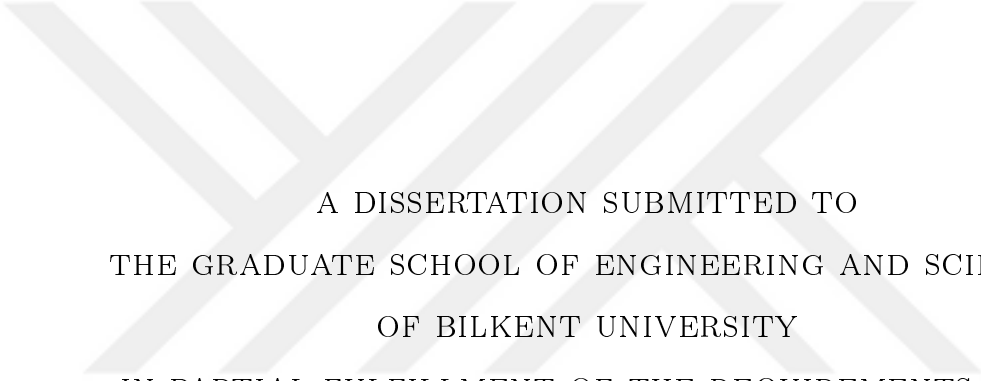


CODE DESIGN FOR INTERFERENCE CHANNELS



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By
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November 2019

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ABSTRACT

CODE DESIGN FOR INTERFERENCE CHANNELS

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As the number of wireless devices dramatically increases, they experience more interference in their communications. As a result, managing interference in wireless networks is an important challenge in future wireless communication systems, which can be tackled in different layers of communications. Designing good channel codes, which can enable reliable communication close to the information theoretic limits in the presence of interference, is one of the ways to increase the quality of service.

With the above motivation, in this research, we focus on code design for interference channels (ICs). We, first consider classical two-user fading IC and study implementation of different encoding/decoding schemes with low-density parity-check (LDPC) codes for both quasi-static and fast fading scenarios. We adopt the Han-Kobayashi (HK) type encoding, derive stability conditions on the degree distributions of LDPC code ensembles, and obtain explicit and practical code designs. In order to estimate the decoding thresholds, a modified form of the extrinsic information transfer (EXIT) chart analysis based on binary erasure channel (BEC) approximation for the incoming messages from the component LDPC decoders to state nodes is developed. The proposed code design is employed in several examples for both fast and quasi-static fading cases. A comprehensive set of examples demonstrates that the designed codes perform close to the achievable information theoretic limits. Furthermore, multiple antenna transmissions employing the Alamouti scheme for fading ICs are studied; a special receiver structure is developed, and specific codes are explored. Finally, advantages of the designed codes over point-to-point (P2P) optimal ones are demonstrated via both asymptotic and finite block length simulations.

Next, we consider cognitive interference channels (CICs), a variant of classical two-user ICs in which one of the transmitters (cognitive transmitter) has non-causal knowledge of the other's (private user's) message. Prompted by the information theoretical results, we design an explicit coding scheme for CIC in the primary decodes cognitive regime. We present a novel joint decoder and design

LDPC codes for our set-up. Simulation results demonstrate that the proposed joint decoder and the designed codes outperform the conventional maximum ratio combining type decoder and the point-to-point optimal codes, respectively. Later, we propose and evaluate the idea of flexible modulation for P2P communication with available channel side information at the transmitter. This technique does not perform as well as dirty paper coding (DPC); however, its simplicity is a major advantage. Also the flexible modulation technique shows more robustness to inaccuracy in the channel state information. Finally, we consider a multiple access channel (MAC) in which the non-causal knowledge of one of the users' message is available at the other user. We consider both Gaussian channel and fading scenarios. We propose the idea of joint encoding, and study its performance via simulations demonstrating that the proposed approach outperforms the classical coding scheme.

Keywords: Low-density parity-check codes, interference channels, code design, iterative joint decoding, Han-Kobayashi encoding, multiuser systems, dirty paper coding.

ÖZET

GİRİŞİM KANALLARI İÇİN KOD TASARIMI

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Kablosuz cihazların sayısının önemli ölçüde yükselmesiyle kablosuz cihazlar iletişimlerinde daha fazla girişim sinyaline maruz kalmaktadır. Kablosuz ağlarda girişimi yönetmek iletişimin değişik katmanlarında ele alınabilecek, kablosuz iletişimin geleceği için önemli bir zorluktur. Kanalda girişimin varlığında güvenilir iletişim sağlayabilecek, bilgi teorik limitlere yakın kanal kodlarının tasarımı servis kalitesini artırmanın yollarından birisidir.

Yukarıdaki motivasyonla, bu araştırmada girişim kanalları (IC) için kod tasarımına odaklandık. İlk olarak iki kullanıcılı sönümlemeli IC'yi göz önünde bulundurduk ve düşük-yoğunluklu parite-kontrol (LDPC) kodlarıyla neredeyse statik ve hızlı sönümleme senaryoları için değişik kodlama/kod çözme şemalarını uyguladık. Han-Kobayashi (HK) kodlama şemasını kullandık, LDPC kod grupları için stabilite koşulları türettik ve açık ve pratik kod tasarımları elde ettik. Kod çözme eşiklerini kestirebilmek amacıyla, dışsal bilgi transferi (EXIT) analizinin komponent LDPC kod çözücülerden durum düğümlerine gelen mesajlar için ikili silinti kanalları (BEC) yaklaştırmaya dayalı değiştirilmiş bir versiyonunu kullandık. Önerilen kod tasarımı hızlı ve neredeyse statik sönümlemeli durumlar için birçok örnekte kullanılmıştır. Kapsamlı örnekler dizayn edilen kodların ulaşılabilir bilgi teorik limitlere yakın performansla çalıştığını göstermiştir. Ayrıca, Alamouti şemasını sönümlemeli girişim kanalları için kullanan çoklu anten iletimleri üzerinde çalışılmıştır; özel bir alıcı yapısı geliştirilmiş ve belirli kodlar incelenmiştir. Son olarak, dizayn edilmiş kodların noktadan-noktaya (P2P) en iyi kodlara göre avantajları asimtotik ve sonlu blok uzunluklu simülasyonlarla gösterilmiştir.

Sonraki aşamada klasik iki kullanıcılı girişim kanallarının vericilerden birisinin (bilişsel verici) diğerlerinin (özel kullanıcı) nedensel olmayan mesaj bilgisine sahip olduğu bir versiyonu olan bilişsel girişim kanallarını (CIC) göz önünde bulundurduk. Bilgi teorik sonuçların yardımıyla CIC için birincil kod çözücü bilişsel alanında açık bir kodlama şeması tasarladık. Özgün bir birleşik kod çözücü

sunduk ve yapımız için düşük yoğunluklu parite kodları tasarladık. Simülasyon sonuçları önerilen birleşik kod çözücünün ve tasarlanan kodların geleneksel maksimum oran birleştirmesi kod çözücüsü ve noktadan-noktaya en iyi kodlardan daha iyi çalıştığını göstermiştir. Daha sonra, vericide kanal yardımcı bilgisinin mevcut olduğu P2P iletişim için esnek modülasyon fikrini önerdik ve değerlendirdik. Bu yöntem kirli kağıt kodlaması (DPC) kadar iyi çalışmasa da basitliği tekniğin ana avantajıdır. Ayrıca, esnek modülasyon tekniği kanal durum bilgisindeki yanlışlıklara daha çok dayanıklılık göstermektedir. Son olarak, kullanıcılardan birinin nedensel olmayan mesaj bilgisinin diğer kullanıcıda bulunduğu bir çoklu erişim kanalı (MAC) göz önünde bulundurduk. Birleşik kodlama fikrini önerdik ve simülasyon sonuçları bu tekniğin klasik kodlama şemasından daha iyi çalıştığını göstermiştir.

Anahtar sözcükler: Düşük-yoğunluklu parite-kodları, girişim kanalları, kod dizaynı, yinelemeli birleşik kod çözme, Han-Kobayashi kodlaması, çok kullanıcıli sistem, kirli kağıt kodlaması.

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Abbreviations

5G	5 th generation wireless systems
ARR	achievable rate region
AWGN	additive white Gaussian noise
BC	broadcast channel
BEC	binary erasure channel
BER	bit error rate
BI-AWGN	binary-input additive white Gaussian noise
BP	belief propagation
BPSK	binary phase shift keying
CIC	cognitive interference channel
CN	check node
DE	density evolution
DL	deep learning
DMIC	discrete memoryless interference channel
DPC	dirty paper coding
EXIT	extrinsic information transfer
FER	frame error rate
GIC	Gaussian interference channel
HK	Han-Kobayashi

IC	interference channel
i.i.d.	independent and identically distributed
INR	interference to noise ratio
IRA	irregular repeat accumulate
LDPC	low-density parity-check
LLR	log-likelihood-ratios
MAC	multiple access channel
MIMO	multiple-input multiple-output
MRC	maximum ratio combiner
P2P	point-to-point
PDF	probability density function
PMF	probability mass function
QPSK	quadrature phase shift keying
SNR	signal to noise ratio
SUD	simultaneous unique decoding
TCQ	trellis coded quantization
TIN	treating interference as noise
VN	variable node

Chapter 1

Introduction

Limited bandwidth combined with the ever-growing requests for higher data rates drives the need for higher spectral efficiencies. The broadcast and superposition nature of the wireless medium make interference a major limiting factor of reliable communication in wireless networks. The ever increasing number of wireless devices makes the problem of managing the interference more vital. To avoid interference, in the past, communication resource division schemes have been widely used. These schemes allocate a slot of time, frequency, or code to a specific user, i.e., the other users are not allowed to communicate in that resource. Although these schemes are quite simple and practical, they do not perform well in terms of spectral efficiency in many scenarios, and there is a need for more advanced solutions.

Many studies have been conducted to manage interference, including beam-forming (to focus the signal toward the intended receiver), millimeter wave communication (to reduce the number of users using the shared spectrum by increasing the utilized communication bandwidth), interference alignment (to manage the interfering signals in the network). Besides these techniques, channel coding plays an important role to improve the spectral efficiency in wireless networks, which is the focus of this research.

Interference channel (IC) is one of the basic multiuser channel models, which consists of several pairs of users communicating through a shared channel. Although the existing communication networks are usually more complicated than the classical ICs, the studies on this model can shed light on the methods to manage interference in large networks. Also, such models can be used as building blocks of practical wireless networks.

Despite a rich set of information theoretical results, there are only a limited number of investigations in the literature regarding practical (explicit and implementable) coding schemes for multiuser channels. Low-density parity-check (LDPC) codes are designed for broadcast channels (BCs) which achieve rate pairs near the boundary of the achievable rate region (ARR) in [1]. Similarly, well performing LDPC codes have been designed, which achieve rates close to the information theoretic limits, for multiple access channels (MACs) [2], relay channels [3], and ICs with fixed channel gains [4]. However, explicit and implementable code design problem for many multiuser scenarios is still an open problem.

The principal question that we attempt to address in this work is to find efficient and practical coding schemes to communicate close to the information theoretic limits of interference channels. Motivated by the relevance of fading interference channels to wireless communications of multiple users utilizing the same spectrum, we consider the code design for (fast and quasi-static) fading ICs under the assumption that the transmitters have only the statistics of the channel gains. Prompted by the information theoretical results, we implement a Han-Kobayashi (HK) type coding scheme in our set-up. Then, we extend our study to unfaded cognitive interference channel (CIC) which is a variant of classical IC, and propose a novel joint decoder for a specific set of channel parameters called *primary decodes cognitive* regime. We also propose a flexible modulation technique for a dirty paper channel (i.e., an additive white Gaussian noise (AWGN) channel corrupted by an additive interference signal which is known to the transmitter but not to the receiver). Additionally, we consider fading scenario for a particular MAC for which one of the transmitters has non-causal knowledge of the other user's message, and propose a joint encoding scheme. The main

contributions along these lines as are explained in more detail in the following.

1.1 Contributions

We first consider two-user Gaussian ICs with fast fading and study implementation of all public and Han-Kobayashi coding schemes with LDPC codes. Stability conditions for the studied coding schemes are derived, and a modified form of the extrinsic information transfer (EXIT) chart analysis is implemented to estimate the decoding thresholds of LDPC code ensembles. The proposed code design is employed in several examples and the resulting rate pairs are compared with the ARR boundaries demonstrating that rate pairs very close to the ARR boundaries are attained. Performance of finite block length codes are also studied through simulations of specific codes picked from the optimized LDPC code ensembles in order to verify the analysis. Finally, advantages of the designed codes over point-to-point (P2P) optimal ones are demonstrated via both asymptotic and finite block length simulations. The results of this line of investigation have been published in [5,6].

We next study two-user Gaussian ICs with quasi-static fading and implement different encoding/decoding schemes with LDPC codes. We derive stability conditions on the degree distributions of LDPC code ensembles, and obtain explicit and practical code designs. In order to estimate the outage probability, we utilize a modified form of the EXIT chart analysis. The proposed code design is employed in several examples, which demonstrate that the designed codes perform close to the achievable information theoretic limits. Furthermore, multiple antenna transmissions employing the Alamouti scheme are studied; a special receiver structure is developed, and specific codes are explored. The advantage of the designed codes over P2P optimal ones is observed via finite block length simulations. The results of this line of investigation have been published in [6].

We then investigate CICs. We consider the unfaded scenario, and prompted by the information theoretical results, we design an explicit coding scheme for

CIC in the primary decodes cognitive regime. We present a novel joint decoder and design LDPC codes for our set-up. Simulation results demonstrate that the proposed joint decoder and the designed codes outperform the conventional maximum ratio combining type decoder and the P2P optimal codes, respectively. We next consider a dirty paper channel and develop a flexible modulation technique, and show that by utilizing this simple technique we can have a significant gain due to the knowledge of the interfering signal at the cognitive transmitter. Also, the proposed technique shows more robustness to inaccuracies in channel state information compared to dirty paper coding (DPC) implemented using LDPC codes and trellis coded quantization as the channel and source coding components [7]. We also consider the fading scenario for a particular form of MAC, for which one of the users has non-causal knowledge of the message of the other one. We develop and study the idea of joint encoding at the cognitive transmitter. The numerical results demonstrate that along with the improvement in the bit error rate (BER) performance of the primary user, the technique improves the cognitive user's BER performance for some regimes. Part of the results of this line of investigation have been published in [8].

1.2 Thesis Outline

The rest of this thesis is organized as follows. In Chapter 2, we review the existing literature on information theoretic limits for ICs, and briefly discuss LDPC codes. We also go over the methods used to analyze the performance of LDPC code ensembles. Chapter 3 investigates LDPC code design for fast fading ICs where we specifically study implementation of (explicit) all public and Han-Kobayashi (HK) coding schemes. In Chapter 4, we turn our attention to quasi-static fading ICs, and after developing coding approaches for the single antenna case, we study multiple antenna transmissions employing the Alamouti scheme. In Chapter 5, we examine the existing coding schemes for CIC for the primary decodes cognitive regime, and propose a new decoder structure and design a practical code for the set-up. Furthermore, we investigate simple methods, robust to changes in the channel, that exploit the side information of the other signal available at the

cognitive transmitter. Finally, we summarize our results, and provide conclusions and directions for future research in Chapter 6.



Chapter 2

Preliminaries and Literature Review

In this chapter, we present the system model for a general interference channel, and review the existing information theoretic limits for different coding schemes. Later, we describe the system model for a cognitive interference channel. After discussing several transmission schemes for the CIC, we discuss the known results on the capacity region of Gaussian CICs. Finally, since much of our development relies on low density parity check codes, a brief summary of them is provided.

2.1 Interference Channel Model

Two-user discrete memoryless IC (DMIC) consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, two output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$ and a channel with transition probability $P(y_1, y_2 | x_1, x_2)$ which illustrates the effects of noise and the interference (see Fig. 2.1). A length- n code $(2^{nR_1}, 2^{nR_2}, n)$ consists two sets of messages M_i which are distributed over¹ $[1 : 2^{nR_i}]$, for $i = 1, 2$, and two encoding functions that map

¹ $[a : b]$ denotes the interval of integer numbers between a and b including them, $[a : b] = \{x \in \mathbb{Z} : a \leq x \leq b\}$.

M_i to a codeword X_i^n with coding rate R_i performed independently. At the receivers, two decoding functions that map the observed signal Y_i^n to the message estimates $\hat{M}_i \in [1 : 2^{nR_i}]$ are employed. The average probability of error for the i^{th} transmitter/receiver pair can be defined as

$$P_{e,i}^n = \frac{1}{2^{n(R_1+R_2)}} \sum_{(m_1, m_2)} \Pr(\hat{M}_i(Y_i^n) \neq M_i | M_1 = m_1, M_2 = m_2), \quad i = 1, 2. \quad (2.1)$$

A rate pair (R_1, R_2) is call achievable if there exist an encoding/decoding function with $P_{e,1}^n, P_{e,2}^n \rightarrow 0$ as $n \rightarrow \infty$.

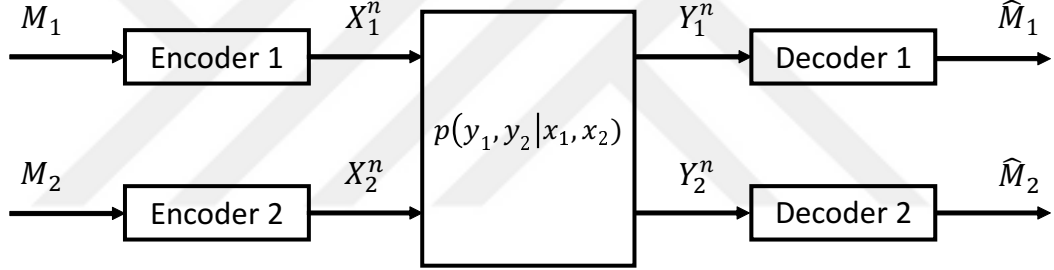


Figure 2.1: General two-user interference channel.

As the communication occurs over a shared channel, the transmission suffers not only from the noise in the channel, but also from the interference due to the other user's transmitted signal. This induces a tradeoff between the rates that the users can reliably communicate.

Gaussian Interference Channel Model: Gaussian interference channel (GIC) depicted in Fig. 2.2 is a particular IC which can be represented by the following set of input-output relationships

$$\begin{aligned} Y_1 &= h_{11}X_1 + h_{21}X_2 + Z_1, \\ Y_2 &= h_{12}X_1 + h_{22}X_2 + Z_2, \end{aligned} \quad (2.2)$$

where h_{ij} is the channel gain between transmitter i and receiver j . The noise term Z_i is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian with zero mean and $\frac{N_0}{2}$ variance per dimension. X_i is the transmitted

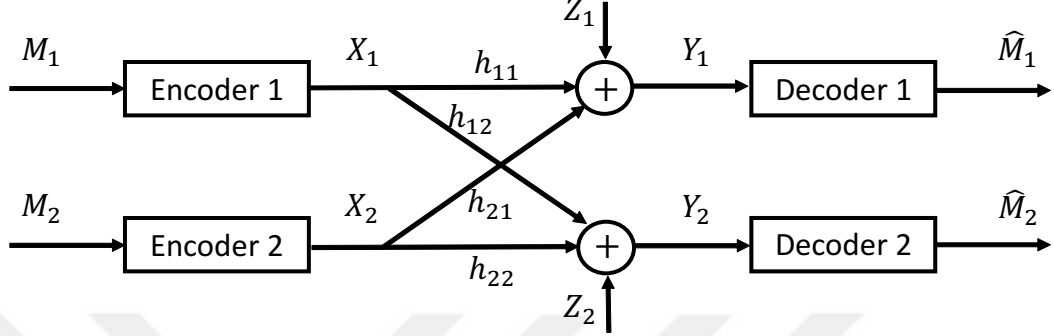


Figure 2.2: Two-user Gaussian interference channel.

signal from transmitter i subject to the power constraint $E\{|X_i|^2\} \leq P_i$, for $i, j = 1, 2$. Signal to noise ratio (SNR) and interference to noise ratio (INR) at receiver i are defined as

$$\begin{aligned} \text{SNR}_i &= \frac{|h_{ii}|^2 P_i}{N_0}, \\ \text{INR}_i &= \frac{|h_{ji}|^2 P_i}{N_0}, \end{aligned} \quad (2.3)$$

where $i, j = 1, 2$ and $i \neq j$. Based on the SNR and INR levels, the interference can be classified as strong ($\text{INR}_i > \text{SNR}_j$), weak ($\text{SNR}_i > \text{INR}_j$), or mixed ($\text{INR}_i > \text{SNR}_j, \text{INR}_j < \text{SNR}_i$).

In the fading scenario, if the channel gains change from one symbol to the next, we have a fast fading IC. If the channel gains change after each codeword transmission and remain constant for the entire codeword, the channel is called quasi-static fading IC. The SNR and INR definitions for the fading scenario are modified as follows:

$$\begin{aligned} \text{SNR}_i &= E \left[\frac{|h_{ii}|^2 P_i}{N_0} \right], \\ \text{INR}_i &= E \left[\frac{|h_{ji}|^2 P_i}{N_0} \right], \end{aligned} \quad (2.4)$$

where $i, j = 1, 2$ and $i \neq j$, where $E[\cdot]$ denotes expectation over the channel statistics.

Interference channels, first introduced by Shannon in 1961 [9], have been receiving significant attention from an information theoretic point of view for decades. In [10] and [11], it is shown that interference does not degrade the performance in the very strong interference regime and the capacity of the channel is the same as that of the P2P channel for each user, i.e.,

$$\begin{aligned} R_1 &< I(X_1; Y_1 | X_2, Q), \\ R_2 &< I(X_2; Y_2 | X_1, Q), \end{aligned} \tag{2.5}$$

for some probability mass function (PMF) $p(q)p(x_1|q)p(x_2|q)$ with $|Q| \leq 2$. The capacity of ICs for some other regimes are also known precisely. These include the strong interference regime [10, 11], classes of degraded ICs [12, 13] and classes of deterministic and semi-deterministic ICs [14, 15]. Some tight inner and outer bounds for different regimes also exist in the literature (see, for example, [16–22]). The best known inner bound for ICs is reported by Han and Kobayashi in [23]. The HK inner bound is demonstrated to be asymptotically optimal at high SNR for GIC [24]. These studies have produced an extensive set of interesting results on various aspects of the problem of communicating over an IC.

2.2 Transmission Over ICs

2.2.1 Time Division

Interference can be avoided simply by time division. The inner bound on the transmission rate by time division with power control, consists of all the rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &\leq \alpha C_1, \\ R_2 &\leq \bar{\alpha} C_2, \end{aligned} \tag{2.6}$$

for $\alpha \in [0, 1]$, which is the ratio of the time that transmitter 1 is active. The ratio of time for which user 2 is active is $\bar{\alpha} = 1 - \alpha$, and C_1 and C_2 are defined as follows

$$\begin{aligned} C_1 &= \max_{p(x_1)} I(X_1, Y | X_2 = x_2), \\ C_2 &= \max_{p(x_2)} I(X_2, Y | X_1 = x_1), \end{aligned} \quad (2.7)$$

For Gaussian signaling, C_1 and C_2 are given by

$$\begin{aligned} C_1 &= C(\text{SNR}_1), \\ C_2 &= C(\text{SNR}_2), \end{aligned} \quad (2.8)$$

where $C(x) = \log_2(1 + x)$. Similarly, ICs can be turned into two P2P Gaussian channels via frequency division.

2.2.2 Treating Interference as Noise

Another simple scheme, which has been used in practice for a long time, is treating interference as noise (TIN). The ARR of this scheme consists of all rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &< I(X_1; Y_1 | Q), \\ R_2 &< I(X_2; Y_2 | Q), \end{aligned} \quad (2.9)$$

for some PMF $p(q)p(x_1|q)p(x_2|q)$, and Q is time sharing random variable [25]. For GIC and without time sharing, the ARR can be characterized as follows [25]

$$\begin{aligned} R_1 &< C\left(\frac{\text{SNR}_1}{1 + \text{INR}_1}\right), \\ R_2 &< C\left(\frac{\text{SNR}_2}{1 + \text{INR}_2}\right). \end{aligned} \quad (2.10)$$

Clearly, as the INRs increase, the ARR of the TIN scheme shrinks. At low INRs, the TIN scheme does not suffer from the interference significantly. It has been

proved that if the interference is weak enough, the structure of the interference is not useful from an information theoretic point of view. In other words, the TIN scheme achieves the maximum possible throughput when the interference parameters are below certain thresholds [26]. It is proved that for the asymmetric Gaussian IC (with real channel coefficients) satisfying

$$\left| \frac{h_{21}}{h_{11}} \left(1 + \left(\frac{h_{12}}{h_{22}} \right)^2 \text{SNR}_1 \right) \right| + \left| \frac{h_{12}}{h_{22}} \left(1 + \left(\frac{h_{21}}{h_{11}} \right)^2 \text{SNR}_2 \right) \right| \leq 1, \quad (2.11)$$

the TIN scheme achieves the sum-capacity given by [26]

$$C_{\text{sum}} = \log_2 \left(1 + \frac{\text{SNR}_1}{1 + \left(\frac{h_{21}}{h_{11}} \right)^2 \text{SNR}_2} \right) + \log_2 \left(1 + \frac{\text{SNR}_2}{1 + \left(\frac{h_{12}}{h_{22}} \right)^2 \text{SNR}_1} \right). \quad (2.12)$$

2.2.3 Simultaneous Unique Decoding

At the other extreme, both receivers decode both messages. The ARR of simultaneous unique decoding (SUD) can be characterized as all the rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &< \min \{ I(X_1; Y_1 | X_2, Q), I(X_1; Y_2 | X_2, Q) \}, \\ R_2 &< \min \{ I(X_2; Y_1 | X_1, Q), I(X_2; Y_2 | X_1, Q) \}, \\ R_1 + R_2 &< \min \{ I(X_1, X_2; Y_1 | Q), I(X_1, X_2; Y_2 | Q) \}, \end{aligned} \quad (2.13)$$

for some PMF $p(q)p(x_1|q)p(x_2|q)$.

Similarly, for the GIC and without time sharing, the ARR can be characterized as follows

$$\begin{aligned} R_1 &< \min \{ C(\text{SNR}_1), C(\text{INR}_2) \}, \\ R_2 &< \min \{ C(\text{INR}_1), C(\text{SNR}_2) \}, \\ R_1 + R_2 &< \min \{ C(\text{SNR}_1 + \text{INR}_1), C(\text{SNR}_2 + \text{INR}_2) \}. \end{aligned} \quad (2.14)$$

As it is observed in Eq. (2.14), the SUD-ARR does not suffer from the interference if the INRs are large enough. In [11], it has been shown that, the SUD inner bound approaches the capacity under the strong interference condition.

A comparison of the achievable rate regions of the time division, treating interference as noise and simultaneous unique decoding is given in Fig. 2.3 for different values of the INR. As it is observed from Fig. 2.3, when the interference is weak, the TIN scheme can outperform the time-division and SUD schemes. As the interference becomes stronger SUD scheme and time division begin to outperform the TIN scheme. As the interference becomes stronger, the SUD scheme outperforms the other two coding schemes.

2.2.4 Han-Kobayashi Coding Scheme

HK scheme can be considered as a scheme in-between TIN and SUD. It provides an inner bound which is the best known bound on the capacity region of the two-user IC [23]. In this scheme, transmitters split their messages into public (denoted by W) and private (denoted by U) parts. The public messages are decoded by both receivers, while the private messages are decoded by the corresponding receivers only (Fig. 2.4). By decoding the public messages, part of the interference is canceled out, and the receiver treats the rest of the interference as noise.

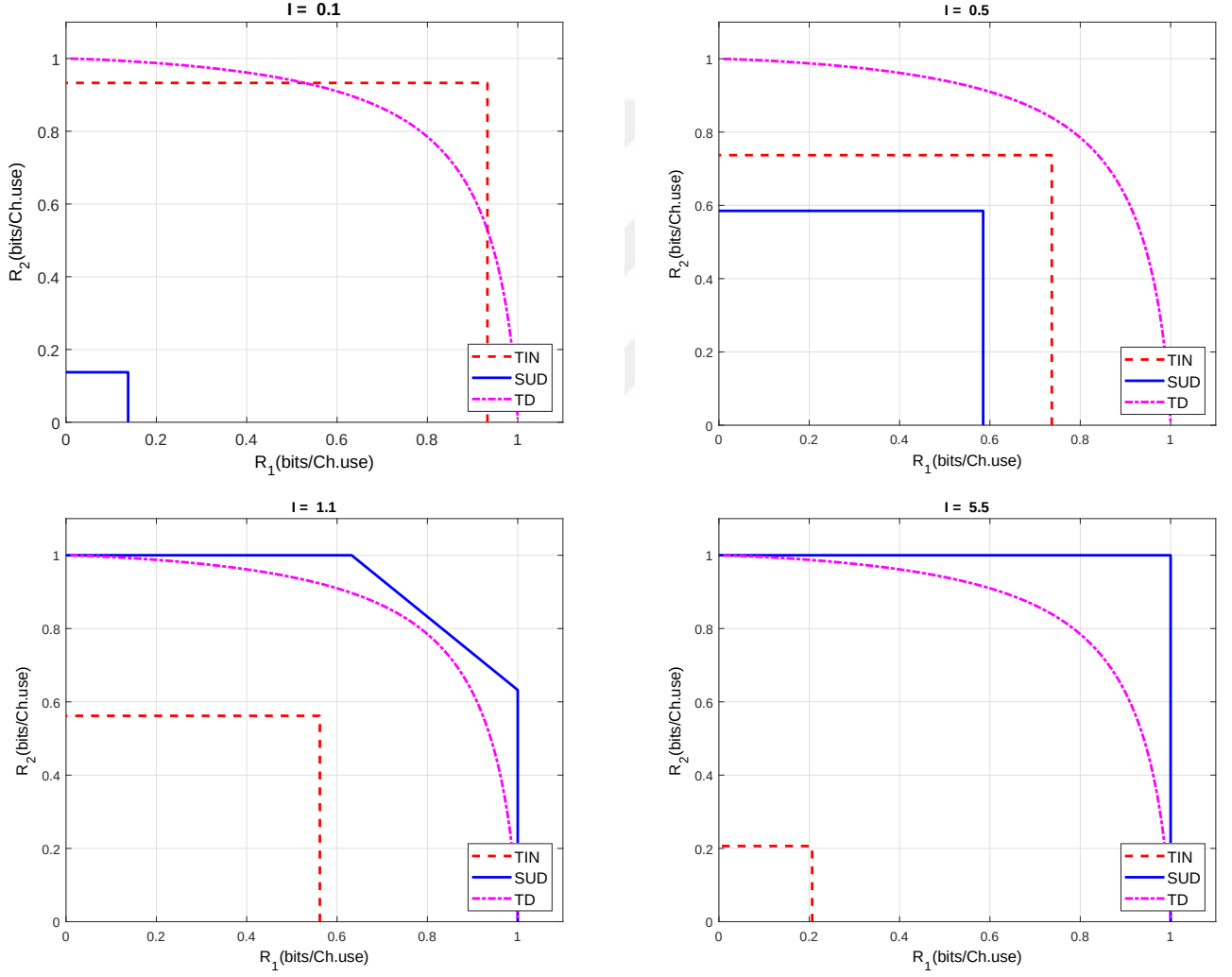


Figure 2.3: Comparison of achievable region by treating noise as interference (region TIN), simultaneous unique decoding (region SUD), and time division (region TD) for $S = 1$ and different values of I .

