by the proposed matheuristic approach

Table 4.3 The computational results of the proposed MIP and matheuristic algorithm

	Problem No	MIP				Proposed SA-based Matheuristic approach			
		Best	LB	Solution Time (sec.)	Gap	Best	Worst	Avg.	Solution Time (sec.)
(4P-3T-4M)	1	18181	18181	0,36	0,00	18181	18181	18181	9,35
	2	16892	16892	0,41	0,00	16892	16892	16892	9,12
	3	14814	14814	0,27	0,00	14814	14814	14814	9,87
	4	16631	16631	0,39	0,00	16631	16631	16631	9,01
	5	20074	20074	0,48	0,00	20074	20074	20074	9,65
	6	16554	16554	0,38	0,00	16554	16554	16554	10,05
	7	18003	18003	0,56	0,00	18003	18003	18003	10,76
	8	20028	20028	0,59	0,00	20028	20028	20028	9,32
	9	16772	16772	0,41	0,00	16772	16772	16772	10,95
	10	18176	18176	0,33	0,00	18176	18176	18176	10,36
(5P-4T-6M)	11	26674	26674	10,21	0,00	26674	28369	27638	25,67
	12	29773	29773	11,45	0,00	29773	31321	30215	26,82
	13	22145	22145	9,55	0,00	22145	22926	22624	24,28
	14	30711	30711	10,05	0,00	30711	31252	30925	27,73
	15	31057	31057	9,87	0,00	31057	32521	31865	27,57
	16	33685	33685	10,39	0,00	33685	34218	33923	25,46
	17	21296	21296	10,67	0,00	21296	21953	21621	27,28
	18	22663	22663	10,99	0,00	22663	23422	23193	24,83
	19	33550	33550	11,08	0,00	33550	34535	34987	25,47
	20	23727	23727	12,15	0,00	23727	25019	24158	23,91
(7P-5T-7M)	21	43396	43396	52,12	0,00	43396	44988	44135	58,35
	22	45149	45149	51,34	0,00	45149	46475	45824	59,65
	23	45580	45580	59,87	0,00	45580	47249	46371	58,98
	24	46641	46641	397,53	0,00	46641	48103	47228	57,32
	25	49464	49464	64,32	0,00	49464	50149	49863	56,12
	26	47522	47522	51,67	0,00	47522	48867	47999	59,54
	27	38785	38785	253,98	0,00	38785	40111	39436	54,98
	28	46723	46723	318,69	0,00	46723	48180	47347	57,98
	29	41217	41217	65,87	0,00	41217	42184	41678	55,43
	30	40304	40304	96,07	0,00	40304	41544	40932	56,85
(9P-6T-8M)	31	59145	59145	1258,65	0,00	59294	61127	60031	83,91
	32	66567	65541,87	3600,00	1,54	67114	69843	68347	84,25
	33	74465	74122,46	3600,00	0,46	74921	78256	76244	83,76
	34	69134	69134	602,81	0,00	70002	72012	70903	84,35
	35	66765	66765	1805,12	0,00	66784	69267	68077	83,24
	36	76947	76947	2841,54	0,00	77813	80141	78824	88,56
	37	62458	62120,73	3600,00	0,54	62702	65302	64109	87,12
	38	58349	58349	1021,84	0,00	58368	61354	59905	86,94
	39	69108	69108	2954,96	0,00	69592	72145	70456	85,15
	40	66375	66375	2348,03	0,00	66709	68455	67667	83,07
(10P-6T-10M)	41	80511	79891,07	3600,00	0,77	80999	83368	82123	116,07
	42	76368	75153,75	3600,00	1,59	77403	79676	78430	117,74
	43	79667	78336,56	3600,00	1,67	80974	83769	82096	113,64
	44	77381	75918,49	3600,00	1,89	78123	80057	79101	116,47
	45	76803	75067,25	3600,00	2,26	78368	81357	79864	114,54
	46	69820	69394,09	3600,00	0,61	70557	73442	71860	113,98
	47	75624	73771,21	3600,00	2,45	77523	79621	78456	114,09
	48	69380	66993,33	3600,00	3,44	70751	73077	72005	119,35
	49	72773	71113,78	3600,00	2,28	74297	76399	75090	118,43
	50	79050	76923,56	3600,00	2,69	80380	83286	81873	117,65
(12P-8T-8M)	51	120455	116504,07	3600,00	3,28	121781	125156	123438	137,95
	52	128521	124254,11	3600,00	3,32	130292	134115	131551	136,12
	53	121575	117939,91	3600,00	2,99	122863	126535	124399	138,39
	54	118836	115223,38	3600,00	3,04	120469	123703	121533	139,45
	55	112120	109843,96	3600,00	2,03	113694	116536	115008	134,84
	56	122440	119978,96	3600,00	2,01	124960	127794	126626	135,38
	57	116358	113507,23	3600,00	2,45	117982	122201	119794	133,74
	58	121516	117651,79	3600,00	3,18	122129	125511	123581	132,87
	59	130627	126198,74	3600,00	3,39	131818	135887	133968	131,22
	60	134034	132345,17	3600,00	1,26	136105	140028	137836	135,83
(15P-10T-7M)	61	221342	213506,49	3600,00	3,54	223623	227464	225715	147,03
	62	207893	201697,78	3600,00	2,98	211703	217547	214619	148,53
	63	224225	220973,73	3600,00	1,45	226109	230584	228745	147,75
	64	197322	186883,66	3600,00	5,29	197607	202294	199784	146,56
	65	222843	214776,08	3600,00	3,62	225527	233085	228864	147,98
	66	200971	193253,71	3600,00	3,84	205104	212611	208672	148,15
	67	224925	216625,26	3600,00	3,69	228051	235429	230596	143,69
	68	188883	180968,79	3600,00	4,19	190692	194682	192097	146,02
	69	216069	209975,85	3600,00	2,82	222248	227748	224479	145,62
	70	215030	209675,75	3600,00	2,49	217692	223845	220323	145,39

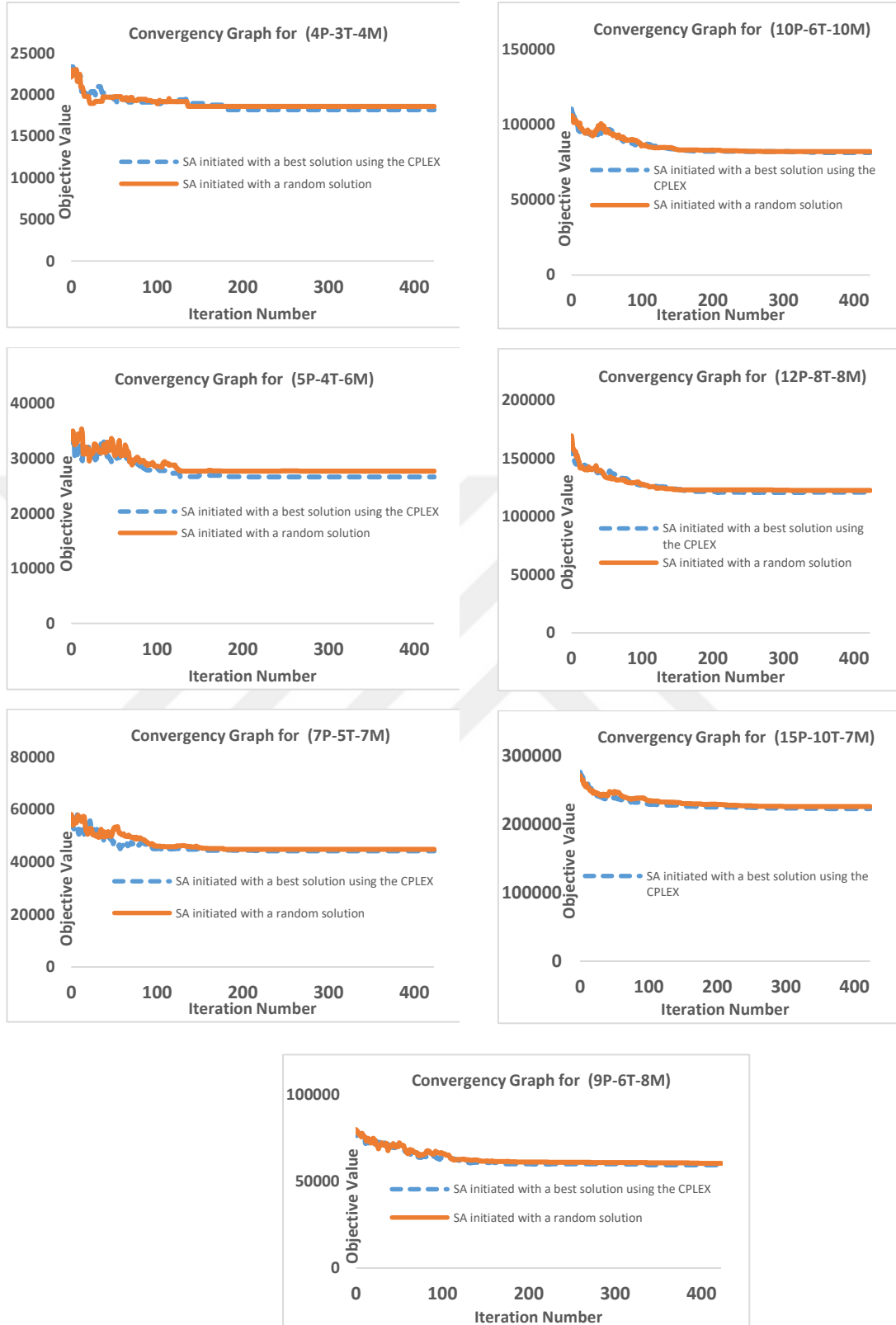


Figure 4.10 Convergence graphs of the proposed matheuristic algorithm for the first problem in each problem instances

