

DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**STATIC AND DYNAMIC ANALYSIS OF PRE-
STRESSED COMPOSITE AND ISOTROPIC
CURVED PLATES WITH CRACKS**

by
Can GÖNENLİ

October, 2020
İZMİR

STATIC AND DYNAMIC ANALYSIS OF PRE-STRESSED COMPOSITE AND ISOTROPIC CURVED PLATES WITH CRACKS

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Program**

**by
Can GÖNENLİ**

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Ph.D. THESIS EXAMINATION RESULT FORM

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STATIC AND DYNAMIC ANALYSIS OF PRE-STRESSED COMPOSITE AND ISOTROPIC CURVED PLATES WITH CRACKS

ABSTRACT

Plates, which are frequently used in engineering structures, are generally used in curved form because of their higher stiffness values. Some of the plates in the assembly stage are produced in a curved form, and some of them are bent during assembly due to their usage places and used in the pre-stressed state. However, creating a realistic mathematical model covering different situations before production and knowing the critical free vibration values are important for the structure's durability and cost.

Within the scope of the thesis, it is aimed to examine the free vibration of pre-stressed composite and isotropic curved plates with cracks. An external distributed vertical load is applied at the free end of a cantilever plate, and then the free edge where the load is applied is fixed to obtain a pre-stressed curved thin plate. The non-linear deflection curve of the large deflected flexible plate is obtained from the large deflection equation. The finite element model used in the thesis comprises the combination of classical plate theory and four-node quadrilateral element. In addition, the effect of curvature on free vibration and mode shapes is examined.

The present theory and the ANSYS results are compared for the same load parameters to verify the present model's accuracy and validity. The different mode shapes for the curve model depending on the changing of load parameter and the different effects of crack depending on the changing of mode shapes are presented.

Keywords: Finite element method, Large deflected plate, Pre-stress, Crack, Natural frequency, Free vibration analysis

ÖN GERİLMELİ ÇATLAKLI KOMPOZİT VE İZOTROPİK EĞRİ PLAKALARIN STATİK VE DİNAMİK ANALİZİ

ÖZ

Mühendislik yapılarında sıklıkla kullanılan plakalar, daha yüksek direngenlik değerleri nedeniyle genellikle eğri formda kullanılır. Montaj aşamasındaki plakaların bir kısmı eğri formda üretilirken, bir kısmı da kullanım yerleri nedeniyle montaj esnasında eğri hale getirilerek ön gerilmeli olarak kullanılır. Bununla birlikte, üretim öncesi farklı durumları kapsayan gerçekçi bir matematiksel model oluşturmak ve kritik serbest titreşim değerlerini bilmek, yapının dayanıklılığı ve maliyeti açısından büyük önem taşımaktadır.

Tez kapsamında çatlaklı ön gerilmeli kompozit ve izotropik eğri plakaların serbest titreşiminin incelenmesi amaçlanmıştır. Ön gerilmeli ince eğri bir plaka elde etmek için tek tarafı sabit bir plakanın serbest ucuna yayılı dikey bir yük uygulanır ve ardından yükün uygulandığı serbest kenar sabitlenir. Eğri forma getirilen esnek plakanın doğrusal olmayan eğrisi, büyük çökme denkleminde elde edilir. Tezde kullanılan sonlu eleman modeli, klasik plaka teorisi ile dört düğümlü dörtgen elemanın birleşiminden oluşmaktadır. Ayrıca eğriliğin serbest titreşim ve mod şekilleri üzerindeki etkisi incelenmiştir.

Mevcut teori ve ANSYS sonuçları, mevcut modelin doğruluğunu ve geçerliliğini doğrulamak için aynı yük parametreleri için karşılaştırılır. Yük parametresine bağlı olarak değişen eğri modeli için farklı mode şekilleri ve mode şekillerinin değişimine bağlı olarak farklı çatlak etkisi grafikleri sunulmuştur.

Keywords: Sonlu elemanlar yöntemi, Büyük çökmeli plaka, Ön gerilme, Çatlak, Doğal frekans, Serbest titreşim analizi

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

With technology growth, the need for more lightweight, higher stiffness, and more economical materials is increasing daily. For this reason, in addition to the use of various materials, there is a need for more stiffer geometries. Composite materials are used much more as in the aviation industry, where the need for lightness and durability comes to the fore, instead of the materials frequently used in engineering structures such as steel and aluminum. In this manner, plates with various materials and geometries are used for structural elements in various engineering structures like aircraft bodies, lightweight engineering structures, ship industry, aerospace vehicles, etc, and they are especially designed in a curved structure due to their higher stiffness. Figure 1.1 shows examples of curved plate geometries used in different areas of engineering.



(a) Smith (2019)



(b) Arpem (2012)

Figure 1.1 Curved structures in different areas. (a) plane, (b) car, (c) bridge, (d) ship, (e) dam, (f) space ship, (g) Submarine, (h) Structure



(c) Structurae (2019)



(d) Wall (2018)



(e) Frank (2015)



(f) Gonton (2020)



(g) Pubby (2019)



(h) Maskus (2020)

Figure 1.1 continues

In some cases, plates may be set up under initial conditions of displacement unless reaching their elastic limits, for design purposes. In the introductory areas and related examples, a large deflection problem of plates evinces itself, and it is essential to determine their static and dynamic characteristics. Correspondingly, any error that may occur in the plates exposed to different variable dynamic loads directly affects the dynamic sensitivity of the structure.

It is necessary to create a reliable mathematical model of flat and curved plates to predict the critical frequency values in the modeling stage before production,

notwithstanding the material. Critical free vibration values should be predicted, especially in engineering structures exposed to variable dynamic loads to prevent many fatal errors. In the scope of the thesis, it is aimed to create a reliable curved plate model covering pre-stressed isotropic, and anisotropic materials. The free vibration analysis of the presented mathematical model is performed with or without cracks in different curvatures for both isotropic and anisotropic material types. In addition, the variation of the free vibration values, and mode shapes of the plate only depending on the radius of curvature is studied. Mode shapes and the first five natural frequency parameter values are given in graphs and tables for all examined geometries.

1.2 Literature Review

There are many studies in the literature about plates in many different areas, such as cracked, uncracked, pre-stressed, non-prestressed, curved, straight, isotropic, anisotropic. Accordingly, in many types of research, different mathematical approaches for flat and curved plates have been used for under different boundary conditions. Prakash et al. (2012) investigated the large magnitude bending vibration behaviours of functionally graded material (FGM) plates employing a shear flexible finite element type. The first-order shear deformation theory is the selected theory for the FGM plate, depend on the Von Karman's equations for large displacement. The harmonic balance method is used for the solution of the governing equations of motion to understand the vibration magnitude of the FGM plates under supersonic air flow. Thenceforth, Newmark's time integration technique is employed for the solution of the nonlinear equations of motion to understand the bending vibration behavior of the FGM plates in the time domain. Amabili (2013) studied a consistent higher-order shear deformation nonlinear theory. The approach is that it has been improved for plates of general shape allowing for thickness changing by using six variables with taking geometric failures. The geometrically nonlinear strain-displacement equations are obtained from retaining full nonlinear terms in the planar displacements. This novel approach is enforced to laminated cylindrical plates. Linear and nonlinear vibrations are worked by using the present study and results are compared with the refined Amabili-Reddy higher-order shear deformation nonlinear shell theory.

Chandrashekhhar & Ganguli (2014) used a different composite plate finite element theory that is employed for the dynamic analysis of large deformation on delaminated composites. The contact between laminations is modeled with the Lagrangian method. Numerical results showed that the b-Newmark method introduces instability issues in the simulation of large deformed delaminated composite plate structures. Momentum and energy preserving composite implicit time integration scheme is used to overthrow this instability problem. A fitting selection of the penalty parameter is very important in the contact simulation, according to the study. Lui et al. (2015) obtained the analytical result of the free vibrations of straight thin plate by the symplectic superposition method. Also analytical results were compared and mode shapes were obtained. Tornabene et al. (2015) used the Dirac-delta function to analyze loads on beams, doubly-curved shells, and plates for the efficiency of the present method. In addition, simulating a point load using a continuous and smooth normalized Gaussian distribution is introduced for the first time with this study. Hamilton's Principle is used for the solution of the equations of motion. Senjanovic et al. (2015) used an uncomplicated analytical procedure for the evaluation of vibration results of thin rectangular plates. Some additional natural modes, which are described as sum and difference of the multiplication of beam mode shapes are seen depend on a detailed finite element analysis. The frequency behavior is very similar in many special cases demonstrating in such a way double frequency circumstance. These families of mode shapes create a total natural frequency spectrum of a rectangular plate as a novelty. A thin square and a rectangular plate are used for the developed method. Rabende & Michalak (2015) studied a thin dense system of ribs with functionally graded plates. The plates are designed, considering that the plates are exposed to normal forces applied in their midplane. In the center of the plate, the width of the ribs can vary gently. This situation grants get a desirable natural frequency of vibrations of the functionally graded plate. The tolerance averaging method is used for the formulation of the averaged model. The results are emphasized using the examination of vibrations of the plates. The effect of initial stress, shape, and characteristics of the material of the plate on natural frequencies is investigated. Vescovini & Dozio (2016) presented an advanced approximate method for the natural frequency and buckling examination of variable stiffness thin plates. A variable-kinematic method is employed to develop

formulation in the background of a variational framework and Ritz's approach. The results are verified according to finite element methods, and solutions found in the literature and showed the efficiency of the theory used in terms of eigenvalues and modal shape definitions. A novel series of vibration and buckling data are granted through this study. Zhang et al. (2016) investigated the large deformation action of functionally graded composite plates augmented with triangular carbon nanotube under distributed loads. The Improved Moving Least Square-Ritz theory is employed for the analysis. The first-order shear deformation theory and von Kármán assumption are used to examine for rotary inertia, shear strains, and rotations. A convergence study is managed by changing the number of nodes and the supporting size. The effects of bending shear deformation, carbon nanotube distribution, and carbon nanotube volume on the bending behaviors under various fixing situations are also investigated. Dey et al. (2016) modeled a composite rectangular plate according to the Mindlin-Reissner plates theory between classical plates theory and high-order differential equations. Triangular finite elements are used as mesh elements. Mass and stiffness matrices obtained natural frequency values of the plate calculated according to the theory. Li et al. (2016) studied analytic vibration solutions of thin rectangular plates with or without an elastic foundation. The symplectic superposition method is employed for the solution method because many boundary value problems are generally solved by the iterative methods. The solution of the selected method is conducted in the symplectic space, The advantage is where the symplectic eigen expansion is accurate, and the estimation of the solution form is evaded. This situation extremely extends the method to the numerical solutions of identical issues. Benchouaf & Boutyour (2016) used an asymptotic analytical method and harmonic balance method to examine nonlinear vibrations of the Von Karman plate under buckling. The finite element method is employed for the solution of the coupled static and dynamic problems. The static characteristic of the buckled plate is computed, and then the natural frequency of vibrations of the plate is calculated, about any balance state. Padé approximants are used to develop the correction of the validity scope of power. Amabili (2016) studied nonlinear vibrations of thin viscoelastic plates. The von Kármán nonlinear strain–displacement relationship is employed for harmonic excitation in the spectral neighborhood of the fundamental resonances, and also geometric errors are taken into

account. The 16 and 22 degrees of freedom models are used to obtain numerical results for the fundamental modes of a simply supported square plate. Nyugen et al. (2017) used a polygonal finite element method. The static and natural frequency analyses of laminated anisotropic plates are managed through the polygonal finite element method, which is extended and merged with the higher-order shear deformation theory. The Hamilton principle is used to obtain the weak forms of static and natural frequency problems. Analytical cases are given to confirm the rightness and reliability of the current method. Wang et al. (2017) developed a semi-analytical solution for functionally graded doubly-curved sandwich panels and shells. Natural frequency analysis was performed with the Hamilton principle of equations of motion and the Ritz variational energy method. Chen et al. (2017) examined planar vibration problems of different anisotropic formed plates with different fixing conditions. Non-uniform rational B-spline basis function is employed to link the gap between the designs of form with the analysis of different field, and construct the displacement field and form of various anisotropic shaped plates. The differential equations of these plates are obtained using the virtual work principle. The refinement schemes are used to show the convergence of this approach in analytical results. Then the correctness and accuracy are approved through comparisons with the available open literature. Li et al. (2017) obtained accurate numerical solutions for the vibration of thick plates. The Hamiltonian system-based governing equation is employed. The eigenvalue problems of two fundamental natural frequencies are formed for an edge-free thick plate, and the other edges are fixed. The fundamental solutions are obtained by symplectic expansion. The superposition of those results is equal to that of the original thick plate, which yields the numerical frequency equation. Gupta & Talha (2017) studied the nonlinear vibrations answer of functionally graded material (FGM) plates by using displacement depends on hyperbolic higher-order shear and normal deformation theory. The suggested theory consists of in-plane and bending displacement through the plate thickness. The number of unknown parameters involved in the suggested theory is only four, but in the case of other well-known shear deformation theories have five or higher unknown parameters. The basic equations of FGM plate are calculated using Von Karman equations, which is employed for large deflection. The validity of the suggested method is ascertained by comparing it with different available

data in the literature. Kumar et al. (2017) studied linear and nonlinear vibration investigation of thin and thick arbitrary flat-sided quadrilateral plates with employing the finite element method. The Mindlin-Reissner plate theory is discretized with quadrilateral background cells. Then the stiffness matrices of context quadrilateral cells are assessed employing edge-based smoothed finite element technique. Liao et al. (2017) merged theoretical, experimental, and analytical analysis using the Timoshenko shell theory to investigate the vibration behaviors of a thin plate oriented horizontally at the underside of a rectangular pot loaded with fluid. Behrami et al. (2017) presented an improvement of the spectral element technique for the natural frequency analysis of thin rectangular shells exposed to impact loads. A thin, shallow shell with two edges is simply supported, and the other two edges with optional boundary conditions are used for the examination, and the dynamic formulation is improved. Comparing works are enforced to evaluate the performance of the suggested method. In addition, the effect of boundary conditions is debated. Yu et al. (2018) studied various bearing loadings of a plate stand on various linear and nonlinear basics exposed to various fixing conditions such as the circled clamped, simply supported, and mixed boundary conditions. The novel wavelet-homotopy technique is suggested for obtaining accurate results of the plate deflection and the stress under various working conditions. Li et al. (2018) used a rational superposition method in the symplectic space to obtain correct numerical free vibration solutions of rectangular thin cantilever plates. The governing equation is obtained on the Hamiltonian principle. The eigenvalue problems of vibration problems are made for a one-side fixed plate. The basic solutions are obtained by symplectic expansion. The improved method ensures numerical solutions with fast convergence and satisfactory correctness without assuming any test solutions. Thence, this method is feasible for solving partial differential equations represented by plates' vibration, bending, and buckling. Yuan et al. (2018) obtained a largely correct technique for static deflections of a thin circular plate. The suggested method is useful for designing complicated structures effectively. There are plenty of techniques to achieve such a target. So, it is also aimed to perform a comparison of optimization method (OM), finite element method (FEM), and harmonic differential quadrature (HDQ) method. Ai et al. (2018) investigated an analytical research on the dynamic interaction problem between a pre-stressed plate

and a transversely isotropic multilayered half-plane subjected to a moving load. Van Do & Lee (2018) studied a replaced mesh-free radial point interpolation technique that utilizes a novel radial basis function-able to create the shape functions without any supporting fixing parameters improved for the bending analyses of functionally graded material (FGM) plates. Minh et al. (2018) investigated the stability in a rectangular functionally graded material (FGM) plate with a crack in the middle. The plate thickness is varied linearly with the length of the plate. The Reissner-Mindlin first-order shear deformation theory is employed to carry out the stability of the fracture of the plate is determined. Xue et al. (2018) studied the vibration characteristics of functionally graded plates taking into consideration in-plane material inhomogeneity. A refined plate theory for isogeometric investigation is used to complete the examination. Senjanovic et al. (2018) studied a circular rotating plate is modeled by the finite element method. The obtained data is compared with the analytical results and the results were consistent with each other. In the modeling part, the equations of motion are obtained with the Hamiltonian principle and the vibration responses are obtained by the Rayleigh-Ritz method. Also, the mode shapes of the modeled cylinder are studied.

There are different studies especially on curved or cylindrical plate model. Iyengar & Naqvit (1966) presented close solutions for the flexural behavior of thin plates taking into consideration large deflections for different fixing conditions and analytical data for plates. Olson & Lindberg (1969) have studied cylindrical shells both for understanding the fundamental shell behavior and designing shell structures for industrial applications. Morris & Dave (1980) presented the finite strip method. This suggested method is used for estimation of the natural frequencies of vibration of longitudinally invariant, rigidly attached mounting of circularly arched and straight (flat) strips having diaphragm end bracket. Hinton & Rao (1993) studied the free vibration analysis of prismatic folded plate and shell structures. These structures supported on two across edges with the other two edges optionally restrained for curved. The plate is modeled on the Mindlin-Reissner shell theory. Wang & El-Sheikh (2005) presented a numerical analysis of large-deflected plates. The plates are under uniform loading conditions, and the study ensures a numerical operation that utility

from the software computing abilities. Yongqiang & Jian (2007) is performed a natural frequency analysis of annular and circular plates using a curve strip Fourier p-element. Wong et al. (2010) studied with free vibration characteristics and natural frequency of horizontally curved plate girder bridges. Kim et al. (2014) performed a set of elastic and elastoplastic large deformation investigation to show how the curvature effects the strength of the plates. Two-dimension analytical approximations, which are the generalized differential quadrature and the finite element methods, are checked by Tornabene et al. (2015) with an exact three-dimension shell answer in the case of the natural frequency analysis of functionally graded material (FGM) shells and plates. Rawat et al. (2016) investigated the thin circular cylindrical shell can vibrate in different modes. Mercan et al. (2016) investigated the free natural frequency answer of cylindrical shells with functionally graded material (FGM). Raffo & Quintana (2017) offer a general algorithm to get approximate numerical answers. A rectangular anisotropic thin plate with an internal arched line hinge and general limitation is subjected to the study of the natural frequency analysis. An ideally anisotropic elastic core, and Sandwich plates and curved shells composed of two composite laminated faces are noted in the study of Hwu et al (2017). Xie et al. (2017) studied elastically coupled thin annular plate and cylindrical shells. These structures are subjected under elastic boundary conditions through wave based method. Qin et al. (2017) studied isogeometric analysis is used as a new approach that establishes an uninterrupted connection between Computer-Aided Design (CAD) and Computer-Aided Engineering (CAE). This method, which employs Uniform Rational B-Splines (URBS) as a geometric image of the item, is a discretization technology for analytical investigation. The Non-Uniform Rational B-Splines (NURBS) depend on the isogeometric analysis method is employed to examine the static and dynamic characteristics of curvilinear plates. This method is employed to examine planar deformation, vibration, and curvilinear characteristics of curvilinear plates. Also, the large deformation of the curvilinear plates and the natural frequency are examined on the base of von Karman's equations. Belalia (2018) investigated the vibration behavior using a curved hierarchical finite element for sandwich elliptic plates. Shi et al. (2018) studied the dynamic prediction model of an annular plate-cylindrical shell structure with complicated fixing conditions, and coupling terms are established. Rezaiefar &

Galal (2018) investigated first three natural frequency modes for flat plates with and without lateral pressure. The vibration frequency increases with increasing the applied lateral pressure. Park et al. (2018) studied the structural behavior of curved plate under axial compression and according to the results, initial deflection, the effects of curvature, and boundary conditions are discussed. Eisenberger & Deutsch (2019) suggested a novel high-efficiency numerical results that covers all the potential combinations for thin rectangular plate that solve the partial differential equations of motion. Their study also gives examples of their new solution and compared with other approximate solutions. Spagnoli et al. (2019) presented an experimental test under fixed loading on the geometrically nonlinear bending behavior of plates. The study shows the critical plate displacement almost independent on the plate size, but linearly depends on the plate thickness.

Studies that offer solution approaches for cracks related to isotropic or anisotropic plates are as follows. Srivatsa & Murty (1992) considered the effect of different boundary conditions, lay-up details, fiber directions, cut-out lengths on laminated composite plates with using thirty-six degrees of freedom triangular element, based on the classical lamination theory. Tran et al. (2015) presented a new and efficient mathematical formulation that combines the extended isogeometric approach and higher-order shear deformation theory. The theory is suggested to investigate the free vibration of functionally graded material plates with crack. Tornabene et al. (2015) examined the free vibration nature of different geometric structures such as laminated composite thick and moderately thick elliptic cones, cylinders, and plates. Joshi et al. (2015) presented a numerical model for vibration analysis of a thin anisotropic plate having a partial crack. The crack is parallel to one of the edges of the plate, spotted at the center of the plate. The Classical Plate Theory is employed for the analysis. Zeng et al. (2016) investigated the vibration and stability analyses of a rectangular plate with a side crack based. The selected theory is based on classical thin plate theory using the famous Ritz method. Admissible functions that are established by the moving least square technique with enriched principle functions. Sahoo et al. (2016) analyzed the geometrically nonlinear deformation answers of glass/epoxy composite flat/curved shell panel structure. Different displacement field kinematics and Green–Lagrange

strain–displacement relation are employed for the examination. Civalek (2017) studied free natural frequency analysis of laminated composite panels and curved plates with functionally graded materials using Love's shell theory and first-order shear deformation theory. Damnjanovic et al. (2017) investigated the dynamic stiffness approach depend on the higher-order shear deformation plate theory (HSDT) and first-order shear deformation plate theory (FSDT) to study natural frequency behaviors of composite stiffened and plate with crack assemblies. The previously derived dynamic stiffness matrices for bending vibration of the laminated composite plate have been coupled with the dynamic stiffness matrix of laminated composite plate in-plane vibration. In order to check the validity and accuracy of the present method in the natural frequency analysis of stiffened plate assemblies, the analytical analysis is carried out through with several examples. Nasirmanesh & Mohammadi (2017) investigated the natural frequency behavior of functionally graded material shells with cracks. The effect of length, density, material stiffness, and angle of the crack are examined. 8-noded shell element is employed for the shell. The shell theory is based on the Mindlin-Reissner shell theory. Azam et al. (2017) used the Mindlin theory of plate to do free vibration analysis of rhombic plate with pre-existing middle crack with using the finite element method. Based on the classical thin plate theory, Huang et al. (2018) proposed novel series of enriched principle functions for planar and flexural displacements of square plates that can yield acceptable functions for the Ritz method using the moving least-squares. The technique used to display the discontinuities of displacement improves the Ritz method's capability to diagnose the presence of a pre-existing crack in a plate. Kormanikova (2018) presented an analytical approximation of mode-frequency analysis of a simply-supported laminated doubly-curved shell. The first-order shear deformation theory is expressed that exactly forecasting shell behavior for the laminated shell. Doan et al. (2018) studied on confirmation phase-field technique for simulation of buckling and free vibration of plates with crack. The used formula in the study is obtained from the Reissner-Mindlin plate theory. Wang et al. (2019) analyzed composite fiber curve laminates and then obtains a damping ratio of each lamina based on a modal experiment using the classical plate theory. Ye et al. (2019) developed a high-performance semi-analytical numerical model. Using the scaled boundary finite element method, this analytical model analyzes the flexural

behavior answers of the angle-ply laminated curved shells with the fiber-reinforced layers. Vidal et al. (2019) used a variable separation technique based on Reissner's Variational Mixed Theorem to form a laminated composite and sandwich shells. Aurojyoti et al. (2019) suggested a polygonal finite element for nonlinear investigation of laminated plates using Reddy's third-order shear deformation theory. Qin et al. (2019) presented a numerical Jacobi-Ritz method. The suggested technique is used for the free vibration analysis of laminated composite rectangular plates. The plates are subjected to arbitrary boundary conditions using the first-order shear deformation theory. Anamagh & Bediz (2019) focused on the dynamics of doubly-curved functionally graded and laminated composite structures. The structures are formed with arbitrary geometries and different boundary conditions. The analyzes are carried out using an energy-based approach where the structure's strain energy is expressed using three-dimensional elasticity equations. Jhanji et al. (2019) studied the composite material's reaction to mechanical loads in presence of multiple cut-outs. Tan et al. (2019) studied the effect of matrix cracking on the delamination morphology. The subjected delamination is inside carbon fiber reinforced plastics during low-velocity impact by using cross-ply laminates. Nguyen et al. (2019) investigated the vibration characteristics of functionally graded microplates with cracks. These plates are obtained from a simple yet rigorous version of Mindlin's generalized continuum and the extended isogeometric analysis. Xue & Wang (2019) studied the free vibration of a straight plate with a side crack of Mode I fracture. The straight plate is also reinforced by one stiffener parallel to the edges of the plate. Narayana & Bhargav (2019) analyzed the effect of cut-outs on buckling both in isotropic and composite cylindrical plates.

1.3 Research Aims and Objectives

This research introduces the free vibration analysis of pre-stressed isotropic and anisotropic curved thin plates. An outer vertical distributed load is applied to the free end of a one-edge fixed plate; after that, the free edge where the load is applied is fixed, for model a pre-stressed curved thin plate. The natural frequency parameter results of both the present and ANSYS models are compared for the same load parameters to confirm the correctness and effectiveness of the present model. The effect of the load

parameter on the first five natural frequency parameters and mode shapes is investigated. An inspection of the literature shows that there have not been any reported studies on this specific case. In addition, the effect of crack on the natural frequency parameter for each modeling situation is investigated.

1.4 Thesis Outline

Chapter 2 provides information on the theory used for plate vibration. In this section, detailed information is given about creating a pre-stressed plate model using the finite element method for both isotropic and anisotropic material type.

Chapter 3 describes creating isotropic and anisotropic curved plate forms using a flat finite element type. The large deflection equation is introduced, and the comparative deflection results of all plate types are given.

Chapter 4 presents the free vibration results of both isotropic and anisotropic plates modeled from flat form to curved form.

Chapter 5 gives the natural frequency analysis of the fixed-length isotropic thin square plate, starting from the flat plate model to the semicircle model fixed on both edges, is performed. In addition, the mode shapes affected by the first five natural frequency changes are shown.

Chapter 6 gives the change of the natural frequency parameter of the isotropic and anisotropic plate according to the position of the crack is examined. The fixed-length crack is added separately to different locations on the plate to see the effect of crack clearly, and the free vibration analysis is repeated. Mode shapes are also examined for the first five natural frequencies to see the effect of crack due to the mode shape.

Chapter 7 draws general conclusions and presents the future work of this thesis.

CHAPTER 2

VIBRATION OF PLATE

2.1 Introduction

Vibration analyses are performed with the help of different finite element types (three-node triangular, four-node quadrilateral, eight-node quadrilateral, etc.) using different approaches for plates. In this research, the frequently used four-node quadrilateral element is chosen for the finite element type. Two different material types, which are isotropic and anisotropic, are used. Although the mathematical equations used for both isotropic plate and anisotropic plate are similar, they differ from each other in terms of their usage, and their material matrices.

2.2 Flexural Vibration (Out-of-plane) of Plate for Isotropic Material

The Kirchhoff–Love plate theory is a two-dimensional mathematical model. This theory is employed to define the stresses and deformations in thin plates exposed to moments and forces (Donnell, 1976). The Kirchhoff–Love plate theory is an extension of the Euler-Bernoulli beam theory. Love used assumptions proposed by Kirchhoff. The approach accepts that a mid-surface plane can mean a three-dimensional plate in a two-dimensional form (Vinson, 2005).

The out-of-plane vibration theory is based on the bending vibration of the plate. The finite element model type for the bending vibration is quadrilateral. The strain (U_1) and kinetic (T_1) energy expressions for a thin plate bending element are:

$$U_1 = \frac{1}{2} \int_A \frac{h^3}{12} \{\chi\}^T [D] \{\chi\} dA \quad (2.1)$$

$$T_1 = \frac{1}{2} \int_A \rho h \dot{w}^2 dA$$

Where ρ is the density of the material, h is the thickness, and $[D]$ is a material matrix and given in equation (2.2).

$$[D] = \frac{Eh^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2.2)$$

E represents Young modulus, ν represents Poisson's ratio. The matrix $\{\chi\}$ represents the strain-displacement relationship and given in equation (2.3).

$$\{\chi\} = \begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial x \partial y \end{bmatrix} \quad (2.3)$$

The four-node quadrilateral finite element is used for this study. This finite element has a total of twelve corner displacements corresponding to three generalized coordinates, which are w , dw/dy , and dw/dx at each corner and given in Figure 2.1 (Belounar & Guenfoud, 2005; Petyt, 2010).

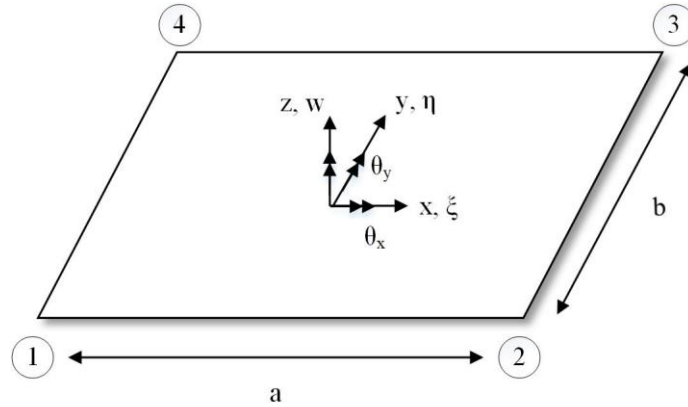


Figure 2.1 Degrees of freedom for the quadrilateral rectangular plate model for out-of-plane vibration

The displacement function is given in equation (2.4).

$$w = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3 \quad (2.4)$$

The flexural displacement, w , is written in terms of natural coordinates (ξ, η) instead of cartesian coordinates (x, y) . Hence, dw/dy and dw/dx become:

$$\begin{aligned}\frac{\partial w}{\partial y} &= \frac{2}{b} \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial x} &= -\frac{2}{a} \frac{\partial w}{\partial \xi}\end{aligned}\tag{2.5}$$

The matrix form of displacement function is:

$$\{w\} = [A_1]\{q_1\}\tag{2.6}$$

Substituting equation (2.4) in terms of ξ and η into equation (2.6) gives the quadratic shape function, and it is given for the flexural vibration of the plate (2.7).

$$N_{1j}^T(\xi, \eta) = \begin{bmatrix} \frac{1}{8}(1 + \xi_j\xi)(1 + \eta_j\eta)(2 + \xi_j\xi + \eta_j\eta - \xi^2 - \eta^2) \\ \frac{b}{2}(1 + \xi_j\xi)(\eta_j + \eta)(\eta^2 - 1) \\ -\frac{a}{2}(\xi_j + \xi)(\xi^2 - 1)(1 + \eta_j\eta) \end{bmatrix}\tag{2.7}$$

(ξ_j, η_j) are the coordinates of node j . The strain displacement matrix B_1 is obtained by equation (2.8).

$$B_1 = \begin{bmatrix} \frac{4}{(a)^2} \frac{\partial^2}{\partial \xi^2} \\ \frac{4}{(b)^2} \frac{\partial^2}{\partial \eta^2} \\ \frac{8}{(a)(b)} \frac{\partial^2}{\partial \xi \partial \eta} \end{bmatrix} N_1(\xi, \eta)\tag{2.8}$$

Element stiffness matrix $[K_1]$ can be calculated, and it is given in equation (2.9).

$$[K_1] = \int_{-1}^1 \int_{-1}^1 [B_1]^T [D] [B_1] d\xi d\eta \quad (2.9)$$

The mass matrix is also calculated using equation (2.10).

$$[M_1] = \rho h \int_{-1}^1 \int_{-1}^1 [N_1]^T [N_1] d\xi d\eta \quad (2.10)$$

2.3 In-plane Vibration of Plate for Isotropic Material

The finite element model to be used for the in-plane vibration type must have a total eight corner displacements, including u and v , at each corner, and this finite element model is given in Figure 2.2.

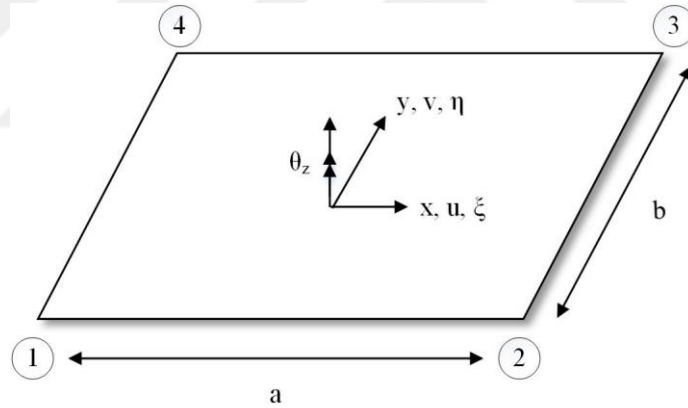


Figure 2.2 Degrees of freedom for the quadrilateral rectangular plate model for in-plane vibration

The shape function of in-plate theory is given in equation (2.11) where ξ_j, η_j are the coordinates of node j (Petyt, 2010; Chandrupatira & Belegundu, 2002).

$$N_{2j} = \frac{1}{4} (1 + \xi_j \xi) (1 + \eta_j \eta) \quad (2.11)$$

The element displacement components u and v are given in equation (2.12).

$$u = N_{21}q_{21} + N_{22}q_{23} + N_{23}q_{25} + N_{24}q_{27} \quad (2.12)$$

$$v = N_{21}q_{22} + N_{22}q_{24} + N_{23}q_{26} + N_{24}q_{28}$$

where q_a represents the denotes the element displacement vector and can be written in matrix form as,

$$v = N_2 q_2 \quad (2.13)$$

where,

$$N_2 = \begin{bmatrix} N_{21} & 0 & N_{22} & 0 & N_{23} & 0 & N_{24} & 0 \\ 0 & N_{21} & 0 & N_{22} & 0 & N_{23} & 0 & N_{24} \end{bmatrix} \quad (2.14)$$

The strain-displacement relations are:

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \end{Bmatrix} \quad (2.15)$$

Subsequently, it needs to express the derivatives of the displacement functions, which are in x and y coordinates, in terms of its derivatives in ξ, η coordinates.

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} = J \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} \quad (2.16)$$

$$\begin{Bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{Bmatrix} = J \begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{Bmatrix}$$

These equations can be inverted as:

$$\begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{pmatrix} = J^{-1} \begin{pmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{pmatrix} \quad (2.17)$$

$$\begin{pmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{pmatrix} = J^{-1} \begin{pmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{pmatrix}$$

where J is the Jacobian matrix.

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (2.18)$$

Equations yield:

$$\varepsilon = A_2 \begin{pmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{pmatrix} \quad (2.19)$$

where A is given by:

$$A_2 = \frac{1}{\det J} \begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \quad (2.20)$$

From the interpolation equations, it can be written as:

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{Bmatrix} = G q_2 \quad (2.21)$$

By substituting equation (2.21) into (2.19), we obtain,

$$\varepsilon = A_2 G q_2 \quad (2.22)$$

where,

$$B_2 = A_2 G \quad (2.23)$$

The stiffness matrix for the quadrilateral rectangular element is obtained from the strain energy, given by:

$$U_2 = \int_V \frac{1}{2} \sigma^T \varepsilon dV \quad (2.24)$$

where σ is given by:

$$\sigma = D B_2 q_2 \quad (2.25)$$

D is a (3x3) material matrix, which is given in equation (2.2) before. The strain energy becomes:

$$U_2 = \frac{1}{2} q_2^T \left[h \int_{-1}^1 \int_{-1}^1 B_2^T D B_2 \det J d\xi d\eta \right] q_2 \quad (2.26)$$

where $[K]_2$ is the stiffness matrix of dimension (8x8).

$$[K_2] = h \int_{-1}^1 \int_{-1}^1 B_2^T D B_2 \det J d\xi d\eta \quad (2.27)$$

The element mass matrix $[M_2]$ also be found from the kinetic energy.

$$T_1 = \frac{1}{2} \dot{q}_2^T \left[\rho h \int_{-1}^1 \int_{-1}^1 N_2^T N_2 \det J d\xi d\eta \right] \dot{q}_2 \quad (2.28)$$

The element mass matrix is given in equation (2.29).

$$[M_2] = \rho h \int_{-1}^1 \int_{-1}^1 N_2^T N_2 \det J d\xi d\eta \quad (2.29)$$

2.4 Laminated Composite Plate

Figure 2.3 shows a laminated composite plate model.

