HEALTHCARE NETWORK DESIGN AND INVENTORY DECISIONS FOR DISASTER MANAGEMENT UNDER UNCERTAINTY

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PhD Dissertation

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ABSTRACT

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Large-scale disasters have occurred in many parts of the world in recent years. A large number of victims suffer from these disasters. Survival of victims after the disaster is largely based on humanitarian assistance. Focusing on these humanitarian assistances, firstly, we consider healthcare location decisions for providing efficient service in a disaster situation and mobile hospitals can be utilized for this purpose. We introduce a network design model considering the location and re-location decisions of mobile hospitals using a two-stage stochastic programming model with various disaster scenarios. We apply our models to a real-life case study for an expected earthquake in Istanbul and extract several managerial insights. Numerical experiments demonstrate that mobile hospitals can provide significant improvements in healthcare service. Secondly, we aim to decide on the amount of relief aid material to procure before and after the disaster considering the budget limitations and focus on how to allocate the available budget among different materials. For this purpose, we first develop single product models with and without budget constraints. Then, we build a two-stage stochastic model with budget constraint for the two-product case and determine its optimal solution. Then, we extend our analysis for multiple product cases and propose approximation algorithms. We compare the results of the models via detailed numerical studies and extract managerial insights for disaster preparedness. Finally, in the last part of this thesis, we focus on the multi period stocking decisions in case of a disaster. We present a dynamic programming model to decide on the optimal stocking and ordering policy considering the budget limitations, random timings and quantities of donations and random demand for materials. The managers need to make a decision at any point in time considering the available budget and available stock at that time. Due to the dynamic nature of this system, the budget and the stock changes over time due to arriving donations and use of materials during disasters. We develop dynamic programming models for this system and extract managerial insights through the solutions of these models.

Keywords: Disaster, Healthcare, Inventory, Newsvendor, Dynamic Programming

ÖZET

AFET YÖNETİMİNDE BELİRSİZLİK ALTINDA SAĞLIK AĞI TASARIMI VE ENVANTER KARARLARI

Müge ACAR

Endüstri Mühendisliği Anabilim Dalı
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Danışman: Prof. Dr. Onur KAYA

Son yıllarda dünyanın birçok yerinde büyük çaplı felaketler meydana gelmektedir. Çok sayıda insan bu felaketlerden etkilenmektedir. Afet sonrası, afetzedelerin hayatta kalması büyük ölçüde insani yardıma dayanmaktadır. İnsani yardımlar olarak, öncelikle bir afet durumunda verimli sağlık hizmeti sunmaya yönelik mobil hastanelerin yerleşim kararları incelenmiştir. Çeşitli afet senaryolarına sahip iki aşamalı stokastik bir programlama modeli kullanarak mobil hastanelerin konum ve yeniden yerleştirme kararlarını dikkate alan bir ağ tasarım modeli sunulmuştur. Modellerimiz İstanbul'da beklenen bir deprem için gerçek hayattaki bir vaka çalışmasına uygulanmıştır ve çeşitli yönetim anlayışları elde edilmiştir. Sayısal deneyler, mobil hastanelerin sağlık hizmetlerinde önemli iyileşmeler sağlayabildiğini göstermektedir. İkinci olarak, bütçe kısıtlamaları ve bütçenin farklı malzemeler arasında nasıl tahsis edileceği göz önünde bulundurularak, afet öncesi ve sonrasında stoklanacak yardım malzemesi miktarlarını belirlemek amaçlanmıştır. Bu amaçla, bütçe kısıtlamaları olan ve olmayan tek ürün modelleri geliştirilmiştir. Ardından, iki ürünlü vaka için bütçe kısıtı altında iki aşamalı stokastik bir model oluşturulup, optimal çözüm elde edilmiştir. Daha sonra, analizimiz çok ürünlü durum için genişletilip ve yaklaşık algoritmalar önerilmiştir. Modellerin çözümüyle elde edilen sonuçlar detaylı sayısal çalışmalar ile karşılaştırılmış ve afete hazırlık için yönetsel bilgiler elde edilmiştir. Son olarak, bir afet durumunda çok dönemli stok kararlarına odaklanılmıştır. Bütçe sınırlamaları, rasgele zamanlamalar, bağış miktarları ve malzeme talebi dikkate alınarak optimal envanter siparişi politikasına karar vermeye odaklanan dinamik bir programlama modeli sunulmuştur. Sayısal analizlerle yönetsel bilgilere ulaşılmıştır.

Anahtar Sözcükler: Afet, Sağlık, Envanter, Gazete Bayii Problemi, Dinamik Programlama

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I would like to dedicate this dissertation to my husband, Özgür, and my son, Çınar. Without Özgür's invaluable support, patience, and encouragement, it would be very difficult to complete this work.

Müge ACAR

22/07/2020

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with “scientific plagiarism detection program” used by Eskisehir Technical University, and that “it does not have any plagiarism” whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.

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1. INTRODUCTION

Disasters that have occurred since the existence of the world affect the lives of societies, environment, economies of countries, and cause loss of life. For example, the Covid-19 pandemic, which appeared in early 2020, has affected many people and covered the whole world. According to the report published by the world health organization in June 2020, about 7 million people were infected, 400.857 people died, and many people are receiving treatment. (WHO, 2020). Disaster management involves performing some operations before and after a disaster occurs to reduce its harmful effects (Govindan et al., 2020). These operations can be listed under four main categories: mitigation, preparedness, response and recovery (McLoughlin, 1985). Mitigation involves providing activities to reduce the risk of a long-term effect of disaster or its potential consequences. Preparedness includes all activities fulfilled before the disaster in order to get a more effective response. The response phase includes activities related to the deployment of vital resources to serve the affected population, after the disaster occurs. And finally, recovery refers to short and long-term activities to restore the normal functioning of the community. (Galindo and Batta, 2013).

In this study we discuss preparedness and response activities of the disaster life-cycle considering mainly location and inventory decisions. Basically, our research consists of three main parts. Firstly, we evaluate facility location decisions to provide efficient healthcare service to the affected people. Secondly, we focus on decisions regarding inventory planning of relief supplies to be distributed to affected people. Finally, we consider inventory decisions in a continuous dynamic setting. When evaluating these issues, we consider operations and decisions both before and after the disaster.

In Section 2, we present a literature review with three subsections. In Section 2.1 we summarize related literature in location decisions for disaster relief. Section 2.2 focus on the literature for inventory decisions in case of a disaster. In Section 2.3 concludes the literature review by briefly outlining multi-period inventory decisions for disaster management.

In Section 3, we introduce a network design model considering the location and re-location decisions of mobile hospitals using a two-stage stochastic programming model with various disaster scenarios. We propose to utilize advanced mobile hospitals for disaster preparation as shown in Figure 1.1. Since these hospitals are easily to move and set up, they can be used before the disaster as regular emergency medical centers (EMCs) and when a disaster happens, they can be easily transported to the disaster areas to provide immediate medical response for casualties. These mobile hospitals include an emergency department, surgical suite, critical care beds, and general treatment and admission area. They can provide comprehensive diagnostic and definitive patient care. Thus, they can be used both before the disaster as regular hospital units and after the disaster for efficient response. Detailed explanations about the operations and structure of these facilities can be found in Blackwell and Bosse (2007) and Cheng et al. (2015). In Section 3, we analyze the pre and post-disaster location, capacity and service decisions for such mobile hospitals considering pre and post-disaster response. This problem addresses critical decisions of governments and relief organizations for disaster preparedness, in designing health care network and improving quality of health care service for the affected people.

(a) (b)

Figure 1.1 *Mobile hospitals in a) operation b) movement (Blackwell and Bosse, 2007)*

In the literature, we observe that such location decisions are made only either before or after the disaster. Since the locations and the effects of a disaster are not known before it happens, making the location decisions before the disaster might not suitably meet the demand in the post-disaster environment. The demand at some regions might be very high and the capacity at those regions might be insufficient to cover that demand, while the capacity at other regions might be high while there is little demand if the disaster did not affect that region significantly. In addition, if these facilities are only built for disaster response, they are kept idle until a disaster happens, which is a serious loss of resources. On the other hand, making the EMC location decisions after the disaster also has serious drawbacks. First of all, advanced centers can not be built at the affected regions in a very short time and generally basic first aid medical centers or temporary emergency centers are located in the affected regions after the disaster happens. In such situations, many people who need advanced treatment are routed to regular hospitals after their first aid treatment at these centers, leading to a surge of demand at the regular hospitals, and long waiting times due to insufficient capacity. In addition, supplying the necessary equipment, materials and personnel to these centers after the disaster requires a certain time and leads to significant delays in providing rapid response. The literature on location models for disaster preparedness can be described as either pre-disaster or post-disaster location models. Different from the literature, when mobile hospitals are used, location and capacity decisions are made before the disaster, and re-location decisions are made after the disaster. Thus, our model is a combination of pre-disaster, and post-disaster location models in the literature. Our model helps to eliminate the disadvantages of either models in the literature due to the mobile characteristic of the analyzed hospitals and the re-location decisions made in the model.

Majority of the location decision literature considers, only the travel times between the demand sites and the selected facility locations waiting times are ignored. However, in addition to access times to the EMCs, the waiting times at these centers also pose a serious problem in case of disasters when demand is much higher. In this study, we consider the waiting times at the EMCs and integrate them into our model. We also consider possible damages of the disasters to the existing EMCs, when making the location and allocation decisions. We introduce a network design problem in order to determine the number and capacities of mobile hospitals that need to be acquired before the disaster, the re-location and allocation decisions in post-disaster situations considering the travel times, expected waiting times, and possible facility damages in the post-disaster environment. We propose a two-stage stochastic programming model for this purpose considering different disaster scenarios, in order to minimize the total response time in both pre and post-disaster environments considering the movements of the mobile hospitals when a disaster happens.

In Section 4, we evaluate the inventory decisions for efficient disaster response. It is one of the duties expected from Non-governmental organizations (NGOs) to assess the inventory decisions and to meet the demand of certain aid materials using their budget. However, due to the unpredictability of the disaster and insufficient foreknowledge of demand, the number of materials to be needed cannot be predicted precisely and situations such as missing or excessive materials may be encountered. In such cases, it becomes necessary to purchase the missing relief items after the disaster to eliminate these shortcomings, if possible. Thus, NGOs need to decide on the stocking quantities of pre-disaster relief materials and/or the purchase of the missing items after the disaster under their budget limitations. In the literature (Silva, 2010; Donahue and Tuohy, 2006), it is specified that there is a limited budget in order to carry out aid activities like stocking relief materials and supplying missing items after the disaster. In the humanitarian relief literature, most of the models ignore budget limitations and assume that any relief material can be purchased as it is needed. However, in reality, there exists budget constraints and all investments to the aid activities are made with that budget; both stocking before the disaster and providing materials after the disaster. Thus, the amount of relief aid materials to be kept in stock is not just dependent on demand but budget constraints have to be considered in making these decisions.

We also evaluate the inventory decisions for efficient disaster response. We aim to decide how many of each type of material to procure before and after the disaster considering the budget limitations and how to allocate the budget among different materials. For this purpose, we first develop single product models with and without budget constraints. Then, we build a two-stage stochastic model with budget constraint for the two-product case and determine its optimal solution. We extend our analysis for multiple product cases and propose approximation algorithms. In Section 5, we evaluate inventory decisions in a dynamic setting considering multiple periods. NGOs do not make one-time decisions they operate and make decisions about stock management in a continuous manner. During these operations, they have to manage their budget effectively. Many studies in the literature examining inventory management decisions in case of disasters make their inventory decisions assuming an unlimited budget, which might be an unrealistic assumption. In real life, aid materials are generally supplied using the income and the budget of the related organization.

Even though different NGOs might have different sources of income, donations made by the public is one of their main sources and donations made at different times change their budget. With the changing budget, the decisions about the stocking quantities of aid materials might also change. In addition, since the timing of the disasters are also unknown, the stocked materials might be required at any time when the disaster occurs. The amount of the stocked materials may decrease due to the demand in the case of a disaster. For this reason, NGOs need to decide to place new orders based on their budgets on hand and the amount of stock available at any time considering future needs and possible changes in the budget in a dynamic manner.

Finally, the last section, Section 6, concludes the study and points out some possible future research directions.

2. LITERATURE REVIEW

We examine the literature in three parts as facility location in disaster environment (related to Section 3), inventory systems for humanitarian relief literature (related to Section 4) and inventory management in multi period perspective (related to Section 5).

2.1. Facility Location Decisions for Disaster Relief

In this section, we provide the literature related to facility location decisions in disaster management. There are many studies that discuss different types of facilities which have importance for casualties. There are numerous studies that discuss location decisions for warehouses in which the crucial relief items are stored, like tents, water or medicine (e.g. Rawls and Turnquist, 2010; Mete and Zabinsky, 2010; Doyen et al., 2012). In these studies, facility location decisions are made in order to maximize the expected demand covered when resources are limited (e.g. Balcik and Beamon, 2008; Salman and Yucel, 2015). Also, some studies focus on meeting the demand with minimum fixed and transportation costs (e.g. Rawls and Turnquist, 2011; Noyan, 2012). There are also other studies considering the minimum weighted distance in which demand also depends on the distance (e.g. Gormez et al., 2011; Malek et al., 2016). Lu (2013), Bell et al. (2014) and Espejo et al. (2015) consider p-center facility location problems. Javadi and Lee (2016) propose a routing problem that minimizes travel time and latency time which is defined as the time of victims waiting to reach the relief materials they need. Loree and Aros-Vera (2018), in addition to travel time, suggest deprivation cost. All these types of facilities are assumed to be stationary which cannot be moved according to the circumstances of the disaster. Since disasters create an uncertain environment, many studies may have to face additional costs with stationary facilities.

When facility location literature on disaster management is considered number of studies that take into account the location of the emergency medical centers are less than the studies on warehouse location and allocation. Revelle and Snyder (1995) address emergency medical center location problems considering maximum demand covered. Salman and Gul (2014) consider minimum weighted response time to determine ambulance station locations. Gunes and Nickel (2015) review the healthcare facility location problems considering various facility types.

Emergency events happen suddenly and unexpectedly but drive a surge of demand for all types of emergency services (Javid, 2017). After the effects of the disaster vanish, there is no need to run such emergency facilities. Temporary emergency facilities (TMC) are adequate for such events. In the literature some studies consider casualty collection points (CCP), points of dispensing (POD) as parks, schools, public buildings or tents. These types of facilities are used for gathering or for first aid to casualties or/and for distribution of some drugs. Drezner (2004) and Drezner and Drezner (2006) apply p-median, p-center and some variants of them to CCP location problem. Apte et al. (2014) suggest utilization of personnel and decontamination units and ambulances, and provide a general resource allocation plan with predetermined CCP locations as a p-center problem. Murali et al. (2012) consider POD location decisions in order to process catastrophic bio-terror attack. Huang et al (2010) study locations of TMCs in case there is a disruption in existing hospitals without considering uncertainty. They maximize the number of people covered within a given distance. Chen (2016) utilizes a heuristic for CCP location problems in a post-disaster environment. They assume that the hospital capacities are unlimited in post-disaster phase. Although it is less costly and convenient to meet the high demand on PODs or CCPs like public parks and schools, advanced treatment cannot be done at these locations and many patients still need to be directed to regular hospitals after their first-aid treatment at these locations, leading to high demand and waiting times at the hospitals. This results in insufficient healthcare response and significant discomfort for the patients. In addition, the affected areas may not be reached in a short period of time from the PODs or CCPs if their locations are determined at the pre-disaster phase. The need for medical equipment for extra demand may not be met on time from other facilities, which may incur additional costs.

In the literature, most of the studies assume that existing facilities are not affected by the disaster (e.g. Mete and Zabinsky, 2010; Mohammadi et al., 2016; Rabbani et al., 2015). However, this assumption might be unrealistic. A few studies in the literature consider possible damages to these facilities. Huang et al. (2010) suggest to use medical centers that are closest to the demand points considering whether those facilities are failed or not. Galindo and Batta (2013) study prepositioning relief supplies in a warehouse in order to provide minimum response time for delivering materials on post-disaster considering partial damage to warehouses. Paul and MacDonald (2016) discuss location decisions for warehouses of relief supplies evaluating the risk of facility damages. In this study, we consider possible damages to the existing healthcare facilities, too.

Disaster management concerns with actions that need to be performed to meet the needs of people affected by disaster. Disaster management is generally examined in two phases; pre-disaster and post-disaster. In order to reduce the impact of the disaster preparedness and response activities are executed according to these phases. Even though there are not many studies considering emergency healthcare operations in pre and post-disaster phase, there are many models using two stage stochastic models considering aid material stockpile and their distribution (e.g. Balcik and Beamon, 2008; Rawls and Turnquist, 2010; Noyan, 2012; Hamdan and Diabat, 2019). The studies in the literature may consider different decisions in this two phase modeling according to the content of the studies. Rawls and Turnquist (2011) consider location and material prepositioning decisions in the first stage and transportation of unused inventory decisions in the second stage. Bozorgi-Amiri (2012) address location decisions in the first stage and transportation, inventory and shortage decisions in the second stage. Noyan (2012) consider location decisions and prepositioning items in the first stage and transportation, shortage decisions in the second stage. Hu et al. (2015) propose a two stage model in which location is the first stage decision; transportation and delayed time are the second stage decisions. Fahimnia et al. (2017) consider blood supply operation decisions in which the number of blood supply facilities is the first stage decision while location allocation and inventory decisions are given in the second stage. Hamdan and Diabat (2019) consider the same decisions about blood supply via mobile blood collection facilities.

In Section 3, we propose a two stage stochastic optimization model that determines the locations and capacities of mobile hospitals for pre and post-disaster phases. We present a case study for large scale emergency due to an expected earthquake in Istanbul and analyze the impact of uncertainty by evaluating possible earthquake scenarios. We consider the cost of mobile hospitals and the travel time of patients in addition to their waiting times at these centers considering the demand and capacities of the hospitals.

Table 2.1. *Comparison of Section 3 with the most related studies in the literature*

	Uncertainty	Two stage	Pre-disaster Location	Post-disaster Location	Waiting Time	Mobile	Facility Damage	Case Study	Equity
This Study	+	+	+	+	+	+	+	+	+
Drezner (2004)				+				+	+
Drezner et al. (2006)				+				+	+
Balcik and Beamon (2008)	+	+	+						
Paul and Batta (2008)	+		+				+	+	+
Huang (2010)	+		+				+		+
Rawls and Turnquist (2010)	+	+	+					+	
Gormez et al. (2011)	+	+	+	+				+	
Bozorgi and Amiri (2012)	+		+				+		
Doyen et al. (2012)	+	+	+	+					
Noyan (2012)	+	+	+					+	
Apte et al. (2014)				+				+	+
Salman and Gul (2014)				+	+			+	
Salman and Yucel (2015)	+	+	+					+	
Javadi and Lee (2016)				+	+				
Malek et al. (2016)	+	+	+				+	+	
Mohammadi et al. (2016)	+		+						+
Paul and MacDonald (2016)	+		+				+	+	

In Table 2.1, we present the comparison of our study in Section 3 with the most related papers in the literature. The main differences of our study compared to the above studies and our contributions to the literature can be summarized as below:

i) In our model, due to the displacement feature of mobile hospitals, location and allocation decisions are made at both pre and post-disaster stages considering the re-location ability of the mobile hospitals. However, in the literature, facilities are not allowed to move and facility location decisions are generally only made either before the disaster under uncertainty for disaster preparedness (e.g. Balcik and Beamon, 2008; Rawls and Turnquist, 2010; Noyan, 2012; Hamdan and Diabat, 2019) or after the disaster for disaster response (e.g. Drezner, 2004; Drezner and Drezner, 2006; Lee et al., 2009; Murali et al., 2012; Apte et al., 2014; Nafarrate-Ramirez et al., 2015). Our model includes facility location decisions for both disaster preparedness and disaster response. In our best knowledge, this is the first study that considers location decisions in the first stage and re-location decisions in the second stage considering movable facilities.

ii) We consider the waiting times at the hospitals explicitly, in addition to the travel times of the patients since waiting times of casualties at the EMCs are critical and we believe that it has great importance in making healthcare facility location decisions. It is commonly seen that some of the hospitals and EMCs are overcrowded, leading to long waiting times, while others have idle capacity. In order to obtain a balanced workload, we also consider equity among the hospitals in making the location and allocation decisions. The imbalance between the workloads of the hospitals are penalized in our model. Consideration of equity among hospitals allows the model to provide more balanced location and allocation decisions. As a result, the decisions will be affected by the workloads of the regular hospitals and not all casualties will be allocated to the closest hospital if it leads to long waiting times at those hospitals. Instead, some casualties are allocated to the hospitals with idle capacity, which might cause a longer transportation time but a shorter waiting time, if the savings exceed the costs.

iii) Our model includes the existing facilities at their current locations, in addition to new ones that will be built at potential locations. Most of the studies in the literature either assume that the existing facilities are not affected by the disaster and perform in full capacity after the disaster happens, or they do not consider these facilities at all. However, it is widely observed in reality that the existing facilities are also affected by the disasters and their capacity might be decreased or they might be fully dysfunctional after the disaster. In this study, we consider facility damages as well and assume that some of the existing hospitals may have decreased capacities depending on their location and the effect of the disaster.

2.2. Inventory Systems for Humanitarian Relief Literature

In disaster management literature, Donahue and Tuohy (2006) present a study about the lessons learned from disasters such as Hurricane Andrew 1992, Oklahoma City bombing 1995, September attack 2001, Hurricane Katrina 2005. They report that some managerial difficulties were experienced on those disasters such that, inventory management was not done well before disaster. It is stated that there were an abundance of materials that are unusable on the depots after disaster. The reason for this problem may be that budget, which is spent for pre-disaster stocking, is more than necessary. In addition, it may have been less costly to make pre-disaster stocking with less budget and procure the missing materials after the disaster. In this way wasted investments could be prevented.

Silva (2010) presents an analysis of Aceh earthquake that happened in 2004. It is reported that since some relief materials were stocked less than the required quantity, scarcity was experienced. For this reason, those relief products were sold at high prices in Aceh in the post-disaster phase. This leads to a significant increase in inflation in the economy of Indonesia. The country fell into a rough economic condition after the disaster because of this situation. As can be seen here, keeping stocks at the right amount has a great importance and it is crucial to manage the budget effectively for expenditures at both pre-disaster and post-disaster stages.

In both studies (Silva, 2010; Donahue and Tuohy, 2006), it is specified that there is a limited budget in order to carry out aid activities like stocking relief materials and supplying missing items after the disaster. In the humanitarian relief literature, most of the models ignore budget limitations and assume that all relief materials can be purchased as it is needed. However, in reality, budget constraints exist and all investments of the aid activities are made with that budget; both stocking before the disaster and providing materials after the disaster. Thus, the amount of relief aid materials to be kept in stock is not just dependent on demand but budget constraints have to be considered in making these decisions.

In the humanitarian relief literature, inventory systems have been considered by several studies (Balcik and Beamon, 2008; Galindo and Batta, 2013; Döyen et al., 2012; Klibi et al., 2013; Loree and Aros-Vera, 2018). Balcik et al. (2016) classify studies about inventory management in humanitarian supply chains based on disaster management phases as pre-disaster and post disaster. There are many studies that examine the decisions in pre-disaster stage, such as locations of distribution centers, optimal quantities of inventory to be stocked, etc. Decisions made in pre-disaster stage are long term and can be considered as strategic decisions. Studies considering this kind of decisions generally utilize stochastic programming methods. (Duran et al. 2011; Hong et al. 2015; Mohammadi et al. 2016). Specifically, two-stage stochastic programming is the most common modeling approach (Balcik and Beamon, 2008; Galindo and Batta, 2013; Klibi et al., 2013). On the other hand, studies dealing with post-disaster decisions generally use deterministic methods (Loree and Aros-Vera; 2018). A few studies focus on finding optimal inventory amounts through the use of newsvendor models. Lodree and Taskin (2008) utilize a newsvendor model in order to determine the stock quantities for disaster situations. Campell and Jones (2011) use a newsvendor based model in order to evaluate where and how much to pre-position items. Yao et al. (2018) examine optimal ordering quantities with a newsvendor model by taking into consideration outsourcing strategies considering perishable and imperishable commodities. Coskun et al. (2018) evaluate a newsvendor model with two NGOs considering a game theoretic approach. Toyasaki et al. (2017) consider an umbrella organization providing relief materials and focus on NGOs' stocking decisions under the umbrella organization. Chen et al. (2018) consider one NGO with multiple products and develop a newsvendor model with and without budget limitations. However, they assume that the materials can only be acquired before the disaster and possible post-disaster acquisitions are neglected. In their model, budget is only limiting in the acquisition of materials in the pre-disaster stage and budget is not considered for post disaster activities, which is not very realistic. NGOs need to use their budget for both pre-disaster and post-disaster acquisitions and thus need to decide how much of their budget to use before the disaster and how much to keep for post-disaster requirements. Stocking the items before the disaster might be cheaper per unit, however due to the uncertainty of demand, especially when multiple products are considered, not using all the budget and keeping a certain amount for post-disaster usage might be more beneficial.

