

ATTITUDE AND ORBIT CONTROL OF A SOLAR SAIL SPACECRAFT FOR
RESPONSIVE OPERATIONAL CONCEPT

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY



BY
HALİS CAN POLAT

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
AEROSPACE ENGINEERING

JUNE 2021

Approval of the thesis:

**ATTITUDE AND ORBIT CONTROL OF A SOLAR SAIL SPACECRAFT
FOR RESPONSIVE OPERATIONAL CONCEPT**

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ABSTRACT

ATTITUDE AND ORBIT CONTROL OF A SOLAR SAIL SPACECRAFT FOR RESPONSIVE OPERATIONAL CONCEPT

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Doctor of Philosophy, Aerospace Engineering
Supervisor : Prof. Dr. Ozan Tekinalp

June 2021, 210 pages

Utilization of the propellant-free thrust capability of Solar Sail Spacecraft (SSS) is addressed. For this purpose, an elliptical orbit with a very low perigee altitude and an apogee altitude high enough to have solar sail acceleration is proposed for Earth observation mission. In the mission part, regional observation tasks are carried out that take place in very Low Earth Orbit region. The orbit maintenance and control are handled in the high altitude regions. The baseline SSS design is accomplished and presented. Orbital maintenance for station keeping and orbital maneuvers for carrying out specific missions are analyzed, and capabilities of SSS for changing orbital parameters are investigated. A constellation of observation satellites is also proposed addressing the weaknesses in rapid response of SSS to mission needs. Attitude control for smooth attitude maneuvers is also addressed. It is found that the novel quaternion-based attitude tracking control approach with time-dependent attitude trajectory offers smooth, jerk-free and accurate attitude tracking.

Keywords: Solar Sail, Low Earth Orbit, Attitude Control, Orbit Control, Mission Orbit Performance

ÖZ

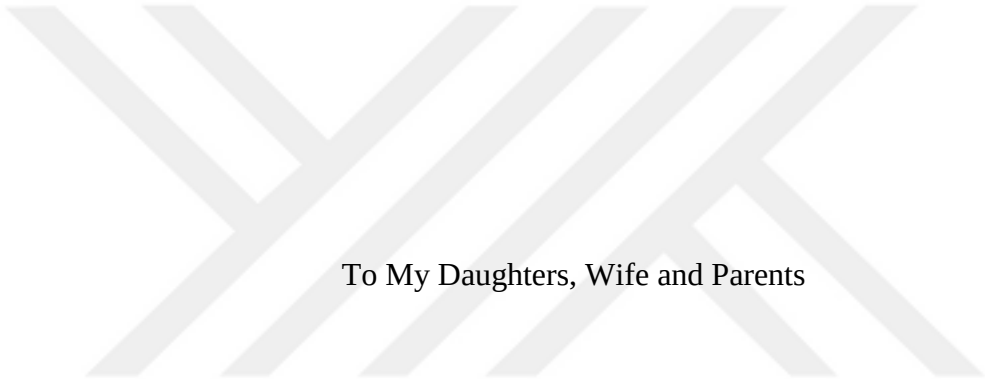
GÜNEŞ YELKENLİ UZAY ARACININ ELASTİK OPERASYONEL KONSEPT ÇERÇEVESİNDE YÖNELİM VE YÖRÜNGE KONTROLÜ

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Tez Yöneticisi: Prof. Dr. Ozan Tekinalp

Haziran 2021, 210 sayfa

Güneş Yelkenli Uzay Araçlarının (GYUA) yakıtsız itki üretebilme özelliğinin kullanımı çalışılmıştır. Bu amaçla, çok düşük irtifalı yerberi noktası ile güneş yelkeninden kaynaklı ivmeyi yaratabilecek yükseklikteki yeröte noktasına haiz bir eliptik yörünge, Dünya gözlemi maksadıyla önerilmiştir. Bölgesel gözlem görevleri eliptik yörüngenin düşük irtifalı kısımlarında gerçekleştirilirken yörünge bakımı ve kontrolü yüksek irtifalı kısımlarında ele alınmıştır. Referans teşkil edecek şekilde GYUA tasarımı ortaya konmuştur. Yörüngenin bakımı ve belirli görevler için gerekli yörünge manevraları analiz edilmiş olup yörünge parametrelerinin değiştirilmesine yönelik GYUA kabiliyetleri araştırılmıştır. Gözlem uydularından oluşan bir takım uydu tasarımı ise, GYUA'nın görev ihtiyaçlarına hızlı reaksiyon verme konusundaki zayıflığına karşılık olarak önerilmiştir. Yumuşak yönelim manevrası sağlayacak yönelim kontrolü ayrıca ele alınmıştır. Zamana bağlı yönelim rotası kullanan kuaterniyon temelli yönelim takip kontrolü yaklaşımının, yumuşak, sarsıntısız ve hassas bir yönelim takibine olanak kıldığı bulunmuştur.

Anahtar Kelimeler: Güneş Yelkeni, Düşük İrtifa Dünya Yörüngesi, Yönelim Kontrolü, Yörünge Kontrolü, Görev Yörüngesi Performansı



To My Daughters, Wife and Parents

ACKNOWLEDGMENTS

This thesis would not be complete without the enlightening and critical advice from my supervisor Prof. Dr. Ozan Tekinalp from the start of my graduate study in METU. This thesis is the final and most extensive product of my doctoral education. I would also state my gratefulness to my thesis advisory committee members Prof. Dr. Mustafa Kemal Özgören and Asst. Prof. Dr. Ali Türker Kutay for reviewing my work at all steps from the thesis proposal to thesis defense.

In addition, it was a privilege for me to further my education with the distinguished professors in Aerospace, Mechanical, Electrical and Electronics Engineering Departments. I would like to thank all of them for their sincere and professional advice.

I would also like to thank İsmail Ergün Kocacıçak and Mesut Özkan Kahraman for their encouragement to my graduate research study.

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LIST OF ABBREVIATIONS

ABBREVIATIONS

AAS	American Astronautical Society
ACM	Attitude Control Mechanism
AIAA	American Institute of Aeronautics and Astronautics
AN	Access Number
AOA	Angle of Attack
Au	Astronomical Unit
CCL	Cambridge Consultants Ltd.
CMG	Control Moment Gyro
CoM	Center of Mass
CoP	Center of Pressure
DCM	Direction of Cosine Matrix
DLR	German National Aerospace Center
DOY	Day of the Year
ECEF	Earth-Centered Earth-Fixed
ECI	Earth-Centered Inertial
EGM	Earth Gravitational Model
FoM	Figure of Merit
GST	Greenwich Sidereal Time
GVE	Gaussian Variational Equations

HCI	Heliocentric Inertial
IKAROS	Interplanetary Kite-Craft Accelerated by Radiation of the Sun
JAXA	Japan Aerospace Exploration Agency
JD	Julian Date
JHU	John Hopkins University
JPL	Jet Propulsion Laboratory
LEO	Low Earth Orbit
LOS	Line-Of-Sight
LQR	Linear Quadratic Regulator
MLTAN	Mean Local Time of Ascending Node
MRT	Mean Revisit Time
N/A	Not Applicable
NASA	National Aeronautics and Space Administration
PD	Proportional-Derivative
PFM	Performance Formulation Metric
PID	Proportional-Integral-Derivative
R2S2	Rapid Response Solar Sail
RAAN	Right Ascension of Ascending Node
RAST	Recent Advances in Space Technologies
RTN	Radial-Transverse-Normal
RW	Reaction Wheel
SA	Sailing Availability

SR	Mean Slant Range
SRP	Solar Radiation Pressure
SSP	Sub-Satellite Point
SSS	Solar Sail Spacecraft
STK	Systems Tool Kit
TRL	Technology Readiness Level
U3P	The Union pour la Promotion de la Propulsion Photonique
UTC	Coordinated Universal Time

LIST OF SYMBOLS

SYMBOLS

Solar Sailing Symbols

a_0	Characteristic Acceleration
a_{abs}	Absorptivity
$\vec{a}_{SRP}$	Acceleration due to Solar Radiation Pressure
a_n	Solar Acceleration along the Normal Axis
a_t	Solar Acceleration along the Transverse Axis
α_{sail}	Sail Pitch Angle
ϕ	Centerline Angle
Φ	Sail Cone Angle
B_b	Non-Lambertian Coefficient of the Back Surface
B_f	Non-Lambertian Coefficient of the Front Surface
c_{light}	Speed of the Light
ε_b	Emissivity Coefficient of the Back Surface
ε_f	Emissivity Coefficient of the Front Surface
$\vec{f}_a$	Force due to Absorption
$\vec{f}_e$	Force due to Emission
$\vec{f}_i$	Force along the Incident Light Direction

$\vec{f}_n$	Force along the Normal Direction
$\vec{f}_r$	Force along the Reflected Light Direction
$\vec{f}_{ref}$	Force due to Reflection
$\vec{f}_{rs}$	Force due to Scattered Reflection
$\vec{f}_{ru}$	Force due to Uniform Reflection
$\vec{f}_{SRP}$	Force due to Solar Radiation Pressure
$\vec{f}_t$	Force along the Transverse Direction
$\vec{f}_{trms}$	Force due to Transmission
η	Efficiency Factor
L_S	Solar Luminosity
m_{ACM}	Mass of the Attitude Control Mechanism
m_p	Mass of the Payload
m_{sat}	Total Satellite Mass
$m_{sail+struc}$	Mass of the Sail and Supporting Structure
$\hat{m}$	Non-Ideal Force Direction Unit Vector
$\hat{n}$	Normal Unit Vector of the Sail
σ	Sail Loading
P_{SRP}	Solar Radiation Pressure
$\tilde{r}$	Reflectivity

s	Specular Reflection Coefficient
$\hat{t}$	Transverse Unit Vector of the Sail
τ	Transmissivity
$\hat{u}_i$	Incident Unit Vector of the Light
$\hat{u}_r$	Reflecting Unit Vector of the Light
W	Energy Flux
W_E	Energy Flux at Earth Distance

Orbital Mechanics Symbols

A	Area
a	Semi-Major Axis of the Orbit
$\vec{a}_{2-body}$	Acceleration due to Two-Body Gravitational Attraction of the Earth
$\vec{a}_{aero}$	Acceleration due to Aerodynamic Drag
$\vec{a}_{asphericity}$	Acceleration due to Asphericity Effects of the Earth
$\vec{a}_{ThirdBody}$	Acceleration due to Third Bodies
$\vec{a}_{total}$	Total Acceleration Experienced by the Satellite
b_i	Coefficients of the Regression Formulation ($i = 1, 2, 3, 4, 5$)
β	The Angle between Sun Direction and Orbital Plane
C_D	Drag Coefficient
C_{lm}	Spherical Harmonic Coefficients of EGM2008
D	Slant Range

δ	Solar Declination
E	Eccentric Anomaly of the Satellite
e	Eccentricity of the Orbit
$\hat{e}$	Unit Vector of Orbit's Eccentricity
ε	Obliquity of the Ecliptic Plane
ε_{sat}	Satellite Elevation Angle
$eqtime$	Solar Equation of Time in Minutes
F	In-Plane Net Force
$\vec{f}_{aero}$	Aerodynamic Force
h	Specific Angular Momentum of the Orbit
$\hat{h}$	Unit Vector of Orbit's Angular Momentum
ha	Solar Hour Angle
H_{alt}	Altitude
H_{switch}	Mode Switching Altitude
λ	Longitude
$\lambda_{M_{sun}}$	Mean Longitude of the Sun
$\lambda_{ecliptic}$	Sun's Longitude of the Ecliptic
$\lambda_{ecliptic_{moon}}$	Moon's Latitude of the Ecliptic
λ_0	Initial Longitude
$\lambda_{ascending}$	Longitude of the Ascending Node

λ_{region}	Angular Radius of the Region seen by the Satellite Measured at the Center of Earth
λ_{SSP}	Angular Distance from the Sub-Satellite Point to the Target
λ_{sat}	Longitude of the Satellite
i	Inclination of the Orbit
JD	Julian Date
J_l	Dimensionless Coefficients of the EGM2008
Γ	Ecliptic Longitude of the Sun
M	Mean Anomaly of the Satellite
M_{sun}	Mean Anomaly of the Sun
μ	Standard Gravitational Parameter of the Earth
μ_{sun}	Standard Gravitational Parameter of the Sun
μ_{moon}	Standard Gravitational Parameter of the Moon
N	Force per Unit Mass-Normal Direction
$\eta_{off-nadir}$	Off-Nadir Angle of the Satellite
p	Semi-Latus Rectum
P	Period of the Orbit
$P_{lm} [x]$	Associated Legendre Polynomials
ρ_{sat}	Angular Radius of Earth seen from the Satellite
ρ	Density of the Air

ϕ_{site}	Latitude of the Site
ϕ_{sat}	Latitude of the Satellite
$\phi_{ecliptic_{moon}}$	Moon's Longitude of the Ecliptic
$\wp_{moon}$	Parallax of the Moon
R	Force per Unit Mass-Radial Direction
r_a	Radius of the Apogee
R_E	Radius of the Earth
$\vec{r}_{sun}$	Position Vector of the Sun
R_l	Disturbing Function of l^{th} zone
$\vec{r}_{moon}$	Position Vector of the Moon
r_p	Radius of the Perigee
$\vec{r}_{sat}$	Position Vector of the Satellite
$\vec{r}_{site}$	Position Vector of the Site
$\hat{R}_{Sun_Sat}$	Position Vector of the Sun with respect to Satellite
S_{lm}	Spherical Harmonic Coefficients of EGM2008
ς	Normal Force Direction Angle
T	Force per Unit Mass-Transverse Direction
t	Time Variable
T_{local}	Local Sidereal Time

T_{solar}	Local Solar Time
T_{UT1}	Julian Centuries
T_{UTC}	Coordinated Universal Time
$t_{sin\ ceperigee}$	Time since Perigee Point
U	Gravitational Potential
θ	The Angle between Sun Direction and Eccentricity Vector
θ_{air}	The Angle between Relative Wind and Surface Normal Directions
υ	True Anomaly of the Satellite
$\vec{V}_{sat}$	Velocity Vector of the Satellite
$\vec{V}_{wind}$	Relative Wind Vector
$\vec{V}_{co-rot}$	Co-rotation of the Atmosphere
$\vec{V}_{hor_wind}$	Horizontal Winds
$\varpi_{\oplus}$	Earth's Angular Speed
ω	Argument of Perigee of the Orbit
$\dot{\omega}$	Time Rate of Change of Argument of Perigee
ω_0	Initial Argument of Perigee
$\vec{\omega}_{\oplus}$	Angular Velocity of the Earth
Ω	Right Ascension of the Ascending Node of the Orbit
$\dot{\Omega}$	Time Rate of Change of RAAN

Ω_0	Initial RAAN
$\hat{x}_N, \hat{y}_N, \hat{z}_N$	Unit Vectors of the ECI Frame
$\hat{x}_\oplus, \hat{y}_\oplus, \hat{z}_\oplus$	Unit Vectors of the ECEF Frame
$\hat{x}_{sun}, \hat{y}_{sun}, \hat{z}_{sun}$	Unit Vectors of the HCI Frame
$\hat{x}_O, \hat{y}_O, \hat{z}_O$	Unit Vectors of the Orbital Frame
$\hat{x}_P, \hat{y}_P, \hat{z}_P$	Unit Vectors of the Perifocal Frame
$\hat{x}_{RTN}, \hat{y}_{RTN}, \hat{z}_{RTN}$	Unit Vectors of the Radial-Transverse-Normal Frame
γ	The Angle between Sun Direction and Velocity Vector
γ_{year}	Fraction of the Year
ξ	Angle Corresponding to the In-Plane Force Components
χ_i	Coefficients of the PFM Formulation ($i = 0, 1, 2, 3, 4$)

Attitude Mechanics Symbols

κ	Angular Jerk
α_{ang}	Angular Acceleration
$\vec{B}_{Earth}$	Magnetic Field of the Earth
c_i	Coefficients of the 5 th -order Polynomial Guidance ($i = 1, 2, 3, 4, 5, 6$)
$\vec{c}_{p_i}$	Center of Pressure Vector of the i^{th} Surface
$\vec{d}$	Desired Attitude Quaternion
H_i	Shadowing Coefficient of the i^{th} Surface

H_{ang}	Angular Momentum
h_w	Angular Momentum of the Reaction Wheels
$\hat{I}$	Identity Matrix
J_{sat}	Satellite's Moment of Inertia
$\hat{J}$	Inertia Matrix
k_{mag}	Magnetic Torque Gain
$\vec{m}_{torquers}$	Magnetic Dipole Moment of the Magnetic Torquers
S_i	Exposed Area of the i^{th} Surface
$\mathbf{s}$	Parameterization of the Desired Attitude Quaternion
ψ	Rotational Displacement Angle
q	Quaternion
τ	Time Constant in Low-Pass Filter
T_{ang}	Torque
$\vec{T}_d$	Disturbance Torques
$\vec{T}_c$	Control Torques
$\vec{T}_{aero}$	Aerodynamic Torque
$\vec{T}_{mag}$	Torque due to Magnetic Forces
$\vec{T}_{MED}$	Torque due to Momentum Exchange Devices
$\vec{T}_{solar}$	Solar Torque

$\vec{T}_{gg}$	Gravity Gradient Torque
T_{req}	Required Control Torque
T_w	Torque due to Reaction Wheels
$\vec{t}$	To-Go Quaternion
$\hat{t}$	Thrust Vector Control Direction
ω_{ang}	Angular Velocity
$\vec{\omega}_{B/N}$	Relative Angular Velocity of Body Frame with respect to Inertial Frame
$\hat{x}_B, \hat{y}_B, \hat{z}_B$	Unit Vectors of the Satellite's Body Frame
$\hat{x}_T, \hat{y}_T, \hat{z}_T$	Unit Vectors of the Target Frame
$\hat{Z}$	Transformation Matrix for the Reaction Wheel Configuration

Control Theory Symbols

$\bar{\Lambda}$	State Error
$\dot{\bar{\Lambda}}$	Time Rate of Change of the State Error
$\hat{G}$	State Matrix
$\hat{K}$	Gain Matrix
k and c	Positive Scalar Constants
$\vec{u}$	Control Input
V	Lyapunov Candidate Function
ω_n	Natural Frequency
$\bar{x}$	State Vector

$\dot{\bar{x}}$ Time Rate of Change of the State Vector

$\xi_{damping}$ Damping Ratio



CHAPTER 1

INTRODUCTION

Satellites are used for various missions with capable sensors to provide information. The information may be in different forms; e.g. communication signal, position-time tag for navigation, images of the Earth at a number of wavelengths. The demand for satellite images has been present since the start of space age. The information can be used for various applications such as disaster monitoring or environmental surveillance for forestry and agriculture, to name a few.

In order to achieve desired performance from observation satellites, mission designers try to optimize the resolution and coverage (or revisit time in other perspective). Low Earth Orbit (LEO) provides the utmost performance for such considerations. Even though, mission designers generally choose sun-synchronous or polar orbits at LEO for global coverage resulting in constant revisit/coverage performance, the observation performance in terms of revisit frequency can be increased by using inclined prograde orbits at LEO. However, the orbit is then successful only for the given region of interest not for the new sites due to the fact that changing the orbital parameters is too expensive in space in terms of propellant mass, even with the electric propulsion. However, the cost of the propellant for orbital maneuvers is not the case for Solar Sail Spacecraft (SSS). They can be utilized for their propellant-free maneuver capability thanks to their large and light sails with an ability of transforming the Solar Radiation Pressure (SRP) into a desired linear acceleration.

This study aims to propose a concept that combines the advantages of LEO region and solar sailing by demonstrating the utilization of the concept's ideas throughout the disciplines of baseline satellite design determination, processes of mission orbit

parameter selection, mission performance evaluation, constellation design, and eventually orbit and attitude control.

1.1 Motivation

The low but continuous acceleration attribute of SSS attracted many researchers. The use of SSS for variety of mission applications ranging from interplanetary and interstellar missions to Non-Keplerian orbits showed great potential for scientific exploration of the universe and the solar system.

On the other hand, as of today, more realistic and applicable missions of most space-based applications occur at near-Earth region, specifically at LEO altitudes (100-2000 km). The use of SSS at LEO are very limited, if not impossible, due to aerodynamic drag that suppresses the exploitation of SRP acceleration. Therefore, most of the SSS research areas at LEO are starting from high altitudes (greater than 700 km) or carrying out deorbiting operations of the satellites at the end of their lifetimes.

The key physical property of SSS is the very low sail loading, which is the ratio of the total mass to the sail area. The required sail loading levels for an effective SSS make the worst case scenario (very low ballistic coefficient) for aerodynamic drag effects which in most cases unfeasible to compensate with SRP acceleration.

In this study, the utilization of SSS at LEO for the flexibility to attain and alter orbital parameters for mission needs is achieved by employing an elliptical orbit with a proposed concept (Rapid Response Solar Sail-R2S2) despite the drag disadvantages. The in-depth analysis was needed and therefore addressed regarding the baseline satellite design, mission orbit determination process, maneuver planning strategy and relevant station-keeping and orbital transfer applications, attitude control methodology implementation and lastly constellation design for the responsiveness of the concept. This thorough problem solving iteration was necessary to prove the stability of the concept in various disciplines of SSS application.

The next section of literature survey is provided to layout the overall research field of SSS and its mission applications. After that, the research questions will be listed with the aimed contribution to the current literature.

1.2 Literature Survey

The historical background of solar sailing both in literature and accomplished missions can be presented in five main sub-sections: Early Pioneers, Case Studies, On-ground Deployments, In-flight Demonstrations and Mission Applications.

1.2.1 Early Pioneers

The fundamental theory of electromagnetic waves and their momentum were established by Maxwell (1865) and quantitatively proven by Lebedev (1900). Later, the studies of Einstein about the energy of photons in 1905 prepared the mathematical background for the idea of the solar sail use in space.

The Russian space pioneers Tsander and Tsiolkovsky were the earliest academicians to propose the concept of solar sailing in space in 1921-1924 [1]. However, the solar sailing earned its fame with the Arthur Clarke's first short story about the solar sail principle named as "Sunjammer" in 1964 and similar short stories later collected at the book named as "The Wind from the Sun" in 1972 [2].

After that, NASA (especially Jet Propulsion Laboratory-JPL) gave high importance to the utilization of SRP. The missions of ECHO II (1964) and Mariner 10 (1974) took the SRP into consideration and used it to alter their orbital positions and trajectories [3].

1.2.2 Case Studies

NASA JPL started a prestigious project for solar sails in the early 1970s. The project was to rendezvous with next Comet Halley near-Earth pass in 1986. The preliminary design case studies of that period up to 1976 prepared a common ground for solar sail structural design parameters, deployment methodologies and overall mission reliability. The project is then cancelled due to high risks considering the achieved technology readiness level and budgetary reasons [3].

The next big steps were the Moon (1981) and Mars (1986) races, in paper obviously, with solar sails. The design case studies and very limited laboratory testing were performed by different institutes and corporations. These case studies drew attention to the use of solar sail as an alternative to chemical propulsion [3].

1.2.3 On-Ground Deployments

All of the above primary studies showed one important common risk for solar sail use in space which is the deployment process. Very thin and light solar sail material, which should be folded and kept in a contained environment and then deployed in space with only one chance available, puts many projects away from the agendas of institutions, corporations and entities. However, on-ground deployment test studies brought the confidence to the solar sails again starting from the end of the 1990s.

The successful on-ground deployment tests of Cosmos-I with 600 m² sail area (2000), ATK System's S⁴ with 400 m² sail area (2005) and L'garde with 400 m² sail area (2006) brought the solar sail to the public agenda again [4].

1.2.4 In-Flight Demonstrations

For space applications, the best testing is always the actual mission itself and solar sail projects fit to this statement better than any other application due to their risky deployment process.

There are several in-flight demonstrations of solar sail in space. These technological demonstration missions are presented below:

- Znamya with a 20-meter diameter sail by Russian Space Agency (1993) [4],
- Inflatable Antenna Experiment with a 14-meter diameter sail by NASA (1996) [3],
- Nanosail-D with a 10 m² sail area by NASA (2010) [5],
- IKAROS with a 14 m² sail area by JAXA (2010) [6],
- Lightsail-A with a 32 m² sail area by the Planetary Society (2015) [7],
- Inflatesail with a 10 m² sail area by Surrey Space Center (2017) [8],
- Cubesail with a 20 m² sail area by University of Illinois via NASA's educational launch of nanosatellites program (2018) [9],
- Lightsail-2 with a 32 m² sail area by the Planetary Society (2019) [10].

All of these missions are helping to raise the solar sailing's technology readiness levels in space environment and paving the way to achieve increased maneuverability in spaceflight.

1.2.5 Mission Applications

In parallel to the historical development to the technology readiness level of solar sailing, wide range of mission applications and relevant research areas have been proposed by researchers.

Regarding the mission applications, the propellant-free maneuvering capability (theoretically infinite specific impulse) of solar sailing guides the researchers into mostly interstellar/interplanetary mission concepts and their trajectory planning strategies [3, 11]. Visiting Near-Earth asteroids and deflection missions were studied [12-15]. The continuous thrust attribute was utilized such that the Keplerian nature of the orbital mechanics was left and Non-Keplerian orbit concepts, which includes station-keeping and transfer orbit designs, were also proposed [16-20].

All of these mission applications avoided very low altitudes (specifically below 700 km) due to excessive aerodynamic drag. This region is commonly seen as “*No Man’s Land*” for SSS [11]. However, there were few research papers that utilized the avoided LEO region with material-based adjustments to the commonly-used SSS designs [21-22] and accomplished escape trajectories starting from 600 km altitude [23-24]. To obtain the opposite outcome from the solar sailing, there were also great number of studies for deorbiting LEO satellites with solar sail deployments at the end of mission lifetime [25-29].

Apart from the mission applications and orbit control/station-keeping study fields, when publications were analyzed in terms of their research topics, structure was leading the ranking due to the risky deployment process and the urge to come up with a light material that can be folded and deployed later. Sail and support structure design variations, deployment mechanisms, flexibility and non-rigidity effects, thermal and radiation effects and material selections were some examples of research field areas of solar sail structures [3, 11, 30-36].

Following that the attitude control research area was studied extensively due to the complex problem of controlling the attitude of a large and flexible/pseudo-rigid structure with high values of moments of inertia. The history on the attitude control problem for SSS were well summarized in [3, 12]. Furthermore, several attitude control methodologies and mechanisms were studied in the literature such as control tip vanes, refractive surface control, internal moving masses, control boom, reaction wheels and magnetic torquers [3, 12, 37-42]. Depending on the mission type, one or multiple of these attitude control mechanisms can be used. To further point out the research field spectrum of attitude control on SSS, following literature survey is provided in detail.

In Wie’s work, sail attitude control issues, solar radiation pressure models, dynamic modelling and control analysis of a sailcraft at elliptical orbit around the Earth were mostly discussed [12, 38, 42-47]. Specifically, gimballed control boom with control vanes at the tips was proposed for attitude control employing multi-body dynamics

with thrust vector control [38, 43]. This methodology of attitude control was also studied with a flight validation mission analysis [47] and further analysis was improved to a near-Earth asteroid mission [46]. A MatLab based control toolbox with the relevant modelling of mathematical expressions was also published [45].

A number of other approaches to the attitude control problem of the SSS were also proposed. Tekinalp proposed a tracking control methodology based on the quaternion feedback controller over time-dependent desired attitude trajectory in order to obtain a smooth reorientation and low-jerk slewing maneuver in regard to the flexibility attributes of SSS [48]. For interplanetary missions, Bolle used in-plane moving masses to control the orientation in two axes and ballasts at the tips for the roll axis rotation [49]. Ramognali also used moving masses at the sail's boom for time-efficient reorientation maneuvers with the combination of feedforward and feedback control loop algorithms [50]. A similar study was also proposed by Scholz and Ramognali [51].

The general approach to the attitude control problem with shifting masses uses the distance between Center of Pressure (CoP) for Solar Radiation Pressure and Center of Mass (CoM). The aim is mostly then to align them together so that no residual torque over the satellite remains after the reorientation to the desired attitude. This was fundamentally explained in Diedrich's Thesis [52]. Diedrich proposed a linearized system at the equilibrium point with an LQR controller that exploits the distance between CoP and CoM [51]. The same approach was improved with the addition of vibrational dynamics of flexible solar sail structure in Liu's work [53].

Baculi proposed a Fuzzy Control Logic based on the Takagi-Sugeno fuzzy model with linearized system controlled by PID controllers that steer moving masses [54]. Sperber, by using the same gimballed mass methodology, proposed a two-tier control strategy [55]. Sperber, first obtained a gimbal angle on principle rotation axis to minimize the attitude error and then used a regulator to move the gimballed mass to acquire the desired attitude orientation. Sperber uniquely implemented the 1-2-1 and

2-1-2 sequences of rotation to overcome the under-actuated nature of the aforementioned attitude control strategy [55].

Choi used a similar methodology but this time used rotating tip vanes to control the attitude by allocation of control torque to the relevant tip [56]. Also, a novel method of changing/deforming the satellite geometry to exploit the resulted torque changes for attitude control was proposed by Felicetti [57]. A NASA Report [40], made a comparison between spectrally differentiated stability control methods with the movable masses methods.

The literature survey details about the orbit control methodologies, mission orbit parameters determination and constellation design will be given in the relevant chapters of this thesis.

1.3 Contributions to the Literature and Research Questions

When the gaps of the literature of solar sail research area are analyzed, it has been found that the aforementioned concept (R2S2) of utilization of SSS in LEO and relevant mission orbit study along with the orbit and attitude control methodologies are the added-value of this PhD to the current literature. The contributions to the literature are given below in detail:

- Orbit Control/Transfer sub-category of the SSS literature;
 - Adaptive mission response by successive orbital transfer nature of the R2S2 concept,
 - Modification and the further utilization of the Gaussian Variational Equations in favor of the R2S2 concept,
 - Consideration of the seasonal effects of the Sun's position and the parameterization of the effectiveness of orbital maneuvers as a function of day of year and orbital parameters,
 - Elliptical orbit station-keeping with respect to mission requirements through case-based control logic,

- Mission Applications sub-category of the SSS literature;
 - LEO mission application reaching very low altitudes,
 - Mission orbit performance evaluation approach,
 - Determination of the mission orbit performance as a function of day of year and mission orbit parameters in regards to the limitations of R2S2 concept and electro-optical observation payloads,
 - Solar Sail Constellation optimization for responsive reconnaissance missions,
- Attitude Control sub-category of the SSS literature;
 - Implementation of the limitations of the R2S2 concept into the attitude control algorithms by decision-making processes that switches between different modes of attitude orientations,
 - Low-jerk and smooth large angle rotation of solar sail with time-dependent desired attitude trajectory.

The contribution to the current literature stemming from the motivation of this PhD was accomplished by asking the right research questions and accomplishing the challenges of addressing them. Reminding the motivation of the study, LEO has great benefits intrinsically, since it is closer to the Earth. Earth monitoring satellites perform better at LEO with smaller telescopes fulfilling better resolution requirements. Issue of delay time at communication satellites are also taken care of at LEO. Finally, elastic tasking of satellites for regions of interest with more frequent observation opportunities may be applicable with appropriately attained orbital parameters.

To point out the goal of this work, it is to employ SSS at LEO's low altitude portion via an elliptical orbit with a perigee altitude of 400 km and an apogee altitude of 1400 km. By employing such elliptical orbit with a very low perigee altitude and an apogee altitude high enough to have SRP acceleration, responsive operational use of SSS constellation is aimed. The responsive part of the mission is to have regional reconnaissance tasks that require an orbit of the SSS with specific parameters. Then,

SRP acceleration will be used for three main goals: to preserve the mission orbit from environmental disturbances, to transfer (raise the perigee altitude) from mission orbit to standby orbit (at altitude where SRP acceleration is feasible at all points of the orbit) and finally to transfer (decreasing the perigee altitude and limited out-of-plane maneuvers to fulfill reconnaissance task requirements) from standby orbit to the new mission orbit that has new parameters.

In order to prove the feasibility of the concept and relevant orbit and attitude control methodologies, following research questions are answered in this PhD study:

- Can a mission with SSS utilize the advantages of Low Earth Orbit? If yes, what is the suitable orbit design?
- Can SSS answer the regional response reconnaissance requests within an acceptable amount of time? If yes what would be the optimal standby and mission orbits and transfer trajectories between those? Moreover, what would be the optimal constellation design to fulfill the mission requirements?
- Can SSS keep its mission orbit against the orbital perturbations?
- What is the technological readiness level of such SSS? What is the highest TRL value which ensures a graceful degradation?
- What kind of attitude control methodology and actuators should be used to realize such mission? What is the level of sensitivity of the effect of attitude control methodology to the mission success?

1.4 Outline of the Thesis

The thesis work is divided into nine chapters and total work is provided with a continuous flow of the results. Before the presentation of the work, the theory and the mathematical/physical foundation are provided in Chapter 2 for solar sailing and in Chapter 3 for orbital and attitude dynamics. After that the proposed concept of R2S2 is defined with its detailed mode hierarchy in Chapter 4. Later, the baseline satellite design with the considerations of mass/volume budget, characteristic

acceleration, attitude control mechanisms and technology readiness level for the validation of the further attitude/orbit control methodologies is explained in Chapter 5.

After these four chapters, mission related work is commenced and the mission orbit performance evaluation approach along with the determination of the mission orbit parameters for high performance of observation tasks for a given region of interest under the limitations of the R2S2 concept is proposed in Chapter 6. In Chapter 7, maneuver planning strategy with an output of seasonal effectiveness of the in- and out-of-plane maneuvers due to the relative position of the Sun is provided as a function of day of year and orbital parameters.

In Chapter 8, the orbit control of the SSS is implemented for station-keeping of the mission orbit and transfer orbit trajectories. In that chapter, Gaussian Variational Equations are modified according to the needs of the R2S2 concept and two methods of thrust vector control direction determination are used. It is followed by the attitude control of the SSS (Chapter 9) by employing reaction wheels using a nonlinear quaternion feedback control with the limitations of the R2S2 concept. In particular, tracking control of the solar sail satellite with smooth attitude maneuvers is addressed. In Chapter 10, a constellation is proposed to improve the responsiveness of the R2S2 concept as a result of the effectiveness of the orbit control capabilities determined in Chapter 8. Finally, the conclusion is given in Chapter 11.



CHAPTER 2

SOLAR SAILING THEORY

McInnes' approach is followed for the formulation of SRP and adopted terminology. The following is provided for the general formulation of the SRP and acceleration stemming from it [3].

2.1 The Physical Foundation of Solar Radiation Pressure

The pressure resulting from the momentum exchange of sunlight for an ideal sail (fully reflective surface) is determined as:

$$P_{SRP} = \frac{W}{c_{light}} \quad (1)$$

where c_{light} is the speed of the light and W is the energy flux. After that, energy flux is formulated as:

$$W = W_E \left(\frac{(1Au)}{|\vec{r}_{sun}|} \right)^2 \quad (2)$$

where W_E is the energy flux at one astronomical unit ($1 Au = 149,597,871$ km) distance from the Sun and $\vec{r}_{sun}$ is the Sun's position vector. Energy flux at $1 Au$ can be calculated as:

$$W_E = \frac{L_s}{4\pi (1Au)^2} \quad (3)$$

where L_s is the solar luminosity. The pressure value can be accepted as a constant value of $4.56e-6$ N/m² at $1 Au$ for the average values of energy flux (1354 W/m² with

a variation of $\pm 3.5\%$ due to the ellipticity of the Earth's orbit around the Sun) and solar luminosity (3.828×10^{26} W with a variation of $\pm 0.1\%$ due to 11-year-cycle of solar activity).

Having determined the solar pressure on an ideal sail, the alignment of the sail surface with respect to the incident light is analyzed. Figure 2.1 shows incident ($\hat{u}_i$) and reflecting ($\hat{u}_r$) unit vectors of the sunlight from the sail surface. The normal unit vector ($\hat{n}$), also known as sail normal, shows the direction of the resultant SRP force ($\vec{f}_{SRP}$), is illustrated in Figure 2.1.

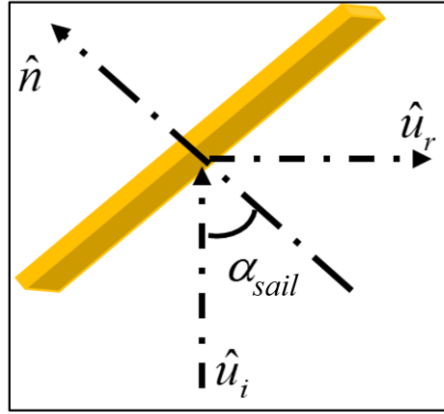


Figure 2.1. The Interaction of the Light with Perfectly Reflecting Flat Sail

Equations (4-7) shows the formulation of the net SRP force on the ideal sail with a sail area of A , by the incident ($\vec{f}_i$) and reflecting ($\vec{f}_r$) forces, which are the outputs of momentum exchange of the incident and reflecting photons, respectively.

$$\vec{f}_i = P_{SRP} A_{sail} (\hat{u}_i \cdot \hat{n}) \hat{u}_i \quad (4)$$

$$\vec{f}_r = -P_{SRP} A_{sail} (\hat{u}_i \cdot \hat{n}) \hat{u}_r \quad (5)$$

$$\hat{u}_i - \hat{u}_r = 2(\hat{u}_i \cdot \hat{n}) \hat{n} \quad (6)$$

$$\vec{f}_i + \vec{f}_r = \vec{f}_{SRP} = 2P_{SRP} A_{sail} (\hat{u}_i \cdot \hat{n})^2 \hat{n} \quad (7)$$

Finally, the acceleration due to SRP force ($\vec{a}_{SRP}$) can be determined as:

$$\vec{a}_{SRP} = 2 \frac{\eta P_{SRP} A_{sail}}{m_{sat}} (\cos \alpha_{sail})^2 \hat{n} \quad (8)$$

where m_{sat} is the satellite's mass and α_{sail} is the angle between sail's normal and the direction of the incident sunlight. The common terminology used for this angle is sail pitch angle. η is defined as the efficiency parameter (selected from a range of values of 85-100%) and is introduced for modelling the non-flatness and non-idealness of the sail [3].

Finally, two other common parameters can be listed as sail loading (σ) and characteristic acceleration (a_0) [3]. Sail loading, given in Equation (9), is the ratio of the satellite's mass and the sail's area. The characteristic acceleration, given in Equation (10), is the acceleration achieved under the assumption of sail pitch angle as zero.

$$\sigma = \frac{m_{sat}}{A_{sail}} \quad (9)$$

$$a_0 = 2 \frac{\eta P_{SRP}}{\sigma} \quad (10)$$

2.2 Non-Ideal Solar Sail Force Model

In Equation (8), the acceleration due to SRP was computed under the assumption of fully reflective sail (ideal sail), which is not possible in real life. The optical features of the film are then addressed to have a non-ideal sail formulation throughout the study.

When light (electromagnetic wave in general) encounters an object, there are three interactions that can happen: Reflection ($\vec{f}_{ref}$), absorption ($\vec{f}_a$), and transmission ($\vec{f}_{trms}$). Moreover, since some thermal energy is also stored, there will also be a radiation ($\vec{f}_e$) from the object (emission).

$$\vec{f}_{SRP} = \vec{f}_a + \vec{f}_{ref} + \vec{f}_{trms} + \vec{f}_e \quad (11)$$

These four phenomena, also illustrated in Equation (11), take place according to the surface properties of the material. In this study, McInnes' approach [3] is used to obtain the mathematical model of the non-ideal solar sail force.

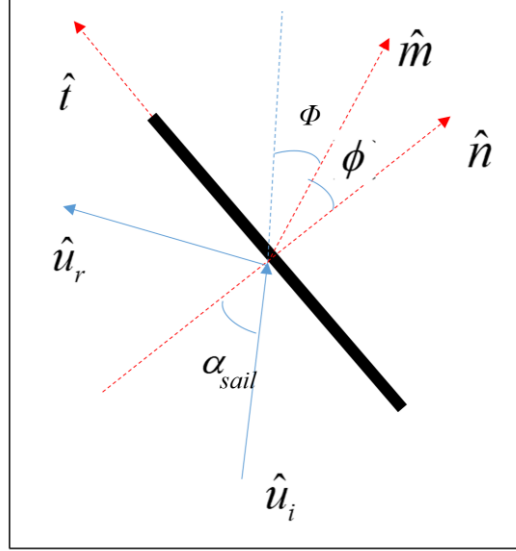


Figure 2.2. The Interaction of the Light with Non-Ideal Sail [3]

The unit vectors of $\hat{u}_i$ and $\hat{u}_r$ show the direction of the incident and reflected light directions, respectively. The unit vectors of $\hat{n}$ and $\hat{t}$ show the normal and transverse directions of the sail film, respectively. The relations between these unit vectors are as follows:

$$\begin{aligned} \hat{u}_i &= \cos(\alpha_{sail})\hat{n} + \sin(\alpha_{sail})\hat{t} \\ \hat{u}_r &= -\cos(\alpha_{sail})\hat{n} + \sin(\alpha_{sail})\hat{t} \end{aligned} \quad (12)$$

The ideal sail results in a solar force along the unit vector $\hat{n}$, but non-ideal sail has a deviation from the sail's normal, which is along the unit vector called $\hat{m}$. The deviation is called centerline angle (ϕ), and the angle relative to the incident solar light is called sail cone angle (Φ). The formulation of each optical force parameter defined in Equation (11) is then illustrated as follows [3]:

$$\tilde{r} + a_{abs} + \tau = 1 \rightarrow \tau = 0 \rightarrow a_{abs} = 1 - \tilde{r} \quad (13)$$

$$\vec{f}_{ref} = \vec{f}_{ru} + \vec{f}_{rs} \quad (14)$$

$$\begin{aligned} \vec{f}_a &= P_{SRP} A_{sail} (\cos^2(\alpha_{sail}) \hat{n} + \cos(\alpha_{sail}) \sin(\alpha_{sail}) \hat{t}) \\ \vec{f}_{ru} &= B_f \tilde{r} (1-s) P_{SRP} A_{sail} \cos(\alpha_{sail}) \hat{n} \\ \vec{f}_{rs} &= -(\tilde{r}s) P_{SRP} A_{sail} \cos(\alpha_{sail}) \hat{s} \\ \vec{f}_e &= P_{SRP} A_{sail} (1-\tilde{r}) \frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b} \cos(\alpha_{sail}) \hat{n} \end{aligned} \quad (15)$$

where $\vec{f}_{ru}$ and $\vec{f}_{rs}$ are used to formulate the uniform and scattered reflection of the photons from the sail film surface. As a reference, the NASA JPL's case study was used for the optical parameters [3] (Table 2.1).

Table 2.1 Optical Properties of Selected Sail Material [3]

Optical Property	Nomenclature	Value
Reflectivity of the Sail Film	$\tilde{r}$	0.88
Specular Reflection Coefficient	s	0.94
Emissivity Coefficient of the Front Surface	ε_f	0.05
Emissivity Coefficient of the Back Surface	ε_b	0.55
Non-Lambertian Coefficient of the Front Surface	B_f	0.79
Non-Lambertian Coefficient of the Back Surface	B_b	0.55
Transmissivity of the Sail Film	τ	0

Finally, Equations (12-15) were used to calculate the solar force acceleration and relevant defined angles as shown below.

$$\vec{f}_{SRP} = \vec{f}_n + \vec{f}_t \quad (16)$$

$$f_{SRP} = \sqrt{f_n^2 + f_t^2} \quad (17)$$

$$\vec{f}_{SRP} = f_{SRP} \hat{m} \quad (18)$$

$$\cos(\alpha_{sail}) = \hat{u}_i \cdot \hat{n} \quad (19)$$

$$\tan(\phi) = f_t / f_n \quad (20)$$

$$\Phi = \alpha_{sail} - \phi \quad (21)$$

The resultant effect of having a non-ideal sail imposes a constraint on the directing capability of the solar force. As can be seen in Figure 2.3, which is generated by the formulations provided in [3], the ideal sail can divert the solar force for almost 90 degrees from the direction of the Sun, even if the magnitude is getting too small as it is proportional to cosine square of that angle. But this is not the case for non-ideal sail, which ends up having a limitation of 55 degrees of directing capacity from the direction of the Sun. This information will be used in Chapter 6 for maneuver planning strategy.

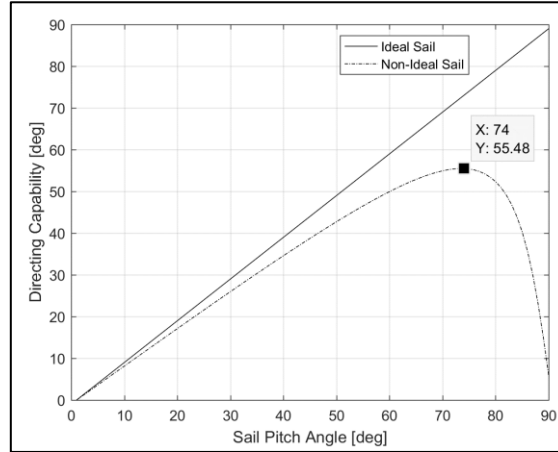


Figure 2.3. The Comparison of Directing Capability of the Sunlight with Ideal and Non-Ideal Sail Properties [3]

CHAPTER 3

FOUNDATIONS FOR ORBITAL AND ATTITUDE DYNAMICS

Having established the foundations for solar sailing mechanics, the necessity for a baseline orbital and attitude dynamics is inevitable. Even though orbital and attitude dynamics subject is well described in numerous books, journals and research papers, the methodologies and nomenclature adopted in this thesis will be explained in the following sections. Thus, this chapter is divided into four categories: Reference Frames, Time-Keeping, Orbital Dynamics, and Attitude Dynamics and Kinematics. In each category, baseline foundations will be laid down according to the well-known and well-accepted references [12, 58-60].

3.1 Reference Frames

Before the explanation of the dynamics, it is beneficial to describe the reference frames adopted in this work. Eight different reference frames have been used in this thesis: Earth-Centered Inertial – $\text{ECI}(\hat{x}_N, \hat{y}_N, \hat{z}_N)$, Earth-Centered Earth-Fixed – $\text{ECEF}(\hat{x}_\oplus, \hat{y}_\oplus, \hat{z}_\oplus)$, Heliocentric Inertial – $\text{HCI}(\hat{x}_{\text{sun}}, \hat{y}_{\text{sun}}, \hat{z}_{\text{sun}})$, Orbital $(\hat{x}_O, \hat{y}_O, \hat{z}_O)$, Perifocal $(\hat{x}_p, \hat{y}_p, \hat{z}_p)$, Radial- Transverse-Normal – $\text{RTN}(\hat{x}_{\text{RTN}}, \hat{y}_{\text{RTN}}, \hat{z}_{\text{RTN}})$, Body $(\hat{x}_B, \hat{y}_B, \hat{z}_B)$, and Target $(\hat{x}_T, \hat{y}_T, \hat{z}_T)$. Co-illustration of ECI, Orbital and Body Frames is provided in Figure 3.1, sole description of Body Frame including sail's normal unit vector direction and surface numeration is shown in Figure 3.2, co-illustrations of ECI-HCI, ECI-ECEF and ECI-Perifocal-RTN Frames are demonstrated in Figure 3.3, Figure 3.4 and Figure 3.5 respectively, and finally Body and Target Frames are illustrated in Figure 3.6.

The ECI and HCI Frames are the inertial (non-accelerating) frames in this work. The equations of translational and rotational motion, described later in the following

chapters, are based on ECI Frame. Moreover, the positions of the Sun and Moon will be calculated using the angles described in HCI Frame. The First Point of Aries direction from Earth through Sun at the first day of spring (vernal equinox) defines the x-axis for both frames. The z-axis is pointing through the North Pole direction (of Earth for ECI and of Sun for HCI) and y-axis is determined by the right-hand rule (Figure 3.3).

The Orbital Frame is used for mission application-oriented (Earth observation) purposes. The z-axis is towards Earth's center and the y-axis is along the opposite direction of the angular momentum vector of the orbit. Then, the x-axis is defined by the right-hand rule, note that x-axis is along the velocity vector at perigee and apogee. The Orbital Frame is attached to the satellite and orients as the satellite travels along the orbital path (Figure 3.1).

The Body Frame is fixed to the satellite and moves with it. The direction of the axes can be defined to any direction according to the needs of particular study; but generally, the principal axes of the satellite are commonly used to simplify the equations of rotational motion. Moreover, the sail's normal is defined as the opposite direction of the z-axis of the Body Frame for mission application-oriented reasons (Figure 3.2).

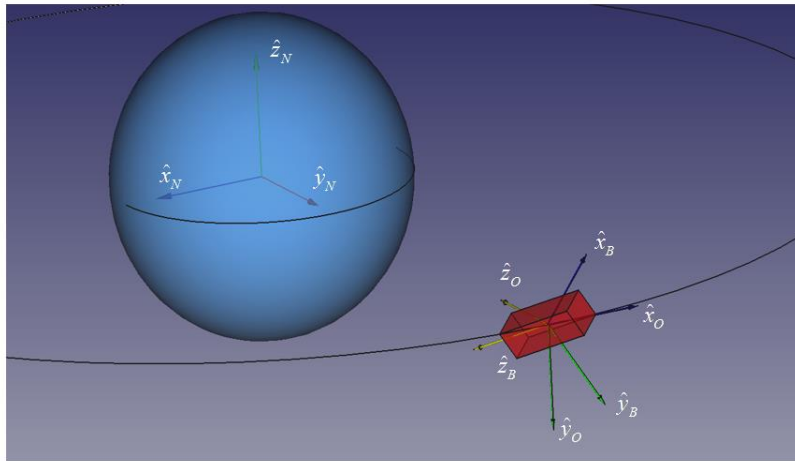


Figure 3.1. ECI, Orbital and Body Frames

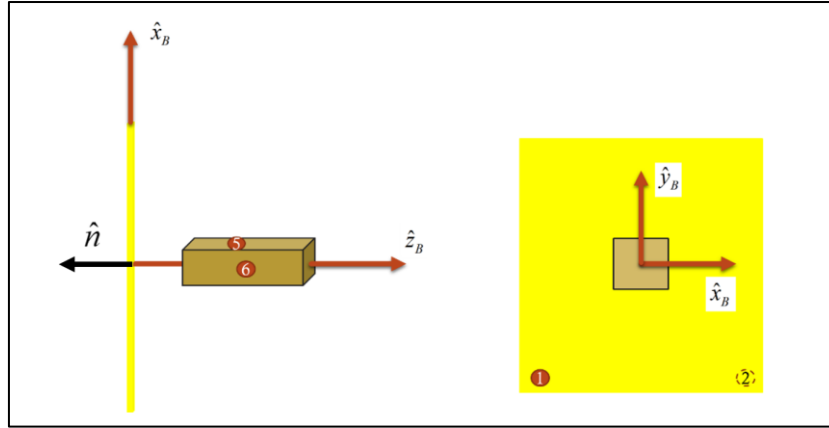


Figure 3.2. Solar Sail Satellite Body Frame and Surface Numbers

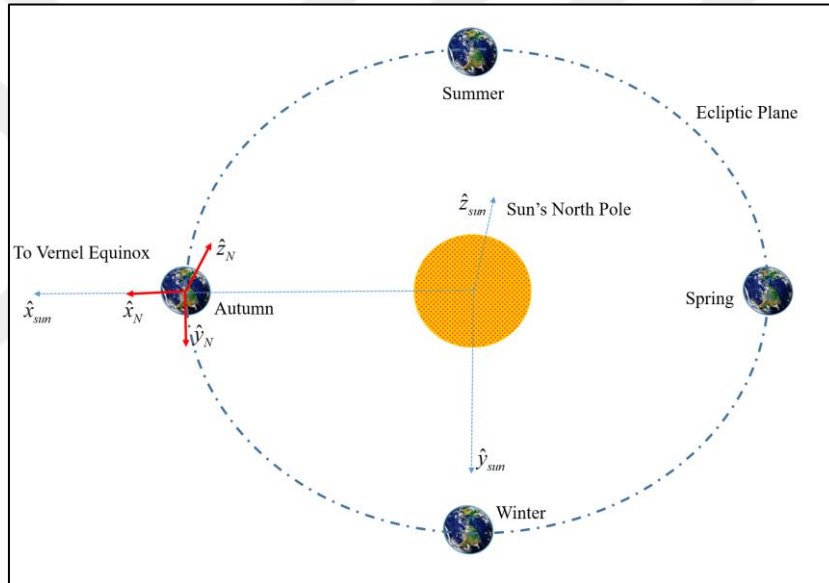


Figure 3.3. ECI and HCI Frames

The acknowledgment of an ECEF Frame is necessary for this study due to both orbital mechanics and mission application-oriented reasons (tracking the site's position). The x and y axes lay on the Equatorial plane while x-axis is drawn through the meridian of Greenwich and z-axis is pointing the North Pole. The ECEF Frame rotates with Earth (Figure 3.4).

Regarding the orbital parameters variation dynamics, perturbation calculations and relevant orbital maneuver strategy constructions, more specific frames (which are

attached to the orbit and/or orbital position) are necessary other than Orbital Frame. Within the context of this study, Perifocal and RTN Frames are used for such purposes. Perifocal Frame is attached to the employed orbit and moves as the orbit is distorted with orbital perturbations while RTN Frame is attached to the orbital position and moves as the position and orbit trajectory of the satellite change. For both frames z-axis is pointing the angular momentum of the orbit (orbit's normal). The x-axis of Perifocal frame is along the eccentricity vector of the orbit while for RTN it is along the position vector of the satellite. For both frames y-axis is calculated by the right-hand rule (Figure 3.5).

Finally, Target Frame is defined for thrust vector control purposes. For the sailing maneuvers, the sail's normal is rotated towards the direction which is determined by the orbit control algorithms with thrust vector control methodology. Since thrust vector control methodologies find a direction, a construction of a target frame based on this direction is found necessary under the consideration of generating low-drag (minimum energy loss principle). At the end, the aim is to come up with an identity matrix as the rotation matrix between Body and Target Frames. Then, z-axis is determined as the opposite direction of the thrust vector control direction ($-\hat{t}$), y-axis is along the angular momentum of the orbit and x-axis is found by the right-hand rule (Figure 3.6).

