A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF ÇANKIRI KARATEKIN UNIVERSITY

FUZZY SET VALUED INTERVAL SOFT SETS

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMATICS

BY
MOHAMMED ABBOOD HUSSEIN AL-FURAIJI
ÇANKIRI
2023

FUZZY SET VALUED INTERVAL SOFT SETS

By Mohammed Abbood Hussein AL-FURAIJI

January 2023

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Advisor : Assoc. Prof. Dr. Faruk KARAASLAN

Examining Committee Members:

Chairman : Prof. Dr. Aslihan SEZGIN
Mathematics Education
Amasya University

Member : Assoc. Prof. Dr. Faruk KARAASLAN
Mathematics
Çankırı Karatekin University

Member : Asst. Prof. Dr. Celalettin KAYA
Mathematics
Çankırı Karatekin University

Approved for the Graduate School of Natural and Applied Sciences

Prof. Dr. İbrahim ÇİFTÇİ
Director of Graduate School

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Mohammed Abbood Hussein AL-FURAIJI

ABSTRACT

FUZZY SET VALUED INTERVAL SOFT SETS

Mohammed Abbood Hussein AL-FURAIJI
Master of Science in Mathematics
Advisor: Assoc. Prof. Dr. Faruk KARAASLAN
January 2023

This thesis consists of seven sections. In section 1, a detailed literature review and the motivation of the thesis are presented. A few fundamental definitions and procedures for soft sets and fuzzy sets are introduced in Section 2. In Section 3, the definition of set-valued intervals is given and their operations are presented. In Section 4, the definition and operations of set-valued interval soft sets in light of the literature review are explained. In Section 5, the concept of fuzzy set-valued intervals is described, and their operations are explained with examples. In Section 6, fuzzy set-valued interval soft sets are discussed, along with their operations, and the properties of the operations are further investigated with examples. In Section 7, information about the thesis's overall framework is provided and the thesis's findings are discussed.

2023, 41 pages

Keywords: Fuzzy set, Soft set, fuzzy set-valued interval, Fuzzy set-valued interval soft sets, Set operations

ÖZET

BULANIK KÜME DEĞERLİ ARALIK ESNEK KÜMELER

Mohammed Abbood Hussein AL-FURAIJI
Matematik, Yüksek Lisans
Danışman: Doç. Dr. Faruk KARAASLAN
Ocak 2023

Bu tez yedi bölümden oluşmaktadır. Bölüm 1'de ayrıntılı bir literatür taraması ve tezin motivasyonu sunulmaktadır. Bölüm 2'de bulanık kümeler ve yumuşak kümeler için bazı temel tanımlar ve işlemler tanıtılmaktadır. Bölüm 3'te küme değerli aralıkların tanımı verilmiş ve işlemleri sunulmuştur. Bölüm 4'te literatür taraması ışığında küme değerli aralıklı esnek kümelerin tanımı ve işlemleri açıklanmıştır. Bölüm 5'te bulanık küme değerli aralıklar kavramı anlatılmış ve işlemleri örneklerle açıklanmıştır. Bölüm 6'da, bulanık küme değerli aralıklı esnek kümeler kavramı ve bunların işlemleri ele alınmakta ve işlemlerin özellikleri incelenerek örnekler verilmektedir. Bölüm 7'de tezin genel çerçevesi hakkında bilgi verilmiş ve tezin bulguları tartışılmıştır.

2023, 41 sayfa

Anahtar Kelimeler: Bulanık küme, Esnek küme, Bulanık küme değerli aralık, Bulanık küme değerli aralık esnek küme, Küme işlemleri

PREFACE AND ACKNOWLEDGEMENTS

I would like to thank my thesis advisor, Assoc. Prof. Dr. Faruk KARAASLAN and my friend Abdulrasool Hasan Sultan AL-HUSSAINAWI, for his patience, guidance and understanding. I would like to thank my father and mother, my wife and my children Adam and Habeeb-allah. Finally my best thanks to my best friend Salim Nadher.

Mohammed Abbood Hussein AL-FURAIJI
Çankırı-2023

CONTENTS

LIST OF SYMBOLS

$[\tilde{\wp}]$	Absolute fuzzy set-valued interval.
$[\wp]$	Absolute set-valued interval
$[\bar{\wp}]$	Absolute set-valued interval soft
$S[\wp]$	All set-valued soft sets over $\wp$.
$\wr$	Difference of set-valued intervals
$\tilde{\emptyset}$	Empty fuzzy set.
$\widetilde{[\tilde{\Phi}]}$	Empty fuzzy set-valued interval soft set.
$\sqcap_e$	Extended intersection of set-valued interval soft sets.
$\sqcup_e$	Extended union of set-valued interval soft sets.
$\tilde{\Cap}$	Intersection of fuzzy set-valued interval.
$\Cap$	Intersection of set-valued intervals
$[\Phi]$	Null set-valued interval
$[\bar{\Phi}]$	Null set-valued interval soft set.
$[\tilde{\Phi}]$	Null fuzzy set-valued interval.
$\sqcap_r$	Restricted intersection of set-valued interval soft sets.
$\sqcup_r$	Restricted union of set-valued interval soft sets.
$\sqsubseteq_i$	Set-valued subinterval
$\wp$	Set of all set-valued intervals on $\wp$
$\mathbb{L}(\wp)$	Set of all fuzzy sets over $\wp$.
$\mathbb{L}[\wp]$	Set of all fuzzy set-valued interval.
$\mathbb{L}S[\wp]$	Set of all fuzzy set-valued interval soft set.
$S[\wp]$	Set of all set-valued soft sets over$\wp$.
$\tilde{\Cup}$	Union of fuzzy set-valued interval.
$\Cup$	Union of set-valued intervals
$\tilde{\wp}$	Universal fuzzy set
$\widetilde{[\tilde{\wp}]}$	Universal fuzzy set-valued interval soft set.
$\wp$	Universal set

LIST OF ABBREVIATIONS

FS	Fuzzy Set
FSVI	Fuzzy set-valued interval
FSVISs	Fuzzy set-valued interval soft set
SVI	Set-valued interval
SVISs	Set-valued interval soft set

1. INTRODUCTION

Molodtsov (1999) suggested "soft set theory" as a universal technique for handling uncertain problems as an alternative approximation to fuzzy sets defined by Zadeh (1965). In 2002, Maji *et al.* (2002) used rough mathematics and the theory of soft set to solve a decision-making problem. Maji *et al.* (2003) defined the operations on soft sets found in set theory, including union, intersection, and complement. They then demonstrated a few of the suggested operations' qualities. Chen *et al.* (2005) gave a procedure connected to the soft set parameterizations reduction utilized to address a decision-making issue. In 2007, the first study on soft algebra started with the soft group structure defined by Aktaş and Çağman (2007). Ali *et al.* (2009) draw attention to various flaws in (Maji *et al.* 2003) and defined some novel operations such as extended of union and intersection, restricted of union and intersection of two soft sets. Çağman and Enginoğlu (2010) developed a decision-making approach known as the uni-int decision-making method under a soft environment and redefined the set theoretical operations of soft sets as more effective in decision-making challenges. Sezgin and Atagun (2011) applied De Morgan's law to some soft set operations. Additionally, they looked at a few characteristics of the businesses that were done by Ali *et al.* (2009) and defined the restricted difference and restricted symmetric difference operations between two soft sets.

The fuzzy soft sets notion was explained by Maji *et al.* (2001a). Roy and Maji (2007) developed a new decision making for object identification from imprecise multi-observer information. The concepts of interval-valued fuzzy soft sets and interval-valued fuzzy sets with soft sets were blended by Yang *et al.* (2009). Chetia and Das (2010) demonstrated the potential of interval-valued fuzzy soft sets for medical diagnosis. For other studies related to "interval-valued fuzzy soft sets" and "fuzzy soft sets", see (Basu *et al.* 2012, Çağman *et al.* 2011, Çağman and Enginoğlu 2012, Kalaichelvi *et al.* 2012, Kong *et al.* 2011, Li and Xie 2015, Liu *et al.* 2017, Ma and Sulaiman 2011, Neog and Sut 2011, Peng and Garg 2018, Rehman *et al.* 2013, Sooraj and Tripath 2018, Sulaiman and Mohammed 2012, Tripathy *et al.* 2016, Xue *et al.* 2017, Zhang 2013, Zhou *et al.* 2012).

Yao (1993) established the idea of interval sets and its operations. He also examined fundamental properties of interval set algebra and discussed the relationship between interval set, rough set (Pawlak *et al.* 1995) and fuzzy sets (Zadeh 1965). For other works related to interval sets, see (Yao and Li 1996, Yao and Wong 1997, Yao and Wang 1997, Yao 2009). Zhang and Fu (2012) introduced the concept of interval soft set. Then, the interval soft sets tabular form was explained by Zhang (2014), the innovative interval choice value notions were made clear, and two methods were employed to apply the theory of interval soft sets to a decision-making problem. Additionally, he constructed several lattice structures and explained certain interval soft set procedures. In the area of interval soft set theory, he also established the concept of "soft equality." The lattice-valued interval soft sets were introduced by Zhang and Wang (2014) which include several soft set extensions like soft sets and interval soft sets (Zhang and Fu 2012), f-soft sets (Maji *et al.* 2001a), L-fuzzy soft sets (Li *et al.* 2012), interval-valued f-soft sets (Yang *et al.* 2009), vague soft sets (Xu *et al.* 2010), intuitionistic f-soft sets (Maji *et al.* 2001b), interval-valued intuitionistic f-soft sets (Jiang *et al.* 2010), rough soft sets (Pawlak *et al.* 1995) etc. He also detailed how interval soft sets functioned and, using the defined procedures, he was able to determine some of the lattice structure of interval soft sets. (Yaylali *et al.* 2021) employed tabular representation of the soft interval set, a decision-making algorithm was built.

An interval is an important tool to represent the possible values of a variable. In a closed real interval set, boundaries are any real numbers such that its lower bound is less than or equal upper bound. Set valued intervals have set valued boundaries. Namely, lower and upper bound of a set valued interval are the classical sets. For the modelings of some problems classical set and logic are not sufficient. With this point-wise, in this thesis we define the concept of fuzzy set-valued intervals and their operations. Then, as we combine the ideas of soft set and fuzzy set-valued interval, we introduce the notion of fuzzy set-valued interval soft set.

Following is how the rest of the thesis is structured: Some basic definitions and procedures for fuzzy sets and soft sets are presented in Section 2. Section 3 gives related concepts to set-valued intervals and operations of them. In Section 4, "set-valued interval soft sets" are explained in light of the study in the literature. In Section 5, the notion of the fuzzy set-valued intervals is defined and their operations are introduced by supporting them with examples. The idea of fuzzy set-valued interval soft sets and associated operations are covered in section 6. Also, the properties of the operations are investigated and examples are given. In Section 7, information general frame of the thesis is given and the results of the thesis are discussed.

2. PRELIMINARIES

In this section, the notions of fuzzy set and soft sets are recalled and some of their properties are given. Throughout this paper $\wp$, E and $P(\wp)$ denote initial universe, set of parameters and power set of $\wp$, respectively.

2.1 Fuzzy Sets

Definition 2.1. (Zadeh 1965) Let $\mathscr{S}$ be a non-empty universal set. A fuzzy set (FS) Ψ on $\mathscr{S}$ is described by a membership function as follows:

$$\mu_\Psi : \mathscr{S} \to [0,1]$$

An FS can be expressed as a group of pairs as follows:

$$\Psi = \{(\mathfrak{S}, \mu_\Psi(\mathfrak{S})) : \mathfrak{s} \in \mathscr{S}\},$$

where $\mu_\Psi(\mathfrak{S})$ indicates the level of the element's membership $\mathfrak{s} \in \mathscr{X}$ to FS Ψ.

Example 2.2. A student studies in a high school. For his level in English language, he takes a course to improve it for two months. During the course, he takes 5 English tests. His level changes from weak to high. Test results can be expressed as a fuzzy set as follows.

$$\Psi = \{(test_1, 0.5), (test_2, 0.6), (test_3, 0.7), (test_4, 0.8), (test_5, 0.9)\},$$

Definition 2.3. Operations on Fuzzy Sets (Zadeh 1965)
Given $\wp$ to be universe of discourse, Ξ and $\aleph$ are two fuzzy sets with membership function $\mu_\Xi(s)$ and $\mu_\aleph(s)$ then,

Union: The union of two fuzzy sets Ξ and $\aleph$ on $\wp$ is a new fuzzy set $\Xi \cup \aleph$ also on $\wp$ which membership function defined as follows:

$$\mu_{\Xi \cup \aleph}(s) = \max(\mu_\Xi(s), \mu_\aleph(s))$$

Intersection: The intersection of two fuzzy sets Ξ and $\aleph$ on $\wp$ is a new fuzzy set $\Xi \cap \aleph$ also on $\wp$ which membership functions defined as follow:

$$\mu_{\Xi \cap \aleph}(s) = \min(\mu_{\Xi}(s), \mu_{\aleph}(s))$$

Complement: Complement of a fuzzy set Ξ is Ξ^c which membership function defined as follow:

$$\mu_{\Xi^c}(s) = 1 - \mu_{\Xi}(s)$$

Equality: Two fuzzy sets Ξ and $\aleph$ are said to be equal i.e, $\Xi = \aleph$ if and only if $\mu_{\Xi}(s) = \mu_{\aleph}(s)$ which means their membership values must be equal.

Product of two fuzzy sets:The product of two fuzzy sets Ξ and $\aleph$ is a new fuzzy set $\Xi\aleph$ with membership function:

$$\mu_{\Xi\aleph}(s) = \mu_{\Xi}(s).\mu_{\aleph}(s)$$

Product of Fuzzy Sets with a Crisp Number

$$\mu_{n.\Xi}(s) = n.\mu_{\Xi}(s)$$

Power of a Fuzzy Set:The fuzzy set's alpha power of Ξ is a new fuzzy set Ξ^{α} whose membership function is:

$$\mu_{\Xi^{\alpha}}(s) = [\mu_{\Xi}(s)]^{\alpha}$$

that is, individual memberships power of α.