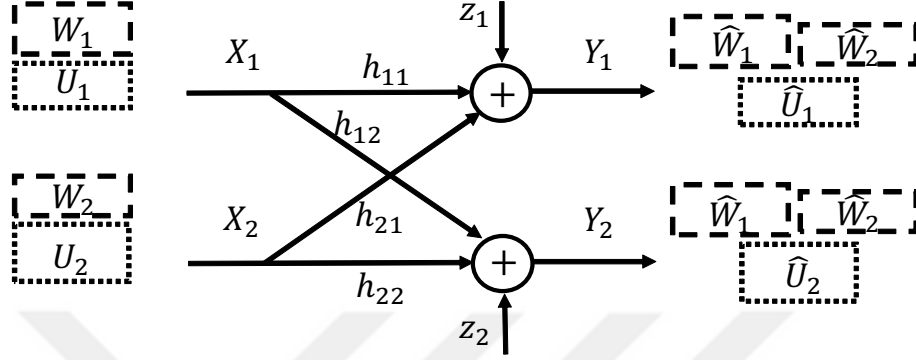


Figure 2.4: HK coding scheme.

The HK inner bound is characterized as follows [27]

$$\begin{aligned}
R_1 &< I(X_1; Y_1 | W_2, Q), \\
R_2 &< I(X_2; Y_2 | W_1, Q), \\
R_1 + R_2 &< I(X_1, W_2; Y_1 | Q) + I(X_2; Y_2 | W_1, W_2, Q), \\
R_1 + R_2 &< I(X_2, W_1; Y_2 | Q) + I(X_1; Y_1 | W_1, W_2, Q), \\
R_1 + R_2 &< I(X_1, W_2; Y_1 | W_1, Q) + I(X_2, W_1; Y_2 | W_2, Q), \\
2R_1 + R_2 &< I(X_1, W_2; Y_1 | Q) + I(X_1; Y_1 | W_1, W_2, Q) + I(X_2, W_1; Y_2 | W_2, Q), \\
R_1 + 2R_2 &< I(X_2, W_1; Y_2 | Q) + I(X_2; Y_2 | W_1, W_2, Q) + I(X_1, W_2; Y_1 | W_1, Q),
\end{aligned} \tag{2.15}$$

for some PMF $p(q)p(w_1, x_1|q)p(w_2, x_2|q)$, where $|W_1| \leq |X_1| + 4$, $|W_2| \leq |X_2| + 4$, Q is time sharing random variable and $|Q| \leq 6$.

For the Gaussian signaling and without time sharing, the ARR of the HK scheme can be characterized as follows

$$\begin{aligned}
R_1 &< C \left(\frac{\text{SNR}_1}{1 + \alpha_2 \text{INR}_1} \right), \\
R_2 &< C \left(\frac{\text{SNR}_2}{1 + \alpha_1 \text{INR}_2} \right), \\
R_1 + R_2 &< C \left(\frac{\text{SNR}_1 + \bar{\alpha}_2 \text{INR}_1}{1 + \alpha_2 \text{INR}_1} \right) + C \left(\frac{\alpha_2 \text{SNR}_2}{1 + \alpha_1 \text{INR}_2} \right), \\
R_1 + R_2 &< C \left(\frac{\text{SNR}_2 + \bar{\alpha}_1 \text{INR}_2}{1 + \alpha_1 \text{INR}_2} \right) + C \left(\frac{\alpha_1 \text{SNR}_1}{1 + \alpha_2 \text{INR}_1} \right), \\
R_1 + R_2 &< C \left(\frac{\alpha_1 \text{SNR}_1 + \bar{\alpha}_2 \text{INR}_1}{1 + \alpha_2 \text{INR}_1} \right) + C \left(\frac{\alpha_2 \text{SNR}_2 + \bar{\alpha}_1 \text{INR}_2}{1 + \alpha_1 \text{INR}_2} \right), \\
2R_1 + R_2 &< C \left(\frac{\text{SNR}_1 + \bar{\alpha}_2 \text{INR}_1}{1 + \alpha_2 \text{INR}_1} \right) + C \left(\frac{\alpha_1 \text{SNR}_1}{1 + \alpha_2 \text{INR}_1} \right) \\
&\quad + C \left(\frac{\alpha_2 \text{SNR}_2 + \bar{\alpha}_1 \text{INR}_2}{1 + \alpha_1 \text{INR}_2} \right), \\
R_1 + 2R_2 &< C \left(\frac{\text{SNR}_2 + \bar{\alpha}_1 \text{INR}_2}{1 + \alpha_1 \text{INR}_2} \right) + C \left(\frac{\alpha_2 \text{SNR}_2}{1 + \alpha_1 \text{INR}_2} \right) \\
&\quad + C \left(\frac{\alpha_1 \text{SNR}_1 + \bar{\alpha}_2 \text{INR}_1}{1 + \alpha_2 \text{INR}_1} \right),
\end{aligned}$$

where α_i is the ratio of the power allocated to the private message at transmitter i and $\bar{\alpha}_i = 1 - \alpha_i$, for $i = 1, 2$.

2.3 Other Signaling Approaches for GIC

In addition to the aforementioned schemes, some other signaling schemes have also been developed in the literature. These schemes are important due to their practicality and/or efficiency. In the following we review some of them.

2.3.1 Channel Capacity Within Half Bit

Although the HK scheme is very natural, it is not clear how to split the messages and how much power to allocate to each part. The required optimization makes it difficult to achieve the HK bound in practice. Etkin et al. [24] propose a special

case of power allocation to the public and private messages and show it achieves the capacity of the two-user GIC within half a bit per Hertz per dimension for all set of channel parameters. In this scheme, the powers allocated to private messages are such that these messages are received at the level of Gaussian noise at the unintended receivers. This scheme is especially interesting in the high SNR regime where it is asymptotically optimal. The ARR of this scheme for Gaussian signaling and no time sharing can be characterized as follows

$$\begin{aligned}
R_1 &< C\left(\frac{\text{SNR}_1}{2}\right), \\
R_2 &< C\left(\frac{\text{SNR}_2}{2}\right), \\
R_1 + R_2 &< C\left(\frac{\text{SNR}_1 + \text{INR}_1 - 1}{2}\right) + C\left(\frac{\alpha_2 \text{SNR}_2}{2}\right), \\
R_1 + R_2 &< C\left(\frac{\text{SNR}_2 + \text{INR}_2 - 1}{2}\right) + C\left(\frac{\alpha_1 \text{SNR}_1}{2}\right), \\
R_1 + R_2 &< C\left(\frac{\alpha_1 \text{SNR}_1 + \text{INR}_1 - 1}{2}\right) + C\left(\frac{\alpha_2 \text{SNR}_2 + \text{INR}_2 - 1}{2}\right), \\
2R_1 + R_2 &< C\left(\frac{\text{SNR}_1 + \text{INR}_1 - 1}{2}\right) + C\left(\frac{\alpha_1 \text{SNR}_1}{2}\right) \\
&\quad + C\left(\frac{\alpha_2 \text{SNR}_2 + \text{INR}_2 - 1}{2}\right), \\
R_1 + 2R_2 &< C\left(\frac{\text{SNR}_2 + \text{INR}_2 - 1}{2}\right) + C\left(\frac{\alpha_2 \text{SNR}_2}{2}\right) \\
&\quad + C\left(\frac{\alpha_1 \text{SNR}_1 + \text{INR}_1 - 1}{2}\right), \tag{2.16}
\end{aligned}$$

where $\alpha_i = \frac{1}{\text{INR}_j}$ and $j \neq i$.

2.3.2 Interference Alignment

Interference alignment is one of interference management methods, which is introduced for ICs with more than two users in [28]. In this technique, the transmitted signals are linearly precoded over multiple dimensions (such as time, frequency or space) such that the interfering signals are observed at each receiver projected

onto a low dimensional subspace. Therefore, it leaves some dimensions free of interfering signals that can be used for desired message transmission [28–30]. However, the main drawback factor of this method is that, perfect global channel side information is required at all the transmitters, which may not be very practical and may limit its application. A modified form of this method for the imperfect channel state information case has been studied in [31–33]. Interference alignment also has been developed for cellular networks [34].

An example of interference alignment is demonstrated in Fig. 2.5. As it is observed in Fig. 2.5, by using the precoders F , the two interfering signals, observed at each receiver, are aligned onto a single dimension and the other dimension is free of interference.

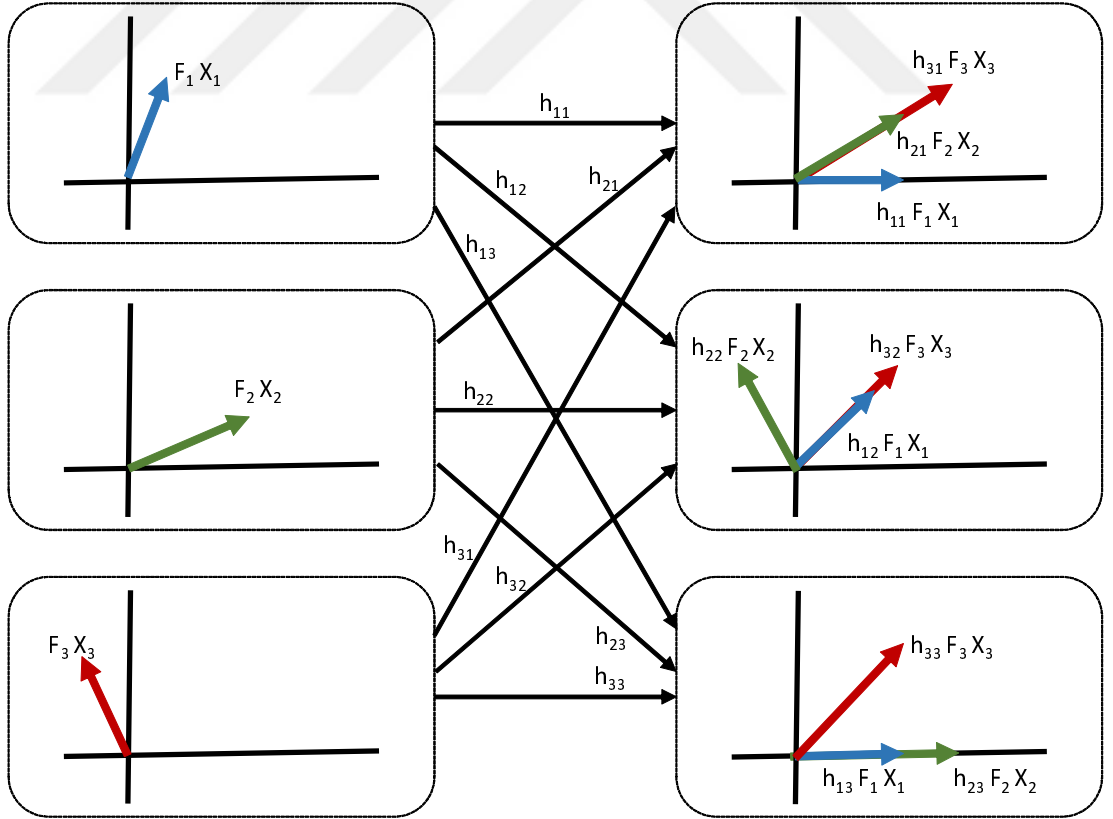


Figure 2.5: Interference alignment for three-users and two dimensions. Three precoders F_i are chosen by three equations $\text{span}(h_{ji}F_jX_j) = \text{span}(h_{ki}F_kX_k)$, for $i \neq j \neq k \in \{1, 2, 3\}$.

Consider a K -user interference channel in which the signal of user j denoted by X_j are precoded by matrix F_j and observed at the receiver i can be represented as

$$Y_i = h_{i,i}F_iX_i + \sum_{j \neq i} h_{ji}F_jX_j + Z_i, \quad (2.17)$$

where $h_{i,j}$ are the channel gains. Interference alignment algorithm chooses a set of precoders F_i such that the receivers, by employing a simple linear transformation W_i , can cancel the interference signals, i.e., $W_i^*H_{j,i}F_l = 0, \forall j \neq i$, without nulling or destroying its desired signal $W_i^*H_{i,i}F_i$. The problem of the feasibility of interference alignment has been studied extensively, see, for example, [35, 36].

2.4 Cognitive Interference Channel Model

Cognitive interference channel is a variant of the classical IC, in which the cognitive transmitter (user 1) has non-causal knowledge of the primary (user 2) message (Fig. 2.6). This model can be justified in practice when the primary message is public and it is available to the other transmitters, e.g., due to a good link from the primary transmitter to the cognitive transmitter. The cognitive transmitter can use this additional information to perform precoding subject to non-causal knowledge of the interference. It can also help the transmission of the primary message.

Similar to the the classical IC model, the observed symbols Y_1, Y_2 drawn according to the conditional probability distribution $P(y_1, y_2 | x_1, x_2)$ which indicate the effects of noise and interference. However, by providing the additional information to the cognitive transmitter, the transmitters have a kind of cooperation in this model. A $(2^{nR_1}, 2^{nR_2}, n)$ code consists of two messages $M_i \in [1 : 2^{nR_i}], i = 1, 2$, and the encoding function of the primary user generates the codeword X_2^n for the the input M_2 while the encoding function of the cognitive user generates the codeword X_1^n according to both messages (M_1, M_2) . Also at the receivers, two decoding functions that map the observed signal Y_i^n to the

estimated message $\hat{M}_i \in [1 : 2^{nR_i}]$ are employed. The average probability of error at receiver i is defined as

$$P_{e,i}^n = \frac{1}{2^{n(R_1+R_2)}} \sum_{(m_1, m_2)} Pr(\hat{M}_i(Y_i^n) \neq M_i | M_1 = m_1, M_2 = m_2), \quad i = 1, 2. \quad (2.18)$$

A rate pair (R_1, R_2) is call achievable if there exist an encoding/decoding function with $P_{e,1}^n, P_{e,2}^n \rightarrow 0$ as $n \rightarrow \infty$.

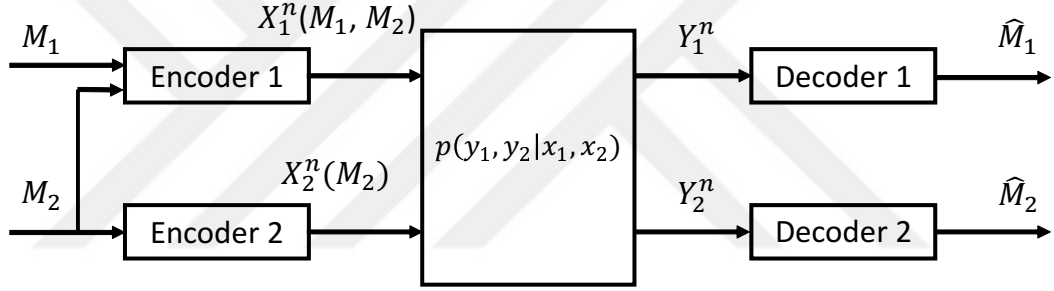


Figure 2.6: General two-user cognitive interference channel.

Gaussian Cognitive Interference Channel Model: Similar to IC, a two-user Gaussian CIC in standard form can be represented as follows (Fig. 2.7):

$$\begin{aligned} Y_1 &= h_{11}X_1 + h_{21}X_2 + Z_1, \\ Y_2 &= h_{12}X_1 + h_{22}X_2 + Z_2. \end{aligned} \quad (2.19)$$

However, in the literature, the two-user (unfaded) Gaussian CIC usually is represented in an equivalent model as

$$\begin{aligned} Y_1 &= X_1 + aX_2 + Z_1, \\ Y_2 &= bX_1 + X_2 + Z_2, \end{aligned} \quad (2.20)$$

where a and b are the constant channel gains, known by all the terminals. The noise term z_i is i.i.d. circularly symmetric complex Gaussian with zero mean and $\frac{N_0}{2}$ variance per dimension. X_i is the transmitted signal from transmitter i subject to the power constraint $E\{|X_i|^2\} \leq P_i$, for $i = 1, 2$. Several specific cases

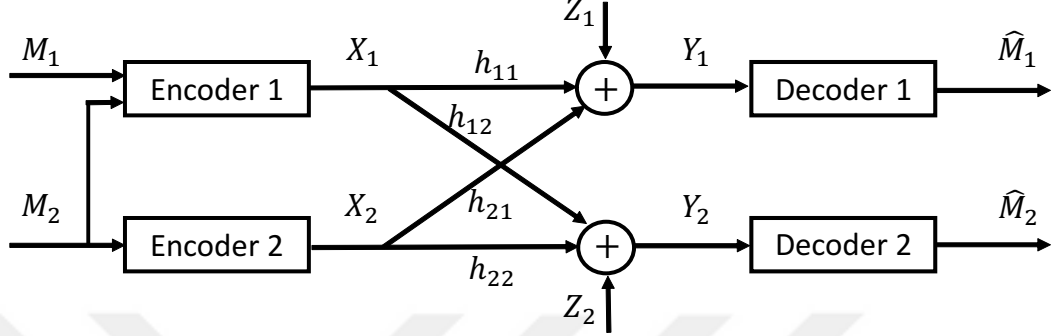


Figure 2.7: Two-user Gaussian cognitive interference channel.

of CIC have been studied in the literature:

Z-channel: if $|b| = 0$, which means that the primary receiver does not experience interference. The capacity region of the Z-channel is given by [37]

$$\begin{aligned} R_1 &= C(\text{SNR}_1), \\ R_2 &= C(\text{SNR}_2). \end{aligned} \tag{2.21}$$

S-channel: if $a = 0$ which means that the cognitive receiver does not experience interference. The capacity of this channel is known only in the weak interference regime ($|b| \leq 1$) [38].

Degraded channel: if $a|b| = 1$. In this case the one channel output is a degraded version of the other one. The capacity of degraded channel is only known in the weak interference regime [38].

As the CIC encompasses IC, MAC and broadcast channel (BC) as special cases, a mixture of the transmission techniques developed with these channel models can be used.

Rate splitting: Similar to the HK scheme in IC, both transmitters can split their messages to private and public parts. However, it has been shown that this technique is unneeded for some channel parameters [39] [40].

Superposition coding: the aim of superposition coding is to encode two messages in the same signal in two layers. Then, the better receiver can recover the messages on both layers, while the worse receiver can recover the message on the coarse layer. Superposition coding achieves the capacity on a Gaussian broadcast channel [41].

Binning: Gel'fand–Pinsker coding, also known as dirty paper coding for Gaussian channels, enables the cognitive transmitter to “pre-cancel” a portion of the interference known to be experienced at the cognitive receiver.

DPC first proposed in [42], where Costa proved that by providing the interference as side information to the transmitter (Fig. 2.8), the capacity of the channel is the same as the case with no interference. Therefore, the interference does not deteriorate the capacity of the channel, even though it is unknown to the receiver. While Costa proved this result for Gaussian channels, it has also been extended to more general cases in [43].

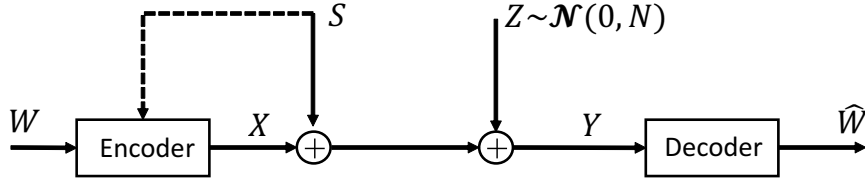


Figure 2.8: Dirty paper coding.

DPC has been utilized in different applications in multiple-input multiple-output (MIMO) broadcast channel [44], data hiding [45, 46], precoding for interference channel [47], and cooperative diversity schemes [48].

2.5 Gaussian CIC Capacity

Providing the primary user’s message to the cognitive transmitter effects the capacity of the Gaussian CIC in such a way that it contains features of both the

interference and the broadcast channels. The capacity of the Gaussian CIC is known only for some special cases; for other cases, some inner and outer bounds have been derived [39, 49–53]. The cases with known capacity regions include the “weak interference” regime, the “very strong interference” regime, and the “primary decodes cognitive” regime. In the following, these regimes and their capacity regions are given [53].

2.5.1 The Weak Interference Regime

The weak interference regime in Gaussian CIC corresponds to the regime in which the cognitive receiver enjoys a better link than the primary receiver for the cognitive transmitter, i.e.,

$$\frac{|b|}{\sqrt{N_2}} \leq \frac{1}{\sqrt{N_1}}. \quad (2.22)$$

The capacity region of the CIC in the weak interference regime can be characterized as [40, 54]

$$\begin{aligned} R_1 &\leq C\left(\frac{\alpha P_1}{N_1}\right), \\ R_2 &\leq C\left(\frac{(|b|\sqrt{\bar{\alpha}P_1} + \sqrt{P_2})^2}{N_2 + |b|^2\alpha P_1}\right), \end{aligned} \quad (2.23)$$

where $\alpha \in [0, 1]$, $\bar{\alpha} = 1 - \alpha$. The capacity achieving strategy for this regime is that, the cognitive transmitter applies DPC against the total interference caused by the primary transmitter at the cognitive receiver. The cognitive receiver applies DPC decoder to decode its own message while the primary receiver treats interference as noise. Therefore, in this strategy, the receivers only decode their own messages, and as a result P2P optimal codes perform well.

2.5.2 The Very Strong Interference Regime

The very strong interference regime refers to the regime in which the channel parameters can be expressed by the following inequalities

$$\begin{aligned} \frac{|b|}{\sqrt{N_2}} &\geq \frac{1}{\sqrt{N_1}}, \\ \left(\frac{P_1}{N_1} - \frac{P_1 b^2}{N_2}\right) + \left(\frac{P_2 a^2}{N_1} - \frac{P_2}{N_2}\right) - \left|\frac{a}{N_1} - \frac{b}{N_2}\right| 2\sqrt{P_1 P_2} &\geq 0. \end{aligned} \quad (2.24)$$

The capacity region of the CIC in the very strong interference regime is given by [55]

$$\begin{aligned} R_1 &\leq C\left(\frac{\alpha P_1}{N_1}\right), \\ R_2 &\leq C\left(\frac{|b|^2 P_1 + P_2 + 2\sqrt{\alpha}|b|^2 P_1 P_2}{N_2}\right), \end{aligned} \quad (2.25)$$

where $\alpha \in [0, 1]$. The capacity of the channel in this regime reduces to that of the compound MAC and the capacity achieving strategy is to use superposition coding at the cognitive transmitter, and let both receivers decode both messages.

2.5.3 The Primary Decodes Cognitive Regime

The last regime that the channel capacity is known is the primary decodes cognitive regime. The channel parameters in this regime can be expressed as follows

$$\begin{aligned} \frac{|b|}{\sqrt{N_2}} &\geq \frac{1}{\sqrt{N_1}}, \\ \frac{\alpha P_1 N_2 b^{-2}}{\alpha P_1 + N_2 b^{-2}} - \frac{\alpha P_1 N_1}{\alpha P_1 + N_1} + P_2 &\left(\left(\frac{\alpha P_1 b^{-1}}{\alpha P_1 + N_2 b^{-1}} - \frac{\alpha P_1 a}{\alpha P_1 + N_1}\right) \right. \\ &\left. + \sqrt{\frac{\alpha P_1}{P_2}} \left(\frac{\alpha P_1}{\alpha P_1 + N_2 b^{-2}} - \frac{\alpha P_1}{\alpha P_1 + N_1}\right)\right)^2 \geq 0, \end{aligned} \quad (2.26)$$

where $0 \leq \alpha \leq 1$. The capacity region of the Gaussian CIC in the primary decodes cognitive regime is [38]

$$\begin{aligned} R_1 &\leq C\left(\frac{\alpha P_1}{N_1}\right), \\ R_2 &\leq C\left(\frac{|b|^2 P_1 + P_2 + 2\sqrt{\alpha}|b|^2 P_1 P_2}{N_2}\right), \end{aligned} \quad (2.27)$$

where $\alpha \in [0, 1]$. The optimal coding strategy for this regime is as follows: the cognitive transmitter performs DPC against the total interference caused by the primary transmitter, while the primary transmitter uses conventional channel coding to generate X_2 . At the receiver side, the cognitive receiver decodes its message by utilizing a DPC decoder, and the primary receiver performs joint decoding of the both messages.

A plot of the known capacity regions for Gaussian CIC for $P_1 = P_2 = 10$, $N_1 = N_2 = 1$ is depicted in Fig. 2.9. As it is observed from the figure, part of the very strong interference regime has an overlap with the primary decodes cognitive regime, which means that both coding schemes are achieve the capacity for those set of channel parameters.

2.6 Review of Low Density Parity Check Codes

LDPC codes are error correcting codes introduced by Gallager [56]. However, due to the complexity of encoding and decoding schemes, they were almost forgotten for some decades until MacKay reintroduced them in [57]. They offer excellent performance for many communication channels. For instance, they are shown to approach the capacity of the binary-input additive white Gaussian noise (BI-AWGN) channel within a tiny fraction of a decibel away from the channel capacity [58].

LDPC codes can be described through their parity check matrices (denoted by $\mathbf{H}$). The parity check matrices of LDPC codes contain only a very small number

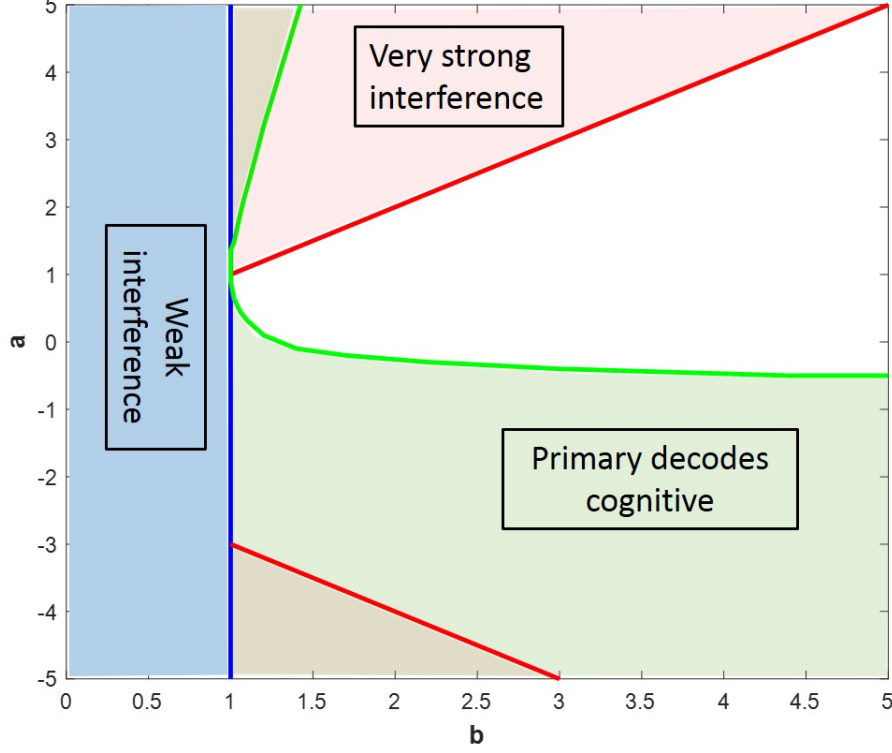


Figure 2.9: Representation of the regions for $P_1 = P_2 = 10$ and $N_0 = 1$.

of non-zero entries which enables efficient iterative decoding. An LDPC code is called regular if the $\mathbf{H}$ matrix has constant column weight and also constant row weight. If the column and row weights are not constant, the code is an irregular LDPC code.

LDPC codes can be completely represented by their Tanner graph. A Tanner graph is a bipartite graph that contains two types of nodes, called variable nodes (VNs) and check nodes (CNs), and edges connecting only the nodes of different types (see Fig. 2.10). In a Tanner graph, the node VN i is connected to CN j if and only if the i^{th} row and the j^{th} column of the matrix $\mathbf{H}$ is 1. Following the notation in [59], the degree distribution of edges connected to VNs is denoted by $\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1}$, and similarly $\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1}$ represents the degree distribution of edges connected to CNs, where d_v is the maximum degree of

variable nodes, and d_c is the maximum of degree of check nodes. The code rate of LDPC codes can be expressed by

$$r = 1 - \frac{\int_0^1 \rho(x) dx}{\int_0^1 \lambda(x) dx}. \quad (2.28)$$

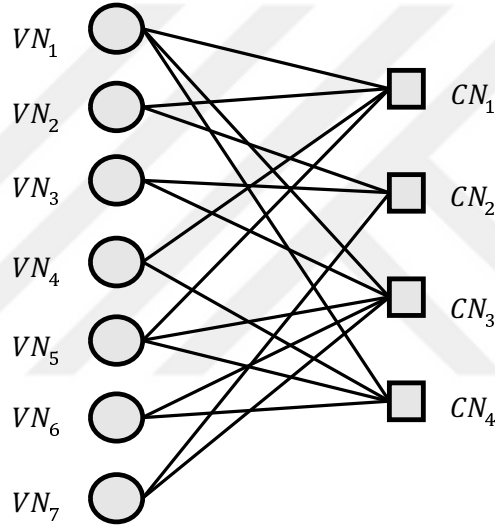


Figure 2.10: The Tanner graph of a LDPC code.