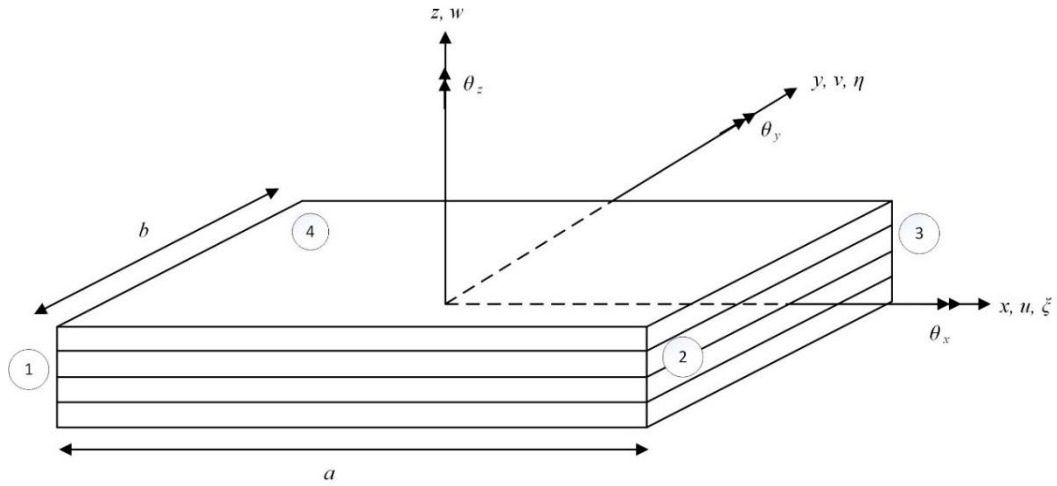


Figure 2.3 Degrees of freedom for the quadrilateral rectangular plate model

The strain energy equation for the thin plate element is given in equation (30) (Petyt, 2010).

$$U = \frac{1}{2} \int_V \sigma^T \varepsilon dV \quad (2.30)$$

where σ is a stress matrix and ε is a strain matrix of the plate. Stress-strain relation can be given in the form of equation (2.31) for a thin plate.

$$\sigma = D\varepsilon \quad (2.31)$$

The strain energy is formed in equation (2.32).

$$U = \frac{1}{2} \int_A \sum_{k=1}^{NL} \int_{z_{k-1}}^{z_k} \{\varepsilon\}^T [D] \{\varepsilon\} dz dA \quad (2.32)$$

where NL is the number of laminae, z_k, z_{k-1} are the top and bottom coordinates of the k^{th} lamina, and D is the material matrix. The stress-strain relation for the laminated composite plate contains three displacements (u_0, v_0, w_0) and can be expressed in matrix form:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (2.33)$$

That is:

$$\{\varepsilon\} = \{\varepsilon_2\} - z\{\varepsilon_1\} \quad (2.34)$$

The kinetic energy equation for the thin plate element is given in equation (2.35).

$$T = \frac{1}{2} \int_A \rho h (\dot{u}^2 + \dot{v}^2) dA \quad (2.35)$$

Two different shape function is necessary to satisfy the stress-strain and displacement relations. The first shape function satisfies the flexural vibration, and the second one satisfies the in-plane vibration of the plate.

2.4.1 Flexural Vibration (Out-of-plane) of Plate for an Anisotropic Plate

The flexural vibration theory is based on the bending vibration of the plate. The finite element model to be used for the bending vibration type must have a total twelve corner displacements corresponding to three generalized coordinates, which are w , dw/dy , and dw/dx at each corner. The strain energy equation derived from equation (2.30) for bending vibration is given in equation (2.36).

$$U_{out} = \frac{1}{2} \int_A \{\varepsilon_1\}^T \sum_{k=1}^{NL} [D^k] (z_k^3 - z_{k-1}^3) \{\varepsilon_1\} dA \quad (2.36)$$

where ε_1 is strain relation for the flexural vibration and given in equation (2.37):

$$\varepsilon_1 = B_{out} q \quad (2.37)$$

Substituting equation (2.37) into equation (2.36):

$$U_{out} = \frac{1}{2} q^T \left[\int_{-1}^1 \int_{-1}^1 [B_{out}]^T [D_B] [B_{out}] \det J d\xi d\eta \right] q \quad (2.38)$$

The matrix B is given in equation (2.39).

$$B_{out} = \begin{bmatrix} \frac{4}{(a)^2} \frac{\partial^2}{\partial \xi^2} \\ \frac{4}{(b)^2} \frac{\partial^2}{\partial \eta^2} \\ \frac{8}{(a)(b)} \frac{\partial^2}{\partial \xi \partial \eta} \end{bmatrix} N_j(\xi, \eta) \quad (2.39)$$

The shape function is given in equation (2.40) for satisfy the second order derivative in equation (2.39).

$$N_{out} = N_j^T(\xi, \eta) = \begin{bmatrix} \frac{1}{8}(1 + \xi_j \xi)(1 + \eta_j \eta)(2 + \xi_j \xi + \eta_j \eta - \xi^2 - \eta^2) \\ \frac{b}{2}(1 + \xi_j \xi)(\eta_j + \eta)(\eta^2 - 1) \\ -\frac{a}{2}(\xi_j + \xi)(\xi^2 - 1)(1 + \eta_j \eta) \end{bmatrix} \quad (2.40)$$

(ξ_j, η_j) are the coordinates of node j in rectangular plate model. Flexural element stiffness matrix $[K]_B$ is calculated and is given in equation (2.41).

$$[K]_B = \int_{-1}^1 \int_{-1}^1 [B_{out}]^T [D_B] [B_{out}] \det J \, d\xi d\eta \quad (2.41)$$

The expanded kinetic energy equation is given in equation (2.42) with using Jacobian matrix, J .

$$T_{out} = \frac{1}{2} \dot{q}^T \left[\rho h \int_{-1}^1 \int_{-1}^1 [N_{out}]^T [N_{out}] \det J \, d\xi d\eta \right] \dot{q} \quad (2.42)$$

Element mass matrix $[M]_B$ is calculated from equation (2.42) and is given in equation (2.43).

$$[M]_B = \rho h \int_{-1}^1 \int_{-1}^1 [N_{out}]^T [N_{out}] \det J \, d\xi d\eta \quad (2.43)$$

2.4.2 In-Plane Vibration of Plate for an Anisotropic Plate

The in-plane vibration theory contains eight corner displacements which are u and v . The strain energy equation derived from equation (2.30) for in-plane vibration is given in equation (2.44).

$$U_{in} = \frac{1}{2} \int_A \{\varepsilon_2\}^T \sum_{k=1}^{NL} [D^k] (z_k - z_{k-1}) \{\varepsilon_2\} dA \quad (2.44)$$

where ε_2 is strain relation for in-plane vibration and given in equation (2.45):

$$\varepsilon_2 = B_{in} q \quad (2.45)$$

Substituting equation (2.45) into equation (2.14):

$$U_{in} = \frac{1}{2} q^T \left[\int_{-1}^1 \int_{-1}^1 [B_{in}]^T [D_L] [B_{in}] \det J d\xi d\eta \right] q \quad (2.46)$$

The matrix B_{in} is written in the form (Chandrupatra & Belegundu, 2002):

$$B_{in} = AG \quad (2.47)$$

where A is given by:

$$A = \frac{1}{\det J} \begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \quad (2.48)$$

where J is the Jacobian matrix. From the interpolation equations, we have:

$$Gq = \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{Bmatrix} \quad (2.49)$$

The element displacement components u and v are given in equation (2.50).

$$u = N_1 q_1 + N_2 q_3 + N_3 q_5 + N_4 q_7 \quad (2.50)$$

$$v = N_1 q_2 + N_2 q_4 + N_3 q_6 + N_4 q_8$$

where q_i ($i=1$ to 8) represents the denotes the element displacement vector. The shape function for staisfy the stress-strain relation is given in equation (2.51), where ξ_i , η_i are the coordinates of node i in rectangular plate model.

$$N_{in} = N_i = \frac{1}{4} (1 + \xi_i \xi) (1 + \eta_i \eta) \quad (2.51)$$

Using matrix B_{in} from equation (2.47) and in-plane material matrix D_L , in-plane element stiffness matrix $[K]_L$ and in-plane element mass matrix $[M]_L$ are calculated similar to $[K]_B$ and $[M]_B$ through equation (2.41) and equation (2.43).

2.4.3 Material Matrix of an Anisotropic Material

For an anisotropic material, which has different material properties in each direction the material matrix is given in form (Petyt, 2010).

$$[D^*] = \begin{bmatrix} E'_x & E'_x \nu_{xy} & 0 \\ & E'_y & 0 \\ Sym & & G_{xy} \end{bmatrix} \quad (2.52)$$

E'_x and E'_y are calculated through equation (2.53) with modulus of elasticity in x direction (E_x), modulus of elasticity in y direction (E_y) and poisson ratios ν_{xy} and ν_{yx} :

$$E'_x = \frac{E_x}{1 - \nu_{xy}\nu_{yx}}$$

$$E'_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}}$$
(2.53)

A laminated composite has fibers, and these fibers can be located at different angles in each lamina. The material matrix is calculated with considering the angles and the transformation matrix which includes the angles is given in equation (2.54).

$$[R^*] = \begin{bmatrix} \cos^2 \beta & \sin^2 \beta & 0.5 \sin 2\beta \\ \sin^2 \beta & \cos^2 \beta & -0.5 \sin 2\beta \\ -\sin 2\beta & \sin 2\beta & \cos 2\beta \end{bmatrix}$$
(2.54)

The material matrix for each lamina is calculated through the equation (2.55).

$$[D^k] = [R^*]^T [D^*] [R^*]$$
(2.55)

2.5 Finite Element Model

2.5.1 Isotropic Model

Flexural and in-plane element stiffness and the mass matrices are combined. The relation is given in equation (2.56).

$$[Flexural, K_1 \& M_1]_{12 \times 12} + [In\ plane, K_2 \& M_2]_{8 \times 8} = \begin{bmatrix} 12 \times 12 & 0 \\ 0 & 8 \times 8 \end{bmatrix}_{20 \times 20}$$
(2.56)

These combined stiffness and mass matrices have (20x20) size matrices, although they carry the effect of θ_z . The bottom 4x4 part of the matrix has to be added formally into matrices for the drilling effect. Thus, a rotatable finite element model with (24x24)

dimensions is formed. The revised version of equation (2.56) is given in equation (2.57).

$$K_e, M_e = \begin{bmatrix} 12 \times 12 & 0 \\ 0 & 8 \times 8 \end{bmatrix}_{20 \times 20} + \theta_z = \begin{bmatrix} 12 \times 12 & 0 & 0 \\ 0 & 8 \times 8 & 0 \\ 0 & 0 & 4 \times 4 \end{bmatrix}_{24 \times 24} \quad (2.57)$$

The value of θ_z has taken absolute value that $1/1000$ of the minimum value in (20x20) stiffness and mass matrices (Niyogi et al., 1999).

2.5.2 Anisotropic Model

When substituting equation (2.34) into equation (2.32), the other two matrices give the coupling effect between flexural and in-plane strain relation. However, in non-axisymmetric materials, the value of these matrices is equal to zero. In this study, both of the axisymmetric and the non-axisymmetric orientations are examined. The element stiffness and the mass matrices are obtained with flexural and in-plane matrices are combined. The relation is given in equation (2.58).

$$[Flexural, K_B \& M_B]_{12 \times 12} + [In\ plane, K_L \& M_L]_{8 \times 8} = \begin{bmatrix} 12 \times 12 & K_{BL} \\ K_{LB} & 8 \times 8 \end{bmatrix}_{20 \times 20} \quad (2.58)$$

where K_{LB} and K_{BL} are coupling effect in stiffness matrix of the axisymmetric orientations.

$$[K]_{LB} = \int_{-1}^1 \int_{-1}^1 [B_{in}]^T [D_C] [B_{out}] \det J \, d\xi d\eta \quad (2.59)$$

$$[K]_{BL} = \int_{-1}^1 \int_{-1}^1 [B_{out}]^T [D_C] [B_{in}] \det J \, d\xi d\eta$$

where D_C is:

$$D_C = [D^k](z_k^2 - z_{k-1}^2) \quad (2.60)$$

The process of obtaining the total twenty-four degree of freedom element matrix for anisotropic plates is done as in isotropic plates in equation (2.57). The value of θ_z has taken absolute value that $1/1000$ of the minimum value in (20x20) stiffness and mass matrices, to form a rotatable finite element model with (24x24) dimensions.

2.6 Initial Stress

The curved plate has initial stress as a result of the vertical load. After creating the finite element model and the curve form, the stress effect of the force, which shapes the form of the curved plate, must be added to the model. The work done by the in-plane loads when they move through the bending displacements is given in equation (2.61) (Yang, 1973).

$$V = \frac{1}{2} N_x \int_A \left(\frac{dw}{dx} \right)^2 dx dy \quad (2.61)$$

where N_x is an initial axial force acting in the local x direction. As seen in Figure 2.4, axial force N_x is obtained as:

$$N_x = \frac{F_y}{\sin \theta} \quad (2.62)$$

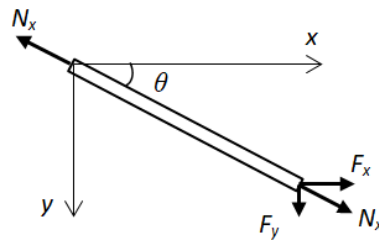


Figure 2.4 Total distributed axial forces in plate element

where F_y is total distributed vertical load. These forces can be retrieved from statics. Eq (2.61), V , can be written in the matrix form:

$$V = \frac{1}{2} q S_e q \quad (2.63)$$

Stiffness matrix (K_e) and mass matrix (M_e) from equation (2.57), and initial stress matrix (S_e) from equation (2.63) are used to form global stiffness (K), mass (M) and initial stress (S) matrices for each plate element. The dynamic response of a plate for the total system can be formulated via equation of motion (2.64).

$$[M]\{\ddot{q}\} + [[K] + [S]]\{q\} = 0 \quad (2.64)$$

Natural frequencies can be obtained via eigenvalue equation (2.65), where ω is natural frequency.

$$(K + S) - \omega^2 M = 0 \quad (2.65)$$

CHAPTER 3

STATIC ANALYSIS

3.1 Introduction

Isotropic and anisotropic plates examined within the scope of the thesis are exposed to a static load before to perform free vibration analysis, and they are brought into curved form. A distributed load is enforced at the free edge of a thin cantilever plate to create a pre-stressed curved plate. After that, the loading edge of the large deflected plate is fixed. While performing this process, the deflection equation obtained from force-moment equations is used. In the static analysis, the deflection values are compared with the ANSYS, and the reliability of the values obtained from the deflection equation is checked.

3.2 Deflection Equation and Transformation Matrix

In many such applications, the deformation can be analyzed by using a thin plate (or beam) theories (Jairazbhoy et al., 2012). The curvature of the non-linear deflection curve under loading at any point depends on the magnitude of the bending moment. At that point, under the assumption that the material remains linearly elastic. The curvature form of the plate has been shown in Figure 3.1a and Figure 3.1b.

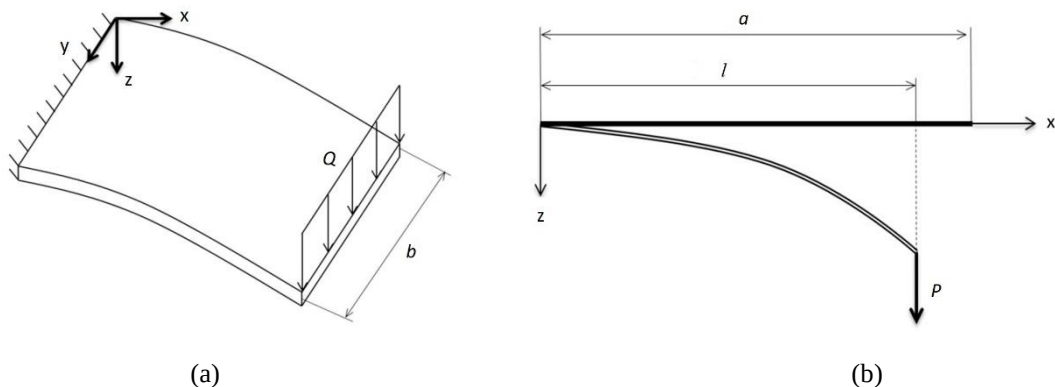
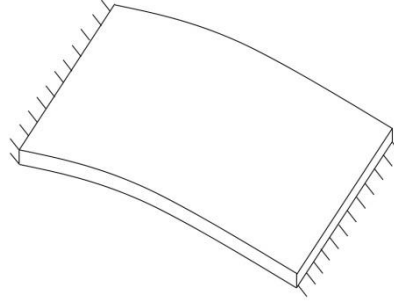


Figure 3.1 The curvilinear form of the thin plate. (a) perspective view, (b) exact length (l) of the plate, (c) two side fixed curved model



(c)

Figure 3.1 continues

Q represents a uniformly distributed load. Ang et al. (1993); Öztürk (2011, 2015) used the deflection equation for the curved model, and it is given in equation (3.1).

$$\begin{aligned}
 z(x) = & \frac{P}{2EI} \left(-\frac{x^3}{3} + lx^2 \right) + \frac{1}{2} \left(\frac{P}{2EI} \right)^3 \left(-\frac{x^7}{7} + lx^6 - \frac{12}{5} l^2 x^5 + 2l^3 x^4 \right) \\
 & + \frac{3}{8} \left(\frac{P}{2EI} \right)^5 \left(-\frac{x^{11}}{11} + lx^{10} - \frac{40}{9} l^2 x^9 + 10l^3 x^8 - \frac{80}{7} l^4 x^7 \right. \\
 & \left. + \frac{16}{3} l^5 x^6 \right) + \dots
 \end{aligned} \quad (3.1)$$

E represents young modulus, P represents force, I represents the moment of inertia for the beam, and l represents the projected length of the beam after the force has been applied. P (equals F_y , see Figure 2.4) has been obtained from equation (3.2).

$$P = Qb \quad (3.2)$$

b represents the width of the plate. The projected plate length l is still an unknown. The projected plate length l can be analytically solved utilization an iterative approach beginning with an initial guess of l close to a with using equation (3.3).

$$a = \int_0^l \sqrt{1 + \left(\frac{dz}{dx} \right)^2} dx \quad (3.3)$$

Rotation angles are required to model a curved plate model. The first derivative of equation (2.4) respect to x gives these rotation angles. Afterward, these angles are placed into a transformation matrix, which is given in Table 3.1. It should be noted that the curvilinear plate has modeled by the assembly of the rotated finite element with respect to the y -axis by using the transformation matrix. Figure 3.2 shows the rotation of the coordinate system.

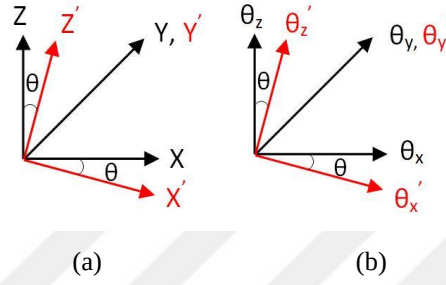


Figure 3.2 Rotation of the coordinate system. (a) the rotation of the axes of translation, (b) the rotation of the axes of rotation

Table 3.1 Transformation matrix

θ_x	θ_y	w	u	v	θ_z	
$\cos(\theta)$	0	0	0	0	$\sin(\theta)$	θ_x
0	1	0	0	0	0	θ_y
0	0	$\cos(\theta)$	$-\sin(\theta)$	0	0	w
0	0	$\sin(\theta)$	$\cos(\theta)$	0	0	u
0	0	0	0	1	0	v
$-\sin(\theta)$	0	0	0	0	$\cos(\theta)$	θ_z

The stiffness and mass matrices can be obtained through the transformation matrix is given in equation (3.4).

$$K_r = T^T \times K_e \times T$$

$$M_r = T^T \times M_e \times T$$
(3.4)

where K_e and M_e represent stiffness and mass matrices of the reference plate. Then rotated matrices K_r and M_r can be obtained with the multiplication of transformation and reference matrices to use in global matrices.

3.3 Large Deflection of an Isotropic Plate

The deflection values are calculated and compared both through the deflection equation and the finite element software ANSYS to check the deflection representation in curve plate modeling. The selected plate dimensions are 1000x1000x3 mm to investigate the effect of a large deflection on free vibration for an isotropic plate. While calculating the deflection values, it is ensured that the stress value does not exceed 85% of the yield limit. The yield stress value of the selected steel is 180 MPa. 85% of this value is 153 MPa. The upper limit value of the stress created by the load on the plate is determined in this way. The critical yield limit value is found with the help of the ANSYS program, and the critical load value is taken as 250 N for the plate of 1000x1000x3 mm. The data obtained from the deflection equation and the values obtained from the ANSYS program are given together in Figure 3.3.

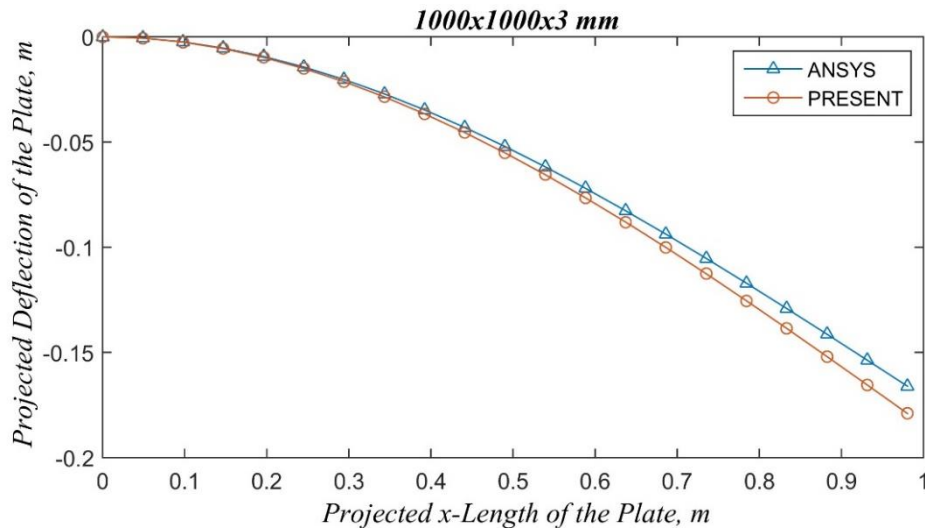


Figure 3.3 Deflection of 1000x1000x3 mm isotropic plate

Three different aspect ratios ($a/b = 1, 0.75$, and 0.5) for plates are selected to examine the effect of a crack in isotropic plates. The dimensions of the plate, whose aspect ratio is equal to 1, is selected as 800x800x3 mm. The deflection equation and

the ANSYS values are compared in Figure 3.4 under the force of 230 N, which corresponds to the 85% yield limit value.

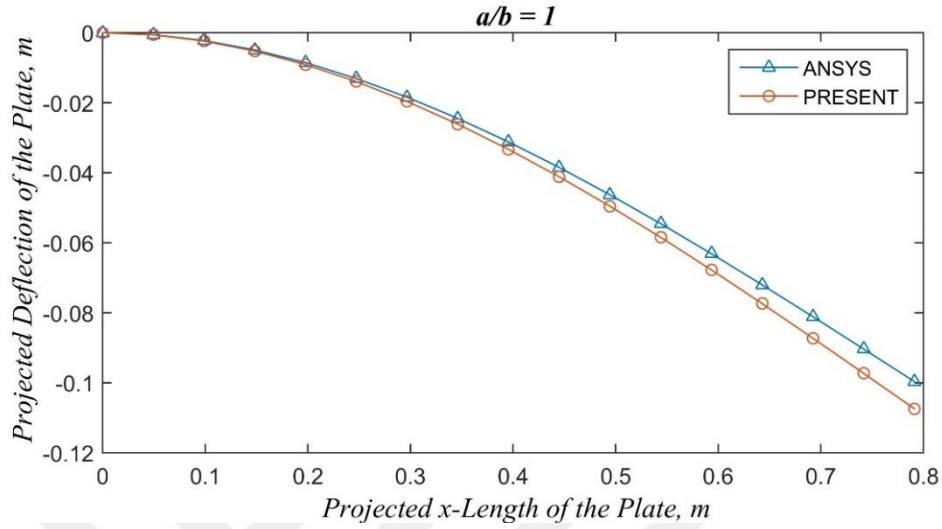


Figure 3.4 Deflection of 800x800x3 mm isotropic plate

Another plate used to examine the effect of crack is 800x600x3 mm and its aspect ratio is 0.75. The critical load for this plate is considered to be 175 N for the yield limit, and the graph of deflections is given in the Figure 3.5.

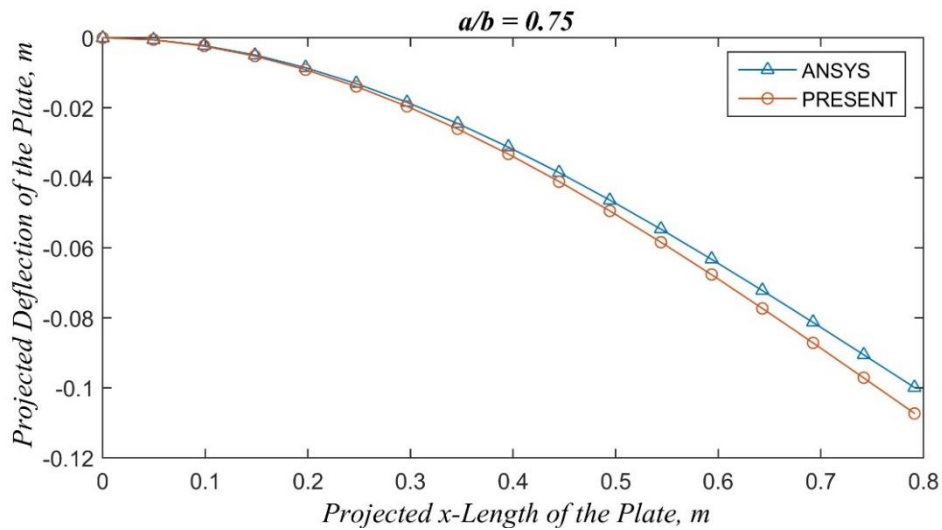


Figure 3.5 Deflection of 800x600x3 mm isotropic plate

The dimensions of the last plate selected when examining the effect of crack are 800x400x3 mm. The critical load for this plate is considered as 115 N. Deflection values are given in Figure 3.6.

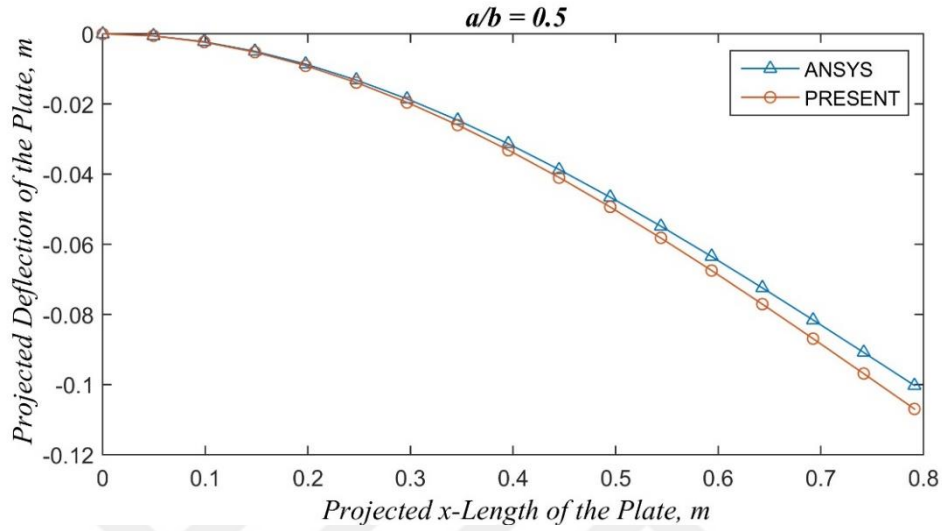


Figure 3.6 Deflection of 800x400x3 mm isotropic plate

The deflection equation represents the curve pattern properly in plates considering all of the isotropic plate deflections. The plates' curvilinear shape is mostly in the same way as ANSYS.

3.4 Large Deflection of Laminated Composite Plate

For the modulus of elasticity (E) in the deflection equation, an effective modulus of elasticity should be used for anisotropic materials. The bending stiffness matrix for each lamina is needed for the effective modulus of elasticity calculation. Equation (3.5) gives the bending stiffness matrix for anisotropic plates.

$$D_b = \frac{1}{3} \sum_{k=1}^N D_k (z_{k+1}^3 - z_k^3) \quad (3.5)$$

Here N gives the number of laminates in the composite plate, z gives the thickness of each lamina. Effective modulus of elasticity equation is given in the equation (3.6) (Öztürk, 2015).

$$E_{ef} = \frac{12}{b^3 D_b^{-1}} \quad (3.6)$$

Using E_{ef} , which is replaced with E in the deflection equation, the deflection values can be calculated in anisotropic plates. The size of all these plates is 800x800x5 mm. 200 mm deflection value is chosen as the maximum deflection limit by considering the mesh density so that the results could be more consistent. Also, according to Tsai-Hall equations, this deflection limit is well below the calculated value for all of the plates.

The anisotropic plate deflection figures related to the present study are given below. All deflection values are calculated according to the deflection equation (3.1) and are given in graphics under the name present study. Within the scope of the study, eight different laminated composite plates with different lamination orientations are examined to show the reliability of the present model. The explanation of these plates is as follows:

- C1 (0 0 0 0)
- C2 (0 90 90 0) (Cross-ply laminate)
- C3 (0 45 -45 0) (Angle-ply laminate)
- C4 (0 60 -60 0) (Angle-ply laminate)
- C5 (0 90 0 90)
- C6 (0 45 0 -45)
- C7 (90 0 0 90) (Cross-ply laminate)
- C8 (90 90 90 90)

Also the material properties are given in Table 3.2.

Table 3.2 Material properties and the geometry of the anisotropic plate (Gay et al., 2013)

Symbol	Property	Quantity
E_x	Elasticity modulus in x-direction	45 GPa
E_y	Elasticity modulus in y-direction	12 GPa
ρ	Mass denstiy	2080 kg/m ³
ν_{xy}	Poisson ratio	0.30
a	Length of the plate	0.8 m
b	Width of the plate	0.8 m
h	Thickness of the plate	5 mm

The glass-fiber epoxy consists of four laminae. Therefore each lamina has 1.25 mm thickness. Effective elasticity modulus for C1 is calculated as 45 GPa, and the maximum load corresponding to 200 mm deflection is 470 N. In order to check the representation of the deflection equation on the plate, the static analysis is performed in ANSYS with the same parameters, and deflection values are found. The comparison between the deflection values obtained from the deflection equation, and the deflection values obtained from ANSYS are given in Figure 3.7.

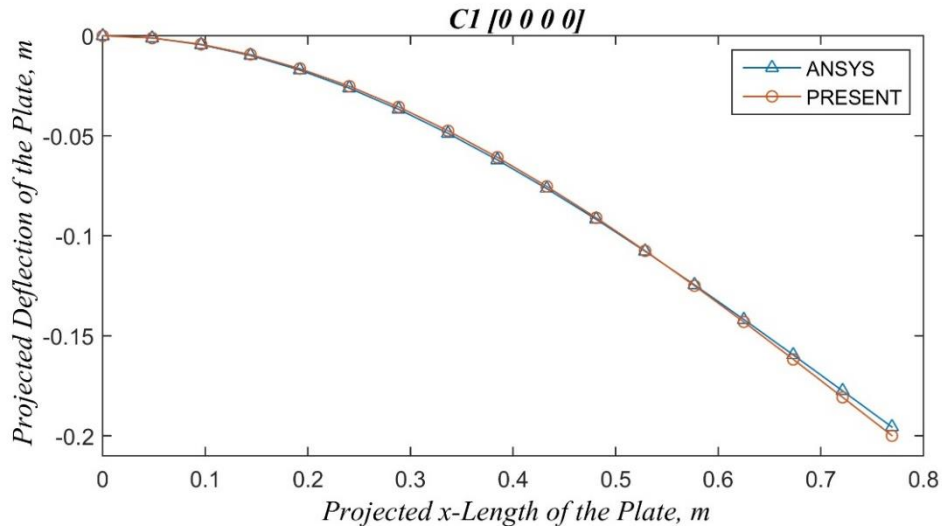


Figure 3.7 Deflection of C1 composite

The effective modulus of elasticity for C2 is calculated as 41.0566 GPa. The maximum load corresponding to the 200 mm deflection value is 430 N. Figure 3.8 gives ANSYS values and deflection equation values for C2.

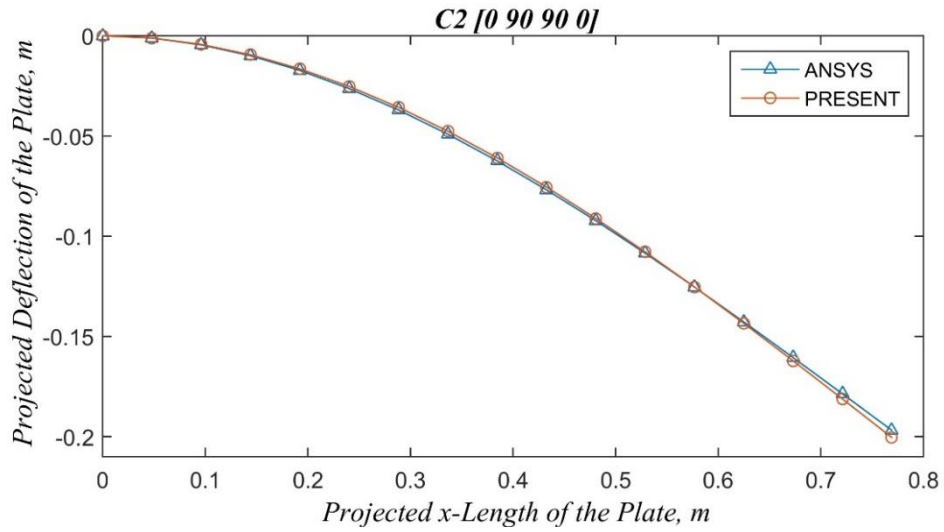


Figure 3.8 Deflection of C2 composite

For C3, the effective modulus of elasticity is calculated as 41.898 GPa and the force value corresponding to 200 mm deflection value is 440 N. The comparison of deflection equation values and ANSYS results is given in Figure 3.9.

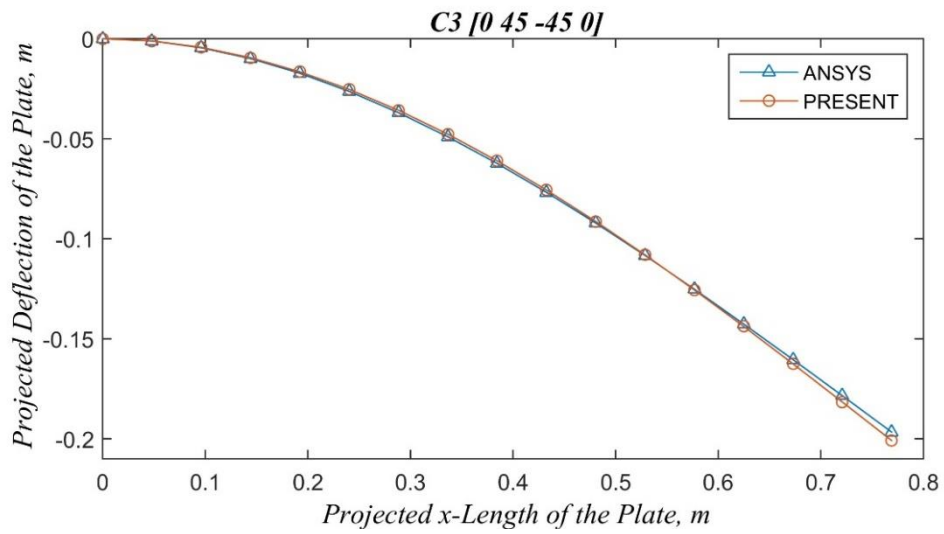


Figure 3.9 Deflection of C3 composite

The effective modulus of elasticity for C4 is calculated as 41.2301 GPa. The force is taken as 435 N for 200 mm deflection. Figure 3.10 gives the deflection graphs of C4.