4.6.3.2 Managerial Insights

In this subsection, we analyze the effect of parameters of the proposed MIP model on the value of the objective function and its cost elements. We make a sensitivity analysis on problem class 1 (4P-3T-4M). We conducted a sensitivity analysis on three main parameters: capacity, setup time, and procurement cost. The remaining parameters are taken the same as that in Section 4.6.2.

4.6.3.3 Impact of Capacity

The available machine capacity has no significant effect on total cost amount. Since, there is interaction between capacity and backlog. Figure 4.11 describes the total cost value for ten instances under different capacity levels.

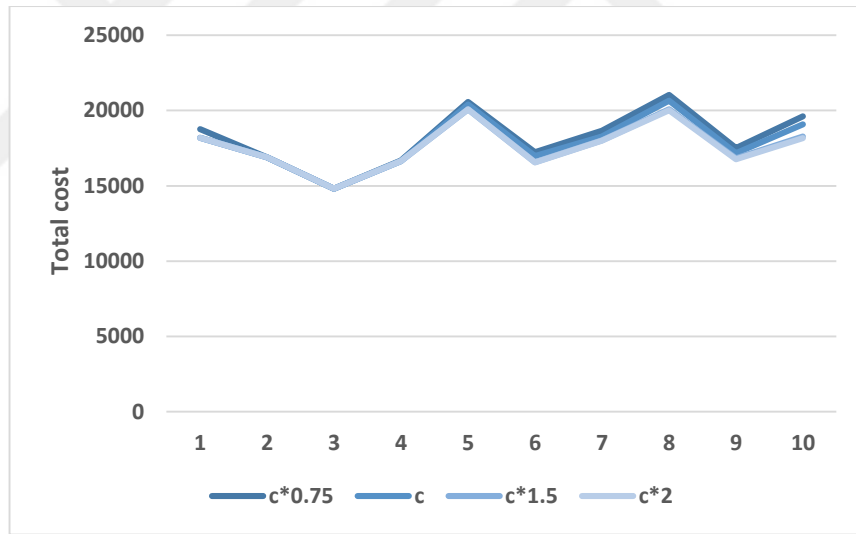


Figure 4.11 Impact of different capacity levels on the total cost

4.6.3.4 Impact of Setup Time

Figure 4.12 shows that the increasing setup level has no significant effect on total cost amount. Figure 4.13 shows that greater setup levels ($s*1.5$ and $s*2$) result in a higher overall procurement amount than lower setup levels ($s*0.75$ and s). However, Figure 4.14 represents lower setup levels ($s*0.75$ and s) result in the same or slightly

higher total rework amount than higher setup levels ($s*1.5$ and $s*2$). Figure 4.15 shows that the increasing setup level has no significant effect on total good quality amount. Figure 4.16 represents, higher setup levels ($s*1.5$ and $s*2$) result in the same or slightly higher total backlog amount (not including instance 4 and 5) than lower setup levels ($s*0.75$ and s).