Difference of Fuzzy Sets:

The difference of two FSs Ξ and $\aleph$ is a new FS $\Xi - \aleph$ which is defined as

$$\Xi - \aleph = \Xi \cap \aleph^c$$

Disjunctive Sum of Ξ and $\aleph$: It is a new FS defined as follows:

$$\Xi \oplus \aleph = (\Xi^c \cap \aleph) \cup (\Xi \cap \aleph^c)$$

Proposition 2.4. Let Ξ and $\aleph$ two fuzzy sets over $\wp$. Then,

1. $(\Xi \cup \aleph)^c = \Xi^c \cap \aleph^c$
2. $(\Xi \cap \aleph)^c = \Xi^c \cup \aleph^c$

2.2 Soft Sets

Definition 2.5. (Molodtsov 1999) Let $\wp$ be an initial universe and E be a set of parameters, $\Xi \subseteq E$ and $P(\wp)$ be a power set of universal set. A pair (L, Ξ) is called a soft set over $\wp$, where L is a mapping given by $L : \Xi \to P(\wp)$.

Çağman and Enginoğlu (2010) redefined the concept of soft sets and its set operation which is more effective for decision-making problems. Therefore, we use definitions and operations given by Çağman and Enginoğlu (2010) in our thesis.

Definition 2.6. (Çağman and Enginoğlu 2010) Let $\wp$ be a universal set, E be a parameter set, $\Xi \subseteq E$. A soft set L_Ξ over $\wp$ is defined set of ordered pairs as follows:

$$L_\Xi = \{(e, l_\Xi(e)) : e \in E, l_\Xi(e) \in P(\wp)\}$$

where $l_\Xi : E \to P(\wp)$ such that $l_\Xi(e) = \emptyset$ if $e \in E \setminus \Xi$. Here, l_Ξ is called approximate function of the soft set L_Ξ.

Note that if $L(e) = \emptyset$, then the element $(e, l_\Xi(e))$ is not appeared in L_D. $\mathbb{D}_E^{\wp}$ denotes the set of all soft sets over $\wp$ and parameter set E.

Definition 2.7. (Çağman and Enginoğlu 2010) Let $L_\Xi, L_\aleph \in \Xi_E^{\wp}$, Ξ, $\aleph \subseteq E$. Then,

1. If $l_\Xi(e) = \emptyset$ for all $e \in E$, it is said that L_Ξ is a null soft set, denoted by Φ.
2. If $l_\Xi(e) = \wp$ for all $e \in E$. L_Ξ, it is said that L_Ξ is an absolute soft set, denoted by $\tilde{\wp}$.
3. L_Ξ is soft subset of $L_\aleph$, denoted by $L_\Xi \tilde{\subseteq} L_\aleph$ if $l_\Xi(e) \subseteq l_\aleph(e)$ for all $e \in E$.

4. $L_{\Xi} = L_{\aleph}$ if $L_{\Xi} \tilde{\subseteq} L_{\aleph}$ and $L_{\aleph} \tilde{\subseteq} L_{\Xi}$.

5. Soft union of L_{Ξ} and $L_{\aleph}$, denoted by $L_{\Xi} \tilde{\cup} L_{\aleph}$, is a soft set over $\wp$ and defined by $L_{\Xi} \tilde{\cup} L_{\aleph} : E \to P(\wp)$ such that $l_{\Xi \tilde{\cup} M}(e) = l_{\Xi}(e) \cup l_{\aleph}(e)$ for all $e \in E$.

6. Soft intersection of L_{Ξ} and $L_{\aleph}$, denoted by $L_{\Xi} \tilde{\cap} L_{\aleph}$, is a soft set over $\wp$ and defined by $L_{\Xi} \tilde{\cap} L_{\aleph} : E \to P(\wp)$ such that $l_{\Xi \tilde{\cap} \aleph}(e) = l_{\Xi}(e) \cap l_{\aleph}(e)$ for all $e \in E$.

7. $L_A^{\tilde{c}}$ denotes the soft complement of L_{Ξ} and defined by $L_{\Xi}^{\tilde{c}} : E \to P(\wp)$ such that $l^{\tilde{c}}(e)_{\Xi} = \wp \backslash l_{\Xi}(e)$ for all $e \in E$.

3. SET VALUED INTERVALS

Set valued intervals and related operations are discussed in this section. Additionally, some of their attributes are listed.

Definition 3.1. (Yao 1993) Let $\wp$ be a crisp set, $P(\wp)$ be the power set of $\wp$ and $\Xi, \aleph \subseteq \wp$. Then, set valued interval (SVI.) $\mathfrak{g}$ over $\wp$ is defined by:

$$\mathfrak{g} = [\Xi, \aleph]; \text{ such that } \Xi \subseteq \aleph \tag{3.1}$$

Note that, for $\Xi, \aleph \subseteq \wp$, expression (3.1) may be rewritten in more concise form as:

$$\mathfrak{g} = \{Y \subseteq \wp : \Xi \subseteq Y \subseteq \aleph\}$$

Thus, $\mathfrak{g}$ is a subset of $P(\wp)$. In particular, if $\Xi = \aleph$ the set valued interval $\mathfrak{g}$ reduces to the classical set.

$$\Xi = [\Xi, \Xi]$$

which is called a set interval or a set singleton. For example $\phi = [\phi, \phi]$, $\wp = [\wp, \wp]$

Form now on, $\wp$ denotes the family of all set valued intervals.

Example 3.2. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}\}$ be a non-empty set, $\mathfrak{g} = [\{\ell_2, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4, \ell_7, \ell_8\}]$, since $\{\ell_2, \ell_3, \ell_4\} \subseteq \{\ell_2, \ell_3, \ell_4, \ell_7, \ell_8\} \subseteq \wp$ then, $\mathfrak{g}$ is a set valued interval over $\wp$.

Definition 3.3. An SV-interval $\mathfrak{g} = [\Xi_L, \Xi_U]$ over $\wp$ is said to be null-SV-interval denoted by $[\Phi]$, if $\Xi_L = \phi$ and $\Xi_U = \phi$.

Definition 3.4. An SVI $\mathfrak{g} = [\Xi_L, \Xi_U]$ over $\wp$ is said to be absolute-SVI denoted by $[\wp]$, if $\Xi_L = \wp$ and $\Xi_U = \wp$.

Definition 3.5. (Yao 1993) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ are two SVIs over $\wp$. Then, $\mathfrak{g} = [\Xi_L, \Xi_U]$ is an SV-subinterval of $\aleph = [\aleph_L, \aleph_U]$, denoted by $\mathfrak{g} \sqsubseteq_i \aleph$ if

$\Xi_L \subseteq \aleph_L$ *and* $\Xi_U \subseteq \aleph_U$.

Example 3.6. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ and $\mathfrak{g} = [\{\ell_2, \ell_3\}, \{\ell_2, \ell_3, \ell_4, \ell_7\}]$ and $\aleph = [\{\ell_2, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}]$ be two SV-intervals. Since $\{\ell_2, \ell_3\} \subseteq \{\ell_2, \ell_3, \ell_4\}$ and $\{\ell_2, v_3, \ell_4, \ell_7\} \subseteq \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}$, then

$$\mathfrak{g} \sqsubseteq_i \aleph$$

Definition 3.7. (Yao 1993) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ are two SVIs over $\wp$. if $\mathfrak{g} \sqsubseteq_i \aleph$ and $\aleph \sqsubseteq_i \mathfrak{g}$ then, $\mathfrak{g} = \aleph$.

Proposition 3.8. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ and $\mathfrak{Y} = [Y_L, Y_U]$ are three SVIs over $\wp$. Then,

1. $[\Phi] \sqsubseteq_i \mathfrak{g}$
2. $\mathfrak{g} \sqsubseteq_i [\wp]$
3. $\mathfrak{g} \sqsubseteq_i \mathfrak{g}$
4. $\mathfrak{g} \sqsubseteq_i \aleph$ and $\aleph \sqsubseteq_i \mathfrak{Y} \Rightarrow \mathfrak{g} \sqsubseteq_i \mathfrak{Y}$

Definition 3.9. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ be an SVI. Then, width of SVI $\mathfrak{g}$, indicated by $\omega(\mathfrak{g})$ is specified as:

$$\omega(\mathfrak{g}) = |\Xi_U \backslash \Xi_L|$$

Example 3.10. Let us consider SVI $\mathfrak{g}$ in Example 3.2. Then,

$$\omega(\mathfrak{g}) = |\{\ell_7, \ell_8\}| = 2$$

Definition 3.11. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ be an SV-interval. Then, image of SVI $\mathfrak{g}$ denoted by $\mathfrak{g}^-$ is defined as:

$$\mathfrak{g}^- = [\Xi_L, \Xi_U]^- = [\wp \backslash \Xi_U, \wp \backslash \Xi_L]$$

Example 3.12. Let us consider SVI $\mathfrak{g}$ in Example 3.2. Then,

$$\mathfrak{g}^- = [\{\ell_1, \ell_5, \ell_6, \ell_9, \ell_{10}\}, \{\ell_1, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}\}]$$

Proposition 3.13. Let $\mathfrak{g}$ and $\aleph$ be two SV-intervals over $\wp$. Then,

1. $(\mathfrak{g}^-)^- = \mathfrak{g}$
2. $[\Phi]^- = [\wp]$
3. $[\wp]^- = [\Phi]$
4. $\mathfrak{g} \sqsubseteq_i \aleph \Rightarrow \aleph^- \sqsubseteq_i \mathfrak{g}^-$

Proof. The proof is obvious from the definition of image.

□

Definition 3.14. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ be an SVI. Then, magnitude of an SVI $\mathfrak{g}$ denoted by $|\mathfrak{g}|$ is defined as:

$$|\mathfrak{g}| = 2^{|\Xi_U \setminus \Xi_L|}$$

Example 3.15. Let us consider SVI $\mathfrak{g}$ in Example 3.2. Then,

$$|\mathfrak{g}| = 2^{|\{\ell_7,\ell_8\}|} = 2^2 = 4$$

Definition 3.16. (Yao 1993) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals. over $\wp$. Then, $\mathfrak{g} \Cup \aleph$ is the union of $\mathfrak{g}$ and $\aleph$ and defined as:

$$\mathfrak{g} \Cup \aleph = [Y \subseteq \wp : \Xi_L \cup \aleph_L \subseteq Y \subseteq D_U \cup \aleph_U]$$

Definition 3.17. (Yao 1993) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals. over $\wp$. Then, $\mathfrak{g} \Cap \aleph$ is the intersection of $\mathfrak{g}$ and $\aleph$ and defined as:

$$\mathfrak{g} \Cap \aleph = [Y \subseteq \wp : \Xi_L \cap \aleph_L \subseteq Y \subseteq \Xi_U \cap \aleph_U]$$

Example 3.18. Given two SVIs $\mathfrak{g} = [\{\ell_1,\ell_2\},\{\ell_1,\ell_2,\ell_3,\ell_5,\ell_6\}]$, and $\aleph = [\{\ell_2,\ell_3,\ell_4\},\{\ell_2,\ell_3,\ell_4,\ell_5,\ell_7\}]$ over $\wp = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7\}$. Then,

$$\begin{aligned}
\mathfrak{g} \Cup \aleph &= [\{\ell_1,\ell_2,\ell_3,\ell_4\},\wp] \\
&= \{\{\ell_1,\ell_2,\ell_3,\ell_4\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_7\}, \\
&\quad \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_7\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_7\},\wp\}
\end{aligned}$$

and

$\mathfrak{g} \Cap \aleph = [\{\ell_2\}, \{\ell_2, \ell_3, \ell_5\}] = \{\{\ell_2\}, \{\ell_2, \ell_3\}, \{\ell_2, \ell_5\}, \{\ell_2, \ell_3, \ell_5\}\}$

The following proposition gives some properties of union and intersection operation of SVIs.

Proposition 3.19. (Zhang 2014) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ and $\mathfrak{Y} = [Y_L, Y_U]$ be three SV-intervals over $\wp$. Then,

1. $\mathfrak{g} \Cap \mathfrak{g} = \mathfrak{g}$ and $\mathfrak{g} \Cup \mathfrak{g} = \mathfrak{g}$
2. $\mathfrak{g} \Cap \aleph = \aleph \Cap \mathfrak{g}$ and $\mathfrak{g} \Cup \aleph = \aleph \Cup \mathfrak{g}$
3. $\mathfrak{g} \Cap \Phi = \Phi$ and $\mathfrak{g} \Cap \wp = \mathfrak{g}$
4. $\mathfrak{g} \Cup \Phi = \mathfrak{g}$ and $\mathfrak{g} \Cup \wp = \wp$
5. $\mathfrak{g} \Cap (\aleph \Cap \mathfrak{Y}) = (\mathfrak{g} \Cap \aleph) \Cap \mathfrak{Y})$ and $\mathfrak{g} \Cup (\aleph \Cup \mathfrak{Y}) = (\mathfrak{g} \Cup \aleph) \Cup \mathfrak{Y})$
6. $\mathfrak{g} \Cap (\aleph \Cup \mathfrak{Y}) = (\mathfrak{g} \Cap \aleph) \Cup (\mathfrak{g} \Cap \mathfrak{Y})$ and $\mathfrak{g} \Cup (\aleph \Cap \mathfrak{Y}) = (\mathfrak{g} \Cup \aleph) \Cap (\mathfrak{g} \Cup \mathfrak{Y})$

Proof. 1. Since $\Xi_L \cap \Xi_L = \Xi_L$ and $\Xi_U \cap \Xi_U = \Xi_U$ then, $\mathfrak{g} \Cap \mathfrak{g} = \mathfrak{g}$and since $\Xi_L \cup \Xi_L = \Xi_L$ and $\Xi_U \cup \Xi_U = \Xi_U$ then, $\mathfrak{g} \Cup \mathfrak{g} = \mathfrak{g}$.
2. Since $\Xi_L \cap \Xi_L = \aleph\_L \cap \Xi_L$ and $\Xi_U \cap B_U = B_U \cap A_U$ and $\Xi_L \cup \aleph_L = \aleph_L \cup \Xi_L$ and $\Xi_U \cup \aleph_U = B_U \cup A_U$ then, $\mathfrak{g} \Cap \aleph = \aleph \Cap \mathfrak{g}$ and $\mathfrak{g} \Cup \aleph = \aleph \Cup \mathfrak{g}$, respectively.
3. Since $\Xi_L \cap \phi = \phi$ and $\Xi_U \cap \phi = \phi$ then, $\mathfrak{g} \Cap \Phi = \Phi$. Since $\Xi_L \cap \wp = \Xi_L$ and $\Xi_U \cap \wp = \Xi_U$ then, $\mathfrak{g} \Cap \wp = \mathfrak{g}$.
4. Since $\Xi_L \cup \phi = \Xi_L$ and $\Xi_U \cup \phi = \Xi_U$ then, $\mathfrak{g} \Cup \Phi = \mathfrak{g}$. Since $\Xi_L \cup \wp = \wp$ and $\Xi_U \cup \wp = \wp$ then, $\mathfrak{g} \Cup \wp = \wp$.
5.