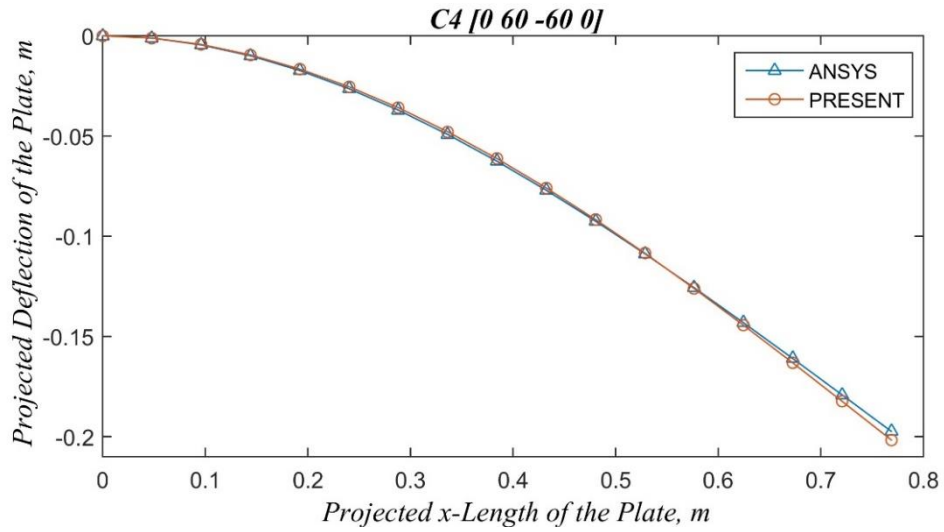


Figure 3.10 Deflection of C4 composite

Due to the outermost 90 degree laminae in C5, the modulus of elasticity is calculated lower. The effective modulus of elasticity is 28.7349 GPa and the maximum force is 300 N. Figure 3.11 gives the C5 deflection graphs for these values

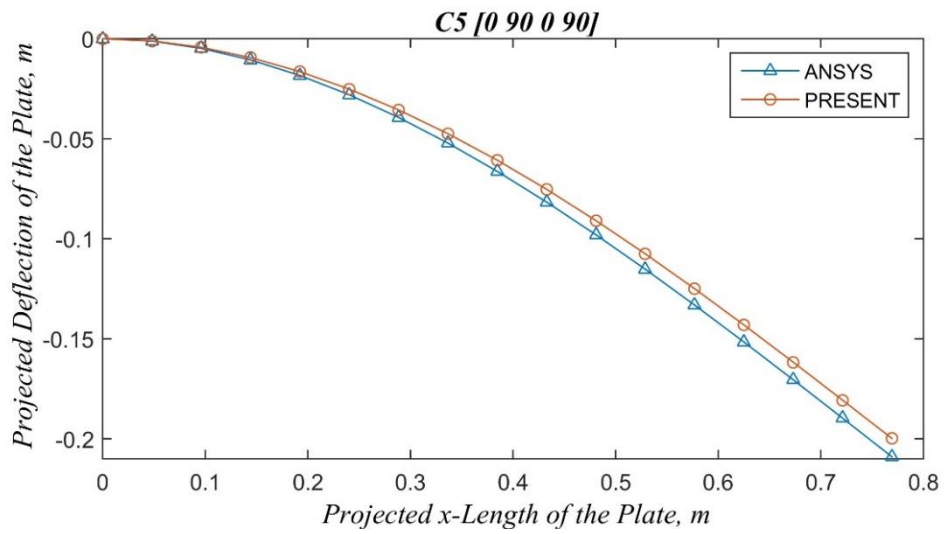


Figure 3.11 Deflection of C5 composite

Another orientation of laminae, the effective modulus of elasticity for C6 is calculated as 32.111 GPa and maximum force is 340 N. Figure 3.12 gives the deflection graphs of C6.

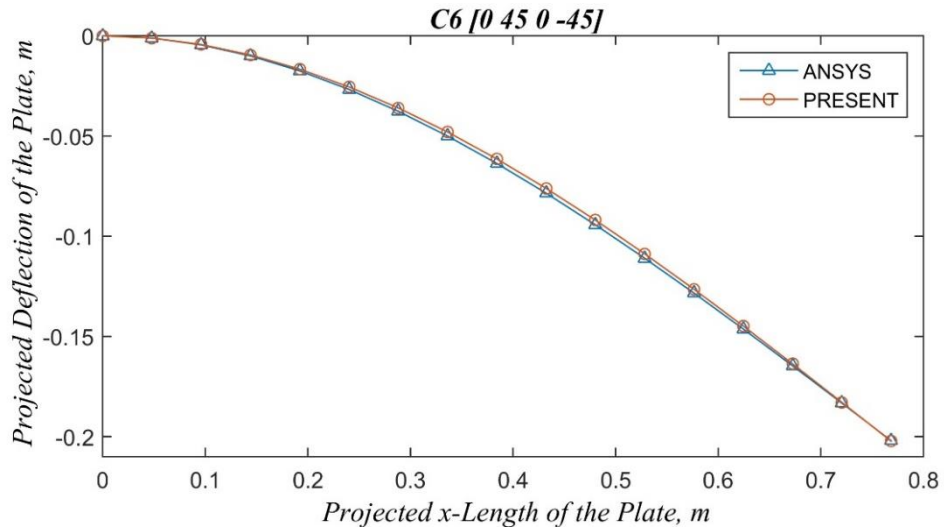


Figure 3.12 Deflection of C6 composite

Figure 3.13 shows the deflection plots of 7th orientation, which is called C7. The effective modulus of elasticity is calculated as 16.1967 GPa due to the fact that two 90 degree laminates are located on the outside. Also, the maximum force is 170 N.

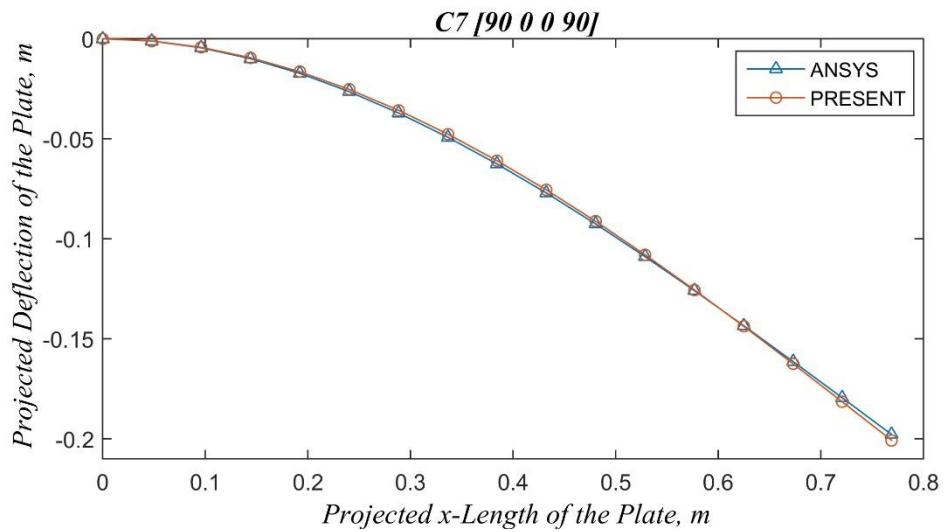


Figure 3.13 Deflection of C7 composite

The last lamination orientation is called C8, and Figure 3.14 shows the deflection plots. Since the fibers in all the laminates in the composite are taken 90 degrees, E_y , which is 12 GPa, is the effective modulus of elasticity for this orientation.

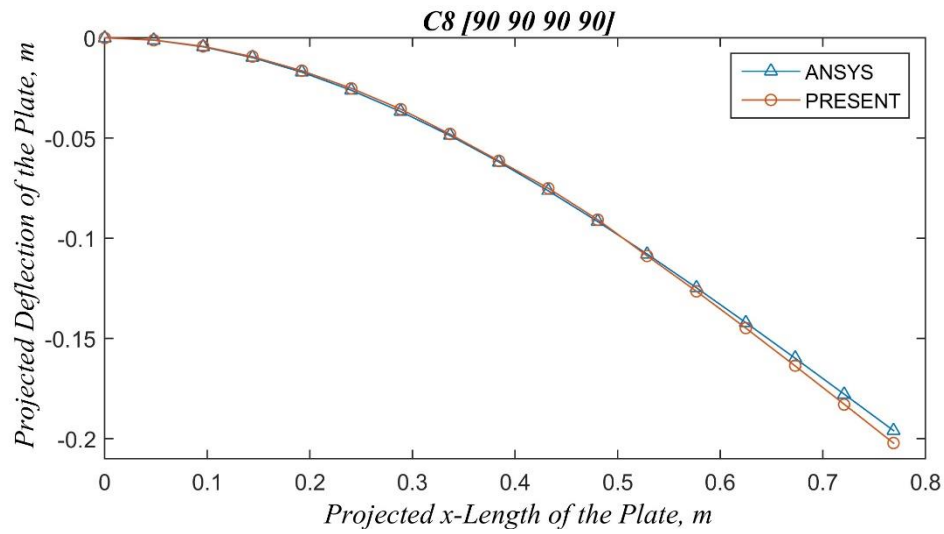


Figure 3.14 Deflection of C8 composite

3.5 Conclusion

Within the thesis's scope, the deflection values of four different isotropic plates and eight different anisotropic plates are calculated. Although there are little differences between the comparison results between the deflection equation and ANSYS results due to the non-linear deflection equation, the deflection results are very similar to ANSYS. With this phenomenon, the deflection equation can be used for both isotropic and anisotropic plates, and the deflection equation accurately represents the plate deflection pattern in the geometric modeling stage.

CHAPTER 4

DYNAMIC ANALYSIS

4.1 Introduction

This chapter presents the free vibration analysis of two edges clamped pre-stressed isotropic and anisotropic curved plates obtained from a cantilever plate with applying a distributed vertical load on its free end. The effects of distributed vertical load on the vibration for the first five modes are investigated, and the results are given in tables and graphics. The results of the present theory are compared with ANSYS results for the same load parameters as used in the theoretical analysis to verify the correctness and effectiveness of the current model. The present model is suitable for vibration analysis of pre-stressed isotropic and anisotropic curved plates when the results are compared.

4.2 Straight and Curved Isotropic Plate

In this section, the free vibration of pre-stressed curved plate is investigated. The four-node flat plate element is used to obtain the pre-stressed plate structure, which is large deflected. The local coordinates of the flat plate element are transformed into global coordinates via the transformation matrix. The plate is discretized into 20x20 finite elements. The material properties are given in Table 4.1.

Table 4.1 Material properties and the geometry of the isotropic plate

Symbol	Property	Quantity
E	Elasticity modulus	200 GPa
ρ	Mass density	7800 kg/m ³
ν	Poisson ratio	0.30
a	Length of the plate	1 m
b	Width of the plate	1 m
h	Thickness of the plate	3 mm

The analysis of the natural frequency parameter (λ) and mode shapes for the first five modes are performed to investigate the effects of vertical non-dimensional load

parameter. The non-dimensional load parameter (β) is considered between 0 and 1.38, which corresponds to almost 85% of the maximum load of the material's yield strength. λ and β are given in equation (4.1).

$$\lambda = \omega \sqrt{\frac{\rho A a^4}{EI}} \quad (4.1)$$

$$\beta = \frac{Pa^2}{vEI}$$

Figure 4.1 shows the deformation of one edge fixed plate subjected to a vertical distributed end load P varies according to the non-dimensional load parameter between 0 and 1.38.

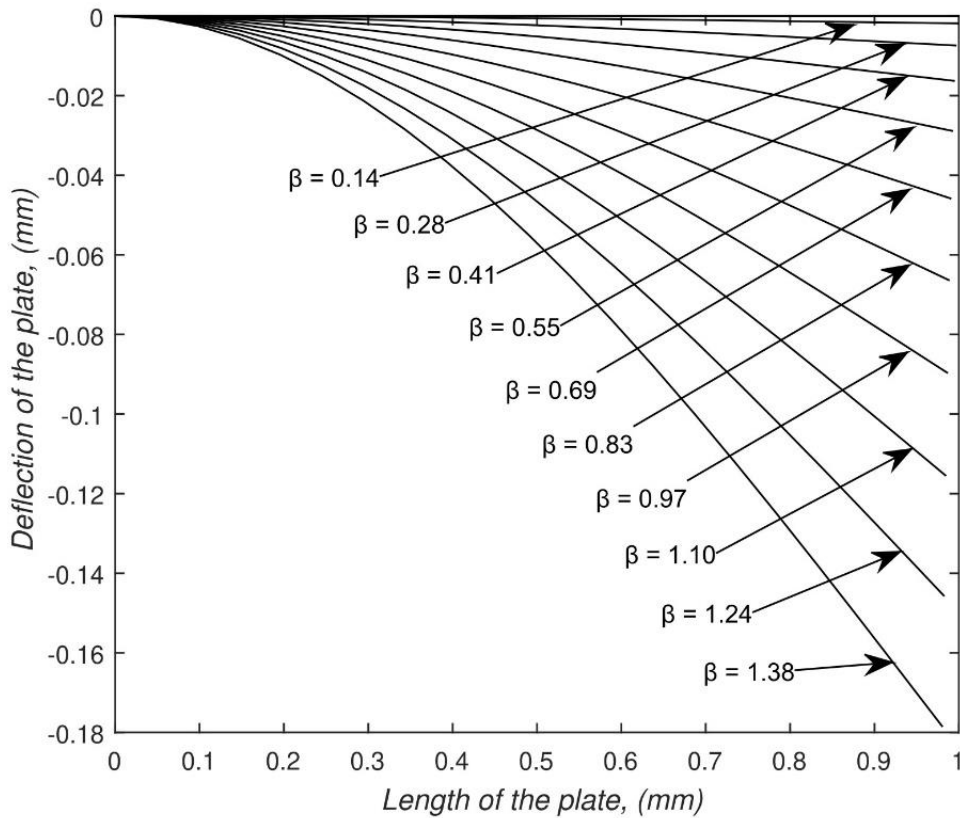


Figure 4.1 Deformation of a plate under different load parameters

After the plate is deflected by vertical distributed loading, the large deflected plate is fixed at the loading point. Thus, pre-stressed curved plate is obtained, and free vibration analysis is performed for all calculated geometric forms. To verify the reliability and validity of the present model, the natural frequency parameters obtained from this study are compared with the results obtained from ANSYS software. Table 4.2 shows the changing of the first five natural frequency parameters due to load parameters.



Table 4.2 Comparison of the present isotropic model and ANSYS software results

β	λ_1 Present	λ_1 ANSYS	Error λ_1	λ_2 Present	λ_2 ANSYS	Error λ_2	λ_3 Present	λ_3 ANSYS	Error λ_3	λ_4 Present	Error λ_4	λ_4 ANSYS	Error λ_4	λ_5 Present	Error λ_5	λ_5 ANSYS	Error
0.00	3.691	3.718	0.73%	4.401	4.424	0.52%	7.251	7.288	0.51%	10.189	0.51%	10.397	2.00%	11.191	2.00%	11.387	1.72%
0.05	4.037	4.037	0.00%	4.701	4.710	0.19%	7.470	7.486	0.22%	10.218	0.22%	10.423	1.97%	11.237	1.97%	11.430	1.69%
0.11	4.904	4.846	1.21%	5.477	5.450	0.50%	8.074	8.038	0.46%	10.313	0.46%	10.508	1.85%	11.380	1.85%	11.561	1.57%
0.16	5.992	5.849	2.44%	6.493	6.391	1.59%	8.943	8.814	1.47%	10.510	1.47%	10.671	1.51%	11.645	1.51%	11.789	1.23%
0.21	7.052	6.868	2.68%	7.540	7.376	2.22%	9.939	9.720	2.25%	10.879	2.25%	10.960	0.74%	12.073	0.74%	12.144	0.59%
0.27	7.895	7.748	1.91%	8.467	8.290	2.14%	10.945	10.662	2.66%	11.518	2.66%	11.453	0.57%	12.711	0.57%	12.665	0.36%
0.32	8.440	8.379	0.72%	9.199	9.047	1.68%	11.886	11.567	2.76%	12.439	2.76%	12.197	1.98%	13.558	1.98%	13.369	1.41%
0.37	8.755	8.774	0.23%	9.752	9.635	1.22%	12.742	12.40	2.76%	13.511	2.76%	13.142	2.81%	14.536	2.81%	14.227	2.18%
0.42	8.938	9.011	0.80%	10.186	10.089	0.96%	13.517	13.157	2.74%	14.576	2.74%	14.164	2.91%	15.526	2.91%	15.157	2.44%
0.48	9.053	9.157	1.14%	10.553	10.458	0.91%	14.231	13.847	2.77%	15.520	2.77%	15.153	2.42%	16.422	2.42%	16.069	2.20%
0.53	9.129	9.252	1.33%	10.887	10.778	1.01%	14.906	14.489	2.88%	16.280	2.88%	16.031	1.55%	17.166	1.55%	16.893	1.62%
0.58	9.183	9.317	1.44%	11.206	11.070	1.22%	15.556	15.096	3.05%	16.850	3.05%	16.757	0.55%	17.746	0.55%	17.588	0.90%
0.64	9.224	9.364	1.49%	11.520	11.350	1.50%	16.190	15.679	3.26%	17.262	3.26%	17.324	0.36%	18.184	0.36%	18.146	0.21%
0.69	9.255	9.398	1.52%	11.834	11.623	1.82%	16.812	16.244	3.50%	17.557	3.50%	17.750	1.09%	18.516	1.09%	18.580	0.34%
0.74	9.280	9.421	1.49%	12.152	11.880	2.28%	17.423	16.769	3.90%	17.772	3.90%	18.057	1.57%	18.773	1.57%	18.905	0.70%
0.80	9.301	9.444	1.52%	12.474	12.167	2.52%	17.931	17.335	3.44%	18.027	3.44%	18.305	1.52%	18.977	1.52%	19.179	1.05%
0.85	9.319	9.460	1.49%	12.80	12.442	2.88%	18.055	17.864	1.07%	18.614	1.07%	18.485	0.70%	19.147	0.70%	19.388	1.24%
0.90	9.334	9.472	1.46%	13.131	12.716	3.27%	18.151	18.375	1.22%	19.190	1.22%	18.623	3.04%	19.293	3.04%	19.557	1.35%
0.96	9.348	9.482	1.42%	13.465	12.992	3.65%	18.228	18.731	2.69%	19.423	2.69%	18.871	2.93%	19.751	2.93%	19.699	0.27%
1.01	9.361	9.490	1.37%	13.802	13.270	4.01%	18.290	18.818	2.80%	19.544	2.80%	19.355	0.98%	20.296	0.98%	19.821	2.40%
1.06	9.372	9.497	1.31%	14.139	13.548	4.36%	18.343	18.888	2.89%	19.660	2.89%	19.822	0.82%	20.823	0.82%	19.929	4.49%
1.11	9.383	9.502	1.25%	14.476	13.826	4.70%	18.387	18.946	2.95%	19.774	2.95%	20.028	1.27%	21.335	1.27%	20.273	5.24%
1.17	9.393	9.506	1.19%	14.811	14.104	5.01%	18.424	18.994	3.00%	19.889	3.00%	20.120	1.15%	21.830	1.15%	20.708	5.41%
1.22	9.403	9.510	1.12%	15.141	14.381	5.29%	18.457	19.034	3.03%	20.007	3.03%	20.208	0.99%	22.308	0.99%	21.128	5.59%
1.27	9.412	9.512	1.05%	15.465	14.654	5.53%	18.486	19.068	3.06%	20.132	3.06%	20.294	0.80%	22.773	0.80%	21.531	5.77%
1.33	9.421	9.514	0.98%	15.780	14.924	5.74%	18.511	19.098	3.07%	20.265	3.07%	20.380	0.57%	23.224	0.57%	21.918	5.96%
1.38	9.430	9.516	0.90%	16.085	15.189	5.90%	18.533	19.123	3.08%	20.408	3.08%	20.468	0.30%	23.664	0.30%	22.290	6.16%

Table 4.2 indicates that the error rates are quite consistent. The error rate does not exceed 6.16% for the first five frequencies. When the maximum load is taken as 60% of the yield strength of the material, ($\beta < 1.01$), the error rate remains below 4%. This situation proves the accuracy of the model applied in the present study. The changing of the natural frequency parameters with respect to the load parameter are given in Figure 4.2.

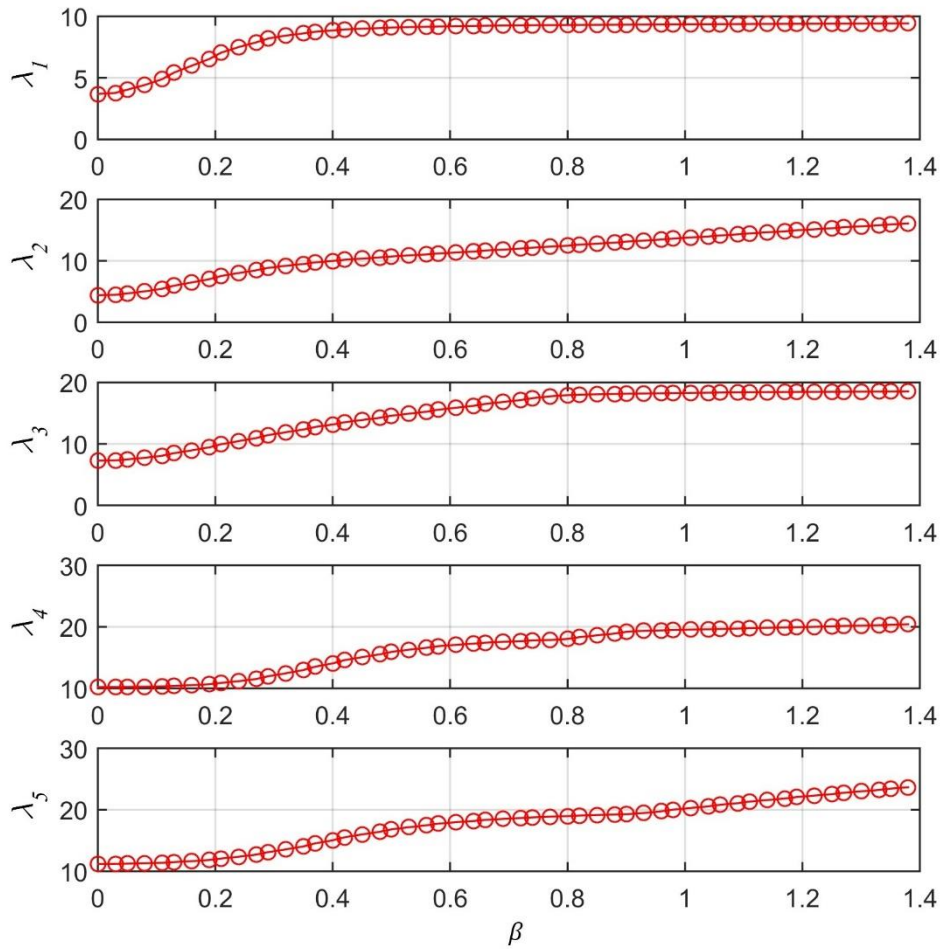


Figure 4.2 Effect of the load parameter on the first five natural frequency parameters for isotropic plate

As seen from Figure 4.2 and Table 4.1, the curve between the natural frequency parameter and the load parameter differs for each natural frequency. The parametric natural frequency values changes in different trends for each load parameter interval. λ_1 , λ_2 , λ_3 increase apparently in the 0-0.2 range, while λ_4 , λ_5 slightly increase. The increase of the first natural frequency parameter between 0.4-1.4 is negligible. This is

seen from $\beta > 0.8$ at the third parametric natural frequency, and from $\beta > 1$ at the fourth natural frequency. There is no such situation in other natural frequencies. Accordingly, the second natural frequency increases almost linearly when $\beta > 0.2$. In the fifth natural frequency, while $\beta = 0.2-0.6$, the increase is more apparent than the 0-0.2 range. This increase in the range of 0.6-0.9 started to decrease and became negligible. However, there is a significant increase in the 0.9-1.4 range again.

In addition, the effect of the changes in curves on the mode shapes is investigated. The first difference in mode shapes is observed at $\beta = 0.54$ as shown in Figure 4.3.

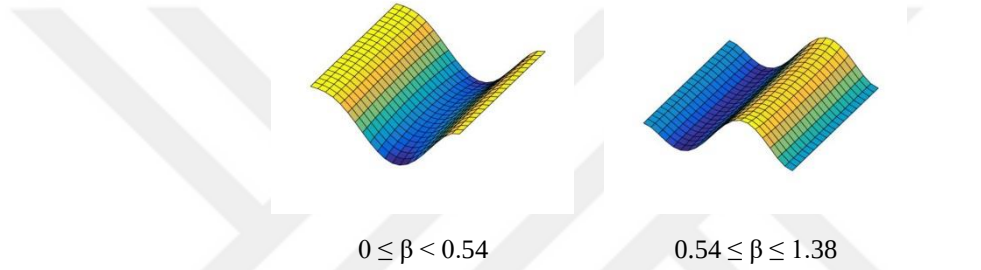


Figure 4.3 Mode shape of first natural frequency parameter (λ_1) with respect to load parameter, β

As the load parameter reaches to 0.57, the second mode changes. Figure 4.4 shows the second mode shapes of the plate structure are between 0-0.57, and 0.57-1.38.

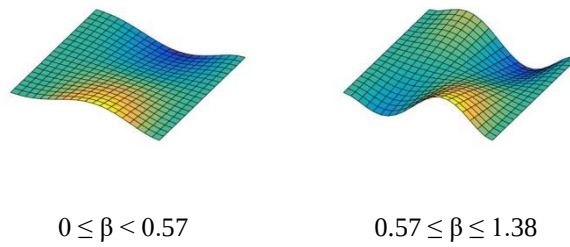


Figure 4.4 Mode shape of second natural frequency parameter (λ_2) with respect to load parameter, β

Three different mode shapes in the third mode as the load parameter is between 0-0.66, 0.66-1.07, and 1.07-1.38, respectively, and these mode shapes are shown in Figure 4.5.

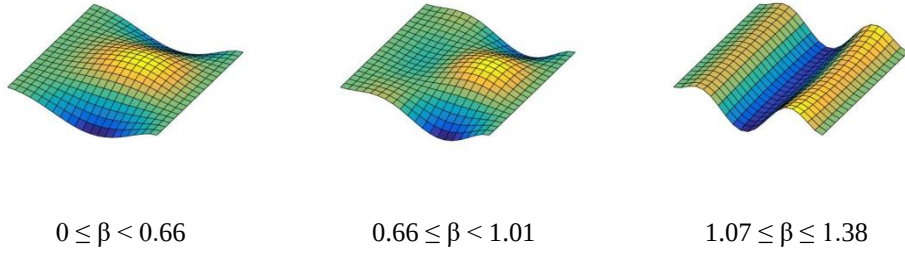


Figure 4.5 Mode shapes of third natural frequency parameter (λ_3) with respect to load parameter, β

The fourth mode has four different mode shapes for four different load parameter intervals, 0-0.79, 0.79-1.07, 1.07-1.15, and 1.15-1.38, shown in Figure 4.6.

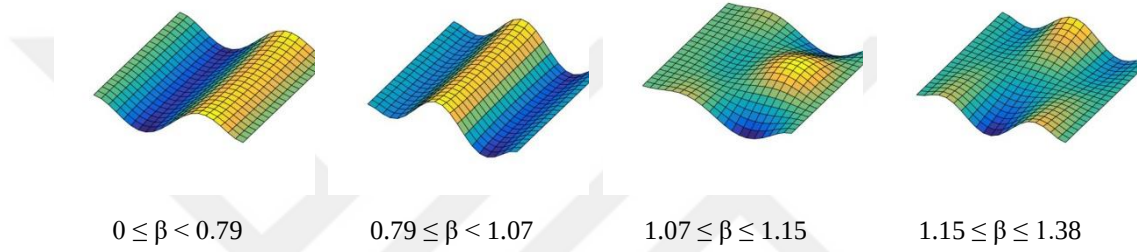


Figure 4.6 Mode shapes of fourth natural frequency parameter (λ_4) with respect to load parameter, β

The fifth mode comprises three different mode shapes as the load parameter changes. The critical points for those changes are $\beta=0.79$, and $\beta=1.15$. The mode shapes that change at these specified load parameter points are given in Figure 4.7.

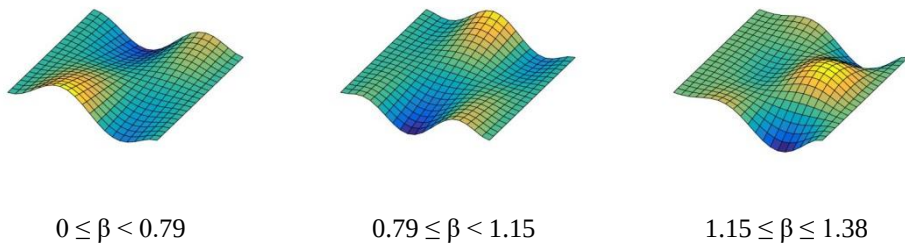


Figure 4.7 Mode shapes of fifth natural frequency parameter (λ_5) with respect to load parameter, β

In a nutshell, two different mode shapes are observed in the first and second natural frequencies, three different mode shapes in the third and fifth natural frequencies, and four different mode shapes in the fourth natural frequency. For fundamental frequencies, mode shapes are in a balanced distribution in symmetric geometries. However, the curved plate examined in this study is deflected on the loading edge.

This reveals asymmetry. This asymmetry shifts the balanced distribution of the mode shape towards the loading edge and creates new mode shapes. The asymmetry, which remains as the load parameter increases, leads to the emergence of new mode shapes. These new mode shapes cause differences in the changing of natural frequency values, as seen in Figure 4.2.

The first mode, as seen from Figure 4.3-a, shifted towards the loading edge as the asymmetry increases. Hence, a new mode shape is emerged. This new mode shape emerges where the loading parameter value reaches to 0.54 at which the trend curve of λ_1 changes, as seen in Figure 4.2. The first mode shape (Figure 4.4-a) of the second natural frequency parameter turns into a different mode shape (Figure 4.4-b) by shifting towards the loading edge when the loading parameter value exceeds 0.57. The mode shape (Figure 4.5-a) of the third natural frequency parameter shifts to the new mode shape (Figure 4.5-b) as the loading parameter value reaches to 0.66. However, as the loading parameter exceeds 1.07, another new mode shape, shown in Figure 4.5-c, is occurred. Four different modes are observed in the fourth mode as the load parameter reaches to 0.79, 1.07, and 1.15, respectively. The critical loading parameter values are 0.79, 1.07, and 1.15. There are three different mode shapes for the fifth mode, and the critical loading parameter values that change the mode shapes are 0.79 and 1.15, respectively.

4.3 Straight and Curved Composite Plate

Within the scope of the subchapter, the analysis of laminated composite plates is performed for eight different lamination orientations. While these analyses are carried out, the deflection values are checked with the help of ANSYS, the natural frequencies of flat plate states are compared, and then the natural frequency parameters under maximum load are examined. It has been seen in the previous section that the deflection values mostly represent the curve model. In this section, the natural frequency parameter values are compared for each composite plate under no load, under 50% load, and under maximum load.

The material properties have been given in Section 3.4 with Table 3.2. The effects of vertical non-dimensional load parameter on the natural frequency (λ) and mode shapes for the first five modes are considered. The non-dimensional load parameter (β) is taken to be between the no-load condition and the load parameter value that corresponding to deflection in the free end up to 25% of the plate length value, of each composite, examined. λ and β are given in equation (4.1) where ω is natural frequency, A is the cross-section area of the plate, ν is the Poisson ratio. Only E_y should be written instead of E in equation (4.1) to obtain equation (4.2).

$$\lambda = \omega \sqrt{\frac{\rho A a^4}{E_y I}} \quad (4.2)$$

$$\beta = \frac{P a^2}{\nu E_y I}$$

In Table 4.3, the comparison of the first five natural frequency parameters calculated with both the present study and ANSYS, then the differences between them are given for C1. The most significant difference is in the state of the third natural frequency parameter at the rate of 7.49% under maximum load. All values except this value are 3% and below.

Table 4.3 Comparative results of C1 composite

C1, 0 0 0 0		$\beta = 0$	$\beta = 1.50$	$\beta = 3.00$
λ_1	ANSYS	7.05	17.47	17.92
	Present	6.99	17.32	18.02
	Error	0.85%	0.83%	0.56%
λ_2	ANSYS	7.63	18.87	21.87
	Present	7.58	18.80	22.70
	Error	0.66%	0.34%	3.78%
λ_3	ANSYS	10.01	22.47	29.39
	Present	9.97	22.79	31.59
	Error	0.40%	1.44%	7.49%
λ_4	ANSYS	15.46	27.49	35.95
	Present	15.32	28.10	35.21
	Error	0.87%	2.23%	2.07%

Table 4.3 continues

C1, 0 0 0 0		$\beta = 0$	$\beta = 1.50$	$\beta = 3.00$
λ_5	ANSYS	19.83	29.83	36.61
	Present	19.42	29.92	36.21
	Error	2.08%	0.32%	1.08%

Comparative results of the first five natural frequency parameters of C2 are given in Table 4.4. The biggest difference is seen in the third natural frequency parameter under the maximum load. All other error values are calculated at 4% and below.

Table 4.4 Comparative results of C2 composite

C2, 0 90 90 0		$\beta = 0$	$\beta = 1.38$	$\beta = 2.75$
λ_1	ANSYS	6.72	16.28	17.01
	Present	6.66	16.21	17.13
	Error	0.85%	0.45%	0.74%
λ_2	ANSYS	7.33	17.48	20.75
	Present	7.29	17.49	21.62
	Error	0.64%	0.09%	4.21%
λ_3	ANSYS	10.02	20.88	28.02
	Present	9.98	21.24	30.22
	Error	0.39%	1.71%	7.84%
λ_4	ANSYS	16.42	25.74	33.56
	Present	16.27	25.89	33.08
	Error	0.96%	0.56%	1.45%
λ_5	ANSYS	18.90	26.04	34.27
	Present	18.51	26.54	34.00
	Error	2.07%	1.90%	0.79%

C3 is a type of composite called cross-ply. Table 4.5 gives the natural frequency parameter results compared with ANSYS. The biggest error rate is seen in the third natural frequency parameter and in the case of maximum load.

Table 4.5 Comparative results of C3 composite

C3, 0 45 -45 0		$\beta = 0$	$\beta = 1.41$	$\beta = 2.82$
λ_1	ANSYS	6.80	16.64	17.26
	Present	6.77	16.59	17.43
	Error	0.50%	0.28%	1.01%
λ_2	ANSYS	7.45	18.08	21.89
	Present	7.44	18.18	23.02
	Error	0.07%	0.56%	5.18%
λ_3	ANSYS	10.03	21.71	29.31
	Present	10.09	22.23	31.80

Table 4.5 continues

C3, 0 45 -45 0		$\beta = 0$	$\beta = 1.41$	$\beta = 2.82$
	Error	0.61%	2.37%	8.52%
λ_4	ANSYS	15.83	26.67	34.26
	Present	15.88	27.20	33.80
	Error	0.31%	2.00%	1.34%
λ_5	ANSYS	19.13	26.95	35.02
	Present	18.81	27.48	34.85
	Error	1.72%	1.97%	0.51%

In C4, another composite called cross-ply, the biggest error rate is seen in the third natural frequency parameter. Table 4.6 gives the results of the first five natural frequency parameters of C4 compared to ANSYS.

Table 4.6 Comparative results of C4 composite

C4, 0 60 -60 0		$\beta = 0$	$\beta = 1.39$	$\beta = 2.78$
λ_1	ANSYS	6.75	16.59	17.11
	Present	6.70	16.52	17.25
	Error	0.73%	0.38%	0.84%
λ_2	ANSYS	7.39	18.16	21.49
	Present	7.37	18.27	22.57
	Error	0.30%	0.62%	5.04%
λ_3	ANSYS	10.02	22.05	28.86
	Present	10.06	22.63	31.32
	Error	0.42%	2.64%	8.54%
λ_4	ANSYS	16.06	27.18	33.83
	Present	16.09	27.61	33.37
	Error	0.21%	1.61%	1.39%
λ_5	ANSYS	18.99	27.37	34.58
	Present	18.62	28.08	34.37
	Error	1.95%	2.58%	0.61%

It is seen that the representation rate is better in C5, which is another composite type. The highest calculated error rate value is 4.49%, at the second natural frequency parameter under the maximum load, and the comparative values are given in Table 4.7.

Table 4.7 Comparative results of C5 composite.