2.6.1 Sum-Product Algorithm

The class of algorithms employed for decoding LDPC codes are called message passing algorithms, since their operations are on messages passing along the edges of a Tanner graph. The sum-product algorithm is a soft decision message passing algorithm that calculates the log-likelihood-ratios (LLRs) of the message and coded bits, and passes them to the other nodes connected by an edge. The extrinsic message $L_{j \rightarrow i}$ from VN j to CN i , which is connected to VN j , is calculated as follows [56]

$$L_{j \rightarrow i} = L_j + \sum_{i' \in N(j) \setminus \{i\}} L_{i' \rightarrow j}, \quad (2.29)$$

where L_j denotes the LLR value computed from the channel sample $L_j = L(c_j|y_j) = \log \left(\frac{Pr(c_j=0|y_j)}{Pr(c_j=1|y_j)} \right)$, and $N(j)$ represents the CNs connected to the VN j . On the other side, the LLR to be sent from CN i to VN j is calculated as follows

$$L_{i \rightarrow j} = 2 \tanh^{-1} \left(\prod_{j' \in N(i) \setminus \{j\}} \tanh \left(\frac{1}{2} L_{j' \rightarrow i} \right) \right), \quad (2.30)$$

where $N(i)$ represents the VNs connected to the CN i . At the last iteration, VNs generate a decision based on the incoming LLRs as follows

$$L_j^{total} = L_j + \sum_{i \in N(j)} L_{i \rightarrow j}. \quad (2.31)$$

The estimated codebit j is 0 if $L_j^{total} > 0$ and it is 1 in the other case.

2.6.2 Decoding Threshold Analysis

LDPC codes show a threshold effect which determines when the decoding error probability can be arbitrary small in terms of the channel parameters. In this section, some techniques introduced in the literature in order to determine the decoding threshold of a given LDPC code ensemble are discussed.

2.6.2.1 Density Evolution Analysis

Density evolution (DE) is the most accurate technique that can be used to calculate the decoding threshold of a code ensemble by tracking analytically the probability density function (PDF) of the exchanged LLRs between the VNs and CNs [60]. Assume that the all-zero codeword is transmitted over a binary-input symmetric channel. The PDF of the outgoing LLRs from the VNs at the l^{th} iteration can be expressed as follows

$$P_l^{(v)} = P_0 * \sum_{d=1}^{d_v} \lambda_d \cdot (P_{l-1}^{(c)})^{*(d-1)}, \quad (2.32)$$

where $*$ denotes the convolution operation, and $P_{l-1}^{(c)}$ denotes the PDF of the LLRs outgoing from the CNs at the iteration $l-1$. P_0 is the PDF of the channel observation.

Let us define $\Phi(x) = \Phi^{-1}(x) = -\ln \tanh(x/2)$. The PDF of the outgoing LLRs from the CNs at the l^{th} iteration can be represented as follows

$$P_l^{(c)} = \Gamma^{-1} \left(\sum_{d=1}^{d_c} \rho_d \cdot \left(\Gamma \left(P_l^{(v)} \right)^{*(d-1)} \right) \right), \quad (2.33)$$

where $\Gamma(\cdot)$ corresponds to the “change of density” due to the transformation $\Phi(\cdot)$. Substitution of (2.33) into (2.32) then gives

$$P_l^{(v)} = P_0 * \sum_{d=1}^{d_v} \lambda_d \cdot \left(\Gamma^{-1} \left(\sum_{d=1}^{d_c} \rho_d \cdot \left(\Gamma \left(P_{l-1}^{(v)} \right)^{*(d-1)} \right) \right) \right)^{*(d-1)}. \quad (2.34)$$

The DE recursion (2.34) describes how the PDF of LLRs passing from the VNs evolves as a function of channel parameter, iteration number, VNs and CNs degree distributions. After some iterations, if $P_l^{(v)}(x) = 0$ for all $x < 0$ we say that the LDPC code ensemble can be reliably decoded (recall that all zero codeword has been assumed).

2.6.2.2 EXIT Chart Analysis

EXIT chart analysis is a prominent technique for estimating the decoding thresholds of LDPC code ensembles [61]. In this technique, the exchanged LLRs are assumed to have symmetric Gaussian distributions with a single parameter. Hence, the mutual information between the transmitted bits and the LLRs passed to the check nodes can be calculated as

$$I_{v \rightarrow c} = \sum_i \lambda_i J \left((i-1) J^{-1}(I_{c \rightarrow v}) + J^{-1}(I_{ch}) \right), \quad (2.35)$$

where $I_{c \rightarrow v}$ and I_{ch} represent the mutual informations of the LLRs from the check nodes to the variable nodes and the mutual informations of the LLRs from the channel with the code bits, respectively. $J(x)$ is defined as

$$J(x) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi x}} \exp\left(-\frac{(l - \frac{x}{2})^2}{2x}\right) \log_2(1 + e^{-l}) dl.$$

In addition, the mutual information of the outgoing LLRs from the check nodes

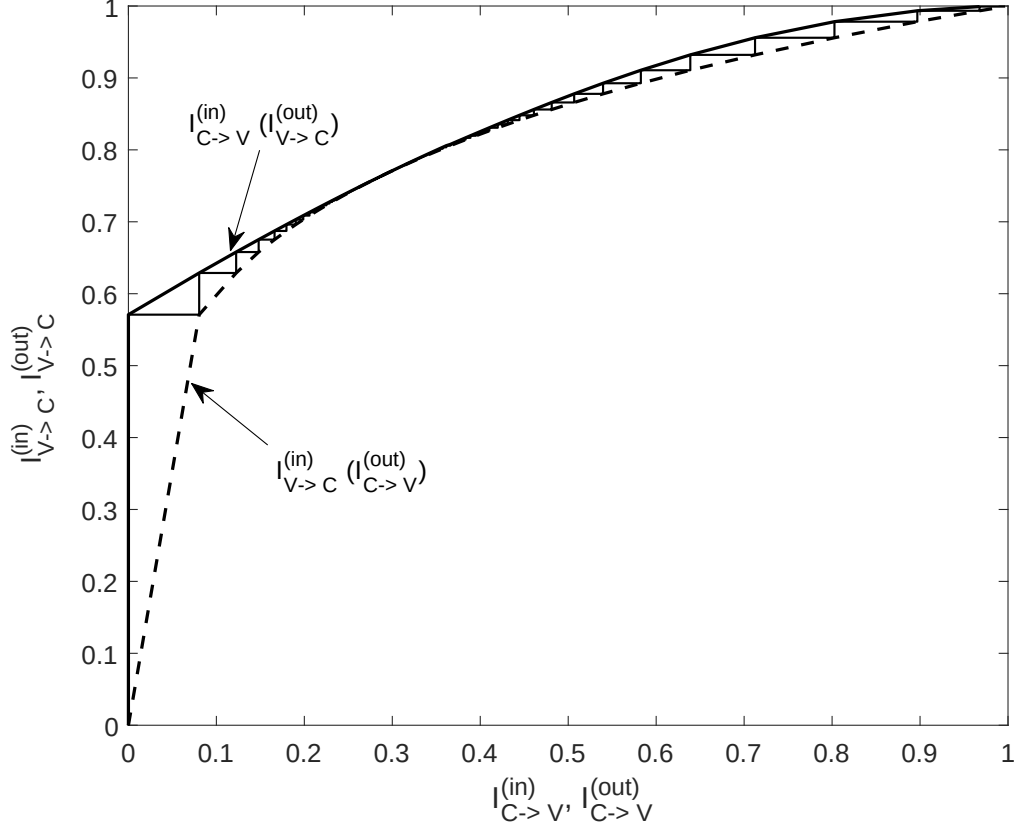


Figure 2.11: An EXIT chart analysis example for the $(d_v, d_c) = (3, 6)$ regular LDPC code ensemble at SNR=1.102 dB.

to the variable nodes with the code bits can be calculated as

$$I_{c \rightarrow v} = \sum_i \rho_i \left(1 - J((i-1)J^{-1}(1 - I_{v \rightarrow c})) \right), \quad (2.36)$$

$$I_{v \rightarrow s} = \sum_i \tilde{\lambda}_i J(iJ^{-1}(I_{c \rightarrow v})), \quad (2.37)$$

where $\tilde{\lambda}(x)$ stands for the variable node degree distribution polynomial from the node perspective [62]. If after some iterations $I_{v \rightarrow c}$ and $I_{c \rightarrow v}$ in the recursive equations (2.35)-(2.37), converges to 1, shows that the LDPC code ensemble can be reliably decoded. Fig. 2.11 demonstrates the values of $I_{c \rightarrow v}$ and $I_{v \rightarrow c}$ after

each iteration for a regular LDPC code with $(d_v, d_c) = (3, 6)$ at $\text{SNR} = 1.102\text{dB}$. As it is observed in the figure, after some iterations $I_{c \rightarrow v}$ and $I_{v \rightarrow c}$ converge to 1, which shows that the code ensemble can be decoded at that SNR.

2.6.2.3 Stability Condition Analysis

An important part of the asymptotic analysis of LDPC codes involves the analysis of the convergence of the code to a zero error state when a low error probability of error has already been achieved. This is essential for eliminating error floor. In the literature, this is called the stability condition analysis.

In [63], it has been shown that for a degree distributions pair (λ, ρ) and a symmetric density P_0 , and assuming that $\int_{\mathbb{R}} \exp(sx) d(P_0)(x) < \infty$, for all s in some neighborhood of zero, if $\lambda'(0)\rho'(1) > \exp(r)$ then there exists a constant $\zeta > 0$, such that for all $l \in \mathbb{N}$, $P_e(P_l) > \zeta$, where $r = -\ln(\int_{\mathbb{R}} P_0(x) \exp(-\frac{x}{2}) dx)$ and P_e is the ratio of erroneously detected bits. The condition on $\lambda'(0)\rho'(1)$ is called stability condition and it can easily shown that for BI-AWGN, the stability condition is $\lambda'(0)\rho'(1) < \exp(-\text{SNR})$.

2.7 Chapter Summary

In this chapter, we have described the system models for ICs and CICs, and reviewed the related literature from an information theoretic point of view. LDPC codes are also briefly explained and some techniques to find the decoding threshold of code ensembles are described. We note that although there are extensive information theoretic results on these settings, a very limited amount of work has been done on development of practical coding solutions, which motivates us to study this problem further as detailed in the subsequent chapters.

Chapter 3

Code Design for Fast Fading Interference Channels

In this chapter, we consider LDPC code design for fast fading two-user ICs. We implement the HK coding scheme and propose an explicit and practical LDPC code design. We propose a modified form of EXIT chart analysis, and also derive the stability condition for the LDPC code ensembles. The proposed code design is employed in several examples, and the obtained rate pairs are compared with the ARR boundaries demonstrating that rate pairs very close to the information theoretic bounds are attained. Performance of finite block length codes are also studied through simulations of specific codes picked from the optimized LDPC code ensembles in order to verify the analysis.

This chapter is organized as follows. In Section 3.1, we review the existing studies on LDPC code design for different multiuser channels in the literature. In Section 3.2, we present the system model. In Section 3.3, we derive the stability condition for joint decoding of LDPC codes over fast fading ICs. We also present our modified EXIT chart analysis to estimate the decoding threshold of the LDPC code ensembles when two public and one private messages are jointly decoded. In Section 3.4, we elaborate on the code optimization procedure utilizing random perturbations. In Section 3.5, examples of achieved rate pairs and finite block

length simulation results are provided. Finally, we conclude the paper in Section 3.6.

3.1 Introduction

Despite a rich set of information theoretical results, there are only a limited number of investigations in the literature regarding practical (explicit and implementable) coding schemes for interference channels. In [64], the authors design LDPC codes for the Gaussian IC by assuming symmetric rate pairs and partial joint decoding. In [65], the authors present a concrete coding scheme to approach the theoretically achievable rate pairs by using LDPC codes concatenated with convolutional codes. Implementation of the HK coding scheme for a general GIC based on LDPC codes is studied in [4] where the authors perform decoding threshold analysis by utilizing Monte Carlo simulations, and show that with their proposed code design method, rate pairs close to the HK inner bounds can be attained. Furthermore, the authors in [66] propose a polar coding scheme for ICs, and show theoretically that a fully joint decoder for the HK coding scheme can be simplified by employing partial joint decoders at the receivers, which is more favorable to polar coding. However, all of these studies are restricted to the case of fixed channel gains, and to the best of our knowledge, there is no work on code design for fading ICs in the literature.

Motivated by the relevance of fading ICs to wireless communications of multiple users utilizing the same spectrum, in this paper, we focus on designing practical coding schemes for the two-user IC with fast fading. In particular, due to their special features, we consider LDPC codes as the channel coding solution. LDPC codes are expected to be widely used in the 5th-generation wireless systems due to their adaptability to the new demands in wireless communications such as high throughput and wide range of block lengths (100-8k information bits) and a variety of coding rates [67]. LDPC codes offer performance very close to the channel capacity for P2P communications [63]. We also note that while there are

many LDPC code constructions for fading channels [68], designs with joint decoding of interfering messages are very scarce. As LDPC code constructions are sensitive to the decoder architecture, new constructions are needed under joint decoding with a nonlinear state node. Towards this goal, the authors in [1] utilize LDPC codes in fast fading broadcast channels by employing superposition coding and joint decoding at the better receiver, and they show that the new designs perform close to the boundary of the achievable rate region. The need for new LDPC code designs for fading MACs are also reported [62].

In our development of LDPC code design for fast fading ICs, we first elaborate on the information theoretical limits, and derive stability conditions for the adopted iterative decoder. We then utilize random perturbations for code search and perform decoding threshold analysis with EXIT charts relying on a binary erasure channel (BEC) approximation for the state nodes. We perform code design for several instances of fast fading ICs. For SUD, we demonstrate a performance close to the asymptotic achievable SNR limits. With HK encoding, we design new code ensembles, which perform beyond the limits of TIN and SUD schemes clearly illustrating the need for HK based practical channel codes for fading ICs.

3.2 System Model

A two-user fading IC is represented by the set of equations

$$\begin{aligned} Y_1 &= h_{11}X_1 + h_{21}X_2 + z_1 \\ Y_2 &= h_{12}X_1 + h_{22}X_2 + z_2, \end{aligned} \tag{3.1}$$

where X_i is the transmitted signal from the transmitter i subject to the power constraint $E\{|X_i|^2\} \leq P_i$, h_{ij} is the channel gain between the transmitter i and receiver j , Y_j is the signal at the receiver j , for $i, j = 1, 2$. The noise term z_j is i.i.d. circularly symmetric complex Gaussian with zero mean and $\frac{N_0}{2}$ variance per dimension. We assume that the channel gains are mutually independent i.i.d. zero

mean complex Gaussian distributed as well (i.e., Rayleigh fading). The channel coefficients change independently from one symbol to the next. We use the signal to noise ratio and interference to noise ratio definitions $\text{SNR}_i = E\{\frac{\|h_{ii}\|^2 P_i}{N_0}\}$ and $\text{INR}_i = E\{\frac{\|h_{ji}\|^2 P_j}{N_0}\}$ with $i, j = 1, 2, i \neq j$, respectively. Without loss of generality, we take $P_1 = P_2 = 1$ and $N_0 = 1$.

With HK type encoding, the message is split into private and public parts at each transmitter. The private (U_i) and public (W_i) messages are separately encoded by the component LDPC codes and transmitted via binary phase shift keying (BPSK), i.e., 0 is transmitted as +1 and 1 is transmitted as -1. The transmitter sends a linear superposition (sum) of the modulated signals. In other words,

$$X_i = \sqrt{\alpha_i} X_{u_i} + \sqrt{1 - \alpha_i} X_{w_i}, \quad i = 1, 2,$$

where $\alpha \in [0, 1]$ is a parameter employed for adjusting the power allocated to the private message. Fig. 3.1 shows a detailed block diagram of the transmitter under consideration.

At each receiver, we employ a joint decoder with three component LDPC decoders that work in parallel as depicted in Fig. 3.2. The LLRs are exchanged by the help of the state nodes, which show the dependencies of the transmitted signals from both users to the channel output.

At receiver j , the outgoing LLR from the state node corresponding to the i^{th} bit of the message k is calculated as [4]

$$L(c_k(i), Y_j(i)) = \log \left(\frac{\sum_{C_i \in S_i^{k+}} f_{Y_j}(Y_j(i)|C_i) P(C_i)}{\sum_{C_i \in S_i^{k-}} f_{Y_j}(Y_j(i)|C_i) P(C_i)} \right), \quad (3.2)$$

where $c_k(i)$ represents the i^{th} coded bit of the message, which can be public or the intended private one. C_i is the vector consisting of the i^{th} bit of all the codewords, i.e., $C_i = \{c_{u_1}(i), c_{w_1}(i), c_{u_2}(i), c_{w_2}(i)\}$. $P(C_i)$ is the probability that C_i is transmitted, and it is determined by the outputs of the component LDPC decoders. f_{Y_j} is the PDF of Y_j , and S_i^{k+} (S_i^{k-}) is the set of all codewords with

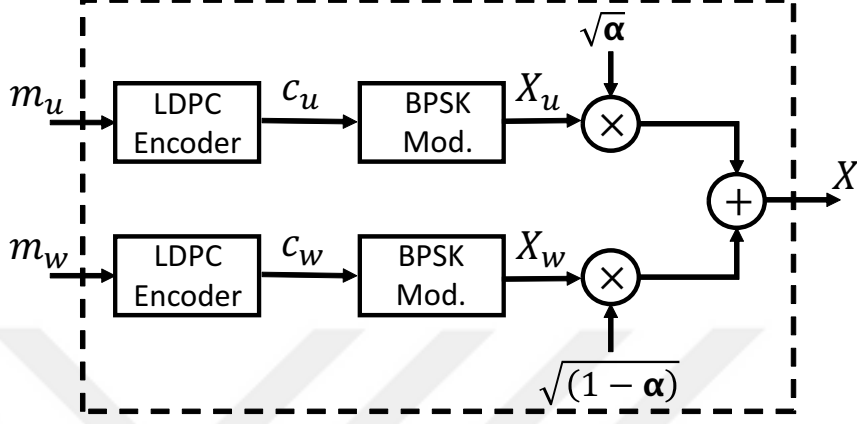


Figure 3.1: The block diagram of the transmitter with the HK encoding.

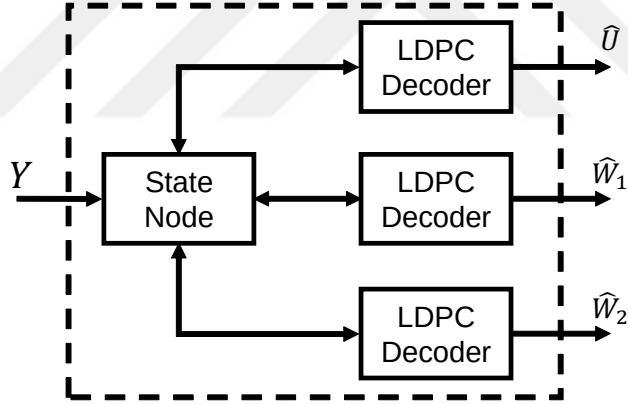


Figure 3.2: The block diagram of the receiver with joint decoding.

$$c_k(i) = 0 \text{ } (c_k(i) = 1).$$

3.3 Performance Analysis

In this section, our objective is to find the decoding thresholds of the candidate LDPC code ensembles in the code optimization process. For this purpose, we first derive the stability conditions for the code ensembles, which give lower bounds on the decoding threshold. We then introduce a modified version of the EXIT analysis to estimate the decoding thresholds of the ensembles upon which the

code design procedure will be based.

3.3.1 Stability Condition

The stability condition has been studied previously for joint iterative decoding in multiuser scenarios [1] and [4] for broadcast and unfaded IC with real channel gains, respectively. Here, we extend the results to fading ICs under joint iterative decoding. We assume asymptotic conditions, which impose cycle free LDPC codes, and almost zero decoding error, and via small perturbations of the degree distribution, we investigate the behavior of the joint decoder.

3.3.1.1 All Public Scheme

In this special case of the HK encoding scheme, both messages are decoded at both receivers, i.e., this case can be considered as a compound MAC. As it is shown in [59], in MAC channels, in order to find the stability condition on the degree distributions of the LDPC codes used, one can assume that the other message has been decoded completely and its effect is subtracted from the channel observations. Hence, the condition boils down to two single user stability conditions, and the modified channel becomes $Y_j' = h_{ij}X_{w_i} + z_j$. By applying the stability condition of the P2P channel to the compound MAC, the stability condition for the degree distributions of public messages for a fast fading IC is expressed as follows:

$$\frac{1}{\lambda'_{w_i}(0)\rho'_{w_i}(1)} > \max \left\{ E_{h_{ii}}[e^{(-\|h_{ii}\|^2)}], E_{h_{ij}}[e^{(-\|h_{ij}\|^2)}] \right\},$$

$$i, j = 1, 2 \text{ and } i \neq j, \quad (3.3)$$

where $\lambda'_i(0)$ denotes the first derivative of the variable node degree distribution polynomial $\lambda(x)$ at $x = 0$, and $\rho'_i(1)$ denotes the first derivative of the check node degree distribution polynomial $\rho(x)$ at $x = 1$, respectively. $E_{h_{ii}}$ ($E_{h_{ij}}$) indicates expectation with respect to the random variable h_{ii} (h_{ij}).

3.3.1.2 The HK Scheme

In this case, each receiver decodes both public messages and its intended private message, however, the unintended private message is not decoded. Therefore, to derive the stability condition on the degree distribution of the LDPC code ensembles, we assume that the other two messages of interest have been completely decoded and the unintended message is treated as noise. Using (3.1) and the superposition formulation, the modified channel outputs Y'_j and Y''_j for the public and intended private messages are obtained by cancelling out the completely decoded messages:

$$Y'_j = \sqrt{1 - \alpha_k} h_{kj} X_{w_k} + \sqrt{\alpha_i} h_{ij} X_{u_i} + z_j, \\ i, j, k = 1, 2 \text{ and } i \neq j, \quad (3.4)$$

$$Y''_j = \sqrt{\alpha_j} h_{jj} X_{u_j} + \sqrt{\alpha_i} h_{ij} X_{u_i} + z_j, \\ i, j = 1, 2 \text{ and } i \neq j. \quad (3.5)$$

Eq. (3.4) expresses the modified channel outputs for the public message of user k at receiver j . In order to calculate the stability condition, we employ an auxiliary function f given by (3.6) where E_{N_1, N_2} indicates expectation with respect to the random variables $N_1, N_2 \sim \mathcal{N}(0, \frac{1}{2})$, as derived in Appendix A.

$$f(h_1, h_2) = E_{N_1, N_2} \left[\left(e^{(-\|h_1\|^2 - \|h_2\|^2)} \right) \times \right. \\ \left. \sqrt{\frac{\cosh(4N_1 \operatorname{Re}(h_2) + 4N_2 \operatorname{Im}(h_2)) + \cosh(4\operatorname{Re}(h_1) \operatorname{Re}(h_2) + 4\operatorname{Im}(h_1) \operatorname{Im}(h_2))}{2}} \right]. \quad (3.6)$$

In fact, (3.6) is the Bhattacharyya constant of the channel $f_{Y|s}(y) \sim h_1 s + h_2 X + N$, where $X \in \{\pm 1\}$ equally likely and the channel gains are complex and constant. As the stability condition can be determined using the Bhattacharyya constant, we obtain the following inequality for the intended private message over a fast fading IC

$$\frac{1}{\lambda_{u_i}(0)\rho'_{u_i}(1)} > E_{h_{ii}, h_{ji}} [f(\sqrt{\alpha_i}h_{ii}, \sqrt{\alpha_j}h_{ji})], \quad i, j = 1, 2 \text{ and } i \neq j. \quad (3.15)$$

Similarly, the stability condition for the public messages can be expressed as

$$\begin{aligned} \frac{1}{\lambda'_{w_i}(0)\rho'_{w_i}(1)} > \\ \max (E_{h_{ii}, h_{ji}} [f(\sqrt{1-\alpha_i}h_{ii}, \sqrt{\alpha_j}h_{ji})], E_{h_{ij}} [f(\sqrt{1-\alpha_i}h_{ij}, \sqrt{\alpha_i}h_{ij})]) \\ i, j = 1, 2 \text{ and } i \neq j. \end{aligned} \quad (3.16)$$

3.3.2 Modified EXIT Chart Analysis

EXIT chart analysis is a prominent technique for estimating the decoding thresholds of LDPC code ensembles [61]. In this technique, the exchanged LLRs are assumed to have symmetric Gaussian distributions with a single parameter. Hence, the mutual information of LLRs incoming to the check node can be calculated as

$$I_{v \rightarrow c} = \sum_i \lambda_i J \left((i-1)J^{-1}(I_{c \rightarrow v}) + J^{-1}(I_{s \rightarrow v}) \right), \quad (3.17)$$

where $I_{c \rightarrow v}$ and $I_{s \rightarrow v}$ represent the mutual informations of the LLRs from the check nodes to the variable nodes and the mutual informations of the LLRs from the state nodes to the variable nodes, respectively. $J(x)$ is defined as

$$J(x) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi x}} \exp \left(-\frac{(l - \frac{x}{2})^2}{2x} \right) \log_2(1 + e^{-l}) dl.$$

In addition, the mutual information of the outgoing LLRs from the check nodes to the variable nodes can be calculated as

$$I_{c \rightarrow v} = \sum_i \rho_i \left(1 - J((i-1)J^{-1}(1 - I_{v \rightarrow c})) \right), \quad (3.18)$$

$$I_{v \rightarrow s} = \sum_i \tilde{\lambda}_i J(iJ^{-1}(I_{c \rightarrow v})), \quad (3.19)$$

where $\tilde{\lambda}(x)$ stands for the variable node degree distribution polynomial from the node perspective [62]. We model the links for incoming LLRs from component LDPC decoders as BECs with erasure probability $1 - I_{v \rightarrow s}$, as employed in [1] for joint decoding of two messages. Via simulations it is observed the BEC approximation model is suitable for estimating the decoding thresholds when multiple messages are jointly decoded (see Fig. 3.3). Under this BEC modeling, with probability $I_{v_a \rightarrow s}$ the value of the interfering signal a is known by the state node, and with probability $1 - I_{v_a \rightarrow s}$ there is no information about its value. By averaging over all the possibilities on the knowledge of the interference signals, $I_{s \rightarrow v}$ for the state node connected to three component LDPC decoders can be obtained as

$$\begin{aligned}
I_{s \rightarrow v} = & I_{v_a \rightarrow s} I_{v_b \rightarrow s} I(X; Y | X_a, X_b) \\
& + (1 - I_{v_a \rightarrow s}) I_{v_b \rightarrow s} I(X; Y | X_b) \\
& + (1 - I_{v_b \rightarrow s}) I_{v_a \rightarrow s} I(X; Y | X_a) \\
& + (1 - I_{v_a \rightarrow s}) (1 - I_{v_b \rightarrow s}) I(X; Y), \tag{3.20}
\end{aligned}$$

where a and b indicate the other two component LDPC decoders connected to the state node.

3.4 LDPC Code Optimization

The proposed LDPC code optimization procedure relies on both the modified EXIT chart analysis and the derived stability condition. While the former is utilized for analyzing the asymptotic decoding performance of the code ensembles, the latter is used for eliminating code ensembles that can potentially produce high error floors. For specific SNR and INR values, we initialize the code optimization with P2P optimal code ensembles. We generate new code ensembles using random perturbations, and we aim at improving the code rates while satisfying the stability condition and providing successful decoding. Once we improve upon the code rates, we assign the corresponding code ensembles as the updated ones,

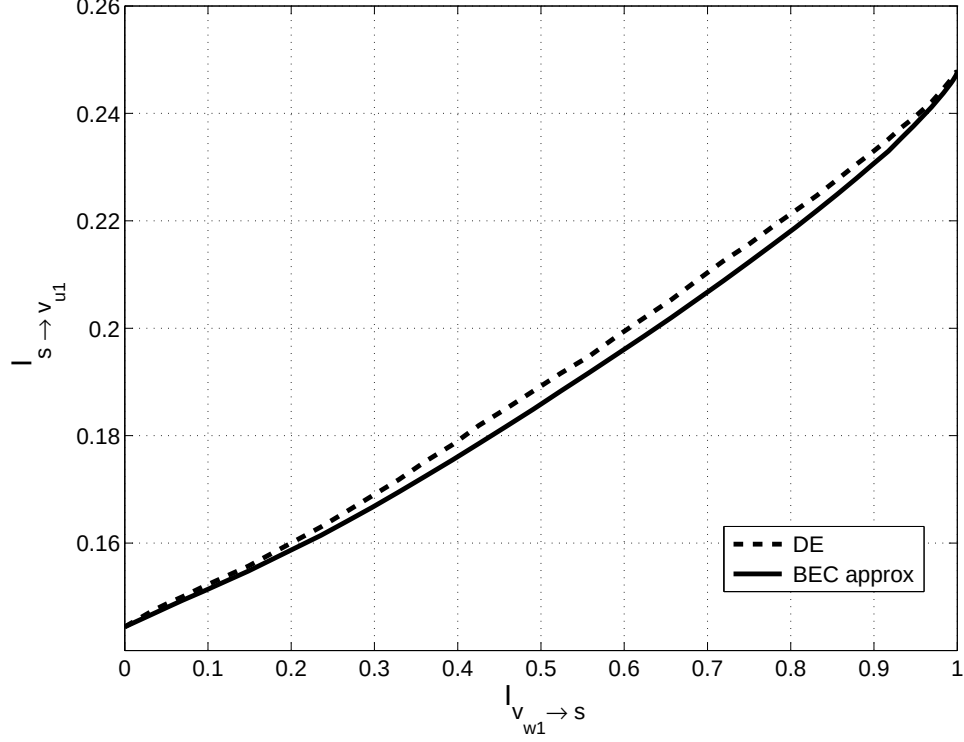


Figure 3.3: The evolution of private message at the state node for the LDPC code ensembles given in Table V in [4] with rate $(0.307, 0.258)$ and channel with $I(X_u; Y) = 0.1444$ and $I(X_u, Y|X_{w_1}, X_{w_2}) = 0.2477$.

and we continue the evolution. The whole procedure is terminated when the code rates cannot be further improved [69].