$$\begin{aligned}
\mathfrak{g} \Cap (\aleph \Cap \aleph) &= [\Xi_L \cap (\aleph_L \cap Y_L), \Xi_U \cap (\aleph_U \cap Y_U)] \\
&= [(\Xi_L \cap \aleph_L) \cap Y_L, (D_U \cap \aleph_U) \cap Y_U] \\
&= [\Xi_L \cap \aleph_L, (\Xi_U \cap \aleph_U)] \Cap [Y_L, Y_U] \\
&= (\mathfrak{g} \Cap \aleph) \Cap \mathfrak{Y}
\end{aligned}$$

and

$$
\begin{aligned}
\mathfrak{g} \Cup (\aleph \Cup \mathfrak{Y}) &= [\Xi_L \cup (\aleph_L \cup Y_L), \Xi_U \cup (\aleph_U \cup Y_U)] \\
&= [(\Xi_L \cup \aleph_L) \cup Y_L, (\Xi_U \cup \aleph_U) \cup Y_U] \\
&= [\Xi_L \cup \aleph_L, (\Xi_U \cup \aleph_U)] \Cup [Y_L, Y_U] \\
&= (\mathfrak{g} \Cup \aleph) \Cup \mathfrak{Y}
\end{aligned}
$$

6.

$$
\begin{aligned}
\mathfrak{g} \Cap (\aleph \Cup \mathfrak{Y}) &= [\Xi_L \cap (\aleph_L \cup Y_L), \Xi_U \cap (\aleph_U \cup Y_U)] \\
&= [(\Xi_L \cap \aleph_L) \cup (\Xi_L \cap Y_L), (\Xi_U \cap \aleph_U) \cup (\Xi_U \cap Y_U)] \\
&= [\Xi_L \cap \aleph_L, \Xi_U \cap \aleph_U] \Cup [\Xi_L \cap Y_L, \Xi_U \cap Y_U] \\
&= (\mathfrak{g} \Cap \aleph) \Cup (\mathfrak{g} \Cap \mathfrak{Y})
\end{aligned}
$$

The proof of $\mathfrak{g} \Cup (\aleph \Cap \mathfrak{Y}) = (\mathfrak{g} \Cup \aleph) \Cap (\mathfrak{g} \Cup \mathfrak{Y})$ can be easily obtained as similar in above. $\square$

Proposition 3.20. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$. Then,

1. $(\mathfrak{g} \Cup \aleph)^- = \mathfrak{g}^- \Cap \aleph^-$
2. $(\mathfrak{g} \Cap \aleph)^- = \mathfrak{g}^- \Cup \aleph^-$

Proof. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$.

1.

$$
\begin{aligned}
(\mathfrak{g} \Cup \aleph)^- &= ([\Xi_L \cup \aleph_L, \Xi_U \cup \aleph_U])^- \\
&= [\wp \setminus (\Xi_U \cup \aleph_U), \wp \setminus (\Xi_L \cup \aleph_L)] \\
&= [(\Xi_U \cup \aleph_U)^c, (\Xi_L \cup \aleph_L)^c] \\
&= [\Xi_U^c \cap \aleph_U^c, \Xi_L^c \cap \aleph_L^c] \\
&= [\Xi_U^c, \Xi_L^c] \Cap [\aleph_U^c, \aleph_L^c] \\
&= [\wp \setminus \Xi_U, U \setminus \Xi_L] \Cap [\wp \setminus \aleph_U, \wp \setminus \aleph_L] \\
&= [\Xi_L, \Xi_U]^- \Cap [\aleph_L, \aleph_U]^- \\
&= \mathfrak{g}^- \Cap \aleph^-
\end{aligned}
$$

2.

$$\begin{aligned}
(\mathfrak{g} \Cap \aleph)^- &= ([\Xi_L \cap \aleph_L, \Xi_U \cap \aleph_U])^- \\
&= [\wp \setminus (\Xi_U \cap \aleph_U), \wp \setminus (\Xi_L \cap \aleph_L)] \\
&= [(\Xi_U \cap \aleph_U)^c, (\Xi_L \cap \aleph_L)^c] \\
&= [\Xi_U^c \cup \aleph_U^c, \Xi_L^c \cup \aleph_L^c] \\
&= [\Xi_U^c, \Xi_L^c] \Cup [\aleph_U^c, \aleph_L^c] \\
&= [\wp \setminus \Xi_U, U \setminus \Xi_L] \Cup [\wp \setminus \aleph_U, \wp \setminus \aleph_L] \\
&= [\Xi_L, \Xi_U]^- \Cup [\aleph_L, \aleph_U]^- \\
&= \mathfrak{g}^- \Cup \aleph^-
\end{aligned}$$

□

Definition 3.21. (Yao 1993) Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$. Then, $\mathfrak{g} \wr \aleph$ denotes the difference of $\mathfrak{g}$ and $\aleph$ and defined as:

$$\mathfrak{g} \wr \aleph = [(\Xi_L \setminus \aleph_U), (\Xi_U \setminus \aleph_L)]$$

Example 3.22. Given two SVIs $\mathfrak{g} = [\{\ell_1, \ell_2\}, \{\ell_1, \ell_2, \ell_3, \ell_5, \ell_6\}]$ and $\aleph = [\{\ell_2, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}]$ over $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}$. Then,

$$\begin{aligned}
\mathfrak{g} \wr \aleph &= [(\Xi_L \setminus \aleph_U), (\Xi_U \setminus \aleph_L)] \\
&= [(\{\ell_1, \ell_2\} \setminus \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}), (\{\ell_1, \ell_2, \ell_3, \ell_5, \ell_6\} \setminus \{\ell_2, \ell_3, \ell_4\})] \\
&= [\{\ell_1\}, \{\ell_1, \ell_5, \ell_6\}]
\end{aligned}$$

Proposition 3.23. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$. Then,

$$\mathfrak{g} \wr \aleph = \mathfrak{g} \Cap \aleph^-$$

Proof. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$

$$
\begin{aligned}
(\mathfrak{g} \wr \aleph) &= ([\Xi_L \setminus \aleph_U, \Xi_U \setminus \aleph_L]) \\
&= [\wp \cap (\Xi_L \setminus \aleph_U), \wp \cap (\Xi_U \setminus \aleph_L)] \\
&= [\wp \cap (\Xi_L \cap \aleph_U^c), \wp \cap (\Xi_U \cap \aleph_L^c)] \\
&= [\Xi_L \cap (\wp \cap \aleph_U^c), \Xi_U \cap (\wp \cap \aleph_L^c)] \\
&= [\Xi_L \cap (\wp \setminus \aleph_U), \Xi_U \cap (\wp \setminus \aleph_L)] \\
&= [\Xi_L, \Xi_U] \Cap [(\wp \setminus \aleph_U), (\wp \setminus \aleph_L)] \\
&= \mathfrak{g} \Cap \aleph^-
\end{aligned}
$$

□

Definition 3.24. Let $\mathfrak{g} = [\Xi_L, \Xi_U]$ and $\aleph = [\aleph_L, \aleph_U]$ be two SV-intervals over $\wp$. Then, symmetric difference of $\mathfrak{g}$ and $\aleph$, denoted by $\mathfrak{g} \triangle \aleph$ is defined as:

$$\mathfrak{g} \triangle \aleph = [(\Xi_L \setminus \aleph_U) \cup (\aleph_L \setminus \Xi_U), (\Xi_U \setminus \aleph_L) \cup (\aleph_U \setminus \Xi_L)].$$

Example 3.25. Given two SV-intervals over $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}$,
$\mathfrak{g} = [\{\ell_1, \ell_2\}, \{\ell_1, \ell_2, \ell_3, \ell_5, \ell_6\}]$, and $\aleph = [\{\ell_2, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}]$. Then,

$$
\begin{aligned}
\mathfrak{g} \triangle \aleph =& [(\Xi_L \setminus \aleph_U) \cup (\aleph_L \setminus \Xi_U), (\Xi_U \setminus \aleph_L) \cup (\aleph_U \setminus \Xi_L)] \\
=& [(\{\ell_1, \ell_2\} \setminus \{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\}) \cup (\{\ell_2, \ell_3, \ell_4\} \setminus \{\ell_1, \ell_2, \ell_3, \ell_5, \ell_6\}), \\
& (\{\ell_1, \ell_2, \ell_3, \ell_5, \ell_6\} \setminus \{\ell_2, \ell_3, \ell_4\}) \cup (\{\ell_2, \ell_3, \ell_4, \ell_5, \ell_7\} \setminus \{\ell_1, \ell_2\})] \\
=& [(\{\ell_1\} \cup \{\ell_4\}), (\{\ell_1, \ell_5, \ell_6\} \cup \{\ell_1, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\})] \\
=& [\{\ell_1, \ell_4\}, \{\ell_1, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}]
\end{aligned}
$$

4. SET VALUED INTERVAL SOFT SETS

We define the idea of set valued interval soft set (SVISS) in this section. After that, we discuss set valued interval soft set operations and look into how they relate to one another.

Definition 4.1. (Zhang 2014) Let E be a parameter set, $\wp$ be an universal set, and $\Xi \subseteq E$, $\wp$ be set of SVIs over $\wp$. Then, set valued interval soft set (SVIS-set) denoted by $(\mathbf{L}, \Xi)$, is defined by:

$$(\mathbf{L}, \Xi) = \{(e, l(e)) : l(e) \in \wp\}$$

here $l : \Xi \to \wp$ is the e-approximation of SVIS-set $(\mathbf{L}, \Xi)$ and $l(e) = [l_1(e), l_2(e)]$, such that $l_1(e), l_2(e) \in P(\wp)$ and $l_1(e) \subseteq l_2(e)$. From now on, we will use $S[\wp]$ instead of the set of all SVIS-sets and approximation function of an SVIS-set will be denoted by lowercase related to SVISS over $\wp$.

Example 4.2. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ be an universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If,

$$\begin{aligned}
l(e_1) &= [\{\ell_1, \ell_3\}, \{\ell_1, \ell_2, \ell_3, \ell_5\}] \\
l(e_2) &= [\{\ell_2, \ell_4\}, \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_6, \ell_8\}] \\
l(e_3) &= [\{\ell_5, \ell_6\}, \{\ell_1, \ell_2, \ell_5, \ell_6, \ell_7\}] \\
l(e_4) &= [\{\ell_1, \ell_2, \ell_7, \ell_8\}, \{\ell_1, \ell_2, \ell_3, \ell_5, \ell_7, \ell_8\}] \\
l(e_5) &= [\phi, \{\ell_1, \ell_4, \ell_5, \ell_6\}]
\end{aligned}$$

then the SVISS $\mathbf{L}$ is written by

$$\begin{aligned}
\mathbf{L} =& \{(e_1, [\{\ell_1, \ell_3\}, \{\ell_1, \ell_2, \ell_3, \ell_5\}]), (\ell_2, [\{\ell_2, \ell_4\}, \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_6, \ell_8\}]), \\
& (e_3, [\{\ell_5, \ell_6\}, \{\ell_1, \ell_2, \ell_5, \ell_6, \ell_7\}]), (e_4, [\{\ell_1, \ell_2, \ell_7, \ell_8\}, \{\ell_1, \ell_2, \ell_3, \ell_5, \ell_7, \ell_8\}]) \\
& (e_5, [\phi, \{\ell_1, \ell_4, \ell_5, \ell_6\}])\}
\end{aligned}$$

The tabular representation of the SVISS $\mathbf{L}$ is as follow:

	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
e_1	[1,1]	[0,1]	[1,1]	[0,0]	[0,1]	[0,0]	[0,0]	[0,0]
e_2	[0,1]	[1,1]	[0,1]	[1,1]	[0,0]	[0,1]	[0,0]	[0,1]
e_3	[0,1]	[0,1]	[0,0]	[0,0]	[1,1]	[1,1]	[0,1]	[0,0]
e_4	[1,1]	[1,1]	[0,1]	[0,0]	[0,1]	[0,0]	[1,1]	[1,1]
e_5	[0,1]	[0,0]	[0,0]	[0,1]	[0,1]	[0,1]	[0,0]	[0,0]

Here, uncertainty elements related to parameter e_1 are ℓ_2 and ℓ_5.

Definition 4.3. Let $\mathbf{L} \in S[\wp]$. Then, element(s) of $l_2(e) \backslash l_1(e)$ is(are) called uncertainty element(s) related to the parameter $e \in E$.

Example 4.4. Let us consider Example 4.2. Then, uncertainty elements related to parameter e_1 are ℓ_2 and ℓ_5.