In Section 4, we present newsvendor models in order to decide on the optimal stocking quantities in a two-period setting under a limited budget, which has not been studied in the literature before. Due to uncertainty in demand, after the first period stocking decisions, there might be some excessive products or missing relief materials, which is revealed after the disaster. In the post-disaster phase, it may be needed to purchase the missing materials if there is remaining budget for that stage. We determine the optimal allocation policy for the remaining budget after the disaster, if the remaining budget is not sufficient to cover the unmet demand of all types of products. So, considering the limited budget, some part of that budget has to be allocated to be used before the disaster for stocking the materials and the rest to be used to purchase the required materials after the disaster. We determine the optimal stocking quantities for relief materials before the disaster, considering both single and two product cases, and also find out the optimal allocation policy for spending the budget to satisfy the unmet demand after the disaster. We then extend our analysis to multiple product cases and propose approximation algorithms for multi-product models under budget constraints. Finally, detailed numerical results considering different parameter settings are presented and significant managerial insights are extracted from these results.

2.3. Inventory Management in Multi Period Perspective

In this subsection, we focus on the literature related to multi period inventory decisions about meeting the demand for aid materials in case of disasters. In Section 5, we model a dynamic programming problem in order to find the optimal ordering and stocking amounts based on available budget and stock on hand at each period. In our model budget may change every period by the donations and the model decides on the ordering amounts considering disaster risk for every period. When we evaluate the literature, there are not many studies on dynamic programming and inventory decisions in disaster management. Lodree and Taskin (2008) present a dynamic programming model considering inventory control problem.

Even though dynamic inventory models are not extensively analyzed in disaster management literature, there are several studies in the inventory management literature that have similarities to our study. When there is no product in stock to meet the demand, the excess demand can either be backordered or lost. Our problem can be considered to be in the context of inventory decisions with lost-sales without fixed cost and without lead time. For this reason, we focus on especially the literature that considers inventory literature with lost sales. Vijayan and Kumaran (2008) review the inventory literature with lost sales considering: continuous review (Johansen and Thorstenson 2004; Mohebbi and Posner 2002) and periodic review (Huh and Janakiraman 2010b; Janakiraman et al. 2007) models. As examples of periodic review studies, Xu et al. (2010) consider optimal (s,S) policy for finite and infinite horizon inventory problem with lost sales. They assume that there is a fixed ordering cost in order to evaluate the cost function. Zhang et al. (2019) present an algorithm taking into account periodic review inventory problem with nonzero lead times. Babiloni and Guijarro (2020) assess continuous review inventory system with discrete demand and lost sales, aiming to meet the customer service level. Li (2015) introduces dynamic programming model considering stochastic demand, lost sales and positive lead time. Bensousan et al. (2008) develop a dynamic programming inventory model with partial demand and lost sales. They consider available stock level and demand for current period in order to calculate the optimal inventory level. Chao et al. (2008) build a dynamic programming model that decides on the optimal stock policy with lost sales considering the capital on hand. They decide to purchase the products or evaluate the capital in a bank account at every period. Similar to our study, considering the random nature of demand, this study also evaluates the tradeoff between using the budget to increase the inventory now, and keeping it as cash to be able to buy more in the future. However, due to the nature of the disaster events, in our study, the increase in the pre-disaster budget with the incoming donations, and the possible increased donations after the disaster are also evaluated. In addition, in our model, the excess demand can be satisfied with the available budget after the disaster at a higher cost. But, if the available budget is not enough to satisfy all the excess demand, then such demand can not be satisfied and the cost of unmet demand will be incurred in those cases. Thus, our study differs from the literature due to this cost structure that occurs in disaster situations and the consideration of donations that affect the budget limitations in the system.

3. A HEALTHCARE NETWORK DESIGN MODEL WITH HOSPITALS FOR DISASTER PREPAREDNESS

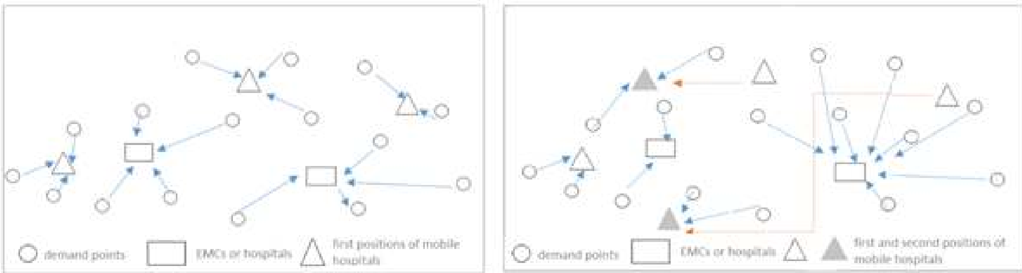
In this section, we introduce a network design model considering the location and re-location decisions of mobile hospitals using a two-stage stochastic programming model with various disaster scenarios. We present three additional model in order to compare the results. We apply our models to a real life case study for an expected earthquake in Istanbul and extract several managerial insights. Numerical experiments demonstrate that mobile hospitals can provide significant improvements in healthcare service.

3.1. Problem Description and Mobile Hospital Location and Casualty Allocation (MHLCA) Model

We consider determining the capacities and locations of mobile hospitals to be established in the pre-disaster phase and re-location of these resources to the appropriate regions at the post-disaster phase, with allocation of demand to these locations, in order to provide better healthcare service to patients affected from disasters. Since mobile hospitals are high level facilities that include almost the same equipment as regular hospitals and since they are thought to be operated just like a department of the regular hospitals, we assume that mobile hospitals can be operated at the same quality level as regular hospitals. One of the important properties of our problem is about the uncertainties of the disaster damages. For example, after a disaster, the number of injured people can vary significantly depending on the damage. In addition some of the existing hospitals can be unusable due to the damage. In order to provide efficient healthcare service, we employ different types of disaster scenarios, considering different values for the affected populations and affected hospitals at different regions.

For this purpose, a scenario-based two stage stochastic programming model is constructed, named Mobile Hospital Location and Casualty Allocation (MHLCA). In this model, N different demand regions are considered, each denoted with index $i \in 1,2,\ldots,N$ and S different disaster scenarios are used, such that each disaster scenario $s \in 1,2,\ldots,S$ has a different effect in each region i. For example, considering an earthquake, each scenario s denotes a different epicenter of the earthquake and a different magnitude at that epicenter. The regions that are closer to the epicenter are affected more than the regions that are far away and an earthquake with a larger magnitude has a larger effect in each region. Each region i has a healthcare demand with rate $p1_i$ in the pre-disaster phase and $p2_{is}$ if a disaster based on scenario s happens. In addition, the travel time metric t_{ij} denotes the travel time between locations i and j. Since obtaining the mobile hospitals requires a certain period of time, the number and capacities of these hospitals need to be decided in the first stage before the disaster happens. In addition, we assume that it is appropriate to use mobile hospitals before the disaster in order to provide additional capacity for healthcare services at certain regions, depending on the demand and capacity of healthcare services in the first stage. However, after the disaster strikes, in the second stage, these mobile hospitals need to be relocated to their new locations that are closer to disaster regions. Thus, considering the relocation times and distances, the second stage scenarios and possible location decisions also affect the first stage location and capacity decisions, based on different disaster scenarios.

An EMC can be an already existing hospital denoted with the location index h and a mobile hospital location indexed by k in the first stage or m in the second stage. Note that the potential locations of the mobile hospitals can be different in the first and second stage. Existing EMC locations and potential locations for mobile hospitals are pre-defined and we use the index j to denote the locations of all EMCs. For mobile hospitals, we consider different types of capacities with index c, that are available to be acquired with different costs, b_c. We assume that a budget κ is used for investing in this system. We use the binary variable $y1_{jc}$ to be equal to 1 if a mobile hospital with type c is located at location j before the disaster and the binary variable $y2_{jsc}$ is equal to 1 if a mobile hospital with type c is relocated at location j after a disaster of scenario s is realized. We also use the binary variable z_{kjsc} for the movement of a mobile hospital from its location in the first stage to its location in the second stage. z_{kjsc} is defined to be equal to 1 if a mobile hospital with type c is at location k before the disaster and at location j after the disaster scenario s. Let $x1_{ij}$ denote the rate of demand at region i assigned to EMC j before the disaster in the first stage and $x2_{ijs}$ denote the rate of demand at region i assigned to EMC j in the second stage after a disaster scenario s. Figures 3.1.a and 3.1.b present examples of locations and allocations in the first and second stage and movements of mobile hospitals when a disaster happens.



(a) (b)

Figure 3.1. *Demonstration of networks in a) pre-disaster and b) post-disaster phases*

Service rate at EMC *j* is denoted by μ_j and arrival rate to EMC *j* is denoted by $\lambda 1_j$ in the first stage and $\lambda 2_{js}$ in the second stage in scenario *s*. We note that, in our model, the main decision variables are facility location *y* variables. However, decisions about facility locations affect allocations and arrival rates to each facility. For example, when a new hospital is opened near a region, some of the patients in that region will go to the new hospital and the arrival rates of all other hospitals will be changed accordingly. Arrival rates are the results of location and allocation decisions. Since locations of mobile hospitals are decision variables, arrival rates are defined as decision variables, as well.

Considering the capacities of the EMCs, we focus on the expected waiting times of patients at these locations in addition to the travel times to these centers. We assume that the inter-arrival times and service rates are exponentially distributed, and the expected waiting time of a patient at EMC *j* is calculated based on an *M/M/1* queueing model. For an M/M/1 queueing model with service rate μ and demand rate $\lambda<\mu$, the expected time in the system is calculated as $1/(\mu - \lambda)$. In our model, we aim to have an expected time less than θ in each EMC, thus we plan to use a constraint as given in equation (3.1) below. However, after a disaster happens, it is widely seen that the demand rate can become much higher than the service rates of the EMCs, and thus the assumption $\lambda<\mu$ may be violated, leading to a congested systems and infeasible solutions. In order to overcome this situation and avoid such infeasibilities, we modify the constraints in our model as given in equation (3.2), in which the variable α denotes the missing capacity of the EMC in order to achieve the target expected time, θ. In overcrowded situations, it is widely observed that either additional capacity needs to be incurred at much higher costs or the existing personnel need to work at increased capacity, which leads to a decrease in service quality. Thus, we use α as a penalty term in our objective function, such that any deviation from the required capacity to achieve the expected waiting times is penalized. We note that α can also correspond to the unmet demand concept used in the literature (e.g. Salman and Gul, 2014; Javadi and Lee, 2016; Loree et al., 2018) at the target waiting level. Finally, since α, λ and μ are decision variables, to avoid the nonlinearities in the model, we rewrite equation (3.2) as in equation (3.3) and use variations of this linear form in the constraints of our model.

$$E[t] = \frac{1}{\mu - \lambda} \leq \theta \tag{3.1}$$

$$\frac{1}{\mu + \alpha - \lambda} \leq \theta \tag{3.2}$$

$$\lambda - \mu + \frac{1}{\theta} \leq \alpha \tag{3.3}$$

As a secondary objective, we also aim to have a balanced workload between the EMCs, thus similar to the penalty terms for missing capacities, we include penalty terms denoted by $\beta 1$ and $\beta 2_s$ that reflect the maximum idle capacities among all EMCs in the pre-disaster and post-disaster phases, respectively. The notations used in our paper are summarized as below:

Indices and Sets

$i \in I$: set of demand nodes.

$h \in H$: set of existing hospital locations

$k \in K$: set of mobile hospital locations in the first stage

$m \in M$: set of mobile hospital locations in the second stage

$j \in J = H \cup K \cup M$: union of sets H, K and M denoting set of all hospital locations

$c \in C$: set of mobile hospital types

$s \in S$: set of scenarios

Scenario-Independent parameters

b^c: unit cost of a mobile hospital of type c

t_{ij}: travel time from region i to EMC location j

$\mu 1_h$: service rate for EMC at location h in pre disaster

δ: setup time for mobile hospitals

γ: rate of flow from mobile hospitals to regular hospitals

$p1_i$: demand rate at region i in the first stage

μ^c: service rate for mobile hospitals with type c

w_1: weight of first stage costs

w_2: weight of second stage costs

w_3: weight of travel times

w_4: weight of missing capacities in order to reach the target expected waiting times

w_5: weight of workload imbalance

w_6: weight of travel times for patients flowing from mobile to existing hopitals

w_7: weight of setup and travel time of mobile hospitals from their first to second position

θ: target value for the expected waiting times at the hospitals

κ: total budget for purchasing mobile hospitals

Scenario Dependent Parameters

$\mu 2_h^s$: service rate for hospital at location h in scenario s.

p^s: probability of scenario s

$p2_i^s$: demand rate at region i in the second stage based on scenario s.

η_k^s: 1, if mobile hospital k will not be damaged by disaster on scenario s, and 0 otherwise.

First Stage Decision Variables

$x1_{ij}$: rate of demand from region i to hospital at location j

$$y1_k^c = \begin{cases} 1, & \text{if a mobile hospital with type c is opened at location } k \\ 0, & \text{otherwise} \end{cases}$$

$$d1_h = \begin{cases} 1, & \text{if a regular hospital at location } h \text{ has positive demand rate} \\ 0, & \text{otherwise} \end{cases}$$

$\lambda 1_j$: arrival rate of patients to hospital at location j

$\alpha 1_j$: missing capacity for a hospital at location j to achieve the target expected waiting time

β1: maximum unused capacity among all hospitals in the first stage

Second Stage Decision Variables

$\mathrm{x2_{ij}^s}$: demand rate from region i to hospital at location j in scenario s.

$$\mathrm{y2_m^{sc}} = \begin{cases} 1, & \text{if a mobile hospital with type } c \text{ is opened at location m in scenario } s \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{z_{km}^{s\,c}} = \begin{cases} 1, & \text{if a mobile hospital with type } c \text{ is moved from location k to m in scenario } s \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{d2_h^s} = \begin{cases} 1, & \text{if a regular hospital at location } h \text{ has positive demand rate in scenario } s \\ 0, & \text{otherwise} \end{cases}$$

$\mathrm{\alpha 2_j^s}$: missing capacity for a hospital at location j in scenario s.

$\mathrm{\lambda 2_j^s}$: arrival rate of patients to hospital j in scenario s.

$\mathrm{\beta 2^s}$: maximum unused capacity among all hospitals in scenario s

The objective of our model is composed of the expected costs in the pre and post-disaster phases with weights w_1 and w_2, respectively. For the pre-disaster phase, the objective function includes the total travel times of patients to their allocated EMC locations, the penalty terms for the missing capacities of the EMCs in order to achieve the target expected waiting times and the maximum idle capacity among all EMCs, with weights w_3, w_4 and w_5 respectively. We note that decreasing the maximum idle capacity among all EMCs will lead to a more balanced workload among the EMCs and thus β values are used as the measure for the cost of imbalance between the workloads of the EMCs. In the post-disaster phase, based on the disaster scenario that takes place, the mobile hospitals are relocated to their new locations and re-allocations are made based on the realized demand structure. Similar to the first stage costs, the objective function for the second stage also includes the expected total travel times, penalty terms for the missing capacities and the cost of imbalance between the workloads, with the same weights w_3, w_4 and w_5 respectively, as in the first stage. However, the objective function for the second stage also includes the travel times for the patients who can not be treated fully at the mobile hospitals and are required to be transported from the mobile hospitals to the regular hospital locations after their initial treatment, with weight w_6. In addition, the relocation times of the mobile hospitals to be transported from their initial locations and set-up in their new locations are included with weight w_7.

$$Min\, w_1 \left[w_3 \sum_i \sum_j \left[t_{ij}\, x1_{ij} \right] + w_4 \left(\sum_k \alpha 1_k + \sum_h \alpha 1_h \right) + w_5 \beta 1 \right] \tag{3.4}$$

$$+ w_2 \sum_s p_s \, [w_3 \sum_i \sum_j \left[t_{ij}\, x2_{ij}^s \right] + w_4 \left(\sum_m \alpha 2_m^s + \sum_h \alpha 2_h^s \right) + w_5 \beta 2^s$$

$$+ w_6 \sum_h \sum_m (t_{mh} x2_{mh}^s) + w_7 \sum_c \sum_k \sum_{m \neq k} \left((t_{km} + \delta) z_{km}^{s\,c} \right)]$$

s.t.

$$\lambda 1_k = \sum_i x1_{ik} \quad \forall k \tag{3.5}$$

$$\lambda 1_h = \sum_i x1_{ih} \quad \forall h \tag{3.6}$$

$$x1_{ik} \leq M y1_k^c \;\; \forall i, k, c \tag{3.7}$$

$$\sum_k \sum_c b^c y1_k^c \leq \kappa \qquad \forall k \tag{3.8}$$

$$\sum_c y1_k^c \leq 1 \;\; \forall k \tag{3.9}$$

$$\lambda1_h - \mu1_h + \frac{d1_h}{\theta} \leq \; \alpha1_h \quad \forall h \tag{3.10}$$

$$\lambda1_k - \sum\nolimits_c \mu^c y1_k^c + \frac{\sum_c y1_k^c}{\theta} \leq \; \alpha1_k \qquad \forall k \tag{3.11}$$

$$\lambda1_k \; \leq M \sum_c y1_k^c \qquad \forall k \tag{3.12}$$

$$\lambda1_h \leq \; Md1_h \qquad \forall h \tag{3.13}$$

$$\sum_k x1_{ik} + \sum_h x1_{ih} = p1_i \quad \forall i \tag{3.14}$$

$$\mu1_h - \; \lambda1_h \leq \; \beta1 \qquad \forall h \tag{3.15}$$

$$\sum_c \mu^c y1_k^c - \lambda1_k \leq \beta1 \quad \forall k \tag{3.16}$$

$$\sum_m z_{km}^{sc} = y1_k^c \; \eta_k^s \quad \forall k, m, c\, , s \tag{3.17}$$

$$\sum_k z_{km}^{sc} = y2_m^{sc} \;\; \forall k, m, s, c \tag{3.18}$$

$$\sum_m x2_{im}^s + \sum_h x2_{ih}^s = \; p2_i^s \quad \forall i, s \tag{3.19}$$

$$\lambda2_h^s = \sum_i x2_{ih}^s + \sum_m x2_{mh}^s \quad \forall h, s \tag{3.20}$$

$$\lambda2_m^s \; = \sum_i x2_{im}^s \;\; \forall m, s \tag{3.21}$$

$$\sum_h x2_{mh}^s \; = \sum_i x2_{im}^s \, \gamma \qquad \forall m, s \tag{3.22}$$

$$x2_{im}^s \; \leq M \sum_c y2_m^{sc} \quad \forall i, m, s \tag{3.23}$$

$$\sum_c y2_m^{sc} \leq 1 \qquad \forall m, s \tag{3.24}$$

$$\lambda 2_h^s - \mu 2_h^s + \frac{d2_h^s}{\theta} \leq \alpha 2_h^s \quad \forall h, s \tag{3.25}$$

$$\lambda 2_m^s - \sum_c \mu^c y2_m^{cs} + \frac{\sum_c y2_m^{cs}}{\theta} \leq \alpha 2_m^s \quad \forall m, s \tag{3.26}$$

$$\alpha 2_m^s \leq M \sum_c y2_m^{cs} \qquad \forall m, s \tag{3.27}$$

$$\lambda 2_h^s \leq M d2_h^s \qquad \forall h, s \tag{3.28}$$

$$\mu 2_h^s - \lambda 2_h^s \leq \beta 2^s \qquad \forall h, s \tag{3.29}$$

$$\sum_c \mu^c y2_m^{cs} - \lambda 2_m^s \leq \beta 2^s \qquad \forall m, s \tag{3.30}$$

$$y1_k^c \in \{0,1\}, x1_{ij}, \alpha 1_h, \beta 1, \lambda 1_j \geq 0 \quad \forall i, j, k, h, c \tag{3.31}$$

$$z_{km}^{sc}, y_{2m}^{sc}, d2_h^s, d1_h \in \{0,1\}, x2_{ij}^s, \alpha 2_j^s, \beta 2^s, \lambda 2_j^s \geq 0 \quad \forall i, j, k, m, h, c, s \tag{3.32}$$

In the above model, constraints (3.5), (3.6) define the demand rate that arrives to mobile and regular hospitals, respectively. Constraint (3.7) ensures that there will be a flow from region i to mobile hospital at location k only if a mobile hospital with some type c is established at that location. Budget constraint is defined in (3.8). Constraints (3.9), (3.24) ensure at most one mobile hospital can be opened at any location in the first and second stage, respectively. Constraints (3.10)-(3.11) and (3.25)-(3.26) define penalty values α for the missing capacities to reach the target expected waiting times at the regular and mobile hospitals in the first and second stages, respectively, as explained above in equations (3.1)-(3.3). Together with these, constraints (3.12) and (3.27) ensure that if a mobile hospital is not opened at a location, then the corresponding α variables are forced to be zero. Similarly, constraints (3.13) and (3.28) ensure that if there is no demand for a hospital, then the corresponding variables $d1_h$, $d2_h^s$, $\alpha 1_h$ and $\alpha 2_h^s$ will be zero when considered with constraints (3.10) and (3.25). Constraints (3.14) and (3.19) ensure that the total demand of region i is allocated to either regular or mobile hospitals for the first and second stages, respectively. Constraints (3.15)-(3.16) and (3.29)-(3.30) define the maximum unused capacities at the hospitals in the first and second stages, respectively. Note that β values are used in the objective function as a penalty term to avoid some hospitals being overcrowded while others have high unused capacities, in order to achieve equity among the hospitals. Constraints (3.17), (3.18) state that if a mobile hospital is moved from a location k in the first stage to another location m in the second stage, then the corresponding variable z_{km}^{sc} is equal to 1 and it is 0, otherwise. These two constraints (3.17)-(3.18) also make sure that a mobile hospital with type c can only be used in the second stage if it is acquired and used in the first stage, too. In constraint (3.17), we also state that if a mobile hospital at location k is damaged in scenario s, then the corresponding value for the parameter η_k^s will be equal to 0 and that mobile hospital cannot be used in the second stage. Otherwise, η_k^s will be equal to 1 and the mobile hospital is not damaged in that scenario and can be moved to and used in its new location. Constraints (3.20)-(3.21) define the demand rates for existing and mobile hospitals in the second stage for each scenario s. Constraint (3.22) states that a portion γ of the demand arriving to mobile hospitals is routed to regular hospitals for more advanced operations. Constraint (3.23) ensures that there is a flow to a mobile hospital at location m only if a mobile hospital is opened at that location in the second stage.

3.2. Alternative models to the MHLCA model

In this section, we consider three different models in order to compare the results of the MHLCA model with similar models in the literature.

3.2.1. Stationary Hospital Location and Casualty Allocation (SHLCA) Model

In the literature, it is observed that some models assume that non-mobile hospitals are built instead of mobile ones such that they are built before the disaster, and thus can not be relocated after the disaster (e.g. Paul and Mac Donald, 2016). We present a modification of our model to reflect this situation. We assume that during the pre-disaster phase extra capacity is built at certain locations. The location and allocation decision is made in the first stage but the newly built capacity can not be relocated even though the demand can be reallocated after the disaster.

In this SHLCA model, all the constraints will be the same with MHLCA, except that the constraints (3.17) and (3.18) are not used. Also we utilize constraint (3.34) as below which makes the additional hospitals stationary in their first position. The objective function is also modified as in equation (3.33) such that the relocation cost of hospitals does not exist. Since these hospitals are not mobile, the traveling cost of casualties from mobile to existing hospitals are removed from the model. The main disadvantage of this model is that since the hospitals are not mobile, due to the uncertainty of the disasters, they can not be relocated to disaster regions and the demand for certain hospitals might be much higher than the capacity while some hospitals have unused capacities. By comparing this model with the model MHLCA, we aim to investigate the benefits of being mobile.

$$Min\ w_1 \left[w_3 \sum_i \sum_j [t_{ij}\ x1_{ij}] + w_4 \left(\sum_k \alpha 1_k + \sum_h \alpha 1_h \right) + w_5 \beta 1 \right] \tag{3.33}$$

$$+w_2 \sum_s p_s [w_3 \sum_i \sum_j [t_{ij}\ x2_{ij}^s\ ] + w_4 \left(\sum_k \alpha 2_m^s + \sum_h \alpha 2_h^s \right) + w_5 \beta 2^s$$

$$+ w_6 \sum_h \sum_m (t_{mh} x2_{mh}^s)\ ]$$

s.t.

$$y1_k^c \eta_k^s = y2_k^{sc} \quad \forall k, s, c \tag{3.34}$$

3.2.2. Collection Point Location and Allocation (CPLCA) Model

In this case we analyze CCP (casualty collection point) or POD (points of dispensing) models as widely mentioned in the literature (e.g. Drezner, 2004; Drezner and Drezner, 2006; Lee el al., 2009; Murali et al., 2012; Apte et al., 2014; Nafarrate-Ramirez et al., 2015). CCPs and PODs are lower level health care centers and they can be built in a short time after the disaster is realized. They are cheaper than mobile hospitals and are generally available for first aid and guidance. Since they are not advanced centers as mobile hospitals, mostly, patients whose wounds require additional intervention are transported to higher level hospitals. These facilities are generally simple places that help casualties for first aid and then transport them to a hospital. CCPs and PODs are utilized during only at the aftermath of the disaster based on the realized demand. Thus, in our model, we do not make any location decision for CCPs or PODs in the pre-disaster stage. Location and allocation decisions are only made at the post-disaster phase.

$$Min\ w_1\left[w_3\sum_i\sum_j[t_{ij}\ x_{ij}] +\ w_4\left(\sum_h \alpha 1_h\right) + w_5\beta 1\right] \tag{3.35}$$

$$+w_2\ \sum_s p_s[w_3\sum_i\sum_j[t_{ij}x2^s_{ij}\ ] + w_4\left(\sum_m \alpha 2^s_m + \sum_h \alpha 2^s_h\right) + w_5\beta 2^s$$

$$+w_6\sum_h\sum_m(t_{mh}x2^s_{mh})\ ]$$

s.t.

$$\sum_m\sum_c e^c y2^{sc}_m \leq \kappa \qquad \forall s \tag{3.36}$$

$$\sum_h x1_{ih} =\ p1_i \quad \forall i \tag{3.37}$$

In addition to constraints (3.6), (3.10), (3.13), (3.15), (3.19)-(3.32) in this model constraints are the same with MHLCA except that the constraints (3.5), (3.7), (3.8), (3.9), (3.11), (3.12), (3.14), (3.16), (3.17), (3.18) are not used, which are related with location decisions in the first stage. We modify the objective function as in equation (3.35) since CCPs and PODs are only used at the second stage. We also modify the budget constraint with cheaper facility costs, denoted with e^c for a facility of type *c*, and update the parameter that denotes the flow of patients from mobile hospitals to regular hospitals, since CCPs and PODs are lower level facilities requiring more patients to be directed to regular hospitals after their first aid is made.