In each perturbation, we generate code ensembles with a specific rate increment ΔR . For this purpose, we perturb both the check node and the variable node polynomials. The i th coefficient of the check node polynomial is perturbed as $\bar{\rho}_i = \rho_i + e_{ci}$, where e_{ci} is the i th element of the perturbing vector $\mathbf{e}_c$. We assume $e_{ci} = 0$ if $\rho_i = 0$. We notice that the perturbing vector should satisfy $\sum_i \bar{\rho}_i = 1$, which results in

$$\sum_i e_{ci} = 0. \quad (3.21)$$

In order to control the amount of perturbation we impose

$$\sigma_e^2 = \sum_i e_{ci}^2. \quad (3.22)$$

For the variable node polynomial, we employ a perturbing vector $\mathbf{e}_v$, which satisfies (3.21) and (3.22) as well. Let Δr_1 be the rate increment after perturbing the check node polynomial. In order to have an overall rate increment of ΔR , the increment after perturbing the variable node polynomial should be $\Delta r_2 = \Delta R - \Delta r_1$. This enforces an extra constraint on $\mathbf{e}_v$ as

$$\sum_i \frac{e_{vi}}{i} = \frac{\Delta r_2 \sum_i \frac{\bar{\rho}_i}{i}}{(1 - (R_0 + \Delta r_1))^2 - \Delta r_2(1 - (R_0 + \Delta r_1))}, \quad (3.23)$$

where R_0 is the initial rate before perturbation. It follows that, two elements of $\mathbf{e}_c$ and three elements of $\mathbf{e}_v$ are dependent coefficients. The remaining free coefficients are generated according to a normal distribution. For simplicity we assume singleton distribution for the check nodes, i.e., $\rho(x) = x^{d_c-1}$, and we provide the design procedure (with $\Delta r_1 = 0$) in Algorithm 1.

3.5 Numerical Results

In this section, we design LDPC codes for fast fading ICs, and investigate their performance via simulations. We optimize LDPC codes with the objective of rate maximization such that the rate increments are along a straight line. Similar to [4], we select the nonzero variable node degrees as $\{2, 3, 4, 9, 10, 19, 20, 49, 50\}$. Our code design parameters are $\sigma_e^2 = 0.005$ and $\Delta R = 0.005R_0$. For comparisons we also simulate P2P codes which are optimized using the conventional EXIT chart analysis [69].

3.5.1 Real Channel Gain

In this example, we consider the all public (SUD) transmission scheme over the channel with parameters $\text{SNR}_1 = -3$ dB, $\text{INR}_1 = -2.25$ dB, $\text{SNR}_2 = -2.5$ dB

Algorithm 1 Code design algorithm for fast fading IC

Input: SNR, INR and d_c values for users, σ_e^2 , Δr_2 and T .

- 1: Initialize degree distribution polynomials for users: $\lambda_j(x)$, $j = 1, 2$.
- 2: Set $t = 0$.
- 3: **while** $t < T$ **do** ▷ main iteration starts
- 4: **for** $j = 1, 2$ **do**
- 5: Generate a random perturbing vector satisfying (3.21),
 (3.22) and (3.23).
- 6: Construct the polynomial representation for the random
 perturbing vector: $e_j^t(x)$.
- 7: Calculate $\lambda_j^t(x) = \lambda_j(x) + e_j^t(x)$.
- 8: **end for**
- 9: **if** Every $\lambda_j^t(x)$ satisfies the stability condition and the
 successful asymptotic decoding **then**
- 10: $\lambda_j(x) = \lambda_j^t(x)$, $j = 1, 2$. ▷ improve degree
distribution polynomials
- 11: **go to 2.**
- 12: **else**
- 13: $t = t + 1$.
- 14: **end if**
- 15: **end while** ▷ main iteration ends
- 16: Output: $\lambda_j(x)$, $j = 1, 2$.

and $\text{INR}_2 = -1.75$ dB. In Table 3.1, we provide the designed degree distributions, and in Fig. 3.4, we illustrate the achieved rate pairs along with the information theoretical ARR. We observe from Fig. 3.4 that the achieved rate pairs are at the outside of TIN-ARR boundary and they are close to the boundary of SUD-ARR. We also note that For highly asymmetric rate pairs, however, we observe that the design advantage is small.

For this instance, we consider an HK encoding scheme over fading channel with parameters $\text{SNR}_1 = \text{SNR}_2 = -3.5$ dB and $\text{INR}_1 = \text{INR}_2 = -6.5$ dB. In the code optimization process, the initial code ensembles have singleton check node degrees, however, during the evolution we let the check nodes have one degree higher and lower as well. In Table 3.2, we provide the designed degree distributions, and in Fig. 3.5 we show an achieved rate pair along with the

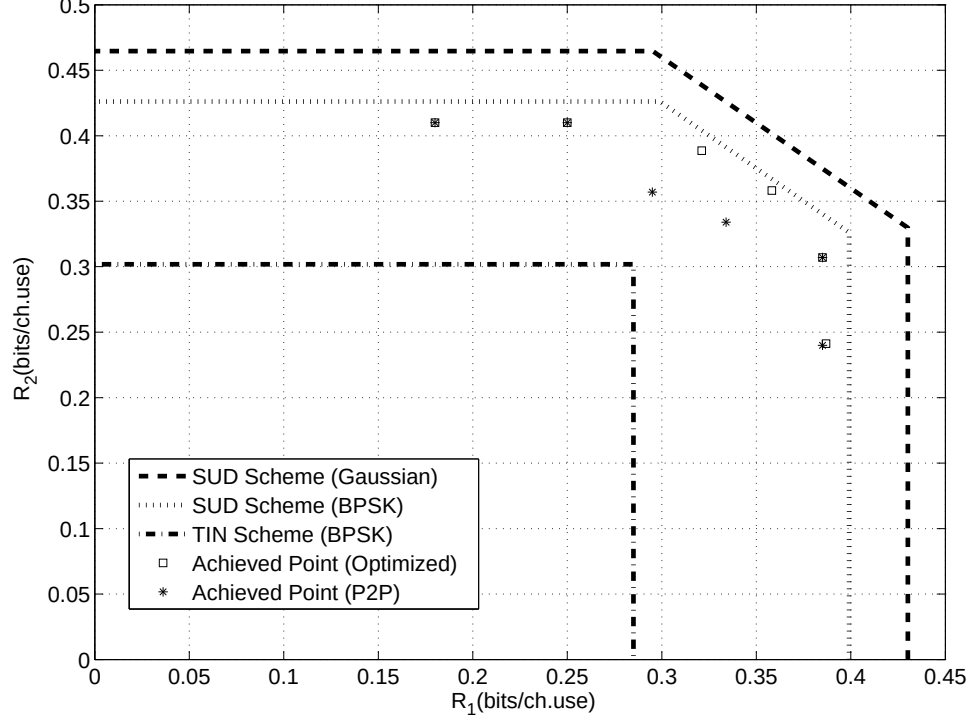


Figure 3.4: Achieved points and achievable rate regions for fast Rayleigh fading IC with real channel gain and parameters $\text{SNR}_1 = -3$ dB, $\text{INR}_1 = -2.25$ dB, $\text{SNR}_2 = -2.5$ dB and $\text{INR}_2 = -1.75$ dB.

corresponding ARR. We observe that the achieved rate pair is close to the HK ARR boundary and it is outside of the TIN-ARR, SUD-ARR and SND-ARR and it surpasses the P2P codes.

3.5.2 Complex Channel Gain

In the first example, we consider the SUD scheme over the channel with parameters $\text{SNR}_1 = -3$ dB, $\text{SNR}_2 = \text{SNR}_1 + 0.5$ dB, $\text{INR}_1 = \text{SNR}_1 + 0.75$ dB and $\text{INR}_2 = \text{SNR}_1 + 1.25$ dB. For this set of parameters, we perform code optimization and provide the resulting degree distributions in Table 3.3. In Table 3.4 we provide the decoding thresholds of both optimized and P2P code ensembles

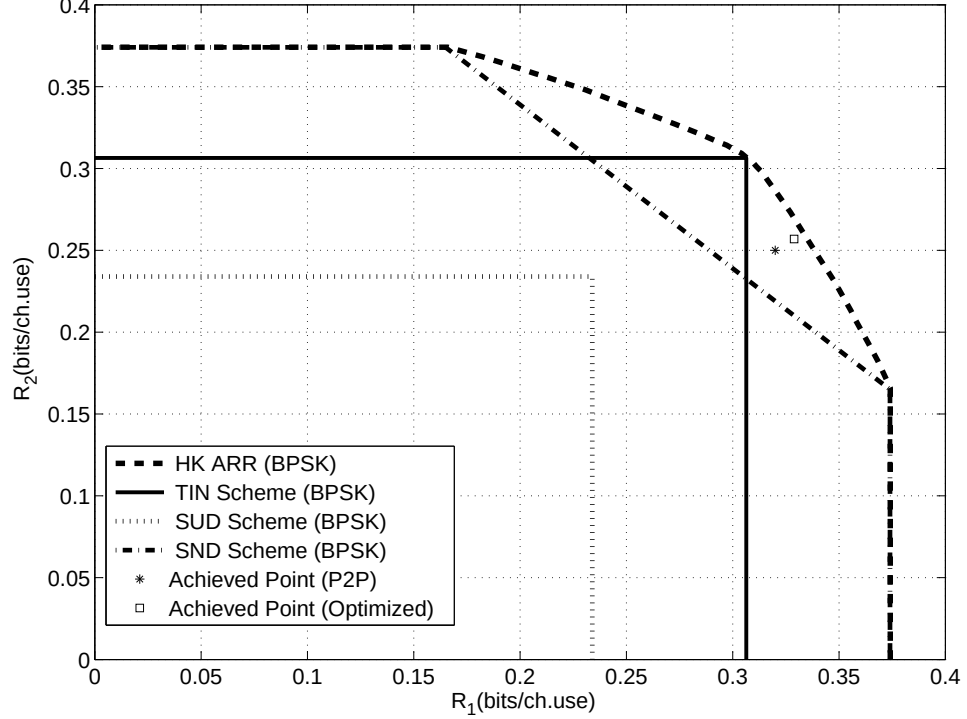


Figure 3.5: Achieved point and achievable rate regions for fast Rayleigh fading IC with real channel gain and parameters $\text{SNR}_1 = \text{SNR}_2 = -3.5\text{dB}$ and $\text{INR}_1 = \text{INR}_2 = -6.5\text{dB}$.

calculated via Monte Carlo simulations. We also provide information theoretical achievable SNR limits calculated by adapting the simple approach developed in [27]. Notice that our designed code ensembles perform about 0.3 dB away from them. Furthermore, all of the codes perform beyond the achievable limits with TIN. In addition, more than 0.3 dB design advantage can be attained with respect to the P2P code ensembles. For highly asymmetric rate pairs, however, we notice that the design advantage is negligible. This is because, for these rate pairs, one of the messages is decoded in few iterations and the channel resembles a P2P link for the other message.

We now consider a HK coding scheme over the channel with parameters $\text{SNR}_1 = -2.5\text{ dB}$, $\text{SNR}_2 = \text{SNR}_1 - 0.5\text{ dB}$, $\text{INR}_1 = \text{SNR}_1 - 1\text{ dB}$ and $\text{INR}_2 = \text{SNR}_1 - 1.5\text{ dB}$. We give the optimized degree distributions in Table

3.5, and provide the decoding thresholds of both optimized and P2P code ensembles in Table 3.6 along with the information theoretical achievable SNR limits for various schemes. We observe that our designed code ensembles perform about 0.4 dB away from the achievable SNR limits of HK coding scheme, and the operate beyond the achievable limits with TIN and SUD schemes. In addition, more than 0.3 dB design advantage can be attained with respect to the P2P codes, which indicate the clear need for designing new codes for fading ICs.

Table 3.1: Optimized degree distributions for real channel gain and SUD scheme.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.3869	6	0.277619	0.233538	0.06663	0.271047	0.018887	0.082542	0.039863	0.002013	0.007863
W_2	0.2412	5	0.303078	0.122145	0.182813	0.02924	0.160099	0.040231	0.033075	0.001155	0.128161
W_1	0.385	6	0.283716	0.224631	0.058075	0.269403	0.037447	0.091953	0.017816	0.002833	0.014128
W_2	0.307	6	0.257774	0.139652	0.136525	0.095943	0.137377	0.034667	0.024896	0.001359	0.171807
W_1	0.3582	6	0.277138	0.171823	0.155583	0.144162	0.00653	0.073998	0.031496	0.00781	0.131459
W_2	0.3582	6	0.298246	0.176194	0.099708	0.139798	0.007465	0.046815	0.116553	0.027135	0.088085
W_1	0.3211	5	0.35899	0.190692	0.093998	0.073942	0.140638	0.05911	0.032727	0.031156	0.018748
W_2	0.3886	6	0.297686	0.145391	0.21113	0.132244	0.00156	0.062982	0.044762	0.042783	0.061463
W_1	0.25	5	0.304003	0.133438	0.170489	0.071632	0.136125	0.051564	0.019649	0.082394	0.030706
W_2	0.41	6	0.295879	0.232994	0.066842	0.268901	0.07288	0.025542	0.030665	0.004143	0.002152
W_1	0.18	4	0.382555	0.119569	0.208389	0.080133	0.066469	0.048408	0.055122	0.013484	0.02587
W_2	0.41	6	0.295879	0.232994	0.066842	0.268901	0.07288	0.025542	0.030665	0.004143	0.002152

Table 3.2: Optimized degree distributions for real channel gain and HK scheme.

Msg.	R	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
U_1	0.3289	0.268408	0.103947	0.204066	0.042862	0.153914	0.006212	0.101346	0.04997	0.069275
U_2	0.2035	0.272296	0.181992	0.109596	0.141035	0.032012	0.034161	0.039141	0.034097	0.155671
W_2	0.0534	0.51391	0.134063	0.155149	0.023417	0.055269	0.008101	0.016638	0.010507	0.082946

Msg.	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7
U_1	0	0	0	0.003393	0.97694	0.019666
U_2	0	0	0.011697	0.959861	0.028442	0
W_2	0.004418	0.981465	0.014117	0	0	0

3.5.3 Finite Block Length Results

Finally, we provide BER performance analysis for some of the designed codes via finite block length simulations of specific codes picked from the ensembles. We use the random construction method to build the parity check matrices [70]. For the real channel gain scenario, we consider block length of 5k and 50k and demonstrate the resulting bit error rates in Fig. 3.7. The maximum number

Table 3.3: Optimized degree distributions for complex channel gain and SUD scheme.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.394	8	0.172489	0.234395	0.035092	0.225446	0.006194	0.004258	0.024904	0.061975	0.235248
W_2	0.3518	7	0.222048	0.143843	0.115085	0.209143	0.025527	0.001216	0.035363	0.128079	0.119696
W_1	0.3879	7	0.233243	0.10733	0.215338	0.115348	0.041379	0.038024	0.107015	0.034526	0.107795
W_2	0.3879	7	0.245681	0.100373	0.196623	0.115048	0.044966	0.044935	0.107476	0.072281	0.072617
W_1	0.3674	7	0.226061	0.199873	0.066646	0.118809	0.096977	0.011083	0.013204	0.021915	0.24543
W_2	0.4163	8	0.217262	0.108635	0.16151	0.112415	0.099672	0.010327	0.001427	0.189408	0.099345
W_1	0.31	6	0.261265	0.094317	0.214035	0.061744	0.104821	0.031201	0.076642	0.071036	0.084937
W_2	0.425	8	0.199757	0.191086	0.075019	0.123773	0.143391	0.005096	0.04852	0.023703	0.189655

Table 3.4: Decoding thresholds in terms of SNR_1 for complex channel gain and SUD scheme.

Rate pair (R_1, R_2)	P2P code ensemble	Optimized code ensemble	Achievable limit (SUD)	Achievable limit (TIN)
(0.394,0.352)	-2.70 dB	-2.72 dB	-3.10 dB	-2.02 dB
(0.388,0.388)	-2.66 dB	-2.84 dB	-3.10 dB	-2.15 dB
(0.367,0.416)	-2.38 dB	-2.76 dB	-3.05 dB	-2.06 dB
(0.310,0.425)	-2.73 dB	-2.73 dB	-3.03 dB	-1.85 dB

Table 3.5: Optimized degree distributions for complex channel gain and HK scheme.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
U_1	0.3085	6	0.247233	0.160895	0.136835	0.073672	0.142451	0.047803	0.025683	0.094171	0.071258
W_1	0.0915	3	0.51823	0.195265	0.111174	0.08633	0.030592	0.009713	0.024965	0.011482	0.012248
U_2	0.1487	4	0.368714	0.128519	0.172771	0.098244	0.079759	0.025089	0.016697	0.102447	0.007761
W_2	0.206	5	0.28792	0.132107	0.159936	0.085127	0.089978	0.014282	0.000788	0.179696	0.050166
U_1	0.2045	5	0.280208	0.132278	0.179354	0.103725	0.041653	0.00182	0.045874	0.014092	0.200995
W_1	0.1713	4	0.376407	0.177955	0.11046	0.158397	0.055449	0.004026	0.027599	0.004899	0.084808
U_2	0.1713	4	0.365051	0.187842	0.12563	0.140829	0.061966	0.002475	0.026849	0.044926	0.044431
W_2	0.2045	5	0.285428	0.131186	0.167797	0.114655	0.034684	0.003364	0.043889	0.174081	0.044915
U_1	0.3728	7	0.231534	0.147394	0.113944	0.179444	0.072756	0.00268	0.064363	0.117768	0.070117
W_1	0.057	3	0.507098	0.179974	0.090527	0.088258	0.031726	0.033972	0.044318	0.011838	0.012288
W_2	0.3224	6	0.268153	0.097446	0.205879	0.071591	0.142721	0.034838	0.00844	0.095648	0.075284

Table 3.6: Decoding thresholds in terms of SNR_1 for complex channel gain and HK scheme.

Rate pair (R_1, R_2)	P2P code ensemble	Optimized code ensemble	Achievable limit (SUD)	Achievable limit (TIN)	Achievable limit (HK)
(0.4,0.3547)	-1.84 dB	-2.18 dB	-1.55 dB	-2.05 dB	-2.63 dB
(0.3758,0.3758)	-1.91 dB	-2.15 dB	-1.97 dB	-2.12 dB	-2.60 dB
(0.4298,0.3224)	-2.05 dB	-2.22 dB	-0.98 dB	-1.45 dB	-2.54 dB

Table 3.7: Optimized degree distributions utilized for comparisons with AR4JA codes.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.5	9	0.20044	0.213231	0.093849	0.116405	0.018885	0.133335	0.037869	0.023627	0.162357
W_2	0.5	9	0.20044	0.213231	0.093849	0.116405	0.018885	0.133335	0.037869	0.023627	0.162357

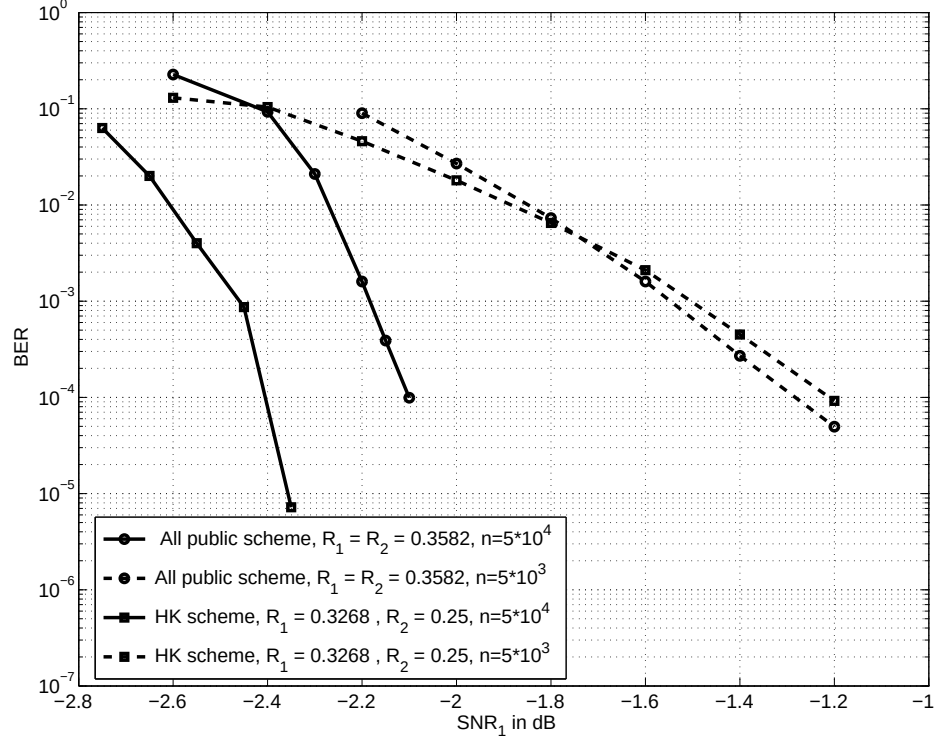


Figure 3.6: Finite block length results for the designed codes with real channel gain and SUD and HK coding.

of iterations for are set to 500 and the SNR to INR ratios are kept constant, throughout the simulations. The results are for the messages which have the worst error rate (bottleneck). We deduce that a BER of 10^{-4} is achieved with 50k length codes at about 1 dB away from the decoding thresholds measured with the modified EXIT chart analysis. Also, in order to achieve the same BER with 5K block length about 1 dB more power is needed.

For the complex channel gain scenario, we consider 10k block length and set the maximum number of iterations to 250. In Fig. 3.7 the performance of the designed codes over fading IC with 10k block length is given. We note that the BER results are for the messages with the worst error rate (bottleneck). From Fig. 3.7, we observe that the optimized codes provide significant performance

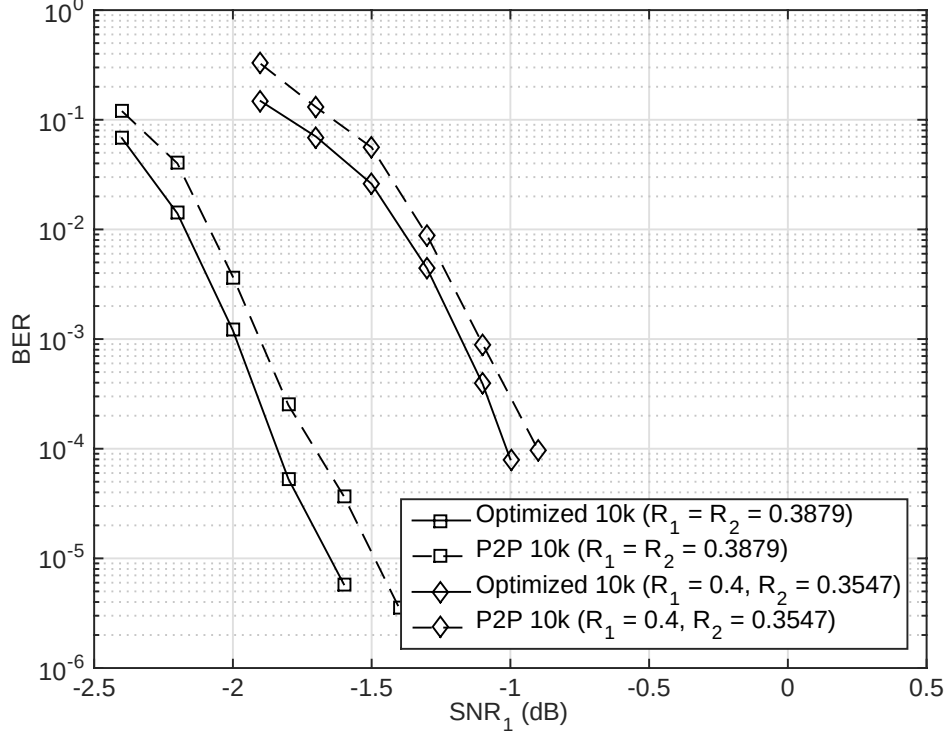


Figure 3.7: Finite block length results for the designed codes with complex channel gain and SUD and HK coding.

improvements over the P2P ones.

Finally, we consider an instance of a fast fading for three-user symmetry IC with complex channel gains and equal SNRs, and $\text{INR} = \text{SNR} + 0.75$. We only consider all public scenario and perform code design for $\text{SNR} = -1.5$. In this simulation, we assume the codes have same degree distribution, so during the perturbations in optimization process we perturb the codes similarly. The BER performance of the optimized codes (Table 3.8) and P2P optimized (with the same rate) codes for 10k block length are demonstrated in Fig. 3.8. As it is demonstrated in Fig. 3.8, the optimized codes outperforms around 0.4 dB at BER 10^{-4} .

Table 3.8: Optimized degree distributions for complex channel gain for three-user IC.

Msg.	R	dc	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.4465	8	0.232692	0.140176	0.172118	0.046987	0.039072	0.097827	0.000492	0.001054	0.269583
W_2	0.4465	8	0.232692	0.140176	0.172118	0.046987	0.039072	0.097827	0.000492	0.001054	0.269583
W_3	0.4465	8	0.232692	0.140176	0.172118	0.046987	0.039072	0.097827	0.000492	0.001054	0.269583

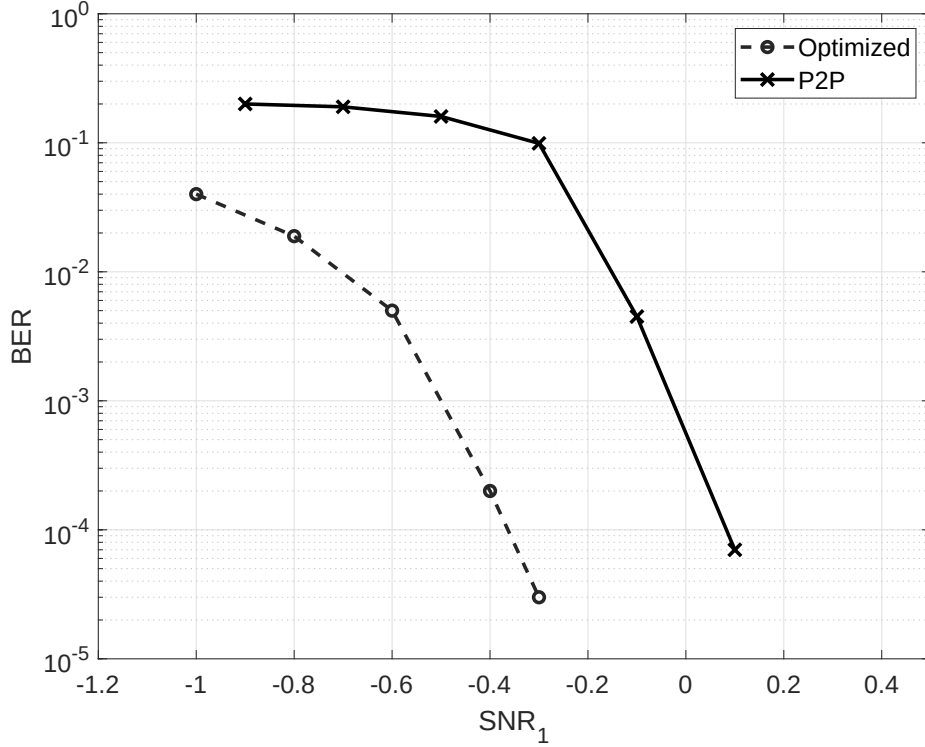


Figure 3.8: Finite block length results for the designed codes for three-user IC with 10k block length.

3.5.4 Structured Code Constructions

For moderate block lengths ($\approx 2k$), randomly constructed codes can suffer from the existence of short cycles in parity check matrix resulting in high error floors [71, 72]. Protograph based code is a structured code enjoying a lifted graph from a so called base graph or protograph [68, 73]. In [74], a construction method was proposed utilizing non-commuting permutation matrices and non-abelian groups. It was shown that the resulting derived code has no short cycles. A non-abelian group of order $m = pq$ is characterized by (p, q, s) where p and q are prime numbers, $\text{mod}(q, p1) = 0$, and $\text{mod}(s^q, p) = 1$. In the following, we consider a

fast Rayleigh fading IC and evaluate the performance of the optimized LDPC codes with moderate block lengths. The parity check matrices are constructed utilizing the method given in [74].

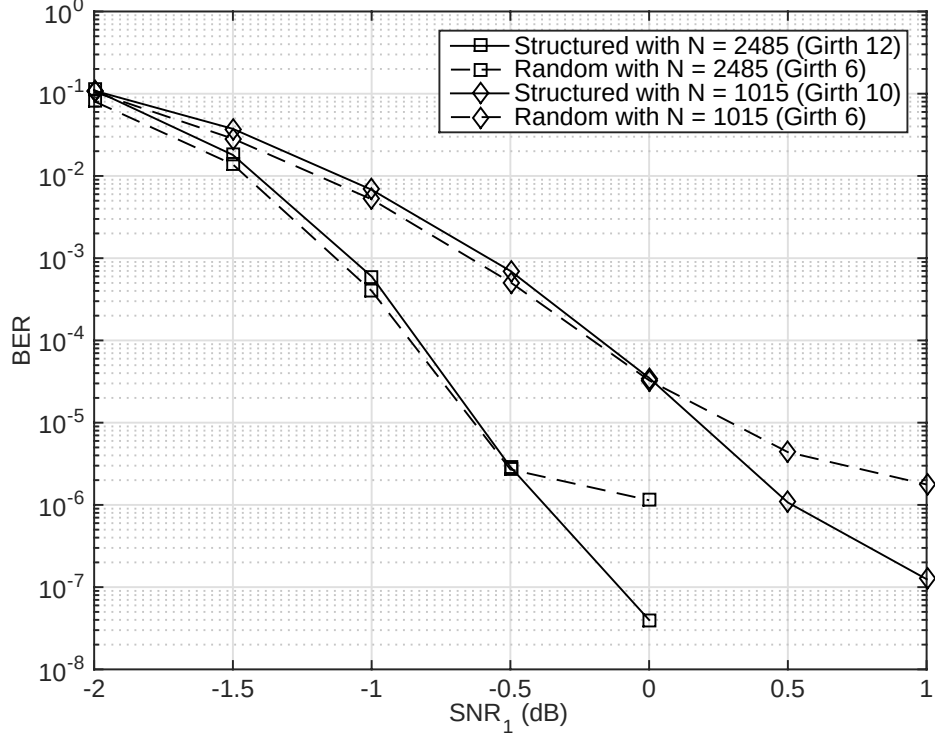


Figure 3.9: BER results for the structured and random codes.