Definition 4.5. Let $\mathbf{L} \in S[\wp]$. If $l(e) = [\Phi]$ for all $e \in E$. Then, $\mathbf{L}$ is called an empty SVISS denoted by $[\bar{\Phi}]$:

Definition 4.6. Let $\mathbf{L} \in S[\wp]$. If $l(e) = [\wp]$ for all $e \in E$. Then, $\mathbf{L}$ is called a universal SVISS denoted by $[\bar{\wp}]$:

Definition 4.7. (Zhang and Fu 2012) The extended intersection of two SVISS $(\mathbf{L}, \Xi)$ and $(\mathbf{M}, \aleph)$ over a universe $\wp$ is the SVISS $(\mathbf{N}, Y) = (\mathbf{L}, \Xi) \sqcap_e (\mathbf{M}, \aleph)$, where $Y = \Xi \cup \aleph$, and for all $e \in Y$,

$$n(e) = \begin{cases} l(e) & if\ e \in \Xi - \aleph, \\ m(e) & if\ e \in \aleph - \Xi, \\ l(e) \Cap m(e) & if\ e \in \Xi \cap \aleph \end{cases}$$

Example 4.8. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ be an initial universe and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. $\Xi, \aleph \subseteq E$ and $\Xi = \{e_1, e_2, e_3\}$ and $\aleph = \{e_3, e_4, e_5\}$. Now we can consider the following SVISS

$$
\begin{aligned}
(\mathbf{L},\Xi) &= \{(e_1,[\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3,\ell_5\}]),(e_2,[\{\ell_2,\ell_4\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_3,[\{\ell_5,\ell_6\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}
\end{aligned}
$$

and

$$
\begin{aligned}
(\mathbf{M},\aleph) &= \{(e_3,[\{\ell_1,\ell_5\},\{\ell_1,\ell_2,\ell_4,\ell_5\}]),(e_4,[\{\ell_2,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_5,[\{\ell_2,\ell_7\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
(\mathbf{N},Y) &= (\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph)\\
&= \{(e_1,[\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3,\ell_5\}]),(e_2,[\{\ell_2,\ell_4\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_3,[\{\ell_5\},\{\ell_1,\ell_2,\ell_5\}]),(e_4,[\{\ell_2,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_5,[\{\ell_2,\ell_7\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}.
\end{aligned}
$$

Definition 4.9. (Zhang and Fu 2012) The extended union of two SVISSs $(\mathbf{L},\Xi)$ and $(\mathbf{M},\aleph)$ over a universe $\wp$ is the SVISS $(\mathbf{N},Y=\Xi\cup\aleph)=(\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph)$, for all $e\in Y$,

$$
n(e)=\begin{cases} l(e) & if\ e\in\Xi-\aleph,\\ m(e) & if\ e\in\aleph-\Xi,\\ l(e)\Cup m(e) & if\ e\in\Xi\cap\aleph \end{cases}
$$

Example 4.10. Let $\wp=\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7,\ell_8\}$ be an initial universe, $E=\{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters, $D,\aleph\subseteq E$ and $\Xi=\{e_1,e_2,e_3\}$ and $\aleph=\{e_3,e_4,e_5\}$. Let us consider the following SVISSs

$$
\begin{aligned}
(\mathbf{L},\Xi) &= \{(e_1,[\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3,\ell_5\}]),(e_2,[\{\ell_2,\ell_4\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_3,[\{\ell_5,\ell_6\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}
\end{aligned}
$$

and

$$
\begin{aligned}
(\mathbf{M},\aleph) &= \{(e_3,[\{\ell_1,\ell_5\},\{\ell_1,\ell_2,\ell_4,\ell_5\}]),(e_4,[\{\ell_2,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),\\
&\quad (e_5,[\{\ell_2,\ell_7\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}
\end{aligned}
$$

Then,

$$
\begin{aligned}
(\mathbf{N},Y) &= (\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph) = \{(e_1,[\{\ell_1,\ell_3\},\{\ell_1,\ell_2,\ell_3,\ell_5\}]),(e_2,[\{\ell_2,\ell_4\},\\
&\quad \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),(e_3,[\{\ell_1,\ell_5,\ell_6\},\{\ell_1,\ell_2,\ell_4,\ell_5,\ell_6,\ell_7\}]),\\
&\quad (e_4,[\{\ell_2,\ell_6\},\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_6,\ell_8\}]),(e_5,[\{\ell_2,\ell_7\},\{\ell_1,\ell_2,\ell_5,\ell_6,\ell_7\}])\}
\end{aligned}
$$

Theorem 4.11. (Zhang and Fu 2012) Let $(\mathbf{L},\Xi)$, $(\mathbf{M},\aleph)$ and $(\mathbf{N},Y)$ be a SVISSs over the same universe $\wp$. Then,

1. $(\mathbf{L},\Xi)\sqcap_e(\mathbf{L},\Xi) = (\mathbf{L},\Xi)$;
2. $(\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph) = (\mathbf{M},\aleph)\sqcap_e(\mathbf{L},\Xi)$;
3. $((\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph)\sqcap_e(\mathbf{N},Y) = (\mathbf{L},\Xi)\sqcap_e((\mathbf{M},\aleph)\sqcap_e(\mathbf{N},Y))$;
4. $(\mathbf{L},\Xi)\sqcup_e(\mathbf{L},\Xi) = (\mathbf{L},\Xi)$;
5. $(\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph) = (\mathbf{M},\aleph)\sqcup_e(\mathbf{L},\Xi)$;
6. $((\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph)\sqcup_e(\mathbf{N},Y) = (\mathbf{L},\Xi)\sqcup_e((\mathbf{M},\aleph)\sqcup_e(\mathbf{N},Y))$.

Definition 4.12. (Zhang and Fu 2012) The "restricted intersection" of two SVISSs $(\mathbf{L},\Xi)$ and $(\mathbf{M},\aleph)$ over a universe $\wp$ is the SVISS $(\mathbf{N},Y) = (\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph)$, where $Y = \Xi\cap\aleph$ and for all $e\in Y$, $n(e) = l(e)\Cap m(e)$.

Example 4.13. For the SVISSs over $\wp$ given in Example 4.8, we get:

$$(\mathbf{N},Y) = (\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph) = \{(e_3,[\{\ell_5\},\{\ell_1,\ell_2,\ell_5\}])\}.$$

Definition 4.14. (Zhang and Fu 2012) The restricted union of two SVISSs $(\mathbf{L},\Xi)$ and $(\mathbf{M},\aleph)$ over the universe $\wp$ is the SVISS $(\mathbf{N},Y) = (\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph)$, where $Y = \Xi\cap\aleph\neq\phi$ and for all $e\in Y$, $n(e) = l(e)\Cup m(e)$.

Example 4.15. Let us consider the SVISSs given in Example 4.8. We get:

$$(\mathbf{N},Y) = \mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph) = \{(e_3,[\{\ell_1,\ell_5,\ell_6\},\{\ell_1,\ell_2,\ell_4,\ell_5,\ell_6,\ell_7\}])\}.$$

Theorem 4.16. (Zhang and Fu 2012) Let $(\mathbf{L},\Xi)$, $(\mathbf{M},\aleph)$ and $(\mathbf{N},Y)$ be a SVISSs over the same universe $\wp$. Then,

1. $(\mathbf{L},\Xi)\sqcap_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$;
2. $(\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph)=(\mathbf{M},\aleph)\sqcap_r(\mathbf{L},\Xi)$;
3. $((\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph)\sqcap_r(\mathbf{N},Y)=(\mathbf{L},\Xi)\sqcap_r((\mathbf{M},\aleph)\sqcap_r(\mathbf{N},Y))$;
4. $(\mathbf{L},\Xi)\sqcup_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$;
5. $(\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph)=(\mathbf{M},\aleph)\sqcup_r(\mathbf{L},\Xi)$;
6. $((\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph)\sqcup_r(\mathbf{N},Y)=(\mathbf{L},\Xi)\sqcup_r((\mathbf{M},\aleph)\sqcup_r(\mathbf{N},Y))$.

For SVISSs, operations $\sqcap_r$ and $\sqcup_r$ satisfy commutativity, associativity, idempotent and absorption rules. Using these properties, the following results can be get

Theorem 4.17. (Zhang 2014) Let $(\mathbf{L},\Xi)$, $(\mathbf{M},\aleph)$ and $(\mathbf{N},Y)$ be three SVISSs over the same universe $\wp$. Then,

1. $((\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph))\sqcap_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$;
2. $((\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph))\sqcup_e(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

Proof. 1. Suppose that $(\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph)=(\mathbf{N},\Xi\cup\aleph)$ and $((\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph))\sqcap_r(\mathbf{L},\Xi)=(\mathbf{O},(\Xi\cup\aleph)\cap\Xi)=(\mathbf{O},\Xi)$.
For all $e\in(\Xi\cup\aleph)\cap\Xi$, $e\in\Xi$, $\mathbf{O}(e)=\mathbf{N}(e)\sqcap\mathbf{L}(e)=(\mathbf{L}(e)\sqcup\mathbf{M}(e))\sqcap\mathbf{L}(e)=\mathbf{L}(e)$;
Thus, $((\mathbf{L},\Xi)\sqcup_e(\mathbf{M},\aleph))\sqcap_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

2. Suppose that $(\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph)=(\mathbf{N},\Xi\cap\aleph)$ and
$((\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph))\sqcup_e(\mathbf{L},\Xi)=(\mathbf{O},(\Xi\cap\aleph)\cup\Xi)=(\mathbf{O},\Xi)$.
For all $e\in(\Xi\cap\aleph)\cup\Xi$, $e\in\Xi$, $\mathbf{O}(e)=\mathbf{N}(e)\sqcup\mathbf{L}(e)=(\mathbf{L}(e)\sqcap\mathbf{M}(e))\sqcup\mathbf{L}(e)=\mathbf{L}(e)$;
Therefore, $((\mathbf{L},\Xi)\sqcap_r(\mathbf{M},\aleph))\sqcup_e(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

$\square$

Theorem 4.18. (Zhang 2014) Let $(\mathbf{L},\Xi)$, $(\mathbf{M},\aleph)$ and $(\mathbf{N},Y)$ be three SVISSs over the same universe $\wp$. Then,

1. $((\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph))\sqcap_e(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$;
2. $((\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph))\sqcup_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

Proof. 1. Let $(\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph)=(\mathbf{N},\Xi\cap\aleph)$ and $((\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph))\sqcap_e(\mathbf{L},\Xi)=(\mathbf{O},(\Xi\cap\aleph)\cup\Xi)=(\mathbf{O},\Xi)$. For all $e\in\Xi\cap\aleph$, $\mathbf{O}(e)=\mathbf{N}(e)\sqcap\mathbf{L}(e)=(\mathbf{L}(e)\sqcup\mathbf{M}(e))\sqcap\mathbf{L}(e)=\mathbf{L}(e)$; for all $e\in\Xi\setminus(\Xi\cap\aleph),\mathbf{O}(e)=\mathbf{N}(e)\sqcap\mathbf{L}(e)=(\mathbf{L}(e)\sqcup\mathbf{M}(e))\sqcap\mathbf{L}(e)=\mathbf{L}(e)$.
Therefore, $((\mathbf{L},\Xi)\sqcup_r(\mathbf{M},\aleph))\sqcap_e(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

2. Suppose that $(\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph)=(\mathbf{N},\Xi\cup\aleph)$ and $((\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph))\sqcup_r(\mathbf{L},\Xi)=(\mathbf{O},(\Xi\cup\aleph)\cap\Xi)=(\mathbf{O},\Xi)$. For all $e\in(\Xi\cup\aleph)\cap\Xi$, $e\in\Xi$, $\mathbf{O}(e)=\mathbf{N}(e)\sqcup\mathbf{L}(e)=(\mathbf{L}(e)\sqcap\mathbf{M}(e))\sqcup\mathbf{L}(e)=\mathbf{L}(e)$.
Therefore, $((\mathbf{L},\Xi)\sqcap_e(\mathbf{M},\aleph))\sqcup_r(\mathbf{L},\Xi)=(\mathbf{L},\Xi)$.

$\square$

5. FUZZY SET VALUED INTERVAL SETS

In this section, we define concept of fuzzy set valued interval (FSVI) and their operations.

Definition 5.1. Let $\wp$ be a crisp set, $\mathbb{L}(\wp)$ be the collection of fuzzy set over $\wp$ and $\tilde{\Xi}_L, \tilde{\Xi}_U \in \mathbb{L}(\wp)$. Then, fuzzy set valued interval (FSVI) Ξ over $\wp$ is defined by:

$$\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]; \quad \tilde{\Xi}_L \subseteq \tilde{\Xi}_U$$

and $\mathbb{FSVI}_U$ the family of all fuzzy set valued intervals over $\wp$.
Thus, Ξ is a fuzzy subset of $\wp$. In particular, if $\tilde{\Xi_L} = \tilde{\Xi_U}$ the SVI Ξ reduces to the classical FS

$$\Xi = [\tilde{\Xi}, \tilde{\Xi}]$$

which is called a FS-interval or a FS-singleton. For example $\phi = [\phi, \phi]$, $\wp = [\tilde{\wp}, \tilde{\wp}]$, here $\tilde{\wp}$ and $\tilde{\phi}$ are universal fuzzy set and empty fuzzy set over $\wp$.

Example 5.2. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ be a non-empty set, $\Xi = [\{\ell_2/.4, \ell_3/.5\}, \{\ell_2/.5, \ell_3/.6, \ell_4/.7, \ell_7/.8\}]$, since $\{\ell_2/.4, \ell_3/.5\} \subseteq \{\ell_2/.5, \ell_3/.6, \ell_4/.7, \ell_7/.8\} \subseteq \tilde{\wp}$ then, Ξ is a FSVI over $\wp$.

Definition 5.3. An FSV-interval $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ over $\wp$ is said to be null- FSVI denoted by $[\tilde{\Phi}]$, if $\tilde{\Xi}_L = \tilde{\phi}$ and $\tilde{\Xi}_U = \tilde{\phi}$.

Definition 5.4. An FSVI $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ over $\wp$ is said to be absolute-FSVI denoted by $[\tilde{\wp}]$, if $\tilde{\Xi}_L = \wp$ and $\tilde{\Xi}_U = \wp$.

Definition 5.5. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ are two FSVIs over $\wp$. If $\tilde{\Xi}_L \subseteq \tilde{\aleph}_L$ and $\tilde{\Xi}_U \subseteq \tilde{\aleph}_U$, it is said that $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ is an FSVSI of $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ and it is denoted by $\Xi \sqsubseteq_i \aleph$.

Example 5.6. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ and $\mathbb{D} = [\{\ell_2/.4, \ell_3/.4\}, \{\ell_2/.5, \ell_3/.6, \ell_4/.7, \ell_7/.8\}]$ and $\aleph = [\{\ell_3/.5, \ell_4/.7, \ell_5/.7\}, \{\ell_2/.5, \ell_3/.6, \ell_4/.7, \ell_5/.9,$

$\ell_7/.8\}]$ are two FSVIs. Then, since $\{\ell_2/.4,\ell_3/.4\} \subseteq \{\ell_3/.5,\ell_4/.7,\ell_5/.7\}$ and $\{\ell_2/.5,\ell_3/.6,\ell_4/.7,\ell_7/.8\} \subseteq \{\ell_2/.5,\ell_3/.6,\ell_4/.7,\ell_5/.9,\ell_7/.8\}$ then, $\mathbb{D} \sqsubseteq_i \aleph$.

Definition 5.7. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ are two FSVIs over $\wp$. if $\Xi \sqsubseteq_i \aleph$ and $\aleph \sqsubseteq_i \Xi$ then, $\tilde{\Xi} = \aleph$.

