GENİŞLETİLMİŞ (G'/G)-AÇILIM YÖNTEMİ İLE MANYETO-OPTİK DALGA KILAVUZLARINDA OPTİK SOLİTON PERTÜRBASYONU

ERHAN KOÇ

YÜKSEK LİSANS TEZİ

MATEMATİK ANABİLİM DALI

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MATEMATİK ANA BİLİM DALI

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Tez Danışmanı

Dr. Öğr. Üyesi MEHMET EKİCİ

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TEZ BEYANI

Tez yazım kurallarına uygun olarak hazırlanan bu tezin yazılmasında bilimsel ahlak kurallarına uyulduğunu, başkalarının eserlerinden yararlanılması durumunda bilimsel normlara uygun olarak atıfta bulunulduğunu, tezin içerdiği yenilik ve sonuçların başka bir yerden alınmadığını, kullanılan verilerde herhangi bir tahrifat yapılmadığını, tezin herhangi bir kısmının bu üniversite veya başka bir üniversitedeki başka bir tez çalışması olarak sunulmadığını beyan eder, aksi bir durumda aleyhime doğabilecek tüm hak kayıplarını kabullendiğimi beyan ederim.

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ÖZET

YÜKSEK LİSANS

GENİŞLETİLMİŞ (G'/G)-AÇILIM YÖNTEMİ İLE MANYETO-OPTİK DALGA KILAVUZLARINDA OPTİK SOLİTON PERTÜRBASYONU

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MATEMATİK ANABİLİM DALI

TEZ DANIŞMANI: DR. ÖĞR. ÜYESİ MEHMET EKİCİ

Optik soliton bilimlerinin en geçerli uygulamalarından biri, harici bir manyetik alanın soliton dağınıklığı etkisini kontrol ettiği manyeto-optik dalga kılavuzlarıdır. Bu teknoloji harikası, kıtalar arası mesafelerde soliton darbelerin aerodinamik akışını sağlamak için endüstri genelinde uygulanır. Bu nedenle, koruma yasaları, nümerik analiz ve benzeri ek özelliklere bakmak için, manyeto-optik dalga kılavuzlarında ana model denklemi üzerinde çalışmak ve bir soliton çözümü sağlamak zorunludur. Bu çalışmada, çeşitli pertürbasyon terimleri ile manyeto-optik dalga kılavuzlarındaki optik soliton çözümler elde edildi. Grup-hız dağılımına ek olarak, uzay-zamansal dağılım terimi de dikkate alındı. Bu çözümleri ortaya çıkarmak için kullanılan analitik şema, genişletilmiş (G'/G) −açılım yaklaşımıdır. Bu çalışmada ele alınan on nonlineerlik türü vardır. Bunlar, Kerr yasası, kuvvet yasası, kuadratik-kübik yasası, parabolik yasası, çift-kuvvet yasası, logaritma yasası, anti-kübik yasası, kübik-kuintik-septik yasası, üçlü kuvvet yasası nonlineerlikleri ve zayıf yerel-olmayan nonlineerliğe sahip parabolik yasasıdır. Çalışma sonucunda parlak, koyu ve tekil soliton çözümler elde edildi. Ayrıca bu solitonların varlığı için kısıtlama koşulları da sunuldu.

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ANAHTAR KELİMELER: Solitonlar; Manyeto-Optik Dalga Kılavuzları; Kerr-Olmama Yasası; İntegrallenebilirlik; Genişletilmiş (G'/G) −Açılım Yaklaşımı.

ABSTRACT

MASTER THESIS

OPTICAL SOLITON PERTURBATION IN MAGNETO-OPTIC WAVEGUIDES BY EXTENDED (G'/G)-EXPANSION METHOD

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One of the most viable applications of optical soliton sciences is in magneto-optic waveguides where an external magnetic field controls the effect of soliton clutter. This technological marvel has been implemented across the industry to enable a streamlined flow of soliton pulses across inter-continental distances. Therefore, it is imperative to study the governing model equation in magneto-optic waveguides and secure a soliton solution for looking into additional features such as conservation laws, numerical analysis and so on. This study obtains optical soliton solutions in magneto-optic waveguides with several perturbation terms. In addition to group-velocity dispersion, spatio-temporal dispersion term is taken into consideration. The analytical scheme employed to recover these solutions is extended (G'/G) −expansion approach. There are ten types of nonlinearities that are considered in this work. They are Kerr law, power law, quadratic-cubic law, parabolic law, dual-power law, log law, anti-cubic law, cubic-quintic-septic law and triple-power law nonlinearities and parabolic law with weak non-local nonlinearity. Bright, dark and singular soliton solutions are revealed. The constraint conditions for the existence of these solitons are also presented.

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KEYWORDS: Solitons; Magneto-Optic Waveguides; Non-Kerr law; Integrability; Extended (G'/G) −Expansion Approach.

ÖNSÖZ

“Genişletilmiş (G'/G)-Açılım Yöntemi İle Manyeto-Optik Dalga Kılavuzlarında Optik Soliton Pertürbasyonu” konulu tez çalışmasının seçiminde, yürütülmesinde, sonuçlandırılmasında ve sonuçlarının değerlendirilmesinde değerli zamanını feda ederek, her türlü destek ve yardımlarını esirgemeyen, engin bilgisi ile beni yönlendiren değerli tez danışmanım Dr. Öğr. Üyesi Mehmet EKİCİ’ye ve tez çalışmaları sırasında bilgi ve tecrübelerinden yararlandığım Doç. Dr. Anjan BISWAS’a teşekkürlerimi sunarım.

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İÇİNDEKİLER

Sayfa

ŞEKİLLER LİSTESİ

SİMGELER VE KISALTMALAR

Bu çalışmada kullanılmış simgeler ve kısaltmalar, açıklamaları ile birlikte aşağıda sunulmuştur.

Simgeler	**Açıklamalar**
α	Alpha
β	Beta
χ	Chi
δ	Delta
η	Eta
γ	Gamma
κ	Kappa
μ	Mu
ν	Nu
ω	Omega
ϕ	Phi
σ	Sigma
$\sum$	Toplam
θ	Theta
ζ	Zeta
ξ	Xi

Kısaltmalar	**Açıklamalar**
DWDM	Dense wavelength division multiplexing (Yoğun dalgaboyu bölmeli çoğullama)
GVD	Group velocity dispersion (Grup hız dağılımı)
NLS	Nonlineer Schrödinger denklemi
PCF	Photonic crystal fibre (Fotonik kristal fiber)
STD	Spatio-temporal dispersion (Uzay-zamansal dağılım)

1. GİRİŞ

Nonlineer Schrödinger denklemi (NLS) fizik, biyoloji ve mühendislik bilimlerinin çeşitli alanlarında hayati bir rol oynar. Bu denklem, akışkanlar dinamiğinde, nonlineer optikte, plazma fiziğinde ve protein kimyası da dahil olmak üzere birçok uygulamalı alanda ortaya çıkar. Bu çalışmada,

$$iq_t + \frac{1}{2}q_{xx} + F\left(|q|^2\right)q = 0 \tag{1.1}$$

ile verilen genelleştirilmiş nonlineer Schrödinger denklemi (GNLS) olarak bilinen NLS denkleminin önemli bir genelleştirmesi incelenecektir (Akhmediev, 1997,1998; Akhmediev ve ark., 1999). Eşitlik 1.1.'de F bir reel değerli cebirsel fonksiyondur ve $F\left(|q|^2\right)q \,:\, C \longmapsto C$ kompleks fonksiyonunun düzgünlüğüne gereksinim duyulur. C kompleks düzlemini R^2 iki boyutlu lineer uzayı olarak göz önüne alınırsa $F\left(|q|^2\right)q$ fonksiyonunun k kez sürekli olarak türevlenebilir olduğu söylenebilir ve böylece

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m)\,;R^2\right) \tag{1.2}$$

yazılabilir. Eşitlik 1.1. ile verilen modelde q bağımlı değişkendir, x ve t bağımsız değişkenlerdir ve alt indisler ilgili değişkene göre q'nun kısmi türevlerini temsil eder. Buna göre q_t, $\partial q/\partial t$ anlamına gelirken q_{xx}, $\partial^2 q/\partial x^2$ anlamına gelir. Eşitlik 1.1.'deki ilk terim zaman gelişimi terimini temsil ederken ikinci terim grup hız dağılımı içindir ve üçüncü terim nonlineerliğe sebep olmaktadır. Böylece bu tipteki denklemler bazen nonlineer gelişim denklemleri olarak bilinir. Bu, genel olarak integrallenemeyen bir nonlineer kısmi diferansiyel denklemdir. İntegrallenememe durumunun Eşitlik 1.1.'deki nonlineer terim ile ilgili olması gerekmez. Örneğin, yüksek mertebeden dağılımda, hâlâ Hamilton (Hamiltonian) olarak kalsa dahi sistemi integrallenemez yapabilir. Eşitlik 1.1.'in zayıf nonlineer ve dağıtıcı ortamda bir dalga paketinin gelişimini yönettiği ve böylece dalgalar, plazma fiziği ve nonlineer optik gibi değişik alanlarda ortaya çıktığı gösterilmiştir. Bu denklemin diğer bir uygulaması da bazı denge dışı model oluşturma sistemlerini modellemek için kullanılmakta olduğu model oluşumudur. Özel olarak, bu denklem artık optik alanında, nonlineer fiberlerde optik darbe yayılması için iyi bir model olarak yaygın olarak kullanılmaktadır. Eşitlik 1.1.'in nonlineerliğin sonraki bölümlerde ele alınacak çeşitli türleri için solitonları veya soliton çözümleri desteklediği bilinmektedir. Soliton terimi, niteliklerini değiştirmeksizin yayılan ve kimliklerini muhafaza eden diğer solitonlarla karşılıklı çarpışmalara karşı kararlı olan bir nonlineer dalgayı belirtir. Solitonlar matematiksel fiziğin çeşitli alanlarında kapsamlıca çalışılmıştır. Optik fiberler bağlamında solitonlar yalnızca temel bir ilgi alanı olmakla kalmaz, aynı zamanda optik fiber iletişim alanında potansiyel uygulamalara sahiptir.

Optik fiberler, fotonik kristal fiber (PCF) ve metamalzemeler yoluyla soliton yayılımının dinamikleri, son birkaç on yıldır doğrusal olmayan optik alanında devam eden önemli bir araştırma alanıdır. Çeşitli dergilerde yeni sonuçların ifade edildiği ve ortaya atıldığı önemli çalışmalar bulunmaktadır (Biswas, 2004; Biswas ve Konar, 2006; Liu, 2008; Vega-Guzman ve ark., 2014; Savescu ve ark., 2014; Arnous ve ark., 2015,2016,2017a,b; Mirzazadeh ve ark., 2015; Ekici ve ark., 2016; Zhou ve ark., 2016; El-Borai ve ark., 2017; Izdebskaya ve ark., 2017; Biswas ve ark., 2018). Bunlardan bazıları, çift kırılmalı lifler, gömülü solitonlar, Thirring solitonları, yoğun dalga boyu bölmeli çoğullama (DWDM) sistemleri, kademeli sistemler, nematiconlar ve diğer birkaçından elde edilen sonuçlar şeklinde sıralanabilirler (Jisha ve ark., 2014; Kwasny ve ark., 2014; Azmi ve Marchant, 2014; Panayotaros ve Rivero, 2014; Sciberras ve ark., 2014; Alberucci ve ark., 2014; Sala ve ark., 2014; Mirzazadeh ve ark., 2015; Smyth ve ark., 2016; Smyth ve Tope, 2016; Martinez-Galicia ve Panayotaros, 2016; Pal ve ark., 2016; Latiff ve ark., 2016; Nath ve Roy, 2016; Zhang ve ark., 2016; Biyoghe ve ark., 2017; Christian ve Lundie, 2017; Li ve Zhang, 2017; Ekici ve ark., 2017; Huang ve ark., 2017).

İnce lal (garnet) filmlerden yapılan optik dalga kılavuzları, ilk kez örneklerine rastlanan 1972 yılından beri ilgi odağıdır. Ek olarak garnet filmler yüksek GHz frekans aralığına iyi işledikleri için, çoğunlukla manyeto-statik dalgaların varlığının akusto-optik uygulamalara göre kayda değer avantajlara sahip olan cihazları gerektirdiği öne sürülmektedir. Bu önemli bir mikrodalga frekans aralığıdır ve garnet filmleri manyetik ayarlanabilirlikten kaynaklı ilave bir esnekliği sunar. Gerçekten de, filtreler, ilişkilendiriciler, spektrum analizör anahtarları, modülatörler, frekans değiştiriciler ve ayarlanabilir filtreler gibi mikrodalga sinyal işleme cihazının her çeşidi ya kullanımdadır ya da kullanımı ufukta görünmektedir. Bu tür cihazlara, diğer bir esneklik derecesi olarak güç de eklenebilirse çok daha kullanışlı olacaklardır. Başka bir ifadeyle, nonlineer manyeto-optik etkileşimler üzerindeki çalışmalar birincil derecede önemlidir. Kanal dalga kılavuzu olan temel entegre optik yapı bloğunu yapmak artık kolaydır. Ayrıca, lazer dedektörleri ve amplifikatörler gibi aktif cihazlar içeren yarı iletken substratlar ile manyeto-optik cihazların gelecekteki entegrasyonunu öngörmek de mümkündür. Bu durumda, nonlineerlik ve manyeto-optiği bir araya getirilmenin peşine düşmek için bundan daha uygun bir zamanın hiç gelmediği ortaya çıkacaktır. Bu, manyeto-optik son zamanlarda entegre optiğin üvey çocuğu olarak tanımlanmaya başlanmış olsa bile doğrudur. Bu izlenimlerin bir kısmı, materyallerin büyüleyici karmaşıklığını ele alıp kontrol etmek yerine, manyeto-optiği bilinen tasarımlara yerleştirme isteğinden kaynaklanmaktadır (Moloney, 1998).

Bu çalışmada, yapılan çalışmalardan farklı olarak yeni optik solitonlar ele alınacaktır. Manyetik alanın etkisi dahil edilecek ve böyle bir manyetik alan varlığında soliton pertürbasyonunun dinamikleri elde edilecektir. Bu çalışmada incelenecek olan on tür nonlineer ortam vardır. Bunlar, Kerr yasası, kuvvet yasası, parabolik yasası, çift-kuvvet yasası, logaritma yasası, kuadratik (ikinci derece)-kübik yasası, anti-kübik yasası, kübik-kuintik-septik yasası, üçlü kuvvet yasası ve zayıf yerel-olmayan nonlineerliğe sahip parabolik yasasıdır. Modele soliton çözümleri elde etmek için uygulanacak olan integrasyon şeması genişletilmiş $(G'/G)-$açılım yöntemidir (Guo ve Zhou, 2010; Hayek, 2010; Zhou ve ark., 2016). Bu integrasyon algoritması parlak (bright) solitonlar, koyu (dark) solitonlar ve tekil(singular) solitonlar verir. Bu solitonların varlığı için kısıtlama koşulları da sunulmaktadır.

2. MATEMATİKSEL MODEL

Manyeto-optik alan varlığında optik fiberler boyunca soliton yayılımının dinamiklerini tanımlayan matematiksel model, aşağıdaki çiftlenmiş (coupled) NLS denklem sistemi tarafından verilmektedir (Biswas, 2004; Biswas ve Konar, 2006; Vega-Guzman ve ark., 2014; Savescu ve ark., 2014; Biswas ve ark., 2018; Zayed ve ark., 2020):

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 F\left(|q|^2\right) + \eta_1 F\left(|r|^2\right)\right\} q = Q_1 r + i\left\{\alpha_1 q_x + \beta_1 \left(F\left(|q|^2\right) q\right)_x \right. \\ \left. + \nu_1 \left(F\left(|q|^2\right)\right)_x q + \theta_1 F\left(|q|^2\right) q_x\right\} \tag{2.1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 F\left(|r|^2\right) + \eta_2 F\left(|q|^2\right)\right\} r = Q_2 q + i\left\{\alpha_2 r_x + \beta_2 \left(F\left(|r|^2\right) r\right)_x \right. \\ \left. + \nu_2 \left(F\left(|r|^2\right)\right)_x r + \theta_2 F\left(|r|^2\right) r_x\right\}. \tag{2.2}$$

Eşitlik 2.1. ve eşitlik 2.2. ifadelerinde, $j = 1, 2$ için a_j, grup hız dağılımı (GVD) katsayılarını temsil ederken, b_j, uzay-zamansal dağılım (STD) katsayılarını temsil eder. F fonksiyoneli, dikkate alınacak nonlineerlik türüdür. Sağ tarafta, Q_j, soliton karmaşasının oluşumunu önleyen manyetik alan etkisini temsil eder. Pertürbasyon terimlerinden α_j, intermodal dağılımın katsayılarıdır. Ayrıca β_j, şok dalgası oluşumunu önlemek için kendi kendine dikleşen terimlerin katsayılarını temsil eder, ν_j doğrusal olmayan dağılımın katsayılarıdır, θ_j ise doğrusal olmayan dağılım verir. Sağ tarafta, hepsi güçlü tedirginlik terimleri olarak değerlendirilir. Bu çalışmada, soliton çözümlerini elde etmek için eşitlik 2.1. ve eşitlik 2.2.'de verilen modelin integrasyonu gerçekleştirilecektir. Nonlineerlik türünün bilinmesi şartıyla bu mümkün olacaktır. Ele alınacak olan F fonksiyonelinin on formu vardır. Bu nonlineerlik tiplerinin her biri aşağıdaki bölümlerde detaylı bir şekilde incelenecektir.

3. GENİŞLETİLMİŞ $(G'/G)-$AÇILIM YÖNTEMİ

Lineer olmayan kısmi diferansiyel denklemin kapalı formu

$$P(u_l, u_{lt}, u_{lx}, u_{lxx}, u_{ltt}, u_{ltx}, ...) = 0 \tag{3.1}$$

şeklinde ifade edilir. Burada $u_l = u_l(x,t)$ bilinmeyen bir fonksiyon ve P, $u_l = u_l(x,t)$'de ve onun kısmi türevlerinde, en yüksek mertebeden türevlerin ve lineer olmayan terimlerin dahil olduğu bir polinomdur. Genişletilmiş $(G'/G)-$açılım yönteminin ana basamakları aşağıdaki gibi sunulur (Guo ve Zhou, 2010; Hayek, 2010; Zhou ve ark., 2016):

1. Adım : Öncelikle

$$u_l(x,t) = U_l(\zeta), \quad \zeta = x - vt \tag{3.2}$$

ile verilen dalga dönüşümünü yapalım. Burada v daha sonra belirlenecek olan bir sabittir. Bu dönüşüm yardımıyla, eşitlik 3.1.

$$O(U_l, U_l', U_l'', ...) = 0 \tag{3.3}$$

nonlineer adi diferansiyel denkleme indirgenir.

2. Adım : Eşitlik 3.3. ile verilen denklemin

$$U_l(\zeta) = \alpha_0^{(l)} + \sum_{i=1}^{M} \left\{ \alpha_i^{(l)} \left(\frac{G'}{G}\right)^i + \beta_i^{(l)} \left(\frac{G'}{G}\right)^{i-1} \sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} \right.$$
$$\left. +\gamma_i^{(l)} \left(\frac{G'}{G}\right)^{-i} + \delta_i^{(l)} \frac{\left(\frac{G'}{G}\right)^{-i+1}}{\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}} \right\} \tag{3.4}$$

formunda çözümü olduğu kabul edelim. Burada $\alpha_0^{(l)}$, $\alpha_i^{(l)}$, $\beta_i^{(l)}$, $\gamma_i^{(l)}$, $\delta_i^{(l)}$ $(i = 1, \ldots, M)$ daha sonra belirlenecek olan sabitler, $\sigma = \pm 1$ ve M pozitif bir tamsayıdır. Ayrıca, $G = G(\zeta)$ fonksiyonu

$$G'' + \mu G = 0 \tag{3.5}$$

ikinci mertebeden lineer adi diferansiyel denklemini sağlar. Burada μ daha sonra belirlenecek olan sabittir.

3. Adım : Eşitlik 3.1. veya eşitlik 3.3.'teki en yüksek mertebeden türevler ile lineer olmayan terimler balans (dengeleme) edilerek M pozitif tam sayısı belirlenir.

4. Adım : Eşitlik 3.4. ve eşitlik 3.5., eşitlik 3.3.'te yerine yazılarak ve $\left(\frac{G'}{G}\right)^j$ ve $\left(\frac{G'}{G}\right)^j \sqrt{\sigma\left\{1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right\}}$ terimlerinin aynı dereceli tüm terimleri bir araya getirilerek, bu taktirde eşitlik 3.3.'ün sol tarafı $\left(\frac{G'}{G}\right)^j$ ve $\left(\frac{G'}{G}\right)^j \sqrt{\sigma\left\{1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right\}}$ terimlerinde bir polinoma dönüşür. Bu polinomun tüm katsayıları yok edildiğinde, v, μ, $\alpha_0^{(l)}$, $\alpha_0^{(l)}$, $\alpha_i^{(l)}$, $\beta_i^{(l)}$, $\gamma_i^{(l)}$, $\delta_i^{(l)}$ $(i=1,\ldots,M)$ hakkında bir cebirsel denklem elde edilir.

5. Adım : v, μ, $\alpha_0^{(l)}$, $\alpha_0^{(l)}$, $\alpha_i^{(l)}$, $\beta_i^{(l)}$, $\gamma_i^{(l)}$, $\delta_i^{(l)}$ $(i=1,\ldots,M)$ sabitlerinin bir önceki adımda elde edilen cebirsel denklem sisteminin çözülerek elde edilebildiğini kabul edelim. Böylece, bu sabitler ve eşitlik 3.5. ifadesinin bilinen genel çözümleri eşitlik 3.4.'te yerine yazılarak eşitlik 3.1.'in çözümlerine ulaşılabilir.

Açıklama: Eşitlik 3.5.'teki μ parametresi çözüm tiplerinin belirlenmesinde önemli bir rol oynamaktadır.

$\mu < 0$ iken, hiperbolik-tip çözümler elde edilir:

$$\frac{G'}{G} = \sqrt{-\mu}\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right] \tag{3.6}$$

burada $A_1 \neq A_2$ olmak üzere A_1 ve A_2 keyfi sabitlerdir.

$\mu > 0$ için, trigonometrik-tip çözümler bulunur:

$$\frac{G'}{G} = \sqrt{\mu}\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right] \tag{3.7}$$

burada A_1 ve A_2 keyfi sabitlerdir.

$\mu = 0$ ise, rasyonel-tip çözümler elde edilir:

$$\frac{G'}{G} = \frac{A_1}{A_1\zeta + A_2} \tag{3.8}$$

burada A_1 ve A_2 keyfi sabitlerdir.

4. ÇİFTLENMİŞ NLS DENKLEM SİSTEMİNİN GENİŞLETİLMİŞ $(G'/G)-$AÇILIM YÖNTEMİ İLE ÇÖZÜMÜ

Bu bölümde, çeşitli nonlineer formlarla ele alınacak olan çiftlenmiş NLS denklem sistemine genişletilmiş $(G'/G)-$açılım algoritması uygulanacaktır. Nonlineer formların her biri için detaylar sonraki alt bölümlerde numaralandırılır.