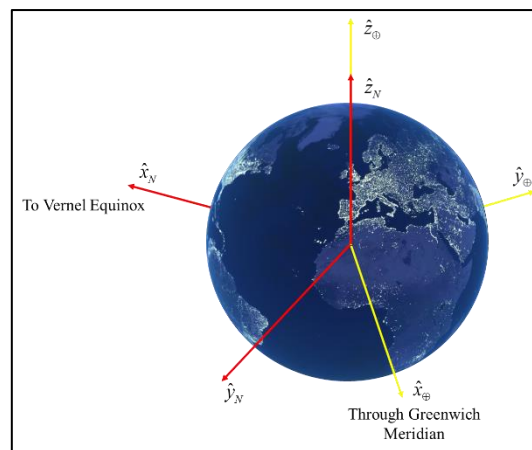


Figure 3.4. ECI and ECEF Frames

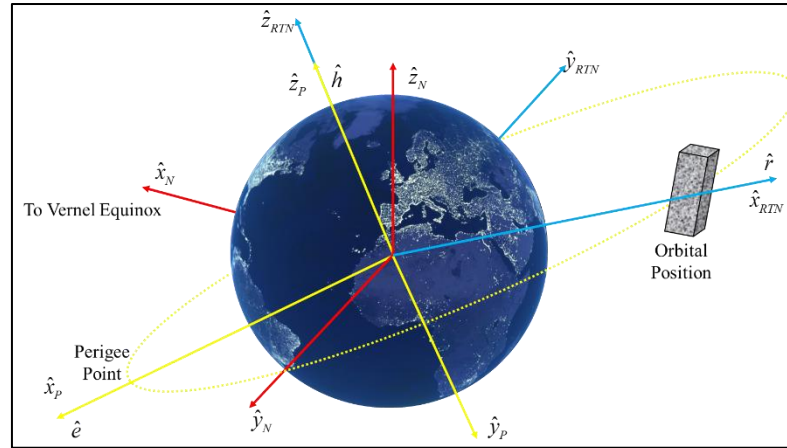


Figure 3.5. ECI, Perifocal and RTN Frames

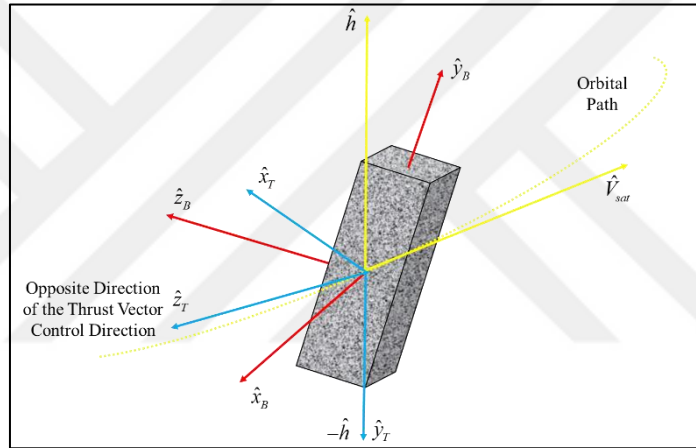


Figure 3.6. Body and Target Frames

3.2 Time-Keeping

The time-keeping is necessary for the determination of the positions of the Sun and Moon, the transformations between ECI and ECEF frames and finally mission application-oriented purposes. The list of definitions is provided for the adopted time units [59] in Table 3.1.

Table 3.1 Definitions of the Time Units

Time Units	Symbol	Definitions
Coordinated Universal Time	T_{UTC}	Standard time used worldwide derived from atomic clocks, requires the consideration of leap seconds.
Local Solar Time	T_{solar}	Time derived from the relative motion of Sun and Earth. Successive overhead passage of Sun on a position on Earth defined as 1 Solar Day.
Local Sidereal Time	T_{local}	Time derived from the relative motion of the revolving Earth with respect to the distant stars (J2000 epoch is used).
Julian Date	JD	Number of days since 1 st of January 4713 B.C. at noon.
Julian Centuries	T_{UT1}	Number of centuries calculated from Julian Date at a pre-determined epoch.

3.3 Orbital Dynamics

The foundations for orbital dynamics are adopted from the books of Battin [60], Vallado [59] and Curtis [58]. The satellite's motion around the Earth is first characterized as a two-body problem as below:

$$\vec{a}_{2-Body} = -\mu \frac{\vec{r}_{sat}}{|\vec{r}_{sat}|^3} \quad (22)$$

where μ is the Earth's gravitational parameter and $\vec{r}_{sat}$ is the position of the satellite in ECI Frame. Then, orbital perturbations are implemented and total force acting on the satellite is shown as below:

$$\vec{a}_{total} = \vec{a}_{2-body} + \vec{a}_{ThirdBody} + \vec{a}_{asphericity} + \vec{a}_{aero} + \vec{a}_{SRP} \quad (23)$$

Remaining terms in the right hand side other than the two-body acceleration are then modelled. The calculation of the acceleration due to SRP is explained earlier in Chapter 2. The Sun and the Moon are used for the third body gravitational attractions by the formulation [59] below:

$$\begin{aligned} \vec{a}_{ThirdBody} = & \frac{-\mu_{sun}}{r_{sun}^3} \left[\vec{r}_{sat} - 3\vec{r}_{\square} \frac{(\vec{r}_{sat} \cdot \vec{r}_{sun})}{r_{sun}^2} - 15\vec{r}_{\square} \left(\frac{(\vec{r}_{sat} \cdot \vec{r}_{sun})}{r_{sun}^2} \right)^2 \right] \dots \\ & \frac{-\mu_{moon}}{r_{moon}^3} \left[\vec{r}_{sat} - 3\vec{r}_{moon} \frac{(\vec{r}_{sat} \cdot \vec{r}_{moon})}{r_{moon}^2} - 15\vec{r}_{moon} \left(\frac{(\vec{r}_{sat} \cdot \vec{r}_{moon})}{r_{moon}^2} \right)^2 \right] \end{aligned} \quad (24)$$

where μ_{sun} and μ_{moon} are the Sun's and Moon's gravitational parameters, $\vec{r}_{sun}$ and $\vec{r}_{moon}$ are the positions of the Sun and the Moon in ECI Frame, respectively. In order to determine the positions the Sun and the Moon by utilization of the astronomical almanac formulas, Julian Date and Julian Century are used [59].

$$T_{UT1} = \frac{JD - 2451545}{36525} \quad (25)$$

First of all, mean longitude ($\lambda_{M_{sun}}$) and mean anomaly (M_{sun}) of the Sun, longitude ($\lambda_{ecliptic}$) and obliquity (ε) of the ecliptic are found:

$$\begin{aligned} \lambda_{M_{sun}} &= 280.4606184^0 + 36000.77005361T_{UT1} \\ M_{sun} &= 357.5277233 + 35999.05034T_{UT1} \\ \lambda_{ecliptic} &= \lambda_{M_{sun}} + 1.914666471^0 \sin(M_{sun}) + 0.019994643 \sin(2M_{sun}) \\ \varepsilon &= 23.439291^0 - 0.0130042T_{UT1} \end{aligned} \quad (26)$$

Then, the Sun's position vector, starting from Earth's center and ending at the Sun's center, is calculated in ECI Frame in the unit of Au , as below [59]:

$$r_{sun} = 1.000140612 - 0.016708617 \cos(M_{sun}) - 0.000139589 \cos(2M_{sun}) \quad (27)$$

$$\vec{r}_{sun} = \begin{bmatrix} \cos(\lambda_{ecliptic}) \\ \cos(\varepsilon) \sin(\lambda_{ecliptic}) \\ \sin(\varepsilon) \sin(\lambda_{ecliptic}) \end{bmatrix} r_{sun} \quad (28)$$

The same methodology is applied for the determination of the Moon's position. First, ecliptic latitude ($\lambda_{ecliptic_{moon}}$), longitude ($\phi_{ecliptic_{moon}}$) and parallax ($\wp_{moon}$) of the Moon are found [59]:

$$\begin{aligned}
\lambda_{ecliptic_{moon}} &= 218.32^0 + 481267.8813T_{UT1} + 6.29\sin(134.9 + 477198.85T_{UT1})... \\
&-1.27\sin(259.2 - 413335.38T_{UT1}) + 0.66\sin(235.7 + 890534.23T_{UT1}) ... \\
&+0.21\sin(269.9 + 954397.7T_{UT1}) - 0.19\sin(357.5 + 35999.05T_{UT1})... \\
&-0.11\sin(186.6 + 966404.05T_{UT1}) \\
\phi_{ecliptic_{moon}} &= 5.13\sin(93.3 + 483202.03T_{UT1}) + 0.28\sin(228.2 + 960400.87T_{UT1}) ... \quad (29) \\
&-0.28\sin(318.3 + 6003.18T_{UT1}) - 0.17\sin(217.6 - 407332.20T_{UT1}) \\
\wp_{moon} &= 0.9508^0 + 0.0518\cos(134.9 + 477198.85T_{UT1})... \\
&+0.0095\cos(259.2 - 413335.38T_{UT1}) ... \\
&+0.0078\cos(235.7 + 890534.23T_{UT1}) + 0.0028\cos(269.9 + 954397.7 T_{UT1})
\end{aligned}$$

Then, the Moon's position vector in Earth Radius, from Earth's center to Moon's center, is found in ECI Frame as follows [59]:

$$r_{moon} = \frac{1}{\sin(\wp_{moon})} \quad (30)$$

$$\vec{r}_{moon} = \begin{bmatrix} \cos(\lambda_{ecliptic_{moon}}) \cos(\phi_{ecliptic_{moon}}) \\ \cos(\varepsilon) \cos(\phi_{ecliptic_{moon}}) \sin(\lambda_{ecliptic_{moon}}) - \sin(\varepsilon) \sin(\phi_{ecliptic_{moon}}) \\ \sin(\varepsilon) \cos(\phi_{ecliptic_{moon}}) \sin(\lambda_{ecliptic_{moon}}) + \cos(\varepsilon) \sin(\phi_{ecliptic_{moon}}) \end{bmatrix} r_{moon} \quad (31)$$

In order to model the asphericity of Earth, it is needed to address the methodology here. As a common knowledge, the Earth is not sphere. The shape of Earth has a specific name, "Geoid". This shape has distinct features such as bulges at the equator, oblateness at the poles, varying highs and lows with respect to land/water distribution. This shape is represented by various techniques, however the spherical harmonic representation is the most common one. There are coefficients, which are derived from empirical campaigns both on ground or from satellite measurements, to model the shape. These coefficients show the variations with respect to the perfect sphere model. Earth is divided into regions to increase the fidelity of the results, however one can choose the grid model or size appropriate to his/her study. These gridding methods are called Zonal, Sectoral and Tesseral (Figure 3.7) [59].

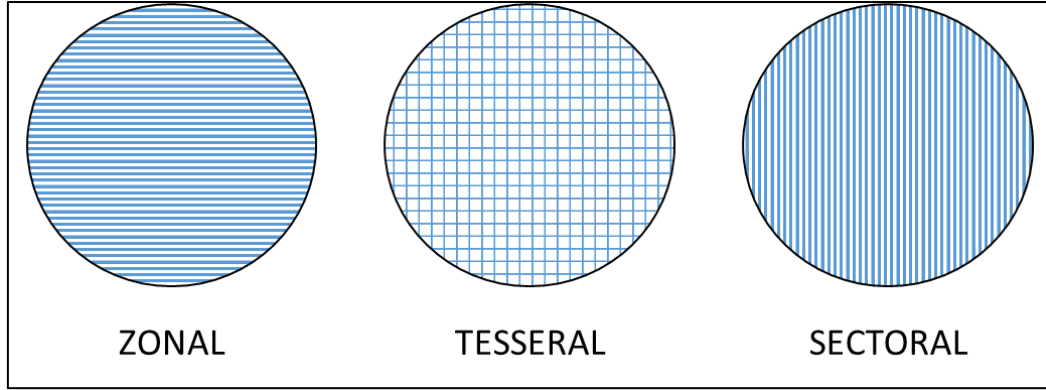


Figure 3.7. Zonal, Tesseral and Sectoral Harmonic Representation [59]

The gravitational potential (U) of the Earth can be described with spherical harmonics as follows [59]:

$$U = \frac{\mu}{r} \left[1 - \sum_{l=2}^{\infty} J_l \left(\frac{R_E}{r} \right)^l P_l[\sin(\phi_{sat})] + \sum_{l=2}^{\infty} \sum_{m=1}^l \left(\frac{R_E}{r} \right)^l P_{lm}[\sin(\phi_{sat})] \{ C_{lm} \cos(m\lambda_{sat}) + S_{lm} \sin(m\lambda_{sat}) \} \right] \quad (32)$$

where R_E is the mean radius of the Earth (6371 km); ϕ_{sat} and λ_{sat} are the geocentric latitude and longitude of the satellite, respectively; $P_{lm}[x]$ is representing the Associated Legendre Polynomials of degree l and order m ; C_{lm} and S_{lm} are the spherical harmonic coefficients of degree l and order m from Earth Gravitational Model 2008 (EGM2008) [61]; J_l is the dimensionless coefficients of the EGM2008 [61]. The model can provide the order/degree numbers up to 2190. However, for this study the need of data accuracy is limited as the computational burden is also important. Therefore, there are some assumptions to have a moderate accuracy with acceptable computational burden. These are listed below;

- All tesseral and sectoral terms are excluded,
- Only first four zonal harmonics are included,
- Disturbing function for each zone is obtained,

- Finally, with gradient operator the effect of that disturbing function on specific axis is calculated.

This approach [59] is useful for computational efficiency. The mathematical derivation for only the zone 2 and x-axis of the ECI Frame is shown below as an example in Equations (33-34), and resultant acceleration is formulated in Equation (35):

$$R_2 = -\frac{3J_2\mu}{2r}\left(\frac{R_E}{r}\right)^2\left(\sin^2(\phi_{sat}) - \frac{1}{3}\right) \quad (33)$$

$$R_2 = -\frac{3J_2\mu R_E^2}{2r^3}\left(\frac{r_K^2}{r^2}\right) + \frac{J_2\mu R_E^2}{2r^3} \quad (34)$$

$$\vec{a}_{J_2} = \frac{\partial R_2}{\partial \vec{r}} = -\frac{3J_2\mu R_E^2}{2r^5} \begin{bmatrix} r_I \left(1 - \frac{5r_K^2}{r^2}\right) \\ r_J \left(1 - \frac{5r_K^2}{r^2}\right) \\ r_K \left(3 - \frac{5r_K^2}{r^2}\right) \end{bmatrix} \quad (35)$$

where R_2 is the disturbing function of the second zone; I , J and K are axes of the ECI Frame. The same methodology is done for zones up to 4.

$$\vec{a}_{J_3} = -\frac{5J_3\mu R_E^3}{2r^7} \begin{bmatrix} r_I \left(3r_K - \frac{7r_K^3}{r^2}\right) \\ r_J \left(3r_K - \frac{7r_K^3}{r^2}\right) \\ 6r_K^2 - \frac{7r_K^4}{r^2} - \frac{3}{5}r^2 \end{bmatrix} \quad (36)$$

$$\vec{a}_{J_4} = \frac{15J_4\mu R_E^4}{8r^7} \begin{bmatrix} r_I \left(1 - \frac{14r_K^2}{r^2} + \frac{21r_K^4}{r^4} \right) \\ r_J \left(1 - \frac{14r_K^2}{r^2} + \frac{21r_K^4}{r^4} \right) \\ r_K \left(5 - \frac{70r_K^2}{3r^2} + \frac{21r_K^4}{r^4} \right) \end{bmatrix} \quad (37)$$

Hughes' approach is used for the modeling of aerodynamic drag [62]. The continuum flow for the modelling of aerodynamic forces, drag in this study, cannot be used as the molecular mean free path is way larger than the dimensions of the satellite. For this reason, free molecular flow model is utilized by calculating the forces according to the momentum exchange only. Figure 3.8 shows the air molecule and the surface interaction.

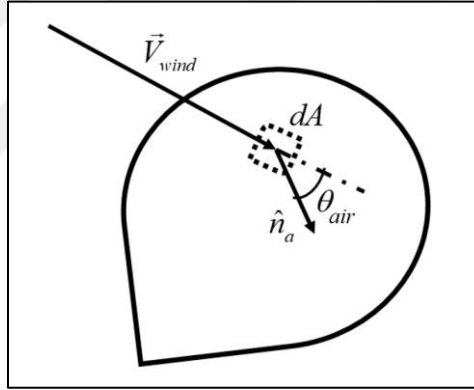


Figure 3.8. Air Molecule and the Surface Interaction

The momentum exchange can be divided to two sections, as in the formulation of acceleration due to solar radiation pressure, incident and leaving. The leaving phase is not considered in this work due to low values which are eligible for neglecting. Regarding the incident air molecules, the force element can be defined as:

$$d\vec{f}_{aero} = \frac{1}{2} \rho C_D |\vec{V}_{wind}| \vec{V}_{wind} (\hat{V}_{wind} \cdot \hat{n}_a) dA \quad (38)$$

where C_D is the drag coefficient, ρ is the density of air, $(\hat{\mathbf{V}}_{wind} \cdot \hat{\mathbf{n}}_a)$ is used to define the exposed area element (dA) by the relation of $\cos \theta_{air}$. Equation (38) is then integrated over the whole surface to find the aerodynamic drag. Eventually, the deceleration due to aerodynamic drag can be found as below.

$$\vec{a}_{aero} = \frac{1}{2BC} \rho \cos \theta_{air} |\vec{V}_{wind}| \vec{V}_{wind} \quad (39)$$

where BC is the ballistic coefficient, $BC = \frac{m_{sat}}{C_D A}$. The air density is found by

implementing the mathematical model of the atmosphere [63]. In the model, direction and the magnitude of horizontal winds (below 500 km) are found by using the 2007 U.S. Naval Research Laboratory Horizontal Wind Model [64]. Another factor in the mathematical model of the atmosphere as the co-rotation of the atmosphere is formulated in ECI frame as below:

$$\vec{V}_{co-rot} = \vec{\omega}_{\oplus} \times \vec{r}_{sat} \quad (40)$$

where $\vec{\omega}_{\oplus}$ is the angular velocity of the Earth. At the end, the resultant relative velocity vector of the incident air molecules in the ECI frame can be found:

$$\vec{V}_{wind} = \vec{V}_{co-rot} + \vec{V}_{hor_wind} - \vec{V}_{sat} \quad (41)$$

The motion of the satellite around the Earth is now represented with Equation (23). However, the state vector of the satellite consisting of position and velocity vectors with respect to the ECI Frame is not useful for analysis of the orbit in three dimension. Therefore, classical orbital elements [58] are used for such purposes which will be frequently applied throughout the study. The nomenclature for the classical orbital elements can be found in Table 3.2. The three dimensional illustration of the classical orbital elements relative to the ECI Frame is given in Figure 3.9 [58].

Table 3.2 Nomenclature

a	Semi-major axis	h	Specific Angular Momentum
e	Eccentricity	p	Semi-Latus Rectum
i	Inclination	r_{sat}	Magnitude of Position Vector
Ω	RAAN	R	Force per Unit Mass-Radial Direction
ω	Argument of Perigee	T	Force per Unit Mass-Transverse Direction
ν	True Anomaly	N	Force per Unit Mass-Normal Direction

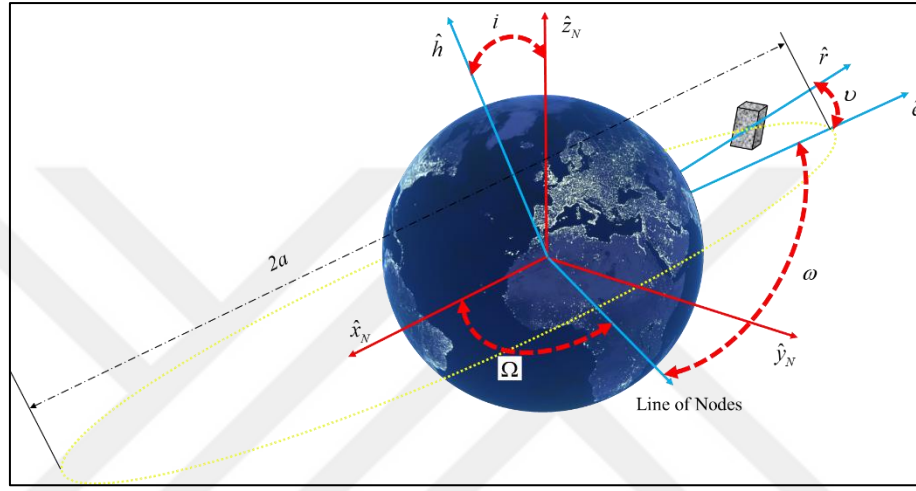


Figure 3.9. Classical Orbital Elements [58]

The dynamics of the classical orbital elements are vital to the R2S2 concept. Therefore, they are implemented by utilizing the Gaussian Variational Equations [60]. GVEs, listed in Equation (42), provides the dynamics for classical orbital elements under the influence of two-body Earth's gravitational attraction. All of the other disturbances/control forces are treated as control input. Nomenclature for Equation (42) is also given in Table 3.2.

The dynamics of the classical orbital elements given in Equation (42) depend only on themselves with no control input. For variables h , p and r , Equation (43) is given for demonstrating the strict dependence on classical orbital elements.

$$\begin{aligned}
\dot{a} &= \frac{2a^2}{h} \left[eR \sin \nu + \frac{pT}{r_{sat}} \right] \\
\dot{e} &= \frac{1}{h} \left(pR \sin \nu + [(p + r_{sat}) \cos \nu + r_{sat} e] T \right) \\
\dot{i} &= \frac{r \cos(\omega + \nu)}{h} N \\
\dot{\Omega} &= \frac{r \sin(\omega + \nu)}{h \sin i} N \\
\dot{\omega} &= -\frac{r_{sat} \sin(\omega + \nu)}{h \tan i} N + \frac{1}{eh} [-pR \cos \nu + (p + r_{sat}) T \sin \nu] \\
\dot{\theta} &= \frac{h}{r_{sat}^2} + \frac{1}{eh} [pR \cos \nu - (p + r_{sat}) T \sin \nu] \\
r_{sat} &= \frac{a(1-e^2)}{1+e \cos \nu} \\
p &= a(1-e^2) \\
h &= \sqrt{\mu a(1-e^2)}
\end{aligned} \tag{42}$$

$$\begin{aligned}
r_{sat} &= \frac{a(1-e^2)}{1+e \cos \nu} \\
p &= a(1-e^2) \\
h &= \sqrt{\mu a(1-e^2)}
\end{aligned} \tag{43}$$

The equations regarding the RAAN, argument of perigee and true anomaly cannot be used for circular and/or equatorial orbits since the equations become undefined in those values. However, this is not the case for R2S2 Concept. The orbits will be inclined and elliptical. Therefore, GVEs are applicable for the R2S2 Concept. The detailed explanations relevant to the orbits of the R2S2 Concept will be given in Chapter 4 and utilization of the GVEs for orbit control purposes will be given in Chapter 8.

3.4 Attitude Dynamics and Kinematics

The general approach in Wie [12] is adopted for attitude dynamics and kinematics formulizations (exceptions will be stated). The dynamics for the rotational motion are based on the Euler's Equation. The following relation between the time rate of change of the angular momentum and the net torque in the body frame is then solved for angular acceleration for each axis. The following compact form is the result of the selection of the principal axis for the body frame so that the inertia matrix is diagonal [12]. The resultant equations of rotational motion are nonlinear due to the

products of angular velocity terms. The rotational equations of motion are then controlled by nonlinear control methods or controlled by linear control methods when linearization is applicable. In this thesis, the nonlinear methods based on the quaternion feedback control will be applied due to the existence of large angle rotations and their lack of linearization performance.

$$\dot{\vec{H}}_{ang} + \vec{\omega}_{ang} \times \vec{H}_{ang} = \vec{T}_{ang} \quad (44)$$

$$\begin{aligned} J_x \dot{\omega}_x + (J_z - J_y) \omega_z \omega_y &= T_x \\ J_y \dot{\omega}_y + (J_x - J_z) \omega_x \omega_z &= T_y \\ J_z \dot{\omega}_z + (J_y - J_x) \omega_y \omega_x &= T_z \end{aligned} \quad (45)$$

The net torque in Equations (44-45) is then divided into two categories as control and disturbance torques. The disturbance torques are then divided into three components (magnetic dipole moment torque as a disturbance is neglected, but as a control torque is included) as aerodynamic torque, solar torque and gravity gradient torque.

$$\begin{aligned} \vec{T}_{ang} &= \vec{T}_d + \vec{T}_c \\ \vec{T}_d &= \vec{T}_{aero} + \vec{T}_{solar} + \vec{T}_{gg} \end{aligned} \quad (46)$$

The aerodynamic and solar torques are calculated for each surface shown in Figure 3.10 (1-2: Sail Surfaces, 3-6 sides of the payload, bottom and top surfaces of the payload are neglected).

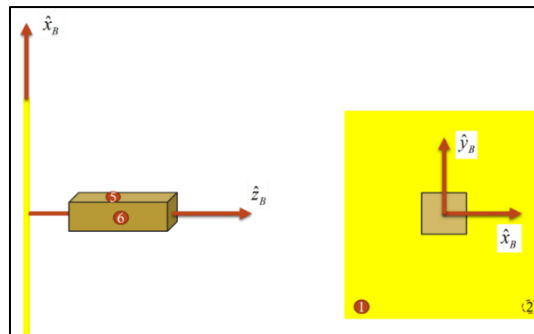


Figure 3.10. Solar Satellite Body Frame, where $\hat{x}_B$ and $\hat{y}_B$ are in the solar sail plane, and Surface Numbers

The determination of relevant forces, solar radiation pressure over a surface and aerodynamic drag force, are described earlier in Chapter 2 and Chapter 3.3, respectively. In order to acquire the torque relevant to those forces, the moment arms from the center of each surface to the center of mass of the satellite system are used. Moreover, a shadowing methodology is implemented with a Heaviside function to determine which surface is in effect. The following equations are used for implementing the determination of the torques of aerodynamic drag and solar radiation pressure [62].

$$\vec{T}_{aero}^B = \sum_{i=1}^6 \vec{c}_{p_i} \times \vec{f}_{aero_i}^B \quad (47)$$

$$\vec{f}_{aero_i}^B = \frac{1}{2} C_D \rho V^2 S_i \hat{V}^B \quad (48)$$

$$S_i = A_i (\hat{V}^B \cdot \hat{n}_i) H_i \quad (49)$$

$$\vec{T}_{solar}^B = \sum_{i=1}^2 \vec{c}_{p_i} \times \vec{f}_{solar}^B \quad (50)$$

The aerodynamic torque is calculated for all six surfaces, however solar torque is calculated for only two surfaces. Therefore, the cosine-square relation of solar force is adequate to determine the solar torque. On the other hand, a shadowing algorithm based on the dot product of velocity vector and surface normal vector is used for aerodynamic torque calculations. Moreover, if the H function for surface number 1 is 1, all other surfaces are considered to be in the shadow (aerodynamic force is zero for those surfaces). Otherwise, the H function for each surface has to be checked in order to determine the contribution of each surface to the aerodynamic force and torque calculations.

$$H_i = \begin{cases} H_i = 0 & \text{if } (\hat{V}^B \cdot \hat{n}_i) \geq 0 \\ H_i = 1 & \text{if } (\hat{V}^B \cdot \hat{n}_i) < 0 \\ H_{2...6} = 0 & \text{if } (H_1 = 1) \end{cases} \quad (51)$$

The last term of the disturbance torques is the gravity gradient torque. The following equation is used to determine the gravity gradient torque in Body Frame [12]. The gravity gradient torque is a critical disturbance for solar sail satellite, since the moment of inertia values are large, resultant gravity gradient torques are also large and they result in quick saturation of momentum exchange devices if not handled well in attitude control strategy. However, as it will be explained in Chapter 4, the division of the employed orbit of the R2S2 concept and the orientation of the satellite below 800 km is favoring the minimization of gravity gradient torques and eventually avoiding the saturation of the momentum exchange devices prematurely.

$$\vec{T}_{gg}^B = \frac{3\mu}{R^5} \begin{bmatrix} (J_3 - J_2)R_y R_z \\ (J_1 - J_3)R_z R_x \\ (J_2 - J_1)R_x R_y \end{bmatrix} \quad (52)$$

Regarding the control torques in Equation (46), the formulation for momentum exchange devices and magnetic torquers in the use for momentum dumping are implemented. Individual reaction wheel contribution applied by the matrix $\hat{Z}$ to the change of the angular momentum of the system is given in Equation (53) [65].

$$\vec{T}_{MED} = -\left(\vec{\omega}_{B/N} \times \hat{Z}\vec{h}_w + \hat{Z}\dot{\vec{h}}_w\right) \quad (53)$$

In Equation (54), the formulation of the $\hat{Z}$ matrix is given for a pyramidal configuration of a four-reaction wheel system each employed at trisectrix of each surface. The $\hat{Z}$ matrix can also be interpreted as a transformation matrix of each wheels' individual axis of rotation into the body frame.

$$\hat{Z} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (54)$$

The change of the angular momentum of the wheels is the torque applied by the wheels themselves and they result in the reorientation of the attitude of the satellite thanks to the conservation of the angular momentum. The required angular

acceleration of the reaction wheel is then found by applying pseudo-inverse distribution law [65].

$$\dot{\bar{h}}_w = \bar{T}_w \quad (55)$$

$$\dot{\bar{h}}_w = \text{pinv}(\hat{Z}) \left(-T_{req} - \bar{\omega}_{B/N} \times \hat{Z} \bar{h}_w \right) \quad (56)$$

The angular momentum buildup in reaction wheels are then dumped using magnetic torquers. Magnetic torquers generate a magnetic dipole moment in the direction such that the interaction with the Earth's magnetic field results in a momentum dumping magnetic torque direction [62].

$$\vec{T}_{mag} = -k_{mag} \bar{h}_w \quad (57)$$

$$\vec{T}_{mag} = \vec{m}_{torquers} \times \vec{B}_{Earth} \quad (58)$$

$$\vec{m}_{torquers} = \frac{\vec{B}_{Earth} \times \vec{T}_{mag}}{(\vec{B}_{Earth} \cdot \vec{B}_{Earth})} \quad (59)$$

After determining the net torque over the system, angular acceleration is solved and integrated to find the angular velocity. Then, the angular velocity with the previous quaternion state of the body is used to determine the time rate of change of the quaternion as follows [12]:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_4 & -q_3 & q_2 & q_1 \\ q_3 & q_4 & -q_1 & q_2 \\ -q_2 & q_1 & q_4 & q_3 \\ -q_1 & -q_2 & -q_3 & q_4 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ 0 \end{bmatrix} \quad (60)$$

The time rate of change of the quaternion is then integrated to find the next quaternion state. Other than defining the attitude state of the satellite, the required rotational maneuver can also be described with quaternions, which is also called as to-go-quaternions (Figure 3.11). This rotation is also called Eigenaxis rotation [12].

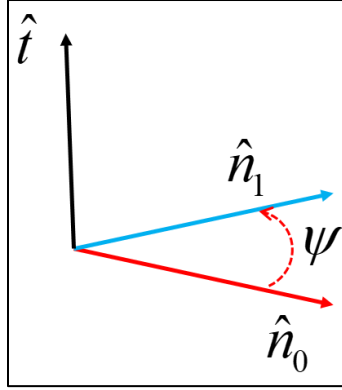


Figure 3.11. Eigenaxis Rotation with To-Go-Quaternion

The rotation angle and axis are found using the initial and final states of the defined unit vector direction.

$$\hat{t} = \hat{n}_0 \times \hat{n}_1 \quad (61)$$

$$\cos(\psi) = \hat{n}_0 \cdot \hat{n}_1 \quad (62)$$

Then, the to-go-quaternion is built as below.

$$\vec{t} = \begin{bmatrix} t_1 \sin\left(\frac{\psi}{2}\right) \\ t_2 \sin\left(\frac{\psi}{2}\right) \\ t_3 \sin\left(\frac{\psi}{2}\right) \\ \cos\left(\frac{\psi}{2}\right) \end{bmatrix} \quad (63)$$

The dynamics of the to-go-quaternion are given below. Notice that they are the compact form of Equation (60) [12].

$$2\dot{\vec{t}} = t_4 \vec{\omega} - \vec{\omega} \times \vec{t} \quad (64)$$

$$2\dot{t}_4 = -\vec{\omega}^T \vec{t} \quad (65)$$

Even though the quaternions are enough to represent the attitude of the body, the conventional Euler Angles are also derived from the Direction of Cosine Matrix (DCM- $\hat{C}$) in 3-2-1 (Yaw-Pitch-Roll) sequence as follows [12]:

$$\left[\hat{C}\right]_{\psi_3-\psi_2-\psi_1} = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2+q_3q_4) & 2(q_1q_3-q_2q_4) \\ 2(q_1q_2-q_3q_4) & 1-2(q_1^2+q_3^2) & 2(q_2q_3+q_1q_4) \\ 2(q_1q_3+q_2q_4) & 2(q_2q_3-q_1q_4) & 1-2(q_2^2+q_1^2) \end{bmatrix} \quad (66)$$

$$\left[\hat{C}\right]_{\psi_3-\psi_2-\psi_1} = \begin{bmatrix} c\psi_2c\psi_3 & c\psi_2s\psi_3 & -s\psi_2 \\ s\psi_1s\psi_2c\psi_3 - c\psi_1s\psi_3 & s\psi_1s\psi_2s\psi_3 + c\psi_1c\psi_3 & s\psi_1c\psi_2 \\ c\psi_1s\psi_2c\psi_3 + s\psi_1s\psi_3 & c\psi_1s\psi_2s\psi_3 - s\psi_1c\psi_3 & c\psi_1c\psi_2 \end{bmatrix} \quad (67)$$

$$\begin{aligned} \psi_1 &= \text{atan2}(C_{23}, C_{33}) \\ \psi_2 &= -\sin^{-1}(C_{13}) \\ \psi_3 &= \text{atan2}(C_{12}, C_{11}) \end{aligned} \quad (68)$$

Moreover, the attitude control methodologies and analyses do need an extra attention on the angular velocity information. Hence, angular momentum derivation formulations [12] from the defined unit vector direction, DCM and to-go-quaternion, and their time rate of change values are given below:

$$\vec{\omega} = \left(\dot{\vec{b}}_2 \cdot \vec{b}_3\right)\vec{b}_1 + \left(\dot{\vec{b}}_3 \cdot \vec{b}_1\right)\vec{b}_2 + \left(\dot{\vec{b}}_1 \cdot \vec{b}_2\right)\vec{b}_3 \quad (69)$$

$$\vec{\omega} = \begin{bmatrix} \dot{C}_{21}C_{31} + \dot{C}_{22}C_{32} + \dot{C}_{23}C_{33} \\ \dot{C}_{31}C_{11} + \dot{C}_{32}C_{12} + \dot{C}_{33}C_{13} \\ \dot{C}_{11}C_{21} + \dot{C}_{12}C_{22} + \dot{C}_{13}C_{23} \end{bmatrix} \quad (70)$$

$$\vec{\omega} = 2 \begin{bmatrix} \dot{t}_1t_4 + \dot{t}_2t_3 - \dot{t}_3t_2 - \dot{t}_4t_1 \\ \dot{t}_2t_4 + \dot{t}_3t_1 - \dot{t}_1t_3 - \dot{t}_4t_2 \\ \dot{t}_3t_4 + \dot{t}_1t_2 - \dot{t}_2t_1 - \dot{t}_4t_3 \end{bmatrix} \quad (71)$$

Finally, the relation between DCM and to-go-quaternion is provided [12]:

$$\hat{C} = (t_4^2 - \vec{t}^T \vec{t}) \hat{I} + 2\vec{t}\vec{t}^T - 2t_4\vec{t} \quad (72)$$

$$t_4 = \frac{1}{2} \sqrt{\text{trace}(\hat{C}) + 1} \quad (73)$$

$$\vec{t} = \frac{1}{4t_4} \begin{bmatrix} C_{23} - C_{32} \\ C_{31} - C_{13} \\ C_{12} - C_{21} \end{bmatrix} \quad (74)$$





CHAPTER 4

RAPID RESPONSE SOLAR SAIL CONCEPT

The proposed mission concept is named as “Rapid Response Solar Sail-R2S2”. The critical feature of R2S2 concept is the harvesting of sunlight as an unlimited source of propellant. In this vein, the alteration of the orbital parameters with respect to the needs of the mission is investigated. In the following sections, detailed definitions of the concept, mission orbit and mode hierarchy are given.

4.1 Concept Definition

To repeat, R2S2 synthesizes the gains of solar sailing and low altitudes by using an elliptic orbit. The mission orbit is then divided into two parts. Below the mode switching altitude, the payload mission is carried out while the sail is kept tangent to the orbit primarily to reduce aerodynamic drag (Figure 4.1) and eventually minimizing the disturbances to the gravity gradient torques. Above the mode switching altitude, solar sailing is used to resize, reshape and/or reorient the orbit, either for station-keeping or transferring to a different orbit, as needed. Moreover, the perigee altitude can be raised above mode switching altitude in order to have the ability to use solar sail maneuvers everywhere along the orbit trajectory. This capability has the potential to have region of interest specific orbit selection, and transfer to orbits with best mission performance for the next region of interest.

The division of the elliptical orbit is allowing the use of SSS at very low altitudes down to 400 km. In this approach, the advantages of the LEO is utilized in the favor of the payload working principles (improved performance when it is at the close proximity to Earth) while preserving the generation of propellant-free thrust by using SRP. Hence, low but continuous solar acceleration is pointed along the direction which the mission requires for altering/maintaining orbital parameters.

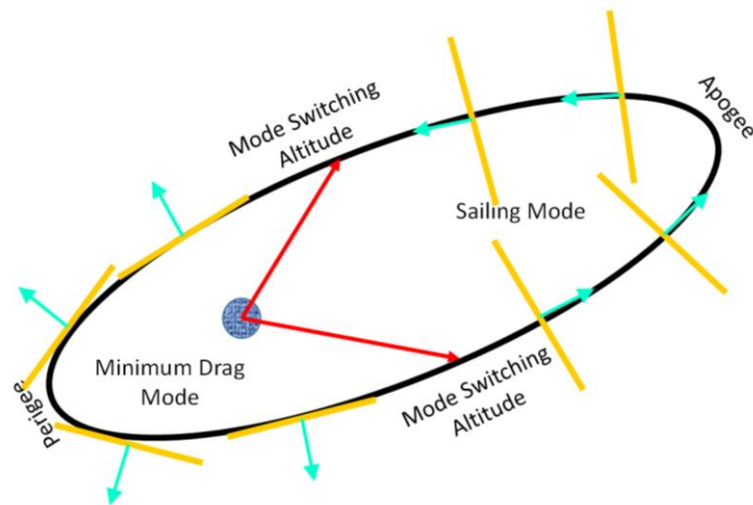


Figure 4.1. Illustration of the R2S2 Concept with Switching Modes

R2S2 concept consists of three phases: Mission, Standby and Transfer (Figure 4.2). The transitions between the aforementioned phases continue until the end of the lifetime of the satellite. Lifetime of the satellite, in the case of solar sails, can be formulated by the reliability properties of the electronic components of the satellite (specifically solar panel cells and battery) or the degradation of the sail material. There are studies on the degradation of the sail material and development of more durable sail materials [26, 34, and 66].

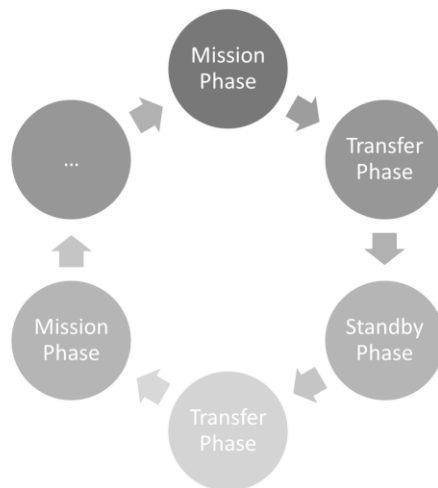


Figure 4.2. Phases of the R2S2 Concept

At the end of the mission, whether by environmental disturbances or the completion of the mission, the perigee altitude is increased to an altitude where solar acceleration is dominant over the aerodynamic drag and solar sailing can be applied in the entire orbit trajectory (above 800 km), as long as sunlight reaches to satellite (no Earth shadowing). This orbit is named as “Standby Orbit”. When the new needs of the mission are brought up, the perigee altitude is then lowered to an altitude down to 400 km, with extra considerations over the argument of perigee, right ascension of ascending node (RAAN) and mean local time of ascending node (MLTAN). During these transfer orbits, solar sailing is applied above the mode switching altitude and sail alignment is kept such that aerodynamic drag is minimized. Both increasing and lowering phases are called “Transfer Phase”. The trajectories can be numerous depending on the varying initial and final conditions. The details about the phases of the R2S2 concept and relevant modes are provided in the next section.

4.2 Mode Hierarchy

The R2S2 concept with continuous phase transitions and solar sailing maneuvers uses 4-tier mode hierarchy (Figure 4.3) for mission control. The first mode (Mission Mode) is for defining the orbit type, then Task Mode is defined with respect to the other goals (station-keeping, minimum energy loss, raising or decreasing the orbit). It is followed by the Orientation Mode is determined according to the altitude of the satellite with respect to the mode switching altitude. This allows us to give green flag to make sail rotations for desired attitude with negligible aerodynamic drag effects. The last tier is used for only station-keeping. The definitions are listed in Table 4.1.

The switches between mission modes depend on mission lifetime. The mission lifetime depends on two reasons: First one is mission-specific lifetime, which will not be discussed further here since it is coming from the end user. Second reason is the disturbance-specific lifetime that is taken into consideration in this study.

Table 4.1 Mode Definitions

Mode Name	Definition
Mission Orbit	This mode is the indication of the orbit type. The selected elliptical orbit is employed for the Earth observation mission and pre-determined orbital parameters are maintained by station-keeping.
Transfer Orbit	This mode is the indication of the orbital transfers between mission and standby orbits.
Station-Keeping	This mode is active when mission orbit is employed and triggering conditions for selected orbital parameters are activated. Necessary orbital maneuvers are performed to maintain the mission-specific orbital parameters.
Minimum Energy Loss	This mode is the indication of that there is no maneuvering needs, therefore minimum energy loss is aimed. This aim is accomplished by the Minimum Drag and Minimum Solar modes.
To Standby	This mode, belonging the Transfer Orbit, is the indication of necessary maneuvers to be performed in order to reach the standby orbit by raising the perigee altitude above the mode switching altitude.
To Mission	This mode, belonging the Transfer Orbit, is the indication of necessary maneuvers to be performed in order to reach the mission orbit by altering the perigee altitude, inclination, right ascension of ascending node and argument of perigee.
Sailing	This mode is the indication of that the altitude of the satellite is above the mode switching altitude and the satellite is free to orient its sail to generate required thrust for maneuvering needs.
Minimum Drag	This mode is the indication of that the altitude of the satellite is below the mode switching altitude and sail orientation should be kept tangent to orbit in order to minimize the aerodynamic drag.
Minimum Solar	This mode is the indication of that the altitude of the satellite is above the mode switching altitude and there is no maneuvering needs. Therefore, the sail orientation should be kept tangent to ecliptic plane in order to minimize the Solar Radiation Pressure effects on the orbit.

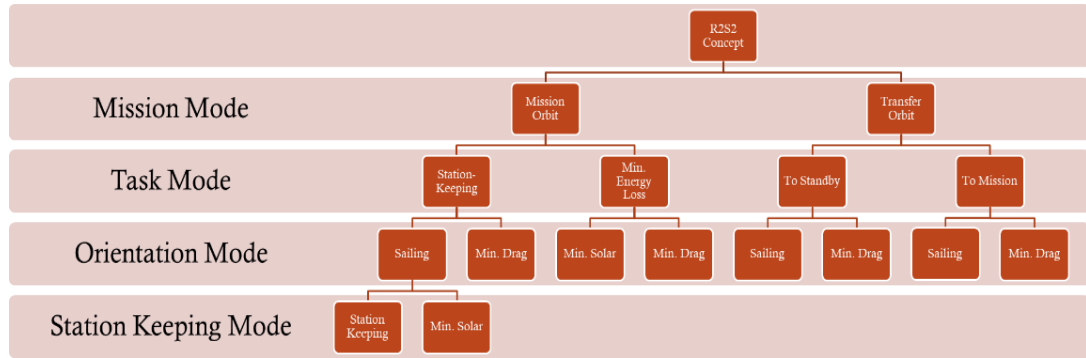


Figure 4.3. Mode Hierarchy of the R2S2 Concept

Four orbital parameters (out of six) are taken into consideration for the disturbance-specific lifetime: semi-major axis, eccentricity, argument of perigee and RAAN. Since the ellipticity of the mission orbit is vital to the concept, semi-major axis and eccentricity are tracked in terms of perigee and apogee altitudes. Moreover, perigee and apogee altitudes are crucial for the safety of the concept, because the failure of maintaining these altitudes will result in the decay of the orbit exponentially. Without station-keeping, lifetime in the mission orbit may vary between 20 days to 3 months depending on the Sun-Earth-Satellite orientation (Chapter 4.4). As it will be seen in Chapter 8, proper station-keeping of the perigee and apogee altitudes may raise the mission lifetime to 6-12 months.

Argument of perigee and RAAN values are important for mission success (revisit frequency and access number within the finite mission lifetime). The details of these effects are presented in Chapter 6. In summary, there are some intervals of values for argument of perigee effecting the mission performance relevant to the altitude criteria for an acceptable access of the satellite to the observation site. Similar intervals are also present for RAAN values for given day of the year effecting the mission performance relevant to daylight condition. Changes due to J2 effects (apsidal and nodal precessions) are to be reduced by solar sailing to extend the mission lifetime with high performance observation metrics. The aforementioned intervals shall be optimized for the mission orbit at the beginning with the trends of J2 perturbation. Although it is impossible to compensate for the asphericity effects of the Earth continuously, through solar sailing this tendency may be reduced.

To better understand, three conditions of the R2S2 concept conditions for an acceptable access are given below:

- The site is illuminated by the Sun (Daylight condition is needed for electro-optical satellite to view the site),
- The altitude of the satellite is below the mode switching altitude,
- Small Line-of-Sight angle between site and satellite is achieved (i.e. 20 degrees for half sensor angle for pointing off-nadir).

4.3 Forces Acting on the Satellite

A study was carried out on the forces in the equations of motion of the orbiting SSS within the altitude envelope of R2S2 concept. The goals of the analysis were;

- To determine the relative order of magnitudes of forces that will enable us to propose some simplifications and assumptions at the further strategies of R2S2 concept (for example maneuver planning strategy in Chapter 6),
- To define the relation between solar and aerodynamic accelerations with varying altitudes and angles that eventually allow the determination of the mode switching altitude.

The total acceleration experienced by a satellite around Earth can be remembered here again:

$$\vec{a}_{total} = \vec{a}_{2-body} + \vec{a}_{ThirdBody} + \vec{a}_{asphericity} + \vec{a}_{aero} + \vec{a}_{SRP} \quad (75)$$

Table 4.2 shows the order of magnitudes of gravitational acceleration contributors. Figure 4.4, on the other hand, gives the comparison of the gravitational accelerations in the logarithmic scale.

Based on the relative contribution of each gravitational acceleration term, it has been observed that the third body accelerations due to the Moon and Sun are relatively too small and acceleration due to Earth asphericity is dominated by the J2 effect.

Table 4.2 Order of Magnitudes of Gravitational Accelerations

Gravitational Acceleration	Order of Magnitude [m/s ²]
Two-Body Acceleration	10^1
Asphericity Acceleration	10^{-3} - 10^{-2}
J2 Effect	10^{-3} - 10^{-2}
J3 Effect	10^{-5}
J4 Effect	10^{-4}
Third Body Acceleration	10^{-7} - 10^{-6}

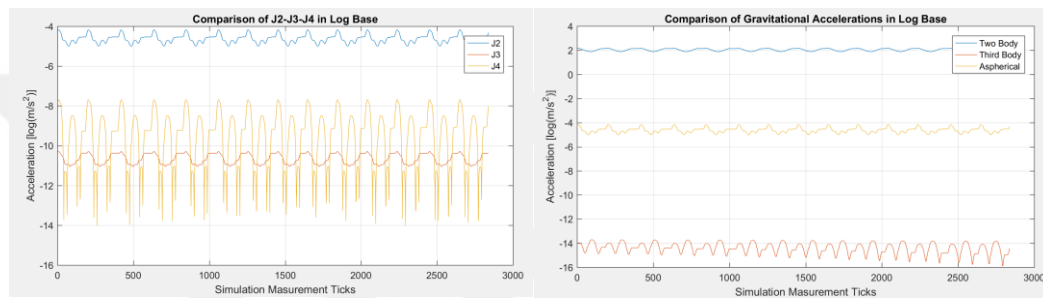


Figure 4.4. Graphical Representation of Conservative Accelerations

In addition to those gravitational conservative forces, non-conservative forces of aerodynamic drag and solar radiation pressure are the two main factors needed for the successful realization of the R2S2 concept. Figure 4.5 shows the variation of the solar acceleration with respect to sail pitch angle, whereas Figure 4.6 shows the aerodynamic acceleration with respect to altitude and angle of attack, defined as the angle between the satellite velocity and sail normal. The mesh results in Figure 4.6 are tabulated in Figure 4.7.

As it may be observed from Figure 4.6 and Figure 4.7, aerodynamic acceleration varies quite a lot with altitude. Table 4.3 shows the order of magnitude of non-conservative accelerations. The sail pitch angle results in one order of magnitude change on solar acceleration, on the other hand the altitude results in five orders of magnitude change on aerodynamic drag due to rapid density drop and velocity decrease as altitude gets higher.

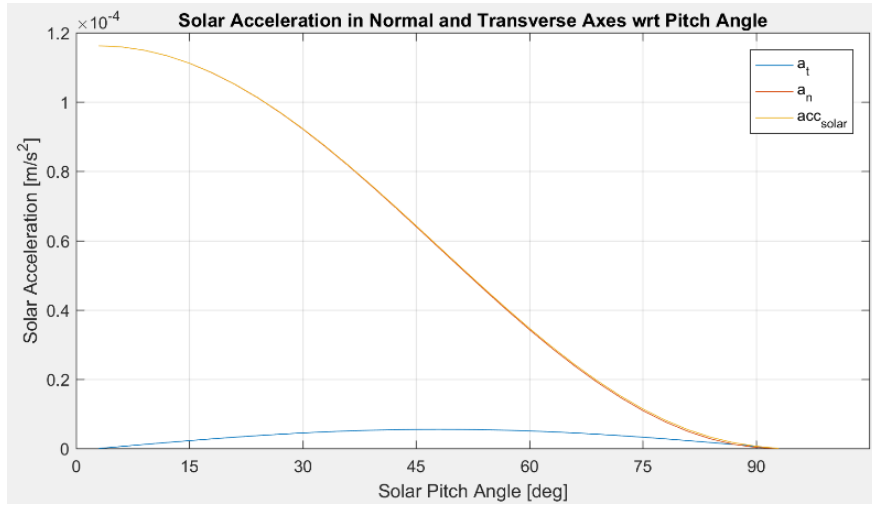


Figure 4.5. Solar Acceleration Variation as a Function of Sail Pitch Angle

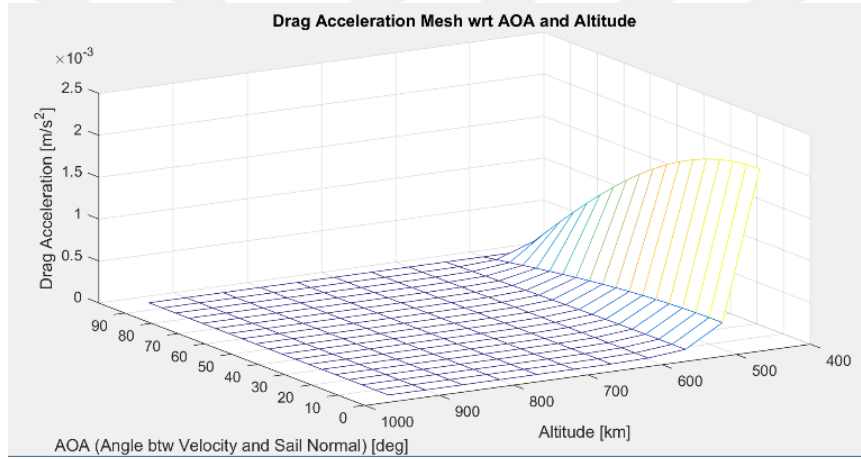


Figure 4.6. Aerodynamic Drag as a Function of Altitude and Sail Angle of Attack

		ANGLE OF ATTACK ANGLE [DEG]																		
		0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
ALTITUDE [KM]	400	2.1E-03	2.1E-03	2.1E-03	2.0E-03	2.0E-03	1.9E-03	1.8E-03	1.7E-03	1.6E-03	1.5E-03	1.3E-03	1.2E-03	1.0E-03	8.8E-04	7.1E-04	5.4E-04	3.6E-04	1.8E-04	0.0E+00
	500	3.2E-04	3.2E-04	3.1E-04	3.1E-04	3.0E-04	2.9E-04	2.7E-04	2.6E-04	2.4E-04	2.2E-04	2.0E-04	1.8E-04	1.6E-04	1.3E-04	1.1E-04	8.2E-05	5.5E-05	2.8E-05	0.0E+00
	600	5.8E-05	5.7E-05	5.7E-05	5.6E-05	5.4E-05	5.2E-05	5.0E-05	4.7E-05	4.4E-05	4.1E-05	3.7E-05	3.3E-05	2.9E-05	2.4E-05	2.0E-05	1.5E-05	1.0E-05	5.0E-06	0.0E+00
	700	1.6E-05	1.6E-05	1.5E-05	1.5E-05	1.5E-05	1.4E-05	1.3E-05	1.3E-05	1.2E-05	1.1E-05	1.0E-05	8.9E-06	7.8E-06	6.6E-06	5.3E-06	4.0E-06	2.7E-06	1.4E-06	0.0E+00
	800	5.8E-06	5.7E-06	5.7E-06	5.6E-06	5.4E-06	5.2E-06	5.0E-06	4.7E-06	4.4E-06	4.1E-06	3.7E-06	3.3E-06	2.9E-06	2.4E-06	2.0E-06	1.5E-06	1.0E-06	5.0E-07	0.0E+00
	900	3.2E-06	3.2E-06	3.2E-06	3.1E-06	3.0E-06	2.9E-06	2.8E-06	2.6E-06	2.5E-06	2.3E-06	2.1E-06	1.8E-06	1.6E-06	1.4E-06	1.1E-06	8.3E-07	5.6E-07	2.8E-07	0.0E+00
	1000	1.9E-06	1.9E-06	1.9E-06	1.9E-06	1.8E-06	1.8E-06	1.7E-06	1.6E-06	1.5E-06	1.4E-06	1.2E-06	1.1E-06	9.7E-07	8.2E-07	6.6E-07	5.0E-07	3.4E-07	1.7E-07	0.0E+00
	1100	1.3E-06	1.3E-06	1.3E-06	1.2E-06	1.2E-06	1.2E-06	1.1E-06	1.0E-06	9.8E-07	9.1E-07	8.2E-07	7.3E-07	6.4E-07	5.4E-07	4.4E-07	3.3E-07	2.2E-07	1.1E-07	0.0E+00
	1200	8.7E-07	8.6E-07	8.5E-07	8.4E-07	8.1E-07	7.8E-07	7.5E-07	7.1E-07	6.6E-07	6.1E-07	5.6E-07	5.0E-07	4.3E-07	3.7E-07	3.0E-07	2.2E-07	1.5E-07	7.5E-08	0.0E+00
	1300	6.0E-07	6.0E-07	5.9E-07	5.8E-07	5.6E-07	5.4E-07	5.2E-07	4.9E-07	4.6E-07	4.2E-07	3.8E-07	3.4E-07	3.0E-07	2.5E-07	2.0E-07	1.5E-07	1.0E-07	5.2E-08	0.0E+00
1400	4.3E-07	4.3E-07	4.2E-07	4.1E-07	4.0E-07	3.9E-07	3.7E-07	3.5E-07	3.3E-07	3.0E-07	2.8E-07	2.5E-07	2.1E-07	1.8E-07	1.5E-07	1.1E-07	7.4E-08	3.7E-08	0.0E+00	

Figure 4.7. Aerodynamic Drag at Different Altitudes and Sail Angle of Attack

Table 4.3 Order of Magnitudes of Non-Conservative Accelerations

Non-Conservative Acceleration	Order of Magnitude [m/s ²]
Aerodynamic Drag	10^{-3} - 10^{-8}
Solar Acceleration	10^{-4} for $\alpha_{sail}:[0^0$ - $24^0]$, otherwise $<10^{-5}$

When Figure 4.7 and Table 4.3 are synthesized, following deductions can be made;

- Aerodynamic deceleration is always greater than solar acceleration at altitudes below 400 km regardless of angle of attack,
- At altitudes around 500 km, the same relation holds up to 75 degrees of angle of attack.
- Above 600 km altitude, solar acceleration is substantially greater than aerodynamic deceleration for high solar acceleration values (sail pitch angle is below 24 degrees),
- Above 800 km altitude on the other hand, solar acceleration is always greater than aerodynamic deceleration regardless of sail pitch angle.