3.2.3. Depot and Mobile Hospital Location and Casualty Allocation (DMLCA) Model

In our model, we assume that mobile hospitals are used both before and after the disaster. However, in some cases it might not be feasible to use them before the disaster and rather they are only bought for disaster preparedness. In this case we assume that mobile hospitals are bought at the pre-disaster phase but they are stored in a single warehouse with location index 0, and not used before the disaster happens. They are located at the necessary places after the disaster strikes. This model resembles the models in the literature in which supplies are bought at the pre-disaster phase but used only at the post-disaster phase according to the realized demand (eg. Malek et al., 2016). Thus, we generate DMLCA model in order to compare the results of our model with the results of such models in the literature. In this model, definition of $y1^c$ is changed as a positive variable which denotes the acquired number of mobile hospitals of type *c*.

$$Min\ w_1\left[w_3\sum_i\sum_j\left[t_{ij}\ x1_{ij}\right]+\ w_4\left(\sum_h \alpha 1_h\right)+w_5\beta 1\right] \tag{3.38}$$

$$+w_2\sum_s p_s[w_3\sum_i\sum_j\left[t_{ij}\ x2_{ij}^s\right]+w_4\left(\sum_m \alpha 2_m^s+\sum_h \alpha 2_h^s\right)+w_5\beta 2^s$$

$$+w_6\sum_h\sum_m(t_{mh}\ x2_{mh}^s)\ +w_7\sum_c\sum_m(t_{0m}+\delta)y2_m^{sc}\ ]$$

s.t.

$$\sum_c b^c y1^c\leq\kappa \tag{3.39}$$

$$y1^c\geq 0\quad\forall c \tag{3.40}$$

$$\sum_h x1_{ih}\ =\ p1_i\qquad\forall i \tag{3.41}$$

$$\sum_m y2_m^{sc}\ =\ y1^c\eta_0^s\qquad\forall s,c \tag{3.42}$$

In addition to constraints (3.6), (3.10), (3.13), (3.15), (3.19)-(3.32), in this model, constraints (3.5), (3.7), (3.8), (3.9), (3.11), (3.12), (3.14), (3.16), (3.17), (3.18) and the first stage costs related with the mobile hospitals are not used since the mobile hospitals are not utilized before the disaster. We add constraint (3.40) which states that the number of MHs of type *c* is greater than or equal to zero. Constraint (3.39) is the new budget constraint implying that in the first stage demand is satisfied by only existing hospitals. Constraint (3.41) implies first stage demand is met by only existing hospitals. Constraint (3.42) shows that total number of mobile hospitals of each type used in the second stage should be equal to the number of mobile hospitals at the depot in the first stage.

3.3. Application and Results for Istanbul Earthquake Case Study

In this section, we conduct a case study for the earthquake preparedness of Istanbul by applying the above mathematical models. In order to mitigate the possible damages of a disaster, Istanbul Metropolitan Municipality has developed a disaster prevention and mitigation plan for Istanbul in 2002 with the cooperation of Japan International Cooperation. This study describes a detailed risk analysis for the metropolitan city (JICA & IMM, 2002). We utilize JICA report for generating most of our input data. Also, we use GIS (Geographic Information System) in order to generate distances between the locations and obtain some data from the studies examining Istanbul as a case study in the literature (e.g. Salman and Gul, 2014; Kinay et al., 2018; Yucel et al., 2018). For example, hospital service rates are generated from Gul (2008). We generate varying scenarios according to different earthquake magnitudes and epicenters according to Parsons et al. (2000). For each scenario, we also generate the healthcare demand using the results in the JICA report (JICA & IMM, 2002). We explain our data in more detail as below.

3.3.1. Data generation and parameter setting

There are some studies discussing the likelihood and the effects of a possible Istanbul earthquake, its time and epicenter (e.g. Parsons et al., 2000; Ambraseys and Jackson, 2000; Ambraseys, 2002). While examining the earthquake scenarios in Istanbul, we consider the North Anatolian fault line in our study. Breaking points in this fault line are presented by many publications. Parsons et al. (2000) study the fault lines and epicenters which strongly affect Istanbul. They present 4 epicenters that have the highest earthquake risks and also evaluate some earthquakes that happened in the past in relation with the epicenters. In our study we evaluate the effects of possible earthquakes with different magnitudes and epicenters to different regions in Istanbul. We use these four epicenters in our study, namely Ganos, Marmara, Izmit and CP (a location in the Marmara sea is referenced as CP in Parsons et al., 2000). The exact coordinates of each epicenter can be found in Parsons et al. (2000) as seen in Figure 3.2. We observe that there are several ways that are used to predict the intensity of the earthquakes and their probabilities in the literature (e.g. Unal et al., 2014; Erdik et al., 2003; Ambraseys 2002). In order to analyze the system performance we utilize 6 different possible magnitudes from 6 to 8.5 with 0.5 increments for all potential epicenters, with a total of 24 scenarios. We analyze past earthquakes that happened in these 4 epicenters and determine the scenario probabilities by analyzing the frequencies of these data for each epicenter and intensity combination.

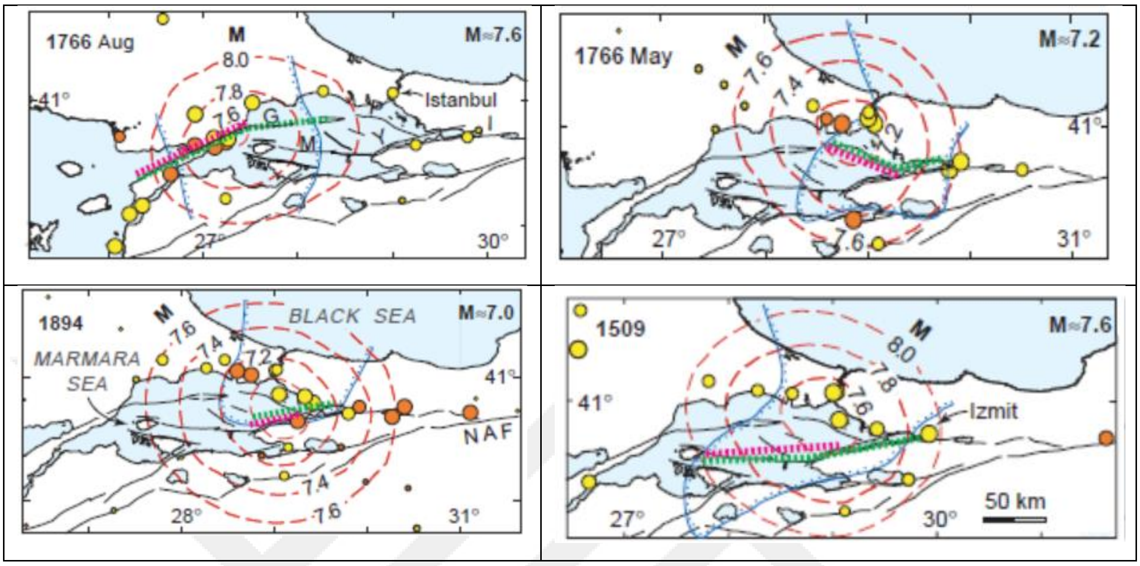
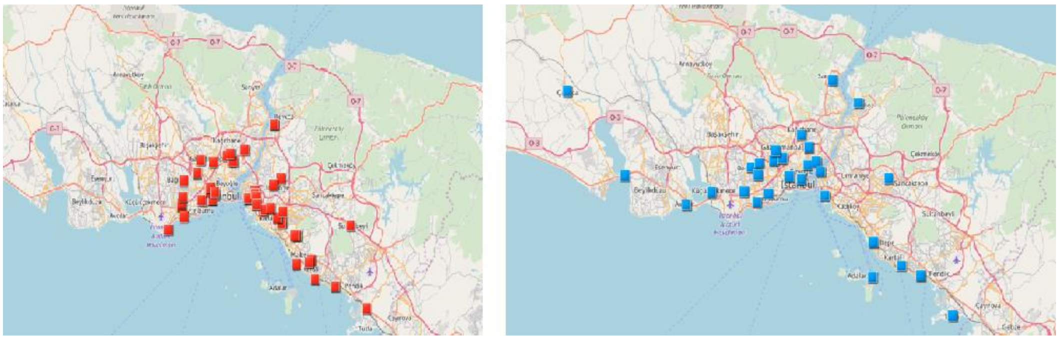


Figure 3.2. *Epicenters of earthquakes for Istanbul case study (Parsons et al., 2000)*

Ambraseys and Jackson (2000) emphasize the factors affecting epicentral intensity. These factors are specified as magnitude, focal depth and distance from epicenter. In addition, Battara et al. (2018) calculate the intensity of an earthquake according to the distance from the epicenter in order to generate scenarios and apply to a case study for Turkey accordingly. In the JICA report, the casualty rates in different districts in Istanbul are given for an expected earthquake. Motivated by these studies, we create the scenarios in the same way. First of all, we generate the number of casualties based on data from the JICA report for several magnitudes (6, 6.5, 7, 7.5, 8, 8.5) by taking into account the distance between the epicenter and the districts of Istanbul. Then, we predict the number of casualties for different epicenters assuming an earthquake with the same tectonic characteristics (like seismic energy etc). We assume that the realized earthquake magnitude reduces by 0.5 units per 20 km from the epicenter (Battara et al., 2018). For example, an earthquake with the epicenter at Izmit and a magnitude 7.5 will be felt as 6.5 in the Tuzla district. We generate different magnitudes for each epicenter and calculate the casualty numbers accordingly. Then, we estimate the demand rate for each district of Istanbul for healthcare services considering their adjusted 2017 populations and the effects of different earthquake scenarios on each district.

In our numerical experiments, we consider 29 districts of Istanbul as the demand points. Before the disaster, the demand rates in each region are estimated using the population of each district and the actual demand data of the hospitals in that region. After the disaster, they are estimated based on the effect of the realized scenario in each district, as described above. The number of estimated casualties in each district in each scenario is used to estimate the demand rate for healthcare services.

The gathering points in Istanbul are determined by the municipality in case of a disaster. Simsek and Yeniay (2012) present 35 gathering points. We use these points as candidate locations for mobile hospitals in addition to the locations of the existing 48 hospitals in Istanbul. Their locations are shown in Figure 3.3. and we make the location selection among these points.



(a) (b)

Figure 3.3. *a) Locations of existing hospitals,b) Potential locations for mobile hospitals*

For our case study, during the pre-disaster phase, we assume that the mobile hospitals can only be located next to the locations of the existing hospitals and they operate just like a department of the regular hospitals. There are mainly two reasons for this location selection. Firstly, since the time until the disaster is unknown, the collection points are operational places in daily life by the metropolitan municipality, such as hippodromes, football fields, parking lots, etc. It would not have been realistic to place mobile hospitals in the first stage at these designated collection points. Another reason for placing mobile hospitals next to the existing hospitals is about operational restrictions. It would not be easy to operate the mobile hospitals on its own in the long term, however they can easily be operated as additional capacity at the existing hospital locations. In addition, the cost of supplying materials from existing hospitals to mobile hospitals will be lowered in the pre-disaster phase. Thus, we make location decisions from 48 candidate points which are the points next to the existing hospital locations for pre-disaster phase.

For post-disaster phase we decide the locations for mobile hospitals among 83 candidate points which are composed of 35 gathering points determined by the Istanbul municipality in addition to the locations next to the 48 existing hospitals. We note that the main decision maker in this system is thought to be the government and the Ministry of Health. Most of the hospitals and vehicles are owned by the government and they are under the control of the Ministry of Health. Thus, the borders of Istanbul are not critical factors and vehicles can be deployed to other regions, if needed.

We determine the distances between the locations in our model through GIS and then convert them to transportation times using 30km/h as the average velocity. Then, we generate service rates of the existing hospitals from the data given in Gul (2008). In addition, we consider the case of disruption for the existing hospitals in each scenario. If the effect of a disaster at a district has the magnitude between 6.5-7.5, a hospital in that region is assumed to be moderately damaged and service rate is multiplied by 0.5. For different realized magnitudes at each hospital location, the coefficients used for realized service rates are given in Table 3.1. In our base case setting, we assume that mobile hospitals are not affected by the disaster since they are mobile trucks and built to be resistant to disasters. However, we also analyze the system results when mobile hospitals are also affected by the disasters in various settings in Section 3.6.

Table 3.1 *Disruption levels for existing hospitals*

Magnitudes	7.5 up	6.5-7.5	5.5-6.5	5.5 down
Coefficients	0.25	0.50	0.75	1
Status	Heavily	Moderately	Partially	Not damaged

There are three types of mobile hospitals with different capacities and prices, as provided from a supplier of these hospitals. Service rates of mobile hospitals are μ^1=32, μ^2=48, μ^3=64 patient per hour and the prices of mobile hospitals are b^1 =\$128500, b^2 =\$170000, and b^3 =\$210000 for each capacity, respectively. The total budget for obtaining mobile hospitals is assumed to be \$3.000.000. We also use $\gamma = 0.05$ to denote the ratio of patients flowing from the mobile hospitals to regular hospitals for more advanced operations in case of a disaster and the target value for the expected waiting times, θ, is assumed to be 0.5 hours. We use the weight parameters w_1=1, $w_2 = 2$, $w_3 = 1$, $w_4 = 1$, $w_5 = w_6 = w_7 = 1$ in our base case model in order for the cost values to be compatible with each other. We also provide a sensitivity analysis on these weights in the next section.

3.3.2. Analysis of the results

3.3.2.1. MHLCA model

For our case study, we analyze the results of the MHLCA model under the parameter settings as explained above. All the results of our numerical experiments are obtained by using GAMS 24.8.5 and MIP solver CPLEX 12.7.0 on a 2.80 GHz Intel i7 server with 32GB RAM. In order to assess the results of the model we report the total travel time, missing capacities to achieve the target expected waiting times, cost of equity and the total cost values for both the first and second stages in Table 3.2. We note that the total travel time in the second stage includes the weighted transportation and set-up time for the mobile hospitals and the flow of patients from the mobile to the regular hospitals.

Table 3.2. Results of the MHLCA model

	MHLCA model
First stage	
Total Travel Cost	671.49
Cost of Missing Capacity and Equity	386.86
Total Cost for First Stage	**1058.35**
Second stage	
Expected Total Travel Cost (including the travel time of patients from mobile to regular hospitals and the setup time of mobile hospitals)	3170.45
Expected Cost of Missing Capacity and Equity	13354.11
Expected Total Cost for Second Stage	**16524.56**
Expected Total Cost	**17582.91**

Figure 3.4. *Mobile hospital locations a) before disaster b) after disaster at Ganos. c) at Marmara Sea*

In Figure 3.4.(a), we present the number and locations of mobile hospitals before the disaster based on the MHLCA model results. It is also seen that mobile hospitals with different capacities are used depending on the demand. Then, after a disaster occurs, depending on the realized scenario, these mobile hospitals are relocated to their new locations. For example, if a disaster happens on the epicenter of Ganos, which is at the south-west of Istanbul, it is observed that some of the mobile hospitals are moved to their second stage positions that are closer to the west side, as seen in Figure 3.4.(b). Similarly, if another scenario is realized, such as an earthquake with the epicenter of Marmara Sea, the second stage positions will be the ones given in Figure 3.4.(c). We can observe that since the epicenter of Marmara Sea is at the south side of Istanbul, some of the mobile hospitals are relocated to their new positions that are closer to the epicenter. The ability of the mobile hospitals to move according to the realized demand in each scenario provides faster and more effective healthcare response to injured people.

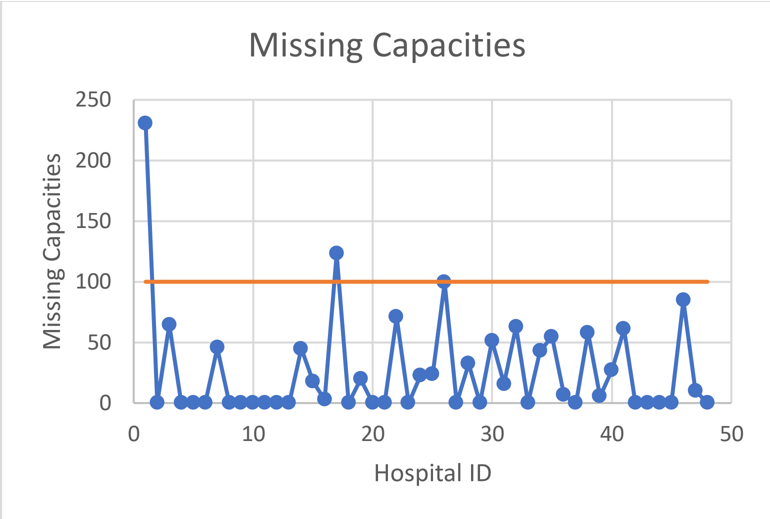


Figure 3.5 Missing capacities according to MHLCA model

In this section, our analysis also helps us gain insights about the sufficiency of the hospitals in case of a disaster. In Figure 3.5., we present the expected values of the missing capacities for each existing hospital in order to achieve the target expected waiting times. We observe that the missing capacities for some of the hospitals are zero, denoting that the capacities of these hospitals will be sufficient to meet the demand at the target expected waiting time limit. However, some hospitals have very large missing capacity values, denoting that these hospitals will be overcrowded in case of a disaster and additional capacities are required in these regions. For example, hospitals 1 and 17 are the hospitals with the largest missing capacity values, meaning that their capacities will not be enough to satisfy the demand. Either their capacities need to be increased or new hospitals need to be built around these regions for improved disaster preparedness.

3.3.2.2. Comparison of Model Results

We compare the results of our model with similar models in the literature as explained in Section 4 and aim to analyze the benefits of using mobile hospitals compared to other approaches in the literature. Mobile hospitals are upper level hospitals which are fully equipped as regular hospitals and can provide any kind of treatment that casualties need. For instance, mobile hospitals have all healthcare devices required to fulfill an important surgery. But CCPs are low level healthcare centers, as mentioned in literature (e.g. Apte et al., 2014), that are used to provide mostly first aid. Thus, mobile hospitals are much more expensive than CCPs or PODs. We did a basic research about the costs and prices by collecting information from the providers of these facilities and we use the prices that are obtained from the related firms. It is seen that the smallest type of a mobile hospital is almost eight times more expensive than a basic CCP. Thus, the prices of facilities are decreased to e^1=\$15000, e^2 =\$20000, e^3=\$28000 for the service rates μ^1=16, μ^2=24, μ^3=32 patient per hour for the model CPLCA. Also the parameter considering flows from mobile hospitals to existing hospitals is increased from $\gamma = 0.05$ to 0.5 in that model.

Table 3.3 *Comparison of models*

	Results of Models			
First stage	MHLCA	SHLCA	CPLCA	DMLCA
Total Travel Cost	671.49	675.85	650.10	650.10
Cost of Missing Capacity and Equity	386.86	386.86	1252.86	1252.86
Total Cost for First Stage	**1058.35**	**1062.71**	**1902.96**	**1902.96**
Second stage				
Expected Total Travel Cost	3170.45	5364.48	5416.83	3228.53
Expected Cost of Missing Capacity and Equity	13354.11	12998.58	12440.41	13349.14
Expected Total Cost for Second Stage	**16524.56**	**18363.06**	**17857.24**	**16577.67**
Expected Total Cost	**17582.91**	**19425.77**	**19760.2**	**18480.63**
Expected Cost Increase (%) vs. MHLCA	**-**	10.4%	12.3%	5.1%
Computation Time (sec.)	171.45	62.58	3.87	5.95

As seen in Table 3.3, the model MHLCA has lower total costs compared to the models SHLCA, CPLCA and DMLCA with 10.4%, 12.3% and 5.1% improvement values, respectively. One of the reasons for these differences is the use of mobile hospitals in the first stage, because when CPLCA or DMLCA models are used, all the demand in the first stage is met by existing hospitals alone and the resources bought for disaster preparedness are kept idle until a disaster happens, which is one of the most important deficiencies of disaster preparedness models in the literature. For this reason, missing capacity values are higher in these two models. When we compare the SHLCA model with the MHLCA model, since the built capacity for medical response in the first stage can not be moved in the second stage according to the realized demand in the SHLCA model, it leads to higher travel times and higher total costs. In the MHLCA model, because of the new positions of mobile hospitals, the travel time for casualties decreases. As a result, we can state that using mobile hospitals might improve the system performance significantly for healthcare services in case of a disaster, compared to the approaches in the literature. When we consider the run times of the algorithms, we observe that the model MHLCA requires a longer computation time, and this is mainly because of the additional complexities arising in the model due to re-location and re-allocation decisions in the second stage. We also note that this problem is NP-Hard since it can be reduced to the classical p-median problem when only the first stage transportation costs are considered in the objective function (weight parameters other than w_1 and w_3 are all 0), only mobile hospitals are considered (number of regular hospitals is 0), budget (κ) is set to p, a single type of mobile hospital is used with cost equal to 1 and all other constraints except constraints (3.7), (3.8) and (3.14) are removed from the model MHLCA. Since the classical p-median problem is known to be NP-Hard (Kariv and Hakimi, 1979), so is the model MHLCA. However, the computation times for the analyzed case study are seen to be reasonable to solve this real life problem. Similar computation times are also seen when the parameters are changed in the next section in our sensitivity analysis.

3.3.3. Sensitivity Analysis

In order to assess the robustness of the solutions, we conduct sensitivity analysis by changing some of the parameters. First of all, we analyze the effect of the budget constraint on our results since mobile hospitals are costly items. In Figure 3.6., we present the objective function values of different models for varying budget amounts, κ. Based on this figure, the objective function of the CPLCA model does not change much as the budget increases, since the CPLCA model generally utilizes lower level facilities with lower costs and having much higher budgets only improve the system performance slightly. However, since the other models utilize mobile hospitals, which are much more costly, having more budget affects the system results more significantly, especially when the budget is tight. However, as the budget becomes more than a certain level, increasing the budget even more, only slightly improves the system performance since additional mobile hospitals can only bring decreased returns to the system. Thus, it can be said that in a low budget plan, the CPLCA model can be more advantageous, whereas as the budget increases, utilizing mobile hospitals and using the MHLCA model becomes much more preferable.

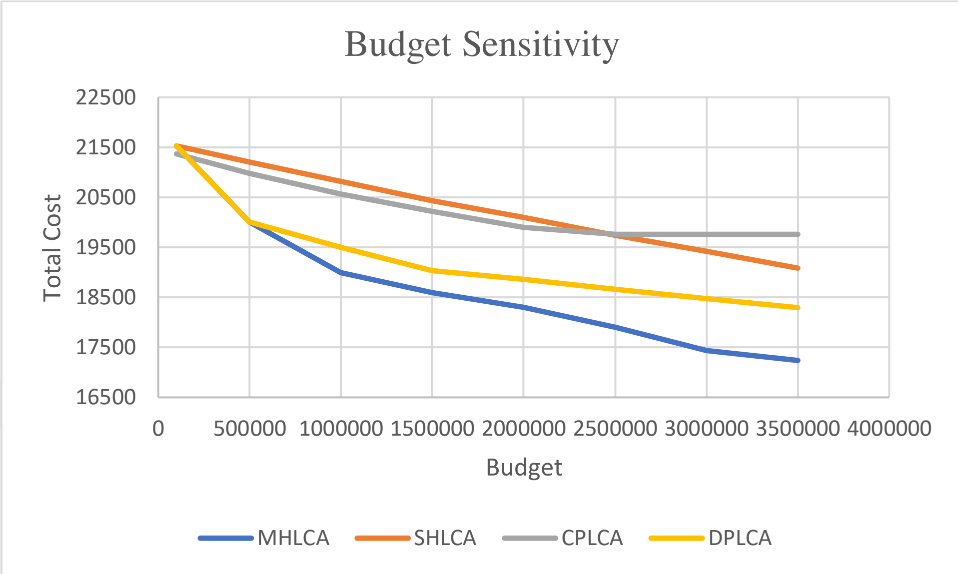


Figure 3.6 Sensitivity for budget amount

We also employ a sensitivity analysis on the parameters of the model, such as the weights used in the objective function and the health care demand rate factors. Recall that, we utilize different objectives as travel time, missing capacities, equity etc. and we give different weights to each objective in our model. In the sensitivity analysis, we examine the results by changing these weights. We report the total cost and improvement values for each model for different parameters in Table 3.4. in order to compare the models under varying conditions. The first row of Table 3.4. presents the results under the base case parameters as given above. Then, in each row we change one of the parameters at a time while all others remain constant and aim to analyze the effect of that parameter. The parameter in consideration and its new value are given in the first column in Table 3.4..