C5, 0 90 0 90		$\beta = 0$	$\beta = 0.96$	$\beta = 1.92$
λ_1	ANSYS	5.43	13.57	13.81
	Present	5.56	13.79	14.34
	Error	2.42%	1.57%	3.82%
λ_2	ANSYS	6.18	15.43	18.67
	Present	6.30	15.52	19.50
	Error	1.98%	0.54%	4.49%
λ_3	ANSYS	9.81	20.23	27.13
	Present	9.98	20.31	28.02
	Error	1.74%	0.38%	3.25%
λ_4	ANSYS	15.28	24.27	27.92
	Present	15.45	23.81	28.71
	Error	1.14%	1.89%	2.84%
λ_5	ANSYS	16.30	25.18	29.03
	Present	16.48	24.74	29.27
	Error	1.11%	1.74%	0.83%

Comparative results are given in the Table 4.8 for C6 which is the last type of the study. The maximum difference in this plate occurred in the maximum load state of the third natural frequency parameter.

Table 4.8 Comparative results of C6 composite

C6, 0 45 0 -45		$\beta = 0$	$\beta = 1.09$	$\beta = 2.18$
λ_1	ANSYS	5.92	14.81	15.14
	Present	6.04	15.01	15.62
	Error	2.10%	1.34%	3.23%
λ_2	ANSYS	6.72	16.77	20.60
	Present	6.93	17.09	21.91
	Error	3.10%	1.89%	6.37%
λ_3	ANSYS	9.87	21.22	28.33
	Present	10.27	22.00	30.51
	Error	4.06%	3.67%	7.68%
λ_4	ANSYS	16.44	25.83	30.39
	Present	16.77	25.93	30.97
	Error	1.95%	0.38%	1.90%
λ_5	ANSYS	16.74	26.77	31.89
	Present	17.14	27.05	32.40
	Error	2.35%	1.02%	1.59%

The effective modulus of elasticity of the C7 composite is much lower than the first six lamination orientations, and due to this module, the natural frequency values are

more affected. In this lamination orientation, two 90 degrees laminates are on the outside. Table 4.9 gives the comparative results of the current study with ANSYS.

Table 4.9 Comparative results of C7 composite

C7, 90 0 0 90		$\beta = 0$	$\beta = 0.54$	$\beta = 1.09$
λ_1	ANSYS	4.22	10.67	10.77
	Present	4.18	10.53	10.81
	Error	0.90%	1.23%	0.38%
λ_2	ANSYS	5.15	12.95	16.06
	Present	5.13	13.01	17.07
	Error	0.55%	0.40%	6.31%
λ_3	ANSYS	10.00	18.86	21.93
	Present	9.96	19.22	21.33
	Error	0.42%	1.92%	2.74%
λ_4	ANSYS	11.87	20.29	23.50
	Present	11.62	19.92	23.23
	Error	2.15%	1.81%	1.14%
λ_5	ANSYS	13.18	21.41	24.95
	Present	12.96	21.16	26.77
	Error	1.67%	1.16%	7.27%

The C8 composite consists of four 90-degree laminae and the effective modulus of elasticity is equal to the modulus of elasticity in the y-direction of the composite material whose properties are given. Table 4.10 gives the numerical differences between ANSYS and the current study.

Table 4.10 Comparative results of C8 composite

C8, 90 90 90 90		$\beta = 0$	$\beta = 0.40$	$\beta = 0.80$
λ_1	ANSYS	3.64	9.05	9.26
	Present	3.60	8.97	9.30
	Error	0.93%	0.97%	0.45%
λ_2	ANSYS	4.69	11.09	13.93
	Present	4.67	11.21	14.93
	Error	0.52%	1.10%	7.19%
λ_3	ANSYS	9.99	15.71	18.58
	Present	9.95	15.71	18.16
	Error	0.34%	0.03%	2.27%
λ_4	ANSYS	10.24	16.48	20.11
	Present	10.00	16.85	20.00
	Error	2.26%	2.27%	0.57%
λ_5	ANSYS	11.74	17.10	21.50
	Present	11.55	17.10	23.23
	Error	1.60%	0.01%	8.05%

4.4 Conclusion

The maximum vertical distributed load is taken a value that brings the isotropic plate up to 85% of its yield stress value, due to the operating conditions. In anisotropic plates, the deflection value is limited to 200 mm. All results obtained from the present model are in a close agreement with ANSYS software results. It is essential to understand the dynamic properties of pre-stressed plate structures under several loading conditions. This chapter reveals the change of dynamic behaviour of the pre-stressed curved plate structure as the parametric load, which is the function of the distributed vertical load, increases. Based on all results presented within this chapter, the conclusions are as follows:

- Knowing dynamic behaviours under operating conditions is important in terms of predicting the damages that may occur in the structure due to concepts such as resonance and stability. In this chapter, the dynamic behaviour of the pre-stressed curved thin plate structure is examined.
- In all loading cases, up to 85% of the isotropic plates' yield stress value and the deflection point limited to 200 mm for the anisotropic plates; the present model represents the pre-stressed curved thin plate model accurately.
- The error rates are acceptable and partially better for the first, third, and fourth natural frequency parameters, where they are mostly tended to remain stable than the second and fifth natural frequency parameters in isotropic plates.
- Error rates are acceptable for all composite plates. The maximum error rate did not exceed 8% even in C7 and C8 composites, which are considered more challenging to represent due to their stacking order.
- Various curvature values of the structure occur as the applied load increases. This causes mode shape differences in the same modes.
- The asymmetry of the curved plate shifts the pattern of the mode shape to the edge on which the loading is present.
- As the asymmetry of the curved plate becomes apparent, different mode shapes emerge.

- In the light of the isotropic natural frequency parameter results, the method used in this study provides a simple and fast alternative way for modelling of curved plates.
- In the light of all composite natural frequency parameter values, the flat and curved models that are examined show great similarity with ANSYS results.



CHAPTER 5

EFFECT OF CURVATURE ON VIBRATION

5.1 Introduction

Flat and curved plates with different geometries are used as structural elements in many engineering fields. Different theories of plates for a cylindrical structure are also the subject of different studies.

It is well-known knowledge that the stiffness of the plates increases with increasing the curvature. This information is used in bridge models, dam structures, vehicle designs. However, the mode shapes that change with the change of curvature should also be considered as a design parameter. Furthermore, the continued increase in curvature sharply reduces the stiffness of the geometry. In this chapter, the natural frequency analysis of the fixed-length isotropic thin square plate, starting from the flat plate model to the semicircle model fixed on both edges, is performed. The results are confirmed by the finite element program. In addition, the mode shapes affected by the first five natural frequency changes are shown.

5.2 Curve Model

To examine the effect of the radius of curvature, firstly, the flat plate fixed on both sides. At this point, the radius of curvature is theoretically infinite. For the transition to the curved plate, the radius of curvature is then reduced from infinite to 160 mm, where the structure is exactly a semicircle. The length of the plate is kept constant so that the natural frequency change can be seen during the radius change. Figure 5.1 shows a linear representation of curved plate models ranging from flat model to semicircle.

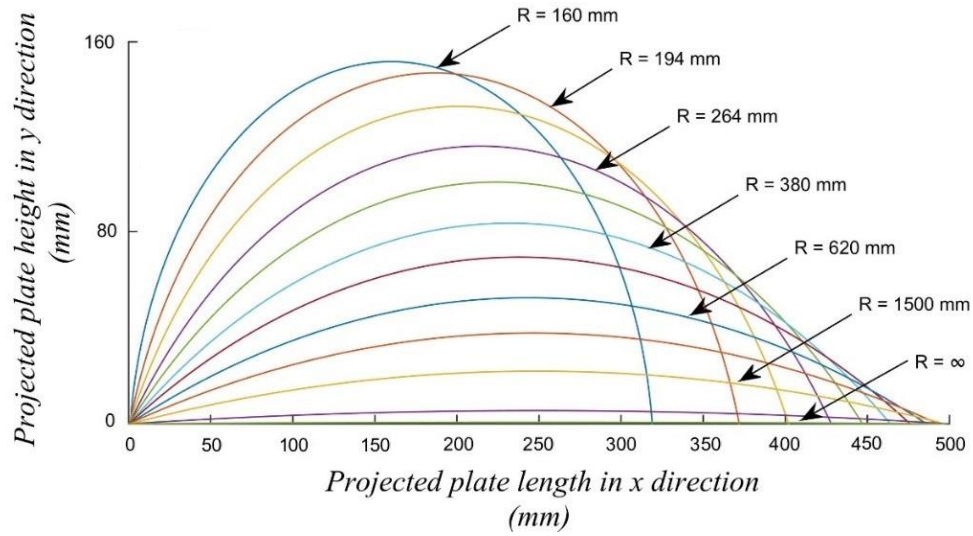


Figure 5.1 Changing of flat model to semicircle model with constant length

5.3 Results

In order to analyze the effect of curvature, steel is chosen as the material, which is frequently used in engineering structures. The geometric and material properties are given in Table 5.1.

Table 5.1 Material properties and the geometry of the isotropic plate for the effect of curvature

Symbol	Property	Quantity
E	Elasticity modulus	200 GPa
ρ	Mass denstiy	7870 kg/m ³
ν	Poisson ratio	0.29
a	Length of the plate	0.5 m
b	Width of the plate	0.5 m
h	Thickness of the plate	5 mm

The effects of radius of curvature on the natural frequency, λ and mode shapes for the first five modes are investigated. λ has been given in equation (4.1).

The natural frequency parameters of the present model and ANSYS are compared to check the accuracy of the present model. Representative flat and semicircle models with two sides fixed are shown in Figure 5.2. Table 5.2 shows a comparison of the results between the present study and ANSYS.

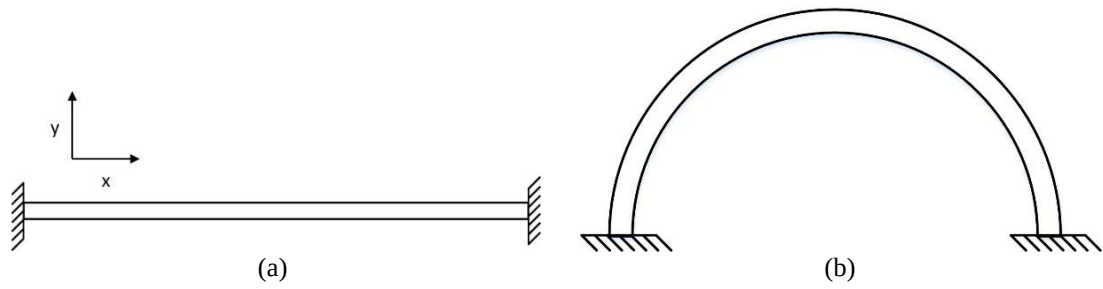


Figure 5.2 Plate models. (a) flat plate model, (b) semicircle plate model

Table 5.2 Comparison results between the present model and ANSYS model for the effect of curvature

Radius (mm)	λ_1		λ_2		λ_3		λ_4		λ_5	
	ANSYS	Present	ANSYS	Present	ANSYS	Present	ANSYS	Present	ANSYS	Present
Flat	3.70	3.69	4.41	4.40	7.25	7.25	10.18	10.19	11.18	11.19
5000	5.90	5.95	6.37	6.41	8.77	8.76	10.18	10.21	11.32	11.35
4500	6.30	6.35	6.74	6.79	9.08	9.08	10.18	10.21	11.36	11.39
4000	6.81	6.88	7.23	7.29	9.51	9.52	10.17	10.21	11.40	11.44
3500	7.49	7.57	7.88	7.95	10.09	10.11	10.17	10.21	11.47	11.51
3000	8.42	8.51	8.77	8.86	10.17	10.21	10.93	10.96	11.58	11.62
2500	9.72	9.83	10.04	10.15	10.17	10.20	11.75	11.79	12.18	12.23
2000	10.17	10.19	11.60	11.73	11.93	12.06	12.06	12.11	14.15	14.22
1500	10.15	10.18	12.68	12.76	14.28	14.42	14.72	14.87	17.49	17.57
1000	10.10	10.12	14.25	14.40	16.91	17.02	17.73	17.84	21.58	21.71
500	9.80	9.82	17.83	17.91	18.76	18.85	19.69	20.00	29.10	29.23
160	7.18	7.22	15.75	15.88	15.92	16.08	25.36	25.73	29.20	29.58

Table 5.2 shows that the natural frequency parameter results of the current model and the error rates with respect to ANSYS results are quite consistent. The results of the first five natural frequency parameters of the curved plate model starting from 5000 mm radius until the semicircle changes are given in Figure 5.3.

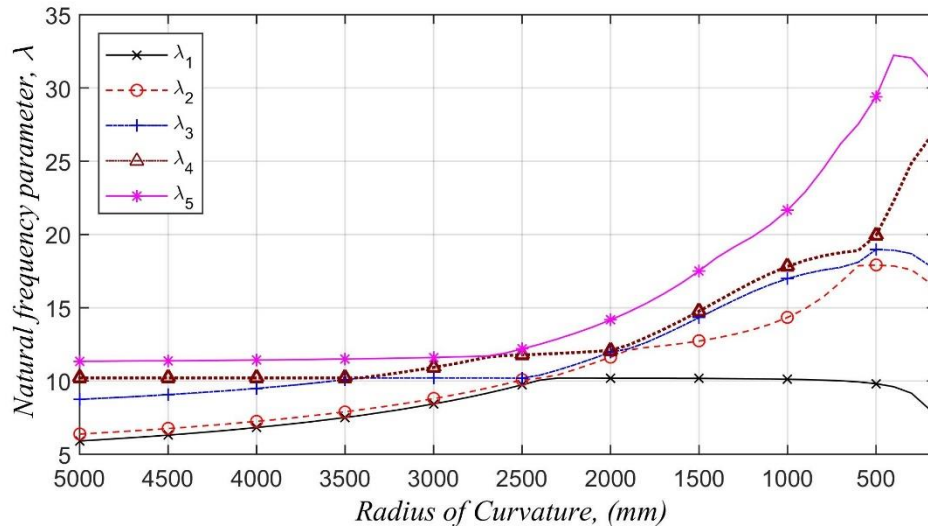


Figure 5.3 Effect of the radius of the curvature on the natural frequencies

When the results of the natural frequency parameters from the flat plate model to the semicircle plate model are examined, it is seen that from Figure 5.3, each natural frequency parameter curve changes its behavior at a different radius of curvature. Different mode shapes are observed with these natural frequency parameter curve changes. Moreover, each natural frequency parameter shows a sharp decrease when the radius of curvature is close to the semicircle.

When the mode shapes belong to the first five natural frequencies are examined, the constant mode shape occurs up to a radius of curvature of 2400 mm for the first natural frequency parameter, and then a different mode shape occurs. The two different mode shapes are given in Figure 5.4. Also, all mode shape graphs are given in the rectangular plate plane.

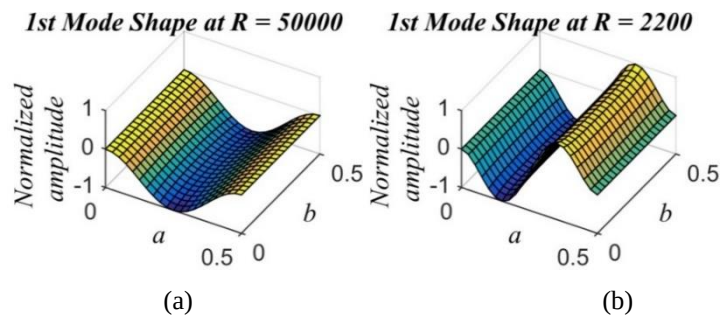


Figure 5.4 The two different mode shapes for the first natural frequency for the effect of curvature

Figure 5.4-a represents the mode shape for the radius of curvature value between the flat plate and the 2400 mm, Figure 5.4-b represents the mode shape in the section where the radius varies between 2400 mm and 160 mm in the first natural frequency parameter.

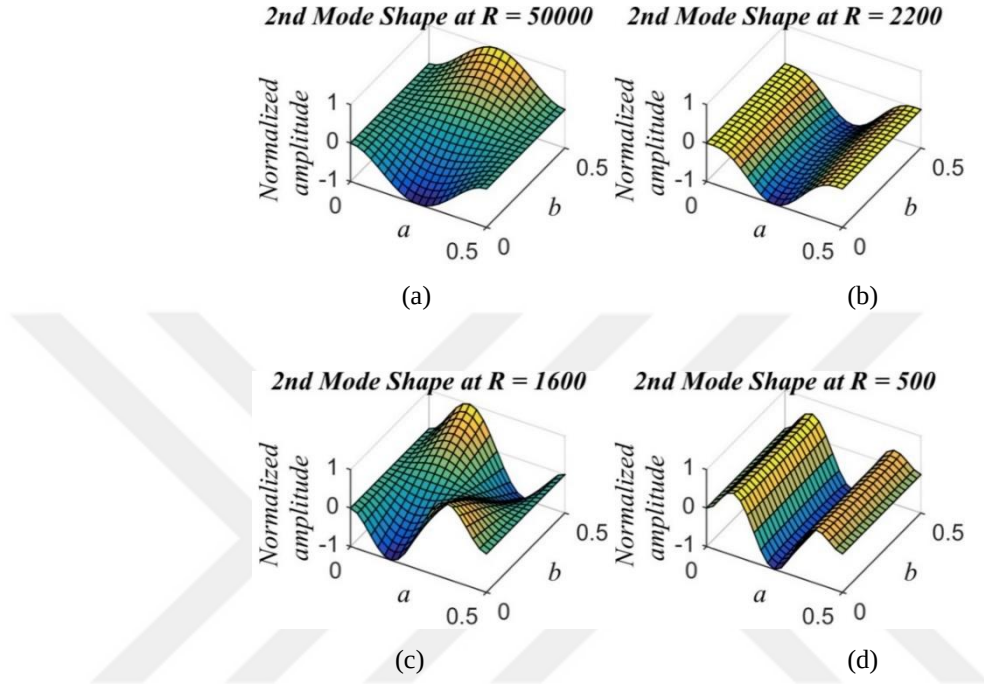


Figure 5.5 The four different mode shapes for the second natural frequency for the effect of curvature

Four different mode shapes are seen for the second natural frequency parameter. Figure 5.5-a represents the mode shape for the radius of curvature value between the flat plate and the 2500 mm. Figure 5.5-b, Figure 5.5-c, and Figure 5.5-d display the mode shapes for the radius of curvature in the range of 2500 to 2000 mm, 2000 to 600 mm, and 600 to 160 mm, respectively.

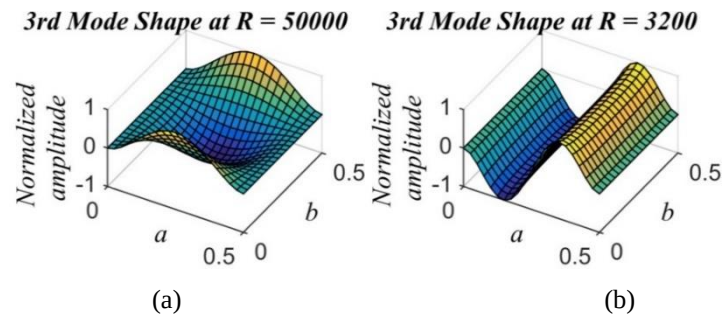


Figure 5.6 The six different mode shapes for the third natural frequency for the effect of curvature

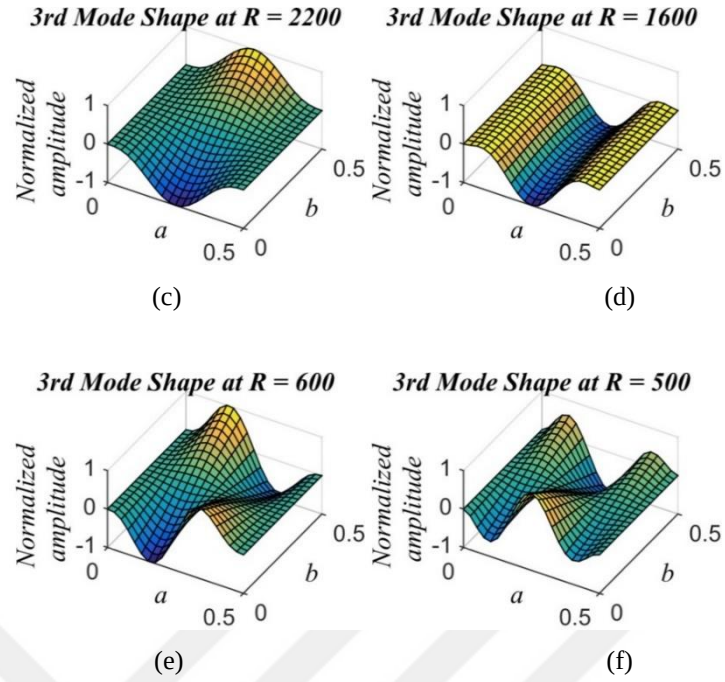


Figure 5.6 continues

Six different mode shapes are seen for the third natural frequency parameter. Figure 5.6-a represents the mode shape for the radius of curvature value between the flat plate and the 3500 mm. Figure 5.6-b, Figure 5.6-c, Figure 5.6-d, Figure 5.6-e, and Figure 5.6-f display the mode shapes for the radius of curvature in the range of 3500 to 2500 mm, 2500 to 2000 mm, 2000 to 800 mm, 800 to 500 mm, and 500 to 160 mm, respectively.

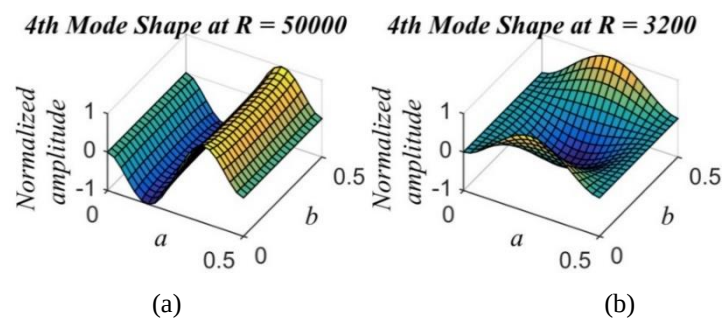


Figure 5.7 The six different mode shapes for the fourth natural frequency for the effect of curvature

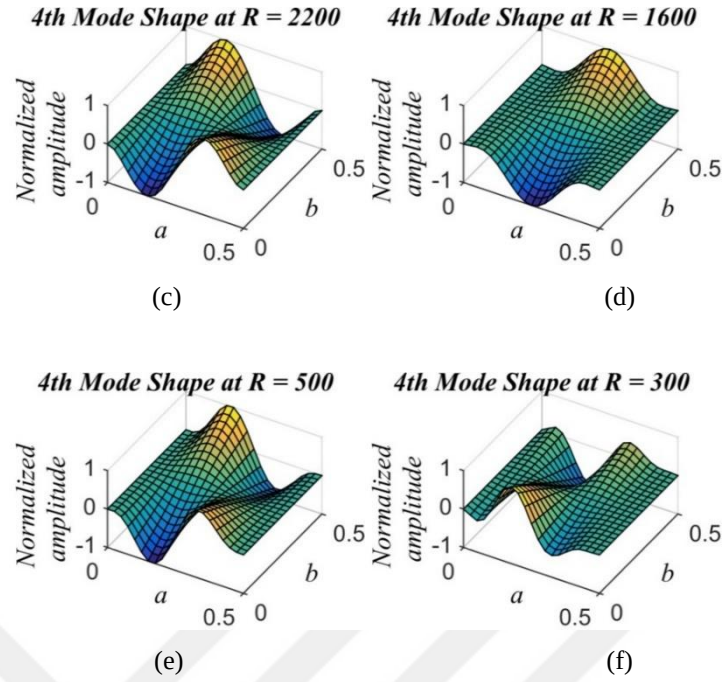


Figure 5.7 continues

Six different mode shapes are seen for the fourth natural frequency parameter, as for the third natural frequency parameter. Figure 5.7-a represents the mode shape for the radius of curvature value between the flat plate and the 3500 mm. Figure 5.7-b, Figure 5.7-c, Figure 5.7-d, Figure 5.7-e, and Figure 5.7-f display the mode shapes for the radius of curvature in the range of 3500 to 2700 mm, 2700 to 2000 mm, 2000 to 1600 mm, 1600 to 600 mm, and 600 to 160 mm, respectively.

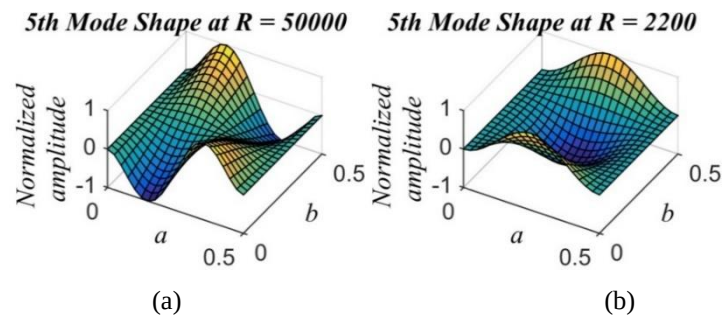


Figure 5.8 The four different mode shapes for the fifth natural frequency for the effect of curvature

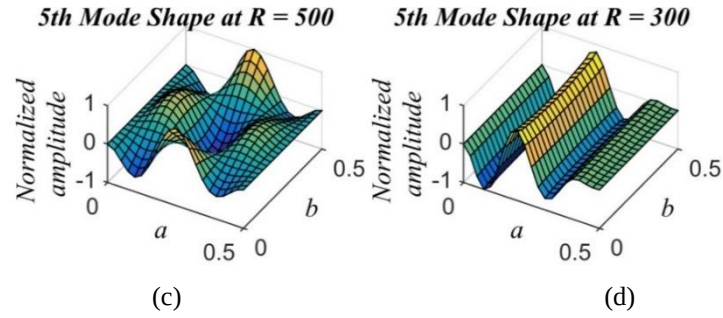


Figure 5.8 continues

Four different mode shapes are seen for the fifth natural frequency parameter, as for the second natural frequency parameter. Figure 5.8-a represents the mode shape for the radius of curvature value between the flat plate and the 2700 mm. Figure 5.8-b, Figure 5.8-c, and Figure 5.8-d display the mode shapes for the radius of curvature in the range of 2700 to 1400 mm, 1400 to 400 mm, and 400 to 160 mm, respectively.

Figures 5.4-5.8 show the mode shapes resulting from changes in the natural frequency parameter curves given in Figure 5.3. Consequently, different mode shapes occur at different points where the shape of the natural frequency parameter curvature changes. In addition, it is seen that the natural frequency parameter value shows a sudden decrease for all five natural frequencies where the geometry is closer to the semicircle.

5.4 Conclusion

In this chapter, the effect of the radius of curvature on the natural frequency parameter is investigated by keeping the plate length constant. The first five natural frequencies are examined for all radius of curvature from the flat plate model to the semicircle plate model. Moreover, the mode shape changes due to the radius of curvature are also examined. According to the results obtained:

- As the curvature of the plate increases with constant length, the natural frequency values also increase to a certain level for all the natural frequency parameters.

- By changing the radius of the curvature, more than one mode shape occurs for each natural frequency parameter.
- With the change of the radius of curvature, the natural frequency parameter curve behaves differently for each natural frequency parameter. Each change in the curves shows that a different mode shape occurs.
- As the radius of curvature moves the plate closer to the semicircle geometry, the increase in all the natural frequency parameters stops, and then all values begin to decrease sharply.



CHAPTER 6

EFFECT OF CRACK ON VIBRATION

6.1 Introduction

Flat and curved plates with various geometries are fundamental structural elements for many engineering fields. Some of the plates in the assembly stage are produced in a curved form, and some of them are curved during assembly due to their usage places and used in a pre-stressed state. Therefore, any error that may occur in the plates that are exposed to different variable dynamic loads directly affects the vibration sensitivity of the structure. In this section, the change of the natural frequency parameter of the plate according to the position of the crack is examined.

6.2 Straight and Curved Isotropic Plate with Crack

The crack length is kept constant at two times the mesh size. The thickness of the crack is equal to the plate thickness. Figure 6.1 shows the position of the crack, where c_x and c_y represent the location of the crack.

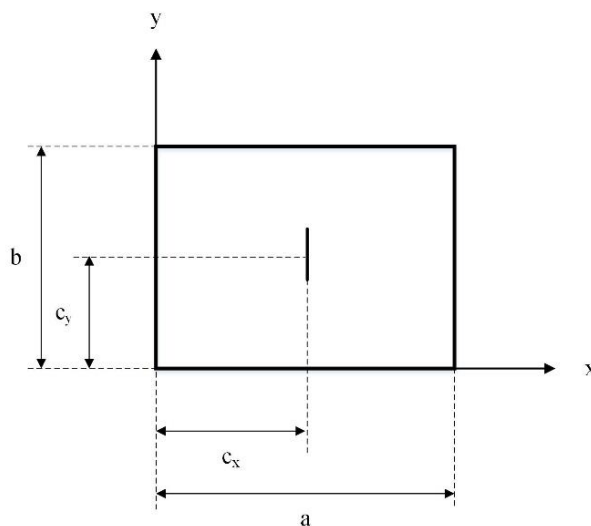


Figure 6.1 Crack position on the plate

The existing mesh layout is preserved so that the crack is correctly attached to the plate, and the effect of crack is clearly understandable. The mesh pattern and revised mesh pattern for crack is shown in Figure 6.2.

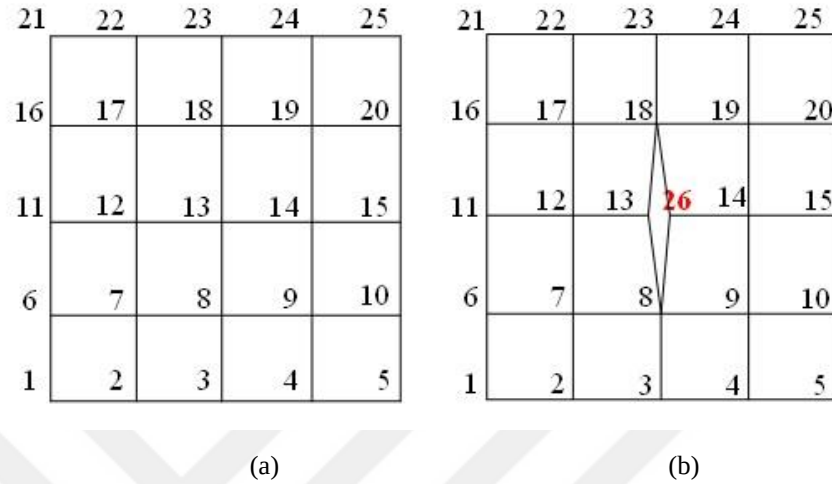


Figure 6.2 Mesh layout for the crack. (a) mesh pattern on the plate, (b) mesh pattern on the plate with crack

In this section, pre-stressed curved thin plates with a crack that are fixed on both sides are examined, as shown in Figure 3.1-c. In order to observe the crack and mode shape behaviors at different aspect ratios, three different ratios ($b/a=1$, $b/a=0.75$, $b/a=0.5$) are selected. The material properties are given in Table 6.1.

Table 6.1 Material properties and the geometry of the isotropic plate and the crack

Symbol	Property	Quantity
E	Elasticity modulus	200 GPa
ρ	Mass densitiy	7870 kg/m ³
ν	Poisson ratio	0.29
a	Length of the plate	0.8 m
b_1	Width of the plate	0.8 m
b_2	Width of the plate	0.6 m
b_3	Width of the plate	0.4 m
h	Thickness of the plate	3 mm

The non-dimensional load parameter (β) is considered to be between 0 to 1.41, which corresponds almost 85% of the maximum load of the yield strength of the material. λ and β have been given in equation (72).

In addition, the plate is discretized into 16x16, 16x12, and 16x8 finite elements. While selecting the finite element number, the mesh density graph obtained by increasing the number of mesh, and the graph is shown in Figure 6.3 for the aspect ratio equals 1.

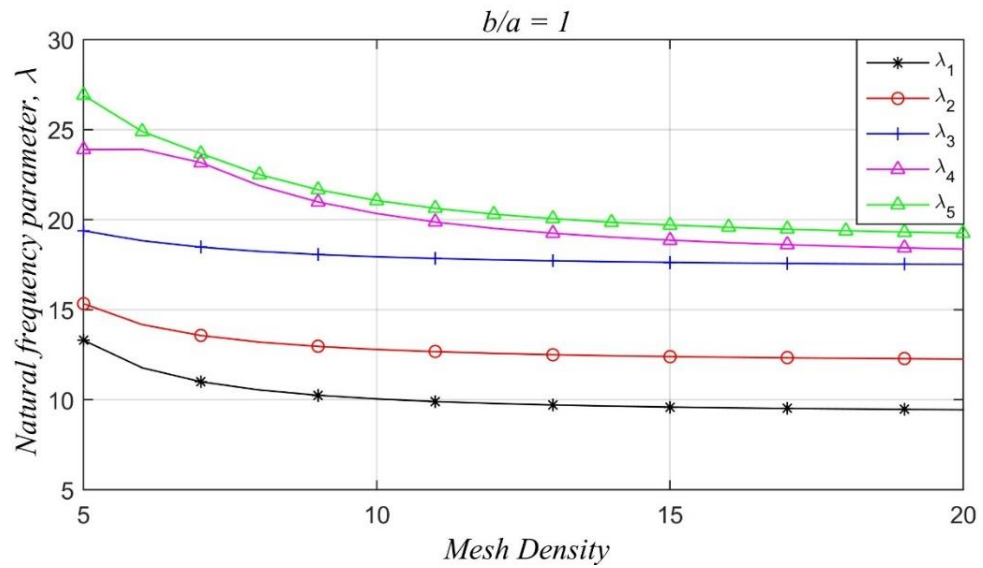


Figure 6.3 Mesh density graph

After the plate is deflected by vertical distributed loading, the large deflected plate is fixed at the loading point to model pre-stressed form. Thus, the pre-stressed curved plate is received, and free vibration analysis is performed for all calculated geometric forms. ANSYS analyzes are carried out for the same model.

Table 6.2 Comparison results of the natural frequency parameter between the present model and the ANSYS model for different aspect ratios

			λ_1	λ_2	λ_3	λ_4	λ_5
$\beta = 0$	$b/a = 1$	<i>Present</i>	3.69	4.40	7.24	10.19	11.19
		<i>Ansys</i>	3.73	4.44	7.30	10.52	11.50
		<i>Error</i>	1.15%	0.80%	0.79%	3.12%	2.69%
	$b/a = 0.75$	<i>Present</i>	3.69	4.88	10.05	10.18	11.91
		<i>Ansys</i>	3.72	4.90	10.15	10.50	12.20
		<i>Error</i>	0.81%	0.41%	0.99%	3.05%	2.38%
	$b/a = 0.5$	<i>Present</i>	3.68	6.01	10.15	13.74	18.16
		<i>Ansys</i>	3.71	6.02	10.46	14.00	18.54
		<i>Error</i>	0.81%	0.17%	2.96%	1.86%	2.05%
$\beta = 1.41$	$b/a = 1$	<i>Present</i>	9.33	12.66	18.00	18.35	19.08
		<i>Ansys</i>	9.55	12.34	17.56	18.71	19.56
		<i>Error</i>	2.31%	-2.59%	-2.51%	1.90%	2.47%
	$b/a = 0.75$	<i>Present</i>	9.32	14.09	18.01	19.87	21.72
		<i>Ansys</i>	9.54	13.55	18.71	20.23	20.65
		<i>Error</i>	2.31%	-3.99%	3.74%	1.78%	-5.18%
	$b/a = 0.5$	<i>Present</i>	9.29	16.25	17.94	21.75	28.70
		<i>Ansys</i>	9.53	15.63	18.68	21.99	27.82
		<i>Error</i>	2.52%	-3.97%	3.96%	1.09%	-3.16%

Table 6.2 shows that the natural frequency parameter results of the present model and the error rates with respect to ANSYS results are entirely consistent. According to the results of the three separate plates studied, the most significant difference between ANSYS and the present model is %5. In addition to checking the present model's validity without crack for the free-case and pre-stressed case, a comparison analysis for the free-case and the pre-stressed case with crack also performed. The comparison analysis includes the plates' first five natural frequency parameters for the aspect ratio equals 1 for three different crack position, and given in Table 6.3. In Table 6.3, P1 represents c_x/a and c_y/b equal 0.5; P2 represents c_x/a and c_y/b equal 0.25; P3 represents c_x/a equal 0.25 and c_y/b equal 0.5.

