DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

FREE VIBRATION ANALYSIS OF CURVED SANDWICH BEAMS

by
Yusuf BERKTAŞ

August, 2015
İZMİR

FREE VIBRATION ANALYSIS OF CURVED SANDWICH BEAMS

A Thesis Submitted to the
Graduate School of Natural and Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Degree of Master of Science
in Mechanical Engineering, Mechanics Program

by
Yusuf BERKTAŞ

August, 2015
İZMİR

M. Sc. THESIS EXAMINATION RESULT FORM

We have read the thesis, entitled "**FREE VIBRATION ANALYSIS OF CURVED SANDWICH BEAMS**" completed by **YUSUF BERKTAŞ** under supervision of **ASSOC. PROF. DR. BİNNUR GÖREN KIRAL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

....

Assoc. Prof. Dr. Binnur GÖREN KIRAL

Supervisor

....

Jury Member

Doç. Dr. Hasan ÖZTÜRK

....

Jury Member

Doç. Dr. Hasan SELİM

Prof. Dr. Ayşe OKUR

Director

Graduate School of Natural and Applied Sciences

ACKNOWLEDGMENTS

I would like to express my thanks and appreciations to my supervisor, Assoc. Prof. Dr. Binnur GÖREN KIRAL, for giving me this great opportunity to work on this project. Her invaluable guidance, patience and encouragement will always be appreciated. I have been extremely lucky to have a supervisor who cared so much about my work, and who responded to my questions and queries so promptly.

Also, I am very grateful to my parents for their understanding, support and love. They worked very hard to support me all over past years. They are always my biggest motivation to overcome any obstacles that I encountered.

I would like to express my heartfelt thanks to my girlfriend and my friends. Thanks for your love and encouragement. Thanks for your understandings and for giving me so much freedom to pursue my research dream that now is finally complete.

Yusuf BERKTAŞ

FREE VIBRATION ANALYSIS OF CURVED SANDWICH BEAMS

ABSTRACT

Composite sandwich materials have found a place in many industrial applications because they have high rigidity and strength properties besides their lightness in recent years. Thanks to highly developed production technology, sandwich composites which their areas of use expand have been used in aviation, space and defense industries. Considering the area of usage, it is important to determine the mechanical properties for the safe design. One of the most important structural features is definitely vibration response of the system.

The aim of this thesis is to investigate effect of the curvature on the vibration response of the composite sandwich beams experimentally and theoretically. Within the scope of this study, natural frequency values and mode shapes of the curved sandwich composite beams are examined for different geometrical parameters. In the numerical analyses, natural frequency values of and associated mod shapes of the curved sandwich composite beam are determined using ANSYS program based on the finite element method. Numerical natural frequency values are compared with the experimental results. Experimental studies are carried out for the clamped-free (cantilever) boundary condition. It is observed that there is good agreement between the experimental and numerical results. Numerical analyses are also performed to investigate the effect of the boundary condition on the free vibration response of the curved sandwich composite beams.

Keywords: Sandwich composite, curved beam, natural frequency, free vibration, finite element method.

EĞRİSEL SANDVİÇ KİRİŞLERİN SERBEST TİTREŞİM ANALİZİ

ÖZ

Sandviç kompozit malzemeler, hafiflikleri yanı sıra yüksek rijitlik ve mukavemet özelliklerine sahip oldukları için son yıllarda birçok endüstriyel uygulamada yer bulmaktadırlar. Gelişen üretim teknolojileri sayesinde kullanım alanları oldukça genişleyen sandviç kompozitler özellikle havacılık, uzay ve savunma sanayinde kullanılmaktadırlar. Kullanım alanları dikkate alındığında, güvenli tasarımlar için mekanik özelliklerinin belirlenmesi önemlidir. Yapısal özelliklerinin en önemli göstergelerinden biri şüphesiz sistemin titreşim cevabıdır.

Bu tezin amacı, eğriliğin sandviç kompozit kirişin titreşim cevabı üzerine etkisini deneysel ve teorik olarak incelemektir. Bu çalışma kapsamında; eğri sandviç kompozit kirişlerin doğal frekans ve titreşim biçimleri farklı geometrik parametreler için incelenmiştir. Sayısal analizlerde, eğrisel sandviç kompozit kirişin doğal frekans değerleri ve ilgili titreşim biçimleri, sonlu elemanlar yöntemini temel alan ANSYS paket programı kullanılarak belirlenmiştir. Sayısal doğal frekans değerleri, deneysel sonuçlarla karşılaştırılmıştır. Deneysel çalışmalar sabit-serbest (ankastre) sınır koşulu için gerçekleştirilmiştir. Deneysel ve sayısal sonuçlar arasında uyum olduğu gözlemlenmiştir. Sayısal analizler ayrıca sınır koşulunun eğri sandviç kompozit kirişlerin serbest titreşim cevabına etkisini incelemek için gerçekleştirilmiştir.

Anahtar kelimeler: Sandviç kompozit, eğri kiriş, doğal frekans, serbest titreşim, sonlu elemanlar yöntemi.

CONTENTS

LIST OF FIGURES

Page

LIST OF TABLES

CHAPTER ONE
INTRODUCTION

1.1 Literature Survey

So far, it has been reviewed numerous studies on free vibration of curved beams. As it can be seen from the studies of Petty (1971) and Rossettos (1978) even with very little curvature has a significant influence on the dynamic behavior of the system such as natural frequencies and mode shapes. However, studies which have been performed generally examine the vibration of curved beams made of isotropic material (Gunsmith & Self, 1998).

In curved beams made of sandwich material, Ahmed (1971) studied on the determination of effect of the changing geometric parameters to the resonance frequency by using the finite element method in 1971. It is possible to find used theoretical models for the analysis of sandwich structures in the book of Zenkert (1995) and the detailed classification about the creation of sandwich model of panel and shell in the article of Noor (1996). While these resources reveal the importance of curvature in the responses of vibration of sandwich composites, they also show that there aren't so many studies on the subject recently.

Sakiyama (1997) studied vibration analysis of curved sandwich with three layered elastic and viscoelastic core. Similarly Vaswani (1998) carried out vibration and damping analysis of curved sandwich. On the other hand, Kwon (2007) examined general trends of cylindrical composite panels with different damping properties of hybrid natural frequencies.

Dynamic behavior of sandwich beams and panels has been an object of many studies. To investigate free vibrations of sandwich structures both classical and shear plate theories have been exploited (Reddy & Wang, 2000). It is a well-established fact that the lowest vibration frequency of the beam belongs to the flexural bending mode, and this mode has been investigated thoroughly (Vinson, 1999). It is also known that there exist other vibration modes that involve shear deformations (Lee &

Chang, 1979). Nilsson considered sound transmission through infinite sandwich plates that were modeled on the basis of the three dimensional classical theory of elasticity (Nilsson, 1990). In addition to the low-frequency waves that correspond to the flexural bending mode, in-phase and out-of-phase waves propagating longitudinally in the sandwich panel as well as non-propagating through the thickness waves were identified in his model. This result implies that one should expect four different types of eigenmodes of vibration in a finite sandwich beam. In order to describe these four types of modes, one has to include the flexibility of the core in modeling sandwich structures, particularly for the thick core sandwiches.

These four types of modes could not be predicted on the basis of the theory of laminates, which has been used extensively to model the dynamics of sandwich beams (Fasana & Marchesielo, 2001). Laminate theory does not take into account the transverse flexibility of the core nor its longitudinal deformations. This last omission is also characteristic for a high-order shear deformation model developed by one of the authors earlier (Bozhevolnaya & Frostig, 2001).

Analysis with the help of the finite element method does not reveal the whole range of the eigenmodes in the case of sandwich beams and plates. This is due to the fact that the finite element method (FEM) modeling is usually performed on the basis of plate/shell elements (Singh, 1999). Such an approach excludes shear modes from consideration, while a modeling with the volume elements seems to be technically challenging and computationally intensive (Burton & Noor, 1997).

The growth in practical applications of sandwiches in the high performance vehicles needs a more attentive investigation of all vibration modes of the structure. This is due to the fact that ultrasound debonding detection exploits the through the thickness vibration mode, and it is widely used for the maintenance of the structures (Jensen & Irgens, 1999). The purpose of this paper is to study dynamic behavior of the cylindrically curved sandwich beams on the basis of the first order shear deformation theory which includes longitudinal deformation of the core taken into consideration and thus ensures a determination of all types of the vibration modes in the sandwich beam.

In the literature, the constrained layer damper lends itself to a sandwich construction and is often referred to as sandwich structures (Soovere & Drake, 1985). In this paper, the sandwich structure is referred to as a construction made of two thin and stiff skins (faces) and a thick soft foam core. Nevertheless, many modeling approaches for the constrained layer damper are very similar to the ones in the sandwich literature.

Another important factor that largely affects the results of the sandwich beam analysis is the assumed vibration behavior of the layers. The simplest sandwich beam model utilizes Euler-Bernoulli theory for the face layers and only allows the core to deform only in shear. This assumption has been widely used in several dynamic stiffness methods (DSM) and finite element method (FEM) studies such as those by several researcher and scientist (Banerjee, 2003; Mead & Markus, 1968; Fasana & Marchesiello, 2001; Adique & Hashemi, 2007; Abrate, 2011). In more recent publications, Banerjee derived two new dynamic stiffness method models which exploit more complex displacement fields. In the first and simpler of the two (Banerjee & Sobey, 2005), the core bending is governed by Timoshenko beam theory, whereas the face plates are modeled as Rayleigh beams.

In 1989, Ha went over finite elements applied to the sandwich plate (Ha, 1989), and also Bert (1991) conducted a review study of the analysis of sandwich plates. Being recently introduced, the reports containing references to more than 800 of Zenkert (1995), Noor & Burton and Bert (1996) as a complementary bibliography are available.

1.2 Historical Development of Sandwich Materials

Historical development of sandwich materials, are summarized in a book which is published by Vinson (1999). According to information’s which take place in this book, the history of the structure extends until the study of Fairbaim (1849) as reported in the article of Noor, Burton and Bert (1996). Feichtinger (1988) refers that the concept of the sandwich structure, in the United States during World War II, consists of two reinforced plastic shell and low-density from the core. In 1943

Wright Patterson Air Force Base has been designed and manufactured the body of the aircraft Vultee BT - 15 by using glass / fiber reinforced polyester as shell, honeycomb and balsa wood as the core (Rheinfrank, 1944).

The first research article on sandwich structures was associated with the burden of the press of the plane and belonged to Marguerite (1944). Hoff (1950) produced differential equations for deflection and buckling of sandwich plates by using the principle of virtual displacement. Flugge (1949) has made broadcast on structural optimization studies which are determined head load and minimum weight and also core features for weight and maximum press load, its geometrical dimensions and strength of sandwich which are given. For all occasions which are worked, the materials are accepted as isotropic and the damages such as wrinkle, sprains are considered. The unloaded edges are taken free or simple headstock. Flugge (1952) then by continuing this work published.

Bijlaard (1951), interested in the topic of optimization sandwich which influences maximum buckling load, is calculated the ratio of the modules of elasticity of the core and shells and lastly is examined the weight is applied to the surface area of the sheet. He also has enlarged his optimization studies for the rate between shell / core thickness which belongs to the sandwich plate.

Eringen (1951) has published a report which he calculates sandwich panels, consisted of the isotropic core and shells, strike with the effect of shear deformation. In this study, the bending rigidity of shells has been also taken into account. In the representation in distortion, developments and results which Fourier series are used are presented. At the same time, for the shape changing elements in sandwich panels, which have orthotropic shell and core, the general expressions are presented. Eringen (1952) found differential equations of rectangular sandwich plates, which have the shell and core edge under the conditions of varying load and isotropic, for buckling and bending by utilizing the theory of minimum potential energy. March (1952), he has worked on sandwich panels under lateral loading head. This study is basically isotropic shell and core for sandwiches. The statement of Warping, bending moment and reaction are rarely encountered. Ericksen (1956) examined distortion of panels by using the Fourier series again. March (1955) has reported the work of the differential equations of rectangular sandwich plates, which have the shell and core edge under the conditions of varying load and isotropic. The final report for design and optimization is offered by Ruville (1955). Rectangle, in simply supported panels for warpage and residual stresses theoretical expressions are developed by using the Fourier series. Gerard (1956) presented the optimization of the plate of sandwiches as a summary of a section of a book called minimum weight analysis of compression structures.

Kaechele (1957) has published a report on the minimum weight design of sandwich panels. The plain, simply supported panels under uniaxial load press when their load, width, the stress-strain curves are given for the shell material, a method is found to determine the optimum configuration. Besides, he has examined the effects of edge conditions on the maximum acceptable shell stretching, core strength and density. He has applied the method to the panels which have hexagonal and square cell with honeycomb cores and has studied the effects of high temperature. The other serial solution which contains distortion, shear and moment expressions is presented by Cheng (1962). Heath (1960) published an article upon corrected and expanded the present theory which belongs to exposed the pressure to the flat panels. Second part of this article deals with the optimal design of flat sandwich panels.

The first theoretical studies about the behavior of rectangular sandwich panels subjected to lateral loads is limited with uniform load and simply supported edge conditions. After the initial period of the World War II, American forest products laboratory (USFPL) has made a big contribution group in the development of methods in design and analysis of sandwich structures. Until the period in the mid - 1960s, the studies which have been made on sandwich structures has been extended. Plantema (1966) published his first and famous book for sandwiches. The book of Allen (1969) followed this. These are the most fundamental books in this area.

In the middle of the 1960s, the Naval Air Engineering Center supported the studies which had done for the development of fiber-reinforced sandwich structures that can compete with conventional aluminum structures used for aero planes. The majority of these studies, for the same loading conditions, focus on minimum weight optimization of sandwich structures used fiber-reinforced sandwich which has less weight than aluminum structure.

The report which took place of a publication exceeding 250 related to the sandwich structure belonged to before 1966 was prepared by Vinson (1967). Since 1960s, the use of sandwich structures has increased rapidly and developed publications on the topic. The book of collected articles of Hoff as a guide which is about sandwich structures (Hoff, 1986).

In this thesis, investigation of the effect of the curvature on the vibration response of the curved composite sandwich beams is aimed. To this end, natural frequency values and mode shapes of the curved sandwich composite beams are examined for different geometrical parameters experimentally and theoretically. In the numerical analyses, natural frequency values of and associated mod shapes of the curved sandwich composite beam are determined using ANSYS program based on the finite element method. Numerical natural frequency values are compared with the experimental results. Experimental studies are carried out for the cantilever beams. It is seen that there is good agreement between the experimental and numerical results. Numerical analyses are also performed to investigate effect of the boundary condition on the free vibration response of the curved sandwich composite beams.

CHAPTER TWO
SANDWICH COMPOSITE MATERIALS

2.1 Composite Material

Two or more materials in the same or different materials of groups, in order to get together the best characteristics or get new features, the new features which are composed by combining at the level of macro are called as composite material, shown in Figure 2.1. Fiber glass polyester panels, steel reinforced concrete structures and vehicle tires are the main examples (Kaw, 2006).

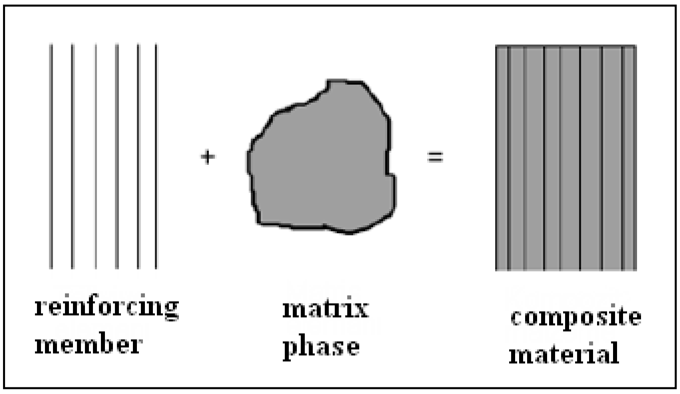


Figure 2.1 Composite materials

The union between groups of materials which form composite material is significant to be in macroscopic sizes. It is out of the question to constitute of chemical reactions or the situation of establishing a chemical bond between materials. That is, while composite material shows feature of heterogeneous material in terms of microscopic, it is a homogeneous supply in terms of macroscopic.

2.2 Classification of Composite Materials

Composite materials are basically comprised of two separate phases which are named as matrix and the reinforcing element. Matrix phase which holds together the reinforcement phase and shares load is main phase. Reinforcement phase is the secondary phase within matrix and it increases the strength and rigidity of matrix shown in Figure 2.2.

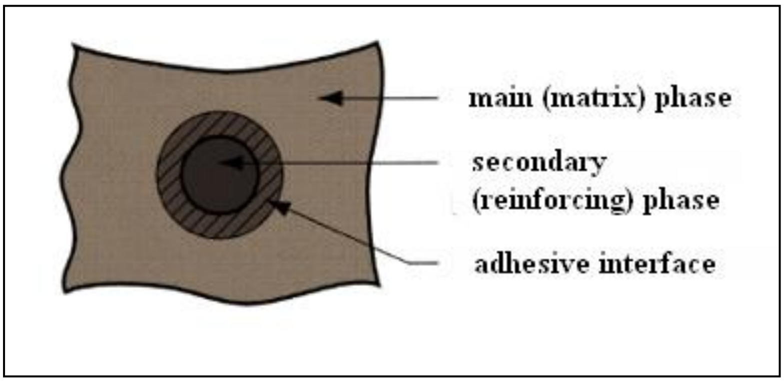


Figure 2.2 Basic components

Although there aren't any certain rules in the classification of composite materials, this basis is able to make classification according to phases.

2.2.1 Classification According to Matrix Phase

- Polymer matrix composite materials
- Metal matrix composite materials
- Ceramic matrix composite materials

2.2.1.1 Polymer Matrix Composite Materials

Polymer composite materials form nearly 90 percent of composite materials. Polymer matrices constitutively are divided into two groups as thermosets and thermoplastics. Polyester and epoxy resins are the most used thermosets in the industry of composite.

Mostly fibers are constantly used in polymer composite materials as reinforcing element. The major of these are glass fiber, Kevlar fiber, boron fiber and carbon fiber.

The main areas of application of polymer matrix composites can be shown as the areas of marine applications due to its corrosion resistance, also automotive and other transportation industries and sporting equipment’s (See, Figure 2.3) because of its lightness, and lastly like automotive interior decoration which is requested with fire resistance property.

Figure 2.3 Polymer matrix composite materials manufacturing by using fiberglass and epoxy resin

Within the scope of this study our sample materials that we perform the production to use for experiments are included in the group of polymer composite materials. In our samples, constantly braided glass fiber as a reinforcing element and polyester resin from thermoset as matrix material are used.

2.2.1.2 Metal Matrix Composite Materials

Metal matrix composite materials are the composed materials which its main structure (matrix phase) is formed of metal and are usually used the reinforcement phase of a ceramics or metals (See, Figure 2.4). There is almost no limitation in the selection of these materials.

Wear-resistant, high fracture toughness and compressive stress materials are obtained with the combination of high elastic modulus of ceramics and plastic deformation of metal properties. These composites are widely used in the aerospace and defense industries.

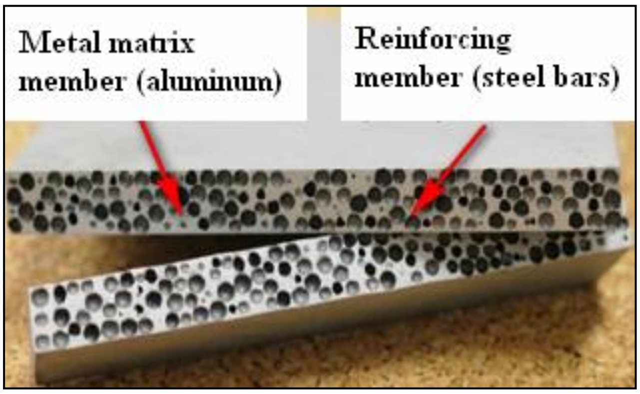


Figure 2.4 Metal matrix composite material

2.2.2 Classification According to the Reinforcement Phase

- Fiber-reinforced composite materials
- Particle-reinforced composite materials
- Layered composite materials

2.2.2.1 Fiber-Reinforced Composite Materials

It's a kind of composite materials which is obtained with the distribution of fine staple fibers within matrix (See, Figure 2.5). The distribution of fine staple fibers within matrix is an important factor which majorly influences its strength. While providing maximum strength in the direction of fiber, minimum strength values are seen in perpendicular to the fibers.

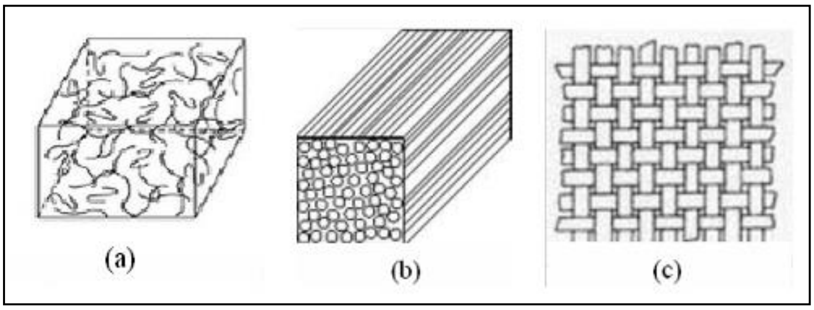


Figure 2.5 Schematic representations of the fiber-reinforced composite materials (a) particle-reinforced (b) Continuous fiber reinforced (c) knitted fabric for continuous fiber reinforced

2.2.2.2 *Particle-Reinforced Composite Materials*

Reinforcement particles dispersed within the matrix phase. Composite particles dispersed in the matrix depend on the hardness of the strength of the structure.

The most common example is metal particulate structures which take part in the elastic matrix shown in Figure 2.6. While metal particles provide electrical and thermal conductivity, it also increases the strength of structure.