Table 3.4 *Sensitivity Results*

	MHLCA	SHLCA		CPLCA		DMLCA	
	TC	TC	%imp	TC	%imp	TC	%imp
Base Case	17582.91	19425.77	10.48%	19760.21	12.38%	18480.63	5.11%
w_1=0	16520.68	18357.16	11.12%	17857.26	8.09%	16577.68	0.35%
w_1=0.5	17052.85	18892.17	10.79%	18808.73	10.30%	17529.14	2.79%
w_1=2	18637.01	20485.9	9.92%	21663.18	16.24%	20383.59	9.37%
w_2=0	1056.16	1056.16	0.00%	1902.96	80.18%	1902.96	80.18%
w_2=1	9318.86	10242.95	9.92%	10831.59	16.24%	10191.79	9.37%
w_2=3	25839.08	28592.13	10.65%	28688.83	11.03%	26769.30	3.60%
w_3=0	13343.24	13354.45	0.08%	13696.65	2.64%	14309.44	7.23%
w_3=0.5	15709.72	16397.57	4.38%	16758.1	6.67%	16623.77	5.82%
w_3=2	21228.58	25440.57	19.84%	25321.66	19.28%	22104.05	4.12%
w_4=0	3639.02	5909.52	62.39%	4048.78	10.10%	3651.14	0.33%
w_4=0.5	10717.3	12749.43	18.96%	12847.71	19.88%	11186.49	4.38%
w_4=2	31254.21	32750.67	4.79%	33393.82	6.85%	33013.89	5.63%
w_4=w_5=0	3465.27	5780.58	66.81%	3883.39	12.07%	3475.46	0.29%
w_5=0	17510.96	19355.95	10.54%	19705.42	12.53%	18412.53	5.15%
w_5=0.5	17550.51	19397.74	10.53%	19736.89	12.46%	18449.17	5.12%
w_5=2	17631.68	19468.34	10.42%	19798.03	12.29%	18532.93	5.11%
w_6=0	17448.73	19425.77	11.33%	19636.07	12.54%	18387.43	5.38%
w_6=0.5	17526.86	19425.77	10.83%	19704.17	12.42%	18433.79	5.17%
w_6=2	17669.46	19425.77	9.94%	19843.12	12.30%	18574,19	5.12%
w_7=0	17544.31	19425.77	10.72%	19760.21	12.63%	18347.54	4.58%
w_7=0.5	17566.31	19425.77	10.59%	19760.21	12.49%	18425.73	4.89%
w_7=2	17607.17	19425.77	10.33%	19760.21	12.23%	18570.86	5.47%
$\forall p1_i$ down by 20%	17095.76	18942.1	10.80%	19085.51	11.64%	17805.89	4.15%
$\forall p1_i$ up by 20%	18272.86	20114.86	10.08%	20463.39	11.99%	19183.80	4.99%
$\forall p2_i^s$ down by 20%	13701.56	15174.39	10.75%	15552.05	13.51%	14601.94	6.57%
$\forall p2_i^s$ up by 20%	21484.52	23708.59	10.35%	24014.24	11.77%	22383.45	4.18%

We observe that the weight parameters w_2, w_3 and w_4 have the highest impact on the results. When we compare MHLCA with SHLCA, as the post-disaster costs become more important than the pre-disaster costs (i.e. w_1 decreases or w_2 increases), we observe that the mobility of the hospitals provides more benefits. Thus, mobile hospitals are especially useful for disaster preparation rather than for regular healthcare demand. On the other hand, when w_1 is increased or w_2 is decreased, MHLCA performs much better than CPLCA and DMLCA models since these models only focus on the second stage and mobile hospitals are not used in the first stage. As the weight of the travel times (w_3) increases, MHLCA performs much better than SHLCA since the mobile hospitals can be located to much closer locations to the higher demand regions in the second stage. MHLCA model seems to have a better performance than SHLCA when the other weight parameters w_4, w_5, w_6 and w_7 get smaller since the costs related to mobility decreases when these parameters are small. When compared with CPLCA, we observe that the percentage improvement of MHLCA increases when $w_{3,} w_5$, w_6 or w_7 decreases but no monotonic structure is observed in percentage improvements with respect to w_4. Finally, it is observed that the percentage difference between MHLCA and DMLCA increases as w_4 and w_7 increase, or w_3, w_5 and w_6 decrease. As a result, we can state that using mobile hospitals can provide significant improvements in satisfying the healthcare needs in case of a disaster and they become especially useful if more importance is given to post-disaster response and to the travel times to reach the hospitals.

It is also observed that if only the post-disaster situation is considered and the first stage costs are ignored (i.e. $w_1 = 0$), MHLCA becomes much better than SHLCA, however the performances of CPLCA and DMLCA get closer to MHLCA, since these two models focus mostly on post-disaster response and the benefits of using mobile hospitals in the first stage are eliminated in this case. On the other hand, if post-disaster response is totally ignored and only regular demand is considered (i.e. $w_2 = 0$), SHLCA performs as good as MHLCA since no action is needed based on different disaster scenarios. Not using any mobile hospital in the first stage (as in models CPLCA and DMLCA) leads to about 80% worse performance in this case. When transportation times are neglected and only waiting times and capacities of the hospitals are considered in the model (i.e. $w_3 = 0$), SHLCA performs almost as good as MHLCA because the locations of the mobile hospitals would not be important in that case since patients can be assigned to much distant locations at no cost. On the other hand if waiting times and missing capacities are neglected (i.e. $w_4 = 0$), transportation times become much more dominant and MHLCA performs much better than SHLCA. However, not using the mobile hospitals in the first stage becomes less important and the performance of the model

DMLCA is close to MHLCA. The other weight parameters w_5, w_6 and w_7 only slightly affect these results and thus not considering them in the model does not lead to as much significant changes in the results. To have more interpretable results, we also consider a case in which $w_4=w_5=0$ such that all capacity related costs are removed and all remaining terms in the objective function are in time units. In this case, the performance of DMLCA is seen to be close to MHLCA but they perform much better than SHLCA.

We also note that the demand rate might be varying depending on various factors and thus we analyze the effects of demand rate in the pre and post disaster stages on the system results. It is observed that even when the demand rates change, MHLCA model still performs significantly better than the other models and performance improvements are not affected much by these changes. Even though the costs values are significantly affected, especially by the post-disaster demand, the benefits of MHLCA and the main results are pretty robust against demand variations.

3.4. Effect of Mobile Hospitals Being Prone to Damages

In this section, we analyze the effects of disruption risks of mobile hospitals on the system results. In our base case experiments, we assumed that mobile hospitals are not damaged by the disaster, however due to the unknown nature of the disaster, mobile hospitals may also be thought to be affected by the disaster. In that case, they can not be used in the second stage after the disaster. We design a set of experiments to analyze the effects of disruption risks of the mobile hospitals. If a mobile hospital is located at a certain location k in the first stage and if the effect of the disaster at that location is above a certain value τ, depending on the epicenter and the intensity of the realized scenario, then that mobile hospital is assumed to be damaged and can not be used in the second stage. For example, if the critical disruption level τ is chosen to be 7 and if a disaster happens with an effect of 7.5 at a certain location, then the mobile hospital located at that location will be damaged and can not be used in the second stage.

We analyze the results considering different critical disruption levels, τ and present the results in Table 3.5. We observe that as the critical level τ decreases, in other words, as the mobile hospitals become more prone to damages, all the costs increase (except CPLCA costs since they are not used in the first stage and thus can not be damaged) and the benefit of the model MHLCA compared to SHLCA and CPLCA decreases. The main reason for this situation is that as the mobile hospitals become more prone to damages, the disasters will affect them more and since they can not be used in the second stage due to these damages, the benefit of being mobile decreases. However the difference between these models are still above 8% and MHLCA still performs significantly better than these two models. When we consider DMLCA, we observe that as τ decreases, the benefit of MHLCA compared to DMLCA increases.

Table 3.5 Results of the models when mobile hospitals are damaged

	MHLCA TC	SHLCA TC	%imp	CPLCA TC	%imp	DMLCA TC	%imp
Base Case (No damage)	17582.91	19425.77	10.48%	19760.21	12.38%	18480.63	5.11%
$\tau = 6.5$	18144.96	19672.5	8.42%	19760.21	8.90%	19269.41	6.20%
$\tau = 7.0$	17779.85	19535.14	9.87%	19760.21	11.14%	18857.29	6.06%
$\tau = 7.5$	17626.41	19460.89	10.41%	19760.21	12.11%	18614.75	5.61%
$\tau = 8.0$	17591.16	19433.48	10.47%	19760.21	12.33%	18550.31	5.45%

3.5. An Extension of the Model: All patients are assumed to choose to go to the closest hospitals

In the model presented above, the allocation of patients to the hospitals are assumed to be made by a central agency such that the patients are allocated to the hospitals in order to minimize the defined objective function. The model presented above reflects many disaster and emergency situations in which patients are directed to the most available hospital (in terms of distance and capacity) by “Emergency Medical Services” and “Ambulance Control Centers” that exist in many countries. These control centers directly make the allocation decisions of patients to hospitals and thus control the arrival rates as their decisions. Our model provides managerial insights to these controllers about how these allocations need to be made among the hospitals. In addition, even in cases in which a central agency does not make this allocation and patients make their own choice, it is commonly observed that if the patient’s choice of hospital is too congested, the patients are directed to the most available hospital in the region when they reach to the hospital, or the patients themselves choose to do so. In addition, if the arrival rates to a certain hospital is much higher than its capacity, it would lead to high waiting times and some of the patients would choose to go to another hospital with lower waiting times, even if that hospital is slightly far away than the current one, leading to a similar result in our model with a lower total cost. Our model also reflects such behaviors by including travel and waiting times combined in the objective function.

In the literature, Zhang et al. (2009) report that there are various studies that evaluate the patient assignments in different ways. Some of them (e.g. Mitropoulos et al. 2006; Guerriero et al. 2016) assume that allocation of patients to facilities is controlled by the policy maker, similar to our model. This assumption leads to the best allocation of patients to hospitals with the lowest cost possible for the society and thus provides a lower bound for the system-wide costs. In this section, in order to analyze the effects of this model assumption, we also provide an extension of our model, in which the patient allocations are not freely made by the model but instead the patients in each region are thought to go to the nearest hospital in that region as assumed by Berman (1995), Verter and Lapierre (2002), Mestre et al. (2015) in the literature. Thus, we consider closest facility allocation as the manner of patient behavior. This assumption seems especially reasonable in case of emergency situations.

In order to model this phenomenon, we adapt the set of constraints (3.43) - (3.52) from Espejo et al. (2012) and modify our MHLCA model as stated below. The other models are also modified similarly. Note that, even though it is known that each patient will go to the nearest hospital, the allocation decisions and arrival rates to hospitals still have to be defined as decision variables, since the nearest hospital to a region might be changed by the location decisions. If a new hospital is opened in a region, the patients in that region would go to that hospital and the arrival rates of all hospitals will be affected.

Additional decision variables for closest assignment

$\xi1_{ij}$: $fraction\ of\ demand\ in\ region\ i\ that\ goes\ to\ hospital\ j\ in\ the\ first\ stage$

$$v1_j = \begin{cases} 1, & if\ there\ is\ a\ hospital\ at\ location\ k\ in\ the\ first\ stage \\ 0, & otherwise \end{cases}$$

$\xi2_{ij}^s$: $fraction\ of\ demand\ in\ region\ that\ goes\ to\ hospital\ j\ in\ scenario\ s\ in\ the\ second\ stage$

$$v2_j^s = \begin{cases} 1, & if\ there\ is\ a\ hospital\ at\ location\ j\ in\ scenario\ s\ in\ the\ second\ stage \\ 0, & otherwise \end{cases}$$

Then, we add the following constraints to the model (3.4)-(3.32). With the addition of these constraints to the model, all the patients will be allocated to the closest hospitals by the model.

$$\sum_{n:\, d_{in}=d_{ij}} \xi 1_{in} + \sum_{n:\, d_{in}<d_{ij}} v1_n \geq v1_j \quad \forall i,j \tag{3.43}$$

$$\xi 1_{ij}\, p1_i = x1_{ij} \qquad \forall i,j \tag{3.44}$$

$$v1_h = 1, \quad v1_k = \textstyle\sum_c y1_k^c \qquad \forall h,k \tag{3.45}$$

$$\sum_j \xi 1_{ij} = 1 \qquad \forall i \tag{3.46}$$

$$0 \leq \xi 1_{ij} \leq 1 \qquad \forall i,j \tag{3.47}$$

$$\sum_{n:\, d_{in}=d_{ij}} \xi 2_{in}^s + \sum_{n: d_{in}<d_{ij}} v2_k^s \leq v2_j^s \quad \forall i,j,s \tag{3.48}$$

$$\xi 2_{ij}^s p2_i^s = x2_{ij}^s \qquad \forall i,j,s \tag{3.49}$$

$$v2_h^s = 1, \;\; v2_m^s = \sum_c y2_m^{cs} \qquad \forall h,m,s \tag{3.50}$$

$$\sum_j \xi 2_{ij}^s = 1 \qquad \forall i,s \tag{3.51}$$

$$0 \leq \xi 2_{ij}^s \leq 1 \qquad \forall i,j,s \tag{3.52}$$

In the above model, Constraint (3.43) denotes that if there is a hospital at location *j* and if there does not exist any hospital that is closer to region *i* than location *j* (or with equal distance), then all the patients in region *i* will go to hospital *j* (if there are other hospitals with the same distance as j, then the patients in region *i* can go to any of these hospitals). Constraint (3.44) converts the fractions ($\xi 1$) to actual numbers (*x1*) as used in the original model by multiplying the fraction by the total demand in that region (*p1*). Constraint (3.45) denotes whether there exists a hospital at each location or not and relates the variables *v1* with the variables *y1* in the original model. Constraints (3.46) and (3.47) state that the fractional values should be between *0* and *1* and they have to add up to *1* since all the demand in each region should be assigned to a hospital. Constraints (3.48)-(3.52) are the equivalent of the constraints (3.43)-(3.47) for the second stage decisions.

The results of this model are presented in Tables 3.6. and 3.7.. It is observed that even when the patients are not freely allocated by a central agency and they just choose to go the nearest hospital, the model results are consistent with the previous results stated above. The model MHLCA still performs significantly better than the other models under this assumption and similar benefits can be obtained. Total travel costs for all models in the first stage are the same in Table 3.6. since mobile hospitals are located next to regular hospitals in the pre-disaster phase and the travel times of the patients are not affected by these locations since they all go to the nearest hospital in all models. Even though the travel costs are decreased in Table 3.6. compared to Table 3.3. due to patients being assigned to closest hospitals, the total costs are increased due to the resulting imbalance between the hospitals and increased waiting and missing capacity costs.

Table 3.6. *Result of the models when all patients are assigned to closest hospitals*

	MHLCA	SHLCA	CPLCA	DMLCA
First stage				
Total Travel Cost	567.75	567.75	567.75	567.75
Cost of Missing Capacity and Equity	1546.27	1548.80	2412.27	2412.27
Total Cost for First Stage	**2114.02**	**2116.55**	**2980.02**	**2980.02**
Second stage				
Expected Total Travel Cost	3070.43	5212.83	5177.13	3191.35
Expected Cost of Missing Capacity and Equity	14468.44	14445.49	14396.95	14512.08
Expected Total Cost for Second Stage	**17538.87**	**19658.32**	**19574.08**	**17703.43**
Expected Total Cost	**19652.89**	**21774.87**	**22554.10**	**20683.45**
Expected Cost Increase (%) vs. MHLCA	-	10.80%	14.76%	5.24%

Table 3.7. *Sensitivity Analysis for the models when all patients are assigned to closest hospitals*

	MHLCA	SHLCA		CPLCA		DMLCA	
	TC	TC	%imp	TC	%imp	TC	%imp
Base Case	19652.89	21774.87	10.80%	22554.86	14.77%	20683.47	5.24%
w_1=0	17052.85	18892.17	10.79%	18808.73	10.30%	17529.14	2.79%
w_1=0.5	17835.53	19641.74	10.13%	19574.02	9.75%	19595.23	9.87%
w_1=2	21834.95	23891.42	9.42%	25534.06	16.94%	23130.34	5.93%
w_2=0	2114.02	2114.02	0.00%	2980.02	40.96%	2980.02	40.96%
w_2=1	10898.49	11945.71	9.61%	12767.11	17.15%	11811.08	8.37%
w_2=3	28536.28	31604.03	10.75%	32341.04	13.33%	29450.81	3.20%
w_3=0	14683.82	15993.65	8.92%	16886.76	15.00%	17098.48	16.44%
w_3=0.5	17947.42	18884.58	5.22%	19757.77	10.09%	18971.53	5.71%
w_3=2	23006.61	27555.45	19.77%	27686.79	20.34%	24086.61	4.69%
w_4=0	3760.45	6003.19	59.64%	4131.51	9.87%	6196.58	64.78%
w_4=0.5	11815.02	13892.77	17.59%	14193.91	20.13%	12254.27	3.72%
w_4=2	35524.67	37539.06	5.67%	39292.21	10.61%	37391.67	5.26%
w_5=0	19374.96	21544.77	11.20%	22354.88	15.38%	20302.44	4.79%
w_5=0.5	19588.37	21659.82	10.57%	22461.07	14.67%	20543.22	4.87%
w_5=2	20106.94	22004.97	9.44%	22934.65	14.06%	21218.44	5.53%
w_6=0	19606.28	21774.87	11.06%	22253.25	13.50%	20583.7	4.99%
w_6=0.5	19617.11	21774.87	11.00%	22430.83	14.34%	20633.76	5.18%
w_6=2	19994.80	21774.87	8.90%	22815.73	14.11%	22815.73	14.11%
w_7=0	19521.75	21774.87	11.54%	22554.86	15.53%	20463.87	4.83%
w_7=0.5	19597.56	21774.87	11.11%	22554.86	15.09%	20534.71	4.78%
w_7=2	19967.28	21774.87	9.05%	22554.86	12.96%	21080.49	5.58%

When we compare the results in Tables 3.6. and 3.7. with the results in Tables 3.3. and 3.4., it is observed that all the total cost values in Table 3.6. are more than 10% higher than the corresponding total cost values with centralized allocation as given in Table 3.3. Thus, it can be stated that centralizing the allocation decisions and obtaining a better balanced system can improve the system performance significantly. Directing the patients to more available hospitals rather than everyone going to the closest hospital seems to provide significant benefits to the system. Even when the weight parameters are changed, similar results are obtained as seen in Tables 3.4. and 3.7.

4. INVENTORY DECISIONS FOR HUMANITARIAN AID MATERIALS CONSIDERING BUDGET CONSTRAINTS

In this section, we aim to decide amount of materials to be provided before and after the disaster, considering the budget limitations and focus on how to distribute the budget among different materials. We first develop single product models with and without budget constraints. Then, we build a two-stage stochastic model with budget constraint for the two-product case and determine its optimal solution. Also, we evaluate the problem for multiple product cases and propose approximation algorithms. We conduct numerical experiments and make comparisons for all models and algorithms.

4.1. Problem Definition and Model

Many organizations have a limited budget, denoted as B in our model, to use for disaster preparedness and they use a portion of this budget to stock certain aid materials that might be needed in case of a disaster. Since the aid material demand is uncertain, the stocking quantities of different types of materials need to be decided under this uncertainty. A general idea may be that all the needs of the injured people should be met by buying as much material as possible with the given budget. But this may not be wise in reality. Especially, when we consider multiple products and a limited budget, it is important to use the budget correctly. If all the budget is used before the disaster, due to uncertain demand, some products might be overstocked while others are understocked. If the stocked materials are insufficient to meet the demand, it may become a need to purchase some of these aid materials in the post-disaster stage. But, if all the budget is consumed before the disaster, this option cannot be used and injured people may suffer from scarcity of some materials due to understocking of those materials while some others might be overstocked and remain unused. We let b denote the cost of unmet demand per unit and h denote the cost of overstocking per unit for the products that are unused after the disaster. In this point of view, spending the entire budget at the pre-disaster stage may not be optimal. If there were some remaining budget after the disaster, the understocked items might have been purchased after the disaster with that budget and cost of unmet demand might be avoided.

A second approach might be to use all the budget after the disaster when the uncertainty is resolved. In this approach, none of the materials will be overstocked and materials will be bought as needed. However, after the disaster, due to scarcity of resources and increased demand, the cost of the materials will increase. Purchasing the same material after the disaster will be much more costly than purchasing and stocking the same material before the disaster. In addition, the materials might not be ready immediately when needed and procuring them after the disaster might lead to waiting costs for the injured people. We let c denote the purchase cost of materials before the disaster and $w>c$ denote the procurement cost after the disaster. Thus, reserving all the budget for post-disaster usage might not be optimal, either. For this reason, it may be wise to allocate some amount of the budget for pre-disaster and some for post-disaster purchases.

In our study, we decide how much of the budget should be spent before the disaster, how much should be reserved for post-disaster and how they should be allocated among different types of materials. As preliminaries, we first develop a single product model without budget (SP) and then a single product model with budget (SPB). Then, we extend our study by developing a two product newsvendor model with budget constraint and then propose approximation algorithms for multiple product cases.

4.1.1. Single Product Model without budget constraint (Model SP)

Model SP is the classical newsvendor model considering a single product to minimize expected total cost without any budget limitation in disaster management. In this model, D is used as the random demand for the product with density function $f_D(x)$ and cumulative probability function $F_D(x)$. We let Q denote the decision variable for optimal stock amount and $E[c(Q)]$ is defined as the expected total cost in terms of the decision variable Q. Then, the problem can be stated as Model SP below.

$(Model\ SP)$

$$\text{Min } Z^{SP} = \text{ Min E[c(Q)]} = cQ + h\int_{0}^{Q}(Q-x)f_D(x)dx + w\int_{Q}^{\infty}(x-Q)f_D(x)dx \qquad (4.1)$$

subject to: $Q \geq 0$

According to model SP, since there is no budget limitation, Q units are stocked before the disaster at a unit cost *c*, and in case of need, additional materials can be purchased after the disaster at a unit cost *w*.

Proposition 1: *The optimal solution of model SP is* $Q^* = F_D^{-1}\left(\frac{\mathrm{w-c}}{\mathrm{h+w}}\right)$.

All the proofs are given in the Appendix.

4.1.2. Single product model with budget limitation (Model SPB)

Model SP assumes that there is unlimited budget and it is always possible to meet the whole demand after the disaster. However, this may be an unrealistic assumption. Thus, in this section we develop a single product model with limited budget, named as model SPB.

When a budget constraint exists, it might be impossible to satisfy the whole demand, even after the disaster. In this model, we let *B* denote the budget amount and b denote the unit penalty cost for unmet demand, where $b > w > c$. In model SPB the decision maker needs to allocate the usage of the budget for pre and post disaster stages. Since the price of the product is cheaper before the disaster, some amount of the budget will be used for the purchase of products to be stocked before the disaster. However, some of the budget might not be used at this stage since overstocking also has a certain cost due to unnecessary storage and excessive inventory costs. Thus, some of the budget can be spared to be used after the disaster if additional products will be needed at that stage, even though the purchase price is higher at that time. Note that when Q units are stocked before the disaster, *B*-cQ is left as the remaining budget for the post-disaster stage and at most $\frac{B-cQ}{w}$ units can be bought after the disaster. If the demand happens to be even more than what could be bought by the budget, those demand can not be met and there will be penalty costs associated with those unmet demand. Thus, the expected total cost function is presented by equation (4.2) under the budget constraint.

$(Model\ SPB)$

Min Z^{SPB} = Min $E[c(Q)]$

$$= cQ + h\int_0^Q (Q-x)f_D(x)dx + w\int_Q^{Q+\frac{B-cQ}{w}} (x-Q)f_D(x)dx + \int_{Q+\frac{B-cQ}{w}}^{\infty} \left[\left[w\frac{B-cQ}{w} + b\left(x-Q-\frac{B-cQ}{w}\right)\right] f_D(x)dx\right] \tag{4.2}$$

subject to: $B - cQ \geq 0$ and $Q \geq 0$

Proposition 2: *The optimum solution of model SPB is*

$$Q = \begin{cases} q^*, & B - cq^* \geq 0 \\ \frac{B}{c}, & otherwise \end{cases}$$

where q^* is the solution of the equation below:

$$\frac{cb}{w} - b + (h+w)F_D(q^*) + \left(-w + c + b - \frac{cb}{w}\right)F_D\left(q^* + \frac{B - cq^*}{w}\right) = 0$$

Observe that the solution of the above problem reduces to the result stated in Proposition 1 when *B* goes to infinity.

4.1.3. Two product newsvendor model with budget (TPB)

In a disaster situation, different amounts of different products might be needed. Since the effects of a disaster is uncertain, the demand for the necessary supplies cannot be known exactly. Thus, it is not easy to determine how much of these supplies should be stored. The NGO responsible for coordinating these processes is required to make these decisions. The responsibility of an NGO is to make the right amount of procurement using the available budget. Under a limited budget, stocking decisions for multiple products might be more challenging. Certain quantities will be stocked before the disaster, but when a disaster happens, one type of product might be overstocked while the other is understocked. Thus, the utilization of the budget becomes much more important in this case and if some budget were saved for post-disaster usage, the required materials could be bought by that budget after the actual demand is realized. The quantity decision of one product does not only effect its own results but it also effects the remaining budget and the results related to the other products. Thus, there exists a tradeoff and a competition effect between the quantity decisions of different products and the usage of budget. We also note that if the budget were unlimited, the decisions about multiple products can be separated from each other and the optimal solution for each product can be obtained by the solution of the model SP developed for each product separately.