4.1. Kerr Yasası

Buna kübik nonlineerlik de denir ve bilinen en basit nonlineerlik kuralı şeklidir. Günümüzde mevcut ticari optik fiberlerin çoğu bu Kerr nonlineerlik kuralına uymaktadır. Kerr yasası nonlineerliği için $F(s) = s$ (Biswas ve ark., 2012). Bu yüzden, Kerr yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 |q|^2 + \eta_1 |r|^2\right\} q = Q_1 r + i\left\{\alpha_1 q_x + \beta_1 \left(|q|^2 q\right)_x \right.$$
$$\left. +\nu_1 \left(|q|^2\right)_x q + \theta_1 |q|^2 q_x\right\} \tag{4.1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 |r|^2 + \eta_2 |q|^2\right\} r = Q_2 q + i\left\{\alpha_2 r_x + \beta_2 \left(|r|^2 r\right)_x \right.$$
$$\left. + \nu_2 \left(|r|^2\right)_x r + \theta_2 |r|^2 r_x\right\} \tag{4.2}$$

ile verilir. Eşitlik 4.1. ve eşitlik 4.2. ile verilen modeli integre etmek için

$$q(x,t) = P_1(\zeta) e^{i\phi(x,t)} \tag{4.3}$$

$$r(x,t) = P_2(\zeta) e^{i\phi(x,t)} \tag{4.4}$$

formunda bir çözüm yapısı olduğu kabul edilir. Burada ζ dalga değişkeni

$$\zeta = x - vt \tag{4.5}$$

ile verilir. Ayrıca, $l = 1, 2$ ve $\bar{l} = 3 - l$ için $P_l(\zeta)$ solitonun genlik kısmı ve v solitonun hızı iken solitonun faz faktörü

$$\phi\left(x,t\right) = -\kappa x + \omega t + \theta \tag{4.6}$$

şeklinde tanımlanır. Burada κ solitonun frekansı iken, ω dalga sayısı ve θ faz sabitidir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.1. ve eşitlik 4.2.'de yerine yazılarak ve elde edilen ifade reel ve imajiner kısımlarına ayırılarak aşağıdaki gibi bir ilişki çifti bulunur:

Reel kısım:

$$(a_l - b_l v)P_l'' + \left(b_l\omega\kappa - \omega - a_l\kappa^2 - \alpha_l\kappa\right) P_l - \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) P_l^3 + \eta_l P_l P_{\bar{l}}^2 = Q_l P_{\bar{l}}. \tag{4.7}$$

İmajiner kısım:

$$-v(1 - b_l\kappa)P_l' + \left(b_l\omega - 2a_l\kappa - \alpha_l\right) P_l' = \left(3\beta_l + 2\nu_l + \theta_l\right) P_l^2 P_l'. \tag{4.8}$$

Şimdi

$$3\beta_l + 2\nu_l + \theta_l = 0 \tag{4.9}$$

kısıtlama şartı geçerli kaldığı sürece, eşitlik 4.8.'den solitonun hızını

$$v = \frac{b_l\omega - 2a_l\kappa - \alpha_l}{1 - b_l\kappa}, \quad b_l\kappa \neq 1 \tag{4.10}$$

olarak elde etmek mümkündür. Solitonun hızının elde edilen farklı iki değeri birbirine eşitlendiğinde,

$$a_1 = a_2, \quad b_1 = b_2, \quad \alpha_1 = \alpha_2 \tag{4.11}$$

bulunur. Sonuç olarak, eşitlik 4.10. yeniden yazılarak

$$v = \frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa} \tag{4.12}$$

ifadesine ulaşılır. Burada $l = 1, 2$ için $a_l = a$, $b_l = b$ ve $\alpha_l = \alpha$ olduğu kabul edilir. Bu nedenle, pertürbe edilmiş manyeto-optik dalga kılavuzu için Kerr yasası nonlineerliğine sahip çiftlenmiş NLS denklemi

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + \left\{\xi_1|q|^2 + \eta_1|r|^2\right\} q = Q_1 r + i\left\{\alpha q_x + \beta_1\left(|q|^2 q\right)_x\right. \\ &\left.+\nu_1\left(|q|^2\right)_x q + \theta_1|q|^2 q_x\right\} \end{aligned} \tag{4.13}$$

$$\begin{aligned} &ir_t + ar_{xx} + br_{xt} + \left\{\xi_2|r|^2 + \eta_2|q|^2\right\} r = Q_2 q + i\left\{\alpha r_x + \beta_2\left(|r|^2 r\right)_x\right. \\ &\left.+\nu_2\left(|r|^2\right)_x r + \theta_2|r|^2 r_x\right\} \end{aligned} \tag{4.14}$$

şeklinde yeniden yazılır ve sonuç olarak eşitlik 4.7. ile verilen reel kısım

$$(a - bv)P_l'' + \left(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa\right) P_l - \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) P_l^3 + \eta_l P_l P_{\bar{l}}^2 - Q_l P_{\bar{l}} = 0 \tag{4.15}$$

denklemine dönüşür. Dengeleme prensibi kullanılarak

$$P_{\bar{l}} = P_l \tag{4.16}$$

olur ve eşitlik 4.15.

$$(a-bv)P_l'' + \left(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa - Q_l\right)P_l + \left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)P_l^3 = 0 \qquad (4.17)$$

şeklinde yeniden yazılır.

Bu bölümde parlak, koyu ve tekil soliton çözümler elde etmek için genişletilmiş $(G'/G)-$açılım algoritması detaylı bir şekilde incelenecektir. Homojen dengeleme prensibine göre, eşitlik 4.17. ifadesinde en yüksek mertebeden türevler ile lineer olmayan terimlerin yani P_l'' ile P_l^3 terimlerinin dengelenmesi ile $M = 1$ bulunur. Böylece eşitlik 4.17.'nin

$$\begin{aligned} P_l(\zeta) &= \alpha_0^{(l)} + \alpha_1^{(l)}\left(\frac{G'}{G}\right) + \beta_1^{(l)}\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + \gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1} \\ &\quad + \delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}} \end{aligned} \qquad (4.18)$$

formunda çözümü vardır. Eşitlik 3.5. ve eşitlik 4.18. ile verilen formal çözüm, eşitlik 4.17.'de yerine yazılarak ve sonra $\left(\frac{G'}{G}\right)^j$ ve $\left(\frac{G'}{G}\right)^j\sqrt{\sigma\left\{1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right\}}$ terimlerinin katsayıları sıfıra eşitlenerek bir cebirsel denklem sistemi elde edilir. Bu denklem sistemi çözülerek aşağıdaki çözüm setleri bulunur:

1. Set:

$$\alpha_0^{(l)} = \beta_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_1^{(l)} = \alpha_1^{(l)},$$

$$\omega = \frac{2\kappa(\alpha + bv\kappa) + 2Q_l + \left(\alpha_1^{(l)}\right)^2\left(\kappa^2 - 2\mu\right)\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2(b\kappa - 1)}, \qquad (4.19)$$

$$a = \frac{2bv + \left(\alpha_1^{(l)}\right)^2\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2}.$$

2. Set:

$$\alpha_0^{(l)} = \alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \beta_1^{(l)} = \beta_1^{(l)},$$

$$\omega = \frac{2\kappa\mu(\alpha + bv\kappa) + 2\mu Q_l + \sigma\left(\beta_1^{(l)}\right)^2(\kappa^2 + \mu)\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2\mu(b\kappa - 1)}, \tag{4.20}$$

$$a = \frac{2bv\mu + \sigma\left(\beta_1^{(l)}\right)^2\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2\mu}.$$

3. Set:

$$\alpha_0^{(l)} = \alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0, \quad \gamma_1^{(l)} = \gamma_1^{(l)},$$

$$\omega = \frac{2\kappa\mu^2(\alpha + bv\kappa) + 2\mu^2 Q_l + \left(\gamma_1^{(l)}\right)^2(\kappa^2 - 2\mu)\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2\mu^2(b\kappa - 1)}, \tag{4.21}$$

$$a = \frac{2bv\mu^2 + \left(\gamma_1^{(l)}\right)^2\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2\mu^2}.$$

4. Set:

$$\alpha_0^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_1^{(l)} = \pm\frac{\beta_1^{(l)}\sqrt{\sigma}}{\sqrt{\mu}}, \quad \beta_1^{(l)} = \beta_1^{(l)},$$

$$\omega = \frac{\kappa\mu(\alpha + bv\kappa) + \mu Q_l + \sigma\left(\beta_1^{(l)}\right)^2(2\kappa^2 - \mu)\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{\mu(b\kappa - 1)}, \tag{4.22}$$

$$a = \frac{bv\mu - 2\sigma\left(\beta_1^{(l)}\right)^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)}{\mu}.$$

5. Set:

$$\alpha_0^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_1^{(l)} = \alpha_1^{(l)}, \quad \gamma_1^{(l)} = \alpha_1^{(l)}\mu$$

$$\omega = \frac{2\kappa(\alpha + bv\kappa) + 2Q_l + \left(\alpha_1^{(l)}\right)^2(\kappa^2 + 4\mu)\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2(b\kappa - 1)}, \tag{4.23}$$

$$a = \frac{2bv + \left(\alpha_1^{(l)}\right)^2\left(\kappa\left(\beta_l + \theta_l\right) - \eta_l - \xi_l\right)}{2}.$$

Burada κ ve μ keyfi sabitlerdir. Sonuç olarak, eşitlik 4.19. - eşitlik 4.23. ile verilen çözüm setleri ve eşitlik 3.5.'in genel çözümleri eşitlik 4.18.'de yerine yazılarak ve çıkan sonuç eşitlik 4.3. ve eşitlik 4.4. dalga dönüşümünde yerine konularak Kerr yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki tam çözümler türetilir:

$\mu < 0$ iken, hiperbolik hareketli dalga çözümler aşağıdaki gibi elde edilir:

$$q(x,t) = \alpha_1^{(1)}\sqrt{-\mu}\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.24}$$

$$r(x,t) = \alpha_1^{(2)}\sqrt{-\mu}\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.25}$$

$$\begin{aligned} q(x,t) =&\beta_1^{(1)}\sqrt{\sigma\left(1 - \left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2\right)} \\ &\times\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.26}$$

$$\begin{aligned} r(x,t) =&\beta_1^{(2)}\sqrt{\sigma\left(1 - \left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2\right)} \\ &\times\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.27}$$

$$q(x,t) = \frac{\gamma_1^{(1)}}{\sqrt{-\mu}}\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-1}\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.28}$$

$$r(x,t) = \frac{\gamma_1^{(2)}}{\sqrt{-\mu}}\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-1}\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.29}$$

$$\begin{aligned} q(x,t) =&\beta_1^{(1)}\sqrt{\sigma}\left(\pm i\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right. \\ &\left.+\sqrt{1 - \left[\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2}\right)\exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.30}$$

$$r(x,t) = \beta_1^{(2)}\sqrt{\sigma}\left(\pm i\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} + \sqrt{1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.31}$$

$$q(x,t) = \alpha_1^{(1)}\sqrt{-\mu}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} - \left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.32}$$

$$r(x,t) = \alpha_1^{(2)}\sqrt{-\mu}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} - \left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.33}$$

Burada $\zeta = x - \left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.19. - eşitlik 4.23. çözüm setlerinde verilir.

$\mu > 0$ iken, trigonometrik hareketli dalga çözümler aşağıdaki gibi bulunur:

$$q(x,t) = \alpha_1^{(1)}\sqrt{\mu}\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.34}$$

$$r(x,t) = \alpha_1^{(2)}\sqrt{\mu}\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.35}$$

$$q(x,t) = \beta_1^{(1)}\sqrt{\sigma\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right)}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.36}$$

$$r(x,t) = \beta_1^{(2)}\sqrt{\sigma\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right)}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.37}$$

$$q(x,t) = \frac{\gamma_1^{(1)}}{\sqrt{\mu}}\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-1}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.38}$$

$$r(x,t) = \frac{\gamma_1^{(2)}}{\sqrt{\mu}}\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-1}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.39}$$

$$q(x,t) =\beta_1^{(1)}\sqrt{\sigma}\left(\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.40}$$

$$r(x,t) =\beta_1^{(2)}\sqrt{\sigma}\left(\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.41}$$

$$q(x,t) =\alpha_1^{(1)}\sqrt{\mu}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-1}\right)$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.42}$$

$$r(x,t) =\alpha_1^{(2)}\sqrt{\mu}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-1}\right)$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.43}$$

Burada $\zeta = x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.19. - eşitlik 4.23. çözüm setlerinde verilir.

Son olarak, $\mu = 0$ için, rasyonel fonksiyon çözümler elde edilir. Eşitlik 4.19. ve eşitlik 4.23. çözüm setleri kullanılarak,

$$q(x,t) = \alpha_1^{(1)}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.44}$$

$$r(x,t) = \alpha_1^{(2)}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.45}$$

çözümlerine ulaşılır. Bununla birlikte, eşitlik 4.21. çözüm seti kullanılarak

$$q(x,t) = \gamma_1^{(1)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^{-1}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.46}$$

$$r(x,t) = \gamma_1^{(2)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^{-1}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.47}$$

çözümleri elde edilir. Burada $\zeta = x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.19. ve eşitlik 4.21. çözüm setlerinde verilir.

Elde edilen tüm çözümlerin özel durumları ise aşağıdaki gibi verilir:

$\mu < 0$ ve $A_1^2 > A_2^2$ iken eşitlik 4.24. - eşitlik 4.33. ile verilen çözümlerden sırasıyla, aşağıdaki koyu solitonlar

$$q(x,t) = \alpha_1^{(1)}\sqrt{-\mu}\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.48}$$

$$r(x,t) = \alpha_1^{(2)}\sqrt{-\mu}\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.49}$$

parlak solitonlar

$$q(x,t) = \beta_1^{(1)}\sqrt{\sigma}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu(\alpha+bv\kappa)+2\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(\kappa^2+\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.50}$$

$$r(x,t) = \beta_1^{(2)}\sqrt{\sigma}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu(\alpha+bv\kappa)+2\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(\kappa^2+\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.51}$$

tekil solitonlar

$$q(x,t) = \frac{\gamma_1^{(1)}}{\sqrt{-\mu}}\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu^2(\alpha+bv\kappa)+2\mu^2 Q_l+\left(\gamma_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.52}$$

$$r(x,t) = \frac{\gamma_1^{(2)}}{\sqrt{-\mu}}\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right] \\ \times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu^2(\alpha+bv\kappa)+2\mu^2 Q_l+\left(\gamma_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.53}$$

complexiton çözümler

$$q(x,t) = \beta_1^{(1)}\sqrt{\sigma}\left(\pm i\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\left.+\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa\mu(\alpha+bv\kappa)+\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(2\kappa^2-\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{\mu(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.54)$$

$$r(x,t) = \beta_1^{(2)}\sqrt{\sigma}\left(\pm i\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\left.+\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa\mu(\alpha+bv\kappa)+\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(2\kappa^2-\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{\mu(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.55)$$

ve ikinci tipi tekil solitonlar

$$q(x,t) = -2\alpha_1^{(1)}\sqrt{-\mu}\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2+4\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.56)$$

$$r(x,t) = -2\alpha_1^{(2)}\sqrt{-\mu}\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2+4\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.57)$$

elde edilir. Burada $\zeta_0 = \tanh^{-1}(A_2/A_1)$. Ayrıca, eşitlik 4.24. - eşitlik 4.33. çözümlerinde $A_1 = 0$, $A_2 \neq 0$ yada $A_2 = 0$, $A_1 \neq 0$ kabul edilirse, daha bir çok soliter (solitary) dalga çözüm elde edilebilir ancak bu çözümler kolaylık açısından ihmal edildi.

$\mu > 0$ ise, eşitlik 4.34. - eşitlik 4.43 çözümlerinden sırasıyla, aşağıdaki periyodik dalga çözümleri elde edilir:

$$q(x,t) = -\alpha_1^{(1)}\sqrt{\mu}\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.58}$$

$$r(x,t) = -\alpha_1^{(2)}\sqrt{\mu}\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.59}$$

$$q(x,t) = \beta_1^{(1)}\sqrt{\sigma}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu(\alpha+bv\kappa)+2\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(\kappa^2+\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.60}$$

$$r(x,t) = \beta_1^{(2)}\sqrt{\sigma}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu(\alpha+bv\kappa)+2\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(\kappa^2+\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.61}$$

$$q(x,t) = -\frac{\gamma_1^{(1)}}{\sqrt{\mu}}\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu^2(\alpha+bv\kappa)+2\mu^2 Q_l+\left(\gamma_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.62}$$

$$r(x,t) = -\frac{\gamma_1^{(2)}}{\sqrt{\mu}}\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa\mu^2(\alpha+bv\kappa)+2\mu^2 Q_l+\left(\gamma_1^{(l)}\right)^2\left(\kappa^2-2\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2\mu^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.63}$$

$$
\begin{aligned}
q(x,t) =&\beta_1^{(1)}\sqrt{\sigma}\left(\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right.\\
&\left.\mp\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right)\\
&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa\mu(\alpha+bv\kappa)+\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(2\kappa^2-\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{\mu(b\kappa-1)}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.64}
$$

$$
\begin{aligned}
r(x,t) =&\beta_1^{(2)}\sqrt{\sigma}\left(\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right.\\
&\left.\mp\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right)\\
&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa\mu(\alpha+bv\kappa)+\mu Q_l+\sigma\left(\beta_1^{(l)}\right)^2\left(2\kappa^2-\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{\mu(b\kappa-1)}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.65}
$$

$$
\begin{aligned}
q(x,t) =&-2\alpha_1^{(1)}\sqrt{\mu}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-2\zeta_0\right]\\
&\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2+4\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.66}
$$

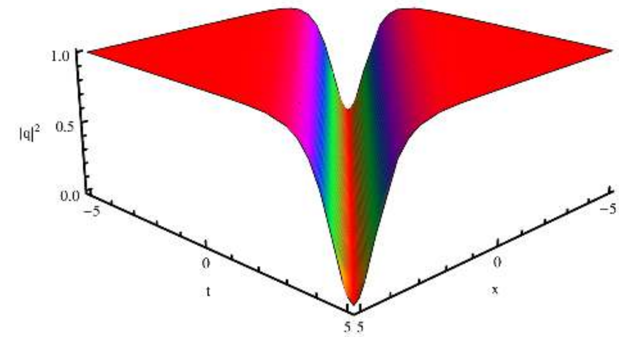
$$
\begin{aligned}
r(x,t) =&-2\alpha_1^{(2)}\sqrt{\mu}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-2\zeta_0\right]\\
&\times\exp\left[i\left(-\kappa x+\left(\frac{2\kappa(\alpha+bv\kappa)+2Q_l+\left(\alpha_1^{(l)}\right)^2\left(\kappa^2+4\mu\right)(\kappa(\beta_l+\theta_l)-\eta_l-\xi_l)}{2(b\kappa-1)}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.67}
$$

burada $\zeta_0 = \tan^{-1}(A_1/A_2)$. Ayrıca, eşitlik 4.24. - eşitlik 4.33. ile verilen çözümlerde $A_1 = 0$, $A_2 \neq 0$ yada $A_2 = 0$, $A_1 \neq 0$ kabul edilirse, daha bir çok periyodik dalga çözüm elde edilebilir ancak bu çözümler kolaylık açısından ihmal edildi.

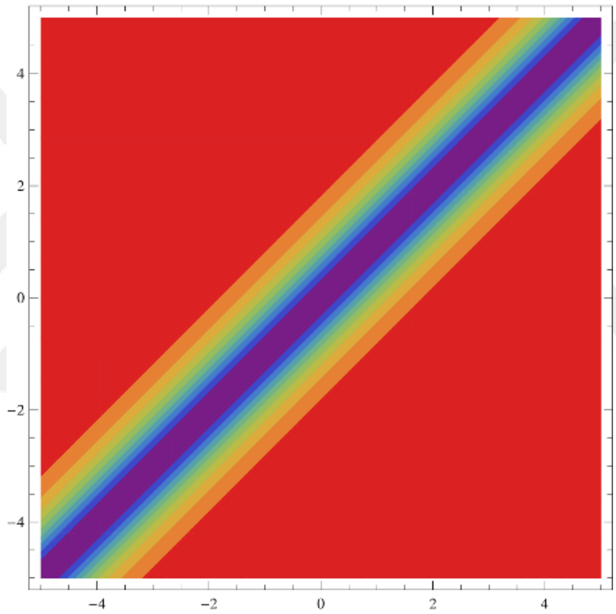
Aşağıdaki grafikler, parlak ve koyu solitonların profillerini temsil eden birkaç nümerik simülasyondur. Seçilen parametre değerleri de sırasıyla gösterilir.

İlk iki grafik, koyu solitonların profilleridir. Seçilen parametre değerleri: $a = 0.25,\ b = 1.5,\ \alpha = 0.15,\ \zeta_0 = 0,\ \kappa = 0.1,\ \mu = -1,\ \omega = 0.7,\ \alpha_1^{(1)} = 1,\ \alpha_1^{(2)} = 2.$

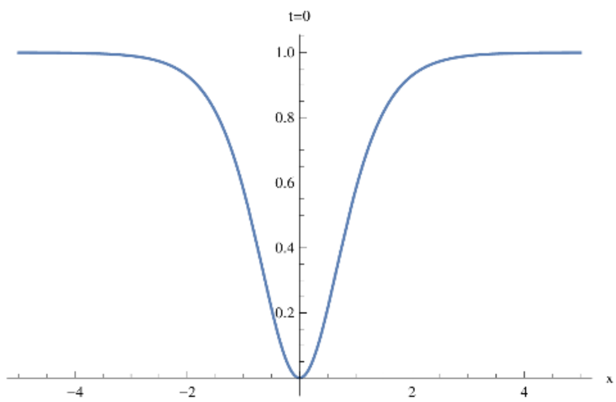
Son iki grafik ise parlak solitonların profilleridir. Bu durumda, alınan parametre değerleri: $a = 0.25,\ b = 1.5,\ \alpha = 0.15,\ \zeta_0 = 0,\ \kappa = 0.1,\ \mu = -1,\ \omega = 0.7,\ \beta_1^{(1)} = 1,\ \beta_1^{(2)} = 2.$



(a) 3 boyutlu grafik

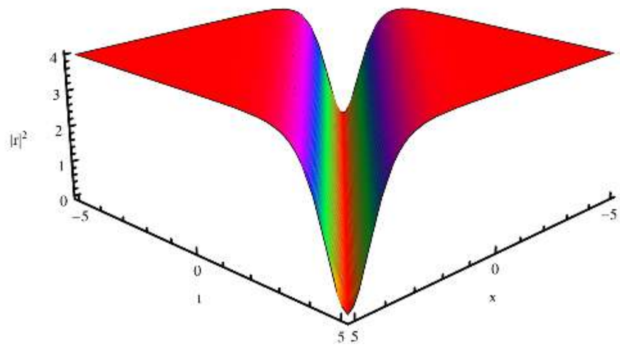


(b) Kontur grafiği

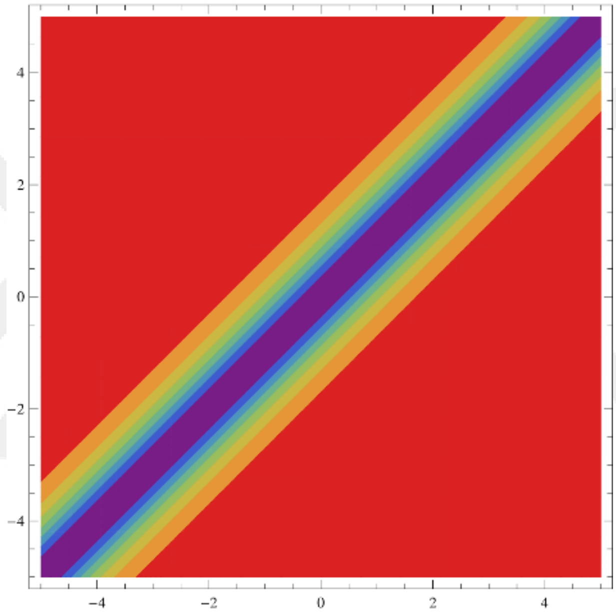


(c) 2 boyutlu grafik

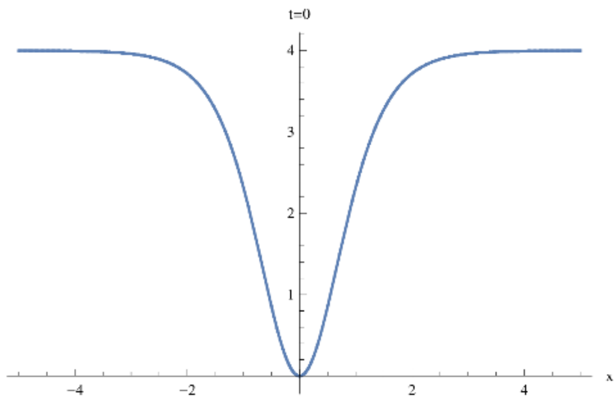
Şekil 1: Eşitlik 4.48. ile verilen koyu solitonun nümerik simülasyonu.