As an example, we consider the complex channel gain scenario and channel parameters $\text{SNR}_2 = \text{SNR}_1 + 0.5$ dB and $\text{INR}_i = \text{SNR}_i + 0.75$ dB for $i = 1, 2$. We fix the size of the base matrix as 3×5 . In order to optimize the base matrix, we consider a set of all possible base matrices with rate 0.4 and with non-zero rows and columns. We note that for each base matrix, there possibly exists a different degree distribution. Among these base matrices, the one with the lowest decoding threshold level is picked as the protograph. Using the design procedure introduced in the previous section, we obtain optimized degree distributions for both users as $\lambda(x) = 0.3077x + 0.6923x^2$ and $\rho(x) = 0.6154x^3 + 0.3846x^4$. Finally, we construct the codes having the optimized degree distributions and the block lengths $N = 1015$ and $N = 2485$ with non-abelian groups $(29, 7, 7)$ and $(71, 7, 20)$, respectively and we set the maximum iteration number to 50. Fig. 3.9 shows

the BER performance of the random and structured codes. For the randomly constructed codes with girth 6, we observe an error floor around 10^{-6} . However, by using the protograph based construction method, we generate codes with girth equal to or larger than 10. For the constructed structure code with the girth 10, the error floor occurs around 10^{-7} , and no error floor is observed for girth 12 all the way down to 5×10^{-8} .

3.6 Chapter Summary

We study code design for two user fading ICs assuming two different signaling schemes, namely, all public scheme and the HK encoding scheme. A practical method for designing LDPC codes over fast fading ICs is proposed. The proposed method is evaluated for several channel parameters, and the achieved rates are compared with the corresponding achievable rate regions. We observe that for the all public scheme, rates close to the boundary of the SUD-ARR can be achieved which are outside TIN-ARR. For the HK encoding scheme, we achieve rate pairs outside the TIN, SUD, and SND-ARRs. We also notice that significant improvements over P2P designs can be attained. Simulation results are provided using specific codes picked from the optimized code ensembles to confirm the asymptotic results as well. Finally, we consider structured code construction via protographs and obtain results with lower error floors.

Chapter 4

Code Design for Quasi-Static Fading Interference Channels

In this chapter, we investigate LDPC code design for two-user quasi-static fading ICs. In this model the fading is so slow that regardless of the codeword length, the channel gains remain fixed during the transmission of one codeword. We investigate different decoding schemes, namely, TIN, SUD, HK, and flexible decoding for which the receivers decide the decoding policy with respect to the channel parameters. For the quasi-static fading scenario, we consider the common outage probability as the performance measure, that is, if any of the receivers fails to decode its message, an outage event is declared. We utilize the modified EXIT chart analysis, introduced in Chapter 3, and derive the stability condition for achieving the desired outage probability P_o . Furthermore, with the motivation that many wireless transceivers are equipped with multiple antennas, we also consider multiple antenna transmissions over quasi-static fading ICs. Specifically, we combine LDPC coding with the Alamouti scheme [75] to obtain spatial diversity for transmitters equipped with two antennas. Specifically, we describe the structure of the receivers, and perform LDPC code design for this scenario as well. We compare the performance of the designed codes with that of P2P optimal off-the-shelf codes and the results demonstrate that there are significant improvements possible in the resulting frame error rates (FERs).

This chapter is organized as follows. In Section 4.1, we describe the system model. In Section 4.2, we study the outage rate region of different coding schemes, and develop the idea of flexible decoding. In Section 4.3, we derive the stability condition for joint decoding of LDPC codes over quasi-static fading ICs. Next, in Section 4.4, we consider multiple antenna transmission, and design the receiver structure for this model. The code optimization procedure is elaborated, utilizing random perturbations, in Section 4.5. In Section 4.6, the achieved rate pairs and finite block length simulation results are provided. Finally, we conclude the paper in Section 4.7.

4.1 System Model

A two-user fading IC is represented by the set of equations

$$\begin{aligned} Y_1 &= h_{11}X_1 + h_{21}X_2 + z_1, \\ Y_2 &= h_{12}X_1 + h_{22}X_2 + z_2, \end{aligned} \tag{4.1}$$

where X_i is the transmitted signal from the transmitter i subject to the power constraint $E\{|X_i|^2\} \leq P_i$, h_{ij} is the complex channel gain between the transmitter i and receiver j , Y_j is the observed signal at the receiver j , for $i, j = 1, 2$. The noise term z_j is i.i.d. circularly symmetric complex Gaussian with zero mean and $\frac{N_0}{2}$ variance per dimension. We assume that the channel gains are mutually independent i.i.d. zero mean complex Gaussian distributed as well (i.e., Rayleigh fading). The channel coefficients change independently from one codeword to the next and remain constant during the entire codeword. We use the signal to noise ratio and interference to noise ratio definitions $\text{SNR}_i = E\{\frac{\|h_{ii}\|^2 P_i}{N_0}\}$ and $\text{INR}_i = E\{\frac{\|h_{ji}\|^2 P_j}{N_0}\}$ with $i, j = 1, 2, i \neq j$, respectively. Without loss of generality, we take $P_1 = P_2 = 1$ and $N_0 = 1$.

With HK type encoding, the message is split into private and public parts at each transmitter. The private (U_i) and public (W_i) messages are separately encoded by the component LDPC codes and transmitted via BPSK, i.e., 0 is

transmitted as +1 and 1 is transmitted as -1. The transmitter sends a linear superposition (sum) of the modulated signals. In other words,

$$X_i = \sqrt{\alpha_i}X_{u_i} + \sqrt{1 - \alpha_i}X_{w_i}, \quad i = 1, 2,$$

where $\alpha \in [0, 1]$ is a parameter employed for adjusting the power allocated to the private message. Fig. 4.1 shows a detailed block diagram of the transmitter under consideration.

At each receiver, we employ a joint decoder with three component LDPC decoders that work in parallel as depicted in Fig. 4.2. The LLRs are exchanged by the help of the state nodes, which show the dependencies of the transmitted signals from both users to the channel output.

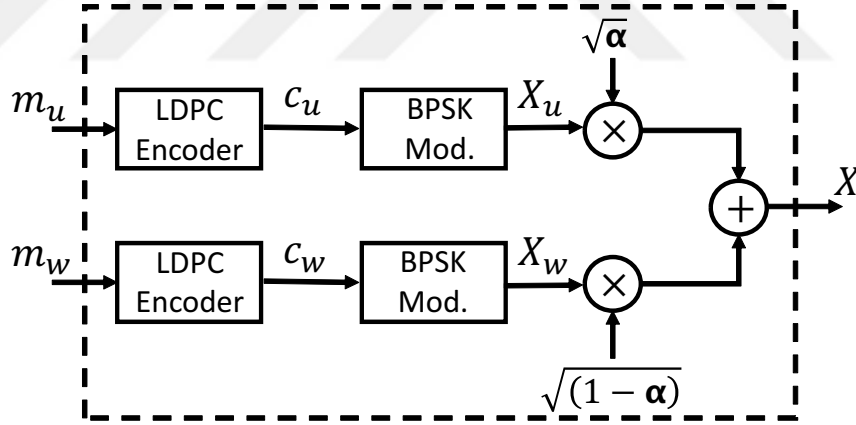


Figure 4.1: The block diagram of the transmitter with the HK encoding.

At receiver j , the outgoing LLR from the state node corresponding to the i^{th} bit of the message k is calculated as [4]

$$L(c_k(i), Y_j(i)) = \log \left(\frac{\sum_{C_i \in S_i^{k+}} f_{Y_j}(Y_j(i)|C_i) P(C_i)}{\sum_{C_i \in S_i^{k-}} f_{Y_j}(Y_j(i)|C_i) P(C_i)} \right), \quad (4.2)$$

where $c_k(i)$ represents the i^{th} coded bit of the message, which can be public or the intended private one. C_i is the vector consisting of the i^{th} bit of all the codewords, i.e., $C_i = \{c_{u_1}(i), c_{w_1}(i), c_{u_2}(i), c_{w_2}(i)\}$. $P(C_i)$ is the probability that

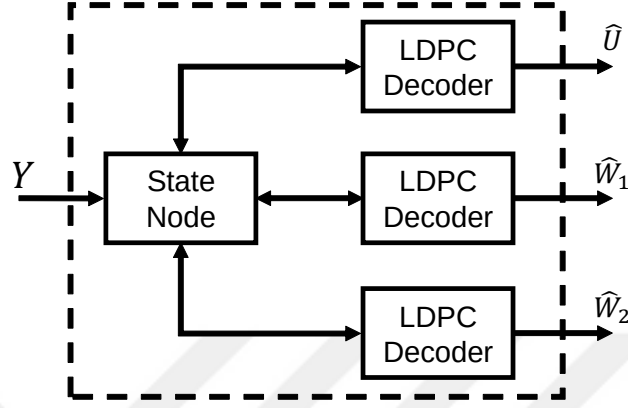


Figure 4.2: The block diagram of the receiver with joint decoding.

C_i is transmitted, and it is determined by the outputs of the component LDPC decoders. f_{Y_j} is the PDF of Y_j , and S_i^{k+} (S_i^{k-}) is the set of all codewords with $c_k(i) = 0$ ($c_k(i) = 1$).

4.2 Outage Region

In this section, we explicitly identify the boundaries of the outage rate region for different decoding schemes, which refers to the set of rate vectors such that the outage constraints are satisfied [76]. We call an event as outage when at least one of the users fails to decode its message using its own decoding policy. We want an outage event probability of less than P_o . Noting that the outage rate region is the set of all rate pairs falling inside the ARR (which is a function of the channel gains, hence random) with probability higher than $1 - P_o$, we elaborate on its computation for different decoding schemes in the remainder of this section.

4.2.1 Without Rate Splitting

We first assume that the transmitters do not split their messages, i.e., they only use one codeword. The messages can be considered as either private or public as

explained below.

TIN decoding scheme: In this scheme, each receiver only decodes its own message and treats the interference signal as noise. Hence, the receivers can decode their own messages if

$$\begin{aligned} R_1 &< I(X_1; Y_1), \\ R_2 &< I(X_2; Y_2). \end{aligned} \tag{4.3}$$

where $I(X; Y)$ stands for mutual information between X and Y .

SUD scheme: In this scheme, the receivers decode both messages, i.e., the set-up can be considered as a compound MAC. Therefore, the messages can be successfully decoded at the receivers if

$$\begin{aligned} R_1 &< \min\{I(X_1; Y_1|X_2), I(X_1; Y_2|X_2)\}, \\ R_2 &< \min\{I(X_2; Y_1|X_1), I(X_2; Y_2|X_1)\}, \\ R_1 + R_2 &< \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\}. \end{aligned} \tag{4.4}$$

Flexible decoding scheme: Since the random channel gains are known by the receivers in our model, the receivers can choose their own decoding strategy between TIN and SUD schemes with respect to the observed channel realizations. Hence, the receivers can decode their own messages if

$$(R_1, R_2) \in (\mathcal{R}_{TIN1} \cup \mathcal{R}_{SUD1}) \cap (\mathcal{R}_{TIN2} \cup \mathcal{R}_{SUD2}), \tag{4.5}$$

where $\mathcal{R}_{TINj}$ (and $\mathcal{R}_{SUDj}$) represents the TIN-ARR (and SUD-ARR) at receiver j . Fig. 4.3 demonstrates the decoding regions of the above schemes for a specific set of channel parameters.

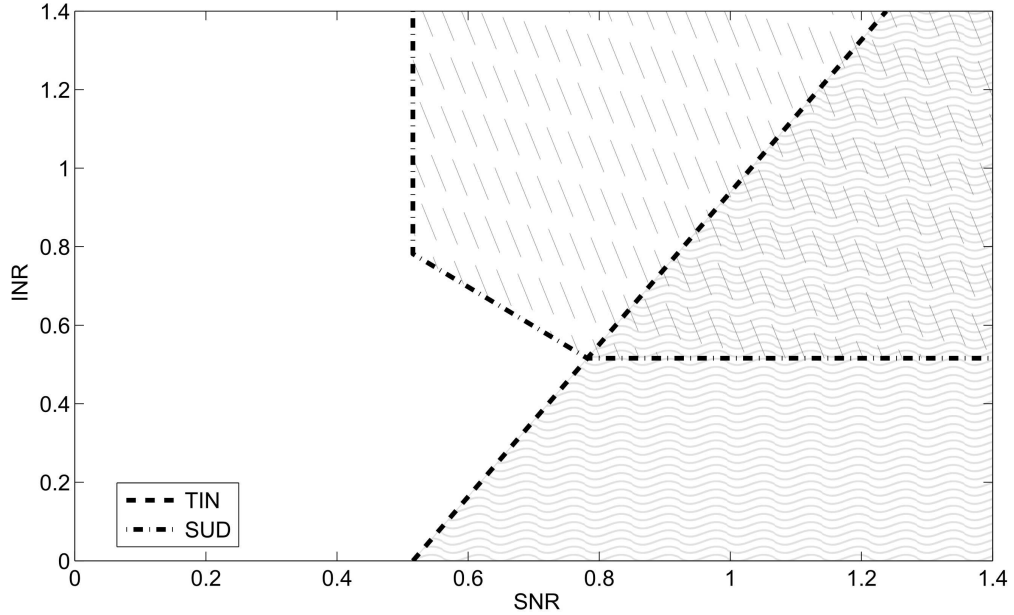


Figure 4.3: An example of decodable region for TIN and SUD schemes without rate splitting, at receiver one for unfaded channel with Gaussian channel input and rate pair $(R_1, R_2) = (0.6, 0.6)$. The decodable region of the flexible decoding is the union of the decodable region of TIN and SUD schemes (the shaded region).

4.2.2 With Rate Splitting

In this case, we assume that the transmitters split their messages with a fixed rate into two messages. The channel can be modeled as a compound K -user MAC, with respect to the decoding scheme, and the ARR for each scheme can be obtained by the following formulations.

TIN decoding scheme: In this scheme each receiver decodes both messages of the intended transmitter and treat the interfering signals from the unintended transmitter as noise. Hence, the ARR is characterized by ¹

¹We use this notation for compactness. Note that i and j cannot be equal to zero at the same time.

$$\begin{aligned}
iR_{u_1} + jR_{w_1} &< I(iX_{u_1}, jX_{w_1}; Y_1 | (1-i)X_{u_1}, (1-j)X_{w_1}), \\
iR_{u_2} + jR_{w_2} &< I(iX_{u_2}, jX_{w_2}; Y_2 | (1-i)X_{u_2}, (1-j)X_{w_2}), \\
i, j &= 0, 1.
\end{aligned} \tag{4.6}$$

SUD scheme: In this scheme each receiver decodes all four messages of the transmitters. The receivers are able to decode successfully if (4.7) is satisfied.

$$\begin{aligned}
iR_{u_1} + jR_{w_1} + kR_{u_2} + lR_{w_2} \\
&< I(iX_{u_1}, jX_{w_1}, kX_{u_2}, lX_{w_2}; Y_1 | (1-i)X_{u_1}, (1-j)X_{w_1}, (1-k)X_{u_2}, (1-l)X_{w_2}), \\
iR_{u_1} + jR_{w_1} + kR_{u_2} + lR_{w_2} \\
&< I(iX_{u_1}, jX_{w_1}, kX_{u_2}, lX_{w_2}; Y_2 | (1-i)X_{u_1}, (1-j)X_{w_1}, (1-k)X_{u_2}, (1-l)X_{w_2}), \\
i, j, k, l &= 0, 1.
\end{aligned} \tag{4.7}$$

HK decoding scheme: The receivers decode the two messages of the intended transmitter and only the second message of the unintended transmitter. So, the receivers can successfully decode if (4.8) is satisfied.

$$\begin{aligned}
iR_{u_1} + jR_{w_1} + kR_{w_2} \\
&< I(iX_{u_1}, jX_{w_1}, kX_{w_2}; Y_1 | (1-i)X_{u_1}, (1-j)X_{w_1}, (1-k)X_{w_2}), \\
iR_{u_2} + jR_{w_1} + kR_{w_2} \\
&< I(iX_{u_2}, jX_{w_1}, kX_{w_2}; Y_2 | (1-i)X_{u_2}, (1-j)X_{w_1}, (1-k)X_{w_2}), \\
i, j, k &= 0, 1.
\end{aligned} \tag{4.8}$$

Flexible decoding scheme: Similar to the previous case of no rate splitting, the receivers can choose their decoding policy with respect to the instantaneous channel gains. Hence, the receivers are able to decode their own messages by utilizing one of the TIN, SUD and HK decoding schemes. Hence, we have

$$(R_1, R_2) \in (\mathcal{R}_{TIN1} \cup \mathcal{R}_{SUD1} \cup \mathcal{R}_{HK1}) \cap (\mathcal{R}_{TIN2} \cup \mathcal{R}_{SUD2} \cup \mathcal{R}_{HK2}), \tag{4.9}$$

where $\mathcal{R}_{TINj}$, $\mathcal{R}_{SUDj}$, and $\mathcal{R}_{HKj}$ represent the TIN-ARR, SUD-ARR, and HK-ARR at receiver j , respectively. Fig. 4.4 indicates the rate regions with the

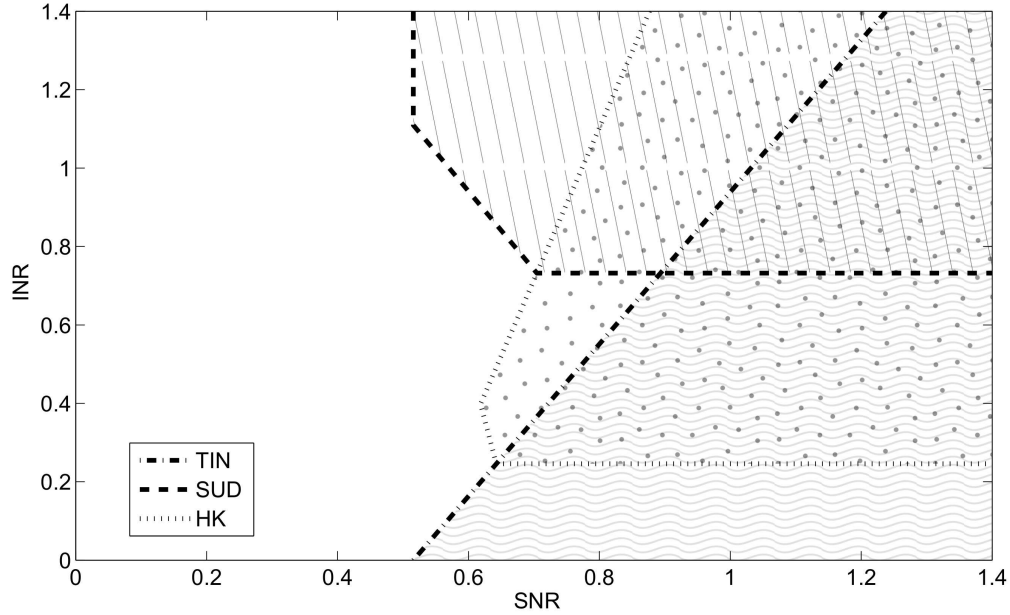


Figure 4.4: An example of decodable region at receiver one for TIN, SUD, and HK decoding scheme and unfaded channel with Gaussian channel input and rate splitting with $(R_{u_1}, R_{w_1}) = (0.3, 0.3)$, $(R_{u_2}, R_{w_2}) = (0.45, 0.15)$, and $(\alpha_1, \alpha_2) = (0.5, 0.5)$. The decodable region of the flexible decoding is the union of the decodable region of TIN, SUD, and HK decoding scheme (the shaded region).

specified decoding approaches for a specific set of channel parameters. As it is observed in Figs. 4.3 and 4.4, the decodable region of flexible decoding scheme changes by splitting the messages. We deduce that splitting messages may be favorable or not regarding the fading distribution.

4.3 Stability Conditions

4.3.1 All Public Scheme

Following a similar approach to the fast fading scenario in Chapter 3, we derive the stability condition for the quasi-static channel as well². In this case, since we

²However, different from the fast fading case, the stability condition for the quasi-static fading scenario serves as a looser lower bound on the decoding threshold. Hence, we only utilize it just for speeding up our design procedure. For eliminating error floors, we employ a robust design algorithm as explained in Section 4.5.

work with an outage probability formulation, the codewords should be decoded error free with a probability higher than $1 - P_o$. Also, as it is discussed above, the modified channel for the SUD case is similar to a P2P channel. We first define a level H such that, with probability $1 - P_o$ the channel power is higher than this threshold. Then, we introduce the stability condition as:

$$\frac{1}{\lambda'_{w_i}(0)\rho'_{w_i}(1)} > \max\{e^{(-H_{ii}^2)}, e^{(-H_{ij}^2)}\}, \quad (4.10)$$

for $i, j = 1, 2$ and $i \neq j$. Here H_{ii} and H_{ij} should satisfy the condition that the union of four events $\bigcup_{i,j} \{H_{ij} \geq \|h_{ij}\|\}$ has probability less than or equal to P_o .

4.3.2 The HK Scheme

In this case, each receiver decodes both public messages and its intended private message, however, the unintended private message is not decoded. Therefore, to derive the stability condition, we assume that the other two messages of interest have been completely decoded and the unintended message is treated as noise. Using (4.1) and the superposition formulation, the modified channel outputs Y'_j and Y''_j for the public and intended private messages are obtained by cancelling out the completely decoded messages:

$$Y'_j = \sqrt{1 - \alpha_k} h_{kj} X_{w_k} + \sqrt{\alpha_i} h_{ij} X_{u_i} + z_j, \quad (4.11)$$

$$Y''_j = \sqrt{\alpha_j} h_{jj} X_{u_j} + \sqrt{\alpha_i} h_{ij} X_{u_i} + z_j, \quad (4.12)$$

for $i, j, k = 1, 2$ and $i \neq j$. Eq. (4.11) expresses the modified channel outputs for the public message of user k at receiver j . In order to calculate the stability condition, we employ an auxiliary function f given by (3.6). The stability conditions for quasi-static fading interference channels for the private messages can be obtained as

$$\frac{1}{\lambda'_{u_i}(0)\rho'_{u_i}(1)} > S_{u_i}, \quad (4.13)$$

where S_{u_i} should satisfy $Pr\left\{\bigcup_i \{S_{u_i} \leq f(\sqrt{\alpha_i}h_{ii}, \sqrt{\alpha_j}h_{ji})\}\right\} \leq P_o$ for $i, j = 1, 2$ and $i \neq j$. On the other hand, the stability condition for public messages can be expressed as

$$\frac{1}{\lambda'_{w_i}(0)\rho'_{w_i}(1)} > \max(S_{1,w_i}, S_{2,w_i}). \quad (4.14)$$

Here S_{1,w_i} and S_{2,w_i} should satisfy the condition that $Pr\left\{\bigcup_i E_i\right\} \leq P_o$ where the event E_i is defined as $\{S_{1,w_i} \leq f(\sqrt{1-\alpha_i}h_{ii}, \sqrt{\alpha_j}h_{ji})\} \cup \{S_{2,w_i} \leq f(\sqrt{1-\alpha_i}h_{ij}, \sqrt{\alpha_i}h_{ij})\}$ for $i, j = 1, 2$ and $i \neq j$.

4.4 Multiple Antenna Transmission

One of the techniques to combat the undesirable effects of fading is to construct different links between the transmitter and the receiver by employing multiple antennas, which can be used to obtain spatial diversity. With this motivation, in this section, we consider fading ICs when the transmitters are equipped with multiple antennas. Specifically, we address LDPC code optimization for quasi-static fading ICs with two transmit antennas and one receive antenna for each user. In the following, we design the corresponding transmitter and receiver structures, and provide a formulation of the messages passed between different decoding blocks.

The block diagram of the transmitter with two antennas is depicted in Fig. 4.5. The output symbols of the HK encoder are fed to an Alamouti encoder. Two symbols are considered at a time, say x_a and x_b , and they are transmitted in two consecutive time slots. In the first time slot, the first antenna transmits the symbol x_a and the second antenna transmits the symbol x_b . In the second time slot, the first antenna transmits $-x_b^*$ and the second antenna transmits x_a^* .

We now formulate the SUD scheme corresponding to the above Alamouti based transmission. We set $E\{|X_i|^2\} \leq \frac{P_i}{2}$ to satisfy the total power constraint. The

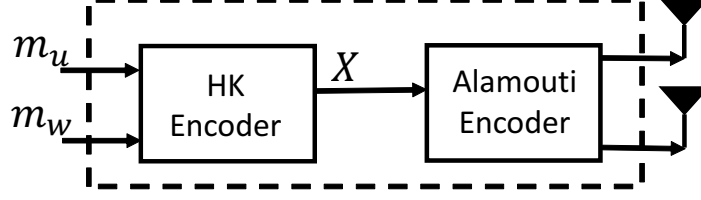


Figure 4.5: The block diagram of the transmitter for Alamouti technique.

channel observation for receiver 1 can be expressed as (4.23) where $h_i^k[n]$ represents the channel gain between the k^{th} antenna of user i and receiver 1 at time n .

$$\begin{aligned}
 Y[2n] &= h_1^1[2n]X_1[2n] + h_1^2[2n]X_1[2n+1] \\
 &\quad + h_2^1[2n]X_2[2n] + h_2^2[2n]X_2[2n+1] + z[2n], \\
 Y[2n+1] &= h_1^2[2n]X_1^*[2n] - h_1^1[2n]X_1^*[2n+1] \\
 &\quad + h_2^2[2n]X_2^*[2n] - h_2^1[2n]X_2^*[2n+1] + z[2n+1].
 \end{aligned} \tag{4.23}$$

The decoder produces the combined signals (see Fig. 4.6)

$$\begin{aligned}
 \tilde{Y}_1[2n] &= h_1^{1*}[2n]Y[2n] + h_1^2[2n]Y^*[2n+1], \\
 \tilde{Y}_1[2n+1] &= h_1^{2*}[2n]Y[2n] - h_1^1[2n]Y^*[2n+1],
 \end{aligned} \tag{4.24}$$

and

$$\begin{aligned}
 \tilde{Y}_2[2n] &= h_2^{1*}[2n]Y[2n] + h_2^2[2n]Y^*[2n+1], \\
 \tilde{Y}_2[2n+1] &= h_2^{2*}[2n]Y[2n] - h_2^1[2n]Y^*[2n+1].
 \end{aligned} \tag{4.25}$$

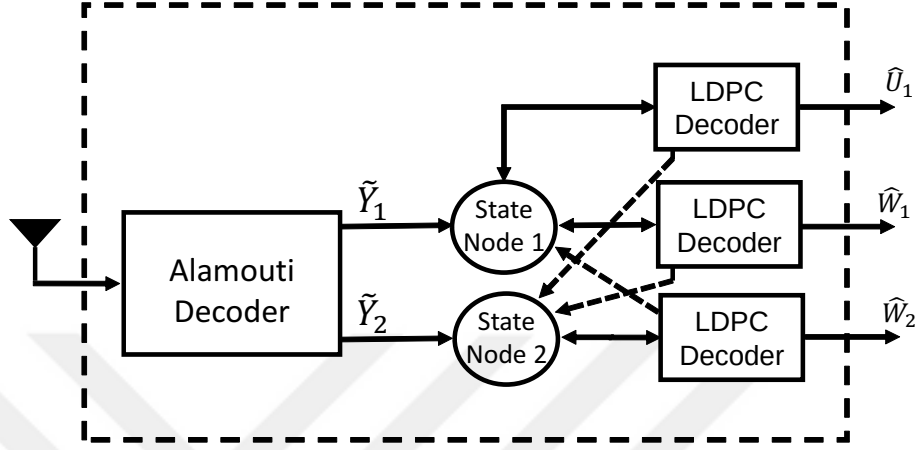


Figure 4.6: The block diagram of the receiver 1 for Alamouti transmission scheme.

Notice that (4.24) can be written as

$$\begin{aligned}
 \tilde{Y}_1[2n] &= (\|h_1^1[2n]\|^2 + \|h_1^2[2n]\|^2)X_1[2n] \\
 &\quad + (h_1^{1*}[2n]h_2^1[2n] + h_1^2[2n]h_2^{2*}[2n])X_2[2n] \\
 &\quad + (h_1^{1*}[2n]h_2^2[2n] - h_1^2[2n]h_2^{1*}[2n])X_2[2n+1] \\
 &\quad + h_1^{1*}[2n]z[2n] + h_1^2[2n]z^*[2n+1], \\
 \tilde{Y}_1[2n+1] &= (\|h_1^1[2n]\|^2 + \|h_1^2[2n]\|^2)X_1[2n+1] \\
 &\quad + (h_1^{2*}[2n]h_2^1[2n] - h_1^1[2n]h_2^{2*}[2n])X_2[2n] \\
 &\quad + (h_1^{2*}[2n]h_2^2[2n] + h_1^1[2n]h_2^{1*}[2n])X_2[2n+1] \\
 &\quad + h_1^{2*}[2n]z[2n] - h_1^1[2n]z^*[2n+1].
 \end{aligned} \tag{4.26}$$

It is clear that only the signal corresponding to the n^{th} bit of the message of user 1 appears in $\tilde{Y}_1[n]$. Applying the same procedure, we can obtain $\tilde{Y}_2$ similarly. As shown in Fig. 4.6, the decoder outputs $\tilde{Y}_1$ and $\tilde{Y}_2$ are fed to the state nodes 1 and 2, respectively. Then the state nodes calculate the LLRs for the component decoders using their incoming LLRs and the decoder outputs. The calculated LLRs are exchanged between the state nodes and the component LDPC decoders in an iterative fashion.

It is evident from these formulations that after passing the received signal through the decoder for the Alamouti based transmission, the distribution of the

coefficients of the intended and unintended signals are not complex Gaussian anymore, hence new code designs may be needed to improve the system performance. We also note that the analysis developed above can be extended to the case of HK based coding as well.