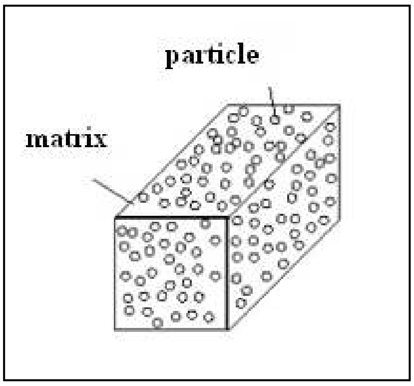


Figure 2.6 Schematic representation of particle-reinforced composite material

2.2.2.3 *Laminated Composite Materials*

Laminated composite material is a type of material which has the most common usage. It is a composite structure which is obtained by gathering layers which have fiber with variable angles and provide really high strength values (See, Figure 2.7). In comparison to metals, it is preferred due to its light weight and strength of materials. Sandwich composites are also in this group.

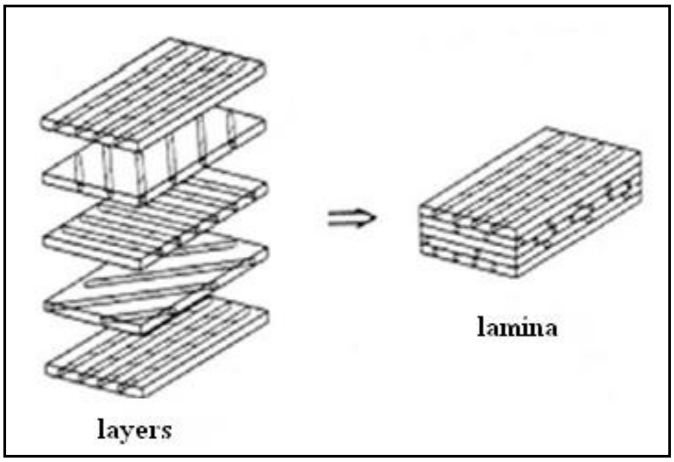


Figure 2.7 Schematic representation of layered composite material

2.3 Sandwich Structure

The needs of high performance of today's industrial applications have made necessary to search for new materials. Sandwich materials are the materials that we can include into these new material groups. Sandwich composite materials, since they have a high flexural rigidity/weight ratio, they are used in many industrial applications nowadays.

Sandwich materials are composed of basically three components shown in Figure 2.8. These are; face / skin, adhesive interface and core materials.

- Layers called as shell on the top and bottom surfaces have a thinner and more durable structure than core. Shell carries a large part of normal stress on structure.
- Although core is much thicker than shells, it is light and flexible component. It carries shear stress through thickness. Their outer surfaces (shells) keep away from the natural axis (this distance ensures to be high that moment of inertia which belongs to the cross-sectional area of sandwich material and also increases the bending rigidity) and thus increase the flexural rigidity.
- The adhesive element which is used interface of adhesion is layer which ensures the core and the shells stay together and its thickness is so little that it can be neglected.

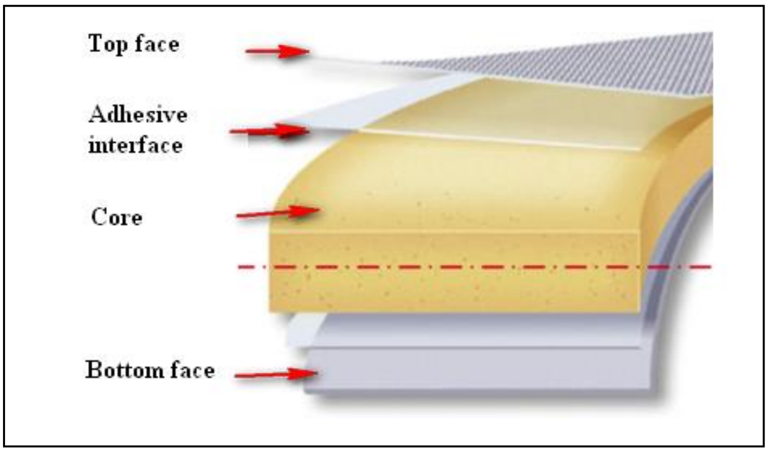


Figure 2.8 Basic elements of the sandwich structure

The most basic features that are expected from sandwich materials;

- A core with a low density and shell plates with strength
- It’s a link to paste the rigid and durable.

2.3.1 The Materials Used in Sandwich Structures

Sandwich composite structures as we mentioned frequently are consisted of two basic elements which are named as core and shell (surface). The materials which are used for the purpose of obtaining these two elements are as follows.

2.3.1.1 The Materials Used to Obtain the Shell Layer (Surface)

The most basic features that are looking for in sandwich structure are lightness and rigidity (different specifications can be looking for according to the place of use). Since shell plates on the top and the bottom surfaces provide the main rigidity, the materials which are used to obtain these surfaces should have a structure as lightweight and rigid as possible.

Although traditional materials are rigid such as steel, they aren't generally preferred due to their high weights in sandwich structures. Composites which are our today's materials come into prominence with their lightweight as well as their rigidity are the elements which are the most used shell elements in sandwich structure.

The most widely used reinforcing materials and the matrix elements in these composites are as following;

a. Reinforcing Elements;

- Glass Fiber: It is one of the most commonly used reinforcing elements nowadays. In the scope of this project, reinforcing element which is used in the shell layer is glass fiber. Detailed information about glass fiber will be given in Chapter 4.
- Graphite Fiber: It is the most widely used reinforcing element with glass fiber.
- Aramid (Kevlar) Fiber
- Boron Fiber

b. Matrix Elements;

- Polyester Resin: It is the one of the most widely used element of matrix nowadays. In the scope of this project the polyester resin, an element of matrix, is used in the shell layer. Detailed information about polyester resin will be given in Chapter 4.
- Epoxy Resin: It is the most widely used polyester resin with matrix element.
- Vinyl-ester Resin

2.3.1.2 Core Materials Varieties

- Polyurethane Foams: It is also referred as polyvinyl chloride (PVC) and has a fairly common of usage. It can be found in 40 - 300 kg/m^3 concentrations. In the scope of this project, polyurethane foam which has 90 kg/m^3 with a density is selected as the core. Detailed information about this material will be given in Chapter 4.
- Balsa Tree: It is one of the most widely used "wood" core types thanks to its lightness. Balsa sandwich cores provide very good rigidity. However; its breakage would be sudden and large. In addition, the standardization of

materials of natural origin is impossible or very difficult. Relatively its high density doesn't give a wide choice like PVC foam.

- Honeycomb: Honeycomb can be based on metallic or composite. Metallic honeycomb is made from very thin aluminum, and composite honeycomb is made from paper or nylon/aramid fibers and epoxy or phenolic resin. Flame-resistant nomex honey comb is one of the most popular systems.

2.3.2 Appropriate Material Selection in the Sandwich Design

In the design of a sandwich structure, a plan and a specific procedure must be created. A systematic approach is needed with the combination of strength of materials, structural, mechanical, and material sciences. By considering the working conditions of the structure it should be targeted to ensure efficiency which is expected from construction. All these will be possible with detailed study and approach of engineering. In every engineering application, they input and output as they are and optimal designs must be created between defined processes with feedback (Mallick, 2007).

In order to create the optimum structure, what is expected from the material should carefully be determined as the need of design, environmental conditions should be well evaluated and should be taken measures for external factors, besides economic factors should be taken into consideration.

For this purpose in Table 2.1, the reinforcing element (fiber), matrix material (resin) and the core material of types the advantages (+) and disadvantages (-) of structural and environmental factors has been compiled (Atas & Sevim, 2010). Within the scope of this study, it is selected glass fiber as reinforcing element, polyester resin as the matrix material and polyurethane foam as the core material due to the advantages of machinability and cost.

Table 2.1 Optimum material selection

	FIBER			RESIN					CORE					
	<u>Glass</u>	Kevlar	Carbon	**<u>Polyester</u>**	Vinyl-ester	Epoxy	Phenolic	Thermo Plastic	Balsa	Crosswise Pvc	Linear Pvc	Aluminum Honeycomb	Thermo Plastic Honeycomb	**<u>Synthetic Foam</u>**
Tensile Strength	+	+	+	-	-	+	-	-	+	+	-	-	-	-
Tensile Stiffness	-	+	+	-	-	-	-	-	+	-	-	+	-	-
Comp-ression Strength	+	-	-	-	-	-	-	-	+	-	+	+	-	-
Comp-ression Stiffness	-	-	+	-	-	-	-	-	+	-	-	+	-	-
Fatigue Strength	-	+	+	-	+	+	-	+	+	-	+	-	+	-
Failure Strength	+	+	-	-	+	+	-	+	-	+	+	-	-	-
Water proofing	+	-	-	-	+	+	-	+	-	+	+	-	-	-
Flame Resistant	+	-	-	-	-	-	+	-	+	-	-	+	-	-
<u>Machina bility</u>	+	-	-	+	-	-	-	-	+	-	-	-	-	+
<u>Cost</u>	**+**	**-**	**-**	**+**	**-**	**-**	**-**	**+**	**+**	**-**	**-**	**-**	**+**	**+**

2.4 The Usage Area of Sandwich Materials

Undoubtedly the most important and outstanding features of high rigidity is lightweight sandwich structures. In this way, especially in the aviation industry since the 1940’s they are used in a wide range of different industrial areas.

Lightness, low cost and high rigidity are the most decisive competitive elements of todays. These three fundamental elements can be achieved with appropriate sandwich materials in the shell (surface), the core and adhesive by the selection of the materials. The usages of sandwich structures will undoubtdly continue to increase in future years with the researches which are done on composite materials. If it's necessary to sort of other properties what make important these structures;

- High bending strength and stiffness
- Consist of lightweight materials
- Obtaining appropriate materials with a combination of low-cost
- Thermal insulation properties
- Sound insulation properties
- Radar waves doesn't get caught
- Appropriate aerodynamic surfaces to create or modify
- Strength to high speed

We can sum up the usage areas of the sandwich materials which is widespread in the industry field of application as follows.

Lightness and stiffness are the most critical factors in aviation and space industry applications. For this reason, sandwich constructions are used such as the regions in today's aircraft designs in floor coverings, wing and tail parts and rotor parts in helicopters.

Honeycomb core materials provide high performance to sandwich structures which they are used mechanically in a lot of ways (vibration damping, high crush strength/weight ratio, heat/insulation etc.). For this reason, the honeycomb core sandwich structures are used in various fields. Wind tunnels, fans, air conditioners, air conditioning systems and heaters are some of the area of usage. In addition to these examples, honeycomb sandwich structures are used at the place of the radio wave shield, energy dissipation, air flow rectifier, satellites, solar panels, etc.

Sandwich composite structures which are produced from fiber-glass reinforced shell material and polyurethane foam core has a wide usage area in all marine vessels, both military and civilian (boat, racing boat, boat, yacht, sailing, etc.) thanks to its high resistance against corrosion and not to sink into the water.

Recently, sandwich materials are increasingly used in the construction sector and infrastructure projects. The usage in the building's exterior wall and roof coatings is increasing day by day thanks to the two outer plates (upper/lower shell) and the high thermal performance of the building structural elements which answer to the description of sandwich material filled with the insulation material inside (core). The materials such as; mineral wool, glass wool, polyurethane and polystyrene as inner core insulation material and steel, aluminum, reinforced concrete, reinforced plastic and wood as the outer coating (shell) material are used. However, it should be noted, the use of sandwich materials as the insulating material for the purpose of enhancing the thermal performance is more common rather than lightness and rigidity.

In addition, in recent years, the use of sandwich materials is becoming common in the body parts of passenger cars and sports utility vehicle, in sports equipment (race biking, snow and water skiing, canoeing, tennis racket, etc.) in the manufacture of machinery and wind turbines.

CHAPTER THREE
MECHANICAL VIBRATIONS

3.1 Basic Concepts to Vibration

Any repetitive motion is called vibration or oscillation. The motion of a guitar string, motion felt by passengers in an automobile traveling over a bumpy road, swaying of tall buildings due to wind or earthquake, and motion of an airplane in turbulence are typical examples of vibration. The theory of vibration deals with the study of oscillatory motion of bodies and the associated forces. The oscillatory motion shown in Figure 3.1 is called harmonic motion and is denoted as;

$$x\,(t) = Xcos\omega t \tag{3.1}$$

Where X is called the amplitude of motion, ω is the frequency of motion, and t is the time.

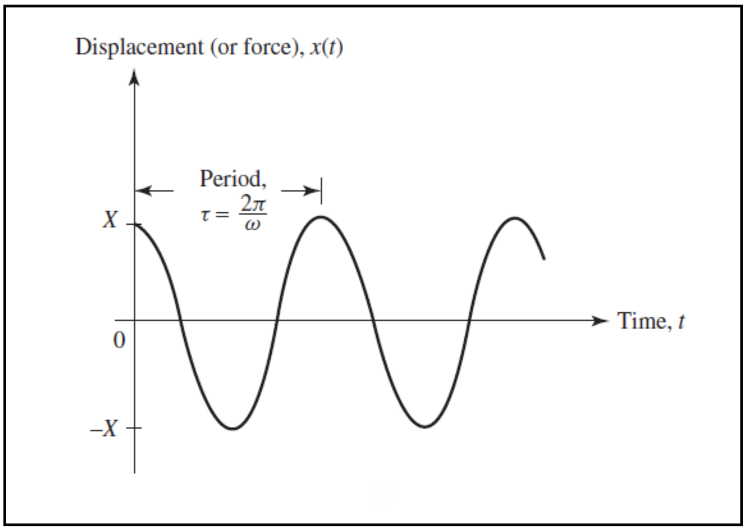


Figure 3.1 Types of displacements for periodic simple harmonic motion

Anybody having mass and elasticity is capable of oscillatory motion. In fact, most human activities, including hearing, seeing, talking, walking, and breathing, also involve oscillatory motion. Hearing involves vibration of the eardrum, seeing is associated with the vibratory motion of light waves, talking requires oscillations of the laryng (tongue), walking involves oscillatory motion of legs and hands, and breathing is based on the periodic motion of lungs. In engineering, an understanding of the vibratory behavior of mechanical and structural systems is important for the safe design, construction, and operation of a variety of machines and structures.

The failure of most mechanical and structural elements and systems can be associated with vibration. For example, the blade and disk failures in steam and gas turbines and structural failures in aircraft are usually associated with vibration and the resulting fatigue. Vibration in machines leads to rapid wear of parts such as gears and bearings, loosening of fasteners such as nuts and bolts, poor surface finish during metal cutting, and excessive noise. Excessive vibration in machines causes not only the failure of components and systems but also annoyance to humans. For example, imbalance in diesel engines can cause ground waves powerful enough to create a nuisance in urban areas. Supersonic aircraft create sonic booms that shatter doors and windows. Several spectacular failures of bridges, buildings, and dams are associated with wind-induced vibration, as well as oscillatory ground motion during earthquakes.

In some engineering applications, vibrations serve a useful purpose. For example, in vibratory conveyors, sieves, hoppers, compactors, dentist drills, electric toothbrushes, washing machines, clocks, electric massaging units, pile drivers, vibratory testing of materials, vibratory finishing processes, and materials processing operations such as casting and forging, vibration is used to improve the efficiency and quality of the process.

3.1.1 Discrete and Continuous Systems

The degrees of freedom of a system are defined by the minimum number of independent coordinates necessary to describe the positions of all parts of the system at any instant of time. For example, the spring–mass system shown in Figure 3.2 is a single degree of freedom system since a single coordinate, $x(t)$, is sufficient to describe the position of the mass from its equilibrium position at any instant of time.

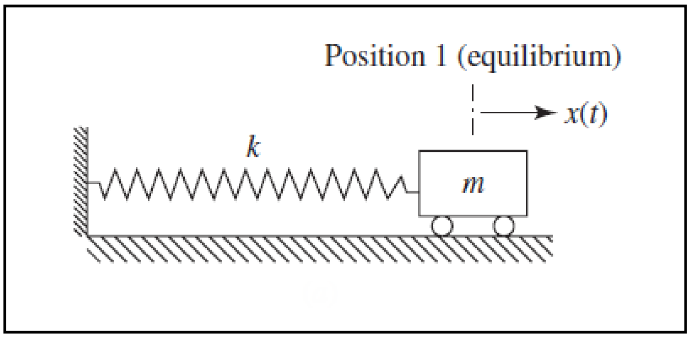


Figure 3.2 Vibratory motion of a spring–mass system, system in equilibrium

Similarly, the simple pendulum shown in Figure 3.3 also denotes a single-degree of freedom system. The reason is that the position of a simple pendulum during motion can be described by using a single angular coordinate, θ. Although the position of a simple pendulum can be stated in terms of the Cartesian coordinates x and y, the two coordinates x and y are not independent; they are related to one another by the constraint $x^2 + y^2 = l^2$, where l is the constant length of the pendulum. Thus, the pendulum is a single degree of freedom system.

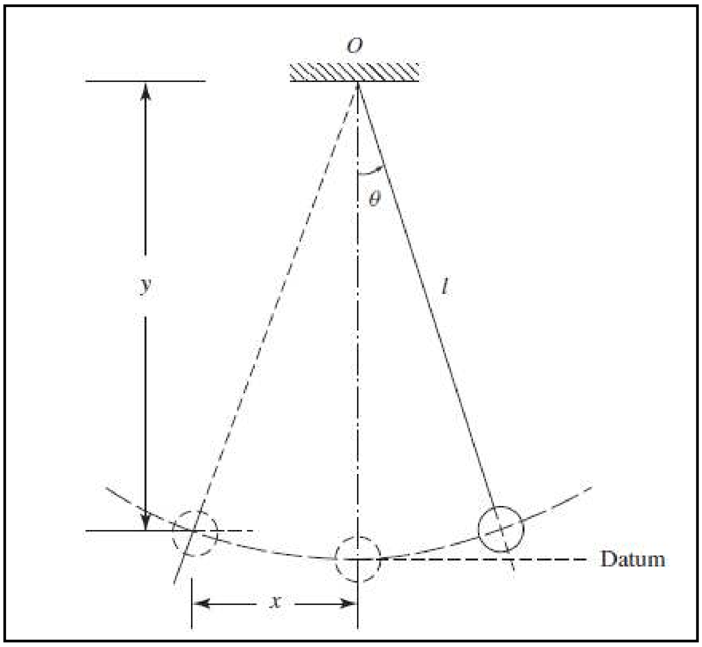


Figure 3.3 Simple pendulum

On the other hand, in a continuous system, the mass, elasticity (or flexibility), and damping are distributed throughout the system. During vibration, each of the infinite number of point masses moves relative to each other point mass in a continuous fashion. These systems are also known as distributed, continuous, or infinite-dimensional systems. A simple example of a continuous system is the cantilever beam shown in Figure 3.4. The beam has an infinite number of mass points, and hence an infinite number of coordinates are required to specify its deflected shape. The infinite numbers of coordinates, in fact, define the elastic deflection curve of the beam. Thus, the cantilever beam is considered to be a system with an infinite number of degrees of freedom. Most mechanical and structural systems have members with continuous elasticity and mass distribution and hence have infinite degrees of freedom (Singiresu, 2007).

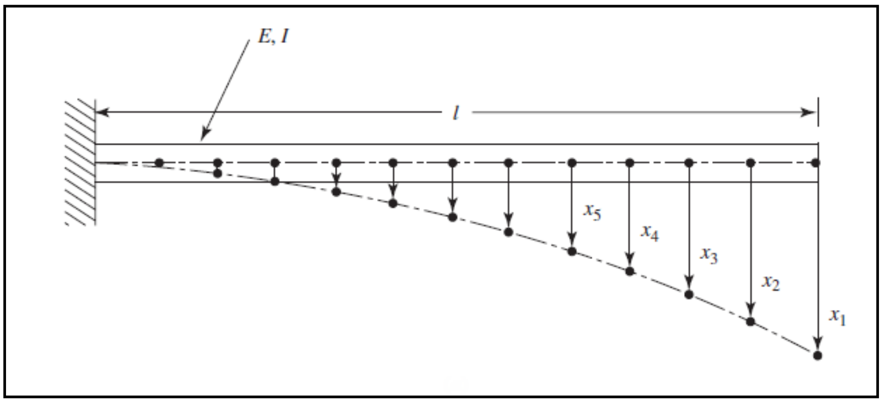


Figure 3.4 modeling of a cantilever beam as a continuous system

3.1.2 Free Vibration

The oscillatory motion of a mechanical system may be generally characterized as one of two types, free vibration or forced vibration. Vibratory motions that occur without the action of external dynamic forces are referred to as free vibrations (Timoshenko, Young & Weaver, 1974). A free vibration only depends on the mass and the flexibility of the object. If the damping element doesn't exist in the system, the vibration continues forever (Figure 3.5). But this is theoretical idea because in real systems the energy is dissipated to the surroundings over time and the amplitude decays away to zero shown in Figure 3.6. This dissipation of energy is called damping. Friction is one of the main causes of damping in engineering systems. And fluid friction, air resistance, magnetic damping, internal damping are some examples of damping systems.