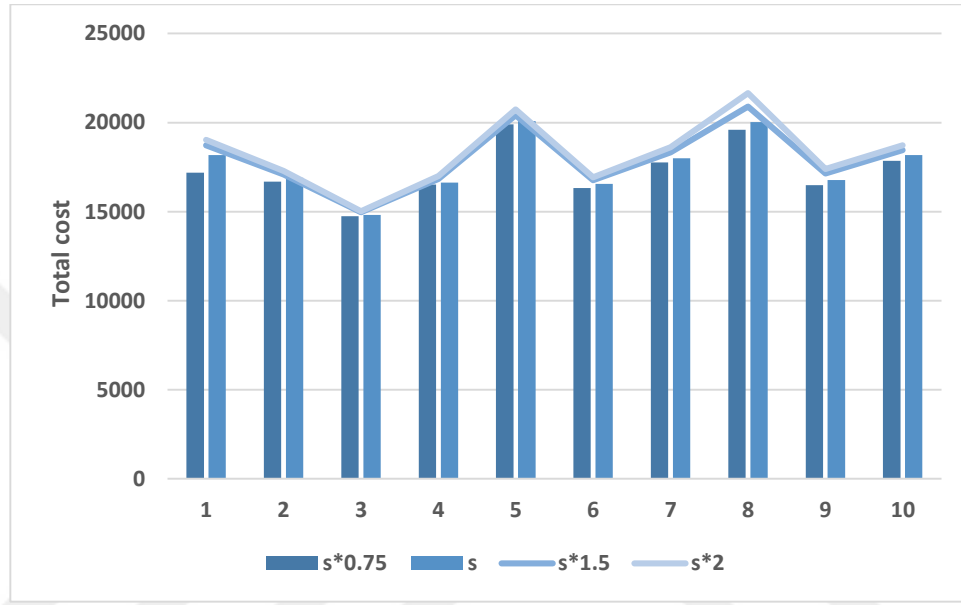


Figure 4.12 Impact of different setup levels on the total cost

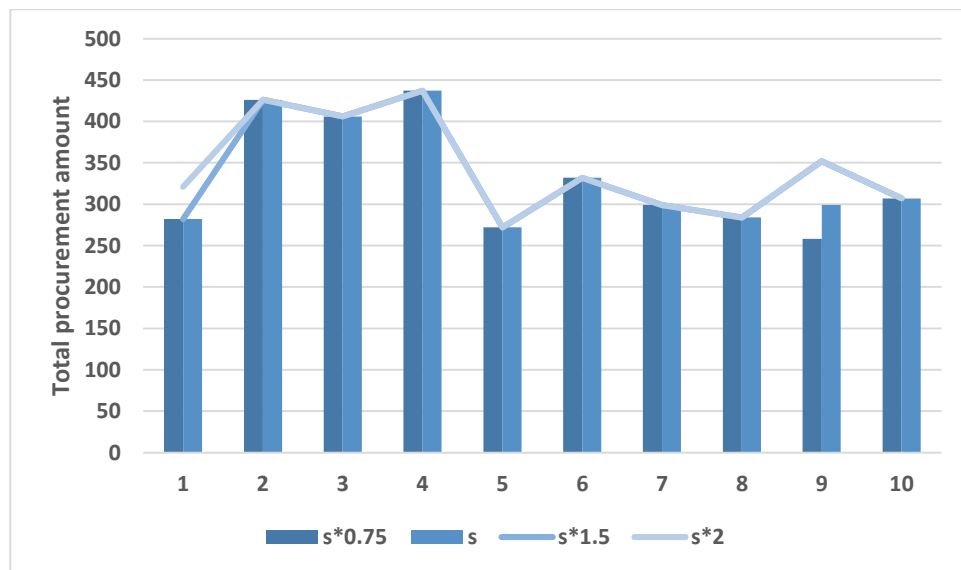


Figure 4.13 Impact of different setup levels on the total procurement amount

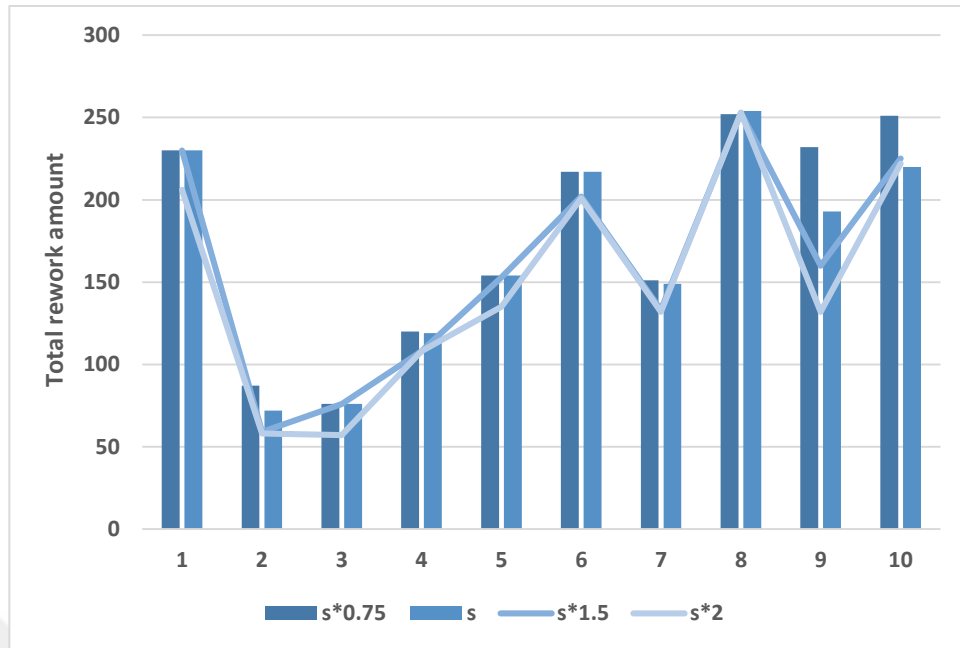


Figure 4.14 Impact of different setup levels on the total rework amount

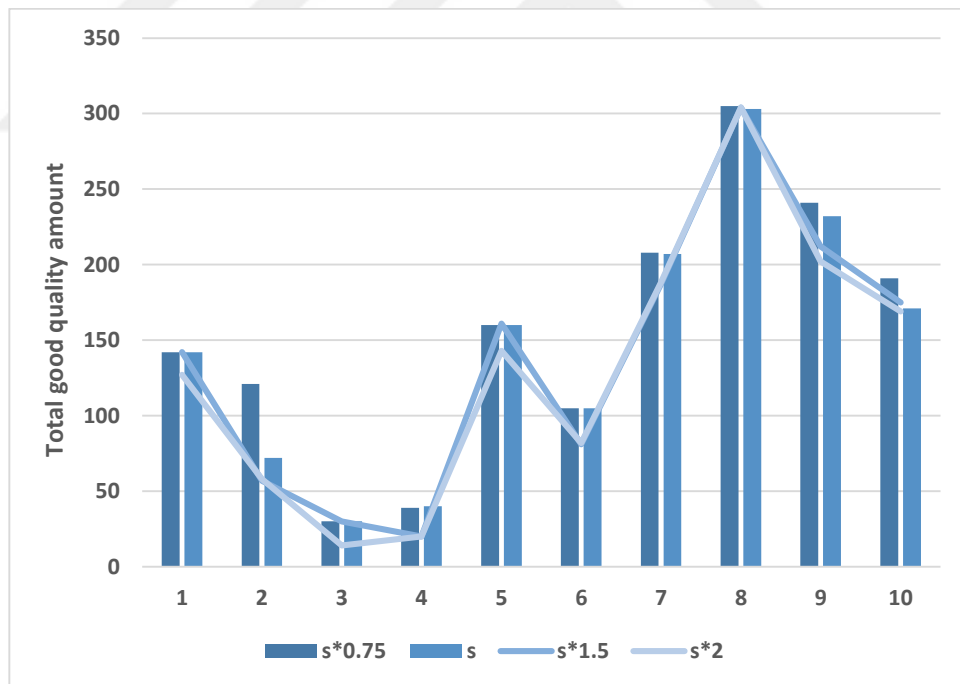


Figure 4.15 Impact of different setup levels on the total good quality amount

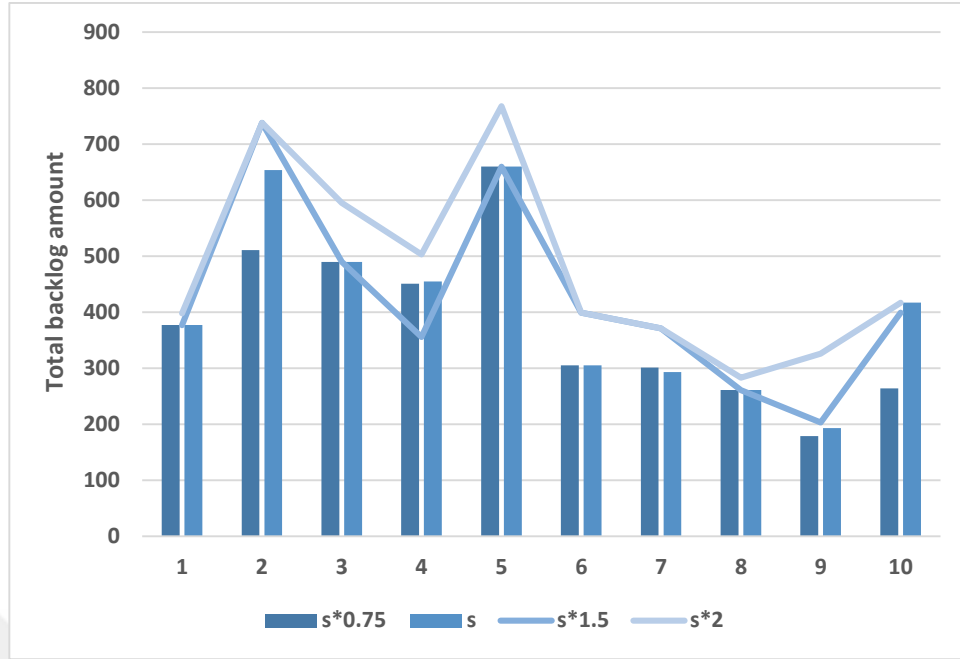


Figure 4.16 Impact of different setup levels on the total backlog amount

4.6.3.5 Impact of Procurement Cost

In order to understand the impact of procurement costs on the problem, we analyzed four different levels of procurement costs. In Figure 4.17 describes the total cost value for all periods under different procurement cost levels. As it is expected, if procurement cost levels increase, the total cost will increase. Total procurement amount can be seen in Figure 4.18. It stands to reason that as procurement costs rise, the overall amount spent on purchases will fall. Total good quality amount can be seen in Figure 4.19. From this figure, it can be seen that an increase in procurement costs will result in an increase in the total amount of produced good quality. Parallel to increasing to amount good quality item, the rising total rework amount can be seen in Figure 4.20. Increasing procurement costs results in increasing the total backlog amount, which can be seen in Figure 4.21.