To understand the relationships between the stocking decisions of multiple products, we first consider a two-product setup under a limited budget, named as model TPB. We firstly note that this two product model can be used as a basis for the more general multi-product models. We develop approximation algorithms for multiple product cases in Section 4.2., using the results of the model TPB. Secondly, even when more than two products exist in the system, model TPB can be used by grouping the products according to their features such that two types of grouped materials are analyzed and their stocking quantities are decided. For example, Klibi et al. (2018) group the products according to their types and analyze stocking decisions considering these groups of products. Thus, the model TPB provides significant results for the analysis of multiple product models, too.

The notations used in this model are outlined as below:

Parameters:

c_i: unit acquisition cost of product i in the pre disaster stage

h_i: unit holding cost of product i

w_i: unit acquisition cost of product i in the post disaster stage

b_i: unit penalty cost of unsatisfied demand for product i in post disaster

B: total amount of budget

D_i: random demand for product i

$f_i(x)$: probability density function of random variable D_i

$F_i(x)$: cumulative distribution function of random variable D_i

Decision Variables:

Q_i: prepositioning quantity of product i

$E[c(Q_1, Q_2)]$: expected total cost when Q_i units are stocked from product i

In a disaster environment, when there exists multiple products in the system, in addition to deciding how much to stock from each product before the disaster under uncertainty, the managers also need to decide how to allocate the remaining budget among the additional material needs after the disaster when the demand uncertainty is resolved. Thus, we develop a two-stage model such that the decisions are divided as the first stage and second stage decisions. In the first stage, the managers need to decide on the quantities of each product to stock under uncertainty of demand. After the disaster happens, in the second stage of the problem, the actual demand will be realized and if there is enough budget to satisfy all the excess demand, then the managers can buy all the required products at that time. However, if there is not enough budget to meet the demand of all types of products, the managers need to decide how much of each product to satisfy under the budget constraint. In this model, we let B denote the total budget and $B_2(\mathrm{Q}_1, \mathrm{Q}_2) = B - c_1 Q_1 - c_2 Q_2$ denote the remaining budget for the second stage. We also let D_i denote the random demand for product *i* with $f_i(x)$ and $F_i(x)$ functions as pdf and cdf of demand distributions. Q_i denotes the decision variables in our model and $E[c(Q_1, Q_2)]$ denotes expected total cost when Q_i units are stocked.

In order to solve this problem, we first need to solve the second stage problem. In the second stage, when the uncertainty disappears, the problem can be modeled as a deterministic mathematical model that turns into a classical knapsack problem that aims to decide how to allocate the budget among the excess demand of required products, i.e. how much of each product to buy with the remaining budget. We assume without loss of generality that $\frac{b_1}{w_1} \geq \frac{b_2}{w_2}$, i.e. the product with the higher *b/w* ratio is named as product 1, while the other is named as product 2. Then, the optimal solution in the second stage problem is characterized in the following lemma.

Lemma 1: *Assuming without loss of generality that* $\frac{b_1}{w_1} \geq \frac{b_2}{w_2}$, *optimal amounts of products bought at the second stage,* $(Q_1^s,\ Q_2^s)$, *can be characterized as below:*

$$
\begin{array}{ll}
(0,0) & if\ D_1 \leq Q_1 \text{and } D_2 \leq Q_2 \\
\left(0, \min\left(D_2 - Q_2, \dfrac{B - c_1 Q_1 - c_2 Q_2}{w_2}\right)\right) & if\ D_1 \leq Q_1 \text{and } D_2 > Q_2
\end{array}
$$

$$
(Q_1^s, Q_2^s) = \begin{cases} \left(\min\left(D_1 - Q_1, \dfrac{B - c_1Q_1 - c_2Q_2}{w_1}\right), 0\right) & if\ D_1 > Q_1 \text{and } D_2 \leq Q_2 \\ (D_1 - Q_1, D_2 - Q_2) & if \quad D_1 > Q_1, D_2 > Q_2, w_1(D_1 - Q_1) + w_2(D_2 - Q_2) \\ & \qquad \leq B - c_1Q_1 - c_2Q_2 \\ \left(\min\left(D_1 - Q_1, \dfrac{B - c_1Q_1 - c_2Q_2}{w_1}\right), \min\left(D_2 - Q_2, \dfrac{[B - c_1Q_1 - c_2Q_2 - w_1Q_1^s]^+}{w_2}\right)\right) & o.w. \end{cases}
$$

After solving the second stage problem, we then model and solve the first stage problem. The solution of the second stage problem states that the remaining budget in the second stage is used to buy the product 1 first as much as needed (since $\frac{b_1}{w_1} \geq \frac{b_2}{w_2}$), and then if the budget remains, product 2 can be bought. Thus, in the post-disaster stage, the first product has priority over the second product. With this in mind, to model the first stage problem, we use the auxiliary variables $\mathrm{A_i}$ and $\mathrm{A(D_1)}$ as defined in equations (4.3)-(4.5). $\mathrm{A_i}$ denotes the sum of $\mathrm{Q_i}$ with the maximum amount of product *i* that can be purchased with the remaining budget in the second stage. On the other hand, $A(D_1)$ denotes the maximum amount of product 2 that can be purchased after $D_1 - Q_1$ units of product 1 are purchased in the second stage, when $D_1 > Q_1$.

$$
A(D_1) = Q_2 + \frac{B - c_1Q_1 - c_2Q_2 - w_1(D_1 - Q_1)}{w_2} \tag{4.3}
$$

$$
A_1 = Q_1 + \frac{B - c_1Q_1 - c_2Q_2}{w_1} \tag{4.4}
$$

$$
A_2 = Q_2 + \frac{B - c_1Q_1 - c_2Q_2}{w_2} \tag{4.5}
$$

In this model, all possible cases of demand realizations should be taken into account when evaluating the optimal stock amount. Depending on the possible demand realizations, there are 8 cases that can occur. These 8 cases, as shown in Figure 2, vary according to the relation of three values, which are the stock quantities for product i (Q_i), demand for product i (D_i) and product quantities that can be purchased with the remaining budget in the second stage (A_i).

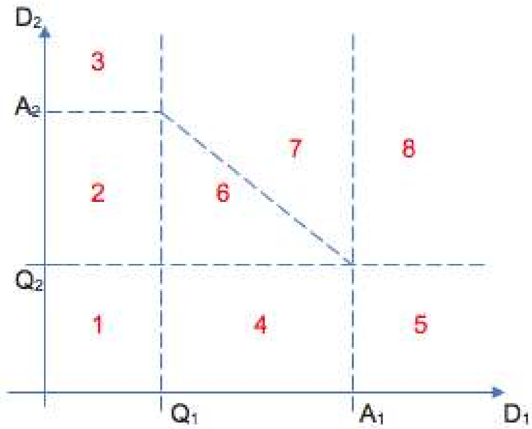


Figure 4.1 *Cases according to uncertain demand*

Case 1: In this case, we evaluate the situation in which the demand for both products are less than the pre-stock quantity ($D_1 < Q_1$ and $D_2 < Q_2$). In the first stage, demands for both products are met from the stock, while in the second stage, no purchase is made. In this model, first $c_1Q_1 + c_2Q_2$ is paid to buy the products in the first stage. Then, if case 1 happens, the cost function for this case is shown by equation (4.6), which consists of only the holding costs of unused products of both types.

Case 2: In this case, it is considered that demand for the first product is less than the initial stock amount while the demand for the second product is greater than its initial stock amount, but less than the maximum quantity that can be purchased with the remaining budget in the second stage ($D_1 < Q_1$ and $Q_2 < D_2 < A_2$). Thus, all the demand of second product can be satisfied in the second stage with the remaining budget. For this case, the cost function is defined as in equation (4.7). Cost of this case consists of holding cost for the first product and purchase cost of the excessive demand of the second product.

Case 3: In this case, it is considered that stock amount for the first product is enough for the demand but the demand for the second product can't be completely satisfied even when all the remaining budget in the second stage is used to buy the second product ($D_1 < Q_1$ and $A_2 < D_2$). The cost function for this case is given in equation (4.8), which include the holding cost for the first product, purchase cost of the second product that can be bought by the remaining budget and the cost of unmet demand for the second product.

Case 4: In this case, the demand for the first product is greater than its initial stock amount but less than the maximum quantity that can be purchased with the remaining budget in the second stage, while the demand for the second product is less than its stock ($Q_1 < D_1 < A_1$ and $D_2 < Q_2$). In this case, some of the demand of the first product will be met from the initial stock and the rest will be satisfied by the budget allocated for post disaster. The cost function for this case is presented in equation (4.9), which includes cost of post disaster purchases of the first product and holding cost of unused second products.

Case 5: We consider the case that the demand for the second product is less than its initial stock but the demand for the first product can not be completely satisfied, even when all the remaining budget in the second stage is used to buy the first product ($A_1 < D_1$ and $D_2 < Q_2$). The cost function in this case is given as in equation (4.10), which includes cost of post disaster purchase of the first product, cost of unmet demand for the first product and holding cost of unused second products.

Case 6: This case happens when the demand of both products are higher than their initial stock quantities but all of the excess demand of both products can be bought by the budget allocated for post disaster ($Q_1 < D_1 < A_1$ and $Q_2 < D_2 < A(D_1)$). Note that, since the first product has priority over the second one, first $D_1 - A_1$ units of the first product are bought and then the second product is bought by the remaining budget. This case reflects the situation when the remaining budget is enough to satisfy the demand of both products. The related cost function is defined as in equation (4.11), which includes the cost of purchase of both products in the post-disaster stage.

Case 7: Similar to the previous case, this case also happens when the demand of both products are higher than their initial stock quantities. Since the first product has priority, the budget allocated to the post-disaster stage is first used to buy the first product and the excess demand of the first product can again be satisfied with this remaining budget. However, different than the previous case, the excess demand of the second product can only be partially satisfied after the purchase of the first product ($Q_1 < D_1 < A_1$ and $A(D_1) < D_2$). Observe that the line $A(D_1)$, which denotes the maximum amount of the second product that can be bought after $D_1 - Q_1$ units are bought in the second stage, separates this case from *Case 6.* The related cost function is given in equation (4.12), which includes the purchase cost of both products in the second stage and the cost of unmet demand of the second product.

Case 8: This case reflects the situation when the demand for both products are higher than their initial quantities, but the demand for the first product is so high that the demand for the first product cannot be completely satisfied even when all the budget allocated to the post-disaster stage is used to buy the first product. In this case, there will be no budget remaining for the second product after the purchase of the first product in the second stage ($D_1 > A_1$, $D_2 > A_2$). The cost function for this case is given in equation (4.13), which includes the purchase cost of the first product and the cost of unmet demand for both product types.

$$M_1 = \int_0^{Q_1} \left[\int_0^{Q_2} [h_1(Q_1 - D_1) + h_2(Q_2 - D_2)] f_2(D_2) dD_2 \right] f_1(D_1) \mathrm{d}D_1 \tag{4.6}$$

$$M_2 = \int_0^{Q_1} \left[\int_{Q_2}^{A_2} [w_2(D_2 - Q_2) + h_1(Q_1 - D_1)] f_2(D_2) dD_2 \right] f_1(D_1) \mathrm{d}D_1 \tag{4.7}$$

$$\begin{aligned} M_3 = \int_0^{Q_1} \Bigg[\int_{A_2}^{\infty} [& w_2(A_2 - Q_2) + b_2(D_2 - A_2) \\ & + h_1(Q_1 - D_1)] f_2(D_2) dD_2 \Bigg] f_1(D_1) \mathrm{d}D_1 \end{aligned} \tag{4.8}$$

$$M_4 = \int_{Q_1}^{A_1} \left[\int_0^{Q_2} [w_1(D_1 - Q_1) + h_2(Q_2 - D_2)] f_2(D_2) dD_2 \right] f_1(D_1) \mathrm{d}D_1 \tag{4.9}$$

$$\begin{aligned} M_5 = \int_{A_1}^{\infty} \Bigg[\int_0^{Q_2} [& w_1(A_1 - Q_1) + b_1(D_1 - A_1) \\ & + h_2(Q_2 - D_2)] f_2(D_2) dD_2 \Bigg] f_1(D_1) \mathrm{d}D_1 \end{aligned} \tag{4.10}$$

$$M_6 = \int_{Q_1}^{A_1} \left[\int_{Q_2}^{A(D_1)} [w_1(D_1 - Q_1) + w_2(D_2 - Q_2)] f_2(D_2) dD_2 \right] f_1(D_1) dD_1 \tag{4.11}$$

$$\begin{aligned} M_7 = \int_{Q_1}^{A_1} \Bigg[\int_{A(D_1)}^{\infty} \big(& w_2(A(D_1) - Q_2) + b_2(D_2 - A(D_1)) \\ & + w_1(D_1 - Q_1) \big) f_2(D_2) dD_2 \Bigg] f_1(D_1) dD_1 \end{aligned} \tag{4.12}$$

$$\begin{aligned} M_8 = \int_{A_1}^{\infty} \Bigg[\int_{Q_2}^{\infty} \big(& w_1(A_1 - Q_1) + b_1(D_1 - A_1) \\ & + b_2(D_2 - Q_2) \big) f_2(D_2)\, dD_2 \Bigg] f_1(D_1) dD_1 \end{aligned} \tag{4.13}$$

The sum of all the cost functions given above for the stated 8 cases forms the expected total cost function as given in equation (4.14). Our aim is to find the optimal stock quantities, ${Q_1}^*$ and ${Q_2}^*$, that minimize this expected cost function under the budget constraint given in equation (4.15). Thus, the problem can be stated as Model TPB below:

$(Model\ TPB)$ (4.14)

$$Min\ M = E[c(Q_1, Q_2)]$$
$$= c_1Q_1 + c_2Q_2 + M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8$$

subject to

$$B - c_1Q_1 - c_2Q_2 \geq 0 \quad (4.15)$$
$$Q_1, Q_2 \geq 0$$

Lemma 2. *The cost function $E[c(Q_1, Q_2)]$ given in equation (14) is jointly convex with respect to Q_1 and Q_2.*

Since the objective function is proven to be jointly convex with respect to Q_1 and Q_2, the global optimal solution of Model TPB can be obtained by using the Karush-Kuhn-Tucker (KKT) conditions. The Lagrangean function and the KKT conditions for this problem are given in equations (4.16) and (4.17)-(4.23), respectively, as below.

$$L(q_1, q_2, \lambda_1, \lambda_2, \lambda_3) = E[c(q_1, q_2)] + \lambda_1 q_1 + \lambda_2 q_2 + \lambda_3(B - c_1q_1 - c_2q_2) \quad (4.16)$$
$$\frac{\partial L}{\partial Q_1} = 0 \quad (4.17)$$
$$\frac{\partial L}{\partial Q_2} = 0 \quad (4.18)$$
$$\lambda_1 Q_1 = 0 \quad (4.19)$$
$$\lambda_2 Q_2 = 0 \quad (4.20)$$
$$\lambda_3(B - c_1Q_1 - c_2Q_2) = 0 \quad (4.21)$$
$$B - c_1Q_1 - c_2Q_2 \geq 0 \quad (4.22)$$
$$\lambda_j \leq 0\ and\ Q_i \geq 0 \quad (4.23)$$

In order to obtain the optimal solution $({Q_1}^*, {Q_2}^*)$ that satisfies all the KKT conditions, we analyze and solve the above equations and obtain the optimal solution as stated in Theorem 1.

Theorem 1.

$$
(Q_1{}^*, Q_2{}^*) = \begin{cases}
(q_1^a, q_2^a) & if \quad c_1 q_1^a + c_2 q_2^a \le B,\ q_1^a \ge 0, q_2^a \ge 0 \\
(q_1^b, \dfrac{B - c_1 q_1^b}{c_2}) & if \quad c_1 q_1^b \le B, q_1^b \ge 0,\ \lambda_3^b \le 0 \\
(q_1^c, 0) & if \quad c_1 q_1^c \le B, q_1^c \ge 0, \lambda_2^c \le 0 \\
(0, q_2^d) & if \quad c_2 q_2^d \le B, q_2^d \ge 0, \lambda_1^d \le 0 \\
(0,0) & if \quad \lambda_1^e \le 0, \lambda_2^e \le 0 \\
\left(0, \dfrac{B}{c_2}\right) & if \quad \lambda_1^f \le 0, \lambda_3^f \le 0 \\
\left(\dfrac{B}{c_1}, 0\right) & if \quad \lambda_2^g \le 0, \lambda_3^g \le 0
\end{cases}
$$

where,

(q_1^a, q_2^a) is the solution of equations (4.24) and (4.25).

$$
(c_1 + h_1)F_1(q_1^a) + \frac{c_1(b_2 - w_2)}{w_2}[1 - F_2(A_2)]\, F_1(A_1) - \left(\frac{b_1(w_1 - c_1)}{w_1}\right)[1 - F_1(A_1)]
$$
$$
-\left(\frac{b_2(w_1 - c_1)}{w_2}\right)[F_1(A_1) - F_1(q_1^a)] + \int_{q_1^a}^{A_1} \left(\frac{(b_2 - w_2)(w_1 - c_1)}{w_2}\right) F_2\big(A(x_1)\big) f_1(x_1)\,dx_1 = 0 \tag{4.24}
$$

$$
c_2 F_1(q_1^a)F_2(A_2) + h_2 F_2(q_2^a) + w_2[F_2(q_2^a)F_1(A_1) - F_2(A_2)F_1(q_1^a)] - (b_2[1 - F_2(q_2^a)]
$$
$$
- \frac{b_1 c_2}{w_1})[1 - F_1(A_1)]
$$
$$
+\left(-b_2 + \frac{b_2 c_2}{w_2}\right)[F_1(A_1) - F_1(q_1^a)F_2(A_2)] - (w_2 - c_2 - b_2 + \frac{b_2 c_2}{w_2}) \int_{q_1{}^a}^{A_1} [F_2(A(x))]\, f_1(x_1)\,dx_1 = 0 \tag{4.25}
$$

(q_1^b, λ_3^b) is the solution of equations (4.26) and (4.27)

$$
(c_1 + h_1)F_1(q_1^b) + \frac{c_1(b_2 - w_2)}{w_2}\left[1 - F_2\left(\left(\frac{B - c_1 q_1^b}{c_2}\right)\right)\right] F_1(q_1^b) - \frac{b_1(w_1 - c_1)}{w_1}[1 - F_1(q_1^b)] - c_1\lambda_3^b
$$
$$
= 0 \tag{4.26}
$$

$$
[(c_2 + w_2)F_1(q_1^b) + h_2] F_2\left(\frac{B - c_1 q_1^b}{c_2}\right) + \left(-b_2 + \frac{b_2 c_2}{w_2} F_1(q_1^b)\right)\left[1 - F_2\left(\frac{B - c_1 q_1^b}{c_2}\right)\right]
$$
$$
+ \frac{b_1 c_2}{w_1}[1 - F_1(q_1^b)] - c_2\lambda_3^b = 0 \tag{4.27}
$$

(q_1^c, λ_2^c) is the solution of equations (4.28) and (4.29)

$$
(c_1 + h_1)F_1(q_1^c) + \frac{c_1(b_2 - w_2)}{w_2}\left[1 - F_2\left(\frac{B - c_1 q_1^c}{w_2}\right)\right] F_1\left(q_1^c + \frac{B - c_1 q_1^c}{w_1}\right) \tag{4.28}
$$
$$
- \frac{b_1(w_1 - c_1)}{w_1}\left[1 - F_1\left(q_1^c + \frac{B - c_1 q_1^c}{w_1}\right)\right] - \left(\frac{b_2(w_1 - c_1)}{w_2}\right)\left[F_1\left(q_1^c + \frac{B - c_1 q_1^c}{w_1}\right) - F_1(q_1^c)\right]
$$

$$+\int_{q_1^c}^{q_1^c+\frac{B-c_1q_1^c}{w_1}} \left(\frac{(b_2-w_2)(w_1-c_1)}{w_2}\right)F_2\left(\frac{B-c_1q_1^c-w_1(x_1-q_1^c)}{w_2}\right)f_1(x_1)dx_1=0$$

$$(c_2+b_2-w_2-\frac{b_2c_2}{w_2})F_1(q_1^c)F_2\left(\frac{B-c_1q_1^c}{w_2}\right)-b_2+\frac{b_1c_2}{w_1}+\left(\frac{b_2c_2}{w_2}-\frac{b_1c_2}{w_1}\right)\left[F_1\left(q_1^c+\frac{B-c_1q_1^c}{w_1}\right)\right] \tag{4.29}$$

$$-(w_2-c_2-b_2+\frac{b_2c_2}{w_2})\int_{q_1^c}^{q_1^c+\frac{B-c_1q_1^c}{w_1}} F_2\left(\frac{B-c_1q_1^c-w_1(x_1-q_1^c)}{w_2}\right)f_1(x_1)dx_1+\lambda_2^c=0$$

(q_2^d,λ_1^d) is the solution of equations (4.30) and (4.31)

$$\frac{c_1(b_2-w_2)}{w_2}\left[1-F_2\left(q_2^d+\frac{B-c_2q_2^d}{w_2}\right)\right]F_1\left(\frac{B-c_2q_2^d}{w_1}\right)-\frac{b_1(w_1-c_1)}{w_1}+(w_1-c_1)(\frac{b_1}{w_1}-\frac{b_2}{w_2})\left[F_1\left(\frac{B-c_2q_2^d}{w_1}\right)\right] \tag{4.30}$$

$$+\int_0^{\frac{B-c_2q_2^d}{w_1}} \left(\frac{(b_2-w_2)(w_1-c_1)}{w_2}\right)F_2\left(q_2^d+\frac{B-c_2q_2^d-w_1x_1}{w_2}\right)f_1(x_1)dx_1+\lambda_1^d=0$$

$$h_2F_2(q_2^d)+[-b_2+\frac{b_2c_2}{w_2}+w_2F_2(q_2^d)]F_1\left(\frac{B-c_2q_2^d}{w_1}\right)+[\frac{b_1c_2}{w_1}-b_2(1-F_2(q_2^d))]\left[1-F_1\left(\frac{B-c_2q_2^d}{w_1}\right)\right] \tag{4.31}$$

$$-(w_2-c_2-b_2+\frac{b_2c_2}{w_2})\int_0^{\frac{B-c_2q_2^d}{w_1}}\left[F_2\left(q_2^d+\frac{B-c_2q_2^d-w_1x_1}{w_2}\right)\right]f_1(x_1)dx_1=0$$

$(\lambda_1^e,\lambda_2^e)$ is the solution of equations (4.32) and (4.33)

$$\frac{c_1(b_2-w_2)}{w_2}\left[1-F_2\left(\frac{B}{w_2}\right)\right]F_1\left(\frac{B}{w_1}\right)-\frac{b_1(w_1-c_1)}{w_1}\left[1-F_1\left(\frac{B}{w_1}\right)\right]$$

$$-\left(\frac{b_2(w_1-c_1)}{w_2}\right)F_1\left(\frac{B}{w_1}\right)+\int_0^{\frac{B}{w_1}}\left(\frac{(b_2-w_2)(w_1-c_1)}{w_2}\right)F_2\left(\frac{B-w_1x_1}{w_2}\right)f_1(x_1)dx_1+\lambda_1^e=0 \tag{4.32}$$

$$\left(\frac{b_2c_2}{w_2}-\frac{b_1c_2}{w_1}\right)F_1\left(\frac{B}{w_1}\right)+\frac{b_1c_2}{w_1}-b_2-\left(w_2-c_2-b_2+\frac{b_2c_2}{w_2}\right)\int_0^{\frac{B}{w_1}}\left[F_2\left(\frac{B-w_1(x_1)}{w_2}\right)\right]f_1(x_1)dx_1+\lambda_2^e=0 \tag{4.33}$$

$(\lambda_1^f,\lambda_3^f)$ is the solution of equations (4.34) and (4.35)

$$-\frac{b_1(w_1-c_1)}{w_1}+\lambda_1^f-c_1\lambda_3^f=0 \tag{4.34}$$

$$(h_2+b_2)F_2\left(\frac{B}{c_2}\right)-b_2+\frac{b_1c_2}{w_1}-c_2\lambda_3^f=0 \tag{4.35}$$

$(\lambda_2^g,\lambda_3^g)$ is the solution of equations (4.36) and (4.37)

$$(c_1+h_1+\frac{c_1(b_2-w_2)}{w_2})F_1\left(\frac{B}{c_1}\right)-\left(\frac{b_1(w_1-c_1)}{w_1}\right)\left[1-F_1\left(\frac{B}{c_1}\right)\right]-c_1\lambda_3{}^g=0 \tag{4.36}$$

$$(\frac{b_1c_2}{w_1}-b_2)\left[1-F_1\left(\frac{B}{c_1}\right)\right]+\left(-b_2+\frac{b_2c_2}{w_2}\right)F_1\left(\frac{B}{c_1}\right)+\lambda_2{}^g-c_2\lambda_3{}^g=0 \tag{4.37}$$

Observe that as B goes to infinity, the first case (a) will be satisfied and equations (4.24)-(4.25) reduce to the result stated in Proposition 1, because under unlimited budget, the products can be analyzed separately and the optimal solution will be the same as the solution of the single product model without budget constraint.