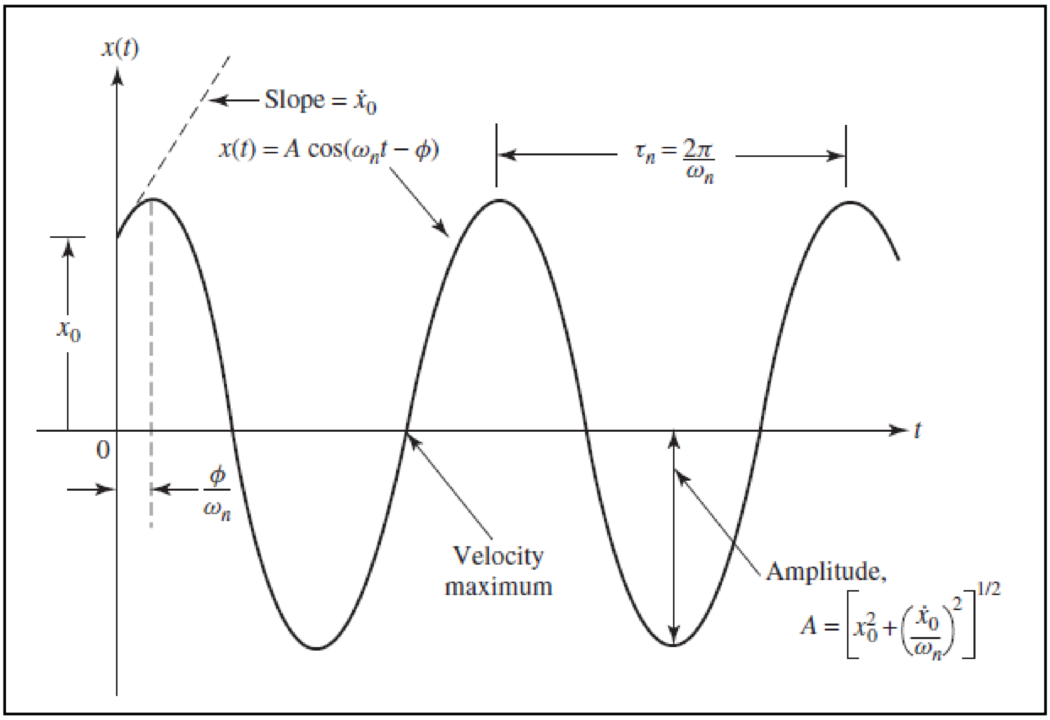


Figure 3.5 Free vibration responses (undamped vibration)

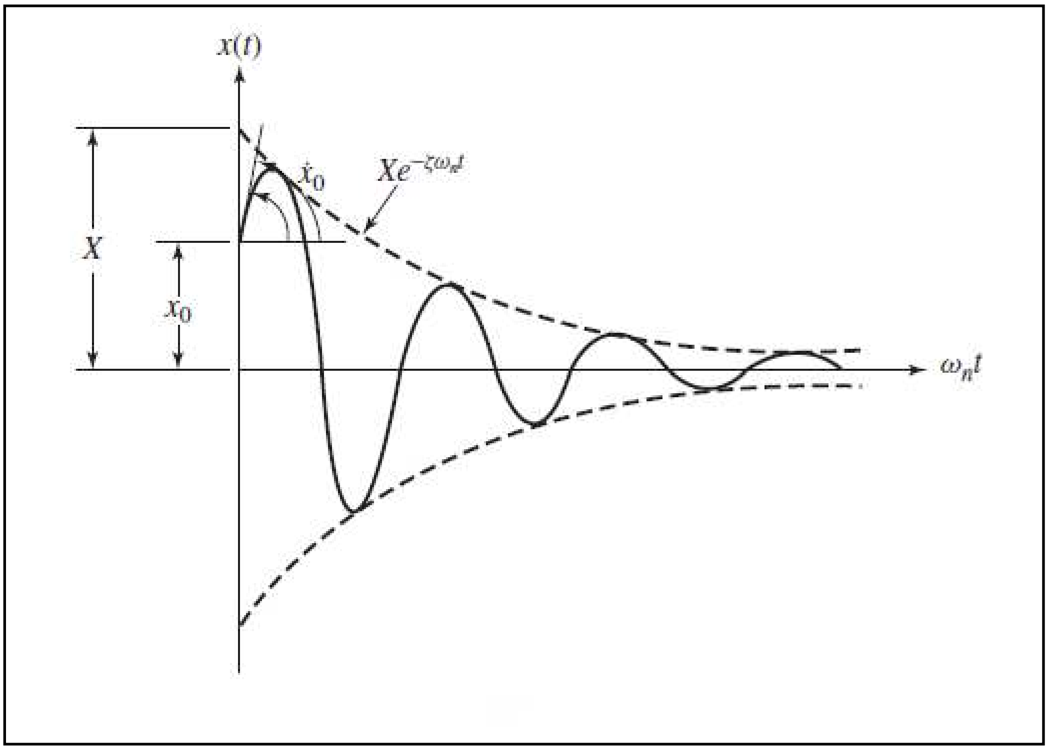


Figure 3.6 Free vibration response (damped vibration, ζ < 1)

Here, ζ is the damping ratio, given by;

$$\zeta = \frac{c}{2m\omega_n} \tag{3.2}$$

Here, *c* is damping coefficient.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{3.3}$$

Here, *k* is stiffness and *m* is mass.

3.1.3 Forced Vibration

The most practical way to obtain the governing differential equations for forced vibrations is to apply the second law of Newton by taking full advantage of free body diagrams. Free vibration equation of the rate obtained by the addition of an external driving force the equality of Equation (3.4) is also presented.

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{3.4}$$

Here, $F(t)$ is external driving force (If the external force is destroyed, the equation equals zero).

3.2 The Method of Analysis Frequency and Mode Shape

In dynamic problems, the displacement, velocity, strain, stress and load depend on the time. The procedure of a dynamic problem to obtain the finite element equations is as follows;

$$\bar{U}(x,y,z,t) = \begin{Bmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{Bmatrix} = [N(x,y,z)]\bar{Q}^{(e)}(t) \tag{3.5}$$

Here $\overline{\mathrm{U}}$ is displacement vector, [N] is the matrix of shape functions and $\overline{\mathrm{Q}}^{(\mathrm{e})}(\mathrm{t})$ is the time dependent displacement vector. Strain and stress are as follows;

$$\bar{\bar{\varepsilon}} = [B]\bar{Q}^{(e)} \tag{3.6}$$

$$\bar{\bar{\sigma}} = [D]\bar{\bar{\varepsilon}} = [D][B]\bar{Q}^{(e)} \tag{3.7}$$

If the time dependent variation of Equation (3.5) is taken, speed equation is obtained.

$$\dot{\bar{U}}(x, y, z, t) = [N(x, y, z)]\dot{\bar{Q}}^{(e)}(t) \tag{3.8}$$

Here $\dot{\overline{\mathrm{Q}}}^{(\mathrm{e})}$ is the nodal velocity vector. Lagrange equations which are used in obtaining the equations of dynamic motion of structures are as follows:

$$\frac{d}{dt}\left\{\frac{\eth L}{\eth \dot{\bar{Q}}}\right\} - \left\{\frac{\eth L}{\eth \bar{Q}}\right\} + \left\{\frac{\eth R}{\eth \dot{\bar{Q}}}\right\} = \{0\} \tag{3.9}$$

Here;

$$L = T - \Pi_p \tag{3.10}$$

It is called as the Lagrangian function. Where, T is kinetic energy, Π_P is potential energy, R is the loss function, $\overline{\mathrm{Q}}$ is nodal displacement, $\dot{\overline{\mathrm{Q}}}$ and is nodal speed. The volume of the element 'e' kinetic and its potential energies;

$$T^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \rho \dot{\bar{U}}^T \dot{\bar{U}} dV \tag{3.11}$$

$$\Pi_p^{(e)} = \frac{1}{2}\iiint_{V^{(e)}} \bar{\bar{\varepsilon}}^T \bar{\bar{\sigma}} dV - \iint_{S_1^{(e)}} \bar{U}^T \Phi dS_1 - \iiint_{V^{(e)}} \bar{U}^T \Phi dV \tag{3.12}$$

Here V^e is volume, ρ is density and $\dot{\bar{U}}$ element is the velocity vector of 'e'.

$$R^{(e)} = \frac{1}{2} \iiint_{V^{(e)}} c\, \dot{\bar{U}}^T \dot{\bar{U}} dV \tag{3.13}$$

Here c is the damping coefficient. If T, Π_P and R is to be written by using Equation (3.5) - (3.7);

$$T = \sum_{e=1}^{E} T^{(e)} = \frac{1}{2} \dot{\bar{Q}}^T \left[\sum_{e=1}^{E} \iiint_{V^e} \rho [N]^T [N] dV \right] \dot{\bar{Q}} \tag{3.14}$$

$$\Pi_p = \sum_{e=1}^{E} \Pi_p^{(e)} = \frac{1}{2} \bar{Q}^T \left[\sum_{e=1}^{E} \iiint_{V^e} [B]^T [D][B] dV \right] \bar{Q}$$

$$-\bar{Q}^T \left(\sum_{e=1}^{E} \iint_{S_1^{(e)}} [N]^T \Phi(t) dS_1 + \iiint_{V^e} [N]^T \Phi(t) dV \right) - \bar{Q}^T \bar{P}_c(t) \tag{3.15}$$

$$R = \sum_{e=1}^{E} R^{(e)} = \frac{1}{2} \dot{\bar{Q}}^T \left[\sum_{e=1}^{E} \iiint_{V^e} c [N]^T [N] dV \right] \dot{\bar{Q}} \tag{3.16}$$

Here $\bar{Q}$ is the global nodal displacement vector, $\dot{\bar{Q}}$ is global nodal velocity vector and $\bar{P}_c(t)$ is nodal force vector concentrated on the object. Matrix notation of integrals;

$$[M^{(e)}] = \text{mass matrix} = \iiint_{V^e} \rho [N]^T [N] dV \tag{3.17}$$

$$[K^{(e)}] = \text{stiffness matrix} = \iiint_{V^e} [B]^T [D][B] dV \tag{3.18}$$

$$[C^{(e)}] = \text{damping matrix} = \iiint_{V^e} c [N]^T [N] dV \tag{3.19}$$

$P_s^{(e)}$ = the vector of element nodal forces created by surface forces

$$= \iint_{S_1^{(e)}} [N]^T \Phi dS_1 \tag{3.20}$$

$P_b^{(e)}$ = the vector of element nodal forces created by body forces

$$= \iiint_{V^e} [N]^T \Phi dV \tag{3.21}$$

The complete systems of equations of motion are obtained by incorporating element matrices and mass matrices.

$$T = \frac{1}{2} \dot{\bar{Q}}^T [M] \dot{\bar{Q}} \tag{3.22}$$

$$\Pi_p = \frac{1}{2} \bar{Q}^T [K] \bar{Q} - \bar{Q}^T \bar{P} \tag{3.23}$$

$$R = \frac{1}{2} \dot{\bar{Q}}^T [C] \dot{\bar{Q}} \tag{3.24}$$

Here;

$$[M] = \text{ total mass matrix } = \sum_{e=1}^{E} \left[M^{(e)}\right] \tag{3.25}$$

$$[K] = \text{ total stiffness matrix } = \sum_{e=1}^{E} \left[K^{(e)}\right] \tag{3.26}$$

$$[C] = \text{ total damping matrix } = \sum_{e=1}^{E} \left[C^{(e)}\right] \tag{3.27}$$

$$\bar{P}(t) = \text{ the vector of total forces } = \sum_{e=1}^{E} \left(P_s^{(e)} + P_b^{(e)}\right) + \bar{P}_c(t) \tag{3.28}$$

Instead of the Equation (3.9), the Equation (3.22) - (3.24) is to be written to the equations of motion of dynamic objects obtained as follows;

$$[M]\ddot{\bar{Q}}(t) + [C]\dot{\bar{Q}}(t) + [K]\bar{Q}(t) = \bar{P_c}(t) \tag{3.29}$$

Here, $\ddot{\bar{Q}}(t)$ is the nodal acceleration vector in the global system. If the damping is not present in the system, the equations of motion can be written as follows;

$$[M]\ddot{\bar{Q}} + [K]\bar{Q} = \bar{P_c} \tag{3.30}$$

The equation of motion of undamped structures which are rigidity and mass effect are constant and the absence of external influences such as force or displacement;

$$[M]\ddot{\bar{Q}} + [K]\bar{Q} = 0 \tag{3.31}$$

Undamped harmonic movements of the structure;

$$\{Q\} = \{\bar{Q}\}\sin(wt) \tag{3.32}$$

And

$$\{\ddot{Q}\} = -w^2\{\bar{Q}\}\sin(wt) \tag{3.33}$$

Here $\{\bar{Q}\}$ is multi-degree of freedom vibration amplitude, ω is circular frequency. If equations (3.31) and (3.33) are combined,

$$([K] - \lambda[M])\{\bar{Q}\} = \{0\} \tag{3.34}$$

$\lambda = \omega^2$ at Equation (3.34). The solution of eigenvalue problems;

$$det([K] - \lambda[M]) = \{0\} \tag{3.35}$$

K and M are stiffness and mass matrices, respectively.

If the natural frequency f_n is expressed in terms of the circular frequency of ω_n can be written as follows;

$$f_n = \frac{w_n}{2\pi} \tag{3.36}$$

CHAPTER FOUR
MATERIALS AND TEST METHODS

4.1 Material Properties and Test Sample

Sandwich composite samples studied within the scope of the project, polyurethane foam as the core material, the shell (surface) glass fiber reinforced laminated composites are selected as the material. Glass fiber which is used as a reinforcing element is weave fabric in the form of ±45° orientation angle.

4.1.1 Core Material

Polyurethane foam shown in Figure 4.1 has been selected as the core material. Polyurethane foam is obtained by reaction with the exothermic material as chemically, an organic di-isocyanate or poly-isocyanate, a polyol (two or more hydroxyl alcohol containing type). Polyurethane is a general name for the family of a polymer that can contain a combination of different features. It is possible to obtain all types of polyurethane by using a few basic di-isocyanates with a polyol using a number of different molecular weights and different characteristics. Mainly of polyurethane types, it is also listed to Tale 4.1, along with the features that they have and their use of areas (Okutan Baba, Thoppul, & Gibson, 2010).

Figure 4.1 Polyurethane foam

Table 4.1 Types and properties of polyurethane foam

Polyurethane Type	**Features**	**Area of Usage**
Rigid foam	As a thermal insulation material is used. It can be applied by casting or spraying technique (Density: 30 - 100 kg/m^3).	The polyurethane layer, polyurethane sandwich panel, etc.
Flexible foam	Density: 30-60 kg/m^3	Seats, furniture, etc.
Semi-rigid foam	With the absorption feature of impact and energy of the materials that are found wide applications (Density: 100 - 300 kg/m^3).	Complete the front panel of the car, helmet, the outer coverings of boat, etc.
Elastomer foam	Strokes that can receive the very hard and big materials (Density: 800 kg/m^3).	Car bumpers, etc.
Casting elastomer foam	They are materials that are very resistant to abrasion and impact.	Machine parts that can be worn, wheels and rollers.
Rigid integral skin foam	Highly tough, durable and lightweight materials (Density: 300 - 700 kg/m^3).	Table, window profiles, machine tools, welding equipment, etc.
Flexible integral skin foam	They are durable materials but flexible when they contact with hand (Density: 300 - 600 kg/m^3).	The steering wheel, the arm of the chair etc.
Integral skin foam	They are flexible and durable materials (Density: 400 - 550 kg/m^3).	Shoes, shoe soles, etc.
Coating	Abrasion, impact, chemical-resistant materials.	Industrial floor coating, durable furniture pieces.
Systems for Sports Grounds	Sports surface coating materials on the ground.	Tennis courts, athletics track, etc.

4.1.2 Shell (surface) Material

4.1.2.1 Glass Fiber

In polymer-based composite materials, the most common and cheapest material is glass fiber as a reinforcing element shown in Figure 4.2. Fiber glass can be manufactured in many types and quality such as ordinary glass bottle, high purity quartz glass. According to the desired properties in reinforcing element, it is possible to obtain different quality and types of glass fibers by adding determined proportions to oxides of the following elements to the silica sand (SiO_2) which is the main component of glass fibers. These elements are;

- Sodium (Na)
- Calcium (Ca)
- Magnesium (Mg)
- Aluminum (Al)
- Barium (Ba)
- Iron (Fe)

The types of the glass obtained by mixing different proportions of the different components;

a. Glass of A: It's a kind of glass that is used in the manufacture of window and bottle glass. It is not used a lot in the production of composite materials.

b. Glass of E: The most used type of glass reinforcing fibers in the production. Low cost, good insulation and low water absorption rate are one of the most important features.

c. Glass of S: It is a high-cost, high-performance lens. It is mostly used in the aerospace industry. The wires inside the fiber are half of the diameter e glass. In this way, more glass fiber fit inside the same diameter of fiber. Therefore, glass fibers which is reinforcing element combine more powerful with the matrix material and obtain composite materials with high resistance.

d. Glass of C: It shows high chemical resistance. It is used in places such as storage tanks. Their mechanical properties are low.

The general superior properties of glass fibers according to the other types of fibers;

- Their cheapness
- Their high tensile strength
- Their high chemical resistance

Disadvantages;

- Their heat resistance is low
- They don't burn but they get soft at high temperatures
- They don't transmit electricity

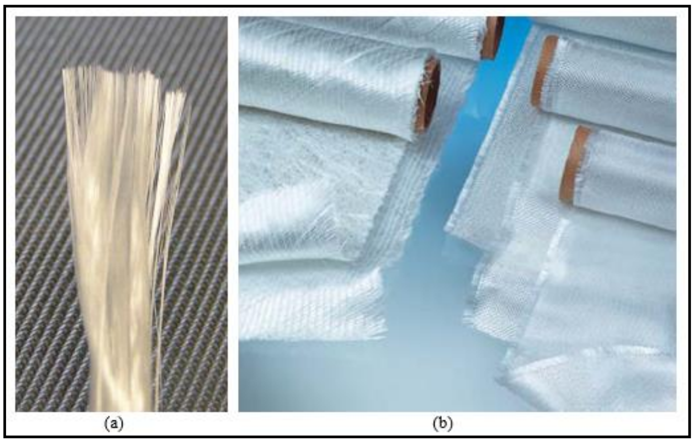


Figure 4.2 (a) Glass fiber (b) Glass fiber mat and weaving

4.1.2.2 Polyester Resin

The word of polyester is comprised of the word "very" meaningful "poly" and expressing an organic salt "ester". It can be expressed in the form of "Numerous organic salt". They are formed from a chain of ester molecules chemically. They occur through the polymerization of terephthalic acid with ethylene glycol. Polyester resin is the most widely used both in Turkey and in the world. Under the temperature in 100^{o} C of polyester resins mechanical and chemical resistance are good.

Some areas of usage;

- They are used as protective against moisture in painting, rubber industry, and corrosion in metals and in wood.
- In shipping (the skeleton of the ship) and in the construction sector (panels).
- In the manufacture of fiber reinforced polyesters such as pipe, tank and automotive body parts.

Some advantages;

- Because of the low viscosity of polyester, it can easily wet the reinforcing element which is used in composite materials and generates high quality composite with the glass fiber in particular.
- It's easy to use and low in cost.
- It can be obtained some different features using in polyesters with the aid of changing the shape of bond.
- It is suitable for basic molding techniques which are from hand lay-up to complex molding techniques.

Some disadvantages:

- It occurs during erection the high rate tensile (% 5...12). In this case, it causes sprain and fracture of the fibers under compressive stress.
- There is the difficulty of obtaining a smooth surface.
- They emit toxic gas.
- Their shelf life is short.
- The corrosion resistance is low in alkaline and basic environments.
- They disintegrate by taking water in their structure.

4.2 Preparation of Samples

Composite sandwich beams which are analyzed in the study are manufactured for the company of Berk Composite in Ankara. In all samples, as the core material polyurethane foam which is thickness 10 mm and density 90 kg/m^3 and the composites selected in the form of single layer glass fiber mesh fabrics (500 gr/m^2) as the shell material are used. In Figure 4.3, the materials that belong to the sandwich beam are shown.

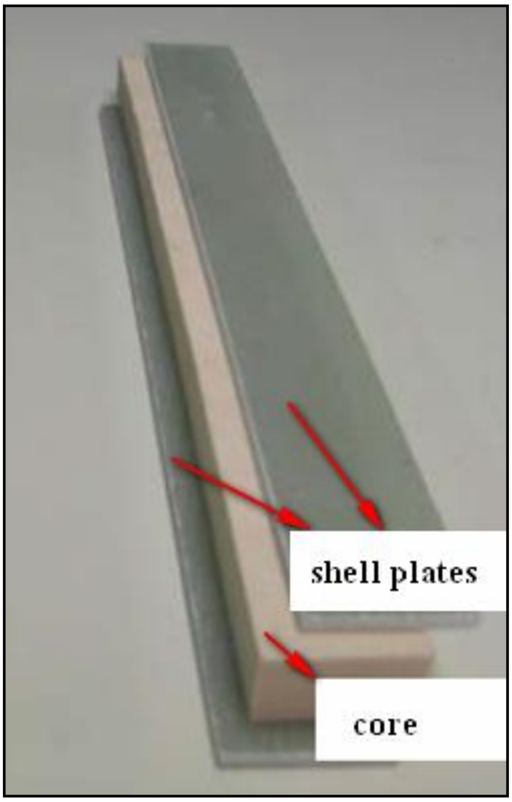


Figure 4.3 Glass fiber–polyester / polyurethane / glass fiber–polyester flat sandwich beam

The schematic picture which shows the geometric dimensions belonging to flat sandwich beam is shown in Figure 4.4. All flat sandwich beam has length L = 300 mm and width w = 25 mm. The thickness of the top and bottom of the foam layer shells are t_s = 0.7 mm and the average thickness of completed manufacture of the sandwich material is 11.4 mm. Polyurethane foam is put in between the fabric (fiber glass) wetted with polyester resin and in mold, length of 350 mm and width of 80 mm in order to produce samples. It is left at a room temperature for up to 24 hours for the curing process by closing the mold.

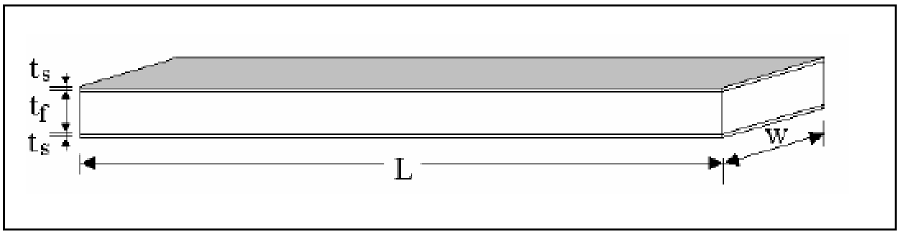


Figure 4.4 Geometry of flat sandwich beam

For curved sandwich beams shown in Figure 4.5, three different radius of curvature are chosen which have radii of curvature R = 294 mm, 594 mm and 1194 mm. The angle of curvature (θ) is selected as 61.35, 29.25 and 14.43 degree so as to keep constant the length of the beam L flat. For manufacturing of these beams, the molds which made of wood (radii of curvature of 294 mm, 594 mm and 1194 mm) have been used. All other operations are the same with the production of flat sandwich beam. All the prepared samples are shown in Figure 4.6.