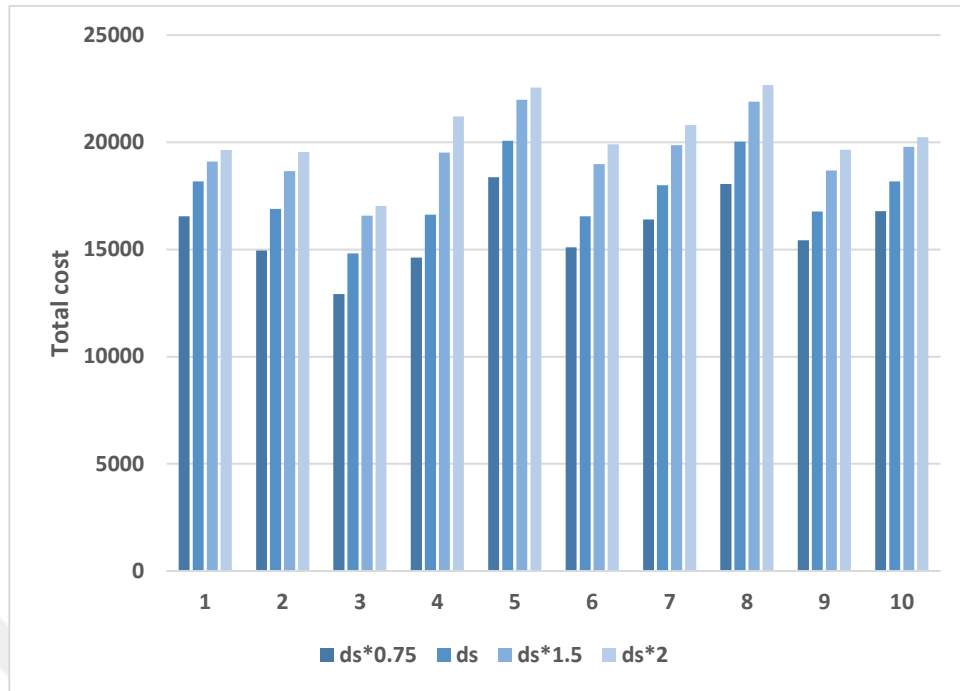


Figure 4.17 Impact of procurement cost on the total cost

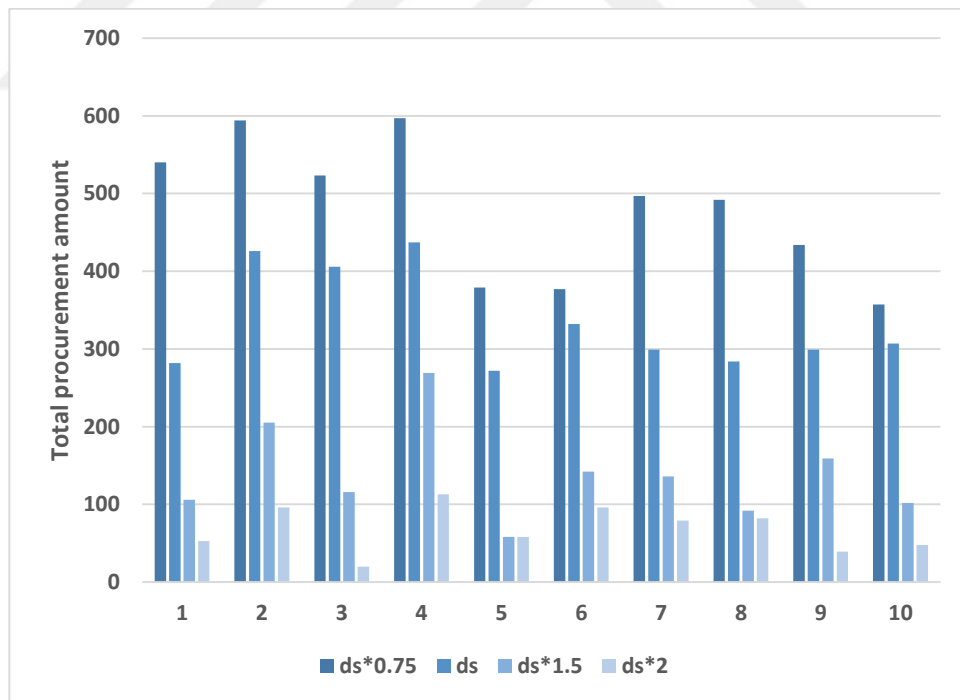


Figure 4.18 Impact of procurement cost levels on the total procurement amount

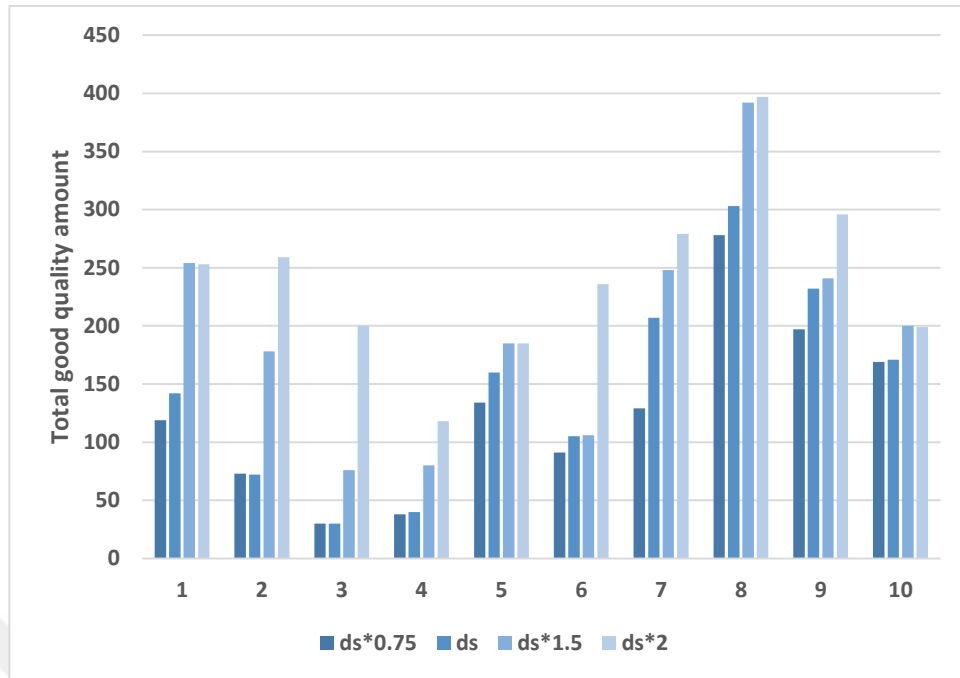


Figure 4.19 Impact of procurement cost on the total good quality amount

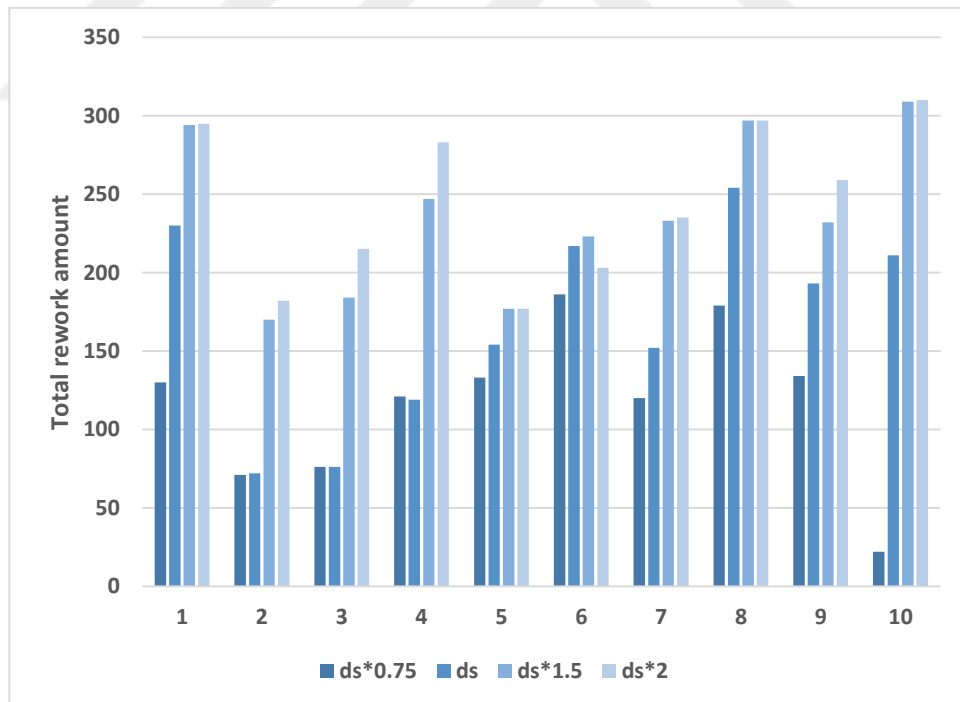


Figure 4.20 Impact of procurement cost on the total rework amount

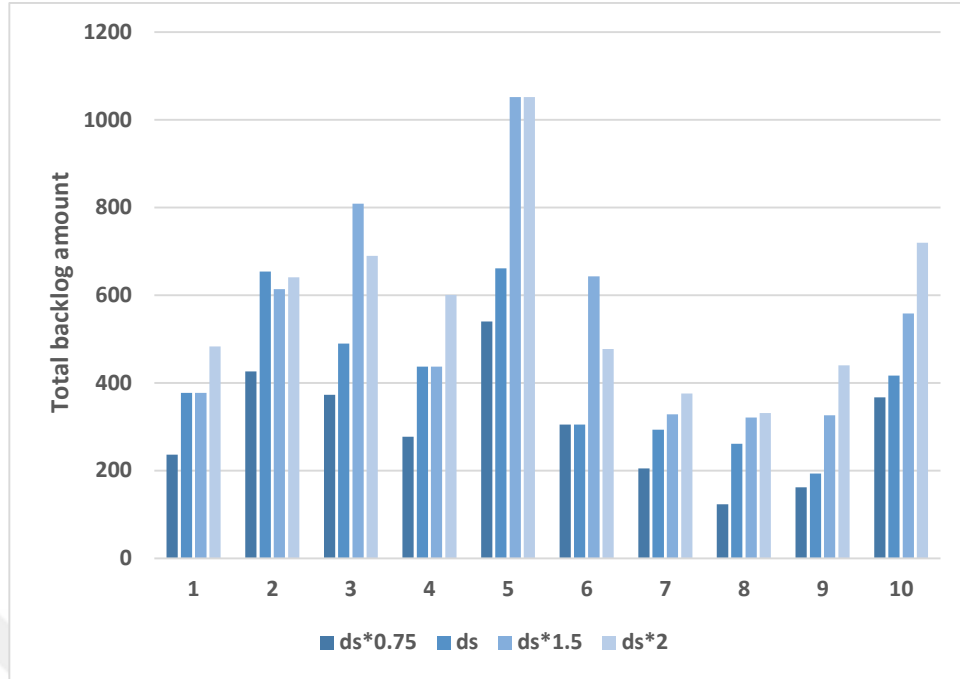


Figure 4.21 Impact of procurement cost on the total backlog amount

4.7 Conclusion

In this chapter, a new integer programming model for the capacitated lot sizing and scheduling problem with external procurements, defective/backordered items, setup times and cutting plan assignment is proposed. The objective of the proposed model is to minimize the total cost, which includes holding cost of good quality and defective items, backorder, procurement, production, and sequence-dependent setup costs. The proposed model is actually developed for a paper bag production company, but the proposed model can also be applied to other production environments with similar production characteristics. In order to be able to solve larger size practical problems in reasonable times a new solution methodology is also proposed. The proposed solution method is matheuristic that is based on simulated annealing algorithm incorporating integer programming. Actually, the proposed integer programming model is divided into two sub-problems, as cutting plan assignment and lot sizing sub-problems to be solved simultaneously. A SA algorithm is devised for generating cutting plan assignments as input to the LSSP that is optimally solved by using an integer

programming solver. LSSP feedbacks the objective function value back to the SA algorithm and the solution process iterates in this manner until convergence.

Extensive computational tests are carried out on the test problems that are derived from the original paper bag production problem. It is observed that the proposed matheuristic based solution approach is able to generate optimal or near optimal solutions in short computational times. The present study can also be expanded in a variety of ways. For example, new constraints related to disposal option can be incorporated into the model. The proposed model can also be extended to multi machine or multi objectives cases. Finally, different versions of matheuristic algorithms can also be devised.

CHAPTER FIVE

UNRELATED PARALLEL MACHINE CAPACITATED LOT SIZING AND SCHEDULING PROBLEM WITH MACHINE ELIGIBILITY CONSTRAINTS AND A COMMON SERVER OPERATOR

5.1 Introduction

In general, production planning and scheduling problem continue to be one of the major challenges that researchers have been trying to solve effectively in today's competitive environment. Firms seek to obtain the production plan that will maximize the profits of the firms with lowest cost. In order to achieve this purpose, planning of production factors such as stock quantities, backorder amounts, capacity of machines and setup times must be updated according to the dynamic structure of the production system. Furthermore, various decisions which are related to production planning problem, such as capacity planning, ordering, lot sizing and scheduling, have to be considered together (Stadtler et al., 2015). In this study, considered capacitated lot sizing and scheduling problem contains multiple single-level items that produced on unrelated parallel machines over several time periods. The class of lot sizing problem with multiple parallel resources appears in several applications, such as tile production (De Matta & Guignard, 1994), pharmaceutical production (De Matta & Guignard, 1995), the tire industry (Jans & Degraeve, 2004), packing for yogurt production (Marinelli, Nenni & Sforza, 2007), and chipset production (Xiao, Yang, Zhang, Zheng & Gupta, 2015). This problem also frequently encountered in pool equipment manufacturer industry. Pool equipment is the tools used in the maintenance and cleaning processes of pools, starting from the construction of pool projects. The effective use of pool equipment enables more reliable, healthy and trouble-free pools.

In this study, the problem of determining lot size and scheduling in the plastic injection department of a company that produces pool equipment will be discussed. It is aimed to reduce costs such as production and lot sizing. The production of different product groups in the factory is carried out upon request. The company's product catalog has nearly 3000 product types. The product group to be discussed in this study