Proposition 5.8. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ and $\mathbb{Y} = [\tilde{Y}_L, \tilde{Y}_U]$ are three FSVIs over $\wp$. Then,

1. $[\tilde{\Phi}] \sqsubseteq_i \Xi$
2. $\Xi \sqsubseteq_i [\tilde{\wp}]$
3. $\Xi \sqsubseteq_i \Xi$
4. $\Xi \sqsubseteq_i \aleph \quad \text{and} \quad \aleph \sqsubseteq_i \mathbb{Y} \Rightarrow \Xi \sqsubseteq_i \mathbb{Y}$

Definition 5.9. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ be an FSVI. Then, width of FSV-interval ℓ, denoted by $\omega(\Xi)$ is defined as:

$$\omega(\Xi) = |\tilde{\Xi}_U \backslash \tilde{\Xi}_L| = \sum_{\ell \in \tilde{\Xi}_L} \mu(\ell)$$

Example 5.10. we will consider FSVI. Ξ in Example 5.6. Then,

$$\omega(\Xi) = |\{\ell_2/.5,\ell_3/.6,\ell_4/.7,\ell_7/.8\}| = 2.6$$

Definition 5.11. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ be FSVI. Then, image of FSVI Ξ denoted by Ξ^- is defined as:

$$\Xi^- = [\tilde{\Xi}_L, \tilde{\Xi}_U]^- = [\tilde{\wp} \backslash \tilde{\Xi}_U, \tilde{\wp} \backslash \tilde{\Xi}_L]$$

Example 5.12. Let us consider FSVI Ξ in Example 5.6. Then,
$\Xi^- = [\{\ell_1/1,\ell_2/.5,\ell_3/.4,\ell_5/1,\ell_6/.1,\ell_7/.2,\ell_8/1\},\{\ell_1/1,\ell_2/.6,\ell_3/.6,\ell_4/1,\ell_5/1, \ell_6/1,\ell_7/1,\ell_8/1\}]$.

Proposition 5.13. Let Ξ and $\aleph$ be two FSVIs over $\wp$. Then,

1. $(\Xi^-)^- = \Xi$
2. $[\tilde{\Phi}]^- = [\tilde{\wp}]$
3. $[\tilde{\wp}]^- = [\tilde{\Phi}]$

4. $\Xi \sqsubseteq_i \aleph \Rightarrow \aleph^- \sqsubseteq_i \Xi^-$

Definition 5.14. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ be an FSVI. Then, magnitude of FSVI Ξ denoted by $|\Xi|$ is defined by:

$$|\Xi| = 2^{|\tilde{\Xi}_U \setminus \tilde{\Xi}_L|}$$

Example 5.15. Let us consider SVI Ξ in Example 5.6. Then,

$$|\Xi| = 2^{2.6} = 6.06$$

Definition 5.16. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then, union of Ξ and $\aleph$, indicated by $\Xi \widetilde{\Cup} \aleph$ is specified as:

$$\Xi \widetilde{\Cup} \aleph = [\tilde{\Xi}_L \cup \tilde{\aleph}_L, \tilde{\Xi}_U \cup \tilde{\aleph}_U]$$

Definition 5.17. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\Xi = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then, intersection of Ξ and $\aleph$, indicated by $\Xi \widetilde{\Cap} \Xi$ is defined as:

$$\Xi \widetilde{\Cap} \aleph = [\tilde{\Xi}_L \cap \tilde{\aleph}_L, \tilde{\Xi}_U \cap \tilde{\aleph}_U]$$

Example 5.18. Given two FSVIs over $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}$,
$\Xi = [\{\ell_1/.2, \ell_2/.3\}, \{\ell_1/.2, \ell_2/.4, \ell_3/.5, \ell_5/.6, \ell_6/.7\}]$ and
$\aleph = [\{\ell_2/.3, \ell_3/.4, \ell_4/.5\}, \{\ell_2/.4, \ell_3/.6, \ell_4/.5, \ell_5/.4, \ell_7/.8\}]$ then,

$$\Xi \widetilde{\Cup} \aleph = [\{\ell_1/.2, \ell_2/.3, \ell_3/.4, \ell_4/.5\}, \{\ell_1/.2, \ell_2/.4, \ell_3/.6, \ell_4/.5, \ell_5/.6, \ell_6/.7, \ell_7/.8\}]$$

and

$$\Xi \widetilde{\Cap} \aleph = [\{\ell_2/.3\}, \{\ell_2/.4, \ell_3/.5, \ell_5/.4\}]$$

Proposition 5.19. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ and $\mathbb{Y} = [\tilde{Y}_L, \tilde{Y}_U]$ be three FSVIs over $\wp$. Then,

1. $\Xi \widetilde{\Cap} \Xi = \Xi$ and $\Xi \widetilde{\Cup} \Xi = \Xi$
2. $\Xi \widetilde{\Cap} \aleph = \aleph \widetilde{\Cap} \Xi$ and $\Xi \widetilde{\Cup} \aleph = \aleph \widetilde{\Cup} \Xi$
3. $\Xi \widetilde{\Cap} [\tilde{\Phi}] = [\tilde{\Phi}]$ and $\Xi \widetilde{\Cap} [\tilde{\wp}] = \Xi$
4. $\Xi \widetilde{\Cup} [\tilde{\Phi}] = \Xi$ and $\Xi \widetilde{\Cup} [\tilde{\wp}] = [\tilde{\wp}]$
5. $\Xi \widetilde{\Cap} (\aleph \widetilde{\Cap} \mathbb{Y}) = (\Xi \widetilde{\Cap} \aleph) \widetilde{\Cap} \mathbb{Y})$ and $\Xi \widetilde{\Cup} (\aleph \Cup \mathbb{Y}) = (\Xi \widetilde{\Cup} \aleph) \widetilde{\Cup} \mathbb{Y})$

6. $\Xi\widetilde{\Cap}(\aleph\widetilde{\Cup}\mathbb{Y}) = (\Xi\widetilde{\Cap}\aleph)\widetilde{\Cup}(\Xi\widetilde{\Cap}\mathbb{Y})$ and $\Xi\widetilde{\Cup}(\aleph\widetilde{\Cap}\mathbb{Y}) = (\Xi\widetilde{\Cup}\aleph)\widetilde{\Cap}(\Xi\widetilde{\Cup}\mathbb{Y})$

Proof. 1. Since $\tilde{\Xi}_L \cap \tilde{\Xi}_L = \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cap \tilde{\Xi}_U = \tilde{\Xi}_U$ then, $\Xi\widetilde{\Cap}\Xi = \Xi$ and since $\tilde{\Xi}_L \cup \tilde{\Xi}_L = \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cup \tilde{\Xi}_U = \tilde{\Xi}_U$ then, $\Xi\widetilde{\Cup}\Xi = \Xi$.

2. Since $\tilde{\Xi}_L \cap \tilde{\aleph}_L = \tilde{\aleph}_L \cap \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cap \tilde{\aleph}_U = \tilde{\aleph}_U \cap \tilde{\Xi}_U$ and $\tilde{\Xi}_L \cup \tilde{\aleph}_L = \tilde{\aleph}_L \cup \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cup \tilde{\aleph}_U = \tilde{\aleph}_U \cup \tilde{\Xi}_U$ then, $\Xi\widetilde{\Cap}\aleph = \aleph\widetilde{\Cap}\Xi$ and $\Xi\widetilde{\Cup}\aleph = \aleph\widetilde{\Cup}\Xi$, respectively.

3. Since $\tilde{\Xi}_L \cap \tilde{\phi} = \tilde{\phi}$ and $\tilde{\Xi}_U \cap \tilde{\phi} = \tilde{\phi}$ then, $\Xi\widetilde{\Cap}[\tilde{\Phi}] = [\tilde{\Phi}]$. Since $\tilde{\Xi}_L \cap [\tilde{\wp}] = \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cap [\tilde{\wp}] = \tilde{\Xi}_U$ then, $\Xi\widetilde{\Cap}[\tilde{\wp}] = \Xi$.

4. Since $\tilde{\Xi}_L \cup \tilde{\phi} = \tilde{\Xi}_L$ and $\tilde{\Xi}_U \cup [\tilde{\Phi}] = \tilde{\Xi}_U$ then, $\Xi\widetilde{\Cup}[\tilde{\Phi}] = \Xi$. Since $\tilde{\Xi}_L \cup [\tilde{\wp}] = [\tilde{\wp}]$ and $\tilde{\Xi}_U \cup [\tilde{\wp}] = [\tilde{\wp}]$ then, $\Xi \Cup [\tilde{\wp}] = [\tilde{\wp}]$.

5.

$$\begin{aligned}
\Xi\widetilde{\Cap}(\aleph\widetilde{\Cap}\mathbb{Y}) &= [\tilde{\Xi}_L \cap (\tilde{\aleph}_L \cap \tilde{Y}_L), \tilde{\Xi}_U \cap (\tilde{\aleph}_U \cap \tilde{Y}_U)] \\
&= [(\tilde{\Xi}_L \cap \tilde{\aleph}_L) \cap \tilde{Y}_L, (\tilde{\Xi}_U \cap \tilde{\aleph}_U) \cap \tilde{Y}_U] \\
&= [\tilde{\Xi}_L \cap \tilde{\aleph}_L, (\tilde{\Xi}_U \cap \tilde{\aleph}_U)]\widetilde{\Cap}[\tilde{Y}_L, \tilde{Y}_U] \\
&= (\Xi\widetilde{\Cap}\aleph)\widetilde{\Cap}\mathbb{Y}
\end{aligned}$$

and

$$\begin{aligned}
\Xi\widetilde{\Cup}(\aleph\widetilde{\Cup}\mathbb{Y}) &= [\tilde{\Xi}_L \cup (\tilde{\aleph}_L \cup \tilde{Y}_L), \tilde{\Xi}_U \cup (\tilde{\aleph}_U \cup \tilde{Y}_U)] \\
&= [(\tilde{\Xi}_L \cup \tilde{\aleph}_L) \cup \tilde{Y}_L, (\tilde{\Xi}_U \cup \tilde{\aleph}_U) \cup \tilde{Y}_U] \\
&= [\tilde{\Xi}_L \cup \tilde{\aleph}_L, \tilde{\Xi}_U \cup \tilde{\aleph}_U]\widetilde{\Cup}[\tilde{Y}_L, \tilde{Y}_U] \\
&= (\Xi\widetilde{\Cup}\aleph)\widetilde{\Cup}\mathbb{Y}
\end{aligned}$$

6.

$$\begin{aligned}\Xi\widetilde{\Cap}(\aleph\widetilde{\Cup}\mathbb{Y}) =&[\tilde{\Xi}_L\cap(\tilde{\aleph}_L\cup\tilde{Y}_L),\tilde{\Xi}_U\cap(\tilde{\aleph}_U\cup\tilde{Y}_U)]\\ =&[(\tilde{\Xi}_L\cap\tilde{\aleph}_L)\cup(\tilde{\Xi}_L\cap\tilde{Y}_L),(\tilde{\Xi}_U\cap\tilde{\aleph}_U)\cup(\tilde{\aleph}_U\cap\tilde{Y}_U)]\\ =&[\tilde{\Xi}_L\cap\tilde{\aleph}_L,\tilde{\Xi}_U\cap\tilde{\aleph}_U]\widetilde{\Cup}[\tilde{\Xi}_L\cap\tilde{Y}_L,\tilde{\Xi}_U\cap\tilde{Y}_U]\\ =&(\Xi\widetilde{\Cap}\aleph)\widetilde{\Cup}(\Xi\widetilde{\Cap}\mathbb{Y})\end{aligned}$$

The proof of $\Xi\widetilde{\Cup}(\aleph\widetilde{\Cap}\mathbb{Y})=(\Xi\widetilde{\Cup}\aleph)\widetilde{\Cap}(\Xi\widetilde{\Cup}\mathbb{Y})$ can be easily obtained as similar in above. □

Proposition 5.20. Let $\Xi=[\tilde{\Xi}_L,\tilde{\Xi}_U]$ and $\aleph=[\tilde{\aleph}_L,\tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then,

1. $(\Xi\widetilde{\Cup}\aleph)^-=\Xi^-\widetilde{\Cap}\aleph^-$
2. $(\Xi\widetilde{\Cap}\aleph)^-=\Xi^-\widetilde{\Cup}\aleph^-$

Proof. 1.

$$\begin{aligned}(\Xi\widetilde{\Cup}\aleph)^- =&([\tilde{\Xi}_L\cup\tilde{\aleph}_L,\tilde{\Xi}_U\cup\tilde{\aleph}_U])^-\\ =&[\tilde{\wp}\setminus(\tilde{\Xi}_U\cup\tilde{\aleph}_U),\tilde{\wp}\setminus(\tilde{\Xi}_L\cup\tilde{\aleph}_L)]\\ =&[(\tilde{\Xi}_U\cup\tilde{\aleph}_U)^c,(\tilde{\Xi}_L\cup\tilde{\aleph}_L)^c]\\ =&[\tilde{\Xi}_U^c\cap\tilde{\aleph}_U^c,\tilde{\Xi}_L^c\cap\tilde{\aleph}_L^c]\\ =&[\tilde{\Xi}_U^c,\tilde{\Xi}_L^c]\widetilde{\Cap}[\tilde{B}_U^c,\tilde{\aleph}_L^c]\\ =&[\tilde{\wp}\setminus\tilde{\Xi}_U,\tilde{\wp}\setminus\tilde{\Xi}_L]\widetilde{\Cap}[\tilde{\wp}\setminus\tilde{\aleph}_U,\tilde{\wp}\setminus\tilde{\aleph}_L]\\ =&[\tilde{\Xi}_L,\tilde{\Xi}_U]^-\widetilde{\Cap}[\tilde{\aleph}_L,\tilde{\aleph}_U]^-\\ =&\Xi^-\widetilde{\Cap}\aleph^-\end{aligned}$$

2.

$$\begin{aligned}
(\Xi \widetilde{\Cap} \aleph)^- =&([\tilde{\Xi}_L \cap \tilde{\aleph}_L, \tilde{\Xi}_U \cap \tilde{\aleph}_U])^- \\
=&[\tilde{\wp} \setminus (\tilde{\Xi}_U \cap \tilde{\aleph}_U), \tilde{\wp} \setminus (\tilde{\Xi}_L \cap \tilde{\aleph}_L)] \\
=&[(\tilde{\Xi}_U \cap \tilde{\aleph}_U)^c, (\tilde{\Xi}_L \cap \tilde{\aleph}_L)^c] \\
=&[\tilde{\Xi}_U^c \cup \tilde{\aleph}_U^c, \tilde{\Xi}_L^c \cup \tilde{\aleph}_L^c] \\
=&[\tilde{\Xi}_U^c, \tilde{\Xi}_L^c] \widetilde{\Cup} [\tilde{\aleph}_U^c, \tilde{\aleph}_L^c] \\
=&[\tilde{\wp} \setminus \tilde{\Xi}_U, \tilde{\wp} \setminus \tilde{\Xi}_L] \widetilde{\Cup} [\tilde{\wp} \setminus \tilde{\aleph}_U, \tilde{\wp} \setminus \tilde{\aleph}_L] \\
=&[\tilde{\Xi}_L, \tilde{\Xi}_U]^- \widetilde{\Cup} [\tilde{\aleph}_L, \tilde{\aleph}_U]^- \\
=&\Xi^- \widetilde{\Cup} \aleph^-
\end{aligned}$$