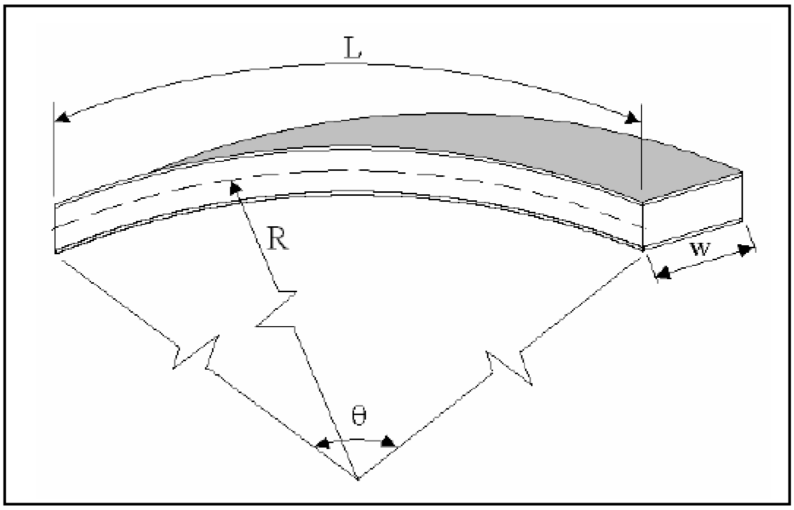


Figure 4.5 Geometry of curved sandwich beam

Figure 4.6 Flat and curved beams model

4.3 Mechanical Properties of Materials

The mechanical properties of materials, which are used to manufacture the sandwich beams to perform the free vibration tests, are given in Table 4.2.

Table 4.2 Material properties of the sandwich beam

	Materials	
	Skin material *Glass / polyester*	Core material *Polyurethane*
E_1 **(MPa)**	10160	115
E_2 **(MPa)**	10160	-
E_3 **(MPa)**	10160	-
υ_{12}	0.3	0.3
υ_{13}	0.3	-
υ_{23}	0.3	-
G_{12} **(MPa)**	3000	-
G_{13} **(MPa)**	3000	-
G_{23} **(MPa)**	3000	-
ρ **(kgm^{-3})**	1869	90

4.4 Vibration Test

Free vibration tests of flat and curved sandwich beams are carried out for clamped-free (C-F) boundary condition. During the test, clamped end condition is provided through a clamp as shown in Figure 4.7. The vibration signals generated by a PCB 086C01 type impact hammer are detected using three PCB 352A24 type miniature accelerometers as shown in Figure 4.8, each of which has a sensitivity of 100 mV/g, weight of 0.8 gr and is suitable for vibration measurements within a range of 0.4-12000 Hz. All the vibration signals acquired from accelerometers were sampled at 4000 Hz and recorded on a computer using a NI PXI 4496 (which is a trade mark of National Instrument) data acquisition system and LabVIEW software, as seen in Figure 4.7. Vibration tests have been repeated at least 2 times for each sample. The frequency contents of the time domain vibration signals are calculated using the “fft.m” function in Matlab environment.

Types, dimensions and geometrical boundary conditions of the considered sandwich beams are shown in Table 4.3. Only the Clamped-Free boundary condition is considered in the experimental tests for each sample.

Table 4.3 Specimen material and test conditions

Material	Radius of curvature R(mm)	Boundary condition	Length L (mm)	Width W (mm)	Face sheet thickness t_s (mm)	Core thickness t_f (mm)
Glass Polyurethane Glass	Flat R1194 R594 R294	Clamped / Free	260	25	0.7	10
		Clamped / Clamped				

Note: In order to properly interpret the results, all samples are produced with exactly the same technique and same materials (fiber glass, polyurethane foam, and polyester resin). That is, it is strongly given importance to be only the radius of curvature is the single variable between samples. Complete adhesion between the core material and face sheets is also provided. There is no any debond surface.

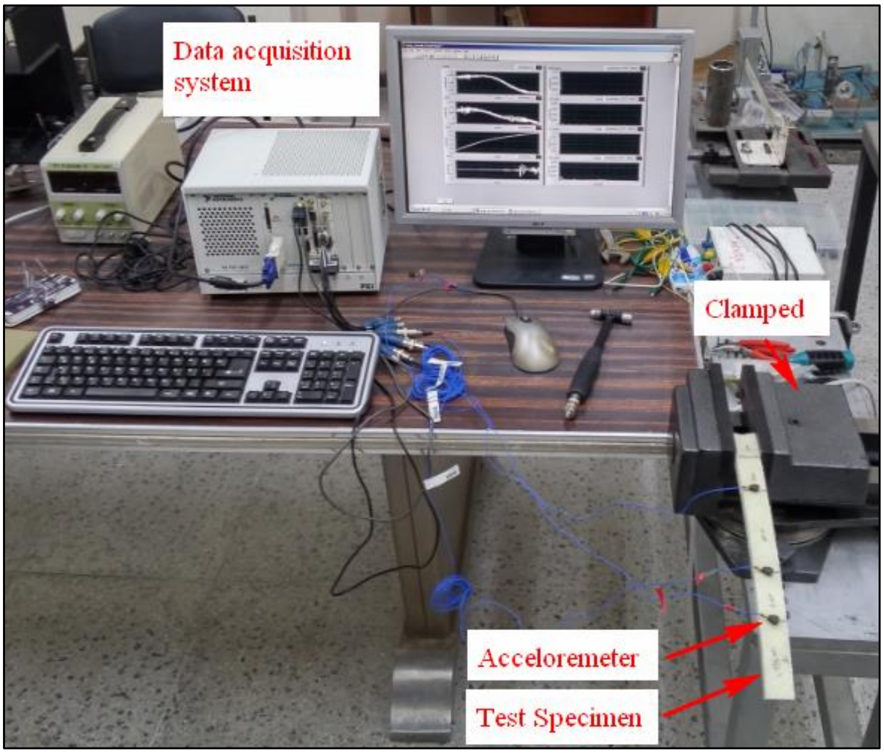


Figure 4.7 Experimental setup for vibration testing

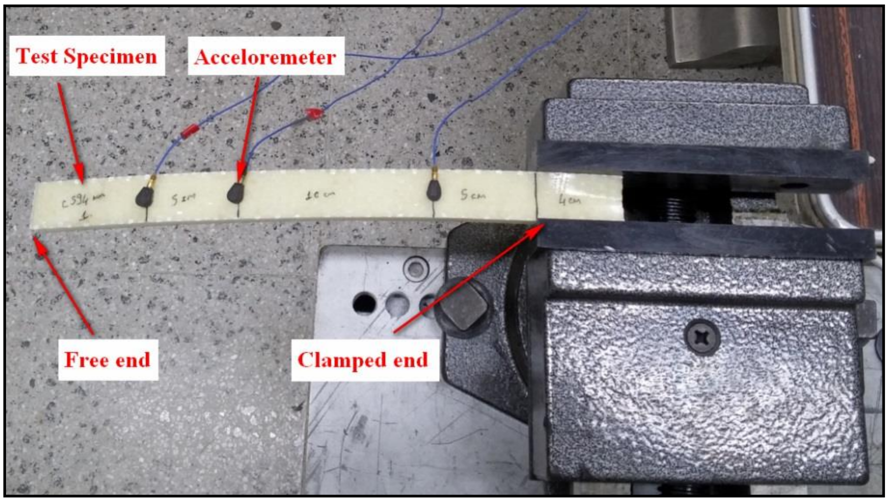


Figure 4.8 Clamped-free boundary condition

4.4.1 The Natural Frequency of Sandwich Beams For the Clamped - Free Boundary Condition

The time domain acceleration signals are collected by using the experimental setup mentioned in the previous section. The natural frequencies of the sandwich beams are determined by means of fft transform of the time domain signals. The fft analysis is carried out in Matlab for three accelerometer signals. Figure 4.9 illustrates the results of the fft analysis. In this figure, the first window shows the time domain signal for the accelerometer near the free end of the beam, and the other three windows show the fft transform of the acceleration signals collected from three accelerometers. The peaks, which arise in the frequency plots corresponds to a natural frequency of the considered sandwich beam. Each accelerometer captures different vibrational modes due to their locations on the beam. The magnitude of the peak, which corresponds to the lower natural frequency of the beam, is higher for the accelerometer located near the free end of the beam than those obtained for the accelerometers near to the clamped end of the beam. This situation is contrary for higher natural frequencies, since the accelerations are more sensible at the clamped end of the beam.

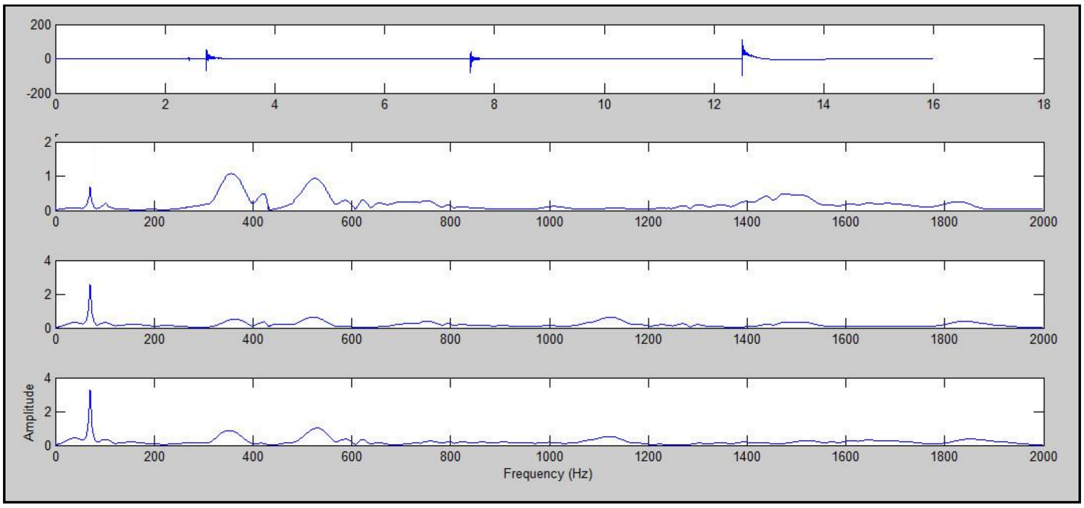


Figure 4.9 Acceleration–frequency graphs

The natural frequencies of the flat and curved sandwich beams are presented in Figures from 4.10 to 4.13. The peaks, which correspond to a natural frequency of the sandwich beam, are marked on the graph in order to read the frequency value.

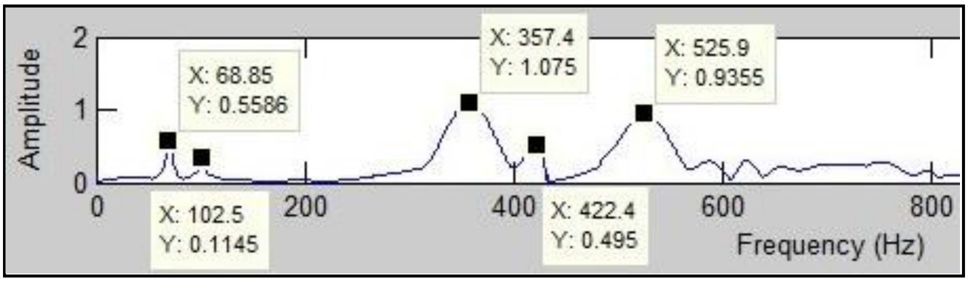


Figure 4.10 Frequency content for flat beam

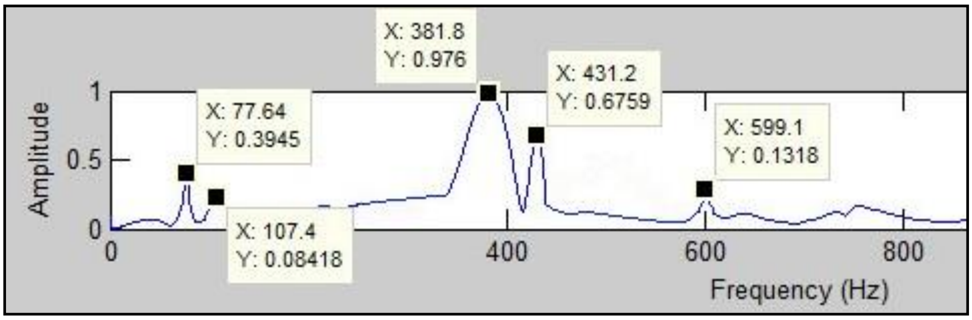


Figure 4.11 Frequency content for R1194 curved beam

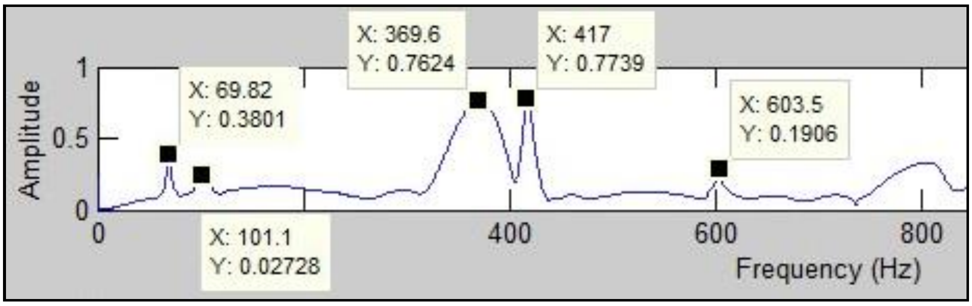


Figure 4.12 Frequency content for R594 curved beam

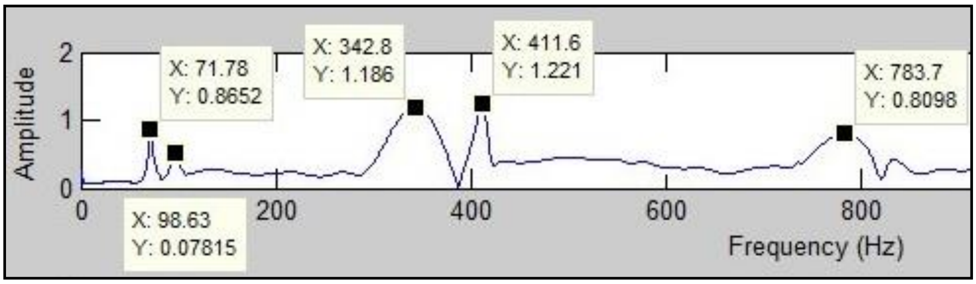


Figure 4.13 Frequency content for R294 curved beam

Table 4.4 shows the effect of the curvature on the first five natural frequencies of the sandwich beam. The experimental natural frequencies show that the radius of curvature has measurable effect on the natural frequencies for all modes. The natural frequencies are given also in graphical form in Figure 4.14.

Table 4.4 Effect of curvature on the natural frequencies (experimental results)

Mode Number	Natural Frequency (Hz)			
	Radius of curvature (R - mm)			
	Flat	**R1194**	**R594**	**R294**
1	68.85	77.64	69.82	71.78
2	102.5	107.4	101.1	98.63
3	357.4	381.8	369.6	342.8
4	422.4	431.2	417	411.6
5	525.9	599.1	603.5	783.7

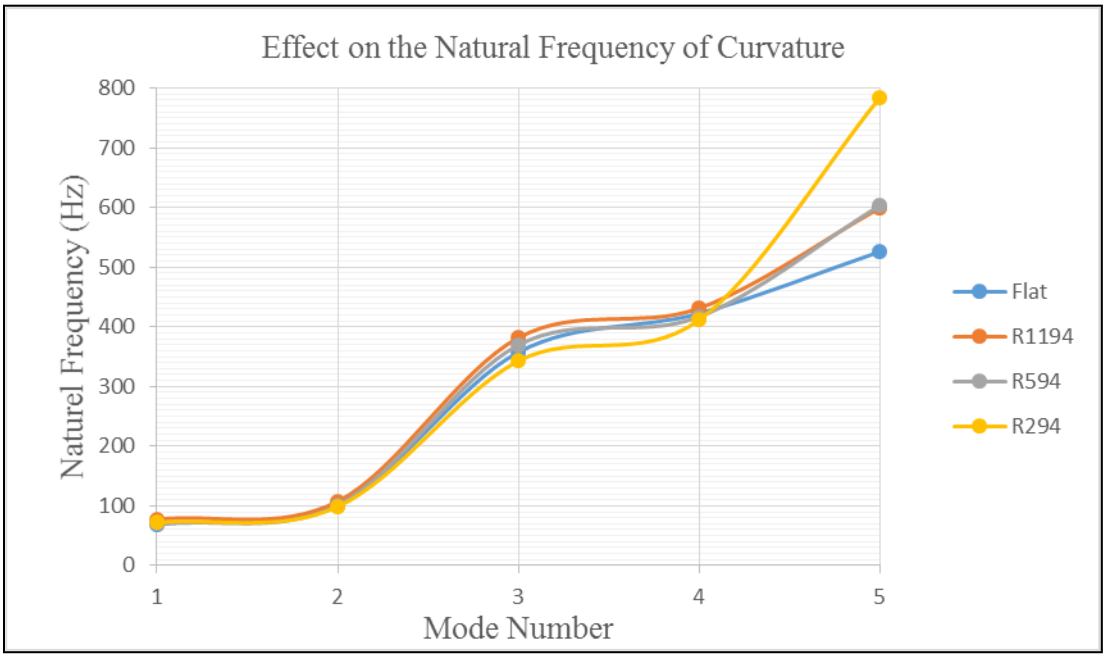


Figure 4.14 Effect of curvature on the natural frequency for clamped - free boundary condition (experimental results)

The natural frequency values for first five natural vibration modes versus the curvature of the sandwich beam are given in Figure 4.15 in more detail.

The natural frequencies of the sandwich beam are calculated in ANSYS by using the finite element method in order to make a comparison between the experimental and numerical results. The results of this comparison are presented in Chapter 5.

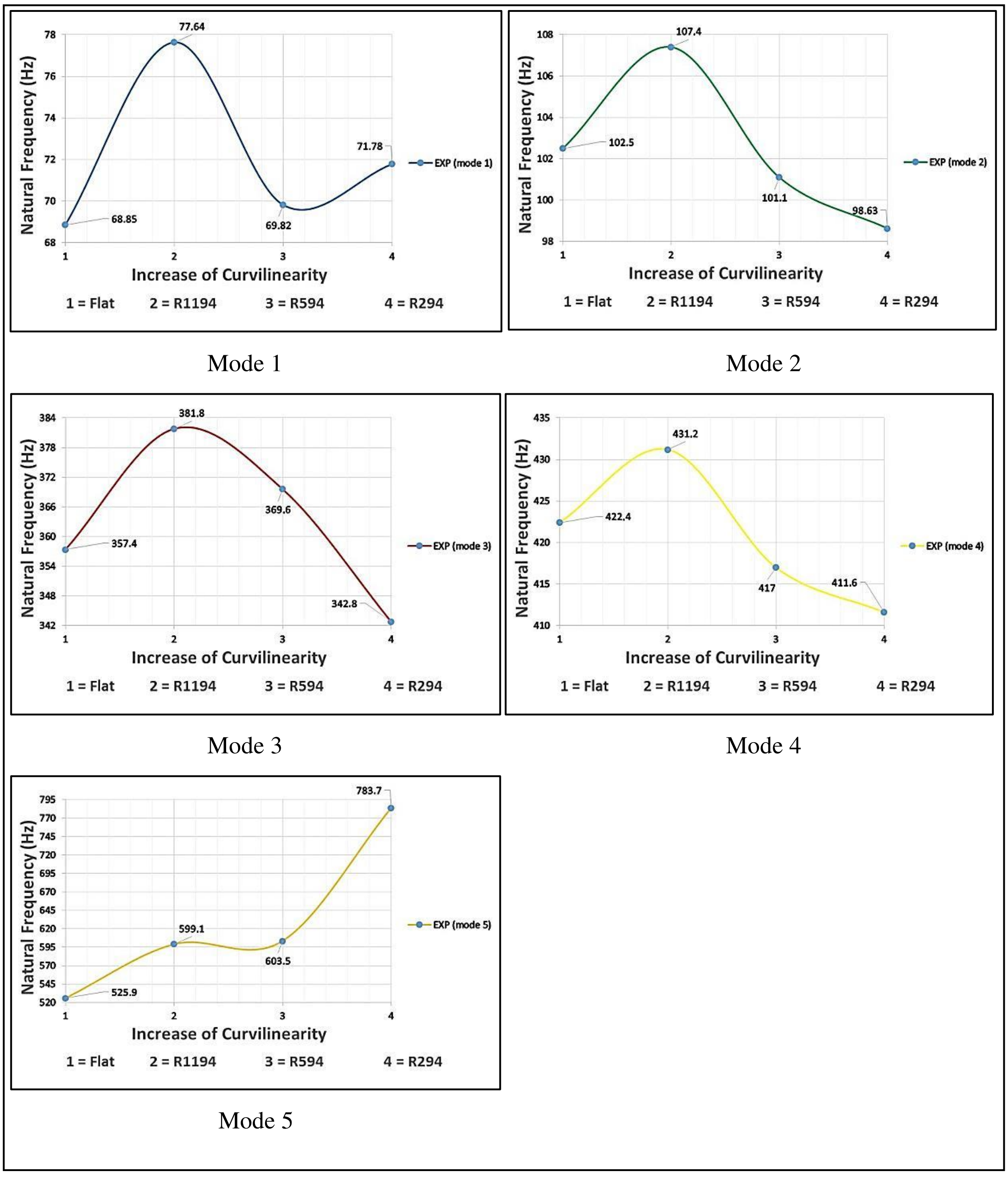


Figure 4.15 Effect of curvature on the natural frequency for clamped-free boundary condition (experimental results)

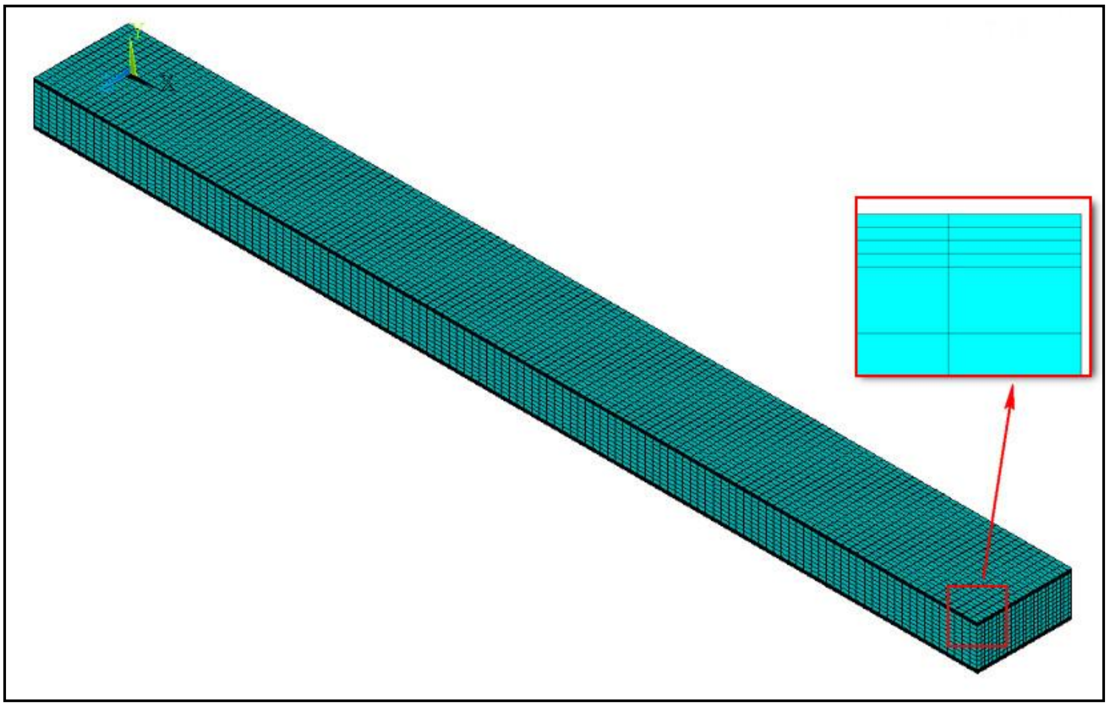
CHAPTER FIVE
COMPARISON OF FINITE ELEMENT ANALYSIS AND TEST RESULTS

In this chapter, free vibration analyses of flat and curved sandwich beams are carried out in order to make a comparison between the numerical and experimental results. The procedure for the numerical analysis and its findings are given in the following section.