(a) 3 boyutlu grafik

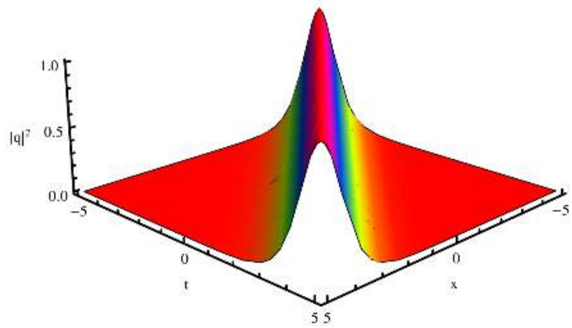


(b) Kontur grafiği

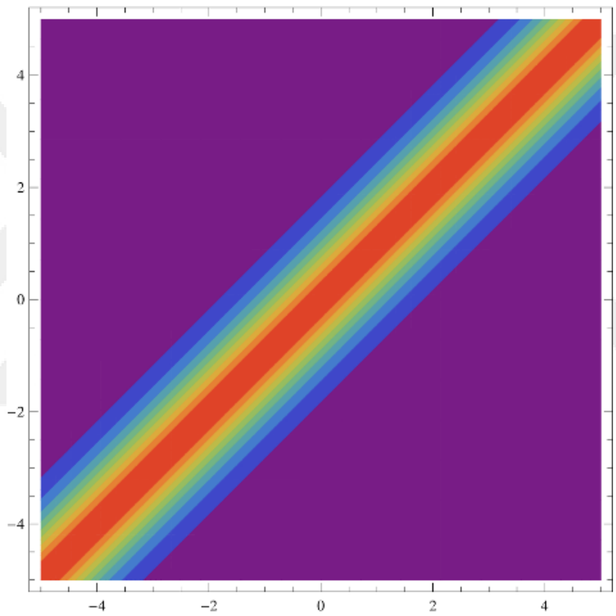


(c) 2 boyutlu grafik

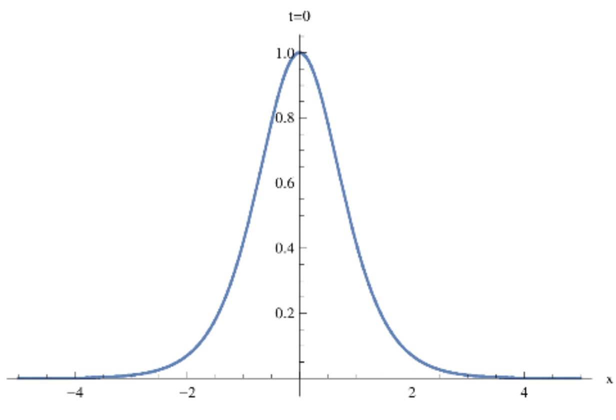
Şekil 2: Eşitlik 4.49. ile verilen koyu solitonun nümerik simülasyonu.



(a) 3 boyutlu grafik

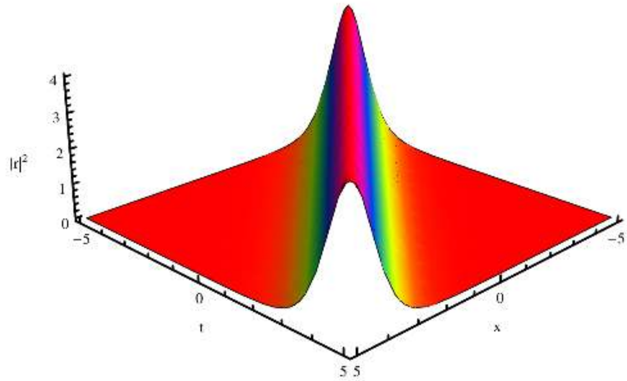


(b) Kontur grafiği

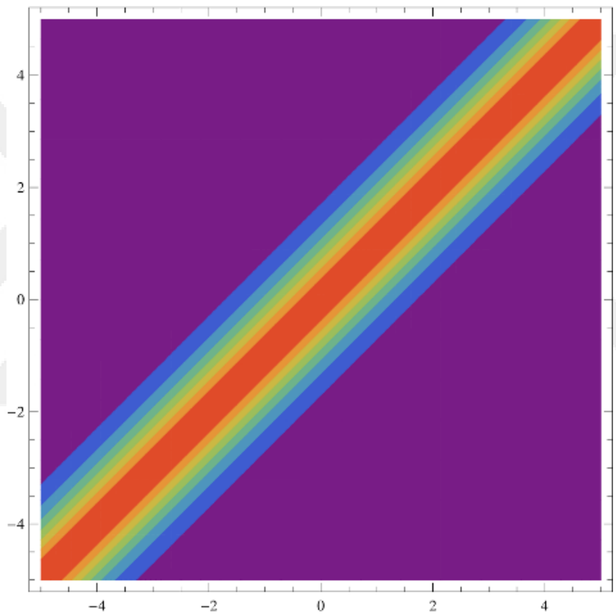


(c) 2 boyutlu grafik

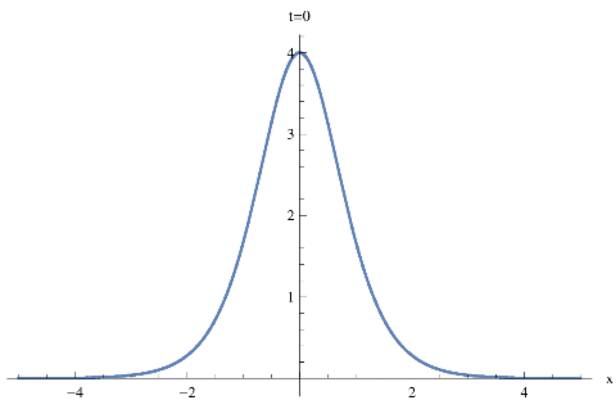
Şekil 3: Eşitlik 4.50. ile verilen parlak solitonun nümerik simülasyonu.



(a) 3 boyutlu grafik



(b) Kontur grafiği



(c) 2 boyutlu grafik

Şekil 4: Eşitlik 4.51. ile verilen parlak solitonun nümerik simülasyonu.

4.2. Kuvvet Yasası

Bu durum altında, dalga çöküşünden kaçınmak için $0 < n < 2$ olması gereklidir. Burada n parametresi, kuvvet yasası nonlineerliği parametresini temsil eder. Gerçekten, kendiliğinden-odaklanma tekilliğini önlemek için $n \neq 2$ olması zorunludur. Bu nonlineerlik kuralı nonlineer plazmalarda ortaya çıkar ve zayıf türbülans teorisinde küçük K-yoğunlaşması problemini çözer. Buna ek olarak, nonlineer optik bağlamında da ortaya çıkar. Fiziksel olarak, yarıkondüktörleri de içeren çeşitli materyaller kuvvet kuralı nonlineerlikleri sergilerler. Nonlineerliğin bu durumu çok ölçekli analiz ile pertürbasyon terimini de içerecek şekilde çalışılmaktadır. $n = \frac{1}{2}$ olma durumu soliton türbülansı bağlamında çalışılır. Kuvvet yasası nonlineerliği için, $F(s) = s^n$ (Biswas ve ark., 2012). Bu yüzden, kuvvet yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 |q|^{2n} + \eta_1 |r|^{2n}\right\} q = Q_1 r + i\left\{\alpha_1 q_x + \beta_1 \left(|q|^{2n} q\right)_x \right.$$
$$\left. + \nu_1 \left(|q|^{2n}\right)_x q + \theta_1 |q|^{2n} q_x\right\} \tag{4.68}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 |r|^{2n} + \eta_2 |q|^{2n}\right\} r = Q_2 q + i\left\{\alpha_2 r_x + \beta_2 \left(|r|^{2n} r\right)_x \right.$$
$$\left. + \nu_2 \left(|r|^{2n}\right)_x r + \theta_2 |r|^{2n} r_x\right\} \tag{4.69}$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.68. ve eşitlik 4.69.'da yerine yazıldıktan sonra, elde edilen reel ve imajiner kısımlar aşağıdaki gibidir:

Reel kısım:

$$(a_l - b_l v) P_l'' + \left(b_l \omega \kappa - \omega - a_l \kappa^2 - \alpha_l \kappa\right) P_l + \left(\xi_l P_l^{2n} + \eta_l P_{\bar{l}}^{2n}\right) P_l$$
$$= Q_l P_{\bar{l}} + \kappa \left(\beta_l + \theta_l\right) P_l^{2n+1}. \tag{4.70}$$

İmajiner kısım:

$$v(b_l \kappa - 1) P_l' + \left(b_l \omega - 2a_l \kappa - \alpha_l\right) P_l' = \left\{(2n+1)\beta_l + 2n\nu_l + \theta_l\right\} P_l^{2n} P_l'. \tag{4.71}$$

Şimdi,

$$(2n+1)\beta_l + 2n\nu_l + \theta_l = 0 \tag{4.72}$$

kısıtlama şartı geçerli olduğu sürece, eşitlik 4.71.'den solitonun hızını, eşitlik 4.10.'da verildiği gibi elde etmek mümkündür. Sonuç olarak, eşitlik 4.11. ve eşitlik 4.12.'de sağlanır ve eşitlik 4.70. ile verilen reel kısım

$$(a-bv)P_l''+\left(b\omega\kappa-\omega-a\kappa^2-\alpha\kappa\right)P_l+\left(\xi_l P_l^{2n}+\eta_l P_{\bar{l}}^{2n}\right)P_l$$
$$=Q_l P_{\bar{l}}+\kappa\left(\beta_l+\theta_l\right)P_l^{2n+1} \tag{4.73}$$

denklemine dönüşür. Eşitlik 4.73.'de balans prensibi kullanılarak

$$P_{\bar{l}}=P_l \tag{4.74}$$

elde edilir ve eşitlik 4.73.

$$(a-bv)P_l''+\left(b\omega\kappa-\omega-a\kappa^2-\alpha\kappa-Q_l\right)P_l+\left(\xi_l+\eta_l-\kappa\left(\beta_l+\theta_l\right)\right)P_l^{2n+1}=0 \tag{4.75}$$

denklemine indirgenir. Kapalı form çözümler türetmek için

$$P_l=U_l^{\frac{1}{2n}} \tag{4.76}$$

dönüşümü eşitlik 4.75.'de uygulanır ve böylece, eşitlik 4.75.

$$(a-bv)\left(2nU_lU_l''-(2n-1)\left(U_l'\right)^2\right)-4n^2\left(\alpha\kappa+a\kappa^2+\omega-b\kappa\omega+Q_l\right)U_l^2$$
$$+4n^2\left(\eta_l+\xi_l-\kappa\left(\beta_l+\theta_l\right)\right)U_l^3=0 \tag{4.77}$$

denklemine dönüşür.

Bu bölümde, genişletilmiş $(G'/G)-$açılım mekanizması, kuvvet yasası nonlineerliğine sahip çiftlenmiş NLS denklemine parlak, koyu ve tekil soliton çözümler araştırmak için ayrıntılı bir şekilde kullanılacaktır. Homojen balans prensibine göre, eşitlik 4.77.'de en yüksek mertebeden türevler ile lineer olmayan terimlerin, yani U_lU_l'' veya $(U_l')^2$ ile U_l^3 terimlerinin dengelenmesi ile $M=2$ bulunur. Böylece eşitlik 4.77'nin çözümü

$$U_l(\zeta)=\alpha_0^{(l)}+\sum_{i=1}^{2}\left\{\alpha_i^{(l)}\left(\frac{G'}{G}\right)^i+\beta_i^{(l)}\left(\frac{G'}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}\right.$$
$$\left.+\gamma_i^{(l)}\left(\frac{G'}{G}\right)^{-i}+\delta_i^{(l)}\frac{\left(\frac{G'}{G}\right)^{-i+1}}{\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}\right\} \tag{4.78}$$

formunda yapılanır. Eşitlik 3.5. ve eşitlik 4.78., eşitlik 4.77.'de yerine yazılarak ve sonra $\left(\frac{G'}{G}\right)^j$ ve $\left(\frac{G'}{G}\right)^j\sqrt{\sigma\left\{1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right\}}$ terimlerinin katsayıları sıfıra eşitlenerek bir cebirsel denklem sistemi bulunur. Bu denklem sistemi çözülerek aşağıdaki çözüm setleri türetilir.

1. Set:

$$\begin{aligned}
&\alpha_1^{(l)} = \beta_1^{(l)} = \beta_2^{(l)} = \gamma_1^{(l)} = \gamma_2^{(l)} = \delta_1^{(l)} = \delta_2^{(l)} = 0,\\
&\alpha_0^{(l)} = -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\alpha_2^{(l)} = -\frac{(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\omega = \frac{\mu(a-bv) + n^2(a\kappa^2 + \alpha\kappa + Q_l)}{n^2(b\kappa - 1)}.
\end{aligned} \tag{4.79}$$

2. Set:

$$\begin{aligned}
&\alpha_1^{(l)} = \alpha_2^{(l)} = \beta_1^{(l)} = \beta_2^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = \delta_2^{(l)} = 0,\\
&\alpha_0^{(l)} = -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\gamma_2^{(l)} = -\frac{\mu^2(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\omega = \frac{\mu(a-bv) + n^2(a\kappa^2 + \alpha\kappa + Q_l)}{n^2(b\kappa - 1)}.
\end{aligned} \tag{4.80}$$

3. Set:

$$\begin{aligned}
&\alpha_1^{(l)} = \beta_1^{(l)} = \gamma_1^{(l)} = \gamma_2^{(l)} = \delta_1^{(l)} = \delta_2^{(l)} = 0,\\
&\alpha_0^{(l)} = -\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\alpha_2^{(l)} = -\frac{(n+1)(a-bv)}{2n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},\\
&\beta_2^{(l)} = \pm\frac{\sqrt{\mu}(n+1)(a-bv)}{2n^2\sqrt{\sigma\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)^2}},\\
&\omega = \frac{\mu(a-bv) + 4n^2(a\kappa^2 + \alpha\kappa + Q_l)}{4n^2(b\kappa - 1)}.
\end{aligned} \tag{4.81}$$

4. Set:

$$\alpha_1^{(l)} = \beta_1^{(l)} = \beta_2^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = \delta_2^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{2\mu(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},$$

$$\alpha_2^{(l)} = -\frac{(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)}, \tag{4.82}$$

$$\gamma_2^{(l)} = -\frac{\mu^2(n+1)(a-bv)}{n^2\left(\eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\right)},$$

$$\omega = \frac{4\mu(a-bv) + n^2(a\kappa^2 + \alpha\kappa + Q_l)}{n^2(b\kappa - 1)}.$$

Burada κ ve μ keyfi sabitlerdir. Sonuç olarak, eşitlik 4.79. - eşitlik 4.82. ile verilen çözüm setleri ve eşitlik 3.5.'in genel çözümleri eşitlik 4.78.'de yerine yazılarak ve çıkan sonuç eşitlik 4.3. ve eşitlik 4.4. dalga dönüşümünde yerine yazılarak kuvvet yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki tam çözümleri bulunur:

$\mu < 0$ iken, hiperbolik hareketli dalga çözümler aşağıdaki gibi listelenir:

$$q(x,t) = \left\{ -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1 + \xi_1 - \kappa\left(\beta_1 + \theta_1\right)\right)} \right. \\ \left. \times \left(1 - \left[\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right]^2 \right) \right\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.83}$$

$$r(x,t) = \left\{ -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2 + \xi_2 - \kappa\left(\beta_2 + \theta_2\right)\right)} \right. \\ \left. \times \left(1 - \left[\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right]^2 \right) \right\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.84}$$

$$
\begin{aligned}
q(x,t) = \Bigg\{ & -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)} \\
& \times \left(1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-2}\right)\Bigg\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.85}
$$

$$
\begin{aligned}
r(x,t) = \Bigg\{ & -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)} \\
& \times \left(1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-2}\right)\Bigg\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.86}
$$

$$
\begin{aligned}
q(x,t) = \Bigg\{ & -\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)} \Bigg(1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{2} \\
& \pm i\ \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} \\
& \times \sqrt{1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{2}}\Bigg)\Bigg\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.87}
$$

$$
\begin{aligned}
r(x,t) = \Bigg\{ & -\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)} \Bigg(1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{2} \\
& \pm i\ \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} \\
& \times \sqrt{1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{2}}\Bigg)\Bigg\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.88}
$$

$$
\begin{aligned}
q(x,t) = \Bigg\{ & -\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)} \Bigg(2-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{2} \\
& -\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-2}\Bigg)\Bigg\}^{\frac{1}{2n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.89}
$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(2-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2\right.\right.$$
$$\left.\left.-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^{-2}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.90}$$

Burada $\zeta=x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.79. - eşitlik 4.82. çözüm setlerinde verilir.

$\mu>0$ ise, trigonometrik hareketli dalga çözümler aşağıdaki gibi çıkarılır:

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.91}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.92}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.93}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.94}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right.\right.$$
$$\left.\left.\mp\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.95}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right.\right.$$
$$\left.\left.\mp\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.96}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(2+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right.\right.$$
$$\left.\left.+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-2}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.97}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(2+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2\right.\right.$$
$$\left.\left.+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^{-2}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.98}$$

Burada $\zeta=x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.79. - eşitlik 4.82. çözüm setlerinde verilir.

Son olarak, $\mu=0$ için, rasyonel fonksiyon çözümler elde edilir. Eşitlik 4.79. ve eşitlik 4.82. çözüm setleri kullanılarak

$$q(x,t)=\left\{-\frac{(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^2\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.99}$$

$$r(x,t)=\left\{-\frac{(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^2\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.100}$$

çözümlerine ulaşılır. Ayrıca, eşitlik 4.81. çözüm setinin yardımıyla

$$q(x,t)=\left\{-\frac{(n+1)(a-bv)}{2n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^2\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.101}$$

$$r(x,t)=\left\{-\frac{(n+1)(a-bv)}{2n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\left(\frac{A_1}{A_1\zeta+A_2}\right)^2\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.102}$$

çözümleri elde edilir. Burada $\zeta=x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t$ ve ω değerleri her bir çözüm çifti için sırasıyla eşitlik 4.79. ve eşitlik 4.81. çözüm setlerinde verilir.

Bu çözümlerin özel durumları ise aşağıdaki gibi verilir:

$\mu<0$ ve $A_1^2>A_2^2$ iken eşitlik 4.83. - eşitlik 4.90. çözümlerinden sırasıyla, aşağıdaki parlak solitonlar

$$\begin{aligned}q(x,t)=&\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\operatorname{sech}^2\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2n}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.103}$$

$$\begin{aligned}r(x,t)=&\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\operatorname{sech}^2\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2n}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.104}$$

tekil solitonlar

$$\begin{aligned}q(x,t)=&\left\{\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\operatorname{csch}^2\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2n}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.105}$$

$$\begin{aligned}r(x,t)=&\left\{\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\operatorname{csch}^2\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2n}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.106}$$

complexiton çözümler

$$\begin{aligned}q(x,t)=&\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\\&\times\left.\left(\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\pm i\tanh\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+4n^2(a\kappa^2+\alpha\kappa+Q_l)}{4n^2(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.107}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\times\left.\left(\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\pm i\tanh\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+4n^2(a\kappa^2+\alpha\kappa+Q_l)}{4n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.108)$$

ve tekil solitonlar

$$q(x,t)=\left\{\frac{4\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\operatorname{csch}^2\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.109)$$

$$r(x,t)=\left\{\frac{4\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\operatorname{csch}^2\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.110)$$

çıkarılır. Burada $\zeta_0 = \tanh^{-1}(A_2/A_1)$. Ayrıca, eşitlik 4.83. - eşitlik 4.90. ile verilen çözümlerde $A_1 = 0$, $A_2 \neq 0$ yada $A_2 = 0$, $A_1 \neq 0$ kabul edilirse, daha bir çok solitary dalga çözümler elde edilebilir ancak bu çözümler kolaylık için ihmal edildi.

$\mu > 0$ ise, eşitlik 4.91. - eşitlik 4.98.'deki çözümlerden sırasıyla, aşağıdaki periyodik dalga çözümler türetilir:

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\sec^2\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.111)$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\sec^2\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.112)$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\csc^2\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.113)$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\csc^2\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.114)$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\sec\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right.$$
$$\left.\times\left(\sec\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\pm\tan\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+4n^2(a\kappa^2+\alpha\kappa+Q_l)}{4n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.115}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{2n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\sec\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right.$$
$$\left.\times\left(\sec\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\pm\tan\left[\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+4n^2(a\kappa^2+\alpha\kappa+Q_l)}{4n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.116}$$

$$q(x,t)=\left\{-\frac{4\mu(n+1)(a-bv)}{n^2\left(\eta_1+\xi_1-\kappa\left(\beta_1+\theta_1\right)\right)}\csc^2\left[2\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-2\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.117}$$

$$r(x,t)=\left\{-\frac{4\mu(n+1)(a-bv)}{n^2\left(\eta_2+\xi_2-\kappa\left(\beta_2+\theta_2\right)\right)}\csc^2\left[2\sqrt{\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}-2\zeta_0\right]\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+n^2(a\kappa^2+\alpha\kappa+Q_l)}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.118}$$

burada $\zeta_0 = \tan^{-1}(A_1/A_2)$. Ayrıca, eşitlik 4.91. - eşitlik 4.98. ile verilen çözümlerde $A_1 = 0$, $A_2 \neq 0$ yada $A_2 = 0$, $A_1 \neq 0$ kabul edilirse, daha bir çok periyodik dalga çözümler elde edilebilir ancak bu çözümler kolaylık için ihmal edildi.