We now derive a stability condition for the SUD scheme with Alamouti based transmission. We assume that the other message has completely been decoded. Hence, with respect to (4.26), the modified channel for user i at receiver j becomes $Y'_j = (\|h_{ij}^1\|^2 + \|h_{ij}^2\|^2)X_{w_i} + \sqrt{\|h_{ij}^1\|^2 + \|h_{ij}^2\|^2}z_j$. Similar to the single antenna case, the stability conditions for the degree distributions of public messages over a quasi-static fading interference channel can then be obtained as

$$\frac{1}{\lambda'_{w_i}(0)\rho'_{w_i}(1)} > \max\{e^{(-H_{ii}^2)}, e^{(-H_{ij}^2)}\}, \quad (4.27)$$

for $i, j = 1, 2$ and $i \neq j$. Here H_{ii} and H_{ij} should satisfy the condition that the union of four events $\bigcup_{i,j} \left\{ H_{ij} \geq \frac{\|h_{ij}^1\|^2 + \|h_{ij}^2\|^2}{2} \right\}$ has probability less than or equal to P_o . Comparing this result with the single-antenna case in (4.10), we observe that the new stability condition takes into account the arithmetic mean of the channel powers.

4.5 LDPC Code Optimization

We initialize the code optimization with P2P optimal code ensembles. We generate new code ensembles using random perturbations, and we aim at improving the code rates while satisfying the stability condition and providing successful decoding. Once we improve upon the code rates, we assign the corresponding code ensembles as the updated ones, and we continue the evolution. The whole procedure is terminated when the code rates cannot be further improved [69].

We consider a large number of channel realizations (taken as 2×10^4 in the numerical results) for checking the code ensembles under consideration. For each channel realization, we analyze the ARR, stability condition and modified EXIT

in order. If any of the analyses fails, then we declare the decoding result for that channel realization as failure. If the overall failure rate is below P_o , we assign the corresponding code ensembles as the updated ones. Here we note that our aim is to reach an FER less than or equal to P_o .

In each perturbation, we generate code ensembles with a specific rate increment ΔR . For this purpose, we perturb both the check node and the variable node polynomials. The i th coefficient of the check node polynomial is perturbed as $\bar{\rho}_i = \rho_i + e_{ci}$, where e_{ci} is the i th element of the perturbing vector $\mathbf{e}_c$. We assume $e_{ci} = 0$ if $\rho_i = 0$. We notice that the perturbing vector should satisfy $\sum_i \bar{\rho}_i = 1$, which results in

$$\sum_i e_{ci} = 0. \quad (4.28)$$

In order to control the amount of perturbation we impose

$$\sigma_e^2 = \sum_i e_{ci}^2. \quad (4.29)$$

For the variable node polynomial, we employ a perturbing vector $\mathbf{e}_v$, which satisfies (4.28) and (4.29) as well. Let Δr_1 be the rate increment after perturbing the check node polynomial. In order to have an overall rate increment of ΔR , the increment after perturbing the variable node polynomial should be $\Delta r_2 = \Delta R - \Delta r_1$. This enforces an extra constraint on $\mathbf{e}_v$ as

$$\sum_i \frac{e_{vi}}{i} = \frac{\Delta r_2 \sum_i \frac{\bar{\rho}_i}{i}}{(1 - (R_0 + \Delta r_1))^2 - \Delta r_2 (1 - (R_0 + \Delta r_1))}, \quad (4.30)$$

where R_0 is the initial rate before perturbation. It follows that, two elements of $\mathbf{e}_c$ and three elements of $\mathbf{e}_v$ are dependent coefficients. The remaining free coefficients are generated according to a normal distribution. For simplicity we assume singleton distribution for the check nodes, i.e., $\rho(x) = x^{d_c-1}$, and we provide the design procedure (with $\Delta r_1 = 0$) in Algorithm 2.

Algorithm 2 Code design algorithm for quasi-static fading IC

Input: SNR, INR and d_c values for users, σ_e^2 , Δr_2 and T .

- 1: Initialize degree distribution polynomials for users: $\lambda_j(x)$, $j = 1, 2$.
- 2: Set $t = 0$.
- 3: **while** $t < T$ **do** ▷ main iteration starts
- 4: **for** $j = 1, 2$ **do**
- 5: Generate a random perturbing vector satisfying (4.28),
 (4.29) and (4.30).
- 6: Construct the polynomial representation for the random
 perturbing vector: $e_j^t(x)$.
- 7: Calculate $\lambda_j^t(x) = \lambda_j(x) + e_j^t(x)$.
- 8: **end for**
- 9: **if** $\lambda_j^t(x)$ satisfies the stability condition, the successful asymptotic
 decoding, and the code rate is inside ARR for more than $1 - P_o$ **then**
- 10: $\lambda_j(x) = \lambda_j^t(x)$, $j = 1, 2$. ▷ improve degree
 distribution polynomials
- 11: **go to** 2.
- 12: **else**
- 13: $t = t + 1$.
- 14: **end if**
- 15: **end while** ▷ main iteration ends
- 16: Output: $\lambda_j(x)$, $j = 1, 2$.

4.6 Numerical Results

In this section, we design LDPC codes for quasi-static fading ICs with complex channel gains, and investigate their performance via simulations. Similar to [4], we select the nonzero variable node degrees as $\{2, 3, 4, 9, 10, 19, 20, 49, 50\}$. We assume $\sigma_e^2 = 0.005$, $\Delta r_2 = 0.005R_0$ and $T = 1000$ in the code design. Unless otherwise stated, we consider single transmit antenna users. For comparisons we also simulate P2P codes which are optimized using the conventional EXIT chart analysis [69].

We provide code design examples for quasi-static fading ICs with different decoding approaches. We observe that, in general, rate splitting does not improve the outage rate region significantly. Hence, we do not consider that strategy. As

it is not appropriate to mention about the asymptotic performance in the quasi-static fading case, we validate our code designs using only finite block length code simulations.

First we consider a Rayleigh fading IC with parameters $\text{SNR}_1 = 8$ dB, $\text{SNR}_2 = \text{SNR}_1 - 1$ dB, $\text{INR}_1 = \text{SNR}_1 + 3$ dB and $\text{INR}_2 = \text{SNR}_1 + 2$ dB. We consider an outage probability of $P_o = 0.1$. Optimized degree distributions with SUD are given in Table 4.1. We also perform code design for the case of two transmit antennas considering an IC with the same channel parameters. The resulting designed degree distributions are provided in Table 4.2.

In the second example, we consider another Rayleigh fading IC with parameters $\text{SNR}_1 = 5$ dB, $\text{SNR}_2 = \text{SNR}_1$ dB, $\text{INR}_1 = \text{INR}_2 = \text{SNR}_1 + 3$ dB and an outage probability of $P_o = 0.1$. Optimized degree distributions for flexible decoding are given in Table 4.3.

4.6.1 Finite Block Length Simulation Results

Finite block length performance of the designed codes over quasi-static Rayleigh fading IC with SUD is depicted in Fig. 4.7 and Table 4.4. For comparison, we also show the performance of P2P codes. The results suggest that, at an FER of 0.1, our codes with block lengths 10k operate about 0.55 dB away from the outage limit. We also observe that the designed codes outperform the P2P codes by about 0.35 dB. The FER performance of designed codes for the Alamouti transmission scheme is depicted in Fig. 4.8, indicating that, at an FER of 10^{-2} , the designed codes with block lengths 10k and 2k operate about 0.5 and 0.8 dB away from the outage limit, respectively. We also deduce that the designed codes offer a performance enhancement up to 0.7 dB when compared to the P2P codes with the same block length. However, we also notice that the designed codes for the two antenna case have almost the same performance with the ones designed for single antenna case.

Table 4.1: Optimized degree distributions for SUD.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.2628	5	0.298697	0.144636	0.185787	0.022514	0.191288	0.058976	0.019543	0.003286	0.075274
W_2	0.2155	5	0.28608	0.12891	0.176329	0.07531	0.112404	0.021402	0.001853	0.148091	0.049622
W_1	0.3642	7	0.217104	0.166379	0.11869	0.162814	0.05443	0.001543	0.058521	0.17184	0.048678
W_2	0.102	4	0.324211	0.195939	0.128127	0.099471	0.029218	0.001508	0.013505	0.130795	0.077227
W_1	0.1314	4	0.357801	0.119989	0.19361	0.093057	0.04632	0.003388	0.053987	0.076639	0.055209
W_2	0.3153	6	0.260758	0.121757	0.18376	0.102342	0.084807	0.024197	0.031441	0.022821	0.168117

Table 4.2: Optimized degree distributions for Alamouti transmission.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.2654	5	0.307358	0.131746	0.190356	0.077811	0.119491	0.030343	0.067058	0.047555	0.028282
W_2	0.2176	5	0.293478	0.115053	0.181472	0.131295	0.040833	0.050643	0.003052	0.016766	0.167408

Table 4.3: Optimized degree distributions for flexible decoding.

Msg.	R	d_c	λ_2	λ_3	λ_4	λ_9	λ_{10}	λ_{19}	λ_{20}	λ_{49}	λ_{50}
W_1	0.204	5	0.275952	0.18641	0.093157	0.164057	0.040362	0.007804	0.017464	0.034803	0.17999
W_2	0.204	5	0.275952	0.18641	0.093157	0.164057	0.040362	0.007804	0.017464	0.034803	0.17999
W_1	0.255	5	0.30101	0.137732	0.182577	0.063301	0.136805	0.046525	0.019292	0.068336	0.044422
W_2	0.15	4	0.354227	0.159814	0.16456	0.087873	0.086675	0.021908	0.016184	0.028572	0.080189
W_1	0.15	4	0.354227	0.159814	0.16456	0.087873	0.086675	0.021908	0.016184	0.028572	0.080189
W_2	0.255	5	0.30101	0.137732	0.182577	0.063301	0.136805	0.046525	0.019292	0.068336	0.044422

Table 4.4: SNR₁ for achieving 0.1 FER with SUD under quasi-static fading.

Rate pair (R_1, R_2)	P2P 10k code	Optimized 10k code	Outage (SUD)
(0.2628,0.2155)	8.85 dB	8.5 dB	7.8 dB
(0.3642,0.102)	8.75 dB	8.4 dB	7.95 dB
(0.1314,0.3153)	8.7 dB	8.35 dB	7.85 dB

Table 4.5: SNR for achieving 0.1 FER with flexible decoding under quasi-static fading.

Rate pair (R_1, R_2)	P2P 10k code	Optimized 10k code	Outage (SUD)	Outage (TIN)	Outage (Flexible)
(0.204,0.204)	6.15 dB	5.75 dB	6.5 dB	5.3 dB	4.75 dB
(0.255,0.15)	5.9 dB	5.6 dB	6.6 dB	5.35 dB	4.85 dB
(0.15,0.255)	5.9 dB	5.6 dB	6.6 dB	5.35 dB	4.85 dB

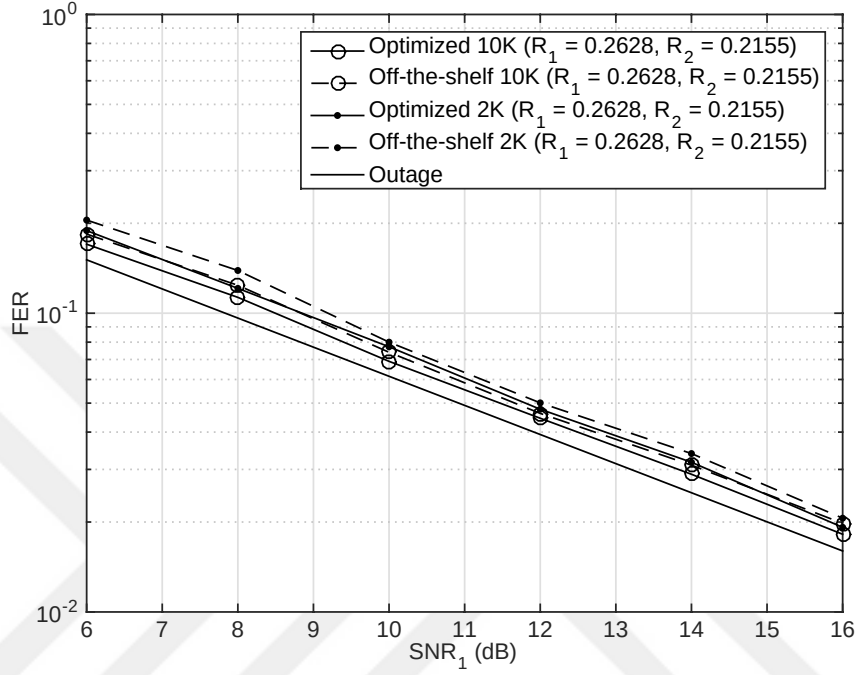


Figure 4.7: Performance results with SUD.

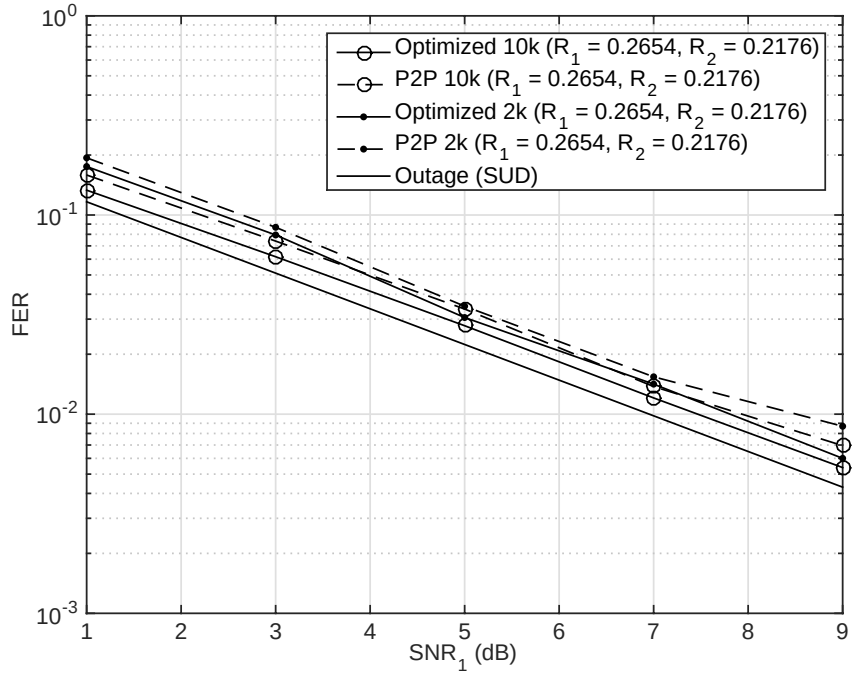


Figure 4.8: Performance results with SUD and two transmit antennas.

The performance of the designed codes for quasi-static Rayleigh fading IC with flexible decoding is depicted in Fig. 4.9 and Table 4.5. It is observed that the outage probability for flexible decoding is smaller than those for the TIN and SUD. In addition, the desired outage probability is achieved about 0.65 dB away from the target SNR indicating the efficacy of our designs. The superiority of the optimized codes are observed in this example, as well. We also report that all of our designed codes provide better performance than the naive time-sharing between the users.

We further note that, while SUD scheme requires two component LDPC decoders and a state node, TIN scheme requires only a single component LDPC decoder. Hence the decoding complexity for the SUD scheme is more than twice of the decoding complexity for TIN. On the other hand, complexity of the flexible decoding scheme is between those for SUD and TIN. For the specific example we consider, at an SNR_1 of 5 dB, only in 4.9% of the channel realizations the flexible receiver performs joint decoding, making the complexity very close to that of TIN scheme.

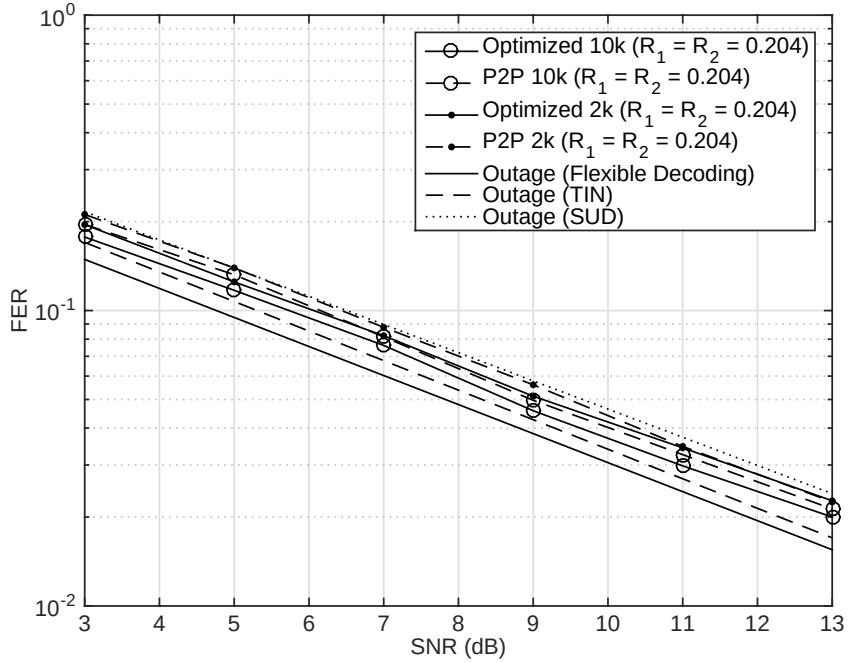


Figure 4.9: Performance results with flexible decoding.

4.7 Chapter Summary

We study code design for two-user quasi-static fading ICs assuming two different signaling schemes, namely, with rate splitting and without rate splitting. An implementable method for designing LDPC codes over quasi-static fading ICs is proposed. The proposed method is evaluated for several examples, and the achieved rates are compared with the corresponding achievable rate regions. For quasi-static fading, we perform code design for SUD and flexible decoding schemes. For both decoding approaches, a performance close to the outage limit is attained. We also observe that by performing flexible decoding, we can obtain an improvement in the outage behavior of the system. Additionally, multiple antenna transmissions employing the Alamouti scheme are investigated, and after designing a new receiver structure, LDPC code design is also performed. For all the numerical examples considered, we notice that improvements over the P2P optimal codes can be attained.

Chapter 5

Coding for Cognitive Interference Channel

In this chapter, we focus on transmission schemes for cognitive interference channels, variant of classical ICs in which the cognitive transmitter has non-causal knowledge of the primary message. First, we study an explicit coding scheme for CIC in the primary decodes cognitive regime. Considering the optimal decoding strategy, the cognitive receiver decodes only its own message, which is pre-coded against the interference observed at the cognitive receiver, and the primary receiver jointly decodes both primary and cognitive messages. We present a novel decoder that combines soft information using a state node and allows for iterative processing. We also design LDPC codes by utilizing EXIT charts armed with a novel asymptotic stability analysis. Simulation results indicate that the proposed decoder and the designed codes provide improvements over the conventional maximum ratio combining type decoder and the optimized P2P codes, respectively.

We then propose a simple transmission scheme for the dirty paper channel dubbed as flexible modulation. We show that with a very simple operation at the transmitter, we can achieve a significant performance improvement due to the

additional information about the interfering signal for some set of channel parameters. We also show that the proposed flexible modulation scheme is robust to inaccuracies in the channel state information of the unintended transmitter. Later, we consider the fading scenario for a particular form of MAC, for which one of the users has non-causal knowledge of the message of the other one. We assume the case where the channel state information is not available at the transmitters. We propose a joint encoding based scheme in which the cognitive transmitter encodes the two messages jointly, and compare the resulting performance with the use of conventional convolutional codes with the same decoder complexity. Simulations show that the joint encoding approach can improve the BER performance of the primary message. Also, we observe that decoding the primary message and canceling it out can improve the BER performance of the cognitive message in some instances as well. This technique can be applied for both Gaussian channels and fading channels, and even without channel state information at the transmitter, it performs well.

The of the chapter is organized as follows. In Section 5.1, we review the existing studies on CIC, and present one of the efficient implementations of DPC. In Section 5.2, we present the system model for a CIC. In Section 5.3, we present a practical and efficient coding scheme for CICs in primary decodes cognitive regime. In Section 5.4, we focus on the DPC framework, and propose a simple coding scheme that exploits the side information of interfering signal improving the BER performance of the system. In Section 5.5, we consider a particular form of MAC and investigate the idea of joint encoding. Finally, we conclude the chapter in Section 5.6.

5.1 Introduction

CIC is a type of interference channel with primary and cognitive transmitter-receiver pairs. In this channel model, the cognitive transmitter has non-causal knowledge of the primary message (that can be obtained with the aid of a good link between the two transmitters), which is frequently encountered in modern

wireless communications systems [77]. The capacity region of CIC is not known in general, however, the “primary decodes cognitive” regime is one of the regimes with a known capacity region [38]. Information theoretical results show that, in the primary decodes cognitive regime, the optimal strategy is DPC against the interference caused by the primary message at the cognitive receiver. At the receivers’ side, the cognitive receiver decodes only its own message while the primary receiver decodes both messages. To the best of our knowledge, [78] is the only work in the literature on the practical and explicit coding schemes for this challenging regime.

In [78], the primary receiver combines three different transformations of the channel observation using a maximum ratio combiner (MRC). The output of MRC is fed to a BCJR decoder, which works in parallel with two single-user decoders. We make the observation that the method of combining the transformations of the channel observation is not optimal and may degrade the performance of the receiver; and hence, we design a new structure for the primary receiver, in which the primary and cognitive messages are decoded in parallel with the aid of an advanced state node decoder. Due to their promising performance in multiuser communications [1, 4, 5], we consider LDPC codes as the channel coding solution and develop a code design procedure. Simulation results demonstrate that the performance of our receiver surpasses that of the receiver proposed in [78]. Moreover, our designed codes outperform the P2P optimal ones.

5.1.1 Review of Implementation of Dirty Paper Coding

Motivated by the applications of DPC, different studies have been published for its efficient and practical implementation. Ref. [79] is the first practical implementation of DPC under the name of quantization index modulation. In [80], the authors utilize trellis coded quantization (TCQ) [81] as the source code and trellis coded modulation as the channel code [82] to implement DPC. However, these implementations are several dBs away from the information theoretic limits. As an alternative implementation, [25] is based on nested turbo codes which

performs 1.42dB away from the information theoretic limit at a rate of 1 b/s.

The aforementioned designs do not perform well in the low SNR regime. Refs. [83] and [84] target the low SNR regime, and obtain a performance only 1.3 dB away from the information theoretic limit at a rate 0.25 b/s. The authors in [85] utilize irregular repeat accumulate (IRA) codes as the channel code and TCQ as the source code, and their proposed scheme performs 0.638 dB away at the same rate. Finally, in [7] the authors utilize an LDPC code and TCQ as the channel and source coding components, respectively. Their design performs 0.95 dB away from the theoretical limits, moreover it is robust in the sense that it performs close to the information theoretic limit in the high, medium, and low rate regimes. In this work, we pick the implementation of [7] due to its efficiency and simplicity, and in the following, we discuss it in detail.

5.1.1.1 DPC Encoder

The block diagram of the DPC encoder is depicted in Fig. 5.1, where S is the interference signal observed at the cognitive receiver, and known non-causally by the cognitive transmitter. This signal is scaled by the Costa scaling factor ($\lambda_{Costa} = \frac{P_1}{P_1 + N_1}$) and after adding to a pseudo random dither sequence D , it is forwarded to a TCQ unit. Also, the cognitive message is encoded by a component LDPC encoder with rate R_1 , and the encoded message W_1 (with binary elements) is feeded to the TCQ. The TCQ uses the Viterbi algorithm over a rate $\frac{1}{2}$ convolutional code with modulo 4Δ distance metric and chooses a coset from the alphabet $\{0, \Delta, 2\Delta, 3\Delta\}$ corresponding to a two-bit output (d_1, d_2) . The selected coset is shifted by $\frac{\delta}{2}(1 - 2W_1)$. Therefore, the elements of TCQ codeword are in the set $A = \{-\delta/2, \delta/2, \Delta - \delta/2, \Delta + \delta/2, 2\Delta - \delta/2, 2\Delta + \delta/2, 3\Delta - \delta/2, 3\Delta + \delta/2\}$. Finally, the primary codeword is obtained by the quantization error between $(\lambda S + D)_{mod4\Delta}$ and the TCQ codeword, namely,

$$\begin{aligned}\sqrt{P_1}X_c &= [\lambda S + D, W_1]_{TCQ} - (\lambda S + D)_{mod4\Delta}, \\ &= \Lambda - (\lambda S + D)_{mod4\Delta}.\end{aligned}\tag{5.1}$$

We notice that the transmitted power is adjusted by the use of the Δ parameter [78].

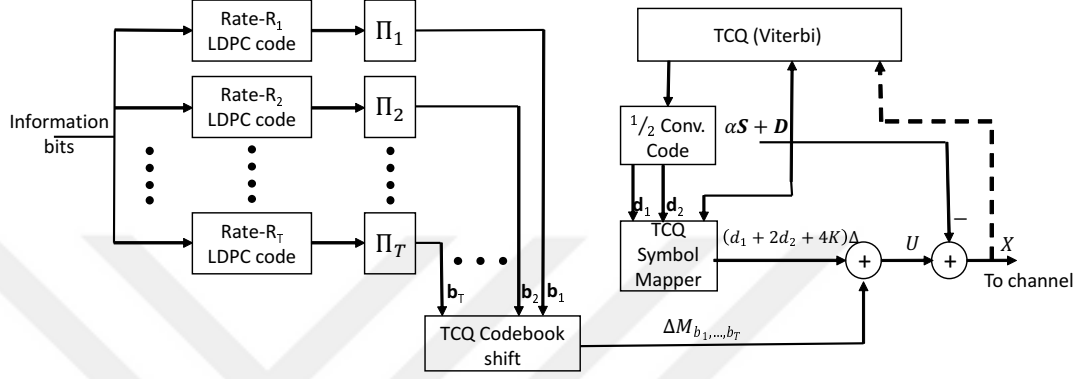


Figure 5.1: Multi-level DPC encoder block diagram.

5.1.1.2 DPC Decoder

As depicted in Fig. 5.2, the cognitive message is decoded by employing a belief-propagation (BP) algorithm wherein the soft-information about the primary message is exchanged in an iterative fashion between a BCJR algorithm on the TCQ trellis and a component LDPC decoder. In order to remove the dither, the receiver considers the following channel observation transformation

$$\begin{aligned}
 Y_1 &= \sqrt{P_1}X_c + S + z_1, \\
 Y_1' &= \left(\lambda_{Costa}Y_1 + D \right)_{\text{mod}4\Delta} \\
 &= \left(\underbrace{\sqrt{P_1}X_c + (\lambda_{Costa}S + D)_{\text{mod}4\Delta}}_{\Lambda} + \underbrace{(\lambda_{Costa} - 1)\sqrt{P_1}X_c + \lambda_{Costa}z_1}_{\text{noise } z_1'} \right)_{\text{mod}4\Delta}.
 \end{aligned} \tag{5.2}$$

The channel Y_1' is equivalent to a P2P transmission of codeword Λ through a modulo AWGN channel. Simulations show the distribution of the cognitive codeword X_c is very close to a Gaussian distribution. As a result, the noise z_1' in the equivalent channel in (5.2) has Gaussian distribution with zero mean and variance

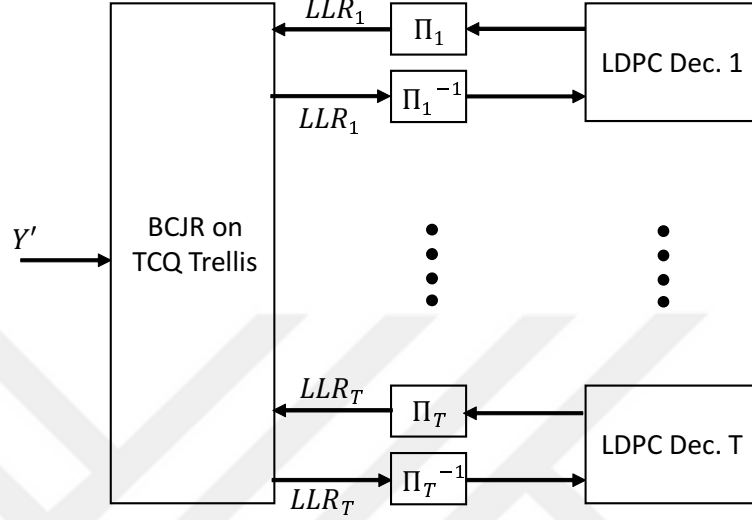


Figure 5.2: Multi-level DPC decoder block.

$P_{z_1'} = \lambda_{Costa}^2 N_1 + (\lambda_{Costa} - 1)^2 P_1$. The BCJR algorithm calculates the extrinsic LLR of the cognitive codeword by considering Y_1' and the respective LLRs coming from the component LDPC decoder. The iterations between the BCJR algorithm and the LDPC decoder components continue until a valid codeword is found, or a predetermined number of iterations is reached.

5.2 System Model

A two user Gaussian CIC is described by the following input-output relationship

$$\begin{aligned} Y_1 &= h_{11}X_1 + ah_{21}X_2 + z_1, \\ Y_2 &= bh_{12}X_1 + h_{22}X_2 + z_2, \end{aligned}$$

where X_i is the transmitted signal from transmitter i , which satisfies the power constraint $E[X_i^2] \leq P_i$, and z_i denotes the Gaussian noise component at the receiver i with zero mean and variance N_i , $i = 1, 2$. The channel gains are real and for the fixed channel scenario, it is assumed they are known at all terminals.

Armed with a non-causal knowledge of the primary message, the cognitive transmitter can boost the primary message transmission by utilizing power allocation as $X_1 = \sqrt{\alpha P_1} X_c + \sqrt{(1-\alpha) P_1} X_p$ and $X_2 = \sqrt{P_2} X_p$ where X_c and X_p denote the modulated cognitive and primary codewords, respectively. Notice that X_p is binary phase shift keying (BPSK) modulated, i.e., 0 and 1 are mapped to $+1$ and -1 , respectively. α is the power sharing parameter in the interval $(0, 1]$. Hence the received signals can be represented as follows

$$\begin{aligned} Y_1 &= \sqrt{\alpha P_1} X_c + \left(a + \sqrt{\bar{\alpha} P_1 / P_2}\right) \sqrt{P_2} X_p + z_1 \\ Y_2 &= b \sqrt{\alpha P_1} X_c + \left(1 + b \sqrt{\bar{\alpha} P_1 / P_2}\right) \sqrt{P_2} X_p + z_2, \end{aligned} \quad (5.3)$$

where $\bar{\alpha} = 1 - \alpha$.

5.3 Coding for CIC in the Primary Decodes Cognitive Regime

In this section, we propose a novel joint decoder structure for CIC for primary decodes cognitive regime. Furthermore, we present an LDPC code design for the set-up.