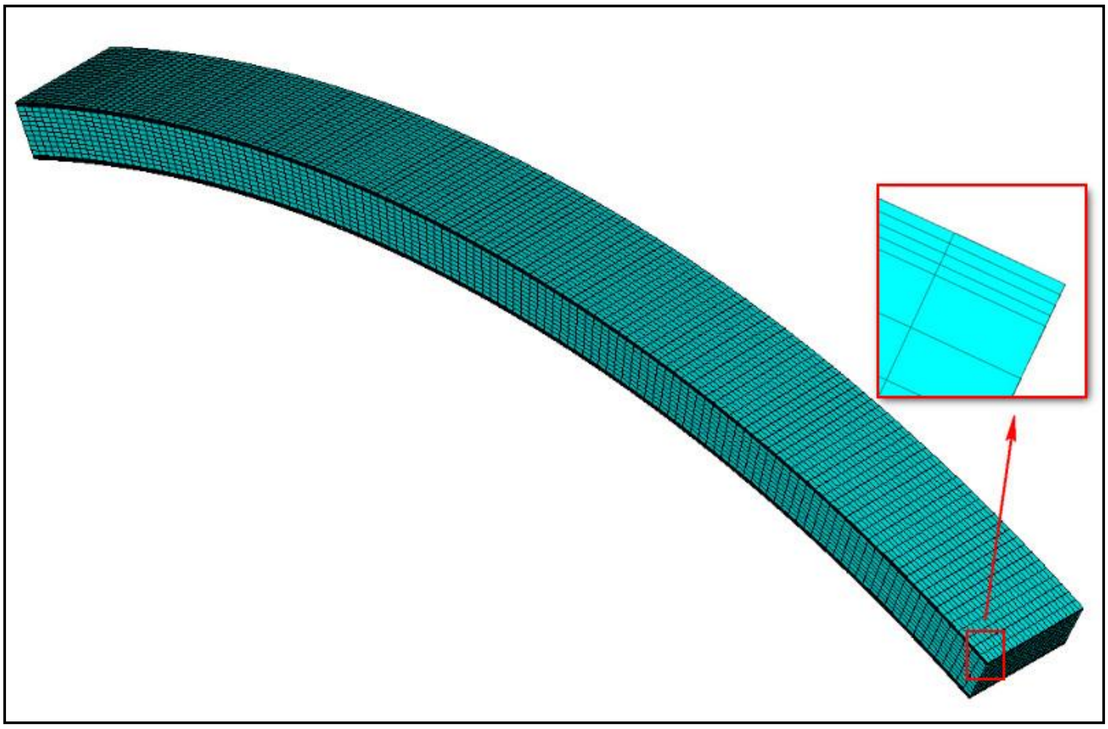
5.1 Finite Element Analysis

The numerical analyses for the free vibration behavior of the sandwich beam are carried out using the finite element method. For this purpose, the solid models of the flat and curved beams are prepared in SOLIDWORKS program. Then, the modal analyses are performed in the finite element package ANSYS 14.5. The mechanical properties used as input for the material model, have been provided by the company Berk Composite, which have produced the test samples. Material properties used in the finite element models of sandwich beams were given in Section 4.3, Table 4.2. The face sheet material is accepted as linear, elastic and orthotropic and foam material (core) is accepted as linear, elastic and isotropic.

The finite element modal analyses of the sandwich beams are performed for two boundary conditions namely Clamped-Free and Clamped-Clamped. In the finite element model, three-dimensional “8-node Brick 185” element is used. Cantilever boundary condition is provided by the limitation of displacements and rotations of the areas at both end surfaces of the beam in all directions. The previous studies have been reviewed in order to determine the most appropriate mesh size for the finite element modeling. As a result of these studies, it is seen that the use of ten parts through the thickness of the foam, four parts through the thickness of the shell and 130 parts along the length of the beam are sufficient to obtain precise results. Finite element models have got 56250 elements and 62244 nodes. Figure 5.1 shows the detailed view of the finite element model. It is assumed that there is 100% adhesion between the shell layer and the core material.



(a)



(b)

Figure 5.1 Mesh view of the finite element model (a) flat beam (b) curved beam

5.2 Result of the Finite Element Analysis

In this section, the results of the finite element results are given together with the experimental results for clamped-free boundary condition. Table 5.1 shows the first five natural frequencies of the flat and curved sandwich beams. The percentage deviations between numerical end experimental results are also given in the table. From the results presented in this table, it has been seen that, there is a good correlation between the numerical and experimental results. The maximum difference between the results is found as 19.93%. Apart from that, the general difference is seen in the range of ± 10%. The effect of the curvature on the natural frequencies is also seen for numerical and experimental cases from Figures from 5.2 to Figure 5.5 at first five free vibration modes. The overlap between the experimental and numerical natural frequency curves can be seen in these figures.

Figures 5.6-5.10 illustrate the natural frequency variation versus the change in the beam curvature for each free vibration mode. For the first mode, the experimental natural frequency values increases as the curvature of the beam increases. This behavior can be observed from the ANSYS results. For 2nd, 3rd and 4th modes, natural frequency values decreases as the beam curvature increases from both experimental and numerical results. The 5th natural frequency increases as the beam curvature increases for both result groups.

Table 5.1 Effect of the beam curvature on the natural frequencies of sandwich beams (clamped-free boundary conditions, EXP: experimental, FEA: finite element analysis)

		Natural Frequencies of Sandwich Beams (Hz)				
		Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**
Flat	**EXP**	68.85	102.5	357.4	422.4	525.9
	FEA	66.763	93.544	375.14	480.51	564.96
	%Dif	- 2.92	- 8.73	+ 4.79	+ 13.75	+7.42
R1194 mm	**EXP**	77.64	107.4	381.8	431.2	599.1
	FEA	68.173	93.728	380.08	467.25	630.87
	%Dif	- 12.19	- 12.73	- 0.45	+ 8.36	+5.30
R594 mm	**EXP**	69.82	101.1	369.6	417	603.5
	FEA	70.596	91.363	383.68	428.25	723.82
	%Dif	+ 1.10	- 9.63	+ 3.80	+ 2.69	+ 19.93
R294 mm	**EXP**	71.78	98.63	342.8	411.6	783.7
	FEA	72.717	79.395	364.69	377.76	895.41
	%Dif	+ 1.30	-19.50	+ 6.38	- 8.22	+ 14.25