4.3. Kuadratik-Kübik Yasası

Bu nonlineerlik ilk olarak 2011 yılında ortaya çıktı (Fujioka ve ark., 2011). Genel form, c_1 ve c_2 sabitler olmak üzere $F(s) = c_1\sqrt{s} + c_2 s$. Bu yüzden, kuadratik-kübik yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1(c_1|q| + c_2|q|^2) + \eta_1(c_1|r| + c_2|r|^2)\right\} q = Q_1 r$$
$$+\, i\left\{\alpha_1 q_x + \beta_1\left((c_1|q| + c_2|q|^2)q\right)_x + \nu_1\left(c_1|q| + c_2|q|^2\right)_x q + \theta_1(c_1|q| + c_2|q|^2)q_x\right\} \tag{4.119}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2(c_1|r| + c_2|r|^2) + \eta_2(c_1|q| + c_2|q|^2)\right\} r = Q_2 q$$
$$+\, i\left\{\alpha_2 r_x + \beta_2\left((c_1|r| + c_2|r|^2)r\right)_x + \nu_2\left(c_1|r| + c_2|r|^2\right)_x r + \theta_2(c_1|r| + c_2|r|^2)r_x\right\} \tag{4.120}$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.119. ve eşitlik 4.120. ile verilen modelde yerine yazılarak ve ortaya çıkan denklemler reel ve imajiner kısımlarına ayrılarak aşağıdaki ilişki çifti elde edilir:

Reel kısım:

$$(a_l - b_l v)\, P_l'' + \left(b_l \kappa\omega - a_l \kappa^2 - \alpha_l \kappa - \omega\right) P_l + c_1 \left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^2$$
$$+ c_2 \left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3 + \left(c_1 + c_2 P_{\bar{l}}\right) \eta_l P_l P_{\bar{l}} - Q_l P_{\bar{l}} = 0. \tag{4.121}$$

İmajiner kısım:

$$\left(2a_l\kappa + \alpha_l - b_l(\kappa v + \omega) + v\right) P_l' + c_1 \left(2\beta_l + \theta_l + \nu_l\right) P_l' P_l$$
$$+ c_2 \left(3\beta_l + \theta_l + 2\nu_l\right) P_l' P_l^2 = 0. \tag{4.122}$$

Eşitlik 4.122.’den solitonun hızı eşitlik 4.10.’daki gibi bulunur. Bununla birlikte, ortaya çıkan kısıtlama şartları

$$2\beta_l + \theta_l + \nu_l = 0 \tag{4.123}$$

$$3\beta_l + \theta_l + 2\nu_l = 0 \tag{4.124}$$

şeklindedir. Eşitlik 4.123. ve eşitlik 4.124. ifadelerinden

$$\beta_l + \nu_l = 0 \tag{4.125}$$

$$\beta_l + \theta_l = 0 \tag{4.126}$$

$$\nu_l = \theta_l \tag{4.127}$$

elde edilir. Eşitlik 4.11.’deki sonuçlar kullanılarak, eşitlik 4.121. ile verilen reel kısım

$$(a - bv)\, P_l'' + \left(b\kappa\omega - a\kappa^2 - \alpha\kappa - \omega\right) P_l + c_1 \left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^2$$
$$+ c_2 \left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3 + \left(c_1 + c_2 P_{\bar{l}}\right) \eta_l P_l P_{\bar{l}} - Q_l P_{\bar{l}} = 0 \tag{4.128}$$

denklemine dönüşür. Balans prensibi kullanılarak

$$P_{\bar{l}} = P_l \tag{4.129}$$

eşitliğine ulaşılır ve eşitlik 4.128.

$$(a - bv)\, P_l'' + \left(b\kappa\omega - a\kappa^2 - \alpha\kappa - \omega - Q_l\right) P_l + c_1 \left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^2$$
$$+ c_2 \left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3 = 0 \tag{4.130}$$

denklemine indirgenir. Şimdi, eşitlik 4.130.'deki P_l'' ile P_l^3 terimleri balans edilerek $M = 1$ bulunur. Böylece eşitlik 4.130.'in formal çözümü

$$P_l(\zeta) = \alpha_0^{(l)} + \alpha_1^{(l)} \left(\frac{G'}{G}\right) + \beta_1^{(l)} \sqrt{\sigma \left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + \gamma_1^{(l)} \left(\frac{G'}{G}\right)^{-1}$$
$$+ \delta_1^{(l)} \frac{1}{\sqrt{\sigma \left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}} \tag{4.131}$$

halini alır. Kerr yasası ve kuvvet yasası nonlineerliklerinde olduğu gibi ilerleyerek aşağıdaki çözüm setleri türetilir:

1. Set:

$$\beta_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},$$

$$\alpha_1^{(l)} = \pm\frac{i\alpha_0^{(l)}}{\sqrt{\mu}},$$

$$\omega = \frac{\kappa(a\kappa + \alpha) + 4\mu(a - bv) + Q_l}{b\kappa - 1}, \tag{4.132}$$

$$\xi_l = \frac{2\mu(a - bv) + (\alpha_0^{(l)})^2 c_2 \left(\kappa\left(\beta_l + \theta_l\right) - \eta_l\right)}{(\alpha_0^{(l)})^2 c_2},$$

$$c_1 = -3\alpha_0^{(l)} c_2.$$

2. Set:

$$\alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},$$

$$\beta_1^{(l)} = \pm\frac{\sqrt{2}\alpha_0^{(l)}}{\sqrt{\sigma}},$$

$$\omega = \frac{a\kappa^2 + \alpha\kappa - 2a\mu + 2b\mu v + Q_l}{b\kappa - 1}, \tag{4.133}$$

$$\xi_l = \frac{-a\mu + (\alpha_0^{(l)})^2 c_2 \left(\kappa\left(\beta_l + \theta_l\right) - \eta_l\right) + b\mu v}{(\alpha_0^{(l)})^2 c_2},$$

$$c_1 = -3\alpha_0^{(l)} c_2.$$

3. Set:

$$\alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},$$

$$\gamma_1^{(l)} = \pm i\alpha_0^{(l)}\sqrt{\mu},$$

$$\omega = \frac{\kappa(a\kappa + \alpha) + 4\mu(a - bv) + Q_l}{b\kappa - 1}, \tag{4.134}$$

$$\xi_l = \frac{2\mu(a - bv) + (\alpha_0^{(l)})^2 c_2 \left(\kappa\left(\beta_l + \theta_l\right) - \eta_l\right)}{(\alpha_0^{(l)})^2 c_2},$$

$$c_1 = -3\alpha_0^{(l)} c_2.$$

4. Set:

$$\gamma_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},$$

$$\alpha_1^{(l)} = \pm\frac{i\alpha_0^{(l)}}{\sqrt{\mu}},$$

$$\beta_1^{(l)} = \frac{i\alpha_0^{(l)}}{\sqrt{\sigma}},$$

$$\omega = \frac{\kappa(a\kappa + \alpha) + \mu(a - bv) + Q_l}{b\kappa - 1}, \tag{4.135}$$

$$\xi_l = \frac{\mu(a - bv) + 2(\alpha_0^{(l)})^2 c_2 \left(\kappa\left(\beta_l + \theta_l\right) - \eta_l\right)}{2(\alpha_0^{(l)})^2 c_2},$$

$$c_1 = -3\alpha_0^{(l)} c_2.$$

5. Set:

$$\begin{aligned}
&\beta_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},\\
&\alpha_1^{(l)} = \pm\frac{i\alpha_0^{(l)}}{2\sqrt{\mu}},\\
&\gamma_1^{(l)} = \pm\frac{1}{2}i\alpha_0^{(l)}\sqrt{\mu},\\
&\omega = \frac{\kappa(a\kappa+\alpha)+16\mu(a-bv)+Q_l}{b\kappa-1},\\
&\xi_l = \frac{8\mu(a-bv)+(\alpha_0^{(l)})^2c_2\left(\kappa\left(\beta_l+\theta_l\right)-\eta_l\right)}{(\alpha_0^{(l)})^2c_2},\\
&c_1 = -3\alpha_0^{(l)}c_2.
\end{aligned} \tag{4.136}$$

6. Set:

$$\begin{aligned}
&\beta_1^{(l)} = \delta_1^{(l)} = 0, \quad \alpha_0^{(l)} = \alpha_0^{(l)},\\
&\alpha_1^{(l)} = \pm\frac{\alpha_0^{(l)}}{\sqrt{2}\sqrt{\mu}},\\
&\gamma_1^{(l)} = \pm\frac{\alpha_0^{(l)}\sqrt{\mu}}{\sqrt{2}},\\
&\omega = \frac{a\kappa^2+\alpha\kappa-8a\mu+8b\mu v+Q_l}{b\kappa-1},\\
&\xi_l = \frac{(\alpha_0^{(l)})^2c_2\left(\kappa\left(\beta_l+\theta_l\right)-\eta_l\right)-4\mu(a-bv)}{(\alpha_0^{(l)})^2c_2},\\
&c_1 = -3\alpha_0^{(l)}c_2.
\end{aligned} \tag{4.137}$$

Burada κ ve μ keyfi sabitlerdir. Böylece, elde edilen bu çözüm setleri yardımıyla kuadratik-kübik yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için tam çözümler aşağıdaki gibi listelenir.

$\mu < 0$ iken, aşağıda verilen hiperbolik hareketli dalga çözümler çıkarılır:

$$q(x,t) = \alpha_0^{(1)} \left(1 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right) \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.138}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right) \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.139}$$

$$\begin{aligned} q(x,t) = &\alpha_0^{(1)} \left(1 \pm \sqrt{2 - 2\left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^2}\right) \\ &\times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.140}$$

$$\begin{aligned} r(x,t) = &\alpha_0^{(2)} \left(1 \pm \sqrt{2 - 2\left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^2}\right) \\ &\times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.141}$$

$$\begin{aligned} q(x,t) = &\alpha_0^{(1)} \left(1 \mp \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right) \\ &\times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.142}$$

$$\begin{aligned} r(x,t) = &\alpha_0^{(2)} \left(1 \mp \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right) \\ &\times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.143}$$

$$\begin{aligned} q(x,t) = &\alpha_0^{(1)} \left(1 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right. \\ &\left. + i\sqrt{1 - \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^2}\right) \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \end{aligned} \tag{4.144}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right.$$
$$\left. + i\sqrt{1 - \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right)^2} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.145}$$

$$q(x,t) = \frac{\alpha_0^{(1)}}{2} \left(2 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right.$$
$$\left. \mp \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.146}$$

$$r(x,t) = \frac{\alpha_0^{(2)}}{2} \left(2 \pm \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right.$$
$$\left. \mp \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.147}$$

$$q(x,t) = \alpha_0^{(1)} \frac{\sqrt{2}}{2} \left(\sqrt{2} \mp i \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right.$$
$$\left. \pm i \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.148}$$

$$r(x,t) = \alpha_0^{(2)} \frac{\sqrt{2}}{2} \left(\sqrt{2} \mp i \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right.$$
$$\left. \pm i \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right]. \tag{4.149}$$

$\mu > 0$ ise, trigonometrik hareketli dalga çözümler aşağıdaki gibi bulunur:

$$q(x,t) = \alpha_0^{(1)} \left(1 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.150}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.151}$$

$$q(x,t) = \alpha_0^{(1)} \left(1 \pm \sqrt{2 + 2\left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^2} \right)$$
$$\times \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.152}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm \sqrt{2 + 2\left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^2} \right) \times \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.153}$$

$$q(x,t) = \alpha_0^{(1)} \left(1 \pm i \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.154}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm i \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.155}$$

$$q(x,t) = \alpha_0^{(1)} \left(1 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} + i\sqrt{1 + \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^2} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.156}$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} + i\sqrt{1 + \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^2} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.157}$$

$$q(x,t) = \frac{\alpha_0^{(1)}}{2} \left(2 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \pm i \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.158}$$

$$r(x,t) = \frac{\alpha_0^{(2)}}{2} \left(2 \pm i \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \pm i \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.159}$$

$$q(x,t) = \alpha_0^{(1)} \frac{\sqrt{2}}{2} \left(\sqrt{2} \pm \frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \pm \left(\frac{A_1 \cos(\sqrt{\mu}\zeta) - A_2 \sin(\sqrt{\mu}\zeta)}{A_1 \sin(\sqrt{\mu}\zeta) + A_2 \cos(\sqrt{\mu}\zeta)} \right)^{-1} \right) \exp\left[i\left(-\kappa x + \omega t + \theta \right) \right] \tag{4.160}$$

$$r(x,t) =\alpha_0^{(2)}\frac{\sqrt{2}}{2}\left(\sqrt{2}\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.\pm\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.161}$$

Son olarak, $\mu = 0$ iken rasyonel fonksiyon çözümler elde edilebilir. Ancak, bu nonlineerlik için rasyonel fonksiyon çözümler mevcut değildir.

Özel durumlar aşağıdaki gibi sunulur:

$\mu < 0$ ve $A_1^2 > A_2^2$ iken, eşitlik 4.138. - eşitlik 4.149. ile verilen çözümlerden sırasıyla, aşağıdaki koyu solitonlar

$$q(x,t) =\alpha_0^{(1)}\left(1\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.162}$$

$$r(x,t) =\alpha_0^{(2)}\left(1\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.163}$$

parlak solitonlar

$$q(x,t) =\alpha_0^{(1)}\left(1\pm\sqrt{2}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.164}$$

$$r(x,t) =\alpha_0^{(2)}\left(1\pm\sqrt{2}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.165}$$

tekil solitonlar

$$q(x,t) =\alpha_0^{(1)}\left(1\mp\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.166}$$

$$r(x,t) =\alpha_0^{(2)}\left(1\mp\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.167}$$

complexiton çözümler

$$
\begin{aligned}
q(x,t) =& \alpha_0^{(1)} \left(1 \pm \tanh\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + \zeta_0\right]\right. \\
&\left. + i \operatorname{sech}\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + \zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + \mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.168}
$$

$$
\begin{aligned}
r(x,t) =& \alpha_0^{(2)} \left(1 \pm \tanh\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + \zeta_0\right]\right. \\
&\left. + i \operatorname{sech}\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + \zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + \mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.169}
$$

tekil solitonların diğer tipi

$$
\begin{aligned}
q(x,t) =& \alpha_0^{(1)} \left(1 \mp \operatorname{csch}\left[2\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + 16\mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.170}
$$

$$
\begin{aligned}
r(x,t) =& \alpha_0^{(2)} \left(1 \mp \operatorname{csch}\left[2\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + 16\mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.171}
$$

ve diğer complexiton çözümler

$$
\begin{aligned}
q(x,t) =& \alpha_0^{(1)} \frac{\sqrt{2}}{2} \left(\sqrt{2} \pm 2i \operatorname{csch}\left[2\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{a\kappa^2 + \alpha\kappa - 8a\mu + 8b\mu v + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.172}
$$

$$
\begin{aligned}
r(x,t) =& \alpha_0^{(2)} \frac{\sqrt{2}}{2} \left(\sqrt{2} \pm 2i \operatorname{csch}\left[2\sqrt{-\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right) \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{a\kappa^2 + \alpha\kappa - 8a\mu + 8b\mu v + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]
\end{aligned} \tag{4.173}
$$

elde edilir. Burada $\zeta_0 = \tanh^{-1}(A_2/A_1)$.

$\mu > 0$ için, eşitlik 4.150. - eşitlik 4.161. ile verilen çözümlerden sırasıyla, aşağıdaki periyodik dalga çözümler bulunur:

$$\begin{aligned}q(x,t)=&\alpha_0^{(1)}\left(1\mp i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.174}$$

$$\begin{aligned}r(x,t)=&\alpha_0^{(2)}\left(1\mp i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.175}$$

$$\begin{aligned}q(x,t)=&\alpha_0^{(1)}\left(1\pm\sqrt{2}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.176}$$

$$\begin{aligned}r(x,t)=&\alpha_0^{(2)}\left(1\pm\sqrt{2}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.177}$$

$$\begin{aligned}q(x,t)=&\alpha_0^{(1)}\left(1\mp i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.178}$$

$$\begin{aligned}r(x,t)=&\alpha_0^{(2)}\left(1\mp i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.179}$$

$$\begin{aligned}q(x,t)=&\alpha_0^{(1)}\left(1\mp i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\\&\left.+i\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.180}$$

$$\begin{aligned}r(x,t)=&\alpha_0^{(2)}\left(1\mp i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\\&\left.+i\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\&\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]\end{aligned}\tag{4.181}$$

$$q(x,t) = \alpha_0^{(1)} \left(1 \mp i\csc\left[2\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right)$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + 16\mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right] \quad (4.182)$$

$$r(x,t) = \alpha_0^{(2)} \left(1 \mp i\csc\left[2\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right)$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{\kappa(a\kappa + \alpha) + 16\mu(a - bv) + Q_l}{b\kappa - 1}\right)t + \theta\right)\right] \quad (4.183)$$

$$q(x,t) = \alpha_0^{(1)} \frac{\sqrt{2}}{2}\left(\sqrt{2} \mp 2\csc\left[2\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right)$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{a\kappa^2 + \alpha\kappa - 8a\mu + 8b\mu v + Q_l}{b\kappa - 1}\right)t + \theta\right)\right] \quad (4.184)$$

$$r(x,t) = \alpha_0^{(2)} \frac{\sqrt{2}}{2}\left(\sqrt{2} \mp 2\csc\left[2\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1 - b\kappa}\right)t\right\} + 2\zeta_0\right]\right)$$
$$\times \exp\left[i\left(-\kappa x + \left(\frac{a\kappa^2 + \alpha\kappa - 8a\mu + 8b\mu v + Q_l}{b\kappa - 1}\right)t + \theta\right)\right]. \quad (4.185)$$

Burada $\zeta_0 = \tan^{-1}(A_1/A_2)$.

4.4. Parabolik Yasası

Bu kural yaygın olarak kübik-beşinci dereceden nonlineerlik olarak bilinmektedir. p-toluen sülfonat kristalleri için olan durumda ikinci terim büyüktür. Bu kural Langmuir dalgaları ile elektronlar arasındaki etkileşimde ortaya çıkar; yüksek frekanslı Langmuir dalgaları ile iyon akustik dalgalar arasındaki nonlineer etkileşimi pondermotive kuvvetler açıklar. Kübik-beşinci dereceden nonlineerliğin bu durumu çok ölçekli analiziyle de çalışılmıştır. Parabolik yasası nonlineerliği için, c_1 ve c_2 sabitler olmak üzere $F(s) = c_1 s + c_2 s^2$ (Biswas ve ark., 2012). Bu yüzden, parabolik yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1(c_1|q|^2 + c_2|q|^4) + \eta_1(c_1|r|^2 + c_2|r|^4)\right\} q = Q_1 r$$
$$+ i\left\{\alpha_1 q_x + \beta_1\left((c_1|q|^2 + c_2|q|^4)q\right)_x + \nu_1\left(c_1|q|^2 + c_2|q|^4\right)_x q + \theta_1(c_1|q|^2 + c_2|q|^4)q_x\right\}$$
$$(4.186)$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2(c_1|r|^2 + c_2|r|^4) + \eta_2(c_1|q|^2 + c_2|q|^4)\right\} r = Q_2 q$$
$$+ i\left\{\alpha_2 r_x + \beta_2\left((c_1|r|^2 + c_2|r|^4)r\right)_x + \nu_2\left(c_1|r|^2 + c_2|r|^4\right)_x r + \theta_2(c_1|r|^2 + c_2|r|^4)r_x\right\}$$
$$(4.187)$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.186. ve eşitlik 4.187.'de yerine yazılarak ve elde edilen denklemler reel ve imajiner kısımlarına ayrılarak aşağıdaki ilişki çifti bulunur:
Reel kısım:

$$\left(a_l - b_l v\right) P_l'' - \left(\omega + \kappa\left(a_l\kappa + \alpha_l - b_l\omega\right)\right) P_l + c_1\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3$$
$$+ c_2\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^5 + c_1\eta_l P_l P_{\bar{l}}^2 + c_2\eta_l P_l P_{\bar{l}}^4 - Q_l P_{\bar{l}} = 0. \tag{4.188}$$

İmajiner kısım:

$$\left(v + 2a_l\kappa + \alpha_l - b_l(v\kappa + \omega)\right) P_l' + c_1\left(3\beta_l + \theta_l + 2\nu_l\right) P_l^2 P_l'$$
$$+ c_2\left(5\beta_l + \theta_l + 4\nu_l\right) P_l^4 P_l' = 0. \tag{4.189}$$

İmajiner kısımdan solitonun hızı eşitlik 4.10.'daki gibi bulunur. Ayrıca, eşitlik 4.189.'den, kısıtlama koşuları aşağıdaki gibi oluşur:

$$3\beta_l + \theta_l + 2\nu_l = 0 \tag{4.190}$$

$$5\beta_l + \theta_l + 4\nu_l = 0. \tag{4.191}$$

Bu taktirde, eşitlik 4.190. ve eşitlik 4.191.'dan

$$\beta_l + \nu_l = 0 \tag{4.192}$$

$$\beta_l + \theta_l = 0 \tag{4.193}$$

$$\nu_l = \theta_l \tag{4.194}$$

elde edilir. Dolayısıyla eşitlik 4.11.'de verilen eşitlikler kullanılarak, eşitlik 4.188 ile verilen reel kısım

$$\left(a - bv\right) P_l'' - \left(\omega + \kappa\left(a\kappa + \alpha - b\omega\right)\right) P_l + c_1\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3$$
$$+ c_2\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^5 + c_1\eta_l P_l P_{\bar{l}}^2 + c_2\eta_l P_l P_{\bar{l}}^4 - Q_l P_{\bar{l}} = 0 \tag{4.195}$$

denklemine dönüşür. Balans prensibi kullanılarak

$$P_{\bar{l}} = P_l \tag{4.196}$$

ifadesine ulaşılır ve eşitlik 4.195.

$$\left(a - bv\right) P_l'' - \left(\omega + \kappa\left(a\kappa + \alpha - b\omega\right) + Q_l\right) P_l + c_1\left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^3$$
$$+ c_2\left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right) P_l^5 = 0 \tag{4.197}$$

denklemine indirgenir. Kapalı form çözümler elde etmek için

$$P_l = U_l^{\frac{1}{2}} \tag{4.198}$$

dönüşümü eşitlik 4.197.'te uygulanır ve böylece eşitlik 4.197.

$$\begin{aligned}&2(a-bv)U_l U_l'' - (a-bv)\left(U_l'\right)^2 - 4\left(\omega + \kappa(a\kappa + \alpha - b\omega) + Q_l\right)U_l^2 \\ &+ 4c_1\left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right)U_l^3 + 4c_2\left(\xi_l + \eta_l - \kappa\left(\beta_l + \theta_l\right)\right)U_l^4 = 0\end{aligned} \tag{4.199}$$

halini alır. Şimdi, eşitlik 4.199.'de $U_l U_l''$ veya $\left(U_l'\right)^2$ ile U_l^4 terimleri balans edilerek $M = 1$ bulunur. Böylece, eşitlik 4.199.'in çözümü

$$\begin{aligned}U_l(\zeta) =& \alpha_0^{(l)} + \alpha_1^{(l)}\left(\frac{G'}{G}\right) + \beta_1^{(l)}\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + \gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1} \\ &+ \delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}\end{aligned} \tag{4.200}$$

şeklinde yapılanır. Daha önceki bölümlerde olduğu gibi ilerleyerek çözüm setleri aşağıdaki gibi listelenir:

1. Set:

$$\begin{aligned}&\beta_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \\ &\alpha_0^{(l)} = -\frac{2\mu(a-bv)}{c_1\chi_l}, \\ &\alpha_1^{(l)} = \pm\frac{2i\sqrt{\mu}(a-bv)}{c_1\chi_l}, \\ &\omega = \frac{\kappa(a\kappa + \alpha) + \mu(a-bv) + Q_l}{b\kappa - 1}, \\ &c_2 = \frac{3c_1^2\chi_l}{16\mu(a-bv)}.\end{aligned} \tag{4.201}$$

2. Set:

$$\alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = \frac{2\mu(a-bv)}{c_1\chi_l},$$

$$\beta_1^{(l)} = \pm\frac{2i\mu(a-bv)}{\sqrt{-c_1^2\sigma\chi_l^2}}, \tag{4.202}$$

$$\omega = \frac{4\kappa(a\kappa+\alpha)-5\mu(a-bv)+4Q_l}{4(b\kappa-1)},$$

$$c_2 = -\frac{3c_1^2\chi_l}{16\mu(a-bv)}.$$

3. Set:

$$\alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{2\mu(a-bv)}{c_1\chi_l},$$

$$\gamma_1^{(l)} = \pm\frac{2i\mu^{3/2}(a-bv)}{c_1\chi_l}, \tag{4.203}$$

$$\omega = \frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1},$$

$$c_2 = \frac{3c_1^2\chi_l}{16\mu(a-bv)}.$$

4. Set:

$$\gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{\mu(a-bv)}{2c_1\chi_l},$$

$$\alpha_1^{(l)} = \pm\frac{\sqrt{-\mu}(a-bv)}{2c_1\chi_l},$$

$$\beta_1^{(l)} = \frac{i\mu(a-bv)}{2\sqrt{c_1^2\sigma\chi_l^2}}, \tag{4.204}$$

$$\omega = \frac{4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l}{4(b\kappa-1)},$$

$$c_2 = \frac{3c_1^2\chi_l}{4\mu(a-bv)}.$$

5. Set:

$$\beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = \frac{8\mu(a-bv)}{c_1\chi_l},$$

$$\alpha_1^{(l)} = \pm\frac{4\sqrt{\mu}(a-bv)}{c_1\chi_l},$$

$$\gamma_1^{(l)} = \pm\frac{4\mu^{3/2}(a-bv)}{c_1\chi_l}, \tag{4.205}$$

$$\omega = \frac{\kappa(a\kappa+\alpha)-5\mu(a-bv)+Q_l}{b\kappa-1},$$

$$c_2 = -\frac{3c_1^2\chi_l}{64\mu(a-bv)}.$$

6. Set:

$$
\begin{aligned}
&\beta_1^{(l)} = \delta_1^{(l)} = 0, \\
&\alpha_0^{(l)} = -\frac{8\mu(a-bv)}{c_1\chi_l}, \\
&\alpha_1^{(l)} = \pm\frac{4i\sqrt{\mu}(a-bv)}{c_1\chi_l}, \\
&\gamma_1^{(l)} = \mp\frac{4i\mu^{3/2}(a-bv)}{c_1\chi_l}, \\
&\omega = \frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}, \\
&c_2 = \frac{3c_1^2\chi_l}{64\mu(a-bv)}.
\end{aligned}
\tag{4.206}
$$

Burada κ ve μ keyfi sabitlerdir. Ayrıca

$$\chi_l = \eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right) \tag{4.207}$$

ile verilir. Sonuç olarak, bu çözüm setleri yardımıyla parabolik yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sisteminde aşağıdaki çözümler elde edilir:

$\mu < 0$ iken, aşağıdaki hiperbolik hareketli dalga çözümler türetilir:

$$
\begin{aligned}
q(x,t) = &\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1 \mp \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2}} \\
&\times \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned}
\tag{4.208}
$$

$$
\begin{aligned}
r(x,t) = &\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1 \mp \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2}} \\
&\times \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned}
\tag{4.209}
$$

$$
\begin{aligned}
q(x,t) = &\left\{\frac{2\mu(a-bv)}{c_1\chi_1}\left(1 \pm \sqrt{1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2}} \\
&\times \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned}
\tag{4.210}
$$

$$r(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\sqrt{1-\left[\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.211}$$

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.212}$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.213}$$

$$q(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_1}\left(1\pm\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.214}$$

$$r(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_2}\left(1\pm\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.215}$$

$$q(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_1}\left[2\mp i\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.\pm i\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.216}$$

$$r(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_2}\left[2\mp i\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.\pm i\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.217}$$

$$q(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_1}\left[2\mp \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\mp\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.218}$$

$$r(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_2}\left[2\mp \frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\mp\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.219}$$

$\mu > 0$ için, aşağıdaki trigonometrik hareketli dalga çözümler elde edilir:

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.220}$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.221}$$

$$q(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.222}$$

$$r(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\sqrt{1+\left[\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right]^2}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.223}$$

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.224}$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.225}$$

$$q(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_1}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.-i\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.226}$$

$$r(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_2}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.-i\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.227}$$

$$q(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_1}\left[2\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.228}$$

$$r(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_2}\left[2\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.229}$$

$$q(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_1}\left[2\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.\pm i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.230}$$

$$r(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_2}\left[2\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right.\right.$$
$$\left.\left.\pm i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right]\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \quad (4.231)$$

Son olarak, $\mu = 0$ ise, rasyonel fonksiyon çözümler elde edilebilir. Ancak bu nonlineerlik için rasyonel fonksiyon çözüm yoktur.