Thus, mode switching altitude is selected to be 800 km in order to avoid exposing the large sail area to aerodynamic drag. In this configuration, the duration of the orbital interval is 42% of the orbital period for the minimum drag mode and 58% for the sailing mode, if the perigee altitude is 400 km and apogee altitude is 1400 km. As mentioned earlier, the mode switching altitude is critical to the R2S2 concept. The improper selection of that altitude may result in the failure of the concept. If it is selected as lower than it should be, then attitude changes for the maneuvers of solar sailing result in excessive aerodynamic drag and orbit will decay faster. On the other hand, if it is selected as a higher value, then maneuverable orbit positions will be diminished such that certain type of maneuvers will be ineffective (especially for those maneuvers aiming to alter the apogee altitude value).

4.4 Preliminary Feasibility of the Concept

Prior to the in-depth analyses and strategy buildups, a preliminary feasibility study was carried out. This feasibility study mainly concentrated on the subject of transfer capability of solar sailing in such orbit altitude envelope of R2S2 concept. The results of the feasibility study was presented at RAST 2019 conference in Istanbul/Turkey [67]. In this thesis, the results are then fed into Chapter 5 for baseline satellite design.

For the preliminary feasibility study, successive transfer phases between mission and standby orbits are simulated. For this purpose, perigee altitude was increased to 800 km with solar sailing. After that, the altitude was lowered to the perigee altitude at the beginning. In this way, successive transfer maneuvers were carried out.

The rationale behind the feasibility simulations is given in Table 4.4. The decision-making process for switching modes between raising and decreasing was carried out by the criteria shown in Table 4.4. Then, the orientation of the sail is determined with the selected mode. For instance, Line of Sight (LOS) between satellite and the Sun should be present (LOS=1) to activate the Sailing Mode since solar acceleration needs the exposure of the sunlight to the sail surface.

Table 4.4 Mode Switching Process

Raising Mode	Sailing Mode	LOS=1 Altitude>800 km $\vec{V}_{sat} \cdot (\vec{r}_{sat} - \vec{r}_{\odot}) > 0$
	Minimum Drag Mode	When above conditions are not met
Decreasing Mode	Sailing Mode	LOS=1 Altitude>800 km $\vec{V}_{sat} \cdot (\vec{r}_{sat} - \vec{r}_{\odot}) < 0$
	Minimum Drag Mode	When above conditions are not met

When the satellite's conditions met the criteria for minimum drag mode, the sail was slewed such that normal unit vector was perpendicular to the satellite's velocity vector. Thus, the aerodynamic drag effects were minimized. The normal unit vector of the sail was then given as below:

$$\hat{n} = \left(\frac{\vec{r}_{sat} \times \vec{V}_{sat}}{|\vec{r}_{sat} \times \vec{V}_{sat}|} \right) \times \left(\frac{\vec{V}_{sat}}{|\vec{V}_{sat}|} \right) \quad (76)$$

For the increasing mode, sail's normal was pointed towards the velocity vector of the satellite. Similarly, it was put along the opposite direction of the velocity vector for the decreasing mode. The use of velocity vector direction maximized the rate of change of the orbital energy for both modes. This switch on-off control approach was also applied by various studies [3, 24-25, 68].

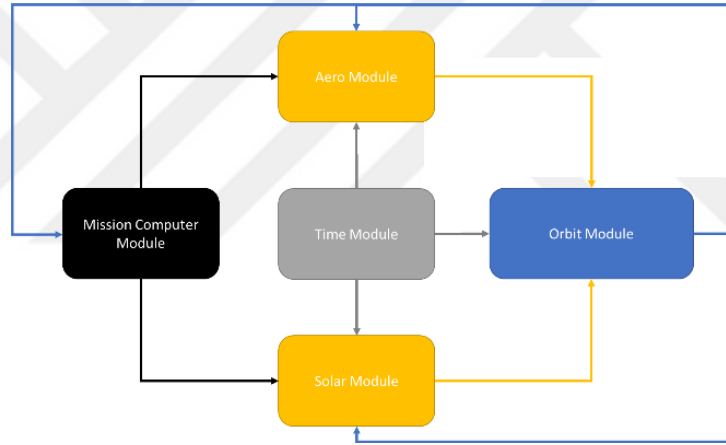


Figure 4.8. Block Diagram of the Feasibility Simulation

As can be observed in Figure 4.8, the feasibility simulation model was consisted of five modules. Orbit module was in charge of propagation of the satellite's motion while only gravitational attraction of the Earth as a perfect sphere was taken into consideration. The output of this module were the state vector (position and velocity information) and orbital elements.

Aero module calculated aerodynamic drag and relevant deceleration by taking the attitude information into account. Solar module determined the position of the Sun and propagated it, then, the solar acceleration was calculated by using the direction

information of the sail normal under the assumption of an ideal sail as in Equation (8). Time module was accounted for the timekeeping and transformation between different units of time such as Barycentric Dynamical Time, Julian Date, Julian Centuries and Greenwich Sidereal Time.

At last, the module of mission computer decided the mode with respect to Table 4.4 and generated the information of the direction of the sail's normal for the other modules.

For the SSS starting attributes, three various designs were chosen and implemented in the simulations as provided in Table 4.5: John Hopkins University-JHU (late 1980s) [69], Cambridge Consultants Ltd-CCL (early 1990s) [3] and ODISSEE mission by DLR/JPL [68].

Table 4.5 Different Satellite Configuration Parameters

Parameter	JHU	CCL	ODISSEE
Payload Mass (kg)	80 kg	60 kg	36 kg
Sail and Supporting Structure Mass	100 kg	240 kg	41 kg
Total Mass (kg)	180 kg	300 kg	77 kg
Sail Area (m ²)	22700	60000	1600
Coefficient of Drag	3	3	3
Satellite Bus Dimensions	N/A	4 m height 4 m diam.	0.6x0.6x0.8 m
Efficiency Factor	0.85	0.85	0.85
Sail Loading (g/m ²)	7.9	5.0	48.1
Characteristic Acceleration (mm/s ²)	0.98	1.55	0.16

These various SSS designs were chosen by taking three main factors into consideration. First, they were commonly referenced and used by other studies as a common baseline designs for solar sailing. Second, their characteristic acceleration levels were in a wide range of spectrum; i.e. $<1 \text{ mm/s}^2$, $\approx 1 \text{ mm/s}^2$ and $>1 \text{ mm/s}^2$. This was beneficial for the study in terms of seeing different performances for the R2S2 concept and analyzing the achieved response times to the tasks of altitude change.

Third, their choice of geometry for the sails were all different. JHU utilized a petal design which was the combination of heliogyro and flat plate, CCL used a disk type geometry and ODISSEE used a square geometry [3].

Initial conditions for orbital parameters and timing were shown in Table 4.6. The predetermined numbers for the orbit and timing were chosen to analyze various SSS designs depicted earlier.

Table 4.6 Initial Simulation Conditions

Parameter	Initial Value
Perigee Altitude	400 km
Apogee Altitude	1200 km
Mode Switching Altitude	800 km
Inclination	75 ⁰
RAAN	0 ⁰
Argument of Perigee	0 ⁰
True Anomaly	0 ⁰
Initial Time in UTC	23/02/2019 00:00:00.000

The results were analyzed in two sections. First, a standard initial orbit and time attributes were set. The applicability of successive transfer phases of raising and decreasing were run in the simulation and the performances of three SSS designs were shown. Then, JHU design was used for different inclination and orbital altitude values (various perigee and apogee altitudes). Moreover, the perigee and apogee altitude alterations and cycle durations (one cycle means a total transfer from mission to standby orbit and back to mission orbit again) received extra attention for the validation of the feasibility of the concept.

The maneuvering of the perigee altitude was presented in Figure 4.9. JHU and the CCL designs demonstrated a 9 cycle/year performance while ODISSEE performed a 1.2 cycle/year value.

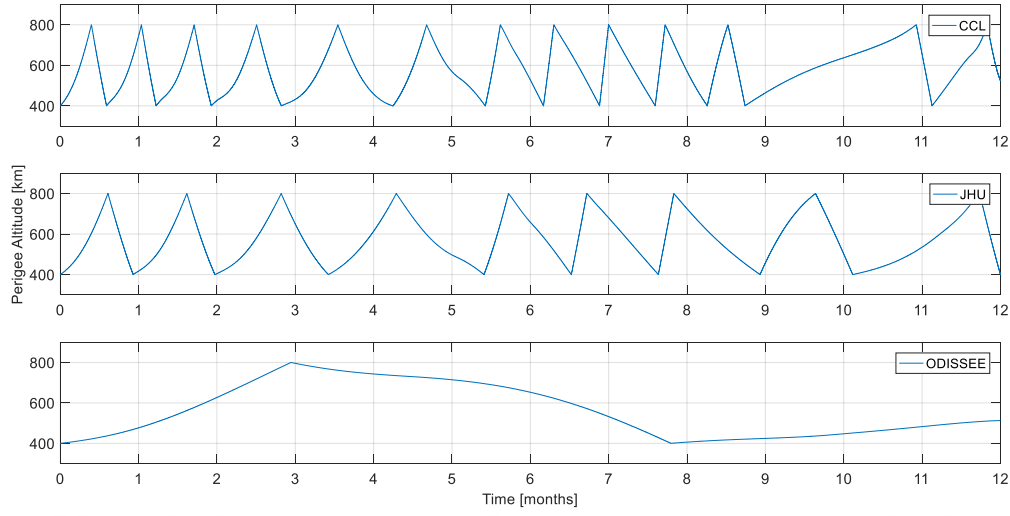


Figure 4.9. Perigee Altitude Change During Successive Transfer Phases

The outputs for various inclination values can be observed in Figure 4.10. After the second cycle, the initial inclination values changed the performance of the altitude maneuvers. Nevertheless, prograde orbits performed similarly regarding the number of the cycles, while retrograde orbits demonstrated differently as Sun's relative positioning and eventually illumination of the satellite varied (Figure 4.11). This behavior will be addressed in Chapter 7 in detail.

To see the boundaries of the altitude of perigee, different starting values of altitudes for perigee and apogee were tested (Figure 4.12-Figure 4.15). When the outputs were analyzed, it was deduced that control solutions were not obtained under 300 km of perigee altitude. Nonetheless, the increased value of the altitude of apogee caused a boost on the number of the cycle/year (Figure 4.13-Figure 4.15) and applicability of the orbital altitude maneuvers (Figure 4.13). For instance, the orbital conditions with a perigee altitude of 300 km and an apogee altitude more than 1300 km were seen feasible, because the increase of the apogee altitude resulted in a rise over the available time for the sailing maneuvers.

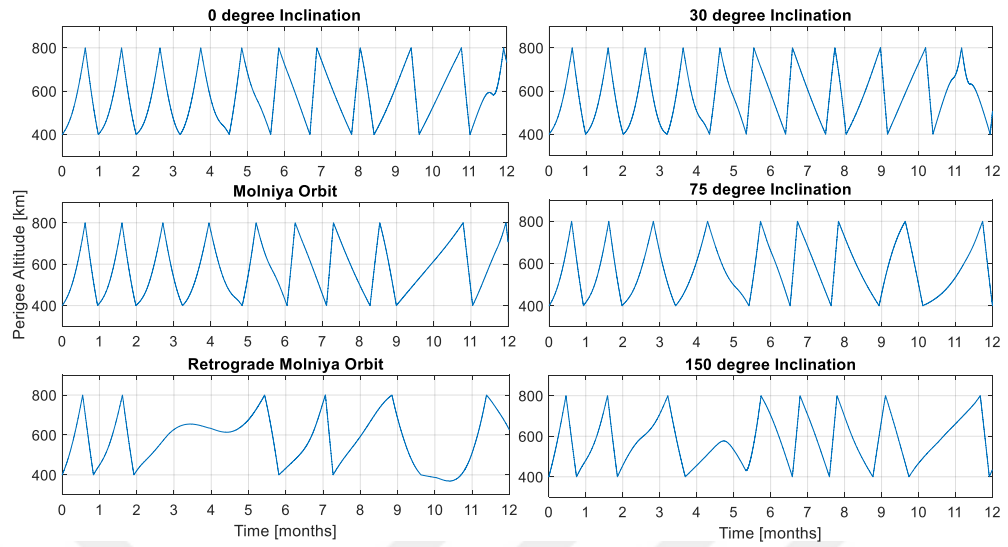


Figure 4.10. The Results for Different Inclination Values

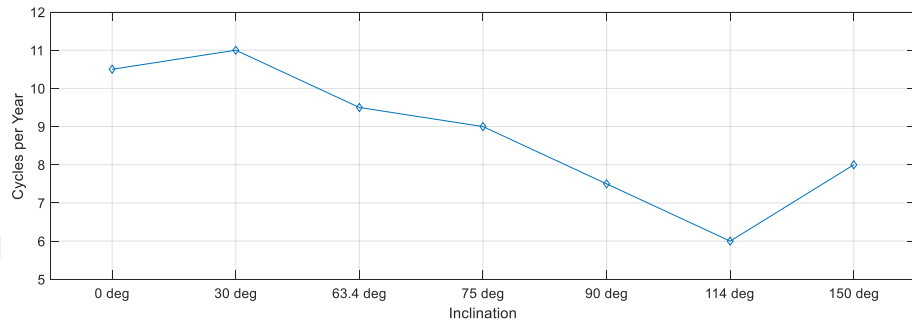


Figure 4.11. Cycles/Year Performance for Various Inclination Values

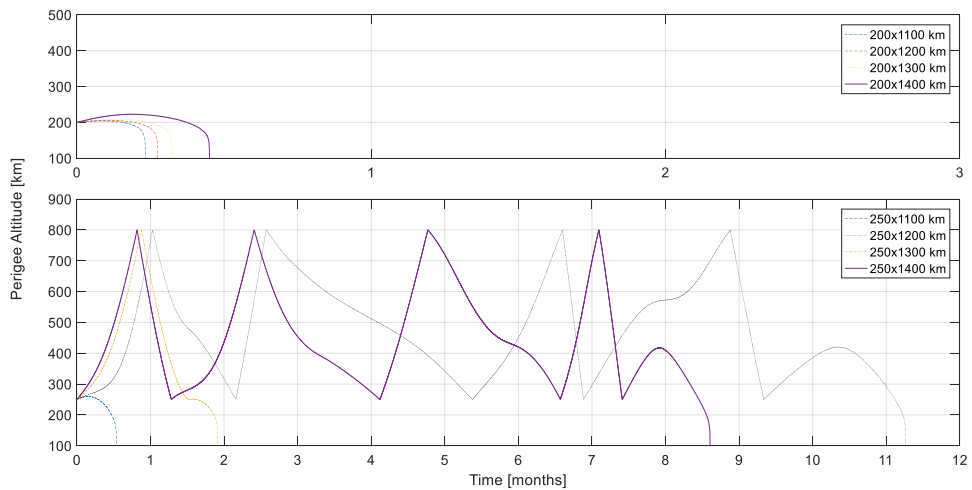


Figure 4.12. The Demonstration of Unfeasible Orbital Altitude Regimes

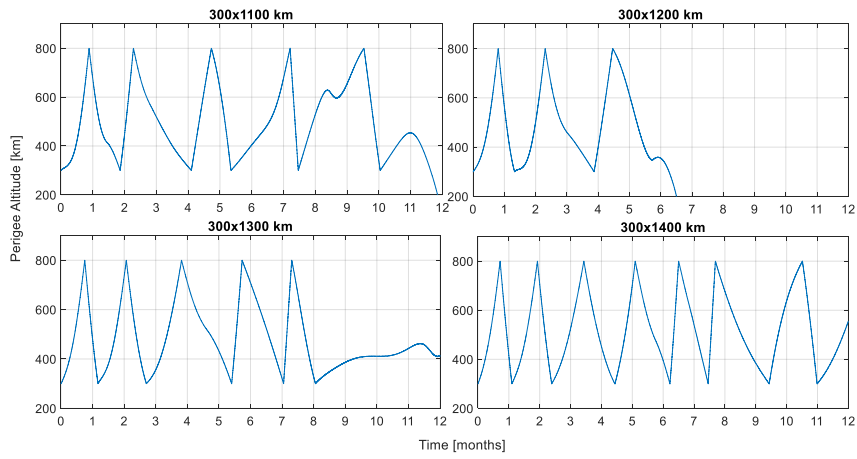


Figure 4.13. Changing Apogee Altitude with 300 km of Perigee Altitude

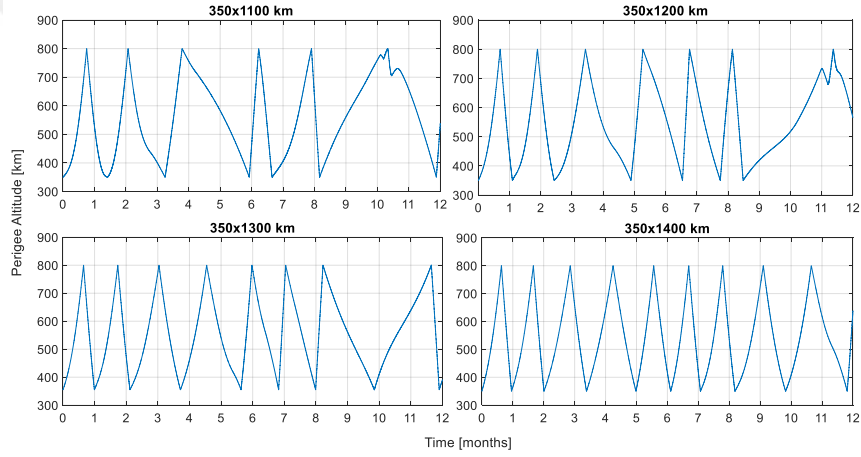


Figure 4.14. Changing Apogee Altitude with 350 km of Perigee Altitude

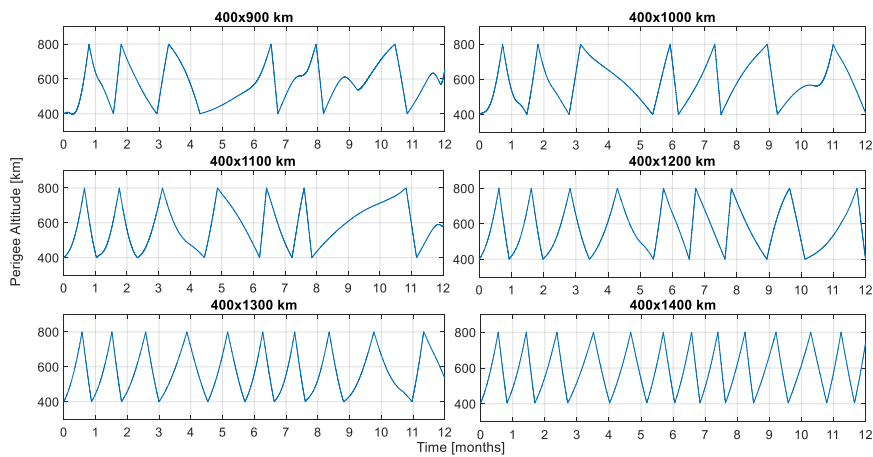


Figure 4.15. Changing Apogee Altitude with 400 km of Perigee Altitude

Applicable lower limits for the apogee altitude were also studied (Figure 4.16). An altitude of 900 km did not result in an unfeasible maneuvering performance, however it caused an inefficient utilization of the R2S2 concept.

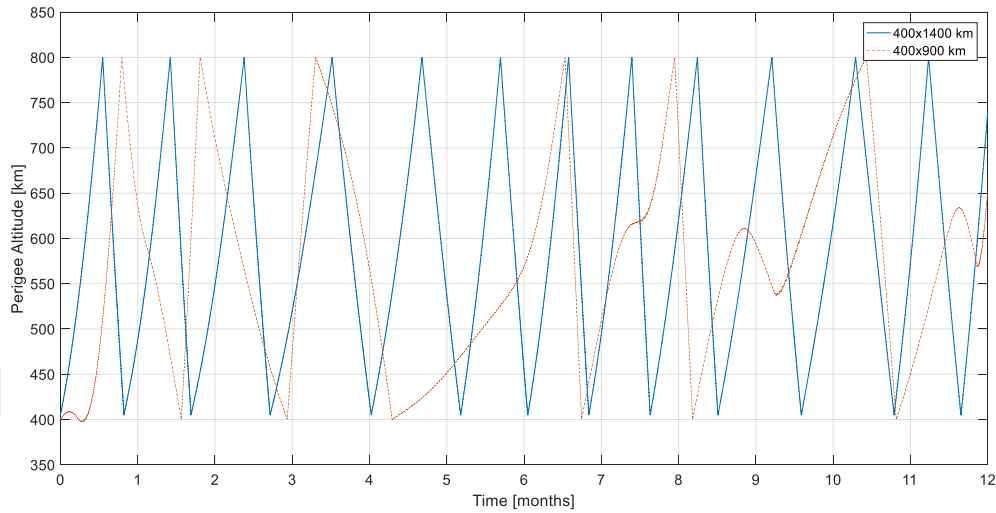


Figure 4.16. Comparison of Upper and Lower Limits for Apogee Altitude

The results of the feasibility study are then used in the remaining part of the study as the initial stepping stone for the determination of the parameters regarding the orbit size and shape, baseline satellite design and eventually constellation optimization.

4.5 Limitations of the Concept

The final portion about the proposed R2S2 concept is its limitations of application. These limitations are stemming from different reasons and disciplines. However, the in-depth demonstration of the R2S2 concept in the following chapters should take these limitations into consideration for the sake of applicability. The limitations of the R2S2 concept listed below were divided into two categories of maneuverability and applicability.

Maneuverability:

- The sail should be kept tangent to the orbital plane below the mode switching altitude in order to minimize the aerodynamic drag. For this reason, the sail cannot be maneuvered along the computed thrust vector. This no-maneuver region of the orbit is taken into consideration by computing a true anomaly interval from the orbit equation.
- The moment of inertia of SSS are very large and slewing times can be quite long. Therefore, returning to the minimum drag mode orientation should be taken into consideration. This is done by quitting the thrust vector control trajectory before crossing the 800 km altitude limit and allowing the necessary slew time to acquire the minimum drag mode orientation.
- Reorientation of the orbit is too expensive even for a propellant-free thrust capability with solar sailing. The secular perturbation of asphericity effects of the Earth on RAAN and argument of perigee make the control authority of R2S2 concept over these orbital parameters ineffective. This effect is periodic on the inclination, therefore R2S2 concept has the ability of changing the inclination with a small amount. Therefore, responsiveness of the R2S2 concept should be met by a proper employment of an optimal constellation.

Applicability:

- The angle between computed thrust vector and sunlight direction should be below 90 degrees. Otherwise, solar radiation pressure cannot be utilized in the direction of control input. Moreover, if the satellite is oriented along the control input direction in such cases, an acceleration in the opposite direction is achieved. Therefore, the sail orientation should track the control input direction if only the aforementioned angle is below 90 degrees. This is a seasonal effect on the orientation of the orbit and apriori maneuver planning strategy should be determined.

- The satellite should not be shadowed by Earth. It is obvious that sunlight should reach the satellite in order to cause a solar acceleration. This is stemming from diurnal and seasonal positioning of the Sun, Earth and orbit orientation. A proper maneuver planning strategy should be laid down.
- When optical parameters (absorption, reflection, transmission, emission) of the sail film material are taken into consideration, the solar acceleration direction is deviated from the sail normal by an angle called centerline angle as described in Chapter 2.2. This deviation is then resulting in a limitation over the directing capability of the sunlight direction into the desired control input direction. This limitation is effecting the efficiency of the orbital maneuvers depending on the day of the year (seasonal effects) and this can be formulated with the angle between orbital plane and sunlight direction. The details about the seasonal effects on the in-plane and out-of-plane maneuvers will be provided in Chapter 6.



CHAPTER 5

BASELINE SATELLITE DESIGN

The R2S2 concept is proposed to bring together the advantages of LEO for Earth observation and solar sailing for propellant-free maneuvering. Therefore, the baseline satellite configuration selection is critical. The criticality of the selection is not only for determining geometrical and mass properties but also for setting the design on foot with achievable Technology Readiness Level (TRL) in the short/mid-term future.

The feasibility study of the R2S2 concept, which was presented in Chapter 4.4, included three different sail designs. However, these different configurations were only used for geometrical and mass properties for the simulation. In the following subsections, the baseline satellite configuration selection is extended to include the TRL of these sail designs and a mission payload assessment to get the maximum advantage of being in the LEO region. After having decided the desired baseline satellite configuration, one more iteration has been done in regard to attitude control considerations.

5.1 Characteristic Acceleration Analysis

If the characteristic acceleration formula in Equation (10) for solar sailing is recalled, a requirement table for sail loading parameter, which was formulated in Equation (9), can be populated. The maximum values of sail loading for given characteristic acceleration are then tabulated in Table 5.1.

Table 5.1 Characteristic Acceleration Analysis

$a_0 \cdot \sigma$ [mm/s ²].[g/m ²]	η	$a_0=1$ mm/s ²	$a_0=0.5$ mm/s ²	$a_0=0.1$ mm/s ²	$a_0=0.01$ mm/s ²
7.296	80%	$\sigma=7.296$ g/m ²	$\sigma=14.6$ g/m ²	$\sigma=72.96$ g/m ²	$\sigma=729.6$ g/m ²
7.752	85%	$\sigma=7.752$ g/m ²	$\sigma=15.5$ g/m ²	$\sigma=77.52$ g/m ²	$\sigma=775.2$ g/m ²
8.208	90%	$\sigma=8.208$ g/m ²	$\sigma=16.4$ g/m ²	$\sigma=82.08$ g/m ²	$\sigma=820.8$ g/m ²
8.664	95%	$\sigma=8.664$ g/m ²	$\sigma=17.3$ g/m ²	$\sigma=86.64$ g/m ²	$\sigma=866.4$ g/m ²
9.12	100%	$\sigma=9.12$ g/m ²	$\sigma=18.24$ g/m ²	$\sigma=91.2$ g/m ²	$\sigma=912$ g/m ²

5.2 Mass Analysis

After calculating the sail loading requirements for different characteristic acceleration values (higher acceleration results in more responsive sail) and different sail efficiency factors, five different sail configuration sizes are introduced into the mass analysis, which are IKAROS [6], ODISSEE [68], U3P [3], JHU [69] and CCL [3]. By merging the sail loading requirements with the sail configurations, the maximum payload mass in kg is calculated and presented in Table 5.2. “Not Applicable-N/A” values in Table 5.2 mean that the sail configuration cannot carry a payload for that characteristic acceleration since the sail is too heavy.

Table 5.2 Sail and Payload Mass Analysis

Sail Area [m ²]	Reference Sail	η	$m_{sail+struc}$ [kg]	$a_0=1$ mm/s ²	$a_0=0.5$ mm/s ²	$a_0=0.1$ mm/s ²	$a_0=0.01$ mm/s ²
400	IKAROS	0.9	25	N/A	N/A	$m_p = 7.8$	$m_p = 303$
1600	ODISSEE	0.9	77	N/A	N/A	$m_p = 54$	$m_p = 1236$
4000	U3P	0.85	146	N/A	N/A	$m_p = 164$	$m_p = 2955$
22700	JHU	0.85	100	$m_p = 76$	$m_p = 252$	$m_p = 1660$	$m_p = 17400$
60000	CCL	0.85	240	$m_p = 197$	$m_p = 636$	$m_p = 4138$	$m_p = 43536$

5.3 Technology Readiness Level Analysis

To repeat, the aim was to have a baseline payload and sail selection with an adequate and usable mission performance (also relevant with R2S2) at an acceptable TRL.

Therefore, TRL assessments of those five sail configurations are added to the analysis and presented in Table 5.3. TRL values were assessed according to the NASA standard definitions [70] presented in Appendix A.

The feasibility study has shown that accelerations of $0.1\text{-}1.5\text{ mm/s}^2$ are feasible, moreover a threshold with a TRL value of 5 is found adequate for the concept. With these considerations, maximum values of payload masses for three different sail configurations are found and shown as underlined in Table 5.3.

Table 5.3 TRL Analysis

Sail Area [m ²]	Reference Sail	TRL	$a_0=1$ mm/s ²	$a_0=0.5$ mm/s ²	$a_0=0.1$ mm/s ²	$a_0=0.01$ mm/s ²
400	IKAROS	8	N/A	N/A	<u>$m_p = 7.8$</u>	$m_p = 303$
1600	ODISSEE	6-7	N/A	N/A	<u>$m_p = 54$</u>	$m_p = 1236$
4000	U3P	4-5	N/A	N/A	<u>$m_p = 164$</u>	$m_p = 2955$
22700	JHU	3	$m_p = 76$	$m_p = 252$	$m_p = 1660$	$m_p = 17400$
60000	CCL	3	$m_p = 197$	$m_p = 636$	$m_p = 4138$	$m_p = 43536$

5.4 Mission Payload

As stated earlier, the mission altitudes (approximately 400-800 km) are to be exploited in terms of close proximity to Earth. Thus, electro-optic telescopes are the main candidate here. As the telescope is closer to Earth, one can achieve better resolution with same telescope or same resolution with smaller telescope, which are both beneficial.

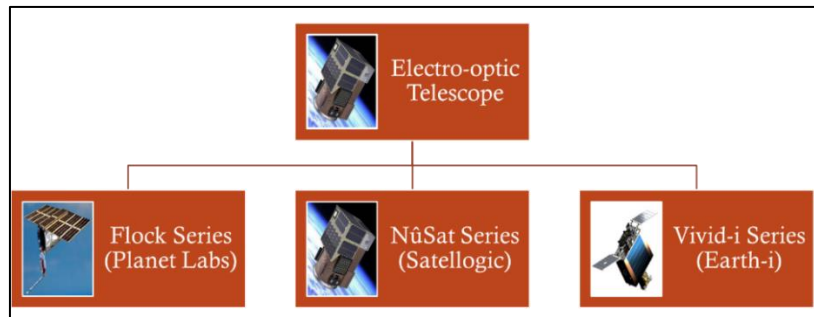


Figure 5.1. Candidates for Electro-Optic Mission Payload

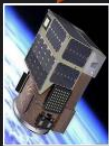
There are numbers of examples and projects that may be considered. However, those presented in Figure 5.1 are selected due to their low mass and relatively high mission performance in LEO region in terms of image resolution with different imaging modes. The Flock Series by the company Planet Labs, are employing a 3U CubeSat (0.1x0.1x0.3 m) with a mass of 6 kg and achieving a 3-5 m resolution at 500 km altitude with employed reaction wheels and magnetic torquers for attitude control and magnetometers and star tracker for attitude determination [71]. The NûSat Series by the company Satellogic, are employing 0.4x0.43x0.75 meter-sized satellite with a mass of 37 kg and achieving 1 m resolution at 500 km altitude [72]. Finally, the Vivid-i Series by the company Earth-i, are employing 0.65x0.65x0.72 meter-sized satellite with a mass of 100 kg and achieving a 60 cm resolution at 500 km altitude [73]. The information of these candidate electro-optical satellites are then integrated into Table 5.3 and results are shown in Figure 5.2.

All three payload-sail pairs shown in Figure 5.2 are valid for the R2S2 concept. However, the pair of NûSat-ODISSEE is selected as the baseline configuration due to two main reasons: TRL of the sail and sub-meter resolution capability of the payload. All the remaining part of this study will use the mass and geometrical properties of the selected configuration (NûSat-ODISSEE).

Sail Area [m ²]	Reference	TRL	$a_0=1 \text{ mm/s}^2$	$a_0=0.5 \text{ mm/s}^2$	$a_0=0.1 \text{ mm/s}^2$	$a_0=0.01 \text{ mm/s}^2$
400	IKAROS	8	N/A	N/A	$m_p=7.8 \text{ kg}$	$m_p=303 \text{ kg}$
1600	ODISSEE	6-7	N/A	N/A	$m_p=54 \text{ kg}$	$m_p=1236 \text{ kg}$
4000	U3P	4-5	N/A	N/A	$m_p=164 \text{ kg}$	$m_p=2955 \text{ kg}$
22700	JHU	3	$m_p=76 \text{ kg}$	$m_p=252 \text{ kg}$	$m_p=1660 \text{ kg}$	$m_p=17400 \text{ kg}$
60000	CCL	3	$m_p=197 \text{ kg}$	$m_p=636 \text{ kg}$	$m_p=4138 \text{ kg}$	$m_p=43536 \text{ kg}$



Flock



NûSat



Vivid-i

Figure 5.2. Mission Payload Analysis

5.5 Design Iteration for Attitude Control Considerations

The defined concept of operation at Low Earth Orbit requires attitude maneuvers with high slewing performances. Therefore, capable attitude control mechanisms should be employed in order to achieve timely transition between sailing and minimum drag modes. For this reason, the baseline satellite design efforts described above are re-iterated with the consideration of attitude control performance. In order to analyze the effect of the mass, the characteristic acceleration formulation in Equation (10) is revisited.

Total satellite mass is then categorized as sail mass, payload satellite mass and attitude control mechanism mass. The selected sail configuration for R2S2 concept uses a 40x40 m² sail area (Chapter 5.4) with a sail film thickness level corresponding to 0.048 kg/m² mass to area ratio. The selected sail mass is 77 kg and payload satellite mass is 37 kg (Chapter 5.4). Solar radiation pressure is taken as a constant value of 4.56x10⁻⁶ N/m² and 90% for sail efficiency is used. With all these values substituted into Equation (10), resultant characteristic acceleration as a function of attitude control mechanism's mass is obtained and the relevant curve of the characteristic acceleration is shown in Figure 5.3.

$$a_0(m_{ACM}) = \frac{0.0131}{(114 + m_{ACM})} \quad (77)$$

The feasibility of the R2S2 concept (Chapter 4.4) and the baseline satellite design reached at Chapter 5.4 were built upon some performance metrics that limits the drop of the characteristic acceleration down to 0.01 mm/s². However, for an effective attitude control performance, this lower limit is stretched down to 0.07 mm/s², which corresponds to 73 kg of attitude control mechanism mass budget limit as can be calculated from Equation (77) and observed in Figure 5.3.

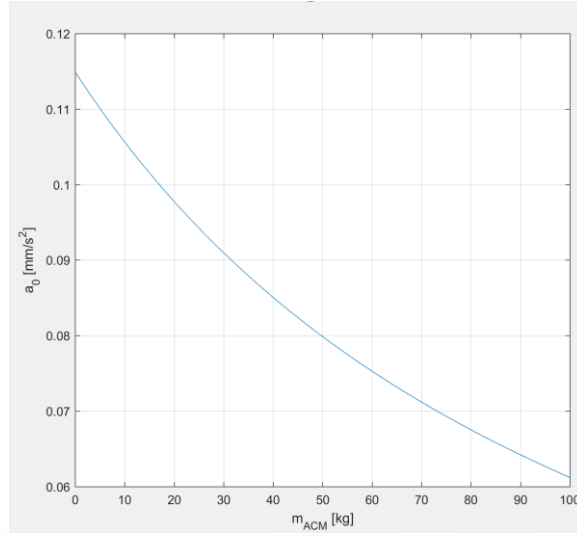


Figure 5.3. The Trend of the Characteristic Acceleration with m_{ACM}

The next part is to analyze the slewing time performance. As shown in Figure 4.1, the aim is to track required directions and to perform two large angle rotations between attitude modes in one orbit revolution. This rotations should be achieved in timely transition between sailing and minimum drag modes. Thus, slewing time performance of the rotations are needed to be analyzed.

Preliminary performance analysis starts with some simplifications. We will consider a single axis rotation carried out in minimum time about a principal axis. This results in a constant acceleration and constant deceleration motion as observed in Figure 5.4. Then, the final rotation angle can be formulated as below.

$$\psi_f = \alpha_{ang_{max}} t_{1/2}^2 \quad (78)$$

Let's recall the relation of the angular momentum and moment of inertia.

$$\begin{aligned} H_{ang} &= J_{sat} \omega_{ang} \\ \dot{H}_{ang} &= T_{ang} = J_{sat} \alpha_{ang} \end{aligned} \quad (79)$$

Then, Equation (79) can be substituted back into Equation (78).

$$\psi_f = \frac{T_{ang_{max}}}{J_{sat}} t_{1/2}^2 \quad (80)$$

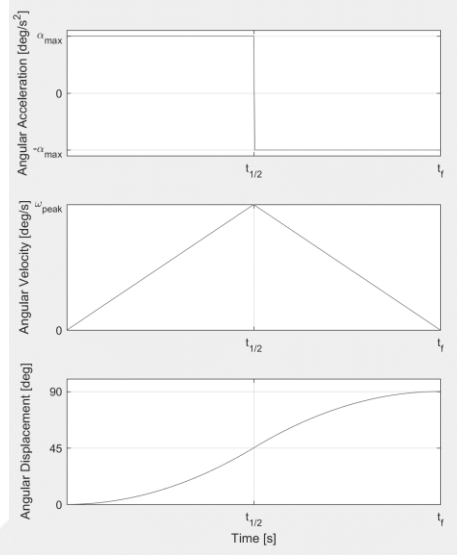


Figure 5.4. Minimum Time Maneuver Motion

After that, Equation (80) is put into graphics (Figure 5.5) in terms of total time to slew and maximum angular momentum value as a function of maximum torque value with total displaced angle as 90 degrees (1.5708 radians) and moment of inertia as 10326 kg-m².

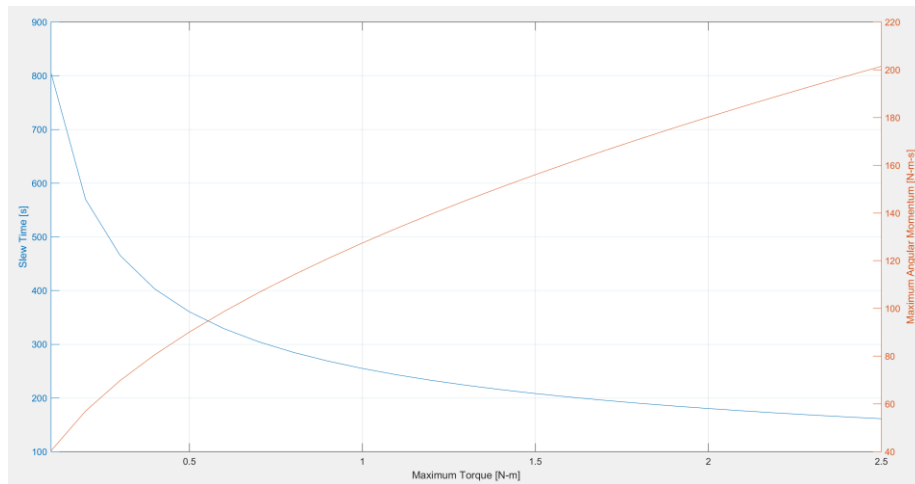


Figure 5.5. The Trend of the Slew Time and Maximum Angular Momentum with Changing Maximum Torque

Now, Equation (78) is rewritten under the consideration of constant maximum angular acceleration cannot be supplied to a system due to the limitation of angular momentum capacity of attitude control systems. Therefore, Equation (78) is modified such that maximum angular momentum is respected and satellite coasts at this angular velocity for a while as may be seen in Figure 5.6.

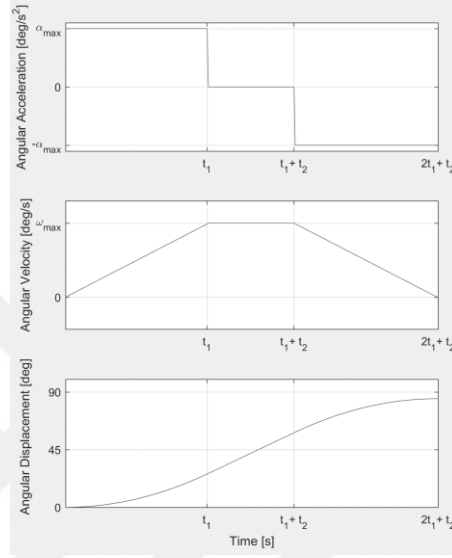


Figure 5.6. Minimum Time Maneuver Motion with Limitation on Maximum Angular Momentum

$$\psi_f = \psi_{2t_1+t_2} = \frac{1}{2} \alpha_{ang_max} t_1^2 + \omega_{ang_max} t_2 + \frac{1}{2} \alpha_{ang_max} t_1^2 \quad (81)$$

where t_1 is the time to reach the maximum angular momentum capacity and t_2 is the time that is spent during at constant angular rate. Then, Equation (79) is substituted into Equation (81).

$$\begin{aligned} \psi_f &= \frac{T_{ang_max}}{J_{sat}} \left(\frac{H_{ang_max}}{T_{ang_max}} \right)^2 + \frac{T_{ang_max}}{J_{sat}} \left(\frac{H_{ang_max}}{T_{ang_max}} \right) t_2 \\ \psi_f &= \frac{H_{ang_max}}{J_{sat}} (t_1 + t_2) \end{aligned} \quad (82)$$

After that, Equation (80) and Equation (82) are put into graphics (Figure 5.7) in terms of total time to slew as a function of maximum torque and maximum angular

momentum capacity value with total displaced angle as 90 degrees (1.5708 radians) and moment of inertia as 10326 kg-m². The total time to slew is computed with Equation (80) when maximum angular momentum is not reached or with Equation (82) when maximum angular momentum is exceeded.

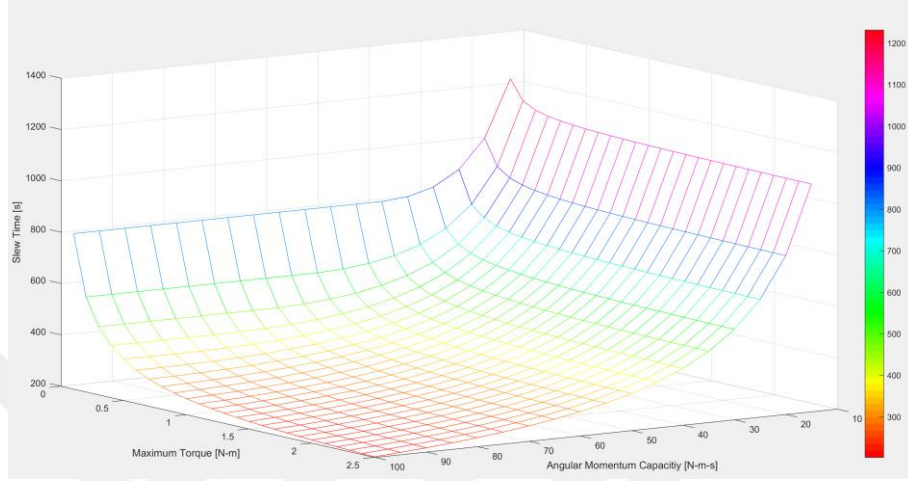


Figure 5.7. The Trend of the Slew Time with Changing Maximum Torque and Angular Momentum Capacity

When all these mass budget (altering the characteristic acceleration) and torque/angular momentum budget (altering the slew time performance) are synthesized under the conditions of R2S2 concept with performance metrics accepted as $a_0 \geq 0.07 \text{ mm/s}^2$ and $t_{90^\circ} \leq 300 \text{ s}$, following baseline satellite properties (Figure 5.8 and Table 5.4) are selected for the remaining part of the study.

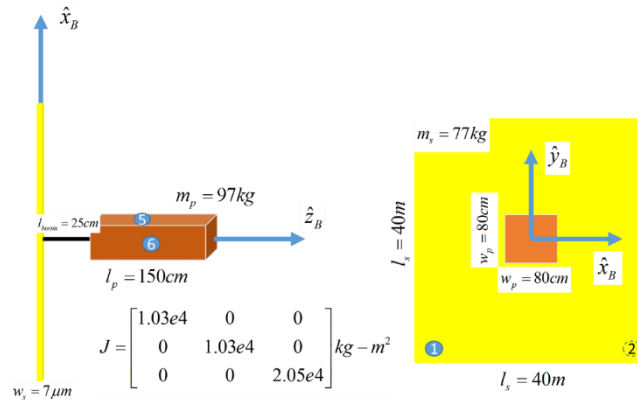


Figure 5.8. Baseline Satellite Dimensions

Table 5.4 Baseline Satellite Properties

Parameter	Initial Value
Sail Mass	77 kg
Payload Satellite Mass	37 kg
Attitude Control Mechanism Mass	45 kg (4 RWs and 1 Electronic Unit)
Sail Area	40x40=1600 m ²
Moments of Inertia	[10326 ; 10326 ; 20542] kg-m ²
RW's Maximum Torque	1.25 N-m
RW's Angular Momentum Capacity	68 N-m-s
Payload Satellite and ACM Total Dimensions	0.8x0.8x1.5 m ³

However, the above analysis does not consider the jerk-level of the satellite system during the maneuvers. In fact, the aim of having low-jerk and smooth transition to the maximum angular acceleration level is crucial for light-weight and flexible solar sail structures. The motion of the minimum time maneuver under the low-jerk condition is then shown in Figure 5.9. Thus, Equation (80) and Equation (82) are revisited and modified accordingly.

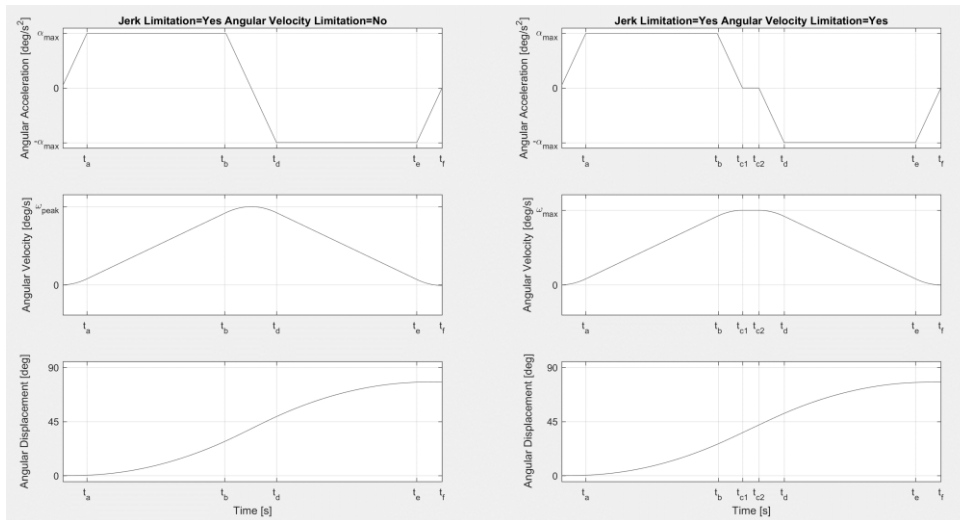


Figure 5.9. Minimum time maneuver motion (a): with limitation on maximum jerk-level (b): with limitation on maximum jerk-level and maximum angular momentum/velocity

First, Equation (80) is modified with the limitation of jerk-level. The maximum allowable jerk-level ($\kappa = \frac{d^3\psi}{dt^3}$) can be described as a function of rise time to maximum angular acceleration (t_a):

$$\tan \kappa_{\max} = \frac{\alpha_{\text{ang}_{\max}}}{t_a} \quad (83)$$

With the introduction of maximum allowable jerk level parameter, Equation (80) is then rewritten:

$$\psi_f = 2 \frac{\left(\frac{T_{\text{ang}_{\max}}}{J_{\text{sat}}} \right)^3}{\tan^2 \kappa} + 3 \frac{\left(\frac{T_{\text{ang}_{\max}}}{J_{\text{sat}}} \right)^2}{\tan \kappa} (t_b - t_a) + \left(\frac{T_{\text{ang}_{\max}}}{J_{\text{sat}}} \right) (t_b - t_a)^2 \quad (84)$$

where $(t_b - t_a)$ is the time duration spent during the constant acceleration phases of the motion. With the new formulation, Figure 5.5 is replotted in Figure 5.10. The jerk-level is introduced in terms of rise time variable (t_a).

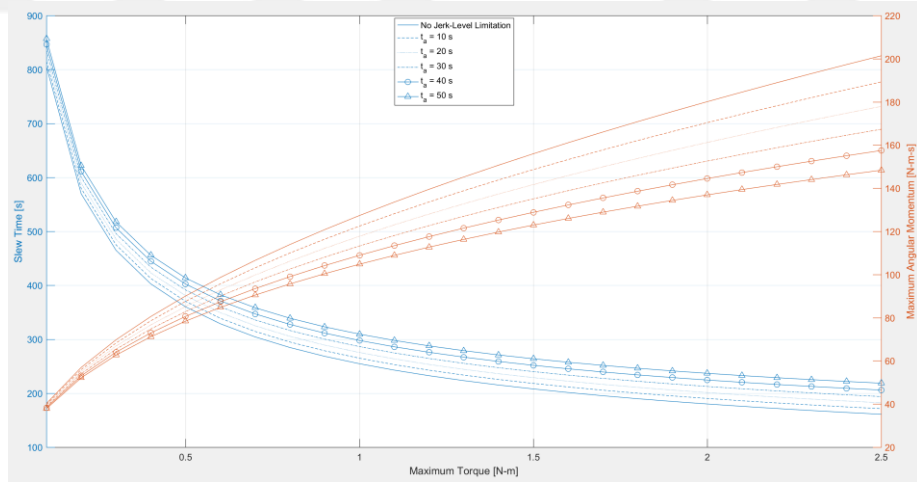


Figure 5.10. The Trend of the Slew Time and Maximum Angular Momentum with Changing Maximum Torque and Jerk-Level

As expected, slew time is increased as the jerk-level is diminished and on the other hand reached maximum angular momentum value is decreased.

The next modification is on Equation (82) over the jerk-level limitation:

$$\psi_f = 2 \frac{\left(\frac{T_{\text{ang_max}}}{J_{\text{sat}}} \right)^3}{\tan^2 \kappa} + 3 \frac{\left(\frac{T_{\text{ang_max}}}{J_{\text{sat}}} \right)^2}{\tan \kappa} (t_b - t_a) + \left(\frac{T_{\text{ang_max}}}{J_{\text{sat}}} \right) (t_b - t_a)^2 + \left(\frac{H_{\text{ang_max}}}{J_{\text{sat}}} \right) (t_{c2} - t_{c1}) \quad (85)$$

where $(t_b - t_a)$ is again the time duration spent during the constant acceleration phases of the motion, whereas $(t_{c2} - t_{c1})$ is the time duration spent during constant maximum angular velocity. Now, Figure 5.7 is regenerated twice (with Equation (85) if maximum angular momentum exceeded or with Equation (84) if maximum angular momentum is not reached) for constant rise time of 20 seconds as jerk-level limitation (Figure 5.11) and constant maximum angular momentum value as 68 N-m-s (Figure 5.12).