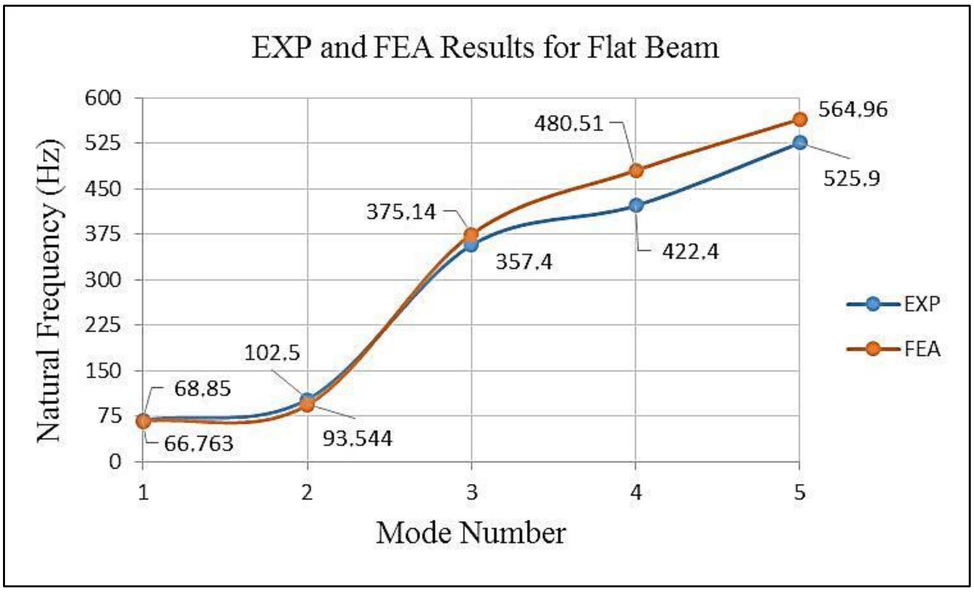


Figure 5.2 Experimental and finite element results for flat beam

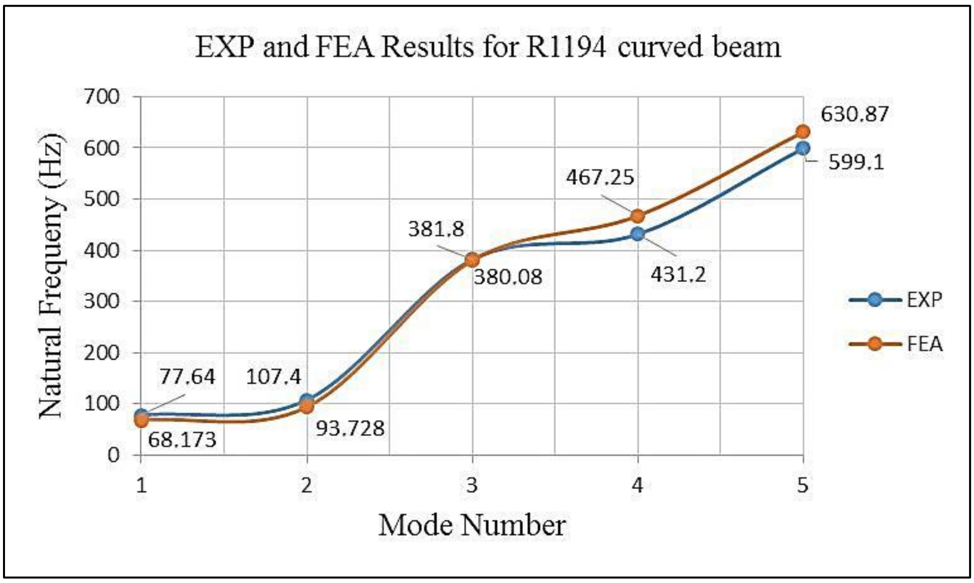


Figure 5.3 Experimental and finite element results for R1194 curved beam

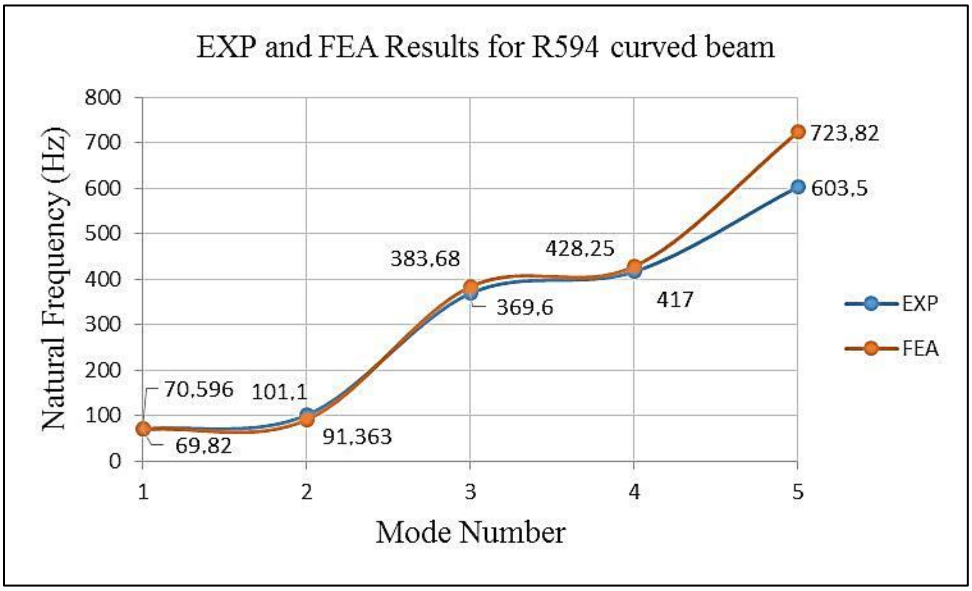


Figure 5.4 Experimental and finite element results for R594 curved beam

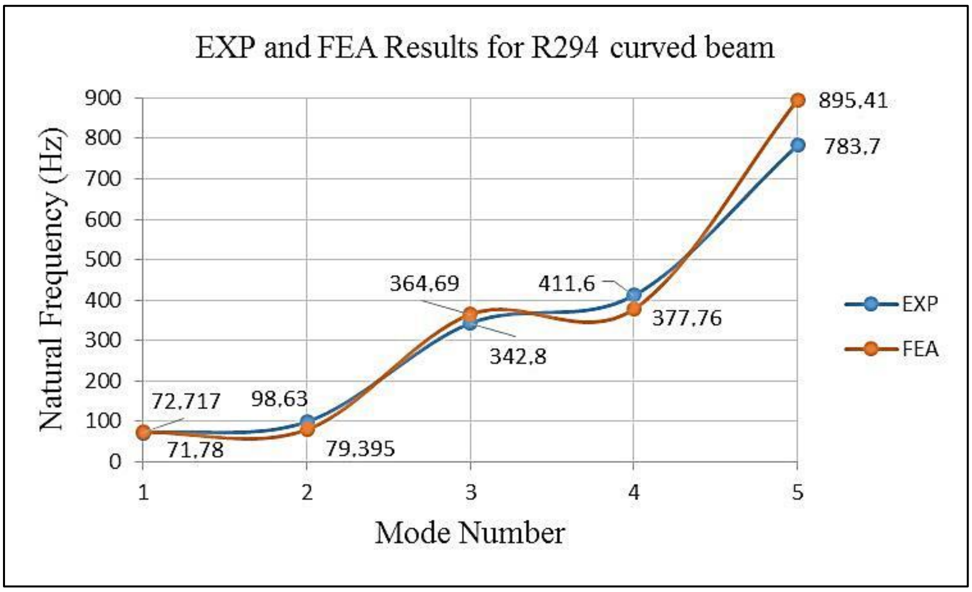


Figure 5.5 Experimental and finite element results for R294 curved beam

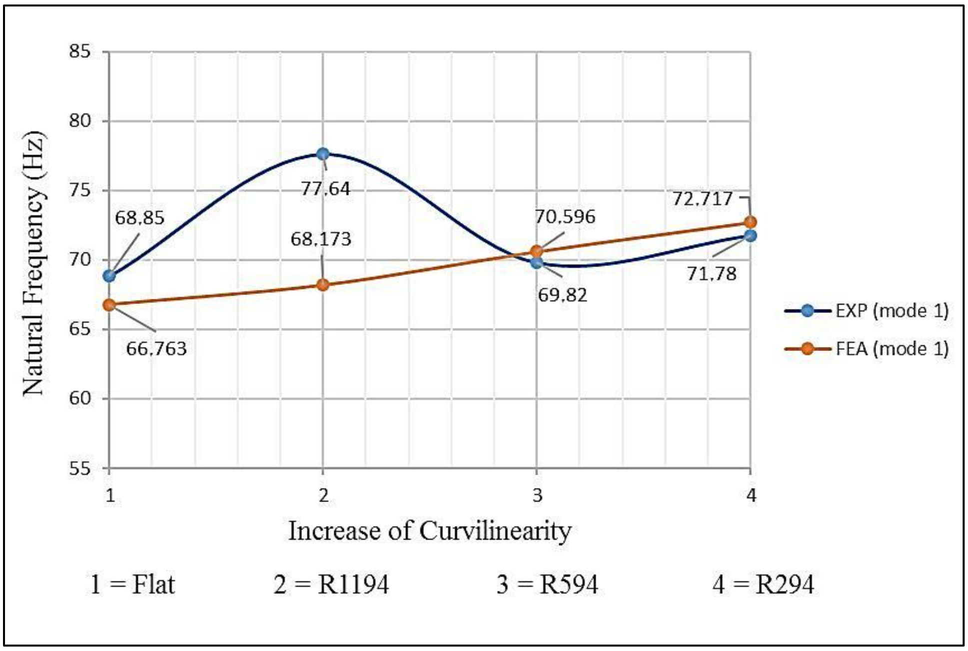


Figure 5.6 Experimental and finite element results for mode 1

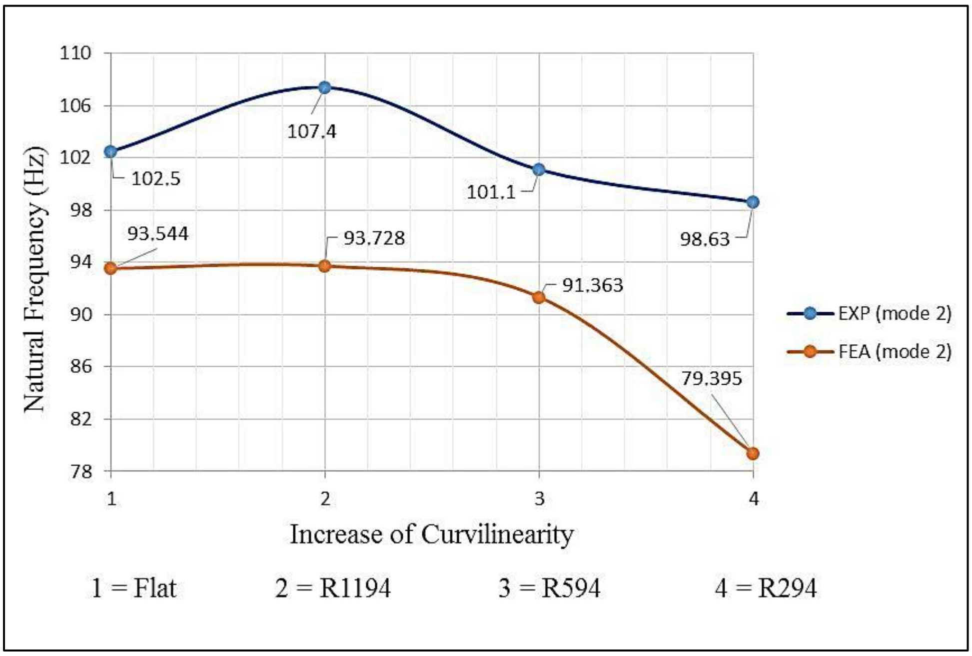


Figure 5.7 Experimental and finite element results for mode 2

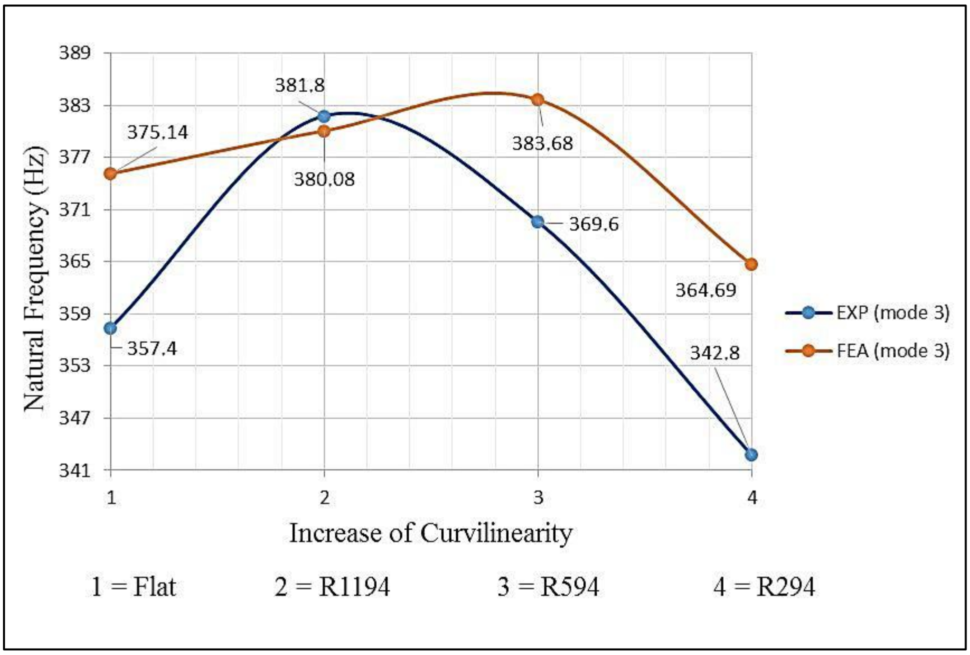


Figure 5.8 Experimental and finite element results for mode 3

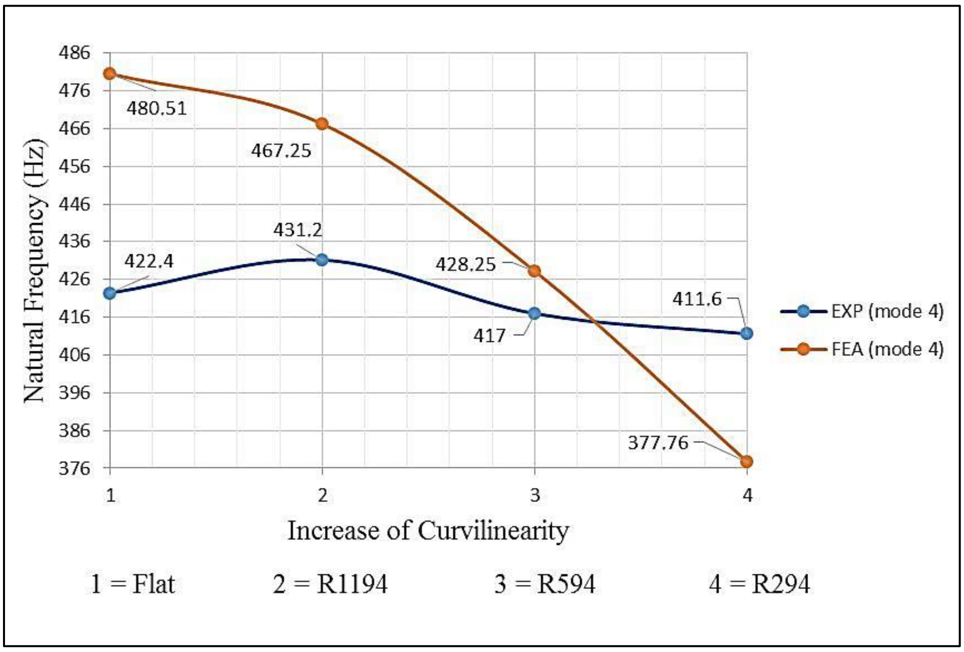


Figure 5.9 Experimental and finite element results for mode 4

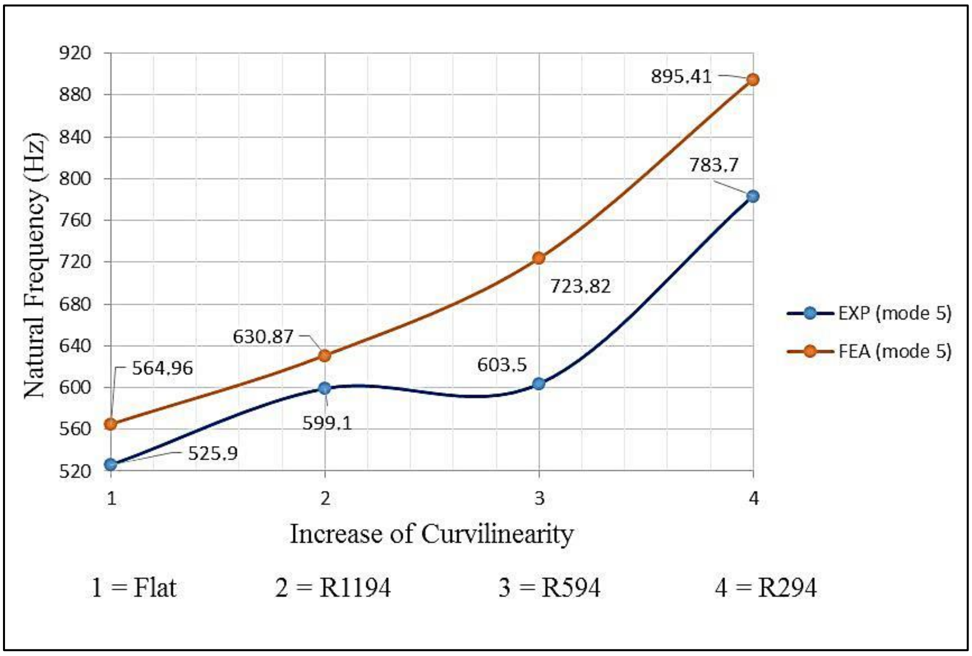


Figure 5.10 Experimental and finite element results for mode 5

The shapes of first five vibration modes of flat sandwich beam are shown in Figure 5.11 for clamped–free boundary condition. The number of nodes, which is the zero displacement point, is mode number-1 as expected.

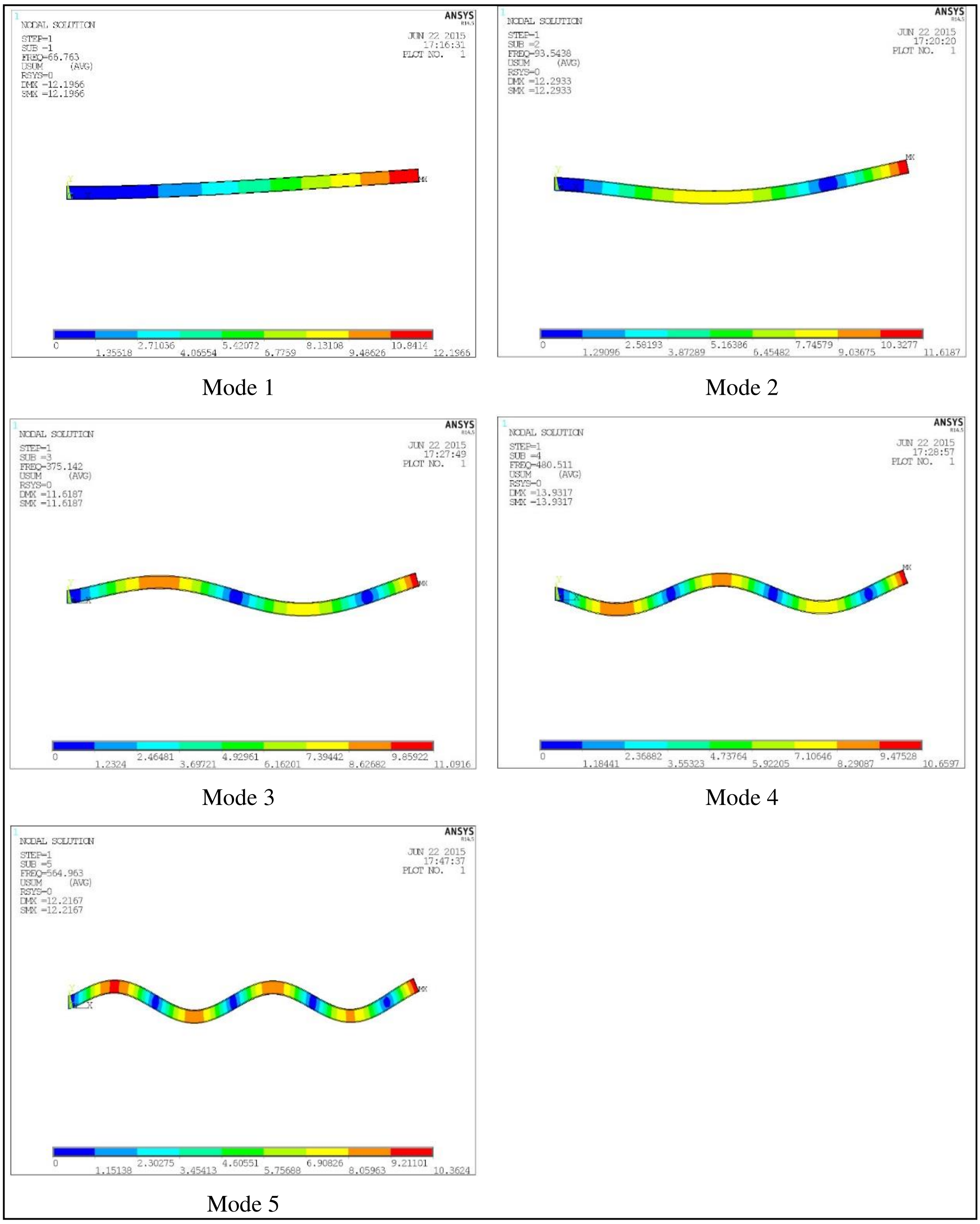


Figure 5.11 Mode shapes for flat beam, clamped–free boundary condition

The shapes of first five free vibration modes, regarding R1194 curved beam, are shown in Figure 5.12.

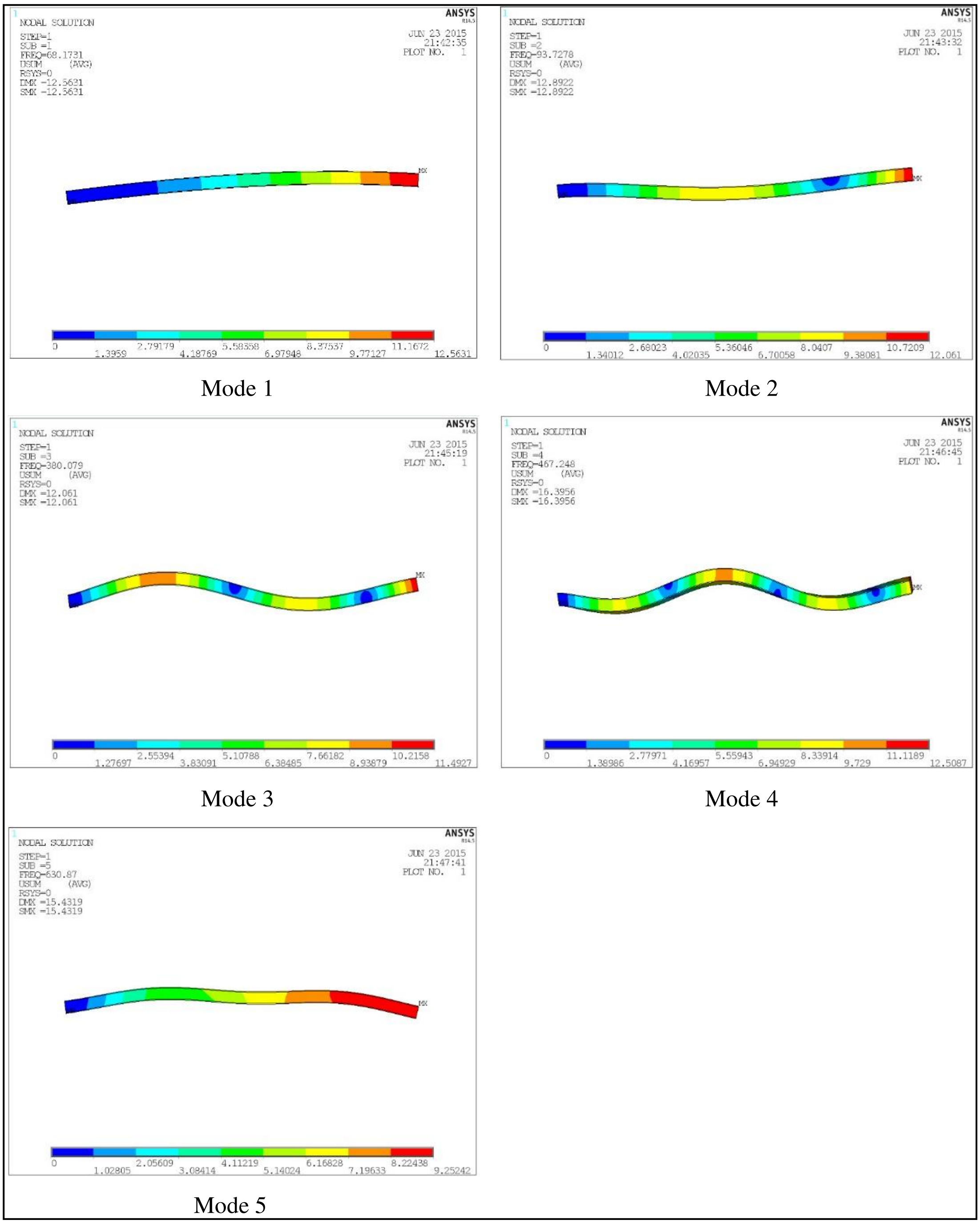


Figure 5.12 Mode shapes for R1194 curved beam, clamped–free boundary condition

The shapes of first five vibration modes, regarding R594 curved beam, are shown in Figure 5.13.

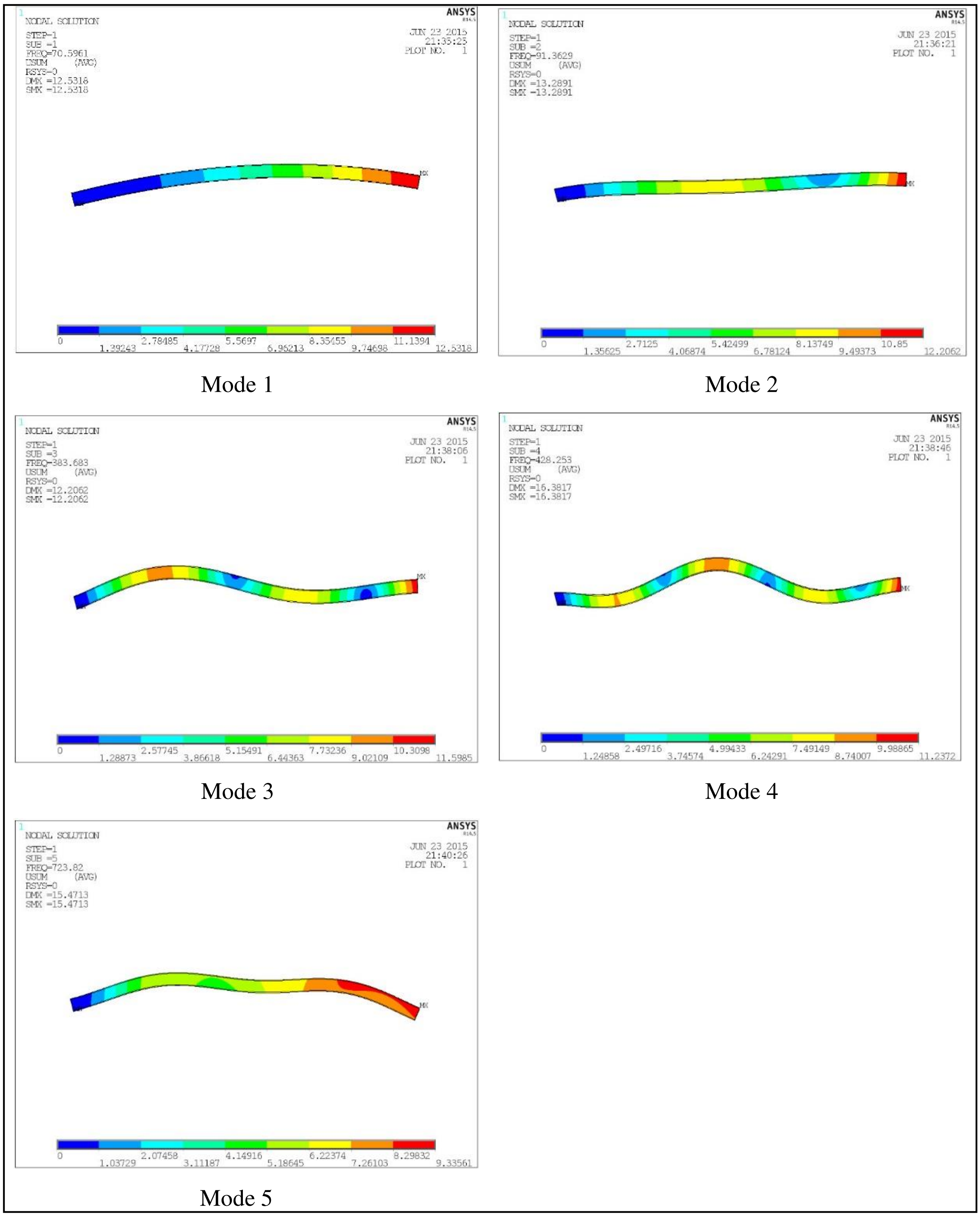


Figure 5.13 Mode shapes for R594 curved beam, clamped–free boundary condition

The mode shapes of first five free vibration modes of the curved beam R294 are shown in Figure 5.14.

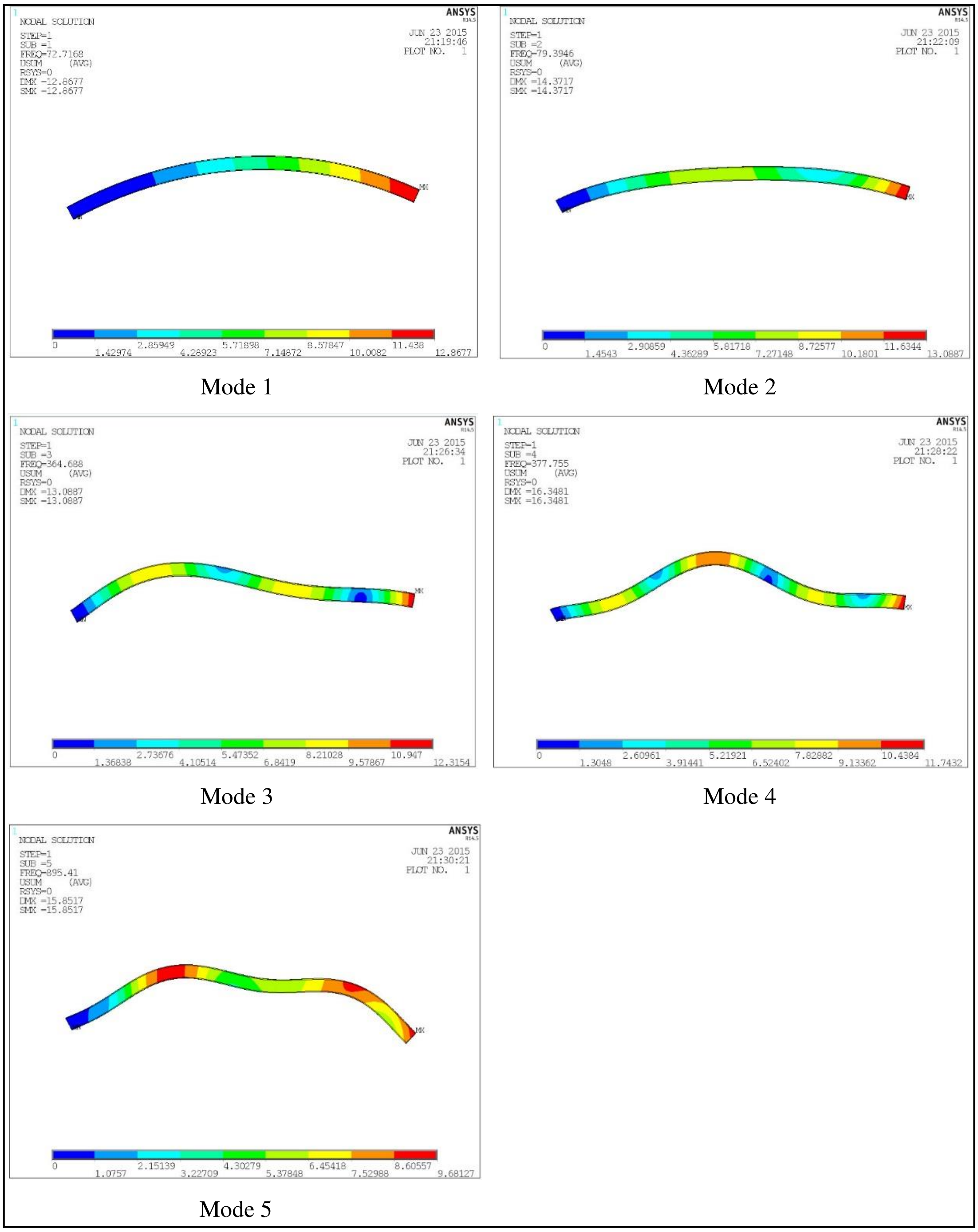


Figure 5.14 Mode shapes for R294 curved beam, clamped–free boundary condition

5.3 Natural Frequency Analyses for Clamped – Clamped Boundary Condition

The finite element modal analyses are performed for clamped-clamped boundary condition in order to show the effect of the change in the end conditions. Table 5.2 shows the first five natural frequencies of the flat and curved sandwich beams with clamped-clamped boundary condition. In this table, the percentage differences with respect to flat beam are also given. The natural frequency values for different beam curvatures versus mode number are given in graphical form in Figure 5.15. It is seen from the numerical results that, the clamped-clamped boundary condition increases the natural frequencies of the sandwich beams due to the increase in the structural stiffness. The natural frequency value increases as the radius of the curvature decreases for the first free vibration mode. The variation in the natural frequencies for other modes has fluctuating characteristics.