□

Definition 5.21. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then, difference of Ξ and $\aleph$, indicated by $\Xi \wr \aleph$ is specified as:

$$\Xi \wr \aleph = [(\tilde{\Xi}_L \setminus \tilde{\aleph}_U), (\tilde{\Xi}_U \setminus \tilde{\aleph}_L)]$$

Example 5.22. Let us consider two FSVIs on $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}$ given as:
$\Xi = [\{\ell_1/.2, \ell_4/.3, \ell_5/.6\}, \{\ell_1/.7, \ell_2/.4, \ell_4/.7, \ell_5/.8, \ell_6/.2\}]$ and
$\aleph = [\{\ell_2/.4, \ell_5/.6, \ell_7/.3\}, \{\ell_1/.3, \ell_2/.5, \ell_4/.8, \ell_5/.9, \ell_7/.4\}]$.
Then, $\Xi \wr \aleph = [\{\ell_1/.2, \ell_4/.2, \ell_5/.1\}, \{\ell_1/.7, \ell_2/.4, \ell_4/.7, \ell_5/.4, \ell_6/.2\}]$

Proposition 5.23. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then,

$$\Xi \wr \aleph = \Xi \widetilde{\Cap} \aleph^-$$

Proof.

$$
\begin{aligned}
(\Xi \wr \aleph) &= ([\tilde{\Xi}_L \setminus \tilde{\aleph}_U, \tilde{\Xi}_U \setminus \aleph_L]) \\
&= [\wp \cap (\tilde{\Xi}_L \setminus \tilde{\aleph}_U), \wp \cap (\tilde{\Xi}_U \setminus \tilde{\aleph}_L)] \\
&= [\tilde{\wp} \cap (\tilde{\Xi}_L \cap \tilde{\aleph}_U^c), \tilde{\wp} \cap (\tilde{\Xi}_U \cap \tilde{\aleph}_L^c)] \\
&= [\tilde{\Xi}_L \cap (\tilde{\wp} \cap \tilde{\aleph}_U^c), \tilde{\Xi}_U \cap (\tilde{\wp} \cap \tilde{\aleph}_L^c)] \\
&= [\tilde{\Xi}_L \cap (\tilde{\wp} \setminus \tilde{\aleph}_U), \tilde{\Xi}_U \cap (\tilde{\wp} \setminus \tilde{\aleph}_L)] \\
&= [\tilde{\Xi}_L, \tilde{\Xi}_U] \widetilde{\Cap} [(\tilde{\wp} \setminus \tilde{\aleph}_U), (\tilde{\wp} \setminus \tilde{\aleph}_L)] \\
&= \Xi \widetilde{\Cap} \aleph^-
\end{aligned}
$$

□

Definition 5.24. Let $\Xi = [\tilde{\Xi}_L, \tilde{\Xi}_U]$ and $\aleph = [\tilde{\aleph}_L, \tilde{\aleph}_U]$ be two FSVIs over $\wp$. Then, symmetric difference of Ξ and $\aleph$, indicated by $\Xi \triangle \aleph$ is specified as:

$$\Xi \triangle \aleph = [(\tilde{\Xi}_L \setminus \tilde{\aleph}_U) \cup (\tilde{\aleph}_L \setminus \tilde{\Xi}_U), (\tilde{\Xi}_U \setminus \tilde{\aleph}_L) \cup (\tilde{\aleph}_U \setminus \tilde{\Xi}_L)]$$

Example 5.25. Let us consider FSVIs Ξ and $\aleph$ given in Example 5.22. Then,

$$\Xi \triangle \aleph = [\{\ell_1/.2, \ell_2/.4, \ell_4/.2, \ell_5/.2, \ell_7/.2\}, \{\ell_1/.7, \ell_2/.4, \ell_4/.7, \ell_5/.4, \ell_6/.2, \ell_7/.4\}]$$

6. FUZZY SET VALUED INTERVAL SOFT SETS

We present the concept of fuzzy set valued interval soft set (FSVIS-set) in this section. After that, we describe the operations of FSVISS and look into how they relate to one another.

Definition 6.1. Let E be a parameter set, $\wp$ be an initial universe and $\mathscr{L}[\wp]$ be set of FSVI over $\wp$. Then, fuzzy set valued interval soft set (FSVIS-set) denoted by $(\tilde{l}, D)$ is defined by

$$(\tilde{l}, \Xi) = \{(e, \tilde{l}(e)) : \tilde{l}(e) \in \mathscr{L}[\wp]\}$$

here $\tilde{l}(e) = [\tilde{l}_1(e), \tilde{l}_2(e)]$, such that $\tilde{l}_1(e), \tilde{l}_2(e) \in \mathscr{L}(\wp)$ and $\tilde{l}_1(e) \subseteq \tilde{l}_2(e)$. From now on, we will use $\mathscr{L}S[\wp]$ instead of the set of all FSVISSs over $\wp$.

Definition 6.2. Let $\tilde{l} \in \mathscr{L}S[\wp]$. Then, element(s) of $\tilde{l}_2(e) \backslash \tilde{l}_1(e)$ is(are) called uncertainty element(s) related to the parameter $e \in E$.

Example 6.3. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ be an initial universe and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If,

$$\begin{aligned}
\tilde{l}(e_1) &= [\{\ell_1/.5, \ell_3/.4\}, \{\ell_1/.6, \ell_2/.2, \ell_3/.5, \ell_5/.7\}] \\
\tilde{l}(e_2) &= [\{\ell_2/.1, \ell_4/.6\}, \{\ell_1/.9, \ell_2/.3, \ell_3/.5, \ell_4/.7, \ell_6/.2, \ell_8/.3\}] \\
\tilde{l}(e_3) &= [\{\ell_5/.4, \ell_6/.7\}, \{\ell_1/.2, \ell_2/.4, \ell_5/.8, \ell_6/.9, \ell_7/.3\}] \\
\tilde{l}(e_4) &= [\{\ell_1/.3, \ell_2/.4, \ell_7/.5, \ell_8/.6\}, \{\ell_1/.4, \ell_2/.6, \ell_3/.2, \ell_5/.4, \ell_7/.6, \ell_8/.9\}] \\
\tilde{l}(e_5) &= [\tilde{\phi}, \{\ell_1/.1, \ell_4/.7, \ell_5/.5, \ell_6/.4\}]
\end{aligned}$$

then the FSVISS is written by

$$\begin{aligned}\tilde{l} =& \{(e_1,[\{\ell_1/.5,\ell_3/.4\},\{\ell_1/.6,\ell_2/.2,\ell_3/.5,\ell_5/.7\}]),\\
&(e_2,[\{\ell_2/.1,\ell_4/.6\},\{\ell_1/.9,\ell_2/.3,\ell_3/.5,\ell_4/.7,\ell_6/.2,\ell_8/.3\}]),\\
&(e_3,[\{\ell_5/.4,\ell_6/.7\},\{\ell_1/.2,\ell_2/.4,\ell_5/.8,\ell_6/.9,\ell_7/.3\}]),\\
&(e_4,[\{\ell_1/.3,\ell_2/.4,\ell_7/.5,\ell_8/.6\},\{\ell_1/.4,\ell_2/.6,\ell_3/.2,\ell_5/.4,\ell_7/.6,\ell_8/.9\}])\\
&(e_5,[\tilde{\phi},\{\ell_1/.1,\ell_4/.7,\ell_5/.5,\ell_6/.4\}])\}\end{aligned}$$

The tabler represents the FSVISS $\tilde{l}$ is as follow:

	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
e_1	[.5,.6]	[0,.2]	[.4,.5]	[0,0]	[0,.7]	[0,0]	[0,0]	[0,0]
e_2	[0,.9]	[.1,.3]	[0,.5]	[.6,.7]	[0,0]	[0,.2]	[0,0]	[0,.3]
e_3	[0,.2]	[0,.4]	[0,0]	[0,0]	[.4,.8]	[.7,.9]	[0,.3]	[0,0]
e_4	[.3,.4]	[.4,.6]	[0,.2]	[0,0]	[0,.4]	[0,0]	[.5,.7]	[.6,.9]
e_5	[0,.1]	[0,0]	[0,0]	[0,.7]	[0,.5]	[0,.4]	[0,0]	[0,0]

Definition 6.4. Let $\tilde{l} \in \mathscr{L}S[\wp]$. If $\tilde{l}(e) = [\tilde{\Phi}]$ for all $e \in E$. Then, $\tilde{l}$ is called an empty FSVISS, denoted by $\widetilde{[\tilde{\Phi}]}$.

Definition 6.5. Let $\tilde{l} \in S[\wp]$. If $\tilde{l}(e) = [\tilde{\wp}]$ for all $e \in E$. Then, $\tilde{l}$ is called an universal FSVISS, denoted by $\widetilde{[\tilde{\wp}]}$.

Definition 6.6. The extended intersection of two FSVISSs $(\tilde{l},\Xi)$ and $(\tilde{m},\aleph)$ over an universe $\wp$ is the FSVIS-set denoted by $(\tilde{n},Y) = (\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph)$, where $Y = \Xi \cup \aleph$, $(\Xi, \aleph \subseteq E)$ and for all $e \in Y$,

$$\tilde{n}(e) = \begin{cases} \tilde{l}(e) & if\ e \in \Xi - \aleph, \\ \tilde{m}(e) & if\ e \in \aleph - \Xi, \\ \tilde{l}(e) \Cap \tilde{m}(e) & if\ e \in \Xi \cap \aleph \end{cases}$$

Example 6.7. Let $\wp = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7,\ell_8\}$ be an initial universe and $E = \{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters. $\Xi, \aleph \subseteq E$ and $\Xi = \{e_1,e_2,e_3\}$ and $\aleph = \{e_3,e_4,e_5\}$. Let us consider the following FSVIS-sets.

$$\begin{aligned}(\tilde{l},\Xi)) &= \{(e_1,[\{\ell_1/.5,\ell_3/.4\},\{\ell_1/.6,\ell_2/.2,\ell_3/.5,\ell_5/.7\}]),(e_2,\\ &[\{\ell_2/.1,\ell_4/.6\},\{\ell_1/.9,\ell_2/.3,\ell_3/.5,\ell_4/.7,\ell_6/.2,\ell_8/.3\}]),(e_3,\\ &[\{\ell_5/.4,\ell_6/.7\},\{\ell_1/.2,\ell_2/.5,\ell_5/.8,\ell_6/.9,\ell_7/.3\}])\}\end{aligned}$$

and

$$\begin{aligned}(\tilde{m},\aleph)) &= \{(e_3,[\{\ell_1/.1,\ell_5/.4\},\{\ell_1/.2,\ell_2/.5,\ell_4/.6,\ell_5/.8\}]),(e_4,[\{\ell_2/.8,\ell_6/.7\},\\ &\{\ell_1/.2,\ell_2/.9,\ell_3/.5,\ell_4/.5,\ell_6/.8,\ell_8/.7\}]),(e_5,[\{\ell_2/.2,\ell_7/.3\},\\ &\{\ell_1/.4,\ell_2/.3,\ell_5/.4,\ell_6/.6,\ell_7/.4\}])\}.\end{aligned}$$

Then,

$$\begin{aligned}(\tilde{n},Y) &= (\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph)\\ &= \{(e_1,[\{\ell_1/.5,\ell_3/.4\},\{\ell_1/.6,\ell_2/.2,\ell_3/.5,\ell_5/.7\}]),(e_2,[\{\ell_2/.1,\ell_4/.6\},\\ &\{\ell_1/.9,\ell_2/.3,\ell_3/.5,\ell_4/.7,\ell_6/.2,\ell_8/.3\}]),(e_3,[\{\ell_5/.4\},\\ &\{\ell_1/.2,\ell_2/.5,\ell_5/.8\}]),(e_4,[\{\ell_2/.8,\ell_6/.7\},\\ &\{\ell_1/.2,\ell_2/.9,\ell_3/.5,\ell_4/.5,\ell_6/.8,\ell_8/.7\}]),\\ &(e_5,[\{\ell_2/.2,\ell_7/.3\},\{\ell_1/.4,\ell_2/.3,\ell_5/.4,\ell_6/.6,\ell_7/.4\}])\}.\end{aligned}$$

Definition 6.8. The extended union of two FSVIS-sets $(\tilde{l},\Xi)$ and $(\tilde{m},\aleph)$ over a universe $\wp$ is the FSVIS-set denoted by $(\tilde{n},Y) = (\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph)$, where $Y = \Xi\cup\aleph$, and for all $e \in Y$ ($\Xi, \aleph \subseteq E$),

$$\tilde{n}(e) = \begin{cases} \tilde{l}(e) & if\ e\in\Xi-\aleph,\\ \tilde{m}(e) & if\ e\in\aleph-\Xi,\\ \tilde{l}(e)\mathbb{U}\tilde{m}(e) & if\ e\in\Xi\cap\aleph.\end{cases}$$