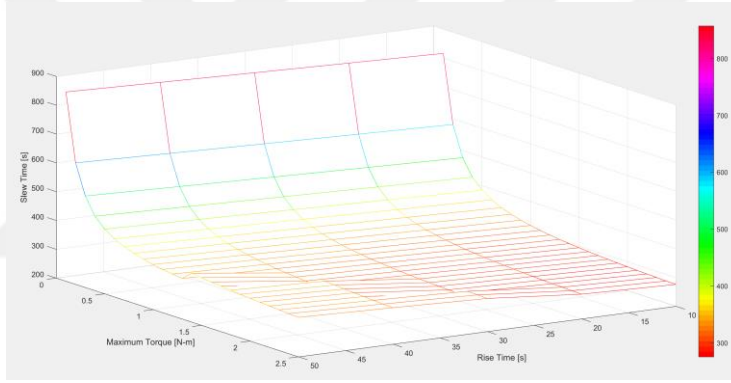


Figure 5.11. The Trend of the Slew Time with Changing Maximum Torque and Jerk-Level (Rise Time)

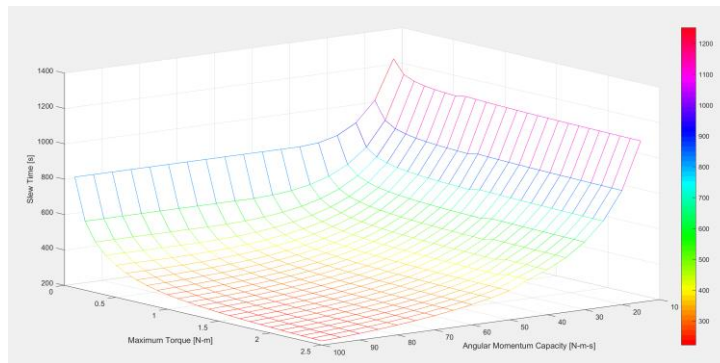


Figure 5.12. The Trend of the Slew Time with Changing Maximum Torque and Angular Momentum Capacity

The introduction of the low-jerk motion to the sizing analysis of the attitude control system increased the slew time of large angle maneuvers. However, consideration of the jerk-level of the solar sail satellites are inevitable due their pseudo-fixed structural nature. Therefore, torque/angular momentum budget (altering the slew time performance) are re-synthesized under the condition of a jerk-level limitation as a 20 seconds of rise time. If baseline satellite properties were kept same as Table 5.4, then t_{90° would be 315 seconds unlike 300 s. Maximum torque level would be increased 1.6 N-m if $t_{90^\circ} \leq 300$ criteria should be met. At the end, properties in Table 5.4 will be kept as they are and 15 seconds of surplus time is considered to be negligible within the scope of R2S2 concept to avoid an increase on attitude control system mass that would be introduced when torque levels are increased by 28%.



CHAPTER 6

MISSION ORBIT DETERMINATION

R2S2 concept's aim is to combine the advantages of observation at LEO altitudes together with propulsion using solar sailing. For this purpose mission orbit shall be optimized for given region of interest (latitude and longitude). Since the employed orbit is elliptical, inclined and prograde, the selection of the initial orbit parameters under the influence of Earth's major asphericity effect, J2 effect, plays an important role. Moreover, the use of the term "performance" comes with the questions: "How good is the performance? What is the level of acceptable performance?" These questions are answered with the generalized performance evaluation formula. Finally, a quick algorithm to find the access number for a given initial orbital parameters and given target's latitude and longitude is proposed. This algorithm is useful to test and validate the relations proposed in the initial orbit parameter determination part. The overall study on the mission orbit determination was presented at 2020 AAS/AIAA Astrodynamics Specialist Conference in Lake Tahoe, California/USA [74].

6.1 Viewing Geometry and Access Criteria

The selected baseline satellite configuration of the R2S2 concept is employed with an electro-optic payload. This means that in order to accomplish the observation mission, some criteria should be met. These are;

- The region of interest (site) should be in the field of view of the satellite (Geometry Criteria),
- The site should be illuminated by the Sun (Daylight Criteria),
- The altitude of the satellite should be below mode switching altitude of 800 km (Altitude Criteria).

All these criteria should be met simultaneously for the access to be valid. The terminology and formulation for the orbit geometry given in [75] are adopted. The reference geometry of viewing Earth from space can be shown in Figure 6.1. The terminology is listed in Table 6.1.

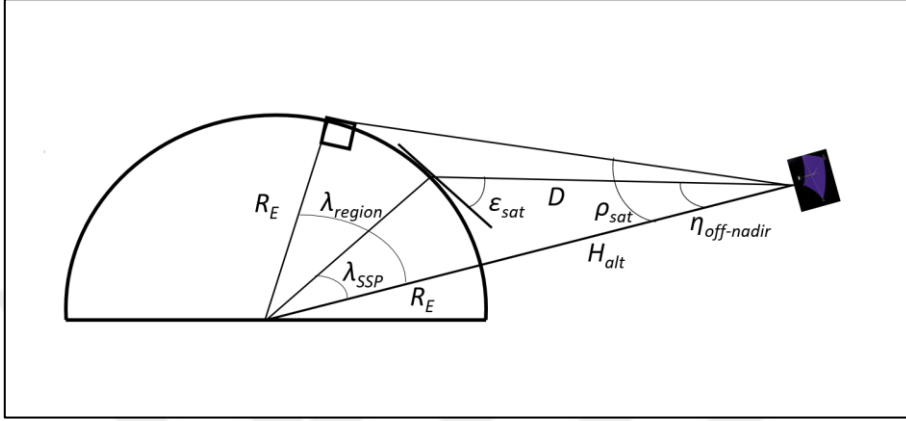


Figure 6.1. Viewing Geometry [75]

Table 6.1 Orbit Geometry Terminology

Symbol	Meaning	Symbol	Meaning
R_E	Radius of Earth	ϵ_{sat}	Satellite elevation angle
λ_{SSP}	Angular distance from the sub-satellite point to the target	$\eta_{off-nadir}$	Off-nadir angle of the satellite
λ_{region}	Angular radius of the region seen by the satellite measured at the center of Earth	ρ_{sat}	Angular radius of Earth seen from the satellite
D	Slant Range	H_{alt}	Altitude

The formulation to derive the viewing condition is as follows [75]:

$$\sin \rho_{sat} = \cos \lambda_{region} = \frac{R_E}{R_E + H_{alt}} \quad (86)$$

Two angles in Equation (86) are functions of the altitude of the satellite. However, the angle ρ_{sat} is valid if the satellite can see up to true horizon, which is not the case for electro-optical satellites. There is a limitation for the off-nadir maneuvering. For

the purpose of this study, the maximum off-nadir angle ($\eta_{\text{off-nadir}_{\text{max}}}$) is set to 20 degrees. By determining the maximum off-nadir angle, we can calculate the minimum satellite elevation angle for a site to be accepted as in view of the satellite:

$$\cos \varepsilon_{\text{sat}_{\text{min}}} = \frac{\sin \eta_{\text{off-nadir}_{\text{max}}}}{\sin \rho_{\text{sat}}} \quad (87)$$

If the satellite elevation angle is larger than this minimum elevation angle the site is in the field of view of the satellite. The satellite elevation angle is calculated as follows [75]:

$$\varepsilon_{\text{sat}} = \frac{\pi}{2} - \cos^{-1} \left(\frac{(\vec{r}_{\text{sat}} - \vec{r}_{\text{site}}) \cdot \vec{r}_{\text{site}}}{D \cdot |\vec{r}_{\text{site}}|} \right) \quad (88)$$

The other two criteria are tracked with conditions listed below:

- The site should be in daylight ($\cos^{-1}(\vec{r}_{\text{site}} \cdot \vec{r}_{\odot}) > 0$),
- The altitude of the satellite should be below 800 km ($H_{\text{alt}} < 800$ km).

6.2 Mission Orbit Performance Evaluation

In this section, the question of “How good is the performance?” is answered. Regarding the observation mission performance, there are couple of metrics that may be measured. These metrics are called Figure of Merit (FoM) [75-76]. The list of FoM definitions used in this study are given below:

- Access Number: The cumulative number of accesses during the simulation time,
- Maximum Response Time: The longest duration between two consecutive accesses,
- Minimum Response Time: The shortest duration between two consecutive accesses,

- Time Average Gap: $\frac{\sum_{i=0}^n (\text{duration}_{\text{gap}(i)})^2}{\text{SimulationTime}}$,
- Mean Response Time: The half of the Time Average Gap,
- Mean Revisit Time: $\frac{\sum_{i=0}^n (\text{duration}_{\text{gap}(i)})}{n}$,
- Mean Access Duration: The arithmetic average of the access durations,
- Mean Slant Range: The arithmetic average of the distance between the satellite and the site during accesses,
- Mean Depointing: The arithmetic average of the off-nadir angle of the satellite during the accesses.
- Sailing Availability: The percentage of the available time for sailing mode to be eligible over the simulation time.

The list of FoM contains metrics in three different categories: *accesses* (Access Number, Mean Access Duration, Mean Depointing and Mean Slant Range), *gaps* (Maximum Response Time, Minimum Response Time, Mean Response Time, Time Average Gap and Mean Revisit Time) and *sailing* (Sailing Availability).

These FoMs are calculated for an orbit with respect to the given site during the mission lifetime of one month. Before finalizing the performance determination, an overall simulation (satellite's motion is under the influence of gravitational forces of Sun, Moon and Earth including asphericity effects up to 4th order, also aerodynamic drag and solar radiation pressure effects are modelled) for various initial orbit parameters (Table 6.2) and 25 different sites (Figure 6.2) all over the Earth is carried out.

Table 6.2 Simulation Plan

RAAN	Argument of Perigee	Inclination	Initial Date	#Simulation
10 sets of Data (36 ⁰ intervals)	10 sets of Data (36 ⁰ intervals)	6 sets of Data (40 ⁰ -65 ⁰)	12 sets of Data (1 st day of each month)	7200 runs

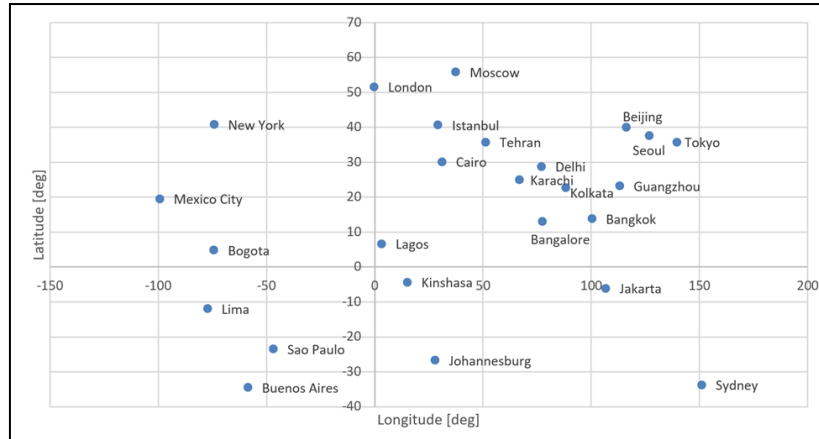


Figure 6.2. Site Distribution over the Globe

In order to get the overall access information relevant to the regions of the world, different methods can be applied, such as creating a grid on the globe and calculating accesses to each grid. However, this method is infeasible due to excessive computational load. For that reason, a different approach is used in this study. First, a site list is created, consisting of cities with large population and also distributed over the globe. The reasoning beside the criteria of the population is coming from the concept. The R2S2 concept aims to respond to reconnaissance or disaster monitoring requests of the end-user/customer. Statistically, the regions of interest to be near of these highly populated cities are probable. Moreover, the cities that are too close to each other are eliminated from the top 25 list, and new cities are added which have different latitude and longitude values. With that adjustment, the distribution of the cities over the globe (Figure 6.2) is guaranteed at this level of the study. Details about the regions of interest can be found in Appendix B.

The results of the overall simulation are analyzed in terms of FoM values with varying initial orbit parameters with respect to different site locations. The aim is to decrease the number FoMs to assess the performance of the mission orbit. According to the simulation results and common knowledge about the observation performance, the following deductions have been made:

- Access Number is the most important FoM since each access means new information about the region of interest,

- Mean Revisit Time is the next important FoM since frequent accesses allows in-depth analysis over the region,
- Mean Revisit Time adequately represents other *gap* FoMs,
- Mean Slant Range information is related to the Mean Depointing and Access Duration,
- Sailing Availability is a concept-specific FoM and considered for the performance assessment.

With these conclusions, number of FoMs to be used in performance assessment is reduced to four: Access Number (*AN*), Mean Revisit Time (*MRT*), Mean Slant Range (*SR*) and Sailing Availability (*SA*). The range of values of these FoMs at the end of the total simulation runs are presented in Table 6.3.

Table 6.3 The Range Information of FoMs

<i>AN</i>	<i>MRT</i>	<i>SR</i>	<i>SA</i>
[0-43]#	[15-520] hours	[350-850] km	[0-60]%

The information in Table 6.3 is then synthesized with the weight coefficients (Table 6.4) assigned due to their effects in the mission performance.

Table 6.4 Weight Coefficients for FoMs

<i>AN</i>	<i>MRT</i>	<i>SR</i>	<i>SA</i>
60%	20%	10%	10%

Then, following formulation is proposed to assess the performance of the mission orbit. In order to equalize the ranges of the FoM values (Table 6.3), adjust their contribution according to weight coefficients (Table 6.4) and reflect their proportionality (directly or inversely), the coefficients in Equation (89) are determined. The output of this formulation is called *Performance Formulation Metric (PFM)*. As one can also notice, the MRT and SR do not have zero values. Hence, a constant (χ_0) is added to limit the PFM output to be between 0-100. After

all, PFM is used as a percentage value for the mission performance rating for the R2S2 concept.

$$PFM = \chi_1 AN + \chi_2 SA + \chi_3 SR + \chi_4 MRT + \chi_0$$

$$\begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \\ \chi_0 \end{bmatrix} = \begin{bmatrix} 1.33 \\ 16.67 \\ -0.02 \\ -0.04 \\ 37.70 \end{bmatrix} \quad (89)$$

The resultant performance values of each mission orbit candidate in the simulation is then calculated with Equation (89) and presented in Figure 6.3.

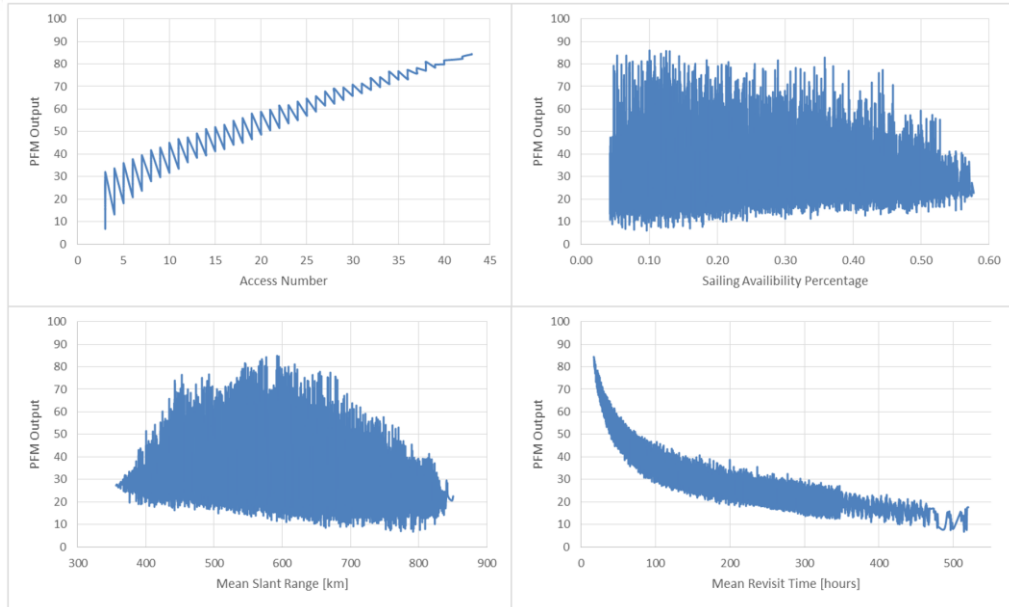


Figure 6.3. Selected Formula Output with respect to FoMs

From Figure 6.3, the following can be concluded:

- Access Number is directly proportional with performance output and the range of possible values of performance output for specific Access Number is decreasing as the Access Number is higher (this is due to the selection of 60% weight for Access Number),

- Mean Revisit Time is inversely proportional with the performance output and above 70-100 hours the performance output gets very low (this is due to the fact that when Mean Revisit Time is getting higher, accesses to the site are getting less frequent),
- Slant Range and Sailing Availability do not have a direct effect on the performance output since they have 10% weight over the formula,
- Sailing Availability results indicates that as the percentage value gets higher, the range and the maxima of performance outputs decrease (This is due to the Sun's position over the target area. For high values of performance output, the site should be illuminated and the altitude of the orbit over the target should be lower. This results in the shadowing of the high altitude portions of the orbit by the Earth which means less sailing availability),
- Slant Range plot shows a normal distribution for the performance output (this is due to the relatively small contribution of the Slant Range to the performance output. Thus, the mean value is the expected value for the high performance output).

Finally, the second question (What is the level of acceptable performance?) can be answered in the light of the acquired results above. As Sailing Availability and Mean Slant Range do not change the performance output dramatically, the acceptable performance is selected based on the trends of Access Number and Mean Revisit Time. As can be seen in Figure 6.3, 48-72 hours (2-3 days) of Mean Revisit Time corresponds to a performance value of 50 approximately. Moreover, same performance value is obtained with 17-20 accesses. These results are found adequate to determine the acceptable performance as 50 for a mission orbit candidate with respect to the given site's location on Earth.

6.3 Determination of the Mission Orbit Parameters

In this section mathematical expressions to assess the performance of the candidate orbit for a given site's latitude and longitude under the influence of Earth's

gravitational acceleration (including J2 effects) are obtained. Through these expressions, orbital parameters (RAAN, Argument of Perigee, Inclination and Day of Year-DOY) for maximum observation performance of given site information (latitude and longitude) are calculated. Remembering the criteria for access to be established between the satellite and the site (Chapter 6.1), three relations are the ultimate outputs of the initial orbit parameter determination process:

- The Geometry Criteria-----Relation with Inclination,
- The Altitude Criteria-----Relation with Argument of Perigee,
- The Daylight Criteria-----Relation with RAAN based on DOY.

The inclination has the potential to be determined in advance for a given target latitude. Let's assume that the target is in the Northern Hemisphere. Then the maximum latitude region of the ground track of the satellite will be most visited location in terms of longitude interval for a specific latitude.

The first relation based on the geometry criteria is developed by revisiting Figure 6.1 and Equations (86-87). The off-nadir maneuver capability of the R2S2 baseline satellite configuration is set to 20 degrees. With this information and Equation (90), Table 6.5 is populated that shows the angular distance from sub-satellite point in terms of latitude value.

$$\lambda_{SSP} = \frac{\pi}{2} - \eta_{off-nadir} - \epsilon_{sat} \quad (90)$$

Table 6.5 Latitude Envelope for the Sub-Satellite Point

H_{alt} [km]	λ_{region} [deg]	λ_{SSP} [deg]
400	19.78	1.31
500	21.98	1.64
600	23.33	1.97
700	25.7	2.31
800	27.31	2.61

From Figure 6.3 and Table 6.5, the mean value of the slant range (600 km) is used to determine the ± 2 degrees of latitude envelope. From Figure 6.4, the region for the largest longitude coverage for a specific latitude may be deduced. Thus, the probability to view the target is the highest. This analysis can be verified by Figure 6.5. The histogram shows that the inclination, which is same as the maximum achievable latitude, minus two degrees is the most viewed region of the ground track of the satellite. So, if the longitude of the target is between lower and upper boundaries, the target is in view by the satellite for that orbital revolution. For this to be true, the inclination should be selected as follows (1st Relation):

$$i = \phi_{site} + 2^\circ \quad (91)$$

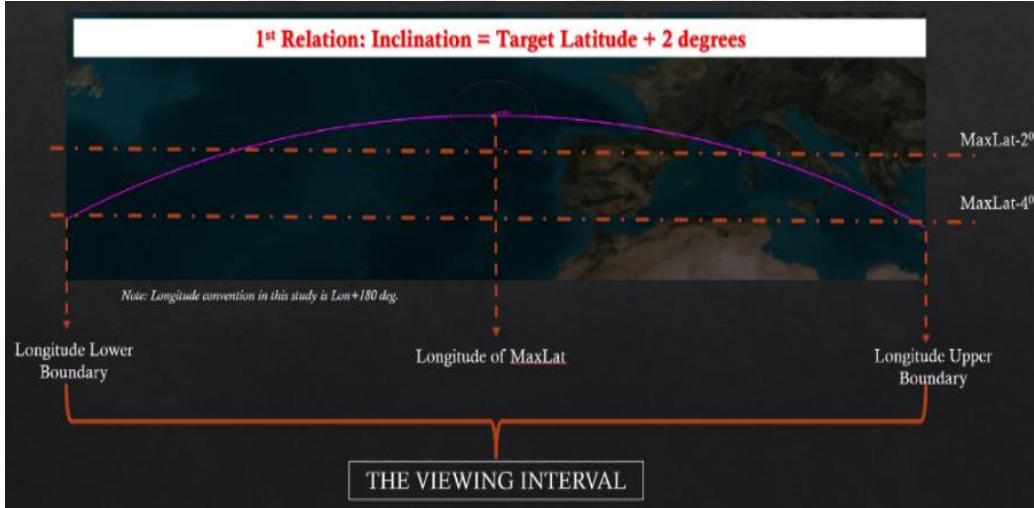


Figure 6.4. The Viewing Interval of the Maximum Latitude Region

The next criteria to be examined is the Altitude Criteria. The R2S2 concept limits the mission to be carried out below 800 km, because the portion of the orbit above 800 km is allocated to sailing maneuvers. Altitude of 800 km corresponds to two true anomaly values. The calculation is done by using orbit equation:

$$r_{sat} = \frac{a(1-e^2)}{1+e \cos \nu} \quad (92)$$

where r_{sat} is the radial position of the satellite, a is the semi-major axis, e is the eccentricity and ν is the true anomaly.

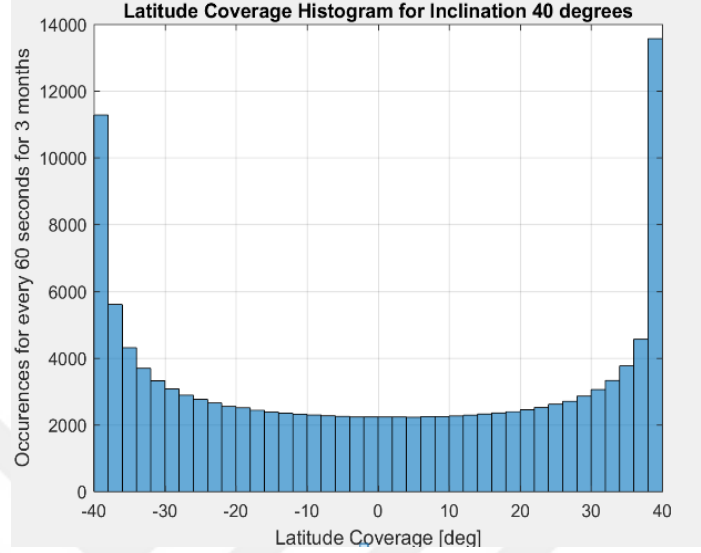


Figure 6.5. Latitude Coverage Histogram

For an altitude of 800 km (H_{switch}), the true anomaly value is $\pm 82.35^\circ$ which means that the satellite's instantaneous true anomaly value should be between $\pm 82.35^\circ$ in order to comply with the Altitude Criteria. Moreover, the value of the argument of perigee (ω) is also important since the viewing of the site will be accomplished only around the maximum latitude region due to the implementation of Equation (91). The change of the argument of perigee ($\dot{\omega}$) due to the J2 effect is formulated below [77]:

$$\dot{\omega} = 1.03237 \times 10^{14} \times a^{-3.5} (4 - 5 \sin^2 i) (1 - e^2)^{-2} [\text{deg/day}] \quad (93)$$

As can be seen in Equation (93) the change of the argument of perigee is a function of semi-major axis, eccentricity and inclination. However, as stated in the beginning of this section, satellite's motion is only under the influence of Earth's gravitational attraction (conservative forces), meaning that semi-major axis and eccentricity will be constant. Thus, the change in argument of perigee can be shown as below:

$$\omega = \omega_0 + \dot{\omega}(i)t = \omega(i, t) \quad (94)$$

where ω_0 is the initial argument of perigee value which is the focus of the initial orbit parameter determination process regarding the Altitude Criteria. Time (t) parameter in Equation (94) can be used in different units depending on the change of argument of perigee unit, such as deg/sec, deg/rev or deg/day. The next step is to determine the true anomaly value of the maximum latitude region:

$$\nu_{\max lat} = 90 - \omega_0 - \dot{\omega}(i)t = \nu_{\max lat}(i, t) \quad (95)$$

If we combine the Equations (92 and 95), we can derive the following relations between initial argument of perigee and the Altitude Criteria:

$$r_{\max lat} = \frac{a(1-e^2)}{1+e \cos \nu_{\max lat}} \leq R_E + H_{\text{switch}} \quad (96)$$

$$\cos \nu_{\max lat} \geq \frac{a(1-e^2) - (R_E + H_{\text{switch}})}{(R_E + H_{\text{switch}})e} \equiv C_1 \quad (97)$$

$$\cos(90 - \omega_0 - \dot{\omega}(i)t) - C_1 \geq 0 \quad (98)$$

$$\sin(\omega_0 + \dot{\omega}(i)t) - C_1 \geq 0 \quad (99)$$

$$f(\omega_0, i, t) \geq 0 \quad (100)$$

The function in Equation (100) depends on the initial argument of perigee, inclination and time. The graphics of this function for various inclination values and time as an orbit revolution number is during one month of mission lifetime are shown in Figure 6.6. Figure 6.6 shows the function sign (positive or negative) for initial argument of perigee (horizontal axis) and orbit revolution number (vertical axis).

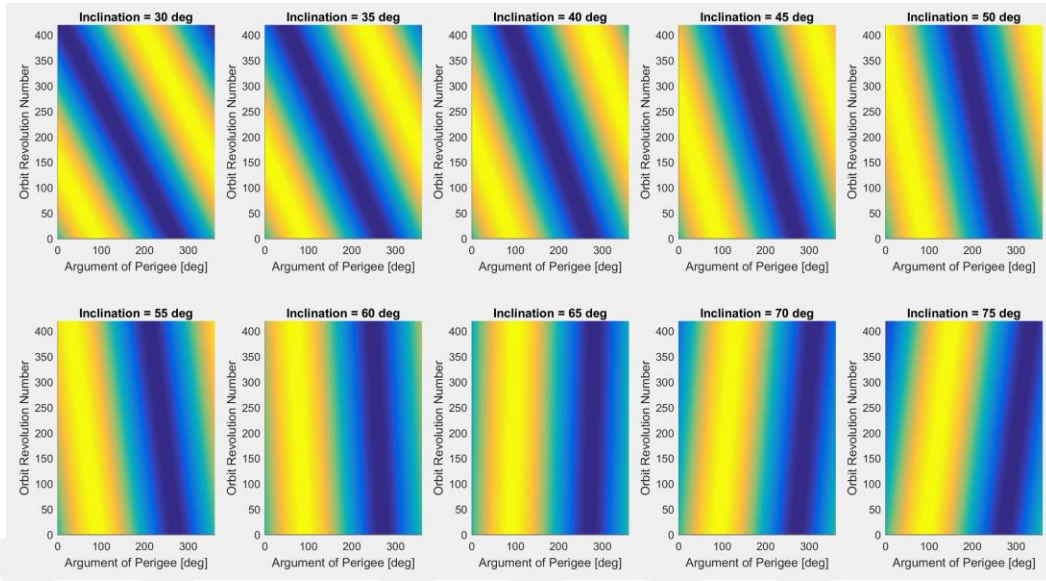


Figure 6.6. Equation (100)'s Output (*blue means function is negative, yellow means positive*)

Figure 6.6 shows different values of the function in Equation (100) starting from the orbit revolution number zero (initial condition). Each orbit revolution changes the value of the function and eventually the sign of the function changes. The argument of perigee change is the key driver for the aforementioned change in the value of the function.

In order to create boundaries for initial condition for argument of perigee, let's assume the function value as 1 if the function is positive and 0 if the function is non-positive. After that, all values for each initial argument of perigee with changing orbit revolution number are summed. In other saying, each column regarding to separate initial argument of perigee is summed. The total sum value for each initial argument of perigee is then divided by 419 (total revolution number in one month). The resultant values are then plotted in Figure 6.7.

It may be observed from Figure 6.7 that the initial argument of perigee has boundaries for a successful mission with given inclination. When the output value for an individual initial argument of perigee is 1, we can deduce that the orbit agrees to altitude criteria during its entire mission lifetime of one month. In contrary, if the

output value for an initial argument of perigee is 0, the orbit never complies with the altitude criteria during that period of time of one month.

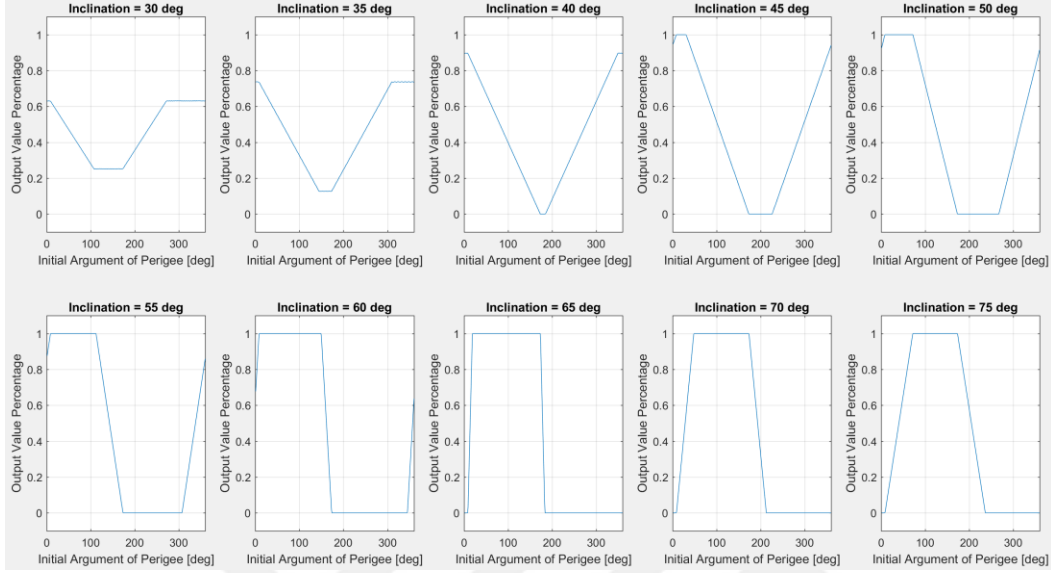


Figure 6.7. Argument of Perigee Relation for Different Inclination Values

Since, according to the Equation (91) inclination is set to a value with respect to Site's Latitude, initial argument of perigee is also bounded with given Site's Latitude.

The final relation is between the RAAN and DOY with the Daylight Criteria. Similar to the Altitude Criteria of being below the altitude of 800 km, maximum latitude of the orbit (the viewing region) should be in daylight in order to observe the site. This means that the local time of the maximum latitude should be between Sunrise and Sunset times. However, the local time of the maximum latitude is changing in every revolution. In order to relate the local time of the maximum latitude with the initial RAAN value, we have to revisit the change of the RAAN ($\dot{\Omega}$) formulation [77]:

$$\dot{\Omega} = -2.06474 \times 10^{14} \times a^{-3.5} \cos i (1 - e^2)^{-2} [\text{deg/day}] \quad (101)$$

Having fixed the semi-major axis and eccentricity as in the change of argument of perigee, the change of the RAAN will only depend on the inclination. Moreover, in

every revolution (P is the period of the orbit) the longitude of the ascending node changes due to the Earth's angular speed ($\varpi_{\oplus}$) and the change of the RAAN.

$$\Delta\lambda = (\dot{\Omega}(i) - \varpi_{\oplus})P \quad (102)$$

With these information, local time difference between successive ascending nodes (ΔT_{local}) starting from revolution number 0 ($T_{0_{local}}$) to 1 ($T_{1_{local}}$) can be derived as below. Time since perigee is added to the Equation (103) due to the assumption that simulation starts from the perigee point with true anomaly is equal to zero as midnight at UTC.

$$T_{0_{local}} = \left[24 - t_{sin\ ceperigee} + \frac{\lambda_0}{\varpi_{\oplus}} \right]_{\text{mod } 24} \quad (103)$$

$$\Delta T_{local} = T_{1_{local}} - T_{0_{local}} = P + \frac{\Delta\lambda}{\varpi_{\oplus}} \quad (104)$$

$$\Delta T_{local} = P + \frac{(\dot{\Omega}(i) - \varpi_{\oplus})P}{\varpi_{\oplus}} \quad (105)$$

$$\Delta T_{local} = \frac{\dot{\Omega}(i)}{\varpi_{\oplus}} P \quad (106)$$

In each revolution, the ascending node crossing local time difference would be zero if RAAN change does not exist. But for inclinations other than 90 degrees, the RAAN change exists. For example, for 45 degrees of inclination, the local time for the next ascending node crossing is approximately -2 minutes from the previous node crossing. Thus, for a one month mission lifetime (419 revolutions), if the first ascending node crossing local time is just before the Sunset time, all of the following ascending nodes will be in daylight. This result can also be paraphrased as; if the first ascending node crossing local time is just before the Sunrise time, all of the following ascending nodes will be in night time. In order to relate the initial RAAN and local time, the following derivation is accomplished:

$$\lambda_{ascending_0} = \lambda_0 + \Omega_0 - (DOY - 1) \times 86400 \times \varpi_{\oplus} \quad (107)$$

where λ_0 is the initial longitude at 01 January 2020 midnight UTC with a RAAN value of zero ($\lambda_0 = -99.866$ degrees).

$$\lambda_{ascending_0} = \lambda_{ascending_0}(\Omega_0, DOY) \quad (108)$$

$$T_{ascending_{local}} = \frac{\dot{\Omega}(i)}{\varpi_{\oplus}} Pt + 24 - t_{sin\ cepergee}(\omega_0) + \frac{\lambda_{ascending_0}(\Omega_0, DOY)}{\varpi_{\oplus}} \quad (109)$$

$$T_{ascending_{local}} = T_{ascending_{local}}(\omega_0, i, \Omega_0, DOY, t) \quad (110)$$

$$T_{max\ lat_{local}} = T_{ascending_{local}}(\omega_0, i, \Omega_0, DOY, t) + \frac{P}{4} \frac{\dot{\Omega}(i)}{\varpi_{\oplus}} + \frac{90^\circ}{\varpi_{\oplus}} \quad (111)$$

$$T_{max\ lat_{local}} = T_{max\ lat_{local}}(\omega_0, i, \Omega_0, DOY, t) \quad (112)$$

$$T_{sunrise} \leq T_{max\ lat_{local}}(\omega_0, i, \Omega_0, DOY, t) \leq T_{sunset} \quad (113)$$

As can be seen from the derivation, the local time of the maximum latitude point is a function of initial argument of perigee, inclination, initial RAAN, DOY and time (revolution number). Next, the Sunrise and Sunset times [76] are related with the following derivation which has some worst case assumptions to decrease the number of dependent variables in the end.

$$\gamma_{year} = \frac{2\pi(DOY-1)}{366} \quad (114)$$

$$eqtime = 229.18 \left(\begin{array}{l} 0.000075 + 0.001868\cos(\gamma_{year}) - 0.032077\sin(\gamma_{year}) \dots \\ -0.014615\cos(2\gamma_{year}) - 0.040849\sin(2\gamma_{year}) \end{array} \right) \quad (115)$$

$$\delta = 0.006918 - 0.399912\cos(\gamma_{year}) + 0.070257\sin(\gamma_{year}) - 0.006758\cos(2\gamma_{year}) + \dots \\ 0.000907\sin(2\gamma_{year}) - 0.002697\cos(3\gamma_{year}) + 0.00148\sin(3\gamma_{year}) \quad (116)$$

$$ha = \left| \cos^{-1} \left[\frac{\cos(90.833)}{\cos(\phi_{site}) \cos \delta} - \tan(\phi_{site}) \tan \delta \right] \right| \quad (117)$$

$$SunRise = 720 - 4(\lambda_{site} + ha) - eqtime \quad (118)$$

$$SunSet = 720 - 4(\lambda_{site} - ha) - eqtime \quad (119)$$

where γ_{year} is the fraction of the year, δ is the solar declination, $eqtime$ is the solar equation of time in minutes, ha is the solar hour angle, and finally *Sunrise* and *Sunset* values are obtained in UTC minutes. The remaining part of the derivation is as follows:

$$ha(i, DOY) = \left| \cos^{-1} \left[\frac{\cos(90.833)}{\cos(i - 2^\circ) \cos(\delta(DOY))} - \tan(i - 2^\circ) \tan(\delta(DOY)) \right] \right| \quad (120)$$

$$12 - \frac{ha(i, DOY)}{\varpi_\oplus} + P + 0.25 \leq T_{\max lat_{local}}(i, \Omega_0, DOY, t) \leq 12 + \frac{ha(i, DOY)}{\varpi_\oplus} - 0.25 \quad (121)$$

$$12 + \frac{ha(i, DOY)}{\varpi_\oplus} - 0.25 - 24 - \frac{\lambda_{ascending_0}(\Omega_0, DOY)}{\varpi_\oplus} - \frac{P}{4} \frac{\dot{\Omega}(i)}{\varpi_\oplus} - \frac{90^\circ}{\varpi_\oplus} = 0 \quad (122)$$

$$f(\Omega_0, DOY, i) = 0 \quad (123)$$

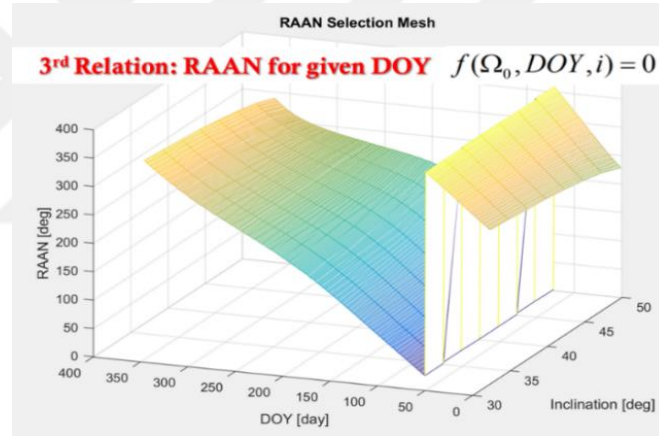


Figure 6.8. RAAN Relation for Different Inclination and DOY Values

As can be seen from the derivation, final function in Equation (123) is depending on only three variables: initial RAAN, DOY and inclination. The initial argument of perigee information is added to the function as the period time for a worst case condition. Moreover, as explained earlier, the first local time equated to the Sunset time to have the most illuminated condition. This leads to the elimination of the time (revolution) variable. The function is then illustrated as a mesh of DOY and inclination (Figure 6.8). The z-axis of Figure 6.8 (the mesh value) is the best RAAN selection for this study.

As can be seen in Figure 6.8, initial RAAN has a relation for a successful mission with given inclination and DOY. Since, according to the 1st Relation inclination is set to a value with respect to Site's Latitude, initial RAAN is then also bounded with given Site's Latitude. As a result, three relations regarding the three criteria are obtained and final equations are given in Equations (91, 100 and 123).

6.4 Access Number Estimation Algorithm

As mentioned before, a quick algorithm to determine the access number for given initial orbital parameters and site location is built. The algorithm is then used for testing the relations in Equations (91, 100 and 123).

The algorithm is constructed upon the reference revolution definition. One Reference Revolution is defined as the satellite's ground track from the ascending point to the next ascending point. The critical points are shown in Figure 6.9. For each revolution; time, longitude and true anomaly information of these critical points are needed for the algorithm. In order to acquire those information, estimation formulas or physical relations are used.

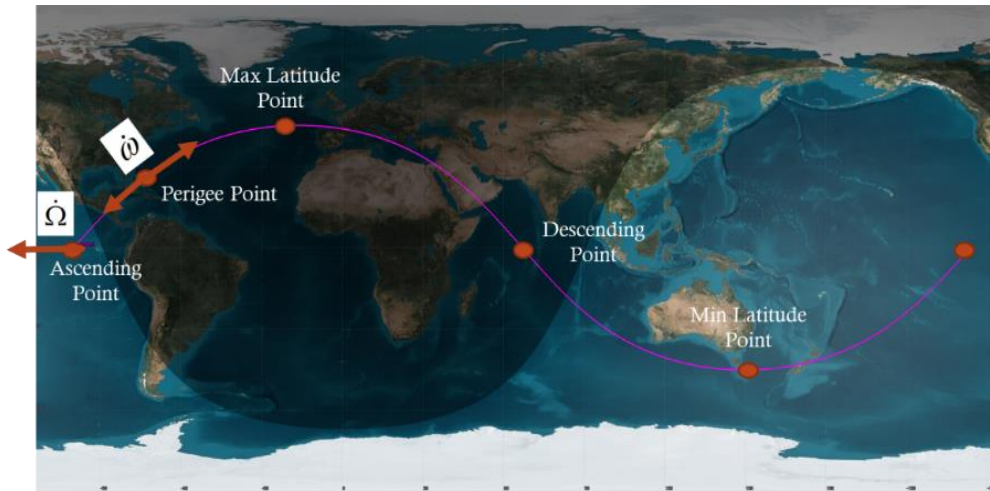


Figure 6.9. Reference Revolution and Critical Points

The perigee point travels along the reference revolution track due to J2 effects. The location of perigee point is important, as it determines the time differences of each point and also the altitude of each critical point.

The max/min latitude points are the 90° angle points of the sinusoidal-like reference revolution track. Depending on the hemisphere of the target, these critical points in the reference revolution track will determine whether the target is in view or not, as illustrated in Figure 6.4.

The time between successive perigee points is constant during the propagation. Since the aerodynamic drag and solar radiation pressure forces are neglected in the algorithm, the orbital energy does not change. It means that the semi-major axis does not change and eventually the orbital period is constant. However, due to the shifts in argument of perigee and RAAN stemming from the J2 effects, the time between perigee points is deviated from the orbital period with seconds. This shift is constant since the changes of true anomaly, RAAN and argument of perigee are constant due to the fact that the orbital energy is not varying. A constant shift is estimated between the orbital period and the time between perigees by the following relation:

$$\Delta T_{perigee} = P + b_1 \dot{\omega} + b_2 \dot{\Omega} + b_3 \quad (124)$$

After a linear least square regression process, the parameters (b_1 , b_2 and b_3) in Equation (124) are found. Time differences for other critical points are found from the time between perigees. However, these time differences are not constant but vary as perigee point travels along the reference revolution track. These time differences are estimated by the following relations:

$$\begin{aligned} \Delta T_{ascending} &= \Delta T_{perigee} + b_4 \dot{\omega} + b_5 \dot{\omega} \cos(\omega) \\ \Delta T_{descending} &= \Delta T_{perigee} + b_4 \dot{\omega} + b_5 \dot{\omega} \cos(\omega + \pi) \\ \Delta T_{max\ lat} &= \Delta T_{perigee} + b_4 \dot{\omega} + b_5 \dot{\omega} \sin(\omega) \\ \Delta T_{min\ lat} &= \Delta T_{perigee} + b_4 \dot{\omega} + b_5 \dot{\omega} \sin(\omega + \pi) \end{aligned} \quad (125)$$

The parameters (b_4 and b_5) are then found by regression analysis. The vital issue in this approach is to find the first values of these critical points. Since the simulation

trials are considered to be started from perigee point ($\nu=0$), the time differences from perigee point to other critical points are essential.

Kepler's formula [58] is used to determine the time differences between perigee and other critical points. Prior to that, true anomaly value of each critical point is determined from the argument of perigee value as follows:

$$\begin{aligned} \nu_{ascending} &= 2\pi - \omega \\ \nu_{descending} &= \pi - \omega \\ \nu_{\max lat} &= \pi/2 - \omega \\ \nu_{\max lat} &= 3\pi/2 - \omega \end{aligned} \quad (126)$$

After determining the true anomaly difference from the first perigee point, Kepler's formula is used to determine the first time stamp of the relevant critical point (t as the time since perigee value).

$$M = \frac{2\pi}{P} t = E - e \sin E \quad (127)$$

$$E = 2 \tan^{-1} \left(\sqrt{\frac{1-e}{1+e}} \tan \frac{\nu}{2} \right) \quad (128)$$

where M is the mean anomaly for the ellipse and the E is the eccentric anomaly. After determining the time differences and true anomaly information of each critical point, longitude information is addressed. The nodal shift value is used as a constant value during the study. The nodal shift is the summation of the RAAN shift and the rotation of the Earth. For the ease of calculations, 180 degrees are added to the longitudes to bring them to an interval of 0-360 degrees. Then, each longitude information is found by multiplying the nodal shift change with the time between relevant critical points.

$$\begin{aligned}
\Delta\lambda_{ascending} &= \Delta T_{ascending} (\dot{\Omega} - \varpi_{\oplus}) \\
\Delta\lambda_{descending} &= \Delta T_{descending} (\dot{\Omega} - \varpi_{\oplus}) \\
\Delta\lambda_{max\,lat} &= \Delta T_{max\,lat} (\dot{\Omega} - \varpi_{\oplus}) \\
\Delta\lambda_{min\,lat} &= \Delta T_{min\,lat} (\dot{\Omega} - \varpi_{\oplus})
\end{aligned} \tag{129}$$

For the first iteration, the longitude information is found from the ascending node by multiplying the nodal shift value with the time difference between ascending node and relevant critical point, then adding a phase angle (90 degrees for max latitude, 180 degrees for descending node, and 270 degrees for min latitude). The first longitude value of the ascending is found using Equation (107).

$$\begin{aligned}
\lambda_{max\,lat_0} - \lambda_{ascending_0} &= (T_{max\,lat_0} - T_{ascending_0}) (\dot{\Omega} - \varpi_{\oplus}) + 90 \\
\lambda_{descending_0} - \lambda_{ascending_0} &= (T_{descending_0} - T_{ascending_0}) (\dot{\Omega} - \varpi_{\oplus}) + 180 \\
\lambda_{min\,lat_0} - \lambda_{ascending_0} &= (T_{min\,lat_0} - T_{ascending_0}) (\dot{\Omega} - \varpi_{\oplus}) + 270
\end{aligned} \tag{130}$$

The access value is then calculated by looking at three criteria:

- Viewing Geometry
- Altitude
- Daylight Condition

For the viewing geometry, the longitude of the target should be within the lower and upper longitude boundaries of max latitude point. This is also same for the altitude criteria. Recall that the true anomaly information is propagated for only max latitude point. However, since the viewing interval includes the lower and upper longitude points, the interval for true anomaly should be determined. Both true anomaly and longitude intervals are estimated with the following 4th order polynomial based on the inclination (the estimation can also be seen in Figure 6.10):

$$\begin{aligned}
\pm \Delta\nu/2 &= 9 \times 10^{-6} i^4 - 0.002 i^3 + 0.1726 i^2 - 7.0297 i + 160.56 \\
\pm \Delta\lambda/2 &= 1 \times 10^{-5} i^4 - 0.0026 i^3 + 0.2013 i^2 - 7.1084 i + 152.54
\end{aligned} \tag{131}$$

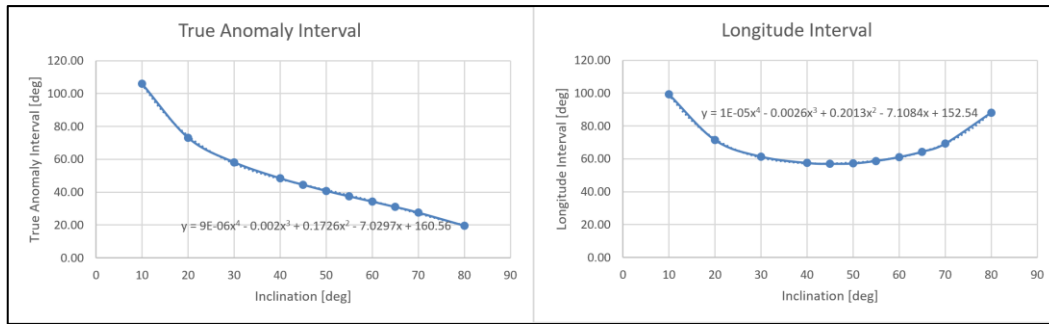


Figure 6.10. True Anomaly and Longitude Interval Estimation

For the daylight condition to be checked, *Sunrise* and *Sunset* information are calculated for the given site's longitude by using Equations (114-119). If the time value of max latitude point is within the *Sunrise* and *Sunset*, then region of interest (site) is considered to be in daylight condition.

The algorithm then uses the above information to calculate the successive information of critical points and in each reference revolution checks the three criteria for the access. The results are verified with the simulation outputs (described in Table 6.2) and STK software outputs.

The result of an initial orbit access number information for a given site's latitude and longitude is acquired way much faster (from 2-3 minutes to 0.02 seconds) than the simulation results obtained previously. Therefore, more initial orbit trials are performed. In the following graphics, the results for the inclination value of 45 degrees are shown for simplicity. The results of this trial are analyzed, then 2nd and 3rd relations, which are the results of Equations (100 and 123), are verified. The trials are done for various initial conditions as stated in Table 6.6.

Table 6.6 Algorithm Trial Plan

RAAN	Argument of Perigee	Site's Longitude	Initial Date	#Simulation
360 sets of Data (1 ⁰ intervals)	360 sets of Data (1 ⁰ intervals)	12 sets of Data (30 ⁰ intervals)	4 sets of Data (1 st day of each 3 months)	6220800 runs

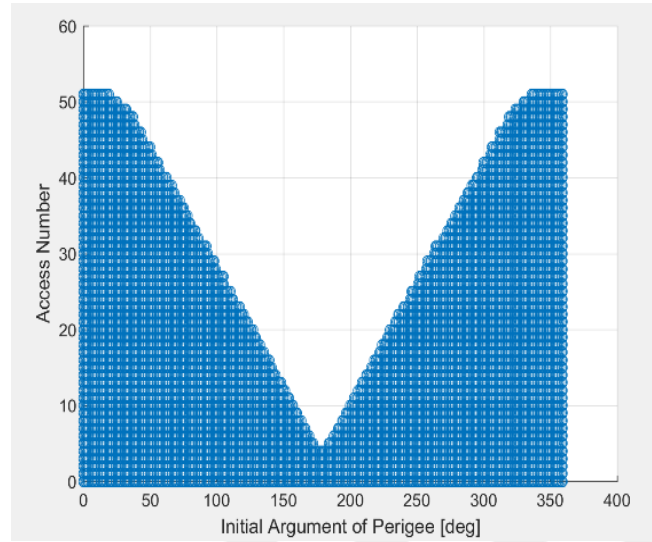


Figure 6.11. The Access Number Distribution for Initial Argument of Perigee

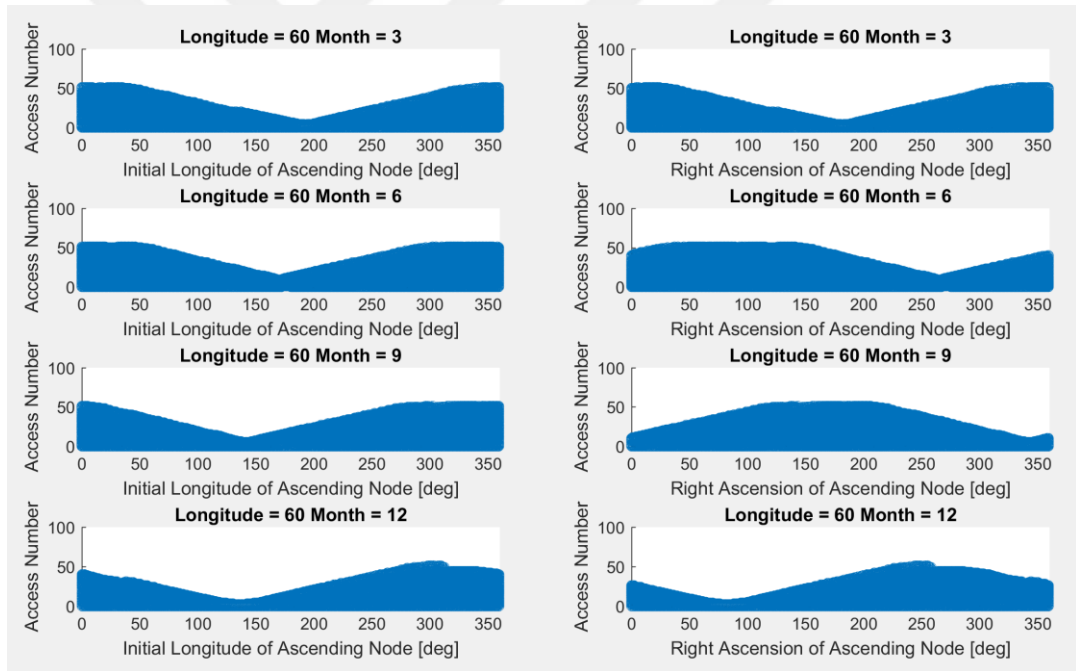


Figure 6.12. First Longitude of Ascending Node and RAAN Valleys for Access Numbers

Figure 6.11 and Figure 6.12 show the valleys in the performance of the mission orbit regarding the initial orbit parameters of argument of perigee and RAAN (at a given DOY). The perigee valley in Figure 6.11 and RAAN valleys in Figure 6.12 should

be avoided in order to assure the high performance for the R2S2 concept. With the help of Equations (91, 100 and 123), these valleys can be obtained without extensive simulations and/or algorithm runs.



CHAPTER 7

MANEUVER PLANNING STRATEGY

The proposed R2S2 concept utilizes the solar radiation pressure for maneuvering needs. Depending on the particular case, R2S2 concept requires resizing (altering semi-major axis), reshaping (altering perigee and apogee altitudes) and/or reorienting maneuvers (altering inclination, argument of perigee and right ascension of ascending node). For example, transfer orbits need resizing and reshaping maneuvers for raising of the perigee altitude (eventually the semi-major axis) if transfer is from mission orbit to standby orbit or decreasing of the perigee altitude (eventually the semi-major axis) if transfer is from standby to mission orbit. Another example might be the station keeping of the mission orbit which is optimized for the region of interest. In this case, all kinds of maneuvers (resizing, reshaping, and reorienting) are in consideration.

All these maneuvering needs can be categorized as in-plane and out-of-plane maneuvers. The efficiency of these maneuvers highly depends on the relation between Sun's relative position with respect to the satellite and orientation of the orbital plane with respect to ecliptic plane. Therefore a maneuver planning strategy has to be constructed. The overall study on the maneuver planning strategy was presented at 2021 AIAA SciTech Forum in Nashville, Tennessee/USA [79].

7.1 Significant Parameters

Solar sailing in LEO is affected by the Sun-Earth-Satellite relative geometry. For this reason, the need for a parameterization of this relative geometry and examination of the relations for the effectiveness of orbital maneuvers with solar sailing are inevitable. Therefore, some orbital elements, directions and relevant angles are taken into consideration for maneuver planning. Table 7.1 shows the list of the significant

parameters. Selected orbital elements and vector directions' definitions are well-known [3, 59].