Özel durumlar aşağıdaki gibi sunulur:

$\mu < 0$ ve $A_1^2 > A_2^2$ iken, eşitlik 4.208. - eşitlik 4.219. ile verilen çözümlerden sırasıyla, aşağıdaki optik soliton çözümler elde edilir:

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\mp\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.232)$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\mp\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.233)$$

$$q(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)-5\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.234)$$

$$r(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)-5\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.235)$$

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.236)$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.237)$$

$$q(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_1}\left(1\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.-i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.238}$$

$$r(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_2}\left(1\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.-i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.239}$$

$$q(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_1}\left(2\pm2i\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)-5\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.240}$$

$$r(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_2}\left(2\pm2i\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)-5\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.241}$$

$$q(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_1}\left(2\mp\coth\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.242}$$

$$r(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_2}\left(2\mp\coth\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]. \tag{4.243}$$

Burada $\zeta_0=\tanh^{-1}(A_2/A_1)$.

$\mu>0$ için, eşitlik 4.220. - eşitlik 4.231. ile verilen çözümlerden sırasıyla, aşağıdaki periyodik dalga çözümler çıkarılır:

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.244}$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.245}$$

$$q(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)-5\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.246}$$

$$r(x,t)=\left\{\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)-5\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.247}$$

$$q(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_1}\left(1\pm i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.248}$$

$$r(x,t)=\left\{-\frac{2\mu(a-bv)}{c_1\chi_2}\left(1\pm i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.249}$$

$$q(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_1}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.-i\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.250}$$

$$r(x,t)=\left\{-\frac{\mu(a-bv)}{2c_1\chi_2}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.-i\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l}{4(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.251}$$

$$q(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_1}\left(2\mp 2\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)-5\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.252)$$

$$r(x,t)=\left\{\frac{4\mu(a-bv)}{c_1\chi_2}\left(2\mp 2\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)-5\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.253)$$

$$q(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_1}\left(2\mp 2i\cot\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \quad (4.254)$$

$$r(x,t)=\left\{-\frac{4\mu(a-bv)}{c_1\chi_2}\left(2\mp 2i\cot\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]. \quad (4.255)$$

Burada $\zeta_0=\tan^{-1}(A_1/A_2)$.

4.5. Çift-Kuvvet Yasası

Bu model nonlineer kırılma indisinin doygunluğunu açıklamak için kullanılır ve tam soliton çözümleri bilinmektedir. Bu ikili kuvvet kuralı nonlineerliği ile birlikte etkili GNLS denklemi, lityum niyobat gibi fotovoltaik-fotorefraktif materyallerde uzaysal solitonları açıklamak için basit bir model olarak işlev görür. Birçok organik ve polimer materyallerdeki optik nonlineerlikler, nonlineerliğin bu şekli kullanılarak modellenebilir. Bu modelin solitonları $1\le n<2$ kararsızlık bölgesinde kararsız hâle gelir ve bozulurken, $n\ge 2$ için solitonlar sonlu bir zaman zarfında çöker. Çift-kuvvet yasası nonlineerliği için c_1 ve c_2 sabitler olmak üzere $F(s)=c_1s^n+c_2s^{2n}$ (Biswas ve ark., 2012). Bu yüzden, çift-kuvvet yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t+a_1q_{xx}+b_1q_{xt}+\left\{\xi_1(c_1|q|^{2n}+c_2|q|^{4n})+\eta_1(c_1|r|^{2n}+c_2|r|^{4n})\right\}q=Q_1r$$
$$+i\left\{\alpha_1q_x+\beta_1\left((c_1|q|^{2n}+c_2|q|^{4n})q\right)_x\right.$$
$$\left.+\nu_1\left(c_1|q|^{2n}+c_2|q|^{4n}\right)_xq+\theta_1(c_1|q|^{2n}+c_2|q|^{4n})q_x\right\} \quad (4.256)$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2(c_1|r|^{2n} + c_2|r|^{4n}) + \eta_2(c_1|q|^{2n} + c_2|q|^{4n})\right\} r = Q_2 q$$
$$+ i\left\{\alpha_2 r_x + \beta_2\left((c_1|r|^{2n} + c_2|r|^{4n})r\right)_x\right.$$
$$\left. +\nu_2\left(c_1|r|^{2n} + c_2|r|^{4n}\right)_x r + \theta_2(c_1|r|^{2n} + c_2|r|^{4n})r_x\right\} \tag{4.257}$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.256. ve eşitlik 4.257.'te yazılarak, aşağıdaki reel kısım

$$(a_l - b_l v)\,P_l'' - (\omega + \kappa\,(a_l\kappa + \alpha_l - b_l\omega))\,P_l + c_1\,(\xi_l - \kappa\,(\beta_l + \theta_l))\,P_l^{2n+1}$$
$$+ c_2\,(\xi_l - \kappa\,(\beta_l + \theta_l))\,P_l^{4n+1} + c_1\eta_l P_l P_{\bar{l}}^{2n} + c_2\eta_l P_l P_{\bar{l}}^{4n} - Q_l P_{\bar{l}} = 0 \tag{4.258}$$

ve imajiner kısım

$$(v + 2a_l\kappa + \alpha_l - b_l(v\kappa + \omega))\,P_l' + c_1\,(\beta_l + \theta_l + 2n(\beta_l + \nu_l))\,P_l^{2n}P_l'$$
$$+ c_2\,(\beta_l + \theta_l + 4n(\beta_l + \nu_l))\,P_l^{4n}P_l' = 0 \tag{4.259}$$

elde edilir. İmajiner kısımda lineer bağımsız fonksiyonların katsayıları sıfıra eşitlenerek solitonun hızı, eşitlik 4.10.'daki gibi bulunur ve ayrıca

$$\beta_l + \theta_l + 2n(\beta_l + \nu_l) = 0 \tag{4.260}$$

$$\beta_l + \theta_l + 4n(\beta_l + \nu_l) = 0 \tag{4.261}$$

kısıtlama şartları elde edilir. Bu taktirde, 4.260. ve eşitlik 4.261.'den

$$\beta_l + \theta_l = 0 \tag{4.262}$$

$$\beta_l + \nu_l = 0 \tag{4.263}$$

$$\nu_l = \theta_l \tag{4.264}$$

bulunur. Dolayısıyla, eşitlik 4.11. ile, eşitlik 4.258. ile verilen reel kısım

$$(a - bv)\,P_l'' - (\omega + \kappa\,(a\kappa + \alpha - b\omega))\,P_l + c_1\,(\xi - \kappa\,(\beta + \theta))\,P_l^{2n+1}$$
$$+ c_2\,(\xi - \kappa\,(\beta + \theta))\,P_l^{4n+1} + c_1\eta P_l P_{\bar{l}}^{2n} + c_2\eta P_l P_{\bar{l}}^{4n} - Q_l P_{\bar{l}} = 0 \tag{4.265}$$

olarak yeniden yazılabilir. Balans prensibinin yardımıyla

$$P_{\bar{l}} = P_l \tag{4.266}$$

eşitliği elde edilir ve böylece eşitlik 4.265.

$$(a-bv)\,P_l''-(\omega+\kappa\,(a\kappa+\alpha-b\omega)+Q_l)\,P_l+c_1\,(\xi+\eta-\kappa\,(\beta+\theta))\,P_l^{2n+1}$$
$$+\,c_2\,(\xi+\eta-\kappa\,(\beta+\theta))\,P_l^{4n+1}=0 \tag{4.267}$$

şeklini alır. Kapalı form çözümler türetmek için

$$P_l=U_l^{\frac{1}{2n}} \tag{4.268}$$

dönüşümü uygulanır. Bu durumda, eşitlik 4.267.

$$2n(a-bv)U_lU_l''-(2n-1)(a-bv)\left(U_l'\right)^2-4n^2\left(\omega+\kappa(a\kappa+\alpha-b\omega)+Q_l\right)U_l^2$$
$$+\,4c_1n^2\left(\xi_l+\eta_l-\kappa\left(\beta_l+\theta_l\right)\right)U_l^3+4c_2n^2\left(\xi_l+\eta_l-\kappa\left(\beta_l+\theta_l\right)\right)U_l^4=0 \tag{4.269}$$

denklemine dönüşür. Şimdi, son denklemdeki U_lU_l'' veya $\left(U_l'\right)^2$ ile U_l^4 terimleri balans edilerek $M=1$ elde edilir. Böylece, eşitlik 4.269.'ün çözümü aşağıdaki formdadır:

$$U_l(\zeta)=\alpha_0^{(l)}+\alpha_1^{(l)}\left(\frac{G'}{G}\right)+\beta_1^{(l)}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}+\gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1}$$
$$+\,\delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}. \tag{4.270}$$

Önceki bölümlerde olduğu gibi, benimsenen metodolojinin ana adımları izlenerek aşağıdaki çözüm setleri bulunur.

1. Set:

$$\beta_1^{(l)}=\gamma_1^{(l)}=\delta_1^{(l)}=0,$$

$$\alpha_0^{(l)}=-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_l},$$

$$\alpha_1^{(l)}=\pm\frac{i\sqrt{\mu}(n+1)(a-bv)}{c_1n^2\chi_l}, \tag{4.271}$$

$$\omega=\frac{\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)},$$

$$c_2=\frac{c_1^2n^2(2n+1)\chi_l}{4\mu(n+1)^2(a-bv)}.$$

2. Set:

$$\alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_l},$$

$$\gamma_1^{(l)} = \pm\frac{i\mu^{3/2}(n+1)(a-bv)}{c_1 n^2 \chi_l}, \tag{4.272}$$

$$\omega = \frac{\mu(a-bv) + \kappa n^2(a\kappa + \alpha) + n^2 Q_l}{n^2(b\kappa - 1)},$$

$$c_2 = \frac{c_1^2 n^2(2n+1)\chi_l}{4\mu(n+1)^2(a-bv)}.$$

3. Set:

$$\gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{\mu(n+1)(a-bv)}{4c_1 n^2 \chi_l},$$

$$\alpha_1^{(l)} = \pm\frac{i\sqrt{\mu}(n+1)(a-bv)}{4c_1 n^2 \chi_l},$$

$$\beta_1^{(l)} = \frac{i\mu(n+1)(a-bv)}{4\sqrt{c_1^2 n^4 \sigma \chi_l^2}}, \tag{4.273}$$

$$\omega = \frac{\mu(a-bv) + 4\kappa n^2(a\kappa + \alpha) + 4n^2 Q_l}{4n^2(b\kappa - 1)},$$

$$c_2 = \frac{c_1^2 n^2(2n+1)\chi_l}{\mu(n+1)^2(a-bv)}.$$

4. Set:

$$
\begin{aligned}
&\beta_1^{(l)} = \delta_1^{(l)} = 0 \\
&\alpha_0^{(l)} = -\frac{4\mu(n+1)(a-bv)}{c_1 n^2 \chi_l}, \\
&\alpha_1^{(l)} = \pm\frac{2i\sqrt{\mu}(n+1)(a-bv)}{c_1 n^2 \chi_l}, \\
&\gamma_1^{(l)} = \pm\frac{2i\mu^{3/2}(n+1)(a-bv)}{c_1 n^2 \chi_l}, \\
&\omega = \frac{4\mu(a-bv) + \kappa n^2(a\kappa + \alpha) + n^2 Q_l}{n^2(b\kappa - 1)}, \\
&c_2 = -\frac{c_1^2 n^2 (2n+1)\chi_l}{16\mu(n+1)^2(a-bv)}.
\end{aligned} \tag{4.274}
$$

Burada κ ve μ keyfi sabitlerdir. Ayrıca

$$
\chi_l = \eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right) \tag{4.275}
$$

ile verilir. Bu durumda, çift-kuvvet yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki çözümler keşfedilir:

$\mu < 0$ iken, aşağıda verilen hiperbolik hareketli dalga çözümlere ulaşılır:

$$
q(x,t) = \left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_1}\left(1 \mp \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2n}} \times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.276}
$$

$$
r(x,t) = \left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_2}\left(1 \mp \frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2n}} \times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.277}
$$

$$
q(x,t) = \left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_1}\left(1 \pm \left(\frac{A_1 \sinh(\sqrt{-\mu}\zeta) + A_2 \cosh(\sqrt{-\mu}\zeta)}{A_1 \cosh(\sqrt{-\mu}\zeta) + A_2 \sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}} \times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.278}
$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\pm\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.279}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_1}\left(1\mp\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.280}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_2}\left(1\mp\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.281}$$

$$q(x,t)=\left\{-\frac{2\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(2\mp\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.282}$$

$$r(x,t)=\left\{-\frac{2\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(2\mp\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.283}$$

$\mu>0$ ise, aşağıda verilen trigonometrik hareketli dalga çözümlerine ulaşılır:

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.284}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.285}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.286}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.287}$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_1}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.288}$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_2}\left(1\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.-i\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.289}$$

$$q(x,t)=\left\{-\frac{2\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(2\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.290}$$

$$r(x,t)=\left\{-\frac{2\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(2\mp i\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\mp i\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.291}$$

Son olarak, $\mu = 0$ iken rasyonel fonksiyon çözümler elde edilebilir. Ancak, bu nonlineerlik için rasyonel fonksiyon çözümler mevcut değildir.

Özel durumlar aşağıdaki gibi listelenir:

$\mu < 0$ ve $A_1^2 > A_2^2$ iken, eşitlik 4.276. - eşitlik 4.283. ile verilen çözümlerden aşağıdaki çözümler elde edilir:

$$\begin{aligned} q(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_1}\left(1 \mp \tanh\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2 Q_l}{n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.292}$$

$$\begin{aligned} r(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_2}\left(1 \mp \tanh\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2 Q_l}{n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.293}$$

$$\begin{aligned} q(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_1}\left(1 \pm \coth\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2 Q_l}{n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.294}$$

$$\begin{aligned} r(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{c_1 n^2 \chi_2}\left(1 \pm \coth\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2 Q_l}{n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.295}$$

$$\begin{aligned} q(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{4c_1 n^2 \chi_1}\left(1 \mp \tanh\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right.\right. \\ &\left.\left. -i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + 4\kappa n^2(a\kappa+\alpha) + 4n^2 Q_l}{4n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.296}$$

$$\begin{aligned} r(x,t) = &\left\{-\frac{\mu(n+1)(a-bv)}{4c_1 n^2 \chi_2}\left(1 \mp \tanh\left[\sqrt{-\mu}\left\{x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right.\right. \\ &\left.\left. -i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x - \left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right)\right\}^{\frac{1}{2n}} \\ &\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + 4\kappa n^2(a\kappa+\alpha) + 4n^2 Q_l}{4n^2(b\kappa-1)}\right)t + \theta\right)\right] \end{aligned} \tag{4.297}$$

$$q(x,t)=\left\{-\frac{4\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(1\pm\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.298)$$

$$r(x,t)=\left\{-\frac{4\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\pm\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right]. \quad (4.299)$$

Burada $\zeta_0=\tanh^{-1}(A_2/A_1)$.

$\mu>0$ için, eşitlik 4.284. - eşitlik 4.291. ile verilen çözümler sırasıyla, aşağıdaki çözümlere dönüşür:

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.300)$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.301)$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_1}\left(1\pm i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.302)$$

$$r(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{c_1n^2\chi_2}\left(1\pm i\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+\kappa n^2(a\kappa+\alpha)+n^2Q_l}{n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.303)$$

$$q(x,t)=\left\{-\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_1}\left(1\pm i\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.-i\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+4\kappa n^2(a\kappa+\alpha)+4n^2Q_l}{4n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.304)$$

$$
\begin{aligned}
r(x,t) = &\left\{ -\frac{\mu(n+1)(a-bv)}{4c_1n^2\chi_2} \left(1 \pm i\tan\left[\sqrt{\mu}\left\{ x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa} \right) t \right\} + \zeta_0 \right] \right.\right. \\
&\left.\left. -i\sec\left[\sqrt{\mu}\left\{ x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa} \right) t \right\} + \zeta_0 \right] \right) \right\}^{\frac{1}{2n}} \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{\mu(a-bv) + 4\kappa n^2(a\kappa+\alpha) + 4n^2Q_l}{4n^2(b\kappa-1)} \right) t + \theta \right) \right] \qquad (4.305)
\end{aligned}
$$

$$
\begin{aligned}
q(x,t) = &\left\{ -\frac{4\mu(n+1)(a-bv)}{c_1n^2\chi_1} \left(1 \pm i\csc\left[2\sqrt{\mu}\left\{ x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa} \right) t \right\} + 2\zeta_0 \right] \right) \right\}^{\frac{1}{2n}} \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{4\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2Q_l}{n^2(b\kappa-1)} \right) t + \theta \right) \right] \qquad (4.306)
\end{aligned}
$$

$$
\begin{aligned}
r(x,t) = &\left\{ -\frac{4\mu(n+1)(a-bv)}{c_1n^2\chi_2} \left(1 \pm i\csc\left[2\sqrt{\mu}\left\{ x - \left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa} \right) t \right\} + 2\zeta_0 \right] \right) \right\}^{\frac{1}{2n}} \\
&\times \exp\left[i\left(-\kappa x + \left(\frac{4\mu(a-bv) + \kappa n^2(a\kappa+\alpha) + n^2Q_l}{n^2(b\kappa-1)} \right) t + \theta \right) \right]. \qquad (4.307)
\end{aligned}
$$

Burada $\zeta_0 = \tan^{-1}(A_1/A_2)$.

4.6. Logaritma Yasası

Bu kural güncel fiziğin çeşitli aranlarında ortaya çıkar. Sabit Gauss ışınları (gaussonlar) yanında periyodik ve kuasi-periyodik ışın gelişim rejimleri için de kapalı formda tam ifadeleri mümkün kılar. Bu modelin avantajı, lineerize edilmiş problemde sadece ayrık spektrum bulunduğu için, periyodik solitonda radyasyona rastlanmamasıdır. Logaritmik yasası nonlineerliği için $F(s) = \ln s$ (Biswas ve ark., 2012). Böylece, eşitlik 2.1. ve eşitlik 2.2. ile verilen model

$$iq_t + a_1q_{xx} + b_1q_{xt} + \left\{ \xi_1 \ln|q|^2 + \eta_1 \ln|r|^2 \right\} q = Q_1r + i\alpha_1q_x \qquad (4.308)$$

$$ir_t + a_2r_{xx} + b_2r_{xt} + \left\{ \xi_2 \ln|r|^2 + \eta_2 \ln|q|^2 \right\} r = Q_2q + i\alpha_2r_x \qquad (4.309)$$

olarak yeniden yazılabilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.308. ve eşitlik 4.309.'de yazılarak reel ve imajiner kısımlar sırasıyla aşağıdaki gibi bulunur:

Reel kısım:

$$(a_l - b_lv)\,P_l'' + \left(b_l\omega\kappa - \omega - a_l\kappa^2 - \alpha_l\kappa \right) P_l + 2\xi_lP_l\ln P_l + 2\eta_lP_l\ln P_{\bar{l}} = Q_lP_{\bar{l}}. \quad (4.310)$$

İmajiner kısım:

$$v(b_l\kappa - 1)P_l' + (b_l\omega - 2a_l\kappa - \alpha_l)\,P_l' = 0. \qquad (4.311)$$

Eşitlik 4.311.'dan solitonun hızı eşitlik 4.10.'daki gibi elde etmek mümkündür. Sonuç olarak, bu durumda da eşitlik 4.11. ve eşitlik 4.12. sağlanır ve böylece eşitlik 4.310.

$$(a-bv)\,P_l'' + \left(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa\right)P_l + 2\xi_l P_l \ln P_l + 2\eta_l P_l \ln P_{\bar{l}} = Q_l P_{\bar{l}} \tag{4.312}$$

şeklinde yazılabilir. Balans prensibi kullanılarak

$$P_{\bar{l}} = P_l \tag{4.313}$$

ifadesine ulaşılır ve bu durumda eşitlik 4.312.

$$(a-bv)\,P_l'' + \left(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa\right)P_l + 2\left(\xi_l + \eta_l\right)P_l \ln P_l = Q_l P_l \tag{4.314}$$

biçiminde yeniden yazılabilir. Logaritma yasasına sahip matematiksel modele analitik çözümler elde etmek için

$$P_l = \exp\frac{1}{U_l} \tag{4.315}$$

dönüşümü eşitlik 4.314.'da uygulanır ve bu taktirde

$$\begin{aligned}&(a-bv)\left\{(U_l')^2 + 2U_l(U_l')^2 - U_l^2U_l''\right\} + 2(\xi_l+\eta_l)U_l^3\\ &+\left(b\omega\kappa - \omega - a\kappa^2 - \alpha\kappa - Q_l\right)U_l^4 = 0\end{aligned} \tag{4.316}$$

elde edilir. Eşitlik 4.316.'de $U_l(U_l')^2$ veya $U_l^2U_l''$ ile U_l^4 terimleri balans edilerek $M = 2$ bulunur. Böylece, eşitlik 4.316'in çözümü aşağıdaki formdadır:

$$\begin{aligned}U_l(\zeta) =& \alpha_0^{(l)} + \sum_{i=1}^{2}\left\{\alpha_i^{(l)}\left(\frac{G'}{G}\right)^i + \beta_i^{(l)}\left(\frac{G'}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}\right.\\ &\left. + \gamma_i^{(l)}\left(\frac{G'}{G}\right)^{-i} + \delta_i^{(l)}\frac{\left(\frac{G'}{G}\right)^{-i+1}}{\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}\right\}.\end{aligned} \tag{4.317}$$

Eşitlik 3.5. ve eşitlik 4.317. ile verilen formal çözüm, eşitlik 4.316.'de yerine yazılarak ve sonra $\left(\frac{G'}{G}\right)^j$ ve $\left(\frac{G'}{G}\right)^j\sqrt{\sigma\left\{1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right\}}$ terimlerinin katsayıları sıfıra eşitlenerek bir cebirsel denklem sistemi elde edildi. Ancak, bu denklem sistemi çözülerek istenen katsayılar belirlenemedi. Bunun iki sebebi olabilir: Birincisi, kullanılan bilgisayarlar yetersiz olabilir. Diğeri ise bu integrasyon şeması logaritma yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için uygun bir yöntem olmayabilir.