5.3.1 Joint Decoding Scheme

For the regime under consideration, the primary receiver decodes both primary and cognitive messages. As an alternative to the joint decoder with MRC [78], we propose a novel joint decoder as depicted by Fig. 5.3(b). In what follows each decoding block is explained in detail.

The cognitive codeword decoder block: As depicted in Fig. 5.3(b), this decoder block consists of a BJCR decoder connected to a component LDPC

decoder where λ_c is the scaling factor of the block. We consider a transformation of Y_2 such that

$$\begin{aligned} Y_2' &= (\lambda_c Y_2 + D)_{\text{mod}4\Delta} \\ &= \left(\Lambda + \underbrace{(\lambda_c h_p - \lambda_{Costa} h_s) X_p}_{\text{interference}} + \underbrace{(\lambda_c b - 1) \sqrt{\alpha P_1} X_c + \lambda_c z_2}_{\text{noise } z_2'} \right)_{\text{mod}4\Delta}, \end{aligned} \quad (5.4)$$

where h_p and h_s are the coefficients determining the contributions of X_p to the signals Y_1 and Y_2 , respectively. W_2 is the primary codeword, d_1 and d_2 are the quantized bits corresponding to $\lambda S + D$. The noise signal z_2' is approximately a Gaussian random variable with zero mean and variance $P_{z_2'} = (\lambda_c b - 1)^2 \alpha P_1 + \lambda_c^2 N_2$. In order to minimize the sum of the interference and noise powers, λ_c is optimized as

$$\hat{\lambda}_c = \arg \min_{\lambda_c \in \mathbb{R}} (\lambda_c h_p - \lambda_{Costa} h_s)^2 + P_{z_2'}. \quad (5.5)$$

BCJR algorithm is used to calculate the extrinsic LLRs of the cognitive codewords represented by $L(W_1)$. In addition, it calculates the log of the probabilities for the bits (d_1, d_2) which are sent to the X_2 decoder. On the other hand, the component LDPC decoder improves the LLRs of the cognitive codewords and supplies them to the X_2 decoder.

The primary codeword decoder block: As depicted in Fig. 5.3(b), this decoder block consists of a state node connected to a component LDPC decoder, where λ_p is the scaling factor of the block. The state node updates $L(W_2)$ using the transformed received signal $\tilde{Y}_2$ and $L(W_1)$ coming from the DPC decoder, whereas the component LDPC decoder decodes the primary message. $\tilde{Y}_2$ is simply defined by

$$\begin{aligned} \tilde{Y}_2 &= (\lambda_p Y_2 + D)_{\text{mod}4\Delta} \\ &= \left(\Lambda + \underbrace{(\lambda_p h_p - \lambda_{Costa} h_s) X_p}_{\text{interested message.}} + \underbrace{(\lambda_p b - 1) \sqrt{\alpha P_1} X_c + \lambda_p z_2}_{\text{noise } \tilde{z}_2} \right)_{\text{mod}4\Delta}. \end{aligned} \quad (5.6)$$

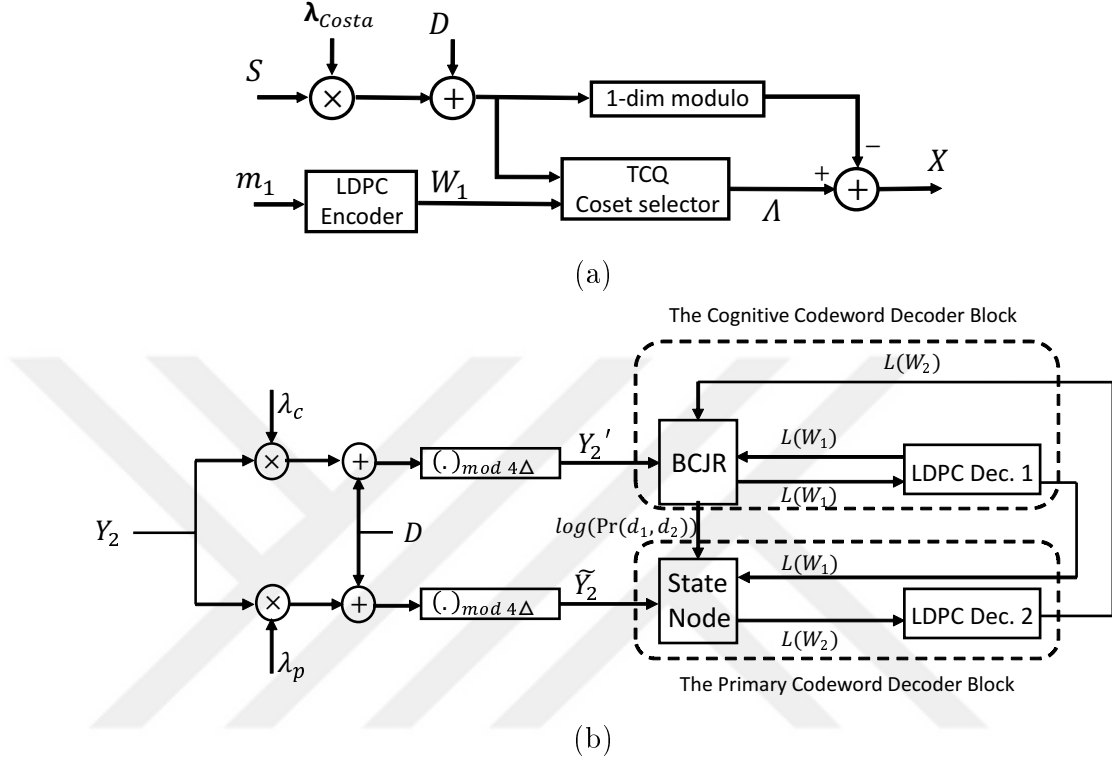


Figure 5.3: (a) The block diagram of the DPC encoder, (b) The block diagram of the proposed receiver.

At the state node, $L(W_2)$ is calculated as

$$L(W_2[i]) = \log \left(\frac{f_{\tilde{Y}_2}(\tilde{Y}_2[i] | W_2[i] = 0)}{f_{\tilde{Y}_2}(\tilde{Y}_2[i] | W_2[i] = 1)} \right),$$

where the conditional PDFs are calculated by marginalizing over Λ , as similar to the procedure described in [4]. In order to maximize the signal-to-noise ratio in (5.6), we optimize λ_p as

$$\hat{\lambda}_p = \arg \max_{\lambda_p \in \mathbb{R}} \frac{(\lambda_p h_p - \lambda_{Costa} h_s)^2}{(\lambda_p b - 1)^2 \alpha P_1 + \lambda_p^2 N_2}. \quad (5.7)$$

We notice that, while λ_c is optimized by minimizing interference and noise powers, λ_p is optimized under the assumption that the interference signal Λ is known and can be cancelled out perfectly. Hence this approach results in a suboptimal solution, however, it simplifies the parameter optimization considerably.

5.3.2 Performance Analysis

5.3.2.1 Stability Condition

In the following, we derive the stability condition for the cognitive message decoded at both receivers, and for the primary message decoded at the primary receiver. In the derivations, we assume asymptotic approximate analysis of LDPC codes and error-free BJCR decoding on the TCQ trellis.

The Cognitive Message: We assume that the interference signal and the convolutional code (coset selector) have been completely decoded and cancelled out from the received signals. Under this assumption, (5.2) and (5.4) turn into $Y_1' = (\delta/2 - \delta W_1 + z_1')_{mod4\Delta}$ and $Y_2' = (\delta/2 - \delta W_1 + z_2')_{mod4\Delta}$, respectively. These modified channels are actually modulo P2P channels. In Appendix B, the Bhattacharyya constant of modulo P2P channel is derived for BPSK modulation. The overall stability condition for the cognitive LDPC code ensemble is that the first derivative of the variable node degree distribution polynomial $\lambda(x)$ evaluated at $x = 0$ multiplied by the first derivative of the check node degree distribution polynomial $\rho(x)$ evaluated at $x = 1$ should be smaller than the minimum of the two Bhattacharyya constants derived from the analysis for the cognitive and primary receiver, respectively.

The Primary Message: We derive the stability condition for the primary code ensemble under the assumption that the convolutional code and cognitive message are completely decoded and cancelled out. Hence, the channel in (5.6) turns into $\tilde{Y}_2 = (\lambda_p h_p X_p - \lambda_{Costa} h_s X_p + \tilde{z}_2)_{mod4\Delta}$. The stability condition of the primary code ensemble is obtained similarly by utilizing the Bhattacharyya constant of this modulo P2P channel.

5.3.3 LDPC Code Design

For code design, we aim at finding the highest rate pair for which reliable communication can take place for the given set of channel parameters. We employ a recursive systematic convolutional code at the TCQ and fix the generator polynomial in octal form as $[625, 242]^1$ [81]. Then, we consider optimization of the LDPC code ensembles. We utilize small perturbations to find the code ensembles with higher rates gradually. The decoding thresholds of the candidate ensembles are examined by an EXIT chart analysis powered by the stability condition, where the latter is checked for eliminating error floors. While utilizing EXIT chart analysis, we make the Gaussianity assumption for the LLRs exchanged among the blocks, except for the LLRs incoming to the state node. Due to the non-linearity of the state node function, we model the links of the incoming LLRs to the state node as binary erasure channels [1]. We calculate the mutual information of the LLRs outgoing from the BCJR decoder by utilizing a look-up table which is constructed using Monte-Carlo simulations.

If the ensembles satisfy the stability condition, and if their decoding tunnels are open, which is examined using the EXIT chart analysis, we consider these ensembles as admissible. The process of searching for new ensembles continues until admissible code ensembles cannot be improved any further in terms of rate.

5.3.4 Numerical Results

We first consider the example in [78] with parameters $P_1 = 1.1$, $P_2 = 1$, $N_1 = 1.325$, $N_2 = 3.15$, $a = -1.2$, $b = 1.53$ and assume an MRC type decoding at the primary receiver. Fig. 5.4 shows the capacity region and the achieved rate pair reported in [78] with the off-the-shelf IRA codes with 40k block length at a bit error rate of 10^{-5} . Assuming the same block length and bit error rate, our code design results in rate improvements such that $(0.239, 0.37)$ and $(0.265, 0.288)$ rate pairs can be achieved with the proposed joint decoder structure. Table 5.1 shows

¹We choose this generator polynomial owing to its good performance in DPC [7].

Algorithm 3 Code design algorithm for CIC in primary decodes cognitive regime

Input: The channel parameters and d_c values for users, σ_e^2 , Δr_2 and T .

- 1: Initialize degree distribution polynomials for users: $\lambda_j(x)$, $j = 1, 2$.
- 2: Set $t = 0$.
- 3: **while** $t < T$ **do** ▷ main iteration starts
- 4: **for** $j = 1, 2$ **do**
- 5: Generate a random perturbing vector satisfying (3.21),
 (3.22) and (3.23).
- 6: Construct the polynomial representation for the random
 perturbing vector: $e_j^t(x)$.
- 7: Calculate $\lambda_j^t(x) = \lambda_j(x) + e_j^t(x)$.
- 8: **end for**
- 9: **if** Every $\lambda_j^t(x)$ satisfies the stability condition and the
 successful asymptotic decoding **then**
- 10: $\lambda_j(x) = \lambda_j^t(x)$, $j = 1, 2$. ▷ improve degree
 distribution polynomials
- 11: **go to 2**.
- 12: **else**
- 13: $t = t + 1$.
- 14: **end if**
- 15: **end while** ▷ main iteration ends
- 16: Output: $\lambda_j(x)$, $j = 1, 2$.

the designed LDPC degree distributions.

Next we consider an instance of CIC with parameters $P_1 = 1.1$, $P_2 = 1$, $N_1 = 1.325$, $N_2 = 3.15$, $a = -0.38$, $b = 2$ and $\alpha = 0.68$. For this example, we perform LDPC code design for both the proposed joint decoder and the MRC-type decoder. We provide the resulting degree distributions in Table 5.1. In Fig. 5.5, we illustrate the error-rate performance of the designed LDPC codes for the primary message with blocklength 40k. In the same figure, we also show the performance of P2P optimal codes with the same rate and block length. We observe that, at a bit error rate of 10^{-4} , the advantage of joint decoding is at least 0.3 dB. Furthermore, the code design provides an improvement of at least 0.7 dB. However, we note that, the joint decoding advantage depends on the channel parameters, i.e., there exist channel parameters for which the MRC-type

decoder surpasses the proposed joint decoder.

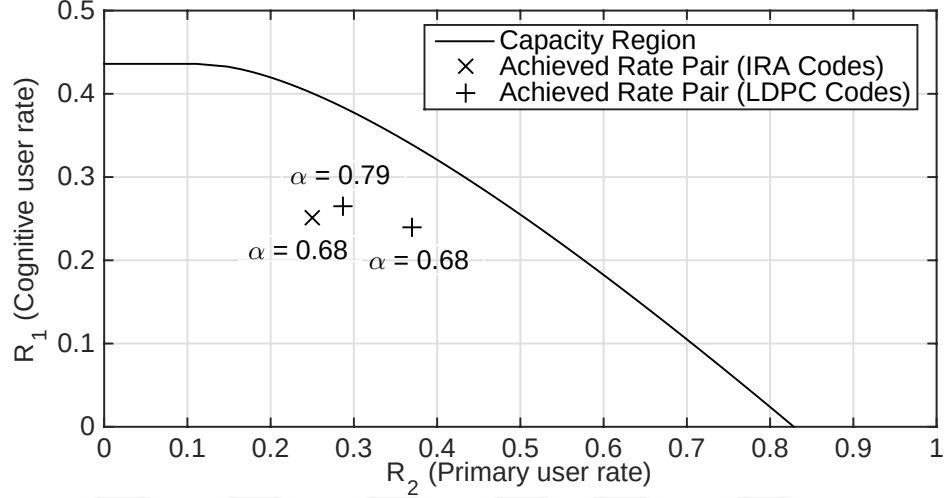


Figure 5.4: The channel capacity and the achieved points.

5.4 Flexible Modulation for the DPC Problem

Considering the high complexity of the DPC scheme, it is also important to study alternatives for which the transmitter applies some simpler techniques to take the advantage of the knowledge of the interfering signal. We develop one such technique, which we call flexible modulation, in this section.

Let us consider the following P2P channel

$$Y_1 = h_1 X_1 + h_2 X_2 + z_1, \quad (5.8)$$

where the information of the interference signal $S = h_2 X_2$ is available at the transmitter, and z_1 represents zero mean white Gaussian noise.

The idea of flexible modulation is to use the knowledge of the interference signal to increase the minimum distance of two constellation points with different intended bit values. In this technique, the transmitted signal $X_1[i]$ is a function

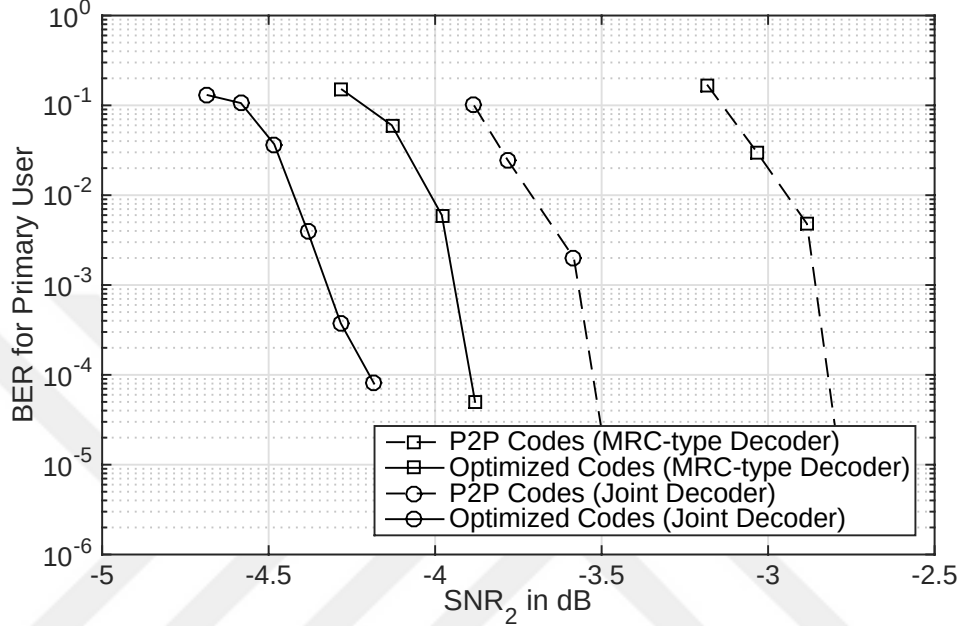


Figure 5.5: Finite block length decoding results. SNR_i is defined as P_i/N_i and $\text{SNR}_2/\text{SNR}_1$ is kept fixed.

of both coded bits $c_1[i]$ and $c_2[i]$ in contrast to standard modulation for which the transmitted signal $X_1[i]$ is determined based on only $c_1[i]$. The mapping function is chosen based on the channel gains h_1 and h_2 . As both the transmitter and receiver know the channel gains, they determine the mapping function in the same manner independently at no extra communication cost.

As an instance, we assume BPSK modulation for both signals X_1 and X_2 , and consider real channel gains with $\text{SNR}_1 = \text{INR}_1 + 0.5\text{dB}$. We devise a specific mapping function for the flexible modulation scheme to maximize the minimum distance ($d_{\min}$) between two constellation points with different intended bit values. We consider the following mapping function $X_1[i] = 1 - 2(c_1[i] \oplus c_2[i])$, where $\oplus$ denotes the *XOR* operation. Additionally, in the regular modulation, the mapping function is $X_1[i] = 1 - 2c_1[i]$ (Fig. 5.6). The $d_{\min}$ in the regular modulation is $d_{\min} = \min(2||h_1| - |h_2||, 2|h_1|)$, while in the proposed flexible modulation scheme, it is $d_{\min} = \min(2|h_1|, 2|h_2|)$. Therefore, if

Table 5.1: Optimized LDPC degree distributions.

Msg	R	λ_2	λ_3	λ_4	λ_6	λ_7	λ_{12}	λ_{13}	λ_{25}	λ_{100}	λ_{200}
W_1 (Fig. 5.4)	0.239	0.2376	0.1258	0.1279	0.0260	0.0748	0.0141	0.0533	0.0288	0.0954	0.2163
W_1 (Fig. 5.4)	0.265	0.2443	0.1307	0.1213	0.0200	0.1170	0.0072	0.0496	0.0301	0.0687	0.2110
W_2 (Fig. 5.4)	0.288	0.3209	0.1256	0.0462	0.0027	0.0235	0.0019	0.0096	0.1851	0.0021	0.2824
W_1 (Fig. 5.5)	0.239	0.2377	0.1258	0.1279	0.0260	0.0748	0.0141	0.0533	0.0288	0.0954	0.2163
W_2 (Fig. 5.4)	0.370	0.2407	0.0715	0.1885	0.0375	0.0749	0.0113	0.0112	0.2367	0.0947	0.0330
W_2 (Fig. 5.5)	0.394	0.3162	0.1448	0.0868	0.0101	0.0114	0.0241	0.0004	0.1426	0.0944	0.1692
W_2 (Fig. 5.5)	0.394	0.2299	0.1238	0.1868	0.0227	0.0222	0.0521	0.0018	0.3235	0.0215	0.0159

Msg.	R	ρ_2	ρ_3	ρ_6	ρ_7	ρ_8
W_1 (Fig. 5.4)	0.239	0.0126	0	0.7228	0.2646	0
W_1 (Fig. 5.4)	0.265	0.0106	0	0.7399	0.2494	0
W_2 (Fig. 5.4)	0.288	0.0051	0	0.7280	0.2669	0
W_1 (Fig. 5.5)	0.239	0.0126	0	0.7228	0.2646	0
W_2 (Fig. 5.4)	0.370	0	0.0035	0	0.7395	0.2570
W_2 (Fig. 5.5)	0.394	0	0.0420	0	0.6893	0.2688
W_2 (Fig. 5.5)	0.394	0	0.0016	0	0.7062	0.2922



Figure 5.6: Demonstration of constellation points for BPSK modulation and real channel gain scenarios for regular mapping (left hand side) and flexible mapping (right hand side). The labels represent (c_1, c_2) .

$\min(2|h_1|, 2|h_2|) > \min(2||h_1| - |h_2||, 2|h_1|)$, the minimum distance can be increased by employing the proposed approach.

We evaluate the BER performance of uncoded signals for flexible modulation and standard schemes in Fig. 5.7. It is observed that the flexible modulation technique increases the minimum distance of the constellation points which results in more than a 20 dB BER improvement at a BER of 10^{-5} .

As another instance of flexible modulation, we consider quadrature phase shift keying (QPSK) modulation and complex channel gains. In this example we take $h_2 = h_1 \exp(j\frac{\pi}{16})$. We find a mapping function to maximize the d_{min} which is demonstrated in Fig. 5.8. We evaluate and compare the BER performance of the proposed technique with the specific mapping function compared with the regular modulation for uncoded transmissions (Fig. 5.9). The simulations show that flexible modulation provides more than 10 dB improvement at a BER of 10^{-5} compared to regular modulation for the chosen set of channel parameters.

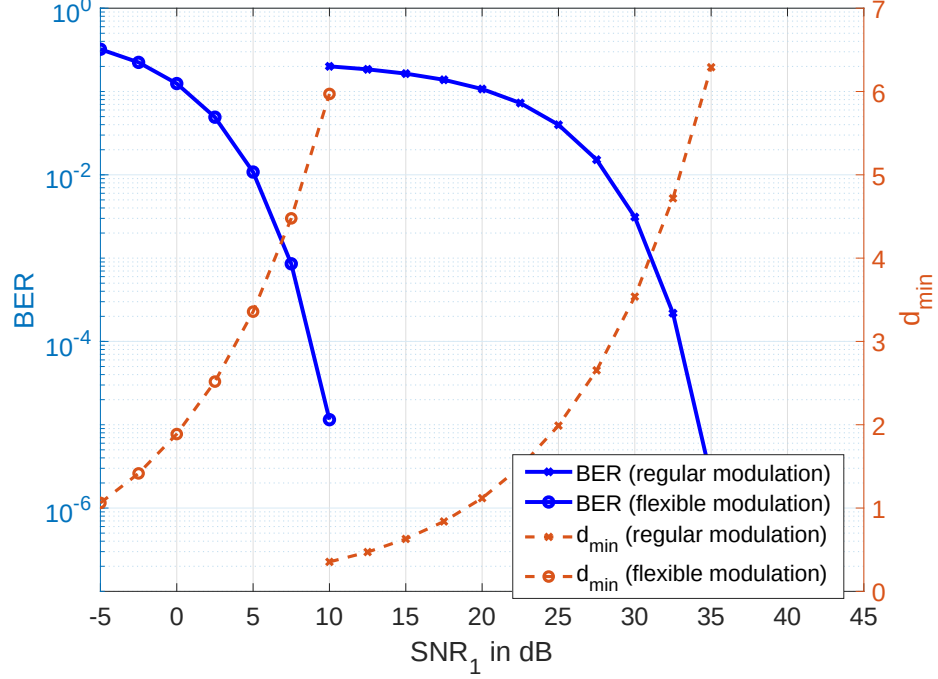


Figure 5.7: The comparison of the BER performance of flexible modulation and regular modulation for uncoded transmission with BPSK, real channel gains and $\text{SNR}_1 = \text{INR}_1 + 0.5$.

Next, we evaluate the performance of flexible modulation with channel coding. We pick LDPC codes designed for P2P transmission with blocklength 10k, and rates 0.4 and 0.5 for the intended and unintended messages, respectively. We compare the performance of flexible modulation with regular modulation, DPC scheme, and also with the no interference at the cognitive receiver case. As it is depicted in Fig. 5.10, the flexible modulation outperforms the regular modulation by around 5 dB at $\text{BER} = 10^{-4}$. Also, it is observed that the DPC outperforms the flexible modulation at $\text{BER} = 10^{-4}$, by about 1 dB. However, considering the complexity of DPC, the proposed flexible modulation approach can still be a better option in some applications.

To investigate the performance of DPC and flexible modulation when the side information on the interference signal is not accurate, we consider the following

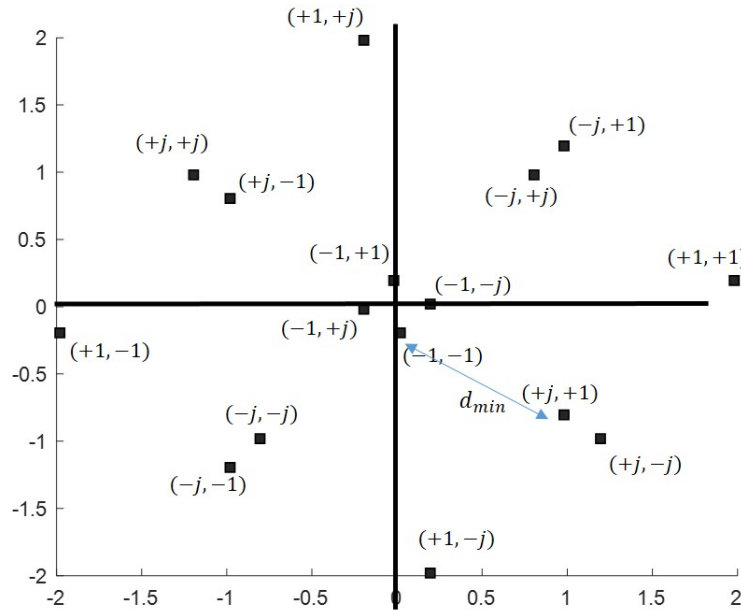
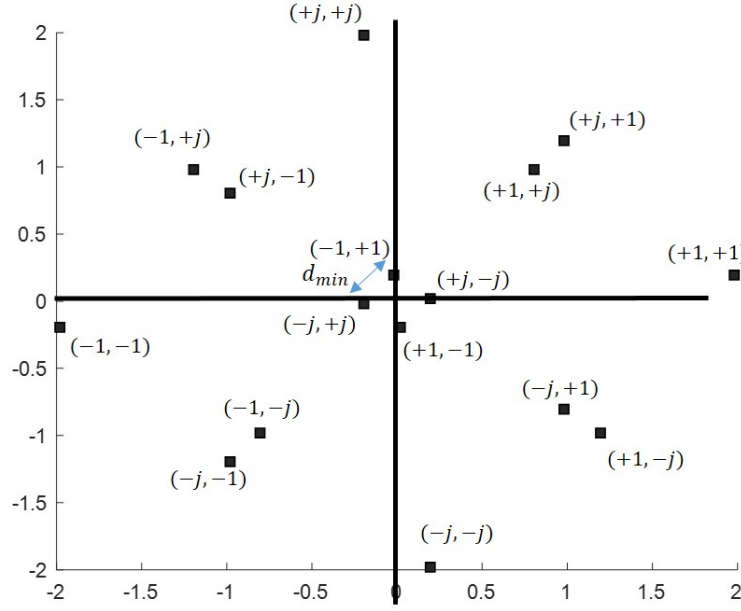


Figure 5.8: Demonstration of equivalent constellation points for QPSK modulation, complex channel gains and $h_2 = h_1 \exp(j\frac{\pi}{16})$ for regular mapping (top) and flexible mapping (bottom). The labels represent (c_1, c_2) .

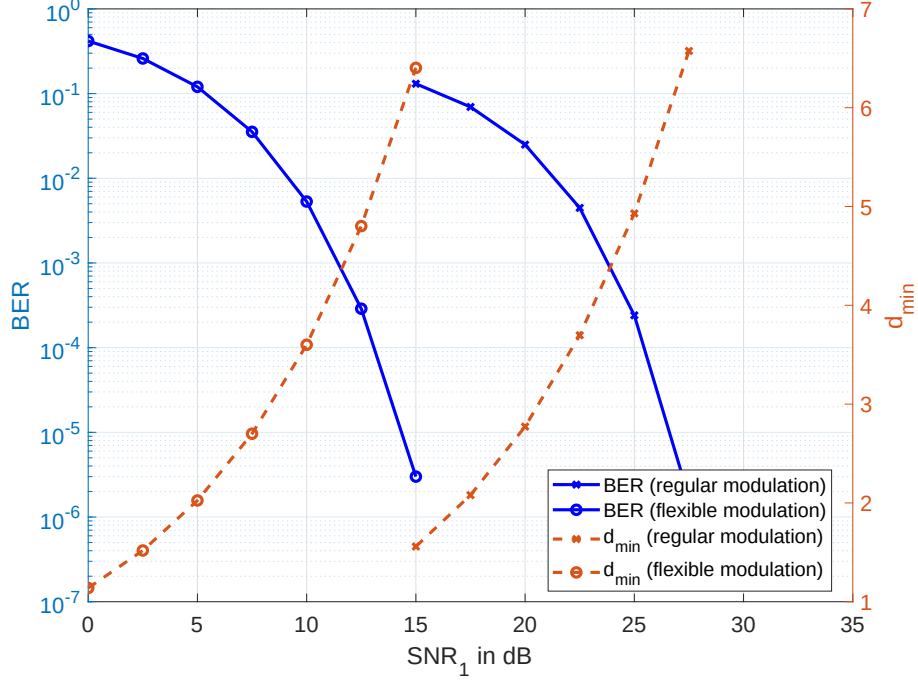


Figure 5.9: Comparison of the BER performance of flexible modulation and regular modulation for uncoded bits with QPSK and complex channel gain and $h_2 = h_1 \exp(j\frac{\pi}{16})$.

channel model

$$Y = h_1 X_1 + (1 + w) h_2 X_2 + z_1, \quad (5.9)$$

where w represents the inaccuracy in the unintended transmitter channel measurement which is modeled as a Gaussian random variable with σ_w^2 variance and zero mean. Same as the previous simulation set-up, we take $\text{SNR}_1 = \text{INR}_1 + 0.5 \text{ dB}$, and consider length 10k, P2P optimal LDPC codes for the cognitive and primary messages with rates 0.4 and 0.5, respectively. As it is demonstrated in Fig. 5.11, for $\sigma_w^2 = 0.02$, the DPC scheme outperforms the flexible modulation by 1 dB. However, for $\sigma_w^2 = 0.05$ the flexible modulation outperforms the DPC by 0.75 dB. The results demonstrate that the flexible modulation shows more robustness compared to the DPC to the inaccuracies in the channel state information.