Table 7.1 Significant Parameters

Significant Orbital Elements	Significant Directions	Significant Angles
Inclination	Sun's Position Vector Direction with respect to the Satellite	Sail Pitch Angle- α_{sail}
RAAN	Velocity Vector Direction	Angle between Sun Direction and Orbital Plane - β
Argument of Perigee	Perigee Vector Direction	Angle between Sun Direction and Eccentricity Vector - θ
True Anomaly	Sail's Normal Direction	Angle between Sun Direction and Velocity Vector- γ
DOY	-	-

First significant angle is the sail pitch angle as discussed in Chapter 2.1. The next one is the angle between Sun direction and orbital plane, called beta angle in the literature [80], and it helps to derive the seasonal intervals showing the efficiency of orbital maneuvers with solar sailing. In order to derive relationships between orbital parameters and Sun's direction, two more angles are defined: Angle between Sun direction and eccentricity vector (θ), and angle between Sun direction and velocity vector (γ). The first is defined to analyze the altitude criteria, of the proposed concept of R2S2, which does not allow solar sailing below a certain altitude. The latter is defined to determine whether the Sun direction is along the same or in the opposite direction of the velocity vector. The geometry for these angles are presented in Figure 7.1, where sail pitch angle is already shown in Figure 2.1, and related mathematical expressions are provided in Equation (132).

$$\begin{aligned}
\cos(\alpha_{sail}) &= \hat{n} \cdot \hat{R}_{Sun_Sat} \\
\sin(\beta) &= -\hat{R}_{Sun_Sat} \cdot \hat{h} \\
\cos(\theta) &= \hat{R}_{Sun_Sat} \cdot \hat{e} \\
\cos(\gamma) &= \hat{R}_{Sun_Sat} \cdot \hat{V}_{Sat}
\end{aligned} \tag{132}$$

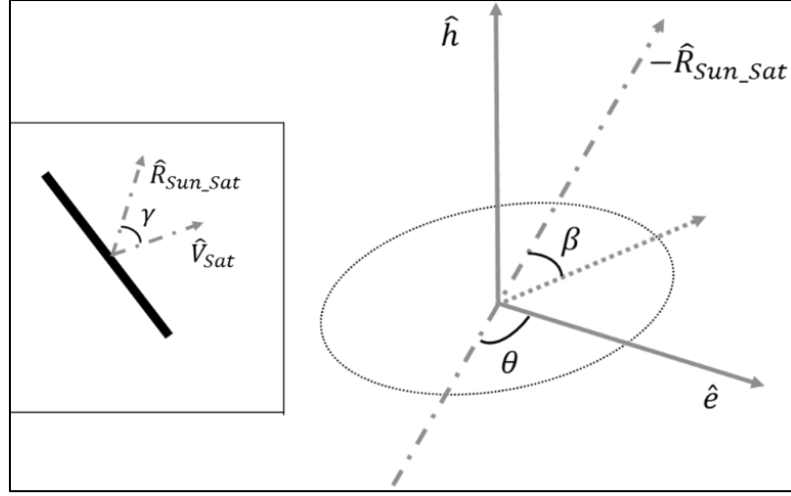


Figure 7.1. Five Significant Angles

For the analyses on the maneuver planning, the angle between Sun direction and orbital plane is important. According to the Equation (132), the angle between Sun direction and orbital plane can be calculated if we know the unit vector of the angular momentum of the orbit and Sun direction measured from the satellite. These two unit vectors can be derived using rotation matrices, given in Equations (133-134), populated by obliquity of the ecliptic plane, ecliptic longitude of the Sun, and orientation parameters of the satellite orbit (RAAN and inclination) [80]. Here, the direction of the Sun is formulated based on the center of the Earth since the change due to the shift of base point from satellite to Earth center is negligible for the direction of the Sun ($\pm 5.2 \times 10^{-5}$ rad or ± 10.8 arc-seconds).

$$-\hat{R}_{Sun_Sat} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\mathcal{E} & -s\mathcal{E} \\ 0 & s\mathcal{E} & c\mathcal{E} \end{bmatrix} \begin{bmatrix} c\Gamma & -s\Gamma & 0 \\ s\Gamma & c\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \quad (133)$$

$$\hat{h} = \begin{bmatrix} c\Omega & -s\Omega & 0 \\ s\Omega & c\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & ci & -si \\ 0 & si & ci \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad (134)$$

When we perform matrix operations in Equations (133-134) and substitute the results into Equation (132), following relation for the angle between Sun direction and orbital plane is obtained:

$$\sin \beta = \cos \Gamma \sin \Omega \sin i - \sin \Gamma \cos \varepsilon \cos \Omega \sin i + \sin \Gamma \sin \varepsilon \cos i \quad (135)$$

The angle between Sun direction and orbital plane, in Equation (135), depends on four parameters: ecliptic longitude of the Sun, obliquity of the ecliptic, inclination, and RAAN. Ecliptic longitude of the Sun can be approximated as in Equation (136) and obliquity of the ecliptic can be considered as constant. Then, the angle between Sun direction and orbital plane becomes a function of DOY, RAAN, and inclination as in Equation (137).

$$\Gamma = 280^\circ + \frac{360^\circ}{365}(\text{DOY} - 1) \quad (136)$$

$$\beta = f(\text{DOY}, \Omega, i) \quad (137)$$

Next, the angle between Sun direction and eccentricity vector is parameterized. According to the Equation (132), the angle between Sun direction and eccentricity vector can be calculated if we know the unit vector of the eccentricity and Sun's direction. The eccentricity vector direction can be determined by elemental rotation matrices as shown in Equation (138).

$$\hat{e} = \begin{bmatrix} c\Omega & -s\Omega & 0 \\ s\Omega & c\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & ci & -si \\ 0 & si & ci \end{bmatrix} \begin{bmatrix} c\omega & -s\omega & 0 \\ s\omega & c\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (138)$$

When we perform matrix operations in Equations (133, 138) and substitute the results into Equation (132), parameterization of the angle between Sun direction and eccentricity vector is obtained:

$$\theta = f(\text{DOY}, \Omega, i, \omega) \quad (139)$$

Similar approach is adopted for the parameterization of the angle between Sun direction and velocity vector. According to the Equation (132), the angle between

Sun direction and velocity vector can be calculated if we know the velocity direction and Sun's direction. The velocity vector direction may be calculated by using Equations (140-141). Equation (132) may again be used for the parameterization of the angle between Sun direction and velocity vector and the resultant relation is provided in Eq. (142).

$$\hat{V}_{Sat} = \hat{h} \times \hat{r}_{sat} \quad (140)$$

$$\hat{r} = \begin{bmatrix} c\Omega & -s\Omega & 0 \\ s\Omega & c\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & ci & -si \\ 0 & si & ci \end{bmatrix} \begin{bmatrix} c\omega & -s\omega & 0 \\ s\omega & c\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} cv \\ sv \\ 0 \end{Bmatrix} \quad (141)$$

$$\gamma = f(\text{DOY}, \Omega, i, \omega, v) \quad (142)$$

In summary, we have derived the three significant angles by using three significant vector directions (except sail's normal since it will be used in the strategy sections, Chapters 7.2 and 7.3) based on the five significant orbital parameters. The parameterized functions of angles in Equations (137, 139, and 142) will be used to predict the applicability of the out-of-plane and in-plane maneuvers. Since these angles are functions of significant directions, which are also functions of significant orbital parameters, the Significant Parameters listed in Table 7.1 are the key factors for the maneuver planning strategy. The method of utilization of these significant parameters to predict the applicability of the maneuvers will be provided in the following sections.

7.2 Strategy for Out-of-Plane Maneuvers

Out-of-plane maneuvers are used to reorient the orbit, i.e. altering the inclination, RAAN, and argument of perigee. The required control vector direction is along the orbit normal. Therefore, the relationship between Sun's direction and orbit normal should be analyzed to determine the efficiency of the out-of-plane maneuvers. With the angle between Sun direction and orbital plane (equivalently indicating the

angular relation with respect to the orbit normal), the directing capability of sunlight direction into required control vector direction is analyzed. This capability is limited to an angle of 55 degrees as shown in the non-ideal solar force model section (Chapter 3.2). The resulting geometrical situation regarding the out-of-plane maneuvers is then shown in Figure 7.2.

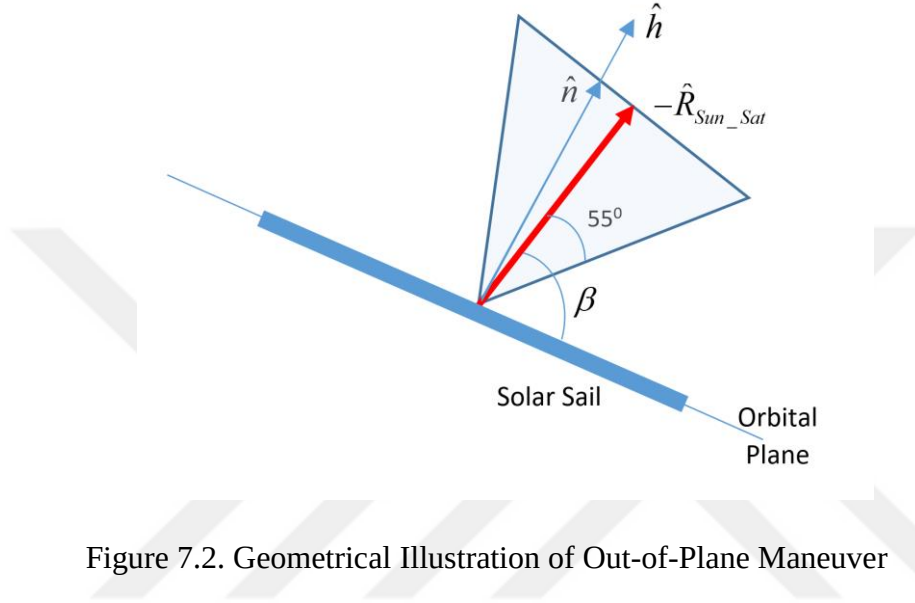


Figure 7.2. Geometrical Illustration of Out-of-Plane Maneuver

As can be seen in Figure 7.2, achievable directions for the required control vector are within the cone with a half angle of 55 degrees from the Sun's direction. This means that orbit normal should be within the achievable envelope shown in Figure 7.2. Following relations are then proposed for successful out-of-plane maneuver for R2S2 satellite:

$$|\beta| \geq 35^\circ \quad (143)$$

In addition to Equation (143), a further relation is proposed regarding the direction of the required control vector. In order to obtain a thrust along the orbit normal, the angle between Sun direction and orbital plane should be negative. If the required thrust vector is in the opposite direction of the orbit normal vector, the angle then should be positive as given in Equations (144-145). The angle between Sun direction and orbital plane is considered as positive when the Sun is in the northwards direction

of the orbit in heliocentric inertial frame, conversely considered as negative when the Sun is in the southwards direction [80].

$$\beta \leq -35^\circ \quad (144)$$

$$\beta \geq 35^\circ \quad (145)$$

This relationship based on β for successful out-of-plane maneuvers with solar sailing will be discussed further in Chapter 7.4. At the end of the evaluation, seasonal (DOY-based) limitations will be provided for given inclination and RAAN of the orbit.

7.3 Strategy for In-Plane Maneuvers

In-plane maneuvers are used to resize and reshape the orbit, i.e., changing the semi-major axis, perigee and apogee altitudes. The required control vector direction is along the velocity vector for maximum efficiency in terms of orbital energy increase. Therefore, the angle between Sun direction and velocity vector is important. This angle should be less than 55 degrees due to the solar sail's directing capability of the sunlight as mentioned earlier. The resulting geometry is shown in Figure 7.3.

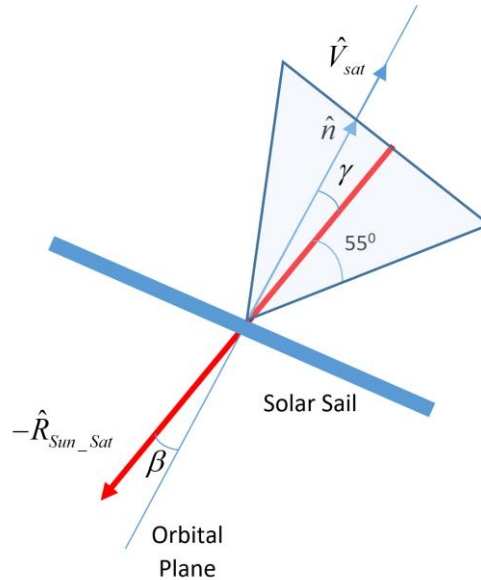


Figure 7.3. Geometrical Illustration of In-Plane Maneuver

Similar to the approach adopted regarding the out-of-plane maneuver strategy, velocity vector should be within the achievable envelope of directions in Figure 7.3. In contrast with the orbit normal, velocity vector scans all 360 degrees during one orbit revolution. This means that, the minimum value of the angle between Sun direction and velocity vector is equal to the angle between Sun direction and orbital plane. Similarly, maximum value of the angle between Sun direction and velocity vector is equal to $180-|\beta|$. Following relation is then proposed for successful in-plane maneuver for R2S2 satellite:

$$|\beta| \leq 55^\circ \quad (146)$$

As stated in the out-of-plane maneuver strategy section, the relationship in Equation (146) will be evaluated in Chapter 7.4.

The approach for the in-plane maneuvers has one more factor to be taken into consideration: It is the angle between Sun direction and eccentricity vector. Similar to the velocity vector, eccentricity vector also scans all 360 degrees at a rate that depends on the apsidal precession. This means that the minimum value of the angle between Sun direction and eccentricity vector is equal to the angle between Sun direction and orbital plane, and maximum value of the angle between Sun direction and eccentricity vector is equal to $180-|\beta|$.

The factor of sailing mode region is taken into consideration with simplifications first, then a generalization is made. The first simplification is to assume that the angle between Sun direction and orbital plane is zero. The geometrical illustration of various values of the angle between Sun direction and eccentricity vector and velocity vector direction at apogee may be observed from Figure 7.4.

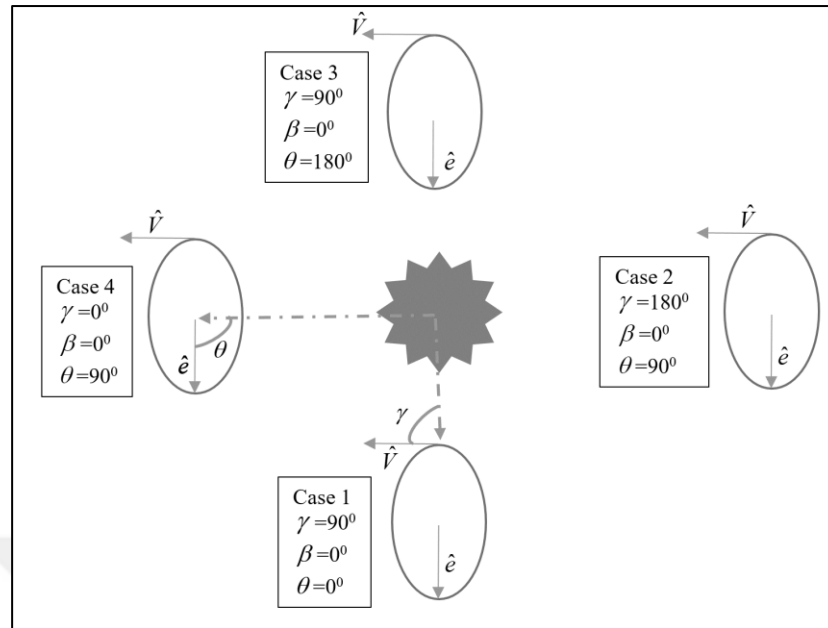


Figure 7.4. Orbit Orientation and Sun Directions for Sailing Mode Considerations

Velocity vector is illustrated at apogee since it is the most efficient point of the sailing mode part of the orbit. The second simplification is to assume that the orbit altitude is zero. In this way, the shadowing properties of the Earth on the sail becomes simpler (Figure 7.5).

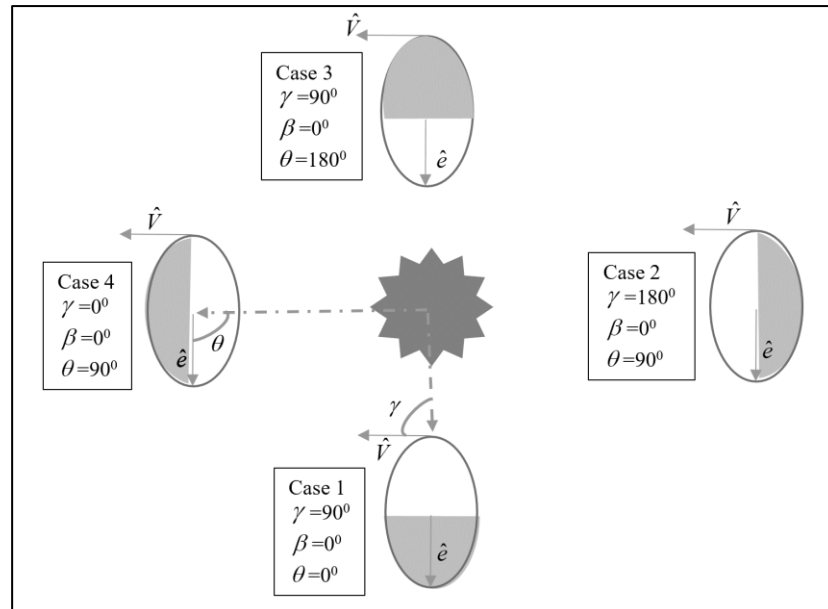


Figure 7.5. Orbit Orientations and Shadowing on the Sail

It may be observed from Figure 7.5, that there are four cases to examine. Due to the second simplification of assuming the altitude as zero, half of the orbit is always in Earth's shadow. If we combine this result with the sailing mode region of the orbit as shown in Figure 4.1, then four cases can be examined in terms of in-plane maneuver effectiveness for the R2S2 satellite:

- Case 1: Sailing mode region of the orbit (altitudes above 800 km, corresponding to the true anomaly interval from 82 degrees to 278 degrees) is not in the Earth's shadow. In that region sailing maneuvers can be performed by orienting the sail to generate the solar acceleration. Therefore, in-plane maneuvers in this orientation will be possible.
- Case 2: Half of the sailing mode region of the orbit is not in the Earth's shadow (true anomaly interval is from 180 degrees to 278 degrees). However, that region always corresponds to $\gamma \geq 90^\circ$ as shown in Figure 7.6, which means that Sun direction is along the opposite direction of the velocity vector which is not suitable for in-plane maneuvers. Sail's normal direction in this case will be in the direction to decrease the orbital energy. Thus, in-plane maneuvers in this orientation will be inefficient.
- Case 3: Sailing mode region of the orbit is in the Earth's shadow. Hence, maneuvers will be impossible.

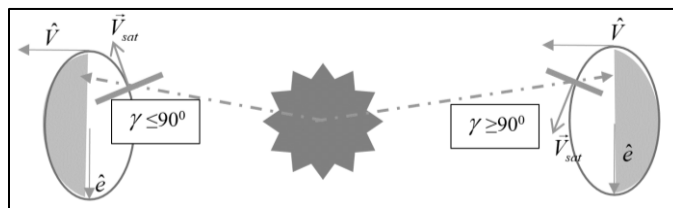


Figure 7.6. The Effect of the Angle between Sun Direction and Velocity Vector

- Case 4: Half of the sailing mode region of the orbit is not in the Earth's shadow. Due to the 55 degrees of directing capability of the sunlight and the regions of $\gamma \leq 90^\circ$ as shown in Figure 7.6, which means that Sun direction is

along the velocity vector for in-plane maneuvers will be efficient, but not as much as Case 1.

If we keep the angle between Sun direction and orbital plane at zero, and span all possible angles between Sun direction and eccentricity vector, it can be deduced that all the orientations between Case 2 and Case 3 will not be suitable for in-plane maneuvers. On the contrary all orientations between Case 4 and Case 1 will be feasible. Transition from an inefficient case to an efficient case between Case 2 and Case 1 is determined with the 55 degrees of directing capability of the sunlight. Then, we can say that inefficiency will be valid from Case 2 to an orientation such that the angle between Sun direction and eccentricity vector is 55 degrees. The resultant relation is shown in Figure 7.7 and can be expressed in Equation (147).

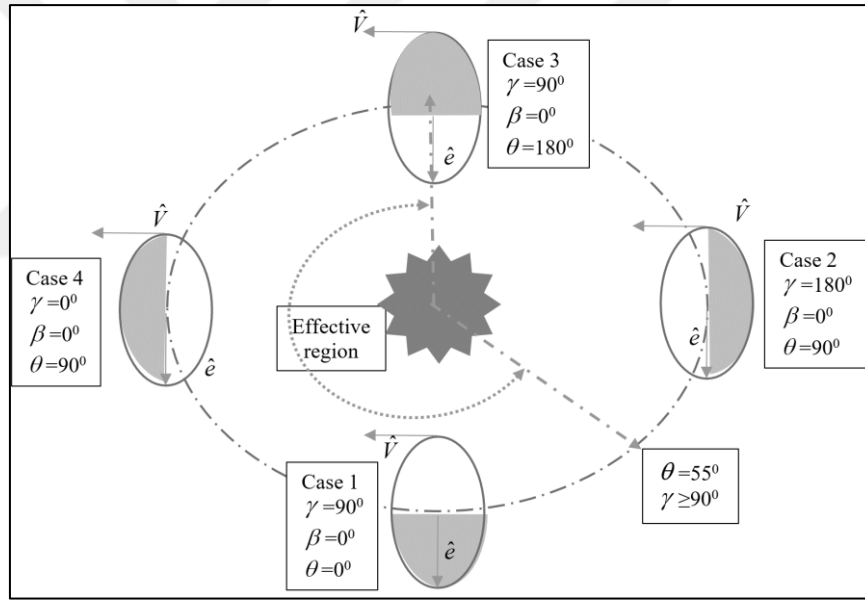


Figure 7.7. Effective Regions for In-Plane Maneuvers

$$\begin{cases} 55^\circ \leq \theta \leq 180^\circ \ \& \ \gamma_{apogee} > 90^\circ & \text{Inefficient} \\ \text{Otherwise} & \text{Efficient} \end{cases} \quad (147)$$

Now, two simplifications are taken into consideration. First, the altitude simplification is cancelled. This will affect the portion of the orbit in shadow. Due to the altitude of the satellite, eclipse exit point of the orbit will not be at 90 degrees

from the ecliptic plane. The angle separation from the ecliptic plane is getting smaller as the altitude gets higher. The shadow geometry and eclipse exit condition are shown in Figure 7.8, which is based on the “*Earth’s Cylindrical Shadow Model*” described in [102].

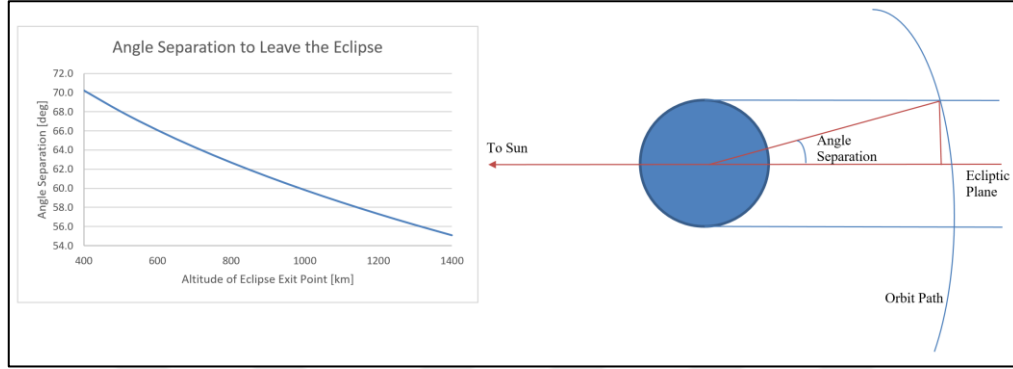


Figure 7.8. Shadow Geometry and Eclipse Exit Condition [102]

Next simplification to be taken into consideration is the angle between Sun direction and orbital plane. The angle separation shown in Figure 7.8 can be achieved by the movement of the satellite. Another reason for angle separation is the angle between Sun direction and orbital plane. For the extreme condition of the angle between Sun direction and orbital plane is equal to 90 degrees, all points of the orbit are exposed to the sunlight. However, in-plane maneuvers for the R2S2 satellite force the angle between Sun direction and orbital plane to be below 55 degrees as in Eq. (146). This value of the angle between Sun direction and orbital plane is same as the angle separation for 1400 km altitude. Hence, eclipse exit due to orientation of the orbit (combination of inclination and RAAN) is not the case for the in-plane maneuvers of R2S2 satellite. For these reasons, the simplification regarding the angle between Sun direction and orbital plane does not need to be generalized. The analysis in Figure 7.5 is still valid except the modifications due to the altitude simplification. When altitude of the orbit is included in the shadow analysis the illustration given in Figure 7.9 is obtained.

As can be seen in Figure 7.9, the efficiency status of the cases change in the positive direction. Case 3 is not inefficient anymore due to the angle separation brought by

the altitude of the satellite and this is valid until $\theta = 150^\circ$. The resultant relation for the in-plane maneuver strategy is provided below.

$$\begin{cases} 55^\circ \leq \theta \leq 150^\circ & \& \gamma_{apogee} > 90^\circ & \text{Inefficient} \\ \text{Otherwise} & & \text{Efficient} \end{cases} \quad (148)$$

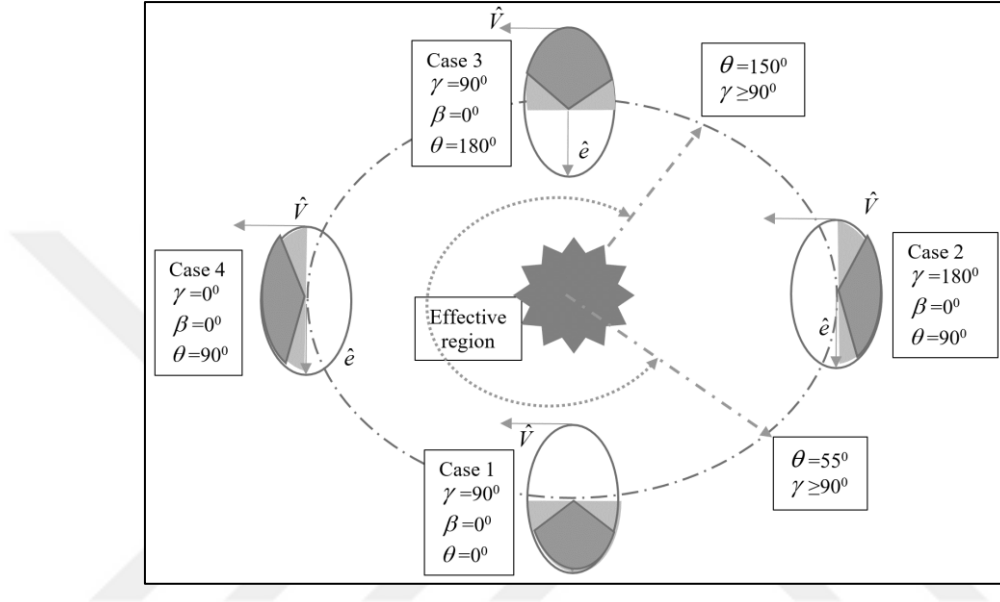


Figure 7.9. Orbital Orientations and Shadowing on the Sail without the Altitude Simplification (Dark grey area is the resultant shadowing condition when the altitude of the satellite is taken into consideration)

The relationship based on the angles between Sun direction with eccentricity and velocity vectors in Equation (148) and the angle between Sun direction and orbital plane in Equation (146) for successful in-plane maneuvers with solar sailing will be evaluated in Chapter 7.4. At the end of the evaluation, seasonal (DOY-based) limitations will be provided for given inclination, argument of perigee and right ascension of the ascending node of the orbit.

7.4 Results and Discussion about the Maneuver Planning Strategy

As the strategies are provided with relations depicted in Equation (143) for out-of-plane maneuvers and Equations (146, 148) for in-plane maneuvers, now it is time to evaluate the outcomes of these strategies.

In order to layout the outputs of this maneuver planning strategy, the angle between Sun direction and orbital plane is then calculated with Equation (137) at various inclination values for the entire range of values of DOY and RAAN. Since the angle between Sun direction and orbital plane is a function of inclination, DOY and RAAN, a four-dimensional data matrix is obtained. The generation of figures at specific inclination angles reduces the dimension of the data to three. The gray-coloring of the values of the angle between Sun direction and orbital plane allows us to show the three-dimensional data in a two-dimensional plot. The resultant figures are then modified considering the limitations given in Equations. (144-145) and the angle between Sun direction and orbital plane is obtained. The remaining data is presented in Figure 7.10 for out-of-plane maneuvers with thrust necessities along the angular momentum vector (positive direction). On the other hand a similar data is presented in Figure 7.11, however this time for generating thrust in the opposite direction of the angular momentum vector (negative direction).

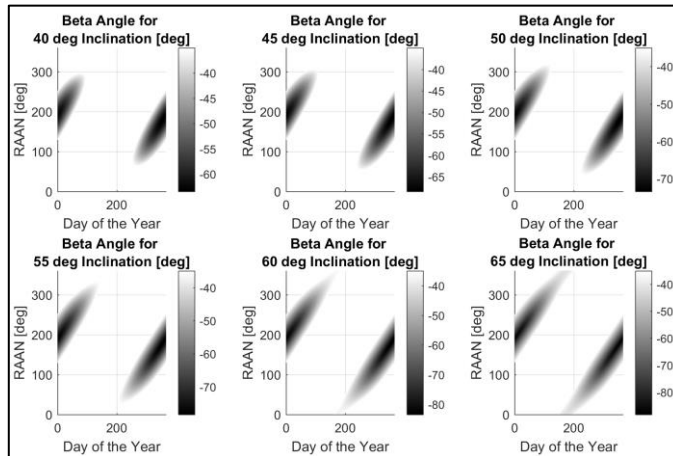


Figure 7.10. Seasonal Limitations for Out-of-Plane Maneuvers in the Positive Direction of the Angular Momentum Vector

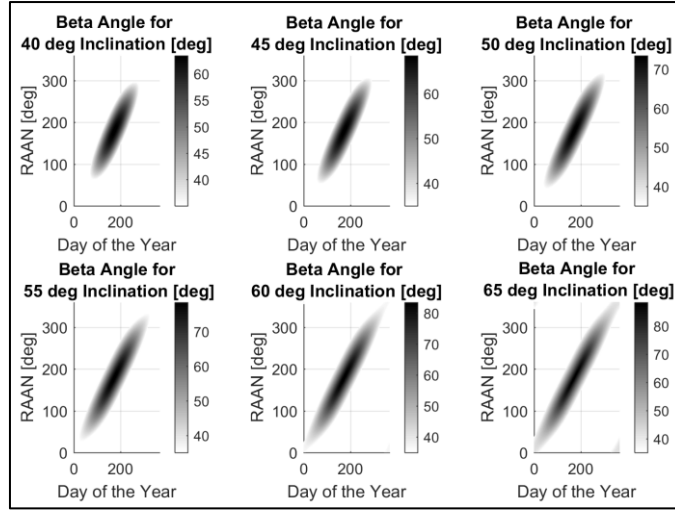


Figure 7.11. Seasonal Limitations for Out-of-Plane Maneuvers in the Negative Direction of the Angular Momentum Vector

The dark grey portions of the data presented in Figure 7.10 and Figure 7.11 are the most effective regions for out-of-plane maneuvers, as the sunlight comes to the orbital plane with a steeper angle closer to 90 degrees. It may also be observed from these figures that as the inclination increases, seasonal envelope expands as expected since the inclination has a direct effect on the boundaries of the angle between Sun direction and orbital plane. This effect can be seen in equation below [80]:

$$-(\varepsilon + i) \leq \beta \leq \varepsilon + |i| \quad (149)$$

With the limitation in Equation (146), figures of the angle between Sun direction and orbital plane are generated again this time for in-plane maneuver effectiveness (Figure 7.12). The most effective seasonal interval for the in-plane maneuvers is achieved when the value of the angle between Sun direction and orbital plane gets closer to zero.

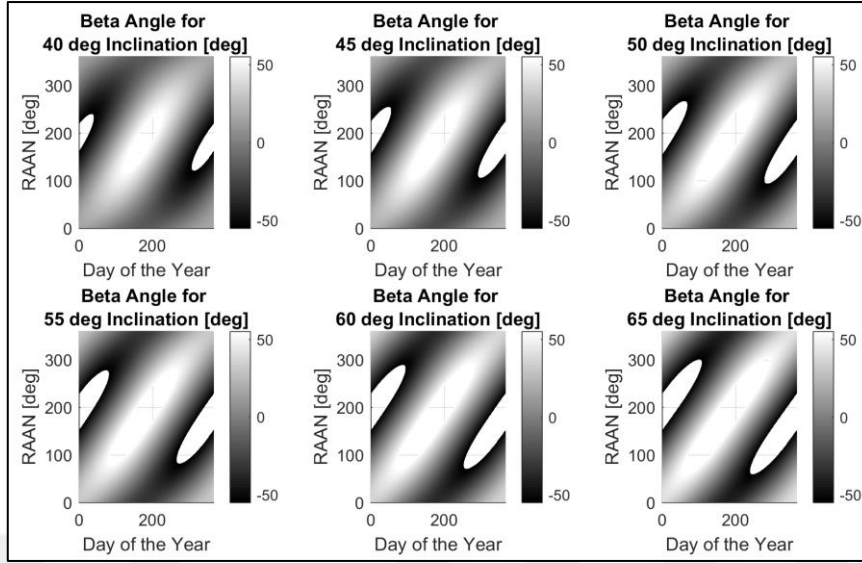


Figure 7.12. Seasonal Limitations for In-Plane Maneuvers

As mentioned in Equation (148), in-plane maneuvers need another factor (sailing mode region of the orbit) to be taken into consideration. For this reason the limitation for the angle between Sun direction with eccentricity and velocity vectors given in Equation (148) is then analyzed by using calculations based on Equations (139, 142).

As expected the calculation of these angles results in five-dimensional data, which can be exploited in terms of effectiveness assessment with respect to the Equation (148). However, to visualize a five-dimensional data, volume illustration approach is adopted.

Initially, plotting these angles for various inclination angles reduces the dimension of five to four. The resultant volumes of the angle between Sun direction and eccentricity vector (Figure 7.13) and the angle between Sun direction and velocity vector (Figure 7.14) illustrated with horizontal and vertical slices. Finally, the coloring of the data shows the value of that angle. Values of these angles are then analyzed with respect Equation (148).

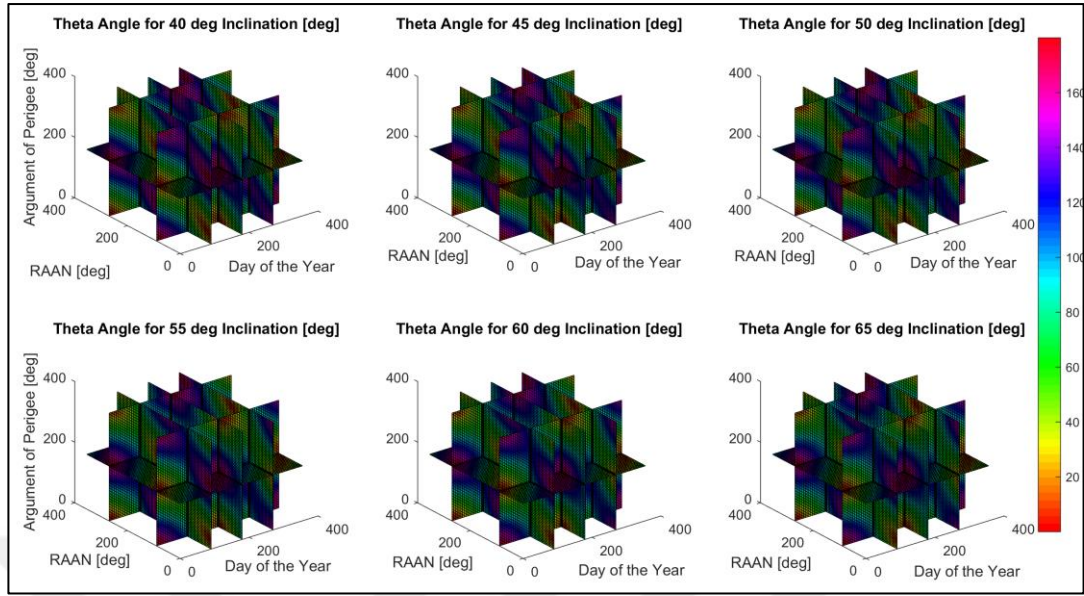


Figure 7.13. Values of the Angle between Sun Direction and Eccentricity Vector, θ , with Varying Orbital Parameters

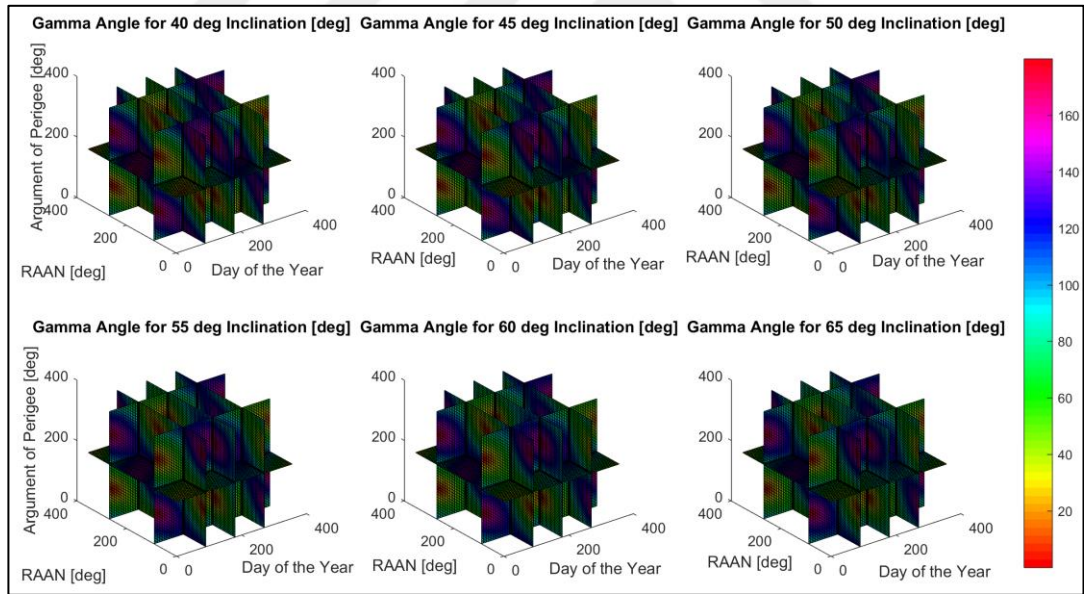


Figure 7.14. Values of the Angle between Sun Direction and Velocity Vector, γ , with Varying Orbital Parameters

For example, the regions in Figure 7.13 with a value in the interval of 55 to 150 degrees will be ineffective if the same region's value in Figure 7.14 is above 90

degrees. This is an example for the interpretation of Equation (148). When ineffective regions are excluded from this four-dimensional data of the angle between Sun direction and eccentricity vector as exemplified, the resultant volume of data is used for maneuver planning. The illustration of the four-dimensional data with the excluded regions are not practical, hence an example with pre-determined RAAN value of 90 degrees (data dimension is then reduced to three) is shown in Figure 7.15. Note that, as θ gets closer to zero (darker areas in Figure 7.15) the effectiveness of the in-plane maneuvers increases and the excluded regions do not comply with Equation (148).

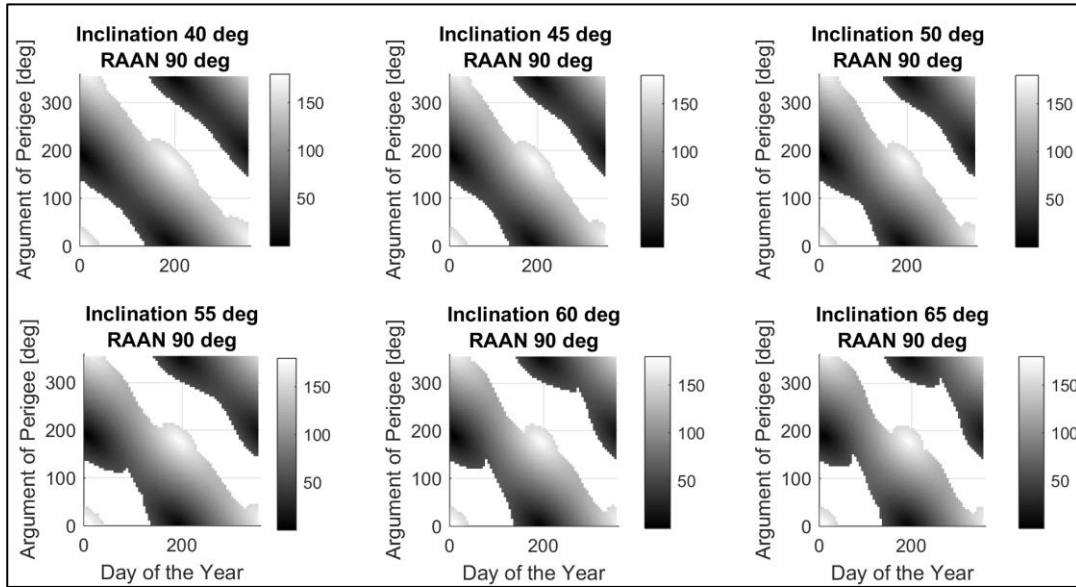


Figure 7.15. The Regions for the Determination of In-Plane Maneuvers Effectiveness (*Excluded regions with no data shown on the graphs mean that in-plane maneuvers are not effective*)

As a result, seasonal intervals for the effectiveness of in-plane maneuvers can be obtained with the given orbital parameters with the exploitation of Equations (146, 148).

R2S2 satellite gets its unique advantage from the utilization of the propellant-free thrust capability of the solar sail at LEO. However, this brings dependence on the orientation of the Sun-Earth-Satellite relative geometry. This orientation-based

efficiency analysis is accomplished by providing seasonal limitations for given orbit inclination, argument of perigee and RAAN values. Planners of the R2S2 maneuvers will then use this information to alter start time and/or lifetime of the mission orbit. For instance, if an ineffective seasonal interval for in-plane maneuvers is about to start, the current standby orbit, transferring to the new mission orbit should be delayed to a day that is within the efficient seasonal interval. Moreover, if strict station-keeping requirements are put in place for a mission orbit, start time and lifetime of that orbit should be selected such that not suitable seasonal interval for out-of-plane maneuvers are avoided.





CHAPTER 8

ORBIT CONTROL STRATEGY

The proposed concept of R2S2 requires the orbital maneuvers for station-keeping of the mission orbit and transferring to standby or new mission orbits. Until this chapter, all relevant sub-topics of R2S2 Concept has been explained. Furthermore, on-off switch-type control methods were applied during the feasibility and mission orbit determination simulations. In this chapter, the control approach is improved by taking the dynamics of the orbital elements into consideration. For this purpose, the Gaussian Variational Equations (GVE) in Equation (42) are modified and utilized by employing two types of approaches to generate a solution to the thrust vector control problem. Then, the orbit control implementation is made for two different kind of mission modes: Station-Keeping and Transfer Orbit. In the Station-Keeping mission mode, only the first control approach is applied, while in Transfer Orbit mission mode both approaches are used.

8.1 Modification of the Gaussian Variational Equations

In GVE, as shown in Equation (42), the force inputs (R - T - N) can be the source of any acceleration experienced by the satellite other than two-body gravitational attraction of the Earth, for example; thrust of chemical/electrical propulsion systems, aerodynamic drag, asphericity effects of the Earth (J2-J3-J4), third body gravitational attractions of the Sun and Moon or in our case solar acceleration.

Since the R2S2 concept highly depends on the altitudes of perigee and apogee, GVEs are modified accordingly. The dynamics of the radii of perigee (r_p) and apogee (r_a) are found:

$$\begin{aligned} r_p &= a(1-e) \\ \dot{r}_p &= \dot{a}(1-e) - a\dot{e} \end{aligned} \quad (150)$$

$$\begin{aligned} r_a &= a(1+e) \\ \dot{r}_a &= \dot{a}(1+e) + a\dot{e} \end{aligned} \quad (151)$$

Substituting the time rate of changes of the semi-major axis and eccentricity in Equation (42) with Equations (150-151) forms Equation (152) which will be used in the determination of the thrust vector direction.

$$\begin{aligned} \dot{r}_p &= \frac{2a^2}{h} \left[e \sin \nu (1-e) - \frac{(1-e^2) \sin \nu}{2} \right] R + \left[\frac{2a^2 p (1-e)}{h r_{sat}} - \frac{a}{h} [(p + r_{sat}) \cos \nu + r_{sat} e] \right] T \\ \dot{r}_a &= \frac{2a^2}{h} \left[e \sin \nu (1+e) + \frac{(1-e^2) \sin \nu}{2} \right] R + \left[\frac{2a^2 p (1+e)}{h r_{sat}} + \frac{a}{h} [(p + r_{sat}) \cos \nu + r_{sat} e] \right] T \\ \dot{i} &= \frac{r_{sat} \cos(\omega + \nu)}{h} N \\ \dot{\Omega} &= \frac{r_{sat} \sin(\omega + \nu)}{h \sin i} N \\ \dot{\omega} &= -\frac{r_{sat} \sin(\omega + \nu)}{h \tan i} N + \frac{1}{eh} [-pR \cos \nu + (p + r_{sat})T \sin \nu] \\ \dot{\theta} &= \frac{h}{r_{sat}^2} + \frac{1}{eh} [pR \cos \nu - (p + r_{sat})T \sin \nu] \end{aligned} \quad (152)$$

The modified version of GVEs in Equation (152) is then used for mission design. The importance of this modification is more evident when the perigee and apogee altitude values of the elliptic mission orbit should be maintained separately as it is the case for R2S2 concept. The resizing methodologies based on the semi-major axis and eccentricity are lacking in this manner and eventually giving a poor performance for intuitive analysis over the elliptic mission orbits.

8.2 Determination of the Thrust Vector Control

Equation (152) will be used in both approaches of determination of the thrust vector direction.

8.2.1 First Approach

The first approach approximates the Equation (152) for the current state of the orbital parameters. In this context, the current values of those elements are substituted to the dynamics equations continuously. This leads to having time rate of changes of the orbital elements under the influence of force inputs and true anomaly.

$$\begin{aligned}\dot{r}_p &= f(\nu, R, T) \\ \dot{r}_a &= f(\nu, R, T) \\ \dot{i} &= f(\nu, N) \\ \dot{\Omega} &= f(\nu, N) \\ \dot{\omega} &= f(\nu, R, T, N)\end{aligned}\tag{153}$$

Furthermore, the radial and transverse force inputs are replaced with an orbital plane force magnitude, F , and a direction identified by an angle ξ ($\tan \xi = T/R$):

$$\begin{aligned}\dot{r}_p &= f(\nu, \beta, F) \\ \dot{r}_a &= f(\nu, \beta, F) \\ \dot{i} &= f(\nu, N) \\ \dot{\Omega} &= f(\nu, N) \\ \dot{\omega} &= f(\nu, \xi, F, N)\end{aligned}\tag{154}$$

In order to decrease the number of variables in Equation (154) the in-plane and out-of-plane forces are decoupled. Regarding the argument of perigee, control input is limited to be only in normal direction as in the out-of-plane forces. This assumption permits neglecting the magnitudes of the control inputs and accept them as unity, then the dynamics are transformed as follows:

$$\begin{aligned}\dot{r}_p &= f(\nu, \xi) \\ \dot{r}_a &= f(\nu, \xi) \\ \dot{i} &= f(\nu, \varsigma) \\ \dot{\Omega} &= f(\nu, \varsigma) \\ \dot{\omega} &= f(\nu, \varsigma)\end{aligned}\tag{155}$$

where ξ can be in the range between 0 and 360 degrees, ς can be either 0 (positive N) or 180 (negative N) degrees. By using the approximated relations in Equation

(155) based on the current state of the orbital parameters, the thrust vector direction, which gives the maximum rate of change (positive or negative, depends on the desired orbital parameter value) for the specific orbital parameter, is found as a function of true anomaly. The approximation is considered as valid during the timespan of one orbit revolution due to the small changes of orbital parameters. This approach will be used in Transfer Orbit control trials in Chapter 8.3.2.

However, the constant need for orbital parameter feedback and the assumption of decoupled dynamics should also be addressed. For this reason, the below modification is made for the first control approach.

First of all, both types of GVEs are used for this modification (The original GVE in Equation (42) and the modified version according to the needs of R2S2 concept in Equation (152)). Approximations to these formulations are calculated with respect to the mission orbit parameters (400 km of perigee altitude and 1400 km of apogee altitude), for once. No need for constant feedback of orbit parameters are necessary since this modified version of first control approach will be used in the Station-Keeping mission mode only. However, this brings an issue of an increase in the number of variables to be taken into consideration for control purposes. As can be observed from Equation (156), approximated dynamics of the classical orbital elements now depend on angles (argument of perigee, true anomaly and inclination) and force inputs. In Equation (156), in-plane forces, T and R are replaced by force F and angle ξ as can be seen in Figure 8.1.

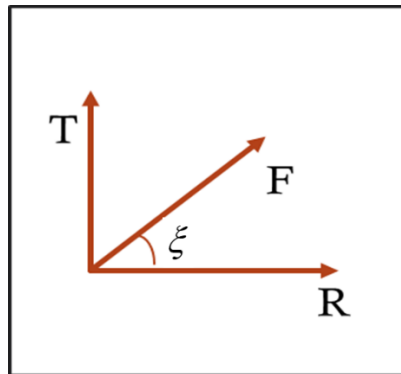


Figure 8.1. In-Plane Force Component Diagram

$$\begin{aligned}
\dot{a} &= 135.44F \cos \xi \sin \nu - 1971.54F \sin \xi (0.068 \cos \nu - 1) [m/s] \\
\dot{e} &= 1.35 \times 10^{-4} F \cos \xi \sin \nu \dots \\
&+ 1.35 \times 10^{-4} \left(\frac{2 \cos \nu + 0.068 \cos^2 \nu + 0.068}{1 + 0.068 \cos \nu} \right) F \sin \xi [1/s] \\
\dot{i} &= \frac{1.35 \times 10^{-4} \cos(\omega + \nu)}{1 + 0.068 \cos \nu} N [rad/s] \\
\dot{\Omega} &= \frac{1.35 \times 10^{-4} \sin(\omega + \nu)}{\sin i (1 + 0.068 \cos \nu)} N [rad/s] \\
\dot{\omega} &= -\frac{1.35 \times 10^{-4} \sin(\omega + \nu)}{\tan i (1 + 0.068 \cos \nu)} N - 1.98 \times 10^{-3} F \cos \xi \cos \nu \dots \\
&+ 1.35 \times 10^{-4} \left(\frac{29.4 \sin \nu + 0.5 \sin 2\nu}{1 + 0.068 \cos \nu} \right) F \sin \xi [rad/s]
\end{aligned} \tag{156}$$

8.2.2 Second Approach

The second approach, on the other hand, uses the Equation (152) without any further approximation. In regards to the second method, designing a controller based on GVE has been well studied [81-84]. These studies either find optimal points in the orbit to make maneuvers or provide continuous thrust options. But the common point is to form the time rate of change of the state vector with GVE, except [84], since they use Cartesian form to solve for the control problem.

Here we modify the approach in [81]. The common part is to control the orbital elements by implementing of a Lyapunov-based nonlinear feedback controller, but instead of semi-major axis and eccentricity, we will use perigee and apogee altitudes as formulated in Equation (152). State vector $\bar{x}$ and its dynamics $\dot{\bar{x}}$ are formulated as below:

$$\dot{\bar{x}} = g(\bar{x})\bar{u}$$

$$\bar{x} = \begin{bmatrix} r_p \\ r_a \\ i \\ \Omega \\ \omega \end{bmatrix} \dots \bar{u} = \begin{bmatrix} R \\ T \\ N \end{bmatrix} \quad (157)$$

$$\bar{\Lambda} = \begin{bmatrix} r_p - r_p^* \\ r_a - r_a^* \\ i - i^* \\ \Omega - \Omega^* \\ \omega - \omega^* \end{bmatrix} = \begin{bmatrix} \delta r_p \\ \delta r_a \\ \delta i \\ \delta \Omega \\ \delta \omega \end{bmatrix} \quad (158)$$

where $\bar{\Lambda}$ is the state error with respect to the desired state values (with asterisk symbols), the time rate of change of the state error is then populated using the Equation (152):

$$\dot{\bar{\Lambda}} = \hat{G}\bar{u} = \begin{bmatrix} \frac{2a^2}{h} \left[e \sin \nu (1-e) - \frac{(1-e^2) \sin \nu}{2} \right] & \frac{2a^2 p (1-e)}{h r_{sat}} - \frac{a}{h} [(p + r_{sat}) \cos \nu + r_{sat} e] & 0 \\ \frac{2a^2}{h} \left[e \sin \nu (1+e) + \frac{(1-e^2) \sin \nu}{2} \right] & \frac{2a^2 p (1+e)}{h r_{sat}} + \frac{a}{h} [(p + r_{sat}) \cos \nu + r_{sat} e] & 0 \\ 0 & 0 & \frac{r_{sat} \cos(\omega + \nu)}{h} \\ 0 & 0 & \frac{r_{sat} \sin(\omega + \nu)}{h \sin i} \\ \frac{-p \cos \nu}{h e} & \frac{(p + r_{sat}) \sin \nu}{h e} & -\frac{r_{sat} \sin(\omega + \nu)}{h \tan i} \end{bmatrix} \begin{bmatrix} R \\ T \\ N \end{bmatrix} \quad (159)$$

With the defined state matrix G , a positive definite gain matrix K and a Lyapunov candidate function based on the error vector are then used to derive the control input that makes the system as globally asymptotically stable at the origin [85] selecting a positive definite Lyapunov function as follows:

$$V(\Lambda) = \frac{1}{2} \Lambda^T \Lambda$$

$$\dot{V}(\Lambda) = \Lambda^T \dot{\Lambda} = -\Lambda^T K \Lambda \leq 0$$

$$\Rightarrow u = -G^T K \Lambda \quad (160)$$

The gain matrix K in Equation (161) is a diagonal matrix, which is used to give relative importance between orbital elements and also equalize the order of magnitude based on the units of the parameters. For example, the inclination is measured in degrees from 0 to 360, on the other hand perigee altitude is in kilometers ranging from 400 to 1000 kilometers. The control sensitivity in terms of the magnitude of control is not needed to be efficiently adjusted since magnitude of the solar acceleration is not controllable. The only controllable part of the solar acceleration is its application direction. Nevertheless, an optimization has been made for the values of the diagonal of the gain matrix K based on trial and error approach.

$$K = \begin{bmatrix} K_{r_p} & 0 & 0 & 0 & 0 \\ 0 & K_{r_a} & 0 & 0 & 0 \\ 0 & 0 & K_i & 0 & 0 \\ 0 & 0 & 0 & K_\Omega & 0 \\ 0 & 0 & 0 & 0 & K_\omega \end{bmatrix} \quad (161)$$

Table 8.1 Gain Selection Table

Diagonal of K	First Tier		Second Tier	
	Condition	Value	Condition	Value
K_{r_p}	$\delta r_p > 1 \text{ km}$	10^{-5}	$\delta r_p \leq 1 \text{ km}$	10^{-7}
K_{r_a}	$\delta r_a > 1 \text{ km}$	10^{-5}	$\delta r_a \leq 1 \text{ km}$	10^{-7}
K_i	$\delta i > 0.01 \text{ deg}$	10	$\delta i \leq 0.01 \text{ deg}$	10^{-2}
K_Ω	$\delta \Omega > 0.01 \text{ deg}$	10	$\delta \Omega \leq 0.01 \text{ deg}$	10^{-2}
K_ω	$\delta \omega > 0.01 \text{ deg}$	10	$\delta \omega \leq 0.01 \text{ deg}$	10^{-2}

During the aforementioned trial and error approaches for determining the optimal diagonal values of gain matrix K , a need for a gain scheduling has been detected. Thus, a two-tier gain selection has been made as the resultant diagonal values of gain matrix K is tabulated in Table 8.1.