4.7. Anti-Kübik Yasası

Anti-kübik yasası ilk olarak 2003 yılında ortaya çıktı (Fedele ve ark., 2003; Triki ve ark., 2016). Daha sonra, bu nonlineerlik ile ilgili birçok gelişme oldu ve sonuçlar rapor edildi. Bu durumda, c_1, c_2 ve c_3 sabitler olmak üzere $F(s) = c_1 s^{-2} + c_2 s + c_3 s^2$. Bu yüzden, anti-kübik yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$
\begin{aligned}
&iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 \left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right) + \eta_1 \left(\frac{c_1}{|r|^4} + c_2|r|^2 + c_3|r|^4\right)\right\} q \\
&= Q_1 r + i\left\{\alpha_1 q_x + \beta_1 \left(\left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right) q\right)_x \right. \\
&\left. + \nu_1 \left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right)_x q + \theta_1 \left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right) q_x \right\}
\end{aligned} \tag{4.318}
$$

$$
\begin{aligned}
&ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 \left(\frac{c_1}{|r|^4} + c_2|r|^2 + c_3|r|^4\right) + \eta_2 \left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right)\right\} r \\
&= Q_2 q + i\left\{\alpha_2 r_x + \beta_2 \left(\left(\frac{c_1}{|r|^4} + c_2|r|^2 + c_3|r|^4\right) r\right)_x \right. \\
&\left. + \nu_2 \left(\frac{c_1}{|r|^4} + c_2|r|^2 + c_3|r|^4\right)_x r + \theta_2 \left(\frac{c_1}{|r|^4} + c_2|r|^2 + c_3|r|^4\right) r_x \right\}.
\end{aligned} \tag{4.319}
$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.318. ve eşitlik 4.319.'de yazılarak reel ve imajiner kısımlar sırasıyla aşağıdaki gibi bulunur:

$$
\begin{aligned}
&c_1\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_{\bar{l}}^4 + c_1 \eta_l P_l^4 - P_l^3 P_{\bar{l}}^4 \left(P_l \left(\kappa\left(a_l \kappa + \alpha_l - b_l \omega\right)\right.\right. \\
&\left. - \eta_l P_{\bar{l}}^2 \left(c_3 P_{\bar{l}}^2 + c_2\right) + \omega\right) + \left(b_l v - a_l\right) P_l'' + c_3 P_l^5 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) \\
&\left. + c_2 P_l^3 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + Q_l P_{\bar{l}}\right) = 0
\end{aligned} \tag{4.320}
$$

$$
\begin{aligned}
&P_l' \left(P_l^4 \left(2a_l\kappa + \alpha_l - b_l(\kappa v + \omega) + v\right) + c_1\left(-3\beta_l + \theta_l - 4\nu_l\right) + c_3 P_l^8 \left(5\beta_l + \theta_l + 4\nu_l\right)\right. \\
&\left. + c_2 P_l^6 \left(3\beta_l + \theta_l + 2\nu_l\right)\right) = 0.
\end{aligned} \tag{4.321}
$$

Eşitlik 4.321.'dan lineer bağımsız fonksiyonların katsayıları sıfıra eşitlenerek solitonun hızı eşitlik 4.10.'daki gibi elde edilir ve ayrıca

$$-3\beta_l + \theta_l - 4\nu_l = 0 \tag{4.322}$$

$$5\beta_l + \theta_l + 4\nu_l = 0 \tag{4.323}$$

$$3\beta_l + \theta_l + 2\nu_l = 0 \tag{4.324}$$

kısıtlama şartları bulunur ve buradan

$$\beta_l + \nu_l = 0 \tag{4.325}$$

$$\beta_l + \theta_l = 0 \tag{4.326}$$

$$\nu_l = \theta_l \tag{4.327}$$

eşitliklerine ulaşılır. Bu taktirde, eşitlik 4.11. ile, eşitlik 4.320.

$$\begin{aligned} &c_1\left(\xi_l - \kappa\left(\beta_l + \theta_l\right)\right) P_{\bar{l}}^4 + c_1\eta_l {P_l}^4 - {P_l}^3 P_{\bar{l}}^4 \left(P_l\left(\kappa\left(a\kappa + \alpha - b\omega\right)\right.\right. \\ &-\eta_l P_{\bar{l}}^2\left(c_3 P_{\bar{l}}^2 + c_2\right) + \omega\big) + \left(bv - a\right) P_l'' + c_3 {P_l}^5\left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) \\ &+c_2 {P_l}^3\left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + Q_l P_{\bar{l}}\big) = 0 \end{aligned} \tag{4.328}$$

olarak yazılabilir. Balans prensibinin yardımıyla

$$P_{\bar{l}} = P_l \tag{4.329}$$

ifadesine ulaşılır ve eşitlik 4.328.

$$\begin{aligned} &c_1\left(-\beta_l\kappa + \eta_l - \theta_l\kappa + \xi_l\right) - {P_l}^3\left(P_l\left(a\kappa^2 + \alpha\kappa - b\kappa\omega + Q_l + \omega\right) + (bv - a)P_l''\right. \\ &\left.+c_3 {P_l}^5\left(\beta_l\kappa - \eta_l + \theta_l\kappa - \xi_l\right) + c_2 {P_l}^3\left(\beta_l\kappa - \eta_l + \theta_l\kappa - \xi_l\right)\right) = 0 \end{aligned} \tag{4.330}$$

şeklini alır. Kapalı form çözümler türetmek için

$$P_l = U_l^{\frac{1}{2}} \tag{4.331}$$

dönüşümü yapılır ve bu durumda, eşitlik 4.330.

$$\begin{aligned} &-4{U_l}^2\left(a\kappa^2 + \alpha\kappa - b\kappa\omega + Q_l + \omega\right) + 2(a - bv)U_l U_l'' - 4c_1\left(\beta_l\kappa - \eta_l + \theta_l\kappa - \xi_l\right) \\ &-4c_3 {U_l}^4\left(\beta_l\kappa - \eta_l + \theta_l\kappa - \xi_l\right) - 4c_2 {U_l}^3\left(\beta_l\kappa - \eta_l + \theta_l\kappa - \xi_l\right) + (bv - a)(U_l')^2 = 0 \end{aligned} \tag{4.332}$$

denklemine dönüşür. Şimdi, eşitlik 4.332.'de $U_l U_l''$ veya $(U_l')^2$ ile U_l^4 terimlerinin dengelenmesi ile $M = 1$ bulunur. Böylece eşitlik 4.332.'nin çözümü aşağıdaki gibi verilir:

$$\begin{aligned} U_l(\zeta) =&\alpha_0^{(l)} + \alpha_1^{(l)}\left(\frac{G'}{G}\right) + \beta_1^{(l)}\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + \gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1} \\ &+ \delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}. \end{aligned} \tag{4.333}$$

Bu durumda aşağıdaki çözüm setleri elde edilir:

1. Set:

$$\beta_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{3c_2}{8c_3},$$

$$\alpha_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{a-bv}}{2\sqrt{-c_3\chi_l}}, \tag{4.334}$$

$$\omega = \frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)},$$

$$c_1 = -\frac{3\left(16c_3\mu(a-bv)-3c_2^2\chi_l\right)^2}{4096c_3^3\chi_l^2}.$$

2. Set:

$$\alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{3c_2}{8c_3},$$

$$\beta_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{\mu(a-bv)}}{2\sqrt{c_3(-\sigma)\chi_l}}, \tag{4.335}$$

$$\omega = \frac{8c_3\left(4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)},$$

$$c_1 = \frac{9c_2^2\left(-\frac{16c_3\mu(a-bv)}{\chi_l}-3c_2^2\right)}{4096c_3^3}.$$

3. Set:

$$\alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{3c_2}{8c_3},$$

$$\gamma_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{\mu^2(a-bv)}}{2\sqrt{-c_3\chi_l}}, \tag{4.336}$$

$$\omega = \frac{16c_3\left(2a\kappa^2 + 2\alpha\kappa - a\mu + b\mu v + 2Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa - 1)},$$

$$c_1 = -\frac{3\left(16c_3\mu(a-bv) - 3c_2^2\chi_l\right)^2}{4096c_3^3\chi_l^2}.$$

4. Set:

$$\gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_0^{(l)} = -\frac{3c_2}{8c_3},$$

$$\alpha_1^{(l)} = \frac{\sqrt{3}\sqrt{a-bv}}{4\sqrt{-c_3\chi_l}},$$

$$\beta_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{\mu(a-bv)}}{4\sqrt{c_3(-\sigma)\chi_l}}, \tag{4.337}$$

$$\omega = \frac{4c_3\left(8a\kappa^2 + 8\alpha\kappa - a\mu + b\mu v + 8Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa - 1)},$$

$$c_1 = -\frac{3\left(4c_3\mu(a-bv) - 3c_2^2\chi_l\right)^2}{4096c_3^3\chi_l^2}.$$

5. Set:

$$
\begin{aligned}
&\beta_1^{(l)} = \delta_1^{(l)} = 0,\\
&\alpha_0^{(l)} = -\frac{3c_2}{8c_3},\\
&\alpha_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{a-bv}}{2\sqrt{-c_3\chi_l}},\\
&\gamma_1^{(l)} = \pm\frac{\sqrt{3}\sqrt{\mu^2(a-bv)}}{2\sqrt{-c_3\chi_l}},\\
&\omega = \frac{16c_3\left(2\kappa(a\kappa+\alpha)+2Q_l+\ell_2\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)},\\
&c_1 = -\frac{3\left(2048c_3^2\mu\ell_1(a-bv)+9c_2^4\chi_l^2+96c_3c_2^2\ell_2\chi_l\right)}{4096c_3^3\chi_l^2}.
\end{aligned}
\tag{4.338}
$$

Burada κ ve μ keyfi sabitlerdir. Ayrıca

$$
\begin{aligned}
&\chi_l = \eta_l + \xi_l - \kappa\left(\beta_l + \theta_l\right)\\
&\ell_1 = \mu(a-bv) - \sqrt{a-bv}\sqrt{\mu^2(a-bv)}\\
&\ell_2 = \mu(-a+bv) + 3\sqrt{a-bv}\sqrt{\mu^2(a-bv)}
\end{aligned}
\tag{4.339}
$$

şeklinde verilir. Sonuç olarak anti-kübik yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki çözümler keşfedilir:

$\mu < 0$ iken, hiperbolik fonksiyon çözümler aşağıdaki gibi elde edilir:

$$
\begin{aligned}
q(x,t) = &\left\{-\frac{3c_2}{8c_3} \pm \frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2}}\\
&\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned}
\tag{4.340}
$$

$$
\begin{aligned}
r(x,t) = &\left\{-\frac{3c_2}{8c_3} \pm \frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)\right\}^{\frac{1}{2}}\\
&\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned}
\tag{4.341}
$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}i\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\right.$$
$$\left.\times\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.342}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}i\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\right.$$
$$\left.\times\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.343}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.344}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.345}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.346}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.347}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.348}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.349}$$

$\mu > 0$ için, trigonometrik fonksiyon çözümler aşağıdaki gibi çıkarılır:

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.350}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.351}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.352}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.353}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.354}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.355}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]$$
$$\tag{4.356}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.\pm\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]$$
$$\tag{4.357}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.358}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.359}$$

Son olarak, $\mu=0$ ise, rasyonel fonksiyon çözümler elde edilir. Eşitlik 4.334. ve eşitlik 4.338.'ten

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{a-bv}{c_3\chi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.360}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{a-bv}{c_3\chi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.361}$$

çözümlerine ulaşılırken, eşitlik 4.337.'den

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{-\frac{a-bv}{c_3\chi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.362}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{-\frac{a-bv}{c_3\chi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{2}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.363}$$

çözümleri elde edilir.

Özel durumlar aşağıdaki gibi verilir:

$\mu<0$ ve $A_1^2>A_2^2$ iken, eşitlik 4.340. - eşitlik 4.349. ile verilen çözümlerden sırasıyla, aşağıdaki optik soliton çözümler oluşur:

$$\begin{aligned}q(x,t)=&\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\tanh\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.364}$$

$$\begin{aligned}r(x,t)=&\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\tanh\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.365}$$

$$\begin{aligned}q(x,t)=&\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{8c_3\left(4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.366}$$

$$\begin{aligned}r(x,t)=&\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}\\&\times\exp\left[i\left(-\kappa x+\left(\frac{8c_3\left(4\kappa(a\kappa+\alpha)+\mu(a-bv)+4Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right]\end{aligned} \tag{4.367}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\coth\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.368}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\coth\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.369}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.\pm i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4c_3\left(8a\kappa^2+8\alpha\kappa-a\mu+b\mu v+8Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.370}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}+\frac{\sqrt{3}}{4}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.\pm i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4c_3\left(8a\kappa^2+8\alpha\kappa-a\mu+b\mu v+8Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.371}$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\sqrt{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\coth\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2\kappa(a\kappa+\alpha)+2Q_l+\ell_2\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.372}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\pm\sqrt{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\coth\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2\kappa(a\kappa+\alpha)+2Q_l+\ell_2\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.373}$$

Burada $\zeta_0 = \tanh^{-1}(A_2/A_1)$.

$\mu > 0$ ise, eşitlik 4.350. - eşitlik 4.359. ile verilen çözümlerden sırasıyla, aşağıdaki periyodik dalga çözümler oluşur:

$$q(x,t) = \left\{ -\frac{3c_2}{8c_3} \mp \frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}} \tan\left[\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right\}^{\frac{1}{2}} \times \exp\left[i\left(-\kappa x + \left(\frac{16c_3\left(2a\kappa^2 + 2\alpha\kappa - a\mu + b\mu v + 2Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t + \theta\right)\right] \tag{4.374}$$

$$r(x,t) = \left\{ -\frac{3c_2}{8c_3} \mp \frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}} \tan\left[\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right\}^{\frac{1}{2}} \times \exp\left[i\left(-\kappa x + \left(\frac{16c_3\left(2a\kappa^2 + 2\alpha\kappa - a\mu + b\mu v + 2Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t + \theta\right)\right] \tag{4.375}$$

$$q(x,t) = \left\{ -\frac{3c_2}{8c_3} \pm \frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}} \sec\left[\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right\}^{\frac{1}{2}} \times \exp\left[i\left(-\kappa x + \left(\frac{8c_3\left(4\kappa(a\kappa + \alpha) + \mu(a-bv) + 4Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t + \theta\right)\right] \tag{4.376}$$

$$r(x,t) = \left\{ -\frac{3c_2}{8c_3} \pm \frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}} \sec\left[\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right\}^{\frac{1}{2}} \times \exp\left[i\left(-\kappa x + \left(\frac{8c_3\left(4\kappa(a\kappa + \alpha) + \mu(a-bv) + 4Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t + \theta\right)\right] \tag{4.377}$$

$$q(x,t) = \left\{ -\frac{3c_2}{8c_3} \mp \frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}} \cot\left[\sqrt{\mu}\left\{x - \left(\frac{b\omega - 2a\kappa - \alpha}{1-b\kappa}\right)t\right\} + \zeta_0\right]\right\}^{\frac{1}{2}} \times \exp\left[i\left(-\kappa x + \left(\frac{16c_3\left(2a\kappa^2 + 2\alpha\kappa - a\mu + b\mu v + 2Q_l\right) + 9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t + \theta\right)\right] \tag{4.378}$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\mp\frac{\sqrt{3}}{2}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.379)$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}-\frac{\sqrt{3}}{4}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.\mp\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4c_3\left(8a\kappa^2+8\alpha\kappa-a\mu+b\mu v+8Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.380)$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}-\frac{\sqrt{3}}{4}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\right.$$
$$\left.\left.\mp\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4c_3\left(8a\kappa^2+8\alpha\kappa-a\mu+b\mu v+8Q_l\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.381)$$

$$q(x,t)=\left\{-\frac{3c_2}{8c_3}\mp\sqrt{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2\kappa(a\kappa+\alpha)+2Q_l+\ell_2\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.382)$$

$$r(x,t)=\left\{-\frac{3c_2}{8c_3}\mp\sqrt{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{2}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{16c_3\left(2\kappa(a\kappa+\alpha)+2Q_l+\ell_2\right)+9c_2^2\chi_l}{32c_3(b\kappa-1)}\right)t+\theta\right)\right]. \quad (4.383)$$

Burada $\zeta_0=\tan^{-1}(A_1/A_2)$.

4.8. Kübik-Kuintik-Septik Yasası

Bu yasa, parabolik yasası nonlineerliğinin bir genişlemesidir. Bu nonlineer optik ortam türü, c_1, c_2 ve c_3 sabitler olmak üzere $F(s)=c_1s+c_2s^2+c_3s^3$ şeklini alır (Kaup ve Yang, 2000; Biswas ve Milovic, 2009). Bu yüzden, kübik-kuintik-septik yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$\begin{aligned}&iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1(c_1|q|^2 + c_2|q|^4 + c_3|q|^6) + \eta_1(c_1|r|^2 + c_2|r|^4 + c_3|r|^6)\right\} q \\ &= Q_1 r + i\left\{\alpha_1 q_x + \beta_1\left((c_1|q|^2 + c_2|q|^4 + c_3|q|^6)q\right)_x\right. \\ &\left.+\nu_1\left(c_1|q|^2 + c_2|q|^4 + c_3|q|^6\right)_x q + \theta_1(c_1|q|^2 + c_2|q|^4 + c_3|q|^6)q_x\right\}\end{aligned} \tag{4.384}$$

$$\begin{aligned}&ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2(c_1|r|^2 + c_2|r|^4 + c_3|r|^6) + \eta_2(c_1|q|^2 + c_2|q|^4 + c_3|q|^6)\right\} r \\ &= Q_2 q + i\left\{\alpha_2 r_x + \beta_2\left((c_1|r|^2 + c_2|r|^4 + c_3|r|^6)r\right)_x\right. \\ &\left.+\nu_2\left(c_1|r|^2 + c_2|r|^4 + c_3|r|^6\right)_x r + \theta_2(c_1|r|^2 + c_2|r|^4 + c_3|r|^6)r_x\right\}\end{aligned} \tag{4.385}$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.384. ve eşitlik 4.385.'de yerine yazılarak reel ve imajiner kısımlar sırasıyla

$$\begin{aligned}&(a_l - b_l v) P_l'' + \left(b_l\kappa\omega - \omega - a_l\kappa^2 - \alpha_l\kappa\right) P_l + (\xi_l - \kappa(\beta_l + \theta_l))\left(c_1 P_l^3 + c_2 P_l^5 + c_3 P_l^7\right) \\ &+ (c_1 + c_2 P_{\bar{l}}^2 + c_3 P_{\bar{l}}^4)\eta_l P_l P_{\bar{l}}^2 - Q_l P_{\bar{l}} = 0\end{aligned} \tag{4.386}$$

$$\begin{aligned}&(v + 2a_l\kappa + \alpha_l - b_l(v\kappa + \omega)) P_l' + c_1(3\beta_l + \theta_l + 2\nu_l) P_l^2 P_l' \\ &+ c_2(5\beta_l + \theta_l + 4\nu_l) P_l^4 P_l' + c_3(7\beta_l + \theta_l + 6\nu_l) P_l^6 P_l' = 0\end{aligned} \tag{4.387}$$

olarak elde edilir. İmajiner kısımdan, solitonun hızı eşitlik 4.10.'daki gibi bulunur ve ayrıca

$$3\beta_l + \theta_l + 2\nu_l = 0 \tag{4.388}$$

$$5\beta_l + \theta_l + 4\nu_l = 0 \tag{4.389}$$

$$7\beta_l + \theta_l + 6\nu_l = 0 \tag{4.390}$$

kısıtlama şartları elde edilir ve bu taktirde eşitlik 4.388. - eşitlik 4.390.'den

$$\beta_l + \nu_l = 0 \tag{4.391}$$

$$\beta_l + \theta_l = 0 \tag{4.392}$$

$$\theta_l = \nu_l \tag{4.393}$$

eşitlikleri çıkarılır. Dolayısıyla, eşitlik 4.11.'in yardımıyla, eşitlik 4.386. ile verilen reel kısım

$$\begin{aligned}&(a-bv)\,P_l''+\left(b\kappa\omega-\omega-a\kappa^2-\alpha\kappa\right)P_l+\left(\xi_l-\kappa(\beta_l+\theta_l)\right)\left(c_1P_l^3+c_2P_l^5+c_3P_l^7\right)\\&+(c_1+c_2P_{\bar{l}}^2+c_3P_{\bar{l}}^4)\eta_lP_lP_{\bar{l}}^2-Q_lP_{\bar{l}}=0\end{aligned}\tag{4.394}$$

biçiminde yeniden yazılabilir. Balans prensibinden

$$P_{\bar{l}}=P_l\tag{4.395}$$

bulunur ve böylece eşitlik 4.394.

$$\begin{aligned}&(a-bv)\,P_l''+\left(b\kappa\omega-\omega-a\kappa^2-\alpha\kappa\right)P_l+\left(\xi_l+\eta_l-\kappa(\beta_l+\theta_l)\right)\\&\times\left(c_1P_l^3+c_2P_l^5+c_3P_l^7\right)-Q_lP_l=0\end{aligned}\tag{4.396}$$

halini alır. Kapalı form çözümler keşfetmek için

$$P_l=U_l^{\frac{1}{3}}\tag{4.397}$$

dönüşümü uygulanır ve böylece, eşitlik 4.396.

$$\begin{aligned}&3(a-bv)U_lU_l''-2(a-bv)(U_l')^2-9\left(\omega+\kappa(a\kappa+\alpha-b\omega)+Q_l\right)U_l^2\\&+9c_3\left(\eta_l+\xi_l-\kappa(\beta_l+\theta_l)\right)U_l^4=0\end{aligned}\tag{4.398}$$

denklemine dönüşür. Homojen balans prensibine göre, son denklemin çözümü (eşitlik 4.398.)

$$\begin{aligned}U_l(\zeta)=&\alpha_0^{(l)}+\alpha_1^{(l)}\left(\frac{G'}{G}\right)+\beta_1^{(l)}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}+\gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1}\\&+\delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}\end{aligned}\tag{4.399}$$

formundadır. Bu durumda çözüm setleri aşağıdaki gibi listelenir:

1. Set:

$$
\begin{aligned}
&\alpha_0^{(l)} = \alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,\\
&\beta_1^{(l)} = \pm\frac{2\sqrt{\mu(a-bv)}}{3\sqrt{c_3\sigma\chi_l}},\\
&\omega = \frac{9\kappa(a\kappa+\alpha)+\mu(a-bv)+9Q_l}{9b\kappa-9}.
\end{aligned} \tag{4.400}
$$

2. Set:

$$
\begin{aligned}
&\alpha_0^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,\\
&\alpha_1^{(l)} = \pm\frac{2\sqrt{a-bv}}{3\sqrt{c_3\chi_l}},\\
&\gamma_1^{(l)} = \pm\frac{2\mu\sqrt{a-bv}}{3\sqrt{c_3\chi_l}},\\
&\omega = \frac{9\kappa(a\kappa+\alpha)+4\mu(a-bv)+9Q_l}{9b\kappa-9}.
\end{aligned} \tag{4.401}
$$

Burada κ ve μ keyfi sabitlerdir. Ayrıca

$$\chi_l = \kappa\left(\beta_l+\theta_l\right) - \eta_l - \xi_l \tag{4.402}$$

ile verilir. Böylece, kübik-kuintik-septik yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki çözümler türetilir:

$\mu < 0$ iken, aşağıdaki hiperbolik fonksiyon çözümler elde edilir:

$$
\begin{aligned}
q(x,t) = &\left\{\pm\frac{2}{3}i\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3}}\\
&\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.403}
$$

$$
\begin{aligned}
r(x,t) = &\left\{\pm\frac{2}{3}i\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3}}\\
&\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]
\end{aligned} \tag{4.404}
$$

$$q(x,t)=\left\{\pm\frac{2}{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.405}$$

$$r(x,t)=\left\{\pm\frac{2}{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.\right.$$
$$\left.\left.-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.406}$$

$\mu>0$ için, aşağıdaki trigonometrik fonksiyon çözümler bulunur:

$$q(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.407}$$

$$r(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.408}$$

$$q(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.409}$$

$$r(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.410}$$

Son olarak, $\mu=0$ ise, aşağıdaki rasyonel fonksiyon çözümler keşfedilir:

$$q(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{a-bv}{c_3\chi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.411}$$

$$r(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{a-bv}{c_3\chi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{3}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.412}$$

Özel durumlar aşağıdaki gibi sunulur:

$\mu<0$ ve $A_1^2>A_2^2$ iken, eşitlik 4.403. - eşitlik 4.406. ile verilen çözümler, aşağıdaki complexiton çözümlere

$$q(x,t)=\left\{\pm\frac{2}{3}i\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.413}$$

$$r(x,t)=\left\{\pm\frac{2}{3}i\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.414}$$

ve tekil soliton çözümlere

$$q(x,t)=\left\{\mp\frac{4}{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_1}}\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+4\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.415}$$

$$r(x,t)=\left\{\mp\frac{4}{3}\sqrt{-\frac{\mu(a-bv)}{c_3\chi_2}}\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+4\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.416}$$

dönüşür. Burada $\zeta_0=\tanh^{-1}(A_2/A_1)$.