is PVC fittings. The production of this product group is carried out by plastic injection. Plastic injection is a production method that follows the process of shaping the granular or powder plastic raw material by injecting it into the mold after heating it until it becomes liquid, and cooling and removing it from the mold. Today, many products in different sizes and categories are produced by plastic injection. The company produces PVC fittings, consisting of 13 main groups, in different colors and sizes in order to better respond to customer needs. Thus, it offers approximately 85 types of PVC fittings. It is of great importance to make high quality and fast production in the market where customer needs are at the forefront and it is desired to respond to these needs as soon as possible. Material, mold, process and machine play a role in obtaining quality products in plastic injection. In case of problems with any of these factors, the part is scrapped.

In this manufacturing industry, not all parallel machines in the facility are eligible for every product type and each product type can only be produced by a certain subset of eligible machines. Also, in some cases alternative machine sets can be used to produce. However, this case cost higher than producing via eligible machine set. Since, it requires more equipment than the eligible machines require. For solving this problem, we developed MIP model by using existing models (Xiao et al., 2015; Bektur & Saraç, 2019) according to our considered problem. In our objective function, instead of machine preference, alternative machine selection is case and this case generates extra cost.

5.2 Related Literature

Most of the existing studies in the literature have either ignored the setup times in the Capacitated Lot Sizing Problem (CLSP) problems or included only order-independent setup times. Karimi, Ghomi & Wilson (2006), Özdamar, Bilbil & Portmann (2002) and Hindi, Fleszar & Charalambous (2003) addressed the single-machine lot sizing problem with sequence-independent setup times. Özdamar & Barbarosoğlu (1999), Stadtler (2003), Sahling, Buschkühl, Tempelmeier & Helber (2009) and Wu, Shi, Geunes & Akartunalı (2011) examined the multi-stage CLSP

problem with sequence-independent setup times and proposed various heuristic algorithms to solve such a problem. González-Ramírez, Smith & Askin (2011), Degraeve & Jans (2007), and Wu, Shi & Duffie (2010) used the Dantzig-Wolfe decomposition method to solve the multi-product capacitated lot sizing problem with sequence-independent setup times. Porkka, Vepsäläinen & Kuula (2003) examined the two modifications of carry over models and found that setup costs could be reduced in models with setup carry over case. Sox & Gao (1999) proposed a heuristic algorithm for the solution of the Capacitated Lot Sizing Problem with Linked Lot Sizes (CLSPL) problem, which has no backorder and setup times, by formulating it as the shortest route problem. Tabu search algorithm, Gopalakrishnan, Ding, Bourjolly & Mohan (2001) for the single-machine CLSPL problem with sequence-dependent setup time.

Haase (1996) investigated the CLSD problem for single-machine situations, ignoring setup times. Kovács, Brown & Tarim (2009) transformed the single machine problem with sequence-dependent setup times into TSP and obtained optimum sequencing using dynamic programming. The relax-and-fix heuristic algorithm for the identical parallel machine and single machine CLSP problem with sequence dependent setup time was developed by Beraldi, Ghiani, Grieco & Guerriero (2008). Lang & Shen (2011) proposed MIP-based relax-and-fix and fix-and-optimize heuristic solutions for the single-machine CLSP problem with sequence-dependent setup time. James & Almada-Lobo (2011) developed MIP based heuristics by addressing the CLSP problem with a single and parallel machine with sequence-dependent setup time and cost, machine availability constraints.

Lot sizing and scheduling issues typically entail fluctuating demand within a constrained planning horizon, but rarely impose limitations on production and/or delivery time intervals. It is difficult to solve CLSP problems involving backward ordering (Karimi et al., 2003). A dynamic programming algorithm for solving the lot sizing problem with one machine and one piece for problems involving backorder was proposed by Song & Chan (2005). Akartunalı & Miller (2009) studied the sequence-independent setup time and multi-level lot sizing problem. Wu, Shi & Song (2012) combines the relax-and-fix and nested partition algorithms to introduce an

optimization system for the row-independent setup-time and multi-level lot sizing problem, thus extending the solution for time-lapse lot sizing problems with both under capacity and fixed capacity. they have provided. The heuristic algorithm for solving the CLSP problem with sequence-independent setup times and generation time interval Brahimi, Dauzere-Peres & Wolsey (2010) suggested. Absi, Kedad-Sidhoum & Dauzère-Pérès (2011), on the other hand, carried out their studies on the problem of lot sizing, which has a production time interval, where lost sales are possible, early production and delayed orders.