8.3 Application of the Thrust Vector Control

Having laid down the baseline equations and approaches for the determination of thrust vector control direction, now it is time to apply these approaches on different modes of missions. The application of the thrust vector control will be divided into two categories as mentioned earlier: Station-Keeping and Transfer Orbit. For both mission modes, principles and simulation results will be provided in the following sub-sections with further discussion of the results.

8.3.1 Station-Keeping

The station-keeping strategy of the mission orbit of the R2S2 concept is explained below in detail. First, adopted principles are provided with an analysis over the control authority of the solar acceleration for the orbital elements. Then, mathematical formulation of the strategy is given with case-based control logic for the entire combination of possible cases. Finally, simulation results are presented with discussion. The overall study on the station-keeping strategy was presented at 2020 Southeast Europe Space Conference in Belgrade, Serbia [86].

8.3.1.1 Principles

Two-tier control is applied to perigee and apogee altitudes to avoid chattering. 350 km is defined as the lower limit to trigger the perigee altitude station-keeping mode and is active until 400 km is achieved. Similar approach is applied for apogee altitude with 1350 km as lower limit resulting in an active station-keeping until reaching 1400 km of apogee altitude.

Depending on the initial orbital parameters and analysis on the observation performance of the mission orbit with requested mission lifetime, station keeping may be triggered for argument of perigee and/or RAAN as well. However, this

should be done at the beginning of the particular observation task and should be continuously carried out.

When possible, in-plane and out-of-plane force components shall be used separately for controlling the orbit size (in-plane for perigee and/or apogee altitudes) and orientation (out-of-plane for argument of perigee and/or RAAN) control.

For cases with multiple orbital parameters, the priority order is as follows: perigee altitude, apogee altitude, argument of perigee and RAAN. The rationale for the priority is straight-forward: First mission safety, then mission performance.

Regarding the analysis over the control authority of the solar acceleration for the orbital elements, rates of changes of orbital elements are plotted with respect to true anomaly, argument of perigee and in plane force angle ξ as shown in Figure 8.2 and Figure 8.3. In those graphs, all force per unit mass inputs are treated as unity. In these plots, the current inclination, which appears in argument of perigee and RAAN rate equations, is taken as 55° . Because, the R2S2 concept always uses prograde orbits which results in the situation that inclination does not flip the sign of the formula output. The magnitude changes are negligible in the scope of this study. However, for information, as inclination decreases the changes in argument of perigee and RAAN increase.

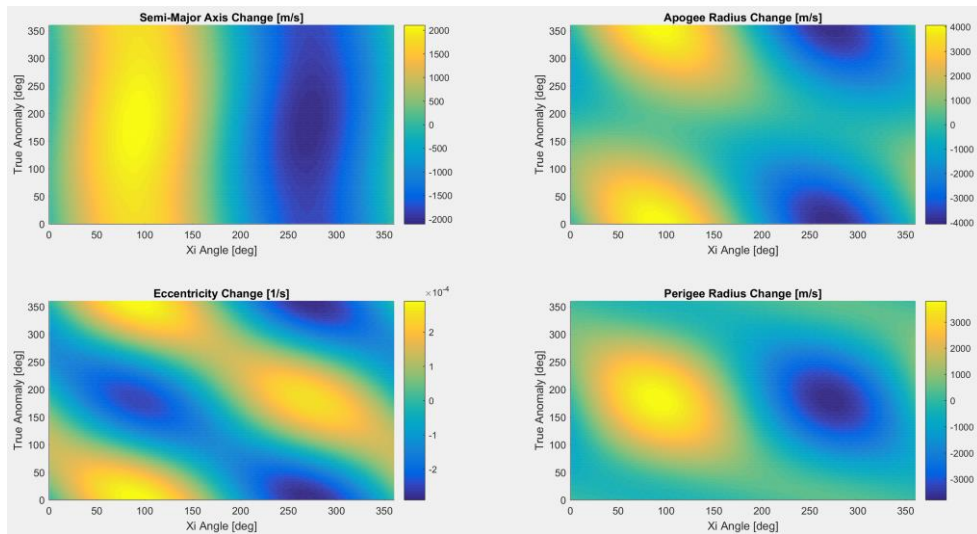


Figure 8.2. Orbit Resizing Graphs

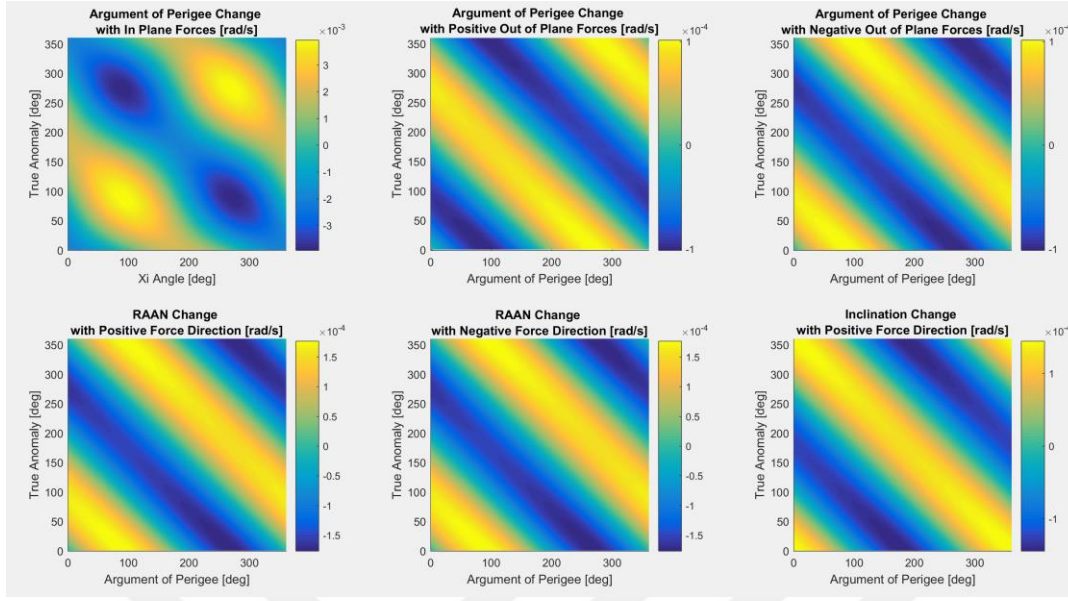


Figure 8.3. Orbit Reorienting Graphs

Since, the characteristic acceleration (achieved in the R2S2 concept and its baseline satellite design) is in the order of 0.1 mm/s^2 , these graphs can be interpreted as the change times 10^{-4} . This implies that approximated changes for orientation angles (Figure 8.3) are minor relative to the size parameters (Figure 8.2). This results in only 1% improvement over the mission lifetime due to the argument of perigee or RAAN changes by the asphericity of the Earth. As mentioned earlier, the tendency of shifts of argument of perigee and RAAN by the asphericity effects of the Earth cannot be balanced by solar sailing maneuvers, only reducing the shift rate is possible. This reduction rate over shifts of the argument of perigee and RAAN at the selected characteristic acceleration of the R2S2 concept is at 1%. But, this improvement can be increased if a larger sail area is used (resulting an increase on the characteristic acceleration value to the orders of 1 mm/s^2 or more and eventually an increase on the reduction rate over the shifts).

8.3.1.2 Case-Based Control Logic

With four parameters to be considered for station-keeping a list of cases is generated and given in Table 8.2. All 16 combinations are considered for station-keeping

strategy including Case Zero where no station-keeping condition is triggered. Each case is indicating the need for a time rate of change of a specific orbital parameter or combination of those depending on the triggering conditions. The need to generate the time rate of change of these orbital parameters are then met by solar sailing maneuvers.

Table 8.2 Case List

Case Code	Condition	Case Code	Condition
Case A	$\dot{r}_p$	Case I	$\dot{r}_p + \dot{r}_a + \dot{\Omega} + \dot{\omega}$
Case B	$\dot{r}_a$	Case J	$\dot{r}_p + \dot{\Omega}$
Case C	$\dot{\Omega}$	Case K	$\dot{r}_p + \dot{\omega}$
Case D	$\dot{\omega}$	Case L	$\dot{r}_a + \dot{\Omega}$
Case E	$\dot{r}_p + \dot{r}_a$	Case M	$\dot{r}_a + \dot{\omega}$
Case F	$\dot{r}_p + \dot{r}_a + \dot{\Omega}$	Case N	$\dot{r}_p + \dot{\Omega} + \dot{\omega}$
Case G	$\dot{r}_p + \dot{r}_a + \dot{\omega}$	Case O	$\dot{r}_a + \dot{\Omega} + \dot{\omega}$
Case H	$\dot{\Omega} + \dot{\omega}$	Case Zero	No Condition

Due to the principles of the station-keeping strategy of the R2S2 concept, triggering condition for the argument of perigee and/or RAAN will be initially set and kept continuous. For that reason, the need for the time rate of changes of argument of perigee and/or RAAN will always be the same (they exist or not). Only perigee and apogee altitude needs may change. As triggering conditions for perigee and apogee altitudes occur, case codes change. Similarly, as perigee and apogee altitudes are increased to their mission parameters by station-keeping maneuvers, case codes change again. This behavior results in four different cycles for case transitions (Figure 8.4, the subplots are ordered as; No argument of perigee and RAAN station-keeping, Only argument of perigee station-keeping, Only RAAN station-keeping, Both argument of perigee and RAAN station-keeping). During the mission lifetime, station-keeping cases transformed into a different case within the cycles shown in Figure 8.4. No interchange for case codes is possible between case cycles.

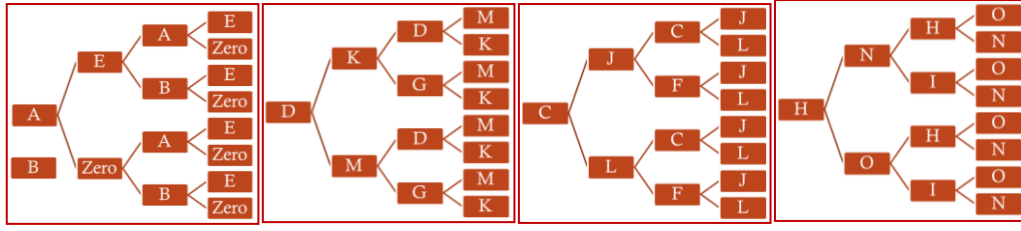


Figure 8.4. Case Cycles

The approximated dynamics output of the classical orbital elements, in Equation (153), is used to determine the control force direction. This process is carried out for each case with their own properties and requirements.

Case A: The perigee altitude dynamics depending on the true anomaly and xi angle are manipulated excluding the true anomaly region from -83^0 to $+83^0$ (due to the no Sailing Zone from the concept itself), values above -10^{-4} s^{-1} of eccentricity change (due to the aim of decoupling the effects of perigee and apogee altitudes) and values below 1000 m/s of perigee altitude change are discarded. The remainder region for maneuvers are shown in Figure 8.5. The optimal xi angle for given true anomaly is then used for controlling the sail orientation when Case A is triggered.

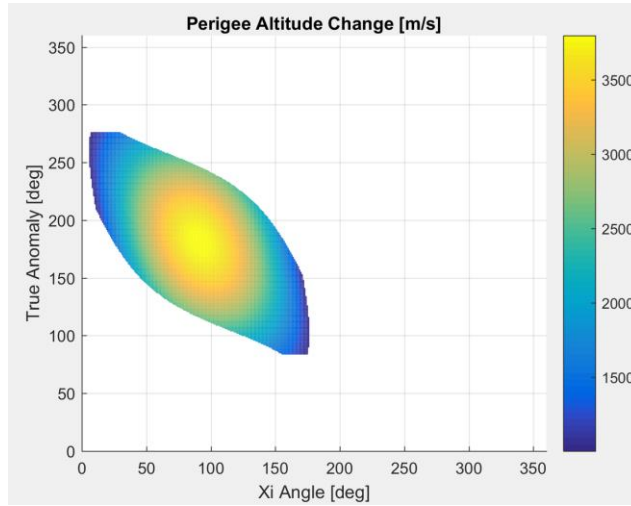


Figure 8.5. Perigee Altitude Change

Case B: Similarly, apogee altitude dynamics are given under the consideration of eccentricity and no Sailing Zone true anomaly region. The resultant graph is given

in Figure 8.6. As may be observed from the graph, the region for maneuvers are limited. This is due to the fact that portions of the orbit around the perigee altitude, which are the most effective region for altering the apogee altitude, are treated as no sailing zone. When these portions of the orbit are discarded, there exist very limited opportunities to carry out necessary maneuvers for increasing the apogee altitude. These opportunities are tried to be increased by lowering the limit of 1000 m/s change to 100 m/s.

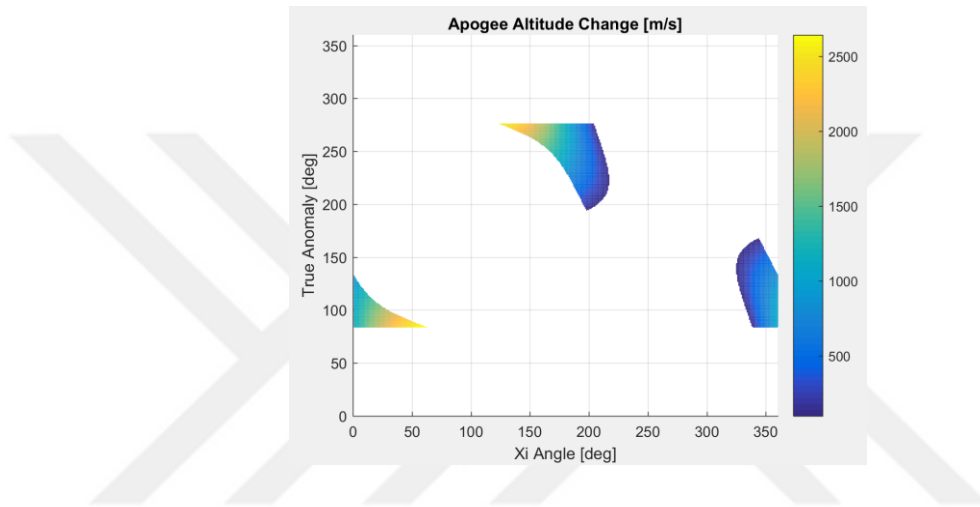


Figure 8.6. Apogee Altitude Change

Case C and Case D: The dynamics for RAAN and argument of perigee are given in Figure 8.7 and Figure 8.8. In here, only out-of-plane force components are used for argument of perigee due to the principles given above.

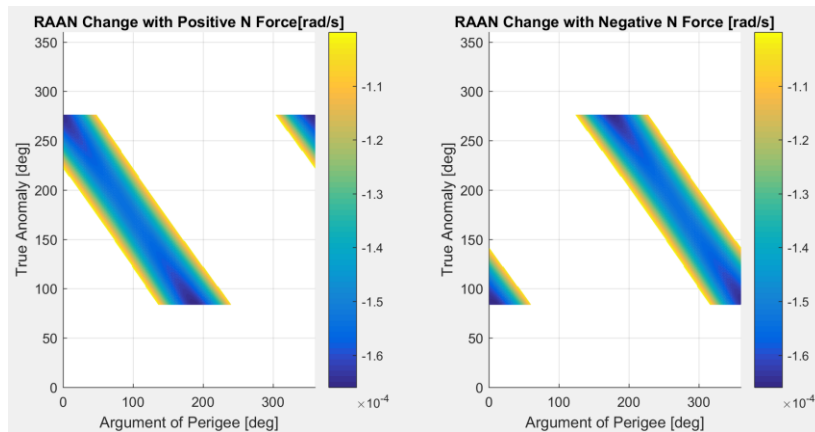


Figure 8.7. RAAN Change

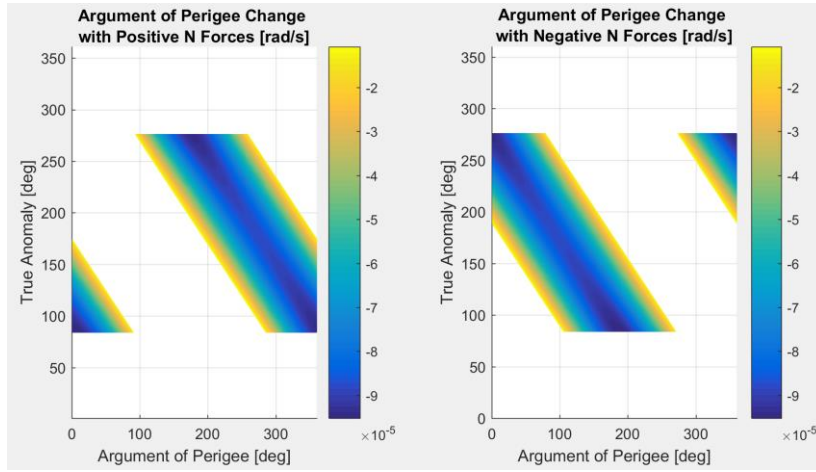


Figure 8.8. Argument of Perigee Change

Case E: This case requests the station-keeping of perigee and apogee altitudes together. Since semi-major axis is the half of the sum of the apogee and perigee radii, the case request is well-met with station-keeping of semi-major axis. The dynamics of the semi-major axis is given in Figure 8.9. The xi angle for optimal change for semi-major axis follows the velocity vector as expected.

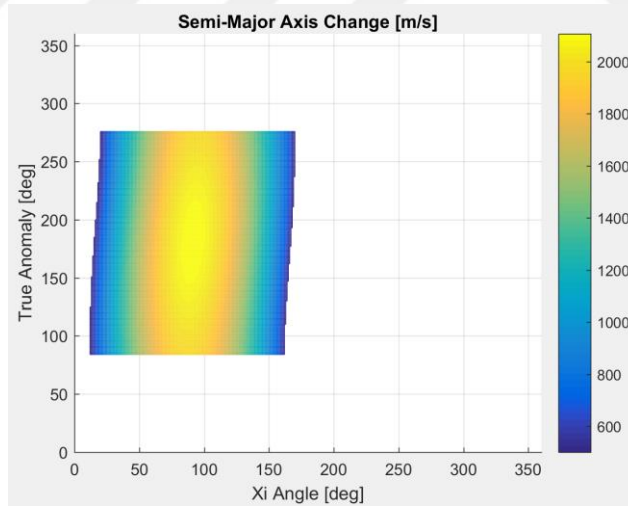


Figure 8.9. Semi-Major Axis Change

Case F: Since the primary cases are covered for individual orbital elements (Case A-E), from this point, combination of these strategies will be seen. Case F is the first example of this. The control force direction is given in Figure 8.10. F force is

determined from Case E and N force is determined from Case C. Then, these directions are combined with weighting parameter which gives more force component to the in-plane forces due to the urgent need for station-keeping of the orbit size as it is described earlier in the prioritization principle among the orbital elements.

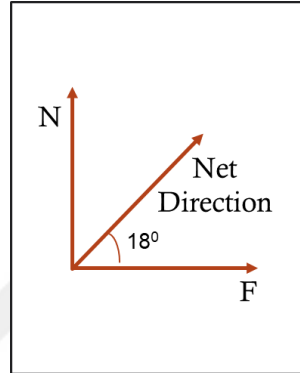


Figure 8.10. Weighting between In-Plane and Out-of-Plane Forces

Case G: In this case, out-of-plane forces are not considered for the argument of perigee, since the portion of the semi-major axis graph affecting the argument of perigee is an order of magnitude larger than the changes resulting from out-of-plane forces. Therefore, that portion in semi-major axis is discarded and in-plane forces are used for argument of perigee as well. Final status is given in Figure 8.11.

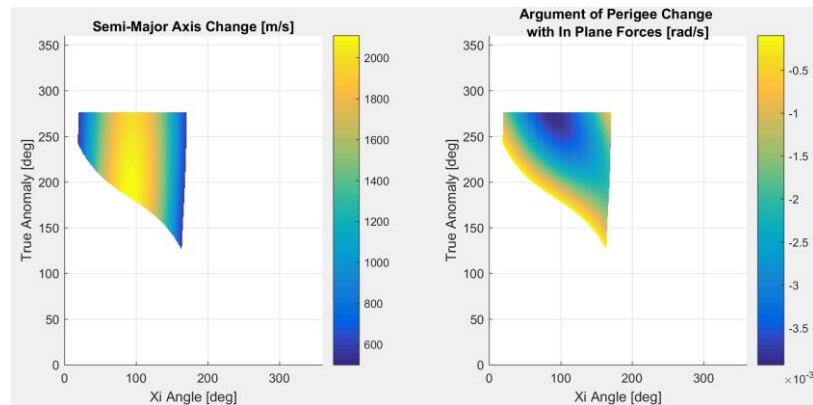


Figure 8.11. Co-Illustration of Semi-Major Axis and Argument of Perigee Changes

Case H: When Equation (156) and Figure 8.3 are analyzed together, the dynamics of argument of perigee and RAAN show that they have the same periodic dynamics with varying magnitude and there is a 90-degree phase shift. This means a particular change for the principles of the R2S2 concept. Hence, RAAN and argument of perigee cannot be controlled simultaneously with only out-of-plane forces. Therefore, in-plane forces are considered for argument of perigee. While applying in-plane forces for argument of perigee, regions, which affect the perigee and apogee altitudes in large amounts, are discarded in order to avoid unnecessary altering of the altitude of the orbit. The resultant graphs for the determination of the control input are given in Figure 8.12.

Case I: The control force direction is given in Figure 8.10. F force is determined from *Case G* and N force is determined from *Case C*.

Case J: The control force direction is given in Figure 8.10. F force is determined from *Case A* and N force is determined from *Case C*.

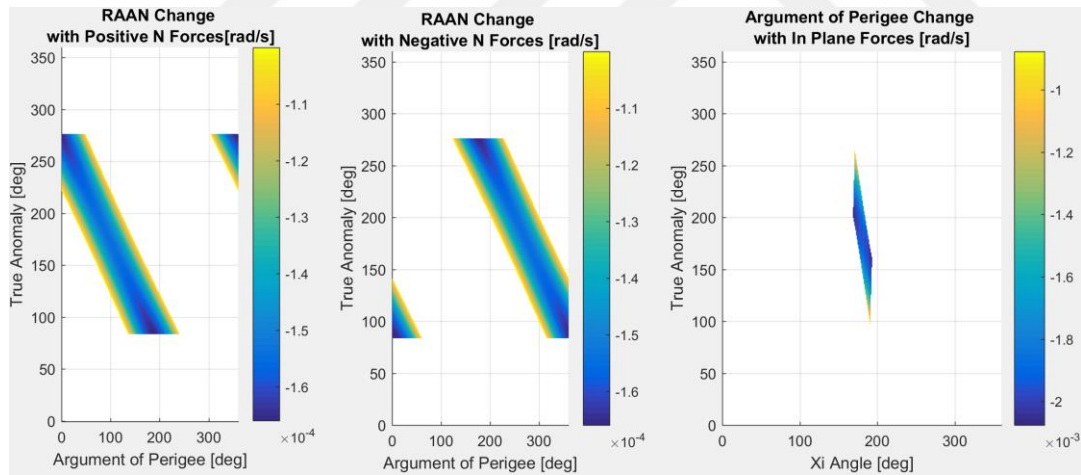


Figure 8.12. RAAN as a function of Argument of Perigee and Argument of Perigee with respect to ξ

Case K: Similar to *Case H*, out-of-plane forces for argument of perigee is out of consideration in *Case K*. In-plane forces are used by considering the coupled dynamics of perigee altitude and argument of perigee. The resultant graph for determination of the control force direction is given in Figure 8.13.

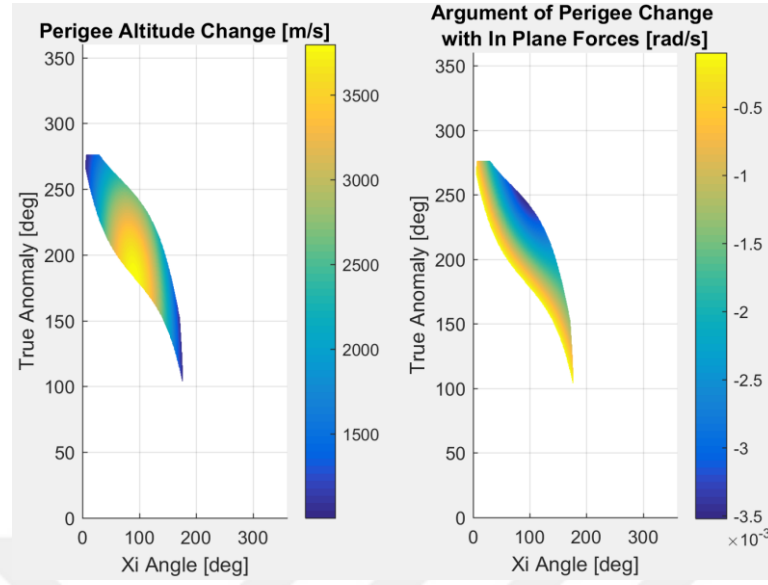


Figure 8.13. Perigee Altitude and Argument of Perigee with respect to ξ

Case L: The control force direction is given in Figure 8.10. F force is determined from *Case B* and N force is determined from *Case C*.

Case M: The control force direction is given in Figure 8.10. F force is determined from *Case B* and N force is determined from *Case D*.

Case N: The control force direction is given in Figure 8.10. F force is determined from *Case K* and N force is determined from *Case C*.

Case O: The control force direction is given in Figure 8.10. F force is determined as given in Figure 8.14 and N force is determined from *Case C*. The graph given in Figure 8.14 is generated by considering the coupled dynamics of apogee altitude and argument of perigee.

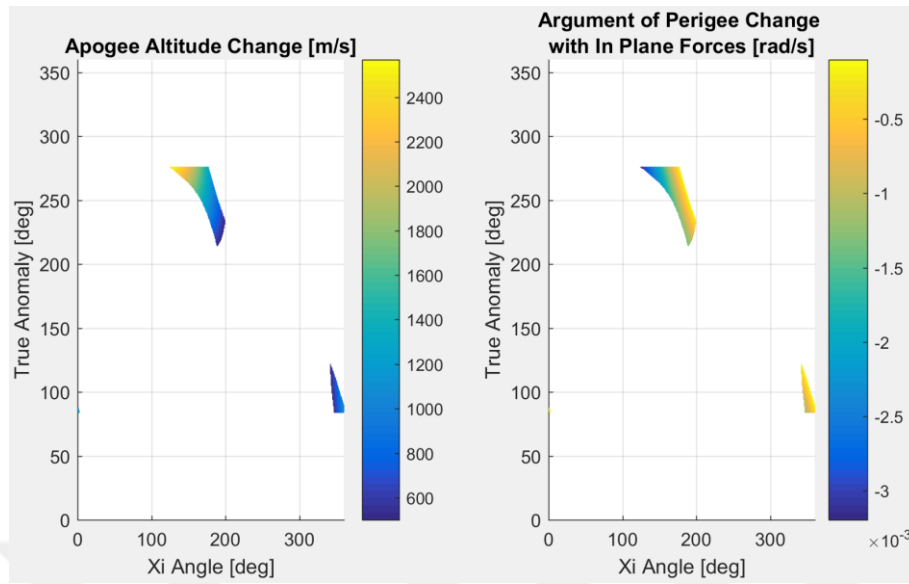


Figure 8.14. Co-Illustration of Apogee Altitude and Argument of Perigee Changes

Case Zero or Attitude Mode=0 Condition: This case is valid when no station-keeping condition is triggered. The sail direction is oriented such that minimum energy loss is obtained.

8.3.1.3 Simulation Results

Finally, an extensive simulation is done. Simulation conditions given in Table 8.3 result in 384 different combinations. Force models for orbit propagation is modelled as listed in Table 8.3. Moreover, the satellite properties used in the simulation are also given in Table 8.3.

Table 8.3 Simulation Setup and Satellite Properties

Simulation Conditions	Number of Different Sets	Force Models	Satellite Properties
Different Case Cycles	4 sets	Earth's Gravity	Sail Mass: 77 kg
Initial Date of Year	4 sets	Earth's Asphericity Effects (up to 4 th Order)	Payload Mass: 37 kg
Inclination	3 sets	Third Body Effects (Sun and Moon)	Sail Area: 40x40 m ²
RAAN	4 sets	Aerodynamic Drag	Payload: 40x40x75 cm
Argument of Perigee	2 sets	Solar Radiation Force	Sail Efficiency = 90%
Total	384 combinations		

The results of the simulation are given in four graphs to demonstrate the aimed goal of station-keeping strategy of the R2S2 concept:

- Case switching status showing the case transitions based on the triggering conditions,
- Attitude mode showing that there is sailing maneuvers or not,
- Perigee altitude trends,
- Orbit altitude trends.

First example resultant graphs are given in Figure 8.15. One can see that successive perigee altitude control between 350 and 400 km. Moreover, apogee altitude drop is diminished.

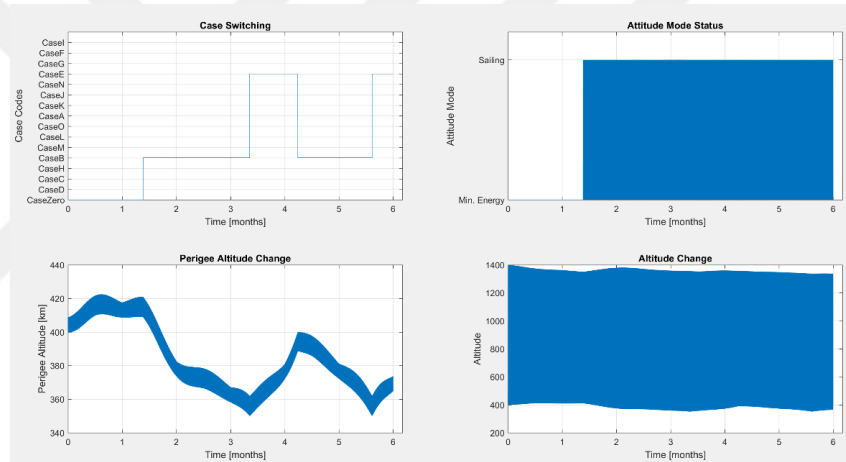


Figure 8.15. An Example of Simulation Results for First Case Cycle

The example in Figure 8.16 demonstrates the continuous station-keeping effort on argument of perigee. The switching up and down of cases implies an effective station-keeping effort. One more thing to notice here is the gaps within the attitude mode graph. These are the results of Sun-Earth-Satellite orientation that provides no valid condition for sailing maneuver. Thus, providing the requested control force direction is geometrically impossible as it was explained in Chapter 7.

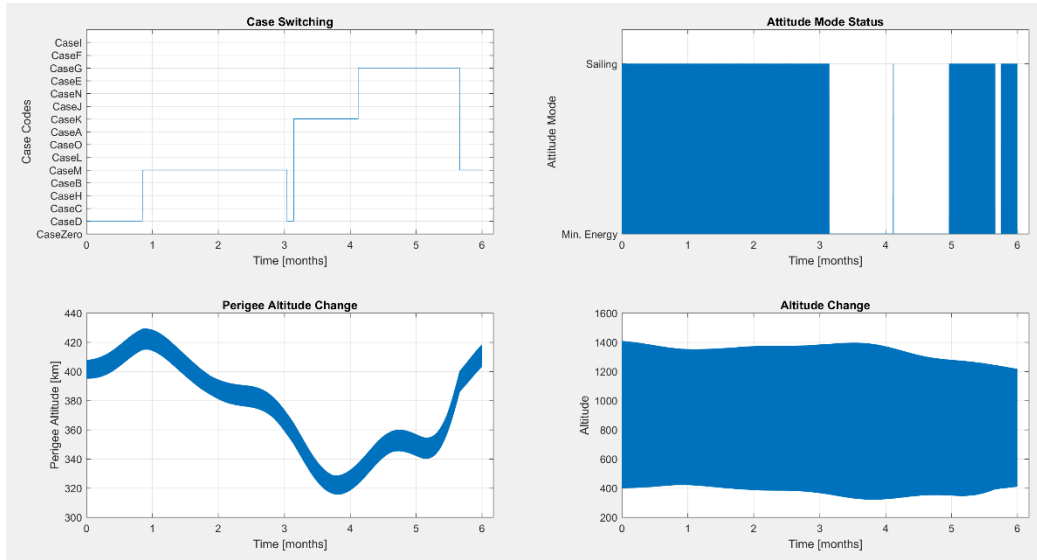


Figure 8.16. An Example of Simulation Results for Second Case Cycle

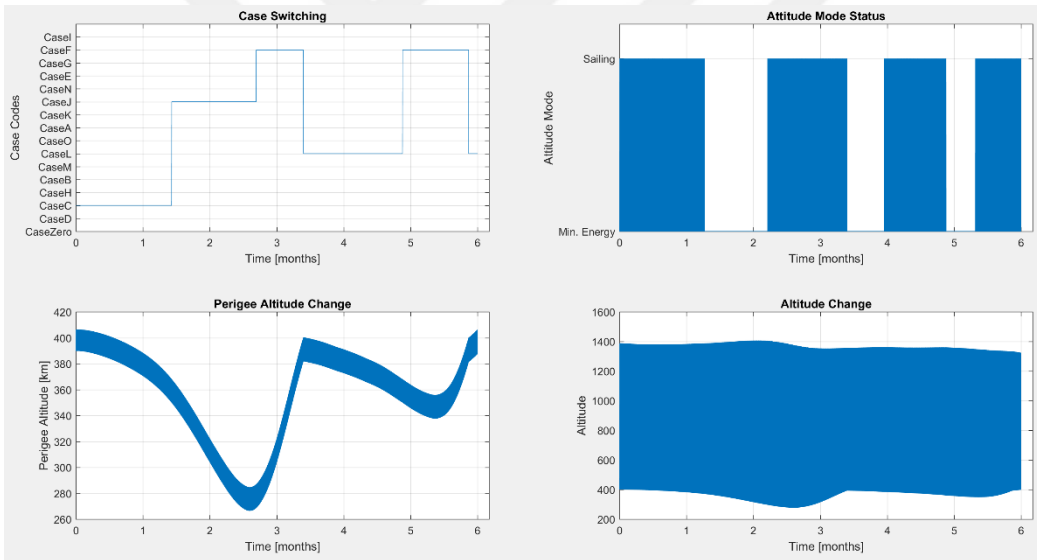


Figure 8.17. An Example of Simulation Results for Third Case Cycle

Similarly, RAAN station-keeping is continuous in Figure 8.17 and successful perigee altitude restoration is achieved even with the gaps within the attitude mode graph.

Last example is from a case with continuous argument of perigee and RAAN station-keeping efforts and results are given in Figure 8.18. Case transitions and restoring of

the perigee and apogee altitudes are the implications of successful station-keeping strategy.

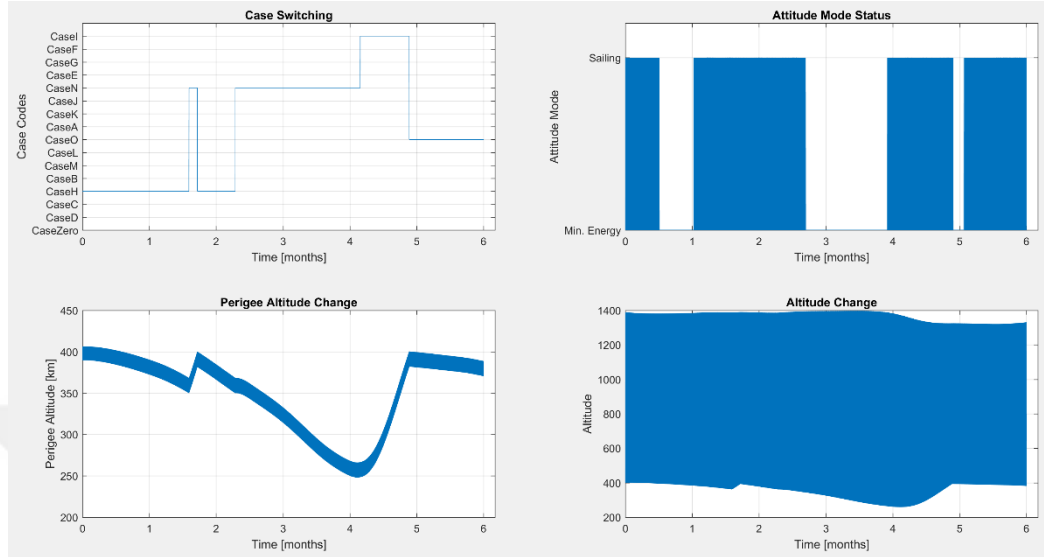


Figure 8.18. An Example of Simulation Results for Fourth Case Cycle

As a result, station-keeping strategy for the mission orbit of the R2S2 concept is provided with mathematical formulizations and simulation results. With four orbital parameters in consideration (perigee and apogee altitudes, argument of perigee and right ascension of ascending node), 16 different station-keeping cases are analyzed and solution to the control force direction determination problem for each case is explained.

The coupled dynamics of orbital parameters are handled with the approximation of modified Gaussian Variational Equations. The original Gaussian Variational Equations are modified to better exploit for the elliptical orbital dynamics and this modification allows us to determine the control force direction under the coupled dynamics of four orbital parameters in consideration.

Moreover, principles for station-keeping strategy of the proposed concept of R2S2 is also respected when dealing with coupled dynamics of orbital parameters. Finally, simulation results show that solar sailing at LEO is adequate for station-keeping purposes. However, for the parameters of argument of perigee and right ascension

of ascending node, the selected sail dimensions and properties do not give high performance for extending the mission lifetime.

8.3.2 Transfer Orbit

Thrust vector control of the sail's normal direction is used to harvest the solar radiation pressure for generating the necessary acceleration in order to alter the orbital elements according to the needs of the defined concept of R2S2. For the transfer orbit, the required control vector direction is determined with two approaches defined in Chapter 8.2.1 and 8.2.2. The first approach is realized by approximating the Gaussian Variational Equations at the employed orbit properties and deriving the optimal force direction in the RTN Frame as a function of true anomaly. The second method is accomplished by implementing a Lyapunov-based nonlinear feedback controller that outputs continuous force direction in the RTN frame. The simulation results for the orbit control methods are provided and remarks for Solar Sail Satellite capability for altering the orbital elements at LEO are stated. It is demonstrated that both methods give satisfactory results.

8.3.2.1 Simulation Results

The simulation results for the both approaches described in the previous section are provided in terms of the desired and achieved outputs for specific orbital elements. The simulation conditions and baseline satellite properties are provided in Table 8.4.

Table 8.4 Simulation Conditions and Baseline Satellite Properties

Force Models in Effect	Satellite Property	Value/Parameter
Earth's Two Body Gravitational Attraction	Sail Mass	77 kg
Earth's Asphericity Effects (J2 Only)	Payload Mass	82 kg
Aerodynamic Drag	Sail Area	40x40=1600 m ²
Solar Radiation Force	Payload Dimensions	0.8x0.8x1.5 m ³

The simulation runs are performed for both approaches by performing four different modes of orbital transfer maneuvers. The first orbital transfer maneuver simulation is carried out from mission orbit to standby orbit by raising the perigee altitude. The second orbital transfer maneuver simulation shows how the standby orbit's perigee is decreased to mission orbit's perigee. These transfer modes require in-plane maneuvers. The controls required for these transfers are also presented. However, the applicable controls are different than the control demands due to the availability of solar radiation pressure both in magnitude and direction. Consequently the applied solar radiation pressures are also presented in the figures. The third orbital transfer maneuver simulation is carried out to change the inclination at the standby orbit. The last orbital transfer maneuver simulation focuses on the controllability of the RAAN and argument of perigee values under the influence of the J2 effects.

Figure 8.19, Figure 8.20, Figure 8.21, Figure 8.22 and Figure 8.23 belong to the approximated-GVE derived control as the first approach.

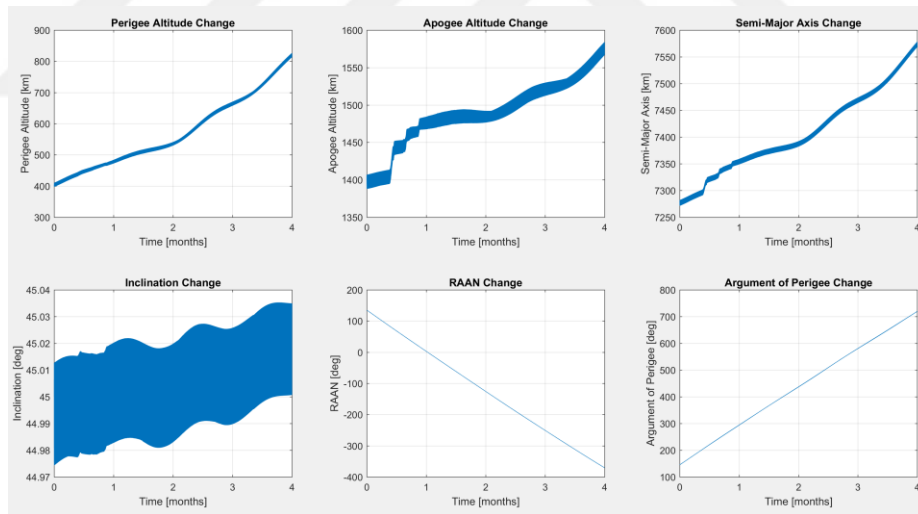


Figure 8.19. Time History of the Orbital Elements During Perigee Altitude Raise Maneuver

As it is observed from Figure 8.19, the control over the perigee altitude results in a change of 100 km per month. However, the first approach does not take the coupled dynamics of the orbital elements into account. This is observed in the apogee altitude

deviation from the 1400 km, which will not be the case in the second approach. In Figure 8.20, the achieved control levels are illustrated with the demanded control direction.

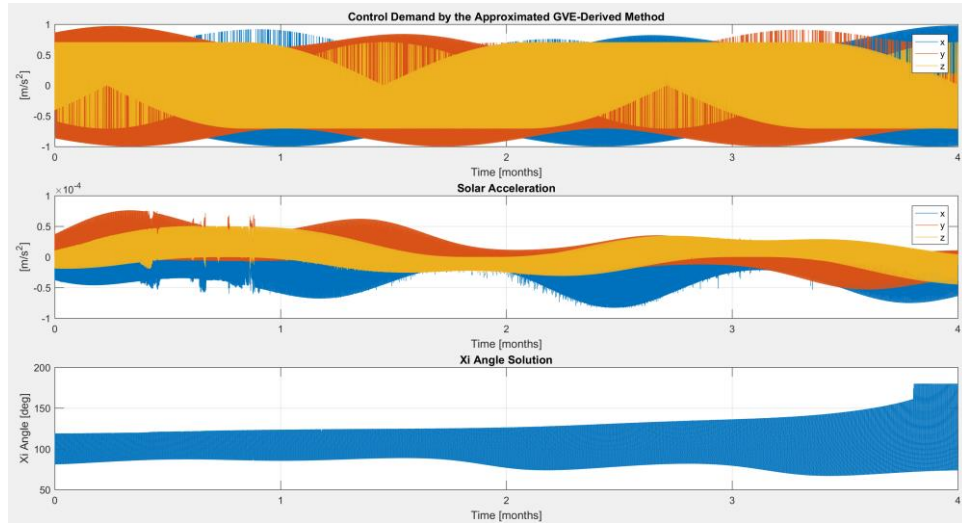


Figure 8.20. Control Demand History and Solar Acceleration History During Perigee Altitude Raise Maneuver

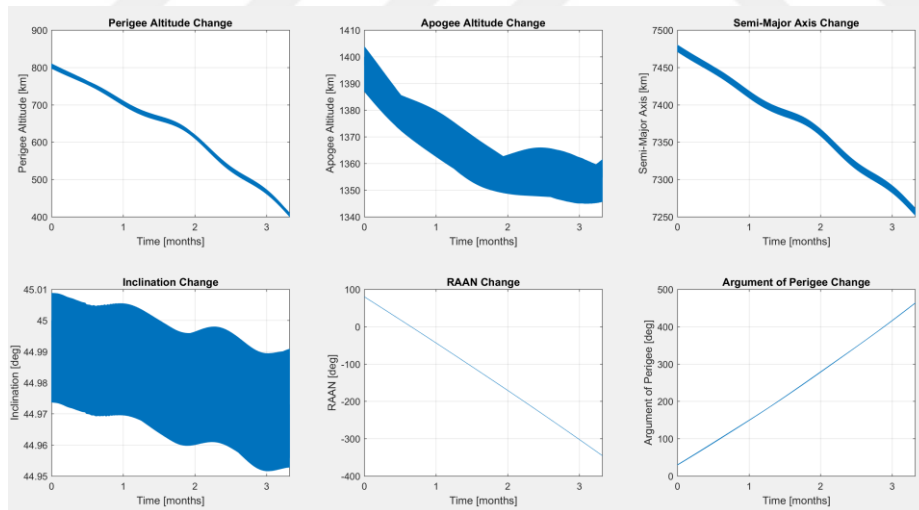


Figure 8.21. Time History of the Orbital Elements During Perigee Altitude Decrease Maneuver

The orbital element trends for the second orbital maneuver transfer simulation are given in Figure 8.21. The control over the perigee altitude is improved in the

decreasing maneuver to the level of 125 km per month (a 25% raise relative to the increasing maneuver). The behavior of the apogee altitude is similar with the rationale explained for Figure 8.19. In Figure 8.22, the achieved control levels are demonstrated.

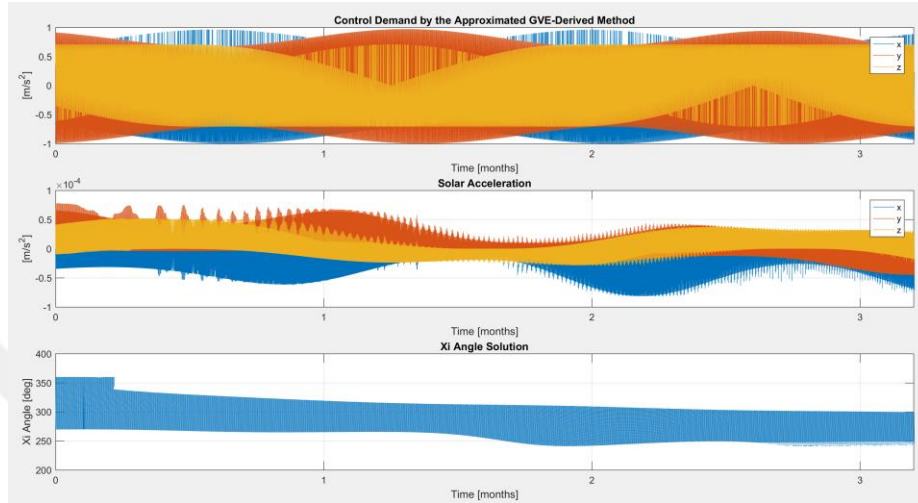


Figure 8.22. Control Demand History and Solar Acceleration History During Perigee Altitude Decrease Maneuver

The third orbital transfer maneuver simulation regarding the inclination change at the standby orbit is performed and the results are shown in Figure 8.23. The control over inclination is at the level of 0.5 deg for the duration of simulation of 4 months. Since only out-of-plane forces are used for control purposes, the changes in the perigee and apogee altitudes stay at the minimal values.

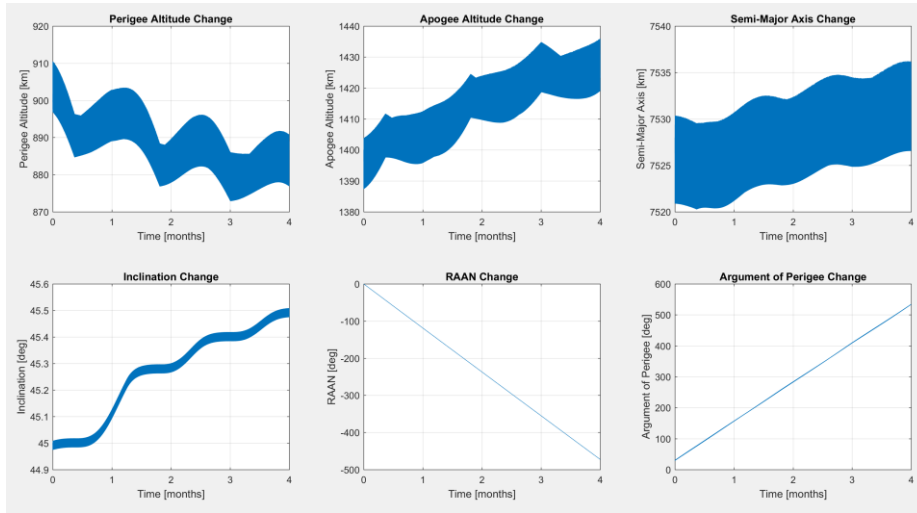


Figure 8.23. Time History of the Orbital Elements During Inclination Change Maneuver

The orbit control using the second approach, namely the Lyapunov-based feedback controller, is also carried out as mentioned earlier. The change in orbital elements during the control to raise and decrease the perigee altitude are presented in Figure 8.24 and Figure 8.26, respectively. The demanded and realized solar radiation accelerations are also presented in Figure 8.25 and Figure 8.27. Figure 8.28, on the other hand, presents the control to change the inclination at the standby orbit.

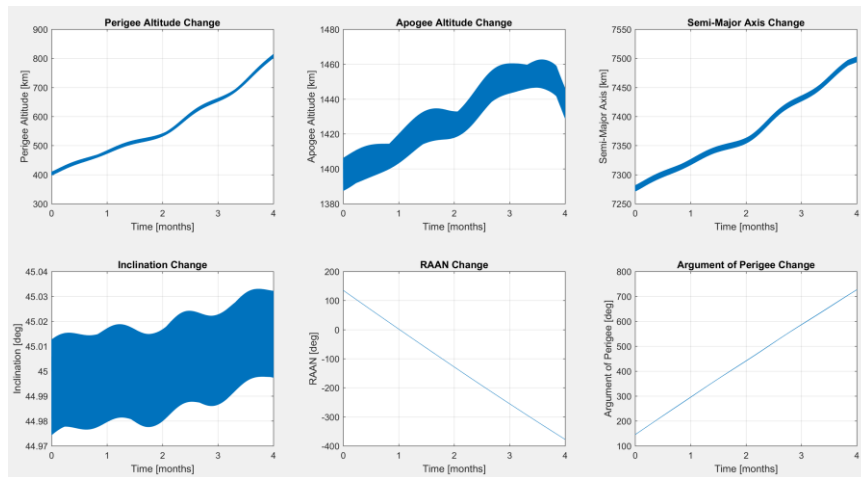


Figure 8.24. Time History of the Orbital Elements During Perigee Altitude Raise Maneuver

The control over the perigee altitude at the level of 100 km per month is equal to the performance of the first approach. Please note the control authority over the apogee altitude. The coupled dynamics are taken into consideration unlike the first approach. The second approach then is more successful by keeping the apogee altitude changes in minimal values. In Figure 8.25, the achieved control levels are illustrated with the demanded control direction. The second approach is not only giving the control direction, it also gives the magnitude.

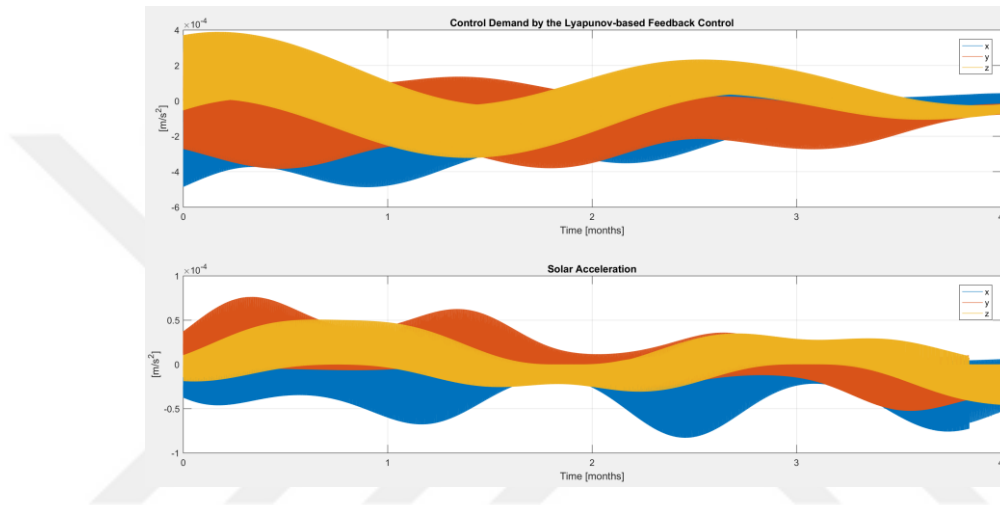


Figure 8.25. Control Demand History and Solar Acceleration History During Perigee Altitude Raise Maneuver

By properly selecting the gain matrix values, the control demand magnitudes given in Figure 8.25 are made proportional to the solar acceleration output magnitudes as mentioned earlier in Chapter 8.2.2.