$\mu>0$ ise, eşitlik 4.407. - eşitlik 4.410. ile verilen çözümler

$$q(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\,\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.417}$$

$$r(x,t)=\left\{\pm\frac{2}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\,\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.418}$$

$$q(x,t)=\left\{\mp\frac{4}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_1}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+4\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.419}$$

$$r(x,t)=\left\{\mp\frac{4}{3}\sqrt{\frac{\mu(a-bv)}{c_3\chi_2}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{9\kappa(a\kappa+\alpha)+4\mu(a-bv)+9Q_l}{9b\kappa-9}\right)t+\theta\right)\right] \tag{4.420}$$

periyodik dalga çözümlere dönüşür. Burada $\zeta_0=\tan^{-1}(A_1/A_2)$.

4.9. Üçlü-Kuvvet Yasası

Bu yasa, çift-kuvvet yasası nonlineerliğinin bir genişlemesi ve kübik-kuintik-septik yasası nonlineerliğinin bir genelleştirmesidir. Bu durumda c_1, c_2 ve c_3 sabitler olmak üzere $F(s)=c_1s^n+c_2s^{2n}+c_3s^{3n}$ (Kaup ve Yang, 2000; Biswas ve Milovic, 2009). Bu yüzden, üçlü-kuvvet yasası nonlineerliğine sahip çiftlenmiş NLS denklem sistemi

$$iq_t+a_1q_{xx}+b_1q_{xt}+\left\{\xi_1(c_1|q|^{2n}+c_2|q|^{4n}+c_3|q|^{6n})\right.$$
$$\left.+\eta_1(c_1|r|^{2n}+c_2|r|^{4n}+c_3|r|^{6n})\right\}q=Q_1r+i\left\{\alpha_1q_x\right.$$
$$+\beta_1\left((c_1|q|^{2n}+c_2|q|^{4n}+c_3|q|^{6n})q\right)_x+\nu_1\left(c_1|q|^{2n}+c_2|q|^{4n}+c_3|q|^{6n}\right)_xq$$
$$\left.+\theta_1(c_1|q|^{2n}+c_2|q|^{4n}+c_3|q|^{6n})q_x\right\} \tag{4.421}$$

$$ir_t+a_2r_{xx}+b_2r_{xt}+\left\{\xi_2(c_1|r|^{2n}+c_2|r|^{4n}+c_3|r|^{6n})\right.$$
$$\left.+\eta_2(c_1|q|^{2n}+c_2|q|^{4n}+c_3|q|^{6n})\right\}r=Q_2q+i\left\{\alpha_2r_x\right.$$
$$+\beta_2\left((c_1|r|^{2n}+c_2|r|^{4n}+c_3|r|^{6n})r\right)_x+\nu_2\left(c_1|r|^{2n}+c_2|r|^{4n}+c_3|r|^{6n}\right)_xr$$
$$\left.+\theta_2(c_1|r|^{2n}+c_2|r|^{4n}+c_3|r|^{6n})r_x\right\} \tag{4.422}$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.421. ve eşitlik 4.422.'de yerine yazılarak sırasıyla, reel ve imajiner kısımlar

$$(a_l-b_lv)P_l''+\left(b_l\kappa\omega-\omega-a_l\kappa^2-\alpha_l\kappa\right)P_l+(\xi_l-\kappa(\beta_l+\theta_l))$$
$$\times\left(c_1P_l^{2n+1}+c_2P_l^{4n+1}+c_3P_l^{6n+1}\right)+(c_1+c_2P_{\bar{l}}^{2n}+c_3P_{\bar{l}}^{4n})\eta_lP_lP_{\bar{l}}^{2n}-Q_lP_{\bar{l}}=0 \tag{4.423}$$

$$(v + 2a_l\kappa + \alpha_l - b_l(v\kappa + \omega))\, P_l' + c_1\left((2n+1)\beta_l + \theta_l + 2n\nu_l\right) P_l^{2n} P_l'$$
$$+ c_2\left((4n+1)\beta_l + \theta_l + 4n\nu_l\right) P_l^{4n} P_l' + c_3\left((6n+1)\beta_l + \theta_l + 6n\nu_l\right) P_l^{6n} P_l' = 0 \quad (4.424)$$

elde edilir. İmajiner kısımdan, solitonun hızı eşitlik 4.10.'daki gibi bulunur ve ayrıca kısıtlama şartları

$$(2n+1)\beta_l + \theta_l + 2n\nu_l = 0 \quad (4.425)$$

$$(4n+1)\beta_l + \theta_l + 4n\nu_l = 0 \quad (4.426)$$

$$(6n+1)\beta_l + \theta_l + 6n\nu_l = 0 \quad (4.427)$$

biçiminde ortaya çıkar. Bu taktirde, eşitlik 4.425. - eşitlik 4.427.'den

$$\beta_l + \nu_l = 0 \quad (4.428)$$

$$\beta_l + \theta_l = 0 \quad (4.429)$$

$$\theta_l = \nu_l \quad (4.430)$$

ilişkileri keşfedilir. Dolayısıyla, eşitlik 4.11. ile, reel kısım

$$(a - bv)\, P_l'' + \left(b\kappa\omega - \omega - a\kappa^2 - \alpha\kappa\right) P_l + (\xi_l - \kappa(\beta_l + \theta_l))$$
$$\times \left(c_1 P_l^{2n+1} + c_2 P_l^{4n+1} + c_3 P_l^{6n+1}\right) + (c_1 + c_2 P_{\bar{l}}^{2n} + c_3 P_{\bar{l}}^{4n})\eta_l P_l P_{\bar{l}}^{2n} - Q_l P_{\bar{l}} = 0$$
$$(4.431)$$

şeklinde yeniden yazılabilir. Balans prensibinden

$$P_{\bar{l}} = P_l \quad (4.432)$$

eşitliğine ulaşılır ve böylece eşitlik 4.431.

$$(a - bv)\, P_l'' + \left(b\kappa\omega - \omega - a\kappa^2 - \alpha\kappa\right) P_l + (\xi_l + \eta_l - \kappa(\beta_l + \theta_l))$$
$$\times \left(c_1 P_l^{2n+1} + c_2 P_l^{4n+1} + c_3 P_l^{6n+1}\right) - Q_l P_l = 0 \quad (4.433)$$

denklemine indirgenir. Benimsenen modele kapalı form çözümler araştırmak için

$$P_l = U_l^{\frac{1}{3n}} \quad (4.434)$$

dönüşümü yapılır ve bu durumda, eşitlik 4.433.

$$3n(a-bv)U_lU_l''-(3n-1)(a-bv)(U_l')^2-9n^2\left(\omega+\kappa(a\kappa+\alpha-b\omega)+Q_l\right)U_l^2$$
$$+9c_3n^2\left(\eta_l+\xi_l-\kappa(\beta_l+\theta_l)\right)U_l^4=0 \qquad (4.435)$$

denklemine dönüşür. Homojen balans prensibinden

$$U_l(\zeta)=\alpha_0^{(l)}+\alpha_1^{(l)}\left(\frac{G'}{G}\right)+\beta_1^{(l)}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}+\gamma_1^{(l)}\left(\frac{G'}{G}\right)^{-1}$$
$$+\delta_1^{(l)}\frac{1}{\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}} \qquad (4.436)$$

sonucuna ulaşılabilir. Bu durumda çözüm setleri aşağıdaki formda numaralandırılır:

1. Set:

$$\alpha_0^{(l)}=\alpha_1^{(l)}=\gamma_1^{(l)}=\delta_1^{(l)}=0,$$

$$\beta_1^{(l)}=\pm\frac{\sqrt{\mu(3n+1)(a-bv)}}{3\sqrt{c_3\left(-n^2\right)\sigma\chi_l}}, \qquad (4.437)$$

$$\omega=\frac{\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}.$$

2. Set:

$$\alpha_0^{(l)}=\beta_1^{(l)}=\delta_1^{(l)}=0,$$

$$\alpha_1^{(l)}=\pm\frac{i\sqrt{(3n+1)(a-bv)}}{3\sqrt{c_3n^2\chi_l}},$$

$$\gamma_1^{(l)}=\pm\frac{i\mu\sqrt{(3n+1)(a-bv)}}{3\sqrt{c_3n^2\chi_l}}, \qquad (4.438)$$

$$\omega=\frac{4\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}.$$

Burada κ ve μ keyfi sabitlerdir. Ayrıca

$$\chi_l=\eta_l+\xi_l-\kappa\left(\beta_l+\theta_l\right) \qquad (4.439)$$

ile verilir. Sonuç olarak, bu çözüm setlerinin yardımıyla, üçlü-kuvvet yasası nonlineerliğine sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki çözümler elde edilir:

$\mu < 0$ iken

$$q(x,t) = \left\{ \pm\frac{1}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}} \times\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.440}$$

$$r(x,t) = \left\{ \pm\frac{1}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}} \times\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.441}$$

$$q(x,t) = \left\{ \mp\frac{i}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}} \left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} - \left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.442}$$

$$r(x,t) = \left\{ \mp\frac{i}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}} \left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)} - \left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3n}} \exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.443}$$

$\mu > 0$ ise

$$q(x,t) = \left\{ \pm\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3n}} \times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.444}$$

$$r(x,t)=\left\{\pm\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.445}$$

$$q(x,t)=\left\{\pm\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.446}$$

$$r(x,t)=\left\{\pm\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.\right.$$
$$\left.\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\right\}^{\frac{1}{3n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.447}$$

Son olarak, $\mu=0$ için

$$q(x,t)=\left\{\pm\frac{i}{3n}\sqrt{\frac{(3n+1)(a-bv)}{c_3\chi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{3n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.448}$$

$$r(x,t)=\left\{\pm\frac{i}{3n}\sqrt{\frac{(3n+1)(a-bv)}{c_3\chi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\right\}^{\frac{1}{3n}}\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \tag{4.449}$$

Özel durumlar aşağıdaki gibi sunulur:

$\mu<0$ ve $A_1^2>A_2^2$ iken, eşitlik 4.440 - eşitlik 4.443. ile verilen çözümler, aşağıdaki parlak soliton çözümlere ve complexiton çözümlere dönüşür:

$$q(x,t)=\left\{\pm\frac{1}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.450}$$

$$r(x,t)=\left\{\pm\frac{1}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \tag{4.451}$$

$$q(x,t)=\left\{\mp\frac{2i}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.452)$$

$$r(x,t)=\left\{\mp\frac{2i}{3n}\sqrt{-\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\operatorname{csch}\left[2\sqrt{-\mu}\left\{x-\left(\tfrac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right]. \quad (4.453)$$

Burada $\zeta_0=\tanh^{-1}(A_2/A_1)$.

$\mu>0$ ise, eşitlik 4.444. - eşitlik 4.447. ile verilen çözümler sırasıyla

$$q(x,t)=\left\{\mp\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.454)$$

$$r(x,t)=\left\{\mp\frac{i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.455)$$

$$q(x,t)=\left\{\pm\frac{2i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_1}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.456)$$

$$r(x,t)=\left\{\pm\frac{2i}{3n}\sqrt{\frac{\mu(3n+1)(a-bv)}{c_3\chi_2}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\right\}^{\frac{1}{3n}}$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{4\mu(a-bv)+9\kappa n^2(a\kappa+\alpha)+9n^2Q_l}{9n^2(b\kappa-1)}\right)t+\theta\right)\right] \quad (4.457)$$

periyodik dalga çözümlere dönüşür. Burada $\zeta_0=\tan^{-1}(A_1/A_2)$.

4.10. Zayıf Yerel-Olmayan Nonlineerliğe Sahip Parabolik Yasası

Bu yasa c_1, c_2 ve c_3 sabitler olmak üzere $F(s)=c_1s+c_2s^2+c_3s_{xx}$ şeklinde ifade edilir (Zhou ve ark., 2013). Bu yüzden, zayıf yerel-olmayan nonlineerliğe sahip parabolik yasası için çiftlenmiş NLS denklem sistemi

$$
\begin{aligned}
& iq_t + a_1 q_{xx} + b_1 q_{xt} + \left\{\xi_1 (c_1|q|^2 + c_2|q|^4 + c_3(|q|^2)_{xx})\right.\\
& \left. +\eta_1 (c_1|r|^2 + c_2|r|^4 + c_3(|r|^2)_{xx})\right\} q = Q_1 r + i \left\{\alpha_1 q_x\right.\\
& +\beta_1 \left((c_1|q|^2 + c_2|q|^4 + c_3(|q|^2)_{xx})q\right)_x + \nu_1 \left((c_1|q|^2 + c_2|q|^4 + c_3(|q|^2)_{xx})\right)_x q\\
& \left. +\theta_1 (c_1|q|^2 + c_2|q|^4 + c_3(|q|^2)_{xx}) q_x\right\}
\end{aligned} \tag{4.458}
$$

$$
\begin{aligned}
& ir_t + a_2 r_{xx} + b_2 r_{xt} + \left\{\xi_2 (c_1|r|^2 + c_2|r|^4 + c_3(|r|^2)_{xx})\right.\\
& \left. +\eta_2 (c_1|q|^2 + c_2|q|^4 + c_3(|q|^2)_{xx})\right\} r = Q_2 q + i \left\{\alpha_2 r_x\right.\\
& +\beta_2 \left((c_1|r|^2 + c_2|r|^4 + c_3(|r|^2)_{xx})r\right)_x + \nu_2 \left((c_1|r|^2 + c_2|r|^4 + c_3(|r|^2)_{xx})\right)_x r\\
& \left. +\theta_2 (c_1|r|^2 + c_2|r|^4 + c_3(|r|^2)_{xx}) r_x\right\}
\end{aligned} \tag{4.459}
$$

ile verilir. Eşitlik 4.3. ve eşitlik 4.4., eşitlik 4.458. ve eşitlik 4.459.'de yerine yazılarak reel ve imajiner kısımlar sırasıyla aşağıdaki gibi bulunur:

$$
\begin{aligned}
& P_l \left(a_l\kappa^2 + \alpha_l\kappa - b_l\kappa\omega + 2\beta_l c_3 \kappa P_l'^2 - 2c_3\eta_l P_{\bar{l}}'^2 - 2c_3\eta_l P_{\bar{l}} P_{\bar{l}}'' - \eta_l P_{\bar{l}}^2 \left(c_2 P_{\bar{l}}^2 + c_1\right)\right.\\
& \left. +2c_3\theta_l\kappa P_l'^2 - 2c_3\xi_l P_l'^2 + \omega\right) + (b_l v - a_l) P_l'' + 2c_3 {P_l}^2 P_l'' \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right)\\
& + c_2 {P_l}^5 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + c_1 {P_l}^3 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + Q_l P_{\bar{l}} = 0
\end{aligned} \tag{4.460}
$$

$$
\begin{aligned}
& P_l' \left(2a_l\kappa + \alpha_l - b_l(\kappa v + \omega) + P_l \left(2c_3 \left(4\beta_l + \theta_l + 3\nu_l\right) P_l'' + c_2 {P_l}^3 \left(5\beta_l + \theta_l + 4\nu_l\right)\right.\right.\\
& \left.\left. +c_1 P_l \left(3\beta_l + \theta_l + 2\nu_l\right)\right) + v\right) + 2c_3 \left(\beta_l + \theta_l\right) P_l'^3 + 2c_3 \left(\beta_l + \nu_l\right) {P_l}^2 {P_l}^{(3)} = 0.
\end{aligned} \tag{4.461}
$$

İmajiner kısımdan, solitonun hızı eşitlik 4.10.'daki gibi keşfedilir ve ayrıca ortaya çıkan kısıtlama şartları

$$4\beta_l + \theta_l + 3\nu_l = 0 \tag{4.462}$$

$$5\beta_l + \theta_l + 4\nu_l = 0 \tag{4.463}$$

$$3\beta_l + \theta_l + 2\nu_l = 0 \tag{4.464}$$

eşitlikleri ile verilir ve buradan

$$\beta_l + \theta_l = 0 \tag{4.465}$$

$$\beta_l + \nu_l = 0 \tag{4.466}$$

$$\nu_l = \theta_l \tag{4.467}$$

ilişkilerine ulaşılır. Dolayısıyla, eşitlik 4.11.'in yardımıyla, eşitlik 4.460 ile verilen reel kısım aşağıdaki gibi yeniden yazılabilir:

$$\begin{aligned}
&P_l \left(a\kappa^2 + \alpha\kappa - b\kappa\omega + 2\beta_l c_3 \kappa P_l'^2 - 2c_3\eta_l P_{\bar{l}}'^2 - 2c_3\eta_l P_{\bar{l}} P_{\bar{l}}'' - \eta_l P_{\bar{l}}^2 \left(c_2 P_{\bar{l}}^2 + c_1\right)\right. \\
&\left. +2c_3\theta_l\kappa P_l'^2 - 2c_3\xi_l P_l'^2 + \omega\right) + (bv - a) P_l'' + 2c_3 {P_l}^2 P_l'' \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) \\
&+ c_2 {P_l}^5 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + c_1 {P_l}^3 \left(\kappa\left(\beta_l + \theta_l\right) - \xi_l\right) + Q_l P_{\bar{l}} = 0.
\end{aligned} \tag{4.468}$$

Balans prensibinden

$$P_{\bar{l}} = P_l \tag{4.469}$$

ifadesine ulaşılır ve eşitlik 4.468.

$$\begin{aligned}
&P_l \left(a\kappa^2 + \alpha\kappa - b\kappa\omega - 2c_3\left(\eta_l + \xi_l\right) P_l'^2 + Q_l + \omega\right) + (bv - a) P_l'' \\
&- 2c_3\left(\eta_l + \xi_l\right) {P_l}^2 P_l'' - c_2\left(\eta_l + \xi_l\right) {P_l}^5 - c_1\left(\eta_l + \xi_l\right) {P_l}^3 = 0
\end{aligned} \tag{4.470}$$

denklemine indirgenir. Eşitlik 4.470.'deki $P_l^2 P_l''$ ile P_l^5 terimlerinin dengelenmesi ile $M = 1$ bulunur. Böylece eşitlik 4.470.'in formal çözümü

$$\begin{aligned}
P_l(\zeta) =& \alpha_0^{(l)} + \alpha_1^{(l)} \left(\frac{G'}{G}\right) + \beta_1^{(l)} \sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + \gamma_1^{(l)} \left(\frac{G'}{G}\right)^{-1} \\
&+ \delta_1^{(l)} \frac{1}{\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}}
\end{aligned} \tag{4.471}$$

formundadır. Bu durumda elde edilen çözüm setleri aşağıdaki gibidir:

1. Set:

$$\begin{aligned}
&\alpha_0^{(l)} = \beta_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0, \\
&\alpha_1^{(l)} = \pm\frac{\sqrt{2bv - 2a}}{\sqrt{\left(8c_3\mu + c_1\right)\left(\eta_l + \xi_l\right)}}, \\
&\omega = \frac{c_1\left(a\kappa^2 + \alpha\kappa - 2a\mu + 2b\mu v + Q_l\right) + 4c_3\mu\left(2\kappa(a\kappa + \alpha) - 3\mu(a - bv) + 2Q_l\right)}{\left(b\kappa - 1\right)\left(8c_3\mu + c_1\right)}, \\
&c_2 = \frac{3c_3\left(8c_3\mu + c_1\right)\left(\eta_l + \xi_l\right)}{a - bv}.
\end{aligned} \tag{4.472}$$

2. Set:

$$\alpha_0^{(l)} = \alpha_1^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\beta_1^{(l)} = \pm \frac{\sqrt{2}\sqrt{\mu(a-bv)}}{\sqrt{-\sigma\left(c_1 - 4c_3\mu\right)\left(\eta_l + \xi_l\right)}},$$

$$\omega = \frac{\kappa(a\kappa + \alpha) + \mu(a-bv) + Q_l}{b\kappa - 1}, \tag{4.473}$$

$$c_2 = \frac{3c_3\left(c_1 - 4c_3\mu\right)\left(\eta_l + \xi_l\right)}{a - bv}.$$

3. Set:

$$\alpha_0^{(l)} = \alpha_1^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\gamma_1^{(l)} = \pm \frac{\sqrt{2}\sqrt{\mu^2(bv-a)}}{\sqrt{\left(8c_3\mu + c_1\right)\left(\eta_l + \xi_l\right)}},$$

$$\omega = \frac{c_1\left(a\kappa^2 + \alpha\kappa - 2a\mu + 2b\mu v + Q_l\right) + 4c_3\mu\left(2\kappa(a\kappa + \alpha) - 3\mu(a-bv) + 2Q_l\right)}{\left(b\kappa - 1\right)\left(8c_3\mu + c_1\right)},$$

$$c_2 = \frac{3c_3\left(8c_3\mu + c_1\right)\left(\eta_l + \xi_l\right)}{a - bv}. \tag{4.474}$$