In the following years, Xiao et al. (2015) developed an extension of this CLSP problem, which Haase addressed in 1996, in a single model by including four new constraints (where setup times on independent machines can be moved to the next period and reverse ordering). This problem is encountered by many companies operating in the manufacturing sector while scheduling production in practice. In the presence of all these constraints, existing heuristic algorithms are not sufficient to solve the problem under consideration (Xiao et al., 2015). For example, the model proposed by Quadt & Kuhn (2009) cannot be used because the processing times of different product types on individual machines differ. For the unrelated parallel machine CLSP problem, Meyr (2002) suggested methods but did not consider cases where there is backordering.

A scheduling and lot sizing problem with a common setup operator (server) is first studied in Tempelmeier & Buschkühl (2008). The setups performed by the operator are considered to be scheduled without overlaps. The setups are associated only with the following task, making them sequence-independent. The proposed ILP formulations can solve instances of tens of tasks, usually under a 1-minute time limit.

This rest of the paper is organized as follows. In section 5.3, the mathematical model is given. In section 5.4, numerical experiments are given. In section 5.5, The proposed multi-objective mixed-integer mathematical model will be explained and related numerical experiments are given. In section 5.6, results of the computational experiments, the conclusions and outlook for future research are presented.

5.3 Model Formulation

The mathematical model is described based on the practice in pool equipment manufacturing factory.

5.3.1 Mixed-Integer Mathematical Programming Model

The notations of the mathematical model are introduced as follows:

Sets and Indices:

J	: Number of items
M	: Number of machines
T	: Number of periods
K	: Number of positions
j, q, i, h	: Items indices, where $j, q, i, h \in J$
m	: Machine indices, where $m \in M$
t	: Periods indices, where $t \in T$
k, o	: Position indices, where $k, o \in K$

Parameters:

p_{mj}	: Process time of item j on machine m
α_{qj}	: Sequence dependent setup time/cost to process item j after item q on the same machine
MM_{mj}	: 1 if machine m is eligible for item j , 0 otherwise
d_{jt}	: Demand of item j in period t
AO_{mj}	: 1 if item j is preferred to be processed on alternative machine m , 0 otherwise
Cap_{mt}	: Capacity of machine m in period t
h_j	: Holding cost of item j
b_j	: Backorder cost of item j
pc_{mj}	: Production cost of item j on machine m
BM	: Big M

Decision Variables:

C_{mjtk} : Completion time of item j in position k in period t on machine m

A_{mjtk} : Start time of setup operation for item j in position k in period t on machine m

ZS_{mjtk} : Completion time of setup operation for item j in position k in period t on machine m

V_{mjtk} : Wait time of item j for the server to be available in position k in period t on machine m (Idle time)

Q_{mjtk} : Process quantity of item j on machine m in position k in period t

I_{jt}^+ : Inventory amount of item j in period t

I_{jt}^- : Backorder amount of item j in period t

S_{mjtk} : Sequencing variable indicating the processing sequence of item j in position k in period t on machine m

X_{mjtk} : 1 if item j is processed in position k in period t on machine m , 0 otherwise

Y_{mjtk} : 1 if item j immediately precedes item q on the machine m in position k in period t , 0 otherwise

E_{mjtk} : 1 if the completion time of the setup for item j is less than the start time of the setup for item q on the machine m in position k in period t ; 0 If the completion time of the setup for item q is less than the start time of the setup for item j on the machine m in position k in period t

The problem is formulated as follows:

The objective of the model (5.1) is to minimize the cost of holding, backorder, sequence dependent setup and production.

$$\begin{aligned} \text{Min } Z = & \sum_{j=1}^J \sum_{t=1}^T (h_j I_{jt}^+ + b_j I_{jt}^-) + \sum_{m=1}^M \sum_{j=1}^J \sum_{\substack{h=1 \\ h \neq j}}^J \sum_{t=1}^T \sum_{k=1}^K (\alpha_{jh} Y_{mjhtk}) \\ & + \sum_{j=1}^J \sum_{m=1}^M \sum_{t=1}^T \sum_{k=1}^K (p c_{mj} Q_{mjtk}) \end{aligned} \quad (5.1)$$

subject to

$$C_{mjtk} + BM(1 - X_{mjtk}) \geq Q_{mjto}p_{mj} + V_{mjtk}, \quad \forall m, j, t \text{ and } k = 1 \quad (5.2)$$

$$C_{mjtk} - BM(1 - X_{mjtk}) \leq Q_{mjto}p_{mj} + V_{mjtk}, \quad \forall m, j, t \text{ and } k = 1 \quad (5.3)$$

$$C_{mjtk} + BM(2 - X_{mht(k-1)} - X_{mjtk}) \geq \alpha_{hj} + \sum_{o=1}^K Q_{mjto} p_{mj} + C_{mhtk-1} + V_{mjtk}, \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.4)$$

$$C_{mjtk} - BM(2 - X_{mht(k-1)} - X_{mjtk}) \leq \alpha_{hj} + \sum_{o=1}^K Q_{mjto} p_{mj} + C_{mhtk-1} + V_{mjtk}, \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.5)$$

$$\sum_{j=1}^J X_{mjtk} \leq \sum_{h=1}^J X_{mht(k-1)}, \quad \forall m, t \text{ and } k > 1 \quad (5.6)$$

$$\sum_{j=1}^J X_{mjtk} \leq 1, \quad \forall m, t \text{ and } k \quad (5.7)$$

$$\sum_{k=1}^K X_{mjtk} \leq 1, \quad \forall m, j \text{ and } t \quad (5.8)$$

$$\sum_{m=1}^M X_{mjtk} \leq 1, \quad \forall j, t \text{ and } k \quad (5.9)$$

$$A_{mjtk} \geq C_{mht(k-1)} - BM(2 - X_{mjtk} - X_{mht(k-1)}) + V_{mjtk}, \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.10)$$

$$A_{mjtk} \leq C_{mht(k-1)} + BM(2 - X_{mjtk} - X_{mht(k-1)}) + V_{mjtk}, \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.11)$$

$$A_{mjtk} \geq V_{mjtk} - BM(1 - X_{mjtk}), \quad \forall h, j, t, k = 1 \text{ and } m \quad (5.12)$$

$$A_{mjtk} \leq V_{mjtk} + BM(1 - X_{mjtk}), \quad \forall h, j, t, k = 1 \text{ and } m \quad (5.13)$$

$$ZS_{mjtk} \geq A_{mjtk} - BM(1 - X_{mjtk}), \quad \forall j, m, t \text{ and } k = 1 \quad (5.14)$$

$$ZS_{mjtk} \leq A_{mjtk} + BM(1 - X_{mjtk}), \quad \forall j, m, t \text{ and } k = 1 \quad (5.15)$$

$$ZS_{mjtk} \geq A_{mjtk} + \alpha_{hj} - BM(2 - X_{mht(k-1)} - X_{mjtk}), \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.16)$$

$$ZS_{mjtk} \leq A_{mjtk} + \alpha_{hj} + BM(2 - X_{mht(k-1)} - X_{mjtk}), \quad \forall h, j, t, k > 1, m \text{ and } h \neq j \quad (5.17)$$

$$ZS_{mqtk} \leq A_{mjtk} + BME_{mjqt}, \quad \forall m, j, q, t, k \text{ and } j < q \quad (5.18)$$

$$ZS_{mjtk} \leq A_{mqtk} + BM(1 - E_{mjqt}), \quad \forall m, j, q, t, k \text{ and } j < q \quad (5.19)$$

$$I_{j(t-1)}^+ - I_{j(t-1)}^- + \sum_{m=1}^M \sum_{k=1}^K Q_{mjtk} - I_{jt}^+ + I_{jt}^- = d_{jt}, \quad \forall j \text{ and } t \quad (5.20)$$

$$\sum_{j=1}^J \sum_{k=1}^K p_{mj} Q_{mjtk} + \sum_{j=1}^J \sum_{i=1}^J \sum_{k=1}^K \alpha_{ji} Y_{mjtk} \leq CAP_{mt}, \quad \forall m \text{ and } t \quad (5.21)$$

$$Q_{mjtk} \leq \min\left\{\frac{C_{mt}}{p_{mj}}, \sum_{t=1}^T d_{jt}\right\} \cdot \left(\sum_{\substack{i=1 \\ i \neq j}}^J Y_{mijtk} + X_{mj(k-1)}\right), \quad \forall m, j, t \text{ and } k \quad (5.22)$$

$$\sum_{\substack{i=1 \\ i \neq j}}^J p_{mi} Q_{mitk} \leq CAP_{mt} (2 - X_{mj(k-1)} - X_{mjtk}), \quad \forall m, j, t \text{ and } k \quad (5.23)$$