4.2. Approximation Algorithms for Multiple Products

In the previous sections, we obtain the optimal solutions of the models when single or two products are considered. However, when the number of products in the system becomes more than 2, it becomes much more difficult to write the cost of the system explicitly and becomes almost impossible to determine the optimal solution in closed form. In this section, we consider $N \geq 2$ products and extend the solutions developed in the previous sections to multi-product cases and propose two different approximation algorithms. In the first approximation, which can be considered as a first-order approximation, we use the solution of the model SP as the basis. The second approximation uses the solution of the model TPB as the basis of the algorithm and this can be considered as the second-order approximation. In our models, we let i=1,2,..,N denote the index of the product, c_i and w_i denote the purchase cost of product i before and after the disaster, respectively, b_i denote the cost of stock-out for product i and D_i denote the random demand of product i with density function $f_i(x)$ and cumulative distribution function $F_i(x)$.

4.2.1. Single Product Model Approximation (SPA)

In our first heuristic, we first consider to solve the problem by assuming an unlimited budget and determine Q_i values accordingly without the budget constraint. Under the unlimited budget assumption, the problem can be decomposed into single product models and we determine Q_i values for each product *i* by solving the model SP for each product independently as described in Section 2.1. If these Q_i values satisfy the budget constraint (i.e. $\sum_{i=1}^{N} c_i Q_i \leq B$), then we use these Q_i values as the approximate solutions. However, the resulting Q_i values might violate the budget constraint (i.e. $\sum_{i=1}^{N} c_i Q_i > B$) and may not be feasible. In that case, we propose two different approaches to satisfy feasibility.

In the first approach, denoted as algorithm SPA-I, we use $Q_i^I = \frac{Q_i B}{\sum_{i=1}^{N} c_i Q_i}$ instead of the original Q_i values. Observe that, in this algorithm, each Q_i value is decreased by the same ratio such that $\sum_{i=1}^{N} c_i Q_i^I = B$ will be satisfied at the end. In the second approach, denoted as algorithm SPA-II, we consider the priorities of the products when decreasing their quantities. Note that since the budget constraint is violated, decreasing one unit of product *i* brings a saving of c_i in the budget but might lead to a stock-out cost of b_i. In this heuristic, we order the products according to the ratio of $\frac{b_i}{c_i}$. The product *i* with the highest ratio has the highest priority, and we do not want to decrease the quantity of high priority products. Thus, in this heuristic, we start from the highest priority product and buy as much as we can from that product *i*, up to the quantity Q_i. Then, we continue in the same manner going from the higher priority products to the lower ones. As a result, for each product i, the resulting quantity can be denoted as $Q_i^{II} = \min\{Q_i, \frac{B - \sum_{j \in S_i} c_j Q_j^{II}}{c_i}\}$, where S_i denotes the set of products *j* with a higher priority than product *i*. The nominator of the second term inside the brackets denotes the remaining budget for product *i* after higher priority products are bought, and when we divide that remaining budget by c_i we obtain the maximum feasible quantity of product *i* that can be bought at that time. If that quantity is lower than Q_i, that quantity is bought but if it is higher than Q_i, then Q_i is bought.

4.2.2. Two Product Model Approximation (TPA)

The first-order approximation algorithm SPA considers the products in isolation from each other and do not consider the effect of the quantity decision about any product on the others. However, due to the budget limitation, such effects might be significant in the optimal decisions. In this section, we extend the result of model TPB to multi-product case and model the multi product problem as a two-product model such that when deciding on the quantity Q_i for product *i*, all the other products are grouped and considered as a single representative product, denoted as product *j*. The demand of product *j* will be the total demand of the grouped products, thus utilizing the central limit theorem, we assume that the representative product *j* has a normal distribution with mean μ_j and standard deviation σ_j. The characteristics of the representative product *j* are then calculated through the following equations, similar to the equations used for the EMSR-b heuristic proposed by Belobaba (1987, 1989, 1992) for airline seat inventory control in revenue management systems.

$$\mu_j = \sum_{k=1,k\neq i}^{N} \mu_k, \quad \sigma_j = \sqrt{\sum_{k=1,k\neq i}^{N} \sigma_k^2},$$

$$c_j = \frac{\sum_{k=1,k\neq i}^{N} c_k \mu_k}{\sum_{k=1,k\neq i}^{N} \mu_k}, \quad h_j = \frac{\sum_{k=1,k\neq i}^{N} h_k \mu_k}{\sum_{k=1,k\neq i}^{N} \mu_k}, \quad w_j = \frac{\sum_{k=1,k\neq i}^{N} w_k \mu_k}{\sum_{k=1,k\neq i}^{N} \mu_k}, \quad b_j = \frac{\sum_{k=1,k\neq i}^{N} b_k \mu_k}{\sum_{k=1,k\neq i}^{N} \mu_k}.$$

In the above equations, the mean and standard deviation of the representative product *j* is calculated using the sum of the means and variances of the grouped products. Then, the cost parameters are calculated by taking the weighted average of the individual cost values considering the mean demands as the weight factors.

Then, the problem with these two products *i* and *j* are solved by the model TPB and the quantity decision Q_i about product *i* is made as a result of the solution of this model. Note that we solve model TPB for each product *i*=1,2,..,*N* separately for N times and we decide on Q_i while all other products are grouped as product *j*. As a result of the solution of these N models, we obtain all Q_i values. However, similar to the situation in SPA, the resulting Q_i values might still violate the budget constraint. In that case, we follow the same approach as stated in the previous section and develop two different algorithms. In algorithm TPA-I, we use $Q_i^I = \frac{Q_i B}{\sum_{i=1}^{N} c_i Q_i}$ values and in algorithm TPA-II, we use $Q_i^{II} =$

$\min\{Q_i, \frac{B-\sum_{j\in S_i} c_j Q_j^{II}}{c_i}\}$ values, as explained in the previous section. Note that the algorithm TPA provides the optimal result for *N*=2 and we present the performances of the algorithms TPA and SPA for $N \geq 2$ in the numerical results section.

4.3. Numerical Experiments

In this section, we perform several numerical experiments in order to analyze the optimal solutions under different parameter settings, present the performances of the proposed algorithms and aim to extract managerial insights about the system.

4.3.1. Single product model results (Models SP and SPB)

We first present the results of the single product models. In our experiments, we use the parameters as $h = 1, c = 2, w = 6, b = 7$ and demand is uniformly distributed between 0 and 10,000 (i.e. $D \sim U(0{,}10000)$). When the model SP under unlimited budget is solved, the optimal stock quantity turns out to be 5714 units. For model SPB, we present the optimal stock quantities under varying budget (B) values in Figure 4.2. We observe that when the budget is very limited, all the budget is used in the first stage and no budget is left for the post-disaster stage. As the budget increases, since more units can be bought with that budget, the stocking quantity increases up to a certain value. The maximum stocking value turns out to be 6074 units when the budget is 13000. After that point, as the budget increases, the optimal stocking quantity decreases and converges to the optimal quantity found for model SP as the budget becomes very high. Under a limited budget, when it is possible, the optimal stocking quantity turns out to be higher than the optimal quantity under unlimited budget. The optimal stocking quantity increases as the budget decreases, until a certain point, which seems counter-intuitive at first. The main reason for this structure is that, when the budget is limited, there is a potential risk of unsatisfied demand at the post-disaster stage, with a higher cost (b>w). Because of this risk, the model chooses to stock higher quantities in the first stage when the products are cheaper (c<w). As the budget increases, since the products can be bought with the remaining budget in the second stage, the risk of unsatisfied demand decreases and the model tends to stock lower quantities. It can be observed that the budget has a significant effect on stock quantities. Thus, decision makers (NGOs) are required to consider their budget when determining their optimal stock amounts.

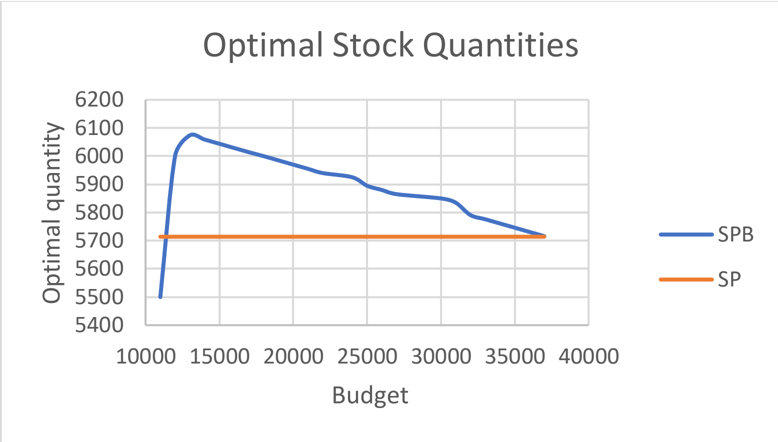


Figure 4.2 *Optimal stock quantities for models SP and SPB*

4.3.2. Two product model results (Model TPB) and their comparison with SPA

In this section, we present the numerical results for model TPB with the following parameters:
$c_1 = 8, c_2 = 6, h_1 = 4, \quad h_2 = 3, \quad w_1 = 15, w_2 = 12, \quad b_1 = 45, b_2 = 24.$ We use uniformly distributed demand functions where D_1 ~U (0, 40000) and D_2 ~U (0, 20000). We present the optimal stock quantities for both products with varying budget values in Table 4.1. In Table 4.1., we also present the used budget in the pre-disaster phase under column B' and the expected total cost of the system under column TC. We also compare these results with the single product approximation results in order to analyze the effect of budget consideration in these decisions. Recall that when the budget is unlimited, two product model can be solved through the solution of the single product model without budget. Thus, when the budget is ignored and the solution of the model SP is used for both products independently, it is found that $Q_1 = 14737$ and $Q_2 = 8000$ with total cost 344421. In the last four columns, we provide the results of the algorithms SPA-I and SPA-II. Note that when the budget is higher than a certain value SPA-I and SPA-II provide the same quantities such that $Q_1 = 14737$ and $Q_2 = 8000$, which are also the optimal solutions under unlimited budget. However, when the budget is tight, the resulting values under SPA-I and SPA-II are given separately in the last four columns, the first value belonging to SPA-I and the second value belonging to SPA-II. For this set of parameters,

we observe that SPA-II performs better than SPA-I and the percentage difference between the optimal solution (TC under TPB) and SPA-II ranges between 0% up to 5.32% in the worst case. Thus, we can state that making stocking decisions without considering the budget limitations might lead to significant losses, especially when the budget is at moderate levels. When the budget is very tight or when the budget is very high, the optimal decisions under TPB and SPA converge to each other.

The results of model TPB show that as the budget is tight, all of the budget is used at the pre-disaster stage and allocated between the products based on their costs and demand structures. As the budget increases, after a certain threshold, some of the budget is left over for the post-disaster stage. As B increases to higher values above the level of 250000, the usage of the budget in the pre-disaster phase B_1 decreases and similar to model SPB, higher values are left over for the post-disaster stage. As the budget increases even further and reaches to high enough levels, the optimal Q_1 and Q_2 values and the optimal budget usage converge to the SP model results as expected. The optimal Q_1 values are also seen to increase at first due to tight budget limitations, but after a certain threshold, similar to model SPB results, it starts to decrease as the budget increases. However, the optimal Q_2 values do not behave in similar monotonic structure. Due to the changes in B and Q_1, Q_2 behaves in a non-monotone manner in a certain region when the budget is moderate, but it then converges to model SP result as the budget becomes increases. The reason for this is that since the model wants to allocate a budget for the first product, it buys less from the second product when the budget is low. As the budget increases, it increases the quantity of the second product at first, but when the budget becomes higher than a certain level, since the post-disaster budget would be enough to buy both the first and second product excess demand, the model decreases the quantity of the second product again and converges towards the unlimited budget case. The behaviors of Q_1, Q_2 values with respect to budget can be seen in Figure 4.3. In summary, numerical results imply that as the budget increases, total cost decreases and the usage of budget during the pre-disaster phase first increases and then decreases. The optimal solution of the model TPB converges to the optimal solution of the model SP as the budget becomes high enough.

Table 4.1 *Results of Model TPB and comparison with SPA-I and SPA-II*

	TPB				SPA-I SPA-II			
B	Q_1	Q_2	B_I	TC	Q_1	Q_2	TC	% diff
100000	11900	800	100000	772468	8883	4822	788552	2.08%
					12500	0	773203	0.10%
150000	15975	3700	150000	647876	11104	6028	680785	5.08%
					14737	350	674261	4.07%
200000	20136	6485	200000	554966	13325	7233	588241	6.00%
					14737	5350	578821	4.30%
250000	22238	7880	225184	491213	14737	8000	516496	5.15%
300000	22209	7785	224382	441982	14737	8000	465492	5.32%
350000	22002	7561	221382	405593	14737	8000	425177	4.83%
400000	20785	7601	211886	381380	14737	8000	393270	3.12%
450000	18985	7904	199304	363911	14737	8000	369697	1.59%
500000	17186	8234	186892	352550	14737	8000	354457	0.54%
550000	15603	8468	175632	347222	14737	8000	347519	0.09%
600000	15094	8201	169958	345181	14737	8000	345233	0.02%
650000	14809	8040	166712	344487	14737	8000	344489	0.00%
700000	14737	8000	165896	344421	14737	8000	344421	0.00%
Unlimited	14737	8000	165896	344421	14737	8000	344421	0.00%

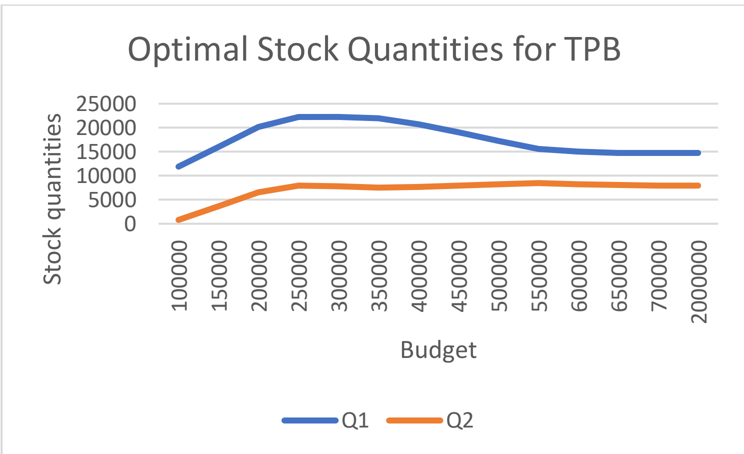


Figure 4.3 *Optimal stock quantities according to model TPB*

Even though it is observed in Table 4.2. that SPA-II performs better than SPA-I for the given parameter settings, SPA-I might also have a better performance for different parameters. For example, when we change the value of b_1 and set b_1=32 instead of 45, everything else remaining the same, we obtain the results in Table 4.2. It can be seen that when b_1=32, SPA-I performs better than SPA-II.

Table 4.2 *Performance analysis of the heuristics SPA-I and SPA-II*

	TPB			SPA-I SPA-II			
B	Q_1	Q_2	TC	Q_1	Q_2	TC	% diff
100000	9101	4532	631136.5	8883	4822	631211.1	0.01%
				12500	0	650312.5	3.03%
150000	13650	6800	545057.3	11104	6028	560301.4	2.79%
				14737	350	586323.2	7.57%
200000	18000	9333	484600.1	13325	7233	499707.7	3.11%
				14737	5350	500675.4	3.31%
250000	19338	7870	445529.2	14737	8000	453712.2	1.83%

4.3.3. Sensitivity Analysis for Model TPB and SPA results

To analyze the robustness of the solutions and the effects of different parameters on the system results, we perform a sensitivity analysis by changing some of the parameters, as shown in Table 4.3. We use the same parameters stated in the previous section and B=250000 in our base case stated in the first row of Table 4.3. Then, in each row of Table 4.3, we change one of the parameters at a time while all others remain constant and aim to analyze the effect of that parameter on the system results.

It is observed that when c_1 is decreased to 6.4 from its base case level of $c_1 = 8$, both Q_1 and Q_2 are increased and there is higher budget remained for the post-disaster phase. However, when c_1 is increased to 9.6, Q_1 is decreased but Q_2 is increased and all the budget is consumed at the pre-disaster phase. When h_1 or h_2 is increased, the model decreases the amounts of both Q_1 and Q_2 since higher quantities would lead to higher holding costs. But the effect of holding costs on the system results are much lower than the effects of purchase prices. As the post-disaster purchase price, w_1 or w_2, is increased, both the optimal stock quantities, Q_1 and Q_2, are seen to be increased. The main reason for this effect is that, as it becomes more costly to buy one of the products after the disaster, the model chooses to buy more of that product before the disaster. In addition, since this leads to less budget for the post-disaster phase, the model prefers to buy more of the other product before the disaster, too. When we analyze the effect of b_1, we observe that the optimal Q_1 increases while the optimal Q_2 decreases as b_1 increases. As the stock-out cost for the first product increases, the model prefers to buy more of that product but since this leads to less budget remained for the second product. Since the first product becomes much more important, the model prefers to leave a certain budget for post-disaster usage and buy the second product after the disaster if needed. Similar effect is also observed for b_2, and as b_2 increases, optimal Q_1 decreases while the optimal Q_2 increases.

When we compare the total cost values under model TPB and SPA, we observe that TPB provides significantly better results than SPA. Note that in all cases, the budget amount of B=250000 is enough to cover the quantities obtained under SPA, thus SPA-I and SPA-II provide the same result. In Table 4.3, the average percentage difference between total cost values of TPB and SPA is seen to be around 5.2%, with the maximum difference at 7.97% when b_1=54. It can be seen that TPB is especially better to use rather than SPA when h_1, w_2, b_1 are high or c_1, c_2, h_2, w_1, b_2 are low.

Table 4.3 *Sensitivity results*

	TPB				SPA				
	Q_1	Q_2	*TC*	*B1*	Q_1	Q_2	TC	*B1*	*%diff*
Base Case	22238	7880	491213	225184	14737	8000	516496.00	165896	5.15%
c_1=6.4	25715	8441	413920	215222	18106	8000	439822.20	163878	6.26%
c_1=9.6	20697	8550	565133	250000	11369	8000	583231.83	157142	3.20%
c_2=4.8	22738	10069	468800	230235	14737	9600	496043.83	163976	5.81%
c_2=7.2	21757	5592	508415	214318	14737	6400	532225.00	163976	4.68%
h_1=3.2	22741	7967	486156	229730	15385	8000	510048.27	171080	4.91%
h_1=4.8	21754	7797	496050	220814	14142	8000	522542.26	161136	5.34%
h_2=2.4	22284	8094	490256	226836	14737	8333	516003.93	167894	5.25%
h_2=3.6	22195	7675	492120	223610	14737	7692	516993.97	164048	5.05%
w_1=12	18106	5865	476361	180038	10000	8000	504238.13	128000	5.85%
w_1=18	24218	9251	493735	249250	18182	8000	512465.59	193456	3.79%
w_2=9.6	21743	6706	486505	214180	14737	5716	507970.63	152192	4.41%
w_2=14.4	23199	8670	493151	237612	14737	9656	523969.61	175832	6.25%
b_1=36	20516	9158	459826	219076	14737	8000	473030.27	165896	2.87%
b_1=54	23516	6458	518641	226876	14737	8000	559961.48	165896	7.97%
b_2=19.2	23138	6211	474901	222370	14737	8000	505620.09	165896	6.47%
b_2=28.8	21538	9223	504167	227642	14737	8000	527371.66	165896	4.60%

It can also be seen in Table 4.3 that Q_1 increases when c_1 is decreased or w_1 is increased, because the model wants to stock more products before the disaster when its pre-disaster price decreases or post-disaster price increases. When h_1 is reduced, keeping inventory will be less costly and thus Q_1 increases. However, since the budget is not considered by SPA, Q_2 is not affected by the change of c_1, w_1 or h_1. Similar results are also observed when the related parameters for the second product are changed. Finally, it can be observed that the cost of unmet demand (b_1 or b_2) does not affect the quantity decisions under SPA since budget is thought to be unlimited under this model and all required demand is thought to be met after the disaster under this model.

4.4. Multiple Product Approximation Algorithms

In this section, we first analyze the results of the algorithms TPA and SPA considering N=3 products. In this case, we assume that the demand of each product is normally distributed with parameters as given in Table 4.4.

Table 4.4 *Parameters for N=3 products*

Product #	B	w	C	h	μ	σ
1	34	12	6	4	12000	2700
2	35	15	5	3	10000	2500
3	38	18	6	3	8000	2300

The quantities obtained by the algorithms TPA and SPA are given in Table 4.5 for varying budget values. Observe that the initial quantities obtained by SPA are 8137 and 10349, but when the budget is 150000, these quantities violate the budget constraint and they need to be decreased according to SPA-I or SPA-II algorithms. The resulting quantities are given separately in the third row of Table 4.5. In the other cases, the budget constraints are not violated and the resulting quantities are stated in the appropriate cells in Table 4.5 for the algorithms SPA and TPA.

In order to compare the performances of the algorithms, we need to calculate the total costs of the systems, however for N>2, it is almost impossible to calculate the exact cost function due to exponentially increasing number of cases that might occur and the complex cost functions. Because of this, we employ a Monte-Carlo simulation study and generate the realized demand values for each product based on their demand distributions for 1000 replications. We calculate the realized cost value in each replication using the generated demand in that replication and the stocking quantities obtained by the algorithms TPA and SPA. We report the average cost of 1000 replications for both TPA and SPA algorithms under the column TC in Table 4.5. We observe that contrary to the results in Table 1, SPA-I performs much better than SPA-II when the budget is tight under the given parameters. Besides, TPA performs significantly better than SPA with the percentage difference ranging from 1.04% to 13.26%. Thus, we can state that considering the interactions between the products is important when deciding on the stocking quantities and making decisions independently for each product might lead to significantly worse results.

Table 4.5 *Results with the algorithms TPA and SPA for N=3 products*

		Q_1	Q_2	Q_3	*TC*	*%diff*	*B1*
B=150000	TPA	8610	10100	7900	109360	-	149560
	SPA-I	8230	10185	8281	110500	1.04%	150000
	SPA-II	10349	8362	6585	145886	33.40%	150000
B=250000	TPA	8436	11003	8541	182781	-	156877
	SPA	8137	10349	8362	207015	13.26%	150739
B=350000	TPA	8364	10351	8416	189881	-	152435
	SPA	8137	10349	8362	198366	4.47%	150739

We also analyze the performances of the algorithms TPA and SPA for *N*=5 and *N*=10 products with the budget *B*=250000 and the other parameters as given in Tables 4.6 and 4.7.

Table 4.6. *Parameters for N=5 products*

Product #	1	2	3	4	5
B	33	35	36	33	34
W	12	15	17	16	18

C	6	7	7	8	8
H	3	4	3	3	5
μ	5000	6000	5000	7000	4300
σ	1400	1500	1200	1100	1000

Table 4.7 *Parameters for N=10 products*

Product #	1	2	3	4	5	6	7	8	9	10
B	35	31	35	37	32	36	36	33	34	31
W	13	12	14	17	15	17	18	17	18	17
C	8	7	8	7	10	6	6	10	6	6
H	3	4	3	4	2	2	3	4	2	4
μ	2000	3000	3500	2000	5000	4500	2000	3000	2500	2500
σ	800	1000	875	700	900	850	1000	1000	1000	900

Table 4.8 *Results with the algorithms TPA and SPA for N=5 products*

	Q_1	Q_2	Q_3	Q_4	Q_5	*TC*	*%diff*	*B₁*
SPA	4397	5702	4850	6781	4025	61147	-	186694
TPA	4542	5917	4971	6854	4067	56574	%8.08	190836

We note that the total costs of the systems stated under column TC in Tables 4.8 and 4.9 reflect the average cost observed in 1000 replications, where the realized demand values for each product in each replication are obtained through the Monte-Carlo simulation as explained above. We observe that TPA performs significantly better than SPA both for *N*=5 and *N*=10 products. The percentage difference between the total costs under TPA and SPA turns out to be 8.08% when *N*=5 and 10.58% when *N*=10. Thus, it can be stated that the two-product approximation model provides significantly better results than the single product approximation even when larger number of products are considered in the system.

Table 4.9 *Results with the algorithms TPA and SPA for N=10 products*

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	TC	*%diff*	B_I
SPA	1608	2511	3170	1958	4513	4669	2569	2181	2753	2553	115517	-	211711
TPA	1694	2232	3285	2044	4614	4756	2313	2656	2826	2613	104455	%10.58	217512

4.5. An Extension of the Model TPB with Cash Donations after the Disaster

In a disaster situation, sometimes NGOs are provided with cash donations from the community and these donations can also be used to satisfy the demand of the products. In this section, we present an extension of the model TPB, denoted as TPBd (Two product model with budget and donations), in which we consider cash donations after the disaster.

When cash donations of amount d are made after the disaster, the budget that can be used after the disaster increases by that amount. However, note that since these donations are made after the disaster, they cannot be used to make purchases in the pre-disaster phase. The amount of donations are not known in advance, thus d is random with density function $f_d(x)$ and cumulative probability function $F_d(x)$. Note that model TPB reflects the case when d=0. When d is random, the objective function of the model TPBd can be written as below:

Min $M_d = \int_0^\infty M(x)\, f_d(x)dx$

where M(x) is the objective function given in equation (14) for model TPB with x units of cash donation. When there is a donation of x units, the remaining budget equations in the second stage needs to be updated as below instead of the equations $\mathrm{A}(D_1), A_1$ and A_2 given in equations (4.3)-(4.5), everything else remaining the same.

$$A^d(D_1) = Q_2 + \frac{B + d - c_1Q_1 - c_2Q_2 - w_1(D_1 - Q_1)}{w_2}$$

$$A_1^d = Q_1 + \frac{B + d - c_1Q_1 - c_2Q_2}{w_1}$$

$$A_2^d = Q_2 + \frac{B + d - c_1Q_1 - c_2Q_2}{w_2}$$

Since the objective function of the model TPB, M, is shown to be jointly convex, $M(x)$ with a fixed donation x, and $M_d = E[M(d)] = \int_0^\infty M(x)\, f_d(x)dx$ are also jointly convex since expectation operator preserves convexity, where M_d is the expected function of $M(d)$ with the donation d as the random variable. Thus, the optimal solution of this problem can be obtained by using the first order derivatives with the help of KKT conditions, similar to the equations obtained in Theorem 1.