4. Set:

$$\alpha_0^{(l)} = \gamma_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_1^{(l)} = \pm \frac{\sqrt{bv-a}}{\sqrt{2}\sqrt{(2c_3\mu + c_1)(\eta_l + \xi_l)}},$$

$$\beta_1^{(l)} = \frac{\sqrt{b\mu v - a\mu}}{\sqrt{2}\sqrt{\sigma(2c_3\mu + c_1)(\eta_l + \xi_l)}},$$

$$\omega = \frac{2c_1(2a\kappa^2 + 2\alpha\kappa - a\mu + b\mu v + 2Q_l) + c_3\mu(8\kappa(a\kappa + \alpha) - 3\mu(a - bv) + 8Q_l)}{4(b\kappa - 1)(2c_3\mu + c_1)},$$

$$c_2 = \frac{3c_3(2c_3\mu + c_1)(\eta_l + \xi_l)}{a - bv}. \tag{4.475}$$

5. Set:

$$\alpha_0^{(l)} = \beta_1^{(l)} = \delta_1^{(l)} = 0,$$

$$\alpha_1^{(l)} = \pm \frac{\sqrt{2}\sqrt{a-bv}}{\sqrt{(16c_3\mu - c_1)(\eta_l + \xi_l)}},$$

$$\gamma_1^{(l)} = \pm \frac{\sqrt{2}\mu\sqrt{a-bv}}{\sqrt{(16c_3\mu - c_1)(\eta_l + \xi_l)}}, \tag{4.476}$$

$$\omega = \frac{\kappa(a\kappa + \alpha) + 4\mu(a - bv) + Q_l}{b\kappa - 1},$$

$$c_2 = \frac{3c_3(c_1 - 16c_3\mu)(\eta_l + \xi_l)}{a - bv}.$$

Burada κ ve μ keyfi sabitlerdir. Sonuç olarak, bu çözüm setlerinin yardımıyla, zayıf yerel-olmayan nonlineerliğe sahip parabolik yasasına sahip manyeto-optik dalga kılavuzlarında çiftlenmiş NLS denklem sistemi için aşağıdaki çözümler keşfedilir:

$\mu < 0$ iken

$$q(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu + c_1)(\eta_1 + \xi_1)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta) + A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta) + A_2\sinh(\sqrt{-\mu}\zeta)}\right)$$
$$\times \exp\left[i\left(-\kappa x + \omega t + \theta\right)\right] \tag{4.477}$$

$$r(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.478}$$

$$q(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_1+\xi_1)}}\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.479}$$

$$r(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_2+\xi_2)}}\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.480}$$

$$q(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.481}$$

$$r(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.482}$$

$$q(x,t) =\sqrt{\frac{\mu(a-bv)}{(4c_3\mu+2c_1)(\eta_1+\xi_1)}}\left(\pm\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.$$
$$\left.+i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.483}$$

$$r(x,t) =\sqrt{\frac{\mu(a-bv)}{(4c_3\mu+2c_1)(\eta_2+\xi_2)}}\left(\pm\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.$$
$$\left.+i\sqrt{1-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \tag{4.484}$$

$$q(x,t)=\pm\sqrt{-\frac{2\mu(a-bv)}{(16c_3\mu-c_1)(\eta_1+\xi_1)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.$$
$$\left.-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.485)$$

$$r(x,t)=\pm\sqrt{-\frac{2\mu(a-bv)}{(16c_3\mu-c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right.$$
$$\left.-\left(\frac{A_1\sinh(\sqrt{-\mu}\zeta)+A_2\cosh(\sqrt{-\mu}\zeta)}{A_1\cosh(\sqrt{-\mu}\zeta)+A_2\sinh(\sqrt{-\mu}\zeta)}\right)^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right].$$
$$(4.486)$$

$\mu>0$ ise

$$q(x,t)=\pm\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.487)$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.488)$$

$$q(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_1+\xi_1)}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.489)$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_2+\xi_2)}}\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.490)$$

$$q(x,t)=\pm\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.491)$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}$$
$$\times\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.492)$$

$$q(x,t) = \sqrt{\frac{\mu(bv-a)}{(4c_3\mu+2c_1)(\eta_1+\xi_1)}}\left(\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.493)$$

$$r(x,t) = \sqrt{\frac{\mu(bv-a)}{(4c_3\mu+2c_1)(\eta_2+\xi_2)}}\left(\pm\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\sqrt{1+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.494)$$

$$q(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(16c_3\mu-c_1)(\eta_1+\xi_1)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.495)$$

$$r(x,t) = \pm\sqrt{\frac{2\mu(a-bv)}{(16c_3\mu-c_1)(\eta_2+\xi_2)}}\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right.$$
$$\left.+\left(\frac{A_1\cos(\sqrt{\mu}\zeta)-A_2\sin(\sqrt{\mu}\zeta)}{A_1\sin(\sqrt{\mu}\zeta)+A_2\cos(\sqrt{\mu}\zeta)}\right)^{-1}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right]. \quad (4.496)$$

Son olarak, $\mu = 0$ için, rasyonel fonksiyon çözümler elde edilir. Eşitlik 4.334. ve eşitlik 4.338. ile verilen çözüm setlerinden

$$q(x,t) = \pm\sqrt{\frac{2(bv-a)}{c_1(\eta_1+\xi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.497)$$

$$r(x,t) = \pm\sqrt{\frac{2(bv-a)}{c_1(\eta_2+\xi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.498)$$

elde edilir ve ayrıca eşitlik 4.475.'in kullanılmasıyla

$$q(x,t) = \pm\sqrt{\frac{bv-a}{2c_1(\eta_1+\xi_1}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.499)$$

$$r(x,t) = \pm\sqrt{\frac{bv-a}{2c_1(\eta_2+\xi_2}}\left(\frac{A_1}{A_1\zeta+A_2}\right)\exp\left[i\left(-\kappa x+\omega t+\theta\right)\right] \quad (4.500)$$

bulunur.

Özel durumlar aşağıdaki gibi sergilenir:

$\mu < 0$ ve $A_1^2 > A_2^2$ iken, eşitlik 4.477. - eşitlik 4.486. ile verilen çözümlerden sırasıyla, aşağıdaki optik soliton çözümler oluşur:

$$q(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.501}$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.502}$$

$$q(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_1+\xi_1)}}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.503}$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_2+\xi_2)}}\operatorname{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.504}$$

$$q(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.505}$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\coth\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.506}$$

$$q(x,t)=\sqrt{\frac{\mu(a-bv)}{(4c_3\mu+2c_1)(\eta_1+\xi_1)}}\left(\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\left.+i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\tfrac{2c_1\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+c_3\mu(8\kappa(a\kappa+\alpha)-3\mu(a-bv)+8Q_l)}{4(b\kappa-1)(2c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.507}$$

$$r(x,t)=\sqrt{\frac{\mu(a-bv)}{(4c_3\mu+2c_1)(\eta_2+\xi_2)}}\left(\pm\tanh\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\left.+i\,\mathrm{sech}\left[\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\tfrac{2c_1\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+c_3\mu(8\kappa(a\kappa+\alpha)-3\mu(a-bv)+8Q_l)}{4(b\kappa-1)(2c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.508}$$

$$q(x,t)=\mp\sqrt{-\frac{8\mu(a-bv)}{(16c_3\mu-c_1)(\eta_1+\xi_1)}}\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.509}$$

$$r(x,t)=\mp\sqrt{-\frac{8\mu(a-bv)}{(16c_3\mu-c_1)(\eta_2+\xi_2)}}\,\mathrm{csch}\left[2\sqrt{-\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]. \tag{4.510}$$

Burada $\zeta_0=\tanh^{-1}(A_2/A_1)$.

$\mu>0$ ise, eşitlik 4.487. - eşitlik 4.496.'deki çözümler sırasıyla, aşağıdaki periyodik dalga çözümlere dönüşür:

$$q(x,t)=\mp\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\tfrac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.511}$$

$$r(x,t)=\mp\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.512}$$

$$q(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_1+\xi_1)}}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.513}$$

$$r(x,t)=\pm\sqrt{\frac{2\mu(a-bv)}{(4c_3\mu-c_1)(\eta_2+\xi_2)}}\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right] \tag{4.514}$$

$$q(x,t)=\mp\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_1+\xi_1)}}\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.515}$$

$$r(x,t)=\mp\sqrt{\frac{2\mu(bv-a)}{(8c_3\mu+c_1)(\eta_2+\xi_2)}}\cot\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{c_1\left(a\kappa^2+\alpha\kappa-2a\mu+2b\mu v+Q_l\right)+4c_3\mu(2\kappa(a\kappa+\alpha)-3\mu(a-bv)+2Q_l)}{(b\kappa-1)(8c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.516}$$

$$q(x,t)=\sqrt{\frac{\mu(bv-a)}{(4c_3\mu+2c_1)(\eta_1+\xi_1)}}\left(\mp\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.$$
$$\left.+\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)$$
$$\times\exp\left[i\left(-\kappa x+\left(\frac{2c_1\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+c_3\mu(8\kappa(a\kappa+\alpha)-3\mu(a-bv)+8Q_l)}{4(b\kappa-1)(2c_3\mu+c_1)}\right)t+\theta\right)\right] \tag{4.517}$$

$$
\begin{aligned}
r(x,t) = & \sqrt{\frac{\mu(bv-a)}{(4c_3\mu+2c_1)(\eta_2+\xi_2)}}\left(\mp\tan\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right.\\
& \left.+\sec\left[\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+\zeta_0\right]\right)\\
& \times\exp\left[i\left(-\kappa x+\left(\frac{2c_1\left(2a\kappa^2+2\alpha\kappa-a\mu+b\mu v+2Q_l\right)+c_3\mu(8\kappa(a\kappa+\alpha)-3\mu(a-bv)+8Q_l)}{4(b\kappa-1)(2c_3\mu+c_1)}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.518}
$$

$$
\begin{aligned}
q(x,t) = & \mp\sqrt{\frac{8\mu(a-bv)}{(16c_3\mu-c_1)(\eta_1+\xi_1)}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\\
& \times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right]
\end{aligned}
\tag{4.519}
$$

$$
\begin{aligned}
r(x,t) = & \mp\sqrt{\frac{8\mu(a-bv)}{(16c_3\mu-c_1)(\eta_2+\xi_2)}}\csc\left[2\sqrt{\mu}\left\{x-\left(\frac{b\omega-2a\kappa-\alpha}{1-b\kappa}\right)t\right\}+2\zeta_0\right]\\
& \times\exp\left[i\left(-\kappa x+\left(\frac{\kappa(a\kappa+\alpha)+4\mu(a-bv)+Q_l}{b\kappa-1}\right)t+\theta\right)\right].
\end{aligned}
\tag{4.520}
$$

Burada $\zeta_0 = \tan^{-1}(A_1/A_2)$.

5. SONUÇ VE ÖNERİLER

Nonlineer ortamın on formu ile manyeto-optik dalga kılavuzları için soliton çözümlerin geniş bir spektrumu elde edilmiştir. Çalışmanın sonuçlarının optoelektronik alanında çeşitli uygulamaları vardır. Özet bölümünde bahsedildiği gibi, parlak soliton çözümler, soliton dağınıklığını kontrol etmede büyük bir varlık olacaktır. Bu, solitonların, dağınıklığı temizleme anlamına gelen bir çekim durumundan ayrılma durumuna dönüştürülebileceği anlamına gelir. Bu nedenle, bu, hayatta kalmak için internetin her gün gerekli olduğu günümüz telekomünikasyon endüstrisi için büyüyen bir sorun olan internet darboğazına bir kolaylık faktörü getirecektir. Tüm iş faaliyetlerinin çevrimiçi yürütüldüğü COVID-19'un mevcut salgın sürecinde, kesintisiz internet iletişimi için düzgün ve sürekli bir darbe akışının olması zorunludur. Aynı şekilde, bir arka plan dalgası mevcut olduğunda koyu solitonlar da soliton iletimine fayda sağlayacaktır. Bununla birlikte, tekil soliton çözümler, solitonların sadece süslü bir formudur ve yalnızca modelden elde edilen soliton çözümlerinin tam bir spektrumunu göstermek için listelenmiştir. Bu çözümler fiber optik iletişimde uygulanabilir değildir. Bu solitonlar, bu yolda daha ileri çalışmalar için temel unsurlardır. Gelecekte, soliton çözümler, çarpan yaklaşımı yardımıyla korunan yoğunluklar geri kazanıldıktan sonra korunan miktarları ortaya çıkarmayı sağlayacaktır. Ardından, soliton pertürbasyon teorisi, yarı monokromatiklik ile parametrelerin adiyabatik (adiabatic) parametre gelişimini verecektir. Daha sonra, ek çalışmalar da bu tür dalga kılavuzlarındaki yarı-durağan soliton çözümlerini ortaya çıkaracaktır. Bunlar uzun dönemde keşfedilecek sadece birkaç yoldur.

KAYNAKLAR

Akhmediev, N. N. (1998). Spatial solitons in Kerr and Kerr-like media. *Optical and Quantum Electronics,* 30(7-10), 535-569.

Akhmediev, N. N., ve Ankiewicz, A. (1997). *Solitons: nonlinear pulses and beams,* İngiltere: Chapman and Hall.

Akhmediev, N., Ankiewicz, A., ve Grimshaw, R. (1999). Hamiltonian-versus-energy diagrams in soliton theory. *Physical Review E,* 59(5), 6088-6096.

Alberucci, A., Assanto, G., MacNeil, J. M. L., ve Smyth, N. F. (2014). Nematic liquid crystals: An excellent playground for nonlocal nonlinear light localization in soft matter. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450046.

Arnous, A. H., Mirzazadeh, M., Moshokoa, S., Medhekar, S., Zhou, Q., Mahmood, M. F., Biswas, A. ve Belic, M. (2015). Solitons in optical metamaterials with trial solution approach and Backlund transform of Riccati equation. *Journal of Computational and Theoretical Nanoscience*, 12(12), 5940-5948.

Arnous, A. H., Mirzazadeh, M., Zhou, Q., Moshokoa, S. P., Biswas, A., ve Belic, M. (2016). Soliton solutions to resonant nonlinear schrödinger's equation with time-dependent coefficients by modified simple equation method. *Optik,* 127(23), 11450-11459.

Arnous, A. H., Ullah, M. Z., Moshokoa, S. P., Zhou, Q., Triki, H., Mirzazadeh, M., ve Biswas, A. (2017a). Optical solitons in birefringent fibers with modified simple equation method. *Optik,* 130, 996-1003.

Arnous, A. H., Ullah, M. Z., Moshokoa, S. P., Zhou, Q., Triki, H., Mirzazadeh, M., ve Biswas, A. (2017b). Optical solitons in nonlinear directional couplers with trial function scheme. *Nonlinear Dynamics,* 88(3), 1891-1915.

Azmi, A., ve Marchant, T. R. (2014). Dispersive shock waves in colloids with temperature dependent compressibility. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450043.

Biswas, A. (2004). Solitons in magneto-optic waveguides. *Applied Mathematics and Computation,* 153(2), 387-393.

Biswas, A., Arnous, A. H., Ekici, M., Sonmezoglu, A., Seadawy, A. R., Zhou, Q., Mahmood M. F., Moshokoa S. P. ve Belic, M. (2018). Optical soliton perturbation in magneto-optic waveguides. *Journal of Nonlinear Optical Physics and Materials,* 27(01), 1850005.

Biswas, A., Fessak, M., Johnson, S., Beatrice, S., Milovic, D., Jovanoski, Z., Kohl, F. ve Majid, F. (2012). Optical soliton perturbation in non-Kerr law media: traveling wave solution. *Optics and Laser Technology,* 44(1), 263-268.

Biswas, A., ve Konar, S. (2006). *Introduction to non-Kerr law optical solitons,* Amerika: CRC press.

Biswas, A., ve Milovic, D. (2009). Travelling wave solutions of the non-linear Schrödinger's equation in non-Kerr law media. *Communications in Nonlinear Science and Numerical Simulation,* 14(5), 1993-1998.

Biyoghe, S. N., Ekogo, T. B., ve Moubissi, A. B. (2017). Collective variable analysis of the nonlinear Schrödinger equation for soliton molecules in fibers. *Journal of Nonlinear Optical Physics and Materials,* 26(02), 1750023.

Christian, J. M., ve Lundie, M. J. (2017). Bistable Helmholtz dark spatial optical solitons in materials with self-defocusing saturable nonlinearity. *Journal of Nonlinear Optical Physics and Materials,* 26(02), 1750024.

Ekici, M., Mirzazadeh, M., Sonmezoglu, A., Ullah, M. Z., Zhou, Q., Moshokoa, S. P., Biswas, A. ve Belic, M. (2017). Nematicons in liquid crystals by extended trial equation method. *Journal of Nonlinear Optical Physics and Materials,* 26(01), 1750005.

Ekici, M., Mirzazadeh, M., ve Eslami, M. (2016). Solitons and other solutions to Boussinesq equation with power law nonlinearity and dual dispersion. *Nonlinear Dynamics,* 84(2), 669-676.

El-Borai, M. M., El-Owaidy, H. M., Ahmed, H. M., Arnous, A. H., Moshokoa, S. P., Biswas, A., ve Belic, M. (2017). Dark and singular optical solitons with spatio-temporal dispersion using modified simple equation method. *Optik,* 130, 324-331.

Fedele, R., Schamel, H., Karpman, V. I., ve Shukla, P. K. (2003). Envelope solitons of nonlinear Schrödinger equation with an anti-cubic nonlinearity. *Journal of Physics A: Mathematical and General,* 36(4), 1169-1173.

Fujioka, J., Cortés, E., Pérez-Pascual, R., Rodríguez, R. F., Espinosa, A., ve Malomed, B. A. (2011). Chaotic solitons in the quadratic-cubic nonlinear Schrödinger equation under nonlinearity management. *Chaos: An Interdisciplinary Journal of Nonlinear Science,* 21(3), 033120.

Guo, S., ve Zhou, Y. (2010). The extended G′/G-expansion method and its applications to the Whitham–Broer–Kaup–Like equations and coupled Hirota–Satsuma KdV equations. *Applied Mathematics and Computation,* 215(9), 3214-3221.

Hayek, M. (2010). Constructing of exact solutions to the KdV and Burgers equations with power-law nonlinearity by the extended G′ / G-expansion method. *Applied Mathematics and Computation,* 217(1), 212-221.

Huang, J. G., Christian, J. M., ve McDonald, G. S. (2017). Spontaneous spatial fractal pattern formation in dispersive systems. *Journal of Nonlinear Optical Physics and Materials,* 26(01), 1750009.

Izdebskaya, Y., Shvedov, V., Assanto, G., ve Krolikowski, W. (2017). Magnetic routing of light-induced waveguides. *Nature Communications,* 8(1), 14452.

Jisha, C. P., Alberucci, A., ve Assanto, G. (2014). PT-symmetric nonlocal gap solitons in optical lattices. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450041.

Kaup, D. J., ve Yang, J. (2000). Stability and evolution of solitary waves in perturbed generalized nonlinear Schrödinger equations. *SIAM Journal on Applied Mathematics,* 60(3), 967-989.

Kwaśny, M., Laudyn, U. A., Rutkowska, K. A., ve Karpierz, M. A. (2014). Nematicons routing through two types of disclination lines in chiral nematic liquid crystals. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450042.

Latiff, A. A., Shamsudin, H., Tiu, Z. C., Ahmad, H., ve Harun, S. W. (2016). Switchable soliton mode-locked and multi-wavelength operation in thulium-doped all-fiber ring laser. *Journal of Nonlinear Optical Physics and Materials,* 25(03), 1650034.

Li, X., Yu, R., ve Zhang, N. (2017). Surface solitons supported by PT-symmetric lattice with spatially modulated nonlinearity. *Journal of Nonlinear Optical Physics ve Materials,* 26(01), 1750001.

Liu, C. S. (2008). Representations and classification of traveling wave solutions to Sinh-Gordon equation. *Communications in Theoretical Physics,* 49(1), 153-158.

Martinez-Galicia, R., ve Panayotaros, P. (2016). Coherent states and localization in a quantized discrete NLS lattice. *Journal of Nonlinear Optical Physics and Materials,* 25(04), 1650047.

Mirzazadeh, M., Arnous, A. H., Mahmood, M. F., Zerrad, E., ve Biswas, A. (2015). Soliton solutions to resonant nonlinear Schrödinger's equation with time-dependent coefficients by trial solution approach. *Nonlinear Dynamics,* 81(1-2), 277-282.

Mirzazadeh, M., Eslami, M., Savescu, M., Bhrawy, A. H., Alshaery, A. A., Hilal, E. M., ve Biswas, A. (2015). Optical solitons in DWDM system with spatio-temporal dispersion. *Journal of Nonlinear Optical Physics and Materials,* 24(01), 1550006.

Moloney, J. V. (1998). *Ultrafast intense probes of nonlinear optical material interactions, In Nonlinear Optical Materials,* New York: Springer,177-204.

Nath, D., ve Roy, P. (2016). Exact localized solutions of (1+ 1)-dimensional nonlinear Schrödinger equation with complex PT symmetric potentials and power-law nonlinearity. *Journal of Nonlinear Optical Physics and Materials,* 25(03), 1650036.

Pal, R., Goyal, A., Loomba, S., Raju, T. S., ve Kumar, C. N. (2016). Compression of optical similaritons induced by cubic-quintic nonlinear media in a graded-index waveguide. *Journal of Nonlinear Optical Physics and Materials,* 25(03), 1650033.

Panayotaros, P., ve Rivero, F. (2014). Multi-peak breather stability in a dissipative discrete Nonlinear Schrödinger (NLS) equation. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450044.

Sala, F. A., Karpierz, M. A., ve Assanto, G. (2014). Spatial routing with light-induced waveguides in uniaxial nematic liquid crystals. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450047.

Savescu, M., Bhrawy, A. H., Hilal, E. M., Alshaery, A. A., ve Biswas, A. (2014). Optical solitons in magneto-optic waveguides with spatio-temporal dispersion. *Frequenz,* 68(9-10), 445-451.

Sciberras, L. W., Minzoni, A. A., Smyth, N. F., ve Assanto, G. (2014). Steering of optical solitary waves by coplanar low power beams in reorientational media. *Journal of Nonlinear Optical Physics and Materials,* 23(04), 1450045.

Smyth, N. F., Piccardi, A., Alberucci, A., ve Assanto, G. (2016). Highly nonlocal optical response: Benefit or drawback?. *Journal of Nonlinear Optical Physics and Materials,* 25(04), 1650043.

Smyth, N. F., ve Tope, B. (2016). Beam on beam control: Beyond the particle approximation. *Journal of Nonlinear Optical Physics and Materials,* 25(04), 1650046.

Triki, H., Kara, A. H., Biswas, A., Moshokoa, S. P., ve Belic, M. (2016). Optical solitons and conservation laws with anti-cubic nonlinearity. *Optik,* 127(24), 12056-12062.

Vega-Guzman, J., Alshaery, A. A., Hilal, E. M., Bhrawy, A. H., Mahmood, M. F., Moraru, L., ve Biswas, A. (2014). Optical soliton perturbation in magneto-optic waveguides with spatio-temporal dispersion. *Journal of Optoelectronics and Advanced Materials,* 16(9-10), 1063-1070.

Zayed, E. M., Alngar, M. E., Shohib, R. M., Biswas, A., Ekici, M., Alzahrani, A. K., ve Belic, M. R. (2020). Solitions in magneto–optic waveguides with anti–cubic nonlinearity. *Optik,* 222, 165313.

Zhang, J., Xiang, Y., Li, Y., Lei, D., ve Zhang, L. (2016). Role of anomalous self-steepening in controlling self-accelerating Airy pulses in nonlinear metamaterials. *Journal of Nonlinear Optical Physics and Materials,* 25(01), 1650010.

Zhou, Q., Ekici, M., Sonmezoglu, A., Mirzazadeh, M., ve Eslami, M. (2016). Optical solitons with Biswas–Milovic equation by extended trial equation method. *Nonlinear Dynamics,* 84(4), 1883-1900.

Zhou, Q., Ekici, M., Sonmezoglu, A., ve Mirzazadeh, M. (2016). Optical solitons with Biswas–Milovic equation by extended G′/G-expansion method. *Optik,* 127(16), 6277-6290.

Zhou, Q., Yao, D., Ding, S., Zhang, Y., Chen, F., Chen, F., ve Liu, X. (2013). Spatial optical solitons in fifth order and seventh order weakly nonlocal nonlinear media. *Optik,* 124(22), 5683-5686.