$$\sum_{\substack{i=1 \\ i \neq j}}^J Y_{mijtk} + X_{mj(k-1)} = \sum_{\substack{h=1 \\ h \neq j}}^J Y_{mhjtk} + X_{mjtk}, \quad \forall m, j, t \text{ and } k \quad (5.24)$$

$$S_{mitk} \geq S_{mjtk} + 1 - J(1 - Y_{mjtk}), \quad \forall m, i, j, t \text{ and } k \quad (5.25)$$

$$\sum_{i=1}^J Y_{mijtk} \leq MM_{mj}, \quad \forall m, j, t \text{ and } k \quad (5.26)$$

$$\sum_{i=1}^J Y_{mjitk} \leq MM_{mj}, \quad \forall m, j, t \text{ and } k \quad (5.27)$$

$$X_{mjtk} \leq MM_{mj}, \quad \forall m, j, t \text{ and } k \quad (5.28)$$

$$BM * X_{mjtk} \geq Q_{mjtk}, \quad \forall m, j, t \text{ and } k \quad (5.29)$$

$$Q_{mjtk} \geq X_{mjtk}, \quad \forall m, j, t \text{ and } k \quad (5.30)$$

$$I_{j0}^+ = I_j^{0+}, \quad \forall j \quad (5.31)$$

$$I_{j0}^- = I_j^{0-}, \quad \forall j \quad (5.32)$$

$$Y_{mijtk}, E_{mjqt}, X_{mjtk} \in \{0,1\}, \quad \forall m, i, j, t \text{ and } k \quad (5.33)$$

$$C_{mjtk}, A_{mjtk}, ZS_{mjtk}, V_{mjtk}, Q_{mjtk}, I_{jt}^+, I_{jt}^- \geq 0, \quad \forall m, j, t \text{ and } k \quad (5.34)$$

Constraints (5.2) and (5.3) calculate the completion time of each items in position $k=1$. Constraints (5.4) and (5.5) calculate the completion time of each items in position $k > 1$. Constraint (5.6) states that if job h is processed on machine m in period t , it will be immediately followed by at most one other job j on the same machine. Constraint

(5.7) ensures that only one job can be processed at each position k and period t . Constraint (5.8) ensures that each job can be processed only one position in each period t . Constraint (5.9) ensures that each job can be processed only one machine m in each position k at each period t . Constraints (5.10) - (5.13) calculate the start times of the set up operations. Constraints (5.14) - (5.17) calculates the completion times of the setup operations. Constraints (5.18) - (5.19) ensures that the server is processing only one setup operation at any time. Constraint (5.20) is the typical inventory balance constraints. Constraints (5.21) - (5.23) satisfy that the capacity constraints limit of each machine in each periods. In constraint (5.22), it is guaranteed that the production of item type j only occurs in case the setup of production is ready in each machine m during period t . In this case, it is no matter whether setup state change to other item or not. Constraint (5.24) is the typical setup balance constraints. Constraint (5.25) is the typical subtours elimination constraints. Constraints (5.26) - (5.28) are the machine eligibility constraints. Constraints (5.29) - (5.30) are the logic constraints of decision variables. Constraints (5.31) - (5.32) set the initial value for inventory, backorder. Finally, constraint sets (5.33) - (5.34) define the domain of the variables.

5.4 Test Instances

The number of machines (m) are 3, 4, and 5. The number of products depends on the number of machines and is taken as between m and $2*m$. The processing times (p_{mj}) are derived from a uniform distribution between 5 and 20. Setup times (α_{qj}) are derived from $U(5,10)$. The machine eligibility matrix is constructed with probability $P_H=70\%$ - 90% . Due to the considerable volatility of market demand, the demand is set at three levels: low, medium, and high $U(5,50)$, $U(20,100)$, and $U(50,200)$. The machine capacity (Cap_{mt}) in each period are taken fixed as 300. The production cost pc_{mj} is taken as $p_{mj}/10$. The backorder and holding cost derived from uniform distribution between 1 and 3. The planning horizon is fixed at $T=4$ and the position number is taken as equal to number of products.

5.4.1 Computational Results of MIP

The MIP model is coded using IBM ILOG CPLEX version 12.10. The nine problem instances are allowed to run only 1 hours. The obtained results for the problem instances are given in Table 5.1.

Table 5.1 The results of problem instances

Nm/ Demand	Objective	LB	Gap	Time (sec)
3/L	3518	3518	0,00	1802,30
3/M	6496	6487	0,14	3600
3/H	14592	14592	0,00	292,77
4/L	3017	2331	22,73	3600
4/M	10133	9730	3,97	3600
4/H	18440	18338	0,55	3600
5/L	4801	4801	0,00	2641,44
5/M	10809	10495	2,9	3600
5/H	22401	21191	5,4	3600

5.5 Multi Objective Model

In the mathematical model of the multi-objective model, the same notation of the single-objective model mentioned in section (5.3) is used. The model has new second objective Z_2 which minimizes the total production amount on alternative machines.

$$\begin{aligned}
 \text{Min } Z_1 = & \sum_{j=1}^J \sum_{t=1}^T (h_j I_{jt}^+ + b_j I_{jt}^-) + \sum_{m=1}^M \sum_{j=1}^J \sum_{\substack{h=1 \\ h \neq j}}^J \sum_{t=1}^T \sum_{k=1}^K (\alpha_{jh} Y_{mjhtk}) \\
 & + \sum_{j=1}^J \sum_{m=1}^M \sum_{t=1}^T \sum_{k=1}^K (pc_{mj} Q_{mjtk})
 \end{aligned} \tag{5.35}$$

$$\text{Min } Z_2 = \sum_{j=1}^J \sum_{m=1}^M \sum_{t=1}^T \sum_{k=1}^K (AO_{mj} Q_{mjtk}) \tag{5.36}$$

5.5.1 Weighted Sum Scalarization Method

The Weighted Sum Scalarization (WSS) method, each objective function c is assigned to a weight (w_c) and the total objective is minimized. Different objective functions are integrated into a single objective function as follows:

$$\text{Minimize } \sum_{c=1}^c w_c f_c(x) \text{ s. t. } x \in \Omega \quad (5.37)$$

In this method, non-dominated solutions are calculated by trying different weights for the objective functions. The w_c parameters are determined by decision makers. The weighting method can also be used with normalization.

5.5.2 Computational Results of Multi-Objective MIP

CPLEX results of the multi-objective mathematical model for the problem instances are shown in Tables 5.2 - 5.4. In the weighted sum method, the objectives are evaluated by giving a certain weight to each objective. For instance, when $w_1=1$ and $w_2=9$, this means that the second objective function is 9 times more important than the first objective function. Therefore, in 3/L problem instances, decreasing w_2 results in increasing the second objective function z_2 value. Similarly, in the same problem instances, increasing w_2 values results in decreasing the first objective function z_1 value.

Table 5.2 CPLEX results of the multi-objective mathematical model for the three machine problem instances

	W₁	W₂	Z₁	Z₂	OBJ	Time (sec)
3/L	1	9	3889	8	3961	497,36
	2	8	3679	52	7774	178,94
	3	7	3621	70	11353	689,72
	4	6	3532	117	14830	966,27
	5	5	3532	117	18245	383,70
	6	4	3532	117	21660	789,13
	7	3	3532	117	25075	3600
	8	2	3518	159	28462	1174,61
	9	1	3518	159	31821	1365,64
3/M	1	9	6510	9	6591	3600
	2	8	6496	12	13088	3600
	3	7	6496	12	19572	3600
	4	6	6496	12	26056	3600
	5	5	6496	12	32540	3600
	6	4	6496	12	39024	3600
	7	3	6496	12	45508	3600
	8	2	6496	12	51992	3600
	9	1	6496	12	58476	3600
3/H	1	9	14606	9	14687	3600
	2	8	14592	12	29280	3600
	3	7	14592	12	43860	3600
	4	6	14592	12	58440	3600
	5	5	14592	12	73020	3600
	6	4	14592	12	87600	3600
	7	3	14592	12	102180	3600
	8	2	14592	12	116760	3600
	9	1	14592	12	131340	3600

Table 5.3 CPLEX results of the multi-objective mathematical model for the four machine problem instances