Table 6.3 Comparison results of the natural frequency parameter with and without crack between the present model and the ANSYS model

$b/a = 1$			λ_1	λ_2	λ_3	λ_4	λ_5
$\beta = 0$	Without Crack	Present	3.692	4.401	7.244	10.195	11.194
		Ansys	3.734	4.437	7.302	10.523	11.503
	With Crack (P1)	Present	3.674	4.401	7.215	10.193	11.194
		Ansys	3.707	4.437	7.257	10.516	11.502
	With Crack (P2)	Present	3.691	4.401	7.243	10.144	11.155
		Ansys	3.732	4.435	7.299	10.431	11.442
	With Crack (P3)	Present	3.691	4.401	7.237	10.148	11.194
		Ansys	3.734	4.437	7.288	10.457	11.502
$\beta = 0.70$	Without Crack	Present	8.900	10.073	13.296	14.248	15.223
		Ansys	9.027	10.014	12.945	13.927	14.954
	With Crack (P1)	Present	8.884	10.071	13.205	14.157	15.220
		Ansys	9.027	10.014	12.872	13.867	14.954
	With Crack (P2)	Present	8.852	10.044	13.290	14.185	15.168
		Ansys	8.855	9.884	12.842	13.865	14.925
	With Crack (P3)	Present	8.855	10.072	13.258	14.189	15.221
		Ansys	8.870	9.953	12.777	13.887	14.954
$\beta = 1.41$	Without Crack	Present	9.330	12.660	18.005	18.354	19.077
		Ansys	9.550	12.340	17.564	18.709	19.560
	With Crack (P1)	Present	9.323	12.648	17.873	18.298	19.077
		Ansys	9.550	12.317	17.509	18.672	19.560
	With Crack (P2)	Present	9.281	12.632	17.940	18.344	19.030
		Ansys	9.394	12.171	17.361	18.477	19.444
	With Crack (P3)	Present	9.282	12.650	17.942	18.342	19.077
		Ansys	9.415	12.237	17.380	18.555	19.560

Figures 6.4, 6.5, and 6.6 show the variation of the first five natural frequency parameters to the load parameter value for aspect ratio equal to 1, 0.75, and 0.5, respectively.

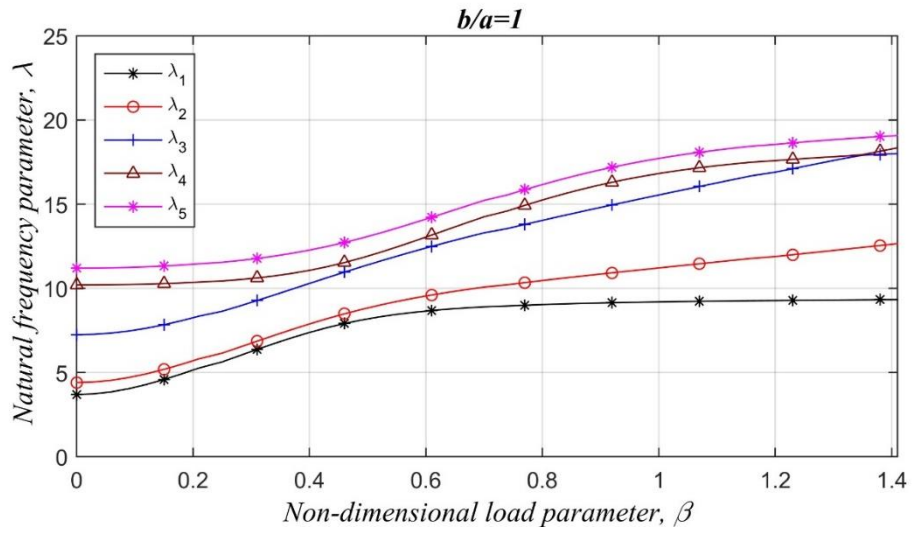


Figure 6.4 The effect of the load parameter on the first five natural frequency parameters for $b/a = 1$

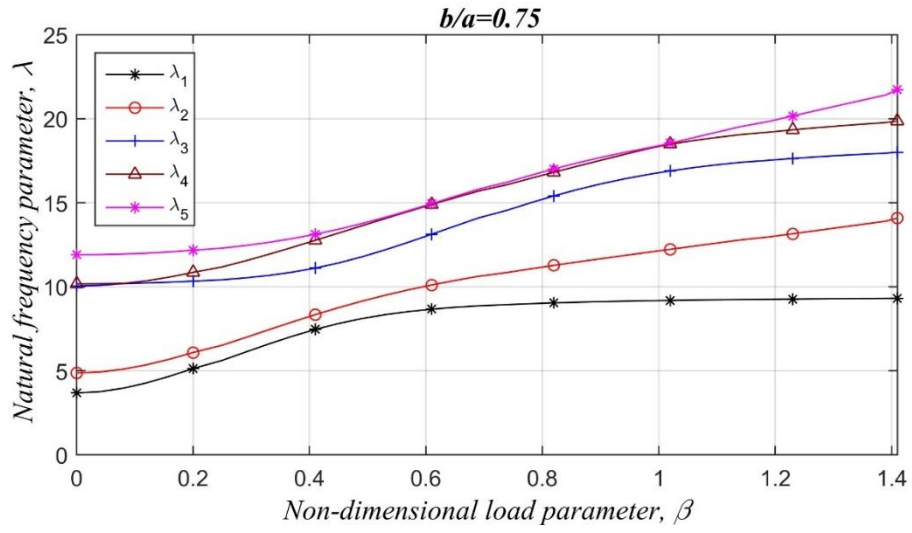


Figure 6.5 The effect of the load parameter on the first five natural frequency parameters for $b/a = 0.75$

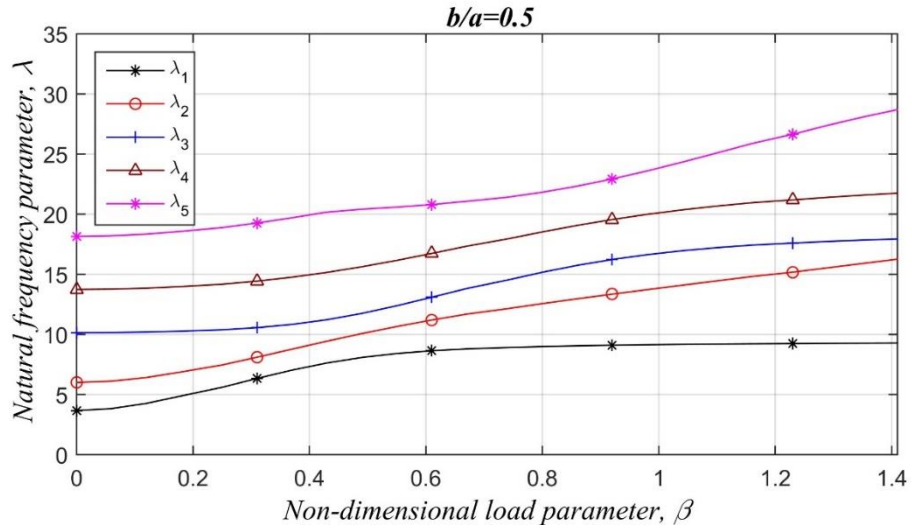


Figure 6.6 The effect of the load parameter on the first five natural frequency parameters for $b/a = 0.5$

The curve between the natural frequency parameter and the load parameter differs for each natural frequency. With the increase of the load parameter, the curvature of each natural frequency parameter progresses in different ways. In all aspect ratios, the first natural frequency parameter increases up to a load parameter value, 0.6. Beyond this value, although the initial stress raises the stiffness of the curved plates depending on the load parameter increment, λ_1 has a minimal increment for all the curved plates. The second natural frequency parameter increases rapidly up to a load parameter value, 0.5, depending on the load parameter in all aspect ratios but shows a slower increase after this value. The third natural frequency parameter shows a very slow increase in plates with an aspect ratio equal to 0.75 and 0.5, followed by a rapid increase and then a slow increase in the end. For the aspect ratio equals to 1, the transition from a rapid increase to a slower increase is smooth. In the fourth natural frequency parameter, the plates with an aspect ratio equal to 1 and 0.5 show a very slow increase and continue with a decreasing rate of increase. The plate with an aspect ratio equals to 0.75; the slow increase at the beginning that appears on the other plates does not exist. In the fifth natural frequency parameter, different behavior observed for each plate. The plate with an aspect ratio of 1 shows a slow increase, then a rapid increase and then a slow increase, again. The plate with an aspect ratio of 0.75 shows a slow increase and then a rapid increase. On the plate with an aspect ratio of 0.5, the rate of increase is slow, rapid, slow, and rapid, respectively. Besides, each curve change causes the changing of the mode shape. Figure 6.7 shows the first mode shapes and their effect of crack.

The letter ‘a’ in these graphs represents aspect ratio equals 1, the letter ‘b’ represents aspect ratio equals 0.75, and the letter ‘c’ represents aspect ratio equals 0.5. The changing of the effect of crack depends on the mode shape and the figures of the effect of crack positioned under each mode shape.

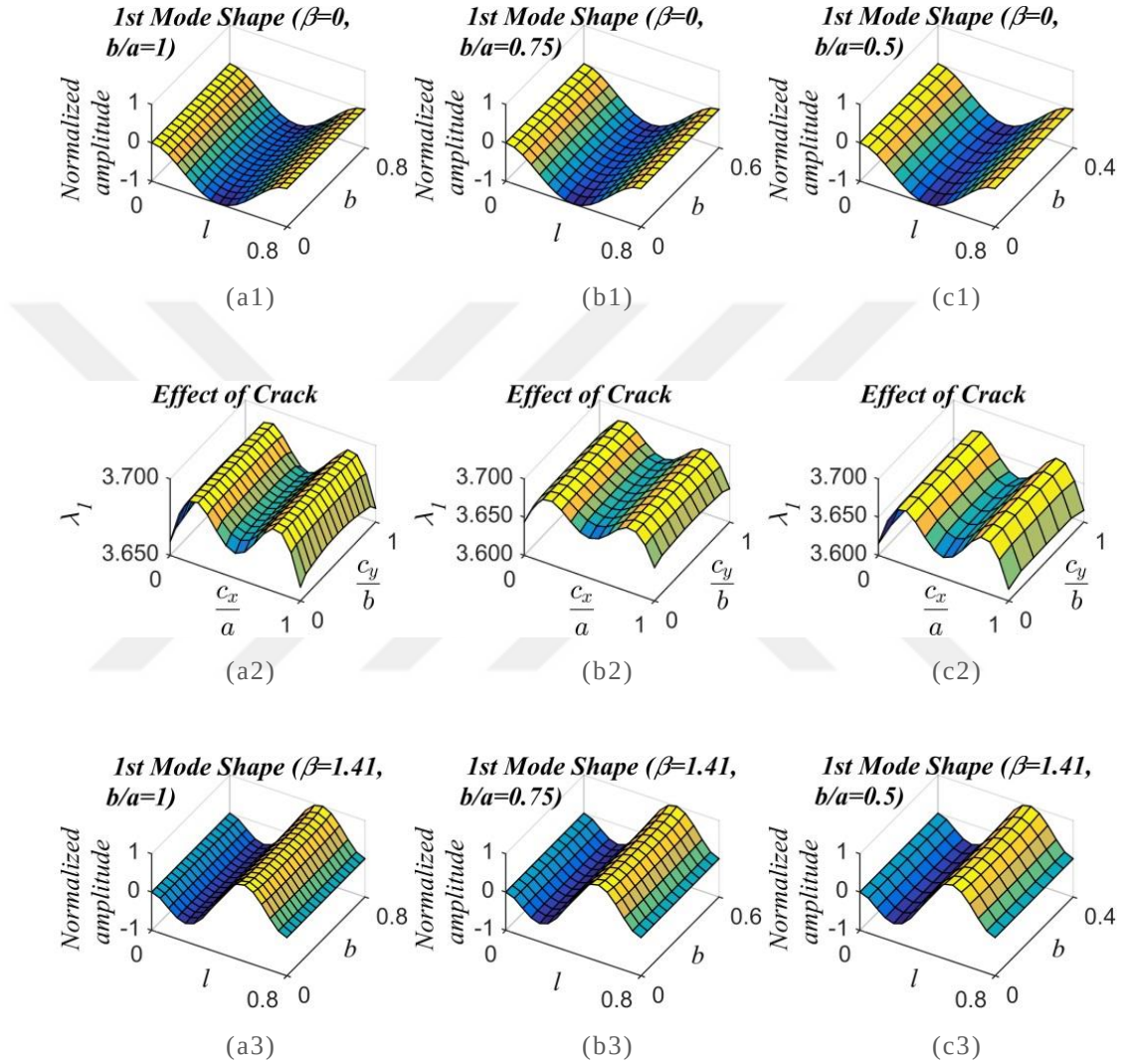


Figure 6.7 Mode shapes and the effect of crack on the first natural frequency parameter for $b/a = 1$ (a1, a2, a3, a4), for $b/a = 0.75$ (b1, b2, b3, b4) and for $b/a = 0.5$ (c1, c2, c3, c4)

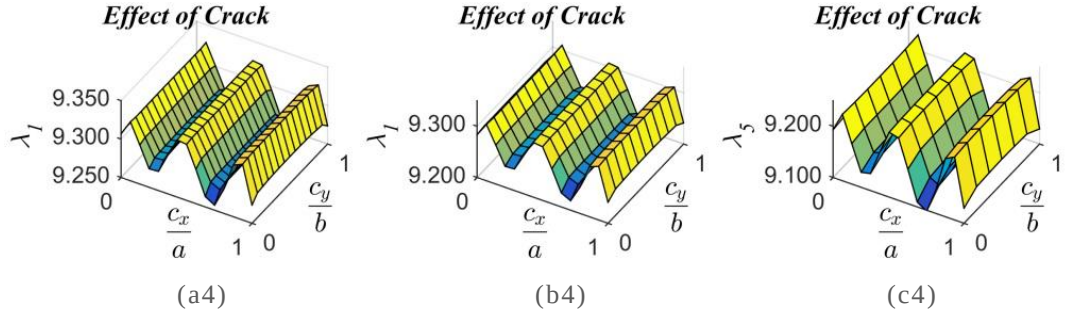


Figure 6.7 continues

The effect of crack is higher where the strain is higher in the plate due to the mode shape. For example, the strain is higher on the fixed edges and the midline of the plate when Figure 6.7-a1 is examined. Accordingly, the natural frequency parameter is more affected by the crack in these areas, as can be seen in Figure 6.7-a2. Also, in the areas where the strain is less, the effect of crack on vibration decrease considerably. The second mode shapes and their effect of crack for the plate with an aspect ratio equal to 1, 0.75, and 0.5 are shown in Figure 6.8, respectively.

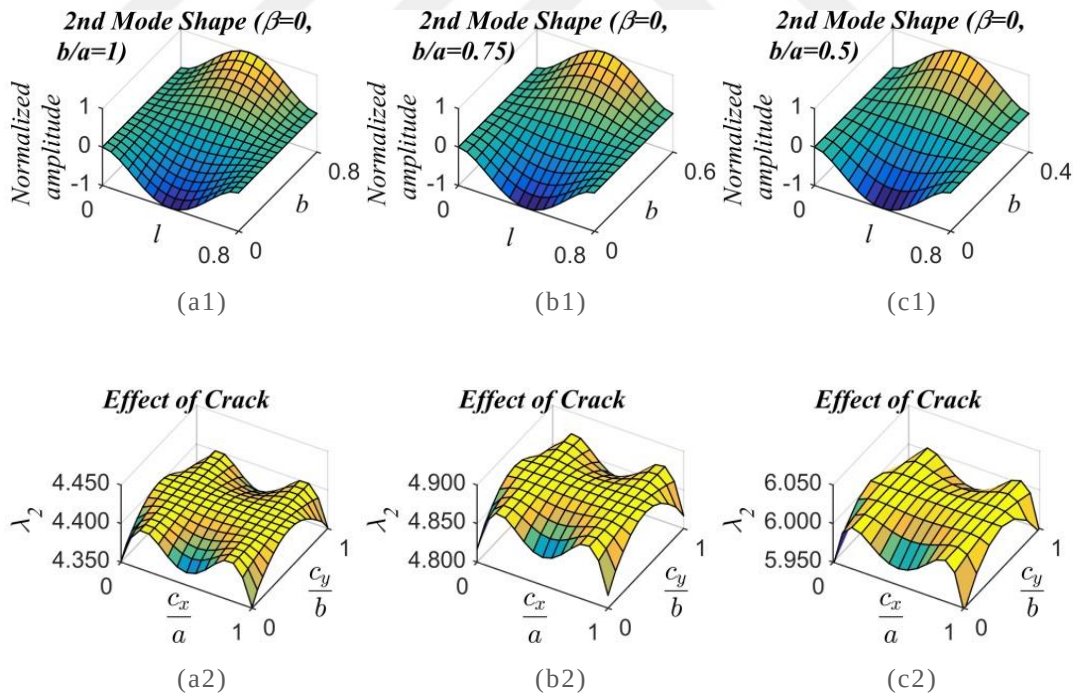


Figure 6.8 Mode shapes and the effect of crack on the second natural frequency parameter for $b/a = 1$ (a1, a2, a3, a4), for $b/a = 0.75$ (b1, b2, b3, b4), and for $b/a = 0.5$ (c1, c2, c3, c4)

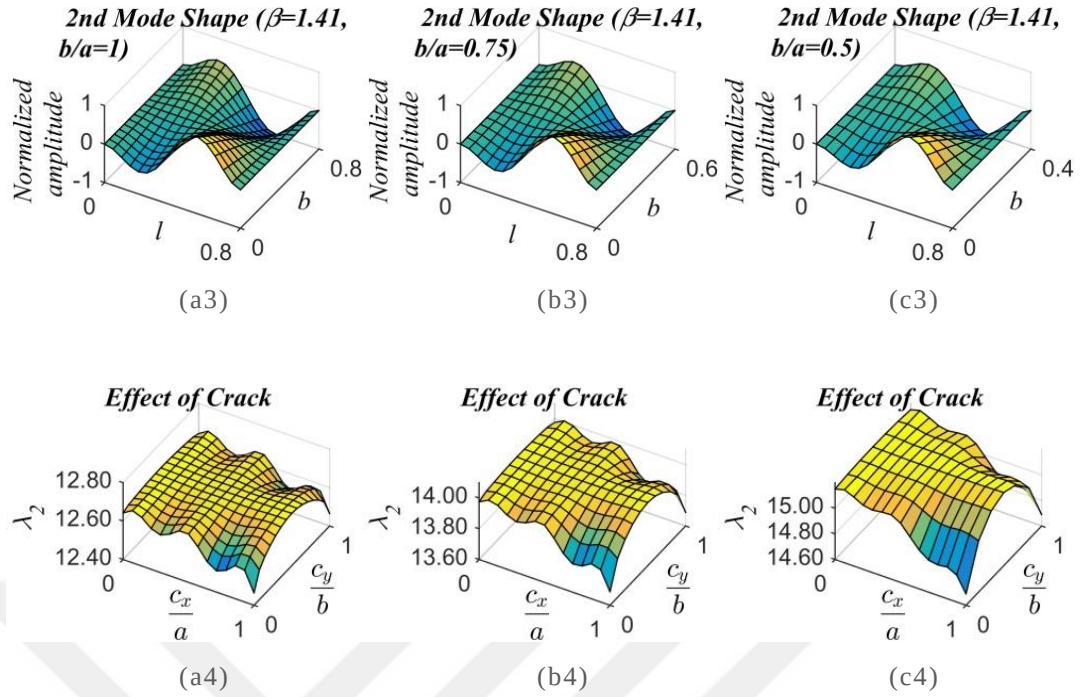


Figure 6.8 continues

The first and second mode shapes in the unloaded state and the loaded state are the same for all aspect ratios. Depending on these mode shapes, the effects of crack are the same, too. Third mode shapes and their effect of crack for the plate with an aspect ratio equal 1, 0.75, and 0.5 are shown in Figure 6.9. Unlike the first and second natural frequencies, three different mode shapes occur at aspect ratio equal 1 and 0.75 with increasing of the load parameter.

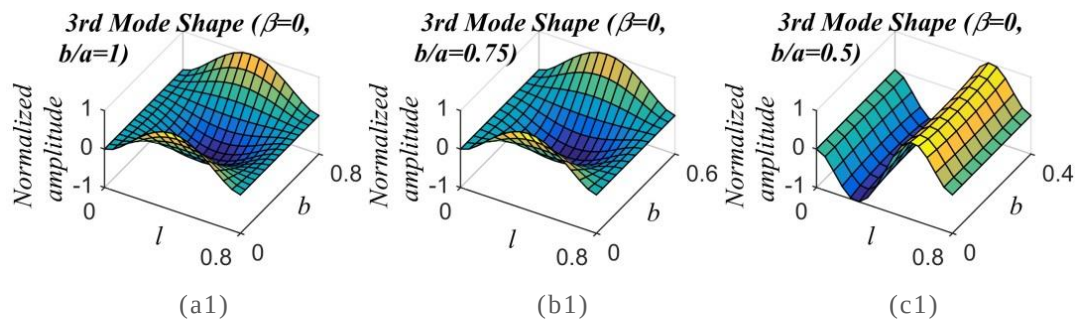


Figure 6.9 Mode shapes and the effect of crack on the third natural frequency parameter $b/a = 1$ (a1, a2, a3, a4, a5, a6), $b/a = 0.75$ (b1, b2, b3, b4, b5, b6) and for $b/a = 0.5$ (c1, c2, c3, c4)

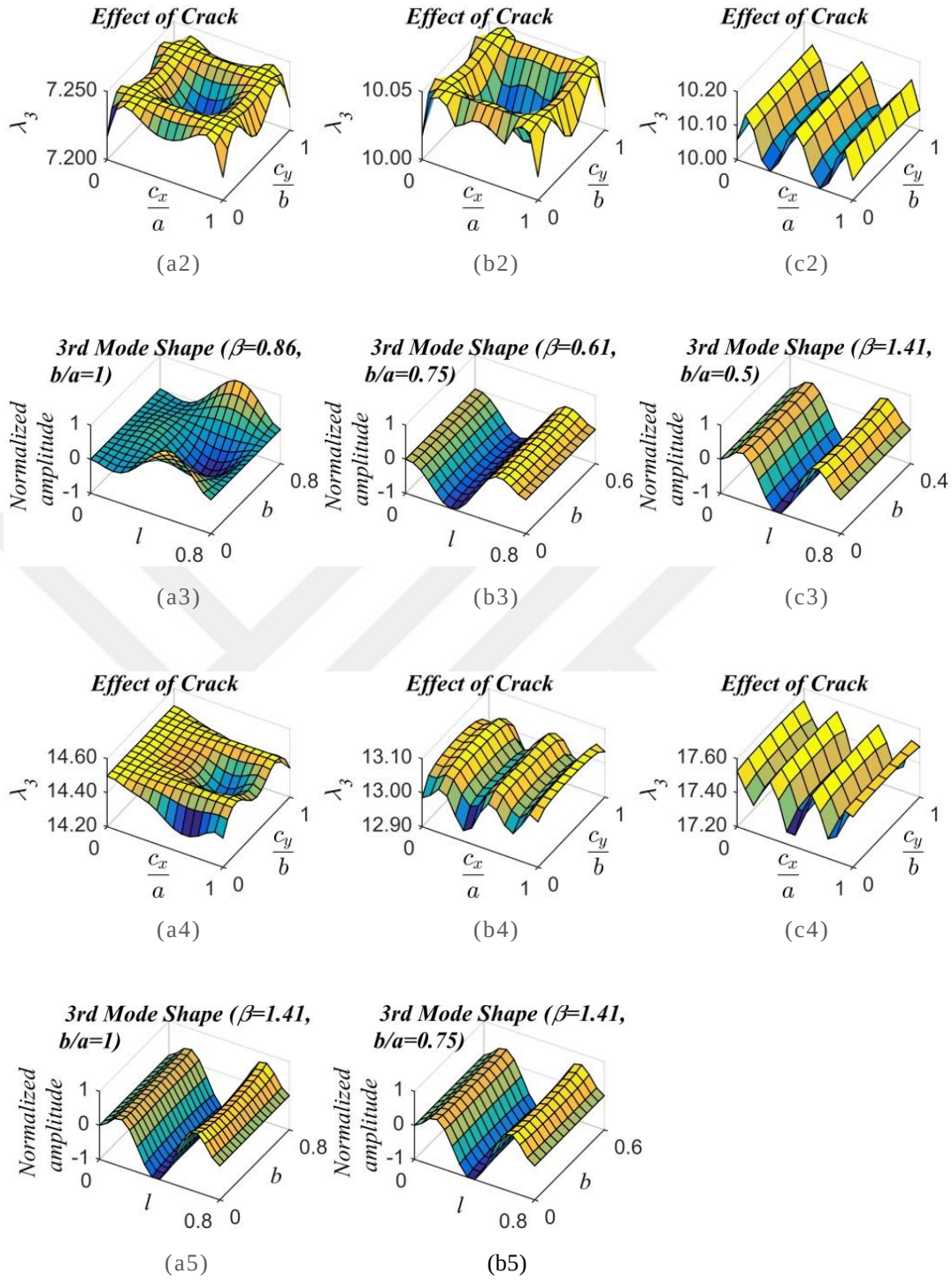


Figure 6.9 continues

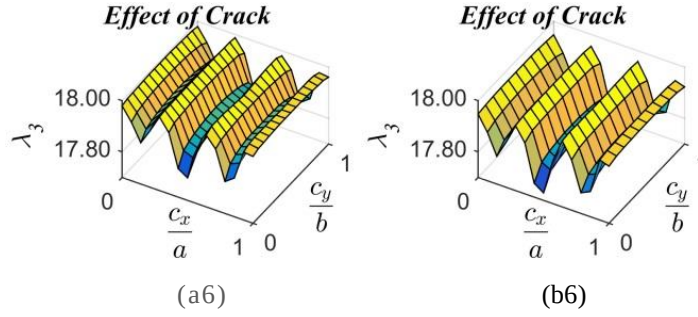


Figure 6.9 continues

The plates with the aspect ratio equal to 1 and 0.75 are the same in the third mode shapes, and the plate with the aspect ratio equals to 0.5 is different for the unloaded state. Apart from the plate with an aspect ratio equals to 0.5, an additional mode shape is seen in the other two plates due to the increase in load. Figure 6.10 shows the fourth mode shapes and their effect of crack for each plate.

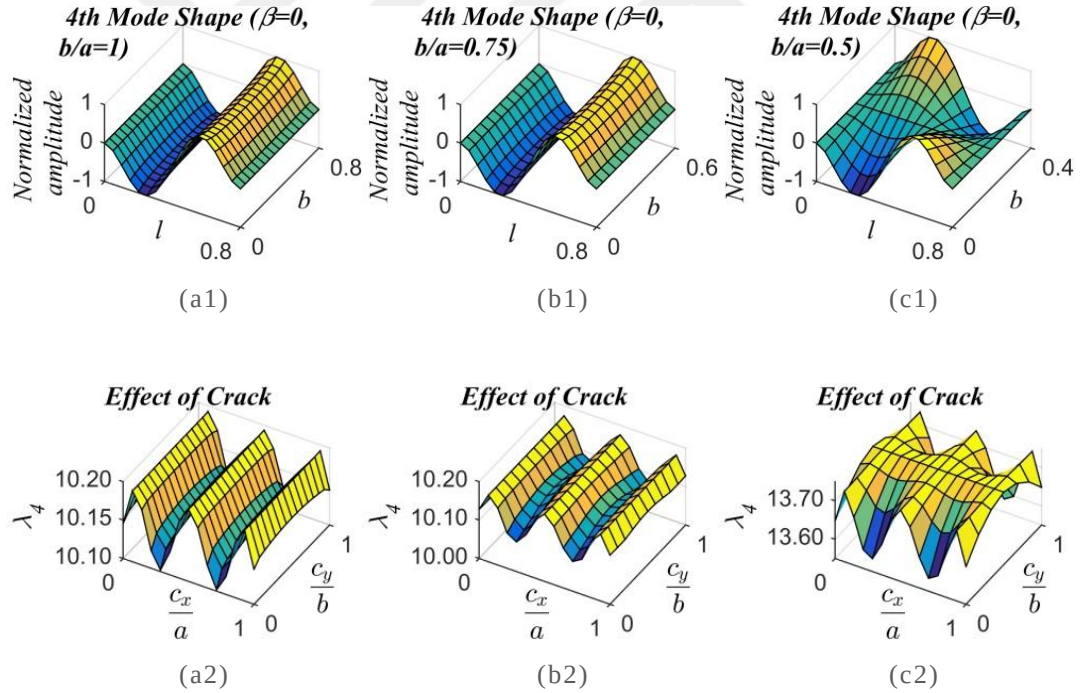


Figure 6.10 Mode shapes and the effect of crack on the fourth natural frequency parameter for $b/a = 1$ (a1, a2, a3, a4), for $b/a = 0.75$ (b1, b2, b3, b4, b5, b6) and for $b/a = 0.5$ (c1, c2, c3, c4)

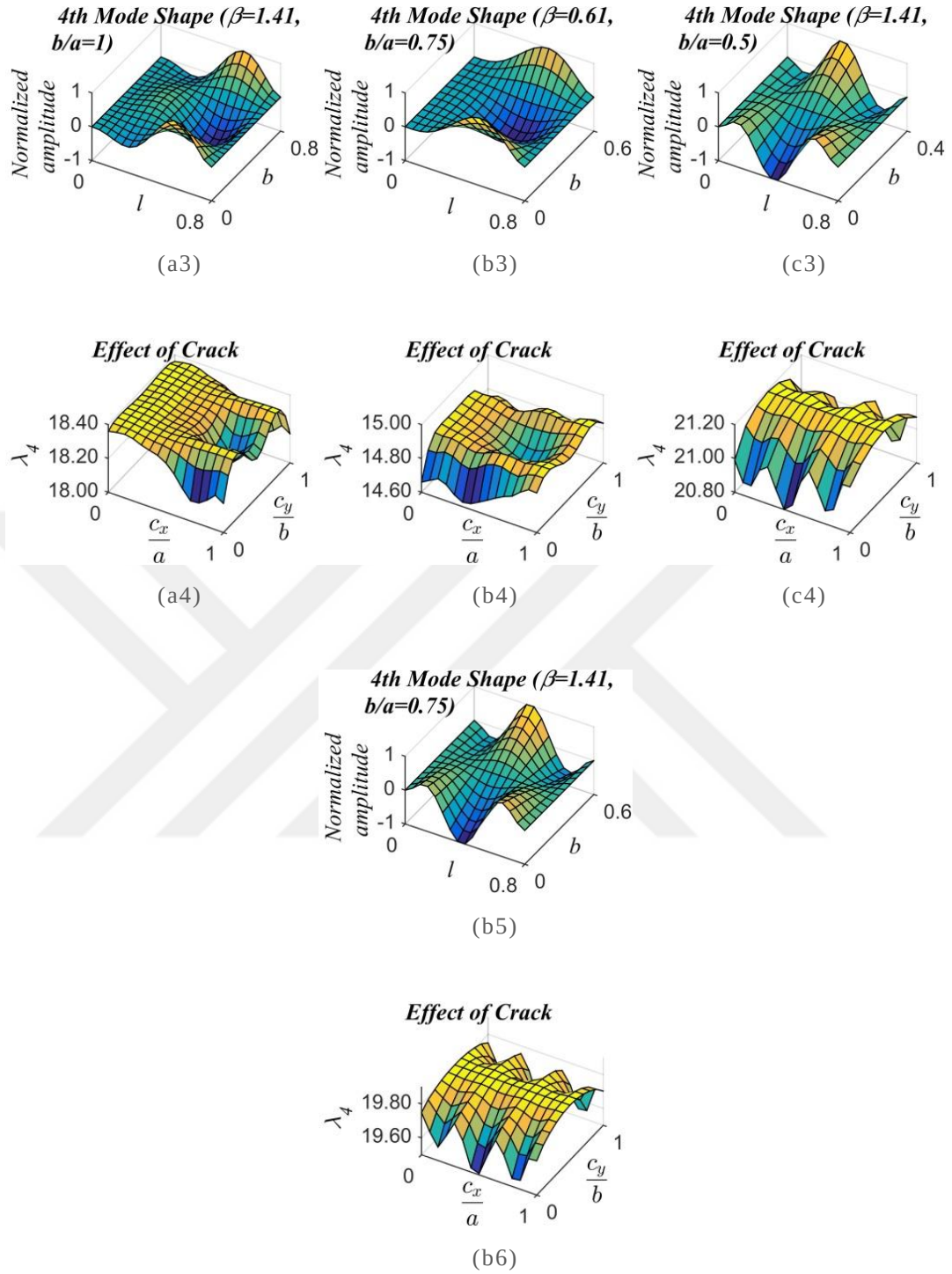


Figure 6.10 continues

In the fourth natural frequencies, while the load parameter is zero, the same mode shape is seen in plates with an aspect ratio equal 1 and 0.75, and when the load parameter value reaches a maximum, plates with an aspect ratio equal 0.75 and 0.5 appear to be in the same mode shape. An extra mode shape is shown for $b/a = 0.75$.

Fifth mode shapes and their effect of crack for the plate with an aspect ratio equal 1, 0.75, and 0.5 are shown in Figure 6.11.

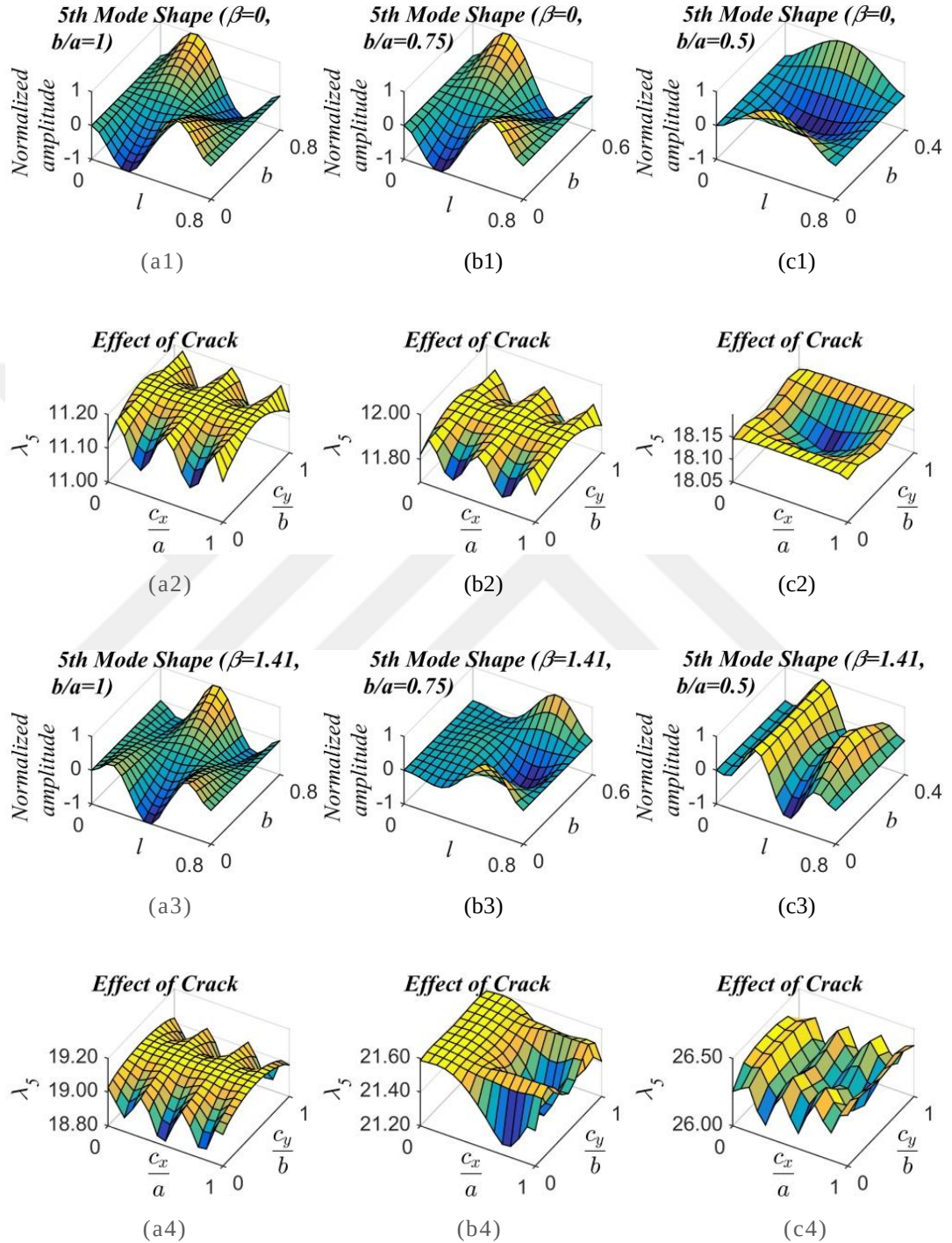


Figure 6.11 Mode shapes and the effect of crack on the fifth natural frequency parameter for $b/a = 1$ (a1, a2, a3, a4), for $b/a = 0.75$ (b1, b2, b3, b4) and for $b/a = 0.5$ (c1, c2, c3, c4)

In the fifth natural frequency parameter, the same mode shape is seen in plates with aspect ratios equal to 1 and 0.75 under the unloaded state. Also, all mode shapes are different for each plate under maximum load.