Example 6.9. Let $\wp = \{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7,\ell_8\}$ be an initial universe and $E = \{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters. $\Xi,\aleph \subseteq E$ and $\Xi = \{e_1,e_2,e_3\}$ and $\aleph = \{e_3,e_4,e_5\}$. Let us consider the following FSVIS-sets

$$
\begin{aligned}
(\tilde{l},\Xi) \;=\; & \{(e_1,[\{\ell_1/.5,\ell_3/.4\},\{\ell_1/.6,\ell_2/.2,\ell_3/.5,\ell_5/.7\}]),(e_2,\\
& [\{\ell_2/.1,\ell_4/.6\},\{\ell_1/.9,\ell_2/.3,\ell_3/.5,\ell_4/.7,\ell_6/.2,\ell_8/.3\}]),\\
& (e_3,[\{\ell_5/.4,\ell_6/.7\},\{\ell_1/.2,\ell_2/.5,\ell_5/.8,\ell_6/.9,\ell_7/.3\}])\}
\end{aligned}
$$

and

$$
\begin{aligned}
(\tilde{m},\aleph) \;=\; & \{(e_3,[\{\ell_1/.1,\ell_5/.4\},\{\ell_1/.2,\ell_2/.5,\ell_4/.6,\ell_5/.8\}]),(e_4,[\{\ell_2/.8,\ell_6/.7\},\\
& \{\ell_1/.2,\ell_2/.9,\ell_3/.5,\ell_4/.5,\ell_6/.8,\ell_8/.7\}]),(e_5,[\{\ell_2/.2,\ell_7/.3\},\\
& \{\ell_1/.4,\ell_2/.3,\ell_5/.4,\ell_6/.6,\ell_7/.4\}])\}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
(\tilde{n},Y) \;=\; & ((\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph))\\
=\; & \{(e_1,[\{\ell_1/.5,\ell_3/.4\},\{\ell_1/.6,\ell_2/.2,\ell_3/.5,\ell_5/.7\}]),(e_2,[\{\ell_2/.1,\ell_4/.6\},\\
& \{\ell_1/.9,\ell_2/.3,\ell_3/.5,\ell_4/.7,\ell_6/.2,\ell_8/.3\}]),(e_3,[\{\ell_1/.1,\ell_5/.4,\ell_6/.7\},\\
& \{\ell_1/.2,\ell_2/.5,\ell_4/.6,\ell_5/.8,\ell_6/.9,\ell_7/.3\}]),(e_4,[\{\ell_2/.8,\ell_6/.7\},\\
& \{\ell_1/.2,\ell_2/.9,\ell_3/.5,\ell_4/.5,\ell_6/.8,\ell_8/.7\}]),(e_5,[\{\ell_2/.2,\ell_7/.3\},\\
& \{\ell_1/.4,\ell_2/.3,\ell_5/.4,\ell_6/.6,\ell_7/.4\}])\}.
\end{aligned}
$$

Theorem 6.10. Let $(\tilde{l},\Xi),(\tilde{m},\aleph)$ and $(\tilde{n},Y)$ be three FSVIS-sets over the same universe $\wp$. Then,

1. $(\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
2. $(\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph)=(\tilde{m},\aleph)\tilde{\sqcap}_e(\tilde{l},\Xi)$;
3. $((\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph))\tilde{\sqcap}_e(\tilde{n},Y)=(\tilde{l},\Xi)\tilde{\sqcap}_e((\tilde{m},\aleph)\tilde{\sqcap}_e(\tilde{n},Y))$;
4. $(\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
5. $(\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph)=(\tilde{m},\aleph)\tilde{\sqcup}_e(\tilde{l},\Xi)$;
6. $((\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph))\tilde{\sqcup}_e(\tilde{n},Y)=(\tilde{l},\Xi)\tilde{\sqcup}_e((\tilde{m},\aleph)\tilde{\sqcup}_e(\tilde{n},Y)$.

Definition 6.11. The restricted intersection of two FSVIS-sets $(\tilde{l},\Xi)$ and $(\tilde{m},\aleph)$ over a universe $\wp$ is the FSVIS-set denoted by $(\tilde{n},Y)=(\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph)$, where $Y=\Xi\cap\aleph\,(\Xi,\aleph\subseteq E)$ and for all $e\in Y$, $\tilde{n}(e)=\tilde{l}(e)\widetilde{\Cap}\tilde{m}(e)$.

Example 6.12. Let $\wp=\{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7,\ell_8\}$ be an initial universe and $E=\{e_1,e_2,e_3,e_4,e_5\}$ be a set of parameters. $\Xi,\aleph\subseteq E$ and $\Xi=\{e_1,e_2,e_3\}$ and

$\aleph = \{e_3, e_4, e_5\}$. Let us consider the following FSVIS-sets

$$\begin{aligned}(\tilde{l}, \Xi) &= \{(e_1, [\{\ell_1/.5, \ell_3/.4\}, \{\ell_1/.6, \ell_2/.2, \ell_3/.5, \ell_5/.7\}]), (e_2,\\ &\quad [\{\ell_2/.1, \ell_4/.6\}, \{\ell_1/.9, \ell_2/.3, \ell_3/.5, \ell_4/.7, \ell_6/.2, \ell_8/.3\}]),\\ &\quad (e_3, [\{\ell_5/.4, \ell_6/.7\}, \{\ell_1/.2, \ell_2/.5, \ell_5/.8, \ell_6/.9, \ell_7/.3\}])\}\end{aligned}$$

and

$$\begin{aligned}(\tilde{m}, \aleph) &= \{(e_3, [\{\ell_1/.1, \ell_5/.4\}, \{\ell_1/.2, \ell_2/.5, \ell_4/.6, \ell_5/.8\}]), (e_4, [\{\ell_2/.8, \ell_6/.7\},\\ &\quad \{\ell_1/.2, \ell_2/.9, \ell_3/.5, \ell_4/.5, \ell_6/.8, \ell_8/.7\}]), (e_5, [\{\ell_2/.2, \ell_7/.3\},\\ &\quad \{\ell_1/.4, \ell_2/.3, \ell_5/.4, \ell_6/.6, \ell_7/.4\}])\}.\end{aligned}$$

Then,

$$\begin{aligned}(\tilde{n}, Y) &= (\tilde{l}, \Xi) \tilde{\sqcap}_r (\tilde{m}, \aleph)\\ &= \{(e_3, [\{\ell_5/.4\}, \{\ell_1/.2, \ell_2/.5, \ell_5/.8\}])\}.\end{aligned}$$

Definition 6.13. The restricted union of two FSVISSs $(\tilde{l}, \Xi)$ and $(\tilde{m}, \aleph)$ over a universe $\wp$ is an FSVISS denoted by $(\tilde{n}, Y) = (\tilde{l}, \Xi) \tilde{\sqcup}_r (\tilde{m}, \aleph)$, where $Y = \Xi \cap \aleph \neq \phi$ and $\forall\ e \in Y$, $n(e) = \tilde{l}(e) \tilde{\mathbb{U}} \tilde{m}(e)$.

Example 6.14. Let $\wp = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ be an initial universe and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. $\Xi, \aleph \subseteq E$ and $\Xi = \{e_1, e_2, e_3\}$ and $\aleph = \{e_3, e_4, e_5\}$. Let us consider the following,

$$\begin{aligned}(\tilde{l}, \Xi) &= \{(e_1, [\{\ell_1/.5, \ell_3/.4\}, \{\ell_1/.6, \ell_2/.2, \ell_3/.5, \ell_5/.7\}]), (e_2,\\ &\quad [\{\ell_2/.1, \ell_4/.6\}, \{\ell_1/.9, \ell_2/.3, \ell_3/.5, \ell_4/.7, \ell_6/.2, \ell_8/.3\}]),\\ &\quad (e_3, [\{\ell_5/.4, \ell_6/.7\}, \{\ell_1/.2, \ell_2/.5, \ell_5/.8, \ell_6/.9, \ell_7/.3\}])\}\end{aligned}$$

and

$$
\begin{aligned}
(\tilde{m},\aleph) &= \{(e_3,[\{\ell_1/.1,\ell_5/.4\},\{\ell_1/.2,\ell_2/.5,\ell_4/.6,\ell_5/.8\}]),(e_4,[\{\ell_2/.8,\ell_6/.7\},\\
&\quad \{\ell_1/.2,\ell_2/.9,\ell_3/.5,\ell_4/.5,\ell_6/.8,\ell_8/.7\}]),(e_5,[\{\ell_2/.2,\ell_7/.3\},\\
&\quad \{\ell_1/.4,\ell_2/.3,\ell_5/.4,\ell_6/.6,\ell_7/.4\}])\}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
(\tilde{n},Y) &= (\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph)\\
&= \{(e_3,[\{\ell_1/.1,\ell_5/.4,\ell_6/.7\},\{\ell_1/.2,\ell_2/.5,\ell_4/.6,\ell_5/.8,\ell_6/.9,\ell_7/.3\}])\}.
\end{aligned}
$$

Theorem 6.15. Let $(\tilde{l},\Xi)$, $(\tilde{m},\aleph)$ and $(\tilde{n},Y)$ be three FSVISSs over the same universe $\wp$. Then,

1. $(\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
2. $(\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph)=(\tilde{m},\aleph)\tilde{\sqcap}_r(\tilde{l},\Xi)$;
3. $((\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph))\tilde{\sqcap}_r(\tilde{n},Y)=(\tilde{l},\Xi)\tilde{\sqcap}_r((\tilde{m},\aleph)\tilde{\sqcap}_r(\tilde{n},Y))$;
4. $(\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
5. $(\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph)=(\tilde{m},\aleph)\tilde{\sqcup}_r(\tilde{l},\Xi)$;
6. $((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))\tilde{\sqcup}_r(\tilde{n},Y)=(\tilde{l},\Xi)\tilde{\sqcup}_r((\tilde{m},\aleph)\tilde{\sqcup}_r(\tilde{n},Y)$.

FSVISS operations $\tilde{\sqcap}_r$ and $\tilde{\sqcup}_e$ satisfy commutativity, associativity, idempotent and absorption. Using these properties, we can get following results.

Theorem 6.16. Let $(\tilde{l},\Xi)$, $(\tilde{m},\aleph)$ and $(\tilde{n},Y)$ be three FSVISSs over the same universe $\wp$. Then,

1. $((\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph))\tilde{\sqcap}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
2. $((\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph))\tilde{\sqcup}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$

Proof. 1. Suppose that $(\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph))=(\tilde{n},\Xi\cup\aleph)$ and $((\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph)))\tilde{\sqcap}_r(\tilde{l},\Xi)=(\tilde{o},(\Xi\cup\aleph)\cap\Xi)=(\tilde{o},\Xi)$. For all $e\in(\Xi\cup\aleph)\cap\Xi$, $e\in\Xi$. Then, $\tilde{o}(e)=\tilde{n}(e)\tilde{\Cap}\tilde{l}(e)=(\tilde{l}(e)\tilde{\Cup}\tilde{m}(e))\tilde{\Cap}\tilde{l}(e)=\tilde{l}(e)$. Therefore, $((\tilde{l},\Xi)\tilde{\sqcup}_e(\tilde{m},\aleph)))\tilde{\sqcap}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$.

2. Suppose that $(\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph))=(\tilde{n},\Xi\cap\aleph)$ and

$((\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph)))\tilde{\sqcup}_e(\tilde{l},\Xi)=(\tilde{o},(\Xi\cap\aleph)\cup\Xi)=(\tilde{o},\Xi)$. For all $e\in(\Xi\cap\aleph)\cup\Xi$, $e\in\Xi$.
Then, $\tilde{o}(e)=\tilde{n}(e)\tilde{\Cup}\tilde{l}(e)=(\tilde{l}(e)\tilde{\Cap}\tilde{m}(e))\tilde{\Cup}\tilde{l}(e)=\tilde{l}(e)$.
Therefore, $((\tilde{l},\Xi)\tilde{\sqcap}_r(\tilde{m},\aleph))\tilde{\sqcup}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$.

□

Theorem 6.17. Let $(\tilde{l},\Xi)$, $(\tilde{m},\aleph))$ and $(\tilde{n},Y)$ be three FSVISSs over the same universe $\wp$. Then,

1. $((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))\tilde{\sqcap}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$;
2. $((\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph))\tilde{\sqcup}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$.

Proof. Let $(\tilde{l},\Xi)$, $(\tilde{m},\aleph)$ and $(\tilde{n},Y)$ be three FSVISSs over the same universe $\wp$.
1. Let $(\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph)=(\tilde{n},Y)$ and
$((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))\tilde{\sqcap}_e(\tilde{l},\Xi)=(\tilde{o},(\Xi\cup\aleph)\cap\Xi)=(\tilde{o},\Xi)\forall e\in D$
(a) if $e\in\aleph$, then $\tilde{o}(e)=\tilde{n}(e)\tilde{\Cap}\tilde{l}(e)=(\tilde{l}(e)\tilde{\Cup}\tilde{m}(e))\tilde{\Cap}\tilde{l}(e)=\tilde{l}(e)$;
(b) if $e\in\Xi\setminus\aleph$, then $\tilde{o}(e)=\tilde{n}(e)\tilde{\Cap}\tilde{l}(e)=\tilde{l}(e)\tilde{\Cap}l(e)=\tilde{l}(e)$;
Therefore, $((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))\tilde{\sqcap}_e(\tilde{l},\Xi)=(\tilde{l},\Xi)$.

2. Suppose that $(\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph)=(\tilde{n},\Xi\cap\aleph)$ and
$((\tilde{l},\Xi)\sqcap_e(\tilde{m},\aleph))\tilde{\sqcup}_r(\tilde{l},\Xi)=(\tilde{o},(\Xi\cap\aleph)\cup\Xi)=(\tilde{o},\Xi)$. For all $e\in\Xi$
(a) if $e\in\aleph$, then $\tilde{o}(e)=\tilde{n}(e)\sqcup\tilde{l}(e)=(\tilde{l}(e)\sqcap\tilde{m}(e))\sqcup\tilde{l}(e)=\tilde{l}(e)$;
(b) if $e\in\Xi\setminus\aleph$, then $\tilde{o}(e)=\tilde{l}(e)$;
Therefore, $((\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph))\tilde{\sqcup}_r(\tilde{l},\Xi)=(\tilde{l},\Xi)$.