	W_1	W_2	Z_1	Z_2	OBJ	Time (sec)
4/L	1	9	3104	17	3257	3600
	2	8	3405	22	6266	3600
	3	7	3041	24	9291	3600
	4	6	3046	27	12436	3600
	5	5	3029	33	15310	3600
	6	4	3021	33	18258	3600
	7	3	3015	36	21213	3600
	8	2	3016	37	24202	3600
	9	1	3002	34	27052	3600
4/M	1	9	10297	12	10405	3600
	2	8	10177	37	20650	3600
	3	7	10035	83	30686	3600
	4	6	10027	87	40630	3600
	5	5	10028	87	50575	3600
	6	4	10029	87	60522	3600
	7	3	10047	86	70587	3600
	8	2	10012	116	80328	3600
	9	1	10027	115	90358	3600
4/H	1	9	18446	12	18554	3600
	2	8	18448	12	36992	3600
	3	7	18451	12	55437	3600
	4	6	18434	16	73832	3600
	5	5	18432	16	92240	3600
	6	4	18435	16	110674	3600
	7	3	18431	16	129065	3600
	8	2	18431	16	147480	3600
	9	1	18433	16	165913	3600

Table 5.4 CPLEX results of the multi-objective mathematical model for the five machine problem instances

	W₁	W₂	Z₁	Z₂	OBJ	Time (sec)
5/L	1	9	4581	14	4707	3600
	2	8	4579	14	9270	3600
	3	7	4663	17	14108	3600
	4	6	4451	68	18212	3600
	5	5	4801	0	24005	3600
	6	4	4627	46	27946	3600
	7	3	4801	0	33607	3600
	8	2	4801	0	38408	3600
	9	1	4428	69	39921	3600
5/M	1	9	10809	0	10809	3600
	2	8	10809	0	21618	3600
	3	7	10809	0	32427	3600
	4	6	10809	0	43236	3600
	5	5	10809	0	54045	3600
	6	4	10809	0	64854	3600
	7	3	10809	0	75663	3600
	8	2	10809	0	86472	3600
	9	1	10809	0	97281	3600
5/H	1	9	22542	12	22650	3600
	2	8	22530	12	45156	3600
	3	7	22769	0	68307	3600
	4	6	22397	65	89978	3600
	5	5	22394	65	112295	3600
	6	4	22454	44	134900	3600
	7	3	22537	42	157885	3600
	8	2	22414	65	179442	3600
	9	1	22463	44	202211	3600

5.6 Conclusion

In this study, the multi-machine capacitated lot sizing and scheduling problem with machine eligibility constraints in a company which produces pool equipment is discussed. A MIP model that exist in the literature has been adopted to present the considered problem. The developed model aims to minimize the total cost of the sequence dependent, inventory holding, backorder, purchase and alternative machine selection. The computational results demonstrate that the proposed model can be verified by testing the operating data and can create realistic and applicable optimal production plans that can provide all constraints. The considered problem is in the NP-hard class. Therefore, it is not possible to obtain results in reasonable time when the size of the problem is increased. For this reason, metaheuristic algorithms can be proposed to solve the problem in the future.

CHAPTER SIX

CONCLUSION

6.1 Summary of the Thesis

The production planning problem is crucial in the context of production economics. Lot sizing and scheduling problems are one of the important production planning problems which aims to minimize the total cost of production, setup, and inventory. In these problems, the decision on the production period and quantities of each production with respect to demand are determined. Depending on the application area, several variants of the problem have been proposed in the literature, and most of the lot sizing and scheduling problems are known as combinatorial and hard to solve. Even today, manual handling of this complex problems in many industries would not only result in increased expenses but would also impair both company and customer satisfaction. This dissertation focuses on two crucial operational level decision making problems in paper bag and pool equipment industry. The selection of these two problems is motivated by the fact that both problems are common encountered problems. Effective production plans which include lot sizing and scheduling decision would improve industry's economy and satisfaction level in this field of study.

Therefore, four chapters have been designated to tackle these valuable lot sizing and scheduling problems. Chapter 2 is devoted to the GLSP problem and Chapters 3 and 4 are devoted to the paper bag industry problems. Chapter 5 is devoted to the pool equipment industry problems.

In Chapter 2, GLSP in single level capacitated environments with sequence dependent item changeovers. The proposed model simultaneously determines the production sequence of multiple items with capacity-constrained dynamic demand and lot size to minimize overall costs. First, a MIP model for the GLSP is developed in order to solve smaller size problems. Afterwards, a matheuristic algorithm that integrates SA algorithm and the proposed MIP model is devised for solving larger size problems. The proposed matheuristic approach decomposes the GLSP into

subproblems. The proposed SA algorithm plays the controller role. It guides the search process by determining values for some of the decision variables and calls the MIP model to identify the optimal values for the remaining decision variables at each iteration. Extensive numerical experiments on randomly generated test instances are performed in order to evaluate the performance of the proposed matheuristic method. It is observed that the proposed matheuristic based solution method outperforms the MIP and SA, if they are used alone for solving the present GLSP.

In Chapter 3, Capacitated lot sizing is a frequently encountered production-planning problem in manufacturing companies. However, many real life factors are usually not taken into account during the model development phase for these problems. In this chapter, an integrated lot sizing and scheduling problem of a paper bag production firm is considered. A MIP model is proposed for modeling and solving the problem. Setup cost/time between product and period changes are considered in the proposed model. It is aimed to minimize the total cost of production, holding inventory, backorder, procurement and setup for product type change and product period change. It is observed that productive utilization of manufacturing resources and generation of more effective production plans is possible by the application of the proposed model.

In Chapter 4 addresses a practical production issue in the paper bag industry that contains the problem of capacitated lot sizing and scheduling with cutting plan assignment. We propose a MIP model for the integrated problem. The proposed model considers specific constraints such as backordered and defective items, external procurement, and sequence dependent setup times. There are numerous applications exist in industrial settings such as furniture, molded pulp, wood, foundry and commonly in paper industries. The model under consideration is based on a real-world case study from the paper bag industry. To handle the industrial case, a problem-based matheuristic approach that resolves the lot sizing and assignment problems separately is provided based on this mathematical formulation. Random instances have been generated, and the performance of the models is tested. In addition, we conducted sensitivity analyses on setup time, capacity, and procurement cost. The results

demonstrate the validity of the proposed model and offer practitioners valuable insights.

In Chapter 5, the lot sizing and scheduling problem with the additional considerations of machine eligibility, sequence-dependent setups, and common server operator is considered. This study examines the multi-machine capacitated lot sizing and scheduling problem with machine eligibility constraints. In this problem, not all machines in the facility can be eligible for every product and product types can be produced on several alternative machines. Such problems appear in several industrial applications and one of them is quite common in the pool equipment manufacturer industry. Therefore, a MIP model is constructed for the problem. In order to show the efficiency of the model, we perform several analyses. The considered problem is also formulated as a multi-objective MIP model. Despite the prevalence of metaheuristics in this type of multi-objective problem, we attempt to solve it by transforming the two objectives into a single objective via weighted sum scalarization approach. Computational experiments indicate that the constructed model meets the practical requirements for the pool equipment manufacturer industry.

6.2 Contributions

This dissertation contributes to the literature in a variety of ways. The original contributions are summarized in the following paragraphs.

First of all, a matheuristic approach is proposed for solving the GLSP better than MIP and SA. Furthermore, micro period analysis on the randomly generated test instances is presented and new valid inequalities are proposed for the GLSP.

Secondly, we extend existing models to include new parameters such as external procurements, setup times, defective items, and cutting plan assignment. These criteria increase the problem's applicability to the real-world paper bag production business. To our knowledge, this is the first time that all of these parameters are considered at the same time. We propose a new MIP model for the capacitated lot sizing and

scheduling problem. The proposed new model can assign cutting plans and scheduling of these products according to the inventory balancing constraints to meet demands and capacity constraints. The assignment of cutting plan configurations can change the total cost and time. Significant inventory and waste material reductions may be possible due to the flexibility of these configurations' selection and their interaction with scheduling and lot sizing decisions (Boas et al., 2021). In terms of the solution approach, a problem-based matheuristic is proposed to solve the problem efficiently. We offer managers practical management insights through a thorough sensitivity study of the model's parameters.

The last contribution to the literature is that, multi objective lot sizing and scheduling with additional considerations of machine eligibilities, sequence dependent setups, and one common server is firstly successfully formulated.

6.3 Future Research Directions

From applying the solution strategies to additional models to extending existing models with extra constraints and objectives, there exist several future research directions originated from this thesis.

On the solution approaches, the proposed matheuristic methods can be hybridized with other MIP-based heuristics or metaheuristic. The problems under consideration can be studied in an uncertain environment using stochastic, fuzzy, or random variable parameters. Future study could also focus on metaheuristic techniques for solving the multi-objective optimization problem.

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