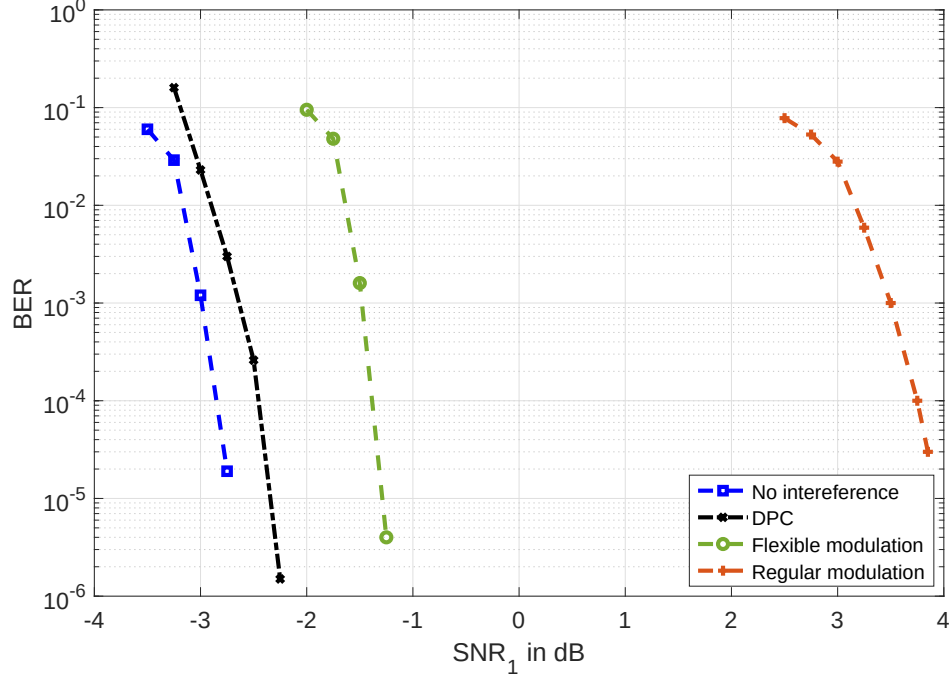


Figure 5.10: Comparison of the performance of flexible mapping with channel coding with other techniques for BPSK modulation and $\text{SNR}_1 = \text{INR}_1 + 0.5$.

In this subsection, we introduce flexible modulation as a simple and practical technique to exploit the side information about the interfering signal at the transmitter side. Simulations demonstrate that the performance of this technique is not as good as DPC. However, its simplicity and robustness to inaccuracies in the channel side information may make it an attractive solution for some applications.

5.5 Joint Encoding Scheme for MAC with One Transmitter Having Access to Both Messages

In this section, we consider a variant form of classical two-user MAC in which the cognitive (user 1) transmitter has non-causal knowledge of the primary (user 2) message. We propose a new approach called the joint encoding scheme. We consider both Gaussian channels and (fast and quasi-static) fading scenarios and

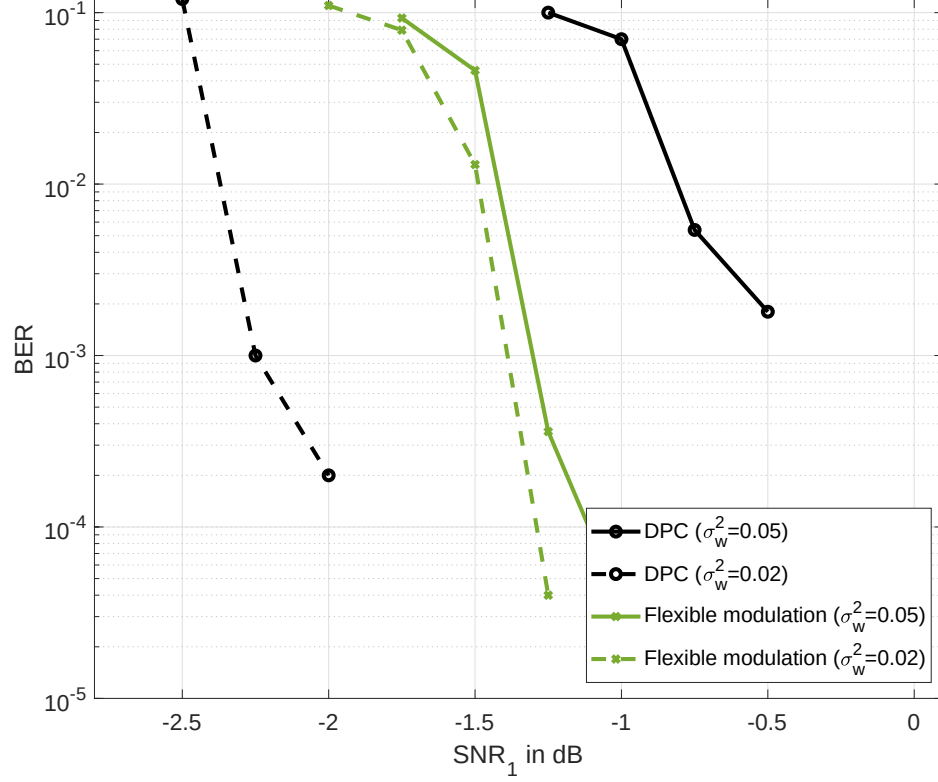


Figure 5.11: Comparison of the performance of flexible mapping with DPC for inaccurate channel state information for BPSK modulation and $\text{SNR}_1 = \text{INR}_1 + 0.5$.

evaluate the BER performance of the new scheme and compare it with the BER performance of conventional channel coding for standard MAC to demonstrate how much can be gained by providing the additional information to the cognitive transmitter and utilizing the proposed approach.

The input-output relationship for the channel can be expressed as follows (see Fig. 5.12):

$$Y_1 = h_1 X_1 + h_2 X_2 + z_1, \quad (5.10)$$

where h_i is the real channel gain between transmitter i and the receiver, for $i = 1, 2$, and z_1 represents zero mean white Gaussian noise. In the joint encoding

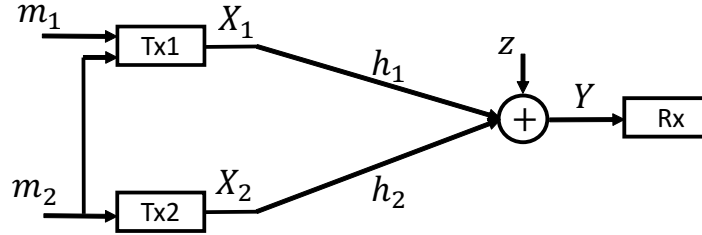


Figure 5.12: The variant form of MAC.

scheme, the cognitive transmitter first interleaves the cognitive and primary information bits (e.g., the odd bits are for the cognitive message and the even bits are for the primary message), and then encodes the output sequence. Also, the primary transmitter encodes its message by a conventional channel code. The receiver decodes the messages jointly, and exploits the primary message information in the cognitive transmitter's signal. We would like to note that the cognitive transmitter uses the information of the primary message regardless of the channel gains, potentially making this technique more robust to changes in the channel.

We evaluate the performance of the joint encoding scheme by several examples, where we employ convolutional codes with rate 0.25 and memory size 5 as the channel coding components. The blocklength of the codes are $4k$. The receiver decodes the messages jointly by utilizing a product trellis with 2^{10} states. For comparison, we consider the classical two-user MAC, where the transmitters independently encode their messages, with the same convolutional codes.

For the unfaded channel case, as it is expected, the BER performance of the primary message is improved with the joint encoding scheme. More interestingly, as observed from Figs. 5.13 and 5.14, in the high SNR regime, both the primary and cognitive users' BERs for the variant MAC are improved compared to the classical MAC. We explain this observation as follows: in the high SNR regime the performance of the system is limited by interference rather than the noise. In the variant MAC, when the primary message is reliably decoded and canceled out from the observed signal, it helps the receiver to decode the cognitive message.

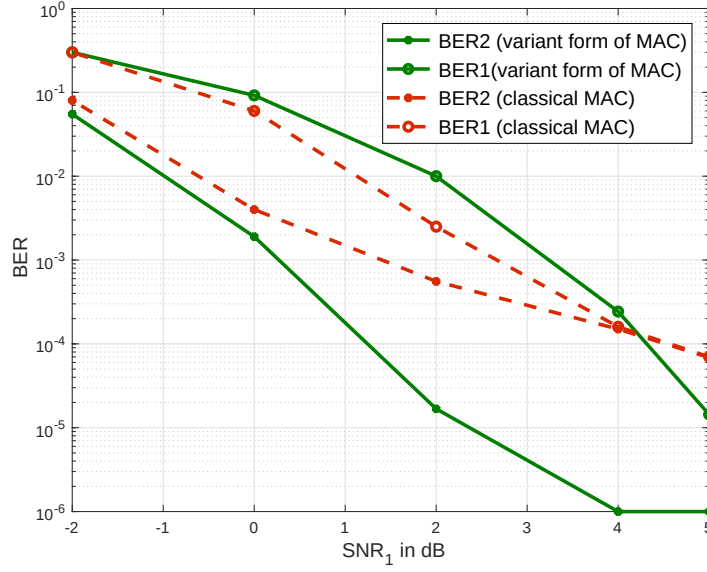


Figure 5.13: BER results for the unfaded channel with $\text{SNR}_1 = 0.5 \times \text{SNR}_2$.

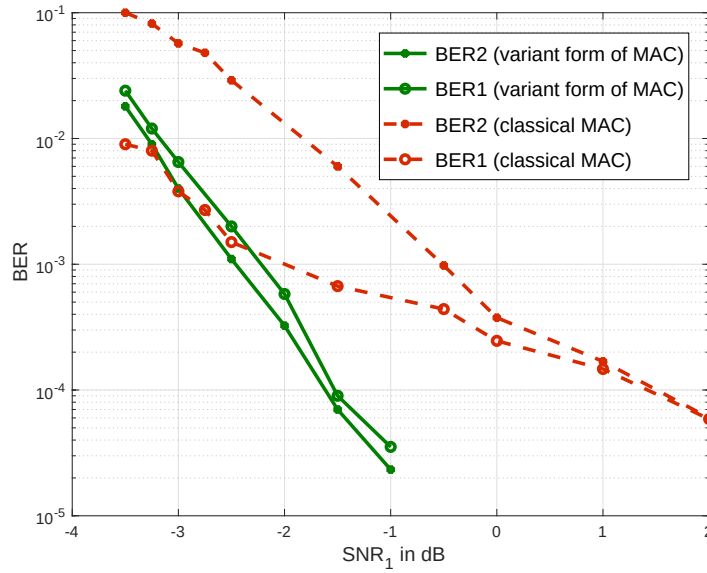


Figure 5.14: BER results for the unfaded channel with $\text{SNR}_1 = 2 \times \text{SNR}_2$.

For the fast fading scenario, we observe that the primary users' BERs for the variant MAC are better than the classical MAC (Fig. 5.15). Unlike the unfaded

case, in this scenario, the cognitive users' BER curve for the classical MAC is better than the variant form.

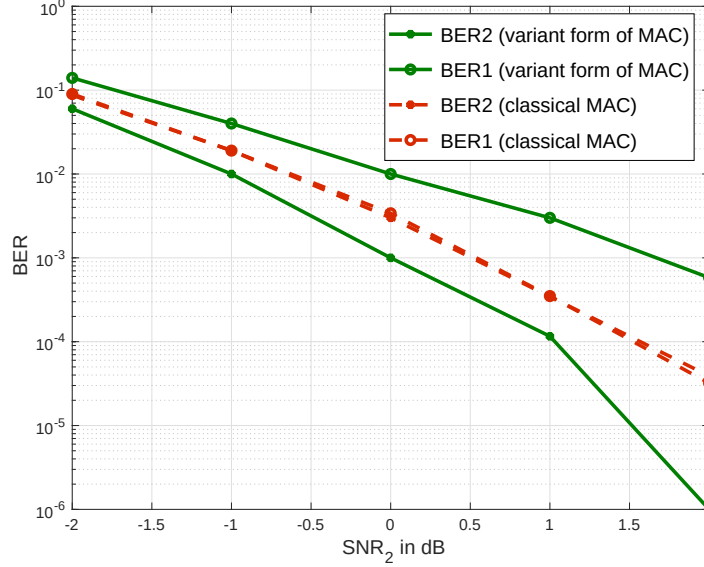


Figure 5.15: BER results for the fast fading channel with $\text{SNR}_1 = \text{SNR}_2$.

For the quasi-static fading scenario, we observe that the primary user enjoys a diversity gain, as the primary message is transmitted over two independent channels (Figs. 5.16 and 5.17). In addition, as the primary message is decoded reliably and canceled out in the medium and high SNR regimes, the cognitive message does not suffer from interference. As a result, both the primary and cognitive messages have better BER performance for the variant form of MAC.

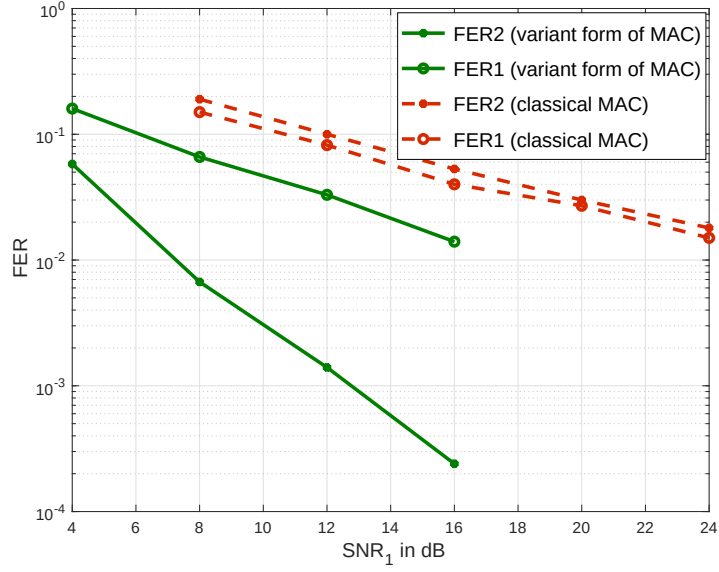


Figure 5.16: BER results for the quasi-static fading channel with $\text{SNR}_1 = 2 \times \text{SNR}_2$.

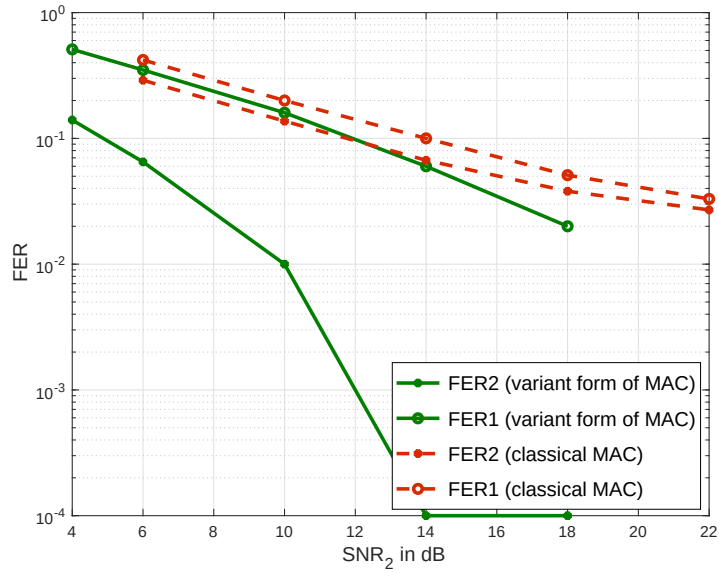


Figure 5.17: BER results for the quasi-static fading channel with $\text{SNR}_1 = 0.5 \times \text{SNR}_2$.

As it is demonstrated via simulations for different cases, the joint encoding

technique can be used for both fixed channels and fading scenarios. Besides improving the BER performance of the primary user's transmission, in some cases, it also improves the BER performance of the cognitive user's transmission. We also note that while we only consider convolutional codes in the above examples, the idea can be extended to employ other channel codes in a straightforward manner as well. This technique can also be employed in CIC and DPC channel with the assumption that the receiver has access to the unintended user's codebook. By joint encoding scheme the transmitter can utilize the side channel information to improve its BER performance even in the fading scenarios.

5.6 Chapter Summary

In this chapter, we study channel coding for cognitive interference channels. First, we consider an explicit coding scheme in the primary decodes cognitive regime. We propose a novel joint decoding scheme and develop an LDPC code design procedure. Simulation results demonstrate that the proposed joint decoder and the designed codes outperform the conventional maximum ratio combining type decoder and the point-to-point optimal codes, respectively. Then, we consider the DPC problem, in which, the intended transmitter has non causal knowledge of the interference caused by the unintended transmitter. We develop a coding scheme for this scenario, called flexible modulation, in which the transmitted bit is chosen based on the bit value of the codeword and the interference signal. Although, the DPC scheme that exists in the literature shows better performance, the simplicity of flexible modulation makes it a good choice in some practical scenarios with decoder complexity concerns. Moreover, the flexible modulation scheme shows more robustness to inaccuracies in the channel state information provided to the transmitter compared to the DPC. Finally, we consider a particular form of MAC, for which one of the users (the cognitive user) has non-causal knowledge of the message of the other one (the primary user). We propose a joint encoding scheme, which can be used for both fixed channels and fading channels. The simulation results show that the proposed scheme can improve the BER performance of the primary message compared to the classical one. Also, for some channel

parameters, the cognitive user can also gain from this type of transmission, as decoding and canceling out the primary message make the cognitive user effectively interference free.



Chapter 6

Summary and Conclusions

In this dissertation, we have studied the problem of LDPC code design for fading interference channels. We focus on fading ICs as they are practical wireless communication models for many scenarios, and they are becoming more and more important as the bandwidth efficiency becomes increasingly important. Moreover, the results of this study can be extended to more complex multiuser channel models (i.e., with larger number of users, transmissions with energy harvesting, etc.). We consider LDPC codes as the channel coding approach since they show promising performance for a wide variety of communication models, and they are being utilized in different communication systems including the upcoming 5th Generation (5G) Wireless Systems.

We start with the problem of LDPC code design for fast fading ICs and motivated by information theoretic results, we investigate implementation of all public and HK based practical and explicit coding schemes. We propose a modified form of the EXIT chart analysis to estimate the decoding threshold of LDPC code ensembles. Also stability conditions for the code ensembles are derived to avoid the error floors and to accelerate the code design process. We also consider short block length communications by focusing on structured LDPC code constructions via protographs. The proposed code designs are employed in many examples and the achieved rate pairs are compared with the ARR boundaries showing that

rate pairs very close to the ARR limits can be attained. Performances of finite block length codes are also simulated for specific codes picked from the optimized LDPC code ensembles in order to verify the analysis results. For all the instances, we notice that significant improvements over the P2P optimal codes can be attained. Furthermore, extensions of the design methodology for three-user ICs are considered, and performance of the optimized codes are compared with that of P2P optimal ones.

We then study the problem of LDPC code design for quasi-static fading ICs. We consider different decoding schemes and derive the corresponding stability conditions for each of them. We observe that rate splitting for quasi-static fading with Rayleigh distribution does not significantly improve the performance of the system. Therefore, we do not split the messages at the transmitters. Also, information theoretical studies demonstrate that by performing flexible decoding, we can obtain an improvement in the outage behavior of the system. We perform code designs by utilizing a random perturbation technique via a modified EXIT chart analysis and the derived stability condition. We consider examples for both SUD decoding and flexible decoding schemes, and show that a performance close to the outage limits can be attained. Furthermore, multiple antenna transmissions with the Alamouti scheme for fading ICs are studied, a special receiver structure is developed, and specific codes are explored. Finally, advantages of the designed codes over P2P optimal ones are demonstrated via finite block length simulations.

In another line of investigation, we consider cognitive multiuser channels. We start with the problem of designing an explicit and practical coding scheme for CIC in the primary decodes cognitive regime. We propose a novel joint decoder structure and perform LDPC code designs. The superiority of the proposed decoding approach and optimized codes is observed compared to the the only joint decoding solution available in the literature and over the P2P optimal codes. We also investigate the idea of using side information at the transmitter (in the form of the knowledge of the interference signal) to choose the constellation points in order to maximize the minimum distance of the observed signal constellation at the receiver. Although the performance of this scheme is not as good as that

of the DPC, the simplicity of the approach makes it a good candidate when good complexity is of concern. We also observe that this flexible modulation scheme shows more robustness to the inaccuracies in the knowledge of the channel gains. Finally, we investigate the idea of joint encoding for a particular form of MAC with fading, for which one of the users (the cognitive user) has non-causal knowledge of the message of the other one (the primary user). That is, the cognitive user assists the primary one by jointly encoding the primary and cognitive message bits. We demonstrate that this technique provides a better BER performance for the primary user, and for some channel parameters, it also improves the BER performance of the cognitive user. The robustness of this technique to the changes in channel is another advantage.

These are promising extensions of the work developed in this thesis. In the LDPC code design for fading IC, we focus on binary LDPC codes. However, non-binary LDPC codes provide higher spectral efficiencies in high SNRs [86–88]. Therefore, designing efficient and practical non-binary LDPC codes for ICs could be interesting. Also design of low complexity joint decoders is important, specifically, for ICs with larger number of users (e.g., in the range of 3 to 5). On the other hand, one of the main goals of the upcoming 5G is to provide low latency communications due to some new applications (e.g., tele-surgery, factory automation, autonomous driving, etc.) which dictates the use of very short blocklength codes [89–95]. With this motivation, extending these P2P code designs with short block lengths to the case of multiuser communications, specifically, to ICs is another possible direction for future work.

Another possible extension of the work in this thesis is in the area of cognitive multiuser channels with fading. In most of this thesis, we assume that the interference signal observed at the receiver is completely known at the transmitter. However, for fading channels, this is not a practical assumption due to channel variations and imperfect knowledge of the channel gains. Noting that DPC does not perform well in presence of channel state information inaccuracies, investigating new approaches that exploit the (imperfect) side information at the transmitter is important. Furthermore, all the existing DPC implementations in the literature suffer from high complexity. That is perhaps one of the reasons

that they are not being employed in current communication systems. Therefore developing practical DPC solutions is also necessary.

Polar codes are a class of capacity-achieving codes introduced in [96]. Owing to this favorable characteristics such as explicit constructions, and modest encoding and decoding complexities with no error floors, they have been adopted in the ongoing 5G standard. Polar codes have been studied for some multiuser scenarios, such as relay channels [97], MACs [98], BCs [99] and unfaded ICs [66, 100]. However, studies on their explicit and practical implementations for fading ICs and CICs are missing from the literature, and could be interesting to explore.

In recent years, advantages of using deep learning (DL) for designing communication systems have been reported [101–113]. In [101, 102], it is demonstrated for AWGN channels that neural codes can achieve a performance similar to good classical codes for very short block lengths. DL has been used to design decoders for well known classical codes, i.e., BCH codes, high density parity-check codes, turbo codes, and polar codes [105–108]. These recent results show the high performance and robustness of neural decoders over the existing ones for certain communications systems. Therefore, the problem of utilizing DL to design efficient codes and transmission schemes for multiuser channels, specifically, for the interference channel should be explored.

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Appendix A

Derivation of the Bhattacharyya Constant of Additive Binary and Gaussian Noise Channel

In this appendix we provide a derivation of the Bhattacharyya constant of channel with $f_{Y|x_1}(y) \sim h_1x_1 + h_2x_2 + N$, where $x_2 \in \{\pm 1\}$ and $N \sim \mathcal{CN}(0, \sigma^2)$. The LLR of the observed signal can be written as in (A.30). As there is no information of the signal x_2 at the receiver, $P(x_2 = 1) = \frac{1}{2}$. Hence we have

$$LLR(y) = \log \left(\frac{\exp \left(-\frac{\|y-h_1-h_2\|^2}{\sigma^2} \right) + \exp \left(-\frac{\|y-h_1+h_2\|^2}{\sigma^2} \right)}{\exp \left(-\frac{\|y+h_1+h_2\|^2}{\sigma^2} \right) + \exp \left(-\frac{\|y+h_1-h_2\|^2}{\sigma^2} \right)} \right). \quad (\text{A.31})$$

After some manipulations, the $LLR(y)$ can be written as

$$LLR(y) = \frac{4y \cdot h_1}{\sigma^2} + \log \left(\frac{\cosh \left((y - h_1) \star \frac{2h_2}{\sigma^2} \right)}{\cosh \left((y + h_1) \star \frac{2h_2}{\sigma^2} \right)} \right), \quad (\text{A.32})$$

where the operator $\star$ is defined as $a \star b = \text{Re}(a)\text{Re}(b) + \text{Im}(a)\text{Im}(b)$. In order to derive the Bhattacharyya constant of the channel $\mathcal{B}(P_L)$, we follow the approach developed in [63]. Defining P_L as the LLR -density of the channel output, we

calculate

$$\begin{aligned}\mathcal{B}(P_L) &= \int_{\mathbb{R}} P_L(l) \exp\left(-\frac{l}{2}\right) dl \\ &= \int_{\mathbb{C}} P_Y(y) \exp\left(-\frac{L(y)}{2}\right) dy,\end{aligned}\tag{A.33}$$

where $P_Y(y)$ is the PDF of the channel output. Since

$$P_Y(y|x_1) = \frac{1}{2\pi\sigma^2} \left(\exp\left(-\frac{\|y - h_1 - h_2\|^2}{\sigma^2}\right) + \exp\left(-\frac{\|y - h_1 + h_2\|^2}{\sigma^2}\right) \right),\tag{A.34}$$

we obtain the following expression after simplifications

$$\begin{aligned}\mathcal{B}(P_L) &= \int_{\mathbb{C}} \frac{1}{\pi\sigma^2} \exp\left(-\frac{\|y\|^2}{\sigma^2}\right) \exp\left(-\frac{\|h_1\|^2}{\sigma^2} - \frac{\|h_2\|^2}{\sigma^2}\right) \\ &\quad \times \sqrt{\cosh\left((y + h_1)\frac{2h_2}{\sigma^2}\right) \cosh\left((y - h_1)\frac{2h_2}{\sigma^2}\right)} dy.\end{aligned}\tag{A.35}$$

In a more convenient form, the Bhattacharyya constant of the channel can be expressed as in (A.36) where $N_1, N_2 \sim \mathcal{N}(0, \frac{\sigma^2}{2})$.

$$\begin{aligned}LLR(y) &= \frac{P(y|x_1 = 1)}{P(y|x_1 = -1)} \\ &= \log \left(\frac{\frac{1}{\pi\sigma^2} \exp\left(-\frac{\|y - h_1 - h_2\|^2}{\sigma^2}\right) P(x_2=1) + \frac{1}{\pi\sigma^2} \exp\left(-\frac{\|y - h_1 + h_2\|^2}{\sigma^2}\right) P(x_2=-1)}{\frac{1}{\pi\sigma^2} \exp\left(-\frac{\|y + h_1 - h_2\|^2}{\sigma^2}\right) P(x_2=1) + \frac{1}{\pi\sigma^2} \exp\left(-\frac{\|y + h_1 + h_2\|^2}{\sigma^2}\right) P(x_2=-1)} \right).\end{aligned}\tag{A.30}$$

$$\begin{aligned}\mathcal{B}(P_L) &= E_{N_1, N_2} \left[\exp\left(-\frac{\|h_1\|^2}{\sigma^2} - \frac{\|h_2\|^2}{\sigma^2}\right) \times \right. \\ &\quad \left. \sqrt{\frac{\cosh\left(\frac{4N_1 \operatorname{Re}(h_2) + 4N_2 \operatorname{Im}(h_2)}{\sigma^2}\right) + \cosh\left(\frac{4\operatorname{Re}(h_1) \operatorname{Re}(h_2) + 4\operatorname{Im}(h_1) \operatorname{Im}(h_2)}{\sigma^2}\right)}{2}} \right].\end{aligned}\tag{A.36}$$

Appendix B

Derivation of the Bhattacharyya Constant for the Modulo Channel

In this appendix, we consider the modulo channel $f_{Y|x}(y) \sim (hx + n)_{\text{mod } K}$, where $x \in \{-1, 1\}$ and $n \sim \mathcal{N}(0, \sigma^2)$. The LLR of the observed signal which is the logarithm of $P(y|x = 1)/P(y|x = -1)$ can be obtained as

$$LLR(y) = \frac{2yh}{\sigma^2} + \log \left(\frac{\sum_{l \in \mathbb{Z}} \exp \left(-\frac{l^2 K^2 - 2l(y-h)K}{2\sigma^2} \right)}{\sum_{l \in \mathbb{Z}} \exp \left(-\frac{l^2 K^2 - 2l(y+h)K}{2\sigma^2} \right)} \right). \quad (\text{B.1})$$

In order to find the Bhattacharyya constant of the channel $\mathcal{B}(P_L)$, we calculate

$$\mathcal{B}(P_L) = \int_{\mathbb{R}} P_L(l) \exp \left(-\frac{l}{2} \right) dl = \int_{\mathbb{R}} P_Y(y) \exp \left(-\frac{L(y)}{2} \right) dy,$$

where $P_L(l)$ is the PDF of LLR of the channel output and $P_Y(y)$ is the PDF of Y . By using the symmetry of the channel, we utilize

$$P_Y(y|x = 1) = \frac{1}{\sqrt{2\pi}\sigma^2} \sum_{l \in \mathbb{Z}} \exp \left(-\frac{(y - h - lK)^2}{2\sigma^2} \right) \quad (\text{B.2})$$

with $y \in [-K/2, K/2]$. Finally, we obtain the Bhattacharyya constant as

$$\begin{aligned}
\mathcal{B}(P_L) = & \int_{-K/2}^{K/2} \frac{1}{\sqrt{2\pi}\sigma^2} \left(\sum_{l \in \mathbb{Z}} \exp \left(-\frac{(y-h-lK)^2}{2\sigma^2} \right) \right) \\
& \times \exp \left(-\frac{yh}{\sigma^2} \right) \left(\sqrt{\frac{\sum_{l \in \mathbb{Z}} \exp \left(-\frac{l^2 K^2 - 2l(y+h)K}{2\sigma^2} \right)}{\sum_{l \in \mathbb{Z}} \exp \left(-\frac{l^2 K^2 - 2l(y-h)K}{2\sigma^2} \right)}} \right) dy, \tag{B.3}
\end{aligned}$$

which can be computed numerically.