□

Definition 6.18. Let $(\tilde{l},\Xi)$ be an FSVISS over $\wp$. Then, image of $(\tilde{l},\Xi)$ denoted by $(\tilde{l},\Xi)^{\sim}$ is an FSVISS defined as follows:

$$(\tilde{l},\Xi)^{\sim}=\{(e,[\tilde{\wp}\setminus\tilde{l}_U(e),\tilde{\wp}\setminus\tilde{l}_L(e)]):e\in\Xi\}$$

Theorem 6.19. Let $(\tilde{l},\Xi)$ and $(\tilde{m},\aleph)$ be two FSVISSs over the same universe $\wp$. Then,

1. $((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))^{\sim}=(\tilde{l},\Xi)^{-}\tilde{\sqcap}_e(\tilde{m},\aleph)^{\sim}$;
2. $((\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph))^{\sim}=(\tilde{l},\Xi)^{\sim}\tilde{\sqcup}_r(\tilde{m},\aleph)^{\sim}$;

Proof. Let us consider FSVISSs;

$$(\tilde{l},\Xi)=\left\{(e,[\tilde{l}_L(e),\tilde{l}_U(e)]) : e\in D, \tilde{l}_L(e),\tilde{l}_U(e)\in \mathscr{L}(\wp)\right\}$$

and

$$(\tilde{m},\aleph)=\left\{(e,[\tilde{m}_L(e),\tilde{m}_U(e)]) : e\in \aleph, \tilde{m}_L(e),\tilde{m}_U(e)\in \mathscr{L}(\wp)\right\}.$$

1. Assume that $(\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph)=(\tilde{n},Y)$ here

$$\tilde{n}(e)=\tilde{l}(e)\Cup\tilde{m}(e)=[\tilde{l}_L(e)\cup\tilde{m}_L(e),\tilde{l}_U(e)\cup\tilde{m}_U(e)]$$

for all $e\in Y=\Xi\cap\aleph\neq\emptyset$.

$$\begin{aligned}
\tilde{n}^{-}(e) &= [\wp\setminus(\tilde{l}_U(e)\cup\tilde{m}_U(e)),\wp\setminus(\tilde{l}_L(e)\cup\tilde{m}_L(e))]\\
&= [\tilde{\wp}\cap(\tilde{l}_U(e)\cup\tilde{m}_U(e))^c,\tilde{\wp}\cap(\tilde{l}_L(e)\cup\tilde{m}_L(e))^c]\\
&= [\tilde{\wp}\cap(\tilde{l}_U^c(e)\cap\tilde{m}_U^c(e)),\tilde{\wp}\cap(\tilde{l}_L^c(e)\cap\tilde{m}_L^c(e))]\\
&= [(\tilde{\wp}\cap\tilde{l}_U^c(e))\cap(\tilde{\wp}\cap\tilde{m}_U^c(e)),(\tilde{\wp}\cap\tilde{l}_L^c(e))\cap(\tilde{\wp}\cap\tilde{m}_L^c(e))]\\
&= [(\tilde{\wp}\setminus\tilde{l}_U(e))\cap(\tilde{\wp}\setminus\tilde{m}_U(e)),(\tilde{\wp}\setminus\tilde{l}_L(e))\cap(\tilde{\wp}\setminus\tilde{m}_L(e))]\\
&= [(\tilde{\wp}\setminus\tilde{l}_U(e)),(\tilde{\wp}\setminus\tilde{l}_L(e))]\Cap[(\tilde{\wp}\setminus\tilde{m}_U(e)),(\tilde{\wp}\setminus\tilde{m}_L(e))]\\
&= [\tilde{l}_L(e),\tilde{l}_U(e)]^{-}\Cap[\tilde{m}_L(e),\tilde{m}_U(e)]^{-}\\
&= \tilde{l}^{-}(e)\Cap\tilde{m}^{-}(e).
\end{aligned}$$

Thus, $((\tilde{l},\Xi)\tilde{\sqcup}_r(\tilde{m},\aleph))^{\sim}=(\tilde{l},\Xi)^{\sim}\tilde{\sqcap}_e(\tilde{m},\aleph)^{\sim}$.

2. Assume that $(\tilde{l},\Xi)\sqcap_e(\tilde{m},\aleph)=(\tilde{o},Y)$ here

$$\tilde{n}(e)=\tilde{l}(e)\Cap\tilde{m}(e)=[\tilde{l}_L(e)\cap\tilde{m}_L(e),\tilde{l}_U(e)\cap\tilde{m}_U(e)]$$

for all $e\in Y=\Xi\cap\aleph\neq\emptyset$.

$$
\begin{aligned}
\tilde{o}^{-}(e) &= [\tilde{\wp}\setminus(\tilde{l}_U(e)\cap\tilde{m}_U(e)),\tilde{\wp}\setminus(\tilde{l}_L(e)\cap\tilde{m}_L(e))] \\
&= [\tilde{\wp}\cap(\tilde{l}_U(e)\cap\tilde{m}_U(e))^c,\tilde{\wp}\cap(\tilde{l}_L(e)\cap\tilde{m}_L(e))^c] \\
&= [\tilde{\wp}\cap(\tilde{l}^c_U(e)\cup\tilde{m}^c_U(e)),\tilde{\wp}\cap(\tilde{l}^c_L(e)\cup\tilde{m}^c_L(e))] \\
&= [(\tilde{\wp}\cap\tilde{l}^c_U(e))\cup(\tilde{\wp}\cap\tilde{m}^c_U(e)),(\tilde{\wp}\cap\tilde{l}^c_L(e))\cup(\tilde{\wp}\cap\tilde{m}^c_L(e))] \\
&= [(\tilde{\wp}\setminus\tilde{l}_U(e))\cup(\tilde{\wp}\setminus\tilde{m}_U(e)),(\tilde{\wp}\setminus\tilde{l}_L(e))\cup(\tilde{\wp}\setminus\tilde{m}_L(e))] \\
&= [(\tilde{\wp}\setminus\tilde{l}_U(e)),(\tilde{\wp}\setminus\tilde{l}_L(e))]\tilde{\Cup}[(\tilde{\wp}\setminus\tilde{m}_U(e)),(\tilde{\wp}\setminus\tilde{m}_L(e))] \\
&= [\tilde{l}_L(e),\tilde{l}_U(e)]^{-}\tilde{\Cup}[\tilde{m}_L(e),\tilde{m}_U(e)]^{-} \\
&= \tilde{l}^{-}(e)\tilde{\Cup}\tilde{m}^{-}(e).
\end{aligned}
$$

Hence, $((\tilde{l},\Xi)\tilde{\sqcap}_e(\tilde{m},\aleph))^{\sim} = (\tilde{l},\Xi)^{\sim}\tilde{\sqcup}_r(\tilde{m},\aleph)^{\sim}$.

$\square$

7. CONCLUSION

In this thesis, a novel concept called as fuzzy set-valued interval soft sets and set-theoretical operations between two fuzzy sets valued interval soft set are defined. Then, some of their properties are obtained. A fuzzy set valued interval soft set is an important tool for the representation of the information in environments including uncertainty. Because, in a fuzzy set valued interval soft set, both parameterized family of a set and fuzzy set valued intervals are used. This allows us to represent the information involving uncertainty more effective. Also, in the light of definitions and results, some generalization of the proposed concepts such as intuitionistic fuzzy set valued interval and intuitionistic fuzzy set valued interval soft sets can be studied by researchers.

REFERENCES

Ali, M. I., Feng, F., Liu, X., Min, W. K., and Shabir, M. (2009). On some new operations in soft set theory. Computers and Mathematics with Applications, 57(9):1547-1553.

Aktas, H., and Çağman, N. 2007. Soft sets and soft groups. Information Sciences, 177(13): 2726-2735.

Basu, T. M., Mahapatra, N. K., and Mondal, S. K. (2012). On some new operations in fuzzy soft set and intuitionistic fuzzy soft set theory. International Journal of computer Science and information technology and security, 2(2): 408-417.

Çağman, N. and Enginoğlu, S. 2010. Soft set theory and uni-int decision making. European Journal of operational research, 207: 848-855.

Çağman, N., Enginoğlu, S.and Çıtak, F. 2011. Fuzzy soft set theory and its applications. Iranian Journal of fuzzy systems, 8(3): 137-147.

Çağman, N. and Enginoglu, S. 2012. Fuzzy soft matrix theory and its application in decision making. Iranian Journal of Fuzzy Systems, 9(1): 109-119.

Chen, D., Tsang, E.C.C., Yeung, D.S. and Wang, X. 2005.The parameterization reduction of soft sets and its applications, Comput. Math. Appl. 49: 757–763.

Chetia, B. and Das, P. K. 2010. An application of interval-valued fuzzy soft. International Journal contemp. Math. Sciences, 5(38): 1887-1894.

Jiang, Y., Tang, Y., Chen, Q., Liu, H., and Tang, J. 2010. Interval-valued intuitionistic fuzzy soft sets and their properties. Computersand Mathematics with Applications, 60(3): 906-918.

Kalaichelvi, A., Malini, P. H., and Janofer, K. 2012. Application of Interval Fuzzy Number Matrices and Interval-Valued Fuzzy Soft Sets in the Analysis of the factors influencing High Scores in Higher Secondary Examinations. International Journal of Mathematical Sciences and Applications, 2(2): 777-780.

Kong, Z., Wang, L. and Wu, Z. 2011. Application of fuzzy soft set in decision making problems based on grey theory. Journal of Computational and Applied Mathematics, 236(6): 1521-1530.

Li, Z., Zheng, D.,and Hao, J. 2012. L-fuzzy soft sets based on complete Boolean lattices. Computers and Mathematics with Applications, 64(8): 2558-2574.

Li, Z., Wen, G. and Xie, N. 2015. An approach to fuzzy soft sets in decision making based on grey relational analysis and Dempster–Shafer theory of evidence: An application in medical diagnosis. Artificial Intelligence in Medicine, 64(3): 161-171.

Liu, Z., Qin, K. and Pei, Z. 2017. A method for fuzzy soft sets in decision-making based on an ideal solution. Symmetry, 9(10): 246.

Ma, X., and Sulaiman, N. 2011. An interval-valued fuzzy soft set approach for normal parameter reduction. In International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing (pp. 211-214). Springer, Berlin, Heidelberg.

Maji, P.K., Roy, A.R. and Biswas, R. 2001a. Fuzzy soft sets, J. Fuzzy Math. 9(3):589-602.

Maji, P. K., Biswas, R., and Roy, A. R. 2001b. Intuitionistic fuzzy soft sets. Journal of fuzzy Mathematics, 9(3): 677-692.

Maji, P.K., Roy, A.R. and Biswas, R. 2002. An application of soft sets in a decision making problem, Comput. Math. Appl. 44:1077-1083.

Maji, P.K., Biswas, R. and Roy, A.R. 2003. Soft set theory, Comput. Math. Appl. 45: 555-562.

Molodtsov, D.A. 1999. Soft set theory-first results, Comput. Math. Appl. 37: 19-31.

Neog, T. J.,and Sut, D. K. 2011. Theory of fuzzy soft sets from a new perspective. International Journal of Latest Trends in Computing, 2(3): 439-450.

Pawlak, Z., Grzymala-Busse, J., Slowinski, R., and Ziarko, W. 1995. Rough sets. Communications of the ACM, 38(11): 88-95.

Peng, X.,and Garg, H. (2018). Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure. Computers and Industrial Engineering, 119: 439-452.

Rehman, A., Abdullah, S., Aslam, M., and Kamran, M. S. 2013. A study on fuzzy soft set and its operations. Ann. Fuzzy Math. Inform, 6(2): 339-362.

Roy, A. R. and Maji, P. K. 2007. A fuzzy soft set theoretic approach to decision making problems. Journal of computational and Applied Mathematics, 203(2): 412-418.

Sezgin, A., and Atagun, A. O. 2011. On operations of soft sets. Computers and Mathematics with Applications, 61(5): 1457-1467.

Sooraj, T. R., and Tripathy, B. K. 2018. An interval valued fuzzy soft set based optimization algorithm for high yielding seed selection. International Journal of Fuzzy System Applications (IJFSA), 7(2): 44-61.

Sulaiman, N. H., and Mohamad, D. 2012. Multi-aspect fuzzy soft sets and their properties. Applied Mathematical Sciences, 6(115): 5709-5718.

Tripathy, B. K., Sooraj, T. R. and Mohanty, R. K. 2016. A new approach to fuzzy soft set theory and its application in decision making. In Computational Intelligence in Data Mining—Volume 2 (pp. 305-313). Springer, New Delhi.

Xu, W., Ma, J., Wang, S.,and Hao, G. (2010). Vague soft sets and their properties. Computers and Mathematics with Applications, 59(2): 787-794.

Xue, B., Wang, L. and Qin, K. 2017. An interval-valued fuzzy soft set based triple I method. In 2017 12th International Conference on Intelligent Systems and Knowledge Engineering (ISKE) (pp. 1-7). IEEE.

Yang, X., Lin, T. Y., Yang, J., Li, Y. and Yu, D. 2009. Combination of interval-valued fuzzy set and soft set. Computers and Mathematics with Applications, 58(3):521-527.

Yao, Y. Y. 1993. Interval-set algebra for qualitative knowledge representation. In Proceedings of ICCI'93: 5th International Conference on Computing and Information (pp. 370-374). IEEE.

Yao, Y. Y. and Li, X. 1996. Comparison of rough-set and interval-set models for uncertain reasoning. Fundamenta Informaticae, 27(2-3): 289-298.

Yao, Y. Y. and Wong, S. M. 1997. Interval approaches for uncertain reasoning. In International Symposium on Methodologies for Intelligent Systems (pp. 381-390). Springer, Berlin, Heidelberg.

Yao, Y. Y. and Wang, J. 1997. Interval based uncertain reasoning using fuzzy and rough sets. Advances in Machine Intelligence and Soft-Computing, 4: 196-215.

Yao, Y. 2009. Interval sets and interval-set algebras. In 2009 8th IEEE International Conference on Cognitive Informatics (pp. 307-314). IEEE.

Yaylali, G., Çakmak Polat, N. and Tanay, B. 2021. A Soft Interval Based Decision Making Method and Its Computer Application.

Zadeh, L. A. 1965. Fuzzy sets. Info. Comput., 8: 338-353.

Zhang, X.H. and Fu, Q. 2012. Interval soft sets. Proceedings of 2nd IEEE International Conference on Cloud Computing and Intelligence Systems, 74–78.

Zhang, Z. 2013. The parameter reduction of fuzzy soft sets based on soft fuzzy rough sets. Advances in Fuzzy Systems, 2013.

Zhang, X. 2014. On interval soft sets with applications. International Journal of Computational Intelligence Systems, 7(1): 186-196.

Zhang, X., and Wang, W. 2014. Lattice-valued interval soft sets–A general frame of many soft set models. Journal of Intelligent and Fuzzy Systems, 26(3): 1311-1321.

Zhou, X., Li, Q. and Guo, L. 2012. On generalised interval-valued fuzzy soft sets. Journal of Applied Mathematics, 2012.

CURRICULUM VITAE

Personal Information

Name Surname : Mohammed Abbood Hussein AL-FURAIJI

Education

MSc	Çankırı Karatekin University Graduate School of Natural and Applied Sciences Department of Mathematics	2022
Undergraduate	Tikrit University College of Computer Science and Mathematics	2019