In Table 4.10, we present the results of the model TPBd with the base case parameters as stated in Section 4.2 and random donation parameter d that is uniformly distributed between 0 and 100000. When we compare the results in Table 10 with the results in Table 1, in which there is no donation, we observe that similar results are obtained in both cases when the budget is tight. As the budget increases, slightly less quantities are bought in Table 4.10, since the required amounts can be bought after the disaster with the donations. As the budget becomes high enough, the model results converge to the same point as in Table 1 since additional donations will have no effect when the budget is already high enough.

Table 4.10. *Optimal stocking quantities and total cost for the model TPBd*

B	Q_1	Q_2	B_I	TC
100000	11900	800	100000	693182
150000	15975	3700	150000	577893
200000	20140	6480	200000	495761
250000	22182	7767	224058	444129
300000	21879	7590	220572	407772
350000	20804	7611	212098	382773
400000	18982	7905	199286	365300
450000	17243	8201	187150	353901
500000	15881	8336	177064	347845
550000	15146	8222	170500	345426
600000	14852	8063	167194	344597
650000	14789	8018	166420	344443
700000	14737	8000	165896	344421
Unlimited	14737	8000	165896	344421

5. MULTI-PERIOD INVENTORY DECISIONS FOR HUMANITARIAN AID MATERIALS CONSIDERING BUDGET LIMITATIONS

In this section, we present a dynamic programming model that focus on the optimal stocking and ordering policy considering budget limitations, random timings and quantities of donations and random demand for materials. The managers need to make a decision at any point in time considering the available budget and available stock at that time. Due to the dynamic nature of this system, the budget and the stock changes over time because of arriving donations and usage of materials during disasters. We develop dynamic programming models for this system and extract managerial insights through the solutions of these models. In order to show the effect of budget considerations, model results with and without budget limitations are compared while determining the optimal order policy.

5.1. Problem Definition and Model

NGOs need to decide how to use their budget for disaster preparedness. Due to the uncertainty of disasters, they need to stock certain amount of aid materials to satisfy the demand when the disaster happens. However, two situations may come up in this system. The first is that when they stock products with all their budget, there may not be a disaster and those materials will remain inactive for a long time. Thus, in this case the budget will be used inefficiently. The organizations might incur high holding costs as the products wait for long times in the depot. Also, since there is no budget left, it may not be possible to purchase later. For this reason, it may not be reasonable to buy large number of products with the whole budget. Secondly, if only a small number of aid materials are purchased with a small part of the budget, these materials may not be sufficient to meet the demand when the disaster occurs. In this case, if the available budget is sufficient, purchases can be made after the disaster. However, purchasing the materials after the disaster will be much more costly than purchasing them before the disaster due to the imbalance that will occur between supply and demand in case of a disaster. In addition, there will be the cost of waiting since it will take a certain time to acquire these materials after the disaster happens. Due to the increased cost of these materials after the disaster, lesser quantities can be bought after the disaster with the available budget and some of the demand may not be met just because enough quantities are not bought before the disaster. Because of these tradeoffs it is an important decision for NGOs to stock right amount of aid materials with the limited budget they have to prepare for disasters.

Donations are the main sources of income for many NGOs. Donations are made from different sources to help the needs of the injured people. These donations are used by NGOs for assistance in necessary actions. Each donation arrives at random times at random quantities, which are additional amounts to the budget and updates the state of the budget on hand. As the budget is updated, NGOs need to decide whether they should buy additional materials with the increased budget or keep it in cash to use in the future.

In this study, we analyze the dynamic stocking strategy of an NGO for disaster preparedness, considering a single product in an infinite horizon setting. We consider a continuous time model and try to determine the optimal ordering quantities depending on the amount of the budget and stock on hand at every state. In our model NGO decides whether to order new products based on the budget and the number of stocks available. In our model, we assume that donations arrive according to a Poisson process with rate λ_1, and disasters occur with exponential interarrival times with rate λ_2. When a donation arrives, it will be of the random quantity *db* with the density function $f_{db}(x)$. When a disaster happens, it leads to the demand *D* with the density function $f_D(x)$. In addition, every time a disaster happens, it leads the society to make more donations. Thus, we assume that every time a disaster happens, it also leads to a random donation quantity *db* with the density function $f_{db}(x)$.

5.2. Multi period inventory model with budget constraint (MPIB)

In this section, we develop a continuous time infinite horizon dynamic programming model for inventory decisions with budget considerations (MPIB). We define the state at any time with two state variables *(q, B)* where *q* denotes the quantity at hand and *B* denotes the available budget at that time. Then, $V(q, \mathrm{B})$ denotes the value function for that state. We let *h* denote the holding cost per unit, *c* denote the purchase cost per unit before the disaster, *w* denote the purchase cost per unit after the disaster and b denote the cost of unmet demand per unit.

When making preparation plans for disasters, another factor to be considered is donation. In real life, people donate a certain amount of money before or after a disaster. Therefore, we take into account cash donation as a factor in our model.
We summarize the notation used in this study as follows.

Parameters:

h: holding cost per unit product per unit time

c: purchasing cost per unit product before the disaster

w: purchasing cost per unit product after the disaster

b: cost of unmet demand per unit

x: random demand for the product in case of a disaster

$f_D(x)$: probability density function of demand D

da: random donation amount if there is a disaster

$f_{da}(x)$: probability density function of donation da

db: random donation amount when there is no disaster

$f_{db}(x)$: probability density function of donation db

λ_1: rate of donation arrivals per unit time

λ_2: rate of disaster occurrences per unit time

System States

q: stock quantity at hand

B: available budget at hand

Decision variables

$Q(q, B)$: purchase quantity at state (q, B)

$V(q, B)$: total cost to go function at state (q, B)

$$A(\boldsymbol{x}) = (Q + q) + \frac{B + x - cQ}{w} \tag{5.1}$$

In our study, we aim to decide on the optimal quantity that needs to be ordered by considering the available stock and the budget on hand. For this purpose, we develop a continuous time infinite horizon discounted cost model with discount factor β. Without loss of generality, we normalize the parameters such that $\lambda_1 + \lambda_2 + \beta = 1$. We note that all the continuous time dynamic programming models stated below are developed under this normalization assumption. Bellman's equation to be used for this problem can be stated as below. Please refer to Bertsekas (2001) for details about Bellman's equation for continuous time discounted cost dynamic programming formulations.

$(Model\ MPIB)$

$$
\begin{aligned}
&V(q,B) \\
&= \min_{Q\geq 0, cQ\leq B} \Bigg[h(q+Q) + cQ + \lambda_1 \int_{x=0}^{\infty} V(q+Q, B-cQ+x) f_{db}(x)dx \\
&\qquad + \lambda_2 (\int_{x=0}^{\infty} (\int_{y=0}^{q+Q} V_k(q+Q-y, B-cQ+x) f_D(y)dy) \\
&\qquad + (\int_{y=q+Q}^{A(x)} w(y-(q+Q)) \\
&\qquad + V\big(0, B-cQ+x-w(y-(q+Q))\big) f_D(y)dy) \\
&\qquad + (\int_{y=A(x)}^{\infty} V(0,0) + w(B-cQ-(q+Q)) \\
&\qquad + b\left(y-\left(q+Q+\frac{(B-cQ+x)}{w}\right)\right) f_D(y)dy) f_{da}(x)dx \Bigg]
\end{aligned} \tag{5.2}
$$

We define the cost-to-go function $V(q,B)$ as the function which is equal to the sum of the discounted costs of all periods. When Q units are purchased, the instantaneous purchasing cost will be cQ. Since there will be $(Q+q)$ units of stock on hand until the state changes, the holding cost until the next transition becomes $h(Q+q)$ due to the normalized transition rates. If the next transition is to a donation occurrence, which happens with rate λ_1, the stock at hand remains $(Q+q)$, and the budget in the next state will be *B-cQ+x* since cQ is paid to buy Q units and an additional random donation *x* has arrived. On the other hand, if the next transition is to a disaster occurrence, which happens with rate λ_2, three different situations might occur depending on the demand. If the random demand is low such that it can be met by the stock at hand, the next state will be such that the stock at hand is reduced by that amount, while the budget increases with the incoming donation. If the demand is more than the available stock, but can be satisfied with the available budget, then the next state will be such that the stock decreases to 0 since all the available stock will be used to satisfy the demand, while the budget is reduced by the cost of additional purchases made after the disaster to meet the excess demand. The cost of this purchase is also added to the function with the usage of *w* per unit. Finally, if the demand is too high such that the available budget is not sufficient to satisfy them all, all the available quantity and all the budget will be used to satisfy the portion of demand and the next state will be (0,0). The cost of these purchases after the disaster and the cost of unmet demand are added to this cost function.

5.3. Multi period inventory model without budget constraint (MPI)

In this section, in order to analyze the effect of budget limitations, we modify the model MPIB by eliminating the budget limitations We develop a model, named MPI, as stated below. In this model, since it is assumed that there is unlimited budget, we only use q that denotes the available stock at hand and use it as the only state variable in this model. The decision variable $Q(q)$ denotes the purchase quantity when there are q units at hand. In this model, we aim to decide on the optimal order amount $Q(q)$ which minimizes the value function $V(q)$. The Bellman's equation for this problem can be stated as Model MPI below.

$$(Model\ MPI)$$

$$V(q) = \min_{Q\geq 0}\Bigg[h(q+Q) + cQ + \lambda_1 \mathrm{V}(q+Q) + \lambda_2 \int_0^{q+Q} V(q+Q-x)\, f_D(x)dx \\ + (\int_{q+Q}^{\infty} V(0) + \big(x-(q+Q)\big)w) f_D(x)dx\Bigg] \tag{5.3}$$

In this model, when Q units of the product is purchased at the unit purchasing price c, the cost cQ will be incurred, and $q + Q$ units of products will be kept in stock until the next transition, leading to the related holding cost h $(q+Q)$, due to normalized transition rates. The next transition can be a donation with rate λ_1, as in model MPIB, but since there is no budget limitation in this model, the next state can simply be denoted with the new state $(q+Q)$ in that transition. However, if the next transition is a disaster occurrence with rate λ_2, there can be two situations. If the random demand quantity, x, is less than the available stock, the demand is met from the stock and the next state will be $(q+Q\text{-}x)$.

If the demand is higher than the stock on hand, the quantity at the beginning of the next state will be 0 and the amount of excess demand needs to be purchased at the unit cost *w,* leading to the cost *w*(*x*-*q*-*Q*). So, total expected cost is shown as equation (5.3).

5.4. Computational Results

In this section, we present our computational results and discuss on the inferences of these results under different settings. Also, we conduct sensitivity analysis to show the robustness of our results. We solve the infinite horizon dynamic programming models presented in the previous sections by the value iteration method.

We first present the numerical results for the base case models with the following parameters: $\lambda_1 = 0.88, \lambda_2 = 0.1, h = 0.01, c = 3, w = 6, b = 20$. We use demand (x) as uniform distribution between 0-19. And donations are used as uniform distribution between 0-2 for before disaster (*da*) and uniform distribution between 0-9 for after disaster (*db*). We express the results through different figures. For example, the order levels of the model according to states are shown in Figure 5.1. In this figure optimal order levels, varying according to two "system states" which are the budget *(B)* and the stock amount available *(q),* are indicated. However, in order to see the results in more detail, we fixed the budget and the stock at hand in turn and interpret it as in Figure 2a-b. In Figure 2a, the stock at hand is fixed as *q = 0, q = 5, q = 10.* It shows the optimal order-levels changing according to the budget. In Figure 2b, to be able to present the structure of the optimal solution more clearly, budget is fixed to *B = 50, B = 75, B = 247.* This figure implies the order up to levels vary according to stock at hand. As can be seen here in Figure 5.2a, the model buys as many products as the budget can buy in cases where the budget is low. This means that model tries to complete stock to high order levels when the budget is low. The risk of not meeting demand is high, when the budget is low. However, when the budget continues to increase, we see that the order levels decreases. The reason is that, as the budget increases, even though the products are more costly after the disaster, they can be bought with the remaining budget if needed, and the risk of not meeting the demand decreases. Thus, model reduces order up to levels to lower values. Similar effect is observed in Figure 5.2b. Optimum order levels decrease as the amount of stock at hand increases. The reason for this is that as the amount of stock at hand increases, the risk of not meeting the demand decreases, so the order levels decrease.

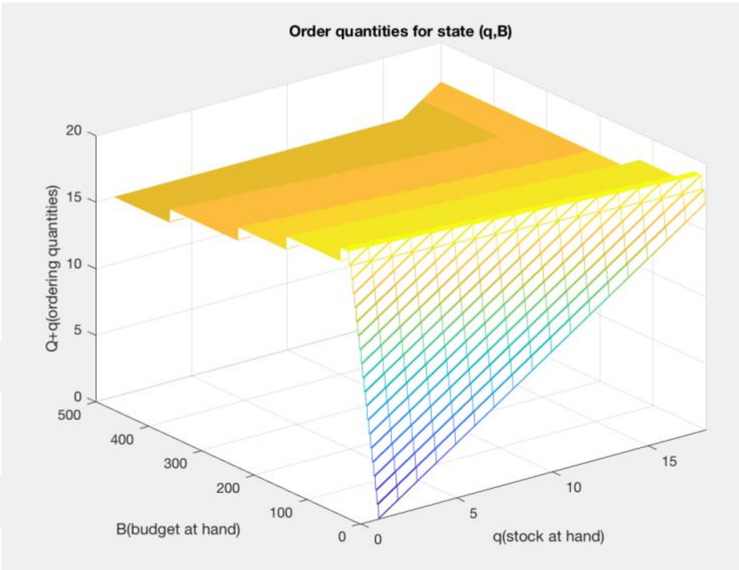
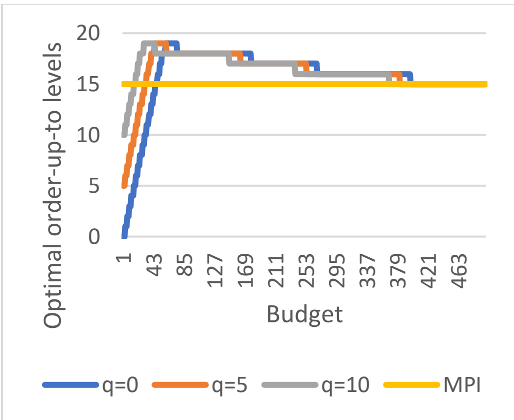
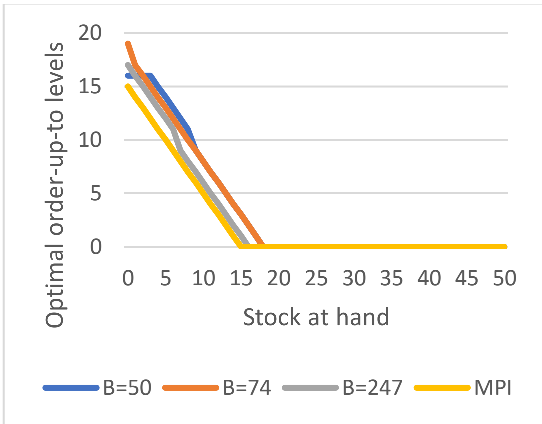


Figure 5.1 *Order-up to levels for each state (q,B))*



(a)



(b)

Figure 5.2 *Order-up-to levels a) stock amount is fixed as q, b) budget is fixed as B*

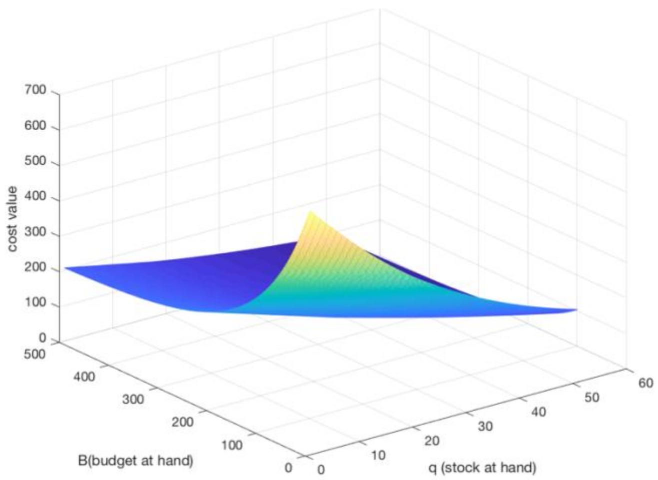


Figure 5.3 *Cost values for each state (q,B)*

Figure 5.3 shows how the cost value changes for each state. In order to see the results in more detail, we fixed the budget and the stock at hand in turn and interpret it as in Figure 5.4, 5.5. In Figure 5.4, the stock at hand is fixed as $q = 0$, $q = 5$, $q = 10$. It shows the cost changing according to the budget. In Figure 5.5., budget is fixed to $B = 50$, $B = 100$, $B = 200$. It can be seen in both figures that the cost convexly decreases when both B, q values increase. The reason is that the risk of not meeting the demand decreases when q or B increases. Therefore, the cost also decreases. Furthermore, for the low budget amounts, cost decreases fast, because budget is important at first. Then when the budget gets higher, it becomes less important. Therefore, it can be observed that budget has impact on the costs.

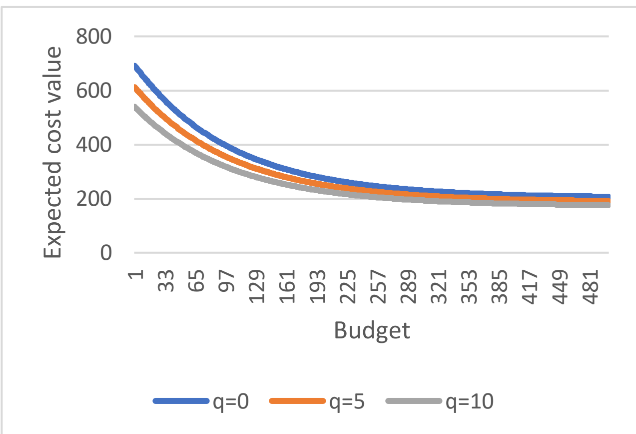


Figure 5.4 *Cost values varying according to budget for MPIB model*

Also, we conduct some experiments that compares the results of the MPI (the model that does not take budget into consideration) and MPIB models. For the comparison, firstly we obtain optimal order up to levels by MPI model, not considering budget. Then we fix those decisions to calculate the cost results according to MPIB model. While obtaining these results, for some cases, the order levels, found optimally with MPI model, turn out to be infeasible since they cannot actually be purchased with the budget. For those cases, it is assumed that the maximum amount that the budget could buy is purchased. For the comparison metric, we use the percentage difference between the costs of the models MPI and MPIB. The percentage improvement values for $q = 0$ state can be seen in Figure 5.6. Also, some inferences can be made for Figure 5.6. For example, when the budget is low, it is not important to consider the budget. The reason for this is that the model wants to purchase product with entire budget. For this reason improvement rate is low at first. When the budget amount increases a little more, the effect of the budget becomes more important. The budget becomes important as the risk of not meeting the demand increases. Thus, improvement rate increases. When the budget increases further, it is seen that the same results are obtained with the MPI model, improvement rate decreases.

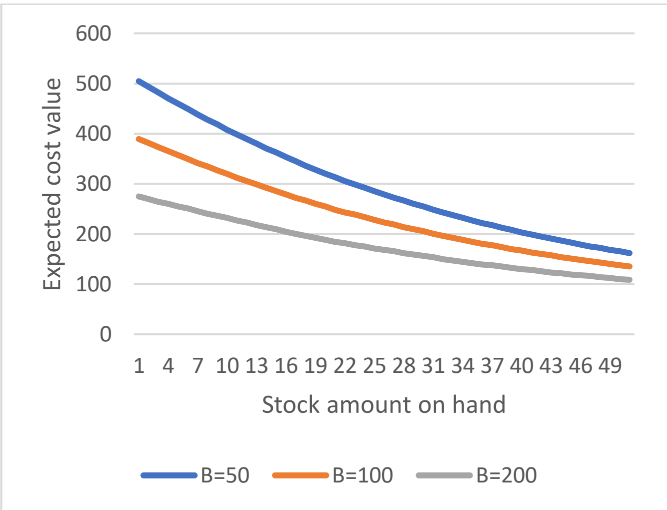


Figure 5.5 *Cost values varying according to available stock for MIPB model*

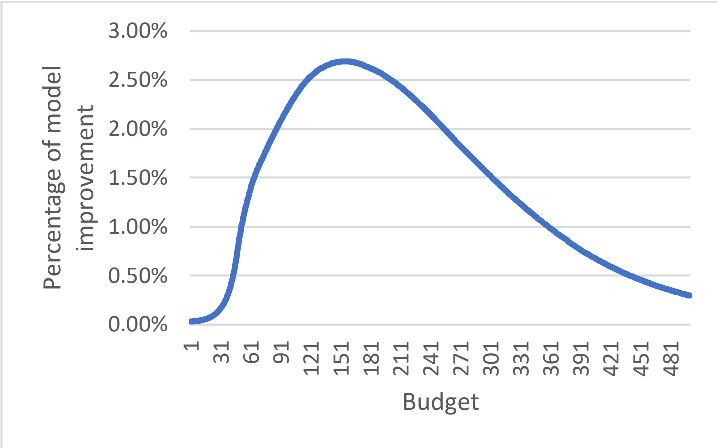


Figure 5.6 *Cost differences between MPI and MPIB with q=0*

5.5. Sensitivity Analysis

In order to show the robustness of the model we also conduct a sensitivity analysis. We evaluate holding cost (h), unit purchasing price before the disaster (c), unit purchase price after the disaster (w) and unit cost for unmet demand (b), in order to observe the effects of these parameters on the optimal order levels and costs. We define the base case parameters as $h = 0.01,\ c = 3,\ w = 6,\ b = 20$ and increase or decrease each parameter at a time to observe the effects of that parameter. In Figure 5.7, when we change the unit purchase cost before disaster, we examine how the optimal order-up-to levels change, for $q = 0$. It is observed that the order levels decrease as the c value increases. The reason for this is that as the product gets expensive, overstocking becomes more costly and the model wants to order less. The cost value also increases with c, which can be seen in Figure 5.8. In addition, it is seen in Figure 5.9 that as the c value increases, the consideration of budget becomes more important and the MPIB model performs much better than the model MPI.

We analyze the effect of the cost of unmet demand in the following three figures, Figure 5.10, 5.11, 5.12. If b increases, the order quantities increase and the model wants to keep more inventory to avoid the high cost of unmet demand. Cost values also increase for the same reason as observed in Figure 5.11. In Figure 5.12 it seen that, when the b value increases, MPIB model performs much better than the model MPI.

We use Figure 5.13, 5.14, 5.15 to examine the effect of unit purchase price after disaster (*w*). When the unit purchase price (*w*) increases, the order quantities increase because the model does not want to buy products after the disaster at a higher price. Therefore, it increases the order quantities before the disaster. When w decreases, although the costs also decrease a little, there will not be a significant cost reduction, as seen in Figure 5.14. If we consider the percentage improvement rates in Figure 5.15, as the value of *w* decreases, since the order quantities also decrease and become more dependent on budget, MPIB model provides much higher improvements compared to model MPI.