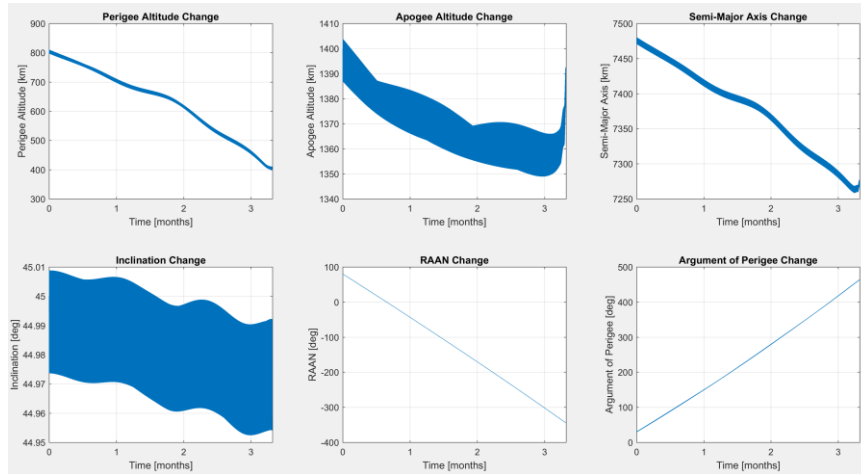


Figure 8.26. Time History of the Orbital Elements During Perigee Altitude Decrease Maneuver

The control over the perigee altitude during the second orbital transfer maneuver simulation stays at the same level of 125 km per month as the first approach. However, the station-keeping of the apogee altitude during a large change of perigee altitude is still an important factor regarding the second approach. In Figure 8.27, the achieved control levels are illustrated with the demanded control direction.

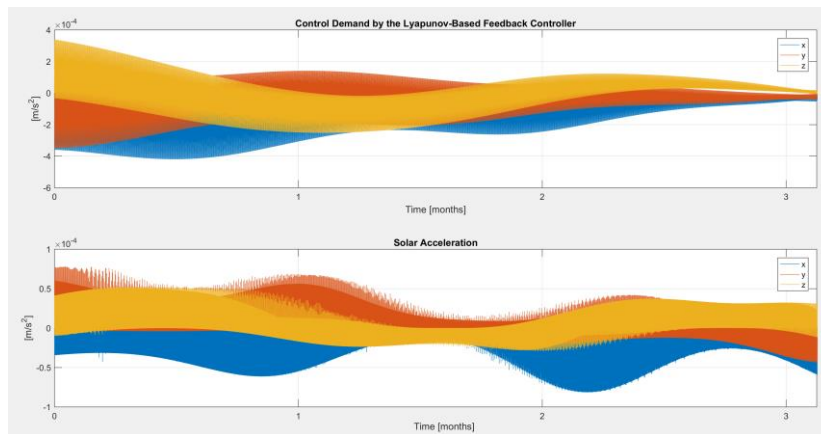


Figure 8.27. Control Demand History and Solar Acceleration History During Perigee Altitude Decrease Maneuver

The third orbital transfer maneuver simulation regarding the inclination change at the standby orbit is performed for the second approach and the results are shown in Figure 8.28. The control over inclination is at the level of 0.5 deg for the duration of

simulation of 4 months as it was in the first approach results. The changes in the perigee and apogee altitudes stay at the minimal values as out-of-plane maneuvers are utilized for inclination change.

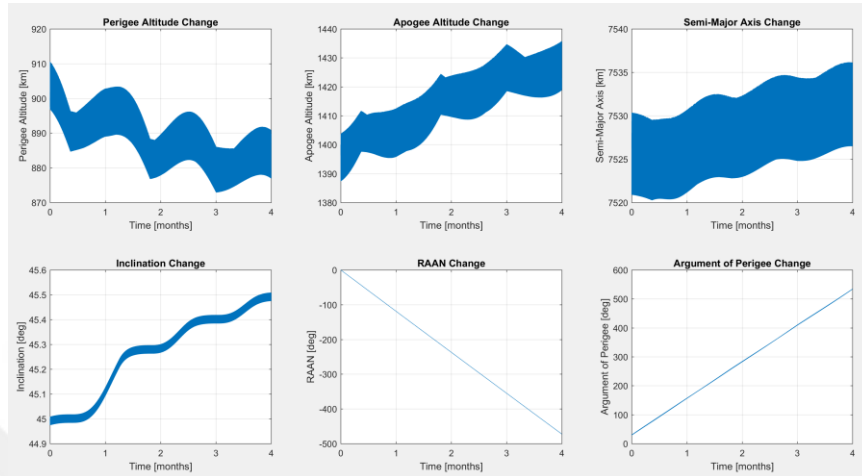


Figure 8.28. Time History of the Orbital Elements During Inclination Change Maneuver

Regarding the controllability of the RAAN and argument of perigee values under the influence of the J_2 effects, the results are same for both control methods and they are provided in Figure 8.29.

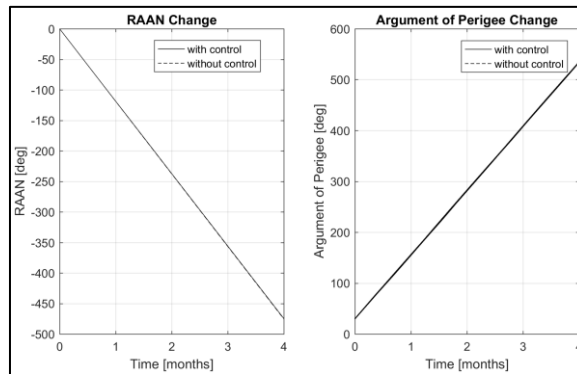


Figure 8.29. Argument of Perigee and RAAN Change Trends with Out-of-Plane Forces Only

However, the changes desired for the argument of perigee and RAAN could not be achieved. Continuous J_2 effects (which are two orders of magnitude larger than solar

accelerations) do not allow us to fix the RAAN or argument of perigee values. Even the deviation from the trend of these values under J2 effects (Figure 8.29) are not applicable with control force as solar radiation. In the option of using in-plane force components for argument of perigee control, the deviation from J2 trends are applicable in the expense of changing perigee and apogee altitudes, which is not desired in the R2S2 concept. This was not the case for inclination as J2 effects of the Earth has only periodic perturbations to inclination, thus allowing us to add secular drift with solar acceleration.

8.3.2.2 Discussion

First of all, both approaches give similar results since the main concentration is on the decoupled dynamics of the orbital elements. When the apogee altitude trends of both methods are compared, the difference about the coupled effects can be seen. In this regard, Lyapunov-based control method (second approach) is more successful. Nevertheless, closed- (Lyapunov-based) and open- (approximated-GVE derived) loop control approaches are applicable for the maneuvers of the defined R2S2 concept's needs. Both of them give adequate direction information (magnitude also in the case for Lyapunov-based control method) to implement the thrust vector control for harvesting the solar radiation pressure.

To summarize the results, the performance metrics can be listed as 4 months to raise the perigee altitude of 400 km to 800 km (Figure 8.19 and Figure 8.24), 3.2 months to decrease from 800 km to 400 km (Figure 8.21 and Figure 8.26) and 0.5 degrees of inclination change in 4 months (Figure 8.23 and Figure 8.28). However, the changes desired for the argument of perigee and RAAN could not be achieved as mentioned earlier and observed in Figure 8.29.

These maneuver performances (even for the argument of perigee and RAAN) can be increased by either reducing the payload mass (mission satellite), increasing the sail area or reducing the sail loading (total mass per sail area). The miniaturization

technology for the satellites and the improvements on the super-light materials will obviously increase the performance results of this study.





CHAPTER 9

ATTITUDE CONTROL STRATEGY

Attitude control strategy for a Solar Sail Satellite with a large moment of inertia is studied with high slewing performance required by the concept of operation at LEO. The attitude maneuvers between directions of solar sailing and minimum drag modes are simulated under the disturbance torques of aerodynamic drag, solar radiation pressure and gravity gradient. Simulation results showed that the high slewing performance needs of the defined concept of operation at LEO can be met by the employment of high momentum-high torque reaction wheels.

For large angle slew maneuvers, quaternion error feedback is the most common adopted methodology [12]. Even though quaternion error feedback controller provides the direct solution for generating eigenaxis rotation of large angles, the resultant trajectory and applied angular torque levels might not result in minimum energy rotations.

Smooth attitude control plays an important factor when the focus is rotating a large and low-weight structure such as solar sail satellite. A solar sail satellite is quite flexible that requires low-jerk maneuvers to avoid structural coupling. In order to avoid exciting vibration modes of flexible solar sail satellite structure, angular maneuvers should follow a trajectory that ensures low-jerk and smooth attitude reorientation.

The detailed mathematical formulations of the necessary orbital maneuvers and their resultant thrust vector control directions are given in Chapter 8 and the attitude limitations of the R2S2 concept are given in Chapter 4.5.

The list of principles for the attitude control for the R2S2 Concept can be summarized below:

- Below the mode switching altitude (<800 km), the desired attitude of the sail normal is along $-\left(\hat{h} \times \hat{V}_{sat}\right)$,
- Above the mode switching altitude (≥ 800 km), the desired attitude of the sail normal is along the direction that is the solution of the thrust vector control problem,
- There should be enough allocated time to reach the minimum drag mode attitude before 800 km of altitude due to long slewing times, hence the eigenaxis rotation angle between the current attitude and minimum drag mode attitude is always tracked and the time to start the slewing maneuver to the minimum drag mode attitude is determined by solving the Kepler's Problem,
- The relative position of the Sun is always tracked and the satellite is oriented along the solution of the thrust vector control problem if only the angle between desired direction of the sail's normal and sunlight direction below 90 degrees,
- Finally, the line-of-sight between the Sun and Solar Sail Satellite should be achieved (not shadowed by Earth).

These principles are implemented into the attitude control algorithm by rules that enables switches between sailing and minimum drag modes.

In the following sections, attitude control methodology buildup will be provided with simulation results and comparison of slewing performance of each control approach.

9.1 Nonlinear Quaternion Feedback Control Implementation

Based on the results in Chapter 5.5 about the design iteration for the determination of the baseline satellite properties in regard to attitude control considerations, a nonlinear quaternion feedback controller is implemented. The relevant methodology is provided by [12]. This methodology, commonly used as the benchmark of

nonlinear quaternion feedback controllers, uses the control input based on the PD state-feedback control by considering the gyroscopic term:

$$\vec{u} = k\hat{J}\vec{t} - c\hat{J}\vec{\omega}_{B/N} + \vec{\omega}_{B/N} \times \hat{J}\vec{\omega}_{B/N} \quad (162)$$

where $\vec{t}$ is the to-go quaternion (conjugate of the error quaternion), k and c are properly determined positive scalar constants that assures globally asymptotic stability, $\hat{J}$ is the inertia matrix, and $\vec{\omega}_{B/N}$ is the relative angular velocity of Body frame with respect to the Inertial frame. Positive scalar constants, k and c , will be determined by $2\omega_n^2$ and $2\xi_{damping}\omega_n$, respectively as in [12].

The attitude maneuver simulations for the nonlinear quaternion feedback controller based on the Equation (162) use the baseline satellite properties given in Table 5.4 and orbital parameters given in Table 9.1. As it is observed in Table 9.1, both in-plane and out-of-plane maneuvers' attitude control simulations are used.

Table 9.1 Simulation Orbital Parameters

For In-Plane Maneuvers		For Out-of-Plane Maneuvers	
Parameters	Values	Parameters	Values
Perigee Altitude	400 km	Perigee Altitude	900 km
Apogee Altitude	1400 km	Apogee Altitude	1400 km
Inclination	45 ⁰	Inclination	45 ⁰
RAAN	135 ⁰	RAAN	0 ⁰
Argument of Perigee	145 ⁰	Argument of Perigee	30 ⁰

For in-plane maneuvers, the target direction is rotating in the Inertial Frame at a similar rate with velocity vector. On the other hand, out-of-plane maneuvers require the thrust vector direction along the orbit normal (orbital angular momentum vector direction) which is pseudo-fixed in the Inertial Frame. This is allowing us to test the capability of the attitude control methodology with various conditions of thrust vector direction.

For the in-plane maneuvers, the perigee altitude is aimed to be raised. The thrust vector control solution, which maximizes the rate of the perigee value, is the

direction that is tracked by the sail's normal. On the other hand, the inclination is aimed to be raised from the initial inclination value for the out-of-plane maneuvers. The thrust vector solution is along or opposite of the orbit normal direction depending on the orbital position of the satellite. All of the simulations start with zero displacement angle and zero angular velocity between Target and Body Frames. Furthermore, aerodynamic drag, solar radiation pressure and gravity gradient effects as disturbance torques are taken into consideration. The simulations last for 4 hours which is more than two orbit revolutions.

The following figures show the outputs of the simulations runs for the nonlinear quaternion feedback controller based on Equation (162).

As may be observed from Figure 9.1, two large angle rotations per one orbit revolution has been accomplished with a slewing time performance under 300 seconds per 90 degrees of rotation.

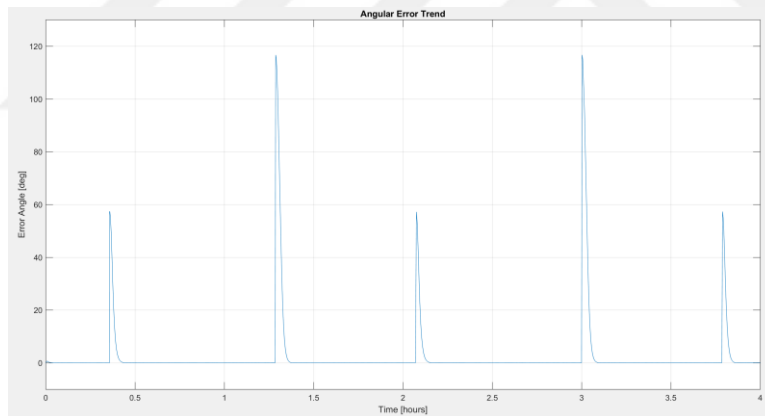


Figure 9.1. Angular Error Trend during In-Plane Maneuvers

However, the step-wise introduction of the error to the control logic results in calculations of high control torque solutions. The instant change of the desired thrust vector control direction at the mode switching points in the orbit causes the spikes in the angular error in tracking (Figure 9.1). This behavior is also observable in difference between current and desired attitude information (Figure 9.2). Moreover,

this step-wise introduction of high angular error to the control logic increases the possibility of saturation of the wheels and high angular velocity values (Figure 9.3).

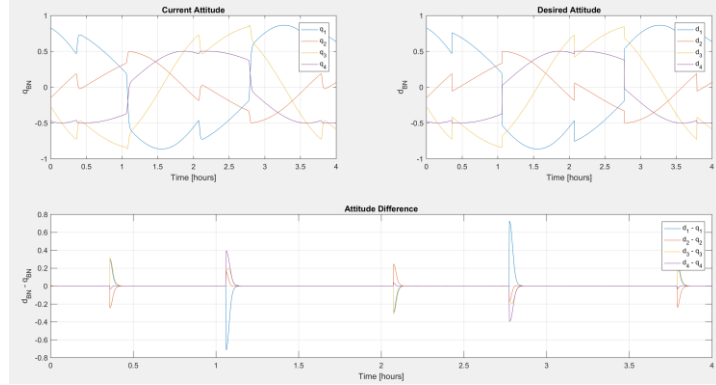


Figure 9.2. Desired and Current State of the Attitude during In-Plane Maneuvers

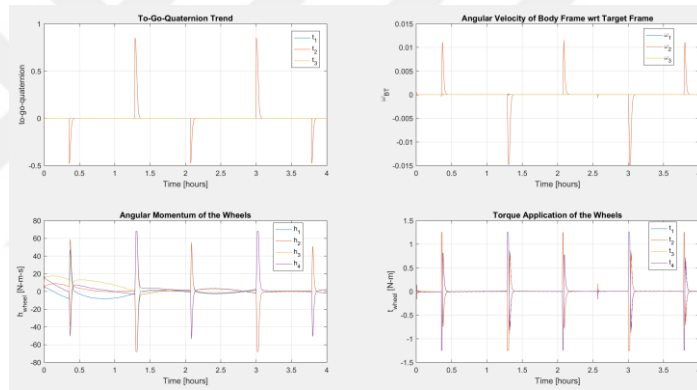


Figure 9.3. Attitude Control Performance during In-Plane Maneuvers

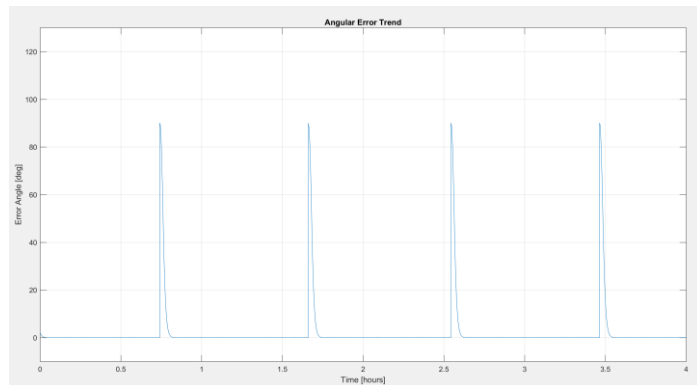


Figure 9.4. Angular Error Trend during Out-of-Plane Maneuvers

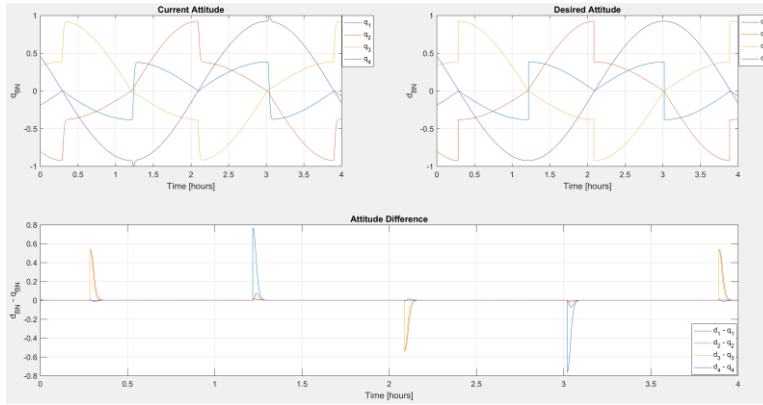


Figure 9.5. Desired and Current State of the Attitude during Out-of-Plane Maneuvers

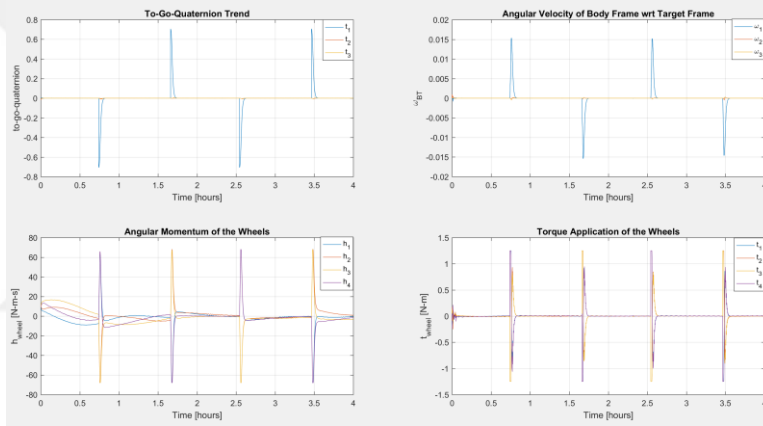


Figure 9.6. Attitude Control Performance during Out-of-Plane Maneuvers

Similar results are obtained for out-of-plane maneuvers as shown in Figure 9.4, Figure 9.5 and Figure 9.6. The nonlinear quaternion feedback controller based on Equation (162) are meeting the needs of the R2S2 concept in terms of overall slewing time performance and tracking accuracy.

When we analyze more of the attitude maneuvers (Figure 9.1 and Figure 9.4), two large angle attitude maneuvers for each orbit are present as expected. Regarding the in-plane maneuvers, the first maneuver (minimum drag mode to sailing mode) is initiated at crossing up the 800 km altitude with appropriate sunlight direction, the second maneuver is initiated at necessary time (which is the required slewing time

to acquire the minimum drag mode attitude) before crossing down the 800 km altitude, and this is periodical with only one varying parameter of sunlight direction affecting the switching mode attitude initialization point in the orbit other than the altitude itself. Same analysis can be made for the out-of-plane maneuvers, but this time mode switching is only initiated by the sunlight direction since the altitude is always greater than 800 km at all points within the orbit.

However, Equation (162) provides a control input without any consideration on the desired attitude trend. This might cause some problems on systems susceptible to high-jerk maneuvers due to their flexible structures. Solar Sail Satellites are the perfect example for such disadvantage on maneuvering performance. Therefore, in the following section, modified version of Equation (162) is implemented such that smooth and low-jerk slewing of the Solar Sail Satellite is achieved by avoiding possible excitement of sail structure with relatively high accuracy on the tracking performance.

9.2 Smooth Attitude Control Realization

This second approach [48] modifies the Equation (162) by taking the time dependent trajectory of the desired attitude quaternion into account to assure low level of tracking attitude error and to enable a smooth and low-jerk eigenaxis rotation. The modified version of Equation (162) is as follows [48]:

$$\vec{u} = k\hat{J}\vec{t} - c\hat{J}\vec{\omega}_{B/N} + \vec{\omega}_{B/N} \times \hat{J}\vec{\omega}_{B/N} + 2(c\hat{J}\vec{s} + \hat{J}\dot{\vec{s}}) \quad (163)$$

where $\vec{s}$ is the parameterization of the desired attitude quaternion $\{\vec{d}; d_4\}$ and time rate of changes of each quaternion parameter $\{\dot{d}_1; \dot{d}_2; \dot{d}_3; \dot{d}_4\}$, since the derivative of the to-go quaternion may be written as:

$$\begin{Bmatrix} \dot{\vec{t}} \\ \dot{t}_4 \end{Bmatrix} = \begin{bmatrix} -\vec{s}^\times & \vec{s} \\ -\vec{s}^T & 0 \end{bmatrix} + \hat{I}S_4 + \frac{1}{2} \begin{bmatrix} -\vec{\omega}_{B/N}^\times & -\vec{\omega}_{B/N} \\ \vec{\omega}_{B/N}^T & 0 \end{bmatrix} \begin{Bmatrix} \vec{t} \\ t_4 \end{Bmatrix} \quad (164)$$

where $(\)^x$ indicates the cross-product matrix, $\hat{I}$ is the identity matrix and $\mathbf{s}$ is calculated as below:

$$\mathbf{s} = \begin{Bmatrix} \dot{d}_1 d_4 + \dot{d}_2 d_3 - \dot{d}_3 d_2 - \dot{d}_4 d_1 \\ -\dot{d}_1 d_3 + \dot{d}_2 d_4 + \dot{d}_3 d_1 - \dot{d}_4 d_2 \\ \dot{d}_1 d_2 - \dot{d}_2 d_1 + \dot{d}_3 d_4 - \dot{d}_4 d_3 \end{Bmatrix} \quad (165)$$

$$s_4 = \dot{d}_1 d_1 + \dot{d}_2 d_2 - \dot{d}_3 d_3 - \dot{d}_4 d_4 \quad (166)$$

where s_4 is zero. The relation for the to-go quaternion for the desired attitude is also given below:

$$\begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{Bmatrix} = \begin{bmatrix} -d_4 & -d_3 & d_2 & d_1 \\ d_3 & -d_4 & -d_1 & d_2 \\ -d_2 & d_1 & -d_4 & d_3 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}_{Current} \quad (167)$$

The control input given in Equation (163) makes the system asymptotically stable with the following choice of Lyapunov candidate function. Detailed prove for the asymptotic stability of the rotational motion of the satellite is given in [48].

$$V(\mathbf{s}, \vec{\omega}_{B/N}, (1-t_4)) = \frac{1}{2} (-2\mathbf{s} + \vec{\omega}_{B/N})^T \frac{1}{k} (-2\mathbf{s} + \vec{\omega}_{B/N}) + 2(1-t_4) \quad (168)$$

The proposed method in Equation (163) requires the information of the desired attitude quaternion as a function of time. This information is obtained for each orbit by approximating the Gaussian Variational Equations and finding the attitude orientation required by the R2S2 concept. The derivation of the thrust vector problem solution based on the approximated version of the Gaussian Variational Equations can be found Chapter 8. Then, the first and second derivatives of the derived desired attitude information are obtained numerically and used in Equation (163). Following figures are then generated based on the same simulation conditions and satellite properties given in Table 9.1 and Table 5.4, respectively.

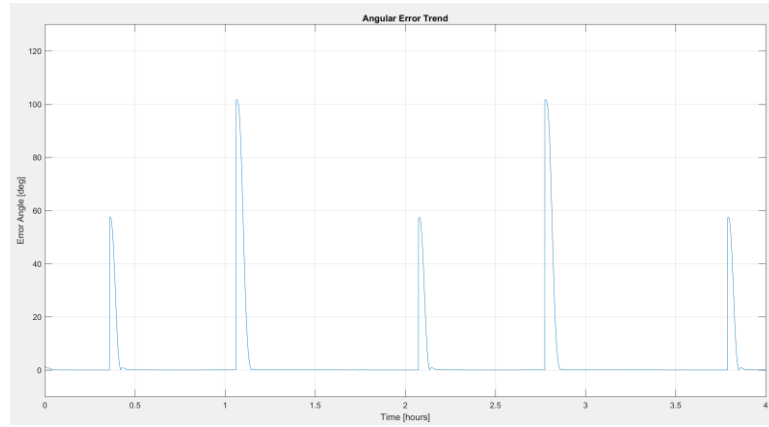


Figure 9.7. Angular Error Trend during In-Plane Maneuvers with Time-Dependent Attitude Control Logic

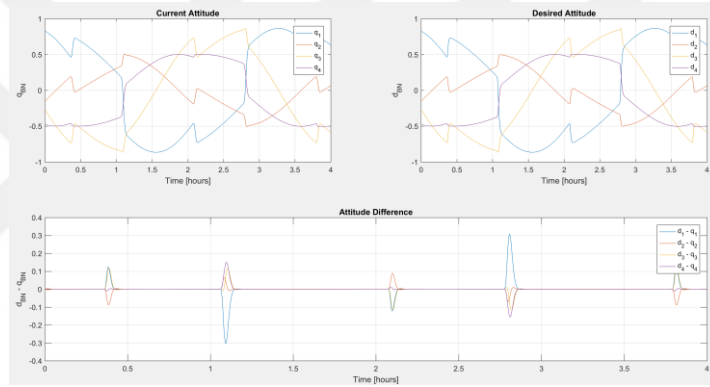


Figure 9.8. Desired and Current State of the Attitude during In-Plane Maneuvers with Time-Dependent Attitude Control Logic

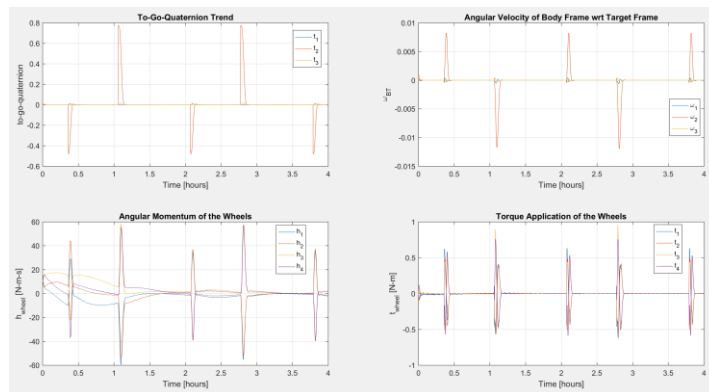


Figure 9.9. Attitude Control Performance during In-Plane Maneuvers with Time-Dependent Attitude Control Logic

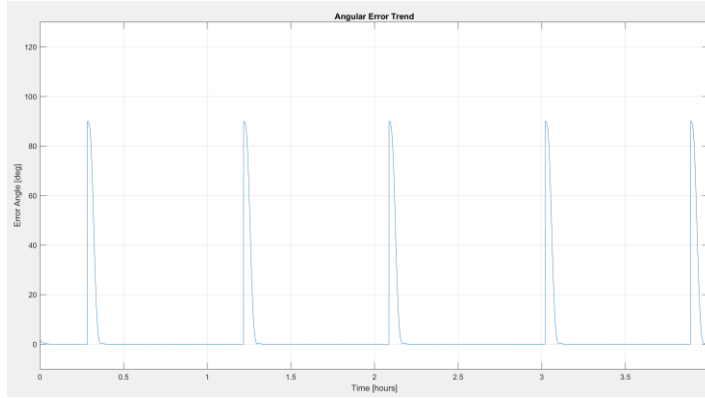


Figure 9.10. Angular Error Trend during Out-Of-Plane Maneuvers with Time-Dependent Attitude Control Logic

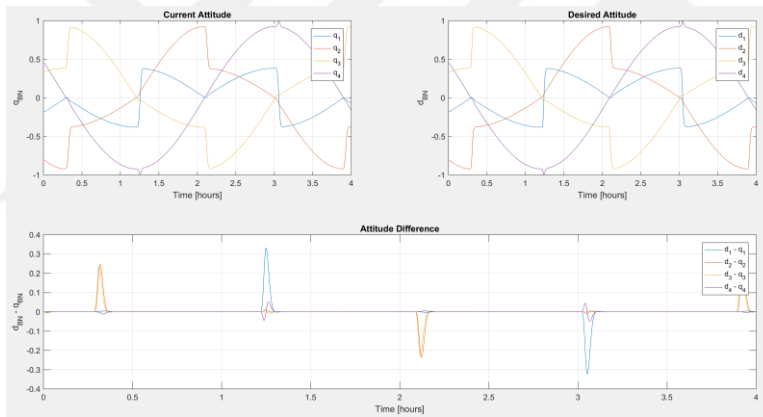


Figure 9.11. Desired and Current State of the Attitude during Out-Of-Plane Maneuvers with Time-Dependent Attitude Control Logic

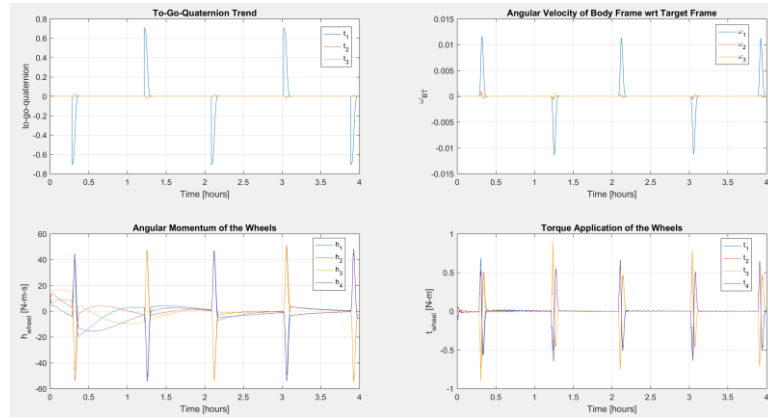


Figure 9.12. Attitude Control Performance during Out-Of-Plane Maneuvers with Time-Dependent Attitude Control Logic

The attitude control strategy based on the implementation of both nonlinear quaternion feedback control schemes, Equation (162) and Equation (163) are successful to meet the needs of the R2S2 concept. This can be seen in Figure 9.1 for thrust vector directions at in-plane maneuvers and in Figure 9.4 for thrust vector directions at out-of-plane maneuvers for the nonlinear quaternion feedback controller based on Equation (162). For the modified version of the nonlinear quaternion feedback controller taking the time-dependent desired attitude information into account, attitude maneuver history for in-plane and out-of-plane maneuvers can be found in Figure 9.7 and Figure 9.10, respectively. Similar slewing times are experienced in both attitude control approach types. Moreover, zeroing of *to-go-quaternion*, *angular velocity* of Body frame with respect to Target frame and the *angular error* between sail's normal and thrust vector control direction are all reached in both attitude control approach types with same positive scalar constants of k as 0.0018 and c as 0.054 (Figure 9.1, Figure 9.3, Figure 9.4, Figure 9.6, Figure 9.7, Figure 9.9, Figure 9.10 and Figure 9.12). The tracking capabilities of both attitude control approach types are shown in Figure 9.2, Figure 9.5, Figure 9.8 and Figure 9.11.

As explained above, the tracking of the desired attitude within the acceptable accuracy limitations is well achieved by both attitude control approach types under

the conditions and requirements of in- and out-of-plane maneuvers. The critical difference between both attitude control approaches can be observed by comparing Figure 9.3 and Figure 9.9 for in-plane maneuvers, Figure 9.6 and Figure 9.12 for out-of-plane maneuvers. The angular momentum and torque capacities of the wheels are used at their limitations with nonlinear quaternion feedback control law based on Equation (162) as may be seen in Figure 9.3 and Figure 9.6. This results in relatively high values of angular velocity of the satellite system at the maximum levels of 0.01-0.015 rad/s. The use of desired attitude trend within the control law as in Equation (163) enables a smoother slewing maneuver and results in an approximately 50% drop in the reaction wheel torque values and 10-35% decrease in angular momentum values of the reaction wheels (Figure 9.9 and Figure 9.12). Hence, the angular velocities of the satellite system stays at relatively lower values of 0.008-0.001 rad/s.

However, the numerical derivation of the time-dependent desired attitude trajectory is not adequate for reaching better tracking accuracies and smoother attitude re-orientation due to the step-wise introduction of high angular error values to the control logic. Thus, an attitude trajectory planning is carried out. Let's assume that initial attitude condition is such that the x -axis of body frame in Figure 3.2 is aligned with velocity vector, y -axis is aligned with the opposite direction of the orbit normal, and finally z -axis is determined from the right hand's rule. As mentioned earlier, the employed elliptical orbit is divided into two attitude modes: minimum drag and sailing. During the minimum drag mode, the initial attitude alignment is maintained by tracking the velocity vector. Within the sailing mode, the sail's normal ($-z$ -axis) is oriented towards the velocity vector and kept in such orientation (maximum orbital energy increase is aimed in this case). When the eigenaxis ($\hat{t}$) is selected as the y -axis of the body frame, the rotation angle (ψ) is defined with its first and second derivatives for minimum drag mode in Equation (169) and for sailing mode in Equation (170).

$$\begin{aligned}
\psi &= \gamma_{fp} - \nu \\
\dot{\psi} &= \dot{\gamma}_{fp} - \dot{\nu} \\
\ddot{\psi} &= \ddot{\gamma}_{fp} - \ddot{\nu}
\end{aligned} \tag{169}$$

$$\begin{aligned}
\psi &= -\pi/2 + \gamma_{fp} - \nu \\
\dot{\psi} &= \dot{\gamma}_{fp} - \dot{\nu} \\
\ddot{\psi} &= \ddot{\gamma}_{fp} - \ddot{\nu}
\end{aligned} \tag{170}$$

where γ_{fp} is the flight path angle, which is between the velocity vector and transverse axis of the RTN frame. The formulations in [58] are used to derive required time-dependent relations of flight path angle and true anomaly with their first and second derivatives in Equations (171-175).

$$\gamma_{fp} = \tan^{-1} \left(\frac{e \sin \nu}{1 + e \cos \nu} \right) \tag{171}$$

$$\dot{\nu} = \frac{h}{r^2} \tag{172}$$

$$\dot{\gamma}_{fp} = \frac{d\gamma_{fp}}{d\nu} \frac{d\nu}{dt} = \frac{e^2 + e \cos \nu}{1 + 2e \cos \nu + e^2} \cdot \frac{h(1 + e \cos \nu)^2}{a^2(1 - e^2)^2} \tag{173}$$

$$\ddot{\nu} = -\frac{2h}{r^3} \cdot \frac{\mu}{h} e \sin \nu \tag{174}$$

$$\ddot{\gamma}_{fp} = \frac{d^2\gamma_{fp}}{d\nu^2} \left(\frac{d\nu}{dt} \right)^2 + \frac{d\gamma_{fp}}{d\nu} \frac{d^2\nu}{dt^2} = \frac{e \sin \nu (e^2 - 1)}{(1 + 2e \cos \nu + e^2)^2} \tag{175}$$

With the computations in Equations (171-175), the rotation angle is defined as a function of time in Equations (169-170). Then, the desired attitude quaternion is formed with the selected eigenaxis of *y-axis* of the body frame.

The next part is to solve for the large angle rotation problem at the transitions between minimum drag and sailing attitude modes. A 5th-order polynomial for the rotation angle is then generated as shown below:

$$\begin{aligned}
\psi(t) &= c_1 t^5 + c_2 t^4 + c_3 t^3 + c_4 t^2 + c_5 t + c_6 \\
\dot{\psi}(t) &= 5c_1 t^4 + 4c_2 t^3 + 3c_3 t^2 + 2c_4 t + c_5 \\
\ddot{\psi}(t) &= 20c_1 t^3 + 12c_2 t^2 + 6c_3 t + 2c_4
\end{aligned} \tag{176}$$

where coefficients $(c_1, c_2, c_3, c_4, c_5, c_6)$ are determined from initial and final conditions of rotation angle and its first and second derivatives. When all initial and final conditions of angular displacement, velocity and acceleration information are aimed to be taken into consideration, 5th-order polynomial guidance with 6 parameters is the optimal selection that avoids under- or over-determined systems. For example, if the transition from minimum drag to sailing mode is in discussion, then initial conditions are computed with Equation (169) at $t = t_i$ and final conditions are computed with Equation (170) at $t = t_f$.

Having determined the time-dependent attitude trajectory based on a 5th-order polynomial guidance, attitude simulations are performed again for the modified nonlinear quaternion feedback controller based on Equation (163). Orbital parameters and initial attitude conditions of the simulations are shown in Table 9.2 and illustrated in Figure 9.13. Regarding the satellite properties, Table 5.4 is used again.

Attitude propagation is carried out under the disturbances of gravity gradient torque, solar radiation torque and aerodynamic drag torque due to the orbital altitude regime and large sail area properties. Simulations last for one orbital revolution, therefore only two-body gravitational attraction of the Earth is taken into account for orbital propagation.

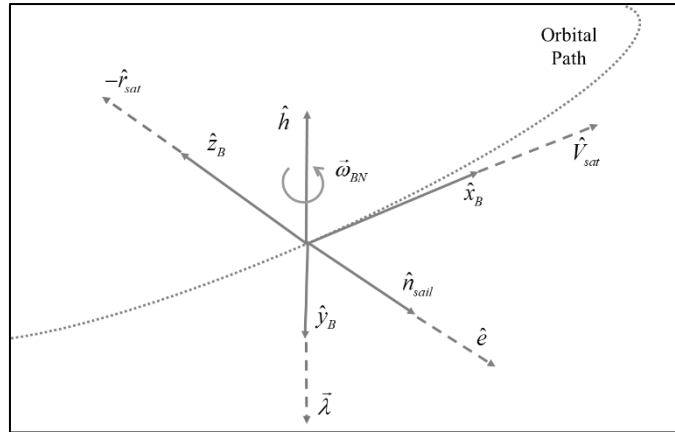


Figure 9.13. Initial Attitude Condition

Table 9.2 Simulation Orbital Parameters and Initial Attitude Condition

Parameter	Value	Parameter	Value
Perigee Altitude	400 km	q_{BN0}	0.8247
Apogee Altitude	1400 km		-0.1504
Inclination	45^0		-0.2642
RAAN	135^0		-0.4769
Argument of Perigee	145^0	ω_{BN0} [rad/s]	0
True Anomaly	0^0		-0.0011
			0

Tracking of the two attitude mode directions (minimum drag and sailing) and transitions between modes of attitude consisting of large angle rotations are controlled during the simulation. The results are shown in the following figures (Figure 9.14, Figure 9.15 and Figure 9.16).

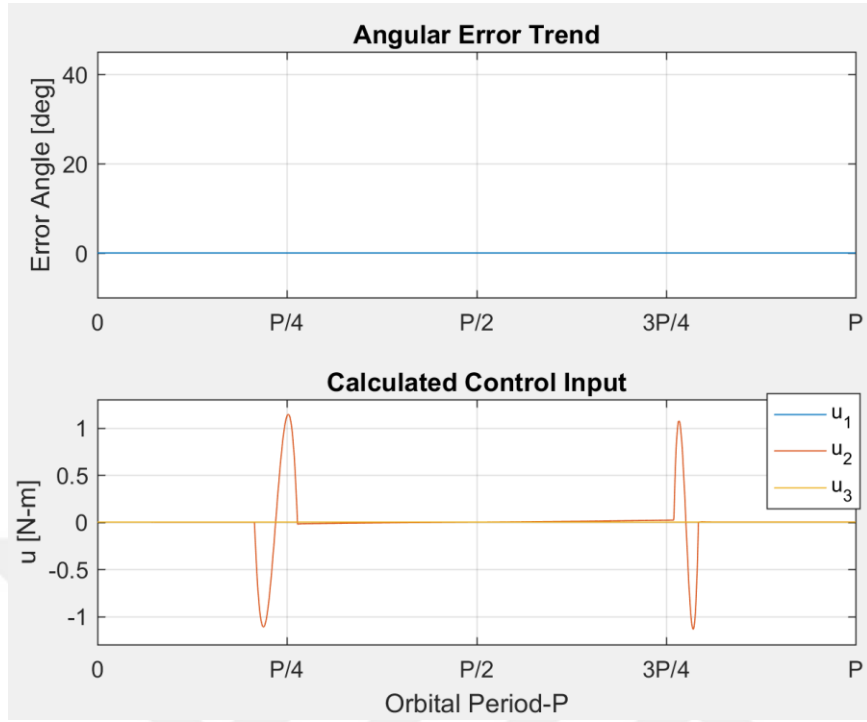


Figure 9.14. Angular Error Trend between Desired and Current Attitudes and Calculated Control Input with Proposed Smooth Attitude Control in Equation (163)

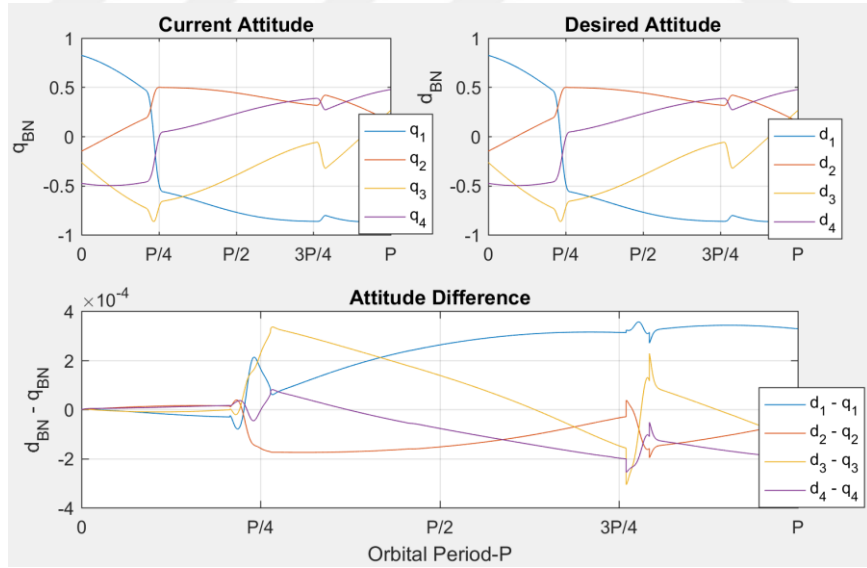


Figure 9.15. Illustration of the Desired and Current Attitude Quaternions with Proposed Smooth Attitude Control in Equation (163)

After an analysis over the simulation results shown in Figure 9.14, Figure 9.15 and Figure 9.16, the first outcome might be the dramatic decrease on attitude tracking errors.

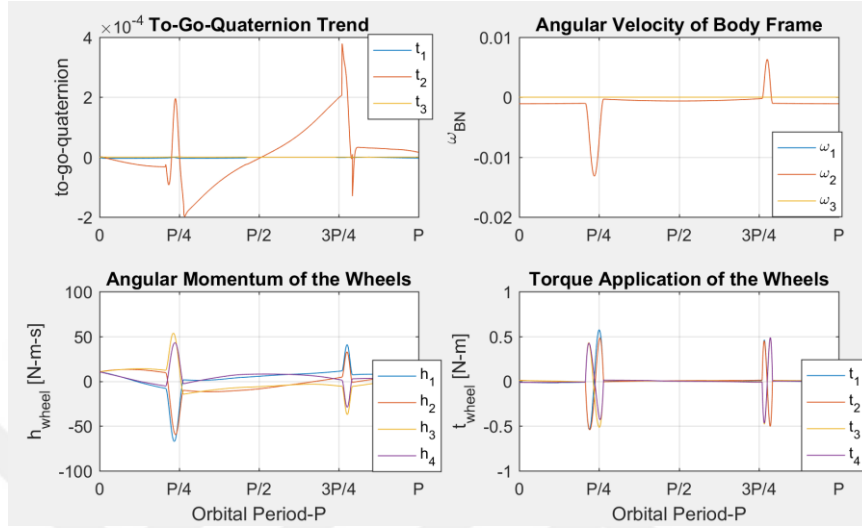


Figure 9.16. Trends of To-Go-Quaternion, Angular Velocity, Momentum and Torque Levels of Reaction Wheels with Proposed Smooth Attitude Control in Equation (163)

Figure 9.14 is showing the angular error value between desired and current attitudes. In addition, the calculated control input is illustrated based on the to-go-quaternion, angular velocity and additionally desired attitude trend in Equation (163). The smooth attitude control approach based on Equation (163) performs very well in terms of tracking with low error values and avoids excessive lag for the desired attitude input to the control logic. These values are relatively high in the first approach based on the nonlinear quaternion feedback controller of Equation (162) and in the second approach which uses a numerically derived attitude information with modified nonlinear quaternion feedback controller of Equation (163).

The tracking error is important especially when we deal with Earth-orbiting satellites and their observation mission. Moreover, the attitude direction and the generation of required thrust highly depend on the relation between Sun's direction and sail normal, therefore angular errors bring orbital trajectory errors as well. Finally,

smaller angular error values and avoidance of excessive lag result in spike-free control input generation. This is another factor when the smooth attitude control is aimed.

Figure 9.15 is illustrating the trends of desired and current attitude along with the difference of both attitude quaternions. As may be observed, the difference between current and desired attitude quaternions in Figure 9.2 and Figure 9.8 are three-order of magnitude larger than the smooth attitude control. This is emphasizing the tracking capability with high accuracy of the smooth attitude control approach in comparison to the nonlinear quaternion feedback controller.

Figure 9.16 is showing four subplots consisting of to-go-quaternion, angular velocity of body frame with respect to inertial frame, angular momentum of the reaction wheels and torque application of the reaction wheels. Due to its high performance over the error values, the to-go-quaternion levels are three-order of magnitude lower than the previous approaches (Figure 9.3 and Figure 9.8). This is stressing the same issue of tracking the desired attitude with low error levels. Angular velocities of body frame with respect to inertial frame, angular momentum of the reaction wheels and applied torque levels of the reaction wheels are more preferable than the results obtained with first approach (Figure 9.3). However, they are similar with the results of second approach (Figure 9.9).

Even though a 5th-order polynomial guidance provides a smooth transition, a low-pass filter is implemented over the obtained rotation angle trajectory in Equation (176) in order to assure a low-frequency rotation operation of the solar sail satellite. The following transfer function is used as the low-pass filter and relevant bode plot of this transfer function with $\tau = 10$ is given in Figure 9.17.

$$H(s) = \frac{1}{(\tau s + 1)^3} \quad (177)$$

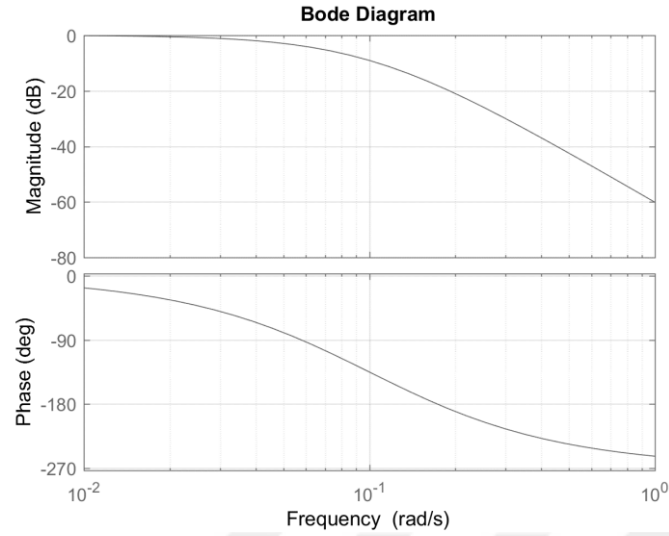


Figure 9.17. Bode Plot of the Low-Pass Filter

As may be observed in Figure 9.17, the selection of $\tau = 10$ results in a cut-off frequency at 0.05 rad/s and a corner frequency at 0.1 rad/s. This ensures a low-frequency trajectory tracking and the attenuation of the possible high-frequency rotation angle trajectory with the implementation over the 5th-order polynomial guidance.

The resultant rotation angle as a function of time is then aimed to be derived. First formulation is given in *s-domain* including the transfer function definition as provided below:

$$Y(s) = H(s)X(s) \quad (178)$$

where $Y(s)$ represents the resultant rotation angle in *s-domain* and $X(s)$ represents the 5th-order polynomial trajectory in *s-domain*. Then, the Laplace transformation of the rotation angle trajectory given in Equation (176) is done and the result is provided below:

$$X(s) = \frac{120c_1 + 24c_2s + 6c_3s^2 + 2c_4s^3 + c_5s^4 + c_6s^5}{s^6} \quad (179)$$

After that, the response of the filter for the 5th-order polynomial trajectory in *s-domain* is obtained by substituting Equation (177) and Equation (179) into Equation (178).

$$Y(s) = \frac{120c_1 + 24c_2s + 6c_3s^2 + 2c_4s^3 + c_5s^4 + c_6s^5}{s^6(\tau s + 1)^3} \quad (180)$$

Finally, inverse Laplace transformation is performed on Equation (180) and resultant rotation angle trend as a function of time is obtained.

Similarly, the first and second derivatives of rotation angle are also found by taking the time-derivative of Equation (181) in order to populate the time-dependent desired attitude quaternion trajectory. The resultant response of the 5th-order polynomial trajectory and its filtered path may be seen in Figure 9.18.

$$\begin{aligned} L^{-1}(Y(s)) &= y(t) = \psi(t) \\ \psi(t) &= t^2 \exp\left(-\frac{t}{\tau}\right) (60c_1\tau^3 - 12c_2\tau^2 + 3c_3\tau - c_4 + 0.5c_5\tau^{-1} - 0.5c_6\tau^{-2}) + \dots \\ &\quad \exp\left(-\frac{t}{\tau}\right) (2520c_1\tau^5 - 360c_2\tau^4 + 60c_3\tau^3 - 12c_4\tau^2 + 3c_5\tau - c_6) + \dots \\ &\quad t \exp\left(-\frac{t}{\tau}\right) (720c_1\tau^4 - 120c_2\tau^3 + 24c_3\tau^2 - 6c_4\tau + 2c_5 - c_6\tau^{-1}) + \dots \\ &\quad c_1t^5 - t^4(15c_1\tau - c_2) + t^3(120c_1\tau^2 - 12c_2\tau + c_3) - \dots \\ &\quad t^2(600c_1\tau^3 - 72c_2\tau^2 + 9c_3\tau - c_4) + \dots \\ &\quad t(1800c_1\tau^4 - 240c_2\tau^3 + 36c_3\tau^2 - 6c_4\tau + c_5) - \dots \\ &\quad (2520c_1\tau^5 - 360c_2\tau^4 + 60c_3\tau^3 - 12c_4\tau^2 + 3c_5\tau - c_6) \end{aligned} \quad (181)$$

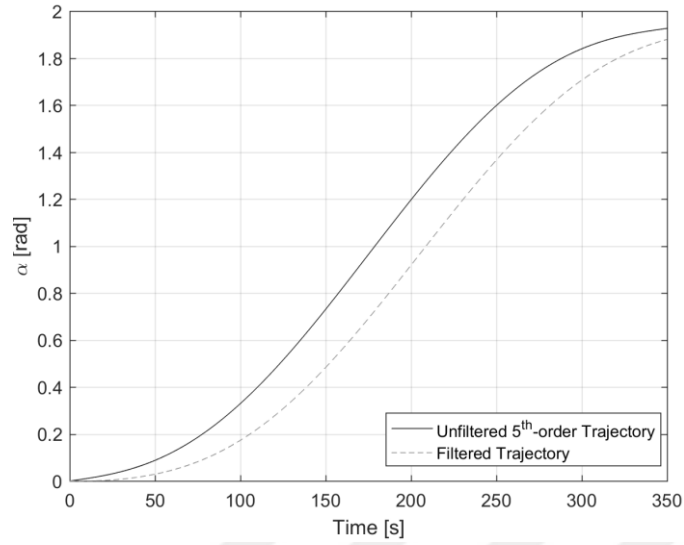


Figure 9.18. Rotation Angle Trajectory and Filter Application

In the following figures, attitude simulations with the introduction of low-pass filter to the 5th-order polynomial guidance trajectory are shown based on the modified nonlinear quaternion feedback controller given in Equation (163). Same simulation conditions and satellite properties are used.