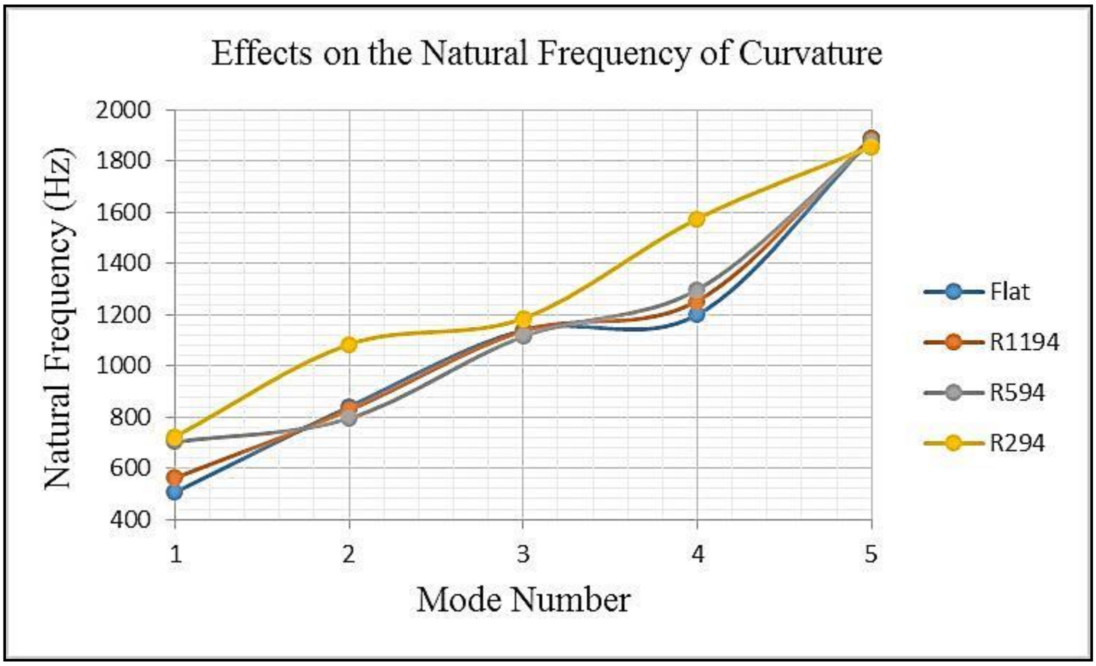


Figure 5.15 Effects on the natural frequency of curvature for clamped - clamped boundary condition

Table 5.2 Effect of curvature on the natural frequencies of the sandwich beams with clamped-clamped boundary condition (ANSYS).

Natural Frequencies of Sandwich Beams (Hz)						
		Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**
Flat	**FEA**	506.06	840.44	1140.2	1200.6	1887.6
R1194	**FEA**	562.39	827.46	1136.9	1254.9	1891.2
	Percentage Increase %	11.13	- 1.54	- 0.29	4.52	0.20
R594	**FEA**	702.93	795.54	1116.2	1298.6	1875.9
	Percentage Increase %	38.90	- 5.34	- 2.10	8.16	- 0.62
R294	**FEA**	723.55	1084.5	1186.0	1575.6	1858.2
	Percentage Increase %	42.98	29.04	4.02	31.23	- 1.56

Table 5.2 presents the comparison of natural frequencies of the curved sandwich beam with respect to the flat beam. The deviations are given in percentage error. The maximum increase is 42.98% for R294 curved beam, and first vibration frequency increases with the increase in the curvature. The variation exhibits a wavy characteristic for other vibration modes as mentioned before.

Figure 5.16 shows the mode shapes of first five free vibration modes, regarding flat beam, for clamped–clamped boundary condition.

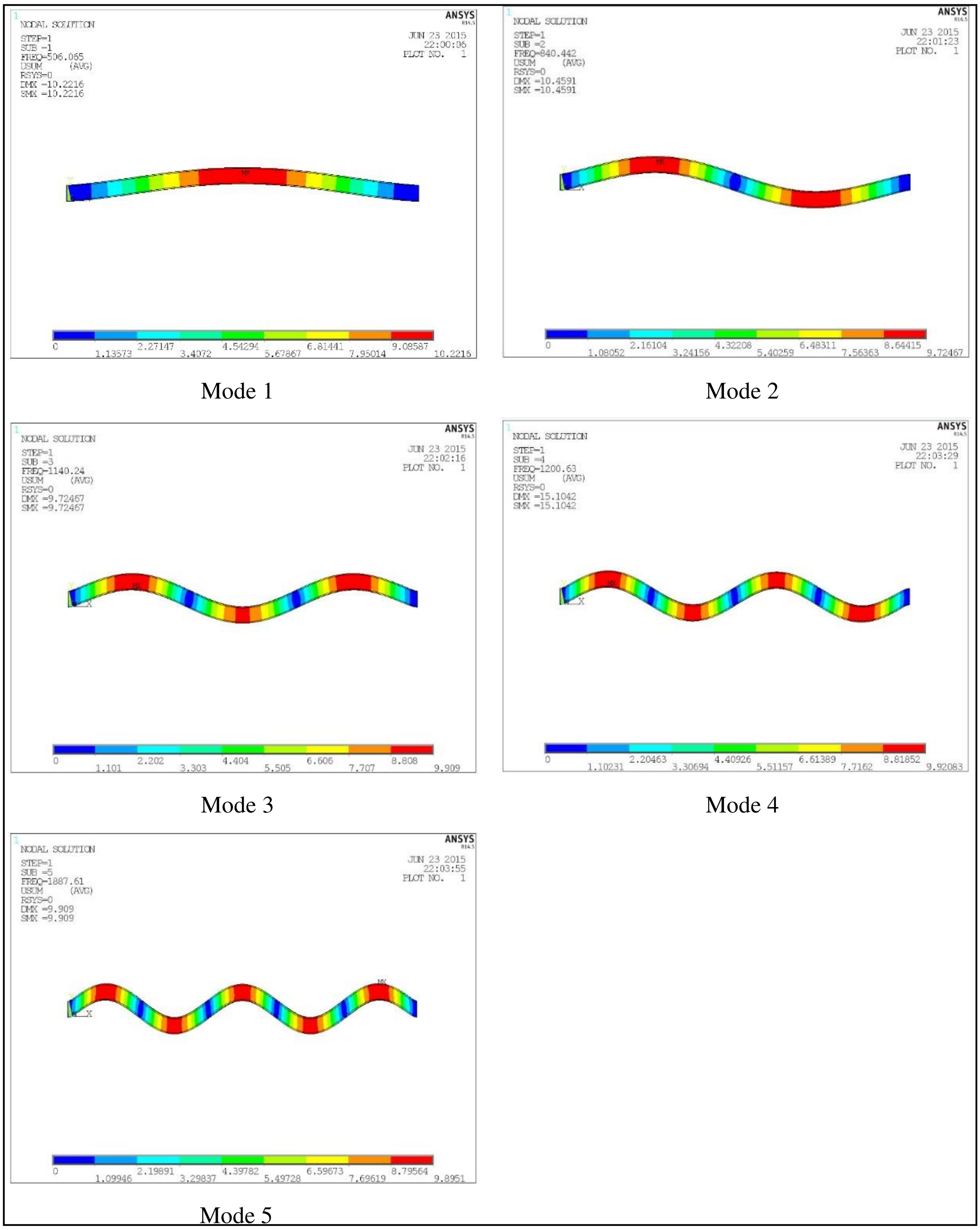


Figure 5.16 Mode shapes for flat beam, clamped–clamped boundary condition

Figure 5.17 shows the mode shapes of first five free vibration modes, regarding R1194 curved beam, for clamped–clamped boundary condition.

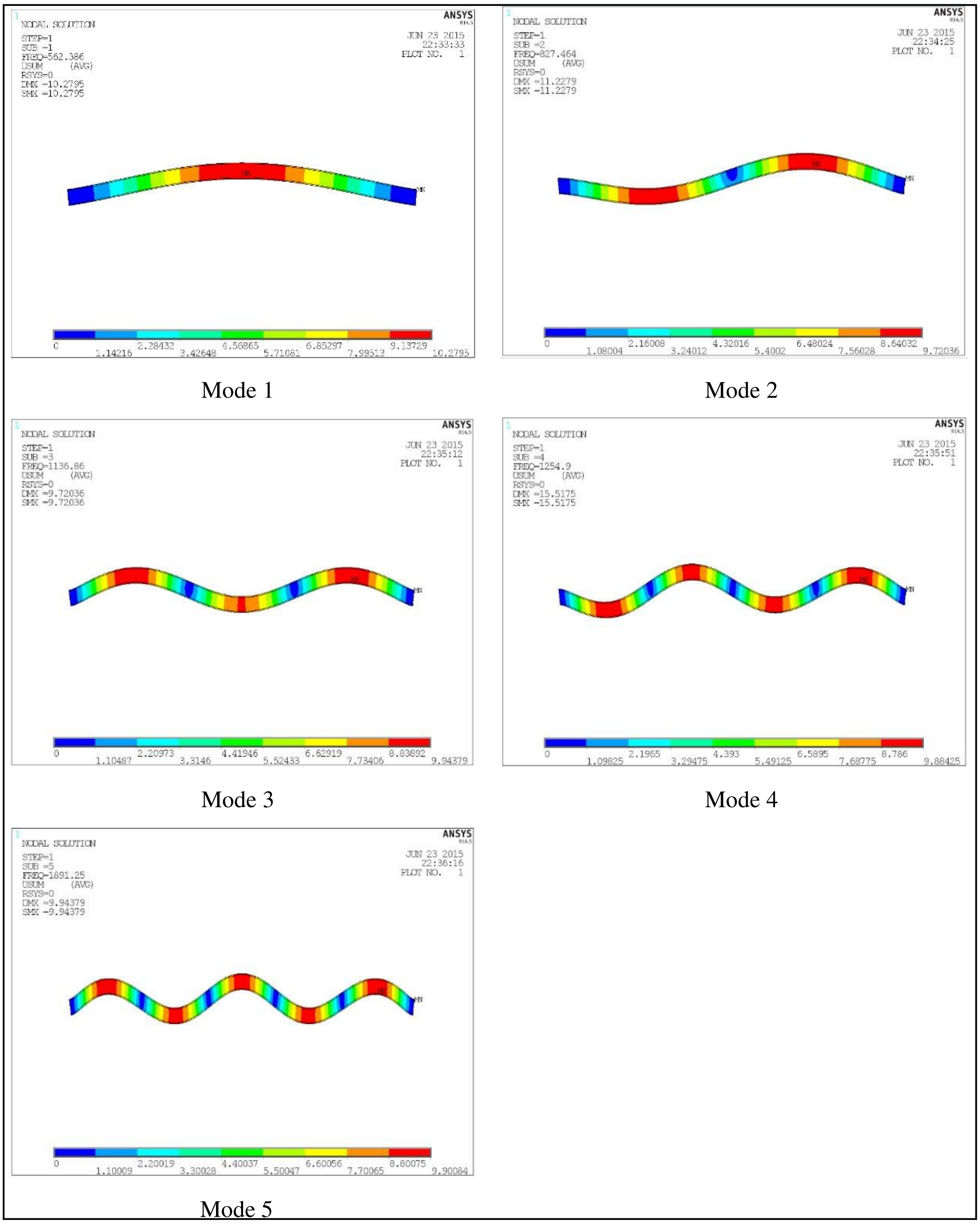
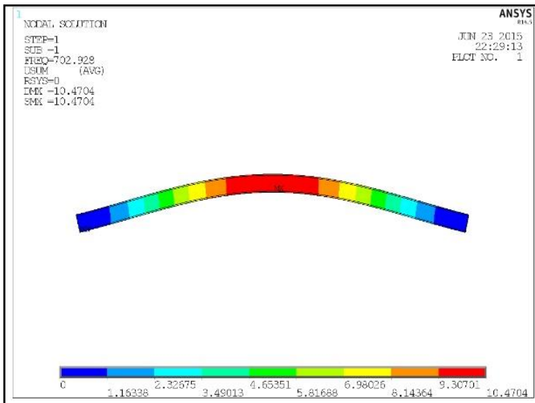
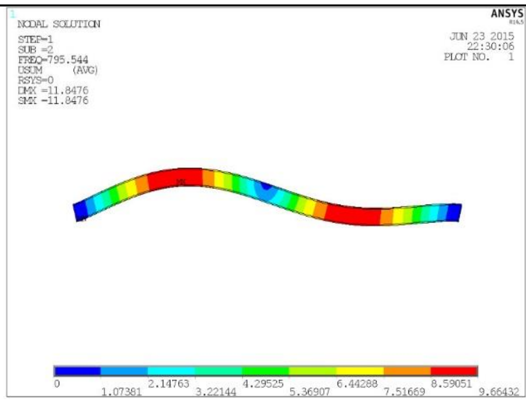


Figure 5.17 Mode shapes for R1194 curved beam with clamped–clamped boundary condition

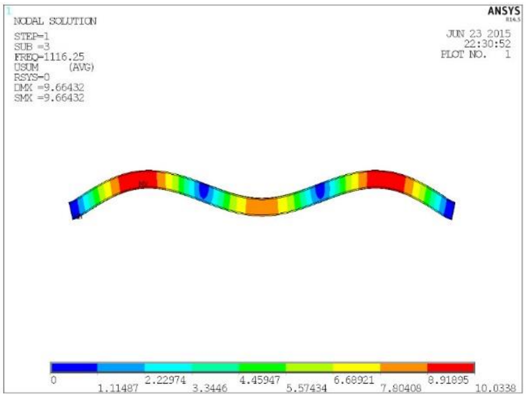
Figure 5.18 shows the mode shapes of first five free vibration modes, regarding R594 curved beam, for clamped–clamped boundary condition.



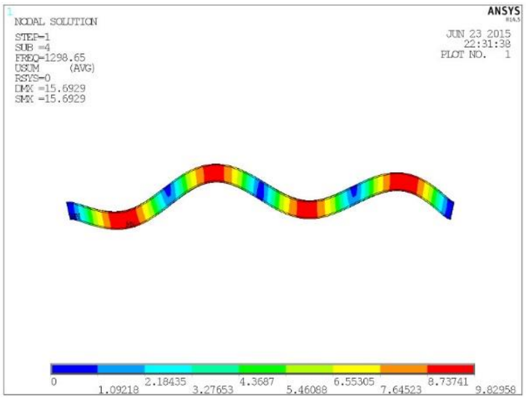
Mode 1



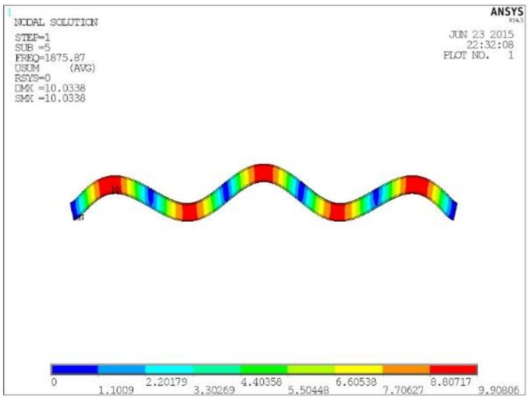
Mode 2



Mode 3



Mode 4



Mode 5

Figure 5.18 Mode shapes for R594 curved beam with clamped–clamped boundary condition

Figure 5.19 illustrates the mode shapes of first five free vibration modes, regarding R294 curved beam, for clamped–clamped boundary condition.

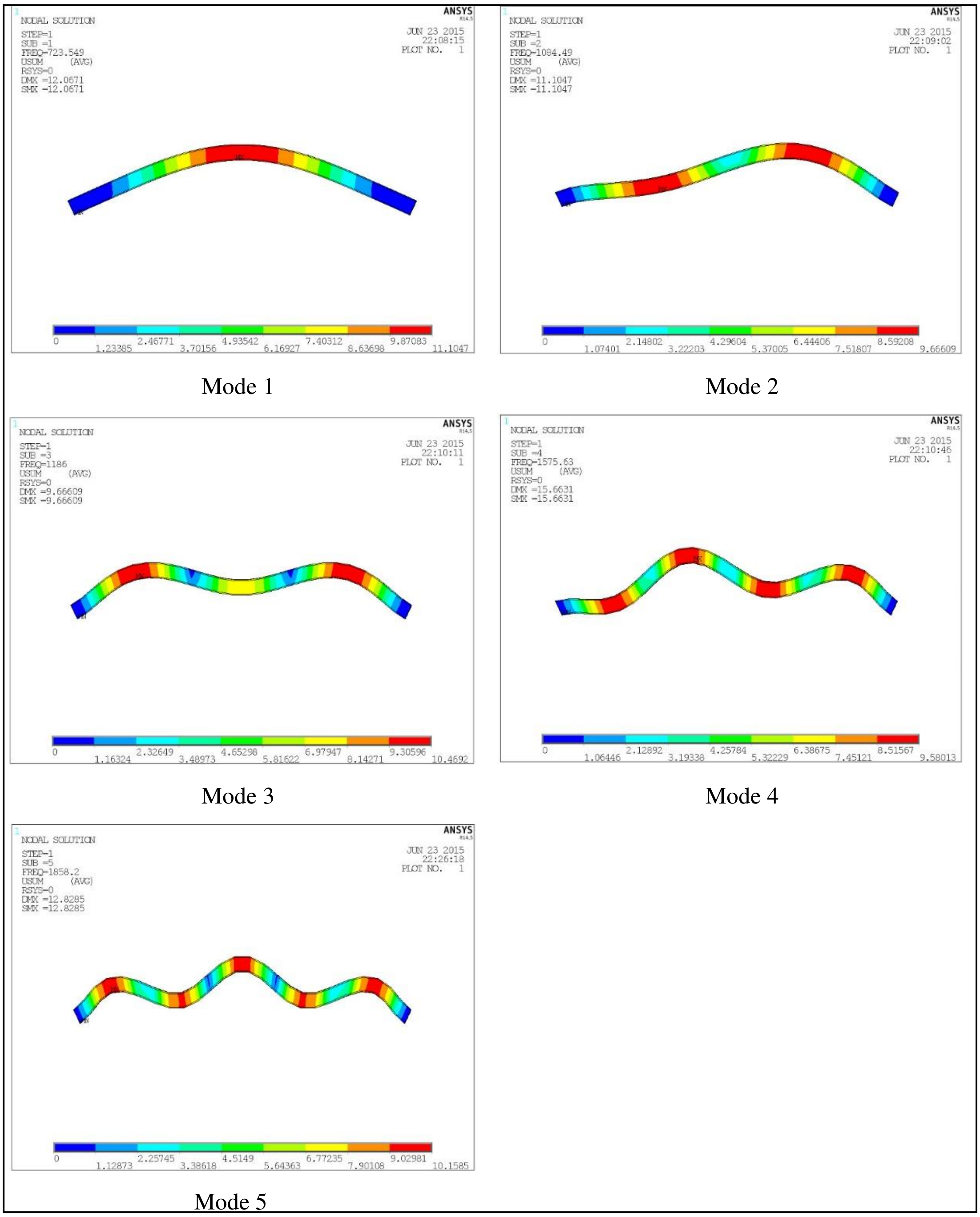


Figure 5.19 Mode shapes for R294 curved beam with clamped–clamped boundary condition

5.4 Effect of Beam Length on the Natural Frequency of the Curved Beam

In this section, the effect of the beam length on the natural frequencies of the curved beam is investigated for R1194 beam both experimentally and numerically considering the clamped-free boundary condition. The variation in the natural frequencies due to the change in the beam length is also investigated for clamped-clamped curved beam, numerically. Table 5.3 summarizes the scenarios used to investigate the effect of the beam length on the natural frequencies of the curved sandwich beam.

Table 5.3 Parameters used in investigating the effect of the beam length.

Material	Length	Boundary condition	Radius of curvature	Width	Face sheet thickness	Core thickness
	L(mm)		R (mm)	W (mm)	t_s (mm)	t_f (mm)
Glass Polyurethane Glass	L110 L260 L420	Clamped / Free	R1194	25	0.7	10
		Clamped / Clamped				

5.4.1 Clamped – Free Boundary Condition

The free vibration tests for R1194 curved beam are performed for three different beam lengths namely, 110 mm, 260 mm and 420 mm. Frequency contents of the free vibration signals are obtained after taking the fft transform in Matlab. The frequency spectra of three curved beams are given in Figures 5.20-5.22.

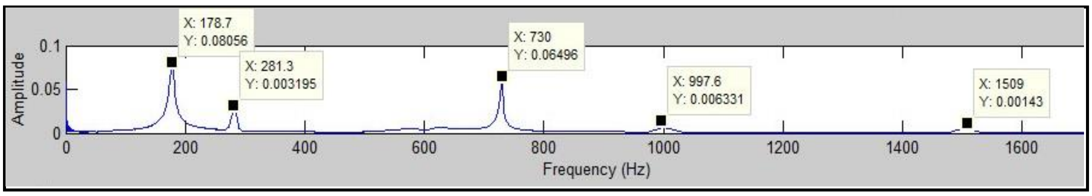


Figure 5.20 Frequency spectrum of R1194_L110 curved beam

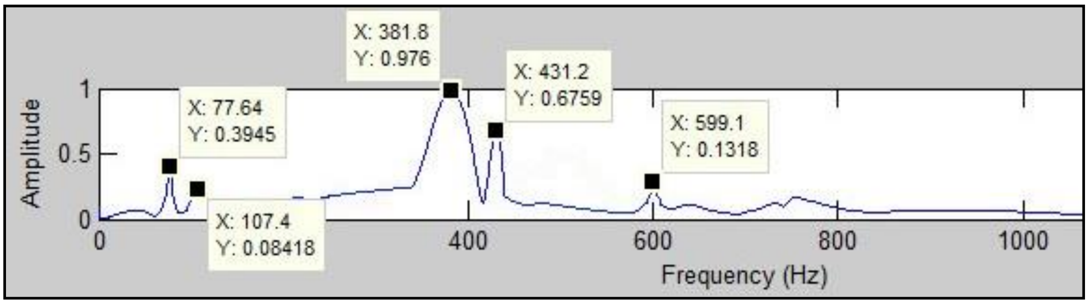


Figure 5.21 Frequency spectrum of R1194_L260 curved beam

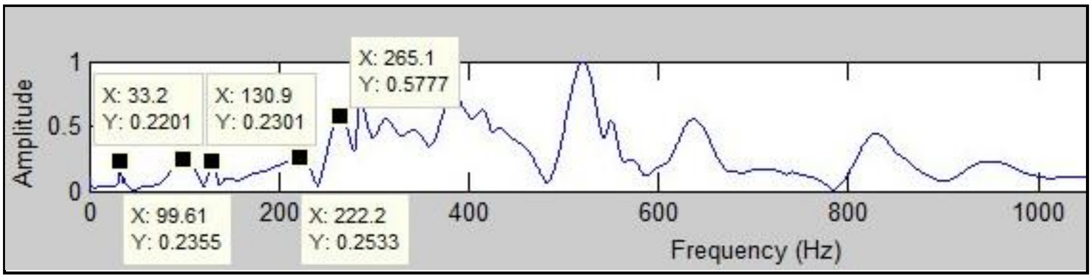


Figure 5.22 Frequency spectrum of R1194_L420 curved beam

The peaks in the frequency spectrum corresponds a natural frequency value. These frequency values are summarized in Table 5.4.