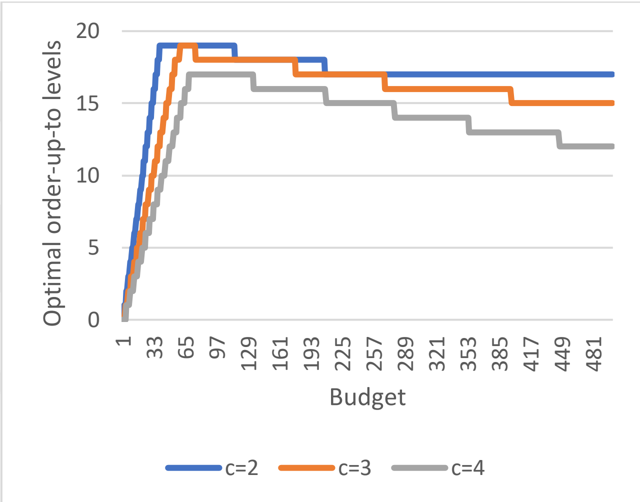


Figure 5.7 *Order levels varying according to purchasing price before disaster for q=0*

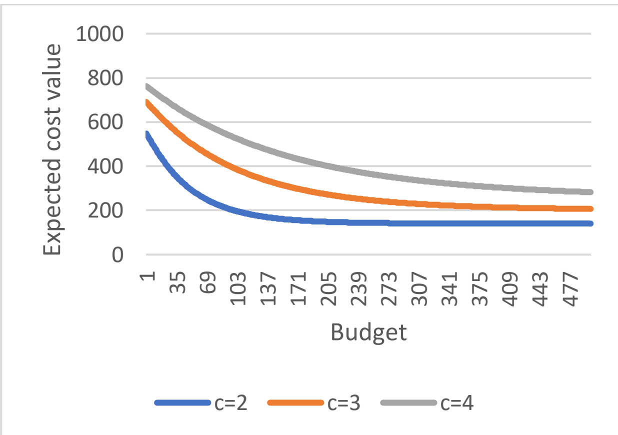


Figure 5.8. *Costs varying according to purchasing price before disaster for q=0*

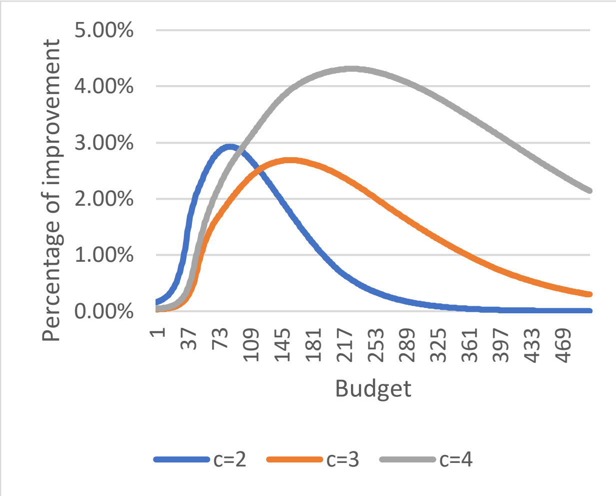


Figure 5.9 *Improvement percentage between models varying according to purchasing price*

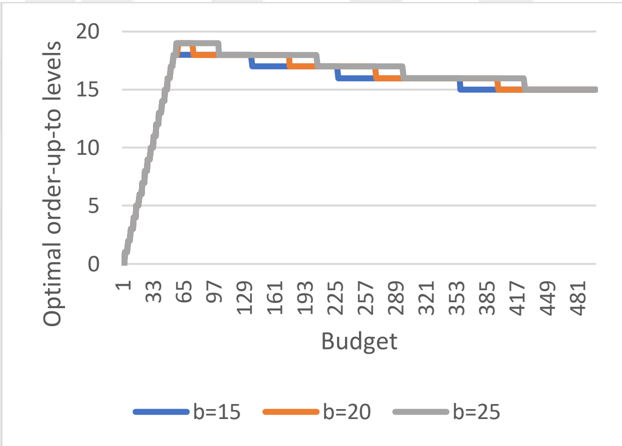


Figure 5.10 *Optimal order quantity varying according to unmet demand cost for q=0*

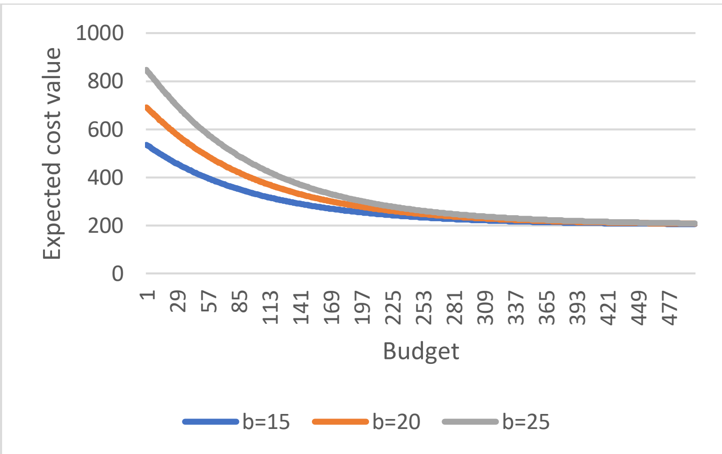


Figure 5.11 *Cost values varying according to unmet demand cost for q=0*

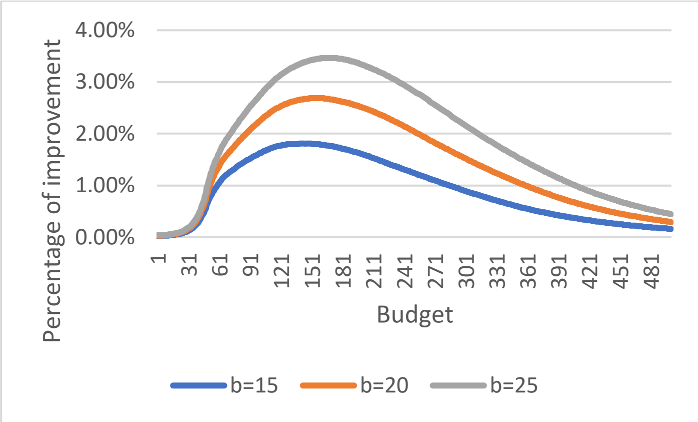


Figure 5.12 *Improvement of percentages varying according to unmet demand cost for q=0*

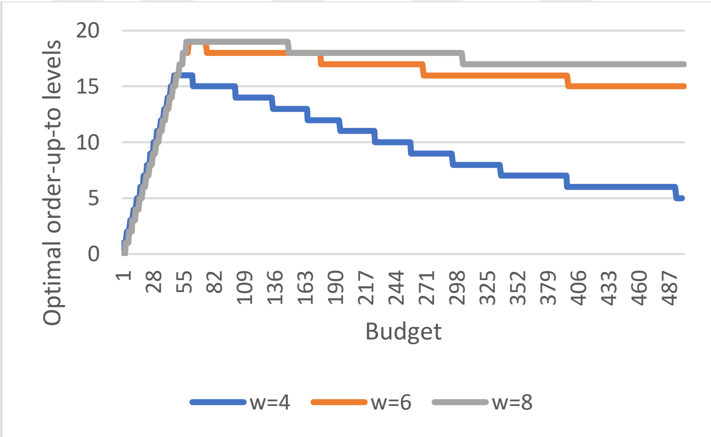


Figure 5.13 *Optimal order levels varying according to purchasing price after disaster for q=0*

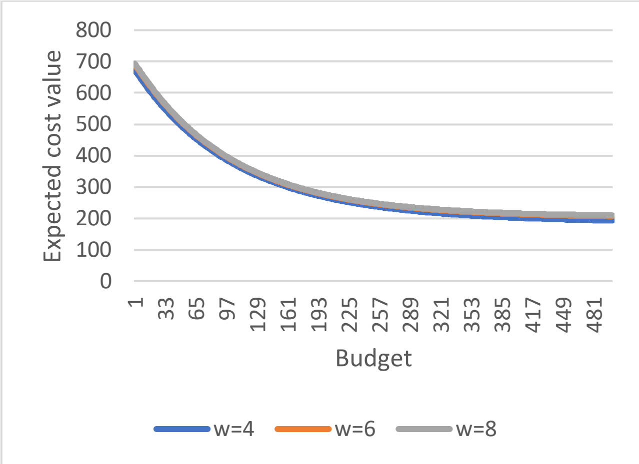


Figure 5.14 *Cost value varying according to purchase price after disaster for q=0*

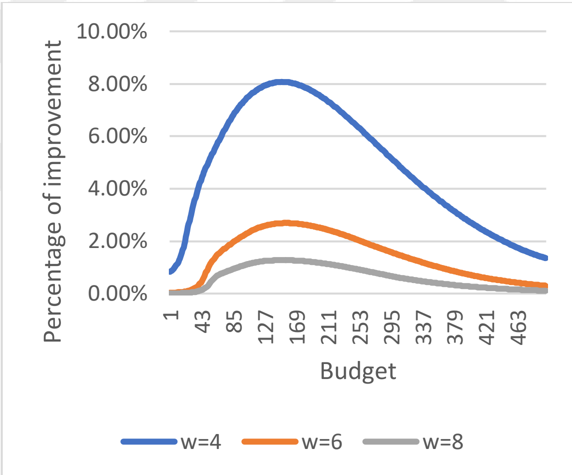


Figure 5.15 *Improvement percentages varying according to purchase price after disaster for q=0*

6. CONCLUSION

In this study, we focus on location and inventory decisions for disaster management. In the first part of the study, we consider using mobile hospitals for disaster preparedness, to improve healthcare services in case of a disaster. We focus on the facility location and allocation decisions considering mobility of hospitals such that they can be moved from their initial location to another one after a disaster happens. We develop a two-stage stochastic optimization model in order to minimize the expected total response times by taking possible facility damages into consideration. In order to compare our results, we also develop alternative models similar to the ones used in the literature. We apply our model for a case study of a possible earthquake in Istanbul and compare the results of our model with the alternative models proposed in the literature. We observe that using mobile hospitals can provide significant improvements in healthcare services in case of a disaster, provided that a sufficient budget is allocated for this type of investment. Our case study results also provide an important managerial insight, stating which regions of Istanbul require additional capacity investment for healthcare services in order to provide sufficient response in case of a disaster.

In second part of the study, the optimal stocking decisions of relief aid materials are analyzed for disaster preparedness. Humanitarian aid organizations have a certain budget and they need to use this budget appropriately to satisfy the material demand in case of a disaster. This demand can be either satisfied by stocking certain quantities of the required materials before the disaster, or by procuring them as needed after the disaster. Using all of the budget before the disaster might lead to overstocking and understocking costs due to demand uncertainty. On the other hand, procuring the required materials after the disaster when the demand uncertainty is resolved might be more costly due to imbalance between demand and supply at that time. In addition procuring the materials after the disaster might require some time and thus lead to waiting costs for the society. Considering the budget limitations, we develop newsvendor models and determine the optimal solutions for single and two-product cases using a two-stage model. We then extend our analysis and propose approximation algorithms for multiple products. The results of these models provide managerial insights about how the budget should be allocated for pre-disaster and post-disaster usage. In addition, we determine how much of each type of material should be stocked before the disaster and how should the remaining budget be used after the disaster. We also extend our analysis by considering donations that might occur after the disaster.

It is observed through our numerical studies that considering the interactions between the materials are important in making the optimal stocking decisions in multi-product environments. Ignoring the budget limitations or making the stocking decisions for each material type separately might lead to significantly worse results in certain situations, especially when the budget is at moderate levels. When the budget is high enough, the materials can be handled separately but as the budget decreases, the decision about one type of product affects the other one more significantly. However, when the budget becomes too tight, all the budget is seen to be used at the pre-disaster stage and allocation of budget for pre and post-disaster stages becomes irrelevant. It is also observed that when budget limitations are considered, contrary to expectations, more products are stored compared to unlimited budget case. The main reason for this behavior is that since it is more costly to procure the materials after the disaster, the model prefers to stock more before the disaster due to limited budget. The optimal stocking decisions of the materials and the optimal allocation of budget for pre and post-disaster usage are seen to be affected significantly by the system parameters and material characteristics. We present a sensitivity analysis to present the effects of the parameters on the system results.

In the third part of the study, we present a dynamic programming model that focus on the optimal stocking policy considering the budget limitations, random timings and quantities of donations and random demand for materials. In order to observe the contribution of our model, we examine the MPI model, in which the budget is not considered and compare the results of this model with the results of the MPIB model. By conducting a sensitivity analysis, we evaluate the robustness of the model and extract managerial insights.

In our study, for disaster management, we evaluate healthcare network design decisions and the inventory decisions for static and dynamic systems. In future studies, this thesis can be extended in several ways. Firstly, for Section 3 casualty priority status can be taken into consideration, casualties can be divided into different classes according to their priorities and different allocation criteria can be evaluated. Secondly, for Sections 4 and 5, the interaction between multiple humanitarian aid organizations, their coordinated stocking decisions and material exchanges between these organizations can be incorporated into the models. Thirdly, improved solution algorithms can be designed for the multiple product systems analyzed in Section 4. Finally, for Section 5, multiple products can be analyzed in a dynamic setting.

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APPENDIX: PROOFS OF THE RESULTSIN SECTION 4

Proof of Proposition 1

Since the second order derivative of $\mathrm{E}[\mathrm{c}(\mathrm{Q})]$ with respect to Q is positive, $\mathrm{E}[\mathrm{c}(\mathrm{Q})]$ is convex with respect to Q and the optimal solution can be obtained by the equation $\frac{\partial \mathrm{E}[\mathrm{c}(\mathrm{Q})]}{\partial \mathrm{Q}} = \mathrm{c} + \mathrm{h}\, F_D(\mathrm{Q}) - \mathrm{w}[1 - F_D(\mathrm{Q})] = 0$, which results in $Q^* = F_D^{-1}\left(\frac{\mathrm{w}-\mathrm{c}}{\mathrm{h}+\mathrm{w}}\right)$.

Proof of Proposition 2

Let $\mathrm{A} = \mathrm{Q} + \frac{\mathrm{B}-\mathrm{cQ}}{\mathrm{w}}$. Then, the first order derivative of $\mathrm{E}[\mathrm{c}(\mathrm{Q})]$with respect to Q is:

$$\frac{\partial \mathrm{E}[\mathrm{c}(\mathrm{Q})]}{\partial \mathrm{Q}} = \mathrm{c} + \mathrm{h}[F_D(\mathrm{Q})] - \mathrm{w}[F_D(\mathrm{A}) - F_D(\mathrm{Q})] - \left(\mathrm{c} + \mathrm{b} - \frac{\mathrm{cb}}{\mathrm{w}}\right)(1 - F_D(\mathrm{A}))$$

Since the second order derivative of $\mathbf{E}[\mathbf{c}(\mathbf{Q})]$ with respect to Q is seen to be positive, $\mathbf{E}[\mathbf{c}(\mathbf{Q})]$ is convex. Thus, the optimal solution will be q* as given in Lemma 2 if q* satisfies the constraint $\mathbf{B} - \boldsymbol{cq}^* \geq \mathbf{0}$, and the optimal solution will be the boundary value, $\frac{\boldsymbol{B}}{\boldsymbol{c}}$, otherwise.

Proof of Lemma 1

If $D_1 \leq Q_1$and $D_2 \leq Q_2$, then all the demand will already be satisfied with the initial inventory and $Q_1^s = Q_2^s = 0$. If $D_1 \leq Q_1$and $D_2 > Q_2$, then $D_2 - Q_2$units of product 2 will be needed at the second stage and $(Q_1^s,\ Q_2^s) = (0, \min\,(D_2 - Q_2, \frac{B-c_1Q_1-c_2Q_2}{w_2}))$ units will be bought, where the second term denotes the maximum amount of product 2 that can be bought by the remaining budget in the second stage. Similarly, if $D_1 > Q_1$and $D_2 \leq Q_2$, then $(Q_1^s,\ Q_2^s) = (\min\,(D_1 - Q_1, \frac{B-c_1Q_1-c_2Q_2}{w_1}, 0)$ units will be bought at the second stage. Finally, when $D_1 > Q_1$and $D_2 > Q_2$, the remaining budget needs to be allocated between two products. In this case, if the remaining budget can satisfy all the required products of both types, i.e. if $w_1(D_1 - Q_1) + w_2(D_2 - Q_2) \leq B - c_1Q_1 - c_2Q_2$, then all the required products will be bought at the second stage, i.e. $Q_1^s = D_1 - Q_1$ and $Q_2^s = D_2 - Q_2$. On the other hand, if the remaining budget is insufficient to satisfy all the demand, the objective function of the problem can be written as given in Equation (C.1) below. Then this model can be rewritten as Equation $(C.3)$ since the sum $b_1D_1 + b_2D_2 - b_1Q_1 - b_2Q_2 + B - c_1Q_1 - c_2Q_2$ is constant and can be deleted from the objective function without affecting the optimal values of the decision variables.

$$Min\ M = b_1(D_1 - Q_1 - Q_1^s) + b_2(D_2 - Q_2 - Q_2^s) + w_1 Q_1^s + w_2 Q_2^s \tag{C.1}$$

subject to

$$w_1 Q_1^s + w_2 Q_2^s = B - c_1 Q_1 - c_2 Q_2$$
$$0 \le Q_1^s \le D_1 - Q_1$$
$$0 \le Q_2^s \le D_2 - Q_2$$

$$Min\ M = -b_1 Q_1^s - b_2 Q_2^s + b_1 D_1 + b_2 D_2 - b_1 Q_1 - b_2 Q_2 + B - c_1 Q_1 - c_2 Q_2 \tag{C.2}$$

subject to

$$w_1 Q_1^s + w_2 Q_2^s = B - c_1 Q_1 - c_2 Q_2$$
$$0 \le Q_1^s \le D_1 - Q_1$$
$$0 \le Q_2^s \le D_2 - Q_2$$

$$Max\ b_1 Q_1^s + b_2 Q_2^s \tag{C.3}$$

subject to

$$w_1 Q_1^s + w_2 Q_2^s = B - c_1 Q_1 - c_2 Q_2$$
$$0 \le Q_1^s \le D_1 - Q_1$$
$$0 \le Q_2^s \le D_2 - Q_2$$

We observe that Model $(C.3)$ is a model of the well known continuous knapsack problem. Thus, the optimal solution of this problem can be obtained by ordering the products according to the ratio of $\frac{b_i}{w_i}$ and it will be optimal to purchase as much as possible from the product with the highest ratio. Since $\frac{b_1}{w_1} \ge \frac{b_2}{w_2}$, first the demand of product 1 is satisfied as much as possible, thus $Q_1^s = \min\left(D_1 - Q_1, \frac{B - c_1 Q_1 - c_2 Q_2}{w_1}\right)$. Then the remaining budget, if any, which is equal to $[B - c_1 Q_1 - c_2 Q_2 - w_1 Q_1^s]^+$, is used to satisfy the demand of product 2 as much as possible such that $Q_2^s = \min\left(D_2 - Q_2, \frac{[B - c_1 Q_1 - c_2 Q_2 - w_1 Q_1^s]^+}{w_2}\right)$.

Proof of Lemma 2

The Hessian Matrix for the objective function M can be stated as below.

Hessian matrix= $\begin{bmatrix} \frac{d^2 M}{d{Q_1}^2} & \frac{d^2 M}{dQ_1 dQ_2} \\ \frac{d^2 M}{dQ_2 dQ_1} & \frac{d^2 M}{d{Q_2}^2} \end{bmatrix}$

$$\frac{d^2 M}{d{Q_1}^2} = (c_1 + h_1) f_1(Q_1) + \frac{c_1(b_2 - w_2)}{w_2}\left[f_1(Q_1)[1 - F_2(A_2)] + \frac{c_1}{w_2} f_2(A_2) F_1(Q_1)\right]$$

$$+\left(\frac{(b_2-w_2)(w_1-c_1)^2}{w_1w_2}\right)F_2(Q_2)f_1(A_1)+\int_{Q_1}^{A_1}\left(\frac{(b_2-w_2)(w_1-c_1)^2}{{w_2}^2}\right)f_2\big(A(x_1)\big)f_1(x_1)dx_1$$

$$+\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)\frac{(w_1-c_1)^2}{w_1}f_1(A_1)+\left(\frac{w_1-c_1}{w_2}\right)\Big(b_2\big(1-F_2(A_2)\big)+w_2F_2(A_2)\Big)f_1(Q_1)\geq 0$$

$$\frac{d^2M}{d{Q_2}^2}=F_1(Q_1)f_2(A_2)\left(\frac{(w_2-c_2)^2(b_2-w_2)}{{w_2}^2}\right)+(h_2+b_2(1-F_1(A_1)))f_2(Q_2)$$
$$+f_2(Q_2)F_1(A_1)w_2$$
$$+\frac{c_2^2}{w_1}f_1(A_1)[F_2(Q_2)\left(\frac{b_2-w_2}{w_2}\right)+\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)]$$
$$+\frac{(w_2-c_2)^2(b_2-w_2)}{{w_2}^2}\int_{Q_1}^{A_1}f_2\big(A(x_1)\big)f_1(x_1)dx_1\geq 0$$

$$\frac{d^2M}{dQ_1dQ_2}=\frac{d^2M}{dQ_2dQ_1}$$
$$=-\left(\frac{c_1(b_2-w_2)(w_2-c_2)}{{w_2}^2}\right)f_2(A_2)F_1(Q_1)-\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)\frac{(w_1-c_1)c_2}{w_1}f_1(A_1)$$

$$+(b_2-w_2)(w_1-c_1)\left(\frac{-c_2F_2(Q_2)f_1(A_1)}{w_1w_2}+\int_{Q_1}^{A_1}\frac{(w_2-c_2)}{{w_2}^2}f_2\big(A(x_1)\big)f_1(x_1)dx_1\right)$$

The determinant of the Hessian matrix can also be written as below:

$$det|M|=\frac{(b_2-w_2)^2}{w_2^3w_1}\big(c_1c_2-(w_1-c_1)(w_2-c_2)\big)^2f_1(A_1)F_2(Q_2)f_2(A_2)F_1(Q_1)$$

$$+\frac{(b_2-w_2)}{w_2^2w_1}\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)(c_1c_2-(w_1-c_1)(w_2-c_2))^2f_1(A_1)f_2(A_2)F_1(Q_1)$$

$$+\frac{(b_2-w_2)^2(w_2-c_2)^2}{w_2^4}\big(c_1-(w_1-c_1)\big)^2f_2(A_2)F_1(Q_1)\int_{Q_1}^{A_1}f_2(A(x_1))f_1(x_1)\,dx_1$$

$$+\frac{(b_2-w_2)(w_1-c_1)^2}{w_2^2w_1}\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)\big((w_2-c_2)-c_2\big)^2f_1(A_1)\int_{Q_1}^{A_1}f_2(A(x_1))f_1(x_1)\,dx_1$$

$$+\frac{(b_2-w_2)^2(w_1-c_1)^2}{w_2^3w_1}\big((w_2-c_2)-c_2\big)^2F_2(Q_2)f_1(A_1)\int_{Q_1}^{A_1}f_2(A(x_1))f_1(x_1)\,dx_1$$

$$+\left(1-\frac{c_1}{w_1}\right)\left(\frac{(b_2-w_2)(w_1-c_1)}{w_2}\right)(h_2+F_1(A_1)w_2+b_2(1-F_1(A_1)))f_2(Q_2)f_1(A_1)F_2(Q_2)$$

$$+\frac{(b_2-w_2)(w_1-c_1)^2}{w_2^2}(h_2+F_1(A_1)w_2+b_2(1-F_1(A_1)))f_2(Q_2)\int_{Q_1}^{A_1}f_2(A(x_1))f_1(x_1)\,dx_1$$

$$+\left(\frac{(b_2-w_2)(w_2-c_2)^2}{w_2^2}\right)\left(h_1+F_2(A_2)w_1+\frac{b_2w_1}{w_2}(1-F_2(A_2))\right)f_1(Q_1)F_1(Q_1)f_2(A_2)$$

$$+\left(\frac{(b_2-w_2){c_2}^2}{w_2w_1}\right)\left(h_1+F_2(A_2)w_1+\frac{b_2w_1}{w_2}(1-F_2(A_2))\right)f_1(Q_1)f_1(A_1)F_2(Q_2)$$

$$+\left(\frac{(b_2-w_2)(w_2-c_2)}{w_2}\right)\left(1-\frac{c_2}{w_2}\right)\left(h_1+F_2(A_2)w_1+\frac{b_2w_1}{w_2}(1-F_2(A_2))\right)f_1(Q_1)$$

$$+\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)\frac{{c_2}^2}{w_1}\left(h_1+F_2(A_2)w_1+\frac{b_2w_1}{w_2}(1-F_2(A_2))\right)f_1(A_1)f_1(Q_1)$$

$$+f_1(Q_1)f_2(Q_2)\big(h_2+F_1(A_1)w_2+\big(1-F_1(A_1)\big)b_2\big)(h_1w_2+w_1w_2)\frac{b_2w_1}{w_2}(1-F_2(A_2))$$

$$+\left(\frac{c_1(b_2-w_2)}{w_2}\frac{c_1}{w_2}\right)(h_2+F_1(A_1)w_2+(1-F_1(A_1))b_2)f_2(Q_2)f_2(A_2)F_1(Q_1)$$

$$+\left(\frac{b_1}{w_1}-\frac{b_2}{w_2}\right)(w_1-c_1)\left(1-\frac{c_1}{w_1}\right)(h_2+F_1(A_1)w_2+(1-F_1(A_1))b_2)f_2(Q_2)f_1(A_1)$$

In the Hessian matrix, it can be observed that $\frac{d^2M}{d{Q_1}^2}\geq 0$ and $\frac{d^2M}{d{Q_2}^2}\geq 0$ since all the elements in these functions can be seen to be positive in the above equations. Each term of the determinant as written above can also be seen to be positive and thus the determinant of the Hessian is also positive. Thus, it can be concluded that the objective function M is jointly convex in Q_1 and Q_2.

Proof of Theorem 1

When the KKT conditions are analyzed, considering the conditions (19)-(21), there are 8 different possible cases as given in Table E.1, depending on the possible values of the variables.

Table E.1. Variables according to cases

Case	Q_1	Q_2	λ_1	λ_2	λ_3
1	≥ 0	≥ 0	0	0	0
2	≥ 0	≥ 0	0	0	≤ 0
3	≥ 0	0	0	≤ 0	0
4	0	≥ 0	≤ 0	0	0
5	0	0	≤ 0	≤ 0	0
6	0	≥ 0	≤ 0	0	≤ 0
7	≥ 0	0	0	≤ 0	≤ 0
8	0	0	≤ 0	≤ 0	≤ 0

When each case is solved using the KKT conditions with the properties of the variables given for that case in Table E.1, the associated equations given in Theorem 1 is obtained. We note that, Case 8 in Table E.1 has no feasible solution (assuming $B>0$), thus the solution of that case is not included in Theorem 1, leading to 7 different possible cases as given in Theorem 1.