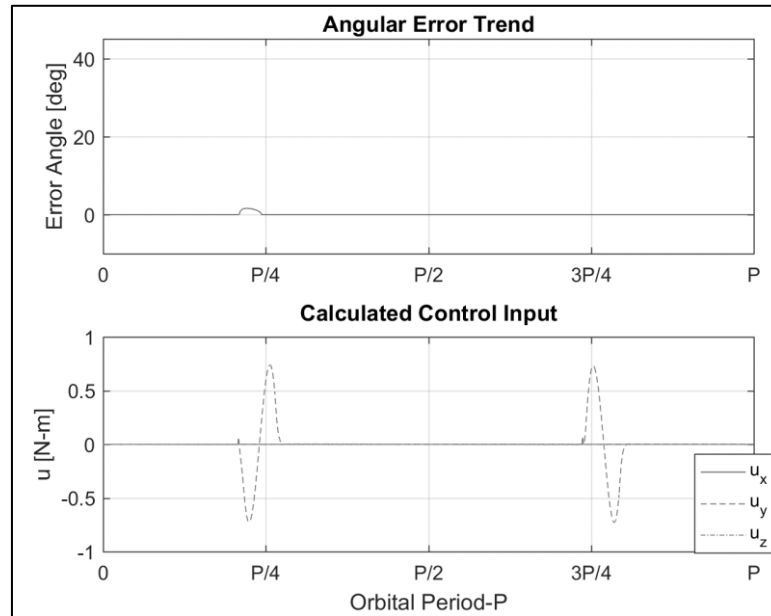


Figure 9.19. Angular Error Trend between Desired and Current Attitudes and Calculated Control Input with Proposed Smooth Attitude Control in Equation (163)

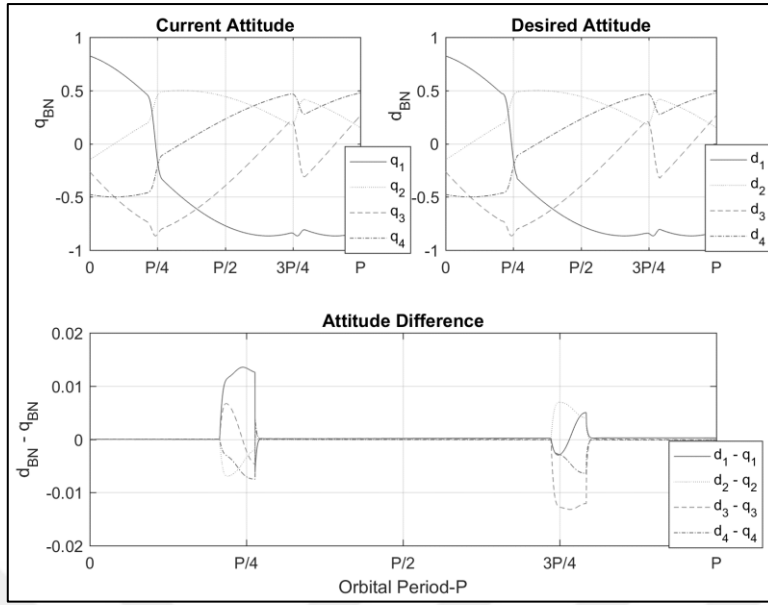


Figure 9.20. Illustration of the Desired and Current Attitude Quaternions with Proposed Smooth Attitude Control in Equation (163)

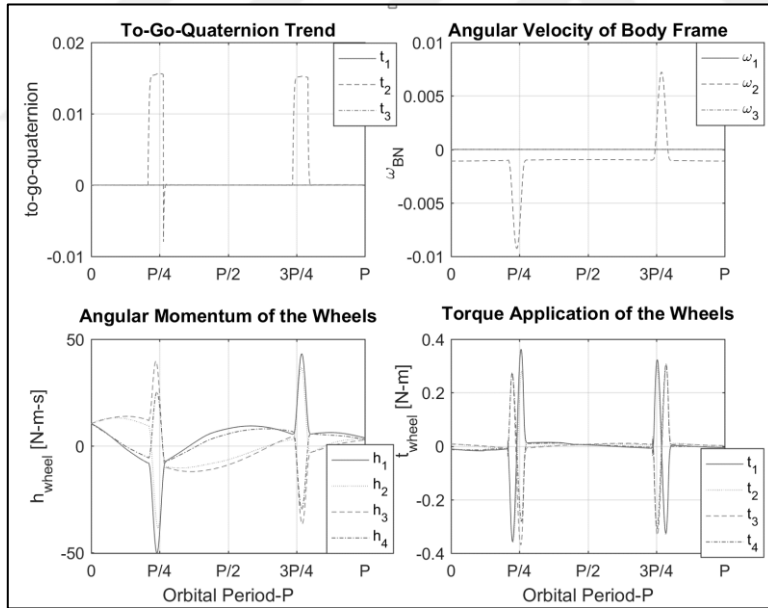


Figure 9.21. Trends of To-Go-Quaternion, Angular Velocity, Momentum and Torque Levels of Reaction Wheels with Proposed Smooth Attitude Control in Equation (163)

When the figures are obtained as a result of simulation, the comparison with respect to previous simulation results are performed. While assuring a low-frequency large angle rotation, the last method with a 5th-order polynomial guidance accompanied by a low-pass filter causes lag in tracking. This results in an increase over angular error and to-go-quaternion values (Figure 9.19 and Figure 9.21). However, there is a significant relief on the calculated control input values that allows 20% drop on applied torque levels and reached angular momentum values of reaction wheels. Eventually, the achieved angular velocity of the satellite system is diminished. This result is critical in terms of smooth, low-jerk and accurate attitude tracking performance.

9.3 Discussion on the Results of Attitude Simulations

To summarize the evolution of the adopted approach to the large angle attitude rotation for a smooth and low-jerk tracking, discussion of the results of each four attitude control approach is provided below.

Four approaches can be defined as:

1. Nonlinear quaternion feedback controller based on Equation (162) with control input as desired attitude trajectory without any time-dependence (Figure 9.1, Figure 9.2, Figure 9.3, Figure 9.4, Figure 9.5 and Figure 9.6),
2. Modified nonlinear quaternion feedback controller based on Equation (163) with control input as the numerically derived time-dependent attitude trajectory (Figure 9.7, Figure 9.8, Figure 9.9, Figure 9.10, Figure 9.11 and Figure 9.12),
3. Modified nonlinear quaternion feedback controller based on Equation (163) with control input as time-dependent attitude trajectory obtained from a 5th-order polynomial guidance (Figure 9.14, Figure 9.15 and Figure 9.16),
4. Modified nonlinear quaternion feedback controller based on Equation (163) with control input as time-dependent attitude trajectory obtained from a 5th-

order polynomial guidance accompanied by a low-pass filter (Figure 9.19, Figure 9.20, Figure 9.21).

The first approach gives successful results in terms of slewing time performance. However, it lacks in tracking error and moreover applied torque and angular momentum levels of the reaction wheels result in inefficient rotation maneuver which is not desired for solar sail satellites due to their flexible structure issues.

The second approach improves the performance for achieving similar slewing time performance with lower applied torque and angular momentum levels of reaction wheels, and eventually resulting in a decrease over the angular velocity of the satellite system. Nevertheless, the tracking error values are not at the desired levels.

The third approach performs well in terms of tracking error values while keeping the applied torque and angular momentum levels of the reaction wheels at the same regime.

Finally, the fourth approach brings additional improvement of the achieved torque and angular momentum levels of the reaction wheels while resulting in an increase over the tracking error values.

In the end, the fourth approach (the modified nonlinear quaternion feedback controller that takes the trend of the desired attitude trajectory into account based on a 5th-order polynomial guidance accompanied with a low-pass filter) is evaluated as the optimal approach. Even though the tracking error performance of the fourth approach is worse than the third approach, the importance of smooth and low-jerk maneuver necessity of a large solar sail satellite makes the fourth approach preferable rather than all other three approaches.

The simulation results are successful for each large angle attitude maneuvers. Moreover, successful implementation of the modified version of the nonlinear quaternion feedback control method is fulfilled and resultant slewing performance is also within the limits of maneuver time for the R2S2 concept. With an acceptable slewing performance, the modified version is more successful than the original

nonlinear quaternion feedback control method due to its smooth and low-jerk maneuver capability through tracking of the desired attitude with a time-dependent, pre-determined trajectory. The smoothness of the maneuvers results in use of low torques on reaction wheels and eventually low levels of angular momentum of reaction wheels and angular velocity of the satellite system.

In summary, the proposed smooth attitude control approach is more successful in tracking the desired attitude trajectory with high accuracy while its assurance for low-frequency rotation operation is in place. Moreover, all these precautions relevant to the flexibility of the solar sail satellite (5th-order polynomial guidance and low-pass filter) are taken into consideration while large angle rotations are accomplished within an appropriate amount of time aimed at Chapter 5.5.

Smooth attitude control of a solar sail satellite is achieved by implementing a 5th-order polynomial guidance and a low-pass filter. This desired attitude trajectory generation ensures low-frequency rotational motion along with a smooth large angle reorientation of the attitude. Then, the time-dependent desired attitude information along with its first and second derivatives are fed into a novel nonlinear quaternion feedback controller which takes the desired attitude trend into account. This allows us to acquire accurate tracking performance while attenuating of high frequency portions of the rotational motion. The entire rotational operation of the solar sail satellite during one orbit revolution is demonstrated with two large angle rotations. The simulation results aiming a comparison of the proposed smooth attitude control approach with respect to the commonly known quaternion feedback controller are given and discussed that the proposed method performs well in terms of tracking accuracy and smoothness of the rotational motion.

For further work, the control mass boom methodology and internal moving masses may be used to decrease the effects of the aerodynamic and gravity gradient torques, on the other hand control tip vanes or refractive sail attitude control may be used to compensate for the torques stemming from solar radiation pressure. With these

alternative methods of attitude control, the momentum buildup in the reaction wheels may be diminished.

This attitude control scheme is also used for long term simulations up to 4 months in Chapter 8. The long term simulation results show that this attitude control strategy and attitude control mechanism sizing are adequate for the defined concept at Low Earth Orbit. From this result, we can also deduce that Control Moment Gyros (CMGs) are not the optimal solution to employ in such mission configuration, saving extra mass and eventually relief from the extra drop in the characteristic acceleration.



CHAPTER 10

CONSTELLATION DESIGN

The final part of this thesis is to design a constellation that meets the requirements of the R2S2 Concept, especially the responsiveness. As it was observed, the reorientation of the orbit was not feasible for RAAN and argument of perigee values and was in small amount for inclination value. Even though solar sailing brings an utmost advantage on resizing and reshaping the orbit, a constellation is necessary for closing the gaps stemming from the lack of capability on reorientation of the orbit. Moreover, the transfer times between mission and standby orbits are taken into consideration during the constellation design.

The work on constellation design for R2S2 Concept is provided by first laying out the literature on the foundations of constellation designs. Then, principles coming from the R2S2 Concept are given. Having noted the principles, the determination process for the constellation parameters (number of satellites, number of planes, phase angle separations etc.) is designed and finally relevant simulation results are discussed.

10.1 Fundamentals on Constellation Design

The design of satellite constellations has been in the research focus for many years. Constellation designers are often intrigued for global coverage, frequent revisit times and geometrical distribution for scientific reasons. In order to consider all different aspects of the subject a thorough decision making process was structured by researchers [87-88]. In these algorithms, factors such as launch and satellite development costs, constellation maintenance and replenishments, spare satellite employments and use of specialized orbits were also taken into consideration in addition to main goal of mission-oriented requirements.

Regarding the mission-oriented requirements, main idea has always been on the subjects of coverage and revisit performance. However, depending on the mission application some other parameters were also under analysis, such as transmission delay, Doppler Effect, link budget, inter-satellite links, and Dilutions of Precision (Horizontal, Vertical, Geometric, Position and Time) [88]. Thus, the question of an optimal constellation is an open-ended question with no absolute answer [87].

The struggle to come up with a successful algorithm for designing a constellation is divided into three categories for coverage requirements: Global, Zonal and Regional [89]. A numerical solution to Global Coverage was given by Walker [90], then improved with an analytical solution by Rider to both Global and Zonal Coverages [91-92]. The most commonly used algorithms of Walker Delta Patterns [90] and Streets of Coverage [92-93] were introduced during that period. The common properties of these concepts were the use of symmetry and circularity. Moreover, the inclination of different planes of the constellations were also set to a pre-determined value to equalize the drifts due to the asphericity of the Earth.

Later, these subjects were improved and extended by many researchers. Palmerini extended the circular and symmetric concepts of Global Coverage constellations to elliptical orbits under the consideration of J2 effects [94]. Then, Soccorsi proposed methods to the problem of Regional Coverage [89]. With the help of improving genetic algorithm technologies, researchers such as Ely [95] and Confesore [96], developed methodologies for Zonal and Regional Coverages. Ulybyshev also drew attention to the elliptical orbit's (Molniya-type) coverages for illuminated regions of the world [97] and proposed analyses methodologies of 2D graphs of RAAN and argument of perigee [98]. A recent study also emphasized a different aspect of the analysis of the constellation design process by using relative frames rather than inertial frames [99].

All the above concepts and methodologies concentrate on the constellation use over the common region of interest. However, in this study, the reason for a constellation use is different. In R2S2 concept, a single satellite is always used for single region

of interest. In other saying, if there were no limitations, which will be listed in Chapter 10.2, there would be no need for a constellation design. For this reason, the design of a constellation for the R2S2 Concept differs from the literature. The concentration will be given to the limitations coming from the solar sailing incapacities, relative Earth-Sun-Satellite geometry and R2S2 mission orbit attributes. The number of satellites to be used and number of planes with their orientation properties will be determined to guarantee the responsiveness and extend the mission performance over a region of interest.

10.2 Limitations over the Mission Performance

The performance of the R2S2 concept is affected by four main factors:

- Time spent during transfer;
 - Perigee altitude transfers take 3-4 months between mission and standby orbits,
 - If an inclination change (of 2.5 degrees) is necessary (which should be completed at the standby orbit), total transfer time from standby orbit to mission orbit extends to 2 years.
- Orbit orientation valleys on observation performance;
 - Depending on the inclination, argument of perigee value at the start of the mission phase is effecting the mission performance,
 - Depending on the inclination and DOY, RAAN value at the start of the mission phase is effecting the mission performance,
 - The dependence on the inclination is stemming from the latitude of the region of interest (the longitude of the site has no effect over the mission performance).
- Seasonal effects on maneuvers;
 - Depending on the inclination and DOY, specific RAAN and argument of perigee values are not efficient for in- and out-of-plane maneuvers.

- Lack of reorientation capability of the orbit;
 - Argument of perigee and RAAN changes via solar sailing are not feasible with the proposed baseline satellite design (achieved characteristic acceleration level) due to the secular contribution of J2 effect,
 - Small amount of change capability over the inclination (1.5 deg/year).

10.3 Determination of the Constellation

The requirements of the constellation design for R2S2 concept are twofold: Minimization of the response time to a new region of interest and the extension of the mission over an existing region of interest.

As mentioned and demonstrated earlier, single satellite in the mission orbit with optimized orbital parameters is adequate for a finite time of mission duration (Chapter 6). The constellation buildup for the limitations of the R2S2 concept is then accomplished by following the decision-making process described in Figure 10.2. The following process of determination of the constellation will be explained in the order given in Figure 10.2.

As mentioned during the introduction of the literature survey about the constellation designs, varying inclinations of the orbital planes within constellations were avoided by the designers. The reason is that different inclination values bring varying changes on the orientation of the orbit (RAAN and argument of perigee) mainly due to J2 effects. This varying changes to the orbit orientation are not feasible to counteract by propulsion means as it is the case for R2S2 concept. However, the problem arising from varying orientation shifts is not applicable to R2S2 concept since one satellite at a time is allocated to a region of interest rather than a collective observation tasks aimed by the literature of constellation designs. Therefore, the use of planes with varying inclination values is not a problem for R2S2 concept, conversely it is a must. As it is demonstrated in Chapter 6.3, maximum latitude regions of the elliptic

mission orbits are allocated to the mission execution. Therefore, a small intervals of inclination values between constellation planes are necessary. If we remember the results of mission orbit determination process, inclination value is set to 2 degrees above the site's latitude in Equation (91). The value of that inclination still provides good observation performance over sites with latitude values of 5 degrees below as can be observed in Figure 6.5. This value of 5 degree-interval is also comparable to the transfer capability limitation over the inclination value of ± 2.5 degrees given in Chapter 10.2. Therefore, 5-degree interval of inclination values of successive orbital planes will be used as the first stepping stone.

The next issue is the range of these inclination values. In regard to the concentration of R2S2 concept, the population and landmass densities by the latitude information are two factors for determining the range. According to the research papers of [100] and [101], the population densities are high between 35-50 degrees of latitude in northern hemisphere and landmass densities are high between 30-60 degrees of latitude in northern hemisphere, respectively. Thus, the range of inclination values are selected as 40-65 degrees. Therefore, 6 different planes for specified inclination values are determined (Figure 10.1).

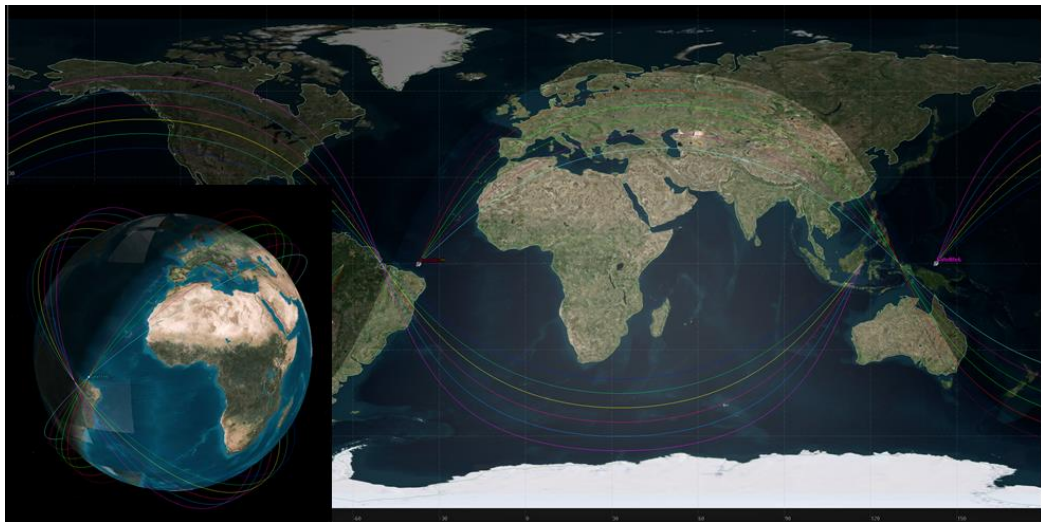


Figure 10.1. Orbital Planes with Varying Inclination Values

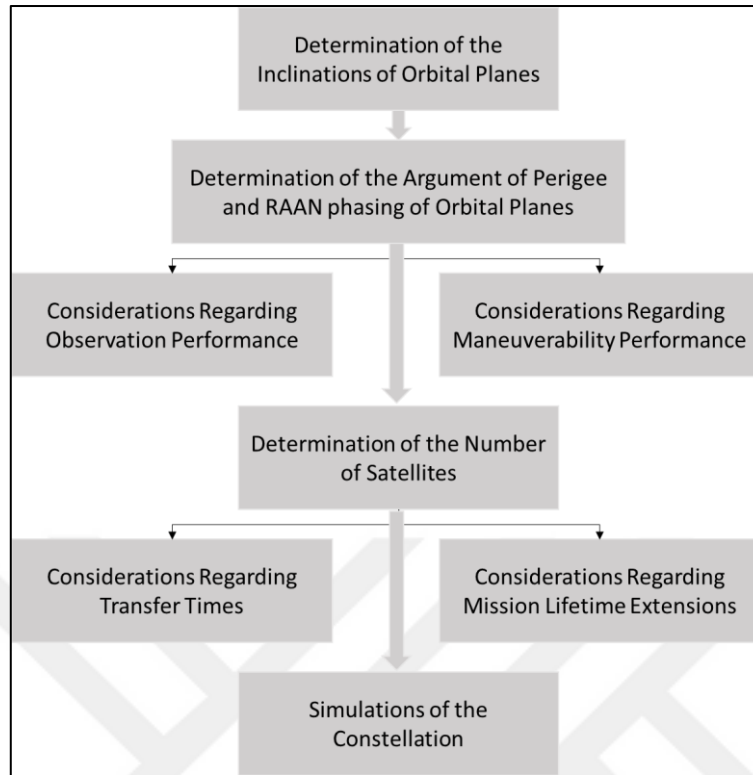


Figure 10.2. Constellation Design Decision-Making Process

With the inclination of the planes determined, the work on the argument of perigee and RAAN is provided. Since each plane with different inclination values is going to be used for specific latitude interval (zonal coverage), the co-dependence of planes for coverage purposes are not the case for R2S2 Concept. Therefore, RAAN and argument of perigee considerations will be deduced for each plane separately.

To give an insight about the effect of inclination to argument of perigee and RAAN drifts, a phase-plane is given in Figure 10.3. Then, Figure 6.7, which illustrates the argument of perigee value effects on the altitude criteria of mission performance, is tabulated below in Table 10.1. In Table 10.1, the information of time duration to pass through an argument of perigee valley is also provided. This time value is used to determine the argument of perigee phasing for each orbital plane. The process starts with adding an 180^0 initial phase to the argument of perigee and add denser phases while the number of orbits in the not-allowed interval is more than one. Eventually,

120⁰ phase is found feasible for all orbital planes, since range of not-allowed intervals are approximately 196 degrees.

Table 10.1 Argument of Perigee Intervals for Mission Performance

Orbital Planes	Argument of Perigee		Time to Travel through Not- Allowed Interval
	Allowed Intervals [deg]	Not-Allowed Intervals [deg]	
<i>i=40 deg</i>	0-81 & 277-360	81-277	32 days
<i>i=45 deg</i>	0-100 & 299-360	100-299	42 days
<i>i=50 deg</i>	0-121 & 318-360	121-318	58 days
<i>i=55 deg</i>	0-142 & 338-360	142-338	96 days
<i>i=60 deg</i>	0-161 & 357-360	161-357	247 days
<i>i=65 deg</i>	14-178	0-14 & 178-360	578 days

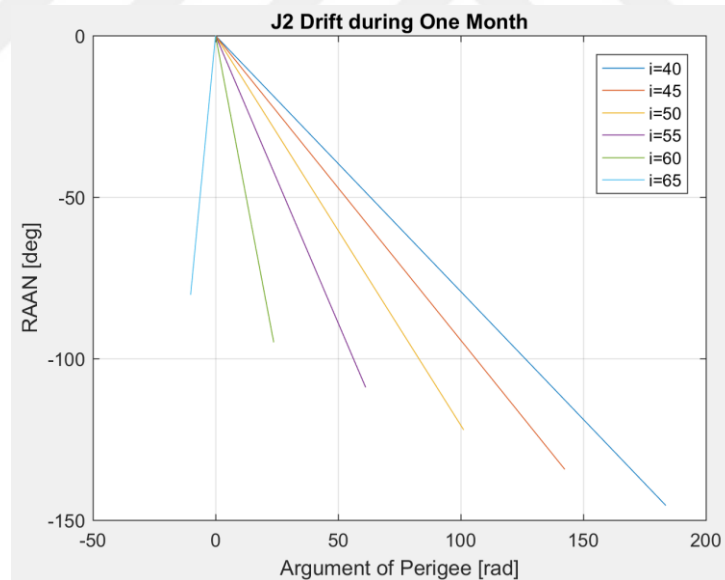


Figure 10.3. Drifts of RAAN and Argument of Perigee Values

In Figure 6.8, a similar parameter determination result can be observed for initial RAAN based on given inclination and DOY. That selection is stemming from the daylight criteria of the R2S2 Concept by setting the first maximum latitude point

local time to Sunset time. Then, the local time of the maximum latitude point of the orbit is shifted to the Sunrise time in each revolution. The times for travelling through Sunrise and Sunset times are tabulated in Table 10.2 for different inclination values. The local times of the Sunrise and Sunset differ with the DOY and latitude, therefore the worst conditions (shortest day duration) are used for determining Table 10.2. As can be observed there is a 134.7-day-period of the travel of the local time of the maximum latitude. The process then starts with adding an 180^0 initial phase to the RAAN and add denser phases as the number of orbits in the not-allowed interval is more than one. Eventually, 120^0 phase is found feasible for orbital planes with an inclination of 50 degrees and below, 90^0 phase is found feasible for orbital planes with an inclination of 55 and 60 degrees and finally 72^0 phase is found feasible for orbital plane with an inclination of 65 degrees.

Table 10.2 Time to Reach the Not-Allowed Region for RAAN

Orbital Planes	Shortest Day Duration	Time to Reach the Not-Allowed Region	Time to Reach back to Allowed Region
<i>i=40 deg</i>	<i>9 hours</i>	<i>51 days</i>	<i>84 days</i>
<i>i=45 deg</i>	<i>8.5 hours</i>	<i>48 days</i>	<i>87 days</i>
<i>i=50 deg</i>	<i>8 hours</i>	<i>45 days</i>	<i>90 days</i>
<i>i=55 deg</i>	<i>7 hours</i>	<i>39 days</i>	<i>95 days</i>
<i>i=60 deg</i>	<i>6 hours</i>	<i>34 days</i>	<i>101 days</i>
<i>i=65 deg</i>	<i>5 hours</i>	<i>28 days</i>	<i>107 days</i>

At this point the number of satellites become 66 as can be calculated from Table 10.3. This number is the result of an approach with no tolerance to the observation gaps. If some intervals for no-observation periods are acceptable, then this number will be smaller. For example, periods of two and four months of no-observation leads to the drop of the total number of satellites to 34 and 28, respectively. The requirements of the R2S2 concept should be well thought to come up with a

requirement for the no-observation period. This is going to be a tradeoff between observation performance and constellation buildup/maintenance costs.

Table 10.3 Orbital Parameters for Constellation Design

Orbital Planes	Argument of Perigee Phase Angle	RAAN Phase Angle
<i>i=40 deg</i>	<i>120 deg</i>	<i>120 deg</i>
<i>i=45 deg</i>	<i>120 deg</i>	<i>120 deg</i>
<i>i=50 deg</i>	<i>120 deg</i>	<i>120 deg</i>
<i>i=55 deg</i>	<i>120 deg</i>	<i>90 deg</i>
<i>i=60 deg</i>	<i>120 deg</i>	<i>90 deg</i>
<i>i=65 deg</i>	<i>120 deg</i>	<i>72 deg</i>

The architecture of constellation design based on the phasing of inclination, RAAN and argument of perigee is covering the needs of the R2S2 concept for the mission orbits and compensating the relevant limitations described in Chapter 10.2. Only exception is the limitation of the time spent during perigee altitude transfers. This value of 3-4 months for one transfer maneuver or 6-8 months for a full cycle from mission to standby and back to mission orbit can be compensated by two different methodologies: Full back up or ad-hoc spare satellite employment. The first strategy is straightforward. A mirror constellation is employed at the standby orbit and this doubles the required total satellite number. The latter strategy is employing one more satellite at each orbital plane at the altitudes of standby orbit and increasing the required total satellite number by only six. These extra satellites for each orbital plane will then tasked to perform transfer maneuvers to the next mission orbit while a satellite at the not-allowed region is tasked to take the transfer to standby orbit.

10.4 Simulation Results and Discussion

The rationale given in the previous section is simulated for the inclination at 45 degrees as an example and the discussion about the constellation buildup is also given in this section.

As can be recalled from Chapter 6.1, R2S2 concept requires three criteria to be met in order to consider an access as valid. These three criteria are Geometry, Altitude and Daylight.

The first criteria is the most intuitive one, because it is obvious that the site should be in the field of view of the orbiting satellite. The other two criteria are caused by the proposed concept of R2S2. The concept does not allow a mission execution above 800 km. Finally, the employment of an electro-optic satellite as a mission payload does not permit accesses in the night time. All these criteria effects over the orbital parameters and DOY were analyzed and relevant relations were found in Chapter 6.3. Then, a constellation determination has been done in Chapter 10.3 according to the limitations described in Chapter 10.2, which is stemming from Chapters 6.3 and 7. The simulations are performed at STK software with a propagator including asphericity effects up to J4 perturbation.

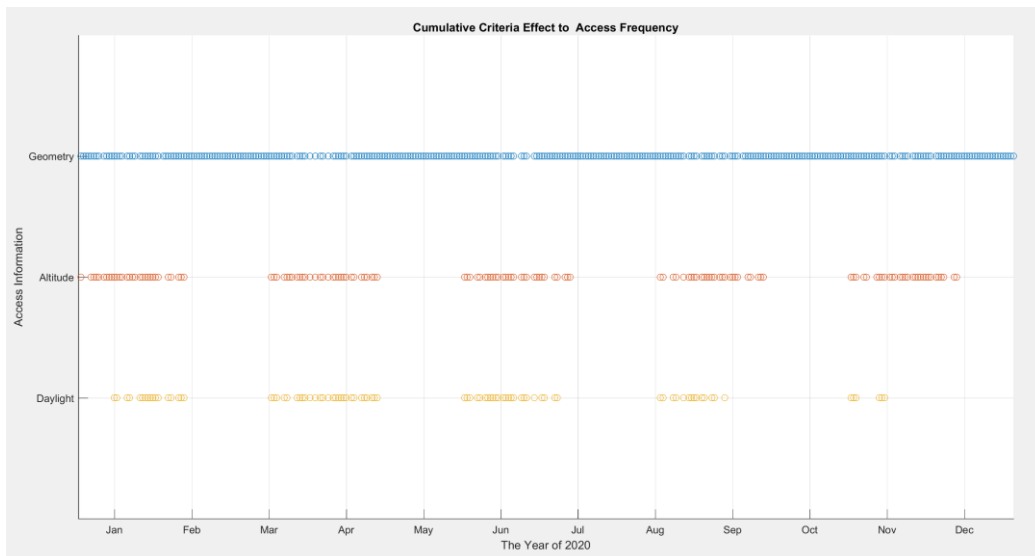


Figure 10.4. Valleys of No Access Intervals due to the Access Criteria

In Figure 10.4, the effects of three criteria are shown. As calculated before, the altitude and daylight criteria bring some intervals of no-observation periods. The entire idea of constellation buildup for R2S2 concept is caused by this regular intervals of no-observation periods.

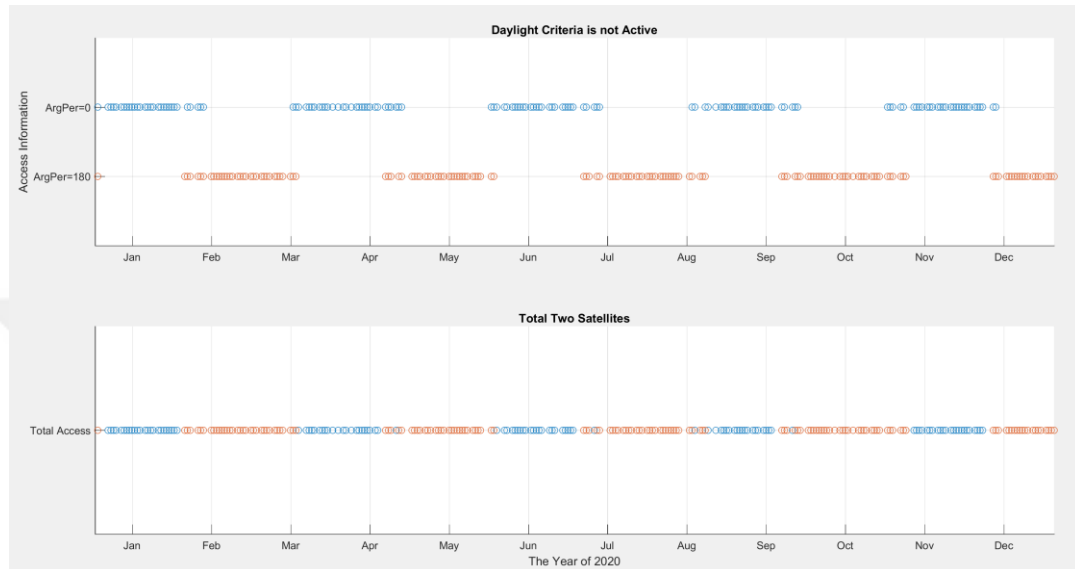


Figure 10.5. Phasing of Argument of Perigee by 180 degrees

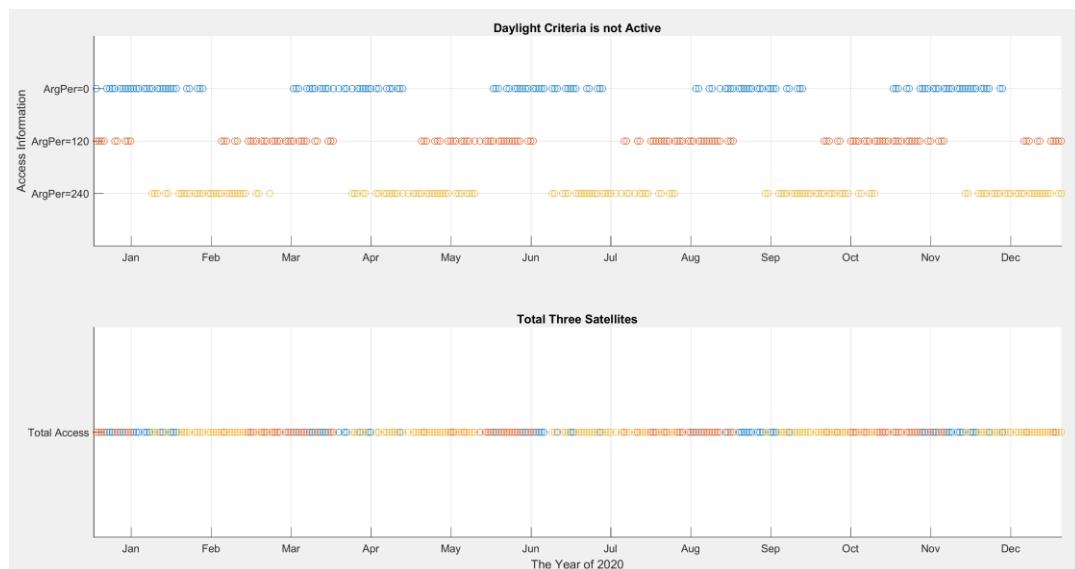


Figure 10.6. Phasing of Argument of Perigee by 120 degrees

In Figure 10.5 and Figure 10.6, the daylight criteria is not active, meaning that night time accesses are also counted. In this situation, only criteria for the valleys of no-observation periods is the altitude criteria. The simulation results are also comparable with the calculation results which were tabulated in Table 10.1. Therefore, simulations of 180^0 and 120^0 degree-phasing for argument of perigee are performed and results are shown in Figure 10.5 and Figure 10.6, respectively. 180^0 degree-phasing with two satellites give very good observation performance (Figure 10.5), however there are small amounts of no-observation periods. If the requirement for allowed no-observation period is stretched, then two-satellite constellation can be considered as acceptable. If a strict requirement on no-observation period is present, then three-satellite constellation should be selected with a 120^0 degree-phasing of argument of perigee (Figure 10.6). This rationale on requirement of allowed no-observation period given in Chapter 10.3 for the determination of number of satellites is also comparable with the simulation results.

After including the daylight criteria to the simulation results, phasing of argument of perigee become inadequate as demonstrated in Figure 10.7 and Figure 10.8. This also complies with the rationale given in Chapter 10.3 with the no-observation valleys due to RAAN and DOY tabulated in Table 10.2.

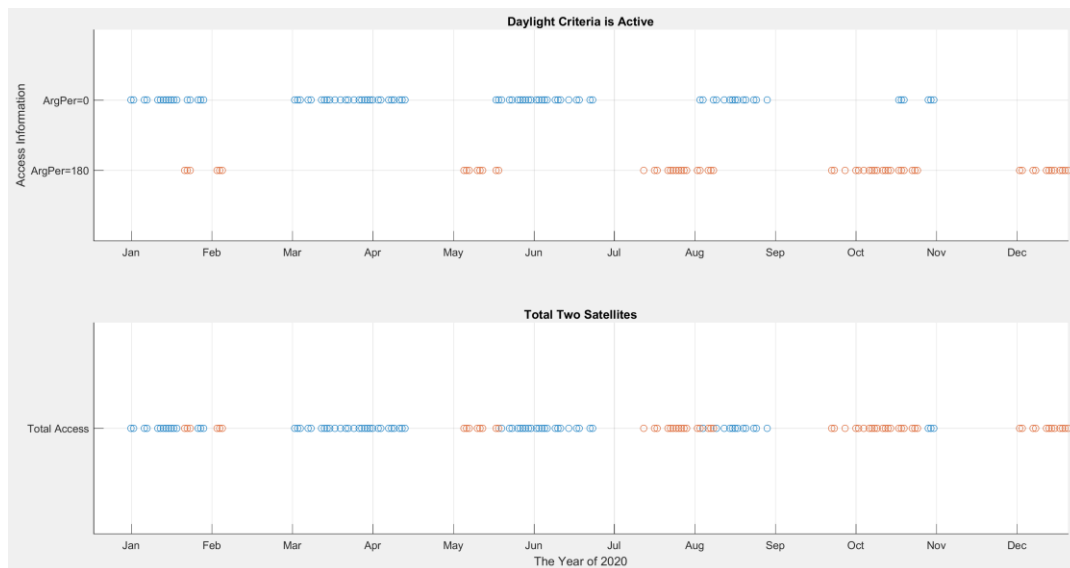


Figure 10.7. Phasing of Argument of Perigee by 180 degrees



Figure 10.8. Phasing of Argument of Perigee by 120 degrees

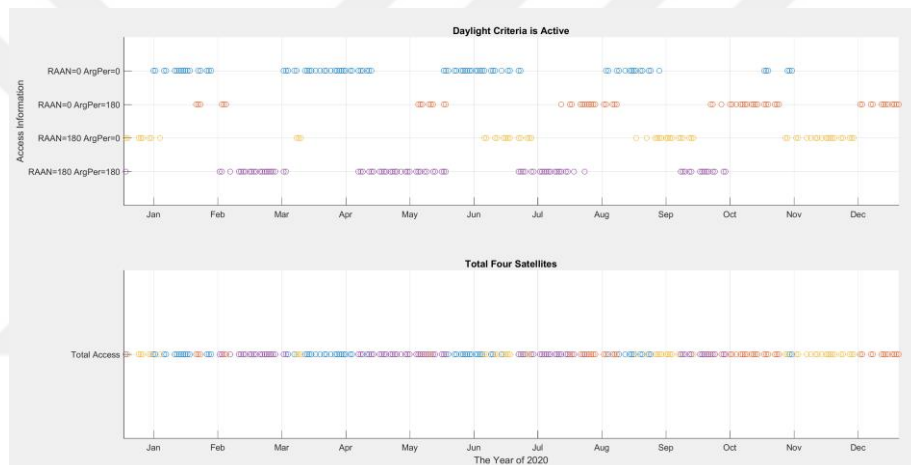


Figure 10.9. Phasing of Argument of Perigee and RAAN by 180 degrees

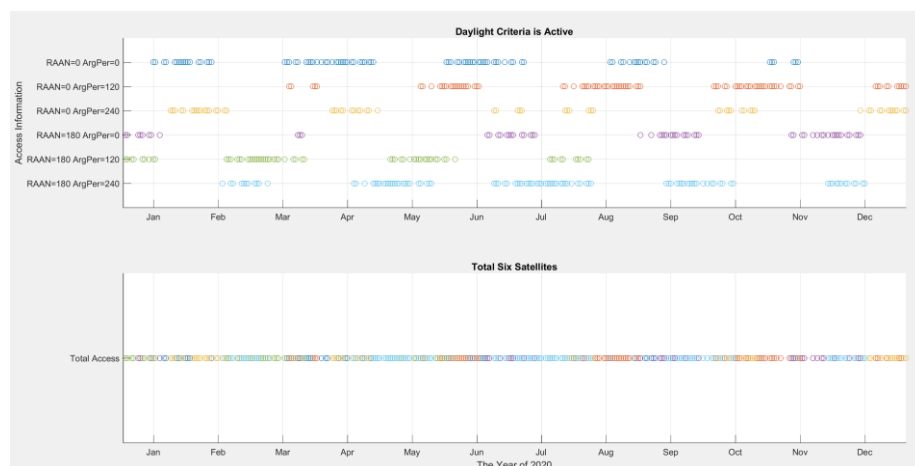


Figure 10.10. Final Phasing of Argument of Perigee and RAAN

Finally, RAAN phasing of 180 degrees is implemented for two different types of argument of perigee phasing. As can be observed in Figure 10.9, total number of four satellites (180 degree of phasing for argument of perigee and RAAN) provide very good observation performance, but there are still small no-observation periods. This is solved in six-satellite constellation (120 degree of phasing for argument of perigee and 180 degree of phasing for RAAN) and demonstrated in Figure 10.10.

As mentioned in Chapter 10.3, the determination process for constellation design and decision on the number of satellites depend on the requirement of allowed no-observation period. This requirement changes the necessary number of satellites. Other than this mission-specific requirement, the limitations stemming from the R2S2 concept should be compensated with the constellation as it has been designed in this chapter. The determination process based on the calculations given in Chapter 10.3 and simulation results for access information illustrated in Chapter 10.4 proved the constellation necessity and laid down the strategy about the constellation structure for R2S2 concept.

CHAPTER 11

CONCLUSION

Combining the intrinsic capabilities of solar sailing and Earth observation from LEO region was accomplished with the proposed concept of R2S2 despite the unfavorable interaction of large sail area and relatively dense air. Employing an elliptical orbit and division of that orbit into two parts to circumvent the risks of altitude decay due to drag are the key starting points for the fulfilment of the original idea of R2S2 concept. Unprecedented trial of the use of large solar sails at the very low altitudes (down to 300-400 km) with the proposed concept of R2S2 were tested for various mission modes.

The R2S2 concept validation with its physical and geometrical limitations was realized by carrying out analysis and computations on various disciplines of satellite orbit and attitude control research field. After having the preliminary confidence over the proposed concept of R2S2 with the feasibility study simulations, an iterative process for a baseline satellite design was completed while giving a special attention to technology readiness levels. A solar sail spacecraft with an optimized mission payload reached characteristic acceleration values of 0.07 mm/s^2 and a TRL of 6-7 at the end of the iterative baseline satellite design process.

The proposed concept of R2S2 was successful under various simulation conditions and enabling a safe and reliable mission performance thanks to the detailed hierarchy of mission mode structure (Mission, Task, Attitude and Station-Keeping). This four-level mode hierarchy allowed an elastic operation of the satellite for the observation mission over the region of interest, maintaining the mission orbit, avoiding excessive drag exposure of the large sail area, carrying out necessary orbital maneuvers to/from standby orbit and fulfilling the transfer orbit requirements of thrust vector control pointing.

Indigenous mission orbit parameter selection formulations were also derived during the study in order to determine the mission orbit with a high observation performance for the given region of interest. Inclination, argument of perigee and RAAN based on DOY were bounded with the given site's latitude and valleys of low observation performance for argument of perigee and RAAN were found giving the mission designers the elasticity of selecting proper initial orbital parameters for the region of interest. Those outputs were derived by the utilization of access criteria of Geometry, Altitude and Daylight Conditions. The results were tested with an algorithm (developed during this study for the relief on computational burden and achieving high speeds with moderate accuracy) of access number estimation and the performances of the mission orbit candidates were evaluated with *PFM* formulation (developed and parameterized by FoM values of mission orbit candidates during this study).

The high dependence on the relative positioning of Sun-Earth-Satellite system for the maneuverability of solar sailing was taken into consideration by conducting a thorough analysis over the maneuver planning strategy. The effects of relative geometry of Sun-Earth-Satellite system were under evaluation with the help of *Significant Parameters* consisting of angles, directions and orbital parameters. The relations were then deduced with an output of seasonal intervals (DOY-based) for the efficiency of in- and out-of-plane maneuvers based on the orientation of the employed orbit (inclination, argument of perigee and RAAN). The results gave an option to the mission designers to properly select the orbital parameters within the time of the year for their specific needs of orbital maneuvers.

Finally, three critical problems for the validation of the proposed concept of R2S2 were addressed: Orbit Control, Attitude Control and Constellation Design.

Regarding the orbit control, the key development was the modification of GVEs in the favor of elliptic orbit mission dynamics by using apogee and perigee radii/altitudes instead of semi-major axis and eccentricity. The modification of the GVEs were then utilized by two proposed approaches. First approach was realized

by approximating the GVEs and deriving a relation with the control force direction for the specific orbital parameter change with respect to true anomaly of the orbit. The approximation of the GVEs in the first approach was accomplished by two different methods: constantly feeding the current orbital parameters (used in transfer orbit with uncoupled dynamics of orbital elements) and using the preset orbit parameters only at the beginning (used in the station-keeping of the mission orbit with coupled dynamics of orbital elements). In the latter, the sole dependence on true anomaly was stretched by adding variables of inclination and argument of perigee. The second approach was accomplished by using a Lyapunov-based orbital element feedback nonlinear control method. This approach was used in transfer orbit maneuvers of all kinds. The results were simulated and performance of 100 km/month orbit size changing capability and 1.5 deg/year inclination variation attribute were achieved. However, the control authority over argument of perigee and RAAN was not enough due to the secular effect of J2 perturbation of the asphericity of Earth with the accepted characteristic acceleration levels of R2S2 concept.

Later, the attitude control methodology was implemented by using a quaternion feedback control law. The attitude control mechanism with high torque-high momentum capacity (250-300 seconds for scanning a 90 degree attitude trajectory) was needed to achieve the necessary and frequent (twice in one orbit revolution) large angle slewing maneuvers which were commonly avoided in the literature of solar sailing. Thanks to the proper selection of the mode switching altitude and calculation of the in advance time of the start of slewing maneuver, the risks of exposing to excessive air drag and limited allowed sailing time were all handled and proposed elasticity of the mission concept was consistent with the attitude control mechanisms at running.

Moreover, an attitude trajectory planning based on the 5th-order polynomial guidance accompanied with a low-pass filter assured a low-frequency operation of rotational maneuvers for large angles. With the modification of nonlinear quaternion feedback

control methodology that takes the desired attitude trend into account, the smooth and low-jerk reorientation of the large solar sail structure have been accomplished.

Overall results depicted above also brought some limitations of the R2S2 Concept onto table. Most of them were stemming from the achieved characteristic acceleration levels under the consideration of high TRL values and some of them were caused by the relative geometry of Sun-Earth-Satellite system. Nonetheless, the responsiveness of the concept should be assured by proposing a constellation. Even though the concept of R2S2 aims and relies on single satellite solution, a constellation of satellites were designed due to the reasons mentioned above. The resultant system of satellites were placed in the Earth orbit with phasing on inclination, argument of perigee and RAAN. Depending on the requirements of mission designers on allowed time for low observation performance, the number of satellites vary. However, the responsiveness of the R2S2 Concept was assured with orbital planes separated from each other by 5-degree inclination phasing ranging from 40 to 65 degrees of latitude based on the analysis over the land/water distribution and population density information of the Earth. Later phasing of argument of perigee to compensate for the low observation performance valleys due to altitude criteria and phasing of RAAN to circumvent the valleys due to daylight criteria were also analyzed and tested depending on the requirement levels of allowed time for low observation performance intervals.

The R2S2 concept operating at LEO may be used for post-disaster monitoring of regions of interest with a high observation performance. Moreover, the flexibility of altering orbital parameters in request will bring the mission designers to task fewer satellites to more various locations over the globe. This concept of operation at LEO is also orbital debris friendly since the limitless orbit maneuvering capability is allowing to deorbit the satellite at the end of its mission lifetime.

For further improvements of the study, there should be some technological improvements and eventually an increase over the TRL of solar sail systems with high characteristic acceleration. The utmost advantage of the proposed concept of

R2S2 will be achieved with characteristic acceleration levels greater than 1 mm/s^2 . Moreover, the material based modifications to the classical solar sail geometry, allowing an extension on the directing capability of the sunlight to the close proximity of the transverse axis, will also be a critical milestone on achieving lower altitudes and even solar sailing maneuvers below mode switching altitudes. Finally, shape deforming technologies and reliable folding mechanisms might lead the way of using deployed solar sails in the cases of maneuvers only and stowing the sails otherwise, in successive manner. This feature, if succeeded and brought to the high values of TRL, can change the tendency on the use of conventional propulsion systems for orbital maneuvering needs at LEO.



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APPENDICES

A. Definitions of Technology Readiness Levels

TRL 1 Basic principles observed and reported: Transition from scientific research to applied research. Essential characteristics and behaviors of systems and architectures. Descriptive tools are mathematical formulations or algorithms.

TRL 2 Technology concept and/or application formulated: Applied research. Theory and scientific principles are focused on specific application area to define the concept. Characteristics of the application are described. Analytical tools are developed for simulation or analysis of the application.

TRL 3 Analytical and experimental critical function and/or characteristic proof-of-concept: Proof of concept validation. Active Research and Development (R&D) is initiated with analytical and laboratory studies. Demonstration of technical feasibility using breadboard or brassboard implementations that are exercised with representative data.

TRL 4 Component/subsystem validation in laboratory environment: Standalone prototyping implementation and test. Integration of technology elements. Experiments with full-scale problems or data sets.

TRL 5 System/subsystem/component validation in relevant environment: Thorough testing of prototyping in representative environment. Basic technology elements integrated with reasonably realistic supporting elements. Prototyping implementations conform to target environment and interfaces.

TRL 6 System/subsystem model or prototyping demonstration in a relevant end-to-end environment (ground or space): Prototyping implementations on full-scale realistic problems. Partially integrated with existing systems. Limited documentation available. Engineering feasibility fully demonstrated in actual system application.

TRL 7 System prototyping demonstration in an operational environment (ground or space): System prototyping demonstration in operational environment. System is at or near scale of the operational system, with most functions available for demonstration and test. Well integrated with collateral and ancillary systems. Limited documentation available.

TRL 8 Actual system completed and "mission qualified" through test and demonstration in an operational environment (ground or space): End of system development. Fully integrated with operational hardware and software systems. Most user documentation, training documentation, and maintenance documentation completed. All functionality tested in simulated and operational scenarios. Verification and Validation (V&V) completed.

TRL 9 Actual system "mission proven" through successful mission operations (ground or space): Fully integrated with operational hardware/software systems. Actual system has been thoroughly demonstrated and tested in its operational environment. All documentation completed. Successful operational experience. Sustaining engineering support in place.

Figure A.1. NASA Standards for Technology Readiness Levels [70]



B. Region of Interest List

Table B.1 Locations of the Regions of Interest

City Name	Latitude [deg]	Longitude [deg]	Altitude [meter]	Position Vector in ECEF [x-y-z] [m]		
Tokyo	35.7	139.8	15	4724855	2865420	3174609
Delhi	28.7	77.1	264	-2179088	4388013	4070177
Sao Paulo	-23.5	-46.6	769	2262539	5329949	2665622
Mexico City	19.4	-99.1	2236	2755372	-4475467	-3601629
Cairo	30.0	31.2	25	168221.1	5889816	2433878
Beijing	39.9	116.4	34	6326071	373511.1	720191.5
Karachi	24.9	67.0	19	6135211	1674376	-490504
Buenos Aires	-34.6	-58.4	31	-2317662	5391391	2490317
Kolkata	22.6	88.4	101	2849175	2195269	5249812
Lagos	6.5	3.4	18	1335816	6072103	1422591
Kinshasa	-4.4	15.3	465	1745169	-6115120	520882.5
Guangzhou	23.1	113.3	117	-1837418	6068976	-684911
Moscow	55.8	37.6	207	1398867	-6079974	-1321974
Bangalore	13.0	77.6	910	-1129298	6092551	1506993
Bogota	4.7	-74.1	2573	-3044686	4043811	3867596
Jakarta	-6.2	106.8	16	3977957	-8817.43	4968935
Lima	-12.0	-77.0	131	3236893	4053190	3700986
Bangkok	13.8	100.5	15	1334058	-4653945	4138408
Seoul	37.6	127.0	78	5029207	2679258	-2859434
London	51.5	-0.1	24	-4646476	2553471	-3533800
Tehran	35.7	51.4	1159	4226338	2381419	4127137
New York	40.7	-74.0	10	4724855	2865420	3174609
Johannesburg	-26.8	28.0	1738	-2179088	4388013	4070177
Sydney	-33.9	151.2	96	2262539	5329949	2665622
Istanbul	40.6	29.4	40	2755372	-4475467	-3601629

Table B.2 Populations and Identifiers of the Regions of Interest

City Name	Population [million]	Identifier
Tokyo	37.4	1
Delhi	29.3	2
Sao Paulo	21.8	3
Mexico City	21.7	4
Cairo	20.4	5
Beijing	20	6
Karachi	15.7	7
Buenos Aires	15	8
Kolkata	14.7	9
Lagos	13.9	10
Kinshasa	13.7	11
Guangzhou	12.9	12
Moscow	12.4	13
Bangalore	11.8	14
Bogota	10.7	15
Jakarta	10.6	16
Lima	10.5	17
Bangkok	10.3	18
Seoul	9.9	19
London	9.1	20
Tehran	9	21
New York	8.6	22
Johannesburg	5.6	23
Sydney	4.8	24
Istanbul	14.9	25

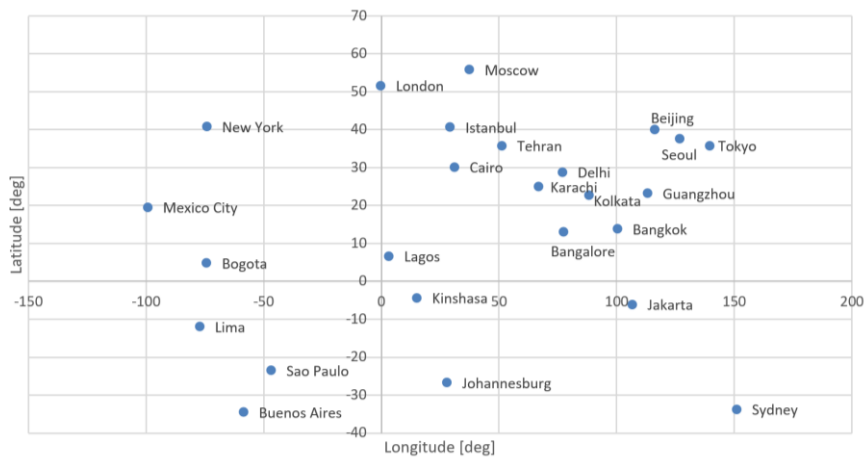


Figure B.2. Regions of Interest Distribution over the Globe

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Polat, Halis Can

EDUCATION

Degree	Institution	Year of Graduation
MS	Naval Postgraduate School, Space Systems Operations, Monterey, CA, USA	2016
BS	Turkish Air Force Academy Aeronautical Engineering, İstanbul	2009
High School	Kuleli Military High School, İstanbul	2005

WORK EXPERIENCE

Year	Place	Enrollment
2009-Present	Turkish Air Forces	Officer

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

1. Polat, H. C., "Prototype Design and Mission Analysis for a Small Satellite Exploiting Environmental Disturbances for Attitude Stabilization," Master's Thesis, Naval Postgraduate School, Monterey, CA, 2016.
2. Polat, H. C., Virgili-Llop, J., and Romano, M., "Survey, Statistical Analysis and Classification of Launched CubeSat Missions with Emphasis on the Attitude Control Method," *Journal of Small Satellites*, Vol. 5, 2016, pp. 513–530.

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