When the first five natural frequencies of the plate with three different aspect ratios are examined together, a total of thirty-three mode shapes are seen. However, only eight similar mode shapes are seen. The figures for the effect of crack show the symmetrical distribution in the unloaded state, and this symmetry disappears with increasing the load parameter. Besides, any mode shape that occurs in plates at all aspect ratios may occur again in different natural frequencies by increasing the load parameter. This phenomenon provides a better opportunity to observe the effect of crack on the plate with an increase in the load parameter.

6.3 Straight and Curved Composite Plate with Crack

In this section, large deflected pre-stressed laminated composite thin plates that are fixed on both sides are examined, as shown in Figure 3.1c. The use of the four-node quadrilateral plate element approach is convenient since the radius of the curvature of the pre-stressed curved plate is large. The local coordinates of the straight plate element are transformed into global coordinates using its angle of rotation. Eight different configurations are examined in order to understand and interpret the effect of large deflection on the laminated composite plates. For simplicity, the configurations are denoted by C1 ([0 0 0 0]), C2 ([0 90 90 0]), C3 ([0 45 -45 0]), C4 ([0 60 -60 0]), C5 ([0 90 0 90]), C6 ([0 45 0 -45]), C7 ([90 0 0 90]), and C8 ([90 90 90 90]). C2 called cross-ply laminate, C3, and C4 are called angle-ply laminates. In addition, the plate is discretized into 16x16 finite elements. While selecting the finite element density, the mesh density graph obtained by increasing the number of mesh is used, and the graph is shown in Figure 6.12. λ and β have been given in equation (73) before.

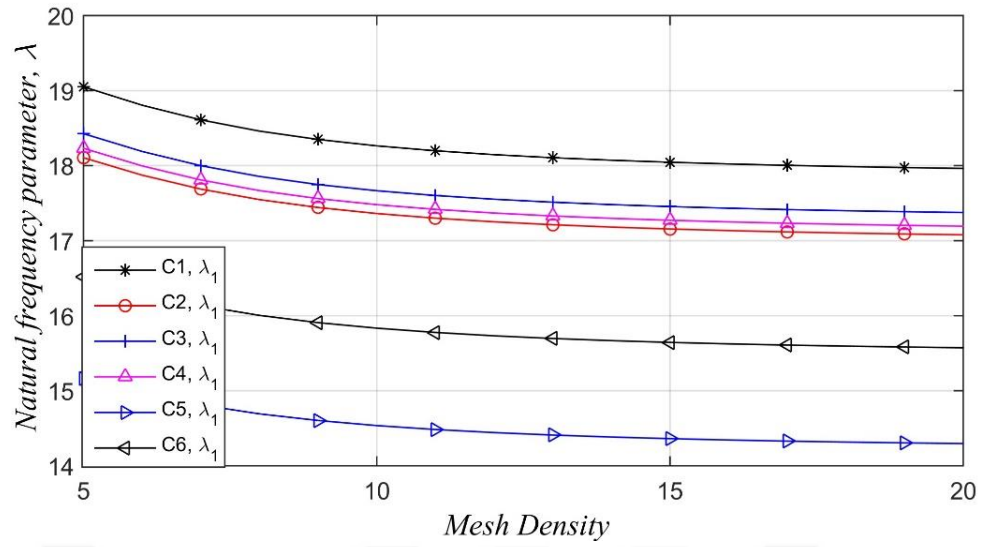


Figure 6.12 Mesh density graph for composite plates

After the plate is deflected by distributed loading, the large deflected plate is fixed at the loading point. Thus, the pre-stressed curved plate is obtained, and free vibration analysis is performed for all calculated geometric forms. ANSYS analyzes are performed for eight different composites, and the results are compared with the present study.

Tables 4.2 to 4.9 show that the natural frequency parameter results of the present model and the error rates with respect to ANSYS results are quite consistent for each laminated composite configuration. For each composite material, the effect of the load parameter on the natural frequency parameter is examined for the first five natural frequencies. Figure 6.13-6.20 shows the effect of the load parameter on the natural frequency parameter for C1, C2, C3, C4, C5, C6, C7, and C8, respectively. The increment of the first two natural frequency parameter curves is similar in the examined eight different composites. However, there are differences in the increases of the third, fourth, and fifth natural frequency parameters. For example, the third natural frequency parameter continues to increase at the maximum loading point in C1, C2, C3, and C4. The third natural frequency parameter's curvature has changed in C5 and C6 just before reaching the maximum loading point. In C7 and C8, the third natural frequency parameter is horizontal as it approaches the maximum loading point. As a result of these differences, different mode shapes are seen in each composite.

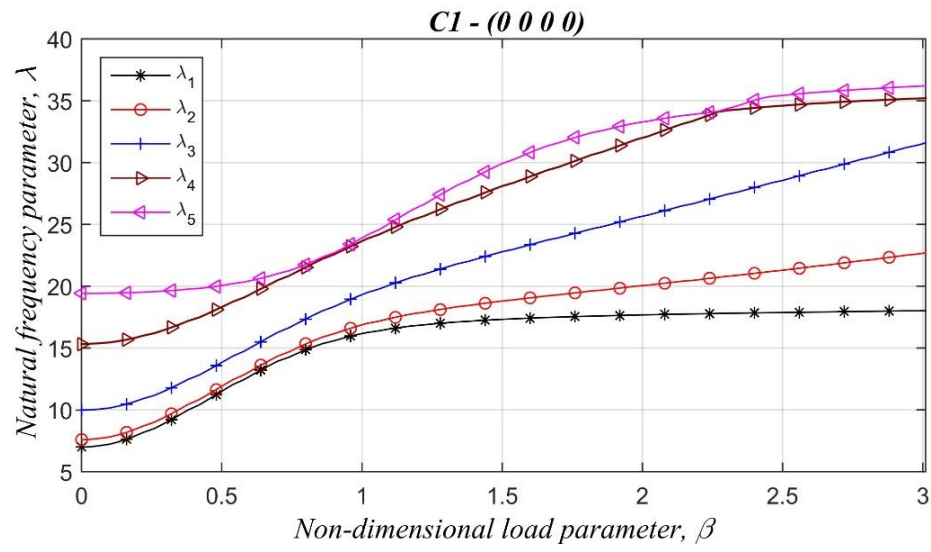


Figure 6.13 Effect of load parameter on the first five natural frequency parameters for C1 composite

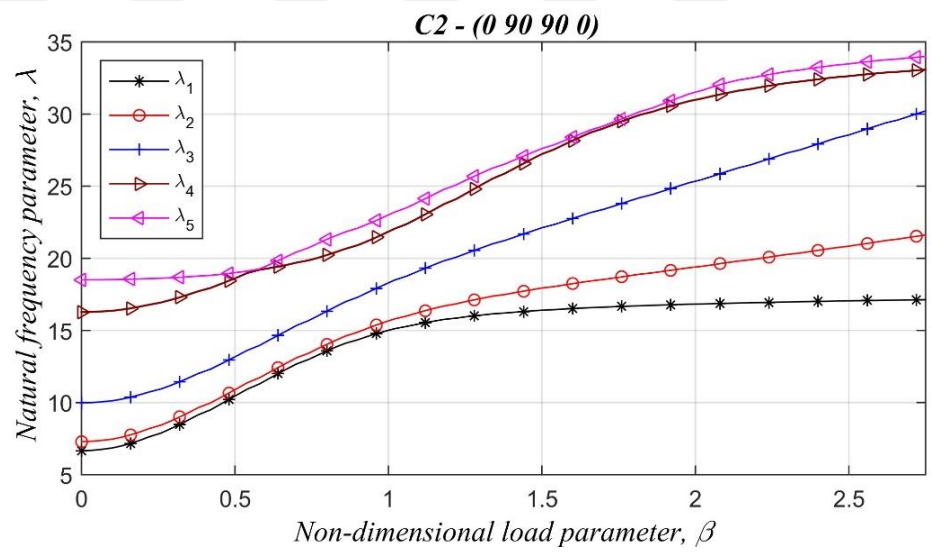


Figure 6.14 Effect of load parameter on the first five natural frequency parameters for C2 composite

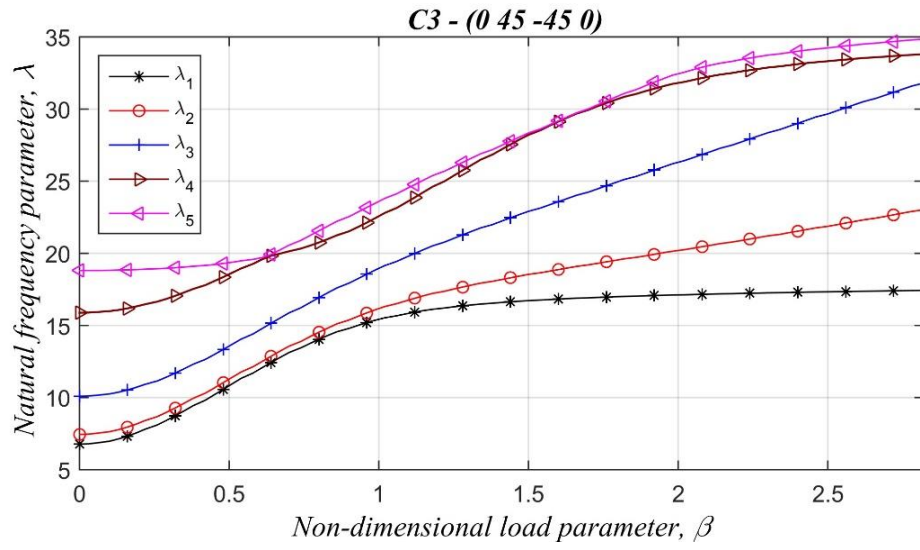


Figure 6.15 Effect of load parameter on the first five natural frequency parameters for C3 composite

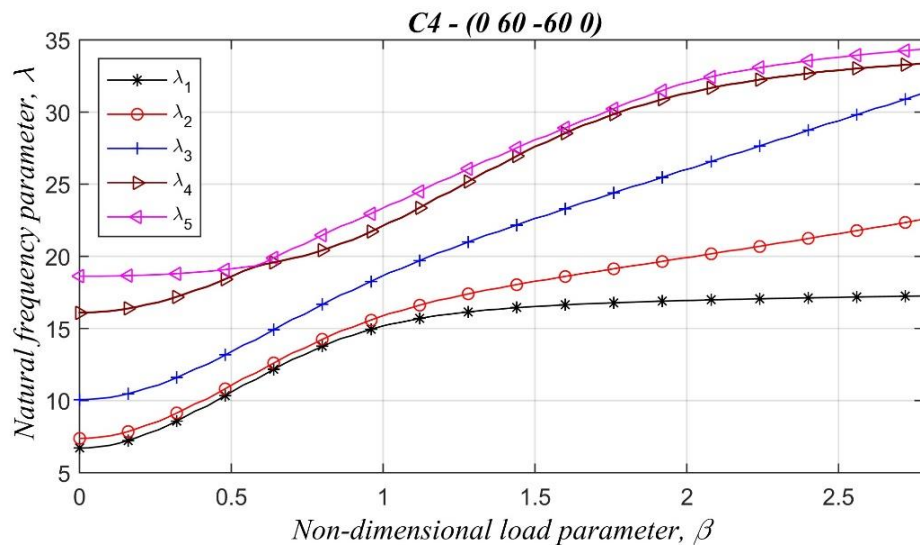


Figure 6.16 Effect of load parameter on the first five natural frequency parameters for C4 composite

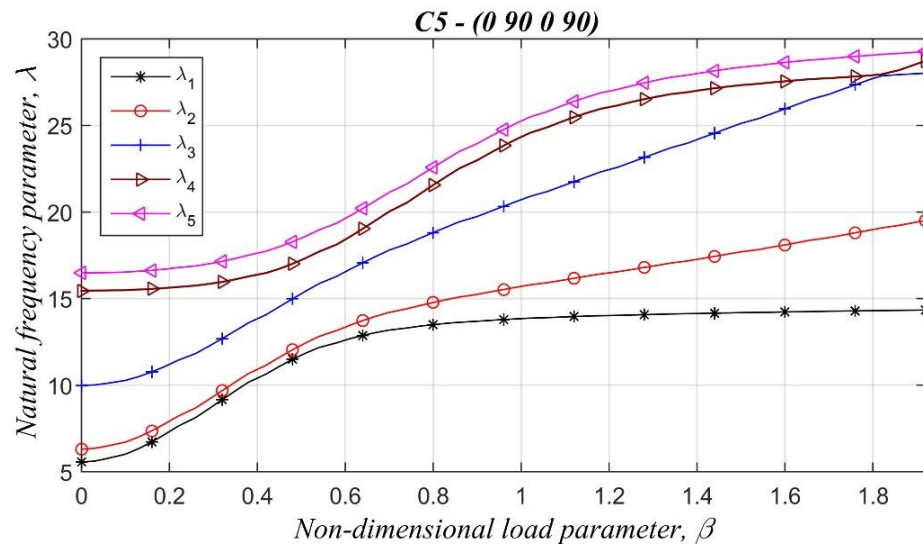


Figure 6.17 Effect of load parameter on the first five natural frequency parameters for C5 composite

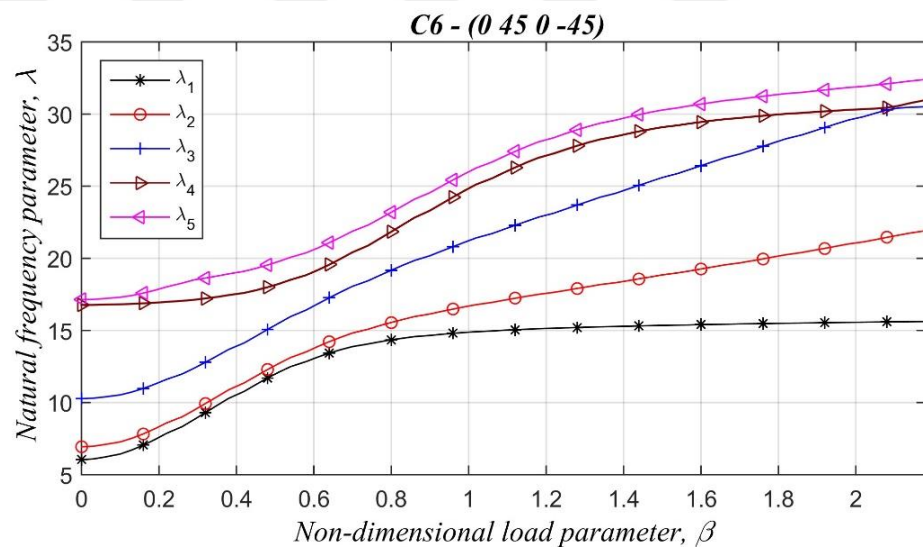


Figure 6.18 Effect of load parameter on the first five natural frequency parameters for C6 composite

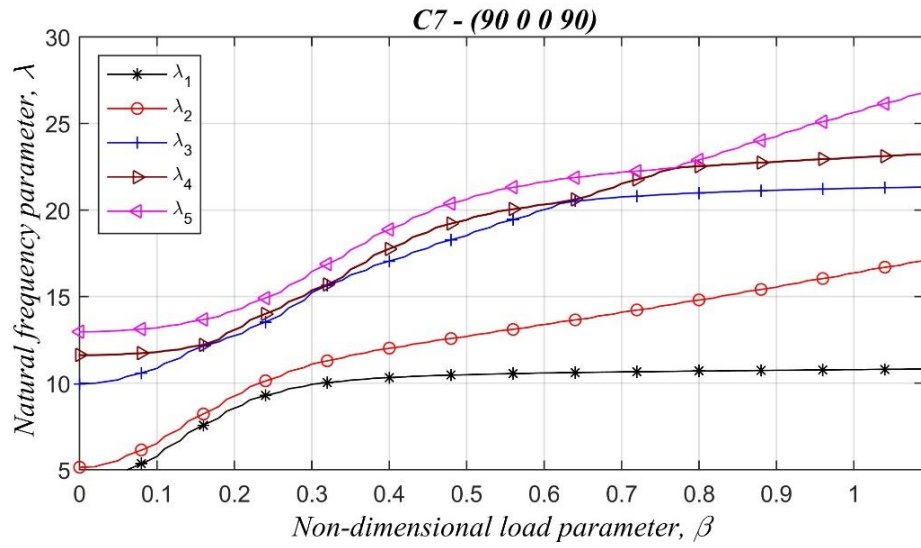


Figure 6.19 Effect of load parameter on the first five natural frequency parameters for C7 composite

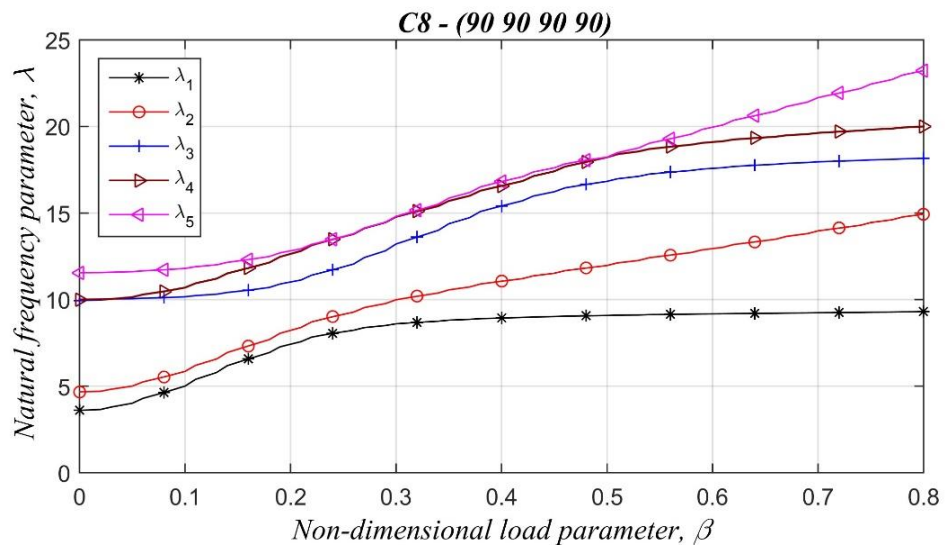


Figure 6.20 Effect of load parameter on the first five natural frequency parameters for C8 composite

The curve between the natural frequency parameter and the load parameter differs for each natural frequency on each laminated composite configuration. With the increase of the load parameter, the curvature of each natural frequency parameter shows similarity but progresses by showing different behaviors at different points. Different curves or changes in curvature for each configuration of laminated composites reveal different mode shapes.

In all lamination orientations, the first mode shapes are in the same shapes in both no-load and maximum load conditions. The most crucial difference that can be generalized is that as the effective modulus of elasticity increases, the natural frequency parameter value increases. Figure 6.21 shows the first mode shapes.

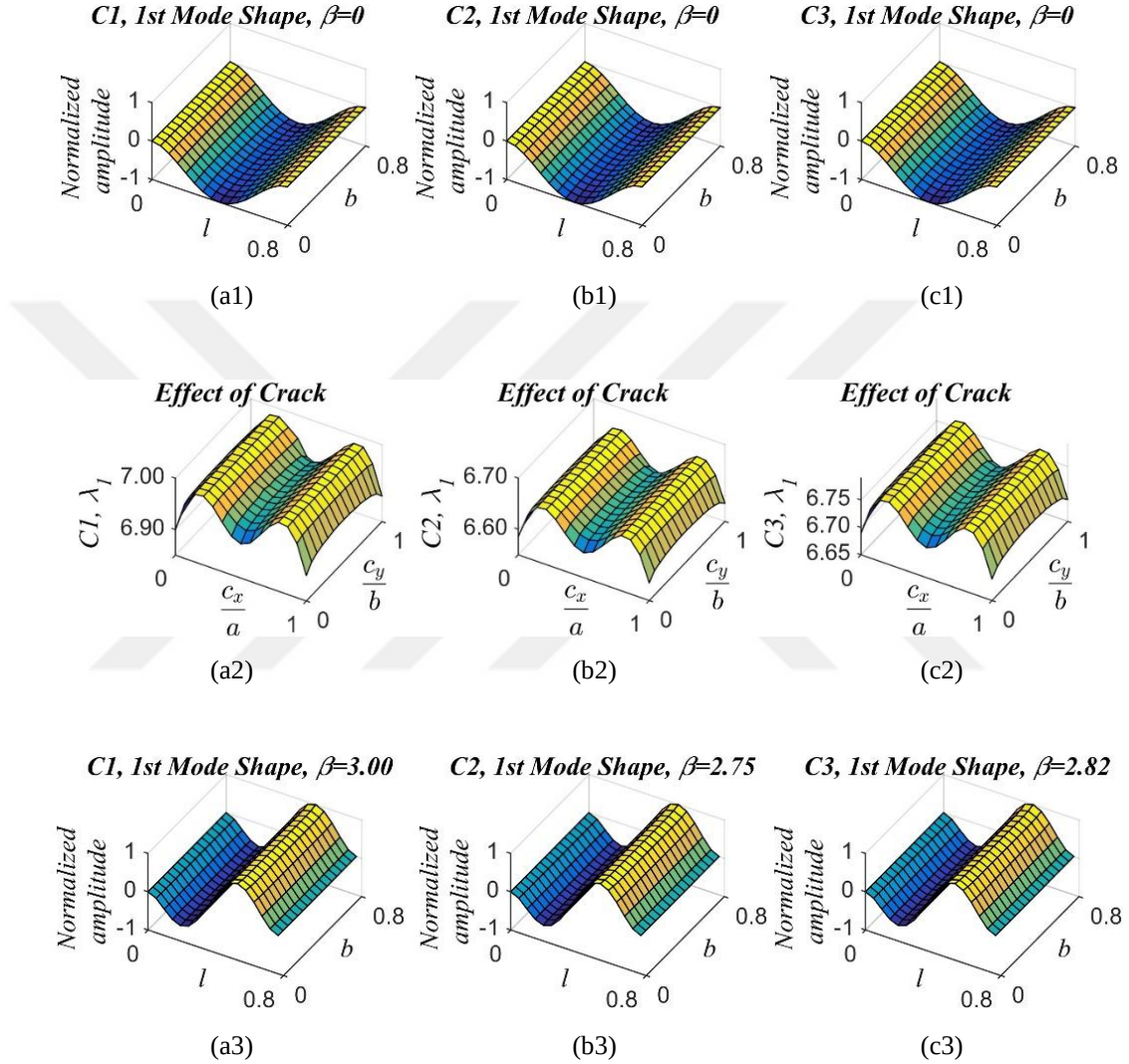


Figure 6.21 Mode shapes and the effect of crack on the first natural frequency parameter for C1 (a1, a2, a3, a4), C2 (b1, b2, b3, b4), C3 (c1, c2, c3, c4), C4 (d1, d2, d3, d4), C5 (e1, e2, e3, e4), C6 (f1, f2, f3, f4), C7 (g1, g2, g3, g4), C8 (h1, h2, h3, h4)

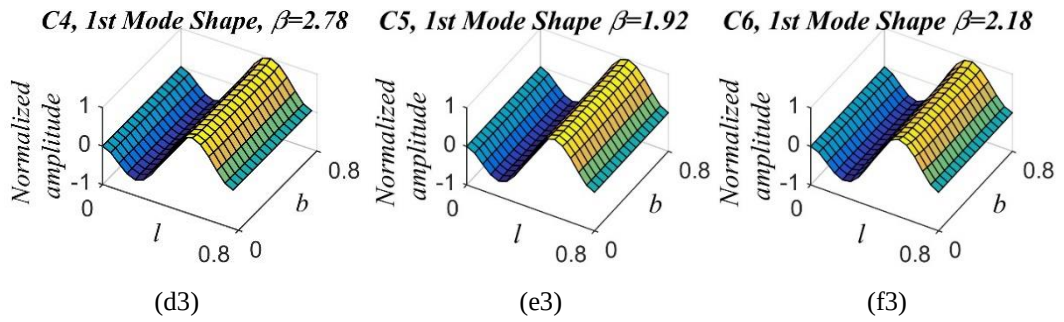
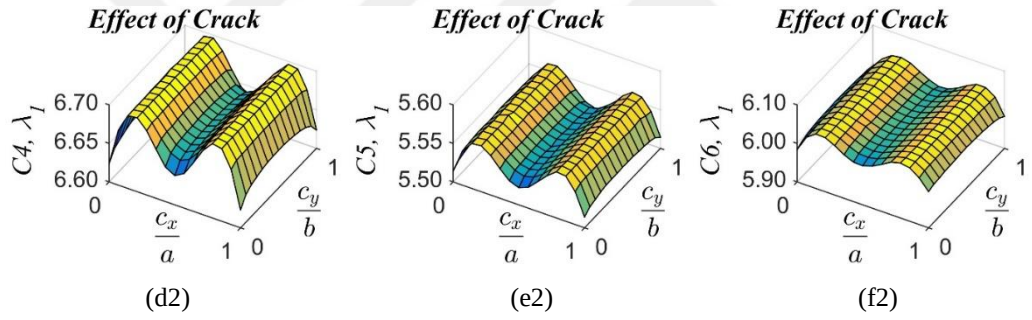
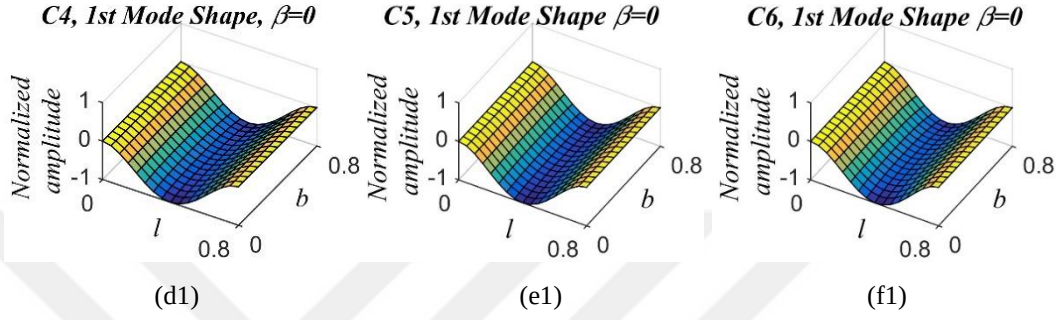
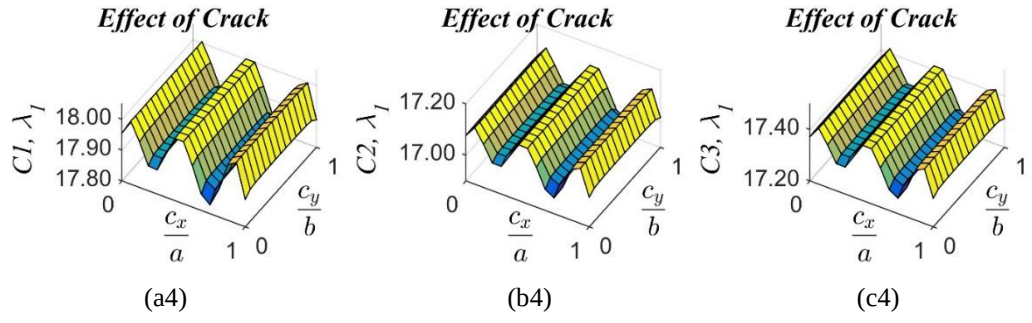


Figure 6.21 continues

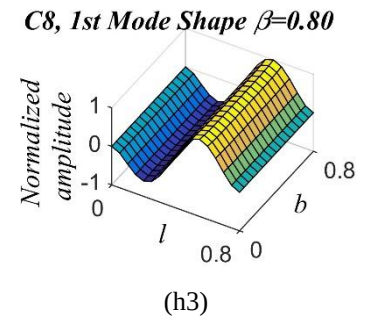
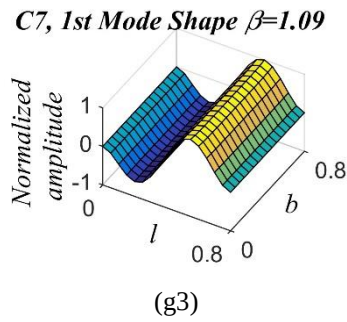
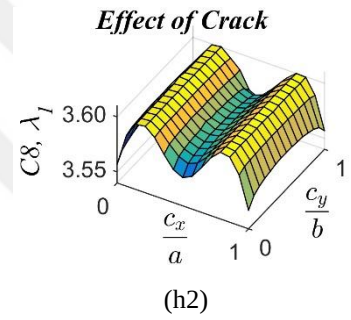
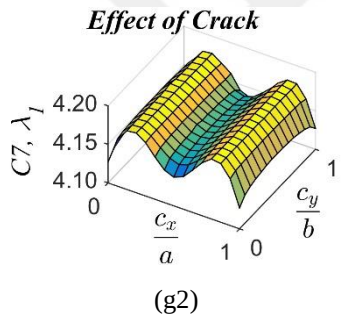
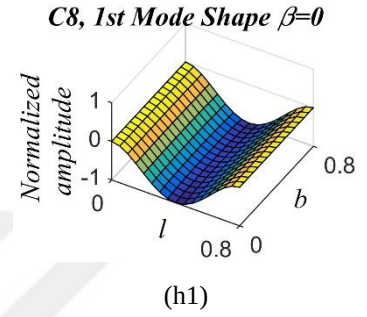
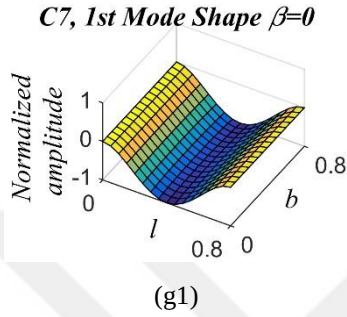
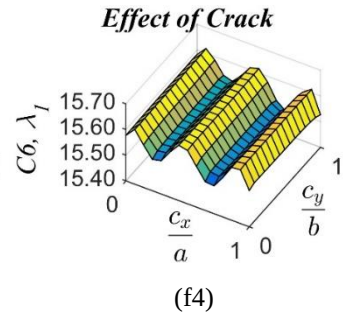
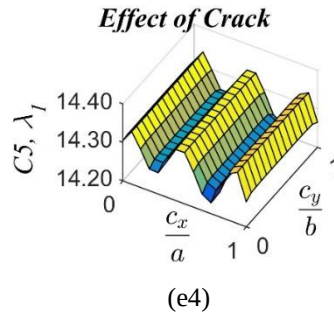
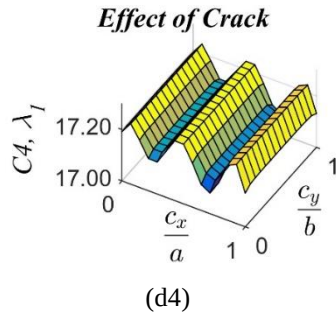


Figure 6.21 continues

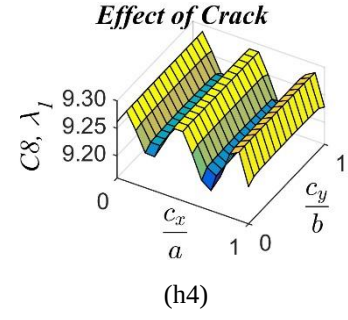
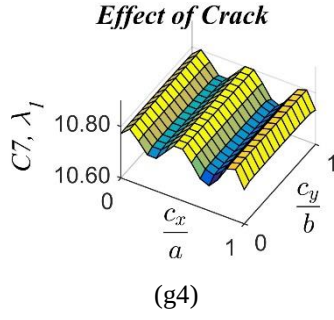


Figure 6.21 continues

In the second mode shapes, the same situation occurs as in the first mode shapes. For the composite plate with eight different lamination orientations, the same mode shapes and the same effects of crack are formed in the no-load condition and the maximum load case. The second mode shapes are given in Figure 6.22.