Table 5.4 Effect of beam length on natural frequencies for clamped–free boundary condition

Natural Frequencies of Sandwich Beams (Hz)						
R1194		**Mode 1**	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**
L110	**EXP**	178.7	281.3	730	997.6	1509
	FEA	186.48	265.71	840.12	913.38	1535.2
	%Dif	+ 4.35	- 5.54	+ 15.08	-8.44	+ 1.73
L260	**EXP**	77.64	107.4	381.8	431.2	599.1
	FEA	68.173	93.728	380.08	467.25	630.87
	%Dif	- 12.19	- 12.73	- 0.45	+ 8.36	+5.30
L420	**EXP**	33.2	99.61	130.9	222.2	265.1
	FEA	39.7	84.156	138.76	238.63	276.95
	%Dif	+ 19.57	-15.51	+ 6	+ 7.39	+ 4.48

The experimental and numerical natural frequency values are given in Table 5.4 for R1194 curved beam with three different lengths. Generally, the discrepancies between the experimental and numerical results are within ± 10% error band, but for some cases, namely L420-1st and 2nd mode, L110 3rd mode, the difference is greater than 15%. But it can be said that the correlations between the experimental and numerical results is within the acceptable limits. The experimental natural frequency values presented in Table 5.4 are also given in graphical format in Figure 5.23. It can easily be seen from this figure that, the natural frequency values decrease as the beam length increases due to the decreasing flexural rigidity. The rate of decrease also decreases as the beam length increases.

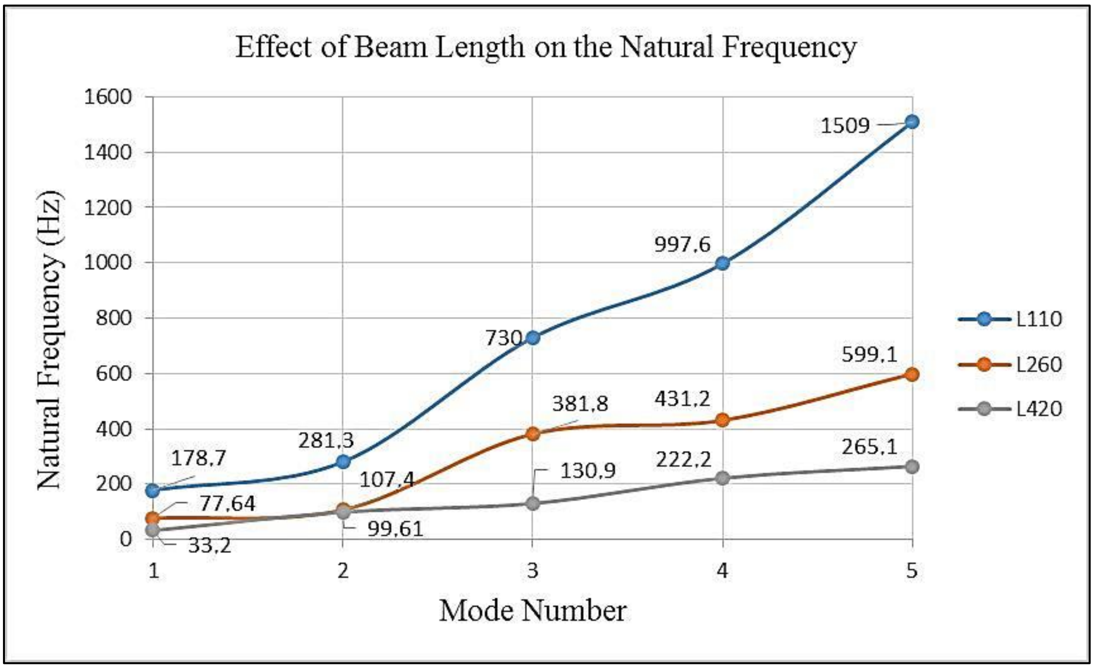


Figure 5.23 Effect of beam length on the natural frequency of the clamped-free beam (experimental results)

The comparison presented in Table 5.4 is also given in graphical format between Figures 5.24 - 5.26. The harmony between the experimental and numerical natural frequency values can also be seen from these figures.

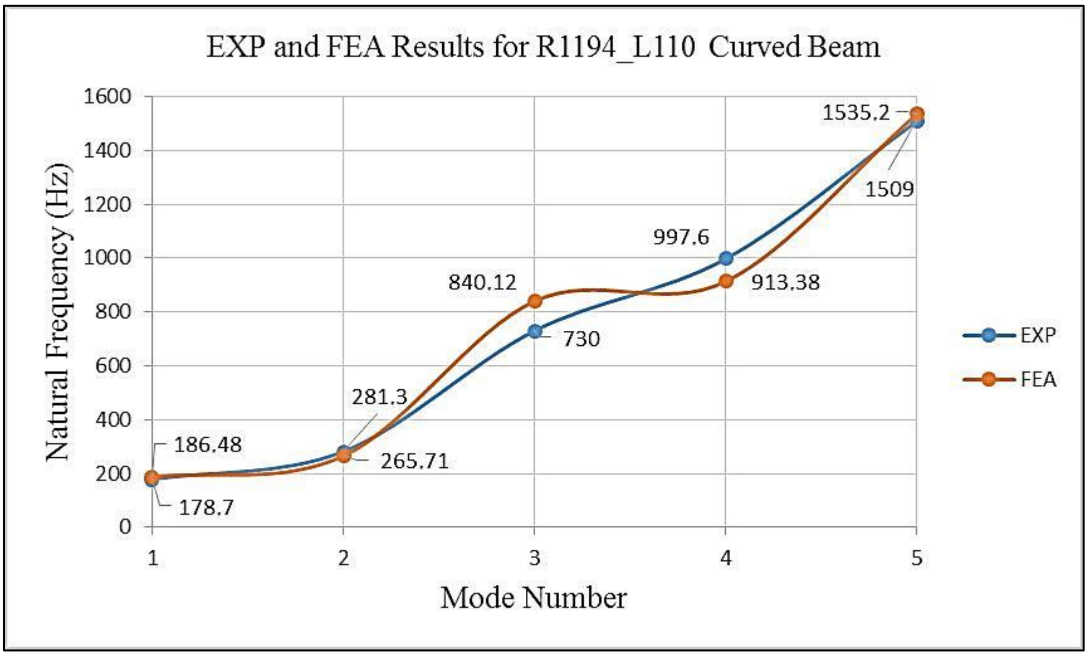


Figure 5.24 Experimental and numerical natural frequencies for R1194_L110 curved beam (clamped–free boundary condition)

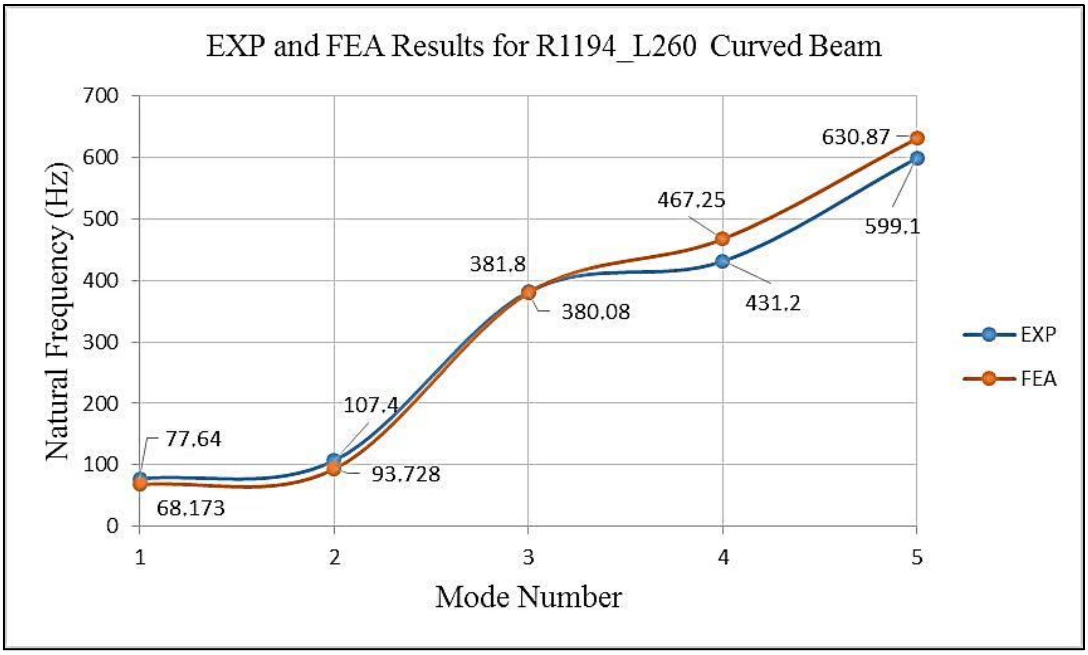


Figure 5.25 Experimental and numerical natural frequencies for R1194_L260 curved beam (clamped–free boundary condition)

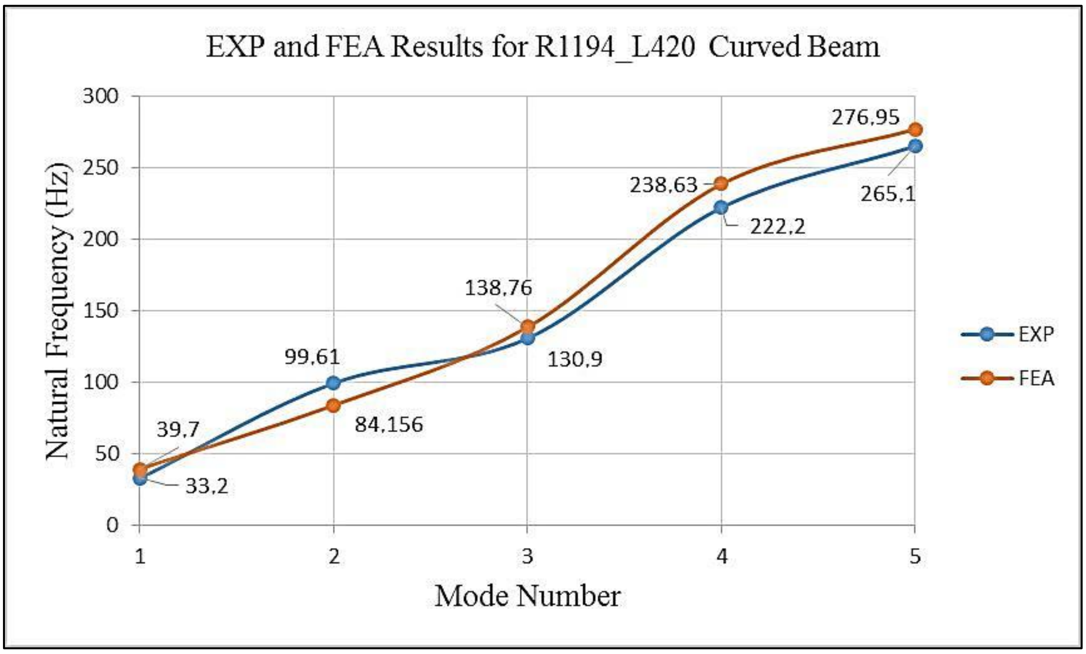


Figure 5.26 Experimental and numerical natural frequencies for R1194_L420 curved beam (clamped–free boundary condition)

5.4.2 Clamped – Clamped Boundary Condition

The effect of the beam length on the natural frequencies of R1194 curved beam is analyzed for clamped - clamped boundary condition. Numerical results are obtained using ANSYS. The numerical natural frequency values of three different beams are given in Table 5.5 and Figure 5.27. It is seen from the numerical results that, the natural frequency value increases as the beam length decreases, due to the increasing flexural rigidity of the beam. On the other hand, the rate of increase in the natural frequencies decreases as the beam length increases.

Table 5.5 Effect of beam length on the natural frequencies for clamped–clamped boundary condition

Natural Frequencies of Sandwich Beams (Hz)						
R1194 mm		**Mode1**	**Mode2**	**Mode3**	**Mode4**	**Mode5**
L110	**FEA**	1812.8	3246.5	3647.9	3992.4	5749.7
L260	**FEA**	562.39	827.46	1136.9	1254.9	1891.2
L420	**FEA**	304.08	321.33	515.74	782.51	816.52

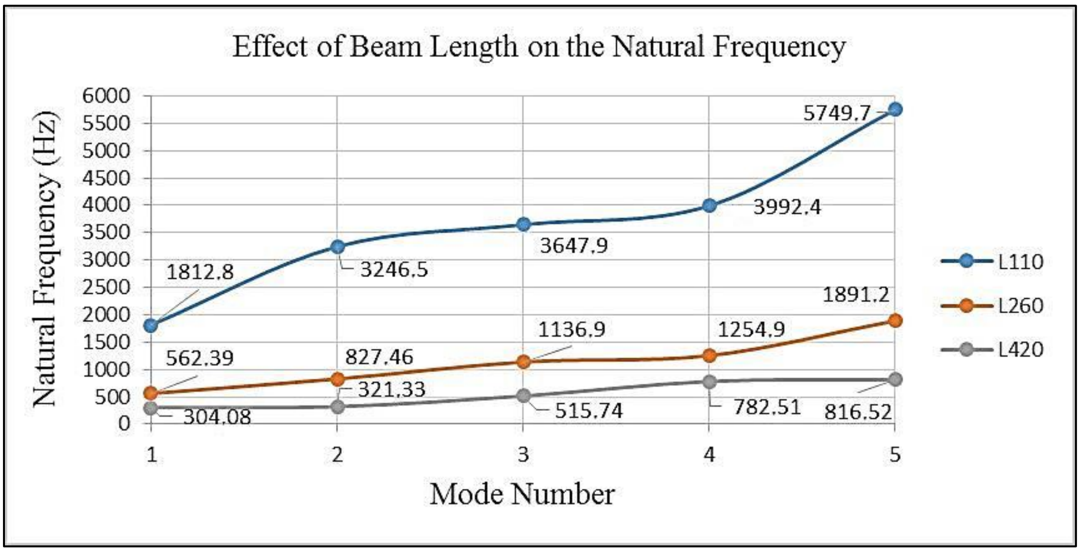


Figure 5.27 Effect of beam length on the natural frequencies for the clamped-clamped boundary condition, (FEA results)

CHAPTER SIX
CONCLUSIONS

6.1 Results

In this study, free vibration response of flat and curved sandwich beams have been presented both experimentally and numerically. The effects of radius of curvature, boundary condition and length of the beam are taken into consideration. In the numerical part of the study, the finite element method is employed and ANSYS software is used. From the experimental and numerical results obtained in this thesis, following conclusions can be drawn:

1- Generally, similar results are obtained for natural frequencies from the experimental and numerical studies. The discrepancies are found within ±10% error band except some cases.

2- The 1st and 5th natural frequency values increase as the radius of curvature of the beam decreases for clamped-free boundary condition. For the other modes, the natural frequencies tend to decrease with decreasing radius of curvature.

3- For clamped-clamped boundary condition, the 1st natural frequencies increase as the beam curvature increases. For higher modes, the change in the natural frequencies depending on the beam curvature exhibits a wavy nature.

4- The natural frequencies increase as the beam length decreases for two boundary conditions. Additionally, the rate of increase in the natural frequencies decreases for greater beam lengths.

6.2 Suggestions

In the light of the results and observations obtained from this study, some suggestions for future work are listed below;

1- The discrepancies between the experimental and numerical results may be decreased by enhancing the material properties and boundary conditions.

2- The experimental and numerical results can be expanded by considering different radius of curvatures.

3- The effect of delamination on the natural frequencies can be taken into consideration.

4- The experimental and numerical natural frequency analyses can be performed for flat and curved plates.

REFERENCES

Allen, H. G. (1969). *Analysis and design of structural sandwich panels.* Oxford: Pergamon Press.

Ahmed, K.M. (1971). Free vibration of a curved sandwich beam by the finite element method. *Journal of Sound and Vibration, 18* (1), 61-74.

Adique, E., & Hashemi, S.M. (2007). Dynamic finite element formulation and free vibration analysis of a three - layered sandwich beam. *Proceedings of the 7th Joint Canada - Japan Workshop on Composite Materials*, 93-100.

Atas, C., & Sevim, C. (2010). On the impact response of sandwich composite with cores of balsa wood and PVC foam. *Composite Structures, 93,* 40-48.

Abrate, S. (2011). *Impact engineering of composite structures*. Italy: Springer Wien.

Bijlaard, P. P. (1951). Analysis of elastic and plastic stability of sandwich plates by method of split rigidities. *Journal of the Aeronautical Sciences.*

Bert, C. W. (1991). Part 1. *Shock and Vibration Digest, 2*3, 3-14.

Burton, W.S., & Noor, A.K. (1997). Assessment of continuum models for sandwich panel honeycomb cores. *Computer Methods in Applied Mechanics and Engineering*, 145, 341–360.

Bozhevolnaya, E., & Frostig, Y. (1997). Nonlinear closed-form high-order analysis of curved sandwich panels. *Composite Structures*, 38, 384–393.

Bozhevolnaya, E., & Frostig, Y. (2001). Free vibrations of curved sandwich beams with a transversely flexible core. *Journal of Sandwich Structures and Materials*, 3(4), 311–342.

Banerjee, J. R. (2003). Free vibration of sandwich using the dynamic stiffness method. *Computers & Structures*, 81, 18-19.

Banerjee, J. R., & Sobey, A. J. (2005). Dynamic stiffness formulation and free vibration analysis of a three-layered sandwich beam. *International Journal of Solids and Structures*, 42, 8.

Cheng, S. (1962). On the theory of bending of sandwich plates. *Proceedings of the Fourth U.S. National Congress of Applied Mechanics*.

Feichtinger, K. A. (1988) Test methods and performance of structural core materials-l, Static Properties, *4th Annual ASM International Engineering Society of Detroit - Advanced Composites Conference/Exposition,* 13-15.

Fasana, A., & Marchesielo, S. (2001). Rayleigh-Ritz analysis of sandwich beams. *Journal of Sound and Vibration*, 241(4), 643–652.

Ha, K. H. (1989). Finite element and sandwich construction a critical review. Olsson K. A. & Reichard R. P. (Ed.). *Sandwich Constructions I-Proceedings of the First International Conference on Sandwich Construction* (69-84). United Kingdom: EMAS Publications.

Jensen, A.E., & Irgens, F. (1999). Thickness vibrations of sandwich plates and beams and delamination detection. *Journal of Intelligent Material Systems and Structures*, 10, 46–55.

Kaw, A. K. (2006). *Mechanics of composite materials*, (2nd ed.). Taylor & Francis Group, LLC.

Lee, C.Y.P., & Chang, N. (1979). Harmonic waves in elastic sandwich plates. *Journal of Elasticity*, 9(1), 51–69.

Mead, D. J., & Markus, S. (1968). The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions. *Journal of Sound and Vibration,* 10 (2), 163-175.

Mallick, P.K. (2007). *Fiber-Reinforced composites materials, manufacturing and design*. Taylor & Francis Group.

Noor, A. K., Scott Burton W., & Bert C. W. (1996). Computational models for sandwich panels and shells. *Applied Mechanics Reviews,* 49 (3), 155-199.

Okutan Baba, B., Thoppul S., & Gibson R.F. (2010) Experimental and numerical investigation of free vibrations of composite sandwich beam with curvature and debonds. *Experimental Mechanics*, 51, 857-868.

Petty, M., & Fleischer, C.C. (1971) Free vibration of a curved beam. *Journal of Sound and Vibration*, 18, 17-30.

Reddy, J.N., & Wang, C.M. (2000). An overview of the relationships between solutions of the classical and shear deformation plate theories. *Composite Science and Technology*, 60, 2327–2335.

Sakiyama, T., Matsuda, H. & Morita, C. (1997). Free vibration analysis of sandwich arches with elastic or viscoelastic core and various kinds of axis – shape and boundary conditions. *Journal of Sound and Vibration*, 203(3), 505–522.

Singh, A.V. (1999). Free vibration analysis of deep doubly curved sandwich panels. *Computers and Structures*, 73(1–3), 385–394.

Singiresu S. R. (2007). *Vibration of continuous systems*. New Jersey: John Wiley and Sons.

Timoshenko, S., Young, D.H., & Weaver, W. Jr. (1974). *Vibration problems in engineering*. New York: John Wiley and Sons.

Vinson, J. R., & Shore S. (1965) Bibliography on methods of structural optimization for flat sandwich panels. *Naval Air Engineering Center Report*.

Vinson, J. R., & Shore S. (1967) Structural optimization of corrugated core and web core sandwich panels subjected to uniaxial compression. *Naval Air Engineering Center Report*.

Vaswani, J., Asnani, N.T., & Nakra, B.C. (1998) Vibration and damping analysis of curved sandwich beams with an elastic core. *Composite Structures,* 10 (3) 231-245.

Vinson, J.R. (1999). *The behavior of sandwich structures of isotropic and composite materials*. Pennsylvania: Technomic Publishing Co. Inc.

Zenkert D. (1995). *An investigation to sandwich construction*. London: Chameleon Press.