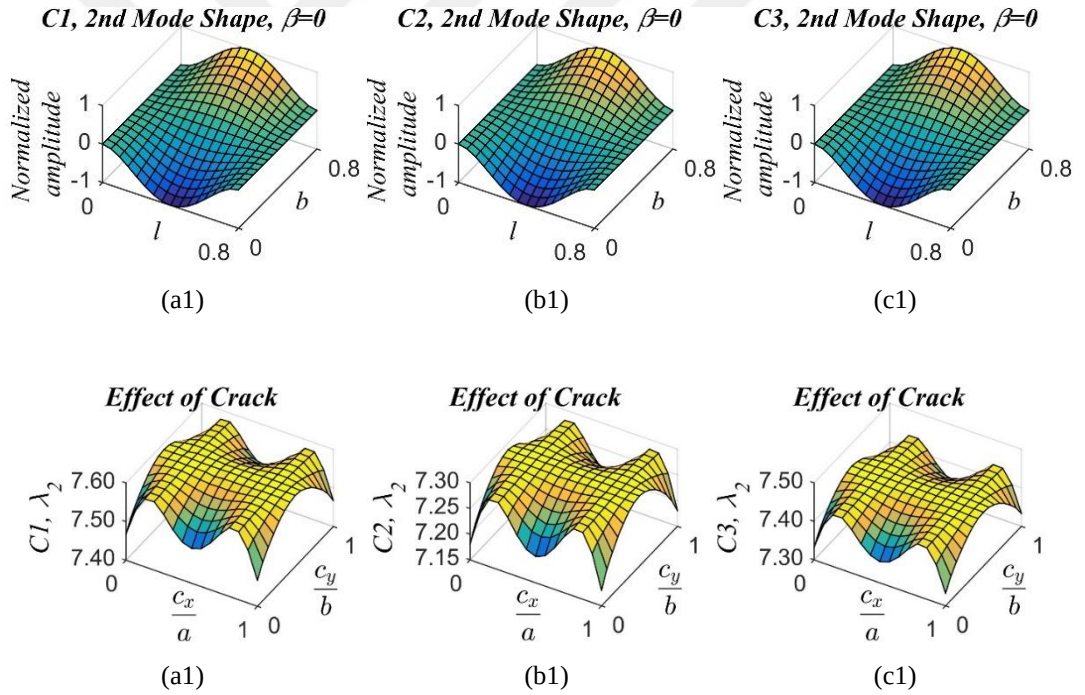


Figure 6.22 Mode shapes and the effect of crack on the second natural frequency parameter for C1 (a1, a2, a3, a4), C2 (b1, b2, b3, b4), C3 (c1, c2, c3, c4), C4 (d1, d2, d3, d4), C5 (e1, e2, e3, e4), C6 (f1, f2, f3, f4), C7 (g1, g2, g3, g4), C8 (h1, h2, h3, h4)

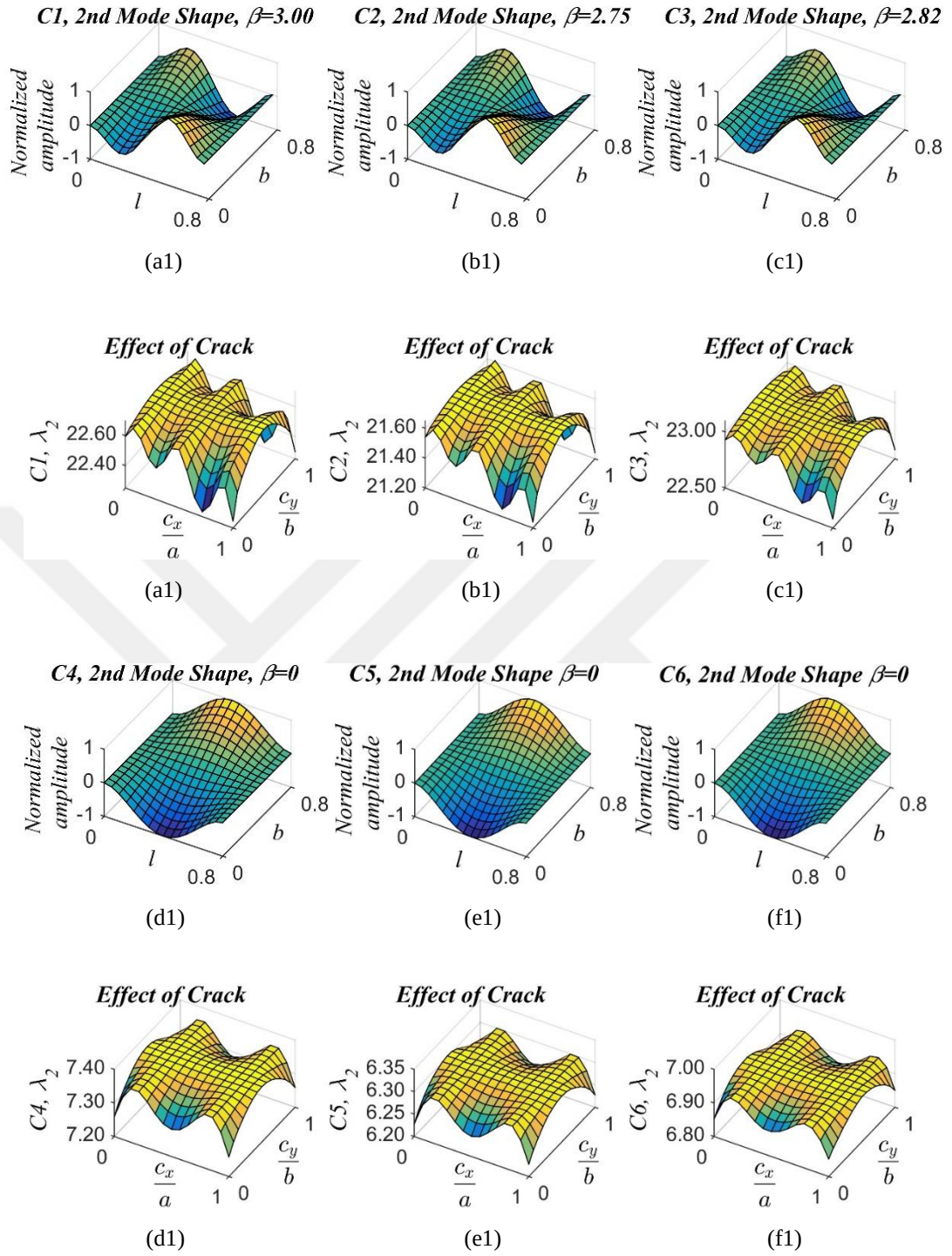


Figure 6.22 continues

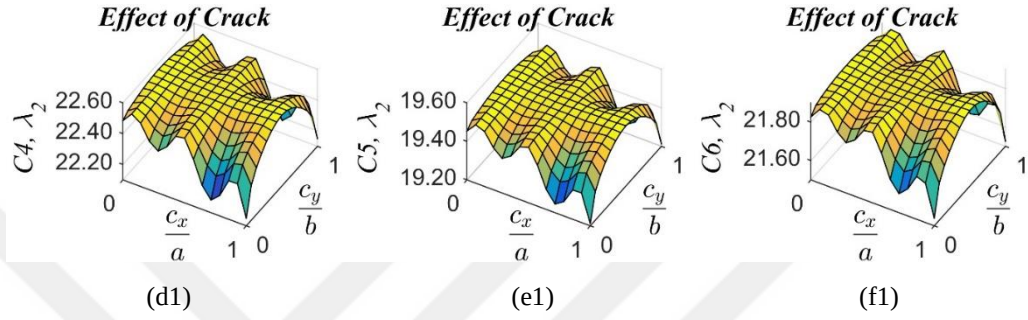
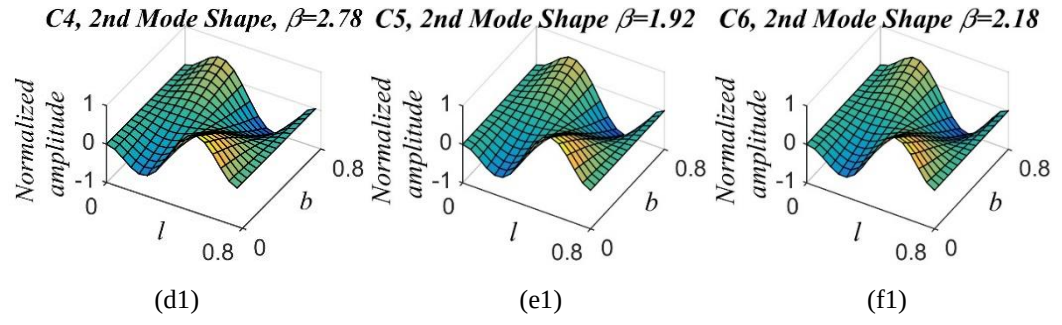
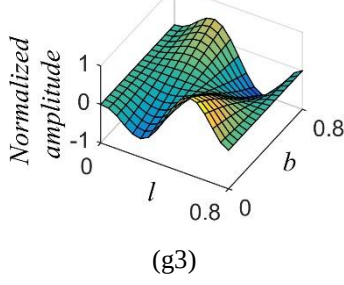
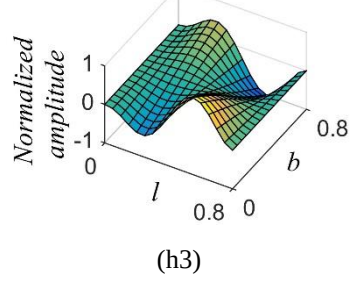


Figure 6.22 continues

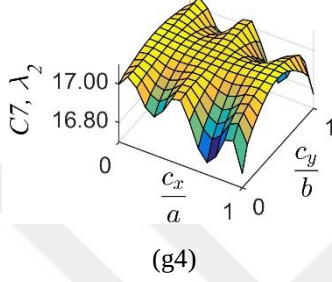
C7, 2nd Mode Shape $\beta=1.09$



C8, 2nd Mode Shape $\beta=0.80$



Effect of Crack



Effect of Crack

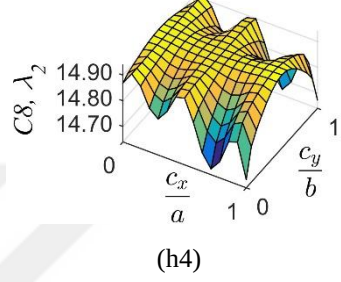
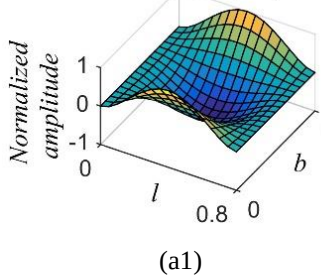


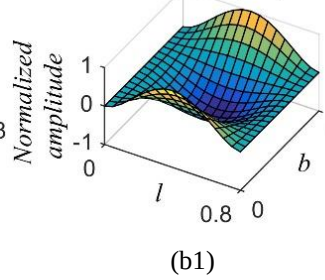
Figure 6.22 continues

According to the graphs of the third mode shapes, C5, and C6 have an extra one mode shape, and C7 has an extra two mode shapes compared to other composites. The extra mode shape in C5, and C6 composites are the same. The graphics of the third mode shapes are given in Figure 6.23.

C1, 3rd Mode Shape, $\beta=0$



C2, 3rd Mode Shape, $\beta=0$



C3, 3rd Mode Shape, $\beta=0$

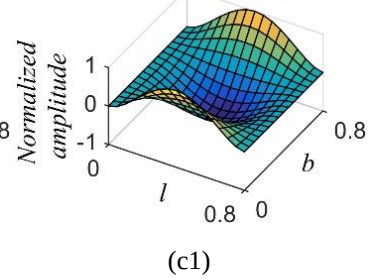


Figure 6.23 Mode shapes and the effect of crack on the third natural frequency parameter for C1 (a1, a2, a3, a4), C2 (b1, b2, b3, b4), C3 (c1, c2, c3, c4), C4 (d1, d2, d3, d4), C5 (e1, e2, e3, e4, e5, e6), C6 (f1, f2, f3, f4, f5, f6), C7 (g1, g2, g3, g4, g5, g6, g7, g8), C8 (h1, h2, h3, h4)

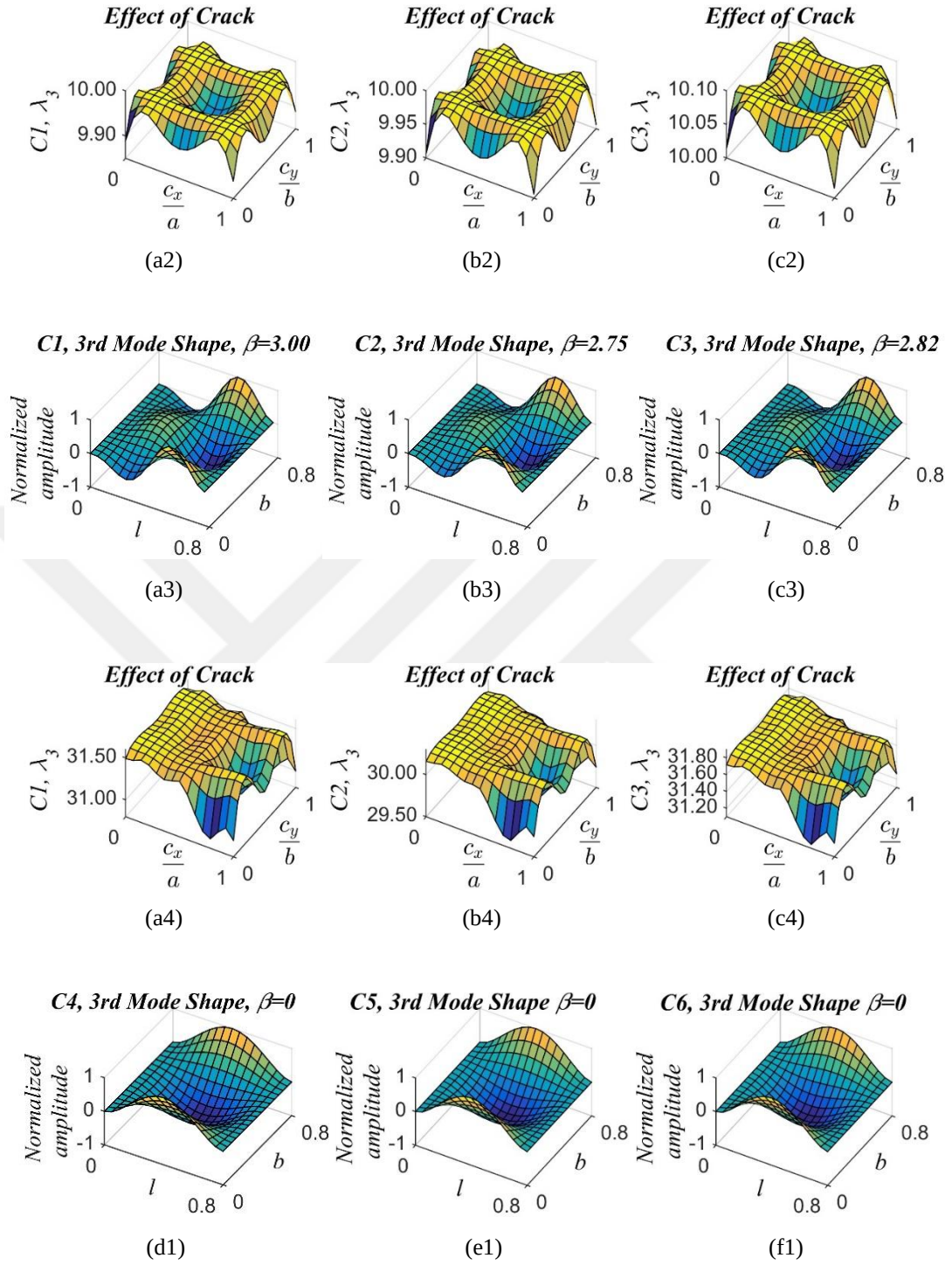


Figure 6.23 continues

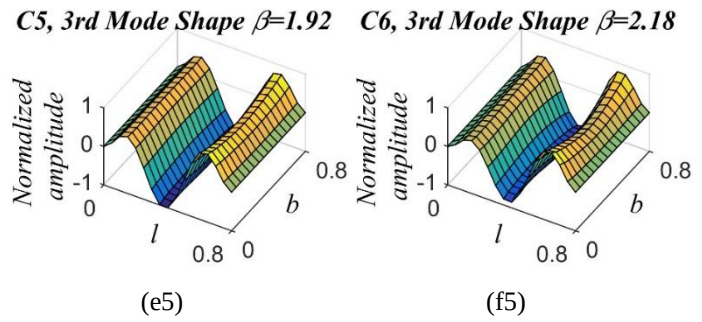
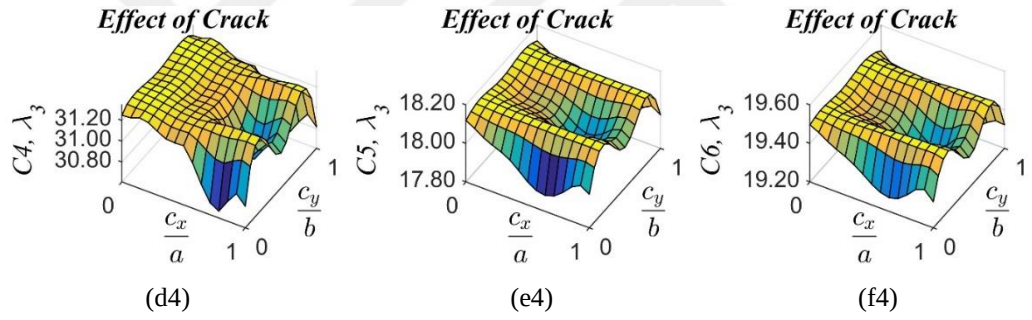
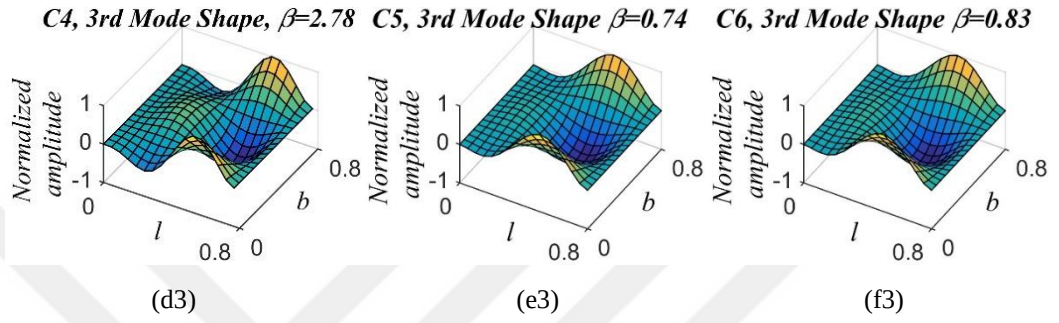
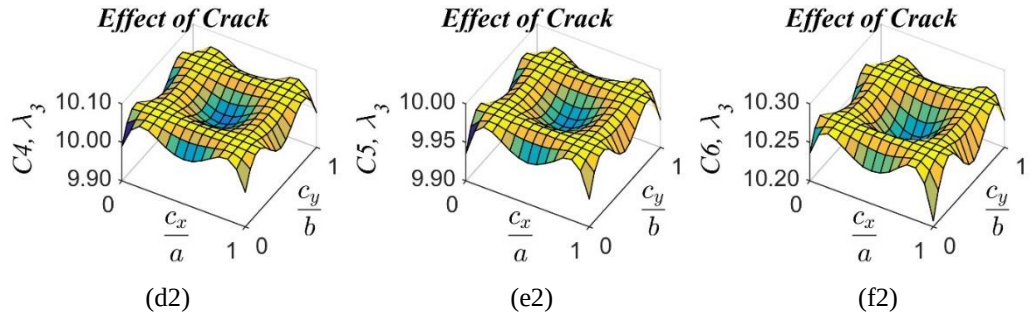


Figure 6.23 continues

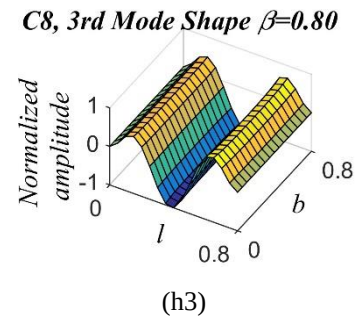
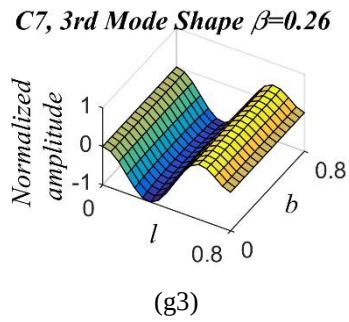
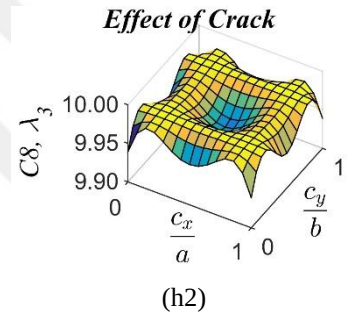
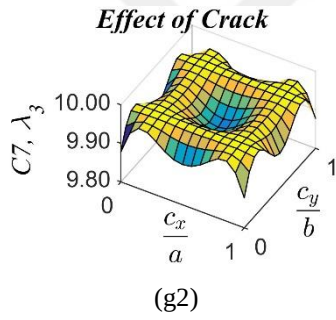
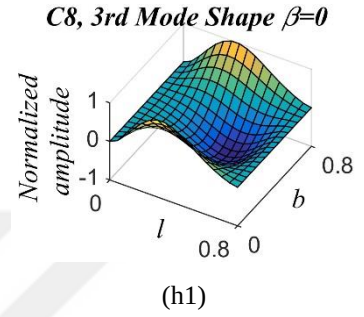
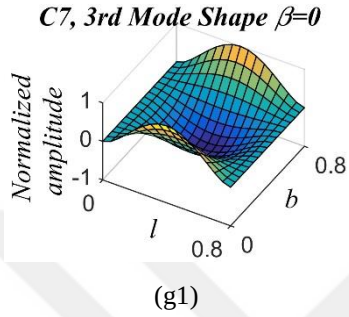
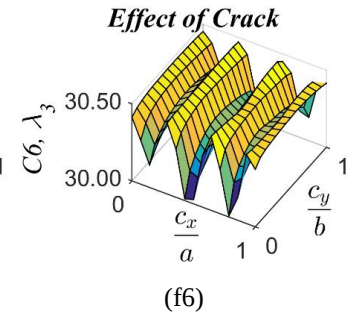
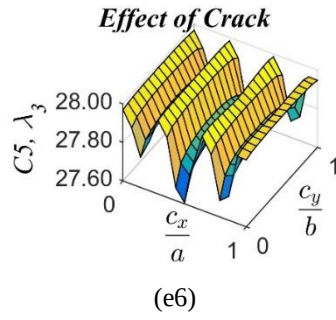


Figure 6.23 continues

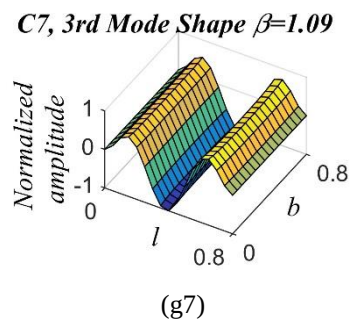
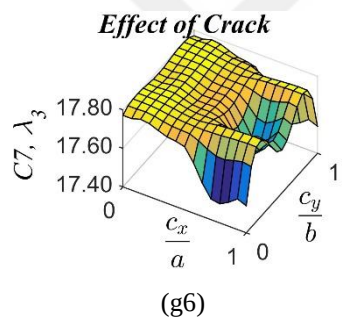
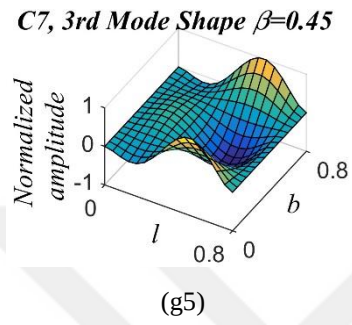
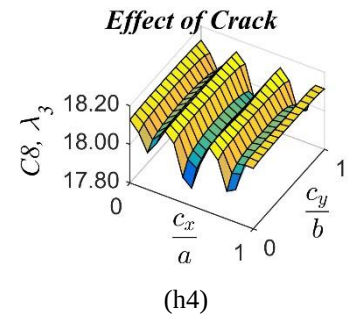
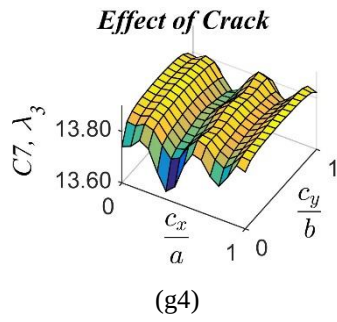


Figure 6.23 continues

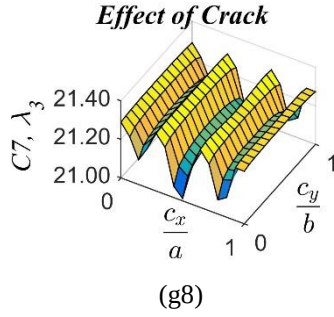


Figure 6.23 continues

The fourth mode shapes of each configuration are shown in Figure 6.24. While there are three different mode shapes in all composites except C6, and C7, there are four different mode shapes in C6, and five different mode shapes in C7. C2, C3, and C4 are identical in all mode shapes and the effect of crack. The second mode shape of C1 is different from these three. Except for the extra mode shape in C6, C5 and C6 are similar to each other.

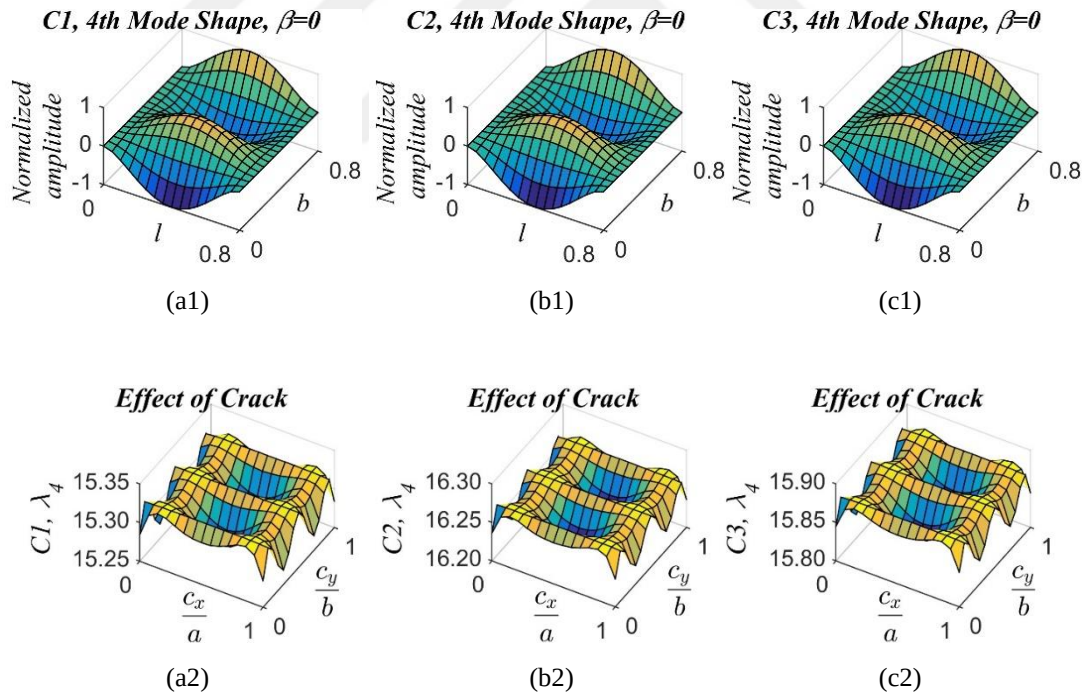


Figure 6.24 Mode shapes and the effect of crack on the fourth natural frequency parameter for C1 (a1, a2, a3, a4, a5, a6), C2 (b1, b2, b3, b4, b5, b6), C3 (c1, c2, c3, c4, c5, c6), C4 (d1, d2, d3, d4, d5, d6), C5 (e1, e2, e3, e4, e5, e6), C6 (f1, f2, f3, f4, f5, f6, f7, f8), C7 (g1, g2, g3, g4, g5, g6, g7, g8, g9, g10), C8 (h1, h2, h3, h4, h5, h6)

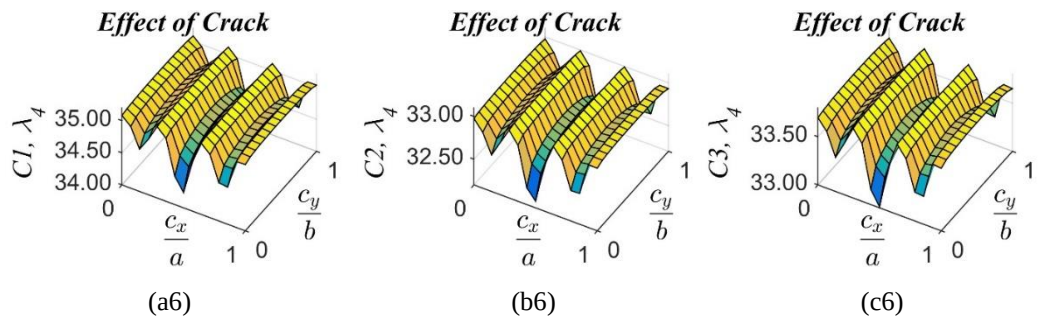
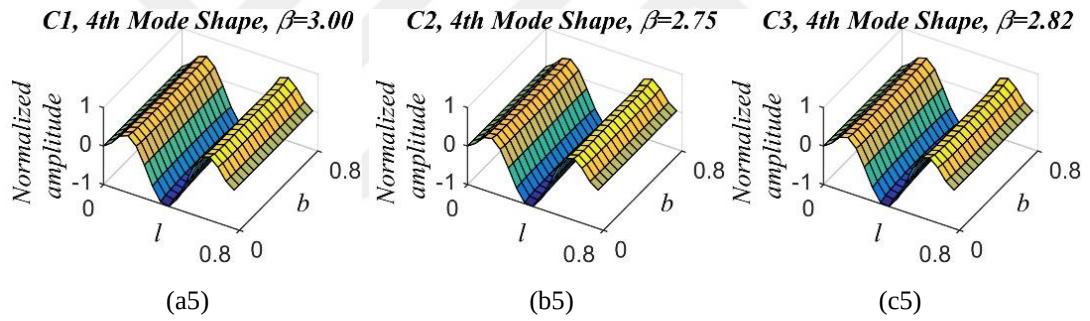
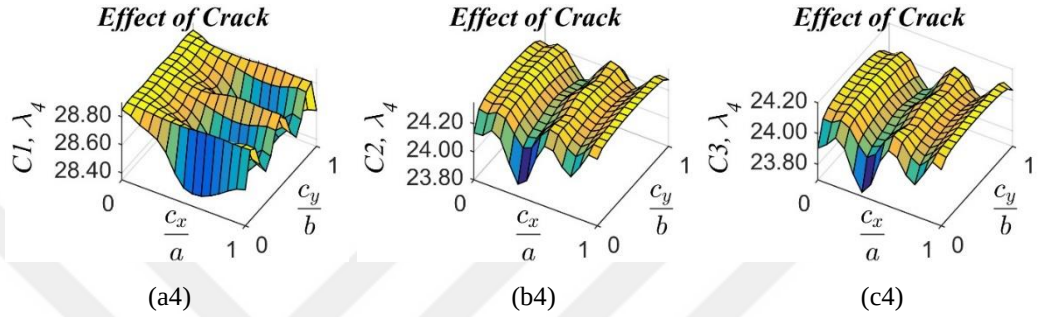
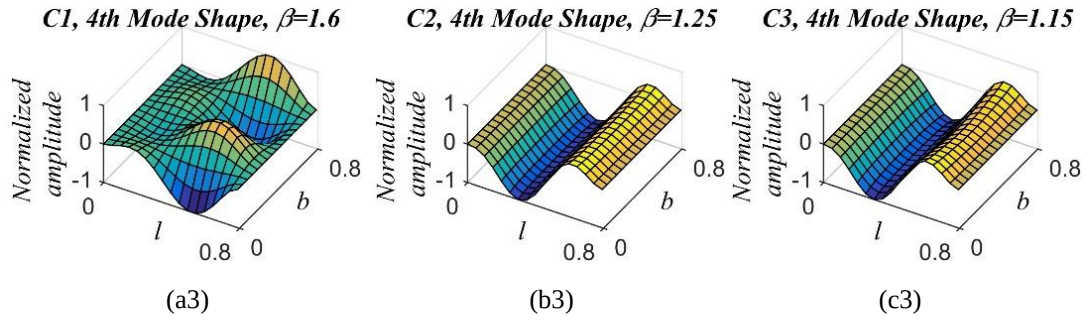


Figure 6.24 continues

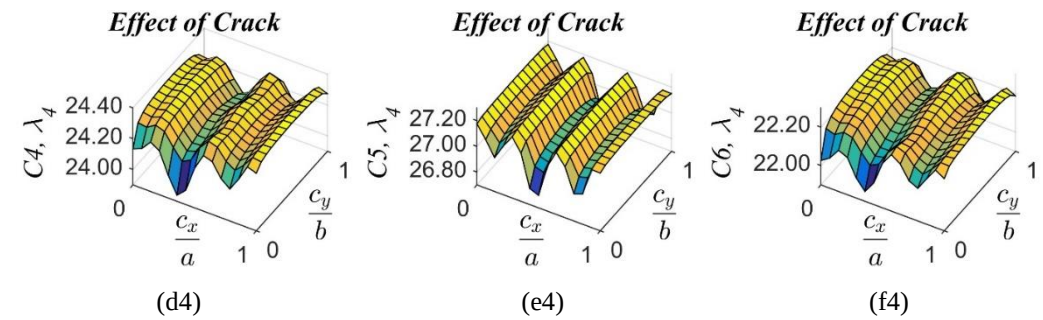
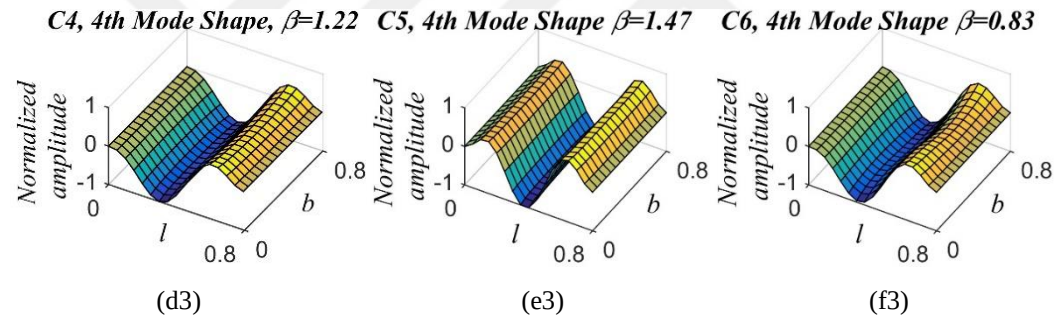
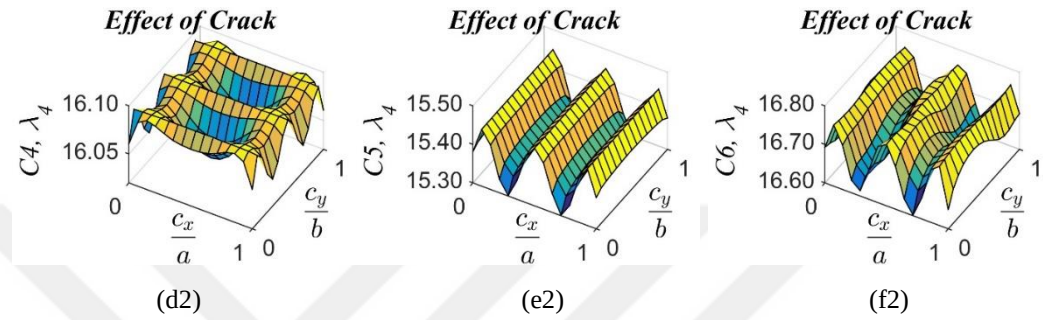
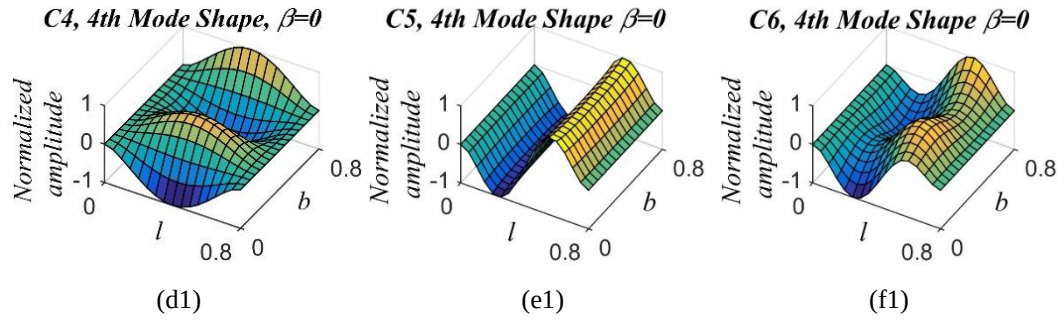
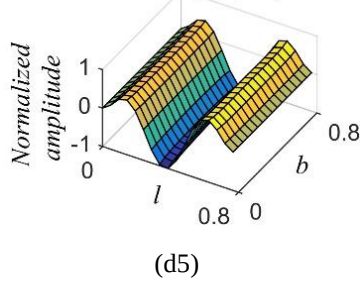
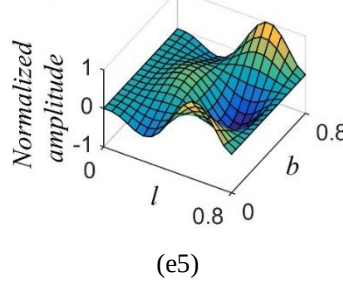


Figure 6.24 continues

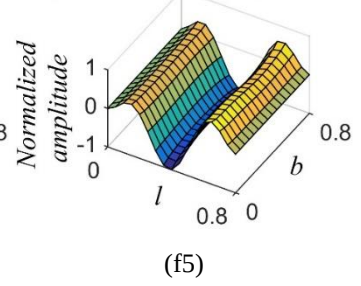
C4, 4th Mode Shape, $\beta=2.78$



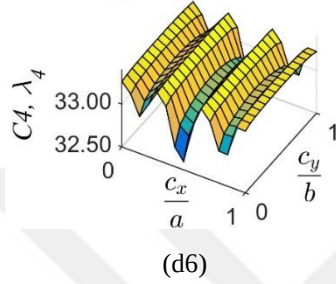
C5, 4th Mode Shape $\beta=1.92$



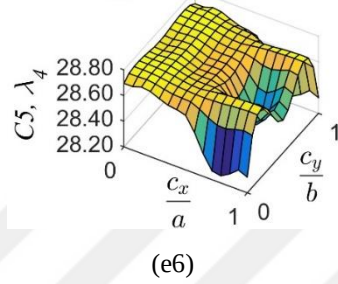
C6, 4th Mode Shape $\beta=1.66$



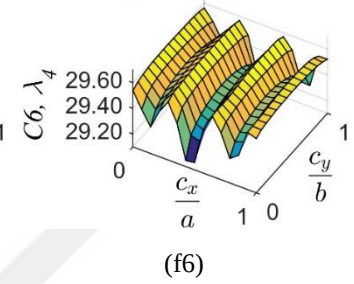
Effect of Crack



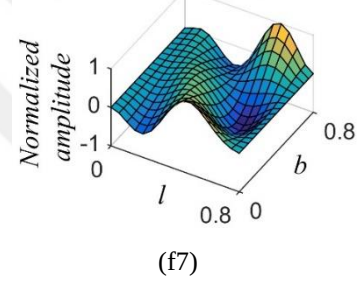
Effect of Crack



Effect of Crack



C6, 4th Mode Shape $\beta=2.18$



Effect of Crack

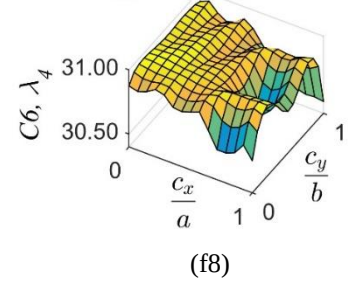


Figure 6.24 continues

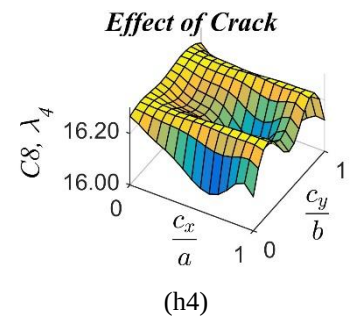
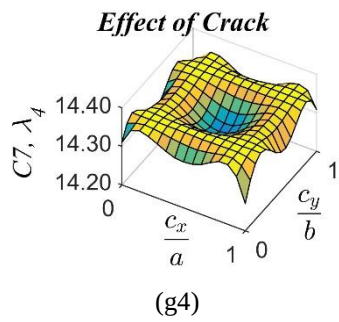
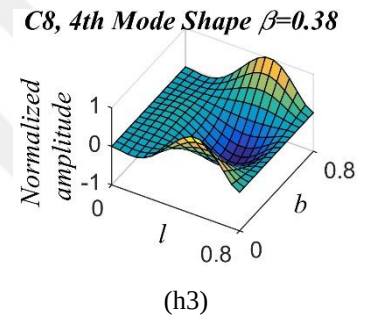
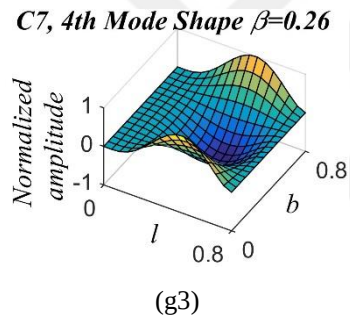
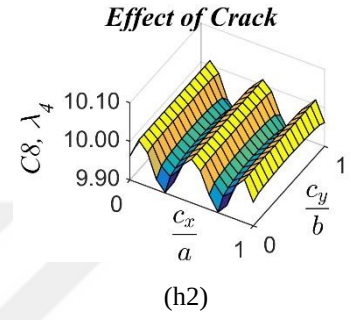
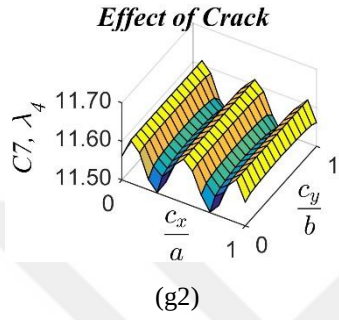
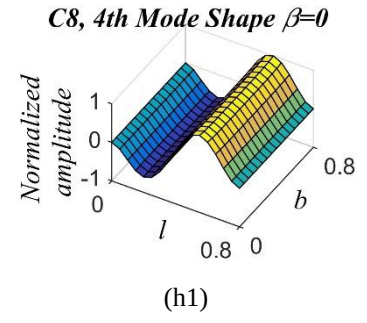
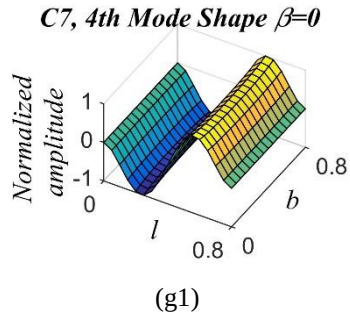
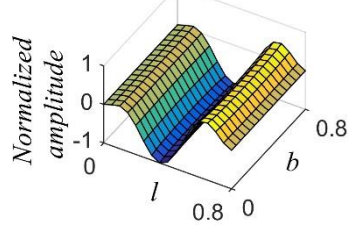


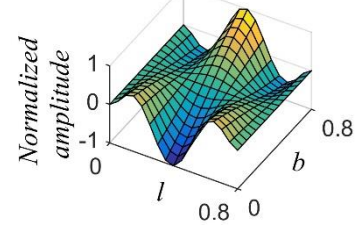
Figure 6.24 continues

C7, 4th Mode Shape $\beta=0.45$



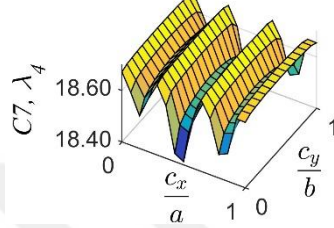
(g5)

C8, 4th Mode Shape $\beta=0.80$



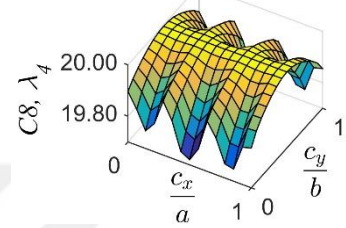
(h5)

Effect of Crack



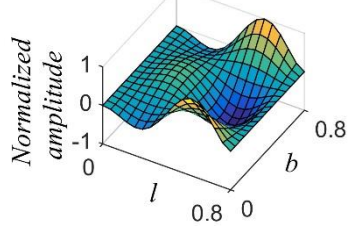
(g6)

Effect of Crack



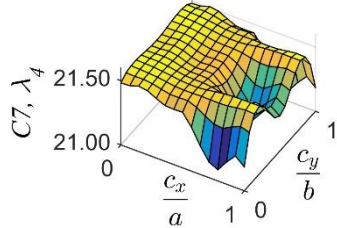
(h6)

C7, 4th Mode Shape $\beta=0.70$



(g7)

Effect of Crack



(g8)

Figure 6.24 continues

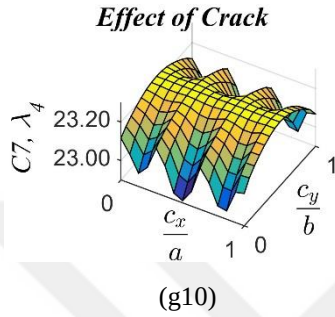
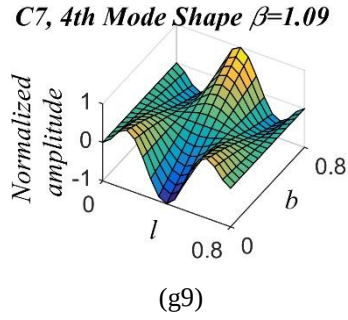


Figure 6.24 continues

There are two different mode shapes in C5, three different types in C6, C7, and C8, four different types in C1, C3 and C4, and five different types in C2, for the mode shapes in the fifth natural frequencies. While C3 and C4 are identical in all mode shapes and their effect of crack, C1 differs from these two only in the second mode. In C2, there is an extra mode shape in addition to all the mode shapes seen in C3 and C4. All composites except C7, and C8, have the same mode shape under their maximum loads. The fifth mode shapes of each configuration and their effect of crack are shown in Figure 6.25.

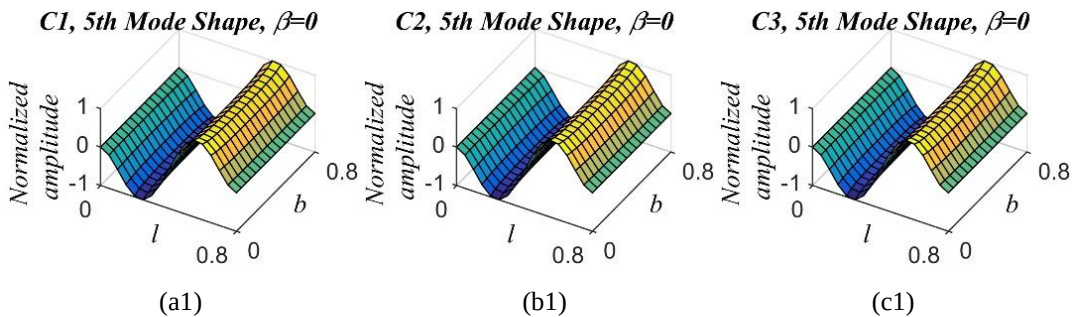


Figure 6.25 Mode shapes and the effect of crack on the fifth natural frequency parameter for C1 (a1, a2, a3, a4, a5, a6, a7, a8), C2 (b1, b2, b3, b4, b5, b6, b7, b8, b9, b10), C3 (c1, c2, c3, c4, c5, c6, c7, c8), C4 (d1, d2, d3, d4, d5, d6, d7, d8), C5 (e1, e2, e3, e4), C6 (f1, f2, f3, f4, f5, f6), C7 (g1, g2, g3, g4, g5, g6), C8 (h1, h2, h3, h4, h5, h6)

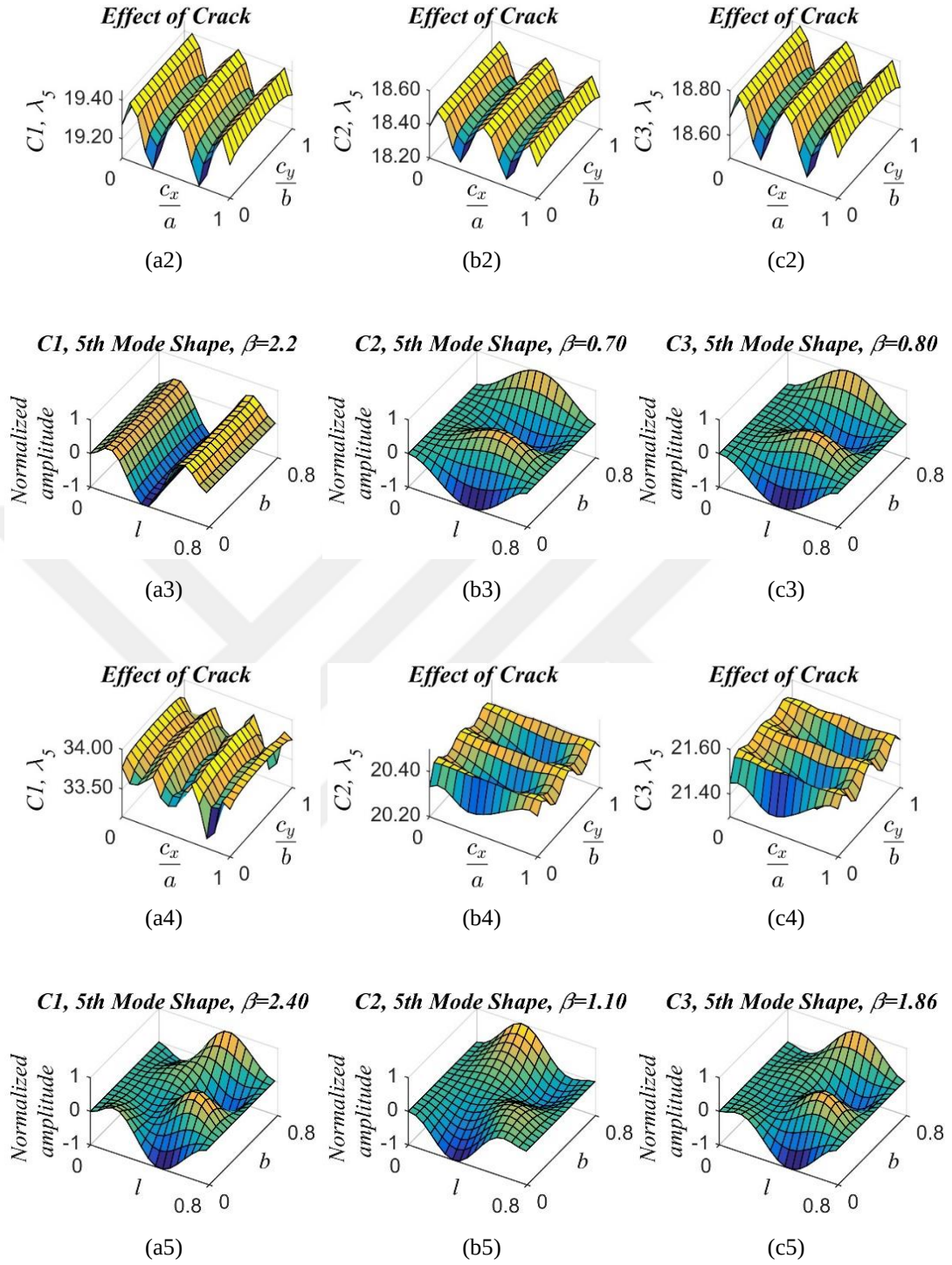


Figure 6.25 continues

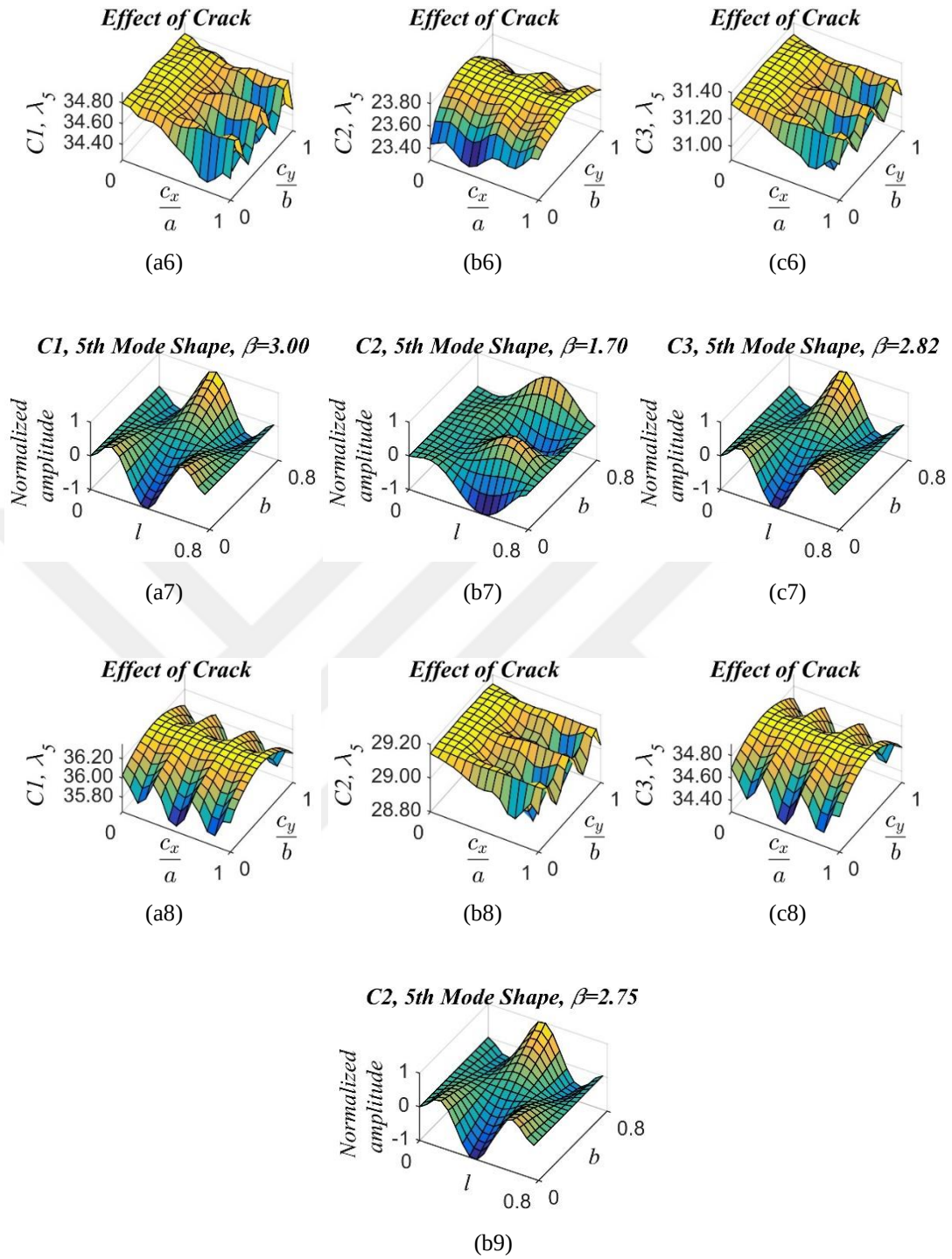
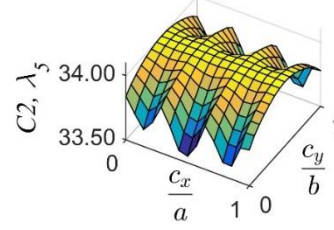


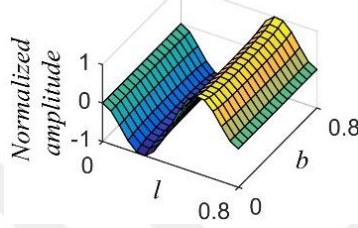
Figure 6.25 continues

Effect of Crack



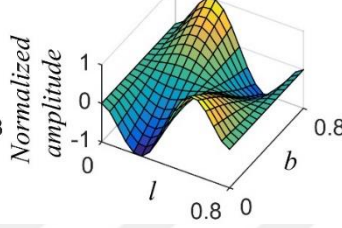
(b10)

C4, 5th Mode Shape, $\beta=0$



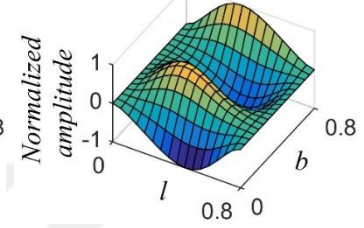
(d1)

C5, 5th Mode Shape $\beta=0$



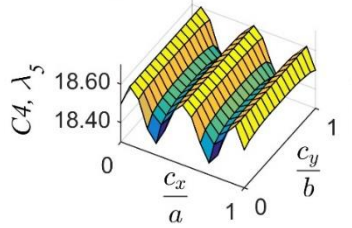
(e1)

C6, 5th Mode Shape $\beta=0$



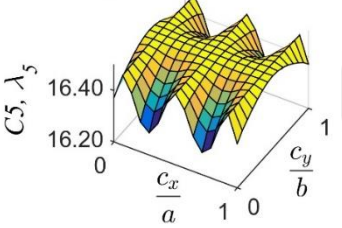
(f1)

Effect of Crack



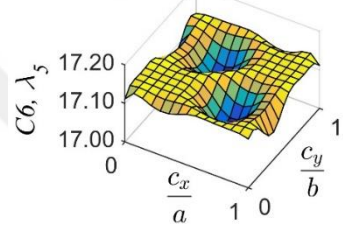
(d2)

Effect of Crack



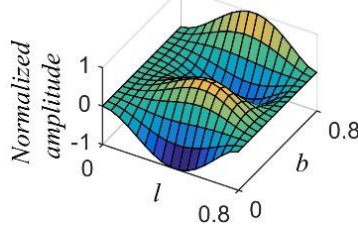
(e2)

Effect of Crack



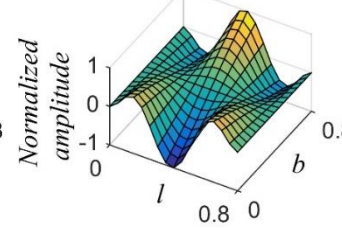
(f2)

C4, 5th Mode Shape, $\beta=0.70$



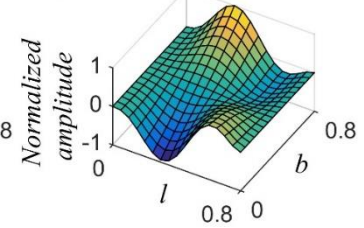
(d3)

C5, 5th Mode Shape $\beta=1.92$



(e3)

C6, 5th Mode Shape $\beta=0.83$



(f3)

Figure 6.25 continues

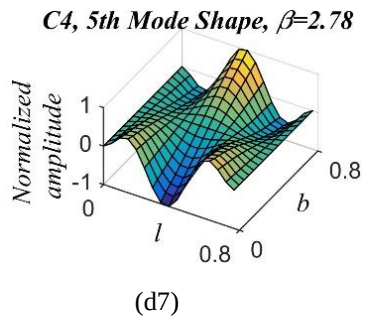
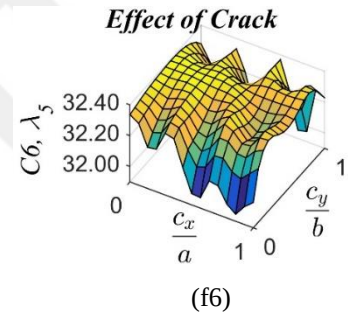
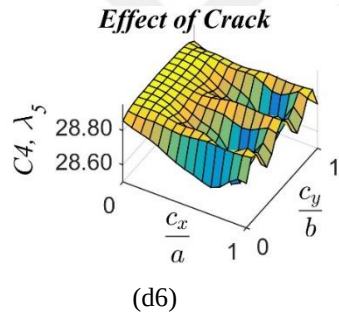
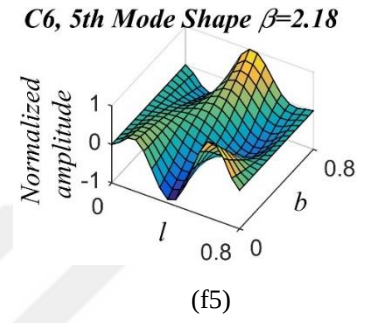
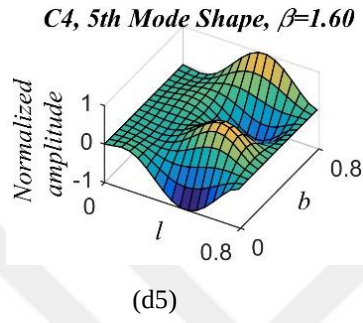
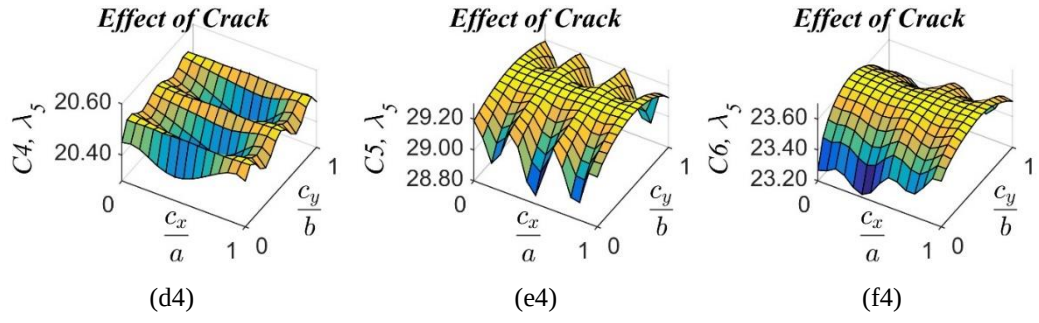


Figure 6.25 continues

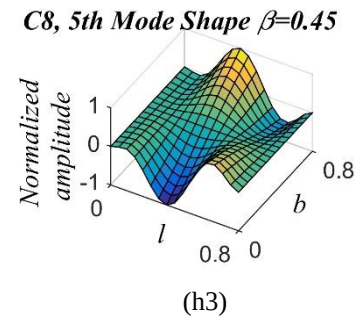
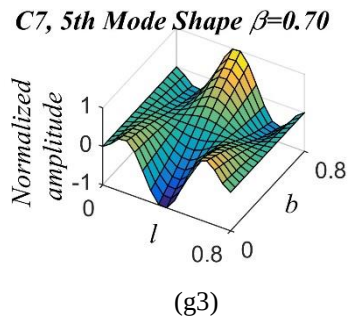
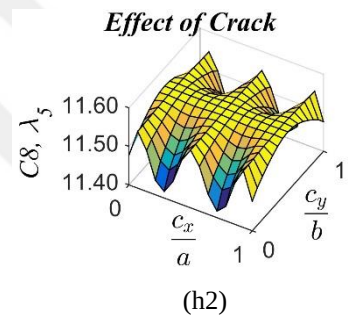
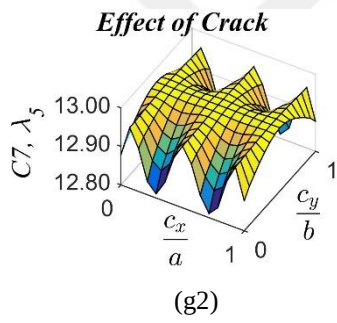
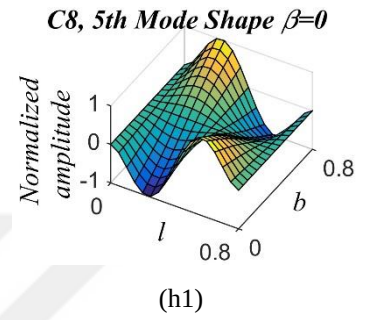
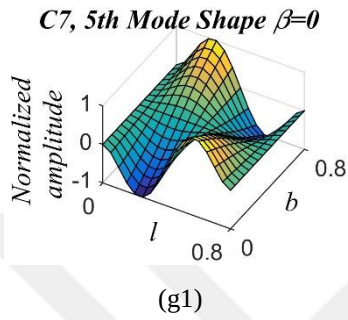
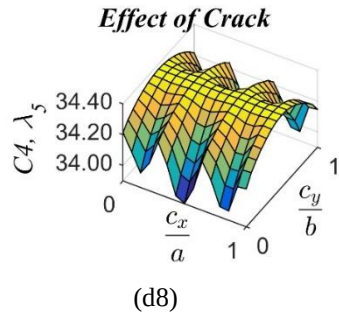


Figure 6.25 continues

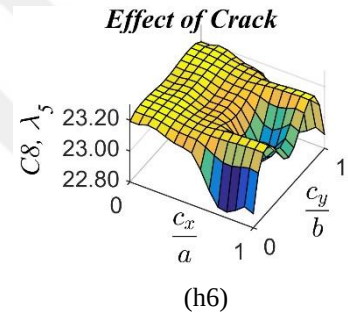
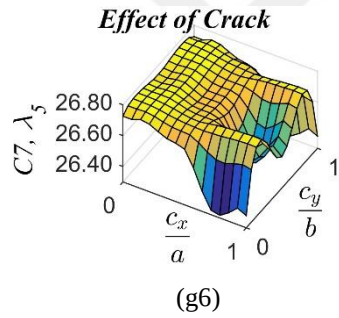
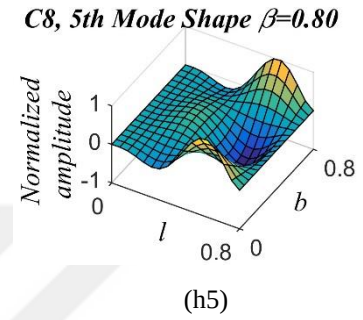
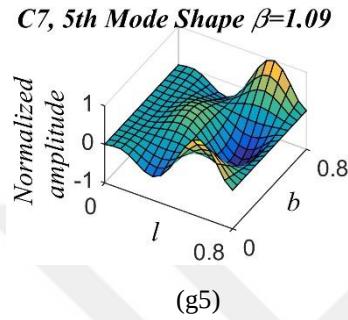
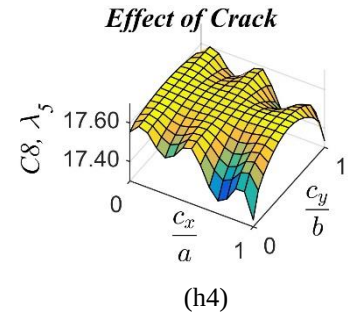
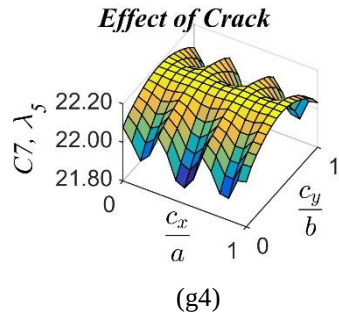


Figure 6.25 continues

Different numbers of mode shapes are seen for the first five natural frequencies in each composite. There are 13 mode shapes for C1, C3, and C4, 14 for C2 and C6, 12 for C5 and C8, and 16 for C7. Depending on the increase of the load parameter, any plate can have more or less mode shape in any natural frequency parameter than the others. Mode shape transitions can occur within the first five natural frequencies of the same composite. For example, Figure 6.21-b3 is one of the first mode shapes of C2 composite, and it is shown again in Figure 6.24-b3, which is one of the fourth mode shapes of C2. For the third and fourth mode shapes of C3 composite, Figure 6.23-e3 and Figure 6.24-e5 are the same. On the other hand, although the same mode shapes can be shown in different sequences, the strain differences can be seen in the mode

shapes. However, regardless of the plate type and load condition, the variability of the effect of crack depends only on the strain of the mode shape.

6.4 Conclusion

In this chapter, the effect of crack on the natural frequency parameter and mode shapes of the pre-stressed curved isotropic and anisotropic plates is investigated. Three different plates with an aspect ratio equal to 1, 0.75, and 0.5 are examined for the isotropic plates. Eight different configurations of the laminated composite plate are examined. In order to observe the effect of crack, the crack is separately added in all nodes. According to the results obtained:

- In all loading cases up to 85% of the yield stress value of the isotropic plates, and in all loading cases up to deflection value as 25% of the length value of the anisotropic plates, it is found that the present model consistently represented the pre-stressed curved plate model in different aspect ratios.
- As loading increases, the curvatures in natural frequency parameter (λ) – load parameter (β) graphs, vary at different points for each natural frequency and mode shape changes are seen at these points.
- In the case of plates with different aspect ratios, mode shapes can appear in different shapes as loading increases. However, this does not affect the relationship between crack and mode shape.
- The increase in the loading parameter and the curved structure of the plate makes the effect of crack on the plates asymmetric.
- While the mode shapes of the first two natural frequencies are the same in all aspect ratios, different mode shapes appear as the aspect ratio decreases in the third, fourth, and fifth natural frequencies. The appearance of different mode shapes causes the effect of crack to change in different plates under the same load.
- When the load parameter value is equal to zero, the same mode shape is displayed for all types of configurations examined.

- As the load parameter value increases, the natural frequency parameter value increases. However, this happens in different forms of curvature for each natural frequency.
- In different laminated composite configurations, different mode shapes may occur in a different order and in different numbers, especially in higher natural frequencies.
- The effective modulus of elasticity is the decisive factor in the fact that plates of different configurations show differences at different points as the load parameter value increases.
- Considering all of the composite plates, C5 and C8 have the least number of mode shapes for the first five natural frequencies during the load parameter's changing.

CHAPTER 7

CONCLUSIONS

7.1 Review of the Thesis

Within the scope of the thesis, a cantilever plate fixed on one side is shaped into a curved form by applying a load from its free end. An external distributed vertical load is applied at the free edge of the cantilever plate when the opposite edge is clamped. Then the loading edge of the deflected plate is fixed to obtain a curved plate. The plate fixed from the loading edge becomes pre-stressed as it is fixed with force on it. The curvilinear geometry, which is a result of applying distributed force, is investigated by free vibration analysis.

The plate geometry designed as pre-stressed is studied for different material types and different aspect ratios. The first five natural frequency values of the plates with isotropic and anisotropic material type, mode shapes for each plate, and the effect of load conditions on each natural frequency are investigated. Considering the plate size and thickness for the isotropic plate, the critical load for each plate is determined according to the stress value, which is formed the plate. Steel, which is the type of material frequently used in engineering structures, is accepted as the main material in isotropic plate studies. While calculating the deflection values, it is ensured that the stress value does not exceed 85% of the yield limit. The critical load value that constitutes the stress value is found with the help of the ANSYS program, and this load value is used in the deflection equation. In the static analysis section, the comparisons of the deflection graphs under the maximum load of all plates examined within the scope of the thesis are given.

Apart from the pre-stressed structure, the effect of curvature on the natural frequency and mode shapes is examined for only the isotropic plate. In section 5, the radius of curvature is reduced for the isotropic plate whose aspect ratio is equal to 1, provided that the plate length remains constant. The lowest curvature radius value for the isotropic plate with dimensions of 500x500x5 mm is accepted as the value that

makes this plate a semicircle. For the geometries from the flat plate model to the semicircle plate model, first five natural frequency parameters and their mode shapes are examined.

The laminated composite plates are shaped with the deflection equation as in the isotropic plate. Due to the composite material selected, the material needs to be curved a lot to reach the yield limit. This situation causes an increase in the difference between ANSYS and the theoretical approach due to the density of mesh used. To eliminate this problem, the criterion that is taken into consideration when determining the deflection limit for composite materials is the deflection value, not the yield limit. The maximum deflection value is accepted as 25% of the length value of the plate. Different estimation methods, such as increasing the density of the mesh, changing the theoretical approach, or machine learning, can be used for the current theory in order to obtain more accurate results in larger deflection values.

In order to use the deflection equation in the analysis of laminated composite plates, an effective bending elasticity modulus is needed. The effective elasticity modulus calculation is given in the static analysis section. The static analysis of the anisotropic plates with eight different lamination orientations is carried out to check the accuracy of the deflection equation with the effective elasticity modulus. Comparative graphs of curved models show the accuracy of the curved model created for composite plates. The effect of the load parameter on the first five natural frequency values and their mode shapes are given for the eight different composite plates.

Both the isotropic plates and the anisotropic plates are statically and dynamically examined, and their models are revised by adding cracks on them. The crack is added to the plates geometrically in the form of an crack, twice the size of the mesh, taking into account the current mesh density. In each plate, the effect of crack on each node in the x and y plane is recorded separately, and the the effect of crack is drawn. In this way, the curvature effect on the plates with crack is observed.

7.2 Main Findings of the Studies in the Thesis

The order of examining all plates and all models examined under the thesis is given below:

- Flexural vibration of the isotropic plate
- In-plane vibration of the isotropic plate
- Merged model for the isotropic plate
- The curved isotropic model with using the transformation matrix
- Examination of the effect of the radius on plates
- Using the deflection equation for the curved isotropic model
- Adding initial stress to isotropic plate model
- Adding a crack to isotropic plate model
- Creating mode shapes and their effect of crack for isotropic model
- Laminated composite model
- The curved laminated composite model with the transformation matrix
- Adaption of the deflection equation to anisotropic model
- Using the deflection equation for the curved anisotropic model
- Adding a crack to anisotropic plate model
- Creating mode shapes and their effect of crack for anisotropic model

Based on all of the data in this research, the theoretical approaches proposed for both isotropic and anisotropic material types related to pre-stressed curved plates can be used with a good representation rate. Considering all the plate types, mode shapes, the effect of cracks, and plate geometries investigated, the following generalized results can be achieved.

- The literature review shows that there have not been any published studies on the pre-stressed curved plate, which is obtained from a cantilever plate with applying a vertical distributed load on its free end.
- The deflection equation, which has been used for beams in previous studies, has good representation on thin plates.

- Various curvature values of the structure occur as the applied load increases in pre-stressed models. This causes mode shape differences at the same modes.
- Different mode shapes can be seen depending on the aspect ratio in isotropic plates whose curve model changes depending on the load parameter.
- The error rates between the present model and ANSYS are within acceptable limits.
- In laminated composite plates, the main parameter on which the mode shapes depend is the effective elasticity modulus.
- The curved plate's asymmetry shifts the pattern of the mode shape to the edge on which the loading is present. As the asymmetry of the curved plate becomes apparent, different mode shapes emerge.
- The effective elasticity modulus affects the load-dependent variation of the first five natural frequency parameters in different ways. This can cause transitions between modes.
- The increase of curvature in the plates causes an increase in all natural frequency parameters. As the curvature continues to increase, the first five natural frequency parameter values suddenly decrease as they approach the semicircular model.
- Depending on the changing of the curvature, each natural frequency parameter increases in different ways. This variety causes different mode shapes to appear.
- The effect of crack depends only on the mode shape in both isotropic and anisotropic plates.
- While the first two mode shapes and the effect of crack depending on the mode shape are the same for all isotropic and anisotropic plates, mode shapes can be formed in different numbers and shapes beginning from the third natural frequency parameter.
- Mode shapes of the plate can change depending not only on the load parameter but also on the aspect ratio. However, this situation does not affect the relationship between crack and mode shape.

- The effect of crack can be seen more clearly in regions where strain value increases due to the mode shape. However, in the neutral regions that are not affected by the mode shape, the effect of crack can go down to zero.
- The fiber orientation is the decisive factor in that laminated composite plates show differences in natural frequency parameter (λ) – load parameter (β) graphs.
- Considering all of the composite plates, C5 and C8 have the least number of mode shapes, and C7 has the most number of mode shapes for the first five natural frequencies during the load parameter's changing.

7.3 Suggestions for Future Works

The presented model supports free vibration in both isotropic and anisotropic curvilinear plates and gives quite good results. The presented model's role in the static and dynamic analysis can also be confirmed experimentally in future works. In light of the results, many correct and valid data can be collected, and free vibration analysis of different models can be made more practical. Since the presented model is also a wide-scale model that includes models of different curvatures, thicknesses, and additional material types, both the free vibration and the dynamic stability analysis of different and special geometries can be carried out based on these limits. By improving the deformation method used, free vibration analysis can also be performed within the plastic zone.

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